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Parametric Investigation on the Coalescence of Coplanar Fatigue Cracks

by

Christopher J. Bayley

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Masters of Engineering

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of
Mechanical and Aerospace Engineering
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The Faculty of Graduate Studies and Research
acceptance of the thesis

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in partial fulfilment of the requirements for
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A handwritten signature in black ink, appearing to read "Robert Bell", written over a horizontal line.

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Thesis Supervisor

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May 1997

Abstract

This thesis presents a parametric investigation on the coalescence of similar coplanar fatigue cracks. The investigation involves a numerical simulation of coalescence for both applied loads of bending and tension. Two hundred and forty four finite element simulations were carried out with variations in the crack aspect ratio, depth and percentage completion of coalescence. Coalescence was simulated by incrementally increasing the coalescence depth from close to the surface to 80% of the maximum crack depth.

It was determined that the aspect ratio of the two cracks has the greatest effect on the stress intensity factor at the coalescence plane. Increasing the coalescence depth, caused a dramatic decrease in the stress intensity factor, while increasing the crack depth increased slightly the stress intensity factor.

Results from this investigation are in agreement with an earlier experimental program and confirm that crack growth during coalescence is a stable crack growth process.

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List of Symbols

γ	Interaction Factor
γ_A	Interaction Factor Coalescence Plane
γ_B	Interaction Factor Deepest Point
K_I	Mode I stress intensity factor (SIF)
a	Crack depth
a_{coal}	Coalescence depth
c	Half surface crack length
web	Dimension of focused Mesh around Crack Front
$E(k)$	Complete Elliptic integral
t	Plate thickness
b	Half plane width
e	Crack tip to edge distance
t_s	Inter crack tip spacing
A	Coalescence point
A'	Free surface point of single crack
B	Deepest point coalescing crack
B'	Deepest point independent crack
ϕ	Elliptical angle
a/c	Aspect ratio
a/t	Depth ratio
E	Young's Modulus
ν	Poisson's ratio
S_t	Uniform tensile stress
S_b	Outer fiber bending stress
σ_y	Stress in Y direction close to crack tip
σ_x	Stress in X direction close to crack tip

τ_{xy}	Shear Stress close to crack tip
σ_{ys}	Yield Strength
r_y	Plastic zone size
H	Boundary correction factor
F	Finite width correction factor
Q	Shape factor
da/dN	Crack growth rate
C	Material growth rate constant
m	Material growth rate exponent
ΔK	Stress intensity factor range mode I
J	J integral
G	Strain energy release rate
k	Coalescence coefficient
N_{coal}	Computed crack life
N_{exp}	Experimental crack life
Γ	Integration Path defined for J integral
W	Strain Energy Density
s	incremental distance along Γ
Π	Potential Energy of a body
u	displacement
T	Traction

Chapter 1

Introduction

As the average age of most large engineering structures continues to increase, fatigue crack growth is becoming an increasingly important subject. Fatigue is a stable crack growth process that occurs under cyclic loading over the lifetime of nearly all engineering structures. Crack growth occurs even when the mean stress level is well below the yield strength of the material and can cause the crack to propagate until either the critical stress intensity factor is reached causing sudden failure or until net section yielding takes place. Crack initiation occurs commonly in localized areas of stress concentration such as fillets, notches and other discontinuities in the structure. In welded structures, cracks often result from the multitude of initiation sites caused by welding process. Since crack initiation can occur at multiple sites, the number of cracks along a meter of weldment can exceed 100-200 cracks per meter [6]. During the lifetime of the component, these initiated cracks grow together and form a dominant crack which then can lead to catastrophic failures of the structure.

In this investigation the coalescence of fatigue cracks is examined with a particular interest in determining the variation in the stress intensity factor around coalescing coplanar surface cracks. Fatigue crack coalescence is the process by which several small adjacent cracks increase in size and eventually grow together forming a single larger crack. As the adjacent cracks grow together, an interaction effect

occurs due to their mutual effect on each other. After coalescence has occurred the crack behaves once again as a single crack and continues to grow in the surface and depth directions.

To date the interaction process has been widely studied and it is known that interaction does not occur until distance between the leading crack tips is small. However the process that leads to the formation of the larger crack remains relatively unknown. The present investigation is aimed at determining the stress intensity factors for two similar and coplanar coalescing cracks throughout the entire coalescence period. The study uses the finite element method to extract the stress intensity factor around the coalescing crack. Coalescence is modeled by increasing the crack depth along the plane of the intersection of the two surface cracks. In this way, only the variation in the crack shape during coalescence is studied and not the complete process. However using the stress intensity factor values determined from this investigation, it would be possible to determine the crack growth at any point along a coalescing crack.

1.1 Objectives of the Investigation

The objectives of this investigation are as follows:

1. To determine the effect of the aspect ratio of the cracks during coalescence.
2. To determine the effect of the maximum crack depth during coalescence.
3. Provide the stress intensity factors at the coalescence, deepest and surface point which can be incorporated into a predictive life model for fatigue crack growth.

Chapter 2

Literature Review

This chapter presents an overview of the background information used in this investigation. The concepts of linear elastic fracture mechanics (LEFM) is presented along with the definition of the stress intensity factor (SIF). A description of fatigue crack growth and the coalescence process is described with a review of the current literature on the topic of coplanar fatigue crack interaction and coalescence. Finally the tools used in this investigation, the finite element method (FEM) and J integral are presented.

2.1 Linear Elastic Fracture Mechanics

In nearly all engineering structures discontinuities caused by flaws result in a localized inhomogeneous stress distribution. The stress distribution increases rapidly towards the material surrounding the flaw. Neglecting the material adjacent to the crack front, the stress distribution remains within the elastic portion of the materials stress strain behaviour. For most metallic and ceramic materials, the size of the inelastic region adjacent to the crack tip is small compared to the crack size or any other characteristic length of the body. Therefore, the relative amount of material experiencing inelastic behaviour is small compared to the overall elastic state of the structure. Linear elastic fracture mechanics attempts to describe the stress state in all regions of a structure which contain flaws.

Depending on the external loading conditions applied to the structure, three

independent crack extension modes can be considered (Illustrated in Figure 2.1).

- Opening Mode I Crack surface open symmetrically about the crack plane
- Sliding Mode II Crack faces slide relative to one another normal to front surface
- Tearing Mode III Crack faces slide relative to one another parallel to front surface

Figure 2.1: Three Modes of Stress Intensity

In this investigation Mode I will be used exclusively since it is the most common fracture mode, and promotes the quickest crack growth through the thickness of a material.

Many researchers have proposed solutions to describe the stress distribution around cracks. Westergaard [32] proposed the following stress distribution using polar coordinates centered on the crack front. Equations 2.1 describe a crack in an infinite plate with a through thickness crack of length $2a$, subjected to an applied uniform stress.

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (2.1)$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (2.2)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \left(\sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) \quad (2.3)$$

$$\sigma_z = 0 \quad \text{Plane Stress} \quad (2.4)$$

$$\sigma_z = \nu(\sigma_x + \sigma_y) \quad \text{Plane Strain} \quad (2.5)$$

In the above equations, r is the radial distance from the crack front and K_I is the stress intensity factor. It is seen from all of the equations that as the crack front is approached in all direction the stress increases. The stress is proportional to $1/\sqrt{r}$ which is termed a stress singularity. Since the material adjacent to the crack front cannot withstand an infinite stress, a plastic zone surrounding the crack front exists. The size of the plastic zone can be calculated by substituting 2.1 into a suitable yielding criteria such as the VonMises solution. Under monotonic loading the plastic zone size can be determined as:

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \quad \text{Plane Stress} \quad (2.6)$$

$$r_y = \frac{1}{6\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \quad \text{Plane Strain} \quad (2.7)$$

while under cyclic loading the plastic zone is further reduced.

$$r_y = \frac{1}{8\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \quad \text{Plane Stress} \quad (2.8)$$

$$r_y = \frac{1}{24\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \quad \text{Plane Strain} \quad (2.9)$$

The further reduction in the plastic zone found under cyclic loading is another reason why LEFM can be used for fatigue analysis. Plane strain conditions have a smaller yield volume due to the increased constraints imposed by the surrounding material.

The K_I , *Stress Intensity Factor* found in equations 2.1-2.9 is a fundamental quantity used to describe the stress field near the crack front. The value of the

stress intensity factor (SIF) depends on the geometry of the crack, boundary conditions and the applied load. Due to the near infinite variations of the variables used to define the stress intensity factor, exact solutions for all cases are often unavailable. Various methods have been introduced to define the SIF for a given geometry and crack type. These include; photo-elasticity, weight functions and numerical approximations. The use of numerical approximations such as the finite element and boundary element methods are attractive due their relative efficiencies and the availability of inexpensive computational power.

In three dimensions, Irwin calculated an exact stress intensity factor along the periphery of an embedded elliptical crack within an infinite solid [7]:

$$K_I = \frac{\sigma\sqrt{\pi b}}{E(k)} \left(\sin^2 \phi + \frac{b^2}{a^2} \cos^2 \phi \right)^{1/4} \quad (2.10)$$

where ϕ is the elliptical angle and $E(k)$ is the complete elliptic integral of second kind. Equation 2.10 is satisfactory in defining the SIF around an embedded elliptical crack, but at a surface when the crack is only half of an ellipse, the solution is no longer valid. The inclusion of boundary correction and finite width correction factors have been added to the above equation by various authors in order to predict the SIF around a surface crack.

2.2 Stress intensity solutions for surface cracks

In 1979, Newman and Raju [23][26] determined an SIF solution for an elliptical surface cracks using the finite element method. Their solution defines the SIF for $0 \leq a/c \leq 1$; $0 \leq a/t \leq 1$; $0 \leq \phi \leq \pi$ where variables are defined in figure 2.2. The solution is similar in form to the exact solution for an embedded elliptical crack proposed by Irwin [7] and is given by:

$$K_I = (S_t + H S_b) \sqrt{\pi \frac{a}{Q}} F \left(\frac{a}{t}, \frac{a}{c}, \frac{c}{b}, \phi \right) \quad (2.11)$$

where S_t and S_b are the uniform tension and outer fiber bending stress respectively. The function Q is the solution for the elliptical integral of the second kind

Figure 2.2: Single Surface Crack

which can be estimated as:

$$Q = 1 + 1.464 \left(\frac{a}{c} \right)^{1.65} \quad (2.12)$$

with only a 4% decrease in accuracy from the exact solution [28]. The function F is the boundary correction factor and accounts for the influence of the front surface, back surface and finite width of the plate. The functions F and H are defined so that in tension, the boundary correction factor is equal to F , and in bending is equal to the product of F and H .

In a comparison of SIF solutions for semi elliptical surface cracks to experimentally determined fracture data, the Newman Raju solution ranked the highest [13][23]. This solution was able to correlate 95 percent of the data analyzed within ± 10 percent error. The experimental data was determined from a large number of surface crack tension specimens within a brittle epoxy material. The aspect ratios of the crack varied from: $0.3 \leq a/c \leq 0.84$ and thickness ratios $0.15 \leq a/t \leq 1$. These tests were all carried out under uniform tension.

Various other solutions have been proposed to calculate the SIF around a crack front. In a survey of crack tip stress intensity factors, Scott and Thorpe [28], fitted polynomial expressions to published SIF values and recommended different solutions for bending and tension. They recommended the solution of Koterazawa and Minamisaka [28] for bending and the Newman Raju solutions for tension. Both of these solutions were obtained from FEM analysis.

Figure 2.3: Comparison of Newman Raju [23] and Koterazawa and Minamisaka [28] solution for different aspect ratios and depth ratios at the deepest point of a surface crack in bending

Figure 2.3 compares the SIF at the deepest point of an elliptical crack with varying aspect ratio and depth ratios for both the Koterazawa & Minamisaka and Newman & Raju solution. At low depth ratios (a/t), both solutions converge to the same values while increasing the depth ratio to 0.5 the solutions yield very different results. At an aspect ratio of 0.2, and $a/t = 0.5$, the percentage difference is around 20%. This illustrates the large discrepancies found between solutions. McFadyen, Bell and Vosikovsky [20] compared their experimental data to both solutions. They found that the Koterazawa and Minamisaka solution predicted better crack growth starting from defects with low aspect ratio, while the Newman Raju solution was better for starter cracks with higher aspect ratio, although both solutions fit the data well.

Wu [33] compared his solution for an aspect ratio of 0.2 and a thickness ratio of $a/t=0.5$ to the Newman Raju solution. Under pure bending his solutions were 8% lower than those predicted by the Newman Raju solution.

Figure 2.4: SIF Factors around elliptical crack front in Tension from the Newman Raju solution

Figure 2.5: SIF Factors around elliptical crack front in Bending from the Newman Raju solution

The Newman Raju equation has been the most widely used and accepted solution in both bending and tension. For this reason it will be used to compare the SIF around crack fronts for single cracks. Results of the Newman Raju expression are plotted and shown in Figures 2.4-2.5 Under a uniform tension load, the variation of SIF around the crack front varies as a function of both aspect ratio and depth ratio. The deeper the crack, the higher the SIF value will be at any point along the crack front. For low aspect ratio cracks, the deepest point along the crack front has the maximum SIF value, while the free surface is close to the minimum. Increasing the aspect ratio the SIF around the crack front tends to one at the deepest point and has a more uniform stress intensity distribution around the crack front.

Under bending a different pattern is observed. Most noticeable is the effect of crack depth on the SIF value. Increasing the crack depth causes an inflection in the SIF values with deeper cracks having a lower SIF than smaller cracks. The decrease in the SIF values with increasing crack depth is seen particularly at the deepest point of the crack. Generally the highest SIF value is seen at the surface of the crack.

The SIF variation for cracks in bending and tension are not equivalent. At the deepest point in tension, increasing the crack depth increases the SIF, while in bending the SIF is reduced. At all points along the crack front, the crack experiences a more severe stress intensification under tensile loading than under an equivalent bending load.

2.3 Fatigue Crack Growth

Fatigue crack growth is a process of localized damage accumulation [30]. In metallic materials, the crack growth involves intense localized deformation around the crack front, with the extent of deformation related to the applied external load. Stable fatigue crack growth is observed whenever the plastic zone around the crack tip (equation 2.6 and 2.7), is similar in size to a specific micro-structural parameter [30][10]. This stage of crack growth is of the most interest since a majority of the fatigue crack's life is spent in this region.

The Paris-Erdogan [25] growth law describes the crack growth during this stage:

$$\frac{da}{dN} = C (\Delta K)^m \quad (2.13)$$

where ΔK is the cyclic stress intensity factor. C and m are material constants and a and N are the crack depth and number of cycles. The Paris growth law predicts the crack extension for each cycle based on the stress intensity value of that point.

2.4 Crack shape variation during Fatigue

Cracks will naturally grow in any direction to assume an iso-K configuration. The Paris law equation 2.13 predicts that cracks will propagate in the direction of maximum stress intensity. Figure 2.4 and 2.5 illustrates the variation of SIF around a crack front for various crack shapes and depths. It is seen that large differences exist between the SIF variation for tension and bending load. The loading condition has the greatest effect on the aspect ratio of the observed crack [5].

In an investigation of fatigue crack shape development, Dwivedi and Green [5] found that regardless of the initial starter crack size, the cracks would grow towards the equilibrium crack shape, before growth in other directions would occur. In bending, they found that the equilibrium crack shape varied over the thickness of the specimen, while in tension, the aspect ratio varied less over the same growth period. Similar findings have been documented by McFayden [20], Raviachandran [16], Lin [19] and many others. This variation is in agreement with the predictions of crack shape using the Paris law.

Cracks will naturally grow to obtain their equilibrium shape. This shape is determined from the applied load and the geometry of the specimen [19]. Local effects caused by stress concentrators and in the presence of inhomogeneities or complex loading conditions, surface cracks experience constant variation in crack shape. In the presence of deformed layers introduced to improve fatigue life through processes such as shot peening as well as the elongated grains formed through the

material processing, surface cracks are found to develop shallow crack configurations [16]. However larger cracks, with plastic zones greater than the limiting micro-structural unit are relatively insensitive to the local effects of surface conditions and tend to grow towards their equilibrium shape [10].

In general, greater bending loads will cause the crack to grow in the surface direction, while tensile loads cause the crack to propagate in the depth direction. An iso-K configuration can be reached for all cracks, but are generally not maintained due to either effects of the finite boundaries or bending components in the load.

2.5 Fatigue Crack Interaction and Coalescence

Fracture mechanics assumes that many small sub critical flaws are present within the structure at all times. During the lifetime of the structure, adjacent cracks can grow together and form a single larger cracks. The process of the two independent cracks growing together and joining to form one crack is know as interaction and coalescence. Many researchers; Harrington (1995) [8], Leek (1990) [17], Sobojevo (1990) [29], Kishimoto (1989) [15], Twaddle (1988) [31] and O'Donoghue (1985) [24] have all examined the coalescence of twin coplanar surface cracks.

Interaction between the adjacent cracks indicate an influence of each other on their growth rates. Determining the distance between adjacent crack tips before any influence is felt has been widely researched. However once the leading crack tips have begun to interact, the coalescence process begins and the two previously independent cracks begin to form as a single crack. This process is not widely understood and earlier researchers proposed that the two cracks recharaterize instantaneously into the new larger crack. This instantaneous re-characterization would not allow for stable crack growth to occur.

In both the ASME Pressure Vessel Code Section XI [2] and BSI PD6493 [12] crack interaction and coalescence are assumed well before the leading crack tips touch. Re-characterization of the two independent cracks into a single enveloping crack occurs whenever the maximum of either the depth or the crack length are met. The enveloping crack is considered to be semi elliptical having the dimensions

of a rectangle that includes both the coalescing cracks. When the above conditions are not satisfied, the cracks are considered to be independent. All of the above researchers find that both of the design codes are highly unrealistic and yield unnecessary conservative crack growth predictions.

Table 2.1. shows the interaction and coalescence conditions proposed by various researchers and the two design codes. The interaction is for two coplanar symmetric cracks with the following dimensions: $a=10\text{mm}$ and $c=25\text{mm}$. The value t_s is the distance between adjacent crack tips necessary to achieve interaction proposed by the authors. Figure 2.6 defines the variables.

Researcher	Interaction Solution	t_s/c	Coalescence
ASME[2]	$t_s \leq 2 \times (a_1 \text{ or } a_2)$	0.8	Instantaneous
BSI[12]	$t_s \leq 2 \times (c_1 \text{ or } c_2)$	2	Instantaneous
Murakami [22]	$t_s/c = 1$	1.0	NA
Heath & Grant [9]	$t_s/c \geq 0.5$	0.5	NA
Twaddle [31]	No Interaction	0	Instantaneous
Kishimoto [15]	NA	NA	Stable Growth
Soboyejo [29]	$t_s/c \geq 0.5$	0.5	Stable Growth
Kim [14]	$t_s = (\sigma/\sigma_{ys})^2(2cQ)$	0.45	Instantaneous
Harrington [8]	$0.8 \leq t_s \leq 1.0$	0.8–1.0	Stable Growth
Leek [17]	$t_s/c \times t_s/a > 3.38$ $t_s/a \geq 2.49$	1.16	Instantaneous
Frise [6]	No Interaction		Stable Growth

Table 2.1: Interaction and Coalescence conditions proposed by various researchers and design codes. Based on two equal coplanar cracks. $a=10\text{mm}$ $c=25\text{mm}$

It is seen that a wide range of interaction solutions exist for a single geometry. They range from the crack tips touching to a distance equal to the crack length. During coalescence many of the researchers assume that the cracks will instantaneously re-characterize into a larger crack.

The ASME design code [2], assumes that during the interaction and coalescence process, the aspect ratio of the crack remains constant. This assumption holds irrespective of the loading or the initial flaw shape. The cracks are assumed to undergo an instantaneous coalescence process whenever the distance between the leading crack tips is less than or equal to twice the depth of the deeper crack. The depth and length of the resulting crack have the dimensions of a rectangle encompassing the original cracks.

The BSI PD 6493 [12] code suggests that adjacent cracks are assumed to form a single larger crack whenever the distance between the inside tips of the crack is less than twice that of the original surface length of the original cracks. When the bending component exceeds more than twenty percent, the cracks are assumed to re-characterize as an edge crack. For bending components less than twenty percent, the surface length of the crack is held constant and the crack is assumed to grow in the thickness direction until either a semi-circular aspect ratio is obtained, or until the defect penetrates through the thickness.

Heath and Grant [9] examined the interaction of coplanar corner cracks along a hole bore. They used a finite element alternating technique to determine the stress intensity factors around the cracks. Interaction effects along the hole bore increased as the separation distance between the cracks decreased. Secondly, the localized effect of coalescence is found to depend on the crack size (a/t) and weakly on the aspect ratios of the cracks. However as the cracks become more semi-circular (larger a/c) a slight increase in the interaction parameter was seen.

Murakami and Nemat-Nasser [22] used a body force method to examine the stress intensification effects of dissimilar colinear cracks. They determined that the larger and deeper the crack, the greater the intensification at the leading crack tips. Secondly, only the interacting crack tips experienced the intensification. They concluded that the distance over which the stress intensification occurs is limited to the surface length of the smallest crack.

Leek and Howard [17] conducted both an experimental and numerical simulation of the interaction and coalescence of cracks. Crack growth predictions for coplanar and non-coplanar cracks were proposed with a considerable increase in accuracy

relative to the design code predictions. Their interaction and coalescence predictions take into account the inter planar spacing of the parallel cracks. They concluded that neglecting this distance can give unsafe crack growth predictions.

Kim [14] suggested changing the interaction criteria to within the plastic zone of the leading crack tips. Crack interaction was assumed to have occurred whenever a five per cent increase in the crack tip stress intensity factor was obtained. A comparison between his FEM results and the new crack interaction factor showed a much better correlation than either the design codes.

Twaddle and Hancock [31] conducted an experimental and numerical examination of coalescing cracks. They showed that the interaction between adjacent semi-elliptical cracks is small, but that a large magnification in the SIF occurs during the initial stages of coalescence. This decreases rapidly as the coalescence depth approaches the depth of the previous single crack. They noted that the coalescence process occurs so rapidly that it can be ignored in the development of predictive models of fatigue life.

Kishimoto [15] carried out an investigation of the variation in stress intensity during the coalescence process. This was done in conjunction with Sobojevo [29] who carried out an experimental investigation. Kishimoto used the finite element method to determine the stress intensity variation during coalescence. The coalescence process was broken up into a number of stages, with each stage representing an equal number of loading cycles. The crack shape was estimated from the Paris growth law and using the incremental extension the new crack shape was determined. Initially the two coplanar cracks had an aspect ratio (a_o/c_o) of 1.0, a crack depth ratio (a_o/t) of 0.0625 and a crack tip spacing of t_s/c_o of 0.4. The natural growth of the crack was simulated during coalescence since the coalescence depth was determined by the stress intensity solution at that point. They found a sudden increase in the SIF as the crack tips came into contact which was approximately twice the value found at the deepest point. The SIF at the coalescence point then decreases as the crack evolves and forms a smooth convex profile. They attributed the sudden increase in SIF at the beginning of coalescence to the rapid change in crack shape at this point. At the surface point and deepest point, the SIF increased

linearly with the increased crack area. The eventual trend of the stress intensity factors was towards a single value representing an iso-K configuration.

Frise [6] conducted an experimental investigation of the coalescence of coplanar cracks at a notch. It was found that a large percentage of the fatigue life (28% to 87%) is spent in the coalescence stage. At the coalescence plane, increased crack growth was observed indicating a magnified stress intensity factor at this point. Based on the experimental results, an empirical crack shape development model was developed to relate the variation in the aspect ratio to the crack depth during coalescence. The model however does not take into account the crack shape change during coalescence, and secondly the model assumed that the crack does not grow in the surface length direction during coalescence.

Harrington [8] conducted an experimental investigation of crack shape development during coalescence. He observed that interaction between the two leading crack tips is not observed until just prior to the coalescence process. An approximate fifteen percent increase in the stress intensity factor was determined at this point relative to a single crack. During coalescence the SIF at the coalescence point was approximately twice that found at the deepest point of the two coalescing cracks. This value was seen to decrease quickly as the coalescence depth increased.

Zhang [34] wrote a numerical predictive program to determine the lifetime of interacting and coalescing cracks. Using the experimental results of Harrington, Zhang was able to predict the fatigue life to within $\pm 5\%$. The predictive model used a coalescence coefficient k :

$$k = \frac{N_{coal}}{N_{exp}}$$

where N_{coal} and N_{exp} are the computed and experimental coalescence lives respectively. The coalescence coefficient modified the crack growth rate along the coalescence plane to account for the accelerated growth in this region. Since Zhang's predictive program used an experimental result to account for the interaction factor, the life time predictions are dependent on the geometry and loading conditions of the experimental results.

All of the above researchers have shown that the interaction conditions used in

both design codes are very conservative. The research shows that crack interaction for coplanar surface cracks does not start until the leading tips are nearly touching. It has been clearly demonstrated that the coalescence of two cracks is a stable crack growth process and does not occur instantaneously when the two crack tips meet.

Figure 2.6: Elliptical crack growth modeled as continuously increasing ellipses
 a) Before leading crack tips meet (Interaction) b) After tips meet (Coalescence)

Based on the experimental results of Frise and Harrington, crack growth is stable throughout the entire interaction and coalescence process. Crack growth occurs in the surface and depth directions with the magnitude of the crack growth each cycle being proportional to the SIF value at that point. Crack growth can be visualized as a continuous outward expansion of an ellipse (Figure 2.6). Along the surface the distance separating the two ellipses t , shrinks until the two ellipses begin to touch. At this point, any further increase in size, causes the ellipses to overlap. This defines the beginning of coalescence since the two previously independent ellipses are now linked and form a continuous profile. Any further increase in radial direction causes this intersection point to be driven back in the depth direction. The coalescence process commences when the two ellipses intersect and is completed when the coalescence

depth is the same as the semi-minor axis of the ellipse.

To date the effect of the crack geometry during coalescence has not been investigated. Kishimoto [15] modeled the complete coalescence process including growth in both the surface and maximum depth directions, but no knowledge of the effect of the aspect ratio or the crack depth on the coalescence process has been determined. This is the aim of this investigation.

2.6 The Finite Element Method

The finite element method is a versatile method for determining values of interest within a body. In elastic static analyses the FE method attempts to solve the equilibrium between the external and the internal work due to a loading condition.

$$\int_{\Omega} \sigma_{ij} \delta \epsilon_{ij} d\Omega = \int_{\Omega} b_i \delta u_i d\Omega \quad (2.14)$$

Where $\sigma_{ij} \delta \epsilon_{ij}$ and Ω are the local value of stress, virtual strain and the domain of the continuum body. These values constitute the internal strain energy within the body while b_i and δu_i comprise the forces and displacements imposed on the body. In the FE method, the continuum body is broken down into discrete areas in 2D and volumes in 3D. These sections form elements and within an element the displacement and forces are calculated at nodes. Interpolation functions are used to estimate the value of the displacement and forces at any point within the element. Based on the compatibility of the nodes within different elements, a global stiffness matrix can be constructed, which describes the relationship between displacements and forces at all points within the body. Together with the boundary conditions this system of equations can be solved and specific quantities at any point within the continuum can be determined.

In order to correctly estimate the fracture criteria of the body, the stress and strain singularity observed at the crack tip must be modeled correctly. Historically this was achieved through a highly refined mesh which attempted to capture the rapidly varying stress gradient [18]. This method yielded highly inaccurate results

with large variations in the fracture criteria. Various researchers presented solutions to address this problems which varied from incorporating displacement fields within the crack tip elements to adding weighting functions into the standard shape functions of the elements. By far the most popular approach is to displace the mid side nodes of an eight noded quadrilateral element to the quarter point position. This causes the element strain field to naturally exhibited a square root singularity. This method has several advantages. First the method employs existing elements (8 node in 2D and 20 node in 3D) and second the square root singularity can be made to emanate from all rays surrounding the crack tip. In elastic problems the $1/\sqrt{r}$ strain singularity can be modeled by moving all the mid side nodes surrounding the crack tip to the quarter node position and furthermore by collapsing the element into a triangle in 2D and a wedge in 3D, the singularity is experienced throughout the element. In the formation of collapsed elements, 3 nodes occupy the same point in space. To correctly model the $1/\sqrt{r}$ strain singularity, these three nodes should be tied together so that they move as one. By not restraining the degenerated nodes, a combined $1/\sqrt{r}$ and $1/r$ singularity is formed. The latter singularity is particularly useful for power law hardening materials in the study of plasticity [11][1].

The strain singularity in these elements come from moving the mid side nodes to the quarter point and the mapping from the local coordinate system of the element to the global one.

Figure 2.7: Quadrilateral iso-parametric element with mid side nodes moved to quarter points

Consider an 8 node quadrilateral iso-parametric element, figure 2.7 with mid side nodes moved to the quarter point positions. ξ and η define the local coordinate system within the element. x and y are the global coordinate system. The shape function for describing nodes 1,2 and 5 on the bottom of the element can be written as:[1]

$$N_1 = -\frac{1}{2}\xi(1 - \xi) \quad (2.15)$$

$$N_2 = \frac{1}{2}\xi(1 - \xi) \quad (2.16)$$

$$N_5 = (1 - \xi^2) \quad (2.17)$$

In the global coordinate system, $\mathbf{x} = \sum_{i=1}^m N_i \mathbf{x}_i$ and setting $x_1 = 0$, $x_2 = L$ and $x_5 = L/4$ result in:

$$x = \frac{1}{2}\xi(1 - \xi)L + (1 - \xi^2)\frac{L}{4} \quad (2.18)$$

where L is the length of the element between nodes 1 and 2. Solving for ξ gives:

$$\xi = -1 + 2\sqrt{\frac{x}{L}} \quad (2.19)$$

Within an element, the displacements can be interpolated through the shape functions, $\mathbf{u} = \sum_{i=1}^m N_i u_i$ where u_i are the nodal displacements. Substituting, the shape functions (N) and the value for ξ into the expression for the displacements yields:

$$\mathbf{u} = -\frac{1}{2} \left(-1 + 2\sqrt{\frac{x}{L}} \right) \left(2 - 2\sqrt{\frac{x}{L}} \right) u_1 + \left(-1 + 2\sqrt{\frac{x}{L}} \right) \left(2\sqrt{\frac{x}{L}} \right) u_2 + \left(\sqrt{\frac{x}{L}} - \frac{x}{L} \right) u_5 \quad (2.20)$$

Solving for the strain along the x direction leads to the desired $1/\sqrt{r}$ singularity.

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial \xi}{\partial x} \frac{\partial u}{\partial \xi} \quad (2.21)$$

$$\epsilon_x = -\frac{1}{2} \left(\frac{3}{\sqrt{xL}} - \frac{4}{L} \right) u_1 + \frac{1}{2} \left(-\frac{1}{\sqrt{xL}} + \frac{4}{L} \right) u_2 + \left(\frac{2}{\sqrt{xL}} - \frac{4}{L} \right) u_5 \quad (2.22)$$

From equation 2.22, it is obvious that by moving the mid side node to the quarter point the appropriate strain singularity is produced along the entire length of the element [1].

Regular 8 node Isoparametric Element 2D

Collapsed Singular Element 2D

20 Node Isoparametric Element 3D

Collapsed 20 Node Singular Element 3D

Figure 2.8: Iso-parametric Elements, conventional and collapsed singular in 2D and 3D

A similar derivation in three dimensions results in the same strain singularity along all planes normal to the crack front. This relationship holds even when the crack front is curved as in an elliptical crack [3]. Degenerating the cubic isoparametric element, into a wedge causes the singularity to exist within the elements as well as along its boundaries. This further increases the accuracy of the calculated crack strain singularity.

2.7 J Integral

A common method for extracting the stress intensity at the crack tip from a finite element analysis is through the J integral. The J integral is based on the potential energy difference during the extension of a crack. For linear elastic fracture mechanics, the J integral is equivalent to the energy release rate G and related to the SIF through:

$$J = G = \frac{E}{(1 - \nu^2)} K^2 \quad (2.23)$$

The J integral was initially developed by Rice [27] as a path independent contour integral. When x is taken parallel to the direction of crack propagation and the crack is straight, the expression for J becomes:

$$J = \int_{\Gamma} \left(W dy - T \frac{\partial u}{\partial x} ds \right) \quad (2.24)$$

where W is the strain energy density, $W = \int \sigma_{ij} d\epsilon_{ij}$, T is the traction along the contour, u the displacement and s is an incremental distance along the contour Γ . For any closed contour, the value of J is zero, from which it is concluded that the J integral is path independent[7].

The J integral can be physically interpreted in terms of the rate of change of the potential energy with respect to an incremental increase in crack length. With the direction of the crack fixed parallel to the x direction of a Cartesian coordinate system, the potential energy of the cracked body can be written as:

$$\Pi(a) = \int_A W dA - \int_{\Gamma} T_k u_k ds \quad (2.25)$$

where A is the area of body and Γ is it's boundary. The potential energy of the body as a function of crack length (a), is equal to the integral of the strain energy (W) over the area (A), and the traction (T) along it's boundary (Γ). Differentiating $\Pi(a)$ with respect to the crack length (a) yields a solution equivalent to the J integral defined earlier. Therefore

$$J = -\frac{d\Pi}{da} \quad (2.26)$$

which expresses the J integral as the rate of change in potential energy as a function of the crack length for self similar crack growth.

Through the use of Green's theorem and the divergence theorem the contour integral can be written as an equivalent area integral. This is advantageous in the use of the FE method, where the integration over the domain is commonly carried out. Rewriting the contour integral as an equivalent area integral for two dimensions:

Figure 2.9: J Integral represented as an area integral

$$J = \int_A \left(\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W \delta_{1j} \right) \frac{\partial q}{\partial x_j} dA \quad (2.27)$$

The term q is a sufficiently smooth function such that it is unity on the first contour and zero on the second. The function q can be interpreted as a normalized virtual crack extension. This function is merely a mathematical device that enables the generation of an area integral which is better suited to numerical calculations. Although both J integral methods are analytically equivalent, when written in the language of FEM, Banks-Sills and Sherman [4] found that the line and area integrals yielded different results. The difference stems from the Gaussian integration method for higher order elements. The area J integral yields much higher accuracy than the analytically equivalent line integral. Equation 2.27 neglects plastic and thermal strains, as well as any traction on the crack faces.

Figure 2.10: Interpretation of the function q in 3D as a virtual crack advance

For three dimensions, the change in potential energy per unit crack advance is no longer uniform and varies as a function of position along the crack front. Defining the virtual crack extension as a local crack advance results in a local measure of the J integral. The formulation remains the same, with only the expression for q changing. In Figure (2.10), Δa_{max} denotes the maximum extension of the virtual crack advance. The function q interpolates this depth from zero at either extremes of the incremental length along the crack front to a maximum value. Rewriting the closed surface integral around the crack front as volume integral allows simple numerical implementation.

For FEM formulation of the J integral in two dimensions, the area of the integral is partitioned into elements. Most of the terms in the J integral can be computed directly from the output of the FEM program. The area or volume of the integral coincides with the boundaries of the elements. In the ABAQUS FEM code, the J integral is evaluated as an energy domain method. This method is found to be extremely accurate and numerically efficient to calculate. The accuracy of this method over other methods lies in the relatively small strain energy contribution at the crack tip [11]. The domain independence of the contours, allows the J integral to be evaluated away from the crack tip region allowing coarser meshes and is therefore relatively mesh independent.

ABAQUS allows the contour integral to be evaluated over a number of contours emanating from the crack front. The volume or area of the integral coincides with the elements found within this contour. Variations in the J integral estimate found from different contours should be slight, with a strong variation in the estimates indicating a need for mesh refinement.

Chapter 3

Models and Methods

The current investigation is a parametric study to determine the effect of the crack geometry on the stress intensity factor for coplanar crack coalescence. The FE models used in this investigation are all permutations of a surface crack in a finite plate with each model having variations in the crack geometry. In order to create these unique models, a two stage process was designed. The preliminary stage was to create the 'control nodes' which define the crack geometry and the boundaries of the FE model. These control nodes, were then exported in a standard format and read by the FE pre-processor. Using the control nodes the pre-processor was used to fully populate the model. In total, around one thousand elements were used to simulate a quarter of a crack. The FE package used in this investigation was ABAQUS Standard 5-5.

3.1 Crack Growth During Coalescence

Previously fatigue crack growth was described as a continuously increasing ellipse. This process results in stable crack growth throughout the interaction and coalescence of the coplanar cracks. Growth occurs along the entire crack front with the crack advance being proportional to the SIF at that point. This results in a continuously changing crack shape throughout the entire growth process.

The localized region where the intersection of the ellipses takes place is not a well defined point but occurs over a smooth transition. In this study two smoothing

approaches were considered: a parabolic and a circular region which link the two ellipses. Examination of laboratory results [8] revealed that the region between the coalescing crack profiles was parabolic in shape. The parabola would form an arc linking the two ellipses and pass through a minimum value along the plane of intersection. Initial models used a parabolic shape to smooth the transition from the elliptical region of the unaffected crack to the coalescent region. As the model progressed, this smoothing region, became circular in order facilitate the modeling.

In this investigation the coalescence process was not entirely modeled, but rather the growth of the crack along the coalescence plane. The crack geometry in this study was held constant while the crack depth along the coalescence plane increased from the surface to the deepest point. Modeling the coalescence in this way is advantageous since no prior knowledge of the crack shape is needed. Secondly, SIF values are determined for a whole range of aspect ratios and the effect of the crack shape during the coalescence process can be determined.

Figure 3.1: Schematic of the method used to model the growth of the crack along the coalescence plane

Figure 3.1 shows schematically the method used to grow the crack along the coalescence plane. The coalescing region was modeled as an arc extending towards the elliptical crack. As the coalescence depth increased the radius of the arc increased as well. Towards the end of the coalescence process the segment of the arc between the two ellipses would approach a straight line. Modeling the coalescence region in this way ensured that the direction of crack growth at the coalescence plane was normal to the free surface of the plate.

The crack growth along the coalescence plane was broken down into fractional

stages. Crack growth in this direction was assumed to begin when the two leading tips of the adjacent cracks touched. The fractional stage was defined as a_{coal}/a where a_{coal} is the coalescence depth and a is the crack depth.

Figure 3.2: Defining the variables used a) Coalescing cracks b) Single crack

Over the course of the study one of the three independent variables:

- aspect ratio (a/c)
- crack depth (a/t)
- coalescence depth a_{coal}/a

was varied while the other two were kept constant. Varying a single variable allowed a parametric evaluation of the crack growth along the coalescence plane. Table 3.1 shows an outline of the numerical calculations conducted. At each aspect

ratio (a/c) each of the depth ratios (a/t) were carried out at all of the coalescence stages. In total 244 FEM analysis were completed for both tension and pure bending.

a/c	a/t	Completion of Coalescence
0.1	0.05	0.1
0.2	0.10	0.2
0.3	0.15	0.3
0.4	0.25	0.4
0.5	0.50	0.5
0.6		0.6
0.7		0.7
		0.8

Table 3.1: Variation of independent variables

At low aspect ratios, runs were not conducted at the largest crack depth. The models at these aspect ratios and depths produced questionable results and were omitted from the data sets. This is further discussed in the results. Secondly, not all of the coalescence depths were carried out since the rate of change in the SIF decreased as the coalescence depth approached the maximum crack depth.

3.2 Solution of a Single Crack

The results of a single crack were compared directly with the Newman Raju solution. The models were constructed in a similar manner to the coalescence models. A summary of the aspect ratios and crack depths checked is shown in Table 3.2.

Comparison of the SIF values around the single crack were compared to the Newman Raju solution. This provided a convenient check for the validity of the coalescence models since a very similar mesh was used in both cases. For most of the models the percentage difference between the two solutions was less than the error on the Newman Raju solution of $\pm 5\%$. The results of the single crack analysis are presented in the results section.

a/c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	1.0
a/t		0.05		0.05			0.05	
				0.10		0.10		
		0.15						
	0.25	0.25	0.25					
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50

Table 3.2: Summary of single crack models used to verify coalescence models

3.3 Matlab Program used to Generate Nodes around the Crack Front

A matlab script file was written to produce the control nodes necessary for the description of the mesh in ABAQUS. The use of the matlab environment greatly simplified the mesh production since the coordinates of the nodes could be plotted and viewed as a preprocessor. Based on the values of the crack geometry, the crack front was calculated including the smoothing of the elliptical crack around the coalescence plane. This smoothing was either in the form of a circular arc or a parabola.

3.3.1 Circular Smoothing

Modeling the stages of coalescence by smoothing the region between two abutted ellipses with a circular arc is analogous to placing an increasingly larger circle centered on the coalescence plane. By varying the radii of the circle, the coalescence depth increases. The point of contact between the circle and the ellipses was the inflection point needed to model the crack front as a continuous curve.

The inflection point was determined through an iterative procedure by incrementing the distance x away from the plane of symmetry. At each value of x along the crack front a vector normal to the crack front was determined. Using this normal vector, the radial distance to the plane of symmetry was calculated. The distance along the crack front was incremented until the Height (Figure 3.3) minus the radial

Figure 3.3: Schematic representation of the Matlab program to determine the Control Nodes

distance was within 0.5% of the required coalescence depth. When this condition was satisfied, the inflection point between the elliptical and circular regions had been determined.

Along the elliptical portion of the crack, nodes were placed at even increments of ϕ , where ϕ is the elliptical angle defined earlier. A circular to elliptical transformation of the following form was used to move the nodes onto the elliptical crack front.

$$Z = a \cos \phi \quad (3.1)$$

$$X = \sqrt{1 - \frac{Z^2}{a^2}} \cdot c \quad (3.2)$$

X and Z are the Cartesian coordinates of a node along the crack front. Beyond the inflection point, the circular arc was divided evenly by the number of nodes required. Nodes at the inflection point coincided with the edges of elements so that the distance from either element side to the mid-side nodes was equidistant.

A second band of nodes were placed on either side of the crack front. The nodes along the secondary band were positioned such that they were orthogonal to the the

tangent of the crack front. The distance between this secondary band of nodes and the crack front was termed the *web*.

Figure 3.4: Group of collapsed elements which make up the region around the crack front

This distance coincided with the dimensions of the collapsed element region and was kept constant along the crack front. The size of the *web* was limited to the radius of curvature of the circular arc region. The size of the web varied from $5.6 \times 10^{-3} \leq L/a \leq 0.16 \times 10^{-3}$. The ratio L/a is the ratio of the distance from the quarter point node to the crack front, with respect to the depth of the crack. The effect of this ratio on the numerical results is described later.

The outer boundaries of the mesh varied as a function of the crack geometry. The plate width was roughly kept to twice the size of the coalescing crack although this ratio did vary from $1.5 \leq b/2c \leq 5$. This variation was caused by an attempt to hold the outer plate dimensions fixed in the early development of the analysis. The length of the specimen coincided with the width of the plate.

3.3.2 Parabolic Smoothing

In the earliest models, the crack front used a parabolic arc to smooth the region between the two ellipses. The placement of the parabola was similar to the above circular smoothing except that the minima of the parabola was fixed at the coalescence depth. The inflection point was determined as the point where the slope of both tangents were equivalent and shared the same point.

$$\frac{x_1^2}{c^2} + \frac{y_1^2}{a^2} = 1 \quad \text{Position along ellipse} \quad (3.3)$$

$$\frac{dy_1}{dx_1} = \frac{-x_1}{a\sqrt{1 - x_1^2/c^2}} \cdot \frac{a^2}{c^2} \quad \text{Slope at any point} \quad (3.4)$$

$$y_2 = m(x_2 - c)^2 + b \quad \text{Position along parabola} \quad (3.5)$$

$$\frac{dy_2}{dx_2} = 2m(x_2 - c) \quad \text{Slope at any point} \quad (3.6)$$

An iteration process was developed along the crack front which incremented the distance from the coalescence plane. Two sets of equations were developed, one representing the equality of slopes of the crack tangent, the other the equality of the position along the crack front. Both of these equations were rearranged for the value m . By comparing this variable at each increment along the crack front the point of inflection was found whenever the percentage difference reached 0.5%.

Nodes were positioned along the crack front in a similar fashion as in the circular smoothing models. The major difference between the two smoothing functions lay in the node placement beyond the inflection point where the nodes along the crack front were moved from the ellipse to the parabola. In the parabolic smoothing approach, the x coordinate of the nodes were held constant, and the y coordinate was displaced to coincide with the parabola. Consequently, the spacing between the crack tip nodes are unequal in this region and have a concentration towards the coalescence plane. Figure 3.3 shows schematically the parabolic crack front smoothing and the placement of nodes in this region.

In the coalescence study, the circular smoothing was preferred since it facilitated the placement of equally spaced nodes along the crack front. The effect of the node placement along the crack front is dealt with in the results.

3.4 FEM Model

Figure 3.8 shows a typical mesh used in the investigation. It consists a quarter of the two cracks since the coalescence plane is a plane of symmetry. Depending on the geometry of the crack, the model was comprised of either 1094 or 1904 elements. The lower the aspect ratio of the crack, the greater the number of elements needed to effectively model the crack region. Along the longer crack front, 14 elements were used while smaller cracks were generally modeled with only 8 elements. The crack was simulated by using a focused mesh comprised of iso-parametric singular elements. Furthermore the mid side nodes were moved to the $1/4$ position to simulate the strain singularity experienced at this point. Nodes along the crack front were unconstrained and therefore able to move independently. Unconstrained crack tip nodes produce a combined $1/\sqrt{r}$ and $1/r$ strain singularity, while fully constrained nodes along the crack front produce a $1/\sqrt{r}$ singularity. A comparison of these two crack tip constraints resulted in only a minor effect on the SIF values around the crack front. This slight difference is attributed to the mesh independence of the J integral for the determination of the stress intensity factor. In the model, reduced integration 20 node iso-parametric bricks were used throughout.

The boundary conditions used coincide with the quarter symmetry of the model. Along the base of the model, a y plane of symmetry was invoked as a boundary condition. This constrains any movement in the y direction, as well as constrains rotation about the x and z axis. This boundary condition was applied everywhere on the base of the model except below the free surface of the crack. Along the plane of coalescence an x plane of symmetry exists to simulate the other half of the coalescing crack. The x plane of symmetry constrains all motion in the x direction and rotation about that face. Finally the model was 'pinned' to the page by removing all the degrees of freedom of a node along the back edges of the model.

Two modes of loading were applied; pure bending and tension. Tension was achieved by applying a uniform pressure to the top of the model through the *DLOAD option in ABAQUS. Bending was simulated by specifying a user defined variable pressure through the *USER SUBROUTINE option. This allowed a simple method of specifying the appropriate stress distribution for four point bending.

Figure 3.5: Virtual Crack advance q defined around the coalescing crack front.

Note at the coalescence plane, q is directed along the plane

The *CONTOUR INTEGRAL option was used to determine the 3D J integral around the crack. Four contours were specified with the last contour outside of the collapsed element region. The virtual crack advance, (q figure 3.5) was specified as being in a direction that was normal to the crack front. This is calculated based on a parabolic interpolation between the nodes on either side of each node along the crack front. ABAQUS specifies that the direction of the virtual crack advance at the intersection of a free surface or at a plane of symmetry must lie along this plane. By incorporating a smoothing function close to the coalescence plane this requirement was satisfied.

3.5 Post-Processing Results

The output from the *CONTOUR INTEGRAL was written to a file. It consisted of the J values for each contour at each node on the crack front. The mode I stress intensity factor was determined from the last three J integral values. It is well documented that the first contour produces erroneous results, due to the quarter noded elements. When calculating the K_I values plane strain was assumed throughout since the sensitivity of the mesh was felt to be inadequate to feel the rapid transition from plane stress to plane strain which occurs at the free surface. Omitting

the plane stress correction at the surface does not effect the results at either the coalescence plane or at the deepest point since the plane stress is limited to only the free surface. The mode I stress intensity factor was determined from:

$$K_I = \sqrt{\frac{J E}{1 - \nu^2}} \quad (3.7)$$

Where E is the elastic modulus of the material used in the analysis ($E=207$ GPa) J is the average contour integral and a Poisson's ratio of $\nu=0.3$. As a comparison, the nodal displacement method was attempted. This method was found to be too mesh sensitive and not appropriate for the calculation of the SIF in this investigation.

3.6 Comparison of Crack Shape Development

In order to verify the simulation of fatigue crack coalescence a comparison to experimental data was undertaken. An experimental sample FPB6-6, with two coplanar cracks was obtained from Harrington's investigation [8]. This sample was selected since it displayed the best beach-marking and the smallest cusp region. The configuration of the sample is illustrated in figure 3.6 and table 3.3.

Figure 3.6: Dimensions of Experimental Sample FPB6-6

Number of Notches	Starter Notch Depth (mm)	Starter Notch Length $2c$ (mm)	Initial Notch Spacing (mm)	Stress Range $\Delta\sigma$ MPa
2	2.1	18.6	42.1	175

Table 3.3: Geometry of experimental sample FPB6-6 [8]

Examination within the coalescence region revealed seven clearly defined beach-marks. The beach-marks are viewed as lines engraved into the fracture surface and provide an accurate history of the crack front during the fatigue process. They are formed by rapidly cycling the specimen at a reduced stress ratio. The beach marks in the coalescence region were measured in order to accurately simulate the coalescence process. The following measurements were taken with the aid of a Nikon image projector:

- Semi-Major axis of ellipse. Distance measured from center of ellipse to the non-coalescing side of the crack.
- Semi-Minor axis of ellipse. Distance measured from center of ellipse to the deepest point.
- Coalescence Depth. Distance along plane of intersection from free surface to deepest point.

The fractional completion of coalescence for this model was taken as the partial coalescence depth divided by the corresponding crack depth. The length ($2c$) of the crack was determined from the non-coalescing side of the crack. The increase in surface length from the initial starter notch to the beach-mark was measured and based on this value, the crack length was determined. This method assumed that the cracks would grow equivalently in both surface directions. The coalescence plane was determined to coexist with the cusp zone and the distance from the center of the starter notch to the cusp was recorded and termed the critical surface crack length.

Figure 3.7a shows the measured aspect ratio variation during the coalescence process. Superimposed on the graphs are the points used in the FEM model. Over the course of the coalescence the two cracks maintain a near constant aspect ratio with a rapid decrease seen towards the end of the process. This rapid change towards the end of coalescence is thought to be caused by the crack edges approaching the free surface of the specimen, rather than an effect of the coalescence process.

Figure 3.7b shows the crack depth along the coalescence plane throughout the coalescence process. The elliptical overlap crack depth was determined from the

Figure 3.7: a) Aspect ratio variation during Coalescence b) Variation in Coalescence depth

intersection of two similar ellipses with the same geometry as the beach-marks. Crack growth rates at the surface points of the cracks were found to be unequal, especially as the leading crack tips grew together. Since the measurements were taking on the non-coalescing side of the cracks, the accelerated crack growth found on the coalescing side was not taking into account. For this reason the elliptical overlap model predicts a much lower coalescence depth than was measured. Along the coalescing side of the cracks, coalescence had commenced before the measured crack length was equal to the distance between the starter notch and the coalescence plane. In both graphs it is seen that the FEM models attempt to model the crack shape as accurately as possible.

In order to model the crack coalescence, incremental stages of the process were selected between the measured bounds of the coalescence depth. The crack profile used was a blend of an elliptical crack with parabolic smoothing close to the coalescence plane. A parabolic regime was used since it represented the crack shape more accurately in the localized coalescence region. Twelve elements were positioned along the length of the crack front with a concentration of elements towards

the coalescence plane. In total 1265 elements were used to model one quarter of the two coalescing cracks. The model was loaded in pure bending through a user subroutine similar to the one discussed earlier. Results of this comparison are given in section 4.2 Comparison of FEM to Experimental Results.

The coalescence investigation could not use a similar procedure since the relationship between the crack geometry and the coalescence depth is undocumented. Extrapolating the results from the experimental observations would be insufficient to describe the wide variety of crack geometries used. For these reasons this investigation simulated coalescence by increasing the crack depth along the coalescence plane while keeping the crack geometry fixed.

Figure 3.8: FEM Mesh $a/c=0.4$ $a/t=0.5$ $a_{coal}/a=0.3$ and Close up of the Plane of Symmetry

Chapter 4

Results

This chapter presents the results of the coalescence investigation, single crack models and the comparison of the FEM results to the experimentally determined results of Harrington [8]. For brevity only typical plots of the SIF variation around the crack front are presented. The values of the normalized SIF at the coalescence plane and deepest point are included in the appendix.

4.1 Results of the Coalescence Study

In total 244 FEM analyses were conducted. These simulations had aspect ratios varying from 0.1 to 0.7 and depth ratios varying from 0.05 to 0.50. Within each one of the crack sizes the coalescence depth was modeled from 10% to 80% completion. Beyond 80% completion the rate of change of SIF is very small and the nearly coalesced crack behaves as a large crack. Both pure bending and uniform tension loads were carried out for all of the tests.

Within each crack shape group (similar a/c and a/t) the variation of the SIF around the crack front is displayed for the various stages of coalescence. Figure 4.1 is a typical plot of the variation in the stress intensity factor (SIF) around the crack front.

The SIF values are all normalized by $\sigma\sqrt{\pi a}$ to account for the loading and crack depth. The SIF values are given along the entire crack length starting from the surface point ($\phi=0$) to the coalescence plane. Due to the symmetry of the

Figure 4.1: Typical SIF variation around crack front for both coalescence and a similar shaped single crack. Stages of crack growth along coalescence plane ranging from 10% to 80% completion.

problem only the results from one half of the crack are displayed. The other side of the coalescing crack would demonstrate the same results. The elliptical angle ϕ is determined from the depth of the crack along the crack front. For this reason each stage terminates at a unique angle related to the coalescence depth of the intersection between the crack front and the coalescence plane.

The effect of coalescence on the stress intensity factor is located mainly around the coalescence plane. The increase found here is far greater than at any other position along the crack front.

Both the Newman Raju and FEM solutions are presented for the SIF of the single crack. The solution of the single crack provides a reference line for the comparison of the coalescence models. The FEM model of the single crack used a mesh similar to those used in the coalescence study with dimensions equal to one of the coalescing cracks. Comparing the FEM with the accepted solutions confirms the validity of the model, and secondly shows that there exists an increase in the SIF around the

$$\gamma_A = \frac{\text{SIF at A}}{\text{SIF at A'}} \quad \gamma_B = \frac{\text{SIF at B}}{\text{SIF at B'}}$$

Figure 4.2: Defining the interaction factors. SIF at free surface and deepest point normalized by similar positions on a single cracks. SIF of single cracks determined from Newman Raju solution. [23]

entire crack front. The validity of the models is confirmed by the excellent agreement between the FEM result of the single crack and the solution of Newman and Raju.

4.1.1 SIF at the Coalescence Plane

Figures 4.11 - 4.14 show the interaction factor at the coalescence plane for two identical coplanar cracks. Since the solutions of the interaction factor for the lowest aspect ratio cracks are unique, they are dealt with individually.

The interaction factor is defined as the SIF of the point along the coalescing crack normalized by the SIF of the surface point of a single crack. The dimensions of the single crack are equivalent to those of one of the coalescing cracks. Figure 4.2 defines the interaction factor and the dimensions used to compare the two cracks. The results for both the bending and uniform tension loads are presented.

Examination of the plots reveal that all of the factors: coalescence depth; aspect ratio and crack depth have an effect on the interaction factor.

It is seen from all of the plots that the interaction factor decreases as the coa-

lescence depth increases. In all of the plots except at the lowest aspect ratio, the initial stage of coalescence results in the highest interaction factor.

Interaction factors at the initial stages of coalescence for all of the aspect ratios, shows a much wider range of values than at the maximum coalescence depth. The range in interaction factors at the initial stages of coalescence for both tension and bending ranges from 2 to 6 while at completion the range is between 1 to 2. Therefore the main effect of the aspect ratio is to modify the interaction factor at the initial stages of coalescence.

The interaction factor is a normalized function which accounts for the maximum crack depth of the coalescing cracks. The SIF of a crack along the coalescence plane is normalized by the free surface of a single crack. Since deeper single cracks have a larger SIF value, variation in the interaction factor due to the crack size should be taken into account. Therefore any increase in the interaction factor found with varying crack depth can be attributed to crack coalescence. Generally increasing the crack depth causes an increase in the interaction factor.

4.1.2 SIF at Deepest Point

The interaction factor at the deepest point is included to show the relative zone of influence of the coalescence depth. Examining Figure 4.1 it is clearly seen that the effect of coalescence on the SIF at the deepest point ($\phi = 90^\circ$) is less than at the coalescence plane.

The effect of increasing the coalescence depth increases the interaction factor. (Figures 4.15 - 4.18). This is seen to occur under both tension and bending loads. Increasing the crack depth (greater a/t) increases the SIF experienced at the deepest point of the crack. A similar analysis was not conducted at the free surface due to the inaccuracies of the solution at this point.

4.1.3 Coalescing Cracks with Low Aspect Ratios

It is seen from the results that the SIF values for the lowest aspect ratios do not follow the above trends. At an aspect ratio of 0.1, the interaction factor is seen to

decrease with increasing crack depth at both the coalescence plane and the deepest point. As well the interaction factor increases towards the completion of coalescence. Both of these observations are opposite of what was found at all the other aspect ratios.

Figure 4.3: Normalized SIF variation around the crack front of a low aspect ratio crack. $a/c=0.1$, $a/t=0.05$ percentages refer to completion of coalescence

Comparing Figure 4.3 to Figure 4.1 it is seen that a significant difference exist between the two. Only the results for the smallest crack depth ($a/t=0.05$) are presented, since the other depths display similar trends. The most noticeable difference between the two is the relatively small increase in the SIF at the coalescence plane. In Figure 4.3 the increase at the 10% completion for both tension and bending is insignificant in comparison to the similar position along a higher aspect ratio crack. For the low aspect ratio cracks, the highest SIF value always coincides with the

Figure 4.4: Effective overlap of leading crack tips vs Crack interaction Factor

deepest point. The relative deviation from the solution of the single crack is also very small. At the deepest point for the tension load, the SIF for the coalescence solutions are even seen to be below the SIF of the single crack. This is particularly noticeable for larger crack depths. The above observations can be attributed to the relatively large SIF value at the deepest point in relation to all other points along the crack front.

Figure 4.5: Interaction and effective overlap defined

4.2 Comparison of FEM to Experimental Results

Figure 4.4 shows both the results of the experimental work carried out by Harrington and the FEM results of the present study. The interaction factor and effective overlap are both defined in Figure 4.5. Both a parabolic and circular crack front smoothing procedure are shown for the same coalescence interval. A very good correlation exists between the experimental and the FEM work based on the parabolic crack shapes of the same specimen while the circular smoothing shows less agreement. These differences will be discussed later.

In the experimental analysis the SIF values were obtained from the crack growth rates using the Paris law expression:

$$\frac{da}{dN} = C_a (\Delta K_a)^m \quad (4.1)$$

where C_a and m are material and loading constants. For the steel used in the experimental study, $C_a = 2.51 \times 10^{-13}$ and $m=3.96$. These values were obtained through a least square linear regression of all of the experimental data obtained in Harrington's investigation. In both the experimental and numerical analysis the SIF value at A' was estimated as the SIF at the surface point of the crack on the non-coalescing side. It was previously shown that the surface point on the non-coalescing side of the crack feels a relatively small increase in the SIF value during coalescence. Due to the limitations on the Newman Raju solution for the stress intensity factor around a single crack it is impossible to determine a value of the

non-interacting stress intensity factor at this point. However due to the excellent agreement between the FEM and experimental results the estimation of the SIF at the free surface of the crack was felt to be acceptable. In the current investigation, the surface point was analyzed in plane stress, while at all other points plane strain was assumed. A published coalescence interaction result from Sobojeyo et al. [29] is presented as well. Their curve lies slightly above the current work for all points. In their study they assumed a constant aspect ratio during coalescence of $a/c=0.67$. Since they did not provide either their crack or plate geometries, it is impossible to make a full comparison between the two sets of results.

Figure 4.6: Experimental and FEM Results at Coalescence point and at Deepest point of crack

Figure 4.6 shows the normalized stress intensity at both the deepest point and the coalescence point over the coalescence depth for both the experimental and the FEM results. The experimental results were normalized by $\Delta\sigma\sqrt{\pi a}$. Data at five points was taken from a graph of stress intensity versus coalescence depth for the experimental sample FPB6-6. The data points were normalized by $\Delta\sigma\sqrt{\pi a}$ and the crack depth so that they could be compared to the FEM results. The five data points selected correspond to the beach marks found on the same sample. It is seen that a fair agreement exists for both the deepest and coalescence point for all coalescence depths. Along the coalescence plane, the FEM analysis predicts a slightly higher normalized stress intensity than the experimental results. At the deepest point the FEM analysis suggests a greater increase of the SIF as the coalescence process progresses. Both the FEM and experimental results predict a larger SIF at the beginning of coalescence with rapid decay as coalescence proceeds. At the deepest point the SIF values increase eventually meeting the SIF value of the coalescence point. At the intersection point observation of the crack front shows that it is almost straight fronted and the coalescence process has completed.

4.3 Parabolic vs Circular smoothing

In Figure 4.4 and Figure 4.6 differences between modeling of the crack front coalescence area as either a parabola or a segment of a circular arc were shown. In both cases similar crack depths, lengths, and coalescence depths were used. The difference in crack shape is quite minimal with only a very small change caused by the smoothing function close to the coalescence plane. The crack tip nodes for both smoothing functions are presented in figure 4.7.

The difference in interaction factors and SIF seen in Figures 4.4 and 4.6 are attributed to the placement of the nodes along the crack front in the smoothing region. In the circular section the nodes are placed equally along the crack front while the parabolic smoothing displaces the crack nodes vertically from the ellipse to the parabola (See figure 3.3) Due to the differences in the crack nodes placement schemes the parabolic smoothing function causes the crack tip nodes to be clustered

Figure 4.7: Nodes displayed along the crack front for 2 smoothing functions

at the coalescence plane while those produced in the circular scheme are equally spaced.

Figure 4.8 compares the normalized stress intensity around the crack front for both of the smoothing schemes. The circular smoothing model consists of only seventeen nodes along the crack front and 1088 elements, while the parabolic model has twenty-five and 1251 elements in total.

The normalized SIF values are presented alongside standard deviation of the J integrals used for the SIF calculation. Very little difference in SIF values exists between either of the two smoothing functions along the entire length of the crack. However, the standard deviation of the J integrals are significantly different. The standard deviation measures the domain dependence of the J integral values at each position along the crack front. As described earlier, each J integral should be similar, with strong domain dependence indicating a need for mesh refinement. The standard deviation values at the coalescence plane for the parabolic smoothing are much higher for all of the stages presented. This indicates that at the coalescence plane the certainty of the SIF value for the parabolic smoothing is far less than the

Figure 4.8: Comparison of parabolic and circular crack smoothing functions.
Standard Deviation of J integral at each node along the crack front

circular smoothing regime.

Although high domain dependence for the J integral is associated with mesh refinement, the domain dependence is thought to arise from the position of the nodes along the crack front. By clustering the nodes towards the coalescence plane the virtual crack advance calculated by ABAQUS was often incorrect and had to be corrected by slightly modifying the nodal positions along the crack front. This error was overcome with the circular smoothing function and the virtual crack advance was always oriented in the correct direction. It is interesting to note that although the parabolic smoothing exhibited very large domain dependence towards the coalescence plane it fitted the experimental data much better than the circular smoothing.

4.4 Meshing Effects on the J integral

Although the J integral is relatively mesh insensitive the distance of the quarter node to the crack front does have an effect on the J estimates. As explained earlier, the

Figure 4.9: Effect of L/a ratio on the SIF around the crack front. Single cracks with $a/c=0.4$ and 0.7 with $a/t=0.05$ for all cracks. Note L/a ratio is $\times 10^{-3}$

size of the focused mesh around the crack front varied to accommodate the radius of the arc used to smooth the crack front around the coalesce region. As the aspect ratio of the cracks increased the radius of the arc decreased significantly causing the distance L from the quarter node to the crack front to shrink. In this investigation the ratio of L to the crack depth varied from $0.16 \times 10^{-3} \leq L/a \leq 5.6 \times 10^{-3}$. At the lowest extremes of this ratio the solution of the J estimates were very inaccurate and differed greatly from the solution of a single crack. Figure 4.9 shows the variation in the SIF around a single crack front for two aspect ratios and various L/a ratios. The results are for bending, but a similar pattern was seen in tension. It is seen that increasing the L/a ratio causes the FEM solution to converge with the Newman Raju solution.

For L/a ratios less than 0.5×10^{-3} , the accuracy of the SIF values around the crack front are questionable especially at the coalescence point. It is seen that an underestimation of the SIF value is predicted when the L/a ratio is less than 0.5×10^{-3} . Above this value the effect of the L/a ratio is limited to within the error of the Newman Raju solution. Due to the inaccuracies in modeling the single crack, two simulations were not included in the coalescence data. They both correspond to the smallest coalescence depths with aspect ratios of 0.6 and 0.7 and $a/t=0.05$.

4.5 Solutions for Single Cracks

The analysis of single cracks was used to verify the meshes of the coalescence study. Table 3.2 presents all of the single crack models studied. Most of these analyses were within $\pm 5\%$ of the Newman Raju solution for an elliptical surface crack. At lower aspect ratios and deeper cracks the percentage difference from the Newman Raju solution became quite large. Figure 4.10 presents the error from the Newman Raju solution for both tension and bending loads.

Due to the poor performance of the single cracks in bending for aspect ratios less than 0.3, the FEM models were not considered sufficient to effectively model the SIF of a coalescing crack. For this reason the coalescence study omitted the deepest cracks ($a/t=0.5$) for aspect ratios of 0.1 and 0.2.

Although geometrically possible, in reality these crack sizes are physically unrealistic due to their prohibitively large size. With an aspect ratio of 0.1 and a depth of 50 mm, the surface length of the crack would be 500 mm. Shallow cracks of this size would quickly change aspect ratio to approach their equilibrium shape. Based on the results of the SIF variation for shallow cracks the cracks would grow preferentially in the depth direction increasing their aspect ratio before the coalescence process would complete.

Figure 4.10: Absolute error around the crack front for aspect ratios $0.1 \leq a/c \leq 0.9$ for $a/t=0.5$. FEM results compared to Newman Raju.

Figure 4.11: Interaction factor at the coalescence point for $0.1 \leq a/c \leq 0.4$
 Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.12: Interaction factor at the coalescence point for $0.5 \leq a/c \leq 0.7$

Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.13: Interaction factor at the coalescence point for $0.1 \leq a/c \leq 0.4$

Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.14: Interaction factor at the coalescence point for $0.5 \leq a/c \leq 0.7$

Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.15: Interaction factor at the coalescence point for $0.1 \leq a/c \leq 0.4$
 Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.16: Interaction factor at the coalescence point for $0.5 \leq a/c \leq 0.7$

Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.17: Interaction factor at the coalescence point for $0.1 \leq a/c \leq 0.4$
Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Figure 4.18: Interaction factor at the coalescence point for $0.5 \leq a/c \leq 0.7$

Graph showing the variation in interaction factor over the coalescence process for all of the crack depths studied

Chapter 5

Discussion and Conclusions

5.1 Discussion

During the investigation, the following parameters were varied:

- aspect ratio a/c
- depth ratio a/t
- coalescence depth a_{coal}/a

All of the above parameters were found to have an effect on the stress intensity factors during coalescence. The most noticeable are the coalescence depth and the aspect ratio. The results also show that regardless of the crack geometry, coalescence involves stable crack growth. This is in agreement with the experimental works of Harrington [8], Frise [6], Sobojevo [29] and the numerical results of Kishimoto [15]. This is determined since there exists a finite value for all of the stress intensity values as the coalescence depth is increased from 10% to 80% of the maximum crack depth.

5.1.1 Effect of Aspect Ratio

Increasing the semi-circular (increasing a/c) nature of the coalescing cracks leads to higher SIF values at the initial stages of coalescence. A similar observation was noted by Heath and Grant [9] during the interaction of semi-elliptical cracks along a hole bore.

Figure 5.1: Normalized stress intensity at the coalescence plane when the coalescence depth is 10% of the maximum crack depth.

Figure 5.1 shows the interactions factor at the beginning of coalescence. It is seen that increases in the aspect ratio are the dominant factor in coalescence. The SIF values are normalized by the free surface of a similarly sized crack. This normalizing accounts for the loading and the maximum crack depth. Since all of the curves apart from the deepest crack ($a/t=0.50$) are clustered together, the size effect is of secondary importance compared to the aspect ratio. Larger cracks will display a higher SIF value, but not substantially larger than predicted by the free surface of the single crack.

However the SIF of the deepest crack, $a/t=0.50$, has a consistently greater interaction factor than the other sized cracks. This is attributed to both a slight increase

in the SIF due to being a larger crack and secondly to some domain dependence of the solution. Examination of the standard deviation of the J integral values reveals that at $a/t=0.5$, all of the points have slightly greater scatter than the other crack depths.

Figure 5.2: Interaction factors for various aspect ratios for all coalescence depths. Filled symbols indicate the predicted γ_A based on the SIF of the free surface and the deepest point of the fully coalesced crack.

Figure 5.2 shows the variation of interaction factor over the coalescence depths for some of the aspect ratios studied. The filled symbols indicate the predicted interaction factor from the SIF of the fully coalesced crack at the deepest point divided by the SIF of the surface point of the independent crack. These values were obtained from the Newman Raju solution and therefore ignore any crack interaction parameters. The results are displayed for a constant crack depth of $a/t=0.10$, but since coalescence is relatively independent of the maximum crack depth Figure 5.2 is fairly typical.

It is seen that regardless of the aspect ratio, the interaction factor along the coalescence plane tends towards the SIF of the deepest point of the fully coalesced crack. The SIF of the deepest point on the coalesced crack varies with the aspect ratio of the crack. Shallow cracks (low a/c) have a much higher SIF value than a similarly sized semi-circular crack.

At the initial stages of coalescence, when the coalescence depth is small, a relatively large interaction factor is seen which then decays rapidly. This is particularly noticeable for higher aspect ratio cracks. For shallow cracks the interaction factor at the beginning of coalescence is relatively small compared to more semi-circular cracks. Physically the shallow cracks will grow in the depth direction changing their aspect ratios before coalescence will occur. For this reason completion of coalescence for the shallow crack observed in Figure 5.2 is unrealistic since it will grow preferentially in the depth direction before coalescence is completed. Once the coalescing cracks have reached their equilibrium shape, coalescence will continue. As the coalescence depth increases, a sharp decay in the SIF is seen for the coalescing cracks. This decay in the SIF translates into slower crack growth. The reduction in crack growth rates towards the completion of coalescence was also noted by Harrington [8], Sobojevo [29] and Kishimoto [15].

The increase in the SIF found for larger aspect ratios can be related to the radius of the concave crack segment of the re-entrant sector of the coalesced crack. Figure 5.3 shows both the aspect ratio and radius of the concave segment of the crack front. The aspect ratio of the coalescing cracks is defined by the initial non-interacting cracks which was held constant over coalescence. Increasing the aspect ratio of the cracks decreases dramatically the radius needed to link the two elliptical crack segments.

Figure 5.3: Profile of Ellipses with varying aspect ratios and constant depths

The sharp decay in the SIF values seen as the coalescence depth increases can also be attributed to the rapidly changing radius of the concave crack segment needed to link the two elliptical regions. This radius increases rapidly as the coalescence depth increases.

At other points along the crack front it was shown that a slight increase occurs as the crack depth increases along the coalescence plane. However the magnitude of the variation over the coalescence plane was far less than experienced at the coalescence plane. An increase in the maximum crack depth lead to an increase in the SIF at the deepest point. This was seen for all the aspect ratios and load cases apart from $a/c=0.1$ loaded under bending. In this case, increases in the maximum crack depth. lead to a decrease in the SIF at that point. Increasing the maximum crack depth from $a/t=0.05$ to $a/t=0.50$ resulted in an increase of about 30% in the interaction factor. When normalized by the deepest point of a single crack, no difference in the interaction factor was seen for the different load cases.

5.1.2 Effect of Crack Depth

The increase in the interaction factor was seen to increase slightly with an increase in the maximum crack depth. Kishimoto [15] observed a similar trend in their FEM simulation of the coalescence process. They attributed the increase in the SIF to the increase in the crack area. As the coalescence depth increases so does the crack area. An increase in the SIF is felt everywhere along the crack front to account for the increased crack area. In the present investigation, the increase in the SIF at the deepest point was seen to increase with the coalescence depth, particularly for the higher aspect ratio cracks. This may be attributed to both the proximity of the deepest point to the coalescence plane, and the proportionally greater increase in crack area caused by the coalescence of the two cracks.

An assumption used in this FEM analysis is that coplanar cracks will remain coplanar throughout the entire coalescence process. It was shown by Melin [21], that it is energetically unfavorable for coplanar fatigue cracks to remain coplanar when they approach. The leading edges of the two cracks are often seen to grow past each other forming a cusp region. Below the surface, the cracks intersect and the remaining ligament of material between the two crack tips shears. Since the FEM analysis assumes coplanar cracks and does not take into account the formation of a cusp zone, the values of the SIF at the beginning of coalescence cannot be

extrapolated to the meeting of the crack tips at the free surface.

5.2 Fitting Data to Polynomial Expression

A polynomial regression was performed on the data to obtain an expression relating the aspect ratio, coalescence depth and maximum crack depth to the normalized SIF value. A multiple regression analysis was performed on the data to obtain first an expression relating the variation in normalized SIF to the coalescence depth for each crack aspect ratio with a crack depth (a/t) of 0.05. A fourth order polynomial regression was performed for both the applied tension and bending cases. The regression correlation coefficient (R^2) values for the regression were very high approaching 1 for most of the analysis performed. In order to determine the effect of the aspect ratio on the normalized SIF, a second regression was performed on the coefficients of the first regression analysis. The order of the polynomial expression relating the effect of the aspect ratios to the SIF varied from a cubic to a linear expression.

The polynomial expression was of the following form:

$$SIF = \left(A_4 \left(\frac{a_{coal}}{a} \right)^4 + A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0 \right) cf \quad (5.1)$$

where A_4 through A_0 are the coefficients of the polynomial terms, cf is a correction factor and a_{coal}/a is the fractional coalescence depth. SIF is the normalized stress intensity $K/\sigma\sqrt{\pi a}$.

The effect of the aspect ratio on the normalized stress intensity was accounted in the terms A_4 through A_0 . These coefficients were regressed with the expressions for A_i taking the form of:

$$A_i = M(i, j)(a/c) + M(i, j) \quad (5.2)$$

where A_i is any one of the primary regression coefficients $M(i, j)$ are values in the coefficient matrix and a/c is the crack aspect ratio. Equation 5.2 is in the form of a linear expression, but any polynomial expression could be used with a matching

sized coefficient matrix. A coefficient matrix was used for the ease of incorporating the equation into a computer program.

Since the above expressions were based on the values of one depth ratio ($a/t=0.05$) a correction factor (cf) was introduced to predict the increase in SIF for various crack depths. The magnitude of the correction factor varies over the coalescence depth and aspect ratios. The correction factor was determined by comparing the predicted solution obtained from the above regressions to the numerically determined values. Solutions for the correction factor were determined for crack depths of $a/t=0.25$ and 0.50 using a similar approach as above. For crack depths (a/t) other than 0.25 and 0.5 the SIF was determined by using a weighting function between the $0.05 \leq a/t \leq 0.25$ and $0.25 \leq a/t \leq 0.50$.

5.2.1 Tension load Expression

Table 5.1 provides the coefficients for the calculation of the normalized SIF for coalescing cracks with aspect ratios in the range of ($0.2 \leq a/c \leq 0.7$) coalescence depths in the range of ($0.1 \leq a_{coal}/a \leq 0.8$) and for crack depths (a/t) of 0.05 . For all other crack depths a correction factor must be used to account for the increased SIF. The subscripts on the coefficients A_i or B_i refer to the i^{th} power of variable a_{coal}/a and a/c respectively. SIF_5 is the value of the normalized SIF for a maximum crack depth (a/t) of 0.05 .

$$SIF_5 = A_4 \left(\frac{a_{coal}}{a} \right)^4 + A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0$$

The coefficients A_4 through A_0 are determined from:

$$A_i = B_1(a/c) + B_0$$

with the values of B_1 and B_0 from table 5.1 which were determined from a linear regression of the coefficients A_4 through A_0

The correction factor expressions used to predict the SIF at crack depths other than $a/t=0.05$ are presented in tables 5.2 and 5.3 SIF_{25} and SIF_{50} are the normalized SIF values for crack depths of $a/t=0.25$ and $a/t=0.50$ respectively.

	B_1	B_0
A_4	124.6454	0.8867043
A_3	-254.2722	-2.001892
A_2	187.2827	1.460053
A_1	-62.10191	0.4523076
A_0	9.026981	0.3888741

Table 5.1: Coefficient Matrix for $a/t=0.05$ Tension Load

$$SIF_{25} = SIF_5 \times cf_{25}$$

$$cf_{25} = A_4 \left(\frac{a_{coal}}{a} \right)^4 + A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0$$

$$A_i = B_3(a/c)^3 + B_2(a/c)^2 + B_1(a/c)^1 + B_0$$

	B_3	B_2	B_1	B_0
A_4	182.72741	-241.907261	93.3835001	-21.6864199
A_3	-302.300737	400.626149	-153.509115	39.2121456
A_2	163.544912	-217.781323	82.6909271	-24.2540421
A_1	0	0	1.790856	3.616566
A_0	0	0	-0.2758328	0.9331943

Table 5.2: Coefficients for correction factor $a/t=0.25$ Tension Load

Correction factors for the deepest crack were determined in a similar approach as above except a linear expression was used to relate the aspect ratio to the SIF values. The SIF at an a/t of 0.50 was determined from:

$$SIF_{50} = SIF_5 \times cf_{50}$$

$$cf_{50} = A_4 \left(\frac{a_{coal}}{a} \right)^4 + A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0$$

	B_1	B_0
A_4	-29.18432	-1.584218
A_3	48.83834	8.059861
A_2	-26.62085	-9.576393
A_1	5.843394	3.604931
A_0	-1.131629	1.549196

Table 5.3: Coefficients for correction factor $a/t=0.50$ Tension Load

$$A_i = B_1(a/c) + B_0$$

The values of B_1 and B_0 are determined from table 5.3.

In order to determine the SIF at crack depths other than a/t equal to 0.05, 0.25 and 0.50, a weighting function was introduced which scales the correction factor between these values. Two different weighting functions were introduced, one for a/t between 0.05 and 0.25, the other for $a/t \geq 0.5$.

$$0.05 \leq a/t \leq 0.25$$

$$SIF_{predict} = SIF_5 \left(\frac{a/t - 0.05}{0.20} \right) + SIF_{25} \left(1 - \frac{a/t - 0.05}{0.20} \right)$$

$$0.25 \leq a/t \leq 0.50$$

$$SIF_{predict} = SIF_{25} \left(\frac{a/t - 0.25}{0.25} \right) + SIF_{50} \left(1 - \frac{a/t - 0.25}{0.25} \right)$$

5.2.2 Bending Load Expression

A bending load expression was determined using a similar method as above. The values of the coefficients matrix are displayed in table 5.4. Two correction factor terms are incorporated both to account for the maximum crack depths. The correction factor coefficients are displayed in tables 5.5 and 5.6. A similar weighting function as above was used to interpolate between the correction functions. SIF_5 predicts the SIF for a crack depth of $a/t=0.05$ with values for B_i obtained from table 5.4.

$$SIF_5 = A_4 \left(\frac{a_{coal}}{a} \right)^4 + A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0$$

$$A_i = B_3(a/c)^3 + B_2(a/c)^2 + B_1(a/c)^1 + B_0$$

	B ₃	B ₂	B ₁	B ₀
A ₄	118.36695	-325.2129	266.75602	-15.03183
A ₃	-238.74804	646.9255	-539.18756	30.11300
A ₂	172.31615	-456.3588	391.05095	-21.80656
A ₁	-51.32761	134.3493	-123.97211	7.66011
A ₀	4.93296	-14.1763	16.14487	-0.49566

Table 5.4: Coefficient Matrix for a/t=0.05 Bending Load

Correction factor expressions to account for the variation in SIF at a maximum crack depth (a/t) of 0.25 was of the following form with the values of the coefficients in table 5.5. The SIF for coalescing cracks with crack depths a/t=0.25 is determined from SIF₂₅:

$$SIF_{25} = SIF_5 \times cf_{25}$$

$$cf_{25} = A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0$$

$$A_i = B_2(a/c)^2 + B_1(a/c)^1 + B_0$$

where the value of B_i are found in table 5.5.

	B ₂	B ₁	B ₀
A ₃	-8.3987918	7.52451473	-0.200120295
A ₂	13.0321673	-11.9203102	0.34740155
A ₁	-5.67925743	5.43377365	-0.375235616
A ₀	1.03333612	-1.05591478	1.21476742

Table 5.5: Coefficients for correction factor a/t=0.25 Bending Load

Finally the correction factor expression used to determine the SIF at the deepest crack depth is presented. SIF₅₀ predicts the SIF for a crack with an a/t of 0.50. It is determined from:

$$SIF_{50} = SIF_5 \times cf_{50}$$

$$cf_{50} = A_3 \left(\frac{a_{coal}}{a} \right)^3 + A_2 \left(\frac{a_{coal}}{a} \right)^2 + A_1 \left(\frac{a_{coal}}{a} \right)^1 + A_0$$

$$A_i = B_1(a/c) + B_0$$

where the values of B_i are found in table 5.6.

	B_1	B_0
A_3	-0.6716807	2.032941
A_2	0.01510084	-2.794452
A_1	0.9061971	0.418021
A_0	-0.8744588	1.59943

Table 5.6: Coefficients for correction factor $a/t=0.50$ Bending Load

5.2.3 Performance of Polynomial Expression

The curve fitting expressions presented perform exceptionally well. Figure 5.4 illustrates the results of the expression for selected crack depths (a/t) for an aspect ratio (a/c) of 0.4. The symbols indicate the SIF values determined from the FEM analysis while the solution of the curve fitting expression is the solid line. It is seen that for both the tension and bending loads, the expression performs very well. The solution for $a/t=0.15$ was determined from the weighting functions between the SIF values of $a/t=0.05$ and $a/t=0.25$ and results in an excellent agreement. The maximum error found between the calculated SIF and the numerical results is 6% with mean error being less than 5%. For shallow cracks, a/c less than 0.3, the solution for deep cracks ($a/t \geq 0.25$) is unreliable since the FEM analysis was not carried out at these depths.

Figure 5.4: Results from curve fitting expression and FEM analysis

5.3 Conclusions

A parametric study of the coalescence process was conducted. In total 244 FEM analysis with variations in aspect ratio (a/c) crack depth (a/t) and coalescence depth (a_{coal}) were simulated. In all of the simulations, the aspect ratio and crack depth were held constant for varying coalescence depths.

The greatest effect on the coalescence process was the aspect ratio of the interacting cracks. Increasing the aspect ratio caused an increasing stress intensity factor particularly at the commencement of coalescence. As the coalescence depth increases, a rapid decrease in the SIF is observed. Towards the completion of coalescence, the SIF along the coalescence plane tend towards the SIF of the deepest point of the larger coalesced crack.

The region adjacent to the coalescence plane experience the greatest change in the SIF. The effect is generally localized to the coalescence plane. The increase in the SIF at this point is attributed to the local curvature of the crack front.

The slight increase in the SIF experienced by deeper cracks is attributed to the increase in crack area caused by the crack coalescence. The effect of the increase in area was experienced around the entire crack front.

Through a series of curve fits, a fourth order polynomial expression was determined that related the normalized stress intensity value as a function of aspect ratio (a/c), crack depth (a/t) and coalescence depth (a_{coal}/a) for two coplanar similarly sized cracks. The expression determines the normalized SIF at the coalescence plane between the two cracks and is accurate within $\pm 5\%$ of the FEM calculations.

5.4 Recommendations for Further Work

A predictive crack growth simulation during coalescence should be carried out using the stress intensity values obtained in this investigation. The incremental extension of the crack each cycle could be determined from the stress intensity values at each point along the crack front. Based on the new crack shape, the process could be repeated until completion of coalescence or complete failure of the part. Comparison of life prediction to experimental data and to other numerical simulations such as Kishimoto [15] could be made. An analysis of this type would allow an accurate assessment of fatigue life for coplanar fatigue cracks and aid in the fatigue life assessment of in service components.

References

- [1] T.L. Anderson. *Computational Fracture Mechanics*, chapter 11, pages 565–598. CRC Press, 1995.
- [2] ASME. Boiler and pressure vessel code, section xi, 1992.
- [3] Leslie Banks-Sills. Application of the finite element method to linear elastic fracture mechanics. *Applied Mechanics Review*, 44(10):447–461, October 1991.
- [4] Leslie Banks-Sills and Dov Sherman. On the computation of stress intensity factors for three-dimensional geometries by means of the stiffness derivative and j-integral methods. *International Journal of Fracture*. 53:1–20, 1992.
- [5] Priya Dwivedi and David Green. Indentation crack shape evolution during subcritical crack growth. *Journal of the American Ceramic Society*, 78(5):1240–1246, 1995.
- [6] P.R. Frise and R. Bell. Modelling fatigue crack growth and coalescence in notches. *International Journal of Pressure Vessel and Piping*, (51):107–126, 1992.
- [7] E.E. Gdoutos. *Fracture Mechanics An Introduction*, volume 14 of *Solid Mechanics and It's Applications*. Kluwar Academic Publishers, 1993.
- [8] D. S. Harrington. Fatigue crack coalescence and shape development an experimental investigation. Master's thesis, Carleton University, Department of Mechanical and Aerospace Engineering, March 1995.

- [9] B.J. Heath and A.F. Grandt. Stress intensity factors for coalescing and single corner flaws along a hole in a plate. *Engineering Fracture Mechanics*, 19(4):665–673, 1984.
- [10] R.W. Hertzberg. *Deformation and Fracture Mechanics of Engineering Materials*. Wiley, third edition, 1989.
- [11] Karlsson Hibbitt and Sorenson. *Contour Integral Evaluation*. Abaqus Standard Users Guide 5.5. Hibbitt, Karlsson and Sorenson, 1997.
- [12] British Standards Institute. Guidance on some methods for assessing the acceptability of flaws in fusion welded joints, 1991.
- [13] Newman J. C. Jr. *A Review and Assessment of the Stress Intensity Factors for Surface Cracks*, pages 16–37. ASTM STP 687. ASTM, 1979.
- [14] K.H. Kim, D.S Lo. Crack interaction criteria in pressure vessels and pipe. *Journal of Offshore Mechanics and Artic Engineering*, 117:260–264, November 1995. Transactions of the ASME.
- [15] K. Kishimoto, W.O. Soboyejo, R.A. Smith, and J.F. Knott. A numerical investigation of the interaction and coalescence of twin coplanar semi elliptical fatigue cracks. *International Journal of Fatigue*, 11(2):91–96, 1989.
- [16] Ravichandran K.S. and J.M. Larsen. An approach to measure the shapes of three dimensional surface cracks during fatigue crack growth. *Engineering Fracture Mechanics*, 16(8):909–930, 1993.
- [17] T.H. Leek and I.C. Howard. An examination of methods of assessing interacting surface cracks by comparison with experimental data. *International Journal of Pressure Vessel and Piping*, 68:181–201, 1996.
- [18] H. Liebowitz and Jr. E.T. Moyer. Finite elements in fracture mechanics. *Computers and Structures*, 31(1):1–9, 1988.

- [19] X.B. Lin and R.A. Smith. Numerical prediction of fatigue crack growth of a surface defect. *Fatigue and Fracture of Engineering Materials and Structures*. 18(2):247,256, 1995.
- [20] R. McFadyen, N.B. Bell and Vosikovsky O. Fatigue crack growth of semi-elliptical surface cracks. *International Journal of Fatigue*, 12(1):43–50, 1990.
- [21] S. Melin. Why do cracks avoid each other? *International Journal of Fracture*. (23):37–45, 1983.
- [22] S. Murakami, Y. Nemat-Nasser. Interacting dissimilar semi-elliptical surface flaws under tension and bending. *Engineering Fracture Mechanics*. 16(3):373–386, 1982.
- [23] J.C. Newman and I.S. Raju. An empirical stress intensity factor equation for the surface crack. *Engineering Fracture Mechanics*, 15(1-2), 1981.
- [24] T. O'Donoghue, P.E Nishioka and N.S. Atluris. Multiple surface cracks in pressure vessels. *Engineering Fracture Mechanics*, 20(3):545–560, 1984.
- [25] P.C. Paris and F. Erdogan. A critical analysis of crack propagation law. *Transactions of ASME Journal of Basic Engineering*, 85(4), 1963.
- [26] I.S. Raju and J.C. Newman. Stress intensity factors for a wide range of semi-elliptical surface cracks in finite width plates. *Engineering Fracture Mechanics*. 11, 1979.
- [27] J.R. Rice. A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Journal of Applied Mechanics*, 35:378–386, 1968.
- [28] P.M. Scott and T.W. Thorpe. Prediction of semi-elliptical crack shape development during fatigue crack growth. January 1981.
- [29] W.O. Soboyejo and J.F. Knott. Fatigue crack propagation of coplanar semi-elliptical cracks in pure bending. *Engineering Fracture Mechanics*, 37(2):323–340, 1990.

- [30] S. Suresh. *Fatigue of Materials*. Cambridge University Press, 1991.
- [31] J.W. Twaddle, B.R. Hancock. *The development of cracks by defect coalescence*, pages 185–198. Engineering Materials Advisory Service, Sept 1988.
- [32] H.M. Westergaard. Bearing pressures and cracks. *Journal of Applied Mechanics*, 16:A49–A53, 1939.
- [33] R.X. Wu. Stress intensity factors for half-elliptical surface cracks subjected to complex crack face loadings. *Engineering Fracture Mechanics*, 19(3):387–405, 1984.
- [34] H. Zhang. Fatigue crack growth and coalescence study. Master's thesis, Carleton University, Department of Mechanical and Aerospace Engineering, August 1993.

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a/c	a/t	Coalescence Depth a_{coal}/a					
		0.1000	0.2000	0.3000	0.4000	0.6000	0.8000
0.1	0.05	0.7481	0.6799	0.7147	0.7678	0.8905	1.0076
	0.10	0.7408	0.6806	0.7156	0.7678	0.8917	1.0097
	0.15	0.7503	0.6838	0.7309	0.7842	0.9117	1.0363
	0.25	0.7814	0.7330	0.7696	0.8257	0.9652	1.1129
0.1	0.05	1.2972	0.9671	0.9037	0.8994	0.9509	1.0462
	0.10	1.3008	0.9904	0.9246	0.9174	0.9665	1.0468
	0.15	1.3860	1.0604	0.9942	0.9861	1.0385	1.1260
	0.25	1.4717	1.1608	1.0803	1.0696	1.1240	1.2222
0.3	0.05	1.8918	1.2183	1.0751	1.0245	1.0126	1.0567
	0.10	1.9900	1.2807	1.1289	1.0732	1.0600	1.1108
	0.15	1.8953	1.3341	1.1652	1.0968	1.0695	1.1114
	0.25	2.0668	1.4836	1.3005	1.2230	1.1895	1.2368
	0.50	2.7643	2.0150	1.8294	1.7112	1.6493	1.7163
0.4	0.05	2.3612	1.4614	1.2334	1.1329	1.0584	1.0649
	0.10	2.4324	1.5285	1.2903	1.1852	1.1047	1.1148
	0.15	2.3957	1.6205	1.3520	1.2299	1.1317	1.1333
	0.25	2.5785	1.7637	1.4894	1.3524	1.2426	1.2446
	0.50	3.4689	2.4046	2.0698	1.8713	1.7044	1.6975
0.5	0.05	2.8140	1.6732	1.3722	1.2278	1.0938	1.0658
	0.10	2.8851	1.7364	1.4290	1.2771	1.1337	1.1148
	0.15	3.0491	1.8517	1.5243	1.3639	1.2133	1.1907
	0.25	3.0278	2.0135	1.6429	1.4614	1.2840	1.2519
	0.50	3.9469	2.6660	2.1895	1.9548	1.6113	1.3272
0.6	0.05	2.7500	1.8652	1.4990	1.3119	1.1261	1.0664
	0.10	2.7839	1.8567	1.5253	1.3465	1.1588	1.1012
	0.15	3.2699	2.0231	1.6418	1.4397	1.2336	1.1756
	0.25	3.4053	2.2291	1.7917	1.5607	1.3335	1.2592
	0.50	4.2625	2.8577	2.3067	2.0141	1.7273	1.6167
0.7	0.05	3.5000	1.9986	1.5910	1.3795	1.1462	1.0587
	0.10	3.5867	2.0463	1.6343	1.4178	1.1819	1.0939
	0.15	3.6883	2.1579	1.7311	1.4985	1.2496	1.1606
	0.25	3.7268	2.4198	1.9242	1.6563	1.3586	1.2668
	0.50	4.4917	3.0006	2.3968	2.0668	1.7283	1.5795

Table A-1: Coalescence Plane Tension Load $K/\sigma\sqrt{\pi a}$

a/c	a/t	Coalescence Depth a_{coal}/a					
		0.1000	0.2000	0.3000	0.4000	0.6000	0.8000
0.1	0.05	0.7611	0.6784	0.7100	0.7583	0.8699	0.9725
	0.10	0.7528	0.6778	0.7061	0.7500	0.8494	0.9380
	0.15	0.7760	0.6998	0.7235	0.7645	0.8559	0.9363
	0.25	0.8580	0.7653	0.7864	0.8233	0.9033	0.9751
0.2	0.05	1.2985	0.9587	0.8913	0.8832	0.9241	1.0012
	0.10	1.2773	0.9615	0.8883	0.8740	0.9007	0.9525
	0.15	1.3475	1.0131	0.9307	0.9119	0.9293	0.9717
	0.25	1.4003	1.0354	0.9489	0.9217	0.9196	0.9421
0.3	0.05	1.8875	1.2029	1.0545	1.0011	0.9792	1.0099
	0.10	1.9309	1.2374	1.0826	1.0215	0.9882	1.0112
	0.15	1.8122	1.2548	1.0785	1.0042	0.9507	0.9538
	0.25	1.9019	1.3067	1.1170	1.0328	0.9578	0.9397
	0.50	2.5517	1.7408	1.4256	1.2963	1.1488	1.0681
0.4	0.05	2.2454	1.4527	1.2104	1.1065	1.0224	1.0167
	0.10	2.3058	1.4731	1.2307	1.1226	1.0262	1.0117
	0.15	2.2695	1.5207	1.2518	1.1212	1.0039	0.9718
	0.25	2.3347	1.5528	1.2694	1.1342	0.9968	0.9431
	0.50	2.9176	1.9064	1.5085	1.3271	1.1152	0.9920
0.5	0.05	2.6640	1.6923	1.3453	1.1983	1.0577	1.0188
	0.10	2.7213	1.6694	1.3650	1.2118	1.0552	1.0082
	0.15	2.8314	1.7337	1.4121	1.2512	1.0832	1.0273
	0.25	2.7172	1.7699	1.4261	1.2310	1.0364	0.9485
	0.50	3.1876	2.0557	1.6188	1.3776	1.0938	0.9528
0.6	0.05	2.7545	1.8492	1.4706	1.2813	1.0879	1.0185
	0.10	2.7234	1.7819	1.4535	1.2745	1.0771	1.0001
	0.15	3.0935	1.8905	1.5221	1.3204	1.1008	1.0136
	0.25	3.0378	1.9610	1.5520	1.3194	1.0688	0.9547
	0.50	3.3705	2.1764	1.6882	1.4208	1.0856	0.9034
0.7	0.05	3.3885	2.0559	1.5587	1.3470	1.1094	1.0131
	0.10	3.3885	1.9647	1.5551	1.3393	1.0967	0.9919
	0.15	3.4204	2.0142	1.6030	1.3708	1.1119	0.9976
	0.25	3.3117	2.1200	1.6606	1.3988	1.0947	0.9616
	0.50	3.5035	2.2728	1.7472	1.4524	1.0771	0.8730

Table A-2: Coalescence Plane Bending Load $K/\sigma\sqrt{\pi a}$

a/c	a/t	Coalescence Depth a_{coal}/a					
		0.1000	0.2000	0.3000	0.4000	0.6000	0.8000
0.1	0.05	1.1144	1.1176	1.1176	1.1187	1.1183	1.1202
	0.10	1.1060	1.1140	1.1140	1.1142	1.1154	1.1184
	0.15	1.1217	1.1217	1.1369	1.1372	1.1391	1.1443
	0.25	1.1867	1.2126	1.2128	1.2134	1.2173	1.2279
0.2	0.05	1.0651	1.0690	1.0716	1.0719	1.0737	1.0832
	0.10	1.0765	1.0819	1.0852	1.0857	1.0881	1.0944
	0.15	1.1004	1.1086	1.1171	1.1178	1.1214	1.1302
	0.25	1.1784	1.2179	1.2187	1.2203	1.2275	1.2446
0.3	0.05	1.0325	1.0171	1.0228	1.0236	1.0276	1.0373
	0.10	1.0539	1.0558	1.0587	1.0599	1.0657	1.0795
	0.15	1.0474	1.0593	1.0657	1.0671	1.0729	1.0861
	0.25	1.1124	1.1408	1.1502	1.1526	1.1626	1.1841
	0.50	1.3307	1.3499	1.3977	1.4058	1.4369	1.5003
0.4	0.05	0.9543	0.9612	0.9652	0.9687	0.9770	0.9910
	0.10	0.9831	0.9920	1.0007	1.0028	1.0112	1.0292
	0.15	0.9908	1.0034	1.0099	1.0164	1.0252	1.0433
	0.25	1.0446	1.0684	1.0834	1.0869	1.0998	1.1259
	0.50	1.2324	1.2545	1.2904	1.3003	1.3346	1.3999
0.5	0.05	0.9033	0.9079	0.9129	0.9173	0.9263	0.9441
	0.10	0.9261	0.9332	0.9386	0.9419	0.9536	0.9819
	0.15	0.9618	0.9770	0.9847	0.9890	1.0036	1.0321
	0.25	0.9762	1.0056	1.0091	1.0228	1.0389	1.0736
	0.50	1.1339	1.1570	1.1733	1.1931	1.1559	1.0369
0.6	0.10	0.8607	0.8795	0.8867	0.8909	0.9042	0.9287
	0.15	0.8786	0.9158	0.9210	0.9288	0.9466	0.9777
	0.25	0.9138	0.9457	0.9546	0.9665	0.9927	1.0270
	0.50	1.0440	1.0710	1.0870	1.1024	1.1543	1.2196
0.7	0.10	0.8012	0.8243	0.8380	0.8440	0.8591	0.8860
	0.15	0.8329	0.8590	0.8655	0.8766	0.8968	0.9297
	0.25	0.8501	0.8984	0.9078	0.9193	0.9450	0.9855
	0.50	0.9629	0.9947	1.0119	1.0276	1.0807	1.1458

Table A-3: Deepest Point Tension Load $K/\sigma\sqrt{\pi a}$

a/c	a/t	Coalescence Depth a_{coal}/a					
		0.1000	0.2000	0.3000	0.4000	0.6000	0.8000
0.1	0.05	1.1144	1.0601	1.0601	1.0602	1.0607	1.0625
	0.10	1.0078	1.0026	1.0026	1.0028	1.0039	1.0068
	0.15	0.9817	0.9818	0.9773	0.9777	0.9796	0.9845
	0.25	0.9782	0.9708	0.9715	0.9725	0.9768	0.9868
0.2	0.05	1.0094	1.0107	1.0118	1.0121	1.0139	1.0210
	0.10	0.9566	0.9558	0.9555	0.9559	0.9581	0.9637
	0.15	0.9189	0.9173	0.9157	0.9163	0.9194	0.9270
	0.25	0.8672	0.8560	0.8567	0.8581	0.8640	0.8774
0.3	0.05	0.9517	0.9596	0.9623	0.9631	0.9670	0.9763
	0.10	1.0539	0.9311	0.9326	0.9338	0.9393	0.9518
	0.15	0.8656	0.8677	0.8673	0.8685	0.8736	0.8850
	0.25	0.7979	0.7950	0.7938	0.7959	0.8039	0.8204
	0.50	0.7081	0.6935	0.6777	0.6841	0.7065	0.7476
0.4	0.05	0.8679	0.9062	0.9090	0.9114	0.9182	0.9317
	0.10	0.8519	0.8742	0.8782	0.8802	0.8881	0.9044
	0.15	0.8136	0.8218	0.8233	0.8248	0.8325	0.8481
	0.25	0.7385	0.7442	0.7424	0.7454	0.7558	0.7757
	0.50	0.5935	0.5864	0.5774	0.5845	0.6074	0.6468
0.5	0.05	0.8182	0.8549	0.8590	0.8624	0.8710	0.8883
	0.10	0.7983	0.8204	0.8247	0.8278	0.8383	0.8591
	0.15	0.7819	0.7999	0.8046	0.8086	0.8217	0.8465
	0.25	0.6830	0.6979	0.7009	0.7026	0.7156	0.7373
	0.50	0.5133	0.5137	0.5157	0.5195	0.5883	0.6736
0.6	0.10	0.7583	0.7708	0.7763	0.7804	0.7928	0.8153
	0.15	0.7201	0.7466	0.7516	0.7577	0.7731	0.8002
	0.25	0.6336	0.6546	0.6598	0.6640	0.6773	0.7036
	0.50	0.4505	0.4569	0.4607	0.4679	0.4856	0.5232
0.7	0.10	0.6869	0.7209	0.7316	0.7368	0.7509	0.7755
	0.15	0.6705	0.6973	0.7034	0.7115	0.7285	0.7571
	0.25	0.5848	0.6172	0.6233	0.6288	0.6476	0.6740
	0.50	0.3978	0.4099	0.4157	0.4237	0.4419	0.4791

Table A-4: Deepest Point Bending Load $K/\sigma\sqrt{\pi a}$

IMAGE EVALUATION TEST TARGET (QA-3)

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