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A STUDY OF BONE DENSITY
MEASUREMENTS USING THE
PHOTON SCATTERING TECHNIQUE

by
MICHAEL W. WHITE

A thesis submitted to the Faculty of
Graduate Studies in partial fulfillment of the
requirements for the degree of Master of Science.

Carleton University
Ottawa, Ontario
May, 1977

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The undersigned recommend to the Faculty of Graduate Studies acceptance of the thesis "A STUDY OF BONE DENSITY MEASUREMENTS USING THE PHOTON SCATTERING TECHNIQUE, submitted by MICHAEL W. WHITE, B.Sc., in partial fulfillment of the requirements for the degree of Master of Science.

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ABSTRACT

A bone density system for the measurement of the radius is described. The technique involves the measurement of the intensity of photons scattered by the bone from a continuous x-ray beam.

Theoretical and experimental studies are made on the inherent sources of error in the bone densities. It is found that multiple scattering is the prime source of error, and a theoretical correction is successfully applied to the experimental data. The first results from human volunteers are presented.

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1 - INTRODUCTION.

The quantitative determination of bone mineral density has become a most active field of medical physics during the past ten years. The large amount of activity stems from the desire to measure accurately the mineral content of bones in routine diagnostic procedures. Although much information can be obtained using apparatus and techniques available today, there still exists the necessity and desire for greater precision and more versatility in the selection of bone location.

In studying the factors which affect the bone mineral content, one must consider the activity and stress, lactation, nutrition, medication, disease, age, sex and race of the subject. There have been a number of clinical studies examining the bone mineral content in people with osteoporosis (47), with recent fractures (49) and with primary hyperparathyroidism (18). As well, many other studies have been conducted to collect "normative data" (13, 30, 35).

These clinical studies have made use of a number of techniques as well as a number of different bone sites. Thus, it would be beneficial to summarize the different techniques employed in bone mineral determination with emphasis on assumptions made in addition to the various advantages and disadvantages of each. It should be emphasized that a high degree of precision and versatility in bone sites are possibly the two most important criteria in any technique.

The first attempts to make in vivo measurements of bone density have been based upon radiographic procedures in which a

photo-densitometric comparison is made between the investigated bone and a standard wedge. There has been much work done on these procedures (9, 31) and it has been shown that standard radiographs can only show changes in bone density greater than 30% (31). This single disadvantage is enough to say that this procedure is not a practical method to objectively study bone mineral content, since a much higher level of accuracy is required.

The most popular commercially available technique is the one developed by Cameron and Sorenson (1, 5), using direct photon absorptiometry. Cameron is quite careful to distinguish between the terms bone mass and bone density (5). This confusion started with the use of the term "bone densitometry" which describes studies in which film densitometry was used to analyze bone images on x-ray film. However, the densitometry of film was confused with the densitometry of bones. Absorptiometry techniques measure the quantity "bone mass". (By other techniques it is possible to measure the "mineral density of bone" which is the number of grams of bone mineral per cubic centimeter of bone volume.) This technique uses a well collimated photon beam which is supplied by a radioactive source, usually iodine -125 at 27.3 keV or americium -241 at 59.6 keV. Thus, for a photon beam of initial intensity I_0 , the intensity after the passage through a thickness T of tissue equivalent material is I_0^* : (1)

$$I_0^* = I_0 e^{-\mu_m^0 T} \quad (1-1)$$

where μ_m is the mass absorption coefficient of tissue and ρ_m is the density. The intensity I after passage through an equal thickness of bone mineral T_b and tissue T_m ; i.e., $T = T_b + T_m$, is given by:

$$\begin{aligned}
 I &= I_0 \exp (-\mu_m \rho_m T_m - \mu_b \rho_b T_b) \\
 &= I_0 \exp (-\mu_m \rho_m (T - T_b) - \mu_b \rho_b T_b) \\
 &= I_0 \exp (-\mu_m \rho_m T) \exp (-\mu_b \rho_b T_b + \mu_m \rho_m T_b) \\
 &= I_0^* \exp (-\mu_b \rho_b T_b + \mu_m \rho_m T_b) \quad (1-2)
 \end{aligned}$$

This can be rewritten in terms of the equivalent thickness of compact bone mineral as:

$$T = \ln (I_0^*/I) / (\mu_b \rho_b - \mu_m \rho_m) \quad (1-3)$$

By measuring I at closely spaced intervals across the bone, one obtains a series of equivalent thicknesses which by summing gives the equivalent cross-sectional area of compact bone mineral in the bone. Cameron states (1) that from the known density of bone one can determine the mass of bone mineral in a unit length of bone. This method requires accurate knowledge of the composition of the bone mineral to determine the absorption coefficients μ_b . As well, it is assumed that all non-bone mineral substances absorb radiation to the same degree. In equation (1-2), the substitution into I_0^* makes use of the assumption that the absorption coefficients for the tissue and tissue-equivalent substance are the same. The reproducibility of the developed apparatus seems to warrant a

large interest but it would seem desirable to have a technique with greater accuracy (4% to 7% error (6)) and more versatility in the bone site location. (The apparatus described for this technique uses the radius about 10 cm from the distal end (1, 6, 16).)

As an extension of the monochromatic absorptiometry method of Cameron, a dual energy technique has been developed. This method was developed hopefully to correct for errors in the Cameron technique due to the presence of adipose tissue (36). The technique gives an expression for bone mineral content, BMC(g/cm) in the following form:

$$\text{BMC (g/cm)} = \frac{\rho_b}{a} L_1 \left(\frac{cd}{ad} \right)^{-1} (1 - bL_2/dL_1) \quad (1-4)$$

where

$$a = \mu_{b_1} - \mu_{s_1}$$

$$b = \mu_{f_1} - \mu_{s_1}$$

$$c = \mu_{b_2} - \mu_{s_2}$$

$$d = \mu_{f_2} - \mu_{s_2}$$

and where the subscripts are defined by: b represents bone mineral, s represents soft tissue, f represents fat, and subscripts 1 and 2 refer to photon energies 1 and 2. L is defined as the experimental

quantity $\delta \sum_i \ln \left(\frac{I_o^*}{I_i} \right)$ (36) for the respective energy.

Although equation (1-4) requires the determination of only two constant ratios, $\frac{c}{a}$ and $\frac{b}{c}$; these ratios are subject to a certain amount of error. For example, an abrupt change in thickness of the sample can cause a resultant error in the ratios of up to 5% (41).

This method has the following substantial error sources when trying to correct for tissue: scattering, geometry and composition, Poisson statistical error and photon integrity (41).

A recent method has been developed by Oikkonen and co-workers (38) which uses the ratio of the peak areas of the coherently scattered photons (A_{coh}) and the Compton scattered photons (A_{compt}). It can be shown that the coherently scattered cross section is given by (21):

$$\sigma_{coh} = \text{const.} |F(q, Z)|^2 \quad (1-5)$$

where $F(q, Z)$ is the form factor. For elastic scattering this is found to be proportional to Z^2 (44). Thus the coherent cross section is proportional to Z^2 . The number of photons (and thus the area) from the incident beam that emerge after coherently scattering is proportional to the number of photons in the incident beam and the number of particles in the target (which is proportional to Z). Thus,

$$A_{coh} \propto Z^3 \quad (1-6)$$

The number of Compton scattered photons depends upon the number of photons in the incident beam, the Compton cross section and the

electron density (which is proportional to Z/A). Thus,

$$A_{\text{compt}} \propto Z/A \quad (1-7)$$

Thus, the ratio of these two peak areas can be used to calculate the density of a substance (38) as shown below:

$$\frac{A_{\text{coh}}}{A_{\text{compt}}} = \text{Const.} \frac{\bar{Z}^3}{\bar{Z}} \bar{A} \quad (1-8)$$

where $\bar{Z}$ and $\bar{A}$ are the average charge and mass numbers of the target substance. It should be noted that the constant in equation (1-8) is dependent upon attenuation coefficients of the coherent and Compton scattered photons. This technique does not account for any change in attenuation coefficients resulting in a change in thickness or material. It would seem important that an accurate determination of attenuation coefficients (both for coherent and Compton scattered photons) be made before it is possible to confidently use this technique irrespective of the amount of tissue overlying the measuring site.

The technique employed in this study (described in the next section) requires no specific knowledge of the attenuation or scattering through the tissue or bone. Since it requires no recalibration, the measurement of the density of bones of varying sizes and shapes is easily obtained. There are a certain number of assumptions required in this technique. These assumptions have to do with such things as multiple and coherent scattering, beam hardening and geometric considerations. This study examines the effects of the above and

wherever possible corrections for these errors are made. As well, an examination of the precision and accuracy of the apparatus is made. Finally, pre-clinical results from human volunteers are presented.

The technique used in this study uses the Compton scattered photons to measure density. This method was developed by Clarke and van Dyk (8) and also by Kennett and Webber (27, 28, 52). They found that measuring density with the use of scattered photons allowed the study of irregular shaped bones. Also, a detailed knowledge of the surrounding material is not required. The use of a continuous x-ray spectrum as the source was introduced by A. Mortimer (37). He found that by using x-rays rather than a radioactive source it would be possible to reduce the exposure time due to the increased source strength; study smaller bones since smaller collimators may be used; and use the x-rays to locate the area under study. The present study is a continuation of Mortimer's work using x-rays. Apparatus was developed to allow the use of human bones along with the necessary detection electronics.

2 - THEORY

(a) Compton Scattering

The Compton scattering process may be assumed to be an elastic collision between a photon and an electron (which is considered as an isolated particle) and is thus governed by the conservation of energy and the conservation of momentum.

Let the energy of the incident photon be $E_0 = h\nu_0$ and the momentum be $p_0 = \frac{h\nu_0}{c}$. It scatters from an electron, initially at rest at point a (see Figure 2-1). The scattered photon is emitted at an angle θ with an energy $E' = h\nu'$ and momentum $p' = \frac{h\nu'}{c}$, while the electron recoils at an angle ϕ with a kinetic energy T and momentum p .

The conservation laws are applicable to this two-body collision, regardless of the details of the interaction at the point of the collision. They give the following relationships:

$$\frac{h\nu_0}{c} = \frac{h\nu'}{c} \cos \theta + p \cos \phi \quad (2-1)$$

$$0 = -\frac{h\nu'}{c} \sin \theta + p \sin \phi \quad (2-2)$$

$$h\nu_0 = h\nu' + T \quad (2-3)$$

Using the relativistic relationship for the total relativistic energy,

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad (2-4)$$

Figure 2-1: Compton Scattering. Trajectories for the incident photon $h\nu_0$, scattered photon $h\nu'$ and scattered electron P, T .

which can be rewritten as:

$$pc = \sqrt{T(T - 2m_0c^2)} \quad (2-5)$$

it is possible to eliminate any two parameters from equations 2-1, 2-2 and 2-3.

The difference in wavelength between the incident and scattered photon, called the Compton shift, is found to be:

$$\frac{c}{\nu'} - \frac{c}{\nu_0} = \lambda' - \lambda_0 = \frac{h}{m_0c} (1 - \cos \theta) \quad (2-6)$$

The energy of the scattered quantum is:

$$h\nu' = \frac{m_0c^2}{1 - \cos \theta + (1/\alpha)} \quad (2-7)$$

where $\alpha = h\nu_0/m_0c^2$

The kinetic energy of the recoiled electron is:

$$T = h\nu_0 \frac{\alpha (1 - \cos \theta)}{1 + \alpha (1 - \cos \theta)} \quad (2-8)$$

The two scattering angles, θ and ϕ , are related through the equation

$$\cot \phi = (1 + \alpha) \frac{1 - \cos \theta}{\sin \theta} = (1 + \alpha) \tan \frac{(\theta)}{(2)} \quad (2-9)$$

The differential cross section for the scattering of the radiation with free electrons is given classically by the Thomson cross

section (derived in 15).

$$\frac{d\sigma_T}{d\Omega} = \frac{r_0^2}{2} (1 + \cos^2 \theta) \quad (2-10)$$

where $r_0 = \frac{e^2}{m_0 c^2}$ is the classical electron radius.

The correct quantum-mechanical calculation for Compton scattering gives the well-known Klein-Nishina cross section (29):

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \left\{ \frac{1 + \cos^2 \theta}{[1 + \alpha(1 - \cos \theta)]^2} + \left[1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right] \right\} \quad (2-11)$$

It should be noted that equation (2-11) reduces to the classical Thomson cross section (equation 2-10) for $\alpha \rightarrow 0$.

In this study, the maximum energy that a photon may have is about 130 KeV. In this case, $\alpha = .25$ which justifies the use of the classical cross section.

(D) Determination of Bone Density

The method used in this study was developed by Clarke and Van Dyk (8) and described as the photon scattering technique. This method is also examined by Kennett, Garnett, and Weber (8, 27, 52). This group has made use of the high-energy variation. This study uses the low-energy method. Both techniques are described by Clarke and Van Dyk (8).

The technique used in this study employs a single photon

source and two detectors, one to measure the count rate of transmitted photons and the other to measure the count rate of scattered photons. The configuration of the source and detectors is shown in figure 2-2(a). As can be seen in this diagram, the projection of the collimator holes defines a small volume within the sample under study. This volume is referred to as the sensitive volume.

The intensity of the incident beam as it impinges on the sensitive volume is $I_1 e^{-\mu_1 w}$. Where

I_1 is the intensity of source 1

μ_1 is the total linear attenuation coefficient for the incident beam

w is the distance indicated in figure 2-2.

The sensitive volume acts as a source of intensity $I_1 e^{-\mu_1 w}$. The transmitted photons will be attenuated over a distance x and will be detected at D_1 with a counting rate given by

$$T_1 = D_1 I_1 e^{-\mu_1 (w + x)} \quad (2-12)$$

where D_1 is a constant depending on the efficiency of detector 1.

The number of scattered photons for a given angle θ from the sensitive volume depends on:

- (a) the Compton cross section, σ_e
- (b) the intensity at the sensitive volume, $I_1 e^{-\mu_1 w}$
- (c) the electron density of the sample.

Since the electron density ρ_e is defined as:

Figure 2-2: Configuration of sources and detectors for bone density measurement. (a) initial configuration; (b) source moved to second location.

$$\rho_e = \bar{Z} \left(\frac{\text{electrons}}{\text{atom}} \right) N_a \left(\frac{\text{atoms}}{\text{mole}} \right) \rho_m \left(\frac{\text{gm}}{\text{cm}^3} \right) \frac{1}{A} \left(\frac{\text{mole}}{\text{gm}} \right) \quad (2-13)$$

where

$\bar{Z}$ is average atomic number.

N_a is Avagadro's number

ρ_m is density

A is average atomic mass

then the number of scattered photons is

$$n = k I_1 e^{-\mu_1 w} \sigma_e \frac{\bar{Z} N_a}{A} \rho_m \quad (2-14)$$

where k is a constant, or

$$n = \frac{\bar{Z}}{A} K I_1 e^{-\mu_1 w} \rho_m \quad (2-14')$$

where K is a more general constant.

There will be attenuation of these photons over the distance Z ,

resulting in counting rate at detector 2 of:

$$S_1 = D_2 I_1 e^{-\mu_1 w} \frac{\bar{Z}}{A} K \rho_m e^{-\mu_2 Z} \quad (2-15)$$

where D_2 is a constant depending on the efficiency of detector 2 and μ_2 is the total linear attenuation coefficient for the scattered beam.

The source is then moved to the second location S_2 in figure 2-2(b). The intensity of the source is the same as in the above consideration. In this configuration, the transmitted counting rate is:

$$T_2 = D_2 I_1 e^{-\mu_1(y+Z)} \quad (2-16)$$

The scattered counting rate is found as in equation (2-15) and becomes:

$$S_2 = D_1 I_1 e^{-\mu_1 y} K Z/A e^{-\mu_2 x} \rho_m \quad (2-17)$$

From equations (2-12), (2-15), (2-16) and (2-17) one obtains the ratio

$$S_1 S_2 / T_1 T_2 = K^2 \rho_m^2 \bar{Z}^2 / A^2 \frac{e^{-\mu_2(x+\bar{Z})}}{e^{-\mu_1(x+\bar{Z})}} \quad (2-18)$$

To simplify the above ratio, it is necessary to have $\mu_1 = \mu_2$. Since total linear attenuation coefficients are energy dependent (1), it is necessary that the transmitted and scattered beams have the same energy distribution. In this situation the ratio gives the relation:

$$(S_1 S_2 / T_1 T_2)^{1/2} = \bar{Z}/A K \rho_m \quad (2-19)$$

The constant in equation (2-19) is measured using a sample of known density, usually water. Care must be taken to include the values of $\bar{Z}$ and A for samples and standards. Thus, the four measured rates result in the desired quantity ρ_m .

(c) Multiple Scattering

The density as measured using equation (2-19) depends upon the number of photons which have undergone single scattering interactions within the sample. Thus, any detectable multiple scattered photon would result in a falsely high scattered count rate. The effect of multiple

scattering is a complex, highly geometric problem. Although this contribution can be evaluated through the use of Monte Carlo simulation, Kennett and Weber (27) point out that anatomical and density variations do not allow this technique to be beneficial. Thus a simplistic approach has been used to provide an estimate of the effect of multiple scattering. Assuming as a first approximation that the total linear attenuation coefficients are the same for the transmitted and scattered beams, and the attenuation distances x and Z are equal, then from equations (2-12) and (2-15) the true density, ρ_t , is proportional to the $S_{1t}:T_1$; i.e.

$$\rho_t \propto S_{1t}/T_1 \quad (2-20)$$

Now, the scattered count rate S_1 includes a multiple scattered count rate, say M , so

$$S_1 = S_{1t} + M \quad (2-21)$$

Thus, the observed density ρ_m is proportional to $S_1:T_1$ or

$$\rho_m \propto S_1/T_1$$

This can be written as:

$$\rho_m \propto \rho_t + \frac{M}{T_1} \quad (2-22)$$

From equation (2-12) it is seen that

$$T_1 \propto e^{-\mu_1 a}$$

where a is the total path length. Then

$$\rho_m \propto \rho_t + M e^{\mu_1 a} \quad (2-23)$$

The multiple scattering rate, M , can be assumed to be proportional to the total scattering within the sample (37). The total scattering within the sample would be $1 - e^{-\mu_1 a}$. The constant of proportionality will in part consist of a constant, say B , which depends on the geometry of the object. Thus,

$$M \propto B(1 - e^{-\mu_1 a})$$

Substituting the above into equation (2-23) gives

$$\rho_m \propto \rho_t + B(e^{\mu_1 a} - 1)$$

The constant of proportionality can be found using water. This would give the effect of multiple scattering by the equation

$$\rho_m = \frac{\rho_t + B(e^{\mu_1 a} - 1)}{1 + B(e^{\mu_1 a} - 1)} \quad (2-25)$$

where μ is the attenuation coefficient for water.

(d) Coherent Scattering

Equation (2-19) shows the density is inversely dependent upon the number of photons which have been transmitted through the sample. In the development of this equation it was stated that the sensitive volume acts as a source of intensity $I_1 e^{-\mu_1 w}$. As well, transmitted counting rate was shown to be proportional to $I_1 e^{-\mu_1 (w+x)}$ (see equation 2-12). In these expressions I_1 is the intensity of the source and μ_1 is the total linear attenuation coefficient for the

incident beam. The total linear attenuation coefficient is defined as the sum of the attenuation coefficients for the Compton process, photoelectric interaction and the pair-production collision (15). But there exists another process which is not included in the total linear attenuation coefficient - coherent scattering. Since a coherently scattered photon is not taken into account, the transmitted count rate is too low.

To see the effect of coherent scattering, it is assumed that the total linear attenuation coefficients are the same for the transmitted and scattered beams, and the attenuation distances x and z are equal (these are identical to the assumptions made for the treatment of multiple scattering). Employing the above conditions on equations (2-12) and (2-15), the true density, ρ_t , is proportional to the ratio $S_1:T_{1t}$; i.e.

$$\rho_t \propto \frac{S_1}{T_{1t}} \quad (2-26)$$

The true transmitted count rate T_{1t} includes a coherent scattered count rate, C . Thus,

$$T_{1t} = T_1 + C \quad (2-27)$$

The observed density ρ is proportional to $S_1:T_1$ which can be written with the use of equation (2-27) as

$$\rho_m \propto \frac{S_1}{T_t - C} \quad (2-28)$$

Equation (2-28) can be rewritten as (with L as the constant of

proportionality):

$$\rho_m = L \rho_t \frac{(T_1 + C)}{T_1} \quad (2-29)$$

The coherent scattering rate, C , is proportional to $1 - e^{-\sigma_c a}$. The constant of proportionality, H , depends upon the geometry of the system and the detection electronics. Thus, C can be written as

$$C = H (1 - e^{-\sigma_c a}) \quad (2-30)$$

where σ_c is the coherent scattering cross section and a is the path length.

With the use of equation (2-30), equation (2-29) can be rewritten as

$$\rho_m = L \rho_t \frac{(T_1 + H(1 - e^{-\sigma_c a}))}{T_1} \quad (2-31)$$

The constants in equation (2-31) are measured using samples of known density.

(e) Geometric Effects

The densities given by equations (2-18) and (2-19) do not take into consideration any effects of geometry. The size of the sensitive volume, although ideally infinitesimally small, is finite and this finite size must be examined for its contribution to the overall error. As well, the resolution of the technique should be studied as a function of both energy and the ratio of the radii of bone to fat.

Knowledge of the actual geometry of the sample at the point of the intersection of the beams (that is the sensitive volume) would have little effect on the amount of information gained by this technique. Unlike electrons, which would produce a much more intense beam for a perpendicular blow than for a glancing blow (significant of a sloping surface), the only difference a glancing blow would make would be the amount of attenuation of the incident beam.

(i) Resolution

The measured density, before any assumption regarding the similarity of the energy distribution of the scattered and transmitted beams is made, is determined from equation (2-18). To examine the resolution of density as a function of both energy and various radii, it is necessary to use equation (2-18) on a beam of total incident intensity I_0 impinging a two-substance body of radii, r_1 and r_2 , as shown in figure (2-3). In this particular problem, r_1 is the radius of the bone under study and r_2 is the total radius of bone and other substances. For simplicity is assumed that the section under study is a two-substance body, consisting of bone mineral and fatty tissue. Under such an assumption, thickness of the fatty tissue (x) is simply the difference $r_2 - r_1$.

Applying equation (2-18) to the situation portrayed in figure (2-3), one finds.

$$\frac{S_1 S_2}{T_1 T_2} = K^2 \frac{Z^2}{A^2} \rho_m^2 \frac{e^{-\mu_3 r_1} e^{-\mu_4 (r_2 - r_1)}}{e^{-\mu_1 r_1} e^{-\mu_2 (r_2 - r_1)}} \frac{e^{-\mu'_3 r_1} e^{-\mu'_4 (r_2 - r_1)}}{e^{-\mu'_1 r_1} e^{-\mu'_2 (r_2 - r_1)}} \quad (2-32)$$

This can be rearranged to give

$$\rho_m = \rho_m^1 \left(\frac{e^{-\mu_3 r_1} e^{-\mu_4 (r_2 - r_1)} e^{-\mu'_3 r_1} e^{-\mu'_4 (r_2 - r_1)}}{e^{-\mu_1 r_1} e^{-\mu_2 (r_2 - r_1)} e^{-\mu'_1 r_1} e^{-\mu'_2 (r_2 - r_1)}} \right)^{\frac{1}{2}} \quad (2-33)$$

where ρ_m^1 is the density using the assumption resulting in equation (2-19).

Before equation (2-33) can be used, an examination of the attenuation coefficients is required. A look at figure (2-3) shows that the difference between the primed coefficients and unprimed coefficients simply results from geometrical configuration. Thus, the primed coefficients can be considered equal to their respective unprimed coefficients, as long as radial symmetry exists. In such a case, equation (2-33) can be rewritten as:

$$\rho_m = \rho_m^1 \frac{e^{-\mu_3 r_1} e^{-\mu_4 (r_2 - r_1)}}{e^{-\mu_1 r_1} e^{-\mu_2 (r_2 - r_1)}} \quad (2-34)$$

This can be rewritten in the form

$$\rho_m = \rho_m^1 e^{-(\mu_2 + \mu_3 - \mu_1 - \mu_4) r_1} e^{-(\mu_4 - \mu_2) r_2} \quad (2-35)$$

The error in this density is found by using elementary techniques to be

(a)

(b)

Figure 2-3: Geometry for resolution study. The geometry for the initial and second configuration. In this particular situation r_1 is the radius of the bone and r_2 is the radius of the bone plus fatty tissue.

$$S_{\rho_m} = e^{-\mu^+ r_1} e^{-\mu^x r_2} (S_{\rho_m}^2 + \rho_m'^2 \mu^{+2} S_{r_1}^2 + \rho_m'^2 \mu^{x2} S_{r_2}^2)^{\frac{1}{2}} \quad (2-36)$$

where

$$\mu^+ = \mu_2 + \mu_3 - \mu_1 - \mu_4$$

$$\mu^x = \mu_4 - \mu_2$$

The fractional error would then be

$$\begin{aligned} \frac{S_{\rho_m}}{\rho_m} &= \frac{1}{\rho_m'} (S_{\rho_m}^2 + \rho_m'^2 \mu^{+2} S_{r_1}^2 + \rho_m'^2 \mu^{x2} S_{r_2}^2)^{\frac{1}{2}} \\ &= \left(\frac{S_{\rho_m}^2}{\rho_m'^2} + \mu^{+2} S_{r_1}^2 + \mu^{x2} S_{r_2}^2 \right)^{\frac{1}{2}} \end{aligned} \quad (2-37)$$

In equation (2-37), the terms μ^+ and μ^x are both constants.

Appendix 1 shows the calculation of these two terms. Placing these terms into equation (2-37)

$$\frac{S_{\rho_m}}{\rho_m} = \left(\frac{S_{\rho_m}^2}{\rho_m'^2} + .0276 (S_{r_1}^2 + S_{r_2}^2) \right)^{\frac{1}{2}} \quad (2-38)$$

The error in measuring the radii from an x-ray picture is small, probably of the order of a few tenths of a millimeter (say three), so that

equation (2-38) can be rewritten as:

$$\frac{S_{\rho_m}}{\rho_m} = \left(\frac{S_{\rho_m}^2}{\rho_m^2} + 4.97 \times 10^{-5} \right)^{\frac{1}{2}} \quad (2-39)$$

As can be seen from equation (2-39), the relative error in ρ_m consists of simply the relative error in ρ_m^1 plus a constant. The value of ρ_m^1 for any measurement is given by equation (2-19):

$$\rho_m^1 = k \left(\frac{S_1}{T_1} \frac{S_2}{T_2} \right)^{\frac{1}{2}}$$

As a first approximation one can assume the ideal situation of, having the scattered count rates equal and the transmitted equal (that is, $S_1 = S_2$ and $T_1 = T_2$). In this case, ρ_m becomes

$$\rho_m^1 = k \frac{S}{T}$$

In this situation, the relative error in ρ_m^1 is:

$$\frac{S_{\rho_m^1}}{\rho_m^1} = \sqrt{\left(\frac{S_s}{S} \right)^2 + \left(\frac{S_T}{T} \right)^2 + \left(\frac{S_k}{k} \right)^2} \quad (2-40)$$

The error in the number of counts of both the transmitted and scattered beams is $\sqrt{N}$, where N is the number of counts. Thus, equation (2-40) becomes

$$\frac{S_{\rho_m^1}}{\rho_m^1} = \sqrt{\frac{1}{S} + \frac{1}{T} + \left(\frac{S_k}{k}\right)^2}$$

which can be substituted into equation (2-39) to give

$$\frac{S_{\rho_m^1}}{\rho_m^1} = \left(\frac{1}{S} + \frac{1}{T} + \left(\frac{S_k}{k}\right)^2 + 4.97 \times 10^{-5}\right)^{\frac{1}{2}} \quad (2-41)$$

It should be remembered that k is a calibration constant which is determined using a sample of known density; thus, the term $(S_k/k)^2$ is a constant number. If the amount of fat is increased the transmitted rate, T , will decrease and the scattered rate, S , will increase. If the transmitted rate decreases by an amount W , the scattered rate increases by an amount $4\pi A r^{-2} W$ (for a detector to sensitive volume distance of 18 cm and a cross sectional area of $.01 \text{ cm}^2$, the new scattered rate becomes $S + 3.88 \times 10^{-4} W$). Note that the transmitted rate decreases faster than the scattered rate increases and that the term T^{-1} dominates in equation (2-41).

(ii) Finite Geometry

The actual size of the sensitive volume would probably effect the measured density. In the previous work, no mention was made of the differences in attenuation resulting from a finite beam size rather than infinitesimally small. This was examined by Kennett and Webber (27) and can be explained qualitatively with the assistance of figure (2-4).

Figure 2-4: A model to study the finite size of the sensitive volume. The average transmission along paths CC' and DD' does not equal the transmission along AOB' .

If one considers a scattering volume of cubic shape (each side of length $2a$), the average attenuation resulting from complementary rays CC^1 and DD^1 does not equal that of the central ray AOB^1 ; even though the average path lengths are equal. This difference occurs from the nonlinear function between attenuation and path length.

Since this effect would increase with absorption, densities greater than that of the standard would appear slightly larger than the true value. It has been predicted (27) that this effect is actually quite negligible - it is expected to increase the density by about 1%.

Not only does the beam produce a nonlinear factor due to its finite size, but an error also results from a difference in size of the transmitted and scattered beams. Kennett and Webber (27) examined the effect of this error. If one considers a cylindrical object as in figure (2-5), it is seen that the volume of the transmitted beam is smaller than that of the scattered. The transmitted beam is defined by the detector collimator used for its measurement. The scattered beam is defined by both the X-ray source and detector collimators. It has been predicted (27) that the measured density increases by less than 1%.

Figure 2-5: A cylindrical sample has a narrower transmitted beam than scattered beam.

3 - EXPERIMENTAL APPARATUS

The apparatus used in this experiment can be classified under two main sections: the detection electronics and the bone density apparatus. The detection electronics includes the source of photons as well as all detection and gating circuitry. The bone density apparatus includes collimators, sample positioning and all other mechanical components. In both cases it must be remembered that the apparatus was designed for easy clinical usage and yet to obtain a high degree of accuracy and reliability.

(a) Detection System

The X-ray source used was a Picker portable unit capable of producing 90 kV rms at a maximum current of 15 ma. The energy spectrum of the unit was shown in figure A - 3. A four millimeter thick aluminum filter was used in the measurement of this spectrum.

This X-ray unit makes use of a self-rectified circuit, which means the X-ray tube supply voltage is half-rectified. The X-ray pulse occurred during the negative portion of primary transformer voltage (as shown in figure 3 - 1 (a) and (b)). The X-ray intensity was observed to be similar to that described by Johns and Cunningham (52) and is shown in Figure 3 - 1 (c).

It was found that the intensity of the X-ray unit was very large even at low current settings. The number of equivalent transmitted pulses was so large that it would have required approximately 85 msec for the photomultiplier tube to count a single X-ray burst. But, due to the half-rectification of the tube circuit, there is only

some 8 msec. between bursts. Thus, one expects severe pile-up in the photomultiplier tube for the transmitted beam. This was observed and was the motivation for measuring total charge rather than using the usual counting techniques.

The detection system can be considered to consist of two major segments: the analog circuit which measured the total charge and logic circuit which gated all pulses and acted as an accurate timer of X-ray exposure. Following is a description of each of the two functions.

All X-ray information was detected using a Cs I (Na) crystal ($3/4" \times 1"$) which was optically coupled to an RCA 4802 photomultiplier tube. This is 0.75" in diameter and 3.8" in length. It is a twelve stage type having an S-11 spectral response. The transmitted beam was attenuated by a one millimeter piece of lead to prevent saturation of the PMT.

The analog electronics consisted of a signal attenuator to reduce the current from the transmitted photomultiplier tube (for the scattered photomultiplier tube this attenuator was off), a constant gain amplifier, an integrator and a digital meter to measure the output. An outline of the circuit is shown in figure 3-2 (a) and the complete circuit can be found in Appendix 2. The circuit is identical for both the transmitted and scattered beams with the addition of the attenuation for the transmitted beam.

The negative pulses from the photomultiplier tube are inverted and amplified using an LH0024⁺ operational amplifier. This amplifier

+ all numbers refer to National Semiconductor components

Figure 3-1: Graphs showing the variation with time of the primary voltage (a), the half-rectified tube voltage (b), and x-ray intensity (c).

was chosen because of its high slew rate ($500 \text{ V}/\mu\text{s}$) and thus its capability to receive the fast pulses from the scattered photo-multiplier tube. The gain of this amplifier was set to approximately 200 (as can be seen from the circuit in Appendix 2) and the output pulses were found to possess structure which resembled individual input pulses and showed no sign of saturation. Figure 3 - 3 shows the output pulse for the transmitted beam and figure 3 - 4 shows the scattered pulse. In both cases, the X-ray beam was incident upon the phantom hand which will be described in the next chapter.

The output of the amplifier is gated by an analog switch (AH0014) which receives its information from TTL pulses which originate from the point labeled (a) on the logic circuit (figure 3 - 2(b)). The importance of this logic pulse will be explained in the discussion of the logic circuit.

After the analog switch the amplifier output passes through a diode after which it is integrated. The diode is included since its high reverse resistance will minimize leakage current from the integrator in that direction. The integrator is followed by a buffer amplifier. The operational amplifier 8007 C was chosen for its high input impedance (one million Mohm).

The output of this amplifier is connected to an HP 3476 A digital multimeter (in the DC voltmeter mode). In this mode its input resistance is 10 Mohm and its quoted accuracy is 0.5% of reading $+ 0.2\%$ of range. This meter is capable of reading to tenths of a millivolt.

Figure 3-2 (a): Outline of the analog circuit.

Figure 3-2 (b): An outline of the logic circuit.

Figure 3-3: Transmitted beam after amplification.

Figure 3-4: Scattered beam after amplification.

The output of the buffer (8007 C) is also connected to the noninverting input of the LH-0024 operational amplifier through a variable resistor to produce a noninverting amplifier of gain one. This positive feedback becomes necessary because of the method of integration that is used. With the integrating capacitor connected to ground, there is a dc level at point (B) in the circuit which increases as more and more data reaches it. But, the diode will only allow to pass through it pulses that are more positive than the dc level at its cathode. Thus, as the data is being integrated, less and less of the following data will pass through the diode and be included. With the inclusion of the positive feedback of unity gain, there is an equal dc level at the anode of the diode compared to that at the cathode. Thus, all the data will pass through the diode and be integrated. It should be mentioned that the sum of the dc level and the amplified pulse height should never be allowed to be large enough to saturate the amplifier (LH 0024) (about 10 V).

Finally, it was found that the fast amplifier has a highly temperature - sensitive dc offset. Since, it is not desirable to integrate this, a compensating circuit had to be added to minimize the offset.

The purpose of the logic circuit is two-fold: to gate all analog data and to act as an accurate exposure timer.

Since it is known that the X-ray pulses occur during the negative portion of the transformer voltage, this voltage can be used to monitor the X-ray pulse and thus only collect the desired data.

The X-ray monitor is shown in the logic circuit diagram, figure 3 - 2(b). It is the input of a 741 operational amplifier, whose saturated output provides a square wave which is used for the logic functions.

This output provides the basis of the logic input pulse for both the variable pulse shaping and the various scalers. The first scaler is a divide by six scaler, which produces an output pulse for every six input pulses. Since there are sixty pulses a second, this allows an output pulse every tenth of a second, which is a convenient choice for timing. The purpose of the skip scaler is to have the first twelve X-ray pulses not included in either the data collection or the collection time. This decision resulted from studies (54) which showed that the first few X-ray pulses were both different in energy spectrum and inconsistent when compared with the pulses obtained after the X-ray unit settled down from any initial shock. All timing was done by comparing (at the toggle switches) with the actual number being counted by the timer scalers. When these are equal a pulse is produced from the comparator which resets the flip flops to prevent any other pulse from being included and also shuts off the x-ray unit.

It was found to be desirable to have control over the shaping of the eight millisecond X-ray pulses. In this way it is possible to have more uniform pulses which approach a square pulse by eliminating any low energy tails. This was made possible by using 555 timers as a variable pulse delay and a variable pulse width. In this study, the eight millisecond pulse which had a two millisecond delay (shown in figure 3 - 5(b)) was generated from the back edge

of the pulse delay.⁺ It was set for a four millisecond width. Thus, all collectable data occurred during this pulse and only this data was integrated by the analog circuit. Figure 3 - 6 shows this four millisecond gate pulse along with the transmitted X-ray pulse. The gating of the transmitted beam after amplification is shown in figure 3 - 7 and that of the scattered beam in figure 3 - 8. The net result of this gating system was the integration of a four millisecond X-ray pulse every sixteen milliseconds. This also allowed only four milliseconds of background to be subtracted.

As well as control over each individual pulse, there is a fixed delay of a fraction of a second after pressing the start button. This delay is to prevent any noise from the push button from reaching the logic circuit which could result in false counts or levels. Finally, there is about a one second delay before the logic circuit is initiated which allows the resetting of all flip flops and scalers.

The single start push button starts both the electronics and the X-ray unit. Also, a light was included as an exposure indicator.

The linearity of the system was examined using an intensity versus $(1/r)^2$ test. A Tc^{99m} source (140 keV) was used for both a counting test and a voltage test. For the counting test, the PMT signal went straight to a dual discriminator (EG & G T105/N) which fed to a counter (Fairchild 8200). Figure 3 - 9 shows the linearity of this test. The electronics used in this study was also tested for linearity. In this case, the PMT signal went straight to its analog input. These results are shown in figure 3 - 10.

⁺ For a complete timing diagram see Appendix III.

Figure 3-5: (a) Delay pulse positions the gate pulse.
(b) Gate pulse enables integrator.

Figure 3-6: The X-ray pulse from the transmitted PMT with the gate pulse.

Figure 3-7: The amplified transmitted beam with gate pulse.

Figure 3-8: The amplified scattered pulse with gate pulse.

Figure 3-9: A linearity test of intensity versus the inverse distance squared, with intensity measured by counting.

Figure 3-10: A linearity test of intensity versus the inverse distance squared, with intensity measured by integration.

(b) Bone Density Apparatus

The bone density apparatus was designed to be functional for a variety of conditions; such as a change in the scattering angle or different bones under examination.

The main mechanical device consists of three pairs of concentric rings supported by means of three posts. These can be seen in figures 3 - 12 and 3 - 13. The outer rings which are 40 cm in diameter are fixed in position by means of the support rods. They are graduated in one degree steps. Figure 3 - 11 shows a top view of the apparatus, in which the scale may be noticed. The inner rings (which can be seen in figure 3 - 11) possess pointers to the scale and also are movable. The purpose of having movable rings is to have the freedom of choice of the scattering angle. Each ring is 16 cm in diameter and attached to the two upper ones is a pair of collimators separated by 180 degrees. In each pair one is a detector collimator and one a source collimator. Figure 3 - 11 shows the tips of the four collimators aligned for a scattering angle of 90° .

Figures 3 - 12 and 3 - 13 are two side views of the apparatus. Figure 3 - 13 shows the phantom hand in position for a measurement. The support rods shown in these figures are 30 cm in height, but provisions have been made to alter this height if it becomes necessary. Such a situation could possibly occur if the desired bone under examination is not easily accessible or of some strange geometry. To complement this property, each of the four collimators with their respective detector or X-ray source are adjustable in height.

Figure 3 - 11: A top view of the bone density apparatus. The outer ring is fixed by means of support rods, while the inner ring (with a pointer) is moveable, thus allowing a change in scattering angle.

Figure 3 - 12: A side view of bone density apparatus.

Figure 3 - 13: A side view of the bone density apparatus with the phantom hand in its proper location.

Figures 3 - 12 and 3 - 13 show the collimators midway between the two upper rings.

The lowest ring possesses the mounting for the X-ray unit. This can probably best be seen in figure 3 - 12. The X-ray unit is mounted to this ring in such a manner as to permit free movement of the unit between the source collimators. The total movement of the X-ray unit is limited by the support rods to just over ninety degrees. Since the X-ray unit must be moved from source detector to source detector smoothly and accurately, a number of times, two movable stops have been placed on the edge of the second ring (figure 3 - 12). These stops allow consistently accurate re-alignment of the X-ray unit with a single careful alignment.

The four collimators are all 8 cm long and 3 cm in diameter. They consist of brass cylinders of wall thickness 0.5 cm. Each collimator contains two lead plugs (which are interchangeable) of thickness 0.8 cm and various aperture sizes (for this study three hole diameters were used, 0.35, 0.5 and 0.65 cm). With this design there is a large cavity between lead plugs which limits multiple scattering within the collimator.

Various accessories are added to the basic bone density apparatus for various situations. Some of these can be seen in figures 3 - 12 and 3 - 13. The basic frame which is used to locate the point of interest in a particular bone is shown in use in figure 3 - 13. This frame is free to move in the x, y, and z direction. There can be seen a plastic frame which is used in conjunction with two rubber straps to keep the hand free of movement. The actual set-up for patients is explained in the next chapter.

4 - EXPERIMENTAL METHOD

A number of experiments were performed to examine the reliability and accuracy of the apparatus as well as the physics of the method.

The physical density of a number of samples was measured first by gravimetric techniques and then by the Compton scattered photons method. Each sample was placed in a plastic vial (diameter 2.7 cm and height 6 cm) and placed in the bone density apparatus. For each measurement the 0.5 cm collimator was used with an exposure time of five seconds. Throughout this study water was used as the standard. The effect of multiple scattering and coherent scattering was examined by using a set of dextrose solutions with densities varying from just greater than 1 to about 1.3 g. cm^{-3} . Dextrose was chosen as a test substance for the effect of multiple scattering since the effect would be enhanced.

The effect of the size of the container in which the sample had been placed was examined. Three sizes of plastic vial were used. The largest was 2.7 cm in diameter and 6 cm in height. The middle vial was 1.6 cm in diameter and 5.6 cm in height; while the smallest had a 1.15 cm diameter and a 2.4 cm height. For each vial the density of glycerin was determined after standardization with water.

Glycerin was again used to study the effect of varying the size of the aperture of the collimator. In each of the three aperture diameters (0.35, 0.5, and 0.65 cm) the largest vial was used.

Different thicknesses of lead were used to examine any effect that beam hardening has on the measurement of the density. In this experiment, glycerin was placed in the largest sample bottle and the 0.5 cm collimator was used.

A Variac was used to vary the line voltage supplied to the X-ray unit. The line voltage was varied between 105 V to 120 V and monitored with a Weston 1240 digital multimeter. Water was placed in the largest vial, and an examination of the constants obtained was made.

A phantom hand (known as Charlie) was placed in the bone density apparatus and a scan was performed. Charlie is a 3M Phantom consisting of adult human bones, embedded in transparent, non-granular plastic. It has been advertised that this plastic has the same absorption and secondary radiation-emitting characteristics as living tissue. The bone marrow was filled with tissue equivalent plastic, thus having Charlie equivalent to a subject's hand. In this experiment, Charlie was placed as shown in figure 3 - 13 with the exception that a clamp was used rather than the straps to allow for an easy scan. The scan was performed in a plane 1.85 cm from cut plane. This plane included the radius and ulna. The scan was carried out in 0.5 cm increments. Due to the awkward shape of Charlie a complete scan was not possible. An idea of the location of the sensitive volume was obtained by using perpendicular locating rods. Figure 4 -- 1 shows a typical example of the positioning of the hand.

The X-ray beam was incident upon the hand from the bottom collimator and the right-hand collimator.

A pre-clinical test was performed on nine healthy university students (all male) who were knowledgeable of the nature of the experiment and freely volunteered. The density of each volunteer was measured each day for five consecutive days. The reason for the daily repetition was to get some idea of the error which results from the repositioning of the wrist.

The density was measured 4 cm from the styloid process of the radius bone. Measurements to locate the part of the bone under study were made mechanically using calipers. A mark was placed on the wrist at this point for the purpose of repositioning. The location of this mark was verified through the use of standard x-ray pictures.

The wrist was placed in a holder which contained foam rubber on either side to minimize lateral motion. This was enhanced by the use of two straps across the wrist. The hand was free to grip a bar which minimized movement of the hand. The positioning of the subject's hand is shown in figure 4-2.

Figure 4 - 1: A top view of the bone density apparatus with the phantom hand in position. The perpendicular rods were used to give some idea of the position of the sensitive volume.

Figure 4 - 2: A view of the bone density apparatus showing the positioning of a volunteer's wrist.

5 - RESULTS AND DISCUSSION

In the previous chapter a number of experiments were discussed. The purpose of these experiments was to examine the accuracy and reproducibility of the apparatus as well as to examine various contributions to the uncertainty in the measurements. As measurement of the accuracy a number of trials were performed for any quoted density. The uncertainty attached to these densities is the statistically determined standard error. Following are the results of the various experiments.

(a) Measurement of Density

Using the previously described apparatus, measurements of the density of various liquids were made, following the method outlined in the last chapter. The actual densities were found using gravimetric techniques.

The measured densities were calculated from equation 2-19. Table 5-1 gives the measured densities for a number of liquid samples. The number between parentheses is the standard error associated with the particular density. In all of these measurements, water was used as the standard. The uncertainty on the value of the constant was always found to be less than two per cent.

None of these densities had any corrections applied to them. It was believed worthwhile to quote these densities before examining any of the potential corrections so that an appreciation of the variety of possible samples as well the accuracy could be obtained. Of the seven samples, three had a measured density more than one standard

TABLE 5-1: MEASUREMENT OF DENSITY USING BOTH GRAVIMETRIC AND SCATTERED PHOTON TECHNIQUES.

<u>Sample</u>	<u>Actual Density (ρ_a)(gcm⁻³)</u>	<u>Measured Density (ρ_a)(gcm⁻³)</u>
$\text{CH}_3\text{CO}\cdot\text{CH}_3$ (acetone)	0.791	0.790 (0.025)
CCl_4	1.587	1.511 (0.027)
$\text{C}_6\text{H}_{12}\text{O}_3$ (ethylene glycol monoethyl ether acetate)	0.971	0.937 (0.020)
H_2SO_4	1.840	1.870 (0.032)
C_6H_6 (benzene)	0.878	0.893 (0.017)
HCl	1.186	1.130 (0.032)
$\text{HOCH}_2\text{CHOHCH}_2\text{OH}$ (glycerin)	1.247	1.288 (0.054)

deviation away from the actual density. Of these, only CCl_4 had its measured density three standard deviations from the actual.

(b) Multiple Scattering

As outlined in the theory (section 2-c) the scattered measurement is falsely high due to multiple scattering. Equation (2-25) gave the effect of the multiple scattering on the measured density in terms of the true density, attenuation coefficients and the path length that the x-ray travels. Qualitatively, equation (2-25) shows that as both the density and sample size increase, the effect of the multiple scattering is enhanced.

To examine the effect of the multiple scattering dextrose solutions of varying densities were used.

The measured densities for the various solutions are shown in Table 5-2 and are plotted against the actual density (again determined by the gravimetric method) in Figure 5-1.

Figure 5-1 shows that the observed densities deviate significantly from the identity line, and the amount of deviation generally increases with increasing density, which is the expected result when examining multiple scattering.

Equation (2-25) was used to determine the predicted measured densities (denoted by ρ_m). The attenuation coefficients were determined as previously. Water was used to determine the constant of proportionality, as explained in Chapter 2. The geometry-dependent constant B was found

TABLE 5-2: Experimentally Measured Densities (ρ_e) and Predicted Measured Densities (ρ_m) of the Dextrose Solutions.

Percentage	Actual Density ρ_t (g cm ⁻³)	Experimentally Measured Density ρ_e (g cm ⁻³)	Predicted Measured Density ρ_m (g cm ⁻³)	μ_m a cm ³ g ⁻¹
5	1.019	1.016 (.015)	1.023	.6414
16	1.066	1.156 (.026)	1.081	.6350
25	1.101	1.135 (.020)	1.123	.6302
35	1.142	1.174 (.023)	1.172	.6246
44	1.181	1.271 (.033)	1.220	.6195
50	1.212	1.277 (.056)	1.265	.6162
56	1.246	1.338 (.017)	1.317	.6129
60	1.293	1.346 (.024)	1.414	.6106

Figure 5-1: A plot of observed (.) and predicted (O) densities (with its corresponding curve) of the dextrose solutions. The continuous line is the identity line.

using $C_6H_{12}O_3$ and had a value of -1.594. Equation (2-25) is rewritten below with the appropriate constants; thus giving the predicted measured density.

$$\rho_m = -2.269 \rho_t + 3.616 (e^{\mu_1 a} - 1) \quad (5-1)$$

These densities are given in Table 5-2 and shown with the predicted curve in Figure 5-1.

The understanding of this model may be clarified with closer examination of equation (5-1). By expanding the exponential term and substituting the mass attenuation coefficient (μ_m) for the linear attenuation coefficient, the equation becomes:

$$\rho_m = -2.269 \rho_t + 3.616 (1 + \mu_m a \rho_t + \frac{1}{2} (\mu_m a \rho_t)^2 + \dots - 1)$$

After rearrangement this becomes

$$\rho_m = (3.616 \mu_m a - 2.269) \rho_t + 3.616 (\frac{1}{2} (\mu_m a \rho_t)^2 + \frac{1}{6} (\mu_m a \rho_t)^3 + \dots)$$

It is seen that the predicted measured density is a series in ρ_t . Since the product $\mu_m a \rho_t$ is always less than one, the series converges after a number of terms. The terms which have the power of ρ_t greater than one are responsible for the slight curvature in the predicted measured density curve of figure 5-1.

(c) Coherent Scattering

The effect of coherent scattering was examined using the same method as for multiple scattering. The correction that was applied for

coherent scattering is given in equation (2-31). The constants in this equation were determined using water and benzene as known standards. The dextrose solutions were again used for this study. The constants were evaluated to give the following revision of equation (2-31):

$$\rho_c = \frac{1.049 T_1 \rho_m}{T_1 + 21.69 (1 - e^{-\sigma_c a})}$$

This equation was applied to the measured densities of the various dextrose solutions. The corrected densities, along with the actual and measured densities are given in Table 5-3. It is seen from this table that all the corrected densities lie within one standard deviation of the measured densities. Thus, it can be concluded that there is no significant effect due to coherent scattering.

(d) Sample Size

Three sizes of vials were used to examine the effect of the size of the sample (these are described in chapter 4). Two effects are predominant when changing the radius of the container. It is obvious that as the thickness of the sample is increased, the resulting attenuation is increased as well as the amount of multiple scattering. Mortimer (37) fitted the ratio of the multiple scattered photons to transmitted photons to the radius of the sample, and suggested that the multiple scattering increases as r^3 (r is the radius of the sample). This author did not repeat these measurements and thus no explanation on

TABLE 5-3: THE DENSITIES OF DEXTROSE SOLUTIONS TO EXAMINE COHERENT SCATTERING.

Percentage	Actual Density ρ_a (gcm ⁻³)	Measured Density ρ_m (gcm ⁻³)	Corrected Density ρ_c (gcm ⁻³)
5	1.019	1.016 (0.015)	1.015
16	1.066	1.156 (0.026)	1.139
25	1.101	1.135 (0.020)	1.133
35	1.142	1.174 (0.023)	1.161
44	1.181	1.271 (0.033)	1.249
50	1.212	1.277 (0.056)	1.256
56	1.246	1.338 (0.017)	1.342
60	1.293	1.346 (0.024)	1.344

the validity of this fit will be made.

Table 5-4 gives the measured densities for the different sample sizes. In each case, the liquid used was glycerin. Qualitatively at least, it is seen from the measured densities that the effect of multiple scattering increases with increasing size of the sample vial (which means increasing path length in equation 2-25). It is also interesting to note that the uncertainty in the measured density increases with increasing path length. This can most likely be attributed to two sources. Firstly, it is possible that as the amount of multiple scattering increases, its random behaviour yields a larger but non-consistent fraction of the scattered beam. Probably the more significant uncertainty results from the larger amount of attenuation which accompanies the larger path lengths. This larger amount of attenuation simply decreases the data arriving at the photomultiplier tubes, thus producing poorer statistics.

(e) Collimator Size

Various collimator sizes were used to measure the density of glycerin. It was found that the intensity was too low to produce any significant rates for the scattered beams for collimators of diameter less than or equal to 0.35 cm.

With the two collimators which possess larger aperture (0.50 and 0.65 cm) results were obtained. The measured densities with uncertainties are shown on page 63.

TABLE 5-4: THE EFFECT OF SAMPLE SIZE

<u>Diameter of vial (cm)</u>	<u>Actual Density ρ_a (gcm⁻³)</u>	<u>Measured Density ρ_m (gcm⁻³)</u>
1.15	1.247	1.261 (0.009)
1.60	1.247	1.276 (0.015)
2.70	1.247	1.288 (0.054)

<u>Collimator size (cm)</u>	<u>Measured density (ρ_m (gcm⁻³))</u>
0.50	1.288 (0.054)
0.65	1.353 (0.019)

The 0.50 cm seems to give reliable results which are within a standard deviation of the actual density. The larger collimator gives a larger value for the measured density which is more than a few standard deviations away from the actual density. The increased value of the measured density seems to arise from a change in the relative rates of transmitted and scattered beams for water and the glycerin.

Kennett and Weber (27) show that these relative rates provide a density that does increase with increasing diameter, until a collimator with a diameter of 0.8 cm is used. For diameters of 0.8 cm and larger the measured density becomes constant to within 1%. For the purposes of this study, it was felt that a 0.8 cm diameter would result in a sensitive that would be too large. As a general rule, it is best to have the diameter of the collimator a minimum size that still yields significant intensities in the scattered beams. With this in mind, a diameter of 0.5 cm seemed to be the optimum size.

(f) Variation of the Attenuation Thickness

It was desirable to examine the effect that the thickness of lead attenuator has over the transmitted rates and thus the measured density.

For any attenuation thickness less than 0.8 mm, the photomultiplier tube saturates and thus any measurement would be meaningless. For the

three other thicknesses (0.8, 0.9, and 1.35 mm) the measured densities of the glycerin are given below.

<u>Attenuation thickness (mm)</u>	<u>Measured density (ρ_m (gcm⁻³))</u>
0.8	1.229 (0.041)
0.9	1.288 (0.054)
1.35	1.312 (0.062)

As can be seen from above, an increase in the attenuation thickness results in an increase in both the measured density and its related error. The increase in the error can most likely be attributed to the decrease in the rate of the transmitted beam. As well, this decrease in the transmitted rate would give an overestimation of the relative importance of the scattered rate. Thus, this would imply that it would be desirable to minimize the attenuation of any part of the electronics. It was found that for an attenuation thickness of 0.8 mm, the fixed amplifier was close to saturation. It was for this reason that an attenuation thickness of 0.9 mm was chosen. This thickness allowed the transmitted intensity to be large enough for reasonable statistics, yet there was no fear of saturation.

(g) Variation of Line Voltage

During the operation of the x-ray unit it was observed that both the kilovoltage and current changed with a change in the line voltage. Table 5-5 shows the value of the standardization constant

(using water) for various line voltages.

As can be seen from this table variation in the constant is quite large, even for a change of a couple of volts in the line. This is especially noticeable for the lower voltages. Below 105V the x-ray unit did not provide sufficient intensity to be beneficial at all.

Since the line voltage must be allowed to have a few volts variation, a monitor and compensator had to be included in the x-ray unit. To accomplish this, the x-ray unit was connected to a Variac which was then monitored with a multimeter. It was then sufficient to compensate any change in the line voltage by changing the Variac.

(h) Phantom Study

Measurements of the density of Charlie were made at various points along a particular plane. Some qualitative comments can be made about the location of each sensitive volume. The position of the hand with respect to an optical bench and the respective measured densities are given in table 5-6. Although not much comment can be made about these densities, a few notes should be made. For the position 6.95 cm, the sensitive volume was totally in the tissue-equivalent substance. The quoted density of this material is 1.14 gcm^{-3} . Position 6.45 cm had its sensitive volume at the edge of the styloid process of the ulna and thus contained mostly tissue. Position 5.95 cm was between the ulna and radius bones and contained a bit of each as well as tissue. Positions 5.45 and 4.80 cm both were within the radius (5.45 on the styloid process of the radius and the 4.80 more at the bone wall).

TABLE 5-5: VARIATION OF LINE VOLTAGELine Voltage (volts)k

105	0.308 (0.024)
110	0.066 (0.001)
112.5	0.027 (0.002)
115	0.019 (0.007)
117.5	0.016 (0.002)
120	0.016 (0.002)

TABLE 5-6: PHANTOM STUDY

<u>Position (cm)</u>	<u>Measured Density (gcm⁻³)</u>
6.95	1.100 (0.011)
6.45	1.335 (0.020)
5.95	1.347 (0.013)
5.45	1.399 (0.012)
4.80	1.710 (0.024)

It was found in this study that the previous water standard gave highly erroneous results for the measured densities. This probably resulted from the differences in density and the effective Z as well as the object size between the standard and the phantom hand. This would seem to indicate that the standard must be similar to the sample with respect to radiation, as well as similar size. To accomplish this, a 5.7 cm diameter lucite cylinder was used as the standard for both the phantom and human hands. This brought the densities to reasonable results, which is verified by the comparison of the measured tissue-equivalent density with the actual (position 6.95 cm).

(i) Preclinical Test

A preliminary test was performed on nine volunteers. In each case the bone under study was the radius bone. The actual position of the intersection of the beams was approximately four centimeters from the styloid process. This was verified using an x-ray picture.

Using the lucite standard, the observed densities of six of the volunteers are given in table 5-7. Three volunteers did not have enough measurements to be significant, thus, they were omitted from the table. This table is to illustrate the range of the observed densities with their standard errors.

The average standard error is five per cent. The size of this error can be attributed to two considerations. It is felt that the most significant error results from the fact that the necessity for the

TABLE 5-7: PRECLINICAL TEST IN HUMANS

<u>Volunteer</u>	<u>Body Weight (kg)</u>	<u>Measured Density (gcm⁻³)</u>
1	72	1.162 (0.059)
3	82	1.145 (0.066)
4	70	1.217 (0.056)
5	71	1.100 (0.057)
6	80	1.091 (0.090)
9	79	1.223 (0.044)

standardization using the lucite cylinder was only discovered on the third day of the test and thus the first two days made use of a constant which had a large uncertainty (for these two days the average of the constants was used). As well, one is limited by the technique of repositioning the hand. The technique used in this study was efficient (it took approximately 2 minutes per volunteer) as well as fairly accurate for measurements over a short term period. The repositioning technique, although adequate for this test, must be modified for clinical use. Suggestions will be made in the following chapter.

6 - FUTURE CONSIDERATIONS

One of the major objectives of this project was to produce the first prototype of a practical bone densitometer which could be adapted for clinical use. Although this objective has been accomplished, there are a number of areas that must be examined for possible improvements.

As was discussed in the previous chapter, the standardization constant and thus all measurements seem to be dependent upon the line voltage. It is for this reason that it would be desirable for the x-ray unit to be powered by a regulated power supply. Since this was not the situation for the unit that was used, it was discovered that careful monitoring and corrections of the voltage (using a Variac) would suffice. It should be emphasized that this voltage must be carefully controlled.

Since it is always beneficial to have the maximum number of diagnostic tests outside the physical electronics box, a check on the integrator would be desirable. To accomplish this, one can use a built-in reference oscillator which would give at a constant rate a pulse of constant height and width. Thus, the actual charge would be known and this could be compared to the value from the integrator. Although this test is desirable, it is not a necessity for a clinical device.

Probably more desirable is an automatic read out of the integrated voltage. The sample and hold unit in the electronics has been designed to be capable of integrating over a long period of time (seconds) and holding the final voltage for as long as possible.

Unfortunately, there is drift in the display resulting from the time constant in the integrator which is inherent in the System. This may be solved by connecting the output of the analog circuit to an analog to digital converter (ADC) which then goes to the desirable read out (display, print out or computer). The ADC would be triggered on the negative going edge of the logic pulse which turns off the data and x-ray unit. Thus, a much more accurate reading of the integrated voltage can be obtained, improving the accuracy of the device.

Some thought must be given to the standard used for measuring the density of human bones. As was previously discussed, the results do depend upon the properties of the standard. It would be most satisfactory to have a standard of approximately the same physical size and geometry as the area under study (in this case the wrist). As well, the standard should have similar properties with respect to x-rays. The most satisfactory standard (using the above criteria) would be a bone encased in tissue-equivalent plastic. A segment of the bone could be used to accurately determine its density by other physical techniques. This would result in a standard having identical attenuation properties as human bones and would minimize the error introduced by the standard.

Finally, the alignment system must be improved. Since an x-ray picture of the wrist requires a short exposure (approximately 1.5 sec) and a small current (of the order of tenths of a milliamperes) one may use a polaroid unit in conjunction with a lead x-y grid as the final verification of the alignment. Preliminary adjustments may be

made with a coarse control over the frame (shown in figure 3-12). A fine adjustment must be available, and its graduations would depend upon the accuracy required for the bone under study.

The choice of the specific bone that should be studied is beyond the scope of this study. Presently the most popularly studied bones are the os calcis and the distal radius. Both of these are easy to locate and are of regular shape. Although the radius was used in this study, the decision of the choice of location for a clinical study must be made in consultation with a knowledgeable physician.

7 - CONCLUSIONS

A prototype of a clinical bone densitometer has been developed (both the mechanical bone density apparatus and the accompanying electronics) and has proven successful in experiments involving liquid samples as well as in vivo measurements. The technique employed in this study makes use of the Compton scattered photons which originate from a continuous x-ray source.

The density of various liquids were found with a standard error of about 2%. Experiments were performed to show that the effect due to coherent scattering was well within instrument uncertainties and thus can be considered negligible. The more significant process of multiple scattering was examined theoretically and a correction was applied on the experimental results. The significance of the sample size, collimator size, thickness of attenuator for the transmitted beam and line voltage was examined. It was found that each of these (with the exception of sample size) is very critical in the operation of the instrument.

A pre-clinical test was performed on a number of volunteers. The measured densities had a standard error of approximately 5%. This large error was attributed to standardization problems and alignment uncertainty. Suggestions to improve both these problems have been made.

It is concluded that the densitometer will provide accurate and precise measurements of the density of the radius and can be used in a clinical study.

Appendix 1:

The attenuation coefficients μ_1 , μ_2 , μ_3 , and μ_4 are all functions of energy, and can be written as:

$$\begin{aligned}\mu_1 &= \mu_1(E) \\ \mu_2 &= \mu_2(E) \\ \mu_3 &= \mu_1(E^1) \\ \mu_4 &= \mu_2(E^1)\end{aligned}\tag{A-1}$$

In equation (A-1), the subscript 1 refers to the attenuation coefficients of bone and 2 refers to fatty substance. As well, E^1 indicates the new energy which results from Compton scattering. For a scattering angle of 90° , this energy is

$$E^1 = \frac{511E}{E + 511}$$

The attenuation coefficients for the energy range of 25 KeV to 150 KeV is given in reference (50) and are plotted in figures A-1 (bone mineral) and A-2 (fat). These two straight lines are fitted to the following equations (μ_1 for bone and μ_2 for fat):

$$\mu_1 = \exp(e^{-2.71 \times 10^{-2} E + 1.80 - 2})\tag{A-2}$$

$$\mu_2 = \exp(10^{-.941 \log E + 1.24 - 2})\tag{A-3}$$

For the Compton scattered x-rays, one simply replaces E by E^1 .

Using these equations and the incident x-ray energy distribution which is reproduced in figure (A-3)⁺ it is possible to calculate the weighted means for each of the above coefficients.

These values are:

$$\mu_1 = 1.394 \text{ cm}^{-1}$$

$$\mu_2 = 0.240 \text{ cm}^{-1}$$

$$\mu_3 = 1.568 \text{ cm}^{-1}$$

$$\mu_4 = 0.248 \text{ cm}^{-1}$$

The two constants μ^+ and μ^x can be evaluated:

$$\mu^+ = \mu_2 + \mu_3 - \mu_1 - \mu_4 = 0.166 \text{ cm}^{-1}$$

$$\mu^x = \mu_4 - \mu_2 = 0.008 \text{ cm}^{-1}$$

⁺ The energy spectrum was obtained using a three inch NaI (Tl) crystal and a multichannel analyzer. The x-ray unit was approximately 120 feet from the detector.

Figure A-1: A fit of equation (A-2) which shows the attenuation coefficients for bone as a function of photon energy.

Figure A-2: A fit of equation (A-3) which shows the attenuation coefficient for fatty tissue as a function of photon energy.

Relative Counts

Figure A-3: The x-ray energy spectrum of the unit used in this study.

Appendix 2:

The circuit diagram for the electronics is given on the following pages. Four boards were used for the circuit, and so the circuit has been subdivided into four; one diagram per board.

Figure A-4: Circuit diagram for card one.
Pin code is on next page.

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Pin Code

A2 + 15V
B2 input from PMT 1
C2 ground
H2 - J2 to discharge capacitor
N2 output from integrator 1
U2 - 15V
V2 TTL input for analog switch (card 3, P1, R1)
P2 input from PMT 2
A1 - B1 discharge capacitor
F1 output from integrator 2
H1 input from x-ray unit
J1 1K resistor on card 2
U1 + 5V

Figure A-5: Circuit for card 2. Pin code on next page.

Pin Code

A2 + 5V
B2 from flip flop 2 (card 3, V1)
C2 ground
D2 reset scalars (card 3, V1)
E2 output of NOR (card 3, K1)
F2 to AND (card 3, L1)
H2 to inverter (I1) pin 2 (card 3, B2)
J2 to I1, pin 4 (card 3, D2)
K2 to I1, pin 6 (card 3, E2)
L2 to I1, pin 8 (card 3, F2)
M2 to I1, pin 10 (card 3, H2)
N2 to I1, pin 12 (card 3, J2)
P2 output C1, to AND (card 3, M1)
R2 to I2, pin 2 (card 3, K2)
S2 to I2, pin 4 (card 3, L2)
T2 to I2, pin 6 (card 3, M2)
U2 to I2, pin 8 (card 3, N2)
V2 to I2, pin 10 (card 3, R2)
A1 to I2, pin 12 (card 3, R2)
B1 output C2, to AND (card 3, N1)
C1 555 delay pulse (card 3, J1)
J1 to card 1, J1

Pin Code

A2 + 5V
B2 - J2 Comparitor 1 (card 2)
K2 - R2 Comparitor 2 (card 2)
U2 Reset scalers (card 2, D2)
V2 + 14V
A1 Probe output
B1 Probe output
C1 to 555 (start x-ray) card 4
D1 - E1 5K Pot
F1 - H1 10K Pot
J1 output NAND (card 2, C1)
K1 reset flip flop (card 2, E2)
L1 output of flip flop (card 2, F2)
M1 output of comparator 1 (card 2, P2)
N1 output of comparator 2 (card 2, B1)
P1 - R1 analog switch
S1 input from pushbutton (card 4, F2)

Pin Code

A2	+ 5V
B2	pushbutton
C2	ground
D2	pushbutton
E2	to input of start delay 555 (card 3, S1)
C1	from card 3, C1
E1	to start x-ray unit

Figure A-7: Circuit for card 4.

Appendix 3:

The timing diagrams for the various analog and logic signals are given on the following pages. The name associated with each timing sequence refers to the point of observation.

The purpose of the letter beside each sequence is to cross reference the timing diagrams and the circuit diagrams (Appendix 2). Thus each letter is shown at the point of observation for a particular timing sequence.

Figure A-8: START-STOP TIMING DIAGRAM
(for an exposure of 3 sec)

Figure A-9: Scaler Timing Diagram.

Figure A-10: Analog and Analog Gating Timing Diagram

REFERENCES

1. Cameron, J.R. and Sorenson, J.; Science, 142, 230 (1963).
2. Cameron, J.R.; Phys. Med. Biol., 15, 184 (1970).
3. Cameron, J.R. and Sorenson, J.; Science, 142, 230 (1963).
4. Cameron, J.R. and Sorenson, J.A.; J. Nucl. Med., 8, 268 (1967).
5. Cameron, J.R.; private communication.
6. Cameron, J.R., Mazess, R.B., and Sorenson, J.A. Invest. Radiol., 3, 141 (1968).
7. Chestnut, C.H. et al.; "Correlation of Total Body Calcium (Bone Mass) as Determined by Neutron Activation Analysis with Regional Bone Mass as Determined by Photon Absorption"; from Int. Conf. on Bone Mineral Measurement, Oct. 12-13, 1973.
8. Clarke, R.L., and Van Dyk, G.; Phys. Med. Biol., 18, 4, 532 (1973).
9. Colbert, C., Mazess, R.B. and Schmidt, P.B.; Invest. Radiol., 5, 338 (1970).
10. Colbert, C., Schmidt, P.B. and Mazess, R.B.; Phys. Med. Biol., 15, 372 (1970).
11. Dalen, N.; "Choice of Measuring Sites"; from Int. Conf. on Bone Mineral Measurement, Oct. 12-13, 1973.
12. De Puey, E.G., Burdine, J.A. and Murphy, P.; J. Nucl. Med., 12, 351 (1970).
13. Donath, A., et al.; Influence of the Natural Calcium and Fluoride Supply and of a Calcium Supplementation on Bone Mineral Content of Healthy Population in Switzerland"; from Int. Conf. on Bone Mineral Measurement, Oct. 12-13, 1973.

14. Eilbert, R.; "In Vivo Calcium Determination by Proton Activation Analysis"; from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
15. Evans, R.D; The Atomic Nucleus, McGraw-Hill Book Company, Toronto, 1972.
16. Evens, R.G. et al.; "Clinical Use of an In Vivo Measurement of Bone Mineral Content"; from Progress in Methods of Bone Mineral Measurement; Bethesda, Md; Feb. 15-17, 1968.
- 17a. Favo, U.; Nucleonics; 11, 8, 1953.
- 17b. Favo, U.; Nucleonics; 11, 55, 1953.
- 18a. Gebhardt, M. and Zwicker, H.; "A New Apparatus for Bone Mineral Measurement In Vivo"; from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
- 18b. Genant, H.K. et al.; "Skeletal Demineralization in Primary Hyperparathyroidism"; from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
19. Gilson, A.J. et al.; J. Nucl. Med., 6, 345 (abstract), 1965.
20. Goldsmith, N.F. et. al.; J. Bone and Joint Surgery; 53A; 85, 1971.
21. Grodstein, G.W.; "X-ray Attenuation Coefficients from 10 KeV to 100 MeV"; National Bureau of Standards Circular 583, 1957.
22. Hanson, J; "Analysis of ^{153}Gd and of $^{125}\text{I}/^{241}\text{Am}$ Sources"; from Int. Conf. on Bone Mineral Measurement, Oct. 12-13, 1973.
23. Judy, P.F. and Cameron, J.R.; Phys. Med. Biol.; 15, 372, (1970).

24. Judy, P.F.; "Physical Aspects of ^{125}I Bone Absorptiometry";
from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
25. Kan, W.C. et al.; "Direct Readout of Bone Mineral Content with
"Dichromatic Absorptiometry"; from Int. Conf. on Bone Mineral
Measurement; Oct. 12-13, 1973.
26. Karjolaenen, P.; Annals of Clin. Res., 5, 231, 1973.
27. Kennett, T.J. and Webber, C.; Phys. Med. Biol.; 21, 5, 770, 1976.
28. Kennett, T.J. et al.; J. Assoc. Can. Rad., 23, 168, 1972.
29. Klein, O. and Nishina, Y.; Physik, 52, 853, 1929.
30. Kuhlencordt, F. et al.; "Bone Mineral Determination of Radius, Ulna, and
Fingerbones by ^{125}I Photon Absorptiometry on Healthy Persons";
from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
31. Lachmann, E. and Whelan, M; Radiology, 26, 165, 1936.
32. Luther, R.; "Correlation of Os Calcis and Spinal Bone by Compton
Scattering"; from Int. Conf. on Bone Mineral Measurement;
Oct. 12-13, 1973.
33. Mazess, R.B. et al.; Science, 145, 388, 1964.
34. Mazess, R.B. et al.; Phys. Med. Biol., 15, 372; 1970.
35. Mazess, R.B. and Cameron, J.R.; "Bone Mineral Content in Normal
U.S. Whites"; from Int. Conf. on Bone Mineral Measurement;
Oct. 12-13, 1973.
36. McDonald, J.M. and Zeitz, L.; "Dual Energy Absorptiometry Technique for
Bone Mineral Content Measurement"; from Int. Conf. on Bone Mineral
Measurement; Oct. 12-13, 1973.

37. Mortimer, A.; The Measurement of Bone Mineral Content Using Scattered and Transmitted X-rays; M.Sc. Thesis, Carleton University, 1974.
38. Olkkonen, H. et. al.; "Measurement of Bone Mineral Density Using Coherent and Compton Scattering"; from Int. Conf. on Bone Mineral Measurement; New Orleans, Jan. 26-28, 1976.
39. Overton, T.R. et. al.; "U of A Bone Mineral Analysis System - Performance and Clinical Application"; from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
40. Piper, D.G. et al.; "Evaluation of Several Nuclides for Bone Density Determinations by Compton Scattering"; from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.
41. Preuss, L.E. et al.; "Investigation of Certain Errors in Dual Photon Tissue Analysis"; from Fourth Int. Conf. on Med. Phys.; Ottawa, July 25-30, 1976.
42. Reed, G.W.; Phys. Med. Biol.; 11, 174, 1966.
43. Richtmyer, F.K., Kennard, E.H. and Lauritsen, T.; Introduction to Modern Physics; McGraw-Hill Book Co.; Toronto, 1955.
44. Roy, R.R. and Nigam, B.P.; Nuclear Physics; John Wiley and Sons, Inc.; New York, 1967.
45. Ruegsegger, P., et al.; "A Method for the Determination of the Compact Area and the Mean Absorption Density of Human Bones"; from Int. Conf. on Bone Mineral Measurement; Oct. 12-13, 1973.

46. Reiss, K.H., et al.; "Dual Photon X-ray Beam Applications"; from Int. Conf. on Bone Mineral Measurement, Oct 12-13, 1973.
47. Smith, D.M., et al.; J. Am. Med. Ass., 219, 3, 1972.
48. Smith, D.M., et al.; J. Clin. Invest., 45, 1073, 1966.
49. Smith, E. and Cameron, J.R.; "Interpretation of Fracture Index Charts" from Bone Mineral Analysis; distributed by Norland Instruments, 1972.
50. Sorenson, J.A. and Cameron, J.R.; "Transmission Scanning Techniques" in Instrumentation in Nuclear Medicine; edited by G. Hine and J. Sorenson, Academic Press, New York, 1974.
51. Spring, E.; Int. J. Appl. Radiat. Isotop.; 18, 709, 1967.
52. Webber, C.E. and Kennet, T.J.; Phys. Med. Biol., 21, 5, 760, 1976.
53. Johns, H.E. and Cunningham, J.R.; The Physics of Radiology; Charles C. Thoman Publisher; Springfield, U.S.A.; 1969.
54. Taylor, K.E.; "The Significance of Errors in X-ray Images" from a talk presented at CAP Congress, York University, Toronto, June 9 - 12, 1975.