**Three-dimensional Fracture Mechanics Analyses
of Single Edge Cracked Plate Specimens**

by

ZHAOYU JIN, M. Eng.

A thesis submitted to the Faculty of Graduate and Postdoctoral
Affairs in partial fulfillment of the requirements
for the degree of

Master of Applied Science

In

Mechanical Engineering

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**Three-dimensional Fracture Mechanics Analyses
of Single Edge Cracked Plate Specimens**

submitted by

ZHAOYU JIN, M. Eng.

In partial fulfillment of the requirements for
The degree of Master of Applied Science (Mechanical)

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December, 2011

ABSTRACT

In this thesis, the stress intensity factor K_I , T_{11} and T_{33} stresses along the crack front of the single edge cracked plate (SECP) specimens with the crack surface subjected to uniform, linear, parabolic or cubic stress distribution are calculated by three-dimensional finite element analyses. The relative crack depth, a/W , is varied by 0.2, 0.4, 0.6 or 0.8. And the relative thickness, t/W , is chosen by 0.1, 0.2, 0.5, 1.0, 2.0 or 4.0, respectively. For engineering applications, empirical equations of normalized stress intensity factor K_I , T_{11} and T_{33} stress at the mid-plane are obtained.

The thickness effect for the three parameters of stress intensity factor K_I , T_{11} and T_{33} stresses at the mid-plane of the SECP specimen is discussed in this thesis. The stress distribution applied on the crack surface is uniform pressure. In order to study the thickness effect completely, additional cases with relative thickness, $t/W=0.0025, 0.025, 0.05, 8, 16$ for crack surface subjected to the uniform stress distribution are calculated by finite element analyses. In addition, the relative crack depth effect of the stress intensity factor K_I , T_{11} and T_{33} stresses at the mid-plane of the SECP specimen is discussed as well. The three parameters variation along the crack front are also investigated.

The stress state around the crack front in the three-dimensional SECP specimens described by the stress intensity factor K_I , T_{11} and T_{33} stresses are compared to the results from the finite element analyses. The differences between the Williams' expansion and finite element analyses varied by angle, radius and thickness are presented.

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CONTENTS

ABSTRACT.....	II
ACKNOWLEDGEMENTS.....	III
CONTENTS.....	IV
LIST of TABLES	VII
LIST of FIGURES	XIII
NOTATION.....	XXIV
CHAPTER 1	
INTRODUCTION	1
1.1 THESIS OUTLINE	5
CHAPTER 2	
BACKGROUND AND LITERATURE REVIEW	6
2.1 LINEAR ELASTIC FRACTURE MECHANICS	6
2.1.1 Fracture Modes and Stress Intensity Factor.....	6
2.1.2 T -Stress	7
2.1.3 Superposition Method for the determination of K , T_{11} AND T_{33}	9
2.2 FINITE ELEMENT ANALYSIS OF CRACKED COMPONENTS	11
2.2.1 Calculation of stress intensity factor K_I	12
2.2.2 Calculation of T_{11} Stress.....	13
2.2.3 Calculation of T_{33} Stress	14
2.3 LITERATURE REVIEW FOR THREE-DIMENSIONAL SOLUTION OF K , T_{11} AND T_{33} AND STRESS STATE AROUND CRACK FRONT	15
2.4 SINGLE EDGE CRACKED PLATE SPECIMEN	19
2.4.1 Two dimensional solutions for single edge cracked plate specimen ...	20
2.4.2 Three dimensional solutions for single edge cracked plate specimen and the thickness effect.....	20

2.4.3 The present research for single edge cracked plate	23
CHAPTER 3	
STRESS INTENSITY FACTOR AND T-STRESSES FOR CRACK IN THREE DIMENSIONAL SINGLE EDGE CRACKED PLATE SPECIMENS	28
3.1 FINITE ELEMENT ANALYSIS FOR CRACK IN THREE-DIMENSIONAL SINGLE EDGE CRACKED PLATE SPECIMENS	28
3.1.1 Finite Element Model	28
3.1.2 Verification of the Finite Element Model	30
3.1.3 Finite Element Results	31
3.2 EMPIRICAL EQUATIONS OF STRESS INTENSITY FACTOR AND T-STRESS	33
3.2.1 Stress Intensity Factor Equations	33
3.2.2 T_{11} Equations	34
3.2.3 T_{33} Equations	37
3.3 SUMMARY	40
CHAPTER 4	
VARIATIONS OF K_I , T_{11} AND T_{33} BY GEOMETRY	90
4.1 Mid-plane K_I , T_{11} and T_{33} Varied by Geometry	90
4.1.1 Effects of plate thickness for K_I , T_{11} and T_{33}	90
4.1.2 Variation of Crack depths	96
4.2 K_I , T_{11} and T_{33} Variation along the Crack Front	97
CHAPTER 5	
STRESS STATE DESCRIPTION BY K_I , T_{11} AND T_{33}	130
5.1 Stress State Description by K_I , T_{11} and T_{33}	130
5.1.1 Stress State Varied by Angle	131
5.1.2 Stress State Varied by Radius	132
5.1.3 Stress State along Thickness	133

5.2 Conclusion	134
CHAPTER 6	
CONCLUSIONS AND RECOMMENDATIONS	176
6.1 Conclusions	176
6.2 Recommendations	178
REFERENCES	179

LIST of TABLES

Table 3.1 Comparison of the K from present FEM calculation and solution form Qu and Wang for surface corner crack, $a/c=1.0$, $a/t=0.2$	41
Table 3.2 Comparison of the T_{11} from present FEM calculation and solution form Qu and Wang for surface corner crack, $a/c=1.0$, $a/t=0.2$	41
Table 3.3 Comparison of the T_{33} from present FEM calculation and solution form Qu and Wang for surface corner crack, $a/c=1.0$, $a/t=0.2$	42
Table 3.4 Comparison of T_{11} from present FEM calculation and solution from Nakamura and Parks (1992) for $a/W=0.5$, $t/W=0.5$	42
Table 3.5 Comparison of stress intensity factor and T_{11} from present FEM calculation and solution from Sham (1991) for $a/W=0.5$	43
Table 3.6 Normalized K_I , T_{11} and T_{33} for $a/W=0.2$, $t/W=0.1$, SECP	44
Table 3.7 Normalized K_I , T_{11} and T_{33} for $a/W=0.2$, $t/W=0.2$, SECP	45
Table 3.8 Normalized K_I , T_{11} and T_{33} for $a/W=0.2$, $t/W=0.5$, SECP	46
Table 3.9 Normalized K_I , T_{11} and T_{33} for $a/W=0.2$, $t/W=1.0$, SECP	47
Table 3.10 Normalized K_I , T_{11} and T_{33} for $a/W=0.2$, $t/W=2.0$, SECP	48
Table 3.11 Normalized K_I , T_{11} and T_{33} for $a/W=0.2$, $t/W=4.0$, SECP	49
Table 3.12 Normalized K_I , T_{11} and T_{33} for $a/W=0.4$, $t/W=0.1$, SECP	50

Table 3.13 Normalized K_I , T_{11} and T_{33} for $a/W=0.4$, $t/W=0.2$, SECP	51
Table 3.14 Normalized K_I , T_{11} and T_{33} for $a/W=0.4$, $t/W=0.5$, SECP	52
Table 3. 15 Normalized K_I , T_{11} and T_{33} for $a/W=0.4$, $t/W=1.0$, SECP	53
Table 3.16 Normalized K_I , T_{11} and T_{33} for $a/W=0.4$, $t/W=2.0$, SECP	54
Table 3.17 Normalized K_I , T_{11} and T_{33} for $a/W=0.4$, $t/W=4.0$, SECP	55
Table 3.18 Normalized K_I , T_{11} and T_{33} for $a/W=0.6$, $t/W=0.1$, SECP	56
Table 3.19 Normalized K_I , T_{11} and T_{33} for $a/W=0.6$, $t/W=0.2$, SECP	57
Table 3.20 Normalized K_I , T_{11} and T_{33} for $a/W=0.6$, $t/W=0.5$, SECP	58
Table 3.21 Normalized K_I , T_{11} and T_{33} for $a/W=0.6$, $t/W=1.0$, SECP	59
Table 3.22 Normalized K_I , T_{11} and T_{33} for $a/W=0.6$, $t/W=2.0$, SECP	60
Table 3.23 Normalized K_I , T_{11} and T_{33} for $a/W=0.6$, $t/W=4.0$, SECP	61
Table 3.24 Normalized K_I , T_{11} and T_{33} for $a/W=0.8$, $t/W=0.1$, SECP	62
Table 3.25 Normalized K_I , T_{11} and T_{33} for $a/W=0.8$, $t/W=0.2$, SECP	63
Table 3.26 Normalized K_I , T_{11} and T_{33} for $a/W=0.8$, $t/W=0.5$, SECP	64
Table 3.27 Normalized K_I , T_{11} and T_{33} for $a/W=0.8$, $t/W=1.0$, SECP	65
Table 3.28 Normalized K_I , T_{11} and T_{33} for $a/W=0.8$, $t/W=2.0$, SECP	66
Table 3.29 Normalized K_I , T_{11} and T_{33} for $a/W=0.8$, $t/W=4.0$, SECP	67

Table 3.30 Parameters of the empirical formulae for stress intensity factor K_I	68
Table 4.1 Finite element result for $a/W=0.2$, subjected to uniform load, SECP	99
Table 4.2 Finite element result for $a/W=0.4$, subjected to uniform load, SECP	99
Table 4.3 Finite element result for $a/W=0.6$, subjected to uniform load, SECP	99
Table 4.4 Finite element result for $a/W=0.8$, subjected to uniform load, SECP	100
Table 4. 5 2D solution, crack subjected to uniform load, SECP, Sham (1991)	100
Table 4.6 Values of B^{2D} and $B^{thin}(0)$ for $a/W=0.2, 0.4, 0.6$ and 0.8 subjected to uniform load on the crack surface	100
Table 4.7 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.2$	101
Table 4.8 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.4$	101
Table 4.9 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.6$	102
Table 4.10 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.8$	102

Table 4.11 Values of B_{33}^{2D} and $B_{33}^{thin}(0)$ for $a/W=0.2, 0.4, 0.6$ and 0.8 subjected to uniform load on the crack surface.....	103
Table 4.12 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.2$	103
Table 4.13 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.4$	104
Table 4.14 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.6$	104
Table 4.15 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.8$	105
Table 4.16 The thickness correction factor ΔK_{3D} for $a/W=0.2$	105
Table 4.17 The thickness correction factor ΔK_{3D} for $a/W=0.4$	106
Table 4.18 The thickness correction factor ΔK_{3D} for $a/W=0.6$	106
Table 4.19 The thickness correction factor ΔK_{3D} for $a/W=0.8$	107
Table 4.20 The thickness correction factor ΔT_{113D} for $a/W=0.2$	107
Table 4.21 The thickness correction factor ΔT_{113D} for $a/W=0.4$	108

Table 4.22 The thickness correction factor ΔT_{113D} for $a/W=0.6$	108
Table 4. 23 The thickness correction factor ΔT_{113D} for $a/W=0.8$	109
Table 4.24 The thickness correction factor ΔT_{333D} for $a/W=0.2$	109
Table 4.25 The thickness correction factor ΔT_{333D} for $a/W=0.4$	110
Table 4.26 The thickness correction factor ΔT_{333D} for $a/W=0.6$	110
Table 4.27 The thickness correction factor ΔT_{333D} for $a/W=0.8$	111
Table 5.1 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=0.1$, Uniform Load, SECP.....	135
Table5.2 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=0.2$, Uniform Load, SECP.....	136
Table 5.3 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=0.5$, Uniform Load, SECP.....	137
Table 5.4 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=1.0$, Uniform Load, SECP.....	138
Table 5.5 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=2.0$, Uniform Load, SECP.....	139
Table 5.6 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=4.0$, Uniform Load, SECP.....	140

Table 5.7 Percent different between Williams formula and FEM, $a/W=0.6$, $t/W=0.5$, Uniform Load, SECP	141
Table 5.8 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, Uniform, SECP.....	142
Table 5.9 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, Uniform, SECP.....	143
Table 5.10 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, Uniform, SECP.....	144
Table 5.11 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=1.0$, $r/a=0.0078$, Uniform, SECP.....	145
Table 5.12 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, Uniform, SECP.....	146
Table 5.13 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, Uniform, SECP.....	147

LIST of FIGURES

Figure 2.1 The three independent modes of movement of crack surfaces.....	24
Figure 2.2 Three-dimensional coordinate system for the region along the crack front	24
Figure 2.3 Principle of superposition, two-dimensional	25
Figure 2.4 Principle of superposition, three-dimensional	26
Figure 2.5 A schematic of elements used in the definition of the interaction integral: (a), (b), (c)	27
Figure 2.6 Single edge cracked specimen.....	27
Figure 3.1 Geometric parameters of single edge cracked plate specimen with crack surface subjected to stress distribution $\sigma(x)$	69
Figure 3.2 C3D20 element and mesh configuration near the crack tip	70
Figure 3.3 Finite element model for $a/W=0.4$, $t/W=0.5$, SECP	71
Figure 3.4 Corner crack in the finite thickness plate	72
Figure 3.5 Surface corner crack model in ABAQUS, $a/c=1.0$, $a/t=0.2$	72
Figure 3.6 Comparison of the T_{11} from present FEM calculation and solution form Qu and Wang (2006) for surface corner crack, $a/c=1.0$, $a/t=0.2$	73

Figure 3.7 Comparison of the T_{33} from present FEM calculation and solution form Qu and Wang (2006) for surface corner crack, $a/c=1.0$, $a/t=0.2$	73
Figure 3.8 Variation of normalized T_{11} along front of an edge-cracked plate for various Poisson's ratios under tension.....	74
Figure 3.9 Normalized Stress Intensity Factor, $a/W=0.2$, Uniform, SECP.....	75
Figure 3.10 Normalized Stress Intensity Factor, $a/W=0.2$, Linear, SECP	75
Figure 3.11 Normalized T_{11} , $a/W=0.2$, Uniform, SECP.....	76
Figure 3.12 Normalized T_{11} , $a/W=0.2$, Linear, SECP	76
Figure 3.13 Normalized T_{33} , $a/W=0.2$, Uniform, SECP	77
Figure 3.14 Normalized T_{33} , $a/W=0.2$, Linear, SECP	77
Figure 3.15 Normalized Stress Intensity Factor, $a/W=0.4$, Uniform, SECP.....	78
Figure 3.16 Normalized Stress Intensity Factor, $a/W=0.4$, Linear, SECP	78
Figure 3.17 Normalized T_{11} , $a/W=0.4$, Uniform, SECP	79
Figure 3.18 Normalized T_{11} , $a/W=0.4$, Linear, SECP	79
Figure 3.19 Normalized T_{33} , $a/W=0.4$, Uniform, SECP	80
Figure 3.20 Normalized T_{33} , $a/W=0.4$, Linear, SECP	80
Figure 3.21 Normalized Stress Intensity Factor, $a/W=0.6$, Uniform, SECP.....	81

Figure 3.22 Normalized Stress Intensity Factor, $a/W=0.6$, Linear, SECP	81
Figure 3.23 Normalized T_{11} , $a/W=0.6$, Uniform, SECP	82
Figure 3. 24 Normalized T_{11} , $a/W=0.6$, Linear, SECP	82
Figure 3.25 Normalized T_{33} , $a/W=0.6$, Uniform, SECP	83
Figure 3.26 Normalized T_{33} , $a/W=0.6$, Linear, SECP	83
Figure 3.27 Normalized Stress Intensity Factor, $a/W=0.8$, Uniform, SECP	84
Figure 3.28 Normalized Stress Intensity Factor, $a/W=0.8$, Linear, SECP	84
Figure 3.29 Normalized T_{11} , $a/W=0.8$, Uniform, SECP	85
Figure 3.30 Normalized T_{11} , $a/W=0.8$, Linear, SECP	85
Figure 3.31 Normalized T_{33} , $a/W=0.8$, Uniform, SECP	86
Figure 3.32 Normalized T_{33} , $a/W=0.8$, Linear, SECP	86
Figure 3.33 Comparison of numerical results and empirical formulae for stress intensity factor: (a) $t/W=0.1$, (b) $t/W=0.2$, (c) $t/W=0.5$, (d) $t/W=1.0$, (e) $t/W=2.0$, and (f) $t/W=4.0$	87
Figure 3.34 Comparison of numerical results and empirical formulae for T_{11} : (a) $t/W=0.1$, (b) $t/W=0.2$, (c) $t/W=0.5$, (d) $t/W=1.0$, (e) $t/W=2.0$, and (f) $t/W=4.0$	88
Figure 3.35 Comparison of numerical results and empirical formulae for T_{33} : (a) $t/W=0.1$, (b) $t/W=0.2$, (c) $t/W=0.5$, (d) $t/W=1.0$, (e) $t/W=2.0$, and (f) $t/W=4.0$	89

Figure 4.1 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.2$	112
Figure 4.2 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.4$	112
Figure 4.3 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.6$	113
Figure 4.4 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.8$	113
Figure 4.5 Mid-plane value of T_{11} stress as a function of plate thickness in SECP, $a/W=0.2$	114
Figure 4.6 Mid-plane value of T_{11} stress as a function of plate thickness in SECP, $a/W=0.4$	114
Figure 4.7 Mid-plane value of T_{11} stress as a function of plate thickness in SECP, $a/W=0.6$	115
Figure 4.8 Mid-plane value of T_{11} stress as a function of plate thickness in SECP, $a/W=0.8$	115
Figure 4.9 Comparison of approximation formula and FEM for T_{11} , $a/W=0.2$	116
Figure 4.10 Comparison of approximation formula and FEM for T_{11} , $a/W=0.4$	116
Figure 4.11 Comparison of approximation formula and FEM for T_{11} , $a/W=0.6$	117
Figure 4.12 Comparison of approximation formula and FEM for T_{11} , $a/W=0.8$	117
Figure 4.13 Mid-plane value of T_{33} stress as a function of plate thickness in SECP, $a/W=0.2$	118

Figure 4.14 Mid-plane value of T_{33} stress as a function of plate thickness in SECP, $a/W=0.4$	118
Figure 4.15 Mid-plane value of T_{33} stress as a function of plate thickness in SECP, $a/W=0.6$	119
Figure 4.16 Mid-plane value of T_{33} stress as a function of plate thickness in SECP, $a/W=0.8$	119
Figure 4.17 Comparison of approximation formula and FEM for T_{33} , $a/W=0.2$	120
Figure 4.18 Comparison of approximation formula and FEM for T_{33} , $a/W=0.4$	120
Figure 4.19 Comparison of approximation formula and FEM for T_{33} , $a/W=0.6$	121
Figure 4.20 Comparison of approximation formula and FEM for T_{33} , $a/W=0.8$	121
Figure 4.21 Mid-plane value of stress intensity factor as a function of crack depth in SECP.....	122
Figure 4.22 Mid-plane value of T_{11} as a function of crack depth in SECP.....	122
Figure 4.23 Mid-plane value of T_{33} as a function of crack depth in SECP.....	123
Figure 4.24 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.2$	123
Figure 4.25 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.4$	124
Figure 4.26 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.6$	124

Figure 4.27 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.8$	125
Figure 4.28 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.2$	125
Figure 4.29 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.4$	126
Figure 4.30 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.6$	126
Figure 4.31 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.8$	127
Figure 4.32 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.2$	127
Figure 4.33 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.4$	128
Figure 4.34 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.6$	128
Figure 4.35 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.8$	129
Figure 5.1 Coordinate system for stress state of crack front.....	148
Figure 5.2 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,	

$t/W=0.1$	149
Figure 5.3 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$, $t/W=0.2$	149
Figure 5.4 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$, $t/W=0.5$	150
Figure 5.5 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$, $t/W=1.0$	150
Figure 5.6 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$, $t/W=2.0$	151
Figure 5.7 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$, $t/W=4.0$	151
Figure 5.8 Percentage different of $f_{11}(\theta)$ varied by angles at three radiuses location, $a/W=0.8$, Uniform	152
Figure 5.9 Percentage different of $f_{22}(\theta)$ varied by angles at three radiuses location, $a/W=0.8$, Uniform: (a) Degree from 0° to 135° , (b) Degree from 0° to 165°	152
Figure 5.10 Percentage different of $f_{33}(\theta)$ varied by angles at three radiuses location, $a/W=0.8$, Uniform	153
Figure 5.11 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 0° , Uniform, SECP	153
Figure 5.12 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 30° , Uniform, SECP	154

Figure 5.13 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 60° , Uniform, SECP154

Figure 5.14 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 90° , Uniform, SECP155

Figure 5.15 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 120° , Uniform, SECP155

Figure 5.16 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 150° , Uniform, SECP156

Figure 5.17 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{11} , Angle= 180° , Uniform, SECP156

Figure 5.18 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 0° , Uniform, SECP157

Figure 5.19 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 30° , Uniform, SECP157

Figure 5.20 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 60° , Uniform, SECP158

Figure 5.21 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 90° , Uniform, SECP158

Figure 5.22 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 120° , Uniform, SECP159

- Figure 5.23 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 150° , Uniform, SECP159
- Figure 5.24 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle= 180° , Uniform, SECP160
- Figure 5.25 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 0° , Uniform, SECP160
- Figure 5.26 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 30° , Uniform, SECP161
- Figure 5.27 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 60° , Uniform, SECP161
- Figure 5.28 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 90° , Uniform, SECP162
- Figure 5.29 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 120° , Uniform, SECP162
- Figure 5.30 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 150° , Uniform, SECP163
- Figure 5.31 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 180° , Uniform, SECP163
- Figure 5.32 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 0° 164
- Figure 5.33 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 30° 164

Figure 5.34 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 60°	165
Figure 5.35 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 90°	165
Figure 5.36 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 120° ...	166
Figure 5.37 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 150° ...	166
Figure 5.38 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, f_{11} , Uniform, SECP	167
Figure 5.39 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, f_{22} , Uniform, SECP	167
Figure 5.40 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, f_{33} , Uniform, SECP	168
Figure 5.41 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, f_{11} , Uniform, SECP	168
Figure 5.42 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, f_{22} , Uniform, SECP	169
Figure 5.43 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, f_{33} , Uniform, SECP	169
Figure 5.44 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, f_{11} , Uniform, SECP	170
Figure 5.45 Stress State along the circular path close to the front varied by four locations	

along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, f_{22} , Uniform, SECP	170
Figure 5.46 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, f_{33} , Uniform, SECP	171
Figure 5.47 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=1.0$, $r/a=0.0078$, f_{11} , Uniform, SECP	171
Figure 5.48 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=1.0$, $r/a=0.0078$, f_{22} , Uniform, SECP	172
Figure 5.49 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=1.0$, $r/a=0.0078$, f_{33} , Uniform, SECP	172
Figure 5.50 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, f_{11} , Uniform, SECP	173
Figure 5.51 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, f_{22} , Uniform, SECP	173
Figure 5.52 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, f_{33} , Uniform, SECP	174
Figure 5.53 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, f_{11} , Uniform, SECP	174
Figure 5.54 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, f_{22} , Uniform, SECP	175
Figure 5.55 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, f_{33} , Uniform, SECP	175

NOTATION

a	crack depth or half length of the major axis of elliptical cracks or half length of the minor axis of quarter-elliptical cracks
$a_n, b_n, c_n, d_n, e_n,$ f_n, g_n, h_n, i_n, j_n	parameters of F_n
c	half length of the minor axis of quarter-elliptical cracks
f	point force
f_{li}	functions for stress state near crack front
n	types of stress distribution
r, θ	polar coordinates of plane normal to the crack front
t	plate thickness
u_l	components of displacement
x, y, z	Cartesian coordinate
x_1, x_2, x_3	Cartesian coordinate
A_l	parameters of V_0 or parameters of W_0
B^{2D}	2D dimensionless “biaxiality” parameter
B_{33}^{2D}	2D dimensionless “biaxiality” parameter relative to T_{33} stress
B^{thin}	local “biaxiality” factor of a thin plate
B_{33}^{thin}	local “biaxiality” factor of a thin plate relative to T_{33} stress
B_l	parameters of V_1 or parameters of W_1
C_l	parameters of V_2 or parameters of W_2

D_i	parameters of V_3 or parameters of W_3
E	Young's Modulus
F_n	functions to represent the normalized K_I
G	strain energy release rate
G_c	critical strain energy release rate
$I(s)$	interaction integral
J	J integral
K	stress intensity factor
$K_{surface}$	stress intensity factor generated by crack pressure
$K_{uncrack}$	stress intensity factor of uncracked specimen
K_I	stress intensity factor of opening mode
K_I^{2D}	2D stress intensity factor of opening mode
K_{II}	stress intensity factor of shearing mode
K_{III}	stress intensity factor of tearing mode
P	general point along the crack front
Q	load system applied to the specimen
T	T stress
T_{11}	T_{11} stress
T_{11}^{2D}	2D T_{11} stress
$T_{11surface}$	T_{11} stress generated by crack pressure
$T_{11uncrack}$	T_{11} stress of uncracked specimen
T_{12}	T_{12} stress
T_{22}	T_{22} stress
T_{33}	T_{33} stress
T_{33}^{2D}	2D T_{33} stress
$T_{33surface}$	T_{33} stress generated by crack pressure

$T_{33uncrack}$	T_{33} stress of uncracked specimen
T_z	out-of-plane constraint factor
V_n	functions to represent the normalized T_{11} stress
$V_{(s)}$	domain of interaction integral
W	plate width
W_n	functions to represent the normalized T_{33} stress
ε_{ij}	components of strain
$\mu_k(s)$	the direction normal to the crack front at point s
σ_0	uniform stress distribution
σ_{ij}	components of stresses
ϕ	parametric angle
ν	Poisson ratio
ΔK_{I3D}	thickness correction factor of stress intensity factor K_I
ΔT_{113D}	thickness correction factor of T_{11} stress
ΔT_{333D}	thickness correction factor of T_{33} stress

CHAPTER 1

INTRODUCTION

Things break every day. A fracture can cost little, if only a broken toy, or a cracked automobile windshield. However, fractures can also lead to catastrophic failure and involve enormous expense. In 1988, the roof of the forward cabin of a 737 tore away during flight, a flight attendant was killed and many passengers were injured. The cause was multiple fatigue cracks linking up to form a large, catastrophic crack (Chandler, 1989). The accident challenged the notion that fracture was well understood and under control in modern structures.

The traditional straightforward design consideration to avoid structural failure is to keep the maximum stress well under the posted material strength. However, engineers found that most failure began with cracks from investigating fallen structures. Fracture mechanics was introduced to analyze the relationship among stresses, cracks, and fracture toughness. Fracture mechanics is a set of theories describing the behavior of solids or structures with geometrical discontinuity at the scale of the structure. The objective of fracture mechanics is to characterize the local deformation around a crack tip in terms of the asymptotic field around the crack tip scaled by parameters that are a function of the loading and global geometry.

Griffith (1921) set up the first milestone for fracture mechanics by his famous paper that quantitatively relates the flaw size to the fracture stresses. It presented that failure happened when stress level was far below the material strength when the tests were conducted on glass. However, Griffith's approach is only good for brittle materials.

Another milestone came out when Irwin (1948, 1958) developed the concept of strain energy release rate G for ductile materials. G is defined as the rate of change in potential energy near the crack area for a linear elastic material. The crack will grow

when the strain energy release rate reaches the critical value G_c . Later, stress intensity factor K replaced the strain energy release rate G . K depends upon the applied stress, the size and placement of the crack, and the geometry of the specimen. K is defined from the elastic stresses near the tip of a sharp crack under remote loading. And K is used to predict the stress state near the tip of a crack.

The plasticity of the crack tips began to attract more attention from researchers after the fundamentals of fracture mechanics were established around 1960. Rice (1968) modeled the plastic deformation as nonlinear elastic behavior and extended the method of energy release rate to nonlinear materials. A path-independent line integral was established to express the energy release rate by Rice, called the J integral. Meanwhile, a parameter called crack tip opening displacement (CTOD) led the fracture mechanics research in Europe. These methodologies are generally regarded to as the “one-parameter approach”.

It is well known that the fracture resistance (toughness), which is the critical value of the crack driving force (K or J), is dependent on the constraint at the crack tip for a given cracked specimen. And the one-parameter fracture analysis is based on the fact that the crack has to be well constrained so that the minimum value of the material toughness can be achieved. This will ensure the fracture assessment is always in the safe side. But it may cause overly conservative assessment for the structures that are not well constrained, and thus raises the cost of product and maintenance.

Therefore, constraint-based failure analysis has been developed in order to describe the stress and strain fields ahead of a crack tip corresponding to different constraint levels. Rice (1974) suggested the T -stresses and stress intensity factor K to be considered as the constraint parameter.

T_{11} stress is defined as the second term of Williams (1957) series expansion of the stress component parallel to the crack plane. It can be used as the parameter to quantify the constraint effect. The Williams' series expand the Mode I stress field about

a sharp crack in a linear elastic body with respect to r , the distance from the crack tip. Larsson and Carlsson (1973) and Rice (1974) showed that the magnitude of the T -stress affects the size and shape of the plastic zone and the region of tensile triaxiality ahead of the crack tip. If the T_{11} stress is positive, J -dominance exists and a single parameter J can be used for a fracture criterion. If the T_{11} stress is negative, a two-parameter approach (J and T_{11}) is required to characterize the stress fields. Betegon and Hancock (1991), Du and Hancock (1991), O' Dowd and Shih (1991) and Wang (1993) indicated that the T_{11} stress and J -integral provide effective two-parameters characterization of plane strain elastic-plastic crack tip fields in a variety of crack configuration and loading conditions.

For three-dimensional elastic crack problem, Rice (1974) suggested that both T_{11} and T_{33} stresses should be considered as the constraint parameters in order to characterize the stress field in the vicinity of crack front. T_{33} stress is the second term of Williams (1957) series expansion of the stress component tangential along the crack front. Gonzalez-Albuixech et.al (2011) recently showed that the consideration of the components T_{11} and T_{33} was necessary to obtain a correct description of the stress state near the crack front.

The research of the three-dimensional solution of three parameters K , T_{11} and T_{33} , and stress state around crack front have gained more attention recently. The thickness effective of the three parameters are also studied due to the various thickness in engineering application.

Raju and Newman (1977), Tracey (1974), Kwon and Sun (2000), and Kacinaauska et.al (2005) calculated the stress intensity factors by three-dimensional finite element analyses. The differences of the stress intensity factors between the three-dimensional along the crack front and two-dimensional were given.

Nakamura and Parks (1988, 1989, and 1992) analyzed the three-dimensional stress intensity factor and T_{11} for thin elastic plate based on finite element analysis. The

thickness effect of the K and T_{11} were described. Wang (2003), Wang and Bell (2004), and Qu and Wang (2006) presented the T_{11} or T_{33} stress for semi-elliptical surface or embedded elliptical cracks in a finite thickness plate under various loading conditions. Zhao et.al (2007) obtained the distributions of normalized K and T_{11} along the crack front for embedded elliptical cracks in elastic plates. And three-parameter K - T_{11} - T_z describe the stress field around the crack front was provided with the corresponding out-of-plane constraint factor T_z . Meshii et. al (2010), Meshii and Tanaka (2010) performed both calculation and experimental for the thickness effect on the toughness affected by T_{33} .

The superposition method is a very powerful well-known tool for calculating stress intensity factor K , T_{11} or T_{33} stress under stress distributions. The superposition formula can be found in papers of Y. Yamamoto and co-workers (1973 and 1983), Wang and Bell (2004), Qu and Wang (2006) and Meshii et.al (2010).

The single edge cracked plate is a common specimen in research and practice of fracture mechanics. Most solutions are for two-dimensional plane-strain or plane-stress cases. However, some three-dimensional solutions of the K , T_{11} or T_{33} for the single edge cracked were studied but not systematic. Meanwhile, the thickness effect of T_{33} is lack of research. The stress state near the crack front described by three-parameters need to be investigated as well.

The objectives of this thesis research are:

- i). Systematic calculation of the fracture mechanics parameters K_I , T_{11} and T_{33} stresses for three-dimensional single edge cracked plate (SECP) specimen for wide range of parameter, relative crack depth a/W , relative thickness t/W and four loading conditions (uniform, linear, parabolic and cubic) applied on the crack surface.
- ii). Investigate the thickness effect (t/W) on the three fracture mechanics parameters in SECP specimen with crack surface subjected to the uniform stress distribution.
- iii). Study the accuracy of three parameter K_I - T_{11} - T_{33} characterizations of stress fields

in three-dimensional SECP specimens under uniform loading condition.

1.1 THESIS OUTLINE

This thesis consists of six chapters. Chapter 1 is an introduction to the current research. In Chapter 2, an overview of the three-parameter definition, finite element method and superposition formula to calculate the stress intensity factor K_I , T_{11} and T_{33} stresses, and the stress state along the crack front are presented. Extensively literature review is also conducted in Chapter 2. The solutions of normalized stress intensity factor K_I , T_{11} and T_{33} stresses are derived in Chapter 3 for a single edged crack subjected to the various stress distribution in three-dimensional SECP specimens. And empirical equations of normalized stress intensity factor K_I , T_{11} and T_{33} stresses at the mid-plane are obtained. In Chapter 4, the relative thickness and relative crack length effect of the mid-plane stress intensity factor K_I , T_{11} and T_{33} stresses are studied. And the stress intensity factor, T_{11} and T_{33} stresses variation along the crack front are presented. The stress state near the crack front described by stress intensity factor K_I , T_{11} and T_{33} stresses are discussed in Chapter 5. Chapter 6 is the conclusion and recommendations of the thesis.

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

In this chapter, the background on fracture mechanics parameters K , T_{11} , T_{33} , and the finite element method of obtaining them are summarized. Meanwhile, the superposition methods of calculating them are presented. Literature review is conducted on the calculation of K , T_{11} , and T_{33} for three-dimensional cracked specimens.

2.1 LINEAR ELASTIC FRACTURE MECHANICS

2.1.1 Fracture Modes and Stress Intensity Factor

The linear elastic fracture mechanics was founded based on the theory of the stress intensity factor K , which was introduced by Irwin (1948, 1958). The objective of linear elastic fracture mechanics (LEFM) is to predict the critical loads that will cause a crack to grow in a brittle material. And the fundamental postulate of LEFM is that the value of the stress intensity factor determined the behavior of cracks. The stress intensity factors associated with three independent modes of movement of crack surfaces, opening mode *I* (K_I), shearing mode *II* (K_{II}), and tearing mode *III* (K_{III}), see Figure 2.1. The stress intensity factor may be used to determine the static strength of a cracked structure, the rate of growth of a crack in a structure subjected to variable loading, and the rate of growth of a crack in a loaded structure in a corrosive environment. Following Williams series expansion (Williams, 1957) for the stress state around the crack tip can be written as:

$$\sigma_{ij} = r^{-\frac{1}{2}} \{ K_I f_{ij}^I(\theta) + K_{II} f_{ij}^{II}(\theta) + K_{III} f_{ij}^{III}(\theta) \} + \text{higher order terms} \quad (2.1)$$

where σ_{ij} are the near tip stresses, and K_I , K_{II} , K_{III} are the stress intensity factors. Only mode I stress intensity factor K_I is considered in this thesis.

2.1.2 T -Stress

It has been realized that the ability of a single parameter such as the stress intensity factor to fully characterize crack tip conditions irrespective of geometry and load level is limited. Two parameters description of the crack tip stress-strain state have been studied. The next higher order, non-singular, stress terms in the Williams (1957) series expansion play an important role in crack behavior.

In an isotropic linear elastic body containing a crack subjected to symmetric (mode I) loading, the Williams (1957) series expansion for linear elastic crack-tip fields can be written as:

$$\begin{aligned}
 \sigma_{11} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + T_{11}, \\
 \sigma_{22} &= \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right), \\
 \sigma_{33} &= \frac{K_I}{\sqrt{2\pi r}} 2\nu \cos \frac{\theta}{2} + T_{33}, \\
 \sigma_{12} &= \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}, \\
 \sigma_{13} &= T_{13}, \\
 \sigma_{23} &= 0.
 \end{aligned}
 \tag{2.2}$$

where r and θ are in-plane polar coordinates of plane normal to the crack front as shown in Fig.2.2, and K_I is the local modal I stress intensity factor at crack front. ν is the Poisson's ratio. The terms T_{11} , T_{33} and T_{13} are together called the T -stress. T_{11} and T_{33} are the amplitudes of the second order terms in the three-dimensional series expansion of the crack front stress field in the x_1 and x_3 directions, respectively. And only T_{11} and T_{33} are discussed in present work.

From Williams' expansion, where higher order terms are neglected since the circular path is very close to the crack front, the following relationship is obtained:

$$\frac{\sigma_{ii} - T_{ii}}{K_I/\sqrt{2\pi r}} = f_{ii}(\theta) \quad i = 1, 2, 3 \quad (2.3)$$

With

$$\begin{aligned} f_{11}(\theta) &= \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ f_{22}(\theta) &= \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ f_{33}(\theta) &= \nu(f_{11}(\theta) + f_{22}(\theta)) = 2\nu \cos \frac{\theta}{2} \end{aligned} \quad (2.4)$$

and $T_{22} = 0$. That is the stress state described by K_I , T_{11} and T_{33} .

Larsson and Carlsson (1973) and Rice (1974) showed that T_{11} stress can substantially affect the size and shape of the crack-tip plastic zone. Cotterell and Rice (1980) showed that the sign of T_{11} stress determines the stability of a straight mode I path is stable if $T < 0$ and unstable if $T > 0$. It means that positive T_{11} stress strengthens the level of crack tip stress triaxiality and leads to high crack tip constraint, while negative T_{11} stress reduces the level of crack tip stress triaxiality and leads to the loss the crack tip constraint.

The T_{11} stress plays an important role in elastic-plastic fracture analysis, even though the T_{11} stress is calculated from the linear elastic material properties of the same solid containing the crack.

Different methods to evaluate the T_{11} stress results have been developed. Larsson and Carlsson (1973) and Wang and Parks (1992) and Coldthorpe (1992) used stress substitution methods to determine T_{11} stress for three-dimensional cracked bodies. Leeves and Radon (1982) developed a computer program to determine the T_{11} stress. Cardew et al. (1984) and Kfoury (1986) developed a method of calculating the T_{11} stress for two-dimensional geometries based on an unpublished theorem due to Eshelby. The

properties of the path-independent J -integral were used in this method. Sham (1991) developed a high accurate method of evaluating the T_{11} stress for two-dimensional cracked geometries, which using so-called higher order weight functions evaluated by means of special integrals within a modified finite element procedure. Nakamura and Parks (1992) developed a method of evaluating T_{11} stress for three-dimensional cracked structures which was essentially the 3D generalization of the Eshelby method. An interaction domain integral involving the stress, strain and displacement fields of the crack body subjected to its normal loading and those fields appropriate to a line load acting along the crack front was formulated. The local value of an interaction integral was used to derive the T_{11} stress at any point of the crack. This method was used to evaluate the T_{11} stress in ABAQUS (2008). The line spring method was used by Wang and Parks (1992) to develop a means of calculate the variation of T_{11} stress along the crack front.

2.1.3 Superposition Method for the determination of K , T_{11} AND T_{33}

Y. Yamamoto and co-workers (1983) developed a method of computing stress intensity factors based on the superposition of analytical and finite element solutions. The method was applied to problems of two-dimensional fracture, through the thickness cracks in plates subject to tension (Y. Yamamoto and N. Tokuda, 1973), and circumferentially cracked cylindrical bars subjected to constant tension and shear (Y. Yamamoto and Y. Sumi, 1974). Wang and Bell (2004) gave the formula to obtain T_{11} stress based on superposition method. Qu and Wang (2006) and Meshii et.al (2010) introduced the formula for T_{33} stress by superposition method.

The superposition method for the stress intensity factor and T -stresses calculation is discussed here. Consider a cracked two dimensional specimen loaded by mode I load system Q , as demonstrated in Figure 2.3. The stress field of this problem can be divided into two parts: the regular field which appears under the same loading

conditions in the uncracked specimen, and a corrective field due to the presence of the crack. The elastic stress intensity factor or T -stresses for the problem can be calculated from the summation of the stress intensity factor or T -stresses for these two problems, respectively.

The stress intensity factor for the crack specimen was given

$$K = K_{uncrack} + K_{surface} \quad (2.5)$$

Since the K for the regular uncracked specimen is zero,

$$K = K_{surface} \quad (2.6)$$

T_{11} stress for the cracked specimen was given

$$T_{11} = T_{11uncrack} + T_{11surface} \quad (2.7)$$

where $T_{11uncrack}$ is the T_{11} stress generated in the regular field and $T_{11surface}$ is the T_{11} stress generated by crack pressure. Applying the two-dimensional case of crack tip stress fields of equation, and the relationship between local and global coordinates, the corresponding T_{11} stress for the regular field can be found to be

$$T_{11uncrack} = \lim_{r \rightarrow 0^+} (\sigma_{11}|_{\theta=0} - \sigma_{22}|_{\theta=0}) = (\sigma_x - \sigma_z)|_{x=a, z=0} \quad (2.8)$$

where σ_x ($=\sigma_{11}$) and σ_z ($=\sigma_{33}$) are the stress components from the uncracked body under applied load system Q , and the location is at the crack tip. The T_{11} stress for the problem is solved by substituting equation (2.8) into equation (2.7),

$$T_{11} = (\sigma_x - \sigma_z)|_{x=a, z=0} + T_{11surface} \quad (2.9)$$

For three-dimensional cracked plate, the T_{11} is changing along the crack front, as shown in Figure 2.4. The counterpart of equation (2.7) is

$$T_{11}(P) = T_{11uncrack}(P) + T_{11surface}(P) \quad (2.10)$$

where P is the general point along the crack front. The uncracked T_{11} is

$$T_{11uncrack}(P) = \left[\lim_{r \rightarrow 0^+} (\sigma_{11}|_{\theta=0} - \sigma_{22}|_{\theta=0}) \right] \Big|_P = (\sigma_{11} - \sigma_{22})|_P \quad (2.11)$$

The T_{11} stress for the problem under consideration is

$$T_{11}(P) = (\sigma_{11} - \sigma_{22})|_P + T_{11surface}(P) \quad (2.12)$$

Similarly, for T_{33} stress

$$T_{33}(P) = T_{33uncrack}(P) + T_{33surface}(P) \quad (2.13)$$

By considering the $\theta = 0$ plane, and by subtracting the singular term in σ_{33} and σ_{22} in Eq. (2.13), T_{33} stress for the uncracked sub-problem is represented by

$$T_{33uncrack}(P) = \left[\lim_{r \rightarrow 0^+} (\sigma_{33}|_{\theta=0} - 2\nu\sigma_{22}|_{\theta=0}) \right] \Big|_P = (\sigma_{33} - 2\nu\sigma_{22})|_P \quad (2.14)$$

The T_{33} stress for the problem under consideration is then

$$T_{33}(P) = (\sigma_{33} - 2\nu\sigma_{22})|_P + T_{33surface}(P) \quad (2.15)$$

2.2 FINITE ELEMENT ANALYSIS OF CRACKED COMPONENTS

In this section, the finite element method used in this thesis to calculate stress intensity factor K_I , T_{11} and T_{33} stresses is summarized.

2.2.1 Calculation of stress intensity factor K_I

Several methods of estimating stress intensity factors are available, such as the alternating, weight function, body force, boundary element analysis and finite element analysis. The finite element method is employed in the simulation technique because of its versatility and generality for complicated cracked structures. Barsoum (1976) introduced a way to improve the numerical results in vicinity of the crack tip by using singular finite elements. One side of the element must be collapsed in order to have its three nodes just in one geometric point. The square-root stress singularity predicted by LEFM is exactly simulated by shifting the mid-side nodes to quarter-positions.

The J -integral, introduced by Rice (1968), is originally a two-dimensional line integral. It is used in rate-independent quasi-static fracture analysis to characterize the energy release associated with crack growth. It can be related to the stress intensity factor if the material response is linear and it also provides a method for analyzing fracture in nonlinear materials. The J -integral is path independent when contours are taken around a crack tip. It assumes that the material is homogeneous in the crack direction and the material is elastic.

The relation between the J and the stress intensity factor for mode I in a linear elastic material is

$$\begin{aligned} J &= \frac{1 - \nu^2}{E} K_I^2 && \text{for plane strain} \\ J &= \frac{1}{E} K_I^2 && \text{for plane stress} \end{aligned} \tag{2.16}$$

The J -integral can also be generalized to three-dimensions by considering a tubular surface around the crack front. This domain integral method is used to evaluate contour integrals automatically in ABAQUS (2008).

2.2.2 Calculation of T_{11} Stress

Following is the method used to calculate T_{11} stress which introduced by Nakamura and Parks (1992). Considering a three dimensional crack front with a continuously turning tangent as shown in Figure 2.2, assume a line-load of magnitude $f_k = f\mu_k(s)$ to be applied along the crack front as shown in Figure 2.5. $\mu_k(s)$ defines the direction normal to the crack front and in the plane of the crack at point s . The solution for this problem is the case of a plane strain semi-infinite crack with a point force f applied at the crack tip in the direction parallel to the crack. Using superscript 'L' to designate the stress and displacement fields, the analytical solution gives (Timoshenko and Goodier, 1970)

$$\begin{aligned}\sigma_{11}^L &= -\frac{f}{\pi r} \cos^3 \theta, & \sigma_{22}^L &= -\frac{f}{\pi r} \cos \theta \sin^2 \theta, & \sigma_{33}^L &= -\frac{f}{\pi r} \nu \cos \theta, \\ \sigma_{12}^L &= -\frac{f}{\pi r} \cos^2 \theta \sin \theta, & \sigma_{13}^L &= \sigma_{23}^L = 0\end{aligned}\tag{2.17a}$$

and

$$\begin{aligned}u_1^L &= -\frac{1-\nu^2}{E} \frac{f}{\pi} \left\{ \ln \left(\frac{r}{d} \right) + \frac{\sin^2 \theta}{2(1-\nu)} \right\}, \\ u_2^L &= -\frac{1+\nu}{2E} \frac{f}{\pi} \{ (1-2\nu)\theta - \cos \theta \sin \theta \}, \\ u_3^L &= 0\end{aligned}\tag{2.17b}$$

Employing the above solution as an auxiliary field, Cardew et al. (1984) and Kfoury (1986) extracted the T_{11} stress for two-dimensional crack problems by introducing an interaction J integral based on Eshelby's theorem. Nakamura and Parks (1992) extended this method to three-dimensional crack problems and provided the domain integral formulations. Combined with finite element method, it was shown to be a simple and effective method for the determination of the T_{11} stress of two-dimensional and three-dimensional crack specimens. The interaction integral is introduced in the following

domain integral form for three-dimensional crack problems,

$$I(s) = \frac{1}{A_c} \oint \left\{ \left(\sigma_{ij} \frac{\partial u_i^L}{\partial x_k} + \sigma_{ij}^L \frac{\partial u_i}{\partial x_k} \right) \frac{\partial q_k}{\partial x_j} - \varepsilon_{ij}^L \sigma_{ij} \frac{\partial q_k}{\partial x_k} \right\} dV \quad (2.18)$$

The integration is in the domain $V(s)$ which encloses the crack front segment between $s - \varepsilon$ and $s + \varepsilon$, $q_k(s)$ defines the virtual extension of the crack front segment $s - \varepsilon \ll s \ll s + \varepsilon$ and A_c is the increase in crack area generated by the virtual crack advance; σ_{ij} , ε_{ij} and u_i are the stress, strain and displacement components of the three-dimensional crack problem under consideration; σ_{ij}^L , ε_{ij}^L and u_i^L are the corresponding components in the line-load auxiliary solution given by (2.17a) and (2.17b).

The crack tip T_{11} stress is related to $I(s)$, the interaction integral, by the following:

$$T_{11}(s) = \frac{E}{1 - \nu^2} \left\{ \frac{I(s)}{f} + \nu \varepsilon_{33}(s) \right\} \quad (2.19)$$

This domain integral method is used to evaluate T_{11} stress automatically in ABAQUS (2008).

2.2.3 Calculation of T_{33} Stress

If the out-of-plane strain ε_{33} is solved, an expression for calculating T_{33} stress sufficiently far from the vertex corners can be obtained in a straightforward manner. The out-of-plane strain ε_{33} at a given position of the crack front can be obtained by application of the Hooke's law for an isotopic material.

$$\varepsilon_{33} = \frac{1}{E} [\sigma_{33} - \nu(\sigma_{11} + \sigma_{22})] \quad (2.20)$$

After substitution of (2.2) in (2.20), obtain

$$\varepsilon_{33} = \frac{1}{E} \left[2\nu \frac{K_I}{\sqrt{2\pi r}} + T_{33} - \nu \left(\frac{K_I}{\sqrt{2\pi r}} + T_{11} + \frac{K_I}{\sqrt{2\pi r}} \right) \right] \quad (2.21)$$

The following relationship is obtained

$$\varepsilon_{33} = \frac{1}{E} [T_{33} - \nu T_{11}] \quad (2.22)$$

It is shown that the out-of-plane strain ε_{33} is dominated by second order terms that cannot be neglected at crack front.

$$T_{33} = E\varepsilon_{33} + \nu T_{11} \quad (2.23)$$

The value of ε_{33} and T_{11} have to be known in advance to calculate T_{33} .

2.3 LITERATURE REVIEW FOR THREE-DIMENSIONAL SOLUTION OF K , T_{11} AND T_{33} AND STRESS STATE AROUND CRACK FRONT

Earlier publications on three-dimensional cracks can be found in the review paper by Sih (1971). Raju and Newman (1977) used the force method and focused on obtaining stress intensity factor without the assumption of plane strain or plane stress at the crack tip. The stress intensity factor in the mid plane of center crack tension, single and double edge crack tension and compact specimens with through-the-thickness crack were calculated using a three dimensional finite element elastic stress analysis. The results were within 3% of the two-dimensional plane strain values.

Tracey (1974) built up an elastic three-dimensional finite element model for the through-the-thickness cracked compact tension specimen and calculated the mode I stress

intensity factor along the crack front. The stress intensity factor variation with the thickness was discussed. It showed that the difference of the two and three-dimensional stress intensity factor solution was less than 1% in the region $0 < z/t < 0.28$, where z is the axis along the thickness direction with original point at mid-plane of the specimen, and t is the thickness. However, the differences greater than 10% only in the region from $z/t = 0.44$ to the free surface $z/t = 0.50$.

Kacianauskas et.al (2005) investigated the three-dimensional effects of the two-dimensional mode I stress intensity factor for the plate with a V-shaped straight through-thickness notch and gave a suggestion for three-dimensional thickness dependent correction of SIF.

Kwon and Sun (2000) performed a global-local finite element technique with sub-modeling to achieve the refinement required to obtain an accurate mode I stress field for plates with a through-the-thickness crack. The effect of plate thickness on the size of the three-dimensional region was studied and an approximate 3D stress intensity factor solution without 3D calculation at the mid-plane was suggested. A formula describes the relation between the 3D stress intensity factor at the mid-plane and the 2D stress intensity factor was given in that paper.

Nakamura and Parks (1988, 1989, and 1992) made great contributions for three-dimensional stress state analysis around the crack front in a thin elastic plate. Nakamura and Parks (1988) analyzed the three-dimensional of first stress intensity field near the crack front of a thin elastic plate based on very detailed full-field finite element analysis. Nakamura and Parks (1989) computed the antisymmetrical three-dimensional stress field in the same geometry. The asymptotic form of stress near the intersection under general loading conditions is presented, along with the corner stress intensity factor. T_{11} stress distributions of single edge cracked plates and surface crack model in the plate are determined by finite element analysis. Nakamura and Parks (1992) gave an approximant formula for T_{11} stress distributions along the single edge cracked plates in

thin plane.

Henry and Luxmoore (1995) used thick three-dimensional finite element models of centre cracked plates to study the variation in the biaxiality factor β with the crack aspect ratio a/W . The boundary layer method and the displacement field method were chosen to calculate the T_{11} stress in three-dimensions. And it was shown that the boundary layer method gave results that compare rather well with the two-dimensional plane strain values than the displacement field method. It also presented that the material's Poisson's ratio and the specimen's thickness affect the three-dimensional evaluation of the biaxiality factor.

Gao (1992) analyzed the variation of T_{11} and T_{33} along a slightly wavy three-dimensional crack front. The results indicated that the T_{11} increased with the curvature of the crack front but T_{33} decreased with the crack front curvature.

Sherry et.al (1995) collected the published solutions for the T_{11} stresses for a range of two and three-dimensional cracked geometries and presented some new results.

Wang and Bell (2004) evaluated the T_{11} stresses solutions for a semi-elliptical surface crack fronts in a finite thickness plate under various loading conditions by using Nakamura and Parks' method. Qu and Wang (2006) also calculated the T_{11} and T_{33} stresses along a quarter-elliptical corner crack in a finite thickness plate subjected to tension and bending.

Meshii and Tanaka (2010) studied the CT specimen thickness effect on the T_{33} stress, and an experimental expression for the thickness effect on the fracture toughness using T_{33} stress is proposed for 0.55% carbon steel S55C. From their finite element results, it noted that small test specimen thickness dependence for the in-plane elastic parameters K_I and T_{11} , and large dependence for the out-of-plane parameter T_{33} . And a suggestion of improving the existing methods for correlating fracture toughness obtained by test specimens with the toughness of actual cracks found in the structure using T_{33} stress was given. Meshii et.al (2010) presented the T_{11} and T_{33} for semi-elliptical

axial surface cracks in a cylinder subjected to mode I non-uniform stress on the crack surface. And the formulation of the superposition principle was given for the T_{33} stress.

Gonzalez-Albuixech et.al (2011) analyzed influence of the T_{33} stress on the three-dimensional stress state around a mode I elliptical corner cracks with different aspect ratios in an elastic plate. It shows that the consideration of the components T_{11} and T_{33} is necessary to obtain a correct description of the stress state near the crack front.

Wang (2003) calculated the elastic T_{11} -stress for semi-elliptical surface cracks in finite thickness plates by three dimensional finite element analyses. The T_{11} -stress solutions at the deepest, the surface and the middle points of the crack front under tension or bending were presented for cracks with relative depths a/t values of 0.2, 0.4, 0.6 or 0.8 and aspect ratio a/c values of 0.2, 0.4, 0.6 or 1.0. The numerical results are approximated by empirical formulae fitted with an accuracy of 1% or better.

Yu and Wang (2005) presented the application of the weight function method for the calculation of elastic T_{11} -stress for semi-elliptical surface cracks. The weight function method was extended from two dimensional to three dimensional crack problem. And the T_{11} -stress weight functions for the deepest point and for any general point along the crack front of semi-elliptical surface cracks in finite-thickness plates for wide ranges of crack aspect ratios a/c and relative depths a/t were derived.

Qu and Wang (2006) calculated the T -stresses (T_{11} and T_{33} stress) for a quarter-elliptical corner crack in a finite thickness plate with a wide range of crack aspect ratios and relative depths. The loads were remote tension and bending. And empirical equations for T -stresses at two points near the free surfaces were also considered.

Zhao et.al (2007) obtained the distributions of normalized stress intensity factor and T_{11} stress along the crack front with aspect ratios of 0.2, 0.4, 0.5, 0.6, 0.8 and 1.0 for embedded elliptical cracks in elastic plates. And far-field tension and the effect of Poisson's ratio were also considered. Empirical formulae were given for the convenience of engineering applications. Combining with the corresponding out-of-plane constraint

factor T_z , the three-parameter $K-T_{11}-T_z$ describe the stress field around the crack front was provided.

In this thesis, continuation of these early works is carried out. In particular, the three-dimensional single edge cracked plate (SECP) specimens are investigated.

2.4 SINGLE EDGE CRACKED PLATE SPECIMEN

The single edge cracked plate (SECP) is a common specimen in research and practice of fracture mechanics, see Figure 2.6. The most common test specimen configurations are single edge notched bend s (SENB or three-point bend), and the compact tension (CT) specimens.

The single edge notched bend (SENB) specimen is one of specimens that are permitted in ASTM standards that characterize fracture initiation and crack growth. There are three important characteristic dimensions for SENB specimen: the crack depth (a), the thickness (t) and the width (W). In ASTM standard test, $W = 2t$ and $a/W = 0.5$. The SENB specimens have been performed vast majority of fracture toughness tests.

The most tests for SENB specimen are three-point bending, which is high constraint test. But the low constraint test can be applied on the SENB specimen by changing t/W , a/W , and loading condition.

The simulation of crack parameters for the single edge cracked plate specimen is most focus on the stress intensity factor K and T_{11} stress. There are some solutions for two and three dimensional geometries. And a few researchers analyzed the thickness effect of the stress intensity factor and T_{11} stress for single edge cracked plate specimen. The three-dimensional of T_{33} stress distribution and the stress state described by three parameters (stress intensity factor, T_{11} and T_{33} stress) near the crack front are not studied systematically. Next, the available solutions are reviewed.

2.4.1 Two dimensional solutions for single edge cracked plate specimen

Gross et al. (1965) calculated the stress intensity factor for single edge tension notch, three point bend and compact tension fracture specimens. Fett (1998) provided the solutions of stress intensity factor and T_{11} stress for rectangular plates with various length-to-width ratios and different types of cracks (single and double-edge cracks) by the boundary collocation method. Sham (1991) gave the numerical values of the elastic T_{11} stress and stress intensity factor for single edge notch specimens under tension, pure bending and three-point bend by using higher order weight functions. Wang (2002) calculated elastic T_{11} stress solutions for cracked test specimens by finite element analysis. And the T_{11} stress solutions were presented for single edge cracked plates, double edge crack plates and centre cracked plates. The stress distributions applied to the crack face were uniform, linear, parabolic or cubic.

2.4.2 Three dimensional solutions for single edge cracked plate specimen and the thickness effect

Some researchers gave the three dimensional solutions of stress intensity factor and T -stresses for single edge cracked plate specimen. Meantime, the thickness effect for stress intensity factor or T_{11} stress was studied.

Kwon and Sun (2000) analyzed the effect of plane thickness on the size of the three-dimensional region and suggested an approximate 3D stress intensity factor solution without 3D calculation at the mid-plane for through thickness crack. It showed that the 3D stress intensity factor at the mid-plane can obtain front the 2D stress intensity factor with the relation

$$K_{I3D}/K_{I2D} = \sqrt{1/(1 - \nu^2)} \quad (2.24)$$

where ν is the Poisson's ratio.

Kacianauskas et.al (2005) assumed the thickness influence is characterized by the single stress intensity parameter $K_{I3D}(z/t)$, which is thickness-dependent variable. The three dimensional stress intensity factor can be presented as two dimensional value corrected by the thickness influence

$$K_{I3D}(z/t) = K_{I2D} \Delta K_{I3D}(z/t) \quad (2.25)$$

where z is the axis along the crack front, t is the thickness of the plate, and $\Delta K_{I3D}(z/t)$ is the thickness correction factor .

Nakamura and Parks (1992) studied the stress intensity factor and T_{11} stress affected by Poisson's ratio, crack length, and plate thickness for single edge cracked plates in thin plane. An approximant formula to predict T_{11} stress distributions along the cracked front in thin plane was given. The following is the detail about the formula.

From the finite element results of the T_{11} stress for single edge cracked plate, it is noticed that when the thickness t became large compared to in-plane dimensions (e.g. crack length a), the thickness-dependence of the T_{11} stress distribution merge to two-dimensional solution, so T_{11} can be expressed as

$$T_{11}(z/t, \nu) = B^{2D} \frac{K_I^{2D}}{\sqrt{\pi a}} \equiv T_{11}^{2D} \quad \text{as } t/a \rightarrow \infty \quad (2.26)$$

where B^{2D} is a dimensionless "biaxiality" parameter depending only on in-plane geometry and loading. Larsson and Carlsson (1973), Leever and Radon (1983), Cardew et al. (1984), Kfoury (1986) and Sham (1991) gave the values of B^{2D} for several crack geometries.

It is proposed that, from considerations of superposition and dimensional analysis, as relative thickness diminished in the boundary value problem posed, the through-thickness variation of the T_{11} stress distribution can be expressed as,

$$T_{11}(z/t, \nu) \cong T_{11}^{2D} + B^{thin}(z/t, \nu) \frac{K_I(z/t, \nu)}{\sqrt{\pi t}} \quad (2.27)$$

where the T_{11}^{2D} is the two-dimensional solution. Here T_{11}^{2D} represents the “remote” value which applies in the plane stress field found at radii large compared to thickness t , but small compared to crack length a . The B^{thin} represents the local “biaxiality” factor of a thin plate. The functional form of (2.27) is motivated from the perspective of small relative thickness. However, a broader range of applicability is anticipated since the thick plate result (2.26) is recovered as $t/a \rightarrow \infty$ at fixed in-plane dimensions. It also notes that the normalized T_{11} stress decreases and approaches the two-dimensional solution as the plate thickness increases from the finite element solutions.

Then when the thickness is large, Eq.(2.25) can be written as,

$$T_{11}(z/t, \nu) = B^{2D}(z/t, \nu) \frac{K_I^{2D}(z/t, \nu)}{\sqrt{\pi a}} \equiv T_{11}^{2D} \quad \text{as } t/a \rightarrow \infty \quad (2.28)$$

Nakamura and Parks (1992) propose that locally there exists a T_{11} stress distribution along the thin plate crack front based on dimensional grounds, the formula of the T_{11} stress distribution is

$$T_{11}(z/t, \nu) = B^{thin}(z/t, \nu) \frac{K_I(z/t, \nu)}{\sqrt{\pi t}} \quad \text{as } t/a \rightarrow 0 \quad (2.29)$$

With the use of Eq. (2.26), (2.27), (2.28) and (2.29), the normalized $T_{11}(z/t, \nu)$ stress can be expressed as

$$\frac{T_{11}(z/t, \nu)}{K_I(z/t, \nu)/\sqrt{\pi t}} \cong \sqrt{\frac{t}{a}} B^{2D} + B^{thin}(z/t, \nu) \quad (2.30)$$

The first term on the right-hand side of Eq. (2.30) dominates for a thick plate, while it vanishes for a thin plate. The mid-plane value of $B^{thin}(0, \nu = 0.3)$ was given as 0.045.

2.4.3 The present research for single edge cracked plate

In summary, there are three main reasons to perform present research for single edge cracked plate:

- i). No systemic study for three dimensional stress intensity factor and T -stresses (T_{11} and T_{33} stress). Former research was either focused on two dimensional geometry or limited relative crack length and relative thickness for three dimensional geometries.
- ii). The thickness effect is not investigated thoroughly. Especially T_{33} stress variation along the crack front for single edge cracked plate was lack of study.
- iii). The three parameters control the stress state around the crack front of the single edge cracked plate need to be analyzed.

The stress intensity factor and T -stresses (T_{11} and T_{33} stress) for different crack depth and thickness of the single edge cracked plate are calculated by finite element analysis in Chapter 3. The thickness effect and the stress state around the crack front described by three parameters are also studied in Chapter 4 and 5.

Figure 2.1 The three independent modes of movement of crack surfaces

Figure 2.2 Three-dimensional coordinate system for the region along the crack front

Figure 2.3 Principle of superposition, two-dimensional

Figure 2.4 Principle of superposition, three-dimensional

Figure 2.5 A schematic of elements used in the definition of the interaction integral: (a), (b), (c)

Figure 2.6 Single edge cracked specimen

CHAPTER 3

STRESS INTENSITY FACTOR AND T-STRESSES FOR CRACK IN THREE DIMENSIONAL SINGLE EDGE CRACKED PLATE SPECIMENS

In this chapter, three-dimensional finite element analyses are conducted to obtain the stress intensity factor K_I and T -stresses (T_{11} and T_{33}) for three-dimensional single edge cracked plate (SECP) specimens for wide range of a/W and t/W ratios to investigate the effect of thickness on these fracture parameters. And empirical equations of normalized stress intensity factor K_I , T_{11} and T_{33} stresses at the mid-plane are obtained for engineering application.

3.1 FINITE ELEMENT ANALYSIS FOR CRACK IN THREE-DIMENSIONAL SINGLE EDGE CRACKED PLATE SPECIMENS

In this section, the three-dimensional finite element model to calculate the three fracture parameters are created and verified. The normalized stress intensity factor K_I , T_{11} and T_{33} stresses along the crack front for different relative crack depth, a/W , and relative thickness, t/W for SECP specimen with crack surface subjected to different stress distributions are obtained.

3.1.1 Finite Element Model

Three-dimensional model of single edge cracked plate (SECP) specimen was built in ABAQUS (2008). The geometry of the specimen is as shown in Figure 3.1. In the

model, the ratio of half-height, H , to the width, W , was chosen to be $H/W = 3$. Geometries through variations in crack depth, a , and plate thickness, t , were considered. The Poisson's ratio was 0.3 and Young's module was 206 GPa. The values of relative crack depth a/W were 0.2, 0.4, 0.6 and 0.8, respectively. The values of t/W were chosen as 0.1, 0.2, 0.5, 1.0, 2.0 and 4.0, respectively.

A quarter of the plate model was built to apply finite element analysis in ABAQUS (2008). The symmetry conditions across the mid-plane ($z/t = 0$) and the crack, and ligament plane ($y/t = 0$) were used in constructing the finite element mesh. Thinner elements were employed away from the mid-plane due to a high stress gradient along the crack front was expected near the free-surface. The number of element layers through the half-thickness was 20 for $t/W=0.1$, 30 for $t/W=0.2$, and 40 for $t/W=0.5$, 1.0, 2.0 and 4.0, respectively.

The 20-noded three-dimensional quadratic elements with reduced integration (C3D20R) were used to construct the quarter-model in ABAQUS (2008). In order to model the square root singularity at the crack tip, three-dimensional prism elements with four midside nodes at the quarter points (a degenerate cube with one face collapsed) were used. The C3D20R element and mesh configuration near the crack tip are shown in Figure 3.2

The loads were applied directly to the crack face, as shown in Figure 3.1. The stress intensity factor and T -stresses (T_{11} and T_{33}) were calculated for the following stress fields:

Uniform stress field

$$\sigma(x) = \sigma_0 \quad (3.1)$$

Linear stress field

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right) \quad (3.2)$$

Parabolic stress field

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right)^2 \quad (3.3)$$

Cubic stress field

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{a}\right)^3 \quad (3.4)$$

A finite element model for $a/W=0.4$, $t/W=0.5$, single edge cracked plate specimen is shown as Figure 3.3.

The stress intensity factors along crack front were calculated by *J-integral* (energy release rate) method. The calculation of T_{11} stress was based on Nakamura and Parks' (1992) interaction integral method. This technique was discussed in section 2.4.2. Five contours were used in the current analysis, the *J-integral* was stable after two contours, and the average value of T_{11} results from the last two outer contours was used to calculate crack T_{11} distributions. T_{33} was calculated by the Eq.2.23.

3.1.2 Verification of the Finite Element Model

In order to verify the finite element model and analysis method for calculating stress intensity factor K_I , T_{11} and T_{33} stresses, a surface corner crack model ($a/c=1.0$, $a/t=0.2$) in plate was built in ABAQUS (2008). The geometry of surface corner crack is shown in Figure 3.4 and the mesh is shown in Figure 3.5.

The results of K_I , T_{11} and T_{33} compared with Qu and Wang's (2006) results are shown in Table 3.1-3.3, respectively. The differences of stress intensity factor K_I for two positions (0° and 90°) between present analysis and Qu and Wang's results are 2.39% and 3.24%, respectively. There is a good agreement for T_{11} stress solutions along the crack front for present analysis and Qu and Wang's model. Most of the T_{11} stress results

differences are less than 3%, except the point of 5° is 3.12%, as shown in Figure 3.6. It shows that the most difference of T_{33} between present research and Qu and Wang's result is less than 3%, except the difference at the point near the deepest point is 7.17%, as shown in Figure 3.7.

In order to verify the finite element model of single edge cracked plate (SECP) specimen, a model was built in ABAQUS (2008) which the relative crack depth and thickness were fixed at $a/W=0.5$ and $t/W=0.5$, respectively. Remote tension loading was applied to the specimen. Poisson ratio ν was chosen from 0, 0.3 and 0.499, respectively, and the straight crack front intersects the free-surfaces orthogonally. This was the geometry analyzed by Nakamura and Parks (1992).

The results of T_{11} for the tension case are shown in Table 3.4 and plotted in Figure 3.8. The initial data and trends are good fit with the results of Nakamura and Parks (1992). And the results of stress intensity factor K_I and T_{11} for $\nu=0$ (or the two-dimensional solution) were compared with Sham's results (1991), as Table 3.5. The differences are within 0.11% for stress intensity factor K_I and 1.91% for T_{11} stress.

Through these calculations, the present finite element procedures are considered appropriate for K_I , T_{11} and T_{33} calculations for three-dimensional SECP specimens.

3.1.3 Finite Element Results

The stress intensity factor K_I and T -stresses (T_{11} and T_{33}) values for cracks in SECP specimens with a/W of 0.2, 0.4, 0.6 or 0.8, and t/W of 0.1, 0.2, 0.5, 1.0, 2.0, or 4.0, subjected to constant, linear, parabolic or cubic stress distribution as expressed in Eq. (3.1-3.4) have been determined.

The stress intensity factor K_I and T -stresses (T_{11} and T_{33}) results have been normalized as follows:

$$\begin{aligned}
\text{Normalized } K_I &= \frac{K_I}{\sigma_0 \sqrt{\pi a}} \\
\text{Normalized } T_{11} &= \frac{T_{11}}{\sigma_0} \\
\text{Normalized } T_{33} &= \frac{T_{33}}{\sigma_0}
\end{aligned}
\tag{3.5}$$

The results of normalized stress intensity factors K_I and T -stresses (T_{11} and T_{33}) are summarized in Table 3.6-3.11 for $a/W=0.2$. The normalized stress intensity factors K_I for $a/W=0.2$ cases subjected to constant or linear are shown in Figure 3.9 and 3.10, respectively. The normalized T_{11} for $a/W=0.2$ cases subjected to constant or linear are shown in Figure 3.11 and 3.12, respectively. And the normalized T_{33} for $a/W=0.2$ cases subjected to constant or linear are shown in Figure 3.13 and 3.14, respectively.

The normalized stress intensity factors K_I and T -stresses (T_{11} and T_{33}) for $a/W=0.4$ are summarized in Table 3.12-3.17, and the stress intensity factors K_I for the cases subjected to constant or linear are shown in Figure 3.15 and 3.16, respectively. The normalized T_{11} for $a/W=0.4$ cases subjected to constant or linear are shown in Figure 3.17 and 3.18, respectively. And the normalized T_{33} for $a/W=0.4$ cases subjected to constant or linear are shown in Figure 3.19 and 3.20, respectively.

The results of normalized stress intensity factors K_I and T -stresses (T_{11} and T_{33}) are summarized in Table 3.18-3.23 for $a/W=0.6$. The normalized stress intensity factors K_I for $a/W=0.6$ cases subjected to constant or linear are shown in Figure 3.21 and 3.22, respectively. The normalized T_{11} for $a/W=0.6$ cases subjected to constant or linear are shown in Figure 3.23 and 3.24, respectively. And the normalized T_{33} for $a/W=0.6$ cases subjected to constant or linear are shown in Figure 3.25 and 3.26, respectively.

The normalized stress intensity factors K_I and T -stresses (T_{11} and T_{33}) for $a/W=0.8$ are summarized in Table 3.24-3.29, and the stress intensity factors K_I for the cases subjected to constant or linear are shown in Figure 3.27 and 3.28, respectively. The normalized T_{11} for $a/W=0.8$ cases subjected to constant or linear are shown in Figure

3.29 and 3.30, respectively. And the normalized T_{33} for $a/W=0.8$ cases subjected to constant or linear are shown in Figure 3.31 and 3.32, respectively.

3.2 EMPIRICAL EQUATIONS OF STRESS INTENSITY FACTOR AND T-STRESS

For important engineering applications, the numerical stress intensity factor K_I and T -stresses results for cracks in SECP at position $z/t=0$ presented in Section 3.1 were approximated by empirical formulae fitted. These empirical formulae are least square fitted using results presented in Tables 3.6-3.29.

3.2.1 Stress Intensity Factor Equations

The stress intensity factor K_I solutions at $z/t=0$ for stress applied on the crack front corresponding to the Eq. (3.1) to (3.4) are expressed in the following forms:

$$K_I = \sigma_0 F_n \left(\frac{a}{W}, \frac{t}{W} \right) \sqrt{\pi a} \quad (3.6)$$

$$\begin{aligned} F_n = & a_n + b_n \ln \left(\frac{t}{W} \right) + c_n \left(\frac{a}{W} \right) + d_n \left(\ln \left(\frac{t}{W} \right) \right)^2 + e_n \left(\frac{a}{W} \right)^2 + f_n \left(\frac{a}{W} \right) \ln \left(\frac{t}{W} \right) \\ & + g_n \left(\ln \left(\frac{t}{W} \right) \right)^3 + h_n \left(\frac{a}{W} \right)^3 + i_n \left(\frac{a}{W} \right)^2 \ln \left(\frac{t}{W} \right) \\ & + j_n \left(\frac{a}{W} \right) \left(\ln \left(\frac{t}{W} \right) \right)^2 \end{aligned} \quad (3.7)$$

where $n=0, 1, 2, 3$, and functions F_n represents the normalized K_I along the crack front subjected to the uniform, linear, parabolic and cubic stress distribution, respectively. The

parameters in the Eq. 3.7 are shown in the Table 3.30 for four different loading conditions. And the comparison of the prediction of stress intensity factor from finite element analysis and empirical equations are shown in Figure 3.33 (a)-(f). The maximum relative difference between the empirical formulae and the finite element results is less than 3%.

3.2.2 T_{11} Equations

The T_{11} stress solutions at $z/t=0$ for stress applied on the crack front corresponding to the Eq. (3.1) to (3.4) are expressed in the following forms:

$$T_{11} = \sigma_0 V_n \left(\frac{a}{W}, \frac{t}{W} \right) \quad (3.8)$$

where $n=0, 1, 2, 3$, and functions V_n represents the normalized T_{11} along the crack front subjected to the uniform, linear, parabolic and cubic stress field, respectively.

For uniform stress field

$$V_0 = A_0 + A_1 \left(\frac{a}{W} \right)^2 + A_2 \left(\frac{a}{W} \right)^4 + A_3 \left(\frac{a}{W} \right)^6 \quad (3.9)$$

where

$$\begin{aligned} A_0 &= 0.2026 + 0.01368 \left(\frac{t}{W} \right)^{-1} + 0.678 \left(\frac{t}{W} \right) - 0.9001 \left(\frac{t}{W} \right)^2 \\ &\quad + 0.4101 \left(\frac{t}{W} \right)^3 - 0.05629 \left(\frac{t}{W} \right)^4 \\ A_1 &= 6.021 - 0.1249 \left(\frac{t}{W} \right)^{-1} - 12.26 \left(\frac{t}{W} \right) + 14.66 \left(\frac{t}{W} \right)^2 - 6.509 \left(\frac{t}{W} \right)^3 \\ &\quad + 0.8869 \left(\frac{t}{W} \right)^4 \\ A_2 &= -36.09 + 0.9008 \left(\frac{t}{W} \right)^{-1} + 54.61 \left(\frac{t}{W} \right) - 55.95 \left(\frac{t}{W} \right)^2 + 22.23 \left(\frac{t}{W} \right)^3 \\ &\quad - 2.842 \left(\frac{t}{W} \right)^4 \end{aligned}$$

$$A_3 = 75.2 - 1.047 \left(\frac{t}{W} \right)^{-1} - 57.8 \left(\frac{t}{W} \right) + 41.57 \left(\frac{t}{W} \right)^2 - 12.09 \left(\frac{t}{W} \right)^3 + 1.222 \left(\frac{t}{W} \right)^4 \quad (3.10)$$

The numerical results are approximated by empirical formulae fitted within an accuracy of 0.2%.

For linear stress field

$$V_1 = B_0 + B_1 \left(\frac{a}{W} \right)^2 + B_2 \left(\frac{a}{W} \right)^4 + B_3 \left(\frac{a}{W} \right)^6 \quad (3.11)$$

where

$$\begin{aligned} B_0 &= 0.1961 + 0.003405 \left(\frac{t}{W} \right)^{-1} + 0.2411 \left(\frac{t}{W} \right) - 0.3853 \left(\frac{t}{W} \right)^2 \\ &\quad + 0.1882 \left(\frac{t}{W} \right)^3 - 0.02664 \left(\frac{t}{W} \right)^4 \\ B_1 &= 3.268 - 0.03388 \left(\frac{t}{W} \right)^{-1} - 4.291 \left(\frac{t}{W} \right) + 5.432 \left(\frac{t}{W} \right)^2 - 2.535 \left(\frac{t}{W} \right)^3 \\ &\quad + 0.3551 \left(\frac{t}{W} \right)^4 \\ B_2 &= -22.25 + 0.4137 \left(\frac{t}{W} \right)^{-1} + 23.32 \left(\frac{t}{W} \right) - 22.42 \left(\frac{t}{W} \right)^2 + 8.624 \left(\frac{t}{W} \right)^3 \\ &\quad - 1.084 \left(\frac{t}{W} \right)^4 \\ B_3 &= 47.99 - 0.5109 \left(\frac{t}{W} \right)^{-1} - 25.95 \left(\frac{t}{W} \right) + 13.93 \left(\frac{t}{W} \right)^2 - 2.451 \left(\frac{t}{W} \right)^3 \\ &\quad + 0.09097 \left(\frac{t}{W} \right)^4 \end{aligned} \quad (3.12)$$

The most numerical results are approximated by empirical formulae fitted within an accuracy of 0.2%, except $a/W=0.6$, $t/W=4$ is 3% and $a/W=0.8$, $t/W=4$ is 1.62%.

For parabolic stress field

$$V_2 = C_0 + C_1 \left(\frac{a}{W} \right)^2 + C_2 \left(\frac{a}{W} \right)^4 + C_3 \left(\frac{a}{W} \right)^6 \quad (3.13)$$

where

$$\begin{aligned}
C_0 &= 0.1505 + 0.00004019 \left(\frac{t}{W}\right)^{-1} + 0.09923 \left(\frac{t}{W}\right) - 0.2013 \left(\frac{t}{W}\right)^2 \\
&\quad + 0.1056 \left(\frac{t}{W}\right)^3 - 0.01539 \left(\frac{t}{W}\right)^4 \\
C_1 &= 2.262 + 0.01314 \left(\frac{t}{W}\right)^{-1} - 1.3 \left(\frac{t}{W}\right) + 1.957 \left(\frac{t}{W}\right)^2 - 1.032 \left(\frac{t}{W}\right)^3 \\
&\quad + 0.1534 \left(\frac{t}{W}\right)^4 \\
C_2 &= -16.49 + 0.1555 \left(\frac{t}{W}\right)^{-1} + 10.19 \left(\frac{t}{W}\right) - 8.77 \left(\frac{t}{W}\right)^2 + 3.209 \left(\frac{t}{W}\right)^3 \\
&\quad - 0.3943 \left(\frac{t}{W}\right)^4 \\
C_3 &= 35.38 - 0.2299 \left(\frac{t}{W}\right)^{-1} - 12.37 \left(\frac{t}{W}\right) + 3.042 \left(\frac{t}{W}\right)^2 + 1.04 \left(\frac{t}{W}\right)^3 \\
&\quad - 0.2923 \left(\frac{t}{W}\right)^4
\end{aligned} \tag{3.14}$$

The most numerical results are approximated by empirical formulae fitted within an accuracy of 0.7%.

For cubic stress field

$$V_3 = D_0 + D_1 \left(\frac{a}{W}\right)^2 + D_2 \left(\frac{a}{W}\right)^4 + D_3 \left(\frac{a}{W}\right)^6 \tag{3.15}$$

where

$$\begin{aligned}
D_0 &= 0.105 - 0.00101 \left(\frac{t}{W}\right)^{-1} + 0.04351 \left(\frac{t}{W}\right) - 0.1168 \left(\frac{t}{W}\right)^2 \\
&\quad + 0.06545 \left(\frac{t}{W}\right)^3 - 0.00978 \left(\frac{t}{W}\right)^4 \\
D_1 &= 1.884 + 0.03089 \left(\frac{t}{W}\right)^{-1} - 0.1371 \left(\frac{t}{W}\right) + 0.5278 \left(\frac{t}{W}\right)^2 - 0.3939 \left(\frac{t}{W}\right)^3 \\
&\quad + 0.06635 \left(\frac{t}{W}\right)^4 \\
D_2 &= -13.71 + 0.02488 \left(\frac{t}{W}\right)^{-1} + 4.163 \left(\frac{t}{W}\right) - 2.576 \left(\frac{t}{W}\right)^2 + 0.787 \left(\frac{t}{W}\right)^3 \\
&\quad - 0.08874 \left(\frac{t}{W}\right)^4
\end{aligned}$$

$$D_3 = 28.3 - 0.07651 \left(\frac{t}{W} \right)^{-1} - 5.731 \left(\frac{t}{W} \right) - 1.867 \left(\frac{t}{W} \right)^2 + 2.465 \left(\frac{t}{W} \right)^3 - 0.4348 \left(\frac{t}{W} \right)^4 \quad (3.16)$$

The most numerical results are approximated by empirical formulae fitted within an accuracy of 0.3%.

The comparison of the prediction of T_{11} stress from finite element analysis and empirical equations are shown in Figure 3.34 (a)-(f).

3.2.3 T_{33} Equations

The T_{33} solutions at $z/t=0$ for stress applied on the crack front corresponding to the Eq. (3.1) to (3.4) are expressed in the following forms:

$$T_{33} = \sigma_0 W_n \left(\frac{a}{W}, \frac{t}{W} \right) \quad (3.17)$$

where $n=0, 1, 2, 3$, and functions W_n represents the normalized T_{33} along the crack front subjected to the uniform, linear, parabolic and cubic stress field, respectively.

For uniform stress field

$$W_0 = A_0 + A_1 \left(\frac{a}{W} \right)^2 + A_2 \left(\frac{a}{W} \right)^4 + A_3 \left(\frac{a}{W} \right)^6 \quad (3.18)$$

where

$$\begin{aligned} A_0 &= 0.1281 - 0.0267 \left(\frac{t}{W} \right)^{-1} + 0.01839 \left(\frac{t}{W} \right) + 0.09548 \left(\frac{t}{W} \right)^2 \\ &\quad - 0.08128 \left(\frac{t}{W} \right)^3 + 0.014 \left(\frac{t}{W} \right)^4 \\ A_1 &= -8.526 - 0.7466 \left(\frac{t}{W} \right)^{-1} + 26.25 \left(\frac{t}{W} \right) - 28.57 \left(\frac{t}{W} \right)^2 + 12.13 \left(\frac{t}{W} \right)^3 \\ &\quad - 1.621 \left(\frac{t}{W} \right)^4 \end{aligned}$$

$$\begin{aligned}
A_2 &= 22.45 + 1.275 \left(\frac{t}{W}\right)^{-1} - 142.8 \left(\frac{t}{W}\right) + 182.9 \left(\frac{t}{W}\right)^2 - 81.35 \left(\frac{t}{W}\right)^3 \\
&\quad + 11.03 \left(\frac{t}{W}\right)^4 \\
A_3 &= -59.84 - 3.621 \left(\frac{t}{W}\right)^{-1} + 320.9 \left(\frac{t}{W}\right) - 379.9 \left(\frac{t}{W}\right)^2 + 162.4 \left(\frac{t}{W}\right)^3 \\
&\quad - 21.58 \left(\frac{t}{W}\right)^4
\end{aligned}
\tag{3.19}$$

The numerical results are approximated by empirical formulae fitted within an accuracy of 2%.

For linear stress field

$$W_1 = B_0 + B_1 \left(\frac{a}{W}\right)^2 + B_2 \left(\frac{a}{W}\right)^4 + B_3 \left(\frac{a}{W}\right)^6 \tag{3.20}$$

where

$$\begin{aligned}
B_0 &= -0.04633 - 0.01167 \left(\frac{t}{W}\right)^{-1} + 0.4873 \left(\frac{t}{W}\right) - 0.4911 \left(\frac{t}{W}\right)^2 \\
&\quad + 0.1835 \left(\frac{t}{W}\right)^3 - 0.02235 \left(\frac{t}{W}\right)^4 \\
B_1 &= -5.613 - 0.3856 \left(\frac{t}{W}\right)^{-1} + 14 \left(\frac{t}{W}\right) - 13.47 \left(\frac{t}{W}\right)^2 + 5.423 \left(\frac{t}{W}\right)^3 \\
&\quad - 0.7085 \left(\frac{t}{W}\right)^4 \\
B_2 &= 16.58 + 0.7248 \left(\frac{t}{W}\right)^{-1} - 87.39 \left(\frac{t}{W}\right) + 105.9 \left(\frac{t}{W}\right)^2 - 46.07 \left(\frac{t}{W}\right)^3 \\
&\quad + 6.185 \left(\frac{t}{W}\right)^4 \\
B_3 &= -40.44 - 2.294 \left(\frac{t}{W}\right)^{-1} + 202 \left(\frac{t}{W}\right) - 232.8 \left(\frac{t}{W}\right)^2 + 98.31 \left(\frac{t}{W}\right)^3 \\
&\quad - 12.99 \left(\frac{t}{W}\right)^4
\end{aligned}
\tag{3.21}$$

The numerical results are approximated by empirical formulae fitted within an accuracy of 2%.

For parabolic stress field

$$W_2 = C_0 + C_1 \left(\frac{a}{W}\right)^2 + C_2 \left(\frac{a}{W}\right)^4 + C_3 \left(\frac{a}{W}\right)^6 \quad (3.22)$$

where

$$\begin{aligned} C_0 &= -0.07497 - 0.0026 \left(\frac{t}{W}\right)^{-1} + 0.5095 \left(\frac{t}{W}\right) - 0.5498 \left(\frac{t}{W}\right)^2 \\ &\quad + 0.2181 \left(\frac{t}{W}\right)^3 - 0.02765 \left(\frac{t}{W}\right)^4 \\ C_1 &= -3.994 - 0.3259 \left(\frac{t}{W}\right)^{-1} + 8.371 \left(\frac{t}{W}\right) - 6.862 \left(\frac{t}{W}\right)^2 + 2.544 \left(\frac{t}{W}\right)^3 \\ &\quad - 0.3202 \left(\frac{t}{W}\right)^4 \\ C_2 &= 13.78 + 0.8233 \left(\frac{t}{W}\right)^{-1} - 61.2 \left(\frac{t}{W}\right) + 69.47 \left(\frac{t}{W}\right)^2 - 29.43 \left(\frac{t}{W}\right)^3 \\ &\quad + 3.905 \left(\frac{t}{W}\right)^4 \\ C_3 &= -31.52 - 1.984 \left(\frac{t}{W}\right)^{-1} + 144.8 \left(\frac{t}{W}\right) - 161.6 \left(\frac{t}{W}\right)^2 + 67.31 \left(\frac{t}{W}\right)^3 \\ &\quad - 8.834 \left(\frac{t}{W}\right)^4 \end{aligned} \quad (3.23)$$

The most numerical results are approximated by empirical formulae fitted with an accuracy of 5%, except $t/W=4.0$ is about 10%.

For cubic stress field

$$W_3 = D_0 + D_1 \left(\frac{a}{W}\right)^2 + D_2 \left(\frac{a}{W}\right)^4 + D_3 \left(\frac{a}{W}\right)^6 \quad (3.24)$$

Where

$$\begin{aligned} D_0 &= 0.01499 - 0.2554 \left(\frac{t}{W}\right)^{0.5} + 0.6875 \left(\frac{t}{W}\right) - 0.5958 \left(\frac{t}{W}\right)^2 \\ &\quad + 0.2332 \left(\frac{t}{W}\right)^3 - 0.02975 \left(\frac{t}{W}\right)^4 \\ D_1 &= -15.6 + 43.73 \left(\frac{t}{W}\right)^{0.5} - 42.05 \left(\frac{t}{W}\right) + 18.84 \left(\frac{t}{W}\right)^2 - 6.124 \left(\frac{t}{W}\right)^3 \\ &\quad + 0.7315 \left(\frac{t}{W}\right)^4 \end{aligned}$$

$$\begin{aligned}
D_2 &= 51.76 - 137.7 \left(\frac{t}{W} \right)^{0.5} + 103.2 \left(\frac{t}{W} \right) - 21.97 \left(\frac{t}{W} \right)^2 + 3.123 \left(\frac{t}{W} \right)^3 \\
&\quad - 0.1584 \left(\frac{t}{W} \right)^4 \\
D_3 &= -102.8 + 269.1 \left(\frac{t}{W} \right)^{0.5} - 183.9 \left(\frac{t}{W} \right) + 21.17 \left(\frac{t}{W} \right)^2 + 3.131 \left(\frac{t}{W} \right)^3 \\
&\quad - 0.9084 \left(\frac{t}{W} \right)^4
\end{aligned}
\tag{3.25}$$

The most numerical results are approximated by empirical formulae fitted within an accuracy of 3%.

The comparison of the prediction of T_{33} stress from finite element analysis and empirical equations are shown in Figure 3.35 (a)-(f).

Using the superposition method discussed in Chapter 2, these empirical equations can be used to calculate K_I , T_{11} and T_{33} for various linear and nonlinear stress distributions.

3.3 SUMMARY

In this chapter, the finite element model of the three-dimensional single edge cracked plate specimens with different relative crack depth and thickness were built in ABAQUS (2008). The correspondence stress intensity factor K_I and T -stresses (T_{11} and T_{33}) for the crack surface subjected to constant, linear, parabolic or cubic stress distribution were obtained. And the approximated empirical formulae were fitted for the numerical stress intensity factor and T -stresses results for cracks at position $z/t=0$.

In next chapter, the influence of the crack depth, plate thickness and the variation of K_I , T_{11} and T_{33} along the thickness are discussed, respectively.

Table 3.1 Comparison of the K from present FEM calculation and solution form Qu and Wang for surface corner crack, $a/c=1.0$, $a/t=0.2$

Location ϕ^0	Normalized K (Qu and Wang, 2006)	Normalized K Present FEM	Difference (%)
0.00	1.1763	1.1382	3.24
90.00	1.1094	1.0828	2.39

Table 3.2 Comparison of the T_{11} from present FEM calculation and solution form Qu and Wang for surface corner crack, $a/c=1.0$, $a/t=0.2$

Location ϕ^0	Normalized T_{11} (Qu and Wang, 2006)	Normalized T_{11} Present FEM	Difference (%)
5.00	-0.4370	-0.4506	3.12
7.50	-0.4229	-0.4155	1.76
10.00	-0.4113	-0.4018	2.31
12.50	-0.4027	-0.3946	2.01
15.00	-0.3967	-0.3900	1.70
17.50	-0.3926	-0.3867	1.51
20.00	-0.3885	-0.3842	1.11
22.50	-0.3824	-0.3823	0.02
30.00	-0.3819	-0.3789	0.79
37.50	-0.3798	-0.3774	0.64
45.00	-0.3790	-0.3770	0.52
52.50	-0.3791	-0.3775	0.41
60.00	-0.3799	-0.3793	0.16
67.50	-0.3834	-0.3831	0.09
75.00	-0.3879	-0.3909	0.78
82.50	-0.4269	-0.4169	2.33
86.25	-0.4727	-0.4616	2.34

Table 3.3 Comparison of the T_{33} from present FEM calculation and solution form Qu and Wang for surface corner crack, $a/c=1.0$, $a/t=0.2$

Location ϕ^0	Normalized T_{33} (Qu and Wang, 2006)	Normalized T_{33} Present FEM	Difference (%)
86.25	-1.3264	-1.23129	7.17
82.50	-0.8885	-0.93139	4.83
75.00	-0.8191	-0.81491	0.51
67.50	-0.7944	-0.77874	1.97
60.00	-0.783	-0.76756	1.97
52.50	-0.7798	-0.76434	1.98
45.00	-0.7793	-0.76381	1.99
37.50	-0.7796	-0.76481	1.90
30.00	-0.7817	-0.76852	1.69
22.50	-0.7829	-0.78016	0.35
20.00	-0.7825	-0.78801	0.70
17.50	-0.7921	-0.79961	0.95
15.00	-0.8057	-0.81677	1.37
12.50	-0.8255	-0.84221	2.02
10.00	-0.8563	-0.87992	2.76
7.50	-0.9004	-0.93387	3.72
5.00	-0.9737	-1.00256	2.96

Table 3.4 Comparison of T_{11} from present FEM calculation and solution from Nakamura and Parks (1992) for $a/W=0.5$, $t/W=0.5$

z/t	$\nu = 0$			$\nu = 0.3$			$\nu = 0.499$		
	Present	Nakamura and Parks (1992)	%	Present	Nakamura and Parks (1992)	%	Present	Nakamura and Parks (1992)	%
0.00	-0.4124	-0.4364	5.50	-0.2079	-0.2105	1.24	0.0983	0.1006	2.29
0.25	-0.4124	-0.4364	5.50	-0.2138	-0.2289	6.60	0.1111	0.1135	2.11
0.40	-0.4124	-0.4364	5.50	-0.2232	-0.2432	8.22	0.1724	0.1973	12.6

Table 3.5 Comparison of stress intensity factor and T_{11} from present FEM calculation and solution from Sham (1991) for $a/W=0.5$

	$K_I/\sigma\sqrt{\pi a}$	Difference (%) $100 \times$ $ F_p - F_s /F_s$	β $\equiv T_{11}\sqrt{\pi a}/K_I$	Difference (%) $100 \times$ $ F_p - F_s /F_s$	T_{11}/σ	Difference (%) $100 \times$ $ F_p - F_s /F_s$
Present calculation (F_p)	2.824	0.11	-0.15301	0.05	-0.42314	1.91
Sham's result (F_s) (1991)	2.821		-0.15293		-0.43142	

Table 3.6 Normalized K_I , T_{II} and T_{33} for $a/W=0.2$, $t/W=0.1$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	1.4535	0.5144	-0.6499	0.6307	0.3297	-0.4241	0.4022	0.2298	-0.2906	0.2816	0.1664	-0.2076
0.0346	1.4534	0.5144	-0.6533	0.6305	0.3295	-0.4258	0.4021	0.2297	-0.2916	0.2816	0.1663	-0.2083
0.0679	1.4530	0.5152	-0.6628	0.6302	0.3297	-0.4306	0.4019	0.2297	-0.2945	0.2814	0.1663	-0.2103
0.1001	1.4524	0.5165	-0.6783	0.6296	0.3298	-0.4384	0.4015	0.2298	-0.2993	0.2811	0.1664	-0.2135
0.1311	1.4516	0.5182	-0.6997	0.6289	0.3301	-0.4490	0.4009	0.2298	-0.3057	0.2807	0.1664	-0.2179
0.1610	1.4505	0.5203	-0.7269	0.6279	0.3303	-0.4624	0.4002	0.2299	-0.3139	0.2802	0.1665	-0.2235
0.1898	1.4492	0.5228	-0.7604	0.6267	0.3307	-0.4788	0.3994	0.2301	-0.3239	0.2796	0.1666	-0.2304
0.2176	1.4475	0.5256	-0.8006	0.6253	0.3311	-0.4984	0.3984	0.2303	-0.3359	0.2789	0.1668	-0.2385
0.2444	1.4455	0.5288	-0.8484	0.6237	0.3315	-0.5214	0.3972	0.2305	-0.3499	0.2781	0.1670	-0.2482
0.2702	1.4431	0.5324	-0.9047	0.6217	0.3320	-0.5484	0.3959	0.2308	-0.3664	0.2771	0.1672	-0.2595
0.2951	1.4402	0.5362	-0.9713	0.6195	0.3325	-0.5798	0.3943	0.2312	-0.3857	0.2760	0.1676	-0.2728
0.3191	1.4368	0.5403	-1.0502	0.6169	0.3331	-0.6167	0.3926	0.2316	-0.4083	0.2747	0.1680	-0.2884
0.3423	1.4327	0.5447	-1.1448	0.6138	0.3337	-0.6603	0.3905	0.2322	-0.4351	0.2733	0.1685	-0.3069
0.3646	1.4276	0.5492	-1.2599	0.6103	0.3343	-0.7128	0.3881	0.2328	-0.4675	0.2716	0.1691	-0.3293
0.3861	1.4214	0.5539	-1.4035	0.6060	0.3349	-0.7773	0.3853	0.2336	-0.5074	0.2696	0.1698	-0.3570
0.4069	1.4136	0.5585	-1.5888	0.6009	0.3355	-0.8594	0.3819	0.2344	-0.5583	0.2673	0.1707	-0.3924
0.4269	1.4035	0.5637	-1.8433	0.5945	0.3363	-0.9705	0.3778	0.2356	-0.6276	0.2644	0.1717	-0.4406
0.4462	1.3899	0.5653	-2.2089	0.5864	0.3356	-1.1279	0.3726	0.2362	-0.7261	0.2608	0.1724	-0.5094
0.4648	1.3716	0.5499	-2.7787	0.5759	0.3281	-1.3705	0.3659	0.2327	-0.8785	0.2561	0.1703	-0.6159
0.4827	1.3423	0.2544	-3.2018	0.5600	0.2055	-1.5530	0.3559	0.1564	-0.9924	0.2492	0.1172	-0.6955
0.5000	1.1833	0.1095	-11.4792	0.4881	0.1594	-4.9707	0.3105	0.1292	-3.1642	0.2175	0.0982	-2.2165

Table 3.7 Normalized K_I , T_{II} and T_{33} for $a/W=0.2$, $t/W=0.2$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	1.4493	0.4804	-0.3115	0.6334	0.3213	-0.2257	0.4053	0.2284	-0.1626	0.2842	0.1668	-0.1190
0.0490	1.4490	0.4808	-0.3165	0.6331	0.3213	-0.2287	0.4051	0.2283	-0.1646	0.2840	0.1667	-0.1204
0.0946	1.4484	0.4816	-0.3287	0.6324	0.3211	-0.2361	0.4045	0.2279	-0.1694	0.2836	0.1663	-0.1237
0.1361	1.4472	0.4832	-0.3485	0.6312	0.3210	-0.2479	0.4036	0.2275	-0.1770	0.2829	0.1659	-0.1290
0.1747	1.4457	0.4850	-0.3740	0.6297	0.3206	-0.2630	0.4024	0.2268	-0.1867	0.2820	0.1652	-0.1357
0.2098	1.4438	0.4875	-0.4064	0.6278	0.3203	-0.2816	0.4010	0.2260	-0.1986	0.2809	0.1645	-0.1438
0.2424	1.4416	0.4899	-0.4443	0.6256	0.3198	-0.3031	0.3993	0.2251	-0.2122	0.2797	0.1636	-0.1532
0.2722	1.4388	0.4931	-0.4896	0.6230	0.3193	-0.3280	0.3973	0.2241	-0.2278	0.2782	0.1628	-0.1638
0.2998	1.4357	0.4961	-0.5413	0.6201	0.3185	-0.3558	0.3950	0.2230	-0.2451	0.2765	0.1618	-0.1757
0.3250	1.4320	0.4998	-0.6021	0.6167	0.3178	-0.3875	0.3925	0.2219	-0.2647	0.2746	0.1610	-0.1891
0.3484	1.4278	0.5031	-0.6715	0.6129	0.3168	-0.4229	0.3897	0.2207	-0.2865	0.2726	0.1601	-0.2039
0.3698	1.4228	0.5071	-0.7536	0.6086	0.3160	-0.4635	0.3866	0.2198	-0.3113	0.2703	0.1593	-0.2209
0.3896	1.4170	0.5103	-0.8489	0.6038	0.3147	-0.5095	0.3831	0.2186	-0.3394	0.2678	0.1585	-0.2401
0.4076	1.4102	0.5145	-0.9649	0.5984	0.3137	-0.5639	0.3793	0.2178	-0.3725	0.2650	0.1581	-0.2629
0.4244	1.4021	0.5170	-1.1041	0.5922	0.3119	-0.6276	0.3750	0.2168	-0.4114	0.2620	0.1575	-0.2896
0.4397	1.3922	0.5211	-1.2846	0.5850	0.3108	-0.7080	0.3701	0.2164	-0.4606	0.2585	0.1574	-0.3235
0.4539	1.3800	0.5213	-1.5176	0.5767	0.3082	-0.8096	0.3645	0.2153	-0.5229	0.2545	0.1569	-0.3666
0.4668	1.3636	0.5249	-1.8624	0.5662	0.3072	-0.9562	0.3575	0.2157	-0.6134	0.2496	0.1576	-0.4294
0.4789	1.3412	0.5137	-2.4130	0.5528	0.3005	-1.1860	0.3488	0.2129	-0.7561	0.2435	0.1561	-0.5287
0.4898	1.2968	0.5015	-3.5112	0.5295	0.2955	-1.6352	0.3340	0.2121	-1.0368	0.2331	0.1560	-0.7243
0.5000	1.1315	-0.0833	-11.7734	0.4547	0.0779	-4.9612	0.2869	0.0781	-3.1319	0.2003	0.0629	-2.1866

Table 3.8 Normalized K_I , T_{II} and T_{33} for $a/W=0.2$, $t/W=0.5$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	1.4346	0.4676	-0.0215	0.6281	0.3131	-0.0187	0.4028	0.2237	-0.0142	0.2827	0.1639	-0.0107
0.0615	1.4343	0.4669	-0.0260	0.6278	0.3127	-0.0221	0.4026	0.2234	-0.0167	0.2826	0.1637	-0.0125
0.1161	1.4334	0.4652	-0.0379	0.6271	0.3117	-0.0309	0.4020	0.2227	-0.0232	0.2822	0.1632	-0.0173
0.1647	1.4320	0.4628	-0.0556	0.6259	0.3102	-0.0441	0.4011	0.2217	-0.0328	0.2815	0.1624	-0.0243
0.2078	1.4301	0.4601	-0.0784	0.6244	0.3083	-0.0608	0.3999	0.2202	-0.0450	0.2806	0.1613	-0.0333
0.2462	1.4277	0.4572	-0.1060	0.6223	0.3061	-0.0808	0.3983	0.2184	-0.0594	0.2794	0.1599	-0.0438
0.2802	1.4247	0.4545	-0.1384	0.6198	0.3036	-0.1038	0.3964	0.2163	-0.0757	0.2779	0.1582	-0.0557
0.3105	1.4212	0.4521	-0.1757	0.6167	0.3008	-0.1296	0.3940	0.2139	-0.0938	0.2761	0.1563	-0.0688
0.3374	1.4171	0.4502	-0.2185	0.6130	0.2979	-0.1582	0.3912	0.2111	-0.1136	0.2740	0.1540	-0.0829
0.3613	1.4123	0.4489	-0.2675	0.6088	0.2946	-0.1898	0.3879	0.2081	-0.1349	0.2716	0.1515	-0.0980
0.3825	1.4069	0.4482	-0.3239	0.6040	0.2912	-0.2245	0.3842	0.2047	-0.1578	0.2687	0.1488	-0.1141
0.4014	1.4007	0.4480	-0.3894	0.5985	0.2874	-0.2628	0.3800	0.2010	-0.1826	0.2656	0.1458	-0.1313
0.4182	1.3937	0.4483	-0.4662	0.5922	0.2834	-0.3056	0.3753	0.1972	-0.2096	0.2621	0.1428	-0.1499
0.4331	1.3857	0.4491	-0.5579	0.5852	0.2790	-0.3539	0.3700	0.1932	-0.2396	0.2582	0.1397	-0.1704
0.4464	1.3763	0.4500	-0.6699	0.5773	0.2744	-0.4099	0.3642	0.1892	-0.2738	0.2539	0.1367	-0.1938
0.4581	1.3653	0.4509	-0.8112	0.5683	0.2694	-0.4771	0.3577	0.1853	-0.3145	0.2491	0.1339	-0.2215
0.4686	1.3519	0.4516	-0.9981	0.5578	0.2643	-0.5617	0.3503	0.1817	-0.3655	0.2438	0.1315	-0.2563
0.4779	1.3347	0.4511	-1.2648	0.5454	0.2586	-0.6773	0.3417	0.1784	-0.4352	0.2376	0.1295	-0.3041
0.4861	1.3110	0.4489	-1.7038	0.5295	0.2527	-0.8599	0.3310	0.1757	-0.5461	0.2301	0.1280	-0.3805
0.4935	1.2737	0.4269	-2.6392	0.5071	0.2402	-1.2365	0.3164	0.1703	-0.7772	0.2198	0.1249	-0.5404
0.5000	1.0900	-0.0266	-10.7843	0.4230	0.0825	-4.4048	0.2636	0.0774	-2.7464	0.1831	0.0614	-1.9073

Table 3.9 Normalized K_I , T_{II} and T_{33} for $a/W=0.2$, $t/W=1.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	1.4057	0.4306	0.0954	0.6097	0.2854	0.0635	0.3899	0.2032	0.0453	0.2734	0.1487	0.0332
0.0615	1.4059	0.4314	0.0929	0.6099	0.2860	0.0617	0.3900	0.2037	0.0441	0.2735	0.1490	0.0323
0.1161	1.4065	0.4336	0.0861	0.6102	0.2877	0.0571	0.3903	0.2049	0.0407	0.2737	0.1499	0.0298
0.1647	1.4071	0.4363	0.0756	0.6106	0.2898	0.0497	0.3906	0.2065	0.0354	0.2739	0.1511	0.0259
0.2078	1.4075	0.4388	0.0616	0.6109	0.2919	0.0398	0.3908	0.2081	0.0282	0.2741	0.1523	0.0206
0.2462	1.4076	0.4404	0.0442	0.6109	0.2935	0.0272	0.3908	0.2094	0.0190	0.2741	0.1533	0.0138
0.2802	1.4072	0.4409	0.0233	0.6106	0.2945	0.0120	0.3905	0.2103	0.0079	0.2738	0.1540	0.0056
0.3105	1.4062	0.4402	-0.0011	0.6097	0.2946	-0.0058	0.3898	0.2105	-0.0052	0.2733	0.1542	-0.0041
0.3374	1.4045	0.4385	-0.0293	0.6083	0.2939	-0.0266	0.3888	0.2100	-0.0204	0.2725	0.1538	-0.0152
0.3613	1.4021	0.4360	-0.0619	0.6063	0.2923	-0.0503	0.3872	0.2088	-0.0376	0.2714	0.1529	-0.0279
0.3825	1.3988	0.4332	-0.0995	0.6035	0.2900	-0.0774	0.3851	0.2069	-0.0570	0.2698	0.1514	-0.0421
0.4014	1.3946	0.4304	-0.1435	0.5999	0.2872	-0.1082	0.3823	0.2044	-0.0788	0.2677	0.1494	-0.0579
0.4182	1.3895	0.4282	-0.1954	0.5954	0.2838	-0.1432	0.3789	0.2013	-0.1030	0.2651	0.1469	-0.0752
0.4331	1.3832	0.4266	-0.2578	0.5899	0.2798	-0.1833	0.3746	0.1975	-0.1300	0.2618	0.1438	-0.0943
0.4464	1.3756	0.4259	-0.3347	0.5831	0.2752	-0.2297	0.3694	0.1930	-0.1604	0.2579	0.1401	-0.1156
0.4581	1.3662	0.4259	-0.4327	0.5748	0.2697	-0.2848	0.3631	0.1878	-0.1954	0.2533	0.1360	-0.1397
0.4686	1.3546	0.4270	-0.5642	0.5647	0.2635	-0.3532	0.3555	0.1822	-0.2377	0.2476	0.1317	-0.1685
0.4779	1.3393	0.4284	-0.7547	0.5520	0.2563	-0.4446	0.3462	0.1763	-0.2930	0.2408	0.1273	-0.2062
0.4861	1.3178	0.4283	-1.0621	0.5352	0.2475	-0.5816	0.3344	0.1702	-0.3752	0.2323	0.1233	-0.2623
0.4935	1.2824	0.3783	-1.5826	0.5106	0.2189	-0.8018	0.3177	0.1538	-0.5071	0.2204	0.1126	-0.3527
0.5000	1.1031	0.2144	-8.8316	0.4224	0.1654	-3.5945	0.2619	0.1279	-2.2307	0.1815	0.0962	-1.5461

Table 3.10 Normalized K_I , T_{II} and T_{33} for $a/W=0.2$, $t/W=2.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	1.3763	0.4079	0.1169	0.5912	0.2709	0.0778	0.3770	0.1931	0.0555	0.2641	0.1413	0.0406
0.0615	1.3768	0.4084	0.1167	0.5915	0.2712	0.0777	0.3773	0.1933	0.0554	0.2643	0.1415	0.0406
0.1161	1.3782	0.4096	0.1159	0.5924	0.2720	0.0772	0.3778	0.1938	0.0551	0.2647	0.1419	0.0403
0.1647	1.3801	0.4118	0.1144	0.5936	0.2733	0.0762	0.3787	0.1948	0.0543	0.2653	0.1425	0.0398
0.2078	1.3825	0.4148	0.1117	0.5951	0.2753	0.0744	0.3798	0.1961	0.0531	0.2661	0.1435	0.0389
0.2462	1.3851	0.4187	0.1073	0.5968	0.2778	0.0714	0.3809	0.1979	0.0509	0.2669	0.1448	0.0373
0.2802	1.3877	0.4234	0.1004	0.5985	0.2809	0.0667	0.3821	0.2001	0.0476	0.2678	0.1464	0.0349
0.3105	1.3901	0.4283	0.0903	0.6000	0.2844	0.0598	0.3832	0.2026	0.0427	0.2686	0.1483	0.0312
0.3374	1.3921	0.4331	0.0766	0.6013	0.2879	0.0503	0.3841	0.2052	0.0357	0.2692	0.1502	0.0261
0.3613	1.3934	0.4368	0.0587	0.6021	0.2909	0.0375	0.3846	0.2075	0.0265	0.2696	0.1519	0.0193
0.3825	1.3938	0.4389	0.0362	0.6023	0.2930	0.0213	0.3848	0.2092	0.0146	0.2697	0.1532	0.0105
0.4014	1.3933	0.4392	0.0086	0.6018	0.2939	0.0012	0.3843	0.2100	-0.0001	0.2694	0.1538	-0.0004
0.4182	1.3916	0.4377	-0.0251	0.6003	0.2934	-0.0236	0.3832	0.2097	-0.0182	0.2685	0.1536	-0.0137
0.4331	1.3887	0.4349	-0.0665	0.5978	0.2916	-0.0537	0.3813	0.2083	-0.0400	0.2671	0.1525	-0.0297
0.4464	1.3842	0.4316	-0.1183	0.5939	0.2886	-0.0905	0.3783	0.2057	-0.0663	0.2648	0.1505	-0.0488
0.4581	1.3778	0.4286	-0.1851	0.5883	0.2844	-0.1359	0.3740	0.2019	-0.0979	0.2615	0.1474	-0.0715
0.4686	1.3689	0.4268	-0.2744	0.5804	0.2789	-0.1928	0.3679	0.1966	-0.1362	0.2569	0.1430	-0.0986
0.4779	1.3564	0.4256	-0.4008	0.5692	0.2712	-0.2668	0.3593	0.1892	-0.1838	0.2506	0.1371	-0.1316
0.4861	1.3378	0.4215	-0.5930	0.5530	0.2590	-0.3688	0.3472	0.1788	-0.2466	0.2416	0.1292	-0.1743
0.4935	1.3058	0.3760	-0.8987	0.5270	0.2268	-0.5179	0.3286	0.1574	-0.3348	0.2281	0.1144	-0.2340
0.5000	1.1304	0.3266	-7.0017	0.4315	0.2035	-2.8799	0.2670	0.1510	-1.7848	0.1849	0.1122	-1.2363

Table 3.11 Normalized K_I , T_{II} and T_{33} for $a/W=0.2$, $t/W=4.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	1.3672	0.4075	0.1211	0.5855	0.2707	0.0805	0.3731	0.1930	0.0574	0.2613	0.1412	0.0420
0.0615	1.3673	0.4075	0.1210	0.5856	0.2707	0.0805	0.3731	0.1930	0.0574	0.2613	0.1412	0.0420
0.1161	1.3676	0.4075	0.1208	0.5858	0.2707	0.0803	0.3733	0.1929	0.0573	0.2614	0.1412	0.0419
0.1647	1.3681	0.4075	0.1204	0.5861	0.2707	0.0801	0.3735	0.1929	0.0571	0.2616	0.1412	0.0418
0.2078	1.3690	0.4076	0.1200	0.5866	0.2707	0.0798	0.3739	0.1930	0.0569	0.2618	0.1412	0.0417
0.2462	1.3702	0.4080	0.1194	0.5874	0.2710	0.0794	0.3744	0.1932	0.0566	0.2622	0.1414	0.0415
0.2802	1.3719	0.4090	0.1186	0.5885	0.2716	0.0789	0.3752	0.1936	0.0563	0.2627	0.1417	0.0412
0.3105	1.3742	0.4106	0.1173	0.5899	0.2726	0.0781	0.3761	0.1943	0.0557	0.2635	0.1422	0.0408
0.3374	1.3769	0.4132	0.1151	0.5916	0.2742	0.0767	0.3773	0.1954	0.0547	0.2643	0.1430	0.0400
0.3613	1.3800	0.4170	0.1113	0.5936	0.2766	0.0741	0.3787	0.1971	0.0529	0.2653	0.1442	0.0387
0.3825	1.3832	0.4220	0.1047	0.5957	0.2799	0.0697	0.3802	0.1994	0.0497	0.2664	0.1459	0.0364
0.4014	1.3863	0.4278	0.0940	0.5977	0.2839	0.0624	0.3816	0.2023	0.0445	0.2674	0.1480	0.0326
0.4182	1.3889	0.4336	0.0780	0.5993	0.2882	0.0512	0.3827	0.2054	0.0364	0.2682	0.1503	0.0266
0.4331	1.3904	0.4381	0.0552	0.6003	0.2919	0.0349	0.3834	0.2082	0.0245	0.2687	0.1524	0.0179
0.4464	1.3906	0.4399	0.0242	0.6002	0.2940	0.0122	0.3833	0.2100	0.0079	0.2686	0.1538	0.0056
0.4581	1.3889	0.4382	-0.0172	0.5987	0.2938	-0.0187	0.3821	0.2100	-0.0147	0.2677	0.1538	-0.0112
0.4686	1.3845	0.4337	-0.0732	0.5950	0.2909	-0.0606	0.3793	0.2077	-0.0452	0.2656	0.1521	-0.0336
0.4779	1.3764	0.4273	-0.1527	0.5879	0.2849	-0.1186	0.3738	0.2025	-0.0864	0.2614	0.1480	-0.0634
0.4861	1.3620	0.4192	-0.2746	0.5750	0.2738	-0.2019	0.3638	0.1926	-0.1426	0.2539	0.1400	-0.1031
0.4935	1.3350	0.3888	-0.4806	0.5503	0.2453	-0.3282	0.3452	0.1701	-0.2213	0.2401	0.1231	-0.1567
0.5000	1.1612	0.3779	-5.3364	0.4444	0.2189	-2.2432	0.2748	0.1591	-1.3910	0.1903	0.1176	-0.9634

Table 3.12 Normalized K_I , T_{II} and T_{33} for $a/W=0.4$, $t/W=0.1$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	2.2452	0.6641	-2.0153	1.1124	0.4148	-1.2047	0.7479	0.2961	-0.8261	0.5486	0.2235	-0.6082
0.0346	2.2446	0.6639	-2.0228	1.1122	0.4146	-1.2086	0.7478	0.2960	-0.8287	0.5485	0.2235	-0.6101
0.0679	2.2440	0.6655	-2.0444	1.1118	0.4152	-1.2197	0.7475	0.2965	-0.8360	0.5483	0.2238	-0.6155
0.1001	2.2427	0.6679	-2.0792	1.1110	0.4163	-1.2377	0.7469	0.2972	-0.8479	0.5479	0.2244	-0.6242
0.1311	2.2414	0.6712	-2.1271	1.1100	0.4176	-1.2624	0.7462	0.2982	-0.8643	0.5474	0.2252	-0.6361
0.1610	2.2395	0.6753	-2.1883	1.1086	0.4193	-1.2938	0.7453	0.2995	-0.8851	0.5467	0.2261	-0.6514
0.1898	2.2369	0.6801	-2.2633	1.1070	0.4214	-1.3323	0.7442	0.3009	-0.9107	0.5459	0.2273	-0.6700
0.2176	2.2337	0.6856	-2.3534	1.1050	0.4237	-1.3785	0.7429	0.3026	-0.9413	0.5449	0.2286	-0.6924
0.2444	2.2304	0.6919	-2.4603	1.1027	0.4263	-1.4330	0.7414	0.3046	-0.9775	0.5438	0.2301	-0.7189
0.2702	2.2259	0.6988	-2.5865	1.1000	0.4292	-1.4972	0.7395	0.3067	-1.0202	0.5425	0.2317	-0.7501
0.2951	2.2207	0.7065	-2.7355	1.0968	0.4325	-1.5728	0.7374	0.3091	-1.0705	0.5409	0.2336	-0.7870
0.3191	2.2142	0.7149	-2.9126	1.0931	0.4360	-1.6623	0.7349	0.3118	-1.1301	0.5391	0.2356	-0.8306
0.3423	2.2070	0.7241	-3.1253	1.0887	0.4400	-1.7695	0.7320	0.3148	-1.2015	0.5370	0.2379	-0.8829
0.3646	2.1985	0.7342	-3.3851	1.0835	0.4443	-1.8997	0.7285	0.3180	-1.2884	0.5345	0.2404	-0.9466
0.3861	2.1880	0.7452	-3.7089	1.0773	0.4491	-2.0615	0.7244	0.3217	-1.3964	0.5315	0.2431	-1.0258
0.4069	2.1748	0.7566	-4.1230	1.0697	0.4541	-2.2674	0.7193	0.3254	-1.5342	0.5278	0.2460	-1.1268
0.4269	2.1575	0.7643	-4.6635	1.0602	0.4573	-2.5353	0.7130	0.3281	-1.7136	0.5232	0.2481	-1.2584
0.4462	2.1354	0.7494	-5.3742	1.0478	0.4497	-2.8864	0.7048	0.3236	-1.9489	0.5172	0.2448	-1.4310
0.4648	2.1048	0.6120	-6.2272	1.0309	0.3825	-3.3073	0.6935	0.2791	-2.2311	0.5090	0.2122	-1.6381
0.4827	2.0612	-0.1896	-6.6403	1.0075	-0.0078	-3.5143	0.6779	0.0172	-2.3691	0.4976	0.0200	-1.7393
0.5000	1.8103	0.1414	-29.2747	0.8817	0.1650	-14.5408	0.5936	0.1337	-9.7924	0.4358	0.1052	-7.1886

Table 3.13 Normalized K_I , T_{II} and T_{33} for $a/W=0.4$, $t/W=0.2$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	2.2407	0.5943	-1.2232	1.1144	0.3859	-0.7767	0.7498	0.2775	-0.5444	0.5501	0.2096	-0.4039
0.0490	2.2407	0.5951	-1.2350	1.1140	0.3861	-0.7831	0.7495	0.2776	-0.5486	0.5498	0.2097	-0.4069
0.0946	2.2388	0.5966	-1.2637	1.1129	0.3864	-0.7987	0.7488	0.2778	-0.5588	0.5493	0.2098	-0.4143
0.1361	2.2369	0.5999	-1.3102	1.1112	0.3874	-0.8238	0.7475	0.2784	-0.5752	0.5483	0.2103	-0.4262
0.1747	2.2337	0.6033	-1.3700	1.1090	0.3882	-0.8559	0.7459	0.2788	-0.5962	0.5472	0.2107	-0.4414
0.2098	2.2304	0.6081	-1.4451	1.1062	0.3896	-0.8961	0.7439	0.2798	-0.6226	0.5457	0.2114	-0.4605
0.2424	2.2259	0.6126	-1.5329	1.1030	0.3907	-0.9427	0.7417	0.2805	-0.6532	0.5440	0.2121	-0.4827
0.2722	2.2207	0.6186	-1.6367	1.0992	0.3924	-0.9975	0.7390	0.2817	-0.6891	0.5420	0.2131	-0.5088
0.2998	2.2142	0.6237	-1.7550	1.0950	0.3936	-1.0594	0.7360	0.2827	-0.7299	0.5398	0.2139	-0.5384
0.3250	2.2077	0.6302	-1.8927	1.0901	0.3955	-1.1310	0.7326	0.2841	-0.7770	0.5374	0.2151	-0.5727
0.3484	2.1992	0.6356	-2.0493	1.0847	0.3968	-1.2117	0.7289	0.2853	-0.8302	0.5346	0.2162	-0.6115
0.3698	2.1900	0.6425	-2.2330	1.0786	0.3989	-1.3057	0.7247	0.2870	-0.8922	0.5315	0.2176	-0.6567
0.3896	2.1794	0.6473	-2.4456	1.0717	0.3999	-1.4136	0.7200	0.2882	-0.9636	0.5281	0.2187	-0.7089
0.4076	2.1668	0.6542	-2.7013	1.0638	0.4020	-1.5423	0.7146	0.2901	-1.0489	0.5241	0.2203	-0.7712
0.4244	2.1528	0.6577	-3.0108	1.0549	0.4025	-1.6970	0.7086	0.2911	-1.1517	0.5197	0.2213	-0.8464
0.4397	2.1354	0.6640	-3.3987	1.0444	0.4044	-1.8894	0.7015	0.2932	-1.2797	0.5145	0.2231	-0.9402
0.4539	2.1143	0.6656	-3.9281	1.0320	0.4043	-2.1501	0.6931	0.2940	-1.4536	0.5085	0.2239	-1.0676
0.4668	2.0862	0.6679	-4.6275	1.0160	0.4048	-2.4926	0.6825	0.2954	-1.6824	0.5007	0.2252	-1.2353
0.4789	2.0500	0.6544	-5.8780	0.9956	0.3982	-3.1011	0.6688	0.2921	-2.0899	0.4907	0.2230	-1.5342
0.4898	1.9791	0.6529	-8.8721	0.9582	0.3995	-4.5486	0.6439	0.2945	-3.0614	0.4724	0.2249	-2.2471
0.5000	1.7222	-0.1601	-30.1771	0.8291	0.0242	-14.8093	0.5574	0.0431	-9.9582	0.4091	0.0403	-7.3088

Table 3.14 Normalized K_I , T_{II} and T_{33} for $a/W=0.4$, $t/W=0.5$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	2.2272	0.5467	-0.5103	1.1137	0.3725	-0.3454	0.7514	0.2745	-0.2518	0.5520	0.2097	-0.1908
0.0615	2.2265	0.5463	-0.5211	1.1130	0.3718	-0.3523	0.7509	0.2738	-0.2565	0.5515	0.2091	-0.1943
0.1161	2.2240	0.5458	-0.5497	1.1110	0.3705	-0.3703	0.7494	0.2724	-0.2690	0.5504	0.2079	-0.2034
0.1647	2.2201	0.5452	-0.5927	1.1081	0.3686	-0.3971	0.7471	0.2704	-0.2873	0.5486	0.2061	-0.2169
0.2078	2.2155	0.5449	-0.6480	1.1043	0.3663	-0.4312	0.7441	0.2680	-0.3106	0.5463	0.2040	-0.2338
0.2462	2.2097	0.5447	-0.7149	1.0996	0.3638	-0.4717	0.7405	0.2653	-0.3379	0.5435	0.2017	-0.2536
0.2802	2.2031	0.5450	-0.7931	1.0942	0.3611	-0.5181	0.7364	0.2625	-0.3690	0.5403	0.1992	-0.2762
0.3105	2.1953	0.5456	-0.8829	1.0881	0.3583	-0.5703	0.7317	0.2596	-0.4038	0.5367	0.1968	-0.3013
0.3374	2.1867	0.5465	-0.9851	1.0811	0.3555	-0.6286	0.7265	0.2567	-0.4424	0.5327	0.1944	-0.3291
0.3613	2.1768	0.5477	-1.1012	1.0735	0.3526	-0.6934	0.7208	0.2539	-0.4850	0.5283	0.1922	-0.3599
0.3825	2.1655	0.5490	-1.2335	1.0650	0.3497	-0.7657	0.7146	0.2513	-0.5324	0.5236	0.1901	-0.3941
0.4014	2.1528	0.5504	-1.3853	1.0558	0.3467	-0.8469	0.7078	0.2488	-0.5855	0.5185	0.1883	-0.4324
0.4182	2.1387	0.5516	-1.5614	1.0457	0.3437	-0.9393	0.7005	0.2466	-0.6459	0.5130	0.1867	-0.4761
0.4331	2.1232	0.5527	-1.7692	1.0346	0.3407	-1.0464	0.6925	0.2445	-0.7159	0.5070	0.1854	-0.5268
0.4464	2.1055	0.5533	-2.0204	1.0223	0.3376	-1.1735	0.6838	0.2427	-0.7991	0.5006	0.1843	-0.5871
0.4581	2.0848	0.5533	-2.3347	1.0085	0.3343	-1.3301	0.6741	0.2411	-0.9017	0.4934	0.1835	-0.6618
0.4686	2.0605	0.5528	-2.7489	0.9926	0.3312	-1.5334	0.6632	0.2399	-1.0354	0.4853	0.1830	-0.7592
0.4779	2.0295	0.5513	-3.3409	0.9737	0.3281	-1.8201	0.6503	0.2391	-1.2246	0.4758	0.1829	-0.8972
0.4861	1.9893	0.5455	-4.2918	0.9495	0.3237	-2.2756	0.6339	0.2379	-1.5264	0.4638	0.1825	-1.1176
0.4935	1.9260	0.3934	-5.8885	0.9141	0.2526	-3.0352	0.6101	0.1928	-2.0307	0.4464	0.1500	-1.4863
0.5000	1.6405	-0.0935	-27.2184	0.7711	0.0458	-13.0640	0.5148	0.0582	-8.7222	0.3767	0.0520	-6.3823

Table 3. 15 Normalized K_I , T_{II} and T_{33} for $a/W=0.4$, $t/W=1.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	2.1966	0.5299	-0.1172	1.0969	0.3605	-0.0776	0.7404	0.2663	-0.0562	0.5441	0.2038	-0.0425
0.0615	2.1959	0.5288	-0.1260	1.0965	0.3598	-0.0836	0.7401	0.2658	-0.0607	0.5438	0.2034	-0.0459
0.1161	2.1946	0.5262	-0.1489	1.0953	0.3582	-0.0995	0.7392	0.2646	-0.0724	0.5431	0.2025	-0.0548
0.1647	2.1920	0.5222	-0.1830	1.0934	0.3556	-0.1229	0.7377	0.2627	-0.0896	0.5420	0.2010	-0.0679
0.2078	2.1880	0.5172	-0.2262	1.0908	0.3521	-0.1525	0.7357	0.2600	-0.1112	0.5404	0.1989	-0.0843
0.2462	2.1841	0.5112	-0.2777	1.0873	0.3477	-0.1874	0.7330	0.2565	-0.1365	0.5383	0.1961	-0.1034
0.2802	2.1781	0.5047	-0.3370	1.0830	0.3426	-0.2271	0.7297	0.2523	-0.1650	0.5357	0.1927	-0.1248
0.3105	2.1715	0.4980	-0.4042	1.0777	0.3368	-0.2713	0.7257	0.2474	-0.1965	0.5326	0.1888	-0.1483
0.3374	2.1635	0.4913	-0.4798	1.0715	0.3305	-0.3201	0.7209	0.2421	-0.2307	0.5289	0.1844	-0.1736
0.3613	2.1542	0.4849	-0.5648	1.0644	0.3238	-0.3736	0.7154	0.2362	-0.2677	0.5246	0.1796	-0.2009
0.3825	2.1434	0.4788	-0.6609	1.0561	0.3168	-0.4324	0.7091	0.2301	-0.3078	0.5197	0.1746	-0.2302
0.4014	2.1313	0.4733	-0.7704	1.0468	0.3095	-0.4974	0.7020	0.2237	-0.3516	0.5143	0.1694	-0.2620
0.4182	2.1177	0.4682	-0.8969	1.0363	0.3019	-0.5701	0.6941	0.2172	-0.4000	0.5082	0.1642	-0.2970
0.4331	2.1020	0.4635	-1.0457	1.0245	0.2942	-0.6530	0.6853	0.2108	-0.4545	0.5015	0.1592	-0.3364
0.4464	2.0841	0.4592	-1.2250	1.0112	0.2864	-0.7497	0.6755	0.2045	-0.5177	0.4940	0.1544	-0.3819
0.4581	2.0633	0.4549	-1.4485	0.9961	0.2784	-0.8665	0.6645	0.1985	-0.5937	0.4857	0.1500	-0.4367
0.4686	2.0387	0.4504	-1.7389	0.9787	0.2702	-1.0140	0.6520	0.1928	-0.6895	0.4763	0.1460	-0.5059
0.4779	2.0081	0.4427	-2.1386	0.9579	0.2606	-1.2118	0.6372	0.1868	-0.8181	0.4654	0.1420	-0.5990
0.4861	1.9675	0.4194	-2.7289	0.9314	0.2443	-1.4976	0.6189	0.1773	-1.0046	0.4518	0.1357	-0.7344
0.4935	1.9042	0.2680	-3.6400	0.8930	0.1703	-1.9322	0.5927	0.1308	-1.2889	0.4326	0.1025	-0.9412
0.5000	1.6231	0.1351	-21.0594	0.7488	0.1277	-9.9763	0.4967	0.1085	-6.6200	0.3625	0.0875	-4.8304

Table 3.16 Normalized K_I , T_{II} and T_{33} for $a/W=0.4$, $t/W=2.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	2.1481	0.4385	0.0997	1.0660	0.2984	0.0682	0.7182	0.2206	0.0506	0.5273	0.1689	0.0388
0.0615	2.1488	0.4411	0.0952	1.0663	0.3001	0.0652	0.7184	0.2218	0.0484	0.5275	0.1699	0.0371
0.1161	2.1502	0.4476	0.0828	1.0670	0.3045	0.0570	0.7189	0.2250	0.0424	0.5278	0.1723	0.0326
0.1647	2.1508	0.4564	0.0631	1.0677	0.3104	0.0438	0.7194	0.2294	0.0327	0.5283	0.1757	0.0252
0.2078	2.1522	0.4658	0.0362	1.0683	0.3169	0.0257	0.7199	0.2342	0.0195	0.5286	0.1793	0.0151
0.2462	2.1522	0.4743	0.0023	1.0686	0.3228	0.0027	0.7201	0.2386	0.0026	0.5287	0.1827	0.0022
0.2802	2.1522	0.4809	-0.0385	1.0683	0.3276	-0.0251	0.7198	0.2422	-0.0179	0.5285	0.1855	-0.0134
0.3105	2.1508	0.4849	-0.0863	1.0672	0.3306	-0.0578	0.7189	0.2444	-0.0420	0.5278	0.1872	-0.0317
0.3374	2.1481	0.4861	-0.1413	1.0651	0.3316	-0.0954	0.7173	0.2451	-0.0695	0.5266	0.1877	-0.0527
0.3613	2.1441	0.4848	-0.2038	1.0620	0.3306	-0.1379	0.7149	0.2441	-0.1006	0.5247	0.1868	-0.0762
0.3825	2.1381	0.4814	-0.2748	1.0576	0.3277	-0.1858	0.7115	0.2416	-0.1351	0.5220	0.1848	-0.1023
0.4014	2.1306	0.4765	-0.3558	1.0517	0.3231	-0.2394	0.7070	0.2377	-0.1735	0.5185	0.1815	-0.1310
0.4182	2.1211	0.4707	-0.4488	1.0443	0.3171	-0.2996	0.7013	0.2324	-0.2159	0.5141	0.1770	-0.1624
0.4331	2.1096	0.4643	-0.5572	1.0351	0.3098	-0.3677	0.6942	0.2258	-0.2630	0.5086	0.1716	-0.1971
0.4464	2.0945	0.4576	-0.6862	1.0239	0.3012	-0.4460	0.6857	0.2183	-0.3161	0.5019	0.1654	-0.2358
0.4581	2.0772	0.4505	-0.8439	1.0102	0.2913	-0.5379	0.6753	0.2097	-0.3774	0.4940	0.1586	-0.2802
0.4686	2.0549	0.4428	-1.0441	0.9934	0.2800	-0.6497	0.6627	0.2004	-0.4508	0.4844	0.1514	-0.3330
0.4779	2.0266	0.4317	-1.3132	0.9723	0.2673	-0.7123	0.6473	0.1898	-0.5440	0.4727	0.1435	-0.4001
0.4861	1.9871	0.4086	-1.7074	0.9445	0.2457	-0.9952	0.6273	0.1761	-0.6742	0.4577	0.1338	-0.4939
0.4935	1.9253	0.3052	-2.3487	0.9036	0.1883	-1.3116	0.5988	0.1398	-0.8784	0.4366	0.1082	-0.6415
0.5000	1.6404	0.2603	-16.0124	0.7507	0.1775	-7.5829	0.4966	0.1408	-5.0185	0.3619	0.1109	-3.6573

Table 3.17 Normalized K_I , T_{II} and T_{33} for $a/W=0.4$, $t/W=4.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	2.1218	0.4209	0.1244	1.0486	0.2868	0.0847	0.7057	0.2122	0.0626	0.5178	0.1626	0.0480
0.0615	2.1218	0.4209	0.1242	1.0488	0.2868	0.0846	0.7058	0.2122	0.0625	0.5179	0.1626	0.0479
0.1161	2.1225	0.4211	0.1237	1.0493	0.2869	0.0842	0.7061	0.2123	0.0623	0.5182	0.1626	0.0477
0.1647	2.1239	0.4220	0.1225	1.0501	0.2875	0.0834	0.7067	0.2127	0.0616	0.5186	0.1630	0.0472
0.2078	2.1252	0.4245	0.1196	1.0512	0.2892	0.0814	0.7075	0.2139	0.0602	0.5192	0.1639	0.0461
0.2462	2.1272	0.4292	0.1137	1.0525	0.2922	0.0775	0.7085	0.2161	0.0574	0.5200	0.1656	0.0440
0.2802	2.1300	0.4366	0.1033	1.0541	0.2972	0.0705	0.7096	0.2197	0.0523	0.5209	0.1683	0.0401
0.3105	2.1327	0.4464	0.0866	1.0557	0.3037	0.0593	0.7108	0.2245	0.0441	0.5218	0.1719	0.0339
0.3374	2.1347	0.4577	0.0621	1.0572	0.3113	0.0429	0.7119	0.2301	0.0320	0.5225	0.1762	0.0247
0.3613	2.1360	0.4688	0.0288	1.0582	0.3190	0.0203	0.7126	0.2358	0.0154	0.5231	0.1805	0.0120
0.3825	2.1367	0.4781	-0.0139	1.0584	0.3256	-0.0089	0.7127	0.2407	-0.0061	0.5232	0.1843	-0.0044
0.4014	2.1360	0.4842	-0.0668	1.0577	0.3301	-0.0453	0.7121	0.2440	-0.0329	0.5227	0.1869	-0.0248
0.4182	2.1333	0.4862	-0.1307	1.0555	0.3317	-0.0894	0.7104	0.2452	-0.0653	0.5213	0.1877	-0.0495
0.4331	2.1279	0.4844	-0.2075	1.0515	0.3302	-0.1421	0.7073	0.2439	-0.1038	0.5189	0.1866	-0.0787
0.4464	2.1198	0.4791	-0.3001	1.0451	0.3256	-0.2049	0.7024	0.2399	-0.1491	0.5151	0.1833	-0.1128
0.4581	2.1075	0.4711	-0.4138	1.0359	0.3179	-0.2802	0.6953	0.2331	-0.2025	0.5095	0.1777	-0.1526
0.4686	2.0910	0.4613	-0.5579	1.0228	0.3071	-0.3723	0.6852	0.2236	-0.2662	0.5017	0.1699	-0.1994
0.4779	2.0668	0.4488	-0.7516	1.0042	0.2923	-0.4898	0.6711	0.2109	-0.3453	0.4908	0.1596	-0.2568
0.4861	2.0316	0.4291	-1.0372	0.9773	0.2707	-0.6523	0.6510	0.1938	-0.4516	0.4755	0.1464	-0.3332
0.4935	1.9726	0.3566	-1.5102	0.9344	0.2210	-0.9045	0.6200	0.1598	-0.6134	0.4522	0.1219	-0.4492
0.5000	1.6758	0.3363	-12.0742	0.7659	0.2073	-5.7673	0.5062	0.1598	-3.8144	0.3687	0.1246	-2.7788

Table 3.18 Normalized K_I , T_{II} and T_{33} for $a/W=0.6$, $t/W=0.1$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	4.3028	1.6223	-5.2629	2.3459	0.9810	-3.1026	1.5317	0.6531	-2.0364	1.0443	0.4483	-1.3892
0.0346	4.3021	1.6220	-5.2812	2.3455	0.9808	-3.1127	1.5317	0.6530	-2.0430	1.0441	0.4482	-1.3937
0.0679	4.3005	1.6252	-5.3332	2.3447	0.9824	-3.1415	1.5311	0.6541	-2.0617	1.0437	0.4490	-1.4065
0.1001	4.2979	1.6303	-5.4171	2.3430	0.9850	-3.1879	1.5298	0.6558	-2.0919	1.0430	0.4502	-1.4271
0.1311	4.2941	1.6371	-5.5324	2.3406	0.9885	-3.2517	1.5286	0.6582	-2.1334	1.0420	0.4518	-1.4553
0.1610	4.2894	1.6456	-5.6794	2.3377	0.9928	-3.3330	1.5267	0.6611	-2.1863	1.0407	0.4538	-1.4914
0.1898	4.2837	1.6557	-5.8596	2.3344	0.9980	-3.4326	1.5242	0.6645	-2.2511	1.0392	0.4562	-1.5355
0.2176	4.2768	1.6673	-6.0756	2.3303	1.0039	-3.5518	1.5217	0.6685	-2.3287	1.0373	0.4589	-1.5884
0.2444	4.2685	1.6805	-6.3313	2.3253	1.0107	-3.6928	1.5185	0.6730	-2.4205	1.0351	0.4621	-1.6510
0.2702	4.2588	1.6950	-6.6326	2.3195	1.0182	-3.8588	1.5147	0.6781	-2.5286	1.0325	0.4655	-1.7246
0.2951	4.2473	1.7111	-6.9877	2.3125	1.0265	-4.0541	1.5102	0.6836	-2.6558	1.0295	0.4693	-1.8113
0.3191	4.2339	1.7289	-7.4087	2.3046	1.0356	-4.2854	1.5051	0.6898	-2.8064	1.0260	0.4736	-1.9140
0.3423	4.2180	1.7485	-7.9132	2.2954	1.0457	-4.5622	1.4988	0.6966	-2.9868	1.0219	0.4783	-2.0369
0.3646	4.1990	1.7700	-8.5276	2.2841	1.0569	-4.8986	1.4917	0.7041	-3.2061	1.0170	0.4834	-2.1864
0.3861	4.1763	1.7937	-9.2916	2.2710	1.0692	-5.3164	1.4833	0.7124	-3.4784	1.0111	0.4891	-2.3721
0.4069	4.1484	1.8183	-10.2658	2.2548	1.0820	-5.8482	1.4722	0.7211	-3.8253	1.0040	0.4951	-2.6086
0.4269	4.1133	1.8344	-11.5348	2.2347	1.0904	-6.5399	1.4591	0.7269	-4.2765	0.9950	0.4990	-2.9162
0.4462	4.0672	1.7987	-13.1998	2.2084	1.0708	-7.4464	1.4426	0.7144	-4.8680	0.9834	0.4906	-3.3195
0.4648	4.0039	1.4788	-15.1967	2.1728	0.8975	-8.5330	1.4191	0.6017	-5.5771	0.9676	0.4137	-3.8030
0.4827	3.9158	-0.3813	-16.1726	2.1232	-0.1090	-9.0672	1.3869	-0.0553	-5.9253	0.9455	-0.0343	-4.0404
0.5000	3.4315	0.4202	-68.7319	1.8577	0.3375	-37.5223	1.2136	0.2363	-24.5141	0.8275	0.1644	-16.7157

Table 3.19 Normalized K_I , T_{II} and T_{33} for $a/W=0.6$, $t/W=0.2$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	4.2990	1.4787	-3.2914	2.3475	0.9087	-1.9918	1.5330	0.6060	-1.3156	1.0452	0.4160	-0.8988
0.0490	4.2976	1.4799	-3.3209	2.3467	0.9091	-2.0084	1.5323	0.6063	-1.3263	1.0452	0.4162	-0.9061
0.0946	4.2941	1.4816	-3.3923	2.3443	0.9096	-2.0487	1.5311	0.6066	-1.3524	1.0451	0.4164	-0.9239
0.1361	4.2882	1.4868	-3.5075	2.3406	0.9117	-2.1136	1.5286	0.6080	-1.3944	1.0448	0.4174	-0.9524
0.1747	4.2806	1.4912	-3.6553	2.3356	0.9133	-2.1966	1.5254	0.6090	-1.4483	1.0446	0.4181	-0.9890
0.2098	4.2712	1.4986	-3.8403	2.3299	0.9163	-2.3003	1.5217	0.6111	-1.5155	1.0442	0.4195	-1.0348
0.2424	4.2604	1.5049	-4.0557	2.3228	0.9186	-2.4209	1.5166	0.6126	-1.5937	1.0438	0.4207	-1.0880
0.2722	4.2473	1.5142	-4.3092	2.3146	0.9224	-2.5623	1.5115	0.6152	-1.6854	1.0432	0.4225	-1.1504
0.2998	4.2328	1.5214	-4.5966	2.3054	0.9251	-2.7222	1.5051	0.6172	-1.7892	1.0427	0.4239	-1.2211
0.3250	4.2159	1.5316	-4.9295	2.2950	0.9294	-2.9069	1.4988	0.6201	-1.9092	1.0421	0.4261	-1.3027
0.3484	4.1970	1.5393	-5.3061	2.2836	0.9323	-3.1153	1.4910	0.6223	-2.0446	1.0414	0.4276	-1.3949
0.3698	4.1754	1.5506	-5.7456	2.2706	0.9372	-3.3576	1.4820	0.6258	-2.2021	1.0407	0.4301	-1.5022
0.3896	4.1512	1.5570	-6.2517	2.2557	0.9395	-3.6360	1.4729	0.6276	-2.3830	1.0399	0.4315	-1.6255
0.4076	4.1231	1.5684	-6.8571	2.2390	0.9445	-3.9678	1.4618	0.6314	-2.5990	1.0390	0.4341	-1.7727
0.4244	4.0913	1.5723	-7.5867	2.2201	0.9456	-4.3666	1.4492	0.6325	-2.8586	1.0382	0.4350	-1.9496
0.4397	4.0530	1.5830	-8.4966	2.1974	0.9506	-4.8625	1.4346	0.6363	-3.1816	1.0372	0.4377	-2.1698
0.4539	4.0080	1.5835	-9.7325	2.1711	0.9501	-5.5341	1.4177	0.6366	-3.6193	1.0362	0.4380	-2.4681
0.4668	3.9493	1.5867	-11.3597	2.1372	0.9517	-6.4165	1.3952	0.6383	-4.1946	1.0352	0.4392	-2.8604
0.4789	3.8739	1.5552	-14.2570	2.0940	0.9350	-7.9835	1.3672	0.6282	-5.2170	1.0341	0.4324	-3.5575
0.4898	3.7327	1.5577	-21.1665	2.0150	0.9391	-11.7109	1.3157	0.6317	-7.6503	1.0329	0.4349	-5.2166
0.5000	3.2361	-0.2671	-70.2157	1.7430	-0.0257	-38.1310	1.1383	0.0021	-24.9039	1.0317	0.0054	-16.9815

Table 3.20 Normalized K_I , T_{II} and T_{33} for $a/W=0.6$, $t/W=0.5$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	4.2664	1.4081	-1.3548	2.3356	0.8846	-0.8512	1.5267	0.5946	-0.5708	1.0410	0.4092	-0.3923
0.0615	4.2640	1.4050	-1.3855	2.3340	0.8823	-0.8697	1.5254	0.5930	-0.5829	1.0403	0.4081	-0.4005
0.1161	4.2579	1.3987	-1.4663	2.3299	0.8774	-0.9180	1.5229	0.5894	-0.6146	1.0383	0.4056	-0.4221
0.1647	4.2486	1.3896	-1.5863	2.3237	0.8702	-0.9895	1.5185	0.5842	-0.6614	1.0354	0.4019	-0.4539
0.2078	4.2364	1.3788	-1.7391	2.3154	0.8616	-1.0800	1.5128	0.5779	-0.7205	1.0315	0.3975	-0.4941
0.2462	4.2216	1.3667	-1.9209	2.3054	0.8517	-1.1870	1.5058	0.5708	-0.7903	1.0269	0.3924	-0.5416
0.2802	4.2041	1.3541	-2.1299	2.2937	0.8412	-1.3090	1.4981	0.5633	-0.8697	1.0214	0.3872	-0.5955
0.3105	4.1839	1.3410	-2.3659	2.2807	0.8302	-1.4458	1.4891	0.5554	-0.9586	1.0152	0.3817	-0.6559
0.3374	4.1613	1.3279	-2.6301	2.2659	0.8190	-1.5976	1.4794	0.5475	-1.0571	1.0084	0.3762	-0.7228
0.3613	4.1361	1.3149	-2.9254	2.2493	0.8078	-1.7658	1.4683	0.5397	-1.1661	1.0007	0.3708	-0.7968
0.3825	4.1079	1.3021	-3.2564	2.2313	0.7966	-1.9528	1.4558	0.5321	-1.2872	0.9924	0.3656	-0.8791
0.4014	4.0771	1.2895	-3.6304	2.2114	0.7857	-2.1624	1.4426	0.5247	-1.4229	0.9833	0.3606	-0.9713
0.4182	4.0428	1.2771	-4.0582	2.1896	0.7749	-2.4003	1.4285	0.5176	-1.5770	0.9734	0.3559	-1.0761
0.4331	4.0051	1.2646	-4.5565	2.1657	0.7644	-2.6754	1.4123	0.5109	-1.7551	0.9626	0.3514	-1.1972
0.4463	3.9627	1.2520	-5.1517	2.1394	0.7541	-3.0016	1.3952	0.5045	-1.9665	0.9507	0.3472	-1.3411
0.4581	3.9148	1.2388	-5.8885	2.1100	0.7437	-3.4029	1.3757	0.4982	-2.2268	0.9375	0.3431	-1.5182
0.4686	3.8592	1.2261	-6.8499	2.0765	0.7341	-3.9234	1.3535	0.4927	-2.5647	0.9224	0.3395	-1.7484
0.4779	3.7919	1.2130	-8.2124	2.0363	0.7250	-4.6573	1.3272	0.4877	-3.0415	0.9044	0.3363	-2.0731
0.4861	3.7048	1.1931	-10.3862	1.9852	0.7135	-5.8229	1.2938	0.4814	-3.7997	0.8817	0.3322	-2.5897
0.4935	3.5749	0.8488	-14.0204	1.9107	0.5316	-7.7663	1.2452	0.3644	-5.0646	0.8485	0.2527	-3.4515
0.5000	3.0268	-0.1890	-62.2295	1.6112	0.0054	-33.4261	1.0500	0.0236	-21.7862	0.7155	0.0207	-14.8468

Table 3.21 Normalized K_I , T_{II} and T_{33} for $a/W=0.6$, $t/W=1.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	4.1967	1.2844	-0.2317	2.2966	0.8137	-0.1523	1.5013	0.5491	-0.1038	1.0241	0.3786	-0.0718
0.0615	4.1956	1.2845	-0.2571	2.2958	0.8135	-0.1682	1.5007	0.5490	-0.1145	1.0237	0.3785	-0.0792
0.1161	4.1919	1.2853	-0.3237	2.2933	0.8135	-0.2099	1.4994	0.5488	-0.1424	1.0225	0.3783	-0.0984
0.1647	4.1862	1.2846	-0.4229	2.2891	0.8123	-0.2718	1.4962	0.5477	-0.1839	1.0206	0.3775	-0.1268
0.2078	4.1779	1.2812	-0.5494	2.2836	0.8091	-0.3505	1.4923	0.5452	-0.2364	1.0179	0.3757	-0.1629
0.2462	4.1673	1.2740	-0.6999	2.2765	0.8031	-0.4437	1.4878	0.5408	-0.2985	1.0145	0.3725	-0.2054
0.2802	4.1542	1.2630	-0.8724	2.2676	0.7945	-0.5498	1.4813	0.5345	-0.3690	1.0102	0.3680	-0.2536
0.3105	4.1380	1.2480	-1.0661	2.2566	0.7830	-0.6680	1.4742	0.5261	-0.4472	1.0050	0.3621	-0.3071
0.3374	4.1186	1.2294	-1.2810	2.2437	0.7688	-0.7980	1.4650	0.5159	-0.5328	0.9988	0.3549	-0.3655
0.3613	4.0962	1.2078	-1.5184	2.2287	0.7523	-0.9400	1.4552	0.5041	-0.6261	0.9917	0.3466	-0.4291
0.3825	4.0701	1.1836	-1.7812	2.2114	0.7338	-1.0954	1.4432	0.4910	-0.7277	0.9836	0.3373	-0.4982
0.4014	4.0404	1.1575	-2.0738	2.1918	0.7137	-1.2662	1.4299	0.4768	-0.8390	0.9744	0.3275	-0.5738
0.4182	4.0068	1.1299	-2.4037	2.1697	0.6925	-1.4563	1.4150	0.4619	-0.9624	0.9640	0.3172	-0.6577
0.4331	3.9685	1.1013	-2.7823	2.1448	0.6704	-1.6715	1.3979	0.4467	-1.1018	0.9524	0.3067	-0.7524
0.4463	3.9249	1.0721	-3.2279	2.1164	0.6479	-1.9216	1.3790	0.4314	-1.2636	0.9395	0.2962	-0.8622
0.4581	3.8749	1.0421	-3.7706	2.0844	0.6249	-2.2223	1.3576	0.4161	-1.4579	0.9248	0.2858	-0.9943
0.4686	3.8164	1.0115	-4.4620	2.0476	0.6019	-2.6009	1.3331	0.4012	-1.7026	0.9080	0.2758	-1.1606
0.4779	3.7455	0.9744	-5.3969	2.0035	0.5759	-3.1075	1.3039	0.3847	-2.0302	0.8881	0.2647	-1.3834
0.4861	3.6536	0.9043	-6.7583	1.9476	0.5335	-3.8388	1.2671	0.3582	-2.5036	0.8630	0.2470	-1.7055
0.4935	3.5169	0.5522	-8.8395	1.8670	0.3447	-4.9499	1.2143	0.2373	-3.2236	0.8269	0.1650	-2.1955
0.5000	2.9701	0.3062	-47.9023	1.5652	0.2393	-25.5402	1.0178	0.1728	-16.6102	0.6931	0.1216	-11.3112

Table 3.22 Normalized K_I , T_{II} and T_{33} for $a/W=0.6$, $t/W=2.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	4.1163	1.0737	0.2522	2.2472	0.6794	0.1578	1.4683	0.4584	0.1061	1.0015	0.3160	0.0930
0.0615	4.1165	1.0783	0.2429	2.2472	0.6824	0.1518	1.4690	0.4604	0.1020	1.0015	0.3174	0.0933
0.1161	4.1170	1.0908	0.2172	2.2476	0.6904	0.1352	1.4690	0.4659	0.0907	1.0016	0.3212	0.0939
0.1647	4.1172	1.1089	0.1747	2.2476	0.7020	0.1079	1.4690	0.4737	0.0722	1.0016	0.3266	0.0943
0.2078	4.1168	1.1305	0.1139	2.2472	0.7156	0.0689	1.4683	0.4829	0.0458	1.0014	0.3330	0.0938
0.2462	4.1154	1.1530	0.0329	2.2459	0.7297	0.0173	1.4677	0.4923	0.0110	1.0009	0.3394	0.0911
0.2802	4.1123	1.1744	-0.0699	2.2437	0.7428	-0.0479	1.4663	0.5010	-0.0330	0.9998	0.3454	0.0851
0.3105	4.1072	1.1918	-0.1958	2.2403	0.7532	-0.1274	1.4637	0.5079	-0.0864	0.9981	0.3500	0.0741
0.3374	4.0997	1.2038	-0.3457	2.2352	0.7598	-0.2215	1.4598	0.5120	-0.1496	0.9955	0.3528	0.0567
0.3613	4.0889	1.2088	-0.5206	2.2278	0.7618	-0.3306	1.4552	0.5130	-0.2225	0.9920	0.3533	0.0316
0.3825	4.0745	1.2064	-0.7219	2.2179	0.7586	-0.4553	1.4479	0.5103	-0.3056	0.9873	0.3513	-0.0024
0.4014	4.0559	1.1963	-0.9517	2.2053	0.7500	-0.5962	1.4392	0.5039	-0.3991	0.9813	0.3467	-0.0460
0.4182	4.0324	1.1790	-1.2138	2.1896	0.7362	-0.7551	1.4285	0.4938	-0.5039	0.9737	0.3396	-0.1000
0.4331	4.0029	1.1549	-1.5142	2.1697	0.7175	-0.9348	1.4150	0.4803	-0.6218	0.9645	0.3301	-0.1651
0.4463	3.9670	1.1246	-1.8630	2.1461	0.6941	-1.1402	1.3986	0.4637	-0.7559	0.9532	0.3184	-0.2423
0.4581	3.9229	1.0882	-2.2775	2.1168	0.6662	-1.3800	1.3792	0.4442	-0.9117	0.9395	0.3049	-0.3333
0.4686	3.8682	1.0457	-2.7875	2.0811	0.6341	-1.6700	1.3551	0.4221	-1.0993	0.9230	0.2897	-0.4421
0.4779	3.7988	0.9917	-3.4514	2.0363	0.5952	-2.0406	1.3251	0.3961	-1.3385	0.9024	0.2720	-0.5773
0.4861	3.7053	0.9088	-4.3953	1.9774	0.5407	-2.5583	1.2860	0.3608	-1.6724	0.8756	0.2481	-0.7606
0.4935	3.5647	0.6467	-5.8930	1.8910	0.3931	-3.3675	1.2290	0.2663	-2.1945	0.8367	0.1843	-1.0447
0.5000	2.9943	0.5747	-36.4877	1.5707	0.3710	-19.4322	1.0202	0.2577	-12.6242	0.6945	0.1793	-6.5329

Table 3.23 Normalized K_I , T_{II} and T_{33} for $a/W=0.6$, $t/W=4.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	4.0679	1.0383	0.3063	2.2166	0.6561	0.1932	1.4479	0.4425	0.1302	0.9874	0.3050	0.0897
0.0615	4.0684	1.0381	0.3058	2.2170	0.6560	0.1928	1.4486	0.4424	0.1300	0.9875	0.3050	0.0896
0.1161	4.0691	1.0383	0.3041	2.2175	0.6561	0.1918	1.4486	0.4425	0.1292	0.9878	0.3050	0.0891
0.1647	4.0703	1.0396	0.3007	2.2183	0.6570	0.1895	1.4492	0.4431	0.1277	0.9881	0.3055	0.0880
0.2078	4.0720	1.0434	0.2940	2.2192	0.6595	0.1850	1.4499	0.4449	0.1246	0.9886	0.3067	0.0859
0.2462	4.0738	1.0508	0.2814	2.2205	0.6644	0.1768	1.4505	0.4482	0.1190	0.9891	0.3090	0.0820
0.2802	4.0760	1.0634	0.2597	2.2218	0.6727	0.1627	1.4512	0.4538	0.1094	0.9897	0.3129	0.0753
0.3105	4.0776	1.0812	0.2245	2.2227	0.6842	0.1398	1.4519	0.4617	0.0939	0.9901	0.3183	0.0646
0.3374	4.0786	1.1040	0.1711	2.2231	0.6989	0.1053	1.4525	0.4716	0.0705	0.9904	0.3252	0.0484
0.3613	4.0783	1.1300	0.0944	2.2231	0.7153	0.0561	1.4519	0.4827	0.0372	0.9902	0.3328	0.0254
0.3825	4.0762	1.1563	-0.0104	2.2214	0.7316	-0.0107	1.4512	0.4936	-0.0080	0.9893	0.3403	-0.0057
0.4014	4.0712	1.1792	-0.1478	2.2179	0.7455	-0.0978	1.4486	0.5027	-0.0666	0.9876	0.3465	-0.0461
0.4182	4.0625	1.1950	-0.3221	2.2118	0.7545	-0.2077	1.4446	0.5085	-0.1404	0.9847	0.3504	-0.0968
0.4331	4.0488	1.2006	-0.5383	2.2023	0.7564	-0.3430	1.4379	0.5093	-0.2309	0.9802	0.3508	-0.1589
0.4463	4.0283	1.1935	-0.8030	2.1882	0.7496	-0.5071	1.4285	0.5040	-0.3402	0.9735	0.3469	-0.2337
0.4581	3.9989	1.1722	-1.1274	2.1684	0.7327	-0.7056	1.4143	0.4916	-0.4714	0.9640	0.3381	-0.3234
0.4686	3.9571	1.1362	-1.5307	2.1408	0.7051	-0.9481	1.3952	0.4718	-0.6304	0.9508	0.3242	-0.4317
0.4779	3.8979	1.0812	-2.0538	2.1013	0.6641	-1.2555	1.3687	0.4431	-0.8305	0.9323	0.3042	-0.5677
0.4861	3.8104	0.9967	-2.7894	2.0439	0.6042	-1.6762	1.3301	0.4023	-1.1025	0.9058	0.2762	-0.7522
0.4935	3.6680	0.7817	-3.9509	1.9535	0.4725	-2.3230	1.2696	0.3165	-1.5187	0.8643	0.2180	-1.0346
0.5000	3.0543	0.7232	-27.6244	1.5998	0.4424	-14.7586	1.0389	0.3036	-9.5845	0.7071	0.2105	-6.5235

Table 3.24 Normalized K_I , T_{II} and T_{33} for $a/W=0.8$, $t/W=0.1$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	12.7838	9.2375	-17.3030	7.7363	5.9242	-10.6747	5.4367	4.2416	-7.4972	4.0728	3.2081	-5.6131
0.0346	12.7781	9.2337	-17.3698	7.7344	5.9218	-10.7154	5.4354	4.2399	-7.5258	4.0728	3.2068	-5.6346
0.0679	12.7725	9.2393	-17.5600	7.7307	5.9250	-10.8312	5.4327	4.2422	-7.6072	4.0710	3.2085	-5.6956
0.1001	12.7612	9.2481	-17.8666	7.7242	5.9298	-11.0181	5.4274	4.2457	-7.7385	4.0675	3.2111	-5.7940
0.1311	12.7499	9.2598	-18.2871	7.7149	5.9364	-11.2742	5.4221	4.2503	-7.9186	4.0621	3.2145	-5.9289
0.1610	12.7330	9.2747	-18.8220	7.7027	5.9448	-11.5999	5.4128	4.2561	-8.1475	4.0568	3.2189	-6.1005
0.1898	12.7103	9.2925	-19.4756	7.6887	5.9548	-11.9978	5.4035	4.2632	-8.4271	4.0479	3.2241	-6.3101
0.2176	12.6819	9.3133	-20.2561	7.6718	5.9666	-12.4727	5.3915	4.2714	-8.7609	4.0390	3.2303	-6.5603
0.2444	12.6478	9.3375	-21.1765	7.6520	5.9804	-13.0325	5.3767	4.2811	-9.1543	4.0283	3.2375	-6.8552
0.2702	12.6136	9.3643	-22.2561	7.6284	5.9956	-13.6889	5.3606	4.2918	-9.6156	4.0176	3.2455	-7.2009
0.2951	12.5678	9.3951	-23.5224	7.6020	6.0130	-14.4583	5.3418	4.3041	-10.1563	4.0032	3.2547	-7.6062
0.3191	12.5161	9.4297	-25.0156	7.5706	6.0329	-15.3651	5.3201	4.3180	-10.7936	3.9851	3.2652	-8.0837
0.3423	12.4584	9.4689	-26.7948	7.5334	6.0553	-16.4448	5.2944	4.3339	-11.5524	3.9670	3.2770	-8.6524
0.3646	12.3888	9.5137	-28.9484	7.4902	6.0813	-17.7509	5.2643	4.3521	-12.4702	3.9433	3.2907	-9.3401
0.3861	12.3071	9.5648	-31.6098	7.4400	6.1109	-19.3638	5.2286	4.3731	-13.6036	3.9177	3.3063	-10.1895
0.4068	12.2071	9.6182	-34.9826	7.3788	6.1419	-21.4064	5.1843	4.3949	-15.0389	3.8844	3.3226	-11.2650
0.4269	12.0885	9.6433	-39.3511	7.3032	6.1559	-24.0501	5.1312	4.4048	-16.8966	3.8453	3.3301	-12.6570
0.4461	11.9277	9.4959	-45.0560	7.2058	6.0662	-27.5009	5.0634	4.3420	-19.3213	3.7934	3.2828	-14.4738
0.4647	11.7162	8.3980	-51.8837	7.0757	5.4033	-31.6299	4.9715	3.8763	-22.2226	3.7248	2.9340	-16.6477
0.4827	11.4241	2.1455	-55.3031	6.8972	1.6315	-33.7032	4.8452	1.2266	-23.6795	3.6305	0.9488	-17.7395
0.5000	9.9582	5.0786	-231.4927	6.0072	3.4217	-139.9995	4.2205	2.4857	-98.3637	3.1623	1.8925	-73.6913

Table 3.25 Normalized K_I , T_{II} and T_{33} for $a/W=0.8$, $t/W=0.2$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	12.7556	9.0117	-9.5737	7.7233	5.8009	-5.9337	5.4274	4.1559	-4.1637	4.0675	3.1438	-3.1139
0.0490	12.7499	9.0040	-9.6904	7.7186	5.7958	-6.0053	5.4248	4.1523	-4.2140	4.0657	3.1411	-3.1517
0.0946	12.7330	8.9823	-9.9725	7.7093	5.7816	-6.1785	5.4181	4.1422	-4.3358	4.0604	3.1335	-3.2430
0.1361	12.7103	8.9575	-10.4257	7.6943	5.7648	-6.4565	5.4075	4.1303	-4.5312	4.0515	3.1246	-3.3895
0.1747	12.6762	8.9201	-11.0026	7.6755	5.7400	-6.8101	5.3941	4.1128	-4.7798	4.0408	3.1115	-3.5759
0.2098	12.6421	8.8843	-11.7185	7.6511	5.7158	-7.2486	5.3767	4.0956	-5.0881	4.0283	3.0986	-3.8070
0.2424	12.5964	8.8386	-12.5450	7.6228	5.6854	-7.7544	5.3566	4.0740	-5.4436	4.0140	3.0825	-4.0735
0.2722	12.5448	8.7985	-13.5062	7.5896	5.6581	-8.3420	5.3337	4.0548	-5.8566	3.9960	3.0680	-4.3831
0.2998	12.4872	8.7485	-14.5850	7.5525	5.6249	-9.0008	5.3079	4.0313	-6.3196	3.9761	3.0504	-4.7302
0.3250	12.4178	8.7065	-15.8190	7.5104	5.5963	-9.7534	5.2780	4.0111	-6.8486	3.9543	3.0352	-5.1266
0.3484	12.3421	8.6553	-17.2000	7.4632	5.5621	-10.5948	5.2451	3.9871	-7.4398	3.9287	3.0172	-5.5698
0.3698	12.2601	8.6164	-18.7901	7.4109	5.5354	-11.5622	5.2065	3.9683	-8.1196	3.9011	3.0030	-6.0793
0.3896	12.1657	8.5613	-20.6011	7.3524	5.4990	-12.6627	5.1662	3.9427	-8.8929	3.8696	2.9838	-6.6589
0.4076	12.0586	8.5249	-22.7396	7.2854	5.4741	-13.9604	5.1186	3.9251	-9.8047	3.8359	2.9707	-7.3422
0.4244	11.9403	8.4653	-25.2874	7.2108	5.4353	-15.5045	5.0662	3.8979	-10.8895	3.7959	2.9503	-8.1551
0.4397	11.8001	8.4320	-28.4290	7.1234	5.4126	-17.4061	5.0047	3.8821	-12.2255	3.7498	2.9384	-9.1562
0.4539	11.6378	8.3686	-32.6452	7.0235	5.3721	-19.9546	4.9336	3.8539	-14.0158	3.6968	2.9172	-10.4977
0.4668	11.4329	8.3231	-38.1498	6.8962	5.3432	-23.2787	4.8452	3.8339	-16.3509	3.6297	2.9021	-12.2473
0.4788	11.1754	8.1729	-47.8469	6.7376	5.2521	-29.1272	4.7339	3.7703	-20.4593	3.5460	2.8544	-15.3253
0.4898	10.7206	8.1811	-70.7431	6.4592	5.2605	-42.9196	4.5380	3.7768	-30.1476	3.3993	2.8593	-22.5833
0.5000	9.2249	2.4834	-232.2721	5.5534	1.8605	-140.1503	3.9011	1.3906	-98.4450	2.9219	1.0723	-73.7460

Table 3.26 Normalized K_I , T_{II} and T_{33} for $a/W=0.8$, $t/W=0.5$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	12.5333	8.4329	-1.2029	7.5934	5.4663	-0.7330	5.3377	3.9250	-0.5036	3.9996	2.9718	-0.3712
0.0615	12.5276	8.4301	-1.3426	7.5887	5.4637	-0.8201	5.3350	3.9229	-0.5649	3.9978	2.9701	-0.4172
0.1161	12.5103	8.4258	-1.7088	7.5782	5.4588	-1.0484	5.3269	3.9188	-0.7257	3.9924	2.9669	-0.5376
0.1647	12.4872	8.4090	-2.2517	7.5611	5.4451	-1.3866	5.3147	3.9084	-0.9637	3.9833	2.9589	-0.7158
0.2078	12.4468	8.3732	-2.9395	7.5382	5.4188	-1.8145	5.2984	3.8890	-1.2647	3.9707	2.9441	-0.9414
0.2462	12.4062	8.3131	-3.7517	7.5094	5.3769	-2.3190	5.2780	3.8585	-1.6196	3.9543	2.9210	-1.2073
0.2802	12.3480	8.2293	-4.6745	7.4739	5.3199	-2.8910	5.2520	3.8174	-2.0220	3.9360	2.8899	-1.5088
0.3105	12.2836	8.1210	-5.7007	7.4323	5.2473	-3.5259	5.2231	3.7654	-2.4685	3.9140	2.8507	-1.8434
0.3374	12.2130	7.9916	-6.8279	7.3837	5.1615	-4.2217	5.1885	3.7042	-2.9577	3.8863	2.8046	-2.2100
0.3613	12.1242	7.8442	-8.0599	7.3278	5.0645	-4.9803	5.1481	3.6351	-3.4910	3.8565	2.7528	-2.6096
0.3825	12.0227	7.6827	-9.4074	7.2646	4.9586	-5.8079	5.1045	3.5601	-4.0725	3.8227	2.6964	-3.0454
0.4014	11.9144	7.5103	-10.8901	7.1938	4.8463	-6.7160	5.0534	3.4805	-4.7105	3.7860	2.6367	-3.5235
0.4182	11.7903	7.3309	-12.5405	7.1153	4.7298	-7.7238	4.9975	3.3981	-5.4185	3.7441	2.5749	-4.0540
0.4331	11.6520	7.1472	-14.4104	7.0276	4.6110	-8.8626	4.9366	3.3143	-6.2182	3.6976	2.5121	-4.6533
0.4463	11.4970	6.9620	-16.5842	6.9296	4.4917	-10.1827	4.8675	3.2303	-7.1451	3.6456	2.4491	-5.3478
0.4581	11.3221	6.7768	-19.2069	6.8195	4.3730	-11.7710	4.7884	3.1468	-8.2602	3.5872	2.3866	-6.1832
0.4686	11.1211	6.5981	-22.5482	6.6937	4.2589	-13.7892	4.7003	3.0668	-9.6769	3.5203	2.3267	-7.2445
0.4779	10.8820	6.4272	-27.1817	6.5445	4.1507	-16.5810	4.5949	2.9912	-11.6365	3.4414	2.2700	-8.7124
0.4861	10.5792	6.2501	-34.4417	6.3557	4.0407	-20.9462	4.4628	2.9146	-14.7001	3.3420	2.2127	-11.0071
0.4935	10.1445	5.0527	-46.4411	6.0882	3.3233	-28.1511	4.2731	2.4121	-19.7568	3.2006	1.8365	-14.7943
0.5000	8.4963	1.9925	-202.8028	5.0889	1.5310	-121.8192	3.5719	1.1585	-85.4935	2.6749	0.8988	-64.0222

Table 3.27 Normalized K_I , T_{II} and T_{33} for $a/W=0.8$, $t/W=1.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	12.2953	7.3713	1.5892	7.4478	4.8049	1.0305	5.2369	3.4609	0.7415	3.9250	2.6250	0.5622
0.0615	12.2953	7.3909	1.5355	7.4468	4.8169	0.9963	5.2355	3.4693	0.7173	3.9232	2.6312	0.5441
0.1161	12.2895	7.4453	1.3854	7.4439	4.8505	0.9010	5.2341	3.4926	0.6498	3.9213	2.6485	0.4935
0.1647	12.2836	7.5241	1.1394	7.4391	4.8985	0.7451	5.2300	3.5258	0.5397	3.9195	2.6732	0.4109
0.2078	12.2719	7.6177	0.7903	7.4303	4.9549	0.5247	5.2231	3.5648	0.3840	3.9140	2.7020	0.2941
0.2462	12.2543	7.7130	0.3295	7.4187	5.0116	0.2345	5.2148	3.6036	0.1793	3.9066	2.7306	0.1407
0.2802	12.2307	7.7986	-0.2491	7.4022	5.0613	-0.1287	5.2024	3.6373	-0.0767	3.8992	2.7553	-0.0511
0.3105	12.2012	7.8600	-0.9498	7.3807	5.0949	-0.5674	5.1871	3.6595	-0.3857	3.8863	2.7714	-0.2826
0.3374	12.1598	7.8878	-1.7747	7.3533	5.1069	-1.0824	5.1662	3.6663	-0.7482	3.8714	2.7760	-0.5541
0.3613	12.1064	7.8749	-2.7259	7.3180	5.0929	-1.6747	5.1411	3.6549	-1.1649	3.8509	2.7668	-0.8663
0.3825	12.0407	7.8181	-3.8074	7.2745	5.0512	-2.3462	5.1101	3.6239	-1.6372	3.8284	2.7431	-1.2200
0.4014	11.9566	7.7181	-5.0275	7.2218	4.9821	-3.1014	5.0719	3.5737	-2.1680	3.8000	2.7052	-1.6176
0.4182	11.8586	7.5775	-6.4025	7.1577	4.8876	-3.9496	5.0262	3.5059	-2.7641	3.7650	2.6540	-2.0641
0.4331	11.7407	7.4009	-7.9621	7.0818	4.7706	-4.9084	4.9729	3.4225	-3.4375	3.7242	2.5913	-2.5686
0.4463	11.6006	7.1937	-9.7589	6.9917	4.6347	-6.0087	4.9088	3.3261	-4.2100	3.6761	2.5189	-3.1472
0.4581	11.4335	6.9603	-11.8852	6.8847	4.4829	-7.3054	4.8333	3.2188	-5.1202	3.6192	2.4383	-3.8290
0.4686	11.2320	6.7064	-14.5056	6.7569	4.3188	-8.8969	4.7415	3.1031	-6.2368	3.5511	2.3517	-4.6653
0.4779	10.9835	6.4175	-17.9295	6.6004	4.1343	-10.9678	4.6308	2.9735	-7.6894	3.4677	2.2547	-5.7532
0.4861	10.6613	6.0143	-22.7602	6.3975	3.8828	-13.8789	4.4886	2.7975	-9.7309	3.3603	2.1229	-7.2819
0.4935	10.1885	4.6873	-29.9718	6.1036	3.0831	-18.2125	4.2815	2.2379	-12.7696	3.2048	1.7042	-9.5571
0.5000	8.4784	4.0152	-158.9940	5.0634	2.7174	-95.2762	3.5498	1.9873	-66.8013	2.6571	1.5181	-50.0011

Table 3.28 Normalized K_I , T_{II} and T_{33} for $a/W=0.8$, $t/W=2.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	12.1242	7.0752	2.0187	7.3376	4.6081	1.3100	5.1579	3.3183	0.9415	3.8658	2.5166	0.7133
0.0615	12.1242	7.0771	2.0121	7.3376	4.6095	1.3056	5.1579	3.3193	0.9384	3.8658	2.5174	0.7109
0.1161	12.1242	7.0836	1.9932	7.3386	4.6141	1.2931	5.1579	3.3227	0.9293	3.8658	2.5200	0.7041
0.1647	12.1242	7.0971	1.9589	7.3396	4.6233	1.2706	5.1592	3.3295	0.9132	3.8658	2.5252	0.6918
0.2078	12.1242	7.1222	1.9020	7.3396	4.6399	1.2335	5.1592	3.3415	0.8866	3.8658	2.5344	0.6718
0.2462	12.1242	7.1625	1.8096	7.3396	4.6661	1.1738	5.1592	3.3603	0.8440	3.8658	2.5484	0.6396
0.2802	12.1242	7.2244	1.6644	7.3376	4.7055	1.0805	5.1579	3.3881	0.7775	3.8640	2.5692	0.5896
0.3105	12.1183	7.3087	1.4428	7.3337	4.7581	0.9390	5.1551	3.4251	0.6772	3.8621	2.5968	0.5142
0.3374	12.1123	7.4139	1.1191	7.3268	4.8231	0.7335	5.1495	3.4703	0.5319	3.8584	2.6304	0.4051
0.3613	12.0944	7.5309	0.6664	7.3160	4.8941	0.4476	5.1411	3.5193	0.3299	3.8528	2.6668	0.2537
0.3825	12.0706	7.6439	0.0600	7.2992	4.9613	0.0663	5.1284	3.5653	0.0610	3.8415	2.7008	0.0521
0.4014	12.0347	7.7331	-0.7220	7.2755	5.0121	-0.4237	5.1115	3.5995	-0.2841	3.8284	2.7259	-0.2063
0.4182	11.9861	7.7774	-1.6997	7.2418	5.0337	-1.0340	5.0861	3.6129	-0.7136	3.8095	2.7352	-0.5280
0.4331	11.9168	7.7595	-2.8949	7.1958	5.0154	-1.7775	5.0534	3.5981	-1.2365	3.7850	2.7235	-0.9196
0.4463	11.8227	7.6675	-4.3378	7.1345	4.9499	-2.6719	5.0090	3.5503	-1.8651	3.7516	2.6871	-1.3903
0.4581	11.6971	7.4935	-6.0781	7.0522	4.8327	-3.7461	4.9511	3.4662	-2.6197	3.7073	2.6237	-1.9554
0.4686	11.5283	7.2356	-8.2043	6.9431	4.6631	-5.0527	4.8734	3.3455	-3.5370	3.6487	2.5330	-2.6424
0.4779	11.2998	6.8765	-10.8985	6.7962	4.4310	-6.6996	4.7688	3.1815	-4.6925	3.5705	2.4100	-3.5078
0.4861	10.9776	6.3628	-14.5700	6.5917	4.1054	-8.9309	4.6246	2.9524	-6.2574	3.4614	2.2385	-4.6796
0.4935	10.4786	5.2029	-20.1275	6.2781	3.3949	-12.2895	4.4026	2.4550	-8.6118	3.2949	1.8664	-6.4424
0.5000	8.6242	4.8817	-122.4056	5.1425	3.2241	-73.3139	3.6036	2.3417	-51.3745	2.6967	1.7832	-38.4432

Table 3.29 Normalized K_I , T_{II} and T_{33} for $a/W=0.8$, $t/W=4.0$, SECP

z/t	Uniform			Linear			Parabolic			Cubic		
	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}	K_I	T_{II}	T_{33}
0.0000	12.0347	7.0297	2.0917	7.2795	4.5768	1.3603	5.1158	3.2952	0.9788	3.8340	2.4988	0.7420
0.0615	12.0347	7.0291	2.0901	7.2795	4.5765	1.3593	5.1172	3.2949	0.9780	3.8340	2.4986	0.7414
0.1161	12.0347	7.0283	2.0858	7.2805	4.5759	1.3562	5.1172	3.2947	0.9758	3.8340	2.4984	0.7397
0.1647	12.0347	7.0273	2.0785	7.2805	4.5754	1.3511	5.1172	3.2943	0.9720	3.8340	2.4981	0.7367
0.2078	12.0347	7.0269	2.0681	7.2815	4.5753	1.3440	5.1186	3.2943	0.9667	3.8359	2.4981	0.7327
0.2462	12.0347	7.0265	2.0534	7.2824	4.5753	1.3339	5.1186	3.2945	0.9593	3.8359	2.4984	0.7270
0.2802	12.0407	7.0293	2.0334	7.2844	4.5777	1.3204	5.1200	3.2963	0.9493	3.8359	2.4998	0.7194
0.3105	12.0407	7.0365	2.0041	7.2854	4.5829	1.3007	5.1214	3.3003	0.9351	3.8378	2.5029	0.7085
0.3374	12.0467	7.0531	1.9584	7.2874	4.5945	1.2704	5.1214	3.3087	0.9131	3.8378	2.5094	0.6919
0.3613	12.0467	7.0861	1.8823	7.2874	4.6163	1.2206	5.1214	3.3245	0.8774	3.8378	2.5213	0.6648
0.3825	12.0467	7.1437	1.7518	7.2864	4.6534	1.1360	5.1214	3.3509	0.8171	3.8359	2.5412	0.6194
0.4014	12.0407	7.2341	1.5302	7.2824	4.7103	0.9939	5.1172	3.3909	0.7161	3.8340	2.5712	0.5434
0.4182	12.0287	7.3585	1.1670	7.2745	4.7873	0.7627	5.1115	3.4446	0.5524	3.8302	2.6111	0.4205
0.4331	12.0107	7.5047	0.6005	7.2606	4.8759	0.4046	5.1003	3.5059	0.2995	3.8208	2.6566	0.2309
0.4463	11.9735	7.6411	-0.2384	7.2348	4.9561	-0.1230	5.0819	3.5605	-0.0724	3.8076	2.6968	-0.0477
0.4581	11.9126	7.7174	-1.4263	7.1938	4.9961	-0.8664	5.0520	3.5865	-0.5957	3.7839	2.7154	-0.4396
0.4686	11.8123	7.6785	-3.0577	7.1274	4.9619	-1.8825	5.0033	3.5597	-1.3103	3.7475	2.6944	-0.9747
0.4779	11.6489	7.4593	-5.2861	7.0205	4.8133	-3.2633	4.9278	3.4527	-2.2803	3.6896	2.6135	-1.7011
0.4861	11.3767	6.9844	-8.4261	6.8448	4.5049	-5.1961	4.8019	3.2337	-3.6371	3.5952	2.4491	-2.7171
0.4935	10.8867	5.8832	-13.1871	6.5324	3.8155	-8.1047	4.5807	2.7491	-5.6771	3.4286	2.0864	-4.2448
0.5000	8.8290	5.2827	-93.1709	5.2630	3.4522	-55.8460	3.6870	2.5005	-39.1241	2.7587	1.9016	-29.2724

Table 3.30 Parameters of the empirical formulae for stress intensity factor K_I

	Uniform (n=0)	Linear (n=1)	Parabolic (n=2)	Cubic (n=3)
a_n	-3.16746	-2.46	-2.15068	-2.025551
b_n	-0.14676	-0.09	-0.06753	-0.05161
c_n	40.54308	27.26	22.48572	20.274559
d_n	0.013603	0.007	0.00455	0.0033272
e_n	-109.323	-73.4	-59.8064	-53.26736
f_n	0.59828	0.396	0.290944	0.2264706
g_n	0.012726	0.008	0.005238	0.0038529
h_n	103.5832	68.5	54.08243	46.548953
i_n	-0.92723	-0.6	-0.43478	-0.333507
j_n	-0.01347	-0.01	-0.00725	-0.005393

Figure 3.1 Geometric parameters of single edge cracked plate specimen with crack surface subjected to stress distribution $\sigma(x)$

Figure 3.2 C3D20 element and mesh configuration near the crack tip

Figure 3.3 Finite element model for $a/W=0.4$, $t/W=0.5$, SECP

Figure 3.4 Corner crack in the finite thickness plate

Figure 3.5 Surface corner crack model in ABAQUS, $a/c=1.0$, $a/t=0.2$

Figure 3.6 Comparison of the T_{11} from present FEM calculation and solution form Qu and Wang (2006) for surface corner crack, $a/c=1.0$, $a/t=0.2$

Figure 3.7 Comparison of the T_{33} from present FEM calculation and solution form Qu and Wang (2006) for surface corner crack, $a/c=1.0$, $a/t=0.2$

Figure 3.8 Variation of normalized T_{11} along front of an edge-cracked plate for various Poisson's ratios under tension

Figure 3.9 Normalized Stress Intensity Factor, $a/W=0.2$, Uniform, SECP

Figure 3.10 Normalized Stress Intensity Factor, $a/W=0.2$, Linear, SECP

Figure 3.11 Normalized T_{11} , $a/W=0.2$, Uniform, SECP

Figure 3.12 Normalized T_{11} , $a/W=0.2$, Linear, SECP

Figure 3.13 Normalized T_{33} , $a/W=0.2$, Uniform, SECP

Figure 3.14 Normalized T_{33} , $a/W=0.2$, Linear, SECP

Figure 3.15 Normalized Stress Intensity Factor, $a/W=0.4$, Uniform, SECP

Figure 3.16 Normalized Stress Intensity Factor, $a/W=0.4$, Linear, SECP

Figure 3.17 Normalized T_{11} , $a/W=0.4$, Uniform, SECP

Figure 3.18 Normalized T_{11} , $a/W=0.4$, Linear, SECP

Figure 3.19 Normalized T_{33} , $a/W=0.4$, Uniform, SECP

Figure 3.20 Normalized T_{33} , $a/W=0.4$, Linear, SECP

Figure 3.21 Normalized Stress Intensity Factor, $a/W=0.6$, Uniform, SECP

Figure 3.22 Normalized Stress Intensity Factor, $a/W=0.6$, Linear, SECP

Figure 3.23 Normalized T_{11} , $a/W=0.6$, Uniform, SECP

Figure 3. 24 Normalized T_{11} , $a/W=0.6$, Linear, SECP

Figure 3.25 Normalized T_{33} , $a/W=0.6$, Uniform, SECP

Figure 3.26 Normalized T_{33} , $a/W=0.6$, Linear, SECP

Figure 3.27 Normalized Stress Intensity Factor, $a/W=0.8$, Uniform, SECP

Figure 3.28 Normalized Stress Intensity Factor, $a/W=0.8$, Linear, SECP

Figure 3.29 Normalized T_{11} , $a/W=0.8$, Uniform, SECP

Figure 3.30 Normalized T_{11} , $a/W=0.8$, Linear, SECP

Figure 3.31 Normalized T_{33} , $a/W=0.8$, Uniform, SECP

Figure 3.32 Normalized T_{33} , $a/W=0.8$, Linear, SECP

Figure 3.33 Comparison of numerical results and empirical formulae for stress intensity factor: (a) $t/W=0.1$, (b) $t/W=0.2$, (c) $t/W=0.5$, (d) $t/W=1.0$, (e) $t/W=2.0$, and (f) $t/W=4.0$

Figure 3.34 Comparison of numerical results and empirical formulae for T_{11} : (a) $t/W=0.1$, (b) $t/W=0.2$, (c) $t/W=0.5$, (d) $t/W=1.0$, (e) $t/W=2.0$, and (f) $t/W=4.0$

Figure 3.35 Comparison of numerical results and empirical formulae for T_{33} : (a) $t/W=0.1$, (b) $t/W=0.2$, (c) $t/W=0.5$, (d) $t/W=1.0$, (e) $t/W=2.0$, and (f) $t/W=4.0$

CHAPTER 4

VARIATIONS OF K_I , T_{11} AND T_{33} BY GEOMETRY

In this chapter, using the finite element results obtained in Chapter 3, K_I , T_{11} and T_{33} distributions of various three-dimensional single edge cracked plate specimens are studied. The influence of the plate thickness, crack depth and the variations of K_I , T_{11} and T_{33} along the thickness are discussed, respectively.

In order to further analyze the thinner or thicker plate effectiveness, additional $t/W=0.0025, 0.025, 0.05, 8.0, 16.0$ cases subjected to the uniform stress were computed by finite element method in ABAQUS. Additional finite element results of K_I , T_{11} and T_{33} at mid-plane for $a/W=0.2, 0.4, 0.6$ and 0.8 , are shown in Table 4.1-4.4, respectively. The same mesh design and method of solutions discussed in Chapter 3 are used in these calculations. In addition, the two-dimensional results of K_I , T_{11} and T_{33} for $a/W=0.2, 0.4, 0.6$ and 0.8 which taken from Sham's paper (1991) are shown in Table 4.5.

4.1 Mid-plane K_I , T_{11} and T_{33} Varied by Geometry

4.1.1 Effects of plate thickness for K_I , T_{11} and T_{33}

By using the finite element analysis results from Chapter 3 and the additional thickness cases, the effect of the relative plate thickness on the three-dimensional K_I , T_{11} and T_{33} stresses for the crack surface subjected to the uniform stress are studied. The relative crack depth of the plate varies by $a/W=0.2, 0.4, 0.6$ and 0.8 , and the Poisson's ratio are fixed at $\nu = 0.3$, respectively. The thickness variations include $t/W=0.0025, 0.025, 0.05, 0.1, 0.2, 0.5, 1.0, 2.0, 4.0, 8.0$ and 16.0 . The three-dimensional

results for mid-plane are taken from Table 3.6-3.29 and Table 4.1-4.4. And the two-dimensional results (Sham, 1991) are listed in Table 4.5.

4.1.1.1 Effects of plate thickness for Stress Intensity Factor

First, the thickness effect on K_I is investigated. The stress intensity factor results K_I v.s. $\log(t/a)$ are plotted in Figure 4.1-4.4, for $a/W=0.2, 0.4, 0.6$ and 0.8 , respectively. It is noticed that the stress intensity factor K_I at the mid-plane decreases when the plane is thicker. When thickness t becomes large compared to the crack length a , the stress intensity factor trends two-dimensional solution.

$$K_I^{3D}(0) = K_I^{2D} \quad \text{as } t/a \rightarrow \infty \quad (4.1)$$

where $K_I^{3D}(0)$ is the stress intensity factor along the crack front, and K_I^{2D} is the two-dimensional solution of stress intensity factor. When the $t/a \rightarrow 0$, the stress intensity factors keep increasing.

4.1.1.2 Effects of plate thickness for T_{11} Stress

Normalized mid-plane value of T_{11} stress as a function of plate thickness ($\log(t/a)$) in single edge through-thickness cracked plate, for $a/W=0.2, 0.4, 0.6$ and 0.8 , are plotted in Figure 4.5-4.8, respectively. Figure 4.5-4.8 show larger mid-plane T_{11} stress in thinner plates which the crack surface subjected to the uniform stress for relative crack depth ratio $a/W=0.2, 0.4, 0.6$ and 0.8 , respectively. It is noticed that the normalized T_{11} stress decreases and approaches the two-dimensional solution as the plate thickness increases. When $t/a \rightarrow 0$, T_{11} stress keeps increasing.

Nakamura and Parks (1992) presented a formula to predict the mid-plane T_{11} stress for SECP specimens, as described in Section 2.4.2.

When the thickness t became large compared to in-plane dimensions, T_{11} can be expressed as

$$T_{11}(z/t, \nu) = B^{2D} \frac{K_I^{2D}}{\sqrt{\pi a}} \equiv T_{11}^{2D}$$

as $t/a \rightarrow \infty$

(2.26)

In these thesis, only T_{11} stress at mid-plane ($z/t=0$) is discussed. The through-thickness variation of the T_{11} stress distribution (Eq.2.27) where only at mid-plane can be expressed as,

$$T_{11}(0) \cong T_{11}^{2D} + B^{thin}(0) \frac{K_I(0)}{\sqrt{\pi t}}$$

(4.2)

where the T_{11}^{2D} is the two-dimensional solution of T_{11} stress. The $B^{thin}(0)$ represents the local “biaxiality” factor at mid-plane of a thin plate.

Then when the thickness is large, Eq. (2.26) for mid-plane can be written as,

$$T_{11}(0) = B^{2D}(0) \frac{K_I^{2D}(0)}{\sqrt{\pi a}} \equiv T_{11}^{2D}$$

as $t/a \rightarrow \infty$

(4.3)

From the Eq. (2.29), the formula of the T_{11} stress distribution at mid-plane is

$$T_{11}(0) = B^{thin}(0) \frac{K_I(0)}{\sqrt{\pi t}}$$

as $t/a \rightarrow 0$

(4.4)

With the use of Eq. (4.2), (4.3), and (4.4), the normalized $T_{11}(0)$ stress can be expressed as

$$\frac{T_{11}(0)}{K_I(0)/\sqrt{\pi t}} \cong \sqrt{\frac{t}{a}} B^{2D} + B^{thin}(0)$$

(4.5)

The first term on the right-hand side of Eq. (4.5) dominates for a thick plate, while it vanishes for a thin plate. The mid-plane values of $B^{thin}(0)$ were calculated for $a/W=0.2, 0.4, 0.6$ and 0.8 by FEM, respectively. The values of B^{2D} and $B^{thin}(0)$ are shown in Table 4.6.

For case of $a/W=0.2$, suppose $B^{thin}(0) = 0.0846$ at $t/a = 0.0125$, the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results are shown in Table 4.7 for the relative thickness range $0.0125 \leq t/a \leq 80$ and plotted in Figure 4.9.

For case of $a/W=0.4$, suppose $B^{thin}(0) = 0.0519$ at $t/a = 0.00625$, the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results are shown in Table 4.8 for the relative thickness range $0.00625 \leq t/a \leq 40$ and plotted in Figure 4.10.

For case of $a/W=0.6$, suppose $B^{thin}(0) = 0.0526$ at $t/a = 0.00427$, the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results are shown in Table 4.9 for the relative thickness range $0.00427 \leq t/a \leq 27$ and plotted in Figure 4.11.

For case of $a/W=0.8$, suppose $B^{thin}(0) = 0.0675$ at $t/a = 0.003$, the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results are shown in Table 4.10 for the relative thickness range $0.003 \leq t/a \leq 20$ and plotted in Figure 4.12.

Overall, it can be seen that Eq. (4.5) gives good approximation of T_{11} stress variation for different t/a ratios.

4.1.1.3 Effects of plate thickness for T_{33} Stress

The normalized mid-plane T_{33} stress varied by thickness ($\log(t/a)$) for $a/W=0.2, 0.4, 0.6$ and 0.8 are shown in Figure 4.13–4.16, respectively. It is noticed that

the normalized T_{33} stress increases and approaches the two-dimensional solution as the plate thickness increases. The three-dimensional results nearly merge with the two-dimensional solution for thicker plate. As t/a decreases, the T_{33} stress keeps decreasing as well.

Similar as definition of “biaxiality” parameter B , parameter B_{33} for mid-plane is defined as

$$B_{33}(0) = \frac{\sqrt{\pi t} T_{33}(0)}{K_I(0)} \quad (4.6)$$

When the thickness is large,

$$T_{33}(0) \equiv T_{33}^{2D} \quad \text{as } t/a \rightarrow \infty \quad (4.7)$$

According to the prediction process of T_{11} stress at mid-plane, also from considerations of superposition and dimensional analysis, as relative thickness diminished in the boundary value problem posed, the through-thickness variation of the T_{33} stress distribution can be expressed as

$$T_{33}(0) \cong T_{33}^{2D} + B_{33}^{thin}(0) \frac{K_I(0)}{\sqrt{\pi t}} \quad (4.8)$$

Here T_{33}^{2D} represents the “remote” value which applies in the plane stress field found at radii large compared to thickness t , but small compared to crack depth a . The function B_{33}^{thin} represents the local biaxiality factor of a thin plane.

From the Figure 4.13-4.16, it is noticed that the T_{33} stress is still markedly decreasing at thinner plate. It is expected that there exists a T_{33} stress distribution at the mid-plane,

$$T_{33}(0) = B_{33}^{thin}(0) \frac{K_I(0)}{\sqrt{\pi t}} \quad \text{as } t/a \rightarrow 0 \quad (4.9)$$

B_{33}^{thin} can be solved from the Eq. (4.9) by calculating $T_{33}(0)$ and $K_I(0)$ at very low t/a ratio plate.

With the use of Eq. (4.6), (4.7), (4.8) and (4.9), the normalized $T_{33}(0)$ stress can be expressed as

$$\frac{T_{33}(0)}{K_I(0)/\sqrt{\pi t}} \cong \sqrt{\frac{t}{a}} B_{33}^{2D}(0) + B_{33}^{thin}(0) \quad (4.10)$$

In order to compare the results between the finite element analysis and prediction, the $B_{33}^{thin}(0)$ for $a/W=0.2, 0.4, 0.6$ and 0.8 with crack surface subjected to uniform load calculated by Eq. (4.9) for the thinner plane, and $T_{33}(0)$ was predicted by Eq. (4.10). The values of T_{33}^{2D} and $B_{33}^{thin}(0)$ for $a/W=0.2, 0.4, 0.6$ and 0.8 are shown in Table 4.11.

For the relative crack length $a/W=0.2$, using the finite element analysis results for $B_{33}(0)$ at $t/a=0.125$ to get the $B_{33}^{thin}(0) = -0.4479$. Then substitute $B_{33}^{thin}(0)$ into Eq. (4.10), the approximate thickness-dependence of T_{33} stresses at mid-plane were obtained. The comparison of approximate and finite element results of T_{33} stresses at mid-plane was shown in Table 4.12, and plotted in Figure 4.17.

For the relative crack depth $a/W=0.4$, using the finite element analysis results for $B_{33}(0)$ at $t/a=0.00625$ to get $B_{33}^{thin}(0) = -0.2331$. Then substitute $B_{33}^{thin}(0)$ into Eq. (4.10), the approximate thickness-dependence of T_{33} stresses at mid-plane were obtained. The comparison of approximate and finite element results of T_{33} stresses at mid-plane was shown in Table 4.13, and plotted in Figure 4.18.

For the relative crack depth $a/W=0.6$, using the finite element analysis results for $B_{33}(0)$ at $t/a=0.0417$ to get $B_{33}^{thin}(0) = -0.2473$. Then substitute $B_{33}^{thin}(0)$ into Eq. (4.10), the approximate thickness-dependence of T_{33} stresses at mid-plane were obtained. The comparison of approximate and finite element results of T_{33} stresses at mid-plane was shown in Table 4.14, and plotted in Figure 4.19.

For the relative crack depth $a/W=0.8$, using the finite element analysis results for $B_{33}(0)$ at $t/a=0.0313$ to get $B_{33}^{thin}(0) = -0.2441$. Then substitute $B_{33}^{thin}(0)$ into Eq. (4.10), the approximate thickness-dependence of T_{33} stresses at mid-plane were obtained. The comparison of approximate and finite element results of T_{33} stresses at mid-plane was shown in Table 4.15, and plotted in Figure 4.20.

From these comparisons, it can be seen that Eq. (4.10) gives reasonable approximation of T_{33} stress variation with different t/a ratios.

4.1.2 Variation of Crack depths

The effects of crack depth are studied by computing mid-plane K_I , T_{11} and T_{33} values for various relative crack depths at $a/W=0.2, 0.4, 0.6$ and 0.8 .

The normalized stress intensity factor K_I in the mid-plane for the relative thickness of the plate varies by $t/W=0.1, 0.5, 1.0$ and 4.0 , and the Poisson's ratio are fixed at $\nu = 0.3$, as shown in Figure 4.21. The stress intensity factors for two dimensional geometries are also shown in Figure 4.21. The values of stress intensity factor all increase with increasing the crack depths whether the plane thickness is thick or thin.

The T_{11} stress normalized by σ_0 in the mid-plane is shown in Figure 4.22. The relative thickness of the plate are varied by $t/W=0.1, 0.5, 1.0$ and 4.0 , and the Poisson's ratio $\nu = 0.3$. Results for two-dimensional geometries are also shown. All the cases show increasing T_{11} at mid-plane for deeper crack. The three-dimensional normalized T_{11} stress values are slightly higher than the corresponding two-dimensional solutions.

The T_{33} stress for the same cases ($t/W=0.1, 0.5, 1.0$ and 4.0 , and $\nu = 0.3$) normalized by σ_0 together with two-dimensional results are shown in Figure 4.23. The T_{33} stress is increasing at mid-plane for deeper crack for thicker plate ($t/W>0.5$) and decreasing for thinner plate ($t/W=0.1$).

4.2 K_I , T_{11} and T_{33} Variation along the Crack Front

The three-dimensional stress intensity factor can be presented as two-dimensional value corrected by the thickness influence (Kwon and Sun, 2000), as discussed in section 2.4.

$$K_I^{3D}(z/t) = K_I^{2D} \Delta K_{3D}(z/t) \quad (4.10)$$

where $\Delta K_{3D}(z/t)$ stands for the thickness correction factor. The $\Delta K_{3D}(z/t)$ along the crack surface of $a/W=0.2, 0.4, 0.6$ or 0.8 are shown in Figure 4.24-4.27, respectively. And the $\Delta K_{3D}(z/t)$ at four locations along the crack front are shown in Table 4.16-4.19 for $a/W=0.2, 0.4, 0.6$ or 0.8 , respectively. In all cases, the value of $\Delta K_{3D}(z/t)$ is in the range of $0.95 < \Delta K_{3D}(z/t) < 1.07$, except in the small region near the free-surfaces where the stress intensity factor must vanish at the free-surfaces ($|z/t| = 0.5$). The three-dimensional stress intensity factors are slightly different than the two-dimensional values along the most of the crack front.

The three-dimensional T_{11} along the crack front varies by the specimen thickness are presented in Chapter 3. It is noticed that the three-dimensional T_{11} is quite different for different relative thickness ratio t/W . The three-dimensional T_{11} decreases along the thickness at some relative thickness ratio t/W , but increases and then decreases along the thickness at other relative thickness ratio t/W . Similar as stress intensity factor, define the thickness influence factor, $\Delta T_{113D}(z/t)$, to present the difference between the three-dimensional and two-dimensional value of T_{11} .

$$T_{11}^{3D}(z/t) = T_{11}^{2D} \Delta T_{113D}(z/t) \quad (4.11)$$

The $\Delta T_{113D}(z/t)$ along the crack front of $a/W=0.2, 0.4, 0.6$ or 0.8 is shown in Figure 4.28-4.31, respectively. And the values of $\Delta T_{113D}(z/t)$ at the four locations along the

crack front are listed in Table 4.20-4.23 for $a/W=0.2, 0.4, 0.6$ or 0.8 , respectively. It shows that the values of $\Delta T_{113D}(z/t)$ are varied by relative thickness ratio t/W , and deviate from the two-dimension results further for thinner plate. For example, $\Delta T_{113D}(z/t)$ is larger than 1.6 along the most of crack front for $a/W=0.6, t/W=0.1$. That means more than 60% of error will occur if use the two-dimensional solution of T_{11} to instead of the three-dimensional solution. So the two-dimension solution of T_{11} is not sufficient for some three-dimensional problems.

According to the three-dimensional T_{33} results obtained in Chapter 3, it is noticed that the three-dimensional T_{33} keeps decreasing along the crack front at all thickness ratio t/W . There is sharp dropping near the free surface. Similarly, the thickness influence factor, $\Delta T_{333D}(z/t)$, was defined to presented the difference between the three-dimensional and two-dimensional value of T_{33} .

$$T_{33}^{3D}(z/t) = T_{33}^{2D} \Delta T_{333D}(z/t) \quad (4.12)$$

The $\Delta T_{333D}(z/t)$ along the crack surface of $a/W=0.2, 0.4, 0.6$ or 0.8 is shown in Figure 4.32-4.35, respectively. The values of $\Delta T_{333D}(z/t)$ at four locations of the crack front for $a/W=0.2, 0.4, 0.6$ or 0.8 are shown in Table 4.24-4.27, respectively. It shows that the three-dimensional T_{33} stresses are quite different than the two-dimensional solutions. So the three-dimensional T_{33} stresses solution must be applied to solve three-dimensional questions.

Table 4.1 Finite element result for $a/W=0.2$, subjected to uniform load, SECP

t/W	Normalized K_I	Normalized T_{11}	Normalized T_{33}
0.0025	1.4729	1.1146	-5.2770
0.025	1.4579	0.6271	-1.847
0.05	1.4562	0.5624	-1.142
8	1.3649	0.4082	0.1224
16	1.3644	0.4083	0.1226

Table 4.2 Finite element result for $a/W=0.4$, subjected to uniform load, SECP

t/W	Normalized K_I	Normalized T_{11}	Normalized T_{33}
0.0025	2.2847	1.4993	-8.0956
0.025	2.2484	0.9035	-4.6880
0.05	2.2472	0.7629	-3.1282
8	2.1116	0.4218	0.1269
16	2.1082	0.4218	0.1268

Table 4.3 Finite element result for $a/W=0.6$, subjected to uniform load, SECP

t/W	Normalized K_I	Normalized T_{11}	Normalized T_{33}
0.0025	4.3723	3.5605	-19.7328
0.025	4.3048	2.1693	-11.6595
0.05	4.3034	1.8447	-7.9499
8	4.0462	1.0358	0.3118
16	4.0357	1.0333	0.3111

Table 4.4 Finite element result for $a/W=0.8$, subjected to uniform load, SECP

t/W	Normalized K_I	Normalized T_{11}	Normalized T_{33}
0.0025	12.9629	15.6582	-68.6645
0.025	12.7781	10.9469	-39.4515
0.05	12.7781	9.8921	-27.1514
8	11.9927	7.0052	2.0996
16	11.9699	6.9943	2.1010

Table 4. 5 2D solution, crack subjected to uniform load, SECP, Sham (1991)

a/W	Normalized K_I	Normalized T_{11}	Normalized T_{33}
0.2	1.3650	0.4091	0.1227
0.4	2.1083	0.4147	0.1244
0.6	4.0254	1.0278	0.3083
0.8	11.926	6.9755	2.0927

Table 4.6 Values of B^{2D} and $B^{thin}(0)$ for $a/W=0.2, 0.4, 0.6$ and 0.8 subjected to uniform load on the crack surface

a/W	0.2	0.4	0.6	0.8
B^{2D}	0.2997	0.1967	0.2553	0.5849
$B^{thin}(0)$	0.0846	0.0519	0.0526	0.0675
t/a for $B^{thin}(0)$	0.0125	0.00625	0.00417	0.00313

Table 4.7 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.2$

t/a	$\text{Log}(t/a)$	$\frac{T_{11}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B^{2D}$ $+ B^{thin}(0)$ (Predict)	$\frac{ FEM - Predict }{FEM}$ $\times 100\%$
0.125	-0.9031	0.1521	0.1906	25.32
0.25	-0.6021	0.1931	0.2345	21.41
0.5	-0.3010	0.2503	0.2965	18.49
1	0.0000	0.3315	0.3843	15.94
2.5	0.3979	0.5154	0.5585	8.37
5	0.6990	0.6850	0.7548	10.19
10	1.0000	0.9372	1.0324	10.15
20	1.3010	1.3331	1.4249	6.89
40	1.6021	1.8915	1.9801	4.69
80	1.9031	2.6769	2.7653	3.30

Table 4.8 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.4$

t/a	$\text{Log}(t/a)$	$\frac{T_{11}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B^{2D}$ $+ B^{thin}(0)$ (Predict)	$\frac{ FEM - Predict }{FEM}$ $\times 100\%$
0.0625	-1.2041	0.1005	0.1011	0.59
0.125	-0.9031	0.1200	0.1214	1.16
0.25	-0.6021	0.1479	0.1502	1.57
0.5	-0.3010	0.1875	0.1910	1.83
1.25	0.0969	0.2745	0.2718	0.97
2.5	0.3979	0.3815	0.3629	4.87
5	0.6990	0.4565	0.4917	7.72
10	1.0000	0.6273	0.6739	7.43
20	1.3010	0.8934	0.9315	4.27
40	1.6021	1.2654	1.2959	2.41

Table 4.9 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.6$

t/a	$\text{Log}(t/a)$	$\frac{T_{11}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B^{2D}$ $+ B^{thin}(0)$ (Predict)	$\left \frac{\text{FEM} - \text{Predict}}{\text{FEM}} \right $ $\times 100\%$
0.0417	-1.3802	0.1029	0.1047	1.77
0.0833	-1.0792	0.1237	0.1263	2.05
0.1667	-0.7782	0.1539	0.1568	1.87
0.3333	-0.4771	0.1986	0.2000	0.70
0.8333	-0.0792	0.3013	0.2856	5.19
1.6667	0.2218	0.3951	0.3822	3.27
3.3333	0.5229	0.4762	0.5187	8.93
6.6667	0.8239	0.6590	0.7118	8.02
13.3333	1.1249	0.9347	0.9849	5.37
26.6667	1.4260	1.3222	1.3711	3.69

Table 4.10 The comparison of the approximate thickness-dependence of T_{11} stress at mid-plane as given by Eq. (4.5) and the finite element results, Uniform, SECP, $a/W=0.8$

t/a	$\text{Log}(t/a)$	$\frac{T_{11}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B^{2D}$ $+ B^{thin}(0)$ (Predict)	$\left \frac{\text{FEM} - \text{Predict}}{\text{FEM}} \right $ $\times 100\%$
0.03125	-1.5051	0.1514	0.1709	11.40
0.0625	-1.2041	0.1935	0.2137	9.46
0.125	-0.9031	0.2555	0.2743	6.87
0.25	-0.6021	0.3532	0.3600	1.87
0.625	-0.2041	0.5319	0.5299	0.38
1.25	0.0969	0.6703	0.7215	7.09
2.5	0.3979	0.9227	0.9923	7.02
5	0.6990	1.3061	1.3754	5.04
10	1.0000	1.8472	1.9171	3.65
20	1.3010	2.6132	2.6833	2.61

Table 4.11 Values of B_{33}^{2D} and $B_{33}^{thin}(0)$ for $a/W=0.2, 0.4, 0.6$ and 0.8 subjected to uniform load on the crack surface

a/W	0.2	0.4	0.6	0.8
B_{33}^{2D}	0.0899	0.0590	0.0766	0.1755
$B_{33}^{thin}(0)$	-0.4479	-0.2331	-0.2473	-0.2441
t/a for $B_{33}^{thin}(0)$	0.125	0.0625	0.0417	0.0313

Table 4. 12 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.2$

t/a	$\text{Log}(t/a)$	$\frac{T_{33}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B_{33}^{2D}$ + $B_{33}^{thin}(0)$ (Predict)	$\left \frac{\text{FEM} - \text{Predict}}{\text{FEM}} \right $ $\times 100\%$
0.25	-0.6021	-0.3921	-0.4029	2.76
0.5	-0.3010	-0.3162	-0.3843	21.56
1	0.0000	-0.2149	-0.3580	66.59
2.5	0.3979	-0.0237	-0.3057	1187.39
5	0.6990	0.1517	-0.2469	262.73
10	1.0000	0.2686	-0.1636	160.90
20	1.3010	0.3962	-0.0458	111.56
40	1.6021	0.5670	0.1207	78.71
80	1.9031	0.8034	0.3563	55.65

Table 4. 13 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.4$

t/a	$\text{Log}(t/a)$	$\frac{T_{33}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B_{33}^{2D}$ $+ B_{33}^{thin}(0)$ (Predict)	$\frac{ \text{FEM} - \text{Predict} }{\text{FEM}}$ $\times 100\%$
0.125	-0.9031	-0.2201	-0.2122	3.57
0.25	-0.6021	-0.2007	-0.2036	1.44
0.5	-0.3010	-0.1726	-0.1914	10.86
1.25	0.0969	-0.1146	-0.1671	45.88
2.5	0.3979	-0.0377	-0.1398	270.58
5	0.6990	0.0464	-0.1012	317.90
10	1.0000	0.0829	-0.0465	156.09
20	1.3010	0.1202	0.0308	74.38
40	1.6021	0.1701	0.1401	17.63

Table 4.14 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.6$

t/a	$\text{Log}(t/a)$	$\frac{T_{33}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B_{33}^{2D}$ $+ B_{33}^{thin}(0)$ (Predict)	$\frac{ \text{FEM} - \text{Predict} }{\text{FEM}}$ $\times 100\%$
0.0833	-1.0792	-0.2385	-0.2251	5.60
0.1667	-0.7782	-0.2233	-0.2160	3.28
0.3333	-0.4771	-0.1977	-0.2030	2.71
0.8333	-0.0792	-0.1296	-0.1773	36.80
1.6667	0.2218	-0.0319	-0.1484	365.48
3.3333	0.5229	0.0500	-0.1074	314.76
6.6667	0.8239	0.0869	-0.0495	156.93
13.3333	1.1249	0.1259	0.0324	74.24
26.6667	1.4260	0.1780	0.1483	16.72

Table 4.15 The comparison of the approximate thickness-dependence of T_{33} stress at mid-plane as given by Eq. (4.10) and the finite element results, Uniform, SECP, $a/W=0.8$

t/a	$\text{Log}(t/a)$	$\frac{T_{33}(0)}{K_I(0)/\sqrt{\pi t}}$ (FEM)	$\sqrt{\frac{t}{a}} B_{33}^{2D}$ $+ B_{33}^{thin}(0)$ (Predict)	$\left \frac{\text{FEM} - \text{Predict}}{\text{FEM}} \right $ $\times 100\%$
0.0625	-1.2041	-0.2376	-0.2002	15.72
0.125	-0.9031	-0.2140	-0.1820	14.94
0.25	-0.6021	-0.1678	-0.1563	6.84
0.625	-0.2041	-0.0339	-0.1054	210.49
1.25	0.0969	0.0646	-0.0479	174.12
2.5	0.3979	0.1177	0.0334	71.66
5	0.6990	0.1738	0.1483	14.69
10	1.0000	0.2476	0.3108	25.53
20	1.3010	0.3510	0.5406	54.01

Table 4.16 The thickness correction factor ΔK_{3D} for $a/W=0.2$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0649	0.0000	1.0618	0.0000	1.0510
0.2573	1.0581	0.2525	1.0555	0.2549	1.0454
0.4069	1.0356	0.4018	1.0349	0.4014	1.0262
0.4555	1.0103	0.4539	1.0110	0.4581	1.0002
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0298	0.0000	1.0083	0.0000	1.0016
0.2549	1.0312	0.2549	1.0152	0.2549	1.0041
0.4014	1.0217	0.4014	1.0207	0.4014	1.0156
0.4581	1.0009	0.4581	1.0093	0.4581	1.0175

Table 4.17 The thickness correction factor ΔK_{3D} for $a/W=0.4$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0650	0.0000	1.0628	0.0000	1.0564
0.2573	1.0567	0.2525	1.0549	0.2549	1.0475
0.4069	1.0315	0.4018	1.0230	0.4014	1.0211
0.4555	1.0042	0.4539	1.0029	0.4581	0.9889
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0419	0.0000	1.0189	0.0000	1.0064
0.2549	1.0350	0.2549	1.0208	0.2549	1.0093
0.4014	1.0109	0.4014	1.0106	0.4014	1.0132
0.4581	0.9786	0.4581	0.9853	0.4581	0.9996

Table 4.18 The thickness correction factor ΔK_{3D} for $a/W=0.6$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0689	0.0000	1.0680	0.0000	1.0599
0.2573	1.0592	0.2525	1.0573	0.2549	1.0477
0.4069	1.0306	0.4018	1.0267	0.4014	1.0129
0.4555	1.0011	0.4539	0.9957	0.4581	0.9725
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0426	0.0000	1.0226	0.0000	1.0106
0.2549	1.0345	0.2549	1.0222	0.2549	1.0122
0.4014	1.0037	0.4014	1.0076	0.4014	1.0114
0.4581	0.9627	0.4581	0.9745	0.4581	0.9934

Table 4.19 The thickness correction factor ΔK_{3D} for $a/W=0.8$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0719	0.0000	1.0696	0.0000	1.0509
0.2573	1.0591	0.2525	1.0548	0.2549	1.0393
0.4069	1.0236	0.4018	1.0141	0.4014	0.9990
0.4555	0.9896	0.4539	0.9758	0.4581	0.9494
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔK_{3D}	z/t	ΔK_{3D}	z/t	ΔK_{3D}
0.0000	1.0310	0.0000	1.0166	0.0000	1.0091
0.2549	1.0270	0.2549	1.0166	0.2549	1.0096
0.4014	1.0026	0.4014	1.0091	0.4014	1.0096
0.4581	0.9587	0.4581	0.9808	0.4581	0.9989

Table 4.20 The thickness correction factor ΔT_{113D} for $a/W=0.2$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.2574	0.0000	1.1743	0.0000	1.1430
0.2573	1.2982	0.2525	1.2004	0.2549	1.1161
0.4069	1.3652	0.4018	1.2532	0.4014	1.0951
0.4555	1.3927	0.4539	1.2744	0.4581	1.1021
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.0525	0.0000	0.9971	0.0000	0.9962
0.2549	1.0771	0.2549	1.0264	0.2549	0.9978
0.4014	1.0522	0.4014	1.0736	0.4014	1.0457
0.4581	1.0412	0.4581	1.0477	0.4581	1.0712

Table 4.21 The thickness correction factor ΔT_{113D} for $a/W=0.4$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.6015	0.0000	1.4331	0.0000	1.3184
0.2573	1.6791	0.2525	1.4827	0.2549	1.3140
0.4069	1.8244	0.4018	1.5699	0.4014	1.3271
0.4555	1.8456	0.4539	1.6051	0.4581	1.3341
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.2779	0.0000	1.0575	0.0000	1.0149
0.2549	1.2290	0.2549	1.1482	0.2549	1.0391
0.4014	1.1412	0.4014	1.1491	0.4014	1.1675
0.4581	1.0970	0.4581	1.0863	0.4581	1.1359

Table 4.22 The thickness correction factor ΔT_{113D} for $a/W=0.6$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.5784	0.0000	1.4387	0.0000	1.3700
0.2573	1.6443	0.2525	1.4678	0.2549	1.3271
0.4069	1.7691	0.4018	1.5203	0.4014	1.2546
0.4555	1.7862	0.4539	1.5407	0.4581	1.2053
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.2497	0.0000	1.0446	0.0000	1.0102
0.2549	1.2374	0.2549	1.1274	0.2549	1.0252
0.4014	1.1262	0.4014	1.1639	0.4014	1.1473
0.4581	1.0139	0.4581	1.0587	0.4581	1.1405

Table 4. 23 The thickness correction factor ΔT_{113D} for $a/W=0.8$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.3243	0.0000	1.2919	0.0000	1.2089
0.2573	1.3417	0.2525	1.2655	0.2549	1.1892
0.4069	1.3789	0.4018	1.2228	0.4014	1.0767
0.4555	1.3781	0.4539	1.1997	0.4581	0.9715
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{113D}	z/t	ΔT_{113D}	z/t	ΔT_{113D}
0.0000	1.0567	0.0000	1.0143	0.0000	1.0078
0.2549	1.1092	0.2549	1.0289	0.2549	1.0074
0.4014	1.1065	0.4014	1.1086	0.4014	1.0371
0.4581	0.9978	0.4581	1.0743	0.4581	1.1064

Table 4.24 The thickness correction factor ΔT_{333D} for $a/W=0.2$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	-5.2966	0.0000	-2.5383	0.0000	-0.1756
0.2573	-7.1428	0.2525	-3.7398	0.2549	-0.9275
0.4069	-12.9485	0.4018	-7.5219	0.4014	-3.1734
0.4555	-19.9968	0.4539	-12.3682	0.4581	-6.6116
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	0.7772	0.0000	0.9528	0.0000	0.9871
0.2549	0.3195	0.2549	0.8627	0.2549	0.9729
0.4014	-1.1694	0.4014	0.0704	0.4014	0.7663
0.4581	-3.5265	0.4581	-1.5084	0.4581	-0.1401

Table 4.25 The thickness correction factor ΔT_{333D} for $a/W=0.4$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	-16.1999	0.0000	-9.8330	0.0000	-4.1023
0.2573	-20.2609	0.2525	-12.5909	0.2549	-5.8980
0.4069	-33.1427	0.4018	-20.9809	0.4014	-11.1356
0.4555	-48.3556	0.4539	-31.5763	0.4581	-18.7680
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	-0.9421	0.0000	0.8016	0.0000	0.9998
0.2549	-2.3454	0.2549	-0.0548	0.2549	0.8926
0.4014	-6.1929	0.4014	-2.8598	0.4014	-0.5366
0.4581	-11.6437	0.4581	-6.7835	0.4581	-3.3260

Table 4.26 The thickness correction factor ΔT_{333D} for $a/W=0.6$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	-17.0708	0.0000	-10.6760	0.0000	-4.3943
0.2573	-21.0026	0.2525	-13.4199	0.2549	-6.3942
0.4069	-33.2982	0.4018	-21.5420	0.4014	-11.7755
0.4555	-47.6661	0.4539	-31.5682	0.4581	-19.0998
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	-0.7515	0.0000	0.8179	0.0000	0.9936
0.2549	-2.4036	0.2549	0.0325	0.2549	0.8943
0.4014	-6.7266	0.4014	-3.0870	0.4014	-0.4794
0.4581	-12.2303	0.4581	-7.3872	0.4581	-3.6567

Table 4.27 The thickness correction factor ΔT_{333D} for $a/W=0.8$

$t/W = 0.1$		$t/W = 0.2$		$t/W = 0.5$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	-8.2683	0.0000	-4.5748	0.0000	-0.5748
0.2573	-10.3662	0.2525	-6.1430	0.2549	-1.8996
0.4069	-16.7165	0.4018	-10.5036	0.4014	-5.2039
0.4555	-23.9532	0.4539	-15.5995	0.4581	-9.1780
$t/W = 1.0$		$t/W = 2.0$		$t/W = 4.0$	
z/t	ΔT_{333D}	z/t	ΔT_{333D}	z/t	ΔT_{333D}
0.0000	0.7594	0.0000	0.9647	0.0000	0.9995
0.2549	0.0931	0.2549	0.8515	0.2549	0.9775
0.4014	-2.4024	0.4014	-0.3450	0.4014	0.7312
0.4581	-5.6794	0.4581	-2.9044	0.4581	-0.6816

Figure 4.1 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.2$

Figure 4.2 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.4$

Figure 4.3 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.6$

Figure 4.4 Mid-plane value of K_I as a function of plate thickness in SECP, $a/W=0.8$

Figure 4.5 Mid-plane value of T_{11} stress as a function of plate thickness in SECP,
 $a/W=0.2$

Figure 4.6 Mid-plane value of T_{11} stress as a function of plate thickness in SECP,
 $a/W=0.4$

Figure 4.7 Mid-plane value of T_{11} stress as a function of plate thickness in SECP,
 $a/W=0.6$

Figure 4.8 Mid-plane value of T_{11} stress as a function of plate thickness in SECP,
 $a/W=0.8$

Figure 4.9 Comparison of approximation formula and FEM for T_{11} , $a/W=0.2$

Figure 4.10 Comparison of approximation formula and FEM for T_{11} , $a/W=0.4$

Figure 4.11 Comparison of approximation formula and FEM for T_{11} , $a/W=0.6$

Figure 4.12 Comparison of approximation formula and FEM for T_{11} , $a/W=0.8$

Figure 4.13 Mid-plane value of T_{33} stress as a function of plate thickness in SECP,
 $a/W=0.2$

Figure 4.14 Mid-plane value of T_{33} stress as a function of plate thickness in SECP,
 $a/W=0.4$

Figure 4.15 Mid-plane value of T_{33} stress as a function of plate thickness in SECP,
 $a/W=0.6$

Figure 4.16 Mid-plane value of T_{33} stress as a function of plate thickness in SECP,
 $a/W=0.8$

Figure 4.17 Comparison of approximation formula and FEM for T_{33} , $a/W=0.2$

Figure 4.18 Comparison of approximation formula and FEM for T_{33} , $a/W=0.4$

Figure 4.19 Comparison of approximation formula and FEM for T_{33} , $a/W=0.6$

Figure 4.20 Comparison of approximation formula and FEM for T_{33} , $a/W=0.8$

Figure 4.21 Mid-plane value of stress intensity factor as a function of crack depth in SECP

Figure 4.22 Mid-plane value of T_{11} as a function of crack depth in SECP

Figure 4.23 Mid-plane value of T_{33} as a function of crack depth in SECP

Figure 4.24 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.2$

Figure 4.25 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.4$

Figure 4.26 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.6$

Figure 4.27 Variations of relative SIF along the crack tip for plates various thickness, uniform, $a/W=0.8$

Figure 4.28 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.2$

Figure 4.29 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.4$

Figure 4.30 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.6$

Figure 4.31 Variations of relative T_{11} along the crack tip for plates various thickness, uniform, $a/W=0.8$

Figure 4.32 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.2$

Figure 4.33 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.4$

Figure 4.34 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.6$

Figure 4.35 Variations of relative T_{33} along the crack tip for plates various thickness, uniform, $a/W=0.8$

CHAPTER 5

STRESS STATE DESCRIPTION BY K_I , T_{11} AND T_{33}

In this chapter, the stress state around the crack front in three-dimensional SECP specimens described by three parameters, stress intensity factor K_I , and T -stresses (T_{11} and T_{33}) from solutions presented in Chapter 3 are studied. The variation of the stress state described by Williams' expansion is compared with data obtained from finite element analyses by angle, radius and thickness, respectively.

5.1 Stress State Description by K_I , T_{11} and T_{33}

The plane strain and plane stress state assumptions are widely used for two-dimensional problems for the development of stress state but cannot directly generalized to three-dimensional crack problem. However, it is accepted that using the description of the singular stress fields in the vicinity of the three-dimensional crack front according to the Williams series expansion (Williams, 1957). The stress state description using K_I , T_{11} and T_{33} is presented in the following. The stress state is studied at points located at a very small (but finite) radial distance to the crack front. The small circle has a radius r and the polar angle θ is varied between 0° and 180° . The coordinate system used at the crack front is shown in Figure 5.1.

From Williams' expansion, where higher order terms are neglected since the circular path is very close to the crack front, and the following relationships are obtained:

$$\frac{\sigma_{ii} - T_{ii}}{K_I/\sqrt{2\pi r}} = f_{ii}(\theta) \quad i = 1, 2, 3 \quad (5.1)$$

With

$$\begin{aligned}
 f_{11}(\theta) &= \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
 f_{22}(\theta) &= \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\
 f_{33}(\theta) &= \nu (f_{11}(\theta) + f_{22}(\theta)) = 2\nu \cos \frac{\theta}{2} \\
 \text{and } T_{22} &= 0
 \end{aligned}
 \tag{5.2}$$

In the following sections, these equations are used to predict the stress state using K_I , T_{11} and T_{33} , and comparisons are made with finite element calculation results for the crack surface subjected to the uniform loading.

5.1.1 Stress State Varied by Angle

The results of finite element analyses for K_I , T_{11} , T_{33} , and σ_{ii} ($i=1, 2, 3$) as described in Chapter 3 were substituted in Eq. (5.1), and compared with Eq. (5.2). The comparisons of $a/W=0.8$ varied by t/W ($t/W=0.1, 0.2, 0.5, 1.0, 2.0$, or 4.0) at location $z/t=0$ (mid-plane) are shown in Figure 5.2-5.7. The angle θ is increased by 7.5° from 0° to 180° . The ratio of the radius from the crack front to the location chosen for stress state analysis between crack length, r/a , is fixed to $4.15E-3$. The results for the difference between the Eq. (5.2) and the finite element results are listed in Table 5.1-5.6. In general, the agreement between finite element results and formulae evaluation is very good at the analyzed angle for the different relative thickness, except the percentage of few values at particular angles are over 10%.

Next, the model of the relative crack depth $a/W=0.8$ is used to analyze the percentage difference of $f_{ii}(\theta)$ s between the finite element results and formulae results varied by angle, and the results are shown in Figure 5.8-5.10. The relative thickness of

the plate is $t/W=0.1, 0.2, 0.5, 1.0, 2.0$ and 4.0 , respectively. Three relative radiuses are chosen ($r/a=1.26559E-4, 7.56476E-5$, or $3.61595E-5$). The percentage difference of $f_{11}(\theta)$ varied by angles at three radiuses location is shown in Figure 5.8. The angle θ is increased by 15° from 0° to 165° . It shows that the peak of the difference is located on 75° and decreasing to both sides. The percentage different of $f_{22}(\theta)$ varied by angles at three radiuses location is shown in Figure 5.9 (a) and (b). The angle θ is increased by 15° from 0° to 135° in Figure 5.9 (a) and 0° to 165° in Figure 5.9 (b). It shows that the maximum difference appears around 165° , and also the difference is high around 0° . And the minimum difference appears around 90° . The Figure 5.10 shows the percentage difference of $f_{33}(\theta)$ varied by angles at three radiuses location. The angle θ is increased by 15° from 0° to 165° . The maximum difference is around 165° .

From Figure 5.8-5.10, it also notices that the percentage different of $f_{22}(\theta)$ is not varied significantly by thickness of the plate. And the percentage different of $f_{11}(\theta)$ and $f_{33}(\theta)$ is not varied significantly by the thickness of the plate when relative thickness $t/W>0.2$.

5.1.2 Stress State Varied by Radius

The stress state varied by radius for $a/W=0.6, t/W=0.5$ at four locations along thickness subjected by uniform stress field are compared, as shown in Figure 5.11-5.31. The angles are $0^\circ, 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ and 180° , respectively. The four locations along thickness are at $z/t=0, 0.11612, 0.26369$ or 0.4182 , respectively. The difference between the finite element analysis results and Eq. (5.2) for $f_{11}(\theta)$ are shown in Figure 5.11-5.17. The finite element analysis gave good results for very small radius from crack front, and deviating from the results of Eq. (5.2) with the radius increasing. Stress State along the circular path close to the crack front varied by radius at four

locations along thickness for $f_{22}(\theta)$ is shown in Figures 5.18-5.24. It is noticed that the difference between the finite element analysis and the Eq. (5.2) for $f_{22}(\theta)$ are increasing with the relative radius ratio r/a increasing. The finite element results of $f_{11}(\theta)$ and $f_{22}(\theta)$ are smaller than the results calculated from Eq. (5.2). Figures 5.25-5.31 show the $f_{33}(\theta)$ along the circular path close to the crack front varied by radius at four locations along thickness. The finite element analysis results are decreasing with the relative radius rate r/a increasing for three locations along the thickness direction ($z/t=0, 0.11612$, or 0.26369). But the values of the finite element analysis for position $z/t= 0.4182$ are increasing and some of angles are large than the results of Eq. (5.2) finally ($\theta = 150^\circ$ or 180°).

The percentage difference of $f_{ii}(\theta)$ s between finite element results and formulae results varied by radius of $a/W=0.6$, $t/W=0.5$, and $z/t = 0$ (mid-plane) are listed in Table 5.7, and shown in Figure 5.32-5.37. The angles are 0° , 30° , 60° , 90° , 120° , and 150° , respectively. It is noticed that the difference between the finite element and the Eq. (5.2) increases when the distance from the crack front increases for all three $f_{ii}(\theta)$ s. The accurate of the stress state description by the three parameters is changed by the angle for fixed radius.

5.1.3 Stress State along Thickness

The stress state varied by four locations along thickness for $a/W=0.6$, $t/W=0.1$, 0.2 , 0.5 , 1.0 , 2.0 and 4.0 , subjected by uniform stress field were compared, shown as in Fig. 5.38-5.55. And the results are shown in Table 5.8-5.13. The relative ratio of radius and crack length is fixed, $r/a = 7.8E-3$. Due to the mesh difference, the four locations along thickness are $z/t=0, 0.1311, 0.2702$ or 0.3861 for $t/W=0.1$, $z/t=0, 0.12277, 0.26265$ or 0.42441 for $t/W=0.2$, and $z/t=0, 0.11612, 0.26369$ or 0.4182 for $t/W=0.5$,

1.0, 2.0 and 4.0. The stress state description by K_I , T_{11} and T_{33} with Williams formulae and finite element results are compared in the figures. For cases of $t/W=0.1$ and 0.2, the results for first three locations along thickness ($z/t=0, 0.1311$, and 0.2702 for $t/W=0.1$, and $z/t=0, 0.12277$, and 0.26265 for $t/W=0.2$) are slight different. But the results for the locations near the free boundary ($z/t= 0.3861$ for $t/W=0.1$, $z/t= 0.42441$ for $t/W=0.2$) are quite different than the others. There is very good agreement for near crack front stress state in mid-plane for $t/W \geq 0.5$. Except $f_{22}(\theta)$ for the case of $t/W=4.0$. The results of the other three locations along crack front are almost same but slightly different from the mid-plane evaluation for $t/W \geq 0.5$.

5.2 Conclusion

The stress state around the crack front for through-thickness single edge crack in plate under uniform loading was described by three parameters, stress intensity factor K_I , and T -stresses (T_{11} and T_{33}). Generally, when r/a is small, the results of the finite element analyses and the Williams' expansion have very good fit for almost all cases. There is very good agreement for near crack front stress state described by K_I , T_{11} and T_{33} in mid-plane. The other locations along crack front are almost same but slight different from the Eq. (5.2).

Table 5.1 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=0.1$, Uniform Load, SECP

Degree (°)	$f_{11}(\theta)$			$f_{22}(\theta)$			$f_{33}(\theta)$		
	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%
0	1.0000	0.9751	2.49	1.0000	0.9554	4.46	0.6000	0.5738	4.37
7.5	0.9851	0.9454	4.03	1.0106	0.9767	3.35	0.5987	0.5714	4.56
15	0.9419	0.9150	2.85	1.0410	0.9989	4.04	0.5949	0.5690	4.34
22.5	0.8745	0.8363	4.37	1.0871	1.0528	3.15	0.5885	0.5619	4.51
30	0.7891	0.7571	4.06	1.1427	1.1073	3.09	0.5796	0.5548	4.27
37.5	0.6938	0.6578	5.20	1.2000	1.1658	2.85	0.5682	0.5431	4.42
45	0.5972	0.5586	6.47	1.2505	1.2243	2.10	0.5543	0.5313	4.15
52.5	0.5078	0.4734	6.78	1.2859	1.2532	2.54	0.5381	0.5151	4.28
60	0.4330	0.3888	10.21	1.2990	1.2817	1.33	0.5196	0.4989	3.98
67.5	0.3784	0.3443	9.01	1.2845	1.2556	2.26	0.4989	0.4785	4.09
75	0.3472	0.3006	13.40	1.2396	1.2286	0.88	0.4760	0.4581	3.77
82.5	0.3397	0.3046	10.34	1.1640	1.1408	2.00	0.4511	0.4337	3.86
90	0.3536	0.3092	12.53	1.0607	1.0522	0.80	0.4243	0.4094	3.51
97.5	0.3839	0.3474	9.51	0.9348	0.9182	1.77	0.3956	0.3814	3.58
105	0.4239	0.3860	8.95	0.7936	0.7839	1.22	0.3653	0.3535	3.21
112.5	0.4655	0.4284	7.95	0.6457	0.6355	1.58	0.3333	0.3225	3.26
120	0.5000	0.4707	5.86	0.5000	0.4872	2.56	0.3000	0.2914	2.86
127.5	0.5197	0.4843	6.82	0.3649	0.3593	1.54	0.2654	0.2577	2.89
135	0.5180	0.4972	4.02	0.2474	0.2320	6.22	0.2296	0.2240	2.43
142.5	0.4905	0.4595	6.33	0.1523	0.1489	2.27	0.1929	0.1882	2.40
150	0.4356	0.4209	3.36	0.0820	0.0666	18.78	0.1553	0.1524	1.84
157.5	0.3542	0.3302	6.77	0.0360	0.0323	10.29	0.1171	0.1151	1.64
165	0.2501	0.2388	4.53	0.0110	-0.0013	112.02	0.0783	0.0778	0.61
172.5	0.1294	0.1146	11.43	0.0014	-0.0043	411.68	0.0392	0.0397	1.09
180	0.0000	-0.0086	Inf.	0.0000	-0.0087	Inf.	0.0000	0.0014	Inf.

Table5.2 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=0.2$, Uniform Load, SECP

Degree (°)	$f_{11}(\theta)$			$f_{22}(\theta)$			$f_{33}(\theta)$		
	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%
0	1.0000	0.9756	2.44	1.0000	0.9568	4.32	0.6000	0.5787	3.56
7.5	0.9851	0.9460	3.97	1.0106	0.9781	3.21	0.5987	0.5762	3.76
15	0.9419	0.9156	2.79	1.0410	1.0002	3.91	0.5949	0.5737	3.55
22.5	0.8745	0.8369	4.29	1.0871	1.0542	3.03	0.5885	0.5664	3.76
30	0.7891	0.7578	3.97	1.1427	1.1086	2.98	0.5796	0.5590	3.54
37.5	0.6938	0.6586	5.08	1.2000	1.1670	2.75	0.5682	0.5469	3.74
45	0.5972	0.5595	6.32	1.2505	1.2254	2.01	0.5543	0.5348	3.53
52.5	0.5078	0.4744	6.58	1.2859	1.2543	2.46	0.5381	0.5181	3.73
60	0.4330	0.3899	9.95	1.2990	1.2827	1.26	0.5196	0.5014	3.51
67.5	0.3784	0.3456	8.68	1.2845	1.2564	2.19	0.4989	0.4803	3.72
75	0.3472	0.3020	13.00	1.2396	1.2293	0.82	0.4760	0.4593	3.50
82.5	0.3397	0.3061	9.89	1.1640	1.1414	1.94	0.4511	0.4343	3.72
90	0.3536	0.3109	12.07	1.0607	1.0527	0.75	0.4243	0.4093	3.52
97.5	0.3839	0.3491	9.07	0.9348	0.9187	1.72	0.3956	0.3808	3.75
105	0.4239	0.3878	8.53	0.7936	0.7843	1.17	0.3653	0.3522	3.57
112.5	0.4655	0.4303	7.55	0.6457	0.6357	1.54	0.3333	0.3206	3.84
120	0.5000	0.4726	5.47	0.5000	0.4874	2.52	0.3000	0.2889	3.70
127.5	0.5197	0.4862	6.45	0.3649	0.3594	1.51	0.2654	0.2547	4.03
135	0.5180	0.4991	3.65	0.2474	0.2321	6.19	0.2296	0.2205	3.98
142.5	0.4905	0.4613	5.96	0.1523	0.1489	2.24	0.1929	0.1843	4.45
150	0.4356	0.4227	2.96	0.0820	0.0666	18.77	0.1553	0.1481	4.64
157.5	0.3542	0.3318	6.31	0.0360	0.0323	10.30	0.1171	0.1106	5.56
165	0.2501	0.2402	3.93	0.0110	-0.0013	112.15	0.0783	0.0730	6.76
172.5	0.1294	0.1159	10.43	0.0014	-0.0044	412.75	0.0392	0.0348	11.34
180	0.0000	-0.0075	Inf.	0.0000	-0.0087	Inf.	0.0000	-0.0036	Inf.

Table 5.3 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=0.5$, Uniform Load, SECP

Degree (°)	$f_{11}(\theta)$			$f_{22}(\theta)$			$f_{33}(\theta)$		
	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%
0	1.0000	0.9831	1.69	1.0000	0.9647	3.53	0.6000	0.5847	2.56
7.5	0.9851	0.9535	3.21	1.0106	0.9860	2.44	0.5987	0.5822	2.76
15	0.9419	0.9231	2.00	1.0410	1.0081	3.16	0.5949	0.5797	2.55
22.5	0.8745	0.8444	3.44	1.0871	1.0620	2.30	0.5885	0.5722	2.76
30	0.7891	0.7652	3.03	1.1427	1.1165	2.29	0.5796	0.5648	2.54
37.5	0.6938	0.6661	4.00	1.2000	1.1749	2.09	0.5682	0.5526	2.75
45	0.5972	0.5670	5.07	1.2505	1.2333	1.38	0.5543	0.5403	2.53
52.5	0.5078	0.4819	5.10	1.2859	1.2621	1.85	0.5381	0.5234	2.73
60	0.4330	0.3975	8.21	1.2990	1.2905	0.66	0.5196	0.5066	2.51
67.5	0.3784	0.3531	6.68	1.2845	1.2642	1.58	0.4989	0.4853	2.71
75	0.3472	0.3097	10.80	1.2396	1.2371	0.20	0.4760	0.4641	2.49
82.5	0.3397	0.3137	7.63	1.1640	1.1491	1.28	0.4511	0.4389	2.70
90	0.3536	0.3186	9.88	1.0607	1.0604	0.03	0.4243	0.4137	2.48
97.5	0.3839	0.3569	7.03	0.9348	0.9262	0.91	0.3956	0.3849	2.70
105	0.4239	0.3957	6.67	0.7936	0.7918	0.23	0.3653	0.3562	2.49
112.5	0.4655	0.4382	5.86	0.6457	0.6432	0.39	0.3333	0.3243	2.71
120	0.5000	0.4806	3.89	0.5000	0.4947	1.05	0.3000	0.2925	2.51
127.5	0.5197	0.4941	4.92	0.3649	0.3667	0.49	0.2654	0.2581	2.75
135	0.5180	0.5070	2.12	0.2474	0.2393	3.26	0.2296	0.2237	2.57
142.5	0.4905	0.4692	4.34	0.1523	0.1561	2.49	0.1929	0.1874	2.84
150	0.4356	0.4306	1.14	0.0820	0.0738	10.03	0.1553	0.1511	2.71
157.5	0.3542	0.3397	4.09	0.0360	0.0394	9.60	0.1171	0.1135	3.06
165	0.2501	0.2480	0.82	0.0110	0.0058	46.98	0.0783	0.0759	3.13
172.5	0.1294	0.1236	4.48	0.0014	0.0028	99.89	0.0392	0.0376	4.14
180	0.0000	0.0001	Inf.	0.0000	-0.0016	Inf.	0.0000	-0.0008	Inf.

Table 5.4 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=1.0$, Uniform Load, SECP

Degree (°)	$f_{11}(\theta)$			$f_{22}(\theta)$			$f_{33}(\theta)$		
	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%
0	1.0000	0.9827	1.73	1.0000	0.9643	3.57	0.6000	0.5842	2.63
7.5	0.9851	0.9531	3.25	1.0106	0.9856	2.47	0.5987	0.5817	2.84
15	0.9419	0.9227	2.04	1.0410	1.0077	3.19	0.5949	0.5793	2.62
22.5	0.8745	0.8440	3.48	1.0871	1.0617	2.34	0.5885	0.5718	2.83
30	0.7891	0.7649	3.07	1.1427	1.1161	2.32	0.5796	0.5644	2.61
37.5	0.6938	0.6657	4.05	1.2000	1.1745	2.12	0.5682	0.5522	2.81
45	0.5972	0.5666	5.13	1.2505	1.2329	1.41	0.5543	0.5400	2.59
52.5	0.5078	0.4816	5.16	1.2859	1.2618	1.88	0.5381	0.5231	2.79
60	0.4330	0.3971	8.28	1.2990	1.2902	0.68	0.5196	0.5063	2.57
67.5	0.3784	0.3528	6.76	1.2845	1.2639	1.61	0.4989	0.4851	2.77
75	0.3472	0.3094	10.89	1.2396	1.2368	0.22	0.4760	0.4639	2.55
82.5	0.3397	0.3134	7.72	1.1640	1.1489	1.30	0.4511	0.4387	2.75
90	0.3536	0.3183	9.97	1.0607	1.0602	0.05	0.4243	0.4136	2.53
97.5	0.3839	0.3566	7.11	0.9348	0.9261	0.93	0.3956	0.3848	2.73
105	0.4239	0.3954	6.74	0.7936	0.7916	0.25	0.3653	0.3561	2.51
112.5	0.4655	0.4379	5.91	0.6457	0.6431	0.41	0.3333	0.3243	2.73
120	0.5000	0.4803	3.94	0.5000	0.4947	1.07	0.3000	0.2924	2.52
127.5	0.5197	0.4939	4.96	0.3649	0.3666	0.48	0.2654	0.2581	2.74
135	0.5180	0.5068	2.16	0.2474	0.2393	3.28	0.2296	0.2238	2.55
142.5	0.4905	0.4691	4.38	0.1523	0.1561	2.47	0.1929	0.1875	2.80
150	0.4356	0.4305	1.18	0.0820	0.0738	10.04	0.1553	0.1512	2.64
157.5	0.3542	0.3396	4.12	0.0360	0.0394	9.58	0.1171	0.1136	2.95
165	0.2501	0.2480	0.85	0.0110	0.0058	46.98	0.0783	0.0760	2.92
172.5	0.1294	0.1236	4.51	0.0014	0.0028	99.86	0.0392	0.0378	3.67
180	0.0000	0.0001	Inf.	0.0000	-0.0016	Inf.	0.0000	-0.0006	Inf.

Table 5.5 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=2.0$, Uniform Load, SECP

Degree (°)	$f_{11}(\theta)$			$f_{22}(\theta)$			$f_{33}(\theta)$		
	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%
0	1.0000	0.9825	1.75	1.0000	0.9641	3.59	0.6000	0.5840	2.67
7.5	0.9851	0.9529	3.27	1.0106	0.9854	2.50	0.5987	0.5815	2.88
15	0.9419	0.9225	2.06	1.0410	1.0075	3.22	0.5949	0.5790	2.67
22.5	0.8745	0.8438	3.51	1.0871	1.0614	2.36	0.5885	0.5716	2.87
30	0.7891	0.7647	3.10	1.1427	1.1159	2.35	0.5796	0.5642	2.65
37.5	0.6938	0.6655	4.08	1.2000	1.1743	2.14	0.5682	0.5520	2.85
45	0.5972	0.5664	5.16	1.2505	1.2327	1.43	0.5543	0.5397	2.63
52.5	0.5078	0.4814	5.20	1.2859	1.2616	1.89	0.5381	0.5229	2.83
60	0.4330	0.3969	8.33	1.2990	1.2900	0.70	0.5196	0.5061	2.61
67.5	0.3784	0.3526	6.81	1.2845	1.2637	1.62	0.4989	0.4849	2.80
75	0.3472	0.3091	10.95	1.2396	1.2367	0.23	0.4760	0.4637	2.58
82.5	0.3397	0.3132	7.78	1.1640	1.1487	1.31	0.4511	0.4386	2.77
90	0.3536	0.3181	10.03	1.0607	1.0600	0.06	0.4243	0.4134	2.55
97.5	0.3839	0.3564	7.17	0.9348	0.9260	0.94	0.3956	0.3847	2.75
105	0.4239	0.3952	6.79	0.7936	0.7915	0.26	0.3653	0.3560	2.53
112.5	0.4655	0.4377	5.96	0.6457	0.6430	0.42	0.3333	0.3242	2.74
120	0.5000	0.4801	3.98	0.5000	0.4946	1.08	0.3000	0.2924	2.53
127.5	0.5197	0.4937	5.00	0.3649	0.3666	0.46	0.2654	0.2581	2.75
135	0.5180	0.5066	2.20	0.2474	0.2392	3.29	0.2296	0.2238	2.55
142.5	0.4905	0.4689	4.42	0.1523	0.1561	2.46	0.1929	0.1875	2.78
150	0.4356	0.4303	1.22	0.0820	0.0738	10.05	0.1553	0.1512	2.61
157.5	0.3542	0.3394	4.16	0.0360	0.0394	9.57	0.1171	0.1137	2.89
165	0.2501	0.2478	0.90	0.0110	0.0058	46.99	0.0783	0.0761	2.83
172.5	0.1294	0.1235	4.58	0.0014	0.0028	99.84	0.0392	0.0379	3.45
180	0.0000	0.0000	Inf.	0.0000	-0.0016	Inf.	0.0000	-0.0005	Inf.

Table 5.6 Comparison of stress state at mid-plane between finite element results and formulae evaluation for $a/W=0.8$, $t/W=4.0$, Uniform Load, SECP

Degree (°)	$f_{11}(\theta)$			$f_{22}(\theta)$			$f_{33}(\theta)$		
	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%	Eq.(5.2)	FEM	%
0	1.0000	0.9825	1.75	1.0000	0.9641	3.59	0.6000	0.5840	2.67
7.5	0.9851	0.9529	3.27	1.0106	0.9854	2.50	0.5987	0.5815	2.88
15	0.9419	0.9225	2.06	1.0410	1.0075	3.21	0.5949	0.5790	2.67
22.5	0.8745	0.8438	3.50	1.0871	1.0614	2.36	0.5885	0.5716	2.87
30	0.7891	0.7647	3.10	1.1427	1.1159	2.34	0.5796	0.5642	2.65
37.5	0.6938	0.6655	4.08	1.2000	1.1743	2.14	0.5682	0.5520	2.85
45	0.5972	0.5664	5.16	1.2505	1.2327	1.42	0.5543	0.5397	2.63
52.5	0.5078	0.4814	5.20	1.2859	1.2616	1.89	0.5381	0.5229	2.83
60	0.4330	0.3969	8.33	1.2990	1.2900	0.70	0.5196	0.5061	2.61
67.5	0.3784	0.3526	6.82	1.2845	1.2637	1.62	0.4989	0.4849	2.80
75	0.3472	0.3091	10.95	1.2396	1.2367	0.23	0.4760	0.4637	2.58
82.5	0.3397	0.3132	7.79	1.1640	1.1487	1.31	0.4511	0.4386	2.77
90	0.3536	0.3181	10.03	1.0607	1.0601	0.06	0.4243	0.4134	2.55
97.5	0.3839	0.3564	7.17	0.9348	0.9260	0.94	0.3956	0.3847	2.75
105	0.4239	0.3951	6.79	0.7936	0.7915	0.26	0.3653	0.3560	2.53
112.5	0.4655	0.4377	5.96	0.6457	0.6430	0.42	0.3333	0.3242	2.74
120	0.5000	0.4801	3.98	0.5000	0.4946	1.08	0.3000	0.2924	2.53
127.5	0.5197	0.4937	5.01	0.3649	0.3666	0.47	0.2654	0.2581	2.75
135	0.5180	0.5066	2.20	0.2474	0.2393	3.29	0.2296	0.2238	2.55
142.5	0.4905	0.4689	4.42	0.1523	0.1561	2.46	0.1929	0.1875	2.79
150	0.4356	0.4303	1.22	0.0820	0.0738	10.05	0.1553	0.1512	2.62
157.5	0.3542	0.3394	4.17	0.0360	0.0394	9.57	0.1171	0.1137	2.90
165	0.2501	0.2478	0.90	0.0110	0.0058	46.99	0.0783	0.0761	2.84
172.5	0.1294	0.1235	4.58	0.0014	0.0028	99.85	0.0392	0.0379	3.47
180	0.0000	0.0000	Inf.	0.0000	-0.0016	Inf.	0.0000	-0.0005	Inf.

Table 5.7 Percent different between Williams formula and FEM, $a/W=0.6$, $t/W=0.5$,
Uniform Load, SECP

r/a	0°(%)			30°(%)			60°(%)		
	f_{11}	f_{22}	f_{33}	f_{11}	f_{22}	f_{33}	f_{11}	f_{22}	f_{33}
0.0044	1.84	3.56	1.64	3.03	2.54	1.61	7.74	1.30	1.52
0.0054	2.12	3.85	1.82	3.44	2.77	1.79	8.62	1.46	1.70
0.0066	2.45	4.20	2.03	3.91	3.04	2.00	9.56	1.67	1.91
0.0079	2.82	4.58	2.26	4.42	3.35	2.23	10.56	1.90	2.13
0.0094	3.22	5.00	2.52	4.95	3.68	2.48	11.60	2.17	2.37
0.0109	3.65	5.44	2.79	5.51	4.05	2.75	12.68	2.44	2.62
0.0126	4.10	5.92	3.08	6.10	4.44	3.04	13.81	2.73	2.89
0.0144	4.58	6.43	3.40	6.73	4.86	3.34	15.04	3.04	3.18
0.0164	5.09	6.97	3.73	7.41	5.29	3.67	16.31	3.37	3.48
0.0234	6.82	8.82	4.88	9.66	6.82	4.79	20.63	4.49	4.50
r/a	90°(%)			120°(%)			150°(%)		
	f_{11}	f_{22}	f_{33}	f_{11}	f_{22}	f_{33}	f_{11}	f_{22}	f_{33}
0.0044	9.41	1.30	1.41	4.26	4.58	1.31	3.21	34.18	1.25
0.0054	10.45	1.51	1.59	4.94	5.04	1.48	3.92	36.98	1.40
0.0066	11.56	1.77	1.79	5.68	5.56	1.67	4.66	39.79	1.56
0.0079	12.74	2.04	2.00	6.46	6.08	1.85	5.42	42.59	1.72
0.0094	14.02	2.32	2.22	7.28	6.60	2.04	6.20	45.39	1.88
0.0109	15.32	2.61	2.45	8.14	7.12	2.24	7.02	48.32	2.04
0.0126	16.73	2.91	2.69	9.04	7.66	2.45	7.87	51.12	2.20
0.0144	18.20	3.23	2.94	10.00	8.20	2.66	8.77	54.17	2.37
0.0164	19.76	3.55	3.21	11.02	8.76	2.88	9.71	57.10	2.55
0.0234	25.02	4.63	4.09	14.44	10.58	3.59	12.85	66.60	3.08

Table 5.8 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, Uniform, SECP

$\theta(^{\circ})$	$f_{11}(\theta)$					$f_{22}(\theta)$					$f_{33}(\theta)$				
	Eq.(5.2)	$z/t=0$	0.1311	0.2702	0.3861	Eq.(5.2)	$z/t=0$	0.1311	0.2702	0.3861	Eq.(5.2)	$z/t=0$	0.1311	0.2702	0.3861
0	1.0000	0.9668	0.9661	0.9619	0.9434	1.0000	0.9468	0.9459	0.9408	0.9231	0.6000	0.5629	0.5607	0.5505	0.5248
7.5	0.9851	0.9371	0.9364	0.9322	0.9139	1.0106	0.9680	0.9670	0.9620	0.9443	0.5987	0.5605	0.5584	0.5484	0.5232
15	0.9419	0.9067	0.9060	0.9018	0.8835	1.0410	0.9900	0.9891	0.9841	0.9664	0.5949	0.5582	0.5561	0.5463	0.5216
22.5	0.8745	0.8279	0.8272	0.8230	0.8049	1.0871	1.0436	1.0427	1.0378	1.0203	0.5885	0.5513	0.5493	0.5400	0.5169
30	0.7891	0.7487	0.7480	0.7437	0.7258	1.1427	1.0977	1.0968	1.0920	1.0747	0.5796	0.5444	0.5425	0.5337	0.5121
37.5	0.6938	0.6494	0.6487	0.6443	0.6265	1.2000	1.1554	1.1546	1.1500	1.1331	0.5682	0.5329	0.5312	0.5232	0.5043
45	0.5972	0.5502	0.5495	0.5450	0.5273	1.2505	1.2132	1.2124	1.2080	1.1915	0.5543	0.5215	0.5199	0.5127	0.4964
52.5	0.5078	0.4651	0.4643	0.4596	0.4419	1.2859	1.2412	1.2405	1.2363	1.2207	0.5381	0.5056	0.5043	0.4982	0.4854
60	0.4330	0.3805	0.3796	0.3749	0.3571	1.2990	1.2688	1.2681	1.2642	1.2493	0.5196	0.4898	0.4887	0.4837	0.4744
67.5	0.3784	0.3360	0.3351	0.3301	0.3120	1.2845	1.2415	1.2409	1.2373	1.2237	0.4989	0.4698	0.4690	0.4653	0.4603
75	0.3472	0.2923	0.2913	0.2862	0.2678	1.2396	1.2135	1.2129	1.2097	1.1972	0.4760	0.4499	0.4493	0.4469	0.4462
82.5	0.3397	0.2960	0.2950	0.2897	0.2708	1.1640	1.1245	1.1240	1.1212	1.1103	0.4511	0.4260	0.4257	0.4248	0.4291
90	0.3536	0.3005	0.2994	0.2939	0.2745	1.0607	1.0349	1.0345	1.0320	1.0226	0.4243	0.4021	0.4021	0.4026	0.4119
97.5	0.3839	0.3381	0.3371	0.3314	0.3114	0.9348	0.9000	0.8996	0.8976	0.8898	0.3956	0.3746	0.3749	0.3770	0.3917
105	0.4239	0.3762	0.3751	0.3693	0.3487	0.7936	0.7648	0.7645	0.7628	0.7566	0.3653	0.3471	0.3478	0.3514	0.3714
112.5	0.4655	0.4178	0.4167	0.4108	0.3899	0.6457	0.6157	0.6154	0.6141	0.6092	0.3333	0.3165	0.3174	0.3226	0.3481
120	0.5000	0.4593	0.4581	0.4522	0.4309	0.5000	0.4667	0.4666	0.4656	0.4621	0.3000	0.2858	0.2871	0.2938	0.3247
127.5	0.5197	0.4716	0.4705	0.4646	0.4435	0.3649	0.3384	0.3383	0.3375	0.3350	0.2654	0.2524	0.2539	0.2620	0.2982
135	0.5180	0.4834	0.4823	0.4765	0.4555	0.2474	0.2107	0.2106	0.2101	0.2085	0.2296	0.2190	0.2208	0.2302	0.2716
142.5	0.4905	0.4443	0.4433	0.4377	0.4175	0.1523	0.1274	0.1273	0.1269	0.1258	0.1929	0.1833	0.1853	0.1958	0.2417
150	0.4356	0.4044	0.4034	0.3982	0.3787	0.0820	0.0450	0.0449	0.0446	0.0440	0.1553	0.1476	0.1498	0.1615	0.2119
157.5	0.3542	0.3122	0.3113	0.3065	0.2883	0.0360	0.0105	0.0105	0.0102	0.0096	0.1171	0.1101	0.1125	0.1249	0.1784
165	0.2501	0.2192	0.2184	0.2141	0.1973	0.0110	-0.0232	-0.0232	-0.0235	-0.0240	0.0783	0.0727	0.0752	0.0883	0.1450
172.5	0.1294	0.0935	0.0928	0.0891	0.0743	0.0014	-0.0262	-0.0263	-0.0265	-0.0271	0.0392	0.0341	0.0366	0.0500	0.1079
180	0.0000	-0.0313	-0.0319	-0.0350	-0.0478	0.0000	-0.0307	-0.0307	-0.0310	-0.0316	0.0000	-0.0046	-0.0020	0.0115	0.0706

Table 5.9 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, Uniform, SECP

$\theta(^{\circ})$	$f_{11}(\theta)$					$f_{22}(\theta)$					$f_{33}(\theta)$				
	Eq.(5.2)	$z/t=0$	0.1228	0.2627	0.4244	Eq.(5.2)	$z/t=0$	0.1228	0.2627	0.4244	Eq.(5.2)	$z/t=0$	0.1228	0.2627	0.4244
0	1.0000	0.9674	0.9671	0.9650	0.9464	1.0000	0.9490	0.9485	0.9459	0.9254	0.6000	0.5713	0.5704	0.5659	0.5330
7.5	0.9851	0.9378	0.9374	0.9353	0.9168	1.0106	0.9702	0.9697	0.9671	0.9467	0.5987	0.5688	0.5679	0.5635	0.5312
15	0.9419	0.9075	0.9071	0.9049	0.8863	1.0410	0.9922	0.9917	0.9891	0.9689	0.5949	0.5663	0.5655	0.5612	0.5293
22.5	0.8745	0.8288	0.8284	0.8262	0.8075	1.0871	1.0457	1.0452	1.0427	1.0229	0.5885	0.5590	0.5582	0.5540	0.5238
30	0.7891	0.7496	0.7492	0.7470	0.7282	1.1427	1.0997	1.0992	1.0968	1.0774	0.5796	0.5517	0.5509	0.5469	0.5183
37.5	0.6938	0.6505	0.6501	0.6478	0.6288	1.2000	1.1574	1.1569	1.1546	1.1360	0.5682	0.5395	0.5388	0.5352	0.5091
45	0.5972	0.5514	0.5510	0.5487	0.5294	1.2505	1.2151	1.2146	1.2124	1.1946	0.5543	0.5274	0.5268	0.5234	0.4999
52.5	0.5078	0.4665	0.4660	0.4636	0.4440	1.2859	1.2430	1.2425	1.2405	1.2238	0.5381	0.5107	0.5102	0.5072	0.4872
60	0.4330	0.3821	0.3816	0.3791	0.3591	1.2990	1.2704	1.2700	1.2681	1.2525	0.5196	0.4941	0.4936	0.4910	0.4744
67.5	0.3784	0.3377	0.3373	0.3347	0.3142	1.2845	1.2429	1.2426	1.2409	1.2267	0.4989	0.4730	0.4726	0.4705	0.4581
75	0.3472	0.2943	0.2938	0.2911	0.2702	1.2396	1.2147	1.2144	1.2129	1.2001	0.4760	0.4520	0.4517	0.4501	0.4419
82.5	0.3397	0.2982	0.2977	0.2950	0.2735	1.1640	1.1257	1.1254	1.1240	1.1127	0.4511	0.4270	0.4268	0.4257	0.4222
90	0.3536	0.3029	0.3024	0.2996	0.2776	1.0607	1.0358	1.0356	1.0344	1.0247	0.4243	0.4020	0.4019	0.4013	0.4026
97.5	0.3839	0.3408	0.3403	0.3375	0.3151	0.9348	0.9008	0.9006	0.8996	0.8914	0.3956	0.3734	0.3734	0.3733	0.3796
105	0.4239	0.3790	0.3785	0.3757	0.3530	0.7936	0.7654	0.7652	0.7644	0.7577	0.3653	0.3448	0.3448	0.3454	0.3567
112.5	0.4655	0.4208	0.4203	0.4175	0.3947	0.6457	0.6162	0.6160	0.6153	0.6099	0.3333	0.3130	0.3132	0.3142	0.3305
120	0.5000	0.4624	0.4619	0.4591	0.4364	0.5000	0.4671	0.4670	0.4664	0.4623	0.3000	0.2812	0.2815	0.2831	0.3043
127.5	0.5197	0.4748	0.4743	0.4717	0.4495	0.3649	0.3387	0.3385	0.3381	0.3349	0.2654	0.2469	0.2472	0.2492	0.2751
135	0.5180	0.4866	0.4861	0.4836	0.4619	0.2474	0.2109	0.2108	0.2104	0.2081	0.2296	0.2125	0.2129	0.2153	0.2458
142.5	0.4905	0.4475	0.4470	0.4447	0.4242	0.1523	0.1275	0.1274	0.1271	0.1253	0.1929	0.1760	0.1765	0.1793	0.2135
150	0.4356	0.4075	0.4071	0.4049	0.3857	0.0820	0.0450	0.0449	0.0446	0.0433	0.1553	0.1396	0.1401	0.1433	0.1813
157.5	0.3542	0.3151	0.3147	0.3128	0.2954	0.0360	0.0105	0.0105	0.0102	0.0089	0.1171	0.1017	0.1023	0.1057	0.1462
165	0.2501	0.2220	0.2217	0.2200	0.2045	0.0110	-0.0232	-0.0233	-0.0235	-0.0246	0.0783	0.0638	0.0644	0.0680	0.1111
172.5	0.1294	0.0960	0.0957	0.0944	0.0813	0.0014	-0.0263	-0.0263	-0.0266	-0.0277	0.0392	0.0251	0.0257	0.0294	0.0733
180	0.0000	-0.0291	-0.0293	-0.0304	-0.0410	0.0000	-0.0307	-0.0308	-0.0310	-0.0322	0.0000	-0.0138	-0.0131	-0.0093	0.0354

Table 5.10 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, Uniform, SECP

$\theta(^{\circ})$	$f_{11}(\theta)$					$f_{22}(\theta)$					$f_{33}(\theta)$				
	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182
0	1.0000	0.9974	0.9677	0.9665	0.9592	1.0000	0.9796	0.9498	0.9486	0.9401	0.6000	0.5926	0.5746	0.5731	0.5618
7.5	0.9851	0.9678	0.9381	0.9369	0.9295	1.0106	1.0008	0.9710	0.9697	0.9614	0.5987	0.5901	0.5721	0.5705	0.5594
15	0.9419	0.9374	0.9077	0.9066	0.8991	1.0410	1.0227	0.9930	0.9917	0.9834	0.5949	0.5876	0.5696	0.5681	0.5571
22.5	0.8745	0.8588	0.8291	0.8279	0.8203	1.0871	1.0762	1.0464	1.0452	1.0372	0.5885	0.5800	0.5621	0.5606	0.5500
30	0.7891	0.7796	0.7500	0.7488	0.7411	1.1427	1.1302	1.1004	1.0993	1.0915	0.5796	0.5725	0.5546	0.5532	0.5430
37.5	0.6938	0.6805	0.6509	0.6497	0.6418	1.2000	1.1879	1.1581	1.1569	1.1495	0.5682	0.5601	0.5422	0.5409	0.5314
45	0.5972	0.5815	0.5519	0.5507	0.5427	1.2505	1.2455	1.2157	1.2146	1.2075	0.5543	0.5478	0.5298	0.5286	0.5198
52.5	0.5078	0.4966	0.4670	0.4658	0.4575	1.2859	1.2733	1.2435	1.2425	1.2359	0.5381	0.5307	0.5128	0.5117	0.5038
60	0.4330	0.4123	0.3827	0.3814	0.3730	1.2990	1.3007	1.2708	1.2699	1.2638	0.5196	0.5136	0.4958	0.4948	0.4878
67.5	0.3784	0.3680	0.3384	0.3372	0.3286	1.2845	1.2732	1.2434	1.2425	1.2369	0.4989	0.4922	0.4743	0.4735	0.4676
75	0.3472	0.3246	0.2951	0.2938	0.2850	1.2396	1.2449	1.2151	1.2142	1.2092	0.4760	0.4708	0.4529	0.4522	0.4474
82.5	0.3397	0.3286	0.2991	0.2978	0.2889	1.1640	1.1556	1.1259	1.1251	1.1207	0.4511	0.4453	0.4275	0.4269	0.4232
90	0.3536	0.3334	0.3039	0.3025	0.2936	1.0607	1.0657	1.0360	1.0353	1.0314	0.4243	0.4198	0.4020	0.4016	0.3991
97.5	0.3839	0.3714	0.3418	0.3405	0.3316	0.9348	0.9305	0.9009	0.9003	0.8968	0.3956	0.3907	0.3730	0.3726	0.3714
105	0.4239	0.4097	0.3801	0.3788	0.3699	0.7936	0.7950	0.7654	0.7649	0.7619	0.3653	0.3616	0.3439	0.3437	0.3437
112.5	0.4655	0.4515	0.4219	0.4206	0.4119	0.6457	0.6457	0.6161	0.6156	0.6130	0.3333	0.3294	0.3117	0.3117	0.3128
120	0.5000	0.4931	0.4635	0.4622	0.4538	0.5000	0.4965	0.4670	0.4665	0.4643	0.3000	0.2972	0.2796	0.2796	0.2820
127.5	0.5197	0.5055	0.4759	0.4747	0.4667	0.3649	0.3680	0.3385	0.3381	0.3362	0.2654	0.2624	0.2448	0.2450	0.2484
135	0.5180	0.5173	0.4877	0.4866	0.4789	0.2474	0.2401	0.2107	0.2103	0.2087	0.2296	0.2277	0.2101	0.2104	0.2148
142.5	0.4905	0.4781	0.4485	0.4475	0.4405	0.1523	0.1566	0.1273	0.1269	0.1254	0.1929	0.1909	0.1733	0.1738	0.1790
150	0.4356	0.4380	0.4085	0.4075	0.4012	0.0820	0.0741	0.0447	0.0444	0.0430	0.1553	0.1542	0.1366	0.1371	0.1432
157.5	0.3542	0.3454	0.3159	0.3151	0.3096	0.0360	0.0396	0.0103	0.0099	0.0086	0.1171	0.1161	0.0986	0.0991	0.1057
165	0.2501	0.2522	0.2227	0.2220	0.2172	0.0110	0.0058	-0.0235	-0.0238	-0.0250	0.0783	0.0780	0.0605	0.0611	0.0683
172.5	0.1294	0.1260	0.0966	0.0960	0.0922	0.0014	0.0028	-0.0265	-0.0268	-0.0281	0.0392	0.0392	0.0217	0.0224	0.0298
180	0.0000	0.0007	-0.0287	-0.0291	-0.0320	0.0000	-0.0017	-0.0310	-0.0313	-0.0326	0.0000	0.0003	-0.0172	-0.0165	-0.0089

Table 5.11 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=1.0$, $r/a=0.0078$, Uniform, SECP

$\theta(^{\circ})$	$f_{11}(\theta)$					$f_{22}(\theta)$					$f_{33}(\theta)$				
	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182
0	1.0000	0.9975	0.9673	0.9669	0.9631	1.0000	0.9799	0.9497	0.9492	0.9449	0.6000	0.5932	0.5751	0.5746	0.5699
7.5	0.9851	0.9679	0.9377	0.9373	0.9334	1.0106	1.0010	0.9708	0.9704	0.9661	0.5987	0.5907	0.5726	0.5721	0.5675
15	0.9419	0.9375	0.9074	0.9070	0.9031	1.0410	1.0230	0.9928	0.9923	0.9881	0.5949	0.5882	0.5701	0.5696	0.5650
22.5	0.8745	0.8588	0.8287	0.8283	0.8244	1.0871	1.0765	1.0463	1.0458	1.0417	0.5885	0.5806	0.5625	0.5620	0.5576
30	0.7891	0.7797	0.7497	0.7493	0.7452	1.1427	1.1305	1.1003	1.0998	1.0958	0.5796	0.5731	0.5550	0.5545	0.5503
37.5	0.6938	0.6806	0.6506	0.6502	0.6461	1.2000	1.1881	1.1579	1.1574	1.1536	0.5682	0.5607	0.5426	0.5421	0.5381
45	0.5972	0.5816	0.5516	0.5512	0.5470	1.2505	1.2457	1.2155	1.2150	1.2113	0.5543	0.5482	0.5301	0.5297	0.5260
52.5	0.5078	0.4967	0.4668	0.4663	0.4621	1.2859	1.2736	1.2433	1.2429	1.2394	0.5381	0.5311	0.5130	0.5126	0.5092
60	0.4330	0.4124	0.3825	0.3820	0.3777	1.2990	1.3009	1.2706	1.2702	1.2669	0.5196	0.5140	0.4959	0.4956	0.4925
67.5	0.3784	0.3682	0.3383	0.3378	0.3334	1.2845	1.2733	1.2431	1.2427	1.2397	0.4989	0.4925	0.4744	0.4741	0.4714
75	0.3472	0.3248	0.2949	0.2944	0.2900	1.2396	1.2450	1.2148	1.2144	1.2116	0.4760	0.4710	0.4529	0.4526	0.4503
82.5	0.3397	0.3289	0.2989	0.2985	0.2940	1.1640	1.1558	1.1256	1.1252	1.1227	0.4511	0.4454	0.4274	0.4271	0.4252
90	0.3536	0.3337	0.3038	0.3033	0.2987	1.0607	1.0658	1.0356	1.0353	1.0330	0.4243	0.4199	0.4018	0.4016	0.4001
97.5	0.3839	0.3716	0.3417	0.3412	0.3367	0.9348	0.9307	0.9005	0.9002	0.8981	0.3956	0.3907	0.3727	0.3725	0.3714
105	0.4239	0.4100	0.3800	0.3795	0.3751	0.7936	0.7951	0.7650	0.7647	0.7628	0.3653	0.3615	0.3435	0.3434	0.3428
112.5	0.4655	0.4518	0.4218	0.4213	0.4170	0.6457	0.6457	0.6157	0.6154	0.6137	0.3333	0.3292	0.3113	0.3112	0.3109
120	0.5000	0.4934	0.4634	0.4629	0.4587	0.5000	0.4965	0.4666	0.4663	0.4647	0.3000	0.2970	0.2790	0.2790	0.2791
127.5	0.5197	0.5058	0.4758	0.4754	0.4713	0.3649	0.3680	0.3381	0.3378	0.3364	0.2654	0.2621	0.2442	0.2442	0.2447
135	0.5180	0.5176	0.4876	0.4871	0.4833	0.2474	0.2401	0.2102	0.2100	0.2087	0.2296	0.2273	0.2094	0.2094	0.2103
142.5	0.4905	0.4784	0.4484	0.4480	0.4444	0.1523	0.1567	0.1268	0.1266	0.1253	0.1929	0.1905	0.1726	0.1726	0.1738
150	0.4356	0.4383	0.4084	0.4079	0.4047	0.0820	0.0741	0.0443	0.0440	0.0428	0.1553	0.1537	0.1358	0.1359	0.1373
157.5	0.3542	0.3457	0.3158	0.3154	0.3126	0.0360	0.0396	0.0098	0.0096	0.0084	0.1171	0.1156	0.0977	0.0978	0.0995
165	0.2501	0.2524	0.2225	0.2222	0.2197	0.0110	0.0058	-0.0239	-0.0242	-0.0253	0.0783	0.0775	0.0596	0.0597	0.0616
172.5	0.1294	0.1262	0.0964	0.0960	0.0940	0.0014	0.0028	-0.0270	-0.0272	-0.0284	0.0392	0.0387	0.0208	0.0209	0.0229
180	0.0000	0.0008	-0.0289	-0.0292	-0.0309	0.0000	-0.0017	-0.0315	-0.0317	-0.0328	0.0000	-0.0003	-0.0181	-0.0180	-0.0159

Table 5.12 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, Uniform, SECP

$\theta(^{\circ})$	$f_{11}(\theta)$					$f_{22}(\theta)$					$f_{33}(\theta)$				
	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182
0	1.0000	0.9974	0.9667	0.9668	0.9655	1.0000	0.9798	0.9491	0.9492	0.9477	0.6000	0.5932	0.5748	0.5748	0.5734
7.5	0.9851	0.9678	0.9371	0.9372	0.9359	1.0106	1.0009	0.9703	0.9703	0.9688	0.5987	0.5906	0.5722	0.5723	0.5709
15	0.9419	0.9374	0.9068	0.9068	0.9056	1.0410	1.0229	0.9922	0.9923	0.9908	0.5949	0.5881	0.5697	0.5698	0.5684
22.5	0.8745	0.8588	0.8281	0.8282	0.8269	1.0871	1.0764	1.0457	1.0457	1.0443	0.5885	0.5806	0.5622	0.5622	0.5609
30	0.7891	0.7796	0.7491	0.7491	0.7478	1.1427	1.1304	1.0997	1.0997	1.0983	0.5796	0.5730	0.5547	0.5547	0.5534
37.5	0.6938	0.6806	0.6500	0.6501	0.6487	1.2000	1.1880	1.1573	1.1573	1.1560	0.5682	0.5606	0.5422	0.5423	0.5411
45	0.5972	0.5815	0.5510	0.5511	0.5497	1.2505	1.2457	1.2149	1.2149	1.2136	0.5543	0.5482	0.5298	0.5299	0.5287
52.5	0.5078	0.4966	0.4662	0.4662	0.4649	1.2859	1.2735	1.2427	1.2428	1.2415	0.5381	0.5311	0.5127	0.5127	0.5117
60	0.4330	0.4123	0.3819	0.3819	0.3805	1.2990	1.3008	1.2700	1.2701	1.2689	0.5196	0.5140	0.4956	0.4956	0.4947
67.5	0.3784	0.3681	0.3377	0.3377	0.3363	1.2845	1.2733	1.2425	1.2425	1.2414	0.4989	0.4924	0.4741	0.4741	0.4733
75	0.3472	0.3247	0.2943	0.2944	0.2929	1.2396	1.2450	1.2142	1.2142	1.2132	0.4760	0.4709	0.4526	0.4526	0.4519
82.5	0.3397	0.3288	0.2983	0.2984	0.2969	1.1640	1.1558	1.1250	1.1250	1.1240	0.4511	0.4454	0.4270	0.4271	0.4264
90	0.3536	0.3336	0.3032	0.3032	0.3017	1.0607	1.0658	1.0351	1.0351	1.0342	0.4243	0.4198	0.4015	0.4015	0.4010
97.5	0.3839	0.3716	0.3411	0.3411	0.3397	0.9348	0.9306	0.9000	0.9000	0.8991	0.3956	0.3907	0.3723	0.3724	0.3720
105	0.4239	0.4099	0.3795	0.3795	0.3780	0.7936	0.7951	0.7645	0.7645	0.7636	0.3653	0.3615	0.3432	0.3432	0.3430
112.5	0.4655	0.4517	0.4212	0.4213	0.4198	0.6457	0.6457	0.6152	0.6152	0.6144	0.3333	0.3292	0.3109	0.3110	0.3108
120	0.5000	0.4933	0.4629	0.4629	0.4614	0.5000	0.4965	0.4660	0.4660	0.4653	0.3000	0.2970	0.2787	0.2787	0.2786
127.5	0.5197	0.5058	0.4753	0.4753	0.4739	0.3649	0.3680	0.3375	0.3375	0.3368	0.2654	0.2621	0.2438	0.2439	0.2439
135	0.5180	0.5175	0.4870	0.4871	0.4857	0.2474	0.2401	0.2097	0.2097	0.2090	0.2296	0.2273	0.2090	0.2091	0.2092
142.5	0.4905	0.4783	0.4479	0.4479	0.4466	0.1523	0.1567	0.1263	0.1263	0.1256	0.1929	0.1905	0.1722	0.1723	0.1725
150	0.4356	0.4383	0.4078	0.4078	0.4066	0.0820	0.0741	0.0437	0.0437	0.0431	0.1553	0.1537	0.1355	0.1355	0.1358
157.5	0.3542	0.3457	0.3153	0.3153	0.3142	0.0360	0.0396	0.0093	0.0092	0.0086	0.1171	0.1156	0.0974	0.0974	0.0977
165	0.2501	0.2524	0.2220	0.2220	0.2210	0.0110	0.0058	-0.0245	-0.0245	-0.0251	0.0783	0.0775	0.0593	0.0593	0.0597
172.5	0.1294	0.1262	0.0958	0.0958	0.0949	0.0014	0.0028	-0.0275	-0.0276	-0.0282	0.0392	0.0387	0.0205	0.0205	0.0209
180	0.0000	0.0009	-0.0295	-0.0295	-0.0303	0.0000	-0.0017	-0.0320	-0.0320	-0.0326	0.0000	-0.0002	-0.0184	-0.0184	-0.0180

Table 5.13 Stress State along the circular path close to the front varied along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, Uniform, SECP

$\theta(^{\circ})$	$f_{11}(\theta)$					$f_{22}(\theta)$					$f_{33}(\theta)$				
	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182	Eq.(5.2)	$z/t=0$	0.1161	0.2637	0.4182
0	1.0000	0.9974	0.9663	0.9664	0.9664	1.0000	1.0009	0.9487	0.9488	0.9488	0.6000	0.5931	0.5745	0.5746	0.5742
7.5	0.9851	0.9678	0.9367	0.9368	0.9368	1.0106	1.0229	0.9699	0.9700	0.9699	0.5987	0.5906	0.5720	0.5720	0.5717
15	0.9419	0.9374	0.9064	0.9065	0.9065	1.0410	1.0764	0.9918	0.9919	0.9919	0.5949	0.5881	0.5695	0.5695	0.5692
22.5	0.8745	0.8587	0.8278	0.8278	0.8278	1.0871	1.1304	1.0453	1.0454	1.0453	0.5885	0.5805	0.5619	0.5620	0.5616
30	0.7891	0.7796	0.7487	0.7488	0.7488	1.1427	1.1880	1.0993	1.0994	1.0993	0.5796	0.5730	0.5544	0.5544	0.5541
37.5	0.6938	0.6805	0.6496	0.6497	0.6497	1.2000	1.2457	1.1569	1.1570	1.1569	0.5682	0.5606	0.5420	0.5420	0.5417
45	0.5972	0.5815	0.5506	0.5507	0.5507	1.2505	1.2735	1.2145	1.2146	1.2145	0.5543	0.5482	0.5295	0.5296	0.5293
52.5	0.5078	0.4966	0.4658	0.4659	0.4658	1.2859	1.3008	1.2423	1.2424	1.2424	0.5381	0.5310	0.5124	0.5125	0.5122
60	0.4330	0.4123	0.3815	0.3816	0.3815	1.2990	1.2733	1.2696	1.2697	1.2697	0.5196	0.5139	0.4953	0.4954	0.4951
67.5	0.3784	0.3681	0.3373	0.3374	0.3373	1.2845	1.2450	1.2421	1.2422	1.2421	0.4989	0.4924	0.4738	0.4739	0.4736
75	0.3472	0.3247	0.2939	0.2940	0.2940	1.2396	1.1558	1.2138	1.2139	1.2138	0.4760	0.4709	0.4523	0.4524	0.4521
82.5	0.3397	0.3288	0.2980	0.2980	0.2980	1.1640	1.0658	1.1247	1.1247	1.1246	0.4511	0.4454	0.4268	0.4268	0.4266
90	0.3536	0.3336	0.3028	0.3029	0.3028	1.0607	0.9306	1.0347	1.0348	1.0347	0.4243	0.4198	0.4012	0.4013	0.4010
97.5	0.3839	0.3715	0.3407	0.3408	0.3408	0.9348	0.7951	0.8996	0.8997	0.8996	0.3956	0.3906	0.3721	0.3721	0.3719
105	0.4239	0.4099	0.3791	0.3792	0.3791	0.7936	0.6457	0.7641	0.7642	0.7641	0.3653	0.3615	0.3429	0.3430	0.3427
112.5	0.4655	0.4517	0.4208	0.4209	0.4209	0.6457	0.4965	0.6148	0.6149	0.6148	0.3333	0.3292	0.3107	0.3107	0.3105
120	0.5000	0.4933	0.4625	0.4625	0.4625	0.5000	0.3680	0.4657	0.4657	0.4656	0.3000	0.2969	0.2784	0.2784	0.2782
127.5	0.5197	0.5057	0.4749	0.4750	0.4749	0.3649	0.2401	0.3372	0.3372	0.3371	0.2654	0.2621	0.2436	0.2436	0.2434
135	0.5180	0.5175	0.4867	0.4867	0.4866	0.2474	0.1567	0.2093	0.2094	0.2093	0.2296	0.2273	0.2088	0.2088	0.2086
142.5	0.4905	0.4783	0.4475	0.4475	0.4474	0.1523	0.0741	0.1259	0.1260	0.1259	0.1929	0.1905	0.1720	0.1720	0.1718
150	0.4356	0.4383	0.4074	0.4075	0.4074	0.0820	0.0396	0.0434	0.0434	0.0433	0.1553	0.1537	0.1352	0.1352	0.1350
157.5	0.3542	0.3457	0.3149	0.3150	0.3148	0.0360	0.0058	0.0089	0.0089	0.0089	0.1171	0.1156	0.0971	0.0971	0.0969
165	0.2501	0.2524	0.2216	0.2217	0.2216	0.0110	0.0028	-0.0248	-0.0248	-0.0249	0.0783	0.0775	0.0590	0.0590	0.0588
172.5	0.1294	0.1262	0.0955	0.0955	0.0954	0.0014	-0.0017	-0.0279	-0.0279	-0.0279	0.0392	0.0387	0.0203	0.0203	0.0201
180	0.0000	0.0008	-0.0298	-0.0298	-0.0299	0.0000	0.0000	-0.0324	-0.0323	-0.0324	0.0000	-0.0003	-0.0187	-0.0187	-0.0189

Figure 5.1 Coordinate system for stress state of crack front

Figure 5.2 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,
 $t/W=0.1$

Figure 5.3 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,
 $t/W=0.2$

Figure 5.4 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,
 $t/W=0.5$

Figure 5.5 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,
 $t/W=1.0$

Figure 5.6 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,
 $t/W=2.0$

Figure 5.7 Stress State along the circular path close to the front at $z/t=0$, $a/W=0.8$,
 $t/W=4.0$

Figure 5.8 Percentage different of $f_{11}(\theta)$ varied by angles at three radiuses location, $a/W=0.8$, Uniform

Figure 5.9 Percentage different of $f_{22}(\theta)$ varied by angles at three radiuses location, $a/W=0.8$, Uniform: (a) Degree from 0° to 135° , (b) Degree from 0° to 165°

Figure 5.10 Percentage different of $f_{33}(\theta)$ varied by angles at three radiuses location, $a/W=0.8$, Uniform

Figure 5.11 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{II} , Angle= 0° , Uniform, SECP

Figure 5.12 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{II} , Angle= 30° , Uniform, SECP

Figure 5.13 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{II} , Angle= 60° , Uniform, SECP

Figure 5.14 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{II}, \text{ Angle}=90^\circ, \text{ Uniform, SECP}$

Figure 5.15 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{II}, \text{ Angle}=120^\circ, \text{ Uniform, SECP}$

Figure 5.16 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{II} , Angle=150°, Uniform, SECP

Figure 5.17 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{II} , Angle=180°, Uniform, SECP

Figure 5.18 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{22}, \text{Angle}=0^\circ, \text{Uniform, SECP}$

Figure 5.19 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{22}, \text{Angle}=30^\circ, \text{Uniform, SECP}$

Figure 5.20 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{22}, \text{ Angle}=60^\circ, \text{ Uniform, SECP}$

Figure 5.21 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{22}, \text{ Angle}=90^\circ, \text{ Uniform, SECP}$

Figure 5.22 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{22}, \text{ Angle}=120^\circ, \text{ Uniform, SECP}$

Figure 5.23 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{22}, \text{ Angle}=150^\circ, \text{ Uniform, SECP}$

Figure 5.24 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{22} , Angle=180°, Uniform, SECP

Figure 5.25 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle=0°, Uniform, SECP

Figure 5.26 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 30° , Uniform, SECP

Figure 5.27 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle= 60° , Uniform, SECP

Figure 5.28 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{33}, \text{ Angle}=90^\circ, \text{ Uniform, SECP}$

Figure 5.29 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6, t/W=0.5, f_{33}, \text{ Angle}=120^\circ, \text{ Uniform, SECP}$

Figure 5.30 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle=150°, Uniform, SECP

Figure 5.31 Stress State along the circular path close to the front varied by radius at four locations along thickness, $a/W=0.6$, $t/W=0.5$, f_{33} , Angle=180°, Uniform, SECP

Figure 5.32 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 0°

Figure 5.33 Percentage different varied by radius, $a/W=0.6$, $t/W=0.5$, Uniform, 30°

Figure 5.34 Percentage different varied by radius, $a/W=0.6, t/W=0.5$, Uniform, 60°

Figure 5.35 Percentage different varied by radius, $a/W=0.6, t/W=0.5$, Uniform, 90°

Figure 5.36 Percentage different varied by radius, $a/W=0.6, t/W=0.5$, Uniform, 120°

Figure 5.37 Percentage different varied by radius, $a/W=0.6, t/W=0.5$, Uniform, 150°

Figure 5.38 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, f_{11} , Uniform, SECP

Figure 5.39 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, f_{22} , Uniform, SECP

Figure 5.40 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.1$, $r/a=0.0078$, f_{33} , Uniform, SECP

Figure 5.41 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, f_{11} , Uniform, SECP

Figure 5.42 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, f_{22} , Uniform, SECP

Figure 5.43 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.2$, $r/a=0.0078$, f_{33} , Uniform, SECP

Figure 5.44 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, f_{11} , Uniform, SECP

Figure 5.45 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, f_{22} , Uniform, SECP

Figure 5.46 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=0.5$, $r/a=0.0078$, f_{33} , Uniform, SECP

Figure 5.47 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=1.0$, $r/a=0.0078$, f_{11} , Uniform, SECP

Figure 5.48 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6, t/W=1.0, r/a=0.0078, f_{22}$, Uniform, SECP

Figure 5.49 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6, t/W=1.0, r/a=0.0078, f_{33}$, Uniform, SECP

Figure 5.50 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, f_{11} , Uniform, SECP

Figure 5.51 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, f_{22} , Uniform, SECP

Figure 5.52 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=2.0$, $r/a=0.0078$, f_{33} , Uniform, SECP

Figure 5.53 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, f_{11} , Uniform, SECP

Figure 5.54 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, f_{22} , Uniform, SECP

Figure 5.55 Stress State along the circular path close to the front varied by four locations along thickness, $a/W=0.6$, $t/W=4.0$, $r/a=0.0078$, f_{33} , Uniform, SECP

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The three-dimensional finite element analyses for a single edge cracked plate specimens with the crack surface subjected to uniform, linear, parabolic or cubic stress distribution were conducted. The stress intensity factor K_I , T_{11} and T_{33} stresses along the crack front were determined for cracked geometries with the relative crack depth a/W of 0.2, 0.4, 0.6 and 0.8, and the relative thickness t/W of 0.1, 0.2, 0.5, 1.0, 2.0 and 4.0. For the convenience of engineering application, the empirical equations of the stress intensity factor K_I , T_{11} and T_{33} stresses at the mid-plane were given for uniform, linear, parabolic or cubic stress distribution applied on the crack surface.

The effect of the thickness on the stress intensity factor K_I , T_{11} and T_{33} stresses at mid-plane was discussed. Additional $t/W=0.0025, 0.025, 0.05, 8, 16$ cases subjected to the uniform stress were computed by finite element method in order to investigate the thickness effect. It is observed that the stress intensity factor K_I at the mid-plane decreases when the plate is thicker, and trends to two-dimensional solution when the thickness becomes large compared to the crack length. It is noticed that the T_{11} stress at mid-plane merge to two-dimensional solution when the thickness becomes large. A formula to predict the mid-plane T_{11} stress developed by Nakamura and Parks (1992) was used to analyze by the finite element results. Very good accuracy was achieved between the finite element analyses and the predicted formula. It shows that the T_{33} stress increases and approaches the two-dimensional solution as the plate thickness

increases, and the three-dimensional results of T_{33} stress gradually merge with the two-dimensional solution for thicker plate. Similar as the formula of T_{11} stress, a formula to predict the mid-plane T_{33} stress was proposed. The finite element analyses solution and the predicted formula of the mid-plane T_{33} stress were compared. Reasonably agreements were achieved.

The effects of crack depth were studied for the stress intensity factor K_I , T_{11} and T_{33} stresses at mid-plane for various relative crack depths at $a/W=0.2, 0.4, 0.6$ and 0.8 . It is noticed that the values of stress intensity factor all increase with increasing the crack depths whether the plane thickness is thick or thin. The T_{11} stress is increasing at mid-plane for deeper crack at all kinds of the thickness cases. And the T_{33} stress is increasing at mid-plane for deeper crack for larger thickness ($t/W>0.5$) and decreasing for smaller thickness ($t/W=0.2$).

The stress intensity factor K_I , T_{11} and T_{33} stresses varying along the crack front was investigated. For stress intensity factor K_I , the differences between the three-dimensional and two-dimensional solutions are within the range 0.95 to 1.07 for all cases of thickness, except in the small region near the free-surfaces. For T_{11} stress, the three-dimensional and two-dimensional solutions are can be significantly different in the range of 1.5 to 1.8. For T_{33} stress, three-dimensional solutions can be different than the two-dimensional in the factor of 10 to 50. So the three-dimensional T_{33} stresses solution must be applied to solve three-dimensional questions.

The stress state around the crack front in three dimensional SECP specimens is described by three parameters, stress intensity factor K_I , T_{11} and T_{33} stresses. The variation of the stress state of the Williams' expansion is compared with solutions of finite element analyses by angle, radius and thickness, respectively. And the percentages of difference between the finite element analyses and the Williams' expansion are also presented. It was observed the three parameters K_I - T_{11} - T_{33} give good approximation of stress state close to the crack front (when r/a is small).

6.2 Recommendations

In the present thesis, the stress intensity factor K_I , T_{11} and T_{33} stresses of the single edge cracked plate specimens were calculated and investigated. However, there are also the other geometries of common specimens need to be studied, such as centre edge through-thickness cracked plate specimen and double edge through-thickness cracked plate specimen. Furthermore, the corner crack or semi-elliptical surface crack in finite thickness plate is also need to be systematically studied.

The thinnest of the plate to calculate the stress intensity factor K_I , T_{11} and T_{33} stress at mid-plane is relative thickness $t/W=0.0025$, which the thinnest mesh by 3D solid element can be solved in ABAQUS for the models in this thesis. This leads to a limit to investigate the value of B^{thin} in the prediction formula for T_{11} stress or B_{33}^{thin} in the prediction formula for T_{33} stress. Only the prediction formula for T_{11} and T_{33} stresses at mid-plane is created, the other location of along the crack front should be analyzed in future. The prediction formula for T_{33} stress is based on the prediction formula for T_{11} . However, the accuracy is not as good as the one for T_{11} . Further study is required on this topic.

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