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CATASTROPHE THEORY AND URBAN DYNAMICS

by

Ian MacLachlan

A thesis submitted to the Faculty of
Graduate Studies in partial fulfillment
of the requirements for the degree of
Master of Arts

Department of Geography
Carleton University
Ottawa, Ontario
1981

ABSTRACT

Catastrophe theory provides new perspectives on change in form and offers a new paradigm for understanding discontinuity in geography. The form which elementary catastrophes take is set in its philosophical context with a discussion of structural stability and morphogenesis. Four catastrophe models of geographical problems are summarized and critically appraised. A new catastrophe model of racial transition is proposed to reconcile conflicting explanations of sudden changes in the racial composition of neighbourhoods.

The undersigned recommend to the Faculty of
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...le modèle des catastrophes est à la fois beaucoup moins, et beaucoup plus qu'une théorie scientifique; on doit le considerer comme un langage, un methode, qui permet de classifier, de systematiser les données empiriques

Thom (1977, p. 615)

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CHAPTER 1

A NEW PARADIGM FOR GEOGRAPHY

All science asks questions about change. Change is the most pervasive occurrence in life. Catastrophe theory has provided science with a new philosophy of form and a new perspective towards an understanding of change. Catastrophe theory is largely the work of René Thom who coined the term in his seminal book *Stabilité Structurale et Morphogénèse* (1972). The objective of this thesis is to show that this new theory is a significant addition to traditional modes of analysis in geography.

1.1 An Overview of Catastrophe Theory

Catastrophe theory is primarily concerned with the nature of changes in potential systems. A system is a group of elements that are functionally related. A potential system operates under some minimizing criterion. The output level or behavior of a potential system will be adjusted so that the system is always at a stable minimum of some quantity. These adjustments are necessary to countervail changes in the parameters controlling the system's structure or environmental perturbations which may displace a system from its stable minimum. The Canadian passenger railway industry, like any commercial enterprise, is a potential system. The system reacts to amended transport commission regulations and subsidies (system parameters) and changes in consumer demand (environmental perturbation) to minimize losses. The executive decision-makers in the railway corporations are a feedback mechanism which (ideally) determines how best to change behavior to maintain a stable minimum of losses.

Potential systems may undergo slow and continuous transformation or rapid and discontinuous conversion to maintain stability. Consider another example: a scree slope with an angle of repose of 30 degrees. Rock fragments are continuously added to the top of the slope tending to increase the angle of the unconsolidated material. The change in slope form will depend on, say two parameters, the angularity of the fragments and their coefficient of friction. If friction was low and the pebbles were smooth, the continual addition of rock fragments at the top of the slope would be accompanied by a continual downward creep of the scree. If the rock fragments were less smooth they would have more inertia. The angle of the slope would gradually increase until it became unstable at 31 degrees. Some perturbation (perhaps the addition of one more rock, high winds, or a hiker's boot) will trigger a mass movement of scree increasing the length of the slope and restoring the angle. A continuous change proceeding ad seriatim thus is arrested by a sudden and catastrophic change. Catastrophe theory provides a set of conceptual constructs known as the 'seven elementary catastrophes' to model the changes in form of potential systems. The construct employed depends on the number of parameters defining the system.

Given a limited number of key parameters, catastrophe theory demonstrates that there are only a limited number of ways that the system may behave. Parameters controlling the system may vary in time. The system will adjust its behavior to remain in a stable state by reacting to marginal changes in controlling parameters with marginal changes in behavior. Behavioral changes are incremental up to some threshold or critical point. Beyond this point behavior must change dramatically to maintain the system's stability. This dramatic change is a catastrophe. The seven elementary catastrophe models illustrate the geometry and qualitative characteristics of systems undergoing catastrophic changes in form.

Systems amenable to catastrophe modelling usually have five

unique characteristics: bimodality, hysteresis, divergence, sudden jumps in behavior, and inaccessible behavior. These will be dealt with in more detail in Chapter 3; however, a short whimsical example is included here to illustrate some of the properties.

The reader is requested to examine the following eight figures in succession, covering the diagram so that only one frame is visible at a time, and noting what he sees in each illustration (fig. 1.1).

The reader is now requested to go backwards through the illustrations, frame by frame, again noting what he sees in each case.

On the first viewing the reader should experience a sudden transition or catastrophic change in perception as the man's face suddenly becomes a reposing woman, at about frame 6. On the second viewing the sudden transition occurs about frame 3. The timing of the sudden transition depends on the direction of approach. This property is hysteresis. Frame 4 may be perceived as a man's face or a woman's body. The viewer's perception is therefore bimodal, it will interpret the same figure in one of two ways. The characteristic of inaccessible behavior is illustrated by the inability of the viewer to see the man's face or the woman's body at one and the same time and the impossibility of perceiving a sort of hermaphrodite at the mid point of transition.

1.2 The Development of Catastrophe Theory

Catastrophe theory was conceived and named by René Thom, the French mathematician. The mathematical basis of catastrophe theory is differential topology, the branch of mathematics dealing with rates of change on multi-dimensional surfaces. Differential topology has its roots in the works of Newton, Laplace, Poincaré, Morse, Whitney and Malgrange. The development of Thom's ideas was also influenced by contemporary researchers in other fields. Christopher Zeeman, a mathematician; Claude Levi-Strauss, an anthropologist; and Noam

Man or Woman?

8

7

6

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4

3

2

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Fig. 1.1 Catastrophic Change in Perception

Source: Attneave, 1971, p. 62

Chomsky, a linguist, all lent the seeds of what was to become structural stability (Woodcock and Davis, 1978, p. 20). Finally biologist D'Arcy Thompson, author of Growth and Form (1917) and embryologist C.H. Waddington contributed towards the concept of morphogenesis. Thom's "Stabilité Structurelle et Morphogénèse", was published in 1972. An English translation by D.H. Fowler of the University of Warwick was printed in 1975 and almost overnight became widely acclaimed.

The best known, most prolific, and most enthusiastic proponent of catastrophe theory is undoubtedly Professor E.C. Zeeman, Director of the Mathematics Institute, University of Warwick. Between 1972 and 1977, Zeeman authored or co-authored some twenty-two articles on the subject of catastrophe theory. These have been compiled into one book (Zeeman, 1977). The topics pursued in this compendium cover a wide range of subjects from biology, mechanics, social science and pure mathematics. Zeeman's applications of Thom's theoretical work have led the way in popularizing catastrophe theory.

Zeeman has actively encouraged work in catastrophe theory among a number of academics. Aimed at a general audience, Catastrophe Theory by Woodcock and Davis (1978) gives an integrated non-mathematical account of the theory and some potential applications. Poston and Stewart (1978) provided a much more rigorous mathematical treatment of the theory. Finally, Colin Renfrew, an archaeologist at Southampton, (where Zeeman lectured, prior to going to Warwick) co-published Transformations: Mathematical Approaches to Culture Change, a selection of readings including applications of catastrophe theory with contributions from Zeeman and Poston. The work of these academics shows a common thread of thought which reflects Zeeman's influence.

Catastrophe theory has firmly embedded itself into the history of science. It has achieved front page status in the New York Times and

merited article space in Newsweek, Scientific American, Science, and Nature¹. It has sparked incredibly bitter controversy in and out of the mathematical literature (cf Zahler and Sussman, 1977). Catastrophe theory has captured the imagination of scientists in all disciplines in a way surpassed only by Einstein and Watson and Crick in the past fifty years. It has been called, "everything from the most important event in mathematics since calculus to the height of scientific irresponsibility" (Brown, 1979, p. 1020).

There number over a score of review articles or book reviews published in journals of nearly every discipline on the subject of catastrophe theory. Most start with a paragraph like the one above. There is universal astonishment about the sudden popularity of catastrophe theory. The reaction in reviews is remarkably consistant; Thom's mathematics is superb, Zeeman's social science applications were somewhat careless, and whether the theory is viable enough to thrive after the initial euphoria and subsequent controversy have died down is still in doubt. A review concludes about Zeeman's ideas, "The question as to whether they are of use to psychology will be answered by psychologists, not opinionated mathematicians" (Ambler, 1979).

The 'opinionated mathematicians' were Hector Sussman and Raphael Zahler (1978) who summarized their ninety-nine page critique in Synthese with a four page article in Nature (Zahler and Sussman, 1977). They concluded that applications of the theory rested on questionable assumptions based on faulty reasoning. This lead to conclusions which were false, ambiguous, trivial, or untestable. Polemic responses streamed in from Christopher Zeeman, Ian Stewart, Michael Berry, Alexander Woodcock and René Thom to name a few (Nature December 1, 1977, pp. 381-384). The responses successfully hoisted Zahler and Sussman with their own critical petard, however subsequent applications of catastrophe theory have been a little more circumspect in their claims and scope.

1. New York Times, Nov. 19, 1977; Newsweek, Jan. 19, 1976; Scientific American, April 1976; Science, Apr. 15, 1977; and Nature, Oct. 27, 1977.

The controversy persists not only among opponents and proponents of the theory but among the catastrophe advocates as well. Thom, the father of catastrophe theory is at loggerheads with Zeeman, champion of applied catastrophe modelling. Thom's objective in Structural Stability and Morphogenesis was to propose a new theory of form that is in some ways closer to philosophy than to mathematics. His basic principle was that any observable morphogenesis is a catastrophe (Farraro, 1978, p. 312). Cascades of elementary catastrophes lead to complex changes in forms.

Thom emphasized that catastrophe theory was only the beginning of a more comprehensive explanation of changes in dynamical systems. As well, he stressed the qualitative aspect of catastrophe theory in describing structural stability and morphogenesis and further that the mathematics did not justify any prediction.

The great advantage of topology is that it allows one to take a qualitative rather than a quantitative approach to phenomena. But for that you have to pay a price. For if you pass from the quantitative to the qualitative in general you lose any ability to predict future events That is the point where I have a little controversy with my friend, Professor Christopher Zeeman at Warwick University. I do not think that from any mathematical theorem you can predict anything in reality.

(Thom quoted in Dickson, 1975, p. 13)

That view is in diametric opposition to the philosophy underlying most of Zeeman's work. He has suggested that the qualitative catastrophe constructs can and should lead to quantitative prediction.

The advantage of catastrophe theory is to give not only qualitative understanding into phenomena involving catastrophic change but also to provide quantitative models for experimental testing eventually it may be possible to design quantitative monitoring that would provide a basis for action.

(Zeeman, 1977, p. 400)

Zeeman's adaptation of catastrophe theory posits that the discontinuous changes in social and natural phenomena can be interpreted as catastrophe

events to which the seven elementary canonical models may be applied.

There is a second feature which distinguishes Zeeman's approach to catastrophe theory. The concept of discontinuous variation of the control variables is emphasized along with the five characteristics of catastrophe theory, i.e. bimodality, divergence, sudden jump, hysteresis and inaccessibility. Catastrophe theory "is particularly applicable where gradually changing forces create sudden effects" (Zeeman, 1977, p. 1). This is in contrast to other writers (Amson, 1975, and Isard, 1977) who emphasize that catastrophe theory seeks to explain how stable dynamic systems retain their structure and change their form.

The most immediate and important feature of catastrophe theory is that it sets out to study the structural singularities of all sufficiently well regulated systems. Quickly said, it is a theory of the morphology of systems and of their morphogenesis (the generation and modification of their forms), based on the study of their singularities (the places where regularities break down or become exchanged for others).

(Amson, 1975, p. 178)

The two views of catastrophe theory are not necessarily incompatible or antagonistic. They are two sides of the same coin. Thom's approach (1975) so integrally combines the mathematics with the philosophy that it is difficult to tell which comes first. The philosophical side of catastrophe theory has been neglected in the most popular literature leading Berlinski (1978, p. 408) to comment that Zeeman "...instinctively turns from the darker claims of catastrophe theory with what I assume is simple British distaste for the ineffable".

There is another dispute in the catastrophe camp. The disagreement between the social scientists and mathematicians questions how rigorous the mathematical treatment of applications should be.

The beauty of catastrophe theory is that its rich geometry is accessible with only passing acquaintance with the deep mathematics.

(Brown, 1979, p. 1020)

It has been said more than once that it is possible

to apply Thom's theorem without understanding the mathematics behind it: we disagree.
(Poston and Stewart, 1978, p. viii)

Whether or not catastrophe theory is indeed a scientific revolution will be best determined by the historians in hindsight. It has at least been the cause of several interdisciplinary border clashes.

1.3 The Catastrophe Paradigm and Geography

Catastrophe theory is not advanced as a panacea for the post-quantitative ailment of geography. It has achieved sufficient attention to merit its inclusion in two introductory texts (Foust and de Souza, 1979, pp. 235-237; and Haggett, 1978, pp. 610-611). The growing literature attests that geographers are gaining rapid acquaintance with the concept.

The late C.H. Waddington gave extensive treatment to system stability in his book Tools for Thought. It seems, at times, incredible that systems with extremely complicated sets of interactions can function and endure with time. Yet, one only has to look at a person walking down the street to realize that extraordinarily complex systems in detail, often operate with great simplicity at the first level of abstraction (Waddington, 1975, p. 75). In the same vein, agents of spatial change, landscape patterns, regional systems, and space-time processes can, at first glance, appear to function very simply. Analysis will show that all of these systems are enormously complex. Hence they are often studied elementally, often with cross fertilization from other disciplines. Geographers have become like the 'Borrowers' from the children's story by Mary Norton, a closely knit family of little people scuttling about in the wainscoting. They appropriate from other disciplines whatever trendy terminology, analytic methods, or subject matter seems most engaging. The multidisciplinary borrowing has been exciting and fruitful but at a price, the usury of specialization. The initial benefit of borrowing from a specialized body of knowledge must be balanced against the long term costs of getting deeper and deeper into a narrow tunnel field.

Amson (1975 p. 178) argued that contemporary urban studies have suffered from a reductionist approach to changes in city systems. Phenomena such as rapid urban growth, depopulation of the urban core, or the exchange of dominance roles of rival cities are broken down into their constituent elements for behavioral, political, economic or demographic explanation. This reductive analysis,

.... obstructs the very thing it sets out to promote by conditioning the research worker to see new phenomena only through fragmentary explanations, each invented to deal with part of the whole but not the whole itself.
(Amson, 1975, p. 178)

One of the favourite devices of human geographers has been the physical analogy. Boyce (1971, p. 104) has stated, "The use of analogy in geography is so widespread as to require little justification". Consider the gravity and potential models, Weberian weight triangle, gas laws, diffusion, waves of migration, isopleths, entropy, life cycles and the geometrically based models. Such analog models of human activity in space are fascinating but apparently unrealistic. Man is not predictably rational. Even if he were, there is nothing to say that human activities should bear any resemblance to the attraction between masses or the activity of gaseous molecules.

It is here that Thom (1977, p. 637) would disagree. "Catastrophe theory - is quite likely - the first coherent attempt (since Aristotelian Logic) to give a theory on analogy." The fantastic variety of forms found on this earth can be classified into a relatively small number of essentially different forms. Can it be just coincidence that we observe a wave of: migration, innovation, diffusion, fighter aircraft, or water? Some would argue that this is merely a metaphorical trap. Thom believes that there is some natural adaptive organizing tendency or principle that results in many diverse phenomena bearing remarkable similarity of form. This organizing principle is structural stability. Systems having this attribute remain stable by undergoing catastrophic changes of form when parameters change sufficiently.

Consideration of the foregoing makes one at least sympathetic to Brown's assertion that,

Catastrophe theory is more than a branch of mathematics that enables the modelling of discontinuous change in cause and effect processes: it is actually a paradigm that suggests a new mode for analysis.
(Brown, 1979, p. 1020)

Geography is a science of form. Any definition of geography that goes beyond 'what geographers do' is bound to excite controversy and disagreement. A definition which includes the notion of geography as a 'science of the form of the spatial distribution of natural and social activities' would find general acceptance. All geography starts with the form of spatial activity and goes on to focus on the particular processes creating the form or a description of the form itself. This fundamental aspect of geography calls to mind a maxim of Kenneth Boulding; "growth creates form but form limits growth" (Berry et al, 1976, p. 477). It was the changes of embryonic form during growth that provided the impetus for Thom's work in catastrophe theory and for him to pronounce that,

.... the universe we see is a ceaseless creation, evolution, and destruction of forms and the purpose of science is to foresee this change of form and if possible to explain it.
(Thom, 1975, p. 1)

- Whether catastrophe theory will be useful for predicting morphogenesis remains a matter of debate. It has shown exceptional promise as an explanatory modelling agent of changes in spatial form.

There has been a steady growth of catastrophe theory articles in the geographical literature (fig. 1.2). " ... The subject is breathing down our necks" (Poston and Stewart, 1978, p. ix). Review articles (Amson, 1975; Wagstaff, 1976; and Wilson, 1979b) have suggested a wide range of geographical problems to which catastrophe theory may be applied. Chapter Four will summarize and assess applications in the fields of central place theory, (Casti and Swain, 1975); structural

changes in urban systems, (Isard, 1977); urban density patterns (Amson, 1974 and Richardson, 1975); and the intra urban marketing geography of commercial facilities, (Poston and Wilson, 1977). To these may be added catastrophe models from historical geography, (Mees, 1975 and Wagstaff, 1978); glaciology, (MacAyeal, 1979); marketing geography, (Smith, 1977); gravity models, (Wilson, 1978); models of city size, (Dendrinos, 1977, 1980); transportation modelling of modal choice, (Wilson, 1976 and Blase, 1979); and traffic flows, (Dendrinos, 1977). This array of material is remarkable considering that Thom's seminal book is only eight years old. Its ready acceptance by geographers suggests that many more applications will appear in the future. Any morphogenetic process possessing the characteristics of bimodality, hysteresis, sudden transition, divergence, and inaccessible behavior is worthy of at least preliminary investigation in terms of catastrophe theory.

This introductory chapter leads to a number of questions. What do the concepts of structural stability and morphogenesis mean and how are they an integral part of catastrophe theory? What are the basic forms of the elementary catastrophes and how do they help to explain discontinuous change? How has catastrophe theory been applied to geographical problems? Is catastrophe theory a useful modelling tool for geography? The succeeding chapters attempt to answer these questions.

Fig 1.2 The Growth of Catastrophe Theory in the Geographic Literature

Note: Only one half of the 1980 journals had arrived at the Carleton University Library at the time these data were compiled.

Source: Author's survey of the literature. Included are journals from Regional Science, Urban Economics, Planning, and selected elements from Archeology, Transportation, and Earth Science.

CHAPTER TWO

STRUCTURAL STABILITY AND MORPHOGENESIS

Structure, stability, form, and change are basic concepts in catastrophe theory. These terms are also recurrent in the literature on urban dynamics. The connection between these concepts and the catastrophic change of the sort demonstrated by the illustrations in fig. 1.1 may not be intuitively obvious, yet Thom found them important enough to form the title of his treatise on catastrophe theory. This chapter is devoted to examining these concepts and explaining their relevance to catastrophic change. This aim is achieved by sketching the philosophy underlying catastrophe theory under the headings of form, structure, and structural stability.

Structure is represented in fig. 1.1 by the pattern formed by ink on paper. The form is the perception of the structure as either a man's face or a woman's reposing figure. The illustrations are structurally stable because under most small changes in structure the perceived form remains invariant. Frames 1 and 2 are slightly different drawings yet they are perceived to be essentially the same. This is not surprising; most phenomena have the property of structural stability. They maintain essentially the same form after small perturbations in their structure. Eventually, however, a threshold is reached when further small changes in structure result in instability and a dramatic change in form. The addition of a few black lines to frame 5 of fig. 1.1 gives rise to a sudden change in perception and the man's face becomes a woman's body. Structural instability leads irrevocably to morphogenesis, a change in form.

2.1 Structure

Common usage of the word 'structure' implies a framework giving strength and determining form. Bourne (1971, p. 4) used 'structure' in its geographical sense to refer to the location, arrangement, and inter-relationships between social and physical elements in the landscape. The temptation is to consider structure as only the interactive channels connecting centres of essential activity. Thus the structure of the human body is composed of bone and vascular and sensory conductors; the urban structure is composed of ways, pipes, and wired services. Form and structure are sometimes so integrated (as in monocoque aircraft construction) that it is difficult to distinguish between them.

Catastrophe theory requires a more precise definition of structure. "One may define structure as that which controls the rate of change of variables characterizing the system" (Smith, 1977, p. 461). The implication is that structure in a system's perspective is bound inseparably to the mechanism of feedback which determines how the parameters governing a system's form may covary. By way of analogy consider a square frame made of four coffee stir sticks attached loosely at the corners by pins (fig. 2.1). The structure of this system is determined by the fixed pivot points and the equal distances between them along each stick. The system's form is uniquely described by the variable angle B A D. As the frame is compressed from State A to State B the form changes in a manner and at an angular rate determined by its structure. Euclidean geometry will serve to determine all other aspects of form given the angle B A D. Structure may be something as concrete as a polygon made of stir sticks or a grid road network. In other frames of analysis it may be more abstract; such as a schedule of freight rates or the line diagram of a multinational corporation.

2.2 Form

In Chapter 1 it was inferred that geography is fundamentally a science of form. The words 'form'; morphology, the study of form; and

STATE A

STATE B

Fig. 2.1 Coffee Stir Stick Structures

morphogenesis, change in form are important themes in geography yet they are seldom addressed directly. In This Scene of Man James Vance (1977, p. 5) takes morphogenesis, "...the origin of shape and form" as his theme but does not go much further with the concept. Lancelot Whyte in Aspects of Form (1968, pp. 1-10) provides a comprehensive definition of form. He argues that 'form' has many meanings; shape, configuration, pattern, organization, and system of relations. These meanings incorporate the notion of 'organized complexity' operating under some unifying principle. Form includes development and transformation - indeed form is that which changes, the structure is that which persists. Whyte's definition of form does not suggest how one can distinguish between forms.

Mathematically, "...two topological spaces X and Y have the same form if they are homeomorphic (this means that there is a map from X onto Y which is bijective and bicontinuous)" (Thom, 1975, p. 12). Outside of the artificial world of mathematics it becomes much more difficult to distinguish whether two forms are the same or whether one form has changed in time. Has Ottawa's form changed in the last minute? If the mathematical distinction is accepted then one must grudgingly concede that it has. For most purposes, however, it is useful to agree that Ottawa's form has not changed in the past minute (assuming of course that there has been no catastrophic occurrence). It is difficult to develop a criterion for judging when forms are similar enough to be considered as the same, or have changed enough to be considered different. Geographers have grappled with a similar problem in an attempt to define formal regions (Bunge, 1962, pp. 14-26). Statistical techniques such as Analysis of Variance have been developed to provide an objective criterion for distinguishing what is the same. Thom's approach is unique in suggesting that the problem of classifying form can be solved from a more qualitative Gestalt viewpoint (1975, p. 13).

Forms are recognizable whether deterministic and regular or

indeterministic and irregular. The coffee stir stick structure presented in fig. 2.1 is an example of a completely deterministic form as a result of its fixed and simple structure. The one-celled amoeba is the antithesis of deterministic form, it has no rigid framework at the macroscale. It has an outer membrane filled with protoplasm and associated organelles and a dense core called the nucleus. Its outline shape is in a perennial state of unpredictable flux, yet it has a recognizable form. It is amorphous (without form) and is constantly changing (fig. 2.2). There is some remote possibility that it could appear in any one outline shape at two different times, or in a form that is recognizably symmetrical to the human eye. Most of the time, however, an amoeba will have an outline shape which we have never seen before. We recognize it because it has a characteristic succession of forms. This notion lies at the root of René Thom's work with catastrophe theory.

* No two objects or processes are exactly the same when measured with a sufficiently high degree of precision. The two letters 'ee' even when typed have different forms though one would have to resort to microscopy to demonstrate the difference. This distinction begins to make nonsense of language; one wants to say, 'for our purposes those two letters are the same form and they appeared on the paper through the same process of typography'. The phrase 'for our purposes' is an expedient to justify an arbitrary distinction. Thom believes that, "...reducing the arbitrariness of description really is the proper definition of scientific explanation" (1977, p. 637).

It is established that no two things are exactly the same. Many things, diverse phenomena arising from diverse processes are approximately the same. Thom argues,

Almost any natural process exhibits some kind of local regularity...which allows one to distinguish recurrent identifiable elements dominated by words. Otherwise the process would be entirely chaotic and there would be

Fig. 2.2 The Changing Amoeba

There is some remote probability that the form may be identical at two different points in time as at t_0 and t_3 or that the form may be recognisably symmetrical as at t_1 .

nothing to talk about.

(Thom quoted in Woodcock and Davis, 1978, p. 9)

D'Arcy Thompson (1961, pp. 95-119) demonstrated how an apparently incongruous set of forms; beer froth, soap bubbles, gastrulating cells, honeycomb, and piles of cannon balls were aggregated into similar forms by similar forces. He concluded, "...the form of an object is a 'diagram of forces' ..." (Thompson 1961, p. 11). Thom has simply taken Thompson's demonstration a step further by linking the 'recurrent identifiable forms' of diverse origin with the property of structural stability. The forms are qualitatively similar although the circumstances affecting their development are never precisely the same. Thus the geometry of beer froth is like the geometry of soap bubbles even though they have different surface tensions and viscosity indices.

Conventional science seems to emphasize homologies between forms. Analogy is often considered coincidental, specious, or purely as a descriptive metaphor. In Thom's view the aim of catastrophe theory is to provide a qualitative way of classifying analogy (1977, p. 637). Forms may be defined as homologous if they have a resemblance of structure and develop from the same evolutionary source. Homologous forms need not serve the same functions or operate in the same way. The Nanticoke Steel Plant and the cities of Cleveland and Chicago are homologous settlement forms (fig. 2.3). They have a similar structure of ways, building types, and piped services and the principles governing their design and construction come from the same tradition. Nanticoke's function as a human settlement for making steel is quite different from that of Cleveland or Chicago. Ancient Baghdad (fig. 2.3) had similar manufacturing and mercantile functions as Cleveland and Chicago but had a much different form and structure. Baghdad is analogous to Cleveland and Chicago.

Forms which are both analogous and homologous but different in shape become identical under suitable mathematical transformation. The dramatic illustration in fig. 2.4 demonstrates how Thompson (1961 p. 301) applied a simple transformation to the porcupine-fish to arrive

Fig. 2.3 Homology and Analogy in Human Settlement Forms

Chicago, Cleveland, and Nanticoke are homologous.

Chicago, Cleveland, and Baghdad are analogous.

Source: A & B Adams, 1976, p. 147,133

C Wilson, 1980, p. 33

D Doxiadis, 1973 p.277

Fig. 2.4 Transformation of Fish Form

A suitable distortion of the orthogonal grid of the porcupine fish yields a new form identical to the sunfish.

Source: Thompson, 1961, p.301, figs. 154-155

at a drawing nearly identical to the sunfish. This was achieved by deforming the vertical lines into concentric circles and by transforming the horizontal lines into a set of hyperbolas. The same relationship has been noted between geographic forms which are both analogous and homologous. The growth history of Cleveland and Chicago was influenced by similar socio-economic forces and geographic circumstances. The patterns of residential differentiation and industrial activity in Chicago is almost perfectly reproduced at a smaller scale in Cleveland (Schwab and Marsh, 1980, p. 389; Uyeki, 1964, p. 492). Fig. 2.3 demonstrates that if a map of Cleveland is rotated 88 degrees clockwise and enlarged by a factor of 1.3 it bears a remarkable resemblance to Chicago, though admittedly not as strong as that between Thompson's fish.

Thom's objective in Structural Stability and Morphogenesis was to devise a theory of form. In Thom's view the concept of form is integrally connected with stability; that which has form is of necessity stable. The most significant features of a form are its singularities, the mathematically defined points where something different happens. When a system is undergoing some sort of change it will eventually reach a singularity beyond which it will become unstable. An unstable form cannot exist for long. It will either collapse and dissolve or it will change to a more stable form. Many of the forms found in nature undergo analogous sequences of form; instability; and change in form. David Berlinski has stated in this regard, "Instabilities can occur stably ..." (1978, p. 409). Such morphogeneses are manifestations of the "...ceaseless creation, evolution and destruction of forms ..." in the universe we see (Thom, 1975, p. 1). A walk along the seashore reveals an unending chain of waves; each one eventually becoming unstable and breaking on the strand to be followed by another in a stable succession.

2.3 Structural Stability

The concept of structural stability lies at the root of Thom's

view of catastrophe theory. If the structure of the stir sticks illustrated in fig. 2.1 is changed slightly, say by moving one of the pivot points one millimetre closer to the centre, the form will remain recognizably the same. The stir stick model has the property of structural stability. Slight changes in structure do not result in any appreciable change in form. A good deal of Thom's work is devoted to defining mathematically what is meant by 'a slight change in structure' and within what tolerances two forms can be accepted as being the same. It would seem that all science operates under the assumption of structural stability.

Suppose that a geographer conducted an experiment to determine the relationship between the spatial distribution of industrial activity and proximity to major population centres. He might draw a map showing the concentration of industry and conclude that 97 percent of all industrial activity was located in urban centres larger than 10,000 population. A second scientist setting out to replicate the experiment using identical methods and definitions might arrive at a map that was slightly different and find that 96 percent of all industrial activity was concentrated in centres larger than 10,000 people. There is a slight difference between the figures. However, the essential form of industrial concentration is qualitatively the same. "In this way the hypothesis of structural stability of isolated scientific processes is implicit in all scientific observation" (Thom, 1975, p. 16).

Poston (1977, pp. 425-427) has suggested that catastrophe theory may be thought of as a new way of measuring simplicity. The flow of water down a perfectly smooth inclined plane (fig. 2.5a) represents a very complicated situation, it is nearly impossible to predict what form the system will ultimately take. Eventually some almost imperceptible perturbation in the material under the water will trigger positive feedback processes culminating in channel formation and a change in form.

It is impossible to specify exactly what form the flowing water

Complicated

Fig. 2.5a

Simple

Fig. 2.5b

Fig. 2.5 Water Flow and Structural Stability

Source: Poston, 1977, p. 426, figs. 18.1-18.2

will take but it is certain that the ultimate state will qualitatively resemble the situation in fig. 2.5b. This dendritic drainage pattern is a recurrent form, even the adjective 'dendritic' connotes a similarity with nerve cell endings. Trees, plant roots, organizational hierarchies and the Linnaean taxonomy all have a similar qualitative appearance of form. The amoeba is constantly changing its shape yet is always recognizable. An urban geographer could examine maps of Manotick and Marrakesh and would recognize them both as analogous urban settlement forms. René Thom believes that the structural similarity of forms arising from different processes indicates a universal natural stability in all processes. "'Almost any natural process', he argues, 'exhibits some kind of local regularity ...which allows one to distinguish recurrent identifiable elements'". (Thom quoted in Woodcock and Davis, 1978, p. 9). This 'local regularity' is structural stability.

The channelized flow shown in fig. 2.6a is extremely simple because it is structurally stable. If the flow of water is increased slightly or a small quantity of sand falls into a channel the form will remain essentially the same. If the perturbation is great enough the system will become structurally unstable (fig. 2.6b). If the flow is increased sufficiently the form of the flow must change as water will not flow above the confining channel: water always seeks its own level. The situation in fig. 2.6c is manifestly impossible. A system which has become structurally unstable cannot retain the same form. The flood stage in fig. 2.6d is a catastrophic change in form due to structural instability and the tendency of water to always minimize its potential energy.

The most acclaimed achievement of catastrophe theory is its classification of the types of changes that potential systems may undergo. In Chapter 3 the elementary catastrophes will be examined as modelling devices.

Fig. 2.6a

Underfit

Fig. 2.6b

Bankfull

Fig. 2.6c

Impossible Condition

Fig. 2.6d

Flood

A B

Fig. 2.6 Water Flow and Catastrophic Change

CHAPTER 3

TOWARDS A TYPOLOGY OF CATASTROPHES

Aggressive dogs, traffic flow, and growing cities have two things in common: they have been modelled using catastrophe theory and they may be considered as potential systems. A potential system's behavior changes in response to external stimuli. It maintains stability by always responding so as to minimize some quantity. This may be a minimum of energy in a physical system, disutility in an economic system, or risk in a cognitive system.

The hypothetical location of a processing plant (P) in relation to its source of raw materials (RM) and market (M) can demonstrate the response of a potential system to exogenous perturbation. It is assumed that the entrepreneur who wishes to build the plant has determined the optimal production and marketing strategy and his only decision now is to decide where to locate in order to minimize the cost of transportation. The best locations for the plant are determined by the set of minima for a potential function of minimum cost. It is assumed that the entrepreneur is in a monopoly position, the real costs of moving are not great compared to the benefits of decreased transportation costs, and the price of the raw material and finished product doesn't change. The entrepreneur calculates all of the transportation costs and arrives at the situation graphed at t_0 in fig. 3.1. The solution in this case is very simple, there is only one minimum cost location at P_0 . The rational entrepreneur will locate his plant at P_0 , all other things being equal. At time t_1 changes in transportation technology or freight rate structures produce a new local minimum at P_1 in fig. 3.1. The entrepreneur's plant is still at

Fig. 3.1 The Effects of Changing Cost Curves on Least Cost Plant Location

the global cost minimum so he stands pat. By t_2 further changes have resulted in P_1 becoming the global minimum and P_0 is now reduced to a local minimum. The entrepreneur will be aware that there is a less costly location at P_1 but will not change locations because of the real and intangible costs of distant moves. At t_3 transportation costs have changed further and P_0 is no longer even a local minimum cost location. The conservative entrepreneur will be more willing to move his plant a short distance to P_2 to take advantage of the local cost minima and delay a decision to make a longer move over the cost maximum to the global minimum. By t_4 the local minimum at P_2 has dwindled to a point of inflexion and by t_5 has vanished altogether. The blandishments of cost savings at P_1 are too great to ignore and the plant must be moved to the new global maximum by the rational entrepreneur. If all of the cost minima could be graphed against all of their controlling factors such as freight rates, transportation technology, and nature of the commodity being shipped, a catastrophe model of these events could be built.

3.1 The Seven Elementary Catastrophes

Catastrophe Theory includes a typology of the seven elementary catastrophes. These are summarized in tabular form in Table 3.1. The elementary catastrophes are graphic archetypes of the seven simplest ways in which most systems can undergo discontinuous change.

The graphs of the seven elementary catastrophes depict seven topologically distinct arrangements of the points representing stable states. Within certain limits, moreover, they are the only possible arrangements.

(Woodcock and Davis, 1978, p. 33)

The catastrophe models are classified according to the number of control and behavioral dimensions they have. These terms are analogous to the independent and dependent variables of regression analysis. The control variables determine the behavioral variables.

TABLE 3.1
A MATHEMATICAL TYPOLOGY OF THE SEVEN ELEMENTARY CATASTROPHES

CATASTROPHE	CONTROL DIMENSIONS	BEHAVIOURAL DIMENSIONS	POTENTIAL FUNCTION
Fold	1	1	$1/3x^3 + ax$
Cusp	2	1	$1/4x^4 + 1/2ax^2 + bx$
Swallowtail	3	1	$1/5x^5 + 1/3ax^3 + 1/2bx^2 + cx$
Butterfly	4	1	$1/6x^6 + 1/4ax^4 + 1/3bx^3 + 1/2cx^2 + dx$
Hyperbolic umbilic	3	2	$x^3 + y^3 + ax + by + cxy$
Elliptic umbilic	3	2	$x^3 - 3xy^2 + ax + by + cx^2 + cy^3$
Parabolic umbilic	4	2	$x^2y + y^4 + ax + by + cx^2 + dy^2$

Note: a, b, c, d are coefficients representing control
parameters and x, y are variables describing
system behaviour

Each of the elementary catastrophes are visually represented by a graph in three dimensional space drawn on a two dimensional plane. The graphs are named after natural or mathematical shapes which they resemble. Continuous changes in behavior are represented as pathways along lines or surfaces while sudden jumps appear as movements between lines or surfaces.

3.2 The Fold Catastrophe

The average commuter is involved in a mathematical catastrophe twice daily in his journeys to and from work. The driver's calculus involves minimizing the time spent in travel (by driving faster) and maximizing his safety (by driving more slowly in response to congestion). At peak travel times volume to capacity ratios increase and speeds decrease. When the volume to capacity ratio reaches unity the flow of traffic becomes unstable (Paquette et al, 1972, pp. 343-346). Any small perturbation such as an accident or road construction will result in a change from a dynamic flow to a static morphology marked by bumper to bumper frustration as both volume and speed drop to zero. The road system is most prone to dysfunction when its full design capacity is being realized.

The fold catastrophe is two dimensional; it has one control axis and one behavior axis. The traffic jam problem discussed above may be modelled as a fold catastrophe as illustrated in fig. 3.2 (Dendrinos, 1978). Stable behavior at volume to capacity ratios of less than one is depicted as a solid line while unstable behavior is shown as a broken line. When volume reaches critical levels the system becomes metastable. If volume does not decrease some slight perturbation will result in a catastrophic decrease in flow and traffic becomes embroiled in a bottleneck graphed as a downward jump from Y to Z. Substantial decreases in volume will be required before the flow will jump back to stable levels, this appears on the graph as a jump from Z' to Y'. The fold catastrophe graph in this example is mathematically defined as,

Fig. 3.2 Fold Catastrophe Model of Traffic Flow

Stable Flow	—————
Unstable Flow	- - - - -
Trajectory	

Source: Adapted from Dendrinos, 1978, p. 193

$$x^2 - a = 0 \quad (3.2.1)$$

where; x = a function of speed

a = a function of volume/capacity. -

This is the first derivative of the function, ,

$$f(x) = 1/3 x^3 - ax \quad (3.2.2)$$

which describes the potential behavior of the aggregate of drivers who are acting to minimize time spent in travel and risk of collision. When the first derivative is set equal to zero its solution will yield all minima for function (3.2.1) which is graphed in fig. 3.2 as a folded curve.

The fold is the simplest elementary catastrophe. The example of traffic flow demonstrates that when a system's controlling parameter reaches critical proportions an infinitesimal change can result in a dramatic alteration in the system's behavior and form.

3.3 The Cusp Catastrophe ✓

Every late night stroller has known the fear of meeting a ferocious dog. Zeeman's cusp model of canine aggression (1976) has done much to popularize catastrophe theory. Zeeman described a model of aggression composed of two conflicting factors, rage and fear. Whether a dog will cower or attack depends on how frightened and how angry he is. Fig. 3.3 shows Zeeman's three dimensional cusp model with rage and fear as the control variables and the degree of aggression as the behavioral variable. A dog who is unfrightened and very angry will attack, while one that is frightened but not annoyed will slink away. An attacking dog who is sufficiently frightened (by a well-aimed stone for example) may suddenly flee. A normally quiescent dog may be provoked into a vicious attack. These four eventualities can be plotted on fig. 3.3 to visualize how slowly changing control variables may suddenly and radically alter behavior.

Fig. 3.3 Cusp Model of Canine Aggression

Source: Zeeman, 1976, p.80

The cusp catastrophe is three dimensional. It has two control variables and one behavioral variable. Fig. 3.3 demonstrates that it is built on a cubic base with a pleated behavioral surface at the top. The projection of the folded edges of the pleat onto the control surface at the base of the figure forms a cusped feature giving rise to the name.

The cusp catastrophe is described by a potential function, $f(x)$ which could be energy, cost, utility, or cognitive dissonance. The function,

$$f(x) = \frac{1}{4} x^4 + \frac{1}{2} ax^2 + bx \quad (3.3.1)$$

maps all possible values of a and b on x . The first derivative of $f(x)$ is,

$$\frac{df}{dx} = x^3 + ax + b \quad (3.3.2)$$

If a and b are known, equation 3.3.2 will yield the instantaneous rate of change with respect to x . Now if we set,

$$x^3 + ax + b = 0 \quad (3.3.3)$$

then we may determine all values of x which for given values of a and b are not changing, that is to say, all values of x for which the tangent of the curve defined by function (3.3.1) is perfectly horizontal. This will yield all maxima and minima of x for different values of a and b in function (3.3.1). When the set of maxima and minima are graphed against their corresponding ordered pair (a, b) a pleated behavior surface is formed. The surface has a one to one correspondence with the control variables over most of its area. The cusp boundary line or bifurcation set is defined by,

$$A = \{(a, b) \in A \times B \mid 4a^3 + 27b^2 = 0\} \quad (3.3.4)$$

There is a two to one correspondence between the behavior surface and the control surface along the bifurcation set. Within the cusp,

$$B = \{(a, b) \in A \times B \mid 4a^3 + 27b^2 < 0\} \quad (3.3.5)$$

there is a three to one correspondence in what is known as the duplex set. The union of the bifurcation set and the duplex set is known as the catastrophe set,

$$C = \{(a, b) \in A \times B \mid 4a^3 + 27b^2 \leq 0\} \quad (3.3.6)$$

The bottom and top branches of the behavior surface represent two stable minima. The middle branch is an unstable maximum. A potential system's behavior can trace any pathway on the behavior surface which is not on the middle branch. Within the cusp the system may occupy the top or bottom branch with the same control parameter values: this is bimodal behavior. The pathway of the system's behavior determines which branch the system is on. The jump from one branch to another will not occur until the behavior reaches the bifurcation set. The relationship between behavioral jumps and minima is shown diagrammatically in fig. 3.4.

The path $u - w$ on fig. 3.5 represents a smooth and continuous change in behavior. If the system then moves from point w to point z there will be a sudden change in behavior as a minima disappears and there is a jump from x to x' . The system's potential seeking behavior will cause it to remain on the upper manifold for as long as possible without entering the unstable middle branch. When further changes in the control variables force the behavior to move beyond the bifurcation set at x behavior suddenly jumps down because the intervening middle branch is unstable. In the same way a behavior path moving from left to right will jump upward from q to q' . Downward jumps are referred to as catastrophes, while upward jumps are referred to as anastrophes (Renfrew, 1979, pp. 497-498).

Fig. 3.4 Minima and Maxima in various regions of the cusp catastrophe model

The vignettes show the nature of the potential function in relation to various values of the a and b parameters. Outside of the cusp the function is unimodal. Within the cusp the function is bimodal with an intervening unstable maximum. Catastrophes occur when a stable minimum vanishes as a point of inflection.

Continuous Behavior —————
 Discontinuous Behavior - - - - -

**Fig. 3.5 Cusp Catastrophe Model Illustrating Hysteresis
 and Continuous and Discontinuous Behavior**

One of the more interesting properties of the cusp model is 'hysteresis'. This is demonstrated in fig. 3.6 which is a two dimensional section of the pleat in fig. 3.5. Hysteresis simply means that the jump in behavior occurs at different values of "b" (holding "a" constant) depending on whether b is increasing or decreasing. The magnitude of the hysteresis is the horizontal distance between the two sides of the cusp or, in the case of fig. 3.6, the distance between x and q.

The cities of Kingston and Ottawa had nearly identical populations in the 1850's and had similar industrial bases in the lumber trade, agricultural services, and river transshipping. A century later Ottawa had grown to be ten times as large as Kingston and their industrial structures had become quite different. This phenomenon of divergence can be modelled using catastrophe theory. Divergence is manifest in the pathways of points m and n on fig. 3.7. These points are very similar in behavior and control variables. With the passage of time they follow almost parallel paths in terms of their control variables. By the time points o and p are reached, they still have similar control variables yet they have very different behavior levels. A cusp model of the divergence of Kingston and Ottawa is not attempted because a major cause of the differentiation was the choice of Ottawa as the capital in 1857. This choice would be an exogenous factor in a cusp model of population, industrial structure and functional level. The example draws attention to the mistaken temptation to use catastrophe models for processes which are plainly influenced by sudden changes in factors outside of the model's parameters.

The cusp catastrophe model can be drawn in two different ways which reflect the relationship between the control variables. If the two variables are opposing or conflicting forces having qualitatively similar effects on behavior they are referred to as conflicting factors. Such is the case in the Zeeman (1976) model of canine aggression. These are shown in fig. 3.7 as factors α and β . If, on

Fig 3.6 The Hysteresis Effect

This is a cross section of Fig. 3.5 with some positive value of a .

The timing of the jump in behavior is governed by the direction of approach, in this case hysteresis depends on whether b is decreasing or increasing.

Left to right path ———

Right to left path - - - - -

Fig. 3.7 Diverging Behavior and Control Factor Axes on the Cusp Catastrophe

Widely diverging behavior can occur in spite of similar control variable changes.

The control factor axes may be oriented in two ways in the cusp catastrophe; either as normal and splitting factors or as competing factors.

- α & β - competing factors
- a - normal factor
- b - splitting factor

the other hand, the factors are qualitatively different they may be designated as normal and splitting factors. These are sketched on fig. 3.7 as a and b respectively. The normal factor controls whether or not a change in behavior will occur. The splitting factor governs whether a change will be rapid and catastrophic or continuous and gradual. The splitting factor is usually relatively static. It is drawn perpendicular to the cusp axis.

3.4 Other Cuspoidal Catastrophes

The swallowtail catastrophe model has three control variables and one behavioral variable. The model cannot be represented completely on a planar surface. The bifurcation set is three dimensional and is drawn in perspective in fig. 3.8. When C is positive, the surface is a folded sheet. When C is negative, the planes intersect forming a third surface and the distinctive swallowtail shape. The swallowtail catastrophe model has not been extensively used. Woodcock and Davis (1978, p. 51) suggest that this is because under a wide range of conditions there is no stable state.

The butterfly catastrophe is defined by a four dimensional control manifold and a one dimensional behavior variable. The bifurcation set is four dimensional and consequently cannot be drawn. Fig. 3.9 represents what Woodcock and Davis (1978, p. 51) have called the "shadow of a cross section". The four control variables have been termed the normal factor, splitting factor, bias factor, and butterfly factor. The butterfly model allows unimodal, bimodal and trimodal behavior. There are three stable minima and two unstable maxima in the bifurcation set. This allows for some form of compromising behavior between the upper and lower surfaces.

Amson has described two further cuspoid catastrophe models with one dimensional behavior manifolds. The 'wigwam' has five control variables and the 'star' has six.

Fig 3.8 The Swallowtail Catastrophe

The vignettes are cross sections of the bifurcation set.

Source: After Amson, 1975, p. 199

Fig. 3.9 The Butterfly Catastrophe

In this 'shadow' of a cross section only two parameters are shown, it is assumed that the others are constant

The complex behavior surface has three cusps and allows trimodal behavior including a 'pocket of compromise'.

3.5 Umbilic Catastrophes

There are three umbilic catastrophe models, each one corresponding to a member of the conic family: hyperbolic, elliptic, and parabolic. They have two behavior surfaces instead of one. They are represented in computer drawn projections on fig. 3.12. The umbilic models have thus far been found too complex for application in the social sciences.

3.6 Conclusion

The seven elementary catastrophe models have been described in varying degrees of detail. The models most commonly used in the social sciences have been the fold, cusp, and butterfly. The elementary catastrophe models describe the most complicated ways that potential systems governed by given numbers of control parameters can change.

This concludes the examination of catastrophe theory. In the following chapter some of the attempts to apply the theory to models of spatial processes will be examined.

Hyperbolic Umbilic
Catastrophe

Elliptic Umbilic
Catastrophe

Parabolic Umbilic
Catastrophe

Fig. 3.10 Computer drawn projections of the umbilic catastrophes

Source: Woodcock and Davis, 1978, pp. 55-56

CHAPTER 4

APPLICATIONS OF CATASTROPHE THEORY

TO GEOGRAPHIC PROBLEMS

Catastrophe models have provided novel perspectives to prosaic geographical problems. This chapter explores four applications of catastrophe theory taken from the geographic literature. The application of catastrophe models to central place theory, urban structural change, urban density functions, and the location of retail services will be explored to gain an appreciation of how the theory may shed new light on old problems. These examples help to illustrate the diversity of topics amenable to catastrophe modelling and the different approaches to the employment of the theory. The applications employ either the cusp or fold catastrophe models described in the previous chapter.

4.1 A Cusp Model of Central Place Theory

Central place theory has fascinated geographers since Walter Christaller propounded it in 1933. Central place theory is a normative hierarchical model of the distribution of urban centres of various sizes and functional levels on a featureless agricultural landscape. The theory's foundation stems from the differentials in demand between generalized and specialized goods and services.

The provision of a specialty service with low demand levels, say violin repair, requires a large market area to provide enough custom to make the business viable. Violin repair is a high order good and has a large range. Automobile repair, on the other hand, is in far greater demand. High demand levels will provide enough customers to support many service stations in the market area of a single violin

repair shop. Car repair is a lower order service and has a small range. Customers will not be willing to travel great distances to patronize the service station. Generally speaking, lower order goods and services require smaller market areas than do higher ones. The minimum population level that can support a given order of good or service is known as its threshold. A higher order function will have a higher threshold than a lower order function.

Christaller proposed a five tier hierarchy of successively higher order central places and showed how they would be distributed on the landscape to provide the most efficient marketing arrangement. This gave rise to a hierarchical lattice of central places, each in the centre of a hexagonal market area (cf Berry, et al, 1976, pp. 228-235). Christaller's model was essentially static, it did not incorporate the effects of changes in population size, supply and demand, or transportation. These are precisely the phenomena that an observer of the dynamic economic landscape is most likely to notice and question. How did the hierarchy of central places develop and how can differential growth rates be explained? Why do some central places thrive and expand while others wither on the vine? The causes of changes in the central place structure are often easy to assign to overall trends such as transportation improvements or specific events such as the closure of a major business. In other cases dramatic shifts in the functional level of a central place may not appear to be related to any single factor.

Casti and Swain (1975, pp. 11-15) approached the problem of functional transition of central places from one order to another using a cusp catastrophe model. The control parameters are population and disposable income per capita (DIPC) in the market area of the central place. The behavioral variable is the functional level of the central place (fig. 4.1). All of the normal assumptions of neoclassical location theory are made.

The upper and lower thresholds of aggregate purchasing power in

Fig. 4.1 A Manifold for Central Place Catastrophes

Source: Casti and Swain, 1975, p. 14

the market area are represented graphically by the upper and lower folds in the pleat. If population declines sufficiently there will be an anastrophic jump to a lower behavior level, and certain higher level goods and services will no longer be distributed from that central place. If population begins a resurgence the functional level will not increase until the upper threshold of aggregate purchasing power (diagramatically the lower fold) is crossed. This is the hysteresis effect. It is explained on the ground that, "...an entrepreneur would choose that combination of good and market area offering maximal spatial monopoly profits (the upper threshold)" Casti and Swain (1975, p. 12). Presumably other centres would offer higher profits until the centre being modelled crosses the upper threshold.

Population is the normal factor in this model while DIPC is the splitting factor. An increase or decrease in the population of the market area is directly responsible for the change in the functional level of the central place. The DIPC determines whether the change will be rapid or slow. There are two problems with this latter parameter. As an average, DIPC says nothing about the distribution of income which would have an important influence on spending patterns and concomitantly on the demand level for goods and services. The service provision for a population with a low standard deviation of DIPC would be quite different from a population with great income disparities. This is a common problem for catastrophe models using averaged social or economic measures as control variables. It must be assumed that DIPC has a normal distribution with a relatively small standard deviation.

As the splitting factor, DIPC controls the rate of change from one level to another. At high values of DIPC the transition from lower to higher orders is graphed as a gradual change while at low levels the change is discontinuous. Casti and Swain (1975) do not account for this key feature of their model. A tentative explanation is proposed here. If population (and therefore aggregate demand) declines the proprietor of an establishment providing

a good or service has three alternatives. He may do nothing and accept a decrease in profits. This strategy becomes increasingly difficult if profits reach zero and losses are incurred. He may abandon his business which is always the last resort. The third alternative is to increase prices to maintain profit levels. A low income population will start to patronize a more distant service as soon as the price differential exceeds the increased cost of transport to the more distant centre. A population with higher incomes will tend to pay the higher prices because the nearer service is more convenient. This would suggest that the drop in functional level occurs earlier for a declining low income population than for a declining high income population. As the high income population decreases the prices of localized services will continue to increase until the price differential clearly exceeds the benefits of convenience. Eventually the functional level will decline as business switches gradually to more distant but cheaper markets. The low income population will switch to a more distant market quite suddenly as it cannot afford the 'convenience surcharge'. The nature of the switch to more distant markets will directly effect the rate of change in functional level of the local central place. It should be emphasized that the foregoing is purely supposition on the rationale for the continuous and discontinuous paths of change in this model.

The cusp model of central place theory gives a graphic explanation of how incremental changes in population or DIPC can result in sudden changes in the functional level of urban centres. It leads to the prediction that centres serving market areas with low income levels will decline suddenly and rapidly, while higher income centres will tend to decline more gradually. The hysteresis effect demonstrates that the threshold for the appearance of a function is higher than for its disappearance. These accomplishments contribute new insights towards an explanation of central place dynamics.

4.2 Catastrophe Modelling of Urban Welfare

Forecasting the future state of city systems presents a vitally important challenge to contemporary geographers. Fundamental changes in technology, energy consumption, and environmental considerations may result in unheralded transformations in the loci of human activity. Isard (1976) discussed the need for restructuring the urban system in light of a forecast 800-2000 nuclear energy centres in the United States by the year 2025. In a later paper, Isard (1977) went on to develop a new theory of urban hierarchy to cope with major structural changes in the future. To this end a catastrophe model of urban welfare was developed.

Welfare of an urban system is defined as the potential energy function of a cusp catastrophe.

$$W = -\frac{1}{4}x^4 + \frac{1}{2}ax^2 + Bx + C \quad (4.2.1)$$

The term ' $-\frac{1}{4}x^4$ ' is negative because in this case the potential system is supposed to maximize rather than minimize welfare. Maxima of this function indicating the optimal welfare of the system are derived by setting $\frac{dW}{dx} = 0$.

$$-x^3 + ax + B = 0 \quad (4.2.2)$$

where; W = welfare of the urban system

x = population of the city

a = agglomeration externalities

b = contribution of marginal population increment to welfare

C = a constant

The splitting factor, a, is a time dependent variable representing the marginal positive agglomeration externalities attained with an incremental increase in population. The positive externalities stem from diversification, increased specialization, greater opportunities for interaction and greater probabilities of innovation. The normal factor,

B, is also time dependent and represents the contribution of the marginal increment of population to total welfare of the urban system. Wilson and Kirkby (1980, p.349) referred to this parameter as 'per capita productivity'. The term ' x^3 ' found in equation 4.2.2 above is said to represent the negative externalities or deglommerative forces due to the marginal population increment. These negative externalities would include such things as congestion, pollution, higher costs of living, and higher crime rates. The normal and splitting factors are specified as time dependent since the direct contribution of a marginal unit of population can vary over time due to social and technological changes. The cusp model is graphed as fig. 4.2.

Isard's model is essentially quite simple. The behavior surface demonstrates that there are infinitely many welfare maximizing paths of population growth or decline. The model shows that population levels may change gradually or discontinuously in response to changes in the control variables. It is assumed that welfare is a potential function and that population always changes to maximize welfare. This is a phenomenon which is neither intuitively expected nor often observed.

Isard makes two fascinating observations about this cusp model. Consider the path of population growth from z to e in fig. 4.2. When the point reaches j there will be a sudden jump from j to j'. The path would then continue smoothly leftward. The alternative is for an oscillation to occur between j and j' or for a sequence of catastrophe and anastrophe from j, j', h', h and back to j. This is analogous to a sort of iteration process to find a stable state under circumstances of catastrophic change.

Isard's second observation (1977, pp. 4-5) is that there is room for a stochastic behavior pattern in a catastrophe model. Consider once again the path from z to e. From z to h the behavior is unimodal and the probability of a jump is zero since there is no other stable state. After the point has passed h the system is bimodal and there

Fig. 4.2 Optimal Time Paths of Population

Source: After Isard, 1977, p. 2

exists some small probability, say 0.1, that the system could jump to the upper surface. As the point crosses the cusp the probability of such a jump increases reaching 0.5 at q and 1.0 at j . This is to say that a jump is possible anywhere within the cusp and is virtually certain at the far edge. This notion takes into account that the macro system is composed of a number of micro units, each responding to maximize welfare in the presence of small random perturbations.

The inescapable problem with this model is that there is no obvious mechanism which makes welfare a potential function. It may be supposed that the population will increase or decrease to maintain a stable maximum of aggregate welfare. As Isard notes (1977, p.4) this assumption of instantaneous reaction and adjustment to perturbation by a population is extremely unrealistic. The move out of a city usually stems from an economic imperative or a desire to maximize personal satisfaction quite apart from aggregate welfare. The other, and more frightening supposition, is that there will be some future central planner who can completely control the economy in order to keep it on an optimal path. This assumption is fortunately as unlikely as it is unpleasant.

Notwithstanding these shortcomings, the model does make an important contribution to an understanding of future urban behavior. Changes in agglomerative externalities could result in a large and rapid increase in urban capacity with no decrease in aggregate welfare. There is no single critical parametric value which will make such changes a possibility or a necessity. The cusp model demonstrates that the relation between two control variables allows for a spectrum of paths to new and possibly greatly enlarged urban forms.

4.3 Catastrophe Analysis of Urban Density

Residential land use comprises the greatest single proportion of urban space. The single most striking characteristic of residential land use is the variation and juxtaposition of very high and very low

density housing. Most of the attempts to model residential density have been based on the distance from the Central Business District (CBD) (Clarke, 1951, Newling, 1971). These involve an exponential curve culminating in a crater as densities drop in the CBD itself (fig. 4.3). The 'volcanic' appearance of these models provides a fair generalization of urban density trends, but fails completely to explain anomalous increases in densities such as those associated with suburban apartment development (Bourne, 1971). It is unlikely in western society that urban centrality will ever be completely divorced from density. The city is first and foremost a centre of human interaction which is greatly facilitated by high densities. Nevertheless, residential density may be explained using factors which have nothing to do directly with measures of centrality. It will be shown that density may be described as a function of income and rent variables in a cusp catastrophe model. Such a model will help to account for large differentials in densities over space and in time.

This model was first presented by Amson (1974) as one of a set of modes of urban growth. It was reworked from a different perspective by Richardson (1975) but was acknowledged as being based on the earlier work of Amson. The two models are considered together. The aim of the work is to model urban population density in terms of rent and income. Both start with a simple formula based on Van der Waal's Ideal Gas Law. The symbol system of Richardson (1975) has been adopted.

$$d = \frac{p}{KY} \quad (4.3.1)$$

where; d = population density
 Y = income per household
 p = rent per acre
 K = positive constant

We may also write;

$$\frac{p}{d} = KY \quad (4.3.2)$$

Fig. 4.3 Urban Population Density Profile with Central Density Crater

Source: After Newling, 1971, p. 331.

$$\frac{1}{d} = \frac{KY}{p} \quad (4.3.3)$$

These equations which ~~Amson~~ (1974, p. 119) refers to as the 'simple city model' state that density varies directly with rent and inversely with income. We would expect that areas with high rents and low incomes such as inner city districts, will have high densities. Areas with low rents and high incomes such as suburbs will have low densities. This is in keeping with commonly observed patterns in western cities and accords with theoretical work (Alonso, 1971).

The simple equation is rendered more complex by the addition of two constants. It is supposed that household decision makers are willing to allocate a larger proportion of household income for rent in order to obtain lower housing densities. The relative value of this reduced density is considered to be a regionally specific constant based on cultural expectations and environmental constraints. This relative value of space is measured by α , and is called the 'density disutility coefficient' (Richardson, 1975, p.310). The coefficient is multiplied by the density and subtracted from the income variable in equation 4.3.2 to yield,

$$\frac{p}{d} = KY - \alpha d \quad (4.3.4)$$

$$\text{or} \quad p = KYd - \alpha d^2 \quad (4.3.5)$$

The second constant is β which is also regionally specific. β sets an upper limit on maximum attainable densities, a point beyond which overcrowding is intolerable.

The inverse of density, $1/d$, is the area per person or 'gross specific space'. It is stated that, "It is not total area per person but the excess over this minimum (or net space = gross space minus β) which is influenced by what happens to rents and incomes." (Richardson,

1975, p. 309). The term ' $1/d - \beta$ ' may be substituted for the density term in equation 4.3.3 to yield,

$$\frac{1}{d} - \beta = \frac{KY}{p} \quad (4.3.6)$$

$$\text{or} \quad p = \frac{KYd}{1 - \beta d} \quad (4.3.7)$$

The equations 4.3.5 and 4.3.7 which were independently derived may now be combined into a new form which Amson (1974, p. 119) christened the urbanitic law for a saccadic (jumping) city,

$$p + \alpha d^2 = \frac{KYd}{1 - \beta d} \quad (4.3.8)$$

This may be manipulated into a cubic equation mapping a three dimensional surface (Richardson, 1975, p. 310).

$$\alpha \beta d^3 + \alpha d^2 + (KY + \beta p) d - p = 0 \quad (4.3.9)$$

This is not in the form of the potential energy function of any of the elementary catastrophes. Amson (1974, p. 124) applies a set of transformations to arrive at the form of the cusp catastrophe. Amson's work contains several elementary mathematical errors, these are corrected below.

We set $d = 1 + t$, $p = 1 + q$ and $Y = 1 + w$ and allocate temporary values to the constants of $\alpha = 3$, $\beta = 1/3$, and $K = 8/3$. Equation 4.3.8 then transforms to,

$$3t^3 + (8w + q)t + 8w - 2q = 0 \quad (4.3.10)$$

The linear transformations; $8w + q = y$ and $8w - 2q = z$ are then applied to equation 4.3.8 yielding,

$$3t^3 + yt + z = 0. \quad (4.3.11)$$

The squared term has vanished and this cubic now defines the behavior surface of a cusp catastrophe which is graphed on fig. 4.4. The last transformation has had the effect of making z the normal factor and y , the splitting factor. By transforming back to the original control factors (rent and income), represented by q and w respectively, a new surface with competing factors is formed.

The model demonstrates that cities having both low rents and low incomes will experience sudden changes in population density, either anastrophes or catastrophes. Cities having parameters falling outside the catastrophe set may be expected to experience smoothly changing densities. Amson allows that 'inertia' will prevent truly instantaneous density changes in real time. He does suggest that income (or opulence as he defined it) may be regarded as a measure of mobility of the urban population.

Many European and North American cities have exhibited 'approximately saccadic' expansions of territory with attendant falls in density, as the mobility increased and housing rentals fell in response to technological innovations in transportation.

(Amson, 1974, p. 126)

Richardson emphasizes the model's importance in either spatial or intertemporal analysis. In the former case, urban spatial units may be defined radially or sectorally with respect to some central point or be defined by some boundary criterion as relatively homogenous zones. Rent and income could then be shown to account for the magnitude of spatial density differentials. "In intertemporal analysis, if the variables are treated as city wide averages, the framework enables changes in city densities to be determined by changes over time in the level of urban income and in the aggregate rent structure." (Richardson, 1975, p. 306). The second advantage that Richardson feels makes this model more realistic than standard neoclassical models is the dis-

Fig. 4.4 Cusp Catastrophe Model of Urban Population Density

Source: After Amson, 1974, p.124

continuity in spatial structure inherent in the cusp. Smooth functions in standard neoclassical models cannot represent the juxtaposition of very high and very low density areas so often found on the fringe of western cities.

Though certain elements of this model are appealing, it cannot be said to be very successful. The very simple linear equation 4.3.1 is made much more complex by the addition of the α and β constants. While arguments for their inclusion are interesting, no effort is made to justify them using real world examples or bibliographic legitimation. On the whole, the constants seem to be an artificial contrivance to bring equation 4.3.1 into the form of the cusp catastrophe. The cusp model does seem preferable to spatially determined constructs in so far as it may explain adjacent high and low density neighbourhoods which have some differences in rents or incomes. Since factors such as the transportation network, age and quality of housing stock, the nature of urban growth, and the distribution of cultural groups are not taken into account one must expect that there will be significant departures from the model in a real world application. Neither Richardson nor Amson demonstrate how the model may be operationalized and tested with empirical data.

The catastrophe model of urban residential densities represents a significant departure from conventional density analysis since it does not incorporate a direct distance measure. It may be argued that the rent parameter is a surrogate for distance from the centre of the city. The rent parameter is in part a measure of accessibility which in turn is related to distance from the CBD. The catastrophe model of residential growth provides for sudden changes in density over time and in space. Consideration of the accelerating increase of population densities in third world cities such as Sao Paulo, or the anomaly of a twenty storey apartment tower in the midst of a five homes to the acre suburb, makes the catastrophe model seem much more realistic than one relying on smooth exponential curves.

4.4 The Fold Model of Shopping Centre Size

One of the most significant developments in the urban retail function in the post World War Two years has been the development of integrated shopping centres. The focus of commercial activity has been dispersed from the CBD to outlying transportation nodes. A hierarchy of shopping centre sizes has developed with neighbourhood, community, and regional centres providing successively greater varieties of goods and services to increasingly larger market areas. This intraurban commercial organizing principle is not unlike the interurban hierarchy proposed by Central Place Theory.

Poston and Wilson (1977, pp. 681-684) apply the fold catastrophe to examine the relationship between the distance travelled and the size of urban service facilities, specifically shopping centres. This is an intraurban model. The average size of shopping facilities (w) is assumed to vary directly with the average distance travelled (L) by consumers to reach the facility. Two factors affect the distance that a consumer will choose to travel to reach a shopping centre. The consumer will have a greater choice at larger centres than at smaller ones. The benefit from centre size (u_1) has a minimum representing some minimum size of shop and an asymptotic maximum where the benefits of facility size 'saturate', not exceeding a certain level (fig. 4.5a). This is the diseconomy of scale found in truly monstrous shopping centres. The second factor (u_2) is the disutility of travelling, this is modelled as a constant increasing with distance as a linear function. The disutility of travel varies among consumers and is seldom linear; however, the constant represents a rough averaging device. Total utility (u) is calculated as the sum of u_1 and u_2 and is graphed against distance in fig. 4.5b. It is assumed that the consumer's choice of shopping location is based on the criterion of maximizing total utility.

The first derivative of the total utility with respect to distance (du/dL) is graphed in fig. 4.5c. The points where utility is at a maximum in fig. 4.5b are graphed as L axis intercepts in fig. 4.5c.

Fig. 4.5 Utility Functions and Gradient with Changing 'ease of travel'

As the disutility of travel increases the maximum of total utility corresponds to increasingly short distances. The situation in graph 5 will result in a travel distance of zero and consumers will patronize local stores.

Source: Poston and Wilson, 1977, p.683

As the slope of the u_2 line increases in a negative direction a new minimum appears at (fig. 4.5b2), the original global maximum subsides into a local maximum (fig. 4.5b3), and finally both the original maximum and the minimum disappear as a point of inflexion (fig. 4.5b4). The sole remaining maximum is the u intercept where average travel distance is zero.

The five different graphs in fig. 4.5c can be combined into a fold catastrophe map (fig. 4.6). The vertical axis is the average travel distance while the horizontal axis is labelled ' a ', the negative of the slope of u_2 . The vertical lines correspond to the five numbered situations shown in fig. 4.5c. Between B and D' the fold is bimodal, the average distance travelled may be high or low, depending on which direction a point on the graph is moving.

To see how the model works, consider a path moving leftwards from point A. The path moves to the left because the disutility of travel is decreasing, perhaps due to improvements in the urban transit system. As soon as point B is reached, the system must jump to the new utility maximum at B' and large shopping centres will develop very rapidly increasing the average distance travelled to shop. Eventually the system will reach point C with further increases in the average distance travelled and decreases in the disutility of travel. If rising costs of energy are now posited which increase transit fares the disutility of travel will begin to increase. The pathway begins to move to the right until point D is reached when there will be an anastrophic jump to D' at the original stable maximum of utility. The corner stores will enjoy an abrupt resurgence of popularity.

The fold model of shopping centre size is of necessity simplified and consequently has limitations. One must assume a very homogenous population in income, location, and consumer tastes to accept that utility of facility size and travel can be modelled as shown. Nothing in the model says that there is sufficient aggregate purchasing

Fig. 4.6 The Fokl Catastrophe Model of Shopping Centre Size

The vertical numbered lines correspond to the graphs on fig. 4.5.

Source: Poston and Wilson, 1977, p. 683

power in the city to make a shopping centre profitable. The model does not take into account the different demand of consumers for different types of purchases. Urban services are not truly bimodal in centralizing behavior. The model cannot account for both centralized and decentralized facilities existing simultaneously in the same city as one may observe in reality.

This model begs comparison with work done by Casti and Swain (1975) discussed earlier in this chapter. Both models use the functional level of the retail centre as the behavior variable. It will be remembered that the Casti and Swain model employs population and disposable income per capita (DIPC) as the control parameters in a cusp model, while the urban facility model uses the disutility of travel as the single factor in a fold model. The former model is based on the actions of the urban retailers, while the latter model is based on consumer choice. The rural resident has to travel longer distances to patronize a central place, the provider of the central place service has a partial monopoly. The consumer will face substantially higher transportation costs if he chooses to use another centre. In other words, the distance travelled becomes so important that it ceases to be a factor as the consumer must use the nearest centre. A competitive centre closer to the rural resident will not be viable because the market would be too small. In areas of marginal farming such as Lanark County to the west of Ottawa where population and DIPC are declining, an anastrophic leap in functional level may be expected when the bifurcation set (fig. 4.1) is surpassed. Consumers then have no choice but to travel greater distances to the next closest supplier of higher order services. The decline in lower order service centres in the prairies has been due, in part, to population declines, but the principal cause has been a decrease in the disutility of travel.

With mechanized farm machinery came automobiles and a better road system. These made it easier to bypass the local grocer and the implement dealer in order

to shop at larger towns where prices were lower and the selection of goods was greater. In time, local merchants felt the profit squeeze and they too moved out.

(MacKenzie, 1979, p. 56)

The intraurban fold model of shopping centre size accentuates consumer sovereignty. Urban retail services are ordinarily not in a strong spatial monopoly position. The consumer has a greater shopping choice and will maximize his utility by making a satisficing trade off between propinquity and variety. This is not to say, however, that population and DIPC don't have an effect on the ~~retailer's~~ decision to stay or go out of business. Changes in population characteristics associated with ghetto formation have demonstrated this effect.

As lower to middle income white neighbourhoods become lower to working class black neighbourhoods, the number of retail outlets serving the area tend to decline.

.....

In the process of change, community-level centres become neighbourhood centres and small shopping-goods (regional) centres tend to become blighted community level centres.

The death knell to continued viability of shopping-goods centres is struck when the major department store in the complex closes its doors for a more attractive location. When this occurs the local population must travel a greater distance to secure basic commercial items that were previously available within a shorter travel radius.

(Rose, 1971, p. 77)

Nothing in the foregoing suggests that the total utility graph of the black consumer has changed, yet the distance travelled to a retail facility has indisputably increased.

Neither the central place cusp model nor the shopping centre

fold model has incorporated all the relevant retail system parameters yet both have an element of plausibility under some circumstances. There is a need for a comprehensive model of catastrophic changes in functional orders of retail service provision incorporating both consumer utility and market area population and DIPC.

4.5 Conclusion

Four representative geographically oriented models have been described and examined. There are undeniable methodological, logical, or operational flaws in each of them. However, they provide good examples of how the modelling of discontinuous behavior with catastrophe theory may be approached and illustrate the diversity of topics amenable to a catastrophe theoretic treatment. By focussing on the discontinuity of system changes, they open new avenues for exploration of geographic problems.

CHAPTER 5

A CUSP CATASTROPHE MODEL OF RACIAL TRANSITION
IN RESIDENTIAL NEIGHBOURHOODS

Residential segregation of blacks and whites is a fact of American urban life. Catastrophic changes in neighbourhood character can occur as a dominant white population is replaced by one substantially black. A discontinuous change may be triggered at a threshold known as the tipping point. Racial succession may also occur gradually and smoothly. The cusp catastrophe model can provide a conceptual rationalization of catastrophic and continuous paths to new racial communities.

The notion of change involved in the tipping point phenomenon has been of recurring interest in urban social geography. Chombart de Lauwe used social and economic measures to study static areal differentiation in Paris (Murdie, 1971, p. 279). The urban ecologists of the Chicago school related social changes in time to patterns of land use in space. The concept of the neighbourhood life cycle may be traced back to Burgess' seminal work of 1925, "The Growth of the City". The concentric zone model was followed by the sector and multiple nuclei theories (Murdie, 1971, pp. 280-282) based on the same ecological concepts but with different views of the orientation of social forces. Attempts to empirically verify these models met with limited success. Social groups did not behave uniformly enough to justify wholly deterministic models. Morrill (1965) focussed the investigation on the factors influencing the formation and expansion of black ghettos. He used these factors in a Monte Carlo simulation of cell by cell racial diffusion.

The term 'tipping point' appears to have originated in the work of Martin Grodzins who predicted in 1957 that many American cities would become black and poor while suburbs would become white and affluent. He contended that, " ...for the vast majority of white Americans a tip point exists" (Grodzins, 1957, p. 34).

5.1 The Tipping Point

American cities have experienced an enormous influx of rural black migrants since the beginning of the second world war. The black growth rate in northern cities averaged 5 per cent annually between 1950 and 1960. This was due, in part, to rural-urban migration and in part to high birth rates among the new arrivals (Rose, 1972, pp. 46-47). Initially the incoming blacks occupied lower income residential neighbourhoods that already had blacks living there. As the housing density in ghettos soared and black family incomes increased, the new urban black population expanded into hitherto exclusively white neighbourhoods. The Civil Rights Act of 1964 provided further stimulus for the movement of blacks to adjacent areas beyond the traditionally black residential sectors but had little effect on city wide patterns of residential segregation. Against some opposition blacks began buying or renting homes in white neighbourhoods, usually the low and middle income areas closest to black neighbourhoods.

From an ecological perspective the process of racial transition occurs in five stages; invasion, early consolidation, consolidation, late consolidation, and piling up (Schwab and Marsh, 1980, p. 386). The invasion stage is a critical prerequisite to racial transition. Whites may be unwilling to be the first to sell or rent to blacks and blacks may be unwilling to live in a predominantly white area. Once black entry has been established there may be some acceleration in the rate of transition. Some of the selling whites are upwardly mobile and will move in any event, others may be reacting to what they regard as an unfavourable racial climate. The piling up stage culminates the transition to an almost totally black population with an increase

in housing density. The five stages are not discretely defined. One blends into another implying a smooth logistic increase in the black proportion of the population. This is not always the case. There is evidence (Schelling, 1972, Steinnes, 1977) to show that the transition from white to black is continuous only up to a critical threshold. Once an area exceeds this critical percentage of blacks there will be an exodus of whites (white flight) and the area will rapidly become all black. This critical threshold is the tipping point.

There is much disagreement on what the tipping point is and how it operates (Wolf, 1963, pp. 218-219). Some writers have demonstrated empirically that the transition from white to black proceeds linearly and that there is no tipping point (Pryor, 1971, Schwab and Marsh 1980). The tipping point has been defined as:

1. The proportion of blacks which is reached when whites become unwilling to move in.
2. The threshold proportion of blacks which triggers a decision by whites to move elsewhere.
3. The threshold proportion of blacks which is reached when whites actually do move elsewhere.

(Wolf, 1963, p. 219)

None of these definitions is entirely satisfactory. The first definition is nearly meaningless without reference to some measurement of general mobility which is not related to racial change. In a very static neighbourhood the unwillingness of whites to move in would not by itself lead to tipping. The timing of the decision to move out is appealing since there are a host of exogenous variables affecting the time span between the decision to move and the actual move. These include: real estate and mortgage market conditions; the effectiveness of the owner of real estate firm as a marketing agency; the length of the lease; the time in the school year and the presence of school-age children; and the career plans of the head of the household. The disadvantage of this definition is that a decision is much more diffi-

cult to measure than racial composition. The variable time span between the decision and the act means that the observed intent is quite different from the observed outcome. "The fact that a person says he is going to move has been shown to be a relatively poor predictor of actual mobility" (Goering, 1978, p. 76). This leaves the definition of tipping point as the threshold when whites actually do move out. All of the whites in a neighbourhood do not move out in concert on a given day. The tipping point of a neighbourhood occurs as the result of the individual decisions of the myriad residents. The tolerance of the remaining whites for black neighbours is not likely to be the same in each case. Schelling, (1972, p. 83) notes: "People may tip and neighbourhoods may tip, but there is no simple translation from the behavior of the individual to the behavior of the system."

Neighbourhoods do tip but the tipping point has proved to be an elusive concept. The phenomenon of white flight is exacerbated by an 'unravelling' or positive feedback process. As whites who are less tolerant of blacks leave newly invaded neighbourhoods they are replaced by blacks. These new blacks increase the black proportion which then exceeds the tolerance threshold of slightly more tolerant whites. They then move out and are in turn replaced by blacks. This is a chain reaction.

There is some 'critical mass' of blacks which starts the chain reaction of black entry and white exit. (The following is based on Schelling, 1972, pp. 162-173). Four hypothetical frequency distributions of personal tipping points or tolerance levels for varying black population proportions are presented on fig. 5.1. Populations A and B have normal frequency distributions with different standard deviations, population C has a negatively skewed distribution, and population D is bimodal. These frequency distributions have been transformed into cumulative distributions in fig. 5.2.

The tipping point for any population occurs when the cumulative

Population A - - - - -
 Population B
 Population C - · - · -
 Population D ————

75

RELATIVE
 FREQUENCY

Fig. 5.1 Hypothetical
 Frequency Distribution
 of Tipping Points

PERCENTAGE
 CUMULATIVE
 FREQUENCY

Fig. 5.2 Hypothetical
 Cumulative Frequency
 Distribution of Tipping
 Points

frequency line crosses the 45 degree line in fig. 5.2. The diagonal is significant as a threshold between stable and unstable states. "It is a watershed - a point of black entry prior to which white residency is a self sustaining condition and beyond which white departure is a self sustaining process" (Schelling, 1972, p. 166). Consider point x in fig. 5.2 as representative of the stable zone. The tolerance level is 70 percent white (up to 30 percent black) but the actual racial proportion of the population is 80 percent white (only 20 percent of the residents are black). We must assume that the less tolerant whites have already moved out. All of the remaining whites are more than satisfied with the existing racial mix. In fact they will tolerate an additional 10 percent black without anyone wanting to move because of dissatisfaction with black entrants. Point y represents an unstable racial situation. The tolerance level is 80 percent white but the actual white proportion of the population is only 40 percent. This means that a great many people are dissatisfied with the racial mixture and consequently are planning to leave. Point z is on the brink. Fifty percent of the population is white and 50 percent of the original population would tolerate up to 50 percent black. If for any reason one more white family moves out and is replaced by blacks the area will become unstable and a chain reaction of white flight and black entry has begun. Only in the unlikely case of population D, with a bimodal distribution of tolerance levels, can the chain reaction be arrested.

The tipping point concept has been widely criticized on essentially three grounds:

- 1) Many neighbourhoods undergoing racial transition do not tip - the rate of racial change remains relatively constant,
- 2) The black proportion of the population is not the only factor affecting the decision of whites to move,
- 3) The tipping point is not a consistent proportion, estimates have ranged from 10 to 30 percent black.

Levine (1968, pp. 477-479) has developed a model to reconcile the criticisms against the tipping point concept with the observed fact that some neighbourhoods do tip. The rate of racial change can be modelled as a continuous or discontinuous process by employing a simple differential equation (Levine, 1968, p. 478).

$$\frac{dR}{dt} = k_0 + k_1 R \quad (5.1.1)$$

where; $\frac{dR}{dt}$ is the rate of change of the proportion of non white residents with respect to time
 k_0 is a parameter independent of the racial proportion
 k_1 is a "parameter sensitive to the prejudicial reactions elicited by the actual occupancy of the area by non whites,"

R is the proportion of non white residents.

If we assume that $k_1 = 0$, which is to say that the rate of change of racial proportion is not influenced by the racial proportion at a point in time, the neighbourhood will not tip. The solution of (5.1) in this case is a straight line.

$$R = k_0 t \quad (5.1.2)$$

This will produce a graph such as fig. 5.3 which represents the linear change in the racial composition of Russel Woods, a Detroit residential neighbourhood. If the parameter, k_1 , is operative and there is a negative response to increasing proportions of blacks in the neighbourhood the solution of (5.1) becomes,

$$R(t) = \left[R(0) + \frac{k_0}{k_1} \right] e^{k_1 t} - \frac{k_0}{k_1} \quad (5.1.3)$$

This leads to exponential rates of black to white transition such as the curve illustrated in fig. 5.4 for a census tract in Syracuse, New York.

Fig. 5.3 Racial transition in Russel Woods , Detroit

Source: Derived from data in Wolf, 1963, p. 219

Fig. 5.4 Racial transition in a Syracuse census tract

Source: Levine, 1968, p.473

This scheme provides for both the discontinuous and constant rates of racial transition observed empirically. Both k_0 and k_1 would be extraordinarily difficult to measure. The assumption that k_1 could equal zero is dubious in light of the prevailing prejudiced attitude of white Americans towards black neighbours (fig. 5.5). It is awkward for a non-mathematician to visualize differential equations and tedious to derive the curve for given k parameters. It is impossible to specify what point on an exponential curve represents the tipping point.

5.2 A CUSP CATASTROPHE MODEL OF RACIAL TRANSITION

Catastrophe theory has been devised to deal with sudden changes in potential systems. It appears to be a natural candidate for a new model of racial transition. The model serves two purposes. First, it is an original illustration of how catastrophe theory may be appropriately employed. Secondly, the model has been built to rationalize a mass of contradictory evidence and explanation of how racial transition occurs. The model is based on the three dimensional cusp catastrophe. Racial composition in American cities is fundamentally bimodal; areas are either black or white. Fig. 5.6 demonstrates that the racial proportion in Chicago census tracts has a bimodal distribution. The butterfly catastrophe with its 'pocket of compromise' and the swallowtail catastrophe have more than two modes and consequently are inappropriate. The fold catastrophe has only one control variable and would suffer from the same fault as the tipping point concept, racial transition cannot be explained in terms of any single factor. This leaves the cusp catastrophe as the only realistic alternative.

5.2.1 The Hallmarks of Catastrophe

Cusp catastrophe models have the characteristics of bimodality, hysteresis, divergence, sudden transition and inaccessible behavior. These traits are shared by neighbourhoods experiencing racial transition. The vast majority of American neighbourhoods are segregated despite the existence of integration policies and open occupancy

Fig. 5.5 American Attitudes Toward Residential Segregation

Statement: White people have a right to keep blacks out of their neighbourhoods if they want to and blacks should respect that right.

Note: These data were compiled from random samples of approximately 1500 Americans

Source: Taylor et al, 1978, p. 43

Fig. 5.6 Histogram of Racial Composition in Chicago

Note: 419 census tract values were suppressed to protect the privacy of extremely small minorities. This suppression accounts for the anomalously small cohorts at the extremes of the histogram.

Source: Adams, 1976, p. 140

legislation. Neighbourhoods are either predominantly white or predominantly black: they are bimodal with respect to race. Schwaub and March (1980, p. 391) found that Cleveland had a bimodal distribution of racial characteristics. Census tracts tended to be either under 10 percent black or over 85 percent black. The tipping point, when observed, is the threshold of a rapid acceleration in the rate of black entry and white flight. This is the sudden transition characteristic. The sudden transition obviously takes some time, measurable at least in months. The catastrophe model is based on the assumption that the jump from one regime to another is virtually instantaneous. To this extent a catastrophe model will be unrealistic. This is typical of most applications of catastrophe theory to the indeterministic sciences. Isnard and Zeeman (1977, p. 308) have justified this inconsistency on the ground that the sudden transition is so much faster than the slow rate of change of the structural parameters that it is convenient and practical to regard the behavioral jump as instantaneous.

The move out of a neighbourhood involves a variety of real and intangible costs unlike the decision not to move in. It is suggested that the same white who waited until an area was 30 percent black before moving out might not move in if the area was more than 10 percent black. The costs of moving might outweigh his racial preferences until the proportion of 30 percent was reached. At anything over 10 percent black the prospective in-mover might speculate that the proportion might soon increase to unattractive levels while other areas are more racially homogenous and therefore more appealing. The difference between 10 and 30 percent is the magnitude of the hysteresis effect. Two white neighbourhoods which are similar by all socioeconomic measures may in time diverge. One may become substantially black, while the other remains white. Schelling (1971, p. 146) suggested, "In some cases small incentives, almost imperceptible differentials, can lead to strikingly polarized results." The inaccessible behavior corresponds to the situation in the unstable

zone identified by Schelling which is graphed on fig. 5.2. The instability results in a rapid racial transition which only becomes self-limiting when the population becomes almost entirely black. "Truly integrated areas are very rare and probably none can be considered stable" (Morrill, 1977, p. 37).

5.2.2 Model Specifications

The model requires that some simplifying assumptions be made. A hypothetical neighbourhood is named for convenience, 'Woodstock'. Woodstock is a clearly bounded lower middle class neighbourhood in a typical American city. Housing is mainly small, modest, and owner occupied. The population of 5,000 is fairly stable and all stages of the family life cycle are represented. The residents share a uniform middle-American culture and value system. There are black neighbourhoods fairly close by. It is assumed that there are only two ethnic/racial groups; whites and blacks. Woodstock's population is initially all white. These assumptions will be relaxed as necessary for the model building.

A system modelled by the cusp catastrophe must behave according to a potential function, there must be a dynamic force which causes the system's behavior to cling to the behavior manifold. Woodstock is a typical stable American neighbourhood. It is stable simply because its residents are content to live there. The decision to move in, move out, or stay put is based on maximizing the net satisfaction associated with the bundle of housing goods. Factors such as the cost, location, size, and style of the physical structure and the qualities of the neighbourhood are weighed against other housing alternatives and the costs of moving to arrive at the net satisfaction. The satisfaction gained from the acquisition and consumption of a bundle of housing goods is utility. The negative of utility is disutility. Thus Woodstock may be described as a potential system, its component residents always act to minimize disutility.

Two controlling factors determine the nature of the neighbourhood's behavior. The attractiveness of the area is the normal factor and is symbolized by the variable 'a'. It is a measure of how attractive the area is perceived to be by the white population in relation to other housing alternatives. The splitting factor, 'b' represents the magnitude of antiblack prejudice by whites. It is assumed that there is a fairly close degree of consensus on the matter of prejudice among Woodstock's white residents.

5.2 The racial composition of the population measured in percentage white is represented by the behavioral variable, 'x'. This racial proportion is a function of the attractiveness and prejudice factors. The disutility function with respect to racial composition is

$$D(x) = \frac{1}{4} x^4 - \frac{1}{2} a x^2 - b x \quad (5.2.1)$$

in the canonical form of the cusp catastrophe. It is understood that residents will make decisions to move in or out based on the criterion of minimizing disutility. The set of minima of the disutility function will determine the system's behavior. This may be calculated by setting the first derivative of function (5.2.1) equal to zero.

$$\frac{dD}{dx} \frac{1}{4} x^4 - \frac{1}{2} a x^2 - b x = 0 \quad (5.2.2)$$

$$x^3 - ax - b = 0 \quad (5.2.3)$$

The behavior of the system measured as percentage white is graphed as a function of attractiveness and prejudice in fig. 5.7 which is the elementary cusp catastrophe. A number of behavior trajectories may be considered.

5.2.3 Model Operation

At point A the population is entirely white; they are very prejudiced and find their neighbourhood attractive. As time passes

other housing alternatives become available, attractiveness declines, whites move out, and some are replaced by blacks. The addition of blacks make the area increasingly unattractive (since the whites are prejudiced) and more whites will leave. At point B the area has reached the tipping point. If the area becomes the slightest bit less attractive a positive feedback process of white flight and black entry will be initiated. Racial composition will plummet to point C and then slowly decline to point D. The pathway from point E to point F proceeds slowly since the white population is unprejudiced and black in-movers do not exacerbate the rate of white departure.

As time passes the neighbourhood may become more attractive to whites. This may be due to locational advantages or perhaps the charm of older houses. If the white in-movers were unprejudiced this might take the form of a move back to point E from point F. The percentage white would increase slowly, possibly in a random pattern. This is observed as 'white painting', the rehabilitation of older homes by upper income groups. If the whites were highly prejudiced the area would have to become much more attractive before they moved in. The pathway leading from point D to point G and then suddenly jumping to point H might be representative of the razing of blocks of formerly black occupied houses and their replacement by large luxury apartment buildings.

The occurrence of catastrophe at points B - C and anastrophe at points G - H illustrates the hysteresis effect. Fig. 5.8 demonstrates the divergence of two initially very similar neighbourhoods at point X. Slight differentials in the values of the control variables result in their having dissimilar racial compositions when points Y and Z are reached. The inaccessible behavior region is represented by the shaded middle sheet of the behavior surface. Behavior represented on this surface is so unstable that it is impossible for any length of time. This would be manifest in a large proportion of the white population being very dissatisfied with their housing because of the magnitude of

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OF / DE

Fig. 5.8 - Divergence of Neighbourhood Racial Composition

Initially the two neighbourhoods at point *x* are nearly identical in terms of attractiveness and prejudice. Almost imperceptible differences between them result in radically different racial compositions at a later point in time; *y* remains mostly white and *z* has a greater proportion of blacks. This is known as divergence.

prejudice and unattractiveness.

It has been suggested that time is a neglected factor in this model (J.J. Gordon, Bowling Green State University, personal communication, 1980). If black entry proceeds slowly enough while white residents are still in the majority the white residents might grow accustomed to having black neighbours and tipping might be arrested. This assumes that whites will begin to feel positively about blacks as they are constantly exposed to them as fellow residents. If this assumption is correct it is tantamount to a decrease in prejudice which is within the parametric confines of the model. If a slow transition results in a decrease in prejudice the model will demonstrate a continuous transition. In fact, however, there is no evidence to suggest that an initially slow rate of black entry will reduce prejudice and hence result in a slow overall transition. The initially slow entry of blacks may actually increase white feelings of prejudice as the whites' worst fears are realized. Unethical real estate agents may orchestrate a 'blockbusting' plan which has the effect of increasing prejudice and decreasing attractiveness. "In fictional treatments the first blacks are planted and instructed to make noise, to see that their children are dirty, to keep a broken down truck parked in front of the house, ...and even attract the attention of the police" (Schelling, 1972, p. 177).

J.T. Darden (Michigan State University, personal communication, 1980) has suggested that the activity of real estate agents is an important factor in racial transition and perhaps should be incorporated in the model. The diverse activities of third parties in the real estate transaction may be represented by their final effects on the factors of prejudice and attractiveness. For many years American real estate salesmen saw themselves as the custodians of American residential heterogeneity. Realtors would 'steer' blacks into neighbourhoods which already had black residents and would generate the expectation among remaining whites that their area would soon become all black; both of these activities have the effect of decreasing the attractiveness factor. Blockbusting techniques such as those

discussed above would have the effect of increasing prejudice. If realtors could cause a significant decline in attractiveness and an increase in prejudice they could trigger a catastrophic change in racial composition. This would result in greater selling activity at a profit to the real estate broker.

The proximity between white areas and the ghetto has often been cited as a factor in racial transition. Distance lay at the heart of Morrill's study (1965) of racial diffusion; Pryor (1971) found that the best predictor of racial turnover was the distance to the nearest black neighbourhood; and Steinnes (1977) concluded that once an area had tipped the rate of black entry was correlated with proximity. Distance is by no means always a factor influencing the probability of transition. Morrill (1973, p. 32) cited the occurrence of very resistant areas of lower, middle, and upper class whites which were adjacent to black areas yet did not undergo transition.

The success of the cusp catastrophe model pivots on the selection of prejudice as a splitting factor. Prejudice is an attitude and very difficult to predict. It has been pointed out that the most staunch integrationist may flee when his neighbourhood is threatened (Greeley and Sheatsley, 1971, p. 19). Tipping may occur because whites are trying to escape the drop in property values expected when an area becomes black rather than escaping from blacks themselves. There is, in fact, no evidence to support this expectation. Alderich (1975, pp. 335-336) cites seven case studies demonstrating that real estate values do not decrease and may in fact increase during black entry. Nevertheless unprejudiced whites may leave because of the mistaken impression that housing values will decrease. In this case the whites would be leaving because the forecast decrease in housing values constitutes a decrease in the attractiveness of the neighbourhood. If the move out of the neighbourhood by unprejudiced whites was rapid enough to appear catastrophic it would contradict the model. In these circumstances the model is unrealistic because it depicts

catastrophic racial change as impossible if whites are sufficiently unprejudiced towards blacks. Data are not available to measure how often tipping may occur due to purely economic factors.

It has been shown that many of the factors said to influence the likelihood and rate of racial transition are reducible to the control factors of attractiveness and prejudice. These factors would prove difficult to quantify and measure. The problem of empirical quantification pales into insignificance when the refractory task of calibrating and operationalizing the cusp model is considered.

Inherent in this model is the hitherto unstated assumption that white urban Americans control the housing market. Blacks usually have much less bargaining power and have no choice but to take housing that whites don't want. Blacks cannot move in until whites have decided to move out, the higher average incomes of whites gives them many more housing options and greater mobility. This situation is slowly changing. Income levels of black Americans are rising more rapidly than white incomes (U.S. Bureau of the Census, 1979) and in some cases blacks are able to compete for housing on a level footing with whites. A series of studies conducted since 1942 have shown that white attitudes towards residential segregation are becoming progressively more liberal (Greeley and Sheatsley, 1971; Taylor, Sheatsley, and Greeley, 1978). Notwithstanding this progress, residential segregation and its corollary racial prejudice are still powerful forces shaping the geography of American cities.

The cusp model described here may only prove useful as a conceptual perspective towards understanding racial transition and neighbourhood succession. The control factors are couched in terms so broad that it can only be of academic interest. In its favour is its internal consistency and the representation of a complex urban social process on a relatively simple graphic model. The characteristics of sudden transition, bimodality, hysteresis, divergence, and inaccessible behavior so often observed in reality are clearly shown. The conflicting observations and opinions on whether a tipping point exists are reconciled in one unified model.

CHAPTER 6

GEOGRAPHY AND CATASTROPHE THEORY

SUMMARY AND PROSPECTS

The philosophy of René Thom and the elementary catastrophe models outlined in chapters two and three provide new insights on the nature of change and form. One of Thom's objectives was to provide a theory of analogy. The elementary catastrophes are themselves a classified set of pre-pasted analogies which may be used to qualitatively describe many discontinuous behavior patterns. The graphic constructs of catastrophe theory are its best known landmarks but not necessarily the most significant. Woodcock and Davis (1978, p. 144) emphasize that the most essential concept of catastrophe theory is structural stability. In their view, the universality of this notion justifies its consideration as a paradigm. In this final chapter short answers to the questions posed in Chapter 1 will be attempted.

Nature is in constant flux as one form succeeds another. Most forms are the physical manifestations of structurally stable systems. A structurally stable system can withstand small amounts of structural change without showing any change in form. Forms arising from diverse processes show striking similarities. These similarities are not coincidental but often defy explanation. The exact form, location, and development of a residential neighbourhood is beyond precise quantitative prediction yet their qualitative aspects recur in a subjectively familiar pattern.

Structurally stable forms are recurrent and identifiable because they are resilient enough to persist in spite of environmental per-

turbation and changes in structure. There is some dynamic property built into structurally stable systems which enables them to counter-vail exogenous disturbances by adjusting behavior according to some minimizing criterion. The form of structurally stable systems is not eternal. There is some limit to the amount of structural change that a system can withstand. If the system's structure changes sufficiently a threshold is reached beyond which it becomes unstable and must undergo a catastrophic change in form.

The number of essentially different forms which occur in nature is remarkably limited yet a classification of all possible forms is beyond the capabilities of science. It is possible to classify the ways in which many systems undergo a change in form. Under certain mathematical conditions forms definable by up to four control variables can change in only seven essentially different ways. The seven elementary catastrophes are the most complicated arrangement of stable equilibrium points representing all of the behavioral possibilities for potential systems under most circumstances.

Zeeman's model of canine aggression was summarized in Chapter 3. Fig. 3.3 demonstrated that different values of fear and rage could result in neutral, attacking, or retreating behavior. The dog could also be hit by a truck and killed, a possibility not described by the model. Woodstock, the residential neighbourhood modelled in Chapter 5 could be zoned by municipal ordinance for blacks only forcing all of the whites to immediately move out. There is no profit in applying catastrophe models to catastrophic occurrences which are obviously the result of violent environmental perturbation or sudden changes in their structural parameters.

The example of stream flooding diagrammed in fig. 2.6 was described as a catastrophe event. In such a physical system it is easy to conceive how structural instability occurring at the bankfull stage ~~must~~ culminate in a sudden change of form. It is less easy to conceive

how social phenomena cross thresholds and are forced to undergo changes in form. Indeed the thresholds separating stable regimes of human behavior are usually much less clear cut than those in the natural sciences. This is due in part to the abstractness of human control factors and in part to the indeterminacy of social behavior. Clearly the applications of catastrophe theory to problems in human geography are imperfect as many are quick to point out. This is not surprising: nearly every new piece of hardware or analytical technique from the digital computer to factor analysis has been taken up with great enthusiasm only to be misapplied. The great advantage of the use of catastrophe theory in the social sciences is that it is a qualitative method of analysis and hence well suited to processes which are notoriously difficult to quantify.

Applications of catastrophe theory have been classified in three categories: 'true applications' where the theory has led to correct prediction; 'illustrations' in which the theory has confirmed results obtained using other methods; and 'invocations' where the identification of control parameters and system potential remains tentative (Michael Berry, cited by Woodcock and Davis, 1978, p. 144). The four applications summarized in Chapter 4 and the model proposed in Chapter 5 fall in the latter category. These invocations of catastrophe theory provide a new language for describing and understanding particular phenomena with discontinuous behavior characteristics. Woodcock and Davis suggest,

Someday it may be as natural to speak of a "cusp situation" or a "butterfly compromise" as it is today to speak of the "point of diminishing returns" or of a "quantum jump".
(1978, p. 145)

Catastrophe models are now established as an appropriate approach to some types of geographical problems. Unfortunately they are often misapplied and seldom understood. At the very least the catastrophe vocabulary has improved the precision of descriptions of change in

potential systems. At most catastrophe theory may represent a new paradigm for geography albeit in the very early stages of development. The rate of its development will depend on how easily it can be adapted to true geographical applications rather than simply metaphorical invocation.

Catastrophe theory has been touted as a new holistic approach to the exploration of systemic singularities. It represents a course away from the piecemeal disaggregation of problems which obscure their essential form. In an attempt to escape the blinders of analytical reductionism catastrophe theory seems to lead right back to a reductionist critique. Kendall has suggested that it may be

...chasing a will-o'-the wisp to suppose
that social behavior is determined by
least something or other....
The pathway of knowledge is littered
with the wreckage of premature
generalization.

(Kendall quoted in Bourne, 1971, p. 213)

Catastrophe modelling demands that the social system under investigation be defined by a potential function minimizing 'something or other'. Catastrophe theory does attempt a universal generalization of the succession of form and many have argued that it is premature. This thesis has shown how catastrophe theory has opened new epistemological avenues for geography. It is too early to tell what form the will-o'-the wisp will ultimately take.

GLOSSARY

Analogy - similarity in function.

Anastrophe - discontinuous upward jump in the behavior of a system.

Behavior manifold - synonymous with behavior surface, the set of all points in the potential system where the behavior represents a stable minimum.

Bicontinuous - having the quality of a continuous transformation in both directions between two point sets.

Bifurcation set - the line enclosing the duplex set in the cusp region of the control space where behavior is bimodal and there are three sheets in the behavior surface.

Bijjective - having the quality of a one to one correspondence between two point sets.

Bimodal - a trait of a system which enables it to occupy two different behavior states for one set of control variables. (C)

Butterfly Catastrophe - an elementary cuspid catastrophe having one behavior dimension and four control dimensions.

Catastrophe - discontinuous downward jump in the behavior of a system in response to continuously changing control parameters.

Catastrophe set - the union of the duplex set and the bifurcation set, the cusp outline and interior. (A)

Catastrophe theory - a method of mathematical analysis that enables the modelling of discontinuous change.

Conflicting factor - one of two factors in a cusp catastrophe having an equal influence on system behavior.

Convention - a rule governing the behavior of a system.

N.B. Letters following definitions refer to a source key at the end of the Glossary

Cusp Catastrophe - an elementary cuspidal catastrophe having one behavior dimension and two control dimensions.

Cuspidal catastrophe - a catastrophe model with one behavior variable.

Delay convention - a rule in system behavior which dictates that the system will evolve continuously for as long as possible before being forced to jump to new behavior regime. (A)

Divergence - a trait of systems which results in a small initial perturbation causing a large difference in its final state. (C)

Duplex set - the cusp interior which has bimodal behavior. (A)

Dynamic - potential seeking behavior that keeps a system at an equilibrium point on a behavior surface.

Elliptic umbilic catastrophe - an elementary umbilic catastrophe model having two behavior dimensions and three control dimensions.

Fold catastrophe - an elementary cuspidal catastrophe model having one behavior dimension and one control dimension.

Homeomorphic - the quality of two point sets which are precisely identical in all respects; the transformation between the two point sets is both bijective and bicontinuous.

Homeorhesis - the systemic process of maintaining a constant rate of change or flow.

Homeostasis - the systemic process of returning to a steady state after a perturbation.

Homology - having a similarity of structure and evolutionary origin.

Hyperbolic umbilic catastrophe - an elementary umbilic catastrophe model having two behavior dimensions and three control dimensions.

Hysteresis - transition of a system at different values depending on the direction of system change. (C)

Inaccessible zone - in a cusp catastrophe the middle sheet representing an unstable energy maximum which is an impossible state for a system to be in.

Manifold - a multi-dimensional surface.

Maxwell's convention - a rule in system behavior which dictates that the system will always move to the global minimum even though there may be some closer local minimum.

Metastable - the condition of a potential system at a local but not absolute minimum; it will remain stable under infinitesimal perturbations but will become unstable under a finite perturbation.

Morphogenesis - the creation and transformation of the form of a system.

Normal factor - a control dimension running perpendicular to a splitting factor.

Parabolic umbilic catastrophe - an elementary catastrophe model having two behavioral dimensions and four control dimensions.

Pleat - a term used informally to refer to that part of the behavior surface which is directly over the cusp of a cusp catastrophe.

Potential function - a function which always returns to a stable equilibrium state after a perturbation.

Saccadic - having a jumping or jerking motion.

Simplex set - all areas on the behavior surface for which there is only one set of behavior variables - the unimodal portion of the behavior surface. (A)

Singularity - the point on a curve where the direction or quality of curvature changes. (B)

Splitting factor - a control dimension which bisects the cusp axis in a cusp catastrophe.

Star catastrophe - a cuspid catastrophe model with one behavior variable and six control dimensions.

Structure - a framework giving strength and determining form which controls the rate of change of variables characterizing the system.

Sudden transition - the jumping behavior of a system undergoing a catastrophic change from one stable minimum to another.

Swallowtail catastrophe - an elementary cuspidal catastrophe having one behavior dimension and three control dimensions.

Topology - a branch of mathematics concerned with the properties of surfaces in many dimensions. (C)

Umbilic catastrophe - a catastrophe model with two behavior variables.

Unimodal - having only one behavior state for a given set of control variables.

Wigwam catastrophe - a cuspid catastrophe model having one behavior variable and five control dimensions.

SOURCE KEY

- (A) Amson (1975)
- (B) Woodcock and Davis (1978)
- (C) Zeeman (1976)

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