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.....
UNIVERSITY..... Carleton University
DEGREE FOR WHICH THESIS WAS PRESENTED..... Ph.D.
YEAR THIS DEGREE GRANTED..... 1972

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NL-91 (10-68)

ELASTOPLASTIC SIMULATION
BY NONLINEAR PHOTOVISCOELASTICITY

by

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A thesis submitted to the Faculty of Graduate
Studies in partial fulfilment of the requirements
for the degree of Doctor of Philosophy in
Mechanical Engineering

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July, 1972

The undersigned hereby recommend to the Faculty of Graduate Studies, Carleton University, acceptance of this thesis, "Elastoplastic Simulation by Nonlinear Photoviscoelasticity", submitted by Shiek Mohideen Sathikh, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering.

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ABSTRACT

Two methods of elastoplastic simulation by nonlinear photoviscoelasticity have been explored. In one method named as "permanent strain method", attempt has been made to employ the permanent birefringence that remained in a loaded-and-unloaded specimen of Lexan polycarbonate, to study the plastic stress concentration factor in a biaxial and triaxial stress state.

In another method, "frozen-creep method", epoxy resin specimens under load were subjected to a thermal cycle in order to freeze the mechanical and optical creep. The resulting nonlinear stress-strain-birefringence relations measured on loaded, unloaded and appropriately cut models have been employed to study stress and strain concentration factors in thin and thick models.

The molecular theory of polymer behaviour and the appropriate photomechanical relations are discussed. The techniques have been applied to study the stress and strain concentration factors in a tension strip with a large central circular hole.

Due to certain practical difficulties in resolving the high order of fringes in the yielded Lexan polycarbonate the permanent birefringence could not be utilized as planned. However, use was made of the non-linear viscoelastic behaviour of Lexan in the low strain level to study the stress concentration in a tension strip with a large central circular hole. Qualitative results of elastic plastic boundary development are included.

Quantitative results of stress concentration factors as a function of the degree of plasticity in models of constant thickness, evaluated from the frozen-creep method are found satisfactory, when compared with some published results. The effect of thickness on the stress concentration factor of these models investigated in this study indicates a decrease in the average stress concentration factor with thickness.

A concept on the simplification of circular polariscope design is discussed. The relative accuracies of Tardy and Senarmont methods of compensation have been analyzed. A complete geometric construction method for large strain analysis has been developed.

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to his adviser Professor G.W. Bigg for his guidance throughout this investigation and for suggesting improvement of the manuscripts.

The author is indebted to Professor J.T. Pindera for his continued interest in this work and inspiring suggestions. Thanks are due to Professor J. Kirkhope from whom the author received great encouragement during his stay at Carleton University.

The author enjoyed the company of Mr. G.J. Wilson and Mr. S.K. Bhan and appreciates the helpful discussions he had with them. The help rendered by Mr. K.H. Sayeed in drafting is appreciated.

The author gratefully acknowledges the financial support provided by Carleton University and by the Canadian National Research Council through Grant A3294.

Finally, the author will be forever grateful to his daughters in India, Basheera and Safoora, and his wife Sultan for their patience and understanding during the period of this investigation.

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LIST OF SYMBOLS

A, A_1 etc.	= strain- or stress-optic coefficients
A	= light intensity factor (see Appendices B and C)
B, B_1 etc.	= strain- or stress-optic coefficients
B	= light intensity factor (see Appendices B and C)
b	= actual width of specimen or model
b_o	= initial (nominal) width of specimen or model
C, C_1, C_2 etc.	= photomechanical coefficients
D	= diameter of the circular hole
d	= actual thickness of the specimen or model
d_o	= initial (nominal) thickness of the specimen or model
E	= Youngs modulus (initial tangent modulus)
E_s	= Secant modulus = $0.7E$
F, F_1, F_2 etc.	= functions of time
f, f_1, f_2 etc.	= photomechanical functions
g, g_1, g_2 etc.	= photomechanical functions
I	= intensity of light (Appendix B)
I_o	= intensity of light corresponding to the light field circular polariscope set-up (Appendix B)

K	= material constant
K_{ϵ}^G	= strain concentration factor based on gross area
K_{ϵ}^N	= strain concentration factor based on net area
K_{σ}^G	= stress concentration factor based on gross area
K_{σ}^N	= stress concentration factor based on net area
K_e	= elastic stress (or strain) concentration factor based on gross area
k	= γ_c/γ_d (Appendix A)
l_f, l_i	= final (f) and initial (i) lengths of a line element
N	= fringe order
N_c	= fringe order in creep
N_c	= configurational unit fringe order
N_d	= distortional unit fringe order
N_F	= frozen fringe order
N_i	= instantaneous fringe order
N_R	= room temperature fringe order of epoxy resin
N_T	= total fringe order

n	=	exponent in Ramberg-Osgood stress-strain relation
QWP	=	quarter wave plate
R	=	relative retardation response
R	=	C_2/C_1 (Appendix A)
T, T_1 , etc.	=	temperature
T_{br}	=	brittle point temperature
T_g	=	glass transition or softening temperature
T_{max}	=	maximum temperature of the thermal cycle
T_p	=	plastic temperature
t, t_1 etc.	=	time
u	=	displacement in x direction
v	=	displacement in y direction
x, y	=	directions, coordinates
x_i, y_i, z_i	=	quantities measured or determined (Appendix F)
α	=	any angle measured from ϵ_1 (counter clockwise)
α_1	=	angle between ϵ_{1d} and ϵ_{1c}
γ	=	secondary maximum shear strain
γ_c	=	configurational (orientational) shear strain
γ_d	=	distortional shear strain

γ_e, γ_p	= secondary maximum elastic, plastic strain
γ_F	= frozen shear strain
γ_n^i, γ_r^i	= instantaneous (ⁱ) nonrecoverable (_n) recoverable (_r) shear strains
γ_n^r, γ_r^r	= retarded (^r), nonrecoverable (_n) recoverable (_r) shear strains
γ_R	= room temperature shear strain response of epoxy resin
γ_{yp}	= maximum shear strain at yield point
$\dot{\gamma}$	= time rate of maximum shear strain
δ, δ_1 etc.	= (see Appendices B and C)
$\Delta b_o, \Delta d_o$	= change in width and thickness
Δ	= (see Appendix C)
ϵ	= normal strain
$\epsilon, \epsilon_1, \epsilon_2$	= errors (see Appendix C)
$\epsilon_{1,2}$	= principal strains
ϵ_i	= instantaneous strain
ϵ_{max}	= maximum tangential strain
ϵ_T	= total strain response
$\epsilon_{x,y}$	= Lagrangian strains in x and y directions

θ	= angle measured from the larger principal λ i.e. λ_1 to that of λ_x (Appendix E) (counter clockwise)
λ	= wave length of light source
$\lambda_{1,2}$	= $(1 + \epsilon_{1,2})^2$
$\lambda_{x,y}$	= $(1 + \epsilon_{x,y})^2$
λ^G	= degree of plasticity based on gross area = σ_{av}^G / σ_s
λ^N	= degree of plasticity based on net area = σ_{av}^N / σ_s
ν	= Poisson's ratio (effective)
ν_e	= elastic Poisson's ratio
ν_p	= plastic Poisson's ratio
ρ_1, ρ_2, ρ_3 etc.	= angles measured from polarizer axis (Appendix B)
σ	= normal stress (true stress)
$\sigma_{1,2}$	= principal stress
σ_{av}^G	= average stress based on gross area
σ_{av}^N	= average stress based on net area
σ_B	= limit of low elasticity (stress)
σ_{max}	= maximum tangential stress

σ_o	= nominal stress
σ_s	= hypothetical yield stress corresponding to a secant modulus $E_s = 0.7E$
σ_{yp}	= yield stress
τ	= secondary maximum shear stress
τ_e, τ_p	= secondary maximum shear stress in elastic and plastic phases
ϕ_ϵ, ϕ_n	= angle measured from the axis of larger principal stress to that of strain and refractive index

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CHAPTER I

INTRODUCTION

A. GENERAL

The majority of the structural materials are metals which are largely characterized by time-independent mechanical behaviour at room temperature. Significant interest has been shown in the analysis of stress and strain of parts made of these materials. The term plasticity is in engineering use to denote the study of stress and strain of such materials loaded beyond the elastic limit.

The terms photoplasticity and photoelastoplasticity have frequently been used in the literature. Photoelastoplasticity should strictly mean the study of stress and strain in a prototype mechanical part made of any material under elastoplastic deformation using a model made of birefringent material in a similar situation. In view of the predominant time-dependent behaviour of most polymeric model materials loaded to the stress levels that produce the required non-linear stress-strain curve, the behaviour of the polymer is not strictly elastoplastic. Since this time-dependent behaviour of the polymeric model material is used to study

the elastoplastic response of metallic prototypes, the use of the terms, "elastoplastic simulation by nonlinear photo-viscoelasticity" seems appropriate; however, the term "photoplasticity" will be used hereafter in this dissertation to mean the simulation mentioned above, because of its short form.

The design of mechanical parts aims at achieving reliable performance and satisfactory function of the parts under specific operating conditions. The object of modern design is to optimize that design. Of the several factors which influence the optimum design of load carrying members (Johnson 1961), material, geometry of the parts, the magnitude of forces and the method of applying these forces on the parts are of interest in solid mechanics. All four factors together play important roles in the optimization. Light weight design is one aspect of the optimum design which largely depends on increasing the magnitude of the applied stress, without impeding the function and reliability of the part. For a given material, geometry of the part and method of applying the loads, the maximum loads that can be applied are restricted by the maximum stress, strain or deflection that results from the loads. Where strength is the criterion of a design, the study of stress and strain distribution in general and of maximum stress and strain in

particular is, therefore, of interest to a designer.

When the stress level is within the elastic range, the problem is relatively simple. When the stress level exceeds the elastic range, the problem becomes one of plasticity and its solution is involved due to the complex nature of the plasticity problems.

B. AVAILABLE METHODS

Theoretical and experimental methods have been advanced to solve the plasticity problems. The classical theories of plasticity aim at closed form solutions by the integration of the partial differential equations of equilibrium subjected to various plasticity and boundary conditions. Exact closed form solutions have been possible for simple geometry, loading and boundary conditions. In certain cases, approximate analytical solutions which do not satisfy one or two of equilibrium, compatibility and boundary conditions have been sought (Nadai 1950, Lubahn 1961, Huang 1972). For more complicated problems numerical methods using finite difference or finite element techniques have been found helpful. The solution, in general, may be capable of providing information on both stress and strain distributions. The solution of the complete boundary value problem

is essential in order to interpret the results at any point of interest. Several mathematical postulates underlying these theories require justification through experiment.

On the other hand, the experimental methods aim at solving the plasticity problems completely by experiments. Equilibrium, compatibility and boundary conditions are automatically satisfied. The use of the theory of plasticity in part may be inevitable to get a complete solution. In some cases, where the integration of the equilibrium equation is not required, one type of plasticity condition has been used (Theocaris 1964, Durelli 1970(1)). Where the equilibrium equations are to be integrated, for instance to separate the principal stresses, the process has been simplified by the known boundary stresses (Frocht 1958, Thompson 1963). Special points or sections of interest can be isolated for analysis, not requiring the solution of the complete boundary value problem. Difficulties in the measurement of the quantities of interest in the experimental methods, such as moire and photoelastic fringes and their interpretation should not, however, be overlooked.

Theoretical and experimental methods have their own merits and demerits and go hand in hand in achieving the solutions to plasticity problems. However, when the

geometry of parts is complex, experimental methods have definite advantages, especially when isolated points on load free boundaries only are of interest, such as the points of stress or strain concentrations.

C. SOME EXPERIMENTAL METHODS

Strain gauge, moire, grid, and photoelastic- and stress-coating techniques are a class of experimental methods wherein the prototype material is directly subjected to the specific test conditions. Most of them furnish information on the surface only and are not suitable for three-dimensional analysis; others pose more difficulty.

Model photoelasticity is a simulation method which has proven to be an effective tool to solve elastic problems, including three-dimensional ones. Though complete solutions are possible, the enormous amount of data to be collected and analyzed has restricted the photoelastician to choose a few isolated points or sections of importance for analysis. Its accessibility for three-dimensional analysis together with the possibility of isolating specific points such as those of stress and strain concentration, has interested several investigators to expand its sphere to include the study of stress and strain distribution under plastic .

deformation and the term photoplasticity has come into use to denote this study. Drucker (1962) stated, "Here photoelastic techniques can make a contribution immeasurably beyond those of other experimental techniques and of analytical prediction". In the following section, the principles of two photoplastic techniques to be explored in this investigation are presented.

D. OBJECT AND SCOPE OF THIS INVESTIGATION

The object of the present investigation is to explore the possibilities of further contribution to the elastoplastic study under triaxial stress state using the birefringent property of model materials. Attention is focused on those methods that permit unloading the model before making measurements. This involves the search for a theory of mechanical and optical characteristics of birefringent polymers in conformity with polymer science and the development of experimental methods for the analysis, interpretation and application of the results in the light of finite strains. This in turn rests on the availability of the equipment and the necessary modifications in them to meet an acceptable level of accuracy.

A photomechanical analysis requires a relatively

long time for measurements. From this point of view, methods which permit the unloading of the models before measurements are made on them, will be ideal.

It has been noticed in several polymers that, even in a uniaxially stressed specimen, some strain and birefringence remains permanently, after the load is removed and the viscoelastic recovery is over. This has been found to be predominant in cold-drawable polymers. Exploring the possibility as to whether the permanent birefringence can be used as a measure of the applied stress at least in load free boundaries of a general model forms one part of this study. This method is named as the "permanent strain" method.

A freezing method named here as a "frozen creep" method forms another part of this investigation. In rigid cross linked polymers whose glass transition temperature T_g is much higher than the room temperature, the time dependent response at room temperature is negligibly small compared to the time independent response at low and moderate stress levels. At higher temperatures less than T_g the mechanical and optical creeps even in the moderate stress levels, are accelerated and the nonlinearity in creep response is significant. When cooled to room temperature, a uniaxial specimen under load shows large strain and birefringence; as well it becomes rigid again. On unloading the specimen, there will be an instantaneous recovery. The creep recovery will be so retarded that it could be considered negligible compared to the total response. There will remain

frozen strain and birefringence which are attributed to the orientation of molecular segments. Exploring the possibility of studying the applied stress, at least at load free boundaries, in terms of the frozen birefringence, is the object of the second part of this investigation. This is a modified form of Hunter's (1966, 1968, 1970) freezing method, as will be explained later.

The evaluation of the stress and strain concentrations in a flat tension bar with a large central circular hole is attempted using these methods of photo-plasticity, with the degree of plasticity and the thickness as variables.

For the purposes of comparison and evaluation of the proposed exploratory techniques, the determination of the stress and strain concentration factors by the conventional method, that is, taking all the measurements before unloading the model, is considered important.

E. SOME DEFINITIONS

The stress and strain concentration factors and the degree of plasticity are defined on the basis of the net and gross cross sectional areas, with superscripts N for "net" and G for "gross" areas.

Stress concentration factors:

$$K_{\sigma}^N = \sigma_{\max} / \sigma_{av}^N \quad (1)$$

$$K_{\sigma}^G = \sigma_{\max} / \sigma_{av}^G \quad (2)$$

where $\sigma_{\max}$ = maximum tangential stress on the hole boundary (averaged through the thickness).

$$\sigma_{av}^N = P / (b - D) d_0 \quad (3)$$

$$\sigma_{av}^G = P / b d_0 \quad (4)$$

where P = axial load; b = width, d_0 = thickness of the strip; D = diameter of the hole.

Strain concentration factors:

$$K_{\epsilon}^N = \epsilon_{\max} / \epsilon(\sigma_{av}^N) \quad (5)$$

$$K_{\epsilon}^G = \epsilon_{\max} / \epsilon(\sigma_{av}^G) \quad (6)$$

where $\epsilon_{\max}$ = maximum tangential strain on the hole boundary (averaged through the thickness); $\epsilon(\sigma_{av})$ = the strain corresponding to σ_{av} from a uniaxial calibration.

Degree of plasticity:

$$\lambda^N = \sigma_{av}^N / \sigma_S \quad (7)$$

$$\lambda^G = \sigma_{av}^G / \sigma_S \quad (8)$$

where σ_S = hypothetical yield stress corresponding to a secant modulus $E_S = 0.7 E$, E being the initial tangent modulus of elasticity.

F. ORGANIZATION OF THE DISSERTATION

The theory of photoplasticity and the experimental design are explained in Chapter II. Chapter III deals with the complete exploratory study of this investigation. Results are presented in Chapter IV followed by discussion in Chapter V. Conclusions are drawn in Chapter VI. The appendices deal with the simplified strain-optic relation, polariscope analysis, geometrical construction method for large strain analysis and error analysis.

CHAPTER II

THEORY OF PHOTOPLASTICITY

A. STATE OF THE ART

a. Techniques of Photoelasticity

There has been no unified photoelastic approach to the elastoplastic problem ever reached in the literature. There are a wide variety of materials, methods of mechanical and photomechanical characterization and techniques. A close examination, however, will disclose that a nonlinear viscoelastic response, in fact, has been used to study an elastoplastic problem. Focusing attention on the quantitative applications, the following broad classification of previous work seems appropriate.

(i) Conventional Methods:

- (a) Isochronous stress-strain and stress-difference-birefringence relations from creep tests; with measurements under live load (Fried 1950, Hiltcher 1955, Frocht 1958, 1963, Thompson 1963).
- (b) Stress-strain and strain-difference-birefringence relations from a constant deformation rate test; with measurements under a live load (Fried 1954, Brill 1965, Whitfield 1969, 1972).

- (c) Stress-strain and stress-difference-birefringence relation from a constant deformation rate test; with measurements both under live load as well as on lapse of a specific time after unloading (Nisida 1956).
- (d) Stress-strain isochronous relation from a creep test. Birefringence as sum of linear functions of stress-difference and nonelastic strain-difference from two measurements, one under load and the other immediately after releasing the load (Javornicky 1968(1), (2)).
- (e) Stress-strain relation from a constant deformation rate test; birefringence measured immediately after unloading the specimen or model related to strain and stress state under load. (Morris 1972).

(ii) Dispersion Method:

Stress-strain and stress-difference birefringence isochronous relations from constant deformation rate tests; measurements from creep loaded model; anomalous optical dispersion property of celluloid used to identify and measure plastic deformation (Mönch 1963). Limitations of this method have been pointed out by Pindera (1966).

(iii) Freezing Method:

Frozen strain-applied stress and frozen birefringence-applied stress-difference relations from creep test under a thermal cycle with the maximum temperature significantly lower than the softening temperature; with measurements after unloading (Hunter 1966, 1968, 1970).

Most of these photoplastic investigations were confined to plane stress problems. A beam under pure bending was studied by Fried (1950) and Nisida (1956). A tension strip with a circular hole was investigated by Frocht (1958), Thompson (1963), Fried (1954), Brill (1965), Whitfield (1969, 1972), Javornicky (1968(1), (2)), Mönch (1963), Hunter (1966) and Hiltscher (1955). In addition to varying the degree of plasticity, a few authors varied the size of the hole (Frocht 1958, Thompson 1963, Javornicky 1968(2)). Frocht (1958) and Thompson (1963) changed also the location of the hole. Open notches of different shapes and sizes in a tension strip have been studied (Hiltscher 1955, Thompson 1963, Mönch 1963, Javornicky 1968(2)). Filleted tension strips were

analyzed by Frocht (1958) and Thompson (1963). Hunter (1970) did a three-dimensional analysis of a notched bar under bending (1970). Comparison of results from these investigations have been very scarce because of several variations such as materials, methods, stress-strain relations, geometry, etc. Javornicky (1968(2)) collected and reported some results found in the literature.

Only two groups of investigators attempted three-dimensional photoplastic study. With an interest in the problems of triaxial stress state these methods are briefly discussed here. Frocht and Cheng (1963) extended the two-dimensional photoplastic method (Frocht 1958, Thompson 1963) to a thin tube under the state of triaxial stress. Because of the thin walls, the stresses in the radial direction were neglected and the problem was analyzed as a two-dimensional one. Under various combinations of individual principal stresses, the birefringence, measured five hours after applying the load, was found to be a unique nonlinear function of the secondary principal stress difference and the isoclinic parameter was found to represent the secondary principal stress direction in celluloid up to eight per cent strain. Optical and mechanical creep has been used to simulate elastoplastic response.

No application of this method to a general three-dimensional problem was reported. The work was not followed up, perhaps due to the difficulty of measuring the birefringence in a three-dimensional body under load.

Hunter and associates made use of a freezing method. Loaded uniaxial specimens and the general models made of an epoxy resin were subjected to a thermal cycle whose maximum temperature was significantly lower than the glass transition temperature of the epoxy. On cooling to room temperature, they were unloaded. Some strain and birefringence were found frozen. The applied stress had a nonlinear relation with the frozen-strain and this relation as obtained from uniaxial specimens was used to represent the prototype behaviour. The birefringence that remained in the uniaxial specimen and the general model was interpreted as frozen birefringence in the light of the two phase theory (Kuske, 1957, 1962) and was considered to be a function of the applied stress only. Information as to whether there was any instantaneous recovery of strain and birefringence when the model was unloaded, any change in the birefringence when the model was cut to slices, or any time-dependent changes in the birefringence in the unloaded general models and also in the slices cut for the three-dimensional analysis were not reported.

This implies that the postulate did not consider the possibility of the macroscopic residual stress and the associated birefringence that can result on unloading a general model of this nature.

b. Problems in Similarity

In order to correctly interpret the prototype behaviour from the model results certain similarity conditions should be satisfied (Hetenyi 1950, Mönch 1964, Frocht 1958). True mechanical similarity is ensured if the trajectories of principal stress in the geometrically similar bodies are families of geometrically similar curves in space (Nadai 1950, p. 87). The various factors that effect the similarity requirements are: (1) Geometric (2) Loading and (3) Physical factors.

Geometric factors involve the choice of the right shape and dimensions of the model and can be easily achieved. Due to large deformation of the model, however, the shape will be extensively distorted, violating the model law. Loading factor stipulates the right magnitude and application of the forces. This is relatively easy for most cases, though in certain cases it may be difficult; for instance, simulation of uniform pressure on a complex

boundary of a plane model (Durelli 1965). The physical factors are most difficult to fulfil. They are: (a) the shape of the stress-strain curve, (b) the law of yielding, (c) the law of continued yielding and (d) Poisson's ratio, ν . It is customary to test the similarity of the stress strain curve using the Ramberg-Osgood (1943) non-dimensional form for the stress-strain curve. Exact matching of this curve over a large strain range might not be possible (Fried 1954, Brill 1965, Whitfield 1972). The very definition of yielding of time sensitive materials such as polymers is different requiring the time rate and duration of stress application in addition to the usual stress magnitudes (Perzyna 1966). In view of such complications, a strict similarity of yield criterion has not been considered to be possible. However, for some plastics the law of yielding has been determined. For celluloid, Mönch (1963) reported von Mises criterion to hold; but Ito (1956) and Javornicky (1968(1)) observed Tresca criterion. Whitfield (1972) studied the yield law for Lexan polycarbonate, which fell close to von Mises law. Little is known of the law of continued yielding even for prototype materials (Frocht 1958). Equal values of Poisson's ratio for both the materials can seldom be possible.

In the plastic range, it is customary to assume $\nu = 0.5$ to imply constancy of volume. It has been shown that ν varies with strain (Stang 1946, Smith, T.F.W. 1954, Bahuaud 1968) even when constancy of volume is assumed. This further complicates the problem. The error due to mismatch of Poisson's ratio in photoelasticity has been discussed in literature (Coker 1931/1957, Monch 1964). No information for photoplasticity is known to the writer.

In the light of the various difficulties in satisfying the physical factors, many authors considered only the stress-strain curve similarity as representative of the material factor (Fried 1954, Hunter 1966, 1970, Brill 1965).

c. Review of Elementary Photomechanical Relations

The question as to the cause of birefringence - stress or strain or both - has remained open ever since the phenomenon of artificial double refraction was noticed in strained transparent bodies. The inadequacy of a relation with a single stress-optic or strain-optic constant to characterize the optical response of the materials in the non-elastic range, has led to the use of a variety of photomechanical relations. The more common relations which

make use of relative retardation as the optical response as applied to elastoplastic study to date fall into one of the following types, in their simplest forms.

- (1) $N/d = A_1 \gamma$ (Coker 1931/1957)
- (2) $N/d = A_2 \tau$ (Coker 1931/1957)
- (3) $N/d = A_3 \tau + B_3 \gamma$ (Coker 1931/1957)
- (4) $N/d = A_4 \tau_e + B_4 \tau_p$ (Coker 1931/1957, Kuske 1957)
- (5) $N/d = A_5 \tau + B_5 \gamma_p$ (Javonicky 1963, 1968)
- (6) $N/d = A_6 \gamma_e + B_6 \gamma_p$ (Heywood 1969)
- (7) $N/d = f_7(\tau)$ (Frocht 1958, 1963)
- (8) $N/d = f_8(\gamma)$ (Brill 1965)
- (9) $N/d = f_9(\tau) + g_9(\gamma)$ (Bayoumi 1953)
- (10) $N/d = f_{10}(\gamma_e) + g_{10}(\gamma_p)$ (Tokuoka 1965)

where

- N/d = number of fringes per unit thickness
 τ = secondary maximum shear stress
 τ_e, τ_p = secondary maximum shear stress in elastic and plastic phases

- γ = secondary maximum shear strain
 γ_e, γ_p = secondary maximum elastic and plastic shear strains
 A_i, B_i = photomechanical constants
 f_i, g_i = photomechanical functions.

It should be emphasized that the above are either relations for materials of time-independent behaviour or isochronous relations for materials of time-dependent behaviour well below the glass transition temperature and the assumption of coincidence of stress and strain directions underlie all these simple forms.

Elaborate reviews can be found in the literature (Coker 1931/1957, ch.3, Hetenyi 1958, Pindera 1959, 1971, Frocht 1963, AmbaRoa 1964, Heywood 1969). Newman's linear strain-optic and Maxwell's linear stress-optic relations i.e. forms (1) and (2) are interrelated for small stress and strain. The relation of form (1) was found to hold even in plastic range for some polymers: nylon copolymer (Hetenyi 1952), Polyethylene (Crawford 1951, Fried 1954), Polycarbonate (Gurtman 1965, Whitfield 1969, 1972).

Fried (1950) and Ito (1956) noticed celluloid to yield the form (2) in the inelastic range.

Coker (1931/1957) discussed the mixed relation of form (3) but did not accept it as a fundamental phenomenon. Alfrey (1948, pp. 220-225) justified this "mixed law". On the assumption that the mechanical state is completely defined by the principal stress and strain only and the birefringence is related to this mechanical state, a mixed relation which permits the noncoincidence of stress, strain and optical axes, has been reported (Bayoumi 1953). In its simplest form it reduces to form (3). After dismissing the "mixed law", Coker settled for a two phase theory of form (4). This was followed up by Kuske (1957, 1962). This concept invoked strong controversy, as purely phenomenological and not capable of predicting the behaviour of material on the basis of its structure and contradictory to known facts (Alfrey 1948, Javornicky 1963, Pindera 1968).

Javornicky (1963) favoured form (5), and called it "the double nature of birefringence in plastics" and applied it successfully (1968(1), (2)). Heywood (1969 pp. 28, 119) discussed form (6). For a linear elastic stress-strain relation, forms (4) - (6) become one and the same.

Brinson (1971) presented a relation

$$N/d = C_1 (\sigma - \sigma_{yp}) + C_2 \sigma_{yp}$$

where σ_{yp} is the yield stress and he suggested that it may be helpful in three-dimensional elastoplastic study, in view of its similarity to the linear strain hardening stress-strain relation commonly used in plasticity. It was pointed out (Sathikh 1972(3)) that this is the same as form (6). Brinson (1972) alternatively preferred

$$N/d = A \gamma_{yp} + B\gamma$$

where γ_{yp} is the maximum shear strain at yield point.

Several investigators presented photomechanical results from creep tests. Pindera (1959, 1962) reported that the total birefringence response contains an elastic component and two or more inelastic components as functions of stress. Daniel (1965) found

$$\begin{aligned} N_c(t)/d &= A\sigma + B\varepsilon(t) && \text{(creep)} \\ N_r(t)/d &= B\varepsilon + A\sigma(t) && \text{(relaxation)} \end{aligned}$$

These are essentially of form (3). Other creep studies (Heywood 1969, p. 119, Pindera 1962, 1966) use

$$N(t)/d = \sigma (A + BF(t))$$

If one recognizes $\sigma F(t)$ as representing creep strain, then the above relation reduces to form (5) or (6). Another relation reported for creep (Coker 1957, p. 271, Theocaris 1961, 1962(1)) is

$$N(t)/d = N_i + A F_1(t) + B F_2(t)$$

and

$$\epsilon(t) = \epsilon_i + C F_1(t) + D F_2(t).$$

It is evident that $N_i \sim \epsilon_i$, $A \sim C$ and $B \sim D$ and this relation, therefore, is a three constant strain-optic type. After Bugakov, Ohashi (1965) made use of a mixed function of the form

$$N(t)/d = C(\sigma_1(t) - \sigma_2(t)) + f(\epsilon_1(t) - \epsilon_2(t))$$

which in effect falls under one of the above.

Photomechanical relations in the functional forms are reported in literature either in mathematical formulations

or in graphical representations. Pindera (1971) presented a most general expression for the relative retardation response in creep as

$$N/d = R(\lambda, T, T_g, \tau, \dot{\gamma}, \gamma)$$

where λ is the wave length of light used, T the temperature, T_g the glass transition temperature, τ the maximum shear stress, $\dot{\gamma}$ the time rate of shear strain and γ the maximum shear strain. For a given light source, constant temperature well below T_g , and measurement at a sufficiently long time after application of load when the time effect should be insignificant, this should reduce to

$$N/d = R(\tau, \gamma)$$

If it is assured that the relation between τ and γ in the same test is unique, then

$N/d = R(\tau)$, form (7) as presented by Frocht (1958, 1963) for celluloid or

$N/d = R(\gamma)$, form (8) after Brill (1965) for polycarbonate. Brill concluded that the isoclinic parameter showed principal strain directions while

Gurtman noticed coincidence of the stress-optic axes. Whitfield (1969, 1972) could not solve this controversy. Bayoumi's (1953) general formulation parallels form (9) and Tokuoaka's (1965) results resemble form (10).

Several mathematical developments in photoviscoelasticity and photoelasticity based on the classical field theory can be found in the literature (Mindlin 1949, Kiesling 1966, Fowlkes 1970, Ramaniah 1966, Rivlin 1971). Phenomenological relations based on simplified models relate birefringence with macroscopic stress and strain. More general models relate changes in electron polarizability underlying the birefringence to permanent deformations and stress induced deformation in the molecular level. (Born 1970, Ramachandran 1961).

B. POLYMER SCIENCE APPROACH

a. Mechanical Behaviour of Polymers

Understanding the mechanism of mechanical behaviour of polymers will be helpful to explain the optical behaviour of polymers. The strain response to constant stress as a function of time consists of an instantaneous part and a time-dependent part for almost all polymers in their glassy state. The instantaneous (elastic) response stems from the deformation of bond angles and changes in bond lengths of molecular chains. The time-dependent behaviour is a diffusion process. (Alfrey 1948)

Polymer molecules (under no external load) are flexible, making a wide range of shapes or configurations possible, because of the capability of rotation about single valence bonds. In the linear polymer, most molecular configurations are partially curled, neither tightly curled nor rigidly extended to maximum length. The elementary process in the diffusion of an amorphous polymer is the jumping of chain segments, rather than the whole molecules as in liquids, to new equilibrium positions. There are two resultant effects of this elementary process: (1) macro-Brownian movement i.e. the whole polymer molecule wanders about and (2) micro-Brownian movement i.e. the molecule changes its shape. In the presence of an external stress, these movements are said to be biased.

The stress-biased macro-Brownian movement in a linear polymer is a true flow and will carry the whole molecules in preferred directions in a mechanically and thermodynamically irreversible manner. This is almost absent in cross linked polymers. The stress-biased micro-Brownian movement appears as a retarded elastic response and will, on the average, stretch and orient the polymer molecules in certain directions. On removal of the stress, they will return to the original equilibrium distribution

and any macroscopic strain resulting from the stretching and orientation of the chains will disappear. This process is reversible mechanically though not thermodynamically; cold-drawable polymers are an exception and will be explained later. The relative dominance of the three parts of the strain response depends upon the stress, time and temperature (Alfrey 1948 pp. 93-104, Staverman 1956, pp. 51-52).

Far below the brittle point temperature T_{br} (Figure 1a), the Brownian movement is essentially "frozen-in" and only the instantaneous response is significant. In the rubbery state, i.e. above the glass transition temperature T_g , however, the biased-micro-Brownian movement also responds instantaneously.

Below a temperature T_p and a stress level σ_p , there is no true flow (Bartenev 1968, Staverman 1956). T_p is generally very much higher than the glass transition point T_g (Figure 1a). In the temperature range $T_{br} - T_g$, only the biased micro-Brownian movement appears with time. At low stress levels the response is very slow. If the stress level reaches σ_p , called the "limit of low elasticity", "forced viscoelastic response" (cold flow) takes place. The uniaxially stressed specimen necks at this point (Figure 1b). It is known that the cold drawn polymer will

return to its original shape when annealed at a temperature near T_g . It has been shown in literature that the cause of the apparent mechanically irreversible response, interpreted often as plastic strain (Brinson 1971, Whitfield 1972), is the same molecular mechanism as that of the retarded elastic response i.e. micro-Brownian movement (Stein 1956, Staverman 1956, p. 160, Nielsen 1965, pp. 106-107).

The strain is the macroscopic result of the accumulation of the geometric changes in the molecular segments. In the glassy state well below T_g , the total strain response of an amorphous polymer at a specific time after applying a constant stress can be expressed as

$$\gamma = f(\tau; t = t_1; T = T_1) = \gamma_d + \gamma_c \quad (9)$$

where γ_d and γ_c are respectively the distortional and configurational strains.

Since the configurational response may have retarded recoverable and apparent nonrecoverable parts and the latter may contain instantaneous and retarded parts, the total strain response from a phenomenological viewpoint can be written as

$$\gamma = \gamma_r^i + \gamma_r^r + \gamma_n^i + \gamma_n^r \quad (10)$$

where superscripts *i* and *r* denote instantaneous and retarded; subscripts *r* and *n* denote recoverable and non-recoverable components (Figure 2). In the above expression

$$\gamma_d = \gamma_r^i \quad (10a)$$

and

$$\gamma_c = \gamma_r^r + \gamma_n^i + \gamma_n^r$$

The viscoelastic strain response to stress as a function of time has been studied in literature. For a one-step stress history (Drescher 1970) $\sigma(t) = 0$, $t \leq 0$; $\sigma(t) = \sigma_0 = \text{constant}$, $t > 0$, the strain response is expressed as

$$\frac{\epsilon(t, \sigma_0)}{\sigma_0} = J_1(t) + J_2(t, t) \sigma_0 + \dots + J_i(t, \dots, t) \sigma_0^{i-1} + \dots$$

For linear viscoelastic material, the above expression for creep compliance should be independent of stress level; then only the first term is relevant (Flügge 1967).

For time independent materials, J_1 , J_2 etc. are constants and the strain response is, in general, nonlinear with stress. When the time $t = t_1 = \text{constant}$ for a nonlinear viscoelastic material, the isochronous strain response to stress is nonlinear. This nonlinear isochronous strain response to stress contains the distortional and the configurational parts in it.

b. Basis of Birefringence

Polymer science relates the birefringence in a polymer in the glassy state to the instantaneous distortion of molecular bonds and to the orientation of the molecular segments brought about by the micro-Brownian movement discussed in the previous section. The total birefringence response to stress can be expressed as

$$N/d = N_d + N_c \quad (11)$$

The signs of N_d and N_c may be the same or opposite depending on material, temperature, stress level, time of observation, etc. (Pindera 1966).

The distortional part N_d has been associated with the applied stress (Javornicky 1963, 1968(1)) or more correctly with the instantaneous recoverable strain (Alfrey 1949, p. 224). Therefore

$$N_d = f(\tau) \quad \text{or} \quad g_1(\gamma_d)$$

Experiments have shown that the orientation birefringence N_c , frequently known as configurational birefringence can be attributed to the associated strain (Heller 1948, Alfrey 1949, Crawford 1951, Gurnee 1954, Andrews 1957). Therefore

$$N_c = g_2(\gamma_c)$$

$$\text{and} \quad N/d = g_1(\gamma_d) + g_2(\gamma_c) \quad (12)$$

The distortional strain γ_d and the birefringence N_d are known to respond linearly to the stress (Pindera 1966, Nielsen 1965, p. 65, Alfrey 1949, p. 173). Therefore N_d can be expressed as

$$N_d = C_1 \gamma_d$$

or

$$N_d = A_1 \tau$$

and

$$N/d = C_1 \gamma_d + g_2 (\gamma_c) \quad (13)$$

Müller (Stein 1956, pp. 115-116) has derived theoretically a nonlinear relation between the configurational birefringence and the corresponding strain as

$$N_c = A\gamma_c + B\gamma_c^2 + C\gamma_c^3 + \dots$$

Alfrey (1949) suggested a form

$$N_c = A\gamma_1 + B\gamma_2 + C\gamma_3 + \dots$$

where γ_i 's are components of the configurational strain, whose relative dominance depends on the stress magnitude, duration of application of stress, temperature, etc.

At least for cold-drawable polymers it has been reported that the orientation birefringence is linearly related to the corresponding strain (Nielsen 1965, pp. 238-240). For such cases

$$N/d = C_1 \gamma_d + C_2 \gamma_c \quad (14)$$

Pindera (1959, 1962, 1966, 1971) observed that more than one component of the configurational birefringence would be required to characterize certain polymers.

Photoviscoelastic response as a function of the history of strain tensor has been discussed in literature. The simplest linear relation between the refractive index tensor $N_{ij}(t)$ and the strain tensor $\epsilon_{ij}(t)$ has been expressed as

$$N_{ij}(t) = N_{ij}^0 + a_{ijrs} \epsilon_{rs}(t) + \int_0^t A_{ijrs}(t, \tau) \epsilon_{rs}(\tau) d\tau$$

where t and τ are present and past time. (Dill 1964).

Even in the linear case, the birefringence response may be positive or negative, depending on the material, temperature, stress level, time of observation, etc. Also change in the ratio of principal stresses even in the linear viscoelastic material may introduce noncoincidence of stress, strain and optical axes (Kiesling 1969). The isochronous optical response, in general, can be considered as a nonlinear function of strain consisting of the distortional and orientational birefringence (Dill 1964).

c. Coincidence of stress, strain, optic axes

In inelastic range, the stress and strain axes may not coincide in all points of the strained body (Nadai 1950). In general, the coincidence of the optic axis with any of the above two axes may not exist except in special cases (Mindlin 1949, Bayoumi 1953, Tokuoka 1965, Kiesling 1969, Fowlkes 1970). The isoclinic parameter representing the principal optic direction, therefore, does not represent the stress or strain directions. Under this circumstance the photomechanical relations should also include ϕ_n and ϕ_ϵ where

ϕ_n = angle measured from the axis of the larger principal stress to that of refractive index and

ϕ_ϵ = angle measured from the axis of the larger principal axis to that of strain.

Equation (12) should be modified to include ϕ_n and ϕ_ϵ :

$$N/d = g_1(\gamma_d, \phi_n, \phi_\epsilon) + g_2(\gamma_d, \phi_n, \phi_\epsilon) \quad (15)$$

Experimental evidences are few and are contradictory with each other (Bayoumi 1953, Frocht 1963 & cross references, Brill 1965, Whitfield 1969)

C. EXPERIMENTAL DESIGN

a. Choice of Photomechanical Relation for this Study

Unfortunately the case of noncoincidence of the principal axes of stress, strain and refractive index, even for a linear viscoelastic material is complex. For a simple stain-optic relation this is illustrated in Appendix A. In view of this complex nature, literatures have suggested to confine

the analysis to points where coincidence could be ensured (Kiesling 1966, Fowlkes 1970). Therefore, it is proposed here to consider only such a case for application study. For this purpose equation (12) is sufficient. It is shown in Appendix A how the noncoincidence can be explained for a simple case of equation (14).

It was seen that the configurational strain can be divided into three components (equation 10a) and therefore for such cases, the total birefringence is expressible as

$$N/d = g_1(\gamma_d) + g_2(\gamma_r^r, \gamma_n^i, \gamma_n^r) \quad (16)$$

A relation for total birefringence in the form

$$N/d = N_d + N_r^r + N_n$$

has been studied by Pindera (1962)

where

N_d = distortional birefringence

N_r^r = retarded recoverable component of
configurational birefringence

N_n = nonrecoverable component of configurational
strain.

Whether there exists a relation of the type

$$N/d = g_1(\gamma_d) + g_2(\gamma_r^r) + g_3(\gamma_n^i) + g_4(\gamma_n^r) \quad (17)$$

is of interest in this study in addition to the one already mentioned:

$$N/d = g_1(\gamma_d) + g_2(\gamma_c) \quad (12)$$

It seems appropriate to quote Kiesling (1966, p. 27): The results of this (his) study indicate that birefringence is, in fact, more closely related to strains than to stress.

b. Principle of Characterization

With the object of developing photoplasticity methods wherein unloading the model before taking the measurements will be convenient, the following two techniques were chosen: (1) the permanent strain method and (2) the frozen-creep method.

(1) Permanent strain method

It was seen that the total strain response to stress, measured at a convenient time after the application

of load can be expressed as

$$\gamma(\tau) = \gamma_r^i + \gamma_r^r + \gamma_n^i + \gamma_n^r \quad (10)$$

Similarly the birefringence to be studied can be expressed as (Bigg 1971)

$$N/d = g_1(\gamma_r^i) + g_2(\gamma_r^r) + g_3(\gamma_n^i) + g_4(\gamma_n^r) \quad (17)$$

The following principle will be used to identify these parts.

A load is applied on a uniaxial specimen at time $t = 0$ and is maintained. Readings of strain and birefringence are made at $t = 0$. After the lapse of enough time, $(t=t_1)$, when the creep increment is insignificant relative to the total response, a second set of readings is taken. The specimen is unloaded and a third set of readings follows immediately. The specimen is permitted to recover for a long enough time until the creep decrement is small compared to the total response and the fourth set of readings is taken.

$$\text{First reading:} \quad \gamma_r^i + \gamma_n^i$$

$$\text{Second reading:} \quad \gamma_r^i + \gamma_n^i + \gamma_r^r + \gamma_n^r$$

Third reading: $\gamma_n^i + \gamma_r^r + \gamma_n^r$

Fourth reading: $\gamma_n^i + \gamma_n^r$

These are sufficient to separate the four parts of strain. Similarly the four parts of birefringence can be separated.

The interest lies in studying whether the final reading of permanent birefringence could be used to study the strain and stress concentration in a general model.

(2) Frozen-creep Method

This method is different from the conventional frozen-stress technique of three-dimensional photoelasticity in that the maximum temperature of the thermal cycle in this method is significantly lower than the so-called critical temperature which is defined in this thesis as the glass transition temperature T_g . It is known that the orientational strain and strain can be easily locked in by hot stretching (Andrews 1957). The higher temperature is utilized here only to accelerate the creep under moderate load. If the specimen under load is subjected to this thermal cycle, the total birefringence when cooled to the room temperature can be expressed as

$$N/d = g_1(\gamma_d) + g_2(\gamma_c) \quad (12)$$

When a rigid cross-linked polymer such as an epoxy resin whose glass transition temperature T_g is much higher

than the room temperature, is subjected under load, to a thermal cycle with the maximum temperature of the cycle $T_{\max}$ significantly lower than T_g , the creep response, $\gamma(\tau) = \gamma_d + \gamma_c$ has been found to be nonlinear with τ . If unloaded at room temperature after the thermal cycle, an instantaneous change in strain and birefringence occurs. The creep recovery of the remaining strain and birefringence is insignificant relative to the total response. The modified forms of strain and birefringence responses are expressed as

$$\gamma(\tau) = \gamma_R + \gamma_F \quad (18)$$

$$N/d = N_R + N_F \quad (19)$$

where subscripts R and F denote "Room temperature" and "Frozen". The interest lies in seeking an expression of the form

$$N/d = g_1(\gamma_R) + g_2(\gamma_F) \quad (20)$$

and in the application of N_F to study strain and stress concentrations in a general model.

The principle of calibration is that a constant load is applied on a uniaxial specimen which is subjected

to the thermal cycle. The strain and birefringence are measured under load, at the end of the cycle at room temperature. The specimen is unloaded and another set of readings is taken. These two sets of readings are sufficient to establish g_1 and g_2 .

Though the method appears to be the same as Hunter's (1966), it will be shown in the following section that the interpretation of results for a general model is different.

c. Theory of the Photoplastic Applications

(1) Permanent strain method

It was seen that the interest lies in interpreting the permanent birefringence in terms of the corresponding strain and the applied stress. If a uniaxial calibration established the relation among the above quantities, would it be directly applicable to the general model? It was already mentioned that the analysis will be confined to load free boundaries. If the principle of calibration mentioned before, is followed for the general model, the third step is to unload. If a uniaxially stressed specimen is unloaded even after reaching nonlinear deformation, there should

be no macroscopic residual stress. However, if a general model in which the stress distribution was not homogeneous, is unloaded after nonlinear deformation, a time-dependent macroscopic residual stress field is possible. The resulting birefringence not only represents the applied stress but also the macroscopic residual stress. Javornicky (1968(1), (2), 1971) suggested that on load free boundaries where the stress state is uniaxial or nearly so, the residual stress could be relieved with suitable cuts. This would make the application of a uniaxial calibration possible at that point after the recovery has been permitted. This is the basic approach to a problem in which the point of interest lies on a load free boundary as applied in this investigation.

(2) Frozen-creep method

All the arguments of the residual stress of the preceding sub-section equally applies to this method. One advantage is that the rigid cross-linked polymer has negligibly small time-dependent response at room temperature at moderate stress levels.

One difference between the present technique and Hunter's lies in that the existence of a macroscopic residual stress field in the unloaded general model is considered in this study. This necessitates the removal of the macroscopic residual stress in the general model in order to apply the uniaxial calibration results to it. Another difference is that in the present study, the total strain is chosen to represent the material behaviour for purposes of transition, whereas Hunter used the frozen strain.

(3) Conventional method

For purposes of comparing the results from the unloaded model with those of conventional techniques i.e. results from the total birefringence from the loaded model, measurements are also to be recorded before unloading the model.

d. Choice of Application Models

A problem which has a point of interest on a load free boundary with nearly uniaxial stress state is of interest in view of the limitations of the methods arising from the non-coincidence of principal axes and

the residual stress resulting from unloading. Since this is an exploratory study, a tension strip with a large central circular hole was chosen. On the basis that some results for comparison are available, width/hole ratio of two ($b/D = 2.0$) was preferred. In addition to the usual variable of degree of plasticity it was proposed to vary the ratio of thickness to the diameter of the hole. With the increase in the ratio d/D , the problem approaches a three-dimensional one. The stress and strain concentration factors averaged through the thickness, at the point lying on the inner boundary of the hole and the narrow cross section, are of interest.

In the following chapter the experimental details and the method of analysis of the exploratory study are presented.

CHAPTER III

EXPLORATORY STUDIES

A. DETAILS OF EXPERIMENTS

1. Permanent Strain Method

a. Preliminary

i) Material choice and preparation

"Lexan" polycarbonate has been chosen in this study of photoplasticity, since in addition to the good photoelastic qualities, it shows a striking similarity of stress-strain curve to that of mild steel and it is capable of cold drawn and retaining permanent strain and birefringence. As received, this polymer had significant residual fringes: about an order of one in $\frac{1}{4}$ inch sheet; four in $\frac{1}{16}$ inch sheet and thirteen in $\frac{1}{32}$ inch sheet (Figure 3). Various annealing procedures were found in literature (Brill 1965, Hunter 1966, Cloud 1969, Whitfield 1969). Trials showed Hunter's annealing method as satisfactory. The sheet was cut into sizes with large allowance for shrinkage, as much as 40 per cent. The strips were

placed on flat aluminum plates coated with Dow Chemical DC-7 mould release compound. This plate in turn was placed in a furnace whose temperature could be controlled by a programmed cam within $\pm 1^{\circ}\text{F}$ (Figure 4). The temperature was raised at a constant rate of 3.5°F per hour to 325°F . This temperature was held constant for five hours before cooled to room temperature at the rate of 3.5°F per hour. The $1/32$ inch sheets so badly curled (Figure 3) that they could not be used.

The annealed sheets were cleaned with carbon-tetrachloride (CCl_4) as acetone damaged the surface.

ii) Intended tests and quantities to be measured

In order to find the range of load carrying capacity and the extent of time-dependent mechanical behaviour of Lexan, constant elongation rate and relaxation tests on uniaxial tensile specimens were run. A series of uniaxial creep-recovery tests as required for photomechanical characterization explained in the preceding chapter, followed. Application models with various degrees of plasticity and different thickness were tested.

The quantities of interest are stress, strain and birefringence. Stress cannot be directly measured; the average stress is calculated from the load and the cross sectional dimensions. Determination of strain involves the measurement of changes in dimensions due to stress. Birefringence measurement requires the determination of order of the fringes and their location. By virtue of the present method of characterization, it was necessary to record simultaneously the informations on strain and birefringence.

b. Equipment

The availability of equipment was one factor in the choice of the measurement techniques and the size and shape of the specimens and models.

Constant deformation rate tests and relaxation tests were conducted in Instron testing machine (Figures 5 and 6), which was also used for photomechanical calibration tests, below 9000 psi. This machine in its present form cannot simultaneously maintain constant load and record the deformation. Dead weight loads only were, therefore, used in the calibration tests. Instron strain-gauge extensometer of 0.5 inch gauge length can measure up to 50 per cent strain.

The electronic amplification provides a least count of 0.00005 inch on a 0.1 inch division of the strip chart up to a strain of 10 per cent. The creep deformation can be recorded as a function of time on the strip chart. Also load deformation curves can be plotted.

In the range of stress higher than 9000 psi, it was feared that the large cold flow might damage the extensometer. For this range, the loading frame of the Riken polariscope was used (Figures 7 and 8). Photographs of grids taken before and after the deformations provide information on strain.

Grids were scribed mechanically. A precise coordinatograph (Figure 9) has the following specifications according to the manufacturer: Plotting accuracy 0.0015 inch for the greatest distance (47.25 inches) in X and Y directions i.e. 0.00003 inch per inch. The least count of the vernier is 0.001 inch. Spring pressure on the steel pin could be adjusted to get the desired depth of lines.

Grid dimensions were measured from the photographic negatives in a Nikon profile projector (Figure 10). It has a 10X magnification with a least count of 0.0001 inch for the X-Y stage and 1/30 degree for the goniometer.

An Asahi Pentax 35 m.m. Spotmatic camera was used with the close-up lens, which provided pictures up to life size of important regions. According to the need, Kodak Tri X-pan, Plus X-pan and High Contrast Copy films were chosen.

The polariscope mostly used was assembled from the components of the Riken polariscope (Figures 5, 6, 7, and 8) of 300 m.m. field. A He-Ne laser source of 5 m.w ($\lambda = 6328 \text{ \AA}$) was employed to provide diffused light. The diffused light polariscope was preferred for the following reasons: the set-up needed to be portable between two laboratories and hence should have access for easy assembling which is not possible in collimated systems because of the precise location and alignment of optical elements required. Formation of good images in the collimated system requires precision optical elements when used with highly coherent sources. In a diffused light polariscope, the surfaces of the object need not be highly polished. When large deformations occur, the surfaces of the plastic become dull and the diffused light polariscope is very convenient for this purpose.

At low stress levels where strains were measured by strain-gauge extensometer and the number of fringes were low, close-up photographs were not necessary. Then the camera was located away from the specimen or model. This permitted the use of the bulky analyzer and analyzer-QWP. In this set-up the QWP is fixed in space which limited the use of the Tardy compensation method only to those points with principal axes horizontal or vertical. Calibration specimens fell into this category.

Two arrangements of polariscope design have been analyzed in Appendix C. Since the QWP's used were designed for green spectrum ($\lambda = 5461 \text{ \AA}$) and the light source had a $\lambda = 6328 \text{ \AA}$, only the conventional one is applicable for this study. In the conventional set-up, there would be no error in the integral fringe order due to the imperfect but balanced QWP's, as explained in Appendix C. However, correction in the fractional fringe order was necessary which was achieved using the analysis shown in Appendix D.

Whenever close-up photographs were necessary, the objective of the camera needed to be located very close to the model. The bulky analyzer-analyzer-QWP

unit was replaced by a miniature unit fitted on to the objective of the camera. This set-up with the adjustable bellows served as a telescopic polariscope, permitting close-up and full view photographs of the models in quick succession.

Smooth loading was achieved by resting the dead weights on the cross-head or a hydraulic jack and allowing them to gradually release the load.

The thickness was measured by a dial gauge of 0.0001 inch least count and the width by a vernier caliper of 0.001 least count.

The dead weights used were measured in the Instron load cells which could be calibrated with standard weights available.

c. Preparation of Specimen and Models

The configuration of the Lexan calibration specimen was as shown in Figure 11. Because the strain-gauge extensometer was used to measure the deformation, uniform section was chosen. The specimens were fabricated in the standard procedure; first stuck to a template and cut over size in a saw; machined in a high speed router (Figure 12) at steps of 0.01 inch.

The edges were smoothened with sand paper. All specimens were of 1/16 inch thick. The holes for fixtures were drilled. The fine square grids of pitch 0.05, 0.025 or 0.02 inch were scribed by means of the coordinatograph. The thickness and width were measured. The fixtures were clamped at each end.

Similarly the application models (Figure 13) were fabricated taking more care in finishing the central circular holes. All models were of 1/16 inch thickness except one of 1/4 inch thickness. The other procedures were same as for a specimen. The diameter of hole was measured by the vernier caliper and cross-checked in the profile projector.

d. Test Procedures

i) Mechanical tests

Six specimens, three at a cross head speed of 0.02 inch/minute and three at 0.2 inch/minute were tested. In each group, the cross head was stopped just at yielding in one case, just after yielding in another and about 60 per cent elongation in the third. The specimens were permitted to relax. The load-deformation curves and the load-time curves were recorded in the

strip chart of the Instron machine. The standard procedure of a tensile test was followed, including the calibration of the load cell of 1000 pound range by the dead weights provided by the manufacturer.

ii) Photomechanical calibration

One set of specimens below 9000 psi stress, were tested in the Instron machine, the other set in the Riken polariscope loading frame.

In the former strain recording was made on strip chart. First the extensometer was calibrated, by fitting a telescopic rod between the cross heads and separating them by known values which could be read from the graduated dial. Repeated trials with different amplifications were employed to calibrate the extensometer.

The prepared Lexan specimen was hung from the upper cross head. Initial fringes were checked and measured by Tardy method. The loading pan was hung from the bottom end of the specimen and was supported on the lower cross head. The calculated loads were added. The camera was fixed in a position so that the two extreme positions of the bellows would give full view and a close-up of the specimen. With the polariscope set for

light field, photographs in these two positions were taken.

The camera was set for full view and the polariscope for dark field. The extensometer was clamped on the specimen. The pen was balanced and the chart started. The load was gradually applied by lowering the cross head at the rate of 2 inch/minute. At about 20 seconds from the beginning of lowering the cross head, a photograph was taken to record the integral fringe orders. The fractional fringe order was measured by Tardy method. A light field full view and a light and dark field close-ups were photographed. The load was left for 8 hours; at irregular intervals, several full view photographs were taken and fractional fringe order measured.

At the end of 8 hours, photographs were taken and the fractional fringe order measured. The load was gradually removed by raising the lower cross head. Again the specimen was photographed and fractional fringe order measured. The specimen was allowed to recover for 18 hours before photographed again; also the fringe order, if any, was measured. This completed the test. This was repeated for several loads with different specimen for each load.

The same procedure was followed when specimens were tested in the Riken polariscope loading frame. The two exceptions were that no extensometer was employed and load was applied and removed through a hydraulic jack. When the neck started in the specimen, a precalculated amount of load was removed to avoid the fracture of the specimen.

iii) Applications

All the application models were tested in the Riken polariscope loading frame. Following the suggestions from the literature (Frocht 1958, Thompson 1963), the full load was applied over 30 minutes with ten approximately equal steps. This was necessary to avoid unduly larger strain that might result due to larger stress that would be developed at the beginning almost according to an elastic distribution. The stepped load permitted gradual redistribution of stress and prevented high concentration of stress at the inner boundary of the hole.

It was proposed to make radial cuts at the narrow cross section in order to relieve the residual stress, when the model was unloaded; but it was given up for reasons explained in the chapter on discussion of the results.

2. Frozen-creep Method

a. Preliminary

i) Material, choice and preparation

A rigid cross-linked polymer was essential for this method. The epoxy resin commonly known as epoxy (standard Hex) developed by Leven (1963) and noted as a good three dimensional photoelastic plastic for freezing method was chosen for this study. This is Bakelite ERL2774, 100 parts by weight cured by 20 parts of hexahydrophthalic and 42 parts of phthalic anhydride, with the glass transition at 338°F . Its preparation is standard and has been perfected in this laboratory (Bhan 1969, Pal 1971). A block of 8 inches diameter and 14 inches long was cast and cured. Discs of 0.05 inch and 0.25 inch thick were machined from the block taking precautions (Pal 1971). They were cut to $1\frac{1}{2}$ inch x 7 inch pieces.

ii) Intended tests and quantities to be measured

Tests to check the time-independent behaviour of the epoxy and to evaluate the modulus of elasticity at room temperature before and after the specimen was subjected to the thermal cycle, were planned. For this

purpose a few uniaxial specimens with uniform cross section were chosen for testing under dead weight loads. The same specimens were intended to be used for exploring the maximum temperature of the thermal cycle and the maximum stress that can be applied.

Based on the informations of these tests, four calibration specimens of tapered configuration were to be tested. Then tests on application models for studying the effects of degree of plasticity and effect of the thickness on the average stress concentration factors were to follow.

Stress, strain and the birefringence were the quantities of interest. In order to evaluate them, the following measurements were necessary: the applied load, the dimensions of the specimens before and after the deformations, the integral and fractional fringes and their location. With the data made available by the above tests, the evaluation of the Poisson's ratio also was planned.

b. Equipment

All the equipment explained before under the same heading for the permanent strain method was pertinent

to this method. The Instron machine was used only to evaluate the modulus of elasticity of the epoxy at room temperature. It was found convenient to measure the frozen strain and the frozen birefringence directly from the unloaded specimens using the Nikon profile projector polariscope (Figure 10) which had been fabricated from standard components in this laboratory (Khalil 1967). This has also been designed for white and green light.

c. Preparation of Specimens and Models

The calibration specimen had a tapered configuration (Figures 14 and 15). This was chosen after Pindera, since it was capable of providing several points in the stress-strain relation from a single specimen. All the four specimens were of 0.05 inch thickness and were fabricated from the strips in the standard way. The specimens for the trial tests were of uniform sections.

Totally 8 models were prepared for application study. First 5 were of 0.05 inch thickness with configuration as shown in Figures 16a and 17. They were for studying the stress-strain concentration factors with the degree of plasticity as variable. Of the remaining three, one was of configuration of

Figure 16a, but of 0.25 inch thickness. The other two were of the configuration of Figure 16b, one 0.05 inch thick and the other 0.25 inch thick. For all these models, $b/D = 2$. All the first five had $d_o/D = 0.1$. The other three had $d_o/D = 0.2, 0.5$ and 1.0 . The last three together with one of the first five provided four models (Figure 18) to study the effect of thickness variation on stress concentration factor.

Square grids of 0.02 inch sides were scribed on the specimens and the models. The end fixtures were a pair of steel plates, with three holes, stuck to each end surface by pressure sensitive tapes and held by two bolts so that the load can be applied through a pin in the third hole.

d. Test Procedure

i) Trial tests

Uniaxial specimens of uniform cross-section were tested adding dead weight and recording the deformation in the strip chart using the Instron strain gauge extensometer. Loads were added in approximately ten equal steps up to a maximum stress of 4000 psi at room temperature. The exact values of the loads were

determined using the Instron load cells. Between each stepload loading, 30 minutes were allowed to check the amount of the time-dependent behaviour. The average value of the modulus was considered for the application.

Three unloaded specimens were taken to the furnace where they were loaded to three stress levels 1000, 2000, and 4000 psi. The thermal cycle was chosen so that the heating and cooling rates were $4^{\circ}\text{F}/\text{hour}$ with no dwell at the maximum temperature of 200°F . Further three specimens with 1000, 2000 and 3000 psi with $T_{\text{max}} = 250^{\circ}\text{F}$ were tried. The ones stressed to 3000 psi and 4000 psi did not survive. The remaining ones were again tested at room temperature for their time-dependent behaviour and for evaluating the elastic modulus as before. The thermal cycle shown in Figure 19 was chosen for future tests. This had a $T_{\text{max}} = 250^{\circ}\text{F}$. The maximum stress for future tests were decided to be up to 2300 psi.

ii) Photomechanical calibration

The four tapered specimens were vertically hung from rods supported on the furnace wall. The polariscope was set up near the furnace. The laser light beam was passed through the polarizer-QWP unit,

magnified and reflected through two vertical mirrors on to the ground glass placed at the back of the specimen. The initial fringe, if any, was checked and the specimen photographed. The polariscope set up was removed. The specimens were loaded and subjected to the thermal cycle. (Figure 4). The maximum stresses were about 1000, 1500, 2000 and 2200 psi. After cooled to room temperature, the polariscope was set up again. One of the specimens was photographed and the maximum fringe order measured. This specimen was unloaded and taken to the Riken polariscope. The load was again applied and the fringe order measured and the specimen photographed. Since the re-loading showed no difference in fringe order or strain from that of the loaded specimen, the remaining ones were unloaded, removed from the furnace, reloaded in Riken polariscope and measurements and photographs taken. These informations were considered to represent the conditions under the load. The difference in the magnifications was corrected by choosing the distance between the two sharp corners of the end fixture as reference.

It was found convenient to measure the frozen strain and birefringence from the unloaded specimens directly in the Nikon profile projector polariscope.

iii) Applications

All the procedures described for the calibration specimens were pertinent to the models. After the measurement of birefringence from the loaded and the unloaded models and their photographing were over, two models were cut along the minimum cross-section in order to study the change, if any, in the fringe order at the point of stress concentration due to possible macroscopic residual stress having been relieved. Birefringence at this point of interest was again measured.

Figure 20 shows the models set inside the furnace for the thermal cycle.

B. SOME DEFINITIONS AND PRINCIPLES OF COMPUTATION OF RESULTS

a) Nominal Stress

Based on the nominal initial dimensions of the specimens the nominal stress σ_0 was computed.

$$\sigma_0 = P/b_0 d_0 \quad (21)$$

where

P = applied load

b_0 = initial width of specimen

d_0 = initial thickness of specimen

b) Uniaxial Strains

The engineering i.e. Lagrangian definition of strain was followed.

$$\epsilon = \left(\frac{l_f - l_i}{l_i} \right) \Big|_{l_i \rightarrow 0} \quad (22)$$

where l_i and l_f are the initial and final length of a line element. This equally holds for small and large uniaxial strains.

The strain measured at about 20 seconds after the application of the load on polycarbonate specimens is denoted as ϵ_i and that measured at 8 hours as ϵ_T .

The strain in epoxy at room temperature was called as ϵ_R , that frozen as ϵ_F and the total strain as ϵ_T .

c) Nominal Unit Fringe Order

The fringe order N and the nominal thickness d_0 were employed to define the nominal birefringence as N/d_0 corresponding to the chosen light source.

d) True Stress

The true stress was calculated for polycarbonate by using the reduced cross-sectional area measured after unloading,

$$\sigma = P/bd$$

For the epoxy a true cross-sectional area was calculated in the following way. The true area

$$\begin{aligned} bd &= (b_o - \Delta b_o)(d_o - \Delta d_o) \\ \text{or} \quad &= b_o d_o (1 - \nu \epsilon_1)^2 \end{aligned} \quad (23)$$

where ϵ_1 = longitudinal total strain and
 ν = effective Poisson's ratio (Nadai 1951)

$$\nu = \frac{\nu_e \epsilon_{1R} + \nu_p \epsilon_{1F}}{\epsilon_1} \quad (24)$$

where $\nu_e = 0.35$ the elastic Poisson's ratio, obtained from literature (Durelli 1965) and ν_p = plastic Poisson's ratio. For the latter a constancy of volume was assumed. An approximate expression for ν_p with respect to large Lagrangian strain was adopted from literature (Smith, T.F.W. 1954)

$$\nu_p = 0.5 - 0.35 \epsilon_1 \quad (25)$$

The validity of this expression has been experimentally verified (Figure 38)

The true stress $\sigma = P/bd$

$$= \frac{\sigma_o}{(1 - \nu \epsilon_1)^2} \quad (26)$$

where $\sigma_o = P/b_o d_o$ the nominal stress.

For the case of stress-frozen strain relations the true area of cross section was computed from

$$bd = b_o d_o (1 - \nu_p \epsilon_{1F})^2 \quad (27)$$

where all the quantities have the same meaning as above.

e) Thickness Corrected Birefringence

The true thickness

$$d = d_o - \Delta d_o = d_o (1 - \nu \epsilon_1) \quad (28)$$

Thickness corrected birefringence

$$N/d = (N/d_o) \times \frac{1}{(1 - \nu \epsilon_1)} \quad (29)$$

where ν and ϵ_i are as defined above.

f) Principal Strain Difference

The principal strain difference is

$$(\epsilon_1 - \epsilon_2) = \epsilon_1 (1 + \nu) \quad (30)$$

where ν and ϵ_1 are as above.

g) Fitting the Stress-Strain Curve

The Ramberg-Osgood (1943) non-dimensional stress-strain relation was considered. This was based on a stress-strain relation

$$\epsilon = \frac{\sigma}{E} + K\sigma^n \quad (31)$$

where

ϵ = strain

σ = stress

E = initial tangent modulus of elasticity

K, n = material constants.

Introducing another constant σ_S of dimension of stress,

$$\frac{\epsilon E}{\sigma_S} = \frac{\sigma}{\sigma_S} + \frac{K}{\sigma_S} \sigma_S^n \left(\frac{\sigma}{\sigma_S}\right)^n \quad (32)$$

Choosing $\sigma = \sigma_S$ corresponding to a secant modulus
 $E_S = 0.7E,$

$$\sigma_S/\epsilon = 0.7 E \quad (33)$$

or

$$\frac{1}{0.7} = 1 + K \frac{\sigma_S^n}{\sigma_S} \quad (34)$$

or

$$K \frac{\sigma_S^n}{\sigma_S} = 3/7 \quad (35)$$

The nondimensional stress-strain relation became

$$\frac{\epsilon \cdot E}{\sigma_S} = \frac{\sigma}{\sigma_S} + \frac{3}{7} \left(\frac{\sigma}{\sigma_S} \right)^n \quad (36)$$

E and σ_S were available from the experimental $\sigma - \epsilon$ data. Taking logarithm the above equation was reduced to a linear algebraic form

$$\ln \left(\frac{\epsilon \cdot E}{\sigma_S} - \frac{\sigma}{\sigma_S} \right) = n \ln \left(\frac{\sigma}{\sigma_S} \right) + \ln(3/7) \quad (37)$$

resembling

$$Y_i = nX_i + C \quad (38)$$

A least square fit was employed to get the value of n .

h) Theoretical Stress Concentration Factors

Recently Huang (1972) made a survey of some theoretical methods. The other related literature can be obtained from his paper.

i. Neuber's empirical rule

$$K_{\sigma} K_{\epsilon} = K_e^2 \quad (39)$$

where

K_{σ} = plastic stress concentration factor $\equiv K_{\sigma}^G$

K_{ϵ} = plastic strain concentration factor $\equiv K_{\epsilon}^G$

K_e = elastic stress (or strain) concentration factor $\equiv K_e^G$

based on the gross section. In a load free boundary of a plane stress body, the stress state is uniaxial. Using the Ramberg-Osgood stress-strain relation the following equation can be deduced.

$$K_{\sigma}^2 = \frac{K_e^2 (1 + \frac{3}{7} \lambda^{n-1})}{1 + \frac{3}{7} \lambda^{n-1} K_{\sigma}^{n-1}} \quad (40)$$

where $\lambda \equiv \lambda^G$.

ii. Stowel's empirical rule

$$K_{\sigma} = 1 + (K_e - 1) \frac{(E_S)_{\sigma_{\max}}}{(E_S)_{\sigma_{\text{av}}^G}} \quad (41)$$

where

$(E_S)_{\sigma_{\max}}$ = the secant modulus corresponding to $\sigma_{\max}$

$(E_S)_{\sigma_{\text{av}}^G}$ = the secant modulus corresponding to σ_{av}^G , the average stress based on the gross section.

This reduces to the following form, when Ramberg-Osgood relation is used:

$$K_{\sigma} = 1 + \frac{(K_e - 1) (1 + \frac{3}{7} \lambda^{n-1})}{1 + \frac{3}{7} \lambda^{n-1} K_{\sigma}^{n-1}} \quad (42)$$

where $\lambda \equiv \lambda^G$.

After Howland (Savin 1961) for a b/D ratio of 2, $K_e = 4.32$ was used in the above two empirical rules chosen for comparison.

i) Experimental Stress and Strain Concentration Factors

In the load free boundary a uniaxial or nearly uniaxial stress state was assumed in the models of tension strip with central circular hole.

1. Loaded model (conventional technique)

At the end of the specific time (8 hours) after applying the load in the Lexan models or at the end of the thermal cycle in the case of the epoxy models, the fringe order at the point of intersection of the hole boundary with the narrow cross section was measured under the loaded condition. The fringe order per nominal thickness N/d_0 was computed. Using the uniaxial calibration results based on nominal dimensions, explained above in section (a), the strain and stress were read. They were $\epsilon_{\max}$ and $\sigma_{\max}$.

Having known the applied load and the nominal gross and net cross sectional areas, σ_{av}^G and σ_{av}^N , the average stresses were computed. From these the stress concentration factors were determined as

$$K_{\sigma}^G = \sigma_{\max} / \sigma_{av}^G \quad (43)$$

and

$$K_{\sigma}^N = \sigma_{\max} / \sigma_{av}^N \quad (44)$$

In the present case $\sigma_{av}^N / \sigma_{av}^G = 2$, since $b/D = 2$ and only K_{σ}^N was considered because of this simple relation between them.

The strain concentration factors do not possess such a simple relation between them, as can be seen below. The strain concentration factors are defined as

$$K_{\epsilon}^G = \epsilon_{\max} / \epsilon(\sigma_{av}^G) \quad (45)$$

and

$$K_{\epsilon}^N = \epsilon_{\max} / \epsilon(\sigma_{av}^N) \quad (46)$$

Even though σ_{av}^G and σ_{av}^N have a fixed relation based purely on geometry as explained above in this section, the strains corresponding to these stresses have a nonlinear relation which stems from the stress-strain curve. These two strains were respectively read against σ_{av}^G and σ_{av}^N computed before.

The nominal unit fringe has been used because a thickness corrected birefringence requires the actual thickness at the point of interest to be measured and thereby, the advantage of the photomechanical method will be lost. This will, however, introduce a small error.

2. Hunter's analysis (unloaded model)

The fringe order at the point of interest was observed from the unloaded model and N/d_o computed. From the uniaxial frozen birefringence N_F/D_o - applied stress σ_o relation, the corresponding σ_{max} was read. The stress concentration factor

$$K_{\sigma}^N = \sigma_{max} / \sigma_{av}^N$$

was calculated.

3. Unloaded and cut model (present technique)

A radial cut along the narrow cross-section was made. The fringe order at the point of stress concentration was measured and N/d_o computed. From the uniaxial frozen birefringence N_F/d_o - applied stress σ_o relation, σ_{max} corresponding to this N/d_o was read to calculate

$$K_{\sigma}^N = \sigma_{max} / \sigma_{av}^N$$

j). Biaxial Strain Analysis

Two dimensional large normal strains with no rotation are defined by λ_x and λ_y in Lagrangian description as

$$\begin{aligned}\lambda_x &= (1 + \epsilon_x)^2 \\ \lambda_y &= (1 + \epsilon_y)^2\end{aligned}\tag{47}$$

The definition of shear strain γ_{xy} and the descriptions of λ_x , λ_y and γ_{xy} in terms of the spatial derivatives of the displacements u and v respectively in x and y directions are given in Appendix E.

A line of symmetry lies along the narrow cross-section of the tension strip with central circular hole and the strains in that line are principal. At four points on this line, the strains were measured to find the maximum values of $(\epsilon_1 - \epsilon_2)$, in the case of loaded epoxy models. For large strains this quantity $(\epsilon_1 - \epsilon_2)$ does not define the maximum shear strain $\gamma_{\max}$ (Appendix E). Therefore $(\epsilon_1 - \epsilon_2)$ has been called as the principal strain difference. A complete geometrical construction method for large strain analysis was not found in the literature and has been devised in Appendix E. Even though this was not applied here in view of the choice of points on the line of symmetry, it is thought as an appropriate information for similar studies.

CHAPTER IV

PRESENTATION OF RESULTS

A. PERMANENT STRAIN METHOD (LEXAN POLYCARBONATE)

a. Mechanical Tests

Shown in Figure 21 are the results from uniaxial tensile tests for two cross head speeds of 0.02 and 0.20 inch/minute. The two pairs of curves beyond the yield point were computed from the initial cross-sectional area and that measured after unloading the model.

Three relaxation test results can be seen in Figure 22. The test was different from the conventional ones in which the specimen is rapidly strained to a level and permitted to relax. In the present study the constant deformation rate tensile tests were stopped after certain deformation to permit relaxation of stress. Shown in Figure 22 are relaxation curves for 60 per cent elongation at 0.20 inch/minute, 60 per cent at 0.02 inch/minute and just at yielding at 0.02 inch/minute.

The specimens after various stages of deformation are shown in Figure 23a. The specimens B and E were annealed after the deformation and are shown along with the other deformed but unannealed specimens in Figure 23b.

b. Photomechanical Calibration

An isochronous stress-strain relation from the constant load creep tests with the strain measured eight hours after the application of the load is shown in Figure 24. The dashed line portion was only approximately deduced from the partly unloaded specimens, which was necessary to prevent the premature fracture of the specimens, when yielding started before 8 hours.

The photographs in Figure 25 show the fringes in the yielded specimens of the above test. The one in Figure 25a was an unannealed 1/16 inch thick Lexan, yielded and unloaded. Figure 25b shows an annealed one unloaded after yielding. An annealed yielded specimen under partly removed load was photographed as shown in Figure 25c.

Accurately measured results shown in Figures 26 and 27 do not extend beyond 6 per cent strain.

The nominal stress and nominal birefringence measured at two instances, one at about 20 seconds subscripted by i and at 8 hours subscripted by T are shown as functions of the strains ϵ_i and ϵ_T in Figure 26.

Making use of the values of Poisson's ratio evaluated by Brill (1965) at different strain levels, the thickness correction for the birefringence was effected and the principal strain difference calculated from Figure 26 to plot curves 1 and 2 of Figure 27. Curve 3 of Figure 27 was derived from curves 1 and 2 by their differences.

c. Applications

Photographed in Figure 28 are the Lexan application models (A - F) and the aluminium template (T). The models A - F were all 1/16 thick and loaded respectively to 4600, 7360, 7820, 8280, 8740, 9200 psi of average stress σ_{av}^N . E and F failed almost within 10 and 1 minutes respectively after the application of the full load.

In Figure 29 shown are the fringe pattern photographed 8 hours after applying the load, of models B(7360 psi) C(7820 psi) and D(8280 psi).

Figures 30a and b show model D(8280) in two views 8 hours after the application of the load. Figure 30c shows the fringe pattern in the same model D immediately after it was unloaded.

Figure 31 shows the three stages during the deformation and fracture of the model E(8740 psi). Photographs of fringe pattern taken 8 hours after the application of load on a thicker model 1/4 are shown in Figures 32a and b. The fringe pattern in this model just after unloading is shown in Figure 32c. The average stress σ_{av}^N was 6900 psi.

From photographs of fringe pattern of models A - D, the fringe distribution along the narrow symmetric cross-section was obtained as shown in Figure 33.

A qualitative comparison of the elastic-plastic boundary progression is represented in Figure 34.

B. FROZEN-CREEP METHOD (EPOXY)

a. Photomechanical Calibration

Shown in Figure 35 are the uniaxial total strain-total birefringence - stress calibration results. Experimental nominal stress (σ_o) - strain curve together with that of $\sigma_o - \epsilon$ curve fitted by Ramberg-Osgood method

is plotted. Both the nominal and thickness corrected birefringences, N/d_0 and N/d are shown. Also shown is the fitted true stress $\sigma - \epsilon$ relation, σ having been computed as explained in the preceding chapter.

The data for the above were obtained from the measurement of strain and fringe order from photographs of loaded specimens at the end of the thermal cycle. A sample photograph of a loaded specimen is shown in Figure 36a. Figure 36b shows the photograph of the fringe pattern of the above specimen, when unloaded.

From data taken from unloaded models of the type shown in Figure 36b, the frozen strain, frozen birefringence and stress relations were plotted in Figure 37. Experimental σ_0 , fitted σ_0 and σ , N_F/d_0 and N_F/d against the frozen axial strain ϵ_F are shown in this figure.

Directly measured values of Poisson's ratio together with the semi-experimental values are shown in Figure 38. The total strain - optic relations i.e. $(\epsilon_1 - \epsilon_2) - N/d_0$ and N/d are shown in Figure 39. Frozen strain - optic relation, $(\epsilon_1 - \epsilon_2)_F - N_F/d$ is plotted in Figure 40.

Thickness corrections and $(\epsilon_1 - \epsilon_2)$ and σ computations were effected as explained in the preceding chapter.

Four points obtained from the biaxial model results also are shown in Figure 39 for $(\epsilon_1 - \epsilon_2) - N/d_o$ relations.

b. Applications

Figures 41 through 45 show the fringe pattern in the application models of 0.05 inch nominal thickness subjected respectively to the following degrees of plasticity $\lambda^N = 0.473, 0.715, 0.945, 1.155$ and 1.43 . The upper photographs are of loaded models and the lower ones of unloaded models, both taken after the thermal cycle was over.

From the data of photographs such as the ones shown in Figures 41 - 45, the fringe distribution across the minimum cross-section under loaded and unloaded conditions was plotted in Figure 46.

The stress concentration factor K_o^N and the strain concentration factors K_ϵ^G and K_ϵ^N as obtained from the loaded model measurements i.e. conventional technique, are shown in Figure 47 as function of degree

of plasticity. For purposes of comparison stress concentration factors K_{σ}^N computed from Neuber's and Stowell's rules, stress and strain concentration factors K_{σ}^N and K_{ϵ}^N from photoplasticity (Javornicky 1968(2)) and from photoelastic coating technique (Theocaris 1964) are plotted in this figure.

The upper part of Figure 48 compares the stress - concentration factor obtained by using Hunter's technique with those obtained using Neuber's and Stowell's rules. In the lower figure comparison is made between the K_{σ}^N values obtained from Figure 47, those of Hunter's method and those obtained from the cut frozen - creep model of the present study. All these are plotted as function of degree of plasticity.

Distribution of principal strain-difference across the narrow section as obtained from Figures 46 and 39 has been plotted for the five levels of plasticity in Figure 49. An inset in this figure was taken from Theocaris (1964) for qualitative comparison of the results. An approximate nominal principal stress difference across the minimum cross-section is shown in Figure 50. Again Theocaris' results are given in the inset.

The effect of the thickness on the average-stress concentration factor corresponding to measurements made on loaded models is illustrated in Figure 51.

CHAPTER V

DISCUSSION

A. PERMANENT STRAIN METHOD (LEXAN)

a. Mechanical Tests

Contrary to some authors' belief (Brill 1965, Whitfield 1969), the time dependency of mechanical behaviour of Lexan polycarbonate is significant as can be seen from Figures 21 and 22. The material shows a small but almost linear strain hardening, justifying Brinson's (1971) suggestion favouring a bilinear approximation of the stress strain curve. The initial (elastic) tangent modulus was found as 0.33×10^6 psi with the ratio of $E_{\text{elastic}}/E_{\text{plastic}}$ varying from 165 to 440 according to the different conditions of tests shown in Figures 21 and 22. This behaviour has a striking resemblance to that of mild steel, with a distinct yield point of about 10000 psi at 0.02 inch/minute elongation rate and 10800 psi at 0.2 inch/minute. The strain at yield, however, was observed to be insensitive to the rate of elongation and was at 6.5 per cent approximately.

Two deformed specimens B and E of Figure 23a, when annealed return to their original form (Figure 23b), even though the deformation in E was very large. This confirms the observations of polymer scientists (Staverman 1956, Stein 1956, Nielsen 1965) that the micro-Brownian movement responsible for the mechanically reversible retarded viscoelastic behaviour is also responsible for this cold flow.

As explained by Stein (1956), the fringes also disappeared completely on annealing.

b. Photomechanical Calibration

The isochronous stress-strain relation obtained 8 hours after loading the specimen, (Figure 24) shows a yield point approximately 9200 psi and interestingly the yield strain is again 6.5 per cent. Around this value of stress, the yielding occurred over a range of time about the eighth hour. Only a mean value is shown. The dashed line portion beyond this yield point is approximate since the two points shown were obtained from a partly unloaded specimen which was necessary to prevent the premature fracture of the specimen when yielding started before 8 hours with

remarkable local reduction in the cross-section. This sudden reduction in the cross-section of the neck impeded the accurate assessment of the stress and strain in the range of high stress levels.

Furthermore, the large difference in the thicknesses of the necked and the unnecked portions left a transition region (Figures 23 and 25). In this region, often stress cracks formed and the fringe density and gradient were very high. This caused difficulty in the correct assessment of the fringe order. Attempts were made to immerse the specimen in an oil of matching refractive index in order to resolve the fringes in the transition region and were found helpless in view of the stress cracks which opened up in the presence of oil, eventually leading to fracture at this region. The cracks again impeded an attempt made to smoothen the gradient in this transition region.

The thinner sheets were thought suitable since the fringe density could be reduced. Unfortunately, 1/32 inch sheets as received contained high order of residual fringes, about order 13, and annealing caused them curl to that extent that they were not useful for modelling (Figure 3).

The investigation was, therefore, restricted to the small strain region which does not extend to the so-called plastic region and therefore the technique of permanent strain is not applicable in this range of stress and strain. Results shown in Figure 26 mostly were obtained from the Instron strain gauge extensometer records. A remarkable nonlinearity in the total strain-stress relation was noticed (Figure 26). This curve fits, with $n = 5 - 6$, a Ramberg-Osgood relation when $E_S = 0.7E$, $\sigma_S = 7600$ psi, $E = 0.33 \times 10^6$ psi. Since recovery after unloading did not leave any residual strain or birefringence, the orientational response is purely retarded recoverable viscoelastic type in this range of stress and strain.

The strain-optic relation, however, was found to be linear in this range (Figure 27) of the type

$$N/d = C_1 \gamma_d + C_2 \gamma_c$$

which is a special case of equation 12, when γ_c/γ_d is constant as in this case in the range of small stress and strain.

c. Applications

In view of the difficulty experienced in evaluating the stress-strain-optic relation beyond six per cent strain, no helpful quantitative informations were possible from the application models such as shown in photographs in Figures 28 - 32. The fringe distribution across the minimum cross-section shown in Figure 33 could not be traced up to the hole boundary. Thus the permanent strain method was not feasible with polycarbonate material.

Because of the remarkable nonlinearity shown by the $\sigma_o - \epsilon_T$ curve (Figure 26), this nonlinear viscoelastic response was used to predict the stress concentration factor in one of the models whose range of fringes fell into that of Figure 26. The fringe order $N = 21.5$ with thickness $d_o = 0.0755$ yielded $N/d_o = 285$. For this $\sigma_{max} = 8500$ psi from Figure 26; the applied stress $\sigma_{av}^N = 4600$ psi; and $\sigma_s = 7600$ psi. This gives a stress concentration factor of $K_o^N = 1.85$, corresponding to a degree of plasticity $\sigma_{av}^N/\sigma_s = 0.605$. This is a satisfactory prediction for $n = 5 - 6$, as Stowell's empirical rule which is known to underestimate K_o^N for such a geometry (Javornicky

1968(2)) gives a value of 1.68 for $n = 6$ and 1.7 for $n = 5$, the differences being 9 per cent and 8 per cent respectively.

Qualitative informations on the elastic-plastic boundaries can be seen from the photographs of Figures 29 - 31, by the abrupt change in the thickness shown by the shadows. The elastic-plastic boundaries derived by two investigations are shown in Figure 34. The elastic-plastic boundaries of Faerberg (Savin 1961) obtained by a curved beam theory for a material whose $E_{\text{elastic}}/E_{\text{plastic}}$ ratio was 60, are very close to the ones observed in Lexan with $E_{\text{elastic}}/E_{\text{plastic}}$ ratio of 220. The material used by Theocaris (1964) had $E_{\text{elastic}}/E_{\text{plastic}}$ ratio of 32; perhaps this may be the reason why the elastic-plastic boundaries derived by him are very much different from those of polycarbonate and the aluminium alloy of Faerberg.

B. FROZEN-CREEP METHOD (EPOXY)

a. Calibration Results

The total strain-stress experimental points in Figure 35 fall on a smooth curve. The fitted $\sigma_0 - \epsilon$ curve shows significant deviations from the experimental points. Variation of parameters E and σ_s for $E_s = 0.7 E$

was tried and the one with $E = 0.89 \times 10^5$ psi and $\sigma_s = 1400$ psi giving $n = 7.15$ was found to be the best. Mönch (1963) and Javornicky (1968(1)) reported a wide variation in n , 9 - 19 for celluloid and the present fit, therefore, can be considered as satisfactory.

Up to a nondimensional strain $(\frac{E\epsilon}{\sigma_s}) = 7.5$, i.e. 12 per cent nominal axial strain of this epoxy, a value of $n < 7.15$ is suggested. Beyond this $n > 7.15$ seems a good approximation for use of this particular epoxy as a model material. Ramberg and Osgood (1943) showed $C_r - N_i$ steel (full hard) had $n = 3 - 5$ and Al alloy 24S-T, $n = 10 - 120$. Javornicky found that Al alloy 24S - T3 fell into the range of $n = 9 - 19$.

Slight variation of n between 7 - 10 for Al alloy 24S - T was considered by Ramberg and Osgood to show that the best fit extends only over a limited range of strain. Similar suggestion was also advanced by Brill (1965). In view of this it becomes very difficult to specify a material which can be simulated by this epoxy subjected to the particular thermal cycle. It is possible however, to change n by changing T_{max} of the thermal cycle (Hunter 1966).

The same remarks on Figure 35 equally apply to Figure 37 which shows the frozen strain and birefringence.

It was found that the stress-optic and strain-optic relations of the epoxy at room temperature before and after the thermal cycle were linear. The usual photoelastic relations will hold for this component. The fringe value f was found as 73 psi-inch/fringe for He-Ne light (6328Å) and the average value of the modulus of elasticity E as 4.54×10^5 psi at room temperature, as has been explained already in the test procedure. This yields

$$N_R/d_O = C_1(\epsilon_1 - \epsilon_2)_R$$

The Poisson's ratio computed semi-experimentally i.e. using an equation after Smith T.F.W. (1954) and the experimentally determined longitudinal strain (Figure 38) was used to get the strain-optic relations in Figures 39 and 40. These strain-optic relations (Figures 39 and 40) between the principal strain difference and the fringe order per unit thickness are nonlinear. From Figure 40 it can be seen that

$$N_F/d = g_2(\epsilon_1 - \epsilon_2)_F$$

where g_2 is a nonlinear function. The total birefringence and strain relation of Figure 39, therefore, can be expressed as

$$N/d = (N_R + N_F)/d = C_1(\epsilon_1 - \epsilon_2)_R + g_2(\epsilon_1 - \epsilon_2)_F$$

which is the same as equation 20, with $g_1 \equiv C_1$ a constant.

b. Applications

1. Plane stress models

The principal strain difference measured along the line of symmetry across the narrow cross-section of the models D and E provided four points on the strain-optic relation shown in Figure 39. In the absence of elaborate biaxial tests which require, by virtue of the present method of photomechanical characterization, one specimen of thin tube (Marin 1964) for each point in this curve, the four points obtained from the application models were thought the best possible (Brill 1965). No attempt was made to obtain similar points for the unloaded model, in view of the slight residual stress suspected.

From data available in Figures 41 through 46, the stress and strain concentrations were computed as presented in Figure 47. These values of K_σ and K_ϵ are

based on the loaded model and therefore represent the conventional methods of measuring birefringence under the load.

The stress concentration factors K_O^N obtained as function of the degree of plasticity show a fair agreement with the results from Neuber's and Stowell's empirical rules. These two rules respectively were originally derived for open notches in prismatic bars under bending and tension strip with small hole. However they were recommended by some investigators (Javornicky 1968(2), Huang 1972) as empirical rules for other situations in view of their "simplicity and good agreement with many experimental results". The stress concentration (K_O^N) from Javornicky's photoplastic results (1968(2)) using celluloid models of Ramberg-Osgood exponent of $n = 9 - 19$ together with photoelastic coating results of Theocaris (1964) for $n = 11 - 13$, for the same geometry as the present models have been included in Figure 47. Javornicky compared his results with Stowell's rule which underestimated K_O^N , for $\lambda^N = 0.8$ to 1.3. At $\lambda^N = 1.0$, the difference was 11.5 - 4.5 per cent. The same trend is seen in the present results of K_O^N evaluated by the conventional technique with a difference of 9.45 per cent. Based on this comparison

as well as on other opinions of investigators who have spent considerable time and effort on photoplasticity, "that photoplastic work will always represent a certain approximation and will never have the accuracy that can be attained in photoelasticity", (Mönch 1963) the results obtained can be considered satisfactory.

The strain concentration factors K_{ϵ}^N are compared in the lower part of Figure 47. A large difference between the results of Javornicky and Theocaris for almost a very close mean value of n , discourages any quantitative comparison. The trend shown by Theocaris' results, available only for lower degrees of plasticity λ^N , is the same as that of the present results. The trend of the present K_{ϵ}^N with higher levels of plasticity λ^N is strikingly the same as that of Javornicky. The large variations of K_{ϵ}^N between the results of different investigators can be attributed to the high sensitivity of strain to changes in stress in the post yielded zone of the stress-strain curve.

The comparison of K_{σ}^N (Figure 48, upper) obtained by Hunter's technique with those of Neuber's and Stowell's rules show the same trend between the experimental and Stowell's results as found for the conventional technique (Figure 47). However, a comparison

of results of the conventional technique and Hunter's technique shows that the latter are lower (Figure 48, lower).

It was shown that the results of the conventional technique at the load free boundary are of the same accuracy of similar methods. Taking this as the basis, one can examine the results of Hunter's technique. According to Hunter, the material behaviour is characterized by the frozen strain for which $n = 5.87$ is appropriate (Figure 37). Smaller the n , stiffer is the material, approaching an elastic material behaviour and the K_O^N values should be higher. But the experimental results by Hunter's technique are lower than that of a material with $n = 7.15$ (Figure 48). Hunter's concept of the "imaginary material behaviour" as represented by the frozen strain-applied stress relation, thus, does not seem suitable. A total strain-applied stress relation seems more appropriate. If these arguments are accepted, then results of Hunter's technique provide an underestimate of K_O^N . This could be attributed to the presence of any macroscopic residual compressive stress and associated birefringence at the point of stress concentration in the present problem. This would lead to slightly lower fringe order and hence lower value of K_O^N .

The frozen-creep method proposed in this investigation, requires the model to be cut in order to relieve any macroscopic residual stress. This was done on two models with $\lambda^N = 0.715$ and 1.43 by cutting radially along the minimum section. A small increase in the fringe orders was observed, this led to the two points shown in lower part of Figure 48. The results still are slightly underestimated even after cutting the model. However this new K_O^N is associated with the material with $n = 7.15$ not with the imaginary material of $n = 5.78$.

Lower the value of λ^N (Figure 48), the difference between the K_O^N results of the conventional technique and Hunter's technique become larger. This can be expected as, at higher levels of plasticity, the frozen strain and birefringence become relatively so large that the change brought about on them by the residual stress becomes relatively insignificant. This parallels the usual assumption of the theory of plasticity that the elastic strain is negligible when compared with the large plastic strain, without affecting the results very much. Thus for large deformations, satisfactory results seem possible without the removal of residual stress effects. However, the result must not be related to the imaginary material behaviour but

should be used with total strain-stress relation as discussed before.

The distribution of the principal strain difference ($\epsilon_1 - \epsilon_2$) across the narrow section in Figure 49 is based on an assumption that the strain-optic relation of Figure 39 is valid. The closeness of the four points from the biaxial model results justify the use of this relation for an approximate estimate of ($\epsilon_1 - \epsilon_2$) in a biaxial model. The present results of Figure 49 obtained from the conventional technique under load, compare qualitatively very well with those of Theocaris (1964) given in the inset.

On the assumption that a total stress-strain relation of the theory of plasticity holds, ($\sigma_1 - \sigma_2$) distributions were computed (Figure 50). The results qualitatively compare very well with the results of Theocaris shown in the inset.

Separation of the principal values of strain and stress were not attempted. Methods to separate large strains are not known. Stress separation was given up because of the approximate nature of the ($\sigma_1 - \sigma_2$) results of Figure 50, as mentioned above.

2. Thick models

The average stress concentration factors K_{σ}^N for constant degree of plasticity $\lambda^N = 0.945$, have been obtained from the conventional technique (Figure 51). The plastic stress concentration factor K_{σ}^N shows a regular pattern of decrease with increase in the thickness. Welbourne (1970) reported a similar trend for the plastic strain concentration factor in a open notch. Such a trend also has been reported for an elastic case of a circular hole under pure shear (Pal 1971) in a photoelastic study using the same epoxy.

The randomness in the choice of the absolute dimensions of the model, only keeping the same geometric proportions, except d_o/D , eliminates the possibility of time edge effect as the cause for this regular pattern, found by the writer and Pal (1971).

Another factor which is known to influence elastic K_{σ}^N is the length of the model relative to other dimensions (Ibrahim 1966). The lengths were such that $L/D \approx 6.5$ or $L/b \approx 3.25$ which possibly cannot affect so much according to St. Venant's principle.

This regular pattern perhaps could be explained as below. It is known that the normal stress σ_z and the stress concentration factor varies roughly parabolically in the hole boundary in the elastic case (Green 1948, Sternberg 1949). The normal stress σ_z becomes almost

flatly distributed except near the surfaces when the thickness increases. If a similar trend is followed by the tangential stress which is also the measure of stress concentration factor in plastic case, the average may show a regular variation. A definite explanation does not seem possible, except that the parabolic variation of strain through the thickness is also commonly assumed for plastic case (Theocaris 1963). Pindera (1967) pointed out that the deviation of the real stress concentration factor from the idealized (plane stress) one is usually well above the experimental error and the thickness/radius of curvature ratio is, therefore, an important parameter which should be reported to make any information on stress concentration factor complete.

The d_o/D ratio of 1.0 with b/D ratio of only 2.0 is a good case for a three-dimensional stress state. However in the hole boundary the stress state is only biaxial with one stress normal to the plane of observation and the other tangential to the hole boundary. The average values of K_o^N obtained for such a case only have been studied and a strict three-dimensional analysis of stress and strain distribution has not been tackled in this exploratory study.

C. ACCURACY OF RESULTS

a) Because of the nature of photoplasticity, there are certain unavoidable errors. The yield strain in plastics is several times (about 10 to 30) greater than in metals. This causes errors due to the changes in the shape of the model.

b) Another important error is due to the redistribution of stress that occurs in a general model. Thus the loading is not carried out exactly in the manner as prescribed for the uniaxial calibration. This, in the present case, causes an over-estimate of stress and strain concentration factor (Smith J.O. 1965).

c) Error may also be possible from the use of the conventional definition of strains. Natural strain $\bar{\epsilon} = \ln(1 + \epsilon)$ seems to be more appropriate for large strains but its application is complex.

d) Error due to the use of incorrect QWP's can be estimated and the maximum error in the present study is found as 8×10^{-3} fringe when green QWP is used with He-Ne laser assuming no isoclinic setting error (Appendix D).

e) Errors in the experimentally measured quantities are defined by the probable errors and are

estimated as shown in Appendix F. The percentage probable errors in K_O^N and K_e^N resulting from measurements alone are estimated to be less than 5 per cent.

f) Violation of similarity requirements for transition to prototype is a major factor affecting the accuracy. The choice of the model material to satisfy all the requirements is almost impossible.

D. LIMITATIONS OF THE PRESENT TECHNIQUES

Determination of stress and strain concentration factors at load free boundaries only are possible in the present techniques. Only an average stress and strain concentration factor can be determined when thick models are involved.

A complete exploration of even the two-dimensional strain and stress is not possible, without much basic work which will consider the changes in the ratios of principal stresses, directions of stresses and strains and their effects on the optical and mechanical response of the material.

Only an approximate distribution of principal strain and stress differences are possible for the two-dimensional problems. The separation of large strains

has not yet been solved satisfactorily and therefore individual strains cannot be determined.

Several models and specimens are necessary to study the effect of even a single variable.

CHAPTER VI

CONCLUSION

Two methods of photoplasticity, more correctly elastoplastic simulation by nonlinear photoviscoelasticity, were explored: 1. Permanent strain method and 2. Frozen-creep method. For the permanent strain method Lexan polycarbonate was employed. Due to experimental difficulties in resolving the high order of fringes in the yielded Lexan, the permanent strain technique was not feasible. The study was therefore confined to small strain range using only the recoverable nonlinear viscoelastic properties of this material. Stress concentration factor for a particular degree of plasticity using the above information was determined. This quantitative information was assessed satisfactory. Qualitative information on the progression of plastic boundary was compared with those in the literature.

In the frozen-creep method specimens and models of an epoxy were subjected to a thermal cycle, under different constant loads in order to generate a nonlinear stress-strain state with the associated birefringence. Using the measurements under load viz. the conventional

technique, stress and strain concentration factors in a tension strip with a large central circular hole were evaluated for various degrees of plasticity and were assessed, from comparison with published results, as satisfactory.

Hunter's technique of determining the stress concentration factors from the unloaded model was evaluated in the light of the above results as satisfactory for very large deformations where the effect of the macroscopic residual stress in the unloaded models on the fringe orders is very negligible. However, the total strain-applied stress relation is found suitable to represent the mechanical behaviour of the material in preference to the "imaginary material behaviour" represented by the frozen strain-applied stress relation suggested by Hunter.

Improvement on results over those of Hunter's technique was attempted by cutting the model to relieve the macroscopic residual stress due to unloading. The difference was not very significant in the absolute value of the stress concentration factor but it is of some significance when the correct stress-strain relation is not used to represent the material behaviour.

Effect of thickness on the average plastic stress concentration showed a decrease of stress concentration with increase in the thickness. No published results in the plastic region are available for comparison.

As by-products of this investigation, the principle of constructing a circular polariscope for any arbitrary position of the polarizer-polarizer QWP unit with relation to any position of the analyzer-analyzer QWP unit was analysed. The advantages of this wellknown principle are not found in text books and hence has been shown in Appendix C. The relative accuracies of Tardy and Senarmont compensation methods of photo-elastic fringe order measurement have been analyzed in detail in Appendix D. A complete geometric construction method for large strain analysis has been developed in Appendix E.

Lexan polycarbonate in the low stress-strain region shows a strain-optic relation of the type

$$N/d = C_1 \gamma_d + C_2 \gamma_c$$

whereas the epoxy resin subjected to the thermal cycle follows

$$N/d = C_1 \gamma_R + g_2 (\gamma_F)$$

The accuracy and limitations of the present techniques were discussed in the preceding chapter. There are several sources of error that affect the results. The present techniques are limited to the determination of the average stress and strain concentration factors at load-free boundaries only.

In the present day of high speed computers, photoplastic study does not seem to be rewarding. The very fact that the problem of the meaning of the isoclinic parameter in inelastic range, that is, whether it represents the direction of principal stress or strain, has not been solved satisfactorily, is enough to reconsider the usefulness of the application of photoplasticity to a general problem. To quote Kiesling (1966), "It appears that time spent in material selection might be profitably invested". Furthermore, very high accuracy cannot be expected as discussed in the preceding chapter. To quote Mönch (1963), "Considering all sources of errors it must be emphasized that photoplastic work will always represent a certain approximation and will never have the accuracy that can be attained in photoelasticity". This method could, therefore, be considered more for qualitative purposes than for quantitative evaluations. Theories using numerical methods could,

perhaps, yield results at least of the same accuracy but much quicker than a photoplastic method, with the availability of high speed digital computers and the advent of the finite element technique(Zienkiewicz 1969).

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a) Temperature-Strength Diagram

b) Tensile Load-Deformation Diagram in Glassy State

Fig. 1. Deformation and strength of amorphous polymers.

Fig. 2. Qualitative isochronous stress-strain relation from creep and recovery tests.

$(^i)$ - instantaneous; $(^r)$ - retarded;
 $(^r)$ - recoverable; $(^n)$ - nonrecoverable.
 $(^d)$ - distortional; $(^c)$ - orientational.

a. Initial fringes in as-received sheet
(about order 13)

b. Annealed strips

Fig. 3. 'Lexan' polycarbonate strips of
1/32 inch thick before and after
annealing at 325°F.

$$\lambda = 6328 \text{ \AA}.$$

Fig. 4. Furnace with programmable temperature control.

Fig. 5. Instron testing machine with polariscope set-up with He-Ne laser source.

Fig. 6. Instron testing machine with travelling telemicroscope on X-Y stage.

Fig. 7. 'Riken' polariscope with travelling tele-microscope on X-Y stage.

Fig. 8. 'Riken' polariscope with He-Ne laser source, camera, synchro control for rotation of optical elements, hydraulic jack for smooth application and removal of load, etc.

Fig. 9. "Haag-Streit Ag" precision coordinatograph used for mechanical scribing of grids on plastic specimens and models.

Fig. 10. 'Nikon' profile projector polariscope.

Fig. 11. Calibration specimen configuration-polycarbonate.

Fig. 12. High-speed router used for machining plastic specimens and models.

Thickness $d_o \approx 0.07; 0.25$
 Dimensions in Inches

Fig. 13. Model configuration-polycarbonate.

Thickness $d_o = 0.05$
 Dimensions in inches

Fig. 14. Calibration specimen configuration-epoxy.

Fig. 15. Tapered epoxy calibration specimens.

Thickness d_o : 0.05; 0.25

Dimensions in Inches

Fig. 16. Model configurations-epoxy.

Fig. 17. Epoxy models used to study the effect of degree of plasticity on stress and strain concentration factors.

Fig. 18. Epoxy models used to study the effect of thickness on the stress concentration factor.

Fig. 19. Thermal cycle for frozen-creep method-epoxy.

Fig. 20. Loaded epoxy models in the furnace.

Fig. 21. Typical uniaxial tensile test results - polycarbonate.

Fig. 22. Typical relaxation results-polycarbonate.

a. Deformed 'Lexan' polycarbonate specimens.

b. Specimens B and E only annealed.

Fig. 23. Comparison of specimens B and E, which were annealed after deformation with the deformed unannealed specimens in a. (T-template) - polycarbonate.

Fig. 24. Isochronous stress-strain relations (8 hours) from uniaxial creep-polycarbonate.

a

b

c

Fig. 25. Fringe pattern in yielded zone in 'Lexan' polycarbonate.

(a) Unannealed specimen unloaded.

(b) Annealed specimen unloaded.

(c) Annealed specimen partly unloaded.

$$\lambda = 6328 \text{ \AA}.$$

Fig. 26. Stress-strain-birefringence relationship-polycarbonate. (He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 27. Strain-optic relation-polycarbonate. (He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 28. 'Lexan' polycarbonate models tested at various degrees of plasticity. E and F broken. T-template.

Fig. 29. Fringe pattern and plastic zone at three levels of plasticity in 'Lexan' polycarbonate models (8 hours after applying load).

$$\lambda = 6328 \text{ \AA}.$$

Fig. 30. 'Lexan' polycarbonate model loaded to $\sigma_{av}^N = 8280$ psi.

(a) Full view (8 hours) - loaded.

(b) Close-up (8 hours) - loaded.

(c) Close-up immediately after unloading.

$\lambda = 6328 \overset{O}{\text{\AA}}.$

Fig. 31. 'Lexan' polycarbonate loaded to $\sigma_{av}^N = 8740$ psi, model fractured when the last stage of load was applied.

(a) At about 20 seconds after the full load application.

(b) Within a few minutes after (a) and

(c) just after fracture.

$$\lambda = 6328 \text{ \AA}.$$

a

b

c

Fig. 32. 'Lexan' polycarbonate thick ($\frac{1}{8}$ inch) model shows large number of fringes when loaded only to $\sigma_{av}^N = 6900$ psi.

- (a) Full view (8 hours) - loaded.
- (b) Close-up (8 hours) - loaded.
- (c) Close-up immediately after unloading.

$$\lambda = 6328 \text{ \AA}.$$

Fig. 33. Fringe distribution across minimum section of model (8 hours) - polycarbonate. (He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 34. Qualitative comparison of elastic-plastic boundary progression.

Fig. 35. Stress-total strain-birefringence relationships-epoxy (He-Ne laser, $\lambda = 6328\text{\AA}$).

a. At the end of thermal cycle with load on.

b. Unloaded (frozen fringes).

Fig. 36. Fringe pattern in tapered epoxy calibration specimen with maximum stress of 2100 psi.

$$\lambda = 6328 \text{ \AA}.$$

Fig. 37. Stress-frozen strain-frozen birefringence relationships-epoxy (He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 38. Poisson's ratio for large strain-epoxy.

Fig. 39. Strain-optic relation - epoxy.
 (He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 40. Frozen strain-frozen birefringence relation - epoxy.
(He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 41. Epoxy model. $\lambda^N = 0.473$.

(a) Loaded.

(b) Unloaded.

$\lambda = 6328 \text{ \AA}$.

Fig. 42. Epoxy model. $\lambda^N = 0.715$.

(a) Loaded.

(b) Unloaded.

$\lambda = 6328 \text{ \AA}$.

a

b

Fig. 43. Epoxy model. $\lambda^N = 0.945$.

(a) Loaded. (b) Unloaded.

$\lambda = 6328 \text{ \AA}$.

a

b

Fig. 44. Epoxy model. $\lambda^N = 1.155$

(a) Loaded.

(b) Unloaded.

 $\lambda = 6328 \overset{\circ}{\text{A}}.$

(a) Loaded. (Maximum integral fringe = 18).

(b) Unloaded. (Maximum integral fringe = 15).

Fig. 45. Epoxy model. $\lambda^N = 1.43$.

$$\lambda = 6328 \text{ \AA}.$$

Fig. 46. Fringe distribution across the minimum cross section - epoxy.
 (He-Ne laser, $\lambda = 6328\text{\AA}$).

Fig. 47. Stress and strain concentration factors based on nominal stress - plane stress - epoxy (conventional method).

Fig. 48. Comparison of stress concentration factors from present and Hunter's method - epoxy.

Fig. 49. Distribution of principal strain difference across minimum section - epoxy.

Fig. 50. Approximate distribution of principal stress difference across minimum section - epoxy.

Fig. 51. Effect of thickness on average stress concentration factor - epoxy (conventional method).

APPENDIX A

INTERPRETATION OF THE SIMPLIFIED
STRAIN-OPTIC RELATION

For time $t = t_1$, the strain response is a function of stress. From equation (9)

$$\gamma(\tau) = \gamma_d(\tau) + \gamma_c(\tau) \quad (A1)$$

This can be written as

$$\gamma(\tau) = \gamma_d(\tau) [1 + k(\gamma)] \quad (A2)$$

where $k(\gamma) = \gamma_c/\gamma_d$. Similarly equation (14) can be written as

$$N/d = C_1 \gamma_d(\tau) [1 + Rk(\gamma)] \quad (A3)$$

where $R = C_2/C_1$. Eliminating γ_d in (A3) from (A2),

$$N/d = C_1 \gamma(\tau) \cdot \frac{1 + Rk(\gamma)}{1 + k(\gamma)} \quad (A4)$$

Some cases of practical interest are:

- $R \leq 1$
- a) $k = \text{constant}$
 - b) k increases with γ .

The possible types of strain-birefringence relations are illustrated qualitatively in Figure A1.

Under certain circumstances the (total) strain-(total) birefringence relation can be linear (Figure A1). Since the stress-strain curve in general is nonlinear in the elastoplastic range, the stress-birefringence relation cannot be linear according to the equations so far discussed. However, if C_1 is very large compared to C_2 , $\tau - N/d$ relation approaches linearity and may be confused to be linear.

In order to explain the stress-strain-optical directions, Bayoumi's (1953) analysis is adopted with modification i.e. instead of stress and total strain differences, γ_d and γ_c are considered as the basis of birefringence.

With $\gamma_d = (\epsilon_1 - \epsilon_2)_d$ and $\gamma_c = (\epsilon_1 - \epsilon_2)_c$ and letting the angle α_1 represent the angle between ϵ_{1d} and ϵ_{1c} and ϕ_ϵ any angle measured from ϵ_{1d} which is assumed coaxial with σ_1 (Nadai 1951), the shear strain in the plane of ϕ_ϵ is

$$\begin{aligned} \gamma = & (\epsilon_1 + \epsilon_2)_d/2 + (\epsilon_1 - \epsilon_2)_d/2 \cdot \cos 2\phi_\epsilon \\ & + (\epsilon_1 + \epsilon_2)_c/2 + (\epsilon_1 - \epsilon_2)_c/2 \cdot [\cos 2\phi_\epsilon \cos 2\alpha_1 \\ & + \sin 2\phi_\epsilon \sin 2\alpha_1] \end{aligned} \quad (A5)$$

a) Linear strain-optic relation assumed for the two parts of total strain $\gamma = \gamma_d + \gamma_c$

b) Typical total strain-total birefringence relation

c) Some γ - $N/d/C_1$ relations possible for various R - k combination

Fig. A1. Qualitative isochronous strain-optic relations.

For maximum of γ ,

$$\tan 2\phi_E = \gamma_d \sin 2\alpha_1 / \gamma_d + \gamma_c \cos 2\alpha_1 \quad (A6)$$

Similarly from equation (14),

$$\begin{aligned} N/d = C_1 \left[\frac{(\epsilon_1 + \epsilon_2)_d}{2} + \frac{(\epsilon_1 - \epsilon_2)_d}{2} \cos 2\phi_n \right] \\ + C_2 \left[\frac{(\epsilon_1 + \epsilon_2)_c}{2} + \frac{(\epsilon_1 - \epsilon_2)_c}{2} (\cos 2\phi_n \cos 2\alpha_1 \right. \\ \left. + \sin 2\phi_n \sin 2\alpha_1) \right] \quad (A7) \end{aligned}$$

and the principal directions of optical axes are given by

$$\tan 2\phi_n = \frac{C_2 \gamma_c \sin 2\alpha_1}{C_1 \gamma_d + C_2 \gamma_c \cos 2\alpha_1} \quad (A8)$$

Accordingly, the isoclinic which denotes the principal direction of the optical axes represents neither the direction of principal stress nor that of the total strain in general.

When $\phi_E = 0$ i.e. stress-strain coincidence exists, $\alpha_1 = \phi_n = 0$; i.e. mechanical and optical coincidence is ensured. In a general case, the nearness of the principal axis of refractive index to that of strain or stress depends on the relative magnitudes of C_1 , C_2 , γ_d and γ_c , according to the above expressions.

Even in the case of linear viscoelasticity, if the ratio of principal stresses at a point does not remain constant, directions of principal stress, strain and optical axes may not coincide (Kiesling 1969).

APPENDIX B

EQUATIONS OF A GENERAL POLARISCOPE

$\delta_1, \delta_2, \delta_3$ = relative retardations of polarizer QWP, model and analyzer QWP (radians)

$\theta, \rho_1, \rho_2, \rho_3$ = angular positions of axes of analyzer, polarizer-QWP, principal stress in model and analyzer-QWP from polarizer axis.

From Mindlin's (1937) equation (1), the following simplified form is obtained for intensity of light emerging from the analyzer:

$$2I/I_0 = 1 + A \sin 2(\theta - \rho_3) + B \cos 2(\theta - \rho_3) \quad (B1)$$

where

$$\begin{aligned}
 A = & -\cos 2\rho_1 \cos 2(\rho_2 - \rho_1) \sin 2(\rho_3 - \rho_2) \cos\delta_3 \\
 & + \sin 2\rho_1 \sin 2(\rho_2 - \rho_1) \sin 2(\rho_3 - \rho_2) \cos\delta_1 \cos\delta_3 \\
 & - \cos 2\rho_1 \sin 2(\rho_2 - \rho_1) \cos 2(\rho_3 - \rho_2) \cos\delta_2 \cos\delta_3 \\
 & + \cos 2\rho_1 \sin 2(\rho_2 - \rho_1) \sin\delta_3 \sin\delta_2 \\
 & - \sin 2\rho_1 \cos 2(\rho_2 - \rho_1) \cos 2(\rho_3 - \rho_2) \cos\delta_1 \cos\delta_2 \cos\delta_3 \\
 & + \sin 2\rho_1 \cos 2(\rho_2 - \rho_1) \cos\delta_1 \sin\delta_2 \sin\delta_3 \\
 & + \sin 2\rho_1 \cos 2(\rho_3 - \rho_2) \cos\delta_3 \sin\delta_2 \sin\delta_1 \\
 & + \sin 2\rho_1 \sin\delta_3 \cos\delta_2 \sin\delta_1
 \end{aligned} \tag{B2}$$

$$\begin{aligned}
 B = & -\sin 2\rho_1 \sin 2(\rho_2 - \rho_1) \cos 2(\rho_3 - \rho_2) \cos\delta_1 \\
 & - \cos 2\rho_1 \sin 2(\rho_2 - \rho_1) \sin 2(\rho_3 - \rho_2) \cos\delta_2 \\
 & - \sin 2\rho_1 \cos 2(\rho_2 - \rho_1) \sin 2(\rho_3 - \rho_2) \cos\delta_1 \cos\delta_2 \\
 & + \sin 2\rho_1 \sin 2(\rho_3 - \rho_2) \sin\delta_1 \sin\delta_2
 \end{aligned} \tag{B3}$$

Reference can be made to a paper by the writer (Sathikh 1972(2)), for formulation of similar equations in matrix notations.

APPENDIX C

POLARISCOPE DESIGN ANALYSIS

When a He-Ne laser ($\lambda = 6328\text{\AA}$) is used with a set of QWP's designed for green spectrum ($5460\text{\AA}$), the retardations of the QWP's differ from $\pi/2$ but they are equal i.e. $\delta_1 = \delta_2 = \delta \neq \pi/2$. With reference to Appendix B, let: $\rho_1 = 45^\circ$; $\theta - \rho_3 = -45^\circ$; ρ_2, ρ_3, δ and δ_2 arbitrary. (Note, $(\rho_3 - \rho_1) = 90^\circ$ in one type of conventional circular system i.e. $\rho_3 = 90 + \rho_1 = 135^\circ$). This yields

$$2I/I_0 = 1 - A \quad (C1)$$

or

$$\begin{aligned} 2I/I_0 = & 1 + \cos^2\delta \cos 2\rho_2 \sin 2(\rho_3 - \rho_2) \\ & + \cos\delta_2 \left[\sin^2\delta - \cos^2\delta \sin 2\rho_2 \cos 2(\rho_3 - \rho_2) \right] \\ & - \sin\delta_2 \sin\delta \cos\delta \left[\sin 2\rho_2 + \right. \\ & \left. \cos 2(\rho_3 - \rho_2) \right] \end{aligned} \quad (C2)$$

The extremum of intensity is given by $dI/d\delta_2 = 0$:

$$\tan\delta_2 = \frac{\left[\sin 2\rho_2 + \cos 2(\rho_3 - \rho_2) \right] \cos\delta \sin\delta}{\sin^2\delta - \cos^2\delta \sin 2\rho_2 \cos 2(\rho_3 - \rho_2)} \quad (C3)$$

The condition that the fringes truly represent the integral wave lengths of retardation, is $\tan\delta_2 = 0$. Here $\tan\delta_2$ is a function of δ , ρ_2 and ρ_3 . For true representation, therefore,

$$\cos\delta \sin\delta \left[\sin 2\rho_2 + \cos 2(\rho_3 - \rho_2) \right] = 0 \quad (C4)$$

This is possible in two ways:

- (i) $\delta = \pi/2$, though ρ_2, ρ_3 may be arbitrary.
- (ii) $\rho_3 = 135^\circ$, though δ, ρ_2 arbitrary.

From (i) it is seen that if perfect QWP's are used ($\delta = \pi/2$), a circular polariscope can have ρ_3 arbitrary. It is enough to have (a) the polarizer and the polarizer-QWP together as one unit with 45° between their axes; (b) the analyzer and analyzer-QWP as one unit with 45° between their axes; and (c) each unit is independently positioned. This is obvious from the nature of the circular polarization.

This set-up has the following advantage. The polarizer and the polarizer-QWP can be left undisturbed in any position as long as they are set with their polarizing axes at 45° with each other. The analyzer and analyzer-QWP forming a similar unit can be set in any position, in order to obtain the correct integral fringe

orders. It is enough if the analyzer and analyzer-QWP are rotatable in order to determine the fractional fringe order by the Tardy method as will be shown later in this Appendix; this simplifies the design of the circular polariscope. In the system (i) i.e. the new set-up with perfect QWP's, there will be no systematic error when Tardy compensation method is used.

Let the polarizer and the polarizer-QWP be undisturbed. Rotate the analyzer and the analyzer-QWP together so as to align the axis of the analyzer with that of the principal axis of the model at the point of interest. At this position $\theta = \rho_2$ and $(\rho_3 - \rho_2) = 45^\circ$. In order to measure the fractional fringe order, rotate the analyzer alone; let θ be arbitrary. From Appendix B

$$2I/I_0 = 1 + A \sin 2(\theta - \rho_3) + B \cos 2(\theta - \rho_3) \quad (C5)$$

where $A = -\cos \delta_2$ and $B = \sin \delta_2$. Now $dI/d\theta = 0$ yields,

$$\tan 2(\theta - \rho_3) = -\cot \delta_2 = \tan(\delta_2 \pm \pi/2)$$

The upper sign corresponds to $I = 0$ i.e. extinction.

Since $\rho_3 = \rho_2 + \pi/4$, the fractional retardation of δ_2 is given by

$$\delta_f = 2(\theta - \rho_2) - \pi \quad (C6)$$

Let the rotation of analyzer measured from the principal axis of model be θ' then $\theta' = \theta - \rho_2$ and therefore

$$\delta_f = 2\theta' - \pi \quad (C7)$$

It is seen that the analyzer rotation θ' alone determines the fractional fringe order and therefore the disposition of the polarizer and polarizer-QWP does not affect the measurement of integral and fractional fringe orders. Further application of this new concept for the measurement of isoclinic has been reported in literature (Srinath 1971).

From (ii) it follows that in case where the retardation plates are not perfect QWP's but still they are balanced i.e. $\delta_1 = \delta_2 = \delta \neq \pi/2$, ρ_3 should be set to 135° so as to ensure that the integral fringe orders represent integral wave lengths of retardation (Mindlin 1937). This means the conventional polariscope arrangement with balanced wave plates. This is the set-up used in the present study.

If Tardy method is used to determine the fractional fringe order, in the system (ii), errors will result and should be corrected using the analysis shown in Appendix D.

APPENDIX D

ON THE ACCURACY OF GONIOMETRIC COMPENSATION
METHODS IN PHOTOELASTIC FRINGE-ORDER MEASUREMENTS

(Sathikh and Bigg 1972(1))

The following additional symbols are used:

N, N_f, N' = apparent integral, apparent fractional
and total fringe orders

Δ = error in the apparent fractional fringe
order

δ_f = apparent fractional retardation of the
model (radians)

δ_f' = error in retardation of model when θ
= 90 deg in compensation method (radians)

ϵ = error in isoclinic setting in compensation
method (radians)

ϵ_1, ϵ_2 = retardation errors in polarizer and
analyzer QWP's (radians)

With reference to equation B1 (Appendix B)
the following quantities define the two methods of
compensation:

Tardy Method:

$$\rho_1 = \pi/4, \rho_3 = 3\pi/4, \rho_2 = \pm \epsilon, \delta_1 = \pi/2 \pm \epsilon_1, \delta_3 = \pi/2 \pm \epsilon_2$$

Senarmont Method:

$$\rho_1 = \rho_3 = 0, \rho_2 = \pi/4 \pm \epsilon, \delta_1 = 0, \delta_3 = \pi/2 \pm \epsilon_2$$

The rotation of the analyzer for extinction for the two methods of compensation is given by:

$$2\theta = \tan^{-1} \frac{Y}{X} \quad (D1)$$

where for the Tardy method

$$\begin{aligned} X &= \cos^2 2\epsilon \sin \epsilon_1 \sin \epsilon_2 + \sin^2 2\epsilon \sin \epsilon_1 \sin \epsilon_2 \cos \delta_2 \\ &\quad - \sin 2\epsilon \sin \delta_2 \sin(\epsilon_1 - \epsilon_2) + \cos \epsilon_1 \cos \epsilon_2 \cos \delta_2 \\ Y &= \cos 2\epsilon [\sin 2\epsilon \sin \epsilon_1 (\cos \delta_2 - 1) + \cos \epsilon_1 \sin \delta_2] \end{aligned}$$

and for the Senarmont method

$$\begin{aligned} X &= \cos^2 2\epsilon \cos \delta_2 + \sin^2 2\epsilon \\ Y &= \cos 2\epsilon [\sin 2\epsilon \sin \epsilon_2 (1 - \cos \delta_2) + \cos \epsilon_2 \sin \delta_2] \end{aligned}$$

For an alternate Senarmont method which uses only the analyzer-QWP (Flynn 1970), a similar expression can be derived.

The apparent fractional fringe order and the error in it are respectively given by

$$N_f = \delta_f / 2\pi \quad (D2)$$

and

$$\Delta = N_f - (\theta - \frac{\pi}{2}) / \pi \quad (D3)$$

where

$$\delta_f = \delta_2 - 2\pi N \quad (D4)$$

It can easily be seen that the results of Jessop (1953) and Chakrabarti (1969) are special cases for $\epsilon = 0$ and $\epsilon_1 = \epsilon_2 = 0$ respectively. A preliminary analysis using the results of these special cases suggests the following criterion for choice of the method, for small values of the errors: for $|\epsilon_1| = |\epsilon_2| < 2|\epsilon|$ choose the Tardy method, and for $|\epsilon_1| = |\epsilon_2| > 2|\epsilon|$ the Senarmont method.

Fig.D1. Ratio of the maximum error in the Tardy method to the maximum error in the Senarmont method for matched QWP's and various values of the isoclinic setting error.

For two typical combinations of errors, the decision line is shown in Figs. D1 and D2. The region for the Tardy method is below and that of the Senarmont method is above this line. These figures show that the criterion for equal maximum error indicated by the preliminary analysis holds only for $\epsilon_1 = \epsilon_2 = 2 |\epsilon|$ but fails for $\epsilon_1 = -\epsilon_2 = 2 |\epsilon|$. An approximate criterion for the latter is $\epsilon_1 = -\epsilon_2 = |\epsilon|$ as can be seen from Figs. D1 and D2. The equal maximum, however, is smaller for $\epsilon_1 = \epsilon_2$ than for $\epsilon_1 = -\epsilon_2$. As shown in Fig. D3, the maximum error occurs at different fractional fringe orders depending on the relative magnitudes of the input errors and this error is not zero, in general, at $N_f = 0.5$ except in the special cases.

Fig. D2. Maximum error in the fractional fringe order for an isoclinic setting error of 4 deg.

For given nonzero values of ϵ and errors in the QWP's, the maximum error is smaller for $|\epsilon_1| < |\epsilon_2|$ than for $|\epsilon_1| > |\epsilon_2|$ as can be seen from Figs. D2 and D3. This means that the better QWP is to be used as the polarizer-QWP for best results. This conclusion is interesting in view of Jessop's conclusion that the better QWP is used as the analyzer-QWP when $\epsilon = 0$.

The practical utility of the present analysis depends on the knowledge of the errors ϵ , ϵ_1 and ϵ_2 . The practical range of the values of the system errors ϵ_1 and ϵ_2 have been reported in the literature (Jessop 1953, Brown 1950, Polaroid 1965). The experimental error ϵ can only be estimated. Definite values for ϵ are not available (Jessop 1953, Chakrabarti 1969, Frotch 1954).

On the assumption that the magnitudes of ϵ , ϵ_1 and ϵ_2 are available, the steps for choosing the better method on the basis of maximum error are as follows:

1. Use the better QWP as the analyzer-QWP and estimate the resultant error for both methods using equation (D1).
2. If the Tardy method gives a smaller error, the choice is over. However, interchange the QWP's for better results, vide conclusion on the choice of QWP's.

3. If the Senarmont method gives the smaller error, the choice is not yet decided, unless $\epsilon = 0$.
- (a) Note the Senarmont error.
- (b) Interchange the QWP's and estimate the error for the Tardy method. If this Tardy error is still higher than the Senarmont error noted in (a), then choose the Senarmont method; otherwise, use the Tardy method.

Fig. D3. Error in fringe order as a function of fractional fringe order for a typical isoclinic error and typical QWP errors.

Figure D3 shows a typical example of all these steps.

In view of the close control possible of ϵ within ± 0.3 deg as reported by Frocht (1954) and the expected range of ϵ_1 and ϵ_2 reported in the literature, it looks reasonable to conclude in favor of the Senarmont method. However, in view of the significant error magnitudes caused by ϵ , ϵ_1 and ϵ_2 in their practical range and that the actual error varies with the fractional fringe order itself, a reasonable way of improving the accuracy would be to use either method and to correct the measured fringe order rather than to establish the better method. The resultant error in either method can be estimated from equation (D1) when ϵ , ϵ_1 and ϵ_2 are known.

The total fringe order is given by

$$N' = N + \delta'_f/2\pi + N_f \quad (D5)$$

The conditions for minimum intensity, when $\theta = 90$ deg. are:

$$\tan\delta_2 = \frac{\sin 2\epsilon \sin (\epsilon_2 - \epsilon_1)}{\cos(\epsilon_2 - \epsilon_1) - \cos^2 2\epsilon \sin \epsilon_1 \sin \epsilon_2} \quad (D6)$$

for the Tardy method and

$$\cos^2 2\epsilon \sin \delta_2 = 0 \quad (D7)$$

for the Senarmont method. Also $\tan \delta_2 = \tan (2\pi N + \delta'_f) = \tan \delta'_f$ and hence $\delta'_f \neq 0$ for the Tardy method in general and $\delta'_f = 0$ for the Senarmont method.

APPENDIX E

A COMPLETE GEOMETRICAL CONSTRUCTION
METHOD FOR LARGE STRAIN ANALYSIS

For two dimensional large strain with no rotation the normal Lagrangian strains are defined as

$$\lambda_x = (1 + \epsilon_x)^2 = (1 + \frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial x})^2 \quad (E1)$$

$$\lambda_y = (1 + \epsilon_y)^2 = (1 + \frac{\partial v}{\partial y})^2 + (\frac{\partial u}{\partial y})^2 \quad (E2)$$

$$\text{The shear strain } \gamma_{xy} = \frac{\pi}{2} - \beta \quad (E3)$$

where β is the final size of an angle which was initially a right angle and is defined as

$$\sin \gamma_{xy} = \frac{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + (\frac{\partial u}{\partial x})(\frac{\partial u}{\partial y}) + (\frac{\partial v}{\partial x})(\frac{\partial v}{\partial y})}{(1 + \epsilon_x)(1 + \epsilon_y)} \quad (E4)$$

The principal values of λ 's are denoted as λ_1 and λ_2 .

The angle measured from the direction of λ_x to that of λ_1 is θ . Then the principal values λ_1 and λ_2 , the maximum value of γ_{xy} and θ can be found (Novozhilov 1953, Meyer 1963, Brill 1965, Durelli 1970(1), Choi 1971) as

$$\lambda_{1,2} = \frac{\lambda_x + \lambda_y}{2} \pm \sqrt{\left(\frac{\lambda_x - \lambda_y}{2}\right)^2 + \lambda_x \lambda_y \sin^2 \gamma_{xy}} \quad (\text{E5})$$

$$\cos 2\theta = (\lambda_x - \lambda_y) / (\lambda_1 - \lambda_2) \quad (\text{E6})$$

$$(\sin \gamma_{xy})_{\max} = (\lambda_1 - \lambda_2) / (\lambda_1 + \lambda_2) \quad (\text{E7})$$

Equation E5 can be rewritten as

$$\lambda_{1,2} = \frac{\lambda_x + \lambda_y}{2} \pm \sqrt{\left(\frac{\lambda_x - \lambda_y}{2}\right)^2 - \lambda_x \lambda_y \cos^2 \gamma_{xy}} \quad (\text{E8})$$

Invariants:

$$\lambda_1 + \lambda_2 = \lambda_x + \lambda_y = \text{I}$$

$$(\lambda_1 - \lambda_2)/2 = \sqrt{\left(\frac{\lambda_x - \lambda_y}{2}\right)^2 - \lambda_x \lambda_y \cos^2 \gamma_{xy}} = \text{II}$$

As $(\lambda_x + \lambda_y)$ is invariant in II,

$$\lambda_x \lambda_y \cos^2 \gamma_{xy} = \text{II(a)}$$

is also an invariant.

If λ_x , λ_y and γ_{xy} are known from experiment, analytically λ_1 , λ_2 , θ and $\gamma_{xy \max}$ can be determined.

However, a geometrical construction method - a Mohr's circle - is thought as a very good aid to explain the transformation of strains. Before a Mohr's circle can be drawn the quantity $\sqrt{\lambda_x \lambda_y} \cos^2 \gamma_{xy}$ should be computed. The following complete geometrical method obviates such a computation.

Geometrical method:

Refer to Figure E1. (1) Draw $OA = \lambda_y$; $OB = \lambda_x$. (2) Find C the midpoint of AB; therefore $OC = (\lambda_x + \lambda_y)/2$; also $CA = \frac{\lambda_x + \lambda_y}{2} - \lambda_y = (\lambda_x - \lambda_y)/2$. Note OC is the first term of equation E5. (3) Draw an arc with C as centre and OC as radius. (4) Draw $AD \perp OA$, D lying on the arc drawn in step 3;

$$\begin{aligned} AD^2 &= CD^2 - CA^2 = OC^2 - CA^2 \\ &= \left(\frac{\lambda_x + \lambda_y}{2} \right)^2 - \left(\frac{\lambda_x - \lambda_y}{2} \right)^2 = \lambda_x \lambda_y \end{aligned}$$

(5) Draw semicircle on diameter AD. (6) With the known value of $\gamma_{xy} = \angle ADE$, draw DE to cut at E the semicircle of step 5. (7) Draw an arc with centre at A and radius AE to meet AD at F. Note $AE = AF = AD \sin \gamma_{xy} = \sqrt{\lambda_x \lambda_y} \sin \gamma_{xy}$. From triangle

$$ACF, CF = \sqrt{AC^2 + CF^2} = \left(\frac{\lambda_x - \lambda_y}{2}\right)^2 + (\sqrt{\lambda_x \lambda_y} \cdot \sin \gamma_{xy})^2$$

which is the second term in equation E5. (8) With C as centre and CF as radius draw circle to cut OX at H_1 and H_2 . Now $OH_1 = OC + CF = \lambda_1$ and $OH_2 = OC - CF = \lambda_2$. This last part only is the Mohr's circle construction. (9) To find θ , join CF. Now $CF_2 = CH_2 = (\lambda_1 - \lambda_2)/2$; $CA = (\lambda_x - \lambda_y)/2$. Therefore $CA/CF = (\lambda_x - \lambda_y)/(\lambda_1 - \lambda_2) = \cos 2\theta$ and hence $\angle OCF = 2\theta$. (10) To find $\gamma_{xy \max}$, draw a tangent from O to the Mohr's circle to touch it at G. Now $CG/CO = (\lambda_1 - \lambda_2)/2 / (\lambda_1 + \lambda_2)/2 = \sin \gamma_{xy \max}$ as given by equation E7; $\angle COG = \gamma_{xy \max}$. This corresponds to $\lambda'_x = \lambda'_y$ and $2\theta = 90^\circ$; $D \equiv D'$ and $F \equiv F'$. This can be viewed in a different way: $\cos \gamma_{xy} = DE/AD$; but $DE = \sqrt{\lambda_x \lambda_y} \cos \gamma_{xy}$ is an invariant = II(a). $\cos \gamma_{xy}$ is minimum when γ_{xy} is maximum. $|DE|$ cannot change during transformation. Therefore γ_{xy} is maximum only when AD is maximum; but the maximum possible value of $|AD| \equiv CD' = OC = (\lambda_1 + \lambda_2)/2$; this corresponds to $\lambda'_x = \lambda'_y$ and $2\theta = 90^\circ$.

The data available may be either

$\lambda_i = (1 + \epsilon_i)^2$ or $(1 + \epsilon_i) = \sqrt{\lambda_i}$, ($i = x, y$). The principal elongation ratio also is obtained in the

form $\lambda_i = (1 + \varepsilon_i)^2$, ($i = 1, 2$) whereas one is interested in $(1 + \varepsilon_i) = \sqrt{\lambda_i}$. A relation between λ_i 's and $\sqrt{\lambda_i}$'s, ($i = x, y, 1, 2$) can be geometrically established. The points A and B which represent λ_y and λ_x in Figure E1 can be fixed geometrically, if $\sqrt{\lambda_y}$ and $\sqrt{\lambda_x}$ only are known; or the latter can be found from the known values of the former. This is illustrated in Figure E2, for a case where $\lambda_y < 1$ i.e. compressive and $\lambda_x > 1$ i.e. tensile; this part of the construction also can be found in literature (Choi 1971).

Assume $(1 + \varepsilon_x)$ and $(1 + \varepsilon_y)$ i.e. $\sqrt{\lambda_x}$ and $\sqrt{\lambda_y}$ are known. Locate K so that $OK = 1$. Draw semicircle on OK and locate L on this semicircle so that $OL = \sqrt{\lambda_y}$. A perpendicular to OK drawn through L to meet OK locates A. OA gives λ_y as verified below.

$$\begin{aligned} (AL)^2 &= OA \cdot AK = OA(OK - OA) \\ &= OA - (OA)^2 ; \quad (OK = 1) \end{aligned}$$

Also

$$(AL)^2 = (OL)^2 - (OA)^2$$

Therefore

$$OA = (OL)^2 = \lambda_y.$$

To locate B, draw the perpendicular KM to OK. Locate M so that $OM = \sqrt{\lambda_x}$. A semicircle drawn through O and M with centre on OK (extended) gives B ie $OB = \lambda_x$; alternatively, it means the drawing of the perpendicular MB to OM at M to meet OK at B. Now

$$(KM)^2 = OK.KB = (OB - 1)$$

Also

$$(KM)^2 = (OM)^2 - (OK)^2 = (OM)^2 - 1$$

Therefore

$$OB = (OM)^2 = \lambda_x.$$

Similarly it can be shown that $ON_1 = \sqrt{\lambda_1}$ when $OH_1 = \lambda_1$ and $ON_2 = \sqrt{\lambda_2}$ when $OH_2 = \lambda_2$. This illustrates a complete geometrical construction method for large strains.

APPENDIX F

ERROR ANALYSIS

Probable error in direct measurement of a single quantity

The normal distribution law of errors of observation is strictly applicable to the case of infinite number of observations i.e. random population, of a particular quantity when there are a number of independent sources of errors of equal importance. However, it is reasonable to assume its validity for practical cases (Wilson 1952, Ch.9; Paradine 1953, Ch.9). Given a moderate number (n) of observed values (x_i 's) of a quantity x , the sample mean $\bar{x}$ is the best available estimate of the true mean μ , and then the best estimate of standard deviation σ is

$$\sigma_e^2 = \Sigma [(x_i - \bar{x})^2 / (n - 1)] \quad (F1)$$

The probable error e is so defined that the probability of an error numerically less than e is 50 per cent in the long run. This means e is 0.6745σ . In the present case, the probable error of the mean $\bar{x}$ for n observations is

$$e_{\bar{x}} = 0.6745 \frac{\sigma_e}{\sqrt{n}} = 0.6745 \sqrt{\frac{\sum (x_i - \bar{x})^2}{n(n-1)}} \quad (F2)$$

Making use of equation (F2), the probable error in the mean value of a particular quantity measured, is estimated.

Compounding of probable errors

If a quantity z must be calculated from m quantities y_i 's whose estimated values are liable to error as discussed above, the probable error in z can be calculated as below. Let z be a known function f of y_i 's.

$$z = f(y_1, y_2, \dots, y_m) \quad (F3)$$

Let y_i 's are independent of each other and the probable errors in them are very small. Then

$$\delta_z = \sum_i \frac{\partial f}{\partial y_i} \cdot \delta y_i \quad (F4)$$

and independency of y_i 's further, leads to

$$(\delta_z)^2 = \sum_i \left(\frac{\partial f}{\partial y_i} \cdot \delta y_i \right)^2 \quad (F5)$$

For the present case of moderate number of observations, δ 's are replaced by the probable errors.

$$e_z = \sqrt{\sum \left(\frac{\partial f}{\partial y_i} \cdot e_{y_i} \right)^2} \quad (F6)$$

The fractional error gets very much simplified as

$$(e_z/z) = \sqrt{\sum (e_{y_i}/y_i)^2} \quad (F7)$$

With the help of the probable errors of individual quantities which are either estimated from several observations as discussed in the preceding section or known from previous records or experience, the combined probable error in z can be estimated.

<u>Estimated Probable Errors</u>			
Quantity measured	Instrument used	Least count (L.C)	Probable error e
Thickness, d_o	Dial gauge	.0001"	.001"
Width b	Vernier caliper	.001"	.005"
Gauge lengths l_i, l_f	Vernier caliper	.001"	.005"
	Profile projector	.0001"	.001"
Elongation Δ	Instron strain gauge Extensometer	.00001 at 1% range $\equiv$ 0.1" on chart	.0001"
Dead weight P	Instron load cells	Variable amplifier provides L.C of 1% of range chosen	a) Direct load 2% b) Lever load 2% + 0.5 pound
Fractional fringe order	"Riken" Polariscope Tardy method	1°	10° (eye judgement to locate minimum intensity at boundary)

Examples of compounding of probable errors

Strain

$$\epsilon_1 = (l_f - l_i) / l_i = \Delta / l_i$$

$$e_\epsilon / \epsilon_1 = \sqrt{\left(\frac{e_\Delta}{\Delta}\right)^2 + \left(\frac{e_{l_i}}{l_i}\right)^2}$$

(a) l_i measured by Vernier caliper and Δ by Instron extensometer

For 1% strain $\Delta = 0.5/100$; $e_\Delta = .0001$

$$l_i = 0.5 \quad ; \quad e_{l_i} = .005$$

$$\begin{aligned} e_\epsilon / \epsilon &= \sqrt{\left(\frac{.0001}{.5} \times 100\right)^2 + \left(\frac{.005}{.5}\right)^2} \\ &= \sqrt{(.02)^2 + (.01)^2} \\ &= \frac{\sqrt{5}}{100} = 2.25\% \end{aligned}$$

For greater strains, the amplification is reduced and so is the least count of Δ measurement. However, the stability of the amplifier improves and therefore, the same level of accuracy is possible.

(b) l_i and l_f are measured by profile projector: $e_{l_i} = .001$

For $l_i = .02$ and $\epsilon = 1\%$, $l_f = .02$

$$e_\epsilon / \epsilon = \sqrt{2} \left(\frac{.001}{.02}\right) \equiv 7\%$$

For larger strains this error will decrease.

Stress

Polycarbonate

Direct loading

$$\sigma = P/bd_o; \quad d_o = 0.07; \quad b = 0.375;$$

$$e_p = 0.01P; \quad e_d = .001; \quad e_b = .005$$

$$e_\sigma/\sigma = \sqrt{(.02)^2 + \left(\frac{.001}{.07}\right)^2 + \left(\frac{.005}{.375}\right)^2}$$

$$= 2.7\%.$$

Lever loading

$$\text{for } \sigma = 1000 \text{ psi, } P = 7 \times 3.75 = 26.25$$

$$e_p = 0.525 + 0.5 = 1.025$$

$$e_\sigma/\sigma = \sqrt{\left(\frac{1.025}{26.25}\right)^2 + \left(\frac{.001}{.07}\right)^2 + \left(\frac{.005}{.375}\right)^2}$$

$$\approx 4.7\%.$$

Epoxy

Direct load only

$$d_o = 0.05; \quad b = 0.1875$$

$$e_\sigma/\sigma = 3.65\%.$$

Young's Modulus:

$$E = \frac{\sigma}{\epsilon} = \frac{Pl_i}{bd_o \Delta} \quad (\text{Direct load})$$

For polycarbonate $\sigma = 1000 \text{ psi}$, $\Delta = .0015''$.

$$e_E/E = \sqrt{(.02)^2 + \left(\frac{.001}{.07}\right)^2 + \left(\frac{.005}{.375}\right)^2 + \left(\frac{.005}{.5}\right)^2 + \left(\frac{.0001}{.0015}\right)^2}$$

$$\equiv 7.2\%.$$

For epoxy $\sigma = 1000 \text{ psi}$, $\Delta = .0022''$ at room temperature.

$$e_E/E = 4.7\%.$$

For epoxy, modulus corresponding to frozen strain

$$E = \sigma/\epsilon_F$$

$$e_E/E = \sqrt{\left(\frac{e_\sigma}{\sigma}\right)^2 + \left(\frac{e_{\epsilon_F}}{\epsilon_F}\right)^2}$$

$$\sigma = 1000 \text{ psi}, \quad \epsilon_F \approx 1\%$$

$$e_E/E = \sqrt{(3.65)^2 + (7.0)^2} \%$$

$$= 7.8\%$$

Poisson's ratio: $\nu = \epsilon_2/\epsilon_1$ (Epoxy, profile projector)

$$\begin{aligned} e_\nu/\nu &= \sqrt{(e_{\epsilon_1})^2 + (e_{\epsilon_2})^2} \\ &= \sqrt{2} \times 7 = 9.9 \text{ per cent.} \end{aligned}$$

Fringe order:

Integral fringes were counted directly from specimen or photograph. In polycarbonate, the integral fringe orders were very high making the error due to fractional fringe order measurement insignificant. However, for small number of integral fringes, error in fractional fringe order forms a significant part. In a typical case of the epoxy model for $\lambda = 0.473$, $N = 2.484$.

$$e_N/N = \frac{10}{180} \times \frac{1}{2.484} = 2.46\%.$$

Though the absolute probable error e_N is only $1/18$, the percentage error may be very high when the fringe order is very low.

Stress concentration factor:

This involves calculated average stress;
observed number of fringes and thickness of the model

i.e. stress from calibration.

$$e/K_{\sigma}^N = \sqrt{\left(\frac{e_{\sigma}}{\sigma_{av}}\right)^2 + \left(\frac{e_N}{N}\right)^2 + \left(\frac{e_d}{d}\right)^2}$$

Typical values: $\sigma_{av} = 1325$; $(\sigma_{max} = 1810)$; $N = 6.4$;
 $d_o = 0.05$.

$$e/K = \sqrt{\left(\frac{3.65}{100}\right)^2 + \left(\frac{10}{180} \times \frac{1}{6.45}\right)^2 + \left(\frac{.001}{.05}\right)^2}$$

$$= 4.25\%$$

For higher K_{σ}^N , the error should decrease.

Strain concentration factor: Similarly for the same case as above,

$$e/K_{\epsilon}^N = 4.25\%$$

This is the same as above because ϵ_{av}^N is obtained only from σ_{av}^N .

VITAE

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