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**Investigating the subcomponents of working memory
in multi-digit arithmetic problems**

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Abstract

Despite the consensus view that working memory and arithmetic are related, researchers have yet to incorporate the function of working memory into their cognitive architecture models of arithmetic. Two new cognitive models are discussed and used as the framework for linking working memory (*Tripartite memory model*) with multi-digit numerical cognition (*Integrated Calculation Encoding Strategy; ICES*). The ICES numerical cognition model assumes that a procedure and other numerical information are extracted from long-term memory and used with higher cognitive functions like encoding and calculation. The assumption of the *Tripartite memory model* is that mental information is temporarily stored and maintained independently in different memory subcomponents. Linking the *Tripartite memory model* with the ICES model, the assumption is made that extraction of numerical information from long-term memory requires working memory resources to temporarily store and maintain this information within their own respective memory subcomponents (phonological, visual or spatial).

Three experiments were designed to test the role of phonological (Experiment 1), visual (Experiment 2) and spatial (Experiment 3) memory components in solving multi-digit addition problems. Depending on the experiment, participants were required to remember a memory load (verbal, visual or spatial) while concurrently solving a problem (e.g., $326 + 49$). Half of the participants saw the problem presented horizontally and the other half saw the same problems presented vertically. After saying the answer out loud the participant then reproduced the memory load item. The results from Experiment 1 (phonological) indicated that people use phonological resources to solve horizontal

problems, thus suggesting people were using a verbal procedure as part of their calculation process. Experiment 2 (visual) results indicate that people use visual memory resources to maintain visual aspects of the intermediate results, which is not format sensitive. Experiment 3 (spatial) shows that people used spatial resources to solve vertical problems, thus suggesting people were using a spatial procedure as part of their calculation process. The present findings incorporate working memory resources with multi-digit number processing and support the assumptions of the ICES numerical model and the assumptions of the *Tripartite memory model*, respectively.

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I would like to also thank Chris Herdman for introducing me to cognitive psychology. I am grateful to have had Chris as a mentor for the first two years of my Ph.D. program and to have worked with him as his teaching assistant for four years. I am greatly honoured and privileged to have had Dr. Ashcraft to take time out of his busy life to come to Ottawa and participate in my thesis defense. I am also to Dr. Power and Dr. West for their time and contributions to my thesis. Finally, I appreciate the encouragement, support and associations that I have made with all of the members of Jo-Anne LeFevre's lab over the years.

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Introduction

Most people can mentally calculate sums of problems such as $5 + 6$, $45 + 29$, or even $459 + 345$. Mental calculation is effortful, however, and when people are confronted with arithmetic problems in real-world situations, they often rely on aids such as calculators. For instance, if a person gave a cashier \$10 for a \$7.17 lunch, they would expect to receive \$2.83 in change. Without using a calculator, it might be difficult for the buyer to know whether the cashier has given correct change if the change consisted of two dollars, three quarters, a nickel and three pennies. The buyer would need to add the values of the coins together, calculate a total sum, and then compare the sum with the change he or she had expected to receive (i.e., \$2.83). Performing this cognitive process requires recognizing the value of each coin (i.e., encoding the information), then adding these values together to get a total sum (i.e., processing the information), followed by deciding whether the person received the correct change (i.e., executing a response). These cognitive processes also require that the person mentally accesses information from long-term memory (i.e., the value of each coin) and is able to temporarily maintain a running total in working memory. This example illustrates that mental arithmetic invokes processes that are common to a wide variety of cognitive activities and thus, it is a good forum within which to study issues of cognition.

The cognitive processes and memory resources relative to numerical cognition also represent the fundamental functions common to all types of information processes, thus making numerical cognition a viable area of research for understanding the cognitive architecture of information processing and related memory functions. In the example

above, the mental calculations used to process the change received will most likely involve working memory resources (e.g., Fürst & Hitch, 2000; Heathcote, 1994; Logie et al., 1994; Noel, Aubrun, Desert, Seron, 2001). The mental organization of simple arithmetic facts in memory has been documented fairly extensively (e.g., Ashcraft, 1987; Campbell, 1987; Dehaene, 1992; Groen & Parkman, 1972; Siegler, 1988), however, the role of working memory in solving multi-digit arithmetic problems has received less attention (see LeFevre, DeStefano, Coleman & Shanahan, in press). The literature on multi-digit arithmetic problems and working memory indicates that working memory is involved with encoding problems (horizontal vs. vertical operands; Heathcote, 1994; Trbovich & LeFevre, 2003), with retaining the partial sum (Logie et al., 1994; Noel et al., 2001) and processing carry problems (Fürst & Hitch, 2000). The involvement of working memory seems to be an integral aspect of numerical cognition (e.g., Ashcraft, 1995; DeStefano & LeFevre, 2004; LeFevre et al., in press), thus making numerical cognition and working memory a viable area of research for understanding the cognitive architecture of information processing and related memory functions. Therefore, the following experiments were designed to examine the relation between working memory and multi-digit arithmetic problems.

Multi-digit Arithmetic Problems and Working Memory

Complex mathematics involves performing mental tasks such as arithmetic, algebra, calculus, and word problems. Although the complexity of these tasks may vary, the underlying cognitive processes are similar. That is, depending on the arithmetic task, people still need to encode (i.e., visually or verbally) the numerical information, and

process this information (i.e., calculation or verification) before producing a response (e.g., Campbell & Clark, 1988, 1992; Dehaene, 1992). For instance, if a person were asked to solve a multi-digit addition problem like $(34 + 58)$, this person would first encode the problem, and then use the encoded information to calculate the sum (e.g., $34 + 58$; $8 + 4 = 12$; retain the partial sum of 2 and add the carry to $3 + 5 + 1 = 9$) before producing an answer of 92. For most people mentally processing a multi-digit problem is slow and intentional, and without assistance from working memory this information may be lost to decay or interference before the calculation is completed (e.g., see Baddeley, 1986, 1990; Hitch, 1980; Logie, 1995). Therefore, temporal mental resources (i.e., phonological and visual-spatial memory subsystems) and careful attention (i.e., central executive control) to processing the numbers should be involved with solving multi-digit arithmetic problems (e.g., Ashcraft 1995; LeFevre et al., in press).

The literature shows that solving arithmetic problems involves central executive control and other potential memory resources depending on the task (see DeStefano & LeFevre, 2004). However, supporting evidence for a role of phonological and visual-spatial resources in multi-digit arithmetic remains contradictory and inconsistent. Researchers have examined the relation between working memory and multi-digit numbers with different presentation formats as shown in Table 1 (i.e., horizontal vs. vertical; Heathcote, 1994; Trbovich & LeFevre, 2003). Trbovich and LeFevre (2003) found that horizontal problems required phonological resources, whereas Heathcote (1994) found that horizontal problems required visual-spatial resources. In contrast, Trbovich and LeFevre (2003) found that *vertical* problems required visual resources.

Thus, the relation of working memory resources with solving multi-digit problems presented in different formats is unclear.

Table 1

Examples of presentation format of multi-digit numbers

Horizontal	Vertical
$357 + 42$	357 $+ 42$

Results from research on different presentation formats (e.g., horizontal vs. vertical) also shows that multi-digit problems presented vertically are solved faster (and with less errors) than the same problem presented than horizontally (Heathcote, 1994; Trbovich & LeFevre, 2003). A reasonable explanation for this result might be that people are more familiar with the calculation procedure for solving multi-digit problems presented vertically than horizontally. For example, a familiar calculation procedure for most people solving $63 + 29$ in a vertical format is to start with adding $3 + 9$, retain the partial sum (2), and then add the carry (1) to $6 + 2$ in order to produce an answer of 92. However, when a problem is presented horizontally, people may be forced to use a less familiar calculation procedure for solving the same problem. For instance, a person solving $63 + 29$ in a horizontal format may add $63 + 30$ (93) and subtract one from this partial sum before answering with 92. Therefore, horizontal problems may take people

longer to solve than vertical problems because they are forced to use a calculation procedure that is not their typical procedure for solving problems presented in the familiar vertical format.

The evidence on multi-digit numbers remains controversial because some researchers suggest that visual-spatial resources are involved with solving carry problems (Hayes, 1973; Heathcote, 1994), whereas others have indicated that visual resources are not involved with solving carry problems (Logie et al., 1994). Other evidence indicates that the phonological resources are associated with calculation by temporarily retaining the partial sum (Fürst & Hitch, 2000; Heathcote, 1994; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). Hence, the evidence on whether phonological and visual-spatial memory resources are related with solving a multi-digit arithmetic problem needs further investigation. Investigating the relation of working memory with multi-digit number processing will provide insight on the cognitive architecture of the working memory system and the numerical cognition system. Furthermore, understanding the association between working memory and number processing is important to our understanding of how to better evaluate cognitive strengths or deficits that people might have when they process multi-digit, real-world tasks.

Multi-component Model of Working Memory

Although the prominent working memory models vary on a number of dimensions (e.g., see Miyake & Shah, 1999), the commonality among these models is that they describe how information is encoded into the working memory system, and how the system temporarily maintains this information. Working memory has been associated

with a variety of everyday tasks that involve the temporary maintenance and processing of information (Baddeley, 1986, 1990, 1996; Baddeley & Hitch, 1974; Baddeley & Logie, 1999; Bull & Johnston, 1999; Hitch & McAuley, 1991; Logie, 1995; Logie & Pearson, 1997; Pearson, Logie, & Green, 1996; Pickering, Gathercole, Hall & Lloyd, 2001; Quinn, 1991; Salamé & Baddeley, 1982; Robbins et al., 1996; Vecchi & Cornoldi, 1999).

The most recent working memory model proposed by Baddeley (2002) includes four components: the central executive, the episodic buffer, the phonological loop, and the visual-spatial sketchpad (see Figure 1). The central executive is assumed to have a variety of functions related to managing the storage and processing of cognitive resources (Baddeley, 1986, 1990; Baddeley & Logie, 1999). Baddeley (1986) suggested that the function of the central executive was to act as a general-purpose system for supervising and coordinating the activities of the memory subcomponents (i.e., phonological loop, visual-spatial component), and to act as a selective filter, possibly controlling the locus of attention for encoding information. Furthermore, the central executive is also assumed to have the capability of manipulating and selecting information from long-term memory. Information retrieved from long-term memory information can be used by the central executive to carry out administrative functions (e.g., using conceptual or procedural algorithms) or to manipulate the visual-spatial codes (VSSP), or verbal codes (Phonological loop), or episodic codes before storing this new information back into long-term memory (e.g., Baddeley, 1996, 2002).

Figure 1. Baddeley's Multi-component model of working memory

Baddeley (2002) suggested that the episodic buffer is responsible for temporarily storing and integrating information from a variety of resources that have been activated by the central executive (e.g., from LTM, phonological loop, visual-spatial sketchpad, perception). Furthermore, Baddeley (2002) explains that the episodic buffer is, "...limited in capacity because of the computational demand of providing simultaneous access to the necessarily wide range of different codes" (p.421). Although the episodic buffer is a memory component that is necessary for integrating and manipulating new information or accessed information (LTM), there is not yet much empirical evidence supporting such a component of working memory.

The literature does show that, within the phonological loop, verbal information is passively stored as phonological codes that require active rehearsal (Baddeley, Lewis, &

Vallar, 1984; Baddeley, 1996). Without rehearsal, these verbal codes rapidly decay from memory (Baddeley, 1990; Baddeley, Chincotta, & Adlam, 2001; Baddeley & Logie, 1999). For example, Baddeley (1986) had participants verbally rehearse multiple sequences of nine digits. The participants were asked to recall and write each sequence on either the left side or right side of the paper. In one condition, the participants were told on each trial the digit sequence followed by a brief delay before being instructed as to which side of the paper to write the digit sequence on. In another condition, before hearing the digit sequence of the first trial, participants were told which side of the paper to write the digit sequence on and then to alternate sides for the remaining trials. Baddeley (1986) found that participants were better at recalling the digit sequence when they could verbally rehearse the digit sequence (i.e., alternating condition) than when verbal rehearsal was disrupted by instructions of where to write the sequence on the paper (i.e., delayed condition). Baddeley (1986) suggested that if phonological codes are not actively rehearsed then they will become subject to rapid decay. Therefore, one function of the phonological loop is to actively rehearse verbal codes that are passively retained in a temporary phonological storage buffer. Conversely, within the visual-spatial sketchpad the visual component is believed to passively store visual codes, whereas the spatial component is assumed to keep these visual codes from decaying from memory (Logie, 1995).

In the visual-spatial sketchpad component both visual and spatial information is maintained and stored as visual-spatial codes (Baddeley & Hitch, 1974; Baddeley, 1986; Logie, 1995). Logie (1995) suggested that visual information has obligatory access to the

VSSP where visual information is passively stored in the visual component and actively rehearsed by a spatial component called the inner scribe. The inner scribe actively maintains the spatial location of objects that are stored within the visual component and keeps the visual codes from decaying. Furthermore, the VSSP memory component is believed to be independent from the phonological loop. For example, Logie (1986) explored visual-spatial processes using a visual suppression technique in conjunction with visual and verbal memory tasks. In one dual-task condition, participants had to remember the previously presented visual pattern while concurrently viewing other randomly presented visual patterns (i.e., visual suppression task). In another dual-task condition, participants were asked to remember a visual pattern while concurrently repeating the word "the" (i.e., articulatory suppression task). In both dual-task conditions participants were asked to reproduce the original visual pattern after the intervening task. Logie (1986) found that the visual suppression task disrupted visual memory processes more than verbal memory processes. This dissociation supports the view that visual-spatial memory processes are independent from phonological memory. Similarly, recent evidence supports a dissociation of the visual component from the spatial component of the VSSP (e.g., Coleman & LeFevre, 2002; Della Sala, Gray, Baddeley, Allamano, & Wilson, 1999; Hecker & Mapperson, 1997; Quinn & McConnell, 1999; McConnell & Quinn, 2000), thus this evidence supports the argument that visual and spatial codes are maintained and stored within different working memory subcomponents.

Fractionating the Visual-spatial Sketchpad

Application of Baddeley and colleagues' multi-component working memory model (e.g., Baddeley, 2002; Baddeley & Hitch, 1974; Baddeley & Logie, 1999) to cognitive arithmetic works reasonably well (see LeFevre et al., in press) as outlined above. As I was formulating a model of how working memory and cognitive arithmetic might be related, it became clear that Baddeley's working memory model needed to be modified. Much of the research on the subcomponents of working memory has used a dual-task paradigm in which selective interference tasks (e.g., verbal, visual, or spatial) are used in combination with content-specific memory tasks (e.g., verbal, visual, or spatial). To the extent that, for example, verbal interference tasks interfere with verbal memory whereas visual interference tasks interfere with visual memory, the two memory components are assumed to be independent. Support for a double dissociation between the phonological loop and the VSSP comes from research on verbal interference tasks in which continuous articulation of a word interferes selectively with the storage of verbal information (Baddeley, 1986; Logie, 1995). In contrast, only a few researchers have explored the relations between visual and spatial memory. The literature indicates that visual and spatial memory components are independent subcomponents of the working memory system (e.g., Coleman & LeFevre, 2002; Della Sala et al., 1999; Logie & Marchetti, 1991; Tresch, Sinnamon, & Seamon, 1993).

Logie (1995) proposed the VSSP included two interdependent components, the *visual cache* and the *inner scribe* (Baddeley & Logie, 1999; Della Sala, et. al., 1999; Hecker & Mapperson, 1997; Logie, 1995; Logie & Marchetti, 1991; Quinn &

McConnell, 1999). According to Logie, the visual cache is a passive store that is vulnerable to temporal decay and to interference from irrelevant visual material. In contrast, the inner scribe provides storage for spatial information plus a rehearsal mechanism for both visual and spatial information (Baddeley & Lieberman, 1980; Quinn, 1994; Smyth & Pendleton, 1989). Logie (1995) suggested that while the visual cache and the inner scribe store visual and spatial information independently of one another, the maintenance of that information through rehearsal involves a single, shared mechanism. On this view, visual interference only affects the contents of the visual component, whereas spatial interference will disrupt the rehearsal mechanism of the inner scribe, resulting in decay of information from both visual and spatial components.

Much of the research on the subcomponents of working memory has used a dual-task paradigm in which selective interference tasks (e.g., verbal, visual, or spatial) are used in combination with content-specific memory tasks (e.g., verbal, visual, or spatial). To the extent that, for example, verbal interference tasks interfere with verbal memory whereas visual interference tasks interfere with visual memory, the two memory components are assumed to be independent. Support for a double dissociation between the phonological loop and the VSSP comes from research on verbal interference tasks in which continuous articulation of a word interferes selectively with the storage of verbal information (Baddeley, 1986; Logie, 1995). In contrast, only a few researchers have explored the relations between visual and spatial memory. The literature indicates that visual and spatial memory components are independent subcomponents of the working memory system (e.g., Coleman & LeFevre, 2002; Della Sala et al., 1999; Logie &

Marchetti, 1991; Tresch, Sinnamon, & Seamon, 1993).

Della Sala et al. (1999) found a double dissociation between spatial and visual working memory. The primary visual memory task required that participants retain a pattern of black and white squares whereas the primary spatial memory task required that they retain a sequence of locations (i.e., the Corsi blocks task; Corsi, 1972). The secondary visual interference task involved viewing colourful abstract pictures and the secondary spatial interference task required tapping by participants on protruding pegs of a wooden board that was placed behind a screen. Under dual-task conditions, the participants viewed the primary memory task for three-seconds, performed one of the interference tasks for 10 s, and then reproduced the primary memory task. Della Sala et al. (1999) found that viewing the abstract pictures interfered more with retaining a visual pattern than with retaining a spatial sequence. Conversely, tapping on the pegboard interfered more with retaining a spatial sequence than with retaining a visual pattern. This evidence suggests that visual and spatial working memory resources can be dissociated from one another. However, Della Sala et al. (1999) reported only a single experiment with a small number of participants, hence their results may not be reliable.

Logie and Marchetti (1991) also explored the relations between visual and spatial memory using a dual-task approach. One group of participants was given a visual memory task in which they retained colours of four squares. Another group was given a spatial memory task in which they retained the sequential order of six squares. Following the primary memory task, the participant either performed active arm movements (i.e., spatial interference), viewed drawings of common objects or animals (i.e., visual

interference), or looking at a blank screen (i.e., control condition) during a 10 s retention interval. Logie and Marchetti found a double dissociation effect that showed remembering colours interfered with visual memory and arm movements interfered with spatial memory. This result supports the view that spatial and visual memories are functionally distinct. However, Logie and Marchetti's visual memory task may have involved verbal coding so that the participants may have labeled the colours and retained them in the phonological loop. On that view, the dissociation may have been between spatial memory and the phonological loop.

Tresch et al. (1993) used a somewhat different paradigm to test whether spatial and visual memories are independent. For the visual memory tasks, participants were required to remember eight common objects (e.g., square, circle) whereas for the spatial memory task, they remembered the location of a dot on the screen. The visual and spatial interference tasks were colour discrimination and movement discrimination, respectively. In accord with Della Sala et al. (1999) and Logie and Marchetti (1991), Tresch et al. (1993) found a double dissociation between visual and spatial memory: Colour discrimination interfered with visual memory performance whereas movement discrimination interfered with spatial memory performance. One limitation of Tresch et al.'s visual task, however, was that the shapes had well-known verbal labels. Thus, although the results of these experiments are in accord with the view that spatial and visual memory stores are independent, they provide only a small body of research.

Coleman and LeFevre (2002) used a dual-task paradigm to examine the relation between the maintenance and retention of visual and spatial information. Logie (1995)

proposed that the inner scribe performed two functions; retention of spatial information and rehearsal of visual information. Spatial interference was expected to interfere with both visual and spatial memory performance because it would disrupt the rehearsal component of the inner scribe. In contrast, visual interference was expected to disrupt only visual memory performance, presumably by overwriting the contents of the visual component. Participants were asked to remember a visual pattern of black and white squares for the primary visual memory task, whereas they were asked to remember the correct sequence of highlighted blocks (Corsi, 1967) for the primary spatial memory task. On each trial the participants were shown a primary task followed by 10 s retention interval. During the retention interval, participants performed a secondary memory task that was either related or unrelated to the primary memory task before being asked to recall the pattern or sequence. In the first two experiments, the visual secondary task required participants to stare at abstract coloured pictures for 10 s, whereas the spatial secondary task required participants to tap on a pegboard with their hand placed behind a screen (cf., Della Sala et al., 1999). In both experiments, participants performed worse on the sequence task while they were tapping, as compared to the no-tapping control, whereas memory for a visual pattern was not affected by the abstract picture. This evidence did not support the idea of a dissociated VSSP component.

In Coleman and LeFevre's (2002) third experiment, however, participants performed the same spatial interference task, but the visual interference task was replaced with a new visual interference task. This task required participants to visually scan a display of colourful dots for the letters X and O hidden amongst the dots under dual-task

conditions. The results from this study show that visual scanning interfered with the participants' retention of visual patterns and haptic tapping interfered with their retention of a sequence of highlighted blocks. This evidence supports a double dissociation effect between visual and spatial working memory components and, furthermore, indicates that both components have independent mechanisms for storing and maintaining visual or spatial codes, respectively. Therefore, based on the results of these experiments (Coleman & LeFevre, 2002), I propose a modified version of Baddeley's multi-component working memory model below.

Tripartite Working Memory Model

Logie (1995) proposed that the visual cache and inner scribe are independent storage components that share a common rehearsal mechanism. This view may be incorrect because some evidence supports a double dissociation between the visual cache and the inner scribe (e.g., Coleman & LeFevre, 2002; Della Sala et al., 1999; Hecker & Mapperson, 1997; McConnell & Quinn, 2000; Quinn & McConnell, 1999). Although the characteristics of these storage and rehearsal mechanisms are not well understood (e.g., Pickering et al., 1999; Quinn & McConnell, 1999; Zimmer & Speiser, 2002), emerging evidence does appear to support dissociable memory components between the visual cache and inner scribe, which is compatible with the evidence for a dissociation between the phonological loop and the VSSP (e.g., Baddeley & Hitch, 1974; Baddeley, 1996; Logie, 1986). On this view, the present hypotheses were based on the *tripartite memory model* shown in Figure 2 below. That is numerical aspects related to the problem solving process that are visual, verbal or spatial are assumed to be temporarily retained within

their own respective memory subcomponents. There are three independent memory subcomponents (i.e., visual, phonological and spatial) under the control of the central executive. These three memory components are assumed to be independent, to the extent that phonological interference tasks interfere with phonological memory, visual interference tasks interfere with visual memory, and spatial interference task interfere with spatial memory. On this point, I will assume that the numerical aspects of briefly presented problems and intermediate results that are verbal, visual or spatial will be stored and maintained within their own respective memory subcomponents. Therefore, in this paper I tested the assumption that all three subcomponents of working memory are differentially involved with solving multi-digit arithmetic problems like $345 + 67$.

Figure 2. Modified version of Baddeley's multi-component model

Working Memory and Multi-digit Arithmetic

In the present experiments I studied multi-digit addition problems consisting of three-digits plus two-digits (e.g., $423 + 59$). Three types of problems were used: no-carry, carry-unit, and carry-ten. No-carry problems (e.g., $423 + 42 = 465$) were matched with carry-unit (e.g., $42\bar{6} + 3\bar{9} = 465$) and carry-ten problems (e.g., $3\bar{7}2 + \bar{9}3 = 465$) by sums. Problems like these are relatively challenging to mentally solve because they require multiple procedural steps to solve the problem. That is, because people are rarely able to retrieve a problem like $372 + 93$ from memory, they must break the problem down into a series of calculation steps. For instance, a person solving $372 + 93$ might begin by adding the units ($2+3$) and then retain the partial sum of five in memory before adding the decades ($7+9$). While rehearsing the partial sum of five, the sum of 16 is retrieved from memory and then partial sum is updated to 65. In the final step, the carry is then added to the hundreds ($1+3$) to give a total sum of 465. Therefore, it is likely that they will rely on working memory resources to help maintain calculated information while the procedural steps are executed.

More research has been done on understanding the working memory system and its influence on other cognitive tasks such as reading (e.g., Daneman & Carpenter, 1980, 1983; Daneman & Merikle, 1996), in comparison to its influence on mental arithmetic (see DeStefano & LeFevre, 2004; LeFevre et al. in press; Smyth, Collins, Morris, & Levy, 1994). Working memory is likely to be involved in the early stages of mental arithmetic where information is encoded verbally or visually (e.g., Heathcote, 1994; Logie et al., 1994; Siegel & Linder, 1984) and in the maintenance of information in

further processes such as calculation (Logie & Baddeley, 1987). Research on the encoding processes of numbers has implicated the central executive when participants process both single-digit (e.g., Adams & Hitch, 1997; Logie & Baddeley, 1987) and multi-digit problems (Fürst & Hitch, 2000; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). Phonological memory resources are assumed to be involved with retaining partial sums (Logie et al., 1994), and the verbal information representing the digits of the problem (i.e., operands; brief vs. extended; Fürst & Hitch, 2000; Heathcote, 1994). Also, both phonological and visual-spatial resources are believed to be necessary when encoding problems presented in different formats (vertical vs. horizontal; Heathcote, 1994; Trbovich & LeFevre, 2003).

Encoding: VSSP and Phonological loop. The format in which arithmetic problems are presented to participants seems to influence their numerical cognition. That is, participants usually make more errors and take longer to respond to multi-digit problems presented horizontally than problems presented vertically (see examples in Table 1 above; Heathcote, 1994; Trbovich & LeFevre, 2003). For example, Heathcote (1994; Experiment 2) attempted to clarify the role of spatial working memory in maintaining the spatial codes of the arithmetic problem. On the assumption that participants might mentally re-align horizontal problems into a vertical format (Hitch, 1978), solving these problems would use visual-spatial resources. The multi-digit problems were double-carry problems (e.g., $357 + 567$) presented horizontally or vertically. Each problem was presented in the middle of the computer screen for approximately one-second. The participants were required to produce a verbal answer.

Heathcote found that participants solved horizontal problems more slowly than vertical problems. From this evidence, he suggested that the VSSP was involved with solving arithmetic problems that required participants' to mentally re-arrange the operands into a more familiar format (i.e., vertical format; also see Hayes, 1973; Hitch, 1978). This finding suggests that when people encode a horizontal problem they use a mental procedure (i.e., possibly re-aligning the operands into a vertical format) that uses visual-spatial resources.

Trbovich and LeFevre (2003) examined phonological and visual-spatial memory performance on multi-digit problems (e.g., $34 + 7$) that were presented either vertically or horizontally to participants under dual-task conditions. The phonological memory loads were either a single consonant-vowel-consonant (CVC) nonsense word (e.g., MUP) or three CVC nonsense words (e.g., FAC MUB LER). Phonological memory probes that were close to the original memory item (MIP if MUP were the original) were used to test whether their participants were using phonological coding. The visual-spatial memory loads were patterns of four asterisks or eight asterisks arranged in a transparent 5×5 grid. Visual-spatial probe patterns were used to ensure that participants were using visual-spatial codes. The probe patterns were either the same as the original pattern or different than the original. At the beginning of each trial, the participant saw the memory load item for a few seconds, followed by a multi-digit addition problem. After the participant answered the problem, the operands disappeared from the screen and the participant was shown a memory probe. They verified whether the probe matched the original memory load. Trbovich and LeFevre found that under phonological memory loads, participants

made more errors when solving horizontal problems than vertical problems. In contrast, vertical problems were only moderately affected by phonological memory loads. This evidence suggests that horizontal problems required phonological resources. However, this evidence conflicts with Heathcote's (1994) suggestion that horizontal problems require visual-spatial resources. Trbovich and LeFevre (2003) suggested that participants may rely on using a verbal procedure (e.g., decomposition) to solve the problem. Therefore, Trbovich and LeFevre's (2003) evidence suggests that when people encode a horizontal problem they are more likely to use a mental procedure that requires phonological resources.

When the memory load was a visual pattern, participants in Trbovich and LeFevre (2003) made more errors when the problems were presented vertically than horizontally. This evidence suggests that visual-spatial memory may be involved with solving vertical problems. Possibly, as Trbovich & LeFevre suggested, vertical format activates a visual procedure (i.e., columnar algorithm) in the VSSP. This evidence also conflicts with the suggestion that VSSP resources are used with processing horizontal problems and not vertical problems (Heathcote, 1994). However, because the problem remained visible on the screen in Trbovich and LeFevre (2003), it was unclear as to why a visual representation of the problem would need to be activated in the VSSP. Furthermore, participants' errors on horizontal problems also increased with visual memory load, thus suggesting that visual-spatial resources are not limited to a particular presentation format or not required at all. Perhaps visual-spatial resources were used if their participants

were visualizing certain aspects of the calculation process, such as the carry (see Hayes, 1973), rather than activating a visual representation of the problem at encoding.

Calculation: VSSP and Phonological loop. After encoding the arithmetic problem, people then must perform further mental calculations to produce the answer to the problem. Calculation of multi-digit problems may involve direct access to basic facts (e.g., $2 + 3$ in $52 + 83$) and stored procedural knowledge (Ashcraft, Donley, Halas, & Vakali, 1992; Hecht, 1999, 2002; LeFevre et al., 1996a, 1996b). Research on mental calculation has implicated the involvement of the central executive in solving single-digit carry problems (De Rammelaere, Stuyven & Vandierendonck, 1996; 2001a, 2001b; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000), and in processing multi-digit carry problems (Fürst & Hitch, 2000; Hitch, 1978; Logie et al., 1994; Noel et al., 2001; Seitz & Schumann-Hengsteler, 2002). Furthermore, calculation is thought to require phonological maintenance of the partial results (Fürst & Hitch, 2000; Logie et al., 1994; Noel et al., 2001), and possibly visual-spatial resources for the carry process (Hayes, 1973; Heathcote, 1994). This evidence suggests that the phonological loop and VSSP may have subsidiary roles in mentally calculating a multi-digit arithmetic problem.

Phonological Loop. Some researchers have found that verbal interference tasks do not disrupt processing of arithmetic problems (e.g., De Rammelaere, 1999; Noel et al., 2000), whereas others have found that verbal interference tasks do disrupt solution processes (Lemaire et al., 1996; Logie et al., 1994). For example, Logie et al. (1994) used a memory span task in which participants had to add a series of two-digit numbers

that were presented verbally while concurrently participating in an articulatory suppression task or in an irrelevant picture task. The arithmetic task required participants to keep track of running totals while maintaining accuracy in their calculations. Participants were shown a double-digit number followed by another double-digit number. The participant was to add these numbers together and then add the sum to the next double-digit number in the sequence (e.g., $13 + 29(42) + 17 + 12 \dots$). Logie et al. (1994) found that concurrent performance of an arithmetic task was disrupted more by the articulatory suppression task than by the irrelevant picture task, therefore, suggesting that the role of the phonological loop was to keep track of running totals and maintaining accuracy in mental calculations.

Fürst and Hitch (2000) investigated the role of the phonological loop in solving multi-digit problems (e.g., $362 + 197$). Their participants were required to solve either briefly presented or continuously presented problems while concurrently processing a secondary interference task. The secondary interference task involved a control condition and a verbal condition (i.e., articulatory suppression) where the participant repeated the word "the" at a rate of one per second. Fürst and Hitch found that under dual-task conditions the articulatory suppression task had an effect when participants were solving briefly presented problems but it did not effect calculation of continuously presented problems. This evidence suggests that phonological resources are required in calculating multi-digit problems, possibly to temporarily maintain problem information (i.e., partial results; Logie et al., 1994). Therefore, the evidence from the studies mentioned above and

Fürst and Hitch's (2000) study suggests that the phonological loop may have a subsidiary role in calculating arithmetic problems.

Visual-Spatial Sketchpad. Few researchers have examined the role of the visual-spatial sketchpad in mathematical processes (e.g., Hayes, 1973; Heathcote, 1994; Logie et al., 1994; Siegel & Linder, 1984; Trbovich & LeFevre, 2003). For instance, Heathcote (1994) tested the role of the visual-spatial sketchpad in solving addition problems. He hypothesized that the visual-spatial sketchpad would act as a mental "workbench" upon which the numerical material could be represented and maintained. In Experiment 1, Heathcote (1994) assumed that the phonological loop and the visual-spatial sketchpad would play different roles in solving carry and no-carry addition problems. Carry problems were defined by Heathcote to be problems where the sum of the units, or tens, or hundreds columns exceeded the value of nine. When the sum of the values in a column exceeded nine, then the carry (which is always a 1 in addition) was added to the first digit of the next column. For example, in the problem $437 + 245$, people may use a conventional algorithm to calculate the sum by adding seven and five to equal twelve, record the two in the unit column and add the carry of "one" to the sum of three plus four. The partial results total is 82, before adding the digits in the hundreds column together, thus making the total 682. Heathcote (1994) predicted that if carry problems involved visual-spatial resources, then concurrent visual or spatial interference tasks would produce more interference on carry than on no-carry problems.

Each addition problem was presented to the participant in vertical format on a computer monitor. Both operands in each problem were triple digits (e.g., $357 + 469$),

which were presented to the participants either auditorally or visually. Half of the problems were double-carry problems (e.g., $357 + 469$) and the other half were no-carry problems (e.g., $357 + 432$). In the visual presentation condition, the participant saw the problem in the middle of the screen for one second. Participants engaged in solving the problem and announced their solutions vocally. In the visual interference condition, the computer randomly generated visual patterns of squares for the participant to see while concurrently solving the addition problem. In the auditory dual-task condition, participants were asked to perform a spatial interference (i.e., Moars Box; Moar, 1978) task concurrently with solving the problem. The Moars Box (Moar, 1978) has a 5×5 array of keys hidden from the participant's sight. Participants were required to press keys along one column and then reverse along the next column and continue until all the keys were pressed in the correct spatial sequences. An articulatory suppression task, that required participant to repeat the word "the" at a rate of 2 to 3 times per second, was used to interfere with sub-vocal rehearsal (Baddeley, 1996).

Heathcote (1994) found that participants always solved carry problems more slowly than no-carry problems. He also found that viewing a visual pattern slowed participants' performance on carry problems but not on no-carry problems. However, spatial and verbal interference tasks did not differentially affect participants' performance on carry and no-carry problems. Thus, Heathcote suggested that the function of the VSSP was to maintain visual codes for carry problems. This finding suggests that the VSSP might be involved with maintaining the visual information of a carry problem

(Hayes, 1973; Hitch, 1978) when mental calculations require producing an answer involving a multi-digit problem.

Summary

In summary, clearly there is some evidence to suggest that the VSSP and the phonological loop have roles to play in encoding and calculating arithmetic problems (Heathcote, 1994; Fürst & Hitch, 2000; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002; Trbovich & LeFevre, 2003). Some researchers suggested that phonological resources are involved with encoding numerical information because briefly presented problems tend use more phonological resources than continuously presented problems (Fürst & Hitch, 2000; Heathcote, 1994; Noel et al., 2001). Perhaps the verbal codes representing a briefly presented problem require active maintenance in the phonological loop for further processing, whereas continuous problems are less reliant upon using phonological resources. As for the VSSP, researchers have suggested that visual-spatial resources are needed to solve vertical problems and not horizontal problems (Trbovich & LeFevre, 2003), whereas other suggestions indicate that horizontal problems rely on VSSP resources. Therefore, these memory subsystems seem to have a role with encoding arithmetic problems; however, the role of the phonological loop is better understood than the role of the VSSP.

Calculation of a multi-digit problem seems to involve phonological and visual-spatial memory resources (Heathcote, 1994; Fürst & Hitch, 2000; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2002). The phonological loop is thought to be involved with retaining the partial results of arithmetic problems (Fürst & Hitch, 2000; Heathcote,

1994; Seitz & Schumann-Hengsteler, 2000), however, the role of the VSSP is still controversial. Some evidence indicates that visual-spatial resources are involved with processing carry problems (Hayes, 1973; Heathcote, 1994), whereas other evidence indicates that carry problems do not need VSSP resources for calculation (Logie et al., 1994). Therefore, there is evidence supporting a role for phonological resources in mental calculation and possibly visual-spatial resources, however, further investigation is needed.

Linking Numerical Cognition with Working Memory

Despite the consensus view that working memory and arithmetic are related (LeFevre et al., in press), researchers have yet to incorporate the function of working memory into their cognitive architecture models of arithmetic (Campbell & Clark, 1988; Campbell & Epp, in press; Dehaene & Cohen, 1995; McCloskey, 1992). Four numerical cognition models (Campbell & Epp, in press; Dehaene & Cohen, 1995; McCloskey, 1992; Siegler & Shipley, 1995) and their potential links with working memory are discussed below.

Abstract code model. According to the *abstract code model* (McCloskey, 1992) number processing is comprised of three cognitive systems: encoding, calculation system, and response production. The encoding system converts different surface features (e.g., $8+5$ vs. eight + five) into a single common abstract representation. The calculation system then uses this abstract representation to compute an answer. Finally, the production system recodes the abstract output from the calculation system into a written or spoken number response. On this view, encoding does not affect calculation.

Consequently, it is reasonable to assume from this numerical model that solving a multi-digit arithmetic problem presented in different formats (i.e., horizontal: $384 + 47$ vs. vertical: $384 + 47$) should not affect mental calculation. Furthermore, if a single common representation (i.e., abstract code) is used with the calculation process, then it is reasonable to assume that general working memory resources are involved with temporarily storing and retaining the common numerical representation. Therefore, in order to support a link between working memory and the *abstract code model*, the following results should show; 1) that arithmetic problems presented in different formats (horizontal vs. vertical) will not affect the solution process, and 2) under dual-task conditions any secondary memory load task (phonological, visual, or spatial) should interfere with retaining the abstract numerical information in working memory during calculation.

Triple code model. The *triple code model* (Dehaene & Cohen, 1995) has a different cognitive architecture than the *abstract code model* (McCloskey, 1992), but is built on a similar premise – that stages of processing are independent. This model has three independent calculation components; auditory-verbal word frame, analog magnitude representation, and the visual-arabic number form. The auditory-verbal component processes written and spoken encoding and production and is the basis for retrieval of simple arithmetic facts (e.g., 2×3). The visual-arabic component is responsible for processing Arabic digit forms ($8+5$), parity judgments, and multi-digit operations. The analog magnitude component is used for approximate calculation, numerical-size comparisons, and subitizing. Information can be translated from one

component to the other without having to transcode information into an abstract representation as in the *abstract code model* mentioned above (McCloskey, 1992). Once all of the encoded information has been delegated to the appropriate cognitive component, then calculation will proceed. According to the *triple code model* (Dehaene & Cohen, 1995) the process of encoding the operands (e.g., $8+5$ or eight + five) will not affect the calculation process. Like the *abstract code model* (McCloskey, 1992), if the encoding process does not affect calculation then format (i.e., horizontal vs. vertical) should not affect the calculation process of multi-digit arithmetic problems.

Furthermore, if three distinct components are used to process arithmetic problems then three different working memory resources may also be involved with the calculation process. For example, the auditory-verbal component could be linked with phonological resources; the visual-arabic component with visual memory resources, and analog magnitude component with spatial memory resources. Consequently, if the visual-arabic component is primarily responsible for processing multi-digit arithmetic problems, then retaining a visual pattern in memory should disrupt the calculation process. Therefore, in order to support a link between working memory and the *triple code model*, the following results should show; 1) that arithmetic problems presented in different formats (horizontal vs. vertical) will not affect the solution process, and 2) under dual-task conditions visual memory loads should interfere more with solving multi-digit arithmetic problems than phonological or spatial memory loads.

Encoding complex hypothesis model. In contrast to the above numerical cognition models, the *encoding complex hypothesis model* (Campbell, 1992, 1994;

Campbell & Clark, 1988; Campbell & Epp, in press) is based on the assumption that the encoding process does influence the calculation process. According to the *encoding complex hypothesis* model when an arithmetic problem is presented, all numerical representations (e.g., visual, spatial, verbal, magnitude) are activated in parallel. These numerical representations are interconnected with other number representations, thus forming a network of associated numerical representations (i.e., the encoding complex). These numerical representations are linked with higher cognitive processes such as encoding, calculation, and production. Problem solving then requires extracting task-specific information (e.g., visual, visual-spatial, verbal, motor) from the encoding complex. The efficiency of the extraction process is assumed to be related to familiarity with the surface features of a problem. Surface features could include the shape, font style, font size, word, Arabic digits, or presentation format (e.g., horizontal or vertical). Problems with familiar surface features (e.g., $8 + 5$) will be extracted more quickly from the encoding complex than problems presented with less familiar features (e.g., eight + five). Accordingly, it is logical to assume then that extracting specific numerical information from the encoding complex should require temporary maintenance in specific working memory components. For example, presenting an arithmetic problem in the familiar vertical format (e.g., $28 + 39$) may be associated with using visual memory resources, whereas a less familiar horizontal format ($28 + 39$) may be associated with verbal memory resources. Assuming that vertical and horizontal problems have different surface features then the numerical representations extracted from the encoding complex would be expected to be associated with different working memory resources. Therefore,

in order to support a link between working memory and the *encoding complex hypothesis model*, the following results should show; 1) that arithmetic problems presented in different formats (horizontal vs. vertical) differentially affect the solution process, and 2) different memory resources are associated with solving vertical and horizontal problems. However, more than just differences in surface features might be involved with solving complex arithmetic problems.

Adaptive Strategy Choice Model. For most people mentally processing a complex problem is slow and intentional, and may involve using a procedure that requires several procedural steps to solve the problem (e.g., Ashcraft, 1995; LeFevre et al., in press; Siegler, 1996). The above numerical cognition models are all silent on explaining how people use procedures to solve multi-digit arithmetic problems. The literature suggests that people use various procedures to solve multi-digit arithmetic problems (Heathcote, 1994; Siegler, 1996, 1999; Trbovich & LeFevre, 2003). An appropriate numerical cognition model that assumes people use procedures to solve arithmetic problems is the *adaptive strategy choice model* (ASCM; Siegler & Shipley, 1995). According to the ASCM model (Siegler, 1996; Siegler & Shipley, 1995) numerical representations are assumed to include various associations between problems and potential answers. The associative strength between a problem and a potential answer is related to the procedure (e.g., min. counting, count-all, decomposition, finger-counting, retrieval) involved with solving the arithmetic problem. A procedure that has been used more often by a person to solve a particular problem will develop a stronger associative tie with that problem than a procedure that has been used less often. When an

arithmetic problem is presented all procedures and their associative strengths are activated. Selection of a procedure is based on the associative strength of each procedure. The procedure with the strongest associative strength will be selected over procedures with weaker associative strengths. If a retrieval procedure is selected and its associative strength exceeds a certain confidence criterion then an answer will be produced, otherwise the search for the correct answer will continue until another procedure is selected. If a non-retrieval procedure (e.g., decomposition: $832 + 59 = 830 + 60$ (890) $+ 2$ (892) $- 1 = 891$) is selected then this procedure will be executed until the answer is produced. The surface features (e.g., font, size, shape, colour, word, Arabic digit) of a problem are linked with developing the associative strength of a particular procedure. As the familiarity of a particular problem increases, the associative strength of the procedure used to solve the problem will also increase. The answer is then assumed to modify the internal representations that are stored in long-term memory, including the past efficiency of the procedure with solving that particular problem.

Researchers have suggested that working memory resources are associated with the procedure that is used to solve a multi-digit arithmetic problem (Heathcote, 1994; Trbovich & LeFevre, 2003). For example, if people use a verbal procedure (e.g., decomposition) to solve horizontal arithmetic problems (Trbovich & LeFevre, 2003), then there should be more demands for phonological resources than other memory resources. Conversely, if people use a spatial procedure (e.g., spatial re-alignment) to solve horizontal problems (Heathcote, 1994), then there should be more demands for spatial resources than other memory resources. On this point, a specific format

(horizontal or vertical) may activate a specific procedure that requires the temporal maintenance and storage of numerical information to be stored in a specific working memory component. However, the *adaptive strategy choice model* is silent on what affect encoding (i.e. format) has on selecting a procedure that will be used in the calculation process. Therefore, in order to support a link between working memory and the *adaptive strategy choice model*, the ASCM model would need to be modified to include an explanation of how encoding might influence procedure selection. The following numerical cognition model is a hybrid model that combines aspects of the ASCM model (Siegler & Shipley, 1995) with the encoding complex model (Campbell & Epp, in press) to explain the processing of multi-digit arithmetic problems and related working memory resources.

Integrated Calculation Encoding Strategy (ICES). As I was formulating a model of how working memory and cognitive arithmetic might be related, it became clear that the *encoding complex hypothesis model* (Campbell & Epp, in press) and the *adaptive strategy choice model* (Siegler & Shipley, 1995) could be modified to account for solving multi-digit arithmetic problems and linked with the tripartite working memory model discussed above. The following ICES model is a description of how working memory could be associated with solving multi-digit arithmetic problems.

An assumption of the ICES model is that all numerical representations (e.g., visual, spatial, verbal, magnitude) with their interconnections and associative strengths are activated in parallel from long-term memory, thus forming an encoding complex (Campbell & Epp, in press). Activation of numerical representations includes, but is not

limited to, the activation of procedures from long-term memory (e.g., Siegler & Shipley, 1995). Another assumption of the *ICES* model is that, depending on the arithmetic task, relevant numerical representations will be extracted from the encoding complex and used with higher cognitive processes. Extraction is assumed to involve facilitating relevant information and inhibiting irrelevant information from interfering with higher cognitive functions. As suggested by Siegler and colleagues (Siegler, 1996, 1999; Siegler & Jenkins, 1988; Siegler & Shipley, 1995), the *ICES* model also assumes that if a retrieval procedure is selected from the encoding complex and its associative strength exceeds a certain confidence criterion then an answer will be produced. If a non-retrieval procedure is selected from the encoding complex then this procedure will be executed until the answer is produced. The answer (right or wrong) is then assumed to modify the internal representations that are stored in long-term memory. The advantage of this hybrid model is that it uses Campbell's (Campbell & Clarke, 1988; Campbell & Epp, in press) *encoding complex hypothesis model* to explain how encoding might influence the calculation process and Siegler's (Siegler, 1996; Siegler & Shipley, 1995) *ASCM* model to explain how procedures might be involved with solving multi-digit arithmetic problems. From the framework of the *ICES* model, the working memory components of the *tripartite memory model* mentioned above could be incorporated with multi-digit number processing.

The literature on the central executive and arithmetic shows that although the central executive is involved with processing multi-digit arithmetic problems (e.g., Fürst & Hitch, 2000; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002), and

simple arithmetic problems (De Rammelaere et al., 1999; Lemaire et al., 1996), however its relation with number processing is not well understood. Some researchers have suggested that the central executive retrieves numerical information from long-term memory and this information is then used to solve the arithmetic problem (Baddeley & Logie, 1999; Logie et al., 1994). Others have suggested that central executive control is required to solve carry problems (Fürst & Hitch, 2000; Hayes, 1973; Heathcote, 1994). Another proposed function of the central executive is to act as a selective filter by controlling the locus of attention when encoding and processing information (Baddeley, 1990; Baddeley & Logie, 1999; Bull & Johnston, 1997). Evidence in support of the central executive as a selective filter comes from research with children with arithmetic disabilities. These children have a particular difficulty with inhibiting irrelevant numerical information (e.g., Bull, Johnston, & Roy, 1999; Bull & Scerif, 2001; Geary, Hoard, & Hamson, 1999; Passolunghi & Siegel, 2001). On this view, the central executive should be involved with retrieving relevant numerical representations from long-term memory while inhibiting irrelevant number representations that were also activated in long-term memory. This assumption is compatible with the view of the *encoding complex hypothesis model* (Campbell & Epp, in press) that suggests extracting relevant numerical information from the encoding complex involves inhibiting irrelevant information from activating higher cognitive functions. Therefore, an assumption of the ICES model with the *tripartite memory model* is that the central executive should inhibit irrelevant numerical representations and extract relevant numerical representations

(including a procedure) from the encoding complex that is associated with a particular procedure.

After the relevant numerical representations have been extracted from the encoding complex this information is then assumed to be temporarily stored within their respective memory subcomponents (i.e., phonological, visual, spatial). For example, solving a problem like $632 + 49$ should involve the visual, phonological, and spatial memory components temporarily storing and maintaining their own respective numerical information. The phonological component should maintain the verbal information of briefly presented problems (phonology: /six-hundred-thirty-two/ and /forty-nine/), and partial sums during calculation (phonology: /eleven/; /eighty/; /six-hundred/). The visual component should maintain the visual information of briefly presented problems (visual form: 3,4,4,5), and partial sums during calculation (e.g., retain visual form of 7 & 9). The spatial component should maintain the relative locations of each digit within the problem (e.g., First operand 34: 3 in the decades and 4 in the units; Second operand 45: 4 in the decades and 5 in the units) as well as maintaining the locations of the sum (e.g., 7 in the decades and 9 in the units). Therefore, the assumption of the ICES model in conjunction with the *tripartite memory model* is that demands for various working memory resources will depend on the procedure that is selected from the encoding complex and used to solve a multi-digit arithmetic problem.

The ICES model with the tripartite model can be used to make certain predictions linking working memory resources with numerical cognition. For example, Figure 3 shows that if a person selects a verbal procedure from the encoding complex to solve a

horizontal problem like $632 + 49$ (e.g., Trbovich & LeFevre, 2003), then it is assumed that the demands for phonological resources during calculation (e.g., decomposition: /six-hundred-thirty/ + /fifty/ + /two/ - /one/ = /six-hundred-eighty-one/), will be greater than the demands for visual or spatial resources.

Figure 3. Selecting a verbal procedure may involve using more phonological resources than visual or spatial memory resources.

In other situations, Figure 4 shows that if a person selects a spatial procedure from the encoding complex to solve a vertical problem ($632 + 49$), then it is assumed that the demands for spatial resources during calculation (e.g., spatial maintenance of positions of the sum: 6 in the hundreds column; 8 in the decades column; 1 in the units column), will be greater than the demands for visual or phonological resources.

Figure 4. Selecting a spatial procedure may involve using more spatial resources than visual or phonological memory resources.

In another situation, Figure 5 shows that if a person selects a visual procedure from the encoding complex to solve a horizontal problem ($632 + 49$; Heathcote, 1994), then it is assumed that the demands for visual resources during calculation (e.g., visual re-alignment of one operand into a vertical format), will be greater than the demands for visual or phonological resources.

Heathcote (1994):
visual re-alignment
into vertical format

$$632 + 49$$

Figure 5. Selecting a visual procedure may involve using more visual memory resources than phonological or spatial memory resources.

In summary, the ICES model provides a good framework for incorporating working memory resources with multi-digit numerical cognition. According to the ICES model, specific predictions on the relation between various working memory resources and number processing can be made. The model is also useful for understanding the cognitive structures of working memory and number processing. Examining the influence of working memory resources on solving multi-digit numbers using the ICES model as a framework should provide interesting predictions on mental structures of the numerical cognition system. Conversely, the way people solve multi-digit arithmetic problems using working memory resources should provide valuable insight into the cognitive structures of the working memory system. Therefore, this ICES model is not only useful as a framework for explaining a relation between working memory and

number processing, but also for investigating the mental components of numerical cognition and working memory.

Predictions

The following experiments were designed to test the ICES model on the relation between phonological, visual and spatial memory resources and multi-digit arithmetic problems. Each experiment used a memory load paradigm, where on each trial the participant was shown a memory load (i.e., verbal, visual or spatial) for two seconds on the computer screen. One third of the memory load trials were hard (e.g., FAC MUB LER), one third were easy trials (e.g., MIP) and one third were no-memory load trials (e.g., *). The memory load then disappeared from the screen and a multi-digit problem (e.g., $426 + 48$) appeared 500 ms later. The participant was expected to rehearse the memory load while concurrently solving the addition problem in their head. Once the participant solved the problem they then said the answer out loud for the experimenter to record. The final step for the participant was to reproduce the memory load item for the experimenter. Three experiments were designed using this procedure. Systematic manipulations to the secondary memory task were made according to the following predictions.

Phonological Memory and Multi-digit Arithmetic. In Experiment 1 the role of the phonological component in solving multi-digit addition problems was tested. A third of the experimental trials contained a hard-memory load (e.g., FAC MUB LER), a third contained an easy-memory load (e.g., GIP), and a third contained a no-memory load (e.g., *). The literature suggests that people use phonological resources to temporarily store

and rehearse verbal information (i.e., verbal codes) when solving horizontal addition problems more than when they are solving vertical problems (Trbovich & LeFevre, 2003). This evidence indicates that people might be more likely to use a verbal procedure to solve horizontal problems (e.g., decomposition: $826 + 59 = 826 + 60 - 1 = 886 - 1 = 885$) than to solve vertical problems. If people use a verbal procedure to solve horizontal problems, then maintaining these verbal codes should increase phonological resource demands under phonological memory load conditions. Conversely, when people solve vertical problems, they should be less likely to use a verbal procedure, and hence, use fewer phonological resources. Therefore, in Experiment 1, one role of the phonological memory component might be to supply phonological resources to cognitive procedures that require these resources to solve multi-digit problems. On this assumption, I predicted that participants in Experiment 1 would solve horizontal problems more slowly and less accurately than vertical problems under phonological memory load conditions.

Briefly presented problems have been shown to disrupt the retention of partial sums under phonological memory load conditions (Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1980; Logie et al., 1994). If briefly presented problems use phonological memory resources to retain partial sums during calculation, then maintaining the phonological codes representing the briefly presented problem should increase demands on using phonological resources concurrently with phonological memory load conditions. In contrast, continuously presented problems should not require phonological resources. Therefore, another role of the phonological loop might be to temporarily store verbal

information (i.e., possibly the partial sums) during mental calculation. On this view, I predict that participants in Experiment 1 should take longer and make more errors on solving briefly presented problems than on continuously presented problems under phonological memory load conditions.

Visual Memory and Multi-digit Arithmetic. In Experiment 2 the role of the visual component in solving multi-digit addition problems was tested. On each trial of the experiment the participant was shown either a hard memory load (e.g., a pattern of 7 black and 7 white squares in a grid), an easy memory load (e.g., a pattern of 4 black and 4 white squares in a grid) or no memory load (e.g., *). Current evidence indicates that visual resources are more likely to be used by people for temporary storage of visual-spatial information when solving vertical problems than horizontal problems (Trbovich & LeFevre, 2003). If people use a visual procedure (e.g., columnar algorithm; Hayes, 1973; Heathcote, 1994) to solve vertical problems, then temporary maintenance of these visual codes should increase demands on visual memory resources, resulting in interference when participants were also concurrently maintaining a visual memory load. In contrast, horizontal problems should be less likely to require visual memory resources. Therefore, I predicted that the participants should take longer and make more errors when solving vertical problems than solving horizontal problems concurrently with a visual memory load.

Multi-digit carry problems have been shown to disrupt the retention of visual information under visual-spatial memory load conditions (Heathcote, 1994). If carry problems use visual memory resources during calculation, then maintaining these visual

codes should increase demands on the available visual resources under visual memory load conditions. However, no-carry problems should be less likely to require visual resources during calculation. The role of the visual component might be, therefore, to temporarily store visual information (i.e., possibly the carry) during mental calculation (Hayes, 1973). On this assumption, I predicted an interaction between load and problem type. Participants always solve carry problems more slowly and less accurately than no-carry problems (ref), but I predicted that carry problems would show much greater interference from a visual memory load than no-carry problems.

Spatial Memory and Multi-digit Arithmetic. In Experiment 3 the role of the spatial component in solving multi-digit addition problems was tested. In each trial of the experiment, the participant was shown either a hard memory load (e.g., a sequence of 4 highlighted squares in a 10 square grid), or an easy memory load (e.g., a sequence of two highlighted squares in a 10 square grid), or no memory load (e.g., *). Researchers have suggested that spatial memory resources may be involved with solving horizontal problems (Hayes, 1973; Heathcote, 1994; Experiment 2). If people use a spatial procedure (e.g., digit re-alignment) to solve horizontal problems, then the requirement to maintain the spatial codes in the memory load condition should increase the demands on spatial component resources. In contrast, vertical problems should be less likely to require spatial memory resources (e.g., Hitch, 1978). Therefore, I predicted that the participants should take longer and make more errors when solving horizontal problems than solving vertical problems under spatial memory load conditions.

Experiment 1

This experiment explored the role of the phonological component in solving multi-digit addition problems. For this experiment I predicted that the participants' latencies on briefly presented problems would be faster under easy load conditions (i.e., GUP) than under hard load conditions (i.e., FAC MUB LER), however their latencies should be fastest under no load conditions (i.e., *). Conversely, on the assumption that presenting problems continuously minimizes the verbal memory load (Fürst & Hitch, 2000) participants' performance on continuously presented problems should not differ with phonological memory load conditions.

Research shows that problems presented horizontally take longer for participants to solve than problems presented vertically (Heathcote, 1994). Trbovich and LeFevre (2003) suggested that people are more likely to use phonological memory resources to solve horizontal problems than they are to solve vertical problems. Therefore, I predict that participants' performance should vary with format and phonological memory load. Namely, participants' performance on solving horizontal problems should vary with phonological memory load, whereas the phonological memory load should have little impact on their mental ability to solve vertical problems.

Finally, researchers (e.g., Ashcraft, 1995; Hayes, 1973; Heathcote, 1996; Hitch, 1978) have suggested that more procedural steps are used to mentally solve carry problems than no-carry problems. Increasing the number of procedural steps of calculation apparently increases the demands on central executive resources (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Lee & Kang, 2001; Logie et al, 1994;

Noel et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002). On this view, I predicted that participants' performance should decrease while solving carry problems, regardless of memory load type (i.e., phonological, visual or spatial), as compared to their performance on solving no-carry problems.

Method

Participants

Fifty-five students from Carleton University's first year psychology course received course credit in exchange for participating in this experiment. The data of eight participants were not included in the analyses either because of computer malfunction or experimenter error. The 47 participants (25 females and 22 males) whose data were analyzed were between the ages of 18 and 28 years old.

Materials

Experimental Measures. The participant's primary task was to solve a multi-digit problem containing a triple-digit number and a double-digit number (e.g., $245 + 32$). Their secondary task was to rehearse and remember a letter string composed of one cvc (MUB) or three cvcs (FAC MUB LER).

Addition Task. There were 180 combinations of triple-digit and double-digit numbers used for experimental trials and 27 used for practice trials. The triple-digit operands, double-digit operands and their sums ranged between 172 and 942, 19 and 96, and 267 to 986, respectively. Addition problems with zeros and ones in the decade or unit positions were avoided because these problems are solved using rules and may require working memory resources (e.g., Ashcraft, 1992, 1995). The 180 problems were

divided into two different groups of 90 problems. In each group, there were 30 no-carry problems matched with 30 carry-unit problems and 30 carry-ten problems. Carry-unit problems required the participant to carry the one from the sum in the unit column and add it to a digit in the decade column (e.g., $34\underline{7} + \underline{2}8$). A carry-ten problem required the participant had to carry the one from the sum in the decade column and add it to a digit in the hundreds column (e.g., $29\underline{3} + \underline{8}2$). The participant did not have to perform a carry operation on no-carry problems, since the sums from the unit and decade columns were less than nine (e.g., $352 + 23$). Every no-carry problem was matched with a carry-unit and carry-ten problem by sum. For example, a no-carry problem (e.g., $352 + 23 = 375$) was matched with a carry-unit problem (e.g., $34\underline{7} + \underline{2}8 = 375$) and a carry-ten problem (e.g., $29\underline{3} + \underline{8}2 = 375$) to form a matched set. Appendix A shows all the problems used in this study.

Problems within a matched set were balanced (Latin square) so that each problem only occurred once with each level of memory load. To balance problems with levels of memory load, three sets of problems were created. For example, the same problem (e.g., $352 + 23$) was matched with a no-load condition (*) in one set of problems, an easy load condition (e.g., GUP) in another set and then with a hard load condition (e.g., FAC MUBLER) in the other set. From each of the three sets there were two lists made where the problems and their matched memory load items were subjected to random ordering. An example in appendix B shows how these six lists were then balanced across participants with the levels of duration (brief vs. continuous), and the levels of format (vertical vs. horizontal).

Phonological Memory Task. Three phonological memory loads were constructed. There were 30 no-load items (e.g., *), 30 easy-load items (e.g., GUP) and 30 hard-load items (e.g., FAC MUB LER). For each memory load condition, there were 10 different no-carry problems, 10 different carry-unit problems and 10 different carry-ten problems for the participant to solve. Pronounceable consonant-vowel-consonant (cvc) non-words were used as the memory load items. Each cvc was used only once during the experiment. There were 34 single cvcs and 34 cvc clusters constructed for the experimental trials. The cvc did not spell a proper name, or common abbreviation to increase the likelihood that cvc rehearsal relied upon sustained phonological loop resources rather than long-term memory.

Individual Difference Measures.

Participants were asked to complete three short-term memory tests, a test of arithmetic fluency (i.e., French kit) and two background questionnaires.

Visual Pattern Test. A participant's visual short-term memory span was measured using a visual pattern test (Della Sala, et al., 1997). The visual pattern test is made of black and white squares arranged in a grid pattern that is mounted on a 14 cm x 20cm cardboard card. Each square measures 1cm². Each pattern consists of filling half of the grid with black squares and the other half with white squares. The patterns did not resemble a common picture, person, or object. There are 42 possible visual patterns for the participant to reproduce, starting with the easiest difficulty level (i.e., 2 white and 2 black squares) and progressing to the most difficult level (i.e., 15 white and 15 black squares).

The participant reproduced the original visual pattern on a blank grid of the same size as the original visual pattern. The participant used a pencil to indicate where the black squares in the visual pattern would go on the blank grid. There were seven sheets of paper with two columns of grids on each sheet. In each column, there were three blank grids of the same size. The grid sizes ranged from the smallest of four blank squares to the largest of 30 blank squares. A large piece of thin cardboard was used to cover the column of blank grids to prevent the participant from using the blank grids to help remember the visual pattern.

Spatial Sequence Test. The spatial sequence task from the Wechsler adult intelligence scale - revised (Kaplan et al., 1991) measured spatial memory capacity. The spatial sequence test consisted of 10 wooden blue cubes (3cm^3) mounted on a white 28 cm x 22 cm wooden board. The cubes are asymmetrically situated on the board. A white number is painted on the back of each cube, for the experimenter to see.

Phonological Span Test. The verbal span test developed to measure phonological memory capacity. Researchers have suggested that rhyming letters (e.g., B, C, D) utilize more phonological resources than non-rhyming letters (Conrad & Hull, 1964; Baddeley, 1990). On this view, the phonological span test contained rhyming letters and non-rhyming letters. Nonsense letters strings are used so that the letter string would be more likely to require use of phonological resources than other resources such as long-term memory. As shown in Appendix G, there are nine levels of nonsense letter strings. The letter strings range from two letters (e.g., G, P) to 10 letters (e.g., C, K, D, N, P, U, V, O, B, Z).

Arithmetic Fluency. A test involving multi-digit addition, subtraction and multiplication problems (French, Ekstrom, & Price, 1963) were used to assess each participant's fluency with arithmetic problems. There are four pages in this test with 60 problems per page. On each page, the problems were arranged in six rows with 10 problems per row. Two pages contained 120 multi-digit addition problems, and the remaining pages contained a combination of 60 subtraction and 60 multiplication problems. A stopwatch was used to time participants.

Questionnaires. Participants were asked to complete the Math Background (see Appendix D) and Interests and the General Background questionnaires (see Appendix C). The Math background and interests questionnaire asked participants about their past experience with mathematics, highest math course taken in high-school, if English was their first language, etc. The general background questionnaire was concerned with past events involving head injuries, use of corrective lenses, etc. Three participants reported having prior head injuries to the visual cortex. Although, this should not have any effect on verbal processing in the temporal lobes, their data were tested. Whether their data were removed or left in the analyses did not change the results reported below. However, one of the three participant's data was removed because of computer malfunctions. Therefore, the data from the other two were analyzed with the data from the other 45 participants.

Apparatus

Addition problems were presented on a 12-inch monochrome monitor that was connected to an IBM-type 286 computer. The participant said the answer to each

problem out loud into a microphone that was connected to the computer. The computer recorded the response latency times to the nearest millisecond and the experimenter recorded the participant's answer and the participant's reproduction of the memory load. The program for this experiment was written in Turbo Pascal.

Design

A 2 (format: vertical vs. horizontal) x 2 (presentation time: brief vs. continuous) x 3 (Problem type: no carry vs. single-carry units vs. single-carry tens) x 3 (memory load: no load vs. easy load vs. difficult load), mixed factorial design was used, with repeated measures on the last three factors.

The dependent measures were correct latencies to solve addition problems and percent of combined errors under dual-task conditions. Latencies were measured in milliseconds starting from the time the addition problem was presented on the screen and ending when the participant responded with an answer out loud into the microphone.

Procedure

Each participant was asked first to read and sign the informed consent form (see appendix E). Next, the participant solved a set of 90 problems under dual-task conditions (see Appendix L). Then the experimenter administered the three short-term memory tests to the participant, before commencing with the second set of 90 problems. After completing the second set of 90 problems, the experimenter tested the participant's arithmetic fluency with the French kit (French et al., 1963), followed by having the participant fill out the questionnaires before debriefing them on the experiment. In the case where a participant did not want to fill out the questionnaires (typically because the

allotted time for the experiment had been exceeded), they were given a debriefing before they left the experiment (see Appendix F).

Experimental procedures. Participants sat about 50 cm from the computer screen. They were instructed that on each trial they would first see the word “READY” flash three times in the middle of the screen. This was their cue to be ready for the phonological memory load. 500 ms after the last “READY” prompt flashed in the middle of the screen, one of three phonological memory loads appeared in the middle of the monitor for 2000 ms. Participants were instructed to say the memory item out loud before it disappeared. After seeing the memory item for 2 s, the screen went blank for 500 ms before a multi-digit problem appeared in the middle of the screen. Half of the participants saw the problem always presented vertically and the other saw the problem always presented horizontally. After solving the problem out loud, there was a 500 ms delay before the word “RECALL” was presented in the middle of the screen. This was the participant’s cue to reproduce the memory item by saying it out loud. The experimenter then recorded their response into the computer. After providing a response, the words “press space bar to continue” appeared on the screen. This allowed participants to regulate their pace from trial-to-trial throughout the experiment. When the space bar was pressed the next trial started. After every 30 trials the participants were required to take a small break (i.e., usually about 1 minute) before proceeding with the next 30 trials. Before the experimental trials began, the experimenter re-emphasized the instructions to the participant to make sure that they understood to rehearse the memory

load in their head while concurrently solving the problem as *quickly* and *accurately* as possible in their head.

Duration conditions. Participants used the above procedure when solving briefly presented problems and continuously presented problems. Half of the participants saw their first set of 90 problems presented briefly and their second set of problems presented continuously. In brief conditions the participants saw the problem in the middle of the screen for 2-seconds. In the continuous conditions the participants saw the problem on the screen until they responded, or until 15 s had expired from the problem onset. If the participant responded with an answer before the 15 s time limit, the problem disappeared from the screen and 500 ms later the word "RECALL" appeared. The other half of the participants solved a set of continuous problems before solving a set of brief problems.

Practice. Eighteen practice trials were used for the first set of 90 problems and nine practice trials were used for the second set of problems. In the first 18 problems, there were six problems with no-loads, six with easy-loads, and six with hard-loads. The nine practice problems for the second set of 90 problems contained three no-load, three easy-load and three hard-load conditions. More practice trials were available and offered to the participant, however, no participant requested more practice. Practice problems were not used in the experimental trials.

Individual differences

Participants were asked to complete three short-term memory tests, a test of arithmetic fluency (i.e., French kit) and two background questionnaires. However, due to

the length of the experiment (approx: 120mins), approximately 17 of the participants chose not to fill out the questionnaires.

Visual Span Task. At the start of each trial, the participant was instructed to look at the back of a blank card. The experimenter would then remove the blank card and reveal a grid of black and white squares for 2-seconds. This grid constituted a visual pattern. The participant was asked to remember as much of the visual pattern before the blank card was replaced in front of the pattern for 10-seconds. During this 10 s interval, the participant was instructed to continue looking at the blank card while rehearsing the pattern in their head. Following this interval, the participant removed a cover from the blank grid. The blank grid was the same size as the original grid. Then with a pencil, the participant placed an X in the blank squares where they thought the black squares from the original pattern should go. The participant had no time limit to reproduce the original pattern. When they were finished replicating the pattern, the experimenter then removed the blank card and determined if the participant's reproduction of the original was correct.

All participants started with the easiest visual pattern level consisting of two white and two black squares in a grid. Each level contains three visual patterns of similar difficulty. To progress from one level to the next, the participant had to reproduce at least one of the three patterns correctly. Termination of this task occurred if the participant failed to reproduce all three visual patterns of the same level of difficulty. The largest visual pattern that the participant could correctly reproduce before terminating the task determined their visual memory span score. For example, if a

participant could only remember six white and six black squares correctly before failing to reproduce three patterns of seven white and seven black squares, their visual span score would be six.

Spatial Span Task. At the start of each trial, the participant was instructed to watch the order in which the experimenter touched certain blue cubes. After touching a sequence of cubes, the experimenter covered the cubes with a large cardboard sheet and the participant was instructed to then stare at an orange dot on a blank wall. While the participant looked at the dot, they were asked to rehearse the sequence in their head. After 10 s, the experimenter removed the cardboard from off of the cubes and indicated to the participant to reproduce the sequence. The participant then touched the cubes and the experimenter recorded whether their reproduced sequence was correct.

All participants started with the easiest spatial sequence level consisting of two cubes. Each level contained two spatial sequences. To progress from one level to the next, the participant had to reproduce at least one of the two sequences correctly. Termination of this task occurred if the participant failed to reproduce both sequences of the same level of difficulty. The largest spatial sequence that the participant could correctly reproduce before terminating the task determined their spatial memory span score. For example, if a participant could only remember a sequence of four cubes before failing to reproduce two sequences of five cubes, then their spatial span score would be four.

Phonological Span Task. According to Baddeley (1990) the phonological loop's capacity to process verbal information is sensitive to the type of information that requires

processing. He found that nonsense letter strings with similar sounding letters (e.g., B T E V) interfered more with phonological processes than did nonsense letter strings with different sounding letters (e.g., S L N M). Using this information, a phonological span task was developed that used similar procedures as the visual pattern task and the spatial sequence task (see Appendix G).

At the start of each trial, the participant asked to listen to a string of letters said out loud by the experimenter. Each letter in the string was pronounced clearly and at a rate of one letter per second. After the experimenter finished saying the letter string the participant then stared at an orange dot on a blank wall. While staring at the orange dot they were told to rehearse the letter string in their head. After 10 s, the experimenter indicated to the participant to recall the letters in the same order they were said previously. The experimenter recorded whether the participant correctly reproduced the letter string.

All participants started with the easiest letter string consisting of two letters (e.g., G, P). Each level contained two letter strings of the same length. To progress from one level to the next, the participant had to reproduce at least one of the two letter strings correctly. Termination of this task occurred if the participant failed to reproduce both letter strings of same length. The largest letter string that the participant could correctly recall before terminating the task determined their phonological memory span score. For example, if a participant could only remember four letters before failing to reproduce two strings of five letters, then their phonological span score would be four.

Practice. For all three of the above span tasks, participants were given at least two practice trials. The experimenter gave the participant an opportunity to have more practice, if they wished to have more. All participants declined further practice. The visual pattern practice trials required participants to retain a pattern of eight squares (i.e., four black and four white). The spatial sequence task required the participant to remember a sequence of four cubes, whereas the phonological task required the participant to remember a letter string of four letters.

Arithmetic Fluency. Arithmetic fluency was used to determine each participant's ability to solve addition, subtraction and multiplication problems (French et al., 1963). There are four pages in this test with 60 problems per page. Participants were given two minutes per page to solve as many problems as possible. They were instructed by the experimenter to correctly solve each problem on the page as quickly and accurately as possible. The participant was told to start with the problem on the left and then continue across the row and down the page until time was called. The number of correct problems on each of the four pages was summed to yield an arithmetic fluency score out of 240 for each participant.

Questionnaires. Participants were asked to complete the Math Background and Interests and the General Background questionnaires (see Appendices C & D).

Results and Discussion

Arithmetic Fluency

Participants' arithmetic fluency scores ($M = 76$, $SD = 27$) were not significantly different from what was expected ($M = 80$) for this population, $t(46) = 1.14$, $p > .05$ (see

LeFevre et al., 2003). The participants' mean arithmetic fluency scores were used to determine whether arithmetic fluency was similar across the between-subjects conditions. Arithmetic fluency scores were analyzed in a one-way ANOVA with format (vertical vs. horizontal) as a between-subjects variable. Arithmetic fluency scores for participants in the vertical presentation group ($M = 75$, $SD = 5.7$) and in the horizontal presentation group ($M = 76$, $SD = 5.5$) were not significantly different, $F(1,45) < 1.00$, $p > .05$. This result suggests that the two format groups of participants did not differ in arithmetic fluency.

Order effects

Half of the participants completed a block of brief problems before solving a block of continuous problems and vice versa. A test for order effects on latencies showed that order did not vary with format, memory load, or problem type, $F_s(4,172) \leq 1.93$, $ps > .05$, $MSEs \leq 159356$. Further, order did not vary with format, memory load or problem type on percent errors, $F_s(4,172) \leq 2.29$, $ps > .05$, $MSE \leq 122$. Therefore, order of presentation duration was not included in the analyses.

Analyses of Solution Latencies and Combined Errors

The participants collectively solved 8640 multi-digit addition problems. Of those 8640 problems, participants solved 7219 (84%) correctly and made errors on 1192 (14%). There were 229 (2%) invalid responses. Invalid responses included those that were too quiet (and thus no response time was recorded), or accidental noises (e.g., clearing throat, sighing, saying umm). The incorrect and invalid responses were not included in the latency analyses.

The correct latencies ranged from 737 ms to 14,992 ms ($Md = 3117$ ms, $SD = 2065$ ms). In the dual-task condition, each correctly answered problem was selected for analysis if: a) the participant had correctly reproduced at least 33% of the concurrent memory load item, and b) the latency did not exceed two standard deviations from the mean for vertical problems or three standard deviations from the median for horizontal problems (see explanation in Appendix H).

The percentage of trials in each condition that the participant either made an error on the primary task (i.e., addition task) or recalled less than 33% correct on the secondary task (i.e., memory load) determined the combined error score values. Other researchers (e.g., Ashcraft & Kirk, 2001; Trbovich & LeFevre, 2002; Seyler et al., 2002) have used a similar procedure on the assumption that reduced performance on either the primary task or secondary task under dual-task conditions reflects working memory demands.

Combined error scores and the median correct latencies to addition problems were analyzed in separate 2 (format: horizontal vs. vertical) x 2 (presentation duration: brief vs. continuous) x 3 (problem type: no carry vs. carry units vs. carry tens) x 3 (memory load: none vs. easy vs. hard) mixed ANOVAs design with repeated measures on the last three factors. The error bars in the following figures represent 95% confidence intervals calculated from the mean-square error (MSE) value for the interaction of interest (c.f., Loftus & Masson, 1994; Masson & Loftus, 2002). For follow-up tests a significant difference between means was considered if the absolute difference between the compared means of interest was larger than the square root of two times the confidence interval (i.e., $|M_1 - M_2| > \sqrt{2} \times \text{confidence interval}$; Loftus & Masson, 1994).

Brief vs. Continuous Problems

I predicted that presentation duration would interact with memory load. On this assumption, brief presentations were expected to increase the likelihood that a participant would have to store the operands in phonological memory, thus increasing the demands for phonological resources under verbal memory loads. However, briefly presented problems were solved just as fast as continuously presented problems (see Appendix K). This result does not support the literature (Fürst & Hitch, 2000; Heathcote, 1994). That is, briefly presented problems were solved faster than continuously presented problems; however, the participants' committed more errors when problems were presented briefly than continuously. This result indicates that there were speed-accuracy tradeoffs between participants' latencies and errors committed on the primary task. Furthermore, in participants' anecdotal reports, they claimed that it was difficult to encode a briefly presented problem and rehearse it while rehearsing a nonsense letter string. Since it is unclear as to what may have caused these speed-accuracy tradeoffs the presentation duration factor was, therefore, not tested in any further experiments. On this note, the following analyses were made on data from continuously presented problems for Experiments 1, 2 and 3 using format, problem type and memory load as factors.

Results and Discussion (Continuously presented problems)

Format x Memory Load

As predicted, participants' solved vertical problems more quickly than horizontal problems (3010 ms vs. 3978 ms), $F(1,45) = 7.00$, $p < .01$, $MSE = 14161933$, and made more errors solving horizontal problems than vertical problems (18% vs.13%),

$F(1,45) = 4.77, p < .05, MSE = 566$. This evidence replicates other studies that have examined the effects of presentation format (Heathcote, 1994; Trbovich & LeFevre, 2003).

I also assumed that participants' performance would vary with memory load. Two orthogonal contrasts (1 df) were made to test this assumption. The linear contrast for the memory load effect (1, 0, -1), shows that participants found solving problems under memory load conditions to be difficult for latencies, $F(1,45) = 18.34, p < .001, MSE = 257043, CI = 147$ ms, and for errors, $F(1,45) = 25.98, p < .001, MSE = 213, CI = 4.25\%$ (see Table 2). Furthermore, the quadratic contrast for the memory load effect (1, -2, 1) indicates that participants' solution times were significantly increased under hard memory load conditions, $F(1,45) = 5.61, p < .05, MSE = 183383, CI = 125$ ms, and the same trend was found for the memory load effect in errors, $F(1,45) = 7.64, p < .01, MSE = 130, CI = 3.32\%$ (see Table 2). This result suggests a verbal memory load in solving multi-digit problems that is also found in the literature (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Lee & Kang, 2002; Logie et al., 1994; Noel et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002).

Table 2

Latencies in milliseconds and percent errors with levels of memory load

	Experiment 1		Experiment 2		Experiment 3	
Memory Load	Latency	Errors	Latency	Errors	Latency	Errors
No Load	3400	11	4068	9	3899	10
Easy Load	3424	13	4202	20	3940	23
Hard Load	3658	20	4473	23	4125	34

I predicted that format should vary with memory load. This interaction was significant for latencies, $F(2,90) = 3.49, p < .05, MSE = 220213, CI = 137$ ms, and approached significance for errors, $F(2,90) = 2.68, p = .07, MSE = 171, CI = 3.81\%$. As shown in Figure 6 (left frame), participants' latencies varied with format and memory load. That is, participants found solving horizontal problems more difficult than solving vertical problems with memory load conditions. Furthermore, participants' performance was affect most by having to solve horizontal problems presented under hard memory load conditions. This result supports the view that phonological resources are more likely to be involved with solving horizontal problems than with vertical problems (Heathcote, 1994; Trbovich & LeFevre, 2003).

Figure 6. Latencies in milliseconds and percent errors with format and memory load in Experiment 1.

Problem Type x Memory Load

Two orthogonal contrasts (1 df) were made to test two effects; the carry effect (1, -2, 1) and the position effect (1, 0, -1). The carry effect was based on the assumption that carry problems would take participants longer to solve than no carry problems (Ashcraft, 1992, 1995; Dehaene, 1992; Heathcote, 1994, Hecht, 1999, 2002; Logie et al., 1994; Noel et al., 2001). The position effect was founded on the assumption that problems with carries coming from the unit position would take longer to solve than problems with

carries from the decade position (e.g., Fürst & Hitch, 2000; Hayes, 1973; Hitch, 1978).

As shown in Table 3, the carry effect in latencies was significant, $F(1,45) = 108.81$, $p < .001$, $MSE = 828509$, $CI = 266$ ms, and in errors, $F(1,45) = 24.44$, $p < .001$, $MSE = 206$, $CI = 4.18\%$. Furthermore, the position effect in latencies was significant, $F(1,45) = 5.33$, $p < .05$, $MSE = 606265$, $CI = 227$ ms, as well as in errors, $F(1,45) = 7.19$, $p < .01$, $MSE = 300$, $CI = 5.05\%$. The carry effect indicates that the participants were faster at solving no-carry problems than carry problems, which supports other evidence in the literature (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Logie et al., 1994; Seitz & Schumann-Hengsteler). However, the position effect indicates that more cognitive resources were needed to process carry-unit problems than carry-ten problems and supports evidence on processing multi-digit problems in the literature (Noel et al., 2001).

Table 3

Latencies in milliseconds and percent errors with levels of problem type

	Experiment 1		Experiment 2		Experiment 3	
Problem Type	Latency	Errors	Latency	Errors	Latency	Errors
No Carry	2840	10	3178	14	2994	16
Carry-ten	3713	14	4665	18	4358	25
Carry-unit	3928	20	4890	20	4610	27

The interaction between problem type and memory load was not significant for latencies, $F(2,90) < 1$, $1.46 < 1$, $MSE = 164372$, $CI = 118$ ms, and for errors, $F(2,90) < 1$, $MSE = 127$, $CI = 3.29\%$. This result shows that phonological resources are not involved with solving multi-digit problems, which is contrary to the suggestion that solving carry problems involves phonological resources for processing (Fürst & Hitch, 2000).

Summary

In summary, solving no-carry multi-digit problems were easier for participants than solving carry problems and this result is consistent with the literature (Ashcraft, 1992; Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Logie & Baddeley, 1987; Logie et al., 1994). Solving a carry-unit problem impeded participants' performance more than solving carry-ten problems. This evidence indicates that the increasing demands on cognitive resources vary with carry problem type. For instance, a carry-unit problem (e.g., $328 + 57$) will require an extra calculation step (carry added to decade: $1+2+5$) than a carry from the decade column ($382 + 75$; carry added to hundred: $1 + 3$). Increasing the calculation steps may also increase the demand for available central executive resources, thus resulting with participants' mental performance to diminish more with solving carry-unit problems than carry-ten problems.

Briefly presented problems appeared to be too difficult for people to solve under a phonological memory load, thus this condition was dropped from the analyses. Aside from the presentation duration manipulation, the phonological component seemed to be

more involved with solving horizontal problems than with solving vertical problems. Consistent with the literature, participants were fast and made fewer errors when solving vertical problems than solving horizontal problems (Heathcote, 1994; Trbovich & LeFevre, 2003). This evidence indicates that the phonological component does play a significant role in solving multi-digit addition problems. Perhaps, as suggested by Trbovich and LeFevre (2003), when people encode a horizontal arithmetic problem they select a verbal procedure to solve the problem. This procedure (e.g., decomposition), in turn, may require the substantial use of phonological memory resources in order to maintain appropriate information needed to solve the arithmetic problem.

In the following experiments, the only methodological difference from Experiment 1 was the memory load. In Experiment 2 the memory load condition was visual (i.e., remembering a pattern of black-and-white squares) instead of verbal (i.e., remembering a nonsense letter string), as in Experiment 1. The following Experiment 2 was, therefore, designed to test the role of the visual component in solving the same multi-digit problems under various visual memory load conditions.

Experiment 2

Hayes (1973) had participants report their procedures for solving carry problems. He reported that a common procedure used by participants was to visualize a "blip" or a "1" occurring near the digit that the carry was to be added too. Others have reported that people use a visual procedure (e.g., visual imagery) to solve carry problems (Heathcote, 1994; Hitch, 1978; 1980). This literature suggests, therefore, that people will engage more visual memory resources to solve carry problems than to solve no-carry problems (see Heathcote, 1994; Logie et al., 1994). On this view, I predicted under visual memory load conditions people's mental performance would vary with solving carry problems and not with solving no-carry problems.

The literature on problem presentation format (i.e., horizontal vs. vertical) indicates that people take longer to solve horizontal problems than vertical problems (see Experiment 1; Heathcote, 1994; Trbovich & LeFevre, 2003). Therefore, I predicted that horizontal problems would take longer to solve than vertical problems. Furthermore, Trbovich and LeFevre (2003) suggested people are more likely to use a visual procedure to solve vertical problems than horizontal problems. On this point, I anticipated that solving a vertical problem would increase people's latencies and errors more than solving a horizontal problem.

Method

Participants

Forty-nine students from Carleton University's first year psychology course were given course credit for participating. The data from two participants were not included

because the participants could not resist using external aids (i.e., scratch paper, fingers) to calculate answers to addition problems, even after being asked not to several times.

There were 47 participants (26 females and 21 males) between the ages of 18 and 25 years old. Four people reported having head injury. Excluding their data from the analyses below did not change the results; therefore, their data were included in the following analyses. The experiment took participants approximately two hours to complete (see Appendix O).

Materials

The materials used for the experimental measures, addition task, and memory spans (visual, spatial & phonological), arithmetic fluency and questionnaire were the same as described in Experiment 1. The phonological memory task was replaced with a visual memory task. Three visual memory loads were constructed. There were 30 no-load items (e.g., *), 30 easy-load items (e.g., 4 white & 4 black squares in a grid), and 30 hard-load items (e.g., 7 white & 7 black squares in a grid). Each visual load pattern was used only once during the experiment. For each memory load condition, there were 10 different no-carry problems, 10 different carry-unit problems, and 10 different carry-ten problems for the participant to solve. There were 34 easy load patterns and 34 hard load patterns constructed for the experimental trials. To reduce the likelihood of involving any verbal processing or visual representations drawn from long-term memory, the patterns were developed so that they did not resemble any common objects (e.g., Mickey mouse), places or things.

Apparatus

The apparatus was the same as described in Experiment 1.

Design

A 2 (format: vertical vs. horizontal) x 3 (problem type: no carry vs. carry-units vs. carry-tens) x 3 (memory load: no load vs. easy load vs. difficult load), mixed ANOVA design was used with the first variable being between-subjects and the latter two variables being repeated measures. The dependent measures were the latencies to correctly solved addition problems and percent of errors on problem under dual-task conditions (see Experiment 1).

Procedure

The procedures used for the general conditions, experimental trials (see Appendix M), practice trials, individual differences, memory spans (visual, spatial & phonological), arithmetic fluency, and questionnaires were the same as described in Experiment 1. However, due to the length of the experimental trials (approx: 120mins), approximately two-thirds of the participants chose not to fill out the questionnaires.

Results and Discussion

Arithmetic Fluency

Arithmetic fluency scores were analyzed in a one-way ANOVA with format (vertical vs. horizontal) as a between-subjects variable. Arithmetic fluency for participants in the vertical presentation group ($M = 78$, $SD = 22.4$) and in the horizontal presentation group ($M = 69$, $SD = 22.4$) were not significantly different, $F(1,45) = 1.94$,

$p > .05$, $MSE = 503$. This result suggests that participants were reasonably similar in arithmetic fluency across format conditions.

Solution Latency

The participants collectively solved 4204 multi-digit addition problems. Of those 4204 problems, 3666 (82%) problems were solved correctly by participants, 458 (11%) problems were incorrectly solved, and 80 (2%) problems were considered to be invalid answers. Trials where the participant responded too quietly or made an accidental noise (e.g., clearing throat, sighing, etc) were considered invalid trials. The incorrect and invalid responses were not included in the latency analyses.

The correct problems (3666) across the levels of format, memory load, and problem type contained latencies that ranged from 973 ms to 14,459 ms ($Md = 4357$ ms, $SD = 2156$ ms). Each correctly answered problem was selected for analysis if it met the criteria discussed in Experiment 1. The table in Appendix I shows the average number of correct problems that participants' solved with format, memory load and problem type.

Percent error

As stated in Experiment 1, the percentage of trials for the experiment that the participant either made an error on the primary task or incorrectly recalled less than 1/3rd of the secondary task determined the combined error scores.

Analyses

Combined error scores and the median correct latencies to addition problems were analyzed in separate 2 (format: horizontal vs. vertical) x 3 (problem type: no carry vs. carry units vs. carry tens) x 3 (memory load: none vs. easy vs. hard) mixed ANOVAs.

Format was the between-groups variable and the remaining variables were repeated measures. As explained in Experiment 1, the error bars in the figures represent 95% confidence intervals.

Format x Memory Load

As predicted, participants were slower at solving horizontal problems than vertical problems (4705 ms vs. 3790 ms), $F(1,45) = 5.60, p < .05, MSE = 15796768$, whereas their error performance did not vary with horizontal and vertical formats (18% vs. 16%),

$F(1,45) < 1.0, MSE = 2491$. This result replicates results from Experiment 1 and evidence from the literature (Heathcote, 1994; Trbovich & LeFevre, 2003).

Participants' latencies varied with memory load, $F(2,90) = 12.97, p < .001, MSE = 462920, CI = 198$ ms; and so did their errors, $F(2,90) = 14.99, p < .001, MSE = 535, CI = 6.75\%$. The memory load effect (linear contrast of no-load, easy-load, hard-load: 1, 0 - 1) indicates that their latencies, $F(1,45) = 16.38, p < .001, MSE = 706045, CI = 245$ ms, and errors, $F(1,45) = 20.12, p < .001, MSE = 737, CI = 7.91\%$, varied with memory load. These results indicate that the visual memory load conditions did effect participant's cognitive processing of multi-digit arithmetic problems.

The interaction shown in Figure 7 (centre panel) between format and memory load was significant, $F(2,90) = 3.69, p < .05, MSE = 462920, CI = 198$ ms. I predicted that the effect of solving vertical problems would be greater than solving horizontal problems under memory load conditions. In contrast, I found that people took longer to respond to horizontal problems than vertical problems while concurrently rehearsing a

visual memory load, $F(1,45) = 4.77, p < .05, MSE = 706045, CI = 245$ ms. Examination of the errors people made (see Figure 7) indicates that their errors increased with memory load, however, there were no differential effects between formats, $F(2,90) < 1, MSE = 535, CI = 6.75\%$. This evidence indicates that visual resources may be involved with solving horizontal problems.

Figure 7. Latencies in milliseconds and percent errors with levels of memory load and format for Experiment 2 (right panel) compared with in Experiment 1 (left panel).

Problem Type x Memory Load

I predicted that carry problems would take participants longer to solve than no-carry problems, and this assumption was supported, $F(2,90) = 116, p < .001, MSE =$

1039426, $CI = 297$ ms. The carry effect (no-carry vs. carry; contrast: 1, -2, 1) is shown in Table 4, where participants solved no-carry problems faster than the carry problems, $F(1,45) = 176.0, p < .001, MSE = 1348164, CI = 338$ ms. Table 4 also shows that participants' committed fewer errors when processing no-carry problems than carry problems, $F(1,45) = 17.1, p < .05, MSE = 139. CI = 3.44\%$. For the position effect (carry-unit vs. carry-ten; contrast: 1, 0, -1), participants solved carry-ten problems faster than carry-unit problems, $F(1,45) = 4.85, p < .05, MSE = 730689, CI = 249$ ms; and their error performance between carry-ten and carry-unit problems was, however, not significant, $F(1,45) = 0.46, p > .05, MSE = 200, CI = 4.13\%$

Table 4

Latencies in milliseconds and percent errors with levels of problem type

Problem Type	Experiment 1		Experiment 2		Experiment 3	
	Latency	Errors	Latency	Errors	Latency	Errors
No Carry	2840	10	3178	14	2994	16
Carry-ten	3713	14	4665	18	4358	25
Carry-unit	3928	20	4890	20	4610	27

The interaction between problem type and memory load was not significant, $F(2,90) = 1.52, p > .05, MSE = 308376, CI = 162$ ms. However, I had predicted that carry problems would utilize more visual working memory resources than the no-carry problems (i.e., carry effect) under memory load conditions. On this prediction, Figure 8 shows that participants' latencies for problem type and memory load do interact, $F(1,45) = 5.54, p < .05, MSE = 247217, CI = 145$ ms. Conversely, participants' errors for this interaction between problem type and memory load was not significant, $F(1,45) < 1, MSE = 111, CI = 3.07\%$. The latency results indicate carry problems use visual memory resources, and no-carry problems do not, thus supporting the suggestion that solving a carry problem involves visual working memory resources (e.g., Hayes, 1973; Heathcote, 1994). Furthermore, participants' errors indicate a differential trend between carry and no-carry problems with memory load (see centre panel in Figure 8 below); however, this trend was not significant.

Figure 8. Latencies in milliseconds and percent errors with levels of memory load and problem type for Experiment 2 (right panel) compared with in Experiment 1 (left panel).

Summary

In summary, carry problems were more difficult for participants to solve than no-carry problems. This evidence is consistent with the results found in Experiment 1 and with the literature that suggests carry problems are more difficult to solve than no-carry problems (Ashcraft, 1995; Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Logie & Baddeley, 1987; Logie et al., 1994).

A result not found in the literature is the position effect (carry-unit vs. carry-ten), where solving a carry-unit problem takes longer to solve than a carry-ten problem. This result was also found in Experiment 1, thus indicating that the “carry process” may

involve more than phonological or visual memory resources. Possibly the differences in cognitive performance is related to different procedural processes that people may use to solve carry-unit and carry-ten problems. For example, when a carry occurs from the unit position (e.g., $34\underline{7} + 2\underline{6}$) the carry is added to two-decade digits (e.g., $1+4+2$), thus increasing the number of calculations to be performed and working memory resources needed to process these calculations. In comparison, when a carry from the decade position (e.g., $3\underline{7}4 + \underline{6}2$) is added to the hundreds digit (e.g., $1+3$), there are fewer calculations to be performed and working memory resources needed to process this calculation. Therefore, the number of calculations needed to solve a carry problem may have contributed to these results and implicates a functional role for the central executive component of working memory in solving carry problems (e.g., Fürst & Hitch, 2000; Logie et al., 1994; Noel et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002).

Format appears to have an effect on how people solve addition problems (Heathcote, 1994; Trbovich & LeFevre, 2003). One suggestion is that different formats may involve participants using different procedures to solve the problems. For instance, Trbovich and LeFevre (2003) suggested that people might use a verbal procedure (e.g., decomposition) when processing horizontal problems, and use a visual procedure (i.e., columnar alignment) when processing vertical problems. However, the results from Experiment 2 do not support this assumption. For example, Figure 7 shows that more visual resources were used for processing horizontal problems than for vertical problems. Perhaps participants were using a visual procedure (i.e., visually realigning the digits into a vertical format) to solve horizontal problems (see also Hayes, 1973; Hitch, 1978).

The evidence from Experiment 2 indicates a couple of roles for the visual component. One possible role of the visual component is to temporarily maintain information about carry problems. It is possible that people do need working memory resources to visualize the carry as suggested by others (e.g., Hayes, 1973; Heathcote, 1994). Another potential role of the visual component is to supply visual resources to processing horizontal problems. Perhaps, people use a mental procedure (e.g., operand re-alignment into a vertical format) that requires visual memory resources; however this experiment was not designed to test this assumption. Therefore, visual working memory resources seem to make a significant contribution in processing multi-digit problems. The following experiment examines the role of spatial working memory in solving multi-digit addition problems.

Experiment 3

Research done on presentation format (horizontal vs. vertical) shows that multi-digit problems presented horizontally take longer for participants to solve than problems presented vertically (see Experiment 1 & 2; Heathcote, 1994; Trbovich & LeFevre, 2003). I assumed that horizontal problems would take longer to solve than vertical problems. Furthermore, researchers have suggested that visual-spatial memory resources may be involved with solving horizontal problems (e.g., Heathcote, 1994; Hitch, 1978). Heathcote (1994) suggested that the participants' use spatial resources to maintain the location of each digit within a problem. Hayes (1973) and Hitch (1978) have suggested that participants use a realignment procedure to solve horizontal problems. That is, they mentally realign the horizontal problem into a vertical format. Therefore, participants who use this vertical realignment procedure should use spatial memory resources to maintain the location of each digit during calculation. On this assumption, I predicted that the demands on spatial memory resources should vary with format and memory load. That is, horizontal problems should take longer to solve than vertical problems under spatial memory load conditions.

Method

Participants

Fifty students from Carleton University's first year psychology course were given course credit for participating. Two participants were unable to complete the experiment. From the remaining forty-eight participants there were (27 females and 21 males) between the ages of 18 and 41 years old. Two people reported having head injury.

Excluding their data from the analyses below did not change the results; therefore, their data were included in the following analyses. The experiment took participants approximately two hours to complete (see Appendix O).

Materials

The materials used for experimental measures, addition task, and memory spans (visual, spatial & phonological), arithmetic fluency and questionnaire were the same as described in Experiment 1 and 2. The spatial memory load task for this experiment was based on the Corsi Blocks task (see Corsi, 1972). Three spatial memory loads were constructed. There were 30 no-load items (e.g., *), 30 easy-load items (e.g., remembering a sequence of 2 highlighted squares on a grid) and 30 hard-load items (e.g., remembering a sequence of 4 highlighted squares on a grid). For each memory load condition, there were 10 different no-carry problems, 10 different carry-unit problems, and 10 different carry-ten problems for the participant to solve. Each spatial load sequence was used only once during the experiment. There were 34 easy load sequences and 34 hard load sequences constructed.

Apparatus

The apparatus was the same as described in Experiment 1.

Design

A 2 (format: vertical vs. horizontal) x 3 (problem type: no carry vs. carry-units vs. carry-tens) x 3 (memory load: no load vs. easy load vs. difficult load), mixed ANOVA design was used, with the former variable being between-subjects and the latter two variables being repeated measures. The dependent measures were the latencies to

correctly solved addition problems and percent of errors on problem under dual-task conditions (see Experiment 1).

Procedure

The procedures used for the general, experimental conditions (see Appendix N), practice, individual differences, memory spans (visual, spatial & phonological), arithmetic fluency, and questionnaire were the same as described in Experiment 1. Due to the length of the experiment (approx: 120mins), approximately two-thirds of the participants chose not to fill out the questionnaires.

Results and Discussion

Arithmetic Fluency

The overall arithmetic fluency scores were not significantly lower ($M = 74$, $SD = 28$) than what was expected ($M = 80$) for this population, $t(47) = 1.14$, $p > .05$ (see LeFevre et al., 2003). Arithmetic fluency scores were analyzed in a one-way ANOVA with format (vertical vs. horizontal) as a between-subjects variable. Arithmetic fluency for participants in the vertical presentation group ($M = 74$, $SD = 24$) and in the horizontal presentation group ($M = 74$, $SD = 32$) were not significantly different, $F(1,47) = 0$, $p > .05$, $MSE = 810$. This result suggests that participants were reasonably similar in arithmetic fluency across format conditions.

Order effects

Half of the participants completed a block of brief problems before solving a block of continuous problems and vice versa. A test for order effects on latencies showed that order did not interact with format, duration, memory load, and problem type,

$F_s(4,172) \leq 1.59$, $ps > .05$, $MSEs \leq 256884$, and the test of order effects on errors did not interact with any of the variables, $F_s(4,172) \leq 1.49$, $ps > .05$, $MSE \leq 129$. These results indicate that order did not contribute to the participants' performance on solving multi-digit problems.

Solution Latency

The participants collectively solved 4273 multi-digit addition problems. Of those problems, 3700 (87%) problems were solved correctly by participants, 473 (11%) problems were incorrectly solved, and 100 (2%) problems were considered to be invalid answers. Invalid answers were made by participants who responded too quietly or made a non-answer noise (e.g., clearing throat, sighing, etc). The incorrect and invalid responses were not included in the latency analyses.

For the correct problems (3700) across the levels of format, memory load, and problem type contained latencies that ranged from 888 ms to 14,964 ms ($Md = 4150$ ms, $SD = 2189$ ms). Each correctly answered problem was selected for analysis according to the criteria explained in Experiment 1.

Percent error

As stated in Experiment 1, the percentage of trials in each condition that the participant either made an error on the primary task or on the secondary task or on both tasks determined the combined error scores.

Analyses

Combined error scores and the median correct latencies to addition problems were analyzed in separate 2 (format: horizontal vs. vertical) x 3 (problem type: no carry vs.

carry units vs. carry tens) x 3 (memory load: none vs. easy vs. hard) mixed ANOVAs. Format was the between-groups variable and the remaining variables were repeated measures. As explained in Experiment 1, the error bars in the figures represent 95% confidence intervals.

Format x Memory Load

Although participants appear to have taken longer to solve horizontal problems than vertical problems (4089 ms vs. 3878 ms), this difference between formats was not significant, $F(1,46) = 0.2, p > .05, MSE = 1751178, CI = 386$ ms. Furthermore, participants made just as many errors when solving horizontal problems as when solving vertical problems, (23% vs. 22%), $F(1,46) < 1, MSE = 1493, CI = 11.27\%$. This result does not support the contemporary research on format effects (Heathcote, 1994; Trbovich & LeFevre, 2003), or the results found in Experiments 1 and 2.

One possibility for this result may be that there was a selection confound with format and arithmetic skill. For instance, there may have been more participants with high arithmetic scores (i.e., French kit scores) in the horizontal conditions than the vertical conditions, thus producing faster latency times in the horizontal condition in Experiment 3. On this view, arithmetic skill should interact with format across experiments, however, this interaction was not significant, $F(1,44) = 2.25, p > .05, MSE = 4781964$. Therefore, the results suggest that participants' arithmetic skill is not likely to be confounded with format and experiments.

Participant's latencies with memory load was significant, $F(2,92) = 5.28, p < .001, MSE = 393045, CI = 182$ ms, as well as their error performance, $F(2,92) = 63.44,$

$p < .001$, $MSE = 336$, $CI = 5.35\%$. The memory load effect shown indicates that participants were slower at solving problems under hard memory loads than under no memory loads, $F(1,46) = 7.39$, $p < .001$, $MSE = 494863$, $CI = 205$ ms. Furthermore, the participants also committed more errors under hard load conditions than under no-load conditions, $F(1,46) = 84.79$, $p < .001$, $MSE = 501$, $CI = 6.53\%$. These results indicate that participants' found problem solving difficult with increasing spatial memory loads.

I assumed that spatial memory resources were more likely to be involved with processing horizontal problems than with vertical problems. Figure 9 (right panel) shows that the memory load effect with format was not significant for latencies, $F(1,46) < 1$, $MSE = 494863$, $CI = 203$ ms, or for errors, $F(1,46) = 3.45$, $p > .05$, $MSE = 501$, $CI = 6.53\%$. Notably, however, the pattern of errors is very different from that found in Experiment 1 (and 2). In Experiment 1 (and 2) the participants always made more errors in the horizontal than in the vertical condition. As shown in Figure 9, however, participants made more errors in the vertical than in the horizontal condition in this experiment. An analysis of format x load errors across Experiments is presented below.

Figure 9. Latencies in milliseconds and percent errors with levels of memory load and format for Experiment 3 (right panel) compared with in Experiments 1 (left panel) and 2 (middle panel).

Problem Type x Memory Load

I assumed that participants' would take longer to solve carry problems than no-carry problems. The main effect of problem type was significant, $F(2,92) = 76.60$, $p < .001$, $MSE = 1420767$, $CI = 347$ ms. The carry effect (no-carry vs. carry; 1, -2, 1) shown indicates that participants processed no-carry problems faster than the carry problems, $F(1,46) = 114.61$, $p < .001$, $MSE = 1859550$, $CI = 397$ ms. Participants' committed fewer errors when processing no-carry problems than carry problems, $F(1,46) = 36.39$, $p < .001$, $MSE = 295$, $CI = 5.01\%$.

The position effect (carry-unit vs. carry-ten), showed that participants' were solving carry-ten problems much faster than carry-unit problems, $F(1,46) = 4.63, p < .05$, $MSE = 981885$, $CI = 289$ ms. The errors for this position effect also indicate that participants' errors increased when solving carry-unit problems as compared to carry-ten problems, but the difference was not significant, $F(1,46) = 1.57, p > .05$, $MSE = 201$. $CI = 4.14\%$. These results support the literature that suggests carry problems require more cognitive resources for processing than no-carry problems (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). Moreover, participants' seemed to require more mental resources to process carry-unit problems than carry-ten problems.

The interaction shown in Figure 10 between problem type and memory load was not significant, $F(1,46) < 1$, $MSE = 206337$, $CI = 131$ ms. However, the participants' errors for this interaction between the memory load effect and carry effect was significant, $F(1,46) = 4.15, p < .05$, $MSE = 170$, $CI = 3.76\%$. Participants made more errors solving carry problems under spatial load conditions than under no spatial load conditions. This result suggests that carry problems require spatial resources, however, given that this effect emerges with all three memory load conditions (phonological, visual, and spatial) it is more likely that other working memory resources (i.e., central executive) are involved with processing carry problems and would support the literature (Fürst & Hitch, 2000; Logie et al., 1994) on multi-digit carry problems.

Figure 10. Latencies in milliseconds and percent errors with levels of memory load and problem type for Experiment 3 (right panel) compared with in Experiments 1 (left panel) and 2 (middle panel).

Summary

In summary, the carry effects shows that participants' were slower and made more errors when solving carry problems than solving no-carry problems. This result is consistent with the results found in Experiments 1 and 2 and with the literature that suggests carry problems require more cognitive resources than no-carry to process (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Logie & Baddeley, 1987;

Logie et al., 1994). Moreover, the position effect shows that participants were slower and made more errors when solving carry-unit problems than carry-ten problems. The consistency of these results across experiments suggests that the “carry effect “ and “position effect” may rely heavily on other working memory resources, such as the central executive. Although the literature indicates that central executive resources are significantly involved with multi-digit carry problems, these experiments were not designed to test this function and this assumed role of the central executive requires further research.

Results from Experiments 1 (and 2) and the literature on problem format (horizontal vs. vertical) suggests that more cognitive resources are devoted to processing horizontal problems than vertical problems (Heathcote, 1994; Trbovich & LeFevre, 2003), and also the results from Experiment 1 (and 2). In Experiment 3, cognitive performance was not different with format. Furthermore, performance did not differentiate with format and memory load. However, participant’s errors (see right panel in Figure 9) indicate trend in more errors being committed when processing vertical problems than horizontal problems. This trend in the result suggests that spatial memory resources may be involved with solving vertical problems. Perhaps, retaining the partial sum in memory requires maintaining the location of each digit that represents the partial sum. For example, when mentally solving a problem (e.g., $347 + 29 = 376$) the location of each digit needs to be retained (i.e., the 3 is located in the hundreds position, the 7 in the decades and the 6 in the units, respectively) in spatial memory until a response is executed. This result indicates that people use spatial memory resources while solving

vertical arithmetic problems and further examination of their errors should show an interaction between format and memory load across experiments (i.e., compare error performance under phonological memory load vs. under spatial memory load).

Combined Experimental Analyses

Examination of the latency and error performance by participants in the visual experiment (Experiment 2) indicated that the participants may have found it difficult to retain the visual memory load while concurrently solving a multi-digit problem, regardless of what format the problem was presented in (see Appendix P). Thus, visual loads were not differentially related to the format conditions. However, in Experiments 1 and 3, the loads appeared to have quite different effects on the vertical and horizontal problems. Possibly the visual memory load task was too difficult to process while concurrently solving a multi-digit problem.

The results show in Appendix P that memory load performance was affected by problem type, memory load size and experiment. The memory load performance varied with the size of the memory load (easy vs. hard) and the type of arithmetic problem (no-carry vs. carry). In Experiment 1 (phonological), people made more errors reproducing the memory load task when they had to solve carry problems, relative to no-carry problems, under hard phonological loads but not under easy phonological loads. In Experiment 3 (spatial), secondary task performance varied with problem type under easy spatial and hard spatial load conditions. These results indicate that the secondary task performance is differentially related with solving carry and no-carry problems. In contrast Experiment 2 shows that problem type does not vary with easy visual or hard visual memory load conditions. This result suggests that the visual load is unrelated to the problem type manipulation. Because it was not clear what effect the visual memory load had on dual-task performance, the data from Experiment 2 (visual) were, therefore,

not included in the combined analysis of Experiment 1 (phonological) and Experiment 3 (spatial).

Combined error scores and the median correct latencies to addition problems were analyzed in separate 2 (format: horizontal vs. vertical) x 2 (experiment: phonological vs. spatial) x 3 (problem type: no carry vs. carry units vs. carry tens) x 3 (memory load: none vs. easy vs. hard) mixed ANOVAs, with repeated measures on the last two factors. As in the separate analyses for each experiment, the problem type effect was broken down in to the carry-effect and the position-effect. As explained in Experiment 1, the error bars in the figures represent 95% confidence intervals (Loftus & Masson, 2002).

Problem Type x Experiment

As shown in Table 5, participants' latencies and errors varied with problem type, $F(2,182) = 138.54, p < .001$, and $F(2,182) = 34.47, p < .001$, respectively. As in the analyses described in the separate experiments, participants solved no-carry problems more quickly and accurately than carry problems, $F(1,91) = 241.56, p < .001$, $F(1,91) = 60.59, p < .001$. Similarly, carry-unit problems were more difficult to solve than carry-ten problems, $F_s(1,91) = 9.68$ and 8.29 , respectively for latencies and errors.

Table 5

Latencies (in milliseconds) and percentage of errors with problem type

Problem Type	Latency (ms)	Error (%)
No carry	2918*	13*
Carry-ten	4036*	20*
Carry-unit	4269*	24*

* $p < .05$

As compared to Experiment 3 (spatial load), participants in Experiment 1 (phonological load) solved problems more quickly (3494 ms vs. 3988 ms), $F(1,91) = 3.28, p = .07, MSE = 15855263$, and accurately (15% vs. 23%), $F(1,91) = 12.95, p < .001, MSE = 1040$. The main effects of experiment and problem type were qualified by the interaction of these variables for latencies, $F(2,182) = 5.79, p < .01, MSE = 1072942, CI = 212$ ms (although not for errors, $F(1,91) = 1.85, p > .05, MSE = 251, CI = 3.25\%$). Figure 11 shows the difference between no-carry problems and carry problems was smaller under phonological load (980 ms) than spatial load (1490 ms), $F(1,91) = 9.17, p < .01, MSE = 1349695, CI = 238$ ms. This result supports the literature that suggests carry problems require more cognitive resources for processing than no-carry problems (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). The larger difference between carry and no-carry problems under spatial memory loads (Experiment 3) than under phonological loads (Experiment 1) indicates that spatial memory may play a significant

role in solving carry problems or that the spatial memory load task is more difficult than the phonological memory load task under dual-task conditions. The position effect (i.e., carry-ten vs. carry-unit) with experiment, shown in Figure 12 below, was not significant, $F(1,91) < 1$, $MSE = 796190$, $CI = 183$ ms.

Carry Effect x Experiment

Figure 11. Latencies in milliseconds and percent errors between no-carry and carry problems for Experiments 1 (phonological) and 3 (spatial)

Figure 12. Latencies in milliseconds and percent errors for no-carry, carry-ten and carry-unit problems for Experiments 1 (phonological) and 3 (spatial)

Format x Memory Load x Experiment

Effects of format and load were similar to those found in the separate experiments. Participants solved horizontal problems more slowly than vertical problems (4035 ms vs. 3446 ms), $F(1,91) = 4.68, p < .05, MSE = 15855263$, although errors did not vary with format (18% vs. 20%), $F(1,91) < 1, MSE = 1039$. Participants' latencies and errors varied with memory load, as shown in Table 6, $F(2,182) = 15.96, p < .001, MSE = 307579$, and $F(2,182) = 77.34, p < .001, MSE = 255$, respectively.

Table 6*Latencies (in milliseconds) and percentage of errors with memory load*

Memory Load	Latency (ms)	Errors (%)
None	3649	11*
Easy	3682	18*
Hard	3891*	27*

* $p < .05$

None of the two-way interactions for latencies were significant (i.e., load x format; load x experiment; format x experiment), $F_s(2,182) < 2.05$, $p > .05$. For errors, however, there was a significant interaction between memory load and experiment, $F(2,182) = 18.41$, $p < .001$, $MSE = 255$, $CI = 3.28\%$. Figure 13 shows that participants' errors did not differentiate with experiment under no memory load conditions; however their errors did differentiate with experiments under easy and hard memory loads. This result suggests that solving a multi-digit arithmetic problem under a spatial memory load was more difficult than under a phonological memory load.

Figure 13. Latencies in milliseconds and percent errors with levels of memory load for Experiments 1 (phonological) and 3 (spatial)

The three-way interaction of format, memory load and experiment was significant for errors, $F(2,182) = 4.91, p < .001, MSE = 255, CI = 3.27\%$ (not for latencies $F(2,182) < 1$). As shown in Figure 14, participants made more errors on horizontal problems than vertical problems under phonological memory load conditions. This result is consistent with the literature that suggests people use a verbal procedure (e.g., decomposition) to solve multi-digit arithmetic problems (Trbovich & LeFevre, 2003). In contrast, participants made more errors on vertical problems than horizontal problems under spatial memory load conditions. This result is not consistent with the current literature (Heathcote, 1994; Trbovich & LeFevre, 2003), and suggests that spatial memory

resources are related with solving vertical problems. Perhaps, people use spatial resources to maintain the location of each digit in the sum.

Figure 14. Latencies in milliseconds and percent errors with levels of memory load and format for Experiments 1 (phonological) and 3 (spatial)

Summary

In summary, the carry effects shows that participants' were slower and made more errors when solving carry problems than solving no-carry problems. This result is consistent with the literature that suggests carry problems require more cognitive resources than no-carry to process (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Logie & Baddeley, 1987; Logie et al., 1994). The consistency of these

results across experiments suggests that the carry effect may rely heavily on other working memory resources, such as the central executive (e.g., Fürst & Hitch, 2000; Seitz & Schumann-Hengsteler, 2000). Furthermore, considering that people made more errors and took longer to respond under spatial memory load tasks than phonological memory load tasks (see Figure 14), and the larger difference between carry and no-carry problems under spatial memory loads than under phonological loads (see Figure 11) suggests that spatial loads were more difficult than phonological loads to maintain while concurrently solving a multi-digit problem.

Although the spatial memory load may have been more difficult than the phonological memory to perform under dual-task conditions, peoples' performance differentiated with format and memory load. Presenting the same multi-digit arithmetic problem horizontally has a differential effect on solving the problem than presenting it vertically. Presenting a problem horizontally requires more phonological memory resources than spatial memory resources. This result is consistent with the literature which suggests that people might use a verbal procedure to solve horizontal problems (Trbovich & LeFevre, 2003). Conversely, the same problem presented vertically requires more spatial memory resources than phonological memory resources. This result contrasts with the suggestion that people use visual memory resources to solve vertical problems (Trbovich & LeFevre, 2003). This result also contrasts with the notion that spatial memory resources are related to using a "columnar alignment" procedure for solving horizontal problems (Hayes, 1971; Heathcote, 1994). The present findings suggest that spatial memory resources may be related to retaining the location of each

digit that represents the partial sum. For example, when mentally solving a problem (e.g., $347 + 29$) the location of each digit in the sum needs to be retained (i.e., the 3 is located in the hundreds position, the 7 in the decades and the 6 in the units, respectively) in spatial memory until a response is executed. Therefore, spatial memory may be related to using a spatial position maintenance procedure, whereas phonological memory may be related with using a verbal procedure to solve multi-digit arithmetic problems.

General Discussion

The goal of the present research was to examine the relation between working memory resources and multi-digit arithmetic problems. The results indicate that phonological and spatial working memory resources are associated with solving multi-digit arithmetic problems, but the nature of the association depends upon presentation format (i.e., horizontal vs. vertical). Phonological memory resources were linked with solving horizontal problems, whereas spatial memory resources were associated with solving vertical problems. Visual memory resources were also involved with problem solving, but were not differentially associated with format. The present findings, therefore, are consistent with the hypothesis that solving multi-digit arithmetic problems requires activation and use of phonological, spatial, and visual memory resources (LeFevre et al., in press).

The presentation format of multi-digit problems influenced numerical cognition. In Experiments 1 (and 2) participants made more errors and took longer to respond to multi-digit problems presented horizontally than the same problem presented vertically. This result is consistent with the literature (Heathcote, 1994; Trbovich & LeFevre, 2003).

However, the source of the link between format and working memory resources is controversial. Researchers have suggested that the possible source might be related to the procedure people use to solve multi-digit problems (Hayes, 1971; Heathcote, 1994; Trbovich & LeFevre, 2003). On the assumption that people might use a procedure that involves verbal codes (e.g., maintenance of partial sums in working memory) to solve horizontal problems (e.g., Trbovich & LeFevre, 2003), it is reasonable to assume that demands for phonological resources should increase relative to demands for visual or spatial memory resources. Conversely, if people use a spatial procedure (e.g., re-alignment into a vertical problem) to solve horizontal problems, as proposed by Hayes (1978) and Heathcote (1994), the demands for spatial memory resources should increase relative to concurrent demands for other memory resources. The present findings (Experiment 1) support the former assumption. That is, both latencies and errors for solving horizontal problems with a verbal memory load indicate that horizontal problems are associated with phonological resources. The pattern of errors across experiments differ (see Figure 14) for format and memory load, thus supporting the assumption that solving horizontal problems demands more phonological resources than visual or spatial resources.

The association between phonological memory resources and horizontal problems may be attributed to the likelihood of people using a reconstructive procedure (e.g., decomposition; Trbovich & LeFevre, 2003) as part of the solution process. For example, mentally decomposing a problem like $828 + 59$ [(e.g., $830 + 60$ (890) $- 2$ (888) $- 1 = 887$)] involves recoding the operands into different verbal representations, and then using

these new representations as part of the solution process. Maintaining these new representations in working memory involves temporarily storing these representations in a phonological form. In general, demands for working memory resources should be related to the procedure used to solve the problem.

The present research indicates that errors in solving multi-digit arithmetic problems did not vary with format under visual memory load conditions. One interpretation of this result is that visual memory resources are not associated with solving arithmetic problems (Logie et al., 1994). Another interpretation is that visual memory resources are involved with processing multi-digit problems in other ways that were not addressed in the present study (e.g., Hayes, 1973; Heathcote, 1994; Noel et al., 2001). For example, people make more errors solving multi-digit problems with similar surface features (e.g., $38 + 69$) than problems with dissimilar surface features (e.g., $87 + 24$; Heathcote, 1994; Noel et al., 2001). This evidence suggests that visual resources may be associated with maintaining the surface features of operands and their sums. Another possibility for the present finding is that visual memory resources maintain the visual characteristics (e.g., font size, font type, shape) of the intermediate values during calculation. In the present study, therefore, format should not interact with visual memory load because the visual aspects of the sums for vertical and horizontal problems were the same.

The current findings show that spatial memory load affects the processing of vertical problems more than horizontal problems. The preferred procedure (e.g., columnar alignment) may involve having to maintain the mental positions of each digit

represented as part of the sum. For example, if the problem $628 + 59$ (in vertical format) is presented continuously on the screen, the relative mental position of the sum's unit value (7), decade value (8) and hundreds value (6) should be stored and maintained within the spatial memory component. Therefore, using a spatial procedure with solving multi-digit problems may demand more spatial resources than phonological or visual resources.

In all three experiments, participants made more errors solving carry problems than solving no-carry problems. Some researchers have implied that visual memory resources are involved with solving carry problems (e.g., Hayes, 1973; Heathcote, 1994). The present findings show that solving carry problems is not *differentially* associated with type of memory load, however. On this point, the literature suggests that carry problems demand more central executive resources than no-carry problems (Fürst & Hitch, 2000; Hitch, 1978; Logie et al., 1994; Noel et al., 2001; Seitz & Schumann-Hengsteler, 2002). Consistent with the literature, I conclude that in the present research, carry problems in the present study required more central executive resources than no-carry problems, but did not differentially require phonological, visual, or spatial codes.

The present findings also show a carry-position effect, such that people's performance was worse on carry-unit problems (e.g., $34\underline{6} + 2\underline{9}$) than on carry-ten problems ($28\underline{2} + \underline{9}3$). One explanation of this result is that central executive demands are related to the number of calculation steps (i.e., problem complexity) used to solve these problems (see DeStefano & LeFevre, 2004). For example, the number of calculation steps for solving a carry-unit problem involves adding three digits

(e.g., $346 + 29 = 1+4+2$), whereas the number of calculations steps solving a carry-ten problem involves adding two digits (e.g., $282 + 93 = 1+2$). Therefore, demands for central executive control may be related to the number of calculation steps used to solve various carry problems.

The problem-complexity explanation for the carry-position effect may be a special case for multi-digit addition problems and not for other operations such as multiplication. In particular, the carry value for addition problems will always be one, whereas the carry value for multiplication problems can vary between one and eight. Manipulating carry problems with different arithmetic operations under a central executive memory load may provide useful insight on the relation between central executive function and multi-digit numerical cognition.

In summary, the current findings indicate that working memory resources are associated with solving multi-digit arithmetic problems. Phonological resources and spatial resources are linked with presentation format; however this does not preclude the possibility that visual memory resources are required for aspects of problem solution (Heathcote, 1994; Noel et al., 2001). Furthermore, carry problems are linked with central executive resources. As suggested by the current literature, working memory resources are assumed to be connected with the procedure people use to solve a multi-digit arithmetic problem (Heathcote, 1994; Trbovich & LeFevre, 2003). Therefore, the present results show that working memory is associated with numerical cognition and also provides some insight into the cognitive structures of working memory and numerical cognition.

Working Memory

The present findings indicate that the visual and spatial components independently store and retain their own information, which is consistent with existing research and the proposed *tripartite memory model* above (Coleman & LeFevre, 2000; Della Sala et al., 1999). For example, examination of participants' errors across experiments (see Figure 14) shows more errors were committed solving horizontal problems than vertical problems under phonological memory loads. Conversely, more errors were made solving vertical problems than horizontal problems under spatial memory loads. Under visual memory load conditions people made just as many errors solving vertical problems as they did solving horizontal problems. If spatial and visual memory components are assumed to share resources (Logie, 1995), then both visual and spatial memory loads should have similar effects on solving vertical problems. In contrast to this prediction, the present results indicate that phonological, visual, and spatial memory have independent storage components, which supports the assumptions of the *tripartite memory model* and other current research (Coleman & LeFevre, 2002; Della Sala et al., 1999).

Numerical Cognition

Several numerical cognition models have emerged from research on solving simple arithmetic problems. The *abstract code model* (McCloskey, 1992) and the *triple code model* (Dehaene & Cohen, 1995) are built upon the assumption that encoded information does not affect calculation, even though the models propose different architectures. Conversely, the *encoding complex hypothesis model* (Campbell & Clark,

1988; Campbell & Epp, in press) assumes that encoded information does affect calculation. The present findings indicate, however, that format may influence selection of solution procedures and the *encoding complex model* not address this issue because it a model of one procedure – retrieval. The *adaptive strategy choice model* (ASCM: Siegler & Shipely, 1995; Siegler, 1996) does accommodate the use of procedure, however, it is silent on how encoding might influence calculation. Therefore, I proposed a hybrid numerical model (*integrated calculation encoding strategy, or ICES*) that combined aspects of the *encoding complex hypothesis* model (Campbell & Epp, in press) with the *adaptive strategy choice model* (Siegler & Shipley, 1995). The ICES model addresses the limitations of the ASCM model (Siegler & Shipley, 1995) and the *encoding complex model* (Campbell & Epp, in press) and incorporates the influence of working memory resources (*tripartite memory model*) with solving multi-digit arithmetic problems.

Integrated calculation encoding strategy

The basic assumption of the *integrated calculation encoding strategy* (ICES) model is that working memory resources are related to procedures people use to solve multi-digit arithmetic problems. Consequently, numerical cognition models should have a procedure mechanism for solving multi-digit arithmetic problems. According to the ICES model, all numerical representations (including procedures) and their associative strengths are activated in parallel (e.g., Campbell & Epp, in press). Under control of the central executive, a procedure is then selected from the encoding complex if its associated strength with the problem is larger than the associated strengths of the other procedures (e.g., Siegler & Shipley, 1995). Since multi-digit arithmetic involves a

combination of procedural steps and retrieval of intermediate results, one could predict the involvement of the memory subcomponents while solving the arithmetic problem. For example, the visual component may be used as a blackboard on which to retain the visual image of the intermediate results (e.g., Hayes, 1978; Heathcote, 1994; Seron et al., 1992). The spatial component could be used to retain the location of each digit of the intermediate results, and the phonological component could be used to retain retrieved information of simple arithmetic facts or verbal information of derived arithmetic facts that come from executing the procedure to solve the problem. Therefore, the ICES model incorporates the function of working memory with number processing and provides a cognitive framework for future research on numerical cognition.

Future Research

The ICES model explained above is an attempt to incorporate working memory resources (*Tripartite memory model*) with processing multi-digit arithmetic problems. The framework of this hybrid model could be used, for example, to empirically examine the role of the central executive in extracting relevant numerical information from the encoding complex. Furthermore, research linking various procedures with working memory resources and multi-digit arithmetic problems needs to be better understood. For example, presenting a vertical arithmetic problem with operands that are misaligned may be linked with using a verbal procedure, thus invoking phonological memory resources rather than spatial resources as indicated in the present findings. One suggestion for linking procedures with working memory would be to collect measures of procedure use (e.g., LeFevre et al., 1998), keeping in mind the limitations of these measures

(see Kirk & Ashcraft, 2001). Understanding how various numerical procedures are associated with working memory could provide better insight on the cognitive strengths or deficits that people might have when they process complex information.

A potential advantage of linking research on arithmetic and working memory involves the implications for better understanding of working memory models. The arithmetic literature shows that depending upon the manipulations used (e.g., format: horizontal vs. vertical or words vs. digits), various numerical representations are activated and used to solve arithmetic problems (Campbell, 1992, 1994; Fürst & Hitch, 2000; Hecht, 2002; Trbovich & LeFevre, 2003). Potentially these numerical manipulations could be used with a working memory paradigm (as in the present study) to explore and better understand the dynamics of the working memory system. For instance, the present findings show that manipulating the problem's format (horizontal vs. vertical) differentially affects working memory demands. This evidence suggests the working memory system is comprised of multiple independent subcomponents that are used for storing and maintaining information. Integrating arithmetic research with working memory, therefore, should provide useful insights into the cognitive architecture of the working memory system.

Another advantage of linking research on cognitive arithmetic with working memory is an enhanced understanding of current numerical models. The current literature shows that under dual-task conditions certain memory tasks show differential effects on retaining information in working memory (Baddeley & Logie, 1999; Coleman & LeFevre, 2002; Della Sala et al., 1999; Zimmerman, 2003). Numerical research

designs that incorporate the use of these memory tasks (i.e., spatial, visual, phonological, central executive) should provide useful information on the cognitive processes involved with solving multi-digit arithmetic problems. For instance, other researchers have suggested that different memory resources are associated with the way in which an arithmetic problem is presented (Trbovich & LeFevre, 2003). Presentation format of the multi-digit arithmetic problem has been suggested to involve using a procedure to help solve the problem (Hayes, 1973; Heathcote, 1994; Trbovich & LeFevre, 2003). Thus, cognitive numerical models of multi-digit arithmetic should include structures (i.e., encoding, calculation, production) that are interactive, and capable of explaining procedure use with solving multi-digit arithmetic problems. The insights to be gained from applying working memory research with numerical cognition are likely to be very important for furthering research in this field.

Finally, across these Experiments there was variability on arithmetic performance and memory load performance that was not addressed in this paper. The variability in mental performance on solving multi-digit problems and retaining memory load items may be related to individual differences on working memory span, or arithmetic skill. However, the focus of this paper was to examine the role of the working memory subcomponents in solving multi-digit problems. Examination of individual differences would provide useful insights on underlying cognitive performance of multi-digit problems.

Limitations of the Present Research

In the present research, a dual-task paradigm was used. In a dual-task paradigm, performance on a primary task is measured while participants' concurrently perform a secondary task that is known to place demands on cognitive resources. As the difficulty of the secondary task increases, performance on the primary task and secondary task are expected to suffer to the extent that both tasks depend on the same cognitive resources. However, the limitation of this methodological paradigm, as well as with the present research, is that an independent measure of the cognitive demands required to process the secondary task cannot be obtained. Recently, a dual-task paradigm has been used that provides independent measures of both the primary and secondary task performance (Seyler et al., 2003). Seyler et al.'s (2003) methodological paradigm involves using two control task conditions (primary & secondary) conditions and one dual-task condition. The method for the dual-task procedure starts with the participant performing the secondary task, followed by performing the primary task. The primary task is performed while the participant concurrently maintains a representation of the secondary task. After performing the primary task the participant recalls the secondary task. For the primary control condition, the participant observes the secondary task, performs the primary task, and is shown the secondary task again at the end of the trial. The secondary control condition requires the participant to view the secondary task, read aloud the given answer to the primary task, and recall the secondary task at the end of the trial. This way, people's cognitive performance on the primary and secondary tasks under control conditions can be made in relation to their performance under dual-task conditions. In

the current study, using this methodology would have provided a clearer understanding of secondary task performance across experiments. This methodology would have enabled the examination of the influence of the secondary task on primary task performance. In particular, it was not clear whether the arithmetic and load tasks were competing for the same cognitive resources, or whether decrements in the tasks occurred due to the difficulty of combining two tasks (i.e., the secondary task being too difficult to process under dual-task conditions). Therefore, for future consideration, researchers using a dual-task paradigm should collect independent measures on primary and secondary task performances.

In the current research, a 'carry effect' and a 'position effect' were consistently found across experiments. I speculated that central executive resources were involved with processing carry problems, because none of the memory load conditions interacted with processing carry problems. Researchers have used various tasks (e.g., trails task, random letter or digit generation) that are assumed to tap central executive resources. Although the present findings seem to be consistent with literature suggesting that carry problems demand central executive resources (Fürst & Hitch, 2000; Hitch, 1978; Logie et al., 1994; Noel et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002), the limitation of this research is that a central executive measure was not included. Therefore, my conclusions linking carry problems with central executive resources require further investigation.

Conclusion

A contribution of this paper to the literature is the integration of a model of working memory with a model of numerical cognition. The present results support the *tripartite working memory model* proposed earlier. This model describes all three memory subcomponents (phonological, visual and spatial) as being independent components. The present results also show that encoding influences calculation, possibly through procedure selection, which supports the proposed ICES model of number processing. The ICES model is a good candidate model for incorporating working memory with number processing; it accommodates the present findings and provides a testable cognitive framework for future research on numerical cognition. Continued research on the relation between working memory and multi-digit number processing should provide further insight on the cognitive structures of the working memory system and the numerical cognition system. However, given the limitations of the present research methodology, it would be prudent to systematically examine the assumptions of the ICES model and the *tripartite memory model* in order to provide further insight into the relation between working memory and multi-digit numerical cognition.

References

- Adams, J. W. & Hitch, G. J. (1997). Working memory and children's mental addition. *Journal of Experimental Child Psychology*, 67, 21-38.
- Adams, J. W., & Hitch, G. J. (1998). Children's mental arithmetic and working memory. In C. Donan (Ed.), *The development of mathematical skills: Studies in developmental psychology* (pp. 153 - 173). United Kingdom: Psychology press.
- Anderson, J. R. (1983). A spreading activation theory of memory. *Journal of Verbal Learn and Verbal Behavior*, 22, 261 - 295.
- Ashcraft, M. H. (1987). Children's knowledge of simple arithmetic: A developmental and simulation. In J. Bisanz, C. J. Brainerd, & R. Kail (Eds.), *Formal methods in developmental psychology: Progress in cognitive developmental research* (pp. 302 - 338). New York: Springer-Verlag.
- Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, 44, 75-106.
- Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition*, 1, 3-34.
- Ashcraft, M. H. & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General*, 130, 224-237.
- Ashcraft, M. H. & Stazyk, E. H. (1981). Mental addition: A test of three verification models. *Memory and Cognition*, 9, 185 - 196.

- Baddeley, A. (1986). *Working memory*. New York, NY, US: Clarendon Press/Oxford University Press.
- Baddeley, A. D. (1990). The role of memory in cognition: Working memory. In A. Baddeley (Ed.), *Human memory theory and practice* (pp. 67 - 95). Hillsdale USA: Lawrence Erlbaum Associates, Publishers.
- Baddeley, A. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 49A, 5-28.
- Baddeley, A. D. (2002). Is working memory still working? *European Psychologist*, 7, 85-97.
- Baddeley, A., Chincotta, D., & Adlam, A. (2001). Working memory and the control of action: Evidence from task switching. *Journal of Experimental Psychology: General*, 130, 641-657.
- Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. H. Bower (Ed.), *The psychology of learning and motivation: Advances in research and theory* (pp. 47 - 89). New York: Academic Press.
- Baddeley, A. D., Lewis, V., & Vallar, G. (1984). Exploring the articulatory loop. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, 36A, 233-252.
- Baddeley, A. D., & Lieberman, K. (1980). *Spatial working memory*. In R. Nickerson (Ed.), *Attention and performance VIII* (pp. 521 - 539). Hillsdale, NJ: Erlbaum

- Baddeley, A. D. & Logie, R. H. (1999). Working memory: The multiple-component model. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 28-61). Cambridge, UK: Cambridge University Press.
- Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, 15, 421-442.
- Bull, R. & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology*, 19, 273-293.
- Campbell, J.I.D., & Clark, J. M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, & Goodman (1986). *Journal of Experimental Psychology: General*, 117, 204 – 214.
- Campbell, J. I. D. (1994). Architectures for numerical cognition. *Cognition*, 53, 1-44.
- Campbell, J.I.D., (1992). In defense of the encoding-complex approach: Reply to McCloskey, Macaruso & Whetstone. In J.I.D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 539 – 556). Amsterdam: Elsevier Science.
- Campbell, J.I.D., & Epp, L. J. (in press). Architectures for Arithmetic. In J.I.D. Campbell (Ed.), *Handbook of Mathematical Cognition*.
- Coleman, B., & LeFevre, J. (2002). *Fractionating the Visuo-Spatial sketchpad: Active processes required!* Unpublished manuscript, Carleton University.

Conrad, R., & Hull, A. J. (1964). Information, acoustic confusion and memory span.

British Journal of Psychology, 55, 429 – 437.

Corsi, P. M. (1972). Human memory and medial temporal region of the brain.

Unpublished Thesis, McGill University, Montreal.

Cowan, N. (1999). An Embedded-Processes Model of working memory. In A. Miyake &

P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 62-101). Cambridge, UK: Cambridge University Press.

Daneman, M. & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior*, 19, 450-466.

Daneman, M. & Carpenter, P. A. (1983). Individual differences in integrating information between and within sentences. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 9, 561-583.

Daneman, M. & Merikle, P. M. (1996). Working memory and language comprehension: A meta-analysis. *Psychonomic Bulletin and Review*, 3, 422-433.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (1999). The contribution of working memory resources in the verification of simple mental arithmetic sums. *Psychological Research/Psychologische Forschung*, 62, 72-77.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001a). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? *Memory and Cognition*, 29, 267-273.

- De Rammelaere, S. & Vandierendonck, A. (2001b). Are executive processes used to solve simple arithmetic production tasks. *Current Psychology Letters: Behavior, Brain and Cognition*, 79-90.
- Dehaene, S. & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, 1, 83-120.
- DeStefano, D. & LeFevre, J. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, 16, 353 – 386.
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44, 1 - 42.
- Della Sala, S. Gray, C., Baddeley, A., Allamano, N., & Wilson, L. (1999). Pattern span: a tool for unwinding visuo-spatial memory. *Neuropsychologia*, 37, 1189 - 1199
- Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. A. (1999). *The effect of memory load on negative priming: An individual differences investigation*. US: Psychonomic Society.
- French, J. W., Ekstron, R. B., & Price, I. A. (1963). *Kit of reference test for cognitive factors*. Princeton, NJ: Educational Testing Service.
- Fürst, A. J. & Hitch, G. J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory & Cognition*, 28, 774-782.
- Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology*, 74, 213-239.

- Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review*, 79, 329 - 343.
- Hayes, J. R. (1973). On the function of visual imagery in elementary mathematics. In W. Chase (Ed.), *Visual Information Processing* (pp. 177-214). New York: Academic Press.
- Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Cahiers de Psychologie Cognitive/Current Psychology of Cognition*, 13, 207-245.
- Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory and Cognition*, 30, 447-455.
- Hecht, S. A. (1999). Individual solution processes while solving addition and multiplication math facts in adults. *Memory and Cognition*, 27, 1097-1107.
- Hecker, R. & Mapperson, B. (1997). Dissociation of visual and spatial processing in working memory. *Neuropsychologia*, 35, 599 - 603.
- Hitch, G. J. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology*, 10, 302-323.
- Hitch, G. J. (1980). Developing the concept of working memory. In G. Claxton (Ed.), *Cognitive psychology: New directions*. London: Routledge Kegan Paul.
- Hitch, G. J. & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, 82, 375-386.
- Lee, K., & Kang, S. (2002). Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition*, 83, B63 – B68.

- LeFevre, J. A., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S. L., & Sadesky, G. S. (1996a). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General*, 125, 284-306.
- LeFevre, J. A., Sadesky, G. S., & Bisanz, J. (1996b). Selection of procedures in mental addition: Reassessing the problem-size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 22, 216-230.
- LeFevre, J., DeStefano, D., Coleman, B., Shanahan, T. (in press) Mathematical Cognition and Working Memory. To appear in J.I.D. Campbell (Ed.), *Handbook of Mathematical Cognition*. Penguin.
- LeFevre, J., Smith-Chant, H., Hiscock, K., Daley, K. E., & Morris, J. (2003). Young adults' strategic choices in simple arithmetic: Implications for the development of mathematical representations. In A. J. Baroody & A. Dowker (Eds.). *The development of arithmetic concepts and skills: Recent research and theory*. Hillsdale, NJ: Erlbaum.
- Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, 8, 73-103.
- Logie, R. H. (1986). Visuo-spatial processing in working memory. *Quarterly Journal of Experimental Psychology*, 38A, 229 - 247.
- Logie, R. H. (1995). *Visuo-spatial working memory*. In R. H. Logie (Ed.), Hillsdale, NJ: Lawrence Erlbaum Associates

- Logie, R. H. & Baddeley, A. D. (1987) Cognitive processes in counting. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 13, 310-326.
- Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory and Cognition*, 22, 395-410.
- Logie, R. H., & Marchetti, C. (1991). Visuo-spatial working memory: Visual, spatial, or central executive? In R. H. Logie & M. Denis (Eds.), *Mental images in human cognition* (pp. 105 – 115). Amsterdam: North Holland Press.
- Logie, R. H., & Pearson, D. G. (1997). The inner eye and the inner scribe of visuo-spatial working memory: Evidence from developmental fractionation. *European Journal of Cognitive Psychology*, 9, 241 - 257.
- McCloskey, M. (1992) Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia, *Cognition*, 44, 107 – 157.
- McConnell, J., & Quinn, J. G. (2000). Interference in visual memory. *Quarterly Journal of Experimental Psychology*, 53, 53 - 67.
- Miyake, A. & Shah, P. (1999). Toward unified theories of working memory: Emerging general consensus, unresolved theoretical issues, and future research directions. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 442-481). Cambridge, UK: Cambridge University Press.
- Moar, I. T. (1978). Mental triangulation and the nature of the internal representation of space. Unpublished PhD Thesis, University of Cambridge.

- Noël, M. Désert, M. Aubrun, A. & Seron, X. (2001). Involvement of short-term memory in complex mental calculation. *Memory and Cognition*, 29, 34 – 42.
- Passolunghi, M. C. & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, 80, 44-57.
- Pearson, D.G., Logie, R. H., & Green, C. (1996). Mental manipulation, visual working memory, and executive processes. *Psychologische Beiträge*, 38, 324 - 342.
- Pickering, S. J., Gathercole, S. E., Hall, M., & Lloyd, S. A. (2001). Development of memory for pattern and path: Further evidence for the fractionation of visuo-spatial memory. *Quarterly Journal of Experimental Psychology*, 54, 397 - 420.
- Quinn, J. G. (1991). Encoding and maintenance of information in visual working memory. In R. H. Logie, & M. Denis (Eds.), *Mental images in human cognition* (pp. 95 – 104). Amsterdam: North Holland Press.
- Quinn, J. G. (1994). Towards a clarification of spatial processing. *Quarterly Journal of Experimental Psychology*, 47, 465 - 480.
- Quinn, J. G., & McConnell, J. (1996). Irrelevant pictures in visual working memory. *Quarterly Journal of Experimental Psychology*, 49, 200 - 215.
- Quinn, J. G., & McConnell, J. (1999). Manipulation of interference in the passive visual store. *Quarterly Journal of Experimental Psychology*, 11, 373 - 389.
- Robbins, T. W., Anderson, E. J., Barker, D. R., Bradley, A. C., Fearnlyhough, C., Henson, R., & Hudson, S. R. (1996). Working memory in chess. *Memory and Cognition*, 24, 83 - 93.

- Salamé, P., & Baddeley, A. D. (1982). Disruption of short term memory by unattended speech: Implications for the structure of working memory. *Journal of Verbal Learning and Verbal Behavior*, 21, 150 - 164.
- Siegel, L. S. & Linder, B. A. (1984). Short-term memory processes in children with reading and arithmetic learning disabilities. *Developmental Psychology*, 20, 200-207.
- Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General*, 117, 258 - 275.
- Siegler, R. S. (1996). Formal models of strategy choice. In R. S. Siegler (Ed.), *Emerging Minds: The process of change in children's thinking* (pp. 143-176). New York, NY: Oxford University Press.
- Siegler, R. S. (1998). Memory development. In R. S. Siegler (Ed.), *Children's Thinking* (3rd ed.), (pp. 173-211). Upper Saddle River, NJ: Prentice Hall.
- Siegler, R. S., & Jenkins, E., (Eds.). (1988). *How children discover new strategies*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Siegler, R., & Shipley, C. (1995). Variation, selection, and cognitive change. In T. Simon & G. Halford (Eds.), *Developing cognitive competence: New approaches to process modeling*. Hillsdale, NJ: Erlbaum.
- Seitz, K. & Schumann-Hengsteler, R. (2000). Mental multiplication and working memory. *European Journal of Cognitive Psychology*, 12, 552-570.

- Seitz, K. & Schumann-Hengsteler, R. (2002). Phonological loop and central executive processes in mental addition and multiplication. *Psychologische Beitrage*, 44, 275-302.
- Seyler, D. J., Kirk, E. P., & Ashcraft, M. H. Elementry subtraction. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, (2003).
- Sereno, M. I., Dale, A. M., Reppas, J. B., Kwong, K. K., Belliveau, J. W., Brady, T. J., Rosen, B. R., & Tootell, R. B. H. Borders of multiple visual areas in humans revealed by functional magnetic resonance imaging. *Science*, 268, 889 - 893.
- Shah, P. & Miyake, A. (1996). The separability of working memory resources for spatial thinking and language processing: An individual differences approach. *Journal of Experimental Psychology: General*, 125, 4 - 27.
- Smyth, M. M., & Pendleton, L. R. (1989). Working memory for movements. *Quarterly Journal of Experimental Psychology*, 41, 235 - 250.
- Trbovich, P. & LeFevre, J. (2003) Phonological and visual working memory in mental addition. *Memory & Cognition*,.
- Tresch, M. C., Sinnamon, H. M., & Seamon, J. G. (1993). Double dissociation of spatial and object visual memory: Evidence from selective interference in intact human subjects. *Neuropsychologia*, 31, 211 - 219.
- Vallar, G., & Baddeley, A. D. (1984). Fractionation of working memory: Neuropsychological evidence for a phonological short-term store. *Journal of Verbal Learning and Verbal Behavior*, 23, 151 - 161.

- Vecchi, T., & Cornoldi, C. (1999). Passive storage and active manipulation in visuo-spatial working memory: Further evidence from the study of age difference. *European Journal of Cognitive Psychology*, 11, 391 - 406.
- Zimmer H. D., & Speiser, H. R. (2002). The irrelevant picture effect in visuo-spatial working memory: Fact or Fiction? *Psychologische Beiträge*, 44, 223-247

Appendix A

Two sets of 90 addition problems were used in the present study. Each row of the table shows a set of problems that shared the same sum but represent different levels of problem complexity. Thus, the output requirements were balanced across problem complexity and memory load type within participants, and across participants for format and presentation duration (the latter in E1 only). Op 1 = Operand 1; Op 2 = Operand 2.

SET 1

Set 1	No Carry		Carry-Unit		Carry-Ten		Sum
	Op 1	Op 2	Op 1	Op 2	Op 1	Op 2	
	245	42	249	38	193	94	287
	264	24	259	29	196	92	288
	344	24	339	29	282	86	368
	343	32	348	27	292	83	375
	332	45	329	48	283	94	377
	353	33	349	37	292	94	386
	345	43	339	49	296	92	388
	422	35	428	29	364	93	457
	445	22	438	29	382	85	467
	424	54	449	29	396	82	478
	454	24	429	49	382	96	478
	524	42	527	39	472	94	566
	525	42	529	38	493	74	567
	522	54	527	49	484	92	576
	643	22	638	27	592	73	665
	643	32	648	27	582	93	675
	653	24	649	28	594	83	677
	732	25	728	29	684	73	757
	732	33	738	27	693	72	765
	742	34	729	47	684	92	776
	763	23	758	28	694	92	786
	832	24	829	27	783	73	856
	842	24	839	27	794	72	866
	844	24	839	29	786	82	868
	843	32	847	28	793	82	875
	933	23	928	28	873	83	956
	942	24	927	39	894	72	966
	925	42	929	38	893	74	967
	934	42	927	49	892	84	976

942   44   928   58   894   92   986

SET 2

Set 2	No Carry		Carry-Unit		Carry-Ten		Sum
	Op 1	Op 2	Op 1	Op 2	Op 1	Op 2	
	244	23	238	29	172	95	267
	263	23	258	28	194	92	286
	243	45	249	39	192	96	288
	325	42	329	38	295	72	367
	343	32	347	28	282	93	375
	352	24	349	27	284	92	376
	342	35	349	28	295	82	377
	443	22	437	28	392	73	465
	436	32	429	39	385	83	468
	454	24	449	29	384	94	478
	454	34	449	39	392	96	488
	523	42	529	36	493	72	565
	532	34	528	38	493	73	566
	544	24	539	29	484	84	568
	546	42	549	39	493	95	588
	624	43	629	38	595	72	667
	623	52	629	46	582	92	674
	632	45	629	48	583	94	677
	664	23	658	29	593	94	687
	736	32	729	39	685	83	768
	742	34	749	27	684	92	776
	724	54	729	49	694	84	778
	735	52	749	38	694	93	787
	852	22	827	47	792	82	874
	842	34	849	27	794	82	876
	824	53	828	49	784	93	877
	863	23	858	28	794	92	886
	923	42	938	27	892	73	965
	923	52	926	49	882	93	975
	932	54	928	58	894	92	986

Appendix B

An example of how the lists of problems and their matched memory load items was balanced across participants with the levels of duration and format (B = Brief & C = Continuous). For example, the first participant solved a set of 90 horizontal problems presented briefly from list 1. The same participant then solved another set of 90 horizontal problems presented continuously from list 4.

Participant #	Format	Duration	List # for the first set of 90 problems	List # for the second set of 90 problems
1	Horizontal	B → C	1	4
2	Horizontal	B → C	2	5
3	Horizontal	B → C	3	6
4	Horizontal	C → B	4	1
5	Horizontal	C → B	5	2
6	Horizontal	C → B	6	3
7	Vertical	C → B	1	4
8	Vertical	C → B	2	5
9	Vertical	C → B	3	6
10	Vertical	B → C	4	1
11	Vertical	B → C	5	2
12	Vertical	B → C	6	3

Appendix C

General Background Questionnaire

This general questionnaire is concerned with any difficulties you may have with visual or spatial performance. Please remember that you may omit any questions that may make you feel uncomfortable in answering.

Date		
Birth date		
Gender		F / M
Do you wear corrective lenses?		Y / N
	If yes, how long have you worn glasses?	
	For what purpose (e.g., reading, driving)?	
Have you ever worn corrective lenses?		Y / N
	If yes, how long did you wear them?	
	For what purpose?	
Are you colour blind or colour weak (i.e., red & green deficiency)?		Y / N
Do you have any peripheral visual problems?		Y / N
Do you have any depth perception problems?		Y / N
Do you have any visual-motor co-ordination problems?		Y / N
Do you believe you have a good sense of direction?		Y / N
Do you get motion sickness?		Y / N
	If yes, how long have you had this problem?	
Have you ever lost sensation or feeling in your hands for an long period of time?		Y / N
	If yes, when did this happen?	
	Do you still have a loss of feeling in your hands?	
Have you ever had seizures?		Y / N
		Y / N
Have had any cardiovascular problems or head injuries?		Y / N
	If yes to head injury, please describe location of injury on head	

Appendix D

MATH BACKGROUND AND INTERESTS
QUESTIONNAIRE**GENERAL BACKGROUND**

1. From which province did you receive your education (or country if not Canada)?

Elementary: _____

Secondary: _____

Post-Secondary: _____

2. Age: _____

3. Sex: male   female

Appendix D (continued)**PROCEDURE USE**

There are many ways to solve simple arithmetic problems. For each of the problems, please explain how you would usually solve it.

Problem	Explanation
1. 9×7	
2. $20 \div 4$	
3. $13 - 6$	
4. $8 + 7$	
5. $72 \div 8$	
6. 9×6	
7. $11 - 2$	
8. $9 + 5$	

Appendix D (continued)

1. How do you solve problems like 7×6 , 9×5 , 3×9 ? Do you switch the numbers around so that the smaller number is placed first? Do you always do this? When did you learn to do this?

---

2. Overall, please estimate what proportion on the time you solve simple multiplication problems in ways *other than* by automatically remembering the answers?

never   rarely   sometimes   often   always

3. Overall, please estimate what proportion of the time you solve simple addition problems in ways *other than* by automatically remembering the answers?

never   rarely   sometimes   often   always

4. Overall, please estimate what proportion of the time you solve subtraction problems in ways *other than* by automatically remembering the answers?

never   rarely   sometimes   often   always

5. Overall, please estimate what proportion of the time you solve division problems in ways *other than* by automatically remembering the answers?

Never   rarely   sometimes   often   always

Appendix D (continued)

MATHEMATICS

Mathematics Education

Highest level math course taken in high school: _____

Grade: _____

Math or related courses in University/College: _____

Grade: _____

1. Do you remember using a calculator when doing simple arithmetic? If so, when did you start using a calculator?

---

2. How would you rate your level of basic mathematical skill (e.g., skill at calculation)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

3. How would you rate your level of mathematical skill in more complex areas of mathematics (e.g., calculus, algebra)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

4. Do you ever actively AVOID situations involving mathematics?

1	2	3	4	5	6	7
always try to avoid			sometimes try to avoid			never try to avoid

5. How often do you find that situations involving mathematics make you nervous?

1	2	3	4	5	6	7
almost always			sometimes			almost never

Appendix D (continued)

6. How important do you feel that mathematics will be in the development of your career?
1	2	3	4	5	6	7
not important			moderately			very important
7. How difficult was mathematics for you in high school?
1	2	3	4	5	6	7
extremely difficult			moderately difficult			not at all difficult
8. How much emphasis was placed on learning basic arithmetic facts (e.g., $3 + 4$, 5×9) by your teachers in elementary school?
1	2	3	4	5	6	7
little emphasis			moderate emphasis			strong emphasis
9. How much emphasis was placed on rote learning (or memorizing) basic arithmetic facts by your teachers in elementary school?
1	2	3	4	5	6	7
little emphasis			moderate emphasis			strong emphasis

Appendix D (continued)**LANGUAGE (Spoken)**

1. What languages do you speak?

English _____ French _____ Other _____
_____

2. Which do you consider your first language (i.e., most proficient speaking)?

English _____ French _____ Other _____
_____

3. If you did French immersion in school, please answer the following questions:

(a) You were in French immersion from grades _____ to _____

(b) When did you start doing math in French? _____

(c) When did you stop doing math in French? _____

Appendix D (continued)

LANGUAGE (Reading & Writing)

Language Education

Highest level language/English course taken in high school: _____

Grade: _____

Language/English related course in University/College: _____

Grade: _____

1. How would rate your level of basic reading skill (e.g., reading an instructional manual for information)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

2. How would you rate your skill in reading complex materials (e.g., reading a novel for an upper-year English course)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

3. How would you rate your spelling skills?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

4. How would you rate your basic written communication skills (e.g., summarizing a set of notes for a friend or writing a letter)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

5. Overall, how would you rate your written language skill (e.g., writing a paper for a university course)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

6. Do you ever actively AVOID situations involving reading?

1	2	3	4	5	6	7
always try to avoid			sometime try to avoid			never try to avoid

7. Do you ever actively AVOID situations requiring writing?

1	2	3	4	5	6	7

always try
to avoid

sometimes try
to avoid

never try
to avoid

Appendix D (continued)

8. How often do you find that situations requiring that you "read for meaning" make you nervous?

1	2	3	4	5	6	7
almost always			sometimes			almost never

9. How often do you find that situations involving writing make you nervous?

1	2	3	4	5	6	7
almost always			sometimes			almost never

10. How important do you feel that language skills (reading and writing) will be in the development of your career?

1	2	3	4	5	6	7
not important			moderately			very important

11. How difficult was English for you in high school?

1	2	3	4	5	6	7
extremely difficult			moderately difficult			not at all difficult

Appendix E

Informed Consent Form

Title: Role of working memory in arithmetic

The purpose of this form is to ensure that you comprehend both the purpose of the study and the nature of your participation in it. An informed consent form must provide enough information to allow you the opportunity to determine whether you wish to participate in the study. Please inquire with the researcher to clarify any doubts or concerns you may have upon completion of this form.

Research personnel. The following people are involved in this research project and may be contacted at any time:

Ben Coleman (Principal Investigator, 520-2600 ext. 2696); Dr. J. LeFevre (Faculty Sponsor, 520-2600 ext. 2693). If you have any ethical or other concerns about this study please contact Dr. M. Gick (Chair, Carleton University Research Ethics Committee for Psychological Research, 520-2600 ext. 2664) or Dr K. Matheson (Chair, Department of Psychology, 520-2600 ext. 2648).

PURPOSE OF EXPERIMENT

The purpose of the experiment is to investigate the role that working memory might play in solving arithmetic problems.

NATURE OF YOUR PARTICIPATION

There are two parts to this experiment. In the first section of the experiment the experimenter will measure your ability to add numbers and your memory capacity. Then you will be asked to fill out a small general background questionnaire regarding any personal history that you might have difficulty in visual and/or spatial performances. In the second section of the experiment you will be asked to perform two cognitive tasks concurrently in your head. For each trial you will be shown a stimulus (i.e., visual pattern, or a spatial sequence, or a string of letters) for a few seconds on the computer screen. Following the stimulus presentation you will then see an addition problem. After solving the addition problem you will be asked to replicate the stimulus shown to you at the start of the trial.

Experiment Duration and Location:

The experiment will take about 90 minutes for 2 course credits. Testing will take place on Carleton University campus in Loeb A826.

Appendix E (continued)**Potential Risk/Discomfort:**

There is no potential physical or psychological risks involved with, or associated with, participation in this study. If you are uncomfortable with performing anything in this study, please bring your concerns to the researcher's attention immediately.

Confidentiality:

All data collected in this study are kept confidential through the assignment of a coded identification number. All information that you provide will be used for research purposes only and you will not be identified by name in any research reports produced from this study. Personal information provided will be made available only to the researchers associated with this experiment (see names below).

Right to Withdraw:

Your involvement in this research is voluntary. You may choose not to answer any question(s) and you will not suffer any penalty as a result. You also have the right to withdraw from this experiment at any time.

I have read the above description of the study examining the role of working memory in solving arithmetic problems, and I understand the conditions of my participation. My signature indicates that I hereby consent to participate in this experiment.

Name: _____

Date: _____ 200__

Signature: _____

Witness: _____

Appendix F

Debriefing

The role of working memory in solving arithmetic problems

Thank you for participating in this study! The purpose of this study is to investigate the role that working memory might play in solving arithmetic problems. There are three components of working memory that I am interested in. You only participated in one of the three components of working memory: 1) visual memory system, 2) spatial memory system, & 3) verbal memory system.

Visual. The visual working memory system serves to maintain visual information on 'what' has been seen. To test your visual working memory you were asked to do two memory tasks at once. One memory task was to remember a visual pattern. Some of the patterns were shown as an asterisk in the middle of the screen (no-load), while other patterns consisted of four black squares and four white squares (easy load) or seven black and seven white squares (hard load). You were asked to remember the pattern while solving the addition problem. Once the problem was solved you then reproduced the visual pattern. These visual patterns served to test your visual working memory while concurrently solving addition problems. I am predicting that the role of the visual working memory system in solving addition problems is to maintain a specific number (i.e., either to the tens number or hundreds unit) that the carry will be added too. This process of adding the carry to either the tens or hundreds digit should require more resources from the visual working memory system than from the other two working memory systems.

Spatial. The spatial memory system serves to maintain information about 'where' objects are located in space. It has been found that addition problems presented vertically are solved faster than problems presented horizontally. This finding suggests that solving horizontal problems requires more spatial memory resources than solving vertical problems. To test your spatial working memory, you were asked to remember a spatial sequence of blocks. Some of the sequence trials were shown as an asterisk in the middle of the screen (no-load), while other sequence trials consisted of highlighting two blocks (easy load) or five blocks (hard load). You were asked to remember the sequence while solving the addition problem. Once the problem was solved you were then asked to reproduce the sequence. These spatial sequences served to test your spatial working memory while concurrently solving addition problems. I am predicting that the role of the spatial memory system in solving addition problems is to maintain 'where' the digits are located in space. That is, it should have taken you longer to solve problems presented horizontally than vertically. The process of having to solve problems presented in a horizontal format should require more resources from the spatial working memory system than from the other two working memory systems.

Verbal. The verbal memory system serves mainly to keep the verbal codes active during processing of information. To test your verbal memory you were asked to remember a string of nonsense letters called CVCs (e.g., JIS). Some of the CVC trials were shown as an asterisk in the middle of the screen (no-load), while other

Appendix F (continued)

CVC trials consisted of showing a single CVC (easy load) or three CVCs (hard load). You were asked to remember the CVC(s) while solving the addition problem. Once the problem was solved you were then asked to reproduce the CVC(s). These CVC trials serve to test your spatial working memory while concurrently solving addition problems. I am predicting that presenting the addition problem on the screen for a short period of time should draw upon more verbal memory resources to maintain the verbal codes than any of the other two systems. However, under conditions where the problem is left on the screen fewer resources should be required from the verbal memory system, thus allowing you to solve the problem faster.

There are five independent variables and one dependent variable in this experiment. The dependent variable was the time it took you to answer the addition problem out loud. The first independent variable contains three types of working memory tasks (i.e., visual pattern vs. spatial sequence vs. CVC) and the second independent variable contains two types of problem formats by which problems are presented on the screen (i.e., vertical vs. horizontal). Today, you only experienced one of the three working memory tasks with the addition problems presented to you in one of the two formats. A third independent variable was to present the problem on the screen for a short period of time or allow it to remain on the screen until a response was made. A fourth independent variable was the type of problem to be solved. Each problem was categorized as either a no-carry problem ($135+52$), a single-carry problem where the carry comes from the ones unit ($157 + 38$), or as a single-carry problem where the carry comes from the tens unit ($783 + 65$). The last independent variable was the type of working memory load. Some of the trials had no memory load, or an easy memory load or a hard memory load. Depending on the working memory task that you were asked to perform, on some of the trials you were asked only to reproduce an asterisk (no-load). On other trials you were asked to reproduce a relatively easy visual pattern, or spatial sequence, or CVC set (easy-load), while on the other trials you were asked to reproduce a more complicated visual pattern, or spatial sequence, or CVC set (hard-load).

Thanks again for your time. If you have any questions or comments about this research, the please feel free to contact:

Dr. Jo-Anne LeFevre	Faculty Sponsor	520-
2600 ext. 2693		
Ben Coleman	Principal Investigator	520-2600
ext.2696		

Any ethical questions about this experiment should be directed to:

Dr. M. Gick (Chair, Carleton University Research Ethics Committee for Psychological Research 520-2600 ext. 2664) or Dr. K. Matheson (Chair, Department of Psychology, 520-2600 ext. 2648)

Appendix G

The participant was told to quietly rehearse each letter the experimenter said and in the same order they were said. The experimenter then read out loud each letter at the rate of one letter per second. After the experimenter read out loud each letter the participant was then to rehearse each letter quietly in their head for 10 seconds while looking at a dot on a blank white wall. After the 10 second delay, the experimenter said "alright" and the participant then repeated back the letters and in the same order that they were said. The largest sequence of letters that a participant could correctly remember was their verbal memory span score.

On a correct trial, the participant advanced to the next level of auditory memory span. On a failed trial, the participant was given a second trial at the same memory span level. Finally, after 2 consecutive failures at the same level, the auditory span task was ended. Half of the participants get set A and the other half with set B.

Level 1A: K W
Level 1B: G P

Level 2A: A J R
Level 2B: D H P

Level 3A: L Y Q I
Level 3B: X N G V

Level 4A: A J K H L
Level 4B: T L P W D

Level 5A: X J D K Y A
Level 5B: Z R N B T E

Level 6A: V T B D F R X
Level 6B: E H C F T R P

Level 7A: F E X T L B H V
Level 7B: F J L X V T C E

Level 8A: C D G T P X W Z K
Level 8B: D G T C E U Z H N

Level 9A: Q D N V P C Y B K G
Level 9B: R Z L F I D T B E C

Appendix H

The latency for each correctly answered problem was selected for analysis if: a) the participant correctly reproduced at least 33% of the memory load item associated with that correct problem, and b) the latency did not exceed two standard deviations from the median for vertical problems and three standard deviations from the median for horizontal problems. On the first condition, I wanted to select the maximum number of problems that represented participants' correctly solving problems under dual-task conditions. One possibility was to select only latencies where participants' correctly solved the problem and 100% of the associated memory load. There were 36 conditions [i.e., 2 (duration) x 2 (format) x 3 (problem type) x 3 (memory load)] with 10 problems per condition to be examined. Using the above criterion (correct problem + 100% memory load) reduced the number of problems per condition (i.e., 10) to about 3 problems per condition. Therefore, another approach was taken and found the following criteria to work best. It was assumed that participants' who solved the problem correctly and recalled at least 33% of the memory load were processing both tasks concurrently. For example, on easy memory loads (e.g., MUP) the participant had to recall at least one of the three nonsense letters in the same order and get the answer to the addition problem correct. There were 7219 problems correctly solved. Of these problems, 7013 latencies for correctly answered problems were retained and 206 problems were discarded because participants' failed to recall at least 33% of the memory load item.

Examination of the latency distributions for the remaining 7013 problems showed an imbalance of problems between formats and the other conditions of the independent variables. There were 3763 vertical problems and 3250 horizontal problems.

This result was not unreasonable since participants were expected to produce more errors when solving and longer latencies for horizontal problems than vertical problems (Heathcote, 1994; Trbovich & LeFevre, 2003). However, this meant that the number of correctly solved problems (i.e., originally 10 problems per condition) were potentially different across conditions (i.e., 18 vertical conditions & 18 horizontal conditions). For example, when correct problems were selected based on 100% recall of the memory load, it was found that the number of horizontal problems correctly solved was reduced to about 2 problems per condition, whereas the numbers of vertical problems were reduced to about 5 problems per condition. From the number of correct problems with at least 33% of the memory load recalled, I found that selecting vertical problems that exceeded two-standard deviations from its median ($Md = 2806ms$, $SD = 1584ms$) and horizontal problems exceeding three-standard deviations from its median ($Md = 3496ms$, $SD = 2318ms$), produced an average of 7.3 problems between formats and across conditions (i.e., range: 6 – 10). The tables shown in Appendices I and J show the number of problems between formats and across conditions for Experiments 2 and 3, respectively.

Appendix H (continued)

Table H

*Experiment 1: Average number of problems correctly solved by participants ($n = 47$)
across the levels of format, problem, duration and load (max. 10 problems / condition).*

	Vertical			Horizontal		
	No Carry	Carry Units	Carry Tens	No Carry	Carry Units	Carry Tens
	Brief					
No Load	9	8	8	8	7	6
Easy Load	9	8	8	7	7	7
Hard Load	9	7	7	7	6	6
	Continuous					
No Load	10	8	8	9	8	8
Easy Load	10	8	8	9	8	8
Hard Load	9	7	7	8	7	7

Appendix I

Table I

Experiment 2: Average number of continuous problems correctly solved by participants ($n = 47$) across the levels of format, problem type, and memory load (max. 10 problems / condition).

Vertical				Horizontal		
	No Carry	Carry Units	Carry Tens	No Carry	Carry Units	Carry Tens
No Load	9	8	8	9	9	9
Easy Load	9	8	8	9	8	8
Hard Load	9	8	7	9	8	8

Appendix J

Table J

Experiment 3: Average number of continuous problems correctly solved by participants (n = 48) across the levels of format, problem type, and memory load (max. 10 problems / condition).

Vertical				Horizontal		
	No Carry	Carry Units	Carry Tens	No Carry	Carry Units	Carry Tens
No Load	9	8	8	9	8	9
Easy Load	9	8	8	9	8	8
Hard Load	9	8	8	9	8	9

Appendix K

Briefly presented problems with various memory load conditions produced speed-accuracy trade offs. This evidence indicates that participants may have found retaining briefly presented operands and the partial sum in working memory to be too difficult while concurrently rehearsing a phonological memory load. The following results are from the performances by participants when solving problems under various problem-type, memory-load, format and duration conditions in Experiment 1.

Duration x Memory Load

Contrary to my expectations, participants solved problems presented continuously equally quickly as problems presented briefly (3494 ms vs. 3415 ms), $F(1,45) < 1$. This result is not consistent with the literature (e.g., Fürst & Hitch, 2000; Heathcote, 1994) in which briefly presented problems were more difficult for participants to solve than continuously presented problems. My participants did, however, make more errors on briefly presented problems than on continuously presented problems (22% vs. 15%), $F(1,45) = 20.80, p < .01, MSE = 513, CI = 3.38\%$.

The predicted interaction between duration and memory load approached conventional significance for latencies, $F(2,90) = 2.80, p = .065, MSE = 240002, CI = 71$ ms, however, the pattern was not consistent with my hypotheses. As shown in Figure 15 below, continuous problems were solved more slowly than briefly presented problems when participants had a hard phonological load. The opposite pattern was predicted. Examination of the errors (also shown in the Figure 15), suggests that errors for increase for solving both brief and continuous problems, however, the interaction with memory load was not significant, $F(2,90) = .75, p > .05, MSE = 107, CI = 1.54\%$. Further

examination of the latencies in Figure 16 indicates that verbal load has no effect on briefly presented problems. This result is not consistent with the literature (e.g., Fürst & Hitch, 2000; Heathcote, 1994), which suggests that briefly presented problems require phonological resources. In the present experiment there appeared to be a speed-accuracy trade-off in the brief condition. One possible conclusion for this result is that brief problems presented under phonological memory loads were possibly too difficult to encode, which would result in having no effect on the phonological memory load. Furthermore, it is possible that brief problems were presented long enough to encode the operands and process them, thus reducing or eliminating the need to maintain these codes in a phonological store. What is clear, however, is that there was a speed-accuracy tradeoff between briefly presented and continuously presented problems, thus making it difficult to interpret this result.

Figure 15. Latencies in milliseconds and percent errors with memory load & duration

Format x Memory Load

As predicted, participants solved horizontal problems more slowly than vertical problems (3993 ms vs. 2916 ms), $F(1,45) = 9.65, p < .01, MSE = 25413606$. They also made more errors solving horizontal problems than vertical problems (28% vs. 24%), but the difference was not significant, $F(1,45) = 1.80, p > .05, MSE = 1891$. These results replicate previously reported format effects (Heathcote, 1994; Trbovich & LeFevre, 2003).

I also assumed that participants' performance would vary with memory load. Two orthogonal contrasts (1 df) were made to test this assumption. The linear contrast represents the memory load effect, where participants' performance was tested under no-load conditions and under hard-load conditions (1, 0, -1). The latencies in Table 7 show that participants were slower to respond, $F(1,45) = 17.06$, $p < .001$, $MSE = 226417$, $CI = 137$ ms, and they made more errors, $F(1,45) = 26.43$, $p < .001$, $MSE = 329$, $CI = 5.46\%$, solving problems under hard load conditions than no-load conditions. The quadratic contrast was used to test between no-load, easy-load, and hard-load conditions (1,-2, 1), respectively. Table 7 shows that participants' were equivalently fast in no-load and easy-load conditions, however, their responses were much slower than these conditions under hard-load, $F(1,45) = 8.01$, $p < .01$, $MSE = 144473$, $CI = 111$ ms. Participants' errors were also equivalent in no-load and easy-load conditions, however, their errors increased under hard-load conditions, $F(1,45) = 10.25$, $p < .001$, $MSE = 157$, $CI = 3.65\%$. This evidence suggests a general verbal memory load in arithmetic and supports findings in the literature (e.g., Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Lee & Kang, 2002; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002).

Table 7

Latencies in milliseconds and percent errors with memory load

Load	Latency (ms)	Error (%)
None	3398	15
Easy	3403	16
Hard	3564	23

Load and format interacted, $F(2,90) = 5.30$, $p < .01$, $MSE = 185445$, $CI = 124$ ms. The memory load effect (no-load vs. hard load; 1,0, -1) shown in Figure 2, indicates that participants' performance on solving horizontal problems varied with levels of load, but their performance on solving vertical problems was unaffected by load, $F(1,46) = 4.77$, $p < .05$, $MSE = 706045$, $CI = 245$. This result supports the view that the phonological load plays a role in solving horizontal problems but not in solving vertical problems (Heathcote, 1994; Trbovich & LeFevre, 2003). Trbovich and LeFevre have suggested that participants possibly use a verbal strategy, such as decomposition (e.g., $352 + 46 = 350 + 40 + 2 + 6 = 398$), to solve horizontal problems, thus increasing the verbal resources required to retain these extra codes in the phonological loop. Although the horizontal errors and vertical errors both increase with levels of load, the interaction of format and load was not significant, $(1,46) < 1$, $MSE = 737$, $CI = 7.91\%$. As shown in Figure 16, participants made more errors on the horizontal problems than on the vertical problems at all three levels of load. Similarly, participants made more errors in the hard load than in the easy or no-load conditions for both formats. Nevertheless, horizontal problems seemed more affected by hard load, a result that is consistent with the pattern of response times.

Figure 16 Latencies in milliseconds and percent errors with levels of memory load and format

Problem Type. I assumed that calculating carry problems would take participants longer to solve and make more errors than calculating no-carry problems. Problem type was broken down into two orthogonal contrasts. First, the carry effect contrast (1, -2, -1) compared participants' performances on solving no-carry problems with the two carry problems. Table 8 shows that participants solved no-carry problems more quickly than carry problems, $F(1,45) = 134.0, p < .001, MSE = 1110754, CI = 157\text{ms}$. Furthermore,

participants' errors also made more errors solving carry problems than solving no-carry problems, $F(1,45) = 47.3$, $p < .01$, $MSE = 373$, $CI = 2.88\%$. This result supports the literature that suggests carry problems require more cognitive resources for processing than no-carry problems (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Logie et al., 1994; Seitz & Schumann-Hengsteler).

The position effect contrast (1, 0, -1) compared the carry-unit problems and carry-ten problems. As shown in Table 8, participants' solved carry-unit problems more slowly than carry-ten problems, $F(1,45) = 7.80$, $p < .01$, $MSE = 731420$, $CI = 127\text{ms}$, and less accurately, $F(1,45) = 5.50$, $p < .01$, $MSE = 454$, $CI = 3.17\%$. This result indicates that processing a carry-unit problem requires more cognitive resources than processing a carry-ten problem.

Table 8

Latencies in milliseconds and percent errors with problem type

Problem Type	Latency (ms)	Error (%)
No carry	2861	12
Carry-ten	3650	19
Carry-unit	3852	24

Summary

In summary, participants' found carry problems more difficult to solve than no-carry problems. This predicted result was consistent with the literature that suggests

carry problems are more difficult to solve than no-carry problems (Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Heathcote, 1994; Logie & Baddeley, 1987; Logie et al., 1994). Furthermore, participants were slower to solve carry-unit problems than carry-ten a problem, which was not expected. This result suggests that carry problems differentially effect cognitive processing depending on where the carry occurs within the problem.

The presentation format (vertical vs. horizontal) of the addition problems was expected to have differential effects on participant performance. Consistent with the literature, horizontal problems slowed participant performance as compared to performance on vertical problems (Heathcote, 1994; Trbovich & LeFevre, 2003). Furthermore, under phonological memory load horizontal problems were more difficult to solve than vertical problems. This result suggests that the demands on the phonological loop are increased when solving horizontal operands, thus supporting the possibility that people use verbal procedures to solve horizontal problems (Trbovich & LeFevre, 2003).

I predicted that presentation duration would interact with memory load (e.g., Fürst & Hitch, 2000; Heathcote, 1994). However the interaction between duration and memory load shows a speed-accuracy tradeoff between participants' latencies and errors committed on the primary task. Briefly presented problems were solved faster than continuously presented problems; however, the participants' committed more errors when problems were presented briefly than continuously. The duration variable was, therefore, not used in further experiments.

Appendix L

The procedure used for presenting the arithmetic problem and phonological memory load. There are three memory load conditions and 90 experimental trials. Therefore, there are 30 no load, 30 easy load, and 30 hard load trials. In easy load conditions one CVC is shown, whereas in hard load conditions three CVCs are shown. The participant has to remember the nonsense letter string for recall.

Appendix M

The procedure used for presenting the arithmetic problem and visual memory load.

There are three memory load conditions and 90 experimental trials. Therefore, there are 30 no load, 30 easy load, and 30 hard load trials. In easy load conditions four squares are highlighted, whereas in hard load conditions seven squares are highlighted. The participant has to remember the pattern of black and white squares for recall.

Appendix N

The procedure used for presenting the arithmetic problem and spatial memory load.

There are three memory load conditions and 90 experimental trials. Therefore, there are 30 no load, 30 easy load, and 30 hard load trials. The grid size remains the same for easy and hard loads. In easy load conditions two squares are highlighted, whereas in hard load conditions four squares are highlighted. The participant has to remember the location and order that these squares were highlighted in for recall.

Appendix O

Descriptive results from Experiments 1 – 3

	Experiment 1	Experiment 2	Experiment 3
Gender:			
<i>f</i> (Male)	22	21	21
<i>f</i> (Female)	25	26	27
Age (years):			
Median	23	22	27
Range	18 - 28	18 – 25	18 – 41
French Kit (arithmetic skill)			
<i>f</i> (Low [$x < 70$])	27	24	24
<i>f</i> (Medium [$70 \leq x < 80$])	4	6	4
<i>f</i> (High [$x \geq 80$])	24	19	22
Head injury:			
<i>f</i> (Yes)	0	4	2
<i>f</i> (No)	40	12	15
<i>f</i> (No report)	7	31	31

Appendix O (continued)

General questionnaire			
<i>f</i> (complete)	41	16	17
<i>f</i> (not complete)	6	31	31

Math questionnaire			
<i>f</i> (complete)	30	17	19
<i>f</i> (not complete)	17	31	29

Median Memory Span			
Phonological (max: 9)	5	5	5
Visual (max: 15)	9	10	10
Spatial (max: 7)	4	4	4

Appendix P

In a dual-task paradigm, performance on a primary task is measured while participants' concurrently perform a secondary task that is known to place demands on cognitive resources. As the difficulty of the secondary task increases, performance on the primary task and secondary task are expected to suffer to the extent that both tasks depend on the same cognitive resources. However, the limitation of the present research is that an independent measure of the cognitive demands required to process the secondary task without the primary task cannot be obtained. Therefore, it was not clear as to whether primary task performance was impeded by sharing common resources with the secondary task or due to difficulty of concurrently performing the secondary task. For example, some of the evidence in Experiment 2 indicates performance on the primary task (i.e., solving the arithmetic problem) may have suffered because the visual memory load task was possibly too difficult to process while solving a multi-digit arithmetic problem. The following is an analysis of peoples' performance on the secondary task across experiments, problem type, memory load, and format.

Percent correct. In all three experiments there were three types of memory load conditions. In the no-memory load condition participants were not required to remember the asterisk (*) as the memory load. In the easy memory load conditions participants had to remember a CVC (GUP; Experiment 1), or a visual pattern (4 white & 4 black squares in a grid; Experiment 2), or a spatial sequence (remembering a sequence of 2 highlighted squares on a grid of 10 squares; Experiment 3) while concurrently solving a multi-digit problem. In the hard memory load conditions participants had to rehearse three CVCs (FAC MUB LER; Experiment 1), or a visual pattern (7 white & 7 black squares in a grid;

Experiment 2), or spatial sequence (remembering a sequence of 4 highlighted squares on a grid of 10 squares; Experiment 3). Measures of secondary task performance were collected as the percent of the secondary task correctly recalled on each trial. For example, a person who reproduced FAC GUB LEP from the original FAC MUB LER would have received a score on the secondary task performance of seven out of nine (77.77%).

Analyses. Percent error on people's secondary task performance was analyzed in separate 2 (format: horizontal vs. vertical) x 3 (experiment: phonological vs. visual vs. spatial) x 3 (problem type: no carry vs. carry units vs. carry tens) x 3 (memory load: none vs. easy vs. hard) mixed ANOVAs. Format and experiment were the between-groups variable and the remaining variables were repeated measures. As explained in Experiment 1, the error bars in the figures represent 95% confidence intervals (Loftus & Masson, 2002).

Problem Type x Memory Load x Experiment

The main effect of problem type was significant, $F(2,270) = 26.67, p < .001, MSE = 87$. The carry-position effect (carry-unit vs. carry-ten) was not significant, $F(1,135) < 1$. The carry effect (no carry vs. carry) shows that people made more errors recalling the secondary task while solving carry problems than no-carry problems (11% vs. 7%), $F(1,135) = 63.60, p < .001, MSE = 71$. This result indicates that people's secondary task performance was affected more when they solved a carry problem than a no-carry problem.

The main effect of memory load, $F(2,270) = 100.70, p < .001, MSE = 543$, shows that secondary task performance decreased monotonically from hard load to easy load to no load (22% vs. 6% vs. 0%), respectively. This result indicates that the size of the memory load does have an affect on a person's secondary task performance.

Performance by participants on the secondary task varied with Experiment, $F(2,135) = 9.44, p < .01, MSE = 1359$. This result shows that recalling the phonological memory load was much easier than recalling the visual or spatial memory loads (3% vs. 12% vs. 13%), respectively.

The interaction between problem type and memory load was significant, $F(4,540) = 10.91, p < .001, MSE = 81, CI = 1.51\%$. Figure 17 shows that solving carry problems under easy and hard memory load conditions disrupted recall of the secondary task more than solving no-carry problems under the same memory load conditions. This result suggests that people's secondary task performance was affected by the size of the memory load (easy vs. hard) and the type of arithmetic problem (no-carry vs. carry) that the participant was asked to solve.

Figure 17 Percent errors on secondary task with levels of memory load and problem type

Figure 18 shows that problem type interacts with experiment, $F(4,270) = 6.35, p < .001, MSE = 87, CI = 1.56$. This result indicates that carry problems relative to no-carry problems had an affect on recall of phonological and spatial memory loads. Conversely, secondary task performance did not differentiate with problem type under visual memory loads. Possibly people found retaining the visual memory load while concurrently solving an arithmetic problem to be more difficult than rehearsing other memory loads while solving arithmetic problems.

Problem Type x Experiment

Figure 18 Percent errors on secondary task with levels of problem type and experiment

The three-way interaction between problem type, memory load and experiment was significant, $F(2,135) = 9.44, p < .01, MSE = 1359$. The results (see Figure 19) show that people made more errors reproducing the secondary task when they had to solve carry problems, relative to no-carry problems, under hard phonological loads but not under easy phonological loads. Secondary task performance (0% errors) was unimpeded by problem type (i.e., carry vs. no-carry) under easy phonological loads (e.g., GUP). Conversely, secondary task performance differentiated with problem type under hard phonological loads (e.g., FAC MUB LER). Furthermore, Figure 19 shows that under dual-task conditions, errors on the secondary task varied with problem type (carry vs. no-carry) and memory load conditions (easy vs. hard). People made more errors

reproducing the secondary task when they had to solve carry problems, relative to no-carry problems, under hard spatial loads than under easy spatial loads.

Examination of secondary task performance with problem type and visual memory load size shows no problem type effect with easy or hard memory loads.

Relative to secondary task performance in Experiments 1 (and 3), performance in Experiment 2 may not have differentiated with problem type under easy or hard visual loads because people may have found retaining a visual memory load to difficult under dual-task conditions. Interpreting the results from Experiment 2 involving memory load are unclear. That is, the results involving visual memory load could be related to the difficulty of the memory load or to interference with processing the primary task. A drawback to the present study is not having a memory load control condition (see Seyler et al., 2003). Using an independent measure of the secondary task performance would have helped to clarify whether the results of Experiment 2 were related to difficulty of the secondary task.

Figure 19 Percent errors on secondary task with levels of memory load, problem type, and experiment

Summary

In summary, secondary task performance was affected by problem type, memory load size and experiment. The results show that secondary task performance varied with the size of the memory load (easy vs. hard) and the type of arithmetic problem (no-carry vs. carry). In Experiment 1 (phonological), people made more errors reproducing the secondary task when they had to solve carry problems, relative to no-carry problems, under hard phonological loads but not under easy phonological loads. In Experiment 3 (spatial), secondary task performance varied with problem type under easy spatial and hard spatial load conditions. These results indicate that the secondary task performance

is differentially related with solving carry and no-carry problems. Experiment 2, in contrast, shows that secondary task performance does not vary with problem type under easy visual or hard visual memory load conditions. This result suggests that the visual load is unrelated to problem type manipulation. Because it was not clear what effect the visual memory load had on dual-task performance, the data from Experiment 2 (visual) were, therefore, not included in the combined analysis of Experiment 1 (phonological) and Experiment 3 (spatial).