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**INVESTIGATION OF WOOD FLEXURAL MEMBERS
REINFORCED WITH GLASS FIBERS**

by

Jean-Guy Joseph Groulx, B. Eng

**A thesis submitted to the Faculty of Graduate Studies and Research in
partial fulfillment of the requirements for the Degree of**

MASTER OF ENGINEERING

Ottawa-Carleton Institute for Civil Engineering

Ottawa, Ontario

May, 1995

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**Investigation of Wood Flexural Members Reinforced with Glass
Fibers**

submitted by

Jean-Guy Joseph Groulx, B.Eng.

in partial fulfillment of the requirements for
the degree of Master of Engineering

Chair, Department of Civil and
Environmental Engineering

Thesis Supervisor

Carleton University
Ottawa, Ontario, Canada
May 1995

ABSTRACT

Despite a diminishing natural resource base, the demand for wood products continues to be strong primarily due to its aesthetic appeal, relatively low cost and ease of construction. To satisfy this demand, the lumber industry is continually seeking innovative approaches to use the available timber resources in a smarter, more efficient manner. In beam applications, the reinforcement of wood with fiber composite materials provides a structural system which delivers equivalent performance using smaller dimension lumber. Traditionally, this reinforcement has been applied in a continuous fashion for the full length of the beam due to its simplicity of design, however this configuration does not provide the optimum beam performance characteristics. In this thesis the effects of different reinforcement configurations are examined both theoretically and experimentally for solid wood and I-beams. A methodology is then developed to analyze the effects of reinforcement configuration on beam strength and stiffness for the general case of reinforced solid wood and I-beams.

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LIST OF SYMBOLS

A_b	Cross-sectional area of bottom flange
A_c	Cross-sectional area of the composite
A_f	Cross-sectional area of the fiber
A_{fpr}	Cross-sectional area of the fiber reinforced plastic
A_m	Cross-sectional area of the matrix
A_t	Cross-sectional area of top flange
A_w	Cross-sectional area of the wood
A_{web}	Cross-sectional area of the web
b	Width of the wood
c	position of the neutral axis expressed as a ratio of the depth, h
E	Modulus of elasticity
E_c	Modulus of elasticity of the composite
E_f	Modulus of elasticity of the fiber
E_{fpr}	Modulus of elasticity of the fiber reinforced plastic
E_m	Modulus of elasticity of the matrix
E_w	Modulus of Elasticity of the wood
E_{web}	Modulus of elasticity of the web
f_c	Stress in wood at the compressive proportional limit

f_m	Failure stress of wood in bending
f_t	Failure stress of wood in tension
G	Shear Rigidity
h	depth of the wood
I	Moment of inertia
k	Shape Factor
k_1	stress distribution parameter
L	Beam length
mE	slope of the falling branch
M	Moment
P	Force
t_b	Thickness of bottom flange
t_f	Thickness of the flange
t_t	Thickness of top flange
t_w	Thickness of the web
V	Shear Force
V_f	Volumetric fraction of fiber
V_m	Volumetric fraction of matrix
x	Distance from end of beam
ϵ_t	Strain at tension face of the wood
ϵ_c	Strain associated with the compressive proportional limit of wood
ψ	Curvature

Δ	Displacement of the beam
σ_c	Stress in the composite
σ_{cf}	Stress in the composite in the zone governed by the fiber strength
σ_{cm}	Stress in the composite in the zone governed by the matrix strength
σ_f	Stress in the fiber
σ_{fu}	Ultimate failure stress in the fiber
σ_m	Stress in the matrix
σ_{mu}	Ultimate failure stress of the matrix
ν_f	Poisson's ratio for the fiber
ν_m	Poisson's ratio for the matrix

LIST OF APPENDICES

Appendix 1	Derivation of fiber reinforced solid wood elastic moment equation from first principles
Appendix 2	Derivation of fiber reinforced solid wood inelastic moment equation from first principles
Appendix 3	Derivation of fiber reinforced I-beam elastic moment equation from first principles
Appendix 4	Derivation of fiber reinforced I-beam inelastic moment equation from first principles

1.0 Introduction

1.1 Problem Definition

Wood has been used as a structural material since the dawn of civilization. This has been due in large part to its natural abundance in many parts of the world and the simplicity of building structures with wood. Throughout the years, mankind has consumed vast quantities of wood to construct all forms of engineering works with little or no regard of the consequences on the environment. The "exploitation" of this resource particularly in the last fifty years without proper conservation and reforestation measures, threatens to jeopardize world timber stocks.

Already large dimension lumber is becoming increasingly scarce since much of the

Figure 1.1 - Photograph of Clear Cut Area

old growth forests have either been harvested or are subject to environmental protection. The few remaining old growth timber stands available to the forestry industry are for the most part in remote, inaccessible areas which will invariably lead to increased extraction costs. It is also expected that the harvest from reforested areas will yield smaller diameter trees than was previously cut from these areas.

Despite these problems, the demand for wood products continues to be strong primarily due to its aesthetic appeal, relatively low cost and ease of construction. To satisfy this demand from a diminishing natural resource base, the lumber industry is continually seeking innovative approaches to use the available timber resources in a "smarter" and more efficient manner.

One initiative towards the more effective use of wood has been the reinforcement of wood beams with stronger materials to produce a composite capable of resisting the applied loads using substantially less material.

1.2 Thesis Organization

1.2.1 General

In reinforced wooden beams, the lay out of the reinforcement is an important consideration in determining the overall beam behavior, yet little or no work has been done in assessing the effects of altering the reinforcement configuration. Most authors on the subject have adopted a pattern of continuous reinforcement due to its simplicity in the

manufacturing process and uncomplicated design.

The selection of a reinforcement material is based on a multitude of factors including strength , stiffness, durability, and resistance to corrosion; however, the one overriding selection criteria is cost. Glass fiber reinforced plastics (GFRP) are well suited as wood reinforcement offering an economical, non-corroding material with excellent mechanical properties.

1.2.2 Objectives

In this thesis, the reinforcement of solid wood as well as I-beams with a low cost glass fiber in a variety of reinforcement configurations is investigated both theoretically and experimentally. Specifically, the objectives of this thesis are:

- to obtain a better understanding of wood-fiber composite beams;
- to develop a theoretical model to describe fiber reinforced wood beam behavior and evaluate the performance of this model through experimentation;
- to examine the cost effectiveness and viability of reinforcing wood with fiber.

1.2.3 Scope of Work

1.2.3.1 Experimental

The experimental portion of the thesis consisted of two phases in order to validate the theoretical models for both fiber reinforced solid wood beams and I-beams. The first phase consisted of twenty "small beams" constructed from the flange material of the I-beams. These beams were reinforced with a variety of reinforcement configurations as shown in the diagrams below where the thickness of the fiber layer has been exaggerated for clarity:

Control Beams

Continuously Reinforced Beams

*Variable Length Midspan
Reinforced Beams*

*Constant Length Midspan
Reinforced Beams*

The second phase consisted of full-scale laboratory tests on 7.315m (24 ft) TJI 421 I-beams which were donated by Trus-Joist Macmillan. To achieve a better understanding of the reinforced I-beam behavior, three of the experimental beams were broken down into their constituent components and examined in closer detail. A number of experimental tests were carried out on these components to determine certain parameters which would be later used in modelling the overall reinforced I-beam behavior. The remainder of the beams were configured with the following reinforcement configurations:

Control Beams

*Constant Length Midspan
Reinforced Beams*

*Constant Length Midspan
Reinforced Beams*

The small beams were loaded with a two point load applied at $L/3$ and $2L/3$ while the 1 beams were subjected to a uniformly distributed load. Both types of beams were tested to failure in flexure and the results were compared against both the theoretical model and the unreinforced control beams.

1.2.3.2 Theoretical

The existing theoretical models describing the elastic and inelastic behavior of continuously reinforced beams were examined in detail and derived from first principles. This review was then used as the foundation to develop a more general relationship which was applicable to solid wood beams with any reinforcement configuration. Finally, the methodology was extended to I-beams where a model was developed to predict the elastic and inelastic behavior for any tensile reinforcement configuration.

1.3 Basic Concepts

1.3.1 Wood

1.3.1.1 The Natural Resource

Canada is endowed with vast natural resources, one of which is a rather large inventory of forests. According to [Keenan 1986] , 44 percent of the total land area of Canada is classified as forest land with the majority being concentrated in British Columbia.

Species	Region						
	1	2	3	4	5	6	7
Softwoods							
<i>Spruce</i>	1	9	7	4	10	0	31
<i>Pine</i>	-	1	3	4	8	0	16
<i>Fir</i>	1	5	1	-	7	0	14
<i>Hemlock</i>	-	-	-	0	6	0	6
<i>Cedar</i>	-	-	-	-	3	0	3
<i>Douglas Fir</i>	0	0	0	-	3	0	3
<i>Unspecified Softwoods</i>	2	-	-	-	1	3	6
Total Softwoods	4	15	11	8	38	3	79
Hardwoods							
<i>Aspen/Poplar</i>	-	1	3	5	2	0	11
<i>Birch</i>	-	3	2	-	-	0	5
<i>Maple</i>	2	1	1	-	-	0	4
<i>Unspecified Hardwoods</i>	-	-	-	-	-	1	1
Total Hardwoods	2	5	6	5	2	1	21
Total	6	20	17	13	40	4	100

Table 1A - Distribution of Growing Stock by Percentage in Canada [Bonnor 1982]

The trees in these forests are subdivided into two principal groups called hardwoods and softwoods. Softwoods consist of coniferous trees which bear cones, have needle like leaves and, with the exception of larches, retain their leaves year round.

Hardwoods are deciduous trees which tend to be broadleafed, change color with the seasons and shed their leaves in the winter. Although there are more than 30 species of softwoods and over 100 species of hardwoods, the softwoods constitute some 79 percent of Canadian inventoried growing stock while the hardwoods make up the remaining 21 percent.

Often in the lumber industry, species which grow and are harvested together are

often marketed as a species combination. There are four species combinations recognized by the Canadian Timber Code, CAN/CSA-O86-M89, as follows:

<i>Species Combination</i>	<i>Species included in the Combination</i>
Douglas Fir-Larch	Douglas Fir, Western Larch
Hem-Fir	Pacific Coast Hemlock, Amabilis Fir
Spruce-Pine-Fir	Spruce (all species except Coast Sitka Spruce), Jack Pine, Lodgepole Pine, Balsam Fir, Alpine Fir
Northern Species	Any Canadian species graded in accordance with the National Lumber Grades Authority(NLGA) rules

Table 1B - Species Combinations Recognized in CAN/CSA-O86-M89

The largest of these species combinations is the Spruce-Pine-Fir (S-P-F) which makes up more than 50 percent of all inventoried Canadian growing stock and is the most widely used material in timber engineering in Canada. In the hardwood category the only species combination to be used structurally to any extent is the Aspen/Poplar species combination primarily in the manufacture of oriented strandboard (OSB). Efforts are currently underway to increase the use of Aspen/Poplar because of its relative abundance (11% of all inventoried stock) and under-utilization in the lumber industry.[Keenan 1986]

1.3.1.2 Structure of Wood

Trees are complex organic structures which are subjected to a variety of imposed loading conditions induced by wind, snow, earthquake and other natural phenomena. To resist these forces, trees have evolved to produce a highly efficient structure which is adaptable to changing load conditions as the tree matures. The examination of the tree

cross-section can provide some valuable insight into the mechanical behavior of trees and wood in general.

The outermost layer of the tree consists of the outer bark which is a collection of dead cells to protect the tree from the external environment. The next layer, the inner bark, serves the vital function of transporting nutrients from the leaves to the remainder of the tree. This is followed by the cambium, a microscopic layer just inside the inner bark, where new wood cells are formed during the growing season. Below the cambium is the sapwood, a physiologically active portion of the tree, which carries sap from the roots to the leaves. The remainder of the tree consists of inactive cells called the heartwood and contains the centermost portion of the tree called the pith along with the juvenile or pith associated wood. The heartwood and sapwood have essentially the same mechanical properties for a given moisture content with the exception of the pith and juvenile wood which generally have poorer mechanical properties. [Keenan 1986]

Figure 1.2 - Cross-Sectional View of a Tree [Keenan 1986]

The analysis of the tree cross-section also reveals a number of concentric circles called "annual rings" due to the fact that one ring is generated for each growth cycle. Closer scrutiny of these annual rings reveals that the ring is actually composed of two rings, an innermost lighter ring and an outer darker ring. The lighter ring is produced early in the growing season and is comprised of fast growing thin walled "earlywood" cells while the darker ring consists thick walled "latewood" cells so named because of their formation in the latter part of the growing season. In addition to the cells which make up the annual rings, "ray cells" which extend radially from the pith to the bark are produced

presumably for the transportation and storage of nutrients in the radial direction. These ray cells make up about 7% of the volume of the longitudinal cells which make up the annual rings.

Figure 1.3 - Tracheid Structure [Keenan 1986]

The microscopic structure of the cells or tracheids which form the annual rings is extremely complex in nature and can best be described as a slender tube with a length on the order of 4mm and a diameter in the vicinity of 0.04mm (softwoods). The walls of the tracheids consist of four layers of microscopic cellulose fibers in a matrix composed of lignin and hemicellulose with each layer possessing a different fiber orientation. This structure is illustrated in Figure 1.3 and an actual photograph is presented in Figure 1.4.[Canadian Wood Council 1991]

Figure 1.4 - Photograph of Tracheids [Canada Wood Council 1991]

An examination of a wedge derived from a tree shown in Figure 1.5 clearly illustrates the three principal directions of wood as being along the longitudinal(L), radial(R) and tangential(T) axes. The longitudinal direction is often referred to as “parallel to the grain” while the radial and tangential directions are together described as “perpendicular to the grain” based on the direction of the tracheids or the “grain”.

Figure 1.5 - Principal Directions of Wood Internal Structure [Keenan 1986]

1.3.1.3 Wood Behavior

The tubular structure of the tracheids leads to substantially better strength and stiffness characteristics parallel to the grain than those measured perpendicular to the grain. This highly anisotropic behavior of wood along with its organic nature distinguishes wood from other construction materials. Consequently, the analysis of wooden structures tends to be more rigorous since it is necessary to examine a number of different failure modes in different directions. This behavior is clearly demonstrated in Table 1C which shows the experimental and specified mechanical properties for the two

types of lumber used in the experimental portion of the thesis.

Machine Stress Rated Lumber S-P-F 2100F-1.8E	<i>Modulus of Elasticity</i>	12816 MPa		12400 MPa
	<i>Modulus of Elasticity (5th Percentile)</i>		11170 MPa	not specified
	<i>Modulus of Rupture</i>	60.93 MPa	40.67 MPa	30.5 MPa
	<i>Tensile Strength (Parallel to the grain)</i>	40.33 MPa	26.03 MPa	17.7 MPa
	<i>Compressive Strength (Parallel to the grain)</i>	not tested	not tested	22.2 MPa
No. 1 S-P-F Lumber 38mm x 89mm	<i>Modulus of Elasticity</i>	9700 MPa		9000 MPa
	<i>Modulus of Elasticity (5th Percentile)</i>		6630 MPa	5500 MPa
	<i>Modulus of Rupture</i>	46.84 MPa	27.81 MPa	15.3 MPa
	<i>Tensile Strength (Parallel to the Grain)</i>	26.63 MPa	12.81 MPa	6.2 MPa
	<i>Compressive Strength (Parallel to the Grain)</i>	27.44 MPa	21.15 MPa	13.1 MPa
	<i>Compressive Strength (Perpendicular to the Grain)</i>	not tested	not tested	5.3 MPa
	<i>Longitudinal Shear</i>	not tested	not tested	1.7 MPa

Table 1C - Experimental and Specified Strength of Wood

There are a number of factors which influence the behavior of wood and its mechanical properties. Specifically these include, but are not limited to, the following items:

- a. specific gravity;
- b. moisture content;
- c. presence of defects;
- d. load duration;
- e. temperature; and
- f. size of the member.

Specific Gravity is a measure of the amount of cell wall material in a particular sample.

Generally speaking the higher the specific gravity or amount of cell wall material the greater the strength and stiffness exhibited by the wood. The standard method of determining the specific gravity in wood is to employ the "basic specific gravity" which is simply the oven-dry mass of the sample divided by the volume of the sample in the green condition (moisture content greater than 19%). Typically the values for specific gravity range from 0.3 to 0.7 depending upon the species. The effects of specific gravity are incorporated into the specified strength values found in the CAN/CSA-O86-M89. [Keenan 1986][Canadian Wood Council 1991]

Moisture Content is one of the most significant factors affecting the performance of wood and the importance of obtaining a good understanding of the effects of moisture content

cannot be overstated. Water is absorbed by wood either as bound water within the cell walls or as free water held in the cell lumen. The moisture content is the mass of the water contained in the wood expressed as a percentage of the mass of the wood containing no water commonly known as the oven dry mass. As the moisture content is increased from the 0% or oven dry condition, the water is totally contained within the cell walls until the cells become completely saturated at a moisture content of about 28% which is known as the "fiber saturation point". Above the fiber saturation point any additional water is held as free water within the cell cavity(lumen).

Figure 1.6 - Integration of Moisture into the Cell Structure [Canada Wood Council 1991]

As wood dries the moisture is expelled in the reverse order to its retention which means that any free water contained in the cell lumen is shed prior to any removal of water from the cells. The drying of wood down to the fiber saturation point has no appreciable effect on shrinkage or the mechanical properties of wood. However as the moisture content of the wood is reduced below the fiber saturation point the bound water is released from the cell walls which results in shrinkage and an increase in the structural properties of the

wood. The compressive strength, modulus of rupture and the modulus of elasticity are subject to large fluctuations with moisture content as can be seen in Figure 1.7. The only mechanical properties which improve with increasing moisture content are the toughness and ductility.[Keenan 1986]

Figure 1.7 - Effect of Moisture Content on Mechanical Properties [Keenan 1986]

Unfortunately the shrinkage of wood is anisotropic in nature with the greatest shrinkage

occurring in the tangential direction and hardly any shrinkage in the longitudinal direction. This differential shrinkage results in a variety of shrinkage induced cross-sectional distortions which are dependent upon the annual ring orientation as shown in Figure 1.8. In addition to those caused by differential shrinkage, distortions can occur due to grain deviations along the longitudinal axis of the wood in the form of a bow, crook or twist.[Keenan 1986]

Figure 1.8 - Cross-Sectional Distortions due to Shrinkage [Keenan 1986]

Wood is generally hygroscopic in nature and absorbs or loses moisture in response to the temperature and relative humidity of the ambient air. The “equilibrium moisture content” of the wood for a given temperature and relative humidity is well documented and presented below in Figure 1.9.

Figure 1.9 - Equilibrium Moisture Content [Keenan 1986]

The CAN/CSA-O86-M89 uses a service condition factor to account for the effects of elevated moisture content. This service condition factor reduces the value of the mechanical properties specified in the code for wood exposed to elevated moisture conditions.

Presence of Defects in the wood undermine the mechanical properties of the wood and cause premature failure. These defects which are listed in Table 1D and shown in Figure 1.10 are carefully regulated and controlled through either a visual grading of lumber or the testing the lumber by machine. The visual grading system is essentially an assessment of the anticipated strength characteristics based on the appearance of the wood and the visual detection of strength reducing defects. The machine graded or machine stress rated (MSR) lumber is loaded in a non-destructive manner and the deflection characteristics of the wood are used to determine the bending strength and modulus of elasticity of the sample. The MSR process is generally more expensive than visually graded lumber but the increased reliability and strength associated with MSR lumber make it attractive for use in applications such as trusses and I-beams.

Figure 1.10 - Defects Commonly Found in Wood [Keenan 1986]

<i>Defect</i>	<i>Definition</i>
<i>Knot</i>	that portion of a branch or limb that has been surrounded by subsequent growth of wood of the trunk or other portion of the tree
<i>Loose Knot</i>	a knot which is not held firmly in place by growth or position and which cannot be relied upon to remain in place
<i>Sound Tight Knot</i>	a knot so fixed by growth or position that it will retain its place in the piece
<i>Spike Knot</i>	a knot sawn in a lengthwise or nearly lengthwise direction
<i>Cross-grain</i>	a pattern in which the direction of the longitudinal wood cells deviates from a line parallel to the sides of the piece
<i>Slope of Grain</i>	the angle between the grain direction and the axis of the piece, expressed as a slope ratio
<i>Split</i>	a separation along the grain which forms a crack or fissure that extends through the piece from one surface to the another
<i>Shake</i>	a separation along the grain, the greater part of which occurs between the rings of annual growth
<i>Wane</i>	the bark or lack of wood from any cause on the face of a piece
<i>Check</i>	a lengthwise separation of the wood which usually extends across the rings of annual growth, commonly resulting from stresses set up in the wood during seasoning
<i>Pitch Pocket</i>	an opening between the growth rings which usually contains or has contained resin or bark

Table 1D - Definition of Wood Defects [Keenan 1986]

The CAN/CSA-O86-M89 code recognizes presence of these defects and incorporates their effects into the specified strength values for the different grades of wood.

Load Duration has been shown to have a significant effect on the strength properties and a slight effect on the modulus. This effect manifests itself in the form of a decrease in most mechanical properties of the wood as the duration of the load is increased with the exception of the modulus of elasticity which is relatively unaffected by load duration.

Load duration is addressed in the CAN/CSA-O86-M89 code by using ten years as a normal load duration and increasing or decreasing the resistance for other load durations. Since the modulus is not influenced to any great extent by this effect, the load duration factor is not applied to the modulus of elasticity.

Temperature has a significant effect on the behavior of wood, generally speaking the structural properties of wood are increased at lower temperatures and reduced in the presence of heat. This influence on the mechanical properties of wood is a function of many parameters including the temperature, type of exposure, moisture content, specific gravity, and species. Short term exposure to temperatures of less than 65°C followed by a return to normal temperatures produces no permanent changes to the structural properties of wood. However wood exposed to temperatures of 65°C or more for extended periods of time experience a permanent decrease in their structural properties even if the temperature is subsequently returned to normal. The CAN/CSA-O86-M89 recognizes this fact and limits application of the code to wood subjected to temperatures below 50°C with occasional exposure to 65°C.

Member Size has a direct bearing on the strength of the wood in that the observed strength decreases as the size of the member increases. This effect has been explained using the "weak link theory" which says that the probability of encountering a defect or a strength reducing flaw is greater in a large member than for a small one. Size effects, as has been found in other mechanical properties are anisotropic in nature with different size

effects being observed for variations in the length(length effect) and depth (depth effect) of wood. The CAN/CSA-O86-M89 timber code addresses this phenomena through the use of a size effect factor which is applied to the specified strength of the member.

[Keenan 1986]

1.3.2 Web Beams

The development of web beams such as the I-beam and box beam evolved to capitalize on a superior geometric shape as can be seen in Figure 1.11 to deliver improved construction performance while using substantially less wood than conventional sawn-lumber.

The most popular of the web beams is the wooden I-beam which as the name suggests consists of two flanges joined by a web. The flanges are generally made from machine stress rated (MSR) lumber or laminated veneer lumber (LVL) while the web can be of either plywood, oriented strandboard(OSB) or metal construction. The flanges are then joined to the web in a variety of methods ranging from mechanical fasteners to glue.

Figure 1.11 - Photograph of I-Beams

The modular nature of I-beams not only permits the effective utilization of smaller

diameter trees for the individual components but enables the use of rapid growth species which are not normally employed as sawn lumber. The resulting decrease in material costs along with the increase in reliability and the possibility of spanning longer distances tends to offset the higher production costs associated with the manufacture of I-beams.

Figure 1.12 - I-Beam Manufacturing Process[Canada Wood Council 1991]

As with all web beams, the cost and performance characteristics of I-beams are of

primary importance and can vary substantially according to the choice of materials and manufacturing process. In the design of I-beams, the selection of web and flange material is largely predicated on the assumption that the web withstands the shear stresses and the flanges resist the axial stresses induced by bending. This design philosophy is reflected in the CAN/CSA-O86.1-M89 timber code which requires that the following criteria be met in the design of I-beams:

- a. the axial stresses in the flanges must not exceed the flange material's tensile or compressive strength;
- b. the shear-through-thickness stress at the neutral axis must not exceed the web shear strength;
- c. the shear stress at the flange-web interface cannot exceed the strength of the web, the flange or the web-flange connection in shear;
- d. the amount of deflection cannot exceed specified deflection limitations;
- e. certain criteria to ensure the lateral stability of the beam must be met;
- f. the provision of web stiffeners at concentrated load and reaction points;
and
- g. the inclusion of web stabilizers to prevent web buckling.

Inherent in these design criteria is the implicit recognition that I-beams are subject to numerous modes of failure. The determination of the governing mode of failure is a function of many parameters including I-beam geometry, span, loading and support conditions. In practice, however, it is found that the bending capacity as determined by

the axial strength of the flanges, or the deflection of the beam govern the design.

Research [Samson 1983] has shown that the performance of I-beams is improved when the modulus of the tension flange is greater than that of the compression flange. This can be achieved either through the utilization of higher strength wood for the tension flange or, alternatively, as explored in this thesis, by reinforcing the tension flange with a higher strength material.

1.3.3 Fiber Reinforced Plastic(FRP)

1.3.3.1 General

The use of plastics or more specifically polymers in structural applications has increased significantly in the past two decades primarily due to advances in polymer technology and the acceptance of plastics as a construction material. The strength and stiffness characteristics of polymers alone are frequently insufficient for the desired application and some reinforcing material must be added to the polymer. The resulting reinforced polymer is commonly referred to as a "composite material" and possesses enhanced strength and stiffness properties.

This reinforcement of a low modulus polymer matrix with a high modulus, high strength fiber uses the plastic flow of the matrix material under stress to transfer the load to the fiber which results in a high strength, high modulus composite. The aim of this combination is to produce a two-phase material where the primary phase, the fibers, determine the strength and stiffness characteristics while the weaker secondary phase, the

polymer matrix acts as a bonding agent between fibers. [Hollaway 1990]

The overall performance of this composite can be directly related to the following three factors:

- a. the materials used to form the composite;
- b. the orientation and amount of the fibers in the matrix;
- c. the method of fabrication.

1.3.3.2 Materials

1.3.3.2.1 Fiber

Glass

Glass fibers are produced from the rapid extrusion of molten glass through a number of small orifices in a platinum bushing to produce filaments about 10 μm in diameter. The filaments are then treated with a protective coating or sizing and gathered together in groups of 204 or more filaments to form "strands" which are then combined to form thicker parallel bundles called "rovings". The rovings are then chopped for use in a mat, or woven in some fashion to form a "woven roving".

There are several different types of glass fibers commercially available as follows:

- a. **E-glass** - most common glass used in the general construction industry;

- b. **Z-glass** - zirconia glass developed for use with cements and mortars due to its high resistance to alkali attack;
- c. **C-glass** - very high corrosion resistance to acids used in environments where chemical resistance is a factor;
- d. **D-glass** - low density, dielectric glass used for electrical applications; and
- e. **S-glass** - high strength, high modulus glass for use in the space and aerospace applications.

The most widely used glass fiber in composite applications is E-glass due the fact that it is by far the most economical of all the commercially available fibers. It possesses good tensile strength(3448 MPa), high chemical resistance and excellent insulating properties but is endowed with a low modulus(72 GPa) and high sensitivity to abrasion with handling. The other types of glass fiber have not achieved the popularity of E-glass mainly due to higher production costs and are for the most part confined to specialized applications.[Hollaway 1990]

Carbon

Graphite and carbon fibers are popular within the aerospace industry and have recently gained some prominence in civil engineering within structural applications.

Although the terms graphite and carbon are used interchangeably, a carbon fiber typically

contains less than 95% carbon while graphite fibers are at least 99% carbon. The success of these fibers is due to its high strength and high modulus to weight ratios and its ability to withstand high temperatures. However, the widespread use of carbon/graphite is hampered by its high cost and low impact resistance.

The manufacturing process for carbon and graphite fibers is the same and consists of subjecting an initial or precursor material to a sequence of heat treatments to convert the precursor to carbon/graphite by pyrolysis. The fibers are then bundled and sold in groups of anywhere from 1000-160,000 parallel filaments called "tows".

There are three types of precursor materials available:

- a. rayons;
- b. pitch derived products; and
- c. polyacrylonitrile (PAN).

The rayon based carbon/graphite fibers are manufactured by subjecting a rayon precursor to an initial heat treatment at 500°C which oxidizes the material to a form suitable for carbonization at temperatures of 1000-2000°C. The final stage of the process is a stretching of the fibers while subjecting them to temperatures of 2500-3000°C. The fibers produced in this manner typically have a tensile strength of 1000 MPa and a modulus of only 41 GPa consequently the poor performance of these fibers and the high

production costs make this method impractical.

Pitch-derived precursors are obtained as by-products of coal-tar and petroleum processing which are relatively easy to melt and spin into fibers. The fibers created directly from the pitch possess a low tensile strength and modulus which is comparable to rayon-based fibers and consequently is of little practical value. This pitch, however, can be converted into a liquid crystalline phase known as "mesophase" when heated to a temperature of 400°C which can then be used to produce high quality fibers. The conversion of this mesophase pitch into carbon/graphite fibers is then achieved through a rather complex method of extruding the fibers at a specified rate and subjecting them to an air quench process. This method produces three different categories of fiber based on the modulus of elasticity as follows:

- a. Low Modulus - tensile strength of 1400 MPa and a modulus of 160 GPa;
- b. High Modulus - tensile strength of 1700 MPa and a modulus of 380 GPa;
and
- c. Ultra-high Modulus - tensile strength of 2200 MPa and a modulus of 725 GPa.

To date the mesophase based fibers have been unable to compete with the PAN based

fibers due to the expense of the fabrication process and the unexpectedly high costs of removing flaw inducing materials from the precursor pitch. Advancements in purification technology may lead to increased use of mesophase based fibers particularly in high modulus applications.

The most common precursor used in the production of carbon/graphite fibers is polyacrylonitrile (PAN). The initial step in the production of carbon/graphite fibers from PAN is to heat the PAN fiber under tension in air at a temperature of 200-300°C to stabilize the fiber for subsequent heat treatment. The fiber is then subjected to a carbonization process which consists of heating the fiber to 1000°C in an inert atmosphere. Finally, the fiber is given a heat treatment also in an inert atmosphere where temperatures of 1000°C are used to produce a low modulus fiber, 1000-1500°C to create an intermediate modulus fiber and 1990-3000°C to form a high modulus fiber. The relationship between the heat treatment temperature, strength and Young's modulus is clearly seen in Figure 1.13 shown below.

Figure 1.13 - Strength/Modulus of PAN based Carbon Fibers [Savage 1994]

Measures such as neutron irradiation, hot stretching, the addition of boron and solid solution hardening have all been found to increase the mechanical properties of PAN based carbon/graphite fibers.[Savage 1994]

Aramid

Aramid is an aromatic polyamide and has the distinction of being the only organic fiber that is used in structural composites. The aramid filaments are produced by extruding an acidic precursor solution from a spinneret. The best known aramid for structural applications is Kevlar® 49 which has a high tensile strength(3792 MPa), a modest modulus(131 GPa) along with excellent ductility, toughness and impact resistance a low compressive strength. [Gibson 1994]

Other

There are a number of other fibers used in composite materials such as boron and metallic fibers. However, the prohibitive cost of producing these fibers precludes them from being used in any significant manner in structural applications.

1.3.3.2.2 Matrix

Matrix materials are generally made from polymers, metals or ceramics depending upon the specific application. The function of the matrix according to Gibson[Gibson 1994] is to:

- a. hold the fibers together as a structural unit and protect them from external damage;
- b. transfer and distribute the applied loads to the fibers; and
- c. in many cases, contribute some needed property such as ductility, toughness or electrical insulation.

Polymers

The most common of the matrix materials are polymers which are classified as being either thermoset or thermoplastic based on their molecular structure and behavior at elevated temperatures. Thermoset polymers form a highly cross-linked three dimensional network of molecules upon curing which remains rigid at higher temperatures.

Thermoplastic polymers, on the other hand, form a linear molecular structure with no chemical cross-links and as a consequence the molecules are free to move relative to each other at higher temperatures then "freeze" into their new positions upon cooling. Thus a thermoplastic polymer can be reshaped or "postformed" with the application of heat whereas a thermoset polymer retains its shape as the temperature is increased.

Thermosets such as epoxies and resins have been the traditional choice of polymer matrix material for composites, however recent research into matrix materials has demonstrated that there may be some advantages in using a thermoplastic for some applications.[Gibson 1994]

Metal and Ceramics

Metal and ceramic matrices are used predominantly within the aerospace industry in high temperature applications. Due to the prohibitive cost in producing these matrices, the use of these products is extremely rare in structural applications.

1.3.3.3 Fiber Content and Orientation

1.3.3.3.1 Fiber Content

The tensile strength and the modulus of elasticity of composites are highly dependent upon the amount of reinforcing fiber present in the matrix. As a general rule, the mechanical properties of the composite improve as the fiber content is increased up to some threshold value. This maximum fiber content is governed by many parameters but is generally limited by the need to provide sufficient matrix material to effectively transfer the stresses between fibers and protect the composite from external damage.

The fiber content can be reported as either a percentage of the total composite weight or a volumetric fraction of the total composite volume. The latter method is preferred since it facilitates the application of the "law of mixtures" which computes the mechanical properties of the composite from the properties of the component materials and their volumetric fractions.

The effects of fiber content and inter-relationship between the mechanical properties of the fiber, the matrix and the resulting composite will be discussed in further

detail in Chapter 2.

1.3.3.2 Fiber Orientation

The orientation of the fibers within the matrix has a large impact on the structural properties of composite materials. There are a many different fiber orientations possible, however these can all be categorized into four distinct groups as follows:

- a. chopped strand mat;
- b. weaves;
- c. unidirectional fibers; and
- d. hybrids.

Chopped Strand Mat as the name suggests consists of chopped strands of glass fibers ranging from 3-50mm in length randomly oriented and mixed with a resin binder to form a "mat". The mat is then used with a thermoset polymer usually a polyester resin to produce an economical, low strength, low modulus isotropic composite.

Weaves are the most common method of achieving fiber orientation in composites due to their versatility and low production costs. Naturally, the configuration of the weave is of considerable importance since it has a significant influence on the structural properties of the composite material. High unidirectional strength and modulus can be achieved through the use of weaves with a high percentage of fiber in the warp or longitudinal direction and a smaller amount in the weft or transverse direction. Similarly bi-directional composites can be created through the use of equal amounts of fiber in the warp and weft

directions.

Figure 1.14 - Kinking of Fibers

A major disadvantage with traditional fiber weaves has been the “kinking” of the fibers shown in Figure 1.14 which results in a reduction in both the tensile strength and the modulus of the composite. Recently the fiber industry has developed three dimensional weaves which hold the fibers in place without kinking through the use of a melt-away nylon stitching as shown in Figure 1.15. This initiative should result in substantial increases in the strength and modulus of weave-based unidirectional composites.

Figure 1.15 - Three Dimensional Weave

Unidirectional Fibers are extremely delicate structures which must be restrained in some fashion to hold the fibers together and maintain their alignment. This can either be achieved through the use of a binding agent with some form of backing material or the pre-impregnation of the fibers with a pre-determined amount of polymer matrix material. The fibers treated with a binding agent are then processed in the normal manner by applying some form of matrix material to the fibers. The pre-impregnated fibers, however, constitute a composite material in a semi-cured state and usually requires only the application of heat and pressure to finalize the curing process. The use of pre-impregnated fibers in this manner tends to produce a more consistent composite with a uniform fiber content and simplifies the manufacturing process in some applications. Despite these obvious advantages, the use of pre-impregnated fibers has to some extent been limited by its substantially higher cost.

Hybrids are comprised of two or more different fiber orientations or fibers in a single composite.

Figure 1.16 - Hybrid System consisting of a Unidirectional Fiber attached to a Chopped Mat

This can be as simple as using a chopped strand mat combined with a unidirectional fiber or as complex as a multi-layered system consisting of several different fibers in a variety of fiber orientations.

1.3.3.4 Fabrication Methods

The fabrication methods used in the construction of composites have a significant impact on the quality and the fiber content of the finished product. Although there is a variety of individual manufacturing techniques, the general processes in the fabrication of composites are generic in nature as shown in Figure 1.17 and can be divided into open mould and closed mould processes.

Figure 1.17 - Fiber Reinforced Plastics Fabrication Process [Holmes and Just 1983]

1.3.3.4.1 Open Mould Process

The open moulding process differs from the closed mould process in that only one mould is used in the open mould process while a pair of moulds are employed in closed mould operations. The open moulding process can be further subdivided as follows:

- a. contact moulding;
- b. pressure bag techniques; and
- c. filament winding.

Contact Moulding consists of the manual, labor intensive techniques of hand lay up and spray up. In the hand lay up method, the fibers are placed in the mould by hand and the resin matrix is usually applied using a roller. Using this technique fiber contents in the range of 30-50% by weight can be readily produced. This technique has a low production

cost and the ability to cater to design changes, however, the quality of the composite in this process depends on the skill of the fabricator and is subject to variations. The spray up technique consists of spraying a mixture of chopped strand fibers and resin into a mould followed by hand rolling to remove air pockets. Typically fiber contents of about 30% by weight are produced using this method. Spray up methods can produce complex shapes easily and economically, but the strength and modulus are limited by the use of the chopped strand material and the quality is also subject to large variations.

Figure 1.18 - Contact Mould Methods [Gibson 1994]

Pressure Bag Techniques exert pressure on the open face of the mould and provide a

better contact between the mould and the fiber. This results in fewer voids, higher fiber contents and better product than can be obtained from contact moulding. There are three basic types of pressure bag techniques as follows:

Vacuum Bag This method entails the creation of a vacuum under a rubber membrane which is placed on the surface of the composite. Pressures of up to 100 KN/m^2 are exerted on the composite resulting in fiber contents of up to 55% by weight;

a) Vacuum bag

b) Pressure bag

Figure 1.19 - Vacuum Bag and Pressure Bag Methods [Holmes and Just 1983]

Pressure Bag This technique is similar to the vacuum bag except that the pressure

is applied directly to the surface of the membrane. The higher pressures of 350 KN/m^2 exerted on the composite yield a fiber content in the vicinity of 65% by weight; and

Autoclave This method applies high pressures in the vicinity of 700 KN/m^2 to the composite to produce fiber contents of about 70% by weight.

Figure 1.20 - Autoclave Process[Gibson 1994]

Despite the fact that pressure bag techniques produce improved composites with higher fiber contents their widespread use has been limited by the higher productions costs and the substantial capital investment associated with this method.

Filament Winding as the name implies consists of winding a unidirectional pre-impregnated filament on a rotating "mandrel". This method is used to form high strength, high modulus symmetrical circular composite cross-sections with extremely high fiber

contents of up to 90% by weight.

Figure 1.21 - Filament Winding Process[Gibson 1994]

1.3.3.4.2 Closed Mould Processes

The closed mould process uses two moulds to create complex shapes with a high quality finish on all surfaces. Due to the high expense involved in closed mould processes it is best suited for mass production and does not easily adapt to design changes or special orders. The closed mould process can be divided into the following manufacturing methods:

- a. Matched Die;
- b. Pultrusion; and

c. Continuous Lamination.

Matched Die processes usually consist of pressing either a shaped pre-impregnated chopped fiber mat called a "preform" or a chopped fiber-resin mixture called a "premix" between two moulds with a pressure of $400\text{--}4000\text{ KN/m}^2$ to form complex shapes. The application of heat on the order of $120\text{--}150^\circ\text{C}$ may be used to accelerate the curing process in a hot press moulding or may be absent as in a cold press moulding. Typically fiber contents of about 25% are obtained for premix moulded shapes while a 50% fiber content is attained with the use of preforms.

Figure 1.22 - Matched Die Process[Holmes and Just 1983]

Pultrusion is used to produce a variety of different cross-sectional shapes. In this process pre-impregnated fibers are pulled through a die to produce complex shapes such as small I sections or channels with a maximum cross sectional dimension of about 150mm. The fiber contents attained in this process are in the vicinity of 60-80%.

Figure 1.23 - Pultruded Fiber Composite Shapes[Gibson 1994]

Continuous Lamination is a method similar to pultrusion which is used to form composite sheets or corrugated panels. In this process a chopped strand mat or a weave is impregnated with resin pulled through a series of rollers.

Figure 1.24 - Continuous Lamination Process[Gibson 1994]

The rollers are then manipulated to vary the thickness of the sheets or introduce corrugations. The maximum sheet thickness of approximately 3mm and the fiber content of about 30% by weight can be expected from this method.

1.4 State of the Art

1.4.1 Wood

The bending strength of wood has been the subject of study since the first effort by Galileo in 1638, albeit unsuccessful, to determine the distribution of flexural stresses in beams. Later, in 1708, Parent derived the correct stress distribution to describe the behavior of elastic beams, however his findings were not generally accepted until they were confirmed 65 years later by Coulomb in 1773. [Buchanan 1990] In the ensuing years there have been a number of researchers who have attempted to predict the behavior of wood in flexure both in the elastic and inelastic regions. According to [Buchanan 1990] these efforts are complicated by:

- a. The nonlinear nature of the stress-strain relationship for wood in compression;
- b. The large variability in the strength properties of wood; and
- c. The major influence of size effects, which result in failure stresses decreasing as stressed volume increases.

Understandably researchers have sought to develop a stress distribution model that would adequately describe the behavior of wood and would be easy to use in practice. One such model was the simplified stress distribution model adopted by some researchers [Bechtel and Norris 1952], [Ramos 1961], [Mazur

1965] and [Nwokoye 1972]. This model shown in Figure 1.25 was easy to use, however it did not conform to the experimental data which clearly showed that the stresses in the compression zone were not constant as this model predicted and resulted in an over estimation of the bending strength.

Other models developed by a number of other researchers [Robinson and Cooper 1958], [Moe 1961] and [Zakic 1973] tended to have similar flaws in that they did not adequately represent the behavior of wood.

[Malhotra and Bazan 1980] In 1980 Malhotra and Bazan proposed a “falling branch model” shown in Figure 1.26 in

which the compressive failure zone was represented by a sloping branch which gradually

decreased from the maximum compressive stress. This model showed a good correlation between the observed experimental data and the theoretical values for small clear specimens. Still one of the limitations on this model was that it did not fully consider the effects of member size and strength reducing defects. This model was later revised by Buchanan[Buchanan 1990] which addressed these deficiencies by:

- a. identifying four distinct modes of failure for wood beams depending upon the ratio of the tensile to compressive strengths;
- b. defining a more comprehensive approach to member size effects;
- c. fixing the slope of the falling branch as a constant material property rather than a variable dependent upon the neutral axis depth; and
- d. developing a methodology to incorporate the in-grade strength of commercial lumber in the stress distribution.

This revised stress distribution model is the model used in this thesis to describe the behavior of wood under elastic and inelastic load conditions.

1.4.2 Web Beams

The use of composite wood sections was developed in support of the aircraft industry in the early 1920s for use as stringers and ribs in wooden aircraft. The original use of I-beams in structural applications can be traced back to the mid 1930s where I beams with hardboard webs were used in some European buildings.[Leichti et al. 1989]. In the 1940s the advantages of I beams were first recognized in North America when the

Forest Products Laboratory(FPL) was commissioned to do research on box beams and I beams for use in aircraft in support of the war effort. Despite this initial contact by the FPL, research into the behavior and use of I beams in North America did not begin until the 1970s. Consequently most of the early research in I-beams was produced by European authors on I beams with hardboard webs.[Falk et al 1990]

The composition of the web material and its effect on the overall performance of the beam was researched by [Percival et al 1977] who found that waferboard webbed I-beams tended to be 20% stiffer than plywood webbed I-beams. This result was confirmed by later researchers [Leichti 1986] and [Chen et al. 1987] who determined that the web material has a statistically significant effect on I-beam stiffness and load capacity [Leichti et al. 1990a]. Research by [Leichti and Tang 1983] and [Booth 1974] demonstrated that the shear deflection was a major component of total deflection and that the lower the shear modulus the greater was the shear deflection.[Leichti et al 1990a]

The influence of the flange material on I beam behavior has been investigated by numerous authors from a variety of perspectives. The flanges are generally manufactured from Machine Stress Rated(MSR) lumber or laminated products since there is a direct correlation between the tensile strength and stiffness of the flange and the I-beam load capacity. Investigations by [Samson 1983] revealed that more than half of the variation in the load capacity of I-beams was caused by variations in the Modulus of Elasticity(MOE) of the tension flange. Samson also determined that the optimum situation occurred when the MOE of the tension flange was approximately 1.25 times that of the bottom flange.

The manufacture of I beams using MSR lumber must employ finger joints to achieve the longer distances. Unfortunately examinations of these joints [Jokerst 1981], [Leichti and Tang 1983] and [Leichti 1986] have shown that although the stiffness of the wood appears to be relatively unaffected by the joint, the compressive strength is reduced by approximately 10% while the tensile strength can be decreased by as much as 50%.

As can be seen the I beam is a complex structural system composed of two different anisotropic materials which behave in a number of different manners when subjected to different loading conditions. Generally speaking the design methodology for I-beams has been to assume that the flanges carry the axial tensile and compressive forces while the web carries the shear forces associated with bending. This simplification reduces the design effort without the introduction of significant errors and is a commonly accepted practice.

1.4.3 Fiber Reinforced Plastics

Fiber reinforcement is effective because many materials exhibit increased tensile strength and stiffness in fiber form when compared to the bulk form. It is believed that in 1920 Griffith first discovered this phenomenon by measuring the tensile strength of glass rods of different diameter [Gibson 1994]. In his experiments, Griffith found that as the diameter of the rods were decreased to very small diameters, the tensile strength increased appreciably.

Unfortunately at the time of Griffith's research the development of plastics had not sufficiently matured to provide a suitable matrix for the use of small diameter glass fibers in structural applications. It was not until 1940 that development in plastics reached a point where the reinforced plastic industry began experiments using glass fibers for reinforcement, however, early attempts to fabricate glass reinforced phenolics and melamines did not show much commercial promise. Spurred on by the war effort considerable resources were devoted to the development of plastics and by 1941, JF Hyde at Corning Glass Works produced fiberglass reinforced silicones for use as heat-resistant electrical insulation but these were also too expensive for structural applications. Later in 1941, Lynn King at the Columbia Chemicals Division of the Pittsburgh Plate Glass Company prepared the first glass fiber reinforced polyester resin with an allyglycol carbonate resin. By 1942 glass reinforced polyester resins were beginning to be used in the construction of radomes, aircraft and boat parts.

The introduction of epoxy resins in the early 1950s which substantially increased the performance of reinforced plastics along with the advent of jet propulsion lead to increased research in fibers. Early laboratory tests on graphite whiskers in the late 1950s demonstrated the potential of this area of research by posting tensile strengths of 20000 MPa with a modulus 1000 GPa. Encouraged by these results, Union Carbide working with the United States Air Force did a considerable amount of research into carbon and graphite fibers throughout the 1960s and laid much of the foundation for present day knowledge.

Interest in other fibers was also growing with research being conducted on a number of metallic, organic and inorganic fibers. In 1965 Stephanie Kwolek first discovered Kevlar®, an aromatic polyamide(aramid) fiber. This organic fiber was initially developed for use in radial car tires and later refined to produce Kevlar® 49, a high modulus, high strength fiber suitable for structural applications. In 1969 a boron fiber reinforced epoxy skin was used for the horizontal stabilizers on the F-14 fighter aircraft. The use of a fiber reinforced plastic in such a critical component dispelled the popular notion that plastics were a cheap substitute material and legitimized the use of reinforced plastics in structural applications.

By the early 1970s all structural plastics were available with glass, carbon and aramid reinforcement.

1.4.4 Reinforcement of Wood

The first practical example of wood reinforcement was the flitch beam developed in the late nineteenth century which consisted of a "flitch" of wrought iron sandwiched between two wooden joists. These components were then assembled using bolted connections to form a single composite structural element. In the ensuing years numerous improvements were made to flitch beams including the relocation of the metal plates to the outside surface but the costly and time consuming process of predrilling bolt holes

continued to hamper the wide spread usage of this construction material.

The advent of lightweight yet strong sheet metals and improved nail technology after the Second World War rekindled interest in the flitch beam. The new nails could easily penetrate the sheet metal and produce a strong, lightweight composite material without the costly step of predrilling bolt holes. Although this new generation of flitch beam represented a vast improvement over unreinforced beams, researchers would later find more efficient methods of utilizing reinforcing material.

One of the early pioneers in finding more efficient methods of reinforcing wood was Boomsliter who in 1948 used spiral drive dowels to reinforce timber beams. Later in 1954, Granholm used square rods placed in grooves that had been cut in the bottom and top surfaces of both rectangular and I-cross-section timber beams[Bulleit 1984]. In the 1960s the reinforcement of wood both vertically and horizontally with metals such as aluminum [Mark 1961] [Sliker 1962] and steel [Lantos 1964b] [Bond 1966] [Borgin et al. 1968] was carefully examined by a number of researchers. During this time period, the idea of prestressing and post tensioning of wood reinforced beams was also examined by many researchers [Bond and Sidwell 1965] [Lantos 1964a] [Bond and Long 1964] [Peterson 1965]. This research was continued into the 1970s with works by numerous authors [Krueger 1973] [Coleman and Hurst 1974] [Krueger and Sandburg 1974] [Krueger and Eddy 1974] [Bohannon 1974] [Hoyle 1975] [Kobetz and Krueger 1976].

By the 1980s interest in metal as a reinforcement material was almost nonexistent

due to its relatively high cost and susceptibility to corrosion. This situation was similar to that of the concrete industry which had been examining the use of fiber composites as an alternative to steel reinforcement with some success. These early investigations into the use of fiber reinforcement with concrete clearly demonstrated that fiber composites had the potential of being a practical alternative to metal as a reinforcing material. As a result, the focus of research into wood reinforcement shifted towards the use of glass, carbon and aramid fibers as a reinforcing material.

1.4.5 Fiber Reinforcement of Wood

Investigations into the use of fiberglass as a reinforcement material for wood began in the 1960s [Wangaard 1964] [Biblis 1965] [Theakston 1965] however this line of research was not pursued with the same vigor as metal reinforcement. As a result, much of the research material related to fiber reinforcement of wood is of a contemporary nature and a large portion of this work stems from research on fiber reinforcement of concrete.

Much of the research in fiber reinforcement of concrete is highly specialized with little or no applicability to timber engineering, however, certain works are of direct relevance. The work of [An et al. 1991], [Saadatmanesh and Ehsani 1991], [Ritchie et al. 1991], and [Chajes et al. 1994] are all particularly significant since they deal with fiber reinforcement externally bonded to the tension face of concrete beams.

This line of research was carried one step further when [Triantafillou and Deskovic

1991] illustrated an innovative prestressing technique using externally bonded carbon fiber reinforcement on both wooden and concrete beams. Triantafillou went on to develop this prestressing technique more fully for wooden beams [Triantafillou and Deskovic 1992] and clearly demonstrated that prestressing timber beams with fiber reinforcement yielded significant increases in strength, stiffness and ductility. Increases in the flexural performance were also found in non-prestressed beams [Plevris and Triantafillou 1992] where Plevris presented a complete analytical and experimental investigation into the behavior of beams reinforced with glass and carbon fibers.

The use of glass fibers to reinforce wood can also be found in a paper by Newhook [Newhook and Gilkie 1993] who investigated the influence of glass fiber reinforced polyester (GFRP) laminates applied to the sides and tension faces of timber beams containing natural defects. Newhook found that the use of GFRP laminates yielded modest improvements in the stiffness and significant increases in the ultimate strength of experimental beams. Newhook also noticed that although wood defects played a significant role in initiating failure in non-reinforced beams, the GFRP laminations had a considerable influence in both minimizing the effect of defects and determining the mode of failure.

Most recently there have been numerous works by Tingley [Tingley 1994a and Tingley 1994b] regarding the use of a commercial product known as a Fiber Reinforced Plastic (FiRP) panel in such applications as open-web joists, I-beams, and glued laminated lumber. Due to the contemporary nature of this product, much of the technical details

surrounding the use of FiRP panels in I-beam applications were not available at the time of thesis publication.

2.0 Theoretical Considerations

2.1 General

The theoretical concepts for continuous fiber reinforcement of solid beams have been well established by numerous authors for both concrete and wood. In this thesis, these concepts form the basis of the elastic and inelastic analysis of wooden beams reinforced with a generalized reinforcement configuration at the tension face.

The analytical approach adopted employs a two stage process consisting of a cross-sectional analysis followed by the application of the relevant bending theory to model the elastic and inelastic beam behavior. The cross-sectional analysis uses incremental methods to determine the bending stiffness (EI) of the section for a given moment while the applicable bending theory is applied using integration methods over small segments. Each of these segments is assigned a bending stiffness established by the cross-sectional analysis based on the moment applied to the segment. This methodology describes the elastic and inelastic beam behavior under any reinforcement configuration and is easily modeled by computer.

After a brief review of the wood and fiber models as well as the role of the wood-fiber bond, these analytical techniques are used to examine solid wood beams, subjected to a two point load, reinforced with a variety of reinforcement configurations. Finally, the theoretical principles are then extended to fiber reinforced I-beams subjected to a uniformly distributed load taking into account shear deformations.

2.2 Wood Model

The stress-strain relationship for wood used throughout this thesis is the falling branch model initially proposed by Malhotra and Bazan in 1980 and later modified by Buchanan in 1990. In this model, tensile behavior was assumed to be linearly elastic up to failure stress, f_t while the compressive portion consisted of a bilinear relationship with a linearly elastic segment up to the maximum compressive stress, f_c and a constant slope, mE falling branch beyond that point.

Figure 2.1 - Buchanan's Falling Branch Model

Buchanan's model adopted a linear strain where plane sections were assumed to remain plane. In his bilinear falling branch model, beams subjected to compressive stresses below the proportional limit (Figure 2.2) behaved in a linear elastic manner with the

centroidal axis coinciding with the neutral axis. For beams stressed beyond this point (Figure 2.3), the stresses decreased gradually from the yield stress accompanied by a downward shift in the position of the neutral axis caused by the compressive yielding of material.

Figure 2.2 - Stress and Strain Distribution for Elastic Bending

Figure 2.3 - Stress and Strain Distribution for Inelastic Bending

The model also incorporated a size effect factor for beams of differing depth and length, however since all the beams under study are of the same dimension this factor will not be employed. Buchanan also described a phenomenon known as the stress distribution effect which accounts for the stress in the extreme tension fiber at failure, f_m being greater in bending than in axial tension. Buchanan proposed the following equation to relate the axial tensile strength, f_t to the extreme fiber-tensile stress in bending, f_m :

$$f_m = \left(\frac{k_3 + 1}{c} \right)^{\frac{1}{k_3}} f_t \quad [1]$$

Plevris and Triantafillou later rearranged this equation into the form:

$$f_m = \left(\frac{c}{1 + k_3} \right)^{-\frac{1}{k_3}} f_t \quad [2]$$

where

c = depth of the neutral axis from the tension face as a ratio of beam depth

k_3 = stress distribution parameter

In Buchanan's model the value of the stress distribution parameter (k) was equal to 10 and the slope of the falling branch was set at 0.02 times the modulus of elasticity.

As can be seen from Buchanan's model, a tensile failure tends to be more or less brittle characterized by a sudden fracture of the wood fibers at the failure stress whereas the compressive failure is of a ductile nature exhibiting considerable yielding of the wood prior to failure. In addition, the behavior of wood in tension tends to be affected to a far greater extent by defects in the wood than the compressive strength, consequently a compressive failure in wood beams is more desirable than a tensile failure.

Many woods exhibit some form of compressive yielding prior to failure, the application of tension reinforcement increases the amount of compressive yielding and hence the ductility of the wood by shifting the neutral axis downward. Although the reinforced wood experiences increased ductility due to compressive yielding, the failure mode usually remains as a brittle tensile failure occurring once the tensile stress reaches the failure point in the extreme tension fibers. Other desirable quantities such as increases in beam strength and stiffness are also caused by the application of tensile reinforcement and the corresponding downward shift in the neutral axis.

2.3 Fiber Model

The fiber model used in this thesis is founded on the premise that a unidirectional fiber subjected to a uniaxial tensile stress behaves in a linearly elastic manner until brittle failure. This premise supported by numerous authors [Holmes and Just 1983], [Mallick 1988], [Hollaway 1990] and [Gibson 1994] is based on the implicit assumption that the strains in the matrix are identical to the strains in the fiber. This assumption leads to an

expression for the axial force in a composite as follows [Holmes and Just 1983]:

$$P = \sigma_c A_c = \sigma_f A_f + \sigma_m A_m \quad [3]$$

where

P Total force on the composite

$\sigma_c, \sigma_f, \sigma_m$ Stress in the composite, fiber and matrix respectively

A_c, A_f, A_m Area of the composite, fiber and matrix respectively

from which expressions for the stress and modulus of the composite as a function of the properties of the fiber, matrix and their respective volumetric ratios can be determined as shown below:

$$\sigma_c = \sigma_f V_f + \sigma_m V_m \quad [4]$$

$$E_c = E_f V_f + E_m V_m \quad [5]$$

where

E_c, E_f, E_m Modulus of the composite, fiber and matrix respectively

$V_f = \frac{A_f}{A_c}$ Volume fraction of fiber

$$V_m = \frac{A_m}{A_c} \quad \text{Volume fraction of matrix}$$

The law of mixtures provides a reasonably accurate estimate of the modulus of elasticity and rigidity, however in using the law of mixtures to compute the tensile strength of a composite a more in depth examination of is required to achieve a better understanding of the fiber-matrix interaction.

In a composite consisting of glass fibers in a polyester resin matrix the failure strain of the fibers is greater than the failure strain of the matrix, consequently the matrix is always the first to fail. As can be seen in Figure 2.4, if the fiber content is sufficiently small that the fibers are unable to

Figure 2.4 - Factors Affecting Ultimate Tensile Strength [Hollaway 1990]

carry the load after the matrix fails then the composite will fail and the matrix is said to control the failure mode. According to Hollaway the tensile strength at the point of failure

can be expressed using judicious use of the law of mixtures as follows:

$$\sigma_{cm} = (\sigma_f)_{fail} V_f + \sigma_{mu} V_m \quad [6]$$

where

σ_{cm} Stress of the composite in the zone governed by matrix strength

$(\sigma_f)_{fail}$ Stress in the fiber when the matrix fails

σ_{mu} Ultimate failure stress of the matrix

$V_f = \frac{A_f}{A_c}$ Volume fraction of fiber

$V_m = \frac{A_m}{A_c}$ Volume fraction of matrix

As the fiber content is increased beyond a certain threshold value the fibers are able to sustain the applied loads even after the failure of the matrix. The composite continues to accept additional load until the fibers eventually fail. In this situation shown in Figure 2.4 the fibers control the mode of failure and the contribution of the matrix to the tensile strength is considered to be negligible and hence the tensile strength is denoted by the following expression also derived from the law of mixtures:

$$\sigma_d = \sigma_f V_f \quad [7]$$

where

σ_d Strength of the composite in the zone governed by fiber strength

σ_f Ultimate failure strength of the fiber

$$V_f = \frac{A_f}{A_c} \quad \text{Volume fraction of fiber}$$

This threshold value of fiber content is the point at which the fibers have attained sufficient strength to withstand the applied loads when the matrix fails. Typically this value is quite low and is not encountered in most applications consequently the fibers almost always govern the failure mode.

It is therefore important to ascertain the physical properties of the constituent components of the composite as well as the fiber content of the composite. Unfortunately the extreme variability of fiber reinforced plastics particularly those prepared using a hand lay-up method as used in the experimental portion of this thesis tends to complicate the analysis. To address this problem a number of fiber composite samples of different thickness' were prepared and tested to failure in tension. Based on a subjective assessment of these tests and on the theoretical strengths as computed by the "law of

mixtures", design values for the modulus of elasticity, modulus of rigidity and tensile strength were assigned and used in assessing the performance of the fiber reinforced wood beams.

In applying the law of mixtures it must be realized that the theoretical strengths of newly drawn fibers are significantly higher than fibers used in practice.

Experimentation[Holmes and Just 1983] has shown that in typical hand lay-up applications the failure strain of the fibers is in the neighborhood of 2% while the failure strain of the resin is in the vicinity of 1.5-2.0%. This strain rate multiplied by the modulus of elasticity produces an approximate strength value for hand lay up fibers. Although it is recognized that this value is approximate in nature it is felt that it provides a better estimate of the strength for use in the law of mixtures than the newly drawn strength which is artificially high. The values used in applying the law of mixtures are as follows[Holmes and Just 1983]:

$$E_f = 72,400 \text{ MPa}$$

$$\sigma_{fu} = 0.02 \times E_f = 1,448 \text{ MPa}$$

$$E_m = 3,220 \text{ MPa}$$

$$\sigma_{mu} = 51.5 \text{ MPa}$$

$$\nu_f = 0.2$$

$$\nu_m = 0.35$$

2.4 Wood-Fiber Bond

The analysis of the cross-sectional properties was based on the assumption that the

wood-fiber bond was sufficiently strong that it would not fail as the load was applied to the member. The determination of this bond strength was based on a comparison of the amount of shear force per unit length, or "shear flow" acting on the wood-fiber interface and the capacity of the bond per unit length as measured by experimental results.

The shearing force per unit length acting on the bond surface can be readily determined from established theory. Consider the element shown below in Figure 2.5 which is within the bond interface and subjected to bending.

Figure 2.5 - Force and Stress on Wood-Fiber Bond Element

As can be seen in Figure 2.5 the moment on the right hand side of the element is larger than that on the left. Consequently the normal stresses associated with this moment are also larger and must be counteracted by a horizontal shear force to maintain

equilibrium. The magnitude of this force can be determined from:

$$F = \int \sigma_R dA - \int \sigma_L dA \quad [8]$$

From bending theory $\sigma = \frac{My}{I}$ therefore

$$F = \int \frac{(M + dM)}{I} y dA - \int \frac{M}{I} y dA \quad [9]$$

$$F = \frac{dM}{I} \int y dA \quad [10]$$

From first principles it is known that $dM = V dx$ and the term $\int y dA$ is arbitrarily defined as Q reducing the equation to

$$F = \frac{VQ}{I} dx \quad [11]$$

where the "shear flow" or shear per unit length is defined as

$$q = \frac{F}{dx} = \frac{VQ}{I} \quad [12]$$

The experimental measurements indicated a shear flow capacity in the vicinity of 50 N/mm of bond was achieved at the wood-fiber interface.

2.5 Fiber Reinforced Solid Wood Beams

2.5.1 Cross-Sectional Analysis

The analysis of the cross-sectional properties was based on the work of [Plevris and Triantafillou 1992] who used a similar approach in their examination of the behavior of continuously reinforced wooden beams. In this paper, it was found that the moment-curvature ($M-\phi$) relationship of the cross-section could be used to fully describe the behavior of fiber reinforced beams in bending. However, due to the influence of the stress distribution effect [Buchanan 1990], a closed form solution of the $M-\phi$ curve was not possible. This was overcome by employing a numerical method which incremented the strain at the wood bottom fiber, ϵ_b and calculated the corresponding moment using the appropriate stress-strain relationship until the wood failed in tension.

In developing their numerical method, Plevris and Triantafillou presented a series of equations to evaluate the "normalized" moment under all possible modes. Given that the failure strain of wood is considerably lower than the failure strain of the E-glass under study, it is reasonable to assume that the fiber reinforcement will not rupture prior to a tensile fracture of the wood. This assumption simplifies matters greatly and leaves only two modes to consider:

- a. the wood and fiber reinforcement are linearly elastic; and
- b. the wood has yielded to some extent in compression and the fiber

reinforcement is linearly elastic.

Based on the work of Plevris and Triantafillou, a series of theoretical equations describing the behavior of fiber reinforced wood cross-sections under both modes is presented below with a more complete derivation from first principles in Appendix 1. Initially both the wood and fiber reinforcement behave in a linearly elastic fashion and the moment equation can be written as follows:

$$M = \left(\frac{C(c+2) - Tc}{3} \right) bh^2 E_w \quad [13]$$

where

$$T = \frac{c}{2} \epsilon_b \quad [14]$$

$$C = \frac{(1-c)^2}{2c} \epsilon_b \quad [15]$$

$$c = \frac{1}{2 \left(\frac{A_{fp} E_{fp}}{A_w E_w} + 1 \right)} \quad [16]$$

where

b = width of the wood

h = height of the wood

ϵ_b = strain in the extreme tension fiber of the wood

A_{fp} = cross-sectional area of the fiber/resin composite

E_{fp} = modulus of elasticity of the fiber/resin composite

A_w = cross-sectional area of the wood

E_w = modulus of elasticity of the wood

As the load is increased, the wood begins to yield compressively which results in the neutral axis shifting downward as the load increases. This downward movement of the neutral axis produces a corresponding reduction in the overall bending stiffness (EI) of the section which results in non-linear behavior. The moment equation which describes this condition is shown below with a more complete derivation in Appendix 2.

$$M = C_2 \left\{ c + \frac{\epsilon_c}{\epsilon_b} c + \left[1 - \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right) c \right] \left[\frac{\left(1 + \frac{2m}{3} \right) \frac{c}{m} \frac{\epsilon_c}{(c-1)\epsilon_b} + \frac{2}{3}}{\left[\frac{(2+m)}{m} \frac{c}{(c-1)\epsilon_b} + 1 \right]} \right] + C_1 \left(1 + \frac{2\epsilon_c}{3\epsilon_b} \right) - \frac{Tc}{3} \right\} h^2 E_w \quad [17]$$

where

$$C_2 = \frac{1}{2} \left[1 - c \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right) \right] \left[(2+m)\epsilon_c + m \left(1 - \frac{1}{c} \right) \epsilon_b \right] \quad [18]$$

$$C_1 = \frac{c \epsilon_c^2}{2 \epsilon_b} \quad [19]$$

$$T = \frac{c}{2} \epsilon_b \quad [20]$$

and c is obtained by solving the following quadratic equation

$$(1+m) \left(1 + \frac{\epsilon_c}{\epsilon_b} \right)^2 c^2 + \left[2 \frac{A_{fp} E_{fp}}{A_w E_w} - 2(1+m) \frac{\epsilon_c}{\epsilon_b} - 2m \right] c + m = 0 \quad [21]$$

where

ϵ_c = strain corresponding to the compressive strength of wood

ϵ_b = strain in the extreme tension fiber of the wood

$m = 0.02$ [Buchanan 1990] slope of falling branch model mE

A_{fp} = cross-sectional area of the fiber/resin composite

E_{fp} = modulus of elasticity of the fiber/resin composite

A_w = cross-sectional area of the wood

E_w = modulus of elasticity of the wood

The curvature relationship with respect to the strain is well known and can be written as:

This relationship provides the elastic and inelastic bending stiffness of different reinforcement configurations in response to an applied bending moment.

2.5.2 Beam Behavior

In evaluating the flexural performance of solid wood sections, the basic assumption of classical bending theory that plane sections remain plane was assumed to hold and that the effects of shear deformations were negligible

The assumed loading for solid beams consisted of a two point load as shown below in Figure 2.8 .

Figure 2.8 - Solid Beam Loading Conditions

$$\psi = \frac{\epsilon_b}{ch}$$

[22]

The linearly elastic and inelastic behavior of the cross-section can now be modeled using computer simulations by incrementing the bottom strain, ϵ_b , and applying these equations to produce a plot of the moment-curvature relationship for a rectangular solid wood-fiber composite as shown in Figure 2.6.

Figure 2.6 - Moment Curvature Relationship for Solid Wood

This moment-curvature relationship can be further manipulated to obtain an expression for the bending stiffness (EI) as a function of the applied moment using the following incremental equation:

$$EI = \frac{\Delta M}{\Delta \psi} \quad [23]$$

where

EI = bending stiffness

ΔM = increment in moment

$\Delta \psi$ = increment in curvature

The bending stiffness can then be plotted against the applied moment to produce a graphical representation of the effect of compressive yielding on the bending stiffness as shown below in Figure 2.7.

Figure 2.7 - Bending Stiffness - Moment Relationship

The corresponding moment equations for this particular condition can be written as follows:

$$M = Px \quad \text{for} \quad 0 \leq x \leq \frac{L}{3} \quad [24]$$

$$M = \frac{PL}{3} \quad \text{for} \quad \frac{L}{3} \leq x \leq \frac{2L}{3} \quad [25]$$

$$M = PL - Px \quad \text{for} \quad \frac{2L}{3} \leq x \leq L \quad [26]$$

These moment equations can also be expressed in terms of singularity functions as follows:

$$M = Px - P\left\langle x - \frac{L}{3} \right\rangle - P\left\langle x - \frac{2L}{3} \right\rangle \quad [27]$$

where the $P\left\langle x - \frac{L}{3} \right\rangle$ and $P\left\langle x - \frac{2L}{3} \right\rangle$ terms are singularity functions which signifies that when the term inside the pointed bracket is positive the brackets have the same effect as normal brackets, but when the term inside the bracket is negative the value inside is equal to zero.

Insertion of this applied moment into the general bending equation, neglecting shear

deformation and rearranging yields the following expression for the curvature of the beam:

$$M = -EI \frac{d\psi}{dx} \quad \text{General Bending Equation} \quad [28]$$

$$\psi = \int \left[\frac{Px}{EI} - \frac{P}{EI} \left\langle x - \frac{L}{3} \right\rangle - \frac{P}{EI} \left\langle x - \frac{2L}{3} \right\rangle \right] \quad [29]$$

where

M = Applied bending moment

P = Applied load (two point load system)

EI = Bending stiffness

ψ = Rotation of the beam due to bending

To accommodate a variable bending stiffness over the length of the beam, the moment equation can be integrated in segments with the appropriate bending stiffness being assigned to each segment. This segmental bending stiffness can be computed using the Moment-Bending Stiffness relationship derived from the cross-sectional analysis and shown in Figure 2.7. The accuracy of this approach can be significantly enhanced by decreasing the thickness of these segments and by increasing the number of segments in the center of the beam as illustrated below in Figure 2.9.

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Figure 2.9 - Typical Beam Segments

Using this approach, the solid beams examined in this thesis were subdivided into strips 10mm thick at the ends of the beam and 5mm thick in the center. A bending stiffness (EI) for each strip was then computed based on the applied moment using the Moment-Bending Stiffness relationship previously derived from the cross-sectional analysis. The curvature of the beam was then computed using:

$$\begin{aligned} \psi = & \left[-\frac{Px^2}{2EI} + \frac{P}{2EI} \left\langle x - \frac{L}{3} \right\rangle^2 + \frac{P}{2EI} \left\langle x - \frac{2L}{3} \right\rangle^2 \right]_0^{x_1} + \\ & \dots + \left[-\frac{Px^2}{2EI} + \frac{P}{2EI} \left\langle x - \frac{L}{3} \right\rangle^2 + \frac{P}{2EI} \left\langle x - \frac{2L}{3} \right\rangle^2 \right]_{x_{n-1}}^L + C_1 \end{aligned} \quad [30]$$

where C_1 is a constant of integration which can be evaluated by applying the boundary

condition of $\psi = 0$ when $x = \frac{L}{2}$ to yield:

$$C_1 = \left[\frac{Px^2}{2EI} - \frac{P}{2EI} \left\langle x - \frac{L}{3} \right\rangle^2 \right]_0^{x_1} + \dots + \left[\frac{Px^2}{2EI} - \frac{P}{2EI} \left\langle x - \frac{L}{3} \right\rangle^2 \right]_{x_{n/2-1}}^{L/2} \quad [31]$$

The deflection of the beam was then obtained by integrating this curvature expression over the thickness of the finite strips again employing the bending stiffness determined from the cross-sectional analysis.

$$\begin{aligned} \Delta = & \left[\frac{Px^3}{6EI} + \frac{P}{6EI} \left\langle x - \frac{L}{3} \right\rangle^3 + \frac{P}{6EI} \left\langle x - \frac{2L}{3} \right\rangle^3 + C_1 x \right]_0^{x_1} + \dots \\ & + \left[\frac{Px^3}{6EI} + \frac{P}{6EI} \left\langle x - \frac{L}{3} \right\rangle^3 + \frac{P}{6EI} \left\langle x - \frac{2L}{3} \right\rangle^3 + C_1 x \right]_{x_{n-1}}^L + C_2 \end{aligned} \quad [32]$$

where the constant of integration C_2 can be found to equal zero through an examination of the boundary condition $\Delta = 0$ at $x = 0$. Hence the maximum deflection which occurs at midspan can be simplified to the following equation:

$$\Delta_{\max} = \left[\frac{Px^3}{6EI} + \frac{P}{6EI} \left\langle x - \frac{L}{3} \right\rangle^3 + C_1 x \right]_0^{x_1} + \dots + \left[\frac{Px^3}{6EI} + \frac{P}{6EI} \left\langle x - \frac{L}{3} \right\rangle^3 + C_1 x \right]_{x_{n/2-1}}^{L/2} \quad [33]$$

This expression completely describes the elastic and inelastic load-deflection characteristics of a solid beam subjected to a two point load with any reinforcement configuration. The effects of both the reinforcement and inelasticity are reflected in value of the finite strip bending stiffness (EI) as determined by the cross-sectional analysis.

2.6 Fiber Reinforced I-Beams

2.6.1 General

The design of I-beams is based on the assumption that the flanges primarily resist the axial stresses while the web withstands mainly the shear stresses induced by bending. In this model, the stress distribution effect used with solid wood beams is not employed since it is assumed that the bottom flange is subjected to mainly axial forces with little stress redistribution.

2.6.2 Cross Sectional Analysis

The method used to analyze the I-beam cross-section was similar to Traintafillou's approach with solid wood sections, however since the web material possessed a different modulus than that of the flange material, a discontinuous stress distribution resulted as shown in Figure 2.10. It is clearly evident from this figure that the flanges are subjected to mainly axial stress with little or no load sharing thereby validating the assumption that the stress distribution effect was not applicable in the case of I-beams.

Figure 2.10 - General Stress-Strain Relationship for Elastic I-Beam Bending

Given that the failure strain of the E-glass under study is considerably higher than the failure strain of the flange material in tension, it was once again assumed that the fiber reinforcement would not rupture prior to a tensile fracture of the wood. This assumption again reduced the failure modes to the following:

- a. the I-beam and fiber reinforcement are both linearly elastic; and
- b. the I-beam has yielded to some extent in compression and the fiber reinforcement remains linearly elastic.

Using the principles established by Plevris and Triantafillou, the theoretical equations describing the behavior of fiber reinforced I-beams under both modes of failure are described below with a more detailed derivation from first principles in Appendix 3. The initial state where the I-beam and the fiber reinforcement behave in a linearly elastic fashion has the following general moment equation:

$$\begin{aligned}
 M = & T_{fp}ch + T_{f_1}\left(c - \frac{t_b}{2h}\right)h + T_{f_2}\left(c - \frac{t_b}{3h}\right)h + T_w\left(\frac{2}{3}\left(c - \frac{t_b}{h}\right)\right)h \\
 & + C_w\left(\frac{2}{3}\left(1 - c - \frac{t_i}{h}\right)\right)h + C_{f_1}\left(1 - c - \frac{t_i}{h}\right)h + C_{f_2}\left(1 - c - \frac{t_i}{3h}\right)h
 \end{aligned} \quad [34]$$

where

$$T_{fp} = \epsilon_b A_{fp} E_{fp} \quad [35]$$

$$T_{f_1} = \left(\frac{c - \frac{t_b}{h}}{c}\right) \epsilon_b A_b E_b \quad [36]$$

$$T_{f_2} = \left(1 - \left(\frac{c - \frac{t_b}{h}}{c}\right)\right) \frac{\epsilon_b A_b E_b}{2} \quad [37]$$

$$T_w = \left[\frac{\left(c - \frac{t_b}{h}\right)^2}{\left(c - \left(\frac{t_b + t_i}{h}\right)c\right)}\right] \frac{\epsilon_b A_{web} E_{web}}{2} \quad [38]$$

$$C_w = \left[\frac{\left(1 - c - \frac{t_i}{h}\right)^2}{\left(c - \left(\frac{t_b + t_i}{h}\right)c\right)}\right] \frac{\epsilon_b A_{web} E_{web}}{2} \quad [39]$$

$$C_{f_1} = \left[\frac{\left(1 - c - \frac{t_t}{h}\right)}{c} \right] \epsilon_b A_t E_t \quad [40]$$

$$C_{f_2} = \frac{t_t}{2ch} \epsilon_b A_t E_t \quad [41]$$

$$c = \frac{\left[A_b E_b \left(\frac{t_b}{h} \right) - A_t E_t \left(\frac{t_t}{h} \right) + 2 A_t E_t + \frac{A_{web} E_{web} \left(\left(\frac{t_t}{h} \right)^2 - \left(\frac{t_b}{h} \right)^2 - 2 \frac{t_t}{h} + 1 \right)}{\left(1 - \left(\frac{t_b + t_t}{h} \right) \right)} \right]}{2 \left[A_{fpr} E_{fpr} + A_b E_b + A_t E_t + \frac{A_{web} E_{web} \left(-\frac{t_t}{h} - \frac{t_t}{h} + 1 \right)}{\left(1 - \left(\frac{t_b + t_t}{h} \right) \right)} \right]} \quad [42]$$

where

c	distance to the neutral axis as a ratio of the depth
h	depth of the beam
t_b, t_t	Thickness of the bottom and top flange respectively
ϵ_b	Strain at the bottom fiber of the wood
A_b, E_b	Area and modulus of the bottom flange
A_t, E_t	Area and modulus of the top flange
A_{web}, E_{web}	Area and modulus of the web
A_{fpr}, E_{fpr}	Area and modulus of the fiber reinforced plastic

This equation to find the position of the neutral axis at first glance is intimidating but it must be remembered that this represents the general case of a fiber reinforced I-beam in which the top and bottom flanges do not necessarily possess the same properties or cross-section. The expression can be greatly simplified if the I-beam is doubly symmetric with identical top and bottom flanges. It is obvious that certain parameters will be equal in the case of doubly symmetric I-beams, specifically:

$$t_b = t_t$$

$$A_b = A_t$$

$$E_b = E_t$$

Substituting these values into the equation [42] reduces the equation to the following form:

$$c = \frac{2A_b E_b + A_{web} E_{web}}{2[A_{fp} E_{fp} + 2A_b E_b + A_{web} E_{web}]} \quad [43]$$

As the load is increased beyond the proportional limit into the inelastic region, the I-beam begins to yield in compression and the falling branch model applies as shown in Figure 2.11. This compressive yielding results in a downward shift of the neutral axis which leads to a reduction in the bending stiffness (EI) and non-linear behavior.

Figure 2.11 - General Stress-Strain Relationship for Inelastic I-beam Bending

The expression for the moment which describes this condition is shown below with a more complete derivation from first principles found in Appendix 4:

$$\begin{aligned}
 M = & T_{fp}ch + T_{f_t}\left(c - \frac{t_b}{2h}\right)h + T_{f_t}\left(c - \frac{t_b}{3h}\right)h + T_w\left(\frac{2}{3}\left(c - \frac{t_b}{h}\right)\right)h \\
 & + C_w\left(\frac{2}{3}\left(1 - c - \frac{t_t}{h}\right)\right)h + C_{f_t}\left(\frac{\epsilon_c}{\epsilon_b}c - \frac{1}{2}\left(\frac{t_t}{h} - \left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)\right)h \\
 & + C_{f_t}\left(\frac{\epsilon_c}{\epsilon_b}c - \frac{1}{3}\left(\frac{t_t}{h} - \left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)\right)h \\
 & + C_{f_t}\left(\frac{\epsilon_c}{\epsilon_b}c + \frac{1}{2}\left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)h + C_{f_t}\left(\frac{\epsilon_c}{\epsilon_b}c + \frac{1}{3}\left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)h
 \end{aligned} \tag{44}$$

where

$$T_{fp} = \epsilon_b A_{fp} E_{fp} \tag{45}$$

$$T_{f_1} = \left(\frac{c - \frac{t_b}{h}}{c} \right) \epsilon_b A_b E_b \quad [46]$$

$$T_{f_2} = \left(1 - \left(\frac{c - \frac{t_b}{h}}{c} \right) \right) \frac{\epsilon_b A_b E_b}{2} \quad [47]$$

$$T_w = \left[\frac{\left(c - \frac{t_b}{h} \right)^2}{\left(c - \left(\frac{t_b + t_i}{h} \right) c \right)} \right] \frac{\epsilon_b A_{web} E_{web}}{2} \quad [48]$$

$$C_w = \left[\frac{\left(1 - c - \frac{t_i}{h} \right)^2}{\left(c - \left(\frac{t_b + t_i}{h} \right) c \right)} \right] \frac{\epsilon_b A_{web} E_{web}}{2} \quad [49]$$

$$C_{f_1} = \left[\frac{\frac{t_i}{h} - \left(1 - c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_i}{h}} \right] \left[\frac{1 - c - \frac{t_i}{h}}{c} \right] \epsilon_b A_i E_i \quad [50]$$

$$C_{f_1} = \left[\frac{\frac{t_1}{h} - \left(1 - c - \frac{\epsilon_c}{\epsilon_b} c\right)}{\frac{t_1}{h}} \right] \left[\frac{\epsilon_c}{\epsilon_b} - \left(\frac{1 - c - \frac{t_1}{h}}{c} \right) \right] \frac{\epsilon_b A_i E_i}{2} \quad [51]$$

$$C_{f_2} = \left[\frac{1 - c - \frac{\epsilon_c}{\epsilon_b} c}{\frac{t_1}{h}} \right] \left[\epsilon_c - m \left(\frac{1 - c}{c} \epsilon_b - \epsilon_c \right) \right] A_i E_i \quad [52]$$

$$C_{f_3} = \left[\frac{1 - c - \frac{\epsilon_c}{\epsilon_b} c}{\frac{t_1}{h}} \right] \left[\epsilon_c - \left(\epsilon_c - m \left(\frac{1 - c}{c} \epsilon_b - \epsilon_c \right) \right) \right] \frac{A_i E_i}{2} \quad [53]$$

The location of the neutral axis is found by solving the following quadratic equation:

$$\left[\frac{(3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)^2}{\frac{t_1}{h}} A_i E_i \right] c^2 + \left[2(A_{fp} E_{fp} + A_b E_b + A_i E_i + A_{web} E_{web}) - \frac{2A_i E_i (3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)}{\frac{t_1}{h}} \right] c + \left[\frac{A_{web} E_{web} \left(\left(\frac{t_b}{h} \right)^2 - \left(\frac{t_i}{h} \right)^2 + 2 \frac{t_i}{h} - 1 \right)}{1 - \frac{t_i + t_b}{h}} + \frac{A_i E_i \left(\left(\frac{t_i}{h} - 1 \right)^2 + 3m \right)}{\frac{t_i}{h}} - 2A_b E_b \frac{t_b}{h} \right] = 0 \quad [54]$$

where

- ϵ_c The strain at which compressive yielding begins
- m The slope of the falling branch set at 0.02

This general quadratic equation used to determine the value of c is applicable to all reinforced I-beams. In the case of doubly symmetric I-beams this equation can be reduced to:

$$\left[\frac{(3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)^2}{\frac{t_b}{h}} A_b E_b \right] c^2 + \left[2(A_{fp} E_{fp} + 2A_b E_b + A_{web} E_{web}) - \frac{2A_b E_b (3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)}{\frac{t_b}{h}} \right] c + \left[\frac{A_b E_b \left(\left(\frac{t_b}{h} - 1 \right)^2 + 3m \right)}{\frac{t_b}{h}} - 2A_b E_b \frac{t_b}{h} - A_{web} E_{web} \right] = 0 \quad [55]$$

The linearly elastic and inelastic behavior of fiber reinforced I-beams can be modeled using computer simulations by incrementing the bottom strain, ϵ_b and applying the above equations to determine the value of the moment for a given strain. The curvature corresponding to this moment can be easily determined using the relationship:

$$\psi = \frac{\epsilon_b}{ch}$$

[56]

This process effectively establishes a moment-curvature relationship which can be plotted as shown below in Figure 2.12.

Figure 2.12 - Moment Curvature Relationship for I-beams

As was done with solid wood beams, an expression for the bending stiffness (EI) as a function of the applied moment can be extracted from this moment-curvature relationship using the following incremental equation:

$$EI = \frac{\Delta M}{\Delta \psi} \quad [57]$$

where

EI = bending stiffness

ΔM = increment in moment

$\Delta \psi$ = increment in curvature

The bending stiffness can then be plotted against the applied moment to produce a graphical representation of the effect of compressive yielding on the bending stiffness as shown below in Figure 2.13.

Figure 2.13 - Bending Stiffness(EI) - Moment Relationship for I-Beams

2.6.3 Shear Rigidity

The modulus of rigidity, G is analogous to the modulus of elasticity, E in that it relates the stresses to the strains in shear and is constant for a linear-elastic material. In assessing the shear deformations of I-beams it was assumed that the G value remained constant as the beam experienced inelastic behavior. Although this premise is not supported by elastic theory the errors introduced by this assumption are relatively small in comparison to the overall deflection.

2.6.4 I-Beam Bending Behavior

In classical beam theory, plane sections perpendicular to the longitudinal axis are assumed to remain both plane and perpendicular after bending and any deformations due to shear are considered negligible. This model of beam behavior is quite acceptable when dealing with solid lumber beams since the shear deformations are quite small in comparison to the deflection due to bending. However, in the case of I-beams shear deformations can account for a significant portion of the total beam deflection [Maly 1987][Triche 1990] which clearly cannot be considered negligible and a different beam theory is required to account for these shear deformations. The Timoshenko beam theory [Timoshenko 1955] which assumes that plane sections perpendicular to the longitudinal axis remain plane but not necessarily perpendicular during bending accounts for both bending and shear deformations.

The general form of the Timoshenko bending moment and shear expressions are as follows:

$$M = -EI \frac{d\psi}{dx} \quad [58]$$

$$V = kGA \left(\frac{dy}{dx} - \psi \right) \quad [59]$$

where

M = Bending moment

EI = Bending stiffness

ψ = Rotation of the beam due to bending

V = Shear force

GA = Shear rigidity of the section

k = Shape factor

$\frac{dy}{dx}$ = Total rotation of the section

The shape factor k is a complex function which is dependent upon the shape of the member. Booth [Booth 1977] developed an expression for a shear shape factor, S which was the inverse of k for wood I-beams using strain energy methods. This factor was later modified by Fridley [Fridley 1991] to produce the following:

$$k = \frac{10(1 + 3m)^2}{12 + 72m + 150m^2 + 90m^3 + 30mn^2 + 30n^2m^2} \quad [60]$$

where

$$m = \frac{2bt_f}{ht_w}$$

$$n = \frac{b}{h}$$

and

b = width of flange

t_f = thickness of the flange

t_w = thickness of the web

h = total depth of the section

An underlying assumption in both the work of Booth and Fridley was that the shear rigidity, kGA could be approximated using the published shear modulus and effective cross-sectional area of the web. This procedure was incorporated into the calculations of shear deflections throughout this thesis.

Figure 2.14 - Load Configuration for I-Beams

The I beams under investigation in this thesis consist of simply supported beams loaded with a uniformly distributed load as shown in Figure 2.14, hence the following equations for the moment and shear apply:

$$M(x) = \left(\frac{w}{2} \right) (Lx - x^2) \quad [61]$$

$$V(x) = \frac{wL}{2} - wx \quad [62]$$

where

$M(x)$ = Bending moment at any point x along the beam

$V(x)$ = Shear at any point x along the beam

w = Uniformly distributed load per unit length

L = Span length

Substituting this equation into the general expression for the Timoshenko beam and rearranging the equation yields:

$$-EI \frac{d\psi}{dx} = \left(\frac{w}{2} \right) (Lx - x^2) \quad [63]$$

To account for the variable nature of the bending stiffness(EI) the curvature was obtained by integrating this expression in segments and assigning the appropriate EI to each individual segment. This produces a curvature equation as follows:

$$EI\psi(x) = \left[\frac{wx^3}{6} - \frac{wLx^2}{4} \right]_0^{x_1} + \dots + \left[\frac{wx^3}{6} - \frac{wLx^2}{4} \right]_{x_{n-1}}^L + C_1 \quad [64]$$

where C_1 is a constant of integration which can be solved by examining the boundary conditions. In this case it is known that $\psi(x) = 0$ at $\frac{L}{2}$ which leads to a value of:

$$C_1 = \left[\frac{wLx^2}{4EI} - \frac{wx^3}{6EI} \right]_0^{x_1} + \dots + \left[\frac{wLx^2}{4EI} - \frac{wx^3}{6EI} \right]_{x_{n/2-1}}^{L/2} \quad [65]$$

Substituting this expression and equation [62] into the general Timoshenko equation for shear produces

$$\frac{wL}{2} - wx = kGA \left(\frac{dy}{dx} - \left\{ \left[\frac{wx^3}{6EI} - \frac{wLx^2}{4EI} \right]_0^{x_1} + \dots + \left[\frac{wx^3}{6EI} - \frac{wLx^2}{4EI} \right]_{x_{n-1}}^L + C_1 \right\} \right) \quad [66]$$

Rearranging this equation yields

$$\frac{dy}{dx} = \left[\frac{wx^3}{6EI} - \frac{wLx^2}{4EI} \right]_0^{x_1} + \dots + \left[\frac{wx^3}{6EI} - \frac{wLx^2}{4EI} \right]_{x_{n-1}}^L + C_1 + \frac{wL}{2kGA} - \frac{wx}{kGA} \quad [67]$$

This equation can now be integrated in segments using the proper bending stiffness for each segment as determined in the cross-sectional analysis to produce an expression for the deflection at any point:

$$y(x) = \left[\frac{wx^4}{24EI} - \frac{wLx^3}{12EI} + C_1x + \frac{wLx}{2kGA} - \frac{wx^2}{2kGA} \right]_0^{x_1} + \dots + \left[\frac{wx^4}{24EI} - \frac{wLx^3}{12EI} + C_1x + \frac{wLx}{2kGA} - \frac{wx^2}{2kGA} \right]_{x_{n-1}}^L + C_2 \quad [68]$$

where C_2 is another constant of integration which can again be solved using the boundary conditions. In this situation since the deflection at $x = 0$ is equal to zero the value of the constant C_2 is also zero.

This equation fully describes the elastic and inelastic load-deflection characteristics of any

reinforced I-beam subjected to a uniformly distributed load in relation to the applied load.

It is clear that this method and the principles involved could be extended to any load condition for any reinforcement configuration.

3.0 Experimentation

3.1 General

The experimental portion of the thesis was conducted essentially to verify the theoretical concepts established earlier. The properties of the wood were determined using data published by the Canadian Wood Council while the behavior of the fiber and the wood-fiber interface was established through experimentation.

Once the component properties were established, a series of small beams composed of flange material were tested in flexure using a two point load with a variety of reinforcement configurations and compared to the theoretical prediction and a set of unreinforced control beams. The results of these tests were then used to assess the effectiveness of each type of reinforcement configuration and formed the basis in deciding what type of reinforcement to employ in the full scale I-beam tests.

The I-beam tests were then conducted by simulating a uniformly distributed load using a series of pneumatic jacks on I-beams strengthened with the selected reinforcement configurations. The test results were then compared to the theoretical model and the results from a set of unreinforced control beams.

3.2 Component Properties

3.2.1 Flange

The flange material of the I-beams consisted of a Machine Stress Rated(MSR)

2100f-1.8E Spruce-Pine-Fir(SPF). The material properties of the flanges were not tested experimentally since these parameters were well documented in other publications. The 1994 "Canadian Lumber Properties" book published by the Canadian Wood Council was used as a reference in obtaining the following mean values based on a moisture content of approximately 12%:

- a. Modulus of Elasticity for MSR 2100f-1.8E SPF = 12,816 MPa**
- b. Modulus of Elasticity No1 SPF = 9,700 MPa**
- c. Tensile Strength of MSR 2100f-1.8E SPF = 40.33 .MPa (mean)**
- d. Modulus of Rupture of MSR 2100f-1.8E SPF = 60.93 MPa (mean)**
- e. Compressive Strength of MSR 2100f-1.8E SPF = 48.77 MPa (computed)**

Although the Canadian Wood Council did not carry out any specific testing to determine the compressive strength of MSR lumber recent data suggests that the ratio of compressive strength to modulus of rupture is between 0.7 and 1.18[Canadian Wood Council 1994] For the purpose of this thesis an arbitrary value of 0.80 was selected for this ratio making the mean compressive strength of MSR 2100f-1.8E SPF equal to 48.77 MPa.

The mechanical properties used to evaluate the performance of the small beams required careful consideration. The modulus assigned to the MSR lumber was based on mechanical tests performed at its original length, however it was felt that once the MSR lumber was cut into smaller pieces this modulus no longer applied since the position of

defects would play a more predominant role and would result in a lower modulus.

Therefore a modulus of 9700 MPa which corresponds to the modulus of a No 1 SPF was used in lieu of the 12816 MPa MSR modulus.

The effect of the OSB insert in the MSR lumber also had a significant effect on the mechanical properties. This insert which measured approximately 11.1 mm wide by 15mm deep represented approximately 5% of the total cross sectional area and possessed a modulus of about half and significantly lower compressive and tensile strength characteristics when compared to the MSR lumber. To compensate for the effects this weaker insert in the compressive region the compressive strength used in the models was reduced by 15% . The presence of this insert also lead to a downward shift in the neutral axis which results in a reduction in the tensile stresses therefore tensile strength was increased by 15% to compensate.

The values used in evaluating the performance of the small beams after all the corrections were applied consisted of the following:

- a. Modulus of Elasticity - 9700 MPa;
- b. Tensile Strength - 41 MPa;
- c. Compressive Strength - 47 MPa;

3.2.2 Web

The properties of the web material which consists of the proprietary Performance

Plus™ web material, are not readily available and difficult to test experimentally since special test setups are required to establish the relevant mechanical properties. The shear rigidity(GA) was derived from data submitted by the manufacturer to the Canadian Construction Materials Center(CCMC) and found to be approximately 60,000N/mm of web depth which was used in the computations of shear deformations.

3.2.3 Fiber

The fiber used throughout the experimental phase was a #81084 unidirectional E-glass fiber manufactured by JB Martin Company. The resin employed with this E-glass was a standard polyester resin combined with a catalyst or initiator in a 1:100 ratio.

In order to determine the modulus of elasticity and the ultimate tensile strength of the E-glass resin composite, a total of 31 samples were tested. The samples were grouped into three batches as follows:

- **Group A** - This group consisted of 9 samples with 1 layer of E-glass fiber. The average fiber content for this group was approximately 22.1 % by volume.
- **Group B** - This group had 11 samples with 2 layers of E-glass fibers. The average fiber content for these samples was about 37.9 % by volume.

- **Group C** - The last batch had 11 samples with 3 layers of E-glass fibers.
The average fiber content by volume was approximately 42.3 %.

The samples were all prepared in the same manner which consisted of following procedures:

- Preparation*** The working surface was first prepared by applying a suitable release agent to ensure that the resin does not bond to the working surface;
- Resin Mixing*** The resin was then combined with the catalyst in a 100:1 ratio, which provides about 30 minutes of working time before the resin sets;
- Initial Wetting*** The working surface was then "wet down" with resin using a roller;
- Fiber Layer*** The glass fibers were then placed on the wet working surface in the form of sheets of unidirectional woven fibers with an increased number of layers at the ends of the samples and the resin was worked into the fibers using the roller. For those samples with multiple layers of glass

fibers additional resin was required to achieve full saturation of the fibers.

e. *Curing* The samples were permitted at least 48 hours to cure at room temperature prior to testing.

f. *Coupons* After curing, the sheets of fiber reinforced plastic were then cut into test coupons and placed in the test jaws wrapped with a piece of coarse grade emery cloth to prevent slippage as shown in Figure 3.1.

The testing was conducted in accordance with ASTM standard D3039/D3039M-93 at a loading rate of 1.25mm/minute (0.05 inches/minute) using an Instron testing machine. The deflection of the sample was measured at a rate of 1 reading per second using an extensometer with a capacity of 50mm (2 in). The control of both the Instron machine and the data acquisition were handled by a personal computer using SCIEMETRIC CMT 200 Version 4 software.

As can be seen from the table of the data collected (Table 3A), the modulus of elasticity

increases as the number of fiber layers in the fiber-resin composite increases.

<i>Fiber Modulus</i>			
<i>Single Layer</i>		<i>Double Layer</i>	<i>Triple Layer</i>
	MPa	MPa	MPa
	14,503	32,976	37,081
	10,772	32,672	31,822
	21,119	26,787	32,248
	16,554	30,708	33,345
	20,849	32,340	34,108
	18,865	35,516	33,448
	19,825	33,724	37,796
	16,107	30,014	37,686
	14,364	34,348	29,814
		36,405	33,054
		32,411	32,874
<i>Mean</i>	16,995	32,537	33,934
<i>Standard Deviation</i>	3,468	2,675	2,563

Table 3A - Fiber Modulus of Elasticity Values

This is due to the fact that in multiple layered composites, the layers tend to “mesh” together as the resin is applied. This results in a marked increase in the fiber content of the sample as can be seen in Figure 3.3 which in turn leads to increased performance.

The theoretical modulus of elasticity can be computed using the law of mixtures and the fiber contents determined through laboratory measurements using the resin ignition methods described in ASTM standard D2584-68 as follows:

Sample	Fiber Modulus (GPa)	Fiber Density (g/cm ³)	Fiber Length (mm)	Matrix Modulus (GPa)	Matrix Density (g/cm ³)	Matrix Volume Fraction	Fiber Content (%)
<u>1 Layer</u>							
<i>S1-1</i>	0.261	2.54	0.1028	0.424	1.19	0.3563	22.4 %
<i>S1-2</i>	0.352	2.54	0.1386	0.561	1.19	0.4714	22.7 %
<i>S1-3</i>	0.133	2.54	0.0524	0.235	1.19	0.1975	21.0 %
<i>S1-4</i>	0.305	2.54	0.1201	0.492	1.19	0.4134	22.5 %
<i>S1-5</i>	0.154	2.54	0.0606	0.254	1.19	0.2134	22.1 %
<u>2 Layers</u>							
<i>S2-1</i>	0.172	2.54	0.0677	0.145	1.19	.1218	35.72 %
<i>S2-2</i>	0.269	2.54	0.1059	0.184	1.19	0.1546	40.65 %
<i>S2-3</i>	0.278	2.54	0.1094	0.200	1.19	0.1681	39.44 %
<i>S2-4</i>	0.253	2.54	0.0996	0.183	1.19	0.1538	39.31 %
<i>S2-5</i>	0.243	2.54	0.0957	0.214	1.19	0.1798	34.37 %
<u>3 Layers</u>							
<i>S3-1</i>	0.586	2.54	0.2307	0.354	1.19	0.2975	43.68 %
<i>S3-2</i>	0.463	2.54	0.1823	0.302	1.19	0.2538	41.80 %
<i>S3-3</i>	0.405	2.54	0.1594	0.254	1.19	0.2134	42.76 %
<i>S3-4</i>	0.485	2.54	0.1909	0.331	1.19	0.2782	40.70 %
<i>S3-5</i>	0.574	2.54	0.2260	0.359	1.19	0.3017	42.83 %

Table 3B - Fiber Content of Samples

$$E_c = E_f V_f + E_m V_m$$

$$E_c = 72,400 \times 0.221 + 3220 \times 0.80$$

1 Layer of E-Glass

$$E_c = 18,509 \text{ MPa}$$

$$E_c = E_f V_f + E_m V_m$$

$$E_c = 72,400 \times 0.379 + 3220 \times 0.621$$

2 Layers of E-Glass

$$E_c = 29,439 \text{ MPa}$$

$$E_c = E_f V_f + E_m V_m$$

$$E_c = 72,400 \times 0.4235 + 3220 \times 0.5765$$

3 Layers of E-Glass

$$E_c = 32,517 \text{ MPa}$$

As can be seen these theoretical results are reasonably close to the experimental data shown in Table 3B and could be used to predict the behavior of the fiber. It was decided, however to use a more conservative value of 25,000 MPa for the fiber modulus due to the variability of the material properties caused by the meshing effect and the difficulty in maintaining consistency between different batches.

3.3 Fiber-Wood Bond

A critical parameter in the determination of reinforced beam behavior is the strength of the wood-fiber bond. To evaluate the performance of this bond a simple test was devised in which two blocks of flange material, linked together only by E-glass and resin (Figure 3.4), were tested to failure in tension using the Instron test machine. A total of 5 samples consisting of 1 layer of fiber reinforcement at an assumed fiber content of 22% by volume were evaluated in this test.

The wood used in this tests was flange material cut from the I-beams in 178mm (7

inch) lengths. Once the OSB web was made flush with the remainder of the flange, both sides of the sample were lightly sanded, then thoroughly cleaned prior to the application of the resin to the wood. The resin was then applied to the wood on one side using a roller and the unidirectional glass weave was placed on the resin soaked wood and worked in with the roller until the fibers were completely saturated. The composite was then cured for 24 hours at room temperature and the process was repeated for the other side of the sample. Once the second side had cured, the excess fibers were cut away so that a wood-fiber bond length of 4 inches remained on each side for each piece of wood.

The sample was then fastened to two aluminum end units by means of two bolts and placed in the Instron test machine where it was loaded at a rate of 0.05 inches/minute until failure. The failure load in tension recorded by the Instron load cell was then divided by the total bond length of 203.2mm to produce the values shown in Table 3C, which leads to a 5th percentile bond strength of 47.15 N/mm.

<i>Wood Fiber Bond</i>	
Sample	Shear Value N/mm
Shear 001	55.395
Shear 002	81.05
Shear 003	69.209
Shear 004	55.18
Shear 005	63.653
Mean	64.8974
Standard Deviation	10.79083386
5th Percentile	47.146

Table 3C - Wood-Fiber Shear Data

3.4 Flexural Experiments

3.4.1 Small Beam Tests

The small beam tests were performed to fulfill two main objectives: firstly to confirm the validity of the theoretical models and secondly to evaluate the performance of different reinforcement configurations. To accomplish these goals, twenty 38mm x 89mm x 762mm beams were tested to destruction in flexure in accordance with the procedures outlined in ASTM standard D198-84. All the beams with the exception of the control beams were reinforced with 685mm of E-glass but in different configurations as outlined below:

- a. Control - This group consisted of five unreinforced beams which were used to gauge the performance of the reinforced beams.
- b. Continuous Reinforcement - The five beams in this sample had its 685mm of reinforcement configured in a single continuous layer.

- c. Midspan Variable Length Reinforcement - The five beams in this category had their reinforcement configured in three layers of differing lengths centered at midspan. The three layers were 305, 228 and 152 mm long respectively.
- d. Midspan Constant Length Reinforcement - The five beams in this group had three layers each 228mm long centered at midspan.

The wood for each beam was obtained by cutting the flanges off of two of the I-beams in 762mm (30 inch) lengths and sanding the surface with the protruding OSB web until it was flush with the remainder of the wood. During this cutting process random samples were taken to determine the specific gravity and moisture content of the 35 samples of flange material tested was found to be 0.46 and 10% respectively. The side without the OSB web insert was considered to be the bottom or tension face of the beam and was lightly sanded and cleaned prior to the application of resin. The resin was applied to the wood using a roller followed by the application of the glass fiber weave which was worked into the resin with the roller until the fibers were completely saturated. The samples were then permitted to cure for at least 48 hours at room temperature prior to testing.

The test apparatus consisted of two aluminum platens with adjustable rollers attached to the top and bottom crossheads of the Instron machine. The rollers on the

bottom platen were adjusted so that they were 685mm (27 inches) apart and one of the rollers was fixed in position while the second was free to move. The rollers on the top platen were both fixed in position 229mm (9 inches) apart and were used to deliver a two point load to the beam. The sample was then placed on two steel bearing plates which were positioned directly above the bottom rollers. Prior to every test the position of the top and bottom rollers were verified by measurement to ensure that no slippage had occurred in the test apparatus and that the sample was properly positioned on the rollers. The mid-span deflection was measured by means of a Linear Voltage Differential Transformer (LVDT) with a capacity of 150mm (6 in).

Figure 3.5 - Small Beam Test Apparatus

The loading of the beams was displacement controlled at a rate of 1.25 mm/minute (0.05 inches/minute) using a personal computer equipped with SCIOMETRIC CMT 200 version 4 software. Data acquisition which was also handled by the computer consisted of measuring the applied load and mid-span deflection at a rate of 1 data point per second.

The data obtained from the tests was analyzed and compared against the theoretical values for deflection obtained using the principles developed earlier. The presentation of these results along with any experimental observations and graphs is shown below for each of the four groups:

Control

The beams in the control group all failed in tension usually at a knot or some other weakness in the wood. Extracts from the test results along with the theoretical prediction are contained in Table 3D and graphed in Figure 3.7. The mode of failure was more or less brittle in nature with an increase in the amount of ductility at elevated load levels just prior to failure presumably caused by some compressive yielding of the wood.

Figure 3.7 - Load vs Midspan Deflection for Control Beams

Beam 1 Load		Beam 2 Load		Beam 3 Load		Beam 4 Load		Beam 5 Load		Desired Load	
N	mm	N	mm	N	mm	N	mm	N	mm	N	mm
24.1	0.1	5.5	0.1	54.8	0.0	51.6	0.0	19.8	0.0	0	0
326.5	1.0	285.9	1.1	290.3	1.1	275.1	1.0	242.1	1.0	500	1.4
677.0	2.1	589.5	2.0	566.5	2.0	671.7	2.1	542.3	2.1	1000	2.9
1101.1	3.1	940.0	3.0	918.1	3.0	1077.0	3.1	837.1	3.0	1500	4.3
1491.2	4.0	1298.3	4.1	1254.5	4.0	1427.7	4.0	1246.8	4.2	2000	5.8
1962.2	5.1	1620.4	5.0	1620.4	5.0	1828.7	5.1	1583.2	5.1	2500	7.2
2360.0	6.0	1966.6	6.0	1999.5	6.1	2190.2	6.0	1916.3	6.0	3000	8.7
2815.8	7.0	2316.1	7.1	2374.2	7.1	2582.5	7.1	2300.8	7.1	3500	10.1
3277.0	8.1	2633.9	8.0	2704.0	8.0	2915.5	8.0	2668.9	8.1	4000	11.7
3669.2	9.0	2973.5	9.0	3064.4	9.1	3315.4	9.0	3059.0	9.1	4500	13.3
4127.2	10.0	3302.2	10.1	3376.7	10.0	3701.1	10.1	3382.2	10.1	5000	15.4
4574.2	11.1	3579.4	11.0	3743.8	11.0	4178.8	11.4	3755.8	11.1	5500	18.3
4952.2	12.0	3898.3	12.0	4084.4	12.1	4494.3	12.3	4116.2	12.2		
5370.8	13.0	4206.1	13.1	4402.2	13.1	4839.4	13.3	4454.8	13.2		
5742.2	14.1	4455.9	14.0	4661.8	14.0	5113.3	14.2	4701.3	14.1		
6066.5	15.1	4710.0	15.1	4907.3	15.1	5356.5	15.2	4956.6	15.1		
6328.3	16.0	4948.9	16.1	5033.3	16.1	5599.7	16.2	5179.0	16.2		
6591.2	17.1	5123.1	17.0	4684.9	17.0	5597.5	17.2	5321.4	17.1		
6800.5	18.1	5295.2	18.1	4878.8	18.2	5721.4	18.0	5456.1	18.0		
6956.1	19.0	5447.4	19.1	4976.3	18.8	5768.6	18.3	5363.1	19.1		
7102.9	20.1	5573.4	20.2			5786.1	18.4	5477.0	20.1		
7175.2	20.6	5660.0	21.1					5530.7	20.5		
7238.7	21.1	5664.4	21.6					5564.7	20.8		
7267.2	21.4	5700.5	22.1					5525.2	21.0		
7305.6	21.6	5735.6	22.6								

Table 3D - Control Beam Data

Continuous Reinforcement

The beams in this category all failed in tension but exhibited increased ductility, stiffness and strength over the control beams. This result is attributed to the downward shift in the neutral axis caused by the presence of the fibers. This downward shift leads to reduction in the tensile stresses and an increase in the compressive stresses in the beam. The reduction of tensile stresses tends to increase the strength, while the increase in compressive stresses produces more compressive yielding in the beam and hence more ductility. The gains in stiffness are caused by the increase in the moment of inertia due to the presence of the fibers.

Figure 3.8 - Graph of Midspan Deflection versus Load for Continuously Reinforced Beams

Beam 1E		Beam 1E		Beam 1E		Beam 1E		Beam 1E		Thermal Load	
N	mm	N	mm	N	mm	N	mm	N	mm	N	mm
269.6	0.4	138.1	0.4	163.3	0.4	153.4	0.4	88.7	0.4	0	0
515.0	1.0	369.2	1.1	393.3	1.1	416.3	1.1	279.4	1.1	500	1.3
887.4	2.1	710.0	2.1	736.3	2.1	799.7	2.1	665.1	2.1	1000	2.6
1209.6	3.0	1095.7	3.0	1072.6	3.0	1234.7	3.2	999.2	3.0	1500	3.9
1610.6	4.1	1430.9	4.1	1437.5	4.0	1614.9	4.2	1379.4	4.1	2000	5.3
1952.3	5.0	1784.8	5.1	1824.3	5.1	2019.2	5.1	1730.0	5.1	2500	6.6
2349.0	6.1	2155.1	6.1	2184.7	6.1	2444.2	6.2	2118.9	6.1	3000	7.9
2738.0	7.1	2516.6	7.1	2542.9	7.0	2857.4	7.2	2457.5	7.1	3500	9.2
3067.8	8.0	2855.2	8.0	2983.4	8.2	3237.5	8.1	2806.9	8.0	4000	10.5
3442.5	9.1	3239.7	9.0	3340.6	9.2	3664.8	9.2	3177.3	9.0	4500	11.9
3812.7	10.1	3603.5	10.1	3653.9	10.1	4049.3	10.1	3529.0	10.1	5000	13.7
4128.3	11.0	3916.9	11.0	3984.8	11.1	4379.2	11.1	3836.9	11.0	5500	15.8
4485.5	12.1	4189.6	12.2	4300.4	12.0	4783.5	12.2	4217.0	12.2	6000	18.5
4774.7	13.0	4462.5	13.3	4638.9	13.1	5112.2	13.1	4556.7	13.3	6120	19.3
5102.4	14.0	4744.1	14.3	4980.7	14.3	5431.0	14.2	4823.0	14.3		
5373.0	15.1	4978.5	15.3	5198.7	15.1	5695.0	15.2	5087.0	15.3		
5609.6	16.0	5195.4	16.3	5428.8	16.1	5928.4	16.1	5317.0	16.3		
5853.9	17.0	5386.1	17.2	5657.8	17.1	6117.9	17.2	5506.6	17.2		
6057.7	18.1	5529.6	18.3	5840.8	18.1	6282.2	18.2	5687.3	18.3		
6203.4	19.0	5645.7	19.4	5990.9	19.1	6422.5	19.3	5867.0	19.4		
6242.9	20.1	5763.0	20.5	6134.4	20.2	6540.8	20.2	6007.3	20.5		
6383.2	21.1	5833.2	21.4	6243.9	21.2	6636.2	21.1	5954.7	21.4		
6499.2	22.0	5884.6	22.4	6333.8	22.1	6738.0	22.2	6046.7	22.4		
6573.8	23.1	5644.7	23.4	6429.2	23.1	6813.7	23.1	6150.8	23.4		
6603.3	23.3	5692.9	24.4	6504.7	24.1	6882.7	24.2	6223.1	24.4		
6625.2	23.6	5741.1	25.1	6281.3	25.1	6974.7	25.2	6273.5	25.1		
		5793.7	25.4	6370.0	26.2	7019.7	26.0	6295.4	25.4		
		5822.2	25.0	6384.2	27.3	7049.2	26.5				
		5867.1	25.6	6456.5	28.2	7078.8	27.0				
		5891.2	26.0	6508.0	29.1	7110.5	27.5				
		5927.3	26.7	6522.3	29.3	7143.5	28.0				
		5952.5	27.3	6565.0	30.0	7160.9	28.5				
		5948.2	28.1	6570.5	30.3	7188.3	29.1				
		5941.6	28.8	6576.0	30.4	7211.4	29.6				
		5924.0	29.0			7189.4	30.1				

Table 3E - Continuously Reinforced Beam Data

Variable Length Midspan Reinforcement

The beams in this category all failed in tension at considerably higher stiffness, strength and ductility than the control beams. The stiffness when compared to the continuously reinforced beams was higher, however the ultimate strength was lower. This can be explained by the fact that the concentration of fiber in the middle of the beam where the moment is the highest is quite effective in reducing deflections, however failure of the beam occurs in the unreinforced portion when the stress in the wood reaches the failure stress which occurs at a lower load than a continually reinforced beam.

Load vs Mid-Span Deflections

Figure 3.9 - Graph of Load versus Midspan Deflections for Variable Length Midspan Reinforcement

Beam 1-1		Beam 2-1		Beam 3-1		Beam 4-1		Beam 5-1		Theoretical	
Load		Load		Load		Load		Load		Load	
N	mm	N	mm	N	mm	N	mm	N	mm	N	mm
224.6	0.4	73.4	0.4	163.2	0.4	145.7	0.4	117.2	0.4	0	0
614.7	1.1	368.1	1.2	518.2	1.3	462.3	1.4	356.0	1.2	500	1.2
1106.6	2.2	728.6	2.2	899.5	2.3	806.3	2.3	628.8	2.1	1000	2.5
1543.8	3.1	1100.0	3.1	1324.5	3.3	1174.4	3.3	975.1	3.1	1500	3.7
2002.9	4.1	1439.6	4.0	1697.1	4.2	1562.3	4.2	1280.8	4.1	2000	4.9
2481.6	5.1	1868.0	5.1	2124.3	5.2	1966.6	5.1	1583.1	5.1	2500	6.2
3022.9	6.2	2276.7	6.1	2542.9	6.3	2420.2	6.2	1912.9	6.0	3000	7.4
3453.4	7.2	2651.4	7.1	2952.7	7.3	2851.9	7.2	2250.4	7.1	3500	8.6
3892.7	8.1	3064.5	8.2	3323.0	8.2	3263.8	8.1	2594.4	8.2	4000	9.9
4317.9	9.0	3440.3	9.2	3742.6	9.3	3676.9	9.2	2886.9	9.2	4500	11.2
4740.8	10.0	3829.2	10.2	4076.8	10.2	4061.4	10.2	3199.2	10.2	5500	14.8
5209.8	11.1	4141.4	11.1	4414.3	11.2	4175.3	11.3	3479.7	11.1	6000	18.2
5230.5	12.2	4523.8	12.2	4695.8	12.3	4417.5	12.2	3744.8	12.1		
5489.1	13.0	4856.9	13.2	4959.8	13.2	4646.5	13.2	4018.7	13.1		
5801.3	14.1	5150.5	14.1	5164.7	14.3	4890.8	14.1	4109.6	14.1		
6019.4	15.0	5429.9	15.2	5165.9	15.3	5200.9	15.2	4196.2	14.5		
5895.6	16.3	5688.5	16.2	5325.7	16.4	4785.6	16.2				
6065.3	17.3	5663.2	17.4	5467.1	17.4	4957.6	17.2				
6173.9	18.1	4015.4	18.2	5563.5	18.2	5163.6	18.1				
6278.0	19.0	4281.7	19.3	5700.5	19.3	5377.2	19.3				
6321.7	20.1	4423.0	20.2	5803.5	20.2	5489.0	20.2				
6388.6	21.1	4600.5	21.3	5896.6	21.3	5587.6	21.3				
6467.5	22.1	4731.9	22.3	5988.7	22.3	5670.9	22.1				
6477.4	23.2	4833.8	23.3	6035.7	23.3	5753.1	23.1				
6546.4	24.1	4924.8	24.3	6074.1	24.4	5806.8	23.9				
6599.0	25.1	4994.9	25.2	6080.6	24.8						
6669.1	26.0	5055.2	26.4								
6701.9	27.0	4903.9	27.1								
6756.7	28.1										
6795.1	29.1										
6829.0	29.7										
6835.6	30.3										
6493.8	31.1										

Table 3F - Data from Beams with Variable Length Midspan Reinforcement

Constant Length Midspan Reinforcement

The beams in this grouping behaved very much like the variable length midspan reinforcement with only nominal differences in the performance characteristics. This reinforcement configuration was easy to manufacture, possessed increased stiffness over continuous reinforcement, however the strength was slightly less than that of continuous reinforcement.

Load vs Mid-Span Deflections

Figure 3.10 -Graph of Load versus Midspan Deflection for Constant Length Midspan Reinforcement

Beam 1		Beam 2		Beam 3		Beam 4		Beam 5		Beam 6	
Load	Deflection	Load	Deflection	Load	Deflection	Load	Deflection	Load	Deflection	Load	Deflection
N	mm	N	mm	N	mm	N	mm	N	mm	N	mm
129.3	0.4	87.7	0.4	90.9	0.4	112.9	0.4	307.8	0.4	0	0
441.5	1.2	268.4	1.1	313.3	1.1	391.2	1.1	712.1	1.4	500	1.2
792.1	2.2	627.8	2.1	705.6	2.0	776.8	2.1	1079.1	2.3	1000	2.5
1162.4	3.2	982.8	3.2	1181.0	3.1	1162.5	3.0	1487.8	3.3	1500	3.7
1531.6	4.1	1331.2	4.1	1606.2	4.0	1618.3	4.1	1883.3	4.2	2000	5.0
1932.6	5.2	1760.7	5.2	2090.4	5.1	2001.7	5.0	2298.6	5.1	2500	6.2
2307.3	6.2	2123.4	6.2	2512.2	6.1	2398.3	6.0	2772.9	6.2	3000	7.5
2649.2	7.2	2483.8	7.1	2972.4	7.1	2786.2	7.0	3183.9	7.1	3500	8.7
2998.7	8.1	2883.6	8.2	3406.3	8.1	3136.8	8.0	3605.6	8.2	4000	10.0
3395.3	9.2	3227.7	9.1	3810.6	9.1	3513.6	9.0	3991.3	9.2	4500	11.3
3701.0	10.1	3613.3	10.2	4165.5	10.0	3874.1	10.0	4334.3	10.2	5000	12.9
4001.2	11.2	3945.3	11.2	4538.1	11.1	4016.5	11.0	4535.9	11.1	5500	15.2
4220.2	12.1	4285.0	12.2	4554.5	12.0	4300.3	12.0	4884.2	12.3		
4406.6	13.1	4560.0	13.2	4826.2	13.1	4589.6	13.1	5102.3	13.2		
4482.2	14.2	4854.7	14.3	5050.8	14.0	4770.3	14.0	5358.7	14.2		
4603.7	15.1	5073.8	15.2	4948.9	15.1	4996.1	15.0	5577.7	15.2		
4759.3	16.1	5232.7	16.2	5077.1	16.1	5215.2	16.1	5800.2	16.3		
4895.2	17.1	5260.1	16.4	5154.8	17.0	5386.1	17.0	5982.1	17.3		
5032.2	18.1	4146.9	17.4	5237.1	18.1	5524.1	18.0	6133.2	18.3		
5165.9	19.2			5227.2	19.1	5697.3	19.1	6258.2	19.4		
5215.1	20.1			5295.1	20.1	5719.1	19.3	6367.7	20.3		
5279.8	21.1			5313.7	21.2	5754.2	19.6	6463.0	21.3		
5291.8	22.3			5356.5	21.9	5763.0	19.7	6536.4	22.2		
5173.5	23.1			5164.7	22.3	5522.0	19.9	6483.8	23.3		
5188.8	24.0							6549.6	24.3		
5195.4	24.6							6608.7	25.3		
								6649.2	26.3		
								6678.8	27.2		
								5685.1	28.1		
								5867.0	29.1		
								5988.6	30.2		
								6019.3	30.5		
								5918.5	30.9		
								5821.0	31.8		
								5578.8	32.8		
								5490.1	33.8		
								4589.5	34.7		
								4676.1	35.7		
								4773.6	36.8		
								4751.6	37.7		
								4785.6	38.8		
								4831.7	39.8		
								4693.6	40.7		
								4739.6	41.8		
								4464.6	42.7		
								3504.8	43.8		

Table 3G - Beams with Constant Length Reinforcement at Midspan

As was observed in the tests, a concentration of reinforcement at midspan proved to be more effective at increasing the stiffness of the beam than continuous reinforcement. A closer examination of the theoretical models in Figure 3.11 demonstrates that this increased stiffness is attained at the price of slightly reduced strength.

Load vs Mid-Span Deflections

Figure 3.11 - Comparison of Load-Displacement for Different Reinforcement Configurations

Based on these results and the technical difficulties associated with manufacturing variable length midspan reinforcement, it was decided to evaluate the theoretical model on I-beams using a constant length midspan reinforcement configuration.

3.4.2 I-Beam Tests

A total of eighteen TJI 421 I-beams with a depth of 241mm (9.5 inches) measuring 7.315m (24 feet) long were tested in flexure to destruction in a pneumatic testing apparatus designed to simulate a uniformly distributed load. The TJI 421 beam consists of two 2100f-1.8E MSR Spruce-Pine-Fir (SPF) flanges measuring 38mm x 89mm (1.5 x 3.5 inches) with an 11.1mm (7/16 inch) thick Performance Plus™ OSB web. The web-flange connection consists of a 15.9mm (5/8 inch) deep tapered groove in the center of the MSR flange where the web is installed using a phenol-resorcinol adhesive. To achieve longer span lengths the MSR lumber is finger joined at irregular intervals and the web is tongue-in-groove joined at 1220mm (4 foot) intervals using the same phenol-resorcinol adhesive.[NRC 1995]

The eighteen I-beams were divided into three groups according to the reinforcement configuration. Since the test results from the small beam tests indicated that a concentration of reinforcement at the midspan would provide the optimum stiffness and performance characteristics, it was decided to place the tension reinforcement for the I-beams in the center portion of the beam. Consequently the three groups tested were as follows:

- a. *Control Beams* - This group consisted of six I-beams with no reinforcement;
- b. *3 Layers of Tension Reinforcement* - This group consisted of six I-beams with three layers of constant length midspan reinforcement 2333 mm long

centered at midspan;

c. *5 Layers of Tension Reinforcement* - The beams in this group consisted of five layers of constant length midspan reinforcement 2333 mm long centered at midspan;

A total of 72 random samples of the I-beams were taken after the testing to determine the specific gravity and moisture content of the flange material which was found to be approximately 0.48 and 8 % respectively. The preparation of the reinforced beams was much the same as that used in the small beams. In applying a delicate fiber weave measuring over 2m long and 90 mm wide a procedure was developed where the fibers were rolled in a bundle and one end of the bundle was fastened to the beam using small nails or staples. The resin was then applied to the wood using a roller after a light sanding and thorough cleaning of the contact area. The fiber bundle was then unrolled over the resin saturated wood surface and worked into the resin with the roller until the fibers were completely saturated. The samples were then permitted to cure for at least 48 hours at room temperature prior to testing.

The test apparatus as shown in Figure 3.12 consisted of a series of 12 pneumatic jacks mounted on a steel test platform 608 mm (2 feet) apart. The yoke of each jack was then attached to an assembly consisting of two back to back angle irons with two oak bearing pads mounted on the angles positioned 304mm (1 foot) apart. The air pressure was delivered to all jacks simultaneously through the use of a manually controlled regulator and was monitored using pressure gauges positioned at the beginning and end of

the test setup. Lateral support was provided every 2.4m (8 feet) by means of a wooden support mechanism attached to the steel frame.

Figure 3.12 - Experimental Test Setup for I-beams

Figure 3.13 - Experimental Test Set-up

The specimen was placed on an assembly consisting of a pair of steel bearing plates with a 2 inch steel ball sandwiched between the two plates at each end of the beam. This assembly which was placed directly onto the load cell permitted the beam freedom to rotate in all directions. To ensure that the web did not fail prematurely at the supports, an appropriate "blocking material" was placed between the two flanges to prevent any web buckling that might occur.

Figure 3.14 - End Support Conditions

The applied load was recorded through the use of two load cells at each of the reaction points and the deflection was obtained by measuring the difference between an LVDT at midspan and an LVDT directly over the support. The data acquisition was handled by computer

Figure 3.15 - Midspan Deflection Measurement

and the rate of data collection was set at one data point per 2 seconds. The load was applied manually using the load readout from load cells as a guide to establish a rate that would result in beam failure in approximately 20-30 minutes.

The data collected from the tests was processed and compared against the theoretical values and the results from the control beams. The control beams all failed in the tension flange at either a defect in the wood such as a knot or at a finger joint. As reinforcement was added failures of the compressive flange occurred with increased frequency presumably due to the effects of shifting the neutral axis in the downward direction.

<i>Beam</i>	<i>Mode of Failure</i>
<u>Control</u>	
C-1	Tensile failure at a finger joint
C-2	Tensile failure at a knot
C-3	Tensile failure at a finger joint
C-4	Tensile failure at a finger joint
C-5	Tensile failure at a finger joint
C-6	Tensile failure at a knot
<u>3 Layers</u>	
B1-1	Tensile failure at a finger joint
B1-2	Compressive failure at a knot
B1-3	Tensile failure at a finger joint
B1-4	Torsional failure at a lateral support(twisted flange)
B1-5	Compressive failure
B1-6	Tensile failure
<u>3 Layers + Web Reinforcement</u>	
B2-1	Compressive failure
B2-2	Compressive failure
B2-3	Compressive failure
B2-4	Tensile failure at a finger joint
B2-5	Compressive failure
B2-6	Tensile failure
<u>5 Layers</u>	
B3-1	Compressive failure
B3-2	Support failure due to twisting
B3-3	Tensile failure at a knot
B3-4	Tensile failure at a knot
B3-5	Tensile failure at finger joint
B3-6	Tensile failure at finger joint

Table 3H - Failure Modes

The most distressing aspect of the tensile failure modes was the number of failures which occurred at finger joints accounting for 33% of all failures which is extremely high given that the position of the finger joint on the beam was entirely random and many of the failures occurred some distance from the mid-span point. This high incidence of finger joint failures suggests that the limiting strength of the tension flange is not in fact the strength of

Figure 3.16 - Finger Joint Failures

the wood but the strength of the finger joint. The remainder of tensile failures were due to some naturally occurring defect which also limited the strength of the wood to some degree. It is interesting to note that all of the tensile failures occurred outside the reinforced zone despite the presence of some obvious strength reducing defects within the zone in some of the samples.

Figure 3.17 - Tensile Failure at a Knot

The compressive failures or more accurately failures of the compression flange were characterized by a splitting of the flange along the longitudinal axis. Ironically, this failure mode is most likely a tensile failure of the flange perpendicular to the grain caused by the web being forced into the flange.

Figure 3.18 - Compressive Flange Failure

Figure 3.19 - Compressive Flange Failure

Figure 3.20 - Compressive Flange Failure

The data collected from the tests was plotted and compared against the theoretical

predictions. As can be seen in the ensuing pages the theoretical models demonstrated a good correlation with the experimental results and provided a reasonably accurate representation of beam behavior.

Control Beams

The control beams exhibited very little ductility and demonstrated elastic behavior to failure. The model over-estimated the actual strength of the beams by a considerable amount, however some of this difference can be attributed to premature failures at finger joints and knots. Another factor accounting for a reduction in beam strength is the effect of the OSB insert in the compression and tension flanges which was not accounted for in the determination of flange properties.

Figure 3.21 - Load versus Midspan Deflection for Control Beams

Beam 1		Beam 2		Beam 3		Beam 4		Beam 5		Beam 6		Beam 7	
Force (kN)		Force (kN)		Force (kN)		Force (kN)		Force (kN)		Force (kN)		Force (kN)	
0.0	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	0
4.7	0.1	5.5	0.1	5.5	0.2	5.1	0.2	4.6	0.1	9.0	0.2	18.0	0.5
10.1	0.3	10.7	0.3	10.1	0.3	10.2	0.3	11.3	0.3	12.1	0.3	36.0	1.0
15.2	0.4	15.4	0.4	15.2	0.4	15.0	0.4	14.2	0.4	14.9	0.4	54.1	1.5
20.3	0.6	20.7	0.5	20.3	0.6	20.2	0.6	20.9	0.6	19.7	0.5	72.1	2.0
25.1	0.7	24.7	0.6	25.0	0.8	25.3	0.7	25.5	0.7	25.2	0.7	90.1	2.5
30.1	0.9	30.1	0.8	30.6	0.9	30.1	0.9	30.1	0.8	30.2	0.8	108.1	3.0
35.0	1.0	35.0	0.9	35.0	1.1	35.0	1.0	35.2	1.0	35.0	1.0	126.1	3.5
40.0	1.2	40.0	1.1	40.4	1.2	40.0	1.2	40.9	1.1	40.0	1.1	144.1	4.0
45.0	1.3	45.5	1.2	45.9	1.4	45.1	1.3	45.4	1.2	45.7	1.3	162.2	4.5
49.9	1.4	50.0	1.3	50.0	1.5	50.0	1.5	49.5	1.3	49.9	1.4	180.5	5.0
55.1	1.6	54.9	1.5	55.0	1.7	54.9	1.6	54.4	1.5	55.0	1.5	194.8	5.4
60.0	1.7	60.9	1.6	60.2	1.8	59.1	1.7	60.1	1.6	60.0	1.7		
65.1	1.9	64.7	1.7	65.0	2.0	65.0	1.9	65.0	1.8	65.0	1.8		
70.1	2.0	70.0	1.9	70.0	2.1	69.5	2.0	70.1	1.9	70.0	2.0		
75.0	2.1	74.0	2.0	75.0	2.3	75.0	2.2	76.0	2.1	75.8	2.1		
80.0	2.3	80.0	2.1	80.0	2.4	80.0	2.3	80.1	2.1	80.2	2.3		
84.3	2.4	85.0	2.3	87.0	2.6	85.2	2.5	85.0	2.3	85.1	2.4		
90.0	2.6	90.3	2.4	90.2	2.7	90.1	2.6	89.1	2.4	90.0	2.5		
95.0	2.7	95.1	2.5	96.7	2.9	91.1	2.7	95.0	2.5	95.0	2.7		
100.0	2.8	100.1	2.6	100.1	3.0			102.3	2.7	100.1	2.8		
105.0	3.0	105.1	2.8	104.0	3.1			106.1	2.8	105.0	2.9		
110.0	3.1	110.0	2.9	109.0	3.3			110.3	2.9	110.1	3.1		
115.0	3.2	117.4	3.1					115.0	3.0	115.0	3.2		
120.0	3.3	118.8	3.1					120.0	3.2	119.7	3.3		
125.0	3.5							125.1	3.3	125.0	3.5		
130.5	3.6							128.4	3.4	129.2	3.6		
135.0	3.7									134.1	3.7		
140.5	3.9												
141.6	3.9												

Table 3I - Control Beam Data

3 Layers of Constant Length Midspan Reinforcement

The beams reinforced with 3 layers displayed increased stiffness and strength over the unreinforced control beams. The theoretical model predicted the behavior of the beams reasonably well but tended to slightly under estimate the stiffness and continued to over-estimate the strength.

Figure 3.22 - Load versus Midspan Deflection for I-Beams Reinforced with 3 Layers of Constant Length Midspan Reinforcement

Beam 1	Beam 2	Beam 3	Beam 4	Beam 5	Beam 6	Beam 7	Beam 8	Beam 9	Beam 10	Beam 11	Beam 12	Beam 13	Beam 14
P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN	P_{max} kN
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.0	0.1	5.0	0.2	6.3	0.2	6.2	0.2	5.1	0.2	3.9	0.1	17.2	0.5
10.0	0.3	10.4	0.3	10.2	0.3	11.0	0.3	10.7	0.4	10.0	0.3	34.4	1.0
14.9	0.4	15.9	0.5	15.0	0.5	15.3	0.4	15.2	0.5	15.1	0.5	51.6	1.5
19.7	0.6	20.8	0.7	19.8	0.6	19.6	0.5	20.0	0.6	20.1	0.6	68.8	2.0
25.2	0.7	25.1	0.8	25.4	0.8	25.1	0.7	25.0	0.8	24.9	0.8	86.0	2.5
30.0	0.9	30.0	1.0	30.2	0.9	30.7	0.9	29.6	0.9	29.8	1.0	103.2	3.0
35.1	1.0	35.1	1.1	34.2	1.0	35.0	1.0	35.9	1.1	35.4	1.2	120.4	3.5
40.1	1.2	40.2	1.3	40.3	1.2	40.8	1.1	40.1	1.3	40.0	1.3	137.6	4.0
45.0	1.3	45.0	1.4	45.3	1.4	45.3	1.3	44.4	1.4	45.0	1.5	154.8	4.5
50.0	1.5	50.0	1.6	49.7	1.5	50.0	1.4	50.0	1.5	50.1	1.6	172.1	5.0
55.0	1.6	54.7	1.7	54.8	1.6	55.8	1.6	55.4	1.7	55.1	1.8	191.9	5.5
60.1	1.8	60.0	1.9	60.0	1.8	60.2	1.7	60.7	1.8	59.4	1.9	225.8	6.0
65.1	1.9	65.1	2.0	65.0	1.9	65.1	1.8	65.2	1.8	64.9	2.0		
68.6	2.0	69.1	2.1	70.9	2.1	71.6	2.0	70.0	2.0	70.1	2.2		
75.0	2.2	75.1	2.3	75.0	2.2	75.4	2.1	75.0	2.1	75.1	2.3		
79.6	2.3	80.2	2.5	80.2	2.3	80.1	2.2	79.8	2.3	80.7	2.5		
85.0	2.5	86.2	2.7	85.1	2.5	85.0	2.4	84.7	2.4	85.0	2.7		
90.9	2.7	90.0	2.8	90.0	2.6	91.0	2.5	89.1	2.5	91.4	2.9		
95.0	2.8	95.1	2.9	95.0	2.7			95.1	2.7	95.0	3.0		
100.0	2.9	100.0	3.1	100.0	2.9			100.0	2.8	100.0	3.1		
105.0	3.0	105.0	3.2	105.0	3.0			105.4	3.0	105.0	3.3		
110.0	3.2	109.6	3.4	109.1	3.1			112.1	3.2	109.2	3.4		
115.0	3.3	115.0	3.5	115.0	3.3			115.1	3.3	115.0	3.6		
120.6	3.5	122.5	3.7	119.3	3.4			120.2	3.4	120.2	3.7		
125.0	3.5	125.1	3.8	125.3	3.6			123.6	3.5	125.7	3.9		
129.2	3.6	130.0	3.9	130.0	3.7			128.9	3.7				
134.1	3.8	135.6	4.1	132.3	3.7			132.2	3.7				
		139.2	4.7	138.2	3.9			138.7	3.9				
				139.0	4.5			142.1	4.0				
								149.7	4.2				
								152.0	4.2				
								153.7	4.2				
								161.5	4.4				
								165.4	4.5				
								173.4	4.7				

Table 3J - I-Beams reinforced with 3 Layers of Fiber at Midspan

5 Layers of Constant Length Midspan Reinforcement

The beam reinforced with 5 layers of fiber not surprisingly showed substantial increases in stiffness, however the strength did not increase in any great measure. This lack of increase in strength coupled the presence of some measure of inelastic behavior seems to indicate that the reinforcement ratio is approaching a "balance" point where the compression and tension flanges fail at the same time. Increases in reinforcement beyond the balance point would in theory result in increases in stiffness but decreases in the ultimate strength due to premature failure of the compression flange.

Figure 3.23 - Load versus Midspan Deflection for I-Beams Reinforced with 5 Layers of Constant Length Midspan Reinforcement

Table 3K - I-beam Reinforced with 5 Layers of Fiber at Midspan													
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5.1	0.2	2.1	0.1	5.2	0.2	5.8	0.2	6.3	0.2	2.4	0.1	16.7	0.5
9.5	0.3	13.9	0.4	9.9	0.3	9.7	0.3	11.6	0.4	7.1	0.2	33.5	1.0
15.6	0.5	17.7	0.6	15.2	0.5	14.6	0.4	16.0	0.5	15.4	0.5	50.2	1.5
20.0	0.6	20.3	0.6	22.4	0.7	20.4	0.6	20.0	0.6	20.7	0.6	66.9	2.0
23.8	0.7	24.3	0.8	26.2	0.8	25.8	0.8	24.9	0.8	25.5	0.8	83.6	2.5
30.6	0.9	30.9	1.0	30.5	0.9	30.6	0.9	30.2	0.9	30.2	0.9	100.4	3.0
35.1	1.0	35.0	1.1	34.9	1.1	35.0	1.0	34.4	1.1	35.9	1.1	117.1	3.5
40.2	1.2	40.0	1.3	40.1	1.2	40.9	1.2	41.5	1.3	41.3	1.3	133.8	4.0
45.2	1.4	45.0	1.4	44.9	1.3	45.8	1.4	45.5	1.5	45.5	1.4	150.5	4.5
50.5	1.5	50.0	1.6	50.1	1.5	51.0	1.5	50.4	1.6	50.1	1.5	167.3	5.0
55.3	1.7	55.6	1.8	55.8	1.7	54.4	1.6	55.2	1.8	55.8	1.7	186.3	5.5
60.0	1.8	59.8	1.9	58.9	1.8	60.6	1.8	60.0	1.9	60.1	1.8	210.3	6.0
65.1	1.9	65.1	2.1	64.4	2.0	65.8	1.9	65.1	2.0	65.9	2.0		
70.0	2.1	70.3	2.2	70.5	2.2	70.2	2.0	71.5	2.3	70.5	2.2		
74.3	2.2	76.6	2.4	75.0	2.3	75.1	2.2	75.0	2.3	75.2	2.3		
80.0	2.4	80.1	2.5	79.7	2.4	80.1	2.3	80.5	2.5	80.6	2.5		
85.0	2.5	85.2	2.7	84.7	2.6	85.1	2.5	85.3	2.7	86.2	2.6		
88.5	2.6	90.1	2.8	90.2	2.7	89.7	2.6	91.8	2.9	90.3	2.7		
94.9	2.8	93.6	2.9	95.0	2.9	95.0	2.7	94.9	2.9	95.0	2.9		
102.2	3.0	100.1	3.1	100.0	3.0	98.9	2.9	100.6	3.1	102.2	3.1		
105.3	3.1	104.2	3.2	105.4	3.2	105.1	3.0	105.5	3.3	105.6	3.2		
110.9	3.3	110.0	3.4	108.3	3.3	110.1	3.2	110.8	3.4	111.7	3.4		
114.7	3.3	115.1	3.6	113.7	3.4	112.8	3.2	115.4	3.5	115.6	3.5		
120.1	3.5	116.4	3.6			118.0	3.4	120.2	3.7	123.5	3.7		
125.0	3.6					124.5	3.5	123.9	3.8	126.1	3.8		
131.8	3.8					130.9	3.7	131.3	3.9	129.5	3.8		
132.5	3.8					135.5	3.8	137.0	4.1	135.3	4.1		
						138.9	3.9	140.4	4.2	140.3	4.1		
						144.2	4.1	143.1	4.3	148.8	4.3		
										153.1	4.4		
										159.4	4.6		
										163.3	4.6		

Table 3K - I-beam Reinforced with 5 Layers of Fiber at Midspan

Figure 3.24 - Theoretical Models for I-Beams

Summary

Generally speaking the theoretical models described the behavior of both the small and I-beams with a reasonable degree of accuracy. The concentration of reinforcement in the midspan region appears to increase the stiffness of reinforced beams by a greater amount than continuous reinforcement, however continuous reinforcement tends to provide a higher ultimate strength than midspan reinforcement. Since beam deflections are related to the stiffness of the beam and not the ultimate strength it would be reasonable to

assume that midspan reinforcement configurations would be more effective in reducing deflections and hence would be of more practical value in beam applications.

4.0 Analysis

4.1 Solid Wood Beams

A closer examination of the test results obtained during the experimental phase confirms that the use of midspan reinforcement reduces deflections more effectively than a continuous reinforcement configuration.

<i>Beam</i>	<i>Displacement @ 3000 N</i>	<i>Maximum Load</i>
C-1	7.41	7267
C-2	9.11	5736
C-3	8.88	4976
C-4	8.17	5077
C-5	9.02	5787
Average	8.52	5670
I-1	8.03	6625
I-2	8.57	5924
I-3	8.23	6576
I-4	7.66	7211
I-5	8.52	6295
Average	8.20	6526
3-1	6.20	6494
3-2	8.00	4903
3-3	7.30	6080
3-4	7.66	5807
3-5	9.66	4196
Average	7.77	5666
4-1	8.10	5292
4-2	8.52	5260
4-3	7.16	5356
4-4	7.70	5763
4-5	6.90	6678
Average	7.68	5670

Table 4A - Solid Wood Beam Comparisons

Although the properties of the wood vary substantially from sample to sample general trends can be observed by examining the mean values. It can be seen from Table 4A that a continuous reinforcement configuration results in a reduction of approximately 4% in the displacement at a load of 3000 N when compared to unreinforced beams whereas the midspan reinforcement configurations resulted in a 10% decrease in displacements.

The ultimate strength characteristics of the beams are also influenced by the reinforcement configuration with the continuously reinforced beams exhibiting an increase in strength of 13% over the unreinforced control beams while the midspan reinforcement configurations both showed slight decreases in the ultimate strength of 4% and 1.7%.

This behavior of reinforced wood beams can be explained by the fact that midspan reinforced beams generally fail in the unreinforced portion of the beam once the stresses reach the failure stress, hence the ultimate strength is governed by the strength of the wood alone. In a two point loading system the applied moment at $L/3$ is the same as the moment at midspan consequently the stress in the unreinforced portions of midspan reinforced beams is nearly the same as the stress occurring at midspan in an unreinforced beam. The slight reductions in strength of the midspan reinforced beams may be due to stress concentration effects caused by the abrupt changes in the moment of inertia and the accompanying shifts in the neutral axis. In continuously reinforced beams the strength of the beam is determined by the strength of the fiber-wood composite consequently the

ultimate strength is increased over both unreinforced beams and beams with midspan reinforcement.

4.2 I-Beams

The behavior of I-beams reinforced with fiber on the tension flange demonstrated similar reductions in the recorded displacements for the reinforced beams when compared to the unreinforced control beams. The displacement at a load of 2.0 KN/m for beams reinforced with three layers of midspan reinforcement showed a 6% decrease in deflections while the reduction for beams reinforced with 5 layers of fiber was approximately 8%.

<i>Beam</i>	<i>Displacement (at 2 kN/m)</i>	<i>Maximum Load</i>
C-1	70.39	3.92
C-2	75.47	3.11
C-3	65.83	3.28
C-4	68.41	2.65
C-5	73.70	3.35
C-6	71.35	3.73
Mean	70.35	3.34
I-1	68.55	3.80
I-2	64.14	4.68
I-3	67.74	4.54
I-4	70.19	2.54 (Torsion)
I-5	67.20	4.48
I-6	61.43	3.89
Mean	66.54	4.22
3-1	67.16	3.79
3-2	62.74	3.59
3-3	64.36	3.44
3-4	68.33	4.06
3-5	63.29	4.25
3-6	65.54	4.64
Mean	65.23	3.96

Table 4B - I-Beam Comparisons

The increase in strength between the reinforced and the unreinforced control beams can be explained by the fact that the reinforced beams all failed in the unreinforced portion of the beam when the stresses in the wood reached the failure stress. In a uniformly distributed loading pattern, the load required to achieve this failure stress in the wood at $L/3$ is greater than the loading required to achieve this stress at the midspan.

On closer examination it can be seen that the ultimate strength of the reinforced beams with 3 layers of fiber increased by approximately 28% while the beams with 5 layers increased by only 19% when compared with the unreinforced beams. Although the available data is limited this decrease in ultimate strength coupled with the occurrence of compression flange failures indicates that the compression flange has become the critical parameter in determining the strength of the beam. Any further increases in the strength would appear to have the effect of increasing the stiffness but reducing the ultimate strength.

4.3 Practical Considerations

4.3.1 Workability

The workability of wood construction materials is an important consideration in the development of any product and must be carefully evaluated to ensure it does not place any undue restrictions on its application. One characteristic of glass fiber composites is that they are extremely abrasive and difficult to cut with conventional wood saws.

A key characteristic of wood construction has always been the ability to cut components to size on site with relative ease and speed. The use of continuous reinforcement along the full length of the beam severely curtails this ability since special saws and most likely respiratory protection would be required to cut any substantial amount of glass reinforced wood product.

This difficulty is for the most part eliminated through the use of midspan reinforcement configurations since there would be no requirement to cut the glass fibers to achieve a good fit.

4.3.2 Manufacturing Process

The manufacturing process is a key element in determining the viability of reinforced wood products. Continuously reinforcement configurations are naturally the easiest to fabricate, however only a small additional effort is required to assemble a constant length midspan reinforced composite. The manufacture of variable length midspan reinforcement would be technically more difficult and would appear to only yield nominal increases in performance over constant length midspan reinforcement.

4.3.3 Economic Considerations

The most important factor in determining the success of any fiber reinforcement configuration must be the cost. There are many factors which determine the ultimate cost of the composite which can be broken down into three components:

- a. fixed costs;
- b. variable costs; and
- c. profit.

The fixed costs are those costs which are constant irrespective of production and include such items as equipment costs, facility costs, administrative overhead etc... while the variable costs are made up of such things as labor, materials, and shipping costs. In order for a fiber reinforcement configuration to be viable it must offer substantive increases in performance so as to offset the initial capital investment, any increases in the fixed and variable costs, and offer some form of profit margin.

The fixed costs and profit margin are difficult items to ascertain and are beyond the scope of this thesis. The variable costs of producing different reinforcement configurations were established by carefully monitoring the consumption of materials and using the retail prices as a guide to the cost of each item. The labor and transportation costs are merely "ballpark" estimates based on experience obtained in fabricating the beams and observing the production times. The unit costs used in establishing the reinforcement costs were as follows:

- a. Polyester resin - \$7.50/liter;
- b. Glass Fiber - \$0.1154/m;
- c. Increment in labor costs - \$0.20/m; and
- d. Increment in transportation costs - \$0.05/m.

The costs were then graphed for a sample width of 38mm for a variety of fiber contents. The largest component of the cost is by far the resin which accounts for about 70% of the total cost of the composite. Fiber reinforcement placed directly in contact with the wood surface tended to require approximately 1/3 more resin because of the absorption of the resin into the wood. For this reason continuous reinforcement configurations were substantially more expensive than midspan reinforcement configurations with an equivalent amount of fiber.

Figure 4.1 - Cost Comparison

These costs can be then compared to the performance of the beam in a cost-benefit analysis to determine if the beam performance relative to the cost is sufficient to merit further investigation. In the case of constant length midspan reinforcement of a TJI

421 joist the cost of adding the reinforcement is projected to be about \$8.12 which equates to about a 10% increase in the overall cost of the beam. This additional cost would yield a decrease in the deflections of about 6% and an increase in the ultimate strength of about 28%. A similar analysis could be performed on any reinforced wood beam to determine its economic viability in relation to its performance.

5.0 Conclusions and Recommendations

5.1 Conclusions

The methodology established to predict the reinforced beam behavior provides a generalized way of determining the elastic and inelastic characteristics of wood-fiber beams. The theoretical models developed using this method appear to have described the beam behavior of both solid wood beams and I-beams with a reasonable degree of accuracy.

The assumptions made in the model with respect to I-beam behavior that the web carries the shear forces and the flanges carry the bending forces appear to have not adversely affected the performance of the model. It is interesting to note that shear deflections accounted for approximately 7% of all deflections which supports the premise that this component of the deflection was not negligible.

The models as well as the experimental data both indicate that midspan reinforcement configurations are more effective at reducing deflections than continuous reinforcement. The test data also suggests that the ultimate strength of beams with midspan reinforcement configurations tends to be lower than the ultimate strength of beams with continuous reinforcement and is highly dependent upon the loading configuration.

5.2 Recommendations

This study provided a brief overview of the effects reinforcing wood beams with different reinforcement as well as establishing a method for analyzing the effects of reinforcement configuration. There are several related avenues of research which show promise and should be explored in further detail:

- a. Optimum Reinforcement Configuration - No effort was made in this thesis to determine the optimum reinforcement configuration to reduce displacements. This is one endeavor which almost certainly be of some practical use in fiber reinforced wood applications.
- b. Utilization of Different Types of Fibers - This thesis focused on the use of glass fibers in a polyester resin, however stronger fibers such as Aramids and Carbon fibers which possess higher modulus and tensile strength could be employed in wood-fiber composites. The analysis of different fiber combinations from a cost-benefit perspective would provide interesting and useful information.
- c. Compressive Reinforcement - This thesis dealt solely with the effects of reinforcing the tension face of wood. No consideration was given to providing compression reinforcement. The use of compressive reinforcement could be useful in increasing the ultimate strength of wood which is weaker in compression than in tension;

- d. **Long Term Performance Test** - The long term performance of fiber reinforced wood has not been explored to any great extent particularly the long term effects of reinforcement configuration.
- e. **Degradation of the Composite** - The effects of degradation of wood fiber composites due to a variety of factors such as moisture, light, construction and handling damage etc... needs to be explored more fully.

5.3 Concluding Remarks

The use of fiber reinforcement with wood is relatively new and has yet to be developed to any great extent. As seen in this thesis, strategically placed fiber reinforcement in different configurations has the ability of enhancing selected mechanical properties of wood-fiber composite beams. Concentrations of reinforcement at the midspan tend to be more effective at reducing deflections while continuous reinforcement increases the ultimate strength.

One of the major impediments to the widespread use of fiber reinforced wood at the present time is the high cost associated with the polyester resins used in the manufacture of composite materials. It is likely that these resin costs will decrease substantially in the future as composite materials gain acceptance and become more widespread. This decrease in resin prices coupled with increasing lumber costs will

eventually reach a point where the use of wood-fiber products will not only become commercially viable but necessary.

The introduction of the FiRP panel in structural applications is an indication that commercial use of wood-fiber composites is possible for new construction. Nevertheless, the main area of applicability for wood-fiber composites continues to be restorations where the versatility and strength of fiber reinforced plastics make it ideal for this application.

APPENDIX 1 - ELASTIC SOLID WOOD CROSS-SECTIONS

In the elastic region where

$$\epsilon_t \leq \epsilon_c$$

$$\epsilon_b \leq \frac{f_m}{E_w} \quad \text{where} \quad f_m = \left(\frac{c}{1+k_3} \right)^{\frac{1}{k_3}} f_t$$

The forces can be determined by examining the strain distribution as follows:

$$C = \left(\frac{1-c}{c} \right) \epsilon_b (1-c) h \left(\frac{1}{2} \right) E_w = \frac{(1-c)^2}{2c} \epsilon_b A_w E_w$$

$$T = c h \left(\frac{1}{2} \right) \epsilon_b E_w = \frac{c}{2} \epsilon_b A_w E_w$$

$$T_f = A_f E_f \epsilon_b$$

Summing these forces in the x direction yields an expression for the position of the neutral axis as can be seen below

$$\begin{aligned}\sum F_x &= 0 \\ &= C - T - T_f \\ &= \frac{(1-c)^2}{2c} \epsilon_b A_w E_w - \frac{c}{2} \epsilon_b A_w E_w - \epsilon_b A_f E_f \\ &= \frac{(c^2 - 2c + 1)}{2c} \epsilon_b A_w E_w - \frac{c}{2} \epsilon_b A_w E_w - \epsilon_b A_f E_f \\ &= \frac{A_w E_w}{2c} - A_w E_w - A_f E_f\end{aligned}$$

From this relationship a expression for the position of the neutral axis can be determined

$$c = \frac{A_w E_w}{2(A_f E_f + A_w E_w)}$$

$$c = \frac{1}{2\left(\frac{A_f E_f}{A_w E_w} + 1\right)}$$

To determine the moment capacity of the cross-section the internal forces are summed about the neutral axis to produce:

$$\sum M_{NA} = C \left[\left(\frac{2}{3} \right) (1-c)h \right] + T \left[\left(\frac{2}{3} \right) ch \right] + T_f ch$$

but $T_r = C - T$ therefore the moment equation reduces to

$$\sum M_{NA} = \frac{C(2-2c)h + 2Tch + 3Cch - 3Tch}{3}$$

$$= \frac{C(2+c) - Tc}{3} h$$

APPENDIX 2 - INELASTIC SOLID WOOD CROSS-SECTIONS

In the inelastic region

$$\epsilon_t \geq \epsilon_c$$

$$\epsilon_h \leq \frac{f_m}{E_w} \quad \text{where} \quad f_m = \left(\frac{c}{1+k_1} \right)^{-\frac{1}{k_1}} f_t$$

The forces can be determined by examining the strain distribution as follows:

$$T = chb \left(\frac{1}{2} \right) \epsilon_h E_w = \frac{c}{2} \epsilon_h A_w E_w$$

$$T_f = A_f E_f \epsilon_h$$

$$C_1 = \epsilon_c \left(\frac{\epsilon_c}{\epsilon_b} chb \right) \left(\frac{1}{2} \right) E_w = \frac{\epsilon_c^2}{\epsilon_b} \left(\frac{c}{2} \right) A_w E_w$$

$$C_2 = \left[\epsilon_c - m(\epsilon_i - \epsilon_c) \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] hb E_w \\ + \left(\frac{1}{2} \right) \left[\epsilon_c - (\epsilon_c - m(\epsilon_i - \epsilon_c)) \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] hb E_w$$

$$C_2 = \left(\frac{1}{2} \right) \left[2\epsilon_c - m(\epsilon_i - \epsilon_c) \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] hb E_w$$

Since $\epsilon_i = \left(\frac{1-c}{c} \right) \epsilon_b$, this equation becomes:

$$C_2 = \left(\frac{1}{2} \right) \left[(2+m)\epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] hb E_w$$

$$C_2 = \left(\frac{1}{2} \right) \left[(2+m)\epsilon_c + m \left(1 - \frac{1}{c} \right) \epsilon_b \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] A_w E_w$$

Summing these forces in the x direction yields an expression for the position of the neutral axis as can be seen below:

$$\sum F_x = 0$$

$$= C_1 + C_2 - T - T_f$$

$$= \frac{\epsilon_c^2}{\epsilon_b} \left(\frac{c}{2} \right) A_w E_w + \left(\frac{1}{2} \right) \left[(2+m)\epsilon_c + m \left(1 - \frac{1}{c} \right) \epsilon_b \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] A_w E_w - \frac{c}{2} \epsilon_b A_w E_w - A_f E_f \epsilon_b$$

Multiplying by $\frac{2c}{\epsilon_b A_w E_w}$ yields

$$= \frac{\epsilon_c^2}{\epsilon_b^2} c^2 + \left[(2+m) \frac{\epsilon_c}{\epsilon_b} c + m(c-1) \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] - c^2 - 2 \frac{A_f E_f}{A_w E_w} c$$

Expanding this equation

$$= \frac{\epsilon_c^2}{\epsilon_b^2} c^2 + (2+m) \frac{\epsilon_c}{\epsilon_b} c + mc - m - (2+m) \frac{\epsilon_c}{\epsilon_b} c^2 - mc^2 + mc - (2+m) \frac{\epsilon_c^2}{\epsilon_b^2} - m \frac{\epsilon_c}{\epsilon_b} c^2 + m \frac{\epsilon_c}{\epsilon_b} c$$

$$= -(1+m) \frac{\epsilon_c^2}{\epsilon_b^2} c^2 - (1+m) c^2 - 2(1+m) \frac{\epsilon_c}{\epsilon_b} c^2 - 2 \frac{A_f E_f}{A_w E_w} c + 2(1+m) \frac{\epsilon_c}{\epsilon_b} c + 2mc - m$$

Therefore the position of the neutral axis can be determined by solving the following quadratic equation:

$$(1+m) \left(1 + \frac{\epsilon_c}{\epsilon_b} \right)^2 c^2 + \left[2 \frac{A_f E_f}{A_w E_w} - 2(1+m) \frac{\epsilon_c}{\epsilon_b} - 2m \right] c + m = 0$$

To determine the moment capacity of the cross-section the internal forces are summed about the neutral axis as follows:

$$\sum M_{NA} = \frac{2Tch}{3} + T_f ch + \frac{2\epsilon_c}{3\epsilon_b} C_1 ch + \left[\frac{\epsilon_c}{\epsilon_b} ch + xh \right] C_2$$

however it is known that $T_f = C_1 + C_2 - T$ therefore the moment equation can be written as

$$\sum M_{NA} = C_1 ch \left(1 + \frac{2\epsilon_c}{3\epsilon_b} \right) - \frac{Tch}{3} + C_2 \left(ch + \frac{\epsilon_c}{\epsilon_b} ch + xh \right)$$

where the distance x can be established by examining the cross section in the inelastic region and finding the centroid of the forces. To simplify the calculation the centroid will be determined from the proportional limit which is already known to be a distance $\frac{\epsilon_c}{\epsilon_b} ch$ from the neutral axis.

Figure A2.2 - Forces in the Inelastic Region

$$C_{2a} = \left[\epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b + m \epsilon_c \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] hb E_u$$

$$\bar{y}_{2a} = \frac{1}{2} \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) h$$

$$C_{2b} = \frac{1}{2} \left[\epsilon_c - \left[\epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b + m \epsilon_c \right] \right] \left[1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right] A_u E_u$$

$$\bar{y}_{2b} = \frac{1}{3} \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) h$$

$$\bar{y}_2 = \frac{C_{2a} \bar{y}_{2a} + C_{2b} \bar{y}_{2b}}{C_2}$$

$$= \frac{\left(\frac{1}{2} \left[\epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b + m \epsilon_c \right] + \frac{1}{6} \left[m \left(\frac{1-c}{c} \right) \epsilon_b - m \epsilon_c \right] \right) \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right)^2 h A_w E_w}{\left(\frac{1}{2} \right) \left[(2+m) \epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b \right] \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) A_w E_w}$$

$$= \frac{\left(\left[\epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b + m \epsilon_c \right] + \frac{1}{3} \left[m \left(\frac{1-c}{c} \right) \epsilon_b - m \epsilon_c \right] \right) \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) h}{\left[(2+m) \epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b \right]}$$

$$= \frac{\left(\left[\epsilon_c - \frac{2}{3} m \left(\frac{1-c}{c} \right) \epsilon_b + \frac{2}{3} m \epsilon_c \right] \right) \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) h}{\left[(2+m) \epsilon_c - m \left(\frac{1-c}{c} \right) \epsilon_b \right]}$$

Multiplying by $\frac{c}{(1-c)m}$ yields

$$= \frac{\left(\left[\epsilon_c \left(\frac{1 + \frac{2m}{3}}{m} \right) \left(\frac{c}{1-c} \right) - \frac{2}{3} \epsilon_b \right] \right) \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) h}{\left[\frac{(2+m)}{m} \epsilon_c \frac{c}{1-c} - \epsilon_b \right]}$$

$$= \frac{\left(\left[\frac{\epsilon_c}{\epsilon_b} \left(\frac{1 + \frac{2m}{3}}{m} \right) \left(\frac{c}{c-1} \right) + \frac{2}{3} \right] \right) \left(1 - \left(1 + \frac{\epsilon_c}{\epsilon_b} \right) c \right) h}{\left[\frac{\epsilon_c}{\epsilon_b} \left(\frac{(2+m)}{m} \right) \left(\frac{c}{c-1} \right) + 1 \right]}$$

Substituting this expression for x back into the moment equation yields:

$$\sum M_{NA} = C_1 ch \left(1 + \frac{2\varepsilon_c}{3\varepsilon_b} \right) - \frac{Tch}{3} + C_2 \left(ch + \frac{\varepsilon_c}{\varepsilon_b} ch + \frac{\left[\frac{\varepsilon_c}{\varepsilon_b} \left(\frac{1 + \frac{2m}{3}}{m} \right) \left(\frac{c}{c-1} \right) + \frac{2}{3} \right] \left(1 - \left(1 + \frac{\varepsilon_c}{\varepsilon_b} \right) c \right) h}{\left[\frac{\varepsilon_c}{\varepsilon_b} \left(\frac{2+m}{m} \right) \left(\frac{c}{c-1} \right) + 1 \right]} \right)$$

APPENDIX 3 - ELASTIC I-BEAM CROSS-SECTIONS

Figure A3.1 - Strain and Force Diagrams for elastic I-Beams

In the elastic region where

$$\epsilon_t \leq \epsilon_{t_e}$$

$$\epsilon_b \leq \frac{f_m}{E_n} \quad \text{where} \quad f_m = \left(\frac{c}{1+k_3} \right)^{\frac{1}{k_3}} f_t$$

The forces can be determined by examining the strain distribution as follows:

$$T_{fp} = A_{fp} E_{fp} \epsilon_b$$

$$T_{f1} = \left(\frac{c - \frac{t_b}{h}}{c} \right) \epsilon_b A_b E_b$$

$$T_{f2} = \left(\epsilon_b - \frac{\left(c - \frac{t_b}{h} \right) \epsilon_b}{c} \right) \frac{A_b E_b}{2}$$

$$T_w = \frac{\left(c - \frac{t_b}{h} \right)}{c} \epsilon_b \left[\frac{\left(c - \frac{t_t}{h} \right)}{\left(1 - \frac{t_b + t_t}{h} \right)} \right] \frac{A_{web} E_{web}}{2}$$

$$C_w = \frac{\left(1 - c - \frac{t_t}{h} \right)}{c} \epsilon_b \left[\frac{\left(1 - c - \frac{t_t}{h} \right)}{\left(1 - \frac{t_b + t_t}{h} \right)} \right] \frac{A_{web} E_{web}}{2}$$

$$C_{f1} = \frac{\left(1 - c - \frac{t_t}{h} \right)}{c} \epsilon_b A_t E_t$$

$$C_{f2} = \frac{\left((1 - c) - \left(1 - c - \frac{t_t}{h} \right) \right)}{c} \epsilon_b \frac{A_t E_t}{2}$$

Summing these forces in the x direction yields an expression for the position of the neutral axis as can be seen below:

$$\sum F_x = 0$$

$$= C_w + C_{f_1} + C_{f_2} - T_{fp} - T_{f_1} - T_{f_2} - T_w$$

$$= \frac{\left(1 - c - \frac{t_1}{h}\right)}{c} \epsilon_b \left[\frac{\left(1 - c - \frac{t_1}{h}\right)}{\left(1 - \frac{t_b + t_1}{h}\right)} \right] \frac{A_{web} E_{web}}{2} + \frac{\left(1 - c - \frac{t_1}{h}\right)}{c} \epsilon_b A_t E_t + \frac{\left((1 - c) - \left(1 - c - \frac{t_1}{h}\right)\right)}{c} \epsilon_b \frac{A_t E_t}{2}$$

$$- A_{fp} E_{fp} \epsilon_b - \left(\frac{c - \frac{t_b}{h}}{c} \right) \epsilon_b A_b E_b - \left(\epsilon_b - \frac{\left(c - \frac{t_b}{h}\right)}{c} \epsilon_b \right) \frac{A_b E_b}{2} - \frac{\left(c - \frac{t_b}{h}\right)}{c} \epsilon_b \left[\frac{\left(c - \frac{t_1}{h}\right)}{\left(1 - \frac{t_b + t_1}{h}\right)} \right] \frac{A_{web} E_{web}}{2}$$

multiplying by $\frac{2c}{\epsilon_b}$ produces

$$= \left(1 - c - \frac{t_1}{h}\right) \left[\frac{\left(1 - c - \frac{t_1}{h}\right)}{\left(1 - \frac{t_b + t_1}{h}\right)} \right] A_{web} E_{web} + 2 \left(1 - c - \frac{t_1}{h}\right) A_t E_t + \frac{t_1}{h} A_t E_t$$

$$- 2 A_{fp} E_{fp} c - 2 \left(c - \frac{t_b}{h}\right) A_b E_b - \left(c - \left(c - \frac{t_b}{h}\right)\right) A_b E_b - \left(c - \frac{t_b}{h}\right) \left[\frac{\left(c - \frac{t_1}{h}\right)}{\left(1 - \frac{t_b + t_1}{h}\right)} \right] A_{web} E_{web}$$

$$\begin{aligned}
&= A_{web} E_{web} \left(\frac{c^2 + 2 \frac{t_t}{h} c - 2c - 2 \frac{t_t}{h} + \left(\frac{t_t}{h} \right)^2 + 1}{\left(1 - \left(\frac{t_b + t_t}{h} \right) \right)} \right) + 2 A_t E_t \left(1 - c - \frac{t_t}{h} \right) + A_t E_t \left(\frac{t_t}{h} \right) \\
&- 2 A_{fp} E_{fp} c - 2 A_b E_b c + 2 A_b E_b \left(\frac{t_b}{h} \right) - A_b E_b \left(\frac{t_b}{h} \right) - A_{web} E_{web} \left(\frac{c^2 - 2 \frac{t_b}{h} c + \left(\frac{t_b}{h} \right)^2}{\left(1 - \left(\frac{t_b + t_t}{h} \right) \right)} \right)
\end{aligned}$$

From this equation an expression for c can be obtained as shown below:

$$c = \frac{A_b E_b \left(\frac{t_b}{h} \right) - A_t E_t \left(\frac{t_t}{h} \right) + 2 A_t E_t + \frac{A_{web} E_{web} \left(\left(\frac{t_t}{h} \right)^2 - \left(\frac{t_b}{h} \right)^2 - 2 \frac{t_t}{h} + 1 \right)}{\left(1 - \left(\frac{t_t + t_b}{h} \right) \right)}}{2 \left[A_{fp} E_{fp} + A_b E_b + A_t E_t + \frac{A_{web} E_{web} \left(-\frac{t_b}{h} - \frac{t_t}{h} + 1 \right)}{\left(1 - \left(\frac{t_t + t_b}{h} \right) \right)} \right]}$$

This general equation can be further reduced if the top and bottom flange are the same size and possess the same mechanical properties as follows:

$$c = \frac{2 A_b E_b + \frac{A_{web} E_{web} \left(1 - 2 \frac{t_b}{h} \right)}{\left(1 - 2 \frac{t_b}{h} \right)}}{2 \left[A_{fp} E_{fp} + 2 A_b E_b + \frac{A_{web} E_{web} \left(1 - 2 \frac{t_b}{h} \right)}{\left(1 - 2 \frac{t_b}{h} \right)} \right]}$$

$$c = \frac{2A_b E_b + A_{web} E_{web}}{2[A_{fp} E_{fp} + 2A_b E_b + A_{web} E_{web}]}$$

To determine the moment capacity of the cross-section the internal forces are summed about the neutral axis to produce:

$$\begin{aligned} \sum M_{NA} = & T_{fp} ch + T_{f1} \left(c - \frac{t_f}{2h} \right) h + T_{f2} \left(c - \frac{t_f}{3h} \right) h + T_w \left(\frac{2}{3} \left(c - \frac{t_f}{h} \right) \right) h \\ & + C_{f1} \left(1 - c - \frac{t_f}{2h} \right) h + C_{f2} \left(1 - c - \frac{t_f}{3h} \right) h + C_w \left(\frac{2}{3} \left(1 - c - \frac{t_f}{h} \right) \right) h \end{aligned}$$

APPENDIX 4 - INELASTIC I-BEAM CROSS-SECTIONS

Figure A4.1 - Strain and Force Diagrams for elastic I-Beams

In the elastic region where

$$\epsilon_t \geq \epsilon_c$$

$$\epsilon_b \leq \frac{f_m}{E_w} \quad \text{where} \quad f_m = \left(\frac{c}{1+k_1} \right)^{1/k_1} f_t$$

The forces can be determined by examining the strain distribution as follows:

$$T_{fp} = A_{fp} E_{fp} \epsilon_b$$

$$T_{f_1} = \left(\frac{c - \frac{t_b}{h}}{c} \right) \epsilon_b A_b E_b$$

$$T_{f_2} = \left(\epsilon_b - \frac{\left(c - \frac{t_b}{h} \right) \epsilon_b}{c} \right) \frac{A_b E_b}{2}$$

$$T_w = \frac{\left(c - \frac{t_b}{h} \right) \epsilon_b}{c} \left[\frac{\left(c - \frac{t_t}{h} \right)}{\left(1 - \frac{t_b + t_t}{h} \right)} \right] \frac{A_{web} E_{web}}{2}$$

$$C_w = \frac{\left(1 - c - \frac{t_t}{h} \right) \epsilon_b}{c} \left[\frac{\left(1 - c - \frac{t_t}{h} \right)}{\left(1 - \frac{t_b + t_t}{h} \right)} \right] \frac{A_{web} E_{web}}{2}$$

$$C_{f_1} = \frac{\left(1 - c - \frac{t_t}{h} \right) \epsilon_b}{c} \left[\frac{\frac{t}{h} - \left(1 - c - \frac{\epsilon_c c}{\epsilon_b} \right)}{\frac{t_t}{h}} \right] A_t E_t$$

$$C_{f_2} = \left(\epsilon_c - \frac{\left(1 - c - \frac{t_t}{h} \right) \epsilon_b}{c} \right) \left[\frac{\frac{t}{h} - \left(1 - c - \frac{\epsilon_c c}{\epsilon_b} \right)}{\frac{t_t}{h}} \right] \frac{A_t E_t}{2}$$

$$C_{f_1} = \left(\epsilon_c - m \left(\frac{1-c}{c} \epsilon_b - \epsilon_c \right) \right) \left[\frac{\left(1-c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_1}{h}} \right] A_1 E_1$$

$$C_{f_2} = \left(\epsilon_c - \left(\epsilon_c - m \left(\frac{1-c}{c} \epsilon_b - \epsilon_c \right) \right) \right) \left[\frac{\left(1-c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_1}{h}} \right] \frac{A_1 E_1}{2}$$

Summing these forces in the x direction yields an expression for the position of the neutral axis as can be seen below:

$$\sum F_x = 0$$

$$= C_w + C_{f_1} + C_{f_2} + C_{f_3} + C_{f_4} - T_{fp} - T_{f_1} - T_{f_2} - T_w$$

$$\begin{aligned} &= \frac{\left(1-c - \frac{t_1}{h} \right)}{c} \epsilon_b \left[\frac{\left(1-c - \frac{t_1}{h} \right)}{\left(1 - \frac{t_b + t_1}{h} \right)} \right] \frac{A_{web} E_{web}}{2} + \frac{\left(1-c - \frac{t_1}{h} \right)}{c} \epsilon_b \left[\frac{\frac{t}{h} - \left(1-c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_1}{h}} \right] A_1 E_1 + \\ &\left(\epsilon_c - \frac{\left(1-c - \frac{t_1}{h} \right)}{c} \epsilon_b \right) \left[\frac{\frac{t}{h} - \left(1-c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_1}{h}} \right] \frac{A_1 E_1}{2} + \left(\epsilon_c - m \left(\frac{1-c}{c} \epsilon_b - \epsilon_c \right) \right) \left[\frac{\left(1-c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_1}{h}} \right] A_1 E_1 \\ &+ \left(\epsilon_c - \left(\epsilon_c - m \left(\frac{1-c}{c} \epsilon_b - \epsilon_c \right) \right) \right) \left[\frac{\left(1-c - \frac{\epsilon_c}{\epsilon_b} c \right)}{\frac{t_1}{h}} \right] \frac{A_1 E_1}{2} - A_{fp} E_{fp} \epsilon_b - \left(\frac{c - \frac{t_b}{h}}{c} \right) \epsilon_b A_b E_b \\ &- \left(\epsilon_b - \frac{\left(c - \frac{t_b}{h} \right)}{c} \epsilon_b \right) \frac{A_b E_b}{2} - \frac{\left(c - \frac{t_b}{h} \right)}{c} \epsilon_b \left[\frac{\left(c - \frac{t_1}{h} \right)}{\left(1 - \frac{t_b + t_1}{h} \right)} \right] \frac{A_{web} E_{web}}{2} \end{aligned}$$

ultimately this expression reduces down to the form:

$$\left[\frac{(3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)^2}{\frac{t_i}{h}} A_i E_i \right] c^2 +$$

$$\left[2(A_{fp} E_{fp} + A_b E_b + A_i E_i + A_{web} E_{web}) - \frac{2A_i E_i (3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)}{\frac{t_i}{h}} \right] c +$$

$$\left[\frac{A_{web} E_{web} \left(\left(\frac{t_b}{h} \right)^2 - \left(\frac{t_i}{h} \right)^2 + 2 \frac{t_i}{h} - 1 \right)}{1 - \frac{t_i + t_b}{h}} + \frac{A_i E_i \left(\left(\frac{t_i}{h} - 1 \right)^2 + 3m \right)}{\frac{t_i}{h}} - 2A_b E_b \frac{t_b}{h} \right] = 0$$

In the case of doubly symmetric I-beams this equation can be reduced to:

$$\left[\frac{(3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)^2}{\frac{t_b}{h}} A_b E_b \right] c^2 +$$

$$\left[2(A_{fp} E_{fp} + 2A_b E_b + A_{web} E_{web}) - \frac{2A_b E_b (3m+1) \left(\frac{\epsilon_c}{\epsilon_b} + 1 \right)}{\frac{t_b}{h}} \right] c +$$

$$\left[\frac{A_b E_b \left(\left(\frac{t_b}{h} - 1 \right)^2 + 3m \right)}{\frac{t_b}{h}} - 2A_b E_b \frac{t_b}{h} - A_{web} E_{web} \right] = 0$$

To determine the moment capacity of the cross-section the internal forces are summed about the neutral axis to produce:

$$\begin{aligned}
 \sum M_{NA} = & T_{fp}ch + T_{f1}\left(c - \frac{t_b}{2h}\right)h + T_{f2}\left(c - \frac{t_b}{3h}\right)h + T_w\left(\frac{2}{3}\left(c - \frac{t_b}{h}\right)\right)h \\
 & + C_w\left(\frac{2}{3}\left(1 - c - \frac{t_t}{h}\right)\right)h + C_{f1}\left(\frac{\epsilon_c}{\epsilon_b}c - \frac{1}{2}\left(\frac{t_t}{h} - \left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)\right)h \\
 & + C_{f2}\left(\frac{\epsilon_c}{\epsilon_b}c - \frac{1}{3}\left(\frac{t_t}{h} - \left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)\right)h \\
 & + C_{f3}\left(\frac{\epsilon_c}{\epsilon_b}c + \frac{1}{2}\left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)h + C_{f4}\left(\frac{\epsilon_c}{\epsilon_b}c + \frac{1}{3}\left(1 - c - \frac{\epsilon_c}{\epsilon_b}c\right)\right)h
 \end{aligned}$$

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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