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**A SEARCH FOR SMALL ANOMALIES IN
THE ^3H β DECAY SPECTRUM USING THE
GAS PROPORTIONAL CHAMBER METHOD**

by

David J Paterson M. Sc.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Physics
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March, 1997

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The undersigned recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis

**“A Search for Small Anomalies in the ^3H β Decay Spectrum
using the Gas Proportional Chamber Method”**

submitted by **David John Paterson. B.Sc., M.Sc.**

in partial fulfillment of the requirements for

the degree of Doctor of Philosophy

Chair, Department of Physics

Thesis Supervisor

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Date: 12 May '97

Abstract

A gas proportional chamber has been employed to make precise observations of the low energy portion of the β spectrum of ${}^3\text{H}$ in methane molecules. The effects on the energy spectrum, due to the screening of the atomic nuclei by the orbital electron cloud, are observed and measured. The screening potential was found to be larger, by $\sim 2\sigma$, than what is predicted by naive Self Consistent Field calculations, but is consistent with a predicted increase in electron density at the ${}^3\text{He}$ daughter atom resulting from the decay. The screening potential was measured to be 90 ± 3 (*stat*) ± 5 (*sys*) eV (68% *CL*), compared with the predicted value of 74 eV . No anomalous monolines were found in the spectrum, to an upper limit of 2% of the underlying β particle distribution (95% *CL*), and no evidence for an admixture of heavy neutrinos was found, to an upper limit of $\sin^2 \theta \leq 3.6 \times 10^{-3}$ (95% *CL*).

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Chapter I

Introduction

The 3H β decay spectrum has been a frequent subject for study, combining the advantages of a simple nucleus within a minimal electron cloud, and releasing very little energy (~ 18.6 keV) relative to that typical of β decays, in its transition to its 3He final state. It is a mixed allowed transition according to Fermi and Gamow-Teller selection rules, thus having a relatively short lifetime, so that high activity is easily achieved. It has been the primary choice for spectrum end-point studies, in searches for evidence of neutrino mass. It has also recently been the subject of attempts to observe discontinuities in the shape of the spectrum, as compared with Fermi's β decay theory, in pursuit of evidence for heavy neutrinos. A small ($\sim 1\%$) heavy neutrino component, with a mass near 17 keV, was suspected as a consequence of the studies of the 3H spectrum by Simpson [1], apparently supported in studies made by other researchers, employing isotopes of heavier elements [2, 3, 4, 5, 6]. In

these studies, discontinuities in the slopes of the spectra of β particle energies were observed to occur at points near 17 keV below the endpoint energies for the isotopes under study. This support proved not to be robust in most of the isotopes being studied, but the careful ^3H observations of Simpson have not yet been repeated and reported with simultaneously improved systematic certainty and statistical precision.

The detectors employed in searches for evidence of the 17 keV neutrino were principally either semiconductors [1, 2, 3, 4, 5, 7, 8], which measured the β particle energy, or electrostatic or magnetic spectrometers [9, 10, 11, 12, 13], which steered the escaping β particles in proportion to their energy. In both cases the recoil energy of the daughter nucleus is not detected. The recoil energy is trivial in comparison to the β particle and neutrino energies, and small in comparison to the atomic and molecular effects which may influence the observations. Two collaborations [6, 14] made observations of internal bremsstrahlung photons emitted on electron capture decays. While this method is statistically limited due to the low intrinsic rates, it is free of systematic effects which have been demonstrated for the other detection methods. Bahran and Kalbfleisch [15, 16] employed a gas proportional chamber, operating at sub-atmospheric pressure, to observe the energies of β particles from ^3H decays. Unfortunately, large corrections were necessary in consideration of the low gas pressures, and the consequently high probability for β particles to scatter to the detector walls and deposit some of their energy in these detector walls. These wall effects can introduce large distortions to the shape of the spectrum, and they are not separately observable and measurable, thus requiring corrections to the spectrum observed based entirely on computer simulations of the distortion of the spectrum.

One of the uncertain factors which has bearing on the analysis of the heavy neutrino experimental results is an approximation taken in the β decay theory. This approximation, initially by Rose [17], attempts to make a correction to the predicted shape of the spectrum in light of the reduction in strength of the Coulomb field of the nucleus, as seen by the escaping β particle, due to the shielding of the nucleus by the cloud of bound electrons. This is the *screening* effect on the distribution of observed energies of the escaping β particle. To make this correction, a further approximation is often taken by assuming that the daughter atom suddenly has the atomic orbital structure of the parent. These approximations are generally believed to result in an inaccuracy in theoretical spectrum at energies of less than ~ 500 eV, depending of course upon the isotope being studied. Unfortunately, studies generally have not attempted to test the validity of these approximations even at such energies, due to the difficulties inherent in gathering reliable high statistics observations in this energy region, where detector systematic difficulties and environmental effects will dominate [18, 19, 20]. Even at energies somewhat greater than ~ 500 eV, these factors may alter the theoretical prediction of the spectrum by a few percent. The screening potential is calculable, but requires a theoretical determination of the final state probabilities for the atomic (or molecular) electron structure. Possible final states for the β decay of the tritium atom include the initial orbital electron being found in the ^3He ground state, or one of the possible excited states (*shake-up*), or the atom may be doubly ionized (*shake-off*). In the case where the tritium is bound in a molecule, these final state probabilities may be altered. In making the theoretical prediction of the energy spectrum, corrections for all such outcomes are accounted for through adjustments in the energy liberated in the decay to account for chemical en-

ergy changes, and adjustments in the energy of the escaping β particle in recognition of the depth of the well from which the β particle must escape prior to observation by a spectrometer. The above mentioned searches for evidence of massive neutrinos concentrated on the spectra above the 1 keV region, which loosely defines the boundary of the region of validity of these approximations, though the absolute magnitude of the correction attributable to these approximations was a matter of some theoretical discussion in response to the possible heavy neutrino observations. [21, 22, 23]

This present work attempts to make higher precision studies of the ${}^3\text{H}$ β decay energy spectrum, employing the thus far underutilized gas proportional chamber method, at gas pressures sufficiently high as to reduce predicted wall effect corrections to small levels. Other systematic effects, which have thus far made direct tests of Rose's screening approximation at or below 1 keV difficult, are reduced to levels which may be deemed small relative to the effects discussed in regard to the hypothesized heavy neutrino. We have employed a cylindrical gas proportional chamber, with a fiducial volume of about $1\frac{1}{4}$ litres, operating at 5 atmospheres pressure. We base our conclusions upon data collection runs of typically 10^8 decays each. This provides us with 1σ statistical uncertainties of $\sim 0.4\%$ in each channel of adc data, in the region between $\sim 1\text{ keV}$ to $\sim 5\text{ keV}$. Backgrounds have been carefully observed, and the external background has been observed exhibit similar rates whether operating with or without ${}^3\text{H}$ in the chamber. The background rate is typically $\sim 1.6\%$ of the data rate, and is distributed similarly to the ${}^3\text{H}$ spectrum. The uncertainty in the background rate is sufficiently small as to allow less than 1% uncertainty in our background subtraction, and will therefore not mask possible evidence of a heavy

neutrino.

While the recoil energy of the daughter nucleus is known to be trivial, and is treated as such in most theoretical predictions of the ${}^3\text{H}$ β decay spectrum, it is noteworthy that this gas proportional chamber method fully captures all ionization in the gas resulting from β decay of nuclei which lend themselves to mixing with counting gases. This results in containment of the nuclear recoil energy within the active volume of the detector during the measurement, and the observation of the recoil energy should any ionization result. It is not clear, though, that such low energy recoil will produce ionization in the gas. However, the advantage of this method is most apparent when considering the absence of particle collimation or steering apparatus, and thus the simplicity and high certainty with which the energy released in the decays may be measured. Such systematic effects are suspected to be the basis for the false positive results in the heavy neutrino searches with isotopes other than ${}^3\text{H}$. [24]

The energy resolution of gas proportional chambers is generally not as good as that of semiconductor detectors. However, in this energy range, gas proportional chambers do not possess a significant energy resolution disadvantage. For example, the resolution of Simpson's Si(Li) detector was $\sim 220\text{eV}$ [1], as measured at $\sim 6\text{keV}$, while we anticipate a 1σ resolution of $\sim 195\text{eV}$ at $\sim 1.7\text{keV}$. Calorimetric semiconductor detectors are believed to present difficulties for the study of low energy spectra. There is concern about possible lattice damage when drifting, for example, ${}^3\text{H}$ atoms into a silicon or germanium lattice. This lattice damage may result in unmeasurable charge trapping effects, which will distort energy measurements. Even in the absence

of such lattice damage, the lattice structure may introduce rather large effects which will contribute greater uncertainty in the energy observations. The alternative of using amorphous silicon is discarded due to its greatly deteriorated energy resolution relative to silicon in crystalline form. For 90% argon, 10% methane (P10) counting gas, the FWHM energy resolution will be, at best, 13% at the energy of the ^{55}Mn $K_{\alpha 1}$ X-ray (5899 eV, from ^{55}Fe electron capture decay), and will vary as $\frac{1}{\sqrt{E}}$, where E is the energy, yielding a 1σ energy resolution of approximately 11% at 1.5 keV, or ~ 164 eV.

For this present work, Monte Carlo simulations have predicted that the wall effects will affect on the order of 0.3% of the events, and that the energy distribution of such wall effects will not greatly distort the ^3H β decay energy spectrum. It is of particular significance that the use of Monte Carlo predictions of the distribution of the detected energy from β particles which encounter the wall cannot result in a discontinuity in the slope of the predicted energy spectrum at the point at which the heavy neutrino mass effects were observed.

The Monte Carlo simulations assume that the ^3H atoms, introduced in the form of tritiated methane in P10 counting gas, will be homogeneously distributed throughout the gas. One concern of this method is that the methane may be concentrated on or near the cathode walls. This would result in increased levels of wall effects, and this is not readily monitored. Since the predicted level of wall effects is not verifiable, the actual wall effect rate is used as a free parameter in the fitting of the experimental data to the theory. The shape of the distortion due to wall effects is also not observable, and is therefore fixed, for fitting purposes, to that predicted

by the Monte Carlo code.

The objectives of this work are twofold. Our first objective is to make an investigation into Simpson's observation of a discontinuity in the slope of the ${}^3\text{H}$ β decay spectrum near 1.6 keV, employing an apparatus with different characteristics than those previously employed by other researchers. Our second objective is to investigate the screening potential for the ${}^3\text{H}$ decay, in order to investigate the validity of the Rose and sudden approximations. To these ends, this thesis will describe the theory of β decay and the associated final state and environmental effects in Chapter II, the gas proportional chamber method in Chapter III, our experimental apparatus in Chapter IV, and the observations and analysis in Chapter V. Conclusions will be drawn from our results in Chapter VI.

Chapter II

Beta Decay Theory

The theory of β decay is attributed to Fermi [25], and has proven to be a robust theory for over six decades. We first consider the phase space for the decay products, and then incorporate the Coulomb correction factor appropriate for relativistic β particles. In this, we follow the approach taken by Behrens and Bühring [26]. We subsequently consider the effects of radiative corrections on the theoretical spectrum [26, 27, 28]. The effects of the atomic environment on the observed spectrum of escaping β particle energies are then discussed, in light of the Rose and sudden approximations [17]. Additional effects due to the molecular environment are then incorporated into the theory, following the approach of Kaplan and Smutney [29].

II.1 Phase Space Integral

According to Fermi's theory of β decay, the shape of the distribution of β particle energies, neglecting Coulomb interactions between the β particle and the daughter nucleus, is described by the phase space integral. Let us determine this as follows, beginning from scattering theory. The amplitude for any scattering interaction which leads to a final state, f , from an initial state, i , is given by the overlap of the initial and final state wave functions under the S-matrix, or scattering operator. The probability of the interaction, P_{fi} , is then given by this amplitude times its complex conjugate.

$$P_{fi} = \langle f | S | i \rangle^* \langle f | S | i \rangle = S_{fi}^* S_{fi} = S_{fi}^\dagger S_{fi} \quad (\text{II.1})$$

where S_{fi} is the scattering matrix operator connecting the initial state $|i\rangle = |p_i, s_i\rangle$, and the final state $|f\rangle = |p_r, s_r; p_e, s_e; p_\nu, s_\nu\rangle$. In this, we have specified the state by its four-momentum (p) and spin (s) observables. In β decay, the subscript i is connected with the initial nuclear state, and the subscript f is connected with the three particle final state, within which the subscripts r , e , and ν are connected with the recoiling daughter nucleus, the β particle, and the neutrino states respectively. The superscript $+$ indicates the hermitian conjugate, or adjoint operator. For determination of the shape of the energy spectrum, summation must be carried out over all possible final states, and all final spins.

The scattering matrix is connected to the nuclear transition matrix, M , according to:

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(p_f - p_i) N M_{fi}$$

We are concerned only with transitions, and so we will consider only the imaginary part of the scattering matrix. We have introduced N for purposes of normalization, defined as:

$$N = \sqrt{V^{-(n_i+n_f)}}$$

with $n_{f(i)}$ being the number of particles in the final (initial) state, and V being the volume occupied by one particle, and equal to $(2\pi)^3$. In β -decay, $n_i = 1$, and $n_f = 3$.

Then:

$$P_{fi} = \frac{[(2\pi)^4 \delta^4(p_f - p_i)]^2}{V^4} M_{if}^+ M_{fi} \quad (\text{II.2})$$

In evaluating the numerator of this expression, we employ four-momentum conservation to replace one factor $\delta^4(p_f - p_i)$ with $\delta(0)$, and integrate over this $(2\pi)^4 \delta(0)$, yielding for the numerator:

$$[(2\pi)^4 \delta^4(p_f - p_i)]^2 = \lim_{VT \rightarrow \infty} (2\pi)^4 \delta^4(p_f - p_i) VT$$

VT is four dimensional space-time. Then summing over spins and all possible final states N_o where:

$$N_o = \frac{1}{(2\pi)^3} \int d^3\mathbf{x} d^3\mathbf{p} = \frac{V}{(2\pi)^3} \int d^3\mathbf{p}$$

yields:

$$\begin{aligned} P_{fi} &= \sum_f \sum_s \int \frac{(2\pi)^4 \delta^4(p_f - p_i) VT}{V^4} M_{if}^+ M_{fi} \frac{d^3\mathbf{p}_r V}{(2\pi)^3} \frac{d^3\mathbf{p}_e V}{(2\pi)^3} \frac{d^3\mathbf{p}_\nu V}{(2\pi)^3} \\ &= (2\pi)^{-5} \sum_f \sum_s \int |M|^2 T \delta^3(\mathbf{p}_r + \mathbf{p}_e + \mathbf{p}_\nu) \delta(E_r + E_e + E_\nu - M_i) d^3\mathbf{p}_r d^3\mathbf{p}_e d^3\mathbf{p}_\nu \end{aligned}$$

where we have split the δ^4 term into momentum and energy dependent parts, and moved to the rest frame of the parent nucleus in which $\mathbf{p}_i$ is 0.

We will investigate the β decay of 3H , which is a mixed allowed β transition, according to Fermi and Gamow-Teller selection rules. It is known that for allowed β decays, the nuclear matrix element does not depend on the lepton wave function. This permits us to employ an approximation in which summation of $|M|^2$ over all final states and spin states will yield a constant. Therefore, eliminating first the matrix element terms and the spins:

$$\begin{aligned} P_{fi} &= \frac{Constant}{(2\pi)^5} \int \delta^3(\mathbf{p}_r + \mathbf{p}_e + \mathbf{p}_\nu) \delta(E_r + E_e + E_\nu - M_i) d^3\mathbf{p}_r d^3\mathbf{p}_e d^3\mathbf{p}_\nu \\ &= \frac{Constant}{(2\pi)^5} \int dR \end{aligned}$$

where we have absorbed T into the constant as well, and we will later normalize the theoretical shape. Here, the integral over dR is the phase space integral. We may now integrate the remaining momentum delta function over the nuclear recoil momentum, simplifying dR to:

$$dR = \delta(E_r + E_e + E_\nu - M_i) d^3\mathbf{p}_e d^3\mathbf{p}_\nu$$

Recoil energy will be observed in an experiment such as ours if it produces ionization in the gas. However, neglecting of the nuclear recoil is justified for simplicity in this theory, in light of the fact that the maximum recoil energy is given by [29]:

$$E_{r,max} = \left(\frac{m_e}{M_{nucl}} \right) \times E_{e,max}$$

For atomic tritium, $E_{e,max}$ is ~ 18.6 keV, yielding ~ 3 eV in maximum nuclear recoil energy. Then neglecting this recoil energy we have:

$$E_0 = E_\nu + E_e$$

where E_0 is the energy released in the transition. and E_ν and E_e are the neutrino and β particle total energies respectively. We may therefore substitute for the above energy δ function. the term $\delta(E_0 - E_e - E_\nu)$. We will now replace the three dimensional momentum differentials by their equivalents in spherical coordinates. yielding:

$$dR = \delta(E_0 - E_e - E_\nu) p_e^2 dp_e d\Omega_e p_\nu^2 dp_\nu d\Omega_\nu$$

Now differentiating both sides of the relativistic relation $E^2 = c^2 p^2 + m_0^2 c^4$. we arrive at expressions for both the β particle and the neutrino:

$$E_e dE_e = c^2 p_e dp_e$$

$$E_\nu dE_\nu = c^2 p_\nu dp_\nu$$

Substituting for the momentum differentials in terms of energy differentials. we may integrate the remaining delta function over the neutrino energy to yield the β particle energy spectrum. correct to within a normalizing constant. as follows:

$$\begin{aligned} P_{fi} &= \frac{Constant}{(2\pi)^5} \int \delta(E_0 - E_e - E_\nu) \frac{p_e^2 E_e dE_e}{c^2 p_e} \frac{p_\nu^2 E_\nu dE_\nu}{c^2 p_\nu} d\Omega_e d\Omega_\nu \\ &= \frac{Constant}{2\pi^3 c^4} \int \delta(E_0 - E_e - E_\nu) p_e E_e dE_e p_\nu E_\nu dE_\nu \\ &= \frac{Constant}{2\pi^3 c^4} p_e E_e dE_e p_\nu E_\nu \end{aligned}$$

We have integrated over β particle and neutrino directions in the second step. and we have integrated the energy delta function over the neutrino energy differential in the last step. Now assuming a vanishing neutrino mass. we may replace the neutrino momentum with its energy. yielding:

$$P(E_e) dE_e = \frac{Constant}{2\pi^3 c^4} p_e E_e E_\nu^2 dE_e = C' p_e E_e (E_0 - E_e)^2 dE_e \quad (II.3)$$

where we have now expressed the transition probability as a function of β particle energy and momentum alone, yielding a normalized β particle spectrum.

Alternatively, we may assume a non-vanishing neutrino mass. In this case, we will substitute for the neutrino momentum a term involving both the neutrino energy and mass.

$$P_{fi} = \frac{Constant}{2\pi^3 c^4} p_e E_e E_\nu (E_\nu^2 - m_\nu^2)^{\frac{1}{2}} dE_e \quad (II.4)$$

$$= C' p_e E_e E_\nu \left(E_\nu - \kappa_\nu(m, E) \frac{m_\nu^2}{E_\nu} \right) dE_e \quad (II.5)$$

We have presumed a system of units in which $c=1$, and used the binomial theorem to expand the square root. In the second step, κ is a term which approximates the contributions of the higher order terms, and depends on the fraction $\frac{m_\nu}{E_\nu}$. For $m_\nu \rightarrow E_\nu$, $\kappa \rightarrow 1$, but as $m_\nu \rightarrow 0$, κ decreases towards $\frac{1}{2}$. Now multiplying through the bracket with E_ν , and substituting $(E_0 - E_e)$ for E_ν again, we have:

$$P(E_e) dE_e = C' p_e E_e \left[(E_0 - E_e)^2 - \kappa_\nu(m, E) m_\nu^2 \right] dE_e \quad (II.6)$$

This expression reduces to Equation II.3 above in the event of a zero mass neutrino. We will be interested in the case where the term multiplying κ_ν is zero, and the case where κ_ν approaches one, that is where the neutrino mass energy is nearly equal to the energy released in the decay. In this latter case, the neutrino and β particle are born nearly at rest in the rest frame of the nucleus. The 3H β decay system provides an unique opportunity to investigate these two possibilities.

II.2 The Coulomb Correction, Fermi's Function

The Coulomb force on the escaping β particle, due to the nuclear charge, is dependent on that nuclear charge and the distance between the β particle and the daughter nucleus. The effect of the Coulomb interaction is greatest for lower energy β particles. The method employed by Fermi was to introduce a Coulomb correcting factor, now known as the Fermi Function. This function of the β particle energy and the charge of the daughter nucleus takes various forms with varying accuracies under different assumptions. The form, correct for relativistic β particles (ignoring the screening effects of the nuclear charge by orbital electrons) is given by [26, 30]:

$$F(Z, E) = 2(1 + s)(2p\rho)^{2s-2} e^{\pi\eta} \frac{|\Gamma(s + i\eta)|^2}{|\Gamma(2s + 1)|^2}$$

where $\rho = \frac{R}{(\hbar/mc)}$, R is the nuclear radius, p is the β particle momentum in units of mc , $\eta = -\frac{\alpha Z}{\beta}$, α is the fine structure constant, β is the relativistic particle velocity in units of c , and $s = [1 - (\alpha Z)^2]^{\frac{1}{2}}$. We therefore rewrite Equation II.3 as follows:

$$P(E_e)dE_e = C'F(Z, E)p_e E_e (E_0 - E_e)^2 dE_e \quad (\text{II.7})$$

II.3 Light-Heavy Neutrino Mixing

In explaining his observed tritium spectra, Simpson [1] hypothesized a mixture of the zero mass and heavy neutrino cases above, which we may describe with the relation:

$$P(E_e)dE_e = C'F(Z, E)p_e E_e (E_0 - E_e)^2 [\cos^2 \theta - \kappa_\nu(m, E)m_\nu^2 \sin^2 \theta] dE_e \quad (\text{II.8})$$

where $\sin^2 \theta \approx 0.03$, $\kappa_\nu \rightarrow 1$, and $m_\nu \approx 17\text{keV}$. Subsequently, $\sin^2 \theta$ was reduced to ~ 0.01 through refinements introduced to the predictions of the potential gap between the nucleus and the orbital electrons, and to predictions of the change in the excitation energy of the daughter atom [21, 22]. Both of these considerations will be discussed below.

II.4 Radiative Corrections

It is known that β decay is a weak charge changing decay, and is therefore mediated by the exchange of a W boson, as shown in Figure II.1. a ii). Radiative corrections to the β particle energy spectrum are corrections to the calculated probability for β particle emission, as a function of β particle energy. These corrections are attributed to real or virtual photon emission or absorption by the β particle or the parent particle which is undergoing decay. Corrections involving one real or virtual photon have a strength proportional to α , the fine structure constant. Corrections which involve two β particle - photon vertices have a strength proportional to $Z\alpha^2$. Figures II.1 b) and c) demonstrate these categories of corrections.

Radiative corrections are classed [31] as either inner or outer corrections. Outer corrections are independent of the weak and strong interactions which govern the decay vertex. They therefore affect the shape of the energy spectrum of the escaping β particles. Inner corrections renormalize the coupling constants at the vertex, thus affecting only the overall normalization of the spectrum. Inner radiative corrections

Figure II.1: Feynman β -decay Diagrams, with Radiative Corrections to Order α^2

can therefore be neglected in this work. since we have chosen to normalize the resulting spectrum experimentally.

Outer corrections are connected with real photon emission (bremsstrahlung). While it is anticipated that some of the decay energy will, at times, take the form of real photons in the detector (eg bremsstrahlung photon emission on β particle interactions with the counting gas), the construction of the detector supports the expectation that such photons will be largely converted into ionization in the counting gas. This will be investigated in connection with our review of the detector response function. What remains here is to determine the degree to which these outer corrections might alter the theoretical shape of the β particle energy spectrum, and also to determine the shape and relative strength of the real photon spectrum for subsequent consideration in connection with the detector response function.

II.4.1 Outer Corrections

Outer corrections, displayed in Figure II.1 b i) and ii), and c), result in bremsstrahlung photon emission. The strength of the photon coupling is proportional to α for leading order corrections (Figure II.1 b), and proportional to $Z\alpha^2$ for corrections of next to leading order (Figure II.1 c). Higher order corrections arise with strengths which diminish by an additional factor of $Z\alpha$ for each higher order, that is by factors of $\sim 10^{-2}$ per order. For this reason we restrict our considerations to the first two orders. Sirlin [31] has described the correction factor attributable to these outer

corrections as:

$$\delta_R(E_\beta, Z) = \delta_1(E_\beta) + \delta_2(E_\beta, Z) + \dots$$

where δ_R is the total outer radiative correction factor, to be applied to correct the β -decay energy spectrum according to the prescription:

$$C_R(E_\beta) = C(E_\beta)[1 + \delta_R(E_\beta, Z)]$$

$C(E_\beta)$ is the uncorrected spectrum, and $C_R(E_\beta)$ is the corrected spectrum. The terms $\delta_1(E_\beta)$ and $\delta_2(E_\beta, Z)$ are the leading order, and next to leading order terms respectively. The leading order correction, $\delta_1(E_\beta)$, is independent of the nucleus, and thus independent of the strong and weak forces present within the nucleus. δ_1 and δ_2 are given by:

$$\begin{aligned}\delta_1(E_\beta) &= \frac{\alpha}{2\pi} g(E_\beta, E_0) \\ \delta_2(E_\beta, Z) &= |Z| \alpha^2 \ln M_N + \dots\end{aligned}$$

where:

$$\begin{aligned}g(E_\beta, E_0) &= 3 \ln M_N - \frac{3}{4} + 4 \left(\frac{\tanh^{-1} \beta}{\beta} - 1 \right) \times \left\{ \frac{E_0 - E_\beta}{3E_\beta} - \frac{3}{2} + \ln [2(E_0 - E_\beta)] \right\} \\ &+ \frac{4}{\beta} L \left(\frac{2\beta}{1+\beta} \right) \\ &+ \frac{1}{\beta} (\tanh^{-1} \beta) \left[2(1 + \beta^2) + \frac{(E_0 - E_\beta)^2}{6E_\beta^2} - 4 \tanh^{-1} \beta \right]\end{aligned}$$

and:

$$E_0 = \text{the total energy released in the decay.}$$

$$E_\beta = \text{the total energy of the } \beta \text{ particle.}$$

$$\beta = \text{the velocity of the } \beta \text{ particle} = \frac{p_\beta}{E_\beta}.$$

$$\begin{aligned}
M_N &= \text{some cut-off mass, typically } M_p \\
\alpha &= \text{the fine structure constant } \sim \frac{1}{137} \\
Z &= \text{the charge of the daughter nucleus.}
\end{aligned}$$

The term:

$$L\left(\frac{2\beta}{1+\beta}\right)$$

is the Spence function, given by:

$$L(x) = \int_0^x \frac{dy}{y} \ln(1-y)$$

For δ_2 , there are higher terms, signified by the and neglected due to the fact that they contribute about 10% to δ_2 , which is already about 10^{-2} of δ_1 . δ_R is typically of order $\sim 2 \times 10^{-2}$.

δ_2 is apparently a small constant term, and has no impact on the shape of the β particle energy spectrum. It can be absorbed into the normalization constant for the overall calculation of the β particle energy spectrum. However, δ_1 describes the change in the shape of the spectrum due to real bremsstrahlung photon emission, which must be considered with respect to detector wall effects.

II.4.2 Magnitude of Corrections

Figure II.2 a) demonstrates the uncorrected β particle energy spectrum for 3H decay, according to the theory described above. Figure II.2 b) displays the spectrum of as-

Figure II.2: ${}^3\text{H}$ β Decay Energy Spectrum, and Associated Radiative Corrections

sociated real bremsstrahlung photons for the decay, normalized in total to a predicted magnitude of 0.018% of the total β spectrum. This photon spectrum is predicted on the basis of the theory of radiative corrections described above. Figure II.2 c) shows the magnitude of these corrections, as a function of β particle energy, to be applied to correct the expected spectrum for both real and virtual photon processes. It is apparent that these corrections will be observable at a precision of 10^{-4} in the β spectrum, and that the recovery of a fraction of real photons in the gas, in coincidence with the β particles being observed, will have a small effect on the predicted shape of the spectrum, and will be observable only at the 10^{-5} level.

We anticipate an experiment which has systematic and statistical precisions of the order of $\sim 0.3\%$ each, and we may therefore safely neglect consideration of radiative corrections in our theoretically predicted spectra, since these corrections will be unobservable in our experiment.

II.5 Screening of β Particles By Orbital Electrons

The effect of the orbital electron cloud on the β spectrum was first discussed by Rose [17]. The method proposed to incorporate this effect involved the use of the WKB approximation method for solution of the electron wave function in the potential well. The electron is born in the potential well created by the nuclear charge. The method of Rose is therefore to calculate the β particle energy probability distribution with the β particle energies individually reduced by the possible screening potentials.

which are due to the probable final states of the orbital electrons. In light atoms such as 3H , there is a large probability for shake-up or shake-off of orbital electrons during β decay, leading to a range of possible final states and a corresponding range of screening potentials. The WKB method is valid in cases where the well depth is either small or large relative to the particle energy. We will consider that a well depth less than $\sim \frac{1}{5}$ of the β particle energy satisfies the first criterion. This will prove to place a lower limit, for the purposes of this work, on the reliable energy spectrum at ~ 500 eV. In this section, we will consider the screening effects for the case of the decay of a single tritium atom. The additional effects which can be predicted for the decay of tritium within a molecule will subsequently be considered in Section II.6.

II.5.1 Shake-up and Shake-off Effects on Screening

We will begin by employing the *sudden approximation*, that is by assuming that the velocity of the β particle in its escape from the atom is large relative to the velocity of any orbital electrons [27, 28]. This implies that an unbound 3H atom, on decaying, instantaneously transforms to the daughter 3He atom with the orbital electron cloud populating the 3He orbitals in proportions given by the overlap between the original 3H orbital and those of the 3He atom. The overlap between the initial orbital and each of the various possible final orbital states will therefore govern the probability for any particular final state, and will also determine the weight of the contributions of these instantaneous descriptions of the orbital electron cloud to the screening potential. In determining these weighted contributions to the screening potential, account must be

taken of the wave functions of the initial and final atomic electrons.

For atomic 3H decay, the initial and final orbital electron wave functions are taken to be spherically symmetric, and therefore are given by their radial parts as:

$$R_{n,l}(r) = -\left(\frac{2Z}{na_0}\right)^3 \sqrt{\frac{(n-l-1)!}{2n[(n+1)!]^3}} e^{-\frac{1}{2}\rho} \rho^l L_{n+1}^{2l+1}(\rho)$$

normalized via:

$$\int_0^\infty R_{n,l}^2 r^2 dr = 1$$

$$\text{where : } Z_{\text{initial}} = 1.$$

$$Z_{\text{final}} = 2.$$

$$n = 1, 2, 3, \dots \text{ (the principal quantum number).}$$

$$a_0 = \text{the first Bohr radius} = 5.29 \times 10^{-11} \text{ m.}$$

$$l = 0 \text{ (for spherical symmetry).}$$

$$\rho = \frac{2Zr}{na_0}. \text{ and}$$

$$L_{n+1}^{2l+1}(\rho) = \sum_{k=0}^{n-l-1} \frac{[(n+l)!]^2 \rho^k}{(n-l-1-k)!(2l+1+k)!k!}$$

Substituting $l = 0$ and simplifying yields the familiar radial wave functions for spherically symmetric electron orbitals. The angular integral contributes a constant factor which will be recovered on normalization. The wave functions are:

$$R_{1,0}(r) = 2\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\frac{Zr}{a_0}} \quad (\text{II.9})$$

$$R_{2,0}(r) = \left(\frac{Z}{2a_0}\right) \left(2 - \frac{Zr}{a_0}\right) e^{-\frac{Zr}{2a_0}} \quad (\text{II.10})$$

$$R_{3,0}(r) = \left(\frac{Z}{3a_0}\right) e^{-\frac{Zr}{3a_0}} \left[\frac{1}{2} - \sqrt{\frac{2}{9}} \frac{Zr}{a_0} + \sqrt{\frac{1}{486}} \left(\frac{Zr}{a_0}\right)^2 \right] \quad (\text{II.11})$$

Table II.1: 3H β -Decay Shake-up/Shake-off Probabilities, and Screening Potentials

Final State ${}^3H\epsilon_n^{(*)}$	Transition Probability α_n	Energy Shift eV	Excitation Energy eV
n=1	0.7023	54.46	0
n=2	0.2504	13.52	40.94
n=3	0.0131	6.02	48.44
n=4	0.0040	3.39	51.07
...	...	...	...
${}^3H\epsilon^{++}$	0.0302	0.0	54.46
Weighted Sum	1.0000	41.73	12.73

Taking the 3H ground state as the initial state, overlap integrals are evaluated for final states including at least the ground state and the lowest three excited states of the daughter 3He (with $\delta_{l,l'}$ always 0). Then:

$$\alpha_n = \left| \int_0^\infty dr R_{1,0}(r)({}^3H) r^2 R_{n,0}(r)({}^3He) \right|^2 \quad (II.12)$$

The α_n shown in Table II.1 therefore determine the probabilities for shake-up to various final states, and completeness requires that the shake off probability is equal to one minus the shake-up probabilities. For each of these possible final states, according to the Rose approximation, the effect of screening is best incorporated by introducing a change in the distribution of β particle energies which reflects the effect of the potential well for the β particle birth location, assumed to be at the centre of the nucleus. This is simply the potential at the centre of the nucleus, for the orbital

electron. calculated according to:

$$V_n(0) = \int_0^\infty dr R_{n,0}(r)_{(^3He)} \left(\frac{\epsilon^2}{r} \right) R_{n,0}(r)_{(^3He)} \quad (II.13)$$

These calculations yield the Energy Shift results shown in the third column of Table II.1. These are weighted by the shake-up and shake-off probabilities (α_n) shown in the second column of Table II.1. for the various final states (n) listed in the first column. in arriving at the mean energy shift (screening potential) at the bottom of the third column. The mean amount of energy which is taken up into excitation of the orbital electron is given in the fourth column.

II.6 Consideration of Molecular Effects

It is not possible to maintain ionized or neutral atomic tritium in our detector, and so we must choose a tritiated molecule to introduce the unstable element into our experiment. There may therefore be small but not insignificant effects on our spectrum, should a change in molecular binding occur coincident with the β decay. We must then include the potential molecular rearrangements into our theoretical considerations.

We will employ tritiated methane (CH_3T), called methyl tritium, as the species to carry the unstable 3H nuclei into the detector. There are particular advantages in this choice. One advantage rests in the fact that the counting gas already contains methane (CH_4) as a quencher, and so no contamination of the counting gas will occur at the trace levels of our additive. Another advantage is that the penetration of the

^3H species into the walls and construction materials of the detector is retarded when it is introduced in a form which is tightly bound in such a large molecule rather than, for example, the T_2 or HT molecules [32]. This will therefore minimize the amount of ^3H decaying in the walls of the detector, and the amount of bremsstrahlung photons entering the chamber as a result of ^3H decays occurring immediately below the surface of the walls. Of concern in the use of this species is the relative complexity of the molecule when compared to the above mentioned hydrogen molecules. The effects on the β particle energy spectrum, due to changes in the tritiated hydrogen molecule, are relatively easily calculated [29, 33, 34, 35, 36], due to the simple homonuclear diatomic structure of the molecule, and its simple 2 electron orbitals. Overlap calculations for the HT decay to $HH\epsilon$ are more readily completed than are overlap calculations for the CH_3T to $CH_3H\epsilon$ decay. None-the-less, there has been substantial progress in the theoretical and experimental study of the methyl tritium molecule in recent years, enabling us to make predictions for the purposes of estimating the shape of our spectrum. To acquire a working understanding of the possible molecular changes which may affect our spectrum, we will consider the quantum chemistry of the molecules.

II.6.1 The Methane Molecule

The methane molecule is assembled from a carbon atom, and four hydrogen atoms. In the tritiated molecule of interest to us, one or more of the hydrogen atoms is replaced with its unstable heavier isotope, tritium. Chemical considerations are essentially

unaffected by the increased mass attributable to the triton neutrons. The molecule may, however, be affected by the recoil of the daughter ${}^3\text{He}$ atom subsequent to the decay, and by the shaking up of the molecular electron orbitals.

The ground state of the neutral carbon atom contains 6 electrons, populating atomic orbitals as follows: two electrons occupy the $1s$ orbital with opposing spins, two electrons occupy the $2s$ orbital with opposing spins, and two electrons occupy two of the six $2p$ orbitals. We will begin with a representation of these six $2p$ orbitals as comprising two opposite spin states in each of three distinct orbits, the distinction being that each orbit has its unit angular momentum vector directed along different Cartesian axes. According to Hund's Rules [37] for the stability of orbitals, the most stable state for these carbon $2p$ electrons is with their spins aligned. The tetrahedral symmetry of the chemical bonds in the CH_4 molecule requires that one of the carbon $2s$ electrons be promoted to the unoccupied $2p$ orbital, acquiring one unit of angular momentum through the bonding. Employing self consistent field calculations, hybrid sp^3 orbitals may then be described by the method of linear combinations of atomic orbitals to form molecular orbitals (SCF-LCAO-MO). In the limit of greatest precision, with a maximal basis set of atomic orbitals, this method approaches the Hartree-Fock limit, and is sometimes called *near* Hartree Fock. The sp^3 orbitals formed from orthogonal and normalized linear combinations of the minimal basis set of $2s$ and $2p$ atomic orbitals, may be written as follows [37]:

$$\begin{aligned}\Psi_1 &= \frac{1}{\sqrt{4}} (\zeta_{2s} + \zeta_{2p_x} + \zeta_{2p_y} + \zeta_{2p_z}) \\ \Psi_2 &= \frac{1}{\sqrt{4}} (\zeta_{2s} + \zeta_{2p_x} - \zeta_{2p_y} - \zeta_{2p_z})\end{aligned}$$

$$\begin{aligned}\Psi_3 &= \frac{1}{\sqrt{4}} (\zeta_{2s} - \zeta_{2p_x} + \zeta_{2p_y} - \zeta_{2p_z}) \\ \Psi_4 &= \frac{1}{\sqrt{4}} (\zeta_{2s} - \zeta_{2p_x} - \zeta_{2p_y} + \zeta_{2p_z})\end{aligned}$$

where, on switching to spherical coordinates and expanding the $Y_l^m(\theta, \phi)$ spherical harmonics in terms of the polar angles θ and ϕ , with the z axis directed along the $\phi = 0$ axis:

$$\begin{aligned}\zeta_{2s} &= \sqrt{\frac{1}{4\pi}} R_{2,0}(r) \\ \zeta_{2p_x} &= \sqrt{\frac{3}{4\pi}} R_{2,1}(r) \sin \theta \cos \phi \\ \zeta_{2p_y} &= \sqrt{\frac{3}{4\pi}} R_{2,1}(r) \sin \theta \sin \phi \\ \zeta_{2p_z} &= \sqrt{\frac{3}{4\pi}} R_{2,1}(r) \cos \theta\end{aligned}$$

$R_{2,0}(r)$ is as given in Equation II.10 above, while:

$$R_{2,1}(r) = \left(\frac{Z}{2a_0} \right)^{\frac{3}{2}} \left(2 - \frac{Zr}{a_0} \right) e^{-\frac{Zr}{2a_0}} \quad (\text{II.14})$$

These sp^3 hybrid carbon orbitals overlap the hydrogen 1s orbital electrons, with opposing spins. The total binding energy of the methane molecule is 18.2 eV per bond [38].

II.6.2 Influence Of Molecular Structure On The β Decay Spectrum

In a formal way, Kaplan and Smutney [29] have described the molecular effects on the decay spectrum through the introduction of an additional factor along with the Fermi theory above, to account for the transitions to the various possible final molecular states. We may demonstrate their approach, beginning with a modification to Equation II.2 above.

$$P_{fi}(0 \rightarrow n) = \frac{[(2\pi)^4 \delta^4(p_f - p_i)]^2}{V^4} M_{if}^+ M_{fi} M_{0 \rightarrow n}^{+,mol} M_{0 \rightarrow n}^{mol}$$

In this, we now describe the transition probability wherein the initial state, i , is the ground state of the methyl tritium molecule, signified by the 0 in the molecular matrix element subscript. The final state is now described by the index n , signifying excitation of the daughter molecular orbitals to the n^{th} excited state. In the hypothetical case where the transition is always to the ground state of the molecule containing the ${}^3H\epsilon$ atom in place of the original 3H atom, n will always be 0, and $M_{0 \rightarrow 0}^{mol}$ will always be exactly 1, reducing to Equation II.2 above. As was demonstrated, however, in Section II.5.1 above, the decay of the triton will cause some shaking up and shaking off of the atomic electron structure for the daughter atom, which will undoubtedly result in a change to the level of excitation of the final state molecular orbitals. Then, for $N - 1$ discrete excited states of the molecule, with state N being the integral of all states of excitation to the continuum, we have:

$$P_{fi} = \sum_{n=0}^N W_{0 \rightarrow n} P_{fi}(0 \rightarrow n) = \frac{[(2\pi)^4 \delta^4(p_f - p_i)]^2}{V^4} M_{if}^+ M_{fi} \sum_{n=0}^N W_{0 \rightarrow n} M_{0 \rightarrow n}^{+,mol} M_{0 \rightarrow n}^{mol}$$

$W_{0 \rightarrow n}$ is the fractional probability for final state n . The second step is made in the knowledge that the nuclear matrix elements and phase space factors are unaffected by the molecular matrix elements, which depend only on the overlap of the hydrogen and helium atomic orbitals, and on the daughter nucleus recoil momentum. We define $\Delta E_{0 \rightarrow n}$ as the chemical energy given up in transition from the ground state of the parent molecule to the n^{th} level of excitation of the electron cloud above the ground state for the daughter molecule. $\Delta E_{0 \rightarrow n}$ is referred to as the chemical shift. A negative value of $\Delta E_{0 \rightarrow n}$ indicates energy transferred to the decay products, and Kaplan and Smutney [29] have shown that this chemical shift always transfers energy to the products in the decay of tritiated hydrocarbons. Then, without reviewing the formal development of the equations as was done in Section II.1 above, we simply rewrite Equation II.5 above as follows:

$$P(E_e)dE_e = \sum_{n=0}^N W_{0 \rightarrow n} \left(C' p_e E_e E_\nu^n \left(E_\nu^n - \kappa_\nu(m, E) \frac{m_\nu^2}{E_\nu^n} \right) \right) dE_e$$

where:

$$E_\nu^n = E_\nu + \Delta E_{0 \rightarrow n}$$

Then multiplying E_ν^n through the brackets, and expanding the summation:

$$P(E_e)dE_e = C' p_e E_e \left[(E_0 - E_e)^2 + 2(E_0 - E_e) \cdot \overline{\Delta E} + \overline{\Delta E^2} - \kappa_\nu(m, E) m_\nu^2 \right] dE_e \quad (\text{II.15})$$

In this last step, we have taken

$$\begin{aligned} \overline{\Delta E} &= \sum_{n=0}^N W_{0 \rightarrow n} \Delta E_{0 \rightarrow n} \\ \overline{\Delta E^2} &= \sum_{n=0}^N W_{0 \rightarrow n} \Delta E_{0 \rightarrow n}^2 \end{aligned}$$

We recall that the N^{th} elements in each of these sums are in fact integrals over the continuum states.

Again returning to the presumption of a trivial neutrino mass, we rewrite Equation II.7 from Section II.2 above, or simply set m_ν equal to zero in Equation II.15, yielding:

$$P(E_e)dE_e = C'F(Z, E)p_eE_e\left[(E_0 - E_e)^2 + 2(E_0 - E_e) \cdot \overline{\Delta E} + \overline{\Delta E^2}\right]dE_e \quad (\text{II.16})$$

In the LCAO-MO (near Hartree-Fock) formalism, electron orbitals are described with respect to the nuclear coordinates and forces, but without consideration of the mutual repulsion effects (correlations) between electrons. As well, since the Hartree-Fock orbitals typically conserve electron spin and angular momentum separately, while total angular momentum is the physically conserved quantity, more precise calculations are achieved by taking account of spin-orbit coupling effects through the configuration interaction method. Ikuta, Iwata, and Imamura [36] have conducted such a calculation, and found a 61% probability of overlap, upon decay, to the $CH_3H\epsilon^+$ ground state, with ~ 50 eV of the decay energy being required to excite the binding as well as to ultimately break the $C - H\epsilon$ bond. Kaplan and Smutney [29] have confirmed their calculation of the overlap, predicting that 62% of transitions will be to the ground state. Claxton *et al.* [35] have also used similar methods to achieve more precise calculations for the methyl tritium molecule, employing a basis set of 376 spin adapted configurations, yielding $\overline{\Delta E} = -52.10$ eV into the bonding, after summing over 99.83 % of final states. They predict 59% overlap to the ground state of $CH_3H\epsilon^+$. While they have not quoted a value for $\overline{\Delta E^2}$, it may be estimated from their figure to within 20% of $(\overline{\Delta E})^2$, thus introducing a still smaller error to the above calculation if this substitution is made. This enables us to use for the square bracketed term in II.16 above, the term $(E_0 - E_e + \overline{\Delta E})^2$, and a similar term less

$\kappa_\nu(m, E)m_\nu^2$ for Equation II.15 above. Lippmaa *et al* [39] have recently measured the ${}^3H \rightarrow {}^3He$ mass defect as $18599 \pm 2 \text{ eV}$. We initially take this for our E_0 , redefining our E_e as the kinetic energy of the escaping β particle. Subsequently ignoring the small error acquired in substituting $(\overline{\Delta E})^2$ in place of $\overline{\Delta E^2}$, we may use equation II.7 to describe our spectrum, substituting in place of E_0 the value $E_0 + \overline{\Delta E} = 18547 \text{ eV}$. In this we have neglected the neutralization energy of $\sim 52 \text{ eV}$ (less any energy used to break the 3H bond). The efficiency for converting this energy into ionization in the gas is uncertain, and it will have little impact on our eventual results. In this way, we move beyond the sudden approximation and account for the energy balance of the orbital electrons with that of the decay products.

The additional uncertainties we have not resolved in our calculations above include the $\leq 3 \text{ eV}$ nuclear recoil energy, the $\pm 2 \text{ eV}$ uncertainty in the Lippmaa *et al.* end point measurement, and the $\sim 5 \text{ eV}$ (before squaring) uncertainty acquired in the use of $(\overline{\Delta E})^2$ for the final term in the sum in Equation II.16 above. The decision to treat the excitation of the orbital electron structure as a perturbation to the β decay must be reviewed here. While in fact $\overline{\Delta E}$ is large compared to the electron binding energies, none-the-less, Kaplan and Smutney [29] have argued that the “instantaneity” of the decay fragmentation and β particle escape, relative to the nuclear recoil, justifies this use of perturbation methods for β particle energies above $\sim 870 \text{ eV}$.

II.6.3 Methyl Tritium Decays, Screening Effects Revisited

We note that the energy transferred is significantly greater than the methane bond energy of 18.2 eV. In the event that the decay results in the dissociation of the daughter ${}^3\text{He}$ atom from the molecule, with the rest of the molecule remaining intact, there are three possible final states:

where the superscripts (*) indicate the inclusion of all possible excited states. Snell and Pleasanton [40] have reported that II.18 is observed for 82% of decays, while II.17 is never observed, despite the SCF calculations which predict $\sim 60\%$ overlap to the $(CH_3\text{He})^+$ ground state. This seeming inconsistency between predictions and observations is acknowledged by Ikuta, Iwata, and Imamura, where they note that the discrepancy with the overlap predictions to the ground state must indicate some probability ($\sim 82\%$ less 60%) that the CH_3^+ ion is left excited. They make no comment on the observation of II.19 (*shake-off*), though it must certainly be a higher order process than is II.17. They have also calculated that there is no minimum in the relationship between the $C - \text{He}$ bond energy and the bond length, and that the binding continues to decrease at large distance. This indicates that the molecule will inevitably not accept the ${}^3\text{He}$ daughter. The energy lost to the decay from the breaking of the $C - \text{He}$ bond is accounted for in the calculations of Kaplan and Smutney, as well as those of Ikuta, Iwata, and Imamura, and those of Claxton *et al.*

though there are small disagreements (~ 2 eV) in the magnitude of this energy loss.

In the sudden approximation, which we employ to describe the atomic and molecular environment encountered by the decay products, the molecule has not had time to dissociate. For an isolated 3H atom, the degree of excitation of the daughter 3He electron orbitals is best described as is done in Section II.5.1 above by the overlap between hydrogen and helium orbitals. In the molecular case similar overlap calculations using LCAO-MO orbitals will determine the degree of excitation of the molecule. The calculation of Claxton *et al* has determined the total atomic and molecular excitation of the $(CH_3He)^+$ molecule, $\overline{\Delta\epsilon^*}$, as 20.4 eV. This may be compared with 12.7 eV of excitation in the case of the decay of the bare atom alone, as shown in the fourth column of Table II.1, agreeing within ~ 8 eV.

Palke and Lipscomb [41] give the occupation number for electrons at the hydrogen atom in methane as 0.867, based on an LCAO-MO theory calculation. This suggests a minimum screening potential, consistent with the sudden approximation, of $0.867 \times 41.7 = 36.2$ eV. Kaplan and Smutney [29] have shown that the molecular rearrangement is largely local to the decaying 3H atom, and that the electron density at the daughter 3He increases by 0.915 in the decay. Taking Palke's and Lipscomb's calculations for the CH_4 molecule as indicative for conditions prior to the decay and increasing this by 0.915 yields a naive estimate of the occupation number for electrons at the 3He daughter of 1.782 after the decay. If we make a naive assumption that the molecular system as a whole, including the orbital electron environment, collapses to the final state products during the decay so as to increase the daughter atom electron density in a time short relative to the escape of the β particle, we extend our under-

standing of the term *sudden approximation*. Our treatment of the nuclear recoil as a perturbation in the decay is still valid, and our description of the orbital electron cloud according to the overlap to ${}^3\text{He}$ daughter orbitals is still appropriate. In so doing, if we further assume that the increased electron occupation goes directly to the ground state, we predict an increase in screening potential of $\sim 0.915 \times 54.46$ eV, for a total of ~ 86 eV. On the other hand, if the weightings shown in Table II.1 above apply to the total electron cloud at the ${}^3\text{He}$ atom, the screening potential will be 74 eV, and the excitation of the electron cloud at the ${}^3\text{He}$ atom will agree with the Claxton *et al* calculation of the excitation energy to within about two electron volts. The calculations of Claxton *et al* actually predict 59% overlap to the ground state of the $(\text{CH}_3\text{He})^+$ ion, compared with 70% from Table II.1 above for the isolated atom, which suggests a somewhat greater level of excitation of the molecule than is evident in our naive extrapolation of the Table II.1 calculations to the increased electron density in the decay. However, while the uncertainties mentioned above in Section II.6.2 pertain to the spectrum endpoint, the ~ 5 eV uncertainty acquired in the use of $(\overline{\Delta E})^2$ in place of $\overline{\Delta E^2}$ will also affect E_s . This uncertainty precludes distinguishing these small excitation variations from our results. A screening potential greater than $\sim 74 \rightarrow 86$ eV would indicate the rapid formation of neutral ground state ${}^3\text{He}$ within the molecule at the onset of the decay, yielding the maximum predicted screening of 109 eV. We therefore predict a range of screening potentials, E_s , of between 36 eV and 109 eV, with 74 ± 12 eV being predicted from the SCF calculations of Claxton *et al*, and 36 eV according to the sudden approximation for the atom alone. A screening potential of near 86 eV will be consistent with the predictions of Kaplan and Smutney of an increased electron density at the decaying atom, and will support the

hypothesis that this increased electron density contributes in the calculation of the screening potential. This then leads to an expectation that there is a high probability for a neutral ${}^3\text{He}$ atom among the decay products. In this last respect, this will also be consistent with the observations of Snell and Pleasanton of $\sim 82\%$ probability of neutral helium in the final states.

We will therefore fit our experimental data to a theoretical β spectrum, where:

$$P(E_e) dE_e = C' F(Z, E) (E_{0,\Delta} - E_e)^2 p_{e,s} E_{e,s} dE_{e,s} \quad (\text{II.20})$$

We will use:

$$\begin{aligned} E_{e,s} &= E_e - E_s = E_e - 74 \pm 12 \text{ eV, and} \\ E_{0,\Delta} &= E_0 + \overline{\Delta E} = 18547 \pm 10 \text{ eV.} \end{aligned}$$

Thus, in addition to our search for a massive neutrino, indicated by a discontinuity in the slope of the spectrum near 17 keV below the endpoint, we will study the low energy end of the spectrum, near the limit of the region of validity for Rose's approximation, for evidence of a screening potential which is closer to 74 (± 12) eV than to 36 eV. This prediction for the screening potential is guided by theoretical investigations of ${}^3\text{H}$ β decay in the CH_3T molecule, and specifically by the calculations of Claxton *et al* for $\overline{\Delta E} = 52.1$ eV and $\overline{\Delta E}^* = 20.4$ eV, and the predictions of Kaplan and Smutney of a 0.915 increase in electron intensity at the decaying atom.

Chapter III

The Theory And Practice Of Gas Proportional Chambers

Gas proportional chambers belong to a class of ionizing radiation detectors which employ gases to intercept and absorb ionizing radiation. In these detectors, ions are liberated in the gas due to collisions of the incident radiation with the gas, and these liberated ions are subsequently drifted under the influence of an electric field to collecting electrodes. The ion cloud which results from the interactions will contain a number of electrons and positive ions (ionized atoms or molecules) depending upon the energy of the incident radiation and on certain parameters of the gas. The energy loss rate, due to a cascade of collisions between the incident radiation and the gas molecules and atoms was described by Bethe and Bloch. The drift velocities of the liberated ions under the influence of the established electric field will depend upon

the type of ion and the strength of the electric field, which is due to the accelerating potential. At very low potential, the probability for recombination of some or all of the ions is quite high, so the number collected is fewer than the number liberated. At sufficient potential, the drift velocity will be large enough to separate the positive and negative ions before recombination can occur, and so virtually all of the ions liberated may be collected. Though the electric field produces acceleration of the ions, elastic collisions between the gas atoms and molecules and the ions limit their velocities. This *ion saturation* region will be reliable for a limited region of potentials. At higher potential, the electrons gain sufficient energy through the acceleration to cause secondary ionization of the gas. In this *proportional range*, the ion multiplication factor which results from these secondary collisions will be a predictable function of the strength of the electric field for the particular gas being employed. At still higher potentials, proportionality in the number of secondary ions to the electric field strength will begin to break down due to saturation effects, eventually leading to full ionization of the gas volume without proportionality between the primary and secondary ionization. This final region is the *Geiger Mueller* region.

We will employ a chamber comprising a cylindrical cathode with a coaxial anode wire, operating within the proportional region for this type of detector. The chamber will be filled with a common counting gas, and the unstable isotope which we will investigate will be introduced into the counting gas mixture. We therefore now proceed to discuss the theory and practice of gas proportional chambers, and the particular gas we have employed.

III.1 Gas Ionization

The interaction of incident electrically charged radiation with a gaseous medium has been discussed by Sauli [42]. The primary interactions are collisions between the electromagnetic fields of the incident particle and the atoms of the gaseous medium, with all other types of interactions usually being negligible.

The Bethe-Bloch formula describes the rate of energy loss of electrically charged radiation incident upon matter [43]:

$$\frac{dE}{dx} = -Kz^2 \frac{Z}{A} \frac{\rho}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right] \quad (\text{III.1})$$

where E is the energy of the incident particle so $\frac{dE}{dx}$ is the energy loss rate per unit distance. K is a constant given by:

$$K = \frac{2\pi N_A \epsilon^4}{m_e c^2} = 0.307075 \text{ MeV g}^{-1} \text{ cm}^2.$$

N_A is Avogadro's number. Z is the (average) atomic number and A is the (average) atomic weight of the gaseous medium, and z is the charge of the incident radiation in units of the fundamental electron charge, e , and therefore is equal to 1 for the β particles of our experiment. $m_e c^2$ is the electron rest mass. ρ is the density of the gaseous medium. β is the velocity of the incident radiation in units of c , and γ is the associated relativistic factor. I is the first ionization potential of the absorbing matter, given as $I = I_0 Z$ where I_0 is approximately ~ 12 eV, but more accurately measured and tabulated for various gases. T_{max} is the maximum kinetic energy transferred to an electron of the medium in any collision, given by:

$$T_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{M} + (\frac{m_e}{M})^2}$$

M is the mass of the incident particle, which in our case is an electron, and β is small so γ is nearly 1. Then we may approximate T_{max} as follows:

$$T_{max} \approx \frac{1}{2} m_e c^2 \beta^2 \gamma^2$$

The final two terms in the square bracketed sum of Equation III.1 contribute to the loss rate at higher energies. The behaviour of this function is such that, for any given incident charged particle, the energy loss rate is highest at lowest incident energy, and decreases with increasing incident particle energy until a minimum is reached. This minimum in the rate is known as the *minimum ionizing region*, and it occurs for all media in the region where $\beta\gamma$ of the incident particle is of order 3.0 to 3.5. At still higher incident energies, the energy loss rate increases gradually as β^2 approaches 1 and the $\beta^2\gamma^2$ term in the logarithm grows more rapidly. This feature is known as the *relativistic rise*. The density correcting term δ contributes to the function only at higher energies, limiting the relativistic rise and bringing the loss rate to a plateau. We will be concerned only with very small incident energies which are well below even the minimum ionizing region. In the experiment we are conducting, the maximum possible velocity for a ${}^3\text{H}$ β particle is $\sim 0.26c$ and so $\beta\gamma_{max}$ is $\sim 0.27c$. (At the lowest energy end of our experiment, near 500 eV, β and $\beta\gamma$ are both ~ 0.04 .) We may therefore safely ignore the density correcting term, δ , since we are interested in interactions of particles which are well below the minimum ionizing region. We therefore rewrite Equation III.1 above in the following way:

$$\frac{dE}{dx} \approx -K \frac{Z}{A} \frac{\rho}{\beta^2} \left[\ln \frac{m_e c^2 \beta^2 \gamma^2}{I} - \beta^2 \right] \quad (\text{III.2})$$

While the theory of Bethe and Bloch describes the average rate of energy loss,

individual collisions of the incident ionizing radiation with the medium may impart energy in a number of ways. At low energy transfer, atoms of the medium may be excited by the field of the passing particle, but are not necessarily ionized. More efficient energy transfers will result in ionization along the path of the incident particle. In close collisions between the incident particle and the orbital electrons of the atoms of the medium, the amount of energy transferred may be very large relative to the binding energy of the atoms of the medium. In such cases, the momentum transferred to the target electrons may be a large fraction of the momentum of the incident particle, and a shower of electrons will develop from a series of such interactions. In such cases the incident particle will be deflected somewhat from its previous path, though there will still be a persistence of the incident momentum governing the development of the cloud of ionized particles, and the shower will broaden in width about the path of incidence. This will proceed until the integral of the Bethe Bloch function over the total path depletes the energy of the incident particle, and all other particles in the shower, to below the ionization potential of the medium. Within this shower region there will result a total amount of ionization, about the path of length given according to the Bethe Bloch relation, consisting of individual ions which have resulted from low momentum transfer collisions and in clusters of ionization which result from minor particle showers associated with higher momentum transfer collisions. Sauli [42] has tabulated the characteristics of typical counting gases, and we have reproduced in Table III.1 those characteristics of interest to us in our choice of 90% argon 10% methane ($P10$) gas for our detector.

For our detector, we will operate at ~ 5 atmospheres pressure, and so the gas

Table III.1: Counting Gas Properties

Gas	Z	A	ρ g/ ℓ	I_0 eV	E_i eV/ion pair	W_i eV/ion pair	dE/dx (min.) keV/cm	Primary ions cm ⁻¹	Total ions cm ⁻¹
Ar*	18	39.9	1.66	15.8	15.7	26	2.44	29.4	94
CH ₄ *	10	16	0.67	13.1	15.2	28	1.48	16	53
P10*	16.6	36.6	1.56	14.7	15.7	26.2	2.34	28.2	90.1
P10**	16.6	36.6	7.80	14.7	15.7	26.2	11.7	141.0	450.5

* at 1 atmosphere pressure

** at 5 atmospheres pressure

density, energy loss rate, and number of primary and total ions created per collision are those shown in the bottom row of Table III.1. This table implies a path length for the most energetic of 3H β particles (~ 18.6 keV) of 12.7 mm. There will be some non-linear dependence of the total incident particle path length on its energy, as is evident in the non-linear contributions of β and γ to the Bethe Bloch formula. The energy transferred in individual collisions is known to exhibit a Landau like distribution, that is an initially Normal distribution, skewed to exhibit an extended high energy tail. [43] Primary ions, which result from collisions directly involving the incident particle, will be liberated in quantities as indicated in the second to last column of the table. Many of these primary electrons will be energetic, and will participate in secondary collisions, ultimately liberating quantities of ion pairs as indicated in the final column of the table. W_i is the average energy per ion pair, connecting the total number of ion pairs liberated with the total energy absorbed from

the incident particle. Then the total number of ion pairs liberated depends linearly on the incident particle energy. The probability of liberation of an ion pair at a point along the path of the incident particle will exhibit fluctuations according to Poisson statistics. However, the statistical fluctuations in the number of electrons liberated as a function of energy deposited exhibits a narrower distribution than is expected for a Poisson distribution, since not all deposited energy is converted into kinetic energy of the liberated ions in the gas. It is a characteristic of Poisson distributions that the variance is equal to the mean, but for this gas ionization process we expect a variance which is narrower than the mean, by a factor F which is called the Fano factor. The Fano factor is a characteristic of the type of gas being used, and for the 90% argon 10% methane ($P10$) gas we will employ the Fano factor is ~ 0.17 , so the width of the distribution of total ions liberated by an incident particle will be 0.17 times the mean number. Then:

$$\left(\frac{\sigma_{n_0}}{n_0} \right)^2 = \frac{F}{n_0}$$

Since, at the low energy end of our spectrum, where the reliability of our description begins to suffer from our theoretical approximations, incident particles will none-the-less liberate ≥ 20 electrons in the gas. We will thus take advantage of the similarity between the Poisson and Gaussian distributions at high means. At such a high mean, we may safely employ the Gaussian distribution to describe the shape of the distribution of statistical fluctuations inherent in the ionization process. We will return to the question of statistical fluctuations in Section III.2 below. This will enable us to discuss detector energy resolution in Section III.4.

III.2 Gas Amplification

We now seek to determine the charge multiplication performance of a proportional chamber, following the approach of Knoll [44]. The positive ions liberated by the incident particle have large masses and low mobility relative to the electrons liberated. The drift of these positive ions will be much slower than that of the electrons. At sufficiently high field strengths ($\sim 10^6$ V/m), the electrons will be accelerated towards the anode to sufficient energies that they will liberate additional electrons on collisions with the gas atoms and molecules. This will result in a proportional multiplication of the electron charge drifting towards the anode, and subsequent collisions will contribute additional gain. This cascade is known as a Townsend avalanche in which the total liberated charge is given by the Townsend equation:

$$\frac{dn}{n} = \alpha dx$$

α is known as the first Townsend coefficient, and is in fact the inverse of the mean free path for the gas. In the case where the field results from flat parallel plate electrodes, the field will be constant and the Townsend coefficient will be everywhere constant as well. Then the number of electrons in the drifting cloud will be given by the integral of the Townsend equation:

$$n(x) = n(0)e^{\alpha x}$$

where $n(0)$ is the total ionization produced by the incident particle and x is the distance over which the charge has drifted.

Elementary theory of electricity and magnetism, however, instructs us that for

the case of two colinear and cylindrical electrodes, with the anode at the central axis, the electric field established between these electrodes varies with radius (from the anode outward), according to:

$$\mathcal{E}(r) = \frac{V}{r \ln \frac{r_c}{r_a}} \quad (\text{III.3})$$

where V is the voltage established between the cylinders, and r_c and r_a are respectively the radii of the cathode and the anode. We have established from the Townsend equation that the multiplication, M , of the liberated charge depends exponentially on the Townsend coefficient, so we write for a varying field that:

$$\ln M = \int_{r_a}^{r_c} \alpha(r) dr \quad (\text{III.4})$$

We make a change of variables, now integrating over the electric field strength in place of the radius, and so we rewrite Equation III.4 as:

$$\ln M = \int_{\mathcal{E}(r_a)}^{\mathcal{E}(r_c)} \alpha(\mathcal{E}) \frac{\partial r}{\partial \mathcal{E}} \partial \mathcal{E} \quad (\text{III.5})$$

Now substituting Equation III.3 for the field between cylindrical electrodes into Equation III.5, we determine:

$$\ln M = \frac{V}{\ln \frac{r_c}{r_a}} \int_{\mathcal{E}(r_a)}^{\mathcal{E}(r_c)} \frac{\alpha(\mathcal{E}) \partial \mathcal{E}}{\mathcal{E}^2} \quad (\text{III.6})$$

We have determined that the total gas gain in a cylindrical proportional tube geometry therefore varies exponentially with the voltage established across the drift region, and with the ratio of the radii of the anode and cathode. Beyond this, we are interested in studying how the Townsend coefficient varies with the electric field. Diethorn [45] assumed a linear relationship between these parameters and derived the expression:

$$\ln M = \frac{V}{\ln \frac{r_c}{r_a}} \frac{\ln 2}{\Delta V} \left(\ln \frac{V}{pr_a \ln \frac{r_c}{r_a}} - \ln K \right) \quad (\text{III.7})$$

where p is the gas pressure. ΔV is the mean accelerating potential which the electrons cross between successive collisions with the gas molecules, and K is the minimum value of reduced field, $\left(\frac{\mathcal{E}}{p}\right)$, at which collisions between electrons and gas atoms and molecules produce additional ions. Thus, in this theory, the gas gain varies inversely with gas pressure, and with both linear and exponential voltage factors. The constants of the gas were measured by Wolff [46] for the 90% argon 10% methane mixture we have chosen. He reports that ΔV is 23.6 eV, and K is 4.8×10^4 V/cm/atm.

Many researchers have attempted to characterize the gas gain with polynomials of various orders in both pressure and voltage. Beingessner *et al* [47] parameterized the gas gain for argon-methane gas according to:

$$\ln M = \frac{V}{\ln \frac{r_c}{r_a}} \left[a_0 + a_1 \frac{\mathcal{E}(r_a)}{p} + a_2 \frac{\mathcal{E}^2(r_a)}{p^2} + a_3 \frac{\mathcal{E}^3(r_a)}{p^3} + a_4 \frac{\mathcal{E}^4(r_a)}{p^4} + b_0 \frac{\mathcal{E}(r_a)}{p^2 r_a} \right] \quad (\text{III.8})$$

where $\mathcal{E}(r_a)$ is as given in Equation III.3, calculated at $r = r_a$, and:

$$a_0 = -0.6313 \times 10^{-2}$$

$$a_1 = 0.3612 \times 10^{-3}$$

$$a_2 = -0.1020 \times 10^{-5}$$

$$a_3 = 0.1670 \times 10^{-8}$$

$$a_4 = -0.9980 \times 10^{-12}$$

$$b_0 = -0.6260 \times 10^{-5}$$

where the units of the individual constants, a_n , are $(V\text{cm}^{-1}\text{Torr}^{-1})^{-n}$, and those of b_0 are $(V\text{cm}^{-1}\text{Torr}^{-2})^{-1}$. We have found this a useful characterization, and confirmed their parameterization within the accuracy of our calibrations.

The number of electrons produced in an avalanche is predicted, on the assumption that each collision for any electron is independent of the electron's history, to be described by the Furry distribution:

$$P(A) = \frac{(1 - \frac{1}{\bar{A}})^{A-1}}{\bar{A}}$$

where $\bar{A}$ is the mean number produced, which is therefore the average gain in an avalanche. At gains greater than ~ 100 , this approaches an exponential distribution given by:

$$P(A) = \frac{e^{-\frac{A}{\bar{A}}}}{\bar{A}}$$

A more sophisticated model by Byrne [48] includes consideration of the history of each electron in the avalanche, to produce a more accurate theory in which the fluctuations in the avalanche process exhibit a Polya distribution:

$$P(A) = \left(\frac{A(1 + \theta)^\theta}{\bar{A}} \right) e^{\left(\frac{-A(1 + \theta)}{\bar{A}} \right)}$$

where θ is a number between 0 and 1, and is dependent on the fraction of electrons which exceed the potential necessary to ionize the gas. For a Polya distribution, the relative variance is given by:

$$\left(\frac{\sigma_A}{\bar{A}} \right)^2 = \frac{1}{\bar{A}} + b$$

where $b = (1 + \theta)^{-1}$, which is typically near 0.5. As $\bar{A}$ becomes large, this expression reduces to

$$\left(\frac{\sigma_A}{\bar{A}} \right)^2 \approx b$$

For our choice of 90% argon and 10% methane gas, b is exactly 0.5.

III.3 Ion Drift and Diffusion

In the cylindrical geometry of our proportional chamber, the strength of the electric field, $\mathcal{E}(r)$, varies inversely with radius (equation III.3). Since the gas amplification is an exponential function of the electric field strength (equations III.7 and III.8), it is understood that the gas multiplication occurs predominantly near the central anode wire. Prior to collection of the electrons on the anode wire, the electrons and positive ions each drift towards their attracting electrodes at rates governed by their respective mobilities. That is:

$$w^{\pm} = \mu^{\pm} \frac{\mathcal{E}}{p}$$

where $w^{\pm}$ is the drift velocity of the positive or negative ions, $\mu^{\pm}$ is the respective mobility, and p is the gas pressure. For electrons, the drift time of importance for our consideration is that time prior to and during the avalanche gain, since the majority of the electrons are liberated in the high electric field very near the anode wire, and collected on the wire immediately. For electrons liberated by the incident radiation in regions of the chamber far from the anode wire, the drift towards the anode will be accompanied by a small amount of diffusion of the charges from the regions of higher concentration to regions of lower concentration. This diffusion will result in some dispersal of the charge both in the direction of and perpendicular to the electric field lines, though the directionality of the electric field will act to minimize diffusion along the direction of increasing field strength. The diffusion of the charge follows a Gaussian distribution [42] away from the centre of the concentration, as given by:

$$\frac{dN}{N} = \frac{1}{\sqrt{4\pi Dt}} e^{-\left(\frac{r^2}{4Dt}\right)} dx$$

where x is the distance from the centre of the charge concentration, t is the diffusion time, and D is the diffusion coefficient. Transverse and longitudinal diffusion coefficients are signified by D_T and D_L respectively. Sauli [42] thus gives the electron diffusion length via:

$$\sigma_x^2 = \frac{2D}{w}$$

Sauli [42] also gives electron drift velocities in argon-methane gas mixtures as a function of the reduced field ($\frac{E}{p}$), from which we have determined for our chamber that electron drift velocities will be near $5 \text{ cm } \mu\text{s}^{-1}$. The maximum drift time for an event in our experiment will therefore be a few μs . Walenta [49, 50] gives a constant relationship between D and μ^- for $P10$ gas, enabling us to predict the diffusion of the electrons. This will result in some predictable variations in arrival times of the electrons at the anode wire. Since it is the collection of the electrons at the anode which produces a signal from the proportional chamber, these arrival time variations will contribute to variations in the rise time of the signal pulse from the chamber. (This signal pulse would theoretically be a step function if all charges arrived simultaneously.) We have used this information to construct a Monte Carlo simulation of the processes, and to numerically integrate the diffusion along the electrons drift paths, in order to predict the distribution of pulse rise times for our experiment. This consideration was necessitated by the concern that amplifier peaking time constants be appropriately set for our signal pulses, and by a concern that we maintain sensitivity to detection of potentially piled up pulses without introducing arbitrary or accidental cuts on our fiducial volume as a result of inappropriate timing settings in our apparatus.

The positive ions on the other hand, predominantly produced in the avalanche near the anode, must drift the full radius of the chamber before being collected at the cathode cylinder. As these positive ions drift away from the anode wire, the potential induced on the anode will diminish with increasing drift distance. The mobility of these heavy positive ions is less than that of the electrons and so their drift times will be long compared to the electrons. From data supplied by Sauli [42] we see that the mobility of positive ions in our chamber will be near $2 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. The collection of these ions will be very slow compared to that for the electrons, and the signals induced on the anode wire and on the cathode will decline slowly as the positive ions drift towards the cathode, and will last many hundreds of μs . In our Monte Carlo simulation we have also predicted the pulse fall times, so that we may ensure appropriate dead times in our experimental chamber and establish the rate for data taking for our experiment. (In fact, the pulse fall times in our system are actually governed by the decay time of our pre-amplifiers, which is short compared to the positive ion drift times.)

III.4 Resolution

Since the initial ionization of the gas due to the incident radiation and the subsequent gas gain are independent processes, we may add the variances from these two processes in quadrature to determine the resolution we may expect from our choice of gas.

according to:

$$\left(\frac{\sigma_Q}{Q}\right)^2 = \left(\frac{\sigma_{n_0}}{n_0}\right)^2 + \frac{1}{n_0} \left(\frac{\sigma_A}{A}\right)^2 = \frac{F}{n_0} + \frac{b}{n_0}$$

For our choice of 90% argon and 10% methane gas, this yields a theoretical best case FWHM resolution, at the $^{55}\text{Mn } K_{\alpha_1}$ X-ray (5899eV) of 12.8% [44]. Replacing n_0 above by the ratio $\frac{E}{W_i}$, and taking the square root, we arrive at a theoretical relationship between the resolution and the incident particle energy, given as:

$$\frac{\sigma_Q}{Q} = \left(\frac{W(F+b)}{E} \right)^{\frac{1}{2}}$$

That is, the resolution varies inversely as the square root of the incident particle energy. This implies a theoretical best case resolution of 44% FWHM at the lowest energy end of our spectrum (500 eV), and $\sim 25\%$ FWHM at the region of the discontinuity in the slope of the spectrum attributable to Simpson's hypothesized heavy neutrino.

III.5 Space Charge Effects

During the charge multiplication which occurs very near the anode wire, the increasing charge density results in some screening of the electrode potentials, thereby diminishing the local strength of the electric field and prematurely limiting the charge gain. This deviation from ideal behaviour for the proportional chamber displays itself in two principal ways. Depending on the proportional chamber geometry, there may be a dependence of the gas gain on the raw event rate, with gas gain diminishing somewhat as rate is increased. As well, there may be a self-induced screening

of individual events, resulting in a small deviation from proportionality between the incident particle energy and the charge collected at the anode wire.

III.5.1 General Space Charge Effects

Sauli [42] has discussed the dependence of gain on rate for typical proportional chambers. He has used the approach taken by Hendricks [51] to describe the dependence of the anode potential on rate as follows:

$$\Delta V = \frac{nM\epsilon RT}{4\pi^2\epsilon_0} = KMR$$

where n is the number of ion pairs created by the incident radiation. M is the gas multiplication factor discussed in Section III.2 above. ϵ is the electron charge. R is the rate of events arriving at each millimetre of the anode wire, and T is the time required for the positive ions to drift to the cathode and be neutralized. This then describes a linear dependence between the anode potential and the chamber event rate, and consequently an exponential dependence of the gas gain on rate, given by:

$$M = M_0 e^{-KM_0 R}$$

Sauli [42] describes the extent of the charge distribution along the anode wire as being typically 0.1 to 0.5 mm . We will operate our proportional chamber at rates near 600 Hz within a fiducial volume containing a 25 cm anode wire. This will yield a rate of order $24s^{-1}mm^{-1}$, which is very small relative to the $\sim 500 s^{-1}mm^{-1}$ at which Sauli observes such rate effects to occur. In the above, K is of order 10^{-10} .

since $n \leq 700$, $\epsilon = 1.6 \times 10^{-19}$, $T \leq 10^{-3}$, and $\epsilon_0 = 8.85 \times 10^{-12}$. Then assuming operation of the chamber at a gas gain near ~ 500 , we will expect the exponential term in the above expression to be of order 10^{-6} , and we expect the resulting gain reduction due to rate to be, to first order, 1 part in 10^6 . As well, since we expect to operate the experiment at constant rate, within statistical fluctuations, any intrinsic rate effect will not be separable in the normalization of the data, except perhaps for some trivial deterioration in resolution connected with the statistical fluctuations in the rate.

III.5.2 Self-Induced Space Charge Effects

Mori, Uno and Watanabe [52] have described the electric field in a proportional chamber, at the avalanche multiplication, as having two components. The first component is described by Equation III.3 above, and the second is a time dependent component proportional to the space charge which results from the earlier positive ions drifting from the anode wire. The result is that the slower portion of the liberated electrons are subjected to a smaller field and thus achieve smaller multiplication than those which arrive at the anode most rapidly. The magnitude of this effect is seen to depend on gas density, anode wire diameter, and on the magnitude of the charge liberated by the incident radiation. This therefore contributes an energy dependent term to the gas gain. Since the effect is to reduce the electric field proportionately with the amount of absorbed energy, and the gain varies exponentially with the field, then we may expect an exponential deviation of the amplitude of the event from propor-

tionality, increasing with increasing amplitude. This deviation will show itself as an exponential increase in the channel width of our multi-channel analyser, which we may usefully simplify through a Taylor series expansion to simply a term proportional to the square of the channel number. We have no theoretical prediction of the magnitude of this term, though it is apparent that a term which is of order 10^{-5} ch^{-1} times the magnitude of the linear term (eV/ch) will contribute a change to channel width of $\sim 1.5\%$ from the first to the 1500^{th} channel of data, and that higher order terms in the Taylor expansion may be safely ignored since they will contribute at two orders of magnitude or more below this.

Chapter IV

The Experiment

IV.1 Detector Description

A proportional chamber was assembled (Figure IV.1), consisting of a $20\ \mu m$ diameter gold coated tungsten anode wire placed along the common axis of three cylindrical cathode pieces. The cathode pieces were made of gold coated copper tubes of $7.8\ cm$ inside diameter. The cathode pieces were held $5\ mm$ apart with Teflon collars. The central cathode piece was $25\ cm$ in length, and each of the two outer cathode pieces, employed to ensure a constant electric field along the fiducial volume, was $8\ cm$ in length. The outer ends of each of the two outer cathode pieces were closed with Teflon end caps, onto which were mounted two each additional cylindrical copper guard rings. The inner diameters of these copper guard rings were arbitrarily chosen

pressure vessel prior to filling with virgin $P10$ gas. The methyl tritium was extracted from a 3.7 GBq source ampoule, acquired in February of 1987, and tested to contain 94.3% CH_3T , 3.2% HT or T_2 gas, 1.2% 3H ethane, 0.4% 3H propane, and 0.3% 3H butane. The chamber was typically filled with ~ 3.5 kBq of methyl tritium, and 5 atmospheres of $P10$ gas, yielding a typical data taking rate of $\sim 650s^{-1}$ in the fiducial volume. When not operating the experiment, the chamber was typically pumped to a pressure of 50 millitorr, and heated to over $40^\circ C$ to facilitate outgassing.

Connections to the anode wire, each of the cathode pieces, and the guard rings were brought through the pressure vessel to the external high voltage and multi channel analyser system. To minimize distortions in the electric field at the 5 mm gaps between the cathode pieces, the outer casing of the detector was held at a small positive potential with respect to the cathode pieces. This established an electric field between the cathodes and the housing, which was adjusted to be equal in magnitude and opposite in direction, at the outer surfaces of the cathode pieces, to the field on the inner cathode surfaces. As a result, ions produced in the gas by events which occur outside of the fiducial volume do not drift into the fiducial volume. The high voltage was thus established between the anode wire and the cathodes, and was typically of order 2500 to 3000 volts. The cathode to housing potential was of the order of 72 volts.

We have employed two Canberra model 2001A low noise preamplifiers for this experiment, which we have capacitively coupled to the anode and the central cathode piece. In this type of detector, the intrinsic detector noise is a function of the capacitance of the elements of the detector. The capacitance of coaxial cylindrical

electrodes is calculated according to:

$$C = \frac{2\pi\epsilon_0\ell}{\ln \frac{r_o}{r_i}}$$

where ℓ is the length of the electrodes, and r_o (r_i) is the outer (inner) cylinder radius respectively. For our geometry, we therefore predict capacitances of 2.6 pF, 31.2 pF, and 10.0 pF for the anode, central cathode, and each of the end cathodes respectively with respect to the grounded chamber case. To these we must add the parallel capacitances of the connectors to which these components are attached, measured at 22pF, and we may then ignore the trivial contributions of the internal wiring. In order to observe the capacitances, we have employed two different preamplifiers for which the intrinsic noise dependence on load capacitance is well known. These are the Tennelec model TC171, and the Ortec model 142. We have used a pulser to generate monoenergetic signals through these preamps into our spectrum analyser, with the detector load attached to the preamps, and we have observed the width of the resulting peak as a measure of the load capacitance. From the preamp characteristics, the load capacitances inferred are 26 pF, 54 pF, and 33 pF for the anode, central cathode, and each of the end cathodes respectively. The Canberra 2001A preamp noise specification is for maximum noise of 5.3×10^{-17} Coulombs with a 100 pF load capacitance. At a nominal gas gain of ~ 500 , this will contribute typically ~ 17 eV in rms noise to the signals. At the capacitances stated above for the anode and cathode pieces, the rms noise will be ~ 7 eV, ~ 14 eV, and ~ 9 eV respectively. For a 1 keV signal, this will change the width of the signal on the anode (cathode) by a multiplicative factor of 1.05 (1.1), while at 15 keV, the factor will be 1.0125 (1.025). The best case FWHM anode resolution predictable at the ^{55}Mn K X-ray will consequently be increased from 12.8% to 13.2%.

IV.2 The Multi-Channel Analyser

We have employed two Tennelec TC241 high performance spectroscopy amplifiers to shape and amplify the signals from the detector anode and cathode preamplifiers. The specified integral non-linearity for these amplifiers is $\leq \pm 0.05\%$ over the 0 to 10 volt output range, at a peaking time of $3\mu s$, which is the peaking time we have used for our experiment. We have directed the bipolar output signals from these amplifiers (see Figure IV.1) to two channels of a twelve channel LRS2249W 11 bit (1980 channel) analog to digital converter, resident in a Camac crate. We have limited the used range of the amplifiers at 0 to 2 volts at the outputs. The anode preamplifier signal is also sampled and attenuated by a factor ~ 6.11 , and presented to a third channel of the analog to digital convertor, in order to monitor the rate of activity in the chamber for events above the energy range of the 3H spectrum we seek to observe. The integral non-linearity of the analog to digital converter is also specified at $\leq \pm 0.05\%$ over the 0 to 2 volt input range. The duration of the gating pulse at the input of the analog to digital converter is specified at between 30 ns and $10\mu s$, and the conversion time for production of the digitized signal is $106\mu s$. We have observed some non-linear behaviour in the analog to digital converter channel widths, in the region between about channels 200 and 250. For this reason, we have adjusted the pedestals on the anode and cathode analog to digital converters to near channel 300. This has reduced the width of the spectrum to 1680 of the 1980 channels specified for the LRS2249W. The attenuated anode signal pedestal was adjusted to near channel 240, the maximum possible, and this was deemed acceptable despite the residual non-linearity evident in the low energy end of the resulting spectra, since this spectrum is

of interest only in the higher ranges above channel 300. The performance of the signal detection, amplification, shaping, and digitization system was subsequently tested, as discussed in Section IV.3 below.

The anode preamplifier signal is also presented to a Tennelec model 203BLR linear amplifier (see Figure IV.1), in order to generate a signal to the data acquisition system at the onset of an event in the detector. This amplifier was operated at high gain (~ 140), and with a $1 \mu s$ peaking time, to minimize the noise triggering. The output of this amplifier was presented to a pulse discriminator, operating at a minimal threshold, the output of which initiated a sequence of timings and delays. Ultimately, a $400 ns$ wide gate pulse was delivered to the analog to digital converter, $3.2 \mu s$ after the linear amplifier began to respond to the formation of the signal, thereby sampling the peak of the shaped signal. Piled up signals were rejected should they occur before the gate pulse arrived at the analog to digital converter, but not within the first $1 \mu s$ of the formation of the timing signal. We selected this rather long pile-up resolving time since we predicted with our Monte Carlo simulation that pulse rise times for events occurring near the walls of the chamber were characteristically approximately $700 ns$ or more. This long mean rise time results from the diffusion of the charge during its slow drift from the outer reaches of the chamber to the anode wire. The arrival times of the clusters of charge at the anode wire are therefore spread out in time, causing the slow rise in the signal. We subsequently observed that the distribution of event rise times in the detector, when freshly filled with virgin gas but without 3H , exhibited rise times near $700 ns$, as we would expect for bremsstrahlung photons or β particles originating on or in the walls. We also inserted a dead time of $330 \mu s$, updating, after

any event in the detector, and before accepting additional events for analysis. This dead time was long relative to the conversion time of the analog to digital converter, and was in excess of five decay time constants for the preamplifiers, ensuring greater than 99.6% return of signal to the baseline for the preamplifier outputs.

The anode preamplifier output was also presented to an Ortec model 474 Timing Filter Amplifier, for initiating pulse rise time measurements. The timing filter amplifier was operated at a gain of ~ 70 , integrated with a 200 ns integration time constant, and not differentiated. The output of the timing filter amplifier was presented to two Ortec model 453 constant fraction discriminators, which output signals at nominally 20% and 80% of the rise of the preamplifier signal respectively. The outputs of these two constant fraction discriminators were used to start and stop the timing sequence of a LeCroy 2228 time to digital convertor, resident in the Camac crate, so that event rise times could be measured. This rise time measurement technique could not be presumed reliable in the very low energy region of the spectrum under study, since the statistical fluctuations in the arrival times of ion clusters at the anode wire were intrinsically large. For example, for events of ≤ 2000 eV, the number of primary clusters were ≤ 24 ignoring the ionization fluctuations. The arrival time fluctuations of the first 20% of these clusters would exhibit a Poisson distribution, in which the variance is large relative to the mean, and so the observed rise time would be uncertain by the first $\sim 20\%$. At lower energies, the same arguments apply for the observation of the 80% point of the leading edge of the pulse, increasing the uncertainty in the measurement at lowest energies. Consequently, this technique was not used in the final data analysis, but only as an aid in confirming the signal rise

time distribution for a restricted range of energies, which was then taken to be characteristic of all energies in the spectrum. In this way, the Monte Carlo prediction of pulse rise times was confirmed, and the pile-up rejection timing was confirmed.

Additional circuitry, not shown in Figure IV.1, was employed to monitor the system live time during data acquisition. This was done in two ways. A gated timer was operated such that it accumulated the total time during which the system was available while operating. This timer gave a visible readout, which was recorded at the end of each data acquisition run. As well, a scaler was mounted in the Camac crate, which was used to count three things. A 30 Hz rectangular pulse oscillator was presented to one channel of the scaler. The cycles of this oscillator were counted throughout the data acquisition run. A second channel of the scaler counted cycles of the oscillator, though gated off by any signal which initiated the timing cycle discussed above, and gated back on at the end of all contiguous 330 μ s dead times. The third channel of the scaler was used to count the number of times the pile up reject circuit was initiated.

The Camac crate controller was connected to a desk top computer through a Kinetic Systems KS2925 IBM PC to Camac crate interface card. Interfacing with this card is normally low speed, since the card is normally polled via software to determine when an event has occurred in the apparatus. For the purposes of this experiment, the interface card was modified in order to employ some unused circuitry on the card to cause the occurrence of an event to generate a hardware interrupt to the computer. Assembly Language and Fortran software were written to enable the computer to operate in an at-rest cycle in which it presented to the screen the spectra acquired

to that point. It was interrupted from that activity by the occurrence of any event in the detector, through the generation of a *look-at-me* signal from the Camac crate controller, and then proceeded to collect and analyse the data present in the analog to digital and time to digital converters in the Camac crate. At the completion of the data acquisition cycle, the computer collected the scaler inputs, and wrote files of histogram data describing the spectra collected. Data collected for each event include the anode signal amplitude, the cathode signal amplitude, the attenuated anode signal amplitude, the ratio of the anode and cathode signal amplitudes, and the rise time measurement. As well, decisions to discard some events were made according to logic coded into the computer programs, in order to effect a fiducial volume cut on the ends of the central cathode region, as will be discussed in Section IV.4.2 below. It was therefore appropriate to record histogram data for both accepted and rejected events, and to save these various histogram files upon completion of the data acquisition runs.

IV.3 Calibration and Linearity Tests

Linearity of the gain of the preamplifier, amplifier, and analog to digital converter system was tested through the injection of a test pulse, from a Berkeley Nucleonics model PB4 precision pulse generator, into the test inputs of the preamplifiers, under various load conditions. These tests also yielded a precise measurement of the location of the pedestal channels. A Berkeley Nucleonics model GL-3 precision ramp pulser was also employed in conjunction with the model PB4 pulser to sweep out a

flat spectrum of pulse heights from the PB4 pulser, so that the differential linearity of the system might also be determined. The results of these two sets of tests provide a measure of the confidence we may have in the linearity and integrity of the experimental apparatus, apart from the actual proportional chamber, for which we have employed activated radioactive sources of known energy.

IV.3.1 Pulser Tests

The Berkeley Nucleonics model PB4 pulser is specified to have integral non-linearity of $\leq \pm 0.005\%$, differential non-linearity of $\leq \pm 0.03\%$, and amplitude jitter of $\leq \pm 0.001\%$. We have conducted a series of tests with the PB4 pulser to investigate the linearity of the multichannel analyser. An example of the results for both the anode and cathode spectra is shown in Figure IV.2. The fitting to a Gaussian distribution did not produce high confidence in the results, as indicated by the poor, though minimized, values of χ^2 per degree of freedom. However, the mean values of the peaks achieved in the tests were very robust, as were the narrow widths of the distributions. Though the fits shown in Figure IV.2 are to a simple Gaussian distribution, the addition of an underlying polynomial of up to third order, or a second Gaussian, does not appreciably change the values of the parameters selected in the fitting, or the low confidence in the fit. This would indicate that the distributions were well defined, though did not correspond well with a normal distribution. The example shown in Figure IV.2 was acquired with the proportional chamber connected to the multichannel analyser electronic system, but with the high voltage turned off.

Figure IV.2: Pulser Test Spectrum

We take the width of the distributions to be a measure of the total noise of the system combined from all sources, including the intrinsic preamplifier noise and the amplifiers and analog to digital converters. The quality of these fits will not prove to be critical to any aspect of our work except for the precision of our measurement of the electronic system noise, and thus will be shown to have an inconsequential effect on our results.

The rms noise for the TC241, at our typical gain setting, is specified at less than $70 \mu\text{v}$, rated at the amplifier input, or equivalently at the preamplifier output. Given the 1pF feedback capacitor of the preamplifier, this is equivalent to a charge

of 7×10^{-17} Coulombs, or 437 ion pairs, at the anode and cathode. At a nominal gas gain of 500, this is 0.874 primary ions, which at 26 eV per ion pair for *P10* gas means 22.7 eV in maximum noise added by the amplifiers. We have observed that the mean width of the distributions collected in this series of pulser tests was 1.83 channels for the anode and 2.42 channels for the cathode, and that these widths deviated very little from these mean values. At a nominal calibration of $10 \text{ eV } ch^{-1}$, these results indicate 18.3 eV of anode channel noise, and 24.2 eV of Cathode channel noise. Subtracting out the intrinsic preamplifier noise determined in Section IV.1 above, yields 16.9 eV of anode amplifier noise, and 19.7 eV of cathode amplifier noise. Both results are within the amplifier specifications. We have repeated the pulser tests with high voltage applied to the proportional chamber, but with live proportional chamber events gated off (except for the approximately 2% which pile up with the pulser in this test). In this test, we have observed 1.99 channels of rms anode noise, and 2.57 channels of rms cathode noise. This indicates that an additional 7.8 eV of rms anode noise, and 8.7 eV of rms cathode noise are acquired upon turning on the high voltage power supply. These noise sources all add to the total system noise in quadrature, and amount to less than one primary ion pair in the gas. The impact of these small sources of noise is therefore only to broaden the resolution of the system by two to three channels. At higher gas gains, two to three channels will equate to a smaller equivalent energy value for this noise.

The resulting histogram data from these pulser tests has been fit with a third order polynomial, in order to determine the intercept, or pedestal channel, for each of the anode and cathode channels. Figure IV.3 indicates that, without high voltage

Figure IV.3: Pulser Pedestal Fits

applied, the pedestal for the anode signal is at channel 299.6 and that for the cathode is at channel 299.1. The uncertainties used for the peak positions from the pulser tests were ± 0.1 channel for the anode, and ± 0.2 channel for the cathode results. On switching on the high voltage, an event rate in the proportional chamber of ~ 600 events per second resulted in a small reduction in these pedestals, due to the residual signal attributable to the slow drift of positive ions to the cathode, and the inability to return fully to the baseline. The resulting pedestal channels are 298.9 ± 0.2 for

the anode, and 298.6 ± 0.2 for the cathode, based upon pulser fits to which we have assigned ± 0.5 channel position accuracies. We have chosen to use pedestals of 299 channels ($\pm \frac{1}{2}$ channel) above the analog to digital converter origin for both the anode and cathode.

The slopes indicated by the fits show that the anode and cathode channel gains are equal, to within 0.3%, which is at the limit of our reading accuracy for the fine gain controls on the amplifiers.

The non-linearities in the anode and cathode analysers, as indicated by the third fitting parameters in Figures IV.3 a) and b), are smaller than 10^{-5} times the linear (slope) term, which is one full order of magnitude or more smaller than each of the preamplifier, amplifier, and analog to digital converter specifications quoted above. It is also significantly smaller than the maximum non-linearity specified for the PB4 pulser. With the high voltage turned on, and the pulser tests repeated, the best-fit non-linearity terms achieved in the fitting of the data were both -1×10^{-6} times the linear term, with fitting uncertainties which were both sufficiently large as to be consistent with a non-linearity of zero. We therefore take the electronic system non-linearity to be equal to $0 \pm 0.1\%$ according to the specifications listed above, and to be $0 \pm 0.005\%$ as observed, where the uncertainty is attributable to the specified non-linearity of the Berkeley Nucleonics PB4 pulser used for the tests.

IV.3.2 Ramp Pulser Tests, Fourier Analysis

Having observed the non-linear behaviour of the spectrum analyser system below data channel 300 in all analog to digital converter channels, we felt it necessary to study in detail the differential non-linearity of the system. To this end, we conducted a number of pulser tests in which the Berkeley Nucleonics PB4 pulser was triggered randomly while a triangular signal was repeatedly applied, from the Berkeley Nucleonics GL-3 ramp pulser, to the external voltage reference input of the PB4. In this way, a flat spectrum of pulser test pulses was generated, and the spectrum was collected by the spectrum analyser. This spectrum was then analysed for evidence of non-linear behaviour in two ways. The spectrum data was tested for its fit to a straight line, in order to look for individual channels which deviate in width from adjacent channels. As well, the resulting *square* spectrum was Fourier transformed into a channel frequency distribution, to look for anomalous cyclic features.

The ramp pulser was operated to generate a flat spectrum of about 500 million events, thereby achieving a standard deviation in each data channel of 0.18%, which is about half of what we expect at the peak of the 3H spectrum we seek to study. This spectrum was analysed for its fit to a straight line, as shown in Figure IV.4 a). We have restricted the range of the fit to at least 100 channels above the pedestal due to the apparent roll off in the spectrum as it approaches the pedestal. We attribute this roll off (primarily in the first 50 channels) to inefficiencies in the trigger circuit at low signal amplitudes. This shape therefore demonstrates our trigger circuit efficiency as a function of channel number. Above this first 100 channels, the χ^2 per degree of

Figure IV.4: Ramp Pulser Test Spectrum

freedom returned from the fit is a disappointing 1.29, indicating some non-statistical deviations from linearity. The slope of the line is small, at about 1.7×10^{-6} times the mean number of counts per channel. This is smaller than the specified integral non-linearity of the ramp pulser. The standard deviations from the fit parameters for each channel were gathered into Figure IV.4 b). We would expect these individual channel errors to exhibit normal statistical fluctuations, fitting well to a Gaussian distribution which has a mean value zero and standard deviation of one. It is clear from the fit that the distribution of the errors for the ramp pulser data has a mean consistent with zero, but has almost 12% extra width.

We have been led to notice small cyclic variations in channel heights in such ramp pulser distributions, and we have recognized these cyclic patterns to have periods which are consistent with a mixture of channel periods of 2^n channels, where n is an integer smaller than the number of bits in the analog to digital converter data word. We have hypothesized that the width of the analog to digital converter channels may be varying with channel position in a way which depends on the binary state of the 11 bit counter internal to the analog to digital converter. We speculate that as a significant number of binary bits simultaneously achieve a binary one state, and then in unison switch to a binary zero state, the fluctuations in current drawn by the components of the device, and the resulting voltage sags and surges on the analog to digital converter modules, cause small variations in the frequency of the clock circuit controlling the conversion. This may produce small variations in the analog to digital converter channel widths.

The Fourier transform of the data displayed in Figure IV.4 a) above is shown in Figure IV.5. On close inspection, deviations at particular channels, from the envelope of the channel frequency - intensity distribution can be seen near channel frequencies of 384, 512, 768, 1024, 1280, and 1536 channels in Figures IV.5 c), and d). Closer inspection of Figure IV.5 b) suggests a feature near a channel frequency of 256 channels as well. Figure IV.5 e) provides a closer view of the feature near the channel frequency of 768 channels. Table IV.1 demonstrates seven states of the 11 bit binary counter, and the adjacent channels in which these anomalies have been observed. Since the first four bits of the binary channel address word can assume sixteen different states, we might expect another nine anomalous features. However,

Figure IV.5: Fourier Transform of Ramp Pulser Test Spectrum

Table IV.1: Anomalous Fourier Transform Elements and ADC Counter States

Nominal Anomalous Feature Location	Initial Binary Counter Value	Next Binary Counter Value	Initial Counter Decimal Location	Anomalous Feature Channel Locations
256	0001111111	0010000000	255	256, 257
384	0010111111	0011000000	383	384, 385
512	0011111111	0100000000	511	512, 513
768	0101111111	0110000000	767	768, 769
1024	0111111111	1000000000	1023	1024, 1025
1280	1001111111	1010000000	1279	1280, 1281
1536	1011111111	1100000000	1535	1536, 1537

close inspection of the other potential feature locations does not reveal significant features beyond the limits of random channel height fluctuations. We have found that we can produce similar anomalies in the Fourier transforms of simulated ramp pulser spectra by introducing, along with random normal statistical fluctuations, very small changes in the spectrum amplitude according to some binary counting algorithm. These variations can be of the order of 0.1% to 0.01% of the underlying data, and produce features in the Fourier spectrum which are similar to those we find in Figure IV.5.

We have attempted to remove these features from the Fourier space spectrum of Figure IV.5 by arbitrarily reducing the excursions of the amplitudes at the anomalous

features observed in the seven pairs of the 2048 Fourier transform data elements which are listed in Table IV.1, so that the amplitudes in these channels lie at the mean between the levels of their neighbouring channels, and applying an inverse transform back into the event spectrum space. The results are shown in Figure IV.6 a). The

Figure IV.6: Fourier Filtered Ramp Pulser Test Spectrum

fit of the filtered spectrum to a straight line now achieves a χ^2 per degree of freedom which supports 99.7% confidence that the data is well described by the parameters reported, which incidentally are consistent with the parameters achieved in the fitting of the original data as shown in Figure IV.4, but with smaller uncertainties. Figure IV.6 b) now indicates that the remaining errors in the data, relative to the fit, are

much more consistent with a random normal distribution of mean zero and standard deviation one. The indicated χ^2 per degree of freedom is near 1.0, and the mean value achieved is consistent with 0.0. The width of the distribution is now 5% smaller than 1σ , and experience has shown us that distributions with widths of exactly 1σ , and a χ^2 which supports good confidence that the reconstituted ramp pulser spectrum is well described within random normal statistical fluctuations, are often easily achieved. In such cases, a less methodical approach to the cutting of the Fourier space data than what we have used here is sufficient to achieve an error distribution of the inverse transformed data which is less centrally peaked than that of Figure IV.6 b). We use this method therefore to demonstrate that cyclic differential non-linearities which are systematic and of the same order in magnitude as our statistical uncertainties have been detected through the Fourier transforms. We do not use this method to remove these systematic effects from our subsequent data since the editing of the Fourier space distribution, while methodical, is still arbitrary. Nonetheless we are now confident that greater statistical precision would not assist us at this point since we are now approaching the systematic limits of the apparatus. While the differential non-linearity of the analog to digital convertor is not specified, these cyclic anomalies of order 0.1% are near the specified integral non-linearity of 0.05% for the unit.

IV.3.3 Calibration Sources

Three particular calibration sources were prepared and employed in order to verify the energy calibration of the system, and to investigate the integral non-linearity of

the chamber itself, which remained untested by the ramp pulser tests.

^{55}Fe

^{55}Fe decays through electron capture to ^{55}Mn , with 90% of captures being from the K shell, and the balance from the L shell. The ^{55}Mn emits X-rays as the atomic structure adjusts to the decay. The ^{55}Mn K_{α_1} X-ray has an energy of 5898.75 eV [53]. The K_{α_2} X-ray has an energy of 5887.65 eV, at 0.505 the intensity of the K_{α_1} X-ray. The K_{β} X-ray, at 6490 eV, has an intensity of 0.203 that of the K_{α_1} X-ray. These three X-ray lines overlap each other, within the expected resolution of the detector, and so the ^{55}Fe source is often quoted as having an energy of 5965.74 eV, which is the weighted mean of the three X-ray energies. The X-rays emitted on electron capture from the L shell have an energy of 769 eV.

The proportional chamber assembly was fitted with an external ^{55}Fe source, deposited in a small well in a Teflon sleeve, which was fitted to a Teflon rod mounted along side the cathode assembly. A 4 mm diameter entrance aperture in the middle of the central cathode allowed the X-rays from the ^{55}Fe electron capture decays to enter the fiducial volume. The aperture was covered on the inside (fiducial volume side) by a 3 μm mylar foil, coated with 18 nm of aluminum, mounted in electrical contact with the cathode. This was done in order to ensure continuity of the field across the aperture, and to obstruct ions produced outside the fiducial volume from drifting inwards and contributing to events generated within the fiducial volume. The thickness of the cathode piece is 1.6 mm. The depth of the well containing the ^{55}Fe

source is 2 *mm* below the surface of the Teflon sleeve. The Teflon sleeve is held fixed to the Teflon rod, allowing it to be rotated, raised, and lowered into alignment with the entrance aperture by an externally driven worm gear at the end of the rod. The separation between the Teflon sleeve carrying the ^{55}Fe and the outer surface of the cathode is taken to be 3.5 ± 0.5 *mm*, so that the column depth of counting gas from the surface of the ^{55}Fe to the mylar barrier is 7.1 ± 0.5 *mm*. The mean free path of ~ 6 keV X-rays is 5.2 *mm* in 5 atmosphere *P10* gas. The source was measured to deliver a beam of X-rays of $\sim 900\text{s}^{-1}$ at the time of fabrication in January of 1992, and given its 2.685 year half life and the *P10* column depth, it can be predicted to deliver a rate of ~ 25 events per second into the fiducial volume at the time of this writing.

At this source strength, the 769 eV X-ray emitted on *L* electron capture will be very infrequent, and will in any event be difficult to observe due to the larger chamber backgrounds in this energy region.

^{37}Ar

^{37}Ar decays to ^{37}Cl through electron capture. Capture of an *L* electron occurs with less than $\frac{1}{10}^{th}$ the frequency of *K* electron capture, and electrons from higher shells at lesser probabilities by more than another order of magnitude. The resulting rearrangement of the atomic structure of the chlorine atom leads to the emission of a total of 2823 eV in X-ray energy for capture of a *K* electron. *L* electron capture will result in 270 eV in X-ray energy. ^{37}Ar has a half life of 35 days [53], making it a convenient

calibration source to introduce into our detector in trace amounts over periods of a few weeks, drawing upon a sample of a few millilitres of neutron activated ^{36}Ar which will remain usefully active for a few months.

Introducing this source in this way has the advantage that it is fully contained within the active region of the detector, so that we will collect a good peak for the total 2823 eV, which is the summed energy for the K and L X-rays emitted in the rearrangement. In the event that the K_{α} X-ray intercepts the chamber wall, the remaining L X-ray will contribute only about 200 eV into our detector, which is below our thresholds. In the less likely event that the L X-ray intercepts the walls, the surviving K_{β} X-ray will contribute to the low energy tail of the distribution, with the mean value of such events at 200 eV below the peak energy. Since the argon K electron binding energy is larger than that of the chlorine source, there will be no argon K X-ray escape to further complicate the calibration. We have prepared quartz ampoules of 5 atmosphere argon gas, frozen at liquid nitrogen temperatures, and then sealed at a volume of 5 millilitres. We have arranged to have the constituent ^{36}Ar neutron activated in the Atomic Energy of Canada Limited Chalk River nuclear reactor, and then transported to our experiment and introduced into our counting gas in trace amounts.

^{79}Kr

^{79}Kr decays to ^{79}Br through electron capture with 93% probability, or through β^+ decay with 7% probability [53]. K , L and M electron capture decays occur in ^{79}Kr .

The rate for K capture decays exceeds all other decay channels by reported factors of up to 10.8 [53]. The K electron capture decays result in a ^{79}Br daughter, which de-excites through the emission of a total of 13474 eV in X-ray energy. In the improbable event of L X-ray intersection with the chamber walls, the event will contribute only the K_{α} and M X-ray energies to the spectral line, which will result in a small asymmetry in the tails of the distribution, with the low energy tail containing slightly more events than the high energy tail. (This will be compounded by wall effects, to be discussed later.) In the event of K_{α_1} (K_{α_2}) X-ray escape, the L and M X-rays will deposit 1550 eV (1596 eV) in the chamber. As well, L electron capture leads to emission of 1782 eV in X-ray energy, so there will be a broad peak associated with both L electron capture and K_{α} escape. The β^+ decays are energetic, possessing ≤ 0.6 MeV of β particle kinetic energy. The underlying β spectrum will therefore be a relatively slowly changing function of energy in the region of the X-rays of interest to us.

^{79}Kr has a half life of only 35 hours [53], though it is still a convenient calibration source to introduce into our detector over periods of a few days, drawing upon a sample of a few millilitres of activated ^{78}Kr which will have a useful life of about a week. This source also has the advantage that it is fully contained within the active region of the detector, so that we will collect a good peak for the total 13474 eV K binding energy, complicated by the L X-ray interactions with the chamber walls, and by other wall effects. However, the short half life relative to the pressure settling time of the chamber makes precise resolution measurements difficult to achieve. We have also prepared quartz ampoules of 5 atmosphere krypton gas, frozen at liquid nitrogen temperatures, and sealed in a volume of 5 millilitres. We have similarly arranged to

have the constituent ^{78}Kr neutron activated in the Chalk River nuclear reactor, and then transported to our experiment and introduced into our counting gas.

IV.3.4 Calibration Tests

Gas gain is a strong inverse function of gas pressure, as is evident on inspection of Equation III.8. Standard temperature and pressure conditions are assumed in this work, both previously (Chapter III) and subsequently, unless otherwise indicated. Thus, the gas gain dependence is in fact an inverse dependence on the gas density, as indicated in Equation III.1. In our work, our proportional chamber is sealed for the durations of our experiments, thereby ensuring constant gas density, apart from considerations of the integrity of the pressure vessel, and considerations of slow uptake of the gas into the vessel materials.

Our apparatus is equipped with both temperature and pressure monitoring instruments, but these each afford only 1% precision. Our tests have shown that the changes in gas gain are nearly an order of magnitude greater than underlying pressure variations, at constant temperature. We have conducted very lengthy tests to simultaneously monitor the temperature and pressure of our apparatus, and found that while they vary by a few percent each, the values observed are consistent with constant density within the combined instrument errors for periods of weeks to months. The time required for collection of a meaningful spectrum is less than forty hours, and so we are confident that gas gain does not vary by more than 0.15%, due to pressure

vessel integrity, during an experiment.

We have observed large variations in gas gain using our ^{55}Fe source, during the first ~ 30 hours after filling the vessel with virgin gas. We attribute this to uptake of the gas into the vessel materials, since the change in gas gain in such cases is always an increase, which is consistent with a decrease in density. Deterioration of the gas due to oxygen infiltration would normally lead to a loss of gain. We have observed that the gas gain remains stable, within 1%, after the first ~ 30 hours and for the next 100+ hours, eventually commencing a slow decline attributable to deterioration of the gas. We therefore expect reliable and constant performance of the apparatus, and anticipate only a small ($\leq 1\%$) deterioration in resolution due to loss of gas density to the pressure vessel materials, over the duration of our experiments.

One difficulty we encounter as a result of the strong gas gain dependence on pressure, and the inadequacy of the temperature and pressure monitoring devices is that gas density is not precisely repeatable from one filling of the apparatus to another. This becomes problematic particularly when attempting to subtract background data collected in one experiment from the raw ^3H spectrum collected in a different experiment. This has necessitated that we employ our ^{55}Fe calibration source immediately prior to and immediately following each data collection run in order to make a determination of the actual gas gain and the stability of the gas gain during the experiment. We have employed our ^{55}Fe source since it can be introduced and removed at will, and without interfering with the vessel integrity. Thus determination of gas gain stability to better than 1% is possible with relatively little calibration data, but for the purposes of analysing the ^3H β spectrum, much higher

precision in the gain calibration is necessary. We anticipate fractional non-linearities below $10^{-4}ch^{-1}$ in the gain, and over the 1680 channels of data we will collect, the non-linearity will potentially contribute a significant error in the widths of the higher channels. Use of any calibration source simultaneously with the experiment was not considered due to the difficulty in trying to separate the calibration data from the β spectrum. We are forced to use a gas gain dependent factor as a free parameter in the fitting of our 3H data, and verify the consistency of this parameter with our pre-spectrum and post-spectrum gas gain measurements as a test of the quality of the fit of the data.

We therefore will employ our activated source calibration tests in order to determine the nonlinearity in the total gain of the apparatus, and the dependence of the resolution on energy, but not as a fixed gas gain calibration test for the apparatus. The actual gas gain determinations made prior to and after data collection will eliminate any concern regarding gas temperature, pressure, or density problems.

^{55}Fe Calibration Spectra

The ^{55}Fe source is a weak source, delivering ~ 25 X-rays per second into the fiducial volume at five atmospheres pressure. There is a background rate, comprising incident cosmic rays which interact within the detector, as well as natural and internal background sources. These combined backgrounds are comparable in rate to that of the ^{55}Fe source. The energy deposited by natural background sources and cosmic rays demonstrates a flat distribution as a function of energy above the region of the 3H

spectrum, and we expect a similar distribution within this region. On analysis, the internal background spectrum appears to demonstrate a $\frac{1}{E}$ bremsstrahlung photon distribution. The most probable source of this distribution would seem to be tritium contamination which has penetrated below the surface of the proportional chamber cathode pieces. We will deal with the proportional chamber backgrounds in Section V.1 below, but for the purposes of analysing the ^{55}Fe source spectrum, we will be aware of these underlying background distributions.

Because the ^{55}Mn X-rays have an energy greater than the binding energy of the argon K shell, it is therefore possible for some of the energy deposited by the ^{55}Mn X-rays to escape the fiducial volume as argon K X-rays escape the chamber. This will result in an escape peak at an energy equal to the ^{55}Mn K X-ray energies (of which there are three) less the argon K_{α} X-ray energies (of which there are two). The argon K_{β} X-ray is less probable, at about $\frac{1}{10}^{th}$ the intensity of the K_{α} X-rays, and so we will ignore argon K_{β} escape in our fitting. For our calibration tests, the nominal energy calibration is $\sim 10 \text{ eV } ch^{-1}$. Since the energy difference between the two argon K X-rays is only 2.07 eV , we will treat the two argon K X-rays as a single X-ray in our fitting of the data, adding less than 0.1% to the width of the peak through this approximation. As well, the two ^{55}Mn K_{α} X-rays are separated by only 11.1 eV , or about one analog to digital converter channel, which will make it difficult to separate these two X-rays in the fitting. While we may attempt to fit the large peak deposited by the three overlapping ^{55}Mn K X-rays (where there are good statistics) using a triple Gaussian function, we will treat both of the pair of escaping argon K_{α} X-rays and the pair of ^{55}Mn K_{α} incident X-rays as single peaks each when analysing

the escape peak distribution. This will effectively reduce the escape peak to a double Gaussian, resulting from argon K_{α} escape subsequent to the deposition of a $^{55}\text{Mn } K_{\alpha}$ or K_{β} X-ray. Ignoring the 11.1 eV energy difference between the $^{55}\text{Mn } K_{\alpha}$ X-rays will result in the broadening of the escape peak width by an additional 0.4%, for a total additional width due to our approximations of only slightly more than 0.4%. In Figure IV.7 a), we show a typical ^{55}Fe source spectrum. We have monitored an increase in the rate of activity in the chamber while collecting such spectra, and we attribute this increase to outgassing of ^3H gas from the chamber construction materials during the data collection, due to absorption of the gas during previous experimental activity. We have subtracted an appropriately scaled experimental ^3H spectrum from that shown in Figure IV.7 a), and removed the data channels below the pedestal, in producing a spectrum suitable for fitting to the theory. We have tested the fit of the resulting spectrum to a combination of distributions, comprising a triple Gaussian at the location of the $^{55}\text{Mn } K$ X-ray peak, a double Gaussian at the location of the argon escape peak, a gently sloping underlying distribution attributable to the cosmic ray and natural background activity, and a $\frac{1}{E}$ distribution characteristic of bremsstrahlung spectra.

The reduced spectrum shown in Figure IV.7 b) corresponds well to the expectations discussed above, achieving a χ^2 per degree of freedom of 0.900. This low value of χ^2 supports 96.6.% confidence that the data corresponds to the combination of distributions discussed above and described by the parameters quoted on the figure. These fit parameters indicate that the best fit is achieved with the $^{55}\text{Mn } K_{\alpha 1}$ X-ray line located at channel 513.6 ± 0.1 of the analog to digital converter, and with the

Figure IV.7: ^{55}Fe Calibration Spectrum

escape peak associated with $^{55}\text{Mn } K_{\alpha}$ X-ray deposition followed by argon K_{α} escape located at channel 263.0 ± 0.3 . This escape peak is 7.2 channels (2.7%) above the location predicted on the basis of the $^{55}\text{Mn } K_{\alpha_1}$ X-ray, while the fitting uncertainty will only explain 0.4 channels (0.1%) of this error. We note that the fitting process has indicated a $^{55}\text{Mn } K_{\beta}$ intensity of 0.361 times that of the K_{α_1} , which is significantly greater than the accepted value of 0.203 [53]. This is not unreasonable considering the relatively long path length from the ^{55}Fe source to the entrance aperture of the fiducial volume, and the resulting higher mortality rate for the lower energy K_{α} X-rays than the more energetic K_{β} X-rays prior to entering the fiducial volume. This same reasoning leads to a conclusion that the $^{55}\text{Mn } K_{\beta}$ X-rays will penetrate further into the fiducial volume and therefore will contribute a smaller intensity to the escape peak than 0.361. In fact, this hypothesized smaller contribution of the K_{β} X-ray to the escape peak further strengthens our expectations for an excess of that component of the $^{55}\text{Mn } K$ peak. As well, the lower intensity of the escape peak makes estimates of the relative strengths of different components of the escape peak much less certain. We therefore use the estimated 0.361 intensity as an upper limit in fitting the components of the escape peak, but otherwise leave the K_{β} fraction as a free parameter. The actual best fit value for the fractional intensity of the $^{55}\text{Mn } K_{\beta}$ X-ray contributed to the $\text{Ar } K$ -escape peak intensity was 0.108. The Full Width at Half Maximum resolution attributed to the $^{55}\text{Mn } K_{\alpha_1}$ X-ray peak was 14.5%. This may be compared to our theoretical best case prediction of 12.8%, revised to 13.2% in consideration of the observed electronic system noise, and further worsened by an additional $\sim 1\%$ in consideration of gain instability due to uptake of the counting gas in the proportional chamber materials subsequent to the initial stabilization period. The Full Width at

Half Maximum resolution attributed to the dominant component of the escape peak was 21.7%. After allowing for the 0.4% broadening due to our simplifications above, and considering the theoretical $\frac{1}{\sqrt{E}}$ variation of resolution across the spectrum, this is equivalent to $15.0 \pm 0.2\%$ at the $^{55}\text{Mn } K_{\alpha_1}$ peak, in reasonably close agreement with the resolution observed at that X-ray. The anticipated linear (with channel number) contribution to the spectrum from natural background and cosmic activity was fit to an intercept of 916 events per channel, declining by 0.62 events per channel for each additional channel across the spectral space.

Figures IV.7 c) and d) demonstrate the quality of this fit to the argon escape peak and the $^{55}\text{Mn } K_{\alpha_1}$ peak respectively, within $\pm 2\sigma$ of the centroids of the peaks. Both of these peaks have been fit with good confidence, with each fit achieving a χ^2 per degree of freedom very near 1.0. The argon escape peak contributes a χ^2 per degree of freedom of 0.975, supporting 56.9% confidence that the fit parameters describe the observations with only random normal statistical fluctuations. The $^{55}\text{Mn } K_{\alpha_1}$ peak contributes 0.902 per degree of freedom to the overall χ^2 result, achieving 81.6% confidence in the fit parameters. Where the deviations between the fit and the data are attributable only to random normal statistical fluctuations, and not to systematic difficulties, we expect the mean value of the χ^2 statistic for numerous similar tests to approach 1 per degree of freedom, thereby yielding near 50% average confidence per test that the theory does describe the observations.

The energy axis calibration achieved for this spectrum is $11.485 \pm 0.024 \text{ eV ch}^{-1}$ according to the $^{55}\text{Mn } K_{\alpha_1}$ X-ray line, and $11.168 \pm 0.057 \text{ eV ch}^{-1}$ according to the argon escape peak location. In this, we have used the peak location uncertainties from

the fitting, and a $\frac{1}{2}$ channel uncertainty in the pedestal location. The possible error introduced into the escape peak location on taking the mean value of the two argon K_{α} X-rays is only 0.004 eV ch^{-1} . If we take the difference in these two calibration values as indicative of a non-linearity in the energy axis calibration, and if we attribute this non-linearity to a second order term in channel number (space charge effects), we calculate a non-linearity of $1.2 \times 10^{-4} \text{ ch}^{-1} (\pm 26.8\%)$ times the linear term, which is $10.844 \text{ eV ch}^{-1}$. This suspected non-linearity is larger than that predicted above in consideration of the electronic equipment specifications and the pulser tests. We are nonetheless reluctant to use this measurement of the analyser non-linearity due to the suspiciously high confidence we have placed in these fits, the limited size of this sample, and other available information to be presented subsequently. We will achieve a more certain estimate of the possible non-linearity in the energy axis of our analyser with spectra obtained by combining our activated krypton and argon sources with our ^{55}Fe source.

Two additional low intensity features are apparent in the spectrum shown in Figure IV.7 a), near channels ~ 1100 and ~ 1280 . Based on the crude energy calibration achieved above, these features could be attributed to a very low intensity background source emitting $\sim 9500 \text{ eV}$ and $\sim 11800 \text{ eV}$ photons. We hypothesize that these two peaks result from X-rays emitted from the gold surfaces of the cathode pieces (9712 eV and 11442 eV) subsequent to excitation upon intercepting ionizing cosmic rays or other energetic backgrounds. We will explore the potential for using these features as a consistency check below.

Combined ^{37}Ar , ^{55}Fe , and ^{79}Kr Sources Calibration Spectra

We have mixed our activated krypton and argon sources with $P10$ gas, and filled our chamber to pressures of one, three, and five atmospheres. We have simultaneously illuminated the entrance aperture with X-rays from the ^{55}Fe source. Typical spectra under such conditions are shown in Figures IV.8 a), b), and c) respectively. We have gathered spectra from these mixed sources under various high voltage conditions at each of the three pressures, and we have repeated our spectra with two distinct sets of activated krypton and argon sources.

The features most obvious in the figures are the prominent Br K peaks, due to release of the total binding energy of electrons capturing into the bromine K shell, and the pairs of peaks indicated as the Br L peaks and the Cl K peaks, attributable to electrons capturing into the bromine L shell (left-most), and the chlorine K shell. Less obvious due to their lower intensities are the ^{55}Fe peaks, between the chlorine and bromine K peaks. As we have previously stated, we expect peaks in energy to correspond to the total binding energies for the captured electrons for both the bromine and chlorine daughter atoms, since these sources are contained within the fiducial volume of our detector. In the case of the ^{55}Fe source, we expect the combination of transition X-ray lines which we have discussed in the previous section. We will now proceed to consider potential contaminations which may overlap these peaks, and to compare our observations with these expectations for each feature.

The binding energy of the bromine K shell is 13474 eV. In descending to this

Figure IV.8: ^{37}Ar , ^{55}Fe , and ^{79}Kr Calibration Spectra

shell, electrons may cascade through the L and M shells, giving up X-ray photons and Auger electrons at each stage. In the event that the decaying atom is located near the walls of our chamber, it is possible for one or more of these X-rays to intercept the walls, resulting in events of lower apparent energy than the K shell binding energy. It is more probable for the L X-ray to intercept the walls than the M X-ray, due to its higher energy and consequently its longer mean free path. Loss of both the L and M X-rays is highly improbable, relative to loss of the L X-ray alone. In the event that the M X-ray intercepts the wall, approximately 200 eV will be lost from the decay energy (less than $\frac{1}{2}\sigma$ at the expected resolution for this detector), and the event will contribute a small amount of skewing of the peak to the lower energy side. In the event of bromine L X-ray escape, the energy lost will be over 1.5 keV (about 3σ), and such events will contribute an excess of counts in the low energy tail of the distribution. We expect that the rate of contamination of the distribution due to M X-ray escape will be insignificant relative to that for L X-ray escape, and that these rates will depend inversely on chamber gas pressure.

We note also in the spectra shown in Figure IV.8 that there is a prominent plateau below the low energy tail of the K energy distribution (and below the $Br L$ escape energy) at low gas pressure, and that this plateau diminishes with increasing gas pressure. We will attempt to understand this feature also as evidence of a wall effect.

We begin our comparison of the observations with the expectations by simply fitting the distribution to a Gaussian shape over $\pm 2\sigma$ about the centroid (see Figure IV.9 for a typical example), to test for the presence of any significant contamination.

Figure IV.9: ^{79}Br K Binding Energy Peak

skewing the shape of the distribution away from the normal form. For the calibration runs analysed, this fitting alone yielded good results, with reasonable confidence of the correspondance between the observations and the expectations. For twelve data sets (four at each pressure), all having gas gains near that which we will employ for our tritium work, the mean confidence level achieved was 38%. This is somewhat below the anticipated $\sim 50\%$, but not by a significant amount. We will use the peak channel numbers and uncertainties achieved in these fits (see Table IV.2 below) in our determination of the non-linearity and resolution performance of the analyser.

Figure IV.10: ^{79}Br Low Energy Tail And Plateau.

We next attempt to extract some information about the escape of L and M X-rays, and about the lower energy plateau, by fitting the low energy tail of the distribution to a more detailed theory. Figure IV.10 is a typical example of our fitting of the lower energy tail of the peak, and the plateau. In this fit, we use a function which includes the Gaussian shape and the centroid of the main peak from the prior fit at the K shell binding energy, and a plateau having a Gaussian low energy roll off centered near 11400 eV. This 11400 eV feature may be attributable to L X-rays given off by the gold coating on the cathode walls (11442 eV) upon excitation by the more energetic bromine K_α X-rays which encounter the walls. Fitting this feature

to a plateau, rather than a Gaussian distribution, is an arbitrary choice, insofar as it generally produces a fit of slightly improved quality over the simple peak. This may be attributable simply to the large overlap of the higher energy tail of this gold line with lower energy tails of the much more dominant bromine K peak and the bromine L escape peak, along with a small contribution from M escape events. There may also be some residual ionization entering the chamber from the cathode wall due to the energy difference between the bromine K_{α} X-ray ($\sim 11908\text{eV}$) and the gold fluorescence X-ray it produces. The quality of these fits for the twelve relevant data sets is indicated by the 35% mean confidence level achieved, though this is attributable mainly to the good fits achieved at higher pressures, as the plateau and the L escape peaks decline and the distribution becomes less contaminated. Nonetheless, the results indicate that the plateau achieves an amplitude in the distribution of about $7.3 \pm 0.5\%$ of the main peak amplitude at one atmosphere pressure, dropping to about $3.1 \pm 0.7\%$ at three atmospheres, and to $1.8 \pm 0.2\%$ at five atmospheres. This trend supports the hypothesis that this plateau is evidence of a wall effect, declining with increasing pressure.

The regions fit in these two exercises ($\pm 2\sigma$ at the K peak centroid, and the plateau region) are not overlapping, being separated by over 1 keV. In a final attempt to analyse the bromine K peak, a fit function was created to include both of these features plus a small underlying peak corresponding to bromine L X-ray escape near 11900 eV, and this was applied over the combined prior fit ranges, including the intervening channels. These fits achieve similar results for the amplitude of the plateau due to gold L X-rays as in the prior fits. Best fit results are achieved when

also allowing for an L escape peak amplitude of $1.3 \pm 0.1\%$, $0.5 \pm 0.1\%$, and $0.1 \pm 0.1\%$ of the main peak amplitude for one, three, and five atmosphere pressures respectively. However, confidence levels in these fits are disappointing, averaging only 2.4% over the twelve data sets, indicating that there is still some misunderstood underlying structure near the bromine K peak. This is probably attributable to some of the negative ions, liberated by the electron capture and cascade photons, ranging out in the cathode walls (wall effects).

Despite the disappointing confidence levels achieved in this last analysis, we have achieved good fits to locate the precise centroids due to the bromine K binding energy, and we have achieved relative measures of two of the observed wall effects (bromine L X-ray escape, and wall gold L X-ray fluorescence on bromine K_α excitation) at the three pressures employed.

Turning our attention next to the pair of peaks in the lower energy end of the spectrum, we attribute these peaks nominally to the 12% krypton L capture probability which will contribute a peak at the bromine L shell binding energy of 1782 eV, and to the full chlorine K binding energy of 2823 eV upon argon K electron capture (see Figure IV.11). We may anticipate little contamination to the chlorine peak. Chlorine L X-ray escape would contribute some skewing of the peak due to the roughly 200 eV loss in such events, but this is less probable than was bromine L escape due simply to the lower energy of the chlorine L X-rays relative to those of bromine. In fact, chlorine L X-ray energies are comparable to that of bromine M X-rays, which we have ignored as trivial in our analysis above. Since our fits were not particularly certain in finding even L escape contamination of the bromine K peak,

and since we have larger contamination effects to explore with the bromine L peak. we will ignore chlorine L escape here. Beyond this, and some ranging out of electrons in the walls, there is little else capable of corrupting the chlorine K binding energy peak.

Figure IV.11: ^{79}Br L X-ray and ^{37}Cl K X-ray Peaks.

Having observed measureable levels of bromine L escape, and of bromine K_{α} excitation of the cathode walls, we must consider the probability of significant contamination of the krypton L capture (bromine L_1 X-ray) peak due to bromine $K_{\alpha 1}$ (11924 eV) or $K_{\alpha 2}$ (11878 eV) escape. We expect the degree of bromine K_{α} escape to be large compared to the apparent bromine L escape probability, since these K_{α}

X-rays will exhibit a substantially longer mean free path due to the much greater energy of the K X-rays. The ratio of $K_{\alpha 2}$ to $K_{\alpha 1}$ X-rays is fixed (see [53], appendix 11), fixing the ratio of L_2 and L_3 X-rays, but the ratio of these X-rays to the L_1 X-ray produced on krypton L electron capture is determined by the K_{α} escape probability. For purposes of our fitting, we have fixed the relative strengths of the L_2 and L_3 components to each other, and their energies relative to that of the L_1 X-ray, but we have allowed the fitting routine to optimize the intensity of the L_1 component to that of the bromine K_{α} escape events (L_2 and L_3 X-rays) in order to determine the best case fit parameters. We have also allowed the fitting routines to optimize the relative strengths of the bromine L peak and the chlorine K peak, and the individual centroids of these two peaks, while fitting from -2σ below the bromine L peak centroid up to $+2\sigma$ above the chlorine K peak. The mean confidence level achieved in these fits was a disappointing 4%, clearly indicating that the systematics of the distributions are not fully understood. Nonetheless, these are best fit results, and the average χ^2 of 1.5 per degree of freedom achieved in these fits implies that the systematic effects considered here account for all but about $\frac{1}{2}\sigma$ of error within the observations. Analysis of the deviations of the observations from the fit function demonstrate that the deviations are randomly distributed, without apparent underlying systematic causes, and with an essentially Gaussian (though somewhat narrow) distribution.

For the ^{55}Fe source, we expect a peak which includes events which result from the Mn $K_{\alpha 1}$, $K_{\alpha 2}$, and K_{β} X-rays, as demonstrated in the discussion of the ^{55}Fe fitting above. As this is a relatively weak source, we may ignore the possible argon escape peak, which may slightly contaminate the region just above the chlorine K peak. The

Figure IV.12: ^{55}Mn K X-ray Peak.

limited statistical weight of the ^{55}Mn sample within the calibration data, and thus the large statistical uncertainty in the data, allows the fitting process to easily achieve good agreement between the observed peak and the fit function (Figure IV.12), which we take to be a simple Gaussian overlying a gently sloping distribution due to the background activity. For the twelve spectra in this analysis, the average confidence level achieved in the fitting of the data to such a function was 41%, implying that the centroids of the peaks are confidently located, though with larger errors than in the cases of the other more intense sources. At the K X-ray weightings determined above, the energy of this peak will be 6010 eV.

IV.3.5 Non-linearity Due To Space Charge Effects

The statistically limited contribution of the ^{55}Mn peak, and the large contamination of the ^{79}Br L peak, make them less ideal for use in measuring the potential non-linearity of the apparatus than do the Cl and Br K peaks. Based on the known energies of these peaks, the pedestal determinations above, and the locations and uncertainties of these two peaks as achieved in fitting the twelve spectra under analysis here, the non-linearities in the energy calibration were determined, and are shown in Table IV.2. The non-linearity expressed here is of the form:

$$E(x) = ax(1 + bx)$$

where $E(x)$ is the energy of events at analyser channel number x (after subtracting channels below the pedestal). a is the linear calibration term (eV/Ch) sometimes referred to as the energy dispersion, and b is the non-linearity expressed as a fraction per channel of the linear term (ch^{-1}). We will therefore expect a non-linearity in our energy calibration (at 5 Atm.) of 2.2×10^{-5} per channel in our subsequent analyses, and we will assign an uncertainty to this non-linearity of $\pm 5\%$ based on the deviations in the values shown in the table.

This result is consistent with the pulser test results, and the ^{55}Fe calibration data above. It supports the conclusion that space charge effects influence the analyser gain only minimally at these pressures and voltages. Nonetheless, the non-linearity demonstrated in the five atmosphere data above will be sufficient to cause a 2% change in channel width over 1000 channels of the spectrum, which may mask or distort our results. It is therefore necessary that we consider this non-linearity in our analysis of

Table IV.2: Multi-Channel Spectrum Analyser Energy Non-Linearity

Run No.	Pressure (Atm.)	<i>Cl K</i> Peak Channel Number (2823 eV)	<i>Br K</i> Peak Channel Number (13474 eV)	a (eV/Ch)	b (Ch^{-1})
323	1	228.03 ± 0.08	1081.39 ± 0.06	12.359 ± 0.006	7.6×10^{-6}
324	1	292.06 ± 0.05	1381.54 ± 0.04	9.642 ± 0.002	8.3×10^{-6}
339	1	241.47 ± 0.03	1145.20 ± 0.03	11.670 ± 0.002	7.1×10^{-6}
340	1	283.49 ± 0.03	1340.76 ± 0.04	9.933 ± 0.001	8.8×10^{-6}
Mean					$8.0 \times 10^{-6} \text{ }^{+10}_{-11}\%$
327	3	175.16 ± 0.03	826.09 ± 0.02	16.065 ± 0.004	1.9×10^{-5}
329	3	321.85 ± 0.05	1510.18 ± 0.04	8.730 ± 0.002	1.5×10^{-5}
345	3	262.32 ± 0.32	1233.33 ± 0.09	10.718 ± 0.017	1.6×10^{-5}
346	3	316.91 ± 0.03	1487.09 ± 0.05	8.867 ± 0.001	1.5×10^{-5}
Mean					$1.6 \times 10^{-5} \text{ }^{+19}_{-6}\%$
332	5	244.97 ± 0.04	1146.47 ± 0.04	11.462 ± 0.003	2.2×10^{-5}
333	5	302.54 ± 0.04	1412.00 ± 0.04	9.273 ± 0.002	2.1×10^{-5}
350	5	245.79 ± 0.03	1149.37 ± 0.04	11.420 ± 0.002	2.3×10^{-5}
351	5	303.65 ± 0.04	1414.59 ± 0.05	9.235 ± 0.002	2.2×10^{-5}
Mean					$2.2 \times 10^{-5} \pm 5\%$

the tritium spectra.

Resolution

One objective of the calibration data was to observe the energy dependence of the analyser energy resolution. The prediction from Section III.4 above is that the resolution will change by a relative factor of $\frac{1}{\sqrt{E}}$ across the spectrum, achieving near 14.5% FWHM at the $^{55}\text{Mn } K$ peak energy. Due to the short half life of our activated krypton source, ($\sim 35\text{hrs}$), we have been required to acquire our calibration data rather quickly, thereby incurring the difficulty that the chamber gas pressure, and therefore gas gain, will not have been stabilized for most of our data taking. The effect will be least noticeable, however, at one atmosphere pressure, and so we will be most confident of the one atmosphere data. The Full Width at Half Maximum resolutions achieved for the calibration sources discussed above are therefore shown in Table IV.3. The table displays the Full Width at Half Maximum resolution observed for each of the X-ray lines, for the one atmosphere data sets, and the equivalent Full Width at Half Maximum resolution for each line, scaled according to the expected $\frac{1}{\sqrt{E}}$ energy dependence and thus translated to the location of the $^{55}\text{Mn } K_{\alpha 1}$ peak, after allowing for the small constant electronic noise contribution. The resolutions determined for the ^{55}Mn and $^{79}\text{Br } K$ peaks overlap each other, and are consistent with those of the ^{55}Fe analysis above, while the results for the $^{37}\text{Cl } K$ peak are 0.985 times the common value of the other two sources. The uncertainty in the Cl peak resolution explains only about half of this discrepancy, but this small inconsistency will have only a trivial impact on our analysis. These results therefore confirm our

expectations, indicating that the analyser resolution is as observed with the ^{55}Fe five atmosphere calibration discussed above, and consistent with the expectations developed in our above considerations of the sources of electronic noise. As well, the $\frac{1}{\sqrt{E}}$ dependence of the detector resolution is demonstrated, and provides no significant indication of asymmetry to either the lower or higher energy side of the peaks.

Table IV.3: Multi-Channel Spectrum Analyser Energy Resolution

	Run No.	Pressure (Atm.)	<i>Cl K</i> FWHM (%)	<i>Mn K</i> FWHM (%)	<i>Br K</i> FWHM (%)
Observed	323	1	20.32 ± 0.09	14.46 ± 0.34	9.78 ± 0.03
	324	1	20.66 ± 0.07	15.26 ± 0.36	9.84 ± 0.03
	339	1	20.72 ± 0.06	14.47 ± 0.31	9.74 ± 0.03
	340	1	20.80 ± 0.06	15.05 ± 0.37	9.87 ± 0.03
	Mean		20.63 ± 0.07	14.81 ± 0.35	9.81 ± 0.03
Scaled and Translated	323	1	14.18 ± 0.13	14.59 ± 0.35	14.58 ± 0.04
	324	1	14.42 ± 0.10	15.40 ± 0.37	14.67 ± 0.03
	339	1	14.46 ± 0.09	14.60 ± 0.31	14.52 ± 0.04
	340	1	14.51 ± 0.08	15.19 ± 0.37	14.71 ± 0.04
	Mean		14.39 ± 0.10	14.95 ± 0.35	14.62 ± 0.04

Gold L X-ray Calibration

We have seen evidence of two possible L X-rays, perhaps attributable to fluorescence from the gold coating on the cathode, in the section on ^{55}Fe Calibration Spectra above. We have also observed a feature near our ^{79}Kr K calibration peak in our discussion above regarding the combined activated calibration sources. In our studies of background spectra, we have determined that the rate of accumulation of such events at the maximum of the higher energy peak is $(2.25 \pm 0.01) \times 10^{-3} \text{ s}^{-1}$, for each live second of detector operation. The lower energy peak exhibits greater variability in magnitude, as well as greater widths and some variation in its relative position. If we presume here some excitation of the chamber walls by cosmic and other natural background radiation, and thus attribute the higher energy peak to the gold $3d_{\frac{5}{2}}$ to $2p_{\frac{1}{2}}$ transition at 11442 eV as we did in fitting the plateau below our ^{79}Br K binding energy peak, we may also expect some $3d_{\frac{5}{2}}$ to $2p_{\frac{3}{2}}$ transitions at 9712 eV. However, this latter peak may be contaminated with some 8 keV to 9 keV events (FWHM at 9712 eV $\approx$ 1100 eV) originating in the copper beneath the gold, and so we may expect greater uncertainties to arise from analysis of the lower energy peak.

At the rate observed in the background data, we may predict the contamination arising from this source, in a 100 million event tritium spectrum, to peak at about 150 events per channel. Underlying this, the tritium spectrum will contain in excess of 40,000 counts per channel, with random normal statistical fluctuations exhibiting a standard deviation of in excess of 200 counts. Increasing the number of events to one billion will reduce the fractional statistical fluctuations only to about one half of

the predicted gold contamination. It will require about three billion events in order to raise the gold X-rays to in excess of 3σ significance above the tritium data. We are aware of two difficulties which will be encountered in such a test. The first is that this will reduce the statistical uncertainty to well below the systematic limits of the analog to digital converter, revealing the channel width variations attributed previously to the binary counters of the adc. The second is that our chamber's gas gain will not be reliable over the ~ 60 days necessary to collect so many events.

We have conducted such a test in order to check our chamber's consistency with prior observations, when operated under such extreme conditions. We have collected 16 contiguous spectra, each of 200 million events, with the chamber filled with tritium, and also with our ^{55}Fe source simultaneously illuminating the chamber. We have not attempted to collect background spectra for an equivalent total live time, but instead have opted to use an analytical description of the other backgrounds for the purpose of this test. We fit the ^{55}Mn peak in each of the 16 spectra, in order to monitor the change in gas gain over the period. We then rebin all spectra to produce coincident ^{55}Mn peaks, sum all rebinned spectra together, search for the gold peaks, and evaluate their location relative to the ^{55}Mn peaks.

In Figure IV.13 a), we have summed the rebinned spectra to produce a 3.2 billion event spectrum. The individual 200 million event spectra were fit, typically over 190 to 200 channels near the ^{55}Mn peak, with a second order polynomial plus a triple Gaussian reflecting the previously found ^{55}Mn $K_{\alpha 1}$, $K_{\alpha 2}$, and K_{β} fractions, with an average χ^2 of 1.3 per degree of freedom. In the rebinning, the peak channel of the ^{55}Mn distribution was observed to have moved down by 3.9% towards the

Figure IV.13: Rebinned and Summed Tritium Data

pedestal, implying 0.12% reduction in gain per 100 million events collected. This decrease in gas gain could not have resulted from a loss in vessel pressure, since such a loss would lead to an increase in gain. We conclude that our vessel is well sealed and stabilized, and that there will be no noticeable change in gas gain during our 100 million event experiment. The rebinned and summed spectrum was fit with a similar function, exhibiting a vastly improved χ^2 of 0.82 per degree of freedom. (A similar attempt to fit the non-rebinned summed spectra resulted in a χ^2 per degree

of freedom exceeding 5.7.) We attribute the exceedingly good fit, which supports about 97% confidence that another similar test would differ from the polynomial and Gaussian functions used in the fit function, to the smoothing of data which occurred in the rebinning. Figure IV.13 b) demonstrates the difference between the rebinned and summed observations, versus a simple sum of the 16 spectra. The systematic periodicity anticipated due to the adc counters is evident in the 16 channel period of the dominant fluctuations in the difference data. The amplitude of these fluctuations is of order 0.14% of the underlying tritium data, near the level we would predict from our prior Fourier analysis of the ramp pulser spectra. The FWHM resolution achieved in the rebinned and summed data was $\sim 16\%$ at the ^{55}Mn peak.

We have also fit the rebinned and summed spectra to a function which describes the β decay spectrum of tritium, particularly in the region between about 7keV and 14keV , and then studied the difference between the fit and the spectrum. Figure IV.14 shows the shape of the deviations of the observations from the fit function in this region. (The region of the spectrum shown in Figure IV.14 begins 613 data channels above the pedestal channel, that is near 7keV β particle energy.) The error bars shown demonstrate the normal deviations due to the underlying tritium events counted, while the peaks appear to be the same two features observed in the background data discussed previously, and appear to comprise approximately 3σ excursions of the underlying data, or about 5000 events at each peak, as we had expected. The two features we are interested in occur at adc channels $818.2 \pm 0.2\%$ and $1005.1 \pm 0.2\%$. The previously fit ^{55}Mn peak was determined to have its $K_{\alpha 1}$ X-ray (5899eV) at channel 515.1 ± 0.1 . These observations are consistent with a conclusion that these two peaks

Figure IV.14: Residual Gold X-rays in Tritium Data

are attributable to the 9712eV and 11442eV gold L X-rays, allowing for a fractional non-linearity of $2.2 \times 10^{-5} \text{ch}^{-1}$ of the linear energy calibration term, if we accept an additional $+2.7\%$ correction in locating the 9712eV peak, and an additional -1.4% correction in locating the 11442eV peak. Note that the lower energy peak is also a substantially wider peak, where theory would lead us to expect a slightly narrower peak. This is therefore consistent with the hypothesis that this lower energy peak is contaminated with X-rays from the copper which underlies the gold. Considering the difficulties encountered in collecting and summing so many events, the simplified fit function employed to achieve this result, the potential contamination of the lower

energy peak. and the apparent deterioration of the counting gas during this test. we are satisfied that the results are consistent with expectations. While these rather large apparent errors in the locations of the gold L X-ray peaks may each imply a potential difficulty in the value of the non-linear term used. they suggest corrections to this term which deviate in opposite ways from the value used in the calculation. As well. any variation in the non-linear term within the range in evidence here would also be within the range expected to result from the specifications for the components of our apparatus. as discussed in Section IV.3.1 above. The principle conclusions available from this analysis are that:

- ◊ the features observed in the background data and in the ^{79}Br calibration data are observed in the tritium spectra. but at levels which are insignificant to our remaining work.
- ◊ the non-linearity observed for these features is very nearly the same as that observed in our calibration work.
- ◊ the locations of these features are sufficiently uncertain that they do not afford a good absolute calibration.
- ◊ the systematic limits of the analog to digital converter are evident in our high statistics data. at the expected level. and
- ◊ the gas gain changes slowly. due apparently to degradation of the gas rather than pressure instability. at a rate which will be trivial for our remaining work.

Similar tests accumulating 40 million events from ^{55}Fe source. along with cham-

ber backgrounds, have shown fractional non-linearities to be at or below $5 \times 10^{-5} ch^{-1}$ of the linear term, when requiring that the peaks observed be treated as the two gold L X-rays hypothesized. In such tests, the position and width of the lower energy peak, aided by higher statistics, have been more consistent with predicted values than for the case with the large underlying tritium spectrum.

IV.4 Fiducial Volume Boundary Effects

We define the boundaries of the fiducial volume of our chamber in two ways. The radial boundary is the inner surface, or wall, of the cylindrical cathode. Below, we give consideration to any potential distortion of our spectrum which may result from decay events which occur at or near this boundary. These we term wall effects.

The longitudinal boundary of our fiducial volume is established through the use of three cathode sections. These are aligned along a common axis at the anode wire. The two outside sections (guard rings), located at the ends of the central cathode piece, serve to maintain a continuous electric field across the $5mm$ gaps separating them from the central cathode. In this way, electrons drifting towards the anode wire will experience the same electric field (as a function of distance from the anode) regardless of where along the length of the chamber the events originate. Then on arriving at the anode wire, the amplified negative charge will collect on the anode and be swept away, leaving the slower moving positive ions to drift towards the cathode pieces. This change in net charge occurring at the anode will induce a response on the

cathode pieces and associated preamplifiers in proportion to the solid angle subtended by each cathode piece. By comparing the signal induced on the central cathode with the charge collected at the anode, we will determine the position on the anode wire at which the charge is collected, and thereby define the fiducial volume longitudinal boundaries. We consider below whether there may be some event energy dependence in our longitudinal boundary cut.

IV.4.1 Wall Effects

We have made mention of wall effects above in our discussion of the activated calibration sources we have employed to study the resolution and linearity of our detector. In those cases, we were concerned with X-ray photon emission, and the possibility that these photons would encounter the detector walls prior to converting to energetic electrons in the counting gas. In the case of the activated krypton source, our concerns included both K and L X-ray escape probabilities, with M X-ray escape considered improbable. For the activated chlorine source, K X-ray escape would produce an inconsequential L X-ray peak well below our region of interest, and so we were not concerned about this probability. For chlorine, L X-ray escape was also highly improbable. Finally, for X-ray photons from either source which converted very near the detector walls, the possibility of event energy degradation due to scattered electrons ranging out at the detector walls was not carefully considered, and not in evidence except perhaps in the tail on the low energy side of the bromine K peak. The level of such effects appears trivial in that distribution, given our ability

to identify fluorescence of the gold on the cathode walls at the level of 1.8%, and to place a simultaneous limit on bromine L X-ray escape of $0.1 \pm 0.1\%$. Nonetheless, our intended 3H work will have much greater statistical certainty than did the calibration data, and so we must consider potential wall effects more carefully. In the case of 3H decay, there are no X-ray photons which may encounter the walls, and so henceforth, we will mean by wall effects only the possibility of energetic electrons ranging out in the detector walls and depositing some event energy in the walls.

We have simulated the process of ionization of the counting gas following β particle emission from tritium nuclei. We have generated 250 million events in this Monte Carlo simulation, exhibiting the correct theoretical distribution of β particle energies. In our simulation, we have included the scattering process and the proportional chamber boundaries so that we have also simulated the effect of β particles ranging out in the detector walls. However, we have neglected inelastic scattering of these β particles off of the detector walls in our simulation, since the strong angular dependence for scattering cross-sections leads us to expect that backscattering will be much less probable than absorption of the β particle energy into the walls. The effect on the spectrum is demonstrated in Figure IV.15, wherein the uncorrected and corrected spectra are shown in the distributions a) and b) respectively. Distribution c) is that which results from subtracting the uncorrected spectrum from the corrected spectrum, and then dividing the results by the latter spectrum. This distribution is therefore useful in determining the correction to be applied to an observed spectrum in order to compare it with theory. In Figure IV.16, we demonstrate that the shape of the correction approximates a smoothly curved (third order) polynomial

Figure IV.15: Simulated Wall Effect

supplementing the birth spectrum at energies below 8400 eV (Figure IV.16 a), and a linearly increasing depletion of the spectrum with energy above 10400 eV (Figure IV.16 c). (The intervening 2 keV, shown in Figure IV.16 b), are depleted less rapidly and in any event are not greatly distorted from the birth spectrum.) The maximum increase in the spectrum occurs near 3.7 keV, with a magnitude of 0.18% of the underlying observed events at that point. This supplementing of the spectrum results from the combination of depletion of this region due to degradation of a small fraction of

Figure IV.16: Wall Effect Shape Corrections

events born into this region and the accumulation of degraded events of higher birth energies. The depletion of the spectrum above 10.4 keV, due to the degradation of events born into this region of the spectrum, increases at the rate of 0.15% per keV. The dot-dashed lines immediately above and below zero in the distributions of Figure IV.16 (and in Figure IV.15 c) indicate $\pm 1\sigma$ statistical uncertainty for the simulated spectrum, while the predicted 0.18% increase in the distribution near the maximum in the observed spectrum will be approximately $\frac{1}{2}\sigma$ for the experiment we intend to conduct.

This wall effect due to β particles ranging out in the detector walls will not be separately observable in our experiment, but the distortion to the spectrum due to this effect is expected to be small in magnitude and to have a smooth shape. This is therefore not expected to have a substantial effect on our analysis of the results. We will nonetheless allow for the level of wall effects based on our simulations in analysing our results.

IV.4.2 End Effects

Events which collect on the anode wire at the same longitudinal coordinate as the gap between the central cathode and an adjacent guard cathode will induce a signal on the central cathode which has somewhat less than $\frac{1}{2}$ the amplitude of the anode signal. This results from the fact that the ends of the cathode pieces are open, and therefore the solid angle subtended by the central cathode for such events is somewhat

less than 2π . The signal induced on the adjacent guard cathode will be smaller in amplitude than that induced on the central cathode because the guard cathodes are shorter in length (8 *cm* vs 25 *cm*) and therefore subtend a smaller solid angle. The signal induced on the opposite guard cathode will be negligible. Events which occur within the volume of the central cathode piece will induce signals on the central cathode which are a greater fraction of the anode signal than will those at or beyond the ends of the central cathode. For our geometry, we predict that events which occur at the exact centre of the volume will induce a signal on the central cathode which is slightly smaller than that observed at the anode, after considering the fractional solid angle subtended by the open ends of the central cathode piece. We predict also that events which occur at the boundary between the central cathode and one of the guard cathodes will induce a central cathode signal which is slightly less than half that of the anode. Then we expect that we may distinguish the longitudinal location of each event by comparing the signal induced on the central cathode with that observed on the anode. Events which occur within the volume of the central cathode should exhibit a cathode signal to anode signal ratio of between $\sim \frac{1}{2}$ and ~ 1 (assuming the anode and cathode circuits have equal gain). There is no reason to expect an energy dependence in this discrimination method for simple bare charges, provided that there are no non-linearities in the event selection techniques. However, there are two potential difficulties with this fiducial volume cut technique for our experiment. First, we note that the statistical nature of the signal digitization process, and the noise generated on the separate anode and cathode channels introduces uncertainty into the determination of the ratio of the cathode and anode signals, which leads to an uncertainty in the volume defined by this cut, and this uncertainty may lead

to an energy dependent bias in the cutting of events deemed outside the fiducial volume. This noise is greater at lower energies, and may therefore corrupt our event selection at low energies. As well, we have already observed a possible small non-linearity in our energy axis resulting from space charge screening of events, an effect which is proportional to energy. Differences in the linearity of the anode and cathode signals will produce an additional energy dependence in the acceptance of events in our detector, and thus in the volume defined by our fiducial cut. We determine the possible energy dependence in our detector's acceptance below, beginning with the possible effects of noise, and following with consideration of the potential residual energy dependence in the fiducial cut itself.

Anode and Cathode Noise

We take the fiducial volume containing any event in our detector to be related to the ratio of the cathode to anode signals for the event. We define this ratio simply as:

$$R = \frac{c}{a}$$

The fractional uncertainty in the volume is related to the fractional uncertainty of the cathode to anode signal ratio, σ_R , given by:

$$\frac{\sigma_R}{R} = \sqrt{\frac{\sigma_a^2}{a^2} + \frac{\sigma_c^2}{c^2}}$$

where σ_a and σ_c are the anode and cathode RMS noise respectively. We add these noise fractions in quadrature since they are largely uncorrelated noise signals. We have measured these noise figures, and reported them in Section IV.3.1 above, as

1.99 channels on the anode and $2.57\bar{c}$ channels on the cathode. Then:

$$\frac{\sigma_R}{R} = \sqrt{\frac{1.99^2}{a^2} + \frac{2.57^2}{c^2}}$$

Now, in selecting events which we determine to be within our preferred volume, we establish a minimum acceptable ratio of the cathode to anode signals, that is some f_{min} , where we require that $R \geq f_{min}$. In selecting events in which $\frac{c}{a} \geq f_{min}$, we accept an ensemble of events into each anode channel over which the average value of the fraction is $\bar{f}_a$, and we expect this to be constant over the entire spectrum, within statistical fluctuations, and to have a value between f_{min} and ~ 1 , neglecting any mismatch in cathode circuit gain relative to that of the anode. That is, in the absence of any variations which affect the anode and cathode signals in different ways, $\bar{f}_a$ will equal the constant R . We have in fact operated our detector with the gain in the cathode circuit matched to within 0.3% of that in the anode circuit, as measured in pulser tests and identified in Section IV.3.1 above. Then for any anode channel, we may replace c with $\bar{c}$, where $\bar{c} = \bar{f}_a a$. Then:

$$\frac{\sigma_R(a)}{R(a)} = \frac{1.99}{a} \sqrt{1 + \frac{1.29^2}{\bar{f}_a^2}}$$

(In this, we have now accommodated the possibility that R and σ_R may have some energy dependence, that is some components which are a function of channel number a .)

The energy dependence in $\sigma_R(a)$ which concerns us here is to be found in the $\frac{1}{a}$ term. At energies in the lower end of our spectra, where a is small, the uncertainty in the fiducial volume becomes larger. Since we intend to use a minimum cathode to

anode fraction, f_{min} , which is sufficiently large as to ensure that the fiducial volume is totally contained within the central cathode. we anticipate an f_{min} greater than $\sim \frac{1}{2}$, and an average fraction, $\bar{f}_a$, approaching ~ 1 (since the ratio declines significantly only near the ends of the central cathode piece). We will use an f_{min} of 0.67.

In collecting our spectra, we also collect the value of $\bar{f}_a$ for each anode channel. We find this to be typically 0.962 ± 0.002 (independent of any energy dependent contributions), near our maximum expected value. This yields:

$$\frac{\sigma_R(a)}{R(a)} = \frac{3.25}{a}$$

Figure IV.17: Typical 3H and Background Cathode:Anode Signal Ratios

We may now determine $\sigma_R(a)$ for each anode channel, a , and a plot of $R(a) \pm$

$\sigma_R(a)$ versus anode channel number, a , will exhibit an envelope of uncertainty above and below $R(a)$ (see Figure IV.17), which is very large at low values of a , and decreases with increasing a . The matter of concern here arises if the uncertainty, $\sigma_R(a)$, is sufficiently large as to result in individual measurements of $R(a)$ falling below f_{min} due to noise rather than due to the location of the event. If we require that $R(a)$ be at least $3\sigma_R(a)$ away from f_{min} , there will be a 0.14% probability of discarding valid events due to noise. At values of $\bar{f}_a$ and f_{min} of 0.96 and 0.67 respectively, this probability will be met or exceeded only for the first 48 anode channels above the pedestal. Then the region in which we have non-trivial uncertainties in the acceptance of the detector, attributable to noise corrupting the fiducial cut, coincides with the region which we will discard due to its substantial trigger inefficiencies.

Fiducial Volume Dependence on Energy, Non-Linear Effects

We have already recognized an energy dependent variation in our energy calibration of the spectrum, due to space charge effects. This will have the effect of increasing the energy value of the mid points of the channels in our spectra in a non-linear way. The energy difference between these midpoints will increase with increasing energy, across the spectrum. Then the widths of these channels will also increase proportionately, and this will result in an equivalent energy dependence in the normalization of the spectrum. This will not affect the cathode to anode signal ratio, $R(a)$, since these space charge effects act to limit the gas gain in an energy dependent way, and will therefore limit both the cathode and anode signals in similar ways.

We note, however, in Figure IV.17 that $\bar{f}_a$ is not a constant, as we had expected it would be. The slope indicated in the fit is of the same order of magnitude as the space charge non-linearity observed in the energy calibration of the anode signal conducted earlier. We hypothesize that an additional small energy dependent attenuation in the anode signal may not be similarly visible in the cathode signals. (This deviation in the anode and cathode signals may result from such difficulties as the 0.3% possible gain mismatch between the two channels.) Then let us redefine the ratio of the cathode to anode signals, at any particular anode channel, as:

$$R(a) = \frac{\frac{1}{N_a} \sum_{i=1}^{N_a} c_i}{a}$$

If we now introduce some energy dependent δ_a , which acts to attenuate the signal at a , without also introducing a similar effect at the cathode, we will generate a deviation in $R(a)$ as a result.

$$R(a) = \frac{\frac{1}{N_a} \sum_{i=1}^{N_a} c_i}{a(1 - \delta_a)} = R + a\delta R$$

where we expect R to have the constant value $\bar{f}_a$ taken at the first channel in the spectrum, and δR describes some change of $R(a)$ with a . We may now expand the bracketed term in the denominator through a Taylor series expansion to yield:

$$R + a\delta R = \frac{\bar{c}}{a}(1 + \delta_a - \delta_a^2 + \dots) = R(1 + \delta_a - \delta_a^2 + \dots)$$

where $\bar{c}$ is the mean of the cathode signals for the particular anode channel represented. It is therefore apparent that:

$$a\delta R = R(\delta_a - \delta_a^2 + \dots)$$

We therefore anticipate a fractional change in $R(a)$, given here by $\frac{\delta R}{R}$, at channel a , which has an absolute value which is of the same order as the deviation $\frac{\delta_a}{a}$, assuming

that δ_a is small. We do observe such a change in $\frac{\delta R}{R}$ in Figure IV.17 above. Then keeping only the leading term in the expansion, and therefore approximating the deviation to an assumed linear rate of change with channel number, we may expect that:

$$\frac{\delta R}{R} \approx \frac{\delta_a}{a}$$

But $\bar{c}$ is simply the mean of the digitized values of the voltages induced on the central cathode, so that:

$$\frac{\delta_a}{a} = \frac{\bar{c} - aR}{a} \approx \frac{\delta v_c}{v_a}$$

(for $R \approx 1$), where v_a is the anode voltage and δv_c is the deviation of the mean cathode voltage from its expected value, that being R times the anode voltage.

Then holding f_{min} constant while $R(a)$ increases with increasing a has the effect of allowing progressively smaller cathode voltages (compared with the simultaneous anode voltage) to satisfy the fiducial cut criterion. This has the same effect as an f_{min} which is not fixed in value, but which decreases slowly with increasing a , so that the detector's acceptance is extended to progressively larger volumes as event energies increase. Then we anticipate that the deviation of δR from zero implies an energy bias in the fiducial volume cut. This was to be expected since we can not be assured that the anode and cathode signal gains are matched to better than 0.3%.

We have studied in detail the behaviour of our detector as a function of f_{min} . We have noted that the rate of change in the acceptance of our detector becomes nearly constant across the entire spectrum at an f_{min} of about 0.67, for both background

spectra and 3H spectra. In doing this, we have compared the fractional reduction in the number of events counted for each fractional increase in f_{min} , averaging across contiguous 50 or 100 channel blocks, and we have taken this as an indication of the minimization of end effects, and of the energy dependence in this fiducial cut. We attribute the deviation from linearity in this cut at fractions lower than 0.67 principally to backgrounds which originate in the end cap assemblies, or possibly to irregularities in the field or the background near the gaps between the cathode pieces, or near the distant ends of the guard cathodes.

Figure IV.18: Event Rate Dependence on f_{min}

In Figure IV.18, we analyse the fractional rate of change in events counted at our selected value of the limiting cut fraction, f_{min} . The first derivative of this

relation, with respect to channel number, is the slope of the line drawn through the observations, which we expect to equal exactly zero when there is no energy dependent bias in the volume cut, but will reveal the magnitude of the variation in the fiducial volume when this volume varies with energy due to differences between the anode and cathode signals. That is:

$$\left. \frac{d}{da} \left(\frac{dN_a}{N_a} \right) \right|_{f_{min}=0.67} = \xi \frac{d}{dE} \left(\frac{dVol}{Vol} \right)$$

ξ is a constant of proportionality, and is in fact the inverse of the energy dispersion for the detector. We observe in Figure IV.18 that, neglecting the first hundred channels of our spectra where we are aware of trigger inefficiencies, the minimized fractional rates of change in events counted does differ across our spectra, and these differences increase with increasing energy. Then the minimized fractional rate of change of volume with energy, as measured in Figure IV.18, is $3.73 \times 10^{-5} \pm 7\% \text{ ch}^{-1}$. It is also apparent that the presumed linearity in this dependence begins to break down at higher channel numbers, which are beyond the limits across which we will fit our ^3H spectra. We view this effect, not as a channel width variation as was done in our discussion of the anode and cathode signal attenuation due to space charge effects above, but rather as an increasing fiducial volume with increasing event energy. This phenomenon does not affect our energy axis calibration as was the case for space charge effects, but does constitute an additional small energy dependence in the acceptance of our detector. We will therefore find it necessary to analyse our ^3H spectra for the presence of this energy dependent effect in our detector's acceptance, near the levels observed in Figure IV.18, in addition to the space charge effects we have previously measured.

Chapter V

Observations and Analysis

V.1 Background Spectra

Sources of background contamination of our spectra were thoroughly studied. As previously discussed, the dominant components of the backgrounds could be successfully described as originating in three ways. One apparent component of the background is a gently sloping (with energy) distribution probably resulting from cosmic and other natural radiation. We have also discussed and analysed above some fluorescence features which we believe result from excitation of the gold cathode surfaces by these cosmic rays and natural backgrounds. We also observe a $\frac{1}{E}$ event distribution, probably attributable to bremsstrahlung photons originating within the walls of the detector, largely from 3H nuclei contaminating the detector walls.

In determining the actual shape and rate of these background components, in order to subtract them from our tritium observations, account must be taken of the raw rate of background events to be expected in the chamber, exclusive of the residual tritium which will outgas back into the detector from the construction materials during a background acquisition experiment. As well, corrections must be applied to the magnitude of the background subtraction in order to compensate for an expected high detector dead time fraction, which will result from our attempts to achieve a high tritium to background event ratio during final spectrum acquisition.

The strong dependence of detector gain on gas density, and uncertainty in our ability to achieve repeatable gas densities from any particular background experiment to a subsequent tritium fill makes gas gain measurement essential before and after each background and tritium spectrum acquisition run.

We will discuss the raw background rate and shape determination as well as the gas gain determination in the next section, and subsequently apply live time corrections to tritium data.

V.1.1 ^3H Accumulation In The Counting Gas vs Gas Gain Stability

It was observed that, at higher gas pressures, the chamber gas pressure was not constant during the first ~ 30 hours after filling. However, it was also observed that

the background event rate in a freshly filled chamber would rise significantly at first, and then more slowly, to approach an equilibrium rate. Study of changes in the shape of the background spectrum indicated that the spectrum was tending towards a more tritium-like shape with time, and it was concluded that there was a buildup of predominantly 3H residue trapped within the detector materials, and exchanging with the fill gas after a fresh filling of the detector. While this buildup of 3H in the counting gas will contribute a large fraction of the events recorded in a background experiment ($\sim \frac{1}{2}$), it will be small in comparison to the number of 3H events we will eventually record and analyse. We therefore attribute the early gas pressure decreases to uptake of the gas into the chamber walls (evacuated to below 50 *millitorr* between fills), and the event rate changes to simultaneous exchange of the 3H species with the fill gas. While the contamination of the background observation could be minimized by acquiring the spectrum immediately after the filling of the chamber, gas density changes would produce significant gas gain changes during this period, thus degrading the resolution of the chamber for the features being observed. The tritium buildup could be easily subtracted from our background spectrum after monitoring the change in the background event rate from its initial value immediately after filling of the chamber, and throughout the collection of the background spectrum. The appropriate fraction of the events to be removed from the spectrum is that which results from the increase in the rate over its initial value, and the distribution to be subtracted is that of an experimental tritium spectrum of similar gas gain. Experiments were conducted at different high voltages to determine the raw initial background rate. These experiments consisted of acquiring numerous sequential background spectra, commencing immediately after filling the chamber. Each spectrum contained 50,000

events, and required from as much as 36 to as little as 27 minutes to collect. This allowed for individual measurements of the background rate which have 1σ statistical uncertainties of only 0.45%. At our preferred gas pressure and high voltage, the rate was observed to increase during these tests by 33% over the initial rate, during the period of the 36 hours and 40 minutes required for the tests. From these we determined an initial background rate of $24s^{-1} \pm 0.5\%$ at our selected gas gain, which covers an interval of from 0 to $\sim 17.5 \text{ keV}$ for the full range of the adc.

A typical background spectrum is shown in Figure V.1 a). The mean rate of event acquisition was recorded with the spectrum. As well, spectra of $\sim 10^5$ events each were recorded immediately prior to and following acquisition of such a spectrum as that shown in Figure V.1, in order to make independent measurements of the *before* and *after* event rates. With the event rate well determined, an appropriate fraction of an experimental 3H β decay energy distribution, acquired with similar gas gain, was subtracted from this contaminated background spectrum, in order to eliminate from the spectrum the 3H which had built up in the detector. This would reduce the number of events by the difference between the mean rate of acquisition of these background events of $47s^{-1}$, and the raw initial background rate of $24s^{-1}$, as measured through our sequential background rate measurement runs. The corrected background spectrum is displayed in Figure V.1 b), which has also been scaled (scale factor 0.722). The scale factor used ensures that we will make an *equal live time* subtraction of the reduced background from the 3H data recorded. Prior to calculating the scaling factor, the measured live time of the background run was proportionately increased to account for the number of 3H events deducted. The revised background

Figure V.1: Background Spectrum

run live time was then compared to the live time of the ^3H collection run, and scaled to make it equal to this ^3H run live time. As a check on this procedure, spectra were collected through another adc, via a signal attenuator, which resulted in spectra having endpoints more than six times the energy of the spectra used for this analysis. These extended energy spectra contained largely constant cosmic and natural background activity, at energies above the ^3H endpoint, and allowed for a verification of the background subtraction. To accomplish this, the higher energy continuum dis-

tributions of both the 3H and background spectra were compared, and their relative scales were determined, independently of the purging of 3H data from the background spectra. This yielded a scaling factor of 0.735, just 1.8% larger than that determined with the prior procedure. The uncertainty indicated for our background subtraction due to the different results from these two methods amounts to less than 0.03σ at the maximum in our 3H spectrum, and a smaller fraction elsewhere. We have therefore used the procedure of eliminating the 3H contamination from the background spectra in producing all background analyses throughout this report, including previous discussions of the components of the backgrounds.

V.1.2 Gas Gain Monitoring

The procedure developed for acquisition of both background and 3H spectra was to fill the chamber with virgin $P10$ gas, plus a small amount of CH_3T if the spectrum desired was 3H rather than backgrounds, and then to wait for in excess of 30 hours. Then a series of spectra was acquired, beginning with a spectrum in which the ^{55}Fe source illuminated the aperture, and then a 50,000 event measurement of the rate with the ^{55}Fe source turned away. The intended background or 3H spectrum was then acquired. This was then followed again by a similar rate measurement test, and finally a spectrum which included the ^{55}Fe source, as at the start. In this way, both the gas gain and the total rate were determined immediately before and after collecting background or 3H data. For the background spectrum shown in Figure V.1, the rates measured immediately before and after collecting the spectrum were

$43s^{-1}$ and $51s^{-1}$ respectively, over the same 0 to 17.5 keV interval as the previous $24s^{-1}$ background rate measurement, which verifies the $47s^{-1}$ mean rate measured. This rate was typical for our background spectra, and included about 50% outgassed 3H and 50% raw backgrounds.

The spectra acquired while the ^{55}Fe source was illuminating the entrance aperture of the chamber were used to check for potential changes in gain during the background acquisition run. After reducing these spectra by that fraction of the events which we have attributed to 3H outgassing into the chamber, we have fit the remaining features to our known analytical descriptions of the bremsstrahlung distribution, plus a gently sloping flat distribution, and the two fluorescence features described by Gaussian distribution functions. As well, two simple Gaussian distributions were used to describe the $^{55}Mn\ K$ peak, and the argon escape peak. The entire spectra were fit in this way, locating the peaks of the various features in the spectra. From the position and uncertainty thus determined for the $^{55}Mn\ K$ peak, and given the energy calibration non-linearity previously determined for this pressure and voltage range, the linear energy calibrations were determined for both the *before* and *after* data acquisition runs. Then using these results, and the positions and uncertainties of the other features in the spectra (the argon escape peak and the gold fluorescence peaks), the mean energies of the other features were determined as a test of the consistency of the linear calibration term. Results from such a test are shown in Table V.1.

In this way, the change in the linear calibration term was determined to be an increase of 0.62% between the two spectra during the background acquisition

Table V.1: Gas Gain Stability Test

Run No.	^{55}Mn Peak	Ar Escape Peak	1 st Fluorescence Peak	2 nd Fluorescence Peak
$E_{K_{\text{known}}}$ (eV)	6010*	2978.13**	9712.	11442.
.494 Mean Ch. eV/ch $E_{\text{Calc.}}$ (eV)	539.953 ± 0.096 10.9999 ± 0.0020	272.037 ± 0.569 3010.30 ± 6.84	866.171 ± 3.456 9709.38 ± 40.47	1013.95 ± 3.016 11402.2 ± 36.0
.499 Mean Ch. eV/ch $E_{\text{Calc.}}$ (eV)	543.253 ± 0.169 10.9323 ± 0.0020	273.45 ± 0.973 3007.43 ± 11.64	871.715 ± 3.666 9712.63 ± 43.88	1021.75 ± 2.969 11421.2 ± 36.8

*Based on 0.361 $\text{Mn } K_{\beta}$ weighting in K peak.

**Based on 0.108 $\text{Mn } K_{\beta}$ weighting in argon escape peak.

run. This small increase in gain implies some continuing uptake of the gas into the chamber walls, and thus a small decrease in gas density, which may result in a somewhat deteriorated resolution of our apparatus, relative to our expectation of near 14.5%. In any event, $\pm 0.6\%$ instability in gas gain implies $\leq 0.1\%$ instability in gas density, which is beyond our ability to measure with the apparatus at our disposal. The success in locating the gold fluorescence features at the correct energies, as shown in Table V.1, supports our confidence in the consistency of the apparatus, and in our

determination of the small non-linearity in the channel widths.

V.2 3H β Decay Spectrum

V.2.1 Gas Gain Matching

The necessity to ensure adequate matching of the gas density and gas gain conditions between background data sets and 3H data sets has led us to also record the position of the peak due to our ^{55}Fe source immediately *before* and *after* acquiring our 3H spectra. For the particular pair of background and 3H spectra to be discussed below, the peaks were located at channels 544.0 ± 0.3 and 545.5 ± 0.4 (above the pedestal) *before* and *after* the background run respectively, and 541.5 ± 0.2 and 544.6 ± 0.4 *before* and *after* the 3H spectrum respectively. Taking the means of these pairs, we determine that the ^{55}Mn $K_{\alpha 1}$ line was 2.9 ± 0.3 channels, or 0.5% lower for the background spectrum than for the 3H spectrum. This difference is of the same order as the change in the peak position during the life of the background run. To rebin the background spectrum prior to subtraction from the 3H data would require careful management of the uncertainties for each channel of background data, and undoubtedly would slightly increase the uncertainty in this data set. (The rebinning would require transferring $\frac{1}{2}\%$ of the bin 2 data to bin 1, 1% of bin 3 data to bin 2, etc...) We have considered the value of doing such rebinning for this pair of spectra, and compared the results of such rebinning with the raw background data. It is

apparent from the differences between the original background data set, and the rebinned data set, that there is little benefit in rebinning this data. The difference between the two data sets is of order ~ 10 to ~ 40 counts per channel over the entire useful range of the data. After subtraction of the 3H contamination of the background spectrum and scaling the balance in order to achieve the *equal live time* subtraction, the difference will be less than ~ 20 counts per channel, which is small in comparison to the 1σ statistical deviations in the background data, and trivial relative to the normal deviations in each channel of the one hundred million event 3H data. As well, the difference between these two data sets is a smooth function, small in comparison with, and similar in shape to the energy dependence in the detector's acceptance. We have chosen to disregard this small mismatch in gas gain, and proceed with the background subtraction in the belief that there will be at worst only a small penalty in the success of our eventual comparisons of the spectrum with theoretical predictions.

V.2.2 The 3H Spectrum

The reduced background shown in Figure V.1 b) above was subtracted from the 3H spectrum recorded and shown in Figure V.2 a). The resulting change in the spectrum is barely apparent in the lower energy portion of the spectrum shown in Figure V.2 b). The background accounts for only $\sim 1.6\%$ of the 3H spectrum, and the uncertainties due to the background subtraction were added to those of the 3H data, increasing the 3H uncertainties by less than $\frac{1}{8}\sigma$ in each channel.

Figure V.2: 3H β Decay Spectrum

We now proceed with the comparison of the observed spectrum to our theoretical predictions, fitting to the appropriate β particle energy distribution for the zero neutrino mass case. We allow in our fitting for the measured energy calibration non-linearity of $2.2 \times 10^{-5} \text{ ch}^{-1}$ times the linear energy calibration term and the associated channel width increase, the detector's additional acceptance non-linearity of $3.7 \times 10^{-5} \text{ ch}^{-1}$ times the normalization constant, the wall effects predicted with our Monte Carlo simulations (at about $\frac{1}{2}\sigma$ near the maximum in the 3H spectrum).

and the screening effect consistent with a well depth of 74eV at the nucleus. Our strategy will be to fit the spectrum in the higher energy region, where the wall effects will be most pronounced, in order to determine the magnitude of these effects and to verify the systematics of our experiment. We will then fit the lower energy portion of the spectrum, where the shape is most sensitive to screening, in order to determine the magnitude of the screening potential. We will then investigate the consistency of both determined parameters across the combined range in the spectrum, and iterate the fitting where necessary. Finally, we will look for evidence of the possible mixing of a heavy neutrino.

We make our initial fits over the approximate β particle energy range of 2.2 keV to 14.5 keV . The spectrum has a maximum near 2.5 keV , and the previously hypothesized heavy neutrino would produce a kink in the β spectrum near 1.6 keV . The spectrum is shown in Figure V.3 a), with expanded views shown in Figures V.3 b), c), and d). The fit function is drawn in the figures, but is barely visible within the data points and error bars plotted. This fit yields a χ^2 of 1181, for the 1150 degrees of freedom which result from fitting over 1152 adc channels, and using two degrees of freedom for free parameters to normalize the amplitude of the data, and to calibrate the energy axis (linear term) in eV/ch . The quality of this fit is indicated by the 25.6% confidence level which results from this χ^2 value. That is, the χ^2 per degree of freedom is 1.027, and there is a 25.6% probability that a similar test would yield a poorer fit between the observations and the theory, due simply to statistical fluctuations in the observations. This therefore implies that there is little corruption of our observations which could be attributed to the small systematic effects we have

Figure V.3: ^3H Spectrum Preliminary Fit Results

chosen to ignore. (Those systematic effects are the 3H and background spectra gain mismatch, and the small amplitude cyclic pattern attributed to the adc systematic limits, which was apparent in our 500 million event ramp pulser tests and our 3 billion event 3H data.) We have also tested the dependence of the quality of this fit on variations in the detector's energy resolution, varying the nominal 14.5% FWHM resolution at the ${}^{55}Mn$ $K_{\alpha 1}$ X-ray between 13.0% and 16.0%, with no apparent effect on χ^2 . We therefore take confidence in our detector system in all respects, excepting that our description of the wall effects is based solely on our Monte Carlo simulations. While there is little room in the uncertainties in this data for the wall effects to be dramatically different from the simulations, we are limited in our ability to confidently confirm these simulations apart from further analysis of the recorded spectra.

We now seek to test the dependence of the quality of the fit on our assumption of the level of wall effects which we have used. We have allowed the magnitude of wall effects to be a third free parameter in the refitting of the data, and sought the value for this parameter which achieves the lowest χ^2 . The refit spectrum is shown in Figure V.4, and this result indicates a higher level of wall effects than we have estimated, by a factor near 2.8, possibly resulting in part from our having neglected backscattering in our simulations, and possibly reflecting the difficulty in describing electron path lengths due to elastic scattering at very low energies [42]. The χ^2 achieved by the refit is 1143 units, over the now 1149 degrees of freedom, yielding 54.4% confidence that random normal variations in the observations would yield less agreement between the theory and the observations in a subsequent similar test. The region of the fit was exactly the same as for the prior fit, and includes about $\frac{2}{3}$ of the

Figure V.4: ^3H Spectrum Refit Results

98.4 million 3H events recorded in the spectrum. The improvement in χ^2 of 38 units, due to the introduction of the wall effect parameter, is a strong indicator that event degradation due to wall effects is more prevalent in the observations than we had predicted, and reaches about 1.5σ near the maximum in the spectrum. (For normally distributed deviations, there is a 1% probability of achieving a measured result for which $\chi^2 [= \frac{\delta^2}{\sigma^2}] \geq 2.58^2 \approx 6.66$ for one degree of freedom [43]. We therefore take an improvement in χ^2 of 6.66, attributed to a change in one parameter, as supporting 99% confidence in the improved value assigned to that parameter over the prior value. In this case, our improvement attributable to the wall effect parameter is almost five times greater than this.)

V.2.3 Wall Effects vs Non-Linear Effects

We have investigated the dependence of the quality of our fits on the non-linear corrections which we have applied to our spectrum. The space charge non-linearity which we have measured is $2.2 \times 10^{-5} \text{ ch}^{-1} \pm 5\%$, and the additional energy dependence in the fiducial volume measured at $3.7 \times 10^{-5} \text{ ch}^{-1} \pm 7\%$, yield a combined correction factor for the normalization of the spectrum of $5.9 \times 10^{-5} \text{ ch}^{-1} \pm 6\%$. We have also been interested in possible correlations between the strength of the wall effect correction and the strength of the correction due to these non-linear effects during the fitting, since there is some residual uncertainty in the non-linear corrections and a large deviation in the wall effect correction from our expectations. The concern arises from the understanding that these two corrections act in opposite ways upon the

higher energy portions of the 3H spectrum, and may blow up if improperly constrained in the analysis. We have therefore fit the 3H spectrum for wall effect corrections of between one and five times the Monte Carlo predicted value, while varying the combined non-linear effects between $2.5 \times 10^{-5} \text{ ch}^{-1}$ and $9.5 \times 10^{-5} \text{ ch}^{-1}$. For each value of the strength of the wall effect correction, we have sought the non-linear correction which produces a minimum in χ^2 , and then sought the strength of the wall effect correction which achieves the minimum χ^2 from all of these tests. The results are shown in Figure V.5. The curves shown in Figure V.5 a) demonstrate the dependence of the quality of the fits on the non-linear correction term for these various strengths of wall effects. The solid curve is for a wall effect of three times the Monte Carlo predicted value, while the dot-dashed curve to the left (right) is for a wall correction of one (five) times the prediction, and the dashed curve is for a wall correction of two (four) times the predicted strength. Clearly from Figure V.5 a), the χ^2 minima at all strengths of wall effects indicate combined non-linear terms near the $5.9 \pm 0.4 \times 10^{-5} \text{ ch}^{-1}$ range which we determined from the measurements discussed in the previous chapter. The wall effect indicated in Figure V.5 b) which minimizes χ^2 is 3.0 ± 0.6 (95% CL), in agreement with the 2.8 indicated in Figure V.4. Figure V.5 c) demonstrates the minimized non-linear correction of $6.00 \pm 0.06 \times 10^{-5} \text{ ch}^{-1}$ (95% CL), which overlaps the measured value almost precisely and coincides with the wall effect strength of 3 times the Monte Carlo prediction. This statistical analysis of the dependence of the quality of the fit on these two systematic effects yields a more certain determination ($\pm 1\%$) of the non-linearities than the $\pm 6\%$ limit established through the direct measurements, reflecting the strong dependence of χ^2 on variations in the non-linearities. We now also have a determination of the

Figure V.5: Non-Linearity and Wall Effect Correlation

wall effect strength of $3.0 \pm 20\%$, based upon this analysis.

V.2.4 The Screening Potential

We have tested our spectrum for evidence of the predicted screening effect. In our initial fits in Section V.2.2 above, we have used the theoretically supported value of 74eV in our fitting routines, and found significantly better fit results at higher levels of simulated wall effects than our Monte Carlo prediction. However, in this we have fit the spectrum only above 2.2keV , in a region which is decidedly less sensitive to the screening effect than is the region below 2keV . We next test the lower energy region for goodness of fit to various values of the screening potential, while employing the normalization, energy calibration, and wall effect fit parameters achieved in our refit above (Figure V.4), as a description of the gross features of the distribution.

In Table V.2, we show the results for fits conducted between 1075eV and 2150eV , for various values of the screening potential. The χ^2 values shown in the table are the totals over the 101 channels employed in our series of screening fits, and the confidence levels demonstrate the goodness of the fits for the one hundred and one channels of data collected, where all of the parameters in the theory have been held fixed.

We have plotted the screening potentials and the χ^2 values of Table V.2 in Figure V.6. The dependence of χ^2 on the screening parameter exhibits a minimum, with the best value for the screening potential being $85.7 \pm 3.8\text{eV}$ (95% *CL*).

Table V.2: Low Energy Screening Potential Tests

Screening Potential ϵV	Normalization Constant	Energy Calibration $\epsilon V \text{ ch}^{-1}$	Wall Effect Strength	χ^2	Goodness of Fit Percentile %
69	967.7	10.68	2.77	181.2	$\ll 0.1$
74				144.1	0.3
79				119.9	8.5
84				108.8	25.7
89				110.8	21.6
94				126.0	4.0
99				154.5	0.0
104				196.2	$\ll 0.1$

In a further test, the fitting routines were modified to treat both wall effects and screening as free parameters, along with the normalization constant and the energy calibration constant. When fitting was carried out over the same 2.2 keV to 14.5 keV range, the result favoured a wall effect strength of 2.9 times the Monte Carlo prediction and a screening potential of $78.4 \pm 5.6 \text{ eV}$. This fit yielded a χ^2 of 1142 for the now 1148 degrees of freedom (also 54.4% CL), and is therefore comparable with our earlier refit. However, using the same technique of testing the goodness of fit for the indicated normalization and calibration parameters, while varying the screening potential, in the 1.1 keV to 2.1 keV range, the results indicate a higher value for the screening potential. The results, shown in Table V.3 and in Figure V.7, indicate

Figure V.6: Minimized χ^2 Fit For Screening Potential

a 95% confidence result of 87.2 ± 3.8 eV. Both results, above and below 2 keV, overlap the prior result in the limits of their uncertainty ranges, but their failure to overlap each other strongly may indicate some sensitivity in the measurement of the screening potential, depending upon the range of the spectrum employed for the analysis. This variation in the results may indicate a dependence on the shape of our wall effect correction, or more probably a residual bias in the parameterization due to our fitting strategy.

As a result of this apparent dependence of the fit results upon the region of the spectrum being studied, the spectrum was fit over the entire 1.1 keV to 14.5 keV

Table V.3: Low Energy Screening Potential Revised Tests

Screening Potential ϵV	Normalization Constant	Energy Calibration $\epsilon V \text{ } ch^{-1}$	Wall Effect Strength	χ^2	Goodness of Fit Percentile %
69	967.9	10.68	2.86	195.0	$\ll 0.1$
74				153.8	$\ll 0.1$
79				125.5	4.3
84				110.5	22.3
89				108.4	26.6
94				119.6	8.9
99				143.9	0.3
104				181.6	$\ll 0.1$

range, allowing the fitting routines to seek a best fit for all four parameters, those being the normalization, the energy calibration, the wall effects, and the screening potential. The resulting fit (Figure V.8) produced a χ^2 of 1252 units, over 1248 degrees of freedom, indicating by its 46.3% goodness of fit that the deviations between the fit and the observations are randomly distributed. The wall effects concluded from this fit are at a strength of 3.1 ± 0.3 times the Monte Carlo level, which is consistent with our earlier analysis of the goodness of fits for the non-linear systematic effects. The best fit value of the screening parameter achieved here is $90.0 \pm 3.2 \text{ } \epsilon V$ (68% CL). When we study the deviations between the data and the function implied by the fit, we see that the region between $1.1 \text{ } keV$ and $2.1 \text{ } keV$ is consistent with this result.

Figure V.7: Revised Minimized χ^2 Fit For Screening Potential

with a minimized χ^2 achieved with a screening potential of 91.4 ± 3.8 eV (95% C.L.). This result is free of the bias of our initial estimate of the screening potential, which may have influenced the determination of the other three parameters in the fit. It is also consistent with our prior observations of the strength of the wall effects, and is therefore our most reliable result. We will use the 90 eV result obtained from fitting across the wider range of the spectrum, and quote an uncertainty of ± 3 eV (68% C.L.).

In Figure V.9, we plot contours of constant χ^2 for various combinations of wall effect strengths and screening potentials. The hatched area and marker identify the screening potential of 90 ± 3 eV, and the optimized wall effect strength determined

Figure V.8: ^3H Spectrum Full Fit Results

Figure V.9: Screening Potential and Wall Effect Correlation

in this fit to be 3.1 ± 0.3 . In arriving at this final screening potential measurement, we have considered 95% confidence limits on the various systematics which we have discussed in this chapter, including the non-linear effects and the wall effects. These uncertainties are all considered in our 68% C/L statistical uncertainty of ± 3 eV. We have also observed that $\pm 10\%$ variations in the energy resolution have no effect on the fitting. We have not considered the single remaining systematic uncertainty which enters our result, that being the pedestal uncertainty. We have previously quoted (see Section IV.3.1 the pedestal uncertainty as $\pm \frac{1}{2}$ channel, or 5.3 eV (1σ) at this gas gain. We therefore assign an additional systematic uncertainty to our result, yielding a final result of 90 ± 3 (stat) ± 5 (sys) eV (68% C/L). The dotted lines in Figure V.9 demonstrate the 95% C/L limits for the wall effect strength of $3.0 \pm 20\%$. (Note that the ± 3 eV uncertainty we assign to the screening potential is a 68% C/L limit, and the ± 0.3 uncertainty in the wall effect which results from the 1.1 keV to 14.5 keV fit is also quoted with 68% confidence. The previously determined 95% confidence

wall effect of 3.0 ± 0.6 is therefore also shown.) The region of the minimum in χ^2 is well defined, and the result we have determined for the screening potential is within this region.

As a final test, we consider the consistency of our spectrum with a theory in which no allowance is made for screening, as an attempt to verify that a correction such as that due to Rose is essential to understanding the distribution of β particle energies. In this test, we have achieved a χ^2 of 1947 with 1249 degrees of freedom. Such a high χ^2 signifies that there is no support for the hypothesis that our unscreened theory and the observations are consistent, thus confirming the necessity for considering the screening effect in analysing the spectrum. While this is, by now, a fully expected result, our objectives with this work require that we explicitly demonstrate the necessity of a correction for the screening effect.

We therefore take our final determination of the screening potential, from Figure V.8, as 90 eV. Our 68% *CL* statistical uncertainty is ± 3 eV, and we have a 1σ systematic uncertainty of another ± 5 eV. This result is consistent with our measurement of the wall effect strength at 3 ± 0.6 times the Monte Carlo prediction (95% *CL*), and with our measured systematic non-linearities, which are now known to be $6.0 \times 10^{-5} \pm 1\%$ *ch*⁻¹ with 95% *CL*.

V.2.5 The Search For A 17 keV Neutrino

Figure V.10: Heavy Neutrino Kink Search

We have analysed the 1.1 keV to 2.1 keV region of our ${}^3\text{H}$ spectrum for evidence of any residual deviations from theory which might be consistent with the anomaly previously attributed to a $< 1\%$ admixture of 17 keV neutrinos. We have adopted the wall effect magnitude and the screening potential as indicated in our best fit results above, those being 3.1 times the Monte Carlo prediction, and 90 eV respectively. We have varied the squared mixing angle from a maximum of greater than 3×10^{-2} down to zero. Figure V.10 demonstrates the fit for a squared mixing angle of 8.1×10^{-3} . The resulting χ^2 values and the associated squared mixing angles are listed in Table V.4, and the results are presented graphically in Figure V.11 below.

Table V.4: Heavy Neutrino Search Test Results

$\sin^2 \theta$	χ^2	Confidence Level %	$\sin^2 \theta$	χ^2	Confidence Level %
0.0169	147.4	0.2	0.0036	111.2	20.8
0.0144	138.0	0.7	0.0025	109.8	23.7
0.0121	130.3	2.2	0.0016	108.7	26.0
0.0100	124.4	5.0	0.0009	108.0	27.5
0.0081	117.6	11.0	0.0004	107.5	28.6
0.0064	115.9	13.2	0.0001	107.3	29.1
0.0049	113.2	17.3	0.0000	107.2	29.3

It is apparent from both Table V.4 and Figure V.11 that χ^2 improves as the mixing angle decreases towards zero. The goodness of fit indicated at $\sin^2 \theta = 0$ by the 29.3% confidence level is respectable, and the 95% confidence interval above the zero squared mixing angle places an upper limit on $\sin^2 \theta$ of 3.6×10^{-3} . This result excludes the value previously presented by Simpson [1].

Fitting the points plotted in Figure V.11 suggests that χ^2 is minimized at a value below zero, as noted in the figure. However, zero is not excluded, being about 1σ away from the minimum in χ^2 . We hypothesize that this non-physical result is simply a statistical artifact. We have conducted a series of similar fits at the limits of the screening potential and wall effect uncertainties which were apparent from our determination of the screening potential. These include screening potentials of 86 eV.

Figure V.11: Heavy Neutrino Mixing Angle Analysis

90 eV, and 94 eV, at wall effect magnitudes of 2.8, 3.1, and 3.4 times the Monte Carlo prediction. The results of these fits are presented graphically in Figures V.12 and V.13. In Figure V.12, we demonstrate the dependence of the quality of the fits on the screening potential and on $\sin^2\theta$, for wall effect strengths of 2.8 (V.12 a and b), 3.1 (V.12 c and d), and 3.4 (V.12 e and f). The left hand figures show the profile of χ^2 as the screening potential is varied from 86 eV through 94 eV (left to right), for various values of $\sin^2\theta$ between 0.018 (back) through -0.01 (front). The right hand figures are top down views, showing profiles of constant χ^2 . From Figure V.12, it is apparent that all three values of the strength of the wall effect correction (2.8, 3.1, and 3.4) produce minimized χ^2 's at marginally negative values of $\sin^2\theta$. The progression

through Figures V.12 b), d), and f) suggests that the correct value of screening potential, consistent with $\sin^2\theta = 0.000$, might be found to be somewhat larger than our best fit value of 90 eV , at a slightly greater wall effect than strength 3.4. Figure V.13 on the other hand shows the relationships between χ^2 , $\sin^2\theta$, and the strength of the wall effects, for screening potentials of 86 eV (V.13 a and b), 90 eV (V.13 c and d), and 94 eV (V.13 e and f). This figure suggests, to the contrary, that a $\sin^2\theta$ consistent with zero occurs at a screening potential near 86 eV , and that larger screening potentials will fit more poorly unless the wall effect strength is smaller. Altogether, Figures V.12 and V.13 support the view that the fit is optimized near a screening potential of 90 eV , with a wall effect strength near 3.1, and with $\sin^2\theta$ insignificantly below zero. The 95% *CL* upper limit is therefore $\sin^2\theta \leq 0.36\%$.

Figure V.12: Mixing Angle Dependence On Screening Potential

Figure V.13: Mixing Angle Dependence On Wall Effects

Chapter VI

Conclusions

We have used the gas proportional chamber technique to study the β decay spectrum of tritium. This technique is uniquely suited to this measurement because it is a technique in which the total energy of the decay products is measured, there are no intervening baffles and collimating apparatus which might interfere with the measurement, it requires only small corrections due to the detector response function, and because at these low energies ($\sim 2 \text{ keV}$) it has a resolution of $\sim 210 \text{ eV}$, which is comparable with other techniques which have been used for similar purposes. We have measured the effect of atomic screening by the tritiated methane molecule on the energy of the outgoing β particle, and carried out a search for a 17 keV neutrino, which was recently postulated by Simpson [1].

The standard Fermi theory was used as to describe the probability distribution

for the energies of β particles in the β decay of 3H nuclei. In our analysis, we have also considered the following effects. The radiative corrections to the theory were found to be negligible at the level of 10^{-3} times the probabilities which result from the simpler theory. The correction normally made in consideration of the screening, by the orbital electron cloud, of the Coulomb field of the nucleus was incorporated by an approximation, due to Rose, commonly known as the Rose Approximation. It involves the reduction of the β particle kinetic energy by an amount equal to the potential energy of the orbital electron cloud at the nucleus, for the purposes of calculating the Coulomb Fermi function for the observable β particle energy distribution. We have predicted the effect of this correction on the β spectrum for 3H atoms. Anticipating an experiment in which the 3H is bound in a molecule of our choosing, we have considered also the additional effects of the molecular environment on the balancing of the decay energy between the products. We have chosen to observe the decay of 3H while it is within the methyl tritium molecule, where the molecular environment contributes substantially to the predictions regarding the final states subsequent to the decay. The effects of this molecular environment on the screening potential were also considered. Our predictions have been influenced by recent calculations published in the literature. We have also considered the possible features in the spectrum which might result from the recently hypothesized mixing of a heavy neutrino with the standard model light electron neutrino.

We have assembled and refined an apparatus for the observation of the β particle energy spectrum for the decay of 3H in methyl tritium. This apparatus employs the gas proportional chamber technique to calorimetrically measure the kinetic energy

of the β particles emanating from methyl tritium. The precision and reliability of this apparatus have been demonstrated. It is capable of recording the spectrum, in whole or in part, depending upon the gas gain which results from the gas pressure and high voltage conditions under which the detector is operated. Electronic noise in the measuring system has been minimized, so that the resolution of the apparatus is nearly at the intrinsic limits of the technique, and behaves in the predicted way across the energies in that portion of the spectrum under study. This behaviour has been confirmed through the use of activated calibration sources, which have facilitated measurement of the detector's resolution as well as other systematic characteristics. Comparisons of the observed 3H spectra with the theory has also been shown to be independent of any possible residual uncertainties in the energy resolution of the apparatus, over the range $\pm 10\%$ of the observed energy resolution.

We have observed the stability of our apparatus over time, and the repeatability of gas conditions from experiment to experiment. The stability has been found to be suitable for our purposes, with gas conditions becoming predictable to within about $\sim \frac{1}{2}\%$ or less by approximately ~ 30 hours after filling our detector with virgin counting gas and the CH_3T which carries the 3H into the experiment. We have determined methods for both ensuring and assuring repeatability from experiment to subsequent experiment.

Activated calibration sources have been used to measure the detector's deviations from linearity in the measurement of the β particle energies. These deviations, for our selected gas gain conditions for this experiment, were observed to be consistent with a space charge non-linearity in the energy axis, which enters the energy

calibration at the level of $2.2 \times 10^{-5} \text{ ch}^{-1} \pm 5\%$ times the linear energy calibration constant. This deviation also enters the normalization of the spectrum at the same fractional level times the normalization constant due to the progressive increase in channel widths which results. We have also observed energy dependent variations in the volume from which β decay events are accepted. An energy dependent increase in the fiducial volume is evident, at the level of $3.7 \times 10^{-5} \text{ ch}^{-1} \pm 7\%$, probably due to mismatches in the anode and cathode signal measurement systems, which are both essential in the discrimination against events in the detector end regions. An energy dependent cyclic systematic error in analog to digital converter channel widths has been observed, which we attribute to the analog to digital converters being used near the limits of their specifications. We have measured this error, and found it to be a concern only at the level of $\sim 0.1\%$ of the individual channel counts, which is below our other systematic and statistical limits. No deterioration in the quality of the fitting of our collected data appears to have resulted from this systematic error.

One concern in this system is the loss of energy by β particles which hit the walls. The apparatus was designed to operate with a small ratio of surface area to gas volume, having a large radius and being run at a high pressure ($\sim 5 \text{ Atm.}$) to keep this effect near the $\sim 0.3\%$ level. This is our predicted *wall effect*. The effect of this contamination on the β spectrum has been simulated, and it is predicted to be small, but not insignificant. The predicted wall effect is equal to 0.18% , or about $\frac{1}{2}\sigma$, near the maximum in the ${}^3\text{H}$ β particle energy spectrum. We have developed a correction to the spectrum to account for this effect, which we apply during our results analysis.

We have collected and studied background contamination of the 3H spectra, which was measured at the level of $\sim 1.6\%$ of our data. The components of the background are well understood, and have been useful in confirming the predictability of the non-linear space charge effects. This background is easily subtracted from our 3H spectra, prior to comparison of our observations with the theory. We have produced 3H spectra, and performed a background subtraction on the collected data. The data has been fit to the theory described, while correcting for the systematic effects previously observed and measured, as well as for the simulated wall effects.

In fitting the resulting spectra to the theory, the screening correction was incorporated through Rose's technique. We have determined that the observed spectrum is consistent with the theory, on the presumption that the wall effects have been underestimated by a factor of about three. We speculate that the underestimate of the wall effects may result from an inadequate treatment of the backscattering of electrons off of the detector walls, and possibly also from the difficulty in accurately modelling electron scattering at very low energies.

Our measured value of the screening potential, due to the final state orbital electron cloud of the daughter 3He atom, is $90 \pm 3 (stat) \pm 5 (sys) eV$ (68% *C.L.*), which is consistent with a high electron occupation probability at the daughter 3He atom. This result is consistent with the calculations of Kaplan and Smutney [29] of an increase in the electron density at the daughter 3He atom in the molecule, and concurs with the observation of Snell and Pleasanton [40] of a neutral He atom in a large fraction ($\sim 82\%$ in their observations) of final states. This result is consistent with the calculations of Kaplan and Smutney [29], Ikuta, Imata, and Imamura [36], and

those of Claxton *et al.* [35], which predict $\sim 62\%$, $\sim 61\%$, and $\sim 59\%$ respectively, for the probability to produce a neutral, ground state, ${}^3\text{He}$ in the products, if we are able to conclude that the CH_3^+ ion is more probably shaken up than is the He atom. Our naive 74 eV screening potential prediction, which arbitrarily assigns the orbital excitation to the ${}^3\text{He}$ electron cloud is just within a 95% confidence level interval below our observed screening potential.

Our fitting of the $\sim 1.1\text{ keV}$ to $\sim 2.2\text{ keV}$ region of the ${}^3\text{H}$ β decay spectrum to the theoretical predictions produces a χ^2 so close to 1 per degree of freedom that there is no suggestion of other anomalous features in this region of the ${}^3\text{H}$ spectrum. Hypothetical anomalies which might deviate by as much as 1σ ($\frac{1}{2}\%$) and might persist over as few as 10 channels of observations are not in evidence. Given our 14.5% FWHM resolution at the ${}^{55}\text{Mn } K_{\alpha 1}$ line, varying as $\frac{1}{\sqrt{E}}$, any monoline anomalies hypothesized in this region, which would deviate in magnitude by 2% or more from the theory, would produce such a 10 channel wide deviation. These are ruled out with 95% confidence.

We have observed no support for the hypothesized heavy neutrino in the final states of this decay. We determine an upper limit on the mixing of such a neutrino, with a mass between 16.4 keV and 17.3 keV , of $\sin^2\theta \leq 3.6 \times 10^{-3}$. We recognize from our own fitting of the data to the theory how the rather high value of the screening potential observed may be misinterpreted as evidence in support of such a heavy neutrino, if the spectrum simultaneously demonstrates a discontinuity in its slope at an appropriate energy. We see no such discontinuity.

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