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MODELLING OF INSULATED-GATE FIELD-EFFECT TRANSISTORS

by

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ABSTRACT

Existing models describing the operation of Insulated-Gate Field-Effect Transistors are briefly reviewed and their limitations and shortcomings are discussed, establishing the need for a simple accurate model of the device. Two approaches for modelling both the "channel region" and the "saturation region" of device operation are introduced. Using both approaches a close correlation is obtained between calculated and measured device characteristics.

In the 'channel region', a simple 'model' for the mobile carrier distribution in the space charge region characterized by the first moment of charge is used. By this means, the potential developed across the channel, which may for high gate voltages (due to quantum mechanical effects) constitute a major part of the surface potential is represented. Simple expressions for various device characteristics are obtained and are able to account for many of the shortcomings of previous models.

To model the saturation region, an approximate two-dimensional model of the 'pinched-off region' is presented. Used in combination with the one-dimensional model, this model enables device operation to be characterized under all operating conditions and for any device geometry.

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TABLE OF CONTENTS

		page
CHAPTER 1	INTRODUCTION	1
CHAPTER 2	THE DEVICE STRUCTURE AND THE DESCRIBING EQUATIONS	6
CHAPTER 3	EXISTING SOLUTIONS	12
CHAPTER 4	PROPOSED MODEL FOR THE CHANNEL REGION	21
4.1	Introduction	21
4.2	Analysis	31
4.2.1	Solution for the Electrostatic Potential	34
4.2.2	Drain Current and Other Device Parameters	41
4.2.3	Device Electrostatic Parameters	45
4.2.4	Discussion	47
CHAPTER 5	THEORETICAL AND EXPERIMENTAL RESULTS	50
5.1	Theoretical Results	50
5.2	Comparison Between Theory and Experiment	54
5.3	Concluding Remarks	59

	page
CHAPTER 6 A TWO DIMENSIONAL MODEL IN THE CURRENT SATURATION RANGE	70
6.1 Introduction	70
6.2 Existing Models	71
6.3 The Proposed Model for the Drain Section	77
6.4 A Simplified Model	89
6.5 Drain Breakdown Voltage	93
CHAPTER 7 CONCLUSIONS AND FUTURE WORK	103
7.1 Conclusions	103
7.2 Model Applications and Future Work	105
APPENDIX A THE SPACE CHARGE REGION THICKNESS	109
APPENDIX B DEFINITION OF THE EFFECTIVE POTENTIAL ϕ	119
APPENDIX C DEFINITION OF THE BOUNDARY BETWEEN SOURCE AND DRAIN SECTIONS	123
APPENDIX D DERIVATION OF THE CHANNEL-SHORTENING MODEL EQUATION FROM THE PRESENT MODEL	125
APPENDIX E THE RELATION BETWEEN V_g' and U_g	127
REFERENCES	129

LIST OF FIGURES

<u>Figure</u>		<u>page</u>
2.1	Cross sections of the device and potentials	7
4.1	Device terminal parameters vs. gate voltage	22
4.2	Device characteristics using variable mobility	24
4.3	Mobile charge density as a function of distance normal to the surface	26
4.4	Charge density distribution in the space charge region	29
4.5	Device structure and potential distribution normal to the surface	33
4.6	Variation of the surface longitudinal field with distance along the channel for a device operating just in saturation	48
5.1	Calculated device parameters vs. gate voltage using the present theory	51
5.2	Calculated device parameters vs. drain voltage using the present theory	53
5.3	Comparison between theoretical and experi- mental gate capacitance characteristics	64
5.4	Comparison between theoretical and experi- mental gate transconductance characteristics	66
5.5	Comparison between theoretical and experi- mental drain conductance characteristics	69

LIST OF FIGURES (CONTINUED)

<u>Figure</u>		<u>Page</u>
6.1	The drain section and its boundaries	78
6.2	Comparison between measured and calculated g_d-V_d characteristics	96
6.3	Comparison between theory and measurements	98
6.4	Comparison of the simplified model and the complete model	101
6.5	I_d-V_d characteristics including breakdown	102
7.1	Element of a general surface device	107
A.1	Channel thickness obtained from quantum and classical statistics.	110
A.2	Variation of the total induced charge as a function of surface potential	112
A.3	Distributions obtained for a spike function, channel and a channel with finite thickness	117
B.1	Conditions at the source junction boundary	122

LIST OF TABLES

<u>Table</u>		<u>page</u>
5.1(a)	Device data for metal gate n-channel devices	61
5.1(b)	Device data for silicon gate p-channel devices	62
6.1(a)	Metal gate device parameters	95
6.1(b)	Silicon gate device parameters	95

LIST OF PRINCIPAL SYMBOLS

It should be noted that unnormalized voltages are represented by upper case V's while for normalized voltages upper case U's are employed. Normalized voltages are in thermal volts (i.e., normalization constant is $\frac{kT}{q}$).

a	$\frac{C_i}{C_D} e^{-U_f/2}$, a constant
C_D	ϵ_s/L_D , Debye Capacitance
C_i	insulator capacitance per unit area
C_{gs}	gate capacitance per unit area
E_{sl}	surface longitudinal electric field at the boundary between source and drain sections
E_{sd}	surface longitudinal electric field at the drain
E_{xs}	electric field normal to the surface
E_{ys}	longitudinal electric field at the surface
D_n	diffusion coefficient of electrons
g_d	drain conductance
g_m	gate transconductance

I_1	drain current at the onset of saturation
I_d	drain current
J_n	electron current density
J_p	hole current density
k	Boltzmann's constant
L	Channel length
L_D	$(\frac{kT\epsilon_s}{2q^2 n_i})^{1/2}$, intrinsic Debye length
ΔL	length of the drain section of the space charge region
n_i	intrinsic carrier concentration (1.45×10^{10} for silicon at room temperature)
n	electron concentration
N	net doping in the substrate
p	hole concentration
q	electronic charge
Q_b	ionized charge in the space charge region per unit surface area
Q_g	gate charge per unit surface area
Q_G	total gate charge
Q_m	mobile charge in the space charge region per unit surface area

Q_{ss}	sheet of charge at semiconductor-insulator interface equivalent to the insulator and surface states charges
t	time
T	absolute temperature
U	normalized electrostatic potential.
U_b	normalized substrate potential
U_f	normalized Fermi potential of the substrate
U_g	normalized effective gate potential
U_{gg}	normalized applied gate potential
U_n	electron quasi-Fermi potential
U_p	hole quasi-Fermi potential
U_s	normalized potential across the space charge region
U_{sl}	value of U_s at the boundary between source and drain sections
U_{sd}	value of U_s at the drain
U_{sp}	value of U_s for zero mobile charges in the source section
U_{sr}	normalized source potential
V_{BD}	drain breakdown potential

V_d	applied drain potential
V_g	effective gate potential
V_{gg}	applied gate potential
v_o	scattering limited velocity of mobile carriers
x	distance normal to the surface measured from the silicon-insulator interface
x_D	thickness of space charge region from depletion approximation
x_i	insulator thickness
x_s	thickness of space charge region
x_l	value of x_s at boundary between source and drain sections
y	distance along the channel measured from the source junction
z	width of the gate
n	$a(U_g - U_s - U_b)$
ζ	separation of quasi-Fermi levels in chapter 3 and effective potential in the space charge region in chapter 4
μ	effective mobility
μ_n	effective mobility of electrons

ρ

charge density in the semiconductor
space charge region

ρ_i

charge density in the insulator

ϵ_s, ϵ_i

permittivities of silicon and insulator
respectively.

CHAPTER 1

INTRODUCTION

Insulated Gate Field Effect Transistors (IGFET's), which are based on the principle of modulating a longitudinal electric conductance by a transverse electric field, were first proposed about forty years ago^[1,2]. However, it was not until three decades after the initial proposal, that the commercial feasibility of these devices was realized^[3]. Since then, extensive research has been done both on the theoretical and technological aspects of the devices^[4,5]. One area that has been investigated in detail is the physical modelling of the devices, viz., relating their terminal parameters to their physical structure. These investigations led to numerous models ranging from simplified ones that do not predict the behaviour of the devices over a wide range of operation to detailed, though complicated, numerical solutions of the physical equations.

Simple models are essentially one-dimensional solutions and are based on the "Gradual Channel Approximation (GCA)" introduced by Shockley^[11]. The range of applicability of such models is restricted to the range of validity of the GCA; i.e., where the

device behaviour can be described by a one-dimensional Poisson's equation. Even within this range, existing models fail to explain the experimentally observed behaviour of certain device parameters.

Numerical solutions are able to encompass a very wide range of operating conditions since these are two-dimensional solutions. However, owing to their complexity and cost, they are impractical for use in many applications. In particular, they are not suitable for implementation in computer circuit analysis programs or for use in circuit design; further they fail to represent certain observed properties of the device.

The purpose of this investigation is, therefore, to develop a model for characterizing IGFET's which is capable of predicting their behaviour accurately, over a wide range of operating conditions, but which is still simple enough for use as a circuit design tool, in process evaluation and in computer circuit analysis programs. If such a model could be developed in a very general manner from basic physical considerations, then the approach could be extended to include more complicated structures than the basic IGFET and other surface devices such as the charge coupled devices.

The difficulty of the modelling problem arises from a combination of several factors; the most important of these are:

- i) The field distribution in the surface space charge region is controlled by the potentials applied to four electrodes. These electrodes lie on the sides of a rectangle; the drain and source

diffusions being parts of two parallel sides while the gate and substrate form the other two sides. The interaction between these two perpendicular sets of electrodes results in a two-dimensional field distribution in that region necessitating the use of the two-dimensional Poisson's equation.

ii) The operation of the device is based on creating a potential well at the surface below the gate electrode where the mobile carrier concentration in this well is modulated by the potentials applied to the gate or substrate electrodes. For normal gate voltages this concentration can become easily degenerate requiring the use of Fermi-Dirac statistics in evaluation of mobile carrier density dependence on potential.

Therefore, an exact solution of the problem would involve the solution of a two dimensional Poisson's equation where the complex Fermi-Dirac statistics are used to evaluate charge density distribution. Since the boundary values are not defined to start with (because of the quantum statistics), it is impossible to obtain an analytical solution let alone a closed form solution (which is the purpose of the present investigation).

The approach we shall use to this modelling problem is to incorporate what is already known about the effects of these factors, making approximations so as to reduce as far as possible the complexity of the analysis while yet retaining the representation of the essential physical nature of the device.

In Chapter 2, the physical equations describing the operation of the IGFET structure under consideration are formulated. These equations are used as the starting point in Chapter 3 where various approximations, made by previous workers, are discussed together with the resulting solutions. The shortcomings of these solutions are pointed out.

In Chapter 4, a new approach in which an approximate model, for the mobile carrier distribution in the space charge region, characterized by the first moment of charge, is used to account for the potential developed across the channel. In this chapter the two-dimensional problem is reduced to a one-dimensional one by using the GCA. For short channel devices, the resulting model applied to a limited range of applied voltages, the limit here is dictated by the failure of the one-dimensional model to describe device operation. Essentially, it is a model of the "channel region" of the device, over the region for which the GCA is reasonably valid. For a long channel device, the model is capable of describing device operation under all operating conditions. Device characteristics based on this model are compared with experimental results in Chapter 5 showing excellent agreement.

An approximate two-dimensional model of the "pinched-off region" of the device is presented in Chapter 6 to compliment the "channel region" model derived in Chapter 4. In the region modelled, the GCA does not apply and it is important that the essentially two-

dimensional nature of the physical relationships be preserved - though in a sufficiently simple form to fulfil the objective of the thesis. Used in combination with the one-dimensional model (of Chapter 4) this model enables device operation to be characterized under all operating conditions, and for any device geometry (specifically, including short-channel devices). Comparison between this model and experiments is shown at the end of the chapter, again with excellent agreement.

In Chapter 7 conclusions drawn from the two models are presented. Model applications and possible extension of the approach to model other structures such as charge coupled devices are discussed.

CHAPTER 2

DEVICE STRUCTURE AND THE DESCRIBING EQUATIONS

In this chapter the device structure and potential distributions within the space charge region are sketched. The device describing equations are then presented with appropriate symbol notation defined. Fig. 2.1 shows cross-sections of an n-channel device* along the channel and normal to it, together with potential distributions in the different regions. The Cartesian system of coordinates is used with its origin at the intersection of the source metallurgical junction with the semiconductor surface. All potentials are normalized by the thermal voltage, kT/q . In these potential plots, the source and the substrate electrodes are grounded.[†] Applied gate and drain potentials are positive with respect to the source. The neutral substrate, viz. the region between the substrate electrode and the space charge region boundary is assumed to be an equipotential. The

* All analysis in the thesis is carried out for an n-channel device.

[†] Note, in the later analysis of Chapter 4, the general situation, where gate, source, drain and substrate potentials are allowed to take any values, is treated.

Figure 2.1 Cross sections of the device and potentials
 (a) along the channel
 (b) normal to the channel

electrostatic potential in the neutral substrate is taken as the reference potential in the device. The Quasi Fermi levels are assumed constant across the source and drain junctions where the separation of these levels across the junction are the external applied voltage (p-n junction theory). Thus the separation between the quasi-Fermi levels in the drain diffusion and the neutral substrate is the applied drain potential while the separation between the gate and the substrate Fermi levels is the applied gate potential. The direction of the drain current is flowing into the drain electrode which is positive for an n-channel device.

The notations used in this Figure are as follows:

- U = the normalized electrostatic potential
- U_s = the normalized electrostatic potential at the semiconductor-insulator interface
- U_p, U_n = the normalized quasi-Fermi potentials for holes and electrons respectively
- ζ = the separation between normalized quasi-Fermi levels of holes and electrons at the interface
- U_{gg}, U_d = the normalized gate and drain potentials, respectively
- U_f, U_{fs}, U_{fd} = the normalized Fermi levels of substrate, source and drain, respectively
- x_i = the insulator thickness
- L = the channel length

I_d = the drain current

The characteristics of the device are determined, mainly, by the behaviour of the space charge region at the semiconductor surface in response to externally applied voltages. Usually, it is assumed that the diffused source and drain islands form one-sided step junctions with the substrate, thus fixing two of the boundaries of the space charge region independent of the applied voltages. The other two boundaries are the semiconductor insulator interface, which is fixed, and the edge of the space charge region in the substrate which is a function of the applied voltages among other parameters. Once the spatial distributions of mobile carriers and electrostatic potential are known within that region, other device terminal parameters such as drain current can be easily obtained. These distributions are uniquely determined by the solution of the following fundamental semiconductor equations*:

i) Poisson's equation in the semiconductor space

charge region

$$\nabla^2 U = - \frac{q^2}{kT\epsilon_s} (p-n-N) \quad (2.1)$$

ii) Current continuity equations in the space

charge region

$$\nabla \cdot J_n = -R$$

$$\nabla \cdot J_p = R$$

* Symbols as yet/undefined follow conventional usage, as specified at the end of the chapter.

where

$$J_n = -qD_n n \nabla U_n$$

$$J_p = -qD_p p \nabla U_p \quad (2.2)$$

iii) Poisson's equation in the insulator region

$$\nabla^2 U = - \frac{q}{kT\epsilon_i} \rho_i$$

The boundary conditions for solving these equations are:

i) At the semiconductor-insulator interface

$$a) U_i = U_s \quad (\text{continuity of potential})$$

$$b) \frac{\epsilon_i}{\epsilon_s} \frac{\partial U}{\partial x} \Big|_i = \frac{\partial U}{\partial x} \Big|_s + \frac{q}{kT\epsilon_s} Q_{ss} \quad (\text{Gauss' law})$$

$$c) \frac{\partial U_n}{\partial x} = \frac{\partial U_p}{\partial x} = 0 \quad (\text{zero normal current})$$

where Q_{ss} is the interface charge density equivalent to the oxide and semiconductor surface states charge.

ii) At the source boundary

$$U_n - U_p = 0 \quad (\text{grounded source})$$

iii) At the drain boundary

$$U_n - U_p = U_d$$

iv) At the gate boundary

$$U = U_{gg}$$

where

p, n = the carrier densities of holes and electrons, respectively

$N = N_A - N_D$ = the net doping density in the substrate

J_p, J_n = the current densities of holes and electrons, respectively.

D_p, D_n = the diffusion coefficients of holes and electrons, respectively.

R = the net generation-recombination rate

ϵ_s, ϵ_i = the permittivities of semiconductor and insulator, respectively.

ρ_i = the insulator charge density distribution

This system of partial differential equations is very difficult to solve. Even if numerical techniques are used, some simplifications have to be made. To get analytical solutions, more assumptions are made. Numerical and analytical solutions of previous workers, together with the simplifying assumptions involved, are discussed briefly in the next chapter.

CHAPTER 3

EXISTING SOLUTIONS

Starting with equations formulated in the previous chapter, existing solutions, together with the approximations involved in getting them, are discussed in this chapter. We start by presenting the more complete solutions, then proceed to simpler models. Of course, as more approximations are made, the simpler but less accurate models we get.

Numerical solutions^[6-10] are especially important in characterizing short channel devices where two-dimensional effects are significant. Most of these solutions make the following assumptions^[10]:

- i) Boltzmann statistics are used to express the relation between mobile carrier concentration and potentials,

i.e.,

$$n = n_i \exp(U - U_n)$$

$$p = n_i \exp(U_p - U) \quad (3.1)$$

where

n_i = the intrinsic carrier concentration

- ii) Current flow in the channel is due only to minority carriers, i.e., for n-channel devices, $J_p = 0$.
- iii) Net generation-recombination of mobile carriers in the space charge region is neglected, $R = 0$.
- iv) The insulator charge is lumped into a thin sheet at the semiconductor-insulator interface, thus giving an insulator free of charge where Laplace equation can be used.
- v) Mobility is either assumed constant or some empirical forms describing its electric field dependence in a qualitative manner are used.

Standard techniques are then used for solving this boundary value problem, e.g., by transforming partial differential equations into difference equations and then using relaxation methods to solve these questions. These solutions can predict device behaviour over a wide range of operation, but, because of their complexity, they are unsuitable for use in many applications such as device modelling in computer-aided design and analysis, small signal equivalent circuit representation and evaluation of effects of processing parameters on device characteristics.

In order to simplify analysis, more assumptions are added to those mentioned above. The most important is the so-called "Gradual Channel Approximation (GCA)"^[11] in which the charge in the space charge region is assumed to be totally due to the field

gradient normal to the semiconductor surface, thus making (2.1) a one-dimensional equation:

$$\frac{d^2 U}{dx^2} = - \frac{q^2 n_i}{kT \epsilon_s} \left[e^{\frac{U_p - U}{kT}} - e^{\frac{U - U_n}{kT}} + e^{\frac{-U_f}{kT}} - e^{\frac{U_f}{kT}} \right] \quad (3.2)$$

where the expressions for p and n given in Eqn. (3.1) are used.

Furthermore, it is assumed that the quasi-Fermi level of majority carriers (holes) U_p is constant throughout the substrate while quasi-Fermi level U_n of the minority carriers (electrons) is constant in the direction normal to the surface (i.e., no majority carrier current anywhere in the substrate, and no minority carrier current normal to the surface). Eqn. (3.2) can then be integrated in the direction normal to the surface, and when the fields in the insulator and the semiconductor are matched at the interface (boundary condition (i)), we get:

$$\begin{aligned} U_g &= U_{gg} - \frac{q}{kT} \phi_{MS} + \frac{q}{kT} \frac{Q_{ss}}{C_i} \\ &= U_s + \frac{C_D}{C_i} F(U_s, \zeta, U_f) \end{aligned} \quad (3.3)$$

and the function F is given as:

$$F(U, \zeta, U_f) = \left[e^{\frac{U - \zeta - U_f}{kT}} + e^{\frac{U_f - U}{kT}} + (U - 1)e^{\frac{U_f}{kT}} - (U + e^{-\zeta})e^{\frac{-U_f}{kT}} \right]^{\frac{1}{2}} \quad (3.4)$$

where

U_g = the effective gate potential with respect to substrate.

ϕ_{MS} = the metal-semiconductor work function difference

Q_{ss} = the sheet of charge at the interface equivalent to the insulator and the surface state charges

C_i = the insulator capacitance per unit area

$C_D = \epsilon_s / L_D$

$L_D = \left[\frac{kT\epsilon_s}{2q^2 n_i} \right]^{1/2} = \text{the intrinsic Debye length}$

The drain current is then obtained by integrating the current density over the cross-sectional area of the channel [12]

$$I_d = \int_0^{\infty} J_n(x,y) Z dx$$

where Z is the channel width.

Substituting for J_n from Eqn. (2.2), and using the approximations mentioned above, we get

$$I_d = \left(\frac{kT}{q} \right)^2 \left(\frac{Z}{2L} \right) \cdot \mu_n \frac{C_i}{C_D} \int_0^{U_d} \int_0^{U_s} \frac{e^{U-\zeta-U_f}}{F(U,\zeta,U_f)} dU d\zeta \quad (4.3)$$

where

μ_n = the electron mobility and is related to D_n by the Einstein relationship $D_n = (kT/q)\mu_n$

L = the channel length, viz., distance between source and drain metallurgical junctions.

Numerical integration is used in evaluating this equation with the limits obtained by numerically solving Eqn. (3.3). Other device parameters such as conductances or gate capacitance can be obtained in the same manner in the form of integrations^[12].

Closed form solutions for the drain current and other device parameters are possible if Poisson's equation and the continuity equation are decoupled. This was done by assuming the mobile charges forming the channel to be localized in an infinitely thin layer at the semiconductor surface^[13-16]. Thus, except at the surface, the space charge region becomes completely depleted of mobile carriers and Eqn. (3.2) becomes:

$$\frac{d^2U}{dx^2} = \frac{q^2 n_i}{kT\epsilon_s} e^{U_f} \quad (3.5)$$

where, in addition to neglecting mobile carriers, donor density is also neglected.

Integrating Eqn. (3.5) twice from the surface to the neutral substrate, we get:

$$U = U_s + \frac{q^2 n_i e^{U_f}}{kT\epsilon_s} \left(\frac{x^2}{2} - xx_D \right) \quad (3.6)$$

where

$$x_D = \left[\frac{2kT\epsilon_s}{q^2 n_i} e^{-U_f/U_s} \right]^{1/2} \quad (3.7)$$

= the space charge region width

The mobile charge density per unit surface area, Q_m , is then obtained by applying Gauss's law to the semiconductor space charge region. The field normal to the semiconductor surface in the insulator, E_i , is given by:

$$E_i = \left(\frac{U_g - U_s}{x_i} \right) \frac{kT}{q}$$

Thus,

$$\epsilon_i E_i = \epsilon_i \frac{kT}{q} \left(\frac{U_g - U_s}{x_i} \right) = -(Q_b + Q_m)$$

where Q_b is the substrate ionized charge per unit surface area.

Therefore,

$$\begin{aligned} -Q_m &= \epsilon_i \frac{kT}{q} \left(\frac{U_g - U_s}{x_i} \right) - q N x_D \\ &= \epsilon_i \frac{kT}{q} \left(\frac{U_g - U_s}{x_i} \right) - q n_i e^{U_f} \left(\frac{2kT\epsilon_s}{q^2 n_i} e^{-U_f/U_s} \right)^{1/2} \end{aligned} \quad (3.8)$$

and

$$I_d = -Z \frac{kT}{q} \mu_n Q_m \frac{d\zeta}{dy} \quad (3.9)$$

A further assumption is made here that the current flow is by drift alone, i.e. $d\zeta/dy = dU_s/dy$, which means that the separation of the quasi-Fermi levels (the channel potential ζ) is linearly related to the surface potential U_s . This linear relation is assumed to be:

$$U_s = \zeta + 2U_f \quad (3.10)$$

In addition to neglecting the diffusion current, Eqn. (3.10) implies two assumptions:

- i) no current conduction starts until the mobile charge concentration at the surface is equal to majority carrier concentration in the substrate, which underestimates the drain current for small gate voltages.
- ii) for zero drain potential, the space charge region width at any point along the channel remains constant after inversion and is not affected by any further increase of gate voltage.

The drain current is then obtained by integrating Eqn. (3.9) from the source to the drain :

$$I_d = \frac{\mu_n C_i Z}{L} \left(\frac{kT}{q}\right)^2 \left[U_g U_d - \frac{(U_d + 2U_f)^2}{2} - \frac{2}{3} \left\{ \frac{2q^2 n_i e^{U_f}}{kT C_i^2} \right\}^{1/2} \right. \\ \left. \left\{ (U_d + 2U_f)^{3/2} - (2U_f)^{3/2} \right\} \right] \quad (3.11)$$

Device parameters such as conductances g_d and g_m can readily be obtained from this equation. This solution is only valid up to the saturation point which is determined by putting $Q_m=0$ at the drain end of the channel. Beyond saturation, the drain current is either assumed constant and equal to the saturation point value or some approximate representation of "channel shortening" is used^[15,17-19] to account for the finite incremental conductance observed experimentally in that region of operation. Solutions in the saturation region of operation together with the approximations involved are discussed in Chapter 6.

To conclude this chapter, we mention briefly the main approximations made and their implications:

i) To get simple or closed form solutions, Poisson's equation has to be reduced to a one-dimensional equation by using the Gradual Channel Approximation, GCA. This has the effect of limiting the resulting solutions to the region of validity of the GCA, namely the nonsaturation region of device operation.

ii) To account for deviations of device drain conductance and gate transconductance from experiments, empirical forms of surface mobility dependence on potential have been used. The verification of these forms by experiments is done through measurements of device terminal parameters. Since there is no independent check on mobility, it is questionable whether mobility variation is the real cause behind these deviations.

iii) In all solutions discussed, classical statistics, rather than the quantum statistics, have been used to express the dependence of mobile carriers on potential. This approximation allows for the mobile carriers to be localized within a very thin layer at the semiconductor surface (the concentration within this layer can become degenerate) resulting in inaccurate formulation of surface potential dependence on charges in the space charge region.

A thorough understanding of the way in which these approximations fail to represent the actual device is a necessary and basic step in any successful modelling attempt. This we do in the next chapter leading to a new model for the mobile carrier distribution in the channel which is the heart of the present work.

CHAPTER 4

THE PROPOSED MODEL FOR THE CHANNEL REGION

4.1 Introduction

All one-dimensional solutions, i.e., solutions using the Gradual Channel Approximation (GCA)*, discussed so far, show significant disagreements with the device behaviour in the different regions of operation. Some of these discrepancies are due to the GCA itself: namely, in the saturation region of operation where this approximation is questionable (in that region $\frac{\partial^2 U}{\partial y^2} / \frac{\partial^2 U}{\partial x^2}$ could become very large). In regions where the GCA is a valid approximation, i.e., below saturation, it was expected that such solutions would predict the behaviour of the different device parameters correctly. This is not the case, however, and even in the most complete one-dimensional solution given by Pao and Sah^[12], significant discrepancies are observed in heavy inversion regions, the most important of which are in the behaviour of gate and drain conductances at high gate voltages, see Fig. 4.1. It is seen from the figure (i.e., in (a)) that measured gate transconductance g_m below

* The GCA reduces the two-dimensional Poisson's equation to a one dimensional equation by neglecting the longitudinal field gradient along the channel i.e.

$$\frac{\partial^2 U}{\partial y^2} / \frac{\partial^2 U}{\partial x^2} \rightarrow 0$$

Figure 4.1 Device terminal parameters vs. gate voltage (a) and (c) after Pao and Sah^[12], (b) after Sah^[16]

saturation decreases with increasing gate voltage while the theory predicts a constant value. Also in (b) for the drain conductance g_d , the theory shows a straight line behaviour as a function of v_g while the measured values exhibit a deviation from that straight line. For the capacitance vs. gate voltage, the Fig. (c) shows poor fitting between theory and experiment. All these discrepancies occur in the region where the GCA is valid, strongly suggesting that they are not due to this approximation but have other causes.

Some authors have tried to account for the difference between theory and experiment in the nonsaturation region of operation by, arbitrarily, varying the inversion region carrier mobility with surface field and inversion charge so that the calculated parameters fit the experimental results^[16,20-21]. Since there is no independent check on mobility (in fact the mobility is obtained through terminal conductance measurements) this approach is, therefore, an arbitrary way of explaining the behaviour of a specific experimental device parameter rather than a general modelling of the device behaviour. In other words, this method can account for the deviation of one parameter (e.g., g_d) from experimental results but it is incapable of explaining the discrepancies of other parameters (e.g. g_m and C_{gs}). Figure 4.2 is a good illustration of this method. In this figure a mobility that varies with gate voltage is chosen

(a) $g_d - V_{gg}$ characteristics showing the values of μ for an exact fit between theory and experiment

(b) $g_m - V_{gg}$ characteristics where the same values of μ in (a) are used to get the theoretical curves

Figure 4.2 Device characteristics using variable mobility.
(Sah and Pao^[16])

to achieve fitting between theoretical and experimental $g_d - v_g$ characteristics. However, using the same values of mobility in the $g_m - v_g$ characteristic does not show much improvement. Thus, besides the lack of rigourousness in this approach, we can see that its usefulness is rather limited in that the resulting mobility variation applies only to a specific device parameter.

The validity of using Boltzmann statistics for the purpose of evaluating the dependence of mobile carrier concentration on potentials has been investigated by many people^[22-25]. It has been shown^[24] that even for moderate gate voltages, the mobile carrier concentration near the surface of the semiconductor can become easily degenerate, and that Fermi-Dirac statistics should be used for proper formulation of space charge region parameters. Gnadinger and Talley^[25] calculated the mobile carrier distribution in the space charge region, taking into consideration quantization of allowed energy levels and using Fermi-Dirac statistics. For their assumed linear electrostatic potential in the space charge region, the mobile carrier distribution obtained from that quantum solution deviates considerably from that using classical statistics particularly for strong inversion - see Fig. 4.3. The quantum solution gives a lower value of the peak carrier concentration near the surface, and the distribution of mobile carriers is spread out more deeply into the space charge region. It is believed that

Figure 4.3 Mobile charge density as a function of distance normal to the surface (Gnadinger and Talley^[25])
 (a) weak inversion
 (b) strong inversion

these properties are responsible for the discrepancies between observed device properties and the predictions of theoretical analysis based on simple Boltzmann statistics.

A successful solution must, therefore, incorporate in some way the consequences of true quantum statistics in strong inversion regions. However, the complexity of analysis in terms of these statistics is prohibitive (the desirable objective here is, in fact, to obtain simple and preferably closed form solutions). The approach which we shall adopt is to include these factors indirectly and approximately by exploiting what is already known about their effects, with the aim of getting closed form solutions.

The theory developed here is in terms of an approximate form of distribution for mobile carriers in the inversion region, with an assumed constant mobility; it is worth noting at the outset that all features of the device characteristics discussed above are accurately predicted by the theory without any need to depart from the constant mobility assumption initially postulated. The theory is the outcome of the following considerations:

- i) potential distribution in the semiconductor space charge region to be as simple a function as is capable of satisfying the known physical requirements; the very least of these requirements is that the potential function satisfies the surface boundary conditions of

potential and normal electric field in accordance with Gauss's theorem (corresponding to the total charge in the semiconductor).

- ii) mobile carrier distribution in the inversion layer to be an appropriate (and preferably simple) approximation to that given by Fermi-Dirac statistics, and necessarily much more uniform with distance into the semiconductor than that of assumed Maxwell-Boltzmann statistics.

As a starting point, the assumption of a constant mobile carrier density in the inversion region, as illustrated in Fig. 4.4(a) suggests itself as a reasonable choice far more representative of the true situation than the distribution of M-B statistics, also illustrated. Such a distribution, as a model, exhibits the essential features of a deeper and more uniform spread below the surface, together with lower peak concentration compared with the M-B distribution; it also has the virtue of simplicity.

The assumption of a constant mobile carrier density in the inversion region results in a parabolic potential distribution, since the density of ionized impurity charge is also constant. A possible 'model' of the total charge density distribution in the space charge region of the semiconductor would thus appear to be as curve (A) in Figure 4.4(b), for which the potential would consist of

Figure 4.4 • Charge density distribution in the space charge region

parabolic segments in both regions (i) and (ii). If now this charge distribution (A) is approximated by a uniform charge density distribution as indicated by curve (B), then for that latter distribution to be representative of the actual device, it must satisfy the surface boundary conditions of Gauss's theorem, namely surface field and surface potential. It is shown in the next section that by satisfying these conditions, the resulting thickness of distribution (B) is related to the first moment of charge of the actual device.

Now let us examine the conditions for which this equivalent distribution (B) is a good representation of the potential-charge relationship of the actual device. To do that, two extreme cases represented by Figs. 4.3(c) and 4.3(d) are considered, where the equivalent distribution (B) can be identified with either the ionized or mobile components of charge respectively. Below cut-off or for the light inversion regime, the mobile carriers are negligible compared to the ionized charges and the electrostatic potential distribution would result mainly from the ionized charges. The space charge region thickness, in this case, is obtained with good accuracy from the depletion approximation. In strong inversion regions, the usual practice is to calculate the space charge region thickness based on the depletion approximation with the mobile carriers assumed to be a 'spike function' at the surface (or a classical distribution of mobile carriers is considered). This results in a zero (or very small) thickness of the inversion region, and therefore a zero (or negligible) potential across the channel. It is shown in appendix A that, by taking

into consideration a uniform spatial distribution of mobile charges as an approximate representation of the quantum-mechanical channel, the resulting potential drop across the channel is non-negligible. In fact, under strong inversion conditions, most of the surface potential is developed across the channel, and the contribution of the ionized charge to that potential has to decrease in order to satisfy Gauss's law with realistic modible carrier densities and a reasonable channel thickness. In strong inversion, therefore, the equivalent distribution and the resulting potential variation are mainly due to mobile charges. The transition from weak inversion to strong inversion is the region where the equivalent distribution cannot be identified with either of the charge components and as a result small errors would occur, in that ionized charge would be somewhat underestimated in that transition region. Thus under almost all conditions of operation the assumption of a uniform distribution of charge would appear to be a reasonable representation of the conditions in the space charge region; the origin of the distribution being the dominant charge. This is the basis of the present approach to device analysis. It is justified by the results obtained, the agreement between the predictions of the theory and the measured device parameters being much closer than for previous theories.

4.2 Analysis

In the following analysis, the general four terminal IGFET will be considered where voltages are applied simultaneously to the source, gate, drain and substrate. This general configuration makes

the resulting solution readily applicable to any device configuration without the need for any modification of the model equations. Also, the general four terminal model is the form usually needed for implementation in computer-aided circuit analysis programs.

Fig. 4.5(a) shows a cross section of an N-channel device with the voltages applied to the different terminals. The potential reference is taken as an arbitrary point outside the device (reference point in the figure). Fig. 4.5(b) is a schematic representation of potential variation at any point along the channel in the direction normal to the surface. The potential ψ in the figure is an effective potential in the space charge region analogous to the separation of Quasi-Fermi levels in the case of classical statistics, and its exact definition will be discussed later in this section.

We now define the different quantities used in the figure:

U_{sr} , U_g , U_d and U_b are the normalized applied potentials of the source, gate, drain and substrate respectively (in the normal model of operation of an n-channel device U_{sr} , U_g and U_d are positive while U_b is negative or less positive than either U_{sr} or U_d).

U_f is the normalized Fermi level of the majority carriers in the substrate.

U_s is the total band bending at semiconductor surface (i.e., total normalized potential across the semiconductor space charge region).

Figure 4.5 Device structure and potential distribution normal to the surface

U is the normalized electrostatic potential at any point in the space charge region.

ζ is an effective potential to be defined later

x and y are the spatial coordinates normal to and along the surface respectively.

The following assumptions will be used in the analysis:

- i) one dimensional analysis, i.e., GCA is used
- ii) the majority carrier Fermi level U_f is constant throughout the semiconductor substrate, i.e., no majority carrier flow anywhere in the substrate.
- iii) charge density (fixed and mobile) is constant in the direction normal to the surface over the space charge region.
- iv) the mobility is constant.

4.2.1 Solution for the electrostatic potential

The electrostatic potential in the space charge region is a function of both space coordinates x and y in addition to the surface potential. A form suitable for mathematical manipulation is

$$U(x, y) = f(x) g(y) + U_b - U_f$$

At the surface $x = 0$, $U = U_s + U_b$

$$U(0, y) = U_s + U_b = f(0) g(y) + U_b = g(y) + U_b, \quad f(0) = 1$$

Now based on the assumed constant charge density in the x-direction, $f(x)$ is a parabolic function of x , i.e.

$$f(x) = \left(1 - \frac{x}{x_s}\right)^2$$

where x_s is the width of the space charge region. Thus

$$U(x, y) = U_s \left(1 - \frac{x}{x_s}\right)^2 + U_b - U_f \quad (4.1)$$

This form of $f(x)$ is chosen so that at $x = x_s$, $\frac{dU}{dx} = 0$ and $U = U_b - U_f$ a necessary boundary condition of a space charge region.

To evaluate the thickness, x_s , we use the fact that the electric displacement vector should be continuous at the insulator-semiconductor interface, i.e.,

$$\epsilon_s \left. \frac{dU}{dx} \right|_{x=0} = \epsilon_i \left. \frac{dU}{dx} \right|_i \quad (4.2)$$

where

$$\epsilon_i \left. \frac{dU}{dx} \right|_i = -\epsilon_i \frac{U_g - U_s - U_b}{x_i} = -C_i (U_g - U_s - U_b) \quad (4.3)$$

and U_g is an effective gate potential defined as

$$U_g = U_{gg} - \frac{q}{kT} \left(\phi_{MS} - \frac{Q_{ss}}{C_i} \right)$$

where

$$U_{gg} = \text{the applied gate potential}$$

ϕ_{MS} = the metal semiconductor work function difference

Q_{ss} = the interface charge equivalent to the oxide and surface states charges

$C_i = \frac{\epsilon_i}{x_i}$ = the insulator capacitance per unit surface area.

Thus, from (4.1), (4.2) and (4.3)

$$x_s = \frac{2U_s}{\frac{C_i}{\epsilon_s} (U_g - U_s - U_b)} \quad (4.4)$$

If the Q_g and Q_b are the gate and ionized* charges per unit surface area, then

$$Q_g = \frac{kT}{q} C_i (U_g - U_s - U_b) \quad (4.5)$$

$$Q_b = -qNx_s = -qn_i e^{U_f} \cdot \frac{2U_s}{\frac{C_i}{\epsilon_s} (U_g - U_s - U_b)} \quad (4.6)$$

The mobile charge per unit surface area, Q_m , is obtained, using the charge neutrality condition, as (note that in the GCA the contribution of the field gradient along the surface to the charge in the space charge region is neglected)

*The error mentioned in section (4.1) is in fact the underestimation of x_s , and therefore Q_b in the transition region from weak to strong inversion conditions: this is on the basis that Q_m is dominant in strong inversion, and channel thickness is rapidly increasing in weak inversion (Fig. A.1).

$$\begin{aligned}
 Q_m &= -(Q_g + Q_b) \\
 &= -\frac{kT}{q} C_i (U_g - U_b - U_s) + qn_i e^{U_f} \cdot \frac{2U_s}{\frac{C_i}{\epsilon} (U_g - U_b - U_s)}
 \end{aligned}
 \tag{4.7}$$

Define

$$\begin{aligned}
 L_D &= \left(\frac{kT\epsilon_s}{2q^2 n_i} \right)^{1/2} \\
 C_D &= \epsilon_s / L_D \\
 a &= \frac{C_i}{C_D} e^{-U_f/2} \\
 n &= a(U_g - U_b - U_s)
 \end{aligned}
 \tag{4.8}$$

Thus, using the definitions in (4.8), equation (4.7) becomes

$$Q_m = -\frac{kT}{q} \cdot \frac{C_i}{a} \cdot \frac{(n^2 + \frac{n}{a} - U_g + U_b)}{n}
 \tag{4.9}$$

and

$$n = n_i e^{U_f} \cdot \frac{n^2 + \frac{n}{a} - U_g + U_b}{U_g - U_b - \frac{n}{a}}
 \tag{4.10}$$

This equation shows the dependence of the mobile charge along the channel on the parameter n (i.e., on the band bending).

In fact, the thickness x_s is related to an important physical quantity of the actual device, namely its first moment of charge.

If we define an effective charge distance x_c as the position (measured from the surface) of the centre of the space charge^[23], then

$$x_c = \frac{\int_0^\infty \rho x dx}{\int_0^\infty \rho dx}$$

where ρ is the space charge density distribution in the space charge region given by the one-dimensional Poisson's equation as

$$\rho = \frac{kT\epsilon_s}{q} \frac{d^2 U}{dx^2}$$

Thus

$$x_c = \frac{\int_0^\infty \frac{d^2 U}{dx^2} x dx}{\int_0^\infty \frac{d^2 U}{dx^2} dx} = \frac{\int x d\left(\frac{dU}{dx}\right)}{\int d\left(\frac{dU}{dx}\right)}$$

Integrating the numerator by parts noticing that

$$\lim_{x \rightarrow \infty} x \frac{dU}{dx} = 0, \text{ we get}$$

$$x_c = \frac{U_s}{\left. \frac{dU}{dx} \right|_{x=0}} = \frac{U_s}{\frac{c_i}{\epsilon_s} (U_g - U_b - U_s)} = \frac{x_s}{2} \quad (4.14)$$

Equation (4.11) shows that the width x_s is twice the centre of charge distance (a quantity that is invariant for given surface conditions) of the actual device. This, of course, is an expected results since x_s is obtained from applying Gauss's law at the surface. An important consequence of this dependence is the invariance of the quantity $\int ndU$, where the integration is carried out from the surface to the neutral substrate, i.e., this quantity for the present distribution is the same as that of a distribution governed by Poisson's equation. To show that, consider the value of n given by equation (4.10)

$$\begin{aligned} \int ndU &= n_i e^{U_f} \int_{U_b - U_f}^{U_s + U_b - U_f} \frac{n^2 + \frac{n}{a} - U_g + U_b}{U_g - U_b - \frac{n}{a}} dU \\ &= n_i e^{U_f} (n^2 + \frac{n}{a} - U_g + U_b) \end{aligned} \quad 4.12(a)$$

For a distribution defined by Poisson's equation, i.e., $n = \frac{kT\epsilon}{q^2} \frac{d^2U}{dx^2} - N$, we have

$$\begin{aligned} \int ndU &= \int \frac{kT\epsilon_s}{q^2} \frac{d^2U}{dx^2} \frac{dU}{dx} dx - \int NdU \\ &= \frac{kT\epsilon_s}{q^2} \int \frac{dU}{dx} \cdot d \left(\frac{dU}{dx} \right) - \int NdU \\ &= n_i e^{U_f} \left[\frac{kT\epsilon_s e^{-U_f}}{2q^2 n_i} \left(\frac{dU}{dx} \right)_s^2 - U_s \right] \\ &= n_i e^{U_f} \left[n^2 + \frac{n}{a} - U_g + U_b \right] \end{aligned} \quad 4.12(b)$$

This property of the present distribution is used in Appendix B for matching the conditions on the channel side and the junction side of the source and drain boundaries.

To obtain the dependence of Q_m on applied voltages, the relation between η and these voltages should be derived. This is done in Appendix B giving the result,

$$\eta^2 + \frac{\eta}{a} - U_g + U_b = e^{-\frac{\eta}{a} + U_g - \zeta - 2U_f} \quad (4.11)$$

where ζ is an effective potential along the channel related to both the channel charge density (per unit surface area) and the band bending (analogous to the majority carrier Quasi-Fermi level) defined in the Appendix and has the values U_{sr} and U_d at the source and the drain ends of the channel respectively. For example if U_g , U_b and U_d are known, equation (4.11) can be used to evaluate η at the drain end of the channel. Applying the chain rule to equation (4.11), we get

$$\begin{aligned} d\eta &= \frac{\partial \eta}{\partial U_g} dU_g + \frac{\partial \eta}{\partial \zeta} d\zeta + \frac{\partial \eta}{\partial U_b} dU_b \\ &= \frac{a}{\eta^2 + (2a + \frac{1}{a})\eta + 1 - U_g + U_b} \left[\left(\eta^2 + \frac{\eta}{a} + 1 - U_g + U_b \right) dU_g \right. \\ &\quad \left. - \left(\eta^2 + \frac{\eta}{a} - U_g + U_b \right) d\zeta - dU_b \right] \quad (4.12) \end{aligned}$$

This equation illustrates the effects of incremental changes in terminal potential on the parameter η . It will prove very useful

when the device terminal parameters are derived in the next section as a function of different applied voltages.

4.2.2 Drain current and other device terminal parameters

Drain current I_d

In an n-channel device, the drain current is essentially an electron current. The electron current density (including both drift and diffusion components) at any point in the channel is

$$J_n(x, y) = -q D_n n \frac{\partial U}{\partial y} + q D_n \frac{\partial n}{\partial y}$$

The drain current I_d is

$$I_d = Z \int_0^{x_s} J_n dx$$

which can be written in terms of the effective potential ζ as (see Appendix B)

$$I_d = -Z D_n \frac{d\zeta}{dy} Q_m \quad (4.15)$$

where Z is the channel width.

Integrating equation (4.15) along the channel, we get

$$I_d = - \frac{Z D_n}{L} \int_{U_{sr}}^{U_d} Q_m d\zeta \quad (4.16)$$

where L is the channel length.

Substituting for Q_m from equation (4.9) and for $d\zeta$ (with $dU_b = dU_g = 0$) from equation (4.14), we get

$$\begin{aligned} I_d &= - \frac{kT}{q} \frac{D_n c_i Z}{L} \cdot \frac{1}{a^2} \int_{\eta_s}^{\eta_d} \left[\eta + 2a + \frac{1}{a} + \frac{(1-U_g+U_b)}{\eta} \right] d\eta \\ &= - \frac{kT}{q} \frac{D_n c_i Z}{L} \cdot \frac{1}{a^2} \left[\frac{\eta^2}{2} + (2a + \frac{1}{a})\eta + (1-U_g+U_b) \log \eta \right]_{\eta_s}^{\eta_d} \end{aligned} \quad (4.17)$$

where η_s and η_d are obtained from equation (4.13) by putting ζ equal to U_{sr} and U_d respectively.

Source-Drain Conductance g_d

The drain conductance may be obtained from the definition

$$g_d = + \frac{q}{kT} \frac{\partial I_d}{\partial U_d} \bigg|_{U_g, U_b}$$

Differentiating equation (4.16) with respect to U_d , we get

$$\begin{aligned}
 g_d &= - \frac{Z D_n}{L} \cdot Q_m \bigg|_{U_d} \\
 &= \frac{D_n c_i Z}{L} \cdot \frac{1}{a} \left(n_d + \frac{1}{a} - \frac{U_g - U_b}{c n_d} \right) \quad (4.18)
 \end{aligned}$$

Gate Transconductance g_m

This is derived from the drain current equation (4.17) as

$$\begin{aligned}
 g_m &= \frac{q}{kT} \frac{\partial I_d}{\partial U_g} \bigg|_{U_d, U_b, U_{sr}} \\
 &= - \frac{D_n c_i Z}{L} \cdot \frac{1}{a^2} \cdot \frac{\partial}{\partial U_g} \left[\frac{n^2}{2} + \left(2a + \frac{1}{a} \right) n + (1 - U_g + U_b) \log n \right]_{n_s}^{n_d}
 \end{aligned}$$

The partial derivative of the quantity between brackets with respect to U_g is

$$\left[n + \left(2a + \frac{1}{a} \right) + \frac{1 - U_g + U_b}{n} \right] \cdot \frac{\partial n}{\partial U_g} - \log n$$

where $\frac{\partial n}{\partial U_g}$ is obtained from equation (4.14) (with $du_b = d\tau = 0$).

Therefore

$$g_m = \frac{D_n c_i Z}{La} \left[n + \frac{1 - U_g + U_b}{n} - \frac{1}{a} \log n \right]_{n_d}^{n_s} \quad (4.19)$$

Gate Capacitance C_{gs}

This is defined as

$$C_{gs} = \frac{q}{kT} \left. \frac{\partial Q_G}{\partial U_g} \right|_{U_d, U_b, U_{sr}} \quad (4.20)$$

where Q_G is the total gate charge given by

$$\begin{aligned} Q_G &= Z \int_0^L Q_g dy \\ &= \frac{kT}{q} c_i Z \int_0^L (U_g - U_b - U_s) dy \\ &= \frac{kT}{q} c_i Z \int_{U_{sr}}^{U_d} \frac{U_g - U_s - U_b}{\frac{d\zeta}{dy}} d\zeta \end{aligned}$$

Substituting for $d\zeta$ and $\frac{d\zeta}{dy}$ from equations (4.12) and (4.15), and integrating we get

$$Q_G = \frac{kT}{q} \frac{c_i Z L}{a} \cdot \frac{M}{I_{d1}} \quad (4.21)$$

where

$$M = \left[\frac{n}{3} + \frac{n}{2} \left(2a + \frac{1}{a} \right) + (1 - U_g - U_b)n \right] \frac{n_s}{n_d}$$

$$I_{dl} = I_d / \frac{kT}{q} \cdot \frac{D_n c_i Z}{L a^2} = \left[\frac{\eta^2}{2} + (2a + \frac{1}{a})\eta + (1 - U_g + U_b) \log \eta \right]_{\eta_d}^{\eta_s}$$

Differentiating (4.21) with respect to U_g and substituting the result in (4.20) we get

$$C_{gs} = c_i Z L \cdot \frac{I_{dl} (\eta_d^2 - \eta_s^2) - M \cdot g_{ml}}{I_{dl}^2} \quad (4.22)$$

where

$$g_{ml} = g_m / \left[\frac{D_n c_i Z}{L a} \right] = \left[\eta + \frac{1 - U_g + U_b}{\eta} - \frac{1}{a} \log \eta \right]_{\eta_d}^{\eta_s}$$

η_d and η_s are the values of η at drain and source respectively.

4.2.3 Device electrostatic parameters

The variation of certain electrostatic parameters along the channel are of special interest. These are the surface potential, the surface field and the effective potential ζ . Expressions for these quantities are now obtained.

Integrating equation (4.15) to any point y along the channel, we get,

$$I_d \cdot y = \frac{kT}{q} \cdot D_n c_i Z \cdot \frac{1}{a} \left[\frac{\eta^2}{2} + (2a + \frac{1}{a})\eta + (1 - U_g + U_b) \log \eta \right]_{\eta_s}^{\eta}$$

where η is the value corresponding to y . Thus

$$\frac{y}{L} = \frac{\left[\frac{n^2}{2} + (2a + \frac{1}{a})n + (1 - U_g + U_b) \log n \right] n^s}{I_{dl}} \quad (4.23)$$

This equation gives n (and hence U_s) as a function of the distance y along the channel.

The longitudinal surface field E_{ys} along the channel can be obtained by differentiating equation (4.23) as

$$E_{ys} = - \frac{dU_s}{dy} = \frac{1}{La} \cdot \frac{I_{dl}}{n + 2a + \frac{1}{a} + (1 - U_g + U_b)/n} \quad (4.24)$$

The field normal to the surface, E_{xs} , is

$$E_{xs} = \frac{\epsilon_i}{\epsilon_s} \cdot \frac{U_g - U_s - U_b}{x_i} = \frac{e^{U_f/2}}{L_D} \cdot n \quad (4.25)$$

The effective potential ζ is obtained from equation (4.13) as

$$\zeta = U_g - 2U_f - \frac{n}{a} - \log \left(n^2 + \frac{n}{a} - U_g + U_b \right) \quad (4.26)$$

4.2.4 Discussion

Before we proceed to the next chapter and present the comparison between the predictions of the theory discussed here and the measured device parameters (which, as will be seen, is so good in all respects as to give strong support to the validity of the assumptions made), it is worthwhile to examine one of the consequences of the main assumption used in the analysis, namely, the uniform carrier distribution. In particular, we want to check how this assumption affects the validity of the GCA, also used in the analysis.

Pao and Sah^[12] have presented the most complete one-dimensional numerical analysis of the device. The GCA and the assumption of constant mobility are common to Pao and Sah solution and the present theory. The two solutions differ only in the distribution of mobile carriers in the space charge region in that Pao and Sah used Maxwell-Boltzmann statistics whereas in the present theory a uniform distribution is used appropriate to Fermi-Dirac statistics. If the smallness of the longitudinal field gradient along the channel is taken as a measure of the validity of the GCA, then the present theory is superior to that given by Pao and Sah. Fig. 4.6 shows a plot of the longitudinal field along the channel, for a device operating just in saturation, using the expressions derived here (equation 4.24) and those of Pao and Sah. It is seen

Figure 4.6 Variation of the surface longitudinal field along the channel for a device operating just in saturation.

that in the present solution the field gradient over most of the channel length is negligible (a necessity for the validity of the GCA) which is not the case in their solution.

Beside achieving more accurate prediction of device behaviour, the present theory has the important advantage that the resulting solutions are in closed form which is, in fact, a necessity in many applications.

CHAPTER 5

THEORETICAL AND EXPERIMENTAL RESULTS

Before proceeding with the comparison between theory and experiment, it is worthwhile presenting the theoretical characteristics of a typical device as described by the model derived in Chapter 4, in order to show the superiority of the present model over previous models in representing basic device properties.

Comparison between the model and experimental results is then carried out for a number of devices with various dimensions and processing parameters. The chapter concludes with remarks on the limitations of the model.

5.1 Theoretical Results

The dependence of the various terminal parameters, derived in the previous chapter, on applied voltages is shown in Figs. 5.1 and 5.2 for an n-channel device with 100μ channel length, 100μ channel width, 0.1μ insulator-thickness, $7 \times 10^{15} \text{ cm}^{-3}$ substrate doping and $370 \text{ cm}^2/\text{V}\cdot\text{sec}$ surface mobility. The parameters plotted are drain current I_d , drain conductance g_d , gate transconductance g_m and gate capacitance c_{gs} . Fig. 5.1 shows the variation of these parameters

Figure 5.1 Calculated device parameters vs. gate voltage using the present theory. The plots are for zero sub bias.

with gate voltage for fixed drain voltages while 5.2 shows the variation with drain voltage for fixed gate voltages. All the plots are for grounded source and substrate (i.e., in the equations of the previous chapter, $U_{sr} = U_b = 0$). These plots illustrate a very important feature of the present theory, namely, any device parameter is predicted over the whole range of device operation from weak inversion (subthreshold region) to strong inversion by a single closed form expression. This is a distinct advantage over previous theories where solutions have to be calculated in three bias ranges (weak inversion, saturation and strong inversion) and then fitted together [8, 14] (often with adjustments to provide continuous solutions). These figures demonstrate certain features of actual device behaviour, predicted by the present model, but not predicted by previous theories using constant surface mobility. These features are:

- i) The exponential dependence of the drain current on gate voltage in the subthreshold region, Fig. 5.1(a) is described here by the same closed form expression used at higher gate voltages.
- ii) The deviation of drain conductance g_d , from 'the straight line behaviour' as a function of gate voltage, Fig. 5.1(b).
- iii) The falling of gate transconductance g_m , for a fixed drain voltages, at high gate voltages, Fig. 5.1(c).

Figure 5.2 Calculated device characteristics vs. drain voltage using the present theory. The plots are for zero substrate bias.

- iv) More accurate prediction of gate capacitance variation with gate voltage, Fig. 5.1(d) and drain voltage, Fig. 5.2(d) over the whole range of operation using a single expression (equation (4.22)).

In fact, these are the main points of disagreement between previous theories and actual device behaviour (refer to Fig. 4.1). Particularly, features (ii) and (iii) above are of special significance. Previous theories [16,20] were able to explain such behaviour (with limited success) only by considering an empirical dependence of surface mobility on applied voltages. The present model being in terms of a constant surface mobility, supports the need to consider Fermi-Dirac statistics (or appropriate approximations to it) for proper evaluation of mobile carrier distribution in the space charge region. Further support is given by the excellent quantitative agreement, shown in the next section, between theoretical and experimental characteristics.

5.2 Comparison between theory and experiment

Measurements were made on devices with rectangular geometry having channel length from 10 to 140 microns. Metal and silicon gate devices were used in these measurements. Table 5.1 summarizes the physical data for the devices studied. Devices C-1, C-3, D-1 are

n-channel metal-gate (Table 5.1(a)), devices C-1S, D-1S, C-2S, D-2S are p-channel silicon-gate (Table 5.1(b)). For each device column 1 lists the data as provided from the process, while column 2 gives the corresponding data actually used in the device assessment.

Some of the parameters needed to calculate the theoretical curves are obtained based on a few experimental values using the following procedure:

- i) there is considerable uncertainty in the value of the substrate doping near the surface of the semiconductor because of the redistribution of the impurity atoms during the high temperature processes of fabricating the devices. Therefore, a procedure to determine the doping near the surface using the measured gate-source capacitance variation with gate voltage at zero drain voltage is developed as follows:

if $c_{\max}$ is the maximum capacitance per unit area

$c_{\min}$ is the minimum capacitance per unit area

c_s is the stray capacitance per unit area

then

$$c_{\max} = c_i + c_s$$

$$c_{\min} = \frac{c_i c_d}{c_i + c_d} + c_s$$

where c_d is the minimum depletion capacitance and c_i is the insulator capacitance per unit area. Therefore,

$$c_{\max} - c_{\min} = \frac{c_i^2}{c_i + c_d}$$

The only unknown in this equation is c_d . With c_d known, the following formula given by Das^[20] for silicon substrates is then used to get the substrate Fermi level U_f , and hence the doping

$$c_d = \frac{305 e^{U_f/2}}{[2(1.6U_f - 1)]^{1/2}}$$

where

$$\frac{U_f}{e} = \frac{N}{n_i}$$

and

N is the substrate doping.

- ii) the difference between the applied gate voltage V_{gg} and the effective gate voltage V_g , due to surface states and oxide charges and metal-semiconductor work function difference, is obtained by matching theoretical

and experimental $g_m - V_g$ curves near zero conductance value.

- iii) the effective surface mobility is obtained by the matching the peak value of theoretical and experimental $g_m - V_g$ curves for a fixed drain voltage.

Figures 5.3 - 5.5 show comparisons between theoretical and experimental curves of the different device parameters. Fig. 5.3 shows the comparison for the gate capacitance (measured between gate and source terminals with substrate and drain terminals a-c short-circuit to the source). The transconductance $g_m - V_g$ characteristics are shown in Fig. 5.4 while Fig. 5.5 shows the behaviour of the drain conductance g_d as a function of both gate and drain voltages. The good agreement between theory and experiment is evident for all parameters. Beside showing the correct behaviour of g_m and g_d as function of gate voltage, which was not predicted by previous theories, the theory fits the capacitance curves (fig. 5.3) much better than previous theories. Apart from the good agreement obtained in the strong inversion region (region (iii)) in Fig. 5.3(a), the theory shows an excellent fit to the nearly constant part (region (ii)) of the capacitance curve. Since this portion of the curve is the gate capacitance in saturation, it seems to suggest that contribution of mobile charges in the so-called pinch-off region to the total charge in the space charge region is neg-

ligible, especially for long channel devices. Also, matching the slopes in region (ii) indicates the uniform mobile carrier distribution approximation gives the proper relation between the total charge in the space charge region and the surface potential. Some mismatch is evident in region (i) of the capacitance curve. This is the region near cut-off where the space charge region capacitance is minimum. The source and drain junction capacitances, which are not accounted for in the analysis, could be the cause of the deviation in this region. In Fig. 5.4, the behaviour of the transconductance g_m is accurately predicted, especially at high gate voltages (this is where the present model is significantly different from previous models). The amount of dropping of g_m , at high gate voltages, as exhibited by actual devices, is directly proportional to the substrate doping; a characteristic shown by the model in Figs. 5.4(c) and 5.4(d) for devices D-1 (doping = $7 \times 10^{15} \text{ cm}^{-3}$) and B-2S (doping = 10^{15} cm^{-3}). Comparison between the model and measured drain conductance g_d is shown in Fig. 5.5. The dashed line illustrates the calculated results based on previous models with constant surface mobility. For short channel device D-2S, (figure 5.4(f)), the deviation of theoretical curves from experiment at high conductance values may be due to several causes. One reason could be the assumption used in the analysis of abrupt boundaries at source and drain while they are probably linearly graded junctions and this has more effect on short

channel devices. In addition, the ohmic resistances of source and drain diffusions are not considered, and these could have significant effects on measured conductances especially at high current levels. Also, the possibility that the carriers could reach their scattering limited velocity is not considered in the analysis. For short channel devices this possibility exists, and its effect is to reduce the conductance values, especially at high gate and drain voltages.

5.3 Concluding Remarks

Having established the ability of the model to accurately describe device operation, especially in the strong inversion region and to explain some aspects of device behaviour obtained previously through curve fitting (i.e., using empirical mobility variations), we now indicate some of the limitations of the present model.

Drain current saturation is obtained in Fig. 5.2(a), without having to terminate the solution at the pinch-off point, by including the diffusion component of the current^[12]. Although complete saturation of current can describe the behaviour of long channel devices with reasonable accuracy, it is inadequate to account for the large drain conductance observed in saturation for short channel devices. The same argument applies to the transconductance g_m which shows complete saturation in Fig. 5.2(c) while short channel devices exhibit an increasing g_m in saturation as a function of drain voltage.

As these effects are more evident in short channel devices, they are attributed to two-dimensional potential distribution in the space charge region (by using the GCA this potential distribution is described by one-dimensional Poisson's equation). Therefore, for proper modelling of device operation in saturation (mainly, the finite drain conductance), the constraints imposed by the GCA have to be removed. This we do in Chapter 6, where a simple two-dimensional model of the space charge region adjacent to the drain (as it is essentially the region where the GCA fails) is presented leading to accurate description of the drain current and drain conductance in the saturation region.

Table 5.1(a) Device Data for Metal Gate n-channel Devices

Parameter	Data from Processing	Data used in Calculation	Units
<u>Device C-1</u>			
N	$(7 - 10)10^{15}$	4×10^{15}	cm^{-3}
x_i	1050 - 1100	1100	$\text{\AA}$
μ		370	$\text{cm}^2/\text{V}.\text{sec}$
Z_i	22.5	22.5	micron
L	137.5	137.5	micron
<u>Device C-3</u>			
N	$(7 - 10)10^{15}$	4×10^{15}	cm^{-3}
x_i	1050 - 1100	1100	$\text{\AA}$
μ		410	$\text{cm}^2/\text{V}.\text{sec}$
Z	22.5	22.5	micron
L	97.5	97.5	micron
<u>Device D-1</u>			
N	$(7 - 10)10^{15}$	7×10^{15}	cm^{-3}
x_i	1050 - 1100	1100	$\text{\AA}$
μ		370	$\text{cm}^2/\text{V}.\text{sec}$
Z	100	100	micron
L	97.5	97.5	micron

Table 5.1(b) Device Data for Silicon Gate p-channel Devices

Parameter	Data from Processing	Data used in Calculations	Units
<u>Device C-1S</u>			
N	10^{15}	1.3×10^{15}	cm^{-3}
x_i	1000	1000	$\text{\AA}$
μ		190	$\text{cm}^2/\text{V}\cdot\text{sec}$
Z	22.5	22.5	micron
L	137.5	137.5	micron
<u>Device D-1S</u>			
N	10^{15}	10^{15}	cm^{-3}
x_i	1000	1000	$\text{\AA}$
μ		190	$\text{cm}^2/\text{V}\cdot\text{sec}$
Z	100	100	micron
L	97.5	97.5	micron
<u>Device B-2S</u>			
N	10^{15}	10^{15}	cm^{-3}
x_i	1000	1000	$\text{\AA}$
μ		170	$\text{cm}^2/\text{V}\cdot\text{sec}$
Z	22.5	22.5	
L	22.5	22.5	micron

Table 5.1(b) continued.....

Device C-2S

N	10^{15}	10^{15}	cm^{-3}
x_i	1000	1000	$\text{\AA}$
μ		170	$\text{cm}^2/\text{V}\cdot\text{sec}$
Z	22.5	22.5	
L	15	15	micron

Device D-2S

N	10^{15}	10^{15}	cm^{-3}
x_i	1000	1000	$\text{\AA}$
μ		150	$\text{cm}^2/\text{V}\cdot\text{sec}$
Z	250	250	micron
L	10	10	micron

(a)

(b)

Figure 5.3 Gate Capacitance vs. gate voltage

(c)

(d)

(e)

Figure 5.3 continued.

(a)

(b)

Figure 5.4 Gate transconductance vs. gate voltage.

Figure 5.4 continued.

(e)

(f)

Figure 5.4 continued.

Figure 5.5(a) Drain conductance vs. gate voltage

Figure 5.5(b) Drain conductance vs. gate voltage

Figure 5.5(c) Drain conductance vs. drain voltage

CHAPTER 6

A TWO DIMENSIONAL MODEL IN THE CURRENT SATURATION RANGE

6.1 Introduction

The IGFET theory presented so far is capable of predicting device parameters accurately over a wide range of operating conditions. However, due to the limitation imposed by the main assumption used in the analysis, namely the Gradual Channel Approximation "GCA", the theory fails to predict the finite drain conductance at high drain voltages. By using the GCA, the longitudinal field gradient contribution to the charge in the surface space charge region is neglected. For a fixed gate voltage, this field gradient increases with increasing drain voltages; and its contribution to the charge cannot be neglected for drain voltages comparable to the gate voltage. In fact, the GCA becomes less valid as one proceeds along the channel towards the drain, where the potential distribution in the space charge region becomes two-dimensional. It is that part of the space charge region, where the gate and drain electrodes interact, that determines the device characteristics in saturation. This interaction between the gate and drain electrodes produces two electric field components: one parallel and one perpendicular to the

insulator-semiconductor interface attributable to the drain junction and gate electrode respectively. Any attempt to properly model the device in saturation must take into consideration both field components. As will be shown later, one field component or the other could become the dominant factor in determining the device behaviour under certain conditions, (e.g. for certain geometry, or over a limited range of applied voltages) but in general misleading conclusions could be reached, unless both field components were considered.

In this chapter, the existing models for characterizing the IGFET in the saturation regime are discussed briefly. The approximations involved in deriving these models as well as their limitations are pointed out. Then, a two-dimensional model for the space charge region near the drain is presented and the device terminal parameters are derived. It is shown that only by including the two field components in the model, is an accurate prediction of the device behaviour achieved. Theory is compared with measured device parameter behaviour, showing excellent agreement over a wide range of operating conditions, device geometry and physical parameters.

6.2 Existing Models

We shall limit the discussion here to models aimed at accurately representing the saturation drain conductance since the

critical test of a saturation model by experiments lies in the behaviour of that conductance as a function of gate and drain voltages. The drain current dependence on applied voltages is not really a critical test since it is relatively independent of the model used. The analysis in this section is for grounded source and substrate.

In the saturation range of device operation, the region between the source and the drain is divided for convenience along the direction of the current flow, into two sections: a source section originating from the source, where the Gradual Channel Approximation is applicable; a drain section which starts from the drain where the GCA is invalid and the two-dimensional Poisson's equation must be solved. The conditions at the boundary separating the two sections are determined by matching the solutions in both sections. The usual way of doing this is to define that boundary as the end of the source section, i.e. as the point where the channel pinches off. If equation (3.8) is used for the mobile charge in the channel, and substituting zero for Q_m , the electrostatic potential at the boundary is given by:

$$U_{sp} = (U_g + U_A) - \sqrt{(U_g + U_A)^2 - U_g^2} \quad (6.1)$$

where

$$U_A = \epsilon_s q^2 N / (kTC_i^2)$$

which is the value of the normalized surface potential at the boundary.

It is seen that this value is a function of gate voltage only and is independent of the drain voltage. The location of the boundary, along the channel is determined from the solution in the drain section and is a function of both gate and drain voltages. The drain current is continuous from the source to the drain, and so it is the same in both sections. Now let us imagine a hypothetical device composed of the source section and having a virtual drain at the boundary between source and drain sections. For a fixed gate voltage, the source to virtual drain voltage of that device would be a constant value, U_{sp} , independent of drain voltage. However, the channel length of that device is the length of the source section and is dependent on both the applied gate and the applied drain voltages. In other words, the source to virtual drain voltage of that device is determined from the solution in the source section, while its channel length is obtained from the solution in the drain section. For the hypothetical device, the GCA is valid throughout the whole range of operation, and the current equation (3.11) can now be used with the value U_{sp} as the (virtual) drain potential, and a modified channel length L_e is used instead of the actual channel length L . The difference, $\Delta L = (L - L_e)$, is the length of the drain section. Thus, the problem becomes a one of calculating ΔL . Approaches to the calculation of ΔL have been proposed by many authors. Ihantola and Moll^[13] have used a simple step p-n junction

theory to characterize the drain section. The length ΔL is taken as the depletion width of a reverse biased junction due to an applied voltage given by the difference between the drain electrostatic potential U_{sd} , and the potential at the boundary U_{sp} . Therefore ΔL is given by

$$\Delta L = \sqrt{\frac{2\epsilon kT}{q^2 N}} (U_{sd} - U_{sp}) \quad (6.2)$$

where

N is the substrate doping

U_{sd} is the normalized potential at the drain

Instead of a step p-n junction, Wang^[26] assumed the drain junction to be a linearly graded one, to obtain

$$\Delta L = \sqrt[3]{\frac{2\epsilon_s kT}{bq^2}} (U_{sd} - U_{sp}) \quad (6.3)$$

where

b is the grading constant

The formulation given by equations (6.2) and (6.3) make the following assumptions:

- 1) The drain section is completely depleted of mobile carriers, which overestimates the value of ΔL at any applied voltage.
- 2) All the field lines in the drain section terminate on the drain electrode, while none are terminated on the

gate, which overestimates the rate of change of ΔL due to change in the applied drain voltage.

The values of drain conductance g_d , obtained using these equations are in gross disagreement with experimental measurements, and especially for short channel devices these expressions do not even predict the shape of the experimental characteristics. Popa^[27] used the same step junction approximation, but he has taken the mobile carriers in the drain section into consideration. His results are not in a closed form; and in order to get agreement with experiments, a very small depth for the drain region is used which contradicts the fact that mobile carriers are pushed away from the surface by the normal component of the field in that region. Frohman-Bentchkowsky and Grove^[19] have developed a more complete model to obtain ΔL . They use an empirical formula, based on simple physical reasoning, which accounts for the modification of the drain junction field due to the presence of the gate electrode. The expression obtained for ΔL could be written as:

$$\Delta L = \frac{U_{sd} - U_{sp}}{\sqrt{\frac{q^2 N}{2\epsilon_s kT} (U_{sd} - U_{sp})} + \alpha \frac{\epsilon_i (U_{sd} - U_g)}{\epsilon_s x_i} + \beta \frac{\epsilon_i (U_g - U_{sp})}{\epsilon x_i}} \quad (6.4)$$

where α, β are empirical parameters obtained by fitting theory and experiments.

Chiu and Sah [28] have solved the two-dimensional Poisson's equation in the drain section assuming complete depletion of mobile carriers. The final solution is in the form of a set of equations to be solved simultaneously. By taking the two field components into consideration, good agreement between the theory and measured drain conductance have been achieved. However due to neglecting mobile carriers in the drain section, the solution is limited to low injection levels only. Shroeder and Muller [7] have carried out a two-dimensional numerical solution of the Poisson's equation in the drain section, assuming complete depletion of mobile carriers. They were able to calculate the potential distribution throughout the drain section, thereby showing the two-dimensional nature of the device in that region. This solution has, however the drawback of all numerical models, in that it cannot be used as a device model in a computer analysis program for analysing a circuit with a large number of devices as elements of the circuit.

It is clear from the preceding discussion that, in general, there are two classes for models:

- 1) Simple closed form solutions which are either inaccurate in describing the device behaviour or need some experimentally determined parameters to achieve the required agreement.

2) More complicated, viz. two-dimensional models which are capable of characterizing the device over a wide range of operating conditions, but these are impractical to use in many applications such as in computer-aided design and analysis.

The aim of the present work is to develop a model that combines both accuracy and simplicity so that it can be used in computer aided design and analysis.

6.3 The Proposed Model For The Drain Section

As in the model presented in Chapter 4, we consider here the general four terminal device . From the discussion in the previous section, it is clear that any successful and accurate modelling of the drain region must take into consideration the two-dimensional nature of the electrostatic potential distribution in that region. The approach we shall adopt here is to treat that region as a unit which should obey basic electrostatic laws and in particular Gauss's law. This way, the total charge (mobile and immobile) contained within that region is directly related to the integral of electric displacement normal to its boundaries, without having to consider the detailed distribution of charges in the region. Figure 6.1 shows the region under consideration. It is taken as a rectangle with the drain metallurgical junction, the semiconductor insulator interface, the end of

Figure 6.1 The drain section and its boundaries.

the source section and the neutral bulk as the boundaries 1 to 4.

Boundary 1 is the end of the source region and is defined as the point along the channel where the Gradual Channel Approximation becomes invalid. If U is the normalized electrostatic potential in the space charge region, then the criterion for the failure of the GCA is

$$\frac{\partial^2 U}{\partial x^2} / \frac{\partial^2 U}{\partial y^2} < \beta \quad (6.5)$$

where β is a number (say 1.0). (Notice that the validity of the GCA is based on an infinite value for β). The derivatives of U are obtained from the solution in the source section, i.e. the solution given in Chapter 4. In Appendix C, it is shown that the solution of equation (6.5) gives a value U_{s1} for the surface electrostatic potential at boundary 1. Defining the boundary this way would yield a value for U_{s1} that is smaller than the pinch-off potential, U_{sp} , corresponding to that potential U_{s1} , there is a space charge region thickness x_1 . The thickness of the drain section is taken to be equal to that value x_1 and is independent of y . Thus the choice of the position of boundary 1 defines also the region thickness x_1 .

The electric field in the y direction within the drain region could become very high and the mobile carriers might well reach their scattering limited velocity. To account for this, the following expression is used for the surface mobility^[31]:

$$\mu_e = \frac{\mu}{1 + \frac{\mu}{v_o} \frac{kT}{q} \frac{dU_s}{dy}} \quad (6.6)$$

where μ_e is the field dependent surface mobility

v_o is the mobile carriers scattering limited velocity

(10^7 cm/sec for electrons, 6×10^6 cm/sec for holes)

μ is the low-field surface mobility

$\frac{dU_s}{dy}$ is the normalized surface field in the y direction

Gauss's theorem states that

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q \quad (6.7)$$

where

S is a closed surface

$\mathbf{D}$ is the electric displacement density normal to S

Q is the total charges contained within S .

The surface S in this case is defined by planes at the boundaries 1, 2, 4 and a plane at distance y parallel to 1. All these planes have unit width normal to the x - y plane.

Define:

E_1 as the normalized electric field normal to the plane at 1

E_y as the normalized electric field normal to the plane at y

E_x as the normalized electric field normal to the plane at 2.

Thus

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \frac{kT}{q} \int_0^{x_1} (\epsilon_s E_y - \epsilon_s E_1) \cdot 1 \cdot dx + \int_0^y \epsilon_s E_x \cdot 1 \cdot dy \quad (6.8)$$

The field at boundary 4 is zero by definition. The total charge contained in the region is

$$Q = -q \int_0^y \int_0^{x_1} (N + n) \cdot dx dy \quad (6.9)$$

where

N is the ionized charge density

n is the mobile charge density

Therefore, from equations (6.8) and (6.9)

$$\int_0^{x_1} \left(\frac{\partial U}{\partial y} + E_1 \right) dx + \int_0^y \frac{c_i}{\epsilon_s} (U_g - U_b - U_s) dy = \frac{q^2}{kT\epsilon_s} \int_0^y \int_0^{x_1} (N+n) dx dy$$

where

U_s is the normalized potential at the surface

$E_x = \frac{c_i}{\epsilon_s} (U_g - U_b - U_s)$ is the normalized field

perpendicular to the surface

U_g is the normalized effective gate potential (applied gate voltages less the flat band voltage)

U_b is the normalized substrate potential

For a parabolic variation of U with x , we have

$$\int_0^{x_1} U dx = U_s \cdot \frac{x_1}{3}$$

giving

$$\left(\frac{dU_s}{dy} + E_{s1}\right) \frac{x_1}{3} + \int_0^y \frac{c_i}{\epsilon_s} (U_g - U_b - U_s) dy = \frac{q^2}{kT\epsilon_s} \int_0^y \int_0^{x_1} (N+n) dx dy$$

Differentiating both sides w.r.t. y , we get

$$\frac{d^2 U_s}{dy^2} = \frac{3c_i}{\epsilon_s x_1} (U_s - U_g + U_b) + \frac{3q^2}{kT\epsilon_s x_1} \int_0^{x_1} (N+n) dx$$

or

$$\begin{aligned} \frac{d}{dU_s} \left(\frac{dU_s}{dy}\right)^2 &= \frac{6c_i}{\epsilon_s x_1} (U_s - U_g + U_b) + \frac{6q^2}{kT\epsilon_s x_1} \int_0^{x_1} (N+n) dx \\ &= \frac{6c_i}{\epsilon_s x_1} (U_s - U_g + U_b) + \frac{6q^2}{kT\epsilon_s} N - \frac{6q}{kT\epsilon_s x_1} Q_m \end{aligned}$$

(6.10)

where

Q_m is the mobile charge per unit surface area at any position y .

Integrating (6.10) from boundary 1 to boundary 3

2 2
OF/DE

$$\int_{E_{s1}}^{E_{sd}} d\left(\frac{dU_s}{dy}\right)^2 = \frac{6c_i}{\epsilon_s x_1} \int_{U_{s1}}^{U_{sd}} (U_s - U_g + U_b) dU_s + \frac{6q^2 N}{kT\epsilon_s} \int_{U_{s1}}^{U_{sd}} dU_s - \frac{6q}{kT\epsilon_s x_1} \int_{U_{s1}}^{U_{sd}} Q_m dU_s$$

or

$$E_{sd}^2 - E_{s1}^2 = \frac{3c_i}{\epsilon_s x_1} \{ (U_{sd} - U_{s1})^2 - 2(U_g - U_b - U_{s1})(U_{sd} - U_{s1}) \} + \frac{6q^2 N}{kT\epsilon_s} (U_{sd} - U_{s1}) - \frac{6q}{kT\epsilon_s x_1} \int_{U_{s1}}^{U_{sd}} Q_m dU_s \quad (6.11)$$

where E_{sd} is the normalized electric field at the drain.

The drain current given by equation (4.15) is written as

$$I_d = -\frac{kT}{q} Z Q_m \mu_e \frac{d\zeta}{dy}$$

where

μ_e is used here instead of μ .

In the drain section, we can write

$$\frac{d\zeta}{dy} = \frac{dU_s}{dy}$$

i.e., diffusion current component is negligible in that section,

which is in fact the case^[8]. Using the expression for μ_e given by equation (6.6), the current equation becomes

$$I_d = -\frac{kT}{q} Z Q_m \frac{\mu}{1 + \frac{kT}{q} \frac{\mu}{v_o} \frac{dU_s}{dy}} \frac{dU_s}{dy} \quad (6.12a)$$

Integrating from the source to the drain we get

$$I_d = I_1 - \frac{kT}{q} Z \frac{\mu}{L} \int_{U_{s1}}^{U_{sd}} Q_m dU_s - \frac{kT}{q} \frac{\mu}{v_o L} I_d (U_{sd} - U_{s1}) \quad (6.12)$$

where

I_1 is the drain current resulting from a surface potential U_{s1} at the drain

From (6.11) and (6.12) obtain

$$E_{sd}^2 = E_{s1}^2 + \frac{3c_i}{\epsilon_s x_1} \{ (U_{sd} - U_{s1})^2 - 2(U_g - U_b - U_{s1})(U_{sd} - U_{s1}) \} + \frac{6q^2 N}{kT \epsilon_s} (U_{sd} - U_{s1}) + \frac{6q^2}{k^2 T^2 \epsilon_s x_1} \left[\frac{L}{\mu Z} (I_d - I_1) + \frac{kT}{q} \frac{I_d}{v_o Z} (U_{sd} - U_{s1}) \right]$$

But from equation (6.12a) with Q_m and $\frac{dU_s}{dy}$ substituted at the drain end, we get

$$I_d = \frac{kT}{q} \cdot g_d \cdot E_{sd} \cdot L$$

(g_d is defined as $\frac{\partial I_d}{\partial U_{sd}}$), therefore

$$g_d = \frac{I_d}{\frac{kT}{q} \cdot L [A^2 (U_{sd} - U_{s1})^2 + B (U_{sd} - U_{s1}) + C]^{\frac{1}{2}}} \quad (6.13)$$

where

$$A^Z = \frac{3c_i}{\epsilon_s x_1}$$

$$B = \frac{6q^2 N}{kT \epsilon_s} + \frac{6q I_d}{kT v_o \epsilon_s x_1 Z} - \frac{6c_i}{\epsilon_s x_1} (U_g - U_b - U_{s1})$$

$$C = E_{s1}^2 + \frac{6q^2}{k^2 T^2} \frac{L}{\epsilon_s x_1^{2u}} (I_d - I_1)$$

But g_d is defined as

$$g_d = \frac{dI_d}{\frac{kT}{q} dU_{sd}}$$

Thus

$$\frac{dI_d}{dU_{sd}} = \frac{I_d}{L} \cdot \frac{1}{[A^2 (U_{sd} - U_{s1})^2 + B(U_{sd} - U_{s1}) + C]^{\frac{1}{2}}}$$

(6.14)

Equation (6.14) is an ordinary first order differential equation with

I_d and U_{sd} as the dependent and independent parameters respectively,

which is solved with the boundary condition

$$I_d = I_1, \quad U_{sd} = U_{s1}$$

Now, we rewrite equation (6.14) as

$$g_d = \frac{I_d}{\frac{kT}{q} \cdot L} \cdot \frac{1}{[E_{s1}^2 + \frac{3c_i}{\epsilon_s x_1} \{ (U_{sd} - U_g + U_b)^2 - (U_{s1} - U_g + U_b)^2 \} + \frac{6q^2 N}{kT \epsilon_s} (U_{sd} - U_{s1})]}$$

-1-

-2-

$$+ \frac{6qI_d}{kT \epsilon_s v_o x_1^2} (U_{sd} - U_{s1}) + \frac{6q^2 L}{k^2 T^2 \epsilon_s x_1^2 u} (I_d - I_1)]^{\frac{1}{2}}$$

-3-

-4-

(6.15)

Rearranging the equation in this form enables us to investigate the contribution of each term in the denominator to the drain conductance, as well as comment on its physical interpretation.

i) Term (1), which is the result of the field normal to the surface, is not a function of the channel length. It is directly proportional to the square root of the doping N (through x_1) and inversely proportional to the insulator thickness.

ii) Term (2), which is the result of the field along the channel, is proportional to the substrate doping but it is independent of the channel length. (notice that this is the term that appears in the simple channel shortening model, see Appendix D).

iii) Term (3) is the result of taking into consideration the effects

of scattering limited velocity and it is inversely proportional to the channel length (through I_d).

iv) Term (4) is the effect of including the mobile carriers in the drain section and it is not dependent on the channel length.

Inspecting these terms and their dependencies on the device geometry and doping, the following conclusions can be drawn:

1. For lightly doped substrates ($< 10^{15} \text{ cm}^{-3}$) and/or thin insulators ($\sim 0.1 \mu$), term, (1), i.e., the field normal to the surface is the dominant factor in characterizing device behaviour for all applied voltages and for any channel length.
2. For heavily doped substrates ($> 10^{16} \text{ cm}^{-3}$), the longitudinal field (term (2)) is the dominant factor over a wide range of applied voltages except at high drain voltages where term (1) becomes dominant.
3. Effects of scattering limited velocity (term (3)) are important for short channel devices with lightly doped substrates (In fact just by numerical substitution it can be shown that the value of term (3) exceeds that of term (2) for a device having doping of $10^{15} / \text{cm}^3$ and channel length of 10μ).
4. The effect of mobile carriers (term (4)) is only important at the immediate vicinity of the saturation

point and its effect becomes negligible for increasing drain voltage.

Equation (6.14) is used to calculate the g_d - V_d and I_d - V_d characteristics for a number of devices having a wide range of doping, channel length and insulator thickness. The experimental curves of the metal gate devices (631011C-26, 621203C-34, 670901K-01) together with their geometrical parameters are obtained from the results published by Chiu and Sah^[28]. These devices are of circular geometry and they have a thick gate insulator (0.2 - 0.6 μ). The devices A-1S and D-2S are p-channel silicon gate devices with rectangular geometry and they have a relatively thin gate insulator (0.1 μ). The parameters of both metal-gate and silicon-gate devices are listed in Table 6.1. Comparison between theory and experiment is shown in Fig. 6.2 and Fig. 6.3 with excellent agreement obtained for all devices over a wide range of applied voltages including high injection levels. Fig. 6.2 shows this comparison for the two silicon gate devices A-2S and D-2S while Fig. 6.3 is for the metal-gate devices. The theoretical drain current and drain conductance below saturation are calculated using the expressions given in Chapter 4.

In ref. [28], an effective gate voltage V'_g defined as

$$V'_g = V_{gg} - V_{gT}$$

has been used where,

V_{gg} is the applied gate voltage

V_{gT} is the threshold gate voltage required to induce a conducting channel, i.e. V_{gT} is the difference between the applied gate voltage at pinch-off point (in the 1-D solution) and the applied drain voltage.

The relation between V'_g and the normalized gate voltage U_g used in the present analysis is given in Appendix E as

$$U_g = \frac{q}{kT} V'_g + \frac{1}{a} \sqrt{V'_g \cdot \frac{q}{kT}}$$

where a is defined by equation (4.8).

6.4 A Simplified Model

In its present form, as derived, equation (6.14) is relatively complicated; it is suitable for use under a wide range of application conditions, though in a given situation some of its denominator terms may be negligible. It is shown in the preceding discussion that the contribution of the mobile carriers in the drain section is only important in the immediate vicinity of the saturation point. Thus, if the effect of the mobile carriers is neglected, then parameters A, B and C in (6.14) become

$$A^2 = \frac{3c_i}{\epsilon_s x_1} = \frac{1}{x_1 x_1} \quad \text{(for silicon } \epsilon_s = 11.7 \epsilon_0, \\ \text{for SiO}_2 \epsilon_i = 3.9 \epsilon_0)$$

$$B = \frac{6q^2 N}{kT \epsilon_s} - \frac{6c_i}{\epsilon_s x_1} (U_g - U_{sl} - U_b) \\ = \frac{6}{\epsilon_s x_1} \left(\frac{q^2}{kT} N x_1 - c_i (U_g - U_{sl} - U_b) \right) \\ = 0 \quad \text{(from charge neutrality at boundary (1))}$$

$$C = \epsilon_s E_{sl}^2$$

Thus (6.14) becomes

$$\frac{dI_d}{dU_{sd}} = \frac{I_d}{L} \cdot \frac{1}{[A^2 (U_{sd} - U_{sl})^2 + E_{sl}^2]^{\frac{1}{2}}}$$

The denominator of the right hand side of this simplified equation does not contain I_d , and the variables I_d and U_{sd} could be separated as follows:

$$\frac{dI_d}{I_d} = \frac{1}{L} \cdot \frac{dU_{sd}}{[A^2 (U_{sd} - U_{sl})^2 + E_{sl}^2]^{\frac{1}{2}}}$$

Integrating between limits (U_{sl}, I_1) and (U_{sd}, I_d) , we get

$$\log \frac{I_d}{I_1} = \frac{1}{LA} \log \left[\sqrt{\left\{ \frac{A}{E_{sl}} (U_{sd} - U_{sl}) \right\}^2 + 1} + \frac{A}{E_{sl}} (U_{sd} - U_{sl}) \right]$$

or

$$I_d = I_1 \left[\sqrt{\left\{ \frac{A}{E_{s1}} (U_{sd} - U_{s1}) \right\}^2 + 1} + \frac{A}{E_{s1}} (U_{sd} - U_{s1}) \right]^{1/LA} \quad (6.16)$$

and

$$g_d = \frac{I_d}{\frac{kT}{q} \cdot L E_{s1} \left\{ \frac{A^2}{E_{s1}^2} (U_{sd} - U_{s1})^2 + 1 \right\}^{1/2}} \quad (6.17)$$

Equations (6.16) and (6.17) give the dependence of the drain current and drain conductance on applied voltages and device parameters in closed form. A plot for the simple expression (6.17) compared with the complete equation (6.14) is shown in Fig. 6.4 for device D-2S. It is seen that both expressions given essentially the same results at low gate voltages. For high gate voltages (high injection levels), values of drain conductance given by the simple expression are higher than those given by equation (6.14). This illustrates the effect of mobile carriers in the drain section, which could have appreciable effect on device characteristics especially at high current levels.

Inspection of equation (6.16) shows that the exponent $\left(= \sqrt{\frac{x_i x_1}{L^2}} \right)$ of the current expression is inversely proportional to the product of the channel length L and the square root of the substrate doping (thru' A) and is directly proportional to the square root of the insulator thickness. Thus for a long channel device and/or heavily

doped substrates, the exponent is very small giving a nearly constant current in the saturation range. On the other hand for short channel devices or lightly doped substrates, there is a large increase of current with increasing drain voltage. This, in fact, is the type of dependence exhibited by actual devices. Also devices with thick gate insulators (small values of A) have high drain conductance in the saturation region. This dependence of drain conductance on insulator thickness was discussed by Bentchowsky and Grove^[19] and demonstrated on a wide range of devices.

Hofestein and Warfield^[18] presented some results showing that the drain conductance in saturation increases with applied substrate bias, though their analysis has not been able to explain such behaviour. The space charge region thickness x_1 in the drain section is given by (see equation (4.4)):

$$x_1 = \frac{2U_{s1}}{\frac{c_i}{\epsilon_s} (U_g - U_b - U_{s1})} \quad (6.18)$$

where U_b is negative for reverse substrate bias. The effect of applying a substrate bias U_b is to increase U_{s1} by the same amount. Thus, the denominator in equation (6.18) remains unchanged while the numerator increases resulting in a higher x_1 . This reduces the parameter A and increases the exponent of the current expression (6.16).

Therefore, the slope of the drain current vs. drain voltage in saturation increases with applying reverse substrate bias.

6.5 Drain breakdown voltage

For the sake of completeness, we consider the voltage limitations of an IGFET structure. This is not intended to be a study of breakdown mechanisms in the device; rather it is an attempt to define the maximum drain voltage that can be applied to the device and its dependence on other voltages and device parameters. At the drain junction, the electric field has two components, one component along the channel and the other normal to it. Both components are position dependent in the direction normal to the surface. Avalanche breakdown occurs in the region of maximum field. Armstrong and Magowan^[10] carried out a detailed two-dimensional numerical solution of the device showing that the maximum field at the drain junction occurs on or just below the semiconductor-insulator interface. At the drain junction, the surface field along the channel is given by (using equation (6.17)):

$$E_{dy}^2 = \frac{I_d^2}{g_d^2 L^2} = \left(\frac{kT}{q}\right)^2 [A^2 (U_{sd} - U_{sl})^2 + E_{sl}^2] \quad (6.19)$$

and the field normal to the surface (for $U_b = 0$) is

$$E_{dx}^2 = \left(\frac{kT}{q}\right)^2 \left(\frac{U_g - U_{sd}}{x_i}\right)^2 \left(\frac{\epsilon_i}{\epsilon_s}\right)^2 \quad (6.20)$$

Thus, the maximum field is

$$E_{\max}^2 = E_{dy}^2 + E_{dx}^2 = \left(\frac{kT}{q}\right)^2 \left[A^2 (U_{sd} - U_{sl})^2 + E_{sl}^2 + \left(\frac{\epsilon_i}{\epsilon_s}\right)^2 \left(\frac{U_g - U_{sd}}{x_i}\right)^2 \right]$$

(6.21)

By setting $E_{\max}$ equal to the avalanche breakdown field, and for a given gate voltage, the value of U_{sd} obtained from equation (6.21) is the normalized breakdown voltage. Fig. 6.5 shows the I_d - V_d characteristics for the device A-2S (listed in Table 6.1) in which the model of Chapter 4, the saturation model (equation (6.16)) and the breakdown voltage given by equation (6.21) are used. The current behaviour in the breakdown region is, arbitrarily, represented by an exponential.

Table 6.1(a)

Metal Gate Device Parameters

Parameter	Device 631001C-26	Device 621203C-34	Device 670901K-01	Unit
doping (N)	3.2×10^{14}	1.3×10^{15}	5×10^{14}	cm^{-3}
Insulator thickness x_i	0.62	0.26	0.208	micron
width Z	1200	1360	1040	micron
channel length L	50	6.6	70	micron
mobility μ	250	65	155	$\text{cm}^2/\text{V}.\text{sec}$

Table 6.1(b)

Silicon Gate Device Parameters

Parameter	Device A2-S	Device D2-S
doping (N)	10^{15}	10^{15}
insulator thickness x_i	0.1	0.1
width Z	520	250
channel length L	10	10
mobility μ	225	225

Figure 6.2(a) Comparison between measured and calculated g_d - V_d characteristics

Figure 6.2(b) Comparison between measured and calculated g_d - V_d characteristics.

Figure 6.3(a) Comparison between theory and measurements for device 631011C-26

exp points (....) are obtained from [28]

Present model (—)

Figure 6.3(b) Comparison between theory and measurements for device 670901K-01

exp. points (.....) are obtained from [28]

Present model (—)

Figure 6.3(c) Comparison between theory and measurements for device 621203C-34

exp. points (....) are obtained from [28]

Present model (—)

Figure 6.4 Comparison of the simple model and the complete model

Figure 6.5 I_d - V_d characteristics including breakdown

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

Two approaches have been presented to model the Insulated-Gate Field-Effect Transistors (IGFETs): the first is a one-dimensional solution which is capable of predicting device characteristics over a wide range of operating conditions ranging from subthreshold to strong inversion, and the second deals with the device behaviour in saturation.

In the first approach, knowledge that mobile carriers in the inversion layer can easily become degenerate and their distribution can only be properly described by quantum statistics is exploited. In particular Fermi-Dirac (FD) statistics predicts an inversion charge which spreads more deeply into the bulk with a lower peak concentration, than would be obtained on the basis of Maxwell-Boltzmann (MB) statistics, resulting in a channel of non-negligible thickness and making the familiar delta function (zero thickness) channel approximation inappropriate. The mobile charge distribution is approximated by a simple function which still retains the essential physical features of the FD distribution. For this charge distribution, employment of the "Gradual-Channel Approximation"

results in a simple solution of Poisson's equation in the space charge region. Closed form expressions are then obtained for various device parameters, describing their behaviour accurately over a wide range of operating conditions. A significant conclusion is the ability of the model to explain the behaviour of g_m and g_d at high gate voltages (namely the falling of g_m and the deviation from the straight line behaviour of g_d) without the need to use any empirical dependence of mobility on surface field (a constant mobility is used in the model).

The analysis of the saturation region involves two-dimensional modelling of the drain section of the space charge region. The drain section is treated as a volume obeying Gauss's law, thereby enabling the total charge contained within the region to be related to the integral of the electric displacement normal to its boundaries without having to consider the detailed distribution of charge. The outcome is an explicit expression for the drain field as a function of applied voltages, drain current and ionized charge in the drain section. The dependence of device characteristics in saturation on device geometry, processing parameters and applied voltages is directly expressed. Closed form solutions of the drain current and drain conductance are obtained by neglecting the mobile charge in the drain section (which is shown to be the case under a wide range of operating conditions). Excellent agreement with measured drain characteristics of I_d and g_d in saturation for various

device geometries is achieved. By accounting for the two dimensional effects at the drain, a simple expression for the drain avalanche breakdown voltage is obtained which exhibits the actual features of devices.

7.2. Model Applications and Future Work

The simplicity of the model makes it suitable for deriving small signal equivalent circuits^[32] and for use in computer-aided design programs where simple and accurate models are needed to provide a sound basis for design. The model can also be extended to characterize more general structures where the doping along the channel is non-uniform, e.g. IGT^[29] and DMOST^[30]. As the starting point of the approach is defining the space charge region parameters, it can be applied to various surface devices using the appropriate boundary conditions. Modelling of Charge Coupled Devices (CCD) presents a useful and important topic for applying the approach.

The approach presented in Chapter 4 offers a means by which a simple partial differential equation can be obtained to describe the distributed nature of any surface device and its transient behaviour. Previously, due to the complexity of the resulting partial differential equations, a quasi-static approach^[33] (in which the instantaneous distributions during transient are assumed to be a sequence of d.c. steady state distributions) was often used^[34] to reduce these equations to ordinary differential equations. To

illustrate how to obtain this equation, consider an element of a surface device shown in Fig. 7.1 where the substrate is taken as a reference.

The mobile charge per unit surface area, Q_m , is given by equation (4.9) as

$$Q_m = - \frac{kT}{q} \cdot \frac{C_i}{a} \cdot \frac{n^2 + \frac{n}{a} - U_g}{n} \quad (7.1)$$

where the different quantities are as defined in Chapter 4.

The continuity equation along the surface is given by

$$\frac{\partial}{\partial t} Q_m = \frac{\partial}{\partial y} I \quad (7.2)$$

where $I = D Q_m \frac{\partial \zeta}{\partial y}$ is the current per unit width along the surface.

Substituting for $\frac{\partial \zeta}{\partial y}$ from equation (4.14) into equation (7.2) we get

$$\frac{\partial^2 F}{\partial n \partial t} = - \frac{D}{a} \frac{\partial^2 F}{\partial y^2} \quad (7.3)$$

where

$$F = \frac{n^2}{2} + (2a + \frac{1}{a})n + (1 - U_g) \ln n$$

and

$$\frac{\partial F}{\partial n} = \frac{n^2 + (2a + \frac{1}{a})n + (1 - U_g)}{n} = \frac{n^2 + \frac{n}{a} - U_g}{n}$$

($a \ll 1$ for usual device parameters)

Equation (7.3) is the resulting partial differential equation which is to be solved for F using boundary conditions appropriate to the

device under consideration. Other parameters can then be obtained from F , e.g., Q_m is given by

$$Q_m = - \frac{kT}{q} \cdot \frac{C_i}{a} \cdot \frac{\partial F}{\partial \eta}$$

Figure 7.1 Element of a general surface device

7.3 Some Physical Implications of the Channel Region Model

The equivalent charge distribution used to represent the total charge in the channel region results in some consequences regarding parameters that depend on the detailed distribution of the individual components of charge. An example is the high frequency capacitance of an MOS capacitor, which is conventionally derived in terms of the depletion width based on the complete depletion approximation. In such derivation, mobile charge redistribution in the channel is neglected. Therefore, a given perturbation of surface potential would result in an equivalent perturbation of the ionized charge at the edge of the depletion region with no response of the inversion charge to that disturbance. By taking a spatial distribution of mobile carriers in the channel, redistribution of the mobile charge has to be treated. Therefore, for a given increment of ionized charge, the resulting change in surface potential will have two components; the first is due to the ionized charge and the second resulting from redistribution of inversion charge. The contribution of the two components will depend on the detailed spatial distribution of the two charge components. Further work is needed to obtain a quantitative analysis of such behaviour. This does not affect the model presented in Chapter 4 because such situation is not encountered in IGFET operation as source and drain diffusions enable the inversion charge to respond to high frequency disturbances.

APPENDIX A

THE SPACE CHARGE REGION THICKNESS

In this appendix we show the effect of having the mobile carriers spatially distributed over a finite thickness in the space charge region (as an approximate model of quantum statistics) as compared to the usual 'spike' function approximation (to classical statistics). Using quantum mechanical calculations in evaluating the carrier distribution and the thickness of the channel results in two considerable deviations from calculations based on classical statistics^[25]. These are:

- 1) Quantum statistics predict a mobile carrier distribution that^{is} much more uniform with distance normal to the surface, suggesting that a uniform carrier distribution is much more representative than a delta function distribution which is a good representation of classical statistics at the surface.
- 2) Channel thickness obtained from quantum statistics shows a weaker dependence on surface potential than the one predicted from classical statistics, Fig. 1. In fact the quantum mechanical thickness approaches a constant value for increasing surface potential, whereas the classical channel thickness decreases monotonically. (In ref. [25] this is explained

Fig. A-1. Channel thickness* obtained from quantum (—) and classical (-----) statistics (Gnadinger and Talley^[25]).

* Defined as the distance perpendicular to the interface in which 99 percent of the total mobile charge per unit area is located.

by the fact that increasing surface field shifts the quantized levels for mobile carriers up rather than narrowing the spatial distribution of mobile carriers).

Therefore, the mobile carrier distribution in the channel can be 'modelled' as a rectangle with variable height (concentration) and a thickness that is weakly dependent on gate voltage at low values and approaches a constant value at higher gate voltages. Across that channel, there is a potential variation (compared to zero voltage across the delta function channel and a very small voltage across the exact classical channel).

We shall evaluate that voltage drop for both the classical channel and the rectangular representation of the quantum mechanical channel for the same total surface charge. Fig. 2 illustrates the variation of the total charge (mobile and immobile) contained in the space charge region as obtained from classical statistics^[12]. At low values of U_s , the ionized charge is the dominant charge, while at higher values of U_s ($> 2U_f$) the channel charge takes over. Consider three points A, B and C on that curve. These points represent a condition where the classical depletion width has reached its maximum value, and any increase in the total charge Q_s is due to an increase in the channel charge. Points B and C are in the region where the quantum mechanical channel thickness is slowly varying as shown in Fig. A-1.

From point A to point B, the total charge Q_s changes from $0.312 \times 10^{12} q$ to $1.138 \times 10^{12} q$, the difference ($0.926 \times 10^{12} q$) being

Fig. A-2, Variation of the total induced charge as a function of surface potential.

the change of mobile charge in the channel. The change in surface potential to accomodate that charge is $(32-29) = 3 \frac{kT}{q}$.

If we now consider a uniform channel, at point B, (as a representation of the quantum mechanical channel) to produce the same change of potential for the same channel charge, then its thickness can be calculated as follows:

The number of mobile charges in this channel is given by:

$$n\chi_{ch} = \frac{Q_m}{q} = \frac{Q_{SB} - Q_{SA}}{q} = 0.826 \times 10^{12} \quad (A-1)$$

The potential drop across that channel is

$$U_{ch} = \frac{q^2}{kT\epsilon} n \frac{\chi_{ch}^2}{2} = U_{SB} - U_{SA} = 3$$

or

$$32 \times 10^{-7} n \chi_{ch}^2 = 3 \quad (A-2)$$

From equations (A-1) and (A-2), χ_{ch} is

$$\chi_{ch} = 113.5 \text{ \AA}.$$

According to Fig. A-1 the value of χ_{ch} at point B is approximately the same as at point C (for the quantum mechanical channel).

From point A to point C, the channel charges changes from $0.312 \times 10^{12} q$ to $5.55 \times 10^{12} q$. There the mobile charge at point C is given by:

$$n \chi_{ch} = \frac{Q_m}{q} = \frac{Q_{SC} - Q_{SA}}{q} = 5.238 \times 10^{12}$$

- The potential drop across that uniform channel is

$$U_{chC} = 32 \times 10^{-7} \text{ n } \frac{2}{ch} = 32 \times 10^{-7} \times 5.238 \times 10^{12} \times 113.5 \times 10^{-8}$$

$$= \underline{19}$$

The change of surface potential in Fig. A-2 is

$$U_{sc} - U_{sa} = 35 - 29 = 6$$

(of course this value is in agreement with classical statistics, since the classical channel thickness at C is much smaller than that at B).

Thus, for the quantum channel, if the space charge region still extends to the full depletion width with a potential drop across the depletion charge of $2U_f (= 25)$, resulting in a surface potential of 44, representing a band bending of $10 \frac{kT}{q}$ below the edge of the valence band, which would correspond to unrealistic mobile carrier concentration, which would violate Gauss's law (a surface potential at 44 corresponds to a surface density of 10^{24} by classical theory and a density of the same order by quantum theory).

Thus, the depletion region must narrow, thus reducing its contribution to the surface potential (reduction is proportional to (width)⁻²) without affecting Gauss's law.

In fact, another way of showing that is as follows:

In the region where the quantum mechanical channel thickness is approaching

a constant value, we can write

$$\begin{aligned} |Q_m| &= Q_g = \frac{kT}{q} C_i (U_g - U_s) \\ &= q n x_{ch} \end{aligned}$$

The potential drop across that channel is

$$U_{ch} = \frac{q^2}{kT\epsilon} n \cdot \frac{x_{ch}^2}{2} = \frac{x_{ch}}{6x_i} (U_g - U_s)$$

For the value of x_{ch} shown in Fig. A-1, i.e., $x_{ch} = 120 \text{ \AA}$, and for an insulator thickness $x_i = 1000 \text{ \AA}$, we have

$$U_{ch} = (U_g - U_s)/50$$

or

$$(U_g - U_s) = 50 \cdot U_{ch}$$

Thus, for U_{ch} to be equal to U_s in strong inversion which means a zero potential drop across depletion region, a gate voltage $U_g = 51 U_s$ is required. This represents a point in Fig. A-2 where $U_g = 40$ volts and $U_s = 36$.

The preceding argument is illustrated by means of schematic diagrams of charge density, electric field and electrostatic potential variations across the space charge region in the direction normal to the surface (i.e., graphical solution of the one-dimensional Poisson's

equation) in Fig. A.3. A number of situations that can arise depending on the applied voltages (basically the gate voltage) are illustrated in the Figure. These are: i) just above flat band condition ii) weak inversion condition and iii), iv) strong inversion condition.

In the 'spike' function approximation, Fig. A-3(a), the surface potential increases with increasing gate voltage until the strong inversion condition is reached at which point the surface potential stays fixed (or increases very slowly). Since the potential variation results only from the ionized fixed charges, the space charge region thickness reaches a maximum value at strong inversion and stays at that value independent of any further increase of gate voltage, this value being given by the depletion approximation.

Fig. A-3(b) illustrates the corresponding variation in the case of a finite channel thickness (i.e., mobile carriers spatially distributed). For near flat band (i) or weak inversion conditions (ii), the resulting potential variations are very nearly the same as in the spike function case, i.e., the space charge region thickness is given by the depletion thickness. However for strong inversion conditions (iii) and (iv) (where the mobile carrier density is high compared to the fixed charge density), major differences result. To get the same surface potential as in case (a), the area under the field distribution curves must be the same (viz. the area under field curve (iii) must be the same in both (a) and (b)). But since the slope is

Figure A-3 Distributions obtained for: (a) a spike function channel.
 (b) a channel with finite thickness.

different (being much higher in case (b) because of the distribution of mobile carriers), the thickness must be different too (smaller thickness in (b)). Thus, the effect of mobile carrier distribution is to result in a smaller space charge region thickness with the potential variations in strong inversion resulting mainly from mobile carriers. In addition, a further increase of gate voltage results in a smaller space charge region thickness (curve (iv) in Fig. A-3(b)).

APPENDIX B

DEFINITION OF EFFECTIVE POTENTIAL ζ

The electron current density at any point along the channel (including both drift and diffusion components) can be written as

$$J_n = -q D_n \cdot n \frac{\partial U}{\partial y} + q D_n \frac{\partial n}{\partial y}$$

The drain current I_d is given by

$$\begin{aligned} I_d &= z \int_0^{x_s} J_n \cdot dx \\ &= -q D_n z \int_0^{x_s} n \frac{\partial U}{\partial y} dx - \int_0^{x_s} \frac{\partial n}{\partial y} dx \end{aligned}$$

Using the expressions for U and x_s given by equations (4.1) and (4.4) in the text, we get

$$\begin{aligned} I_d &= -D_n z Q_m \left(\frac{2}{3} + \frac{x_s}{18x_i} \right) \frac{dU_s}{dy} - \frac{d}{dy} \log n \\ &= -D_n z Q_m \cdot \frac{d\zeta}{dy} \end{aligned}$$

where ζ is an effective potential (analogous to the Quasi-Fermi levels separation in the case of classical statistics) defined by

$$d\zeta = \left(\frac{2}{3} + \frac{x_s}{18x_i} \right) dU_s - d \log n$$

This expression can be integrated to give

$$n = A e^{U_s - \zeta} \cdot e^{\int (\frac{x_s}{18x_i} - \frac{1}{3}) dU_s}$$

Using equation (4.10) for n we get

$$\eta^2 + \frac{\eta}{a} - U_g + U_b = e^{(-\frac{\eta}{a} + U_g - \zeta)} \cdot f(\eta) \quad (B-1)$$

where

$$f(\eta) = \frac{A}{n_i} (U_g - U_b - \frac{\eta}{a}) \cdot e^{\int (\frac{x_s}{18x_i} - \frac{1}{3}) dU_s - U_f - U_b}$$

is a function, to be determined from considering the conditions at the source or the drain boundaries of the space charge region.

Now, to determine that function $f(\eta)$ and to define the potential ζ in terms of other potentials in the structure, account has to be taken of the fact that in the model a constant mobile carrier density has been assumed throughout the surface space charge region, though the potential is position dependent in this region in accordance with Poisson's equation. This is, of course, not in agreement with the Boltzmann relationship of PN junction theory between carrier density and potential. These two 'models' have to be reconciled when defining the conditions at the boundaries between the space charge region and both source and drain junctions.

A procedure involving the averaging of mobile carrier

density as a function of potential is performed by carrying out the integration $\int n dU$ over the limits of the space charge region in the x-direction on both sides of the boundary.* Taking, for example, the source boundary of the junction, the integration gives

$$\begin{aligned} U_s - U_b - U_f & \int_{U_b - U_f}^{U_s - U_{sr}} n_i e^{U - U_{sr}} dU \\ &= n_i e^{U_s - U_{sr} - U_b - U_f} \\ &= n_i e^{-\frac{\eta}{a} + U_g - U_{sr} - U_f} \end{aligned}$$

on the space charge side of the boundary we have

$$\begin{aligned} \int n dU &= \int_{U_b - U_f}^{U_s + U_b - U_f} n_i e^{U_f} \frac{\eta^2 + \frac{\eta}{a} - U_g + U_b}{U_s} dU \\ &= n_i e^{U_f} \left(\eta^2 + \frac{\eta}{a} - U_g + U_b \right) \end{aligned} \quad (B-3)$$

Equating (B-2) and (B-3) we get

$$\eta^2 + \frac{\eta}{a} - U_g + U_b = e^{-\frac{\eta}{a} + U_g - U_{sr} - 2U_f} \quad (B-4)$$

Comparing (B-4) and (B-1), we get the result

$$\begin{aligned} f(\eta) &= e^{-2U_f} \\ \zeta &= U_{sr} \end{aligned}$$

* As discussed in Section 4.2, $\int n dU$ is invariant for given surface conditions so that this quantity is the same for both the actual channel and the model.

Thus at the source boundary of the space charge region the effective potential ζ is equal to the applied source potential, and similarly at the drain boundary it is equal to the drain potential. Of course along the channel it will assume a variation between these two values, depending upon the values of n and U_s at each point (value of y).

Figure b-1 Conditions at the source junction boundary
(a) source junction
(b) potential distribution along B-B

APPENDIX C

DEFINITION OF THE BOUNDARY BETWEEN SOURCE AND DRAIN SECTIONS

The potential distribution in the source section is given by equation (4.1) in the text as:

$$U(x,y) = U_s \left(1 - \frac{x}{x_s}\right)^2$$

Differentiating twice w.r.t. x , we get

$$\frac{\partial^2 U}{\partial x^2} = \frac{2U_s}{x_s^2} = \frac{e^{U_f}}{2L_D^2} \cdot \frac{n^2}{U_s} \quad (C-1)$$

where the value of x_s given by equation (4.4) is used.

The longitudinal surface field is given by equation (4.24) as

$$E_{ys} = \frac{kT}{La} \cdot \frac{I_{dl}}{n + 2a + \frac{1}{a} + (1 - U_g + U_b)/n} = - \frac{kT}{q} \frac{\partial U_s}{\partial y}$$

Differentiating w.r.t. y

$$\frac{\partial^2 U_s}{\partial y^2} = \frac{I_{dl}^2 (n^2 + U_g - U_b - 1)n}{L^2 a [n^2 + (2a + \frac{1}{a})n + (1 - U_g + U_b)]^3} \quad (C-2)$$

The constant B is given by:

$$B = \frac{\partial^2 U_s}{\partial x^2} / \frac{\partial^2 U_s}{\partial y^2} \quad (C-3)$$

Equations (C-1) and (C-2) are substituted in equation (C-3), which can be solved for a given B (say 10) to give U_{s1} . Substituting U_{s1} in equation (4.24) and (4.17) yields E_{s1} and I_1 respectively.

APPENDIX D

DERIVATION OF THE CHANNEL SHORTENING MODEL EQUATION FROM THE PRESENT

MODEL

Starting with equation (6.15) and neglecting those terms that are not considered in the channel shortening model^[13], namely the normal field (Term (1)), effects of velocity saturation (Term (2)), the mobile carriers (Term (3)) and the field E_{s1} , we get

$$\frac{q}{kT} \frac{dI_d}{dU_{sd}} = g_d = \frac{I_d}{\frac{kT}{q} L} \cdot \frac{1}{\left[\frac{6q^2 N}{kT\epsilon_s} (U_{sd} - U_{s1}) \right]^{\frac{1}{2}}} \quad (D-1)$$

Separating the variables I_d and U_{sd} , equation (D-1) becomes

$$\frac{dI_d}{I_d} = \frac{dU_{sd}}{L \left[\frac{6q^2 N}{kT\epsilon_s} (U_{sd} - U_{s1}) \right]^{\frac{1}{2}}}$$

Integrating between the limits (I_1, U_{s1}) and (I_d, U_{sd})

$$\log \frac{I_d}{I_1} = \frac{1}{L \left[\frac{3q^2 N}{2kT\epsilon_s} \right]^{\frac{1}{2}}} (U_{sd} - U_{s1})^{\frac{1}{2}} = \frac{(U_{sd} - U_{s1})^{\frac{1}{2}}}{L \cdot \alpha}$$

where

$$\alpha = \left[\frac{3q^2 N}{2kT\epsilon_s} \right]^{\frac{1}{2}}$$

$$I_d = \frac{I_1}{e^{\frac{-(U_{sd} - U_{sl})}{L \alpha}}} \quad (D-2)$$

For the large values of α (e.g. for $N = 10^{15}$, $\alpha = 10^5$), we get

$$I_d = \frac{I_1}{1 - \frac{(U_{sd} - U_{sl})}{L}} = \frac{I_1}{1 - \frac{\Delta L}{L}}$$

giving

$$\Delta L = \left[\frac{2\epsilon_s kT}{3q^2 N} (U_{sd} - U_{sl}) \right]^{1/2} \quad (D-3)$$

Equation (D-3) is the same expression given by equation (6-2) except for the factor 3 at the denominator. This is due to the variation of potential normal to the surface considered in the present analysis, whereas in deriving equation (6.2) current flow is limited to a layer at the surface.

APPENDIX E

THE RELATION BETWEEN V'_g and U_g

The dependence of drain conductance g_d on the potential V'_g is given in ref. (28) as

$$g_d = \frac{\mu_n C_i Z}{L} (V_d - V'_g) \quad (E-1)$$

where V_d is the drain potential.

In Chapter 4, g_d is given by equation (4.18) (for zero substrate potential) as

$$g_d = \frac{D_n C_i Z}{L} \cdot \frac{1}{a} \left(\eta_d + \frac{1}{a} - \frac{U_g}{\eta_d} \right) \quad (E-2)$$

U_{sp} is defined as the normalized drain potential at which g_d , given by equations (E-1) and (E-2), is equal to zero. Thus from equation (E-1)

$$U_{sp} = \frac{q}{kT} \cdot V'_g \quad (E-3)$$

and from equation (E-2)

$$U_{sp} + \frac{1}{a} (U_{sp})^{\frac{1}{2}} = U_g \quad (E-4)$$

Substituting from equation (E-3) into equation (E-4), we get

$$U_g = \frac{q}{kT} V_g' + \frac{1}{a} \left[\frac{q}{kT} V_g' \right]^{\frac{1}{2}}$$

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