**Accurate Macromodeling of High-Speed
Interconnects Characterized by
Time-Domain Measurements**

by

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Abstract

During recent years, the intense drive for signal integrity has been at the forefront of rapid and new developments in CAD algorithms focussed on microelectronics. As signal frequencies are approaching the GHz range, the interconnect effects such as delay, crosstalk, ringing and distortion become the dominant factors limiting the overall performance of VLSI systems. At relatively higher frequencies, it may not always be possible to have analytical models for interconnects due to topological and heterogeneity constraints. In such a scenario, interconnects are generally modeled with measurements or full-wave simulation. However, simulation of such networks using conventional simulators such as SPICE suffers due to the mixed frequency/time problem, inaccuracy and CPU inefficiency.

Recently, several model-reduction algorithms, such as asymptotic waveform evaluation (AWE) and complex frequency hopping (CFH) have been introduced to enable rapid simulation of large interconnect networks. In this thesis, a new algorithm is presented for extending model-reduction algorithms for subnetworks characterized by time-domain measurements. In addition, a method is presented for accurate and closed-form computation of moments of measured subnetworks. The new method is based on modeling of time-domain data using associate Hermite basis functions. For this purpose, a new adaptive algorithm is presented for accurately approximating time-domain measurements using modified associate Hermite expansion. The proposed technique is useful for accurate and fast response estimation involving linear subnetworks in VLSI chip packaging, multichip modules, printed circuit boards and in VLSI systems design. Also, it can be easily included in industry standard simulators such as SPICE.

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*Dedicated to my late father Menwal,
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and my brother Mike Harb
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Abbreviations

AH	Associate Hermite.
AWE	Asymptotic Waveform Evaluation.
CAD	Computer Aided Design.
CFH	Complex Frequency Hopping.
EM	Electromagnetic (field).
FFT	Fast Fourier Transform.
IFFT	Inverse Fast Fourier Transform.
LMS	Linear Multi-Step.
MAH	Modified Associate hermite.
MCM	Multi-Chip Module.
MNA	Modified Nodal Admittance.
MMT	Moment Matching Techniques
PCB	Printed Circuit Board.
PVL	Padé via Lanczos.
RC	Resistor-Capacitor (Network).
RLC	Resistor-Inductor-Capacitor (Network).
SPICE	Simulation Program with Integrated Circuits Emphasis.
TEM	Transverse Electromagnetic.
TDR	Time-Domain Reflectometry.

Chapter 1

Introduction

1.1 Motivation

The recent trend in the VLSI industry toward miniature designs, low power consumption, and increased integration of analog circuits with digital blocks has made the signal integrity analysis a challenging task. The quest for high-speed applications has highlighted the previously negligible effects of interconnects Fig. 1.1, such as ringing, signal delay, reflections, distortion and cross-talk. It is predicted that interconnects will be responsible for the majority of signal degradation in high-speed systems [1] - [11].

High-speed interconnect problems are not always handled appropriately by conventional circuit simulators, such as SPICE [12]. If not considered during the design stage, these interconnect effects can cause logic glitches that render a fabricated digital circuit inoperable or they can distort an analog signal such that it fails to meet specifications. Since extra iterations in the design cycle are extremely expensive, accurate prediction of

these effects is a necessity in high-speed designs. Hence, it becomes extremely important for designers to simulate the entire design along with interconnect subcircuits as efficiently as possible while retaining the accuracy of simulation [1] - [11], [13] - [53].

Interconnections can be from various levels of design hierarchy, such as on-chip dies, packaging structures, multichip modules (MCM), printed circuit boards (PCB) and backplanes.

Fig. 1.1. Interconnect effects

Depending on the operating frequency and nature of the structure, these interconnects can be modelled as *lumped components* (RC or RLC) [1], [2], *distributed elements* (frequency independent/dependent RLCG parameters, lossy, coupled) [1], [2], [15], [17], *full-wave models* [13], [18] - [20], or *measured linear* subnetworks [16], [21] - [24].

Recently, characterization and simulation of linear subnetworks represented by measured data in a nonlinear simulation environment has become a topic of extensive research. Important applications of measured data include high-frequency micro-devices and high-speed interconnects. At higher frequencies, it may not always be possible to have analytical models for interconnects due to topological and heterogeneity constraints. For example, in chip carriers, interconnections are usually nonuniform due to high-circuit density, complex shapes and geometrical constraints. Nonuniform transmission lines are also used as filters, couplers, impedance matching blocks, equalizers, resonators, and pulse transformers. Also, the recent boom in wireless communications, in combination with advances in silicon technology has made the on-chip inductors very popular in RF front-end integrated circuits (IC's). In general, the tabulated data can be multi-port scattering (S), admittance (Y), impedance (Z), transmission (T) or hybrid (H) parameters. The behaviour of high-speed interconnects can easily be represented by time-domain terminal measurements as they can effectively and accurately model the subnetwork's behaviour for simulation in the presence of nonlinear terminations. These time-domain data could be obtained by time-domain reflectometry (TDR) measurements [23] or electromagnetic techniques [22], [25].

However, transient simulation of interconnects is associated with two major bottlenecks. First, interconnect networks can sometimes consist of thousands of components which make the simulation CPU expensive. Secondly, at the terminations there generally exist some nonlinear elements such as drivers and receivers, which are described only in

the time-domain, while the high-frequency models such as distributed/full-wave elements are best described in the frequency-domain. Such a simultaneous (*mixed frequency/time*) formulation can not be handled by a traditional ordinary differential equation solver such as SPICE. In addition, measured models do not have a direct representation in the time-domain for efficient simulation purposes. These challenges have lead to intense research in the computer aided design (CAD) area during the recent years based on model-order reduction [2] - [4], [8], [10], [11], [27] - [53].

These techniques are based on the logic that interconnect models tend to add large number of nodes to the circuitry, due to which the nonlinear simulation becomes practically difficult under the given design-time and memory constraints. Hence in order to make interconnect simulation CPU efficient, it was found useful to reduce the order of the linear subnetwork prior to performing nonlinear simulation. The process of finding such a *reduced-order frequency-domain* model characterizing the terminal (*port*) behaviour of the original large linear subnetwork is known as *model-reduction*.

Model-reduction techniques originally introduced in the literature are based on *moment-matching* and *Padé approximation* [23], [27] - [29]. The crux of moment-matching techniques (*MMT*) is the extraction of a small set of dominate poles and residues of a large network. The lower-order approximation is obtained using the Taylor series coefficients (*moments*) of the system and performing Padé approximation on the Laplace-domain transfer function. Such reduced-order models can be used to estimate the transient response of a linear network over a predetermined range of excitation frequencies. Reduced-order frequency-domain models can be easily translated into a set of ordinary differential equations, thus enabling a mixed frequency/time simulation, [30] - [32]. Derivation of differential equations from reduced-order models is referred to as *macromodel synthesis*. Another prime advantage of MMTs is the speed improvement which ranges from 100 to

1000 times faster compared to the SPICE. Recently developed important transient simulation techniques based on moment-matching are *Asymptotic form Evaluation (AWE)* [4], [27], *complex frequency hopping (CFH)* [32] - [34], *Padé via Lanczos (PVL)* [35], [36], *pole analysis via congruent transformation (PACT)* [37], *Arnoldi* [38] - [41] and *passive reduced order interconnect macromodeling algorithm (PRIMA)* [42].

AWE is based on the Taylor expansion and Padé approximation at $s = 0$. However, since Padé approximation is accurate only near the point of expansion, reduced-order models obtained through AWE suffers from accuracy and stability problems. This problem was addressed by the introduction of CFH [33]. CFH ensures the accuracy of an approximation for a desired frequency range using multiple expansion points (hops) and corresponding Padé approximations at the frequencies of interest. CFH can also directly handle distributed interconnect models (without discretization) [15], [33], [34], [43], [44].

The alternative approaches, Padé via Lanczos (PVL) and Arnoldi are based on the Krylov-subspace methods. These techniques overcome the ill-conditioning associated with the direct Padé approximation and are very efficient for simulating large *lumped linear networks* in the presence of nonlinear components. But one of the limitations of these methods is that, in order to handle distributed transmission lines, Telegrapher's equations (partial differential equations) are discretized into a set of ordinary differential equations.

1.2 Contributions

The objective of this thesis is to describe an efficient algorithm, based on moment-matching techniques, for simulation of linear subnetworks characterized by measured data in the time domain. The approach used in this thesis is characterized by an accurate technique for the computation of measured subnetwork moments. This is accomplished by fitting

time-domain data of measured subnetworks using Modified Associate Hermite coefficients, and by applying the Complex Frequency Hopping (CFH). The main contributions made in this thesis are as follows:

1. An efficient optimization algorithm has been proposed to represent the time-domain measured data using Modified Associate Hermite (MAH) coefficients.
2. An accurate closed-form algorithm for computing moments of measured time-domain data has been developed and is based on Modified Associate Hermite formulation.
3. A recursive relationship for computation of system moments for circuits with measured subnetworks has been developed.
4. A new algorithm for simulation of measured subnetworks using Complex Frequency Hopping (CFH) based simulation has been developed.

1.3 Organization of the thesis

This thesis is organized as follows. A brief discussion of high-speed interconnect modeling and simulation difficulties is given in chapter 2. Chapter 3 presents the formulation of circuit equations including measured data. Chapter 4 presents a brief review of model-reduction algorithms and Associate Hermite functions. Chapter 5 introduces the new algorithm for modeling time-domain measured data using Modified Associate Hermite coefficients. Also, numerical results are provided to demonstrate the accuracy and efficiency of the proposed algorithm. Chapter 6 presents the new algorithm for closed-form computation of moments using Modified Associate Hermite functions. It also presents numerical examples to demonstrate the validity of the proposed method. The conclusion and suggestions for further research are presented in chapter 7.

Chapter 2

Interconnect Models

Currently, one of the critical challenges facing the analog circuit simulation is the analysis of interconnects. This simulation requires two main components: an appropriate model to represent the interconnect and a solution algorithm. These components are directly related to the frequency of operation and the complexity of the interconnect. Depending on the desired accuracy and the frequency range, various models may be required, and each model presents different levels of challenges for simulation. The purpose of this chapter is to review some of the existing interconnect modeling and simulation techniques, with an emphasis on measured subnetwork analysis.

2.1 Interconnect Models

Designers are reducing the wiring cross-section to pack the lines together, while at the same time innovating on signals with faster rise times. Reducing the wiring dimensions results in appreciably resistive lines. In addition, all of these interconnections may have nonuniform cross-sections caused by discontinuities such as connectors, vias, wire bonds, flip-chip solder balls, redistribution leads and orthogonal lines. Interconnections can be formed at various levels of the design hierarchy, such as on-chip, packaging structures, multichip modules, printed circuit boards (*PCB's*), and backplane as shown in Fig. 2.1.

Fig. 2.1. Interconnect hierarchy

Fig. 2.2. shows the top and cross sectional view of an example interconnect system. On a broader perspective, interconnection technology can be classified into five categories [5], as shown in Table 2.1, namely, on-chip wiring, thin-film wiring, ceramic carriers, PCB's and shielded cables.

Fig. 2.2. Top and Cross-sectional view of an interconnect system.

<i>Interconnection type</i>	<i>Line Width (μm)</i>	<i>Line Thickness (μm)</i>	<i>Line Resistance (Ohm/cm)</i>	<i>Maximum length (cm)</i>
<i>On-chip</i>	0.5 - 2	0.7 - 2	100 - 1000	0.3 - 1.5
<i>Thin-film</i>	10 - 25	5 - 8	1.25 - 4	20 - 45
<i>Ceramic</i>	75 - 100	16 - 25	0.4 - 0.7	20 - 50
<i>Printed-circuit-board</i>	60 - 100	30 - 50	0.06 - 0.08	40 - 70
<i>Shielded cables</i>	100 - 450	35 - 450	0.0013 - 0.033	150 - 500

TABLE 2.1. Interconnect Technologies

An important criterion used for classifying the interconnects is the electrical length of an interconnect. An interconnect is considered to be “*electrically short*” if at the highest operating frequency of interest, the interconnect length is physically shorter than one-tenth of the wave-length (i.e., $(\text{length of the interconnect} / \lambda) < 0.1$, $\lambda = v/f$). Otherwise the interconnect is referred to as “*electrically long*” [1], [6]. Electrically short interconnects can be represented by lumped models whereas electrically long interconnects need quasi-TEM distributed or full-wave models. For the categories shown in Table 2.1, the wave-length is in the order of 1 - 10cm. The propagated signal rise times are in the range 100 - 1000ps. Hence the line lengths are either comparable to or much longer than the signal wavelengths. Depending on the operating frequency, signal rise times and the nature of the structure, these interconnects can be modeled as lumped (RC or RLC), distributed (frequency independent/dependent RLCG parameters, lossy, coupled), full-wave models or measured linear subnetworks.

2.2 Interconnect Models Based on Analytical formulation

2.2.1 Lumped Models

In the past, interconnect models have been generally restricted to RC tree models. RC trees are RC circuits with capacitors from all nodes to ground, no floating capacitors and no resistors to ground. The signal delay through RC trees was often estimated using a form of the Elmore delay [4], [6], [27] which provided a dominant time constant approximation for monotonic step responses. However, with increasing signal speeds, and in diverse technologies such as Bipolar, BiCMOS or MCMs, RC tree models are no longer adequate. In Bipolar circuits, lumped-element interconnect models may require the use of inductors or grounded resistors, which are not compatible with RC trees. Even for MOS circuits operating at higher-frequencies, the effects of coupling capacitances may need to

be included in the delay estimate. RLC circuits with non-equilibrium initial conditions may have responses which are non-monotonic. This typically results in visible signal ringing in the waveform. A single time constant approximation with Elmore delay is not generally sufficient for such circuits. Usually lumped interconnect circuits extracted from layouts contain a large number of nodes which make the simulation highly CPU expensive. Fig. 2.3. shows a general lumped component model for an interconnect, where L_I is the inductance, R_I is the resistance, G_I is the conductance and C_I is the capacitance of the interconnect.

Fig. 2.3. Lumped transmission line model.

2.2.2 Distributed Transmission Line Models

At relatively higher signal-speeds, the electrical length of interconnects becomes a significant fraction of the wavelength giving rise to signal distorting effects that do not exist at lower frequencies. Consequently, the conventional lumped impedance interconnect models become inadequate and distributed transmission line models based on quasi-TEM (Transverse Electromagnetic) assumptions are needed. The TEM approximation represents the ideal case, where both E and H fields are perpendicular to the direction of propa-

gation, and is valid under the condition that the line cross-section is much smaller than the wavelength. However, in practical wiring configurations, the structure has all inhomogeneities mentioned previously. Such effects give rise to E and H fields in the direction of propagation. If the line cross-section remains a small portion of the wavelength for the desired frequency range, the solution to Maxwell's equations are given by the so called quasi-TEM modes and are characterized by distributed R_l , L_l , C_l , and G_l per unit length parameters as shown in Fig. 2.4..

The basic quasi-TEM model is the simple delay-line or lossless line such that ($R_l=G_l=0$). In this case a signal travelling along a line has the same amplitude at all the points, but is shifted in time with a propagation delay per unit length (τ) given by

$$\tau = \sqrt{L_1 C_1} = \sqrt{\epsilon \mu} \quad (2.1)$$

where ϵ, μ are the dielectric permittivity and permeability of the medium, respectively. The characteristic impedance for the lossless case is given by

$$Z = \sqrt{L_1 / C_1} \quad (2.2)$$

More complicated models include per-unit-length loss (either in the travelling direction or due to dielectric substrate loss) or coupling between adjacent transmission lines, where the coupling might be resistive, inductive, capacitive or a combination of these elements [6]. In such cases, the propagation constants (Υ) and the characteristic impedances (Z) are given by

$$\Upsilon = \sqrt{(R_1 + j\omega L_1)(G_1 + j\omega C_1)} \quad (2.3)$$

$$Z = \sqrt{\frac{R_1 + j\omega L_1}{G_1 + j\omega C_1}} \quad (2.4)$$

Fig. 2.4. Distributed transmission line model.

The models may even require frequency dependent parameters (for addressing high-frequency effects such as skin effect, edge effect and proximity effect) [17] or nonuniform per-unit-length parameters to accurately model different geometries [4], [45].

In addition, as operating frequencies increase, electrically long interconnects function as efficient antennas and pick up emissions from nearby electronic systems. This aspect of EMI (susceptibility of interconnects to emission), is therefore becoming an early stage of the design can result in reduced costs, improved product reliability and reduced time to market [7], [46].

2.3 Interconnect Models Based on Measured/Simulated Data

2.3.1 Full-Wave Models

At further sub-nanosecond rise times, the line cross-section or the nonuniformities become a significant fraction of the wave length and, under these conditions, the field components in the direction of propagation can no longer be neglected (Fig. 2.5.). Consequently, even the distributed models based on quasi-TEM approximations become inaccurate in describing the interconnect performance [13], [43]. In such situations, full-wave models, which take into account all possible field components and satisfies all boundary conditions, are required in order to give an accurate estimation of the high-frequency effects. Usually, full-wave solvers yield simulated data in terms of S , Y , Z or T parameters

Fig. 2.5. Cross-sectional view of a multiconductor dispersive system

2.3.2 Models Based on Measured Subnetworks

In practice, it may not be possible to obtain accurate analytical models for interconnects because of the geometric heterogeneity and associated discontinuities. To handle such situations, modeling techniques based on measured data have been proposed in the literature [16], [21], [22], [25]. In general, the behaviour of high-speed interconnects can easily be represented by measured frequency-dependent scattering parameters or time-domain terminal measurements. Time-domain data could be obtained by time-domain reflectometry (TDR) measurements [23] or electromagnetic techniques [22], [25]. The tabulated data, regardless of how they are obtained, will be called *measured data* and will be the focus of this thesis.

There has been a limited number of attempts in the literature to simulate linear subnetworks with measured data. In [47] a technique is described based on the rational function approximation of the measured data and recursive convolution to analyze transmission lines characterized by tabulated S-parameters. A method based on the macromodeling of each measured subnetwork and recursive convolution for SPICE simulation has been described in [23]. Another method was described in [28] for including measured data into moment-matching simulation. In this method, the circuit, excluding the measured subnetworks, is analyzed using moment-matching techniques; the measured and component based subcircuits are then combined as one set of frequency-domain stencils and solved in the frequency-domain. The time-domain impulse response is obtained using inverse fast Fourier transform (IFFT). An alternative method, described in [48], computes the measured subnetwork moments based on numerical differentiation in frequency-domain. However, due to numerical differentiation, the accuracy of higher-order moments may not be

guaranteed in this case. In [16], [49] a technique based on numerical integration in the time-domain is proposed for computation of measured subnetwork moments. However, this technique may suffer in the presence of long, non-decaying transient data.

2.4 Definition of the Highest Frequency f_{MAX}

One of the important parameter in interconnect modeling is the highest frequency of interest. Electrical length of the interconnect represents the most important principle used for interconnect analysis. If the interconnect length is physically shorter than approximately one tenth of the length. i.e. $l/\lambda = 0.1$ and $\lambda = v/f$ (as discussed in section 2.2.1), then the interconnect is called electrically short. In general, the definition of the highest operating frequency of interest is where the interconnect is electrically short, which corresponds to the minimum length λ .

For most high-frequency applications, the desired highest operating frequency of interest is determined by the rise/fall time of the propagation signal. For example, the energy spectrum of the trapezoidal pulse is spread over an infinite frequency range, therefore, most of the signal energy is concentrated beside the low-frequency region and drops rapidly with the increase in frequency. Therefore, ignoring the high-frequency components of the spectrum above f_{MAX} will not significantly affect the overall signal response. Consequently, for all practical purposes, the width of the spectrum can be assumed to be bounded. The practical relationship between the desired maximum frequency f_{MAX} and the rise/fall time of the signal can be stated as [50] - [53]

$$f_{MAX} \approx 0.35/t_r \quad (2.5)$$

This indicates that, for a rise time of 0.2 ns, the maximum frequency of interest (f_{MAX}) is around 1.5 GHz or the minimum length is approximately 20 cm.

2.5 Interconnect Simulation Difficulties

Transient simulation of interconnects is associated with two major bottlenecks. First, interconnect networks can sometimes consist of thousands of components which make the simulation CPU expensive. Secondly, at the terminations there generally exist some non-linear elements such as drivers and receivers, which are generally described in the time-domain, while the high-frequency models such as distributed/full-wave elements are best described in the frequency-domain. Such a simultaneous (*mixed frequency/time*) formulation can not be handled by a traditional ordinary differential equation solver such as SPICE. In addition, measured models do not have a direct representation in the time-domain for efficient simulation purposes. In the rest of this thesis, a new algorithm based on model reduction is presented for efficient simulation of interconnect subnetworks characterized by time-domain data. The next chapter presents the unified formulation of circuit equations including measured data.

Chapter 3

Formulation of Circuit Equations Including Measured Data

In the previous chapter we looked at various interconnect modeling issues, which suggested that there is a need to develop a unified scheme for formulation of circuit equations for the application of model-reduction algorithms. This chapter presents unified formulation of circuit equations and a methodology to extend the modified nodal analysis (MNA) to include measured subnetworks.

3.1 Background on Circuit Simulation

Prior to introducing interconnect simulation algorithms it would be useful to have a glimpse at the basic circuit simulation techniques. Conventional circuit simulators are based on the simultaneous solution of linear equations which are obtained by applying Kirchhoff's current law (KCL) to each node in the network. Either for frequency or time-domain analysis, the first step is to set up the *Modified Nodal Admittance (MNA)* matrix

[54]. For example, consider the simple circuit shown in Fig. 3.1, where its MNA equations are

$$\left(\begin{bmatrix} G_1 & 0 & 1 \\ 0 & G_2 & -1 \\ 1 & -1 & 0 \end{bmatrix} + s \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & -L_1 \end{bmatrix} \right) \mathbf{X}(s) = \begin{bmatrix} J(s) \\ 0 \\ 0 \end{bmatrix} \quad (3.1)$$

Fig. 3.1. Simple linear circuit

The above equation representing a simple two node circuit, has the same form as any other MNA matrix representing a large linear lumped network. Hence, in general, MNA equations for lumped linear networks can be written as

$$(\mathbf{G} + s\mathbf{C})\mathbf{X}(s) = \mathbf{E}(s) \quad (3.2)$$

or, can be written in time domain as

$$\mathbf{G}\mathbf{x}(t) + \mathbf{C}\dot{\mathbf{x}}(t) = \mathbf{e}(t) \quad (3.3)$$

where

- $\mathbf{x}(t) \in \mathcal{R}^n$ and $\mathbf{X}(s) \in \mathcal{R}^n$ represent the vector of node voltage wave-forms in the time and frequency domain, respectively.
- $\mathbf{C} \in \mathcal{R}^{n \times n}$ and $\mathbf{G} \in \mathcal{R}^{n \times n}$ are constant matrices describing the lumped memory-less and memory elements of the network, respectively.
- $\mathbf{E}(s) \in \mathcal{R}^n$ and $\mathbf{e}(t) \in \mathcal{R}^n$ represent a constant vector with entries determined by the independent voltage and current sources in frequency and time domains, respectively.
- n represents the total number of variables in the MNA formulation.

3.2 *Formulation of Circuit Equations Including Measured Subnetworks*

3.2.1 Formulation of the Measured Subnetwork

The terminal time-domain characteristic equations for the measured subnetwork can be written in the form

$$\mathbf{i}_k(t) = \mathbf{y}_k(t) * \mathbf{v}_k(t) \quad (3.4)$$

where $\mathbf{i}_k(t)$, $\mathbf{v}_k(t)$ represent the time-domain terminal current and voltage vectors, $\mathbf{y}_k(t)$ represents the time-domain admittance matrix of the measured subnetwork k . ‘*’ represents the convolution operation. For example, in the case of a two port network, (3.4) will look like,

$$\begin{aligned}
i_1(t) &= y_{11}(t)*v_1(t) + y_{12}(t)*v_2(t) \\
i_2(t) &= y_{21}(t)*v_1(t) + y_{22}(t)*v_2(t)
\end{aligned} \tag{3.5}$$

where $y_{11}(t), y_{12}(t), y_{21}(t), y_{22}(t)$ (the inverse Fourier transform of $Y_{11}(s), Y_{12}(s), Y_{21}(s), Y_{22}(s)$, respectively), are the transient admittance parameters and can be obtained as follows:

$$y_{11}(t) = \frac{\{i_1(t)\}}{\{v_1(t)\}} \bigg|_{v_2(t)=0} \tag{3.6}$$

where $i_1(t), v_1(t)$ are obtained from measurements and $\frac{\{\text{---}\}}{\{\text{---}\}}$ denotes the deconvolution operation [55]. Similar equations can be obtained for the remaining transient admittance parameters.

Consider the linear subnetwork represented by (3.4). The frequency-domain equations of the measured subnetwork k can be written in the form

$$I_k(s) = Y_k(s)V_k(s) \tag{3.7}$$

where V_k and I_k represent the Laplace-domain terminal voltages and currents for subnetwork k , respectively and Y_k represents the admittance matrix of the measured subnetwork.

3.2.2 Formulation of the Linear Subnetwork Including Measured Subnetworks

Consider a linear multiport element π with n_π ports composed of lumped elements and N_e measured subnetworks (Fig. 3.2). Let the number of ports associated with the k^{th} measured device be denoted by n_k . The MNA (modified nodal analysis) equations for the entire linear subnetwork can be written as

$$C\dot{x} + Gx + \sum_{\phi=0}^{N_e} D_k i_k(t) = Bv_\pi; \quad i_\pi = B^T x \quad (3.8)$$

Fig. 3.2. Linear subnetwork ' π ' with N_e measured subnetworks

where,

- $\mathbf{x} \in \mathfrak{R}^n$ is the vector of node voltage waveforms appended by inductor and port currents. ‘ n ’ is the total number of variables in the MNA formulation,
- $\mathbf{C} \in \mathfrak{R}^{n \times n}$ and $\mathbf{G} \in \mathfrak{R}^{n \times n}$ are constant matrices describing the lumped memory and memoryless elements of the network, respectively,
- i_π and v_π denote the port currents and voltages of the linear subnetwork, respectively,
- $\mathbf{B} = [b_{i,j} \in \{0, 1\}]$ where $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n_\pi\}$, is a selector matrix that maps the port voltages into the node space $\mathfrak{R}^n$ of the network and,
- $\mathbf{D}_k = [d_{i,j} \in \{0, 1\}]$ where $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n_k\}$ with a maximum of one non-zero in each row or column, is a selector matrix that maps $\mathbf{i}_k(t) \in \mathfrak{R}^{n_k}$, the vector of terminal currents entering the measured subnetwork k , into the node space $\mathfrak{R}^n$ of the network. The terminal voltages/currents of the measured subnetwork k are related by y-parameters, given by (3.4).

Next, the frequency-domain equations (3.7) of the k^{th} measured subnetwork can be represented as

$$\mathbf{I}_k(s) = \mathbf{Y}_k(s) \mathbf{V}_k(s) = \mathbf{Y}_k(s) \cdot \mathbf{D}_k^T \mathbf{X}(s) \quad (3.9)$$

Using (3.9), the Laplace transform of (3.8) assuming zero initial conditions can be written as

$$\Psi_{\pi}(s)X(s) = \mathbf{B}V_{\pi}; \quad \mathbf{I}_{\pi} = \mathbf{B}^T \mathbf{X}(s) \quad (3.10)$$

where

$$\Psi_{\pi}(s) = \mathbf{G} + s\mathbf{C} + \mathbf{F}(s); \quad \mathbf{F}(s) = \sum_{k=1}^{N_e} \mathbf{D}_k \mathbf{Y}_k(s) \cdot \mathbf{D}_k^T \quad (3.11)$$

Next, an example is given to illustrate the above formulation of MNA equations.

3.3 Example

Consider the circuit shown in Fig. 3.3 which can be described using (3.11) as:

Fig. 3.3. Example circuit for formulation of network equations

$$\mathbf{G} = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & -1 \\ -G_1 & G_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & G_2 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

and

$$\mathbf{X}(s) = \begin{bmatrix} V_1 & V_2 & V_3 & V_4 & I_L & I_E \end{bmatrix}^T \quad (3.12)$$

The measured network is characterized by

$$\mathbf{Y}_1(s) = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix}; \quad \mathbf{Y}_1(s) \cdot \begin{bmatrix} V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix} \quad (3.13)$$

The admittance parameters are mapped into the network using the selector matrix

$$\mathbf{D}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \quad (3.14)$$

to form

$$\mathbf{F}(s) = \mathbf{D}_1 \mathbf{Y}_1(s) \mathbf{D}_1^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{11}(s) & Y_{12}(s) & 0 & 0 & 0 \\ 0 & Y_{21}(s) & Y_{22}(s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.15)$$

3.4 Measurement Set Up and Generalized Stencil for Measured Subnetworks

In this section, a generalized stencil for measured subnetworks is derived. Such a stencil would be useful while extending the model-reduction based analysis to measured subnetworks. Consider a measured subnetwork k with N_k terminals (Fig. 3.4), characterized by m sets of measurements. Let the i^{th} set of excitation (e.g. voltage sources) and the corresponding measurements at the terminals of subnetwork k (e.g. voltage) be represented by column vectors $\mathbf{J}^{(i)}$ and $\mathbf{V}^{(i)}$, respectively, where

$$\mathbf{J}^{(i)} = \begin{bmatrix} J_1^{(i)} \\ \dots \\ J_{N_k}^{(i)} \end{bmatrix} ; \quad \mathbf{V}^{(i)} = \begin{bmatrix} V_1^{(i)} \\ \dots \\ V_{N_k}^{(i)} \end{bmatrix} \quad (3.16)$$

Let the corresponding terminating impedances be represented by an impedance matrix $\mathbf{Z}_L^{(i)}$ as

$$\mathbf{Z}_L^{(i)} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N_k} \\ \dots & \dots & \dots & \dots \\ Z_{N_k1} & Z_{N_k2} & \dots & Z_{N_kN_k} \end{bmatrix} \quad (3.17)$$

Each set of measurement can have a different set of excitation sources and terminating impedances. For the purpose of illustration, consider the circuit shown in Fig. 3.4(b). Here $\mathbf{Z}_L^{(i)}$ is represented by a diagonal matrix given by $\mathbf{Z}_L^{(i)} = \text{diag}[Z_1 \dots Z_{N_k}]$ and the excitation vector is given by $\mathbf{J}^{(i)} = [V_s \dots 0 \ 0 \ 0]^t$.

Fig. 3.4. Illustration of a general measurement setup

With these m sets of measurements, the subnetwork k can be characterized in frequency-domain as

$$Y_k \begin{bmatrix} V^{(1)} & \dots & V^{(m)} \end{bmatrix}_{N_k \times m} = \begin{bmatrix} Z_L^{-1(1)} (J^{(1)} - V^{(1)}) & \dots & Z_L^{-1(m)} (J^{(m)} - V^{(m)}) \end{bmatrix}_{N_k \times m} \quad (3.18)$$

or, (3.18) can be concisely written as

$$Y_k(s) \tilde{V}_k(s) = \tilde{I}_k(s) \quad (3.19)$$

where Y_k is the unknown $N_k \times N_k$ y-parameter matrix of the measured subnetwork k , represented by

$$Y_k = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N_k} \\ \dots & \dots & \dots & \dots \\ Y_{N_k1} & Y_{N_k2} & \dots & Y_{N_kN_k} \end{bmatrix} \quad (3.20)$$

Next, we need to evaluate the unknown y-parameters from the given measurements (so as to stencil the measured subnetwork k into MNA as represented by (3.7)). These values can be obtained as follows. Let the elements of the matrix Y_k be represented by a column vector Y_{v_k} of length N_k^2 as

$$Y_{v_k} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N_k} & Y_{21} & Y_{22} & \dots & Y_{2N_k} & \dots\dots\dots & Y_{N_k1} & Y_{N_k2} & \dots & Y_{N_kN_k} \end{bmatrix}^t \quad (3.21)$$

Next, taking the transpose of (3.19) we have

$$\tilde{V}_k^t Y_k^t = \tilde{I}_k^t \quad (3.22)$$

Expressing Y_k in terms of its vector notation Y_{v_k} , and with little mathematical manipulations on both RHS and LHS, (3.22) can be expressed as

$$\begin{bmatrix} v^{(1)t} & 0 & \dots & \dots & \dots & 0 \\ 0 & v^{(1)t} & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \dots & 0 & v^{(1)t} \\ v^{(2)t} & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & v^{(2)t} & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \dots & 0 & v^{(2)t} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ v^{(m)t} & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & v^{(m)t} & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \dots & 0 & v^{(m)t} \end{bmatrix}_{(N_k \times m) \times N_k^2} \begin{bmatrix} Y_{11} \\ Y_{12} \\ \dots \\ Y_{1N_k} \\ Y_{21} \\ Y_{22} \\ \dots \\ Y_{2N_k} \\ \dots \\ \dots \\ Y_{N_k1} \\ Y_{N_k2} \\ \dots \\ Y_{N_kN_k} \end{bmatrix}_{N_k^2 \times 1} = \begin{bmatrix} Z_L^{-1(1)}(J^{(1)} - v^{(1)}) \\ Z_L^{-1(2)}(J^{(2)} - v^{(2)}) \\ \dots \\ Z_L^{-1(m)}(J^{(m)} - v^{(m)}) \end{bmatrix}_{(N_k \times m) \times 1} \quad (3.23)$$

or

$$W_k Y_{v_k} = B_k \quad (3.24)$$

where

$$W_k = \begin{bmatrix} W^{(1)} \\ W^{(2)} \\ \dots \\ W^{(m)} \end{bmatrix}_{(N_k \times m) \times N_k^2} ; \quad W^{(i)} = \begin{bmatrix} V^{(i)'} & & \\ & V^{(i)'} & \\ & & \dots \\ & & & V^{(i)'} \end{bmatrix}_{N_k \times N_k^2} \quad (3.25)$$

and

$$B_k = \begin{bmatrix} B^{(1)} \\ B^{(2)} \\ \dots \\ B^{(m)} \end{bmatrix}_{(N_k \times m) \times 1} ; \quad B^{(i)} = Z_L^{-1(i)} (J^{(i)} - V^{(i)}) \quad (3.26)$$

Example: Formulation of Measured Data

Consider a measured subnetwork with two terminals. Let the subnetwork be characterized using three sets of measurements in the set up shown in Fig. 3.5. These measurements can be used to obtain the stencil of the subnetwork using (3.24) as follows:

$$J^{(i)} = \begin{bmatrix} V_s^{(i)} \\ 0 \end{bmatrix}; \quad V^{(i)} = \begin{bmatrix} V_1^{(i)} \\ V_2^{(i)} \end{bmatrix}; \quad Z_L^{(i)} = \begin{bmatrix} R_1^{(i)} & 0 \\ 0 & R_2^{(i)} \end{bmatrix} \quad \text{where } i = 1, 2, 3$$

$$\mathbf{W}^{(i)} = \begin{bmatrix} \mathbf{V}^{(i)t} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^{(i)t} \end{bmatrix} = \begin{bmatrix} V_1^{(i)} & V_2^{(i)} & 0 & 0 \\ 0 & 0 & V_1^{(i)} & V_2^{(i)} \end{bmatrix}$$

$$\mathbf{B}^{(i)} = \mathbf{Z}_L^{-1(i)} (\mathbf{J}^{(i)} - \mathbf{V}^{(i)}) = \begin{bmatrix} -(V_1^{(i)} - V_s^{(i)})/R_1^{(i)} \\ -V_2^{(i)}/R_2^{(i)} \end{bmatrix}$$

$$\mathbf{W}_k = \begin{bmatrix} \mathbf{W}^{(1)} \\ \mathbf{W}^{(2)} \\ \mathbf{W}^{(3)} \end{bmatrix}; \quad \mathbf{Y}_{v_k} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \end{bmatrix}; \quad \mathbf{B}_k = \begin{bmatrix} \mathbf{B}^{(1)} \\ \mathbf{B}^{(2)} \\ \mathbf{B}^{(3)} \end{bmatrix}$$

Fig. 3.5. Circuit for illustration of formulation of measured subnetworks

It is to be noted that in general N_k^2 variables are needed to determine the behavior of a N_k terminal network. However, if some prior information about the topology of the measured subnetwork (such as network is passive or symmetric etc.) is available, then the number of independent variables required to determine network's characteristics can be made less than N_k^2 . Let this number be denoted by n_{f_k} and in general $n_{f_k} \leq N_k^2$. This fact could be utilized to reduce the computational cost involved in solving (3.24). Let $\mathbf{Y}_{f_k}$ be the vector of indepen-

dent variables which is a subset of vector Y_{v_k} . In order to establish a relation between Y_{v_k} and Y_{f_k} , define F_k such that

$$Y_{v_k} = F_k Y_{f_k} \quad (3.27)$$

where $F_k = [f_{i,j}]$, $i \in \{1, \dots, N_k^2\}$, $j \in \{1, \dots, n_{f_k}\}$, is a $N_k^2 \times n_{f_k}$ mapping matrix with elements $f_{i,j} \in \{0, 1\}$. Using (3.27), (3.24) can be rewritten in terms of vector of independent variables Y_{f_k} as

$$A_k Y_{f_k} = B_k \quad (3.28)$$

where

$$A_k = W_k F_k \quad (3.29)$$

Example: Formulation of Measured Data

To illustrate the above steps, consider the two terminal, passive and symmetric measured subnetwork shown in Fig. 3.5. Let the subnetwork have $Y_{11} = Y_{22}$ and $Y_{12} = Y_{21}$. Hence Y_{v_k} , Y_{f_k} and F_k for this case can be written as

$$Y_{v_k} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \end{bmatrix}; \quad Y_{f_k} = \begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix}; \quad F_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Correspondingly (3.28) can be formulated as

$$\begin{bmatrix} V_1^{(1)} & V_2^{(1)} \\ V_2^{(1)} & V_1^{(1)} \\ V_1^{(2)} & V_2^{(2)} \\ V_2^{(2)} & V_1^{(2)} \\ V_1^{(3)} & V_2^{(3)} \\ V_2^{(3)} & V_1^{(3)} \end{bmatrix} \begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix} = \begin{bmatrix} -(V_1^{(1)} - V_s^{(1)})/R_1^{(1)} \\ -V_2^{(1)}/R_2^{(1)} \\ -(V_1^{(2)} - V_s^{(2)})/R_1^{(2)} \\ -V_2^{(2)}/R_2^{(2)} \\ -(V_1^{(3)} - V_s^{(3)})/R_1^{(3)} \\ -V_2^{(3)}/R_2^{(3)} \end{bmatrix}$$

Next, solution of (3.28) is needed for obtaining Y_{f_k} . However, in the presence of m different sets of measurements the subnetwork k can be of either types, determined ($N_k \times m = n_{f_k}$) or overdetermined ($N_k \times m > n_{f_k}$). Overdetermined measurements are of practical importance as measurements usually contain noise and hence the use of single set of measurements may yield inaccurate results. Including all available sets of measurements in the simulation helps to reduce the impact of noise on network responses. If the system is overdetermined then the matrix A_k which has a dimension $(N_k \times m) \times n_{f_k}$ may not be square, in which case the LHS and RHS of (3.28) may not be equal (i.e., the error $A_k Y_{f_k} - B_k \neq 0$). To overcome this difficulty, (3.28) is formulated as an *optimization problem based on the least square approximation* [56], [57] such that the least mean square error of (3.28), i.e., $(A_k Y_{f_k} - B_k)^t (A_k Y_{f_k} - B_k)$ is minimized. The solution to this formulation is given by a new set of equations

$$A_{s_k}^t Y_{f_k} = A_k^t B_k \quad (3.30)$$

where

$$A_{s_k} = A_k^t A_k \quad (3.31)$$

is a square matrix of order $n_{f_k} \times n_{f_k}$. (3.30) can be solved for Y_{f_k} and (3.27) in turn yields Y_{v_k} . The stencil of the measured subnetwork, described by (3.7) can then be obtained using (3.20).

3.5 Discussion of CPU Cost in Conventional Simulation Techniques

Frequency-domain simulation is conventionally done by solving (3.2) at each frequency point through LU decomposition and forward-backward substitution. For time-domain simulation, Linear Multi-Step techniques (*LMS*) [58] are used. The most common of these integration formulae is the trapezoidal rule, which gives the following difference formula

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \Delta t \frac{(\dot{\mathbf{x}}(t) + \dot{\mathbf{x}}(t + \Delta t))}{2} \quad (3.32)$$

$$\left[\mathbf{G} + \frac{2}{\Delta t} \mathbf{C} \right] \mathbf{x}(t + \Delta t) = \left[\frac{2}{\Delta t} \mathbf{C} - \mathbf{G} \right] \mathbf{x}(t) + [\mathbf{e}(t) + \mathbf{e}(t + \Delta t)] \quad (3.33)$$

Note that the trapezoidal rule is an integration formula of order 2, which is the highest possible order that could ensure absolute stability [58]. Due to such a low-order, simulators are forced to use small step sizes in order to ensure the accuracy during transient simulation. Transient response computation requires the LU decomposition of (3.33) at each time step Δt . Also, this solution is more complicated in the presence of nonlinear elements such as drivers and receivers, in which case, the (3.3) gets modified as

$$\mathbf{G}\mathbf{x}(t) + \mathbf{C}(\dot{\mathbf{x}}(t)) + \Phi(\mathbf{x}(t)) - \mathbf{e}(t) = 0 \quad (3.34)$$

where $\Phi(\mathbf{x}(t))$ is the nonlinear function of $\mathbf{x}$. Therefore, the difference equation based on the trapezoidal rule becomes

$$\begin{aligned} \left[\mathbf{G} + \frac{2}{\Delta t} \mathbf{C} \right] \mathbf{x}(t + \Delta t) + \Phi(\mathbf{x}(t + \Delta t)) \\ = \left[\frac{2}{h} \mathbf{C} - \mathbf{G} \right] \mathbf{x}(t) + (\mathbf{e}(t) + \mathbf{e}(t + \Delta t)) - \Phi(\mathbf{x}(t)) \end{aligned} \quad (3.35)$$

In the case of nonlinear elements, Newton iterations are used to solve (3.35) at each time step, which requires at least one LU decomposition. However, due to the low order of integration formula, generally a large number of LU decompositions are required at each time step. This causes (note that $\mathbf{G}$ and $\mathbf{C}$ matrices for interconnect networks are usually very large) the CPU cost of a time-domain analysis to be very expensive. This lead to the development of model-reduction algorithms, which effects a reduction in the order of the linear subnetwork before performing a nonlinear analysis so as to yield fast simulation results.

3.6 Mixed Frequency-Time Domain Problem

It is to be noted that, the MNA equations of the lumped section of the nonlinear network can only be expressed in the time domain as per (3.35). On the other hand, high frequency models such as distributed transmission lines are described in terms of partial differential equations, whose solutions are best expressed in the frequency domain or in terms of measured data. This presents the difficulty of mixed frequency/time simulation for SPICE like conventional simulators. In addition, measured models do not have a direct representation in the time-domain for efficient simulation purposes. In order to address this diffi-

culty and also for obtaining higher CPU efficiency, model reduction algorithms were recently proposed. A brief review of the same is given in the next chapter.

Chapter 4

Review of Model-Reduction Techniques and Associate Hermite Functions

This chapter presents a brief review of two successful algorithms in the area of interconnect analysis and Electromagnetic (EM) device modeling. The first one is the model-reduction algorithm for fast and accurate analysis of large interconnect circuits. The second one is the concept of Associate Hermite (AH) functions which would be useful in the developing of our proposed algorithm for analysis of time-domain characterized measured subnetworks.

4.1 Principles of Model-Reduction Algorithms

Interconnect networks generally tend to have large number of poles, spread over a wide frequency range. Even though majority of these poles would normally have very little effect on simulation results, they cause the linear multi-step algorithms to significantly reduce the step size in order to stay within the stability region. The small step size would normally warrant many time point solutions, which in turn forces large number of LU

decompositions and consequently increases the CPU cost. The moment-matching techniques (*MMTs*) capitalize on the fact that irrespective of the presence of large number of poles in a system, only a few of those poles mainly govern the behaviour of the system transfer function and such poles are referred to as *dominant poles*. This effect is demonstrated in Fig. 4.1, where it is clear that the pole p_2 will have little effect on the final transient result. A brief mathematical description of the underlying concepts of moment-matching techniques is given below

Fig. 4.1. Illustration of dominant poles

Consider a single input/single output system and let $H(s)$ be the transfer function. $H(s)$ can be represented in a rational form as

$$H(s) = \frac{P(s)}{Q(s)} \quad (4.1)$$

where $P(s)$ and $Q(s)$ are polynomials in s . Equivalently, (4.1) can be written as

$$H(s) = c + \sum_{i=0}^{N_p} \frac{k_i}{s - p_i} \quad (4.2)$$

where p_i and k_i are the i^{th} pole-residue pair, N_p is the total number system poles and c is the direct coupling constant. Next, the time-domain impulse response can be computed in a closed form using inverse Laplace transform as

$$h(t) = c\delta t + \sum_{i=0}^{N_p} k_i e^{p_i t} \quad (4.3)$$

In case of large networks, N_p , the total number of poles can be of the order of thousands. Generating all the N_p poles will be highly CPU intensive even for a small network and for large networks it is completely impractical. MMTs address the above issue by deriving a reduced-order approximation $\hat{H}(s)$ in terms of dominant poles, instead of trying to compute all the poles of a system. Assuming that only L dominant poles were extracted which give a reasonably good approximation to the original system, equation (4.1) and the corresponding approximate frequency and time responses can be written as

$$H(s) \approx \hat{H}(s) = \frac{\hat{P}(s)}{\hat{Q}(s)} = \hat{c} + \sum_{i=0}^L \frac{\hat{k}_i}{s - \hat{p}_i} \quad (4.4)$$

$$h(t) \approx \hat{h}(t) = \hat{c}\delta t + \sum_{i=0}^L \hat{k}_i e^{\hat{p}_i t} \quad (4.5)$$

MMTs are based on Padé approximation and the steps involved in generating a reduced-order model using Padé approximation are outlined below.

4.2 Padé Approximations

Consider a system-transfer function $H(s)$ which is approximated by a rational function $\hat{H}(s)$ as

$$H(s) \approx \hat{H}(s) = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_L s^L}{1 + b_1 s + b_2 s^2 + \dots + b_M s^M} = \frac{P_L(s)}{Q_M(s)} \quad (4.6)$$

where the coefficients $a_0, a_1, \dots, a_L$, and $b_1, b_2, \dots, b_M$ are the unknowns (total of $L + M$ variables). Next, expanding $H(s)$ using Taylor series coefficients M_i (*moments*) we have

$$H(s) \approx \hat{H}(s) = m_0 + m_1 s + m_2 s^2 + \dots + m_n s^n = \sum_{i=0}^n s^i m_i \quad (4.7)$$

The series (4.7) is then matched to the lower-order rational polynomial given by (4.6) (*hence is the name, moment-matching techniques (MMTs); which is also known as Padé Approximation*). There are $L+M$ unknowns and hence we need to match only the first $L+M$ moments as

$$\frac{a_0 + a_1 s + a_2 s^2 + \dots + a_L s^L}{1 + b_1 s + b_2 s^2 + \dots + b_M s^M} = m_0 + m_1 s + m_2 s^2 + \dots + m_{(L+M)} s^{(L+M)} \quad (4.8)$$

Cross multiplying and equating the powers of s starting from s^{L+1} to s^{L+M} on both sides of (4.8), we can evaluate the denominator polynomial coefficients as

$$\begin{bmatrix} m_{L-M+1} & m_{L-M+2} & \cdots & m_L \\ m_{L-M+2} & \cdots & \cdots & m_{L+1} \\ \cdots & \cdots & \cdots & \cdots \\ m_L & m_{L+1} & \cdots & m_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-2} \\ \cdots \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ \cdots \\ m_{L+M} \end{bmatrix} \quad (4.9)$$

The numerator coefficients a_i 's can be found by equating the remain powers of s , starting from s^0 up to s^L as

$$\begin{aligned} a_0 &= m_0 \\ a_1 &= m_1 + b_1 m_0 \\ a_2 &= m_2 + b_1 m_1 + b_2 m_0 \\ a_3 &= m_3 + b_1 m_2 + b_2 m_1 + b_3 m_0 \\ &\vdots \\ a_L &= m_L + \sum_{i=1}^{\min(L, M)} b_i m_{L-i} \end{aligned} \quad (4.10)$$

Equations (4.9) and (4.10) yield an approximate transfer function in terms of rational polynomials. Alternatively, an equivalent pole-residue model can be found as follows. Poles p_i are obtained by applying a root-solving algorithm on denominator polynomial $Q(s)$. In order to obtain k_i , expand the approximate transfer function given by (4.4) using Maclaurin series as

$$\hat{H}(s) = \hat{c} - \sum_{n=0}^{\infty} s^n \left(\sum_{i=0}^L \frac{\hat{k}_i}{\hat{p}_i^{n+1}} \right) \quad (4.11)$$

Comparing $\hat{H}(s)$ from equations (4.7) and (4.11), we note that

$$\begin{aligned} m_0 &= \hat{c} - \sum_{i=0}^L \frac{\hat{k}_i}{\hat{p}_i} \\ &\dots \quad (0 < i < 2L) \\ m_i &= - \sum_{i=0}^L \frac{\hat{k}_i}{\hat{p}_i^{i+1}} \end{aligned} \quad (4.12)$$

Residues can be evaluated by writing the equations in (4.12) in a matrix form as

$$\begin{bmatrix} \hat{p}_1^{-1} & \hat{p}_2^{-1} & \hat{p}_L^{-1} & -1 \\ \hat{p}_1^{-2} & \hat{p}_2^{-2} & \hat{p}_L^{-2} & 0 \\ & \dots & & \\ & \dots & & \\ \hat{p}_1^{-L-1} & \hat{p}_2^{-L-1} & \hat{p}_L^{-L-1} & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_1 \\ \hat{k}_2 \\ \dots \\ \hat{k}_L \\ \hat{c} \end{bmatrix} = \begin{bmatrix} m_0 \\ m_1 \\ \dots \\ m_{L-1} \\ m_L \end{bmatrix} \quad (4.13)$$

In the above equations, $\hat{c}$ represents the direct coupling between input and output.

4.3 Computation of Moments

Having outlined the concept of MMTs, to proceed further, we need to evaluate the moments of the system. Consider the circuit equation represented by (3.10) and expand the response vector $X(s)$ using Taylor series as

$$X(s) = M_0 + M_1 s + M_2 s^2 + \dots + M_n s^n = \sum_{i=0}^n s^i M_i \quad (4.14)$$

where M_i represents the i^{th} moment-vector. For the purpose of illustration, consider the simple case of network with only lumped models. In this case, the network can be represented in the form of (3.10) as

$$[G + sC] [X(s)] = [E] \quad (4.15)$$

or

$$[G + sC] [M_0 + M_1 s + M_2 s^2 + \dots] = [E] \quad (4.16)$$

Equating powers of s on both sides of (4.16) we obtain the following relationships

$$\begin{aligned} GM_0 &= E \\ GM_i &= -CM_{i-1}; \quad i > 0 \end{aligned} \quad (4.17)$$

Above equations give a closed form relationship for the computation of moments. The moments of a particular output node of interest (which are represented by m_i in equations (4.7) - (4.13), are picked from moment-vectors M_i . As seen, (4.17) requires only one LU decomposition and few forward/backward substitutions during the recursive computation

of higher order moments. Since the major cost involved in circuit simulation is due to LU decomposition, MMTs yield very high speed advantage (100 to 1000 times) compared to conventional simulators.

Next, consider the system represented by (3.10) when a unit excitation is applied at port ' i '

$$\Psi_{\pi}(s)X(s) = \mathbf{b}_i \quad (4.18)$$

where $\mathbf{b}_i$ is a binary vector with non-zero entry at location i . The moments at a frequency point $s = \alpha$ are calculated by expanding $X(s)$ in (4.18) using Taylor series as

$$X(s) = \sum_{n=0}^{\infty} \mathbf{M}_n^i (s - \alpha)^n \quad (4.19)$$

where $\mathbf{M}_j^i$ represents the j^{th} moment vector of $X(s)$. A recursive relation for the evaluation of moments can be obtained in the form [33]

$$[\Psi_{\pi}(\alpha)]\mathbf{M}_n^i = - \sum_{r=1}^n \frac{\frac{\partial^r}{\partial s^r} \Psi_{\pi}(s) \big|_{s=\alpha} \mathbf{M}_{n-r}^i}{r!}; \quad [\Psi_{\pi}(\alpha)]\mathbf{M}_0^i = \mathbf{b}_i \quad (4.20)$$

To evaluate the moments $\mathbf{M}_n^i$ using (4.20), the derivatives of $\Psi_{\pi}(s)$ (denoted by $\Psi_{\pi}^{(r)}$) is needed. They can be obtained using (3.11) as

$$\begin{aligned}
[\psi_\pi]^{(1)} &= \mathbf{C} + \mathbf{F}^{(1)}(s) = \mathbf{C} + \sum_{k=1}^{N_e} \mathbf{D}_k \mathbf{Y}_k^{(1)}(s) \cdot \mathbf{D}_k^T \\
[\psi_\pi]^{(r)} &= \mathbf{F}^{(r)}(s) = \sum_{k=1}^{N_e} \mathbf{D}_k \mathbf{Y}_k^{(r)}(s) \cdot \mathbf{D}_k^T; \quad (r \geq 2)
\end{aligned} \tag{4.21}$$

However, computation of measured subnetwork $\mathbf{Y}_k^{(r)}(s)$ moments is not easy and also not adequately addressed in the literature. A new algorithm for obtaining these moments is presented in *chapter 6*. Fig. 4.2 summarizes the computational steps involved in the MMT algorithm.

Fig. 4.2. Summary of the steps involved in the MMT algorithm

4.4 Limitations of Single Expansion MMT Algorithms

Obtaining a lower-order approximation of the network transfer function using a single Padé expansion is commonly referred to as *Asymptotic Waveform Evaluation (AWE)* in the literature [27]. However, due to inherent limitations of Padé approximates, MMTs based on single expansion often give inaccurate results. The following is a list of those properties which have the most impact on MMTs.

1. The matrix in (4.9) (which is known as Toeplitz matrix) is ill-conditioned if its size is large. This implies that one can only expect to detect 6 to 8 accurate poles from a single expansion.
2. Padé often produces unstable poles on the right hand side of the complex plane.
3. Padé accuracy deteriorates as we move away from the expansion point.
4. Padé provides no estimates for error bounds.

Generally AWE gives accurate results for RC networks, but often fails for non-RC networks. This is due to the fact that the poles of an RC network are all on the real axis (refer to Fig. 4.1) and therefore an expansion at the origin could clearly determine which ones are dominant and which ones are not. However, in the case of general RLC networks, it is possible to have some of the dominant poles outside the radius of convergence of the Padé expansion (Fig. 4.3(a)). In systems containing distributed elements, the number of dominant poles will be significantly higher, and it is very difficult to capture all of them with a single Padé expansion. This lead to the development of multi-point expansion techniques such as *complex frequency hopping (CFH)*, which are summarized in the next section.

4.5 Complex Frequency Hopping

CFH extends the process of Padé approximation and moment matching to multiple expansion points in the complex plane near or on the imaginary axis using a binary search algorithm. With a minimized number of frequency point expansions, enough information is obtained to enable the generation of an approximate transfer function that matches the original function up to a predefined highest frequency of interest. Using the information from all the expansion points, CFH extracts a dominant pole set as illustrated in Fig. 4.3(b). In addition, CFH provides an error criterion for the selection of accurate poles and transfer functions.

Fig. 4.3. Illustration of CFH.

4.5.1 Selection and Minimization of Hops in CFH

A Padé approximation is accurate only near the point of expansion and its accuracy decreases as we move away from *the point of expansion (hop)*. In order to validate the accuracy of such an approximation, at least two expansion points are necessary. Accuracies of these two expansions can be verified by matching the poles generated at these two hops [33] (referred to as *pole-matching based approach*). Alternatively, the two hops can be verified for their accuracy by comparing the value of the transfer functions due to both these hops at a point intermediate to them (referred as *transfer-function based approach*) [44]. CFH relies on a binary search algorithm to determine the expansion points and to minimize the number of expansions. The steps involved in the binary search algorithm for both the above approaches are summarized below.

A. Transfer Function Based Approach

In this approach the transfer functions obtained at various hops (expansions) are used to ensure the accuracy of the reduced-order model up to the highest frequency of interest. Steps involved in the algorithm are given in Fig. 4.4 and Fig. 4.5. It is to be noted that the computational effort needed for a comparison as required by *Step 5* is trivial as the responses can be computed in a closed-form using the transfer functions generated in *Steps 2 & 3*. Hence ϵ_{th} is a pre-defined threshold for the relative error in the transfer functions. At the completion of the binary search algorithm, a set of transfer functions are generated. When evaluating the frequency response at a frequency point α , only the transfer function which is valid in the region containing α is used. This is repeated for all other frequency points to obtain the frequency response of the system.

- Step 1: Set $f_L = 0$ and $f_H = f_{MAX}$ (Fig. 4.6).
- Step 2: Expand system's response at frequency $f_L = 0$. Determine the coefficients of the corresponding transfer function $H_L(s)$ using (4.9) and (4.10).
- Step 3: Expand system's response at $f_H = f_{MAX}$. Determine the coefficients of the corresponding transfer function $H_H(s)$.
- Step 4: Set $f_{mid} = 1/2(f_L + f_H)$. Calculate $H_L(j2\pi f_{mid})$ and $H_H(j2\pi f_{mid})$ using the transfer function coefficients obtained in Steps 2 and 3.
- Step 5: If $|H_H(j2\pi f_{mid}) - H_L(j2\pi f_{mid})| < \epsilon_{th}$, Go to Step 6. Otherwise expand at f_{mid} and obtain $H_{mid}(s)$.
- Step 6: If the threshold condition specified by step 5 is satisfied, STOP. ELSE repeat steps 2-5 between every two consecutive frequency points (e.g., between f_L & f_{mid} and f_{mid} & f_H).

Fig. 4.4. Transfer function based binary search algorithm

Fig. 4.5. Graphical illustration of transfer function based search algorithm

B. Pole-Matching Based Approach

In this approach, poles of the transfer function are explicitly evaluated at each hop and the hops are verified for their accuracy by comparing the poles from two adjacent hops using a binary search algorithm. If a matching pole is found between two adjacent expansions, then the binary search is stopped. The distance between the matching pole and the expansion point under consideration defines the radius of accuracy for the corresponding expansion. All the poles which are within the radius of accuracy are treated as accurate poles and are retained in the final pole-set. The poles which are outside the radius of accuracy are considered as inaccurate poles and are discarded. Fig. 4.6 illustrates the idea of the pole search. Steps involved in this approach are identical to the one given in Fig. 4.4, except for the matching criterion; which is based on poles here. Once a set of dominant poles are obtained, residues of the system are obtained using (4.13). Further details of CFH and its search algorithms can be found in [33] and [44].

Fig. 4.6. Pole-matching based binary search algorithm

Next, we will review the concept of Associate Hermite functions, which would be useful in understanding the proposed algorithm for analysis of time-domain characterized measured subnetworks.

4.6 Associate Hermite (AH) - Theory

Associate Hermite expressions can be used for the simultaneous interpolation in the time-domain and also extrapolation in the frequency-domain, from the partial sampled data of both forms (time and frequency) [59] - [61].

4.6.1 Associate Hermite Expansion

This section will briefly describe how Associate Hermite expansion can be applied for the simultaneous interpolation and extrapolation in time and frequency domains. According to [60], interpolating some portions of the sampled data in the time domain and its Fourier transform into Associate Hermite, (assuming M_1 and M_2 are the number of known sampled data in time and frequency domain respectively), we are able to extrapolate the original signal in both time and frequency domain as

$$\mathbf{X} = \mathbf{H} \mathbf{k} \quad (4.22)$$

where $\mathbf{X}$ is a vector of length $(M_1 + 2M_2)$ and containing the known sampled data in time ($x(t_i)$) and frequency domain (real $\rightarrow X_R(f_i)$ and imaginary $\rightarrow X_I(f_i)$) i.e.

$$\mathbf{X} = \left[x(t_1) \dots x(t_{M_1}) \quad X_R(f_1) \dots X_R(f_{M_2}) \quad X_I(f_1) \dots X_I(f_{M_2}) \right]^T \quad (4.23)$$

$\mathbf{k}$ is a vector of length $(N+1)$ and represents the Hermite expansion coefficients,

$$\mathbf{k} = \begin{bmatrix} k_0 & k_1 & \dots & k_N \end{bmatrix}^T \quad (4.24)$$

and $\mathbf{H}$ is a $(M_1 + 2M_2) \times (N + 1)$ matrix and represents the Associate Hermite expansion as [60]:

$$\mathbf{H} = \begin{bmatrix}
 h_0(t_1, \lambda) & h_1(t_1, \lambda) & h_2(t_1, \lambda) & h_3(t_1, \lambda) & \dots & h_{N-2}(t_1, \lambda) & h_{N-1}(t_1, \lambda) & h_N(t_1, \lambda) \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 h_0(f_{M_1}, \lambda) & h_1(f_{M_1}, \lambda) & h_2(f_{M_1}, \lambda) & h_3(f_{M_1}, \lambda) & \dots & h_{N-2}(f_{M_1}, \lambda) & h_{N-1}(f_{M_1}, \lambda) & h_N(f_{M_1}, \lambda) \\
 \hline
 h_0(f_1, \mu) & 0 & -h_2(f_1, \mu) & 0 & \dots & (-1)^{N/2-1} h_{N-2}(f_1, \mu) & 0 & (-1)^{N/2} h_N(f_1, \mu) \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 h_0(f_{M_2}, \mu) & 0 & -h_2(f_{M_2}, \mu) & 0 & \dots & (-1)^{N/2-1} h_{N-2}(f_{M_2}, \mu) & 0 & (-1)^{N/2} h_N(f_{M_2}, \mu) \\
 \hline
 0 & -h_1(f_1, \mu) & 0 & h_3(f_1, \mu) & \dots & (-1)^{N/2-2} h_{N-3}(f_1, \mu) & (-1)^{N/2-1} h_{N-1}(f_1, \mu) & 0 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 0 & -h_1(f_{M_2}, \mu) & 0 & h_3(f_{M_2}, \mu) & \dots & (-1)^{N/2-2} h_{N-3}(f_{M_2}, \mu) & (-1)^{N/2-1} h_{N-1}(f_{M_2}, \mu) & 0
 \end{bmatrix}$$

(4.25)

where

$$h_n(t, \lambda) = \frac{1}{\sqrt{2^n n!} \sqrt{\pi} \lambda} w_n\left(\frac{t}{\lambda}\right) \exp\left(-\frac{t^2}{2\lambda^2}\right); \quad n \geq 0 \quad (4.26)$$

and

$$h_n(f, \mu) = \frac{1}{\sqrt{2^n n!} \sqrt{\pi} \mu} w_n(f/\mu) \exp(f^2/2\mu^2); \quad n \geq 0 \quad (4.27)$$

where λ and μ are the scaling factors. $w_n(t)$ and $w_n(f)$ are Hermite polynomials. Note that the Fourier transform of Associate Hermite (AH) with a scaling factor λ and order n is a AH function with the scaling factor $\mu = 1/(2\pi\lambda)$ and order n , as indicated by (4.27).

Next, the Hermite polynomials can be defined recursively as

$$\begin{cases} w_0(t) = 1 \\ w_1(t) = 2t \\ \dots \\ w_n(t) = 2tw_{n-1}(t) - 2(n-1)w_{n-2}(t) \end{cases} \quad (4.28)$$

If the scaling factor λ and μ are used, results of (4.28) can be expressed as

$$w_n(t/\lambda) = 2(t/\lambda)w_{n-1}(t/\lambda) - 2(n-1)w_{n-2}(t/\lambda)$$

and

$$w_n(f/\mu) = 2(f/\mu)w_{n-1}(f/\mu) - 2(n-1)w_{n-2}(f/\mu)$$

Using (4.26) and (4.28), a closed-form of the AH function h_n can be expressed as:

$$h_n = \frac{1}{\sqrt{n}}(\sqrt{2}th_{n-1} - \sqrt{n-1}h_{n-2}) \quad (4.29)$$

By substituting (4.23) and (4.25) in (4.22) we can compute the Hermite Expansion coefficient vector $\mathbf{k}$. Next, the response in the time-domain (*interpolated response*) and also in the frequency-domain (*extrapolated response*) can be obtained using

$$X(t) = \sum_{n=0}^N k_n h_n(t, \lambda) \quad (4.30)$$

and

$$X(f) = \sum_{n=0}^N (-j)^n k_n h_n(f, \mu) \quad (4.31)$$

or

$$X_R(f) = \sum_{m=0}^{2m \leq N} (-1)^m k_{2m} h_{2m}(f, \mu) \quad (4.32)$$

$$X_I(f) = - \sum_{m=0}^{2m+1 \leq N} (-1)^m k_{2m+1} h_{2m+1}(f, \mu) \quad (4.33)$$

An illustrative example of the above formulation is given below.

Illustrative Example: Formulation of AH Equations

Consider (4.25) and (4.26), with the following parameters:

let $N=4$, $\lambda=2.0$, time interval: $-1 \leq t \leq 1$ (sec.), and frequency interval: $0 \leq f \leq 0.4$ (Hz).

Let the time and frequency sampling step size be 1sec. and 0.2Hz respectively.

Here we have

$$\begin{aligned} t &= [t_i]; \quad i = 1, 2, 3. & \text{and} & \quad f = [f_i]; \quad i = 1, 2, 3. \\ &= [-1, 0, 1] \cdot & & \quad = [0, 0.2, 0.4] \cdot \end{aligned}$$

By substituting the given values into (4.26), (4.27), (4.28) and (4.29), entries of the Associate Hermite matrix $\mathbf{H}$ can be obtained as follows:

$$\mathbf{H} = \begin{bmatrix} H_{1,0} & H_{1,1} & H_{1,2} & H_{1,3} & H_{1,4} \\ H_{2,0} & H_{2,1} & H_{2,2} & H_{2,3} & H_{2,4} \\ H_{3,0} & H_{3,1} & H_{3,2} & H_{3,3} & H_{3,4} \\ \hline H_{4,0} & H_{4,1} & H_{4,2} & H_{4,3} & H_{4,4} \\ H_{5,0} & H_{5,1} & H_{5,2} & H_{5,3} & H_{5,4} \\ H_{6,0} & H_{6,1} & H_{6,2} & H_{6,3} & H_{6,4} \\ \hline H_{7,0} & H_{7,1} & H_{7,2} & H_{7,3} & H_{7,4} \\ H_{8,0} & H_{8,1} & H_{8,2} & H_{8,3} & H_{8,4} \\ H_{9,0} & H_{9,1} & H_{9,2} & H_{9,3} & H_{9,4} \end{bmatrix} = \begin{bmatrix} H_t \\ \vdots \\ H_R \\ \vdots \\ H_I \end{bmatrix} \quad (4.34)$$

From (4.25), (4.26) and (4.28), we have

$$H_{1,0} = h_0(t_1, \lambda) = h_0(-1, 2) = \frac{1}{\sqrt{2^0 0!} \sqrt{\pi(2)}} (1) \exp\left(-\frac{(-1)^2}{2(2)^2}\right) = 0.4687 \quad (4.35)$$

$$H_{1,1} = h_1(t_1, \lambda) = h_1(-1, 2) = \frac{1}{\sqrt{2^1 1!} \sqrt{\pi(2)}} \left(-\frac{2(1)}{2}\right) \exp\left(-\frac{(-1)^2}{2(2)^2}\right) = -0.3314 \quad (4.36)$$

and

$$\begin{aligned} H_{1,2} &= h_2(t_1, \lambda) = h_2(-1, 2) \\ &= \frac{1}{\sqrt{2^2 2!} \sqrt{\pi(2)}} \left\{ 2(t_1/\lambda) \left(2 \frac{t_1}{\lambda}\right) - 2(1)(1) \right\} \exp\left(-\frac{(t_1)^2}{2(\lambda)^2}\right) \\ &= \frac{1}{\sqrt{2}} \left(\sqrt{2} \left(\frac{t_1}{\lambda}\right) H_{1,1} - \sqrt{1} H_{1,0} \right) \\ &= -0.1657 \end{aligned} \quad (4.37)$$

and so on. From the above results, a recursive relationship for calculation of $H_{i,n}$ when $n \geq 2$ can be obtained as

$$g(t_i, n) = \frac{1}{\sqrt{n}} \left\{ \sqrt{2} \left(\frac{t_i}{\lambda}\right) H_{t_i, n-1} - \sqrt{n-1} H_{t_i, n-2} \right\} \quad (4.38)$$

Next, entries of H_R and H_I in (4.34) are obtained by representing (4.38) in the frequency domain as

$$g(f_i, n) = \frac{1}{\sqrt{n}} \left\{ \sqrt{2} \left(\frac{f_i}{\mu}\right) H_{f_i, n-1} - \sqrt{n-1} H_{f_i, n-2} \right\} \quad (4.39)$$

where f represents frequency and $\mu = 1/(2\pi\lambda)$

Using (4.35) - (4.39) the elements of the Associate Hermite matrix $\mathbf{H}$ can be generated for the time and frequency interval as

$$\mathbf{H} = \begin{bmatrix} 0.4687 & -0.3314 & -0.1657 & 0.3383 & 0.0239 \\ 0.5311 & 0 & -0.3756 & 0 & 0.3252 \\ 0.4687 & 0.3314 & -0.1657 & -0.3383 & 0.0239 \\ \hline 2.6627 & 0 & -(-1.8828) & 0 & 1.6305 \\ 0.1132 & 0 & -(0.9308) & 0 & 2.0049 \\ 0 & 0 & -(0.0003) & 0 & 0.0040 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & -(0.4022) & 0 & 1.5815 & 0 \\ 0 & -(0.0001) & 0 & 0.0012 & 0 \end{bmatrix} \quad (4.40)$$

For this example, we will have $\mathbf{k} = [k_0 \ k_1 \ k_2 \ k_3 \ k_4]^T$

This AH method outlined in [60] uses both time and frequency sampled data in order to interpolate and extrapolate both time and frequency responses. In this thesis, we propose a new algorithm which requires only time-domain data to generate responses both in time and frequency domains. The proposed method is referred to as Modified Associate Hermite (MAH), and is presented in the next chapter.

Chapter 5

Modeling of Time-Domain Measured Data

5.1 General Overview

Depending on the desired accuracy, CPU efficiency and complexity of the data, several methods can be used for modeling measured subnetworks. The time domain approach is preferred for broadband analysis. It is well suited for modeling in the presence of nonlinear elements. Use of Hermite function [59] for this purpose has been well articulated in the literature [60], [61]. It is to be noted that traditional Hermite approaches suffer from two drawbacks. The first drawback is the selection of the order for Modified Hermite expansion; choosing a large order will introduce oscillations while extrapolating the frequency-response and conversely choosing a smaller order will give inaccurate results in both interpolation (time response) and extrapolation (frequency response). The second drawback is the difficulty of choosing the “*origin: t_0* ” and the scaling factor λ which are both crucial in determining the accuracy of the approximation.

To overcome these drawbacks, this chapter presents an optimal simultaneous interpolation and extrapolation method for computing time and frequency responses using an adaptive procedure and Modified Associate Hermite (MAH) functions. This method can obtain simultaneous extrapolation information in the frequency domain by fitting only the time data. The proposed algorithm also suggests how to optimally choose the order N , origin t_0 and the scaling factor λ for the Modified Hermite expansion. Numerical results are presented to show the validity and accuracy of the proposed method.

5.2 Analysis of Time-Domain Data Using Modified Associate Hermite Expansion

Consider the Associate Hermite function (4.26) [61]

$$h_n(t, \lambda) = \frac{1}{\sqrt{2^n n! \sqrt{\pi} \lambda}} w_n\left(\frac{t}{\lambda}\right) \exp\left(-\frac{t^2}{2\lambda^2}\right); \quad n \geq 0 \quad (5.1)$$

where $w_n(t)$ is the Modified Hermite polynomial, n and λ are the order and scaling factor of the Modified Hermite polynomial, respectively.

The Modified Hermite polynomial can be defined recursively as:

$$\begin{cases} w_0(t) = 1 \\ w_1(t) = 2t \Rightarrow w_1(t/\lambda) = 2\left(\frac{t}{\lambda}\right) \\ \dots \\ w_n(t) = 2tw_{n-1}(t) - 2(n-1)w_{n-2}(t) \end{cases} \quad (5.2)$$

By substituting equation (5.2) in (5.1), we have

$$h_0(t, \lambda) = \frac{1}{\sqrt{2^0 0! \sqrt{\pi} \lambda}} w_0(t/\lambda) \exp\left(-\frac{(t)^2}{2(\lambda)^2}\right) \quad (5.3)$$

$$h_1(t, \lambda) = \frac{1}{\sqrt{2^1 1! \sqrt{\pi} \lambda}} w_1(t/\lambda) \exp\left(-\frac{(t)^2}{2(\lambda)^2}\right) \quad (5.4)$$

$$h_2(t, \lambda) = \frac{1}{\sqrt{2^2 2! \sqrt{\pi} \lambda}} \left\{ 2\left(\frac{t}{\lambda}\right) w_1(t/\lambda) - 2(1) w_0(t/\lambda) \right\} \exp\left(-\frac{(t)^2}{2(\lambda)^2}\right) \quad (5.5)$$

or

$$h_2(t, \lambda) = \frac{1}{\sqrt{2}} \left\{ \sqrt{2} \left(\frac{t}{\lambda}\right) h_1(t, \lambda) - (\sqrt{1}) h_0(t, \lambda) \right\} \quad (5.6)$$

Next, we can extend the result of (5.6) into a recursive relationship as

$$h_n(t, \lambda) = \frac{1}{\sqrt{n}} (\sqrt{2} t h_{n-1}(t, \lambda) - \sqrt{n-1} h_{n-2}(t, \lambda)) \quad (5.7)$$

There are two main features of the Modified Associate Hermite (MAH) function that need to be highlighted. One of them is *its finite time support*, (monotonically decaying) due to the fact that $h_n(t, \lambda)$ is a function of $\text{Exp}(-t^2)$, as described in (5.1). The second one is *the isomorphism property between its function and its Fourier transform*, which is given by [59], [62]:

$$\begin{aligned}
F[h_n(t, \lambda)] &= (-j)^n \frac{\sqrt{2\sqrt{\pi}\lambda}}{\sqrt{2^n n!}} w_n(2\pi\lambda f) \exp\left(-\frac{(2\pi\lambda f)^2}{2}\right) \\
&= (-j)^n h_n(f, \mu)
\end{aligned} \tag{5.8}$$

where $j = \sqrt{-1}$ and $\mu = \frac{1}{2\pi\lambda}$. This implies that the Fourier transform given by (5.8) is a MAH function similar to the time domain function of order n (given by (5.1)) but with a scaling factor of μ instead of λ in the time domain.

Next signal $x(t)$ can be expanded by the MAH functions of order zero to N , as follows:

$$x(t) = \sum_{n=0}^N k_n h_n(t, \lambda) \tag{5.9}$$

Using (5.9), $X(f)$ can be expressed in a closed-form as:

$$X(f) = \sum_{n=0}^N (-j)^n k_n h_n(f, \mu) \tag{5.10}$$

In case of casual signals (signal having $x(t) = 0$ for $t \leq 0$), appropriate origin t_0 needs to be chosen. This is essential for fast and accurate convergence of the fitting while using (5.9). If the midpoint of the measured data is not at $t=0$, but $t=t_0$, then (5.9) and (5.10) can be expressed as

$$x(t) = \sum_{n=0}^N k_n h_n(t-t_0, \lambda) \quad (5.11)$$

$$X(f) = e^{-j2\pi f t_0} \sum_{n=0}^N (-j)^n k_n h_n(f, \mu) \quad (5.12)$$

5.3 Adaptive Optimal Simultaneous Interpolation/Extrapolation Algorithm

This section describes an adaptive optimal simultaneous interpolation and extrapolation algorithm, both in time and frequency domains, using the Modified Associate Hermite expansion.

5.3.1 Calculation of Expansion Coefficient Vector “ k ”

Consider the given time-domain data of a signal $x(t)$. Let M be the number of sample points and N be the order of the Modified Hermite expansion. Then, (5.9) can be expressed by a matrix representation:

$$H k = X \quad (5.13)$$

where $H \in \Re^{(M) \times (N+1)}$ matrix,

$$\mathbf{H} = \begin{bmatrix} h_0(t_1, \lambda) & h_1(t_1, \lambda) & \dots & h_{N-1}(t_1, \lambda) & h_N(t_1, \lambda) \\ h_0(t_2, \lambda) & h_1(t_2, \lambda) & \dots & h_{N-1}(t_2, \lambda) & h_N(t_2, \lambda) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_0(t_M, \lambda) & h_1(t_M, \lambda) & \dots & h_{N-1}(t_M, \lambda) & h_N(t_M, \lambda) \end{bmatrix} \quad (5.14)$$

$\mathbf{k} \in \mathfrak{R}^{(N+1)}$ contains the unknown expansion coefficients,

$$\mathbf{k} = \begin{bmatrix} k_0 & k_1 & \dots & k_N \end{bmatrix}^t \quad (5.15)$$

and $\mathbf{X} \in \mathfrak{R}^M$ contains the given data

$$\mathbf{X} = \begin{bmatrix} x(t_1) & x(t_2) & x(t_3) & \dots & x(t_M) \end{bmatrix}^t \quad (5.16)$$

When using the linear formulation (5.13), the number of sampled data M is usually greater than the Modified Hermite Order N (representing N expansion coefficients of $\mathbf{k}$). To address this issue, the least-squares method can be employed to calculate the coefficients [63]. Pre-multiplying both sides of (5.13) by the transpose of $\mathbf{H}$ we get:

$$\mathbf{H}^t \mathbf{H} \mathbf{k} = \mathbf{H}^t \mathbf{X} \quad (5.17)$$

or

$$\mathbf{H}_g \mathbf{k} = \mathbf{H}^t \mathbf{X} \quad (5.18)$$

where

$$H_g = H^t H \quad (5.19)$$

5.3.2 Optimal Choice of Modified Hermite Coefficients

Equation (5.11) shows that using a different origin t_0 and a different scaling factor λ , will result in different expansion coefficients, for a fixed expansion order N . The expansion error at sample points due to certain t_0 , λ and N is given by

$$Er(t_0, \lambda, N) = \sqrt{\frac{\sum_{m=1}^M \left(x(t_m) - \sum_{n=0}^N k_n h_n(t_m - t_0, \lambda) \right)^2}{\sum_{m=1}^M (x(t_m))^2}} \quad (5.20)$$

The choice of Modified Hermite expansion order N is very crucial. Choosing a small order will lead to inaccurate interpolation and extrapolation results, and choosing a large order will introduce oscillations in the extrapolation region.

Next, an adaptive algorithm is presented in this chapter to choose an appropriate value for t_0 , the scaling factor λ and order N . The steps involving in this adaptive algorithm are given below:

Adaptive Algorithm for Selecting of t_0 , λ and N

Step 1: Set an initial origin t_0 such as: $t_0 = (t_1 + t_M) / 2$, assuming that $(t_1 < t_2 < \dots < t_M)$, where:
 t_1 Represents initial sampled time data point.
 t_M Represents the final sampled time data point (usually the point where the time-domain characteristics settles down).

Step 2: Start with an appropriate expansion order N .

Step 3: Select an appropriate value for the Modified Hermite scaling factor λ .

Step 4: Optimize the nonlinear programming problem $\min\{Er(t_0, \lambda, N)\}$ to find the best Modified Hermite scaling factor λ for a given origin t_0 and for the order of approximation N . Calculate the expansion error $Er(t_0, \lambda, N)$ of the optimal MAH expansion of order N by using equation (5.20)

Step 5: Check if
 $Er(t_0, \lambda, N) \leq \epsilon$
 (The predefined tolerance factor)

NO

Optimal values for
 t_0 , λ and N . **Stop**

Step 6: Replace the order N by $N+1$.

Usually convergence will be faster if $N \approx 50$, and λ is chosen in the range $t_0/20 \leq \lambda \leq t_0/15$ (to started with).

5.4 Numerical Examples

This section provides numerical examples to demonstrate the validity and the accuracy of the proposed method. It also discusses the steps involved in detail.

Example 1:

Consider the test function:

$$x(t) = e^{-t} - 2e^{-2t} + e^{-3t}; \quad (t \geq 0) \quad (5.21)$$

This function is chosen as its frequency-domain equivalent is known,

$$X(s) = \frac{1}{(s+1)} - \frac{2}{(s+2)} + \frac{1}{(s+3)} \quad (5.22)$$

The time-domain samples are considered in the time interval $0 \leq t \leq 8$ sec. ($T_i=0$, $T_f=8$ sec.). A sampling step size of $\{(T_i + T_f) / 200\}$ seconds is used to obtain the samples.

Using the proposed algorithm, the data is fitted in the time domain. During this process, the following parameters are used.

From step 1, the origin (t_0) can be determined as:

$$t_0 = \frac{(T_i + T_f)}{2} = 4 \text{ Seconds}$$

From step 2, λ is selected in the range: $\frac{t_0}{20} \leq \lambda \leq \frac{t_0}{15}$

Applying the proposed adaptive algorithm, we get scaling factor $\lambda = 0.26$ and order $N = 136$. The resulting expansion coefficients k_n are listed in Table 5.1. Fig. 5.1 shows the comparison between the exact sample time data and the interpolated time data up to 8 *seconds* and as seen, they are indistinguishable. Fig. 5.2 - Fig. 5.4 show the comparison of the original frequency-domain data with the extrapolated frequency data (for absolute, real and imaginary responses) up to 2 *Hertz* and they match accurately.

Fig. 5.1. Exact and interpolated time data for Example 1

Fig. 5.2. Amplitude of exact and extrapolated frequency data for Example 1

Fig. 5.3. Real part of exact and extrapolated frequency data for Example 1

Fig. 5.4. Imaginary part of exact and extrapolated frequency data for Example 1

n	k_n	n	k_n	n	k_n	n	k_n
0	1.7459e-2	36	2.0687e-2	72	2.0885e-2	108	2.9215e-3
1	-6.1543e-3	37	-2.0233e-2	73	-2.0374e-2	109	-2.3553e-3
2	1.3809e-2	38	2.1102e-2	74	2.0303e-2	110	2.0253e-3
3	-7.912e-3	39	-2.0679e-2	75	-1.9767e-2	111	-1.5231e-3
4	1.3253e-2	40	2.1486e-2	76	1.9649e-2	112	1.2550e-3
5	-8.9925e-3	41	-2.1087e-2	77	-1.9087e-2	113	-8.2658e-4
6	1.3301e-2	42	2.1834e-2	78	1.8924e-2	114	6.3018e-4
7	-1.0014e-2	43	-2.1455e-2	79	-1.8337e-2	115	-2.8311e-4
8	1.3586e-2	44	2.2144e-2	80	1.8129e-2	116	1.6503e-4
9	-1.0937e-2	45	-2.1780e-2	81	-1.7517e-2	117	9.7502e-5
10	1.3988e-2	46	2.2413e-2	82	1.7266e-2	118	-1.3562e-4
11	-1.1796e-2	47	-2.2059e-2	83	-1.6630e-2	119	3.1646e-4
12	1.4458e-2	48	2.2639e-2	84	1.6338e-2	120	-2.7975e-4
13	-1.2610e-2	49	-2.2289e-2	85	-1.5680e-2	121	3.8914e-4
14	1.4967e-2	50	2.2816e-2	86	1.5350e-2	122	-2.9059e-4
15	-1.3389e-2	51	-2.2469e-2	87	-1.4670e-2	123	3.4608e-4
16	1.5501e-2	52	2.2944e-2	88	1.4306e-2	124	-2.0616e-4
17	-1.4139e-2	53	-2.2595e-2	89	-1.3608e-2	125	2.3062e-4
18	1.6048e-2	54	2.3018e-2	90	1.3213e-2	126	-7.4747e-5
19	-1.4863e-2	55	-2.2664e-2	91	-1.2499e-2	127	9.2094e-5
20	1.6601e-2	56	2.3037e-2	92	1.2077e-2	128	5.3890e-5
21	-1.5562e-2	57	-2.2674e-2	93	-1.1352e-2	129	-2.4533e-5
22	1.7154e-2	58	2.2997e-2	94	1.0908e-2	130	1.4010e-4
23	-1.6238e-2	59	-2.2623e-2	95	-1.0177e-2	131	-8.9850e-5
24	1.7703e-2	60	2.2895e-2	96	9.7168e-3	132	1.6475e-4
25	-1.6889e-2	61	-2.2508e-2	97	-8.9842e-3	133	-9.7522e-5
26	1.8242e-2	62	2.2730e-2	98	8.5153e-3	134	1.3375e-4
27	-1.7517e-2	63	-2.2327e-2	99	-7.7882e-3	135	-6.3901e-5
28	1.8770e-2	64	2.2500e-2	100	7.3177e-3	136	7.3270e-5
29	-1.8118e-2	65	-2.2078e-2	101	-6.6037e-3		
30	1.9281e-2	66	2.2201e-2	102	6.1402e-3		
31	-1.8692e-2	67	-2.1759e-2	103	-5.4479e-3		
32	1.9773e-2	68	2.1833e-2	104	5.0013e-3		
33	-1.9238e-2	69	-2.1370e-2	105	-4.3403e-3		
34	2.0243e-2	70	2.1395e-2	106	3.9210e-3		
35	-1.9752e-2	71	-2.0908e-2	107	-3.3019e-3		

Table. 5.1. Coefficient values k_n of the MAH expansion

Example 2:

This example illustrates the accuracy and usefulness of the proposed adaptive simultaneous interpolation/extrapolation algorithm over the conventional Laplace method.

Consider the test function described by (see Fig. 5.5),

$$x(t) = \frac{a}{T_1}t - \frac{a}{T_1}(t - T_1) - \frac{a}{T_1}(t - T_2) + \frac{a}{T_1}(t - T_3) \quad (5.23)$$

Its Laplace transform is of the form:

$$X(s) = \frac{A}{s^2} \left(1 - e^{-sT_1} - e^{-sT_2} + e^{-sT_3} \right) \quad (5.24)$$

where $A = \frac{a}{T_1}$ is a constant representing the slope of the above function. Also, T_1 , T_2 and T_3 are constants, defining the signal as shown in Fig. 5.5.

Let, the time-domain samples are considered in the interval $0 \leq t \leq 1$ *nanoseconds*. From Fig. 5.5, it is clear that the time domain data starts from $T_0 = 0$ to a final time $T_f = T_3$. A sampling step size of $\{(T_0 + T_3) / 500\}$ *nanoseconds* is used to obtain the samples.

It is to be noted that, application of the traditional Laplace Transform on (5.23) to get the frequency response at $s = 0$ fails as the response is undetermined at this point.

However, this kind of problem does not affect the proposed adaptive algorithm, and an exact value for the desired function can be interpolated at $s = 0.0$. This is possible due to the frequency equation of the MAH at $s = 0.0$ not being directly related to $(1/s)$. Thus, this specific frequency point (when $s = 0$) can be treated similar to any other frequency response with $s \neq 0$.

During the optimization process, the following parameters are used. From step 1, the origin (t_0) can be determined as:

$$t_0 = \frac{(T_i + T_f)}{2} = 0.5 \text{ nseconds}$$

From step 2, λ is selected in the range: $\frac{t_0}{20} \leq \lambda \leq \frac{t_0}{15}$

Applying the proposed adaptive algorithm, we get scaling factor $\lambda = 0.029$ and order $N = 159$. Fig. 5.5 shows the comparison between the exact sample time data and the interpolated time data up to $1ns$ and they are indistinguishable. Fig. 5.6 - Fig. 5.8 show the comparison of the exact frequency-domain data with the extrapolated frequency data (for absolute, real and imaginary responses) up to 10 GHz and they match accurately.

This example proves that the proposed adaptive optimal simultaneous interpolation/extrapolation algorithm can be used to handle more sophisticated problems as we will see in the examples that are discussed in the next chapter.

Fig. 5.5. Exact and interpolated time data for Example 2

Fig. 5.6. Amplitude of exact and extrapolated frequency data for Example 2

Fig. 5.7. Real Part of exact and extrapolated frequency data for Example 2

Fig. 5.8. Imaginary part of exact and extrapolated frequency data for Example 2

Chapter 6

Proposed Algorithm for Computing Moments of Measured Data

In this chapter, the details of the proposed algorithm for computing moments from time-domain measured data, and also to compute the moments of the unknown y- parameter matrix, $Y_k^{(r)}$ (4.21) is given. In the ensuing discussion, it is assumed that the time-domain measurements ($v_k(t)$) for the given subnetwork are available (Please refer to the formulation in section 3.4).

6.1 Moments of the admittance Matrix

From (3.20), derivatives of the measured subnetwork, $Y^{(r)}$ (the subscript k is dropped in the rest of the discussion for simplicity) can be expressed as

$$\mathbf{Y}^{(r)} = \begin{bmatrix} Y_{11}^{(r)} & Y_{12}^{(r)} & \dots & Y_{1N}^{(r)} \\ Y_{21}^{(r)} & Y_{22}^{(r)} & \dots & Y_{2N}^{(r)} \\ \dots & \dots & \dots & \dots \\ Y_{N1}^{(r)} & Y_{N2}^{(r)} & \dots & Y_{NN}^{(r)} \end{bmatrix} \quad (6.1)$$

Next, the derivatives represented by individual entries in the RHS matrix of (6.1) are needed. From (3.21), these can be expressed as the derivatives in the vector $\mathbf{Y}_v$ as

$$\mathbf{Y}_v^{(r)} = \left[Y_{11}^{(r)} \ Y_{12}^{(r)} \ \dots \ Y_{1N}^{(r)} \ Y_{21}^{(r)} \ Y_{22}^{(r)} \ \dots \ Y_{2N}^{(r)} \ \dots \dots \ Y_{N1}^{(r)} \ Y_{N2}^{(r)} \ \dots \ Y_{NN}^{(r)} \right]^t \quad (6.2)$$

Derivatives $\mathbf{Y}_v^{(r)}$ can be evaluated using the relation (3.27) as

$$\mathbf{Y}_v^{(r)} = \mathbf{F} \mathbf{Y}_f^{(r)} \quad (6.3)$$

Next, derivatives $\mathbf{Y}_f^{(r)}$ can be obtained using (3.30) as follows

$$\sum_{r=1}^n \binom{n}{r} A_s^{(r)} \mathbf{Y}_f^{(n-r)} + A_s \mathbf{Y}_f^{(n)} = \sum_{r=1}^n \binom{n}{r} A^{t(r)} \mathbf{B}^{(n-r)} + A^t \mathbf{B}^{(n)} \quad (6.4)$$

or

$$A_s \mathbf{Y}_f^{(n)} = - \sum_{r=1}^n \binom{n}{r} A_s^{(r)} \mathbf{Y}_f^{(n-r)} + \sum_{r=1}^n \binom{n}{r} A^{t(r)} \mathbf{B}^{(n-r)} + A^t \mathbf{B}^{(n)} \quad (6.5)$$

(6.5) gives a recursive relationship for the computation of derivatives of vector $\mathbf{Y}_f$ in terms of the derivatives of $\mathbf{A}_s$, $\mathbf{A}$ and $\mathbf{B}$. Further, the derivatives of $\mathbf{A}_s$ can be obtained recursively using (3.31) as

$$\mathbf{A}_s^{(n)} = \sum_{r=1}^n \binom{n}{r} \mathbf{A}^{t(r)} \mathbf{A}^{(n-r)} + \mathbf{A}^t \mathbf{A}^{(n)} \quad (6.6)$$

To proceed further, the derivatives of $\mathbf{A}$ matrix and $\mathbf{B}$ vector are needed. From (3.29) the r^{th} derivatives of $\mathbf{A}$ can be computed as

$$\mathbf{A}^{(r)} = \mathbf{W}^{(r)} \mathbf{F} \quad (6.7)$$

Using (3.25) and (3.26) the derivatives $\mathbf{W}^{(r)}$ and $\mathbf{B}^{(r)}$ can be computed as

$$\mathbf{W}^{(r)} = \begin{bmatrix} \mathbf{W}^{(1)(r)} \\ \mathbf{W}^{(2)(r)} \\ \dots \\ \mathbf{W}^{(m)(r)} \end{bmatrix}; \quad \mathbf{B}^{(r)} = \begin{bmatrix} \mathbf{B}^{(1)(r)} \\ \mathbf{B}^{(2)(r)} \\ \dots \\ \mathbf{B}^{(m)(r)} \end{bmatrix} \quad (6.8)$$

Next, r^{th} derivatives of $\mathbf{W}^{(i)}$ and $\mathbf{B}^{(i)}$, represented by $\mathbf{W}^{(i)(r)}$ and $\mathbf{B}^{(i)(r)}$, respectively, are needed. Again, from (3.25) and (3.26) these can be computed as

$$\mathbf{W}^{(i)(r)} = \text{diag} \left[\left(\mathbf{V}^{(i)t} \right)^{(r)} \left(\mathbf{V}^{(i)t} \right)^{(r)} \dots \left(\mathbf{V}^{(i)t} \right)^{(r)} \right]_{N \times N^2}^t \quad (6.9)$$

and

$$\mathbf{B}^{(i)(r)} = \left(\mathbf{Z}_L^{-1(i)} (\mathbf{J}^{(i)} - \mathbf{V}^{(i)}) \right)^{(r)} \quad (6.10)$$

Derivatives of $\mathbf{B}^{(i)}$ can be recursively computed as

$$\mathbf{Z}_L^{(i)} \mathbf{B}^{(i)(n)} = \mathbf{J}^{(i)(n)} - \mathbf{V}^{(i)(n)} - \sum_{r=1}^n \binom{n}{r} \mathbf{Z}_L^{(i)(r)} \mathbf{B}^{(i)(n-r)} \quad (6.11)$$

Next, to proceed further, (6.9) and (6.11) require derivatives of the measurement vector $\mathbf{V}^{(i)}$. For the purpose of illustration, consider a particular node 'j' with time-domain measurements represented by $\mathbf{v}_j^{(i)}(t)$. To compute $\mathbf{V}(s)$ and its derivatives in frequency-domain (superscript 'i' and subscript 'j' are dropped for simplicity), a new algorithm based on Modified Associate Hermite modeling of time-domain data is presented in the next section.

6.2 Computation of Moments of Measured Data

Let the given time-domain data at a particular node be represented as

$$\mathbf{v}(t) = \begin{bmatrix} \mathbf{v}(t_1) & \mathbf{v}(t_2) & \dots & \mathbf{v}(t_m) \end{bmatrix} \quad (6.12)$$

Using the adaptive algorithm proposed in *chapter 5*, and the MAH fit in the time-domain (5.11) can be obtained as

$$\mathbf{v}(t) = \sum_{n=0}^N k_n h_n(t-t_0, \lambda) \quad (6.13)$$

Using the closed-form relation (5.12), the Laplace transform of the above function can be obtained as:

$$V(f) = e^{-j2\pi f t_0} \sum_{n=0}^N (-j)^n k_n h_n(f, \mu) \quad (6.14)$$

where

$$h_n(f, \mu) = \frac{1}{\sqrt{2^n n!} \sqrt{\pi \mu}} w_n(f/\mu) \exp\left(-\frac{f^2}{2\mu^2}\right); \quad (n \geq 0) \quad (6.15)$$

and $\mu = \frac{1}{2\pi\lambda}$

Next, in order to proceed with the proposed model-reduction algorithm, we need to compute the derivatives of $V(f)$ described by (6.14).

Equation (5.12) can be rewritten using (5.1) as (since the exponential factor is independent of n):

$$V(f) = e^{-j2\pi f t_0} e^{\left(-\frac{f^2}{2\mu^2}\right)} \sum_{n=0}^N (-j)^n k_n \frac{1}{\sqrt{2^n n!} \sqrt{\pi \mu}} w_n(f/\mu) \quad (6.16)$$

Let $\alpha = t_0$, $\beta^2 = \frac{\lambda^2}{2}$ and $s = 2\pi f i$, equation (6.16) becomes:

$$V(s) = e^{-\alpha s} e^{(\beta s)^2} \sum_{a=0}^N (-j)^a k_a \frac{1}{\sqrt{2^a a!} \sqrt{\pi \mu}} w_a(s/(2\pi \mu i)) \quad (6.17)$$

or

$$V(s) = P(s) \cdot Q(s) \quad (6.18)$$

where

$$P(s) = e^{-\alpha s} e^{(\beta s)^2} \quad (6.19)$$

and

$$Q(s) = \sum_{a=0}^N (-j)^a k_a \frac{1}{\sqrt{2^a a!} \sqrt{\pi \mu}} w_a(s/(2\pi \mu i)) \quad (6.20)$$

Next, the n^{th} derivative of $V(s)$ can be recursively computed as

$$V^{(n)} = P^{(0)} Q^{(n)} + \sum_{r=1}^n \binom{n}{r} P^{(r)} Q^{(n-r)} \quad (6.21)$$

To proceed further, we need the derivative of $P(s)$ and $Q(s)$.

6.2.1 Calculation of Moments of $P(s)$

Expanding $P(s)$ in Taylor series yields:

$$P(s) = p_0 + p_1 s + p_2 s^2 + \dots \quad (6.22)$$

From (6.19), expanding $e^{-\alpha s}$ using Taylor series:

$$e^{-\alpha s} = 1 - \frac{\alpha s}{1!} + \frac{(\alpha s)^2}{2!} - \frac{(\alpha s)^3}{3!} + \dots \quad (6.23)$$

and also expanding $e^{(\beta s)^2}$ using Taylor series:

$$e^{(\beta s)^2} = 1 + \frac{(\beta s)^2}{1!} + \frac{(\beta s)^4}{2!} + \frac{(\beta s)^6}{3!} + \dots \quad (6.24)$$

Substituting equation (6.23) and (6.24) in (6.19) we get:

$$P(s) = \left(1 - \frac{\alpha s}{1!} + (-1)^2 \frac{(\alpha s)^2}{2!} + (-1)^3 \frac{(\alpha s)^3}{3!} + \dots \right) \times \left(1 + \frac{(\beta s)^2}{1!} + \frac{(\beta s)^4}{2!} + \frac{(\beta s)^6}{3!} + \dots \right) \quad (6.25)$$

or

$$\begin{aligned}
P(s) = & 1 - \alpha s + \left(\beta^2 + \frac{1}{2!} \alpha^2 \right) s^2 - \left(\alpha \beta^2 + \frac{1}{3!} \alpha^3 \right) s^3 \\
& + \left(\frac{1}{2!} \beta^4 + \frac{1}{2!} \alpha^2 \beta^2 + \frac{1}{4!} \alpha^4 \right) s^4 - \left(\frac{1}{2!} \alpha \beta^4 + \frac{1}{3!} \alpha^3 \beta^2 + \frac{1}{5!} \alpha^5 \right) s^5 \\
& + \left(\frac{1}{3!} \beta^6 + \frac{1}{2!2!} \alpha^2 \beta^4 + \frac{1}{4!} \alpha^4 \beta^2 + \frac{1}{6!} \alpha^6 \right) s^6 \\
& - \left(\frac{1}{3!} \alpha \beta^6 + \frac{1}{2!3!} \alpha^3 \beta^4 + \frac{1}{5!} \alpha^5 \beta^2 + \frac{1}{7!} \alpha^7 \right) s^7 + \dots
\end{aligned} \tag{6.26}$$

By grouping the coefficients of similar order together,

$$\begin{aligned}
P(s) = & 1 - \alpha s \\
& + \sum_{n=1}^r \left\{ \left(\frac{1}{n!} \beta^{2n} + \frac{1}{(2n)!} \alpha^{2n} + \sum_{m=1}^{n-1} \frac{1}{(n-m)!2m!} \alpha^{2m} \beta^{2(n-m)} \right) s^{2n} \right. \\
& \left. - \left(\frac{1}{n!} \alpha \beta^{2n} + \frac{1}{(2n+1)!} \alpha^{2n+1} + \sum_{m=1}^{n-1} \frac{1}{(n-m)!(2m+1)!} \alpha^{(2m+1)} \beta^{2(n-m)} \right) s^{2n+1} \right\}
\end{aligned} \tag{6.27}$$

where 'r' represents the number of moments to be computed.

Equating equation (6.22) with (6.27) we get:

$$p_0 = 1$$

$$p_1 = -\alpha,$$

$$p_2 = \beta^2 + \frac{1}{2!} \alpha^2$$

$$p_3 = -\left(\alpha \beta^2 + \frac{1}{3!} \alpha^3 \right)$$

...

$$p_{2n} = \frac{1}{n!} \beta^{2n} + \frac{1}{(2n)!} \alpha^{2n} + \sum_{m=1}^{n-1} \frac{1}{(n-m)! 2m!} \alpha^{2m} \beta^{2(n-m)} \quad (6.28)$$

and

$$\begin{aligned} p_{2n+1} = & \frac{1}{n!} \alpha \beta^{2n} + \frac{1}{(2n+1)!} \alpha^{2n+1} \\ & + \sum_{m=1}^{n-1} \frac{1}{(n-m)! (2m+1)!} \alpha^{(2m+1)} \beta^{2(n-m)} \end{aligned} \quad (6.29)$$

with: $1 \leq n \leq r$

6.2.2 Calculation of moments of $Q(s)$

Expanding (6.20) we have

$$\begin{aligned} Q(s) = & (-j)^0 \frac{k_0}{\sqrt{\sqrt{\pi}\mu}} w_0(s_k) + (-j)^1 \frac{k_1}{\sqrt{2}\sqrt{\pi}\mu} w_1(s_k) \\ & + (-j)^2 \frac{k_2}{\sqrt{2^2 2!} \sqrt{\pi}\mu} w_2(s_k) + \dots + (-j)^N \frac{k_N}{\sqrt{2^N N!} \sqrt{\pi}\mu} w_N(s_k) \end{aligned} \quad (6.30)$$

where $s_k = \left(\frac{s}{2\pi\mu i} \right)$ and N represents the optimized Modified Hermite order.

$$\begin{cases} w_0(s_k) = 1 \\ w_1(s_k) = 2(s_k) \\ \dots \\ w_n(s_k) = 2(s_k)w_{n-1}(s_k) - 2(n-1)w_{n-2}(s_k) \end{cases} \quad (6.31)$$

Substituting (6.31) in (6.30), we come up with the moments of $Q(s)$ as

$$Q(s) = q_0 + q_1s + q_2s^2 + \dots + q_ns^n$$

where

q_0 Represents the summation of all constant factors,

q_1 Represents the summation of all s coefficients,

...

q_n Represents the summation of all s^n coefficients.

6.3 Summary of the Algorithm

A summary of the steps required in the macromodel computation of linear subnetworks characterized by measured data is shown in Fig. 6.1

Fig. 6.1. Summary of the proposed algorithm for analysis of time-domain characterized networks.

6.4 Unified Transient Simulation

Having computed the derivatives, using model-reduction algorithms, a reduced model for $Y_\pi(s)$ can be computed as

$$Y_\pi(s) = \begin{bmatrix} c^{1,1} + \sum_{i=1}^q \frac{k_i^{1,1}}{s-p_i} & \dots & c^{1,n_\pi} + \sum_{i=1}^q \frac{k_i^{1,n_\pi}}{s-p_i} \\ \dots & \dots & \dots \\ c^{n_\pi,1} + \sum_{i=1}^q \frac{k_i^{n_\pi,1}}{s-p_i} & \dots & c^{n_\pi,n_\pi} + \sum_{i=1}^q \frac{k_i^{n_\pi,n_\pi}}{s-p_i} \end{bmatrix} \quad (6.32)$$

where p_i is the i^{th} pole in the common pole-set, $k_i^{l,j}$ are the corresponding residues for $Y_\pi^{i,j}$, respectively. The poles and residues thus obtained are used to form first-order differential equations as

$$\begin{aligned} \frac{d}{dt}[x(t)] - [A][x(t)] - [B][i_\pi(t)] &= 0 \\ [v_\pi(t)] - [C][x(t)] + [D][i_\pi(t)] &= 0 \end{aligned} \quad (6.33)$$

where x is the state vector, A is the state matrix, B , C are matrices which relate inputs to states; states to the outputs, respectively and D is a matrix relating inputs directly to output. (6.33) can be easily coupled with the differential equations representing nonlinear components and simulated using a conventional circuit simulator (SPICE) to obtain transient results.

6.5 Computational Results

In this section, general numerical examples are presented to demonstrate the efficiency and accuracy of the proposed algorithm for analysis of time-domain characterized subnetworks.

Example 1: Single Transmission Line

Fig. 6.2 shows the test circuit containing a single measured subnetwork. In order to validate the proposed algorithm, the measured subnetwork is chosen to be a single transmission line with known TEM model parameters ($L = 297$ nH/cm, $C = 161$ pF/cm, $R = 193$ Ω /cm, $G = 7900$ nMho/cm, $Length = 8$ cm). The source and terminal load resistances are $50\ \Omega$ each. The input signal $v_i(t)$ is a 1 V pulse with 0.2ns rise/fall time and 0.6ns duration. For the purpose of validation, the tabulated data is obtained by characterizing the subnetwork in the environment given in Fig. 6.2 using HSPICE simulation. The transient response (tabulated data) at *nodes one* and *two* for voltage $v_1(t)$ and current $i_2(t)$ are shown in Fig. 6.4. The extrapolated frequency responses are shown in Fig. 6.5. These tabulated data are then used to model the tabulated subnetwork in Fig. 6.3 and the circuit is simulated using the proposed algorithm.

Fig. 6.2. Single transmission line tabulated subnetwork.

Fig. 6.3. Example characterization environment for the tabulated subnetwork.

(a): Exact and interpolated time data (tabulated data)

(b): Exact and interpolated time data (tabulated data)

Fig. 6.4. Time-domain tabulated data for example 1

(a): Amplitude of Exact and extrapolated frequency data

(b): Amplitude of Exact and extrapolated frequency data

Fig. 6.5. Frequency responses for example 1

It should be noted that the voltage $v_I(t)$ is obtained at node one and the current $i_2(t)$ is obtained at node two when port two is shorted to ground. The aim of these data is to come up with the admittance parameters $Y_{ij}(s)$ as

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0} \quad (6.34)$$

and

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = 0} \quad (6.35)$$

and from Fig. 6.3, I_1 can be obtained as

$$I_1 = \left. \frac{V_i - V_1}{R_1} \right|_{V_2 = 0} \quad (6.36)$$

Assuming that the measured subnetwork shown in Fig. 6.2 is passive and symmetric, (i.e. $Y_{22} = Y_{11}$ and $Y_{12} = Y_{21}$), (6.34) and (6.35) for this can be formulated in a closed form as

$$\begin{bmatrix} Y_{11}^{(n)} \\ Y_{21}^{(n)} \end{bmatrix} \begin{bmatrix} V_1^{(n)} \end{bmatrix} = \begin{bmatrix} \frac{V_i^{(n)} - V_1^{(n)}}{R_1} \\ I_2^{(n)} \end{bmatrix} \quad (6.37)$$

where n indicates the different sets of characterizations.

An analytical solution for Y_{11} and Y_{21} can be obtained by applying the proposed algorithm. For this purpose, time-domain data for $v_i(t)$, $v_I(t)$, and $i_2(t)$ are fitted with MAH functions and the frequency-domain response is obtained analytically for $V_i(s)$, $V_I(s)$ and $I_2(s)$. Corresponding details of Modified Associate Hermite expansion coefficients for voltages and currents are listed in Table 6.1 and Fig. 6.4, Fig. 6.5. The modeling of the source voltage $v_i(t)$, using MAH functions is done similar to the one described in Chapter 5, (2).

Optimized Vector	Order N	Origin t_0 (ns)	Scaling Factor $\lambda * 1e^9$
v_i	159	0.5	0.029
v_I	144	1.96	0.116
i_2	194	1.5	0.082

TABLE 6.1. Results of MAH expansion coefficients for example 1.

From Fig. 6.4 and Fig. 6.5, it is obvious that a good agreement between the measured/interpolated time-responses and the exact/extrapolated frequency responses has been obtained. Also, Fig. 6.6 and Fig. 6.7 show the comparison of admittance parameters from both HSPICE (which represents the exact results), and the proposed algorithm. As seen, they match accurately up to highest frequency of interest f_{MAX} . Table 6.2, gives details about the number of hops and poles related to Y_{11} and Y_{21} during CFH. Fig. 6.8 shows the frequency responses at node V_2 (Fig. 6.2) from both HSPICE and the proposed macro-model and they match accurately.

(a): Frequency response of Y_{II} (magnitude)

(b): Frequency response of Y_{II} (real-part)

(c): Frequency response of Y_{II} (imaginary-part)

Fig. 6.6. Frequency response of Y_{II} for example 1 using proposed moment generation and CFH algorithm.

(a): Frequency response of Y_{21} (magnitude).

(b): Frequency response of Y_{21} (real-part).

(c): Frequency response of Y_{21} (imaginary-part).

Fig. 6.7. Frequency response of Y_{21} for example 1 using proposed moment generation and CFH algorithm.

Y_{ij}	f_{MAX}	Number of Hops	Number of Poles
Y_{11}	2.7 GHz	16	24
Y_{21}	2.7 GHz	16	24

TABLE 6.2. Number of hops and poles required for modeling of admittance parameters for example 1.

Fig. 6.8. Frequency response at node V_2 for example 1 (Fig. 6.2)

Example 2: Coupled Transmission line

In this example, a four port subnetwork characterized by tabulated data is analyzed. The circuit shown in Fig. 6.9 was evaluated with a 5cm long coupled transmission line. The interconnect structures analysed with respect to the performance of quasi-TEM model parameters are known as

$$\mathbf{L} = \begin{bmatrix} 417.353 & 60.262 \\ 60.262 & 417.353 \end{bmatrix} (nH/m), \quad \mathbf{C} = \begin{bmatrix} 112.162 & -16.1953 \\ -16.1953 & 112.162 \end{bmatrix} (pF/m),$$

$$\mathbf{R} = \begin{bmatrix} 12.697 & 0 \\ 0 & 12.697 \end{bmatrix} (\Omega/m),$$

and

$$R1 = R2 = R3 = R4 = 60 \, \Omega.$$

The input signal $v_i(t)$ is a source of 1V pulse with 0.4ns rise/fall time and 1.2ns duration. The subnetwork is characterized in a test environment given in Fig. 6.10 using HSPICE simulation and the transient response (tabulated data) at *nodes one, two, three and four* for voltages v_i and v_l and currents i_2 , i_3 , and i_4 are shown in Fig. 6.11 to Fig. 6.15, respectively. These data are then used to model the tabulated subnetwork in Fig. 6.9 and the circuit is simulated using the proposed technique. Also, Table 6.3 lists the details of Modified Associate Hermite expansion coefficients for the voltages and currents under consideration.

Fig. 6.9. Four port subnetwork characterized by tabulated data.

Fig. 6.10. Example characterization environment for the tabulated subnetwork.

<i>Optimized Vector</i>	<i>Order N</i>	<i>Origin t_0 (ns)</i>	<i>Scaling Factor $\lambda * 1e^9$</i>
v_i	168	1	0.056
v_1	170	1.35	0.0753
i_2	183	1.5	0.0871
i_3	177	1.2	0.0655
i_4	173	1.5	0.0911

TABLE 6.3. Results of the MAH expansion coefficients for example 2..

(a): Exact and interpolated time data

(b): Amplitude of Exact and extrapolated frequency data

Fig. 6.11. Time and frequency responses of V_i for example 2.

(a): Exact and interpolated time data

(b): Amplitude of Exact and extrapolated frequency data

Fig. 6.12. Time and frequency responses of V_I for example 2.

(a): Exact and interpolated time data

(b): Amplitude of Exact and extrapolated frequency data

Fig. 6.13. Time and frequency responses of I_2 for example 2.

(a): Exact and interpolated time data

(b): Amplitude of Exact and extrapolated frequency data

Fig. 6.14. Time and frequency responses of I_3 for example 2.

(a): Exact and interpolated time data

(b): Amplitude of Exact and extrapolated frequency data

Fig. 6.15. Time and frequency responses of I_4 for example 2.

As shown in Fig. 6.10, the voltage $v_1(t)$ and the circuit current $i_2(t)$, $i_3(t)$, and $i_4(t)$ are obtained when *port two, three and four* are shorted to ground. In this case, the original admittance parameters $Y(s)$ can be computed as

$$Y_{11} = \frac{I_1}{V_1} \bigg|_{V_2, V_3, \text{ and } V_4 = 0}, \quad Y_{21} = \frac{I_2}{V_1} \bigg|_{V_2, V_3, \text{ and } V_4 = 0}$$

$$Y_{31} = \frac{I_3}{V_1} \bigg|_{V_2, V_3, \text{ and } V_4 = 0}, \quad Y_{41} = \frac{I_4}{V_1} \bigg|_{V_2, V_3, \text{ and } V_4 = 0}$$

where

$$I_i = \frac{V_i - V_1}{R_1}$$

Also, for the measured subnetwork shown in Fig. 6.9 we have (symmetric and passive)

$$Y_{11} = Y_{22} = Y_{33} = Y_{44}$$

$$Y_{12} = Y_{21} = Y_{34} = Y_{43}$$

$$Y_{13} = Y_{31} = Y_{24} = Y_{42}$$

$$Y_{14} = Y_{41} = Y_{23} = Y_{32}$$

An analytical solution for Y_{11} , Y_{21} , Y_{31} and Y_{41} can be obtained by applying the proposed algorithm. For this purpose, time-data measurements for $v_i(t)$, $v_1(t)$, $i_2(t)$, $i_3(t)$ and $i_4(t)$ are fitted with MAH functions and the frequency-domain response is obtained analytically for $V_i(s)$, $V_1(s)$, $I_2(s)$, $I_3(s)$ and $I_4(s)$. Corresponding details of Modified Associate Hermite expansion coefficients for voltages and currents are listed in Table 6.3.

From Fig. 6.11 - Fig. 6.15, it is obvious that a good agreement between the exact and interpolated time-responses, and the exact and extrapolated frequency-responses is obtained. Also, Fig. 6.16 - Fig. 6.19 show the comparison of admittance parameters from both

(a): Frequency response of Y_{II} (magnitude-part)

(b): Frequency response of Y_{II} (real-part)

(c): Frequency response of Y_{II} (imaginary-part)

Fig. 6.16. Frequency response of Y_{II} for example 2 using proposed moment generation and CFH algorithm.

(a): Frequency response of Y_{21} (magnitude) (b): Frequency response of Y_{21} (real-part) (c): Frequency response of Y_{21} (imaginary-part)

Fig. 6.17. Frequency response of Y_{21} for example 2 using proposed moment generation and CFH algorithm.

(a): Frequency response of Y_{31} (magnitude)

(b): Frequency response of Y_{31} (real-part)

(c): Frequency response of Y_{31} (imaginary-part)

Fig. 6.18. Frequency response of Y_{31} for example 2 using proposed moment generation and CFH algorithm.

(a): Frequency response of Y_{4I} (magnitude) (b): Frequency response of Y_{4I} (real-part) (c): Frequency response of Y_{4I} (imaginary-part)

Fig. 6.19. Frequency response of Y_{4I} for example 2 using proposed moment generation and CFH algorithm.

HSPICE (which represents the exact results), and the proposed algorithm. As seen, they match accurately up to highest frequency of interest f_{MAX} . Table 6.4, gives details about the number of hops and poles related to Y_{11} , Y_{21} , Y_{31} and Y_{41} during CFH. Fig. 6.20 shows the frequency responses at node V_4 (Fig. 6.9) from both HSPICE and the proposed macro-model and they match accurately.

Y_{ij}	f_{MAX} GHz	Number of Hops	Number of Poles
Y_{11}	2.6	26	30
Y_{21}	1.8	16	20
Y_{31}	2.8	24	28
Y_{41}	1.9	22	24

TABLE 6.4. Number of hops and poles required for modeling of admittance parameters for example 2.

Fig. 6.20. Frequency response at node V_4 for example 2 (Fig. 6.9)

Example 3: Distributed Transmission Line Network

In this example, a three port tabulated subnetwork is considered (shown in Fig. 6.21).

Fig. 6.21. Three port subnetwork characterized by tabulated data.

For the same input signal $v_i(t)$ as in the previous example, the tabulated subnetwork is characterized in a test environment given in Fig. 6.22 using HSPICE simulation. Here v_{1a} , i_{2a} and i_{3a} are obtained when *ports* 2 and 3 are shorted to ground in order to get Y_{11} , Y_{21} and Y_{31} . v_{2b} , and i_{3b} are obtained when *ports* 1 and 3 are shorted to ground in order to get Y_{22} and Y_{32} ,

and v_{3c} is obtained when *ports 1* and 2 are shorted to ground in order to get Y_{33} . The corresponding time-domain data are given in Fig. 6.23 and Fig. 6.24. These graphs also provide the comparison of the proposed modeling using MAH functions. Table 6.5 lists the details of Modified Associate Hermite expansion coefficients.

Fig. 6.22. Example characterization environment for the tabulated subnetwork.

<i>Optimized Vector</i>	<i>Order N</i>	<i>Origin t_0 (ns)</i>	<i>Scaling Factor $\lambda * 1e^9$</i>
v_{1a}	129	1.5	0.0918
i_{2a}	139	1.7	0.0914
i_{3a}	108	1.7	0.10034
v_{2b}	184	1.5	0.0828
i_{3b}	175	1.7	0.0982
v_{3c}	143	1.5	0.091

TABLE 6.5. Results of the MAH expansion coefficients for example 3.

Fig. 6.25 - Fig. 6.27 show the comparison between real, imaginary and magnitude values of Y_{11} , Y_{21} , Y_{31} , Y_{22} , Y_{23} , and Y_{33} respectively obtained directly from HSPICE simulation and the proposed algorithm and they match accurately. Moreover, Table 6.6 provides the details about the number of hops and poles related to Y_{11} , Y_{21} , Y_{31} , Y_{22} , Y_{23} , and Y_{33} during CFH. Fig. 6.28 shows the frequency responses at node V_2 (Fig. 6.21) from both HSPICE and the proposed macromodel and they match accurately.

Fig. 6.23. Time and frequency responses for example 3

Fig. 6.24. Time and frequency responses for example 3

Fig. 6.25. Frequency responses (real-part) for example 3 using proposed moment generation and CFH algorithm.

Fig. 6.26. Frequency responses (imaginary-part) for example 3 using proposed moment generation and CFH algorithm.

Fig. 6.27. Frequency responses (magnitude) for example 3 using proposed moment generation and CFH algorithm.

Y_{ij}	f_{MAX} GHz	Number of Hops	Number of Poles
Y_{11}	1.9	7	10
Y_{22}	1.9	7	10
Y_{33}	1.8	7	10
Y_{21}	1.8	10	15
Y_{31}	1.8	6	10
Y_{23}	1.8	6	10

TABLE 6.6. Number of hops and poles required for modeling of admittance parameters for example 3.

Fig. 6.28. Frequency response at *port 2* for example 3 (Fig. 6.21)

Chapter 7

Conclusions and Future Research

This chapter provides a summary of the work that is presented in this thesis. Also, some future research ideas are discussed.

7.1 Summary

This thesis deals with the simulation of high-speed interconnect networks characterized by tabulated data using industry standard simulators such as SPICE. In specific, it focuses on time-domain tabulated data. For this purpose, a new analytical technique is proposed in order to achieve simultaneous interpolation and extrapolation in time and frequency domains using Modified Associate Hermite coefficients. It is to be noted that, fitting of time domain data using MAH coefficients would give accurate analytical representation of the signal in both time and frequency domains. This ensures that the technique is computationally efficient. A new algorithm for computation of moments of subnetworks charac-

terized by tabulated data is presented for simulation using model-reduction algorithms. Following are the key features of the proposed algorithm.

1. Analytical representation of the time-domain data using MAH can be converted analytically to the frequency domain.
2. A novel optimization technique is developed to find the coefficients of Modified Associate Hermite approximation. This algorithm is easy to apply and helps to replace the time-domain data with a proper analytical form. The proposed optimization technique provides an accurate response for the whole frequency range. Unlike Laplace transform, the analytical form of the Modified Associate Hermite in the frequency domain is not being directly multiplied by $(1/s)$. A proof of this situation has been presented by a numerical example in chapter 4.
3. A set of closed-form equations has been developed for computation of moments of time-domain measured data using MAH representation. Complex frequency hopping based model-reduction algorithm has been extended to accurately and quickly capture the frequency response up to the maximum frequency of interest without requiring hundreds of frequency point analysis.

7.2 Suggestions for Future Research

An emerging issue of interest is the passivity of the macromodel. It would be of great interest to couple the accuracy and efficiency of the MAH coefficients while ensuring the passivity of the macromodel. Also, it would be of interest to explore the possibility of generalizing the optimization algorithm in section (5.2) to handle wide-variety of time-domain data. It would also be of interest to find how to minimize the number of data points required to obtain an accurate time-domain fit.

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