Examining the Fourth Dimension:
Using Compression as a Tool to View Higher Dimensions

by

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Abstract

As humans, we experience three dimensions with our two eyes: height, width and breadth. But are there more? The purpose of this thesis is to explore the potential of higher dimensions. This includes learning how to make higher dimensional models. But ultimately, I will investigate how fourth-dimensional theory can be used as a starting point for creating architecture.

While the fourth dimension is too exotic to convey, I will illustrate how we can use compression to see traces of the fourth dimension in our three dimensional world. I will use these fourth dimensional footprints to design a pavilion.

I will demonstrate how higher dimensional theory offers a new way to think about space. Indeed, I believe it is architecture which must rise to the challenge of contributing knowledge to higher dimensional theory.

Acknowledgements

To Yvan, for your enthusiasm and dedication, it is a delight to have had you as an advisor; you are a great mentor and friend. To Rachel, for making me think critically, for showing me that the use of words and punctuation can be exciting - and most of all, for being my biggest fan. To Matt, for your personal interest, remarkable editing, and for being my toughest critic. And to Ralph and Henriette, I would not be where I am, or who I am without you. Thank you.

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Introduction

This thesis is my attempt to illustrate the fourth dimension, and to examine its possible contributions to architecture. As humans we experience three dimensions every day - but are there more? If so, we should investigate them as they no doubt impact our world, even if they exist beyond our senses.

I will first introduce you to the fourth dimension with a summary of its historical treatment. After providing that context, I will outline more current approaches to understanding higher dimensions. I will investigate why they are so difficult to comprehend and illustrate.

I will then explore a new way to view the fourth dimension, through a process of observing the footprints of three-dimensional space as it is compressed to higher dimensions. The process relies heavily on trends followed by the three known dimensions, and the use of analogies.

Once this process is established I will investigate the results of the compression exercise and identify the products, namely: the footprints, the edge, the crossover and the breach. I will also identify how the notion of "point of view" plays a critical role in perceiving higher dimensions. Further, I will discuss how these compressions have the potential to create new space, which occurs when a compression forces its condensed interior through, so the interior space travels into a higher dimension.

After analyzing the components of compression, discussing the importance of "point of view" and revealing the potential of creating new space, I will investigate these conventions architecturally through a process of miniature architectural vignettes.

Finally, I will use these vignette investigations to inform a cohesive architectural installation in the form of a pavilion.

1.0 Introduction to the Fourth Dimension

Edwin Abbott, a pioneer in fourth dimensional understanding, wrote the book *Flatland.* The text showed us how the fictitious two dimensional world in which his characters lived could be applied to understand properties of our third dimensional world. By developing a make-believe setting without a third dimension, Abbott gave us a framework to appreciate the importance of learning as much as we can about space - even if we begin to search beyond the boundaries of our senses.

1.1 So what is the Fourth Dimension?

Let us begin with the dimensions we are capable of experiencing. First, there is zero dimensions. An infinitely small space, represented by a point (Fig. 01). Moving the point in one direction we get a line. This is the first dimension (Fig. 02). Taking the line and pulling it into another direction results in the second dimension - represented here by a square (Fig. 03). Taking the square and extruding it up into the third dimension results in a cube (Fig 04). The next step, the difficult step, is to then extrude the cube. The result is the hypercube. There are multiple models of the hypercube, as it is a difficult image to grasp (Fig. 05 and 06). One of the major issues of illustrating the hypercube in a diagram is that the representation is in a dimension so far removed from its actual dimension.

fig. 01

fig. 02

fig. 03

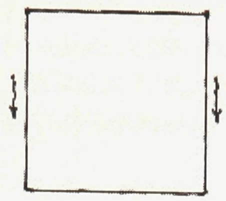
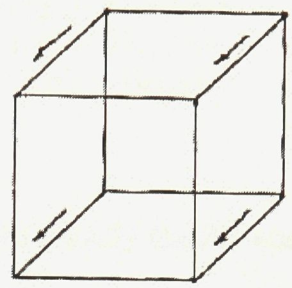


fig. 04



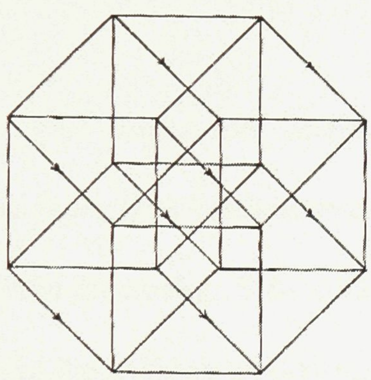


fig. 05

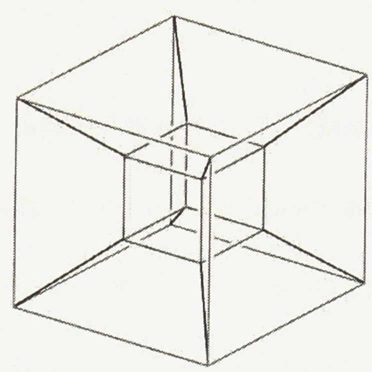


fig. 06

Another important and common 4-D shape is the hypersphere. Despite the hypersphere's importance conceptually, there are few diagrams of this complex shape. Indeed, most scientists believe the space of our universe is in fact the hypersurface of a very large hypersphere.[1]

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fig. 1-5 Rudy Rucker, The Fourth Dimension (Boston: Houghton Mifflin Company, 1984) 32.
fig. 6 Thomas Banchoff, Beyond the third dimension (New York: Scientific American Library, 1990) 115.
[1] Rucker 31.

So why is the fourth dimension so difficult to grasp? Tomas Banchoff, one of the world's most respected fourth dimensional experts, suggests it is due to the limits of our senses. Banchoff says"

> Sight reveals two dimensions directly, the breadth and the height of the object beheld, while the third dimension, the distance of the object, is estimated by means of the muscular turning of the eyes to focus them on it. No sense calls for a fourth direction, perpendicular to the other three; in fact, all of a man's experience leaves him satisfied with three dimensions.[2]

Thus, as Banchoff explains, our sensory limitations clearly hinder our ability to understand the fourth dimension. If our senses do not allow us to see or feel the fourth dimension, it will be a difficult concept to grasp. However, there are a few devices aid in the comprehension of this higher dimension.

One useful tool comes from Edwin Abbott's *Flatland*. While this book was written more than one hundred years ago, it is still the backbone of all books discussing the fourth dimension. Like Abbott, then, we can begin to understand the fourth dimension by using the analogy of lower dimensions looking to higher ones.

Abbott's *Flatland* is the story of a square that lives in a two dimensional world. The square is visited by a three dimensional cube which educates and proves the third dimension's existence to the square (Fig. 07). The square also visits a single dimensional world to better understand how lower dimensions cannot perceive higher ones. By doing so, the square begins to understand its own situation in Flatland.

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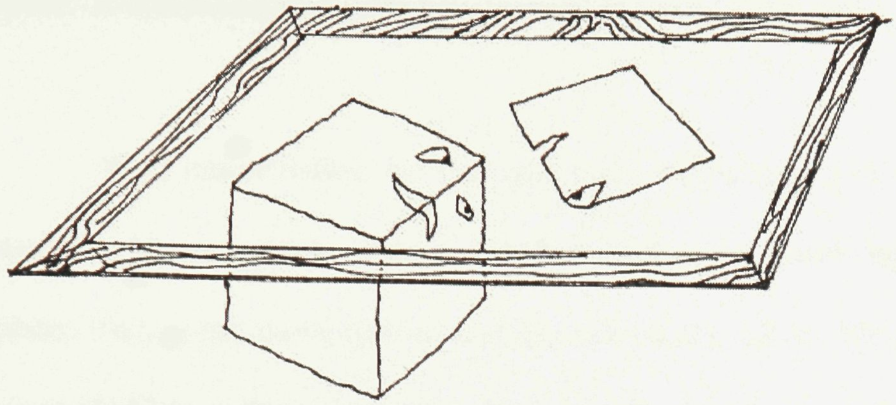
[2] Henry Manning, The Fourth Dimension, simply explained (New York: Dover Publications Inc., 1910) 71.
fig. 7 Rucker 41.



A Cube visits A Square.

fig. 07

It is important to note in this image above that the square does not have the capacity to see the entire cube. The square is only able to see the cube as a square intersecting the square's two-dimensional world.

Abbott's creation of Flatland allows us to put ourselves in the shoes of the square trying to envision higher dimensions. We can apply what devices and difficulties the square would come across in imagining higher dimensions. One of Abbott's most important lessons is that just because we cannot sense a higher dimension does not mean it does not exist.

Another strong tool for understanding the fourth dimension is the use of mathematical models. Higher dimensional geometry, also known as *n*-dimensional geometry, has been used in mathematics since the first half of the nineteenth century. *n*-

dimensional geometry allows for any number of dimensions within Euclidean space or n-space. *n*-dimensional models have proved extremely useful in mathematics.

With mathematics, we can add extra dimensions without the accountability for modeling the procedure. When a mathematician is constructing a timeline along the "x" plane, there is the movement in one direction along a line. He can illustrate any position along that line with one number. If the mathematician is to construct a graph along the "x" and "y" planes, he can then illustrate any position in two dimensions. By adding the "z" plane, he can provide the coordinates in three dimensional space - or, for illustrative purposes, anywhere within or on a cube's surface.[3]

But why stop at three? If coordinates are simply groups of numbers, what happens when we add another number? If adding additional coordinates helps to solve complex mathematical problems, should we not do it? Henry Manning, a professor at Brown University and a pioneer in fourth dimensional philosophy, claims we do not have to believe in the tools we use to arrive at our answers; the important thing is that we arrive at them. Manning says:

> The chemist is permitted to base his investigations on the atomic theory without caring much whether such a thing as an atom exists or not. The physicist may talk of the flow of heat in a rod without believing that heat is a substance or that it flows. The mathematician asked to be allowed to extend his notion of space, and to include in it aggregates of more than three dimensions, even if this lead to physical absurdities.[4]

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[3] Rucker 160.
[4] Manning 161.

Manning supports using facilities that help us understand our world. However, I think his quick dismissal of whether we need to evaluate the validity of the tools we use is incorrect. This is because I am convinced the fourth dimension exists, and I see it as more than simply a mathematical model. In the following section, I will try to support the fourth dimension's spatial presence and justify the need to further understand it, beyond its mathematical utility.

1.2 Usefulness of the Fourth Dimension

Fourth dimensional theory offers answers to problems that we have in our three-dimensional world. For example, it could help us understand movement which cannot be explained in our three dimensions. By analogy I will demonstrate how movement can prove the existence of a higher dimension, and how these movements can be understood if we look at them from higher dimensions.

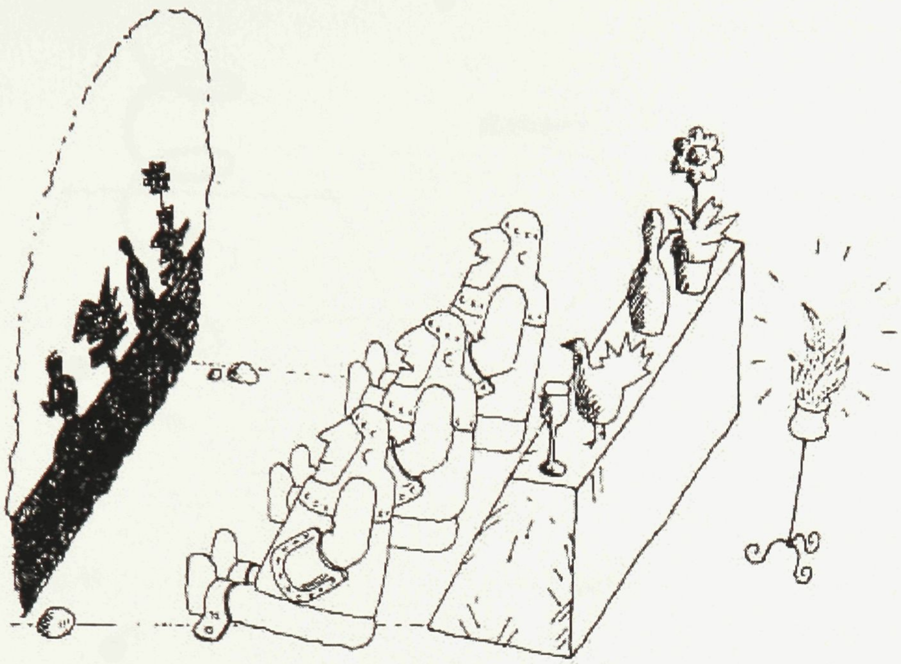

fig. 08

Plato's cave is an early example of venturing to lower dimensions, in order to understand higher ones. In Plato's cave, there is a race of men bound up and facing the cave's wall (Fig.08) with a ramp behind them. Further back, there is a fire creating shadows. Since the men can not see anything except the wall, they believed it to be their true reality. Their conversations with each other reflected off the wall, and as people walked behind them they incorporated them into their world. With this analogy there did exist a higher dimension, only their senses can not see it. We can apply this analogy to our own life. What if we are simply bound in a condition preventing us from sensing the fourth dimension? To say it does not exist solely because we cannot sense it is not sufficiently persuasive.

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fig. 08 Rucker 8.

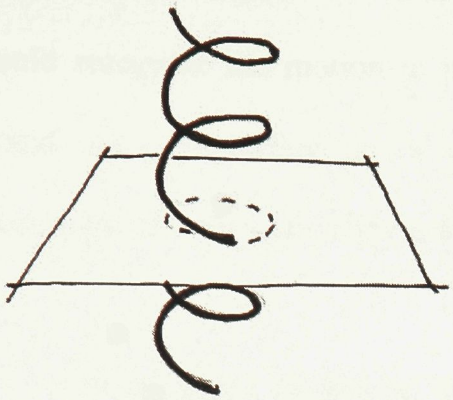

fig. 09

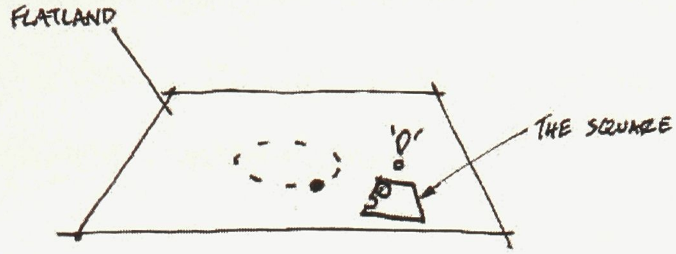

fig. 10

The philosopher Howard Hinton, contributed heavily to the theory surrounding the fourth dimension. One of Howard Hinton's famous models is his *Intersection of a Spiral and a Plane* (Fig. 09)[6]. Hinton speaks of this event,

> If now we suppose a consciousness connected with the film in such way that the intersection of the spiral with the film gives rise to the conscious experience, we see that we shall have in the film a point moving in a circle, conscious of its motion, knowing nothing of that real spiral the record of the successive intersections of which by the film is the motion of the point.

From this model we can extract many 'elements' that are explained from a two dimensional world applied to a higher dimension. To help understand this model, let us assume the film is Abbott's *Flatland* and let us have Abbott's square witness this event.

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[6] Howard Hinton, The Fourth Dimension (New York: Swan Sonnenschein and Co. Ltd., 1904) 25.
fig. 09, Hinton 25.

First, the square would see a point spinning in orbit (Fig. 10). Then the square would recognize the motion to be consistent. The point, as perceived by the square, would not be speeding up or slowing down. The orbit would be consistent and predictable but what drove the point in its orbit would be a mystery.

Furthermore, the square would likely believe he could stop the orbiting point, as the square would seem to appear much larger and embody more mass. However, if he tried to stop the point by standing in front of it, the square would surely be cut. This is because the mass of the force orbiting the point exists in the third dimension. In reality, the spiral is much larger than the square. After the point collided with the square, the point would continue its consistent orbit unfazed by the square's attempt to stop it.

All these events would seem very confusing to the square. However, we exist in the same dimension as the spring. As such, we can see the whole picture of what is happening. This analogy illustrates how an observed, but uneasily explained movement in one dimension can be caused by a higher dimensional act passing through a lower one. The theory of higher dimensions explaining movements in lower ones has proven useful in our three-dimensional world.

In the same way that the orbiting point, as perceived from flatland, had strength that existed in a higher dimension, many believe there are forces that act within our three dimensional world which contain their strength from higher dimensions. Gravity is one of these forces.

Gravity is a force we accept but when one examines the physics behind gravity it seems too weak a force. In fact gravity is the weakest force known. All mass is attracted to all other mass. This level of attraction is represented by the following equation:

$$F = \frac{G \cdot m_1 \cdot m_2}{r^2}$$

fig. 11

In the equation:

F = the force (in newtons)

G = the universal gravitational constant

m1 = mass one

m2 = mass two

r = the distance between the two masses center of gravities

In the equation, it is understood that the greater the values of m1 and/or m2 the greater the force. The strength of the attraction is a function of how much mass there is embodied in the two masses. To witness this force of attraction, you could place a paperclip on the ground. Then hold a 10 pound bowling ball over the paper clip. The paperclip stays on the ground because the mass of the earth exceeds that of the bowling ball. Meaning, the force of the earth pulling on the paperclip exceeds the force of the bowling ball pulling on the paperclip.

If, however, you were to take a small fridge magnet and hover it over the paperclip the paperclip would leave the earth and join the magnet. In this process the force of magnetism acting from the small magnet overcomes the force of gravity acting from the entire mass of the earth. This act reveals the weakness of gravity. Lisa Randal explains how it is possible that magnetism may be bound to our three-dimensional space but gravity extends to higher dimensions as a theory to explain gravity's perplexing weakness. Randall refers to other spaces beyond our three dimensional realm as 'branes' and the empty space between these branes she calls 'bulk'. Randall goes on to explain:

> Because branes could trap most particles and forces, the universe we live in could conceivably be housed on a three-dimensional brane, floating in an extra-dimensional sea. Gravity would extend into the extra dimensions, but stars, planets, people, and everything else that we sense could be confined to a three-dimensional brane. We would then be living on a brane. A brane might be our habitat. The concept of a braneworld is based on this assumption (see Fig. 12)[7]

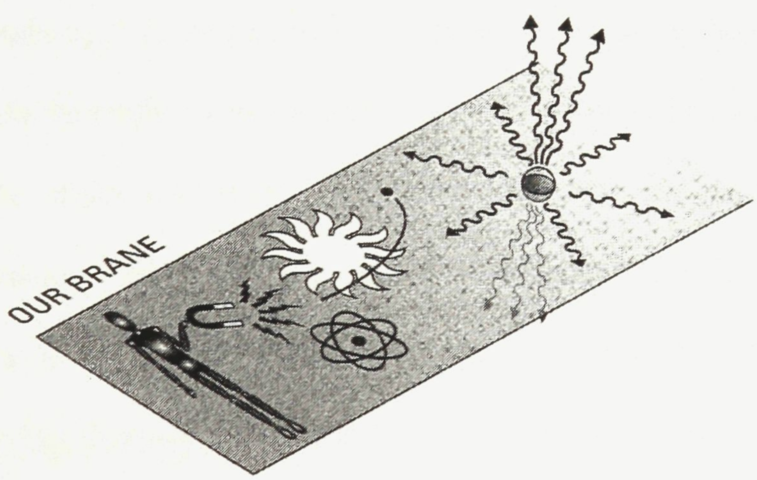

fig. 12

[7] Lisa Randall, Warped Passages (New York: Harper Collins Publishers, 2005) 59.

Although Randall doesn't specifically mention magnetism among the forces bound to our brane. She does show it in her diagram. (Fig. 12) Magnetism is a stronger force than gravity from the perception of our brane. Randall's theory suggests that since gravity can travel beyond our dimension it's force becomes diluted but because magnitism is bound to our brane it remains concentrated and stronger force than gravity. Thus higher dimensional theory offers a solution to the problems that exist in our world.

It is important to understand the concept of the fourth dimension to further understand our universe. Higher dimensions have been an integral part of mathematics since the late nineteenth century[8] and applying fourth dimensional properties has proven useful on the subatomic level as well.

Holographic Theory has been gaining popularity in science as it has begun to answer some of the questions surrounding molecular movement. Holographic Theory is the claim that any volume of space can be represented by just using the boundary of the space. In other words, an entire room's events can be recorded by just using the walls of that room. At the molecular level, particles, atoms, protons and electrons are unpredictable. Quantum Mechanics is the probability of the location of these components at given times. Holographic Theory in tandem with fourth dimensional theory demonstrates that irregular movements of gluon chains can be understood and predictable. Gluons are elementary particles which are indirectly responsible for the binding of protons and neutrons together in atomic nuclei. The article states, "It turns out

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fig. 12 Randall Lisa 59.
[8] Linda Henderson, "Italian Futurism and 'The Forth Dimension'" Art Journal, Vol. 41, No.4, Futurism Winter 1981: 317.

that one type of gluon chain behaves in the four-dimensional spacetime as the graviton, the fundamental quantum particle of gravity." [9] Understanding gluon chains, which exist on the boundary of black holes, is essential for our understanding of black holes. By applying fourth dimensional theory to Quantum Mechanics it is begin to assist the predictability of the movement of these sub-atomic particles, thus helping us to better understand our universe.

This discovery also substantiates the potential movement of particles in the fourth dimension. If there is movement occurring in the fourth dimension, then there must be space in which this movement occurs. Henry Lefebvre, believes each body is space, and has space.[10] The circle's movements on the film, created by Hinton's model (Fig. 10), and the gluon properties on the edge of a black hole are better explained if we evaluate their movements to the result of a 'body' in the fourth dimension moving. Once we establish there is movement of a body in the fourth dimension, it is a logical conclusion that there is also space in a higher dimension.

Architects are generally expected to take full consideration of the surrounding environment, in order to produce truly integrated buildings. Based on the preceding conclusions, it appears both that space exists in the fourth dimension, and that this space interacts with the three, lower dimensions. As such, I believe architects should give the fourth dimension a closer look, in order to best understand and build in our three

[9] Juan Maldacen, "The Illusion of Gravity" Scientific American Nov. 2005: 62
[10] Henri Lefebvre, The Production of Space (Oxford: Blackwell, 1974) 170.
fig. 11 Hinton 25.

dimensions. As I see it, it should be accommodated in the same manner as the other three dimensions.

1.3 Travel and Perception with Regards to the Fourth Dimension

The fourth dimension offers solutions to help explain our world, but it also offers other opportunities. For example, it has created potential for a new system in which to move through our world. In addition to travel, there would be a heightened level of perception.

Traveling in the fourth dimension has been a hot topic in science fiction texts since theories of the dimension began. The notions of traveling great distances in space or traveling through time by use of the fourth dimension have been used extensively, from H.G. Wells' 1895 novel *The Time Machine*, to the 1985 novel *Contact*, made into a motion picture by Carl Sagan.

Unless man reaches a state of complete unrestricted freedom of movement he will always pursue more degrees of freedom. Rudy Rucker, a PhD in mathematics, believes man will always be looking for more freedom. He says, "It could be that mankind's perennial dream of flight is a hunger for more dimensions, for more degrees of freedom."[11]

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[11] Rucker 46.

Tomas Banchoff, a geometer professor at Brown University and author of the book “Beyond the third dimension”, supports man’s desire for more agility. He refers to the engineering term “degrees of freedom,” using an analogy of a woman driving a car. He says:

> A driver finds herself in a tunnel under Baltimore harbor crawling behind a large truck. ‘Do Not Change Lanes,’ she is admonished. She is stuck in one dimension, effectively kept in line, blocked by the vehicle in front of her and the one in back. Once outside the tunnel, she is again able to move in two dimensions, because she now has an additional ‘degree of freedom,’ allowing her to change lanes to the right or left. But a bit later she finds all the lanes blocked by bridge construction in Havre de Grace. Trucks and cars hem her in all sides. She wishes that she could escape into the third dimension, where a police helicopter hovers unconstrained by the traffic on the roadway.[12]

Through this example, Banchoff suggests an additional dimension would offer people, like the hypothetical woman stuck in traffic, more flexibility and efficiency in conducting daily activities.

Travel through the fourth dimension has its advantages. By using the analogy of a two-dimensional model, Rudy Rucker, author of the book “The Fourth Dimension” shows how traversing into a higher dimension has the ability to travel through space via a higher dimensions to another space even if those spaces had solid boundaries. In the image to the lower left (fig. 13) we have a two-dimensional situation where there is the ability to exit the square and enter the circle through the use of an additional dimension.

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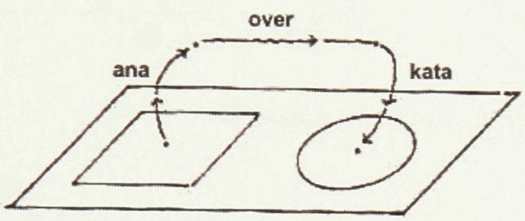
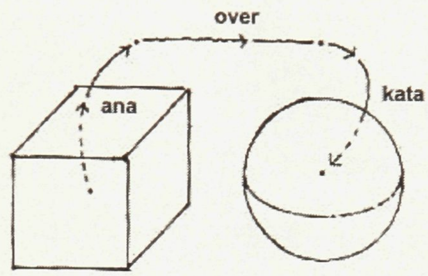
[12] Banchoff 5.

The image to the lower right (Fig. 14) shows how one would move from the centre of a cube to the centre of the sphere by use of the fourth dimension. The diagrams further illustrate how travel through a higher dimension offers great potential for efficiency.





Connecting interior points without cutting through the boundary.

fig. 13 **fig. 14**

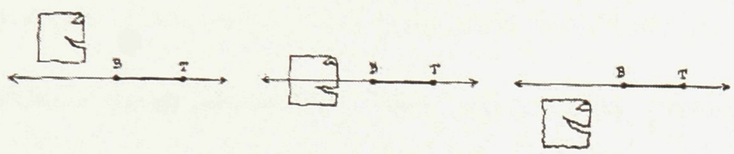
Rucker explains this method of higher-dimensional travel:

> It is a curious feature of 4-D space that we can connect two points in the interiors of two solid 3-D objects without piercing these objects' surfaces. The trick is to use *ana over kata* motions to get in and out of the solid 3-D objects. If you're inside a cubical room and you move *ana* out of it, it's as if you had suddenly dematerialized. You don't go through the walls or the floor or the ceiling, you move over in the *ana* direction to a part of the 4-D space where the room doesn't exist at all.[13]

The ability to travel in and out of the fourth dimension would offer improved freedom and efficiency. It raises questions about how we would create spaces for this new type of travel. If everyone could step in and out of the fourth dimension, how would we protect our valuables? Would we have any use for doors, stairs and elevators

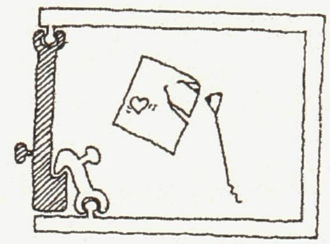
[13] Rucker 28.
fig. 16 - 17 Rucker 15-16.

anymore? No doubt the third dimensional world would need to be designed differently to accommodate this new mode of moving through space.



A Square moves through Lineland.

fig. 15



A Square and his wife in a locked room.

fig. 16

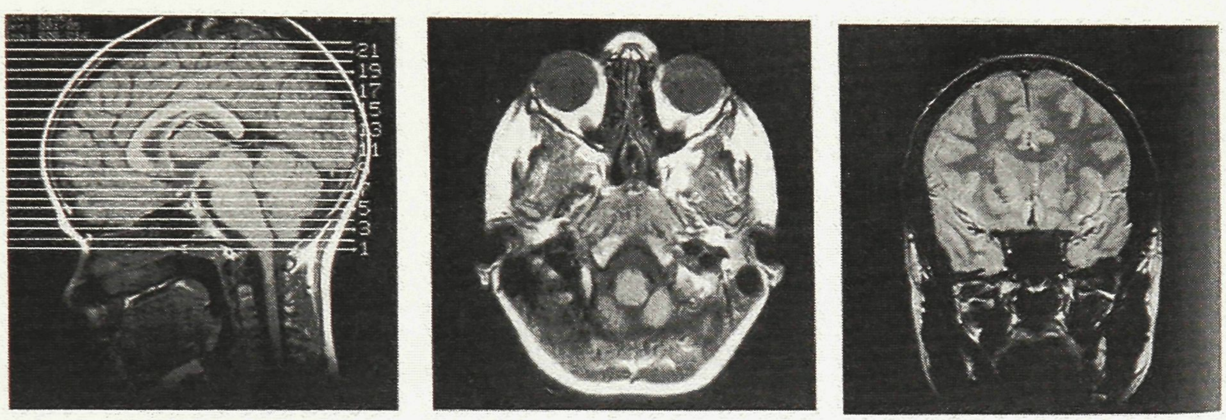


fig. 17 CAT scan of a human head

Not only would travel be different once we mastered the use of the fourth dimension, but how we communicate our world would change as well. Again through analogy, it would seem that if one were in the fourth dimension looking back, you would be able to see all the sides and the inside at once. In the same way that the square, from Abbott's Flatland, perceives the full length of the creatures of Lineland[14] (Fig. 15), and the same way in which we would have the ability to look down to see the square and his

[14] Lineland is a place the Edwin Abbot's fictitious square from the novel "Flatland" traveled to. In Lineland all inhabitants have only one dimension and are bound to a single dimensional world.

wife's insides and outsides in a locked room (Fig. 16), a creature from the fourth dimension would be able to see all of our insides and outsides at once. Perhaps this perception would be similar to an MRI image (Fig. 17) taken of an entire body. Not only could they see all of our insides at once but they would also have the ability to go in (*kata*), to touch and disturb what they wished as well. This offers limitless potential for freedom and efficiency. There are positive implications with regards to surveying, restoration and a more complete examination of our world.

To invert this notion, it would then be possible to create 4-D models to represent a 3-D object as a whole. To design 3-D space using 4-D models would be extremely efficient. A technique for design that is gaining popularity is Computer Assisted Design. This is modeling of the subject, be it a building, chair or automobile, but creating it in a 3D virtual space using a computer. The successes of virtual computer models are that they can be viewed from any angle,, they can be viewed as translucent, and they can be altered in a *kata* fashion without disturbing the integrity of the outside. Such would be the case with a physical model.

Fourth dimensional understanding will alter the way in which we see and move. As perception and movement are married to space, we must therefore rethink the way in which space is affected by the fourth dimension. And it is architecture that must rise to the challenge in creating a world that includes higher dimensions. The intention of this first chapter was to introduce the subject of the forth dimension and how it has begun to demonstrate it's usefulness. In the next chapter we will look at the role that time plays with higher dimensional theory.

2.0 Time and the Fourth Dimension

In this essay, I will consider the notion of time, as it pertains to studying the fourth dimension. Time has been addressed in different ways by philosophers, physicists and artists. Despite their approaches, however, I do not believe time is the fourth spatial dimension. First, I will look at how physicists have considered "spacetime" as fourth-dimensional. I will address the illusion that time creates space, the cubist claim of representing the fourth dimension. I will expose the flaws in the futurist theory that space blurred by time can represent the fourth dimension. Finally, I will discuss how time does not follow trends set by the lower three dimensions. After analyzing the above points, I will explain how time is a useful tool to view the fourth dimension.

2.1 Spacetime as the Fourth Dimension

Spacetime has always been considered fourth dimensional. In physics, spacetime is a mathematical model that uses three coordinates of space and one of time to distinguish certain events from other events.[15] Time is the fourth of the four coordinates, in which spacetime records events. There is some flexibility with regards to the manner in which these coordinates are derived. For instance, if recording an event in Euclidian space – which is the space of solid geometry represented by the three axes x, y and z – the four coordinates would be width, length, height and time. However, if one were using global coordinates to map a spacetime event, the four coordinates would be longitude, latitude, height from sea level and standard time.[16]

As you can see from these examples alone, the way in which spacetime gathers its coordinates is not always consistent. Furthermore, when speaking of mapping events,

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[15] Jürgen, Time, Temporality, Now (Germany: Springer, 1997) 192.
[16] Ehlers 192.

there is no way to distinguish length from width. They are both planer geometries that depend on reference points which are interchangeable.

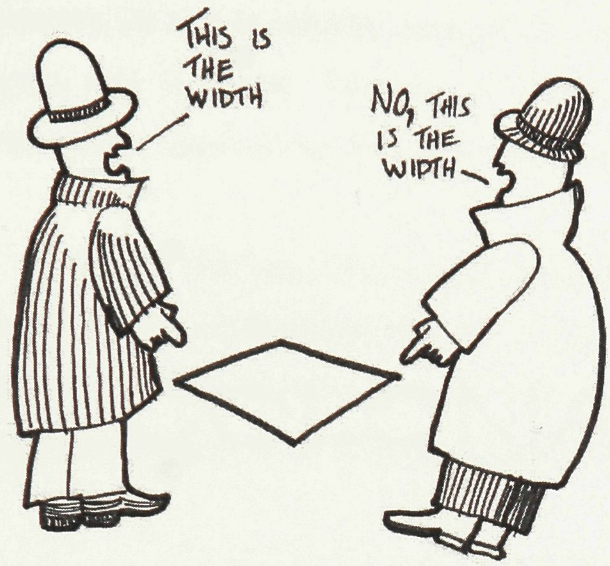

fig. 2.01 which way is width

2.2 How Time Creates Space

In *Spatial Architectonics*, author Henri Lefebvre makes the following argument:

> Time is distinguishable but not separable from space. The concentric rings of a tree trunk reveal the tree's age, just as a shell's spirals, with their 'marvelous' spatial concreteness, reveal the age of that shell's former occupant – this according to rules which only complicated mathematical operations can 'translate' into the language of abstraction.[17]

I agree with Lefebvre. I believe time is associated with space, that time can create, command and alter space. But I also believe the space shaped by time exists within our three-dimensional realm. Time is but a variable that affects space. It is not space itself.

[17] Lefebvre 175.

Time is not the next spatial dimension, but has the ability to affect Euclidian space. Our solar system is a good example of this phenomenon. The boundary of our solar system could be measured by its existence at any moment in time (fig. 2.02). However, its size is actually measured by its furthest planets' orbits (fig. 2.03). In this regard, time is spatial. The planets' predictable orbital paths are used to represent the boundary, or space occupied, within our solar system.

Notice that here, time is used to map the event, and the final geometry of our solar system is represented using Euclidian geometry - x, y and z coordinates. Thus, the notion of time is being represented with the conventions of three dimensions and time is used as a variable to arrive at the final three-dimensional shape.

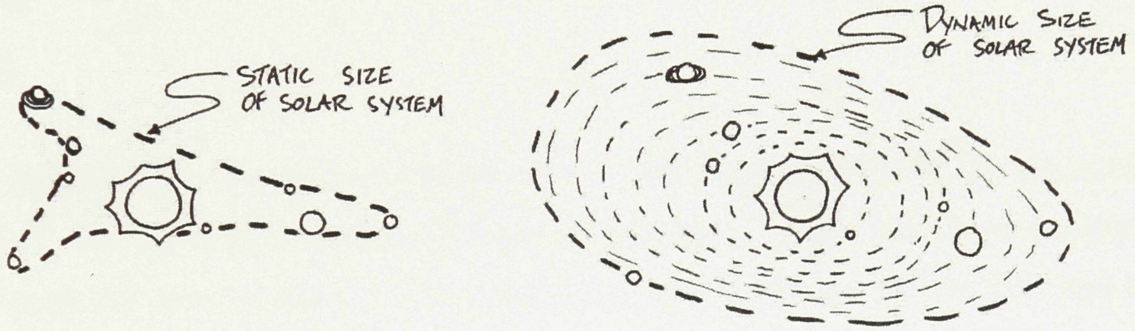

fig. 2.02 and fig. 2.03 our solar system seen differently

2.3 The Cubists' and Futurists' Representation of Time

Cubist and futurist artists have explored the fourth dimension differently. Cubism was an attempt to see the astral plane. To have 'astral sight' or to see the 'astral plane' was to see all sides of an object simultaneously.[18] By using multiple viewpoints and distorting their subjects, cubists would try to achieve astral sight.[19] Futurists, on the other hand, were trying to record events. They sought to capture action within a time frame. They achieved this by overlapping frames of action with added abstract blurring of the frames to capture the event. It is my position that both movements failed to reveal a higher *spatial* dimension; but rather achieved a representation of movement of the point of view for a three-dimensional condition on a two-dimensional surface.

Cubism

Picasso's *Les Demoiselles d'Avignon* (fig. 2.04) is considered to be the first cubist painting.[20] The painting is of five prostitutes in a bordello.[21] The face of the prostitute squatting in the lower-right of the painting is noticeably distorted. The face can be viewed almost as two halves. The left side of the face is facing the viewer, while the right side of the face is in profile looking to the left side. This act of simultaneously representing two points of view makes the face, and the painting, the first example of cubism. It is important to look at the motivations that drove cubism to better understand how it is misinterpreted as four dimensional.

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[18] Arthur I. Miller, Einstien and Picasso (New York: Basic Books, 2001) 104.
[19] Mark Antiliff, and Patricia Leighten, Cubism and Culture (London: Thames and Hudson, 2001) 10.
[20] J Nash, Cubism, Futurism and Constructivism (London: Thames and Hudson, 1974) 7.
[21] Miller 89.

fig. 2.04 Pablo Picasso, *Les Demoiselles d'Avignon*, 1907, painting, NY, MOMA

In the book *Einstein, Picasso Space, Time, and the Beauty That Causes Havoc*, Arthur I. Miller recounts a discussion between the amateur mathematician Maurice Princet, and Picasso. Princet provides Picasso with a detailed explanation of Henri Poincaré's idea about how to represent the fourth dimension. Poincaré suggests:

> "The images of external objects are painted on the retina, which is a plane of two dimensions; these are *perspectives*. But as eye and objects are moveable, we see in succession different perspectives of the same body taken from different points of view...Well, in the same way that we draw the perspective of a three-dimensional figure on a canvas of three (or two) dimensions, so we can draw that of a four dimensional figure from several different points of view. This is only a game for the geometer. Imagine that the different perspectives of one and the same object succeed one another."[22]

[22] The quote is based on a conversation documented in Miller's Book, Miller 105.

Miller considers Poincaré's theories about how to create fourth-dimensional space in the following quotation:

> Poincaré's fourth dimensional space is a spatial representation, and he suggests portraying it as different perspectives in succession on a canvas. This was a mistake. Picasso, with his visual genius, saw that the different perspectives should be shown in spatial simultaneity. Thus emerged *Demoiselles*.

Miller reveals, from the origin of cubism, how the fourth dimension is considered the ability to witness a multiplicity of views simultaneously. However, I do not share the cubists' position. I do not believe that being able to see a three-dimensional subject from all sides is an act of higher dimensional vision. In the image below there is a man looking at a beach ball. The fact that the other side of the beach ball is beyond his vision doesn't mean that it doesn't exist. In fact most can infer what the other side looks like without having seen it. The ability to see all sides of an object simultaneously within a three dimensional realm is not a fourth dimensional act.

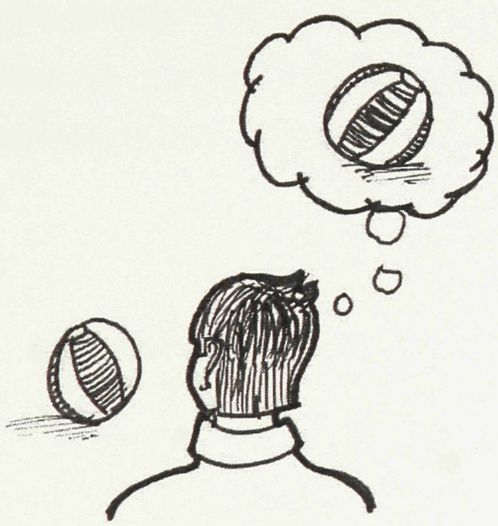


fig. 2.05 Imagining the other side of a beach ball

Around 1915, Picasso responded to a compliment by Ernest Ansermet, a Swiss composer. Ansermet had complimented Picasso's ability of moving from figurative to cubist styles as astonishing. In reply, Picasso immediately said, "But can't you see? The results are the same."[23]

With this remark Picasso was confessing that his images truly were not fourth dimensional because they were ultimately representing the same thing. A fragmented record of all sides is the same as seeing a full image of one side and inferring the other. Both are incomplete images, taken from the same three-dimensional space.

Futurism

Umberto Boccioni's *Unique Forms of Continuity in Space* (fig. 2.06) is an attempt to marry Euclidian geometry with time as space. The futurist statue, cast in bronze, demonstrates the three known dimensions: width, length and height. It also includes time. Time here is represented by the blurred image of a man striding forward. In the same way a photograph blurs when the subject is in motion, Boccioni's sculpture is a three-dimensional snapshot of a man in movement. In a way there are direct parallels to my earlier example of the solar system. The solar system's size is defined by its outermost boundaries, created from the orbital paths of the planets. Boccioni's sculpture is comprised of the boundaries of the body's movement in the time frame of a single stride.

[23] Miller 259.

fig. 2.06 Umberto Boccioni, *Unique Forms of Continuity in Space*, 1913, bronze, NY MOMA[24]

Granted, there is added abstraction in this example. The movement of the calves is bulked in the front and is tapered at the rear to indicate direction. There is also exaggerated thinning, or hollowing out, of areas that move less in the subject's stride. The sculpture is a stunning study of movement in time. However, the image of the fourth dimension fails because it relies on time as the fourth spatial dimension. And as I have already illustrated, time is a variable that can alter our known three dimensions. It is not, however, the next spatial dimension.

If we were to actually see the fourth dimension it would be so exotic that we would not know we were looking at it. This is because we would have nothing to which we could reference it. Although Cubism and Futurism achieved a three-dimensional

[24] Henderson 317.

representation of either movement of the point of view of the subject, or movement of the subject, they did not reveal the fourth spatial dimension.

2.4 Time Does Not Follow the Trends of the Previous Dimensions

Time as the fourth dimension does not follow the 'extrusion trend' common to the lower three dimensions. In standard Euclidian geometry, each dimension can be extruded to achieve the next dimension (see fig. 2.07 – 2.10). First, there are zero dimensions, represented by a point. Moving, or 'extruding' the point in one direction we get a line, then a square and then if we extrude again we get a cube - our final known dimension.[25]

The next step, which enters into the fourth dimension, is to then extrude the cube. The result – which is difficult to grasp – is the hypercube, for which we have multiple models (fig 2.11 – 2.12). One of the major issues of illustrating the hypercube in a diagram is that the representation is in dimensions so far removed from its actual number of dimensions. The images of the hypercubes below are drawn in two dimensions, while the object is to exist in four dimensions. It is the equivalent of trying to draw a three-dimensional image in one dimension.

[25] fig. 2.07 – 2.12 Rucker 32

fig. 2.07

fig. 2.08

fig. 2.09

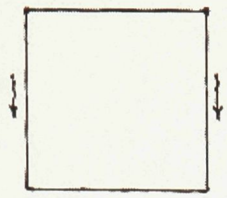
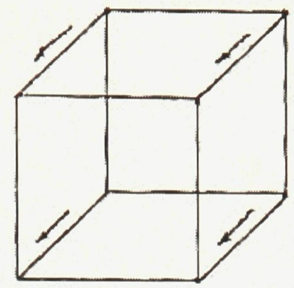


fig. 2.10



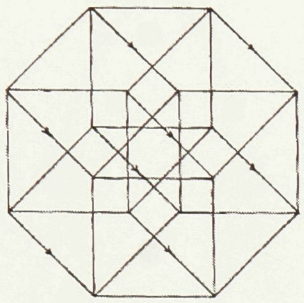


fig. 2.11

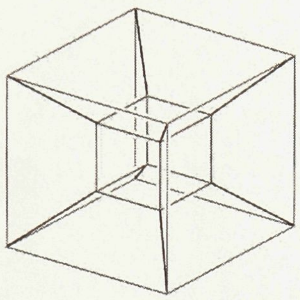


fig. 2.12

Most importantly, however, is the method from which each dimension is achieved. This pattern can also be represented mathematically (fig. 2.13). The diagram below illustrates the mathematical consistency of how each additional dimension's nodes can be reached by a multiple of two for each additional dimension.

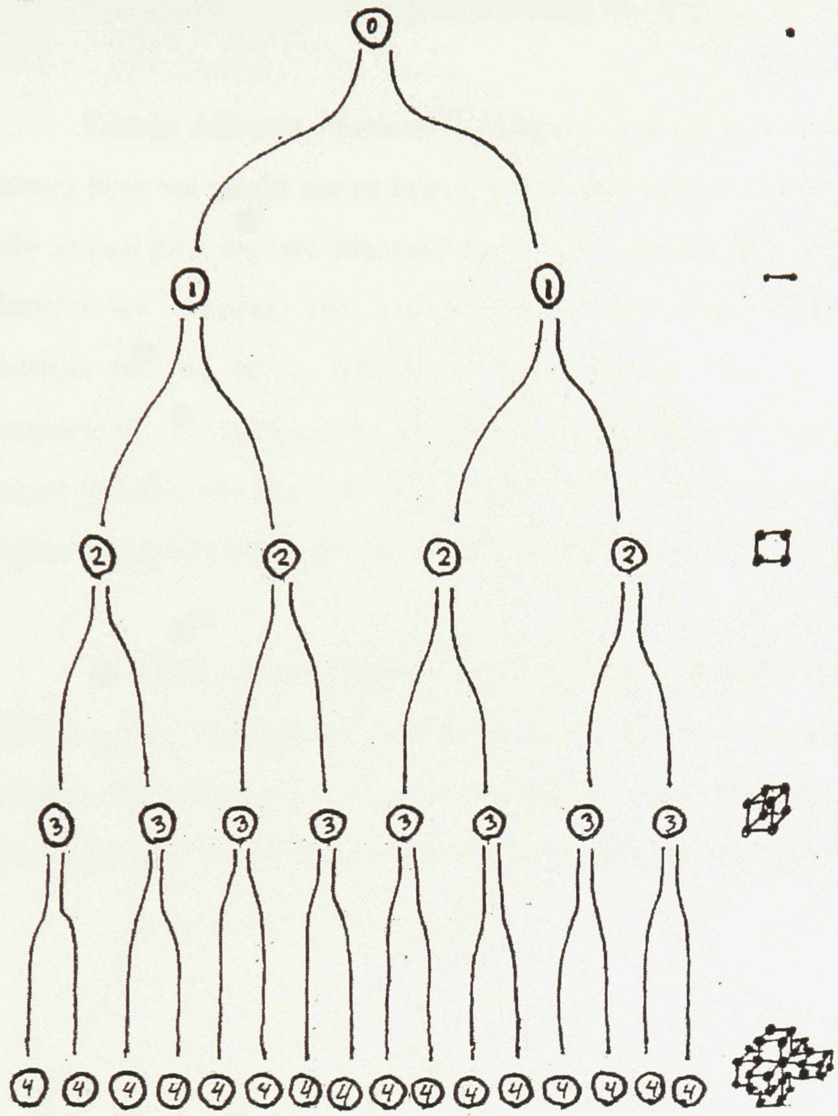


fig. 2.13 a mathematical relationship of dimensions

This brings me back to my concerns about labeling time as the fourth dimension. As you can see from the above models, time does not follow the same trends as the lower dimensions. Time moves forward in a predictable manner and cannot be 'extruded'. It is simply a parallel event to the known x, y and z dimensions that occurs in Euclidian space.

2.5 Time as a Tool to Help in Viewing the Fourth Dimension

Edwin Abbot's *Flatland*[26] embraces the idea of looking to lower dimensions to answer how we might see or begin to interpret higher spatial dimensions. The square can only experience his two dimensional world. Although he frequently brushes with three-dimensional entities, the square cannot see them. His senses are limited to two dimensions and he is forced to think beyond them to grasp the notion of higher dimensions. By analogy we are to project ourselves with similar limitations whereby our senses limit us to only 'see' three dimensions. Abbot's story forces us to ask if there are higher spatial dimensions that exist beyond our senses.

In *Flatland*, the square's first encounter with the third dimension comes when a sphere passes through his two-dimensional world. The square, however, is not able to witness the sphere as a whole. Instead, he sees the event in sections. He sees a very small dot grow into a large circle, then decrease in size and vanish from a dot.

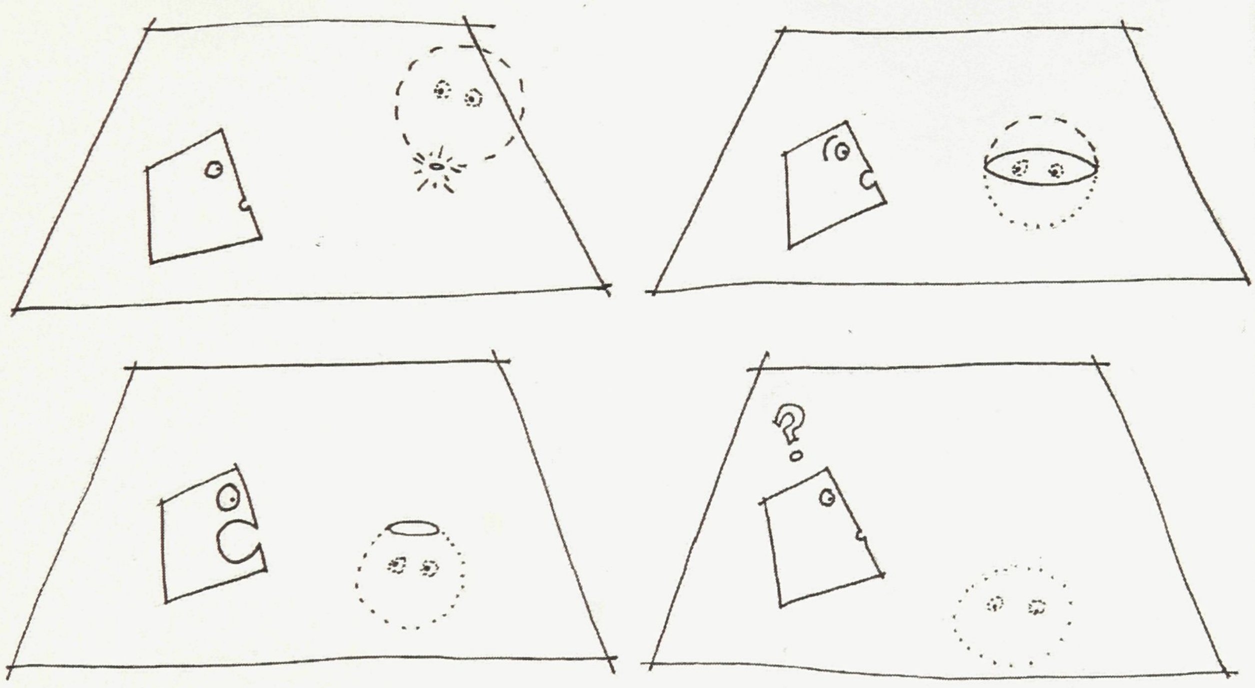


fig. 2.14 – 2.17 Square character witnesses a sphere pass through his two-dimensional world

[26] Edwin Abbot, Flatland (New York, Dover Publications Inc., 1952)

If the square wanted to communicate what he had seen, he only be able to use familiar conventions. He would need to use time as a tool to illustrate this higher-dimensional encounter. He could even speculate that the time segments could have been slices of a higher geometry and it was time that linked them together. This analogy thus raises the possibility that we can use time to witness three-dimensional sections of four-dimensional objects.

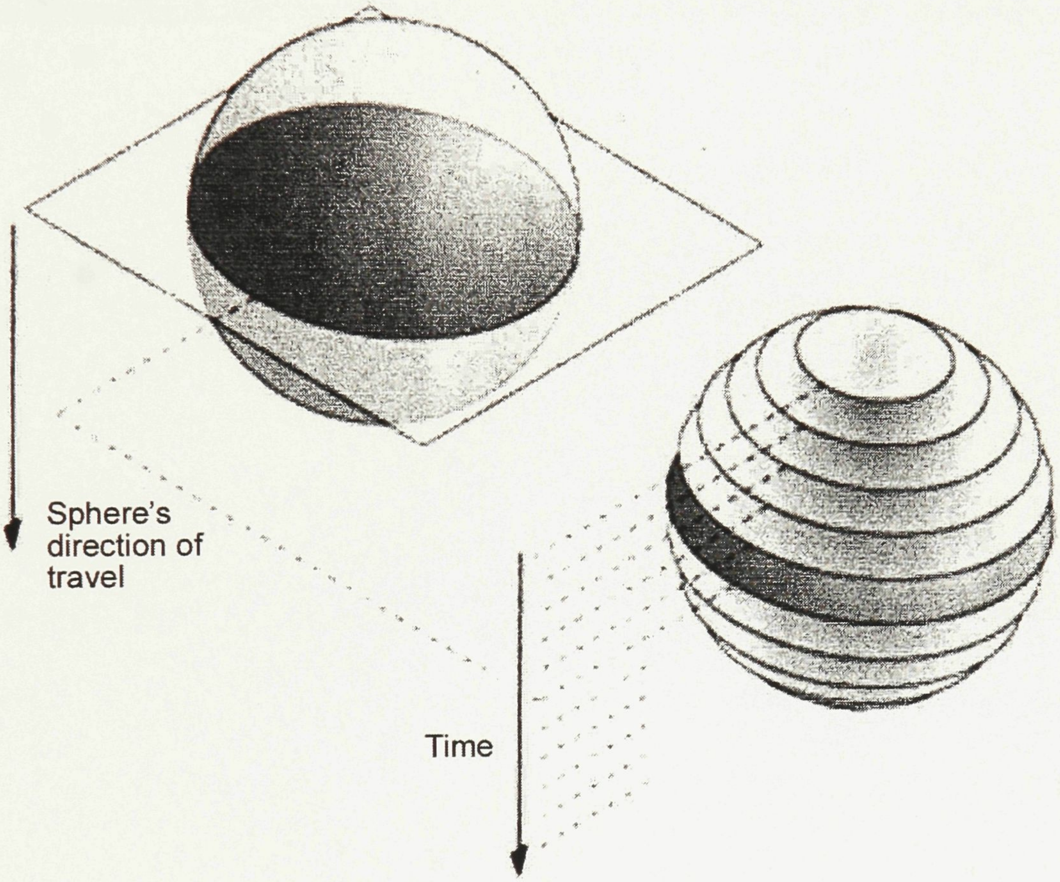


fig. 2.18 Sphere seen in cross-section over time[27]

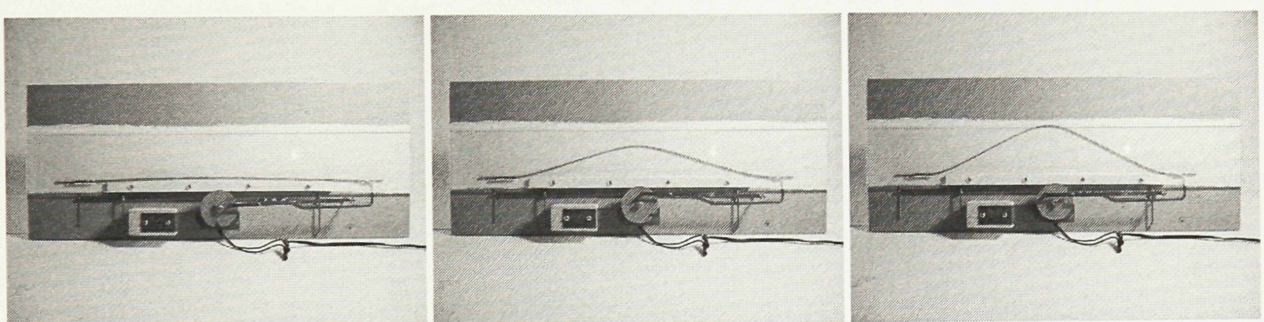
[27] Randall 19.

2.6 Conclusion

Time has been misinterpreted by many as the fourth dimension. Physicists use time as the fourth coordinate to track events. Cubists and futurists claimed to explore the fourth dimension in their work. Although time is present as space is recorded, and time itself can even shape space, time is not a spatial dimension. However, while time is not the fourth dimension, it certainly has a lot to offer as a tool to help view the fourth spatial dimension.

3.0 The Analogy of How to Achieve Higher Dimensions

The ultimate goal of this thesis is to model the fourth dimension. This is a difficult process. Even to successfully represent the fourth dimension, it would be so unlike anything we have seen before that we would not understand what we were looking at. However, it is possible to witness a three-dimensional object become a four dimensional object and, as the object breaks into the fourth dimension, to witness parts stay behind in the three dimensional realm.



a machine which compresses a line into the second dimension

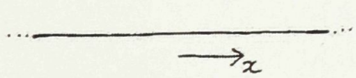


fig. 3.01

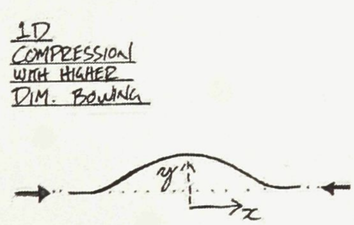


fig. 3.02

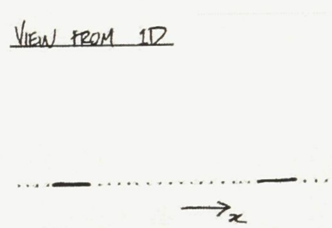


fig. 3.03

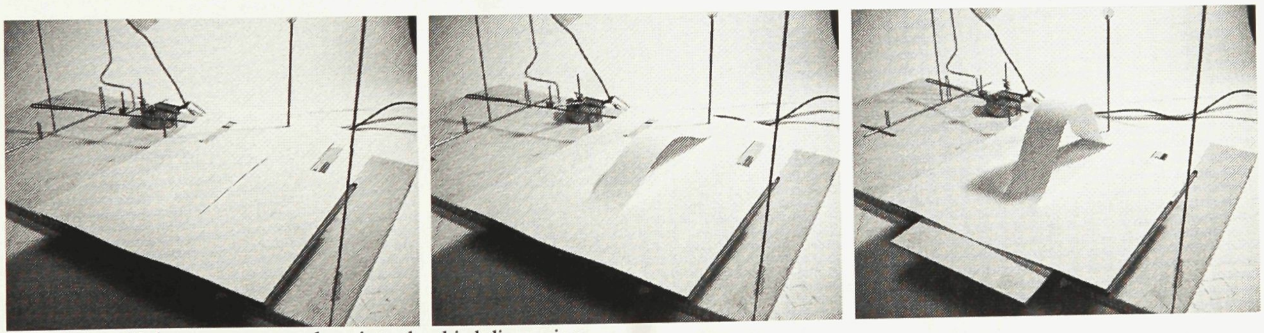
compression of a single dimension into the second dimension

Using time as a tool and following trends from lower dimensions, I have created a process that allows us to glimpse at four-dimensional geometry. The line seen in fig.

3.01 exists in one dimension, on one axis - the “x” axis. Compressing that line would cause it to escape into its higher dimension, it would bow and flex into the “y” axis, shown in fig. 3.02.

This simple act of compression allows the line to break into its higher dimension. The exercise can be performed by holding a piece of wire between your thumb and your index finger. A wire is a good example of a single dimension. Although the wire itself does contain a marginal thickness, it ultimately represents a line and can be used to model a single axis with one dimension. Now, if you squeeze your fingers together, the wire will bow. This act of bowing immediately requires an additional dimension to map. The compression of the wire, which existed solely in the “x” dimension, now represents a curve. As such, it requires both the “x” and “y” dimensions.

Although the wire has breached into the “y” axis, if you were unable to see anything beyond the “x” axis the compressed line would represent something like that in fig. 3.03. The once single line now appears as two shorter lines, separated by a void but still connected through a higher dimension.



a machine which compresses a plane into the third dimension

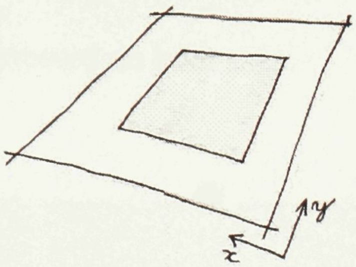


fig. 3.04

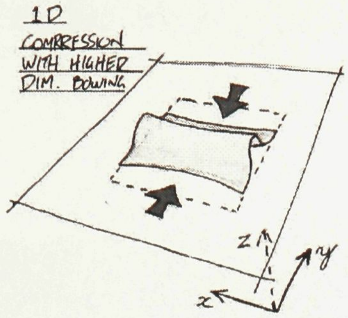


fig. 3.05

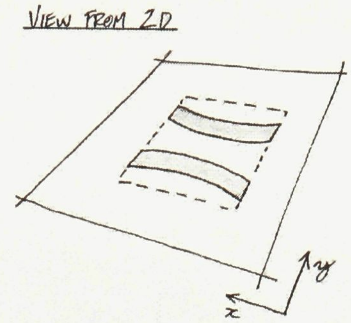


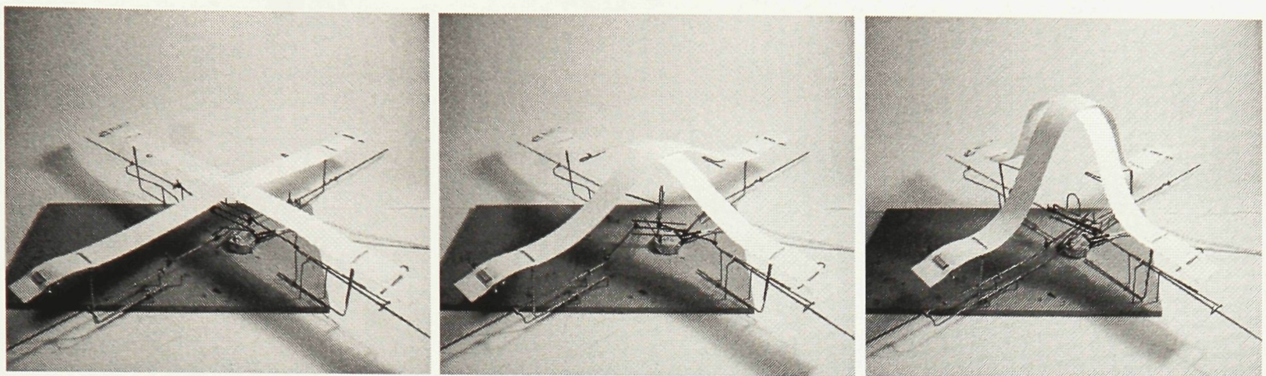
fig.3.06

single axis compression of a two-dimensional plane into the third dimension

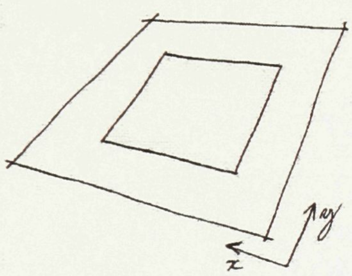
The same procedure can be applied to a two-dimensional figure breaking into the third dimension. Beginning with a square; a square exists in two dimensions on the "x" and "y" axis, seen in fig. 3.04. Compressing the square from opposite sides, the square flexes into the third dimension seen in fig. 3.05.

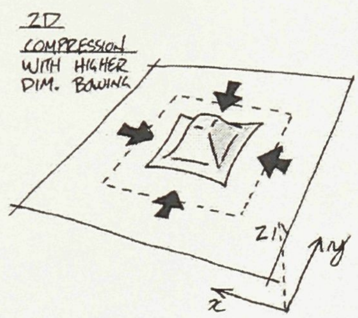
In the first exercise a piece of wire was used to imagine this concept. In this exercise a piece of paper can represent the two-dimensional plane. Compressing the paper would force the paper to bow. Again, the act of compression reveals a higher dimension; in this case, from the second into the third dimension, or the "z" axis. Again if we limit our observations of basic compressions from within the original two dimensions (the "x" and "y" axes), we would see two rectangles where there was once a square. This image is reflected in fig. 3.06.

Since our starting point was a square, by adding a dimension we are also adding to the possibilities of compression. With the addition of a dimension also adds to the possibilities of compression. In our earlier example the line could only be compressed along the "x" axis. The square however, can be compressed along the "x" axis, "y" axis or both, each creating different results. Figures 3.07 to 3.09 illustrate a square being compressed on both axes.



a machine which compresses a plane from two axes into the third dimension





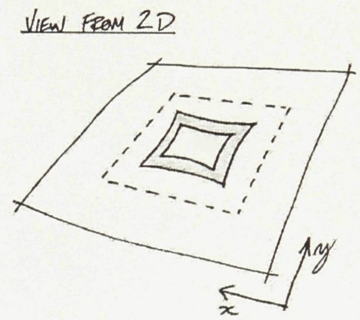


fig. 3.07 fig. 3.08 fig. 3.09

two axis compression of a two-dimensional plane into the third dimension

Further still, there is a myriad of compression combinations and the compressions may not share the same collision axis. Theoretically, there could be multiple compressions on one side with a single opposing force. The way in which the paper breaks into the third dimension may not be a single bow, but rather a wave. Even the 'flexibility' of the paper is a variable. Each of these possibilities would generate different results and thus different images from the restricted vision of the original two-dimension world. The images below show a handful of the different possibilities.

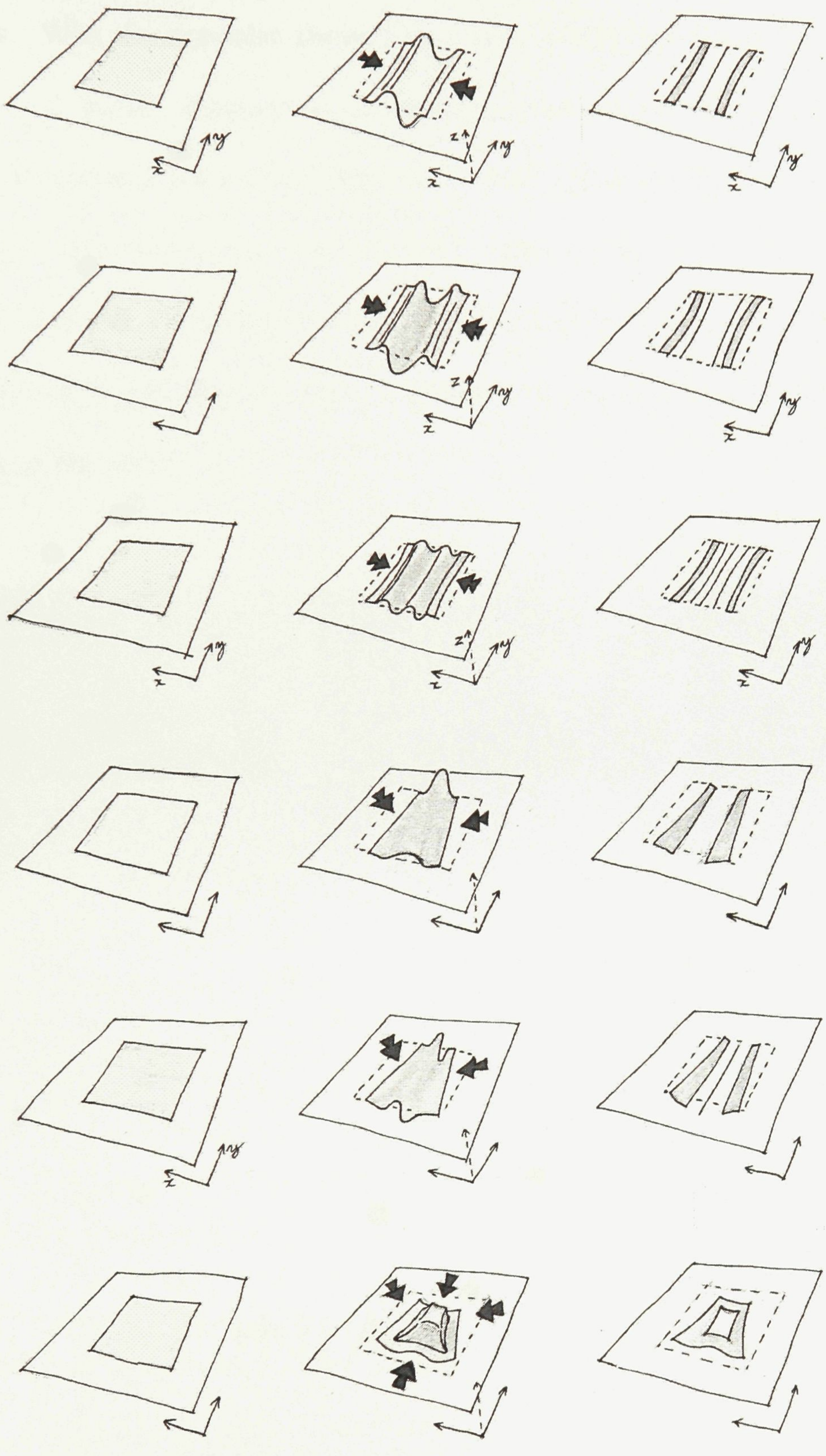
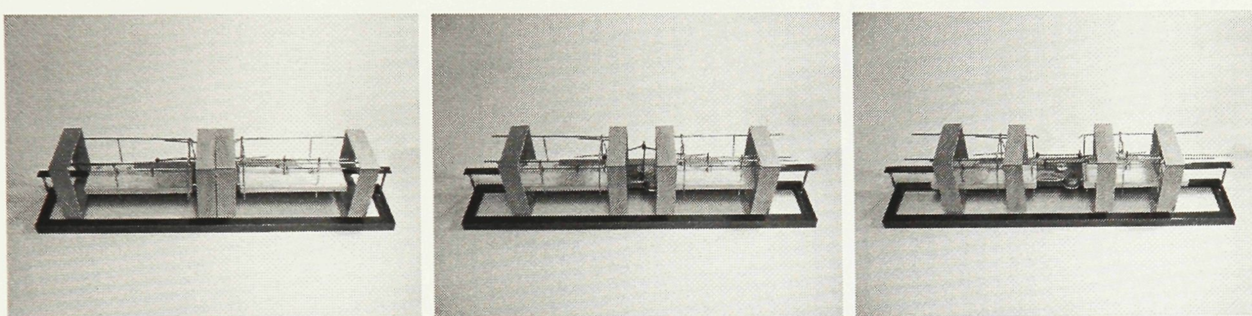


Fig. 3.10 – 3.27 various two dimensional compressed planes which breach the third dimension

We have seen that the fourth dimension is something that exists beyond our senses. With the examples above I have now established that it is possible to see the results of higher dimensional movements from within the original dimension by examining what is left behind. With this in mind, and using the one and two-dimensional breaches discussed above, we can infer what would happen to a cube if it were compressed along a single axis that resulted in a higher dimensional bowing, as seen in figures 3.28 – 3.30. Figures 3.31 – 3.36 show the results of additional axial compression acting on the cube.



a machine which compresses a three dimensional volume into the fourth dimension

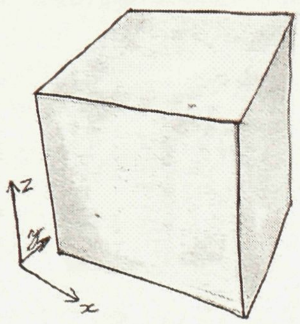


fig. 3.28

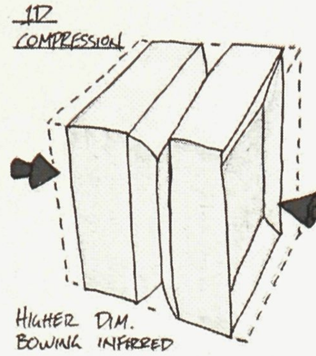


fig. 3.29

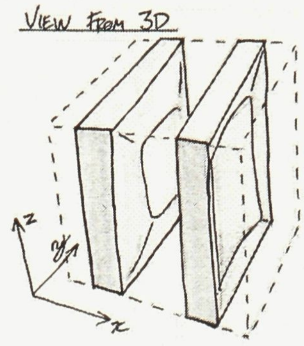


fig. 3.30

single axis compression of a three-dimensional cube into the fourth dimension

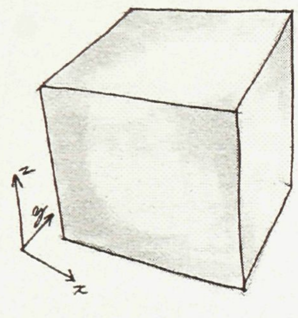


fig. 3.31

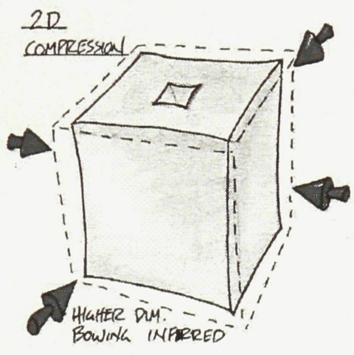


fig. 3.32

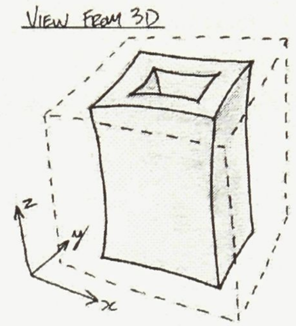


fig. 3.33

two axis compression of a three-dimensional cube into the fourth dimension

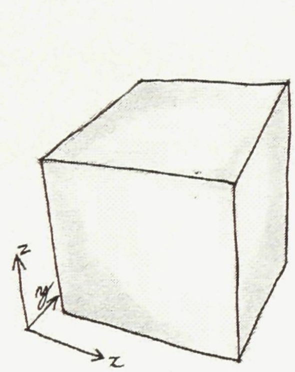


fig. 3.34

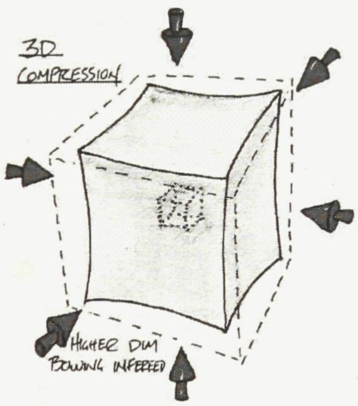


fig. 3.35

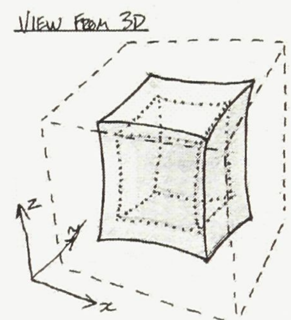


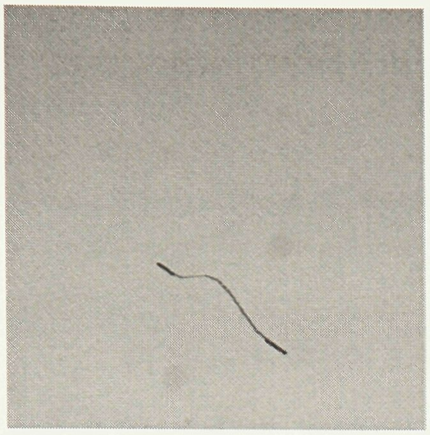
fig. 3.36

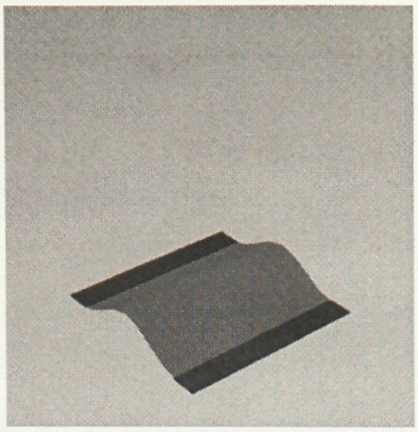
three axis compression of a three-dimensional cube into the fourth dimension

It is important to notice that as I am unable to imagine or communicate what the bowing into the fourth dimension would look like, it is not represented in figures 029, 032 or 035. This does not render this exercise defective. Rather, the tangible – and most important – part of this exercise is to focus on what is left behind in the three-dimensional world.

Earlier, we looked at the results of compression from their original or origin dimensions. Now, we are looking at the footprint a cube leaves behind in the third dimension when it is compressed into the fourth. We are still aware that the centre material has exited to the fourth dimension but it's the footprint that can be measured and studied.

The confirmation of this method reverts back to the trend discussed in Chapter 2. The trend of how each next dimension can be achieved by extruding the current dimension in the next axis. Therefore, we can predict the shape of the fourth-dimensional footprints in three dimensions by extruding the previous. Fig. 38 is an extrusion of Fig. 3.37 and Fig. 3.39 is an extrusion of Fig. 3.38.





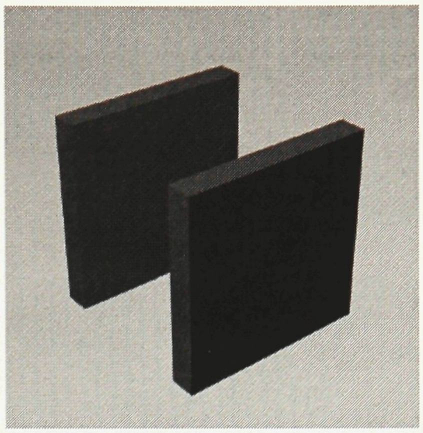


Fig. 3.37 Fig. 3.38 Fig. 3.39

extrusion as a way to achieve the next dimension

In this chapter I have examined a new way to look at higher dimensions. By compressing within the realm of the origin dimension, we can study the footprints left behind.

In the next chapter, I will further examine the components and the architectural significance of these compressed dimensions.

4.0 Examining the Spatial Implications

As an architect, it is critical to engage in discussion about the potential of higher spatial dimensions. Architects are particularly well-equipped to visualize space and their professional experience is advantageous in understanding exotic spaces with perception difficulties. Despite these unique qualifications, the study of higher dimensional space has traditionally been dominated by mathematicians and theoretical scientists. It is time for this to change.

With this sort of investigation comes the responsibility to look at how architects can utilize higher spatial dimensions. This chapter will investigate how higher dimensional theory, and more specifically how higher dimensional breaches, can contribute to architecture. To do this I will break down the components of higher dimensional breaches, and examine how these components can contribute to architecture. In the context of this thesis, to "breach" means to exit a lower dimension and enter a higher one. The breach is the point at while an object makes the upward transition from one dimension to the other.

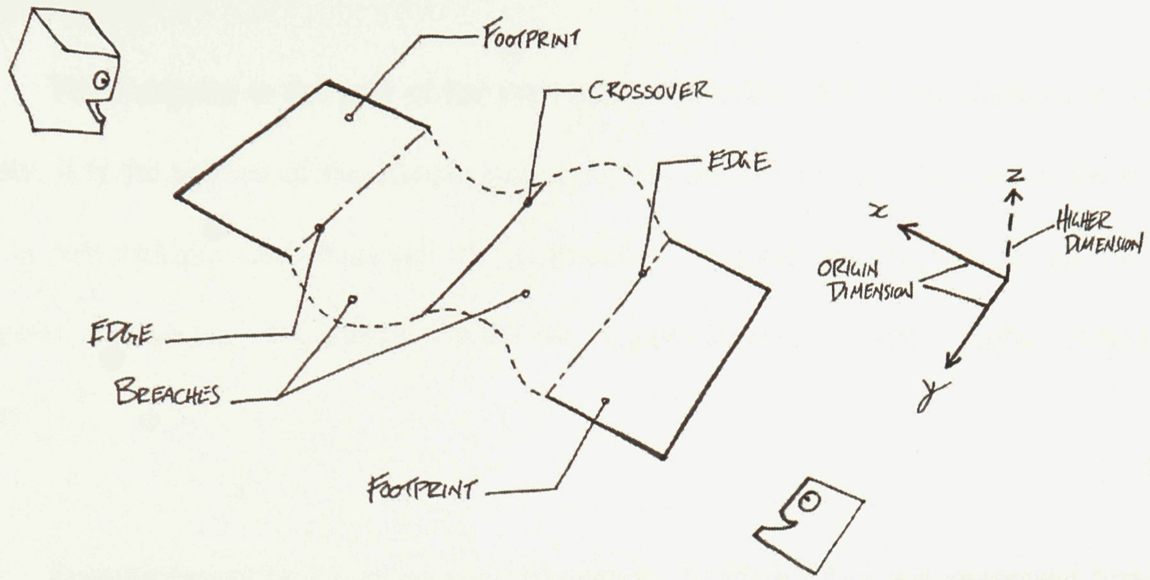


fig. 4.01 diagram of the four components and two levels of perception for higher-dimensional breaching

These compressions are composed of four parts and two points of view. The image above, fig. 4.01, illustrates the four components: the footprint, the edge, the breach and the crossover. I will define and explain each of these components in turn, below. Two points of view are also shown in the image. Here, the origin dimension is two-dimensional. The breach occurs in the higher dimension. In this particular image, that is the third dimension.

The square represents the origin point of view because he, too, exists in the same space as the original two-dimensional rectangle. The cube however, represents the perception from the higher dimension. The cube exists in three dimensions, the same space in which the post-compression breach resides.

The Footprint:

The footprint is the part of the compression stuck to the origin dimension. Quite simply, it is the portion of the compressed that does not breach. It is the part of the object that is left behind, and, because of its inability to reach the higher dimension, the footprint also serves as evidence within the original dimension that a higher dimension exists.

Footprints can be found in any dimension. In the case of a compressed line, the footprint would be two shorter lines (fig. 4.02).

In the example above, fig. 4.01, the footprint was two planes. However, if the original plane was compressed from all four sides the footprint would be a square frame (fig. 4.04). If a cube was compressed, the footprints would either be a pair of rectangles (fig.4.05), a quadrangle (fig. 4.06) or a hollow box (fig. 4.07). Footprints are always visible from both the origin space and the higher space. The "origin space" is space of the original dimension. The "higher space", or "higher dimension", is the original space plus an acquired additional dimension.

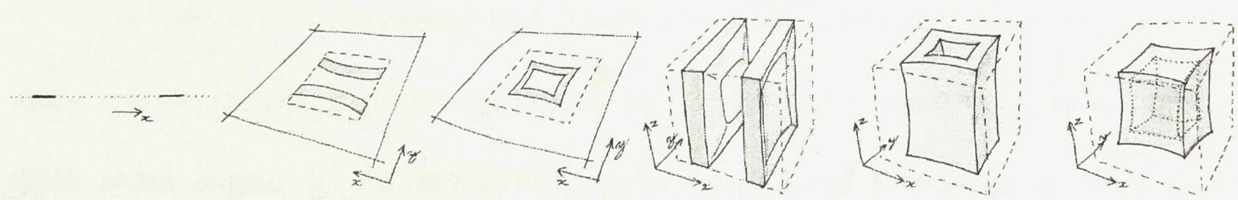


Fig. 4.02 – 4.07 various images of how higher-dimensional compressions would appear from the origin dimensions

A footprint should be considered part of a larger whole, in the same way that a footprint in the sand is evidence of an entire body. A footprint of a building on a map demonstrates its impact on two dimensions. While the third dimensional information is not represented, it is still implied. Footprints, when I speak of higher dimensional breaching, imply that there is more occurring beyond that which we can directly perceive.

In the case of higher dimensional geometry, footprints are invisibly connected across their voids. Architecturally speaking, this generates an interesting question: Are voids simply gaps between two related elements? Should a void boundary respond to being connected to its opposite side? If I were to create a void by digging a trench, the trench now being a void, the two edges are connected because at one time the two edges were seamlessly connected to each other through their homogenous substance. They are still related, because they both became edges at the same time and by the same event. With this in mind, we must consider how a void relates to its boundary's edges, and how the edges relate to each other.

The Edge:

In addition to breaching and footprints creating different ways to think about voids, the boundaries between the breach and the footprint can offer a different way to think about edges. In the context of higher-dimensional breaching, an edge is what connects two dimensions. In essence, it is the bridge between the origin and higher dimension.

Within the limits of the origin dimension, an edge is just what it looks like: the end of an object. However, the way in which that edge is perceived depends entirely on the dimension in which the viewer resides. From the higher dimension, that same "edge" will actually be perceived as a bend. Thus, even though this image looks like an edge of a footprint in the origin dimension, knowledge about a higher dimension enables us to understand there is inherent continuity.

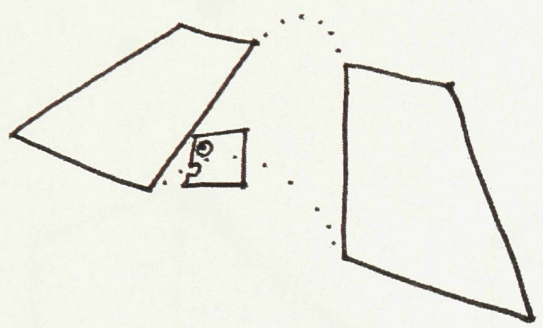


fig. 4.08

a square viewing the edge from the origin dimension

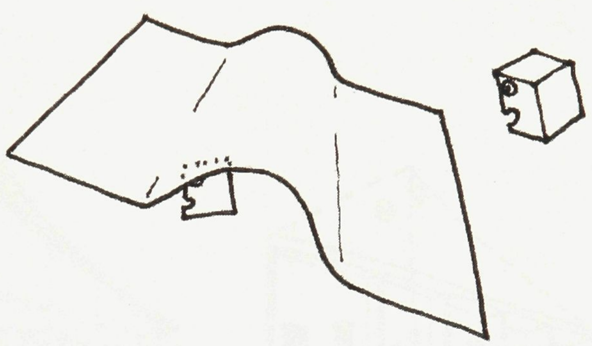


fig. 4.09

a cube viewing the edge from the higher dimension

Fundamentally there are two events occurring from the perception of the square during a compression resulting in a higher dimensional breach. First, there is a notion of more present than is visually available and second; the square witnesses a true sectional viewpoint.

In the example below fig. 4.10 there is a man looking at a series of row houses. He is unable to see the back boundaries of the row houses, but from his past experiences of architecture and form he would understand that there is more to the houses than just the two walls visible. He would project that the row houses probably continued on with the same back boundary and ended square with the last unit (fig. 4.11).

To see the row houses from a sectional viewpoint would allow him to understand the other half of the picture (fig. 4.12). This unique vantage point allows the viewer to see the inside of the row houses. To reveal space in sections is to experience a higher dimensional edge.

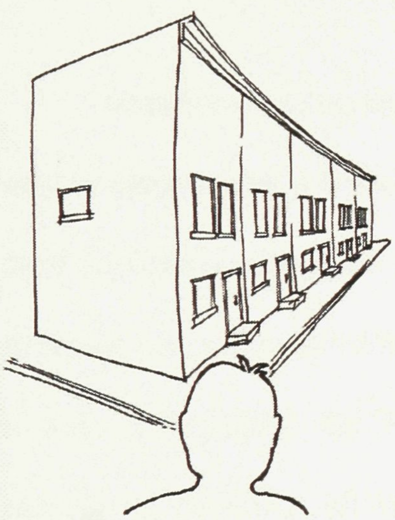


fig. 4.10 viewing a row houses

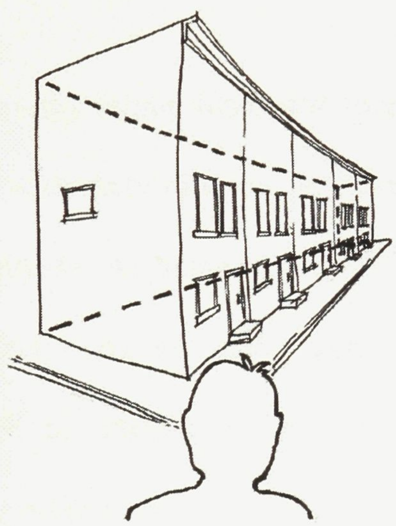


fig. 4.11 projecting what the full boundaries of the row houses are

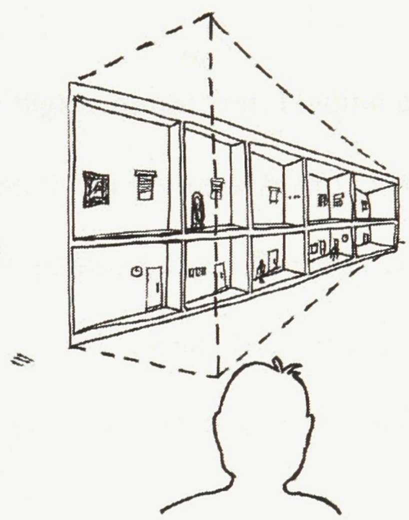


fig. 4.12 sectional view of the row house

The Breach:

The breach is the part of the compression that moves into the next dimension as a result of compression. In fig. 4.21, the breach is the part that becomes invisible to the square while remaining visible to the cube. The level of perception depends on your level of dimensional vision. From the origin perception, which is the square's perception, there is an empty space created. From the perception of the cube, however, there is a bowing bridge created. Regardless of each level of perception, the whole has been transformed and a void has been created in the centre. The space created can be inhabited in both cases. And in both cases, this void is still connected to the whole. The void is just as much a separation between the footprints as it is a bridge.

Breaching can occur from any origin dimension to a higher dimension. Finding a way to demonstrate a breach is extremely difficult, however, when the breach exits the third dimension and enters the fourth. As humans, we easily perceive three dimensions. A breach into the fourth dimension, however, would appear to us as a void. Thus, finding a way to represent the breach is difficult because it requires fourth dimensional perception. Therefore, all we see is the void.

When thinking of these breaches architecturally, we should consider them as containing higher-dimensional spaces simply beyond our vision. In this scenario, a true void would have a higher-dimensional space unobstructed by the limitations of our senses. The void would stare right at us from the fourth dimension, despite the fact that we could not see it.

The Crossovers:

As discussed in the previous chapter, a higher-dimensional breach can take many forms.

Crossovers occur when the breach is not merely a single bow, but when compression forces the breach into a wave.

Crossovers are the result of a non parallel pass-back through the origin dimension. They are infinitely thin, and contain one less dimension than their origin dimension. Below, in fig. 4.13 – 4.15, we can see the compression of a single dimension. Fig. 4.14 illustrates the wave breaching. It is important to notice in this image the criss-crossing through the origin dimension. Fig. 4.15 shows how this criss-crossing, or these crossovers, are perceived from the origin dimension. Another way to look at these crossovers, in this example, is as infinitely thin points. Each point actually represents a zero dimensional point.

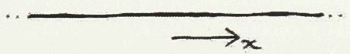


fig. 4.13

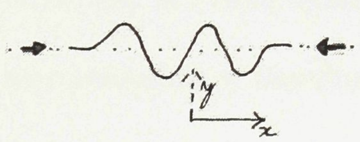


fig. 4.14

fig. 4.15

higher-dimensional breaching of a single dimension

This phenomenon can occur when any compression breaches into a higher dimension. Below, a two-dimensional object breaches into three dimensions. Earlier, I illustrated how a plane can break into the next dimension as a wave rather than a single bow. This creates a standing wave that criss-crosses through the original two-dimensional plane and create a series of one-dimensional lines. Again, we know they are perceived as one-dimensional line from the origin dimension, because the plane that we started with existed in two dimensions. "x" and "y" had a measurement, but "z" is infinitely thin. Therefore, when that infinitely thin wave plane passes through the original infinitely thin plane, the result is a line.

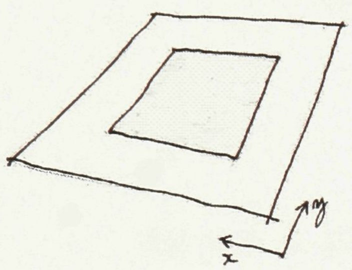


fig. 4.16

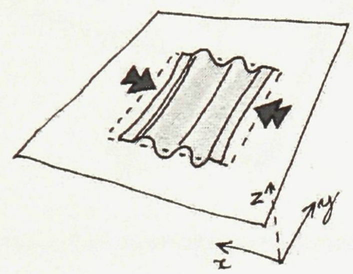


fig. 4.17

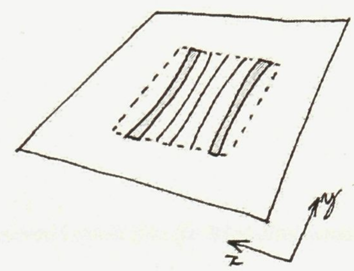


fig. 4.18

higher-dimensional breaching of a two-dimensional plane from one axis

A standing wave that breaches into a higher dimension has an interesting result; we can simultaneously infer space both above and below in a higher dimension. fig. 4.19 represents both fig. 4.17 and fig. 4.18, and is a clear example of this situation. The gray rectangles on either side are the representation of two-dimensions, as they have a length and width. The lines inside the pink rectangles are each a single dimension, and do not contain width or height - only length. These lines are also evidence that the higher dimensional space being occupied by the breaches is both above and below. Another way to put it is that the origin plane is coordinate zero for the higher dimension; the breach affects both positive and negative space from the higher dimension.

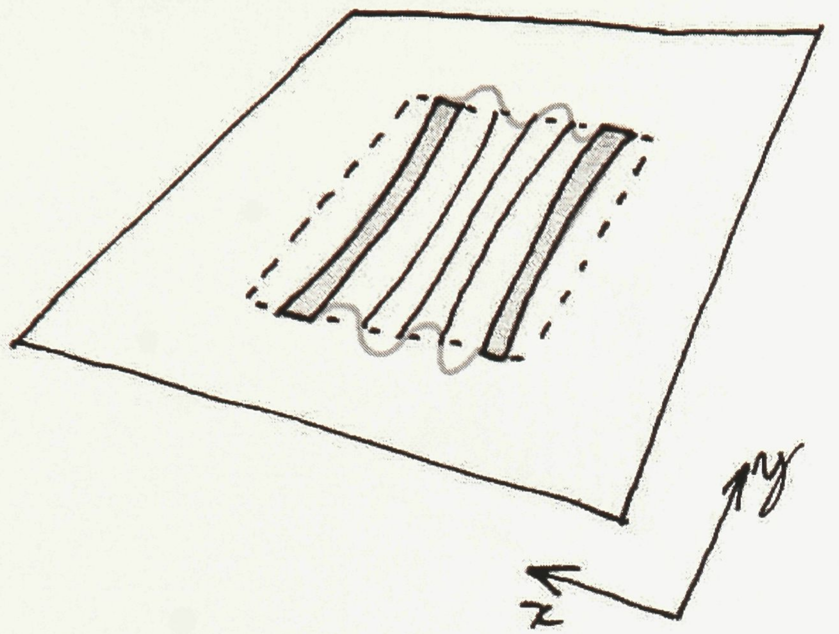


fig. 4.19 two-dimensional plane experiencing a higher-dimensional breach into the third dimension

Let us now examine what crossovers would look like from a breach into the fourth dimension, if viewing from the origin dimension – i.e., the third dimension. If we extrude the image above, we would get the image portrayed in fig. 4.20. Fig. 4.20 is consistent with ideas discussed above; the crossovers contain one less dimension than the origin dimension. In the case represented by fig. 4.20, the origin dimension contains three dimensions while the crossovers contain only two.

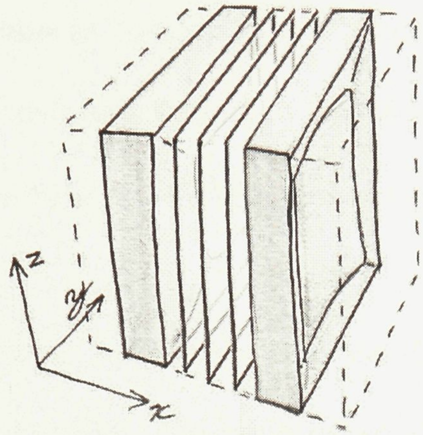


fig. 4.20 three-dimensional cube experiencing a higher dimensional breach into the fourth dimension

Crossovers themselves are miniature footprints of higher dimensions and are evidence of higher dimensions. Even though they are very thin, they are still a successful barrier. Even though they create a partition, their rhythmic spacing and similar size spatially connect them to their opposite ends.

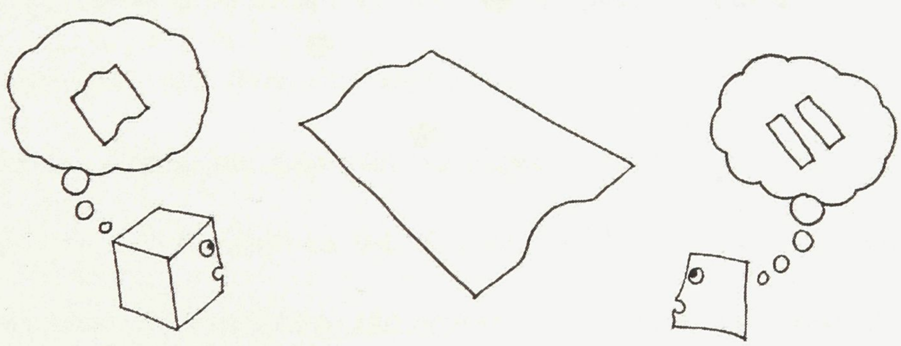


Fig. 4.21 image of a cube and square watching a three-dimensional breach

The perception from the origin dimension:

In fig. 4.18, the square is limited to two dimensions: length and width. He only perceives space within his two-dimensional realm. When the breach occurs, the square does not see the matter breaching into the higher dimension; he sees void.

Humans suffer similar limitations. When I suggest a breach from three dimensions to the fourth dimension, I do so based on the deduction that it occurs, even though I am unable to illustrate it.

As an architect, I have been guilty of limiting my perception to two dimensions and not fully utilizing the third dimension equally. In this way, I am behaving as the square even though I have the perception of the cube. Architects work in plan most frequently, and tend to ignore the third dimension because the medium for design is two dimensional. When planning, an architect needs to avoid ignoring height and elevation – the third dimension. The under use of the third dimension is also attributed to the requirement of stairs, ramps and elevators to physically engage the third dimension. An architect always needs to be careful to not work in plan without always simultaneously considering elevation, the third dimension, height. If we have difficulty in fully utilizing the third dimension, how can we expect to then tackle the utilization of the fourth dimension? It is clear to me that in addition to the exploration of higher dimensions we must still continue to exercise our use of our three known dimensions.

The perception from the higher dimension:

The cube has the advantage of seeing the breach in fig. 013,since the breach is one that enters into the third dimension and within his perception. We share the same three-dimensional sight. We have the ability to see the two-dimensional world and observe breaches that break into the third dimension. However, when it comes to the task of witnessing a breach into the fourth dimension we are unable to witness such an event. Instead we are limited to the footprints left behind. We are like the square for this act. The square cannot witness an object bowing into the third dimension and only sees the void. But also like the square, we have a curiosity to understand what we cannot easily perceive.

Summary

My attempt to catch a glimpse of the fourth dimension was initiated by personal interest and a sense of responsibility to contribute, as an architect, to a discussion dominated by mathematicians and physicists. My explorations have provided me with new insights, not only into the fourth dimension itself, but into how it might serve to influence how we think about architecture. In the next chapter I will discuss how the compression of space can create new space.

5.0 The Creation of New Space

In this chapter I will attempt to illustrate how higher-dimensional breaching can break off space and transport it to a parallel world. As a note I am very aware how much this sounds like science fiction but I ask that you follow me a little further because the philosophy can lend itself not only to theoretical physics but to architecture as well.

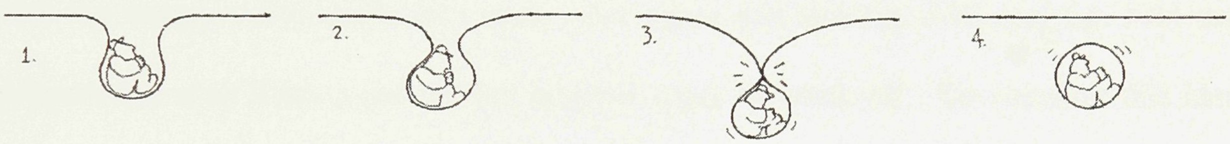


Fig. 5.01 Fig. 5.02 Fig. 5.03 Fig. 5.04

images from the Fourth Dimension *Rudy Rucker*

The image above was the inspiration for my thesis and led to my hypothesis about how higher dimensional geometry can be created, and witnessed, from lower dimensions. The image shows a man squatting into a line. The line accepts his body and the plane dips enough for it to break off.

After some analysis, I determined that the image was problematic. First, the space illustrated by the line is one-dimensional. The man is moving downward, the force of which requires a second dimension. And of course, the man himself exists in three dimensions.

If a higher dimensional force - the man - was to collide with the single dimensional line, it could potential break off a piece of that space and carry it away. This idea is useful, as it raises questions about how new space can be created by taking from another space. But it fails to operate from within the boundaries of its origin space.

The situation in fig. 5.01 could be achieved if the two ends of the line were compressed from within the one-dimensional world where the line exists. Further, it is my position that it is unnecessary to have the body dropped into the line, in order to have the line break off. A different force applied to both ends could theoretically cause the line to breach into the next dimension. If that force continued, it is feasible that the bowing from the compression might find each other again and like fig. 5.03 and fig. 5.04 that compression may force a part of that original space to break off. To visualize this idea, imagine condensation collecting on a ceiling. If a water bead on that ceiling could collect enough moisture from its perimeter, the weight of that water would overcome the water's stickiness to the ceiling, and the drop would fall.

The frames below demonstrate the phenomenon of breaching to create new space, from the second dimension. Unlike the original diagram of the man pushing into the line, the act of breaching, and of space breaking off into a new dimension, occurs within the original two-dimensional world.

When trying to understand higher dimensions, it is important to work as much as possible from within one's known realm. To argue that a force from another dimension creates breaching, and that the breach then breaks off into a new space as indicated from fig. 5.01 – 5.04, is too far removed from the understanding of the origin dimension.

If we are going to examine how a three-dimensional space can escape into four dimensions, it is easier to work from within the three dimensional realm than to use a higher-dimensional collision which may take a piece of our three-dimensional space with it.

The exciting part of this investigation is the potential of breaking off space from our three-dimensional world. To do so, we must apply the analysis in the previous two chapters, where I discussed the trends common to all dimensions and that breaching three dimensions exists beyond our senses.

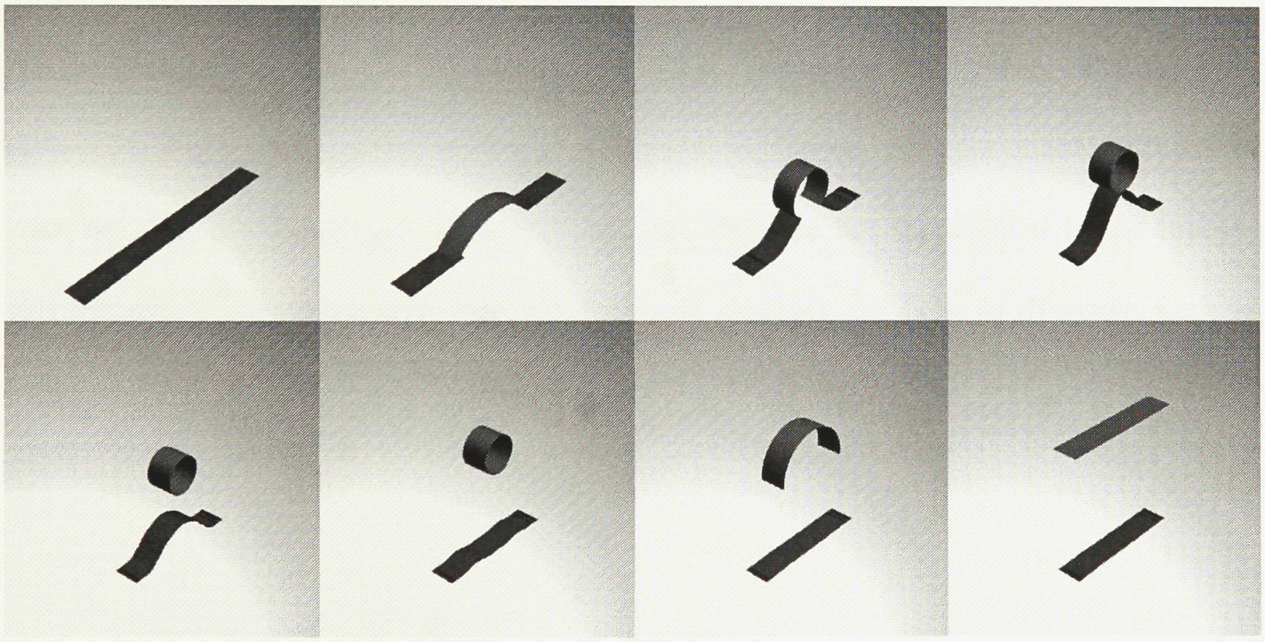


fig. 5.05 – 5.12 the creation of a new two-dimensional space through three-dimensional space

In the case of the two-dimensional plane in fig. 5.05, a new space was created (fig. 5.08), and was sent to a different two-dimensional plane existing in three-dimensional space (fig. 5.12). This means there are at least two, two-dimensional worlds existing in a larger three-dimensional world.

Lisa Randal supports the theory that our three dimensional universe is part of a larger higher dimensional megaverse. Randall suggests:

> Physicists have now returned to the idea that the three-dimensional world that surrounds us could be a three-dimensional slice of a higher-dimensional world. A brane is a distinct region of spacetime that extends through only a (possibly multidimensioal) slice of space.[28]

The images below, fig. 5.11 – 5.15, show how a three-dimensional space could be compressed and broken off to appear somewhere else in the megaverse. The red two-dimensional plane is repeated in this set of images for clarity. As I am unable to illustrate what the fourth-dimensional breaches look like, they are not represented in the images. But the results are consistent with the previous chapters for how to illustrate such an event.

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[28] Randall 52.

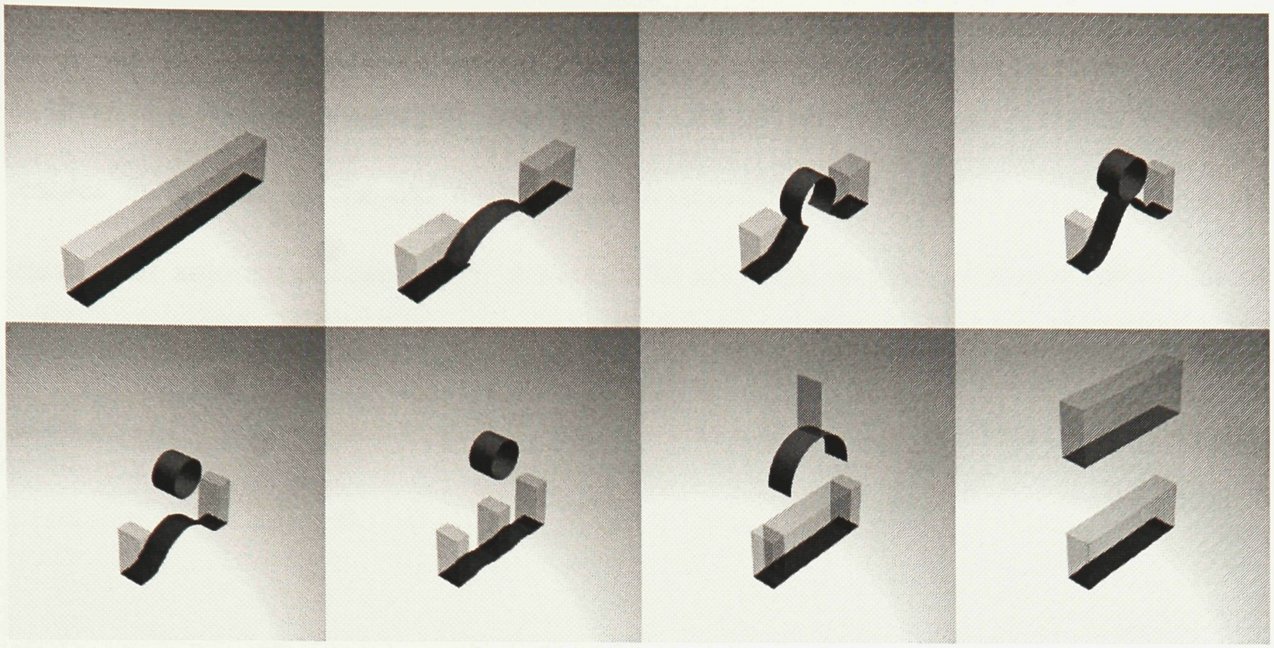


fig. 5.13 – 5.20 the creation of a new three-dimensional space through four-dimensional space

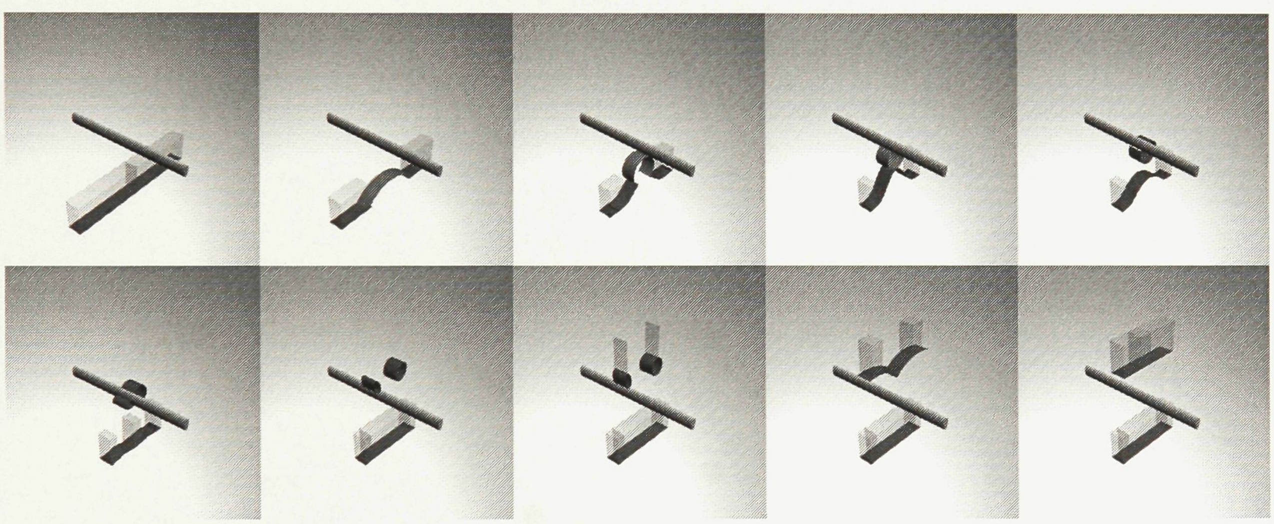


fig. 5.21 - 5.28 the creation of a new three-dimensional space through four-dimensional space with an obstacle

Architecturally, it suggests that the creation of three-dimensional space can occur through four-dimensional space and the creation of a three-dimensional space potentially causes the loss of space somewhere else. There is also a sectional responsibility implied, as there is a vertical connection between new and old space. In fig. 5.20 the new space resides in relation to the old space. The production of new space using other space creates an automatic connection; a benefit to sectional design. Even if there was an obstacle, between the new space and the old space, there would still be an inferred connection.

6.0 Revealing how we currently utilize the third dimension

Our current use of our known three dimensions is underutilized; the third dimension, height, is the most neglected. Before we can look to higher dimensions it is important to fully understand the potential of our known three dimensions.

First, let us examine the first dimension. In a one-dimensional world, there is very little freedom of movement. Like an abacus, the distance between the beads may change: the order of the beads, however, cannot. In the same way that the beads are bound to the rod of an abacus between their neighbors, the creatures of a one dimensional world would also be trapped. In a single-dimensional world there is very little freedom of movement.

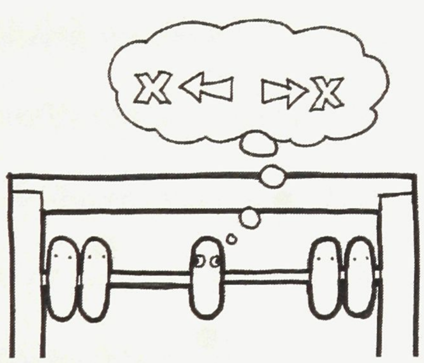


fig. 6.01 image of an abacus bead trapped

A two-dimensional world on the other hand, more closely resembles ours. Edwin Abbots square, from the book *Flatland*, can avoid obstacles. If the square was moving in one direction and came upon an obstacle, he would simple use his second dimension, move sideways, and continue is his original direction. Arguably, our three dimensional existence is superior, my position is that it is, but we do not fully exhaust the potential of the third dimension as we do with the other two dimensions. As humans, we primarily deal with longitude and latitude. On a macro level we navigate through our environments much like Pac-man running through his two dimensional world, traveling, north, east south and west, avoiding obstacles. This similarity forces us to evaluate our own existence. What is the difference between a Pac-man wall and a typical wall sandwiched between two floors?

The third dimension is generally considered superior to the second. While I think that sentiment is correct, I also think its significance is overstated. That is because human movement is typically limited to longitude and latitude. Perhaps gravity is what restricts us from fully capitalizing on the third dimension. In any case, humans have utilized the third dimension mostly through architecture. A hill does not engage the third dimension but a ladder does. In the situation of a hill an individual will gradually change their elevation but their point of view changes very little as they are still traveling and observing space along the horizon. The ladder, however, engages a new point of view. It forces the individual to acknowledge the change in elevation and look up towards the sky, the third dimension.

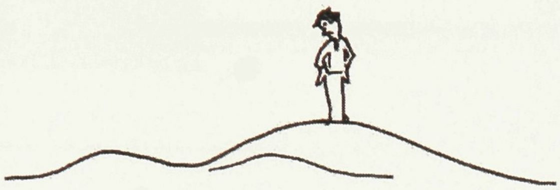


fig. 6.02 man not engaging the third dimension

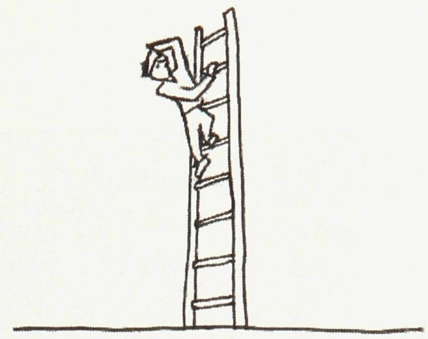


fig. 6.03 man engaging the third dimension

Mies van der Rohe's Barcelona Pavilion is a good example of architecture that utilizes the third dimension. Mies's pavilion embodies a "de-cellurization".[29] It was an architecture unrestricted by corners and celled rooms. Mies's created spaces with free flowing movement which brought freedom to the spaces he created.[30] The Barcelona Pavilion, seen below in fig. 6.04, rarely has spaces with full enclosure. In the design of the pavilion, Mies also denies the distinction between inside and outside. To the right there is almost closure achieved with a backwards 'C' layout. The approach to the space suggests an enclosure as you enter under a canopy and see walls as boundaries, but once inside the space you are aware that the roof is pulled back to allow for a vertical escape to the sky. Mies's Barcelona Pavilion embodies this extension to infinity in all the spaces.

[29] Maritz Vandenberg, New National Gallery, Berlin (London: Phaidon Press Ltd., 1998) 6.
[30] Vandenberg, New National Gallery, Berlin 6.

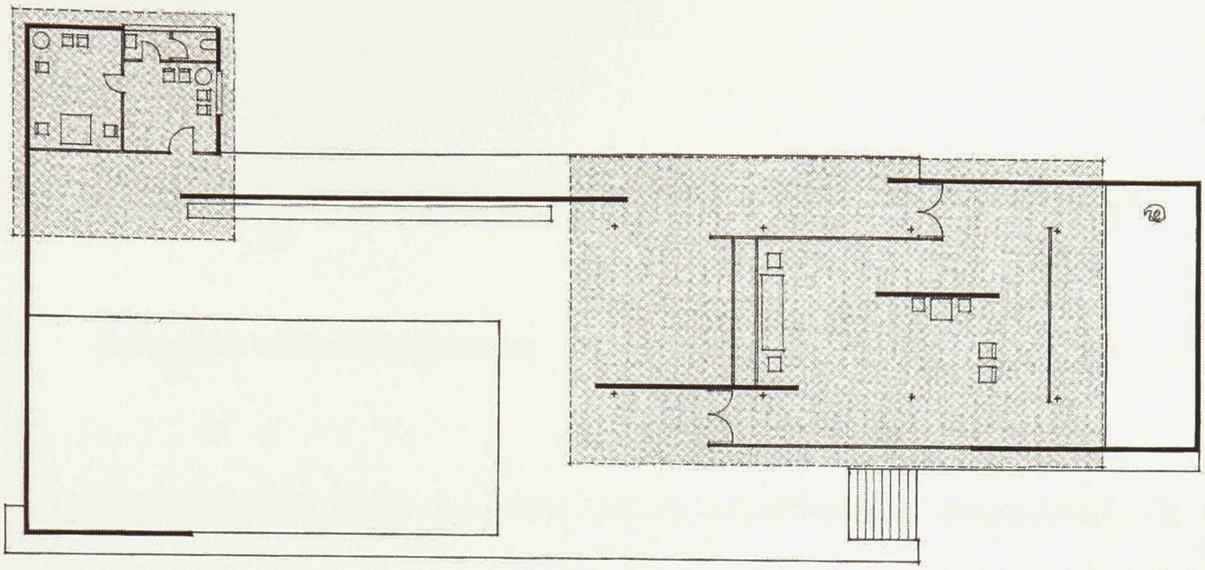


fig. 6.04 floor plan of Barcelona Pavilion

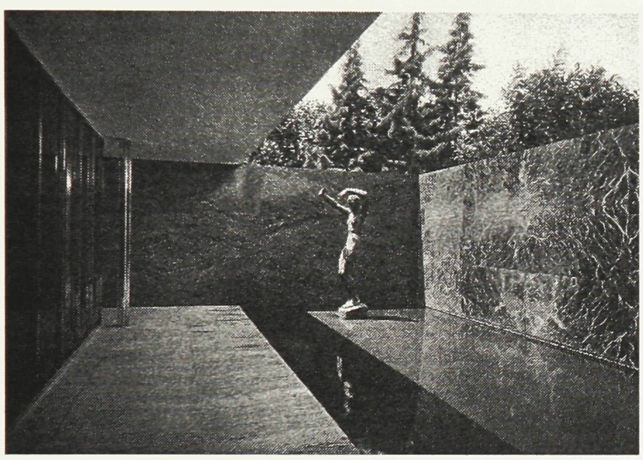


fig. 6.05 small pool at Barcelona Pavilion[32]

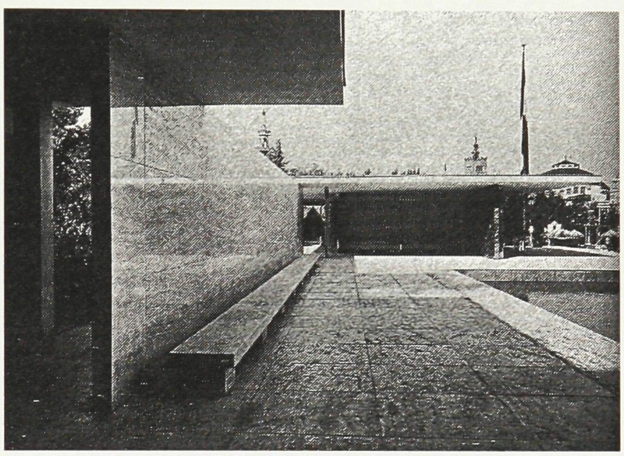


fig. 6.06 larger pool and main house at Barcelona Pavilion[33]

In the same way that Mies van der Rohe utilized the third dimension, I will engage and attempt to utilize the fourth dimension in an architectural speculation. The next chapter will examine how the fourth dimension can be used to sculpt architecture through the design of a pavilion.

fig. 004 Maritz Vandenberg, Farnsworth House (London: Phaidon Press Ltd., 2003) 12.
[32] Josep Quetglas, Fear of Glass, Meis van der Rohe's Pavilion in Barcelona (Boston: Birkhauser, 2001) 135.
[33] Quetglas 96.

7.0 Discussion of the Three Known Dimensions and the Theoretical Fourth

7.1 Extension of the Components

Until now this thesis has been mostly theoretical and diagrammatic. In this chapter, I will explore physical representations of higher dimensional footprints and their components. I have included a series of selected architectural vignettes, which helped inform the architecture of my pavilion design. For this section you will need the 3D glasses attached.

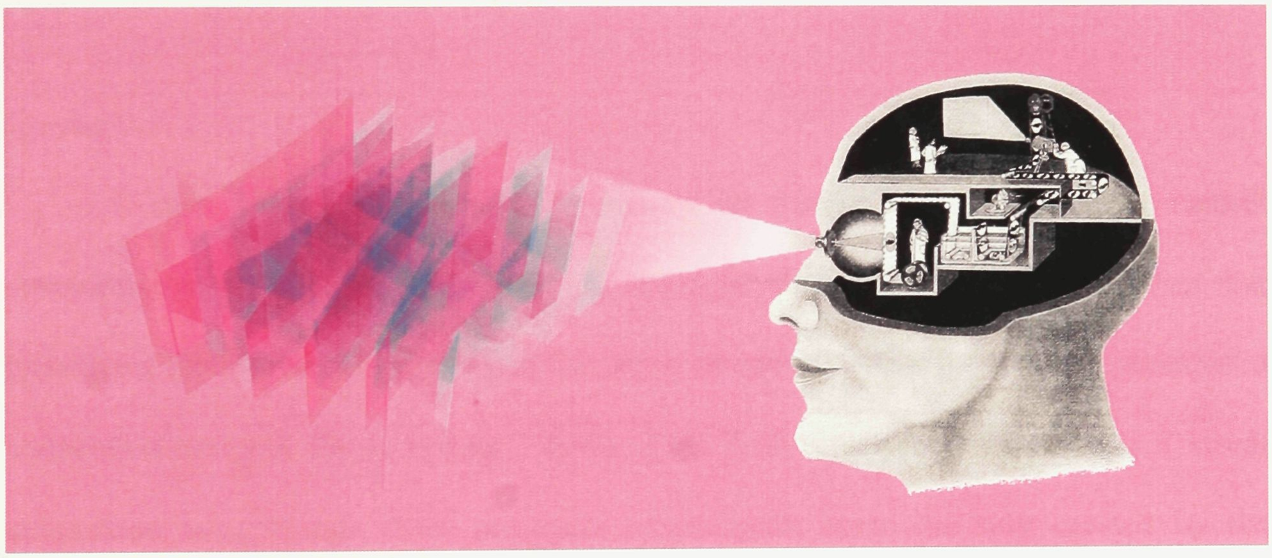


fig. 7.01 The crossovers alone in a double axis compression.

7.2 Exploring the Fourth Dimension Architecturally

This exploration of the fourth dimension has revealed four major components; the breach, the footprint, the edge, the crossover; two points of view; the perspective of the origin dimension and the perception of the higher dimension and the creation of a new plane that occurs as a result of compression. I've explained what happens to them, hypothetically, when interacting with other dimensions. Now, I will explore actual physical representations of these multi-dimensional interactions.

My objective is to create environments that, when visited, will encourage a better understanding of higher dimensions. Thus, the vignettes in the following diagrams directly informed the design of my pavilion.

Crossovers

I have mostly represented crossovers as transparent. Glass is a natural representative material, as it is both clear and a physical barrier. Despite the fact that crossovers are a subdivision, the fact that these crossovers are made of glass preserves a connection between the footprints. In fig. 7.07 from either footprint there is a visual connection even though there are clear subdivisions along the way created by the crossover planes.

Figs. 7.02 – 7.05 show a four-sided space being compressed from four sides into the fourth dimension. In this particular case there is a significant amount of crossovers fig. 7.05. The crossovers themselves pass through each other and create tight vertical cells.

Fig. 7.06 shows a group of crossovers standing alone between its footprints. A quality that distinguishes a crossover from a thin plane is that the former would arguably be self-supporting. A crossover would be supported by its breaches from a higher dimension, and thus would not fall over with any amount of force. In contrast, a plane would respond to three-dimensional properties. Accordingly, it would lean over and could even fall if disturbed.

The Breach

When space is compressed it breaches. In the case of a basic compression to a two-dimensional space the breach would read as 'bowed.' The centre of the plane would be forced into the third dimension. We of course can see the breach as we are able to see the third dimension, however, if a three-dimensional space was compressed the breach would occur into the fourth dimension. That breach reads as a void. The remaining physical space, the footprints, stays connected through the fourth dimension. Fig. 7.16 shows two planes separated by a void. In this image the two planes are clearly related. It is equally clear that they are the products of a compression which led to a new plane.

In fig. 7.13 it is not as clear, but the same relationship can be inferred. If we assume the spaces were created at the same time, then presumably the volume between was also created at the same time. We know that a higher dimension is required for space to exit through and to another space. Therefore, we can infer that the space between them is fourth dimensional space – and, that the space between them is uninhabitable.

The Edge

The edge of a footprint must have a mate opposite to it through the void/breach. It must be a defined and sharp edge. The edge can reconnect with its counterpart via stair, ramp or bridge, but the severed edge must be represented. An edge is best represented with a translucent quality. If the edge occurs in the middle of a inhabitable space, that space must be visible from the void. If the edge occurs in a solid, the solid must be exposed as unfinished. The space behaving as the footprint must be exposed from the edge.

The Footprint

Footprints are opportunities for enclosure. Except for its edge, a footprint follows the rules of it's origin dimension. The footprints of a three-dimensional space that was compressed to the fourth dimension is three dimensional. In fig. 7.09 there are two footprints separated by a void. The footprints are three dimensional spaces and enclosed with the exception of the edges. In this particular case the footprints contain a program of sitting and the void allows for a path.

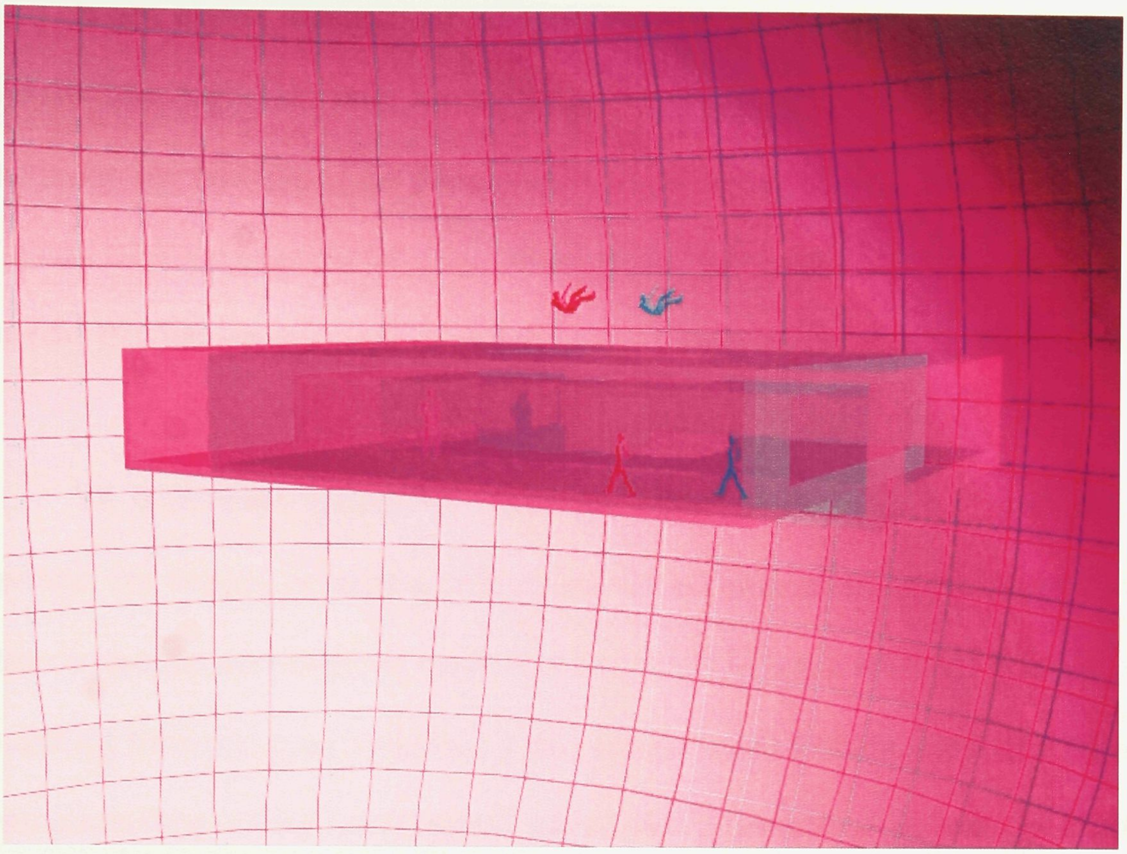


fig. 7.20 (top right) a look at how three spaces from different axes can co exist

Perception from origin and higher dimensions

The point of view is important to establish when examining higher dimensional representations. Depending if you are looking from the perception of the origin dimension or whether you are looking from the perception of the higher dimension there are different understandings of the same space.

To see the world through the eyes of someone in the origin dimension puts oneself at a visual handicap. The constraints of the origin dimension impose perceptual limitations, such that there is more occurring than one can personally see or experience.

When higher dimensional activity is occurring, there is a rift in normal logic whereby that logic can only be explained by applying higher dimensional theory. I mentioned one example of the phenomenon earlier in this chapter: self-supporting planes of glass. Another example of origin-dimensional perception would be an abrupt edge, whereby the interior was revealed and there was a mate to this edge across its void.

To experience perception from the higher dimension, we must attempt to represent the breach, because a void represents the perception of the origin dimension. There is also the advantageous mode of travel of ANA over KATA; the ability to remove something from one space and place it into another space without penetrating boundaries. This penetrating ability goes beyond travel, but is perception as well. In chapter 2, I established that if an object or space can reveal itself inside and out, then that space can be considered to be perceived from a higher dimension. This is true even if the devices to communicate that space are opaque and require the viewer to make assumptions.

Creation of New Space

As I discussed in chapter 5, the act of compression provides an opportunity to create new space. When one creates space from a breach that breaks off to a new plane/brane, there is evidence of its creation. As I explored earlier, the midsection of the space compressed will be torn out of its previous home as it is pushed into the higher dimension, thus leaving it scarred from the event (fig. 7.18).

Furthermore, if there was an obstacle between the origin plane/brane and the new plane/brane, then there is an overlap of space created. This overlap can be resolved in many ways, such as with program or materiality.

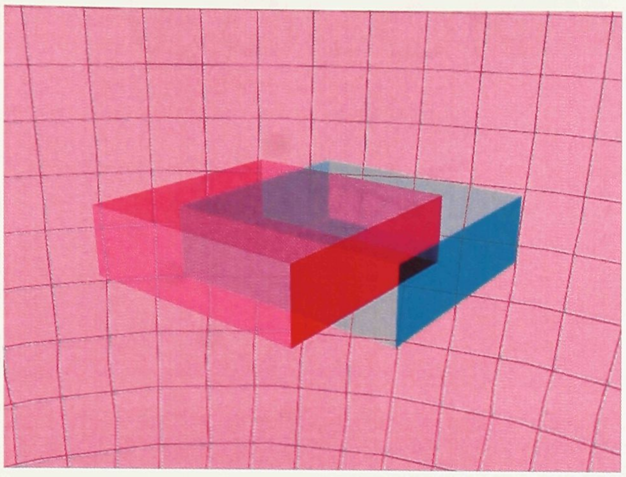
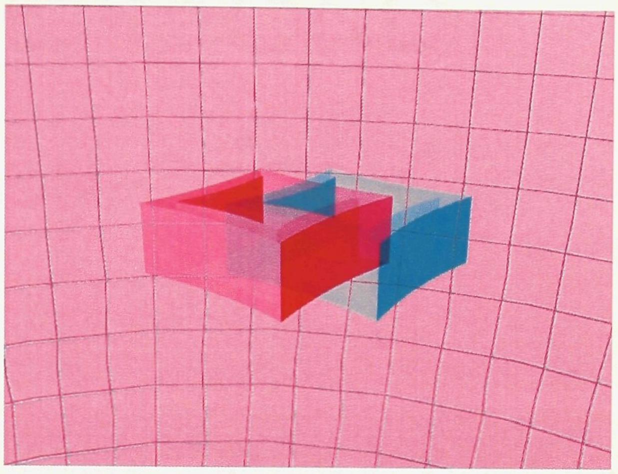
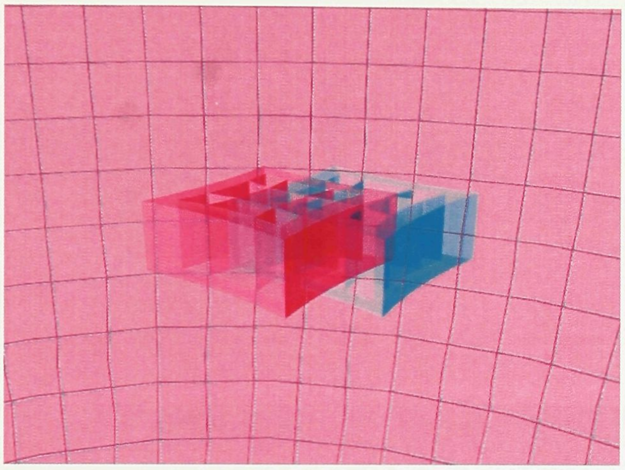
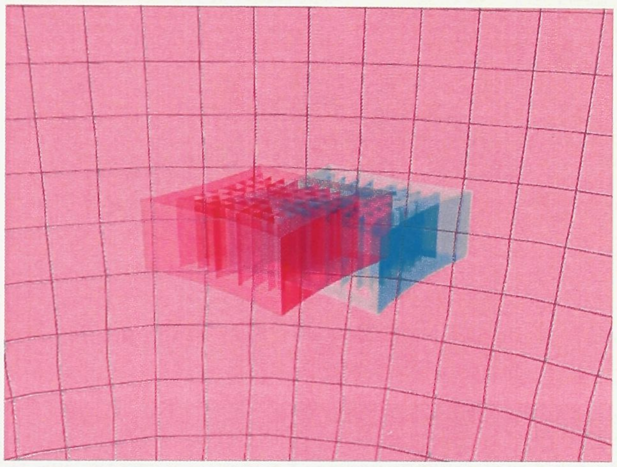
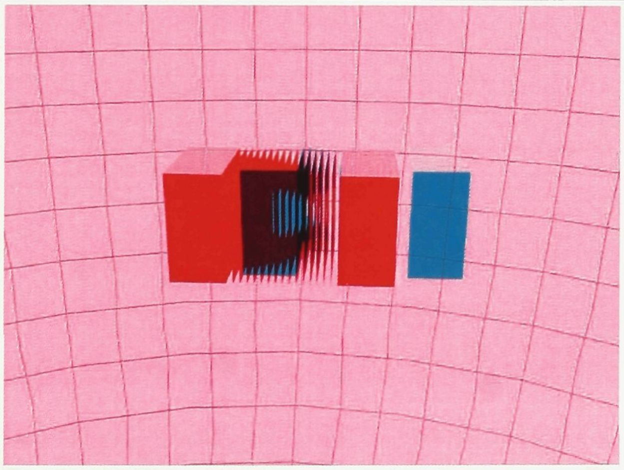
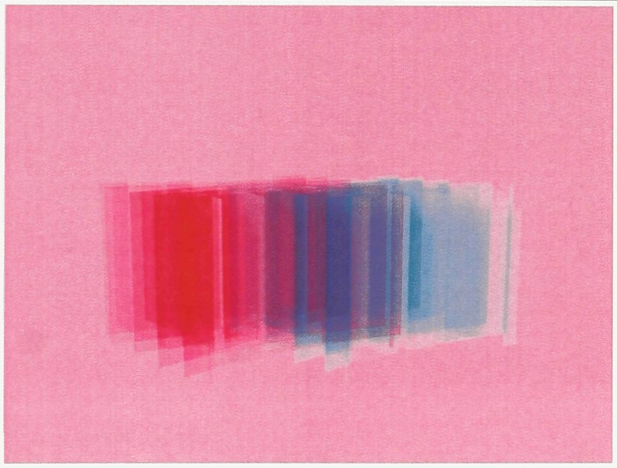







fig. 7.02 (top left) a three dimensional space pre-compression
fig. 7.03 (top right) the three dimensional space post-compression
fig. 7.04 (middle left) the same three dimensional space post-compression with a single crossover
fig. 7.05 (middle right) the same three dimensional space post-compression with multiple crossovers

fig. 7.06 (bottom left) a compressed three dimensional space which has exited it's centre to the fourth dimension. the multiple crossovers create a pattern which bends and reaches from the left side to the right.
fig. 7.07 (bottom right) a pattern of crossovers isolated and translucent

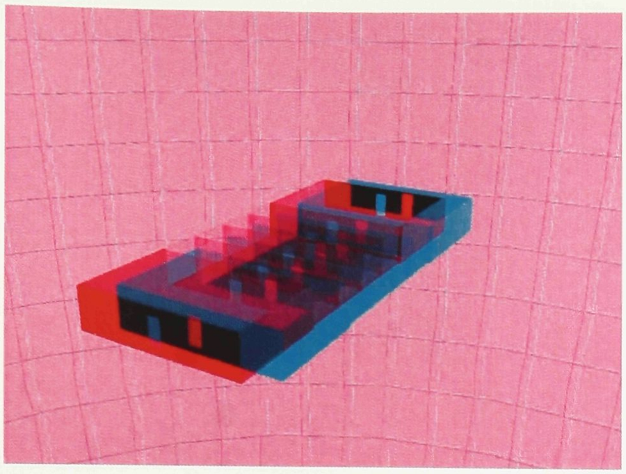
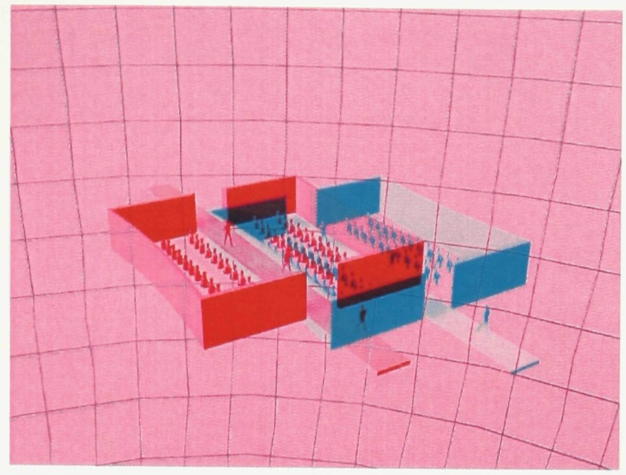
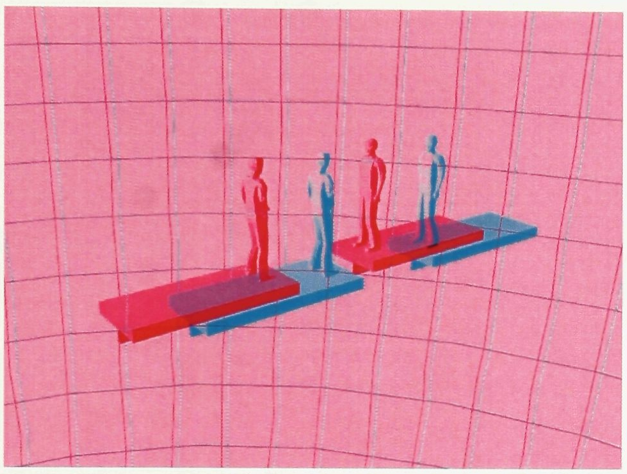
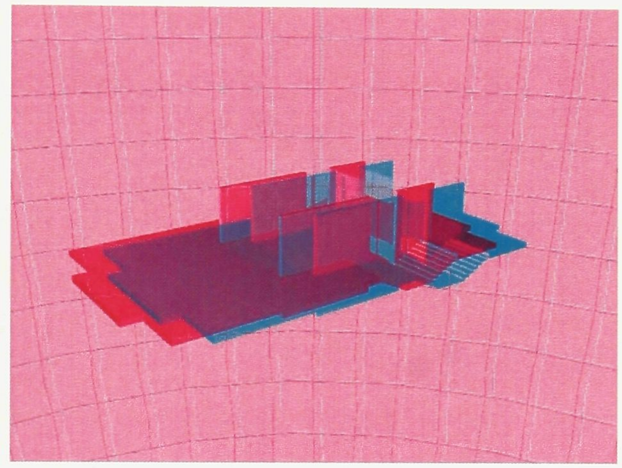
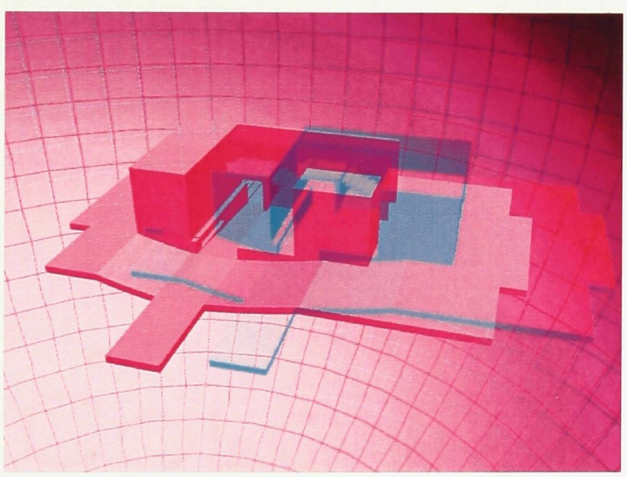
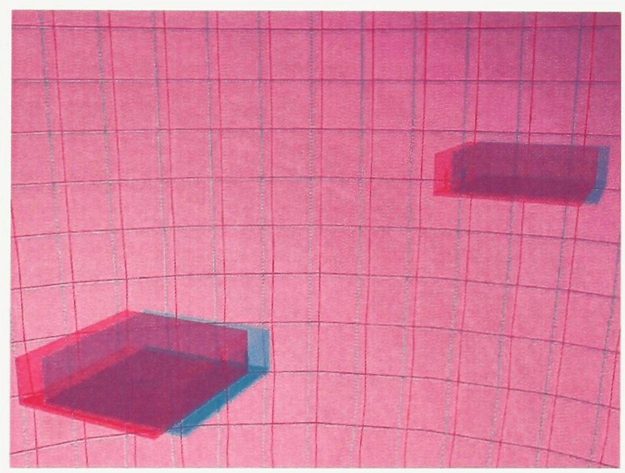







fig. 7.08 (top left) a representation of footprints and crossovers linked by a central passage
fig. 7.09 (top right) a programmatic resolution of a compressed space and it's void. the footprints have a sitting program while the void is a pathways through
fig. 7.10 (middle left) the edges created by a void
fig. 7.11 (middle right) an architectural look at how to represent, travel across and perceive edges
fig. 7.12 (bottom left) another investigation into how to represent the void
fig. 7.13 (bottom right) two three dimensional spaces separated by a fourth dimensional void

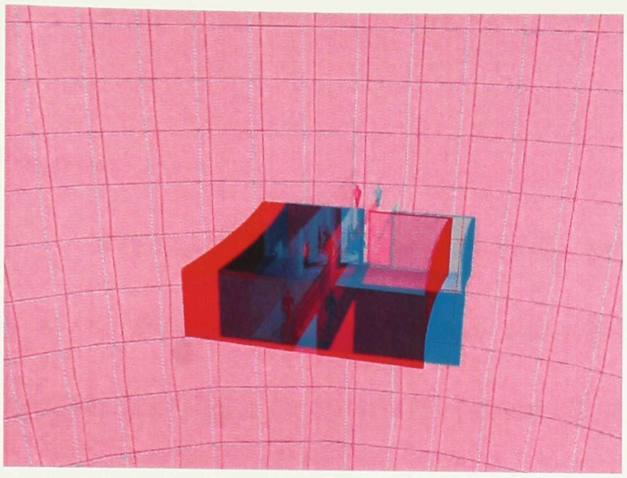
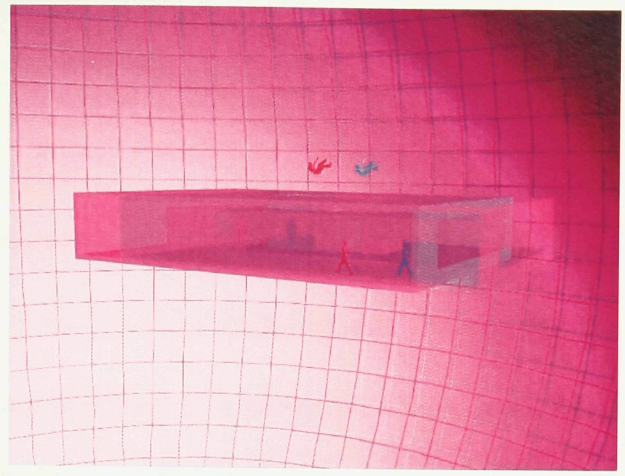
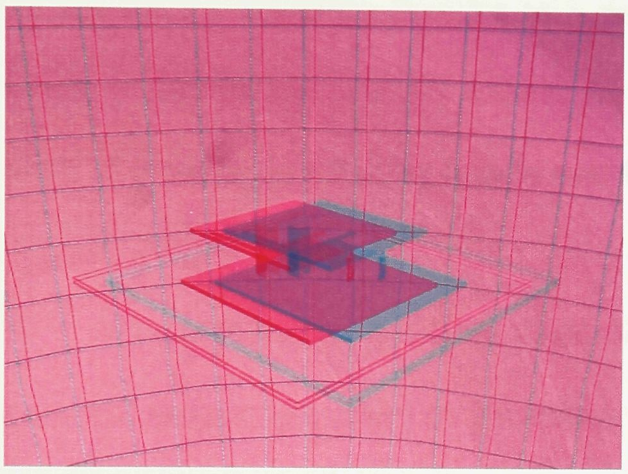
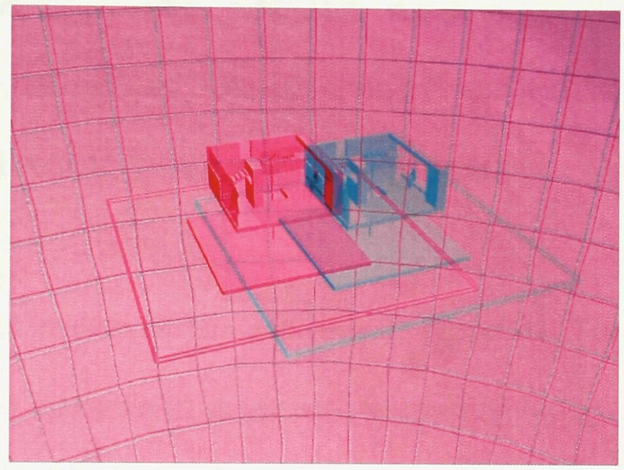
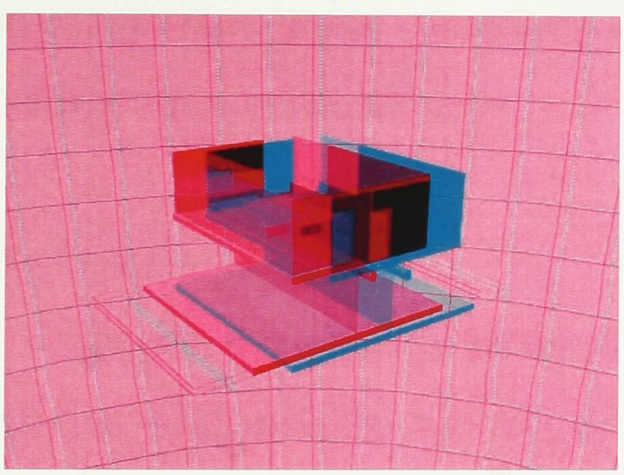
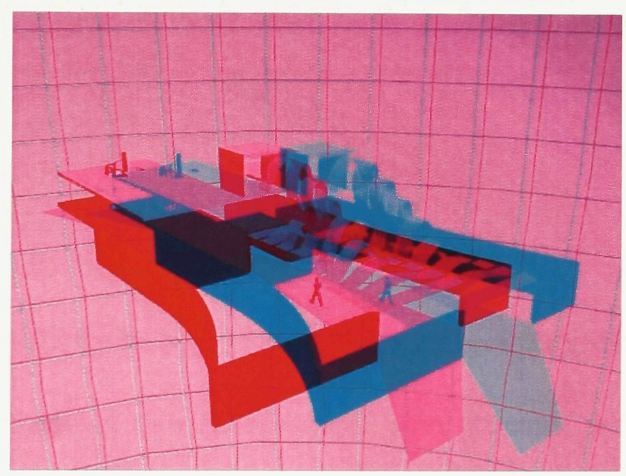







fig. 7.14 (top left) an attempt to synthesize Plato's cave with a compressed space which has breached into a higher dimension
fig. 7.15 (top right) a look at how three spaces from different axes can co exist
fig. 7.16 (middle left) a newly created space post compression
fig. 7.17 (middle right) an investigation of a newly created space that attempts to utilize the four dimensions
fig. 7.18 (bottom left) an investigation of a newly created space which has been influenced by an obstacle
fig. 7.19 (bottom right) a synthesis of newly created space post compression, footprints and a way to inhabit them

7.3 The Motivations Which Led to the Design of the Pavilion

Program

My thesis is an exploration of the physical fourth dimension - what it has to offer us in our three-dimensional realm, and what it has to offer architecture. To convey the concepts learned architecturally I have chosen a pavilion as a program. I have used Mies van der Rohe's Barcelona Pavilion as a precedent. Mies's pavilion was a successful vehicle to deliver and spatially explore his concepts of "de-cellurization" and infinite space. A pavilion served as an equally useful vehicle for my own investigations.

My pavilion is a gathering place, for events or for educational purposes. The space will include an exterior art garden and interior event space with supporting lounge, dining/conference, kitchen and restroom facilities. It will also have a lecture area complete with an intermission space.

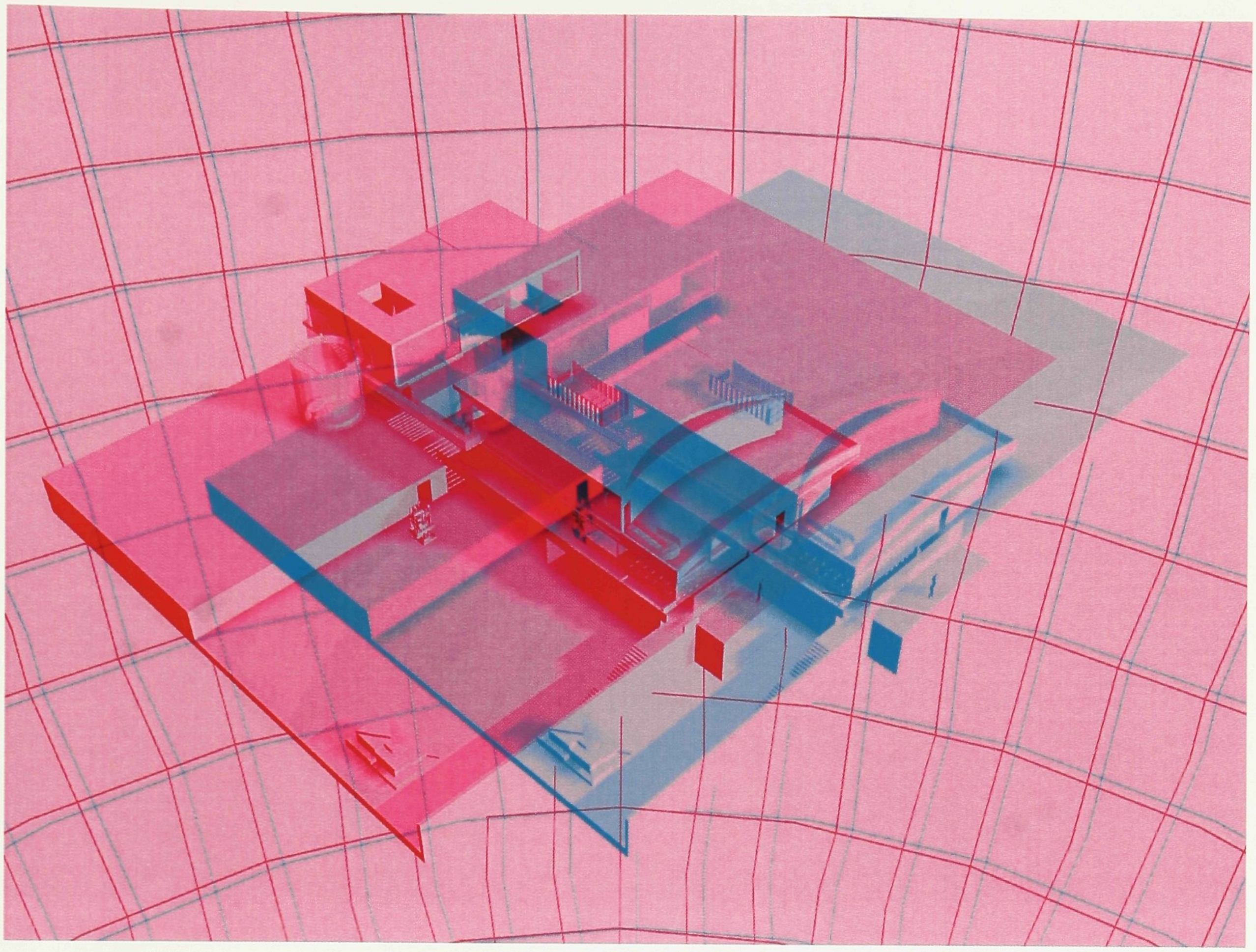
The Pavilion

As you first approach the building, it appears to be a single-storey program raised from the ground plane. The first floor represents a two dimensional environment, not fully engaging the third dimension. It uses conventions from a two-dimensional world such as partitions and edges. It lacks devices that would encourage the use of the third dimension, like ceiling penetrations, elevation changes and vertical elements. The experience of the first floor reflects our current popular use of our environment - that is, mostly using forward, back, left and right movement and visualization and not fully using up and down.

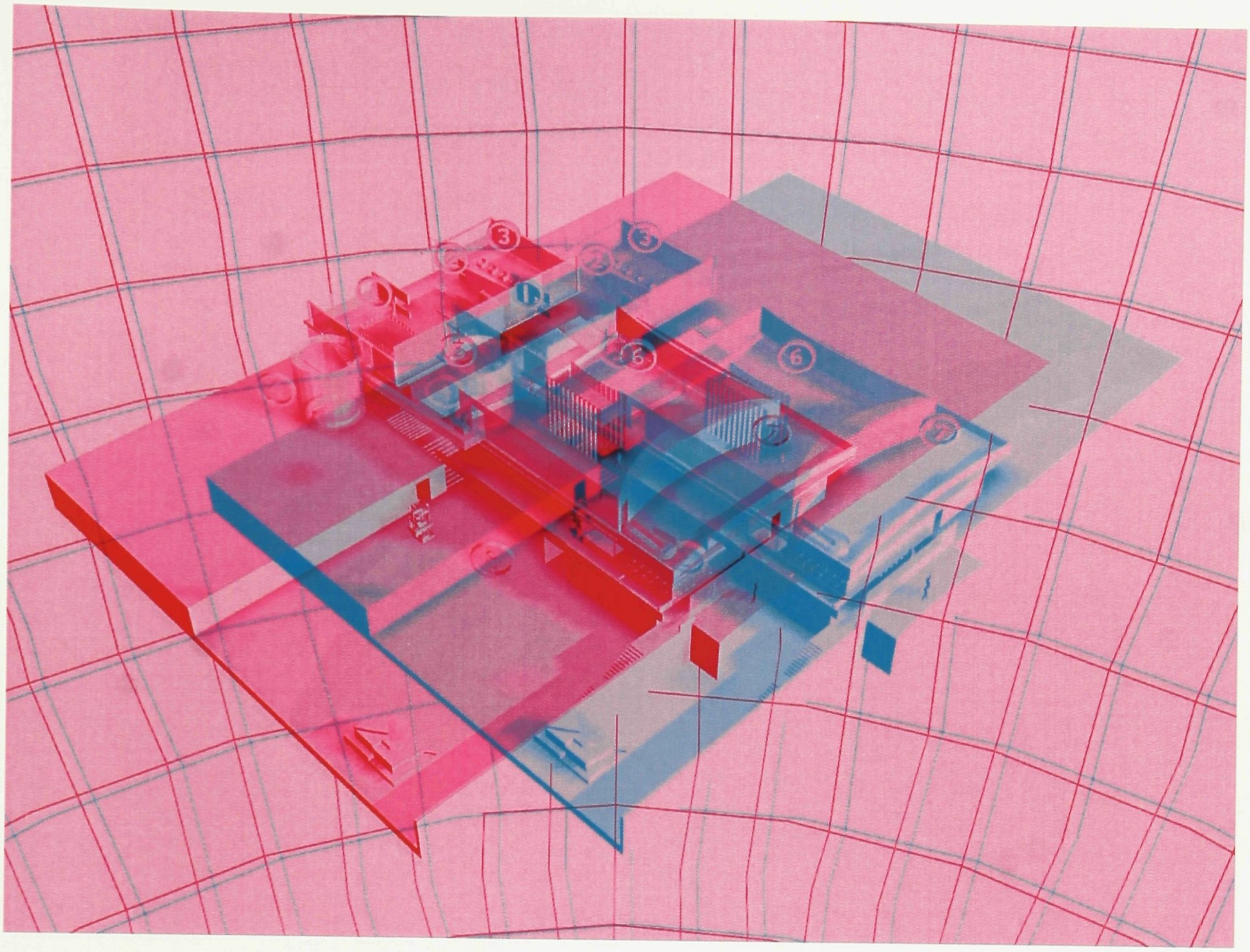
The pavilion's elevated second floor symbolizes many things. First, it creates a contrast to the first floor by utilizing the third dimension through the use of ceiling penetrations, change in elevations and vertical elements. Next, it represents a new brane; a new world, one which begins to explore beyond the third dimension. Third, the act of creating a new plane above an existing one creates the enclosure below. Without the second floor, the space below would not be space but a plane. However, the second floor acts as a second point of enclosure thus creating an inhabitable space.

The below-grade floor only reveals itself upon the exploration of the large void or from the second floor. Below grade is always open to the public and offers the only view by which to see the whole pavilion.

The pavilion not only fully utilizes the three known dimensions, it begins to examine fourth-dimensional implications as well. The pavilion is a model that begins to communicate the very exotic higher dimensional space in terms that are accessible to us – an inhabitable space in our three-dimensional realm.

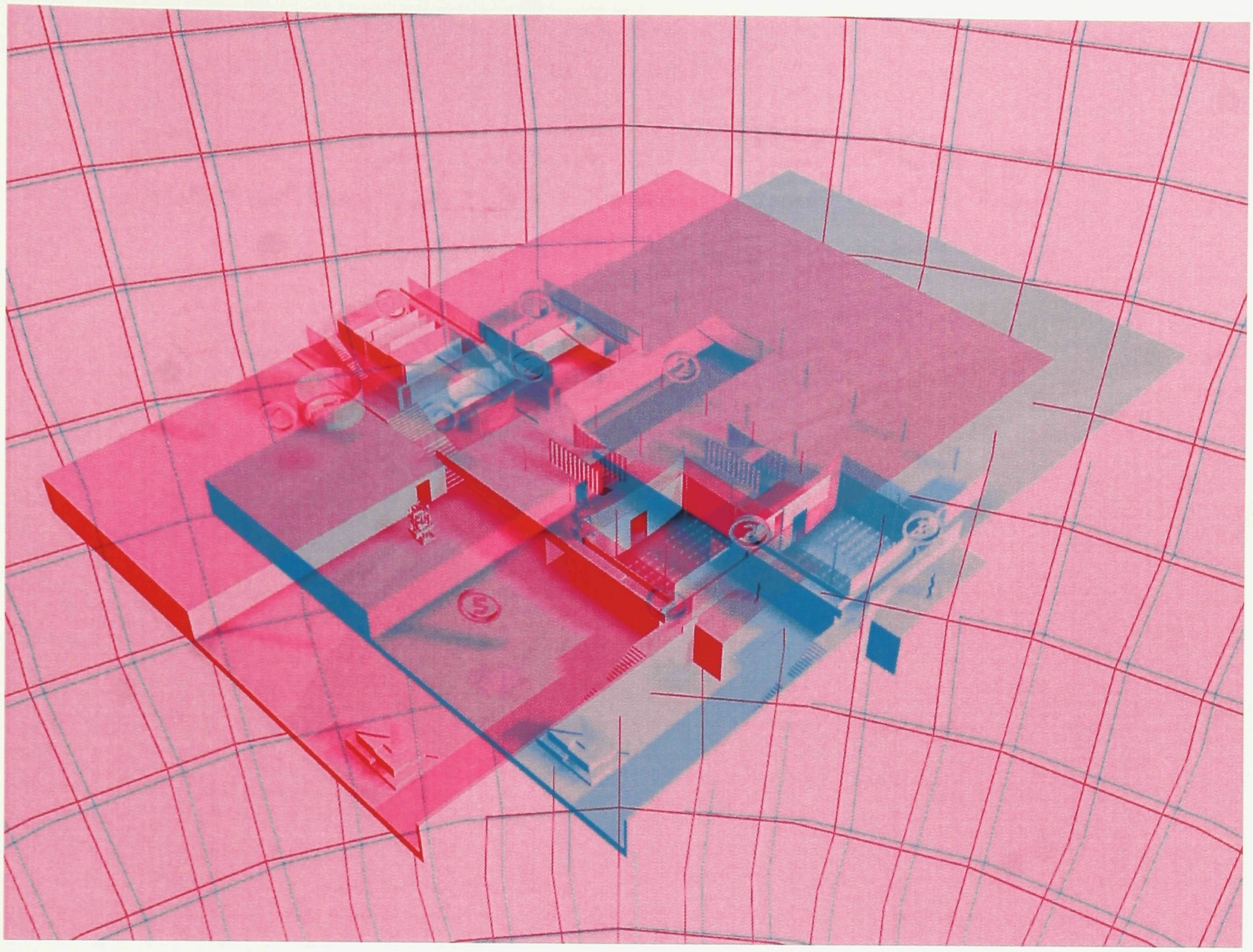


<u>whole building</u>



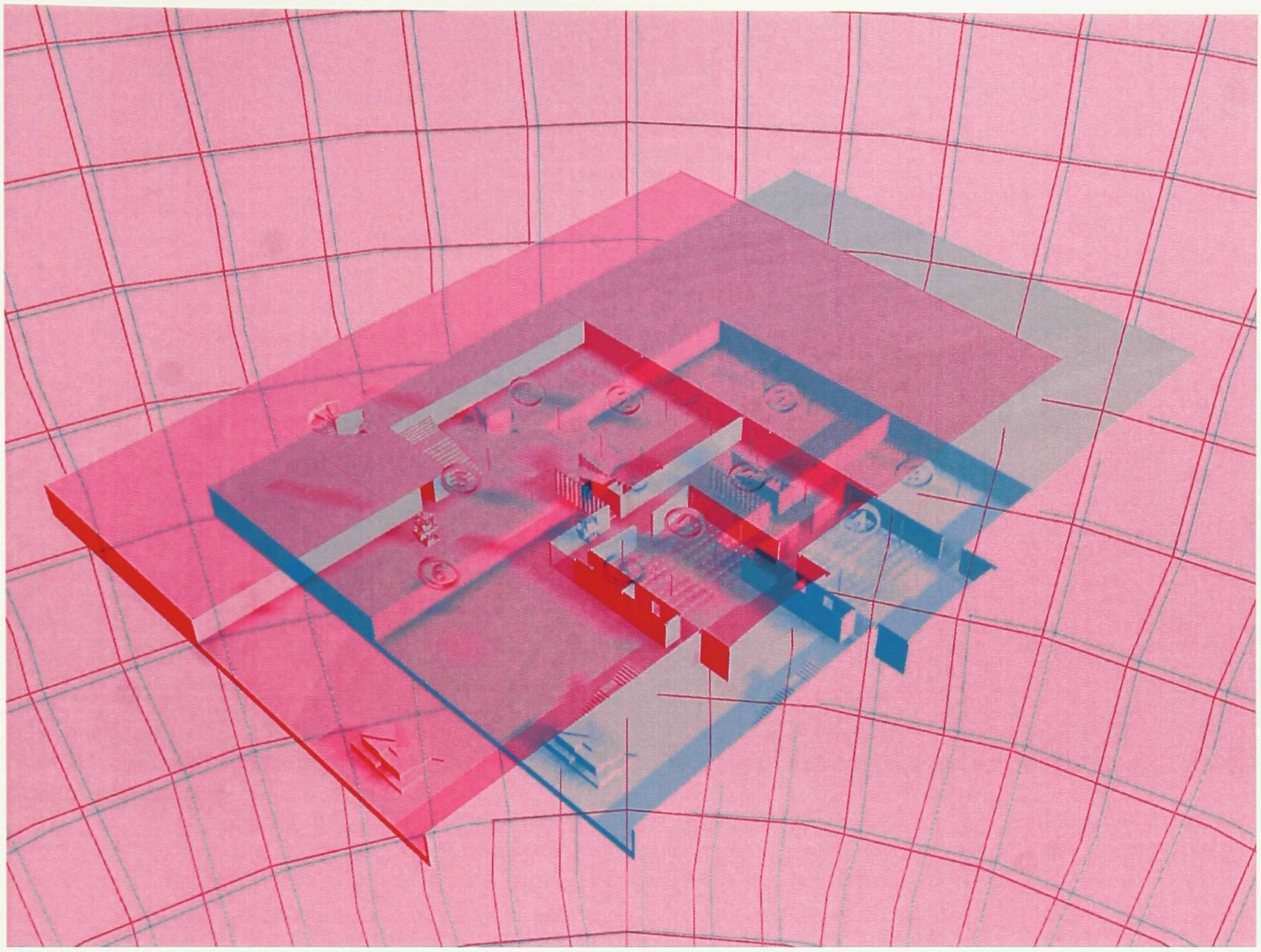
second floor

1) dining/boardroom
2) kitchen
3) lounge
4) sky portal
5) void
6) event space
7) balcony
8) sculpture garden



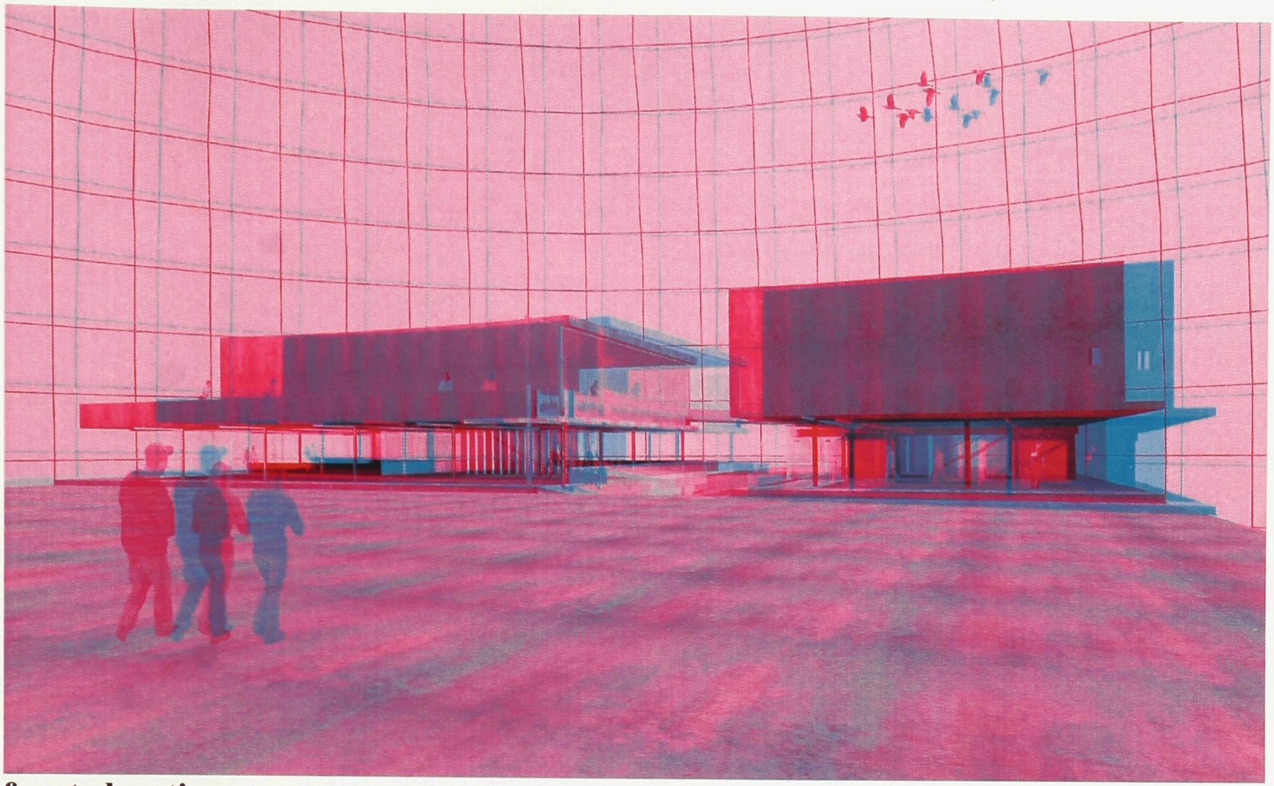
first floor

1) washrooms
2) void
3) Plato's wall
4) sky portal
5) sculpture garden

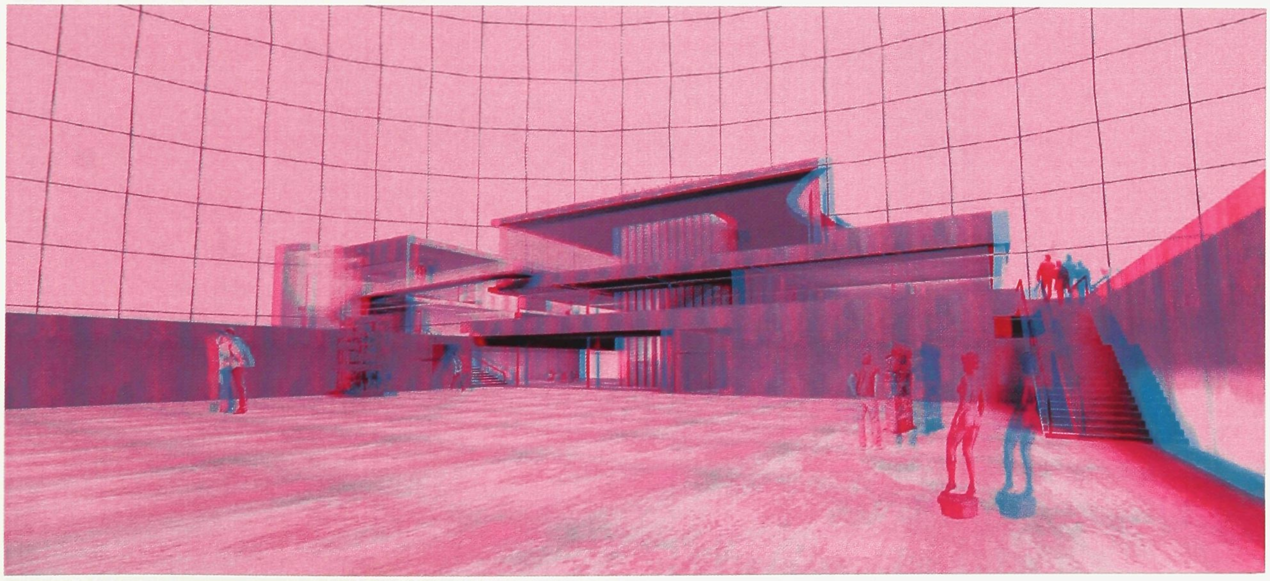


basement

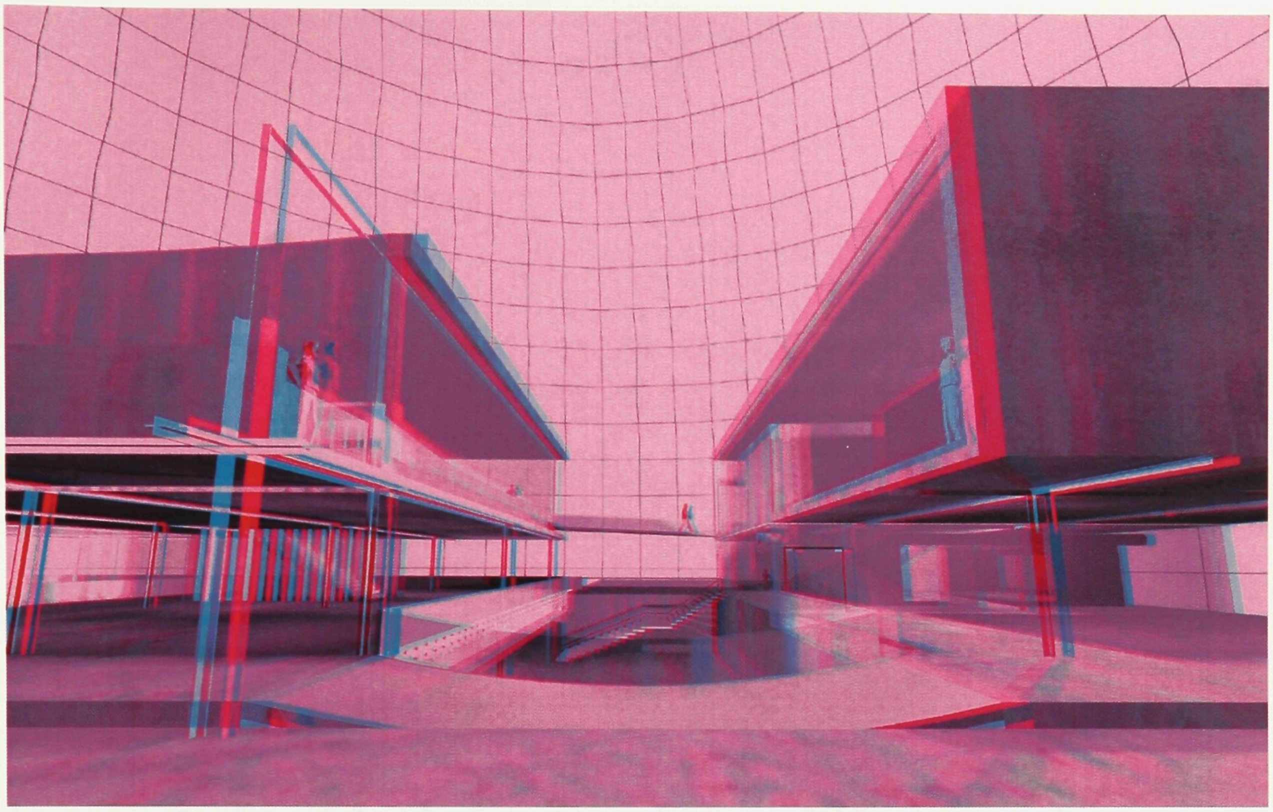
1) void
2) covered gallery
3) intermission space
4) lecture room
5) sky portal entrance
6) sculpture garden



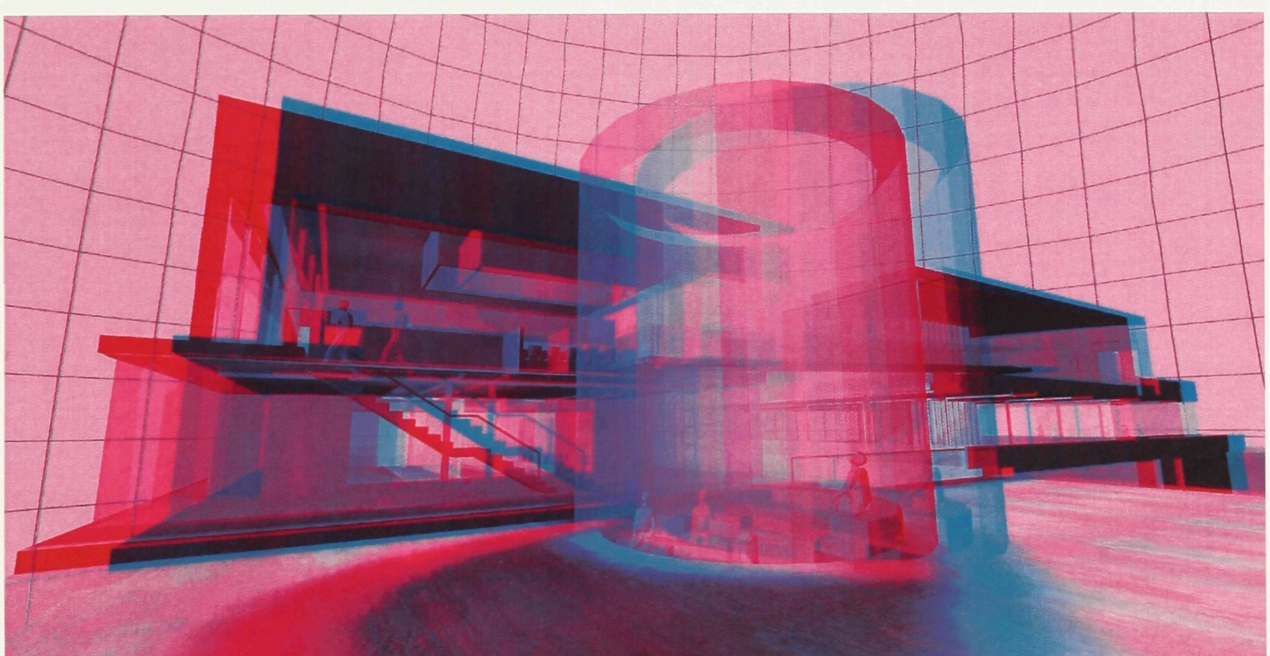
front elevation



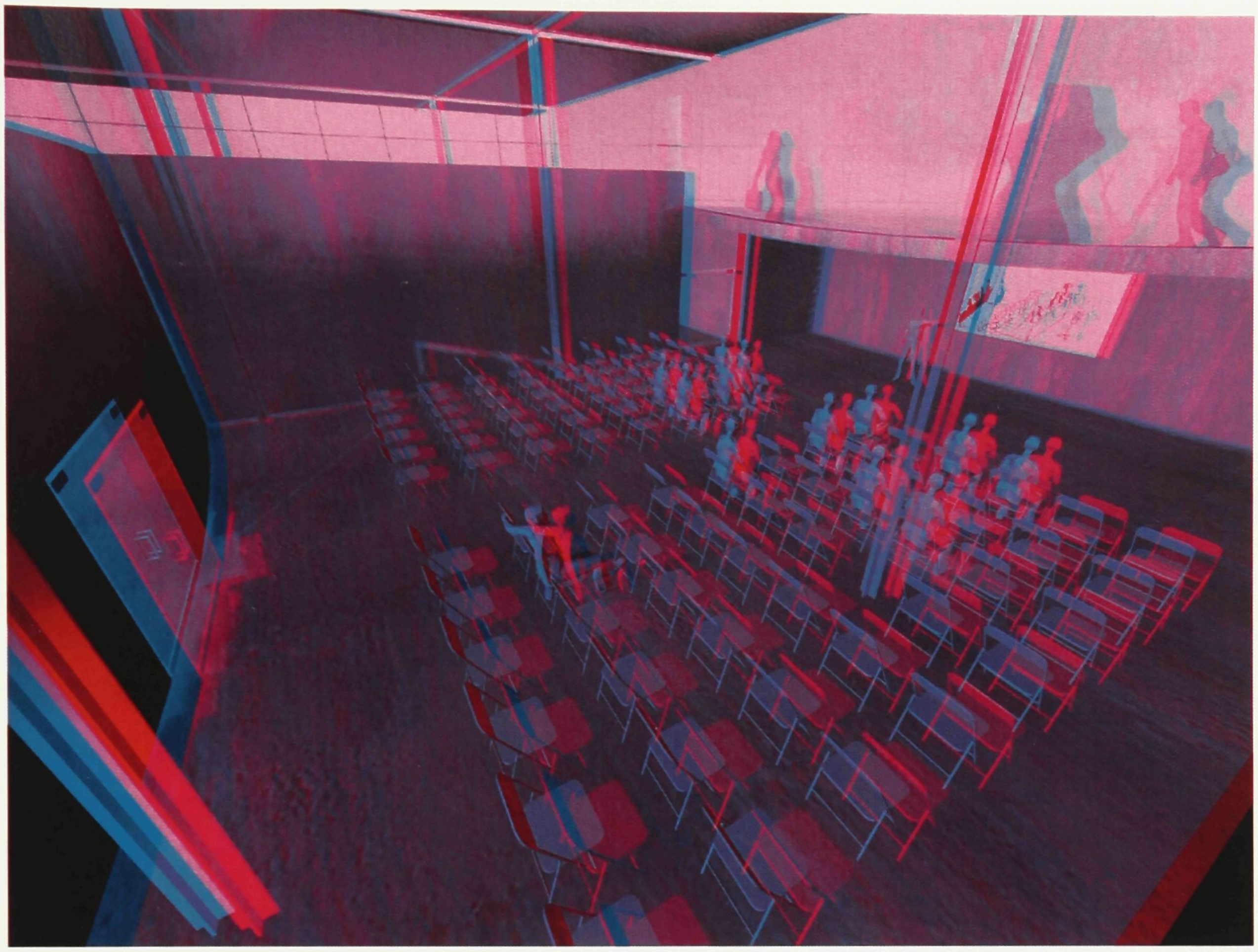
rear elevation view from sculpture garden



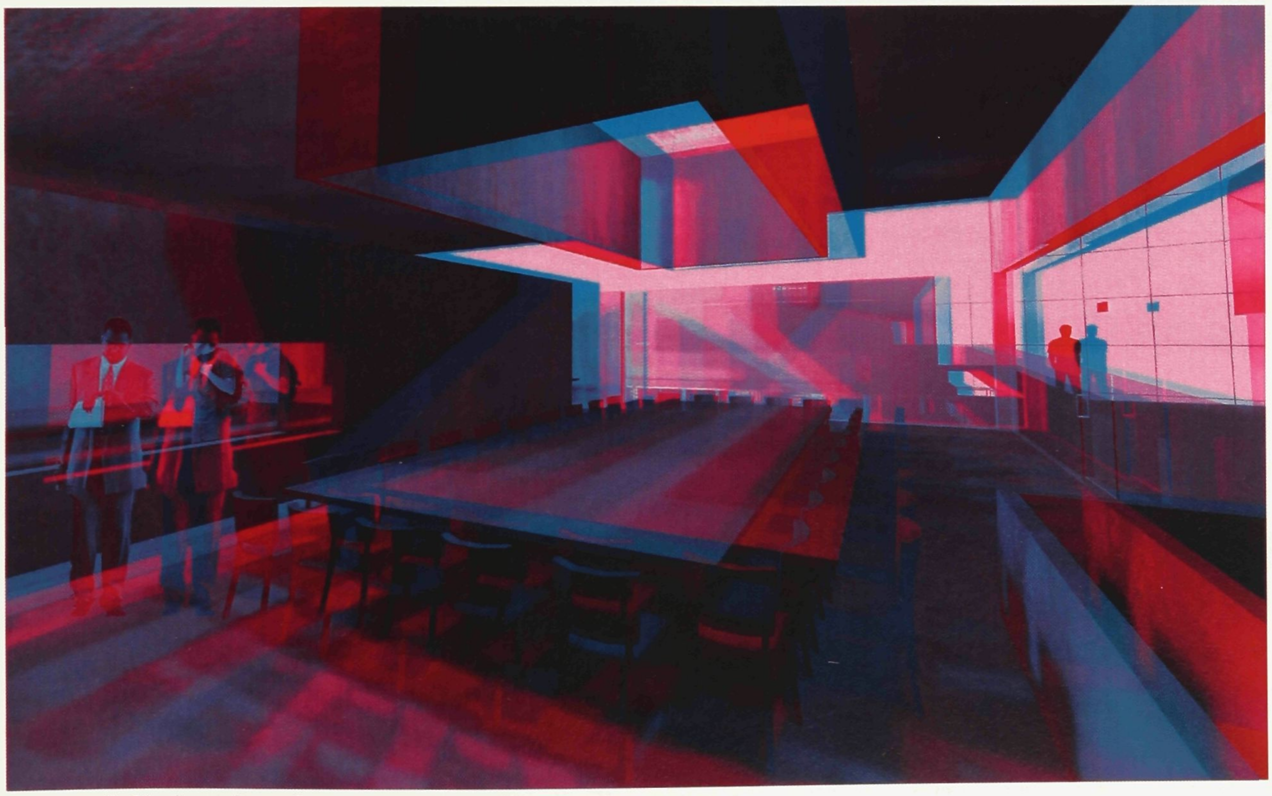
<u>**front view of void (main event space *left* and lounge to *right*)**</u>



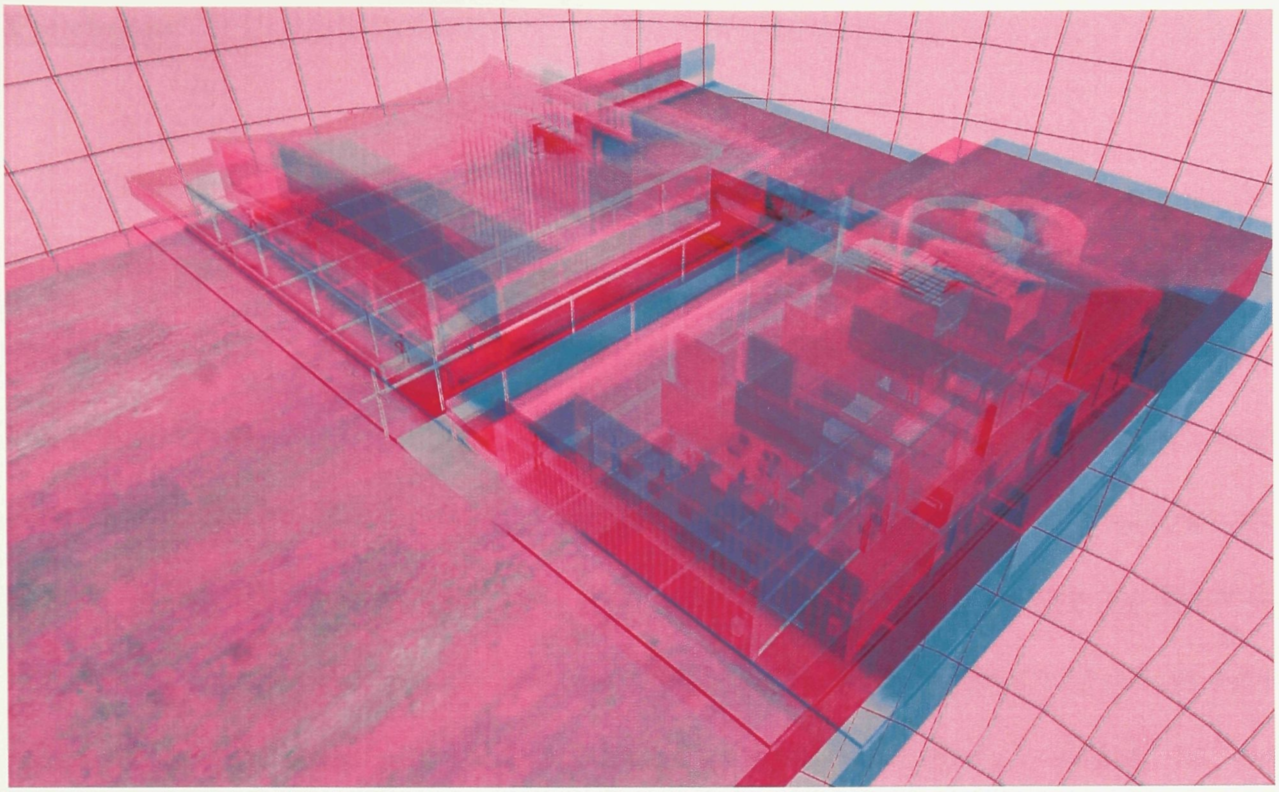
<u>**view of sky portal**</u>



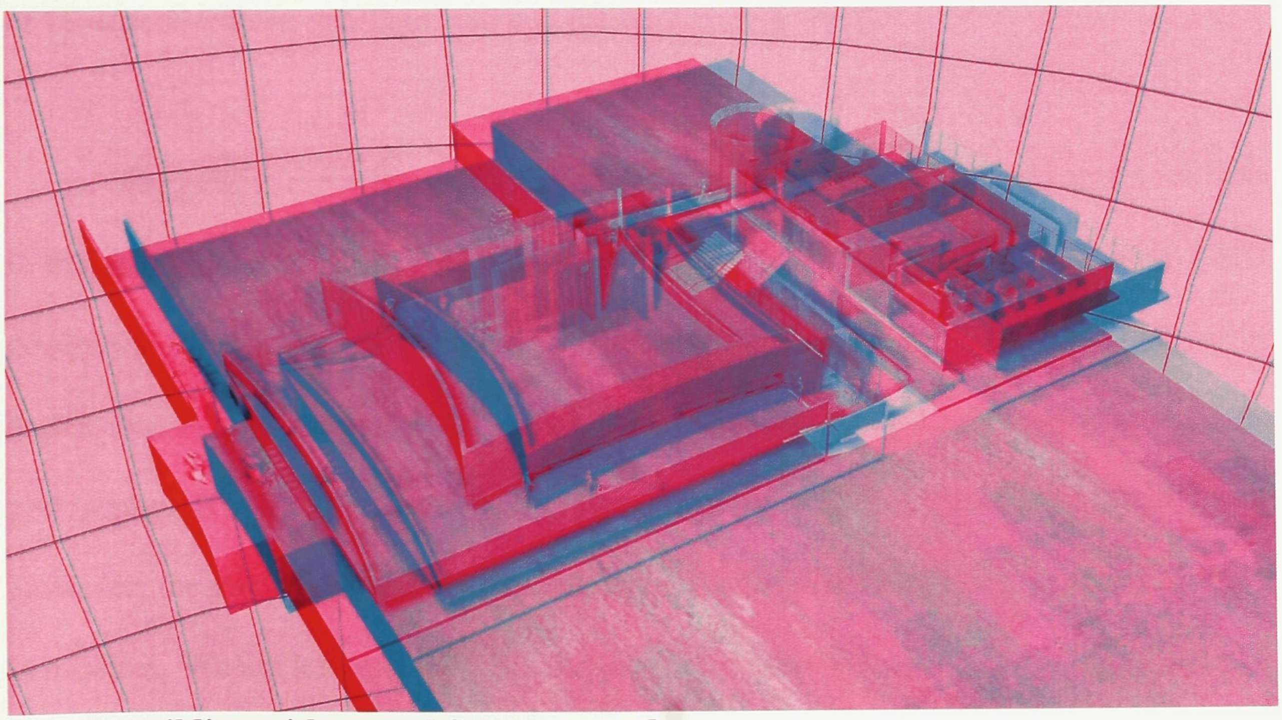
view of lecture hall (Plato's Wall)



view of dining /conference room



translucent view of building



view of building with second floor removed

Conclusion

This thesis began as an exploration of how to model the fourth dimension, as the fourth dimension is a difficult concept to grasp. I explored how to convey this higher spatial dimension from within the constraints of three-dimensional space. My examination revealed compression as a method of observing three-dimensional space as it exited into the fourth dimension, and the components of that event: the footprint, the crossover, the edge, and the breach. I outlined the different points of view by which to observe the compression and I discussed the potential of compression to create new space, of equal dimensions, in a different realm separated by a higher dimension. As an architect, I was duty-bound to examine the architectural implications of these components, points of view, and creations of new space. At its end, this thesis explores architectural applications in the form of architectural vignettes. Finally, the implication of these conventions was then tested in the form of a pavilion.

The pavilion is a successful vehicle that lets us observe space affected by actions in the fourth dimension. The pavilion is the result of compressions which lead to voids, compressed spaces, and new planes. It is a formal representation of the space created by higher dimensional compressions. I do realize that there is a slight irony to this design philosophy. By this I mean the void is intended to reveal the higher dimensional breach. However, the void's presence is by nature an absence. The very thing we are to examine is missing. We are left only with the remnants of the compressions: the translucency into the footprints from the edge, the crossovers and the concave walls, and original footprints all incorporated into the design. A visit to the pavilion would offer a new way to think of three-dimensional space, the theoretical space of the fourth dimension and I would hope to further inspire theories of the fourth dimension to more complex situations.

I limited my exercises to basic shapes and simple compressions. Also, the primary devices used in my design weighed heavily on the components and devices I researched and did not explore the phenomenological. However, I believe the fundamental ideas that carried my thesis would be readily applicable and could be taken further both in theoretical physics and architecture.

Plato's cave showed us that if there was a race of men bound and facing a wall since birth they would understand the shadows on that wall to be their entire world, a world of only two dimensions. We too may suffer the same limitations. Perhaps we are all just shadows of the fourth dimension.

RIGHT