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Design of Plywood Sheathed Wood Floors and Roofs

By

Changjiang Liu

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the
requirements for the degree of
DOCTOR OF PHILOSOPHY

in
CIVIL ENGINEERING

**DEPARTMENT OF CIVIL & ENVIRONMENTAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, CANADA
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The undersigned hereby recommend to
The Faculty of Graduate Studies and Research
acceptance of the thesis.

Design of Plywood Sheathed Wood Floors and Roofs

submitted by

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in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Civil Engineering

Thesis Supervisor

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Carleton University

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Abstract

Existing design methodologies for wood joist floor systems using finite element methods and T-beam models exhibit some short comings. The T-beam model is incapable of predicting deflections of floors subjected to concentrated loads or long-span floors subjected to uniformly distributed loads. On the other hand, the finite element approach is often too complicated to be adopted in practice which is one of the reasons why the T-beam model is still being used in design practice. The design issues of plywood sheathed wood joist floors and roofs are examined using orthotropic plate theory, which provides a simpler approach than finite element analysis and more comprehensive than the simple T-beam method. A simple procedure is proposed for the design of floors subjected to concentrated loads. Using the fact that plywood sheathed floors are stiffer in the direction of the joists than in the transverse direction, the determination of maximum deflections is done using a simplified procedure. The maximum deflection can be calculated from one single formula. This procedure is valid in the scope of current available sizes of joists and plywood panels. The effects of ponding on the deflection of flat roofs is also examined. Roofs are then classified as stable and unstable with respect to ponding. A criterion is derived for design purposes, which defines roofs as stable or unstable depending on the dimensions and rigidity of the roof. Limit states design of wood structures requires not only sound structural analysis procedures to determine the structural behaviour of systems, but also reliable assessment of wood members. A unified theory that combines the effects of size and loading configuration using a total damage accumulation model to assess wood member strength is proposed.

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Chapter 1

Introduction

1.1 Background

Canada has one of the biggest forest inventories in the world. The Canadian forest resource contains a wood volume of about 24 billion m^3 and covers one third of the land in Canada. The forest sector provides during each year (1990 data, [69]):

- 270 000 jobs in logging, lumber, pulp and paper
- \$7.4 billion in salaries and wages
- \$ 32.8 billion worth of products. \$16.2 billion export. \$14 billion balance of trade.
- More than 30% of the world's supply of news-print.

On the other hand, as a renewable material, wood products can be produced without exhausting our forest resources. Wood produces less waste than man-made materials and hence has less environmental impact if it is managed wisely.

Light-frame residential wood buildings represent the largest single construction type in North- America, and they are usually built on the base of experience. These

buildings have traditionally demonstrated high reliability, but the mechanisms behind their behaviour are not always fully understood.

One of the largest sub-systems in light-frame wood construction is the wood joist floor assembly. Although wood joist and sheathing systems are simple in concept, they are complicated systems to analyze. Not only are they usually multi-layered with each layer made of orthotropic materials, but also in nailed and elastometrically glued systems, there exists an interlayer slip of sufficient magnitude to have a major effect on the deflection and stresses in the system. In addition, gaps at joints in the sheathing layers produce discontinuities, which have a significant effect on the composite action of the system. These factors, coupled with the variability of the material in each component, make the analysis of the floor system difficult.

In a plywood sheathed wood joists floor, joists are the primary components which carry much larger proportion of bending in the direction of joists than plywood. However, the contribution of plywood to the bending stiffness of floors in this direction can not be ignored. The composite action between plywood and joist in this direction forms a T-beam cross-section. In the direction perpendicular to the joists, on the other hand, it is the plywood that provides bending strength and stiffness. Connections between the plywood and joist determine the magnitude of the composite action. A glued connection could lead to a much stiffer T-beam than could a loose connection. The stiffness of the floor in the transversal direction is much smaller than in the direction of the joists. Yet, it cannot be ignored in some cases. Gaps between the sheathing panel also constitute a factor that influences the stiffness of floors.

1.2 Literature Review

In the past, safe house construction methods have been found by the inefficient method of trial and error. If modern research efforts, so successful in the solution of other problems, had been applied to housing, houses would be more satisfactory as dwellings and, much more important, the cost would be smaller. A more efficient and economic use of wood materials in housing construction would be an outstanding

contribution to the problem of providing acceptable housing for the low-income groups in this country (Whittemore, 1948 [91] [30]).

1.2.1 Colorado State University, 1971, Mathematical Model and Test, Vanderbilt et. al

In early seventies, it was recognized that the over-simplified design procedures imposed on frame floor systems penalizes wood. There are additive performance effects of the materials and fastenings used in wood frame floor systems. In 1971, a comprehensive study to develop a rational analysis and design procedure for wood joist floor systems began at Colorado State University (Vanderbilt, Goodman, Criswell, 1974 [91]). As the first step, a study of layered beams, the major structural component of the floor system, was carried out. The layered beams or T-beams were considered as having two layers, the sheathing and the joist, connected with mechanical fasteners or glue as shown in Figure 1.1.

Then floors were simplified as a system of crossing beams as shown in Figure 1.2 which ignored the contribution of the torsional stiffnesses of the sheathing and T-beams. A set of equations were developed and solved using matrix structural theory. The mathematical model incorporates two-way action and incomplete composite action as well as variations in material and fastener properties. The two-way action represents the load-distribution effect of sheathing on the transverse direction. Incomplete composite action defines the contribution of sheathing, connections and joists toward the stiffness of the T-beam. The model of an idealized floor under examination contains 11 T-beam and 11 sheathing strips. Simple supports were assumed at all edges.

As demonstrated in Figure 1.3, the model describes the two-way action and composite action well. In the figure, four different cases were demonstrated: 1) joists only, no sheathing; 2) distribution of loads by two-way action but neglecting composite action, two-way and no T-beam; 3) composite action due to 8-inch nail spacing with no two-way action; 4) complete composite and two-way action as predicted by the complete mathematical model.

It can be seen that the composite action improves the stiffness of floors while the two-way action redistributes loads among joists and thus reduces the impact of variations of joist stiffness to achieve a smooth deflection curve.

Experiments were conducted for T-beams and floors (Vanderbilt et. al, 1973, [91]). The floor was tested by applying a concentrated load at the middle of the center joist. Plywood sheathing was glued thus eliminating gaps. It was found that:

- the mathematical model agrees well with the tests at the service load level;
- the floor will not collapse after the failure of the first joist or even a couple of subsequent failures, Figure 1.4.
- the floor failed when the point load punched through the sheathing.

After the initial failure of a joist, the deflection was increased without the increasing the load. The load was re-distributed to adjacent joists. This proved the existence of two-way action and the role of plywood sheathing in a floor.

1.2.2 Colorado State University, 1975, Finite Element Analysis - FEAFL0, Thompson, et. al

This study has already used a concept of finite element analysis by breaking down a continuous floor system into crossing beams and calculating the deflection at the crossing points (Vanderbilt et. al. 1974 [91], Goodman et. al. 1974 [30]). In order to account for gaps, Figure 1.5, the finite element method has to be implemented, (Thompson 1975, [85] [86]).

The gaps were modelled using two different elements: a special element and a soft element. In the first method, a special element of zero length is introduced wherever there is a gap. Gaps can also be modelled using a soft element of finite small length with low modulus of elasticity. No numerical instability occurs when a low MOE is used since at least one layer (the joist) is always continuous at the gap. The result is a program called FEAFL0 (Finite Element Analysis of FL0ors). This program can analyze rectangular wood joist floors subject to any pattern of loading

perpendicular to the plane of the sheathing. The effects of interlayer slip, variable material properties, and gaps at points, are correctly considered. The joints can be in both layers of sheathing independently and both parallel and perpendicular to the floor joists. Input data include a description of floor geometry, number of layers (2 or 3), axial and bending MOE data, gap data, slip moduli, and load data. Output data at each node point include the deflection, extreme fiber stresses in each layer, interlayer shear flows, relative layer slip, nail forces, and the axial force and moment in each layer. The model predicts the deflection of floors with gaps in sheathing very well.

1.2.3 Colorado State University, 1979, Design Procedures Proposed, Sazinski, Vanderbilt

The last stage of the research work at Colorado State University presents a procedure that can be used by code writing committees and design engineers. Three methods were proposed (Sazinski et. al., 1979, [82]). The most accurate and time consuming of the three proposed design methodologies is the use of the finite element program, FEAFL0, which requires computer resources and knowledge of floor behaviour. This method is unlikely to be used in design practice, but could be very useful in research. The second method is to use dimension-less charts. The third method is to use span tables that give complete assemblies of joists, sheathing and connectors that meet specified acceptance criteria. Both the chart and span tables can be produced using FEAFL0.

It was also noticed that when the aspect ratio (joist length divided by the width of the floor) is more than unity, the deflection varies with different aspect ratio as shown in Figure 1.6. The deflection remains the same when the aspect ratio is less than unity while it decreases when the aspect ratio increases. However, no explanation was given. It was assumed that the aspect ratio was less than one, which defines a *short floor*, in all the subsequent studies.

1.2.4 Forest Product Lab, U.S., 1977, T-beam, Simplified Design Procedure, McCutcheon

Research has also been carried out at the Forest Products Laboratory, Forest Service, U.S. Department of Agriculture (McCutcheon, 1977 [62]). The research was aimed at developing a simple procedure that can be used in design practice.

A floor was designed as a T-beam with sheathing attached to the joist either by glue or nails. Gaps between sheathings along the joist were also considered, Figure 1.7. Only two-layer floors were studied. Maximum deflection of the T-beam is then calculated as

$$\Delta = \Delta_R \left[1 + f_{\Delta} \left(\frac{EI_R}{EI_U} - 1 \right) \right] \quad (1.1)$$

$$f_{\Delta} = \frac{10}{(L\alpha)^2 + 10} \quad (1.2)$$

$$EI_R = EI_U + \frac{(EA_1)(EA_2)}{EA_1 + EA_2} h^2 \quad (1.3)$$

where

Δ	deflection of the beam
Δ_R	deflection if the components of the beam are rigidly connected.
EI_R	bending stiffness if the components are rigidly connected.
EI_U	stiffness if the components are completely unconnected.
f_{Δ}	a constant function of $L\alpha$.
L	beam span.
α^2	$\frac{k^2 S}{EI_R - EI_U} \left(\frac{EI_R}{EI_U} \right)$
S	load per unit length which causes a unit slip in the nail or adhesive joint.
EA_1	the axial stiffness of the flange (sheathing)
EA_2	the axial stiffness of the web (joist)
h	the distance between the centroids of the flange and the web.

Table 1.1: Summary of Floor Tests at FPL, USDA, 1974, McCutcheon

Floor	Average E	Deflection at 500 lbs	Deflection at 50 psf	Failure load
Nailed	10^6 lbs/in^2	inch	inch	Lb/ft^2
N-1	1.57	0.155	0.376	128
N-2	2.20	0.126	0.280	190
N-3	2.00	0.115	0.312	120
N-4	1.49	0.152	0.368	140
Glued				
G-1	1.53	0.104	0.246	200
G-2	2.20	0.087	0.189	315
G-3	2.00	0.095	0.190	225

To account for the effects of gaps in the sheathing, L' is used instead of L in calculating f_{Δ} , L' is the distance between gaps along the span of joists.

$$f_{\Delta} = \frac{10}{(L'\alpha)^2 + 10} \quad (1.4)$$

To verify the T-beam formula, seven floors were constructed. Each consisted of nine No. 2 or better 38x184 mm Douglas-fir joists spaced 400 mm on centers. Floors were 3 meters wide and were tested on a span of 3658 mm. This gives an aspect ratio of 1.11 (span/width). Four of them used nail connection, while the other three used glue. Concentrated load as well as uniformly distributed loads were applied to the floors. The results from the T-beam formulation were in good agreement with the tests results of floors subjected to uniformly distributed load. However, the T-beam formulation can not predict the deflection of floors subjected to concentrated load.

The summary of the test results is reproduced in Table 1.1.

1.2.5 Forest Product Lab. USDA, 1984, A Beam-Spring Analog, McCutcheon

In order to account for the two-way action and the variability of joist properties, McCutcheon (1984) [63] proposed a new model. The floor systems were modeled in two steps. First, the sheathing is assumed to be compressed into a narrow beam that spans the simply supported joists and serves as the load-distributing element. Second, the joists are modeled as linear elastic springs that support the load-distributing beams, Figure 1.8. The stiffness of the springs is derived from the bending stiffness of the T-beam, as mentioned before. The beam-spring analog involves just one degree of freedom. A 10-joist floor can be analyzed with just 10 simultaneous equations. Compared to hundreds of equations involved in finite element analysis, the number of equations involved has been greatly reduced and hence the amount of computation. The process can be handled by a simple BASIC program on a hand held calculator. This is of practical implication for design engineers.

The model was accurate as demonstrated by a comparison with finite element analysis and floor tests. However, the process is still complicated since it involves solving multiple complicated equations. It is difficult to predict floor behaviours by simply examining the equations.

McCutcheon (1986) [64] also proposed a simple procedure to compute the composite stiffness of a wood bending member with sheathing attached non-rigidly to one or both edges to form a T-beam or an I-beam. The sheathing consisted of plywood and oriented strand-board (OSB). A so-called "transformed area" procedure was introduced to include the inter-layer slip by modifying the axial stiffness of the flanges and the stiffness of the T-beam or I-beam. 12 T-beams and 24 I-beams were constructed and tested to verify the formulation.

1.2.6 Forintek Canada, Western Lab. 1982, A Combined Fourier Series and Finite Element Analysis, Foschi

Foschi (Forintek Canada, 1982, [20]) used a combined Fourier series and finite element method to analyse floors. It was pointed out that the model in FEAFLOR restricts some degrees of freedom, since it does not consider the torsional deformation. These deformations are small but cannot be neglected. As shown by the effect that joist bridging has on floor deflection. As a consequence, the model is stiffer than the actual structure and, as shown by experimental data ([86]), predicted deflection were found to be consistently smaller than test measurements. An improvement in modelling, including all degrees-of-freedom was introduced to try to avoid the need to introduce parameters, such as flexible gap's properties, which are not well-defined and difficult to estimate.

This permits the consideration of the effect that joist bridging has on the maximum floor deflection and the maximum bending stress. The analysis was still in the linear range, i.e. nonlinearities in the fasteners, inelastic joist or cover behavior were not included. This is the first time a floor is analysed as a complete system.

Four floors were tested. Three of them were beams with three joists each loaded with a pair of concentrated forces located over the central joist. The other one was a floor containing 12 joists, supported on all sides. Gaps were produced on the sheathing. The aspect ratio is 1.19 (Span/Width).

1.2.7 University of British Columbia, 1983, Test of 13 Floors, Foschi

In order to assess the extent of floor nonlinear behaviour, failure mode, nail stiffness information, a series of 13 more floors were tested under uniformly distributed load at the University of British Columbia, Canada (Foschi 1985 [21]). The floors were constructed with 12-38x184 (2"x8") hemlock joists, No.2 and better grade, 15.8mm (5/8") Douglas-fir plywood, and 8d (2-1/2") common nails at 152 mm (6") o.c. Experimental results were presented for floor deflection, first joist failure load, and

subsequent failure loads. Test results were compared with analytical results from a program called FAP developed at UBC.

It was observed that the response of the floors was nearly linear, except for sudden breaks from linearity in coincidence with a major joist failure. Thus even if the nails by themselves had strongly nonlinear load/slip behavior, the effect of this nonlinearity on the overall floor behaviour may be neglected. In particular, this is most evident at load levels below 75 psf (3.6 *kPa*), which corresponds to an upper limit for usual design loads. This suggests that linear analysis programs like FEAFLOR or FAP may be used with sufficient accuracy to predict floor deflections under load, without need to consider fastener nonlinearity, particularly at design load levels. The test results showed that, although first joist failure should be used in reliability analysis, the load carrying capacity of the floor assembly is normally higher and several more joists failures usually occur before total collapse. It was also recognized that the variability in floor strength is substantially lower than the variability in bending strength of individual joists. It was also concluded that the joists which have lower *E* are not generally the ones that fail first in a floor. In only three of the thirteen floors tested was the joist with the lowest modulus of elasticity the first to fail.

1.2.8 Reliability Analysis, 1983, Foschi

In the process of implementing reliability-based structural design for wood structures, it was realized that load duration and loading history plays a role in the assessment of long term strength for wood members. It was also realized that another aspect of reliability assessment in light-frame wood structures is that, because of the usually large variability in material properties, it is important to focus on the structural system rather than the single member.

Foschi (1983, [22]) combined a damage accumulation model with structural analysis of light-frame wood structures, and examined the safety in the wood structural system during the prescribed service life.

Table 1.2: Description of test floors, University of Texas at Austin

Floor groups	Span (ft-in)	Joists, No.2 or better	Joist Spacing (in)	Sheathing
1	11-0	2 x 8, SP	24	3/4-in SP
2	14-4	2 x 10, SP	24	3/4-in. SP
3	14-7	2 x 10, DF	24	3/4-in. SP
4	13-1	2 x 8, DF	16	23/32-in. SP
5	12-10	2 x 8, SP	16	23/32-in. SP

1.2.9 University of Texas at Austin, 1986, Test of 15 Floors, Wheat

In an attempt to study the behaviour of wood-joist floors loaded to their ultimate capacity, 15 full sized wood-joist floors - each having 10 joists - were tested under uniformly distributed loads at the Ferguson Structural Engineering Laboratory of the University of Texas at Austin (Wheat et. al. 1986, [94]). 15 floors covers a variety of spans. Five groups of floors were tested with 3 replications per group. The spans of each of the five groups, measured from the centers of bearing, were 11 feet (3353 mm); 12 feet 10 inch (3912 mm); 13 feet 1 inch (4000 mm); 14 feet, 4 inch (4370 mm) and 14 feet 7 inch (4445 mm). They are listed in Table 1.2.

Behaviour was noted at the design load and at the limit states of first joist rupture and ultimate strength. The general stiffening effect of the sheathing and the two-way action in each system were compared with the assumed bare-joist behaviour at the design load level. It was concluded according to the test of 15 full-size floors that

- The stiffness of the floors at the design load level were greater than required in design. This is due to the contribution of the sheathing with incomplete composite action and two-way action.
- The average strengths at first joist rupture for the floors with 2 x 10 joist were lower than those with 2 x 8 joists.

- The mode of failure at ultimate appeared to depend on the rate of loading.
- Regardless of joist spacing, the floors with 2 x 8 joists had higher average ultimate strengths than those with 2 x 10 joists.

Rosowsky and Ellingwood (1992) [52] realized that strength and serviceability limit states are closely inter-related in wood construction. The design of wood beams is often governed by limits on deflection, and the resulting section is larger than that required for flexure or shear strength.

Members used in light-frame construction are chosen from standard available dimension lumber sizes. The number of available sizes is relatively small for wood compared to the other materials such as steel. The section selected in wood design may be significantly larger than what would otherwise be required by the design equations.

The reliability increased with respect to ultimate strength by including the deflection limit state and by considering the availability of standard sections for individual wood members and parallel-member systems. The significance of system effects from load sharing in serviceability reliability is assessed.

The implications that these effects have for the ongoing development of load and resistance factor design (LRFD) provisions for engineered wood construction were examined.

1.2.10 Ponding

Other than uniformly distributed load and concentrated load, ponding load can be experienced by flat roofs. Flat roofs may fail due to ponding of rainwater as evidenced by reports of failures of flat roofs in rain storms. If the roof was initially deflected due to its dead load or due to lack of initial straightness, and if water get access to fill the resulting depression, then the total load at any point on the roof will be increased by an amount proportional to the deflection. The increase in the load gives rise to more deflection, consequently more load and so on, until the deflection becomes either stable or continues to increase to failure. The roofs which can stand the ponding load with enough stiffness are considered stable structures,

while those which can not stand the ponding load are called unstable structures as far as ponding is concerned. The question is what are the criteria that govern this phenomenon. (Liu and Salinas 1992,[42]).

The amplifying effects of the ponding load were studied by Haussler R.W. (1962, [33]). These studies were motivated by reports of failures caused by instability in wood, steel and aluminum roofs. In this study, the roof was modeled as a beam deflected in a sine curve subjected to ponding load which is proportional to the deflection of the beam, Figure 1.9.

Kuenzi and Bohannon [39] in 1964 derived an approximate solution for the deflection, bending stresses, and increase in load on beams and decking due to ponding of water or other fluids on roofs or floors. Cases considered are simply supported and clamped beams with uniformly distributed and concentrated load, plus load due to ponding. Roofs were simplified as a beam. Tests of aluminium beams were performed to verify the theory and significant ponding effects were observed.

The load per unit length of a beam of unit width is given by the formula:

$$w = w_0 + ky \quad (1.5)$$

where w_0 is uniformly distributed load, k is weight of ponding fluid per unit depth per unit area, and y is deflection.

The beam deflection is approximated by

$$y = A \sin \frac{\pi x}{L} \quad (1.6)$$

where L is span length, x is distance from reaction and A is a constant to be evaluated.

The constant A is therefore determined as

$$A = \frac{4w_0L^4}{\pi^5 EI(1 - \frac{kL^4}{\pi^4 EI})} \quad (1.7)$$

where E is modulus of elasticity of the beam, and I is moment of inertia.

The magnification factor can be defined as

$$f_{amp} = \frac{1}{1 - \frac{kL^4}{\pi^4 EI}} \quad (1.8)$$

A beam is unstable and can produce large deflection with the increase of ponding when

$$k > \frac{\pi^4 EI}{L^4} \quad (1.9)$$

A more detailed study can be found in Sawyer's paper (Sawyer, 1967 [81]). The basic function is described as

$$EI \frac{d^4 y}{dx^4} - kSy = 0 \quad (1.10)$$

in which E = Young's modulus for the material, I the moment of inertia of the beam, k unit weight of the ponding liquid, S the transverse spacing of beams and y is the deflection of the beam as a function of x .

Salama and Moody (1967, [70]) extended the study of the ponding effects using isotropic plate theory, treating the roofs as isotropic plates. Critical stiffness of such a roof was derived.

Folz and Foschi (1990, [16]) studied the structural reliability of roofs of lumber joists that are subjected to ponding loads. An approximate ponding amplification factor was derived and a simple design equation is proposed in the context of reliability based design, which is suitable for implementation in a limit-states design code. The basic model involves simplifying the roof into a beam subjected to ponding load on the tributary area.

1.2.11 The Bending Strength of Wood Members, Damage Accumulation Model

The limit state design of light frame wood structures requires reliability assessment of the structures. This process involves structural analysis of the system to define various limit states and also properties of individual members.

The moment resistance of wood members, e.g. beams, is given by many design standards in terms of the geometric properties of the cross-section and the specified or characteristic bending strength of the material, modified by a resistance factor and by a series of adjustment factors accounting for, among other things, moisture effects, rate of loading, preservative treatment, load sharing, bearing, lateral stability, length effect, volume effect, etc. Each one of these *adjustment factors* is the product of independent research. Studies on the effects of certain parameters on the bending strength of wood flexural members have been adopted by the code-making committees only after serious discussions and deliberations. Adjustment factors concerned with size, rate of loading and duration of load effects have received particular attention by the research community in Canada and the United States. Size effects have also been incorporated into the design formulas as *depth factors*, *length factors* and/or *volume factors*. Load-history effects and creep-rupture phenomena have been studied using damage accumulation models and taken into account by means of a "Load Duration Factor".

Size effects on the strength of lumber have been reported by Madsen (1978, [47]). Madsen and Nielson (1981, [50]) conducted in-grade tests on lumber subjected to bending. These studies showed that:

- longer beams had lower strength
- deeper beams had lower strength
- strength depends on the configuration of the loading scheme
- beams with higher stress volume had lower strength

The size effect was reported by Madsen and Buchanan (1986, [46]) and explained using the modified weakest link theory, assuming that lumber is perfectly brittle and consisting of a series of small, discrete, elements. Failure of any one of these elements - the weakest link - signals the failure of the material.

1.3 Objectives

Limit state design of wood joist structures such as floors and roofs requires a sound structural analysis and an accurate understanding of material properties of the components. The design procedures so derived should be accurate enough to warrant the safe design of a structure and also simple enough to be used in practice and easily adopted by structural engineers.

As described in the literature review, the current approach toward designing wood joist floors has some short comings.

- The simple T-beam approach ignores the two-way action. It is therefore not suitable to describe deflection of floors subject to concentrated load, since the load sharing among adjacent joists is ignored. This load sharing has been observed by a number of tests cited in the literature.
- The finite element analysis was used to account for most of the factors affecting floor deflection. This approach, however, is most suitable to be used in a research environment and too complicated to be adopted as a practical design procedure.

This thesis is, therefore, aimed at finding a practical approach to the design of wood joist floors and roofs. By using orthotropic plate theory, a floor or a roof is treated as a whole structure instead of breaking it into small components. Because of the simplicity of T-beam formula and its wide acceptance, it will be very useful to reach the final result in the form of a T-beam formula with a modification factor. Study focuses on the serviceability of the structures. The study of the strength of the floors and roofs can be carried out in a similar approach.

The formula derived from orthotropic plate theory is verified by tests reported in the literature. The orthotropic plate theory is then compared to the T-beam formula to determine the suitability of using the T-beam formula. If the T-beam formula can not be used directly, then it can be modified. Two loading conditions will be considered for floors: uniformly distributed load and concentrated load. For roofs, ponding and its amplification effects will also be investigated.

Factors that affect the bending strength of wood members will also be discussed. A total damage accumulation model will be proposed to assess the bending strength of wood components. This model and derived formula from structural analysis can be used in a reliability assessment of wood joist floors and roofs.

1.4 Organization of the Thesis

Chapter 1 Background, literature review and objective of the research

Chapter 2 Floors are modeled as an orthotropic plate. Deflection of floors are computed and compared against tests and results from the T-beam formula. Upper and lower bound of floor deflection are defined. The upper bound is also verified using a different approach. Only uniformly distributed load is considered. A parametric study is also carried out. Formulations for other boundary conditions other than simply supported are derived.

Chapter 3 Effects of aspect ratios on deflection of floors are examined in this chapter. The value of the aspect ratios defines the scope in which the T-beam formulation can be used. Long span floors with high aspect ratio, span divided by width of the floor, have to be designed according to plate theory instead of T-beam formula.

Chapter 4 As an extension of last chapter, procedures for designing long span floors are examined.

Chapter 5 A simple procedure to determine the maximum deflection of plywood sheathed wood joist floors subjected to concentrated load is introduced. By realizing that the plywood sheathed floors are much stiffer in the direction of joists than in the transverse direction, determination of the location of maximum deflection is carried out using a simplified approach. Then the maximum deflection can be calculated from one single formula. This procedure is valid for the range of current commercially available sizes of joists and plywood panels.

Chapter 6 Flat roofs subject to ponding load are studied. Criteria are derived to define ponding stable roofs.

Chapter 7 Bending strength of wood members is studied. A total damage accumulation model is proposed to unify size effects, load configuration effects on bending strength of lumber into one single formula. This model can be adopted in reliability analysis of wood joist floors and roofs.

Figure 1.1: Layered beams or T-beams

Figure 1.2: Idealization of a floor system

CSU model

Figure 1.3: CSU result: Composite and two-way behaviour of wood joist floor system with varying joist moduli.

Figure 1.4: Typical load-deflection behaviour of floors.

T-beam

Sheathing strip

Figure 1.5: Crossing beam model of wood joist floor system with gaps.

Figure 1.6: Deflections of floors vs aspect ratio (floor-span/width).

Figure 1.7: T-beam model.

Figure 1.8: Beam-Spring Analog model.

Figure 1.9: Deflection of joists under ponding load.

Chapter 2

Designing Wood Joist Floors as Orthotropic Plates

2.1 Introduction

Wood joist floors consist of joists and sheathing. The sheathing can be plywood panel, waferboard or other panel products. The sheathing can be nailed or glued to the joists. Since several panels are needed for each floor, there are usually gaps between panels. There can be several layers of sheathing, Figure 2.3. The floor is subject to uniformly distributed load, concentrated load or other forms of loads.

Wood joist floors used to be designed as a bare joist subjected to uniformly distributed load on the tributary area. In the 1970s, it was realized that the contribution of the sheathing materials to the stiffness of floors is significant. Research was carried out in the United States and Canada to find ways to design these floors in a more realistic approach. Several models have been developed since. One approach is to design the floor as a T-beam (McCutcheon 1977, [62]). The T-beam consists of a joist as the web, plywood sheathing as the flange. Partial composite action due to non-rigid connections was also considered. In order to consider the variability of modulus of elasticity (MOE) of joists, a beam-spring analog approach was proposed, based on the T-beam theory. Using an analog which represents a floor as a beam supported by elastic springs, the T-beam method is extended to account for variability of joist stiffness and two-way action due to the cross-joist

distributional properties of the sheathing. It is shown that this method computes individual midspan joist deflection, which are virtually identical to those obtained from large-scale finite element program (Thompson et al. [86] 1975, [85] 1977), and is capable of accurately predicting the performance of real floors.

The most comprehensive study is the finite element analysis. Thompson, Vanderbilt and Goodman (1977, [85]) developed a finite element computer program, FEAFL0 for calculating the performance of wood floors. Based on a crossing beam analysis, this model considers some factors not included in the simpler T-beam model, such as joist-to-joist variability and two-way action. Foschi (1982, [20]) presented a finite element analysis technique which includes lateral and torsional joist deformations as degrees of freedom and considers plate action in the sheathing. The finite element analysis is an excellent tool in research to examine the behavior of floors. It can be used to study the details like gaps in the sheathing, multi-layer sheathing, non-rigid connections, variability of joist MOE and different boundary conditions. However, it is not easily used directly by designers in practice. Charts or span tables have to be produced using finite element analysis.

The research at FPL, Forest Product Laboratory U.S., was aimed at finding out design procedures to be used in practice. The proposed beam-spring analog involves solving multiple equations. The simplest approach is the T-beam approach which uses conventional beam theory and accounts for details such as gaps in sheathing, partial composite action between joists and sheathing.

Joists in the floor assembly have different MOE and the sheathing distributes loads so that, the stiffer, and hence stronger joist carries more load, thus guaranteeing a smooth surface of deflection. Every joist in a floor assembly has an equal chance of having a certain stiffness if they are selected from the same grade. Each floor will have the same probability of having a certain behaviour as the rest of the floors assembled using joists and plywood of the same grade. It is the "average" properties of a floor that should be used to assess design procedures. It is therefore the statistical "average" properties that should appear in the LRFD formula for design purpose.

The primary component in a wood-joist floor assembly is the joist. The joists

sustain the major part of the load acting on the floor. The plywood sheathing is a secondary material. It has two major functions. One is to increase the bending rigidity of the floor in the joist direction. This is evidenced by the study of the T-beam. It is also called composite action. The other function of the sheathing is to transfer load laterally from the weaker joists to the stronger ones. In case of a failure of one of the joists, the load will be redistributed to the adjacent of the joists. This is also shown by tests where the floor continued to sustain load even after several joists have failed. The influence of boundary conditions in the transverse direction is directly related to the stiffness of the sheathing.

Instead of breaking the floor up into crossing beams to account for the individual stiffness, the floor assembly can be treated as an orthotropic plate by assuming that all the joists have the same property statistically. The rigidity of the plate in the direction perpendicular to the joists is assumed to be the rigidity of the plywood. The contribution of joists to the torsional stiffness is not considered. The effects of bridging between joists are not studied, but can be a topic of further investigation. Bridging makes a floor stiffer since it increases the torsional stiffness and bending stiffness of the floor in the transverse direction. It is assumed that the floors are not bridged. The rigidity of the plate in the direction of joists is the rigidity of the composite T-beam. The torsional rigidity of the plate is that of the plywood. This orthotropic plate approach gives a reasonable description of floor behavior with joists of the same grade.

In this chapter, deflection of a floor is described using the orthotropic plate theory and then compared against T-beam theory and tests from previous studies.

2.2 Deflection of Floors as Described by Orthotropic Plate Theory

The governing equation for an orthotropic plate is ([40] or [95])

$$(D_x \frac{\partial^4}{\partial x^4} + 2H \frac{\partial^4}{\partial x^2 \partial y^2} + D_y \frac{\partial^4}{\partial y^4})w(x, y) = q(x, y) \quad (2.1)$$

where

- x coordinate in the joist direction
 y coordinate in the direction perpendicular to joists
 D_x bending stiffness in the direction of joists (N mm) taken as the bending rigidity of the T-beam, $D_x = (EI)_e/S$.
 D_y bending rigidity of the floor in the direction perpendicular to joists. It therefore equals the bending rigidity of plywood sheathing, $D_y = B_b$.
 H torsional rigidity, assumed to be $2H = D_y$.
 $(EI)_e$ the effective bending stiffness of the T-beam
 S spacing between two joists

2.3 Wood Joist Floors Subject to Uniformly Distributed Load q_0

For a wood joist floor subjected to uniformly distributed load q_0 , Figure 2.3

$$q(x, y) = q_0$$

Assuming that the floor is simply supported on four sides, the solution for equation 2.1 is

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (2.2)$$

$$w_{mn} = \frac{q_{mn}}{\frac{m^4\pi^4}{L^4} D_x + 2H \frac{m^2 n^2 \pi^4}{L^2 B^2} + \frac{n^4 \pi^4}{B^4} D_y} \quad (2.3)$$

$$q_{mn} = \frac{4q_0}{mn\pi^2} (1 - \cos m\pi)(1 - \cos n\pi) \quad (2.4)$$

The bending rigidity D_x is the effective rigidity of the T-beam, $D_x = (EI)_e/S$. $(EI)_e$ is the bending stiffness of the T-beam.

The following sections are devoted to examining the upper and lower bound of floor deflection so that parametric study can be conducted for the scope of all possible deflection.

2.4 Unfastened Floors - Upper Bound of Floor Deflection

The joists are usually attached to the sheathing using either nails or glue. In an extreme case, when the stiffness of nails becomes zero, the sheathing is not attached to the joists at all. This defines the maximum possible deflection, i.e. the upper bound. The upper bound can also be obtained by assuming the stiffness of connection to be zero in computing the stiffness of the T-beam which is used in the orthotropic plate theory. The lower bound will be defined by floors with plywood glued to the joists. Throughout this thesis, the term "glued" implies a rigid connection, even though in reality a glued connection does not guarantee a perfect bonding.

We will assume the plywood is not attached to the joists at all. Then the floor assembly is considered to be an orthotropic plate (the sheathing itself) sitting on top of a series of joists, as shown in Figure 2.2. The result from this approach should be the same as the result by assuming the nail stiffness being zero in calculating D_x . The solution for the lower bound of deflection is derived as follows.

The deflection of an orthotropic plate $w(x,y)$ is represented in the following differential equations:

$$(D_x \frac{\partial^4}{\partial x^4} + 2H \frac{\partial^4}{\partial x^2 \partial y^2} + D_y \frac{\partial^4}{\partial y^4})w(x,y) = q(x,y) \quad (2.5)$$

$$q(x,y) = Q(x,y) - \frac{Q_j(x)}{dy} \delta(y - y_j) \quad j = 1, 2, \dots, N \quad (2.6)$$

$$Q_j(x) = (EI)_b \frac{d^4 w(x,y)}{dx^4} \Big|_{y=y_j} \quad (2.7)$$

where

- $Q(x, y)$ normal load on the floor
 $Q_j(x)$ reaction from the j th joist
 y_j position of the j th joist
 $(EI)_b$ bending rigidity of the joists
 dy width of supports

For a floor simply supported on four sides, assume

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (2.8)$$

$q(x, y)$ can be expanded to a Fourier series

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (2.9)$$

where

$$q_{mn} = \frac{4}{LB} \int_0^L \int_0^B q(x, y) \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} dx dy \quad (2.10)$$

for a floor supported by joists and subjected to uniformly distributed load q_0 ,

$$Q(x, y) = q_0 \quad (2.11)$$

Substitute the above equation into Equation 2.5 and 2.6

$$\begin{aligned}
 q_{mn} = & \frac{4}{LB} \int_0^L \int_0^B q_0 \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} dx dy \\
 & - \frac{4}{LB} \int_0^L Q_j(x) \sin \frac{n\pi y_j}{B} \sin \frac{m\pi x}{L} dx \quad (2.12)
 \end{aligned}$$

from equation 2.7

$$\begin{aligned}
 q_{mn} = & \frac{4q_0}{mn\pi^2} (1 - \cos m\pi)(1 - \cos n\pi) \\
 & - \frac{4}{LB} \int_0^L (EI)_b \frac{\partial^4 w(x, y)}{\partial x^4} \Big|_{y=y_j} \sin \frac{n\pi y_j}{B} \sin \frac{m\pi x}{L} dx \quad (2.13)
 \end{aligned}$$

from equation 2.8

$$\begin{aligned}
q_{mn} &= \frac{4q_0}{mn\pi^2}(1 - \cos m\pi)(1 - \cos n\pi) \\
&- \frac{4}{LB}(EI)_b \sin \frac{n\pi y_j}{B} \\
&\left(\int_0^L \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} A_{ik} \left(\frac{i\pi}{L}\right)^4 \sin \frac{i\pi x}{L} \sin \frac{k\pi y_j}{B} \sin \frac{m\pi x}{L} dx \right) \quad (2.14)
\end{aligned}$$

Apply orthogonal rule and integrate over the range of 0 to L in the x direction,

$$\begin{aligned}
q_{mn} &= \frac{4q_0}{mn\pi^2}(1 - \cos m\pi)(1 - \cos n\pi) \\
&- \frac{2}{B}(EI)_b \left(\frac{m\pi}{L}\right)^4 \sin \frac{n\pi y_j}{B} \sum_{k=1}^{\infty} A_{mk} \sin \frac{k\pi y_j}{B} \quad (2.15)
\end{aligned}$$

from equation 2.5 2.8 and 2.9

$$\left[D_x \left(\frac{m\pi}{L}\right)^4 + 2H \left(\frac{m\pi}{L}\right)^2 \left(\frac{n\pi}{B}\right)^2 + D_y \left(\frac{n\pi}{B}\right)^4 \right] A_{mn} = q_{mn} \quad (2.16)$$

then

$$D_{mn} A_{mn} + \sum_{i=1}^{\infty} B_{mni} A_{mi} = Q_{mn} \quad (2.17)$$

where

$$D_{mn} = D_x \left(\frac{m\pi}{L}\right)^4 + 2H \left(\frac{m\pi}{L}\right)^2 \left(\frac{n\pi}{B}\right)^2 + D_y \left(\frac{n\pi}{B}\right)^4 \quad (2.18)$$

$$B_{mni} = \frac{2}{B}(EI)_b \left(\frac{m\pi}{L}\right)^4 \sin \frac{n\pi y_j}{B} \sin \frac{i\pi y_j}{B} y_j \quad (2.19)$$

$$Q_{mn} = \frac{4q_0}{mn\pi^2}(1 - \cos m\pi)(1 - \cos n\pi) \quad (2.20)$$

For a floor with N_j joists:

$$B_{mni} = \sum_{j=1}^{N_j} \frac{2}{B}(EI)_b \left(\frac{m\pi}{L}\right)^4 \sin \frac{n\pi y_j}{B} \sin \frac{i\pi y_j}{B} \quad (2.21)$$

The unknowns A_{mn} can be solved from the above equations. The results from this approach and the stiffened plate approach will be compared later when the bending stiffness of the T-beam, D_x , is defined.

2.5 Deflection of Floors as Described by the T-beam Theory

A T-beam is modelled as shown in Figure 2.3. The flange is the plywood sheathing with width S , where S is the spacing between the joists. The web is the joist. The maximum deflection is

$$\Delta = \frac{5}{384} \frac{qL^4}{(EI)_e} \quad (2.22)$$

The upper bound of the maximum deflection can be found by assuming that there is no attachment between the joists and the plywood. This is called an unfastened floor. The lower bound can be found by assuming the plywood is rigidly glued to the joists. The effective rigidity $(EI)_e$ is described in the following sections.

2.5.1 Lower Bound

Assume that the web and flange are rigidly glued having no relative slip to each other. This kind of floors is called a glued floor. The cross-section is shown in Figure 2.3. According to Timoshenko, the neutral axis can be calculated as

$$\begin{aligned} y_0 &= \frac{d}{2} + \frac{d+t}{2} \frac{E_2 A_2}{E_2 A_2 + E_1 A_1} \\ &= \frac{d}{2} + \frac{d+t}{2} \frac{B_s S}{B_s S + E_j db} \end{aligned} \quad (2.23)$$

where

y_0	distance between the neutral axis and the bottom of the web
d	height of the web or joist
t	thickness of the flange or plywood
E_1, E_2	modulus of elasticity of the web and the flange respectively
A_1, A_2	area of the joist and plywood respectively
B_s	tensile stiffness of the plywood
B_e	bending stiffness of the plywood

S joist spacing
 E_j MOE of joists

It can be rewritten as

$$y_0 = \frac{d}{2} + \delta \quad (2.24)$$

$$\begin{aligned} \delta &= \frac{d+t}{2} \frac{E_2 A_2}{E_2 A_2 + E_1 A_1} \\ &= \frac{d+t}{2} \frac{B_o S}{B_o S + E_j b d} \end{aligned} \quad (2.25)$$

The δ represents the effects of the flange or the contribution of the plywood sheathing. The effective rigidity $(EI)_e$ is therefore

$$\begin{aligned} (EI)_e &= (EI)_1 + (EA)_1 \delta^2 + (EI)_2 + (EA)_2 \left(\frac{d+t}{2} - \delta \right)^2 \\ &= E_j \frac{bd^3}{12} + E_j b d \delta^2 + B_o S + B_o S \left(\frac{d+t}{2} - \delta \right)^2 \end{aligned} \quad (2.26)$$

2.5.2 Upper Bound

Assume that the web and the flange are not attached to each other at all. The plywood sheathing simply sits on top of joists. This will give the maximum possible deflection of the T-beam. The cross section is shown in Figure 2.3 .

The effective bending rigidity is

$$\begin{aligned} (EI)_e &= (EI)_1 + (EI)_2 \\ &= E_j \frac{bd^3}{12} + B_o S \end{aligned} \quad (2.27)$$

2.5.3 Composite Action - Comparison Between the Upper and Lower Bound

Let's compare the upper and lower bound of the maximum deflection. Let K_1 represent the effective rigidity of the unfastened floor, K_2 represent the effective

rigidity of the glued floor. Then:

$$\begin{aligned} K_1 = (EI)_e &= (EI)_1 + (EI)_2 \\ &= E_j \frac{bd^3}{12} + B_0 S \end{aligned} \quad (2.28)$$

and

$$\begin{aligned} K_2 &= (EI)_1 + (EA)_1 \delta^2 + (EI)_2 + (EA)_2 \left(\frac{d+t}{2} - \delta \right)^2 \\ &= E_j \frac{bd^3}{12} + E_j bd \delta^2 + B_0 S + B_a S \left(\frac{d+t}{2} - \delta \right)^2 \end{aligned} \quad (2.29)$$

δ is in Equation 2.26, or in a different form:

$$K_2 = K_1 + f_v \quad (2.30)$$

where

$$\begin{aligned} f_v &= (EA)_1 \delta^2 + (EA)_2 \left(\frac{d+t}{2} - \delta \right)^2 \\ &= E_j bd \delta^2 + B_a S \left(\frac{d+t}{2} - \delta \right)^2 \end{aligned} \quad (2.31)$$

is the **adhesive factor** which describes the effects of connection between the joists and sheathing.

An Example.

As an example to illustrate the upper bound and lower bound of the maximum deflection of a T-beam, consider the following floor configuration (Table 2-1, Sherwood and Moody [84]).

joists:

D.Fir, No.1/No.2

E 11000 MPa

38 x 184 or 2" x 8"

spacing S = 16" or 406 mm

span $L = 12'$ or 3660 mm

plywood: (Table 7.3D, CWC [6])

Canadian softwood plywood, standard construction
regular grades, unsanded, 4 plies

orientation of applied force relative to face grain: 0°

plywood thickness: 15.5 mm

$B_b = 430,000 \text{ N.mm}^2/\text{mm}$

$B_a = 75,000 \text{ N.mm}/\text{mm}$

load:

$$\begin{aligned} q &= 40 \text{ lbf/ft}^2 \\ &= 1.9 \text{ kPa} \end{aligned}$$

Results of the calculation is shown below:

From 2.29 and 2.30:

$$K_1 = 217 \text{ kN.m}^2$$

$$K_2 = 434 \text{ kN.m}^2$$

$$f_v = K_2 - K_1 = 217 \text{ kN.m}^2$$

$$\delta = 28.4 \text{ mm}$$

$$\delta/h = 15.4 \%$$

$$K_1/K_2 = 50\%$$

Maximum deflections (Equation 2.22):

$$\text{Upper bound: } \Delta = 8.36 \text{ mm}$$

$$\text{Lower bound: } \Delta = 4.18 \text{ mm}$$

$$\text{Test result: } \Delta = 6.78 \text{ mm (Sherwood and Moody, [84])}$$

In this example, the upper bound is twice as big as the lower bound. The test result of a nailed floor is in the middle of the upper and lower bound. The composite action is significant as evidenced by the value of f_v .

2.5.4 Upper Bound of Deflection - Comparison of Three Approaches

There have been three approaches toward computing the upper bound of deflection. The first one is as described in section 2.3. It can be called the *stiffened plate approach*. The second one is as described in section 2.4. This approach can be called *unfastened floor approach*. The last one is the *T-beam approach*. A computer program has to be written for the unfastened floor approach. Data used in the comparison is listed below:

load:	1.9 kPa	40 psf
joists:	Douglas-fir	No.1/No.2
No. of joists, N_j	8	
MOE	11000 MPa	
size	38 x 184 mm	2" x 8"
spacing	406 mm	16"
span and width of floor	3660 mm	12'
plywood:	Canadian softwood, standard regular, unsanded	4 plies
thickness	15.5 mm	5/8"
bending stiffness, B_b	2,500,000 Nmm^2/mm	@ 0°
bending stiffness, B_b	430,000 Nmm^2/mm	@ 90°
axial stiffness, B_a	75,000 Nmm	

The results from the three approaches are listed below:

Deflection of wood joist floor

T-beam	stiffened floor	unfastened floor
8.36 mm	8.36 mm	8.30 mm

It is shown that the three approaches resulted in similar results. The unfastened floor approach coincides with the upper bound of the stiffened floor with connection

stiffness equal to zero.

2.5.5 Floors with Joist and Plywood Nailed Together

In the last sections, two extreme conditions are considered. One is the upper bound of deflection in which case the joist and plywood are not at all attached to each other. The other is the lower bound. In this case, the plywood sheathing is glued to the joists and we have a solid floor.

In this section, we will consider floors that have plywood nailed to the joists. According to McCutcheon [62], the maximum deflection Δ of a composite beam (T-beam) is

$$\Delta = \Delta_1 \left(\frac{K_1}{K_2} + f_\Delta \frac{f_v}{K_2} \right) \quad (2.32)$$

where K_1 and K_2 are defined in equation 2.29 and 2.30; f_v is defined in equation 2.32 ; and Δ_1 represents the deflection of an unfastened floor with plywood unconnected to the joists. For a simply supported beam subjected to uniformly distributed load, the deflection is

$$\Delta_1 = \frac{5}{384} \frac{qSL^4}{K_1} \quad (2.33)$$

where

- q uniformly distributed load acting on the tributary area of T-beam
L length of the T-beam or span of joists

and

$$f_\Delta = \frac{10}{(L'\alpha)^2 + 10} \quad (2.34)$$

$$\alpha^2 = \frac{\left(\frac{t+d}{2}\right)^2 S_n}{K_2 - K_1} \left(\frac{K_2}{K_1}\right) \quad (2.35)$$

$$S_n = \frac{P/\delta}{s_n} \quad (2.36)$$

where

L'	Length of plywood between gaps
s_n	spacing between nails
P/δ	load/slip ratio

2.6 Interpretation of Test Results using Orthotropic Plate Theory

There have been a number of tests on wood joist floors conducted in the United States and Canada in the last two decades, as listed in Chapter 1. The following is a list of comparisons from the test results and the results calculated from the orthotropic plate theory described in this chapter.

2.6.1 Forest Product Laboratory, U.S., 1977, McCutcheon

Seven floors were constructed at FPL. Each consisted of nine No.2 and better 2-by-8 Douglas-fir joists spaced at (106 mm) (16 in.) on centers. Floors were 4064 mm (13'4") wide and were tested on a span of 3660 mm (12').

Four of the floors (designated N-1 to N-4) used standard grade 15.5 mm (5/8-inch) plywood nailed to the joists with face grain perpendicular to the span. Nails were 8d at 152 mm (6 in.) along the edges and 203 mm (8 in.) over intermediate joists. A 1.5 mm (1/16-inch) gap was left between the plywood sheets. The other three floors (designated G-1 to G-3) had the plywood nail-glued in place using a rigid adhesive. These used tongue-and-groove plywood with the edges glued to give a continuous subfloor. The relevant data is listed below.

McCutcheon 1977	Forest Product Laboratory, U.S.	
load	2.4 kPa	50 psf
floor dimension:		
length	3660 mm	12'
width	4064 mm	13'4"
joists:		
spacing	406 mm	16"
size	38 x 184 mm	2" x 8"
MOE	vary	vary
plywood:		
thickness, t	15.5 mm	5/8"
tensile stiffness, $EA = B_x S$	22240 KN	5.0×10^6 lbs
bending stiffness, B_b	430,000 Nmm^2/mm	
bending stiffness, D_v	3,700,000 Nmm^2/mm	
Nails:		
P/δ , 8d or 2-1/2" nail	1.062×10^6 N/mm	9400 lbs/in.
nail spacing, S_n	203 mm	8"

Using the orthotropic plate theory, the deflections of these floors are calculated and compared with the test results in the following table. The comparison is also illustrated in Figure 2.1. The points should be as close to the 45° line as possible. The deflection from the tests are the average of the middle 5 joists. The deflection from the calculation is at the center point of the floor. It can be proved that for a floor of this configuration, the deflections of the center joists are approximately the same.

McCutcheon 1977	Forest Product	Laboratory,	U.S.	
Floor No.	MOE, 10^6 psi	Test (mm)	Orth. Plate	T-beam, McCutcheon
N-1	1.57	9.6	10.2	10.0
N-2	2.20	7.1	7.3	7.2
N-3	2.00	7.9	8.0	8.0
N-4	1.49	9.3	10.7	10.5
G-1	1.53	6.2	6.0	6.0
G-2	2.20	4.8	4.7	4.7
G-3	2.00	4.8	5.0	5.0

2.6.2 Forintek Canada, 1982, Foschi

One floor consisting of 12 joists was constructed and tested at Forintek Canada, British Columbia. The floor was supported on four sides, had a span of 4.27 m (168 inch), and was uniformly loaded with 1.95 kPa (40 psf). The data is listed below:

Foschi 1982	Forintek Canada	
load	1.95 kPa	40 psf
floor dimension:		
length	4267 mm	168"
width	4470 mm	176"
joists:		
spacing	406 mm	16"
size	38 x 184 mm	2" x 8"
MOE	11171 MPa	1.62×10^6 psi
plywood:		
thickness, t	15.9 mm	5/8"
tensile stiffness, B_a	56,000 N/mm	5.44×10^6 lbs
bending stiffness, B_b	3,230,000 Nmm ² /mm	28,000.0 lb in.
bending stiffness, D_v	1,040,000 Nmm ² /mm	9,000.0 lb in.
Nails:		
P/δ, 8d nail	1.210×10^6 N/mm	6800 lbs/in.
nail spacing, S_n	152 mm	6"

The test data and result of calculation from the orthotropic plate theory is listed below:

average MOE, joists	deflection (mm), test	def., orth. plate	error %
1.62×10^6	10.4	9.8	5%

2.6.3 UBC, Canada, 1985, Foschi

A series of 13 wood floors were tested under uniformly distributed load. The floors were constructed with 12 38x184 hemlok joists, No.2 and better grade, 5/8" or 15.8 mm Douglas-fir plywood, and 8d common nails at 6" or 152 mm o.c. Data is listed below:

Foschi 1985	UBC, Canada	
load	1.95 kPa	40 psf
floor dimension:		
length	3580 mm	141"
width	4470 mm	176"
joists:		
spacing	406 mm	16"
size	38 x 184 mm	2" x 8"
MOE	vary	vary
plywood:		
thickness, t	15.8 mm	5/8"
tensile stiffness, B_a	56,000 N/mm	5.44×10^6 lbs
bending stiffness, B_b	1,016,812 Nmm^2/mm	9,000 lb in.
bending stiffness, D_y	3,163,417 Nmm^2/mm	28,000 lb in.
Nails:		
P/6, 8d or 2-1/2" nail	2.0×10^6 N/mm	11360 lbs/in.
nail spacing, S_n	152 mm	6"

Test data from 10 of the 13 floors are used to compare with the results from orthotropic plate theory assuming no gaps. The list is as follows:

Floor No.	average MOE, 10^6 psi	deflection (mm), tests	calculation	error
1	1.6000	7.0	5.4	21%
2	1.6108	5.8	5.4	7%
3	1.6117	6.3	5.4	14%
4	1.6083	6.2	5.4	12%
5	1.6158	5.0	5.4	-6%
6	1.6158	5.3	5.4	-2 %
7	1.5867	5.0	5.5	-9 %
8	1.5908	6.1	5.5	10%
9	1.5842	5.8	5.5	5%
10	1.5800	4.6	5.5	-20 %
average	1.6017	5.7	5.4	4 %

The average MOE is calculated from the following table: ($E10^6$)

floor No.	Joist Number											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1.85	1.56	1.36	1.69	1.73	1.24	1.66	1.14	1.57	1.96	2.05	1.44
2	1.69	1.52	1.29	1.87	1.87	1.46	1.76	1.16	1.57	2.07	1.68	1.39
3	1.69	1.42	1.20	1.39	1.15	1.84	1.76	2.16	1.61	1.56	1.61	1.95
4	1.43	1.56	1.68	1.61	2.11	1.13	1.61	1.90	1.80	1.72	1.28	1.47
5	1.44	1.68	1.59	1.74	1.36	1.64	2.12	1.79	1.49	1.55	1.22	1.94
6	1.18	1.74	2.06	1.63	1.79	1.97	1.40	1.60	1.49	1.55	1.28	1.70
7	1.74	1.98	1.65	1.54	1.40	1.48	1.72	1.29	1.91	1.17	1.82	1.34
8	1.75	1.64	1.71	1.41	2.06	1.14	1.51	1.87	1.25	1.82	1.55	1.38
9	1.58	1.67	1.39	1.40	1.52	1.54	1.99	1.96	1.77	1.20	1.75	1.21
10	1.31	1.58	1.41	1.48	1.98	1.64	1.17	1.89	1.82	1.28	1.73	1.70

Orthotropic plate theory describes the average behaviour of floors.

2.7 Parametric Study

In this section, we will have a closer look at what is the greatest influence on the deflection of a T-beam. More importantly, how by gluing the plywood and joists together reductions on deflection are made. What affects this improvement the most will be examined. Take the following floor configuration as an example:

$$q = 40 \text{ lbf/ft}^2 = 1.9 \text{ kPa}$$

$$\text{joist span} = 12'$$

D.Fir No.2

$$E = 11000 \text{ MPa}$$

plywood:

$$D_x = 75000 \text{ Nmm}$$

$$D_y = 250000 \frac{\text{Nmm}^2}{\text{mm}} @90^\circ$$

$$2H = D_y$$

2.7.1 Effect of Plywood Thickness on Deflection

Calculation is done by using plywood of different sizes with joists 38x89. The results are listed in Figure 2.8 and Table 2.1. It can be seen that for a thicker plywood, the reduction to deflection by gluing joists to a thicker plywood is larger than with a thinner plywood.

Calculation is also done for floors with different sizes of joists. Referring to Figure 2.9 and Table 2.2, it can be seen that for a floor with larger joists, the influence of the plywood is not so drastic. Joists dominate.

2.7.2 Effect of Joists Size on Deflection

Calculation is done by using plywood of different sizes. The results are listed in Table 2.1 and 2.2.

Refer to Figure 2.10 and Table 2.3, it can be seen that it will achieve better improvement with smaller joist by gluing the plywood and joists together.

Table 2.1: Maximum deflection of T-beam, joist: 38x89

plywood thickness, t	upper bound (u)	lower bound (l)	ratio = (u-l)/l
7.50	74.50	42.87	0.74
9.50	74.48	38.94	0.91
12.50	74.28	29.70	1.50
12.50	74.08	31.88	1.32
15.50	73.99	25.88	1.86
15.50	73.74	30.77	1.40
18.50	72.93	25.06	1.91
18.50	72.93	29.63	1.46
18.50	72.93	25.32	1.88
20.50	71.42	22.64	2.16
20.50	72.23	28.87	1.50
20.50	72.11	24.62	1.93

Table 2.2: Maximum deflection of T-beam, joist: 38x184

plywood thickness, t	upper bound (u)	lower bound (l)	ratio = (u-l)/l
7.50	8.43	6.16	0.37
9.50	8.43	5.79	0.46
12.50	8.43	4.69	0.80
12.50	8.43	5.00	0.69
15.50	8.43	4.20	1.01
15.50	8.42	4.94	0.71
18.50	8.41	4.17	1.02
18.50	8.41	4.87	0.73
18.50	8.41	4.22	1.00
20.50	8.39	3.84	1.19
20.50	8.40	4.83	0.74
20.50	8.40	4.17	1.01

Table 2.3: Deflection ratios $(u-l)/l$

t	38x89	38x114	38x140	38x184	38x235	38x286
7.5 4 plies	0.74	0.58	0.48	0.37	0.29	0.24
9.5 4 plies	0.91	0.72	0.59	0.46	0.36	0.30
12.5 4 plies	1.50	1.21	1.02	0.80	0.64	0.53
15.5 5 plies	1.86	1.51	1.27	1.01	0.81	0.68
18.5 6 plies	1.91	1.55	1.29	1.02	0.81	0.68
20.5 7 plies	2.16	1.77	1.50	1.19	0.96	0.80

2.7.3 Effect of Nail Size on Deflection

It is useful to examine effects of sizes of nails on deflection so that a design decision can be made. The results of calculations are listed in Table 2.4, 2.6, 2.7 and Figure 2.11, 2.12.

It can be seen from Table 2.4 and Figure 2.11 that a 4" nail will achieve quite significant improvements over the unconnected or unfastened floors. With nails bigger than 4", the improvements are not so pronounced. The difference between the deflection of a nailed floor (38.1 mm) and a glued floor (29.7 mm) is only 28% with a 4" nail. The reduction of deflection of a nailed T-beam over an unfastened T-beam (74.28 mm) is more than 48%.

Comparing Table 2.4 with Table 2.6, it can be seen that the differences due to the use of thicker plywood are negligible.

To understand the effects of plywood thickness, nail size and joist size, the following 3 different floors are considered.

1. joist 38x89; plywood thickness 12.5 mm;
2. joist 38x89; plywood thickness 20.5 mm;
3. joist 89x286; plywood thickness 20.5 mm;

The ratios of reduction in deflection of a nailed floor achieved over an unfastened floor $\frac{(W_u - W_n)}{W_u}$ are calculated. The results are tabulated in Table 2.12.

Table 2.4: Effects of nail sizes on floor deflection, joist size 38x89, plywood thickness 12.5 mm

	deflection		
nail diameter	deflection(mm) unfastened	deflection glued	deflection nailed
2d	74.3	29.7	52.4
4d	74.3	29.7	47.3
5d	74.3	29.7	47.3
6d, 2"	74.3	29.7	45.3
8d , 2-1/2"	74.3	29.7	43.2
10d, 3"	74.3	29.7	41.4
16d, 3-1/2"	74.3	29.7	40.2
20d, 4"	74.3	29.7	38.1
30d, 4-1/2"	74.3	29.7	37.3
40d, 5"	74.3	29.7	36.3
60d, 6"	74.3	29.7	35.0

Table 2.5: Conversion of nail sizes between Penny-weight and Inches

Penny-weight	6d	8d	10d	12d	16d	20d	30d	40d	50d	60d
Length in inches	2	2 1/2	3	3 1/4	3 1/2	4	4 1/2	5	5 1/2	6

Table 2.6: Effects of sizes of nails, joist 38x89, plywood 20.5 mm

nail diameter	deflection (mm)		
	unfastened	glued	nailed
2d	74.3	29.7	51.4
4d	74.3	29.7	42.3
5d	74.3	29.7	42.3
6d	74.3	29.7	39.6
8d	74.3	29.7	37.1
10d	74.3	29.7	35.2
16d	74.3	29.7	34.0
20d	74.3	29.7	32.3
30d	74.3	29.7	31.61
40d	74.3	29.7	31.0
60d	74.3	29.7	29.8

Table 2.7: Improvement ratios of nailed floor over unfastened floor

nail diameter	floor 1	floor 2	floor 3
2d	0.29	0.29	0.04
4d	0.36	0.41	0.08
5d	0.36	0.41	0.08
6d	0.39	0.45	0.09
8d	0.42	0.49	0.11
10d	0.44	0.51	0.13
16d	0.46	0.53	0.14
20d	0.49	0.55	0.15
30d	0.50	0.56	0.16
40d	0.51	0.57	0.16
60d	0.53	0.59	0.17

Table 2.8: Comparison between the deflection of floor using orthotropic plate model and T-beam model. joist 38x89, nail size 2d.

t	plate			T-beam			ratio: plate/T-beam		
	unfastened	glued	nailed	loose	glued	nailed	unfastened	glued	nailed
12.5	79.15	31.33	55.93	74.28	29.70	52.40	1.06	1.05	1.06
15.5	76.38	32.95	55.81	73.99	30.81	52.84	1.03	1.06	1.05
18.5	71.93	31.62	54.29	72.93	29.63	52.85	0.98	1.06	1.02
20.5	67.58	26.17	51.11	72.11	24.62	51.42	0.93	1.06	0.99

It can be seen from Figure 2.12 that

- The improvement is less sensitive when the joists are bigger
- It is more sensitive to joist depth than the thickness of plywood

2.8 Comparison and Discussion

In order to compare the results of the plate theory with the T-beam theory, a few floors are used in analysis and comparison between the two theories.

Floor no. 1:

joist: 38x89
 L: 3660 mm
 B: 3660 mm
 E: 11000 MPa
 No. of Joists: 8
 plywood: D.Fir plywood
 t: 12.5 mm - 20.5 mm
 nail size: 2d

Floor no. 2:

Table 2.9: Comparison between the deflection of floor using orthotropic plate model and T-beam model with 38x184 joists and 2d nails.

	plate			T-beam			ratio: plate/T-beam		
t	unfastened	glued	nailed	loose	glued	nailed	loose	glued	nailed
12.5	8.52	4.76	7.16	8.43	4.69	7.10	1.01	1.01	1.00
15.5	8.62	5.00	7.30	8.43	4.94	7.17	1.02	1.01	1.01
18.5	8.75	4.98	7.48	8.41	4.87	7.23	1.04	1.02	1.03
20.5	8.84	4.29	7.50	8.40	4.17	7.17	1.05	1.02	1.04

joist: 38x184
 L: 3660 mm
 B: 3660 mm
 E: 11000 MPa
 No. of Joists: 8
 plywood: D.Fir plywood
 t: 12.5 mm - 20.5 mm
 nail size: 2d

Floor no.3:

joist: 38x89
 L: 3660 mm
 B: 3660 mm
 E: 11000 MPa
 No. of Joists: 8
 plywood: D.Fir
 t: 12.5 mm - 20.5 mm
 nail size: 60d or 6"

Figure 2.4, 2.5 and Figure 2.6 shows that both orthotropic plate theory and T-beam formula agrees well even with different joist and nail sizes. Figure 2.8 and 2.9 shows

Table 2.10: Comparison between the deflection of floor using orthotropic plate model and T-beam model. joist 38x89, nail size 60d.

	plate			T-beam			ratio: plate/T-beam		
t	unfastened	glued	nailed	loose	glued	nailed	loose	glued	nailed
12.5	79.15	31.33	37.06	74.28	29.70	34.98	1.06	1.05	1.05
15.5	76.38	32.95	37.98	73.99	30.81	35.55	1.03	1.06	1.06
18.5	71.93	31.62	36.28	72.93	29.63	34.20	0.98	1.06	1.06
20.5	67.58	26.17	31.34	72.11	24.62	29.80	0.93	1.06	1.04

the effect of plywood thickness on floor deflection. Figure 2.10 shows the effects of joist sizes and plywood thicknesses on floor deflection represented in the form of deflection ratio. The deflection ratio, upper bound (deflection of unfastened floor) minus lower bound (deflection of glued floor) and then divided by lower bound, shows the increase in floor stiffness when composite action is considered. It can be seen that the thicker the plywood and the smaller the joists, the greater increase in floor stiffness. Figure 2.11 shows the effects of nail size on floor deflection. It can be seen that the difference between using a 6d or 2 inch nail and a 10d or 3 inch nail is significant. Considering the low cost of nails, it is advisable that a longer nail be used if possible to achieve higher stiffness on the floor assembly. However, nails bigger than 4" or longer than 4 inches do not show significant differences than 4" nails. Figure 2.12 shows the effects of nail sizes on floor deflection represented in terms of deflection ratios for floors with different plywood thickness and different joist sizes. It can be seen that it shows similar patterns for different joist sizes and different plywood thicknesses.

2.9 Effects of Boundary Conditions: Upper Bound for Deflection of a Floor Simply Supported on Two Opposite Sides

In order to examine the influence different boundary conditions have on deflection, the following procedure can be followed. It assumes that the joists are simply supported and the edge joists may be free to deflect, fixed or on elastic supports. For a stiffened floor, equation 2.5 can be used with relevant boundary conditions. Classical Levy method can be used. To find out the upper bound of a floor simply supported on two opposite sides, it can be assumed that the sheathing simply sits loosely on top of the joists. The procedure to calculate deflection is outlined as follows.

According to equation 2.5 and assume

$$\begin{cases} w(x, y) = \sum_{n=1}^{\infty} w_n(x) \sin\left(\frac{n\pi y}{B}\right) \\ q(x, y) = \sum_{n=1}^{\infty} q_n(x) \sin\left(\frac{n\pi y}{B}\right) \end{cases} \quad (2.37)$$

where

$$\begin{aligned} q_n(x) &= \frac{2}{B} \int_0^B q(x, y) \sin \frac{n\pi y}{B} dy \\ &= \frac{2}{B} \int_0^B q_0 \sin \frac{n\pi y}{B} dy + \frac{2}{B} \left(-\frac{Q_j(x)}{dy} \sin \frac{n\pi y_j}{B} dy \right) \\ &= \frac{2}{B} \left[\frac{Bq_0}{n\pi} (1 - \cos n\pi) - Q_j(x) \sin \frac{n\pi y_j}{B} \right] \end{aligned} \quad (2.38)$$

Equation 2.5 is then reduced by equation 2.37 to

$$\left(D_x \frac{d^4}{dx^4} - 2H \left(\frac{n\pi}{B} \right)^2 \frac{d^2}{dx^2} + D_y \left(\frac{n\pi}{B} \right)^4 \right) w_n(x) = q_n(x) \quad (2.39)$$

where

$$q_n(x) = \frac{2}{B} \left[\frac{Bq_0}{n\pi} (1 - \cos n\pi) - (EI)_b \left(\sum_{i=1}^{\infty} \frac{d^4 w_i(x)}{dx^4} \sin \frac{i\pi y_i}{B} \right) \sin \frac{n\pi y_j}{B} \right] \quad (2.40)$$

for N_j joists, the above equation becomes

$$q_n(x) = \frac{2}{B} \left[\frac{Bq_0}{n\pi} (1 - \cos n\pi) - \sum_{j=1}^{N_j} \left((EI)_b \left(\sum_{i=1}^{\infty} \frac{d^4 w_i(x)}{dx^4} \sin \frac{i\pi y_i}{B} \right) \sin \frac{n\pi y_j}{B} \right) \right] \quad (2.41)$$

Assume the floor is simply supported in the joists (x) direction. The boundary conditions are $w = 0$ @ $x = 0$ and $x = L$. Therefore the following can be assumed:

$$w_n(x) = \sum_{m=1}^{\infty} b_m \sin \frac{m\pi x}{L} \quad (2.42)$$

Substitute equation 2.42 and 2.41 into 2.39, multiply both side by $\sin \frac{m\pi x}{L}$ and integrate. Apply orthogonal rules, the equation becomes

$$\frac{LB}{4} [D_x (\frac{m\pi}{L})^4 + 2H (\frac{n\pi}{B})^2 (\frac{m\pi}{L})^2 + D_y (\frac{n\pi}{B})^4] b_m \quad (2.43)$$

$$= \frac{abq_0}{mn\pi^2} (1 - \cos m\pi)(1 - \cos n\pi) \quad (2.44)$$

$$- \sum_{j=1}^{N_j} ((EI)_y (\sum_{i=1}^{\infty} b_m \frac{L}{2} (\frac{m\pi}{L})^4 \sin \frac{i\pi}{B} y_j) \sin \frac{n\pi}{B} y_j) \quad (2.45)$$

2.10 Summary

In this chapter, plywood sheathed wood joist floors have been modelled as a stiffened orthotropic plate. The deflection of floors calculated using this approach compares favorably with test results obtained at other institutions. It also shows that with the configurations of floors tested, T-beam formula can be used with satisfactory accuracy. The upper bound and lower bound for the deflection of floors are defined. The upper bound is verified through a different approach in which an unfastened floor is modeled as an orthotropic plate (the plywood sheathing) sitting loosely on top of elastic supports (the joists). Parameter study is performed to demonstrate the factors that influence the deflection of floor the most.

In the next chapter, the effects of aspect ratio on floor deflection are examined. Since T-beam formulation ignores boundary conditions in the transverse direction, the orthotropic plate theory can be expected to represent a more accurate approach.

Figure 2.1: Comparison between the test data and calculation from orthotropic plate theory

Plywood subjected to the normal load and reactions from the joists

The j th joist subjected to the reactions from the plywood

Figure 2.2: An unfastened floor - sheathing not fastened to joists

Figure 2.3: A plywood sheathed wood joist floor subject to uniformly distributed load, and its T-beam cross section

Figure 2.4: Deflection of floors with 38x89 joists and 2d nails, orthotropic plate theory vs T-beam formula.

Figure 2.5: Deflection of floors with 38x184 joists, 2d nails, orthotropic plate theory vs T-beam formula.

Figure 2.6: Deflection of floors with 38x89 joists, 60d nails, orthotropic plate theory vs T-beam formula.

Figure 2.7: Tbeam - an example

Figure 2.8: Effects of plywood sheathing thickness on deflection of floors, joist: 38x89.

Figure 2.9: Effects of plywood sheathing thickness on deflection of floors, joist: 38x184

Figure 2.10: Effects of joist sizes and plywood thickness on floor deflection. represented by using deflection ratios $(u-l)/l$

Figure 2.11: Effects of nail sizes on floor deflection

Figure 2.12: Effects of nail sizes on floor deflection shown in terms of deflection ratios for floors with different plywood thickness and different joist sizes.

Chapter 3

The Effect of Aspect Ratio on the Deflection of Floors

3.1 Introduction

In wood-joist floor assembly, the sheathing transfers load in the transverse direction. This is called "two way action" in literature. The thicker the sheathing, the more significant the two way action. A *long span floor* is one that has joists longer than the width of the floor. It has been noticed through tests, (Vanderbilt, 1974, [91]; Foschi, 1983, [21]; Wheat, 1985, [94]), that a floor does not necessarily collapse when one or even several joists fail. The load is often re-distributed to other joists. This shows the significance of two way action. It is also noticed (Sazinski et al, 1979, [82]) that the aspect ratio of floors may be a factor in determining floor deflections. This factor has been ignored up until now. The effects of aspect ratio actually represent the effects of edge supports. We will examine the extent of effect of aspect ratio on floor deflections and subsequent impact on design procedures.

3.2 The Lower Bound - Glued Floors

For the purpose of comparison, let us look at the lower bound of the deflections derived from both the T-beam and plate theories.

3.2.1 The T-beam Formula

The governing equation of a beam in bending is given by:

$$EI \frac{d^4 w_b}{dx^4} = q(x)S \quad (3.1)$$

and the maximum deflection w_{bmax} , for a beam subjected to uniformly distributed load q_0 , is

$$w_{bmax} = \frac{5}{384} \frac{q_0 S L^4}{EI} \quad (3.2)$$

where

L	length of the beam
q	uniformly distributed load per unit area on the floor
S	spacing of joists
EI	bending stiffness of the T-beam
b	subscript denotes the beam

3.2.2 The Maximum Deflection of a Plate Simply Supported on Four Sides

Modelling the floor system as a whole, in Figure 2.3, as an orthotropic plate and assuming that the plate is simply supported on four sides. From the elastic orthotropic plate theory, the governing equation of the deflection of this plate w_{pl} is

$$(D_x \frac{\partial^4}{\partial x^4} + 2H \frac{\partial^4}{\partial x^2 \partial y^2} + D_y \frac{\partial^4}{\partial y^4}) w_{pl}(x, y) = q(x, y) \quad (3.3)$$

The solution for this differential equation is

$$w_{pl} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (3.4)$$

$$w_{mn} = \frac{q_{mn}}{\frac{m^4 \pi^4}{L^4} D_x + 2H \frac{m^2 n^2 \pi^4}{L^2 B^2} + \frac{n^4 \pi^4}{B^4} D_y} \quad (3.5)$$

$$q_{mn} = \frac{4q}{mn\pi^2}(1 - \cos m\pi)(1 - \cos n\pi) \quad (3.6)$$

where

D_x bending stiffness of the plate in x direction (N mm)

H torsional stiffness (N mm)

D_y bending stiffness of the plate in y direction (N mm)

L, B length, width of the plate (mm)

The maximum deflection occurs at the centre point of the plate, $x = L/2$ and $y = B/2$. Then the maximum deflection of the plate w_{pl} is

$$w_{pl} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4q(1 - \cos m\pi)(1 - \cos n\pi) \sin \frac{m\pi}{2} \sin \frac{n\pi}{2}}{mn\pi^6 \left(\frac{m^4}{L^4} D_x + 2H \frac{m^2 n^2}{L^2 B^2} + \frac{n^4}{B^4} D_y \right)} \quad (3.7)$$

3.2.3 The Ratio of Deflections: Plate over T-beam

Let the bending stiffness of the T-beam equal to the bending stiffness of the plate in the joist direction

$$EI = D_x S \quad (3.8)$$

Then the ratio of the maximum deflection of the plate and the beam is

$$\begin{aligned} \alpha &= \frac{w_{pl}}{w_b} \\ &= A \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(1 - \cos m\pi)(1 - \cos n\pi) \sin \frac{m\pi}{2} \sin \frac{n\pi}{2}}{mn[m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4]} \end{aligned} \quad (3.9)$$

where

$$\begin{aligned} A &= \frac{1536}{5\pi^6} \\ \beta_1 &= \frac{2H}{D_x} \end{aligned}$$

$$\begin{aligned}\beta_2 &= \frac{D_y}{D_x} \\ \gamma &= \frac{L}{B}\end{aligned}\quad (3.10)$$

3.2.4 Comparison and Discussion

Assume that the bending stiffness of the plate in the direction of joists is the same as the bending stiffness of the corresponding T-beam, i.e. $EI = D_x S$. Assume the bending stiffness of the plate in the direction perpendicular to the joists D_y and the torsional stiffness H to be that of the plywood sheathing.

Neglecting Bending Stiffness in the y Direction and the Torsional Stiffness

Let $D_y = H = 0$, assume the load is uniformly distributed and compare equations 3.1 and 3.3.:

$$\begin{aligned}EI \frac{d^4 w_b}{dx^4} &= q_0 S \\ D_x \frac{d^4 w_{pl}}{dx^4} &= q_0\end{aligned}\quad (3.11)$$

where q_0 is the uniformly distributed load acting on the floor. $q_0 S$ is the load acting on the tributary area on the T-beam.

Notice that $D_x S = EI$. These two equations are identical. Then

$$w_{plmax} = w_{bmax} \quad (3.12)$$

or

$$\alpha = 1 \quad (3.13)$$

In other form

$$\alpha = A \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(1 - \cos m\pi)(1 - \cos n\pi) \sin \frac{m\pi}{2} \sin \frac{n\pi}{2}}{m^5 n} = 1 \quad (3.14)$$

Table 3.1: Rate of convergence

$\beta_1 = \beta_2 = 0$			$\beta_1 = \beta_2 = 0.01$			$\beta_1 = \beta_2 = 0.1$			$\beta_1 = \beta_2 = 1$		
M	N	α	M	N	α	M	N	α	M	N	α
1	1	1.2782	1	1	1.2531	1	1	1.0651	1	1	.4261
1	3	0.8521	1	3	1.0289	1	3	1.0225	1	3	.4214
1	5	1.1077	1	5	1.0629	1	5	1.0264	1	5	.4218
1	7	0.9251	1	7	1.0558	1	7	1.0257	1	7	.4217
1	9	1.0672	1	9	1.0579	1	9	1.0259	1	9	.4217
1	11	0.951	1	11	1.0571	1	11	1.0258	1	11	.4217
1	13	1.0493	1	13	1.0574	1	13	1.0258	1	13	.4217
1	15	0.9641	1	15	1.0573	1	15	1.0258	1	15	.4217
3	15	0.9601	3	15	1.0532	3	15	1.0217	3	15	.4175
5	15	0.9604	5	15	1.0535	5	15	1.022	5	15	.4179
7	15	0.9604	7	15	1.0534	7	15	1.0219	7	15	.4178
9	15	0.9604	9	15	1.0534	9	15	1.022	9	15	.4178

Rate of Convergence of the Series Solution

The expression for the ratio α is in the form of a series (Equation 3.9). It is important to study the rate of convergence of the series. If it converges too slow, the practical application of the formula will be limited. Fortunately, this is not the case. By examining equation 3.9, it can be noticed that the (m,n)th term of the series decreases in a rate of m^5n , m^3n^3 and mn^5 . This characterises a fast convergence. Table 3.1 shows the rate of convergence.

In Table 3.1, the ratio α is calculated to a certain number of terms of the series. M and N represent the number of terms in the series used. $\beta_1 = \frac{H}{D_1}$ and $\beta_2 = \frac{D_2}{D_1}$ are the stiffness ratios. It can be observed that as the value of β_1 and β_2 increased, the rate of convergence of the ratio α is increased.

Therefore, for practical applications, only a limited number of terms in the series needs to be used to achieve relatively accurate results.

Influences of Torsional and Transverse Bending Stiffness

It has been discussed that the results from plate theory are the same as the results derived from beam theory when ignoring the torsional stiffness and transverse bending stiffness. In this case, $\alpha = 1$, where α is the ratio of maximum deflection of the orthotropic plate and the maximum deflection of the T-beam. The following is a discussion of the significance of ignoring torsional and transverse bending stiffness.

Figure 3.1 shows that for floors that are longer in the direction of the joist than the width of the floor, the T-beam theory cannot be used without incurring large errors. This can be explained by the fact that T-beam theory does not consider the influence in the transverse direction. For floors in such a configuration, the influence cannot be ignored.

It can be seen from Figure 3.2 that the trend is still the same. For floors with joist longer than the width of the floor, the errors caused by using T-beam theory is dramatic. However, it is more sensitive for the loose floors than the glued ones.

Figure 3.3 schematically shows the different layouts of floors. T-beam theory ignores the influence of the other dimension B of the plate on its deflection. When the dimension ratio L/B increases to a certain point, deflection ratio increases dramatically. This is because of the influence of the supports in the transverse direction. The supports limit the deflection. The larger the ratio L/B , the larger the influence of the supports.

This result shows the necessity of limiting the application of T-beam formula in designing timber floors with high dimension ratio L/B .

Figure 3.4 shows the effects of the torsional and transverse bending stiffness of the plywood. The thicker the plywood, the more severe the influence. This in turn proves that the transverse stiffness of the floor will have a drastic effect on floors with joists longer than the width of the floors.

The other obvious observation from the figures is that for a medium span floor, the deflection calculated from plate theory is actually larger than that from the T-beam theory. The difference is as big as 10%. This is shown as the segment of reverse curve. This is also due to the influence of supports in the transverse

direction, except that this time the two sides tries to squeeze the plate and force the center point of the plate to go down even further and therefore yield larger deflection than its T-beam counter-part. Imagine that the floor is not supported in the direction perpendicular to the joists. All the joists will have the same deflection curve which will be the same as the T-beam. Then try to push the unsupported sides up. This will actually push down the center joist of the floor. But this trend stops when the squeezing effect is offset or even overcome by the supporting effect. This supporting effect eventually reduced the deflection of the center point of the floor when it becomes a long span floor from a medium one.

Figure 3.5 shows the short and medium span floors. It can be seen that for a typical floor (short or medium span with 2x8, 2x10 or 2x12 joists), the T-beam theory can predict the maximum deflection of floors without large discrepancy.

Figure 3.1: Effects of aspect ratios on floor deflections for a nailed floor

Figure 3.2: Effects of aspect ratios on deflections of floors with different sizes of nails

$$\alpha = \text{def_plate} / \text{def_Tbeam}$$

Short span floor

Figure 3.3: Long span floor vs short span floor

Figure 3.4: Effects of plywood thickness on floor deflections

Figure 3.5: The most commonly used floor layouts $\gamma < 2$

Chapter 4

Guidelines in Designing Medium and Long Span Floors

For a long span floor, the T-beam theory may introduce significant errors. A long span floor is one that has joists longer than the width of the floor. The design for long span floors becomes an issue, which is investigated in the following section.

4.1 Differences Between T-beam and Plate Theory for Different Floors

Let us examine the differences between deflections calculated by the T-beam theory and the plate theory. The ratio of the maximum deflection of the plate and the beam, as described in last chapter, is

$$\begin{aligned}\alpha &= \frac{w_p l}{w_b} \\ &= A \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(1 - \cos m\pi)(1 - \cos n\pi) \sin \frac{m\pi}{2} \sin \frac{n\pi}{2}}{mn[m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4]}\end{aligned}\quad (4.1)$$

where

$$A = \frac{1536}{5\pi^6}$$

$$\begin{aligned}
 \beta_1 &= \frac{2H}{D_x} \\
 \beta_2 &= \frac{D_y}{D_x} \\
 \gamma &= \frac{a}{b}
 \end{aligned} \tag{4.2}$$

In order to calculate the floor deflection patterns, floors are modelled using the existing range of sizes for all members. The minimum width of joists is 38 mm (2 inches) while the maximum is 89 mm (4 inches). The minimum height of joists are 89 mm (4 inches) and the maximum is 286 (12 inches). Four popular sizes for plywood are chosen with the thinnest as 12.5 mm and thickest as 20.5 mm. Nails 20d (4 inches) are chosen for the reason that any size bigger than 20d does not make a significant difference. Calculations have been done for the following 11 floors:

- joists: 38x89, 38x114, 38x140, 38x184, 38x235, 38x286, 89x114, 89x140, 89x184, 89x235, 89x286
- plywood: 12.5 mm, 4 plies; 15.5 mm 5 plies; 18.5 mm, 6 plies; 20.5 mm 7 plies
- nails: 20d

The results are illustrated in Figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8.

By examining these figures, the following observations can be found:

1. For short span floors, $\gamma = L/B < 1$, the deflection ratio $\alpha = w_{pl}/w_b$ equals approximately to 1. Therefore, the deflection of a short floor can be calculated using the T-beam formula.
2. For long span floors, $\gamma = L/B$ is bigger than 1, the T-beam formula can be used but with modification (to be discussed later).
3. Floors with smaller joists, the D_x is smaller and β_1 and β_2 is larger which leads to smaller α . This suggests that floors with smaller joists tend to be more sensitive with the aspect ratio γ (refer equation 4.1). Floors with thicker plywood which leads to larger β_1 and β_2 are also tend to be more sensitive to aspect ratio.

If a longer span floor is designed using the T-beam formula, which will design the floor as a single T-beam subject to load on the tributary area and ignore the influence of supports transferred through sheathing, the error can not be ignored. The supports at $y=0$ and $y=b$ limit the maximum deflections of the floor in the center. This supporting effect becomes larger with bigger rigidity ratio ($\beta_1 = 2H/D_x$, $\beta_2 = D_y/D_x$). The thicker the plywood, the more supporting effects from the sides are transferred to the center joists and thus the more it limit the maximum deflection. The smaller the joists, or the bigger the rigidity ratios, the less dominant the joists are in the floor system and thus the more influence from the sheathing.

Only simply supported conditions are examined. Floors with fixed boundary conditions can also be examined. However, there are rarely fixed conditions in practice. If a floor is designed according to a simply supported boundary condition, the design is on the safe side since a floor with fixed sides deflects less than a floor with simply supported sides.

4.2 Effects of Connections on the Deflection Ratio α

In last section, all the floors used in discussions are nailed with 20d nails(4 inches in length). In order to examine the influence of connections on the deflection ratio, three kinds of connections are studied: unfastened, glued and nailed with 20d nails.

It can be seen from Figure 4.9, 4.10, 4.11 and 4.12 that the influence of connections on the deflection ratio is not significant especially for floors with larger joists and thinner plywood.

4.3 Design of Long Span Floors

The following sections will be devoted to finding procedures to incorporate the findings into design for long span floors.

4.3.1 Using Charts

To design a long span floor, the T-beam formula can be used to calculate the deflections w_b of a T-beam subjected to the load on the tributary area, then multiplying it by the α value taken from the appropriate figures as described before (Figure 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8).

$$w_{pl} = w_b \alpha \quad (4.3)$$

4.3.2 Using Formula and Approximation

The other way to design a long span floor is to use a formula. To calculate the deflection ratio from Equation 4.1 is too tedious because of the series form. However, if the convergence of the series is fast, a formula of approximation can be derived.

Let us examine the convergence rate for varies floors. The even terms ($n=2,4,6,\dots$; $m=2,4,6,\dots$) equals zero in equation 4.1.

Let

$$\alpha_0 = A \sum_{m=1}^M \sum_{n=1}^N \frac{(1 - \cos m\pi)(1 - \cos n\pi) \sin \frac{m\pi}{2} \sin \frac{n\pi}{2}}{mn[m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4]} \quad (4.4)$$

In Figure 4.13, 4.14, 4.19 and 4.20, α and α_0 with $M=1,3,5$ and $N=1$ are displayed. It can be seen that α converges very fast with m . There is negligible difference between $M=1$, $M=3$ and $M=5$.

In Figure 4.15, 4.16, 4.17 and 4.18, α and α_0 with $M=1$ and $N=1,3,5$ are displayed. It can be seen that α does not converge as fast with n as with m . However, when $\gamma > \gamma_0$, α_0 and α are almost identical. γ_0 represent a critical value beyond which α_0 and α are considered identical.

γ_0 can be calculated as follows:

Let

$$\alpha_{mn} = \frac{4A}{mn[m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4]} \quad (4.5)$$

From equation 4.1, then after neglecting the m terms

$$\alpha = \alpha_{11} - \alpha_{13} + \alpha_{15} - \dots$$

or

$$\alpha = \alpha_{11} - (\alpha_{13} - \alpha_{15} + \dots)$$

If

$$\alpha_{13} < k\alpha_{11} \quad (4.6)$$

then

$$(\alpha_{13} - \alpha_{15} + \dots) < k\alpha_{11}$$

where k is a value determined by the amount of error allowed.

Combining equation 4.5 and 4.6

$$1 + \beta_1\gamma^2 + \beta_2\gamma^4 < 3(1 + 9\beta_1\gamma^2 + 81\beta_2\gamma^4)$$

assuming when $\gamma = \gamma_0$

$$1 + \beta_1\gamma^2 + \beta_2\gamma^4 = 3(1 + 9\beta_1\gamma^2 + 81\beta_2\gamma^4)$$

then

$$\gamma = \gamma_0 = \sqrt{\frac{(1 - 27k)\beta_1 + \sqrt{(27k - 1)\beta_1^2 - (4(3k - 1)(243k - 1)\beta_2)}}{2(243k - 1)\beta_2}} \quad (4.7)$$

γ_0 is calculated for a list of different floors with $k=0.02$. The results are listed in Table 4.1 and 4.2.

For floors with plywood glued to the joists, the γ_0 is listed in Table 4.3 and 4.4.

4.4 Summary

When a designer wants to design for a long span floor with $\gamma = L/B > \gamma_0$, only the first term of equation 4.1 needs to be used. Then

$$\alpha = \alpha_{11}$$

Table 4.1: γ_0 , α and α_0 for different nailed floors, plywood thickness: 12.5, 15.5 mm

t	d	h	nail size	γ_0	α	α_0	$\frac{\alpha - \alpha_0}{\alpha}$
12.5	38	89	20	1.71	1.02	1.06	0.04
12.5	38	114	20	2.00	1.04	1.08	0.04
12.5	38	140	20	2.29	1.05	1.10	0.04
12.5	38	184	20	2.74	1.07	1.11	0.04
12.5	38	235	20	3.23	1.08	1.12	0.04
12.5	38	286	20	3.70	1.08	1.13	0.04
12.5	89	114	20	2.33	1.06	1.10	0.04
12.5	89	140	20	2.67	1.07	1.11	0.04
12.5	89	184	20	3.23	1.08	1.12	0.04
12.5	89	235	20	3.84	1.08	1.13	0.04
12.5	89	286	20	4.42	1.09	1.13	0.04
15.5	38	89	20	1.49	1.00	1.04	0.04
15.5	38	114	20	1.74	1.02	1.06	0.04
15.5	38	140	20	1.98	1.04	1.08	0.04
15.5	38	184	20	2.37	1.06	1.10	0.04
15.5	38	235	20	2.80	1.07	1.11	0.04
15.5	38	286	20	3.20	1.08	1.12	0.04
15.5	89	114	20	2.02	1.04	1.08	0.04
15.5	89	140	20	2.32	1.05	1.10	0.04
15.5	89	184	20	2.80	1.07	1.11	0.04
15.5	89	235	20	3.33	1.08	1.12	0.04
15.5	89	286	20	3.83	1.08	1.13	0.04

Table 4.2: γ_0 , α and α_0 for different nailed floors plywood thickness: 18.5.20.5 mm

t	d	h	nail size	γ_0	α	α_0	$\frac{2-\alpha_0}{\alpha}$
18.5	38	89	20	1.35	0.98	1.02	0.04
18.5	38	114	20	1.57	1.01	1.05	0.04
18.5	38	140	20	1.79	1.03	1.07	0.04
18.5	38	184	20	2.14	1.05	1.09	0.04
18.5	38	235	20	2.52	1.06	1.11	0.04
18.5	38	286	20	2.88	1.07	1.12	0.04
18.5	89	114	20	1.82	1.03	1.07	0.04
18.5	89	140	20	2.09	1.05	1.09	0.04
18.5	89	184	20	2.52	1.06	1.11	0.04
18.5	89	235	20	3.00	1.07	1.12	0.04
18.5	89	286	20	3.45	1.08	1.12	0.04
20.5	38	89	20	1.30	0.97	1.01	0.04
20.5	38	114	20	1.50	1.00	1.04	0.04
20.5	38	140	20	1.70	1.02	1.06	0.04
20.5	38	184	20	2.03	1.04	1.09	0.04
20.5	38	235	20	2.38	1.06	1.10	0.04
20.5	38	286	20	2.72	1.07	1.11	0.04
20.5	89	114	20	1.73	1.02	1.06	0.04
20.5	89	140	20	1.97	1.04	1.08	0.04
20.5	89	184	20	2.37	1.06	1.10	0.04
20.5	89	235	20	2.81	1.07	1.11	0.04
20.5	89	286	20	3.23	1.08	1.12	0.04

Table 4.3: γ_0 , α_0 and α for different glued floors.plywood thickness: 12.5, 15.5 mm

t	d	h	γ_0	α	α_0	$\frac{\alpha - \alpha_0}{\alpha}$
12.5	38	89	1.82	1.03	1.07	0.04
12.5	38	114	2.12	1.05	1.09	0.04
12.5	38	140	2.42	1.06	1.10	0.04
12.5	38	184	2.88	1.07	1.12	0.04
12.5	38	235	3.38	1.08	1.12	0.04
12.5	38	286	3.85	1.08	1.13	0.04
12.5	89	114	2.44	1.06	1.10	0.04
12.5	89	140	2.79	1.07	1.11	0.04
12.5	89	184	3.34	1.08	1.12	0.04
12.5	89	235	3.95	1.08	1.13	0.04
12.5	89	286	4.53	1.09	1.13	0.04
15.5	38	89	1.57	1.01	1.05	0.04
15.5	38	114	1.82	1.03	1.07	0.04
15.5	38	140	2.07	1.04	1.09	0.04
15.5	38	184	2.47	1.06	1.10	0.04
15.5	38	235	2.90	1.07	1.12	0.04
15.5	38	286	3.31	1.08	1.12	0.04
15.5	89	114	2.09	1.05	1.09	0.04
15.5	89	140	2.39	1.06	1.10	0.04
15.5	89	184	2.88	1.07	1.11	0.04
15.5	89	235	3.40	1.08	1.12	0.04
15.5	89	286	3.90	1.08	1.13	0.04

Table 4.4: γ_0 , α and α_0 for different glued floors.plywood thickness: 18.5, 20.5 mm

t	d	h	γ_0	α	α_0	$\frac{\alpha-\alpha_0}{\alpha}$
18.5	38	89	1.43	0.99	1.03	0.04
18.5	38	114	1.65	1.01	1.06	0.04
18.5	38	140	1.87	1.03	1.08	0.04
18.5	38	184	2.23	1.05	1.10	0.04
18.5	38	235	2.61	1.06	1.11	0.04
18.5	38	286	2.98	1.07	1.12	0.04
18.5	89	114	1.89	1.03	1.08	0.04
18.5	89	140	2.16	1.05	1.09	0.04
18.5	89	184	2.59	1.06	1.11	0.04
18.5	89	235	3.06	1.07	1.12	0.04
18.5	89	286	3.51	1.08	1.12	0.04
20.5	38	89	1.39	0.98	1.02	0.04
20.5	38	114	1.60	1.01	1.05	0.04
20.5	38	140	1.82	1.03	1.07	0.04
20.5	38	184	2.15	1.05	1.09	0.04
20.5	38	235	2.51	1.06	1.11	0.04
20.5	38	286	2.85	1.07	1.11	0.04
20.5	89	114	1.82	1.03	1.07	0.04
20.5	89	140	2.07	1.04	1.09	0.04
20.5	89	184	2.48	1.06	1.10	0.04
20.5	89	235	2.91	1.07	1.12	0.04
20.5	89	286	3.33	1.08	1.12	0.04

Figure 4.1: Deflection ratio vs aspect ratio. Joist sizes: 38x89 to 38x286. Plywood thickness: 12.5 mm.

Figure 4.2: Deflection ratio vs aspect ratio. Joist sizes: 38x89 to 38x286. Plywood thickness: 15.5 mm.

Figure 4.3: Deflection ratio vs aspect ratio. Joist sizes: 38x89 to 38x286. Plywood thickness: 18.5 mm

Figure 4.4: Deflection ratio vs aspect ratio. Joist sizes: 38x89 to 38x286. Plywood thickness: 20.5 mm

Figure 4.5: Deflection ratio vs aspect ratio. Joist sizes: 89x114 to 89x286. Plywood thickness: 12.5 mm

Figure 4.6: Deflection ratio vs aspect ratio. Joist sizes: 89x114 to 89x286. Plywood thickness: 15.5 mm

Figure 4.7: Deflection ratio vs aspect ratio. Joist sizes: 89x114 to 89x286. Plywood thickness: 18.5 mm

Figure 4.8: Deflection ratio vs aspect ratio. Joist sizes: 89x114 to 89x286. Plywood thickness: 20.5 mm

Figure 4.9: Influence of connections on deflection ratio, 38x89

Figure 4.10: Influence of connections on deflection ratio, 38x286

Figure 4.11: Influence of connections on deflection ratio, 89x114

Figure 4.12: Influence of connections on deflection ratio, 89x286

Figure 4.13: Convergence of m terms, 38x89, 12.5 mm

Figure 4.14: Convergence of m terms, 38x89, 20.5 mm

and

$$w_{pl} = w_b \alpha = w_b \alpha_{11} \quad (4.8)$$

The errors are negligible.

If $1 < \gamma < \gamma_0$, errors arise. These floors can be described as medium span floors. For a medium span, floor deflection can be calculated using T-beam theory and adjusted upward a maximum of 104.22 and Figure 4.23 illustrate medium span floors with the most commonly used joist sizes: 2x8, 2x10 and 2x12.

It, therefore, can be concluded that to design for a floor subject to uniformly distributed load, the following procedures can be followed:

1. Calculate $\gamma = L/B$.
2. If $\gamma \leq 1$, short span floor, use T-beam formula. This is the most common case.
3. Otherwise, calculate γ_0 according to Equation 4.7.
4. If $\gamma < \gamma_0$, medium span floor, use T-beam formula. The maximum deflection thus calculated is multiplied by an adjustment factor of 1.1 .
5. Otherwise, long span floor, use T-beam formula. The maximum deflection is multiplied by a different adjustment factor of $\alpha = \alpha_{11}$ as in Equation 4.8.

Figure 4.15: Convergence of n terms, 38x89, 12.5 mm

Figure 4.16: Convergence of n terms, 38x89, 20.5 mm

Figure 4.17: Convergence of n terms, 89x286, 12.5 mm

Figure 4.18: Convergence of n terms, 89x286, 20.5 mm

Figure 4.19: Convergence of m terms, 89x286, 12.5 mm

Figure 4.20: Convergence of m terms, 89x286, 20.5 mm

Figure 4.21: Medium span floors, joists: 2x8, 2x10, 2x12, 2-1/2" nails

Figure 4.22: Extreme cases, medium span, 12.5 mm, 2x12 and 20.5 mm, 2x8; 2-1/2" nails

Figure 4.23: Upper and lower limit, medium span

Chapter 5

Design of Plywood Sheathed Floors Subject to Concentrated Load

5.1 Introduction

In many cases, concentrated loads such as heavy machinery in industrial buildings and refrigerators in residential dwellings, have to be considered in the design of floor systems. T-beam theory can not be used to design for concentrated loads acting on plywood sheathed floors. Without sheathing, one of the bare joists will sustain the point load acting on it. If the sheathing is provided to cover the joists, the sheathing becomes a load distribution element. It transfers load from the point of loading to adjacent joists. In addition to the composite action of a T-beam, two-way action becomes an important factor in designing floors subjected to a concentrated load. The thickness of the sheathing and the size of the joists will determine the magnitude of the two-way action. It has been reported (Vanderbilt et al., 1974, [91]) that the load will be distributed to a few neighbouring joists. The joists far from the point of loading will share only a negligible portion of the load.

In this chapter, the deflection of floors subject to a concentrated load is examined using orthotropic plate theory. Results are compared against the T-beam approach and the experimental results. A simple design procedure for wood joist floors subjected to concentrated loads will be proposed.

5.2 Test Results from Forest Product Lab. USDA, McCutcheon

Tests have been done at Forest Products Laboratory, U.S. Department of Agriculture, on floors subject to concentrated loads (McCutcheon 1977, [62]). Four floors (designated N-1 to N-4) used standard grade 5/8-inch plywood nailed to the joists with face grain perpendicular to the span. Nails were 2-1/2" at 6 inches along the edges and 8 inches over the intermediate joists. Another 3 floors (G-1 to G-3) had the plywood nail-glued in place using a rigid adhesive. These used tongue-and-groove plywood with the edges glued to give a continuous sub-floor. The joist stiffness was determined using nondestructive static bending tests before the floors were assembled.

The concentrated point loads were applied by means of a hydraulic jack placed over the center joist at midspan. The concentrated load was twice applied in increments of 100 pounds to a total load of 500 pounds.

The test results are listed below: Concentrated load 500 pounds

Floor No.	Average E, Modulus of Elasticity	500 lbs *
	10^6 lbs/in^2 (10^3 MPa)	inch (mm)
N-1	1.57 (10.82)	0.16 (3.94)
N-2	2.20 (15.17)	0.13 (3.20)
N-3	2.00 (13.79)	0.12 (2.92)
N-4	1.49 (10.27)	0.15 (3.86)
G-1	1.53 (10.55)	0.10 (2.64)
G-2	2.20 (15.17)	0.09 (2.21)
G-3	2.00 (13.79)	0.10 (2.36)
* Deflection at midspan of loaded joist. Concentrated load.		

5.3 An Example Using the T-beam Theory

As an example to illustrate the results from T-beam theory, let us consider the following floor.

plywood:

5/8" standard grade	15.5 mm, 4 plies
$B_a = 110,000 \text{ N/mm}$	@ 0°
75,000	@ 90°
$B_b = 3,700,000 \text{ N} \cdot \frac{\text{mm}^2}{\text{mm}}$	@ 0°
430,000	@ 90°

joists:

$$\begin{aligned} b \times d &= 38 \times 184 \text{ mm} \\ E &= 13,000 \text{ MPa} \end{aligned}$$

spacing of joists:

$$406 \text{ mm}$$

floor layout:

$$\begin{aligned} L &= 3660 \text{ mm} \\ B &= 4060 \text{ mm} \end{aligned}$$

load:

$$P = 2250 \text{ N}$$

Using T-beam theory, recall equation 2.23 and 2.26 in chapter 2:

$$\begin{aligned} y_0 &= \frac{d}{2} + \delta \\ \delta &= \frac{d+t}{2} \frac{(EA)_2}{(EA)_2 + (EA)_1} \\ &= \frac{d+t}{2} \frac{B_a S}{B_a S + Ebd} \\ &= 25 \text{ mm} \\ y_0 &= \frac{184}{2} + \delta \\ &= 92 + \delta \\ &= 117 \text{ mm} \end{aligned} \tag{5.1}$$

The upper limit of the effective rigidity of the T-beam (referring chapter 2):

$$\begin{aligned}
 (EI)_e &= (EI)_1 + (EA)_1 \delta^2 + (EI)_2 + (EA)_2 \left(\frac{d+t}{2} - \delta \right)^2 \\
 &= \frac{Ebd^3}{12} + Ebd\delta^2 + B_b S + B_a S \left(\frac{d+t}{2} - \delta \right)^2 \\
 &= 482.8 \cdot 10^9 (N.mm^2)
 \end{aligned} \tag{5.2}$$

The lower limit of the maximum deflection is therefore:

$$\begin{aligned}
 f_{max} &= \frac{PL^3}{48(EI)_e} \\
 &= 4.75 \text{ mm}
 \end{aligned} \tag{5.3}$$

The lower limit of the effective rigidity:

$$\begin{aligned}
 (EI)_e &= (EI)_1 + (EI)_2 \\
 &= 256 \cdot 10^9 (N.mm^2)
 \end{aligned} \tag{5.4}$$

The upper limit of the maximum deflection:

$$\begin{aligned}
 f_{max} &= \frac{PL^3}{48(EI)_e} \\
 &= 8.96 \text{ mm}
 \end{aligned} \tag{5.5}$$

Comparing the test results, the maximum deflections are between 2.21 mm and 3.937 mm which is much lower than what is predicted by the T-beam theory. This indicates that using the T-beam approach will lead to conservative designs. The T-beam approach ignores the load sharing by adjacent joists and lets the single joist under point load sustain the whole point load. This obviously results in overly conservative design. It is, therefore, necessary to examine the floor using orthotropic plate theory.

5.4 Result From Orthotropic Plate Theory

Assume that the floor is subjected to a concentrated load acting at location $(\xi L, \eta B)$ as shown in figure 5.2, then treat the floor as an orthotropic plate.

$$D_x \left(\frac{\partial^4}{\partial x^4} + 2H \frac{\partial^4}{\partial x^2 \partial y^2} + D_y \frac{\partial^4}{\partial y^4} \right) w(x, y) = q(x, y) \quad (5.6)$$

$$q(x, y) = \begin{cases} \frac{P}{dx dy} & x = \xi L, y = \eta B \\ 0 & \text{else} \end{cases} \quad (5.7)$$

Assume

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (5.8)$$

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (5.9)$$

where

$$q_{mn} = \frac{4}{LB} \int_0^L \int_0^B q(x, y) \sin m\pi \xi \sin n\pi \eta \quad (5.10)$$

substitute equation 5.7 into equation 5.10:

$$q_{mn} = \frac{4P}{LB} \sin m\pi \xi \sin n\pi \eta \quad (5.11)$$

solve the above equation, the result is reached as follows:

$$w_{mn} = \frac{q_{mn}}{D_{mn}} \quad (5.12)$$

$$q_{mn} = \frac{4P}{LB} \sin m\pi \xi \sin n\pi \eta \quad (5.13)$$

$$D_{mn} = D_x \left(\frac{m\pi}{L} \right)^4 + 2H \left(\frac{m\pi}{L} \right)^2 \left(\frac{n\pi}{B} \right)^2 + D_y \left(\frac{n\pi}{B} \right)^4 \quad (5.14)$$

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (5.15)$$

Assume that the concentrated load P is acting at the center of the plate $(\xi = 0.5, \eta = 0.5)$. The deflection at $(x = \frac{L}{2}, y = \frac{B}{2})$ is then

$$w(x, y)|_{x=L/2, y=B/2} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4P}{LB} \frac{\sin^2 \frac{m\pi}{2} \sin^2 \frac{n\pi}{2}}{D_{mn}} \quad (5.16)$$

From McCutcheon's data:

$$P = 2250 \text{ N (500 lbs)}$$

$$L = 3660 \text{ mm}$$

$$B = 4060 \text{ mm}$$

$$D_2 = 3,700,00 \frac{\text{Nmm}^2}{\text{mm}}$$

$$D_1 = 482.8 \times 10^9 / 406 \frac{\text{Nmm}^2}{\text{mm}}$$

(482.8 x 10⁹ is the effective rigidity of the T-beam)

$$2H = D_2$$

The result of calculation: maximum deflection is 2.538 mm which falls within the range of 2.362 to 2.642 mm from McCutcheon's tests of floors with plywood sheathing glued to the joists. This indicates that the orthotropic plate theory describes well the test results on floors subject to a concentrated load.

5.5 Comparison

The following table lists the results from the T-beam theory, orthotropic plate theory and the tests (McCutcheon 1977, [62]). For nailed floors, it was assumed that there are no gaps.

Floor No.	Average MOE	500 lbs *	plate	error %	Tbeam
	10^3 MPa	mm	mm	%	mm
N-1	10.82	3.937	3.73	-5%	8.01
N-2	15.17	3.200	3.06	-4%	6.17
N-3	13.79	2.921	3.28	4%	6.67
N-4	10.27	3.861	3.88	0.5%	8.44
G-1	10.55	2.642	2.99	13%	5.99
G-2	15.17	2.210	2.49	12%	4.70
G-3	13.79	2.362	2.61	10%	5.02
* Deflection at midspan of loaded joist. Concentrated load.					

It can be seen from this table that orthotropic plate theory explains the test results well, especially for the nailed floors which resulted in less than 4% error. Even for the glued floors, and that it is much closer to the test results than what is obtained from T-beam approach. In the next section, detailed investigation on the effects of different parameters on the floor deflections will be conducted.

5.6 Parametric Study

In this section, investigation is made to determine if the discrepancy between the orthotropic plate theory and the T-beam approach is consistent for floors with other joist, nail sizes, connection types and plywood thicknesses. The results are listed in Table 5.1, 5.2 and are also plotted in Figure 5.3, 5.4, 5.5, 5.6 and 5.7. The ratio is obtained by dividing deflections achieved from T-beam approach with deflections from the orthotropic plate theory.

Figure 5.3 shows that for floors with joists of larger sizes the difference tends to be smaller. Figure 5.4 and 5.6 shows that errors caused by using the T-beam formula far out-weigh the differences of using different connections. Figure 5.5 displays the ratio of the deflections computed from T-beam theory and the plate theory. It can be seen that even for the largest size of joists, the difference is more than 50%. The last one, Figure 5.7 shows that even thinner plywood sheathing will not reduce the

Table 5.1: Deflection of different floors calculated using both orthotropic plate theory and T-beam formula with different joist sizes. Joist sizes: 38x89 to 38x286. Plywood thickness: 12.5 mm. 2d nail.

Deflections (mm); plywood: 12.5 mm, nail: 2d, joist 38xh									
h	unfastened floors			glued floors			nailed floors		
	plate	Tbeam	ratio	plate	Tbeam	ratio	plate	Tbeam	ratio
89	26.93	93.11	3.46	13.87	37.36	2.69	20.92	65.71	3.14
114	15.73	44.38	2.82	8.81	20.11	2.28	12.87	33.72	2.62
110	10.03	23.98	2.39	5.98	11.94	2.00	8.49	19.15	2.25
184	5.46	10.57	1.94	3.53	5.90	1.67	4.80	8.90	1.85
235	3.15	5.07	1.61	2.17	3.10	1.43	2.84	4.43	1.56
286	2.01	2.81	1.40	1.45	1.84	1.27	1.85	2.52	1.36

error caused by using the T-beam formula.

Therefore, a wood joist floor with sheathing subjected to concentrated load has to be designed as an orthotropic plate instead of a T-beam. The errors caused by using T-beam approach are not negligible for any floor configuration.

5.7 Location of Maximum Deflections, (x_0, y_0)

It is now established that plywood sheathed joist floors can not be designed using T-beam theory. This section will examine the issues involved in designing floors subjected to concentrated loading.

5.7.1 In the Direction Perpendicular to Joists, y_0

Rewrite equation 5.15

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (5.17)$$

where

Table 5.2: Deflection of different floors calculated using both orthotropic plate theory and T-beam formula with different nail sizes. Joist sizes: 38x89. Plywood thickness: 12.5 mm. Nails: 2d to 60d.

Deflection of floors: plywood: 12.5 mm: nail size: 2d - 60d; joist: 38x89		
plate	Tbeam	ratio
10.92	65.71	3.14
19.44	59.39	3.06
19.44	59.39	3.06
18.84	56.90	3.02
18.19	54.21	2.98
17.64	51.96	2.95
17.27	50.48	2.92
16.61	47.83	2.88
16.35	46.79	2.86
16.05	45.62	2.84
15.62	43.94	2.81

$$w_{mn} = C \frac{\sin m\pi\xi \sin n\pi\eta}{m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4} \quad (5.18)$$

$$C = \frac{4PL^3}{B\pi^4 D_x} \quad (5.19)$$

$$\beta_1 = \frac{2H}{D_x} \quad (5.20)$$

$$\beta_2 = \frac{D_y}{D_x} \quad (5.21)$$

$$\gamma = \frac{L}{B} \quad (5.22)$$

In order to design for the serviceability of a floor, it is necessary to find the maximum deflection. Since equation 5.17 is in series form, it is difficult to find a simple closed form solution for the maximum deflection. Instead of finding the exact location of the maximum deflections, a numerical method was employed with a simple algorithm. By calculating the deflection of a sufficient number of points on the floor, the maximum deflection and its location can be found with satisfactory accuracy. This algorithm is by no means the best one. There are a number of alternative ways to achieve the same result. But, for the purpose of illustrating the discussion herein, this simple algorithm will be used.

Denote (x_0, y_0) as the location of the maximum deflection. The point load is acting at $(\xi L, \eta B)$.

Let us examine the following floor:

<hr/> <hr/>		
joists:		
size	38x184	2" x 8"
L	3660 mm	12', joist span
B	4060 mm	9 joists
S	406 mm	16", joist spacing
<hr/>		
plywood:	4 plies	
t	15.5 mm	5/8 "
<hr/>		
load:	concentrated load	
P	2250 N	500 lbs
<hr/> <hr/>		

Table 5.3: Location of maximum deflections of floors subject to point load.

$\eta \rightarrow$	0.1		0.2		0.3		0.4		0.5	
ξ	x_0	y_0	x_0	y_0	x_0	y_0	x_0	y_0	x_0	y_0
0.1	0.3	0.1	0.35	0.2	0.35	0.3	0.35	0.4	0.35	0.5
0.2	0.35	0.1	0.35	0.2	0.35	0.3	0.35	0.4	0.35	0.5
0.3	0.4	0.1	0.4	0.2	0.4	0.3	0.4	0.4	0.4	0.5
0.4	0.45	0.1	0.45	0.2	0.45	0.3	0.45	0.4	0.45	0.5
0.5	0.5	0.1	0.5	0.2	0.5	0.3	0.5	0.4	0.5	0.5

Table 5.4: Maximum deflections (mm)

$\eta \rightarrow$	0.1		0.2		0.3		0.4		0.5	
ξ	glued	nailed	glued	nailed	glued	nailed	glued	nailed	glued	nailed
0.1	0.77	0.96	0.83	1.09	0.81	1.06	0.81	1.06	0.81	1.06
0.2	1.46	1.82	1.59	2.08	1.56	2.03	1.56	2.03	1.56	2.03
0.3	2.02	2.51	2.21	2.89	2.16	2.82	2.16	2.18	2.16	2.82
0.4	2.39	2.95	2.62	3.43	2.56	3.34	2.56	3.33	2.56	3.34
0.5	2.51	3.11	2.76	3.62	2.70	3.52	2.70	3.52	2.70	3.52

From Table 5.3, it can be seen that $y_0 = \eta B$ which indicates that the maximum deflection occurs along the line $y = y_0$. This line is parallel to the joists and underneath the point load. This conclusion can be further illustrated by Figure 5.11. Deflections of the floor subjected to point load at varies $(\xi L, \eta B)$ are illustrated in Figure 5.8, 5.9, 5.10 and 5.11.

Figure 5.8 shows that when the point load is applied at varies points along the span of the joists and not at the mid-width, the maximum deflection is not at the point of loading. It resembles a single beam subjected to concentrated load. The only exception is that when the load P is applied at mid-span, the maximum deflection is also at mid-span.

Figure 5.9 is the same as Figure 5.8 except that P is acting at the mid-width.

The result is identical in both cases.

Figure 5.10 describes the deflections of a floor along the span when the point load acts at various points along the width at mid-span of the joists. It implies that there is not much difference among the deflection curves along the span under load point when point load is applied in the central part of the floor. As an example, let us assume a floor with 11 joists numbered from 1 to 11. Joist No.1 and No.11 are supported. Joist No.6 is at the center of the floor. The deflection curve of joist No.6, when load is applied at the mid-span of joist No.6, is almost the same as the deflection curve of joist No.5, when the point load is applied to the mid-span of joist No.5.

Figure 5.11 illustrates the deflection curve of the floor at mid-span when a point load is applied to the mid-span. It can be seen that the maximum deflection is under the point of loading for the central joists. It can also be seen that only a few neighboring joists are affected by the point load. It is therefore a local effect. The implication is that for a building built for a special purpose and when the location of the point load is known beforehand, local reinforcement can be applied underneath the point load.

In order to show that $y_0 = \eta B$ is also true for other floors of different joist sizes and plywood thicknesses, let's examine the following floors.

load: P	2250 N	500 lbs
joist span, L	3660 mm	12'
floor width, B	4060 mm	9 joists
joist spacing, S	406 mm	16"
floor 1	joists: 38x89	plywood: t = 12.5 mm, 4 plies
floor 2	joists: 38x89	plywood: t = 20.5 mm, 7 plies
floor 3	joists: 89x286	plywood: t = 12.5 mm, 4 plies
floor 4	joists: 89x286	plywood: t = 20.5 mm, 7 plies

The results are illustrated in Figure 5.12, 5.13, 5.14, and 5.15. This group of 4 floors represent the combinations of thinnest and thickest plywood, smallest and

Figure 5.1: Simply supported beams subject to point load

largest joists. A few observations can be made. First of all, it can be seen that $y_0 = \eta B$. Second, thicker plywood transfers load to other joists further away from point of loading while larger joist will do the opposite. This is apparent by comparing Figure 5.13, which has smaller joists covered by thicker plywood, with Figure 5.14 which has a thinner plywood and larger joists.

Figure 5.16 and 5.12 shows the deflection curves computed by using 3 terms and 15 terms in the joist direction. It is obvious that they are identical. This further shows that the solution converges very fast.

5.7.2 In the Direction of Joists, x_0

It has been observed that the deflection curve of the floor in the direction of the joists is similar to the deflection curve of a simply supported beam subjected to point load, Figure 5.17. The following steps are taken to show that the maximum deflection of the plate can be found by calculating the deflection of the plate at the point where the maximum deflection of the beam occurs.

For a simply supported beam subjected to point load as shown in Figure 5.1, the location of the maximum deflection is at

$$x_b = \sqrt{\frac{L^2 - L_2^2}{3}} \quad L_1 > L_2 \quad (5.23)$$

The deflection of a floor at point $(x_b L, \eta B)$ is

$$\begin{aligned}
w(x_b, \eta) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin m\pi x_b \sin n\pi \eta \\
w_{mn} &= \frac{q_{mn}}{D_{mn}} \\
q_{mn} &= \frac{4P}{LB} \sin m\pi \xi \sin n\pi \eta \\
D_{mn} &= \pi^4 \left[D_x \left(\frac{m}{L} \right)^4 + 2H \left(\frac{mn}{LB} \right)^2 + D_y \left(\frac{n}{B} \right)^4 \right]
\end{aligned} \tag{5.24}$$

Using above equations, the maximum deflection for floors with plywood 12.5 → 20.5 mm, joists (38,64,89) x (89 → 286) are calculated. The result is shown in Figure 5.18. It can be seen that they are almost identical. Therefore, the maximum deflection of a floor can be calculated as the deflection of the floor at point $(x_b, \eta B)$.

5.8 Guidelines in Designing Floors Subject to Point Load

In order to design for a wood joist floors subject to concentrated load, the following procedures can be followed:

1. Deflection of floors at any point (x, y)

$$\begin{aligned}
w(x, y) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \\
w_{mn} &= \frac{q_{mn}}{D_{mn}} \\
q_{mn} &= \frac{4P}{LB} \sin m\pi \xi \sin n\pi \eta \\
D_{mn} &= D_x \left(\frac{m\pi}{L} \right)^4 + 2H \left(\frac{m\pi}{L} \right)^2 \left(\frac{n\pi}{B} \right)^2 + D_y \left(\frac{n\pi}{B} \right)^4
\end{aligned} \tag{5.25}$$

2. Find the location of the maximum deflection of a beam subjected to concentrated load: x_b

$$x_b = \sqrt{\frac{L^2 - L_2^2}{3}} \quad L_1 > L_2 \tag{5.26}$$

3. Calculate the deflection of the floor at point $(x_b, \eta B)$. This deflection will be the approximate maximum deflection.

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin m\pi \frac{x_b}{L} \sin n\pi \eta \quad (5.27)$$

w_{mn} can also be written as

$$\begin{aligned} w_{mn} &= C \frac{\sin m\pi \xi \sin n\pi \eta}{L_{mn}} \\ C &= \frac{4PL^3}{BD_x \pi^4} \\ L_{mn} &= m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4 \\ \beta_1 &= \frac{2H}{D_x} \\ \beta_2 &= \frac{D_y}{D_x} \\ \gamma &= \frac{L}{B} \end{aligned} \quad (5.28)$$

Where $(\xi L, \eta B)$ is the point of loading, P is the point load, L and B is the span and width of the floor, D_x is the bending rigidity of the T-beam and H and D_y the torsional rigidity and bending rigidity of plywood perpendicular to the joists. The convergence of the series is fast.

Figure 5.2: A plywood sheathed floor subjects to concentrated load.

Figure 5.3: Deflection of different floors calculated using both orthotropic plate theory and T-beam formula with different joist sizes. Joist sizes: 38x89 to 38x286. Plywood thickness: 12.5 mm. 2d nail.

Figure 5.4: Deflection of different floors calculated using both orthotropic plate theory and T-beam formula with different joist sizes, upper bound, lower bound and nailed. Joist sizes: 38x89 to 38x286. Plywood thickness: 12.5 mm. 2d nail.

Figure 5.5: Ratio of deflections obtained from Tbeam formula and orthotropic plate theory. Joist sizes: 38x89 to 38x286, Plywood thickness: 12.5 mm, nail size: 2d.

Figure 5.6: Deflection of different floors calculated using both orthotropic plate theory and T-beam formula with different nail sizes. Joist sizes: 38x89. Plywood thickness: 12.5 mm. Nails: 2d to 60d.

Figure 5.7: Deflection of different floors calculated using both orthotropic plate theory and T-beam formula with different plywood thicknesses. Joist sizes: 38x184. Plywood thickness: 12.5 to 20.5 mm. Nails: 8d.

Figure 5.8: Deflections along the span when P acts at varies points along the span at $\eta = 0.3$

Figure 5.9: Deflections along the span when P acts at varies points along the span at mid-width $\eta = 0.5$

Figure 5.10: Deflections along the span when P acts at mid-span

Figure 5.11: Deflections along the width when P acts at mid-span

Figure 5.12: Deflections along the width when P acts at mid-span, 38x89, 12.5 mm

Figure 5.13: Deflections along the width when P acts at mid-span, 38x89, 20.5 mm

Figure 5.14: Deflections along the width when P acts at mid-span, 89x286, 12.5 mm

Figure 5.15: Deflections along the width when P acts at mid-span, 89x286, 20.5 mm

Figure 5.16: Deflections along the width when P acts at mid-span, 38x89, 12.5 mm , $M = 3$

Figure 5.17: Deflections along the span when P acts at $(0.3 L, 0.5 B)$

Figure 5.18: Deflections at $(x_b, \eta B)$ and maximum deflection when P acts at $(0.3 L, 0.5 B)$, plywood 12.5, 15.0, 18.5, 20.5 mm; joists 38,64,89 x 89 $\rightarrow$ 284

Chapter 6

Failure Criteria for Flat Roofs Subject to Ponding

6.1 Introduction

Flat roofs may fail due to ponding of rainwater as evidenced by reports of failures of flat roofs in rain storms. Ponding load initiates and increases the deflection of flat roofs. Increased deflection in turn increases the ponding load. This cycle of deflection-ponding-more deflection- more ponding continues until the structure reaches equilibrium. The roof may stand at equilibrium and the extra water run off the edge, or the roof may experience large deflections before reaching equilibrium which could mean collapse. The roofs which can stand the ponding load with enough stiffness are considered **stable** structures while those which can not stand the ponding load are called **unstable** structures as far as ponding is concerned. The question is, what are the criteria that govern this phenomenon (Liu et al. 1992, [42]).

The amplifying effects of the ponding load were studied by Haussler R.W. [33] These studies were motivated by reports of failures caused by instability in wood, steel and aluminum roofs. In this study, the roof was modeled as a beam deflected in a sine curve subjected to ponding load which is proportional to the deflection of the beam. Kuenzi and Bohannon [39] in 1964 performed tests to verify the theory. Significant ponding effects were observed. More investigations by several

other authors followed. Salama and Moody [70] extended the study of the ponding effects using isotropic plates theory and by treating the roofs as isotropic plates. They derived the critical stiffness of such a plate. Folz and Foschi [16] studied the structural reliability of lumber joists of roofs that are subjected to ponding loads. Roofs were modeled as a beam of sine curve.

However, most roofs are not constructed in a manner that they can be treated as isotropic plates. For example, timber roofs are constructed using plywood, which itself is an orthotropic material, and supported with joists. The roof structure as a whole has to be treated as an orthotropic plate. This kind of structure also has its own characteristics such as having high bending stiffness in one direction and weak in the other.

In this study, the criteria of the instability of orthotropic plates are derived. The criteria are expressed in terms of dimensions and rigidity of the plates and the density of the ponding liquid.

The results can be used to provide provisions for the dimensions and clear spans of flat roofs. Any flat roof which can be treated as an orthotropic plate such as corrugated steel plates can be analyzed by using the derived results.

6.2 Governing Equations for Orthotropic Plates

An orthotropic rectangular plate is considered as shown in Figure 6.1. Small deformation and a linear stress-strain relationship are assumed.

The governing equation is :

$$(D_x \frac{\partial^4}{\partial x^4} + 2H \frac{\partial^4}{\partial x^2 \partial y^2} + D_y \frac{\partial^4}{\partial y^4}) w(x, y) = p(x, y) \quad (6.1)$$

Where

$$H = D_{12} + 2D_k$$

$$D_x = \frac{t^3}{12} \left(\frac{E_1}{1 - \mu_1 \mu_2} \right)$$

$$D_y = \frac{t^3}{12} \left(\frac{E_2}{1 - \mu_1 \mu_2} \right) = \frac{E_2}{E_1} D_x$$

$$D_{12} = \frac{t^3}{12} \left(\frac{E_1 \mu_2}{1 - \mu_1 \mu_2} \right) = \left(\frac{t^3}{12} \frac{E_2 \mu_2}{1 - \mu_1 \mu_2} \right)$$

$$D_k = \frac{t^3}{12} G$$

w	deflection of the plate (mm)
p	loads ($\frac{N}{mm^2}$)
E_1, E_2	effective modulus of elasticity in the x, y direction ($\frac{N}{mm^2}$)
μ_1, μ_2	Poisson's ratios
G	shear modulus ($\frac{N}{mm^2}$)
t	plate thickness (mm)

6.3 Ponding of Orthotropic Plate Simply Supported on Four Sides

Ponding load can be described as kw . Then the total load on the structure is

$$p(x, y) = q(x, y) + kw(x, y) \quad (6.2)$$

where q is the load without ponding, k is water density, w is the deflection of the plate.

Using the classical Navier method and assuming:

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (6.3)$$

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \quad (6.4)$$

where

$$q_{mn} = \frac{1}{LB} \int_0^L \int_0^B q(x, y) \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} dx dy \quad (6.5)$$

For constant load:

$$q = q_0$$

then

$$q_{mn} = \frac{1q_0}{mn\pi^2}(1 - \cos m\pi)(1 - \cos n\pi) \quad (6.6)$$

From equation 6.1, 6.2, 6.3 and 6.4, we get

$$\begin{aligned} & \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn}(D_{mn} - k) \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \\ &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \end{aligned} \quad (6.7)$$

where

$$D_{mn} = \frac{m^4\pi^4}{L^4}D_x + 2H\frac{m^2n^2\pi^4}{L^2B^2} + \frac{n^4\pi^4}{B^4}D_y \quad (6.8)$$

Since the equation 6.7 is true for all x and y, therefore

$$w_{mn}(D_{mn} - k) = q_{mn} \quad (6.9)$$

then

$$w_{mn} = \frac{q_{mn}}{D_{mn} - k} \quad (6.10)$$

or

$$w_{mn} = A_{mn} \frac{q_{mn}}{D_{mn}} \quad (6.11)$$

where

$$A_{mn} = \frac{1}{1 - \frac{k}{D_{mn}}} \quad (6.12)$$

A_{mn} is called the amplification factor and it is function of the dimension of plate (L,B), the rigidity (D_x D_y H), and the density of the liquid or water (k).

It should be noted that in the Equation 6.11, if $q_{mn} = 0$, then $w_{mn} = 0$. This is true because if there is no other external load to develop initial deflection, ponding will not occur. In other words, un-deformed plate will not hold liquid. Therefore ponding can only exist with initial deflection or co-exist with other external loads.

6.4 Critical Dimensions to Develop Instability

Consider the first order of w_{mn} or w_{11} .

$$w_{11} = \frac{16q_0}{\pi^6 \left(\frac{D_x}{L^4} + \frac{2H}{L^2 B^2} + \frac{D_y}{B^4} \right)} \quad (6.13)$$

(1) For a square plate, i.e., $L = B$, so that

$$\frac{\pi^4}{L^4} (D_x + 2H + D_y) - k = 0 \quad (6.14)$$

$$A_{11} = \infty, w_{11} = \infty \quad (6.15)$$

This means that when L approaches a critical dimension the deflection is infinite. In other words, the plate is unstable. The critical dimension is

$$L = \pi \left(\frac{D_x + 2H + D_y}{k} \right)^{\frac{1}{4}} \quad (6.16)$$

(2) For a rectangular plate with unequal sides, $L \neq B$,

To make $w_{11} = \infty$, let

$$\pi^4 \left(\frac{D_x}{L^4} + \frac{2H}{L^2 B^2} + \frac{D_y}{B^4} \right) - k = 0 \quad (6.17)$$

Now solve for L in terms of B , the only real solution is:

$$L = f_{11}(B) = B \sqrt{\frac{H + \sqrt{H^2 - D_x \left(D_y - \frac{k B^4}{\pi^4} \right)}}{-D_y + \frac{k B^4}{\pi^4}}} \\ B > B_{11} = \pi \left(\frac{D_y}{k} \right)^{\frac{1}{4}} \quad (6.18)$$

where B_{11} is the critical value of width B . It can be seen from equation 6.18 that

$$\lim_{B \rightarrow \infty} L = L_{11} = \pi \left(\frac{D_x}{k} \right)^{\frac{1}{4}} \quad (6.19)$$

Similarly, L_{11} is the critical length as shown in figure 6.2.

Define function $f(L, B)$:

$$f(a, b) = \pi^4 \left(\frac{D_x}{L^4} + \frac{2H}{L^2 B^2} + \frac{D_y}{B^4} \right) - k \quad (6.20)$$

When $f(L, B) > 0$, plate is stable; when $f(L, B) < 0$, the plate is unstable; while $f(L, B) \rightarrow 0$, deflection approaches infinite.

6.5 Discussion

6.5.1 Stable and unstable regions.

In Figure 6.2, the region is divided into two sub-regions by the curve $L = f_{11}(B)$, equation 6.18.

Any point (L, B) represents the dimensions of a plate of width B and length L . If a point (L, B) falls within the stable region, such a plate will not deflect infinitely, as shown in Figure 6.5 and 6.6. If a point falls in the unstable region, the deflection of such a plate could be infinite. In other words, the plate may collapse suddenly, as shown in Figure 6.6 and 6.7. Figure 6.5, 6.6 and 6.7 will be discussed in detail in later examples.

6.5.2 Higher order approximation of roof deflection, w

The discussions so far are concerned with the first order of w , or w_{11} . What is the effect of higher order terms. How many terms should one choose for a practical and accurate approximation? From equation 6.6 we find that $q_{mn} = 0$ when m or n is an even number. Therefore the second non-zero term of w is w_{13} and w_{31} followed by w_{33} .

$$w_{13} = \frac{\frac{16q_0}{3\pi^2}}{\pi^4 \left(\frac{D_x}{L^4} + 9 \frac{2H}{L^2 B^2} + 81 \frac{D_y}{B^4} \right) - k} \quad (6.21)$$

By comparing w_{13} , w_{31} , w_{33} with w_{11} , it can be seen that w_{13} , w_{31} are not negligible while w_{33} can be neglected. For instance, when $k = 0$, $\frac{w_{31}}{w_{11}} = \frac{1}{729} = 0.00137$.

$$w_{31} = \frac{\frac{16q_0}{9\pi^2}}{\pi^4(81\frac{D_x}{L^4} + 9\frac{2H}{L^2B^2} + \frac{D_y}{B^4}) - k} \quad (6.22)$$

$$w_{33} = \frac{\frac{16q_0}{9\pi^2}}{81\pi^4(\frac{D_x}{L^4} + \frac{2H}{L^2B^2} + \frac{D_y}{B^4}) - k} \quad (6.23)$$

To study the stable and unstable regions for w_{13} , w_{31} and w_{33} , follow the same procedure used for w_{11} . The conclusions are shown in Figure 6.3.

The critical curves

for w_{31} is

$$\begin{aligned} L &= f_{31}(B) = 3f_{11}(B) \\ L_{31} &= 3L_{11} \end{aligned} \quad (6.24)$$

for w_{13} is

$$\begin{aligned} L &= f_{13}(B) = \frac{B}{3} \sqrt{\frac{H + \sqrt{H^2 - D_x(D_y - \frac{kb^4}{81\pi^4})}}{-D_y + \frac{kb^4}{81\pi^4}}} \\ B_{13} &= 3B_{11} \end{aligned} \quad (6.25)$$

for w_{33} is

$$\begin{aligned} L &= f_{33}(B) = 3B \sqrt{\frac{H + \sqrt{H^2 - D_x(D_y - \frac{kb^4}{81\pi^4})}}{-D_y + \frac{kb^4}{81\pi^4}}} \\ L_{33} &= L_{31} = 3L_{11} \\ B_{33} &= B_{13} = 3B_{11} \end{aligned} \quad (6.26)$$

$$(6.27)$$

It is proved that the stable region of w_{11} is within the stable region of higher order terms. Therefore the criterion of stability of w_{11} governs. In other words, as long as the width B and length L of a plate are smaller than the critical dimensions B_{11} and L_{11} respectively by a finite amount, the deflection of the plate will be finite.

6.6 Practical Applications

Consider a flat roof consisting of plywood sheathing of thickness t , width B and length L and N joists of size $b \times d$, as shown in Figure 6.4. Assume that the sheathing and joists are rigidly attached to each other, i.e. no slippage between them.

This roof system can be analyzed as an orthotropic plate. It can be reasonably assumed that the bending stiffness along the y direction, D_y , is the bending stiffness of the plywood sheathing. The twisting stiffness H is also the twisting stiffness of plywood. The bending stiffness along the x direction can be calculated as following:

Take a T-beam section as shown in Figure 6.4.
where

S spacing of joists
 t thickness of plywood

Then the bending stiffness of this cross-section is

$$D_x = B_b + B_a(d + \frac{t}{2} - y_0)^2 + \frac{E_b b d^3}{12S} + \frac{E_b b d}{S}(\frac{d}{2} - y_0)^2 \quad (6.28)$$

where

B_b bending stiffness of plywood (N mm)
 B_a axial stiffness of plywood (N/mm)
 E_b modulus of elasticity of joists (N/mm²)
 y_0 distance between the bottom of joist and the neutral axis of the T-beam

$$y_0 = \frac{B_a S(\frac{t}{2} + d) + E_b b d \frac{d}{2}}{B_a S + E_b b d} \quad (6.29)$$

The boundary conditions are assumed to be simply supported on four sides. At $y = 0$, b, the sheathing is attached to a joist. Rotation about the x axis is usually not restrained. At $x = 0, L$, joists are the dominant members. They are

simply supported. The assumption of a simply supported boundary condition is reasonable.

Example:

N	= 12 joists
L	= 3657.6 (mm)
B	= 3657.6 (mm)
b	= 38 (mm)
d	= 184 (mm)
Eb	= 11000 ($\frac{N}{mm^2}$)
S	= 406.4 (mm)
t	= 15.5 (mm)
Ba	= 75000 ($\frac{N}{mm}$)
Bb	= 630000 (N mm)
Dy	= 3000000 (N mm)
H	= 3000000 (N mm)
k	= 9.81×10^{-6} ($\frac{N}{mm^3}$)
q_0	= 1.9 (kPa, 40 lbf/ft ²)

(1) Stable Region

For such a structure, Figure 6.5 shows the deflection vs width B of the plate when L is in the stable region. It can be seen that the deflection stabilized after it reaches finite value with the increase of width B. One of the dimensions, L in this case, controls. It is also shown that ponding effect is not dramatic.

This situation is represented by line $L = 5601$ in Figure 6.6 which shows that since value of L is below L_{11} the line stays in the stable region.

(2) Unstable Region

In Figure 6.7, on the other hand, B is larger than B_{11} . In this case, when L approaches the critical value L_{11} , deflection approaches infinity. Ponding effect is dramatic.

This situation can be represented by hline 2 in Figure 6.6. Since B is larger than B_{11} , L is limited by the curve $L = f_{11}(B)$ which is the boundary between stable and

Table 6.1: Effects of sheathing thickness on floor deflection.

t	Ba	Bb	H x1000	Dy x1000	Dx 10 ⁶	z ₀	w ₀	w ponding
7.5	47	300	3.4	17	984	109	7.08	7.33
9.5	47	570	6.6	33	992	109	7.03	7.28
12.5	59	1300	38	190	1080	113	6.49	6.71
15.5	75	2500	86	430	1190	117	5.94	6.12
18.5	95	3400	260	1300	1310	122	5.49	5.64
20.5	100	4300	520	2600	1350	124	5.48	5.63

unstable region.

(3) Influence of plywood thickness Figure 6.8 shows the influence of the thickness of plywood. Deflection decreases hlinearly as thickness of plywood sheathing increases. This shows that the composite action of plywood is not negligible. However, The ponding effect remains the same for different thickness evidenced by the almost constant difference between the two curves.

(4) Influence of joist spacing

Figure 6.9 shows the effects of joist spacing. It is drawn by placing different number of joists under the sheathing. The data is concluded in Table 6.2. It can be seen that the deflection increases almost hlinearly when spacing increases. There is a slight increase in the effect of ponding with the increase of spacing.

(5) Implications in timber code:

The limit state design equation for the ultimate strength of timber can be expressed in the following format:

$$1.25E(D_n) + 1.50E(Q_n) = \Phi R_{0.05} \quad (6.30)$$

where

$E(D_n), E(Q_n)$	design dead and live load effects
Φ	performance factor
$R_{0.05}$	characteristic strength

Table 6.2: Effects of joist spacing on floor deflection

No of joists	spacing	z_0	$D_r 10^6$	w_0	w
1	2135	172	107	18.6	20.39
2	1423	165	568	13.1	13.92
3	1068	159	722	10.3	10.84
4	854	154	842	8.65	9.02
5	712	150	962	7.54	7.82
6	610	146	1070	6.74	6.95
7	534	143	1180	6.13	6.31
8	474	140	1270	5.65	5.80
9	427	137	1370	5.25	5.39
10	388	135	1450	4.93	5.05
11	356	133	1540	4.65	4.76
12	329	131	1620	4.41	4.50
13	305	129	1700	4.20	4.29
14	285	127	1780	4.01	4.09

Table 6.3: Maximum width for different plywood thickness

plywood thickness, t, mm	12.5	15.5	18.5	20.5
Maximum width B_{11} of a stable roof, mm	2027	2462	2600	2801

The amplification factor A_{mn} will appear in the above equation. For ponding stable structures, a different performance factor has to be calibrated using reliability analysis accounting for the variability of the material properties.

6.7 Critical Dimensions That Define Stable and Unstable Roofs

Table 6.3 shows the maximum width a roof can have for a given plywood thickness without incurring ponding instability, i.e., the critical width B_{11} . In Table 6.4, 6.5, 6.6 and 6.7, the upper and lower bound of the maximum length and width a roof can have without incurring ponding instability are listed. The upper bound corresponding to a loose roof in which the plywood sheathing is not connected to the joists by either nail or glued. The lower bound, on the other hand, corresponds to the glued roofs with plywood glued to the joists. It can be seen from the tables that the upper and lower bounds are close .

6.8 Comparison with Other Research

It has been proposed that the following amplifying factor be used in timber design code to design for ponding (B. Folz, R. Foschi, 1990, [16]) :

$$A_p = \frac{1}{1 - \frac{kSL^4}{EI\pi^4}} \quad (6.31)$$

This formula is derived by simplifying the roof as a T-beam and assume that the T-beam deflects into a sine curve. The critical length of the roof is therefore

Table 6.4: Maximum length for different joists, plywood thickness: 12.5 mm

joist width, b	height, d	Max. length.lower bound	upper bound
38	89	4953	5851
38	114	5961	6847
38	140	6953	7825
38	184	8534	9381
38	235	10252	11074
38	286	11879	12678
64	114	6790	7488
64	140	7920	8592
64	184	9721	10359
64	235	11679	12285
64	286	13532	14114
89	114	7373	7958
89	140	8601	9158
89	184	10557	11079
89	235	12682	13175
89	286	14695	15164

Table 6.5: Maximum length for different joists, plywood thickness: 15.5 mm

joist width, b	height, d	Max. length, lower bound	upper bound
38	89	4958	5980
38	111	5964	6975
38	140	6955	7952
38	184	8535	9506
38	235	10253	11196
38	286	11879	12798
64	114	6792	7603
64	140	7921	8703
64	184	9722	10463
64	235	11679	12384
64	286	13533	14208
89	114	7374	8061
89	140	8602	9255
89	184	10557	11168
89	235	12683	13258
89	286	14695	15243

Table 6.6: Maximum length for different joists, plywood thickness: 18.5 mm

joist width. b	height. d	Max. length.lower bound	upper bound
38	89	1976	5911
38	114	5974	6884
38	140	6961	7846
38	184	8538	9387
38	235	10255	11067
38	286	11881	12664
64	114	6799	7515
64	140	7926	8607
64	184	9724	10361
64	235	11681	12279
64	286	13533	14102
89	114	7380	7981
89	140	8605	9170
89	184	10559	11080
89	235	12684	13169
89	286	14696	15154

Table 6.7: Maximum length for different joists. plywood thickness: 20.5 mm

joist width, b	height, d	Max. length.lower bound	upper bound
38	89	4990	6098
38	114	5982	7077
38	140	6966	8041
38	184	8541	9563
38	235	10256	11264
38	286	11882	12859
64	114	6804	7691
64	140	7929	8779
64	184	9726	10526
64	235	11682	12438
64	286	13534	14256
89	114	7384	8139
89	140	8608	9321
89	184	10560	11222
89	235	12685	13303
89	286	14697	15283

$$L = \pi \left(\frac{EI}{kS} \right)^{\frac{1}{3}} \quad (6.32)$$

Since $Dx = EI/S$, the above critical equation is identical to L_{11} .

By examining Figure 6.2 and 6.6, it can be seen that L_{11} is the asymptote of $L = f_{11}(B)$ when B approaches infinity. It is true that for a very wide roof, the T-beam formula can be used with sufficient accuracy since boundary conditions in the transverse direction is negligible. For a roof with regular size, i.e. the width is close to the span, there will be a difference between the critical dimensions derived from orthotropic plate theory and simplified T-beam of sine curve. However, the asymptote and, hence, the result from simplified T-beam theory can be used to define the critical dimension on a conservative side. Thus defined critical dimension will be simpler than using the actual critical dimension defined by curve $L = f_{11}(B)$.

The other point to be made is that for a wood joist roof, the critical width is much smaller than the critical span as can be seen in Table 6.3 and 6.5. Therefore, the governing dimension for a wood joist roof is the span. If the critical span is exceeded, the roof is ponding unstable since the critical width is usually smaller than the actual width of roofs. Therefore, equation 6.32 can be used to define the critical span of wood joist roofs. This coincides with the result of simplified beam theory.

6.9 Conclusions

The findings and implications of this study can be summarized as follows:

1. An amplification factor which describes the ponding effects is derived. It is a function of bending rigidity, dimensions of orthotropic plates and the density of water.
2. Critical dimensions L_{11} and B_{11} governing the stability of structures subjected to ponding load are derived. The L_{11} , B_{11} are the maximum width and length of a stable orthotropic plate subjected to ponding respectively. As long as one of the dimensions is kept smaller than its maximum, the structure will not

deflect infinitely. Ponding effect is not dramatic. However, keeping $L < L_{11}$ or $B < B_{11}$ does not guarantee that it will not develop large deflection due to normal load. A limit of the dimensions has to be further established to restrict the bending stresses from exceeding the strength of the material.

3. Results can be used to calibrate the formulation in the timber code to account for the ponding effects.
4. The procedure developed in this study can be used to analyze floor and roof systems, especially in timber structures. The effects of connections between the plywood sheathing and joists can be found by incorporating the effective rigidity of the T-beam, as described in previous chapters, into the formula.

It should be pointed out that the criterion derived are within the range of small deflection. Roofs may yield large deflection before failure. The behaviour of roofs with large deflection has to be examined using the theory of nonlinear orthotropic plate theory which is beyond the scope of this thesis.

Figure 6.1: An orthotropic plate

Figure 6.2: Critical dimensions of roofs subject to ponding

Figure 6.3: Higher order critical dimensions

Figure 6.4: Plywood sheathed roofs subject to ponding load

Figure 6.5: Deflection in a stable region

Figure 6.6: Regions

Figure 6.7: Deflections when span, L , approaching unstable region

Figure 6.8: Deflection vs plywood thickness

Figure 6.9: Deflection vs joist spacing

Chapter 7

Modeling Bending Strength of Wood Members Using Damage Accumulation Concepts

7.1 Introduction

Size effects on the strength of lumber have been observed by Madsen and Nielson [49]. In 1976, Madsen and Nielsen conducted testing to investigate the size effects on lumber subjected to bending. It was found that

1. longer beams had lower strength
2. deeper beam had lower strength
3. strength depends on load configuration. The beam having higher stress volume tends to have lower strength.

As illustrated in Figure 7.1, beam (1) tends to have higher strength than beam (2) provided both beams have the same cross section dimensions, same species, same grade and subjected to the same loading condition.

In Figure 7.2, beam (2) which is deeper tends to have lower strength. Beams with same dimensions subjected to different loading conditions will have different strengths. The beam in case (2), Figure 7.3, tends to have higher strength.

Figure 7.1: Beams with different length

Figure 7.2: Beams with different depth

Figure 7.3: Beams with different load scheme

This phenomenon was explained later (1986) by Madsen and Buchanan [46] using the modified weakest link theory. In this theory, the lumber is considered perfectly brittle. It consists of a series of elements. Failure of any one of these elements (the weakest link) signals the failure of lumber. In other words, the strength of lumber is governed by the strength of its weakest link.

7.2 Characterization of Size and Load Configuration Effects

The size effects are quantified as follows:

(1) length effects:

$$\frac{r_1}{r_2} = \left(\frac{L_2}{L_1}\right)^{k_1} \quad (7.1)$$

where r_1 and r_2 are the strengths of members of length L_1 and L_2 , respectively, and k_1 is the length effect parameter. The parameter k_1 was found to be approximately 4.3 by calibration of equation 7.1 into Madsen's test data [48] [47] [45] [46].

(2) depth effect:

$$\frac{r_1}{r_2} = \left(\frac{d_2}{d_1}\right)^{\frac{1}{k_2}} \quad (7.2)$$

where d_1 and d_2 are depth of the beams, k_2 is the depth effect parameter.

(3) load configuration effect:

The load configuration effect was quantified by converting the specific type of loading to an equivalent length which can be used in 7.1. For example, for the load configuration in Figure 7.3 (1):

$$L_e = \frac{1 + \frac{x_0}{L} k_1}{1 + k_1} L \quad (7.3)$$

where L_e is the equivalent length and x_0 = distance between two concentrated loads on a simply supported beam of span L .

Foschi [24] included the size effects results into the design formula using reliability analysis.

However, all the studies so far have separately considered the different size effects and load configurations. In this chapter, we will include the size effects, load configuration effects and rate of loading effects into one unified formula by looking at this problem from another point of view, namely, total damage accumulation in the lumber [11].

7.3 The Total Damage Accumulation Model

At any point in a piece of lumber, a parameter α can be used to describe the damage accumulation at that point. Parameter α is a random variable which ranges from zero to one. A value of α equals zero means no damage accumulated at the point described at a specific time, while a value of 1 for α represents failure. Since lumber is full of cracks and flaws, α is one at points where cracks or flaws exist. Flaws are randomly distributed in a piece of lumber. Figure 7.4 shows the initial damage distribution on a cross-section of a beam.

The conventional idea for failure of lumber is that lumber fails when the extreme fibre in a bending member reaches the maximum strength or in the weakest element

PM-1 3½"X4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

Figure 7.4: Pre-existing damage

in the lumber fails. However, this is not true in at least the following two cases: (1) initial damage already exists in every piece of lumber, as shown in Figure 7.4. Its density and distribution depends on grade. (2) in the case when a hole is drilled (or other kind of man made damage), the strength of the broken fibres is zero in this case and hence such fibres become the weakest link. But the strength of the lumber is obviously not zero.

Lumber fails when damage is accumulated to a certain amount in the whole volume of the lumber. Lumber may not fail if one or even a bundle of fibres is broken. This can be expressed as follows:

$$\Omega = \int_V \alpha dv \quad (7.4)$$

where

Ω	total damage accumulated in the lumber
α	damage accumulated at a point, per unit volume
v	volume

The damage accumulation rate depends on the stress level and the damage which has already accumulated at that point. The higher the stress, the higher is the damage accumulation rate. Pre-existing damage accelerates damage accumulation. The damage accumulation rate then can be written as:

$$\frac{d\alpha}{dt} = f_n(\sigma, \alpha) \quad (7.5)$$

where

α	damage accumulation at a point
t	time
σ	stress level at a point

This concept was used to study the long term strength of lumber [48] [47] [46] [23] [24].

The Canadian model to describe the long term strength is [24]:

$$\frac{d\alpha}{dt} = a[\sigma(t) - \sigma_0\sigma_s]^b + c[\sigma(t) - \sigma_0\sigma_s]^n \alpha \quad (7.6)$$

where

a,b,c,n	model parameter
$\sigma(t)$	stress at time t
σ_s	short term strength
σ_0	threshold stress ratio

The U.S. model is [48] [47] [46] [23]:

$$\frac{d\alpha}{dt} = \exp(-a + b \frac{\sigma(t)}{\sigma_s}) \quad (7.7)$$

where

a,b	model parameter
$\sigma(t)$	stress state
σ_s	short term strength

To demonstrate the damage accumulation in a bending member, let's choose the simplest relation:

$$\frac{d\alpha}{dt} = \sigma \quad (7.8)$$

for a beam in bending:

$$\sigma = \frac{M(x)}{I} y \quad (7.9)$$

where

M(x)	moment at a cross section x
I	moment of inertia of the cross section
y	distance from the neutral axis

Figure 7.5: Stress induced damage accumulation

Figure 7.6: Accumulated damage at failure

hence

$$\alpha = \alpha_0 + \frac{M(x)}{I}ty \quad (7.10)$$

After time t , at a cross section x , the damage accumulation distribution will look like in Figure 7.5.

At this stage, the damage accumulated in some of the fibres has reached one. These fibres have already failed. But the total damage in the whole volume has not reached the critical value. The member has not failed yet.

After another time interval has elapsed, the damage distribution would look

Figure 7.7: Beam subjected to 2 points load

like that shown in figure 7.6. At this stage, the damage in the whole volume has accumulated to a critical amount, and the member fails.

7.4 Interpretation of Test Results

Consider a member subjected to loading as illustrate in figure 7.7.

$$\sigma = \frac{M(x)}{I} y \quad (7.11)$$

$$\begin{aligned} M(x) &= \frac{p}{2} x & x < \frac{L-x_0}{2} \\ &= \frac{p}{4} (L - x_0) & \frac{L-x_0}{2} < x < \frac{L+x_0}{2} \end{aligned} \quad (7.12)$$

Assume a simple damage accumulation model:

$$\frac{d\alpha}{dt} = a\sigma^b \quad (7.13)$$

where a and b are model parameters. They are constant with respect to time and size of member.

From equation 7.4, we have

$$\frac{d\Omega}{dt} = \int_V \frac{d\alpha}{dt} dv \quad (7.14)$$

Combine equation 7.11, 7.12, 7.13 and 7.14:

$$\begin{aligned}
 \frac{d\Omega}{dt} &= \int_V \frac{d\alpha}{dt} dv \\
 &= \int_V a\sigma^b dv \\
 &= \frac{a}{(b+1)^2} \sigma_t^b V \left(1 + \frac{x_0 b}{L}\right)
 \end{aligned} \tag{7.15}$$

where

$$\sigma_t = \frac{p(L - x_0)}{4I} \left(\frac{h}{2}\right) \tag{7.16}$$

is the stress at the extreme fibre. To study the short term strength, assume

$$\sigma_t = Kt \tag{7.17}$$

$$K = \frac{k_0(L - x_0)}{4I} \left(\frac{h}{2}\right) \tag{7.18}$$

where

k_0 rate of loading related to the concentrated load

K rate of loading related to stress

then

$$\sigma_s = KT \tag{7.19}$$

where

σ_s short term strength

T total time needed for lumber to fail

then, from equation 7.15

$$\Omega = \frac{a}{(b+1)^3} dh(L + x_0 b) \sigma_s^b T \tag{7.20}$$

Lumber fails when Ω reaches a critical value. From equation 7.20, we can see that for a certain value of Ω

1. σ_s decreases when h increases $\Rightarrow$ depth effect
2. σ_s decreases when L increases $\Rightarrow$ length effect
3. σ_s decreases when x_0 increases $\Rightarrow$ load configuration effect
4. σ_s decreases when T increases $\Rightarrow$ rate of loading effect

These trends have been proven by tests.

The only thing found in the tests that is contrary to what is predicted is the width effect. Strength increases when width increases. This chapter will not discuss width effect.

7.5 A Unified Approach

Equation 7.20 shows the relation between the strength of lumber and the dimensions of lumber, rate of loading and load configuration. But, the accuracy of this relationship depends on the accuracy of the assumption of Equation 7.13. The damage accumulation model (Canadian and U.S.) for long term strength may be used. A more accurate model may be found in order to get a more accurate formulation of Ω .

However, Equation 7.20 can be modified as follows:

$$\Omega = AL^{a_1} h^{a_2} T^{a_3} \left(1 + \frac{a_4 x_0}{L}\right)^{a_5} \sigma_s \quad (7.21)$$

where A , a_1 , a_2 , a_3 , a_4 and a_5 are model parameters which can be determined by calibrating test results into the formulation.

7.5.1 Length Effects

Considering two beams with same dimension of cross sections subjected to a third point loading with the same rate of loading. i.e. $h_1 = h_2$, $T_1 = T_2$ and $x_0 = 0$. The member fails when $\Omega = \Omega_0$. Therefore,

$$\Omega_0 = AL_1^{a_1} h^{a_2} T^{a_3} \sigma_{s1} = AL_2^{a_1} h^{a_2} T^{a_3} \sigma_{s2} \quad (7.22)$$

It can be rewritten as

$$\frac{\sigma_{s1}}{\sigma_{s2}} = \left(\frac{L_2}{L_1}\right)^{a_1} \quad (7.23)$$

Compare equation 7.23 with 7.1. then

$$a_1 = \frac{1}{k_1} \quad (7.24)$$

7.5.2 Depth Effects

Following the same procedure, we obtain

$$a_2 = \frac{1}{k_2} \quad (7.25)$$

7.5.3 Load Configuration Effects

Comparing equation 7.21 with 7.3. we get:

$$a_3 = \frac{1}{k_1} \quad (7.26)$$

$$a_4 = k_1 \quad (7.27)$$

Therefore, the total damage accumulation over the volume is

$$\Omega = Ah^{\frac{1}{k_2}} T^{a_3} (L + x_0 k_1)^{\frac{1}{k_1}} \sigma_s \quad (7.28)$$

The only parameter that needs to be calibrated is the rate of loading parameter a_3 .

The formula can also be written as:

$$\frac{\sigma_s}{\sigma_0} = \left(\frac{h_0}{h}\right)^{\frac{1}{k_2}} \left(\frac{T_0}{T}\right)^{a_3} \left(\frac{L_0}{L + k_1 x_0}\right)^{\frac{1}{k_1}} \quad (7.29)$$

Then, the strength σ_s of any bending member with depth h , length L subjected to two concentrated loads with a distance of x_0 from each other with a rate of loading

$k(T)$ can be determined in relation to the strength of a standard third point loading test ($x_0 = 0$) with parameters σ_0 , h_0 , L_0 and T_0 .

7.6 Summary

This chapter has unified the size effects, load configuration and rate of loading effects into one formula using a total damage accumulation model. This model fits well with the existing length, depth and load configuration effect formula. The rate of loading model parameter a_3 can be determined by calibrating the above formula into rate of loading tests data. This concept has been raised first by professor J.J.Salinas.

Chapter 8

Summary and Future Work

8.1 Summary

The following topics have been discussed:

1. Existing methodologies for studying and designing wood joist floors have been examined and summarized. A different approach from the existing finite element analysis and a simple T-beam design is proposed and examined. The floor is modeled as an orthotropic plate. The bending stiffness of the plate is taken as that of the existing T-beam and, therefore, the composite action of the floor has been included. The bending stiffness in the transverse direction is taken as that of the plywood and hence the two-way action is taken into consideration. This model includes parameters like connection type (nailed, glued or loose) between the joists and sheathing, gaps between sheathing, plywood and joist size . The variability of joists is not included.
2. The results from the proposed approach are compared favorably with test results from previous research by other researchers.
3. Short span floors subject to uniformly distributed load can be designed as a T-beam subjected to load on the tributary area.
4. For floors of medium span, which has aspect ratio γ larger than 1 but smaller than a material related value γ_0 , T-beam formula can still be used. The result should be adjusted by a maximum amount of 10%.

5. Aspect ratio is not a negligible parameter in the design of long span floors, which has a aspect ratio γ larger than γ_0 . The boundary conditions in the transverse direction will have a significant effect on the deflection of the floors.
6. Floors subjected to concentrated load can not be designed using the T-beam theory. They have to be designed as orthotropic plates. A simple formula has been derived to design for concentrated loads.
7. Ponding on flat roofs is examined. A criterion is derived. This criterion divides roofs into two categories: stable and unstable. A stable roof is one that will deflect to a finite amount and reach equilibrium. The unstable roof is one in which deflections will increase when the amount of water increases. It keeps deflecting until collapse.
8. A unified theory of size effect and loading configuration effect on the bending strength of wood members is proposed. This is an attempt to unify various factors that affect the bending strength of wood into a single formulation.

Major contributions of this thesis are summarized in following sections.

8.1.1 Design Procedures for Short Span Floors Subject to Uniformly Distributed Load

Short span floors are defined as those with aspect ratio, $\gamma = L/B < 1$. Then plywood sheathed short span floors can be designed according to T-beam formula by simplifying a floor as a T-beam subject to load on the tributary area. Figure 8.1.

Deflection of floors can then be calculated by

$$\Delta = \Delta_1 \left(\frac{K_1}{K_2} + f_\Delta \frac{f_v}{K_2} \right) \quad (8.1)$$

where K_1 and K_2 represent the composite stiffness of an unfastened and glued T-beam respectively. f_v is the adhesive factor defined in Chapter 2. The adhesive factor defines the degree of composite action in the T-beam.

Figure 8.1: Plywood sheathed floor modeled as a T-beam subjects to load on the tributary area.

$$K_1 = (EI)_e = (EI)_1 + (EI)_2 \quad (8.2)$$

$$K_2 = (EI)_1 + (EA)_1\delta^2 + (EI)_2\left(\frac{d+t}{2} - \delta\right)^2 \quad (8.3)$$

$$\delta = \frac{d+t}{2} \frac{(EA)_2}{(EA)_2 + (EA)_1} \quad (8.4)$$

$$f_v = (EA)_1\delta^2 + (EA)_2\left(\frac{d+t}{2} - \delta\right)^2 \quad (8.5)$$

$$\Delta_1 = \frac{5}{384} \frac{qSL^4}{K_1} \quad (8.6)$$

$$f_\Delta = \frac{10}{(L'\alpha)^2 + 10} \quad (8.7)$$

$$\alpha^2 = \frac{\left(\frac{t+h}{2}\right)^2 S_n}{K_2 - K_1} \left(\frac{K_2}{K_1}\right) \quad (8.8)$$

$$S_n = \frac{P/\delta_0}{s_n} \quad (8.9)$$

where

$(EI)_1$	bending rigidity of the joists
$(EA)_1$	axial rigidity in tension for the joists
$(EI)_2$	bending rigidity of the plywood
$(EA)_2$	axial rigidity in tension for the plywood
q	uniformly distributed load acting on the tributary area of T-beam
L	length of the T-beam or span of joists
L'	Length of plywood between gaps
s_n	spacing between nails
P/δ_0	load/slip ratio
S	joist spacing

K_1 and K_2 can also be written in the form commonly used in Canadian code:

$$K_1 = \frac{bd^3}{12} E_j + B_b S \quad (8.10)$$

$$K_2 = \frac{bd^3}{12} E_j + E_j bd \delta^2 + B_b S \left(\frac{d+t}{2} - \delta \right)^2 \quad (8.11)$$

$$\delta = \frac{d+t}{2} \frac{B_a S}{B_a S + E_j bd} \quad (8.12)$$

$$f_1 = B_a S \delta^2 + B_a S \left(\frac{d+t}{2} - \delta \right)^2 \quad (8.13)$$

where

B_a	axial stiffness in tension for plywood
B_b	bending stiffness of plywood
E_j	modulus of elasticity of joists
b, d	width and depth of joists
t	thickness of plywood

8.1.2 Design Procedures for Medium and Long Span Floors Subject to Uniformly Distributed Load

A long span floor is defined as one with aspect ratio, $\gamma > 1$. To design for a long span floor:

- Calculate the deflection of T-beam, w_b , according to T-beam formula in last section.
- Calculate γ_0 .

$$\gamma = \gamma_0 = \sqrt{\frac{(1 - 27k)\beta_1 + \sqrt{(27k - 1)\beta_1^2 - (4(3k - 1)(243k - 1)\beta_2)}}{2(243k - 1)\beta_2}} \quad (8.14)$$

- If $1 < \gamma < \gamma_0$, then it is termed medium span floor, use T-beam formula. The maximum deflection thus calculated will be adjusted upward by 10%. i.e.

$$w_{pl} = 1.1w_b$$

- If $\gamma = \frac{L}{B} > \gamma_0$, it is termed long span floor, then calculate the deflection using T-beam formula and adjust the result using an adjustment factor:

$$w_{pl} = w_b \alpha_{11}$$

where α_{11} is the first order of the modification factor α

$$\alpha_{11} = \frac{4A}{1 + \beta_1 \gamma^2 + \beta_2 \gamma^4}$$

$$A = \frac{1536}{5\pi^6}$$

$$\beta_1 = \frac{2H}{D_x}$$

$$\beta_2 = \frac{D_y}{D_x}$$

D_x = the bending rigidity of the T-beam D_y = the bending rigidity of the plywood @90° H = the torsional rigidity of the plywood

8.1.3 Design Procedures for Floors Subject to Concentrated Load

In order to design for a wood joist floor subjected to concentrated load, the following procedures can be followed:

- Deflection of floors at any point (x, y)

Figure 8.2: A beam subjects to a concentrated load, P

$$\begin{aligned}
 w(x, y) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \\
 w_{mn} &= \frac{q_{mn}}{D_{mn}} \\
 q_{mn} &= \frac{4P}{LB} \sin m\pi\xi \sin n\pi\eta \\
 D_{mn} &= D_x \left(\frac{m\pi}{L}\right)^4 + 2H \left(\frac{m\pi}{L}\right)^2 \left(\frac{n\pi}{B}\right)^2 + D_y \left(\frac{n\pi}{B}\right)^4 \quad (8.15)
 \end{aligned}$$

- Find the location of the maximum deflection of a beam subjected to concentrated load: x_b , Figure 8.2.

$$x_b = \sqrt{\frac{L^2 - L_2^2}{3}} \quad L_1 > L_2 \quad (8.16)$$

- Calculate the deflection of the floor at point $(x_b, \eta B)$. This deflection will be the approximate maximum deflection.

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin m\pi \frac{x_b}{L} \sin n\pi\eta \quad (8.17)$$

w_{mn} can also be written as

$$w_{mn} = C \frac{\sin m\pi\xi \sin n\pi\eta}{L_{mn}} \quad (8.18)$$

$$\begin{aligned}
 C &= \frac{4PL^3}{BD_x\pi^4} \\
 L_{mn} &= m^4 + \beta_1 m^2 n^2 \gamma^2 + \beta_2 n^4 \gamma^4 \\
 \beta_1 &= \frac{2H}{D_x} \\
 \beta_2 &= \frac{D_y}{D_x} \\
 \gamma &= \frac{L}{B}
 \end{aligned}$$

Where $(\xi L, \eta B)$ is the point of loading, P is the concentrated load. L and B is the span and width of the floor, D_x is the bending rigidity of the T-beam and H and D_y the torsional rigidity and bending rigidity of plywood perpendicular to the joists. The convergence of the series is fast.

8.1.4 Design Procedures for Roofs Subject to Ponding Load

Critical dimensions that define a stable roof under ponding load are as follows:

$$width = B < \pi \left(\frac{D_y}{k} \right)^{\frac{1}{4}}$$

$$span = L < \pi \left(\frac{D_x}{k} \right)^{\frac{1}{4}}$$

where k is the density of fluid, D_y and D_x are rigidity along the width and span respectively.

If any one of the above two conditions is satisfied, the roof is defined as a stable roof under ponding load. The roof will not collapse due to the accumulation of rain water. However, the deflection will be amplified. The amplification factor is defined in Chapter 6.

$$A_{mn} = \frac{1}{1 - \frac{k}{D_{mn}}}$$

The maximum deflection can be calculated according to theories developed in above sections and modified with the amplification factor. D_{mn} is as defined in last section.

8.1.5 Damage Accumulation Model for Bending Strength of Wood Members

Total damage in the volume of bending wood member is defined as

$$\Omega = Ah^{\frac{1}{k_2}} T^{a_3} (L + x_0 k_1)^{\frac{1}{k_1}} \sigma_s$$

The stress of any bending member can be calculated through

$$\frac{\sigma_s}{\sigma_0} = \left(\frac{h_0}{h}\right)^{\frac{1}{k_2}} \left(\frac{T_0}{T}\right)^{a_3} \left(\frac{L_0}{L + k_1 x_0}\right)^{\frac{1}{k_1}} \quad (8.19)$$

where k_1 is the length effect parameter, k_2 depth effect parameter, h depth of joist, T loading time, L length of the beam, x_0 distance between two point loads. The strength σ_s of any bending member with depth h , length L subjected to two concentrated loads with a distance of x_0 from each other with a rate of loading k (T) can be determined in relation to the strength of a standard third point loading test ($x_0 = 0$) with parameters σ_0 , h_0 , L_0 and T_0 .

8.2 Future Work

The topics to be further investigated includes:

- The joists in a wood-joist floor assembly all have different properties such as MOE, modulus of elasticity. The sheathing distributes load from weaker joists to the stronger ones. A formula based on the proposed approach has to be developed to account for the material variability among joists using simulation and reliability analysis.
- The effects of using materials other than plywood as sheathing are yet to be examined.
- Effects of other boundary conditions such as fixed on both sides should be studied in detail.

- Reliability based design formula should be derived that bases on the the design procedures proposed in this thesis.

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¹Not all of the literatures listed here have been referenced directly in the thesis. They are the ones that I came across during the research and relevant to the topic of study. I listed them here hoping that they will be useful to the future study.

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Appendix A

Symbols

α constant, mm^{-1}

α ratio of w_{pl}/w_b

α damage accumulated at a point in lumber, per unit volume

α_0 initial damage accumulated at a point in lumber, per unit volume

β_1 $2H/D_x$

β_2 D_y/D_x

Δ maximum deflection of a T-beam, mm

Δ_1 maximum deflection of a T-beam with loose connection, mm

δ distance between the neutral axis of a T-beam and the centerline of the joists, mm

$(\xi \text{ L}, \eta \text{ B})$ the loading point of a concentrated load

μ_1, μ_2 Poisson's ratios

γ aspect ratio, l/B

σ stress state at a point, N/mm^2

$\sigma(t)$ stress state at a given time, N/mm^2

σ_s short term strength, N/mm^2

σ_0 threshold stress ratio

Φ performance factor, LRFD

Ω total damage accumulated in lumber

A constant

A_{mn} constant in a Fourier series, mm

A_{ik} constant in a Fourier series, mm

A_1 area of cross-section of joists, mm^2

A_2 area of the flange of the T-beam, mm^2

L span of the floor in the direction of the joists, mm

a model parameter

B_s tensile rigidity of plywood, $N \cdot mm/mm$

B_b bending rigidity of plywood, $N \cdot mm^2/mm$

B_{mni} constant, N/mm^3

B width of the floor, mm

b model parameter

b width of joists

C a constant, mm

c model parameter

D_{mn} constant, N/mm^3

D_x bending stiffness of a floor in the direction of joists, $N \cdot mm$

D_y bending stiffness of a floor in the direction perpendicular to joists. $N \text{ mm}$

d height of web of T-beam or a joist. mm

d_y width of supports. mm

d_1, d_2 depths of beams. mm

E modulus of elasticity. N/mm^2

E_1 modulus of elasticity of the joists. N/mm^2

E_2 modulus of elasticity of the plywood in the direction of joists. N/mm^2

$(EI)_e$ effective bending stiffness of a T-beam. $N \text{ mm}^2$

$(EI)_b$ bending stiffness of a joist. $N \text{ mm}^2$

f_v adhesive factor, difference between K_2 and K_1 . $N \text{ mm}^2$

f_{max} maximum deflection of a T-beam. mm

G shear modulus. N/mm^2

H torsional rigidity of floor. $N \text{ mm}$

K rate of loading related to stress

K_1 effective rigidity of a loose floor, N/mm^2

K_2 effective rigidity of a glued floor, N/mm^2

k water density, N/mm^3

k constant representing percentage of errors permitted

k_1 length effect factor

k_2 depth effect factor

k_0 rate of loading related to the concentrated load

L' length of plywood between gaps, mm

L_e equivalent length of beam subjected to two concentrated loads, mm

n model parameter

P/δ load/slip ratio of nails, N/mm

P concentrated load, N

$p(x, y)$ load on roofs including ponding, N/mm^2

$Q(x, y)$ normal load acting on the floor, N/mm^2

$Q_j(x, y)$ reaction from the joists, N/mm

Q_{mn} constant, N/mm^2

$q(x, y)$ load acting on the floor, N/mm^2

q_{mn} the (m,n)th term of the Fourier series expanded from $q(x, y)$, N/mm^2

q_0 uniformly distributed load, N/mm^2

S joist spacing, mm

s_n spacing between nails, mm

T time needed for lumber to fail

t thickness of plywood, mm

v volume of beams, mm^3

$w(x, y)$ deflection of floor, mm

w_{bmax} maximum deflection of a beam subjected to uniformly distributed load, *mm*

w_b deflection of a beam, *mm*

w_{pl} deflection of a plate, *mm*

x_0 distance between two concentrated load acting on a beam, *mm*

x_1, x_2 strength of bending members of length L_1, L_2 respectively, *mm*

x coordinate in the direction of joists, *mm*

y coordinate in the direction perpendicular to joists, *mm*

y_0 distance between the neutral axis of the T-beam and bottom of the web, *mm*

y_j position of the j th joist, *mm*

END

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FIN