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HEMCHAND RAM HEEROO

Date of Birth — Date de naissance

April 22nd 1953

Country of Birth — Lieu de naissance

MAURITIUS

Permanent Address — Résidence fixe

HIGHLANDS, PHOENIX, MAURITIUS

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PROBABILISTIC PRODUCTION COSTING OF SINGLE
SYSTEMS WITH PUMPED STORAGE AND INTERCONNECTED
SYSTEMS IN GENERATION PLANNING

by

Hemchand Rai Heeroo, B.Sc.Eng.

A thesis
submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the
requirements of the degree
of
Master of Engineering

Department of Systems and Computer Engineering

Faculty of Engineering

Carleton University

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Thesis Supervisor

Chairman,
Department of Systems and
Computer Engineering

Carleton University

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ABSTRACT

A method for including pumped storage units into probabilistic simulation for production costing in generation planning is investigated. The method is based on the description of the load by a discrete probability density function (PDF) and the effect of a pumped storage unit is accounted for by suitably shifting the discrete impulses describing the PDF of demand. Forced outage and efficiency of the unit as well as energy limitations inherent in a pumped storage unit are taken into account.

A comparison of the economic benefits of pumped storage in a system is made with the economic benefits of interconnecting the given system to another system. The loads in the two systems are described by a discrete bivariate probability density function (BPDF). By appropriately taking the limits of the BPDF and using the Bivariate Gram Charlier series to represent the impulses, the interconnection benefits are calculated.

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LIST OF COMMONLY USED SYMBOLS
AND SOME DEFINITIONS

C	- Capacity of pumped hydro storage unit (MW)
$d_o(L)$	- Duration on original load duration curve (hours)
$d_i(L)$	- Duration on inflated load duration curve (hours)
$d_r(L)$	- Duration on reduced load duration curve (hours)
$d_c(L)$	- Duration on capacity modified load duration curve (hours)
$D(L)$	- Duration on capacity and energy modified load duration curve (hours)
E	- Energy (MWhrs)
E_b	- Energy demand of system without operation of pumped hydro storage unit (MWhrs)
E_m	- Energy demand of system with operation of pumped hydro storage unit (MWhrs)
E_L	- Energy lost by pumped hydro storage unit (MWhrs)
$f(L)$	- PDF of load
$f_o(L)$	- PDF of original load
$f_i(L)$	- PDF of inflated load
$f_r(L)$	- PDF of reduced load
$f_c(L)$	- PDF of capacity modified load
$f_u(L)$	- PDF of capacity and energy modified load
$f_{g_i}(L)$	- Discrete function during generation time segment
$f_{g_{m_i}}(L)$	- Discrete function $f_{g_i}(L)$ modified by generation
$f_{p_i}(L)$	- Discrete function during pumping time segment
$f_{p_{m_i}}(L)$	- Discrete function $f_{p_i}(L)$ modified by pumping
$F(L)$	- Load probability distribution function

- 2
- M - Moment of distribution
 - n - Number of samples for a particular load level taken during 24 hours
 - N - Number of samples taken during a pumping or a generation time segment
 - N_s - Total number of samples taken during T_s
 - P_i - Probability corresponding to energy level E_i
 - q - Forced outage rate
 - T_s - Time duration of simulation (hours)
 - λ - Incremental cost of generation (\$/MWhr)
 - α - In λ

Spinning Level: The sum of maximum capacities (MW) of generators which have been started and are spinning in a system.

Spinning Cost: The cost per unit time (\$/hour) of spinning machines in a system.

Marginal Cost (cf p.1.2): The sum of unit time running expenses (fuel costs, operation costs, etc.) of a generator

Chapter 1

INTRODUCTION1.1 Critical Review of Literature on Pumped Storage

Sidenblad and Lee [1] have studied the problem of probabilistic production costing for systems with storage. Using the concept of Expected Incremental Generation Cost Curve, the method converts a probabilistic problem to a deterministic one; the curve is a plot of expected incremental cost with capacity of the system assuming that units are added to the system to meet demand in the order of increasing incremental costs. An optimization of thermal costs is carried out subject to constraints of load, energy balance, capacity and energy limits of generating units. The storage problem is presented in linear programming form; pumping and generation from pumped storage units modify the load duration curve.

In discussions of the paper [1] Stremel has pointed out that chronology is lost in the construction of the Load Duration Curve and so there is no capability of modelling reservoir constraints. Stremel also points out that the forced outage rate of the pumped storage unit is not considered in the method.

A mixed integer programming model is developed and optimized by El-Sawy and Bird [2] to generate the daily schedule for a combined thermal and hydroelectric pumped storage power generating system. The cost function consists of startup and spinning costs for committed thermal units, the extra cost of operating the committed thermal units above

spinning level, spinning costs of the converted hydroelectric pumped storage units and the net cost of extra pumped hydrogeneration above the spinning level. The cost function is minimized subject to unit commitment constraints; these include spinning reserve, loading and dispatch constraints. Another constraint considered is startup and availability of thermal generators. The constraints imposed by hydroelectric pumped storage are pumping energy limit, upper reservoir capacity and minimum upper reservoir level.

Paris and Salvaderi [3] optimized the basic characteristics of pumped storage plants. A sequential Monte Carlo Simulation, which calculates the probability of not meeting the demand at each single hour taking into consideration the water level in the preceding hour, is used. The simulation follows very closely the chronological evolution of the state of the system. However, the method assumes that the thermal stations are in two categories (peak duty and base duty units) with considerably different marginal costs. The simulation is repeated one hundred times to give an accurate estimate of risk of power and risk of water. A study of the cost of energy curtailment and savings and how they vary with storage capacity (MWh) and installed capacity (MW) of pumped hydro storage plants is made.

Sekine and Uchida [4] proposed an analytic method which determines the economically optimum expansion plan of generating plants taking into account the operation of pumped storage plants. Two models are used: (i) a KW model also called the main model, and (ii) a KWh model also called the submodel. The object is to minimize the sum of capital costs

of generating units, pumped hydro units and fuel costs, subject to constraints imposed by peak demand, minimum output, reservoir levels, operating capacity and available time. The KW model gives the capital costs of generating plants while the KWh model calculates the fuel costs. The method however treats the probability of failure of a unit only by a constraint of the following type:

$$\sigma_N (X_{N_{\text{new}}} + X_{N_{\text{old}}}) \geq \sum_{j=1} X_{N_j} \quad (1.1)$$

where $X_{N_{\text{new}}}$ - operating capacity of new nuclear plants

$X_{N_{\text{old}}}$ - operating capacity of old nuclear plants

$\sum_{j=1} X_{N_j}$ - summation of operating capacities over number of weeks for which study is being carried out

σ_N - factor correcting for randomness of generators

Instead of using an approach based on convolution justified by the fact that probability of failure of individual generating units are independent of each other, the method uses expected generation capacity. The above equation also assumes that old and new units have the same probabilities of failure and difference in failure probabilities is only brought about by the different ways in which they are started.

The addition of generating units to a power system based on an interpretation of a transportation model which incorporates transshipment nodes to represent pumped storage units motivates Windsor [5] in his study of pumped storage optimization in generation systems. By linear

programming techniques an equation representing the present day worth of costs over a planning horizon is minimized. The costs include the cost of energy and the cost of new capacity installed. Constraints are energy demand, active plant capacity in a year, energy balance of the pumped storage unit and annual capacity increments of new power stations. The problem is analyzed with costs being represented as convex, concave and stepped functions. However, no consideration is given to the randomness of units and to the limitation imposed by reservoir capacity associated with pumped storage units.

Pumped hydro storage reduces peak load and increases the base load of a system. This can also be achieved by load management and the relationship between the two has been examined by Scherer [6]. He shows that as system load factor increases through load management system load costs decrease. The amount of peak period generation and base load pumping by the pumped storage unit decreases as system load factor increases; shifting of generation from high operating cost peaking units to low operating cost base units still take place up to a load factor of 80%. A comparison of pumped storage activity between systems with nuclear fueled plants and those with coal fired plants shows that pumped storage is more important in systems with nuclear fueled plants and Scherer notes that this supports the generally accepted belief that pumped storage and nuclear fueled base load plants are especially complementary in meeting time varying electric power loads. He concludes by saying that ambitious load management programs will diminish the conventional economic viability of pumped storage although not to the point of eliminating the traditional form of energy storage from the power system planner's set of technical options.

Todd et al [7] discuss the cost effectiveness of generating energy from the wind in a system with pumped hydro storage. They note that the crucial element limiting widespread application of wind and solar energy sources is the fluctuation of power output compounded by fluctuation in power demand. The abovementioned point out that while many forms of storage suggest themselves, hydro storage provides a viable option. Exploitation of tidal energy might also require incorporation of pumped hydro storage into a system.

1.2 Thesis Objectives

This thesis will investigate a method for including pumped storage units into probabilistic simulation for production costing in generation planning. The hourly loads for the duration of the simulation shall be converted to a discrete probability distribution function (PDF) of load. Generation by the pumped hydro storage unit shall be simulated by moving discrete impulses of the PDF to the left while pumping shall be taken into account by shifting discrete impulses to the right. Billinton and Harrington [8] have studied the modification of the load duration curve of a system by the use of a hydro unit generating at peak hours. An investigation will be carried out to see if the proposed simulation for generation agree with the results of Billington and Harrington. From the discrete impulses the load probability distribution function will be constructed in a manner which takes into account the chronological constraints on the storage reservoir and the efficiency of the pumped hydro storage unit. Production costing for the system will be done using the load probability distribution function.

The economic benefits of a pumped hydro storage unit will be compared to those resulting from building a tieline to a neighbouring

utility. Studies on interconnected systems have been made by Rau et al [9]. Their method will be used to calculate the economic benefits of interconnection.

1.3 Thesis Organization

1. Chapter 1 is the introduction. It includes a literature survey of work done on pumped storage, the thesis objectives and the thesis organization.

2. Chapter 2 gives an overview of storage systems namely their importance, the different types of storage systems and discusses pumped hydro storage in detail.

3. Chapter 3 explains how to represent the load in a system by a discrete probability density function and how shifting of impulses simulate pumping and generation. The manner in which the forced outage rate of the pumped hydro storage unit is taken into consideration is also discussed. It is shown that shifting of impulses for generation is equivalent to Billinton's and Harrington's method [8] of modifying the load duration curve. Equivalent formulae for the manner in which pumping modifies the load duration curve are derived.

4. In Chapter 4 the type of problem in pumped hydro storage handled and the simulation procedure for calculating the economic benefits of pumped hydro storage are explained. Details on how the simulation takes into account efficiency of the pumped hydro storage unit, its forced outage rate and reservoir constraints are given. Energy balance

calculations are made to estimate the accuracy of the proposed method. It is also demonstrated that during the simulation the appropriate data behave according to equations presented or derived in Chapter 3.

5. Chapter 5 explains how to obtain the economic benefits of interconnection using the method of Rau, Schenk, Misra and Necsulescu [9]. The system described in Chapter 4 is interconnected to another system and the economic benefits are calculated.

6. Chapter 6 compares installation of a pumped hydro storage unit in a system to interconnection to a neighbouring utility. It contains suggestions for future work and the conclusion.

Chapter 2

STORAGE SYSTEMS2.1 Basic Concepts

A storage system enables electric energy to be converted to a form in which it can be stored; conversion of energy from the stored form to electricity should be possible when required. Figure 2.1 shows a plot of system load with time and indicates the desired effects of a storage system. During periods of reduced demand base load units produce energy above that necessary to satisfy the demand: the difference is stored. Energy is taken from the storage system to reduce or eliminate the need to operate peaking units during peak load periods.

The electric energy demand of consumers for a given time period as shown in Figure 2.1 is satisfied without the operation of the storage system; suppose the energy produced by the generating units is E_1 . The same electric energy demand for the same given time period is then satisfied with the storage system in operation and suppose that the energy produced by the generating units is now E_2 . E_2 is greater than E_1 because the storage system is not 100% efficient. Energy is lost during conversion of energy from electric form to stored form and still more energy is lost during conversion of energy from stored form to electric form.

The primary benefit of storage systems is in shifting generation of energy from high cost fuels to low cost ones. The cost function on Figure 2.2 shows how fuel costs per unit time varies with the power output of a typical power system. A 500 MW reduction of peak generating capacity reduces fuel costs/unit time by 2700 \$/hr. while a 500 MW increase of base load generating capacity increases fuel costs/unit time by 200 \$/hr.

A storage system maintains base load units at their maximum possible level of efficiency. Base load units would otherwise have to follow changes in hourly load with loss of economy and increased forced outage rate. In systems with no storage it may happen that efficient base load units may have to be shut down and less efficient units started at periods of low demand because the base load units are operating at their minimum capacity. Nuclear units require a high capital investment and economic benefits are realized only when the units are operated continuously at their maximum possible output over periods of low demand and this is possible only in the presence of a storage system.

Stored energy while meeting peak demand can also provide emergency energy when large base load units are forced out. Even during periods of low demand, feeding of energy into the storage system can be stopped in emergency situations to make thermal generation available without interfering with customer service.

Storage systems will become more important when a significant amount of energy will be produced from the wind and solar energy sources. These forms of energy production are subjected to fluctuations of power output

Figure 2.1: Variation of System Load With Time

Figure 2.2: System Cost Function

which are compounded by fluctuations in power demand. A transmission network connecting diverse energy sources can reduce those to some extent but the problems can be eliminated only by employing some form of energy storage.[7].

2.2 Types of Storage Systems

Pumped hydro storage is at present almost the only form of storage system available to large utilities. Because of geographic scarcity of sites, and environmental constraints associated with pumped hydro storage; compressed air is now beginning to be used as an alternative. This and the other examples of storage systems are described in the following paragraphs.

(1) Compressed Air. A schematic of the set up of a compressed air storage system is shown in Figure 2.3. When demand for electricity is low the generator/motor is used as a motor to drive a two stage compressor. Air at 1250 psi is stored in an air reservoir after being cooled by the intercooler and aftercooler. During periods of high electricity demand, the stored air is released, heated in the HP combustor and then used to drive a two stage high pressure and low pressure turbine. The world's first installation of this type was under construction in West Germany in 1977 and it produces 290 MW during peaking time. A compressed air storage system requires a shorter construction time than a pumped hydro storage system.

Figure 2.3. Air Storage System

(2) Batteries. Existing lead acid batteries do not have all of the characteristics required in a utility storage system. Automotive type batteries cannot withstand more than a few hundred charge and discharge cycles. Industrial batteries cost more than target prices set by utility systems. Batteries have the advantage of distributed storage with its economic and safety benefits. Other advantages are rapid installation and demand responsive capacity growth. A great deal of research work is therefore being done on advanced lead acid concepts.

(3) Flywheels. A spinning flywheel stores kinetic energy. The energy storage density of conventional flywheels is limited by the strength to density ratio of steel and by the large safety factors used in their design. Several advanced flywheel designs which take advantage of modern high strength materials have been proposed but they do not appear to be economically competitive.

(4) Hydrogen. Hydrogen is produced by water electrolysis or thermochemical splitting of water at high temperatures. The hydrogen may be compressed and stored in gaseous form, but there are uncertainties as regards the suitable containment materials. Liquefaction of hydrogen is possible but it carries with it high capital costs and losses inherent in the process. Another method being considered is to chemically bond the hydrogen into metal hydrides. Conversion of the hydrogen to electric energy can be done in fuel cells, combustion devices, boilers or gas turbines.

(5) Superconducting magnets. A future method of storing energy is in superconducting magnets. Such a system will consist of a large magnet

in a solenoidal or toroidal configuration which will be cooled to temperatures where the superconducting effects occur. Alternating current will be transformed to direct current and used to feed power to the magnetic coil. Power required from the magnet at peak load will require inversion to alternating current. The losses of such a system will only be those associated with cryogenic cooling and the inverter-converter system; efficiencies of 90 to 95% are expected. Studies have shown that magnetic storage systems costs approach those of conventional pumped hydro storage only for systems that could supply the entire demand of a typical utility rather than just the peak load.

2.3 Pumped Hydro Storage

2.3.1 Conventional

The diagram of Figure 2.4 shows the main features of a conventional pumped hydro storage system. During periods of high electric demand, water flows from the upper reservoir to the lower reservoir through the penstock and the machine. Conversion of the kinetic energy of water into mechanical energy is done by the machine acting like a turbine; the mechanical energy is then converted to electric energy by the generator attached to the machine. During periods of low electrical energy demand, electrical energy is fed into the machine, the generator now acts like a motor, drives the pump which transfers water from the lower reservoir to the higher reservoir.

Figure 2.4: Conventional Pumped Hydro Storage System

Electric power output of a generator at time $t(P_g)$ and energy output during a period $T(E_g)$ are given by equations (2.1) and (2.2) respectively. [22].

$$P_g = \frac{N_g Q_g H_g}{102} \text{ MW} \quad (2.1)$$

$$E_g = \frac{N_g H_e V_g}{367200} \text{ MW} \quad (2.2)$$

where

N_g = average efficiency of turbine generator set.

Q_g = water flow through the turbine at time t ($\text{m}^3/\text{sec.}$).

H_g = net head of the power station at time t (m).

$H_e = \frac{\int_0^T Q_g H_g dt}{V_g}$ weighted average net head of station (m).

V_g = Volume of water discharged in period T (m^3).

The electric power input into the motor/pump at time $t(P_p)$ and the energy input during period $T(E_p)$ into the motor/pump are given by equations (2.3) and (2.4) respectively.

$$P_p = \frac{Q_p H_p}{102 N_p} \text{ MW} \quad (2.3)$$

$$E_p = \frac{H_p V_p}{367200 N_p} \text{ MW} \quad (2.4)$$

where

Q_p = water flow being pumped at time t ($m^3/sec.$)

H_p = gross pumping head (m)

N_p = average efficiency of motor/pump

$V_p = \int_0^T Q_p dt$ - volume of water pumped in period T (m^3)

2.3.2 Underground Pumped Storage System

Figure 2.5 sketches an underground pumped storage system. It differs from a conventional pumped storage in that the head is developed between a reservoir located at ground surface (the upper reservoir) and a reservoir located in a cavity excavated at a depth below the surface (the lower reservoir), rather than between two reservoirs at the surface separated by a difference in elevation created by topographic features.

The head is dictated only by the necessity to have competent rock for the lower reservoir caverns and limited by the capacity of pumps and turbines. From equations (2) and (4), there is an inverse relationship between the volume of water and head of water. Heads of 1000 m are possible in an underground pumped storage system; at that head $7.5 \times 10^6 m^3$ of water can provide 20,000 MWhrs of storage. The same energy storage for a conventional system at a head difference of 300 m requires $23 \times 10^6 m^3$ of water. Underground pumped storage leads to a reduction in the extent and hence cost of structure and excavation required for upper and lower reservoir.

Figure 2.5: Underground Pumped Storage System

The initial filling can be done from a relatively small stream. Due to the reduced surface area of water exposed to the atmosphere, the underground pumped storage system has less environmental impact. The location of the underground system is independent of topography thus making it no longer necessary to look for natural sites far from load centres. Another consequence of topographic independence is short water conduits between reservoir and units; the hydraulic starting times are reduced with consequent improvement in unit operation.

2.4 Types of Turbines/Pumps

Different types of turbines/pumps are listed below:

- 1) Axial Flow. The flow of water and the axis of the machine are in the same direction. The axis of the machine can be vertical or at an angle to the horizontal and on the axis at the top is the electrical machine. This is desirable for heads of 5 to 10 m and relatively high volume flows.
- 2) Francis (also called radial flow). When the machine is in turbine mode the flow of water is perpendicular to the axis of the machine and after hitting the turbine flows along the axis of the turbine; the reverse is true when the machine is in pumping mode. The electrical machine lies at the top of the turbine/pump and on the same axis. This type of machine can be used for heads up to 500 m.
- 3) Mixed flow. This is a "hybrid" of the radial and axial flow machines. While in turbine mode, the flow of water arrives radially and is gradually turned to an axial flow with the machine. Medium flows and medium heads

are best handled by this type of machine.

4) Separate Francis turbine and Multistage pumps. To be able to cope with heads of 750 m and above which exist in underground pumped storage systems a separate Francis turbine which can be directly coupled to multistage pumps is used.

Chapter 3

SINGLE SYSTEM WITH PUMPED HYDROSTORAGE- THEORETICAL BACKGROUND3.1 Basic Concepts

Consider the load, shown graphically in Figure 3.1, in a power system for 24 hours. It shows how the predicted load varies with time of day. The load duration curve which is a plot of load level equalled or exceeded during the 24 hours against its time duration is shown in Figure 3.3.

The load is sampled every hour and the number of samples (n) for each load level is counted. The total number of samples taken during one day is 24; $n/24$ is calculated for each load level. A plot of $n/24$ against load level which is a histogram as shown in Figure 3.2. A histogram roughly approximates a probability density function (PDF). Henceforth all such histograms shall be called probability density functions (PDF) and the ratios of number of samples to total number of samples (such as $n/24$), shall be termed probabilities. Figure 3.2 can be conveniently represented as follows:

$$f(L) = \frac{5}{24} \delta(L-4) + \frac{2}{24} \delta(L-5) + \frac{2}{24} \delta(L-6.5) + \frac{4}{24} \delta(L-7) + \\ + \frac{1}{24} \delta(L-7.5) + \frac{4}{24} \delta(L-8) + \frac{2}{24} \delta(L-8.5) + \frac{2}{24} \delta(L-9) + \\ + \frac{1}{24} \delta(L-9.5) + \frac{1}{24} \delta(L-10)$$

where $\delta(x)$ is the impulse function specified by its integral property,

$$\int_{-\infty}^{\infty} \phi(x) \delta(x) dx = \phi(0)$$

Where $\phi(x)$ is any function continuous at the origin. As a result expectations and similar calculations can be represented as in equation (3.1) using integration.

The energy demand for the system during the 24 hours is the area under the curve of Figure 3.1. From Figure 3.2 the expected load during the given 24 hour period can be determined thus

$$\begin{aligned}
 \text{Expected load level} &= (p_1 L_1 + p_2 L_2 + \dots + p_n L_n) \\
 &= \int_{-\infty}^{\infty} 1 f_0(l) dl = \left(\frac{5}{24} \times 4.0 + \frac{2}{24} \times 5.0 + \frac{2}{24} \times 6.5 + \right. \\
 &\quad \left. + \frac{4}{24} \times 7.0 + \frac{1}{24} \times 7.5 + \frac{4}{24} \times 8.0 + \frac{2}{24} \times 8.5 + \frac{2}{24} \times 9.0 + \right. \\
 &\quad \left. + \frac{1}{24} \times 9.5 + \frac{1}{24} \times 10.0 \right) \\
 &= 6.875 \text{ MW} \tag{3.1}
 \end{aligned}$$

Figure 3.1: Load Curve

Figure 3.2: Probability Density Function of Load

Figure 3.3: Load Duration Curve

where

p_1 = probability of occurrence of load level L_1 MW

p_2 = probability of occurrence of load level L_2 MW

p_n = probability of occurrence of load level L_n MW

$f_0(l)$ = PDF of Figure 3.2

The energy demand for the system from time t_1 to time t_2 , where the duration of load T is given by $t_2 - t_1$, can also be calculated as follows. ($T=24$ hours for the load profile shown on Figure 3.1).

Energy demand = Expected load level $\times T$

$$= \int_{-\infty}^{\infty} l f_0(l) dl \times T \quad (3.2)$$

$$= 6.88 \text{ MW} \times 24 \text{ hours} = 165 \text{ MWhrs}$$

Integration of the function of Figure 3.2 from L to ∞ yields the load probability distribution curve, $F(L)$, i.e.

$$F(L) = \int_L^{\infty} f(l) dl \quad (3.3)$$

A sketch of $F(L)$ is shown on Figure 3.4. $F(L)$ can be obtained from Figure 3.3 by dividing the duration axis by the maximum duration of the load T (on Figure 3.3, $T=24$ hours), and interchanging the duration and load axes. Therefore if $d(L)$ is the duration in Figure 3.3

$$d(l) = T \times F(L) = T \times \int_L^{\infty} f(l) dl \quad (3.4)$$

Figure 3.4: Load Probability Distribution Curve for Load Curve of Figure 3.1

3.2 Pumped Hydro Storage Unit Acting as a Generator

The pumped hydrostorage unit is brought into operation as a generator of capacity 1 MW whenever the load described graphically in Figure 3.1 exceeds 8 MW. This diminishes the peak loads to be met by other generators in the system and is called peakshaving. The effect of the unit on the load profile and the discrete PDF can be seen in Figures 3.5 and 3.6 respectively. Comparison of Figures 3.2 and 3.6 show that impulses in the discrete PDF for load levels greater than or equal to 9 MW are shifted to the left by 1 MW whereas impulses for load levels greater than 8 MW but less than 9 MW are shifted to 8 MW. The energy demand after peakshaving can be calculated from Figure 3.6 by using equation (3.2). The energy generated by the peakshaving unit, E_g , in time duration T is given by

$$E_g = (\text{Expected load level calculated from original discrete PDF} - \text{Expected load level calculated from left-shifted discrete PDF}) \times T \quad (3.5)$$

The peakshaving unit has an outage rate (q) of 0.1. When the peakshaving unit fails, no leftshifting of impulses of the discrete PDF of Figure 3.2 takes place: the discrete PDF stays like the one of Figure 3.2 with a probability of $q = 0.1$ and transforms to the PDF of Figure 3.6 with a probability of $(1-q) = 0.9$. Multiplying the PDF of Figure 3.2 by 0.1 and that of Figure 3.6 by 0.9 and adding the two products gives Figure 3.7 which is the discrete PDF of the load after peakshaving by the randomly available pumped storage unit i.e.,

Figure 3.5: Load Curve Peak Shaved by Pumped Hydro Storage Unit

Figure 3.6: Probability Density Function of Load Peak Shaved by Pumped Hydro Storage Unit

Figure 3.7: Probability Density Function of Load Peak Shaved by Pumped Hydro Unit with $FOR = 0.1$

$$f_c(L) = f_o(L) \times q + f_r(L) \times (1-q) \quad (3.6)$$

where

$f_o(L)$ = PDF of Figure 3.2

$f_r(L)$ = PDF of Figure 3.6

$f_c(L)$ = PDF of Figure 3.7

Integration of equation (3.6) from L to ∞ gives the following:

$$\int_L^{\infty} f_c(l) dl = \int_L^{\infty} f_o(l) dl \times q + \int_L^{\infty} f_r(l) dl \times (1-q) \quad (3.7)$$

Equation (3.7) is multiplied by T , the maximum duration of the load,

$$T \times \int_L^{\infty} f_c(l) dl = T \times \int_L^{\infty} f_o(l) dl \times q + T \times \int_L^{\infty} f_r(l) dl \times (1-q) \quad (3.8)$$

Comparing equation (3.8) to equation (3.4) leads to:

$$d_c(L) = d_o(L) \times q + d_r(L) \times (1-q) \quad (3.9)$$

where

$d_o(L)$ = duration on load duration curve corresponding to the PDF of Figure 3.2 which is shown on Figure 3.9 as the original curve.

$d_r(L)$ = duration on load duration curve corresponding to the PDF of Figure 3.6 which is shown on Figure 3.9 as the "reduced curve".

$d_c(L)$ = duration on load duration curve corresponding to the PDF of Figure 3.7 which is shown on Figure 3.9 as the "capacity modified curve".

This formula agrees with the formula for capacity modified curves derived by Billinton and Harrington [8].

The pumped hydro storage unit is assumed to be a generator with 100% efficiency and has a reservoir with probabilistic energy model shown in Table 3.1. The discrete PDF corresponding to each energy level is shown in Figures 3.8(a) to 3.8(c). The maximum expected energy output of the pumped hydro storage unit with the restriction that peakshaving takes place only for load levels above 8 MW is 4.5 MWhrs this corresponds to the leftshifted PDF of Figure 3.8(c). The probability of having 4.5 MWhrs or more is found to be 0.4 from Table 3.1. Figure 3.8(a) is multiplied by 0.2, Figure 3.8(b) is multiplied by 0.4, Figure 3.8(c) is multiplied by 0.4 and the products are added to give Figure 3.8(d)

Table 3.1

Energy Model for Reservoir of Pumped Hydro Storage Unit

Energy - MWhrs	Probability
2	0.2
4	0.4
6	0.2
8	0.1
10	0.1

Figure 3.8(a): PDF Of Load After Expected Peak Shaving by 2 MWhrs, Probability = 0.2

Figure 3.8(b): PDF of Load After Expected Peak Shaving by 4 MWhrs, Probability = 0.4

Figure 3.8(c): PDF of Load After Peak Shaving Load Levels Above 8 MW (Expected Energy = 4.5 MWhrs), Probability = 0.4

Figure 3.8(d): PDF of Load After Peak Shaving by Pumped Hydro Storage Unit with Energy Model of Table 3.1

Figure 3.9(a): Load Duration Curves

Figure 3.9(b): Load Duration Curves

The load duration curve corresponding to the discrete PDF of Figure 3.8(d) is shown in Figure 3.9 as the energy and capacity modified curve. Billinton and Harrington [8] calculate this curve by the following formula:

$$D(L) = d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))] \quad (3.10)$$

where

- $D(L)$ = value of duration of final peakshaved curve corresponding to a load of L MW.
- $d_c(L)$ = value of duration on "capacity modified" curve corresponding to a load of L MW.
- $d_o(L)$ = value of duration on original load curve corresponding to a load of L MW.
- $E(L)$ = expected energy output of pumped hydro storage unit if loaded whenever system load equalled or exceeded L MW.
- $P(E(L))$ = probability of energy equalling or exceeding $E(L)$ MW

The following calculations are done to see if the curves in Figure 3.9 agree with equation (3.10).

(i) $L = 9.5$ MW, PDF of load shown in Figure 3.10(a)

$E(L) = 0.45$ MWhrs ; calculated using equation (3.5)

$P(E(L)) = 1.0$; from Table 3.1

$d_c(L) = 0.1$ hours ; from Figure 3.9

$d_o(L) = 1.0$ hours ; from Figure 3.9

$D(L) = 0.1 \times 1.0 + 1.0 \times 0 = 0.1$ hours; using equation (3.10)

$D(L) = 0.1$ hours ; from Figure 3.9

Figure 3.10(a): PDF of Load if Pumped Hydro Storage Unit is Loaded Whenever Load ≥ 9.5 MW

Figure 3.10(b): PDF of Load if Pumped Hydro Storage Unit is Loaded Whenever Load ≥ 8.75 MW

Figure 3.10(c): PDF of Load if Pumped Hydro Storage Unit is Loaded Whenever Load ≥ 8.25 MW

Figure 3.10(d): PDF of Load if Pumped Hydro Storage Unit is Loaded Whenever ≥ 8.1 MW

(ii) $L = 8.75$ MW; PDF of load shown in Figure 3.10(b)

$$\begin{aligned} E(L) &= 2.025 \text{ MWhrs} && \text{; calculated using equation (3.5)} \\ P(E(L)) &= 0.8 && \text{; from Table 3.1} \\ d_c(L) &= 1.3 \text{ hours} && \text{; from Figure 3.9} \\ d_o(L) &= 4.0 \text{ hours} && \text{; from Figure 3.9} \\ D(L) &= 1.3 \times 0.8 + 4.0 \times 0.2 = 1.84 \text{ hours; using equation (3.10)} \\ D(L) &= 1.84 \text{ hours} && \text{; from Figure 3.9} \end{aligned}$$

(iii) $L = 8.25$ MW; PDF of load shown in Figure 3.10(c)

$$\begin{aligned} E(L) &= 3.6 \text{ MWhrs} && \text{; calculated using equation (3.5)} \\ P(E(L)) &= 0.8 && \text{; from Table 3.1} \\ d_c(L) &= 2.4 \text{ hours} && \text{; from Figure 3.9} \\ d_o(L) &= 6 \text{ hours} && \text{; from Figure 3.9} \\ D(L) &= 2.4 \times 0.8 + 6 \times 0.2 = 3.12 \text{ hours; using equation (3.10)} \\ D(L) &= 3.12 \text{ hours} && \text{; from Figure 3.9} \end{aligned}$$

(iv) $L = 8.10$ MW; PDF of load shown in Figure 3.10(d)

$$\begin{aligned} E(L) &= 4.14 \text{ MWhrs} && \text{; calculated using equation (3.5)} \\ P(E(L)) &= 0.4 && \text{; from Table 1.1} \\ d_c(L) &= 2.4 \text{ hours} && \text{; from Figure 3.9} \\ d_o(L) &= 6 \text{ hours} && \text{; from Figure 3.9} \\ D(L) &= 2.4 \times 0.4 + 6 \times 0.6 = 4.56 \text{ hours; using equation (3.10)} \\ D(L) &= 4.56 \text{ hours} && \text{; from Figure 3.9} \end{aligned}$$

Therefore the curves obtained by shifting of impulses agree with Billinton and Harrington's formula.

3.3 Pumped Hydro Storage Unit Acting as a Pump

The load described graphically by Figure 3.1 is modified by a pumped hydro storage unit acting as a pump of capacity 1 MW whenever the load level goes below 6 MW. The effect of the unit on the load profile and its discrete PDF can be seen in Figures 3.11 and 3.12. Comparison of Figures 3.2 and 3.12 show that impulses on the discrete PDF for load levels less than or equal to 5 MW are shifted to the right by 1 MW whereas impulses for load levels greater than 5 MW but less than 6 MW are shifted to 6 MW. The electric energy fed into the pump, E_p , in time duration T is given by

$$E_p = (\text{Expected load level calculated from right shifted PDF} - \text{expected load level calculated from original PDF}) \times T \quad (3.11)$$

Pumping cannot take place if the pump is unavailable (probability q_p) or if there is not enough generating capacity to meet the system demand and the pumping load (probability q_g). If $q = (q_p + q_g) = 0.1$, the PDF of the load modified by the pump is calculated by adding Figure 3.2 multiplied by $q = 0.1$ to Figure 3.12 multiplied by $(1-q) = 0.9$, the resulting PDF is sketched in Figure 3.13. In equation form the PDF's are related thus:

$$f_c(L) = f_0(L) \times q + f_1(L) \times (1-q) \quad (3.12)$$

where

$f_0(L)$ = PDF of Figure 3.2

$f_1(L)$ = PDF of Figure 3.12

$f_c(L)$ = PDF of Figure 3.13

Figure 3.11: Load Curve Modified by Pumped Hydro Storage Unit Acting as a Pump

Figure 3.12: PDF of Load Modified by Pump

Figure 3.13: PDF of Load Modified by Randomly Available Pump

Energy MWhr	Probability
0	0.1
2	0.1
4	0.2
6	0.4
8	0.2

Table 3.2: Probabilistic Model of Amount of Energy Which Can Be Stored in Reservoir

Repeating the steps used for the derivation of equation (3.9) from equation (3.7) the following equation is obtained:

$$d_c(L) = d_o(L) \times q + d_i(L) \times (1-q) \quad (3.13)$$

where

$d_o(L)$ = duration on load duration curve corresponding to the PDF of Figure 3.2 which is shown on Figure 3.15 as the original curve.

$d_i(L)$ = duration on load duration curve corresponding to the PDF of Figure 3.12 which is shown on Figure 3.15 as the "inflated curve".

$d_c(L)$ = duration on load duration curve corresponding to the PDF of Figure 3.13 which is shown on Figure 3.15 as the "capacity modified curve".

The pump is assumed to be 100% efficient and fills up a reservoir; the amount of energy which can be stored in the reservoir has the probabilistic model described in Table 3.2. The discrete PDF corresponding to each energy level is shown in Figures 3.14(a) to 3.14(e). The maximum expected energy which can be delivered by the pump to the reservoir with the constraint that pumping only takes place for load levels below 6 MW is 6.3 MWhrs. From Table 3.2, the probability, of the reservoir being able to take in 6.3 MWhrs more is 0.2. Each of the PDF's depicted in Figures 3.14(a) to 3.14(e) is multiplied by its probability of occurrence and the products are added to give the energy and capacity modified PDF of Figure 3.14(f).

Figure 3.14(a): PDF of Load After Expected Delivery of 0 MWhr By Pump To Reservoir, Probability = 0.1

Figure 3.14(b): PDF of Load After Expected Delivery of 2 MWhr By Pump To Reservoir, Probability = 0.1

Figure 3.14(c): PDF of Load After Expected Delivery of 4 MWhr by Pump to Reservoir, Probability = 0.2

Figure 3.14(d): PDF of Load After Expected Delivery of 6 MWhr by Pump to Reservoir, Probability = 0.4

Figure 3.14(e): PDF of Load if Pump is Used at Load Levels ≤ 6 MW; Expected Energy Delivered by Pump = 6.3 MWhr, Probability = 0.2

Figure 3.14(f): PDF of Load After Pump Delivers Energy to Reservoir Modelled in Table 3.2

- (a)-----Original Curve
- (b).....Original curve modified by pump
with availability = 1.0
- (c)-----Original curve modified by pump
with availability = 0.9
(Capacity Modified Curve)
- (d)-----Energy and capacity modified curve

Figure 3.15(a): Load Duration Curves

Figure 3.15(b) Load Duration Curves

The load duration curve corresponding to the energy and capacity modified discrete PDF is shown in Figure 3.15. The following calculations are done to see if a formula similar to equation (3.5) holds for the curves of Figure 3.15. The notation used for the variables are:

$D(L)$ = value of duration on "energy and capacity modified" curve corresponding to a load of L MW.

$d_o(L)$ = value of duration on original load duration curve corresponding to a load of L MW.

$d_c(L)$ = value of duration on "capacity modified" curve corresponding to a load of L MW.

$E(L)$ = expected energy input into pump if loaded whenever system load equalled or is less than L MW.

$P(E(L))$ = probability of energy equalling or exceeding $E(L)$ MW.

(i) $L = 4.25$ MW, PDF of load shown in Figure 3.16(a)

$d_c(L)$ = 23.5 hours, from Figure 3.15

$d_o(L)$ = 19 hours, from Figure 3.15

$E(L)$ = 1.125 MWhrs, calculated using equation (3.11)

$P(E(L))$ = 0.9, from Table 3.2

$D(L)$ = 23.05, from Figure 3.15

$$= 23.5 \times 0.9 + 19 \times 0.1 =$$

$$= d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))]$$

Figure 3.16(a): PDF of Load if Pump is Loaded Whenever $L \leq 4.25$ MW

Figure 3.16(b): PDF of Load if Pump is Loaded Whenever $L \leq 4.5$ MW

Figure 3.16(c): PDF of Load if Pump is Loaded
Whenever $L \leq 4.9$ MW

Figure 3.16(d): PDF of Load if Pump is Loaded
Whenever $L \leq 5.25$ MW

Figure 3.16(e): PDF of Load if Pump is Loaded
Whenever $L \leq 5.9$ MW

(ii) $L = 4.5$ MW; PDF of load shown in Figure 3.16(b)

$$d_c(L) = 23.5 \text{ hours, from Figure 3.15}$$

$$d_o(L) = 19 \text{ hours, from Figure 3.15}$$

$$E(L) = 2.25 \text{ MWhrs, calculated using equation (3.11)}$$

$$P(E(L)) = 0.8, \text{ from Table 3.2}$$

$$D(L) = 22.6 \text{ hours, from Figure 3.15}$$

$$= 23.5 \times 0.8 + 19 \times 0.2$$

$$= d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))]$$

(iii) $L = 4.9$ MW, PDF of load shown in Figure 3.16(c)

$$d_c(L) = 23.5 \text{ hours, from Figure 3.15}$$

$$d_o(L) = 19 \text{ hours; from Figure 3.15}$$

$$E(L) = 4.05 \text{ MWhrs, calculated using equation (3.11)}$$

$$P(E(L)) = 0.6, \text{ from Table 3.2}$$

$$D(L) = 21.7 \text{ hours, from Figure 3.15}$$

$$= 23.5 \times 0.6 + 19 \times 0.4$$

$$= d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))]$$

(iv) $L = 5.25$ MW, PDF of load shown in Figure 3.16(d)

$$d_c(L) = 18.8 \text{ hours, from Figure 3.15}$$

$$d_o(L) = 17 \text{ hours, from Figure 3.15}$$

$$E(L) = 4.95 \text{ MWhrs, calculated using equation (3.11)}$$

$$P(E(L)) = 0.6; \text{ from Table 3.2}$$

$$D(L) = 18.08 \text{ hours, from Figure 3.5}$$

$$= 18.8 \times 0.6 + 17 \times 0.4$$

$$= d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))]$$

(v) $L = 5.9$ MW, PDF of load shown in Figure 3.16(e)

$$d_c(L) = 18.8 \text{ hours, from Figure 3.15}$$

$$d_o(L) = 17 \text{ hours, from Figure 3.15}$$

$$E(L) = 6.12 \text{ MWhrs, calculated using equation (3.11)}$$

$$P(E(L)) = 0.2, \text{ from Table 3.2}$$

$$D(L) = 17.36; \text{ from Figure 3.15}$$

$$= 18.8 \times 0.2 + 17 \times 0.8$$

$$= d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))]$$

Therefore, the equation,

$$D(L) = d_c(L) \times P(E(L)) + d_o(L) \times [1 - P(E(L))]$$
 (3.14)

holds for pumping.

3.4 Proof of Equations on Capacity and Energy Modified Curves

(1) Generation

The probabilistic energy model of the reservoir is given by:

Probability reservoir contains E_i MWhrs of energy = p_i

where $1 \leq i \leq n$ and $E_1 < E_2 < \dots < E_n$

$L_{\max}$ MW is the maximum load in the system and C MW is the capacity of the generator being powered by the reservoir. The generator has a forced outage rate of q .

E_i MWhrs of energy can shift pulses lying between $(L_i + C)$ MW and $L_{\max}$ MW to the left by C MW and pulses lying between L_i MW and $(L_i + C)$ MW

to L_i MW. The shifted pdf is denoted by $f_{c_i}(L)$.

Because $E < E_2 \dots < E_n$,

$$L_1 > L_2 > \dots > L_n \quad (3.15)$$

Peakshaving is done for load levels equal to or above L_p MW where $L_K > L_p \geq L_{K+1}$ and $1 \leq K < n$. $f_{c_p}(L)$ is the corresponding PDF of load.

The PDF's $f_{c_i}(L)$, $1 \leq i \leq K$, multiplied by the probability of the energy in the reservoir model are summed up and added to $f_{c_p}(L)$ multiplied by the probability of energy equalling or exceeding E_{K+1} MWhrs to give the capacity and energy modified pdf $f_{c_e}(L)$, i.e.

$$\begin{aligned} f_{c_e}(L) = & p_1 f_{c_1}(L) + p_2 f_{c_2}(L) + \dots + p_K f_{c_K}(L) + \\ & + (p_{K+1} + p_{K+2} + \dots + p_n) f_{c_p}(L) \end{aligned} \quad (3.16)$$

Replacing equation (3.6) in (3.16):

$$\begin{aligned} f_{ce}(L) = & p_1 \{q f_o(L) + (1-q) f_{r_1}(L)\} + p_2 \{q f_o(L) + (1-q) f_{r_2}(L)\} + \\ & + p_K \{q f_o(L) + (1-q) f_{r_K}(L)\} + \\ & + (p_{K+1} + p_{K+2} + \dots + p_n) \{q f_o(L) + (1-q) f_{r_p}(L)\} \\ = & (1-q) p_1 \{f_{r_1}(L) - f_o(L)\} + (1-q) p_2 \{f_{r_2}(L) - f_o(L)\} + \\ & + (1-q) p_K \{f_{r_K}(L) - f_o(L)\} + (1-q) (p_{K+1} + p_{K+2} + \dots + p_n) \times \\ & \times \{f_{r_p}(L) - f_o(L)\} + (p_1 + p_2 + \dots + p_n) f_o(L) = \end{aligned}$$

$$\begin{aligned}
&= (1-q) p_1 \{f_{r_1}(L) - f_0(L)\} + (1-q) p_2 \{f_{r_2}(L) - f_0(L)\} + \\
&+ \dots + (1-q) p_k \{f_{r_k}(L) - f_0(L)\} + (1-q) (p_{k+1} + p_{k+2} + \dots \\
&+ \dots p_n) \{f_{r_p}(L) - f_0(L)\} + f_0(L), \quad (3.17)
\end{aligned}$$

Since $(p_1 + p_2 + \dots + p_n) = 1$.

Consider a load level L MW where $L_j > L \geq L_{j+1}$ and $T \leq j \leq K$.

Because of equation (3.15):

$$\text{for } i \leq j; \quad f_{r_i}(L) = f_0(L) \quad (3.18)$$

and

$$f_{r_{j+1}}(L) = f_{r_{j+2}}(L) = f_{r_k}(L) = f_{r_p}(L) \quad (3.19)$$

Replacing equations (3.18) and (3.19) in equation (3.17),

$$\begin{aligned}
f_{ce}(L) &= (p_{j+1} + p_{j+2} + \dots + p_n) (1-q) (f_{r_p}(L) - f_0(L)) + \\
&+ f_0(L) \quad (3.20)
\end{aligned}$$

Equation (3.20) is multiplied by T , the maximum duration of the load, and integrated from L to ∞ :

$$\begin{aligned}
T \times \int_L^{\infty} f_{ce}(L) dL &= T \times (p_{j+1} + p_{j+2} + \dots + p_n) (1-q) \times \\
&\times \left(\int_L^{\infty} f_{r_k}(L) dL - \int_L^{\infty} f_0(L) dL \right) + T \times \int_L^{\infty} f_0(L) dL \quad (3.21)
\end{aligned}$$

Substituting equation (3.4) in equation (3.21) gives

$$D(L) = P(E(L)) (1-q) (dr_p(L) - d_o(L)) + d_o(L) \quad (3.22)$$

where the notations are the same as those explained after equations (3.9) and (3.10). Note that

$$P(E(L)) = p_{j+1} + p_{j+2} + \dots + p_n \quad (3.23)$$

From equation (3.9)

$$(1-q) dr_p(L) = d_c(L) - q d_o(L) \quad (3.24)$$

which when substituted in equation (3.22) gives

$$\begin{aligned} D(L) &= P(E(L)) \{d_c(L) - q d_o(L) - (1-q) d_o(L)\} \\ &\quad + d_o(L) = d_c(L) \times P(E(L)) + d_o(L) \times [1-P(E(L))] \end{aligned} \quad (3.25)$$

Equation (3.25) is similar to equation (3.10), Billinton and Harrington's formula for capacity and energy modified curves.

(2) Pumping

The probabilistic model for the amount of energy which can be stored in the reservoir is given by:

Probability E_s MWhrs of energy can be stored in reservoir = p_i

where $1 \leq s \leq n$ and $E_1 < E_2 < \dots < E_n$

$L_{\min}$ MW is the minimum load in the system and C MW is the capacity of the pump transferring energy into the reservoir. The pump has an unavailability rate of q .

Transferring E_s MWhrs of energy into the reservoir shifts pulses

lying between $L_{\min}$ MW and $(L_s - C)$ MW to the right by C MW and pulses lying between $(L_s - C)$ MW and L_s MW to L_s MW. The shifted pdf is denoted by $f_{c_s}(L)$.

Because $E_1 < E_2 \dots < E_n$,

$$L_1 < L_2 < \dots < L_n \quad (3.26)$$

Pumping is done for load levels equal to or less than L_p MW where $L_{K+1} \geq L_p > L_K$ and $1 < K \leq n$. $f_{c_p}(L)$ is the corresponding pdf of load.

The PDF's $f_{c_s}(L)$, $1 \leq i \leq K$, multiplied by the probability of its energy in the reservoir model are summed up and added to $f_{c_p}(L)$ multiplied by the probability that the amount of energy which can be stored equals or exceeds E_{K+1} MWhrs to give the capacity and energy modified pdf $f_{ce}(L)$, i.e.,

$$\begin{aligned} f_{ce}(L) = & p_1 f_{c_1}(L) + p_2 f_{c_2}(L) + \dots + p_K f_{c_K}(L) + \\ & + (p_{K+1} + p_{K+2} + \dots + p_n) f_{c_p}(L) \end{aligned} \quad (3.27)$$

Replacing equation (3.12) into equation (3.27):

$$\begin{aligned} f_{ce}(L) = & p_1 \{ q f_0(L) + (1-q) f_{i_1}(L) \} + p_2 \{ q f_0(L) + (1-q) f_{i_2}(L) \} + \dots \\ & + p_K \{ q f_0(L) + (1-q) f_{i_K}(L) \} + \\ & + (p_{K+1} + p_{K+2} + \dots + p_n) \{ q f_0(L) + (1-q) f_{i_p}(L) \} \\ = & (1-q) p_1 \{ f_{i_1}(L) - f_0(L) \} + (1-q) p_2 \{ f_{i_2}(L) - f_0(L) \} + \dots \end{aligned}$$

$$\begin{aligned}
& \dots + (1-q) p_k \{f_{i_k}(L) - f_o(L)\} + (1-q) (p_{k+1} + p_{k+2} + \dots + p_n) \times \\
& \times \{f_{i_p}(L) - f_o(L)\} + (p_1 + p_2 + \dots + p_n) f_o(L) = \\
& = (1-q) p_1 \{f_{i_1}(L) - f_o(L)\} + (1-q) p_2 \{f_{i_2}(L) - f_o(L)\} + \\
& + (1-q) p_k \{f_{i_k}(L) - f_o(L)\} + (1-q) (p_{k+1} + p_{k+2} + \dots + p_n) \times \\
& \times \{f_{i_p}(L) - f_o(L)\} + f_o(L) \quad (3.28)
\end{aligned}$$

Since $p_1 + p_2 + \dots + p_n = 1$.

Consider a load level L MW where $L_j < L \leq L_{j+1}$ and $1 \leq j \leq K$.

Because of equation (3.26)

$$\text{for } S \leq J : f_{i_s}(L) = f_o(L) \quad (3.29)$$

$$\text{and } f_{i_{j+1}}(L) = f_{i_{j+2}}(L) = \dots = f_{i_K}(L) = f_{i_p}(L) \quad (3.30)$$

Replacing equations (3.29) and (3.30) in (3.28) gives

$$\begin{aligned}
f_{ce}(L) &= (p_{j+1} + p_{j+2} + \dots + p_n) (1-q) \{f_{i_p}(L) - f_o(L)\} + \\
&+ f_o(L) \quad (3.31)
\end{aligned}$$

Equation (3.31) is multiplied by T , the maximum duration of the load and integrated from L to ∞ :

$$\begin{aligned}
T \times \int_L^{\infty} f_{ce}(1) d1 &= T \times (p_{j+1} + p_{j+2} + \dots + p_n) (1-q) \times \\
&\times \left\{ \int_L^{\infty} f_{i_p}(1) d1 - \int_L^{\infty} f_o(1) d1 \right\} + T \times \int_L^{\infty} f_o(1) d1 \quad (3.32)
\end{aligned}$$

Substituting equation (3.4) in equation (3.32) gives

$$D(L) = P(E(L)) (1-q) (d_{i_p}(L) - d_o(L)) + d_o(L) \quad (3.33)$$

where the notations are the same as those in section 3.3. Note that

$$P(E(L)) = p_{j+1} + p_{j+2} + \dots + p_n \quad (3.34)$$

From equation (3.13),

$$(1-q) d_{i_p}(L) = d_c(L) - q d_o(L) \quad (3.35)$$

which when substituted in equation (3.33) gives

$$\begin{aligned} D(L) &= PE(L) \{d_c(L) - q d_o(L) - (1-q) d_o(L)\} + d_o(L) = \\ &= d_c(L) \times PE(L) + d_o(L) \times [1-P(E(L))] \end{aligned} \quad (3.36)$$

Equation (3.36) is similar to equation (3.14).

Chapter 4

SINGLE SYSTEMS WITH PUMPED HYDRO STORAGE- SIMULATION PROCEDURE AND RESULTS4.1 Load: Sampling and PDF Calculation

The data sampled at regular intervals on the load curve of the power system being studied were rounded off to the nearest multiple of a figure. The load levels expected are of the order of 1000 MW. If the figure chosen is 5 MW, an error of the order of 0.25% is introduced into the data. However, the number of load levels for which the probability of occurrence has to be calculated is reduced. If the figure chosen is 2.5 MW, the error in the data is brought down to 0.125%. But the number of load levels to be dealt with is more numerous when working with 2.5 MW than 5 MW. This means that there are more impulses on a PDF of load rounded in multiples of 2.5 MW than on one rounded to 5 MW.

It will become clear that it is necessary to divide the time duration during which the power system is being studied, T_s , into periods as illustrated in Figure 4.1; a time segment during which pumping takes place is followed by another one during which generation takes place. The number of times that a particular load level occurs during a time segment of T_s has to be counted in order to calculate its probability of occurrence during T_s . If N is the number of data

Figure 4.1: Division of the Time Duration During Which the Power System is Being Studied (T_s) into Time Segments.

sampled during the time segment, this can be done by comparing each datum with the other $(N-1)$ data: about N^2 computations are required. A more efficient method of doing the count involves keeping the data in a binary tree data structure. An algorithm called Heapsort (described in Appendix A) is then used to arrange the data into a non-decreasing order of magnitude. This arrangement is done in about $N \log N$ computations.

The number of times that a load level of L MW occurs in the ordered data is counted; suppose it is n . The probability of occurrence of L MW during T_s is then n/N_s where N_s is the total number of samples taken during T_s .

Plotting the probabilities of load levels against load for each time segment yields a discrete function. Let the functions obtained during generation time segments be denoted by $f_{g_i}(L)$ while those obtained during pumping time segments be denoted by $f_{p_i}(L)$ where i varies from 1 to K as shown in Figure 4.1. Using the concept of indicator functions, $X_{T_i}^g(t)$ and $X_{T_i}^p(t)$ defined as

$$X_{T_i}^g(t) = \begin{cases} 1 & t \in T_i^g \\ 0 & \text{otherwise} \end{cases} \quad (4.0a)$$

$$X_{T_i}^p(t) = \begin{cases} 1 & t \in T_i^p \\ 0 & \text{otherwise} \end{cases} \quad (4.0b)$$

where t is time

T_i^g is generation time segment, $1 \leq i \leq K$

T_i^p is pumping time segment, $1 \leq i \leq K$,

the PDF of load $f_t(L)$ for the time duration, T_s , of the simulation can be expressed as

$$f_t(L) = \sum_{i=1}^K \{X_{T_i}^p(t) f_{p_i}(L) + X_{T_i}^g(t) f_{g_i}(L)\} \quad (4.1)$$

The load probability distribution curve (of equation (3.3)) is given by

$$F_t(L) = \int_L^\infty f_t(L) dL \quad (4.2)$$

4.2 Modelling the Operation of the Pumped Hydro Storage Unit

4.2.1 Loading Strategy of Pumped Hydro Storage Unit

During weekends, the pumped hydro storage unit pumps water into its storage reservoir. Based on predicted load the pump is brought into operation at hours of lowest energy demand. Figure 4.2 shows that the committed capacity of the pump is gradually increased to its maximum as the load goes below P_p MW while a gradual decrease in committed capacity occurs when the load increases towards P_p MW. Assuming that the pump is fully reliable, the way in which the impulses corresponding to Figure 4.2 are shifted is illustrated in Figures 4.3(a) and 4.3(b). Impulses at the extreme left are shifted by the capacity of the pump, C MW. Impulses between $(P_p - C)$ and P_p are shifted to P_p . The shaded area in Figure 4.2 is equal to the electric energy fed into the pump during the weekend, E_p . The energy transferred to the reservoir, E_s , is given by

$$E_s = \eta_p E_p \quad (4.3)$$

where η_p is the efficiency of the pumped hydro storage unit acting as a pump.

On weekdays, the pumped hydro storage unit acts like a generator and uses the energy in its storage reservoir to modify the load shown on Figure 4.4. The capacity of the generator used to peakshave the load is gradually increased from zero to its maximum of C MW as the load goes above P_g MW. After peak hours the capacity of the machine is gradually decreased from C to zero as the load goes below $(P_g + C)$ MW.

— Original load curve
 - - - Load curve after modification by pumping

Figure 4.2: Load Curve During Weekend - Action of Pumped Hydro Storage Unit

Figure 4.3: $f_{p_i}(L)$ of Load Shown on Figure 4.2

———— Original load curve

4.6

----- Load curve after modification by generation

Figure 4.4: Load Curve During Weekdays - Action of Pumped Hydro Storage Unit

Figure 4.5: $f_{g_i}(L)$ of Load Shown on Figure 4.4

The impulses resulting from sampling the original load are drawn in Figure 4.5(a) and if the generator is fully reliable, shifting of impulses are shown in Figure 4.5(b) happens. Impulses at the extreme right are shifted by the capacity of the machine C MW while impulses between P_g MW and $(P_g + C)$ MW are shifted to P_g MW. The electric energy supplied during the weekdays by the pumped hydro storage unit acting as a generator, E_g , is equal to the shaded area of Figure 4.4. If the energy stored in the reservoir decreases by E_s while generating E_g , the following equation relates E_g to E_s

$$E_g = \eta_g E_s \quad (4.4)$$

where η_g is the efficiency of the pumped hydro storage unit acting as a generator.

4.2.2 Random Operation of Pumped Hydro Storage Unit

Consider the following events associated with the operation of the pumped hydro storage unit:

- A_i = reservoir is full on Monday morning of week i
- B_i = reservoir is empty on Friday evening of week i
- C_i = during weekend of week i , generators can supply weekend load plus pumping load
- D = pumped hydro storage unit is in running condition.

It is assumed that the reservoir can only be in two states namely it is either full or empty. $\bar{A}_i$ means the reservoir is empty on Monday morning of week i and $\bar{B}_i$ means the reservoir is full on Friday evening of week i .

equal to the available energy. In this way the hydro unit's capacity rather than the capacity of high cost thermal generating unit is used to compensate for the capacity of a low cost thermal generating unit on outage and therefore leads to lower total system operation cost. Reference [14] describes in more detail how the hydro unit is fitted into the equivalent load probability distribution curve.

The generators are made to satisfy the load model corresponding to the original load profile and the fuel cost F_1 is calculated. The fuel cost F_2 incurred in running the generators to meet the load modified by the pumped hydro storage unit is then found out. The difference ($F_1 - F_2$) gives the savings brought about by the pumped hydro storage unit.

4.4 Results and Calculations

4.4.1 Introduction

A simulation is carried out for a time duration (T_s) of 13 weeks. Pumping takes place during weekends of weeks 1 to 13 while there is generation during weekdays of weeks 2 to 14. An empty reservoir is expected on Friday evening and a full reservoir is expected on Monday morning. The load sampled at intervals of one hour for the system under study is tabulated in Appendix E; the load duration curve for the system is drawn in Figure 4.7. The generators used to meet the load are also described in the same Appendix.

$$\begin{aligned} &\text{Probability (reservoir is full on Monday morning of week } i+1) \\ &= P(A_{i+1}) = P(\bar{B}_i) + P(D) \cdot P(B_i) \cdot P(C_i) \end{aligned} \quad (4.8)$$

If $P(C_i) = 1$.

$$\begin{aligned} &\text{Probability (there is pumping during weekend of week } i) \\ &= P(D) \cdot P(B_i) \end{aligned} \quad (4.7a)$$

$$\begin{aligned} &\text{Probability (reservoir is full on Monday morning of week } i+1) \\ &= P(A_{i+1}) = P(\bar{B}_i) + P(D) \cdot P(B_i) \end{aligned} \quad (4.8a)$$

4.2.3 Chronology of Events

As time elapses, the reservoir has the model shown in Table 4.1. The probabilities are calculated using equations (4.6) and (4.8) and noting that

$$\begin{aligned} &\text{Probability (reservoir empty)} + \text{Probability (reservoir full)} \\ &= 1 \end{aligned} \quad (4.9)$$

Generation of energy by the pumped hydro storage unit during weekdays i leads to shifting of impulses to the left as shown in Figure 4.5(b). The figure is multiplied by the probability that there is generation (equation (4.6)) and added to Figure 4.5(a) multiplied by the probability that there is no generation which is given by

$$\begin{aligned} &\text{Probability (there is no generation during week } i) \\ &= 1 - \text{Probability (there is generation during week } i) \end{aligned} \quad (4.10)$$

Time	Probability (reservoir empty)	Probability (reservoir full)
Week 1		
Monday morning		
Friday evening		
Week i		
Monday morning		
Friday evening		
Week i+1		
Monday morning		
Friday evening		
Week K		
Monday morning		
Friday evening		

Table 4.1: Reservoir Model

The sum of the two products is a function denoted by $f_{g_{mi}}(L)$.

Pumping during weekend i by the pumped hydro storage unit shifts impulses to the right as shown in Figure 4.3(b). The figure is multiplied by the probability that there is pumping (equation (4.7c)) and added to Figure 4.3(a) multiplied by the probability that there is no pumping which is given by

$$\begin{aligned} &\text{Probability (there is no pumping during week } i) \\ &= 1 - \text{Probability (there is pumping during week } i) \end{aligned} \quad (4.11)$$

The sum of the two products is a function denoted by $f_{p_{mi}}(L)$.

Using equations (4.1) and (4.2) the load probability distribution function after modification by the pumped hydrostorage unit is given by

$$F_m(L) = \int_L^\infty \left[\sum_{i=1}^K \{X_{T_i}^p(t) f_{p_i}(L) + X_{T_i}^g(t) f_{g_i}(L)\} \right] dt \quad (4.12)$$

The simulation is started with an empty reservoir on a Friday evening, i.e. in equation (4.6) probability (reservoir is empty on Friday evening of week i) is made equal to 1.0. The probability that generators in the system under study cannot meet the weekend load is found to be negligible, i.e. the second term on the right hand side of equation (4.7a) is found to be negligible. This is shown in Appendix B using the method of cumulants.

The different steps used in obtaining $F_m(L)$ is summarized in the flowchart of Figure 4.6. For this simulation equation (4.8) should be modified to:

$$F_m(L) = \int_L^{\infty} \left\{ \sum_{i=1}^K X_{T_i}^p(t) f_{p_i}(1) \right\} dl + \int_L^{\infty} \left\{ \sum_{i=2}^{K+1} X_{T_i}^g(t) f_{g_i}(1) \right\} dl \quad (4.13)$$

This is justified by the fact that there is no generation during week 1 and no pumping during week $K+1$; examination of the flowchart also clarifies the equation.

4.3 Production Costing

Production costing to calculate the benefits of pumped hydro storage is done using an existing software package. The package simulates how the demand in a power system is met by generating units so as to ensure cost minimization. The inputs to the package are information on the load and generators; the energy demand of the system and the expected fuel cost are the outputs. Explanation of the inputs and outputs and how to obtain the benefits of pumped hydro storage is given in the following paragraphs.

The load is represented as a probability distribution curve. Modification of the load model to take account of a randomly available pumped hydrostorage unit has been described. The package uses the probability distribution curve and the time duration of the simulation, T_s to calculate the energy demand of consumers in the system.

Figure 4.6: Flowchart illustrating Stages of the Calculation of the Load Probability Distribution Curve

Generating units are sometimes not in a condition to supply energy. This information is fed into the package by specifying a forced outage rate for each generator. Generating units can be divided into two categories: thermal and hydro. Thermal units have a fuel cost associated with them whereas hydro units have zero fuel costs but sometimes are energy limited.

The unit cost of energy for a thermal generator is not constant. The relationship between unit cost of energy and capacity as well as how this is brought about by the basic shape of the heat rate curve and the associated input/output curve are described in Appendix C. Also shown is how to take the variation of unit cost of energy with capacity into account by dividing the generating units into capacity blocks each with a different average production cost.

The capacity blocks are placed in ascending order of average production costs. The load is first met by the capacity block with the lowest average production cost followed by the other blocks; the procedure is termed merit order of loading. To take account of the forced outage rate of the generating units, the load probability distribution curve is converted into an equivalent load probability distribution curve, a concept explained in Appendix D. Equations for expected energy and fuel cost of each generating unit are also described in the same Appendix.

The given software package can perform a production costing simulation in a system with one energy limited hydro unit. The unit is fitted into the equivalent load probability distribution curve until a position is found among the thermal units where the amount of energy generated is

equal to the available energy. In this way the hydro unit's capacity rather than the capacity of high cost thermal generating unit is used to compensate for the capacity of a low cost thermal generating unit on outage and therefore leads to lower total system operation cost. Reference [14] describes in more detail how the hydro unit is fitted into the equivalent load probability distribution curve.

The generators are made to satisfy the load model corresponding to the original load profile and the fuel cost F_1 is calculated. The fuel cost F_2 incurred in running the generators to meet the load modified by the pumped hydro storage unit is then found out. The difference $(F_1 - F_2)$ gives the savings brought about by the pumped hydro storage unit.

4.4 Results and Calculations

4.4.1 Introduction

A simulation is carried out for a time duration (T_s) of 13 weeks. Pumping takes place during weekends of weeks 1 to 13 while there is generation during weekdays of weeks 2 to 14. An empty reservoir is expected on Friday evening and a full reservoir is expected on Monday morning. The load sampled at intervals of one hour for the system under study is tabulated in Appendix E; the load duration curve for the system is drawn in Figure 4.7. The generators used to meet the load are also described in the same Appendix.

Figure 4.7: System Load Duration Curve

4.4.2 Verifying Whether During the Simulation Derived Formulae
for Pumping and Billinton's and Harrington's Formulae
for Generation Are Obeyed

(a) The pumped hydro storage unit under consideration has a reservoir capacity of 3000 MWhrs and a machine capacity of 150 MW with an availability rate of 0.9. The efficiency of the machine while pumping is 0.7. Therefore the energy input into the machine needed to fill up the reservoir is $3000/0.7 \text{ MWhr} = 4285.71 \text{ MWhr}$. The study is conducted in the fifth week of the simulation. The chronological evolution of the reservoir model is calculated using equations (4.5) to (4.8) and is tabulated in Appendix F. For the week under consideration the energy model is:

Probability (energy input into pump = 4285.71 MW)

= 0.90909091

Probability (energy input into pump = 0 MW)

= 0.09090909

The following curves are plotted: original load duration curve, load duration curve inflated by capacity of pumped hydro storage unit, load duration curve modified by randomly available pumped hydro storage unit acting as a pump (capacity modified curve) and the energy and capacity modified curve. The graphs are shown in Figure 4.8 while the data are tabulated on Table 4.2.

L MW	d_o (L) Hours	d_i (L) Hours	d_c (L) Hours	D(L) Hours
1320.00	48.00	48.00	48.00	48.00
1322.50	48.00	48.00	48.00	48.00
1325.00	48.00	48.00	48.00	48.00
1327.50	48.00	48.00	48.00	48.00
1330.00	48.00	48.00	48.00	48.00
1332.50	47.00	48.00	47.90	47.82
1335.00	47.00	48.00	47.90	47.82
1337.50	47.00	48.00	47.90	47.82
1340.00	47.00	48.00	47.90	47.82
1342.50	47.00	48.00	47.90	47.82
1345.00	47.00	48.00	47.90	47.82
1347.50	47.00	48.00	47.90	47.82
1350.00	47.00	48.00	47.90	47.82
1352.50	46.00	48.00	47.80	47.64
1355.00	46.00	48.00	47.80	47.64
1357.50	46.00	48.00	47.80	47.64
1360.00	46.00	48.00	47.80	47.64
1362.50	46.00	48.00	47.80	47.64
1365.00	46.00	48.00	47.80	47.64
1367.50	45.00	48.00	47.70	47.45
1370.00	45.00	48.00	47.70	47.45
1372.50	43.00	48.00	47.50	47.09
1375.00	43.00	48.00	47.50	47.09
1377.50	43.00	48.00	47.50	47.09
1380.00	43.00	48.00	47.50	47.09
1382.50	43.00	48.00	47.50	47.09
1385.00	43.00	48.00	47.50	47.09
1387.50	43.00	48.00	47.50	47.09
1390.00	42.00	48.00	47.40	46.91
1392.50	42.00	48.00	47.40	46.91
1395.00	42.00	48.00	47.40	46.91
1397.50	42.00	48.00	47.40	46.91
1400.00	42.00	48.00	47.40	46.91
1402.50	42.00	48.00	47.40	46.91
1405.00	42.00	48.00	47.40	46.91
1407.50	42.00	48.00	47.40	46.91
1410.00	40.00	48.00	47.20	46.55
1412.50	40.00	48.00	47.20	46.55
1415.00	39.00	48.00	47.10	46.36
1417.50	39.00	48.00	47.10	46.36
1420.00	39.00	48.00	47.10	46.36
1422.50	39.00	48.00	47.10	46.36
1425.00	39.00	48.00	47.10	46.36
1427.50	39.00	48.00	47.10	46.36
1430.00	39.00	48.00	47.10	46.36
1432.50	39.00	48.00	47.10	46.36
1435.00	39.00	48.00	47.10	46.36
1437.50	39.00	48.00	47.10	46.36
1440.00	39.00	48.00	47.10	46.36

Table 4.2: Effect of Pumping on Load Duration Curves

L MW	$d_o(L)$ Hours	$d_i(L)$ Hours	$d_c(L)$ Hours	D(L) Hours
1442.50	39.00	48.00	47.10	46.36
1445.00	39.00	48.00	47.10	46.36
1447.50	39.00	48.00	47.10	46.36
1450.00	39.00	48.00	47.10	46.36
1452.50	38.00	48.00	47.00	46.13
1455.00	38.00	48.00	47.00	46.13
1457.50	37.00	48.00	46.90	46.00
1460.00	37.00	48.00	46.90	46.00
1462.50	37.00	48.00	46.90	46.00
1465.00	37.00	48.00	46.90	46.00
1467.50	37.00	48.00	46.90	46.00
1470.00	37.00	48.00	46.90	46.00
1472.50	37.00	48.00	46.90	46.00
1475.00	37.00	48.00	46.90	46.00
1477.50	37.00	48.00	46.90	46.00
1480.00	37.00	48.00	46.90	46.00
1482.50	37.00	47.00	46.00	45.13
1485.00	37.00	47.00	46.00	45.13
1487.50	37.00	47.00	46.00	45.13
1490.00	37.00	47.00	46.00	45.13
1492.50	37.00	47.00	46.00	45.13
1495.00	36.00	47.00	45.90	45.00
1497.50	35.00	47.00	45.80	44.32
1500.00	35.00	47.00	45.80	44.32
1502.50	35.00	46.00	44.90	44.00
1505.00	35.00	46.00	44.90	44.00
1507.50	35.00	46.00	44.90	44.00
1510.00	35.00	46.00	44.90	44.00
1512.50	35.00	46.00	44.90	44.00
1515.00	35.00	46.00	44.90	44.00
1517.50	35.00	45.00	44.00	43.13
1520.00	35.00	45.00	44.00	43.13
1522.50	35.00	43.00	42.20	41.55
1525.00	35.00	43.00	42.20	41.55
1527.50	35.00	43.00	42.20	41.55
1530.00	35.00	43.00	42.20	41.55
1532.50	35.00	43.00	42.20	41.55
1535.00	35.00	43.00	42.20	41.55
1537.50	34.00	43.00	42.10	41.36
1540.00	34.00	42.00	41.20	40.55
1542.50	34.00	42.00	41.20	40.55
1545.00	34.00	42.00	41.20	40.55
1547.50	34.00	42.00	41.20	40.55
1550.00	34.00	42.00	41.20	40.55
1552.50	34.00	42.00	41.20	40.55
1555.00	34.00	42.00	41.20	40.55
1557.50	34.00	42.00	41.20	40.55
1560.00	34.00	40.00	39.40	38.91
1562.50	34.00	40.00	39.40	38.91

Table 4.2: (continued)

L MW	d ₀ (L) Hours	d _i (L) Hours	d _c (L) Hours	D(L) Hours
1565.00	34.00	39.00	38.50	38.09
1567.50	34.00	39.00	38.50	38.09
1570.00	34.00	39.00	38.50	38.09
1572.50	34.00	39.00	38.50	38.09
1575.00	34.00	39.00	38.50	38.09
1577.50	34.00	39.00	38.50	38.09
1580.00	34.00	39.00	38.50	38.09
1582.50	34.00	39.00	38.50	38.09
1585.00	34.00	39.00	38.50	38.09
1587.50	34.00	39.00	38.50	38.09
1590.00	34.00	39.00	38.50	38.09
1592.50	34.00	39.00	38.50	38.09
1595.00	34.00	39.00	38.50	38.09
1597.50	34.00	39.00	38.50	38.09
1600.00	34.00	39.00	38.50	38.09
1602.50	34.00	38.00	37.60	37.27
1605.00	34.00	38.00	37.60	37.27
1607.50	34.00	37.00	36.70	36.45
1610.00	34.00	37.00	36.70	36.45
1612.50	34.00	37.00	36.70	36.45
1615.00	34.00	37.00	36.70	36.45
1617.50	34.00	37.00	36.70	36.45
1620.00	34.00	37.00	36.70	36.45
1622.50	33.00	37.00	36.60	36.27
1625.00	33.00	37.00	36.60	36.27
1627.50	33.00	37.00	36.60	36.27
1630.00	33.00	37.00	36.60	36.27
1632.50	33.00	37.00	36.60	36.27
1635.00	33.00	37.00	36.60	36.27
1637.50	33.00	37.00	36.60	36.27
1640.00	33.00	37.00	36.60	36.27
1642.50	33.00	37.00	36.60	36.27
1645.00	33.00	36.00	35.70	35.45
1647.50	33.00	35.00	34.80	34.64
1650.00	33.00	35.00	34.80	34.64
1652.50	33.00	35.00	34.80	34.64
1655.00	33.00	35.00	34.80	34.64
1657.50	33.00	35.00	34.80	34.64
1660.00	33.00	35.00	34.80	34.64
1662.50	33.00	35.00	34.80	34.64
1665.00	32.00	35.00	34.70	34.45
1667.50	31.00	35.00	34.60	34.27
1670.00	31.00	35.00	34.60	34.27
1672.50	31.00	35.00	34.60	34.27
1675.00	31.00	35.00	34.60	34.27
1677.50	31.00	35.00	34.60	34.27
1680.00	31.00	35.00	34.60	34.27
1682.50	31.00	35.00	34.60	34.27
1685.00	30.00	35.00	34.50	34.09

Table 4.2: (continued)

L MW	d_0 (L) Hours	d_i (L) Hours	d_c (L) Hours	D(L) Hours
1687.50	30.00	34.00	33.60	33.27
1690.00	30.00	34.00	33.60	33.27
1692.50	30.00	34.00	33.60	33.27
1695.00	30.00	34.00	33.60	33.27
1697.50	30.00	34.00	33.60	33.27
1700.00	30.00	34.00	33.60	33.27
1702.50	30.00	34.00	33.60	33.27
1705.00	30.00	34.00	33.60	33.27
1707.50	30.00	34.00	33.60	33.27
1710.00	29.00	34.00	33.50	33.09
1712.50	29.00	34.00	33.50	33.09
1715.00	29.00	34.00	33.50	33.09
1717.50	29.00	34.00	33.50	33.09
1720.00	29.00	34.00	33.50	33.09
1722.50	29.00	34.00	33.50	33.09
1725.00	29.00	34.00	33.50	33.09
1727.50	29.00	34.00	33.50	33.09
1730.00	28.00	34.00	33.40	32.91
1732.50	28.00	34.00	33.40	32.91
1735.00	28.00	34.00	33.40	32.91
1737.50	28.00	34.00	33.40	32.91
1740.00	28.00	34.00	33.40	32.91
1742.50	28.00	34.00	33.40	32.91
1745.00	28.00	34.00	33.40	32.91
1747.50	28.00	34.00	33.40	32.91
1750.00	28.00	34.00	33.40	32.91
1752.50	28.00	34.00	33.40	32.91
1755.00	28.00	34.00	33.40	32.91
1757.50	28.00	34.00	33.40	32.91
1760.00	28.00	34.00	33.40	32.91
1762.50	28.00	34.00	33.40	32.91
1765.00	28.00	34.00	33.40	32.91
1767.50	28.00	34.00	33.40	32.91
1770.00	28.00	34.00	33.40	32.91
1772.50	28.00	33.00	32.50	32.09
1775.00	28.00	33.00	32.50	32.09
1777.50	28.00	33.00	32.50	32.09
1780.00	28.00	33.00	32.50	32.09
1782.50	28.00	33.00	32.50	32.09
1785.00	28.00	33.00	32.50	32.09
1787.50	28.00	33.00	32.50	32.09
1790.00	28.00	33.00	32.50	32.09
1792.50	28.00	33.00	32.50	32.09
1795.00	28.00	33.00	32.50	32.09
1797.50	28.00	33.00	32.50	32.09
1800.00	28.00	33.00	32.50	32.09
1802.50	28.00	33.00	32.50	32.09
1805.00	28.00	33.00	32.50	32.09
1807.50	28.00	33.00	32.50	32.09

Table 4.2: (Continued)

L MW	d ₀ (L) Hours	d _i (L) Hours	d _c (L) Hours	D(L) Hours
1810.00	25.00	33.00	32.20	31.55
1812.50	25.00	33.00	32.20	31.55
1815.00	25.00	32.00	31.30	30.73
1817.50	25.00	31.00	30.40	29.91
1820.00	25.00	31.00	30.40	29.91
1822.50	25.00	31.00	30.40	29.91
1825.00	25.00	31.00	30.40	29.91
1827.50	25.00	31.00	30.40	29.91
1830.00	25.00	31.00	30.40	29.91
1832.50	23.00	31.00	30.20	29.55
1835.00	23.00	30.00	29.30	28.73
1837.50	23.00	30.00	29.30	28.73
1840.00	23.00	30.00	29.30	28.73
1842.50	23.00	30.00	29.30	28.73
1845.00	23.00	30.00	29.30	28.73
1847.50	23.00	30.00	29.30	28.73
1850.00	23.00	30.00	29.30	28.73
1852.50	23.00	30.00	29.30	28.73
1855.00	23.00	30.00	29.30	28.73
1857.50	23.00	30.00	29.30	28.73
1860.00	20.00	29.00	28.10	27.36
1862.50	20.00	29.00	28.10	27.36
1865.00	20.00	29.00	28.10	27.36
1867.50	20.00	29.00	28.10	27.36
1870.00	20.00	29.00	28.10	27.36
1872.50	18.00	29.00	27.90	27.00
1875.00	18.00	29.00	27.90	27.00
1877.50	18.00	29.00	27.90	27.00
1880.00	16.00	28.00	26.80	25.82
1882.50	16.00	28.00	26.80	25.82
1885.00	16.00	28.00	26.80	25.82
1887.50	16.00	28.00	26.80	25.82
1890.00	16.00	28.00	26.80	25.82
1892.50	16.00	28.00	26.80	25.82
1895.00	14.00	28.00	26.60	25.45
1897.50	14.00	28.00	26.60	25.45
1900.00	14.00	28.00	26.60	25.45
1902.50	14.00	28.00	26.60	25.45
1905.00	14.00	28.00	26.60	25.45
1907.50	14.00	28.00	26.60	25.45
1910.00	14.00	28.00	26.60	25.45
1912.50	14.00	28.00	26.60	25.45
1915.00	13.00	28.00	26.50	25.27
1917.50	13.00	28.00	26.50	25.27
1920.00	13.00	28.00	26.50	25.27
1922.50	11.00	28.00	26.30	24.91
1925.00	11.00	28.00	26.30	24.91
1927.50	11.00	28.00	26.30	24.91
1930.00	11.00	28.00	26.30	24.91

Table 4.2: (Continued)

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L MW	d_0 (L) Hours	d_i (L) Hours	d_c (L) Hours	D(L) Hours
1932.50	11.00	28.00	26.30	24.71
1935.00	11.00	28.00	26.30	24.71
1937.50	11.00	28.00	26.30	24.71
1940.00	11.00	28.00	26.30	24.71
1942.50	11.00	11.00	11.00	11.00
1945.00	9.00	9.00	9.00	9.00
1947.50	9.00	9.00	9.00	9.00
1950.00	9.00	9.00	9.00	9.00
1952.50	9.00	9.00	9.00	9.00
1955.00	8.00	8.00	8.00	8.00
1957.50	8.00	8.00	8.00	8.00
1960.00	8.00	8.00	8.00	8.00
1962.50	8.00	8.00	8.00	8.00
1965.00	7.00	7.00	7.00	7.00
1967.50	7.00	7.00	7.00	7.00
1970.00	7.00	7.00	7.00	7.00
1972.50	7.00	7.00	7.00	7.00
1975.00	7.00	7.00	7.00	7.00
1977.50	7.00	7.00	7.00	7.00
1980.00	7.00	7.00	7.00	7.00
1982.50	7.00	7.00	7.00	7.00
1985.00	7.00	7.00	7.00	7.00
1987.50	7.00	7.00	7.00	7.00
1990.00	7.00	7.00	7.00	7.00
1992.50	7.00	7.00	7.00	7.00
1995.00	7.00	7.00	7.00	7.00
1997.50	7.00	7.00	7.00	7.00
2000.00	7.00	7.00	7.00	7.00
2002.50	7.00	7.00	7.00	7.00
2005.00	7.00	7.00	7.00	7.00
2007.50	6.00	6.00	6.00	6.00
2010.00	6.00	6.00	6.00	6.00
2012.50	6.00	6.00	6.00	6.00
2015.00	6.00	6.00	6.00	6.00
2017.50	5.00	5.00	5.00	5.00
2020.00	5.00	5.00	5.00	5.00
2022.50	5.00	5.00	5.00	5.00
2025.00	5.00	5.00	5.00	5.00
2027.50	5.00	5.00	5.00	5.00
2030.00	5.00	5.00	5.00	5.00
2032.50	5.00	5.00	5.00	5.00
2035.00	5.00	5.00	5.00	5.00
2037.50	5.00	5.00	5.00	5.00
2040.00	5.00	5.00	5.00	5.00
2042.50	5.00	5.00	5.00	5.00
2045.00	5.00	5.00	5.00	5.00
2047.50	5.00	5.00	5.00	5.00
2050.00	5.00	5.00	5.00	5.00
2052.50	5.00	5.00	5.00	5.00

Table 4.2: (Continued)

Figure 4.8 : Effect of Pumping on Load Duration Curves

The data agree with equations (3.13) and (3.14), the derived formulae for pumping. This is shown by the following sample calculation for the load of L of 1700 MW:

$$\text{Duration of load on original load duration} = d_o(L) = 30.0$$

$$\text{Duration of load on inflated load duration curve} = d_i(L) = 34.0$$

$$\text{The probability that there is no pumping is} = q = 0.1$$

$$\text{Duration of load on capacity modified curve} = d_c(L) =$$

$$d_o(L) \times q + d_i(L) \times (1-q) = 33.6 \text{ hours}$$

$$P(E(L)) = 0.90909091 \text{ (from energy model above)}$$

$$\text{Duration of energy and capacity modified curve} = D(L) =$$

$$d_c(L) \times P(E(L)) + d_o(L) \times (1-P(E(L))) =$$

$$33.6 \times 0.90909091 + 30.0 \times 0.09090909 = 33.27 \text{ hours}$$

(b) The same pumped hydro storage unit considered for pumping in (a) is now used for generation. The efficiency of the machine while generating is 0.9. Therefore, the energy output of the machine is $3000 \times 0.9 \text{ MW} = 2700 \text{ MWhr}$. The study is conducted in the sixth week of the simulation. From Appendix F the energy model is:

$$\text{Probability (energy output of generator} = 2700 \text{ MWhrs)}$$

$$= 0.90909091$$

$$\text{Probability (energy output of generator} = 0 \text{ MWhrs)}$$

$$= 0.09090909$$

The same curves described in (a) are plotted with the exception that the inflated curve is replaced by a reduced curve. The graphs are shown in Figure 4.9 while the data are tabulated in Table 4.3.

The data agree with equations (3.9) and (3.10) for generation.

This is shown by the following sample calculation for a load L of

2350 MW:

$$\begin{aligned} \text{Duration of load on original load duration curve} &= d_o(L) = \\ &= 24.0 \text{ hours} \end{aligned}$$

$$\begin{aligned} \text{Duration of load on reduced load duration curve} &= d_r(L) = \\ &= 10.0 \text{ hours} \end{aligned}$$

$$\begin{aligned} \text{Duration of load on capacity modified curve} &= d_c(L) = \\ &= d_o(L) \times q + d_r(L) \times (1-q) = 11.4 \text{ hours} \end{aligned}$$

$$P(E(L)) = 0.90909091$$

$$\text{Duration on energy and capacity modified curve} = D(L) =$$

$$= d_c(L) \times P(E(L)) + d_o(L) \times (1-P(E(L))) =$$

$$= 11.4 \times 0.90909091 + 24 \times 0.09090909 =$$

$$= 12.55 \text{ hours}$$

4.4.3 Energy Balance Calculations

Calculated energy demand of the system without a pumped hydro

$$\text{storage unit} = E_o \text{ MWhrs}$$

Calculated energy demand of the system with a pumped hydro

$$\text{storage unit} = E_m \text{ MWhrs}$$

L MW	d_o (L) Hours	d_i (L) Hours	d_c (L) Hours	D(L) Hours
2300.00	35.00	35.00	35.00	35.00
2302.50	35.00	35.00	35.00	35.00
2305.00	35.00	35.00	35.00	35.00
2307.50	35.00	35.00	35.00	35.00
2310.00	32.00	32.00	32.00	32.00
2312.50	29.00	29.00	29.00	29.00
2315.00	29.00	29.00	29.00	29.00
2317.50	29.00	29.00	29.00	29.00
2320.00	29.00	29.00	29.00	29.00
2322.50	29.00	11.00	12.80	14.27
2325.00	29.00	11.00	12.80	14.27
2327.50	29.00	11.00	12.80	14.27
2330.00	29.00	11.00	12.80	14.27
2332.50	29.00	10.00	11.90	13.45
2335.00	24.00	10.00	11.40	12.55
2337.50	24.00	10.00	11.40	12.55
2340.00	24.00	10.00	11.40	12.55
2342.50	24.00	10.00	11.40	12.55
2345.00	24.00	10.00	11.40	12.55
2347.50	24.00	10.00	11.40	12.55
2350.00	24.00	10.00	11.40	12.55
2352.50	24.00	10.00	11.40	12.55
2355.00	24.00	10.00	11.40	12.55
2357.50	19.00	10.00	10.90	11.64
2360.00	19.00	6.00	7.30	8.36
2362.50	19.00	6.00	7.30	8.36
2365.00	19.00	6.00	7.30	8.36
2367.50	19.00	6.00	7.30	8.36
2370.00	19.00	6.00	7.30	8.36
2372.50	19.00	6.00	7.30	8.36
2375.00	19.00	6.00	7.30	8.36
2377.50	19.00	6.00	7.30	8.36
2380.00	19.00	6.00	7.30	8.36
2382.50	18.00	6.00	7.20	8.18
2385.00	18.00	3.00	4.50	5.73
2387.50	18.00	3.00	4.50	5.73
2390.00	18.00	3.00	4.50	5.73
2392.50	18.00	3.00	4.50	5.73
2395.00	18.00	3.00	4.50	5.73
2397.50	18.00	3.00	4.50	5.73
2400.00	18.00	3.00	4.50	5.73
2402.50	18.00	3.00	4.50	5.73
2405.00	17.00	3.00	4.40	5.55
2407.50	15.00	3.00	4.20	5.18
2410.00	15.00	3.00	4.20	5.18
2412.50	15.00	3.00	4.20	5.18
2415.00	15.00	3.00	4.20	5.18
2417.50	15.00	3.00	4.20	5.18
2420.00	15.00	3.00	4.20	5.18

Table 4.3: Effect of Generation in Load Duration Curves

L MW	d_0 (L) Hours	d_i (L) Hours	d_c (L) Hours	D(L) Hours
2422.50	15.00	3.00	4.20	5.16
2425.00	15.00	3.00	4.20	5.13
2427.50	15.00	3.00	4.20	5.13
2430.00	15.00	3.00	4.20	5.13
2432.50	14.00	3.00	4.10	5.00
2435.00	14.00	3.00	4.10	5.00
2437.50	14.00	3.00	4.10	5.00
2440.00	14.00	3.00	4.10	5.00
2442.50	14.00	3.00	4.10	5.00
2445.00	14.00	3.00	4.10	5.00
2447.50	14.00	3.00	4.10	5.00
2450.00	14.00	3.00	4.10	5.00
2452.50	14.00	3.00	4.10	5.00
2455.00	14.00	3.00	3.80	4.45
2457.50	11.00	3.00	3.80	4.45
2460.00	11.00	3.00	3.80	4.45
2462.50	11.00	2.00	2.90	3.64
2465.00	11.00	2.00	2.90	3.64
2467.50	11.00	2.00	2.90	3.64
2470.00	11.00	2.00	2.90	3.64
2472.50	11.00	2.00	2.90	3.64
2475.00	11.00	2.00	2.90	3.64
2477.50	11.00	2.00	2.90	3.64
2480.00	11.00	2.00	2.80	3.45
2482.50	10.00	2.00	2.80	3.45
2485.00	10.00	2.00	2.80	3.45
2487.50	10.00	2.00	2.80	3.45
2490.00	10.00	2.00	1.00	1.82
2492.50	10.00	.00	1.00	1.82
2495.00	10.00	.00	1.00	1.82
2497.50	10.00	.00	1.00	1.82
2500.00	10.00	.00	1.00	1.82
2502.50	10.00	.00	1.00	1.82
2505.00	10.00	.00	1.00	1.82
2507.50	10.00	.00	.60	1.09
2510.00	6.00	.00	.60	1.09
2512.50	6.00	.00	.60	1.09
2515.00	6.00	.00	.60	1.09
2517.50	6.00	.00	.60	1.09
2520.00	6.00	.00	.60	1.09
2522.50	6.00	.00	.60	1.09
2525.00	6.00	.00	.60	1.09
2527.50	6.00	.00	.60	1.09
2530.00	6.00	.00	.60	1.09
2532.50	6.00	.00	.30	.55
2535.00	3.00	.00	.30	.55
2537.50	3.00	.00	.30	.55
2540.00	3.00	.00	.30	.55
2542.50	3.00	.00	.30	.55

Table 4.3: (Continued)

L MW	$d_o(L)$ Hours	$d_i(L)$ Hours	$d_c(L)$ Hours	$D(L)$ Hours
2545.00	3.00	.00	.30	.55
2547.50	3.00	.00	.30	.55
2550.00	3.00	.00	.30	.55
2552.50	3.00	.00	.30	.55
2555.00	3.00	.00	.30	.55
2557.50	3.00	.00	.30	.55
2560.00	3.00	.00	.30	.55
2562.50	3.00	.00	.30	.55
2565.00	3.00	.00	.30	.55
2567.50	3.00	.00	.30	.55
2570.00	3.00	.00	.30	.55
2572.50	3.00	.00	.30	.55
2575.00	3.00	.00	.30	.55
2577.50	3.00	.00	.30	.55
2580.00	3.00	.00	.30	.55
2582.50	3.00	.00	.30	.55
2585.00	3.00	.00	.30	.55
2587.50	3.00	.00	.30	.55
2590.00	3.00	.00	.30	.55
2592.50	3.00	.00	.30	.55
2595.00	3.00	.00	.30	.55
2597.50	3.00	.00	.30	.55
2600.00	3.00	.00	.30	.55
2602.50	3.00	.00	.30	.55
2605.00	3.00	.00	.30	.55
2607.50	3.00	.00	.30	.55
2610.00	3.00	.00	.30	.55
2612.50	3.00	.00	.30	.55
2615.00	2.00	.00	.20	.36
2617.50	2.00	.00	.20	.36
2620.00	2.00	.00	.20	.36
2622.50	2.00	.00	.20	.36
2625.00	2.00	.00	.20	.36
2627.50	2.00	.00	.20	.36
2630.00	2.00	.00	.20	.36
2632.50	2.00	.00	.20	.36
2635.00	2.00	.00	.20	.36
2637.50	2.00	.00	.20	.36
2640.00	2.00	.00	.20	.36
2642.50	.00	.00	.00	.00
2645.00	.00	.00	.00	.00

Table 4.3: (Continued)

Figure 4.9: Effect of Generation on Load Duration Curves

Calculated energy losses due to inefficiency of pumped
hydro storage unit = E_L

Ideally E_L should be equal to $E_m - E_o$

$$\text{Percentage difference in energy balance} = \frac{(E_m - E_o) - E_L}{E_L} \times 100 \% \quad (4.14)$$

The following calculations are done to illustrate how the percentage difference in energy balance is obtained for different simulations:

(a) Pumped hydro storage unit has availability of 1.0. Capacity of machine can fill up reservoir during weekend.

Reservoir capacity	= 3000 MWhr
Machine capacity	= 100 MW
Efficiency of machine while pumping	= 0.7
Electrical energy input into machine	= 3000/0.7 MWhrs
required to fill reservoir during	= 4285.71 MWhrs
one weekend	
Time taken to fill up reservoir	= 4285.71 MWhrs/100 MW
	= 42.86 hours

Therefore reservoir can be filled up by machine during one weekend (48 hours).

Electrical energy input into machine	= 4285.71 MWhrs/week
required to fill reservoir for	x 13 weeks
13 weeks	= 55714.28 MWhrs
Electrical energy output of generator	= 3000 MWhrs/week x 0.9
during 13 weeks	x 13 weeks
	= 35100.00 MWhrs

$$\begin{aligned} \text{Energy loss due to inefficiency of pumped} &= (55714.28 - 35100.00) \text{ MWhrs} \\ \text{hydro storage unit} &= 20614.28 \text{ MWhrs} = E_L \end{aligned}$$

$$\begin{aligned} \text{Energy demand of system without pumped} &= E_o = 4122.834 \text{ GWhrs} \\ \text{hydro storage unit} & \end{aligned}$$

$$\text{Energy demand of system with pumped} = E_m = 4143.484 \text{ GWhrs}$$

$$\begin{aligned} \text{hydro storage unit} \quad E_m - E_o &= 20.65 \text{ GWhrs} \\ &= 20650 \text{ MWhrs} \end{aligned}$$

$$\begin{aligned} \text{Percentage difference in energy balance} &= \frac{(E_m - E_o) - E_L}{E_L} \times 100\% \\ &= 0.17\% \end{aligned}$$

(b) Pumped hydro storage unit has availability of 1.0 capacity of machine cannot fill up reservoir during weekend.

Same reservoir capacity and same machine efficiencies as in (a).
Machine capacity of 50 MW.

$$\text{Time taken to fill up reservoir} = \frac{4285.71 \text{ MWhrs}}{50 \text{ MW}} = 85.72 \text{ hours}$$

The reservoir cannot be filled up by the machine during one weekend (48 hours).

$$\begin{aligned} \text{Electrical energy input into machine} &= 50 \text{ MW} \times 48 \text{ hours} \times 13 \text{ weeks} \\ \text{during 13 weekends} &= 31200 \text{ MWhrs} \end{aligned}$$

$$\begin{aligned} \text{Energy transferred to reservoir} &= 31200 \text{ MWhrs} \times 0.7 \\ &= 21840 \text{ MWhrs} \end{aligned}$$

$$\begin{aligned} \text{Energy generated during 13 weeks} &= 21840 \times 0.9 \text{ MWhrs} \\ &= 19656 \text{ MWhrs} \end{aligned}$$

$$\begin{aligned} \text{Energy losses due to inefficiency of} &= E_L = (31200 - 19656) \text{ MWhrs} \\ \text{pumped/hydro storage unit} &= 11544 \text{ MWhrs} \end{aligned}$$

$$\text{Energy demand of system with pumped hydro} = E_m = 4134.201 \text{ GWhrs}$$

$$\begin{aligned} \text{storage unit} \quad E_m - E_o &= (4134.201 - 4122.834) \text{ GWhrs} \\ &= 11367 \text{ MWhrs} \end{aligned}$$

$$\begin{aligned} \text{Percentage difference in energy balance} &= \frac{(E_m - E_o) - E_L}{E_L} \times 100 \% \\ &= -1.53 \% \end{aligned}$$

(c) Pumped hydro storage unit has availability of 0.9. Reservoir can be filled during weekend.

Same reservoir capacity, machine capacity and efficiency as in (a).

$P_p(i)$ is the probability that there is pumping during week i . The values of $P_p(i)$ for a pumped hydro storage unit with availability of 0.9 can be found in Appendix F.

Expected electrical energy input into pump to fill reservoir during 13 weeks:

$$\begin{aligned} &= \sum_{i=1}^{13} \text{electrical energy input into pump/week} \times P_p(i) \\ &= \text{electrical energy input into pump/week} \times \sum_{i=1}^{13} P_p(i) \\ &= 4285.71 \text{ MW/week} \times 10.7100838 \\ &= 45938.56 \text{ MW} \end{aligned}$$

$P_g(i)$ is the probability that there is generation during week i . The values of $P_g(i)$ for a pumped hydro storage unit with availability of 0.9 can be found in Appendix G.

Expected electrical energy output of generator during 13 weeks

$$\begin{aligned}
 &= \sum_{i=1}^{13} \text{electrical energy output of generator/week} \times P_g(i) \\
 &= \text{electrical energy output of generator/week} \times \sum_{i=1}^{13} P_g(i) \\
 &= 3000 \times 0.9 \text{ MWhrs/week} \times 10.62809919 \\
 &= 28695.87 \text{ MWhrs}
 \end{aligned}$$

Expected energy loss due to inefficiency of pumped hydro storage unit

$$E_L = (45938.56 - 28695.87) \text{ MWhrs} = 17242.69 \text{ MWhrs}$$

Energy demand of system with pumped hydro storage unit

$$E_m = 4139.612 \text{ GWhrs}$$

$$E_m - E_o = (4139.612 - 4122.834) \text{ GWhrs} = 16778 \text{ MWhrs}$$

Percentage difference in energy balance

$$= \frac{(E_m - E_o) - E_L}{E_L} \times 100\% = -2.69\%$$

(d) Pumped hydro storage unit has availability of 0.9. Capacity of machine cannot fill up reservoir during weekend. Same reservoir, same machine capacity and efficiencies as in (b).

Expected electrical energy input into machine during 13 weekends.

$$= \sum_{i=1}^{13} \text{electrical energy input into machine/weekend} \times P_p(i)$$

$$= \text{electrical energy input into machine/weekend} \times \sum_{i=1}^{13} P_p(i)$$

$$= 50 \text{ MW} \times 48 \text{ hours/weekend} \times 10.71900828 \text{ weeks} = 25725.62 \text{ MWhrs}$$

Expected electrical energy output of generator during 13 weeks

$$= \sum_{i=1}^{13} \text{electrical energy output of generator/week} \times P_g(i)$$

$$= \text{electrical energy output of generator/week} \times \sum_{i=1}^{13} P_g(i)$$

$$= 50 \text{ MW} \times 48 \text{ hours} \times 0.7 \times 0.9/\text{week} \times 10.62809919 \text{ weeks}$$

$$= 16069.69 \text{ MWhrs}$$

Expected energy loss due to the inefficiency of pumped hydro storage unit = $E_L = (25725.62 - 16069.69) \text{ MWhrs} = 9655.93 \text{ MWhrs}$

Energy demand of system with pumped hydro storage unit

$$= E_m = 4132.881 \text{ GWhrs}$$

$$E_m - E_o = (4132.881 - 4122.834) \text{ GWhrs} = 10047 \text{ MWhrs}$$

$$\text{Percentage difference in energy balance} = \frac{(E_m - E_o) - E_L}{E_L} \times 100 \% = 4.05\%$$

The results of the simulations are summarized in Tables 4.4 to 4.15. Each table describes results obtained for a given size of the pumped hydro storage unit's reservoir and a given value of its machine's availability. The nearest multiple to which the load levels are rounded off are mentioned on each table. The column "Energy Demand of System" lists values of E_m which are defined above.

The energy losses due to the inefficiency of the pumped hydro storage unit (E_L) mentioned at the top of each table are losses calculated for the 13 weeks duration of each simulation, and is for machine capacities which can fill up the reservoir in the weekend. In case the machine capacity cannot fill up the reservoir in 1 weekend, there is a note in the remarks column and the appropriate value of E_L is given. The values in the "Percentage Difference in Energy Balance" column have been calculated using equation (4.14) and following the steps in the sample calculations described above. The value of E_0 , the energy demand of the system without a pumped hydro storage unit, is 4122.834 GWhrs.

4.4.4 Economic Benefits

The economic benefits of pumped hydro storage obtained from the simulations can be found in the columns entitled "Benefits" on Tables 4.4 to 4.15. The graphs of Figures 4.10 to 4.14 show the relationship between economic benefits and the machine capacity of the pumped hydro storage unit. Each figure is for a different reservoir size while each curve is for a different availability of the pumped hydro storage unit's machine.

The graphs of Figures 4.10 to 4.13 are for systems whose loads are satisfied by the set of generators described in Appendix E. A 500 MW nuclear-fueled unit has been removed from the generators in Appendix E and the remaining generators are made to satisfy the system load. The plot of economic benefits against the machine capacity of pumped hydro storage unit for this case is drawn in Figure 4.14.

Pumped Hydrostorage Unit:

Reservoir Capacity = 1000 MWhrs
 Machine Availability = 1.0
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 6871.43$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4129.719	0.20	37103.43	
75	4129.626	-1.15%	55200.51	
100	4129.876	2.48%	69158.51	
125	4129.521	-2.68%	76288.31	
150	4129.346	-3.92%	85702.94	
200	4129.277	-6.23%	88376.95	
250	4129.277	-6.23%	88376.95	
300	4129.277	-6.23%	88376.95	

Table 4.4

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 1000 MWhrs
 Machine Availability = 0.95
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 1.25 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 6283.835$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4129.873	-4.08	26800.01	
75	4130.399	4.28	41498.95	
100	4130.450	5.10	53847.40	
150	4130.423	4.67	66100.17	
200	4130.478	5.54	70722.47	
250	4130.478	5.54	70722.47	
300	4130.478	5.54	70722.47	

Table 4.5

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 1000 MWhrs
 Machine Availability = 0.90
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 1.25 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 5747.58$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4128.979	-10.7	22025.28	
75	4129.875	4.90	34100.18	
100	4129.897	5.30	46722.28	
120	4129.717	2.15	54199.77	
150	4129.849	4.44	60600.01	
200	4129.919	5.66	64267.89	
250	4129.919	5.66	64267.89	
300	4129.919	5.66	64267.89	

Table 4.6

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 2000 MWhrs
 Machine Availability = 1.0
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 13742.86$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4134.201	-1.53	-30160.36	Machine capacity cannot fill reservoir during one weekend $E_L = 11544.00$ MWhrs
100	4136.504	-0.53	28690.89	
120	4136.219	-2.60	70827.57	
150	4136.379	-1.44	98210.25	
175	4136.476	-0.73	111064.57	
200	4136.523	-0.39	114747.60	
250	4136.475	-0.74	124515.86	
300	4136.495	-0.60	123588.25	
350	4136.495	-0.60	123588.25	

Table 4.7

Results of Simulation

Pumped Hydrostorage Unit :

Reservoir Capacity = 2000 MWhrs
 Machine Availability = 0.95
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 12567.67$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4133.777	3.66	-36578.45	Machine capacity cannot fill reservoir during one weekend $E_L = 10556.84$ MWhrs
100	4135.522	0.96	20050.41	
125	4135.820	3.32	55166.47	
150	4135.659	2.04	92702.54	
200	4135.100	-2.40	112050.48	
250	4135.047	-2.82	113152.47	
300	4135.111	-2.31	113899.34	
350	4135.111	-2.31	113899.34	
400	4135.111	-2.31	113899.34	

Table 4.8

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 2000 MWhrs
 Machine Availability = 0.90
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW
 If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 11495.12$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4132.881	4.05	-35295.95	Machine capacity cannot fill reservoir during one weekend $E_L = 9655.93$ MWhrs
100	4134.179	-1.30	18196.77	
125	4134.730	-3.49	47203.49	
150	4134.573	2.12	72898.87	
200	4133.753	-5.01	91104.03	
250	4133.705	-5.42	97716.51	
300	4133.790	-4.69	95465.93	
350	4133.790	-4.69	95465.93	
400	4133.790	-4.69	95465.93	

Table 4.9

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 3000 MWhrs

Machine Availability = 1.0

Pumping Efficiency = 0.70

Generating Efficiency = 0.90

Load levels rounded to nearest multiple of 2.5 MW

If machine capacity can fill up reservoir, expected energy loss of pumped hydro storage unit =

 $E_L = 20614.29$ MWhrs

Machine Capacity of Pumped-Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4134.201	-1.53	-30160.36	Machine capacity cannot fill reservoir during one weekend $E_L = 11544.00$ MWhrs
100	4143.484	0.17	-53344.59	
135	4143.723	1.33	-10568.60	
160	4143.211	-1.15	29510.59	
190	4142.824	-3.03	71997.87	
220	4142.748	-3.40	102800.29	
250	4142.685	-3.70	120841.45	
300	4142.729	-3.49	129338.96	
350	4142.787	-3.21	134281.00	

Table 4.10

Results of Simulation

Pumped Hydrostorage Unit :

Reservoir Capacity = 3000 MWhrs
 Machine Availability = 0.95
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 18851.51$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4133.777	3.66	-36578.45	Machine capacity cannot fill reservoir during one weekend $E_L = 10556.84$ MWhrs
100	4141.545	-0.75	-48960.60	
150	4141.923	1.26	1895.83	
200	4142.025	1.80	48230.91	
220	4142.066	2.02	60215.23	
250	4142.097	2.18	75116.13	
300	4142.094	2.17	76396.65	
350	4142.152	2.47	76615.55	
400	4142.052	1.94	76292.57	

Table 4.11

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 3000 MWhrs
 Machine Availability = 0.90
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 17242.74$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4132.883	4.05	-35295.95	Machine capacity cannot fill reservoir during one weekend $E_L = 9655.93$ MWhrs
100	4139.612	-2.69	-42446.85	
150	4140.305	1.32	2992.17	
200	4140.721	3.74	40188.43	
220	4140.420	1.99	51243.85	
250	4140.449	2.16	64732.39	
300	4140.445	2.14	65503.90	
350	4140.498	2.44	66108.74	
400	4140.406	1.91	65873.44	

Table 4.12
Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 5000 MWhrs
 Machine Availability = 1.0
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 34357.14$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4134.207	-1.53	-30160.36	Machine Capacity cannot fill reservoir during one weekend $E_L = 11544.00$ MWhrs
100	4145.633	-1.25	-91016.37	Machine capacity cannot fill reservoir during one weekend $E_L = 23088$ MWhrs
125	4151.652	-0.15	-135390	Machine capacity cannot fill reservoir during one weekend $E_L = 28860$ MWhrs
150	4157.104	-0.25	-179958.45	
175	4156.564	-1.83	-159501.55	
200	4157.316	-0.36	-91620.61	
250	4156.629	-1.64	-23473.72	
300	4157.832	1.87	-1784.23	
350	4157.711	1.75	4398.58	

Table 4.13

Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 5000 MWhrs
 Machine Availability = 0.95
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW

If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 31417.96$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4133.777	3.66	-36578.45	Machine capacity cannot fill reservoir during one weekend $E_L = 10566.84$ MWhrs
100	4142.766	-5.60	-80956.31	Machine capacity cannot fill reservoir during one weekend $E_L = 21114.41$ MWhrs
150	4153.300	-3.03	-169116.86	
175	4153.590	-2.11	-144477.73	
200	4154.538	0.91	-114716.66	
250	4153.107	-3.64	-47075.11	
300	4154.825	-1.82	-28171.98	
350	4154.701	1.43	-23661.44	

Table 4.14
Results of Simulation

Pumped Hydrostorage Unit:

Reservoir Capacity = 2000 MWhrs
 Machine Availability = 1.0
 Pumping Efficiency = 0.70
 Generating Efficiency = 0.90

Load levels rounded to nearest
 multiple of 2.5 MW
 If machine capacity can fill up
 reservoir, expected energy loss
 of pumped hydro storage unit =
 $E_L = 13154$ MWhrs

Machine Capacity of Pumped Hydro Storage Unit MW	Energy Demand of System GWhrs	Percentage Difference in Energy Balance %	Benefits \$	Remarks
50	4132.533	-29.42	-151567.53	Reservoir capacity cannot fill reservoir during one weekend $E_L = 11544$ MWhrs
100	4135.293	-9.34	-116734.75	
150	4135.055	-11.07	-78869.37	
200	4135.655	-6.70	-64307.21	
300	4135.988	-4.28	-71591.96	
350	4135.988	-4.28	-71591.96	

The simulations on this table have been done with 1 500 MW nuclear
 unit removed from the set of generators.

Table 4.15
Results of Simulation

Figure 4.10: Relationship Between Economic Benefits and Machine Capacity for a Reservoir Size of 1000 MWhrs

Graph of Benefits v/s Machine Capacity of Pumped Hydro Storage Unit

Figure 4.11: Relationship Between Economic Benefits and Machine Capacity for a Reservoir Size of 2000 MWhrs

Graph of Benefits v/s Machine Capacity of Pumped Hydro Storage Unit

Figure 4.12: Relationship Between Economic Benefits and Machine Capacity for a Reservoir Size of 3000 MWhrs

Figure 4.13: Relationship Between Economic Benefits and Machine Capacity for a Reservoir Size of 5000 MWhrs

Figure 4.14: Effect of Removing One 500 MW Nuclear Unit From Set of Generators on Economic Benefits

4.5 Comments and Conclusion

The losses of the pumped hydro storage unit have been calculated in two ways: (i) by calculating the difference between the energy demands of the system with and without operation of the pumped hydro storage unit ($E_m - E_o$) and (ii) by analysis of the inefficiencies of the pumped hydro storage unit. Method (ii) is more accurate than method (i) because method (ii) involves no approximations. An indication of how the values obtained differ is the "Percentage Difference in Energy Balance" listed on Tables 4.4 to 4.15. The difference arises through (i) round-off errors introduced while approximating load levels to nearest multiples of 5 MW, 2.5 MW or 1.25 MW and (ii) the manner in which the software package doing the energy demand and production costing calculations interprets the load probability distribution curve.

Round-off errors can be explained as follows. During pumping, starting at the left of the function $f_{p_i}(L)$ (defined in section 4.1), the first impulse is shifted by the capacity of the pumped hydro storage unit; the second, third and other impulses are also similarly shifted. There shall be impulses in the middle of the PDF which cannot be shifted by the capacity of the machine or else the reservoir of the pumped hydro storage unit would have to take in more energy than it has room for. Such impulses generally lie between two "round-off" load levels as shown in Figure 4.15 where there is an impulse at 1551.90 MW. For specifying points on the load duration curve the impulse at 1551.90

Figure 4.15: Explaining Round-Off Errors

Figure 4.16: Load Probability Distribution Curve

is taken to 1552.5 MW thus making the reservoir appear to store more energy than it actually can. During generation impulses of function $f_{g_i}(L)$ (defined in section 4.1) are shifted to the left and similar problems arise.

Rounding-off in multiples of 5 MW rather than 2.5 MW gives higher values of percentage difference in energy balance as evidenced by comparisons of Tables 4.15 and 4.7. The two tables have loads which are modified by pumped hydro storage units with the same characteristics but the load levels in Table 4.7 are rounded off to the nearest multiples of 2.5 MW while those in Table 4.15 are rounded off in multiples of 5 MW. Simulations for randomly available pumped hydro storage units with reservoir size of 1000 MWhrs (Tables 4.5 and 4.6), when carried out with load levels rounded off to multiples of 2.5 MW gave percentage differences in energy balance of the order to 25 to 30%; the simulations tabulated were done with load levels rounded off to nearest multiples of 1.25 MW.

The calculation of energy demand was done using a provided software package. The load probability distribution curve obtained from sampling of load levels is a stepped function similar to Figure 4.16. The software package interprets the load probability distribution curve as having straight lines AB, BC, CD, etc.

The relation between economic benefits and machine capacity of the pumped hydro storage unit shown in Figures 4.10 to 4.14 all have the basic shape sketched in Figure 4.17. Point X occurs at machine capacities required to fill the reservoir during 1 weekend (48 hours).

Figure 4.17: Basic Shape of Benefits v/s Machine Capacity Curve

The curves for a reservoir size of 3000 MWhrs shown in Figure 4.12 have point X around 90 MW;

Machine capacity required to fill reservoir in 48 hours

$$= \frac{\text{Energy stored in reservoir}}{48 \times \text{Efficiency of pumped hydro storage unit operation as a pump}}$$

$$= \frac{3000}{48 \times 0.7} = 89.29 \text{ MW}$$

For a reservoir size of 5000 MWhrs (curves shown in Figure 4.13), for X is around 150 MW;

Machine capacity required to fill reservoir in 48 hours

$$= \frac{5000}{48 \times 0.7} = 148.81 \text{ MW}$$

Between W and X the energy loss of the pumped hydro storage unit are determined thus:

Capacity of pumped hydro storage unit	= C MW
Energy fed into pumped hydro storage unit during pumping for 48 hours	= 48 C MWhrs
Efficiency of pump	= η_p
Energy transferred to reservoir	= 48 C η_p MWhrs
Efficiency of pumped hydro storage unit during generation	= η_g
Energy generated by pumped hydro storage unit	= 48 C $\eta_p \eta_g$ MWhrs
Energy loss	= 48 C - 48 C $\eta_p \eta_g$
	= 48 C (1 - $\eta_p \eta_g$) MWhrs

Therefore as C increases, losses increase explaining the decrease in economic benefits between W and X .

For machine capacities greater than machine capacity at X , the losses of the pumped hydro storage unit are:

$$\begin{aligned}
 \text{Energy stored in reservoir} &= S \text{ MWhrs} \\
 \text{Energy fed into pumped hydro storage} &= S/\eta_p \text{ MWhrs} \\
 \text{unit to fill reservoir} & \\
 \text{Energy generated by pumped hydro storage} &= \eta_g S \text{ MWhrs} \\
 \text{unit from stored energy in reservoir} & \\
 \text{Energy losses} &= S/\eta_p - \eta_g S \\
 &= S(1/\eta_p - \eta_g) \text{ MWhrs}
 \end{aligned}$$

Since S , η_p and η_g are constant, the losses are constant. Between X and Y advantage is taken of the shape of the system cost function drawn in Figure 2.2 and explained in section 2.1.

There is a levelling off of economic benefits with increase of machine capacity between Y and Z . As reservoir size increases point Y moves further to the right. Whereas from W to Y the action of the machine modifies the load curves in the way shown in Figures 4.2 and 4.4; from C to D modification of the load curves take place similar to Figure 4.18 and 4.19. The maximum capacity of the machine cannot be used to fill up the reservoir while pumping or to drain the reservoir while generating.

————— Original load
 - - - - - Modification brought about by pumping

4.60

Figure 4.18: Load Curve on a Typical "Pumping" Day to Explain Segment YZ of Curve on Figure 4.17

————— Original load
 - - - - - Modification brought about by generation

Figure 4.19: Load Curve on a Typical "Generating" Day to Explain Segment YZ of Curve on Figure 4.17

Some parts or whole of the graphs on Figures 4.12 to 4.14 show negative economic benefits. This means that the advantage of shifting generation from high cost to low cost fuels brought about by pumped hydro storage unit cannot compensate for the additional generation required to meet the losses of the unit. Comparing Figures 4.14 and 4.11 shows that removal of a baseload unit (the 500 MW nuclear unit) has adverse effects on economic benefits. The average cost of the low cost fuel rises and hence the reduction in costs brought about by pumped storage is not enough to compensate for the additional generation required to meet the losses.

A 100 MW pumped hydrostorage unit results in benefits of \$69,158 with a 1000 MWhr reservoir as compared to \$28,690 for a 2000 MWhr reservoir (of Tables 4.4 and 4.7). Benefits of pumped hydrostorage is the difference between savings due to reduced cost of generation during weekdays and added expenses due to increased cost of generation during weekends. Filling a 2000 MWhr reservoir takes about twice the time to fill a 1000 MWhr reservoir. Added costs during weekends has a bigger effect on benefits for a 100 MW unit.

Economic benefits of pumped hydro storage is influenced by two factors: (i) losses of the pumped hydro storage unit which as they increase have a negative effect; (ii) difference between cost decrease while generating and cost increase while pumping, the difference increases with capacity. The effect of reduced availability of the pumped hydro storage unit is to diminish its number of hours of operation and

how this affects economic benefits can be seen in Figures 4.10 to 4.13. For discussion purposes, Figure 4.20 is drawn to label different segments of the curves. The effect of factor (i) is more marked between P and Q because diminished hours of operation means diminished losses. Between Q and R factor (ii) is more important because diminished hours of operation means less time during which factor (ii) can be taken advantage of. For pumped hydro storage units of practical interest (reasonable reservoir size and machine capacity); the effect of decreased availability (increased forced outage rate) is to decrease economic benefits.

It is recognized that the strategy of always trying to fill the reservoir over a weekend and trying to empty the reservoir during weekdays might not be desirable. The reservoir model, assumed to consist of only two states (full and empty), is not realistic. However, future work can be done to refine the simulation on these two points.

Figure 4.20: Effect of Reduced Availability on Benefits v/s Machine Capacity Curve

Chapter 5

ECONOMIC BENEFITS OF INTERCONNECTED SYSTEMS5.1 Introduction

Another way of shifting generation of energy from high cost to low cost fuels in an electric utility is by interconnection to another utility. Instead of running more expensive units the utility may import power from a neighbouring utility. A model is considered in which two power systems A and B are linked by a tie line. Whenever the incremental cost of generation in A is greater than that in B, A imports power from B, such that either the incremental cost of generation in the two systems are equalized or the tie line transfer capacity is reached. When the incremental cost of generation in B is greater than that in A, B imports from A and similar conditions prevail.

5.2 Incremental Cost and Outages

In a single power system the merit order of loading of generating units based on their average incremental costs yields the stepped function shown in Figure 5.1. As developed by Rau et al [9] the incremental cost (λ) v/s load relation can be approximated by the exponential equation (5.1) obtained by the least squares method of curve fitting

$$\lambda = \beta e^{KL} \quad (5.1)$$

If $\alpha = \ln \lambda$ and $\alpha = \ln \beta$ are substituted in equation (5.1), equation (5.2) results:

$$\theta = \alpha + KL \quad (5.2)$$

Equation (5.2) is sketched in Figure 5.2. If unit no. 1 goes on outage, the load which was being met by unit 1 will have to be met by unit 2 and those loads that were being met by unit 2 will have to be met by unit 3 and so on. The new situation is indicated by the dashed line in Figure 5.2, where all θ values are increased by α_1 .

If unit no.2 goes on outage, the loads that were being met by unit 1 are unaffected, those that were met by unit 2 are now met by unit 3 and so on. θ values for loads greater than capacity of unit 1 are increased by α_2 and is shown by the dotted line in Figure 5.2.

System load is random and owing to the linear relationship between θ and system load, θ is also random. The probability distribution function of θ for the case when all machines are operating $f_0(\theta)$ is shown in Figure 5.3(a) and represented as a block on Figure 5.3(b). Unit 1 has an availability of p_1 and a forced outage rate of q_1 and its failure increases all θ values of α_1 as shown by Figure 5.3(d). The density of θ will be according to Figure 5.3(b), for p_1 percent of the time and according to Figure 5.3(d) for q_1 percent of the time. Multiplying Figure 5.3(b) by p_1 and Figure 5.3(d) by q_1 and then adding the two products gives the probability distribution function of θ as modified by the randomness of unit 1. This is shown in Figure 5.3(e).

Figure 5.1: Incremental Cost v/s Load Relation

Figure 5.2: λ v/s Load Relation

Outage of unit 2 increases the value of a by a_2 for values of a greater than a_1 as shown in Figure 5.3(g). The availability of unit 2 is p_2 and its forced outage rate is q_2 ; therefore to take account of the randomness of unit 2, Figure 5.3(e) multiplied by p_2 has to be added to Figure 5.3(g) multiplied by q_2 yielding the probability distribution function of a shown in Figure 5.3(h).

The procedure can be continued to take into account a third machine as shown in Figure 5.3(i) to 5.3(k) and to any number of machines n giving $f_n(a)$. $f_3(a)$ is given by equations (5.3a) to (5.3d) and as can be seen it includes the outages of all machines.

$$f_3(a) = p_1 f_0(a) \quad \text{for } a \in (0, a_1) \quad (5.3a)$$

$$f_3(a) = p_2 f_1(a) \quad \text{for } a \in (a_1, a_1+a_2) \quad (5.3b)$$

$$f_3(a) = p_3 f_2(a) \quad \text{for } a \in (a_1+a_2, a_1+a_2+a_3) \quad (5.3c)$$

$$\begin{aligned} f_3(a) = & p_1 p_2 p_3 f_0(a) + q_1 p_2 p_3 f_0(a-a_1) \\ & + p_1 q_2 p_3 f_0(a-a_2) + p_1 p_2 q_3 f_0(a-a_3) \\ & + q_1 q_2 p_3 f_0(a-a_1-a_2) + q_1 p_2 q_3 f_0(a-a_1-a_3) \\ & + p_1 q_2 q_3 f_0(a-a_2-a_3) + q_1 q_2 q_3 f_0(a-a_1-a_2-a_3) \\ & \text{for } a \in (a_1+a_2+a_3, a_1+a_2+a_3+a_4) \end{aligned} \quad (5.3d)$$

The probability distribution function of a distribution can be characterized by moments and the use of moments facilitates the

Figure 5.3: Effect of Machine Outages on PDF of a

calculation of $f_n(a)$ from $f_0(a)$. Figure 5.4 shows the moments calculated from $f_0(a)$. a is divided into seven sub-blocks of equal length ΔL . Seven moments (zeroth, first, second, third, fourth, fifth and sixth) of each sub-block are calculated and indicated in Figure 5.4 as vectors $x_{M_1}^n$ etc. Each moment vector is given by

$$x_{M_i}^n = [M_0, M_1, M_2, \dots, M_6]^T \quad (5.4)$$

where $M_0 \dots M_6$ are the zeroth up to sixth moments, n signifies the sub-block number and i is the order of $f_i(a)$. $x_{M_0}^4$ represents the seven moments about the origin of the fourth sub-block of Figure 5.4(b). The meaning of X is explained below.

Figure 5.4(c) corresponds to Figure 5.3(d) where due to outage of unit 1 the sub-blocks characterizing $f_0(a)$ have been displaced to the right by a_1 . Thus, ${}^0M_0^1$ changes to ${}^{a_1}M_0^1$ the superscript indicating the shift (by an amount of a_1). Following shifting of pdf's all cumulants with the exception of the first cumulant remain unchanged; the first cumulant is increased by the amount of the shift. Appendix G gives the relation between moments and cumulants. The moments of the shifted blocks are calculated.

The moments of Figure 5.4(d) are obtained by multiplying the moments of $f_0(a)$ in Figure 5.4(b) by p_1 (availability), multiplying the moments of Figure 5.4(c) by q_1 (forced outage rate) and doing an addition of the two products. Thus

Figure 5.4: Block Representation of the PDF of a

$${}^0M_1^1 = {}^0M_0^1 \times p_1$$

$${}^0M_1^3 = {}^0M_0^3 \times p_1 + {}^a_1 M_0^1 \times q_1 \quad (5.5)$$

$${}^0M_1^8 = ({}^a_1 M_0^6 + {}^a_1 M_0^7) \times q_1$$

The outage of the second machine corresponding to Figure 5.3(g) is shown in Figure 5.4(c). The moment vectors M of the third and subsequent sub-blocks are shifted by a_2 using cumulant-moment relations. The moments of corresponding sub-blocks of Figure 5.4(d) multiplied by p_2 and those of Figure 5.4(e) multiplied by q_2 are added to give the moments of $f_0(a)$

$${}^0M_2^1 = {}^0M_1^1$$

$${}^0M_2^2 = {}^0M_1^2$$

$${}^0M_2^3 = {}^0M_1^3 p_2$$

$${}^0M_2^4 = {}^0M_1^4 p_2$$

$${}^0M_2^5 = {}^0M_1^5 p_2 + {}^a_2 M_1^3 q_2$$

$${}^0M_2^8 = {}^0M_1^8 p_2 + ({}^a_2 M_1^6 + {}^a_2 M_1^7 + {}^a_2 M_1^8) q_2 \quad (5.6)$$

The method is repeated for subsequent machines to obtain the moment of each sub-block for $f_n(a)$. A sum of the moments of each sub-block represents the moments of $f_n(a)$.

All the machines in the system can be considered equivalent to one hypothetical machine. The difference between the cumulants of $f_0(a)$ and $f_n(a)$ say $K_m(a)$ gives the cumulants of the hypothetical machine with outage representation $f_m(a)$. Convolution of $f_m(a)$ with $f_0(a)$ gives $f_n(a)$. From $K_m(a)$, μ_m , the moments of $f_m(a)$ are calculated.

5.3 Interconnected Systems

Consider the daily hourly loads in systems A and B as shown in Figure 5.5. The hourly load profiles are sampled every hour and each sample is given a probability of occurrence of 1/24; a joint probability mass function is obtained and shown in Figure 5.6. A load of 4 MW in system A and 5 MW in system B occurs for 3 hours; this is shown as three coincident impulses in Figure 5.6. The event consisting of 10 MW in system A and 9.5 MW in system B happens for one hour and is shown as one impulse in Figure 5.6. Continuing in this manner, Figure 5.6 is obtained.

The axes of Figure 5.6 are in MW. A change in axes according to equation (5.2) gives the bivariate probability density function (BPDF) of a_a (a for system A) and a_b (a for system B). The BPDF of Figure 5.6 corresponds to $f_0(a)$ of Figure 5.3(b).

The outages of machines in the two systems are independent. Outage of machine 1 in system A leads to an increase of a of 1.5, i.e. a_1 in Figure 5.3(d) for system A is 1.5. The dots in Figure 5.6 are displaced

Figure 5.5: Daily, Hourly Loads

horizontally by 1.5 and are shown by the unfilled dots. The probability of outage of unit 1 in system A is q_1 and the probability of it being available is p_1 ; therefore the probabilities associated with the original dots are $1/24 \times p_1$ while those associated with the unfilled dots are $1/24 \times q_1$.

The method can be continued to include the outages of all machines in system A. The outages of machines in system B lead to a movement of impulses in the vertical direction along a_b in Figure 5.6. The number of impulses obtained when all the machines are considered is large. A mathematical technique to overcome this problem is described in the next section.

5.4 Mathematical Modelling of Randomness of Load and Generating Units in Interconnected Systems

The product moment of order $(r+s)$ for a discrete BPDF as shown in Figure 5.6 is defined by

$$\mu_{rs} = \sum_{(a_a, a_b) \in J} a_a^r a_b^s p_{a_a a_b}(a_a, a_b) \quad (5.7)$$

where the set J contains all values of a_a and a_b such that $p_{a_a a_b}(a_a, a_b) > 0$. In operator E notation μ_{rs} becomes

$$\mu_{rs} = E\{a_a^r a_b^s\} \quad (5.8)$$

The central moments of order $(r+s)$ is defined as

Figure 5.6: Joint Load Probability Mass Function

$$\begin{aligned}\mu_{rs} &= \sum_{-\infty}^{\infty} \sum (\underline{a}_a - \bar{a}_a)^r (\underline{a}_b - \bar{a}_b)^s = f_{\underline{a}\underline{a}_b}(\underline{a}_a, \underline{a}_b) \\ &= E\{(\underline{a}_a - \bar{a}_a)^r (\underline{a}_b - \bar{a}_b)^s\}\end{aligned}\quad (5.9)$$

where $\bar{a}_a$ and $\bar{a}_b$ are the means of the marginal distribution of $\underline{a}_a$ and $\underline{a}_b$ respectively and are given by equations (5.10) and (5.11).

$$\bar{a}_a = E\{\underline{a}_a\} = \sum_{\underline{a}_a \in J_a} \underline{a}_a p_{\underline{a}_a}(\underline{a}_a) \quad (5.10)$$

$$\bar{a}_b = E\{\underline{a}_b\} = \sum_{\underline{a}_b \in J_b} \underline{a}_b p_{\underline{a}_b}(\underline{a}_b) \quad (5.11)$$

J_a and J_b are the sets of values of $\underline{a}_a$ and $\underline{a}_b$ such that $p_{\underline{a}_a}(\underline{a}_a) > 0$ and $p_{\underline{a}_b}(\underline{a}_b) > 0$ respectively.

The joint probability mass function $p_{\underline{a}\underline{a}_b}(\underline{a}_a, \underline{a}_b)$ can be approximated by a continuous function $f_{\underline{a}\underline{a}_b}(\underline{a}_a, \underline{a}_b)$ in terms of the bivariate Gram

Charlier Expansion as follows

$$f_{\underline{a}\underline{a}_b}(\underline{a}_a, \underline{a}_b) = \frac{1}{\sigma_a \sigma_b} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} D_{ij} H_{ij} \phi(z_a, z_b) \quad (5.12)$$

σ_a and σ_b are the standard deviations of the marginal distribution of $\underline{a}_a$ and $\underline{a}_b$ respectively. D_{ij} are coefficients involving the central moments of (5.9) and H_{ij} are Hermite polynomials which are functions of the coordinates $(\underline{a}_a, \underline{a}_b)$ at any point. The coefficients D_{ij} and Hermite polynomials are defined in Appendices H and I. $\phi(z_a, z_b)$ is the bivariate normal function and is given by

$$\phi(z_a, z_b) = \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp \left[-\frac{1}{2(1-\rho^2)} \times (z_a^2 - 2\rho z_a z_b + z_b^2) \right] \quad (5.13)$$

with

$$z_a = \frac{a_a - \bar{a}_a}{\sigma_a} \quad \text{and} \quad z_b = \frac{a_b - \bar{a}_b}{\sigma_b} \quad (5.14)$$

The machines in system A are represented as one hypothetical machine (as described in section 5.2 above) with pdf f_a and moments μ_i^a . The pdf of the hypothetical machine has to be convolved with the marginal distribution of a in system A. This modifies the central moments of the bivariate distribution of a as given by (5.9) to equation (5.15)

$$\mu_{rs}^i = E \{ (a_a - \bar{a}_a + f_a)^r (a_b - \bar{a}_b)^s \} \quad (5.15)$$

Expanding (5.15) as a binomial series, equation (5.16) is obtained

$$\begin{aligned} \mu_{rs}^i &= \mu_{rs} + \binom{r}{1} \mu_{(r-1)s} \mu_1^a + \binom{r}{2} \mu_{(r-2)s} \mu_2^a + \dots \\ &+ \mu_{0s} \mu_r^a = \sum_{i=0}^r \binom{r}{i} \mu_{(r-i)s} \mu_i^a \end{aligned} \quad (5.16)$$

Similarly the machines in system B are represented as one hypothetical machine with pdf f_b and moments μ_i^b , and the central moments in (5.16) have to be modified according to equation (5.17),

$$\mu_{rs}^{ii} = \sum_{i=0}^s \binom{s}{i} \mu_{r(s-i)}^i \mu_i^b \quad (5.17)$$

The final moments given by (5.17) are the moments of the load and outages combined in both systems. Thus they represent the moments of the bivariate density of the two equivalent loads. These moments are used in the evaluation of the D coefficients of equation (5.12).

5.5 Calculation of Economic Benefits

The plan view of the BPDF of a for two interconnected systems is shown in Figure 5.7. The probability associated with any particular values of a is represented by a discrete pulse normal to the plane of the paper; the representation of all pulses is approximated by the continuous function (5.10) whose D coefficients have been appropriately modified to take machine outages in both systems into consideration. When a_a is greater than a_b , energy is transferred from system B to system A whereas the converse is true when a_b is greater than a_a .

The function does not exist in regions where loads are less than the minimum loads in both systems; the regions are shown blacked out in Figure 5.7. At point A the two systems are operating at a_{IC_A} and a_{IC_B} respectively (IC_A is the installed capacity in system A and IC_B the installed capacity in system B). Points such that a_a is greater than a_{IC_A} and a_b is greater than a_{IC_B} represent failure states when neither system can be operated. When a_a is greater than a_b , the shortfall in capacity in system A is determined using equation (5.2) and is given by

$$C_{sa} = \frac{a_a - a_{IC_A}}{K_a} \quad (5.18)$$

Figure 5.7: Plan View of BPDF of a

whereas the spare capacity in system B is given by

$$C_{sb} = \frac{a_{IC_b} - a_b}{K_b} \quad (5.19)$$

System A is operated only if C_{sa} is less than C_{sb} , i.e.

$$\frac{a_a - a_{IC_a}}{K_a} \leq \frac{a_{IC_b} - a_b}{K_b} \quad (5.20)$$

Using equality with $a_b=0$, XY in Figure 5.7 is as follows:

$$XY = (K_a/K_b) a_{IC_b} \quad (5.21)$$

Distance UV in Figure 5.7 can be similarly determined and is given by

$$UV = (K_b/K_a) a_{IC_a} \quad (5.22)$$

As indicated in Figure 5.7, line VY divides the region into a subregion where the function exists and another one in which the function does not exist.

Transfer of energy between systems A and B may be restricted by tieline transfer capacity, let the maximum power which can be transferred be T MW in either direction. According to equation (5.2) this assistance T transforms in the a domain to a value TK_a for system A. If system A is operated at $a = a_{IC_a}$ (line XA of Figure 5.7), the assistance TK_a results in line WCZ. Similarly assistance T to system B results in line KLM. Therefore after taking into consideration minimum load, installed capacity and transfer capacity the region of interest in Figure 5.7, as far as energy transfer between systems A and B are

are concerned, has been restricted to PQLMZC.

Consider a point inside PQLMZC such that a_a is greater than a_b and let the energy transfer from system B to system A be ΔL MW. The net result is the offloading of generators in system A such that λ of system A is reduced from λ_a to λ_{anew} and more generating capacity is committed in system B so that λ in system B increases from λ_b to λ_{bnew} . Avoided costs in system A due to import is given by

$$\$_a = \int_{L_a}^{L_{anew}} \beta_a e^{K_a L_a} dL_a = -(\lambda_{anew} - \lambda_a)/K_a \quad (5.23)$$

and the increase in cost in system B is given by

$$\$_b = \int_{L_b}^{L_{bnew}} \beta_b e^{K_b L_b} dL_b = (\lambda_{bnew} - \lambda_b)/K_b \quad (5.24)$$

The global savings for system A and system B is given by

$$\$_{global} = -(\lambda_{anew} - \lambda_a)/K_a - (\lambda_{bnew} - \lambda_b)/K_b \quad (5.25)$$

There is a probability associated with any (a_a, a_b) . The mathematical expectation of global savings is the summation of expected savings at each point in region PQLMZC.

$$\begin{aligned} \text{Expected benefits} &= \sum \left(\begin{array}{l} \$_{global} \text{ at any} \\ \text{point in PQLMZC} \\ \text{according to} \\ (5.25) \end{array} \right) \times \left(\begin{array}{l} \text{probability} \\ \text{value at} \\ \text{that point} \end{array} \right) \times \left(\begin{array}{l} \text{period} \\ \text{hours} \end{array} \right) \end{aligned} \quad (5.26)$$

λ_{anew} and λ_{bnew} in equations (5.22) to (5.24) need to be determined. When α_a is greater than α_b , impact of ΔL into system A will reduce α_a due to offloading of machines and the loading of more units in system B shall increase α_b . If ΔL is unconstrained by tieline capacity, ΔL can be made such that $\alpha_a = \alpha_b$ to result in maximum gains (the concept of equal incremental cost loading). Then

$$\alpha_a + K_a (L_a - \Delta L) = \alpha_b + K_b (L_b + \Delta L) \quad (5.27)$$

from which one gets

$$\Delta L = (\alpha_a - \alpha_b) / (K_a + K_b) \quad (5.28)$$

Substituting (5.27) into (5.26) the following equation for α_{anew} and α_{bnew} are obtained

$$\alpha_{\text{anew}} = \alpha_{\text{bnew}} = (\alpha_a K_b + \alpha_b K_a) / (K_a + K_b) \quad (5.29)$$

If transfer of energy is limited by tieline capacity the incremental cost in the two systems cannot be equalized. ΔL is equal to T and if α_a is greater than α_b , α_{anew} is greater than α_{bnew} .

$$\alpha_{\text{anew}} = \alpha_a - K_b T \quad (5.30)$$

$$\alpha_{\text{bnew}} = \alpha_b + K_a T \quad (5.31)$$

α_{anew} , α_{bnew} can be calculated from equations (5.28) to (5.30) using the fact that $\alpha = \ln \lambda$ or $\lambda = e^\alpha$.

5.6 Results

The generators present in system A and B are given in Appendix J. The hydraulic units in the two systems are made to drain all their available energy and their position in the loading order is found by probabilistic simulation. Values of α and K calculated for systems A and B are given in Table 5.1.

System	α	K
A	1.641352	0.000527
B	1.869275	0.000545

Table 5.1
Values of α and K

The load model for the two systems is described in Appendix J and the load duration curves for each system are sketched in Figures 5.8 and 5.9. The results obtained are shown in Tables 5.2 and 5.3. In transaction I the two utilities A and B share the global savings equally among them, while in transaction II the importing system pays the export system depending on a value of λ which is the average of the initial incremental costs in the two systems, i.e.

$$\lambda_t = \frac{1}{2} (\lambda_a + \lambda_b) \quad (5.32)$$

The simulations are done for a time period of 2160 hours (3 months).

Figure 5.8: Load Duration Curve for System A

Figure 5.9: Load Duration Curve for System B

Tie Line Capacity MW	Savings x 10 ⁷ dollars		
	System A	System B	Total
200	0.151842	0.151842	0.303684
400	0.238520	0.238520	0.477040
800	0.296976	0.296976	0.593952
1200	0.303822	0.303822	0.607644
1600	0.304212	0.304212	0.608424
2000	0.304222	0.304222	0.608444
2400	0.304220	0.304220	0.608440

Table 5.2

Transaction I Between Utilities A and B

Tie Line Capacity MW	Savings x 10 ⁷ dollars		
	System A	System B	Total
200	0.161024	0.142661	0.303685
400	0.269112	0.207930	0.477042
800	0.369325	0.224633	0.593958
1200	0.390437	0.217211	0.607648
1600	0.392649	0.215784	0.608433
2000	0.392734	0.215712	0.608446
2400	0.392735	0.215712	0.608447

Table 5.3

Transaction II Between Utilities A and B

Figure 5.10: Graph of Economic Benefits v/s Tie Line Capacity for Transactions I and II

Sum of Economic Benefits for System A and System B v/s Tie Line Capacity

Figure 5.11: Graph of Economic Benefits to Both Systems v/s Tie Line Capacity

The production costs for system A in isolation system B in isolation and for the two systems combined as one system are calculated by probabilistic simulation and the results are shown in Table 5.4.

Production cost for system A in isolation(C_A)	\$33250192.00
Production cost for system B in isolation(C_B)	\$58790080.00
$(C_A + C_B)$	\$92040272.00
Production cost for system A and B combined(C)	\$83638784.00
$(C_A + C_B - C)$	\$8401488.00

Table 5.4
Results of Probabilistic Simulation

5.7 Comments and Conclusion

Figure 5.10 shows how economic benefits vary with tie line capacity. Under transaction I, the two systems share the economic benefits equally and as can be seen from Figure 5.10 the curves for the two systems coincide. Under transaction II, the importing system pays the exporting system at a rate given by equation (5.32). As can be seen from Table 5.1, the α and K values for system B are greater than the corresponding values of α and K for system A and therefore system B imports power from system A. The economic benefits to system A are mainly additional revenues due to sale of power while the economic benefits to system B is the difference in cost between the cheaper energy it purchases from

system A and the more expensive energy from its generators. Under transaction II the economic benefits for system A are greater than those to system B.

All the curves in Figure 5.10 tend to the origin as the line capacity is decreased. A situation where no power is transferred over the tie line corresponds to the two systems not being connected and thus no economic benefits can be obtained.

When the tie line capacity tends to infinity, the economic benefits become a constant above a certain tie line capacity. For an infinite tie line capacity any amount of power can be transferred between the two systems and so the two systems effectively become one. The difference between production cost for the two systems combined as one system (C) and the sum of production costs for system A in isolation and system B in isolation ($C_A + C_B$) should give the total economic benefits to system A and system B due to infinite tie line capacity connection.

A plot of total economic benefits against tie line capacity is given in Figure 5.11. In that figure the value of economic benefits for infinite tie line capacity connection obtained by probabilistic simulation is indicated. Probabilistic simulation overestimates the economic benefits by about 38%.

In the main study of benefits of interconnection, the incremental cost of generation, λ , as a function of system load, L_1 is a continuous function described by equation (5.1), i.e., if

$$\lambda = g(L) \quad (5.32)$$

$g(L)$ is a continuous function. In the probabilistic simulation, λ , the incremental cost of generation for each unit is taken to be discrete and therefore the incremental cost of generation, λ , as a function of system load, L , is a stepped function shown in Figure 5.12, i.e. the function $g(L)$ in equation (5.32) is a stepped function.

Calculation of fuel cost from time t_1 to time t_2 , F , can be simply approximated by the integration of the product of incremental cost of generation λ and load L with time, i.e.,

$$F = \int_{t_1}^{t_2} L \lambda \, dt = \int_{t_1}^{t_2} L g(L) \, dt \quad (5.33)$$

Taking $g(L)$ to be a continuous function does not obviously give quite the same value of F as when taking $g(L)$ to be a stepped function. This difference in approach explains the 38% difference in the estimation of economic benefits which involves calculation of fuel costs.

The tie line which has been considered is assumed to be always available. An improvement of the method can be how to study the influence of a randomly available tie line on the economic benefits of interconnection.

Chapter 6

CONCLUDING REMARKS6.1 Comparing the Economic Benefits of Pumped Hydro Storage to
Those of Interconnection to a Neighbouring Utility

The load model for the system on which pumped hydro storage simulations were carried out (which will be simply referred to as system P) is identical to the load model for system B used in the study of benefits of interconnection. The generators in system P and system B are all the same except that system P has one more 500 MW nuclear baseload unit. In fact this 500 MW nuclear unit is purposefully added to the system so that the pumped hydro storage unit would result in benefits rather than losses.

The use of the pumped hydro storage unit in system P takes advantage of the fact that the average price of generating one unit of electric energy during a weekend is lower than the same price during weekdays. Interconnection of system B to system A takes advantages of the differential price of generating one unit of electric energy in system A and system B at all times during a week. The benefits of pumped hydro storage in system P tend to be underestimated because advantage could also have been taken of the difference between the higher price of generating energy during the day in a weekday and the lower price of generating energy during a weekday night.

As has been mentioned the building of the pumped hydro storage unit also necessitates the addition of one more 500 MW nuclear unit to system P. Installation of the nuclear unit brings about a reduction in fuel costs of about \$13,650,000 during 3 months. The studies in Chapter 4 show that the maximum benefits expected from a pumped hydro storage unit in system P is \$130,000 for 3 months; this is for a 300 MW unit with a reservoir size of 3000 MWhrs. An analysis of investments required for the nuclear and pumped hydro storage units is made in Appendix K. It is shown that the reduction in fuel costs are not enough to cover interest and depreciation charges on the investment. Therefore installation of a pumped hydro storage unit and a new nuclear unit is not advisable.

To interconnect system B to system A, a transmission line has to be installed as well as necessary equipment to monitor the difference in price between the two systems, so as to know when and in what direction should power flow occur between the two systems. The graph of Figure 5.11 shows that a 900 MW interconnection would give optimum economic benefits of 6×10^6 \$ in 3 months to the two systems. Reference [16] conducts a study on the present worth of revenue required for building a 900 MW, 500 KV, 30 year lifespan transmission line; present worth of revenue being defined as the sum of levelized annual fixed charges on total line capital investment and annual expenses for line losses. An optimistic value of the present worth of revenue given for the year 1981 is 655,000\$ per mile of transmission line. For breakeven between the present worth of revenue and economic benefits the maximum length of line, L miles, between systems A and B is calculated as follows:

$655,000 \text{ \$/mile} \times L \text{ miles for one year} = \$6 \times 10^6 / 3 \text{ months} \times 12 \text{ months}$

$L \approx 37 \text{ miles}$

This length is unrealistic for a 500 KV line necessary to transfer 900 MW of power; typically their lengths range from 300 miles to 1500 miles. Therefore the economic benefits of interconnection are not sufficient when compared to the annual cost of investment necessary for the interconnecting transmission line.

6.2 Suggestions for Future Work

1. In some power systems, pumping is not only carried out during weekends as has been done in Chapter 4 but also during the periods of low demand at night. This is illustrated in Figure 2.1. An extension of the work in this thesis could be how to apply movement of impulses to simulate pumping during weekends and the nights of weekdays, and generation during peak demand hours of weekdays.

2. The reservoir model used in the pumped hydro storage simulation is quite simple; the reservoir is either full or empty. Improvements in the simulation could be made to cater for more difficult reservoir models like those shown on Tables 3.1 and 3.2.

6.3 Conclusion

1. After describing the load in a system by a discrete probability density function, generation by a pumped hydro storage unit can be simulated by shifting appropriate impulses to the left; pumping by

same can be simulated by shifting appropriate impulses to the right. This approach can take into account the forced outage rate of the pumped hydro storage unit. It has been shown that shifting of impulses to the left during generation is equivalent to Billinton's and Harrington's method of peak shaving the load duration curve. Equivalent formulae on how pumping affects the load duration curve have been derived.

2. From the discrete probability density function whose impulses have been moved to account for the action of the pumped hydro storage unit the load probability distribution function is constructed while obeying the chronological constraints on the storage reservoir. The inefficiencies associated with the pumped hydro storage unit during pumping and generation are also taken care of. The load probability distribution function can be used for production costing studies.

3. The method proposed for simulating the action of a pumped hydro storage unit can be used by a utility to investigate alternative investment proposals with the aim of reducing fuel costs. This thesis has compared two alternatives: (i) installation of a pumped hydro storage unit and (ii) interconnection to a neighbouring utility.

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APPENDIX A

Heapsort Algorithm

The Heapsort Algorithm described in this Appendix is taken from reference [17]. The algorithm arranges a sequence of numbers (the numbers of interest in this thesis are load samples) in a nondecreasing order.

Shown in Figure A.1 is a binary tree. Each circle is called a node. The two nodes attached to the bottom of node A are called the sons of A, B is called the leftson and C is called the rightson. A is called the parent of B and C. Node R has no parent and is called the root of the tree. Nodes which have no sons are called leaves; node B is a leaf. Nodes A, B and C make up a subtree of the tree with root R; the root of the subtree is node A. The height of a tree or subtree is the length of a longest path from the root to a leaf.

The binary tree of Figure A.2 is called a heap. Its special feature is that starting from the root and travelling down any path to a leaf the elements in the nodes are linearly ordered. Such a tree can be kept in an array A as follows:

16	11	9	10	5	6	8	1	2	4
----	----	---	----	---	---	---	---	---	---

$A[1]$ is the element stored as the root. $A[2i]$ and $A[2i+1]$ are the elements stored at the left and rightsons (if they exist) of the vertex in which element $A[i]$ is stored.

Figure A.1: A Binary Tree

Figure A.2: A Heap

If $A(n)$ is the last element in the array, $A[1]$ and $A[n]$ can be interchanged, i.e. the above array looks thus:

4	11	9	10	5	6	8	1	2	16
---	----	---	----	---	---	---	---	---	----

The tree is similar to the one in Figure A.3. Element 16 is excluded from further consideration. The tree in location 1 to $n-1$ is again made into a heap by first exchanging element 4 with 11 the larger of its two sons 11 and 9. In its new position 4 has sons 10 and 5. Since they are larger than 4, 10, the larger son, is interchanged with 4. The new sons of 4 are 1 and 2 and since they are smaller than 4 no more interchanges are required. The resulting heap is shown in Figure A.4.

Let $a_1, a_2, \dots, a_n$ be the sequence of hourly load levels to be put in non-decreasing order. Let $a_1, a_2, \dots, a_n$ be elements of an array A in this order: $A[i] = a_i, 1 \leq i \leq n$. The first step is to build a heap, i.e. the elements in A are rearranged to satisfy the heap property: $A[i] \geq A[2i]$ for $1 \leq i \leq n/2$ and $A[i] \geq A[2i+1]$ for $1 \leq i \leq n/2$.

This is done by starting with the leaves and building larger and larger heaps. Each subtree consisting of a leaf already forms a heap. A subtree of height h is made into a heap by interchanging the element at the root with the larger of the elements at the sons of the root, if it is smaller than either of them. So doing may destroy a heap of height $h-1$ which must then be remade into a heap. The algorithm which converts array A into a heap is called BUILDHEAP and it calls HEAPIFY, a recursive procedure. HEAPIFY has two parameters i and j which give the range of location in the array A having the heap property and i is the root of the heap to be made.

Figure A.3: Result of Interchanging 4 and 16 in the Heap of Figure A.2

Figure A.4: Result After Reconstructing Heap and Excluding Element 16

The procedure HEAPIFY is as follows.

Procedure HEAPIFY (i, j)

1. If i is not a leaf and if a son of i contains a larger element than i does then
begin
2. Let K be a son of i with the larger element;
3. Interchange $A[i]$ and $A[K]$;
4. HEAPIFY (K, j)
- end

The parameter j is used to determine whether i is a leaf and whether i has one or two sons. If $i > j/2$, then i is a leaf and HEAPIFY (i, j) need not do anything since $A[i]$ is a heap by itself.

Algorithm BUILDHEAP is given in flowchart form in Figure A.5. The complete specification of HEAPSORT algorithm is given in Figure A.6. The proof that the sorting of n elements is of the order of $n \log n$ is found in reference [17].

Figure A.5: Algorithm BUILDHEAP

Figure A.6: Algorithm HEAPSORT

APPENDIX C

Thermal Generator Input/Output Characteristics

The energy value of the fuel used to drive a generator is measured in MBtu. The electric energy output is measured in MWhrs. The energy value used to produce 1 MWhr of electric energy in a machine varies with the capacity of the machine as shown in Figure C.1; the curve is called a heat rate curve.

Multiplying each y-axis value by the corresponding x-axis value gives the input/output curve of Figure C.2. Differentiating the input/output curve results in the incremental heat rate curve. The continuous incremental heat rate curve of Figure C.3 is approximated by the dotted lines on the same curve. CB1, CB2, CB3 are three capacity blocks for the machine.

Multiplying the y-axis of Figure C.3 by unit fuel cost in \$/MBtu gives the average production cost of each of the blocks CB1, CB2 and CB3.

$$\text{3rd cumulant} = K_3 = m_3 - 3m_2 m_1 + 2m_1^3$$

$$\text{4th cumulant} = K_4 = m_4 - 4m_3 m_1 - 3m_2^2 + 12m_2 m_1^2 - 6m_1^4$$

$$\begin{aligned} \text{5th cumulant} = K_5 = m_5 - 5m_4 m_1 - 10m_3 m_2 + 20m_3 m_1^2 + 30m_2^2 m_1 \\ - 60m_2 m_1^3 + 24m_1^5 \end{aligned}$$

$$\begin{aligned} \text{6th cumulant} = K_6 = m_6 - 6m_5 m_1 - 15m_4 m_2 + 30m_4 m_1^2 \\ - 10m_3^2 + 120m_3 m_2 m_1 - 120m_3 m_1^3 + 30m_2^3 - 270m_2^2 m_1^2 \\ + 360m_2 m_1^4 - 120m_1^6 \end{aligned}$$

Nuclear Units:

$$\text{No. of machines} = 3$$

$$C = 500 \text{ MW}$$

$$q = 0.18$$

$$m_1 = 90$$

$$m_2 = 45000$$

$$m_3 = 22500000$$

$$m_4 = 1.125 \times 10^{10}$$

$$m_5 = 5.625 \times 10^{12}$$

$$m_6 = 2.8125 \times 10^{15}$$

$$K_1 = 90$$

$$K_2 = 36900$$

$$K_3 = 11808000$$

$$K_4 = 1.05534 \times 10^9$$

$$K_5 = -2.2765824 \times 10^{12}$$

$$K_6 = -1.8766159 \times 10^{15}$$

Coal 7 Units

No. of machines = 2

C = 400

q = 0.13

$$m_1 = 52$$

$$m_2 = 20800$$

$$m_3 = 8320000$$

$$m_4 = 3328000000$$

$$m_5 = 1.3312 \times 10^{12}$$

$$m_6 = 5.3248 \times 10^{14}$$

$$K_1 = 52$$

$$K_2 = 18096$$

$$K_3 = 5356416$$

$$K_4 = 930568704$$

$$K_5 = -3.06123989 \times 10^{11}$$

$$K_6 = -3.9747817 \times 10^{14}$$

Hydro Units

No. of machines = 1

C = 500

q = 0.01

$$m_1 = 5$$

$$m_2 = 2500$$

$$m_3 = 1250000$$

$$m_4 = 625000000$$

$$m_5 = 3.125 \times 10^{11}$$

$$m_6 = 1.5625 \times 10^{14}$$

$$K_1 = 5$$

$$K_2 = 2475$$

$$K_3 = 1212750$$

$$K_4 = 581996250$$

$$K_5 = 2.6716883 \times 10^{11}$$

$$K_6 = 1.1056462 \times 10^{14}$$

Coal 2 Units

No. of machines = 1

C = 150

q = 0.08

$$m_1 = 12$$

$$m_2 = 1800$$

$$m_3 = 270000$$

$$m_4 = 40500000$$

$$m_5 = 6075000000$$

$$m_6 = 9.1125 \times 10^{11}$$

$$K_1 = 12$$

$$K_2 = 1656$$

$$K_3 = 208656$$

$$K_4 = 20805984$$

$$K_5 = 548347968$$

$$K_6 = -4.6776979 \times 10^{11}$$

Coal 3 Units

No. of machines = 2

C = 350

q = 0.13

$$m_1 = 45.5$$

$$m_2 = 15925$$

$$m_3 = 5573750$$

$$m_4 = 1950812500$$

$$m_5 = 6.8278437 \times 10^{11}$$

$$m_6 = 2.3897453 \times 10^{14}$$

$$K_1 = 45.5$$

$$K_2 = 13854.75$$

$$K_3 = 3588380.25$$

$$K_4 = 545482289.6$$

$$K_5 = -1.5701676 \times 10^{11}$$

$$K_6 = -1.7838634 \times 10^{14}$$

011 C Units

No. of machines = 1

C = 350

q = 0.14

$$m_1 = 49$$

$$m_2 = 1.715 \times 10^4$$

$$m_3 = 6.0025 \times 10^6$$

$$m_4 = 2100875000$$

$$m_5 = 7.3530625 \times 10^{11}$$

$$m_6 = 2.5735719 \times 10^{14}$$

$$K_1 = 49$$

$$K_2 = 1.4749 \times 10^4$$

$$K_3 = 3.71675 \times 10^6$$

$$K_4 = 5.01554 \times 10^8$$

$$K_5 = -2.02518 \times 10^{11}$$

$$K_6 = -1.93099 \times 10^{14}$$

Oil 1 Units

No. of machines = 2

C = 200

q = 0.1

$$m_1 = 20$$

$$m_2 = 4000$$

$$m_3 = 800000$$

$$m_4 = 160000000$$

$$m_5 = 3.2 \times 10^{10}$$

$$m_6 = 6.4 \times 10^{12}$$

$$K_1 = 20$$

$$K_2 = 3600$$

$$K_3 = 576000$$

$$K_4 = 66240000$$

$$K_5 = -1843200000$$

$$K_6 = -4.19328 \times 10^{12}$$

GT Units

No. of machines = 4

C = 50

q = 0.21

$$m_1 = 10.5$$

$$m_2 = 525$$

$$m_3 = 26250$$

$$m_4 = 1312500$$

$$m_5 = 65625000$$

$$m_6 = 3281250000$$

$$K_1 = 10.5$$

$$K_2 = 414.75$$

$$K_3 = 12027.75$$

$$K_4 = 4769.625$$

$$K_5 = -29792736.75$$

$$K_6 = -1747815602$$

Addition of Cumulants

The K_1 's of each unit multiplied by the number of machines are all added up to give $\bar{K}_1 = 613$.

The K_2 's of each unit multiplied by the number of machines are all added up to give $\bar{K}_2 = 202340.50$

The K_3 's of each unit multiplied by the number of machines are all added up to give $\bar{K}_3 = 59651860$

The K_4 's of each unit multiplied by the number of machines are all added up to give $\bar{K}_4 = 7354977300$

The K_5 's of each unit multiplied by the number of machines are all added up to give $\bar{K}_5 = -7.6946469 \times 10^{12}$

The K_6 's of each unit multiplied by the number of machines are all added up to give $\bar{K}_6 = -6.8713240 \times 10^{15}$

$$G_1 = \bar{K}_3 / \bar{K}_2^{3/2} = 0.65538993$$

$$G_2 = \bar{K}_4 / \bar{K}_2^2 = 0.17964523$$

$$G_3 = \bar{K}_5 / \bar{K}_2^{5/2} = -4.1781271 \times 10^{-1}$$

$$G_4 = \bar{K}_6 / \bar{K}_2^3 = 8.2945336 \times 10^{-1}$$

$$\text{Mean} = 613$$

$$\text{Standard Deviation} = \bar{K}_2^{1/2} = 449.8227429$$

$$\text{Total Installed Capacity} = 1C = 4600 \text{ MW}$$

Calculation of loss of load probability (LOLP) for weekend peak load which is 2195 MW.

$$\text{Reserve capacity} = RC = IC - 2195 =$$

$$= 4600 - 2195 = 2405 \text{ MW}$$

$$Z = \frac{2405 - \text{Mean}}{\sqrt{K_2}} = \frac{2405 - 613}{432.3661643} =$$

$$= 3.98379146$$

$$\begin{aligned} \text{LOLP} = \int_Z^{\infty} N(Z) dz + \frac{G_1}{3!} N^{(2)}(Z) - \frac{G_2}{4!} N^{(2)}(Z) + \frac{G_5}{5!} N^{(4)}(Z) \\ - \frac{(G + 10G_1^2)}{6!} N^{(5)}(Z) \end{aligned}$$

$$\text{where } \int_Z^{\infty} N(Z) dz = 0.00003392$$

$$N(z) = 0.00014278$$

$$N^{(2)}(Z) = (Z^2 - 1) N(Z) = 0.00212316$$

$$N^{(3)}(Z) = (-Z^3 + 3Z) N(Z) = -0.00732065$$

$$N^{(4)}(Z) = (Z^4 - 6Z^2 + 3) N(Z) = 0.02279445$$

$$N^{(5)}(Z) = (-Z^5 + 10Z^3 - 15Z) N(Z) = -0.06152577$$

$$\text{LOLP} = 0.00067919$$

All other weekend load levels are below 2195 MW. Therefore the loss of load probability for other load levels is less than 0.000679

APPENDIX C

Thermal Generator Input/Output Characteristics

The energy value of the fuel used to drive a generator is measured in MBtu. The electric energy output is measured in MWhrs. The energy value used to produce 1 MWhr of electric energy in a machine varies with the capacity of the machine as shown in Figure C.1; the curve is called a heat rate curve.

Multiplying each y-axis value by the corresponding x-axis value gives the input/output curve of Figure C.2. Differentiating the input/output curve results in the incremental heat rate curve. The continuous incremental heat rate curve of Figure C.3 is approximated by the dotted lines on the same curve. CB1, CB2, CB3 are three capacity blocks for the machine.

Multiplying the y-axis of Figure C.3 by unit fuel cost in \$/MBtu gives the average production cost of each of the blocks CB1, CB2 and CB3.

Figure C.1: Heat Rate Curve

Figure C.2: Input/Output Curve

Figure C.3: Incremental Heat Rate Curve

APPENDIX D

Production Costing Equations

The load model for a power system is represented as a load probability distribution curve, $F(L)$ as shown in Figure D.1. The area under the curve gives the expected load in the system:

$$\text{Expected load} = \int_0^{\theta} F(L) dL \quad (D.1)$$

where θ is the peak load.

The randomness of generators is modelled as outage capacity probability density functions (Figure D.2). The random generators are replaced by fictitious 100% reliable units and fictitious random loads whose probability density functions are the outage capacity probability density functions of the units; the situation is depicted in Figure D.3 for a system with two generating units.

The load to be met by the 100% reliable generating units is called the effective load L_e and is defined by

$$L_e = L + \sum_i L_{oi} \quad (D.2)$$

where L_{oi} is the random outage load of the i th unit. L_e is the sum of independent random variables L and L_{oi} , $i = 1, 2, \dots, G$ where G is the number of generators. $F(L_e)$ is obtained using the recursive convolution equation

$$F^1(L_e) = \int_{L_{oi}} F^{1-1}(L_e - L_{oi}) f_o(L_{oi}) dL_{oi} \quad (D.3)$$

1867.22	2170.02	2397.11	2422.34	2422.34	2397.11
2397.11	2397.11	2346.65	2371.88	2498.04	2523.28
2523.28	2422.34	2296.18	2094.32	1841.99	1589.66
1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2686.07	2713.20
2713.20	2604.67	2469.01	2251.96	1980.64	1709.32
1781.49	1675.13	1595.36	1568.67	1568.77	1595.36
1967.61	2266.68	2525.99	2552.58	2552.58	2525.99
2525.99	2525.99	2472.81	2499.40	2632.35	2658.94
2658.94	2552.58	2419.63	2206.92	1941.02	1675.13
1745.13	1640.94	1562.80	1536.76	1536.76	1562.80
1927.46	2240.02	2474.44	2500.48	2500.48	2474.44
2474.44	2474.44	2422.34	2448.39	2578.63	2604.67
2604.67	2500.48	2370.25	2161.88	1901.41	1640.94
1530.77	1439.38	1370.84	1347.99	1347.99	1370.84
1690.70	1964.87	2170.49	2193.34	2193.34	2170.49
2170.49	2170.49	2124.80	2147.65	2261.88	2294.73
2284.73	2193.34	2079.11	1896.33	1667.85	1439.38
1666.55	1538.45	1452.88	1410.15	1367.42	1338.73
1410.15	1495.49	1709.28	1880.20	1922.94	1944.29
1922.94	1660.20	1858.83	1858.83	1944.30	2136.59
2115.22	2072.49	2008.40	1965.67	1858.83	1730.64

WEEKEND LOAD SAMPLES FOR WEEK 12

1667.25	1539.00	1453.50	1410.75	1368.00	1389.37
1410.75	1496.25	1710.00	1881.00	1923.75	1945.13
1923.75	1861.00	1859.63	1859.63	1945.12	2137.50
2116.13	2073.38	2009.25	1966.50	1859.63	1731.38
1690.59	1589.66	1513.97	1488.73	1488.73	1513.97
1867.22	2170.02	2397.11	2422.34	2422.34	2397.11
2397.11	2397.11	2346.65	2371.88	2498.04	2523.28
2523.28	2422.34	2296.18	2094.32	1841.99	1589.66

WEEKDAY LOAD SAMPLES FOR WEEK 13

1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2686.07	2713.20
2713.20	2604.67	2469.01	2251.96	1980.64	1709.32
1781.49	1675.13	1595.36	1568.67	1568.77	1595.36
1967.61	2266.68	2525.99	2552.58	2552.58	2525.99
2525.99	2525.99	2472.81	2499.40	2632.35	2658.94
2658.94	2552.58	2419.63	2206.92	1941.02	1675.13
1745.13	1640.94	1562.80	1536.76	1536.76	1562.80
1927.46	2240.02	2474.44	2500.48	2500.48	2474.44
2474.44	2474.44	2422.34	2448.39	2578.63	2604.67
2604.67	2500.48	2370.25	2161.88	1901.41	1640.94

Figure D.1: Load Probability Distribution Curve

Figure D.2: Outage Capacity Probability Density Function

where $F^i(L_e)$ is the effective load probability distribution function with the outage capacity of the first i units convolved in. When $i=0$, $F^i(L_e) = F(L)$ and when $i=G$, $F^i(L_e) = F(L_e)$.

$F_{1m}(L_e)$ is the load probability distribution function after commitment of C MW and before commitment of the first capacity block of unit m , which has availability p_m and capacity C_{1m} . The expected energy output of the first capacity block of unit m $E(E_{1m})$ during time T is given by:

$$E(E_{1m}) = T p_m \int_C^{C+C_{1m}} F_{1m}(L_e) dL_e \quad (D.4)$$

If λ_{1m} is the unit fuel cost of capacity block 1 of unit m the expected fuel cost $E(F_{C_{1m}})$ for capacity block 1 of unit m is given by:

$$E(F_{C_{1m}}) = T p_m \lambda_{1m} \int_C^{C+C_{1m}} F_{1m}(L_e) dL_e \quad (D.5)$$

When it becomes necessary to commit the second capacity block of unit m , the first capacity block has to be deconvolved out of the effective load probability distribution curve to give $F^{-1m}(L_e)$ which is then used instead of $F_{1m}(L_e)$ in equation (D.5). When the third capacity block of unit m has to be committed, the same process is applied; the already committed capacity block of unit m are deconvolved out of the effective load probability distribution curve.

Addition of the expected fuel costs of all capacity blocks for the generators in the system gives the total fuel cost. Proofs for equations (D.1), (D.3) and (D.4) can be found in reference [18].

Figure D.3: Fictitious Generating Units and System Load Model

APPENDIX EGenerating and Load Models Used in Chapter 4Generating ModelSystem B

Unit	Number	Capacity MW	FOR	Inc. Cost (\$/MWhr)
Nuclear	3	500	0.18	8.2
Coal	2	400	0.13	17.56
Coal	1	150	0.08	20.85
Coal	2	350	0.13	29.85
Hydro*	1	500	0.01	0
Oil	1	350	0.14	33.17
Oil	2	200	0.10	42.80
Oil	4	50	0.21	65.48

* Hydro unit is to discharge 240 GWhrs in 2184 hours.

Load Model

The load model used for pumped hydro storage simulation is given below. All values are in MW.

WEEKEND LOAD SAMPLES FOR WEEK 1

1523.42	1406.24	1328.11	1289.05	1249.99	1269.52
1289.05	1307.17	1562.48	1718.73	1757.79	1777.33
1757.79	1718.73	1699.20	1699.20	1777.33	1953.10
1933.57	1694.51	1835.92	1796.86	1699.20	1532.01
1483.85	1309.71	1293.61	1255.57	1217.52	1236.54
1255.57	1331.66	1521.90	1674.09	1712.14	1731.16
1712.14	1674.09	1655.07	1655.07	1731.16	1932.36
1883.35	1645.30	1788.23	1750.18	1655.07	1540.92

WEEKDAY LOAD SAMPLES FOR WEEK 2

1672.84	1572.97	1498.06	1473.09	1473.09	1498.06
1847.61	2147.22	2371.93	2396.90	2396.90	2371.93
2371.93	2371.93	2322.00	2346.96	2471.80	2496.77
2496.77	2396.90	2272.06	2072.32	1822.64	1572.97
1798.75	1691.36	1610.82	1583.97	1583.97	1610.82
1986.68	2308.84	2550.46	2577.31	2577.31	2550.46
2550.46	2550.46	2496.77	2523.62	2657.85	2684.70
2684.70	2577.31	2443.08	2228.30	1959.83	1691.36
1762.77	1657.53	1578.50	1552.29	1552.29	1578.50
1946.94	2262.66	2499.46	2525.77	2525.77	2499.46
2499.46	2499.46	2446.84	2473.15	2604.70	2631.01
2631.01	2525.77	2394.22	2183.73	1920.63	1657.53
1726.80	1623.71	1546.39	1520.61	1520.61	1546.39
1907.21	2216.49	2448.45	2474.22	2474.22	2448.45
2448.45	2448.45	2396.90	2422.67	2551.54	2577.31
2577.31	2474.22	2345.35	2139.17	1881.44	1623.71
1690.82	1589.88	1514.17	1488.93	1488.93	1514.17
1867.48	2170.31	2397.44	2422.67	2422.67	2397.44
2397.44	2397.44	2346.96	2372.20	2498.38	2523.62
2523.62	2422.67	2296.49	2094.60	1842.24	1589.88
1612.43	1488.40	1405.71	1364.36	1323.02	1343.69
1364.36	1447.05	1653.77	1819.15	1860.50	1331.17
1860.50	1819.15	1798.48	1798.48	1881.17	2067.22
2046.55	2055.20	1943.19	1901.84	1796.48	1674.45
1570.55	1449.74	1369.20	1328.93	1288.66	1308.79
1328.93	1409.47	1610.82	1771.90	1812.17	1832.31
1812.17	1771.90	1751.77	1751.77	1832.31	2013.52
1993.39	1953.12	1892.71	1852.44	1751.77	1630.95

WEEKEND LOAD SAMPLES FOR WEEK 2

1672.84	1572.97	1498.06	1473.09	1473.09	1498.06
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1847.61	2147.22	2371.93	2396.90	2396.90	2371.93
2371.93	2371.93	2322.00	2346.96	2471.80	2496.77
2496.77	2396.90	2272.06	2072.32	1822.64	1572.97
1798.75	1691.36	1610.82	1583.97	1583.97	1610.82
1986.66	2358.84	2550.46	2577.31	2577.31	2550.46
2550.46	2550.46	2496.77	2523.62	2657.85	2634.70
2684.70	2577.31	2443.08	2228.30	1959.83	1691.36

WEEKDAY LOAD SAMPLES FOR WEEK 3

1762.77	1657.53	1578.50	1552.29	1552.29	1578.60
1946.94	2262.66	2499.46	2525.77	2525.77	2499.46
2499.46	2499.46	2446.84	2473.15	2604.70	2631.01
2631.01	2525.77	2394.22	2183.73	1920.63	1657.53
1726.80	1623.71	1546.39	1520.61	1520.61	1546.39
1907.21	2216.49	2448.45	2474.22	2474.22	2448.45
2448.45	2448.45	2396.90	2422.67	2551.54	2577.31
2577.31	2474.22	2345.35	2139.17	1881.44	1623.71
1690.82	1589.88	1514.17	1488.93	1488.93	1514.17
1867.48	2170.31	2397.44	2422.67	2422.67	2397.44
2397.44	2397.44	2346.96	2372.20	2498.38	2523.62
2523.62	2422.67	2296.49	2094.60	1842.24	1589.88
1612.43	1488.40	1405.71	1364.36	1323.02	1343.69
1364.36	1447.05	1653.77	1819.15	1860.50	1881.17
1860.50	1819.15	1798.48	1798.48	1881.17	2067.22
2046.55	2055.20	1943.19	1901.84	1798.48	1674.45
1570.55	1449.74	1369.20	1328.93	1288.66	1308.79
1328.93	1409.47	1610.82	1771.90	1812.17	1832.31
1812.17	1771.90	1751.77	1751.77	1832.31	2013.52
1993.39	1953.12	1892.71	1852.44	1751.77	1630.95
1722.56	1619.72	1542.59	1516.88	1516.88	1542.59
1902.53	2211.05	2442.44	2468.15	2468.15	2442.44
2442.44	2442.44	2391.02	2416.73	2545.27	2570.98
2570.98	2468.15	2339.60	2133.92	1876.82	1619.72
1852.21	1741.63	1658.70	1631.05	1631.05	1658.70
2045.78	2377.47	2626.27	2653.92	2653.92	2626.27
2626.27	2626.27	2570.98	2598.63	2736.85	2764.50
2764.50	2653.92	2515.70	2294.53	2018.08	1741.63

WEEKEND LOAD SAMPLES FOR WEEK 3

1762.77	1657.53	1578.50	1552.29	1552.29	1578.60
1946.94	2262.66	2499.46	2525.77	2525.77	2499.46
2499.46	2499.46	2446.84	2473.15	2604.70	2631.01
2631.01	2525.77	2394.22	2183.73	1920.63	1657.53
1726.80	1623.71	1546.39	1520.61	1520.61	1546.39
1907.21	2216.49	2448.45	2474.22	2474.22	2448.45
2448.45	2448.45	2396.90	2422.67	2551.54	2577.31
2577.31	2474.22	2345.35	2139.17	1881.44	1623.71

WEEKDAY LOAD SAMPLES FOR WEEK 4

1690.82	1589.84	1514.17	1488.93	1488.93	1514.17
1867.46	2170.31	2397.44	2422.67	2422.67	2397.44
2397.44	2397.44	2346.96	2372.20	2498.38	2523.62
2523.62	2422.67	2296.49	2094.60	1342.24	1589.34
1612.43	1488.40	1405.71	1364.36	1323.02	1343.69
1364.36	1447.05	1653.77	1819.15	1860.50	1381.17
1860.50	1819.15	1798.48	1798.48	1881.17	2067.22
2046.55	2005.20	1943.19	1901.84	1798.48	1674.45
1570.55	1449.74	1369.20	1328.93	1288.66	1308.79
1328.93	1409.47	1610.82	1771.90	1812.17	1832.31
1812.17	1771.90	1751.77	1751.77	1832.31	2013.52
1993.39	1953.12	1892.71	1852.44	1751.77	1630.95
1722.56	1819.72	1542.59	1516.88	1516.88	1542.59
1902.53	2211.05	2442.44	2468.15	2468.15	2442.44
2442.44	2442.44	2391.02	2416.73	2545.27	2570.96
2570.96	2468.15	2339.60	2133.92	1876.82	1819.72
1852.21	1741.63	1658.70	1631.05	1631.05	1658.70
2045.76	2377.47	2626.27	2653.92	2653.92	2626.27
2626.27	2626.27	2570.98	2598.63	2736.85	2764.50
2764.50	2653.92	2515.70	2294.53	2018.08	1741.63
1815.17	1700.80	1625.53	1598.43	1598.43	1625.53
2004.82	2329.92	2573.75	2600.84	2600.84	2573.75
2573.75	2573.75	2519.57	2546.66	2682.12	2709.21
2709.21	2600.84	2465.38	2248.64	1977.72	1706.82
1778.13	1671.97	1592.35	1565.81	1565.81	1592.35
1963.90	2282.37	2521.22	2547.76	2547.76	2521.22
2521.22	2521.22	2468.15	2494.68	2627.38	2653.52
2653.52	2547.76	2415.07	2202.75	1937.36	1671.97

WEEKEND LOAD SAMPLES FOR WEEK 4

1690.82	1589.88	1514.17	1488.93	1488.93	1514.17
1867.46	2170.31	2397.44	2422.67	2422.67	2397.44
2397.44	2397.44	2346.96	2372.20	2498.38	2523.62
2523.62	2422.67	2296.49	2094.60	1842.24	1589.88
1612.43	1488.40	1405.71	1364.36	1323.02	1343.69
1364.36	1447.05	1653.77	1819.15	1860.50	1381.17
1860.50	1819.15	1798.48	1798.48	1881.17	2067.22
2046.55	2005.20	1943.19	1901.84	1798.48	1674.45

WEEKDAY LOAD SAMPLES FOR WEEK 5

1570.55	1449.74	1369.20	1328.93	1288.66	1308.79
1328.93	1409.47	1610.82	1771.90	1812.17	1832.31
1812.17	1771.90	1751.77	1751.77	1832.31	2013.52
1993.39	1953.12	1892.71	1852.44	1751.77	1630.95
1722.56	1819.72	1542.59	1516.88	1516.88	1542.59

1741.06	1637.14	1559.18	1533.19	1533.19	1559.14
1922.99	2234.82	2468.70	2494.68	2494.68	2468.70
2468.70	2468.70	2416.73	2442.71	2572.64	2598.63
2598.63	2494.68	2364.75	2156.86	1897.00	1637.14
1660.36	1532.64	1447.49	1404.92	1362.35	1383.63
1404.92	1490.07	1702.93	1873.22	1915.80	1937.03
1915.80	1673.22	1851.94	1851.94	1937.08	2123.66
2107.36	2064.80	2000.94	1958.37	1851.94	1724.22
1617.23	1492.83	1409.89	1368.43	1326.96	1347.69
1368.43	1451.36	1658.70	1824.57	1866.04	1886.77
1866.04	1824.57	1803.84	1803.84	1886.77	2073.33
2052.64	2011.17	1948.97	1907.50	1803.84	1679.43
1775.83	1669.81	1590.30	1563.79	1563.79	1590.30
1961.37	2279.43	2517.97	2544.48	2544.48	2517.97
2517.97	2517.97	2464.96	2491.47	2623.99	2650.50
2650.50	2544.48	2411.95	2199.91	1934.86	1669.81

WEEKEND LOAD SAMPLES FOR WEEK 6

1852.21	1741.63	1658.70	1631.05	1631.05	1658.70
2045.76	2377.47	2626.27	2653.92	2653.92	2626.27
2626.27	2626.27	2570.98	2598.63	2736.85	2764.50
2764.50	2653.92	2515.70	2294.53	2018.08	1741.63
1815.17	1736.80	1625.53	1598.43	1598.43	1625.53
2004.82	2329.92	2573.75	2600.84	2600.84	2573.75
2573.75	2573.75	2519.57	2546.66	2682.12	2709.21
2709.21	2600.84	2465.38	2248.64	1977.72	1706.80

WEEKDAY LOAD SAMPLES FOR WEEK 7

1778.13	1671.97	1592.35	1565.81	1565.81	1592.35
1963.90	2282.37	2521.22	2547.76	2547.76	2521.22
2521.22	2521.22	2468.15	2494.68	2627.38	2653.52
2653.52	2547.76	2415.07	2202.75	1937.36	1671.97
1741.06	1637.14	1559.18	1533.19	1533.19	1559.18
1922.99	2234.82	2468.70	2494.68	2494.68	2468.70
2468.70	2468.70	2416.73	2442.71	2572.64	2598.63
2598.63	2494.68	2364.75	2156.86	1897.00	1637.14
1660.36	1532.64	1447.49	1404.92	1362.35	1383.63
1404.92	1490.07	1702.93	1873.22	1915.80	1937.03
1915.80	1673.22	1851.94	1851.94	1937.08	2123.66
2107.36	2064.80	2000.94	1958.37	1851.94	1724.22
1617.23	1492.83	1409.89	1368.43	1326.96	1347.69
1368.43	1451.36	1658.70	1824.57	1866.04	1886.77
1866.04	1824.57	1803.84	1803.84	1886.77	2073.33
2052.64	2011.17	1948.97	1907.50	1803.84	1679.43
1775.83	1669.81	1590.30	1563.79	1563.79	1590.30
1961.37	2279.43	2517.97	2544.48	2544.48	2517.97
2517.97	2517.97	2464.96	2491.47	2623.99	2650.50

2650.50	2544.46	2411.95	2199.91	1934.86	1669.31
1909.50	1795.50	1710.00	1681.50	1681.50	1710.00
2109.00	2451.00	2707.50	2736.00	2736.00	2707.50
2707.50	2707.50	2650.50	2679.00	2821.50	2850.00
2850.00	2736.00	2593.50	2365.50	2080.50	1795.50
1871.31	1759.59	1675.80	1647.87	1647.87	1675.80
2066.82	2401.98	2653.35	2681.28	2681.28	2653.35
2653.35	2653.35	2597.49	2625.42	2765.07	2793.00
2793.00	2681.28	2541.63	2318.19	2038.89	1759.59

WEEKEND LOAD SAMPLES FOR WEEK 7

1778.13	1671.97	1592.35	1565.81	1565.81	1592.35
1963.90	2262.37	2521.22	2547.76	2547.76	2521.22
2521.22	2521.22	2468.15	2494.68	2627.38	2653.52
2653.52	2547.76	2415.07	2202.75	1937.36	1671.97
1741.08	1637.14	1559.18	1533.19	1533.19	1559.18
1922.99	2234.82	2468.70	2494.68	2494.68	2468.70
2468.70	2468.70	2416.73	2442.71	2572.64	2572.64
2598.63	2494.68	2364.75	2156.86	1897.00	1637.14

WEEKDAY LOAD SAMPLES FOR WEEK 8

1660.36	1552.64	1447.49	1404.92	1362.35	1333.63
1404.92	1490.07	1702.93	1873.22	1915.80	1937.08
1915.80	1873.22	1851.94	1851.94	1937.08	2128.66
2107.38	2064.80	2000.94	1958.37	1851.94	1724.22
1617.23	1492.83	1409.89	1368.43	1326.96	1347.69
1368.43	1451.36	1658.70	1824.57	1866.04	1886.77
1866.04	1824.57	1803.84	1803.84	1886.77	2073.33
2052.64	2011.17	1948.97	1907.50	1803.84	1679.43
1775.83	1669.81	1590.30	1563.79	1563.79	1590.30
1961.37	2279.43	2517.97	2544.48	2544.48	2517.97
2517.97	2517.97	2464.96	2491.47	2623.99	2650.50
2650.50	2544.48	2411.95	2199.91	1934.86	1669.81
1909.50	1795.50	1710.00	1681.50	1681.50	1710.00
2109.00	2451.00	2707.50	2736.00	2736.00	2707.50
2707.50	2707.50	2650.50	2679.00	2821.50	2850.00
2850.00	2736.00	2593.50	2365.50	2080.50	1795.50
1871.31	1759.59	1675.80	1647.87	1647.87	1675.80
2066.82	2401.98	2653.35	2681.28	2681.28	2653.35
2653.35	2653.35	2597.49	2625.42	2765.07	2793.00
2793.00	2681.28	2541.63	2318.19	2038.89	1759.59
1833.12	1723.68	1641.60	1614.24	1614.24	1641.60
2024.64	2352.96	2599.20	2626.56	2626.56	2599.20
2599.20	2599.20	2544.48	2571.84	2708.64	2736.00
2736.00	2626.56	2489.76	2270.86	1997.28	1723.68
1724.93	1687.77	1607.40	1580.61	1580.61	1607.40
1982.46	2303.94	2545.05	2571.84	2571.84	2545.05

2545.05	2545.05	2491.47	2518.26	2652.21	2679.00
2679.00	2571.84	2437.89	2223.57	1955.67	1687.77

WEEKEND LOAD SAMPLES FOR WEEK 8

1660.36	1532.64	1447.49	1404.92	1362.35	1383.63
1404.92	1490.07	1702.93	1873.22	1915.80	1737.03
1915.80	1873.22	1851.94	1851.94	1937.08	2128.66
2107.36	2064.80	2000.94	1958.37	1851.94	1724.22
1617.23	1492.83	1409.89	1368.43	1326.96	1347.69
1368.43	1451.36	1658.70	1824.57	1866.04	1386.77
1866.04	1824.57	1803.84	1803.84	1886.77	2073.38
2052.64	2011.17	1948.97	1907.50	1803.84	1679.43

WEEKDAY LOAD SAMPLES FOR WEEK 9

1775.83	1669.81	1590.30	1563.79	1563.79	1590.30
1961.37	2279.43	2517.97	2544.48	2544.48	2517.97
2517.97	2517.97	2464.96	2491.47	2623.99	2650.50
2650.50	2544.48	2411.95	2199.91	1934.86	1669.81
1909.50	1795.50	1710.00	1681.50	1681.50	1710.00
2109.00	2451.00	2707.50	2736.00	2736.00	2707.50
2707.50	2736.00	2650.50	2679.00	2821.50	2850.00
2850.00	2736.00	2593.50	2365.50	2080.50	1795.50
1871.31	1759.59	1675.80	1647.87	1647.87	1675.80
2066.82	2401.98	2653.35	2681.28	2681.28	2653.35
2653.35	2653.35	2597.49	2625.42	2765.07	2793.00
2793.00	2681.28	2541.63	2318.19	2038.89	1759.59
1833.12	1723.68	1641.60	1614.24	1614.24	1641.60
2024.64	2352.96	2599.20	2626.56	2626.56	2599.20
2599.20	2599.20	2544.48	2571.84	2708.64	2736.00
2736.00	2626.56	2489.76	2270.86	1997.28	1723.68
1794.93	1667.77	1607.40	1580.61	1580.61	1607.40
1982.46	2303.94	2545.05	2571.84	2571.84	2545.05
2545.05	2545.05	2491.47	2518.26	2652.21	2679.00
2679.00	2571.84	2437.89	2223.57	1955.67	1687.77
1711.71	1580.04	1492.26	1448.37	1404.48	1426.42
1448.37	1536.15	1755.60	1931.16	1975.05	1996.99
1975.05	1931.16	1909.21	1909.21	1996.99	2194.50
2172.55	2128.66	2062.83	2018.94	1909.21	1777.54
1667.25	1539.00	1453.50	1410.75	1362.00	1377.37
1410.75	1496.25	1710.00	1881.00	1923.75	1945.13
1923.75	1881.00	1859.63	1859.63	1945.12	2137.50
2116.13	2073.38	2009.25	1966.50	1859.63	1731.38

WEEKEND LOAD SAMPLES FOR WEEK 9

1775.83	1669.81	1590.30	1563.79	1563.79	1590.30
1961.37	2279.43	2517.97	2544.48	2544.48	2517.97

2517.97	2517.97	2464.96	2491.47	2623.99	2650.50
2650.50	2544.48	2411.95	2199.51	1934.86	1669.51
1909.50	1795.50	1710.00	1631.50	1681.50	1710.00
2109.00	2451.00	2707.50	2736.00	2736.00	2707.50
2707.50	2707.50	2650.50	2679.00	2821.50	2850.00
2850.00	2736.00	2593.50	2365.50	2080.50	1795.50

WEEKDAY LOAD SAMPLES FOR WEEK 10

1871.31	1759.59	1675.80	1647.87	1647.87	1675.80
2066.82	2401.98	2653.35	2681.28	2681.28	2653.35
2653.35	2653.35	2597.49	2625.42	2765.07	2793.00
2793.00	2681.28	2541.63	2318.19	2038.89	1759.59
1833.12	1723.68	1641.60	1614.24	1614.24	1641.60
2024.64	2352.96	2599.20	2626.56	2626.56	2599.20
2599.20	2599.20	2544.48	2571.84	2708.64	2736.00
2736.00	2626.56	2489.76	2270.86	1997.28	1723.68
1794.93	1587.77	1607.40	1580.61	1580.61	1607.40
1982.46	2303.94	2545.05	2571.84	2571.84	2545.05
2545.05	2545.05	2491.47	2518.26	2652.21	2679.00
2679.00	2571.84	2437.89	2223.57	1955.67	1687.77
1711.71	1580.04	1492.26	1448.37	1404.48	1426.42
1448.37	1536.15	1755.60	1931.16	1975.05	1926.79
1975.05	1931.16	1909.21	1909.21	1996.99	2194.50
2172.55	2126.66	2062.83	2018.94	1909.21	1777.54
1667.25	1539.00	1453.50	1410.75	1368.00	1339.37
1410.75	1496.25	1710.00	1881.00	1923.75	1945.13
1923.75	1681.00	1859.63	1859.63	1945.12	2137.50
2116.13	2073.38	2009.25	1966.50	1859.63	1731.33
1690.59	1589.66	1513.97	1488.73	1488.73	1513.97
1867.22	2170.02	2397.11	2422.34	2422.34	2397.11
2397.11	2397.11	2346.65	2371.88	2498.04	2523.28
2523.28	2422.34	2296.18	2094.32	1841.99	1589.66
1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2686.07	2713.20
2713.20	2604.67	2469.01	2251.96	1980.64	1709.32

WEEKEND LOAD SAMPLES FOR WEEK 10

1871.31	1759.59	1675.80	1647.87	1647.87	1675.80
2066.82	2401.98	2653.35	2681.28	2681.28	2653.35
2653.35	2653.35	2597.49	2625.42	2765.07	2793.00
2793.00	2681.28	2541.63	2318.19	2038.89	1759.59
1833.12	1723.68	1641.60	1614.24	1614.24	1641.60
2024.64	2352.96	2599.20	2626.56	2626.56	2599.20
2599.20	2599.20	2544.48	2571.84	2708.64	2736.00
2736.00	2626.56	2489.76	2270.86	1997.28	1723.68

WEEKDAY LOAD SAMPLES FOR WEEK 11

1794.93	1637.77	1607.40	1580.61	1580.61	1607.40
1982.46	2303.94	2545.05	2571.84	2571.84	2545.05
2545.05	2545.05	2491.47	2518.26	2652.21	2679.00
2679.00	2571.84	2437.89	2223.57	1955.67	1637.77
1711.71	1580.04	1492.26	1448.37	1404.48	1426.42
1448.37	1536.15	1755.60	1931.16	1975.05	1996.99
1975.05	1931.16	1909.21	1909.21	1996.99	2194.50
2172.55	2128.66	2062.83	2018.94	1909.21	1777.54
1667.25	1539.00	1453.50	1410.75	1368.00	1389.37
1410.75	1496.25	1710.00	1881.00	1923.75	1945.13
1923.75	1881.00	1859.63	1859.63	1945.12	2137.50
2116.13	2073.38	2009.25	1966.50	1859.63	1731.38
1690.59	1589.66	1513.97	1488.73	1488.73	1513.97
1867.22	2170.02	2397.11	2422.34	2422.34	2397.11
2397.11	2397.11	2346.65	2371.88	2498.04	2523.28
2523.28	2422.34	2296.18	2094.32	1841.99	1599.66
1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2686.07	2713.20
2713.20	2604.67	2469.01	2251.96	1930.64	1709.32
1781.49	1675.13	1595.36	1568.67	1568.77	1595.36
1967.61	2286.68	2525.99	2552.58	2552.58	2525.99
2525.99	2525.99	2472.81	2499.40	2632.35	2658.94
2658.94	2552.58	2419.63	2206.92	1941.02	1675.13
1745.13	1640.94	1562.80	1536.76	1536.76	1562.80
1927.46	2240.02	2474.44	2500.43	2500.48	2474.44
2474.44	2474.44	2422.34	2448.39	2578.63	2604.67
2604.67	2500.48	2370.25	2161.88	1901.41	1640.94

WEEKEND LOAD SAMPLES FOR WEEK 11

1794.93	1637.77	1607.40	1580.61	1580.61	1607.40
1982.46	2303.94	2545.05	2571.84	2571.84	2545.05
2545.05	2545.05	2491.47	2518.26	2652.21	2679.00
2679.00	2571.84	2437.89	2223.57	1955.67	1637.77
1711.71	1580.04	1492.26	1448.37	1404.48	1426.42
1448.37	1536.15	1755.60	1931.16	1975.05	1996.99
1975.05	1931.16	1909.21	1909.21	1996.99	2194.50
2172.55	2128.66	2062.83	2018.94	1909.21	1777.54

WEEKDAY LOAD SAMPLES FOR WEEK 12

1667.25	1539.00	1453.50	1410.75	1368.00	1389.37
1410.75	1496.25	1710.00	1881.00	1923.75	1945.13
1923.75	1881.00	1859.63	1859.63	1945.12	2137.50
2116.13	2073.38	2009.25	1966.50	1859.63	1731.38
1690.59	1589.66	1513.97	1488.73	1488.73	1513.97

1867.22	2170.02	2397.11	2422.34	2422.34	2397.11
2397.11	2397.11	2346.65	2371.88	2498.04	2523.28
2523.28	2422.34	2296.18	2094.32	1841.99	1589.66
1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2686.07	2713.20
2713.20	2604.67	2469.01	2251.96	1980.64	1709.32
1781.49	1675.13	1595.36	1568.67	1568.77	1595.36
1967.61	2266.68	2525.99	2552.58	2552.58	2525.99
2525.99	2525.99	2472.81	2499.40	2632.35	2658.94
2658.94	2552.58	2419.63	2206.92	1941.02	1675.13
1745.13	1640.94	1562.80	1536.76	1536.76	1562.80
1927.46	2240.02	2474.44	2500.48	2500.48	2474.44
2474.44	2474.44	2422.34	2448.39	2578.63	2604.67
2604.67	2500.48	2370.25	2161.88	1901.41	1640.94
1530.77	1439.38	1370.84	1347.99	1347.99	1370.84
1690.70	1964.87	2170.49	2193.34	2193.34	2170.49
2170.49	2170.49	2124.80	2147.65	2261.88	2294.73
2284.73	2193.34	2079.11	1896.33	1667.85	1439.38
1666.55	1538.45	1452.88	1410.15	1367.42	1338.73
1410.15	1495.49	1709.28	1880.20	1922.94	1944.29
1922.94	1860.20	1858.83	1858.83	1944.30	2136.59
2115.22	2072.49	2008.40	1965.67	1858.83	1730.64

WEEKEND LOAD SAMPLES FOR WEEK 12

1667.25	1539.00	1453.50	1410.75	1368.00	1389.37
1410.75	1496.25	1710.00	1881.00	1923.75	1945.13
1923.75	1861.00	1859.63	1859.63	1945.12	2137.50
2116.13	2073.38	2009.25	1966.50	1859.63	1731.38
1690.59	1589.66	1513.97	1488.73	1488.73	1513.97
1867.22	2170.02	2397.11	2422.34	2422.34	2397.11
2397.11	2397.11	2346.65	2371.88	2498.04	2523.28
2523.28	2422.34	2296.18	2094.32	1841.99	1589.66

WEEKDAY LOAD SAMPLES FOR WEEK 13

1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2686.07	2713.20
2713.20	2604.67	2469.01	2251.96	1980.64	1709.32
1781.49	1675.13	1595.36	1568.67	1568.77	1595.36
1967.61	2266.68	2525.99	2552.58	2552.58	2525.99
2525.99	2525.99	2472.81	2499.40	2632.35	2658.94
2658.94	2552.58	2419.63	2206.92	1941.02	1675.13
1745.13	1640.94	1562.80	1536.76	1536.76	1562.80
1927.46	2240.02	2474.44	2500.48	2500.48	2474.44
2474.44	2474.44	2422.34	2448.39	2578.63	2604.67
2604.67	2500.48	2370.25	2161.88	1901.41	1640.94

1530.77	1439.38	1370.84	1347.99	1347.99	1370.84
1690.70	1964.87	2170.49	2193.34	2193.34	2170.49
2170.49	2170.49	2124.80	2147.65	2261.88	2284.73
2284.73	2193.34	2079.11	1896.33	1667.85	1439.38
1666.55	1538.45	1452.88	1410.15	1367.42	1338.73
1410.15	1495.49	1709.28	1880.20	1922.94	1944.29
1922.94	1880.20	1858.83	1858.83	1944.30	2136.59
2115.22	2072.49	2008.40	1965.67	1858.83	1730.64
1623.66	1498.40	1415.14	1373.52	1331.90	1352.71
1373.53	1456.77	1664.88	1831.40	1872.99	1893.80
1872.99	1831.36	1810.56	1810.56	1893.80	2081.10
2060.29	2018.70	1956.24	1914.61	1810.56	1685.70
1645.99	1547.72	1474.02	1449.45	1449.45	1474.02
1817.96	2112.76	2333.86	2358.43	2358.43	2333.86
2333.86	2333.86	2284.73	2309.30	2432.13	2456.70
2456.70	2358.43	2235.60	2039.06	1793.39	1547.72

WEEKEND LOAD SAMPLES FOR WEEK 13

1817.84	1709.32	1627.92	1600.79	1600.79	1627.92
2007.77	2333.35	2577.54	2604.67	2604.67	2577.54
2577.54	2577.54	2523.28	2550.41	2636.07	2713.20
2713.20	2604.67	2469.01	2251.96	1980.64	1709.32
1781.49	1675.13	1595.36	1568.67	1566.77	1595.36
1967.61	2286.68	2525.99	2552.58	2552.58	2525.99
2525.99	2525.99	2472.81	2499.40	2632.35	2658.94
2658.94	2552.58	2419.63	2206.92	1941.02	1675.13

WEEKDAY LOAD SAMPLES FOR WEEK 14

1745.13	1640.94	1562.80	1536.76	1536.76	1562.80
1927.46	2240.02	2474.44	2500.48	2500.48	2474.44
2474.44	2474.44	2422.34	2448.39	2578.63	2604.67
2604.67	2500.48	2370.25	2161.88	1901.41	1640.94
1530.77	1439.38	1370.84	1347.99	1347.99	1370.84
1690.70	1964.87	2170.49	2193.34	2193.34	2170.49
2170.49	2170.49	2124.80	2147.65	2261.88	2284.73
2284.73	2193.34	2079.11	1896.33	1667.85	1439.38
1666.55	1538.45	1452.88	1410.15	1367.42	1338.73
1410.15	1495.49	1709.28	1880.20	1922.94	1944.29
1922.94	1880.20	1858.83	1858.83	1944.30	2136.59
2115.22	2072.49	2008.40	1965.67	1858.83	1730.64
1623.66	1498.40	1415.14	1373.52	1331.90	1352.71
1373.53	1456.77	1664.88	1831.40	1872.99	1893.80
1872.99	1831.36	1810.56	1810.56	1893.80	2081.10
2060.29	2018.70	1956.24	1914.61	1810.56	1685.70
1645.99	1547.72	1474.02	1449.45	1449.45	1474.02
1817.96	2112.76	2333.86	2358.43	2358.43	2333.86
2333.86	2333.86	2284.73	2309.30	2432.13	2456.70

2456.70	2358.43	2235.60	2039.06	1793.39	1547.72
1769.88	1884.22	1584.96	1558.55	1558.55	1534.96
1954.80	2271.78	2509.52	2535.95	2535.95	2509.52
2509.52	2509.52	2456.70	2483.12	2615.20	2641.60
2641.60	2555.95	2403.87	2192.54	1928.38	1664.22
1613.07	1516.77	1444.54	1420.46	1420.46	1444.54
1781.60	2070.51	2287.19	2311.26	2311.26	2287.19
2287.19	2287.19	2239.04	2263.11	2383.49	2407.57
2407.58	2311.26	2190.89	1998.28	1757.52	1516.77

APPENDIX FChronological Evolution of Reservoir Model in Chapter 4

Time	Probability Reservoir Empty	Probability Reservoir Full	Probability Machine Operational
Week 1			
Friday Evening	1.00000000	.00000000	.90000000
Week 2			
Monday Morning	.10000000	.90000000	.81000000
Friday Evening	.91000000	.09000000	.81900000
Week 3			
Monday Morning	.09100000	.90900000	.81810000
Friday Evening	.90910000	.09090000	.81819000
Week 4			
Monday Morning	.09091000	.90909000	.81818100
Friday Evening	.90909100	.09090900	.81818190
Week 5			
Monday Morning	.09090910	.90909090	.81818181
Friday Evening	.90909091	.09090909	.81818182
Week 6			
Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week 7

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week 8

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week 9

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week10

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week11

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week12

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week13

Monday Morning	.09090909	.90909091	.81818182
Friday Evening	.90909091	.09090909	.81818182

Week14

Monday Morning	.09090909	.90909091	.81818182
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APPENDIX G

Relation Between Cumulants and Moments

K_i are the cumulants, μ_i the moments about the mean

$$K_1 = 0$$

$$K_2 = \mu_2$$

$$K_3 = \mu_3$$

$$K_4 = \mu_4 - 3\mu_2^2$$

$$K_5 = \mu_5 - 3\mu_2\mu_3$$

$$K_6 = \mu_6 - 15\mu_4\mu_2 - 10\mu_3^2 + 30\mu_2^3$$

APPENDIX H

Hermite Coefficients for the Bivariate Gram Charlier Expansion

ξ_1 and ξ_2 are functions of normalized variables given by

$$\xi_1 = (x_1 - \rho x_2)/(1-\rho^2)$$

$$\xi_2 = (x_2 - \rho x_1)/(1-\rho^2)$$

where ρ is the correlation coefficient.

The Hermite coefficients are:

$$H_{00} = 1$$

$$H_{01} = \xi_2$$

$$H_{02} = \xi_2^2 - a_{22}$$

$$H_{03} = \xi_2^3 - 3a_{22}\xi_2$$

$$H_{04} = \xi_2^4 - 6a_{22}\xi_2^2 + 3a_{22}^2$$

$$H_{05} = \xi_2^5 - 10a_{22}\xi_2^3 + 15a_{22}^2\xi_2$$

$$H_{06} = \xi_2^6 - 15a_{22}\xi_2^4 + 45a_{22}^2\xi_2^2 - 15a_{22}^3$$

$$H_{14} = \xi_1\xi_2^4 - 6a_{22}\xi_1\xi_2^2 - 4a_{12}\xi_2^3 + 12a_{12}a_{22}\xi_2 + 3a_{22}^2\xi_1$$

$$H_{15} = \xi_1\xi_2^5 - 10a_{22}\xi_1\xi_2^3 + 15a_{22}^2\xi_1\xi_2 - 5a_{12}\xi_2^4 - 15a_{12}a_{22}^2 + 30a_{12}a_{22}\xi_2^2$$

$$H_{23} = \xi_1^2 \xi_2^3 - 6a_{12} \xi_1 \xi_2^2 - 3a_{22} \xi_1^2 \xi_2 - a_{11} \xi_2^3 \\ + 6a_{12} a_{22} \xi_1 + 3a_{11} a_{22} \xi_2 + 6a_{12}^2 \xi_2$$

$$H_{24} = \xi_1^2 \xi_2^4 - 8a_{12} \xi_1 \xi_2^3 - 6a_{22} \xi_1^2 \xi_2^2 + 24a_{12} a_{22} \xi_1 \xi_2 \\ - a_{11} \xi_2^4 + (12a_{12}^2 + 6a_{11} a_{22}) \xi_2^2 + 3a_{22}^2 \xi_1^2 \\ - 12a_{12}^2 \xi_1 a_{22} - 3a_{11} a_{22}$$

$$H_{32} = \xi_1^3 \xi_2^2 - 6a_{12} \xi_1^2 \xi_2 - a_{22} \xi_1^3 + 6a_{12}^2 \xi_1 + 3a_{11} a_{22} \xi_1 \\ + 6a_{11} a_{12} \xi_2 - 3a_{11} \xi_1 \xi_2^2$$

$$H_{33} = \xi_1^3 \xi_2^3 - 9a_{12} \xi_1^2 \xi_2^2 - 3a_{22} \xi_1^3 \xi_2 + 18a_{12}^2 \xi_1 \xi_2 \\ + 9a_{12} a_{22} \xi_1^2 + 9a_{11} a_{22} \xi_1 \xi_2 + 9a_{11} a_{12} \xi_2^2 \\ - 3a_{11} \xi_1 \xi_2^3 - (6a_{12}^3 + 9a_{11} a_{12} a_{22})$$

$$H_{41} = \xi_1^4 \xi_2 - 6a_{11} \xi_1^2 \xi_2 - 4a_{12} \xi_1^3 + 12a_{11} a_{12} \xi_1 + 3a_{11}^2 \xi_2$$

$$H_{42} = \xi_1^4 \xi_2^2 - 8a_{12} \xi_1^3 \xi_2 - 6a_{11} \xi_1^2 \xi_2^2 + 24a_{12} a_{11} \xi_1 \xi_2 \\ - a_{22} \xi_1^4 + (12a_{12}^2 + 6a_{11} a_{22}) \xi_1^2 + 3a_{11}^2 \xi_2^2 \\ - 12a_{11} a_{12}^2 - 3a_{11}^2 a_{22}$$

$$H_{50} = \xi_1^5 - 10a_{11}\xi_1^3 + 15a_{11}^2\xi_1$$

$$H_{51} = \xi_1^5\xi_2 - 10a_{11}\xi_1^3\xi_2 - 5a_{12}\xi_1^4 + 30a_{11}a_{12}\xi_1^2 \\ + 15a_{11}^2\xi_1\xi_2 - 15a_{11}^2a_{12}$$

$$H_{60} = \xi_1^6 - 15a_{11}\xi_1^4 + 45a_{11}^2\xi_1^2 - 15a_{11}^3$$

a_{11} , a_{12} and a_{22} are given by:

$$a_{11} = \frac{1}{1-\rho^2}$$

$$a_{12} = \frac{-2\rho}{1-\rho^2}$$

$$a_{22} = \frac{1}{1-\rho^2}$$

APPENDIX I

Coefficients for the Bivariate Gram Charlier Expansion

$$D_{00} = 1$$

$$D_{01} = D_{02} = D_{10} = D_{12} = D_{20} = 0$$

$$D_{03} = -(1/6) \mu_{03}/\sigma_2^3$$

$$D_{04} = (1/24) (\mu_{04}/\sigma_2^4 - 3)$$

$$D_{05} = \frac{1}{120} \left[\frac{\mu_{05}}{\sigma_2^5} - 10 \frac{\mu_{03}}{\sigma_2^3} \right]$$

$$D_{06} = \frac{1}{720} \left[\frac{\mu_{06}}{\sigma_2^6} - 15 \frac{\mu_{04}}{\sigma_2^4} + 30 \right]$$

$$D_{14} = -\frac{1}{24} \left[\frac{\mu_{14}}{\sigma_1 \sigma_2^4} - 6 \frac{\mu_{12}}{\sigma_1 \sigma_2^2} - 4\rho \frac{\mu_{03}}{\sigma_2^3} \right]$$

$$D_{15} = \frac{1}{120} \left[\frac{\mu_{15}}{\sigma_1 \sigma_2^5} - 10 \frac{\mu_{13}}{\sigma_1 \sigma_2^3} - 5\rho \frac{\mu_{04}}{\sigma_2^4} + 30\rho \right]$$

$$D_{23} = -\frac{1}{12} \left[\frac{\mu_{23}}{\sigma_1^2 \sigma_2^3} - 3 \frac{\mu_{21}}{\sigma_1^2 \sigma_2} - \frac{\mu_{03}}{\sigma_2^3} - 6\rho \frac{\mu_{12}}{\sigma_1 \sigma_2^2} \right]$$

$$D_{24} = \frac{1}{48} \left[\frac{\mu_{24}}{\sigma_1^2 \sigma_2^4} - 6 \frac{\mu_{22}}{\sigma_1^2 \sigma_2^2} - \frac{\mu_{04}}{\sigma_2^4} - 8\rho \frac{\mu_{13}}{\sigma_1 \sigma_2^3} + 24 \rho^2 + 6 \right]$$

$$D_{32} = -\frac{1}{12} \left[\frac{\mu_{32}}{\sigma_1 \sigma_2} - \frac{\mu_{30}}{\sigma_1} - 3 \frac{\mu_{12}}{\sigma_1 \sigma_2} - 6\rho \frac{\mu_{21}}{\sigma_1 \sigma_2} \right]$$

$$D_{33} = \frac{1}{36} \left[\frac{\mu_{33}}{\sigma_1 \sigma_2} - 3 \frac{\mu_{31}}{\sigma_1 \sigma_2} - 3 \frac{\mu_{13}}{\sigma_1 \sigma_2} + 9\rho \frac{\mu_{22}}{\sigma_1 \sigma_2} + 12\rho^3 + 18\rho \right]$$

$$D_{41} = -\frac{1}{24} \left[\frac{\mu_{41}}{\sigma_1 \sigma_2} - 6 \frac{\mu_{21}}{\sigma_1 \sigma_2} - 4\rho \frac{\mu_{30}}{\sigma_1} \right]$$

$$D_{42} = \frac{1}{48} \left[\frac{\mu_{42}}{\sigma_1 \sigma_2} - 6 \frac{\mu_{22}}{\sigma_1 \sigma_2} - \frac{\mu_{40}}{\sigma_1} - 8 \frac{\mu_{31}}{\sigma_1 \sigma_2} + 24\rho^2 + 6 \right]$$

$$D_{50} = -\frac{1}{120} \left[\frac{\mu_{50}}{\sigma_1} - 10 \frac{\mu_{30}}{\sigma_1} \right]$$

$$D_{51} = \frac{1}{120} \left[\frac{\mu_{51}}{\sigma_1 \sigma_2} - 10 \frac{\mu_{31}}{\sigma_1 \sigma_2} - 5\rho \frac{\mu_{40}}{\sigma_1} + 30\rho \right]$$

$$D_{60} = \frac{1}{720} \left[\frac{\mu_{60}}{\sigma_1} - 15 \frac{\mu_{40}}{\sigma_1} + 30 \right]$$

APPENDIX J

Generating and Load Models for Interconnected Systems

Generating ModelsSystem A

Unit	Number	Capacity MW	FOR	Inc. Cost (\$/MWhr)
Nuclear	2	400	0.12	5.592
Coal	4	150	0.04	11.16
Coal	1	350	0.08	11.40
Coal	4	80	0.02	14.882
Hydro*	1	300	0.01	0
Coal	3	200	0.05	19.87
Oil	3	100	0.04	22.08
Oil	5	10	0.02	28.558
Oil	4	20	0.10	37.500

* Hydro unit is to discharge 240 GWhrs in 2160 hours.

System B

Unit	Number	Capacity MW	For	Inc. Cost (\$/MWhr)
Nuclear	2	500	0.18	8.2
Coal	2	400	0.13	17.56
Coal	1	150	0.08	20.85
Coal	2	350	0.13	29.85
Hydro*	1	500	0.01	0
Oil	1	350	0.14	33.17
Oil	2	200	0.10	42.80
Oil	4	50	0.21	65.48

* Hydro unit is to discharge 240 GWhrs in 2160 hours.

Load ModelsSystem A

The following data gives the load model for system A. All values are in MW.

1317.01	1245.32	1174.63	1150.83	1139.03	1103.44
1103.44	1174.63	1441.59	1530.58	1619.56	1655.16
1655.16	1657.36	1619.56	1619.56	1637.36	1672.95
1672.95	1670.75	1779.74	1655.16	1566.17	1423.79
1232.80	1215.45	1444.12	1126.78	1109.45	1074.76
1074.76	1144.12	1404.14	1490.62	1577.50	1612.17
1612.17	1594.83	1577.50	1577.50	1594.83	1629.50
1640.84	1640.84	1733.51	1612.17	1525.49	1386.81
1526.67	1451.27	1333.56	1335.85	1335.85	1333.56
1526.67	1612.94	2075.34	2266.15	2361.60	2335.45
2361.60	2365.45	2385.45	2313.09	2290.03	2290.03
2213.47	2194.61	2194.61	2218.47	2075.34	1717.52
1641.60	1559.00	1467.70	1436.40	1436.40	1437.70
1641.60	1949.45	2231.65	2436.75	2539.35	2565.00
2539.35	2565.00	2565.00	2468.05	2462.40	2462.40
2335.45	2359.80	2359.80	2385.45	2231.55	1346.80
1608.77	1508.22	1457.95	1407.67	1407.67	1457.95
1608.77	1910.41	2136.92	2388.01	2488.56	2513.70
2488.56	2513.70	2513.70	2438.29	2413.15	2413.15
2337.74	2312.60	2312.60	2337.74	2186.92	1809.86
1575.94	1477.44	1428.19	1378.94	1378.94	1428.19
1575.94	1671.42	2142.29	2339.28	2437.78	2462.40
2437.76	2462.40	2462.40	2388.53	2363.90	2363.90
2290.03	2265.41	2265.41	2290.03	2142.29	1772.93
1543.10	1440.66	1398.44	1350.22	1350.22	1378.44
1543.10	1652.44	2097.66	2290.54	2386.99	2411.10
2386.99	2411.10	2411.10	2338.77	2314.66	2314.66
2242.32	2218.21	2218.21	2242.32	2097.66	1735.99
1461.54	1362.53	1303.53	1283.78	1264.03	1224.53
1224.53	1303.53	1599.79	1698.54	1797.29	1336.80
1836.80	1617.05	1797.29	1797.29	1817.05	1356.55
1876.30	1676.30	1975.05	1836.30	1738.04	1580.04
1423.57	1340.62	1269.67	1250.44	1231.20	1192.72
1192.72	1269.67	1356.24	1654.42	1750.61	1739.09
1789.09	1709.85	1750.61	1750.61	1769.85	1808.32
1827.56	1627.56	1923.75	1789.09	1692.90	1539.00
1504.64	1410.60	1363.58	1316.56	1316.56	1363.58
1504.64	1766.75	2045.36	2233.44	2327.43	2350.99
2327.46	2350.99	2350.99	2280.46	2256.95	2256.95
2186.42	2162.91	2162.91	2186.42	2045.36	1692.72
1617.89	1516.77	1466.21	1415.65	1415.65	1466.21

```

NNUNIT=NOUNIT(NO)
DO 574 K=2,NNUNIT
  INSTAL=INSTAL+SIZE(NO)
  DO 574 KZ=1,NNOB
    IFC(N2+K-1,KZ)=IFC(N2,KZ)
574  CONTINUE
  N2=N2+NNUNIT
  NO=NO+1
  GO TO 1111
1227 CONTINUE
C
C  FIND THE UNIT NO OF THE HYDRO UNIT
C
  IF (HENGY.EQ.0.0) GO TO 1233
  IF (IFC(N2,1).NE.0.0E0) GO TO 1233
  PREV=(1.0-FOR(NO))*TIME*SIZE(NO)*1.0D-3
  IF (PREV.LT.HENGY) GO TO 3000
  HINDEX=NO
  HIFC=N2
  IFC(HIFC,1)=1.0E3
1233 N2=N2+1
  NO =NO+1
  GO TO 1111
C
C  END ODF DATA
C
1800 N1=N
  N2=N2-1
  NO=NO-1
  LPEAK=PEAK
  DO 1900 J=1,N1
    XVEC(J)=LXVEC(J)
    YVEC(J)=LYVEC(J)
1900 CONTINUE
C
C  OUTPUT THE BASE LOAD, PEAK LOAD, THE TIME DURATION AND
C  THE ENERGY UNDER THE LDC.
C
  WRITE(6,5000) BASE,PEAK,TIME
  TIME=TIME*1.0D-3
  LDCENG=TIME*ROMBG(PN1,PEAK)
  WRITE(6,5300) LDCENG
C
  MAX=4.0D0*H+INSTAL+PEAK
C
C  START OF LOOP
C
  IF (I.GT.N2) GO TO 99999
  IF (HINDEX.EQ.0) GO TO 2040
  IF (FLAG.EQ.0) GO TO 2040
  IF (I.EQ.N2) GO TO 9999
C
C  TEST TO SEE IF HYDRO UNIT CAN BE FITTED
C
  PRES=(1.0-FOR(HINDEX))*TIME*ROMBG(PN1,PN1+SIZE(HINDEX))
  IF ((PRES.LE.HENGY).AND.(PREV.GE.HENGY)) GO TO 3000
  PREV=PRES
  GO TO 2040
C
C  REACHING 3000 MEANS THAT HYDRO CAN BE FITTED

```


1781.14	1701.96	1742.33	1742.23	1761.98	1800.29
1819.44	1619.44	1915.20	1761.14	1685.38	1532.16
1460.55	1359.25	1323.61	1277.96	1277.96	1323.61
1460.55	1734.38	1935.41	2167.98	2259.26	2282.08
2259.20	2202.03	2232.08	2213.62	2190.80	2190.80
2122.33	2099.51	2099.51	2122.33	1985.41	1643.10
1570.40	1472.31	1423.23	1374.16	1374.16	1423.23
1570.40	1004.93	2134.85	2331.86	2429.31	2453.85
2429.31	2453.85	2453.85	2380.23	2355.70	2355.70
2282.00	2257.54	2257.54	2262.08	2134.85	1766.77
1539.05	1442.86	1394.77	1346.67	1346.67	1394.77
1539.05	1627.63	2092.15	2264.53	2380.72	2404.77
2380.72	2404.77	2404.77	2332.63	2308.58	2308.58
2236.44	2212.39	2212.39	2236.44	2092.15	1731.44
1507.65	1413.42	1366.30	1319.19	1319.19	1366.30
1507.65	1790.33	2049.46	2237.91	2332.14	2355.70
2332.14	2355.70	2355.70	2265.02	2261.47	2261.47
2190.80	2167.24	2167.24	2190.80	2049.46	1696.10
1476.24	1363.97	1337.84	1291.71	1291.71	1337.84
1476.24	1753.03	2006.76	2191.29	2283.55	2306.62
2233.55	2306.62	2306.62	2237.42	2214.35	2214.35
2145.10	2122.09	2122.09	2145.16	2006.76	1660.77
1398.20	1322.62	1247.05	1223.15	1209.26	1171.47
1171.47	1247.05	1530.47	1624.94	1719.41	1757.20
1757.20	1738.31	1719.41	1719.41	1738.31	1776.10
1794.99	1794.99	1889.46	1757.20	1662.73	1511.57
1361.88	1268.27	1214.66	1196.25	1177.86	1141.04
1141.04	1214.66	1490.72	1582.74	1674.75	1711.57
1711.57	1090.16	1674.75	1674.75	1693.16	1729.97
1748.37	1748.37	1840.39	1711.28	1619.55	1472.31
1570.40	1472.31	1423.23	1374.16	1374.16	1423.23
1570.40	1004.93	2134.85	2331.10	2429.31	2453.85
2429.31	2453.85	2453.85	2380.23	2355.70	2355.70
2282.00	2257.54	2257.54	2262.08	2134.85	1766.77
1539.05	1442.86	1394.77	1346.67	1346.67	1394.77
1539.05	1627.63	2092.15	2284.53	2380.72	2404.77
2380.72	2404.77	2404.77	2332.63	2308.58	2308.58
2236.44	2212.39	2212.39	2236.44	2092.15	1731.44
1507.65	1413.42	1366.30	1319.19	1319.19	1366.30
1507.65	1790.33	2049.46	2237.91	2332.14	2355.70
2332.14	2355.70	2355.70	2265.02	2261.47	2261.47
2190.80	2167.24	2167.24	2190.80	2049.46	1696.10
1476.24	1363.97	1337.84	1291.71	1291.71	1337.84
1476.24	1753.03	2006.76	2191.29	2283.55	2306.62
2283.55	2306.62	2306.62	2237.42	2214.35	2214.35
2145.10	2122.09	2122.09	2145.16	2006.76	1660.77
1445.40	1335.13	1309.97	1264.80	1264.80	1309.97
1445.40	1716.51	1964.95	2145.64	2235.97	2258.56
2235.97	2258.56	2258.56	2190.81	2168.21	2168.21

2100.47	2077.88	2077.88	2100.47	1964.95	1626.13
1398.20	1322.62	1247.05	1228.15	1209.26	1171.47
1171.47	1247.05	1530.47	1624.94	1719.41	1757.20
1757.20	1736.31	1719.41	1719.41	1736.31	1776.10
1794.99	1774.99	1839.46	1757.20	1662.73	1511.57
1361.89	1236.27	1214.66	1196.25	1177.85	1141.04
1141.04	1214.66	1490.71	1582.73	1674.75	1711.56
1711.56	1696.16	1674.75	1674.75	1696.16	1729.96
1748.37	1748.37	1840.39	1711.56	1619.54	1472.31
1230.72	1200.68	1160.65	1120.63	1120.63	1160.65
1230.72	1520.60	1740.98	1901.07	1981.12	2001.13
1981.12	2001.13	2001.13	1941.09	1921.08	1921.08
1861.05	1841.04	1841.04	1861.05	1740.98	1440.81
1377.12	1291.05	1249.01	1204.98	1204.98	1248.01
1377.12	1635.33	1872.02	2044.16	2130.23	2151.75
2130.23	2151.75	2151.75	2087.20	2065.68	2065.68
2001.13	1979.61	1979.61	2001.13	1872.02	1549.26
1349.56	1265.23	1223.05	1180.88	1180.88	1223.05
1349.56	1602.62	1834.58	2003.28	2087.53	2108.71
2037.63	2108.71	2108.71	2045.45	2024.37	2024.37
1961.10	1940.02	1940.02	1961.10	1834.58	1518.27
1322.03	1239.41	1198.09	1156.73	1156.73	1198.09
1322.03	1569.92	1797.14	1962.40	2045.02	2065.68
2045.02	2065.68	2065.68	2003.71	1983.05	1983.05
1921.08	1900.43	1900.43	1921.08	1797.14	1487.29
1294.49	1213.59	1173.13	1132.68	1132.68	1173.13
1294.49	1537.21	1759.70	1921.51	2002.42	2022.64
2002.42	2022.64	2022.64	1961.97	1941.74	1941.74
1881.06	1860.83	1860.83	1881.06	1759.70	1456.30
1226.07	1159.79	1073.52	1076.95	1060.38	1027.25
1027.25	1093.52	1342.05	1424.89	1507.73	1540.87
1540.87	1524.30	1507.73	1507.73	1524.30	1557.44
1574.00	1574.00	1656.85	1540.87	1458.03	1325.48
1194.22	1129.67	1055.12	1043.98	1032.84	1000.56
1000.56	1065.12	1307.19	1387.88	1468.57	1500.85
1500.85	1434.71	1468.57	1468.57	1484.71	1516.98
1533.12	1533.12	1613.81	1500.85	1420.15	1291.05
1384.20	1297.68	1254.43	1211.17	1211.17	1254.43
1384.20	1643.73	1831.54	2054.67	2141.18	2162.81
2141.81	2162.81	2162.81	2097.92	2076.30	2076.30
2011.41	1989.78	1939.78	2011.41	1881.64	1557.22
1438.36	1375.36	1348.55	1302.34	1302.34	1348.55
1488.36	1767.46	2023.27	2209.32	2302.34	2325.60
2302.34	2325.60	2325.60	2255.83	2232.58	2232.58
2162.81	2139.55	2139.55	2162.81	2023.27	1674.43
1458.62	1367.45	1321.87	1276.29	1276.29	1321.87
1458.62	1732.11	1982.81	2165.13	2256.30	2279.09
2256.30	2279.09	2279.09	2210.72	2187.92	2187.92
2119.55	2096.76	2096.76	2119.55	1982.81	1640.94

1428.85	1559.55	1294.89	1250.24	1250.24	1294.89
1428.85	1690.76	1742.34	2120.95	2210.25	2232.53
2210.25	2232.58	2232.58	2165.60	2143.27	2143.27
2076.30	2053.97	2052.97	2076.30	1942.34	1607.45
1395.06	1511.64	1267.92	1224.20	1224.20	1267.92
1399.06	1661.41	1901.88	2076.76	2164.20	2186.06
2164.20	2136.06	2186.06	2120.45	2098.62	2098.62
2033.04	2011.15	2011.12	2033.04	1901.88	1573.97
1325.15	1253.50	1181.87	1163.96	1146.06	1110.24
1110.24	1181.87	1450.48	1540.01	1629.55	1665.36
1665.36	1647.45	1629.55	1629.55	1647.45	1683.27
1701.16	1701.18	1790.71	1665.36	1575.83	1432.57
1290.71	1220.94	1151.17	1133.73	1116.29	1081.40
1081.40	1151.17	1412.80	1500.01	1587.22	1622.11
1622.11	1604.66	1587.22	1587.22	1604.66	1639.55
1656.99	1656.99	1744.20	1622.11	1534.90	1395.36
1358.75	1273.83	1231.37	1188.91	1188.91	1231.37
1358.75	1513.52	1847.05	2016.90	2101.82	2123.05
2101.82	2123.05	2123.05	2059.36	2038.13	2038.13
1974.44	1953.21	1953.21	1974.44	1847.05	1528.60
1461.02	1569.71	1324.05	1278.40	1278.40	1324.05
1461.02	1734.97	1986.08	2168.71	2260.02	2282.85
2260.02	2232.85	2282.85	2214.36	2191.54	2191.54
2123.05	2100.22	2100.22	2123.05	1986.08	1643.60
1431.80	1342.32	1297.57	1252.83	1252.83	1297.57
1431.80	1700.27	1946.36	2125.33	2214.82	2237.19
2214.82	2237.19	2237.19	2170.08	2147.71	2147.71
2080.59	2056.22	2058.22	2080.59	1946.36	1610.78
1402.56	1514.92	1271.09	1227.26	1227.26	1271.09
1402.56	1665.50	1906.64	2081.96	2169.62	2191.54
2169.62	2191.54	2191.54	2125.79	2103.87	2103.87
2038.13	2016.21	2016.21	2038.13	1906.64	1577.91
1373.36	1237.53	1244.61	1201.69	1201.69	1244.61
1373.36	1530.87	1866.91	2038.58	2124.42	2145.88
2124.42	2145.88	2145.88	2081.50	2060.04	2060.04
1995.67	1974.21	1974.21	1995.67	1866.91	1545.03
1300.77	1230.46	1160.14	1142.57	1124.99	1089.93
1089.93	1160.14	1423.81	1511.70	1599.59	1634.75
1634.75	1517.17	1599.59	1599.59	1617.17	1652.33
1669.90	1559.90	1757.79	1634.75	1546.86	1406.24
1266.96	1196.50	1130.01	1112.59	1095.77	1061.52
1061.52	1130.01	1336.63	1472.44	1552.04	1592.29
1592.29	1575.17	1558.04	1558.04	1575.17	1609.41
1626.53	1626.53	1712.14	1592.29	1506.68	1369.71
1492.76	1399.46	1352.81	1306.17	1306.17	1352.81
1492.76	1772.65	2029.22	2215.82	2309.12	2332.44
2309.12	2332.44	2332.44	2262.47	2239.14	2239.14
2169.17	2145.84	2145.84	2169.17	2029.22	1679.36
1605.12	1504.80	1454.64	1404.48	1404.48	1454.64

1605.12	1950.05	2181.96	2342.50	2482.92	2508.00
2432.92	2500.00	2500.00	2432.70	2407.68	2407.63
2332.44	2307.36	2307.36	2332.44	2181.96	1835.76
1573.02	1474.70	1425.55	1376.39	1376.39	1425.55
1573.02	1557.94	2138.32	2334.95	2433.26	2457.34
2433.26	2457.84	2457.34	2384.10	2358.53	2359.53
2285.79	2251.21	2261.21	2285.79	2138.32	1769.64
1540.91	1444.61	1396.45	1348.30	1348.30	1396.45
1540.91	1329.84	2094.68	2287.30	2383.60	2407.63
2333.60	2407.63	2407.68	2335.45	2311.37	2311.37
2239.14	2215.07	2215.07	2239.14	2094.68	1733.53
1485.24	1461.66	1414.51	1367.35	1390.94	1532.32
1697.41	2003.89	2239.64	2333.94	2357.52	2333.94
2192.49	2100.92	2121.77	2074.62	2121.77	2168.92
2263.22	2310.37	2263.22	2121.77	1886.02	1650.26
1448.37	1409.75	1332.50	1274.57	1255.25	1255.25
1313.19	1429.06	1602.86	1718.73	1776.67	1815.29
1757.30	1730.04	1733.04	1660.30	1641.49	1699.42
1776.67	1931.16	1373.22	1834.60	1738.04	1641.49
1410.75	1373.13	1297.39	1241.46	1222.65	1222.65
1279.08	1391.94	1561.23	1674.09	1730.52	1768.14
1711.71	1592.90	1692.90	1617.66	1598.85	1655.23
1730.52	1651.00	1324.57	1786.95	1692.90	1598.85
1205.61	1100.47	1148.20	1109.92	1129.06	1243.83
1377.84	1020.61	1817.98	1894.52	1913.66	1894.52
1779.70	1700.57	1722.29	1684.02	1722.29	1760.57
1837.11	1875.39	1837.11	1722.29	1530.93	1339.56
1296.35	1275.77	1234.62	1193.47	1214.04	1337.50
1481.54	1749.04	1954.81	2037.12	2057.70	2037.12
1913.60	1893.08	1851.93	1810.78	1851.93	1893.08
1975.39	2016.55	1975.39	1851.93	1646.16	1440.39
1270.42	1250.26	1209.93	1169.60	1189.76	1310.75
1451.91	1714.06	1915.72	1996.38	2016.55	1996.38
1875.39	1655.22	1814.89	1774.56	1814.89	1255.22
1935.84	1970.21	1935.88	1814.89	1613.24	1411.58
1244.50	1224.74	1185.24	1145.73	1165.48	1286.00
1422.28	1079.08	1876.62	1955.64	1975.39	1955.64
1837.11	1817.36	1777.85	1738.34	1777.85	1817.36
1896.30	1935.88	1896.38	1777.25	1580.31	1382.77
1218.57	1199.23	1160.54	1121.36	1141.20	1257.25
1322.65	1044.10	1337.53	1914.90	1934.24	1914.90
1798.84	1779.50	1740.61	1702.13	1740.81	1779.50
1856.87	1695.55	1856.87	1740.31	1547.39	1353.97
1188.32	1150.63	1093.26	1045.72	1029.88	1029.88
1077.41	1172.48	1315.08	1410.14	1457.67	1489.36
1441.83	1425.99	1425.99	1362.61	1346.76	1394.30
1457.67	1384.43	1536.90	1505.21	1425.99	1346.76
1757.46	1126.59	1064.86	1018.56	1003.13	1003.13
1049.43	1142.02	1280.92	1373.51	1419.81	1450.68

1404.30	1380.95	1388.95	1327.22	1311.70	1358.03
1414.81	1383.27	1490.98	1406.11	1388.95	1311.78
1295.70	1275.21	1234.07	1192.94	1213.50	1336.91
1430.89	1750.27	1953.95	2036.22	2056.79	2036.22
1912.81	1872.24	1851.11	1807.97	1851.11	1892.24
1974.52	2075.55	1974.52	1851.11	1645.43	1439.75
1393.31	1311.79	1320.96	1232.73	1304.84	1437.54
1592.35	1079.36	2101.02	2189.48	2211.60	2189.48
2050.79	2054.57	1990.44	1946.21	1990.44	2034.67
2123.14	2157.37	2123.14	1990.44	1769.28	1548.12
1355.44	1355.77	1300.42	1257.07	1278.75	1408.79
1500.50	1342.26	2059.00	2145.69	2167.37	2145.69
2015.65	1993.93	1950.63	1907.28	1950.63	1993.93
2030.67	2124.02	2080.67	1950.63	1733.89	1517.16
1337.50	1310.34	1275.88	1231.42	1252.65	1380.04
1528.60	1304.67	2016.78	2101.90	2123.14	2101.90
1974.52	1953.29	1910.82	1868.36	1910.82	1953.28
2038.21	2050.57	2038.21	1910.82	1698.51	1486.19
1307.71	1250.92	1247.34	1205.76	1226.65	1351.29
1496.81	1757.07	1974.96	2058.11	2078.90	2058.11
1933.30	1912.50	1871.01	1829.44	1871.01	1912.59
1995.75	2037.33	1995.75	1871.01	1663.12	1455.23
1277.20	1243.14	1175.02	1123.93	1106.91	1106.91
1157.97	1200.17	1413.43	1515.61	1566.70	1600.76
1549.67	1532.64	1532.64	1464.52	1447.49	1498.58
1560.70	1702.93	1651.84	1617.78	1532.64	1447.49
1244.02	1210.35	1144.50	1094.74	1078.15	1078.15
1127.92	1227.44	1376.72	1476.24	1526.00	1559.18
1509.42	1442.83	1492.83	1426.48	1409.89	1459.66
1526.00	1658.70	1608.94	1575.76	1492.83	1409.80
1335.85	1314.65	1272.24	1229.83	1251.04	1378.26
1526.69	1802.34	2014.38	2099.20	2120.40	2099.20
1971.97	1950.77	1908.36	1865.95	1908.36	1950.77
2035.50	2077.99	2035.58	1908.36	1696.32	1484.28
1436.40	1413.60	1368.00	1322.40	1345.20	1482.00
1641.60	1930.00	2166.00	2257.20	2280.00	2257.20
2120.40	2097.60	2052.00	2006.40	2052.00	2097.60
2182.80	2234.20	2188.80	2052.00	1824.00	1596.00
1407.67	1355.33	1340.64	1295.95	1318.30	1452.36
1608.77	1699.24	2122.68	2212.06	2234.40	2212.06
2077.99	2055.55	2010.96	1966.27	2010.96	2055.65
2143.02	2159.71	2145.02	2010.96	1787.52	1564.00
1378.94	1357.06	1313.28	1269.50	1291.39	1422.72
1575.94	1650.48	2079.36	2166.91	2188.80	2166.91
2035.56	2013.70	1969.92	1926.14	1969.92	2013.70
2101.25	2145.02	2101.25	1969.92	1751.04	1532.16
1350.22	1320.78	1285.92	1243.06	1264.49	1393.08
1543.10	1521.72	2036.04	2121.77	2143.20	2121.77
1993.16	1971.74	1928.88	1886.02	1928.88	1971.74

2057.47	2100.34	2057.47	1928.82	1714.56	1500.24
1316.70	1261.59	1211.36	1158.70	1141.14	1141.14
1193.81	1244.14	1457.15	1562.48	1615.15	1650.26
1597.60	1530.04	1530.04	1509.82	1492.26	1544.93
1615.15	1735.60	1702.93	1667.32	1580.04	1492.26
1282.50	1240.30	1179.90	1128.60	1111.50	1111.50
1152.80	1265.40	1419.30	1521.90	1573.20	1607.40
1556.10	1539.00	1539.00	1470.60	1453.50	1504.30
1573.20	1710.00	1658.70	1624.50	1539.00	1453.50
1217.29	1197.97	1159.33	1120.68	1140.01	1255.94
1391.19	1642.38	1835.60	1912.89	1932.21	1912.39
1796.90	1777.64	1738.99	1700.35	1738.99	1777.64
1854.93	1693.57	1854.93	1738.99	1545.77	1352.55
1308.92	1260.14	1246.59	1205.04	1225.81	1350.47
1495.91	1766.00	1973.77	2056.87	2077.65	2056.87
1932.21	1931.44	1869.88	1828.33	1869.88	1911.44
1994.54	2036.10	1994.54	1869.88	1662.12	1454.35
1282.74	1262.38	1221.66	1180.94	1201.30	1323.46
1465.99	1730.68	1934.29	2015.74	2036.10	2015.74
1893.57	1673.21	1832.49	1791.77	1832.49	1873.21
1954.65	1995.38	1954.65	1832.49	1628.88	1425.27
1256.50	1230.62	1196.73	1156.84	1176.78	1296.45
1436.07	1695.36	1894.82	1974.60	1994.54	1974.60
1854.93	1654.98	1795.09	1755.20	1795.09	1834.98
1914.70	1954.65	1914.76	1795.09	1595.63	1396.18

System B

The load model for system B is identical to the first 2160 load samples in the load model of Appendix E.

APPENDIX K

Investment Calculations for Pumped Hydro Storage Unit

Investment/KW for 500 MW nuclear unit (Reference [21]) = \$700/KW.

Total investment necessary for 500 MW unit = \$700/KW x 500,000 KW
= \$350,000,000.

Investment/KW for pumped hydro storage unit (Reference [21]) =
= \$485/KW = \$500/KW.

Total investment for 300 MW pumped hydro storage unit =
= \$500/KW x 300,000 KW = \$150,000,000.

Total investment for nuclear and pumped hydro storage units =
\$350,000,000 + \$150,000,000 = \$500,000,000.

Assuming a rate of yearly interest of 10% on capital,

Interest charges/year on total investment =
= 10% of \$500,000,000 = \$50,000,000.

Depreciation charges for nuclear unit assuming depreciation takes
place over 10 years =

\$350,000,000/10 = \$35,000,000.

Depreciation charges for pumped hydro storage unit assuming
depreciation takes place over 20 years =

\$150,000,000/20 = \$7,500,000.

Sum of interest and depreciation charges =

\$35,000,000 + \$7,500,000 + \$50,000,000
= \$92,500,000.

Reduction in fuel costs for 3 months brought about by installation of 500 MW nuclear units = \$136,41430.57.

Projected over one year the reduction in fuel costs due to the new nuclear unit = $\$13641430.57 \times 4 = \$54,000,000$.

Reduction in fuel costs for 3 months brought about by the installation of a 300 MW pumped hydro storage unit with a reservoir of 300 MWhrs = \$130,000.

Projected over one year the reduction in fuel costs due to the pumped hydro storage units = $\$130,000 \times 4 = \$520,000$.

Reduction in fuel costs for one year due to nuclear and pumped hydro storage units = $\$54,600,000 + \$520,000 = \$55,120,000$.

The reduction in fuel costs for one year (\$55,120,000) is less than the interest and depreciation charges (\$92,500,000).

APPENDIX LPRODUCTION COSTING SOFTWARE PACKAGE

The software package used for production costing is listed below.
 It was provided by Professor Schenk of the Department of Electrical
 Engineering, University of Ottawa, Canada.

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C      AT PRESENT THE PROGRAM CAN HANDLE :
C      (1) 20 BINARY STATE GENERATING STATIONS
C      (2) A TOTAL OF 100 GENERATING UNITS
C      (3) A MAXIMUM OF 5 CAPACITY BLOCKS FOR EACH UNIT
C      (4) 1 ENERGY ASSIGNED GENERATING UNIT
C*****
C      INPUT ORDER AND FORMATS :
C      1. TITLE ---- (18A4)
C      2. *OBJECT TIME FORMAT ---- (18A4)
C      3. BASE,PEAK LOADS,TIME DURATION,STEP SIZE ---- (*OBJECT TIME
C                                     FORMAT)
C      4. *OBJECT TIME FORMAT ---- (18A4)
C      5. THE NO. OF POINTS IN LDC --- (*OBJECT TIME FORMAT)
C      6. *OBJECT TIME FORMAT (FOR X)---- (18A4)
C          *OBJECT TIME FORMAT (FOR Y)----- (18A4)
C      7. THE LDC FIRST X..., THEN Y
C          LXVEC --> (*OBJECT TIME FORMAT)
C          LYVEC --> (*OBJECT TIME FORMAT)
C      8. *OBJECT TIME FORMAT ---> (18A4)
C      9. THE TOTAL HYDRO ENERGY AVAILABLE ---> (*OBJECT TIME FORMAT)
C      10. *OBJECT TIME FORMAT ---> (18A4)
C      10. **OBJECT TIME FORMAT ---> (18A4)
C      12. ***OBJECT TIME FORMAT ---> (18A4)
C-> 13. STATION NAME, NO. OF UNITS,NO OF BLOCKS,UNIT SIZE,
C      FORCED/OUTAGE RATE---> (*OBJECT TIME FORMAT)
C      14. BLOCK SIZE ---> (*OBJECT TIME FORMAT)
C-> 15. IFC --> (**OBJECT TIME FORMAT)
C
C      REAL*8 LXVEC(100),LYVEC(100),XVEC(500),YVEC(500),NYVEC(500),
C      +      SIZE(20),FOR(20),PREV,PRES,
C      +      BASE,PEAK,TIME,H,MENGY,FUNC,ROMBG,MAX,MW,P,Q,
C      +      CAP,LOLP,ENERGY,LPEAK,FCOST
C      REAL*8 BLOCK(20,5)/100*0.000/,
C      +      TB(100)/100*0.0/,PN1/0.000/,PN2/0.000/,TOTAL/0.000/,
C      +      INSTAL/0.000/,TFCOST/0.0000/,
C      +      Q1(100),Q4(100),Q5(100),Q7(100),Q8(100),Q9(100)
C      REAL*8 STAT(20)/20*0.0/,DIFF,LDCENG
C      REAL*4 FMT(18),FMT1(18),FMT2(18),IFC(100,5)/500*1.0E3/,Q6(100)
  
```

```

      INTEGER*4 LNB/0/,IO2(100),IO3(100),KFIT/1/,
      + RINDEX,CINDEX,UNITNO,DUMMY,NOUNIT(20),
      + CARRAY(100)/100*1/,N2/1/,I/1/,N4/1/,FLAG/1/,
      + HIFC,HINDEX/0/,NO/1/
C
C
C
      COMMON XVEC,YVEC,BASE,PEAK,N
C
C
C
      INPUT TITLE
C
      READ(5,FMT) FMT
      WRITE(6,4999) FMT
      READ(5,999) FMT
C
C
C
      INPUT BASE, PEAK LOADS ,TIME DURATION, STEP SIZE
C
      READ(5,FMT) BASE,PEAK,TIME,H
      READ(5,999) FMT
C
C
C
      INPUT NO OF POINTS ON LDC
C
      READ(5,FMT) N
      READ(5,999) FMT
      READ(5,999) FMT1
C
C
C
      INPUT LDC, FIRST X, THEN Y
C
      N=N+3
      READ(5,FMT) (LXVEC(J),J=4,N)
      READ(5,FMT1) (LYVEC(J),J=4,N)
      DO 40 J=1,3
      LXVEC(J)=LXVEC(4)-(4.0D0-J)*50.0D0
      LYVEC(J)=1.0D0
      LXVEC(J+N)=LXVEC(N)+J*50.0D0
      LYVEC(J+N)=0.0D0
40    CONTINUE
      N=N+3
      READ(5,999) FMT
C
C
C
      INPUT HYDRO ENERGY AVAILABLE
C
      READ(5,FMT) HENGY
      READ(5,999) FMT
      READ(5,999) FMT1
      READ(5,999) FMT2
C
C
C
      INPUT STATION NAME,NO. OF UNITS,NO. OF BLOCKS,UNIT SIZE,FOR
      BLOCK SIZE
      IFC
1111 READ (5,FMT,END=1800) STAT(NO),NOUNIT(NO),NNOB,
      + SIZE(NO),FOR(NO)
      READ(5,FMT1) (BLOCK(NO,K),K=1,NNOB)
      READ(5,FMT2) (IFC(N2,K),K=1,NNOB)
C
C
C
      IF (NNOB.GT.LNB) LNB=NNOB
C
C
C
      INSTAL=INSTAL+SIZE(NO)
      IF (NOUNIT(NO).LE.1) GO TO 1227

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NNUNIT=NOUNIT(NO)
DO 574 K=2,NNUNIT
  INSTAL=INSTAL+SIZE(NO)
  DO 574 KZ=1,NNOB
    IFC(N2+K-1,KZ)=IFC(N2,KZ)
574  CONTINUE
    N2=N2+NNUNIT
    NO=NO+1
    GO TO 1111
1227 CONTINUE
C
C  FIND THE UNIT NO OF THE HYDRO UNIT
C
    IF (HENGY.EQ.0.0) GO TO 1233
    IF (IFC(N2,1).NE.0.0E0) GO TO 1233
    PREV=(1.0-FOR(NO))*TIME*SIZE(NO)*1.0D-3
    IF (PREV.LT.HENGY) GO TO 3000
    HINDEX=NO
    HIFC=N2
    IFC(HIFC,1)=1.0E3
1233  N2=N2+1
    NO =NO+1
    GO TO 1111
C
C  END ODF DATA
C
1800  N1=N
    N2=N2-1
    NO=NO-1
    LPEAK=PEAK
    DO 1900 J=1,N1
      XVEC(J)=LXVEC(J)
      YVEC(J)=LYVEC(J)
1900  CONTINUE
C
C  OUTPUT THE BASE LOAD, PEAK LOAD, THE TIME DURATION AND
C  THE ENERGY UNDER THE LDC
C
    WRITE(6,5000) BASE,PEAK,TIME
    TIME=TIME*1.0D-3
    LDCENG=TIME*ROMBG(PN1,PEAK)
    WRITE(6,5300) LDCENG
C
    MAX=4.0D0*H+INSTAL+PEAK
C
C  START OF LOOP
C
    IF (I.GT.N2) GO TO 99999
    IF (HINDEX.EQ.0) GO TO 2040
    IF (FLAG.EQ.0) GO TO 2040
    IF (I.EQ.N2) GO TO 9999
C
C  TEST TO SEE IF HYDRO UNIT CAN BE FITTED...
C
    PRES=(1.0-FOR(HINDEX))*TIME*ROMBG(PN1,PN1+SIZE(HINDEX))
    IF ((PRES.LE.HENGY).AND.(PREV.GE.HENGY)) GO TO 3000
    PREV=PRES
    GO TO 2040
C
C  REACHING 3000 MEANS THAT HYDRO CAN BE FITTED

```

```

C
3000 FLAG=0
      KFIT=0
      IFC(HIFC,1)=0.0E0
C
C      FIND LOADING ORDER OF THE UNIT
C
2040 L=1
      LL=1
      J=1
      DO 2010 II=1,N2
        K=1
        DO 2020 JJ=1,LNB
          IF (IFC(L,J).LE,IFC(LL,K)) GO TO 2020
          L=LL
          J=K
2020 K=K+1
2010 LL=LL+1
      CALL LOCATE (L,NO,NUNIT,RINDEX,UNITNO)
      CINDEX=CARRAY(L)
      TB(L)=TB(L)+BLOCK(RINDEX,CINDEX)
C
C      TEST TO SEE IF DECONVOLUTION IS REQUIRED
C
      IF (CINDEX.EQ.1) GO TO 2070
      N=N1
      PEAK=LPEAK
      DO 2075 J=1,N
        XVEC(J)=LXVEC(J)
        YVEC(J)=LYVEC(J)
2075 CONTINUE
      DO 2080 J=1,N2
        HW=TB(J)
        IF (HW.EQ.0.0D0) GO TO 2080
        IF (J.EQ.L) GO TO 2080
        CALL LOCATE (J,NO,NUNIT,INDEX,DUMMY)
        IF (MAX.LE.XVEC(N)) GO TO 2081
        NN=1+HW/H
        DO 2082 JJ=1,NN
          XVEC(N+JJ)=XVEC(N+JJ-1)+H
2082 N=N+NN
2081 P=1.0D0-FOR(INDEX)
        Q=FOR(INDEX)
        DO 2083 JJ=1,N
          NYVEC(JJ)=P*FUNC(XVEC(JJ))+Q*FUNC(XVEC(JJ)-HW)
2083 DO 2084 JJ=1,N
          YVEC(JJ)=NYVEC(JJ)
2084 PEAK=PEAK+HW
2080 CONTINUE
C
C      NOPE, THEN START INTEGRATION SUBROUTINE
C
2070 HW=TB(L)
      P=1.0D0-FOR(INDEX)
      Q=FOR(INDEX)
      PN2=PN2+BLOCK(RINDEX,CINDEX)
      ENERGY=P*TIME+PN2*(PN1,PN2)
      FCOS1=IFC(L,CINDEX)*ENERGY*1.0D3
      PN1=PN1+BLOCK(RINDEX,CINDEX)

```

```

      IF (N4.EQ.1) GO TO 4000
      IF (KFIT.NE.0) GO TO 4000
C
C      NOW RECALCULATE THE ENERGY DELIVERED BY THE UNIT JUST BEFORE THE
C
      TOTAL=TOTAL-08(N4-1)
      TFCOST=TFCOST-09(N4-1)
      08(N4-1)=08(N4-1)+ENERGY-HENGY
      09(N4-1)=08(N4-1)*08(N4-1)*1.0D3
      TOTAL=TOTAL+08(N4-1)
      TFCOST=TFCOST+09(N4-1)
      ENERGY=HENGY
      FCOST=0.0D0
      KFIT=1
4000  TOTAL=TOTAL+ENERGY
      TFCOST=TFCOST+FCOST
      01(N4)=STAT(RINDEX)
      102(N4)=UNITNO
      103(N4)=CINDEX
      04(N4)=SIZE(RINDEX)
      05(N4)=BLOCK(RINDEX,CINDEX)
      06(N4)=IFC(L,CINDEX)
      07(N4)=FOR(RINDEX)
      08(N4)=ENERGY
      09(N4)=FCOST
C
C      CALCULATE THE NEW ELDC
C
      IF (MAX.LE.XVEC(N)) GO TO 2090
      NN=1+MW/H
      DO 2100 J=1,NN
2100  XVEC(N+J)=XVEC(N+J-1)+H
      N=N+NN
2090  DO 2110 J=1,N
2110  NYVEC(J)=P*FUNC(XVEC(J))+Q*FUNC(XVEC(J))-MW
      PEAK=PEAK+MW
      DO 2120 J=1,N
2120  YVEC(J)=NYVEC(J)
      IFC(L,CINDEX)=1.0E3
      CARRAY(L)=CARRAY(L)+1
      IF (IFC(L,CARRAY(L)).GT.5.0E2) I=I+1
      N4=N4+1
      GO TO 1
99999 CONTINUE
C
C      OUTPUT:
C
      N4=N4-1
      DO 4500 KY=1,N4
4500  WRITE(6,5500) KY,01(KY),102(KY),103(KY),04(KY),05(KY),06(KY),
      + 07(KY),08(KY),09(KY)
C
C      NOW THE LOLP AND THE EXPECTED ENERGY NOT SERVED
C
      LOLP=FUNC(INSTAL)
      ENERGY=TIME*ROMBG(INSTAL,PEAK)
      DIFF=LDCENG-TOTAL-ENERGY
      WRITE(6,5600) INSTAL,LOLP,TOTAL,ENERGY,DIFF,TFCOST
      GO TO 2
8000  WRITE(6,6100)

```

```

GO TO 2
9999 WRITE(6,6200)
CC
CC      WRITE FORMAT STATEMENT:
999  FORMAT (18A4)
4999 FORMAT ('1',18A4)
5000 FORMAT ('-BASE LOAD= ',F9.2,' MW',
+          'PEAK LOAD= ',F9.2,' MW',
+          'TIME DURATION= ',F7.2,' HRS',
5300 FORMAT ('ENERGY UNDER LDC = ',F10.3,' GWH',///
+          '-LOADING',5X,'STATION',5X,'UNIT',5X,'BLOCK',5X,
+          'UNIT SIZE',5X,'BLOCK SIZE',5X,'IFC',8X,'FOR',4X,
+          'EXPECTED ENERGY',5X,'FUEL COST',
+          '2X',ORDER',7X,'NAME',8X,'NO.',8X,'NO.',9X,'(MW)',10X,
+          '(MW)',6X,'$/MWH',18X,'GWH',13X,'($)',
+          '1X,119(''-')')
5500 FORMAT (2X,I3,8X,A8,4X,I3,8X,I2,7X,F7.2,8X,F7.2,5X,F6.3,6X,F4.2,
+          '5X,F12.6,2X,F13.2)
5600 FORMAT ('-INSTALLED CAPACITY = ',F10.2,' MW',
+          'OLOLP = ',D12.6/
+          'OTOTAL GENERATION = ',F12.6,' GWH',
+          'OENERGY NOT SERVED = ',F12.6,' GWH',
+          'OENERGY BALANCE = ',F9.6,' GWH',
+          'OTOTAL FUEL COST = $ ',F13.2)
6100 FORMAT('1*** INPUT ERROR -- MAXIMUM HYDRO ENERGY GENERATED IS',
+          'LESS THAN HYDRO ENERGY AVAILABLE')
6200 FORMAT('1*** DATA ERROR-- MINIMUM HYDRO ENERGY GENERATED IS',
+          'GREATER THAN HYDRO ENERGY AVAILABLE')
2    STOP
END
DOUBLE PRECISION FUNCTION FUNC(X)
CC
CC      LAGRANGIAN INTERPOLATION SUBROUTINE
REAL*8 XVEC(500),YVEC(500),BASE,PEAK,X,C,D,Y
INTEGER*4 I,J,K,KK,L,N
COMMON XVEC,YVEC,BASE,PEAK,N
IF (X.GT.BASE) GO TO 9900
FUNC=1.0D0
RETURN
9900 IF (X.LT.PEAK) GO TO 9910
FUNC=0.0D0
RETURN
CC
CC      ACCELERATION TEST
9910 CONTINUE
DO 8000 KK=1,N,10
IF (X.LE.XVEC(KK)) GO TO 8010
8000 CONTINUE
8010 KK=KK-10
DO 9920 K=KK,N
IF (X.LE.XVEC(K)) GO TO 9930
9920 CONTINUE
9930 L=K-3
IF (YVEC(L).GT.1.0D-15) GO TO 9935
FUNC=0.0D0
RETURN
9935 CONTINUE

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```

DO 9940 J=1,4
IF (X.NE.XVEC(L+J)) GO TO 9940
FUNC=YVEC(L+J)
RETURN
9940 CONTINUE
C=1.0D0
Y=0.0D0
DO 9950 J=1,4
C=C*(X-XVEC(L+J))
9950 CONTINUE
DO 9960 I=1,4
D=1.0D0
DO 9970 J=1,4
IF (I.NE.J) D=D*(XVEC(I+L)-XVEC(J+L))
9970 CONTINUE
Y=Y+(YVEC(I+L)*C)/(D-XVEC(I+L)*D)
9980 CONTINUE
FUNC=Y
RETURN
END
DOUBLE PRECISION FUNCTION KUMBG(A1,B)
C
C ROMBERG INTEGRATION SUBROUTINE
C
REAL*8 XVEC(500),YVEC(500),BASE,PEAK,A,B,T(20,20),
+ A1,FUNC,DX,XX,SUM,ERROR,RECT,DABS
INTEGER*4 I,J,K,L,N,NN
COMMON XVEC,YVEC,BASE,PEAK,N
A=A1
IF (B.GT.BASE) GO TO 9800
ROMBG=B-A
RETURN
9800 RECT=0.0D0
IF (A.GE.BASE) GO TO 8810
RECT=BASE-A
A=BASE
8810 J=3
ERROR=1.0D-5
T(1,1)=((B-A)/2.0D0)*(FUNC(A)+FUNC(B))
T(1,2)=T(1,1)/2.0D0+((B-A)/2.0D0)*FUNC((A+B)/2.0D0)
T(2,1)=(1.0D0/3.0D0)*(4.0D0*T(1,2)-T(1,1))
8820 DX=(B-A)/2.0D0**(J-1)
XX=A-DX
NN=2**(J-2)
SUM=0.0D0
I=1
8830 XX=XX+2.0D0*DX
SUM=SUM+FUNC(XX)
IF (I.EQ.NN) GO TO 8840
I=I+1
GO TO 8830
8840 T(1,J)=T(1,J-1)/2.0D0+DX*SUM
L=2
8850 K=J+1-L
T(L,K)=(4.0D0**(L-1)*T(L-1,K+1)-T(L-1,K))/(4.0D0**(L-1)-1.0D0)
IF (L.EQ.J) GO TO 8860
L=L+1
GO TO 8850
8860 IF ((DABS((T(J,1)-T(J-1,1))/T(J,1))).LT.ERROR) GO TO 8870
J=J+1

```

```

GO TO B820
8870 ROMRG=RECT+T(J,1)
RETURN
END
SUBROUTINE LOCATE (L,NO,NUNIT,RINDEX,UNITNO)
C
C      SUBROUTINE FOR FINDING THE LOCATION OF THE GENERATING
C      STATION IN THE ARRAY 'STAT' AND HENCE FIND THE UN
C      NO OF THE COMMITING UNIT.
C
C      INTEGER*4 NUNIT(20),I,NO,L,RINDEX,UNITNO,NOTOT,LAST
C
C      NOTOT=0
C      LAST=0
C
C      DO 40 I=1,NO
C      NOTOT=NOTOT+NUNIT(I)
C      IF (L.LE.NOTOT) GO TO 50
C      LAST=LAST+NUNIT(I)
40  CONTINUE
50  RINDEX=I
UNITNO=L-LAST
RETURN
END

```


END

1 0 0 6 8 6

FIN