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The Application of Feedback Decoding in the Differential
Detection of Continuous Phase Modulated Signals.

by

Daniel Boudreau, B. Eng.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering

Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

April, 1987

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ABSTRACT

The work reported in this thesis represents a comprehensive treatment of the application of channel coding principles in the processing of the outputs of many levels of differential detection of a continuous phase modulated signal. The differential phase variables of MSK are treated as the output of a convolutional encoder and feedback decoding is applied to perform symbol decisions. The MSK detector analysis makes use of this coding analogy to obtain a procedure leading to the performance prediction. Simulation and analysis are used to obtain the MSK performance on a static AWGN channel and on a slow fading multipath channel. A link is established between the channel parameters and the statistics of the random variables considered by the decoder. An extension of the detection procedure to other full response signals as well as partial response signals is discussed briefly. A practical MSK implementation on a DSP chip is presented and its performance, as measured under laboratory conditions, is compared to the analytical and simulated results.

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Finally, I want to dedicate this thesis to my parents, who have always supported me during my life, my studies and my career.

TABLE OF CONTENTS

	page
CHAPTER 1 INTRODUCTION	1
1.1 INTRODUCTION	1
1.2 DETECTION OF CPM SIGNALS	3
1.3 THESIS OBJECTIVES	6
1.4 THESIS ORGANIZATION	7
CHAPTER 2 BINARY CONTINUOUS PHASE MODULATION	10
2.1 DEFINITIONS	10
2.2 DIFFERENTIAL DETECTION OF CPM SIGNALS WITH $H=1/2$	15
2.3 DIFFERENTIAL PHASE PROPERTIES	24
2.3.1 FULL RESPONSE SIGNALLING	25
2.3.2 FULL RESPONSE SIGNALLING WITH MODULATION INDEX OF $1/2$	31
CHAPTER 3 PHASE FEEDBACK DECODING OF BANDLIMITED MSK	37
3.1 FEEDBACK DECODING PRINCIPLES	38
3.2 PHASE FEEDBACK DECODING PRINCIPLES	59
3.2.1 2-BRANCH PHASE FEEDBACK	

DECODING	60
3.2.2 M-BRANCH PHASE FEEDBACK	
DECODING	62
3.3 PERFORMANCE ANALYSIS	71
3.3.1 CHANNEL CONSIDERATIONS	71
3.3.2 ANALYSIS OF 2-BRANCH	
DECODING	75
3.3.3 ANALYSIS OF 3-BRANCH	
DECODING	85
3.3.4 PERFORMANCE LOWER BOUND	88
3.4 ERROR STATISTICS	
CHARACTERIZATION	94
CHAPTER 4 PERFORMANCE OF PHASE FEEDBACK DECODING OF BANDLIMITED MSK	
	97
4.1 ADDITIVE GAUSSIAN NOISE	
CHANNEL	98
4.1.1 RECEIVE FILTERS	
CONSIDERATIONS IN	
DIFFERENTIAL DETECTION	100
4.1.2 RX FILTERS CONSIDERATIONS	
WITH FEEDBACK DECODING	120
4.1.3 SIMULATION RESULTS	135
4.1.3.1 TWO-BRANCH DECODING	137
4.1.3.2 THREE-BRANCH DECODING	139
4.2 FADING MULTIPATH CHANNEL	146

4.2.1	SIMULATION RESULTS	150
4.2.1.1	TWO-BRANCH DECODING	153
4.2.1.2	THREE-BRANCH DECODING	158
4.3	CONCLUSION	164
CHAPTER 5	APPLICATION TO OTHER BINARY CPM SIGNALS	166
5.1	FULL RESPONSE SIGNALS	166
5.2	PARTIAL RESPONSE SIGNALS	170
5.3	CONCLUSION	173
CHAPTER 6	PRACTICAL IMPLEMENTATION ON A DSP CHIP	174
6.1	2400 BPS TMS32010 MSK MODEM IMPLEMENTATION	174
6.2	MEASURED PERFORMANCE	178
6.3	CONCLUSION	180
CHAPTER 7	CONCLUSIONS AND FUTURE WORK	183
7.1	CONCLUSIONS	183
7.2	FUTURE WORK	184

LIST OF FIGURES

Figure 2.1 (a) A one-bit differential detector for GMSK (b) A two-bit differential detector for GMSK.	16,
Figure 2.2 A GMSK transmitter suited for two-bit differential detection.	16
Figure 2.3 (a) One-bit detector eye diagrams; Gaussian IF filter; $B_r T = 1.25$ (b) Two-bit detector eye diagrams; Gaussian IF filter; $B_r T = 1.25$ (from [21]).	17
Figure 2.4 Performance of one- and two-bit differential detection of GMSK (from [21]).	20
Figure 2.5 One-bit versus two-bit differential detection of MSK with a Gaussian receive filter (from [22]).	21
Figure 2.6 Single Error Correction circuit.	23
Figure 2.7 Differential phase plane for full response signalling with $h=4/7$.	27
Figure 2.8 Differential phase planes for full response signalling with $h=1/4$, (a) $k=1$, $\Delta\Phi_1(n) = a_n \pi/4$ (b) $k=2$, $\Delta\Phi_2(n) = \pi/4 (a_n + a_{n-1}) \bmod 2\pi$ (c) $k=3$, $\Delta\Phi_3(n) = \pi/4 (a_n + a_{n-1} + a_{n-2}) \bmod 2\pi$.	28
Figure 2.9 Decision process on the differential phases for a full response CPM signal with $h=1/4$.	30
Figure 2.10 Demodulator for a full response CPM signal with $h=1/4$.	32
Figure 2.11 Rate $1/M$ convolutional encoder.	35
Figure 3.1 Example of the behaviour of a feedback decoder.	40

Figure 3.2 Encoder and table look-up feedback decoder.	42
Figure 3.3 Trellis diagram corresponding to figure 3.1.	45
Figure 3.4 Feedback decoder for the code of figure 3.1.	45
Figure 3.5 Feedback decoder for the code of figure 3.2 with $L=1$.	48
Figure 3.6 Syndrome register state transitions with $L=1$ and a single error in $e_I(n)$ for (a) $\hat{e}(n-1)=S(n-1)$ (b) $\hat{e}(n-1)=0$.	49
Figure 3.7 Syndrome register state transitions with $L=1$ and a single error in $e_p(n)$ for (a) $\hat{e}(n-1)=S(n-1)$ (b) $\hat{e}(n-1)=0$.	50
Figure 3.8 Probabilistic state transition diagram for the decoder of figure 3.4.	54
Figure 3.9 P_{fd} and P_{fe} versus p .	58
Figure 3.10 Rate 1/2 convolutional encoder.	61
Figure 3.11 (a) Received differential phases for $M=2$ (b) Feedback decoding of the received differential phases for $M=2$.	61
Figure 3.12 Rate 1/M convolutional encoder.	63
Figure 3.13 Trellis diagram for the code with $M=3$.	64
Figure 3.14 Three-branch phase feedback decoder.	69
Figure 3.15 Channel between the modulator and the demodulator.	73
Figure 3.16 2-branch decoder.	77
Figure 3.17 Tree diagram for the 2-branch feedback decoder.	84
Figure 3.18 3-branch decoder differential phase trellis diagram.	87
Figure 3.19 Block of the 3-branch differential phase tree originating from state a.	87
Figure 3.20 Probability of error for binary DECPSK and DPSK.	91

Figure 3.21 Simulation used in the error statistics compilation.	95
Figure 4.1 System model for an AGN channel.	99
Figure 4.2 Phasor diagram for the data sequence [1 1 0 1].	105
Figure 4.3 Example of the eye diagram at the output of (a) the one-bit and three-bit delay detector (b) the two-bit delay detector (from [21]).	107
Figure 4.4 $P(a,b)=1/2 [1-Q(\sqrt{b},\sqrt{a})+Q(\sqrt{a},\sqrt{b})]$.	117
Figure 4.5 Complex phase plane for e_n^{1-1} caused by an important z_{n-1} .	123
Figure 4.6 Complex plane for e_{n-1}^{1-1} caused by an important z_{n-1} and (a) e_{n-1}^{1-1} (b) e_{n-1}^{1-0} .	124
Figure 4.7 Complex plane for e_n^{1-1} caused by an important z_n .	124
Figure 4.8 Noise-free phase plane for the transmitted data "0 1 1 1 0".	127
Figure 4.9 Typical error pattern probability density for uncorrelated noise samples.	127
Figure 4.10 Phase equalized 4 th order Butterworth filter with $BT=1.1$ (a) Frequency response (b) System autocorrelation.	129
Figure 4.11 Error pattern density on an AGN channel at $E_b/N_0=6$ dB with the RX filter characteristics of figure 4.10 (a) 2-branch (b) 3-branch.	130
Figure 4.12 Two-branch error pattern density on an AGN channel for the RX filter of figure 4.10 at (a) $E_b/N_0=8$ dB (b) $E_b/N_0=10$ dB (c) $E_b/N_0=12$ dB.	132
Figure 4.13 Error pattern density on an AGN channel at $E_b/N_0=8$ dB with a $BT=0.9$ phase equalized 4 th order Butterworth receive filter for (a) 2-branch (b) 3-branch.	133

Figure 4.14	Simulated performance of the differential detectors (a) single-bit delay (b) double-bit delay (c) triple-bit delay (d) DPSK.	136
Figure 4.15	Simulated performance (a) output of one-bit detector (b) output of double-bit detector (c) 2-branch feedback decoder (d) first error probability P_{fe} ▲ : Markov model.	138
Figure 4.16	Three-branch error pattern density on an AGN channel for the RX filter of figure 4.10 at (a) $E_b/N_o = -8$ dB (b) $E_b/N_o = -10$ dB (c) $E_b/N_o = -12$ dB.	142
Figure 4.17	First error probability for (a) 2-branch decoding (b) 3-branch decoding	144
Figure 4.18	Simulated performance (a) output of one-bit detector (b) 2-branch feedback decoder (c) 3-branch feedback decoder.	145
Figure 4.19	Fading multipath channel model.	147
Figure 4.20	Magnitude of the multipath shaping filter.	149
Figure 4.21	Simulated performance on a Rician fading channel for: ▲ or △ : single-bit delay ■ or □ : double-bit delay • or ○ : triple-bit delay.	151
Figure 4.22	Two-branch error pattern density on a Rician channel at $E_b/N_o = -8$ dB for (a) $K = -10$ dB (b) $K = -5$ dB (c) $K = 0$ dB.	154
Figure 4.23	Simulated performance on a Rician channel for ----- : 2-branch feedback decoding ——▲—— : the best differential detector.	156
Figure 4.24	Two-branch error pattern density on a Rician channel at $K = 0$ dB for (a) $E_b/N_o = -16$ dB (b) $E_b/N_o = -18$ dB.	157

Figure 4.25 Three-branch error pattern density on a Rician channel for $K=-10$ dB at (a) $E_b/N_0=8$ dB (b) $E_b/N_0=16$ dB.	159
Figure 4.26 Three-branch error pattern density on a Rician channel for $K=-5$ dB at (a) $E_b/N_0=8$ dB (b) $E_b/N_0=16$ dB.	160
Figure 4.27 Three-branch error pattern density on a Rician channel for $K=0$ dB at (a) $E_b/N_0=8$ dB (b) $E_b/N_0=16$ dB.	161
Figure 4.28 First error probability on a Rician channel for —▲— 2-branch decoding -----○----- 3-branch decoding.	162
Figure 4.29 Simulated performance on a Rician channel for —▲— 2-branch decoding -----○----- 3-branch decoding.	163
Figure 5.1 2p-ary rate 1/M convolutional encoder,	167
Figure 5.2 Trellis diagram corresponding to the encoder of figure 5.1.	169
Figure 5.3 Phase feedback decoder for a full response CPM signal with $h=1/4$.	171
Figure 6.1 The MSK modulator algorithm.	175
Figure 6.2 The demodulator algorithm.	177
Figure 6.3 The fading and shadowing model used in the mobile channel simulator.	179
Figure 6.4 Measured performance versus simulated performance ——▲—— simulation -----○----- measurements.	181
Figure C.1 Simulation block diagram.	199
Figure D.1 Phase tree example, $h=2/3$ and $g(t)=1$ for $0 \leq t \leq 3T$ (from [1]).	203
Figure D.2 Optimum receiver for CPM signals ($K=2^{L-1}$).	210

Figure D.3 Normalized squared Euclidean distance versus modulation index for binary CPFSK: upper bound (dashed line) and $d^2(h)$ for $N=1,2,3$ and 4 bit decision intervals (from [2]).	214
Figure D.4 Minimum squared Euclidean distance versus modulation index h for the binary 2RC scheme (from [3]).	215
Figure D.5 Parallel MSK-type receiver.	218
Figure D.6 $\cos\Phi(t,a)$ eye pattern for binary 3RC, $h=1/2$ scheme.	220
Figure D.7 Serial MSK-type receiver.	221
Figure D.8 MSK-type receiver asymptotic performance for some selected CPM schemes. N_T -observation interval (from [10]).	222
Figure D.9 Optimum noncoherent receiver (from [11]).	226
Figure D.10 Upper bound on the minimum normalized squared equivalent distance for CPFSK (from [12]).	228
Figure D.11 Upper bound on the minimum normalized squared equivalent distance for partial response with 2RC frequency pulse (from [28]).	229
Figure D.12 Suboptimum noncoherent CPM detectors (a) Differential detection (b) Frequency discriminator detection.	231
Figure D.13 Calculated and simulated error probabilities for 3RC. (from [16]).	233
Figure D.14 Differential detection structure considered in [14] and [17].	234
Figure D.15 Minimum squared normalized Euclidean distance for differential detection of CPFSK (from [14]).	236

LIST OF TABLES

Table 2.1	Definitions of the frequency pulse function $g(t)$ (from [1]).	12
Table 3.1	Definition of the set $Q=[0,1,2,3]$.	53
Table 3.2	Syndrome matrix corresponding to single and double error patterns in e_{n-2}^1 and any other error variable.	70
Table 3.3	Four possible states of the 2-branch decoder.	76
Table 3.4	Possible error patterns for 2-branch decoding.	79
Table 4.1	Pairs of data patterns giving the same absolute phase shifts at the output of the one-bit delay differential detector.	104
Table 4.2	Data patterns and the corresponding phase shift $\Delta\phi_2(n)$.	106
Table 4.3	Data patterns paired according to the resulting $\Delta\phi_2(n)$.	108
Table 4.4	Data patterns and the corresponding phase shifts $\Delta\phi_3(n)$.	110
Table 4.5	Probability of occurrence of each levels of the 2-bit delay detector eye diagram.	110
Table 4.6	Probability of occurrence of each levels of the 3-bit delay detector eye diagram.	111

LIST OF APPENDICES

	page
APPENDIX A DERIVATIONS OF EQUATIONS (4.16) AND (4.17)	190
APPENDIX B DERIVATIONS OF THE TRANSITION PROBABILITIES	192
APPENDIX C COMPUTER SIMULATION PROCEDURE	197
C.1 GENERAL SRUCTURE	197
C.2 ERROR STATISTICS COMPILATION	200
APPENDIX D DETECTION OF BINARY CPM SIGNALS	202
D.1 OPTIMUM COHERENT DETECTION	202
D.1.1 PHASE TRELLIS DESCRIPTION	202
D.1.2 MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION	204
D.1.3 OPTIMAL RECEPTION - PERFORMANCE EVALUATION	211
D.2 PERFORMANCE OF SUBOPTIMUM DETECTION PROCEDURES	217
D.2.1 COHERENT MSK-TYPE RECEIVERS	217
D.2.2 OPTIMUM NONCOHERENT RECEIVER	223
D.2.3 DIFFERENTIAL DETECTION AND	

FREQUENCY DISCRIMINATOR

DETECTION

230

LIST OF SYMBOLS AND ABBREVIATIONS

MSK:	Minimum Shift Keying.
DMSK:	Differential MSK.
TFM:	Tamed Frequency Modulation.
GMSK:	Gaussian Minimum Shift keying.
CPM:	Continuous Phase Modulation.
$g(t)$:	Frequency pulse in a CPM signal.
FM:	Frequency Modulation.
PSK:	Phase Shift Keying.
DPSK:	Differential PSK.
MLSE:	Maximum Likelihood Sequence Estimation.
AWGN:	Additive White Gaussian Noise.
AGN:	Additive Gaussian Noise
$s(t)$:	Transmitted signal.
E:	Energy per symbol in the transmitted signal.
T:	Symbol duration in seconds.
f_0 :	Carrier frequency.
$\underline{a}$:	Sequence of transmitted binary symbols.
$\Phi(t, \underline{a})$:	Transmitted modulated phase.
Φ_0 :	Constant initial transmitted phase.
h:	Modulation index.
$q(t)$:	Phase pulse (integration over time of $g(t)$).
L:	Number of symbol durations spanned by $g(t)$.
RC:	Raised Cosine function.

LRC: CPM signal with a Raised Cosine frequency pulse spanning LT .
 LSRC: CPM signal with a Spectrally Raised Cosine frequency pulse spanning LT .
 CPFSK: Continuous Phase Frequency Shift Keying.
 LREC: CPM signal with a Rectangular frequency pulse spanning LT .
 θ_n : Accumulated phase, up to time $(n-L)T$, in a CPM signal (phase state).
 $S_t(n)$: Total state vector of a CPM signal.
 $\Delta\phi_k(t)$: Phase difference between the n^{th} interval and the $(n-k)^{\text{th}}$ interval.
 $C_\phi(t)$: Intersymbol interference term in a partial response CPM signal.
 B_t : Transmit filter 3 dB bandwidth.
 B_r : Receive filter 3 dB bandwidth.
 P_e : Probability of bit error.
 SEC: Single Error Correction.
 ISI: Intersymbol Interference.
 $\Delta\phi_k(n)$: Differential phase of order k at time $(n+1)T$.
 d_n^k : Transmitted differential variable of order k at time nT .
 d_{n-1}^k : Transmitted differential variable of order k at time $(n-1)T$.
 y_n^k : Received differential variable of order k at time nT .
 y_{n-1}^k : Received differential variable of order k at time $(n-1)T$.
 e_n^k : Error on d_n^k .
 e_{n-1}^k : Error on d_{n-1}^k .
 S_n^j : j^{th} syndrome at time nT .

S_{n-i}^j : j^{th} syndrome at time $(n-i)T$.
 b_n : Transformed transmitted binary symbol (see equation (2.22)).
 $\underline{D}_n$: Transmitted differential phase vector.
 $\underline{Y}_n$: Received differential phase vector.
 $\underline{E}_n$: Error vector.
 $I(n)$: Transmitted information bits.
 $P(n)$: Transmitted parity bits.
 $g(n)$: Generator polynomial of a convolutional code.
 $e_I(n)$: Errors on $I(n)$.
 $e_P(n)$: Errors on $P(n)$.
 $y_I(n)$: Received information bits.
 $y_P(n)$: Received parity bits.
 $S(n)$: Syndrome in feedback decoding.
 d_f : Convolutional code free distance.
BSC: Binary Symmetric Channel.
 $\sigma(n)$: Syndrome modified by the decoding decisions.
 $\hat{e}_I(n)$: Estimated errors on $I(n)$.
 $\hat{I}(n)$: Estimated information bits.
 z^{-1} : Delay operator.
 P_{fe} : First error probability.
 Π : Transition probabilities matrix.
 $w_I(n)$: State probability at time n .
 $\underline{W}$: State probability vector.
 $P_i(n)$: Error probability when in state i .
 P_{fd} : Error probability in feedback decoder.

$\hat{S}_n^i$:	Syndrome modified by the decoding decisions.
$\hat{e}_n^i$:	Estimated errors.
M_n :	Syndrome matrix at time n .
m_n^{jk} :	Syndrome matrix elements, same as S_{n-k}^j .
M :	Feedback decoder memory.
$f(\phi)$:	Phase probability function.
erf :	Error function.
E_b :	Energy per bit.
N_0 :	White noise two-sided power spectral density.
P_i :	i^{th} error pattern.
DECPSK:	Coherent detection of differentially encoded PSK.
CPSK:	Coherent detection of PSK.
$*$:	Convolution operator.
$\times$:	Vector cross-product operator.
$\cdot$:	Vector dot product operator.
$h_i(t)$:	Received filter impulse response in i^{th} level of differential detection.
$H_i(f)$:	Fourier transform of $h_i(t)$.
$r(t)$:	Complex baseband received signal.
$z_i(t)$:	Filtered complex baseband received signal in i^{th} order of differential detection.
$u(t)$:	Complex white Gaussian noise process.
$\Delta\theta_k(t)$:	Received phase difference.
$\theta(t, \underline{a}, u)$:	Received modulated phase corrupted by the noise process $u(t)$.
$P_1(n)$:	Probability of error given that the data +1 is sent.

$P_{-1}(n)$: Probability of error given that the data -1 is sent.
 ξ : Phase offset introduced by ISI.
 $Q(x,y)$: Marcum Q-function.
 $I_0(t)$: Modified zero-order Bessel function of the first kind.
 t_0 : Eye diagram sampling time.
 K_{ij} : Covariance between $z_i(t_0)$ and $z_j(t_0 - iT)$.
 $\phi_{nn}(\tau)$: Noise autocorrelation.
 $\Phi_{nn}(f)$: Noise power spectral density.
 B_{ni} : Double-sided noise bandwidth of filter in i^{th} level of differential detection.
 $\rho_i(\tau)$: Filter deterministic autocorrelation in i^{th} level of differential detection.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

In many applications for mobile radio and satellite communication systems, digital modulation schemes with efficient power spectra are desirable. This is due to the limited availability of the radio frequency spectrum. The transmitters and repeaters often include nonlinearities, which make a constant envelope modulation scheme desirable in order to avoid spectral regrowth. In recent years, signalling techniques such as Minimum Shift Keying (MSK), Gaussian MSK (GMSK) or Tamed Frequency Modulation (TFM), that exhibit the two desirable characteristics, became popular. These three types of signalling are all members of a more general class of modulation known as Continuous Phase Modulation (CPM).

A CPM modulator can be modeled as a baseband filter with impulse response $g(t)$ followed by a frequency modulator with a certain modulation index. The constant envelope property is assured by the FM modulator, while the phase behaviour is a function of the frequency

pulse $g(t)$ which ultimately influences the spectral content of the output signal. Memory is introduced into the CPM signal by means of the continuous phase. If $g(t)$ is limited to one symbol interval, all the memory is in the phase continuity. This is referred to as full response signalling [2]. Further memory can be built into the CPM signal by choosing a pulse $g(t)$ that spans more than one symbol interval. This is called partial response signalling [3],[43] or correlative encoding of digital FM signals [44],[45].

Over the past decade, an important amount of work was generated in the area of CPM signal detection [1-4],[6-23]. The next section will consider some coherent and noncoherent detection methods proposed in the past few years. In this thesis, a noncoherent detection procedure that was proposed for differentially encoded modulations such as MSK or differential PSK (DPSK) is examined in detail. Note that MSK can be viewed as a differentially encoded two-dimensional linear modulation scheme.

Called "Nonredundant error correction" by some authors [4],[5], this detection procedure can be extended to full response CPM signals and, with certain modifications, to partial response signals. A method to consider and analyse the detection procedure using channel coding principles is presented. Minimum Shift Keying is considered in detail and previously published results are enlightened by this

work. New results concerning the detection procedure and channel effects are also presented.

1.2 DETECTION OF CPM SIGNALS

A brief review of the most important CPM detection procedures described in the open literature is presented in this section in order to put the material of this thesis in perspective. A detailed review of the same subject is given in chapter 2 and appendix D.

In the case of equiprobable modulating sequences on an Additive White Gaussian Noise (AWGN) channel, the receiver that minimizes the probability of sequence error implements the Viterbi algorithm, and suitable metric updates in a bank of matched filters which are sampled every symbol interval (appendix D). This process is known as Maximum Likelihood Sequence Estimation (MLSE) [1].

There are several practical problems in implementing a receiver based on MLSE principles. The main one is the exponential growth, in the complexity, with signal memory. Suboptimum detectors using a reduced maximum likelihood approach were proposed, but they still exhibit a significantly high level of complexity [6],[7]. Optimum detection assumes perfect phase synchronization at the receiver. Two types of suboptimum receivers also assuming phase coherency were

proposed for CPM signal with modulation index of $1/2$. These are the parallel MSK-type receiver and the serial MSK-type receiver. As implied by their names, these detectors are derived from a similar architecture applicable to MSK. In fact, both the parallel offset quadrature receiver [8] and the serial MSK receiver [10], with proper filters, achieve a minimum probability of bit error for MSK. Both types of coherent detectors can be implemented for partial response signals, with frequency pulses spanning less than 5 symbol intervals, and give performance degradation, from MLSE, on the order of one dB [1],[9].

Coherent reception is not always possible, especially on fading channels. Noncoherent detection is therefore used in these cases, where no attempt is made to recover the received signal phase. Optimum and suboptimum (in the minimum error rate sense) noncoherent detection procedures were proposed for CPM signals on an AWGN channel. Optimum noncoherent detection is discussed in [11],[12] and [13]. For CPFSK, where the phase change is linear over one bit interval, it essentially consists of a bank of correlators and amplitude detectors, each one feeding a zero order modified Bessel function of the first kind, followed by a decision device. This type of detector, to be practical, must span a limited memory and therefore must be suboptimum [11].

When $h=1/2$, from equations (2.14) and (2.17), we have

$$\Delta\Phi_k(n) = \pi/2 \sum_{i=0}^{k-1} a_{n-i} \quad \text{modulo } 2\pi \quad (2.20)$$

$$d_n^k = \sum_{i=0}^{k-1} a_{n-i} \quad \text{modulo } 4. \quad (2.21)$$

Equation (2.20) indicates a rotation of $\pm\pi/2$ for any data bit. This implies a four-point general constellation, subject to the modulo-4 arithmetic of equation (2.21). But for any particular d_n^k , the specific constellation contains only 2 points, either at 0 and π or at $\pi/2$ and $3\pi/2$. Therefore the d_n^k 's require only modulo-2 arithmetic and a 2-element alphabet $[0,1]$. Equation (2.21) then becomes

$$d_n^k = \sum_{i=0}^{k-1} b_{n-i} \quad \text{modulo } 2 \quad (2.22)$$

where

$$b_i = 1 \text{ when } a_i = 1$$

$$b_i = 0 \text{ when } a_i = -1.$$

The demodulator is simplified since the decisions are taken on antipodal signal constellations and modulo-2 arithmetic is used in the syndrome calculation.

1.3 THESIS OBJECTIVES

For MSK, the use of a second order of differential detection to improve the error rate performance of a conventional differential detector was proposed by Masamura et al. [4]. The authors show that the quantized output of a 2-bit differential detector constitutes a parity-check signal for the quantized output of the one-bit detector and that these two signals can be processed to reduce the error rate. The analysis performed is similar to the one done by Chow et al. [24] for DPSK and assumes no intersymbol interference and noise correlation properties inherent to matched filter detection. This analysis is misleading since a filter matched to the basic MSK pulse does not meet Nyquist first criterion for ISI-free transmission. It shows, in these artificial channel conditions, a modest improvement with the additional detector. But it is noticed experimentally that the degradation caused by a band-limited channel (channel with memory) in the one-bit detector is reduced significantly with the aid of the two-bit detector. This fact is not explained by the analysis presented. The same detection procedure is studied by simulation in [18] where a few different receive filters are considered. But again, no explanation is given about the procedure called "ISI control by nonredundant error correction".

The main goal of this thesis is to perform the analysis of the procedure proposed in [4] for MSK, taking into account the filter characteristics and the interdependency of the differential detector outputs. An improved version of this detection procedure, for MSK, that uses a three-bit detector is also given. Then an asymptotic lower bound for an infinite number of differential detectors is intuitively derived. These results are extended, for the case of MSK, to fading multipath channels. It is shown explicitly that the proposed detectors are in fact syndrome feedback decoders identical to those used in the decoding of systematic convolutional codes.

Another contribution of this work is to show that this detection method, based on differential phase properties, can be extended to many CPM signals.

1.4 THESIS ORGANIZATION

This thesis is organized as follows;

Chapter 2 reviews the definition of binary CPM signals and differential detection of CPM signals with modulation index of $1/2$ is discussed. Then the differential phase properties of binary CPM signals are presented.

Chapter 3 discusses the application of feedback decoding on the differential phase variables of a minimum shift keying signal. The feedback decoding principles are first reviewed and then are applied in the decoding of a specific number of differential phase variables. The performance of such a procedure is analysed and some techniques are developed to obtain the performance on different types of channels. A performance lower bound is intuitively established, and a procedure to obtain the statistics of the variables considered by the decoder is presented.

Chapter 4 considers the performance of phase feedback decoding of MSK on a static AGN channel and on a slow fading multipath channel. Considerations are given to the receive filter on the AGN channel. A specific filter is chosen and simulated performance results are obtained for the two channels. The joint probability density of the variables considered by the decoder is presented for different channel parameters.

Chapter 5 briefly discusses the possible application of feedback decoding on the differential phase variables of other types of CPM signals.

Chapter 6 presents a practical implementation of phase feedback decoding for MSK, as well as its performance, as measured under laboratory conditions.

Chapter 7 is dedicated to conclusions and proposals for further work in this area of research.

CHAPTER 2

BINARY CONTINUOUS PHASE MODULATION

2.1 DEFINITIONS

Continuous phase modulation refers to a modulation scheme where the amplitude of the modulated signal is constant and the derivative of the phase trajectory is always finite. Using the definitions and notation given in [1]-[3], this sub-section presents some general definitions about binary CPM signals.

The transmitted signal expression is

$$s(t) = \sqrt{2E/T} \cos[2\pi f_0 t + \Phi(t, \underline{a}) + \Phi_0] \quad (2.1)$$

where T is the symbol duration, E is the energy per symbol, f_0 is the carrier frequency, $\underline{a}$ is the sequence of binary data $a_i = \pm 1$ and Φ_0 is a constant initial phase that can be assumed zero without any loss of generality.

The transmitted information is contained in the phase

$$\Phi(t, \underline{a}) = 2\pi h \sum_{i=-\infty}^{\infty} a_i q(t-iT) \quad (2.2)$$

where h is the modulation index and

$$q(t) = \int_{-\infty}^t g(\tau) d\tau \quad (2.3)$$

$$\int_{-\infty}^{\infty} g(t) dt = 1/2. \quad (2.4)$$

The function $g(t)$ is defined as the frequency pulse shape and its integral $q(t)$ is the phase response. The shape of $g(t)$ determines the smoothness of the transmitted information carrying phase and may span a finite time interval $0 \leq t \leq LT$ or may be time infinite. The rate of change of the phase (or instantaneous frequency) is proportional to the modulation index h . The normalization of the pulse shape in equation (2.4) means that for schemes with positive pulses of finite length, the maximum phase change over one symbol interval is $h\pi$. Some of the most popular pulse shapes are given in table 2.1.

It is convenient to see CPM as a generalization of MSK, for which $g(t)$ is a rectangular pulse extending over one bit and the modulation index is $1/2$.

LRC

$$g(t) = \begin{cases} \frac{1}{2LT} \left[1 - \cos \frac{2\pi t}{LT} \right] & 0 \leq t \leq LT \\ 0 & \text{otherwise} \end{cases}$$

TFM

$$g(t) = 1/8 [a g_0(t-T) + b g_0(t) + a g_0(t+T)]$$

$$; a = 1; \quad b = 2$$

$$g_0(t) \approx \sin(\pi t/T) \left[\frac{1}{\pi t} - \frac{2 - 2\pi t/T \cot(\pi t/T) - \pi^2 t^2/T^2}{24\pi t^3/T^2} \right]$$

LSRC

$$g(t) = 1/LT \cdot \frac{\sin(2\pi t/LT)}{2\pi t/LT} \cdot \frac{\cos(\beta 2\pi t/LT)}{1 - (4\beta t/LT)^2} \quad ; \quad 0 \leq \beta \leq 1$$

GMSK

$$g(t) = 1/2T \left[Q \left[2\pi B_t \frac{t-T/2}{\sqrt{\ln 2}} \right] - Q \left[2\pi B_t \frac{t+T/2}{\sqrt{\ln 2}} \right] \right]$$

$$; 0 \leq B_t T < \infty$$

$$Q(t) = 1/\sqrt{2\pi} \int_1^\infty e^{-\tau^2/2} d\tau$$

LREC

$$g(t) = \begin{cases} 1/2LT & ; 0 \leq t \leq LT \\ 0 & ; \text{otherwise} \end{cases}$$

Table 2.1 Definitions of the frequency pulse function $g(t)$
(from [1]).

Memory is introduced into the CPM signal by means of the continuous phase. Each information carrying phase function $\Phi(t, \underline{a})$ is continuous at all times for all combinations of data symbols. Even when the data symbols are uncorrelated, further memory can be built into the CPM signal by choosing $g(t)$ with $L > 1$. These schemes have overlapping shaping pulses and constitute the partial response signalling schemes [3]. CPM signals with $L=1$ correspond to the full response signalling formats [2]. In this case, all the memory is in the continuous phase. Full response signals with a constant frequency pulse are referred to as Continuous Phase Frequency Shift Keying (CPFSK). The phase $\Phi(t, \underline{a})$, during interval $nT \leq t \leq (n+1)T$, can be written

$$\Phi(t, \underline{a}) = 2\pi h \sum_{i=n-L+1}^n a_i q(t-iT) + h\pi \sum_{i=-\infty}^{n-L} a_i \quad \text{modulo } 2\pi \quad (2.5)$$

For given h and $g(t)$ and for any symbol interval n , the phase $\Phi(t, \underline{a})$ is uniquely defined by $\underline{a}_n$, by the correlative state vector $\underline{a}_n = (a_{n-1}, a_{n-2}, \dots, a_{n-L+1})$ and by the phase state θ_n , where

$$\theta_n = h\pi \sum_{i=-\infty}^{n-L} a_i \quad \text{modulo } 2\pi. \quad (2.6)$$

The total state vector $S_t(n)$ is defined as the combination of the correlative state vector and the phase state, i.e.

$$S_t(n) = (\theta_n, a_{n-1}, a_{n-2}, \dots, a_{n-L+1}). \quad (2.7)$$

The phase difference between the n^{th} symbol interval and the $(n-k)^{\text{th}}$ interval is

$$\begin{aligned} \Delta\phi_k(t) &= \phi(t, \underline{a}) - \phi(t-kT, \underline{a}_k) \\ &= 2\pi h \left[\sum_{i=n-L+1}^n a_i q(t-iT) - \sum_{i=n-k-L+1}^{n-k} a_i q(t-iT) \right] + h\pi \sum_{i=0}^{k-1} a_{n-L-i} \end{aligned} \quad (2.8)$$

Equation (2.8) is composed of an intersymbol interference term $C_\phi(t)$, that is inherent to partial response schemes, and of an information bearing term $\Delta\theta_{n+1}$

$$C_\phi(t) = 2\pi h \left[\sum_{i=n-L+1}^n a_i q(t-iT) - \sum_{i=n-k-L+1}^{n-k} a_i q(t-iT) \right] \quad (2.9)$$

$$\Delta\theta_{n+1} = h\pi \sum_{i=0}^{k-1} a_{n-L-i} \quad \text{modulo } 2\pi \quad (2.10)$$

2.2 DIFFERENTIAL DETECTION OF CPM SIGNALS WITH $H=1/2$

This section presents some of the results that appeared in the literature concerning differential detection of CPM signals with modulation index $1/2$. The most popular schemes in this class are MSK, GMSK and TFM. Results for the first two schemes are given here.

Differential detection of GMSK, followed by bit-by-bit decision, was proposed using either a delay of one bit interval or a delay of two bit intervals [21]. The two systems are shown in figure 2.1.

In order for the two-bit detector to be effective, differential encoding must be performed on the modulating data stream. This is illustrated in figure 2.2.

The one-bit detector contains a 90° phase shifter in the delay branch. This forces the multiplier output signal to correspond to the sine of the phase change in the received signal over a one-symbol time interval. The two-bit detector does not contain such a phase shifter and thus the multiplier output signal represents the cosine of the phase change over a two-symbol interval.

These two detectors behave quite differently under severe filtering conditions. Figure 2.3 illustrates the eye diagrams for the

Figure 2.1 (a) A one-bit differential detector for GMSK
(b) A two-bit differential detector for GMSK.

Figure 2.2 A GMSK transmitter suited for two-bit differential detection.

(a)

(b)

Figure 2.3 (a) One-bit detector eye diagrams; Gaussian IF filter; $B_T T = 1.25$
 (b) Two-bit detector eye diagrams; Gaussian IF filter; $B_T T = 1.25$ (from [21]).

one-bit and two-bit differential detector outputs for GMSK with $B_T T = 0.25$ and $B_R T = 1.25$ for a Gaussian receive filter. Note that B_T and B_R are the 3 dB bandwidths of the transmit and receive filters respectively.

This figure shows a symmetrical eye pattern for one-bit detection and an asymmetrical eye with larger opening for two-bit detection. This difference results from a different behaviour in the sine and cosine functions. For one-bit detection, a certain sequence of bits corresponds to the sines of a certain sequence of phase changes. If the input data sequence is complemented, the phase sequence is negated and so is the sine value. Thus a symmetrical eye diagram is obtained. In the two-bit case, because of the differential encoder, the received data pairs "01" and "10" correspond to a transmitted "1" and the received pairs "00" or "11" correspond to a transmitted "0". Comparing the possible values of the cosine of the possible phase sequences for "0" and "1" sent, we observe that they are not the complement of each other. The cosine function can take different values for the same transmitted data bit [21]. This results in an asymmetrical eye diagram. These results are explained further in section 4.1.1.

Therefore, by using two-bit differential detection with a dc bias before decision (the DC bias takes into account the eye asymmetry), performance attained with one-bit differential detection

is exceeded. Figure 2.4 shows the performance of both detection schemes for various B_T products. Under severe filtering ($B_T T = 0.25$), a very important improvement is obtained with two-bit detection.

Since MSK can be thought of as a special case of GMSK ($B_T \rightarrow \infty$), both types of differential detection can be used in an MSK receiver. The curves labelled ∞ in figure 2.4 illustrate the performance of MSK on an AWGN channel. In [22], the relation between the two performance characteristics is given as

$$P_{e|2\text{-bit}} = P_{e|1\text{-bit}} \cdot F(BT) \quad (2.11)$$

where $F(BT)$ reflects the improvement from one-bit to two-bit detection. $F(BT)$ is a complicated function of the BT product, the signal-to-noise ratio and the type of filter used.

Figure 2.5 gives a comparison of the bit error probability of one-bit and two-bit differential detection of MSK with a Gaussian receive filter. An improvement of about 0.5 dB is obtained at $P_e = 10^{-3}$ for $BT = 1.0$. For higher BT products, the advantage of two-bit detection is marginal.

The search for the best receive filter characteristics for one-bit differential detection of MSK was the object of many investigations in the last few years [18], [19], [29]. In [18] and

Figure 2.4 Performance of one- and two-bit differential detection of GMSK (from [21]).

Figure 2.5 One-bit versus two-bit differential detection of MSK with a Gaussian receive filter (from [22]).

[19], the authors conclude that, within the class of conventional analog filters, the best receive filter is a 4th order Butterworth filter with a BT product of 1.1. The degradation from coherent detection, for this filter at a bit error rate of 5×10^{-4} , is approximately 2.9 dB. As a comparison, the Gaussian filter with BT=1 degrades the performance by more than 3.5 dB, as noted in figure 2.5.

Reference [18] also investigated the performance of a detection procedure making use of a two-bit differential detector, in addition to the conventional one-bit detector. It was shown in [4] and [24] that the phase difference between alternate signalling intervals, as obtained in the two-bit detector, provides a parity check for two successive outputs of the one-bit detector. This observation leads to a single error correction (SEC) circuit that improves the overall bit error rate. The SEC circuit is given in figure 2.6 and will be considered in great details in chapter 3.

With a 4th order Butterworth receive filter, this detection procedure improves the performance of the basic one-bit detector by about 1 dB on an AWGN channel [18]. Such a procedure proved to be particularly well suited on bandlimited channels where it was claimed that it has an "ISI control" effect [4]. However, the mechanism by which the ISI is controlled is not clearly explained by the authors [4].

Figure 2.6 Single Error Correction circuit.

By adding a third differential detector, with a 3-bit delay, a detection procedure exploiting more thoroughly the phase properties of MSK on a band-limited channel is obtained [30]. This procedure provides an extra 0.5 dB of gain, which places it at 1.5 dB from coherent detection and at only 0.5 dB from ideal DPSK.

In the next section, the basic differential phase properties of CPM signals that can be exploited in a noncoherent receiver are introduced.

2.3 DIFFERENTIAL PHASE PROPERTIES

The phase difference between the n^{th} symbol interval and the $(n-k)^{\text{th}}$ interval was defined in equation (2.8). $\Delta\Phi_k(t)$ is given by

$$\begin{aligned}\Delta\Phi_k(t) &= \Phi(t, \underline{a}) - \Phi(t-kT, \underline{a}_k) \\ &= 2\pi h \left[\sum_{i=n-L+1}^n a_i q(t-iT) - \sum_{i=n-K-L+1}^{n-K} a_i q(t-iT) \right] + h\pi \sum_{i=0}^{k-1} a_{n-L-i}. \quad (2.8)\end{aligned}$$

In order to have a single random variable that represents the phase variations from one bit transition to the other, the phase difference $\Delta\Phi_k(t)$ is sampled at $t=(n+1)T$ and becomes the "differential phase of order k " $\Delta\Phi_k(n)$:

$$\Delta\Phi_k(n) = 2\pi h \left[\sum_{i=n-L+1}^n d_i q((n+1-i)T) - \sum_{i=n-L-k+1}^{n-k} a_i q((n+1-i)T) \right] + h\pi \sum_{i=0}^{k-1} a_{n-L-i} \quad (2.12)$$

The attention is focused, for the time being, on the full response case, (i.e. $L=1$).

2.3.1 FULL RESPONSE SIGNALLING

For full response signalling, $L=1$, and thus the differential phase is

$$\Delta\Phi_k(n)|_{L=1} = 2\pi h \left[a_n q(T) - a_{n-k} q((k+1)T) \right] + h\pi \sum_{i=0}^{k-1} a_{n-1-i} \quad (2.13)$$

Because of equations (2.3) and (2.4), $q(T) = q((k+1)T) = 1/2$ and the differential phase becomes

$$\Delta\Phi_k(n)|_{L=1} = h\pi \sum_{i=0}^{k-1} a_{n-i} \quad \text{modulo } 2\pi \quad (2.14)$$

The differential phase is therefore rotated by $\pm\pi h$ radians for each new input data bit a_n . In fact, $\Delta\Phi(n)_k|_{L=1}$ represents the behaviour of the continuous phase $\Phi(t, \underline{a})$ during the last k signalling intervals. For a rational modulation index, $h=q/p$ with q and $p \in \mathbb{N}$,

there is a finite set of possible values for $\Delta\Phi_k(n)$. These form a differential phase constellation with a finite number of distinct points. As an example, the differential phase constellation for $h=4/7$ is given in figure 2.7. The differential phase moves by $\pm 4\pi/7$ on this phase plane for each new data bit. For modulation indices of the form $h=1/p$, the differential phase plane is made simpler for some values of k . Figure 2.8 shows the constellations for $h=1/4$ and $k=1,2,3$. Also shown in this figure are the combinations of data bits required to reach each point.

Each constellation of figure 2.8 is a subset of the general 8 equally spaced point constellation.

By examining equation (2.14), the following relation is obvious

$$\Delta\Phi_k(n) = \sum_{i=0}^{k-1} \Delta\Phi_1(n-i) \quad \text{modulo } 2\pi \quad (2.15)$$

which indicates that the $\Delta\Phi_k(n)$'s, for $k \neq 1$, can play the role of parity checks for $\Delta\Phi_1(n)$.

Each point on the general constellation can be associated with a decimal number, from 0 to $(2p-1)$. By considering modulo $2p$ arithmetic, with the alphabet

$$P = [0, 1, 2, \dots, (2p-1)] \quad (2.16)$$

Figure 2.7 Differential phase plane for full response signalling with $h=4/7$.

Figure 2.8 Differential phase planes for full response signalling with $h=1/4$.

(a) $k=1$, $\Delta\Phi_1(n) = a_n \pi/4$

(b) $k=2$, $\Delta\Phi_2(n) = \pi/4 (a_n + a_{n-1}) \bmod 2\pi$

(c) $k=3$, $\Delta\Phi_3(n) = \pi/4 (a_n + a_{n-1} + a_{n-2}) \bmod 2\pi$

the differential phase of equation (2.14) becomes

$$d_n^k = q \sum_{i=0}^{k-1} a_{n-i} \quad \text{modulo } 2p \quad (2.17)$$

where q is the numerator of the rational modulation index $h=q/p$.

Equation (2.15) is transformed into

$$d_n^k = \sum_{i=0}^{k-1} d_{n-i}^1 \quad \text{modulo } 2p. \quad (2.18)$$

The variable d_n^1 can be used to demodulate the data since it is proportional to the transmitted data bit a_n . Then the phase memory of the full response CPM signal can be utilised to validate the d_n^1 through the use of the d_n^k 's for $k>1$. In noiseless conditions, we have from equation (2.18)

$$d_n^k - \sum_{i=0}^{k-1} d_{n-i}^1 = 0 \quad \text{modulo } 2p. \quad (2.19)$$

Therefore, if the random variable y_n^k represents the quantized differential phase in the presence of noise and intersymbol interference, the recovered data y_n^1 could be combined with the y_n^k 's, $k>1$, and possibly improve the final decision on the recovered data. The y_n^k 's are the outputs of decision devices that compare the noisy differential phases with the possible points of the differential phase plane and choose the closest points as y_n^k 's. This process is illustrated in figure 2.9 for a modulation index $h=1/4$ and modulo-8

Figure 2.9 Decision process on the differential phases for a full response CPM signal with $h=1/4$.

arithmetic.

A high level demodulator block diagram for $h=1/4$ is given in figure 2.10. It includes the decision process of figure 2.9 with the parity checks given by equation (2.19). The variables S_n^i , resulting from these parity checks, are called syndromes and are fed to a syndrome decoder where the syndrome information is processed in order to obtain the recovered data.

Although the general principles presented here are applicable to any full response signal with a rational modulation index, they lead to simpler results when $h=1/p$. The case $h=1/2$ is even of more interest since it constitutes one of the most documented type of CPM signal.

2.3.2 FULL RESPONSE SIGNALLING WITH MODULATION INDEX OF 1/2

Much of the work performed on CPM receiver design was concerned with a modulation index of $1/2$ [1]. This section is devoted to this particular case since it leads to some major simplifications in the receiver design and to an easy interpretation of the detection procedure. Furthermore, CPFSK with $h=1/2$ is commonly known as MSK and constitutes the main interest of this work.

Figure 2.10 Demodulator for a full response CPM signal with $h=1/4$.

When $h=1/2$, from equations (2.14) and (2.17), we have

$$\Delta\Phi_k(n) = \pi/2 \sum_{i=0}^{k-1} a_{n-i} \quad \text{modulo } 2\pi \quad (2.20)$$

$$d_n^k = \sum_{i=0}^{k-1} a_{n-i} \quad \text{modulo } 4. \quad (2.21)$$

Equation (2.20) indicates a rotation of $\pm\pi/2$ for any data bit. This implies a four-point general constellation, subject to the modulo-4 arithmetic of equation (2.21). But for any particular d_n^k , the specific constellation contains only 2 points, either at 0 and π or at $\pi/2$ and $3\pi/2$. Therefore the d_n^k 's require only modulo-2 arithmetic and a 2-element alphabet $\{0,1\}$. Equation (2.21) then becomes

$$d_n^k = \sum_{i=0}^{k-1} b_{n-i} \quad \text{modulo } 2 \quad (2.22)$$

where

$$b_i = 1 \text{ when } a_i = 1$$

$$b_i = 0 \text{ when } a_i = -1.$$

The demodulator is simplified since the decisions are taken on antipodal signal constellations and modulo-2 arithmetic is used in the syndrome calculation.

Equation (2.22) gives the differential phase random variable of order one in noiseless conditions. By forming the differential phases of order $k=1,2,\dots,M$, a differential phase vector $\underline{D}_n=[d_n^1, d_n^2, \dots, d_n^M]$ that corresponds to the output of a rate $1/M$ convolutional encoder is obtained [5],[30]. The encoder has the form given in figure 2.11.

In the presence of noise and other impairments, the decision over the differential phase is either right or wrong, since we deal with modulo-2 arithmetic. Therefore, the variable y_n^k is given by

$$\begin{aligned} y_n^k &= d_n^k + e_n^k \quad \text{modulo } 2 \\ &= \sum_{i=0}^{k-1} b_{n-i} + e_n^k \quad \text{modulo } 2 \end{aligned} \quad (2.23)$$

where e_n^k is either 1 or 0 and represents the error in the decision process. Two more vectors are defined, the received vector $\underline{Y}_n=[y_n^1, y_n^2, \dots, y_n^M]$ and the error vector $\underline{E}_n=[e_n^1, e_n^2, \dots, e_n^M]$, so that $\underline{Y}_n$ is the modulo-2 sum of $\underline{D}_n$ and $\underline{E}_n$.

This analogy between the differential phase and a convolutional code is a consequence of the phase properties of the CPM signal. The phase tree description (appendix D) naturally leads to a differential phase tree description, which is again a result of the convolutional code analogy.

Figure 2.11 Rate 1/M convolutional encoder.

In the rest of this thesis, minimum shift keying (MSK) will be considered and the analogy with coding will be used to process the differential phase and to analyse the resulting demodulator.

Finally, a note must be added here. At first glance, a system similar to the one presented in figure 2.10, where many levels of phase comparisons are included, might seem useless for MSK. One can conclude that since the optimum MSK receiver, on an AWGN channel, needs to consider only 2 bit intervals to perform the best possible decision (see appendix D, figure D.3), many levels of phase comparisons are useless (since the signal is considered over more than two intervals in the extra levels).

This conclusion is false for two reasons. First of all, it is true that two bit intervals are enough for optimum decision, IF THE CHANNEL PHASE INFORMATION IS KNOWN AT THE RECEIVER. If this is not the case, then the phase must be estimated, which requires typically more than two bits. So, in a system similar to figure 2.10, by considering many levels of phase comparison, one still obtain useful information about the signal. The second reason why more than two bit intervals could be useful is that the channel, including the phase detectors, is not linear, nor memoryless. Thus the two bit intervals argument, although true on a linear memoryless AWGN channel without any phase shift and delay, does not apply here.

CHAPTER 3

PHASE FEEDBACK DECODING OF BANDLIMITED MSK

Minimum shift keying is well known for its good spectral properties, its constant envelope and its optimum coherent performance, which is identical to coherent PSK (see section D.1.3). As a member of the CPFSK signal class, with a modulation index of $1/2$, the simplified differential phase properties derived in the previous chapter apply to MSK. This fact was used in the "single error correction" (SEC) procedure presented in [4] where the convolutional coding properties, as presented in the last section, were limited to a constraint length of 2. A constraint length of 3 was considered in [30] with some improvement over the constraint length 2 version.

In chapter 3 and 4, MSK is considered because of its popularity and because of the simplified detection procedure inherent to its modulation index. In this chapter, the convolutional coding properties of the differential phase variables are used to exploit the CPM phase memory. A suboptimum decoding procedure for convolutional codes, called feedback decoding, is first introduced. Then this procedure is applied to the differential phase variables. The expression "phase feedback decoding" is used thereafter to qualify

the use of feedback decoding on the differential phase variables. It is shown that the SEC procedure given in [4] is in fact the application of phase feedback decoding. This fact is used to analyse the performance of SEC, and of an extended version [30], on a bandlimited channel. Finally, a simulation procedure to obtain the statistical behaviour of the decoder is presented.

3.1 FEEDBACK DECODING PRINCIPLES

Consider a binary systematic convolutional code on a binary symmetric channel (BSC). The optimum decoding procedure theoretically spans an infinite number of symbols. If, instead of using this true maximum likelihood decoding strategy, a truncated-memory decoder is used, which makes a maximum likelihood decision on a given bit or branch based only on a finite number of received branches beyond this point, a simpler decoder is obtained. The decision is on which half of the tree diagram the next decision will be taken, and the previous part of the tree has no effect on the next decisions. The decoder then operates essentially as a "sliding block decoder" on codes of limited number of codewords over a limited number of branches [32]. This decoder defines the essence of feedback decoding because decisions are "fed back" to the decoder in determining the subset of code paths which are to be considered next. In other words, it utilises the sequence of decoding decisions prior to the actual decision time

to define a decoder starting state (or code tree starting node). For the simple rate 1/2 code whose encoder is given in figure 3.1(a), the sliding box is shown on the tree diagram of figure 3.1(b). Each box has a memory of two branches, which is optimum for this code as will be seen shortly. Note that in this example, u_n and p_n correspond to the differential phase variables d_n^1 and d_n^2 defined in section 2.3.2.

The sliding block decoder concept can be exploited to simplify the feedback decoder implementation on a BSC. For this type of channel, maximum likelihood decoding of a block code reduces to minimum Hamming distance decoding [32]. This kind of decoding, for a systematic block code, can be efficiently implemented by calculating the syndrome of the received vector, and from this, the most likely error vector by consulting a table which contains the minimum weight error vector for each syndrome. Thus for each box, for a systematic convolutional code, syndromes can be calculated and a table consulted to determine if the received data bit of the first branch is in error or not. This process determines which half of the tree diagram will be considered next.

A syndrome is formed for each branch by generating the parity symbols from the received information symbols and adding to them the received parity symbols. Thus, the received information symbols are passed through a replica of the convolutional encoder used for

transmission and the resulting signals are compared with the received parity symbols. Figure 3.2 shows a general syndrome feedback decoder for a rate 1/2 code. In this figure, $g(n)$ represents the generator polynomial of the specific code.

For this example,

$$P(n) = I(n) \cdot g(n) \quad (3.1)$$

$$S(n) = y_I(n) \cdot g(n) + y_p(n) \quad \text{modulo } 2 \quad (3.2)$$

$$= [I(n) + e_I(n)] \cdot g(n) + P(n) + e_p(n) \quad \text{modulo } 2 \quad (3.3)$$

Using equation (3.1) in equation (3.3) and making the necessary simplifications, the following expression is obtained

$$\begin{aligned} S(n) &= I(n) \cdot g(n) + e_I(n) \cdot g(n) + I(n) \cdot g(n) + e_p(n) \\ &= e_I(n) \cdot g(n) + e_p(n) \quad \text{modulo } 2 \end{aligned} \quad (3.4)$$

Therefore the syndrome is only function of the errors introduced by the channel. For each of the L syndromes input to the decision device, there is a minimum Hamming weight error pattern determined by equation (3.4) and stored in a look-up table. In the decoder of figure 3.2, for each code branch, one symbol from each sequence $y_I(n)$ and $y_p(n)$ is shifted in to compute one new symbol of the syndrome sequence $S(n)$ which is routed to the syndrome register. As each new

Figure 3.2 Encoder and table look-up feedback decoder.

syndrome symbol is shifted into this register, the last L syndrome symbols define an address for the decision device look-up table. The decision device then selects the corresponding stored error pattern. If this pattern contains an error in the symbol that is about to be output, then the decision device outputs a 1, thereby correcting the error. The decoder outputs an error otherwise. The correction symbol is fed back to the syndrome register in order to remove its effect on the span of syndrome symbols to be examined next by the decision device. This last action corresponds to the selection of the starting node, in the tree diagram, for the decoding of the next block.

This minimum distance decoder can be generalized for a rate $1/N$ convolutional code using $(N-1)$ syndrome registers and a single decision device. On the BSC, in general, it performs nearly as well as the Viterbi decoder in that it corrects all the most probable error patterns; namely all those of weight $\lfloor (d_f-1)/2 \rfloor$, where d_f is the free distance of the code, if the number L of branches considered is such that all paths (unmerged as well as merged) which diverge from the transmitted path accumulate a weight equal or greater to d_f ($\lfloor x \rfloor$ is the function that assigns to x the largest integer lower or equal to x). Hence, some (though not necessarily all) of the minimum weight error patterns which cause a decision error with this decoder will also cause decision errors with the Viterbi decoder [32].

The code presented in the example of figure 3.1 has a free distance $d_f=3$ as can be observed from its trellis diagram shown in figure 3.3. Recall that the free distance is defined as the minimum Hamming distance between any two unmerged possible paths. If the all-zero sequence is assumed to be transmitted, then a path corresponding to d_f in figure 3.3 is the path $[0\ 1\ 0\ \dots]$. It is obvious that the minimum number of unmerged branches between two paths having a minimum Hamming distance equal to d_f is in this case two. This justifies a sliding block decoder with a memory of two branches as shown in figure 3.1 since the decoder then considers enough branches to detect the minimum weight paths. This decoder will correct all single error patterns in the sequence of errors $[e_I(n)\ e_p(n)\ e_I(n-1)\ e_p(n-1)]$ and can be called a "single error correction" feedback decoder (SEC decoder). Its practical implementation can take the form shown in figure 3.4. If two out of any four consecutive symbols are in error, a decision error occurs that may propagate well beyond a single decision error since this error is in fact fed back to affect further decisions. This error propagation effect is common to some extent to all convolutional decoders. The error propagation is in this case finite, since if no channel errors occur in two consecutive branches, both syndrome stages ($S(n)$ and $\sigma(n)$) cannot simultaneously contain "1" and no decision error is detected or fed back, thus returning the syndrome register to the all-zero state and passing the correct information symbols unchanged.

Figure 3.3 Trellis diagram corresponding to figure 3.1.

Figure 3.4 Feedback decoder for the code of figure 3.1.

The error propagation phenomenon inherent to convolutional codes has been the subject of much attention in the research literature. There are several types of error propagation, each with different causes. Error propagation can be either limited or unlimited. The former cannot be entirely eliminated in a feedback decoder. There are two types of unlimited propagation. Type-I is caused by poor code selection. These codes are the so-called "catastrophic error-propagating codes" and will perform poorly, no matter what decoding technique is used. Systematic codes never have this problem [33]. However, even when good codes are used, improper decoder design can cause type-II unlimited error propagation, which happens when a finite number of channel errors cause the decoder to make an infinite number of information symbol errors. For table look-up feedback decoders, this phenomenon has been shown to occur when the decoding depth L is too short [34].

The example of figure 3.1 uses a decoding depth of $L=2$. It turns out that a reduction of L to one will have a disastrous effect on performance. The error propagation effects can be studied by observing the behaviour of the syndrome register [33]. This register is a finite-state machine and thus its properties can be analysed using a state diagram in which each state is defined by the doublet $[S(n) S(n-1)]$.

Suppose that the decoding depth is reduced to one. Then only a single syndrome symbol to perform error correction is available. With only one syndrome symbol, only two situations can occur. Either $\hat{e}(n-1)=S(n-1)$ or $\hat{e}(n-1)=0$, in which case no error correction is performed. These two possibilities are illustrated in figure 3.5. The state transitions for a single error in $e_I(n)$ is illustrated in figure 3.6 for both settings of the switch of figure 3.5. In both cases, a single error results eventually in a steady error-free output. But if a single error occurs in $e_p(n)$, the error feedback procedure is trapped in state 1, as shown in figure 3.7(a) and produces an infinite length string of errors (type-II unlimited error propagation). There is no error propagation if there is no feedback, as shown in figure 3.7(b). For the L-2 system of figure 3.4, the state transition diagram shows that a single error in either $e_I(n)$ or $e_p(n)$ will eventually bring the decoder in state 0 and therefore that no unlimited error propagation occurs.

Even if the error propagation is limited and therefore not too harmful to the performance, it will produce an average probability of error greater than if the correct decision was always fed back to the syndrome register. Of course, this possibility is only theoretical, but the error probability with no error propagation does have some merit in assessing the performance of a feedback decoder. This type of feedback decoding in which the right decision is always fed back was called "genie decoding" by some authors [26],[31],[33] since the

Figure 3.5 Feedback decoder for the code of figure 3.2 with $L=1$.

Figure 3.6 Syndrome register state transitions with $L=1$ and a single error in $e_I(n)$ for
 (a) $\hat{e}(n-1) = S(n-1)$
 (b) $\hat{e}(n-1) = 0$.

(a)

(b)

Figure 3.7 Syndrome register state transitions with $L=1$ and a single error in $e_P(n)$ for
 (a) $\hat{e}(n-1)=S(n-1)$
 (b) $\hat{e}(n-1)=0$.

right decision can be assumed to be given to the decoder by a magic genie! This fictitious error probability can also be considered as the "first error probability", P_{fe} .

In the case of no error propagation, the decoder of figure 3.4 produces the estimated correction symbol

$$\begin{aligned}\hat{e}_I(n-1) &= S(n) \cdot \sigma(n-1) \\ &= [e_I(n) + e_I(n-1) + e_p(n)] \cdot [e_I(n-1) + e_p(n-1)] \quad (3.5)\end{aligned}$$

since it is assumed that $e_I(n-2)=0$. In this case, the error probability is

$$P_{fe} = p \cdot P[\hat{e}_I(n-1)=0 | e_I(n-1)=1] + (1-p) \cdot P[\hat{e}_I(n-1)=1 | e_I(n-1)=0] \quad (3.6)$$

where p is the probability of making an error on the BSC. It is found

$$\begin{aligned}P[\hat{e}_I(n-1)=0 | e_I(n-1)=1] &= P[1,0,0] + P[1,0,1] + P[1,1,1] \\ &\quad + P[0,1,0] + P[0,1,1] + P[0,0,1] \\ &= 3p(1-p)^2 + 2p^2(1-p) + p^3 \quad (3.7)\end{aligned}$$

where $P[i,j,k]$ refers to $P[e_I(n)=i, e_p(n)=j, e_p(n-1)=k]$ with i, j and k elements of $\{0,1\}$. It is also found

$$P[\hat{e}_I(n-1)=1, e_I(n-1)=0] = P[0,1,1] + P[1,0,1] = 2p^2(1-p). \quad (3.8)$$

Therefore

$$\begin{aligned} P_{fe} &= 3p^2(1-p)^2 + 2p^3(1-p) + p^4 + 2p^2(1-p)^2 \\ &= 5p^2(1-p)^2 + 2p^3(1-p) + p^4 \end{aligned} \quad (3.9)$$

When the error is fed back, the feedback decoder estimates the following correction symbol

$$\hat{e}_I(n-1) = [e_I(n) + e_I(n-1) + e_p(n)] \cdot [e_I(n-1) + e_p(n-1) + e_I(n-2) + \hat{e}_I(n-2)]. \quad (3.10)$$

Because of the effect of $\hat{e}_I(n-2)$ on $\hat{e}_I(n-1)$, a procedure similar to the one given in equations (3.6) to (3.9) does not exist. However, this problem of infinite past history dependence is circumvented by the representation of the feedback decoder of figure 3.4 as a 4-state sequential machine $M=(Q, X, Y, \delta, \beta)$, where Q is the set of four states represented by $(e_I(n-1), \sigma(n))$, X is the set of inputs $(e_I(n), e_p(n))$, Y is the set of output $e_I(n-1)$, $\delta: Q \times X \rightarrow Q$ is the next-state function and $\beta: Q \times X \rightarrow Y$ is the current-output function. The states are defined in table 3.1.

The stochastic behaviour of the machine may be represented by a 4-state Markov process where the transition probabilities between pairs of states are the sums of the probabilities of those input noise vectors that take the machine from one state to the other [31]. Equation (3.11) gives the transition matrix where π_{ij} is the conditional probability that the system is in state j at time n , given that it was in state i at time $(n-1)$.

state	$e_I(n-1)$	$\sigma(n)$
0	0	0
1	0	1
2	1	0
3	1	1

Table 3.1 Definition of the set $Q=[0,1,2,3]$.

$$\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} & \pi_{02} & \pi_{03} \\ \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (1-p)^2 & p(1-p) & p^2 & p(1-p) \\ 1-p & 0 & p & 0 \\ p(1-p) & (1-p)^2 & p(1-p) & p^2 \\ 1-p & 0 & p & 0 \end{bmatrix} \quad (3.11)$$

The corresponding transition diagram is shown in figure 3.8. Since the π_{ij} 's are invariant to a time shift, the process is called homogeneous [35] and the matrix Π is independent of n .

Figure 3.8 Probabilistic state transition diagram for the decoder of figure 3.4.

The probability that the process is in state i at time n is $w_i(n)$ and the state probability vector $\underline{W}$ is defined to be $\underline{W}(n)=[w_0(n) w_1(n) w_2(n) w_3(n)]$. The vector $\underline{W}(n)$ is given by

$$\underline{W}(n) = \underline{W}(n-1)\Pi = \underline{W}(0)\Pi^n \quad (3.12)$$

where $\underline{W}(0)=[1 \ 0 \ 0 \ 0]$ since the feedback decoder is in the all-zero state at the beginning of the decoding process [31].

The steady-state decoder state probability vector, $\underline{W}=[w_0 \ w_1 \ w_2 \ w_3]$ gives the state probabilities when the process reaches an equilibrium. In this case $\underline{W}(n+1)=\underline{W}(n)=\underline{W}$ and $\underline{W}$ is the eigenvector corresponding to the eigenvalue 1 for the matrix Π . Therefore, if $\underline{W}$ exists, it may be found by solving the system given in equation 3.13.

$$\begin{cases} \underline{W} = \underline{W}\Pi \\ \sum_{i=0}^3 w_i = 1. \end{cases} \quad (3.13)$$

The conditions for the existence of $\underline{W}$ are given in [31]. For the BSC case under study, $\underline{W}$ effectively exists.

With each state i is associated a probability of error at time n , $P_i(n)$, given that the process is in state i . $P_i(n)$ is equal to

the sum of the probabilities of $e_I(n)$ and $e_p(n)$ that cause an error at time n . Therefore

$$\begin{aligned} P_0(n) &= P[\text{error} | e_I(n-1)=0, \sigma(n)=0] \\ &= P[\hat{e}_I(n-1)=1 | \sigma(n)=0] \\ &= P[\sigma(n) \cdot S(n)=1] \\ &= P[0 \cdot S(n)=1] = 0 \end{aligned}$$

$$\begin{aligned} P_1(n) &= P[\text{error} | e_I(n-1)=0, \sigma(n)=1] \\ &= P[1 \cdot S(n)=1] \\ &= P[S(n)=1 | e_I(n-1)=0] \\ &= P[e_I(n)=1, e_p(n)=0] + P[e_I(n)=0, e_p(n)=1] \\ &= 2p(1-p) \end{aligned}$$

$$\begin{aligned} P_2(n) &= P[\text{error} | e_I(n-1)=1, \sigma(n)=0] \\ &= P[0 \cdot S(n) - \hat{e}_I(n-1)=0] \\ &= 1 \end{aligned}$$

$$\begin{aligned} P_3(n) &= P[\text{error} | e_I(n-1)=1, \sigma(n)=1] \\ &= P[1 \cdot S(n)=0] \\ &= P[S(n)=0 | e_I(n-1)=1] \\ &= P[e_I(n)=1, e_p(n)=0] + P[e_I(n)=0, e_p(n)=1] \\ &= 2p(1-p) \end{aligned}$$

The overall error probability at time n for the feedback decoder of figure 3.4 is thus

$$P_{fd}(n) = \sum_{i=0}^3 P_i(n) w_i(n) \quad (3.14)$$

If the steady-state vector $\underline{W}$ exists, then the steady-state overall error probability P_{fd} is

$$P_{fd} = \lim_{n \rightarrow \infty} P_{fd}(n) = \sum_{i=0}^3 P_i w_i \quad (3.15)$$

where $P_i = \lim_{n \rightarrow \infty} P_i(n) = P_i$ in this case since the BSC is a stationary channel.

For the decoder of figure 3.4, $\underline{W}$ exists [31] and is found by solving equation (3.13). The solution is

$$\begin{aligned} w_0 &= \frac{1-2p+3p^2-2p^3}{1+3p^2-2p^3} & w_2 &= \frac{3p^2-2p^3}{1+3p^2-2p^3} \\ w_1 &= \frac{p-3p^3+2p^4}{1+3p^2-2p^3} & w_3 &= \frac{p-3p^2+5p^3-2p^4}{1+3p^2-2p^3} \end{aligned} \quad (3.16)$$

and thus

$$P_{fd} = \frac{7p^2-12p^3+10p^4-4p^5}{1+3p^2-2p^3}. \quad (3.17)$$

In figure 3.9, P_{fd} is compared to P_{fe} as given by equation (3.9). Note that for $p < 0.5$, P_{fe} is indeed lower than P_{fd} . P_{fe} is relatively close to P_{fd} , which suggest that it constitutes a fairly tight lower bound for P_{fd} .

This completes this short survey of feedback decoding of convolutional codes. The P_{fd} derivation is due to Morrissey [31] and does not address explicitly the conditions of existence of the steady-state decoder state probability vector $\underline{W}$. A collection of

Figure 3.9 P_{fd} and P_{fe} versus p .

theorems and results, that justify the steady-state assumption, is given in [31].

3.2 PHASE FEEDBACK DECODING PRINCIPLES

It was stated in section 2.3.2 that the set of quantized received differential phase random variables y_n^k for $k=1,2,\dots,M$, produced by subtracting the received signal phase at time $(n-k)$ from the received signal phase at time n , forms a bijection with the set of received data bit and parity bits of a systematic $1/M$ convolutional code. More explicitly we have

$$\left\{ y_n^1, y_n^2, y_n^3, \dots, y_n^M \right\} \leftrightarrow \left\{ y_I(n), y_{p1}(n), y_{p2}(n), \dots, y_{pM-1}(n) \right\} \quad (3.18)$$

where the y_n^i 's are the quantized received differential phase variables and $y_I(n)$ and the $y_{pi}(n)$'s are the received bits over the BSC channel. This bijection and the general demodulator principles introduced in figure 2.10 for full response CPM signals suggest the use of feedback decoding on the received differential phase variables.

3.2.1 2-BRANCH PHASE FEEDBACK DECODING

For $M=2$, the encoder of figure 2.11 reduces to the one shown in figure 3.10. This case corresponds to the example shown in figures 3.1, 3.3 and 3.4 in the previous section. Figure 3.11 is obtained by modifying figure 3.4 using the principles established in section 2.3.1 to generate the variables y_n^1 and y_n^2 in (a) and equation (2.23) to show explicitly the channel errors in (b). Note that figure 3.11(b) shows how the feedback decoder makes use of the variables y_n^1 and y_n^2 and is not a model of the channel through which d_n^1 and d_n^2 are transmitted. In the same figure, the subscript $n-p$ reflects the delay, through the channel, between b_k and its estimate $\hat{b}_{n-p}$.

This decoder will correct any single error in the sequence $[e_n^1, e_n^2, e_{n-1}^1, e_{n-1}^2]$, as explained in the previous section. This procedure was called "nonredundant single error correction" for the differential detection of MSK [4], since error correction is possible in y_n^1 without adding any redundant bits for error correction. For the purpose of this thesis, this technique is called "two-branch phase feedback decoding" in order to emphasize the fact that the structural phase properties of the signal are exploited to perform the best decision allowed by the decoder memory.

Figure 3.10 Rate 1/2 convolutional encoder.

(a)

(b)

Figure 3.11 (a) Received differential phases for M=2
(b) Feedback decoding of the received differential phases for M=2.

The syndrome S_n^1 is given by

$$S_n^1 = y_n^1 + y_{n-1}^1 + y_n^2 \quad \text{modulo 2} \quad (3.19)$$

and the delayed syndrome $\hat{S}_{n-1}^1$ is given by

$$\hat{S}_{n-1}^1 = y_{n-1}^1 + y_{n-2}^1 + y_{n-1}^2 + \hat{e}_{n-2}^1 \quad \text{modulo 2} \quad (3.20)$$

Note that this procedure corresponds to minimum Hamming distance decoding and is not optimum for any other channel than the BSC (i.e. for statistically independent errors).

3.2.2 M-BRANCH PHASE FEEDBACK DECODING

The general rate $1/M$ convolutional encoder of figure 2.11 is reproduced in figure 3.12. A trellis diagram with 2^{M-1} states can be constructed for this code. Each state is represented by the left $M-1$ bits of the delay line. For example, the trellis for the rate $1/3$ code is given in figure 3.13. The two shortest minimum free distance paths are shown on figure 3.13. For this particular code, the minimum free distance is $d_f=6$ and at least three branches are required to reach this distance. The 3-branch path is obtained with the input sequence $[0 \ 1 \ 0 \ 0 \ \dots]$ while the 4-branch path is the result of the input sequence $[0 \ 1 \ 1 \ 0 \ 0 \ \dots]$.

Figure 3.12 Rate 1/M convolutional encoder.

For $M > 3$, the input sequence $[0 \ 1 \ 1 \ 0 \ 0 \ \dots]$ gives the d_f path in the least number of branches. The following description provides the reasoning behind this observation:

- an input "one" is needed to leave the all-zero path and produce M output "ones"; thus $d_f \geq M$ and $L \geq 1$, where L is the least number of branches needed to reach d_f .
- a string of "zeros" following the initial "one" will allow merging in $M-2$ branches, but will produce $(M-1)+(M-2)+\dots+1$ extra output "ones"; i.e. $d_f = M + (M-1) + (M-2) + \dots + 1$.
- a second input "one", followed by a string of "zeros" will produce a single output "one" per branch and will merge in M branches; since the minimum number of output "ones" per unmerged branch is 1, this input sequence will produce the minimum distance path in $M+1$ branches, i.e. $d_f = 2M$ in $L = M+1$ branches.

Thus the following result emerges:

A rate $1/M$ convolutional encoder of the type shown in figure 3.12 has a free distance $d_f = 2M$ and it takes a minimum of $L = M+1$ branches for an unmerged path to reach this distance.

Therefore, a feedback decoder with a memory of $L-M+1$ branches will correct all $\lfloor (d_f-1)/2 \rfloor - M-1$ errors present in these L branches. But if the decoder memory is limited to M branches, the minimum distance path that can exist is at a distance d_f-1 . Therefore $\lfloor (d_f/2) - 1 \rfloor - M-1$ errors can be corrected, which is the same number as with the $(M+1)$ -branch decoder.

A phase feedback decoder accepts as input, at time n , the received phase vector $\underline{y}_n = [y_n^1 \ y_n^2 \ \dots \ y_n^M]$ and computes the syndrome vector $\underline{s}_n = [s_n^1 \ s_n^2 \ \dots \ s_n^{M-1}]$, where the vector components are given by

$$s_n^j = \sum_{i=0}^j y_{n-i}^1 + y_n^{j+1} \quad \text{modulo 2} \quad (3.21)$$

The decoder also keeps in memory the previous $M-1$ corrected syndrome vectors $\underline{s}_{n-k}^j$, $k=1,2,\dots,M-1$, corresponding to the $M-1$ trellis branches that were modified by the previous decoding decisions. The previous corrected syndromes, for $k=1,2,\dots,M-1$, are given by

$$\hat{s}_{n-k}^j = \sum_{i=0}^j y_{n-i-k}^1 + y_{n-k}^{j+1} + \sum_{\substack{p \\ \text{applicable}}} \hat{e}_{n-p}^1 \quad j=1,2,\dots,M-1 \quad (3.22)$$

where the previous decisions $\hat{e}_{n-p}^1$ are added only when y_{n-i-k}^1 goes beyond time $n-M+1$, i.e. when $p-i+k$ is greater than $M-1$.

The syndrome vectors given by equations (3.21) and (3.22) can be combined in a syndrome matrix M_n with elements m_n^{jk} given by

$$m_n^{jk} = \sum_{i=0}^j y_n^{1-i-k} + y_n^{j+1-k} + \sum_{\substack{p \\ \text{applicable}}} \hat{e}_n^{1-p} \quad \begin{matrix} j=1,2,\dots,M-1 \\ k=0,1,\dots,M-1 \end{matrix} \quad (3.23)$$

Using equation (2.23) in the last expression, we get

$$\begin{aligned} m_n^{jk} &= \sum_{i=0}^j b_{n-i-k} + \sum_{i=0}^j e_{n-i-k}^1 + \sum_{i=0}^j b_{n-i-k} + e_{n-k}^{j+1} + \sum_{\substack{p \\ \text{applicable}}} \hat{e}_n^{1-p} \\ &= \sum_{i=0}^j e_{n-i-k}^1 + e_{n-k}^{j+1} + \sum_{\substack{p \\ \text{applicable}}} \hat{e}_n^{1-p} \quad \begin{matrix} j=1,2,\dots,M-1 \\ k=0,1,\dots,M-1 \end{matrix} \end{aligned} \quad (3.24)$$

Equation (3.24) shows again that the syndromes are only function of the errors introduced by the channel and that many error sequences generate the same syndrome. If the channel is a BSC, maximum-likelihood decoding can be realized as before by storing in the decision look-up table the minimum Hamming weight error pattern corresponding to each syndrome matrix. On any channel other than the BSC, this procedure is not guaranteed to be optimum and another type of decision device should be considered.

The 3-branch decoding procedure proposed in [30] is based on such a different decision device. For this case, the syndrome matrix covers 3 branches, i.e. $M=3$, and its elements are

$$m_n^{10} = S_n^1 = y_n^1 + y_{n-1}^1 + y_n^2 = e_n^1 + e_{n-1}^1 + e_n^2$$

$$\begin{aligned}
m_n^{20} - s_n^2 &= y_n^1 + y_{n-1}^1 + y_{n-2}^1 + y_n^3 \\
&= e_n^1 + e_{n-1}^1 + e_{n-2}^1 + e_n^3 \\
m_n^{11} - \hat{s}_{n-1}^1 &= y_{n-1}^1 + y_{n-2}^1 + y_{n-1}^2 \\
&= e_{n-1}^1 + e_{n-2}^1 + e_{n-1}^2 \\
m_n^{21} - \hat{s}_{n-1}^2 &= y_{n-1}^1 + y_{n-2}^1 + y_{n-3}^1 + y_{n-1}^3 + \hat{e}_{n-3}^1 \\
&= e_{n-1}^1 + e_{n-2}^1 + e_{n-1}^3 + e_{n-3}^1 + \hat{e}_{n-3}^1 \\
m_n^{12} - \hat{s}_{n-2}^1 &= y_{n-2}^1 + y_{n-3}^1 + y_{n-2}^2 + \hat{e}_{n-3}^1 \\
&= e_{n-2}^1 + e_{n-2}^2 + e_{n-3}^1 + \hat{e}_{n-3}^1 \\
m_n^{22} - \hat{s}_{n-2}^2 &= y_{n-2}^1 + y_{n-3}^1 + y_{n-4}^1 + y_{n-2}^3 + \hat{e}_{n-3}^1 + \hat{e}_{n-4}^1 \\
&= e_{n-2}^1 + e_{n-2}^3 + e_{n-3}^1 + \hat{e}_{n-3}^1 + e_{n-4}^1 + \hat{e}_{n-4}^1
\end{aligned} \tag{3.25}$$

The free distance is $d_f=2M-6$ and single and double-errors are corrected by such a feedback decoder. Note that these syndromes are not orthogonal in e_{n-2}^1 . There are 64 possible patterns for this 2×3 syndrome matrix. In order to store a specific error pattern in the decision device look-up table, it must be assumed that the previous decisions were correct, that is $e_{n-3}^1 + \hat{e}_{n-3}^1 = 0$ and $e_{n-4}^1 + \hat{e}_{n-4}^1 = 0$. This assumption will be justified later by the fact that on some channels of interest (in particular the AWGN channel), the errors are mostly isolated and therefore corrected most of the time. One possible configuration for this 3-branch feedback decoder is shown in figure 3.14.

This decoder attempts to correct the errors in y_n^1 . In the set $[e_{n-2}^3 e_{n-2}^2 e_{n-2}^1 e_{n-1}^3 e_{n-1}^2 e_{n-1}^1 e_n^3 e_n^2 e_n^1]$, it can detect and correct a single error e_{n-2}^1 and any doublet $[e_{n-2}^1 e_{n-1}^j]$, $i=0,1,2$ and $j=1,2,3$. These error patterns correspond to 9 syndrome matrices that are unique. The look-up table can therefore contain, as a minimum, the syndrome matrix given in table 3.2. If the communication channel is such that more single and double error patterns than multiple error patterns are produced in the set of differential phase variables considered by the decoder, this procedure should assure a net gain in terms of average bit error rate.

	s_n^1	$\hat{s}_{n-1}^1$	$\hat{s}_{n-2}^1$	s_n^2	$\hat{s}_{n-1}^2$	$\hat{s}_{n-2}^2$
e_{n-2}^1	0	1	1	1	1	1
$e_{n-2}^1 e_n^1$	1	1	1	0	1	1
$e_{n-2}^1 e_{n-1}^1$	1	0	1	0	0	1
$e_{n-2}^1 e_n^2$	1	1	1	1	1	1
$e_{n-2}^1 e_{n-1}^2$	0	0	1	1	1	1
$e_{n-2}^1 e_{n-2}^2$	0	1	0	1	1	1
$e_{n-2}^1 e_n^3$	0	1	1	0	1	1
$e_{n-2}^1 e_{n-1}^3$	0	1	1	1	0	1
$e_{n-2}^1 e_{n-2}^3$	0	1	1	1	1	0

Table 3.2 Syndrome matrix corresponding to single and double error patterns in e_{n-2}^1 and any other error variable.

3.3 PERFORMANCE ANALYSIS

3.3.1 CHANNEL CONSIDERATIONS

So far the differential phase variables $y_n^1, y_n^2, \dots$ were considered as the result of the two-level quantization of the phase differences obtained from a bank of phase detectors (see figure 2.10). For a modulation index $h=1/2$, the phase detectors can be replaced by differential detectors (delay and multiply) followed by hard-limiters to produce the differential phase variables. The major difference between the phase detector and the differential detector is that the former normally does not respond to the level of the signal envelope. This is usually accomplished by hardlimiting prior to the phase detector circuit. However, since the envelope is by definition always positive, only the sign of the phase difference is used to make the decision, and the performance of the two detectors is identical [38].

The receive filter precedes the differential detectors in order to reduce the noise level in the detection procedure. On an AWGN channel it would be desirable to have a filter matched to the MSK pulse (assuming that no filtering is performed in the transmitter) in order to obtain the maximum signal-to-noise ratio before detection. However, the half cosine MSK pulse does not satisfy Nyquist first

criterion at the bit rate and intersymbol interference (ISI) degrades the expected performance. The selection of an appropriate receive filter involves a trade-off between ISI and noise rejection capability.

Post-detection filters must also follow the differential detectors in order to remove high frequency components produced by the detectors. A realistic channel between the MSK modulator and the phase feedback decoder will therefore follow the structure shown in figure 3.15.

The propagation channel refers to any medium used for signal transmission. It can be an AWGN channel, a fading multipath channel or any other type of stationary channel.

The channel may therefore exhibit memory. This characteristic affects the statistics of the differential phase variables. Depending on the channel and the nature of the filters considered, more or less correlation is introduced between the y_n^i 's variables. This fact has to be considered in the decoder analysis and the usual simple BSC assumption cannot be used here. The method that was presented in section 3.1, due to Morrissey [31], will therefore be extended in the next section in order to take the channel memory into consideration.

Figure 3.15 Channel between the modulator and the demodulator.

The analysis presented by Masamura et al [4] assumes no intersymbol interference, uncorrelated noise components and a phase probability function at the output of the RX filter given by

$$f(\phi) = \frac{1}{2\pi} \exp(-\rho) + \frac{1}{2} \sqrt{\frac{\rho}{\pi}} \exp(-\rho \sin^2 \phi) \cdot \cos \phi \cdot [1 + \operatorname{erf}(\sqrt{\rho} \cos \phi)] \quad (3.26)$$

where $\rho = E_b/N_0$. This is the density obtained at the output of a channel corrupted by AWGN and exhibiting an overall filtering characteristic satisfying Nyquist first criterion for ISI-free data transmission [25].

The above assumption gives significant results only for DPSK signalling using a Nyquist pulse shape. In this case, differential detection gives the following error probability [25]

$$P_b = 1/2 \exp(-E_b/N_0) \quad (3.27)$$

and any improvement obtained with feedback decoding is over this error probability.

Since the above assumptions are not valid for MSK, any analysis based on these can only provide a performance lower bound. The analysis following in the next sections provides some insight into the performance of phase feedback decoding on a bandlimited channel.

2

Micro

3.3.2 ANALYSIS OF 2-BRANCH DECODING

The two-branch decoder was presented in section 3.2.1. Its structure is shown in figure 3.11. The corresponding syndrome vector contains the two syndromes

$$\begin{aligned} S_n^1 &= y_n^1 + y_{n-1}^1 + y_n^2 \quad \text{modulo 2} \\ \hat{S}_{n-1}^1 &= y_{n-1}^1 + y_{n-2}^1 + y_{n-1}^2 + \hat{e}_{n-2}^1 \quad \text{modulo 2} \end{aligned} \quad (3.28)$$

The estimated error $\hat{e}_{n-1}^1$ is the result of the operation

$$\hat{e}_{n-1}^1 = S_n^1 \cdot \hat{S}_{n-1}^1. \quad (3.29)$$

This decoding procedure is identical to the one presented in section 3.1 for convolutional codes. If the average error statistics on y_n^1 and y_n^2 are known, the Markovian model previously presented can be used to obtain the average error probability at the output of the feedback decoder. This analysis involves state transition probabilities, state probabilities and error probabilities given a specific state. At any time n , the decoder performance is function of the error variables e_n^1 , e_n^2 , e_{n-1}^1 and e_{n-1}^2 . The transition probability π_{ij} of being in state j at time n , given that the state was i at time $n-1$, is only a function of these error variables. Therefore, if the probability of occurrence of each of these variables is known, the

transition probabilities and the error probabilities, given a specific state, can be computed.

For the purpose of this analysis, an all-zero sequence transmission is assumed. It is also assumed that the feedback decoder is represented by a 4-state sequential machine with 2-dimensional random input error vector $\underline{e}_n = [e_n^1 \ e_n^2]$ and single output $\hat{e}_{n-2}^1$. The state at time n is represented by the sequence $[e_{n-1}^1 \ \hat{s}_{n-1}^1]$ and can take the four values given in table 3.3.

state	e_{n-1}^1	$\hat{s}_{n-1}^1$
0	0	0
1	0	1
2	1	0
3	1	1

Table 3.3 Four possible states of the 2-branch decoder.

Figure 3.16 shows the decoder with the different variables labelled.

The Markov transition matrix is

$$\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} & \pi_{02} & \pi_{03} \\ \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \quad (3.30)$$

and the state probability vector is $\underline{w}(n) = [w_0(n) \ w_1(n) \ w_2(n) \ w_3(n)]$ where $w_i(n)$ is the probability that the decoder is in state i at time n . We therefore have

Figure 3.16 2-branch decoder.

$$\underline{W}(n) = \underline{W}(n-1)\Pi = \underline{W}(0)\Pi^n \quad (3.31)$$

where $\underline{W}(0)=[1 \ 0 \ 0 \ 0]$ since the feedback decoder is in the all-zero state at the beginning of the decoding process [31].

With each state i is associated a probability of error P_i , given that the state is in state i . Therefore, the error probability at time n is simply

$$P_e(n) = \sum_{i=0}^3 P_i w_i(n) \quad (3.32)$$

and the average error probability is

$$P_e = \lim_{n \rightarrow \infty} P_e(n) = \sum_{i=0}^3 P_i w_i \quad (3.33)$$

where w_i is the steady-state probability that the decoder is in state i . The steady-state decoder state probability vector, if it exists, is $\underline{W}=[w_0 \ w_1 \ w_2 \ w_3]$ and may be calculated by solving $\underline{W}=\underline{W}\Pi$ since $\underline{W}_{n+1}=\underline{W}_n$ when $n \rightarrow \infty$. The w_i 's must also meet the condition $\sum w_i=1$.

In order to determine P_e , the matrix Π and the probabilities P_i must be calculated. Since the differential phase variables are not independent random variables, as they would be on a BSC, the joint statistics of all the variables considered by the feedback decoder

must be available. In the 2-branch case, these variables are $\{y_n^1, y_n^2, y_{n-1}^1, y_{n-1}^2\}$. Since an all-zero sequence transmission is assumed, the variables to consider are $\{e_n^1, e_n^2, e_{n-1}^1, e_{n-1}^2\}$. There are 16 of these patterns as shown in table 3.4. The probability of occurrence of each can be characterized by a discrete probability density function (PDF) that is function of the average characteristics of the channel considered. The syndrome feedback decoder makes use of these variables by combining e_n^1, e_n^2 and e_{n-1}^1 to form S_n^1 and by combining e_{n-1}^1 and e_{n-1}^2 with the previous variable e_{n-2}^1 and the previous decision $\hat{e}_{n-2}^1$ to form $\hat{S}_{n-1}^1$ (see equation (3.28)).

error pattern	e_{n-1}^2	e_n^2	e_{n-1}^1	e_n^1
P0	0	0	0	0
P1	0	0	0	1
P2	0	0	1	0
P3	0	0	1	1
P4	0	1	0	0
P5	0	1	0	1
P6	0	1	1	0
P7	0	1	1	1
P8	1	0	0	0
P9	1	0	0	1
P10	1	0	1	0
P11	1	0	1	1
P12	1	1	0	0
P13	1	1	0	1
P14	1	1	1	0
P15	1	1	1	1

Table 3.4 Possible error patterns for 2-branch decoding.

The probabilities of error, given one of the possible four states, are determined as function of the probability of occurrence of the different error patterns of table 3.4.

$$\begin{aligned}
 P_0 &= P[\text{error} | e_{n-1}^1=0, \hat{S}_{n-1}^1=0] \\
 &= P[\text{error} | e_{n-1}^1=0, \hat{e}_{n-1}^1=0] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 P_1 &= P[\text{error} | e_{n-1}^1=0, \hat{S}_{n-1}^1=1] \\
 &= P[S_n^1=1 | e_{n-1}^1=0] \\
 &= P[e_n^1=0, e_n^2=1 | e_{n-1}^1=0] + P[e_n^1=1, e_n^2=0 | e_{n-1}^1=0] \\
 &= \frac{P[e_n^1=0, e_n^2=1, e_{n-1}^1=0]}{P[e_{n-1}^1=0]} + \frac{P[e_n^1=1, e_n^2=0, e_{n-1}^1=0]}{P[e_{n-1}^1=0]} \\
 &= \frac{P[p_1] + P[p_4] + P[p_9] + P[p_{12}]}{P[p_0] + P[p_1] + P[p_4] + P[p_5] + P[p_8] + P[p_9] + P[p_{12}] + P[p_{13}]}
 \end{aligned}$$

$$\begin{aligned}
 P_2 &= P[\text{error} | e_{n-1}^1=1, \hat{S}_{n-1}^1=0] \\
 &= P[\text{error} | e_{n-1}^1=1, \hat{e}_{n-1}^1=0] \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 P_3 &= P[\text{error} | e_{n-1}^1=1, \hat{S}_{n-1}^1=1] \\
 &= P[S_n^1=0 | e_{n-1}^1=1] \\
 &= P[e_n^1=0, e_n^2=1 | e_{n-1}^1=1] + P[e_n^1=1, e_n^2=0 | e_{n-1}^1=1] \\
 &= \frac{P[e_n^1=0, e_n^2=1, e_{n-1}^1=1]}{P[e_{n-1}^1=1]} + \frac{P[e_n^1=1, e_n^2=0, e_{n-1}^1=1]}{P[e_{n-1}^1=1]}
 \end{aligned}$$

$$= \frac{P[p_3] + P[p_6] + P[p_{11}] + P[p_{14}]}{P[p_2] + P[p_3] + P[p_6] + P[p_7] + P[p_{10}] + P[p_{11}] + P[p_{14}] + P[p_{15}]}$$

The transition probability π_{ij} is defined as $P[\text{state } j \text{ at time } n | \text{state } i \text{ at time } n-1]$. Based on the pattern probabilities, the π_{ij} 's are expressed as

$$\begin{aligned} \pi_{ij} &= P[e_n^1, \hat{S}_n^1 | e_{n-1}^1, \hat{S}_{n-1}^1] \\ &= P[e_n^1, \hat{S}_n^1 - e_n^1 + e_{n-1}^1 + (S_n^1 \cdot \hat{S}_{n-1}^1) | e_{n-1}^1, \hat{S}_{n-1}^1] \\ &= P[e_n^1, \hat{S}_n^1 - e_n^1 + e_{n-1}^1 + (e_n^1 + e_{n-1}^2 + e_{n-1}^1) \cdot \hat{S}_{n-1}^1 | e_{n-1}^1, \hat{S}_{n-1}^1] \end{aligned} \quad (3.34)$$

The π_{ij} 's are (see appendix B)

$$\pi_{00} = \frac{P[p_0] + P[p_8]}{P[e_{n-1}^1 = 0]}$$

$$\pi_{01} = \frac{P[p_4] + P[p_{12}]}{P[e_{n-1}^1 = 0]}$$

$$\pi_{02} = \frac{P[p_5] + P[p_{13}]}{P[e_{n-1}^1 = 0]}$$

$$\pi_{03} = \frac{P[p_7] + P[p_9]}{P[e_{n-1}^1 = 0]}$$

$$\pi_{10} = \frac{\sum_{i=0}^3 P[p_{4i}]}{P[e_{n-1}^1 = 0]}$$

$$\pi_{11} = 0$$

$$\pi_{12} = \frac{\sum_{i=0}^3 P(p_{4i+1})}{P(e^1_{n-1}=0)}$$

$$\pi_{13} = 0$$

$$\pi_{20} = \frac{P(p_6)+P(p_{14})}{P(e^1_{n-1}=1)}$$

$$\pi_{21} = \frac{P(p_2)+P(p_{10})}{P(e^1_{n-1}=1)}$$

$$\pi_{22} = \frac{P(p_3)+P(p_{11})}{P(e^1_{n-1}=1)}$$

$$\pi_{23} = \frac{P(p_7)+P(p_{15})}{P(e^1_{n-1}=1)}$$

$$\pi_{30} = \frac{\sum_{i=0}^3 P(p_{4i+2})}{P(e^1_{n-1}=1)}$$

$$\pi_{31} = 0$$

$$\pi_{32} = \frac{\sum_{i=0}^3 P(p_{4i+3})}{P(e^1_{n-1}=1)}$$

$$\pi_{33} = 0$$

where

$$P(e^1_{n-1}=0) = P(p_0)+P(p_1)+P(p_4)+P(p_5)+P(p_8)+P(p_9)+P(p_{12})+P(p_{13})$$

$$P[e_{n-1}^1] = P[p_2] + P[p_3] + P[p_6] + P[p_7] + P[p_{10}] + P[p_{11}] + P[p_{14}] + P[p_{15}]$$

Having determined the probabilities P_i 's and the transition matrix Π , the procedure to find P_e is the following:

- 1- solve $\underline{W} \Pi = \underline{W}$, subject to the condition $\sum w_i = 1$

(recall that $\underline{W}$ is defined as $\underline{W} = [w_0 \ w_1 \ w_2 \ w_3]$).

- 2- then $P_e = \sum_{i=0}^3 P_i w_i$.

If a lower bound on the error probability is preferred, then the probability of first error P_{fe} may be computed. This error probability is easily found by considering the tree diagram over which the detector is operating. This diagram is reproduced in figure 3.17 with the two states labelled a and b.

If no error was made previously and the all-zero sequence is transmitted, a minimum Hamming distance decoder will continuously be in state a. This means that the path selection is always performed among the paths of block 1 and that a first error occurs when state b is selected. Thus, the probability of first error is the probability that paths (1 1 0 1) or (1 1 1 0) are selected among the four possible paths.

Figure 3.17 Tree diagram for the 2-branch feedback decoder.

Among the error patterns given in table 3.4, the following patterns will cause an error: $P_3, P_6, P_9, P_{10}, P_{11}, P_{12}, P_{14}$ and P_{15} . Therefore

$$P_{fe} = P[P_3] + P[P_6] + P[P_9] + P[P_{10}] + P[P_{11}] + P[P_{12}] + P[P_{14}] + P[P_{15}]. \quad (3.35)$$

In summary, it was shown that by using the error pattern probability density function, the average error probability and the average first error probability can be computed.

3.3.3 ANALYSIS OF 3-BRANCH DECODING

The 3-branch decoder syndrome matrix is given in equation (3.25) and the decoder implementation is shown in figure 3.14. The same reasoning that applied for the 2-branch analysis is also applicable here. The error pattern density can be used to compute the state transition matrix, the state probabilities and the error probabilities given a specific state. The complexity associated with the analysis of the 3-branch decoder results from the fact that nine received differential phase variables are involved, namely $\{y_{n-2}^3, y_{n-2}^2, y_{n-2}^1, y_{n-1}^3, y_{n-1}^2, y_{n-1}^1, y_n^3, y_n^2, y_n^1\}$. Thus, 512 different patterns must be considered in the analysis.

The sequential machine that simulates the decoder of figure 3.14 has six state variables, $[e_{n-1}^1 \ e_{n-2}^1 \ \hat{S}_{n-1}^1 \ \hat{S}_{n-2}^1 \ \hat{S}_{n-1}^2 \ \hat{S}_{n-2}^2]$. These variables are necessary to describe the state of the decoder while the inputs are $[e_n^1 \ e_n^2 \ e_n^3]$ and the output is $\hat{e}_{n-2}^1$. Thus the machine can assume 64 different states. This means that the state-transition matrix has $64 \times 64 = 4096$ elements! Obviously, the complexity of the analysis based on Markov process theory would attenuate any intuitive comprehension of the error mechanism that could be obtained from it.

The approximation of the error probability by the first-error probability gives a simpler alternative. Following the same approach used in the determination of P_{fe} for the two-branch case, the trellis and tree diagrams spanned by the decoder are first considered. The trellis diagram is given in figure 3.18.

Assuming that the all-zero sequence is transmitted, then if the decoder has not made any error, it is in state "a" and only the block of the tree originating from node "a" needs to be considered. This block is shown in figure 3.19.

The decoder performance is related to the error pattern $[e_{n-2}^3 \ e_{n-1}^3 \ e_n^3 \ e_{n-2}^2 \ e_{n-1}^2 \ e_n^2 \ e_{n-2}^1 \ e_{n-1}^1 \ e_n^1]$. The first error probability is the sum of the occurrence probability of the error patterns producing the selection of the bottom half of the block or producing

Figure 3.18 3-branch decoder differential phase trellis diagram.

Figure 3.19 Block of the 3-branch differential phase tree originating from state a.

$\hat{e}_{n-1}^2=1$ for the first time in the decoder of figure 3.14. Hence, P_{fe} is function of the decoding strategy.

In the decoder where the decision device is based on table 3.2, $\hat{e}_{n-1}^2$ is 1 only if one of the nine listed syndromes is detected. A computer search can be readily done, using equations (3.25) with $e_{n-3}^1 + \hat{e}_{n-3}^1 = 0$ and $e_{n-4}^1 + \hat{e}_{n-4}^1 = 0$, to find the error patterns causing a fault. This procedure will be used in section 4.1.3.2 to obtain P_{fe} for this 3-branch decoder.

In the next section, the asymptotic performance of M-branch decoding, as M goes to infinity, is investigated in order to assess the possible merits of phase feedback decoding.

3.3.4 PERFORMANCE LOWER BOUND

It is of interest to know the best performance that can be achieved when the decoder memory is infinite. Since no redundancy is added to the transmitted message, the asymptotic behaviour is limited to the performance of coherent detection.

As pointed out previously, one of the performance limitations inherent to MSK is related to the fact that its basic pulse shape, when cascaded with a matched filter, does not satisfy Nyquist first

criterion at the data rate and hence introduces harmful intersymbol interference.

In order to establish a lower bound for MSK, it is assumed that the MSK half cosine pulse shape is replaced by a pulse shape that allows matched filtering satisfying Nyquist first criterion without destroying the differential phase behaviour of MSK. i.e.

$$\Delta\phi_k(n) = \pi/2 \sum_{i=0}^{k-1} a_{n-i} \quad (3.36)$$

as given by equation (2.20).

A square-root raised cosine pulse, with a 2-sided noise bandwidth of $1/T$ Hz, satisfies this condition. The resulting signalling scheme is not a member of the CPM class, but it exhibits the phase behaviour of MSK, at the bit transitions. This modulation format, after being passed through an AWGN channel, a matched filter and a single differential detector, gives the well-known DPSK error probability [25]

$$P_{eDPSK} = 1/2 \exp[-E_b/N_0]. \quad (3.37)$$

Since the matched filter is a square-root raised cosine, then its system autocorrelation function satisfies Nyquist first criterion in that the noise samples taken at bit transitions are uncorrelated.

Therefore, every order of differential detectors used in a phase feedback decoder compares two phase samples corrupted by uncorrelated noise samples. Since the noise process is gaussian, the noise samples are uncorrelated and jointly gaussian. Therefore, they are independent and the error probability at the output of each differential detector is given by equation (3.37).

The performance of a single differential detector asymptotically approaches, as E_b/N_0 is increased, the performance of coherent detection of differentially encoded PSK (DECPSK), which is given by

$$P_{eDECPSK} = 2p(1-p) \quad (3.38)$$

where $p = 1/2 \operatorname{erfc}(\sqrt{E_b/N_0})$. The performance curves corresponding to equations (3.37) and (3.38) are given in figure 3.20.

Therefore, as $E_b/N_0 \rightarrow \infty$, the single-bit detector performs as well as the coherent detector and nothing is gained with phase feedback decoding. On the other hand, for a very poor signal to noise ratio, P_{eDPSK} tends toward 0.5 and as can be seen in figure 3.9, a two-branch decoder does not improve the performance of the first order differential detector. As more detectors are added, each with its own probability of error equal to 0.5, the overall error probability remains 0.5.

Figure 3.20 Probability of error for binary DECPSK and DPSK.

In the more practical range of error probabilities, between 10^{-2} and 10^{-6} , the following observations can be made:

The errors are independent, except when a signal is common to some of them. For example, y_n^1 and y_{n-2}^1 are independent, but y_n^1 , y_n^2 , y_n^3 , ... are dependent because they all have the signal at time n as a common reference. This phase reference is noisy, with a phase probability density function given by equation (3.26). The probability of a binary error with a phase offset ϕ is [4],

$$P[\text{error}|\phi] = 1/2 \operatorname{erfc}(\sqrt{E_b/N_0}) \cos \phi. \quad (3.39)$$

The probability of n errors in signals with a common noisy reference with a phase ϕ is therefore

$$P[n \text{ errors with common reference}] = \int_{-\pi}^{\pi} [1/2 \operatorname{erfc}(\sqrt{E_b/N_0}) \cos \phi]^n f(\phi) d\phi. \quad (3.40)$$

When $n=1$, equation (3.37) is obtained. For n greater than one, the probability of error decreases rapidly because of the n^{th} power term inside the integral. For n independent errors (no common reference), the probability of error is

$$P[n \text{ independent errors}] = [1/2 \exp(-E_b/N_0)]^n. \quad (3.41)$$

Therefore the probability of multiple errors decreases rapidly with n and most error sequences contain a single error.

Therefore, the biggest improvement with feedback decoding is obtained with 2-branch decoding. Any further level of decoding attempts to correct error patterns that have a low occurrence probability. Thus, the P_e improvement is very slow, especially around 10^{-6} .

The improvement obtained with more level of decoding is slow but steady.

These observations can be summarized as follows:

For DPSK, the performance of feedback decoding applied on an increasing number of differential detectors is asymptotically that of DECPSK. This performance also constitutes a lower bound for MSK since the performance of coherent detection of MSK is the same as that of DECPSK.

3.4 ERROR STATISTICS CHARACTERIZATION

It was shown in the previous section that by limiting the memory of the feedback decoder to three branches, a 9-variable discrete probability density function needed to be considered in the analysis of the overall detector. This density is affected by channel bandlimiting, by multipath fading or by any other channel impairment.

Several authors have analysed the conventional differential detection procedure (single differential detector without feedback decoding) for bandlimited and multipath channels [20-23], [36] and [37]. The channel covariance matrix is normally used in this analysis. The channel memory may also be considered by taking into account all the bit combinations influenced by this memory and averaging over them to get the channel statistics. Unfortunately, both methods are relatively complex, even for the single variable statistics. The theoretical derivation of a nine-variable density is expected to be almost out of reach and even a four-variable density seems to lead to a very complicated analysis.

In order to keep the problem tractable and to avoid diverging from the main goal of this work, a simple but reliable Monte-Carlo simulation technique is used to gather the error statistics. The simulation block diagram is given in figure 3.21.

Figure 3.21 Simulation used in the error statistics compilation.

The simulation procedure is given in appendix C. The results are presented as bar-graphs where the occurrence probability of each error pattern is given. Each pattern is identified by its decimal equivalent. The error variable e_n^1 is always the least significant bit of the representation. Therefore, for each channel characteristics, the discrete probability densities of the patterns $[e_{n-1}^2 e_n^2 e_{n-1}^1 e_n^1]$ for the two-branch case and $[e_{n-2}^3 e_{n-1}^3 e_n^3 e_{n-2}^2 e_{n-1}^2 e_n^2 e_{n-2}^1 e_{n-1}^1 e_n^1]$ for the three-branch case are obtained.

CHAPTER 4

PERFORMANCE OF PHASE FEEDBACK DECODING OF BANDLIMITED MSK

The theory behind feedback decoding applied to the MSK phase variables has been presented in the last chapter as well as a few methods for evaluating the performance of two-branch and three-branch detection. It was shown that the performance of these detectors is dependent on the error pattern discrete probability density function.

The goal of this chapter is to obtain, by computer simulation, this error pattern PDF for different practical channels. The simulation process also gives the performance at the output of the individual differential detectors. All of these statistics are related to the channel characteristics in order to understand how the decoder performs under various conditions.

Two broad categories of channels are considered. The first one is the additive white gaussian noise channel where the only impairments are additive noise and intersymbol interference introduced by the receive filter. The second one is a fading multipath channel typical of land-mobile satellite communications.

4.1 ADDITIVE GAUSSIAN NOISE CHANNEL

In this section, the system shown in figure 4.1 is considered. White gaussian noise with a one-sided power spectral density N_0 W/Hz is added to the MSK signal. A noise-rejection receive filter is part of the demodulator front-end. Ideally, it is designed to trade-off noise rejection and intersymbol interference effects, in order to give the lowest error probability. A feedback decoder, making use of two or three branches, performs bit decisions.

In this model, it is assumed that all signals and systems are represented by their complex baseband equivalent. In the first section, three different receive filters are assumed, one for each branch, and the error probabilities at the output of each detector is derived. In the next case, a specific filter is chosen and its influence on the error pattern density at the input of the feedback decoder is investigated.

Figure 4.1 System model for an AGN channel.

4.1.1 RECEIVE FILTERS CONSIDERATIONS IN DIFFERENTIAL DETECTION

In this section, the detection scheme involving a different receive filter in front of each differential detector is analysed. The filter in front of the differential detector with delay iT is referred to as "RX filter i ". The goal is to investigate the effects of the receive filters on the detector outputs and to give an optimization method for these filters. The interest is in the individual differential detectors and no feedback decoding is used.

In complex baseband notation, the transmitted signal $s(t)$ in the time interval $nT \leq t \leq (n+1)T$ is given by

$$s(t) = \sqrt{\frac{2E}{T}} \exp \left[j \left[\sum_{i=-\infty}^{n-1} a_i \pi/2 + a_n \pi t/2T \right] \right] \quad nT \leq t \leq (n+1)T \quad (4.1)$$

where a_i can take the values ± 1 and the other variables are defined as in equations (2.1) and (2.2). The output of the RX filter i is

$$\begin{aligned} z_i(t) &= r(t) * h_i(t) \\ &= [s(t) + u(t)] * h_i(t) \end{aligned} \quad (4.2)$$

where $*$ is the convolution operator, $h_i(t)$ is the baseband impulse response of RX filter i with Fourier transform $H_i(f)$ and $u(t)$ is the complex white gaussian noise process with power spectral density N_0 .

W/Hz.

The output of the first detector, under noiseless conditions, must be proportional to the received data bit. Therefore, it must be the sine of the phase difference $\Delta\theta_1(t)$, as given by

$$y_n^1(t) = \sin \Delta\theta_1(t) \\ = \sin[\theta(t, \underline{a}, u) - \theta[(t-T), \underline{a}, u]]. \quad (4.3)$$

where $\theta(t, \underline{a}, u)$ is the phase of the signal $z_1(t)$ while $\theta[(t-T), \underline{a}, u]$ is the phase of its delayed version $z_1(t-T)$. The signal $y_n^1(t)$ can be obtained by taking the vector cross-product of $z_1(t)$ and $z_1(t-T)$ or by adding a 90° phase shift in the delayed branch and taking the real part of the complex product of the two signals. That is

$$y_n^1(t) = z_1(t) \times z_1(t-T) \\ = \text{Re} \left[z_1(t) (z_1(t-T)e^{j\pi/2})^* \right] \quad (4.4)$$

where $\times$ is the complex cross-product operator and $()^*$ is the complex conjugate operator.

Similarly, the output of the second detector $y_n^2(t)$ must be the cosine of the two-bit phase difference and $y_n^3(t)$ must be the sine of

the three-bit phase difference. These relationships are given in equations (4.5) and (4.6).

$$y_n^2(t) = z_2(t) \cdot z_2(t-2T) \\ = \text{Re} \left[z_2(t) z_2^*(t-2T) \right] \quad (4.5)$$

where $\cdot$ is the dot product operator.

$$y_n^3(t) = -(z_3(t) \times z_3(t-3T)) \\ = \text{Re} \left[z_3(t) (z_3(t-3T)e^{-j\pi/2})^* \right] \quad (4.6)$$

Therefore it is easy to reduce the differential detection of any order to a common expression

$$y(t) = \text{Re} [z(t) z_D^*(t)]. \quad (4.7)$$

This signal is sampled at a certain time t_0 before the n^{th} bit decision is performed. Simplifying the notation to $y(t_0)=y_n$, $z(t_0)=z_n$ and $z_D(t_0)=z_{Dn}$, the following general expression is obtained

$$y_n = \text{Re} [z_n z_{Dn}^*]. \quad (4.8)$$

This expression was shown by Stein [38] to lead to a unified formulation of the analysis of certain coherent and noncoherent modulation

systems, namely noncoherent FSK, differential coherent BPSK or coherent BPSK. Since the same expression holds for differential detection of MSK, Stein's unified analysis can also be applied in this case.

The error probability, for the n th bit interval, at any differential detector output is

$$P_e(n) = 1/2 \left[P[y_n < 0 | w_n = 1] + P[y_n > 0 | w_n = -1] \right] \quad (4.9)$$

where

$$\begin{aligned} w_n &= a_n && \text{for the first branch} \\ w_n &= a_n + a_{n-1} \pmod{2} && \text{for the second branch} \\ w_n &= a_n + a_{n-1} + a_{n-2} \pmod{2} && \text{for the third branch.} \end{aligned} \quad (4.10)$$

Let

$$P[y_n < 0 | w_n = 1] = P_1(n) \quad (4.11)$$

$$P[y_n > 0 | w_n = -1] = P_{-1}(n)$$

and then

$$P_e(n) = 1/2 [P_1(n) + P_{-1}(n)]. \quad (4.12)$$

Note that $P_1(n)$ and $P_{-1}(n)$ are not necessarily the same since the effect of intersymbol interference may be different depending on the value of w_n . For example, assume that the ISI is limited to one adjacent bit. Then, in the one-bit delay detector, for each three-bit pattern that produces a positive phase shift $\Delta\phi_1(n)$, there is a corresponding three-bit pattern that produces $-\Delta\phi_1(n)$. These pairs of bit patterns are illustrated in table 4.1 and can be deduced easily from a phasor diagram. Therefore, the eye diagram of the one-bit delay detector is symmetric.

0 0 0	<-->	1 1 1
0 0 1	<-->	1 1 0
1 0 0	<-->	0 1 1
1 0 1	<-->	0 1 0

Table 4.1 Pairs of data patterns giving the same absolute phase shifts at the output of the one-bit delay differential detector.

For the two-bit delay detector, consider the phasor diagram of figure 4.2 where the transmitted data sequence [1 1 0 1] corresponds to the phase shifts $[\Delta\phi_1(n-2), \Delta\phi_1(n-1), \Delta\phi_1(n), \Delta\phi_1(n+1)] = [\pi/2, \pi/2-\xi, -(\pi/2-2\xi), \pi/2]$. Note that a data bit 1 corresponds to a positive phase shift of $\pi/2$ or less while a data bit -1 corresponds to a similar negative phase shift. The amplitude of the phasors has no significance. ξ represents the phase offset introduced by the adjacent transmitted bit, i.e. it corresponds to the closure of the eye

Figure 4.2 Phasor diagram for the data sequence [1 1 0 1].

pattern caused by intersymbol interference. Therefore, instead of always having a full $\pi/2$ phase shift within a bit period, the phase shift is reduced when there is a change in bit polarity.

Note that $\Delta\phi_2(n) = \xi$. For each one of the 16 possible data patterns, $\Delta\phi_2(n)$ is given in table 4.2. From this table, another table can be formed, where the data patterns with the same absolute phase shift $\Delta\phi_2(n)$ but with opposite signs are paired. This is shown in table 4.3 where four patterns are left without any counterpart.

Data pattern	$\Delta\phi_2(n)$
0 0 0 0	$-\pi$
0 0 0 1	$-\pi + \xi$
0 0 1 0	$-\xi$
0 0 1 1	0
0 1 0 0	$-\xi$
0 1 0 1	0
0 1 1 0	$\pi - 2\xi$
0 1 1 1	$\pi - \xi$
1 0 0 0	$-\pi + \xi$
1 0 0 1	$-\pi + 2\xi$
1 0 1 0	0
1 0 1 1	ξ
1 1 0 0	0
1 1 0 1	ξ
1 1 1 0	$\pi - \xi$
1 1 1 1	π

Table 4.2 Data patterns and the corresponding phase shift $\Delta\phi_2(n)$.

This gives rise to an asymmetric eye diagram at the output of the two-bit delay detector since $\cos(0) \neq \cos(-\pi + 2\xi)$. The eye opening is also wider, as can be seen in the example of figure 4.3.

(a)

(b)

Figure 4.3 Example of the eye diagram at the output of
(a) the one-bit and three-bit delay detector
(b) the two-bit delay detector (from [21]).

0 0 0 0	<-->	0 0 1 1
0 0 0 1	<-->	0 0 1 0
0 1 0 0	<-->	0 1 1 1
0 1 0 1	<-->	1 1 1 1
1 0 0 0	<-->	1 0 1 1
1 1 0 1	<-->	1 1 1 0

and

0 1 1 0	-->	$\Delta\phi_2(n) = \pi - 2\xi$
1 0 0 1	-->	$\Delta\phi_2(n) = -\pi + 2\xi$
1 0 1 0	-->	$\Delta\phi_2(n) = 0$
1 1 0 0	-->	$\Delta\phi_2(n) = 0$

Table 4.3 Data patterns paired according to the resulting $\Delta\phi_2(n)$.

The angle $\Delta\phi_3(n)$, for the same type of channel, is given in table 4.4. Observe that every data pattern can be paired with a negative counterpart. This gives rise to a symmetrical eye diagram at the output of the three-bit delay detector, with the same eye opening as in the one-bit delay detector case.

Another aspect of the eye diagram that influences the error probability is the probability of occurrence of each eye level. For the single-bit delay detector, $w_n = a_n$ (equation (4.10)) and each of the eight eye levels have a probability of occurrence equal to $(0.5)^3 = 0.125$.

For the 2-bit and 3-bit delay detector, we can derive the occurrence probability of the different levels as given in tables 4.5 and 4.6 respectively.

Data pattern	$\Delta\phi_3(n)$
0 0 0 0 0	$\pi/2$
0 0 0 0 1	$\pi/2+\xi$
0 0 0 1 0	$-\pi/2+\xi$
0 0 0 1 1	$\pi/2$
0 0 1 0 0	$-\pi/2$
0 0 1 0 1	$-\pi/2+\xi$
0 0 1 1 0	$\pi/2-\xi$
0 0 1 1 1	$\pi/2$
0 1 0 0 0	$-\pi/2-\xi$
0 1 0 0 1	$-\pi/2$
0 1 0 1 0	$\pi/2-2\xi$
0 1 0 1 1	$\pi/2-\xi$
0 1 1 0 0	$\pi/2-\xi$
0 1 1 0 1	$\pi/2$
0 1 1 1 0	$-\pi/2-2\xi$
0 1 1 1 1	$-\pi/2-\xi$
1 0 0 0 0	$\pi/2+\xi$
1 0 0 0 1	$\pi/2+2\xi$
1 0 0 1 0	$-\pi/2$
1 0 0 1 1	$-\pi/2+\xi$
1 0 1 0 0	$-\pi/2+\xi$

1 0 1 0 1	$-\pi/2+2\xi$
1 0 1 1 0	$\pi/2$
1 0 1 1 1	$\pi/2+\xi$
1 1 0 0 0	$-\pi/2$
1 1 0 0 1	$-\pi/2+\xi$
1 1 0 1 0	$\pi/2-\xi$
1 1 0 1 1	$\pi/2$
1 1 1 0 0	$\pi/2$
1 1 1 0 1	$\pi/2+\xi$
1 1 1 1 0	$-\pi/2-\xi$
1 1 1 1 1	$-\pi/2$

Table 4.4 Data patterns and the corresponding phase shifts $\Delta\phi_3(n)$.

$$\begin{aligned}
 P[\cos\Delta\phi_2(n)=1] &= 4 \cdot (0.5)^4 = 1/4 \\
 P[\cos\Delta\phi_2(n)=-1] &= 2 \cdot (0.5)^4 = 1/8 \\
 P[\cos\Delta\phi_2(n)=\cos\xi] &= 4 \cdot (0.5)^4 = 1/4 \\
 P[\cos\Delta\phi_2(n)=\cos(\pi-\xi)] &= 4 \cdot (0.5)^4 = 1/4 \\
 P[\cos\Delta\phi_2(n)=\cos(\pi-2\xi)] &= 2 \cdot (0.5)^4 = 1/8
 \end{aligned}$$

Table 4.5 Probability of occurrence of each level of the 2-bit delay detector eye diagram.

$$\begin{aligned}
 P[-\sin\Delta\phi_3(n)=1] &= 6 \cdot (0.5)^5 = 3/16 \\
 P[-\sin\Delta\phi_3(n)=\sin(\pi/2+\xi)] &= 8 \cdot (0.5)^5 = 1/4,
 \end{aligned}$$

$$P[-\sin\Delta\phi_3(n) = \sin(\pi/2 + 2\xi)] = 2 \cdot (0.5)^5 = 1/16$$

$$P[-\sin\Delta\phi_3(n) = -\sin(\pi/2 + 2\xi)] = 2 \cdot (0.5)^5 = 1/16$$

$$P[-\sin\Delta\phi_3(n) = \sin(\pi/2 + \xi)] = 8 \cdot (0.5)^5 = 1/4$$

$$P[-\sin\Delta\phi_3(n) = -1] = 6 \cdot (0.5)^5 = 3/16$$

Table 4.6 Probability of occurrence of each level of the 3-bit delay detector eye diagram.

Note that the eye levels that are the most vulnerable are also the least frequent. This is especially true in the case of the three-bit delay detector.

From the above considerations, it is obvious that the detection threshold of zero, implied in equation (4.9), is not optimum for the two-bit delay detector. This sub-optimum value is nevertheless assumed in the following derivations in order to ease the calculations.

Generally, the average error probability P_e at the output of a given detector is

$$P_e = 1/2 E[P_1(n)] + 1/2 E[P_{-1}(n)] \quad (4.13)$$

where $E[\cdot]$ denotes the expected value. Stein derived the following expression for $P_1(n)$

$$P_1(n) = P\left[\operatorname{Re}(z_n z_{Dn}^*) < 0 \mid w_{n-1} \right] = 1/2 \left[1 - Q(\sqrt{b}, \sqrt{a}) + Q(\sqrt{a}, \sqrt{b}) \right] - A/2 \exp[-1/2 (a+b)] I_0(\sqrt{ab}) \quad (4.14)$$

where $Q(x,y)$ is Marcum's Q -function given by

$$Q(x,y) = \int_y^\infty \exp \left\{ -\left[\frac{x^2+t^2}{2} \right] \right\} I_0(xt) t dt \quad (4.15)$$

and I_0 is the modified zero-order Bessel function of the first kind, as given in equation (D.36). In appendix A, it is shown that for MSK, a and b are given by

$$\begin{aligned} \begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{0.25}{[K_{11}K_{22} - \text{Im}^2(K_{12})]} \left\{ \begin{aligned} &K_{22}|c_1|^2 + K_{11}|c_2|^2 \\ &\mp 2\sqrt{K_{11}K_{22} - \text{Im}^2(K_{12})} \text{Re}(c_1 c_2^*) + 2\text{Im}(K_{12})\text{Im}(c_1 c_2^*) \end{aligned} \right\} \end{aligned} \quad (4.16)$$

with the upper sign giving a and the lower sign giving b . The parameter A is given by (appendix A)

$$A = \frac{\text{Re}(K_{12})}{\sqrt{K_{11}K_{22} - \text{Im}^2(K_{12})}} \quad (4.17)$$

In these last two equations, c_1 and c_2 are the signal components of z_n and z_{Dn} given in equation (4.8). For the i^{th} detector, c_1 is given by

$$c_1 = s(t) * h_1(t) |_{t=t_0} \quad (4.18a)$$

and c_2 is given by

$$\begin{aligned}
 c_2 &= [s(t) * h_1(t)] e^{j\pi/2} |_{t=t_0-T} \quad ; 1\text{-bit delay} \\
 c_2 &= s(t) * h_2(t) |_{t=t_0-2T} \quad ; 2\text{-bit delay} \\
 c_2 &= [s(t) * h_3(t)] e^{-j\pi/2} |_{t=t_0-3T} \quad ; 3\text{-bit delay}
 \end{aligned}
 \tag{4.18b}$$

K_{pm} is the covariance of the random components of $z_1(t_0)$ and $z_1(t_0-iT)$, i.e.

$$\begin{aligned}
 K_{11} &= 1/2 E[|z_1(t_0) - c_1|^2] \\
 &= 1/2 E[|u(t) * h_1(t)|^2] \quad t=t_0 \\
 &= \phi_{nn}(0)
 \end{aligned}
 \tag{4.19}$$

where

$$n(t) = u(t) * h_1(t) \tag{4.20}$$

and

$$\phi_{nn}(r) = 1/2 E[n(t+r)n^*(t)] \tag{4.21}$$

In term of power spectral densities, the following expression is obtained

$$\Phi_{nn}(f) = \int_{-\infty}^{\infty} \phi_{nn}(\tau) e^{-j2\pi f\tau} d\tau = \Phi_{uu}(f) |H_1(f)|^2 \quad (4.22)$$

where $\Phi_{uu}(f)$ is the baseband noise power spectral density, i.e. $\Phi_{uu}(f) = N_0$ W/Hz. Thus K_{11} is given by

$$\begin{aligned} K_{11} = \phi_{nn}(0) &= \int_{-\infty}^{\infty} N_0 |H_1(f)|^2 df \\ &= N_0 B_{ni} = N_0 \rho_1(0) \end{aligned} \quad (4.23)$$

where B_{ni} is the double-sided noise bandwidth of the equivalent lowpass filter and $\rho_1(\tau)$ is the system autocorrelation [35] of this filter, given by

$$\rho_1(\tau) = \int_{-\infty}^{\infty} h_1(t+\tau) h_1^*(t) dt. \quad (4.24)$$

Similarly,

$$\begin{aligned} K_{22} &= 1/2 E[|z_1(t_0 - iT) - c_2|^2] \\ &= 1/2 E[|n(t_0 - iT)|^2] \\ &= N_0 B_{ni} = N_0 \rho_1(0) \end{aligned} \quad (4.25)$$

because of the stationarity of the noise process. Finally

$$K_{12} = K_{21}^* = 1/2 E[n(t_0) n^*(t_0 - iT)]$$

$$\begin{aligned}
&= \phi_{nn}(iT) \\
&= N_0 \int_{-\infty}^{\infty} |H_1(f)|^2 e^{j2\pi f iT} df \\
&= N_0 \rho_1(iT)
\end{aligned} \tag{4.26}$$

where use was made of the fact that the Fourier transform of $\rho_1(\tau)$ is $|H_1(f)|^2$. It is assumed that $H_1(f) = H_1^*(-f)$ and therefore $\rho_1(\tau)$ is real, as well as K_{11} , K_{22} , K_{12} and K_{21} . Furthermore, in equation (4.16), $\text{Re}[c_1 c_2^*] = \sin \Delta \phi_1(n)$, $\cos \Delta \phi_2(n)$ or $-\sin \Delta \phi_3(n)$ for the one-bit, two-bit or three-bit delay detector respectively.

Using equations (4.23), (4.25) and (4.26) in the expressions for a , b and A ,

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{|c_1|^2 + |c_2|^2}{4N_0 \rho_1(0)} \mp \frac{1}{2N_0 \rho_1(0)} \text{Re}[c_1 c_2^*] \tag{4.27}$$

$$A = \frac{\rho_1(iT)}{\rho_1(0)} \tag{4.28}$$

Define $P(a, b)$ as

$$P(a, b) = 1/2 [1 - Q(\sqrt{b}, \sqrt{a}) + Q(\sqrt{a}, \sqrt{b})]. \tag{4.29}$$

Stein [38] has given a set of curves for $P(a, b)$ that is reproduced in

figure 4.4. $P_1(n)$ now reduces to

$$P_1(n) = P(a,b) = \frac{\rho_1(iT)}{\rho_1(0)} \exp[-1/2 (a+b)] I_0(\sqrt{ab}) \quad (4.30)$$

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{|c_1|^2 + |c_2|^2}{4N_0\rho_1(0)} \mp \frac{\operatorname{Re}\{c_1 c_2^*\}}{2N_0\rho_1(0)}$$

From equations (4.8) and (4.11),

$$P_{-1}(n) = P[\operatorname{Re}(z_n z_{Dn}^*) > 0 | w_n = -1] \quad (4.31)$$

This equation can be transformed into

$$P_{-1}(n) = P[\operatorname{Re}(-z_n z_{Dn}^*) < 0 | w_n = -1] \quad (4.32)$$

which is in the same form as $P_1(n)$. The equations previously introduced can be used by making the following changes

$$c_1' = -c_1 \quad (4.33)$$

$$\begin{aligned} K_{12}' &= 1/2 E[-n(t_0) n^*(t_0 - iT)] \\ &= -K_{12} \end{aligned}$$

Note that c_2 , K_{11} and K_{22} remain the same. Equations (4.27) and (4.28) now become

Figure 4.4 $P(a,b) = 1/2 [1 - Q(\sqrt{b}, \sqrt{a}) + Q(\sqrt{a}, \sqrt{b})]$.

$$\begin{Bmatrix} a' \\ b' \end{Bmatrix} = \frac{|c_1|^2 + |c_2|^2}{4N_0\rho_1(0)} \pm \frac{\operatorname{Re}(c_1 c_2^*)}{2N_0\rho_1(0)} - \begin{Bmatrix} b \\ a \end{Bmatrix}$$

(4.34)

$$A' = \frac{-\rho_1(iT)}{\rho_1(0)} = -A$$

and consequently

$$P_{-1}(n) = P(a', b') + \frac{\rho_1(iT)}{\rho_1(0)} \exp[1/2(a' + b')] I_0(\sqrt{a'b'}) \quad (4.35)$$

Gathering equations (4.13), (4.29), (4.30), (4.34) and (4.35), the average error probability at the output of the i^{th} detector is

$$Pe_i = 1/2 E[P_1(n)] + 1/2 E[P_{-1}(n)]$$

where

$$\begin{aligned} P_1(n) &= P(a, b) - \frac{\rho_1(iT)}{\rho_1(0)} \exp[-1/2(a+b)] I_0(\sqrt{ab}) \\ P_{-1}(n) &= P(b, a) + \frac{\rho_1(iT)}{\rho_1(0)} \exp[-1/2(a+b)] I_0(\sqrt{ab}) \end{aligned} \quad (4.36)$$

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{|c_1|^2 + |c_2|^2}{4N_0\rho_1(0)} \mp \frac{\operatorname{Re}(c_1 c_2^*)}{2N_0\rho_1(0)}$$

From these equations, the following conclusions about the receive filters are obtained:

There is no common filter that minimizes the error probability for

each detector.

The parameters a , b and the correlation coefficient $r = \rho_i(iT)/\rho_i(0)$ are linked together by the impulse response $h_i(t)$. Thus, they cannot be optimized independently.

The effect of the filter autocorrelation is opposite in $P_1(n)$ and $P_{-1}(n)$, i.e. a positive noise correlation is beneficial when $w_n = 1$ and a negative one is better when $w_n = -1$.

Since the term $\text{Re}(c_1 c_2^*)$ represents the sampled noiseless eye diagram, a bigger eye aperture reduces the error probability (surprise!). —

In the two-bit and three-bit delay detectors, since the most vulnerable eye levels are less likely, (which is not the case in the single-bit delay detector) the average error probability is reduced compared to the single-bit average, even when the eye aperture is similar in all cases.

The minimization of P_{e_i} given in equation (4.36), with respect to the i^{th} receive filter, is a fairly complex nonlinear programming problem, even without giving any considerations to the feedback decoding process. It would be of interest to derive the optimum filters characteristics, but it is felt at this point that this

exercise is not essential to the understanding of the phase feedback decoding process. Equation (4.36) will be used in the next sections to explain the behaviour of the differential detectors under certain conditions.

In the next section, the feedback decoding process is considered in detail.

4.1.2 RX FILTERS CONSIDERATIONS WITH FEEDBACK DECODING

It was established in section 3.2.2 that an M-branch phase feedback decoder corrects all patterns with M-1 or less errors within the memory span of the decoder. Ideally, the channel should only introduce this type of error pattern and not any patterns with M or more errors. A fundamental question to ask at this point is: "How do one or many receive filters influence the error patterns generation, which in turn influences the decoder performance?" The present section is devoted to this question.

The attention is focused initially on the two-branch decoder, since it is easier to get an intuitive understanding of its behaviour. The results that are obtained can be extended to three-branch decoding. It was previously established that the error pattern $[e_{n-1}^2 \ e_n^2 \ e_{n-1}^1 \ e_n^1]$ determines the performance of the

two-branch decoder. In noiseless conditions, the signal phasor is rotated a specific angle during each bit period. The noise added on the signal affects the phasor rotation by an additional random phase rotation. Hereafter, this random phase added to the phasor at the i^{th} bit transition is labelled z_i . The conditions for individual errors are:

$$e_n^1 = 1 \quad \text{if } -\pi < \Delta\phi_1(n) + z_n - z_{n-1} < 0, \quad \text{when } \Delta\phi_1(n) > 0$$

$$0 < \Delta\phi_1(n) + z_n - z_{n-1} < \pi, \quad \text{when } \Delta\phi_1(n) < 0$$

$$e_{n-1}^1 = 1 \quad \text{if } -\pi < \Delta\phi_1(n-1) + z_{n-1} - z_{n-2} < 0, \quad \text{when } \Delta\phi_1(n-1) > 0$$

$$0 < \Delta\phi_1(n-1) + z_{n-1} - z_{n-2} < \pi, \quad \text{when } \Delta\phi_1(n-1) < 0$$

$$e_n^2 = 1 \quad \text{if } -\pi/2 < \Delta\phi_2(n) + z_n - z_{n-2} < \pi/2, \quad \text{when } \pi/2 < \Delta\phi_2(n) < 3\pi/2$$

$$\pi/2 < \Delta\phi_2(n) + z_n - z_{n-2} < 3\pi/2, \quad \text{when } -\pi/2 < \Delta\phi_2(n) < \pi/2$$

$$e_{n-1}^2 = 1 \quad \text{if } -\pi/2 < \Delta\phi_2(n-1) + z_{n-1} - z_{n-3} < \pi/2, \quad \text{when } \pi/2 < \Delta\phi_2(n-1) < 3\pi/2$$

$$\pi/2 < \Delta\phi_2(n-1) + z_{n-1} - z_{n-3} < 3\pi/2, \quad \text{when } -\pi/2 < \Delta\phi_2(n-1) < \pi/2$$

where $\Delta\phi_i(k)$ represents the phase shift from the $(k-1)^{\text{th}}$ bit interval to the k^{th} bit interval.

Without loss of generality, assume that $\Delta\phi_1(n) > 0$. If $e_n^1 = 1$, then

$$\Delta\phi_1(n)+z_n < z_{n-1} < \Delta\phi_1(n)+z_n+\pi. \quad (4.37)$$

This possibility is represented by an example in figure 4.5. Inter-symbol interference is accounted for by the fact that $\Delta\phi_1(n)$ is smaller than $\pi/2$. A large noise sample z_{n-1} produces, in this example, an error $e_n^1=1$. Adding $\Delta\phi_1(n-1)-z_{n-2}$ to each term of equation (4.37), we get

$$\begin{aligned} \Delta\phi_1(n)+\Delta\phi_1(n-1)+z_n-z_{n-2} &< \Delta\phi_1(n-1)+z_{n-1}-z_{n-2} < \\ &\Delta\phi_1(n)+\Delta\phi_1(n-1)+z_n-z_{n-2}+\pi \end{aligned} \quad (4.38)$$

which gives the conditions for $e_{n-1}^1=1$. If it is assumed that the noise correlation between sampling times is not too high, then the situation of figure 4.5 is typical of a noise event where an important noise sample perturbs greatly one phasor while leaving the adjacent ones only slightly affected.

Since the complex noise samples are jointly Gaussian with a low correlation, they are close to being independent. Therefore, when $|z_{n-1}|$ is important, $|z_n|$ and $|z_{n-2}|$ are small with high probability. The event $(\Delta\phi_1(n-1)+z_{n-1}-z_{n-2}) > \pi$ is illustrated in figure 4.6(a), which corresponds to $e_{n-1}^1=1$. Figure 4.6(b) illustrates $e_{n-1}^1=0$. This figure shows that when z_{n-1} produces an error, the event $e_{n-1}^1=1$ exhibits a probability similar to the probability of the event $e_{n-1}^1=0$ because the phasor transmitted at time $(n-2)T$ is subject to

Figure 4.5 Complex phase plane for e_n^{1-1} caused by an important z_{n-1} .

Figure 4.6 Complex plane for $e^{j\omega(n-1)T} = 1$ caused by an important z_{n-1} and (a) $e^{j\omega(n-1)T} = 1$
(b) $e^{j\omega(n-1)T} = 0$.

Figure 4.7 Complex plane for $e^{j\omega n T} = 1$ caused by an important z_n

noise rotations approximately equally likely to cause or not an error on y_{n-1}^1 . For $\Delta\phi_1(n-1)$ lower than zero, the same trend to produce $e_{n-1}^1=1$, when an error in y_n^1 is produced by an important noise sample z_{n-1} , shows up. The errors at the output of the one-bit delay differential detector are therefore distributed mainly between single and double error patterns, in approximately the same proportion.

But if the error is produced by z_n , the situation is different. As shown in figure 4.7, e_{n-1}^1 is unlikely to be one if the noise correlation from one bit to the other is small. Thus a single error occurs in y_n^1 and y_{n-1}^1 .

This single error event ($e_n^1=1, e_{n-1}^1=0$) is more likely to happen than the double error event ($e_n^1=1, e_{n-1}^1=1$) since a non-negligible proportion of the events ($e_n^1=1$ caused by an important z_{n-1}) do not produce any error in y_{n-1}^1 while a very small portion of the event ($e_n^1=1$ caused by an important z_n) produces an error in y_{n-1}^1 . Thus the following result is obtained:

When the receive filter is such that the noise correlation between adjacent bits is small, the event ($e_n^1=1, e_{n-1}^1=0$) has a higher probability of occurrence than the event ($e_n^1=1, e_{n-1}^1=1$).

Obviously, a large noise sample z_{n-2} does not affect y_n^1 and, in that case, the event ($e_n^1=0, e_{n-1}^1=1$) has a probability similar to the

probability of the event $P(e_n^1=1, e_{n-1}^1=0)$.

Following the same arguments, some information about the probability of the whole error pattern $[e_{n-1}^2, e_n^2, e_{n-1}^1, e_n^1]$ can be deduced. Consider the noise-free example of a possible complex plane given in figure 4.8. Note that z_n affects $\Delta\phi_1(n)$ (or y_n^1) and $\Delta\phi_2(n)$ (y_n^2) but not $\Delta\phi_1(n-1)$ (y_{n-1}^1) and $\Delta\phi_2(n)$ (y_{n-1}^2). The most probable error patterns for a large isolated noise sample z_n are therefore (0 0 0 1), (0 1 0 0) and (0 1 0 1).

A large isolated noise sample z_{n-1} influences y_n^1 , y_{n-1}^1 and y_{n-1}^2 . The most probable error patterns in this case are (0 0 0 1), (0 0 1 0), (1 0 0 0), (0 0 1 1), (1 0 0 1), (1 0 1 0) and (1 0 1 1).

On the other hand, z_{n-2} affects y_{n-1}^1 and y_n^2 which increases the likelihood of the patterns (0 0 1 0), (0 1 0 0) and (0 1 1 0). Finally, z_{n-3} is mainly responsible for the error pattern (1 0 0 0) since y_{n-1}^2 is affected by this noise sample.

By compiling the likelihood of the different error patterns, the typical pattern distribution of figure 4.9 is obtained, where the patterns are labelled according to table 3.4. Note that in figure 4.9, only the form of the density is important, not the absolute probabilities.

Figure 4.8 Noise-free phase plane for the transmitted data "0 1 1 1 0".

Figure 4.9 Typical error pattern probability density for uncorrelated noise samples.

It is interesting to compare this conclusion to the density obtained by simulation. Consider a phase equalized 4th order Butterworth filter with a BT product of 1.1 as proposed in [18] (B is the filter double-sided 3-dB bandwidth and T is the inverse of the bit rate). The filter frequency response is shown in figure 4.10(a) while its system autocorrelation appears in figure 4.10(b). Note that the condition of low correlation from one bit to the other is met with this filter.

Figure 4.11 gives the error pattern density for both a two-branch and a three-branch detector, at $E_b/N_0 = 6$ dB. The simulated two-branch density is quite similar to the density derived intuitively in figure 4.9. This supports the geometric technique that was adopted to study the problem.

In figure 4.11(b), the error patterns are labelled according to their decimal representation. The patterns that are powers of two correspond to single-error patterns. Note that these ones are the most probable, which tends to show that the three-branch decoder will exhibit a net gain over the single-bit delay differential detector.

Figure 4.12 shows the simulated 2-branch error pattern densities for the same filter at $E_b/N_0 = 8, 10$ and 12 dB. The single-error patterns are still dominant, but note that patterns (0 0 0 1) and (0 0 1 0) become more and more important as the noise level

(a)

(b)

Figure 4.10 Phase equalized 4th order Butterworth filter
with $BT \approx 1.1$ (a) Frequency response
(b) System autocorrelation.

(a)

(b)

Figure 4.11 Error pattern density on an AGN channel at $E_b/N_0 = 6$ dB with the RX filter characteristics of figure 4.10 (a) 2-branch (b) 3-branch.

decreases. This is the case because as the signal-to-noise ratio increases, the intersymbol interference becomes more dominant. Since the output of the single-bit delay detector is more subject to errors than the output of the two-bit and three-bit delay detectors when ISI is present, it follows that more errors are produced in the single-bit delay detector, as can be seen in figure 4.12. A similar effect occurs for three-branch densities as will be shown in the next section.

When the receive filter bandwidth is decreased, intersymbol interference increases as well as the noise correlation between adjacent bits, while the average noise power present at the detector output decreases. The dominant phenomenon is the ISI increase. Its effect is to increase the density of each error pattern, particularly the (0 0 0 1) and (0 0 1 0) patterns. This latter phenomenon is due to the quick reduction of $\Delta\phi_1(n)$ and $\Delta\phi_1(n-1)$ as the filter bandwidth is reduced. Figure 4.13 shows the error pattern densities for the filter of figure 4.10 when $BT=0.9$ and $E_b/N_o=8$ dB. The high level of single errors and the low occurrence probability of multiple-error patterns indicate that feedback decoding is effective in this reduced bandwidth system. In fact, feedback decoding appears to provide an easy way to recover part of the loss in the single-bit detector incurred by filtering at the transmitter, since this loss is dominated by intersymbol interference.

(a)

(b)

(c)

Figure 4.12 Two-branch error pattern density on an AGN channel for the RX filter of figure 4.10 at
 (a) $E_b/N_0 = -8\text{dB}$
 (b) $E_b/N_0 = -10\text{ dB}$
 (c) $E_b/N_0 = -12\text{ dB}$.

Figure 4.13 Error pattern density on an AGN channel at $E_b/N_0 = 8$ dB with a BT-0.9 phase equalized 4th order Butterworth receive filter for (a) 2-branch (b) 3-branch.

The increase in the noise correlation has more impact on the first order detector since the system autocorrelation $\rho(r)$ at $r=2T$ and $3T$ remains low, even when $\rho(T)$ is positive and fairly important. Therefore, any possible detrimental effect of noise correlation in the first detector is in part reduced by phase feedback decoding.

These considerations imply that there is an optimum BT product, for a specific filter, that trades off noise power with noise correlation and ISI to produce an error pattern density where single error patterns are more probable than multiple error patterns.

The answer to the question proposed at the beginning of this section therefore is:

The receive filter is very likely to produce single-error patterns, no matter its bandwidth, which make 2-branch phase feedback decoding a desirable detection technique for most filters. By increasing the span of the decoder memory, further performance improvements are possible, but the biggest gain is obtained by using only two differential detectors. There is undoubtedly an optimum filter that minimizes the occurrence probability of multiple-error patterns which cannot be handled correctly by the phase feedback decoder. Most likely, as equations (4.36) suggest, the optimum filtering strategy consists in a different receive filter, in front of each differential detector, that jointly minimizes the

error probabilities at the output of each detector and thus produces an optimum error pattern density.

4.1.3 SIMULATION RESULTS

In this section, exhaustive simulation results are given for the AGN channel for both 2-branch and 3-branch decoding. Section 4.1.3.1, deals with 2-branch decoding where the analysis procedure developed in section 3.3.2 is applied. In section 4.1.3.2, 3-branch decoding is discussed. In all cases, the phase equalized 4th order Butterworth filter with BT=1.1 is assumed (figure 4.10).

Figure 4.14 displays the simulated performance for each individual differential detectors. As noticed theoretically in section 4.1.1, the two-bit and three-bit delay detectors perform better than the single-delay detector on that type of channel. A gain of 0.7 dB at $BER=10^{-4}$ is possible by using a two bit or a three bit delay detector. The performance of the double and triple-bit delay detectors are almost identical, which indicates that the most vulnerable eye diagram levels have a probability of occurrence low enough to exhibit a net gain over the uniformly distributed eye levels of the single-bit delay detector.

Figure 4.14 Simulated performance of the differential detectors (a) single-bit delay (b) double-bit delay (c) triple-bit delay (d) DPSK.

4.1.3.1 TWO-BRANCH DECODING

The error pattern discrete probability density function for $E_b/N_0 = 6, 8, 10$ and 12 dB is given in figures 4.11 and 4.12 of section 4.1.2. When the signal-to-noise ratio increases, the single-error patterns become dominant because the double-bit and triple-bit delay detectors are less susceptible to ISI. Note also that the error patterns with a decimal equivalent greater or equal to 12 are very unlikely, as was predicted in figure 4.9. These patterns effectively correspond to double errors in the two-bit delay detector (i.e. $e_{n-1}^2 = e_n^2 - 1$), which cannot be caused by an important isolated noise sample.

The error pattern density is used to compute the error probability at the output of the 2-branch feedback decoder in conjunction with the Markov model developed in section 3.3.2. The error rates are given by the triangles in figure 4.15. The values computed with this method are very close to the curve of figure 4.15(c), which is obtained by the Monte-Carlo simulation. Note, in the same figure, the improvement of two-branch decoding over both one-bit and two-bit detection.

Equation (3.35) is used to compute the first error probability. Figure 4.15(d) shows that P_{fe} effectively gives a reasonably tight lower bound for the 2-branch decoder error probability.

Figure 4.15 Simulated performance

- (a) output of one-bit detector
- (b) output of double-bit detector
- (c) 2-branch feedback decoder
- (d) first error probability P_{Fe}
- ▲ : Markov model.

Consequently, equation (3.35) can be used to investigate the critical error patterns for different types of channels. For the AGN channel with the receive filter of figure 4.10, the patterns 3, 6, 9, 10 and 11 are the ones that contribute the most to P_{fe} . Any receive filter or combination of transmit-receive filters that affects these patterns also changes the performance of the decoder.

4.1.3.2 THREE-BRANCH DECODING

The 9-variable error pattern discrete probability density function is given for $E_b/N_0=6$ dB in figure 4.11(b) and for $E_b/N_0=8, 10$ and 12 dB in figure 4.16. The abscissa refers to the decimal equivalent of the error pattern $[e_{n-2}^3 e_{n-1}^3 e_n^3 e_{n-2}^2 e_{n-1}^2 e_n^2 e_{n-2}^1 e_{n-1}^1 e_n^1]$ and it is again assumed that the receive filter has the frequency response shown in figure 4.10.

As with the two-branch density, single-error patterns are predominant because of the low noise correlation from one bit to the other. The errors in the single-bit detector are dominant, mainly at high E_b/N_0 (patterns 1 to 4).

Note that for the particular channel considered, patterns higher than 128 are very unlikely, with the exception of the single-error pattern 256. These unlikely patterns correspond to multiple-errors

4.16 (a)

(c)

Figure 4.16 Three-branch error pattern density on an AGN channel for the RX filter of figure 4.10 at (a) $E_b/N_0 = 8$ dB
 (b) $E_b/N_0 = 10$ dB
 (c) $E_b/N_0 = 12$ dB.

in the three-bit delay detector, which are not frequent because of the low correlation between noise samples that are three bits apart.

Since the multiple-error patterns have a low probability of occurrence, it seems reasonable to conjecture that a three-branch phase feedback decoder will perform better than a single-bit delay differential detector on an AGN channel. Furthermore, since it can correct any double-error pattern, which are the most frequent patterns after the single-error ones (note the cluster on the right hand side of each single-error pattern) it is also reasonable to assume that a three-branch decoder that looks only for the patterns given in table 3.2 will perform better than a two-branch decoder. In order to confirm these ideas, the patterns causing an error for the first time are searched by computer. These patterns are shown in figure 4.16. The sum of these pattern probabilities correspond to P_{fe} , the first error probability. This probability is given in figure 4.17, along with the two-branch first error probability.

An improvement of 0.5 dB is obtained with three-branch decoding at an error rate of 10^{-4} , which tends to confirm the conjecture. This is finally confirmed by true simulation results given in figure 4.18.

Note again that the critical error patterns, for the three-branch decoder considered in this section, are those emphasized

Figure 4.17 First error probability for
(a) 2-branch decoding
(b) 3-branch decoding

Figure 4.18 Simulated performance
 (a) output of one-bit detector
 (b) 2-branch feedback decoder
 (c) 3-branch feedback decoder.

in figure 4.16. If a channel produces more errors in these areas, the performance of the detector is expected to deteriorate.

4.2 FADING MULTIPATH CHANNEL

When a narrowband signal such as MSK is transmitted through a multipath fading channel, then the received signal will, in general, consist of a direct component and a summation of scattered components with random amplitudes and time delays. Assuming a flat (nonfrequency selective) fading model, then the direct component is an amplitude scaled version of the transmitted MSK signal and the sum of the scattered components are modelled by a single diffused component of the transmitted signal with Rayleigh distributed amplitude and uniformly distributed phase.

The channel is thus modeled as a Rician fading one, as shown in figure 4.19. This model assumes that for the bit rate under consideration (2400 bps), the delay of the fading path is negligible relative to the bit period. The bandwidth of the Rayleigh fading process is assumed to be approximately 100 Hz and the normalized spectral shaping filter $H(f)$ is described, in the analog domain, by [39]

$$H(s) = \frac{1}{(0.897s^2 + 0.31s + 1)(1.543s^2 + 0.841s + 1)(1.944s + 1)} \quad (4.39)$$

Figure 4.19 Fading multipath channel model.

The magnitude response of this filter is shown in figure 4.20. It is an approximation of the fading spectral density derived in [40] and given by the expression

$$W(f) = \begin{cases} \frac{k}{\pi \sqrt{f_d^2 - f^2}} & |f| \leq f_d \\ 0 & |f| > f_d \end{cases} \quad (4.40)$$

where k is the variance of the multipath gain and f_d is the maximum Doppler frequency.

The fading channel simulation follows the model of figure 4.19. $H(f)$ is approximated by an FIR filter with a one-sided bandwidth equal to 100 Hz.

The Rician channel is characterized by its K-factor. Let the received signal described by

$$r(t) = s(t) + A(t)e^{j\theta(t)}s(t) + u(t) \quad (4.40a)$$

where $s(t)$ is the transmitted signal, $Ae^{j\theta(t)}$ is the complex gain associated with the Rayleigh fading path and $u(t)$ represents the

Figure 4.20 Magnitude of the multipath shaping filter.

additive white Gaussian noise. Then assuming that the fading process is independent of the signal, the ratio of the fading path to direct path power is given by

$$K = 10 \log \left[\frac{E[|A(t)e^{j\theta(t)}s(t)|^2]}{E[|s(t)|^2]} \right] = 10 \log E[A^2(t)] \quad (4.41)$$

Note that the energy per bit E_b used in the following sections does not include the energy from the reflected path. This allows direct comparisons between the performance with and without fading.

4.2.1 SIMULATION RESULTS

In this section, it is assumed again that the receive filter is a phase equalized 4th order Butterworth filter with $BT=1.1$, whose characteristics are given in figure 4.10.

Figure 4.21 gives the simulated performance of each individual detectors for different K-factors. The set of curves labelled $K=-\infty$ is for a static AGN channel and corresponds to the curves of figure 4.14. For moderate fading ($K=-10$ dB) and low E_b/N_0 , the double and triple-delay detectors still perform better than the single-bit delay detector. The improvement is however marginal and for $K=-5$ dB and $K=0$ dB, the single-bit delay detector gives the best results, followed by the double-bit and triple-bit detectors.

Figure 4.21 Simulated performance on a Rician fading channel for: $\triangle$ or Δ : single-bit delay
 $\blacksquare$ or $\square$: double-bit delay
 $\bullet$ or $\circ$: triple-bit delay.

This loss of performance in the detectors with longer delays is the result of the overall gain applied on the signal by the channel. From equation (4.40a), this gain is

$$G = 1 + A(\tau) e^{j\theta(\tau)} \quad (4.42)$$

When the delay between the phase comparisons ~~is increased~~, the correlation between the samples decreases. This indicates that the phase shifts experienced by the samples further and further apart are likely to be different and therefore likely to produce wrong phase differences, especially at high K-factor. The faster the fading is, the more probable this effect becomes. This tends to discredit differential detection with long delay over a severe fading multipath channel. Note also that the triple-bit delay tends more rapidly toward an irreducible error rate, especially at K=0 dB. Of interest is also the cross-over of the single-bit delay curve by the triple-bit delay curve at K=-10 dB.

4.2.1.1 TWO-BRANCH DECODING

The error pattern density for the Rician channel is shown in figure 4.22 for $E_b/N_0=8$ dB. In comparing this figure with figure 4.12(a), one immediately notices an increase in the average level of occurrence of the error patterns. Pattern 2 is the most probable for the static channel with an occurrence of 4.6×10^{-3} . For $K=-10$ dB, pattern 8 dominates with a probability of 7.7×10^{-3} . For the moderate fading channel ($K=-10$ dB), the single-error patterns (1, 2, 4 and 8) are still the most probable although patterns 1 and 2 are no longer dominant. Patterns 5 and 6, corresponding to $e_{n-1}^2=1$ and either e_n^1 or $e_{n-1}^1=1$, also occur more often. The shifts in the error pattern density function becomes more prevalent as the multipath power is increased. At $K=0$ dB, pattern 8 is clearly dominant, followed by patterns 4, 5 and 6. Furthermore, the multiple-error patterns also occur more frequently.

The increase in the occurrence of patterns 5, 6, 9 to 14 and a decrease of patterns 1 to 3 corresponds to the performance reversal observed in figure 4.21 between the single and the double-delay detectors. The high probability of pattern 8, which correspond to $e_{n-1}^2=1$, is the direct consequence of the high error rate in the double-delay detector.

Figure 4.22 Two-branch error pattern density on a Rician channel at $E_b/N_0 = 8$ dB for (a) $K = -10$ dB
 (b) $K = -5$ dB
 (c) $K = 0$ dB.

Figure 4.23 gives the 2-branch decoder simulated performance, along with the best single detector performance for each K-factors. That is, at K=0 dB and K=-5 dB, the single-bit detector performance is used for comparison, while at K=-10 dB, the double-bit detector is considered. For K=-10 dB, a marginal improvement with 2-branch feedback decoding is noted. For K=0 dB, the feedback decoder performance crosses over the single-bit detector performance at $E_b/N_0 \approx 16$ dB due to the increase of the relative probability of patterns 10, 12 and 14 and the decrease of the occurrence of patterns 1 to 3. This behaviour is illustrated in figure 4.24 and corresponds to the decreasing efficiency of the double-bit delay detector, compared to the single-bit delay detector.

At the end of section 4.1.3.1, it was concluded that any channel that affects the error patterns would also affect the decoder performance. The modification of the pattern density by the fading channel provides a good example of this phenomenon. In the next section, the three-branch pattern density for the same type of channel is considered.

Figure 4.23 Simulated performance on a Rician channel
 for ----- : 2-branch feedback decoding
 —•— : the best differential detector.

(a)

(b)

Figure 4.24 Two-branch error pattern density on a Rician channel at $K=0$ dB for (a) $E_b/N_0 = -16$ dB (b) $E_b/N_0 = -18$ dB.

4.2.1.2 THREE-BRANCH DECODING

The three-branch probability density function is given at E_b/N_0 -8 dB and E_b/N_0 -16 dB in figures 4.25, 4.26 and 4.27 for K -10 dB, -5 dB and 0 dB respectively. Note that, as in the two-branch density case, there is a shift from the low-value patterns to the high-value ones as K increases. Multiple-error patterns occur more frequently as the multipath power increases. This general behaviour, along with the poor performance of the three-bit delay differential detector under fading conditions, suggests that three-branch decoding can only achieve a small improvement over two-branch decoding.

A simple comparison is done using the first error probabilities in order to verify these observations. The results are given in figure 4.28. Note that a small improvement is obtained with three-branch decoding at K -10 dB, but virtually no improvement and even a worse performance is obtained at K -5 dB and K -0 dB. These trends are confirmed with a full simulation of the decoders, the results of which are given in figure 4.29.

Figure 4.25 Three-branch error pattern density on a Rician channel for $K=10$ dB at
 (a) $E_b/N_0=8$ dB
 (b) $E_b/N_0=16$ dB.

Figure 4.27 Three-branch error pattern density on a Rician channel for $K=0$ dB at
 (a) $E_b/N_0 = -8$ dB
 (b) $E_b/N_0 = -16$ dB

Figure 4.28 First error probability on a Rician channel
for —▲— 2-branch decoding
-----○----- 3-branch decoding.

Figure 4.29 Simulated performance on a Rician channel
 for —▲— 2-branch decoding
 - - -○- - - 3-branch decoding.

4.3 CONCLUSION

In this chapter, the performance of three levels of differential detection was considered, both theoretically and by simulation. A specific receive filter was generally assumed, i.e. the filter whose characteristics are given in figure 4.10. It was found that on a static AGN channel, differential detection with delays of two or three bits performs almost identically and shows an improvement over differential detection with a single-bit delay (figure 4.14). The behaviour of the eye diagrams at the output of the three types of differential detectors was considered in details. On a slow fading Rician multipath channel, it was found that the best detector is not the one with the longest delay, even with moderate multipath power ($K = -10$ dB). At $K = -5$ dB and $K = 0$ dB, the double-bit and triple-bit delay detectors perform poorly compared with a differential detector with a single-bit delay (figure 4.21).

On the AGN channel, phase feedback decoding was shown to perform better than a single order of differential detection. The improvement is 1.7 dB at $BER = 10^{-4}$ for the three-branch feedback decoder over single-bit delay detection (figure 4.18). On the slow fading Rician multipath channel, phase feedback decoding provides at best a marginal gain and even degrades the overall performance at high E_b/N_0 for $K = -5$ dB and $K = 0$ dB (figures 4.23 and 4.29).

The error pattern probability densities were given for the different channel conditions considered. On the AGN channel, phasor diagrams were used to show the general behaviour of the two-branch density. It was deduced that the single-error patterns were dominant and that phase feedback decoding would provide a gain with most types of filters. For the fading channel considered, it was argued that the alteration of the error pattern density was responsible for the poor performance of the phase feedback decoder.

In conclusion, all the results that were obtained indicate that phase feedback decoding is beneficial on a static AGN channel, but that it can cause detrimental effects on other types of channels. It seems clear that the so-called "ISI control" or "ISI reduction" properties of the phase feedback decoder are nothing more than the careful exploitation of the relative independence of the error events in different levels of differential detection.

CHAPTER 5

APPLICATION TO OTHER BINARY CPM SIGNALS

In this chapter, the application of phase feedback decoding to binary CPM signals other than MSK is briefly discussed. The phase properties of CPM signals are considered as well as the suitability of phase feedback decoding as a post-detection enhancement technique.

5.1 FULL RESPONSE SIGNALS

It was shown in section 2.4 how the differential phase variables of a general full response signal could be used for its demodulation. Figure 2.10 gives the block diagram of a possible demodulator for this type of signal with $h=1/4$. This type of demodulator can be generalised for any rational modulation index $h=q/p$. The syndrome decoder of figure 2.10 is a feedback decoder operating on a $2p$ -ary code, where $2p$ is the number of points in the differential phase plane constellation. This code corresponds to the output of a rate $1/M$ convolutional encoder operating on a $2p$ -ary alphabet. M is the number of differential phase variables considered by the feedback decoder. The convolutional encoder is given in figure 5.1 with the input $a_n = \pm 1$. The corresponding trellis diagram for $M=3$ is given in

Figure 5.1 $2p$ -ary rate $1/M$ convolutional encoder

figure 5.2, where the state corresponds to the left most $M-1$ symbols and the output, for each branch, is the triplet $[d_n^1, d_n^2, d_n^3]$.

Considering the specific example $h=1/4$ and $M=2$, the decision process illustrated in figure 2.9 leads to the following equations, where the e_k^i 's are the decision errors.

$$y_n^1 = d_n^1 + e_n^1 \quad (5.1)$$

$$y_n^2 = d_n^1 + d_{n-1}^1 + e_n^2$$

The syndrome is

$$\begin{aligned} S_n^1 &= y_n^1 + y_{n-1}^1 - y_n^2 \\ &= d_n^1 + e_n^1 + d_{n-1}^1 + e_{n-1}^1 - d_n^1 - d_{n-1}^1 - e_n^2 \\ &= e_n^1 + e_{n-1}^1 - e_n^2 \end{aligned} \quad (5.2)$$

which is a function of the channel errors only. The delayed syndrome, where e_{n-2}^1 is assumed corrected by the last decision variable, is

$$\hat{S}_{n-1}^1 = e_{n-1}^1 - e_{n-1}^2 \quad (5.3)$$

as with the $h=1/2$ case. If there is only a single error in the set

Figure 5.2 Trellis diagram corresponding to the encoder of figure 5.1.

considered by equations (5.2) and (5.3), then the product of S_n^1 and $\hat{S}_{n-1}^1$ is nonzero only when $e_{n-1}^1 \neq 0$ and an error in y_{n-1}^1 can be detected and corrected. This can be done with the circuit of figure 5.3, which is similar to the MSK two-branch phase feedback decoder. This decoder looks for a single error in y_{n-1}^1 . If this event happens, then e_{n-1}^1 is either +6 or -6 (the difference between the two possible values taken by y_n^1 in figure 2.9. and the product $S_n^1 \cdot \hat{S}_{n-1}^1$ is +36. In this case the switch is closed, allowing the correction of e_{n-1}^1 in both S_n^1 and y_{n-1}^1 . If the product is not +36, no further action is taken.

This example shows how to implement phase feedback decoding for a fairly simple case, but does not address at all the performance of the system. Again, the usefulness of this technique is related to the ability of the channel to mainly produce single-error patterns.

5.2 PARTIAL RESPONSE SIGNALS

In partial response signalling, intersymbol interference is introduced through the term $C_\phi(t)$ given in equation (2.9). Once sampled at bit transitions, this term becomes

$$C_\phi(n) = 2\pi h \left[\sum_{i=n-L+1}^n a_i q[(n-i)T] - \sum_{i=n-k-L+1}^{n-k} a_i q[(n-i)T] \right]. \quad (5.4)$$

Figure 5.3 Phase feedback decoder for a full response CPM signal with $h=1/4$.

3 of/de 3

32

MINI-D
MAGNETIC
RECORDING

This term acts on the signal in a way similar to the action of a transmit filter on a full response signal. That is, it introduces controlled intersymbol interference. One possibility for a receiver would therefore be to use a decision feedback equalization strategy to reduce the effect of this term. Error propagation would be of some concern in this strategy. Then phase feedback decoding could be applied.

Another observation is that, if the signal memory is limited to a few symbol intervals, the error pattern probability density on an AWGN channel, associated with the output of a bank of differential detectors, might be adequate for the application of feedback decoding. That is, single error patterns could be dominating. It seems sensible to conjecture that some improvement over some common noncoherent systems (section D.2.3) could be achieved with this detection procedure. GMSK, when detected differentially, seems particularly suited for this technique.

5.3 CONCLUSION

The phase feedback decoding principles are applicable to a large variety of CPM signals. Whether or not this procedure would be beneficial on certain types of channels remains to be shown. It seems reasonable to think that, at least for some modulation indices and for some intersymbol interference lengths, this procedure would lead to an overall performance that could fit between the optimum noncoherent performance (section D.2.2) and the simple differential or discriminator detection performances (section D.2.3).

CHAPTER 6

PRACTICAL IMPLEMENTATION ON A DSP CHIP

The purpose of this chapter is to present the practical performance of three-branch phase feedback decoding, as it was measured on an implementation based on a TMS32010 DSP chip. The scope of this chapter is intentionally reduced because of the proprietary nature of the algorithms implemented. In the first part, these algorithms are briefly discussed and in the second part, the performance of the modem is given.

6.1 2400 BPS TMS32010 MSK MODEM IMPLEMENTATION

In the course of this work, a series of algorithms were implemented on a TMS32010 in an effort to produce a software based modem for mobile satellite applications [30],[42]. The modulator algorithm is illustrated in figure 6.1. It is composed of two processing layers, one operating at the data rate of 2.4 kHz and the other at 19.2 kHz. Procedures that are performed at the 2.4 kHz processing rate include the differential encoding and alternating the data samples between the inphase and the quadrature channels by

Figure 6.1 The MSK modulator algorithm.

multiplying the real data sequence $[b_n]$ by $\exp(jn\pi/2)$. Note that for any integer sample index n , only one of the real and imaginary parts of this exponential is nonzero and this nonzero part is either +1 or -1. The 19.2 kHz processing includes pulse shaping and up-conversion to a center frequency of 4.8 kHz. Because the sampling frequency is four times the center frequency, the up-converting exponential reduces to $\exp(jk\pi/2)$. This means that only the real part or the imaginary part of the filter output needs to be calculated for each real sample output. The complex pulse shaping filter is an FIR filter which is a combination of the standard MSK pulse shaping filter and a bandlimiting filter.

The demodulator algorithm is illustrated in figure 6.2. It also has two processing layers; the foreground at a 19.2 kHz rate and the background at a 2.4 kHz rate. Procedures that are performed in the foreground include down-conversion to complex baseband, front-end lowpass filtering, and bit timing recovery. The first step in the bit timing recovery is frequency discrimination. The discriminator output is used in the background AFC algorithm as well as in the foreground bit timing recovery. Removing the DC component and passing the discriminated signal through a nonlinearity results in a cyclo-stationary process with a period equal to the bit rate. A narrow-band bandpass filter extracts the energy at that frequency, yielding the recovered clock. This signal is used to decimate the discriminator output to 2.4 ksamples/sec for AFC processing and to

decimate the output of the front-end filter to 4.8 ksamples/sec for the receive filter. This filter is different from the one assumed in the simulations. It is a fixed $T/2$ equalizer that was obtained by programming an adaptive $T/2$ equalizer and by letting its coefficients converge for an AWGN channel with $E_b/N_0 = -8$ dB. The criterion for adaptation was the MSE algorithm [25], which resulted in a filter optimum in the MSE sense.

Three levels of differential detection are then performed, yielding the three differential phase variables used by the three-branch feedback decoder described in chapter 4. The tracking AFC algorithm extracts the median value of the upper and lower envelope of the discriminator output, and generates an error signal that is used by an external VCXO to reduce the frequency offset on the input signal.

6.2 MEASURED PERFORMANCE

The modem has been tested using the CRC mobile channel simulation facility. This facility is capable of simulating an entire RF mobile-satellite link. Fading and shadowing, characterized by the model shown in figure 6.3, can be imposed on the transmitted signal. Shadowing of the direct path is simulated by a voltage-controlled attenuator driven by a lognormal function generator. Multipath

Figure 6.3 The fading and shadowing model used in the mobile channel simulator.

fading is simulated by Rayleigh fading, with an attenuator being used to set the K-factor.

The E_b/N_0 value was measured at the demodulator 4.8 kHz input using a calibrated filter and a true RMS power meter. The BER versus E_b/N_0 curves are given in figure 6.4 along with the simulated results of figure 4.29 for three-branch decoding. Note the very close agreement obtained between the two sets of curves. This implies a very high degree of effectiveness in the AFC and timing recovery algorithms. Furthermore, it substantiates the accuracy of the simulation.

6.3 CONCLUSION

The modem implementation was shown to suffer from very small implementation losses and the feedback decoding procedure was practically assessed. Note that the use of a DSP chip to implement this type of detection procedure is an overkill in the sense that an implementation with digital logic is very straightforward for MSK. On the other hand, the procedure adds virtually no additional computational load on a processor.

Figure 6.4 Measured performance versus simulated performance

—▲— simulation
 - - -○- - - measurements.

In both cases, the implementation is very cheap and effective, which should constitute two good reasons to support its use in any MSK demodulator based on differential detection.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 CONCLUSIONS

This thesis contains a comprehensive treatment of the application of feedback decoding in the differential detection of a subclass of CPM signals. A review of the definition and detection of CPM signals was first given. The differential phase properties of CPM signals were derived and their use for detection purpose was discussed.

The feedback decoding principles were reviewed and applied to the decoding of the quantized differential phase variables of MSK signals. A procedure to analyse the performance of the phase feedback decoder with a 2-branch memory was presented. The first error probability was established as a lower bound on performance for both 2-branch and 3-branch decoding.

A combination of simulation and analysis was used to obtain the performance of 2-branch and 3-branch phase feedback decoding of MSK on an AWGN channel and on a slow fading multipath channel. The detection

procedure was found to be effective on a static channel, but of a limited value for the multipath channel. The effect of filtering was also shown to be attenuated, on a static channel, by the use of more than one differential detector.

The extension of the theory to other CPM signals was discussed. An MSK practical implementation on a DSP chip was presented and showed good agreement with the simulation and theory.

For CPM signals with a modulation index of $1/2$, the implementation of phase feedback decoding using digital logic (modulo-2 arithmetic) is simple and inexpensive and can lead to demodulator performance improvements on static and moderately fading channels.

7.2 FUTURE WORK

Future areas of research include:

The derivation of a closed form formula for the two-branch error pattern density for a specific channel, which would allow a rapid and easy assessment of the detection procedure without the need for computer simulation.

The study of the implications of changing the look-up table in three-branch decoding of MSK.

The derivation of the optimum bank of receive filters or the optimum single receive filter when phase feedback decoding is applied.

The analysis or simulation of the performance of phase feedback decoding for other full response signals.

The study or simulation of the performance of phase feedback decoding on partial response signals such as GMSK or TFM.

The study of the application of decision feedback equalization in phase feedback decoding of partial response signals.

The study of the implementation in VLSI of a CPM demodulator using phase feedback decoding.

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APPENDIX A

DERIVATIONS OF EQUATIONS (4.16) AND (4.17)

Stein [38] (equations (45c) and (46)), shows that parameters a and b are given by

$$\begin{aligned} \begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{1}{4(1-\rho_{NI}^2)} & \left[\frac{\text{Re}^2(c_1) + \text{Im}^2(c_2)}{\sigma_1^2} + \frac{\text{Re}^2(c_2) + \text{Im}^2(c_1)}{\sigma_2^2} \right. \\ & + \frac{2\rho_{NI}}{\sigma_1\sigma_2} (\text{Im}(c_1)\text{Re}(c_2) - \text{Re}(c_1)\text{Im}(c_2)) \\ & \left. \mp \frac{2\sqrt{1-\rho_{NI}^2}}{\sigma_1\sigma_2} (\text{Re}(c_1)\text{Re}(c_2) + \text{Im}(c_1)\text{Im}(c_2)) \right] \end{aligned} \quad (A.1)$$

and parameter A by

$$A = \frac{\rho_{NR}}{\sqrt{1-\rho_{NI}^2}} \quad (A.2)$$

where $\rho_N = \rho_{NR} + j\rho_{NI}$ is the normalized complex crosscorrelation between z_n and z_{Dn} , σ_1^2 and σ_2^2 are the covariance of z_n and z_{Dn} respectively and c_1, c_2 are defined in equation (4.18).

From equations (4.19) and (4.25), we have

$$\sigma_1 = \sqrt{K_{11}}$$

(A.3)

$$\sigma_2 = \sqrt{K_{22}}.$$

ρ_N is defined as [38]

$$\rho_N = \frac{1/2 E[(z_n - E(z_n))(z_{Dn} - E(z_{Dn}))^*]}{\sigma_1 \sigma_2}. \quad (A.4)$$

Using equation (4.26), we then have

$$\rho_N = \frac{K_{12}}{\sigma_1 \sigma_2} = \frac{K_{12}}{\sqrt{K_{11} K_{22}}}. \quad (A.5)$$

Noting that K_{11} and K_{22} are real, equation (A.1) becomes

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = \frac{0.25}{\text{Im}^2(K_{12})} \left[\frac{|c_1|^2}{K_{11}} + \frac{|c_2|^2}{K_{22}} + \frac{2\text{Im}(K_{12})}{K_{11}K_{22}} \text{Im}(c_1 c_2^*) \right. \\ \left. + \frac{2 \sqrt{1 - \frac{\text{Im}^2(K_{12})}{K_{11}K_{22}}}}{\sqrt{K_{11}K_{22}}} \text{Re}(c_1 c_2^*) \right]$$

which leads to equation (4.16).

With the above definitions, equation (A.2) is transformed readily into equation (4.17).

APPENDIX B

DERIVATIONS OF THE TRANSITION PROBABILITIES

The transition probability π_{ij} is defined as $P[\text{state } j \text{ at time } n | \text{state } i \text{ at time } n-1] = P[e_n^1, \hat{S}_n^1 | e_{n-1}^1, \hat{S}_{n-1}^1]$. Therefore

$$\pi_{00} = P[e_n^1=0, \hat{S}_n^1=0 | e_{n-1}^1=0, \hat{S}_{n-1}^1=0]$$

$$= P[e_n^1=0, 0=e_n^2 | e_{n-1}^1=0]$$

$$= \frac{P[e_n^1=0, e_n^2=0, e_{n-1}^1=0]}{P[e_{n-1}^1=0]}$$

$$= \frac{P[p_0] + P[p_8]}{P[e_{n-1}^1=0]}$$

$$\pi_{01} = P[e_n^1=0, \hat{S}_n^1=1 | e_{n-1}^1=0, \hat{S}_{n-1}^1=0]$$

$$= P[e_n^1=0, 1=e_n^2 | e_{n-1}^1=0]$$

$$= \frac{P[e_n^1=0, e_n^2=1, e_{n-1}^1=0]}{P[e_{n-1}^1=0]}$$

$$= \frac{P[p_4] + P[p_{12}]}{P[e_{n-1}^1=0]}$$

$$\pi_{02} = P[e_n^1=1, \hat{S}_n^1=0 | e_{n-1}^1=0, \hat{S}_{n-1}^1=0]$$

$$\begin{aligned}
& - P[e_n^1-1, 0-1+e_n^2 | e_{n-1}^1=0] \\
& - \frac{P[e_n^1-1, e_n^2-1, e_{n-1}^1=0]}{P[e_{n-1}^1=0]} \\
& - \frac{P[p_5]+P[p_{13}]}{P[e_{n-1}^1=0]}
\end{aligned}$$

$$\begin{aligned}
\pi_{03} & - P[e_n^1-1, \hat{S}_n^1-1 | e_{n-1}^1=0, \hat{S}_{n-1}^1=0] \\
& - P[e_n^1-1, 1-1+e_n^2 | e_{n-1}^1=0] \\
& - \frac{P[e_n^1-1, e_n^2=0, e_{n-1}^1=0]}{P[e_{n-1}^1=0]} \\
& - \frac{P[p_1]+P[p_9]}{P[e_{n-1}^1=0]}
\end{aligned}$$

$$\begin{aligned}
\pi_{10} & - P[e_n^1=0, \hat{S}_n^1=0 | e_{n-1}^1=0, \hat{S}_{n-1}^1=1] \\
& - P[e_n^1=0, 0-e_n^2+e_n^2 | e_{n-1}^1=0] \\
& - \frac{P[e_n^1=0, e_{n-1}^1=0]}{P[e_{n-1}^1=0]} \\
& - \frac{P[p_0]+P[p_4]+P[p_8]+P[p_{12}]}{P[e_{n-1}^1=0]}
\end{aligned}$$

$$\begin{aligned}
\pi_{11} & - P[e_n^1=0, \hat{S}_n^1=1 | e_{n-1}^1=0, \hat{S}_{n-1}^1=1] \\
& - P[e_n^1=0, 1-e_n^2+e_n^2 | e_{n-1}^1=0] \\
& - 0
\end{aligned}$$

$$\pi_{12} = P[e_n^{1-1}, \hat{S}_n^{1-0} | e_{n-1}^{1-0}, \hat{S}_{n-1}^{1-1}]$$

$$= P[e_n^{1-0}, 0-1+e_n^2+1+e_n^2 | e_{n-1}^{1-0}]$$

$$= P[e_n^{1-1} | e_{n-1}^{1-0}]$$

$$= \frac{P[e_n^{1-1}, e_{n-1}^{1-0}]}{P[e_{n-1}^{1-0}]}$$

$$= \frac{P[p_1]+P[p_5]+P[p_9]+P[p_{13}]}{P[e_{n-1}^{1-0}]}$$

$$= \frac{P[p_1]+P[p_5]+P[p_9]+P[p_{13}]}{P[e_{n-1}^{1-0}]}$$

$$\pi_{13} = P[e_n^{1-1}, \hat{S}_n^{1-1} | e_{n-1}^{1-0}, \hat{S}_{n-1}^{1-1}]$$

$$= P[e_n^{1-1}, 1-1+e_n^2+1+e_n^2 | e_{n-1}^{1-0}]$$

$$= 0$$

$$\pi_{20} = P[e_n^{1-0}, \hat{S}_n^{1-0} | e_{n-1}^{1-1}, \hat{S}_{n-1}^{1-0}]$$

$$= P[e_n^{1-0}, 0-e_n^2+1 | e_{n-1}^{1-1}]$$

$$= \frac{P[e_n^{1-0}, e_n^2-1, e_{n-1}^{1-1}]}{P[e_{n-1}^{1-1}]}$$

$$= \frac{P[p_6]+P[p_{14}]}{P[e_{n-1}^{1-1}]}$$

$$= \frac{P[p_6]+P[p_{14}]}{P[e_{n-1}^{1-1}]}$$

$$= \frac{P[p_6]+P[p_{14}]}{P[e_{n-1}^{1-1}]}$$

$$\pi_{21} = P[e_n^{1-0}, \hat{S}_n^{1-1} | e_{n-1}^{1-1}, \hat{S}_{n-1}^{1-0}]$$

$$= P[e_n^{1-0}, 1-e_n^2+1 | e_{n-1}^{1-1}]$$

$$= \frac{P[e_n^{1-0}, e_n^2-0, e_{n-1}^{1-1}]}{P[e_{n-1}^{1-1}]}$$

$$= \frac{P[p_7]+P[p_{15}]}{P[e_{n-1}^{1-1}]}$$

$$= \frac{P[p_2] + P[p_{10}]}{P[e^1_{n-1} = 1]}$$

$$\pi_{22} = P[e_n^1 = 1, \hat{S}_n^1 = 0 | e^1_{n-1} = 1, \hat{S}^1_{n-1} = 0]$$

$$= P[e_n^1 = 0, 0 = 1 + e_n^2 + 1 | e^1_{n-1} = 1]$$

$$= \frac{P[e_n^1 = 1, e_n^2 = 0, e^1_{n-1} = 1]}{P[e^1_{n-1} = 1]}$$

$$= \frac{P[p_3] + P[p_{11}]}{P[e^1_{n-1} = 1]}$$

$$\pi_{23} = P[e_n^1 = 1, \hat{S}_n^1 = 1 | e^1_{n-1} = 1, \hat{S}^1_{n-1} = 0]$$

$$= P[e_n^1 = 1, 1 = 1 + e_n^2 + 1 | e^1_{n-1} = 1]$$

$$= \frac{P[e_n^1 = 1, e_n^2 = 1, e^1_{n-1} = 1]}{P[e^1_{n-1} = 1]}$$

$$= \frac{P[p_7] + P[p_{15}]}{P[e^1_{n-1} = 1]}$$

$$\pi_{30} = P[e_n^1 = 0, \hat{S}_n^1 = 0 | e^1_{n-1} = 1, \hat{S}^1_{n-1} = 1]$$

$$= P[e_n^1 = 0, 0 = e_n^2 + 1 + e_n^2 + 1 | e^1_{n-1} = 1]$$

$$= \frac{P[e_n^1 = 0, e^1_{n-1} = 1]}{P[e^1_{n-1} = 1]}$$

$$= \frac{P[p_2] + P[p_6] + P[p_{10}] + P[p_{14}]}{P[e^1_{n-1} = 1]}$$

$$\pi_{31} = P[e_n^1 = 0, \hat{S}_n^1 = 1 | e^1_{n-1} = 1, \hat{S}^1_{n-1} = 1]$$

$$- P[e_n^1=0, 1-e_n^2+1+e_n^2+1|e_{n-1}^1=1]$$

$$- 0$$

$$\pi_{32} = P[e_n^1=1, \hat{S}_n^1=0|e_{n-1}^1=1, \hat{S}_{n-1}^1=1]$$

$$- P[e_n^1=1, 0|1+e_n^2+1+1+e_n^2+1|e_{n-1}^1=1]$$

$$= \frac{P[e_n^1=1, e_{n-1}^1=1]}{P[e_{n-1}^1=1]}$$

$$= \frac{P[p_3]+P[p_7]+P[p_{11}]+P[p_{15}]}{P[e_{n-1}^1=1]}$$

$$\pi_{33} = P[e_n^1=1, \hat{S}_n^1=1|e_{n-1}^1=1, \hat{S}_{n-1}^1=1]$$

$$- P[e_n^1=1, 1-1+e_n^2+1+1+e_n^2+1|e_{n-1}^1=1]$$

$$- 0$$

APPENDIX C

COMPUTER SIMULATION PROCEDURE

The software package used for the simulation of all the detection strategies and for the collection of error pattern statistics is a modified version of a collection of programs developed for the Communications Research Centre by engineers at Miller Communications Systems Ltd. The original package is therefore CRC proprietary and thus only the general approach used in the software is presented. Note that the package was thoroughly tested, both at MCS Ltd and at CRC. The results can therefore be accepted with great confidence.

C.1 GENERAL STRUCTURE

The software simulation of an MSAT land mobile communications channel was developed using a VAX version of Fortran 77 under the VMS operating system on a VAX 11/750. The main constraint of the simulation under this configuration is the time required to simulate the large amounts of data needed to characterize low bit error rates.

The software is a continuous time simulation of the up-link between a fixed station and the satellite and the down-link from the satellite to the mobile station. The nominal bit rate is 2.4 kbps but the majority of the simulation calculations are independent of this figure.

The simulation block diagram of the mobile satellite communications channel is included in figure C.1.

The simulation includes a data generation module which generates a random sequence of independent data bits. The modulator module generates the CPM waveform, which can be filtered by an optional transmit filter. An optional transmit nonlinearity module may also be included, although it was not used in this thesis.

A Rician statistical model is used to simulate the channel behaviour. There is also additive white Gaussian noise at the receiver (see figure 4.19). The receive filter and differential detection modules follow. The feedback decoding process is then applied on the differential phase variables and a module performs data comparisons and error statistics compilation, with and without feedback decoding.

The entire simulation is performed using a complex baseband representation of the signals and systems. A sampling rate of 9.6

Figure C.1 Simulation block diagram.

kHz is used, which gives 4 samples per bit at a bit rate of 2.4 kbps. The simulation is carried out in a block processing mode, passing one block of samples completely through the simulation before starting the next. All the filters are implemented in the frequency domain and, to maintain continuity between blocks, the overlap and save technique is used. All of these filters are also assumed to have finite memory, which provides information on the maximum size of the overlap required.

For module independence, the vector of signal samples is always in the time domain when in the main procedure. Any subroutine called by the main procedure assumes this and returns the signal vector in the time domain.

C.2 ERROR STATISTICS COMPILATION

Since a block processing approach is used, the overall simulation has a loop structure. An outer loop is executed eight times, each iteration representing a subrun. The inner loop corresponding to each subrun is executed the number of times necessary to accommodate the block size (in samples) and the number of bits processed per subrun, as specified by the user. The block size is fixed at 1024 samples, or 256 bits at the 9.6 kHz sampling rate.

The splitting of the simulation into subruns allows the calculation of the standard deviation on the statistics compiled. These statistics include:

- error rate at the output of each level of differential detection
- error rate at the output of the two-branch feedback decoder
- error rate at the output of the three-branch feedback decoder
- rate of occurrence of each error pattern, in two branch decoding and three-branch decoding.

These statistics are given as the mean value of the eight simulation subruns results. The standard deviation on this mean is given by

$$s = \left[\frac{n \sum x_i^2 - \left\{ \sum x_i \right\}^2}{n(n-1)} \right]^{1/2} \quad (C.1)$$

where n is fixed to 8 and x_i is the error rate resulting from the i^{th} subrun. The number of bits per subrun is fixed by trial and error in order to obtain a standard deviation as small as possible, within the computing time constraint of the system used.

APPENDIX D

DETECTION OF BINARY CPM SIGNALS

D.1 OPTIMUM COHERENT DETECTION

D.1.1 PHASE TRELLIS DESCRIPTION

The information carrying phase $\Phi(t, \underline{a})$, as described by equation (2.5), can be plot on a phase tree. This tree shows the phase changes from one bit interval to the other, as well as the smoothness of the phase function. Such a tree is shown in figure D.1, where the total state vector is given at each node. In this figure, $h=2/3$ and the frequency pulse $g(t)$ is rectangular and spans three symbol periods.

If the modulation index is restricted to be rational, i.e.

$$h = \frac{2k}{p} \quad k \text{ and } p \in \mathbb{N} \quad (\text{D.1})$$

there are p different phase states. These take the values from the set

Figure D.1 Phase tree example, $h=2/3$ and $g(t)=1$ for $0 \leq t \leq 3T$ (from [1]).

$$\left\{ \frac{2\pi i}{p} \right\}_{i=0}^{p-1} = \left\{ 0, \frac{2\pi}{p}, \frac{4\pi}{p}, \dots, \frac{2\pi(p-1)}{p} \right\}. \quad (D.2)$$

Therefore, when h is rational, the number of total states defined as in equation (2.7) is $p \cdot 2^{L-1}$ and is finite. The phase tree can therefore be converted into a phase trellis with $p \cdot 2^{L-1}$ different states. This trellis description suggests that the Viterbi algorithm can be used in a receiver that minimizes the bit error probability.

D.1.2 MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION

It is assumed that the CPM signal is transmitted over an additive white and Gaussian noise channel having a one-sided noise power spectral density N_0 W/Hz. Perfect coherent detection is also assumed, so that the phase Φ_0 in equation (2.1) is zero. Thus the receiver observes the signal

$$r(t, \underline{a}) = s(t, \underline{a}) + n(t) \quad (D.3)$$

where $n(t)$ is the zero-mean white Gaussian process.

A detector that minimizes the probability of decision error computes the posterior probabilities

$$\Pr[\text{signal } s(t, \underline{\bar{a}}) \text{ was transmitted} \mid r(t, \underline{a})] \quad (\text{D.4})$$

for all the possible signals $s(t, \underline{a})$ and decides in favor of the signal resulting in the largest of these probabilities [25], ($\underline{\bar{a}}$ denotes here the estimated received sequence). In the case of equiprobable sequences over an AWGN channel, as represented by equation (D.3), the optimum receiver reduces to a correlation receiver where, in theory, an infinite number of correlation

$$J(\underline{\bar{a}}) = \int_{-\infty}^{\infty} r(t, \underline{a}) \cdot s(t, \underline{\bar{a}}) dt \quad (\text{D.5})$$

are computed and the sequence $\underline{\bar{a}}$, corresponding to the maximum correlation value, is selected. This process is known as maximum likelihood sequence estimation (MLSE) and is equivalent to the computation of the squared of the Euclidean distance

$$D^2[r(t, \underline{a}), s(t, \underline{\bar{a}})] = \int_{-\infty}^{\infty} [r(t, \underline{a}) - s(t, \underline{\bar{a}})]^2 dt \quad (\text{D.6})$$

between the received signal $r(t, \underline{a})$ and all the possible transmitted signals $s(t, \underline{\bar{a}})$. The MLSE decision is in favor of the closest transmitted signal.

If the correlation up to time $(n+1)T$ is defined as

$$J_n(\underline{\bar{a}}) = \int_{-\infty}^{(n+1)T} r(t, \underline{a}) s(t, \underline{\bar{a}}) dt \quad (\text{D.7})$$

it is possible to write

$$J_n(\underline{\tilde{a}}) = J_{n-1}(\underline{\tilde{a}}) + Z_n(\underline{\tilde{a}}) \quad (D.8)$$

where

$$\begin{aligned} Z_n(\underline{\tilde{a}}) &= \int_{nT}^{(n+1)T} r(t, \underline{a}) s(t, \underline{\tilde{a}}) dt \\ &= \sqrt{2E/T} \int_{nT}^{(n+1)T} r(t, \underline{a}) \cos[2\pi f_0 t + \Phi(t, \underline{\tilde{a}})] dt \end{aligned} \quad (D.9)$$

$Z_n(\underline{\tilde{a}})$ is a correlation between the received signal and any possible transmitted signal over the interval $[nT, (n+1)T]$.

Equations (D.3) to (D.9) can be converted into a simpler form by using complex baseband notation. Equation (D.5) becomes

$$J_c(\underline{\tilde{a}}) = \operatorname{Re} \left[\int_{-\infty}^{\infty} r_c(t, \underline{a}) s_c^*(t, \underline{\tilde{a}}) dt \right] \quad (D.10)$$

where

$$r_c(t, \underline{a}) = s_c(t, \underline{a}) + \sqrt{2E/T} u(t) \quad (D.11)$$

$$\begin{aligned} s_c(t, \underline{a}) &= I(t, \underline{a}) + j Q(t, \underline{a}) \\ &= \sqrt{2E/T} [\cos \Phi(t, \underline{a}) + j \sin \Phi(t, \underline{a})] \end{aligned} \quad (D.12)$$

$$u(t) = n_I(t) + j n_Q(t) \quad (D.13)$$

The noise quadrature components have a white power spectral density $N_0 \cdot T/2E$ W/Hz, defined within the signal bandwidth.

Equation (D.7) becomes

$$J_{nc}(\underline{\tilde{a}}) = \text{Re} \left[\int_{-\infty}^{(n+1)T} r_c(t, \underline{a}) s_c(t, \underline{\tilde{a}}) dt \right] \quad (D.14)$$

while equations (D.8) and (D.9) change to

$$J_{nc}(\underline{\tilde{a}}) = J_{(n-1)c}(\underline{\tilde{a}}) + Z_{nc}(\underline{\tilde{a}}) \quad (D.15)$$

$$Z_{nc}(\underline{\tilde{a}}) = \text{Re} \left[\int_{nT}^{(n+1)T} r_c(t, \underline{a}) s_c^*(t, \underline{\tilde{a}}) dt \right] \quad (D.16)$$

Inserting equations (D.11) to (D.13) in equation (D.16), we have

$$\begin{aligned} \left[T/2E \right] Z_{nc}(\underline{\tilde{a}}) = & \int_{nT}^{(n+1)T} [\cos\Phi(t, \underline{a}) + n_I(t)] \cos\Phi(t, \underline{\tilde{a}}) dt \\ & + \int_{nT}^{(n+1)T} [\sin\Phi(t, \underline{a}) + n_Q(t)] \sin\Phi(t, \underline{\tilde{a}}) dt. \end{aligned} \quad (D.17)$$

Using the definition of $\Phi(t, \underline{a})$ for $nT \leq t \leq (n+1)T$ (equation (2.5)) and trigonometric identities, alternate expression for $\cos\Phi(t, \underline{\tilde{a}})$ and $\sin\Phi(t, \underline{\tilde{a}})$ can be found. Defining

$$\theta(t, \underline{a}_n) = 2\pi h \sum_{i=n-L+1}^n a_i q(t-iT); \quad nT \leq t \leq (n+1)T \quad (D.18)$$

gives

$$\begin{aligned} \cos\Phi(t, \underline{\tilde{a}}) &= \cos[\theta(t, \underline{\tilde{a}}_n) + \tilde{\theta}_n] \\ &= \cos\theta(t, \underline{\tilde{a}}_n)\cos\tilde{\theta}_n - \sin\theta(t, \underline{\tilde{a}}_n)\sin\tilde{\theta}_n \end{aligned} \quad (D.19)$$

$$\begin{aligned} \sin\Phi(t, \underline{\tilde{a}}) &= \sin[\theta(t, \underline{\tilde{a}}_n) + \tilde{\theta}_n] \\ &= \cos\theta(t, \underline{\tilde{a}}_n)\sin\tilde{\theta}_n + \sin\theta(t, \underline{\tilde{a}}_n)\cos\tilde{\theta}_n \end{aligned} \quad (D.20)$$

where $\tilde{\theta}_n$ is defined in equation (2.6). Inserting equations (D.19) and (D.20) into equation (D.17), gives

$$\begin{aligned} \left[T/2E \right] Z_{nc}(\underline{a}) &= \cos\tilde{\theta}_n \int_{nT}^{(n+1)T} [\cos\Phi(t, \underline{a}) + n_I(t)] \cos\theta(t, \underline{\tilde{a}}_n) dt \\ &+ \cos\tilde{\theta}_n \int_{nT}^{(n+1)T} [\sin\Phi(t, \underline{a}) + n_Q(t)] \sin\theta(t, \underline{\tilde{a}}_n) dt \\ &+ \sin\tilde{\theta}_n \int_{nT}^{(n+1)T} [\sin\Phi(t, \underline{a}) + n_Q(t)] \cos\theta(t, \underline{\tilde{a}}_n) dt \\ &- \sin\tilde{\theta}_n \int_{nT}^{(n+1)T} [\cos\Phi(t, \underline{a}) + n_I(t)] \sin\theta(t, \underline{\tilde{a}}_n) dt. \end{aligned} \quad (D.21)$$

Equation (D.15) is a dynamic programming equation and is a representation of the Viterbi algorithm for finding the most likely received sequence of data bits. $Z_{nc}(\underline{\tilde{a}})$ is the metric that must be computed for each branch entering each node of the trellis. The op-

imum receiver is therefore as shown in figure D.2, where the filters $H_{Ij}(f)$ and $H_{Qj}(f)$ have impulse responses given by

$$h_{Ij}(t) = \begin{cases} \cos\theta(T-t, \underline{\tilde{a}}_j) & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases}$$

$$= \begin{cases} \cos \left[2\pi h \sum_{i=1-L}^0 a_i q((1-i)T-t) \right] & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases} \quad (D.22)$$

and

$$h_{Qj}(t) = \begin{cases} \sin \left[2\pi h \sum_{i=1-L}^0 a_i q(1-i)T-t) \right] & 0 \leq t \leq T \\ 0 & \text{elsewhere} \end{cases} \quad (D.23)$$

where the index j refers to the particular correlative state vector $[a_0, a_{-1}, \dots, a_{1-L}]$ that is selected.

The optimum receiver feeds the complex baseband received signal through a bank of matched filters, each one matched to one of the 2^{L-1} different correlative state vectors. This essentially produces the integrals of equation (D.21). The metric $Z_{nc}(\underline{a})$ is formed by the processor, according to equation (D.21). Dynamic programming is then applied, through the use of the Viterbi algorithm, to save only the

Figure D.2 Optimum receiver for CPM signals ($K=2^{L-1}$).

most probable sequence passing through each node of the phase trellis.

Performance bounds for such a receiver are given in the next section.

D.1.3 OPTIMAL RECEPTION PERFORMANCE EVALUATION

The optimum receiver, as shown in figure D.2, is not practical since an infinite time span is required to compute the correlation of equation (D.5). Therefore, a suboptimum receiver, which observes the received signal $r(t)$ for N -symbol intervals before making a decision about a specific data symbol, must be considered. This suboptimum detector makes easier the study of the optimum receiver. Its limiting case is the MLSE detector as presented in the previous section [2].

The suboptimum correlation is therefore

$$U(\underline{a}) = \int_0^{NT} r(t, \underline{a}) s(t, \underline{a}) dt \quad (D.24)$$

and the squared Euclidean distance between the received signal and each possible transmitted signal is

$$D_s^2 [r(t, \underline{a}), s(t, \underline{\tilde{a}})] = \int_0^{NT} [r(t, \underline{a}) - s(t, \underline{\tilde{a}})]^2 dt \quad (D.25)$$

The probability of an erroneous bit decision can be upper bounded by using the union bound [26]

$$P_b \leq \frac{1}{2^{N-1}} \sum_k \sum_{\substack{p \\ k \neq p}} Q \left[\frac{D_s^2 [s(t, \underline{a}_k), s(t, \underline{a}_p)]}{\sqrt{2N_0}} \right] \quad (D.26)$$

where the squared Euclidean distance is computed for each possible pair of transmitted sequence $\underline{a}_k$ and $\underline{a}_p$ that differ in the first symbol interval. The Q function is defined as [26]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt = 1/2 [1 - \text{erf}(x/\sqrt{2})] \quad (D.27)$$

For high signal-to-noise ratios, a good approximation for the probability of error is [1], [2]

$$P_b = C \cdot Q \left[\sqrt{d_{\min}^2 E_b/N_0} \right] \quad (D.28)$$

where C is a constant independent of E_b/N_0 and $d_{\min}^2$ is the normalized squared minimum Euclidean distance among all the possible pairs of length- N data sequences that differ in the first symbol interval. It is defined as

$$d_{\min}^2 = \min_{\substack{\underline{a}_k \neq \underline{a}_p}} \left\{ \int_0^{NT} [s(t, \underline{a}_k) - s(t, \underline{a}_p)]^2 dt \right\} \frac{1}{2E_b} \quad (D.29)$$

For CPM signals, equation (D.29) can be written as

$$d_{\min}^2 = \min_{\underline{a}_k, \underline{a}_p} \frac{1}{T} \left\{ \int_0^{NT} [1 - \cos[\Phi(t, \underline{a}_k) - \Phi(t, \underline{a}_p)]] dt \right\}. \quad (D.30)$$

The normalization of $d_{\min}^2$ is done such that $d_{\min}^2 = 2$ for BPSK, QPSK and MSK. It is therefore easy to establish the asymptotic gain or loss, compared to optimum detection of MSK, of different CPM forms by plotting $d_{\min}^2$ versus the modulation index h as a function of the observation time N . Such plots are given in [2] and [3] for full and partial response signalling. Some of these are reproduced in figures D.3 and D.4.

Figure D.3 shows the well known minimum squared distance for binary CPFSK for $N=1, 2, 3$ and 4. Also shown in figure D.3 is the upper bound $d_B^2(h)$. For $N=3$, the upper bound is reached for some modulation indices while for $N=4$, $d_{\min}^2(h)$ follows more closely $d_B^2(h)$. This supports the conclusion reached in [11] that little is gained, for CPFSK, by using observation intervals longer than 4 bits. Note that $h=1/2$ corresponds to MSK and gives $d_{\min}^2(1/2)=2$, which is the same as antipodal signalling. The required observation interval for MSK is two-bit intervals. The optimum modulation index for CPFSK is $h=0.715$. This gives the minimum Euclidean distance $d_{\min}^2(0.715)=2.43$ and thus a gain of 0.85 dB in terms of E_b/N_0 is obtained compared to MSK or PSK.

Figure D.3 Normalized squared Euclidean distance versus modulation index for binary CPFSK: upper bound (dashed line) and $d^2(h)$ for $N=1, 2, 3$ and 4 bit decision intervals (from [2]).

Figure D.4 Minimum squared Euclidean distance versus modulation index h for the binary 2RC scheme (from [3]).

Figure D.4 shows the behaviour of $d_{\min}^2(h)$ for the raised cosine frequency pulse spanning two-bit intervals (2RC in table 2.1). For $h=1/2$, the 2RC system yields almost the same distance as the MSK system, i.e. $d_{\min}^2(1/2)|_{2RC} = 1.97$. Hence, MSK and 2RC with $h=1/2$ have almost the same performance in terms of symbol error probability for large SNR, but the optimum detector for the 2RC system must observe one more symbol interval than does the MSK system. However, the 2RC system has a more compact spectrum due to its smoother phase tree [3].

Comparing the two figures, it is apparent that larger values of the minimum squared Euclidean distance can be obtained for the 2RC system. When $h=0.8$, $d_{\min}^2|_{2RC} \approx 2.65$ for $N=4$, compared to a maximum of 2.43 for CPFSK. In the interval $0 < h \leq 1/2$, CPFSK shows a larger upper bound and therefore a better asymptotic behaviour.

D.2 PERFORMANCE OF SUBOPTIMUM DETECTION PROCEDURES

D.2.1 COHERENT MSK-TYPE RECEIVERS

One form of coherent optimum receiver for MSK is described by DeBuda [8]. It consists in two offset quadrature arms where the decision is performed on alternate data bits in each arm. This parallel receiver is illustrated in figure D.5.

The optimum impulse response $a_{\text{opt}}(t)$ is, for MSK

$$a_{\text{MSK}}(t) = \begin{cases} \cos(\pi t/2T) & -T \leq t \leq T \\ 0 & \text{elsewhere} \end{cases} \quad (\text{D.31})$$

For other CPM signals with a modulation index $h=1/2$, this type of coherent receiver can be effective. The outputs of the two filters are $\cos\theta(t, \underline{a})$ and $\sin\theta(t, \underline{a})$, where $\theta(t, \underline{a})$ is the phase resulting from the corrupting noise and the filter memory. Except for MSK, the filter cannot be made optimum, in the minimum error probability sense, because it does not satisfy Nyquist first criterion for intersymbol-free transmission. However, for full response signals an open eye pattern is obtained to perform symbol decisions, while for

Figure D.5 Parallel MSK-type receiver.

partial response, if the frequency pulse $g(t)$ is limited to a few symbol intervals, the inherent ISI produces an eye opening which is also adequate to make symbol decisions. An example of $\cos\Phi(t, \underline{a})$ for a 3RC pulse is given in figure D.6.

The parallel configuration of figure D.5 can be modified in order to give a serial-type detector [10]. The idea behind the serial receiver is to have a single receive data channel instead of the two quadrature channels of the parallel receiver. The structure of the serial MSK-type receiver is given in figure D.7.

In the absence of phase or timing error, the performance of the two MSK-type receivers are the same as long as their respective filters conform to the following relation: [10]

$$\begin{aligned} h_1(t) &= a(t) \cos\pi t/2T \\ h_2(t) &= -a(t) \sin\pi t/2T. \end{aligned} \quad (D.32)$$

It is obvious that these two bit-by-bit detection procedures do not take full advantage of the CPM signal memory. However, for frequency pulses $g(t)$ of limited time span, the small degradation from MLSE makes the simplicity of these detectors attractive. Figure D.8

Figure D.6 $\text{Cos}\Phi(t, \underline{a})$ eye pattern for binary 3RC, $h=1/2$ scheme.

Figure D.7 Serial MSK-type receiver.

Figure D.8 MSK-type receiver asymptotic performance for some selected CPM schemes. N_T —observation interval (from [10]).

shows the asymptotic performance curve for some partial response modulations.

The 2RC scheme performs virtually like MSK with that receiver. For 3RC, the degradation is about 0.16 compared to MLSE, while it is ≈ 0.82 dB for TFM5 and 0.91 dB for 4RC [27].

It was shown in [27] that the serial detector is less sensitive to phase and timing errors than the parallel detector. Therefore, the serial MSK-type detector must be considered as a serious candidate in a coherent detection system for CPM signals.

D.2.2 OPTIMUM NONCOHERENT RECEIVER

The noncoherent receiver does not make any attempt to estimate the phase shift introduced on the transmitted signal by the channel. The received signal is thus

$$r(t) = s(t, \underline{a}, \rho_0) + n(t) \quad (D.33)$$

where $n(t)$ is the usual white Gaussian random process with zero mean and one-sided power spectral density N_0 W/Hz and ρ_0 is an unknown phase shift. This phase shift is assumed to be uniformly distributed between $\pm\pi$.

Due to the continuous phase of the transmitted signal, the optimum observation interval necessary to make a decision about a specific data symbol is larger than one symbol interval. Assuming that the data symbol to be detected is a_0 , corresponding to the interval $[0, T]$, the receiver observes the signal over the interval $[-N_1T, N_2T]$. The optimum receiver must find the sequence of data $a_{-N_1}, a_{-N_1+1}, \dots, a_0, a_1, \dots, a_{N_2}$ which maximizes the likelihood function

$$\Lambda[r(t)] = 1/2\pi \int_{-\pi}^{\pi} \exp \left[2/N_0 \int_{-N_1T}^{N_2T} r(t) \cdot s(t, \bar{a}, \rho_0) dt \right] d\rho_0. \quad (D.34)$$

The average over the random phase yields the zero-order modified Bessel function of the first kind and the resulting likelihood function is [11], [26]

$$\Lambda[r(t)] = I_0 \left[\frac{2}{N_0} \sqrt{\frac{2E}{T}} \sqrt{\left\{ \int_{-N_1T}^{N_2T} r(t) \cdot \cos[2\pi f_0 t + \Phi(t, \bar{a})] dt \right\}^2 + \left\{ \int_{-N_1T}^{N_2T} r(t) \cdot \sin[2\pi f_0 t + \Phi(t, \bar{a})] dt \right\}^2} \right] \quad (D.35)$$

where $I_0(x)$ is defined as [26]

$$I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{x \cos \alpha} d\alpha. \quad (D.36)$$

$\Lambda[r(t)]$ is the likelihood function of one particular sequence $\bar{a} = a_{-N_1}, a_{-N_1+1}, \dots, a_0, a_1, \dots, a_{N_2}$ with a_0 as the estimated bit. There are $M=2^{(N_1+N_2)}$ of these sequences with a_0 as middle bit. Since the

data bits are assumed to be independent, the likelihood function for the set of sequences with middle bit a_0 is

$$\lambda = \sum_{i=1}^{M_1} \Lambda_i[r(t)] \quad (D.37)$$

where each $\Lambda_i[r(t)]$ is defined as in (D.35) for the i^{th} sequence $\underline{\tilde{a}}$. The optimum noncoherent receiver is therefore as shown in figure D.9.

Since $I_0(x)$ is a monotone increasing function of x , an equivalent likelihood function is

$$\begin{aligned} \Lambda[r(t)] = & \left[\int_{-N_1T}^{N_1T} r(t) \cdot \cos[2\pi f_0 t + \Phi(t, \underline{\tilde{a}})] dt \right]^2 \\ & + \left[\int_{-N_1T}^{N_1T} r(t) \cdot \sin[2\pi f_0 t + \Phi(t, \underline{\tilde{a}})] dt \right]^2 \end{aligned} \quad (D.38)$$

which leads to a simpler implementation of the receiver.

The asymptotic probability of error was characterized in [12], [13] and [28] for large E_b/N_0 through the use of a parameter called the equivalent minimum Euclidean distance, $d_{e \min}$. The mathematical relations involved in the calculation of d_e are given in [12] where it is shown that d_e plays the same role as the Euclidean distance d (see equation (D.28)) and that d_e depends on the specific choice of the pair of sequences $\underline{\tilde{a}}$ to be compared. The minimum equivalent Euclidean distance is found by inspection of all the

Figure D.9 Optimum noncoherent receiver (from [11]).

possible pairs of sequences differing in a_0 . Therefore, the asymptotic error probability is given by

$$P_e = C \cdot Q \left[\sqrt{d_{e \min}^2 \frac{E_b}{N_0}} \right] \quad \text{for large } E_b/N_0. \quad (D.39)$$

Figure D.10 shows an upper bound on the minimum normalized (normalization with respect to E_b as in the case of the minimum Euclidean distance) squared equivalent Euclidean distance when $N_2 \rightarrow \infty$ and $N_1=0,1,2,3$ and 10 observed bit intervals for CPFSK [12]. The coherent upper bound is indicated and corresponds to d_B^2 in figure D.3. Comparisons of figures D.3 and D.10 show that for $N_1 \geq 1$, the upper bounds for coherent and noncoherent systems essentially coincide. This similitude in performance was already noted in [11] where it was shown, with tighter bounds, that the two receivers are equivalent in performance to within 0.5 dB.

Figure D.11 shows the behaviour of $d_{e \min}^2$ for the 2RC frequency pulse when $N_2 \rightarrow \infty$ and $N_1=0,1,2,5$ and 10 bit intervals. Again, comparisons with figure D.4 show a very similar asymptotic behaviour between coherent and noncoherent detection. It is stated in [28] that the loss in noncoherent detection compared to coherent is about 0.3 dB if $N_1 > 1$.

Other frequency pulses are considered in [12] and [28] where the derivation of the upper bounds is considered in more details. The

Figure D.10 Upper bound on the minimum normalized squared equivalent distance for CPFSK (from [12]).

Figure D.11 Upper bound on the minimum normalized squared equivalent distance for partial response with 2RC frequency pulse (from [28]).

general conclusion about noncoherent detection of CPM signals is that if the signal is observed for enough symbol intervals, the degradation from coherent detection can be made relatively small.

D.2.3 DIFFERENTIAL DETECTION AND FREQUENCY DISCRIMINATOR DETECTION

Two types of suboptimum noncoherent detection procedures were proposed for CPM signals. They are differential detection and frequency discriminator detection. The two procedures are very similar in nature since differential detection implements the detection of the phase difference between the received signal and a delayed version of it. In noiseless conditions, this phase difference is also the resulting output of the cascade of a frequency discriminator and an integrate-and-dump filter. The two systems are shown in figure D.12.

In noiseless conditions, the output of the differential detection system is

$$y_{\text{diff}}(t) = \cos[2\pi f_0 T + \Delta\phi_1(t) - \beta] \quad (\text{D.40})$$

where $\Delta\phi_1(t)$ is defined as per equation 2.8 with $k=1$. If β is chosen such that $(2\pi f_0 T - \beta) = -\pi/2$, then we have

(a)

(b)

Figure D.12 Suboptimum noncoherent CPM detectors
 (a) Differential detection
 (b) Frequency discriminator detection.

$$\begin{aligned}
 y_{\text{diff}}(t) &= \sin[\Delta\phi_1(t)] \\
 &= \sin[\phi(t, \underline{a}) - \phi(t-T, \underline{a}_1)] \quad (D.41)
 \end{aligned}$$

The noiseless output of the frequency discriminator system is

$$y_{\text{disc}}(t) = \Delta\phi_1(t) = \phi(t, \underline{a}) - \phi(t-T, \underline{a}_1). \quad (D.42)$$

These signals both give an eye pattern that can be used for bit-by-bit detection. Figure D.13 shows the performance of both types of detector for the 3RC scheme and for a few modulation indices. Also shown in the figure is the performance of the discriminator system followed by soft decision Viterbi detection.

A slightly different differential detection scheme was studied in [14] and [17] where the detector was followed by sequence estimation (Viterbi detection). The detector structure is given in figure D.14

The signals $\hat{I}(t, \underline{a})$ and $\hat{Q}(t, \underline{a})$ are

$$\begin{aligned}
 \hat{I}(t, \underline{a}) &= \cos[2\pi f_0 T + \phi(t, \underline{a}) - \phi(t-T, \underline{a}_1)] \\
 \hat{Q}(t, \underline{a}) &= \sin[2\pi f_0 T + \phi(t, \underline{a}) - \phi(t-T, \underline{a}_1)] \quad (D.43)
 \end{aligned}$$

Figure D.13 Calculated and simulated error probabilities for 3RC. (from[16]).

Figure D.14 Differential detection structure considered in [14] and [17].

and can be considered as the quadrature components of a new CPM signal with an information carrying phase $\Delta\Phi_1(t) = \Phi(t, \underline{a}) - \Phi(t-T, \underline{a}_1)$. If the noise added on the signal is assumed white and Gaussian, with double-sided power spectral density N_0 W/Hz, an approximate analysis of this system can be performed by using the tools developed for the coherent case in section D.1.3 [14], [17]. The noise spectral density is doubled because both $\hat{I}(t, \underline{a})$ and $\hat{Q}(t, \underline{a})$ are affected by twice the noise power present on the received signal input [14], [17]. This assumption about the noise constitutes a worse case and implies a degradation of 3 dB compared to the coherent case.

These assumptions therefore lead to the analysis of a coherently detected CPM scheme with frequency pulse $g(t) = g(t) - g(t-T)$ received in additive white gaussian noise. The squared minimum normalized Euclidean distance $d_{D \min}^2$ is given by equation (D.30) with the phase expression replaced by $\Delta\Phi_1(t)$. The asymptotic degradation, from optimum coherent detection, is therefore 3 dB plus any degradation given by $d_{D \min}^2$.

Figure D.15 shows $d_{D \min}^2$ for CPFSK as a function of the modulation index h and the number of bits N used for the metric calculation in the sequence estimation. When these results are compared with those of coherent detection (figure D.3), the maximum distance are found to be approximately the same for both detection procedures. However, the noise-doubling degradation of 3 dB, inherent to

Figure D.15 Minimum squared, normalized Euclidean distance for differential detection of CPFSK (from [14]).

differential detection, must be considered in the asymptotic performance.

References [14] and [17] analyse the behaviour of $d_{D \min}^2$ for other CPM schemes of interest. In particular, $d_{D \min}^2$ is given in [17] as a function of the delay length used in the differential detector.

Some other forms of detectors exist for CPM signals, but the ones presented in this appendix are the most documented and represent the cases of most interests.

END

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