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Finite Element Study of the Interaction and Shielding Effects of Multiple Cracks in Three-Dimensions

by

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A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering
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January, 2000

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The Faculty of Graduate Studies and Research
acceptance of the thesis

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in partial fulfilment of the requirements for
the degree of *Doctor of Philosophy*

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Abstract

This thesis deals with the fracture mechanics study of crack interaction in three-dimensions (3D). The main objective is to investigate the interaction effects between two macro cracks on the stress intensity factors (SIF's). These cracks may be in arbitrary orientations and in this study their configurations are varied in terms of their size, shape aspect ratio, and type (i.e., surface or embedded cracks).

To overcome the modeling difficulty involved in studying arbitrarily oriented interacting cracks, the current work focuses on constructing a finite element model that combines the sub-modeling technique and a mesh generator that can build up a mesh grid based on the size, depth and orientation of the interacting crack sub-models. The capability of the model is extended to the study of the interaction between embedded cracks and the interaction between surface and embedded cracks. Where possible, the results of the current FE model are compared with those available in the literature and good agreement between them is obtained.

The FE model developed is applied to various cases of two asymmetric interacting coplanar or non-coplanar surface or embedded cracks in a thick plate. The results obtained indicated that the SIF's of the coplanar cracks are magnified when the interacting cracks are sufficiently close. For non-coplanar cracks, the SIF's may experience a reduction in values as they overlap. The maximum value of this reduction or shielding effect is found to depend on the position along the crack-front, the shape aspect ratio, the crack depth to material thickness ratio, the type of loading and the relative proximity of the cracks. In case of overlapping cracks, the results have also shown that the assessment rules commonly used in the literature may not be conservative.

Acknowledgments

It is a distinct pleasure for me to have reached the stage at which I am able to express, formally, my gratitude to those who have helped me over the last four years. My advisors, Dr Robert Bell and Dr Choon-Lai Tan, have been my mentors for many years and have given me a tremendous amount of sound advice. It has been their dynamic personalities that have inspired me in much of my work. Others, particularly Ahmed Kamel and my brothers Dr Medhat Mousse and Dr Mohamed Moussa have given me unconditional support. I am also deeply grateful for the full financial support of Carleton University.

It was the smiling faces of my daughter Eman and my wife Lamis and the sweet memories of my grandmother that have fueled me throughout this work. Special thanks to my dear parents for their love and care.

Dedication

To my grandmother.
may she rest in peace.

*When things go wrong, as they sometimes will,
When the road youre trudging seems all up hill,
When the funds are low, and the debts are high,
And you want to smile, but you have to sigh,
When care is pressing you down a bit,
Rest if you can, but dont quit.
Life is a queer with its twists and turns,
As everyone of us sometimes learns,
And many of failure turns about,
When he might have won had he stuck it out;
Dont give up though the pace seems slow,
you may succeed with another blow.
Success is failure turned inside out,
The silver tint of the clouds of doubt,
And you never can tell how close you are,
It may be near when it seems so far,
So stick to the fight when youre hardest hit,
Its when things seem worse,
that you MUST NOT QUIT.*

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List of Symbols

a_Q	The depth of crack Q
a_P	The depth of crack P
a_{max}	Maximum crack depth
a_{new}	Re-characterized crack depth
a	Crack depth
$a:c$	Aspect ratio
$a:t$	Depth ratio
c_Q	Half the length of crack Q
c_P	Half the length of crack P
c_{new}	Re-characterized crack length
c	Half the length of crack Q
d	half-width of plate
h	Relative vertical distance between crack Q and crack P
k	Coalescence coefficient
l	crack length
q	See Eq(3.21)
r	Distance between the center of two arbitrary oriented cracks
r_η	Radial distance from the tip point on the crack front
r_p or r_y	Plastic zone size
s	Relative horizontal distance between crack Q and crack P
s_Γ	incremental distance along Γ
t	Plate thickness
u	Displacement

w	Finite width
E	Young's Modulus
F	Dimensionless correction factor
F_N	Imperial correlation factor
G	Strain energy release rate
G_I	Strain energy release rate for mode I
H	Total height of plate
H'	See Eq(2.8)
J	J-integral
J_{inP}	The J integral for interacting crack P
J_{isP}	The J integral for isolated crack P
J_{inQ}	The J integral for interacting crack Q
J_{isQ}	The J integral for isolated crack Q
K_{Coal}	Stress intensity factor at coalescence
K_I	Stress intensity factor for mode I
K_{II}	Stress intensity factor for mode II
K_{in}	Stress intensity factor for interacting crack
K_{is}	Stress intensity factor for isolated crack
K_{inP}	Stress intensity factor for interacting crack P
K_{isP}	Stress intensity factor for isolated crack P
K_{inQ}	Stress intensity factor for interacting crack Q
K_{isQ}	Stress intensity factor for isolated crack Q
K_θ	Stress intensity factor as a function of θ
$K_{\theta_{max}}$	The maximum value of K_θ

N_{coal}	Computed crack life
N_{exp}	Experimental crack life
M_f	Correction factor to account for the free surface effect
Q'	Shape factor
T	Traction
W	Strain energy density

Greek Symbols

β_Q	See Eq(6.1)
β_P	See Eq(6.2)
γ_Q	Interaction factor for crack Q. see Eq(3.25)
γ_P	Interaction factor for crack P. see Eq(3.26)
λ	A correction exponent for
ν	Poisson's ratio
Ω	The interaction zone
ϕ_Q	The angle along the front of crack Q
ϕ_P	The angle along the front of crack P
ϕ	The angle along the front a crack
ψ_Q	See Eq(5.1)
ψ_P	See Eq(5.2)
σ	Applied uniform direct stress
$\sigma(z)$	Applied direct bending stress
σ_y	Stress in Y direction close to crack tip
σ_x	Stress in X direction close to crack tip
σ_{ys}	Yield Strength

σ_{ij}	Stress tensor
τ_{xy}	Shear Stress close to crack tip
θ	Angle around the crack tip
θ_{max}	The angle of the maximum tangential stress around the crack tip
θ_o	See Fig (3.2)
θ_s	Angle of rotation for arbitrary oriented cracks
Γ	Integration path defined for J-integral
Π	Potential energy of a body

Abbreviations

LEFM	Linear elastic fracture mechanics
SIF	Stress intensity factor
MSD	Multi-site damage
2D	Two dimensional
3D	Three dimensional
PSB	persistent slip bands
SESA	Society for experimental stress analysis
BEM	Boundary element method

Chapter 1

Introduction

The availability of multiple sites for crack initiation makes the multiple cracking problem a common feature in many kinds of material failures, such as fatigue, thermal fatigue, corrosion fatigue and stress corrosion cracking (SCC). As shown in Fig (1.1), stress corrosion cracking is one type of material failure which occasionally occurs on structural surfaces exposed to a corrosive environment. The proposed research is directed to study the elastic stress-field surrounding multiple semi-elliptical interacting surface or embedded cracks and its effect on their growth, coalescence and shielding mechanism using the finite element method (FEM). As a typical case of multiple cracking, SCC often occurs in localized patches similar to that shown in Fig (1.2). These patches typically contain crack arrays that are quite regular in crack length and spacing. Multiple cracks are also found in nearly coplanar or diagonally banded arrays, and in some cases these arrays may consist of only a few cracks.

Analysis of crack nucleation in these arrays has indicated that the inelastic strain and strain rate are key parameters requiring investigation. However, once a threshold crack size is reached, linear elastic fracture mechanics (LEFM) becomes a useful

tool for characterizing the continuous growth of cracks and the eventual fracture. Thus, the need for accurate stress intensity factor (SIF) solutions for such cases of interacting cracks will enable the subcritical growth of these flaws in multi-site damaged structures to be modeled.

When crack colonies are present in a component/structure, its failure is frequently a consequence of their interaction and coalescence. Therefore, a better understanding of the conditions under which interacting cracks may coalesce or shield each other is of importance in assessing the integrity of damaged structures. However, this knowledge of the interaction behavior for surface cracks is limited in the literature. This limitation is due to the lack of rational explanation of the relation between the development of the crack configurations and their interaction behavior in reported experimental observations, Wang et al. (1996). In addition, most material testing methodologies presently used are mainly based on single crack specimens, which limit their applicability to multiple cracking problems.

Recognizing the complexity of the multiple cracking problem is important to understand some of the uncertainties found in the behavior of the problem. It seems difficult at present to derive a simple expression that can describe the overall behavior of all interacting cracks without a knowledge of the effects of their size, shape and relative proximity on their growth mechanism. It might be feasible in some circumstances to represent such behavior by using the characteristics of the largest crack(s). However, this may not always be reliable, as will be discussed in the subsequent chapters. The dominance of coalescence between interacting cracks may not always be sustained due to the *shielding* effects developing between non-coplanar cracks. Such interaction behavior could result in the arrest of smaller cracks and the decrease of the growth rate of the larger cracks.

As have been reported in the literature, multiple cracking sites have been observed in several engineering components, such as pressure vessels, welded joints,

pipes, offshore structures, railroad tracks, die casting and sheet-metal forming dies. Another common problem of multi-site damage is related to aging aircraft structures which came to light after the famous accident of Aloha airlines in 1987, as shown in Fig (1.3). It is believed that this accident was caused by the linking, into one large crack, of numerous small cracks at a number of fastener holes in the longitudinal skin splices located at the front part of the fuselage.

The common features associated with multiple cracking problems have been shown experimentally to be mechanics related. The independence of the interaction behavior from both material and environmental effect implies that knowledge of the stress distribution between the neighboring cracks is a key facet to understanding the phenomena of crack interaction, Wang et al. (1996) and Stonesifer et al. (1993).

1.1 Research Objective

In order to make additional contributions to the work which has been achieved so far by previous researchers, the present research investigates the elastic stress-fields surrounding multiple semi-elliptical interacting surface or embedded cracks using the finite element method (FEM). The common issues associated with crack interaction behavior such as shielding and coalescence are investigated using the following approach :

- (a) Constructing a three-dimensional Finite Element (FE) model for two semi-elliptical surface or embedded cracks. This FE model will implement the sub-modeling technique to construct two separately controlled macro crack sub-models. This approach will result in a FE model that is capable of treating different crack configurations and their proximity and orientation with respect to each other, in a relatively simpler manner.

- (b) Modeling the interaction effect in cases of coplanar or non-coplanar asymmetric surface cracks under remote tension or pure bending loads. Applying symmetry conditions at the plate surface, the same FE model is used to investigate the interaction of non-coplanar asymmetric embedded cracks under remote tension loading. Other cases of crack interaction are also be considered in this study. Some of these cases include the interaction between a surface crack and another embedded crack as well as the interaction of arbitrarily oriented surface cracks under remote tension loading.

1.2 Thesis Outline

The next Chapter contains a review of the literature related to the problem of multiple cracking. The studies that had been carried out to obtain the SIF of a single surface crack are reviewed, so are the general features associated with multiple cracks, or multi-site damage, MSD. Some comments on the use of some of these techniques in developing assessment rules to quantify the integrity of multiple-cracked structures are also presented.

In Chapter 3, the difficulties associated with modeling multiple cracks and some of the approaches to overcome these difficulties are discussed. The sub-modeling approach used in the current work is then presented. This approach combines the sub-modeling technique, with its ability to build separate crack sub-models with different lengths and depths, and a mesh generator that can construct a mesh grid based on the size, depth and orientation of the crack sub-models it contains. The technique used to calculate the stress intensity factors, SIF's, along the fronts of interacting cracks will be described. This will then be followed by a discussion of some test cases to verify the current FE model in 2D and 3D.

Chapter 4 presents the results for the case of interaction between two surface or embedded coplanar cracks. The variations of the interaction factor with the relative proximity of the interacting cracks, the aspect ratio of the semi-elliptical or elliptical cracks modeled and the depth to plate thickness ratio of the cracks are discussed for cases of coplanar surface cracks under remote tension or pure bending loading. Next, the variations of the interaction factor along the front of two asymmetric surface coplanar cracks are discussed.

In Chapter 5, the results for the interaction cases of two surface or embedded non-coplanar cracks in an elastic infinite plate are presented. The variations of the interaction factor with the relative, vertical and horizontal, proximity of the interacting cracks are also discussed. The results dealing with the interaction effect between non-coplanar surface crack and another embedded crack under remote tension are presented in Chapter 6. The second case in this chapter studies the interaction between two arbitrary oriented surface cracks under remote tension.

Chapter 7 presents the conclusion drawn from the work that is accomplished in this thesis dealing with the interaction effect of 2D and in 3D multiple cracks. Some suggestions for future numerical and experimental work are also presented.

(a) gas transmission pipe line

(b) typical corrosion deposits on the pipe

Figure 1.1: Corrosion in gas transmission pipelines, Revie et al. (1998).

Figure 1.2: Crack colonies caused by stress corrosion cracking in pipelines, Revie et al. (1998).

(b) catastrophic failure in aerospace structures caused by multi-site damage

Figure 1.3: Multi-site damage in Aerospace structures, the Aloha (a) multi-site damage in lap joints and its consequences on (b) the catastrophic failure of the Aloha airline flight 243 airframe.

Chapter 2

Literature Survey

This chapter contains a review of the literature deemed most pertinent to the problem of multiple cracking. In Section (2.1), some of the industrial cases where single and multiple cracks exist are discussed. The next section summarizes some of the studies that had been carried out to obtain the SIF of a single surface crack. The third section discusses the general features associated with multiple cracks, or multi-site damage, MSD. Then, in Section (2.4) some of the techniques used to solve for the SIF values of interacting cracks are discussed. Some comments on the use of some of these techniques in developing assessment rules to quantify the integrity of multiple-cracked structures are presented in Section (2.5). Finally, some concluding remarks on the multiple cracking problem are given.

2.1 The multiple cracking problem - Introduction

Surface cracks are perhaps the most common type of flaws found in many practical structures, such as, pressure vessels, pipeline systems, offshore and aerospace structures. The service life of a cracked component in, for example, fatigue loading conditions typically consists of three stages: (i) crack initiation, (ii) stable crack

propagation and (iii) structural instability or complete fracture. Crack initiation generally begins with the formation of micro-cracks at areas of highly localized plastic strain known as persistent slip bands (PSB). The cracks then propagate from these micro-cracks through a multiple slip mechanism created by the repeated loading and unloading of material. Depending upon the size of the crack and the strain amplitudes, most of the component life is spent in the initiation and propagation stages. A surface crack is one which intersects a free surface of a section. It has a depth which is less than the thickness of the section in which it lies, and a surface length which is less than the section width. Throughout history, many structural failures have occurred due to the initiation and growth of surface cracks (Pitt et al., 1999). Among the main sources surface cracks originate from are :

- manufacturing defects present prior to the use of the component
- location of stress concentrations.
- material defects such as voids and clusters

The availability of multiple sites for crack initiation make multiple cracking a common feature in many types of material failures, including stress corrosion cracking (Wang et al., 1994b), fatigue (Suh et al., 1992), corrosion fatigue (Wang et al., 1994a) and thermal fatigue (Winkler et al., 1992). In many cases, multiple cracking is related to the overall aging process of the fractured components and the consequences of their structural degradation. For example, in gas transmission pipelines, multiple cracking takes the shape of crack colonies surrounding longitudinal and spiral welds. As shown in Fig (2.1.a), these welds serve as the source of residual stress that acts as the driving force to propagate small flaws initiated during the welding process. Other sources of multiple cracks are found around dents or in locations that suffer from possible corrosion damage, such as areas surrounding rivets in aircraft fuselages. When these colonies occur, it is standard to document the shape

and spacing of their cracks content. As shown in Fig (2.1.b), the center and the relative location of every crack in the colony is documented. This process helps in understanding the main feature of these cracks and the source that caused them.

Examples of multiple cracks are also found in aging structures of conventional and nuclear power stations. Cases of multi-site damage are also common in manufacturing. Figure (2.2.a) shows a case of MSD in rollers used in the sheet metal forming process. (Kuo et al., 1997b). Cases of MSD caused by contact fatigue are also found in railroad tracks as shown in Fig (2.2.b). Perhaps the most common problem of MSD is related to aging aircraft structures which came to light after the famous accident of Aloha airlines in 1987, as shown in Fig (2.2.c). It is believed that this accident was caused by the linking, into one large crack, of numerous small cracks at a number of fastener holes in the longitudinal skin splices located at the front part of the fuselage.

The key to study sites of multiple cracking is to study the propagation force that cause crack colonies to grow. Fracture mechanics may be used to estimate the propagation life of a component containing an existing defect(s). Linear Elastic Fracture Mechanics (LEFM) is the study of flaws in linear elastic materials and the resulting fracture behavior related to the flaw size, stress level and the material properties. Performing experiments on glass, Griffith (1920) formulated the concept that a crack in a component will propagate to minimize the total energy of the system. Irwin (1949) extended Griffith's theory to include ductile materials through consideration of the energy associated with plastic deformation. In addition to using energy methods, the fracture process may also be analyzed using stress analysis based on the concepts of elastic theory.

A crack in a component may, under loads, suffer one or any combination of the following three different modes of deformation, as shown in Fig (2.3):

- Mode I: opening or tensile mode, where the crack surfaces move directly apart.

- Mode II: sliding or in-plane shear mode, where the crack surfaces slide over one another in a direction perpendicular to the leading edge of the crack.
- Mode III: tearing or anti-plane shear mode, where the crack surfaces move relative to one another and parallel to the leading edge of the crack.

The crack opening mode (Mode I) is the most common as it tends to promote the fastest growth through opening of the crack faces directly. The elastic stresses in the vicinity of the crack tip for Mode I loading are given by Westgaard (1939) as follows:

$$\begin{aligned}\sigma_y &= \frac{K_I}{\sqrt{2\pi r_\theta}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \sigma_x &= \frac{K_I}{\sqrt{2\pi r_\theta}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \tau_{xy} &= \frac{K_I}{\sqrt{2\pi r_\theta}} \left(\sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right)\end{aligned}\quad (2.1)$$

where the notation is explained in Fig (2.4). The singularity in Eq (2.1) indicates a rapid rise in the stress level as r_θ tends to zero at the crack tip. This creates a zone of localized plasticity at the crack tip that is constrained by the surrounding elastic field. The size of this plastic zone under monotonic loading is given by

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \quad \text{for plane stress} \quad (2.2)$$

$$r_y = \frac{1}{6\pi} \left(\frac{K_I}{\sigma_{ys}} \right)^2 \quad \text{for plane strain} \quad (2.3)$$

where σ_{ys} is the yield strength of the material and K_I is referred to as the stress intensity factor (SIF) for Mode I. The parameter K_I relates the stress field in the vicinity of the crack tip to quantities such as the applied stress level, the crack size and the component geometry. For relatively simple geometries, such as through

thickness cracks, classical solutions are available in several handbooks. (Tada et al., 1973; Sih, 1973; Rooke and Cartwright, 1976). For surface cracks with more complicated shapes, exact solutions cannot be easily obtained. In these cases, solutions of the SIF's have been obtained using analytical, experimental and numerical methods. The next section describe some of these solutions.

2.2 Review of solutions for a single surface crack

Early expressions in the literature for the Mode I SIF of a semi-elliptical surface crack, shown in Fig (2.5), originated from the study of a flat elliptical crack embedded in an infinite elastic body which is subject to a uniform stress perpendicular to the plane of the crack, i.e. uniform Mode I loading. Green and Sneddon (1950) analyzed the flat elliptical crack problem and obtained equations for the stresses and displacements at any point in an infinite body. Irwin (1962) used their work together with the crack opening displacements to obtain the exact expression relating the energy release rate, G_I , around the elliptical crack under plane strain conditions to the SIF. Irwin referred to this factor, K_I , as the crack extension force. The relationship between K_I and G_I for plane strain conditions is

$$K_I = \sqrt{\frac{G_I E'}{1 - \nu^2}} \quad (2.4)$$

which leads to the following exact expression for the Mode I SIF around the embedded elliptical crack in an infinite solid subjected to uniform tension.

$$K_I = \sigma_t \sqrt{\frac{\pi a}{Q'}} \left(\frac{a^2}{c^2} \cos^2 \phi + \sin^2 \phi \right)^{\frac{1}{4}} \quad (2.5)$$

where, Q' is the square of the elliptical integral of second kind. Barsom and Rolfe (1987). An approximate expression of Q' is explained next in this section. The above equation indicates that the largest SIF occurs on the minor axis of the ellipse.

This occurrence could be called the ‘curvature’ or ‘geometry’ effect where the bigger the radius of curvature of a crack front, the lower the stiffness and the larger the crack opening, giving a larger SIF. Equation (2.5) further shows that the SIF for an embedded circular crack, ($\frac{a}{c} = 1$), in a uniform stress field is the same all around. If this embedded crack is cut in half to produce a semi-circular crack an increase in SIF will occur near the intersection of the crack front with the free surface due to the reduction of stiffness at this point. For a decreasing $\frac{a}{c}$ the free surface effect will eventually be offset by the ‘geometry’ effect and so at some $\frac{a}{c}$ value the maximum SIF will switch from occurring near the intersection of the crack with the free surface to occurring at the point of deepest penetration.

In real problems, with finite plate thickness, the back face has a pulling effect on the crack, increasing the SIF values around the crack front by an amount depending on $\frac{a}{t}$. Other factors that influence the SIF of a surface crack are finite plate width and varying stress fields such as that due to the bending load. Because of the high level of SIF variation around surface crack fronts (specially under combined tension and bending loading), the solution of 3D cracks is complex. The general form of the Mode I SIF for surface crack comes from equation (2.5) as

$$K_I = F\sigma\sqrt{\frac{\pi a}{Q'}} \quad (2.6)$$

where σ is a measure of the applied stress (tension and /or bending) and F is a dimensionless correction factor that is a function of some or all the variable $\frac{a}{c}$, $\frac{a}{t}$, $\frac{c}{w}$ and ϕ depending on the particular estimate and its range of application. For surface cracks with $0 < \frac{a}{t} < 0.5$ and $0 < \frac{a}{c} < 1.0$, Irwin (1962) estimated that $K_o = \sqrt{1.2}$ at $\phi = \frac{\pi}{2}$. Approximate expressions have been developed for Q' , for which there is no closed form exact solution. The following were first used by Merkle (1973), where

$$\begin{aligned} Q' &= 1.0 + 1.464 \left(\frac{a}{c}\right)^{1.65} \quad \text{for } \frac{a}{c} \leq 1 \\ Q' &= 1.0 + 1.464 \left(\frac{c}{a}\right)^{1.65} \quad \text{for } \frac{a}{c} > 1 \end{aligned} \quad (2.7)$$

These expressions are accepted and used extensively throughout the literature. The maximum error introduced into the SIF calculation by using these empirical expressions for Q is about 0.13 percent for all values of $\frac{a}{c}$, which is well within the error bounds of the currently available estimated SIF solutions. (Newman, 1979).

Although much work has been performed since the Irwin (1962) estimation, an 'exact' solution of the SIF around a surface crack is not yet available. The complexity of the problem suggests that an exact solution would be extremely cumbersome to obtain. Early work by Irwin (1962) included some engineering estimates and incorrect assumptions. (Leek, 1990). This made some of the SIF solutions inaccurate, particularly for cracks in 'real' plates having finite thickness and finite width, where the influences of the boundary were often not correctly accounted for (Leek, 1990). More recently, the availability of computers has enabled direct analysis using various numerical techniques, such as the finite and boundary element methods. The proper use of these tools has led to much improved prediction of the SIF values.

2.2.1 Solutions of the SIF for Semi Elliptical Surface Cracks

The difference between the various SIF estimates lies in the value of F in equation (2.6) under various loading and geometrical conditions. In the literature, estimates of F have been presented in graphical form, analytical form or a combination of both. Newman (1979) has reviewed fourteen such solutions published between 1962 and 1979. Those considered were by Paris and Sih (1965), Smith et al. (1967), Kobayashi and Moss (1969), Masters et al. (1969), Smith and Alavi (1971), Rice and Levy (1970), Anderson et al. (1970), Shah and Kobayashi (1972), Newman (1973), Smith and Sorensen (1974), Kobayashi (1976), Raju and Newman (1979), and a modified Newman and Raju estimate that was originally put forward by Newman (1973) and improved by Raju and Newman (1977). Many of these estimates neglected the effect of finite width; and had limitations on the aspect ratio and depth of the crack that

they can be applied to; and provided the SIF at only the point of maximum depth of a crack, rendering them limited in application.

Comparing some of the solutions mentioned above, Newman (1979) showed that for small values of $\frac{a}{t}$ (up to around 0.3 depending on $\frac{a}{c}$), agreement between the proposed solutions by the above mentioned authors is reasonable with deviations between them of less than 5 percent. However, for larger values of $\frac{a}{t}$, the differences increase greatly, being as large as 80 percent for $\frac{a}{c} = 0.2$, 20 percent for $\frac{a}{c} = 0.6$ and 35 percent for $\frac{a}{c} = 1.0$. The probable reason for this large discrepancies between the estimates is the fact that the further away the aspect ratio from 0.75, the greater the variation of SIF around a surface crack front under tension load. Portch (1977) obtained similar discrepancies in another comparison of earlier estimates.

In an effort to assess the accuracy of the proposed solutions, Newman (1979) compared results from these solutions with experimental fracture data obtained from surface cracks in brittle epoxy material. A statistical analysis of this comparison led to a ranking of the proposed solutions. Top ranking was for Raju and Newman (1979). Smith et al. (1967) was ranked second and the third rank was shared by Newman (1973), Raju and Newman (1977) and Newman (1979). Newman and Raju have done a considerable amount of work to obtain an accurate empirical equation giving the SIF around a surface crack in a plate under tension or bending load. Originally, Newman (1973) fitted an equation to the analytical results of Smith and Alavi (1971) and Gross and Srawley (1965). This equation has since been modified and improved by comparison with experimental data by Raju and Newman (1977) and Newman (1979).

In later work, Raju and Newman (1979); Newman and Raju (1979) performed 3-D elastic FE studies of the surface crack problem, calculating the SIF at a number of points around the front of a wide range of semi-elliptical surface cracks in finite thickness plates subject to uniform tension or bending load. Newman and Raju

(1979) modified and amended their pre-existing equation to fit the results of their FE analysis such that for all combinations of the investigated parameters, the difference was within ± 5 percent. Newman and Raju (1981) gave the following format for the SIF equation :

$$K_I = (\sigma + H'\sigma(z)) \sqrt{\frac{\pi a}{Q'}} \left(\frac{a}{t}, \frac{a}{c}, \frac{c}{w}, \phi \right) \quad (2.8)$$

where, $H' = H' \left(\frac{a}{t}, \frac{a}{c}, \phi \right)$, σ = tensile stress, $\sigma(z)$ = bending stress

This equation was formulated so that the dimensionless correction factor for cracks in plates under bending load is a product of that for cracks in plates loaded in tension and the dimensionless factor H. Thus the standard format of $K_I = F\sigma\sqrt{\frac{\pi a}{Q'}}$ is also maintained. Details of this solution are given by Newman and Raju (1981). Figures (2.6 and 2.7) show the characteristics of the solution by plotting the dimensionless correction factor F against ϕ for a range of $\frac{a}{t}$ and $\frac{a}{c}$ values.

Newman and Raju (1981) used Eq (2.8) and the Paris law to obtain plots of $\frac{a}{c}$ versus $\frac{a}{t}$ for a crack growing by fatigue and compared them with experimental data. A good agreement was obtained between both results. Further validation of Eq (2.8) requires comparison with independently obtained SIF values. A 'benchmark' surface crack problem of a crack with $\frac{a}{c} = 0.5$, $0.2 \leq \frac{a}{t} \leq 0.8$, $\frac{c}{w} \leq 0.2$ in a plate under tension or bending load was studied by a committee of the Society for Experimental Stress Analysis (SESA) and reported by McGowan (1980). Members of this committee each calculated the SIF values around the crack front using different computational methods with $\nu = 0.3$. The results generally agreed to ± 5 percent.

By considering the accuracy of various solutions the 'best' estimates of SIF values, stated to be accurate to ± 3 percent, were obtained. A comparison by Leek (1990) between the values of $F_{\phi=90}$ and $F_{\phi=0}$ from Newman and Raju (1981) equations, with the 'best' estimates from McGowan (1980) showed good agreement.

Hosseini and Mahmoud (1985) compared experimental results obtained from photoelastic studies of surface cracks under tension or bending loading with Eq (2.8). The SIF values from Eq (2.8) were well within the natural error bounds of the photoelastic studies. (Mahmoud and Hosseini, 1986). By examining the available results and solutions Scott and Thorpe (1981) developed another solution in an equation. This solution was a combination of curve fitted results of earlier studies. In their study Scott and Thorpe (1981) concluded that the most accurate SIF values for semi-elliptical cracks in finite thickness plates are by Raju and Newman (1979) obtained using FEA under tension loads, and the results by Koterazawa and Minamisaka (1977) for pure bending loads using the same technique.

Further results of the SIF of a surface crack have been obtained by the use of other numerical techniques such as the body force method (BFM). This technique was originally developed by (Nisitani, 1968) to analyze two dimensional stress problems. The error obtained due to the number of subdivisions used in this analysis was shown to be proportional to $\frac{1}{\sqrt{N}}$, where N is a measure of the number of subdivisions made. This means that to obtain the true result for the limiting case of $\frac{1}{\sqrt{N}} \rightarrow 0$ a straight line extrapolation can be done from results obtained with just two values of N , preventing the need for large values of N to be used and saving analysis effort. The body force method was extended to 3-D stress analysis by Nisitani and Murakami (1974) to obtain the SIF values at the deepest point of a semi-elliptical surface crack under tension loading. The analysis was simplified by the assumption of a semi infinite section and by setting $\nu = 0$. The use of linear extrapolation, as used in 2D, was validated for the 3D analysis. Further, Isida et al. (1984) used the body force method to determine stress intensity factors of semi-elliptical surface cracks under tension or bending load.

Holdbrook and Dover (1979) performed a simple analysis to obtain an equation which approximates the SIF values of a surface crack in a finite plate. They assumed

that the total SIF correction factor, F , of a surface crack could be determined by adding corrections for the following :

- (a) finite section area
- (b) finite second moment of area; and
- (c) free front surface

The first item was addressed by many of the solutions discussed above. For the second item, correction is made due to the induced bending that occurs in a surface cracked finite plate under tensile loading. To account for the free surface, the correction factor (M_f) developed by Kobayashi and Moss (1969) was used. This factor was obtained by parabolic interpolation between an approximate value of 1.03 for $\frac{a}{c} = 1.0$ and a value of 1.12 for an edge crack in a semi-infinite plate i.e. a surface crack with $\frac{a}{c} \simeq 0$. For the third item, although the main SIF values were calculated at the maximum depth point, few values were obtained at the surface. The next section discusses this point in detail.

2.2.2 The Surface Layer Effect

It is an experimental fact that when a fatigue crack is grown from a straight fronted notch in specimens, such as a compact tension specimen, the crack front tends to adopt a thumb nail shape, where less crack growth occurs at the edge of the crack front near the specimen surface than at the central section. Crack growth is governed by the triaxiality of the stress state ahead of the crack front, which varies between the central section and the surface point of the crack front. This can be called the surface layer effect. The presence of a free surface has the following two consequences, which are intrinsically linked :

- (i) Close to the surface, the stress state more closely approximates plane stress in contrast to regions away from a surface where the stress state more nearly approximates plane strain.
- (ii) At the actual point of intersection of a crack front with a free surface, the crack tip singularity is not the usual LEFM one of $\sigma \propto r_\theta^{-0.5}$, where r_θ is the distance from the crack tip

The first point was illustrated for surface cracks by Francis and Davidson (1972). The second point was verified by Benthem (1977) who showed that at the actual point of intersection of a crack front with a free surface, stresses do not vary with $r_\theta^{-0.5}$. Thus, the usual concept of a SIF at this point is not valid. Benthem suggested that at this point a weaker exponent exist, that is, the crack tip singularity is expressed by $\sigma \propto r_\theta^{-\lambda}$ where λ is less than 0.5. The work of Benthem (1977) has since been confirmed by various numerical and experimental investigations as indicated by Yamamoto et al. (1987). Investigations of the SIF at points on the crack front close to a surface show that unless $\nu = 0$ the SIF falls off rapidly to zero at the surface. (Hartranft and Sih, 1970; Sih, 1971; Hilton et al., 1978). Yamamoto and Sumi (1978) estimated the thickness of the layer within which the rapid fall off of SIF occurs to be approximately 0.5 percent of the section thickness.

The plane stress region near the surface 'the surface layer' is more resistant to fatigue crack growth than the region away from the surface. Generally, the constants used in a Paris type growth law, which govern the predicted crack growth rate for a certain SIF value, are obtained under as near plane strain conditions as possible, by following the procedures recommended in ASTM-E647-88 (1988). The use of these constants therefore means that conservative predictions will be made when conditions approaching plane stress occur such as near a free surface. Since conservative predictions are assured the plane strain growth constants are generally

used to predict growth all around the crack front. (Leek, 1990).

Whilst neglecting the surface effect in the analysis of surface cracks also gives errors on the safe side, the effect could be more important. The differing growth rate of the surface point of the crack will affect the aspect ratio of the crack and hence the SIF value all around the crack front. Methods of correcting for the effect of the free surface on surface cracks have been further investigated by Connors (1984), Newman and Raju (1981) and Jolles and Tortoriello (1983).

2.3 Crack interaction

In aging structures, the possibility that multiple cracks can originate in close proximity is relatively high. There is a large amount of evidence indicating that where this occurs, the SIF for each crack is influenced by the presence of the other crack. (Wang et al., 1996). This influence is generally quantified in terms of the interaction factor which is denoted by γ in this thesis. If the SIF at a point on a crack-front in the presence of another crack is K_{in} , and the SIF at the same point on the front of the crack existing alone under the same loading condition is K_s , then γ is defined as

$$\gamma = \frac{K_{in}}{K_s} \quad (2.9)$$

As is intuitively expected, for close cracks that are not 'overlapping', such as cracks C_1 and C_2 in Fig (2.8), the effect of each crack on the other is to increase the SIF values resulting in $\gamma > 1.0$. Interaction factors of less than 1.0 can exist on overlapped portions of the crack fronts. Obviously, an interaction factor of 1.0 at a point on a crack front means that the other crack is having no effect at this point. In this section, some of the main features associated with the problem of multiple cracking are discussed. These features have been reported in the literature. For example, Wang et al. (1996) demonstrated experimentally that the coalescence of

two large cracks resulted in the final failure of an Ni-Cr-Mo-V steel specimen fatigued at low frequency in a concentrated sodium hydroxide solution. These cracks, shown in Fig (2.8), were formed by the joining of several smaller cracks, as indicated by the islands of metal between the cracks. The work of Wang et al. (1996) clearly indicates that the interaction and coalescence of cracks depend strongly upon their relative location and the crack sizes. The common features observed for the coalescence of two offset cracks involves a sequence, where approaching tips often diverge from their previous paths to some extent and propagate beyond one another before turning towards the opposite crack and joining it. If a crack is in a position where it is 'overlapping' with or shielded by another crack, such as (D with respect to B), its further propagation can be retarded or even arrested, as shown in Fig (2.8). Obviously, the frequency of crack coalescence is related to the crack density within the component, that is, the higher the crack density, the higher the probability of two cracks meeting. However, the possibility of whether the cracks will merge or not is largely determined by the relative locations of the cracks.

2.3.1 Conditions for crack coalescence

Although no theoretical formula is available which describes the conditions for two adjacent cracks to merge, a few empirical expressions have been reported. For example, extensive measurements have been made by Parkins and Singh (1990); Parkins et al. (1993, 1994) and Wang et al. (1994b,a) of the lateral distance between adjacent crack tips and the lengths of the cracks, in the context of whether crack pairs showed evidence of coalescence or otherwise. Despite the different material-environment systems involved and various stress levels applied, it has been shown that merging will almost always occur when the closest tips of adjacent cracks have passed one another. Some of the results from measurements for systems involving a Mn-Cr steel in de-ionised water and a Ni-Cr-Mo-V steel in sodium hydroxide solution

is published by Wang et al. (1996). The maximum value of the lateral separation (h) to achieve coalescence is dependent upon the crack length ($2c$) according to

$$h \leq 0.14(2c) \quad (2.10)$$

for both systems. The same expression has been found to hold true for systems involving pipeline steels in two different laboratory environments, service failures (Wang et al., 1994b,a) and high strength spring steel in salt water (Wang et al., 1994a). Similar results have been reported by Ochi et al. (1985) for fatigue in air for mild steel, copper and a 304 stainless steel. Inoue et al. (1994) postulated an equation relating the critical offset distance for coalescence to the crack length as well as to the mean grain-size. The independence of conditions for coalescence from both material and environment implies it must be mechanics related. The stress distribution between the neighboring cracks is therefore the main key to understand the phenomenon of crack interaction.

Although no analytical solutions are available for interacting cracks, numerical results have been reported for cracks with various geometric arrangements. Some of these solutions are discussed in Section (2.4). Most of these solutions have indicated that, depending on the relative location of the two cracks, the stress intensity factors (SIF's) at the interacting crack tips can be either enhanced, have no effect or reduced by comparison with that of an isolated crack. The last effect is known as 'shielding'. These solutions have also indicated that even for systems under Mode I loading, a 'secondary' SIF mode, i.e., II and/or III, can be locally induced in the interaction zone. This locally induced mixed mode effect is discussed in more detail next.

2.3.2 Mixed Mode I/II loading in crack interaction

Stress analysis has shown that the appearance of a second crack in the region of a crack tip, subjected to remote tensile load, could locally induce a Mode II stress

component. In an attempt to understand how Mode II affects the interaction process between cracks, Kamei and Yokobori (1974) obtained the SIF solutions of the nearest tip of two identical offset cracks in an infinite elastic solid subjected to remote tension. In this study, the changes in the ratio of K_{II}/K_I for the nearest tip was obtained as a function of the cracks relative location with respect to their length. These solutions show that Mode I SIF, (K_I) increases as the nearest crack tips approach one another, but then declines as the tips pass. In the meantime, the Mode II SIF, (K_{II}) becomes significant and its sign changes from positive to negative as the crack tips approach and then pass. But, how does that affect the propagation direction of interacting cracks ?

Under linear elastic conditions, the near-tip stress component for a crack subjected remotely to tensile opening and sliding SIF's, K_I and K_{II} , respectively, can be given by

$$\sigma_\theta = \frac{1}{\sqrt{2\pi r_\theta}} \cos \frac{\theta}{2} \left(K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right) \quad (2.11)$$

where r_θ and θ are the polar co-ordinates centered at the crack tip, as shown in Fig (2.4). This tensile stress can be represented by a generalized SIF, K_θ as

$$K_\theta = \sigma_\theta \sqrt{2\pi r_\theta} \quad (2.12)$$

According to the maximum tangential stress theory, the direction of crack growth under a mixed I and II loading can be found by calculating the value of θ , i.e. θ_{max} , that gives the highest value of K_θ , i.e. $K_{\theta_{max}}$. Thus using the condition $(\partial K_\theta / \partial \theta) = 0$, the following result may be obtained:

$$\frac{K_{II}}{K_I} = \frac{\sin \theta_{max}}{1 - 3 \cos \theta_{max}} \quad (2.13)$$

The threshold condition for crack growth was then proposed by Erdogan and Sih (1963) as

$$K_{\theta_{max}} = K_{IC} \quad (2.14)$$

where K_{IC} is the Mode I fracture toughness of the material. Based on Eq (2.13), it is evident that a crack will deviate from its original plane during propagation if Mode II stress is induced. A positive value of K_{II}/K_I will cause cracks to diverge from each other while a negative value will cause them to converge and eventually coalesce. While the extent of variation in SIF's is strongly dependent upon the offset distance of the cracks, the above result indicates that it is the mixed mode stress condition experienced by the nearest tips of the adjacent cracks that leads to the change of crack growth direction and causes coalescence. The same conclusion was reported by Stonesifer et al. (1993) and by Swain and Hagan (1978).

Some of the studies found in literature have also indicated a relation between the existence of Mode II and the invalidity of the LEFM approach. Although the present investigation is concerned with the interaction process under LEFM, it is nevertheless worthwhile to briefly discuss the limitation of LEFM in multiple crack-ing problems and thus the limitation of the results presented in this thesis.

2.3.3 Limitations of LEFM

As shown in Eq (2.11), linear elastic stress analysis of sharp cracks predicts infinite stresses at the crack tip. In real materials, however, stresses at the crack tip are finite because σ_θ must be finite, i.e. $\sigma_\theta = \sigma_y$. Inelastic material deformation, such as plasticity in metals and crazing in polymers, leads to further relaxation of crack tip stresses. The elastic stress analysis becomes increasingly inaccurate as the inelastic region at the crack tip grows. Simple corrections to linear elastic fracture mechanics (LEFM) are available when moderate crack tip yielding occurs. For more extensive yielding, one must apply alternative crack tip parameters that take nonlinear material behavior into account. The limit of LEFM is when the elastic stress intensity factor no longer describes the stress field ahead of the crack tip with sufficient accuracy. This, as shown in Fig (2.9), occurs when the size of the

zone of plastic deformation ahead of the crack, r_p , becomes large relative to crack length. The usual size test of the plastic zone for LEFM validity is

$$r_p \leq \frac{c}{50} \quad (2.15)$$

where c is half the total crack length. In addition to this limitation, Anderson (1995) pointed out that the microscopic events that lead to fracture may not nucleate in the singularity dominated zone. This fact raises an important question: can the SIF be used as a failure criterion in materials that exhibit inelastic deformation at the crack tip? Under certain conditions, K_I can still uniquely characterizes the crack tip conditions when a plastic zone is present. Consider a test specimen and structure loaded to the same K_I level, as illustrated in Fig (2.9). Assume that the plastic zones of both interacting cracks are small compared to all length dimensions in the structure and the test specimen. Let the region surrounding the interacting tips be thought off as a free-body. If this region is sufficiently small to be within the singularity dominated zone, the stresses and displacements at the boundary are defined by Eq (2.11). The interacting zone in Fig (2.9) can be viewed as an independent problem. Imposition of the $1/\sqrt{r_\theta}$ singularity at the boundary results in a plastic zone at the crack tip. The size of the plastic zone and the stress distribution within the interaction zone are a function only of the boundary conditions and the material properties. Therefore, even though the actual stress distribution in the plastic zone is not known, it could be argued that it is uniquely characterized by the boundary conditions: i.e., K_I characterizes the crack tip conditions even though the $1/\sqrt{r_\theta}$ singularity does not apply to the plastic zone.

The third limiting condition of the LEFM is related to the locally induced Mode II. The introduction of Mode II stress component can have two important, but somewhat opposing effects on the crack growth. (Wang et al., 1996).

- (1) An increase in crack tip plasticity, which can accelerate crack growth, but can also invoke plasticity-induced crack closure:
- (2) Crack surface contact and rubbing causing extra crack closure thus reducing the propagation rate.

The results of Shih (1974) show that with increasing Mode II components the plastic zone changes its shape, expands its size and rotates clockwise, centering on its axis near the crack tip. At the same amplitude of far-field loading, the plastic zone size in Mode II is up to five times bigger than that in Mode I. For the situation where crack growth is controlled by crack tip plasticity, such as stage I fatigue crack growth, the increased plasticity associated with the Mode II stress component may facilitate crack development and reduce the validity of LEFM. However, this increased plasticity is accompanied by a decrease in the hydrostatic stress which could enhance the crack closure, (Suresh, 1991). In addition, the Mode II component introduces shear displacement between the crack surfaces, which may cause crack surface contact, rubbing and even interlocking of the two surfaces, depending upon the roughness of the surface. As a result, a shear resistance is introduced between the fracture surfaces: the load is then difficult to transmit to the crack tip, so the driving force at the crack tip decreases. Both of these effects reduce the validity of LEFM. It has been estimated by Zheng et al. (1994) that the shear resistance of pure Mode II can consume 70 to 80 percent of the crack driving force. Also, because of these two opposite effects associated with the Mode II component, the results in regard to the threshold and growth rate of a crack subjected to mixed Mode I and II loading were reported as somewhat inconsistent. For example, experimental results of Suresh et al. (1990); Erdogan and Sih (1963); Gao et al. (1982) on brittle materials and on notched specimens of stainless steel are in good agreement with the criterion of Eq (2.14), while, Zheng et al. (1994); Gao et al. (1979); Baloch and

Brown (1993), show that the threshold was much higher than predicted by Eq (2.14) when the Mode II stress component was introduced.

2.4 Existing solutions of interacting cracks

Solutions for interacting cracks are not abundant in the literature. However, the problem of interacting cracks has received increased attention in the last decade. Techniques used to solve problems of interacting cracks have either employed a hybrid approach, a fully numerical approach or an experimental approach.

2.4.1 Hybrid Approaches

By definition, a hybrid approach is a technique that combines the efficiency provided by the analytical approaches and generality provided by the numerical ones. For example, a hybrid solution which is of special value in solving embedded, surface, and interacting crack problems, is one by Vijayakumar and Atluri (1981). This solution consists of a complete elastic analysis for an embedded elliptical crack in an infinite body subjected to arbitrary order polynomial crack face tractions. Normal and shear tractions are accommodated in this technique, therefore, completely general mixed-mode stress intensity factor solutions are possible. Nishioka and Atluri (1983) took this general solution and adapted it for use with a finite element based alternating method for obtaining mixed-mode SIF solutions. The equations developed by Vijayakumar and Atluri (1981); Nishioka and Atluri (1983) are referred to as the VNA solution.

O'Donoghue et al. (1985, 1986, 1984), developed Mode I SIF's for embedded and surface elliptical cracks in finite bodies (plate and pipe) using the VNA solution and the finite element alternating method. Simon et al. (1987) rederived portions of the VNA solution associated with mixed-mode behavior 'in a more convenient form' and

went on to obtain mixed-mode SIF's for several part elliptical surface flaws. At this point, a few interacting crack solutions were obtained. Nishioka (1985) subsequently used the VNA solution with the alternating method approach to solve interacting embedded crack problems for infinite bodies. Kuo et al. (1992) used the alternating method to combine the VNA solution with the analytical solution for arbitrary surface traction loading of an infinite plate to solve embedded and surface cracked plate problems. The focus of that study was to develop 'accurate, efficient and easy to use' software. However, an interacting crack capability was not achieved.

Heath and Grandt (1984) used (a non-VNA solution based) finite element alternating method to develop solutions for coplanar interacting corner cracks emanating from opposite ends of a hole bore in a plate. These solutions were summarized in tables for various crack sizes and separation distances. Pitt et al. (1999) have also used this technique to study the effect of crack interaction on degrading the damage tolerance of aircraft structures. In pressure vessels, O'Donoghue et al. (1986) used the alternating FE technique to study the interaction of coplanar surface cracks in a thermally shocked cylinder. For non-coplanar cracks, Guozhong et al. (1996) used the boundary element alternating method to study the interaction of embedded elliptical cracks in finite thickness plate under uniform tension.

Stonesifer et al. (1993) used the alternating FE to study a case of two interacting parallel, but not necessarily coplanar, cracks in a plate. However, more general cases of randomly oriented interacting cracks were not discussed. In addition, this study did not discuss cases where more than two cracks are involved and the effect of cracks size, shape and proximity on the calculated SIF. Stonesifer et al. (1993) also reported some numerical problems and a higher percentage error for smaller interacting distance. Inaccurate results were also found at points where crack fronts intersect the surface under a plane stress state. In this study, the advantage in the reduction in the number of degrees of freedom was often offset by a higher portion of

numerical calculation in the analytical part of the solution and the need for several long iterations.

Methods of varying complexity have been developed which employ the representation of cracks by dislocations. In these methods a large or infinite body with N arbitrarily oriented cracks is represented by superimposing N separate embedded crack problems, each with unknown crack surface tractions. The crack surface tractions are then related through a system of integral equations which are solved approximately. Chudnovsky et al. (1987a,b); Kachanov (1987); Kachanov and Laures (1989) and Benveniste et al. (1989) have developed different solution techniques based on this principle. Kachanov (1987) and Kachanov and Laures (1989) greatly simplified this technique by approximating the unknown crack line tractions by their average, which leads to a system of linear algebraic equations. This approximation, however, could lead to inaccurate results for closely spaced cracks. Benveniste et al. (1989) obtained a polynomial expansion for the unknown crack line tractions which allows one to trade-off the computational effort and the accuracy of the solution by changing the order of the approximating polynomials. The constant traction approximation of Kachanov (1987) and Kachanov and Laures (1989) is therefore, a special case of this polynomial approach. All the models by Chudnovsky et al. (1987a,b); Kachanov (1987); Kachanov and Laures (1989); Benveniste et al. (1989) are either limited to 2D cases and/or to problems in which the cracks are far from boundaries. The same technique was also used by Xue and Tzuchiang (1996) to solve for the interaction of multiple cracks in cases where complicated surface conditions exist. In ceramics, Moller and Karihaloo (1995) used this technique to study the effect of collinear array of cracks in transformation toughened ceramic under monotonically increasing load.

Applying the body force method, Murakami and Nemat-Nasser (1982) have presented the Mode I SIF solutions for a number of interacting coplanar surface cracks

in a half space. These solutions are given for a variety of crack shapes and sizes under tension and bending loads. Using the same technique, Isida et al. (1985, 1991) studied the interaction process between embedded cracks and between an array of parallel semi-elliptical surface cracks in a 3D semi-infinite solid under tension, respectively. Isida and Igawa (1992) studied a zigzag array of cracks using the same technique in 2D. Isida et al. (1994) introduced expressions for the body force densities to the technique to replace the stress conditions at the crack surface with the resultant force conditions. This modified technique was used to study the interaction of embedded cracks with arbitrary shapes in an infinite solid under bending, where surface contact take place. The body force method was also used by Kuo et al. (1997a) to investigate the effect of multiple cracking on crack growth in contact fatigue. The interaction cases of coplanar and non-coplanar surface cracks with ($\frac{a}{c} = 1$) were considered in this study.

Ukadgaonker and Naik (1991), applied the superposition technique on a case of two interacting cracks, where

- (a) The problem is solved first for a single crack subjected to the given loading conditions.
- (b) The corresponding boundary condition on the second crack boundary is obtained. However, this does not satisfy the required boundary conditions.
- (c) A second problem of a simply connected region of the second crack is then solved with the required corrected boundary conditions.
- (d) Superposition of these two solutions gives the required solution for the connected multiple region.

This technique, also known as Schwarz algorithm, proved to be convergent for doubly connected regions in 2D by Mikhlin (1934) and in 3D by Soboleff (1936). As another

application to this technique. Gong and Meguid (1991b) also used this technique to study the effects associated with the release of general residual stress-fields as a result of the formation of microcracks in the vicinity of a macro-crack. This study indicated that depending upon the location and orientation of the microcracks, either 'shielding' or amplification effects may become prevalent. Gong and Meguid (1991a) have also used this technique to study the interaction of non-coplanar arbitrarily located elliptical holes near the tip of a main macro-crack. Similar analysis was carried out by Roy and Saha (1995) to study the interaction of penny-shaped crack with an elliptical crack. Also, Galybin (1998) and Suzdalnitsky (1998) used the same technique to study interaction process with periodic crack geometries in 2D and 3D, respectively. Recently, Sumi and Wang (1998) used the Schwarz algorithm to simulate the growth of non-coplanar interacting cracks in a 2D finite heterogeneous elastic solid. In this study, Sumi predicted an incremental crack growth pattern using a first order perturbation solution of slightly kinked and curved extensions of the interacting cracks.

Xiao et al. (1996) studied the interaction of two non-coplanar penny-shaped stacked cracks using the Eshelby (1961) equivalent inclusion method and the superposition principle of the elasticity theory. In this study, Xiao derived a closed-form solution for the SIF's on the crack fronts when the separation distance between the two cracks is much larger than the crack sizes. A numerical method is employed to extract the solution for cracks with small separation distance. Using the same technique, cases of coplanar penny-shaped stacked cracks and asymmetric non-coplanar penny-shaped cracks in 3D have been studied by Xiao et al. (1994, 1995), respectively.

Kushch (1998) used the multipole expansion method to study the SIF variation during the interaction process between two surface cracks, a surface crack and a cavity and a surface and an embedded crack. A recently developed integral equation

method is used by Roy and Chatterjee (1994) to solve the problem of interaction between coplanar elliptical cracks under tension loading. The pair of dual integral equations, in a Cartesian system is first transformed to give four sets of infinite systems of Fredholm integral equations of the second kind in a cylindrical coordinate system. Some analytical expressions for the SIF and strain energy were derived in this study. Also, Ortiz (1988) used a continuum damage-mechanics model to predict the behavior of a macro-crack in the presence of a micro-cracking zone. In this model, the region of the micro-cracks was described by the same constitutive equations, having a reduced effective modulus from that corresponding to the parent material.

2.4.2 Numerical Approaches

Because of their advantage in predicting the effects of external or internal boundaries, numerical techniques such as finite and boundary element methods have been introduced to solve crack interaction problems. Miyazaki et al. (1989) used a combined line-spring and boundary element method to obtain Mode I solutions for coplanar interacting embedded and surface cracks in a plate and compared the results with estimated SIF levels in the ASME design code. The interaction effect, in this study, was reported to be of an amplification type, i.e. $\gamma > 1$. The interaction factor was higher between the twin embedded cracks than the surface ones. It was also concluded that estimates obtained from using the ASME design code were conservative.

Jiang et al. (1990) started a series of studies to investigate the interaction effects between parallel cracks in a finite width sheet. The first study presented an empirical correlation factor, F_Y , for two and three central or edge parallel cracks. The maximum difference between Eq (2.16) and the FEA results was about 3 percent.

$$K_{in} = K_{is} F_N\left(\frac{c}{w}, \frac{h}{c}\right) \quad (2.16)$$

where, w is the plate width. Using the same concept, studies involving groups of more than two cracks were presented by Jiang et al. (1992) and Shu et al. (1993). These studies, which were carried out in 2D, concluded that the stress-field near the crack tips was relaxed, inducing a shielding effect on the SIF values. Shu et al. (1994) also found that the crack size does influence the magnitude of shielding and its dominance on other cracks. Another study by Jiang et al. (1991) discussed the interaction of two parallel identical semi-circular surface cracks, as shown in Fig (2.10.a), subjected to remote tensile stress in 3D. An empirical correlation, similar to Eq (2.16) was also presented. However, the percentage error of this correlated equation with the results of the study along the crack front was as high as 6 percent while at the free surface it remained 3 percent.

In pressure vessels, Kim and Lo (1995) used the elastic-plastic FEM with line spring elements to verify a new interaction criterion between coplanar cracks. To verify the new crack interaction criterion, comparisons of the J-integral values were made for various crack sizes with different distances between cracks and under various loading conditions. Based on these comparisons, the new crack interaction criteria, which compare a physical distance, s , to a characteristic distance $d = (\sigma/\sigma_y)f(c_1, c_2)$, proved to be a reasonable parameter that describes the crack driving force between coplanar interacting cracks. Also, Sun et al. (1997) studied the interaction of surface coplanar cracks in a plate under remote tension. This work used the FE method and considered a wide range of $\frac{a}{c}$ and $\frac{a}{t}$ values.

Denda and Dong (1997) used the boundary element method (BEM) to study cases of coplanar and non-coplanar cracks in 2D. The results of this work were used to study MSD in aging aircraft structures. The same technique was used by Tu and Cai (1988) to study the interaction and propagation of non-coplanar cracks

in an infinite body. Tu and Dai (1994) expanded this work and used the BEM to investigate the interaction behavior of irregularly oriented multiple cracks under uniaxial and biaxial loads. In their work, they compared the numerical results with some experimental results conducted in this same study. This was a good example of a combined numerical and experimental investigation.

Other examples for combined numerical and experimental work were presented by Kishimoto and Soboyejo (1990), Soboyejo et al. (1989) and Soboyejo and Knott (1990), where they obtained Mode I solution for coplanar interacting surface cracks in a plate using classical finite element method. In this study, experimental results of crack coalescence were compared with the finite element results and a change in the fatigue life for the interacting cracks was noticed. Soboyejo and Knott (1991) presented the results for fatigue tests on specimens containing parallel offset and parallel coplanar and non-coplanar cracks. In this study, Soboyejo showed that the fatigue growth of non-coplanar cracks is significantly affected by crack-tip shielding which increases with their overlap.

2.4.3 Experimental Approaches

As mentioned earlier, Soboyejo et al. (1989) carried out an analytical investigation of the variation in stress intensity during the coalescence process. This work was later verified with an experimental investigation by Soboyejo and Knott (1990). Using a new simplified procedure, good engineering estimates of the fatigue life were obtained. While predictions based on re-characterization upon contact of the crack fronts did not account for the changes in crack shape during coalescence, they still yielded acceptable engineering estimates. Soboyejo indicated that the use of ASME Section XI Code results in excessively pessimistic predictions of crack growth due to its premature re-characterization of cracks before their adjacent tips come into contact. The same conclusion was also reached by Leek and Howard (1996) and Tu

and Dai (1994) for cases of uniaxial and biaxial loading of non-coplanar cracks. An experimental technique that combine photoelasticity and acoustic imaging was used by Liu and Smith (1991) to study Mode I SIF at the tip of the center crack in pre-cracked specimens. In this study, experimental results indicated that the existence of the upper and lower cracks produced a shielding effect on the center crack. The degree of shielding depends on the relative size and location of the center crack with respect to the other two cracks.

Swain and Hagan (1978) studied the interaction of two non-coplanar cracks in soda-lime glass under Mode I loading conditions. The main observation made by Swain was that the interacting crack tips are affected by the 'shielding' phenomena, which can reduce their growth rate. Using photoelasticity, the interaction of two parallel cracks with variable overlap distance in a tensile stress field was investigated in this study. The principal stress trajectory patterns and the curves of constant shear stress are presented for the interacting cracks. The conclusion of this study was similar to that by Wang et al. (1996). A similar work was carried out by Soozani et al. (1987) to study the interaction of coplanar and irregularly oriented surface cracks. The result of this study confirmed the trend of K_{II}/K_I , as discussed in section (2.3.2), and its effect on the crack growth direction.

Frise and Bell (1992) conducted an experimental investigation of the coalescence of coplanar cracks at a notch. It was found that a large percentage of the fatigue life (28 percent to 87 percent) is spent in the coalescence stage. At the coalescence plane, increased crack growth was observed indicating a magnified stress intensity factor there. Based on the experimental results, an empirical crack shape development model was developed to relate the variation in the aspect ratio to the crack depth during coalescence. The model, however, did not take into account the change in crack shape during coalescence. Also, the model assumed that the crack does not grow in the surface length direction during coalescence.

Harrington (1995) conducted an experimental investigation of crack shape development during coalescence. He observed that interaction between the two leading crack fronts is not observed until just prior to the coalescence process. An approximate 15 percent increase in the stress intensity factor was determined at this study relative to a single crack. During coalescence the SIF at the coalescence point was approximately twice that found at the deepest point of the two coalescing cracks. This value was seen to decrease quickly as the coalescence depth increased. Zhang (1993) developed a computer code to predict the lives of interacting and coalescing cracks. Using the experimental results of Harrington, Zhang was able to predict the fatigue life to within ± 5 percent. The predictive model used a coalescence coefficient k :

$$k = \frac{N_{coal}}{N_{exp}} \quad (2.17)$$

where N_{coal} and N_{exp} are the computed and experimental coalescence lives respectively. The coalescence coefficient modified the crack growth rate along the coalescence plane to account for the accelerated growth in this region. Since Zhang's predictive program used an experimental result to account for the interaction factor, the life time predictions are dependent on the geometry and loading conditions of the experimental results.

A combined experimental and numerical investigation of the Harrington work was also presented by Bayley and Bell (1997). In this investigation the FEM was used to determine the SIF around the periphery of a coalescing crack. Accurate crack shape development during coalescence was determined from the beach-marks of an experimental sample. Excellent agreement between the SIF values from the experimental and numerical studies was found. Also, the investigation showed that fatigue crack coalescence is a stable crack growth process. Leis and Mohan (1997) conducted a similar study where a criterion to determine whether the conditions between two crack fronts will support the onset of coalescence was developed. This

criterion is based on the values of the SIF at the interacting tips, where the conditions of $K \geq K_{Coal}$ is used as a measure of the material's resistance to cracking. In this work, cases of coplanar and non-coplanar cracks are investigated.

Wu and Sun (1998) investigated the initiation and growth of part-through and through cracks in blunt notched specimens of a medium carbon steel by a replication technique. It was observed that cracks mainly originate from the surface of the notch root. The fatigue life of a notched specimen is controlled mainly by the life in which short surface cracks propagate as part-through cracks before joining up to form a single through-thickness crack. Both part-through cracks and through cracks show in many aspects a different behavior from that predicted on the basis of long crack properties. The original crack profile may be the principal reason causing the anomalous propagation behavior of the through crack in its early stage.

When two interacting cracks grow towards each other, at some point before the cracks meet, either the SIF at the closest tips will reach the fracture toughness of the material leading to failure of the ligament between the cracks and perhaps of the component, or the plastic zones around the crack tips will come together invalidating the elastic analysis. In the former circumstance, failure of the component could be predicted according to the demands of certain safety considerations. Where failure of the component is not predicted, the priority is that the crack growth prediction must continue through the coalescence of the cracks without significant errors being introduced. Thus, some systematic procedures should be used to assess the conditions of interacting cracks in both situations. These procedures must take into account the significant variation of the SIF values as the cracks coalesce and the concave crack front becomes convex. Some of these assessment procedures are discussed in the next section.

2.5 Assessments of interacting cracks

Originally, surface cracks can be of irregular shape and orientation. The standard practice, recommended by both ASME (1992) and BSI-PD6493 (1991), is to characterize a surface crack by projecting it onto the plane normal to the principal stress and to assume the crack to be semi-elliptical, with a major axis ($2c$) and a semi-minor axis (a), having the dimensions of the rectangle that fully encloses the flaw, as shown in Fig (2.11). While it is arguable whether a surface crack will actually achieve a perfectly semi-elliptical shape there is considerable evidence in the literature to show that this characterization is reasonable. (Leek and Howard, 1996). Where two cracks exist, each is first re-characterized as above to obtain two parallel cracks. The distance between the crack planes is taken to be the minimum distance normal to the assumed cracks. The problem then becomes two semi-elliptical cracks that are either coplanar or non-coplanar, i.e., in two parallel planes. Re-characterization of surface cracks as described results in a geometry that is sufficiently simple for the purpose of analyzing crack interaction.

Different assessment methods have been suggested in the literature to re-characterize interacting cracks. In general, the assessment procedure of interacting cracks concentrates upon the situation with just two cracks. This procedure, however, is applicable to situations where more than two cracks exist, as the rules for two cracks are simply followed for each pair. Some of those methods are listed as follows :

- (a) Section XI, ASME (1992)
- (b) BSI-PD6493 (1991)
- (c) Iida (1983)
- (d) Leek and Howard (1994a); Leek (1990)
- (e) Leek and Howard (1994b); Leek (1990)

Methods (a) and (b) are applicable to both coplanar and non-coplanar cracks as is required, whereas Methods. (c), (d) and (e) are currently applicable to coplanar cracks only. The latter methods could be extended to the assessment of non-coplanar cracks by new conditions developed by Leek and Howard (1996).

2.5.1 Coplanar cracks

To assess coplanar surface cracks, Methods (a) and (b) recommend that the interacting cracks should be re-characterized to a single crack, defined as a semi-elliptical crack with the major axis and semi-minor axis having the dimensions of the rectangle that fully envelops the two individual cracks, provided the following condition is satisfied.

$$\begin{aligned} \text{ASME} \quad s &\leq 2 \times (\text{maximum of } \{a_1, a_2\}) \\ \text{BSI} - \text{PD6493} \quad s &\leq 2 \times (\text{minimum of } \{c_1, c_2\}) \end{aligned} \quad (2.18)$$

where, s is the overlapping distance and c_1 , a_1 , c_2 and a_2 are the half length and the total depth of the first and second cracks, respectively, as shown in Fig (2.11). If the condition is not satisfied the cracks are assumed to be independent. This approach is straightforward and is designed to be consistently conservative. As a result, some practicality and flexibility to account for different degrees of interaction with varying crack geometries are sacrificed. Furthermore, the origin and justifications for the above conditions are unknown to the author. While safe crack assessments may be obtained using this approach its overall accuracy is unknown. Also, there is a potential for overly conservative predictions, particularly for cracks under bending load with low $\frac{a}{t}$ or under tension load at any $\frac{a}{t}$. For such cracks, the large reduction in aspect ratio resulting from the application of Eq (2.18) leads to a considerable increase of SIF values and hence of predicted growth, especially in the through thickness direction. Since the SIF value for a crack under tension load increases with $\frac{a}{t}$

and the predicted growth is an exponential function of the SIF, any over-prediction in crack size will be rapidly magnified. For cracks under bending load the SIF at the maximum depth point reduces as $\frac{a}{t}$ increases and so, although over-prediction can occur by an amount dependent upon $\frac{a}{t}$, it does not become magnified. Where combined tension and bending load occur, any spiraling errors in the growth predictions will be dependent upon the ratio of tension to bending load and the values of $\frac{a}{t}$. (Leek and Howard, 1996). Iida (1983) designed Method (c) to enable straightforward and realistic crack growth predictions. He proposed to neglect interaction until the cracks touch. Immediate coalescence is then assumed with the re-characterized semi-elliptical crack drawn through the outer tips of the two coalescing cracks and through the deepest point of the deeper crack. Thus, the dimensions of this assumed crack are:

$$c_{new} = c_1 + c_2 \quad (2.19)$$

$$a_{new} = \text{maximum of } \left\{ \frac{a_i}{\sqrt{1-(1-(c_i/c_{new}))^2}} \right\}_{i=1,2} \quad (2.20)$$

where, a_i and c_i are the dimensions of the two coalescing cracks. This approach has also been used by Kishimoto and Soboyejo (1990), Morgan (1986), McComb et al. (1986) and Soboyejo et al. (1989) who examined a small number of crack geometries using experimental data and computer simulation. However, instead of Eq (2.20) for the depth of the coalesced crack, these latter references used the less conservative choice of an enveloping crack (defined previously) which has dimensions given by Eq (2.19) and

$$a_{new} = \text{maximum of } \{a_i\}_{i=1,2} \quad (2.21)$$

Method (c) is simple to use and it has provided realistic crack growth predictions in the examples reported in the literature. Leek and Howard (1996). However, it is not clear in these examples whether the method safely accounts for interaction on its own or whether safe assessments are obtained due to other factors in the growth

calculations, such as the SIF approximations or the material constants used. For the method to safely account for interaction, any unsafe errors introduced by neglecting interaction must be canceled out by the conservative assumption of immediate transition to a semi-elliptic crack. However, the evidence in support of this being true for all potential crack geometries, and thus for the method to be used as a standard assessment procedure, is not sufficient at present. Although cracks in large sections have smaller interaction factors than the same cracks in small sections, Methods (a) and (b), Method (c) do not include any measure of the $\frac{a}{l}$ effect. There is also disagreement as to which of Eqs (2.20) or (2.21) should be used.

In the last assessment procedure for two coplanar surface cracks considered in this review is where interaction is quantified in some manner and incorporated into the SIF and the crack growth calculations. In Method (d), a continuous method of approximating the interaction factors of the cracks is used and the factors are constantly recalculated as part of the crack growth calculations. When the cracks are predicted to touch, an enveloping crack is immediately assumed. Because of the complexity of this method, a derivative in Method (e) was developed as a compromise intended to be more straightforward in application. In this procedure, a constant interaction factor, which depends on the initial crack geometry only, is applied to all SIF values of the two cracks. When the cracks are predicted to touch, a crack with dimensions given by Eq (2.19) and Eq (2.20) is immediately assumed.

2.5.2 Non-coplanar cracks

The experimental results of Iida (1983), also considered by Iida and Kawahara (1980), show that when two parallel and adjacent surface cracks grow by fatigue, they can (i) curve to meet directly at the surface or (ii) grow past each other and curve to meet under the surface or (iii) because of finite specimen size, grow past each other without meeting until the whole specimen fails. The change in the crack

path, as discussed earlier, is caused by the locally induced Mode II SIF values at the crack fronts.

If the ratio of K_I and K_{II} is known then the direction in which a crack will propagate can be estimated using Eq (2.13). As noted by Leek and Howard (1996) and Tu and Dai (1994), however, there are no suitable range of K_I or K_{II} values presently available for surface cracks in different planes. Alternatively, Leek and Howard (1996) suggested the use of a simpler technique that calculates the crack growth normal to the remote stress direction and follow up by applying the assessment approach recommended by ASME (1992) and BSI-PD6493 (1991). In the latter references, the assessment method assume the cracks to be coplanar and assess them as such, if

$$ASME \quad h \leq 12.7 \text{ mm} \quad (2.22)$$

$$BSI \text{ PD6493} \quad h \leq \frac{1}{2}(a_1 + a_2) \quad (2.23)$$

Otherwise, these cracks are assumed to be independent. In conjunction with the assumption of an enveloping crack, Eqs (2.22) and (2.23) are intended to safely account for the effect that interacting cracks have upon each other. Different conditions have been proposed by Iida and Kawahara (1980) and Iida (1983) for predicting coalescence. Iida and Kawahara (1980) proposed that for parallel surface cracks in adjacent planes to be considered as a coalesced crack

$$s \leq 0 \quad \text{and} \quad h \leq a_{max} \quad (2.24)$$

whereas Iida (1983) concluded that coalescence or imaginary coalescence (rapid curving of a crack path towards an adjacent crack) will occur if the following condition is satisfied :

$$\begin{aligned} h &\leq 5 \text{ mm} \quad \text{for} \quad s \leq 10 \text{ mm} \\ h &\leq 0.5 s \quad \text{for} \quad s > 10 \text{ mm} \end{aligned} \quad (2.25)$$

Iida (1983) also suggested that the conditions of ASME and BSI-PD6493 given above are unnecessarily conservative. Note that the condition for s in Eq (2.24) is similar to the suggestion for coplanar cracks discussed earlier, where the interaction is neglected until ($s = 0$). The value of (a_{max}) in Eq (2.24) refers to the maximum depth of the two cracks under consideration. The work of Iida (1983) raises some doubts about the conditions stated in ASME and BSI PD6493. There is some disagreement between these conditions and a lack of supporting evidence for their application. In particular, those conditions that are dependent upon a fixed crack size. In practical cases, the interaction between cracks depends upon crack shapes, sizes and separation distances, section geometry and the type of loading (tension, bending or combined tension and bending). The assessment rules and conditions for the treatment of multiple cracks in adjacent planes contain few of these variables. Improved assessment methods are therefore required.

2.6 Concluding Remarks

The multiple cracking problem has received significant attention in recent years, because it plays a very important role in predicting the residual service life of ageing structures. Any safety assessment of a multi-site damaged structure must take into account the effect of crack interaction. A survey of the literature deemed most pertinent to the problem of multiple cracking has been summarized in Fig (2.13) and the following conclusions can be drawn :

- (i) Solutions of the SIF for single surface and embedded crack are available in the literature. Most of these solutions deal with cracks in infinite media.
- (ii) Solutions of the SIF distribution along the interacting crack fronts are relatively limited. Many studies only deal with symmetric cracks since they are

easier to model. Also, in most of these studies, the values of the SIF are only reported at the deepest point.

- (iii) Stress intensity factor solutions that take into account the variation of crack shapes, crack sizes, separation distances, section geometry and the type of loading (tension, bending or combined tension and bending) are very limited. This has a significant impact on the validity of the assessment rules and the conditions used to re-characterize non-coplanar interacting cracks.
- (iv) Although many studies used numerical techniques to solve the multiple crack-ing problem, significant improvement on the modeling techniques of cracks that can account for asymmetry of the interacting cracks are very limited.
- (v) There is a vital need for improvements in the experimental techniques used to verify the results of the numerical studies applied to cases of non-coplanar interacting cracks.

Figure 2.1: Multiple stress corrosion cracking in gas transmission pipelines.

Figure 2.2: Multiple cracking in (a) forming rollers, (b) railroad tracks and (c) aircraft fuselage.

Figure 2.3: Three modes of crack deformation.

Figure 2.4: Local stresses in the vicinity of a crack tip.

Figure 2.5: Single surface crack in a plate.

Figure 2.6: The SIF's around an elliptical crack front under remote tension. Newman and Raju (1981).

Figure 2.7: The SIF's around an elliptical crack front under pure bending. Newman and Raju (1981).

Figure 2.8: Highlights of the process of crack interactions in a specimen of high strength spring steel fatigued at $R = -1$ and $\text{Freq} = 30 \text{ Hz}$ in a sodium chloride solution. (Wang et al., 1996).

Figure 2.9: The effect of the plastic zone size on the validity of LEFM - schematic diagram.

Figure 2.10: Different cases reported of interacting stacked cracks (a) Isida et al. (1994) and (b) Jiang et al. (1991).

Figure 2.11: Re-characterization of single and multiple surface cracks.

Figure 2.12: Symmetry in modeling interacting cracks cases (a) alternating crack arrays. (b) single coplanar crack array and (c) two stacked cracks. (Jiang et al., 1991).

Figure 2.13: Summary of the solutions found in the literature for the single and multiple cracking problem.

Chapter 3

Numerical Modeling

The main objective of this study is to investigate the interaction effects between two arbitrary macro cracks. These cracks could be coplanar, non-coplanar or in any arbitrary orientations. The configuration of these cracks may also be different in terms of their size, shape, and type (i.e., surface or embedded cracks). There is a considerable amount of difficulty involved in modeling arbitrarily oriented interacting cracks. To overcome this modeling difficulty, the current work is focused on constructing a finite element model that combines the sub-modeling technique, with its ability to generate crack sub-models of different lengths and depths, and a mesh generator that can build up a mesh grid based on the size, depth and orientation of the interacting crack sub-models. The solution capability of this model will not be limited to study surface cracks only. It will also serve to study the interaction between embedded cracks and the interaction between surface and embedded cracks. Section (3.2) will discuss the geometric details of the sub-models for the interacting cracks. Next, the formulation of the technique used to calculate the stress intensity factors, SIF's, along the fronts of interacting cracks is explained in Section (3.3). Finally, Sections (3.4.1) and (3.4.2) will be devoted to verify the results of the FE model in 2D and 3D, respectively.

3.1 Problem Definition

In the previous chapter, it has been mentioned that although various methods have been employed to solve for the SIF's of multiple cracks in different environments, the modeling techniques applied in these methods have seen relatively limited development. Indeed, about 90 percent of the studies on multiple cracking covered in the literature review in the present study were found to focus on only symmetric cracks. Among the remaining 10 percent, fewer studies have even considered the effect of the variation in relative crack depth, which is an important parameter in assessing the structural integrity of mechanical and aerospace components with multiple cracks. (Pitt et al., 1999).

As a major step, the work of Perl et al. (1996) has included the use of sub-modeling as the base of simulating multiple cracks in a single array configuration. The use of sub-modeling, as indicated by Perl et al. (1997) has clearly opened new horizons for researchers to study interacting cracks with wider ranges of $\frac{a}{c}$ and $\frac{a}{t}$ ratios. A similar study was carried out by Perl and Nachum (1998) for circular crack array in pressurized thick walled cylinder. While this study was a good step forward, the merits of sub-modeling were not fully exploited for treating a wide range of relative crack sizes. Thus, most of the cases studied were for symmetric crack in an array. Another promising technique for modeling interacting cracks was presented by Yoshimura and Yagawa (1997). Using this technique, a geometry model containing one or several 3D cracks is defined using a commercial CAD system. Several local distributions of node density are chosen from the CAD system database, and then automatically superposed on one another over the geometry model by using "fuzzy knowledge processing". Nodes are generated by the bucketing method as explained by Yoshimura and Yagawa (1997). This model used ten-noded quadratic tetrahedral solid elements generated by the Delaunay method. Boruchaki et al. (1999). The

complete finite element model generated is then submitted to the commercial finite element code, MARC, for a stress analysis to be performed. The stress intensity factors are calculated using the displacement extrapolation method. To demonstrate the practical performance of this technique, the problem of two asymmetric semi-elliptical surface coplanar cracks in a plate subjected to uniform tension was solved. In this technique, the concept of fuzzy logic was used to generate a global mesh that can sustain different crack locations. However, a robust model that can provide both variation in the crack size and orientation was not presented in the references mentioned above.

3.2 Methodology

This thesis provides a solution methodology to the above modeling problem of asymmetric interacting cracks in 3D solids. This methodology is focused on constructing a finite element model that combines the sub-modeling technique, with its ability to generate crack sub-models of different lengths and depths, and a mesh generator that can build up a mesh grid based on the size, depth and orientation of the interacting crack sub-models. To construct this model, the following steps are followed:

- (1) As shown in Fig (3.1), a sub-model of the first crack, labeled Q, is generated based on a user defined input of the crack length and depth. A computer code written in C language is used for this purpose.
- (2) The same code is used to generate a second crack sub-model, labeled P, based on its length and depth. While this code is used to generate both crack sub-models, the dimensions of these cracks are not necessary the same. Thus, the principle requirement of asymmetric cracks is achieved.

- (3) As shown in Figs (3.1) and (3.2), the corner nodes of both crack sub-models, parent nodes, points 1 - 4 and 1' - 4', are used to generate four splines, i.e. spline A, A', B, and B'. The x-coordinates of these parent nodes are calculated using the following equations:

$$x_n = f_{xn}(2c_Q, r_o, \theta_o), \quad \text{where, } n = 1 \dots 4 \quad (3.1)$$

$$x_6 = x_{6'} = x_{5'} = x_5 \quad (3.2)$$

$$x_{n'} = f_{xn'}(2c_P, r_o, \theta_o), \quad \text{where, } n' = 1' \dots 4' \quad (3.3)$$

$$x_8 = x_7 = x_{8'} = x_{7'} \quad (3.4)$$

The x-coordinates of the U points, shown in Fig (3.2), are obtained by specifying angle θ_{U_3} as the average of the corresponding angles at the surrounding spline points (e.g. for point U_3 , point 2 and point 1' are the surrounding spline points). For the y-coordinates of the spline points, the following equations are used :

$$y_n = f_{yn}(2c_Q, r_o, \theta_o), \quad \text{where, } n = 1 \dots 4 \quad (3.5)$$

$$y_{n'} = f_{yn'}(2c_P, r_o, \theta_o), \quad \text{where, } n' = 1' \dots 4' \quad (3.6)$$

$$y_5 = f_{y5}(y_4, y_3, r_o, \theta_o) \quad , \quad y_6 = f_{y6}(y_5) \quad (3.7)$$

$$y_{5'} = f_{y5'}(y_{4'}, y_{3'}, r_o, \theta_o) \quad , \quad y_{6'} = f_{y6'}(y_{5'}) \quad (3.8)$$

$$y_7 = f_{y7}(y_4, y_3, r_o, \theta_o) \quad , \quad y_8 = f_{y8}(y_7) \quad (3.9)$$

$$y_{7'} = f_{y7'}(y_{4'}, y_{3'}, r_o, \theta_o) \quad , \quad y_{8'} = f_{y8'}(y_{7'}) \quad (3.10)$$

The y-coordinates of the U points are calculated as the average y values of the surrounding spline points, for example,

$$y_{U_1} = \frac{y_6 + y_{6'}}{2} \quad , \quad y_{U_2} = \frac{y_1 + y_{U_8}}{2} \quad (3.11)$$

$$y_{U_3} = \frac{y_2 + y_{1'}}{2} \quad , \quad y_{U_4} = \frac{y_{U_6} + y_{2'}}{2} \quad (3.12)$$

$$y_{U_5} = \frac{y_6 - y_{6'}}{2} \quad , \quad y_{U_6} = \frac{y_2 - y_6}{2} \quad , \quad y_{U_7} = \frac{y_3 - y_7}{2} \quad (3.13)$$

$$y_{U_8} = \frac{y_{6'} - y_{1'}}{2} \quad , \quad y_{U_9} = \frac{y_{5'} - y_{4'}}{2} \quad (3.14)$$

- (4) The x and y-coordinates of the V points are generated in a similar process to the one used for the U points. However, in this case the coordinate values are controlled by the upper spline lines, i.e., A' and B' and the outer boundary of the interaction global zone Ω
- (5) The two crack sub-models are placed within two strips of elements that are bounded by the splines A, A', B and B' for crack Q and P, respectively. These strips are used to help position the interacting cracks in the model with as limited amount of modification to the whole model as possible. As shown in Fig (3.1), the global interaction zone, Ω , encompassing these strips and the interacting cracks, is divided into nine sub-zones. These sub-zones are labeled U, M and L as for the upper, middle and lower zone, respectively.
- (6) The mesh is finely graded within zones UM, M, and LM, which bound the crack sub-models in the direction of loading. Also, as the interacting cracks are separated further away from each other, the density of the elements in sub-zones UM, LM and M is maintained; and so are their relative positions with respect to the upper and lower cracks.
- (7) The main dimensions of the global model of the plate shown in Fig (3.3) are derived from the length of both crack Q and P, where

$$\frac{d}{\frac{1}{2} \cdot \text{Max}(2c_Q, 2c_P)} \geq 12 \quad : \quad \frac{H}{d} \geq 3 \quad (3.15)$$

Based on this equation, the effect of the plate external boundaries on the interaction process is considered negligible.

- (8) Figure (3.4.a) details the construction of sub-models for crack Q and P. As shown in the figure, the sub-model is for a surface crack in a “packaged” format of a rectangle box. The dimensions of this sub-model is controlled by a number of slave nodes. The coordinates of these nodes are calculated based on the user given values of $\frac{a}{c}$ and $\frac{a}{t}$ for cracks Q and P.
- (9) To build the crack sub-models, a normal fan type meshing style is used to generate the crack front of a semi-elliptical crack. Through a number of trial and error iteration steps, the shape of the elements used in this crack sub-model is adjusted to obtain two main results, namely,
- (i) good agreement between the SIF's calculated using this crack sub-model and the ones estimated by Newman and Raju (1981), deviation percentage is within 10 percent.; and
 - (ii) a flat surface for the outer elements of the crack sub-model , i.e., sub-structure M4, as shown in Fig (3.5.b), should be flat.
- (10) Condition (ii) above is required to enable the use of the crack sub-model as a superelement during the FE solution stage. Therefore, as shown in Fig (3.5.a), a number of sub-structures or zones are required to provide a gradual change of the semi-elliptical edge at the crack front, M2, to the final flat edge at the sub-model outer boundary zone, M4. The elements are mainly concentrated in sub-zone M3 and part of M2. The number of elements in sub-zone M1 is maintained at 48 elements and used to fill in the inside front of the surface crack sub-model.
- (11) To obtain the stress distribution across the plate thickness under bending, a minimum of two element layers are required. Cook et al. (1989). In the current model, four layers of 20-node isoparametric brick elements are employed to

model across the thickness. The total number of elements used in the plate model is 2752 elements with 46377 degrees of freedom.

- (12) To couple all the sub-zones within the global model, bi-quadratic multi-point constraints (MPCs) are used, a feature available in ABAQUS. A special code is written to continuously update the coupling process between the nodes on lines A, A', B, and B' as the relative positions of the two interacting cracks are changed in the parametric study.

The construction and validation process of the finite element model is discussed in more detail in section (3.4).

3.3 Stress Intensity Factor Calculation

Similar modeling techniques are used in both 2D and 3D to build the crack tip or front region. The meshes developed, as shown in Fig (3.6), used second order elements which are 8-node quadrilaterals and 20-node bricks for 2D and 3D models, respectively. One appealing aspect of these second-order elements is that they can be used to model the desired singularity at the crack tip. To obtain a $1/\sqrt{r_\theta}$ stress singularity term, the following conditions must be met:

- (1) One edge of each element at the crack tip or front must be collapsed to zero length, as shown in Fig (3.6).
- (2) The “midside” nodes of the edges radiating out from each of the crack tip elements must be placed at one-quarter of the distance from the crack tip to the other node of the element edge.

If coincident nodes at the crack tip are constrained to displace together, the only singularity term is $1/\sqrt{r_\theta}$; if the crack-tip nodes are free to displace independently,

the crack-tip singularity includes a $1/r_\theta$ term in addition to the $1/\sqrt{r_\theta}$ term. This procedure incorporates the crack tip stress singularity using standard isoparametric elements is explained in detail by Barsoum (1976). The stress intensity factor of mode I for an isolated crack, K_{Is} , is calculated from the J -integral value using Eq (3.16).

$$K_{Is}^2 = E' J \quad (3.16)$$

$$\text{where, } E' = \begin{cases} E & \text{plane stress} \\ E/(1 - \nu^2) & \text{plane strain} \end{cases} \quad (3.17)$$

and ν is the Poisson's ratio and E is the Young's modulus of the material. With reference to Fig (2.5), plane stress conditions are assumed to prevail at $\phi = 0^\circ$ and $\phi = 180^\circ$, the free surface, while plane strain conditions are assumed elsewhere along the crack front. The J -integral method is based on the potential energy difference during the infinitesimal extension of a crack. For linear elastic fracture mechanics, the J -integral is equivalent to the strain energy release rate G and related to the SIF through Eq (3.16).

The J -integral was initially developed by Rice (1968) as a path independent contour integral. When (x) is taken as parallel to the direction of crack propagation and the crack is straight, the expression for J becomes:

$$J = \int_{\Gamma} \left(W dy - T \frac{\partial u}{\partial x} ds_{\Gamma} \right) \quad (3.18)$$

W is the strain energy density, $W = \int \sigma_{ij} d\epsilon_{ij}$, T is the traction along the contour, u the displacement, and s is an incremental distance along the contour Γ . For any closed contour, the J -integral value is zero, from which it is concluded that the J -integral is path independent. (Gdoutos, 1993).

The J -integral can be physically interpreted in terms of the rate of change of the potential energy with respect to an incremental change in crack length. With the direction of the crack fixed parallel to the x direction in a Cartesian coordinate

system, the potential energy of the cracked body can be written as:

$$\Pi(l) = \int_A W dA - \int_{\Gamma} T_k u_k ds_{\Gamma} \quad (3.19)$$

where A is the area of body and Γ is it's boundary. The potential energy of the body as a function of crack length ($l = f(2c, a/c)$), is equal to the integral of the strain energy density W over the area A , and the traction T along it's boundary. Differentiating, $\Pi(l)$ with respect to the crack length l yields a solution equivalent to the J-integral defined earlier. Therefore

$$J = -\frac{d\Pi}{dl} \quad (3.20)$$

which expresses the J-integral as the rate of decrease of the potential energy as a function of the crack length for self similar crack growth.

Through the use of Green's theorem and the divergence theorem the contour integral can be written as an equivalent area integral, see Fig (3.7.a). This is advantageous in the use of the FE method, where the integration over the domain is commonly carried out. Rewriting the contour integral as an equivalent area integral for two dimensions:

$$J = \int_A \left(\sigma_{ij} \frac{\partial u_i}{\partial x_1} - W \delta_{1j} \right) \frac{\partial q}{\partial x_j} dA \quad (3.21)$$

where, σ_{ij} is the stress tensor and u_i is the displacement vector. The term q is a sufficiently smooth function such that it is unity on the first contour and zero on the second. The function q can be interpreted as a normalized virtual crack extension. This function is merely a mathematical device that enables the generation of an area integral which is better suited to numerical calculations. Although both J-integral methods are analytically equivalent, when written in the language of FEM, Banks-Sills and Green (1992) found that the line and area integrals yield different results. The difference stems from the Gaussian integration method for higher order elements. The area J-integral yields much higher accuracy than the analytically

equivalent line integral. The above equation neglects plastic and thermal strains, as well as any traction on the crack faces.

For three dimensions, the change in potential energy per unit crack advance is no longer uniform and varies with position along the crack front. Defining the virtual crack extension as a local crack advance results in a local measure of the J -integral. The formulation remains the same, with only the expression for q changing to S . In Figure (3.7.c), Δa_{max} denotes the maximum extension of the virtual crack advance. The function S interpolates this depth from zero at either extremes of the incremental length along the crack front to a maximum value. Rewriting the closed surface integral around the crack front as a volume integral, the J -integral in 3D, shown in Fig(3.7.b), is defined by Shivakumar and Raju (1992) as :

$$J = \int_{V_{D+1}-V_D} \left[\frac{\partial(W'S)}{\partial x_1} - \frac{\partial}{\partial x_j} \left(\sigma_{ij} \frac{\partial u_i}{\partial x_1} S \right) \right] dV \quad (3.22)$$

For FEM formulation of the J -integral in two dimensions, the area of the integral is partitioned into elements. Most of the terms in the J -integral can be computed directly from the output of the FEM program. The area or volume of the integral coincides with the boundaries of the elements. In the ABAQUS FEM code, the J -integral is evaluated as the equivalent domain integral, EDI. This method is found to be extremely accurate and numerically efficient to calculate. The accuracy of this method over other methods lies in the relatively small strain energy contribution at the crack tip. (Shivakumar and Raju, 1992). The domain independence of the contours allows the J -integral to be evaluated away from the crack tip region allowing coarser meshes and is therefore relatively mesh independent. By using the EDI technique, the J -integral calculated along the crack-front will represent the J -integral average over each domain. Therefore, the effect of mesh sensitivity on the calculated J -integral values is significantly reduced at the crack-front. (Shivakumar and Raju, 1992).

In this thesis, the J-integral values are obtained at different points along each crack-front, with intervals of 15° for the angle ϕ , as defined in Fig (2.5). Although the J-integral values are calculated over three consecutive domains surrounding the crack-front, the values estimated from the first domain of elements abutting the crack-front were found to be less accurate. Therefore, the J-integral values used in the current study are obtained by averaging the values calculated from the second and third domain at each point along the crack front, as shown in Fig (3.7.b).

3.4 Finite Element Model Verification

To check on the accuracy of the FE results obtained using the commercial code ABAQUS and the modeling techniques explained in the previous section, the calculated interaction factor γ (defined in Eq (2.9)) calculated for interacting cracks in different 2D and 3D cases is compared against the results published in literature.

3.4.1 Model Verification in 2D

Validation cases of interacting cracks in 2D were focused on the elastic interaction of two coplanar or parallel cracks in an infinite plane under uniaxial tension loading. In all 2D interaction cases, a plane strain state is assumed where Eqs (3.23) and (3.24) are used to calculate the interaction factors γ_Q and γ_P for crack Q and P, respectively.

$$\gamma_Q = \frac{K_{inQ}}{\sigma \sqrt{\pi c_Q}} \quad (3.23)$$

$$\gamma_P = \frac{K_{inP}}{\sigma \sqrt{\pi c_P}} \quad (3.24)$$

where c_Q and c_P are half the length of cracks Q and P, respectively. As shown in Fig (3.8), s is the horizontal distance between the inner tips of the interacting cracks inner tips and (h) is the vertical distance separating these cracks. If the

interacting cracks are symmetric, γ values are only shown for one crack, usually crack Q. Otherwise, the γ values of both cracks will be discussed.

In Fig (3.9.a), the solution of γ_Q , which is similar to γ_P in this case, is presented for different values of s/c_Q when there are two symmetric coplanar cracks, i.e. $h/2c_Q = 0$, under tension loading. The dashed curves shown in the figure are for the interaction factor at the outer tip of crack Q ($\phi_Q = 0^\circ$). As expected, the value of γ_Q decreases with (s/c_Q) at the inner tip ($\phi_Q = 180^\circ$) more rapidly than that at the outer tip ($\phi_Q = 0^\circ$). At both ends, the maximum deviation between γ_Q values calculated in the current work and those given in Rooke and Cartwright (1976) is 4 percent. It is perhaps worth noting that the values of γ_Q calculated in the current work were always higher than the ones found in literature, which is a conservative result. To validate the accuracy of the J-Integral results, SIF's values calculated using this technique are also compared against those calculated using the displacement method described by Barsoum (1976). As shown in Fig (3.9.a), the maximum difference between the two techniques is 2 percent, with higher SIF values calculated using the J-integral technique in the current study.

Figure (3.9.b), shows the second verification case in 2D. In this case, the interaction factor γ_Q or γ_P is calculated for crack Q or P, respectively, in a single coplanar crack array. At either ends, the maximum deviation between γ_Q values calculated in the current work and the ones given in Rooke and Cartwright (1976) is 3.7 percent.

The solution of γ_Q and γ_P for two asymmetric coplanar cracks ($h/2c = 0$) under tension loading is presented for different values of s/c_Q in Fig (3.10). As can be seen in Fig (3.10.a), γ_P is relatively higher than γ_Q at both inner and outer tips. In Fig (3.10.b), the values of γ_Q and γ_P at the inner tips (i.e. Q_{180} and P_0 , respectively) are compared with the results given in Rooke and Cartwright (1976); the maximum deviations between γ_Q and γ_P values at the inner tips are 3.3 and 5.6 percent, respectively.

While the ratio of γ_Q/γ_P is found to be less than 1.0, based on the results shown in Fig (3.10.a), this does not mean that K_{inP} is higher than K_{inQ} . In fact, using the values of crack length ratio c_Q/c_P , γ_Q and γ_P in Fig (3.10.a), the ratio of K_{inQ}/K_{inP} was found to be higher than 1.0 for all values of s/c_Q studied. This result is consistent with experimental results of Wang et al. (1996), which show a dominance in the growth rate of larger cracks over smaller ones.

Figure (3.11) shows the fourth validation case of crack interaction in 2D. In this case, the interaction of two non-coplanar ($h/2c_Q > 0$) symmetric cracks is studied under uniaxial tension loading. The dashed line at $\gamma_Q = 1$ serves as a reference as it represents the solution for a completely isolated crack and separates the magnification ($\gamma > 1$) and shielding ($\gamma < 1$) effects.

Figure (3.11.a) shows the variations of γ_Q with s/c_Q at the inner tip. As shown in this figure, if the interacting cracks are “not overlapping”, i.e. $s/c_Q > 0$, the value of γ_Q increases at the inner tip as s/c_Q decreases. In this case, the value of γ_Q is larger than unity, which indicates a magnification in the SIF at this tip. However, for “partially overlapped” interacting cracks, i.e. $-2 < s/c_Q < 0$, γ_Q decreases with s/c_Q . At a certain value of s/c_Q , depending on the value of $h/2c_Q$, the value of γ_Q becomes less than unity, which indicates a shielding effect between both cracks inner tip. This effect only occurs when there is sufficient overlap between the interacting cracks. For all the values of $h/2c_Q$ studied in this case, the γ_Q value increases with $h/2c_Q$ for $s/c_Q < 0$ and, as expected, decreases as $h/2c_Q$ increases for $s/c_Q > 0$.

In Fig (3.11.b), the values of γ_Q at the inner tips for $h/2c_Q = 0.5$ and 0.1 are compared with the results published in Rooke and Cartwright (1976). As shown in this figure, the maximum deviations between γ_Q in the two cases are 6.6 and 2.7 percent, for $h/2c_Q = 0.5$ and 0.1, respectively.

A similar trend is seen in Fig (3.12), which shows the variations of γ_Q with s/c_Q at the outer tip under tension. At this tip, however, the decrease in γ occurs at

$s/c_Q < 0$. Also, because the outer tips of both cracks are further away from each other, the maximum magnification effect in this case is lower than the one found at the inner tip. Again, for all values of $h/2c_Q$ analyzed, the γ_Q value increases with $h/2c_Q$ for $s/c_Q < 0$ and decreases as $h/2c_Q$ increases for $s/c_Q > 0$. A comparison between the γ_Q values obtained in the current study and the ones published in Rooke and Cartwright (1976) is shown in Fig (3.12.b). The maximum deviations between γ_Q in both cases shown in this figure are 8.6 and 8.1 percent, for $h/2c_Q = 0.5$ and 0.1, respectively. In all the cases of interaction studied, the value of s/c_Q was varied between -2.0, where the cracks are fully overlapped, and 2.0, at which the interaction effect diminishes. For the inner tips in this range of s/c_Q , the maximum shielding, i.e. minimum γ_Q , is found when there is significant, but not full, overlapping between the interacting cracks. At the outer tips, however, the maximum shielding is found when the cracks are fully overlapped.

The last interaction case studied in 2D is for an array of non-coplanar symmetric cracks shown in Fig (3.13). As shown in this figure, this case was generated using symmetry of the boundary conditions. To the author's knowledge, no similar result for this is available in literature. Thus, to verify the values of γ_Q calculated in this case, the trend of γ_Q variation at $h/2c_Q = 0.1$ is compared with the one published for $h/2c_Q = 0.0$, i.e. an array of coplanar cracks. As can be seen in Fig (3.13), both cases are in good agreement, deviation percentage is less than 10 percent, for $s/c_Q > 0$.

3.4.2 Model Verification in 3D

The verification cases studied in 3D were chosen to validate the crack sub-model used to calculate the SIF along the semi-elliptical crack front. To this end, the numerical solutions obtained by Newman and Raju (1981) and shown earlier in Figs (2.6 and 2.7) and Eq (2.8) are used as for comparison due to its widely accepted accuracy

and range of applicability. Figure (3.14) shows the percentage deviation between the SIF's calculated using crack sub-model Q and the ones found by Newman and Raju (1981). While the percentage deviation in K_I varies along the crack front, the calculated SIF values in the current work were, in general, higher than the ones calculated by Newman and Raju (1981). This result suggests that the results obtained in the present study are more conservative than those of presented by Newman and Raju (1981). The maximum percentage deviation can be seen to vary depending on a_Q/c_Q and a_Q/t . For example, for $a_Q/c_Q = 1.0$ and $a_Q/t = 0.1$, the maximum percentage deviation was 6.5 and 6.6 percent under tension and bending, respectively. For $a_Q/c_Q = 0.5$ and $a_Q/t = 0.3$, however, the maximum deviation decreased to 5.0 and 5.5 percent under tension and bending, respectively.

Figure (3.15.a) shows the first validation case of crack interaction in 3D. In this case, two interacting semi-elliptical surface coplanar cracks are studied. These cracks are subjected to remote uniaxial tension with a crack shape and a crack depth to plate thickness ratios of 0.33 and 0.5, respectively. Throughout this thesis, Eqs (3.25) and (3.26) are used in 3D to calculate the interaction factors γ_Q and γ_P for crack Q and P, respectively.

$$\gamma_Q = \frac{K_{inQ}}{K_{isQ}} = \sqrt{\frac{J_{inQ}}{J_{isQ}}} \quad (3.25)$$

$$\gamma_P = \frac{K_{inP}}{K_{isP}} = \sqrt{\frac{J_{inP}}{J_{isP}}} \quad (3.26)$$

where, J_{isQ} , J_{inQ} , J_{isP} and J_{inP} , are the J-integral values calculated using the current FE model for isolated and interacting cracks, Q and P, respectively. In this interaction case, the variation of the calculated interaction factor γ_Q is compared in Fig (3.15.a) to the values published by Stonesifer et al. (1993). While similar trend of γ_Q is found in both studies, higher values of γ_Q are calculated in the current study. At $s/c_Q = 0.33$, the common data point in both studies, the percentage deviations were 11.2, 15.7 and 1.75 percent at $\phi = 180, 90^\circ$, and 0° , respectively.

Figure (3.15.b) shows the variation of γ_Q with $h/2c_Q$ at $\phi = 180^\circ$ and 0° for a case of two interacting semi-circular fully overlapped surface cracks, i.e. $s/c_Q = -2$. As shown in this figure, good agreement is found between the values of γ_Q and the values published by Jiang et al. (1991) and Isida et al. (1991) for an interaction case with the same a_Q/c_Q and a_Q/t values. It is important to note that Jiang et al. (1991) and Isida et al. (1991) did not use any sub-modeling and relied on symmetry of the boundary conditions to model the second crack. The maximum percentage deviation between γ_Q values of the present study and those published by Jiang et al. (1991) and Isida et al. (1991) was 3.9 and 1.26 percent, respectively.

In Fig (3.16.a), the variation of γ_Q with s/c_Q at $\phi = 180^\circ$ is shown for a case of two non-coplanar interacting symmetric surface cracks. In this figure, the trend of γ_Q in 3D is similar to the one found in 2D and shown earlier in Fig (3.11.a). In this case, the value of γ_Q calculated by Stonesifer et al. (1993) are compared with the one calculated in the current study. At $a_Q/c_Q = 0.33$, $a_Q/t = 0.5$ and $h/2c_Q = 0.33$, the maximum percentage deviation in γ_Q values between both studies was 6.65 percent.

The variation of γ_Q with s/c_Q at the deepest point along crack Q front, i.e. $\phi = 90^\circ$, is shown in Fig (3.16.b). At this angle, the shielding effect is less than the one found at the inner point, i.e. at $\phi = 180^\circ$. Comparing with the γ_Q values obtained by Stonesifer et al. (1993), the trend of both results is similar at this angle. In this case, the maximum percentage deviation in γ_Q values between both studies was 15.5 percent. In Fig (3.17.a), the calculated values of γ_Q at the inner tip, i.e. $\phi = 0^\circ$, are also compared with those published by Stonesifer et al. (1993). The maximum percentage deviation in γ_Q values at this angle was 10.7 percent. Figure (3.17.b) shows the γ_Q values for another case of two non-coplanar surface cracks with $a/c_Q = 1.0$, $h/2c_Q = 0.3$ and $a_Q/t = 0.1$. In this figure, the trends of γ_Q values at $\phi = 180^\circ$, $\phi = 90^\circ$ and $\phi = 0^\circ$ are similar to the ones published by Kuo et al. (1997b). For this case, the maximum percentage deviation in γ_Q was 23.7, 15.2 and

16.7 percent at $\phi = 180^\circ$, $\phi = 90^\circ$ and $\phi = 0^\circ$, respectively.

To study the effect of mesh sensitivity in the interaction zone (Ω) on the numerical results, the value of γ is calculated as the number of elements at sub-zone M, shown in Fig (3.1), is increased from 18 to 32 element. As shown in Fig (3.18), the solution of γ converges as the number of elements in sub-zone M approaches 32 elements. The use of more than 32 elements in sub-zone M showed no significant variation in the FE results, thus, 32 elements will be used in this sub-zone throughout the current study.

The last case studied in this section is for two non-coplanar embedded cracks with $a/c_Q = 1.0$, i.e. penny shaped, and $h/2c_Q = 0.2$. As shown in Fig (3.19), the values of γ_Q are compared at $\phi = 180^\circ$ with those found in the case of 2D non-coplanar cracks. The trends of both results were very similar with a maximum percentage deviation in γ_Q of 17.3 percent.

3.5 Concluding Remarks

A numerical FE model is used to study the interaction factor γ between two cracks in an infinite plate under remote tension or pure bending loading. The methodology used to design and build this model follows two main stages. First, two separate crack sub-models representing the interacting cracks Q and P are automatically meshed. Second, a global 3D mesh of the interaction site, i.e. the thick plate, is constructed to include the two crack sub-models. The coordinates of all nodes used in the global model are calculated based on the size, orientation and relative location of both cracks with respect to each other. Several cases of crack interaction in 2D and 3D have been discussed in this chapter. In these cases, the values of γ are compared with the those published from various studies in literature. The following remarks can be made about these results:

- (i) While no sub-modeling is used in the cases studied in 2D, different configurations of coplanar and non-coplanar interacting cracks are presented in Section (3.4.1). In these cases, good agreement with percentage deviation that is less than 10 percent, is found between the values of γ calculated in the current study and the ones published in literature. This result validated the use of the J-integral method to calculate the SIF for interacting cracks.
- (ii) Good agreement with percentage deviation of less than 7 percent, is found between the calculated Mode-I SIF values and those reported by Newman and Raju (1981). This result validates the use of the sub-models of crack Q and P used in the present 3D analysis under remote tension or pure bending loads.
- (iii) Good qualitative agreement is also found between the trends of γ in the current study and those in the literature along the interacting crack fronts for the plate case under remote tension. This result verifies the validity of the global FE model and indicates that the artificial stiffening owing to the use of MPC's in this model had no significant effect on the calculated SIF's.
- (iv) In the current study, mesh sensitivity in the FE model is reduced by increasing the number of elements used in the interaction sub zone, as shown in Fig (3.19).

Figure 3.1: A schematic diagram of the sub-zones in the global interaction zone Ω . (a) irregularly oriented cracks. (b) non-coplanar cracks.

Figure 3.2: A schematic diagram of the nodes controlling the automatic meshing algorithm of the global interaction zone Ω .

Figure 3.3: The main dimensions of the infinite plate global FE model.

Figure 3.4: A schematic diagram of (a) the FE mesh used to construct crack sub-model Q or P. (b) the volume zones used to evaluate the J-integral around the crack front in 3D.

Figure 3.5: Details of the FE sub-zones used to build crack sub-model Q or P.

Figure 3.6: Elements used to build the crack tip and front in 2D and 3D FE models.

Figure 3.7: Calculation of the J-integral in 2D and 3D.

Figure 3.8: Crack interaction configuration in 2D and 3D.

(a) Two coplanar cracks.

(b) A single array of coplanar cracks.

Figure 3.9: Variations of γ with s/c_Q for coplanar symmetric cracks under remote tension in 2D.

(a) γ values at the inner and outer tips. (b) Verification of γ values.**Figure 3.10:** Variations of γ with s/c_Q for coplanar asymmetric cracks under remote tension in 2D.

(a) γ values for different $h/2c_Q$. (b) Verification of γ values.

Figure 3.11: γ values at the inner tip for two non-coplanar symmetric cracks under tension in 2D.

Figure 3.12: γ values at the outer tip for two non-coplanar symmetric cracks under tension in 2D.

Figure 3.13: The variations of γ with a/c_Q for an array of non-coplanar symmetric cracks under remote tension in 2D.

Figure 3.14: Percentage deviation of K_I for a single crack under remote tension or pure bending, (Newman and Raju, 1981).

(a) Coplanar surface cracks.

(b) Fully overlapped surface cracks.

Figure 3.15: Variations of γ_Q with s/c_Q for surface cracks under remote tension.

Figure 3.16: Variations of γ_Q with s/c_Q for non-coplanar surface cracks under remote tension.

(a) At the outer tip, $\phi_Q = 0^\circ$. (b) At the inner, outer and deepest point, $\phi_Q = 180^\circ, 0^\circ$ and 90° .**Figure 3.17:** Variations of γ_Q with s/c_Q for non-coplanar surface cracks under remote tension.

Figure 3.18: Variations of γ_Q with s/c_Q for non-coplanar embedded cracks under remote tension.

(a) At the inner tip, $\phi_Q = 180^\circ$. (b) At the deepest point, $\phi_Q = 90^\circ$.

Figure 3.19: The effect of mesh sensitivity on the calculation of γ_Q non-coplanar surface cracks under remote tension.

Chapter 4

The interaction between two coplanar cracks in 3D

This chapter presents the results dealing with the interaction of two surface or embedded coplanar cracks in an infinite elastic plate. In the next Section, the variation of the interaction factor γ with the relative proximity of the interacting cracks is discussed for the case of coplanar surface cracks under remote tension or pure bending loading. For this case, the effects of the crack shape aspect ratio a/c and the ratio of crack depth to plate thickness a/t on the values of γ are investigated. The variations of the values of γ along the front of the two asymmetric surface coplanar cracks are also discussed. The effect of asymmetry in crack depth and crack shape aspect ratio on γ values is discussed in Sections (4.1.3) and (4.1.4), respectively. Section (4.2) presents the results obtained for γ values in the case of two interacting embedded cracks under remote tension. The ranges of a/c and a/t used in the case of symmetric and asymmetric embedded cracks are similar to those ranges used in the case of interacting surface cracks. In this Section, the difference between γ values of coplanar embedded cracks and surface cracks are also discussed within the studied ranges of a/c and a/t . Finally, some concluding remarks on the interaction behavior of coplanar cracks are presented in Section (4.3).

4.1 Interaction of two surface coplanar cracks

Surface coplanar cracks are very common in many engineering structures. As shown in Fig (4.1), cases of coplanar surface cracks are found in zones close to longitudinal welds in gas transmission pipelines or marine structures. Other cases of coplanar cracks are also found in aircraft fuselages and landing gears. In this Section the interaction of two coplanar surface cracks Q and P is investigated. As shown in Fig (4.2.a), the same notations used in Chapter 3 are used here for cracks Q and P, where c_Q and c_P are half the length of each crack, respectively. The horizontal distance which separates these cracks is referred to as s , and the thickness of the cracked plate is referred to as t . To provide a quick overview of the cases considered in this study, a table is constructed as shown in Fig (4.3). In this case, for example, the values of different geometric parameters, such as a_P/c_P , c_P/c_Q , a_P/a_Q , a_Q/t , $h/2c_Q$ and s/c_Q used to study the symmetric and asymmetric coplanar surface cracks are as indicated. Also, indicated in this table is the information on where the results are graphically presented. For cases of asymmetric surface cracks, however, the results are presented in two sets of figures that depend on whether the cracks are asymmetric in their depth or in their shape aspect ratio. For the cases of symmetric coplanar cracks studied in this Section, the interaction factor is shown for any of these cracks, usually crack Q, i.e. γ_Q . For the asymmetric crack interaction cases, however, the interaction factor γ is shown for both cracks, i.e. γ_Q and γ_P .

Throughout this thesis, the values of γ_Q or γ_P are usually shown at three points along the crack-front. The first point studied is at the inner point, i.e. $\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$, where high interaction effect is expected. The second one is at the deepest point along the crack-front, i.e. $\phi_Q = 90^\circ$ and $\phi_P = 90^\circ$, where the effect of crack shape, material thickness and loading type on the interaction factor can be shown. The third point is at the outer tip of each crack, i.e. $\phi_Q = 0^\circ$ and

$\phi_P = 180^\circ$, where values of γ might be helpful in understanding the applicability of crack assessment rules mentioned earlier in Section (2.5).

For surface coplanar cracks, interaction cases are presented, as shown in Fig (4.2.a), under remote tension or pure bending loading. The ranges of a/c and a/t are similar in both cases of loading. To distinguish each case of loading, the loading stress under bending will be symbolized as with $\sigma(z)$, which indicates that the stress varies in the z -direction i.e. along the plate thickness. In the case of embedded coplanar cracks, interaction cases are presented as shown in Fig (4.2.b) under remote tension only.

For all cases studied of coplanar cracks, the separation distance is varied so that s/c ranges from a maximum of 2.0, where the variation in the γ values diminish, to a minimum of 0.3. The minimum value of s/c is calculated based on the conditions of $s/r_p \geq 10$, which is recommended by Xin and De Los Rios (1994) for LEFM condition to be valid within the interaction zone between coplanar cracks. Using this range of s/r_p and Eq (2.15), where $r_p/c \leq 1/50$, the value of s/c should be larger than or equal to 0.2. Because of some limitations in the FE model constructed in the current study, $s/c \geq 0.3$ is used, which is a more conservative value. In this chapter, the symbols of s/c_Q and a_Q/t will be used instead of s/c and a/t . In case of asymmetric cracks, values of c_P/c_Q or a_P/a_Q will be specified.

4.1.1 Effect of crack shape aspect ratio

Different studies in the literature have investigated coplanar symmetric surface cracks with varying values of the crack shape aspect ratio a/c . For example, Yoshimura and Yagawa (1997); Leek and Howard (1994a) studied coplanar cracks with $a/c = 0.4, 0.5, 0.6$ and 1.0 . In the current work, the coplanar cracks treated will cover a/c or a_Q/c_Q values of $0.33, 0.5, 0.67$ and 1.0 . In Fig (4.5), the solution of γ_Q variations with s/c_Q under pure tension is presented at the inner point of crack

Q ($\phi_Q = 180^\circ$). At this point the value of γ_Q decreases as a function of s/c_Q in a nonlinear format. The value of γ_Q in this case, as seen in Fig (4.5), and as shown later in other coplanar cracks interaction cases, is higher than 1.0. Thus, for coplanar cracks, the SIF will always be magnified. An increase in the crack shape aspect ratio a_Q/c_Q will, as shown in Fig (4.5), result in an increase in its corresponding value of γ_Q for a given value of s/c_Q . Figures (4.6) and (4.7) show similar trends at the deepest ($\phi_Q = 90^\circ$) and outer point ($\phi_Q = 0^\circ$) along the interacting crack-front, respectively. At the deepest point, however, the effect of crack shape aspect ratio on the variation of γ_Q is found to be on an average of 60 percent less than that found at the inner or outer point along the crack-front.

Figures (4.8), to (4.10) show the results for the same crack interaction case but under pure bending at the inner, deepest and outer point along the interacting crack-front, respectively. The trend of γ_Q under bending is found to be similar to the one found under tension with few exceptions. For example, under bending there is a wider variation of γ_Q with the crack shape aspect ratio at most of the values studied of s/c_Q ; and this variation increases as $a_Q/c_Q \rightarrow 1.0$. While this greater variation in γ_Q is found at all studied points along the crack-front, it is more significant at the deepest point ($\phi_Q = 90^\circ$), as seen in Fig (4.9). In all interaction cases shown in Figs (4.5) to (4.10), the value of a_Q/t was fixed at 0.5. The next Section will investigate the effect of changing the crack depth to material thickness, a_Q/t , on the absolute value of γ_Q and how this effect varies with a_Q/c_Q and s/c_Q .

4.1.2 Effect of material thickness

Figures (4.11) to (4.16) show the variation of γ_Q with a_Q/t for $a_Q/c_Q = 0.33, 0.5, 0.67$ and 1.0. At the inner point, Fig (4.11) shows that the values of γ_Q increase, almost linearly, with a_Q/t at $s/c_Q = 0.3$. While the rate of this increase in γ_Q is similar for different a_Q/c_Q values when $s/c_Q = 0.33$, Fig (4.12) shows that at a higher value of

$s/c_Q = 1.0$, the rate of γ_Q with a_Q/t increases with a_Q/c_Q . This significant change in the rate of change in γ_Q with respect to s/c_Q is only found at the inner point along the crack-front, while at the deepest and outer points, as shown in Figs (4.13, (4.14), (4.15) and (4.16), this change was much less significant. It is important to note that the variation of γ_Q with s/c_Q remains the same for different crack shape aspect ratio while changing the crack depth to material thickness ratio. Similar trends of γ_Q with s/c_Q are found at all points investigated along the interacting crack-front, for a given value of a_Q/t .

Under pure bending loading, Figs (4.17) to (4.22) show the variation of γ_Q with a_Q/t for different values of crack shape aspect ratio, a_Q/c_Q . While there is still a difference in the rate at which γ_Q increases with a_Q/t at different s/c_Q values, this difference is much lower than the one found for the case of tensile loading. In addition, as seen earlier under tension loading, the qualitative trends of γ_Q remains the same for different crack shape aspect ratios while changing the crack depth to material thickness ratio. At the deepest point, the rate of change of γ_Q with a_Q/t under bending, as shown in Figs (4.19) and (4.20), is six times less than the one found under remote tension. This could be related to the introduction of compression across the thickness under bending.

Figures (4.23) and (4.24) show the variations of γ_Q along the interacting crack-front under remote tension at different values of s/c_Q , while keeping a_Q/t at 0.33. As shown in these two figures, the variations in the value of γ_Q increase as s/c_Q decreases. The ratios of γ_Q at $\phi_Q = 180^\circ$ to that at $\phi_Q = 90^\circ$ $\phi_Q = 90^\circ$ are 1.13 and 1.08 under tension and 1.11 and 1.08 under bending, respectively, when $s/c_Q = 0.33$ and $a_Q/t = 0.33$. If the value of a_Q/t is doubled to 0.67 under tension loading, the variations in γ_Q values, shown in Fig (4.25), are found to be more rapid at all values of s/c_Q . Figure (4.24) shows the variation of γ_Q along the interacting crack-front for pure bending. In contrast to the tension case shown in Fig (4.23), at

$a_Q/t = 0.33$, the variation in γ_Q still exist for higher values of s/c_Q . Under either tension or bending, Figs (4.25) and (4.26) show that γ_Q increases at all points along the interacting crack-front with a_Q/t .

4.1.3 Effect of asymmetry in crack depth

In practical situations, multiple cracks may not be symmetric. In fact, in a laboratory environment, it is difficult to regenerate an interaction case of multiple coplanar cracks that are symmetric in their depth and length. This Section will discuss the effect of asymmetry in the crack depth on the variations of the interaction factors γ_Q and γ_P .

- Asymmetry in crack depth of coplanar surface cracks under tension

Figures (4.27) and (4.28) show the variations in γ_P and γ_Q at the inner point of cracks P and Q, respectively, with different ratios of crack length when $a_P/c_P = a_Q/c_Q = 0.33$, $a_P/c_P = 0.33$ and $a_Q/t = 0.3$. Comparing the values of γ_P in asymmetric crack cases ($c_P/c_Q < 1$) with the symmetric cracks case ($c_P/c_Q = 1$). Fig (4.27) indicates that the interaction factor γ_P has increased by 3.4 at the inner point along the front of the smaller crack, while for the larger crack Q, the γ_Q value has decreased by 3.2 at $\phi_Q = 180^\circ$ for $c_P/c_Q = 0.33$, as shown in Fig (4.28). This increase in the γ_P value should not be misinterpreted as an increase in the dominance or in the growth rate of the smaller crack P. In fact, the reduction in the size of crack P reduces the value of K_{isP} compared to that for K_{isQ} , which counters the effect of the increase in the magnification factor. Thus, the resulting value of K_{inQ} is found in this case to be higher than the value of K_{inP} , which indicate that the larger crack Q will have a higher growth rate and show more dominance over crack P. Similar trends in γ_P and γ_Q are also found for higher values of $a_P/c_P = 1.0$, as

shown in Figs (4.29) and (4.30). For other values of a_P/c_P , the variations in γ_P and γ_Q for different crack shape aspect ratio values for cracks P and Q are shown in Figs (4.31) and (4.32), respectively. The ratio of a_P/a_Q or c_P/c_Q in this case is 0.33. In Figs (4.33) to (4.36), it is shown that the variation rate in γ_P and γ_Q with a_P/c_P values at the deepest point ($\phi_P = \phi_Q = 90^\circ$) is lower than that found at the surface.

In all previous cases of asymmetric coplanar surface cracks, the value of $a_Q/t = 0.33$ is used. Figures (4.37) to (4.46) show the effect of changing a_Q/t to 0.5, i.e. deeper cracks. In this case, the trends of γ_P and γ_Q are, qualitatively similar, while the values of both have increased due to the reduction of the crack depth to thickness ratio. For example, the values of γ_P and γ_Q are found to increase by an average of 9.2 percent in this case at $a_P/c_P = 0.33$, $s/c_Q = 0.33$ and $c_P/c_Q = 0.33$ compared to the corresponding case with $a_Q/t = 0.33$. For higher values of $a_P/c_P = 1.0$, similar trends in γ_P and γ_Q are found as shown in Figs (4.39) and (4.40). The variations of γ_Q and γ_P at other values of $a_P/c_P = 0.5$ and 0.67 are shown in Figs (4.41) and (4.42). In addition, Figs (4.43) to (4.46), show that the rate of change in γ_P and γ_Q with a_P/c_P values at the deepest point ($\phi_P = \phi_Q = 90^\circ$) is much lower than that found at the surface. For example, the changing rate of γ_P or γ_Q with c_P/c_Q at the inner point is shown to be on average 5 times larger than that found at the deepest point on each crack-front at $s/c_Q = 0.33$ and $a/t_Q = 0.3$. This result is shown to be similar for higher a/t_Q values.

- Asymmetry in crack depth of coplanar surface cracks under bending

The qualitative trends of γ_Q and γ_P under bending are not significantly different from those found under tension loads when the crack depths are asymmetric. Figures (4.47) to (4.66) cover the results of asymmetry in the depth of coplanar surface cracks under this type of loading. The effects of changing the ratio of crack size (depth or

length) are shown in Figs (4.47) and (4.48) for $a_P/c_P = 0.33$ and Figs (4.49) and (4.50) for $a_P/c_P = 1.0$ at the inner point ($\phi_P = 0^\circ$ and $\phi_Q = 180^\circ$) of both cracks P and Q, respectively. When comparing the results of γ_P or γ_Q for all the values of crack shape aspect ratio investigated, these quantities are shown to increase with a_P/c_P , see Figs (4.51) and (4.52). This result was noticed earlier in the symmetric coplanar surface crack case as well. At the deepest point of both cracks P and Q, Figs (4.53) to (4.56) show that bending loads also increased the magnitude of γ for different crack shape aspect ratios. All the results of γ_Q and γ_P presented in Figs (4.47) to (4.56) are for $a_Q/t = 0.33$.

Figures (4.57) to (4.66) show the results when $a_Q/t = 0.5$. This increase in a_Q/t raises the value of γ_P to its maximum value of 1.7 obtained at $c_P/c_Q = 0.33$ with $a_P/c_P = 1.0$. This effect is also found at the deepest point, as shown in Figs (4.63) to (4.66).

4.1.4 Effect of asymmetry in crack shape aspect ratio

Asymmetric cracks may not always have the same shape aspect ratio. This Section will address this issue by studying the interaction factor of coplanar surface cracks that do not have the same crack shape aspect ratio. In this case, the interacting crack depth is kept the same for both cracks P and Q.

- Asymmetry in crack shape aspect ratio of coplanar surface cracks under tension

Figures (4.67) to (4.74) cover the results obtained for this case of asymmetric coplanar cracks under remote tension. In Figs (4.67) and (4.68) the variations of γ_Q and γ_P at the inner point ($\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$) are shown for different values of c_P/c_Q while keeping $a_P = a_Q$ and $a_Q/t = 0.5$. The effect of asymmetry in crack shape aspect ratio is found to be qualitatively similar to the effect of asymmetry in

crack size as seen in the last Section. Again, the value of γ_Q for the bigger crack Q has decreased at $c_P/c_Q = 0.33$ and $\phi_Q = 180^\circ$ with an average of 5 percent when compared to the case of symmetric cracks where $c_P/c_Q = 1.0$. As shown in Fig (4.68), the average increase in γ_P values is similar to that of γ_Q at a given s/c_Q . Similar trends of γ_Q and γ_P are also found at the deepest and outer points along the crack-front as shown in Figs (4.69) to (4.72), respectively. At the deepest point of both cracks, Figs (4.73) and (4.74) show no significant change in the value of γ_Q and γ_P when lower value of $a_Q/t = 0.33$ is used. A simple reason for this could be because both interacting cracks have the same depth. At other points along the crack-front, the decrease in a_Q/t value also reduces the value of γ_Q and γ_P for a given s/c_Q value.

- **Asymmetry in crack shape aspect ratio of coplanar surface cracks under bending**

The effect of asymmetry in the crack shape aspect ratio of coplanar interacting surface cracks is shown in Figs (4.75) to (4.82) for the case of the infinite plate under pure bending loads. In both cases, when $a_Q/t = 0.33$ or $a_Q/t = 0.5$, the trends of both γ_Q and γ_P with s/c_Q are similar to those found under remote tension at all points along the interacting crack-front. At the deepest point, as shown in Figs (4.81) and (4.82), significant percentage change in the values of γ_Q and γ_P with a_Q/t is found. In this case the percentage change was on average 7 times larger than that found at the same point along both crack-fronts under pure tension load.

4.2 Interaction of two embedded coplanar cracks

Embedded coplanar cracks are common in different types of welds. In some situations, these cracks exist in a coplanar or non-coplanar set of crack arrays. This

Section discusses the interaction factor γ along the fronts of coplanar embedded cracks under remote tension. As indicated in Fig (4.4), the values of γ_Q and γ_P , for asymmetric embedded cracks will be shown in Figs (4.83) to (4.116).

4.2.1 Interaction of symmetric embedded coplanar cracks

Figures (4.83) to (4.88) cover the interaction cases of symmetric embedded coplanar cracks under tension. As shown in this set of figures, the trends of γ_Q are qualitatively similar to the one found for surface cracks in Section (4.1.1). Figures (4.83) to (4.85) show a nonlinear increase in the γ_Q value with respect to s/c_Q at all points along the interacting crack-fronts. In these figures, this rate of increase of γ_Q is shown to depend on the value of a_Q/c_Q , a_Q/t and ϕ_Q . When $a_Q/t = 0.5$, for example, the value of γ_Q is found to increase by -10.8, -3.5 and -10.3 percent at $\phi_Q = 180^\circ$, 90° and 0° , respectively, when a_Q/c_Q is increased from 0.33 to 1.0 at $s/c_Q = 0.33$. On the other hand, Figs (4.86) to (4.88) show that the interaction factor γ_Q has increased almost linearly with a_Q/t when the ratios of a_Q/c_Q and s/c_Q are kept at specific value.

4.2.2 Interaction of asymmetric embedded coplanar cracks

In this Section, the ranges of c_P/c_Q , a_P/c_P and a_Q/t used to model the cases of interacting asymmetric embedded coplanar cracks are similar to those used in Section (4.1.2). As indicated in Fig (4.4), the results of γ_P and γ_Q are shown in Figs (4.89) to (4.108), for cases of asymmetry in crack depth and Figs (4.109) to (4.116), for cases of asymmetry in the crack shape aspect ratio.

- **Asymmetry in crack depth of coplanar embedded cracks under tension**

With reference to the variations of γ_P and γ_Q with c_P/c_Q shown in Figs (4.89) to (4.92), the effect of asymmetry in the depth of embedded coplanar cracks is also found to be similar in trend to the asymmetry effect found in the case of surface coplanar cracks. While the interaction factors γ_Q and γ_P are found to strongly vary with a_P/c_P at the inner and out points along both crack-fronts, Figs (4.95) to (4.98) show that such variation is insignificant at the deepest point ($\phi_Q = \phi_P = 90^\circ$) for all values of s/c_Q investigated. The variation in γ_Q and γ_P with c_P/c_Q at a higher value of a_Q/t is shown in Figs (4.99) to (4.108). In this case the percentage change in γ_P and γ_Q when c_P/c_Q is changed from 1.0 to 0.33 is found to be 3.4 percent and -3.3 at $\phi_P = 0^\circ$ and $\phi_Q = 180^\circ$, respectively, when $a_Q/t = 0.5$ and $a_P/c_P = 1.0$. As shown in Fig (4.101), the value of $\gamma_P = 1.5$ at $\phi_P = 0^\circ$ with $c_P/c_Q = 0.33$ is the highest in all cases studied of coplanar embedded cracks.

- **Asymmetry in crack shape aspect ratio of coplanar embedded cracks under tension**

The last case studied of coplanar embedded interacting cracks investigates the effect of asymmetry in crack shape aspect ratio on the variation of γ_Q and γ_P . Figures (4.109) to (4.116) covers the results found in this case. When c_P/c_Q is decreased from 1.0 to 0.33, the percentage changes of the maximum γ_Q value are found to be within an average of -6.2 percent at $\phi_Q = 180^\circ$, 90° and 0° ; while for γ_P , these percentages are found to be within an average of 3.1 percent, along the front of crack P. Also in this case, Figs (4.115) and (4.116) show that at the deepest point ($\phi_Q = \phi_P = 90^\circ$) no significant changes are found in γ_Q or γ_P when a_Q/t is changed from 0.33 to 0.5.

4.3 Concluding remarks

- (i) For coplanar surface or embedded cracks, the interaction factor γ is always larger than unity for both cracks under remote tension or pure bending. This implies that a magnification of the absolute SIF's along the interacting crack-fronts of these cracks.
- (ii) For all the values of a_Q/t and s/c_Q investigated, γ_Q and γ_P are found to increase with a_Q/c_Q . This rate of increase is shown to depend on the loading and crack type (surface or embedded) as well as the value of c_P/c_Q in the case of asymmetric cracks. Also, keeping s/c_Q constant, the value of γ_Q increases with a_Q/t under tension or bending loads. The rate of increase in γ_Q is a function a_Q/c_Q .
- (iii) For surface or embedded cracks, γ_Q is lowest at the middle zone, i.e. $110^\circ \geq \phi_Q \geq 70^\circ$, along the crack-front. For asymmetric cracks, the range of this zone may differ depending on c_P/c_Q .
- (iv) Asymmetry in interacting crack depth increases the interaction factor γ_P for the smaller crack and decreases the value of γ_Q for the larger crack. However, the decrease in the absolute value of the SIF for the smaller crack and the increase of the corresponding value for the larger crack result in a higher growth rate of the bigger crack, Q.
- (v) Asymmetry in crack shape aspect ratio affects the interaction factors γ_Q and γ_P in a similar qualitative manner as that for interacting crack that are asymmetric in depth. Also, the effects of different a_Q/c_Q and a_Q/t on the values of γ_Q and γ_P for asymmetric coplanar cracks are similar in trends to those for symmetric coplanar embedded or surface cracks.

Figure 4.1: A case of coplanar surface cracks near a longitudinal weld in a gas transmission pipeline, Revie et al. (1998).

Figure 4.2: Crack interaction configuration of two coplanar surface or embedded cracks.

Coplanar surface cracks

results appear in

		Embedded/Embedded					
		Surface/Surface					
		$\frac{a_p}{c_p}$	$\frac{c_p}{c_Q}$	$\frac{a_p}{a_Q}$	$\frac{a_Q}{t}$	$\frac{h}{2c_Q}$	$\frac{S}{c_Q}$
Tension	Fig(4.5-4.16)	0.33, 0.5, 0.67, 1.0	1.0	1.0	0.33, 0.5, 0.67	0.0	0.33 : 2.0
	Fig(4.27-4.46)	0.33, 0.5, 0.67, 1.0	1.0, 0.5, 0.33	1.0, 0.5, 0.33	0.33, 0.5	0.0	0.33 : 2.0
	Fig(4.67-4.74)	0.33, 0.5, 0.67, 1.0	1.0, 0.67, 0.5, 0.33	1.0	0.33, 0.5	0.0	0.33 : 2.0
Bending	Fig(4.17-4.26)	0.33, 0.5, 0.67, 1.0	1.0	1.0	0.33, 0.5, 0.67	0.0	0.33 : 2.0
	Fig(4.47-4.66)	0.33, 0.5, 0.67, 1.0	1.0, 0.5, 0.33	1.0, 0.5, 0.33	0.33, 0.5	0.3, 0.5, 1.0	0.33 : 2.0
	Fig(4.75-4.82)	0.33, 0.5, 0.67, 1.0	1.0, 0.67, 0.5, 0.33	1.0	0.33, 0.5	0.3, 0.5, 1.0	0.33 : 2.0
		Tension or Bending			ϕ_P or ϕ_Q		

Identical cracks
 Assymetry in crack depth
 Assymetry in crack-shape aspect ratio

Figure 4.3: The cases studied for two coplanar surface cracks.

Coplanar embedded cracks

results appear in

Tension

Fig(4.83-4.88)

Fig(4.89-4.110)

Fig(4.111-4.116)

Surface/Surface					
Embedded/Embedded					
$\frac{a_P}{c_P}$	$\frac{c_P}{c_Q}$	$\frac{a_P}{a_Q}$	$\frac{a_Q}{t}$	$\frac{h}{2c_Q}$	$\frac{S}{c_Q}$
0.33, 0.5, 0.67, 1.0	1.0	1.0	0.33, 0.5, 0.67	0.0	0.33 : : 2.0
0.33, 0.5, 0.67, 1.0	1.0, 0.5, 0.33	1.0, 0.5, 0.33	0.33, 0.5	0.0	0.33 : : 2.0
0.33, 0.5, 0.67, 1.0	1.0, 0.67, 0.5, 0.33	1.0	0.33, 0.5	0.0	0.33 : : 2.0
Tension or Bending			Φ_P or Φ_Q		

Identical cracks

Assymetry in crack depth

Assymetry in crack-shape aspect ratio

Figure 4.4: The cases studied for two coplanar embedded cracks.

Figure 4.5: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric surface cracks under tension at $\phi_Q = 180^\circ$.

Figure 4.6: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric surface cracks under tension at $\phi_Q = 90^\circ$.

Figure 4.7: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric surface cracks under tension at $\phi_Q = 0^\circ$.

Figure 4.8: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric surface cracks under bending at $\phi_Q = 180^\circ$.

Figure 4.9: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric surface cracks under bending at $\phi_Q = 90^\circ$.

Figure 4.10: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric surface cracks under bending at $\phi_Q = 0^\circ$.

Figure 4.11: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under tension at $s/c_Q = 0.3$ and $\phi_Q = 180^\circ$.

Figure 4.12: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under tension at $s/c_Q = 1.0$ and $\phi_Q = 180^\circ$.

Figure 4.13: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under tension at $s/c_Q = 0.33$ and $\phi_Q = 90^\circ$.

Figure 4.14: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under tension at $s/c_Q = 1.0$ and $\phi_Q = 90^\circ$.

Figure 4.15: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under tension at $s/c_Q = 0.33$ and $\phi_Q = 0^\circ$.

Figure 4.16: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under tension at $s/c_Q = 1.0$ and $\phi_Q = 0^\circ$.

Figure 4.17: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under bending at $s/c_Q = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.18: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under bending at $s/c_Q = 1.0$ and $\phi_Q = 180^\circ$.

Figure 4.19: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under bending at $s/c_Q = 0.33$ and $\phi_Q = 90^\circ$.

Figure 4.20: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under bending at $s/c_Q = 1.0$ and $\phi_Q = 90^\circ$.

Figure 4.21: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under bending at $s/c_Q = 0.33$ and $\phi_Q = 0^\circ$.

Figure 4.22: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric surface cracks under bending at $s/c_Q = 1.0$ and $\phi_Q = 0^\circ$.

Figure 4.23: Variations of γ_Q with ϕ_Q and s/c_Q for coplanar symmetric surface cracks under tension at $a_Q/t = 0.33$.

Figure 4.24: Variations of γ_Q with ϕ_Q and s/c_Q for coplanar symmetric surface cracks under bending at $a_Q/t = 0.33$.

Figure 4.25: Variations of γ_Q with ϕ_Q and s/c_Q for coplanar symmetric surface cracks under tension at $a_Q/t = 0.67$.

Figure 4.26: Variations of γ_Q with ϕ_Q and s/c_Q for coplanar symmetric surface cracks under bending at $a_Q/t = 0.67$.

Figure 4.27: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_P = 0^\circ$.

Figure 4.28: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.29: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$.

Figure 4.30: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_Q = 180^\circ$.

Figure 4.31: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $c_P/c_Q = 0.33$ and $\phi_P = 0^\circ$.

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Figure 4.35: Variations of γ_P with $a_P/c_P = 0.67$ or 1.0 and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.33$.

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Figure 4.38: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.39: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$.

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Figure 4.41: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $c_P/c_Q = 0.33$ and $\phi_P = 0^\circ$.

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Figure 4.43: Variations of γ_P with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.5$.

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Figure 4.53: Variations of γ_P with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_Q/t = 0.33$.

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Figure 4.59: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$.

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Figure 4.61: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $c_P/c_Q = 0.33$ and $\phi_P = 0^\circ$.

Figure 4.62: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $c_P/c_Q = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.63: Variations of γ_P with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_Q/t = 0.5$.

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Figure 4.70: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 4.71: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 4.72: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 4.73: Variations of γ_Q with $a_Q/t = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$.

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Figure 4.75: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 4.76: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 4.77: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 4.78: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 4.79: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 4.80: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 4.81: Variations of γ_Q with $a_Q/t = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$.

Figure 4.82: Variations of γ_P with $a_Q/t = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$.

Figure 4.83: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric embedded cracks under tension at $\phi_Q = 180^\circ$.

Figure 4.84: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric embedded cracks under tension at $\phi_Q = 90^\circ$.

Figure 4.85: Variations of γ_Q with s/c_Q and a_Q/c_Q for coplanar symmetric embedded cracks under tension at $\phi_Q = 0^\circ$.

Figure 4.86: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric embedded cracks under tension at $s/c_Q = 0.3$ and $\phi_Q = 180^\circ$.

Figure 4.87: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric embedded cracks under tension at $s/c_Q = 0.3$ and $\phi_Q = 90^\circ$.

Figure 4.88: Variations of γ_Q with a_Q/t and a_Q/c_Q for coplanar symmetric embedded cracks under tension at $s/c_Q = 0.3$ and $\phi_Q = 0^\circ$.

Figure 4.89: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_P = 0^\circ$.

Figure 4.90: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.91: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$.

Figure 4.92: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_Q = 180^\circ$.

Figure 4.93: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $c_P/c_Q = 0.33$ and $\phi_P = 0^\circ$.

Figure 4.94: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $c_P/c_Q = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.95: Variations of γ_P with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.33$.

Figure 4.96: Variations of γ_Q with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.33$.

Figure 4.97: Variations of γ_P with $a_P/c_P = 0.67$ or 1.0 and s/C_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.33$.

Figure 4.98: Variations of γ_Q with $a_P/c_P = 0.67$ or 1.0 and s/C_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.33$.

Figure 4.99: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_P = 0^\circ$.

Figure 4.100: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.101: Variations of γ_P with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$.

Figure 4.102: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $c_P/c_Q = 0.33$ and $\phi_P = 0^\circ$.

Figure 4.103: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $c_P/c_Q = 0.33$ and $\phi_Q = 180^\circ$.

Figure 4.104: Variations of γ_Q with c_P/c_Q and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$ and $\phi_Q = 180^\circ$.

Figure 4.105: Variations of γ_P with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.5$.

Figure 4.106: Variations of γ_Q with $a_P/c_P = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.5$.

Figure 4.107: Variations of γ_P with $a_P/c_P = 0.67$ or 1.0 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.5$.

Figure 4.108: Variations of γ_Q with $a_P/c_P = 0.67$ or 1.0 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.5$.

Figure 4.109: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 4.110: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 4.111: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 4.112: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 4.113: Variations of γ_Q with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 4.114: Variations of γ_P with a_P/c_P and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 4.115: Variations of γ_Q with $a_Q/t = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/c_Q = 0.33$ and $c_P/c_Q = 0.33$.

Figure 4.116: Variations of γ_P with $a_Q/t = 0.33$ or 0.5 and s/c_Q for coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/c_Q = 0.33$ and $c_P/c_Q = 0.33$.

Chapter 5

The interaction between two non-coplanar cracks in 3D

This chapter presents the results dealing with the interaction of two surface or embedded non-coplanar cracks in an elastic infinite plate. The variations of the interaction factor γ with the relative vertical and horizontal proximity of the interacting cracks are first discussed for the case of non-coplanar surface cracks under remote tension or pure bending loading. Here, the effects of the crack shape aspect ratio, a/c , relative vertical separation distance, $h/2c$, and the ratio of crack depth to plate thickness, a/t , on the values of γ are investigated. The effects of asymmetry in either crack depth or crack shape aspect ratio on γ values are discussed in Sections (5.1.4) and (5.1.5), respectively. Section (5.2) presents the results obtained for γ values in the case of two interacting embedded non-coplanar cracks under remote tension. The ranges of a/c and a/t used in the case of symmetric and asymmetric embedded cracks are similar to those ranges used in the case of interacting surface cracks. Finally, some concluding remarks on the interaction behavior of non-coplanar cracks are presented in Section (5.3).

5.1 Interaction of two surface non-coplanar cracks

In the last two decades, the existence of surface non-coplanar cracks has been observed to be a feature in numerous mechanical and aerospace structures that have failed in service. Colonies of non-coplanar surface cracks, such as those shown in Fig (5.1), are often found in the vicinity of longitudinal welds used in gas transmission pipelines and marine structures. Other non-coplanar cracks can be found in landing gears of aging aircraft. The interaction behavior between two non-coplanar surface semi-elliptical cracks Q and P is investigated in this Section. Figure (5.2.a) shows the notations used to describe the geometry of those two non-coplanar cracks, where c_Q and c_P are half the length of each crack, respectively; the horizontal and vertical distances separating these cracks are denoted by, s , and, h , respectively. And, the thickness of the cracked plate is denoted by, t . To provide a quick overview of the cases investigated in this chapter, a table is constructed as shown in Fig (5.3).

The results for symmetric non-coplanar surface cracks are covered in Figs (5.6) to (5.29). The effects of asymmetry for non-coplanar surface cracks are shown in Figs (5.30) to (5.83) for interacting cracks with different depths and those for interacting cracks with different crack shape aspect ratios are shown in Figs (5.84) to (5.105). For all the cases of symmetric non-coplanar cracks studied in this chapter, the interaction factor is shown for either of the two cracks (usually crack Q, i.e. γ_Q). For the asymmetric crack interaction cases, however, the interaction factor γ is shown for both cracks, i.e. γ_Q and γ_P . The values of γ_Q or γ_P investigated are usually shown at three locations along the crack-front. These locations are at the inner point, i.e. $\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$, the deepest point along the crack-front, i.e. $\phi_Q = 90^\circ$ and $\phi_P = 90^\circ$, and the outer point of each crack, i.e. $\phi_Q = 0^\circ$ and $\phi_P = 180^\circ$.

For the interaction cases of non-coplanar surface cracks the interaction effect is also discussed, as shown in Fig (5.2.a), under pure bending loading within similar ranges of a/c and a/t . Also in the analysis, the relative horizontal separation distance between the cracks is varied so that s/c ranges from 2.0, where the two cracks are relatively far, to -2.0, where the interacting cracks are fully overlapped, sometimes called stacked. The symbols s/c_Q and a_Q/t will be used here instead of s/c and a/t ; and, in the case of asymmetric cracks, values of c_P/c_Q or a_P/a_Q will be specified. Through this chapter, when needed, percentage changes in the interaction factors γ_Q and γ_P will be presented. In this case, a positive percentage indicate an increase in the interaction factor value while a negative percentage should indicate a decrease in that value in respect to the case compared to.

As discussed in Section (3.4.2), for cracks which are close together but are not 'overlapping', i.e. $2.0 \geq s/c_Q \geq 0.0$, the effect of each crack on the other is to increase the SIF values resulting in an interaction factor that is higher than unity. For overlapping cracks, i.e. $0.0 > s/c_Q \geq -2.0$, interaction factors of less than unity can exist on the overlapped portions of the interacting crack fronts. In the figures showing the results, the dashed horizontal line at $\gamma_Q = 1$ serves to indicate when magnification of the stress SIFs occurs ($\gamma_Q > 1$) and when shielding occurs ($\gamma_Q < 1$). Also, the dashed vertical line is used to distinguish between cases of overlapping, $0.0 > s/c_Q \geq -2.0$, and non-overlapping, $2.0 \geq s/c_Q \geq 0.0$, non-coplanar cracks.

5.1.1 Effect of crack shape aspect ratio

In the few studies on non-coplanar symmetric cracks, such as that by Stonesifer et al. (1993) and Kuo et al. (1997b), only the effect of the relative proximity of the cracks, i.e. $h/2c_Q$ or s/c_Q , has been investigated, and the values of a_Q/c_Q and a_Q/t are being kept constant. In the current work, the cases of non-coplanar cracks analyzed cover a_Q/c_Q ratios of 0.33, 0.5, 0.67 and 1.0 while keeping the values of

$h/2c_Q$ and a_Q/t at 0.3 and 0.2, respectively. Figures (5.6)–(5.8) show the computed solutions of γ_Q at for the case where $a_Q/t = 0.2$ at the three points corresponding to $\phi_Q = 180^\circ$, 90° and 0° , respectively, when the plate is under remote tension. At the inner point ($\phi_Q = 180^\circ$) along the crack front, the value of γ_Q increases nonlinearly as s/c_Q decreases. The value of γ_Q , as shown in Fig (5.6) and will be presented later for the other cases of non-coplanar crack interaction, is higher than unity for $s/c_Q \geq 0$. The interaction behavior of non-coplanar cracks is qualitatively similar to that found for coplanar cracks when $s/c_Q \geq 0.0$. The increase in the crack shape aspect ratio, a_Q/c_Q , is shown in Fig (5.6) to significantly increase the corresponding value of γ_Q for $s/c_Q \geq 0.0$. On the other hand, for $s/c_Q < -0.5$, the starting point of shielding, the effect of increasing a_Q/c_Q is found to decrease γ_Q .

While the effect of a_Q/c_Q on γ_Q is similar in trends at all points along the crack front, the value of s/c_Q at which the shielding effect starts depends on the relative location along the crack-front. Figure (5.7) shows that the point at which γ_Q decreases with an increase in a_Q/c_Q coincides with the shielding starting point only at the deepest point ($\phi_Q = 90^\circ$). At $\phi_Q = 180^\circ$, the maximum and minimum value of γ_Q are calculated as 1.39, and 0.49, respectively; the corresponding values at $\phi_Q = 90^\circ$ are 1.31 and 0.79; and those for $\phi_Q = 0^\circ$ are 1.31 and 0.74, respectively.

Figures (5.9) to (5.11) show that the variations of γ_Q with a_Q/c_Q for the corresponding cases when the plate is subjected to pure bending. The maximum value of γ_Q was found to increase by 5.7 percent under bending, compared to the corresponding case under tension, at $\phi_Q = 180^\circ$ when $a_Q/c_Q = 1.0$. This increase remains about the same for all the values of s/c_Q analyzed. This suggests that interacting cracks will experience higher growth rates under bending as compared to that under tension. At lower values of a_Q/c_Q , however, less significant changes are found in the γ_Q values when compared to those calculated under tension. At $\phi_Q = 180^\circ$, the maximum and minimum value of γ_Q are calculated as 1.47, and 0.58, respectively;

the corresponding values at $\phi_Q = 90^\circ$ are 1.46 and 0.66; and those for $\phi_Q = 0^\circ$ are 1.39 and 0.75, respectively.

5.1.2 Effect of the vertical separation distance

Figures (5.12) to (5.14) show the effect of increasing the relative vertical separation distance, $h/2c_Q$, on the interaction factor γ_Q of non-coplanar asymmetric surface cracks. The cases studied are for $h/2c_Q$ values of 0.3, 0.5 and 1.0, while keeping the values of a_Q/c_Q and a_Q/t at 0.33 and 0.2, respectively. As shown in Fig (5.12), increasing the separation distance, $h/2c_Q$, between the cracks results in a decrease in the interaction factor. This decrease is shown at $\phi_Q = 180^\circ$ as an increase in γ_Q with $h/2c_Q$ for s/c_Q less than approximately -0.1 and a decrease in γ_Q with $h/2c_Q$ for s/c_Q larger than approximately -0.1 . Figures (5.13) and (5.14) show that such an increase in γ_Q with $h/2c_Q$ occurs at $\gamma_Q < -1.0$ and -1.5 for $\phi_Q = 90^\circ$ and 0° , respectively. The maximum value of γ_Q in the case where $h/2c_Q = 0.3$ and $a_Q/c_Q = 0.33$ is calculated at $\phi_Q = 180^\circ$ as 1.21; the corresponding value at $\phi_Q = 90^\circ$ is 1.24; that for $\phi_Q = 0^\circ$ is 1.13.

Figures (5.15) to (5.17) show the variations of γ_Q with $h/2c_Q$ at $a_Q/t = 0.2$ and $a_Q/c_Q = 0.33$, when the plate is subjected to pure bending. While the trends of γ_Q at the inner and outer points along the crack-front are similar to those found under tension, at the deepest point more differences in γ_Q values are found when compared with those found at the corresponding tension cases. For example, under tension, maximum shielding is found at $s/c_Q = -1.8$, as shown in Fig (5.13). Under bending, however, the maximum shielding effect was higher and occurs, as shown in Fig (5.16), when $s/c_Q = -2.0$, i.e. when the interacting cracks are fully overlapped. This suggests a slow down in the crack growth rate at the deepest point for non-coplanar fully overlapped symmetric surface cracks, when under bending. There is also an overall magnification of the SIF under bending loads compared to simple tension load

cases. For example, the maximum value of γ_Q is raised by 4.5 percent, compared to the same case under tension, at $\phi_Q = 90^\circ$ when $h/2c_Q = 0.3$, $a_Q/c_Q = 0.3$ and $a_Q/t = 0.2$. This increase was less than 2 percent at the surface points ($\phi_Q = 180^\circ$ and 0°). At all points along the interacting crack-fronts investigated, the value of s/c_Q at which the effect of $h/2c_Q$ is reversed has increased in the overlapping direction by (s/c_Q) increment of 0.15.

5.1.3 Effect of material thickness

Figures (5.18) to (5.20) present the variations of γ_Q with a_Q/c_Q for a higher ratio of $a_Q/t = 0.5$ under tension while keeping the value of $h/2c_Q$ at 0.3. In this case, at all points along the interacting crack-fronts, some increase is found in the calculated values of γ_Q with the increase in the value of a_Q/t . For example, at $\phi_Q = 180^\circ$, Fig (5.18) shows an increase of 3.0 and 2.5 percent, compared to the case with $a_Q/t = 0.2$, in the maximum and minimum values of γ_Q , respectively. Similar changes in the maximum and minimum γ_Q values are found at the deepest and outer points, as shown in Figs (5.19) and (5.20). While these percentages may indicate that for higher ratios of crack depth to material thickness, the growth rate of interacting cracks is expected to increase at the inner, deepest and outer points, it is important to note that such an increase may not be significant. In this case, the maximum and minimum calculated values of γ_Q along the crack-front were 1.44 and 0.51 at $\phi_Q = 180^\circ$ and $a_Q/c_Q = 1.0$. For another analysis, Figs (5.21) to (5.23) show the effect of varying $h/2c_Q$ at a higher ratio of $a_Q/t = 0.5$ under tension loading with $a_Q/c_Q = 0.33$. In this case, for all the points studied along the crack-front, the maximum and minimum value of γ_Q increased, compared to the corresponding case with lower a_Q/t of 0.2, by 5.2 and 2.9 percent, while maintaining the same trend found previously for $a_Q/t = 0.2$.

Under bending, similar trends of γ_Q variation exist as shown in Figs (5.24) to (5.26): which present the variation of γ_Q with a_Q/c_Q while keeping $h/2c_Q = 0.3$. The maximum and minimum values of γ_Q have increased in this case at $\phi_Q = 180^\circ$ by 3.0 and 2.5 percent, respectively, compared to the same case with $a_Q/t = 0.2$. Also, this increase may not significantly affect the rates at which those two cracks will propagate. At $a_Q/c_Q = 1.0$, the maximum γ_Q value of 1.52 is calculated as the highest magnification ratio reported for the case of two non-coplanar symmetric surface cracks.

When varying the ratio of relative vertical separation distance, $h/2c_Q$, the calculated values of γ_Q , shown in Figs (5.27) to (5.29), were found to increase as $h/2c_Q$ decreases and a_Q/t increases. For example, the maximum value of γ_Q increased by 12.6 percent at $h/2c_Q = 0.3$ and $s/c_Q = 0$ compared to the maximum calculated for $a_Q/t = 0.2$. In all the cases studied for non-coplanar surface symmetric cracks the maximum and minimum values of γ_Q were 1.52 (highest magnification) and 0.5 (highest shielding), as shown in Figs (5.24) and (5.6), respectively.

5.1.4 Effect of asymmetry in crack depth

Figure (5.5) shows an example of a stress corrosion crack colony on the outer surface of a gas transmission pipeline. As shown in this figure, multiple non-coplanar surface cracks found in these colonies are often asymmetric. The difference in the sizes of these interacting cracks plays an important role on the rate they would propagate, hence, the development of these colonies. The effect of asymmetry in the crack depth on the variations of the interaction factor γ is discussed in this Section. Because of the asymmetry in the interacting cracks depth, the values of γ_Q and γ_P will both be presented.

- **Asymmetry in crack depth of non-coplanar surface cracks under tension**

The results of Section (4.1.3) for coplanar surface cracks indicated that asymmetry in crack depth affect the SIFs of interacting cracks by increasing the interaction factor γ_P for the smaller crack, while decreasing the value of γ_Q for the larger crack. This effect was noticed at all the points analyzed along the interacting crack-fronts. Figures (5.30) to (5.56) show that this effect of asymmetry is also true for non-coplanar surface cracks under tension for $s/c_Q \geq 0$. On the other hand, for $s/c_Q < 0$, the reverse was found to be true. In other words, asymmetry is found to enhance the interaction effect, magnification or shielding, for the smaller crack, P, and reduce it for the larger crack, Q.

Figures (5.30) to (5.35) show the variation of γ_P and γ_Q for different values of the crack length ratio c_P/c_Q while keeping a_Q/t , $h/2c_Q$ and a_P/c_P at 0.2, 0.3, and 0.33, respectively. In this case, the percentage change in the maximum γ_P and γ_Q values, from those found for symmetric cracks, was 11.1 and -7.7, respectively. For the same case, the percentage change in the minimum γ_P and γ_Q values was -22.4 and 23.4 percent, respectively. As seen also in the case of coplanar cracks, although $\gamma_P > \gamma_Q$ at the "magnification zone", the absolute values of the SIF at the inner point of crack Q is much higher than that of crack P when no interaction is considered. When interaction is considered, the K_{inQ} value at the inner point of crack Q is shown to be higher than that for crack P, K_{inP} . For example, at $\phi_Q = 180^\circ$, $\phi_P = 0^\circ$, and $c_P/c_Q = 0.33$, K_{isQ}/K_{isP} is 1.74 at $s/c_Q > 2$. In case of shielding, where $s/c_Q = -1.5$, the value of K_{inP} is further reduced with respect to that of K_{inQ} ; and, using the results in Figs (5.30) and (5.31), K_{inQ}/K_{inP} is found to be 2.63. As shown in Fig (5.30), the maximum percentage change in γ_P and γ_Q is calculated as -46.6 at $s/c_Q = -2.0$ and $\phi_P = 0^\circ$. The variations of γ_P and γ_Q

shown in Figs (5.30) to (5.35) are similar in trend at all the studied points along the interacting cracks-fronts. Overall, this result shows that larger cracks, less effected by smaller cracks, will dominate the growth process, while smaller cracks will tend to slow down or remain dormant.

Figures (5.36) to (5.41) present the variation of γ_P and γ_Q with s/c_Q c_P/c_Q for a higher a_P/c_P ratio of 1.0. As the crack shape aspect ratio a_P/c_P increases, the maximum interaction factor at the inner point of both cracks is shown to increase by 10.9 and 21.1 percent, compared to the case with $a_P/c_P = 0.3$, for γ_P and γ_Q , respectively. At the shielding zone, however, the minimum interaction factor is shown to decrease at the inner point by -21.6 and -10.6 percent for γ_P and γ_Q , respectively. Similar changes are shown at the deepest and outer points along the interacting crack-fronts. These changes in the interaction behavior should result in a higher growth rate at the points along the crack-front with $\gamma \geq 1$ and a lower rate at the points with $\gamma < 1$.

The effects of $h/2c_Q$ on asymmetric non-coplanar cracks is can be seen in Figs (5.42) to (5.44), when $a_Q/t = 0.2$, $a_P/c_P = 0.33$ and $c_P/c_Q = 0.5$. For asymmetric cracks, the trends of the γ_P and γ_Q variations with $h/2c_Q$ are shown to be similar to those found for symmetric cracks at all the points along the interacting crack-fronts. These results, also show the rate of change in γ_Q and γ_P with respect to $h/2c_Q$ is maximum at the shielded zone of s/c_Q , i.e. when $\gamma < 1.0$, and that the parameter $h/2c_Q$ has a higher impact on the shielding effect compared to that found at $\gamma \geq 1$. In fact, Fig (5.43) shows that the shielding effect has almost diminished at the deepest point of crack Q when $h/2c_Q$ was increased from 0.3 to 1.0.

The variations of γ_Q and γ_P with s/c_Q and c_P/c_Q are shown in Figs (5.45) to (5.56) for a higher crack depth to material thickness ratio, $a_Q/t = 0.5$. The effect of a_Q/t on the interaction behavior is qualitatively similar to that shown for symmetric cracks. Under tension loads, the maximum and minimum γ_Q and γ_P can be seen in

Figs (5.52), (5.37), (5.51) and (5.36) to be 1.44, 0.498, and 1.54, 0.37 for γ_Q and γ_P , respectively. These results also indicate that the highest shielding effect is expected at the inner point of smaller semi-circle cracks with low a_Q/t and at $s/c_Q \approx -1.5$.

- **Asymmetry in crack depth of non-coplanar surface cracks under bending**

Figures (5.57) to (5.71) show the corresponding cases of asymmetry in crack depth for non-coplanar surface cracks under bending loads when $a_Q/t = 0.2$. The effect of bending is shown in these cases to decrease the magnitude of the interaction factor at the outer and inner point on both cracks Q and P. At the deepest point, however, the reverse is true and the maximum and minimum interaction factors are seen in Figs (5.59) and (5.60) to change by 4.5 and -3.2 percent for crack Q and by 4.5 and -3.4 percent for crack P, compared to the same case under tension with $a_P/c_P = 0.33$. These changes are few percentage higher when a higher crack shape aspect ratio a_P/c_P of 1.0 is used, as shown in Figs (5.65) and (5.66). Also, with reference to Figs (5.65) and (5.66), asymmetry in crack depth is shown to affect the value of s/c_Q at which maximum shielding occurs for cracks P and Q. For example, the minimum value of γ_P was found when $s/c_Q = -2$ for all values of c_P/c_Q analyzed. However, for crack Q, the minimum value of γ_Q was found when $s/c_Q = -2$ for $c_P/c_Q = 1.0$ only. For lower c_P/c_Q values of 0.5 and 0.33, the minimum values of γ_Q were found to be -1.9 and -1.8, respectively.

For higher values of $h/2c_Q$, the interaction effect, shown in Figs (5.69) to (5.71), seems to decrease in a similar manner as that found for tension loading. In this case, as shown in Fig (5.70), the shielding effect has almost diminished for the larger crack Q at $\phi_Q = 90^\circ$ for $h/2c_Q = 1.0$. Figures (5.72) to (5.83) presented the results for the same case of asymmetric cracks at a higher value of $a_Q/t = 0.5$. The effect of raising the crack depth to material thickness ratio is shown in this case to increase the

interaction effect for $\gamma \geq 1.0$ and reduce it for $\gamma < 1.0$. In this case, the maximum value of γ_P and γ_Q (as can be seen in Fig (5.78) and (5.79)) are 1.63 and 1.52, respectively.

5.1.5 Effect of asymmetry in crack shape aspect ratio

In some of the reported cases of multiple cracking caused by stress corrosion, non-coplanar surface cracks are found with different crack shape aspect ratios. Revie et al. (1998). In this Section, asymmetry in the crack shape aspect ratio of the interacting non-coplanar surface cracks is introduced to the geometries of crack Q and P while maintaining the same depth for both cracks. This study is carried to obtain a better understanding of the driving force for multiple cracks with different crack shape aspect ratio a/c .

- Asymmetry in crack shape aspect ratio of non-coplanar surface cracks under tension

Figures (5.84) to (5.94) cover the results obtained for this case of asymmetric non-coplanar cracks under remote tension. In Figs (5.84) and (5.85), the variations of γ_Q and γ_P at the inner point ($\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$) are shown for different values of c_P/c_Q while keeping $a_P = a_Q$ and $a_Q/t = 0.5$. The impact of asymmetry in crack shape aspect ratio is shown to be qualitatively similar to the effect of asymmetry in crack depth shown in the last Section. However, the rate of change in γ_P with respect to c_P/c_Q is found to be lower, in this case, than that found in the case of asymmetry in crack depth at the corresponding s/c_Q , a_Q/t , $h/2c$, a_Q/c_Q and ϕ_P . This reduction in the change rate of γ_P with respect to c_P/c_Q is also found in γ_Q at all the values of ϕ_Q , ϕ_P and s/c_Q studied.

For this case of asymmetry in the crack shape aspect ratio, Figs (5.90) and (5.91) show the effects of changing the a_Q/t on the variation of γ_Q and γ_P with s/c_Q . As

shown in these two figures. γ_Q and γ_P increase with a_Q/t at all points along the interacting crack-fronts. The maximum percentage change in γ_Q and γ_P , compared to the case with $a_Q/t = 0.2$, was found to be 9.84 and 6.0 percent at $\phi_Q = 90^\circ$ and $\phi_P = 0^\circ$, respectively. The effect of $h/2c_Q$ on γ_Q and γ_P is shown in Figs (5.92) to (5.94). The minimum value of γ_P under tension loading is shown to be 0.49 when $s/c_Q = -1.4$ and $h/2c_Q = 0.3$. Figure (5.93) also shows that for crack Q at $\phi_Q = 90^\circ$, the shielding effect almost diminish as $h/2c_Q \rightarrow 1.0$.

- **Asymmetry in crack shape aspect ratio of non-coplanar surface cracks under bending**

Under bending, the effect of asymmetry in the crack shape aspect ratio of non-coplanar interacting surface cracks is shown in Figs (5.95) to (5.105). Compared to the corresponding case under pure tension, the values of γ_Q and γ_P have changed by an average amount of 4.55 and 6.27 percent, respectively. For non-coplanar surface cracks, the maximum effect of the SIF shielding and magnification is found under bending. For the larger crack Q, a maximum γ_Q value of 1.36 is found to occur, as shown in Fig (5.97), at $\phi_Q = 90^\circ$ when $a_Q/t = 0.5$, while the minimum γ_Q value of 0.63 is found to occur at $\phi_Q = 180^\circ$ when $a_Q/t = 0.2$. For the smaller crack P, a maximum γ_P value of 1.45 is found to occur, as shown in Fig (5.102), at $\phi_P = 90^\circ$ when $a_Q/t = 0.5$, while the minimum value of 0.49 is found at $\phi_P = 0^\circ$ when $a_Q/t = 0.2$.

5.2 Interaction of two embedded non-coplanar cracks

In an investigation similar to that done for the surface non-coplanar cracks cases, asymmetries in the interacting crack depth and shape aspect ratio are studied for

embedded non-coplanar cracks with a wide range of c_P/c_Q . Figures (5.4) shows that the interaction factor values are covered in Figs (5.106) to (5.117) for symmetric cracks; and in Figs (5.118) to (5.144) for interacting embedded cracks with different depth and Figs (5.145) to (5.155) for interacting embedded cracks with different crack shape aspect ratio.

5.2.1 Interaction of embedded non-coplanar symmetric cracks

With reference to Figs (5.106) to (5.108), the interaction behavior of non-coplanar symmetric embedded cracks show some similarities with the behavior found for surface non-coplanar cracks under tension loading. As an example, for $s/c_Q \geq 0$, γ_Q is shown to increase as a_Q/c_Q increases and s/c_Q decreases.

The effect of the relative vertical separation distance, $h/2c_Q$, on γ_Q for embedded non-coplanar cracks is shown in Figs (5.109) to (5.111), when the ratios of a_Q/c_Q and a_Q/t are kept at 0.33 and 0.2, respectively. For this case, the percentage changes in the minimum values of γ_Q for $h/2c = 0.3$, compared to the case with $h/2c_Q = 1.0$, are -23.6, -12.7 and -3.5 percent at $\phi_Q = 180^\circ$, 90° and 0° , respectively. For the maximum γ_Q , however, the percentage changes in its values for $h/2c = 0.3$, compared to the case with $h/2c_Q = 1.0$, were 2.96, 3.2 and 1.8 percent at $\phi_Q = 180^\circ$, 90° and 0° , respectively.

Figures (5.112) to (5.114) show the interaction results for two embedded symmetric non-coplanar cracks with a higher crack depth to plate thickness ratio, a_Q/t , of 0.5. In this case, the percentage change in the minimum values of γ_Q , compared to the case with $a_Q/t = 0.2$, $a_Q/c_Q = 0.33$ and $a_Q/c_Q = 1.0$, was found to be less than 3 percent at all points investigated along the interacting crack-fronts. And, for the same case, the percentage change in the maximum values of γ_Q was slightly

higher: 3 percent on average along the front of crack Q. As shown in Figs (5.115) to (5.117), the variations of γ_Q with $h/2c_Q$ are similar in trend for higher a_Q/t ratios. For all the cases studied for embedded non-coplanar symmetric cracks, the maximum magnification effect was found to be 1.44 when $a/t_Q = 0.5$, $a_Q/c_Q = 1.0$, $h/2c_Q = 0.3$ and $\phi_Q = 180^\circ$, while the maximum shielding effect was found to be 0.5 when $a/t_Q = 0.2$, $a_Q/c_Q = 1.0$, $h/2c_Q = 0.3$ and $\phi_Q = 180^\circ$.

5.2.2 Interaction of embedded non-coplanar cracks with asymmetric crack depth

In this Section, the interaction factor of non-coplanar asymmetric embedded cracks, under tension loading, is investigated. Figures (5.118) to (5.143) present the first set of cases discussed when considering asymmetry in the depths of the interacting cracks while maintaining the same aspect ratio of the crack shapes. This type of asymmetry is shown to affect the interaction behavior of embedded non-coplanar cracks by increasing γ_P as c_P/c_Q decreases, while $\gamma_P \geq 1.0$, and decreasing γ_P with c_P/c_Q , for $\gamma_P < 1.0$. While this is the case for the smaller crack P, the effect of asymmetry seems to be the reverse for the larger crack. As explained earlier in the case of non-coplanar asymmetric surface cracks, this change in the value of γ_P and γ_Q is met with a reversed change in the absolute SIF values of each crack along its front. As a result, the overall calculated K_Q is shown to increase while K_P is shown to decrease.

With reference to Figs (5.118) to (5.123), at a size ratio c_P/c_Q of 0.33, the maximum and minimum values of γ_P are found to be 1.37 and 0.45 when $\phi_P = 0^\circ$, while for crack Q, the maximum and minimum values of γ_Q are found to be 1.23 and 0.55 at the inner point ($\phi_Q = 180^\circ$). These values are shown to change by 13.5, -10.2, 9.36 and -21.6 percent when compared to the same interaction case

with higher a_P/c_P value of 1.0, shown in Figs (5.124) to (5.129). Figures (5.130) to (5.141) show the effect of increasing a_Q/t , from 0.2 to 0.5, on the interaction factors γ_P and γ_Q . Comparing this case where $a_P/c_P = 0.33$ with the corresponding case when $a_Q/t = 0.2$, the maximum and minimum γ_P and γ_Q are increased by 6.0 percent and 3.1 percent for both cracks P and Q, respectively. The results in Figs (5.136) to (5.141) show that the highest magnification effect is found at $a_Q/t = 0.5$, $h/2c_Q = 0.3$, $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$. In this case the maximum value of γ_P is calculated as 1.55. The maximum shielding effect, however, as shown in Figs (5.136) to (5.141) and in Fig (5.124), at $a_Q/t = 0.2$, $h/2c_Q = 0.3$, $a_P/c_P = 1.0$ and $\phi_P = 0^\circ$. The minimum value of γ_P in this case is calculated as 0.37.

5.2.3 Interaction of embedded non-coplanar cracks with asymmetric crack shape aspect ratio

The values of the interaction factor for non-coplanar embedded cracks under pure tension with different crack aspect ratio are presented in this Section. Figures (5.145) to (5.150) present the variations of γ_Q and γ_P with s/c_Q and c_P/c_Q when $a_Q/t = 0.5$ and $a_Q/c_Q = 0.33$. As shown in this set of figures, the maximum and minimum values of γ_P were calculated at $\phi_P = 90^\circ$ and $\phi_P = 180^\circ$, respectively. Also, for crack Q, the maximum and minimum values of γ_Q were calculated at the inner point, i.e. $\phi_Q = 180^\circ$. The percentage changes in the maximum and minimum values of γ_Q for $c_P/c_Q = 0.33$ are found to be -7.7 and 12.3 percent, respectively, when compared to those with $c_P/c_Q = 1.0$. In the same comparison, the percentage change in the maximum and minimum values of γ_P . And for crack P, the percentage change in the maximum and minimum values of γ_P its values were -4.7 and -13.3 percent, respectively.

Finally, Figs (5.151) and (5.152) show the effect of a_Q/t in this case of asymmetric embedded non-coplanar cracks. The crack depth to plate thickness ratio is shown to increase the interaction factor for both cracks at all points along their interacting fronts. The effect of $h/2c_Q$ on γ_Q and γ_P is presented in Figs (5.153) to (5.155), when a_Q/t , a_Q/c_Q and c_P/c_Q are kept at 0.2, 0.33 and 0.33, respectively. The interaction effect here is shown to increase as the relative vertical distance decreases between the interacting cracks. Hence, the maximum shielding for this asymmetric case is found at crack P inner point, as shown in Fig (5.153), where γ_P is 0.49.

5.3 Crack interaction and assessment rules

As shown earlier, the values of the SIFs of the interacting cracks are not only affected by the crack shape aspect ratio and the depth to material ratio, but also by their relative horizontal and vertical proximity, $h/2c$ and s/c . This was evident in all the cases studied in this Chapter and also those discussed in Chapter 4. It is thus more realistic to relate the proximity parameters of these cracks to the assessment rules used to re-characterize them. To accomplish this goal, the SIFs calculated at the interacting points along both crack-fronts should be related first to the ones found from the projected length of the re-characterized crack. By doing so, the SIF resulting from this re-characterized crack could be compared with the one calculated in the current study for each interacting crack. In this section an example of this comparison is discussed, for which the parameters ν_Q and ν_P are defined as follows:

$$\nu_P = \frac{K_{inP}}{K_{is}(a_{max}/c_{new}, a_{max}/t)} \quad (5.1)$$

$$\nu_Q = \frac{K_{inQ}}{K_{is}(a_{max}/c_{new}, a_{max}/t)} \quad (5.2)$$

where, $c_{new} = c_P + c_Q + s/2$, and a_{max} is the maximum depth of crack Q or P. Figures (5.156) to (5.161) show the variations of ν_P and ν_Q at the inner, deepest and outer

points along the fronts of two surface non-coplanar cracks with different crack depth, but similar crack shape aspect ratio. As shown in these figures, when $\psi < 1$ this indicates that the SIF of the re-characterized crack is calculated conservatively. For example, Figs (5.156) and (5.157) show that at the inner point, $\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$, the estimated SIF for the re-characterized crack is calculated conservatively, at different values of crack size ratio c_P/c_Q , when there is an overlap between the two cracks. However, when $s/c_Q > -0.5$ the SIF value calculated in this study is higher than that found using the re-characterized crack dimensions. As shown in Figs (5.158) and (5.159), this case is reversed at the deepest point, where the SIF values of the re-characterized crack are shown to be higher than the one calculated in the current study for crack P. On the other hand, the estimated SIF values for the re-characterized crack are shown to be significantly lower for Q, at $\phi_Q = 90^\circ$ and $s/c_Q < 0$; this result indicates that the assessment technique might not be conservative when overlapping exists.

Figures (5.160) and (5.161) show the variations of ψ_P and ψ_Q at the outer point of both crack-fronts. In this case the value of ψ_Q is higher than unity for $s/c_Q > -1, -1.2$ and -1.6 when $c_P/c_Q = 0.33, 0.5$ and 1.0 , respectively. And, while this value decreases with the increase of c_P/c_Q , it is still larger than unity, which is not a conservative estimate for the re-characterized crack SIF. For crack P, however, this case is less severe, where at $c_P/c_Q = 0.5$ and 0.33 , the estimated re-characterized crack SIF is shown to be conservative at this point along its front.

5.4 Concluding remarks

- (i) The interaction effect, magnification or shielding, is found to increase with the increase of the shape aspect ratio of the interacting cracks, a_Q/c_Q . In this case, the rate of change of the interaction factor γ with respect to a_Q/c_Q is

shown to decrease as $a_Q/c_Q \rightarrow 1.0$. As a result, the maximum interaction effect is found around interacting semi-circle crack-fronts, for a fixed value of a_Q/t , s/c_Q and $h/2c_Q$.

- (ii) Keeping a_Q/c_Q constant, the magnification effect is shown to increase for higher values of crack depth to material thickness ratio. On the other hand, the shielding effect is found to decrease as the a_Q/t increases. In addition, keeping the relative overlap distance, s/c_Q , constant, the shielding effect increases, if it exists, as the relative vertical distance of separation, $h/2c_Q$, decreases. Therefore, it is more realistic to relate $h/2c$ and s/c to the assessment rules used to re-characterize the interacting cracks.
- (iii) Along the interacting crack-fronts, the location of maximum interaction effect depends on the relative distance, s/c_Q , between the inner points where $\phi_Q = 180^\circ$. At $\phi_Q = 180^\circ$, the maximum magnification effect is found when $s/c_Q = 0$. The maximum magnification effect at the deepest and outer points, i.e. $\phi_Q = 90^\circ$ and $\phi_Q = 0^\circ$, along the interacting crack-front, respectively, is shown to exist for a higher values of negative s/c_Q .
- (iv) Asymmetry in interacting crack depth increases the interaction effect for the smaller crack, P, and decreases this effect for the larger crack, Q. However, the decrease in the absolute value of the SIF for the smaller crack and the increase of the corresponding value for the larger crack result in a higher overall SIF for the crack Q and a relatively lower SIF for crack P. Therefore, in cases of asymmetric non-coplanar interacting cracks, larger cracks tend to have a higher growth rate while smaller ones should experience growth retardation.
- (v) Asymmetry in the value of crack shape aspect ratio affects the interaction factors γ_Q and γ_P in a similar qualitative manner as that found for interacting

cracks that are asymmetric in depth. Also, the effect of different a_Q/c_Q and a_Q/t on the values of γ_Q and γ_P for asymmetric non-coplanar cracks is similar in trends to those for symmetric non-coplanar embedded or surface cracks.

- (vi) For surface non-coplanar cracks under bending load, the effects of a_Q/t , a_Q/c_Q , $h/2c_Q$ and s/c_Q are shown to be qualitatively similar to that found under pure tension load.
- (vii) The interaction effect calculated for non-coplanar embedded cracks show qualitative similarities to those found for surface non-coplanar interacting cracks.
- (viii) There is a need to relate the proximity parameters of the interacting cracks to the assessment rules used to re-characterize these cracks. This process should result in a more efficient way of re-characterizing multiple cracks in damaged structures.

Figure 5.1: Stress corrosion cracking colony within pipe body, Revie et al. (1998).

Figure 5.2: Crack interaction configuration of two non-coplanar surface cracks.

Non-coplanar surface cracks

results appear in

		Surface/Surface						Embedded/Embedded
		$\frac{a_P}{c_P}$	$\frac{c_P}{c_Q}$	$\frac{a_P}{a_Q}$	$\frac{a_Q}{t}$	$\frac{h}{2c_Q}$	$\frac{S}{c_Q}$	
Tension	Fig(5.6-5.8) Fig(5.12-5.14) Fig(5.18-5.23)	0.33, 0.5, 0.67, 1.0	1.0	1.0	0.2, 0.5	0.3, 0.5, 1.0	-2.0 : : 2.0	Identical cracks
	Fig(5.30-5.56)	0.33, 0.5, 0.67, 1.0	1.0 0.5, 0.33	1.0 0.5, 0.33	0.33, 0.5	0.3, 0.5, 1.0	-2.0 : : 2.0	Assymetry in crack depth
	Fig(5.84-5.94)	0.33, 0.5, 0.67, 1.0	1.0 0.67, 0.5, 0.33	1.0	0.2 0.5	0.3, 0.5, 1.0	-2.0 : : 2.0	Assymetry in crack-shape aspect ratio
Bending	Fig(5.9-5.11) Fig(5.15-5.17) Fig(5.24-5.29)	0.33, 0.5, 0.67, 1.0	1.0	1.0	0.2, 0.5	0.3, 0.5, 1.0	-2.0 : : 2.0	Identical cracks
	Fig(5.57-5.83)	0.33, 0.5, 0.67, 1.0	1.0 0.5, 0.33	1.0 0.5, 0.33	0.2 0.5	0.3, 0.5, 1.0	-2.0 : : 2.0	Assymetry in crack depth
	Fig(5.95-5.105)	0.33, 0.5, 0.67, 1.0	1.0 0.67, 0.5, 0.33	1.0	0.2 0.5	0.3, 0.5, 1.0	-2.0 : : 2.0	Assymetry in crack-shape aspect ratio
		Tension or Bending				ϕ_P or ϕ_Q

Figure 5.3: The cases studied for two non-coplanar surface cracks.

Figure 5.4: The cases studied for two non-coplanar embedded cracks.

Figure 5.5: Non-coplanar asymmetric stress corrosion deep cracks, Revie et al. (1998).

Figure 5.6: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under tension at $\phi_Q = 180^\circ$, $a_Q/t = 0.2$.

Figure 5.7: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under tension at $\phi_Q = 90^\circ$, $a_Q/t = 0.2$.

Figure 5.8: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under tension at $\phi_Q = 0^\circ$, $a_Q/t = 0.2$.

Figure 5.9: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under bending at $\phi_Q = 180^\circ$, $a_Q/t = 0.2$.

Figure 5.10: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under bending at $\phi_Q = 90^\circ$, $a_Q/t = 0.2$.

Figure 5.11: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under bending at $\phi_Q = 0^\circ$, $a_Q/t = 0.2$.

Figure 5.12: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under tension at $\phi_Q = 180^\circ$, $a_Q/t = 0.2$.

Figure 5.13: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under tension at $\phi_Q = 90^\circ$, $a_Q/t = 0.2$.

Figure 5.14: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under tension at $\phi_Q = 0^\circ$, $a_Q/t = 0.2$.

Figure 5.15: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under bending at $\phi_Q = 180^\circ$, $a_Q/t = 0.2$.

Figure 5.16: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under bending at $\phi_Q = 90^\circ$, $a_Q/t = 0.2$.

Figure 5.17: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under bending at $\phi_Q = 0^\circ$, $a_Q/t = 0.2$.

Figure 5.18: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under tension at $\phi_Q = 180^\circ$, $a_Q/t = 0.5$.

Figure 5.19: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under tension at $\phi_Q = 90^\circ$, $a_Q/t = 0.5$.

Figure 5.20: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under tension at $\phi_Q = 0^\circ$, $a_Q/t = 0.5$.

Figure 5.21: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under tension at $\phi_Q = 180^\circ$, $a_Q/t = 0.5$.

Figure 5.22: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under tension at $\phi_Q = 90^\circ$, $a_Q/t = 0.5$.

Figure 5.23: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under tension at $\phi_Q = 0^\circ$, $a_Q/t = 0.5$.

Figure 5.24: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under bending at $\phi_Q = 180^\circ$. $a_Q/t = 0.5$.

Figure 5.25: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under bending at $\phi_Q = 90^\circ$. $a_Q/t = 0.5$.

Figure 5.26: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface cracks under bending at $\phi_Q = 0^\circ$, $a_Q/t = 0.5$.

Figure 5.27: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under bending at $\phi_Q = 180^\circ$, $a_Q/t = 0.5$.

Figure 5.28: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under bending at $\phi_Q = 90^\circ$, $a_Q/t = 0.5$.

Figure 5.29: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface cracks under bending at $\phi_Q = 0^\circ$, $a_Q/t = 0.5$.

Figure 5.30: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 0^\circ$.

Figure 5.31: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 180^\circ$.

Figure 5.32: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 90^\circ$.

Figure 5.33: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 90^\circ$.

Figure 5.34: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 180^\circ$.

Figure 5.35: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 0^\circ$.

Figure 5.36: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 0^\circ$.

Figure 5.37: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 180^\circ$.

Figure 5.38: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 90^\circ$.

Figure 5.39: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 90^\circ$.

Figure 5.40: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 180^\circ$.

Figure 5.41: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 0^\circ$.

Figure 5.42: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$, $\phi_P = 0^\circ$ and $\phi_Q = 180^\circ$.

Figure 5.43: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$, $\phi_P = 90^\circ$ and $\phi_Q = 90^\circ$.

Figure 5.44: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$, $\phi_P = 180^\circ$ and $\phi_Q = 0^\circ$.

Figure 5.45: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.46: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.47: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.48: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.49: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.50: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.51: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.52: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.53: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.54: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.55: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.56: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.57: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 0^\circ$.

Figure 5.58: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 180^\circ$.

Figure 5.59: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 90^\circ$.

Figure 5.60: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 90^\circ$.

Figure 5.61: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 180^\circ$.

Figure 5.62: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 0^\circ$.

Figure 5.63: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 0^\circ$.

Figure 5.64: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 180^\circ$.

Figure 5.65: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 90^\circ$.

Figure 5.66: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 90^\circ$.

Figure 5.67: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 180^\circ$.

Figure 5.68: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 0^\circ$.

Figure 5.69: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$, $\phi_P = 0^\circ$ and $\phi_Q = 180^\circ$.

Figure 5.70: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ $\phi_P = 90^\circ$ and $\phi_Q = 90^\circ$.

Figure 5.71: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ $\phi_P = 180^\circ$ and $\phi_Q = 0^\circ$.

Figure 5.72: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.73: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.74: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.75: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.76: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.77: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.78: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.79: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.80: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.81: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.82: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.83: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under bending at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.84: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.85: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.86: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.87: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.88: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.89: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.90: Variations of γ_Q with $a_Q/t = 0.2$ or 0.5 and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$.

Figure 5.91: Variations of γ_P with $a_Q/t = 0.33$ or 0.5 and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$.

Figure 5.92: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$, $\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$.

Figure 5.93: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$, $\phi_Q = 90^\circ$ and $\phi_P = 90^\circ$.

Figure 5.94: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$, $\phi_Q = 0^\circ$ and $\phi_P = 180^\circ$.

Figure 5.95: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.96: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.97: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.98: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.99: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

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Figure 5.101: Variations of γ_Q with $a_Q/t = 0.2$ or 0.5 and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$.

Figure 5.102: Variations of γ_P with $a_Q/t = 0.2$ or 0.5 and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$.

Figure 5.103: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$. $\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$.

Figure 5.104: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$, $\phi_Q = 90^\circ$ and $\phi_P = 90^\circ$.

Figure 5.105: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q/c_Q \neq a_P/c_P$) under bending at $c_P/c_Q = 0.33$, $\phi_Q = 0^\circ$ and $\phi_P = 180^\circ$.

Figure 5.106: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 180^\circ$, $a_Q/t = 0.2$.

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Figure 5.108: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 0^\circ$. $a_Q/t = 0.2$.

Figure 5.109: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 180^\circ$. $a_Q/t = 0.2$.

Figure 5.110: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 90^\circ$. $a_Q/t = 0.2$.

Figure 5.111: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 0^\circ$. $a_Q/t = 0.2$.

Figure 5.112: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 180^\circ$. $a_Q/t = 0.5$.

Figure 5.113: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 90^\circ$. $a_Q/t = 0.5$.

Figure 5.114: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 0^\circ$. $a_Q/t = 0.5$.

Figure 5.115: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 180^\circ$. $a_Q/t = 0.5$.

Figure 5.116: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 90^\circ$. $a_Q/t = 0.5$.

Figure 5.117: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric embedded cracks under tension at $\phi_Q = 0^\circ$. $a_Q/t = 0.5$.

Figure 5.118: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 0^\circ$.

Figure 5.119: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 180^\circ$.

Figure 5.120: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 90^\circ$.

Figure 5.121: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 90^\circ$.

Figure 5.122: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_P = 180^\circ$.

Figure 5.123: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ and $\phi_Q = 0^\circ$.

Figure 5.124: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 0^\circ$.

Figure 5.125: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 180^\circ$.

Figure 5.126: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 90^\circ$.

Figure 5.127: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 90^\circ$.

Figure 5.128: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_P = 180^\circ$.

Figure 5.129: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.2$ and $\phi_Q = 0^\circ$.

Figure 5.130: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.131: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.132: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.133: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.134: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.135: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.136: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.137: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.138: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.139: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.140: Variations of γ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.141: Variations of γ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 1.0$, $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.142: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ $\phi_P = 0^\circ$ and $\phi_Q = 180^\circ$.

Figure 5.143: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$ $\phi_P = 90^\circ$ and $\phi_Q = 90^\circ$.

Figure 5.144: Variations of γ_P and γ_Q with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q \neq a_P$) under tension at $a_P/c_P = 0.33$, $a_Q/t = 0.2$, $\phi_P = 180^\circ$ and $\phi_Q = 0^\circ$.

Figure 5.145: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric embedded cracks under ($a_Q/c_Q \neq a_P/c_P$) tension at $a_Q/t = 0.5$ and $\phi_Q = 180^\circ$.

Figure 5.146: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 0^\circ$.

Figure 5.147: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 90^\circ$.

Figure 5.148: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 90^\circ$.

Figure 5.149: Variations of γ_Q with a_P/c_P and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_Q = 0^\circ$.

Figure 5.150: Variations of γ_P with a_P/c_P and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $a_Q/t = 0.5$ and $\phi_P = 180^\circ$.

Figure 5.151: Variations of γ_Q with $a_Q/t = 0.2$ or 0.5 and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$.

Figure 5.152: Variations of γ_P with $a_Q/t = 0.33$ or 0.5 and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$.

Figure 5.153: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$. $\phi_Q = 180^\circ$ and $\phi_P = 0^\circ$.

Figure 5.154: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$, $\phi_Q = 90^\circ$ and $\phi_P = 90^\circ$.

Figure 5.155: Variations of γ_Q and γ_P with $h/2c_Q$ and s/c_Q for non-coplanar asymmetric embedded cracks ($a_Q/c_Q \neq a_P/c_P$) under tension at $c_P/c_Q = 0.33$, $\phi_Q = 0^\circ$ and $\phi_P = 90^\circ$.

Figure 5.156: Variations of ψ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.2$, $a_P/c_P = 0.33$, $h/2c_Q = 0.3$ and $\phi_P = 0^\circ$.

Figure 5.157: Variations of ψ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.2$, $a_P/c_P = 0.33$, $h/2c_Q = 0.3$ and $\phi_Q = 180^\circ$.

Figure 5.158: Variations of ψ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.2$, $a_P/c_P = 0.33$, $h/2c_Q = 0.3$ and $\phi_P = 90^\circ$.

Figure 5.159: Variations of ψ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.2$, $a_P/c_P = 0.33$, $h/2c_Q = 0.3$ and $\phi_Q = 90^\circ$.

Figure 5.160: Variations of ψ_P with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.2$, $a_P/c_P = 0.33$, $h/2c_Q = 0.3$ and $\phi_P = 180^\circ$.

Figure 5.161: Variations of ψ_Q with c_P/c_Q and s/c_Q for non-coplanar asymmetric surface cracks ($a_Q \neq a_P$) under tension at $a_Q/t = 0.2$, $a_P/c_P = 0.33$, $h/2c_Q = 0.3$ and $\phi_Q = 0^\circ$.

Chapter 6

Further cases of crack interactions in 3D

The main objective of this chapter is to explore the capability of the FE sub-modeling approach developed here in to deal with other cases of crack interaction in 3D. Therefore, two further cases of crack interaction are discussed here. The first case focuses on the interaction of a surface crack with an embedded non-coplanar crack in an elastic infinite plate. In the first section, the variation in the interaction behavior between these two different symmetric cracks with the relative horizontal and vertical proximity is discussed for a wide range of crack shape aspect ratio, a/c . The second case deals with the interaction of two arbitrarily oriented surface cracks. In this case, the variation of the interaction factor γ with the relative angle and distance between the interacting cracks is discussed. In these two problems, only remote tension loading of the infinite plate is treated.

6.1 Interaction of a surface and an embedded crack

In practice, an embedded crack, nucleated, in for example, a weld might be located in the vicinity of a surface crack that is caused by other surface conditions outside the weld material. This problem may be found in pipelines as well as in welded lap joints in an aircraft fuselage. Figure (6.1.a) shows the physical problem and the

notations used to define the geometric variables that are investigated in this study. A quick overview of the different values of these variables is provided in the table shown in Fig (6.2).

Figures (6.4) to (6.9) show the variations of the interaction factors γ_Q and γ_P with the relative horizontal separation distance s/c_Q and the aspect ratio of crack Q a_Q/c_Q , when $a_Q/t = 0.2$ and $h/2c_Q = 0.3$ at $\phi_Q = 180^\circ, 90^\circ$ and 0° and $\phi_P = 180^\circ, 90^\circ$ and 0° , respectively. The variations of γ_Q are compared with those obtained for the corresponding case of only two surface or two embedded non-coplanar cracks with the same geometric parameters (Figs (5.6) to (5.6) and (5.106) to (5.108) in Chapter 5). No significant change is found in the values of γ_Q and γ_P in both cases. In this set of cases the SIF's of the interacting cracks P and Q are normalized as follows :

$$\beta_Q = \frac{K_{inQ}}{\sigma \sqrt{\pi \frac{a_Q}{Q'}}} \quad (6.1)$$

$$\beta_P = \frac{K_{inP}}{\sigma \sqrt{\pi \frac{a_P}{Q'}}} \quad (6.2)$$

Figures (6.10) to (6.15) show the variations of the factors β_Q and β_P with s/c_Q and a_Q/c_Q at the inner outer and deepest points along the interacting crack-fronts. As indicated in those figures, the normalized SIFs calculated along the front of the surface crack Q is relatively higher than those found at the corresponding points along the front of the embedded crack P. For example, the percentage increases in the maximum and minimum values of β_Q with respect to β_P are calculated as 13.4, 13.3 percent at the inner point and 14.1, 10.3 percent at the outer point along the crack-fronts, respectively. At the deepest point however, this change is shown to be less than 1 percent which is not significant. These findings indicate that the interaction between surface and embedded cracks will result in an increase in the growth rate of the surface crack at the surface point. The percentage of this increase in the growth rate is shown to be higher at the inner point along this crack-front.

The effect of the vertical relative distance between the two cracks on the variations of γ_Q and γ_P in this interaction case is shown in Figs (6.16) to (6.21). The changes in the γ_Q and γ_P values in those figures are also found to be insignificant. However, when the interaction effect is re-represented in terms of β_Q and β_P , as shown in Figs (6.22) to (6.27), this effect is then clarified. In this case, the difference in the relative SIF values are shown to increase as the vertical relative distance decreases, where the percentage increase in the maximum and minimum values of β_Q with respect to β_P was 14 percent on average at the inner and outer points along the crack-fronts. Those results indicate that the decrease in $h/2c_Q$ should also increase the growth rate of the surface crack relative to the embedded crack at the surface points along the front of the surface crack.

6.2 Interaction of two arbitrarily oriented surface cracks

Multiple cracks in arbitrary orientations may be found in electrical and mechanical components made of ceramic materials. In many cases, the orientation of these cracks are often initiated during the manufacturing process. Figure (6.1.b) shows the physical problem that has been idealized here, and the notations used to describe the geometry of the two interacting cracks Q and P and the relative orientation. In this thesis, arbitrarily oriented cracks should always remain on the x-y plane, as shown in Fig (6.3).

The first configuration considered here is for a pair of non-coplanar but parallel surface cracks similar to these shown in Section (5.1). Both cracks are then rotated around their horizontal centerline with an angle θ_S , as shown in Fig (6.3), varying from 0° to 40° . Thus, the first point on the vertical axis of the results presented in every case studied here correspond to the non-coplanar case with $h/2c_Q$ and s/c_Q

parameters when $\theta_S = 0^\circ$.

Figures (6.28) to (6.30) show the variations of the interaction factor γ_Q at the inner, deepest and outer points along the crack front with $a_Q/t = 0.2$ and $a_Q/c_Q = 1.0$. In these figures, four different interaction cases are compared, where the value of s/c_Q when $\theta_S = 0^\circ$ is changed for every case. For these cases, the values of s/c_Q analyzed were -2, -1.5, 0, and 2. When the interacting cracks are overlapping, i.e. $s/c_Q < 0$, the interaction factor is shown in Fig (6.28) to increase with θ_S , which results in a decrease of the shielding effect that exist between these cracks. On the other hand, the interaction factor γ_Q is found to decrease with θ_S for $s/c_Q \geq 0$. Overall, this result shows that any increase in θ_S results in a decrease of the interaction effect. This trend is found at all the points studied along the fronts of the both interacting cracks as shown in Figs (6.29) and (6.30). For this case, the maximum percentage increase in γ_Q is calculated as 60.0, 9.5 and 14.0 percent at $s/c_Q = -1.5$, while the maximum percentage decrease of 27.0, 8.7 and 10.5 percent is calculated at $s/c_Q = 0.0$ for $\phi_Q = 180^\circ$, 90° and 0° , respectively.

Figures (6.31) to (6.33) show the same interaction cases above, but, for a lower a_Q/c_Q value of 0.3. In this case, the interaction effect is shown to be qualitatively lower than that found at $a_Q/c_Q = 1.0$. This effect of the crack shape aspect ratio is similar to that found for non-coplanar surface cracks discussed earlier in section (5.1.1). The effects of crack depth to material thickness ratio on the variation of the interaction factor γ_Q are also shown in Figs (6.34) to (6.36). In this case, an increase in the ratio of a_Q/t is shown to increase the interaction effect along the interacting crack-fronts for all the values of s/c_Q and a_Q/c_Q studied. The maximum percentage increase in γ_Q with θ_S is calculated as 60.6 percent at $s/c_Q = -1.5$ and $\phi_Q = 180^\circ$ while the minimum percentage decrease is calculated as 29.7 percent at $s/c_Q = 0$ and $\phi_Q = 180^\circ$. Also, as a result of increasing the a_Q/t from 0.2 to 0.5, a maximum change of 2.4 and 3.0 percent are found in the maximum percentage

increase and decrease of γ_Q with θ_S , respectively. Finally, Figs (6.37) to (6.39) show the effect on increasing the relative separation distance at $\theta_S = 0^\circ$. In this case the interaction effect is shown to qualitatively decrease similar to the results found for the non-coplanar surface cracks case found in chapter 5.

6.3 Concluding remarks

- (i) The interaction effect of a one non-coplanar surface crack and another embedded crack is shown to be a mix of the two separate cases of either embedded or surface cracks. While the overall trend of the interaction factor is similar for both cracks, the SIF K_{in_Q} of the surface crack is shown to be significantly higher than that of the embedded crack at the surface points. At the deepest point this effect diminishes. This result is repeated for different values of crack aspect ratio and at different levels of the relative vertical separation distance.
- (ii) The introduction of a rotation angle about the horizontal axis of two surface non-coplanar interacting cracks reduces the interaction effect along the fronts of both cracks. This effect holds true for all investigated values of crack depth to plate thickness, crack shape aspect ratio and the relative vertical separation distance.

Figure 6.1: Crack interaction configuration of (a) two surface and embedded non-coplanar cracks (b) two arbitrary oriented surface cracks.

Figure 6.2: The cases studied for two surface and embedded non-coplanar cracks.

Two arbitrary oriented surface crack

results appear in

Tension

Fig(6.28-6.39)

Surface/Surface						
$\frac{a_P}{c_P}$	$\frac{c_P}{c_Q}$	$\frac{a_P}{a_Q}$	$\frac{a_Q}{t}$	$\frac{h}{2c_Q}$	$\frac{S}{c_Q}$	θ_S
0.33, 1.0	1.0	1.0	0.2, 0.5	0.3, 0.5	-2.0, -1.5, 0.0, 2.0	0°, 10°, 20°, 30°, 40°
Tension				ϕ_P or ϕ_Q		

Identical cracks

Figure 6.3: The cases studied for two arbitrary oriented surface cracks.

Figure 6.4: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 180^\circ$.

Figure 6.5: Variations of γ_P with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 0^\circ$.

Figure 6.6: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 90^\circ$.

Figure 6.7: Variations of γ_P with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 90^\circ$.

Figure 6.8: Variations of γ_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 0^\circ$.

Figure 6.9: Variations of γ_P with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 180^\circ$.

Figure 6.10: Variations of β_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 180^\circ$.

Figure 6.11: Variations of β_P with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 0^\circ$.

Figure 6.12: Variations of β_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 90^\circ$.

Figure 6.13: Variations of β_P with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 90^\circ$.

Figure 6.14: Variations of β_Q with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 0^\circ$.

Figure 6.15: Variations of β_P with s/c_Q and a_Q/c_Q for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 180^\circ$.

Figure 6.16: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 180^\circ$.

Figure 6.17: Variations of γ_P with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 0^\circ$.

Figure 6.18: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 90^\circ$.

Figure 6.19: Variations of γ_P with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 90^\circ$.

Figure 6.20: Variations of γ_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 0^\circ$.

Figure 6.21: Variations of γ_P with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 180^\circ$.

Figure 6.22: Variations of β_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 180^\circ$.

Figure 6.23: Variations of β_P with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 0^\circ$.

Figure 6.24: Variations of β_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 90^\circ$.

Figure 6.25: Variations of β_P with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 90^\circ$.

Figure 6.26: Variations of β_Q with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_Q = 0^\circ$.

Figure 6.27: Variations of β_P with s/c_Q and $h/2c_Q$ for non-coplanar symmetric surface and embedded crack under remote tension at $\phi_P = 180^\circ$.

Figure 6.28: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 180^\circ$. $a_Q/c_Q = 1.0$ and $a_Q/t = 0.2$.

Figure 6.29: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 90^\circ$. $a_Q/c_Q = 1.0$ and $a_Q/t = 0.2$.

Figure 6.30: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 0^\circ$. $a_Q/c_Q = 1.0$ and $a_Q/t = 0.2$.

Figure 6.31: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 180^\circ$. $a_Q/c_Q = 0.3$ and $a_Q/t = 0.2$.

Figure 6.32: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 90^\circ$, $a_Q/c_Q = 0.3$ and $a_Q/t = 0.2$.

Figure 6.33: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 0^\circ$, $a_Q/c_Q = 0.3$ and $a_Q/t = 0.2$.

Figure 6.34: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 180^\circ$, $a_Q/c_Q = 1.0$ and $a_Q/t = 0.5$.

Figure 6.35: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 90^\circ$, $a_Q/c_Q = 1.0$ and $a_Q/t = 0.5$.

Figure 6.36: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 0^\circ$, $a_Q/c_Q = 1.0$ and $a_Q/t = 0.5$.

Figure 6.37: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 180^\circ$, $a_Q/c_Q = 0.3$ and $h/2c_Q = 0.5$.

Figure 6.38: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 90^\circ$, $a_Q/c_Q = 0.3$ and $h/2c_Q = 0.5$.

Figure 6.39: Variations of γ_Q with θ_S and r/c_Q for two arbitrary oriented symmetric surface cracks under remote tension at $\phi_Q = 0^\circ$, $a_Q/c_Q = 0.3$ and $h/2c_Q = 0.5$.

Chapter 7

Conclusions and Recommendations for Future Work

7.1 Conclusions

The interaction effects between two macro cracks in an infinite thick plate under tension or bending loadings have been investigated in this study. The different modeling techniques that have been used to analyze the problem of multiple cracking have been reviewed. Some industrial applications where single and multiple cracks exist were used to direct the selection of the interaction cases investigated in the current study.

To meet the objective of this study, a finite element model that applies the concept of sub-modeling was developed to study different cases of interacting cracks that can exist in a coplanar or non-coplanar configuration and in any arbitrary orientation. The sub-modeling technique was used first to develop two sub-models of macro semi-elliptical surface cracks. In the process, the crack shape aspect ratio and the ratio of crack depth to plate thickness, was controlled by a user friendly interface that utilizes this information to generate the sub-model mesh for each crack separately. This approach gives the ability to introduce asymmetry in the depth, length and shape aspect ratio of both cracks.

The stress intensity factor, SIF, was calculated along the interacting crack-fronts using the J -integral. In this study, the commercial code of ABAQUS was used to evaluate the J -integral using the equivalent domain integral approach, EDI. To validate the overall design accuracy of each crack sub-model, various cases of single and multiple cracks were tested under remote tension and pure bending loading. The SIFs in these cases were compared with the corresponding values found in literature. Good agreement was found between the results in the current study and those obtained using other models of multiple cracking. Those validation cases covered coplanar and non-coplanar cracks in 2D. Also, in 3D, the SIF results in the interaction cases of surface and embedded cracks were validated, with good accuracy, at different points along the interacting crack-fronts under tension or bending loading.

In this study, the interaction between coplanar surface or embedded cracks was studied for a wide range of crack shape aspect ratio and crack depth to plate thickness ratio. For symmetric cracks, the interaction factor was found to significantly increase as the relative horizontal distance between the interacting cracks decreases. Also, the interaction factor was shown to increase with the crack depth to plate thickness ratio for all the studied points along the interacting crack-fronts. Overall, the interaction effect was shown to increase with the crack shape aspect ratio for different cases of crack depth to plate thickness ratios. For the majority of the studied cases, maximum interaction effect was calculated at the inner point along the interacting crack-fronts. The effect of crack depth to plate thickness ratio and crack shape aspect ratio on the interaction factor was found to be qualitatively similar for symmetric and asymmetric coplanar cracks. Nevertheless, for asymmetric cracks, the interaction factor at the larger crack was shown to be lower than that at the corresponding point along the front of the smaller crack. This interaction trend was shown to be similar for surface cracks under bending or tension loading and for embedded cracks under tension loading.

The problem of non-coplanar surface or embedded cracks has also been investigated in this study. For non-overlapping symmetric cracks, the interaction effect along both crack-fronts was shown to increase as the relative horizontal distance between these cracks decreases. This interaction trend was sustained to a certain relative horizontal distance where the interaction factor then reaches its maximum at this point along the crack-front. After that, the interaction factor decreases to a value less than unity where the interacting cracks were shown to experience shielding. The point of the maximum shielding varies with the location along the interacting crack-fronts. Also, the interaction effect was shown to decrease with the increase of the relative vertical distance between the interacting cracks. The impact of the shape aspect ratio of both interacting cracks and the crack depth to plate thickness ratio on the interaction effect was shown to be qualitatively similar to that found in the case of coplanar cracks. For asymmetric non-coplanar cracks, although the interaction factor for the larger crack was higher than that of the smaller crack, this effect was found to be counter affected by the increase in the larger crack absolute SIF. Therefore, the overall result was shown to be a relative increase in the SIF value of the larger crack at the corresponding point along its front.

A comparison of the values of the SIFs calculated in this study for interacting non-coplanar surface cracks with those estimated using cracks re-characterization rules was carried out. This comparison indicated that these rules are not always conservative and that there is a need to include the proximity parameters of the interacting cracks to these assessment rules in order to increase their effectiveness.

The interaction between non-coplanar surface crack and embedded crack was also investigated in this thesis. In this case, the trend of the interaction factor with the crack geometry was qualitatively similar to that found for interaction cases of only surface or embedded crack. The current FE model was also used to analyze the interaction case between arbitrary oriented surface cracks under tension loading. In

this case, the interaction effect was shown to decrease with the increase of the relative rotation angle between the horizontal axis of the interacting cracks. The variation of crack shape aspect ratio and crack depth to plate thickness ratio resulted in a similar effect on the interaction factor at all the points studied along both crack-fronts.

7.2 Achievements and Contributions

The achievements and contributions of the present research may thus be summarized below. They represent the application of the finite element method and the sub-modeling technique to solve for the interaction effects between two macro surface or embedded semi-elliptical cracks in a thick plate under remote tension or pure bending loading.

- (i) The development of FE sub-models for two macro cracks in 3D which can be implemented in different global structures with a wide range of crack depth to plate thickness and crack aspect ratios.
- (ii) The development and implementation of a mesh generator that was used to construct the interaction zone containing the sub-models of two arbitrary oriented macro crack in 3D.
- (iii) The determination of new interaction factor solutions for cases of symmetric and asymmetric, coplanar, semi-elliptical surface cracks under tension or bending loading.
- (iv) The calculation of new solutions of the interaction factor for cases of symmetric and asymmetric, coplanar, semi-elliptical embedded cracks under tension loading.
- (v) The establishment of new solutions of the interaction factor for cases of sym-

metric and asymmetric, non-coplanar, semi-elliptical surface cracks under tension or bending loading.

- (vi) The determination of new solutions of the interaction factor for cases of symmetric and asymmetric, non-coplanar, semi-elliptical embedded cracks under tension loading.
- (vii) The establishment of new solutions of the interaction factor for cases of symmetric, non-coplanar, semi-elliptical surface and embedded cracks under tension loading.
- (viii) The calculation of new solutions of the interaction factor for cases of arbitrary oriented semi-elliptical surface cracks under tension loading.

7.3 Recommendations for Future Work

Finally, some suggestions for future work in the analysis of multiple cracks are perhaps in order. They are as follows:

- (i) The results of the current study can be implemented in the study of growth mechanisms of multiple cracks in 3D and their effects on the probability of failure of the multi-site damaged structures.
- (ii) Experimental validation of the results in the current study is strongly needed. A good starting point would be to study the crack tip opening displacement experimentally and compare it with the stress intensity factor found in the current study under different cases of crack interaction. Also, the SIF derived from the experimentally calculated growth rate of the interacting cracks can be used to verify the interaction factor at the inner and outer points along the crack-fronts.

- (iii) The current work can be extended to include non-linear material behavior (e.g. elasto-plasticity) and/or dynamic load situations (e.g. impact loading). The former effect is particularly important in cases such as high load conditions with materials of relatively high ductility. Under very high loads and impact conditions, it may also be necessary to consider geometric non-linearities due to large deformation.
- (iv) The present work can also be extended to study the effect of concentrated force points at the crack surface on the interaction factor of these cracks. This particular case has application to bridging situation in composite materials. The current model can also be extended to investigate the effect of residual stress on the interaction behavior of some of the cases studied.

Appendix A

List of Contributed Publications

To the current date, the research work done in this thesis has contributed the following list of publications, and more are to come....

- (1) Moussa, W., Bell, R., and Tan, C. L., 2000. "On the Interaction Effect Between Two Arbitrary Oriented Symmetric Surface Cracks". submitted to : *ASME PVP Conference*, Seattle, Washington.
- (2) Moussa, W., Bell, R., and Tan, C. L., 1999a. "The Interaction of Two Parallel Semi-Elliptical Surface Cracks Under Tension and Bending". *Trans of ASME, J. Press. Vess. Tech.*, Vol. 121, pp. 111-112.
- (3) Moussa, W., Bell, R., and Tan, C. L., 1999b. "A Parametric Study of Two Parallel Interacting Surface Cracks Under Tension and Bending". *Int. J. Pres. Ves. & Piping*, Vol. 76, pp. 135-145.
- (4) Moussa, W. A., 1998. "Risk-Based Reliability Evaluation of Multi-Site Damage in Pipelines". *Int. J. Computer and Industrial Engng.*, Vol. 35, pp. 595-598.
- (5) Moussa, W., Bell, R., and Tan, C. L., 1998a. "The Interaction of Two Parallel Semi-Elliptical Surface Cracks Under Tension and Bending". in: *Proceedings of ASME PVP Conference*, San Diego, California.

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- (6) Moussa, W., Bell, R., and Tan, C. L., 1998b, "A Parametric Study of Two Parallel Interacting Surface Cracks Under Tension and Bending". in: *Proceedings of ASME OMAE*, Lisbon, Portugal.
 - (7) Moussa, W. A. and Kamel, A., 1997, "Risk-Based Reliability Considerations of Multi-Site Damaged Pipelines". in: *Proceedings Computer and Industrial Engineering*, pp. 427-439.
 - (8) Moussa, W. A., 1997, "Prediction of Multi-Mode Non-Coplanar 2D Crack Interaction". in: *Proceedings of CanCam97*, Quebec, Canada, pp. 115-116.

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