Dark Matter, Higgs portals and Freeze-out

by

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A thesis submitted to the
Faculty of Graduate and Postdoctoral Affairs
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy in Physics

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Ottawa, Canada
August 27, 2019

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Abstract

Based on many astrophysical and cosmological observations, it is clear that the Universe contains some form of non-relativistic, non-interacting form of matter called Dark Matter (DM). The Standard Model of particle physics (SM) does not have a particle with all the suitable properties to explain all of these observations. We look to model the DM through different extensions of the SM ensuring that one or more new particles have all the desired properties. We study bounds on these models through theoretical constraints such as perturbative unitarity and stability of the scalar potential, as well as through experimental constraints such as direct and indirect detection. Of particular interest is the measurement of the relic density which allows us to make a direct connection between the parameters of our models and the amount of DM in the Universe today through a process known as freeze-out. We look at different freeze-out scenarios such as single component and multi-component DM, as well as alternative cosmological scenarios. We find that the parameter space for single component DM is severely constrained, while multi-component DM can escape bounds quite easily because of the many freedoms in the models. We find that changes in the cosmology in the early Universe can significantly change the requirements of a DM model, leading to much more weakly coupled or heavier DM.

Statement of Originality

Chapter 3 contains work originally published in Ref [1]. In this chapter, the author was solely responsible for all the calculations done.

Chapter 4 contains work originally published in Ref [2]. In this chapter, the author was responsible for verifying the perturbative unitarity constraints and the dark matter relic density estimation. The cross section and decay width calculations were done by Heather Logan, Andrea Peterson, and Robyn Campbell. The main ideas behind the paper were conceived by Heather Logan, Andrea Peterson, and Stephen Godfrey.

Chapter 5 contains work originally published in Ref [3]. In this chapter, the choice of model along with all related calculations were performed by the author. The calculations were verified by Stephen Godfrey.

Chapter 6 contains work originally published in Ref [4]. In this chapter, all the theoretical constraints calculations were performed by the author and based on related work by Heather Logan. The code for the numerical scan and the freeze-out calculation was written and run by the author. The calculations for the direct-detection cross sections were performed by Robyn Campbell and verified by the author. The calculations relating to the constraints from measurements Higgs boson properties were done by the author. The model was described by Heather Logan. All calculations were verified by Stephen Godfrey, Heather Logan, and Robyn Campbell.

Acknowledgments

There are many people I want to thank for their contributions to this thesis. Firstly, my graduate supervisor Stephen Godfrey who helped me understand all the topics related in this thesis and who pushed me to make my own choices in my research. I would like to thank Heather Logan who played in significant part in the first papers I was a part of. I want to thank all the others who have played a role in my research, either as a collaborator, or for helpful discussion. These are Robyn Campbell, Roozbeh Yazdi, Andrea D. Peterson, Keith Dienes, Brooks Thomas, Catarina Cosme, Máira Dutra, Thomas Grégoire, and Bruce Campbell. I would also like to thank everyone in the department who has played a positive role in my life at Carleton.

Finally, I would like to dedicate this thesis to my close friends and family who provide, knowingly or not, the motivation for me to strive towards self-improvement and happiness.

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List of acronyms, symbols, and constants

We use Natural units ($\hbar = c = 1$). The metric sign convention is $(+, -, -, -)$. Lowercase greek indices $(\mu, \nu, \dots)$ run from 0 to 3 and go over spacetime coordinates. We employ the Einstein summation convention where repeated indices are summed over ($p_i k^i = \sum_i p_i k^i$). A Lorentz index after a comma represents a derivative with respect to that index, i.e. $A^\mu_{,\nu} = \frac{\partial}{\partial x^\nu} A^\mu = \partial_\nu A^\mu$. A Lorentz index after a semi-colon represents the covariant derivative with respect to that index, i.e. $A^\mu_{;\nu} = A^\mu_{,\nu} + \Gamma^\mu_{\alpha\nu} A^\alpha$ where $\Gamma^\mu_{\alpha\nu}$ are the Christoffel symbols defined in Appendix A.

<u>Acronyms</u>	<u>Description</u>
SM	Standard Model of particle physics
DM	Dark Matter
QFT	Quantum Field Theory
vev	Vacuum expectation value
ATLAS	A Toroidal LHC ApparatuS
CMS	Compact Muon Solenoid
CERN	European Organization for Nuclear Research
LHC	Large hadron collider
BBN	Big Bang Nucleosynthesis
CMB	Cosmic Microwave Background
dSph	Dwarf Spheroidal Galaxies
EWSB	Electroweak Symmetry breaking
WIMP	Weakly interacting massive particle
VBF	Vector boson fusion

Chapter 1

Introduction

How does a typical person experience the world? They may wake up in the morning as the sunlight pierces through their window, get up and make a hot cup of coffee, and go to work. They then come home in the evening, perhaps watch some television, and rest, all to do it all over again the next day. The typical person ever only experiences the world with their own senses on a particular scale. They go through the day without ever truly experiencing the grandeur of the Universe or the strangeness of quantum mechanics; they would be used to Newtonian mechanics. When it was discovered that this is just an approximation of Einstein's theory of general relativity, it was clear that we should not throw out Newtonian mechanics. Similarly, we did not throw out classical electrodynamics when quantum electrodynamics was discovered. Although incomplete, these models turn out to often be an adequate way of describing the world around us. However, to make accurate predictions at different scales, it is important that we have better models for those scenarios.

Today, the two most successful models in physics describe two different scales: the quantum scale at very short distances, and the scales of General Relativity when dealing with very heavy systems or very fast systems. The intuitions we have built

about the classical world no longer hold at these scales, and yet, we are still able to describe events that occur at these scales. Using today's technology, we can probe these scales and gather information to and build up a new intuition about these scales and further probe them to advance our understanding.

So far, we have found four fundamental forces. Gravitation is described by General Relativity, while electromagnetism, the weak nuclear force, and the strong nuclear force are described by the Standard Model (SM) of particle physics. However, just like there are problems with Newtonian mechanics, there are problems with these as well. One such problem which will be a main topic in this thesis is dark matter (DM). DM is some form of energy density which behaves just like ordinary SM particles gravitationally, but which seems to interact very little otherwise. We know it is there from various measurements which we will discuss in section 1.3, but its nature is not yet known. There are many models physicists have concocted to try to explain DM such as modifications to gravity; dark stellar objects such as primordial black holes or brown dwarfs; and particle models, which will be the main focus of this thesis.

Ultimately, the scales in physics are artificial. They merely give us an idea of which models we can use to obtain adequately accurate descriptions of events without absurd amounts of computation. The Universe is expected to have a set of physical laws that apply at all scales. We know that our current models are only approximations, therefore a theory that describes everything at every scale would have as limiting cases the Standard Model, General Relativity, and classical mechanics.

With this being the case, physicists still hunt for more complete models. One's goal might not be to create a model which describes everything, but to work towards a model that can describe a little more than we can now, while still satisfying any

constraints. There are a few glaring problems that poke at our current models such as combining General Relativity with the Standard Model, some internal consistency issues with the SM such as the Hierarchy problem and the strong-CP problem; others are observations that cannot be explained with the current models such as neutrino masses, Dark Energy, and Dark Matter. In this thesis, we will look at different Dark Matter (DM) models and use experimental and theoretical results to put constraints on such models.

In this chapter, we will discuss the details of the SM. We will then introduce Dark Matter and discuss our current experimental results. Finally, we will discuss some cosmological probes of particle physics.

1.1 The Standard Model of Particle Physics

When the idea that the laws of physics should be modified for small distances arose, one of the first successful pictures was the picture of quantum mechanics from Schrodinger through his famous equation. It allowed us to describe measurements on small systems under certain conditions. There were a few problems though: this equation failed to account for the structure of space and time as described by Einstein and therefore could not accurately describe relativistic systems; one could not use this equation to describe particle annihilation or creation; and this equation did not account for a particle's spin. These problems eventually led to the idea of using quantum fields to describe the world. This new framework described particles as quantized excitations of underlying particle fields and interactions between particles and coupling two quantum fields together not so unlike a pair of coupled harmonic oscillators.

When working with a quantum field theory, one typically starts with an equation

called the Lagrangian density, typically referred to simply as the Lagrangian. It is at this point that the decisions are made as to the particle content and forces within the model by writing out all the allowed terms. Thanks to symmetries and theoretical considerations such as renormalizability, there will always only be a finite number of allowed terms. While writing out this equation, one introduces free parameters that must in principle be measured. In the model building process, these are what you can think of as our knobs to match the model to any piece of data. Just like a Gaussian distribution requires the correct mean and standard deviation to properly describe a set of measurements, so too does the parameters of the model need to be adjusted to fit all the data. If no suitable arrangement of parameters accurately describes the measurements, then the model can be abandoned as it cannot accurately describe the Universe.

Through a long process of experimentation and theoretical work, a particular model has arisen which can accurately describe almost all of the observations made in particle physics. This is the so-called Standard Model of particle physics (SM). It should be noted that this model is the culmination of many modifications and extensions which is easily done in the framework of quantum field theory (QFT). This is not unlike what we will do in later chapters when considering extensions to the SM to account for DM.

We will start with what the chemist knows. There are protons, neutrons, and electrons as particles, there is an electrical force keeping the electron bound to the clump of protons and neutrons, as well as a nuclear force binding the protons and neutrons. There must also be a third force called the weak nuclear force responsible for radioactivity. There are still many details which need to be fleshed out, so physicists at the time did what they were good at: they set up some scattering experiments.

What was revealed was that the protons and neutrons were composed of smaller, more fundamental particles called quarks. Motivated by this, experiments began to find other bound states of these quarks known as baryons for three quarks, or mesons, which are quark anti-quark pair bound states. In fact, new fundamental quarks were discovered besides the ones which made up the proton and neutron. A pattern emerged which gave hints of the structure of the force which bound the quarks.

There was another problem with nuclear physics. It was observed that some nuclei would undergo radioactive decay emitting a single electron. The problem was that this electron's energy seemed to follow a spectrum, different from what would be expected from a two-body decay where only a single energy value is allowed by energy conservation. This eventually led to the hypothesis that a third particle was also being emitted and taking away some of the energy in the reaction. Although far-fetched at the time, this would turn out to be exactly the case. The new particle was named the neutrino.

Related to these neutrinos is another particle which was found to be coming from the sky, the muon (μ). This particle seems to have all the same properties as the electron except that it is much more massive and as a result, it has enough energy stored in its mass to decay to lighter particles, making it unstable. It decays to an electron and a neutrino.

So far, the particles which were found were the up quark (u), down quark (d), the strange quark (s), the electron (e), the muon (μ), a neutrino (ν), and the photon (γ) which was known to be related to the electromagnetic force. At this point, it was clear that there seem to be some symmetry between the particles and the forces acting on them. This led to a description of the strong force using representations of the SU(3) symmetry group and the electroweak force, a combination of the weak and

electromagnetic forces, using representations of the $SU(2) \times U(1)$ symmetry group. By imposing that the Lagrangian obey these symmetry groups locally and not just globally, physicists were able to come up with a way to describe forces on the particles in the model by writing down gauge invariant terms in the Lagrangian. In this framework, forces are described by spin-1 particles in an adjoint representation of their respective groups. This ingenious way of writing down the model led to an easy way to expand the model: simply add new particles in the representation of the force that you wish that it would be charged under. When the tau (τ) particle, the various flavours of neutrinos, and the rest of the quarks were found, they were simply added to the model. In fact, because of this structure, some particles were theorized before they were discovered such as the top quark (t) and the tau neutrino (ν_τ).

There is one particle that has yet to be mentioned: the Higgs boson. Because of the symmetries that have been imposed, it is not possible to give a mass to some of the fundamental particles in the usual way. This is a problem because some particles, namely the weak force mediators $W^\pm$ and Z^0 bosons were known to be quite massive. To resolve this issue, the Higgs boson was hypothesized. At low temperatures, the Higgs field would acquire a vacuum expectation value (vev) which would spontaneously break the electroweak symmetry of the SM and give a mass to the weak force mediators. This is similar to the state of a ferromagnet before the individual dipoles pick an overall alignment: initially, the state is spherically symmetric, but then as the ferromagnet cools, they align and pick a particular direction, thus spontaneously breaking the symmetry. The Higgs boson was the last piece of the puzzle and in 2012, it was discovered in the ATLAS and CMS detectors at CERN [5, 6].

The particle content of the SM is summarized in Figure 1.1. Here we see six flavours of quarks, of which only the u and d quarks are commonly encountered every

Three generations of fermions						Bosons	
QUARKS	LEPTONS	I	II	III			
		<div><div><div><div><div>$\frac{2}{3}$</div><div>$\frac{1}{2}$</div></div><div>u</div><div>up</div><div>≈ 2.2 MeV</div></div></div><div><div><div><div>$\frac{2}{3}$</div><div>$\frac{1}{2}$</div></div><div>c</div><div>charm</div><div>≈ 1.28 GeV</div></div></div><div><div><div><div>$\frac{2}{3}$</div><div>$\frac{1}{2}$</div></div><div>t</div><div>top</div><div>≈ 173.1 GeV</div></div></div></div>	<div><div><div><div>0</div><div>1</div></div><div>g</div><div>gluon</div><div>0</div></div></div> <div><div><div><div>0</div><div>0</div></div><div>h</div><div>Higgs Boson</div><div>≈ 124.97 GeV</div></div></div>				
		<div><div><div><div>$-\frac{1}{3}$</div><div>$\frac{1}{2}$</div></div><div>d</div><div>down</div><div>≈ 4.7 MeV</div></div></div> <div><div><div><div>$-\frac{1}{3}$</div><div>$\frac{1}{2}$</div></div><div>s</div><div>strange</div><div>≈ 96 MeV</div></div></div> <div><div><div><div>$-\frac{1}{3}$</div><div>$\frac{1}{2}$</div></div><div>b</div><div>bottom</div><div>≈ 4.18 GeV</div></div></div>	<div><div><div><div>0</div><div>1</div></div><div>γ</div><div>photon</div><div>0</div></div></div>				
		<div><div><div><div>-1</div><div>$\frac{1}{2}$</div></div><div>e</div><div>electron</div><div>≈ 0.511 MeV</div></div></div> <div><div><div><div>-1</div><div>$\frac{1}{2}$</div></div><div>μ</div><div>muon</div><div>≈ 105.66 MeV</div></div></div> <div><div><div><div>-1</div><div>$\frac{1}{2}$</div></div><div>τ</div><div>tau</div><div>≈ 1.7768 GeV</div></div></div>	<div><div><div><div>± 1</div><div>1</div></div><div>W</div><div>W boson</div><div>≈ 80.39 GeV</div></div></div>				
		<div><div><div><div>0</div><div>$\frac{1}{2}$</div></div><div>ν_e</div><div>electron neutrino</div><div>< 2.2 eV</div></div></div> <div><div><div><div>0</div><div>$\frac{1}{2}$</div></div><div>ν_μ</div><div>muon neutrino</div><div>< 0.17 MeV</div></div></div> <div><div><div><div>0</div><div>$\frac{1}{2}$</div></div><div>ν_τ</div><div>tau neutrino</div><div>< 18.2 MeV</div></div></div>	<div><div><div><div>0</div><div>1</div></div><div>Z</div><div>Z boson</div><div>≈ 91.19 GeV</div></div></div>				
				GAUGE BOSONS		SCALAR BOSONS	
<div><div><div>charge</div><div>spin</div><div><i>symbol</i></div><div>name</div><div>mass</div></div></div>							

Figure 1.1: Particle content of the Standard Model.

day. They bind together through the strong force mediated by the gluon (g) to form the protons and neutrons which make up nuclei. They also have somewhat strange looking electric charges of $2/3$ for up type quarks (u , c , or t) or $-1/3$ for down type quarks (d , s , or b). In Figure 1.1, we also see six types of leptons. The commonly seen electron is among them along with its heavier cousins the μ and the τ . In the lepton family, there are also the neutrinos which are electrically neutral and very light. Although there are no mechanisms in the SM to give these particles mass, we know they must be massive thanks to neutrino oscillation experiments. Next, we see that there are four vector particles, the gauge bosons, in the SM. The weak force has a total of three mediators: two charged bosons called the W^+ and W^- bosons, and one neutral boson called the Z boson. The $W^\pm$ allows for interactions between flavours of fermions, e.g. $t \rightarrow bW^+$, and between generations, e.g. $b \rightarrow cW^-$. All the fermions in the SM, as well as the $W^\pm$, Z^0 , and h interact via the weak force. The γ is the well-known photon which mediates the electromagnetic force. It interacts with anything that is charged, i.e. all the quarks, the charged leptons, and the $W^\pm$. The gluon, as mentioned before, is the mediator for the strong force and only couples to the quarks and itself. The Higgs boson couples to every massive particle with a coupling that is proportional to their mass, except for neutrinos since they cannot get their mass from the h . This means that the h couples very strongly to the top quark, itself, and the weak gauge bosons, and then moderately so to the b , c , and τ .

At this point, one may be wondering where the three generations of fermions come from, why the masses of the fermions are different between generations, why the masses of neutrinos cannot be explained in the SM, what determines the values for the masses of any of the particles, why there are not more forces, how gravity can fit in this picture, etc. These are still open questions and we do not have definitive

answers. The values of the SM were measured to be what they are and there is no fundamental reason why they take on their specific values, as far as the SM is concerned. We will not look to answer these questions here. Instead we look to solve the problem of DM.

1.2 Cosmology

For millennia, people around the world have looked up at the sky in wonder. Civilizations around the world have come up with different stories and beliefs, from the Babylonians to the Egyptians to Middle age Europe, the stars in the sky instilled a poetic humbleness on all who gazed up at the sparkling vastness of the unpolluted night sky. Before the telescope, people were not able to come up with scientific theories to explain phenomena we take for granted today: the day/night cycle, seasons, stars, etc. In some ways, this was a beautiful ignorance that allowed people to fill in the gaps with whatever that inspired them. However, living in ignorance is not what scientists are in the business of doing. In the renaissance, when Galileo popularized the telescope, real answers were starting to arise and replace the old beliefs.

We have come a very long way since the times of the flat Earth and geocentric beliefs of the day. We know that the Earth is spherical¹, that orbits around the Sun, which is just a small part of a larger galaxy, which in turn is part of a cluster and supercluster of galaxies. We know enough details about the history of the Universe, gathered through more and more precise observations and with enough statistics, to make meaningful models of its evolution. Where not that long ago we were only able to focus on astrophysics, the study of the machinery of celestial bodies, we now can look at cosmology, the study of the statistical properties of the Universe and its

¹The Earth is actually an oblate spheroid.

evolution.

One good question to ask is how the Universe was created. This used to be the domain of philosophy and religion, but now physics has a say. It was the observations of Edwin Hubble in the early 20th century that showed that the Universe is expanding. This was done by noticing that light coming from further away was redshifted meaning that the wavelengths of the light coming from far away was stretched by the Universe's expansion. Moreover, it appeared that all the far away galaxies were moving away from each other at a rate proportional to their distance. Running time in reverse would suggest that at some point, the Universe started out at a single point which then started expanding. The initial expansion of this point is now called the Big Bang: what we think of as the beginning of space and time itself. The exact physics that led to the Big Bang are still not known and will more likely require a theory of everything to explain. Even if we cannot explain the mechanism behind the initial Big Bang or the few moments immediately following it, we have models that can explain phenomena afterwards. Some key events and periods are listed in Table 1.1.

The Planck epoch is characterized by the period when it is expected that gravity and the SM forces were unified. Because we lack a quantum description of gravity, our models lack any predictive power for this period. After gravity separates from the other forces, there is a period of unification between the forces in the SM. The first of the SM forces to break off is the strong force. After this occurs, there is a period of superluminal expansion of spacetime called inflation. This period was contentious at first, but more recent observations require this period to exist to explain the horizon problem². During inflation, the Universe expanded by a factor of about 10^{26} which

²The horizon problem refers to the fact that although opposite points in the CMB would not have been in thermal contact in the early Universe, they still have the same temperature. This can be explained by inflation by introducing a time period when they would be in thermal contact, followed by a superluminal expansion out of each others causal horizons.

Table 1.1: The history of the very early Universe

Time	Temperature (Energy)	Description
$< 10^{-43}$ s	$> 10^{32}$ K (10^{19} GeV)	The Planck epoch: all currently accepted models of physics break down in this epoch.
$\sim 10^{-36}$ s	$\sim 10^{29}$ K (10^{16} GeV)	End of Grand Unification: the unification of the strong force and the electroweak force ends.
$\sim 10^{-32}$ s	$\sim 10^{28} - 10^{22}$ K ($10^{15} - 10^9$ GeV)	Inflationary epoch: The Universe expands by an order of about 10^{26} cooling the Universe, followed by a period of reheating.
$\sim 10^{-11}$ s	$\sim 10^{15}$ K (10^3 GeV)	Electroweak symmetry breaking: The Higgs bosons gains its vacuum expectation value and gives mass to the particles. The weak force is now distinct from the electromagnetic force.
$\sim 10^1 - 10^3$ s	$\sim 10^9 - 10^7$ K ($100 - 1$ keV)	Big Bang nucleosynthesis: The temperature of the photon bath has cooled below the binding temperature of nucleons. Light nuclei are formed.
$\sim 10^{13}$ s	~ 4000 K (0.4 eV)	Recombination: the temperature of the photon bath has cooled below the binding temperature of electrons. Electrons can now bind to nuclei resulting in the Universe becoming transparent.

led to a quick cooling followed by a reheating period. After inflation, the SM underwent electroweak symmetry breaking where the Higgs boson acquired its vev and the particles of the SM acquired their masses. As the Universe cools, quarks can eventually bind together without being broken apart. It is a similar story once the photons cool past the binding energy of nucleons and electrons, called Big Bang Nucleosynthesis (BBN) and recombination respectively. Once recombination occurred, the Universe became transparent to the photons and they started free streaming. This residual photon background can be measured today as the Cosmic Microwave Background (CMB). This is a uniform measurement of 2.725 K photons coming from all directions. From the time of recombination to today, there was the formation of stars, galaxies, and structure.

Now that we have an idea of what happened in the very first moments of the Universe, we may ask about the dynamics of the Universe throughout its history. We know that the Universe is expanding and that the rate of expansion depends on the energy density in the Universe and how it dilutes. We can imagine that keeping track of distances which keep changing can be a headache, so we introduce a factor that can keep track of that for us: the scale factor $a(t)$. This function of time tells us how distances between objects change as a function of time compared to the distance today, i.e. $a(\text{today}) = 1$, $a(\text{past}) < 1$, $a(\text{future}) > 1$. For example, if a system is 100 Mpc away today, then, at the distance when the Universe was 1 billion years old will be $a(10^9\text{yr})100$ Mpc. We can therefore look at the history of this function $a(t)$ to determine the history of the expansion of the Universe.

There are three main kinds of energy densities after inflation that we categorize by how fast they dilute. For relativistic particles, as the Universe expands, the density of particles will go down by a factor of a^{-3} , and the wavelengths will get stretched

by a factor of a^{-1} , for a total of a^{-4} . Non-relativistic matter, which we can think of as dust, will only dilute by a factor of a^{-3} . This means that radiation will dilute faster than dust. Lastly, there is the mysterious energy density called Dark Energy. This can be thought of as the energy inherent to the vacuum of spacetime itself. As space expands, this energy density does not dilute. Initially, the energy content of the Universe was dominated by radiation. Just before recombination, the energy density of the Universe was about equal parts radiation and non-relativistic matter. This is known as matter-radiation equality. Much later, the energy density of the Universe became dominated by Dark Energy.

Although we have a good idea of how a behaves after BBN, we do not have any observation from before that. We therefore cannot dismiss the possibility that some of the details may differ from what was mentioned so far. Namely, in this thesis, we look at DM which was produced during the periods between electroweak symmetry breaking and BBN. If the scale factor behaved differently than expected, then this will affect the density of DM that we see today. This is explored in [chapter 4](#) of this thesis.

1.3 Dark Matter

Imagine recording measurements and having the data completely disagree with the accepted theories at the time. A physics student may think back to the early years of their undergraduate degrees or a lab experiment gone wrong, where what was supposed to be a linear plot looked more like a scatter plot. To them, the data made no sense and they could not figure out what was wrong: the theory or the experimental methods. In the case of the student, it is no doubt the experimental methods. In an undergraduate lab experiment, the experiment is repeated hundreds, if not thousands

of times and for most of those cases, the correct conclusion was found. In the case of early astrophysicists, obtaining a strange result could very much go either way.

In the 1930s, it was thought the amount of matter in a region of the Universe was proportional to the amount of light coming that region. Not a completely crazy idea as most of the matter was thought to come from light sources such as stars and galaxies. However, when the observations by Jan Oort [7] and Fritz Zwicky [8, 9] on clusters of galaxies revealed hints of additional matter, it was hard to say whether this was an experimental error or a genuine novelty. Their results would be viewed with skepticism, as they should, until more experiments could be done to verify their results. After all, at the time there were no known sources of invisible matter that could account for the missing matter suggested by Oort and Zwicky. It would not be until the observations of Kent Ford and Vera Rubin [10, 11] on the velocity of stars in galaxies that the idea of a non-luminous, or *dark*, component to the matter density would be more accepted. Since the 1970s, the existence of DM has been needed to explain several other observations.

Although there are still many mysteries relating to DM, there have been hints of its properties throughout the decades. The observation of DM in galaxy clusters and in rotation curves of galaxies suggests that it interacts gravitationally. From this, we have even been able to determine the local DM density which stands at about 0.4 GeV/cm^3 . Another consequence of its gravitational interactions is that we expect to see it using gravitational lensing. This is the process in which light is bent due to large masses curving spacetime. Indeed, observations of colliding clusters of galaxies such as the bullet cluster show that the ordinary matter such as stars and dust slow down in the collision while the DM of each cluster seem to pass right through each other [12]. This tells us something about the way that DM interacts,

or more specifically, does not interact, with itself or ordinary matter. We can use these observations to put constraints on how we expect the DM to behave. We now expect the DM to have very little non-gravitational interaction with itself or other SM particles.

In the early Universe where the content of the Universe was a hot soup of subatomic particles, DM was there to leave its mark. One of the earliest observations we have is the CMB. The CMB can tell us quite a bit about what was happening in the Universe while it was still opaque, and itself, has raised some questions when it was found. Because the temperature of the CMB is about the same everywhere, how can one side of the Universe have been in thermal contact with the other side? This was called the horizon problem and has led to the commonly accepted solution of cosmic inflation. This explains the problem by allowing the Universe to be in thermal contact and then expanding at a superluminal rate, pushing everything out of thermal contact, and then having a reheating period. During the inflationary era, quantum fluctuations are believed to have created slightly more/less dense regions. Consider now the dynamics of what happens: the over dense regions have a larger gravitational attraction to the matter around it and grows, in part thanks to the presence of DM. However, things are still very hot, and the compression of baryons and photons create an outward pressure that counters this attraction. The back and forth is analogous to sound waves in the plasma and are called baryon acoustic oscillations (BAO). These oscillations leave their marks in the CMB as very tiny anisotropies which can be measured. With modern measurements of the CMB, we have been able to determine that the amount of DM needed to account for these oscillations is the same as the amount measured in clusters of galaxies. We thus conclude that what is responsible for driving BAO is also responsible for the excess of mass in the clusters

of galaxies. Furthermore, this tells us that DM must have been present at the time of recombination and puts further limits on its particle interactions since it must not significantly contribute to the outward pressure in BAO.

We know that DM was present in the Universe when it was still young. We can now ask what its effect was on structure formation, the process in which large-scale structures such as superclusters of galaxies form. If DM was there, then the additional gravitational interaction should play a role. Using computer simulations, we can see how the Universe would look if we model the DM in various ways. We could imagine changing the density or the temperature, i.e. the velocity, and see what happens. These simulations suggest that the required density be approximately the same as the density found in other measurements. This is a good sanity check of what we have found so far. More interestingly, the simulations show that the DM must be non-relativistic to allow it to be gravitationally bound. Otherwise, it would smear out smaller scale structures. We conclude that DM is cold. This is important because the SM has a family of particles that can fit all the requirements placed on DM so far: the neutrinos. This is the piece of evidence that allows us to say that the DM is not composed of SM neutrinos. We know they were in thermal contact with the SM model in the early Universe and we can calculate their temperature as a function of time after they decoupled. The result is that they are simply too fast to account for the structures observed in the Universe.

There are other pieces of evidence that could be mentioned, but at this point, we have an idea of what DM is: a cold, electrically neutral massive particle which interacts via gravity, and weakly or not at all with the SM particles or itself. Furthermore, we have an idea of the DM abundance in the Universe and locally³. For recent

³It should be mentioned that there are some non-particle based models. We do not discuss these further as they are significantly different than the particle-based models and their experimental searches are quite different.

reviews of DM, see Refs. [13–17]. We should also point out some of the properties we do not know and other questions relating to DM: mass, spin, gauge structure, particle interactions, whether it is composite, how it was created in the early Universe, whether it was involved in other problems like baryon asymmetry, why the density of DM is a factor of order one different than the SM matter⁴, the distribution of DM in galaxies, etc. That is quite a few things we would like to know about DM, and unfortunately at this point, we have more questions than answers. The effort to answer these are two-fold: theoretical modelling of DM to try to find plausible models which can help answer these questions, and experimental efforts to find the answers to these questions directly through measurement. They are by no means independent as theorists take inputs from experimentalists to constrain their models while experimentalists take input from theorists for what to look for.

It is worth looking at the general theoretical avenues for detecting DM. Figure 1.2 shows broadly the types of processes that can lead to a detection. If two DM particles annihilate into SM particles, then we could in principle look for the resulting SM particles. This is known as indirect detection and requires a large density of DM, more than can be found on Earth. For these types of observations, we must look at places like the centre of galaxies, or so-called dwarf spheroidal galaxies (dSph) which are known to contain large amounts of DM. One example of an indirect detection comes from the FERMI-LAT experiment which looks for deviations in the gamma ray spectra of dSph which could have resulted from a possible additional DM contribution [18]. Another example of an indirect detection observation comes from the AMS-02 experiment which looked for an excess of antiprotons, positrons, or gamma rays in our own galaxy which could originate from DM. This is a difficult measurement as it is not always known how much DM is in the observed region, or how much

⁴This is referred to as the cosmic coincidence problem (CCP).

Figure 1.2: Possible avenues for detecting DM.

matter is between us and the observed region, or even if an anomaly is caused by DM. Nevertheless, they are a vital tool when looking for DM as there are no other observations which look at high densities of DM.

Another interesting way to look for DM is if two SM particles were to collide to produce DM. This initially seems like a great way to look for DM since we can control these kinds of processes in colliders. We are capable of colliding vast amount of SM particles together at incredibly high energies so surely this must be the best way for finding DM. Unfortunately, there are several issues that arise when looking for DM this way. The most obvious is that even if you produce DM, it will pass right through your detector! This means that you may have produced DM, but from the point of any kind of data collection system you have, nothing interesting has happened, and the event is not saved. What is needed is a way to trigger the data collection system to save the event, and these conditions are conveniently called triggers. If an event passes a trigger, then it will be saved. We therefore need to find a trigger which will allow us to save these kinds of events and the usual one is to look for large missing transverse momentum. The missing transverse momentum tells us that there was an invisible particle that escaped the detector without interacting with

the detector. This is a good start, but then those pesky neutrinos can behave just like DM in this situation and can contribute significantly to backgrounds. Since DM interacts very weakly if at all, neutrinos have the potential to completely overshadow any kind of signal. However, collider experimentalists are always looking for other triggers, cuts, and analyses that could help in the search for DM. They present a very complementary way of looking for DM and if more was known on the nature of DM, colliders could have much success narrowing down their search and obtaining more precise information on its nature.

Lastly, we could look at the case where the surrounding DM recoils off a SM particle, commonly called direct-detection. This is the case where we simply build a detector and search for collisions. This may seem as though we are just hoping for a stray DM particle to come and interact, but this method turns out to be very promising. Because we know the local DM density and assume that the DM is rotating along with the galaxy, we have an idea of the flux of DM in our detectors and so we can put bounds on the interactions cross sections of the DM and nucleons. A signal in one of these experiments would allow us to determine the mass of the DM and its interaction cross section which could then lead to measurements in other types of experiments. However, there is a catch with these types of experiments: the mass ranges that they can look at are quite limited, even though they look in a very well-motivated mass range. The problem is that we only know the energy density of DM, not its mass. If the mass is much larger than the target, then there is simply not enough DM flux in these detectors to have meaningful constraints. Conversely, if the mass is much lower than the target, they do not transfer enough energy when they recoil to be detected. Improvements for these experiments tend to focus on increasing run times or increasing the target's fiducial region, which also increases

the cost. However, new ideas for direct-detection experiments such as using existing neutrino experiments could prove quite fruitful [19].

Other experiments which may fit in the three categories may look significantly different from those mentioned as specialized searches for specific models arise. For example, some experiments looking for axion DM, a very light particle motivated by the strong CP problem, have their own unique search methods using the Primakoff effect⁵. There are also some searches that do not fit nicely into these three categories such as looking for DM in invisible Higgs decays [20, 21].

One measurement we do have is the DM relic density, $\Omega_{0,CDM}h^2 = 0.120 \pm 0.001$ [22]. This number represents the amount of DM in the Universe and is more precisely defined in Chapter 2. This measurement has been a key part of model building as a robust model must be able to explain this observation with some production mechanism. A popular production mechanism that has been studied is freeze-out. In this scenario, the DM starts in thermal equilibrium with the SM at an early time but eventually decouples once the rate of the reactions maintaining chemical equilibrium become comparable to the expansion rate of the Universe [13–17, 23, 24]. This is the usual mechanism used for Weakly Interacting Massive Particles (WIMPs) models [13–17, 23, 24].

Ensuring that freeze-out produces the right amount of DM can fix some of the couplings in a model. This can lead to a model of DM being ruled out if the required couplings are too big to simultaneously avoid experimental bounds while producing enough DM. This has led to models with additional free parameters such as additional interactions between the dark sector and the SM, including multiple stable DM species, or considering a different production mechanisms altogether.

⁵The Primakoff effect allows for the conversion of photons to pseudoscalars and vice versa. This has historically been used to measure the masses of pseudoscalar mesons but could be used to detect the axion since it is also a pseudoscalar which can couple to photons.

However, WIMPs are very popular because of the so-called WIMP miracle where one could achieve the correct relic density through freeze-out with weak scale masses and couplings. Many extensions to the SM naturally or can easily accommodate a WIMP like particle. Because of this, it is important to understand and study freeze-out and how modifications to the standard cosmological picture could modify the freeze-out results.

This thesis is organized as follows. In Chapter 2, we review the basics of the SM and cosmology. In Chapter 3, we study the consequences of modified cosmological scenarios on DM freeze-out, providing both an analytic and numerical approach. In Chapter 4, we look at the implications of DM self-interaction on simple DM models. We then consider a slightly more complicated model and find that it can escape experimental bounds while generating enough DM through freeze-out in the early Universe. In Chapter 5, we discuss the hidden SU(3) model which has multiple stable DM species which undergo freeze-out. In Chapter 6, we look at the constraints and phenomenology of the Georgi-Machacek (GM) model extended by a real singlet scalar which acts as DM. Finally, in Chapter 7, we summarize all of our results. In the appendices, we provide a short derivation of the Friedmann equations used in Chapter 2 (Appendix A), the details for the choice of $\lambda_4 = -\lambda_5$ for the hidden SU(3) model of Chapter 5 (Appendix B), and the Feynman rules which involved the singlet scalar in our extension of the GM model of Chapter 6 (Appendix D).

Chapter 2

Background

Before jumping into what is new in this thesis, this chapter serves to introduce the necessary background material in particle physics and cosmology.

2.1 Particle physics

In this section, we will be discussing how one typically formulates a particle physics model, along with some of the basic computations that are done. This section follows [\[25–27\]](#). See those references for a more detailed discussion.

2.1.1 The SM Lagrangian

Quantum Field Theory (QFT) is the framework used to describe particle physics. QFT describes the Universe as having quantum fields throughout the Universe, particles as being excitations of these fields, and particle interactions as excitations in one or more fields causing excitations in other fields not unlike a coupled harmonic oscillator. The fundamental object used to describe a particle physics model is an equation called the Lagrangian density, referred to as the Lagrangian. This equa-

tion contains all the information for how the fields propagate in space and how the particles interact. The physical picture is quite complicated with a process being described as an integral over all possible paths a particle could take weighted by the exponential of action, an integral over the Lagrangian. This picture can be very useful as it allows us to make very precise numerical predictions by discretizing space into a lattice. However, this is very computationally intensive and cannot be done accurately for all processes, so it is mainly reserved for non-perturbative processes. In most cases, what is done is a perturbative expansion in the small coupling parameters which are responsible for various particle interactions. The terms in the expansion exactly give us the famous Feynman diagrams, a powerful graphical tool for obtaining analytic results. Using these Feynman diagrams and the Feynman rules, a way to assign mathematical expressions to each part of a Feynman diagram, physicists can calculate a process to any degree of precision they wish, as long as the coupling is perturbative. However, doing so may be quite time consuming.

An important tool that physicists use when constructing a Lagrangian is symmetry. A transformation that leaves the action $S[\mathcal{L}] = \int dx^4 \mathcal{L}$ invariant, either through making the Lagrangian invariant or modified by a total derivative, will introduce a conserved quantity. For example, the Lorentz invariance of the Lagrangian results in conservation of four-momentum. In the SM, the various gauge symmetry lead to various conserved quantities. You will not see a coupling between a quark, an electron and a photon in the SM because this would violate the various gauge symmetries. Similarly, adding discrete symmetries into a model can be used to eliminate complicated terms, or lead to the stability of a particle.

Let us now look at the SM which has as its gauge group $SU(3) \times SU(2) \times U(1)$.

The full covariant derivative for the SM is thus given by:

$$\mathcal{D}_\mu = \partial_\mu - ig' B_\mu Y - ig W_\mu^a T^a - ig_s G_\mu^a t^a. \quad (2.1)$$

Here, g' , g , and g_s are the coupling constant for U(1), SU(2), and SU(3), respectively; B_μ , W_μ^a , and G_μ^a are the gauge bosons for U(1), SU(2), and SU(3), respectively; Y is the hypercharge operator, T^a is $\sigma^a/2$ where σ^a are the Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (2.2)$$

and t^a is $\lambda^a/2$ where λ^a are the Gell-Mann matrices:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.3)$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (2.4)$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (2.5)$$

The different particles in the SM will have different representations for each group. If a particle is not charged under the force, it will be in the trivial representation; if it is a matter particle, it will be in the fundamental representation; and if the particle is a gauge boson, it will be in the adjoint representation of its gauge group. See Table

2.1 for a summary of the field content, charges, and spins.

So far, we have not discussed the masses of these particles. Because of gauge invariance, we cannot just add a mass term to the gauge bosons. However, we know from experiments that many particles have mass. This is where we introduce the Higgs mechanism. Although the overall Lagrangian respects all of the gauge symmetries of the SM at some value of the Higgs field, this will not correspond to the minimum of the scalar potential. This results in the field having a non-zero value everywhere which is reflected by adding a constant to the field, called the vacuum expectation value (vev). When expanding the Lagrangian, this constant generates terms which break $SU(2) \times U(1)$ into $U(1)$ and give masses to all the particles of the SM, except the neutrino. This is typically referred to as electroweak symmetry breaking (EWSB).

Table 2.1: Field content of the Standard Model. The index f goes from 1 to 3 and goes over the generations of the fermions, a from 1 to 3 and b from 1 to 8 and go over the gauge bosons. The L and R next to a spin 1/2 field represents its chirality. In the case of $SU(3)$ and $SU(2)$, the number represents the size of the representation, while for $U(1)$, the value represents hypercharge. The symbols from top to bottom are as follows: the Higgs doublet, the left-handed quark doublets, the right-handed up-type quarks, the right-handed down-type quarks, the left-handed lepton doublets, the right-handed electrons, the hypercharge gauge boson, the weak gauge bosons, the gluons.

	SU(3)	SU(2)	U(1)	Spin
H	1	2	1/2	0
Q_L^f	3	2	1/6	1/2
U_R^f	3	1	2/3	1/2
D_R^f	3	1	-1/3	1/2
L_L^f	1	2	-1/2	1/2
E_R^f	1	1	-1	1/2
B_μ	1	1	0	1
W_μ^a	1	3	0	1
G_μ^b	8	1	0	1

We begin with the definition of the Higgs doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \quad (2.6)$$

Here, each ϕ_i is a real scalar field. The scalar potential in the Lagrangian will be:

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \quad (2.7)$$

where $-\mu^2$ and λ are free parameters. In QFT, particles are treated effectively like harmonic oscillators about the minimum of their potentials. We must therefore find the value of the field which minimizes the potential. To find this, we take derivatives with respect to the ϕ_i :

$$\frac{\partial V}{\partial \phi_i} = \phi_i (-\mu^2 + 2\lambda H^\dagger H). \quad (2.8)$$

In order to satisfy the condition that the partial derivatives are zero, we have two options: set all $\phi_i = 0$ or set $H^\dagger H = \frac{\mu^2}{2\lambda}$. We require that $\lambda > 0$ so that the potential is bounded from below. Otherwise, the field would roll to arbitrarily large values which would lead to all sorts of theoretical problems. Thus the second solution is real only if $\mu^2 > 0$ and it will indeed lead to a global minimum. To take this into account, we will define the vev to be:

$$v = \sqrt{\frac{\mu^2}{\lambda}}. \quad (2.9)$$

We can now expand our field around this vev. We define the H field after EWSB as:

$$H = \begin{pmatrix} G^+ \\ \frac{h+v+iG^0}{\sqrt{2}} \end{pmatrix}. \quad (2.10)$$

The G^+ and G^0 turn out to be Goldstone bosons which we can eliminate using our Gauge freedom. These are thus usually set to zero in the unitarity gauge:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix}. \quad (2.11)$$

To see how this gives rise to the massive gauge bosons, we must look at the covariant derivative acting on H . Note that H is not charged under $SU(3)$ so that part of the covariant derivative is ignored. The kinetic term for H gives:

$$\begin{aligned} \mathcal{D}_\mu H &= (\partial_\mu - ig' B_\mu Y - ig W_\mu^a T^a) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix}, \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2\sqrt{2}} (h+v) \begin{pmatrix} g'(W_\mu^1 - iW_\mu^2) \\ gB_\mu - g'W^3 \end{pmatrix}, \\ (\mathcal{D}^\mu H)^\dagger (\mathcal{D}_\mu H) &= \frac{1}{2} (\partial^\mu h) (\partial_\mu h) + \frac{1}{8} (h+v)^2 \\ &\quad \times [g^2 B^\mu B_\mu - gg' B^\mu W_\mu^3 + g'^2 (W^{1\mu} W_\mu^1 + W^{2\mu} W_\mu^2 + W^{3\mu} W_\mu^3)]. \end{aligned} \quad (2.12)$$

We see that the second term leads to a mass term for the gauge bosons and interactions between the gauge bosons and the Higgs boson. We can diagonalize the mass matrix

and define the usual states:

$$W_\mu^+ = \frac{1}{\sqrt{2}} (W_\mu^1 - iW_\mu^2), \quad (2.13)$$

$$W_\mu^- = \frac{1}{\sqrt{2}} (W_\mu^1 + iW_\mu^2), \quad (2.14)$$

$$Z_\mu = \frac{gW_\mu^0 - g'B_\mu}{\sqrt{g^2 + g'^2}}, \quad (2.15)$$

$$A_\mu = \frac{g'W_\mu^0 + gB_\mu}{\sqrt{g^2 + g'^2}}, \quad (2.16)$$

with masses:

$$M_W = \frac{gv}{2}, \quad (2.17)$$

$$M_Z = \frac{\sqrt{g^2 + g'^2}v}{2}, \quad (2.18)$$

$$M_A = 0. \quad (2.19)$$

Writing the covariant derivative in terms of these gauge fields gives:

$$\begin{aligned} \mathcal{D}_\mu = & \partial_\mu - ig_s G_\mu^a t^a - i \frac{g}{\sqrt{2}} (W_\mu^+ T^+ + W_\mu^- T^-) \\ & - \frac{i}{\sqrt{g^2 + g'^2}} Z_\mu (g^2 T^3 - g'^2 Y) - i A_\mu e Q, \end{aligned} \quad (2.20)$$

where $T^\pm$ are the raising and lowering operators for SU(2), T^3 is the typical third component operator for SU(2), e is the fundamental charge and Q is the charge operator, both given by:

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad (2.21)$$

$$Q = T^3 + Y. \quad (2.22)$$

Now that we have an idea of how the Higgs doublet interacts with the gauge bosons to give them mass, we look at how it gives mass to the fermions. The Yukawa sector of the SM is given by:

$$\mathcal{L} \subset \lambda_{ij}^l \bar{E}_R^i H^\dagger L_L^j H + \lambda_{ij}^d \bar{D}_R^i H^\dagger Q_L^j + \lambda_{ij}^u \bar{U}_R^i \tilde{H}^\dagger Q_L^j + \text{h.c.}, \quad (2.23)$$

where λ_{ij}^l , λ_{ij}^d , and λ_{ij}^u are three by three matrices of Yukawa couplings. The $\tilde{H}^\dagger$ field is the conjugate doublet given by:

$$\tilde{H} \equiv i\sigma^2 H^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix}. \quad (2.24)$$

It should be noted that in the SM, there are no right-handed neutrinos, so we cannot construct a similar term to the up-type quark to give masses to neutrinos. At this point, the Yukawa matrices are general complex three by three matrices and are not convenient to work with. We are always able to diagonalize these using two different unitary matrices. Consider λ^l , we can write:

$$\lambda^l = U^l y^l W^{l\dagger}, \quad (2.25)$$

where y_l is a diagonal three by three matrix and U_l and W_l are unitary matrices. By substituting this back into the Lagrangian, we can do the following field rotations:

$$\bar{E}_R^i \rightarrow U^l{}_{ij} \bar{E}_R^j, \quad (2.26)$$

$$L^i \rightarrow W^{l\dagger}{}_{ij} L_L^j. \quad (2.27)$$

This simplifies things considerably as the weak eigenstates are the same as the mass

eigenstates ¹. However, we run into trouble when doing this with the quarks. Going through a similar process:

$$\lambda^d = U^d y^d W^{d\dagger}, \quad (2.28)$$

$$\lambda^u = U^u y^u W^{u\dagger}. \quad (2.29)$$

Now consider using the field rotation freedom:

$$U_R^i \rightarrow U_{ij}^u U_R^j, \quad (2.30)$$

$$D_R^i \rightarrow U_{ij}^d D_R^j, \quad (2.31)$$

$$Q_L^i \rightarrow W_{ij}^{u\dagger} Q_L^j. \quad (2.32)$$

Herein lies the problem in that you cannot get rid of both W matrices at the same time. Doing one at a time allows us to find well-defined masses for each quark, but when working with the weak force, it is not convenient to keep going from one basis to the other. We typically work in the basis where the up-type quarks' mass eigenstates are the same as their weak eigenstates and leave the down-type quarks in their weak eigenstates. The terms in the Lagrangian are now:

$$\mathcal{L} \subset \lambda_i^l \bar{E}_R^i H^\dagger L_L^i H + y_{ik}^d W_{kl}^{d\dagger} W_{lj}^u \bar{D}_R^i H^\dagger Q_L^j + \lambda_i^u \bar{U}_R^i \tilde{H}^\dagger Q_L^i + \text{h.c.} \quad (2.33)$$

We define the Cabibbo-Kobayashi-Maskawa (CKM) matrix as:

$$V = W^{d\dagger} W^u. \quad (2.34)$$

¹This is the case in the SM. In reality, we know that the neutrinos are massive and so the mass eigenstates are not the same as the weak eigenstates. The mixing between these states is described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

This matrix is the change of basis matrix which changes the down-type quark weak eigenstates to the down-type quark mass eigenstates. Plugging in fields, we can deduce the mass of each fermion:

$$m_l^i = \frac{y_i^l v}{\sqrt{2}}, \quad (2.35)$$

$$m_u^i = \frac{y_i^u v}{\sqrt{2}}, \quad (2.36)$$

$$m_d^i = \frac{y_i^d v}{\sqrt{2}}. \quad (2.37)$$

2.1.2 Theoretical constraints on the Lagrangian

In this section, we will go over some of the commonly used theoretical constraints on a Lagrangian. We will make extensive use of these constraints in later chapters as it is an important tool when constructing new models. These constraints will focus on the scalar potential as the models considered in this thesis all have extended scalar sectors.

2.1.2.1 Unitarity

The first constraint that will be discussed is partial wave unitarity for 2 to 2 scattering. We will consider two incoming particles with momentum k_1 and k_2 and outgoing momenta p_1 and p_2 . First, we define the unitarity matrix S whose elements are given by:

$${}_{\text{out}}\langle p_1 p_2 | k_1 k_2 \rangle_{\text{in}} \equiv \langle p_1 p_2 | S | k_1 k_2 \rangle. \quad (2.38)$$

We can expand out the case where there are no interactions by defining the T-matrix:

$$S = \mathbf{1} + iT. \quad (2.39)$$

Because S is unitary, it is diagonalizable and must have eigenvalues of the form $e^{i\delta}$ for some angle δ . If we look at the partial wave expansion of the invariant matrix element $\mathcal{M}$, which is related to the T-matrix, we get:

$$\mathcal{M} = 16\pi \sum_J (2J+1) a_J P_J(\cos \theta), \quad (2.40)$$

where J is the (orbital) angular momentum, a_J are the J th partial wave amplitudes, and $P_J(\cos \theta)$ are the Legendre polynomials. The unitarity constraint applies to each partial wave because of conservation of momentum and is given by $|\text{Re } a_J| \leq 1/2$. We will only ever look at the zeroth partial wave amplitude a_0 which is easily computed by calculating $\mathcal{M}$. If there are no angular dependencies, then we can simply apply the constraint $|\mathcal{M}| < 8\pi$. Furthermore, because we are working with scalars, by considering only the high energy limit, any Feynman diagram with a propagator will go to zero as the propagator is inversely proportional to the momentum. This allows us to put a restriction on the quartic couplings of our scalar potential.

2.1.2.2 Bounded from below

In QFT, the value of the fields are typically thought to stay around the minimum of their potentials with particles being excitations about that minimum. It should be clear that the potential must have a minimum for this to be the case! This translates to the potential being bounded from below. As the potential is a polynomial of degree at most four of the fields, this is sufficient to ensure the existence of a global

minimum. To determine whether the potential is bounded from below, we look at the quartic terms as they dominate at large field space. Given n fields, we can go to the equivalent of spherical coordinates for n dimensions, and factor out the r^4 part of the potential. Since we are considering only the quartic terms, each term should be proportional to the radial coordinate to the fourth power. What is left is a function purely of the angles. The restriction for the potential to be bounded from below then translates to the restriction that this function always be positive.

2.1.2.3 Avoiding alternative minima

In the models that we will consider, some of the new scalars will get a vev. It is necessary to ensure that the SM Higgs gets the same vev that has been measured, otherwise the model will immediately be excluded. We also want to make sure that if we choose a value for a vev for the new scalars that it actually represents the global minimum of the potential. These are the types of constraints we mean when discussing avoiding alternative minima.

In order to compute this constraint, we need to find the extrema of the potential evaluating it at the vevs and taking the derivatives. At this point, there are three approaches to apply this constraint which depend on the model and how one is scanning the parameter space. The most obvious way to proceed is to find analytic solutions and come up with expressions for the constraints. This is very useful, but not always possible for more complicated models. If scanning over the Lagrangian parameters, one could use the derivative of the potential to numerically find the extrema, test which one gives the deepest minimum, and then select that vacuum. If this vacuum does not correspond with the SM vacuum, then the initial choice of Lagrangian parameters can be excluded. Finally, if one is choosing the vacuum as part of the scan,

either through defining the masses of particles which are typically related to the vevs, or directly, then one must work backwards to find the Lagrangian parameters. Once the Lagrangian parameters are found, all the extrema must be found and tested to see which corresponds to the deepest minimum. If the deepest minimum is not the one that was chosen by the scan, then that point is invalid.

2.2 Cosmology

This section will introduce many of the topics regarding cosmology that will be seen throughout this thesis such as the Friedmann equations and DM freeze-out. This section follows [24] and [28] throughout the section. Refer to those for a more detailed discussion.

2.2.1 The Friedmann equations

In order to understand the Universe on a large scale, we need Einstein's theory of general relativity. This theory models spacetime as a manifold which curves due to the presence of any kind of energy density. Colloquially, energy tells space how to curve, and the curvature tells energy how to move. On large scales, the Universe can be treated as homogeneous and isotropic, so we can model the energy density in the Universe as that of a perfect fluid. These assumptions lead to the Friedmann equations which are given by:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{1}{k^2 a^2} + \frac{\Lambda}{3}, \quad (2.41)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \quad (2.42)$$

where H is the Hubble parameter, G is Newton's gravitational constant, k describes the curvature of the Universe, Λ is the cosmological constant, a is the scale factor, and ρ and p are the energy density and pressure of a perfect fluid. This equation is derived in Appendix A.

Let us discuss the various elements of these equations to get a sense of the underlying physics. The first thing to understand is the scale factor which describes how distances change as a function of time. Let us denote the age of the Universe by t_0 . By convention, we take the scale factor today to be $a(t_0) = 1$. If we measure a distance between two objects to be some distance d at some time t_1 , then the distance that would be measured at some time t_2 would be $d \frac{a(t_2)}{a(t_1)}$. We also note that the scale factor is a strictly increasing function of time so that if $t_1 < t_2$ then $a(t_1) < a(t_2)$. This means that distances are time dependent, which is not very convenient as the scale factor is not a simple function throughout the history of the Universe. Instead of considering just the usual physical coordinates, what is typically used are comoving coordinates. These coordinates factor out the scale factor so that the remaining quantity is independent of the expansion of the Universe.

Next, the curvature term. There are many ways to parameterize the curvature, but written in the form above, k will have units of distance and will be related to the radius of curvature of the Universe. If $k^2 > 0$, then the Universe will be positively curved with a radius k and would therefore be closed. In lower dimensions, we can imagine the surface of a sphere: if we continue along a straight line, we will return to our starting position. If $k^2 < 0$, then the Universe will be negatively curved with a radius of $|k|$. This is like a paraboloid in lower dimensions where two lines which are initially parallel will diverge from one another. If we take $k^2 \rightarrow \infty$, then the Universe is flat. Current observations have set very large lower bounds on the magnitude of

k^2 and so it is typical to just consider the Universe as flat. One way to think about this term is that it is the energy density contained in the inherent curvature of the Universe. As seen in the equation, the radius of curvature expands proportionally to a , so as the energy density is inversely proportional to the square of the radius of curvature, the energy density falls as a^{-2} . To imagine this, consider a small sphere which has a small radius of curvature and so would have a large value for the curvature term. As the sphere expands, the radius expands and eventually, the sphere will look like a flat plane, at which point, the value of the curvature term would tend to zero.

The Λ term corresponds to the cosmological constant term. Although this term is extremely small with $\Lambda = 4.63 \times 10^{-84} \text{ GeV}^2$, this term dominates the behaviour of the scale factor today. The term is typically thought of as the energy density associated with the vacuum of spacetime. As the Universe expands, this form of energy density does not dilute, as seen by the lack of any a factors in the term. There is simply more vacuum, and therefore more net energy. Because of this behaviour, this causes an exponential growth in the scale factor.

The energy density term is typically broken down into two types, depending on how they change with the scale factor:

$$\rho = \frac{\rho_M}{a^3} + \frac{\rho_R}{a^4}. \quad (2.43)$$

Here, the first term corresponds to non-relativistic matter while the second corresponds to relativistic particles, referred to as radiation. Both energy densities will dilute due to the increase in physical volume, but the radiation will also lose energy due to redshift. In the early Universe, the energy density of the Universe was dominated by radiation, but because it dilutes faster than matter, eventually matter took over.

The second equation tells us about the acceleration of the scale factor. If it were not for the cosmological constant, the negative sign on the energy density and pressure term would cause the Universe to eventually collapse back in on itself in what is called the Big Crunch. However, if the Cosmological constant dominates, then the Universe's expansion will keep accelerating and will eventually result in the Heat Death of the Universe. In the case where these terms cancel out, then the Universe would be static with the expansion rate not changing. This was Einstein's original motivation for the Cosmological constant.

Let us define the critical density and the density parameters [22]:

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 8.6 \times 10^{-27} \text{kg} \cdot \text{m}^{-3}, \quad (2.44)$$

$$\Omega_{0,R} = \frac{\rho_R(t_0)}{\rho_c} \approx 0, \quad (2.45)$$

$$\Omega_{0,M} = \frac{\rho_M(t_0)}{\rho_c} = 0.3111 \pm 0.0056, \quad (2.46)$$

$$\Omega_{0,\Lambda} = \frac{\Lambda}{3H_0^2} = 0.6889 \pm 0.0056, \quad (2.47)$$

$$\Omega_0 = \Omega_{0,R} + \Omega_{0,M} + \Omega_{0,\Lambda}, \quad (2.48)$$

$$\Omega_{0,k} = 1 - \Omega_0 = -\frac{1}{k^2 H_0^2} \approx 0, \quad (2.49)$$

where H_0 is the Hubble parameter today, called the Hubble constant, and t_0 is the age of the Universe today. The subscript 0 means that we are looking at these values today. The subscripts R , M , k , and Λ refer to the radiation, matter, curvature, and vacuum terms, respectively. For historical reasons, H_0 is sometimes written as $H_0 = 100h \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ with $h = 0.6766 \pm 0.0042$. Here, the critical density ρ_c is the tipping point between having a positively curved Universe, a flat Universe, or a negatively curved Universe. Note that $\Omega_{0,M}$ is typically divided as $\Omega_{0,b} = 0.0494$

and $\Omega_{0,CMD} = 0.2630$ as the baryonic component and the cold DM component, respectively. The density parameters describe how much of the energy density is in a particular component compared to the critical density. If all the density parameters sum to 1, then it means that the Universe is spatially flat. The density parameters allow us to describe the expansion history of the Universe in terms of measurable quantities today. Using these quantities, we can write the first Friedmann equations as:

$$\frac{H^2}{H_0^2} = \Omega_{0,R}a^{-4} + \Omega_{0,M}a^{-3} + \Omega_{0,k}a^{-2} + \Omega_{0,\Lambda}. \quad (2.50)$$

At its core, this is a non-linear differential equation for the scale factor. Simple solutions exist if we consider one or two terms on the left-hand side, which is useful since it is not a bad approximation to break down the history of the Universe into "eras" where one or two components dominate. Even if we know how the scale factor changes with time, for reasons which will be clear later, it would be useful to consider the temperature as a measure of time, more specifically, the inverse of the temperature. As time increases, the temperature cools so its inverse increases. To use temperature, we will need to look at the thermodynamics involved, which is discussed in the next section.

2.2.2 Equilibrium Thermodynamics

In this section, we will consider various thermodynamic quantities in various equilibrium scenarios in the context of early Universe cosmology. This will help us understand how particles behave when in equilibrium, and later, how they behave when out of equilibrium.

We will start by distinguishing between three kinds of equilibrium: kinetic,

thermal, and chemical. A species of particles is said to be in kinetic equilibrium if their phase space distribution can be described by either a Bose-Einstein distribution if it is a boson, Fermi-Dirac distribution if it is a fermion, or the Maxwell-Boltzmann distribution in the classical limit. It should be noted that this involves only a single particle species. Next, we say that two or more species are in thermal equilibrium if they are all individually in kinetic equilibrium, and that they all share the same temperature. This type of equilibrium relates between different particles species. Finally, we say that two or more species are in chemical equilibrium if the rate of conversion between species is equal to the rate of their reverse reactions. For example, consider four particle species A , B , C , and D , each with their own chemical potentials μ . Let us assume there is a process $A + B \longleftrightarrow C + D$, then the statement that the four species are in chemical equilibrium is equivalent to the statement that $\mu_A + \mu_B = \mu_C + \mu_D$.

We will now focus on a single particle species with mass m and internal degrees of freedom g . Let $f(\vec{p})$ be the phase space density of that species. We will assume that the particle is in kinetic equilibrium and that there is no preferred direction so its phase space distribution can be described by:

$$f(E) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1} \quad (2.51)$$

where E is the energy, μ is the chemical potential, and T is the temperature. The plus corresponds to the fermion case while the minus is the boson case. We now define the number density, energy density and pressure in terms of this distribution:

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3p, \quad (2.52)$$

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3p, \quad (2.53)$$

$$p = \frac{g}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E} f(\vec{p}) d^3p, \quad (2.54)$$

Using $E^2 = |\vec{p}|^2 + m^2$ and Eq 2.51:

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{E(E^2 - m^2)^{1/2}}{e^{\frac{E-\mu}{T}} \pm 1} dE, \quad (2.55)$$

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{E^2(E^2 - m^2)^{1/2}}{e^{\frac{E-\mu}{T}} \pm 1} dE, \quad (2.56)$$

$$p = \frac{g}{6\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{3/2}}{e^{\frac{E-\mu}{T}} \pm 1} dE. \quad (2.57)$$

Let's start with the number density. We will consider the limit where the particle follows a Maxwell-Boltzmann distribution. In that case, Eq 2.55 becomes [29]:

$$n = \frac{gm^2T}{2\pi^2} K_2(m/T) e^{\mu/T}, \quad (2.58)$$

$$\approx \begin{cases} g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T} & \text{for } m, \mu \gg T, \\ g \frac{T^3}{\pi^2} e^{\mu/T} & \text{for } T \gg m, \mu, \end{cases} \quad (2.59)$$

where $K_2(x)$ is the modified Bessel function of the second kind. Next, we will consider the relativistic and non-relativistic limits for ρ and p :

$$\rho = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{for bosons with } T \gg m, \mu, \\ \frac{7}{8} \frac{\pi^2}{30} g T^4 & \text{for fermions with } T \gg m, \mu, \\ mn & \text{for } m, \mu \gg T, \end{cases} \quad (2.60)$$

$$p = \begin{cases} \frac{\rho}{3} & \text{for } T \gg m, \mu, \\ nT & \text{for } m, \mu \gg T. \end{cases} \quad (2.61)$$

Because of the relationship between the energy density and the pressure, it is convenient to treat the species as a perfect fluid with an equation of state of the form:

$$p = w\rho, \quad (2.62)$$

with

$$w = \begin{cases} 0 & \text{for matter,} \\ \frac{1}{3} & \text{for radiation.} \end{cases} \quad (2.63)$$

In the early Universe, the Universe was dominated by radiation. This radiation consisted of any massless particle, as well as any relativistic particles. As the temperature cooled, some species slowed down and stopped contributing to the relativistic part of the energy density. It is typical to write the energy density due to radiation in the early Universe using the effective number of relativistic degrees of freedom g_* :

$$\rho_R = \frac{\pi^2}{30} g_* T^4. \quad (2.64)$$

where g_* is given by:

$$g_* = \sum_{i=\text{particles}} \frac{30}{\pi^2} \rho_i T_\gamma^{-4}, \quad (2.65)$$

$$g_* \approx \sum_{i=\text{boson}} g_i \left(\frac{T_i}{T_\gamma} \right)^4 + \frac{7}{8} \sum_{i=\text{fermion}} g_i \left(\frac{T_i}{T_\gamma} \right)^4, \quad (2.66)$$

where the sum in the first line of Eq. 2.66 is over all particles and the sums in the second line are only over relativistic particles, T_γ is the temperature of the photons, and T_i are the temperature of the particle in question. For a look at the behaviour

Figure 2.1: The evolution of the relativistic degrees of freedom $g_*(T)$ and $g_{*S}(T)$ as a function of time for the SM [30].

of $g_*(T)$, see Fig 2.1.

Next, we look at entropy. When species are in thermal equilibrium, the entropy density S remains constant. A useful quantity to look at is the entropy density per comoving volume:

$$s = \frac{S}{a^3} = \frac{\rho + p - \sum_i \mu_i n_i}{T}. \quad (2.67)$$

Where μ_i is the chemical potential for species i and n_i is the number density per comoving volume. For this thesis, we assume that $\mu_i \ll T$ for all species in the SM and for all the DM species. This is useful since we can relate the change in the entropy

density per comoving volume directly to the Hubble parameter:

$$\dot{s} = -3Hs. \quad (2.68)$$

For a radiation dominated Universe, we have:

$$s = \frac{2\pi^2}{45} g_{*S} T^3, \quad (2.69)$$

where g_{*S} is the effective number of relativistic degrees of freedom for the entropy given by:

$$g_{*S} = \sum_{i=\text{particles}} \frac{45}{2\pi^2} \frac{\rho_i + p_i}{T_i} T_\gamma^{-3}, \quad (2.70)$$

$$g_{*S} \approx \sum_{i=\text{boson}} g_i \left(\frac{T_i}{T_\gamma} \right)^3 + \frac{7}{8} \sum_{i=\text{fermion}} g_i \left(\frac{T_i}{T_\gamma} \right)^3, \quad (2.71)$$

where the sum in the first line of Eq. 2.71 is over all particles and the sums in the second line are only over relativistic particles. See Fig 2.1 for the time evolution of g_{*S} as a function of temperature. For most of the history of the early Universe, the values of g_* and g_{*S} were the same. However, when the neutrinos decoupled and were no longer in thermal equilibrium with the photons, their temperature started to cool at a rate of a^{-1} . As the electrons become non-relativistic, they transfer their entropy to the photons which causes them to cool slower. This discrepancy in the temperature between the neutrinos and the photons is what causes the difference in the values of g_* and g_{*S} .

2.2.3 Out-Of-Equilibrium Thermodynamics

In this section, we discuss how thermodynamic quantities will vary as the Universe expands and cools, namely the number density of a species ϕ with g degrees of freedom. We go over the single particle case with a standard cosmology. In Chapter 3, we will go over the case of modified cosmological scenarios and in Chapter 5, we will go over the multi-species case. We begin with the Boltzmann equation:

$$\hat{L}[f] = C[f] \quad (2.72)$$

where $\hat{L}[f]$ is the Liouville operator, $C[f]$ is the collision term, and f is the phase space density. Here, the Liouville operator describes the changes to the phase space distribution due to forces, diffusion, and dilution, while the collision operator describes changes due to particle collisions. The relativistic form of the Liouville operator is:

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}, \quad (2.73)$$

where $\Gamma^\alpha_{\beta\gamma}$ are the Christoffel symbols which are given in Appendix A for the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. The phase space density will only depend on the energy and time because we are assuming that the Universe is homogeneous and isotropic. This simplifies the equation to:

$$\hat{L}[f] = E \frac{\partial f}{\partial t} - H |\vec{p}|^2 \frac{\partial f}{\partial E}. \quad (2.74)$$

Using the definition of the number density, we can write Eq. 2.72 as:

$$\dot{n} + 3Hn = \frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3p, \quad (2.75)$$

where we have divided both sides by E and integrated over the momentum on both sides. Although the number density is our goal, it is useful to factor out the expansion of the Universe. We do this by defining the number density per comoving volume using the entropy density per comoving volume as a fiducial region. It is also useful to consider temperature instead of time as that is what the collision term will depend on. We define the two dimensionless quantities:

$$Y = \frac{n}{s}, \quad (2.76)$$

$$x = \frac{m}{T}, \quad (2.77)$$

where m is some mass scale. Typically for DM freeze-out, we take m to be the mass of the DM. Using these definitions, we obtain:

$$s\dot{x} \frac{\partial Y}{\partial x} = \frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3p. \quad (2.78)$$

One can find an expression for $\dot{x}$ by looking at $\dot{s}$. For a radiation dominated Universe:

$$\dot{s} = -3Hs = s \frac{\dot{x}}{x} \left(\frac{d \log g_*}{d \log x} - 3 \right) \quad (2.79)$$

$$\dot{x} = 3xH \left(3 - \frac{d \log g_*}{d \log x} \right)^{-1}. \quad (2.80)$$

Now we look at the collision term for the generic process $\phi + a + b + \dots \longleftrightarrow i + j + \dots$:

$$\begin{aligned} \frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3p = & - \int d\Pi_\phi d\Pi_a d\Pi_b \dots d\Pi_i d\Pi_j \dots \\ & \times (2\pi) \delta^4(p_\phi + p_a + p_b + \dots - p_i - p_j - \dots) \end{aligned}$$

$$\begin{aligned} & \times \left[|\mathcal{M}|_{\phi+a+b+\dots \rightarrow i+j+\dots}^2 f_\phi f_a f_b \cdots (1 \pm f_i)(1 \pm f_j) \cdots \right. \\ & \left. - |\mathcal{M}|_{i+j+\dots \rightarrow \phi+a+b+\dots}^2 f_i f_j \cdots (1 \pm f_\phi)(1 \pm f_a)(1 \pm f_b) \right], \end{aligned} \quad (2.81)$$

where

$$d\Pi \equiv \frac{g}{(2\pi)^3} \frac{d^3p}{2E}, \quad (2.82)$$

is the Lorentz invariant phase space of the species specified by the subscript with g being the appropriate number of degrees of freedom, $\mathcal{M}$ is the matrix element for the listed process, and the $\pm$ is $+$ for bosons and $-$ for fermions. It is typical to consider a few simplifying assumptions to this equation. First, we will assume that CP will be conserved in the processes we will consider so that the matrix elements for both the forward and reverse processes are the same. The second simplifying assumption is that we can use Maxwell-Boltzmann statistics for all the species. Using Maxwell-Boltzmann assumes that there is no degenerate Fermi species and no Bose-Einstein condensate. Finally, we will assume that we have $1 \pm f \approx 1$ everywhere it appears. This gives us:

$$\begin{aligned} \frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3p &= - \int d\Pi_\phi d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots \times (2\pi)^4 |\mathcal{M}|^2 \\ &\times \delta^4(p_\phi + p_a + p_b + \cdots - p_i - p_j - \cdots) \times (f_\phi f_a f_b \cdots - f_i f_j \cdots). \end{aligned} \quad (2.83)$$

Because of the delta function, we will have $E_\phi + E_a + E_b \cdots = E_i + E_j + \cdots$. Using this and the Maxwell-Boltzmann distribution, we get:

$$\begin{aligned} f_\phi f_a f_b \cdots - f_i f_j \cdots &= \exp[(\mu_\phi + \mu_a + \mu_b + \cdots)/T] \exp[-(E_\phi + E_a + E_b + \cdots)/T] \\ &- \exp[(\mu_i + \mu_j + \cdots)/T] \exp[-(E_i + E_j + \cdots)/T], \end{aligned}$$

$$\begin{aligned}
&= (\exp [(\mu_\phi + \mu_a + \mu_b + \dots) / T] - \exp [(\mu_i + \mu_j + \dots) / T]) \\
&\quad \times f_\phi^{eq} f_a^{eq} f_b^{eq} \dots, \\
&= \left(\frac{n_\phi n_a n_b \dots}{n_\phi^{eq} n_a^{eq} n_b^{eq} \dots} - \frac{n_i n_j \dots}{n_i^{eq} n_j^{eq} \dots} \right) f_\phi^{eq} f_a^{eq} f_b^{eq} \dots. \tag{2.84}
\end{aligned}$$

where in the last step, we use $n = n^{eq} e^{\mu/T}$ for each species. We define the quantity:

$$\begin{aligned}
A &= - \frac{1}{n_\phi^{eq} n_a^{eq} n_b^{eq} \dots} \int d\Pi_\phi d\Pi_a d\Pi_b \dots d\Pi_i d\Pi_j \dots \times (2\pi)^4 |\mathcal{M}|^2 \\
&\quad \times \delta^4(p_\phi + p_a + p_b + \dots - p_i - p_j - \dots) \times f_\phi^{eq} f_a^{eq} f_b^{eq}. \tag{2.85}
\end{aligned}$$

For k initial state particles, using Eq. 2.76, 2.83, 2.84, and 2.85, we get:

$$\frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3p = s^k \left(\frac{Y_\phi Y_a Y_b \dots}{Y_\phi^{eq} Y_a^{eq} Y_b^{eq} \dots} - \frac{Y_i Y_j \dots}{Y_i^{eq} Y_j^{eq} \dots} \right) Y_\phi^{eq} Y_a^{eq} Y_b^{eq} \dots A. \tag{2.86}$$

Although Eq. 2.86 is general, it is convenient to show the result for one or two initial particles as this is more typical. For the case of a single particle decaying, this reduces to:

$$A = \Gamma_{\phi \rightarrow i+j+\dots} = \frac{1}{n_\phi^{eq}} \int d\Pi_\phi d\Pi_i d\Pi_j \dots \times (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_\phi - p_i - p_j - \dots) f_\phi^{eq}. \tag{2.87}$$

For two particles in the initial state, we have:

$$\sigma_{\phi+a \rightarrow i+j+\dots} v = \int d\Pi_i d\Pi_j \dots \times \frac{1}{4E_\phi E_a} (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_\phi + p_a - p_i - p_j - \dots), \tag{2.88}$$

$$A = \langle \sigma_{\phi+a \rightarrow i+j+\dots} v \rangle = \frac{g_\phi g_a}{n_\phi^{eq} n_a^{eq}} \int \sigma_{\phi+a \rightarrow i+j+\dots} v f_\phi^{eq} f_a^{eq} \frac{d^3p_\phi}{(2\pi)^3} \frac{d^3p_a}{(2\pi)^3}, \tag{2.89}$$

where $v = |v_\phi - v_a|$. We call $\langle \sigma_{\phi+a \rightarrow i+j+\dots} v \rangle$ the thermally averaged cross section. When discussing the evolution of a single DM species, we typically consider 2 to 2 scattering. We therefore usually write Eq. 2.86 as:

$$\frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3p = s^2 \langle \sigma_{\phi+a \rightarrow i+j} v \rangle \left(\frac{Y_\phi Y_a}{Y_\phi^{eq} Y_a^{eq}} - \frac{Y_i Y_j}{Y_i^{eq} Y_j^{eq}} \right) Y_\phi^{eq} Y_a^{eq}. \quad (2.90)$$

In a real model, the collision term will have such an expression for each scattering process. For simplicity, we will only consider $\phi + a \longleftrightarrow i + j$. Equating Eq. 2.78 and 2.90 and using 2.41, 2.64, 2.69, and 2.80 as well as doing some simplifications gives us:

$$\frac{\partial Y_\phi}{\partial x} = - \left(\frac{\pi g_{*S}^2}{45 G g_*} \right)^{1/2} \left(1 - 3 \frac{d \log g_*}{d \log x} \right) \frac{m \langle \sigma_{\phi+a \rightarrow i+j} v \rangle}{x^2} \left(\frac{Y_\phi Y_a}{Y_\phi^{eq} Y_a^{eq}} - \frac{Y_i Y_j}{Y_i^{eq} Y_j^{eq}} \right) Y_\phi^{eq} Y_a^{eq}. \quad (2.91)$$

In a real model, there would be a separate equation governing each different species and this would result in a system of coupled non-linear differential equations.

There are a few things to point out about Eq. 2.91. First is that since the Universe at the time was dominated by photons which were in thermal equilibrium with the rest of the SM, the number density of the photons was equal to their equilibrium number density. Also, because g_* changes very slowly, the $\frac{d \log g_*}{d \log x}$ term is often dropped for simplicity. Finally, the pre-factor is sometimes written using the Planck mass m_p :

$$\left(\frac{\pi}{45 G} \right)^{1/2} = \left(\frac{\pi}{45} \right)^{1/2} m_p. \quad (2.92)$$

In various chapters of this thesis, we will encounter Eq. 2.91 and variations

thereof in the context of specific models. To obtain concrete results, these equations are solved numerically. The inputs used are the thermally averaged cross sections evaluated at various temperatures as well as the masses and the internal degrees of freedom of the species involved. The numerical procedure starts with the initial condition that, for each species, $Y = Y^{eq}$ when x is small ($x \lesssim 3$). The evolution of Y for each DM species is then computed using the classical Runge–Kutta method up to $x = 1000$. The resulting values of $Y^\infty = Y(x = 1000)$ are treated as constant and converted to the cold DM relic density using

$$\Omega_{0,CDM} = \frac{Y^\infty s_0 m}{\rho_c} \quad (2.93)$$

where $s_0 = 2970 \text{ cm}^{-3}$ is the entropy density per comoving volume today and m is the mass of the DM species.

Chapter 3

DM freeze-out in modified cosmological scenarios

3.1 Introduction

In the standard model of cosmology, freeze-out occurs during the radiation dominated era. However, because we do not have any observations before Big Bang Nucleosynthesis (BBN), we cannot say for sure what the expansion history of the universe was before BBN. Namely, we cannot exclude the possibility of an additional contribution to the energy density that cools at a faster rate than radiation. This can come about from alternative cosmological models like alternative Dark Energy models [31, 32], anisotropic expansion [33, 34], brane world cosmology [35–39], some inflation models [40–42], some quintessence models [43–47], or scalar-tensor gravity [48, 49]. There has also been some work done on early matter dominated eras which decay away [50–53]. All these models have a different expansion rate from the standard Λ_{CMD} model and some previous work has been done to understand the physics in these scenarios [54–57].

In this chapter, we develop a general freeze-out equation that can be used for any modified expansion rate and includes the effects of changes in entropy density. In section 3.2, we go through the derivation of the changes to the Boltzmann equations. In section 3.3, we take an analytic approach to obtain a simple way to approximate the relic density. In section 3.4, we look at numerical results from solving the modified Boltzmann equations for the case of a simple change to the energy density and compare these results to our approximations. Finally, in section 3.5, we summarize our conclusions.

3.2 Derivation of the Modifications to the Boltzmann equation

In this section, we derive changes to the Boltzmann equation based on modification to the Hubble Parameter. Many of the definitions and approximations are the same as those of Kolb and Turner [24], namely CP conservation of the matrix elements, no Fermi degeneracy, no Bose-Einstein condensates, and that the temperature of the DM will be the same as the photon temperature until after freeze-out.

3.2.1 Single effective component

In this section, we will derive the Boltzmann equation for a general energy density $\rho(T)$ where T is the photon temperature. We will also use the equation of state $p(T) = w(T)\rho(T)$ which relates the energy density to the pressure $p(T)$. It will be important to consider $w(T)$ as a function of temperature for this treatment to allow for multiple energy components. This energy density can be any well-behaved function of temperature. We start with the Friedman equation for a flat Universe,

neglecting the cosmological constant:

$$H^2 = \frac{8\pi G}{3}\rho(T), \quad (3.1)$$

where G is Newton's constant. To take the expansion of the universe into account, it is typical to use the entropy density as a fiducial quantity. We will assume that the DM has a chemical potential much less than the temperature and use:

$$s = \frac{\rho + p}{T} = \frac{\rho(1 + w)}{T}. \quad (3.2)$$

We will assume that there is no entropy injection during freeze-out so that the entropy per comoving volume S is conserved. This results in $s \propto a^{-3}$ where a is the scale factor, which gives $\dot{s} = -3Hs$.

We now introduce the dimensionless quantity $x = m/T$ where m is some appropriate mass scale, typically the mass of the DM species. It will be important to find an expression for $\dot{x}$. This can be done by looking at $\dot{s}$:

$$\dot{s} = \frac{\dot{x}}{m} \left(\rho(1 + w) + x(1 + w) \frac{d\rho}{dx} + x\rho \frac{dw}{dx} \right), \quad (3.3)$$

$$\dot{x} = -3xH \left(1 + \frac{d \log \rho}{d \log x} + \frac{d \log(1 + w)}{d \log x} \right)^{-1}. \quad (3.4)$$

The usual Boltzmann equation for a single species is written as [24]:

$$\dot{n} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\text{eq}}^2), \quad (3.5)$$

where n is the number density, $\langle \sigma v \rangle$ is the thermally averaged DM annihilation cross-

section times velocity, and n_{eq} is the equilibrium number density given by ¹ :

$$n_{\text{eq}} = \frac{gm^2T}{2\pi^2} K_2\left(\frac{m}{T}\right) \approx g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}, \quad (3.6)$$

where g is the number of internal degrees of freedom, m is the particles mass, and K_2 is the modified Bessel function of the second kind. The approximation is valid for large x or small T . ²

We now make the substitutions

$$Y = \frac{n}{s}, \quad (3.7)$$

$$Y_{\text{eq}} = \frac{n_{\text{eq}}}{s}. \quad (3.8)$$

The derivative of Y with respect to time is:

$$\dot{Y} = \frac{\dot{n}}{s} - \frac{n}{s^2} \dot{s}, \quad (3.9)$$

$$s\dot{x} \frac{dY}{dx} = \dot{n} + 3nH, \quad (3.10)$$

where we used $\dot{s} = -3Hs$. Combining Eq. 3.4, 3.5, and 3.10, we obtain:

$$\frac{dY}{dx} = -\frac{s}{\dot{x}} \langle \sigma v \rangle (Y^2 - Y_{\text{eq}}^2), \quad (3.11)$$

$$\frac{dY}{dx} = \left(\frac{\rho}{24\pi m^2 G}\right)^{1/2} (1+w) \left(1 + \frac{d \log \rho}{d \log x} + \frac{d \log(1+w)}{d \log x}\right) \langle \sigma v \rangle (Y^2 - Y_{\text{eq}}^2). \quad (3.12)$$

Eq. 3.12 can be used for any function of ρ and w . It is currently written for

¹For the case of asymmetric DM, one must include non-zero chemical potentials in the exponential. See [24, 57] for details.

²It is typical to choose the mass scale in the definition of x to be the same as the mass of the particle as it simplifies some of these equations. However, in the case of multi-species DM models where not all the species are the same mass, one must distinguish the mass scale in the definition of x from the mass in the definition of n_{eq} [3].

the case of a single DM species, but extending it to the case of multiple DM species is straightforward. See [3, 58] for more details on the multi-species case.

3.2.2 Multiple energy densities

Equation 3.12 is valid for any initial $\rho(T)$. However, in the case where one simply adds an extra contribution to the energy density, we can write the energy density as multiple contributing energy densities which is not only simpler than treating w as a function of temperature, but also makes the equation clearer. In this section, we look at the case where we can write $\rho = \sum_i \rho_i$ with $p_i = w_i \rho_i$ with the w_i all constant. Following the same steps used to obtain Eq. 3.4, we find

$$\frac{s}{\dot{x}} = -\frac{1}{3mH} \sum_i \rho_i (1 + w_i) \left(1 + \frac{d \log \rho_i}{d \log x} \right). \quad (3.13)$$

Combining Eq 3.5, 3.10, and 3.13, we obtain:

$$\frac{dY}{dx} = \left(\frac{1}{24\pi m^2 G (\sum_i \rho_i)} \right)^{1/2} \left(\sum_i \rho_i (1 + w_i) \left(1 + \frac{d \log \rho_i}{d \log x} \right) \right) \langle \sigma v \rangle (Y^2 - Y_{\text{eq}}^2). \quad (3.14)$$

The radiation energy density in the early universe is given by:

$$\rho_r = g_*(x) \frac{\pi^2}{30} \left(\frac{m}{x} \right)^4, \quad (3.15)$$

where $g_*(x)$ is the number of relativistic degrees of freedom. If we set $\rho = \rho_r$ in Eq 3.14, we obtain the usual result

$$\frac{dY}{dx} = - \left(\frac{\pi g_*(x)}{45G} \right)^{1/2} \left(1 - \frac{1}{3} \frac{d \log g_*}{d \log x} \right) \frac{m \langle \sigma v \rangle}{x^2} (Y^2 - Y_{\text{eq}}^2). \quad (3.16)$$

The $\frac{d \log g_*}{d \log x}$ term is usually ignored because it is small, but we include it in our analysis for completeness.

As a final example, we follow the example from [57]. Let

$$\rho = \rho_r + \rho_D \left(\frac{T}{T_0} \right)^{n_D} = \rho_r \left(1 + \frac{g_*(x_0)}{g_*(x)} \eta \left(\frac{x_0}{x} \right)^{n_D-4} \right), \quad (3.17)$$

where ρ_D , $n_D > 4$, and $\eta = \frac{\rho_D(T_0)}{\rho_r(T_0)}$ are all constants, T_0 is some constant reference temperature and $x_0 = m/T_0$. In our analysis, we will assume that $p_D = w_D \rho_D$ with $3(1 + w_D) = n_D$. Putting this into a similar form as Eq. 3.16 which only included radiation, we obtain:

$$\begin{aligned} \frac{dY}{dx} = & - \left(\frac{\pi g_*(x)}{45G \left(1 + \frac{g_*(x_0)}{g_*(x)} \eta \left(\frac{x_0}{x} \right)^{n_D-4} \right)} \right)^{1/2} \\ & \times \left[1 - \frac{1}{3} \frac{d \log g_*}{d \log x} + \frac{n_D(n_D - 1)}{12} \frac{g_*(x_0)}{g_*(x)} \eta \left(\frac{x_0}{x} \right)^{n_D-4} \right] \frac{m \langle \sigma v \rangle}{x^2} (Y^2 - Y_{\text{eq}}^2). \end{aligned} \quad (3.18)$$

The terms in the square bracket are often neglected because in the radiation dominated case, it is approximately 1. However, if ρ_D dominates resulting in a large value of η , then it is clear that this term becomes an important contribution.

3.3 Analytic Solutions

In this section, we take an analytic approach to solving equation 3.11 in terms of ρ and w . We start by defining $\Delta = Y - Y_{\text{eq}}$ and combine it with Eq. 3.11:

$$\frac{d\Delta}{dx} = -\frac{dY_{\text{eq}}}{dx} - \frac{s}{\dot{x}} \langle \sigma v \rangle \Delta (\Delta + 2Y_{\text{eq}}). \quad (3.19)$$

For small values of x , Δ and $\frac{d\Delta}{dx}$ are small as Y tracks Y_{eq} closely. Using these approximations, as well as equations 3.3 and 3.6, solving for Δ gives:

$$\Delta = \frac{1}{s\langle\sigma v\rangle\left(\frac{\Delta}{Y_{\text{eq}}} + 2\right)}\left(\dot{x} + \frac{3}{2}\frac{\dot{x}}{x} - 3H\right). \quad (3.20)$$

The criterion for freeze-out is given by $\Delta(x_f) = cY_{\text{eq}}$ for some c . This c will define the freeze-out temperature, the point at which we change from the high temperature limit solution to the low temperature limit solution. We give more details about the choice of c in section 3.4 where we choose c to best fit the numerical results. Using the early time solution, we can obtain an implicit equation for x_f :

$$cn_{\text{eq}}(x_f) = \frac{1}{(c+2)\langle\sigma v\rangle}\left(\dot{x}|_{x=x_f} + \frac{3}{2}\frac{\dot{x}|_{x=x_f}}{x_f} - 3H(x_f)\right), \quad (3.21)$$

where H is given in Eq 3.1, $\dot{x}$ is given in Eq 3.4, and n_{eq} is given in Eq 3.6. Without knowing the functional forms of ρ and w , this is as far as we can go in general. Even if Eq. 3.21 is a complicated function, it can always be solved numerically for a value of x_f much faster than solving 3.12.

Once x_f is found, we can look at 3.11 in the large x case where $Y \approx \Delta$ and $Y_{\text{eq}} \approx 0$:

$$\frac{d\Delta}{dx} = -\frac{s}{\dot{x}}\langle\sigma v\rangle\Delta^2. \quad (3.22)$$

Integrating from $x = x_f$ to $x \rightarrow \infty$ gives:

$$Y(x \rightarrow \infty) = \left((cY_{\text{eq}}(x_f))^{-1} + \int_{x_f}^{\infty} \frac{s}{\dot{x}}\langle\sigma v\rangle dx\right)^{-1}, \quad (3.23)$$

where $Y_{\text{eq}} = n_{\text{eq}}/s$, s is given in Eq 3.2, $\dot{x}$ is given in Eq 3.4, and n_{eq} is given in Eq 3.6. It is convenient to neglect the contribution from $(cY_{\text{eq}}(x_f))^{-1}$ as doing so ensures that $Y(x \rightarrow \infty)$ is a strictly increasing function of c . Not ignoring this term can lead to numerical issues such as having two values for c orders of magnitude apart giving the correct relic density, or there being no values of c which give the correct relic density. This is because increasing the value of c increase the value of x_f to the point where the approximation breaks down. For c which results in reasonable values of x_f , the $(cY_{\text{eq}}(x_f))^{-1}$ term stays much smaller than the $\int_{x_f}^{\infty} \frac{s}{\dot{x}} \langle \sigma v \rangle dx$. We shall see in Subsection 3.4.2 that this approximation leads to comparable results to the numerical approach. This gives:

$$Y(x \rightarrow \infty) = \left(\int_{x_f}^{\infty} \frac{s}{\dot{x}} \langle \sigma v \rangle dx \right)^{-1}, \quad (3.24)$$

Using Eq. 3.24, we can find the final relic density:

$$\Omega h^2 = \frac{m s_0 Y(x \rightarrow \infty) h^2}{\rho_C}, \quad (3.25)$$

where

$$s_0 = 2970 \text{ cm}^{-3}, \quad (3.26)$$

is the entropy density today and

$$\rho_C = \frac{3H_0^2}{8\pi G} \approx \left(1.054 \times 10^{-5} \frac{\text{GeV}}{\text{cm}^3} \right) h^2, \quad (3.27)$$

is the critical energy density today. Here, H_0 denotes the value of the Hubble parameter today and is usually given by $H_0 = 100h \text{ km/s/Mpc} \approx 70 \text{ km/s/Mpc}$ [24].

3.4 Numerical Analysis

3.4.1 Solving the Boltzmann equation numerically

In this section, we will investigate numerical solutions to equation 3.18. To do so, we will assume that the DM is composed of scalars and that the thermally averaged cross section is approximately constant with

$$\langle\sigma v\rangle=\sigma_0. \tag{3.28}$$

We will assume that the DM was in thermal equilibrium with the Standard Model well before it froze out.³

The parameters that we are interested in varying are the mass of the DM (m), the value of η , and the thermally averaged cross section σ_0 , as well as investigating the cases of $n_D = 6, 8$. The $n_D = 6$ scenario corresponds to some quintessence models with a kination phase while the $n_D = 8$ corresponds to brane world cosmology or some late inflaton decay models [57]. We are not interested in changing x_0 because any such change can be absorbed into a change in η . For this numerical analysis, we took $x_0 = 25$.

In figure 3.1, we show the required thermally averaged cross section to obtain the observed relic density $\Omega_{\text{CDM}}h^2 = 0.120$ for various values of η . First, we should comment on the shape of the $\eta = 0$ curve, namely why it starts by increasing, followed by a sharp decrease, followed by another increase. It should be noted that the typical value of x for freeze-out is in the range $x = 10$ to $x = 30$, so the important temperatures to consider are on the order of $m/10$. Since $\Omega_{\text{CDM}} \sim mn$ where m is

³If we assumed the DM were fermions, we would just change the number of degrees of freedom from 1 to 2 for Majoranna fermions or 4 for Dirac fermions. However, this does not qualitatively change the results.

Figure 3.1: The thermally averaged cross section required to obtain the observed relic density $\Omega_{\text{CDM}}h^2 = 0.120$ as a function of mass. The black, red, orange, green, blue, and purple (from the top to the bottom) curves represent $\eta = 0, 1, 10, 10^2, 10^3, 10^4$, respectively. The solid coloured curves represent $n_D = 6$ while the dashed curves represent $n_D = 8$. These coincide for the black (top) curve.

the DM mass and n is the DM number density, we see that the general trend for increasing the mass should be to decrease the number density n by increasing the thermally averaged cross section. In the low mass and high mass regimes, this is what happens since $g_*(x)$ does not change significantly for the important values of x . However, around $m = 50$ to $m = 200$ GeV, the DM is freezing out around the same time as the QCD phase transition, resulting in a rapidly changing $g_*(x)$. The increased numbers of degrees of freedom as we increase the mass results in requiring a decrease in the thermally averaged cross section to obtain the correct relic density. The derivative term $\frac{1}{3} \frac{d \log g_*}{d \log x}$ is small at all temperatures and does not significantly influence the above trends. Although it is usually ignored, we include it in our analysis.

As expected from equation 3.12, we see from figure 3.1 that increasing the value of η results in requiring a smaller thermally averaged cross section. Even modest values of η can have large effects on this value. We also see that the $n_D = 8$ case always results in needing a smaller thermally averaged cross section.

In figure 3.2, we show the required thermally averaged cross section as a function of η to obtain the correct relic density $\Omega_{\text{CDM}} h^2 = 0.120$ for various masses. One thing to notice is that the masses do not follow any nice pattern in terms of where they start off for low η . This is understood by imagining picking points from the $\eta = 0$ curve in figure 3.1 at various masses and projecting them onto the σ_0 axis. Because of the irregular shape of the curve, the starting points for the masses will also be irregular.

One thing we do see is that at very small η , there is no effect as expected, but at around $\eta = 0.1$ for $n_D = 6$ and $\eta = 0.01$ for $n_D = 8$, the effects of the modified expansion start to be seen. Between $\eta = 0.1$ and $\eta = 10$ for both cases, we see that

Figure 3.2: The required thermally averaged cross section as a function of η to obtain the correct relic density $\Omega_{\text{CDM}} h^2 = 0.120$. The top figure is for $n_D = 6$ while the bottom figure is for $n_D = 8$. The various colours black, red, orange, brown, green, blue, and purple represent masses from $10^{-3} - 10^3$ GeV by increments of an order of magnitude, respectively.

Figure 3.3: The thermally averaged cross section required to obtain the correct relic density $\Omega_{\text{CDM}} h^2 = 0.120$ as a function of mass and η . The top figure represents $n_D = 6$ while the bottom figure represents $n_D = 8$. The scale represents $\log_{10} \sigma_0$.

Figure 3.4: The ratio of the thermally averaged cross section of the $n_D = 6$ and $n_D = 8$ cases required to obtain the correct relic density $\Omega_{\text{CDM}} h^2 = 0.120$ as a function of mass and η . $\sigma_0^{(6)}$ represents the cross section for $n_D = 6$ while $\sigma_0^{(8)}$ represents the cross section for $n_D = 8$.

the required thermally averaged cross section starts to decrease according to a power law. From equation 3.18, we can deduce that this power law will have the form

$$\sigma_0 \propto \eta^{-1/2}. \quad (3.29)$$

Figure 3.1 and 3.2 give us an idea of how the cross section needs to change to get the correct relic density. In figure 3.3, we show a full map which includes both effects. The various colours represent the value of $\log_{10} \sigma_0$. Here, we see that the cross section gets much smaller as η increases and does not differ much as the mass increases. At first glance, it may be difficult to see any obvious differences between the $n_D = 6$ case on the top and the $n_D = 8$ case on the bottom. In figure 3.4, we plot the ratio $\sigma_0^{(6)}/\sigma_0^{(8)}$ where the superscript indicates the value of n_D . This shows us that the differences are more important at low mass and only differs by a factor of order 1.

At this point it should be noted that if the cross section required to obtain the correct relic density becomes too small, then it will bring the thermal equilibrium assumption into question. Although we do not worry about that in our case, if one was to consider much larger values of η , one would need to keep this in mind. Furthermore, our analysis will only hold if there is no entropy injection at the end of the new cosmological era defined by the dominance of the new energy density. If instead of a rapidly cooling energy density with $n_D > 4$, we choose, for example, a new matter dominated era with $n_D = 3$, the new energy density would need to decay away which would create an entropy injection diluting the DM. Further details about these scenarios can be found in [50–53].

3.4.2 Comparison with analytic approach

When deriving the analytic approach in section 3.3, the value of c was ambiguous. The standard lore suggests that if $\langle\sigma v\rangle \approx \sigma_0 x^{-n}$ for $x \gtrsim 3$, then one should choose c such that $c(c+2) = n+1$ [24]. The value of n is related to the velocity dependence of the cross section and, in practice, accounts for how fast the DM freezes out after it is out of equilibrium. The larger the value of n , the faster the species freezes out. Although we are taking $n = 0$ in our s-wave approximations, the effect of the extra energy density gives the equation a term which looks like $n = n_D/2 - 2$. Depending on the value of η , this term will change what we should choose for c to the point that it is not clear what should be chosen.

We found that if we make fewer simplifications while using this approach, such as using Eq. 3.21 to find x_f and Eq. 3.24 to find $Y(x \rightarrow \infty)$ without neglecting terms, we need much different values of c from the typical $c(c+2) = n+1$ to obtain the same relic density. See figure 3.5 for the values of c needed to obtain the correct relic density using the cross section values obtained in figure 3.3. These were computed by using a binary search for the value of c after ensuring that such a search was applicable. We see that the value of c changes drastically when we change η but does not change much as the mass changes. Using this information, we can fit for c to obtain the simple approximation:

$$c \approx \begin{cases} 0.0165m^{-0.219}\eta^{0.688} & \text{for } n_D = 6, \\ 0.0658m^{-0.233}\eta^{0.774} & \text{for } n_D = 8. \end{cases} \quad (3.30)$$

Using this approximation, we can calculate the relic density and see how it compares

to the actual value. In figure 3.6, we plot

$$\Delta\Omega h^2 = \Omega_{\text{approx}} h^2 - 0.120. \quad (3.31)$$

This approach works well for most of the parameter space explored but can be off by a factor of order one in some places. In particular at low η and low mass, the analytic approximation tends to be too small while at high η and low mass, the result $n_D = 8$ tends to be too large. This does allow one to have the correct order of magnitude for the relic density which, depending on the purpose, can be good enough. In the range considered, the biggest deviations in the $n_D = 6$ case were $\Omega h^2 = 0.0715$ on the low end and $\Omega h^2 = 0.161$ on the high end. For the $n_D = 8$ case, the biggest deviations were $\Omega h^2 = 0.0812$ on the low end and $\Omega h^2 = 0.255$ on the high end.

One may be concerned about the large range of c obtained and the validity of the approximations made in section 3.3. Recall that in the small x regime, we assumed that $\Delta = Y - Y_{\text{eq}}$ was small compared to Y_{eq} and for large x , we assume that Y_{eq} is much smaller than Δ . Inevitably, when we go from one to the other, the assumption will break down and both values will be comparable. Where to choose this point becomes a question of what best fits the data.

3.5 Summary and Conclusions

In this work, we derived a general Boltzmann equation which can be used for any modified expansion history of the universe. This was done by characterizing the energy density as a general function and letting w from the equation of state vary with temperature. We then looked at the specific example where we could write the energy density as a sum of components where all the values of w from the equations of state

Figure 3.5: The value of c needed to obtain the correct relic density using the cross section values from figure 3.3. The top plot has $n_D = 6$ and the bottom plot has $n_D = 8$. Plotted is the value of $\log_{10} c$.

Figure 3.6: The difference in the relic density obtained using the values of c from Eq 3.30 and the observed relic density. The top plot has $n_D = 6$ and the bottom plot has $n_D = 8$.

for each component were constant. We applied these results to obtain the equation specifically for the case of a single extra energy density component parametrized by how fast it cools, n_D , and how abundant it was compared to radiation, η . Finally, using Eq. 3.11, we found an analytic approach to approximate the relic density.

We then solved the modified Boltzmann equation for this last case to compare the effects to the standard picture. We found that for $n_D = 6$ and $n_D = 8$, the required cross section to obtain the measured relic does not change significantly for $\eta \lesssim 10^{-1}$ and follows a power law for $\eta \gtrsim 10$ with some transition region between them. Importantly, a larger value of η required a smaller cross section. This is quite simple to understand if we were to imagine what the expansion history of the universe would be if we ignored radiation. In this hypothetical situation, the scale factor would go as $a \propto t^{2/n_D}$, which gives a Hubble expansion rate of $H = 2t/n_D$, meaning that at the same temperature, the universe would be expanding slower than the radiation dominated case where $H = t/2$. Because of this, it takes more time for the rate of expansion to be comparable to the rate of the reaction keeping the DM in equilibrium, so freeze-out would occur later. A smaller cross section for the equilibrium reaction, which normally results in the DM freezing out earlier, is required to balance this effect.

We also showed that in the $n_D = 8$ case, the cross section needed is comparable to the $n_D = 6$ case. The only significant difference being in the low mass, high η case where they start to differ by a factor of order 1.

This result has some important consequences for model building and DM detection. If one of these modified cosmological scenarios occurred in our Universe and DM went through freeze-out, then it may well be the case that at weak scale masses, the cross sections are orders of magnitude smaller than current bounds. However, this

opens the possibility that heavier DM particles underwent freeze-out. The bound of the mass of DM undergoing freeze-out is about 100 TeV from perturbative unitarity [23], but this bound could be relaxed in these scenarios. This again poses a problem for DM detection as they are not designed to look for DM at these masses. All in all, these results suggest that DM detection may be a bigger challenge than previously expected if there is a significant change to the expansion rate of the early Universe.

Chapter 4

Implications of the observation of DM self-interactions

4.1 Introduction

Evidence for DM self-interactions has been reported [59] based on observations of four elliptical galaxies in the inner 10 kpc core of galaxy cluster Abell 3827. Using gravitational lensing, Ref. [59] reconstructs the DM halos of the four galaxies and observes that at least one of the halos is spatially offset from its stars by a distance of $\Delta = 1.62^{+0.47}_{-0.49}$ kpc. DM self-interactions can lead to such an offset: the DM halos of the infalling galaxies will be impeded as they pass through the DM associated with the cluster. The magnitude of this effect scales with the self-interaction cross section, which can therefore be extracted from the size of the offset.

Kahnhoefer *et al.* [60] consider the case of DM contact interactions, which will be relevant for the scalar DM models we consider below. Whereas the effect on the halo of frequent DM interactions can be described as a drag force, in the case of rare, high-momentum-transfer contact interactions, this picture is inaccurate. Rather, in

the contact-interaction scenario the observed separation is due to the evaporation of DM from the halo. Most DM particles in the halo will not interact, but those that do scatter will be ejected from the halo in the backward direction. Eventually the scattered particles will become substantially separated from the halo, but at short time scales, they remain observationally associated with the galaxy. This leads to a deformation of the halo and an offset between the centroids of the galaxy and the halo. Determination of the size of the offset is fairly sensitive to how the centroid is defined, but Ref. [60] finds that a cross section of

$$\sigma_{\text{DM}}/m \sim 1.5 \text{ cm}^2 \text{ g}^{-1} \quad (4.1)$$

could account for the offset observed in Ref. [59]. This is in slight tension with results from previous studies of other astrophysical systems [12, 60–64], which find an upper bound on the cross section of

$$\sigma_{\text{DM}}/m \lesssim 1 \text{ cm}^2 \text{ g}^{-1}. \quad (4.2)$$

We will use Eqs. (4.1) and (4.2) to set a range for the estimated self-interaction cross section. In particle physics units, this corresponds to

$$\sigma_{\text{DM}}/m \sim (4.7 - 7.0) \times 10^3 \text{ GeV}^{-3}. \quad (4.3)$$

Of course, it is possible that the offset may be an astrophysical artifact. Nevertheless, if a DM self-interaction interpretation holds up, it would represent a profound advance in our knowledge of the nature of DM. It is therefore of great importance to explore its implications for models of DM.

In this chapter we consider models in which the dark matter is a gauge-singlet real scalar particle S that interacts with the rest of the SM through the “Higgs portal” via an operator of the form $SS\Phi^\dagger\Phi$, where Φ is the $SU(2)_L$ -doublet SM Higgs field. The singlet scalar is kept stable by imposing a symmetry $S \rightarrow -S$ on the Lagrangian. In Sec. 4.2 we consider the minimal model in which a single gauge-singlet real scalar is added to the SM [65–84], and show that it cannot explain the observed DM self-interaction cross section once we impose the requirement that the annihilation cross section of the singlet scalars into SM particles yields the correct relic abundance from thermal freeze-out together with the constraints on the invisible branching fraction of the Higgs boson from the CERN Large Hadron Collider (LHC) [85, 86]. The correct relic abundance can, however, be achieved in this model through thermal “freeze-in,” in which the DM is produced slowly in the early universe through extremely weak interactions with the SM Higgs boson [69, 75, 87]. In Sec. 4.3 we attempt to salvage the usual thermal freeze-out picture by considering the real singlet scalar extension of two-Higgs-doublet models (2HDMs) [88–96, 96–98]. We find a small viable region of parameter space in which the singlet scalar mass is tuned to be very close to half the mass of the second, as-yet-undiscovered CP-even neutral Higgs boson of these models. In Sec. 4.4 we conclude.

4.2 Minimal singlet extension of the Standard Model

We write the scalar potential for the SM plus a real singlet as

$$V = -\mu^2\Phi^\dagger\Phi + \frac{\mu_S^2}{2}SS + \lambda_h(\Phi^\dagger\Phi)^2 + \lambda_p SS\Phi^\dagger\Phi + \lambda_S S^4, \quad (4.4)$$

where we have imposed invariance under $S \rightarrow -S$ to keep the singlet stable. The physical SM Higgs mass $m_h \simeq 125$ GeV [99] and the SM Higgs vacuum expectation value (vev) $v \equiv (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV [100] fix two of the scalar potential parameters,

$$\begin{aligned}\lambda_h &= \frac{m_h^2}{2v^2} \simeq 0.129, \\ \mu^2 &= \lambda_h v^2 \simeq (88.4 \text{ GeV})^2.\end{aligned}\tag{4.5}$$

For the potential to be bounded from below we require that $\lambda_S > 0$ and $\lambda_p > -2\sqrt{\lambda_h \lambda_S}$. To keep the singlet from getting a vev and thereby ceasing to be stable, we require that

$$m_S^2 = \mu_S^2 + \lambda_p v^2 > 0,\tag{4.6}$$

where m_S is the physical mass of the singlet scalar in the electroweak-breaking vacuum. This implies that at most one of μ_S^2 or λ_p can be negative. If $\mu_S^2 < 0$ the potential can develop a second minimum in which the singlet gets a vev and the doublet does not; to avoid this possibility we require that $\mu_S^2 > 0$.

The Feynman rules involving the singlet that will be important here are

$$\begin{aligned}SSSS : & \quad -24i\lambda_S, \\ SSh : & \quad -2i\lambda_p v.\end{aligned}\tag{4.7}$$

We will consider DM scattering $SS \rightarrow SS$ in three mass ranges. We work in the minimal real singlet scalar extension of the SM, but what follows will inform the calculations in the extended Higgs sector that we consider later. The relevant Feynman diagrams are shown in Fig. 4.1.

Figure 4.1: Feynman diagrams for the $SS \rightarrow SS$ scattering process in the real scalar singlet extension of the SM.

4.2.1 Heavy DM, $m_S \gg m_h$

When S is very heavy compared to the SM Higgs mass, the Higgs-exchange diagrams in Fig. 4.1 are suppressed by momentum factors compared to the four-point diagram, so we neglect them for simplicity. The scattering cross section per unit DM mass can then be written as

$$\sigma_{\text{DM}}/m_S \simeq \frac{9\lambda_S^2}{2\pi m_S^3}. \quad (4.8)$$

Perturbative unitarity of the $2 \rightarrow 2$ scattering amplitude for $SS \rightarrow SS$ at high energy sets an upper bound on λ_S . Requiring that the zeroth partial wave amplitude a_0 satisfy $|\text{Re } a_0| < 1/2$ and taking into account the identical particle factors of $1/\sqrt{2}$ for the initial and final states yields

$$\lambda_S < 2\pi/3. \quad (4.9)$$

Unfortunately, after imposing this constraint we find that the heavy-singlet scenario is inconsistent with the measured cross section in Eq. (4.3). Indeed, reproducing the observed DM self-interaction cross section would require $m_S \lesssim 0.1$ GeV; we will consider this low-mass scenario below.

4.2.2 DM near the Higgs resonance, $m_S \sim m_h/2$

When the scattering is near resonance, the s -channel Higgs exchange diagram in Fig. 4.1 dominates. Defining the hSS coupling

$$g_{SSH} = -2\lambda_p v, \quad (4.10)$$

we can write the hSS Feynman rule as ig_{SSH} . The s -channel Higgs exchange diagram in Fig. 4.1 then yields a DM self-interaction cross section per unit mass of¹

$$\sigma_{\text{DM}}/m_S \simeq \frac{g_{SSH}^4}{128\pi m_S^3 ((4m_S^2 - m_h^2)^2 + m_h^2 \Gamma_h^2)}, \quad (4.11)$$

where Γ_h is the Higgs width.

Perturbative unitarity of the four $SS \rightarrow \phi_i \phi_i$ scattering amplitudes, where ϕ_i are the four real scalar components of the SM Higgs doublet Φ , constrains

$$|\lambda_p| < 4\pi, \quad \text{or} \quad |g_{SSH}| < 8\pi v. \quad (4.12)$$

Because of the contribution of the $h \rightarrow SS$ decay channel to the Higgs width, the region below half the Higgs mass is severely fine tuned such that $m_h/2 - m_S \lesssim 10^{-6}$ GeV in order to achieve the observed DM self-interaction cross section. For the region above half the Higgs mass, this decay channel is not kinematically allowed, and the restriction reduces to $m_S - m_h/2 \lesssim 0.11$ GeV, where we use the lower value of σ_{DM}/m_S in Eq. (4.3) to obtain an upper bound. This represents a tuning of about two parts per mille.

¹The contribution to the centre-of-mass collision energy of the present-day kinetic energy will be totally negligible compared to the mass splitting that we will find, so we neglect it here.

Figure 4.2: Feynman diagrams for the annihilation of singlet scalars into SM particles in the real singlet scalar extension of the SM.

We now show that this scenario cannot be reconciled with the observed DM relic density because the annihilation cross section into SM particles through the Higgs resonance is several orders of magnitude too large.

4.2.2.1 Relic density

The annihilation cross section required to obtain the correct thermal DM relic density for $m_S \sim m_h/2$ is [30]

$$\sigma v_{\text{rel}} \simeq 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \simeq 1.9 \times 10^{-9} \text{ GeV}^{-2}. \quad (4.13)$$

For $m_S \sim m_h/2$, the singlet scalars annihilate into SM particles predominantly through s -channel Higgs exchange, as shown in Fig. 4.2. The temperature of the thermal bath at the time of freeze-out for a DM particle with the correct relic density is $T \sim m_S/20 \sim 5 \text{ GeV}$ [15]. This is much larger than the mass splitting $|m_S - m_h/2| \lesssim 0.11 \text{ GeV}$ required to obtain a large enough DM self-interaction cross section, so thermal effects during freeze-out cannot be neglected.

The annihilation cross section can be written as

$$\sigma v_{\text{rel}} = \frac{2g_{SSh}^2 \Gamma_{\text{tot}}^{h_{\text{SM}}}}{E_{\text{cm}}((E_{\text{cm}}^2 - m_h^2)^2 + m_h^2 \Gamma^2)}, \quad (4.14)$$

where $E_{\text{cm}} = 2m_S + m_S v_{\text{rel}}^2/4$ is the collision centre-of-mass energy, v_{rel} is the relative velocity of the two DM particles in the centre-of-momentum frame and $\Gamma_{\text{tot}}^{h\text{SM}} \simeq 4.1$ MeV is the SM Higgs total width [101].

The singlet scalar can be heavier or lighter than half the Higgs mass. For a singlet scalar heavier than half the Higgs mass, the thermal motion pushes the annihilation process farther from resonance. Imposing the self-interaction cross section measurement and integrating the annihilation cross section over the Boltzmann distribution with $T \sim m_S/20$ yields

$$\langle \sigma v_{\text{rel}} \rangle \gtrsim 0.03 \text{ GeV}^{-2}, \quad (4.15)$$

where the bound is saturated for $|g_{SSH}|$ as large as possible. This is many orders of magnitude larger than the required cross section for thermal freeze-out given in Eq. (4.13), resulting in a singlet scalar relic abundance way too small to account for the observed DM. If the singlet scalar is lighter than half the Higgs mass, the thermal motion pushes the annihilation process onto the resonance resulting in an even larger thermally-averaged cross section and even smaller relic abundance. These large cross sections are a consequence of the large SSH coupling and the tuning of the singlet mass close to the Higgs resonance that is required to achieve the observed DM self-interaction cross section.

4.2.3 Low-mass DM, $m_S \ll m_h$

We now consider the situation of low-mass DM. In this case all four diagrams in Fig. 4.1 contribute. At low centre-of-mass energies, the Higgs boson h can be inte-

grated out, yielding an effective four- S coupling with Feynman rule $-24i\lambda_{\text{eff}}$, where

$$\lambda_{\text{eff}} = \lambda_S - \frac{g_{SSh}^2}{8m_h^2}. \quad (4.16)$$

$SS \rightarrow SS$ scattering in the low-energy effective theory must also satisfy perturbative unitarity, so as in Eq. (4.9) we obtain

$$|\lambda_{\text{eff}}| < 2\pi/3. \quad (4.17)$$

The DM scattering cross section per unit mass as in Eq. (4.8) becomes

$$\sigma_{\text{DM}}/m_S \simeq \frac{9\lambda_{\text{eff}}^2}{2\pi m_S^3}. \quad (4.18)$$

The range of m_S and λ_{eff} consistent with the cross section measurement from Ref. [60] [Eq. (4.3)] is shown in Fig. 4.3. In particular, the singlet scalar DM must be very light, with mass below about 0.1 GeV.

4.2.3.1 Invisible Higgs decays

When $m_S \ll m_h$, the nonobservation of invisible Higgs boson decays constrains λ_p .

The partial width for $h \rightarrow SS$ is

$$\Gamma(h \rightarrow SS) = \frac{g_{SSh}^2}{32\pi m_h} \sqrt{1 - \frac{4m_S^2}{m_h^2}} \simeq \frac{g_{SSh}^2}{32\pi m_h}, \quad (4.19)$$

where the last expression holds for $m_S \ll m_h$. The invisible Higgs branching fraction is defined as

$$\text{BR}(h \rightarrow \text{invis.}) = \frac{\Gamma(h \rightarrow SS)}{\Gamma_{\text{tot}}^{h_{\text{SM}}} + \Gamma(h \rightarrow SS)}. \quad (4.20)$$

Figure 4.3: Favored region in m_S and the effective $4S$ coupling λ_{eff} defined in Eq. (4.16) for a DM self-interaction cross section in the range $1.0\text{--}1.5\text{ cm}^2\text{ g}^{-1}$. Values of $|\lambda_{\text{eff}}|$ above $2\pi/3 \simeq 2.094$ (horizontal dashed line) are excluded by perturbative unitarity considerations. The vertical dotted lines show the thresholds for SS annihilation into pairs of electrons, muons, and neutral pions. The charged pion threshold is at the right edge of the plot.

The strongest constraint on $\text{BR}(h \rightarrow \text{invis.})$ currently comes from the ATLAS search for invisible Higgs decays in vector boson fusion production, with a 95% confidence level upper limit of $\text{BR}(h \rightarrow \text{invis.}) \leq 0.29$ for a 125 GeV Higgs boson, assuming a SM production cross section [85]. This leads to an upper bound on g_{SSh}^2 , or equivalently $|\lambda_p|$, of

$$g_{SSh}^2 \leq 21\text{ GeV}^2, \quad \text{or} \quad |\lambda_p| \leq 9.3 \times 10^{-3}. \quad (4.21)$$

This constraint is very strong compared to the perturbative unitarity constraint $|\lambda_p| < 4\pi$.

One can also derive an upper bound on g_{SSh}^2 from the measured signal strengths μ_i of the Higgs boson in various SM decay channels. In the singlet scalar extension

of the SM, these signal strengths are all modified by the common factor

$$\mu_i \equiv \mu = \frac{\Gamma_{\text{tot}}^{h_{\text{SM}}}}{\Gamma_{\text{tot}}^{h_{\text{SM}}} + \Gamma(h \rightarrow SS)}. \quad (4.22)$$

Combining all channels, ATLAS [102] and CMS [103] respectively measure an overall signal strength of

$$\begin{aligned} \mu &= 1.18_{-0.14}^{+0.15} \quad (\text{ATLAS}), \\ \mu &= 1.00_{-0.13}^{+0.14} \quad (\text{CMS}). \end{aligned} \quad (4.23)$$

In the real singlet extension of the SM, $\mu \leq 1$, therefore, relative to this best-fit point, the $\Delta\chi^2 = 4$ lower bounds on the signal strength are²

$$\begin{aligned} \mu &\geq 0.85 \quad (\text{ATLAS}), \\ \mu &\geq 0.74 \quad (\text{CMS}). \end{aligned} \quad (4.24)$$

These in turn lead to upper bounds on g_{SSh}^2 , or equivalently $|\lambda_p|$, of

$$\begin{aligned} g_{SSh}^2 &\leq 9.1 \text{ GeV}^2, \quad \text{or} \quad |\lambda_p| \leq 6.1 \times 10^{-3} \quad (\text{ATLAS}), \\ g_{SSh}^2 &\leq 18 \text{ GeV}^2, \quad \text{or} \quad |\lambda_p| \leq 8.6 \times 10^{-3} \quad (\text{CMS}). \end{aligned} \quad (4.25)$$

4.2.3.2 Relic density: Thermal freeze-out

The annihilation cross section required to obtain the correct thermal DM relic density varies slightly with m_S [30]. For m_S around 0.1 GeV, the annihilation cross section

²In the ATLAS case, the $\mu = 1$ point is $\Delta\chi^2 = 1.65$ away from the best fit value of $\mu = 1.18$. We thus apply a $\Delta\chi^2 = 5.65$ lower bound for the signal strength. This is equivalent to looking at about 2.38 standard deviations away.

needs to be [30]

$$\sigma v_{\text{rel}} \simeq 4.8 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \simeq 4.1 \times 10^{-9} \text{ GeV}^{-2}. \quad (4.26)$$

In the low-mass range, the singlet scalars annihilate into SM particles predominantly through s -channel Higgs exchange, as shown in Fig. 4.2. This annihilation cross section can be conveniently expressed for $m_S \ll m_h$ as

$$\sigma v_{\text{rel}} \simeq \frac{4v^2 \lambda_p^2}{m_h^4} \frac{\Gamma_{\text{tot}}^{h\text{SM}}(m = 2m_S)}{m_S}, \quad (4.27)$$

where $\Gamma_{\text{tot}}^{h\text{SM}}(m = 2m_S)$ is the would-be total width of the SM Higgs boson if its mass were equal to $2m_S$. The largest possible values of σv_{rel} are obtained at the high end of the singlet mass range shown in Fig. 4.3, when annihilations into muon pairs are kinematically accessible. In that case, $\Gamma_{\text{tot}}^{h\text{SM}}(m = 2m_S)$ is given to a good approximation by the decay width into muon pairs,

$$\Gamma(h^* \rightarrow \mu\mu) = \frac{1}{8\pi} \frac{m_\mu^2}{v^2} (2m_S) \left[1 - \frac{m_\mu^2}{m_S^2} \right]^{3/2}. \quad (4.28)$$

Inserting this into Eq. (4.27) yields

$$\sigma v_{\text{rel}} \simeq (1.5 \times 10^{-11} \text{ GeV}^{-2}) \lambda_p^2 \left[1 - \frac{m_\mu^2}{m_S^2} \right]^{3/2}. \quad (4.29)$$

Even ignoring the kinematic suppression from the muon mass and the upper bound on $|\lambda_p|$ from LHC Higgs data, it is not possible to obtain a large enough DM annihilation cross section while satisfying the constraint on λ_p from perturbative unitarity; one would need $|\lambda_p| \simeq 5\pi$, in violation of Eq. (4.12). Imposing the constraint on $|\lambda_p|$ from LHC searches for invisible Higgs decays or the measurement of the Higgs signal

strengths yields an annihilation cross section more than six orders of magnitude too small to yield the correct thermal freeze-out DM relic abundance.

We also note that, even if the appropriate annihilation cross section for thermal freeze-out could be obtained, this low-mass scenario with $SS \rightarrow \mu\mu$ is strongly excluded by cosmic microwave background constraints arising from the electromagnetic energy injection at the time of recombination [104] (for an update with Planck data see Ref. [105]).

4.2.3.3 Relic density: Thermal freeze-in

The singlet scalar DM extension of the SM can easily account for the observed DM self-interactions if the singlet mass is below about 0.1 GeV, as shown in Fig. 4.3. The model fails when it attempts to account for the DM relic abundance via thermal freeze-out, because the singlet annihilation cross section into SM particles cannot be made large enough. However, there is an alternative perfectly viable way to obtain the correct DM relic abundance through thermal *freeze-in* [69, 87]. In this scenario, the cross section for two DM particles to annihilate into two SM particles, or vice versa, is extremely small, so that the DM never comes into thermal equilibrium with the SM plasma in the early universe. Instead, DM is slowly produced through annihilations of SM particles, until the universe becomes too cool for DM to be produced. The resulting DM abundance “freezes in” and persists to the present day. Because annihilation of DM particles never becomes important, the kinematic thresholds shown in Fig. 4.3 are irrelevant in this scenario.

This scenario was studied for the singlet scalar extension of the SM in Refs. [69, 75]. In our scenario with $m_S \lesssim 0.1$ GeV, the correct DM relic abundance is obtained for $|\lambda_p| \sim (2 - 3) \times 10^{-11}$ [75].³ This is easily consistent with the upper bounds

³Note that, for this coupling strength, the $\lambda_p SS\Phi^\dagger\Phi$ term contributes about $(1 \text{ MeV})^2$ to m_S^2 [69,

$|\lambda_p| < (6.1 - 9.3) \times 10^{-3}$ from LHC Higgs signal strength measurements and invisible decay branching ratio limits. Such a small $SS\Phi^\dagger\Phi$ interaction strength is even technically natural: if set to zero, λ_p cannot be regenerated through loops involving other couplings in the model. The only disadvantage is phenomenological: the DM becomes impossible to detect through nongravitational interactions with SM particles.

4.3 Singlet extension of the two Higgs doublet model

To try to salvage the thermal freeze-out scenario for singlet scalar DM, we consider extended models with two resonances, the discovered SM-like Higgs boson h with mass 125 GeV and a second CP-even scalar H , which can be lighter or heavier than h . Generic models allow both states to couple to SS ; as a prototype we consider the singlet scalar DM extension of the two-Higgs-doublet model (2HDM) [88–96, 96–98].

As mentioned previously, thermal freeze-out in the low-mass scenario with $m_S \lesssim 0.1$ GeV is already strongly excluded by cosmic microwave background constraints [104]. We therefore focus in what follows on resonant scattering and annihilation through the second scalar resonance H . In the singlet scalar extension of the SM, the resonant-scattering scenario failed because the cross section for DM annihilation via the Higgs resonance was several orders of magnitude too large. In the singlet scalar extension of the 2HDM, we can get around this by having the DM interact resonantly through the H pole while making the coupling of H to SM particles sufficiently weak, thereby suppressing the thermal DM annihilation cross section through the H resonance. As we will see, this will not work in the usual 2HDMs with natural flavour conservation, but it can be made to work in the Yukawa-aligned [106](#).

2HDM extended with a singlet.

Of course, the singlet scalar extension of the 2HDM can also account for the observed DM relic density through thermal *freeze-in* when $m_S \lesssim 0.1$ GeV and the couplings of S to the two Higgs doublets are both extremely small.

4.3.1 2HDM with natural flavour conservation

The scalar potential for the singlet scalar DM extension of the 2HDM can be written as

$$V = \frac{\mu_S^2}{2} S^2 + \lambda_S S^4 + \lambda_{p1} S S \Phi_1^\dagger \Phi_1 + \lambda_{p2} S S \Phi_2^\dagger \Phi_2 + V_{\text{2HDM}}, \quad (4.30)$$

where Φ_1 and Φ_2 are the two Higgs doublets and V_{2HDM} is the usual scalar potential of the 2HDM. We have imposed a second Z_2 symmetry under which $\Phi_1 \rightarrow -\Phi_1$, which can be softly broken by a dimension-two term in V_{2HDM} . This allows us to avoid flavour-changing neutral Higgs interactions through natural flavour conservation [107], by forcing the right-handed fermions to couple to only one Higgs doublet. The transformation properties of the right-handed fermions under the second Z_2 symmetry determine the “Type” of 2HDM (for a review see Ref. [108]).

To set the notation, we write the vacuum expectation values as

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}, \quad (4.31)$$

with $v_1^2 + v_2^2 = v^2 \simeq (246 \text{ GeV})^2$ and $\tan \beta \equiv v_2/v_1$. In terms of the real neutral components of the two doublets $\Phi_i^0 \equiv \phi_i^{0,r}/\sqrt{2}$, the physical CP-even neutral scalars h (identified as the 125 GeV Higgs) and H are defined as

$$h = -\sin \alpha \phi_1^{0,r} + \cos \alpha \phi_2^{0,r},$$

$$H = \cos \alpha \phi_1^{0,r} + \sin \alpha \phi_2^{0,r}, \quad (4.32)$$

where α is a mixing angle.

In terms of these parameters, the Feynman rules for the SSh and SSH vertices are ig_{SSh} and ig_{SSH} , respectively, with

$$\begin{aligned} g_{SSh} &= 2v(\lambda_{p1} \cos \beta \sin \alpha - \lambda_{p2} \sin \beta \cos \alpha), \\ g_{SSH} &= 2v(-\lambda_{p1} \cos \beta \cos \alpha - \lambda_{p2} \sin \beta \sin \alpha). \end{aligned} \quad (4.33)$$

Perturbative unitarity of the $SS \rightarrow \Phi_1 \Phi_1$ and $SS \rightarrow \Phi_2 \Phi_2$ scattering amplitudes separately constrain⁴

$$|\lambda_{p1}| < 4\pi, \quad |\lambda_{p2}| < 4\pi, \quad (4.34)$$

so $|g_{SSH}|$ can be as large as $8\pi v$.

In the 2HDM plus a singlet scalar, the low-energy effective $SS \rightarrow SS$ coupling for $m_S \ll m_h, m_H$ is given by

$$\lambda_{\text{eff}} = \lambda_S - \frac{g_{SSh}^2}{8m_h^2} - \frac{g_{SSH}^2}{8m_H^2}. \quad (4.35)$$

The constraint $m_S \lesssim 0.1$ GeV carries through from the analysis in the previous section, so we do not pursue this low-mass scenario further.

To avoid experimental constraints from h coupling measurements, we work in the alignment limit and tune $\sin(\beta - \alpha) \simeq 1$ [109]. This means that

$$\sin \alpha \simeq -\cos \beta, \quad \cos \alpha \simeq \sin \beta. \quad (4.36)$$

⁴A proper coupled-channel analysis would lead to a tighter constraint.

For $\sin(\beta - \alpha) \simeq 1$, the H coupling to vector boson pairs is heavily suppressed, being proportional to $\cos(\beta - \alpha)$. This allows H to evade searches for invisibly decaying Higgs bosons via H production in vector boson fusion or associated VH production. We focus on the Type-I 2HDM, since in that model all the fermion Yukawa couplings can be suppressed by making $\cot \beta$ small. The H couplings to fermions in the Type-I 2HDM are equal to the corresponding couplings of the SM Higgs boson multiplied by a common scaling factor

$$\kappa_f^H = \frac{\sin \alpha}{\sin \beta} \simeq -\cot \beta, \quad (4.37)$$

where we used Eq. (4.36) to obtain the coupling in the $\sin(\beta - \alpha) \rightarrow 1$ limit. Perturbativity of the top quark Yukawa coupling requires $\cot \beta < 10/3$ [110].

For $m_S \sim m_H/2$, the $SS \rightarrow SS$ scattering is dominated by the s -channel H exchange diagram, yielding a cross section per unit DM mass

$$\sigma_{\text{DM}}/m_S \simeq \frac{g_{SSH}^4}{128\pi m_S^3((4m_S^2 - m_H^2)^2 + m_H^2\Gamma_H^2)}, \quad (4.38)$$

where Γ_H is the width of H . As in the case of the singlet scalar extension of the SM, the singlet mass must be tuned to be extremely close to $m_H/2$ for this scattering cross section to account for the halo displacement observed in Ref. [59].

The DM annihilation cross section to SM particles, which controls the relic density, is similarly given for annihilation through the H resonance by

$$\sigma v_{\text{rel}} \simeq \frac{2g_{SSH}^2\Gamma_{\text{tot}}^{H \rightarrow \text{SM}}}{E_{\text{cm}}((E_{\text{cm}}^2 - m_H^2)^2 + m_H^2\Gamma_H^2)}. \quad (4.39)$$

These expressions take an analogous form to those in Eqs. (4.11) and (4.14) for the singlet scalar extension of the SM. In that model, the problem was that the annihilation cross section was much too large once the DM self-interaction cross section was

fixed to the newly observed value. We therefore attempt to suppress the width of H into SM particles while keeping g_{SSH} large.⁵

We take $m_H > m_h$, so that $m_S > m_h/2$ and there is no need to tune g_{SSH} to be small, since invisible decays $h \rightarrow SS$ will be kinematically forbidden. To suppress $H \rightarrow WW, ZZ$ and keep the h couplings SM-like, we require $\sin(\beta - \alpha) \rightarrow 1$. To suppress H decays to fermions, we take $\cot \beta \ll 1$ in the Type-I 2HDM. Then the decay width of H to SM particles, for $m_H < 2m_t$, can be approximated as

$$\Gamma_{\text{tot}}^{H \rightarrow \text{SM}} \simeq (\kappa_f^H)^2 \left(\frac{m_H}{125 \text{ GeV}} \right) [1 - \text{BR}(h_{\text{SM}} \rightarrow VV)] \Gamma_{\text{tot}}^{h_{\text{SM}}}, \quad (4.42)$$

where $\text{BR}(h_{\text{SM}} \rightarrow VV) \equiv \text{BR}(h_{\text{SM}} \rightarrow WW) + \text{BR}(h_{\text{SM}} \rightarrow ZZ) \simeq 0.24$ [101]. In particular, $\Gamma_{\text{tot}}^{H \rightarrow \text{SM}} \propto (\kappa_f^H)^2 \simeq \cot^2 \beta$.

Comparing Eq. (4.14) with Eqs. (4.39) and (4.42), we see that we need to severely suppress the parameter combination $(\kappa_f^H)^2/g_{SSH}^2$. However, in this model the SSH coupling is

$$g_{SSH} \simeq 2v \sin \beta \cos \beta (-\lambda_{p1} + \lambda_{p2}), \quad (4.43)$$

where we have used $\sin(\beta - \alpha) \simeq 1$. In particular, $g_{SSH} \propto \cos \beta$, so that the ratio $(\kappa_f^H)^2/g_{SSH}^2$ is not at all suppressed at $\cot \beta \ll 1$. Indeed, once the upper bounds $|\lambda_{p1}| < 4\pi$ and $|\lambda_{p2}| < 4\pi$ from perturbative unitarity are imposed, the situation is no better than in the singlet scalar extension of the SM discussed in Sec. 4.2.2.

⁵Note that, if $m_H < m_h$, the $h \rightarrow SS$ invisible decay becomes kinematically accessible and must be avoided by tuning g_{SSH} to be small. Together with Eq. (4.36), this implies that

$$\lambda_{p1} \cot \beta \simeq -\lambda_{p2} \tan \beta, \quad (4.40)$$

and hence

$$g_{SSH} \simeq 2v \lambda_{p1} \cot \beta \simeq -2v \lambda_{p2} \tan \beta. \quad (4.41)$$

4.3.2 Yukawa-aligned 2HDM

In the preceding section our main problem was that suppressing the H couplings to fermions simultaneously suppressed the SSH coupling, so that the factor $(\kappa_f^H)^2/g_{SSH}^2$ in Eq. (4.39) remained no smaller than in the singlet scalar extension of the SM. This could be avoided if the scalar potential contained a term $(SS\Phi_1^\dagger\Phi_2 + \text{H.c.})$. Such a term breaks the Z_2 symmetry imposed to avoid flavour-changing neutral Higgs interactions; however, there is no reason to assume *a priori* that flavour conservation originates from the structure of the scalar potential.

The Yukawa-aligned 2HDM [111] allows both Higgs doublets to couple to all fermions, and avoids flavour-changing neutral Higgs interactions by positing that the (unknown) flavour physics causes the two Yukawa matrices for the up-type quarks to be proportional to each other, and similarly for the down-type quarks and the charged leptons. Then one can add a third term to the scalar potential in Eq. (4.30) coupling the singlet scalar to the doublets,

$$V \supset (\lambda_{p3} SS\Phi_1^\dagger\Phi_2 + \text{H.c.}), \quad (4.44)$$

where for simplicity we will avoid CP violation by choosing λ_{p3} to be real.⁶ The couplings in Eq. (4.33) become

$$\begin{aligned} g_{SSH} &= 2v[\lambda_{p1} \cos \beta \sin \alpha - \lambda_{p2} \sin \beta \cos \alpha \\ &\quad - \lambda_{p3}(\cos \beta \cos \alpha - \sin \beta \sin \alpha)], \\ g_{SSH} &= 2v[-\lambda_{p1} \cos \beta \cos \alpha - \lambda_{p2} \sin \beta \sin \alpha \\ &\quad - \lambda_{p3}(\cos \beta \sin \alpha + \sin \beta \cos \alpha)]. \end{aligned} \quad (4.45)$$

⁶The singlet scalar extension of the Yukawa-aligned 2HDM was considered in Ref. [96], but the term in Eq. (4.44) was not considered there.

Tuning $\sin(\beta - \alpha) \simeq 1$ to make the h couplings SM-like reduces these to

$$\begin{aligned}
g_{SSH} &\simeq 2v[-\lambda_{p1} \cos^2 \beta - \lambda_{p2} \sin^2 \beta - 2\lambda_{p3} \sin \beta \cos \beta], \\
g_{SSH} &\simeq 2v[(-\lambda_{p1} + \lambda_{p2}) \sin \beta \cos \beta \\
&\quad - \lambda_{p3}(\sin^2 \beta - \cos^2 \beta)].
\end{aligned} \tag{4.46}$$

Now a suppression of the $(\kappa_f^H)^2$ factor in Eq. (4.39) need not be counteracted by a suppression of g_{SSH}^2 . We can choose the relative Yukawa couplings of the two Higgs doublets so that the H couplings to fermions are all suppressed while at the same time keeping g_{SSH} unsuppressed. We need a fermionic coupling suppression $\kappa_f^H \sim 10^{-4}$ in order to obtain an annihilation cross section in the right range for thermal freeze-out. For a full treatment one should also take into account the effect of thermal averaging of the annihilation cross section at the time of freeze-out, which may be important due to the severe tuning of the singlet mass relative to the H resonance. We leave this for future work.

We finally remark that, if $m_H > m_h$, this scenario offers the possibility of direct DM detection due to scattering through exchange of h . The SSH coupling need not be small if $h \rightarrow SS$ is kinematically forbidden. However, the couplings of H to fermions (and hence to the nucleons) must be severely suppressed. Thus the direct-detection cross section in this scenario is controlled by different model parameters than the annihilation cross section, so the two are not necessarily correlated. Therefore a direct-detection signal in this model would not necessarily probe the coupling responsible for the relic density.

4.4 Conclusions

In this chapter we use the apparent observation of DM self-interactions in the cluster Abell 3827 to constrain models of singlet scalar DM. We showed that the most natural scenario in this class of models is very light DM, below about 0.1 GeV, whose relic abundance is set by *freeze-in* through extraordinarily weak interactions with the Higgs doublet(s) through a quartic coupling of order 10^{-11} . In the case of thermal freeze-out, we found that no combination of masses and couplings could account for both the self-interaction and the correct relic density.

Thermal freeze-out can be salvaged by extending the Higgs sector with another doublet, so long as flavour conservation is achieved through Yukawa alignment. The solution is, however, quite fine tuned: the singlet mass must be tuned very close (of order a part per mille) to half the mass of the second CP-even neutral Higgs boson.

Chapter 5

Multi-component dark matter from a hidden gauged SU(3)

5.1 Introduction

Up until now, much of the work has focused on simple DM sectors, typically with only one or possibly two DM candidates. These scenarios are increasingly constrained by experimental measurements. Therefore, workers in the field are exploring ever more complex DM scenarios including models with multiple DM species (see, for example, Refs. [112–144]). In this context, there is a growing body of work studying details of “freeze-out” for multiple interacting DM species, e.g. Refs. [58, 115, 123, 125–127, 130–132, 140, 142, 145]. To expand on this theme, we developed a computer program capable of calculating the relic densities of an arbitrary number of interacting DM species and in this work, use it to explore the behavior for several scenarios and the implications for DM direct- and indirect detection experiments.

For a specific example of a model with multiple DM species, we reexamine the hidden gauged SU(3) model of Arcadi *et al.* [112]. This model has the potential

to have numerous DM species depending on the choice of parameter values from the breaking of the new $SU(3)$. The paper of Arcadi *et al.* [112] chose specific values of the parameters that reduced the multispecies DM problem to effectively two DM species to enable them to implement the model in the computer program `MICROMEGAS` [146]. It turns out that different values of the parameters can lead to much more complex and interesting scenarios with multiple DM species to explore. Thus, a main focus of this chapter is to explore different values of the parameters of the hidden gauged $SU(3)$ model that lead to a more complex DM particle spectrum. We found that some scenarios with multiple DM species give the correct relic density, but because the dominant components have a small direct detection cross section, the DM will be difficult to observe in direct detection experiments. However, in some cases when the DM is not observable by direct detection experiments, it could be observable in indirect detection measurements and vice versa. These points were also made by Arcadi *et al.* [112] and by many others for other DM scenarios. A small sampling of examples is given by Refs. [13–17, 117, 135, 139].

The hidden gauged $SU(3)$ model has a very rich phenomenology with details of the phenomenology very much dependent on the choice of parameter values. Some of our results convey a warning that approximations can lead to erroneous conclusions. For example, in a scenario with multiple DM species, a moderate change in the input parameter values can lead to major differences in the relative relic densities of the stable DM species. This would have a major impact on whether DM could be observed by direct or indirect detection experiments. An important lesson from our study is that models with multiple species of DM particles can lead to a very rich phenomenology and that one needs to be careful when studying relic densities in multispecies DM models.

The parameter space of the hidden gauged SU(3) model is large. Integrating the coupled Boltzmann equations to obtain the relic density for multiple DM species is computationally intensive, so it is impractical to perform a complete scan of the parameter space. Instead, we choose a small number of representative benchmark points in the parameter space and examine their phenomenology. For two of the benchmark points, there are four stable DM species, while in the third, there are seven. These cases will be examined in detail in Sec. 5.5.

Section 5.2 gives the details of the coupled Boltzmann equations we use to calculate the relic abundance for multispecies DM scenarios [58]. This is the core ingredient of the chapter needed to calculate the subsequent results for the hidden gauged SU(3) model. Before proceeding to the hidden gauged SU(3) model, we explore two simple multispecies DM scenarios in Sec. 5.3. In that section, we assume three or four scalar DM species with a range of masses and assumptions for the interactions between them. We do so to gain some insight before proceeding to the more complicated hidden gauged SU(3) model. In Sec. 5.4, we give the details of the hidden gauged SU(3) model. In that section, we start by writing down the Lagrangian, then compute the mass eigenstates for the scalar and vector sectors in terms of the Lagrangian parameters, the vacuum expectation values, and the new gauge coupling. We then write down expressions for the theoretical constraints on these parameters. In Sec. 5.5, we explore the phenomenology of three benchmark points for the hidden gauged SU(3) model that satisfy the theoretical constraints and use the relic density to constrain the parameters of the model which are then used to calculate direct detection and indirect detection cross sections. Finally, in Sec. 5.6, we summarize our results.

5.2 Boltzmann Equations for Multispecies DM

Our purpose is to study models of DM that include multiple DM species. The first step in such a study is to calculate the relic density for a DM sector with an arbitrary number of interacting DM species. To calculate the DM relic density, we assume that the dark sector was in thermal and chemical equilibrium at some early time, which allows us to use the coupled Boltzmann equations in the form written down by Dienes, et al. [58], which is a generalization of the Boltzmann equation given in Kolb and Turner [24] (see also Ref. [147]). We need to use the generalized Boltzmann equation because there will be multiple DM species which will interact with each other thus making the number density of each species dependent on each other species present. Multi-component DM and related themes have been explored previously by Edsjo and Gondolo [145] and others, for example, Refs. [122, 126, 130–132, 142, 148].

In what follows, we simply reproduce the coupled Boltzmann equations as given by Dienes et al. [58], which we will use to calculate each of the individual relic densities, and we refer the interested reader to their work for details [58]. We only include the details needed to reproduce our results including some of the equations and definitions that we used from Kolb and Turner [24] and refer the interested reader to that monograph for more complete details.

We wish to calculate the relic density of each of the DM species, ϕ_i , which is given by

$$\Omega_i = \frac{\rho_i}{\rho_C} = \frac{8\pi G m_i n_i}{3H^2}, \quad (5.1)$$

where m_i , n_i , and $\rho_i = m_i n_i$ are the mass, number density, and energy density of particle ϕ_i , ρ_C is the critical density, G is Newton's constant, and H is the Hubble

parameter. For species ϕ_i in kinetic equilibrium, the number density is given by

$$n_i^{\text{eq}} = \frac{g_i}{(2\pi)^3} \int f_i(\vec{p}_i) d^3p, \quad (5.2)$$

where g_i is the number of internal degrees of freedom and the phase space distribution, $f_i(\vec{p}_i)$, is given by

$$f_i(\vec{p}_i) = \frac{1}{e^{(E_i - \mu_i)/T} \pm 1}, \quad (5.3)$$

where $\vec{p}_i$, E_i , and μ_i are the three-momentum, energy, and chemical potential associated with species ϕ_i ; T is the temperature of the photon bath; and the plus and minus signs are for fermionic and bosonic species, respectively. We are working in the comoving frame in which $\langle \vec{v} \rangle = 0$ for all ϕ_i . In a frame in which $\langle \vec{v} \rangle = 0$, the $f_i(\vec{v})$ only depend on the magnitude of $\vec{v}$ and not its direction.

In general, the Boltzmann equations can include an arbitrary number of species participating in reactions and decays [58]. We will restrict ourselves to simplified cases in which the Boltzmann equations for all relevant species can be expressed in terms of number densities, n_i , rather than the phase space distributions, f_i ; we assume CP is conserved so that the matrix elements satisfy $|\mathcal{M}(ij \rightarrow kl)|^2 = |\mathcal{M}(kl \rightarrow ij)|^2$ for cross sections and $|\mathcal{M}(i \rightarrow jk)|^2 = |\mathcal{M}(jk \rightarrow i)|^2$ for decays and that all visible sector particles and all dark-sector particles are in thermal equilibrium due to rapid interactions among themselves until long after the lightest DM species have frozen out. To minimize the complexity of the problem, we restrict ourselves to $1 \rightarrow 2$ and $2 \rightarrow 2$ type interactions. With these simplifications, the Boltzmann equations become [58]

$$\begin{aligned}
\frac{dn_i}{dt} = & -3Hn_i - \sum_{a,b} \Gamma_{i \rightarrow ab} (n_i - n_i^{eq}) \\
& - \sum_{j \neq i} \sum_a \left[\Gamma_{i \rightarrow ja} \left(n_i - n_j \frac{n_i^{eq}}{n_j^{eq}} \right) - \Gamma_{j \rightarrow ia} \left(n_j - n_i \frac{n_j^{eq}}{n_i^{eq}} \right) \right] \\
& - \sum_{j,k} \left[\frac{1}{2} (1 + \delta_{jk}) \Gamma_{i \rightarrow jk} \left(n_i - n_j n_k \frac{n_i^{eq}}{n_j^{eq} n_k^{eq}} \right) \right. \\
& \quad \left. - (1 + \delta_{ik}) \Gamma_{j \rightarrow ik} \left(n_j - n_i n_k \frac{n_j^{eq}}{n_i^{eq} n_k^{eq}} \right) \right] \\
& - \sum_j \sum_{a,b} (1 + \delta_{ij}) \langle \sigma_{ij \rightarrow ab} v \rangle (n_i n_j - n_i^{eq} n_j^{eq}) \\
& - \sum_j \sum_{a,b} \left[\langle \sigma_{ia \rightarrow jb} v \rangle \left(n_i - n_j \frac{n_i^{eq}}{n_j^{eq}} \right) n_a^{eq} - \langle \sigma_{ja \rightarrow ib} v \rangle \left(n_j - n_i \frac{n_j^{eq}}{n_i^{eq}} \right) n_a^{eq} \right] \\
& - \sum_{j,k} \sum_a \left[(1 + \delta_{jk}) \langle \sigma_{ia \rightarrow jk} v \rangle \left(n_i - n_j n_k \frac{n_i^{eq}}{n_j^{eq} n_k^{eq}} \right) n_a^{eq} \right. \\
& \quad \left. - (1 + \delta_{ik}) \langle \sigma_{ja \rightarrow ik} v \rangle \left(n_j - n_i n_k \frac{n_j^{eq}}{n_i^{eq} n_k^{eq}} \right) n_a^{eq} \right] \\
& - \sum_{j,k} \sum_a (1 + \delta_{ij}) \langle \sigma_{ij \rightarrow ka} v \rangle \left(n_i n_j - n_k \frac{n_i^{eq} n_j^{eq}}{n_k^{eq}} \right) \\
& - \sum_{j,k,l} (1 + \delta_{ij})(1 + \delta_{kl}) \langle \sigma_{ij \rightarrow kl} v \rangle \left(n_i n_j - n_k n_l \frac{n_i^{eq} n_j^{eq}}{n_k^{eq} n_l^{eq}} \right), \tag{5.4}
\end{aligned}$$

where in Eq. 5.4, the indices i, j, k, l run over the dark-sector particles and a, b run over visible sector particles. The first term on the right-hand side is the dilution due to the expansion of the Universe, and H is the Hubble parameter. $\langle \sigma_{xy \rightarrow zw} v \rangle$, which will be given below, represents the thermally averaged annihilation cross section of two initial state particles, xy , to two final state particles, zw , where $xyzw$ represent either visible or dark-sector particles. Likewise, $\Gamma_{x \rightarrow yz}$ represents the partial decay width of particle x into particles yz . Equation (5.4) assumes that all particles are in

thermal equilibrium. In Eq. (5.4), the thermally averaged cross sections are given by

$$\langle \sigma_{xy \rightarrow zw} v_{xy} \rangle \equiv \frac{g_x g_y}{n_x^{eq} n_y^{eq}} \int \sigma_{xy \rightarrow zw} v_{xy} f_x^{eq} f_y^{eq} \frac{d^3 p_x}{(2\pi)^3} \frac{d^3 p_y}{(2\pi)^3}, \quad (5.5)$$

where g_i is the number of internal degrees of freedom of particle i , $\sigma_{xy \rightarrow zw}$ is the cross section for the process $\phi_x \phi_y \rightarrow \phi_z \phi_w$ and

$$v_{xy} \equiv |v_x - v_y| = \frac{\sqrt{(p_x \cdot p_y)^2 - m_x^2 m_y^2}}{E_x E_y} \quad (5.6)$$

is the magnitude of the relative velocity of ϕ_x and ϕ_y in the comoving frame. In the absence of a degenerate Fermi species or a Bose condensate, the phase space distributions of the DM species reduce to the classical Maxwell-Boltzmann distribution in the nonrelativistic limit [24]. For sufficiently rapid interactions, the particles approach kinetic and chemical equilibrium depending on the nature of the interaction. This latter situation places constraints on the chemical potentials of the species involved, which depend on the reaction. For example, the reaction $\phi_i + \phi_j \leftrightarrow \phi_k + \phi_l$ would imply the constraint $\mu_i + \mu_j = \mu_k + \mu_l$. For the visible particles, the chemical potential μ is small compared to the temperature or zero, which results in the chemical potential of the DM species being small compared to the temperature or zero in the absence of an asymmetry. We will therefore treat all equilibrium chemical potentials as zero. Under these conditions, we approximate $f_i(v_i)$ by the equilibrium phase space distribution

$$f_i^{eq}(v_i) \propto e^{-E_i(v_i)/T} \quad (5.7)$$

where v_i is the speed of the particle. With these approximations, the equilibrium

number density, n_i^{eq} , is given by

$$n_i^{eq} = g_i \int \frac{d^3 p_i}{(2\pi)^3} e^{-E_i/T} = \frac{g_i m_i^2 T}{2\pi^2} K_2(m_i/T), \quad (5.8)$$

$$\approx \begin{cases} g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T} & \text{for } m_i \gg T, \\ g_i \frac{T^3}{\pi^2} & \text{for } T \gg m_i, \end{cases} \quad (5.9)$$

where $K_2(x)$ is the modified Bessel function of the second kind.

The standard approach for solving Eq. (5.4) is to scale out the effect of the expansion of the Universe by considering the evolution of the number density in a comoving volume. This is done by using the entropy density, s , as a fiducial quantity by defining [24]

$$Y = \frac{n_i}{s}, \quad Y^{eq} = \frac{n_i^{eq}}{s}, \quad (5.10)$$

where the entropy density in the comoving volume in any cosmological epoch is defined by

$$s \equiv \frac{S}{V} = \frac{\rho + p}{T} = \frac{2\pi^2}{45} g_{*S} T^3, \quad (5.11)$$

T is the photon temperature, and ρ and p are the total energy density and the pressure, respectively, expressed in terms of the photon temperature T [24]. g_{*S} represents the number of interacting degrees of freedom in the thermal bath, which can be found by rewriting Eq. (5.11) as

$$g_{*S} = \frac{45}{2\pi^2 T^4} (\rho + p). \quad (5.12)$$

Y_i is the actual number per comoving volume, and Y_i^{eq} is the equilibrium number per comoving volume. In addition, we rescale the Hubble constant $H(T) = H(m)/x^2$ with $x = m/T$ where m is any convenient mass scale [24], which we will take to be the mass of the heaviest stable DM particle. In the radiation dominated epoch, $H(m)$

is given by [24]

$$H(m) = \frac{2\pi^{3/2}}{\sqrt{45}} \frac{g_*^{1/2} m^2}{m_{Pl}}, \quad (5.13)$$

where m_{Pl} is the Planck mass and g_* represents the total number of effectively massless degrees of freedom and is given by

$$g_* = \frac{30\rho_R}{\pi^2 T^4}, \quad (5.14)$$

where ρ_R is the total energy density of all species in equilibrium. Expressions for ρ_R , g_{*S} , and g_* can be found in the work by Kolb and Turner [24]. In computing g_* and g_{*S} , we used the Fermi-Dirac or Bose-Einstein distributions as appropriate in the sum. For the region around the QCD phase transition, the correct degrees of freedom become ambiguous and require lattice QCD to compute. We use the values found in Ref. [30] for this region. With these substitutions, Eq. (5.4) is recast as

$$\begin{aligned} \frac{dY_i}{dx} = & -\frac{xs}{H(m)} \left\{ \frac{1}{s} \sum_{a,b} \Gamma_{i \rightarrow ab} (Y_i - Y_i^{eq}) \right. \\ & + \frac{1}{s} \sum_{j \neq i} \sum_a \left[\Gamma_{i \rightarrow ja} \left(Y_i - Y_j \frac{Y_i^{eq}}{Y_j^{eq}} \right) - \Gamma_{j \rightarrow ia} \left(Y_j - Y_i \frac{Y_j^{eq}}{Y_i^{eq}} \right) \right] \\ & + \frac{1}{s} \sum_{j,k} \left[\frac{1}{2} (1 + \delta_{jk}) \Gamma_{i \rightarrow jk} \left(Y_i - Y_j Y_k \frac{Y_i^{eq}}{Y_j^{eq} Y_k^{eq}} \right) \right. \\ & \quad \left. - (1 + \delta_{ik}) \Gamma_{j \rightarrow ik} \left(Y_j - Y_i Y_k \frac{Y_j^{eq}}{Y_i^{eq} Y_k^{eq}} \right) \right] \\ & + \sum_j \sum_{a,b} (1 + \delta_{ij}) \langle \sigma_{ij \rightarrow ab} v \rangle (Y_i Y_j - Y_i^{eq} Y_j^{eq}) \\ & + \sum_j \sum_{a,b} \left[\langle \sigma_{ia \rightarrow jb} v \rangle \left(Y_i - Y_j \frac{Y_i^{eq}}{Y_j^{eq}} \right) Y_a^{eq} - \langle \sigma_{ja \rightarrow ib} v \rangle \left(Y_j - Y_i \frac{Y_j^{eq}}{Y_i^{eq}} \right) Y_a^{eq} \right] \end{aligned}$$

$$\begin{aligned}
& + \sum_{j,k} \sum_a \left[(1 + \delta_{jk}) \langle \sigma_{ia \rightarrow jk} v \rangle \left(Y_i - Y_j Y_k \frac{Y_i^{eq}}{Y_j^{eq} Y_k^{eq}} \right) Y_a^{eq} \right. \\
& \quad \left. - (1 + \delta_{ik}) \langle \sigma_{ja \rightarrow ik} v \rangle \left(Y_j - Y_i Y_k \frac{Y_j^{eq}}{Y_i^{eq} Y_k^{eq}} \right) Y_a^{eq} \right] \\
& + \sum_{j,k} \sum_a (1 + \delta_{ij}) \langle \sigma_{ij \rightarrow ka} v \rangle \left(Y_i Y_j - Y_k \frac{Y_i^{eq} Y_j^{eq}}{Y_k^{eq}} \right) \\
& + \sum_{j,k,l} (1 + \delta_{ij}) (1 + \delta_{kl}) \langle \sigma_{ij \rightarrow kl} v \rangle \left(Y_i Y_j - Y_k Y_l \frac{Y_i^{eq} Y_j^{eq}}{Y_k^{eq} Y_l^{eq}} \right) \Bigg\}. \tag{5.15}
\end{aligned}$$

5.3 Simple Multicomponent DM Scenarios

Before proceeding to the hidden gauged SU(3) DM model, we start by exploring the relic abundance of two simplified multi species DM scenarios. Models with multiple species and different types of interactions can be complicated. Deconstructing how the different terms in Eq. (5.15) affect the number densities can give us insight into the coupled Boltzmann equations and help us understand which terms are important and which ones can be neglected. Both of the scenarios initially consist of multiple DM species freezing out by only self-annihilating into Standard Model (SM) particles. We then add additional interactions between the DM species and observe the effects. Finally, we include all interactions that would be present due to crossing symmetry to obtain the final result. We assume that the thermally averaged cross sections for all interactions are s-wave, and thus independent of speed, and use the approximation $\langle \sigma v \rangle = 0.1$ pb for all the interactions. In a particular model, these cross sections would not necessarily be equal, but this assumption is reasonable for the purpose of exploring the behavior of Eq. (5.15) for simplified scenarios. In all cases, we assume that the SM particles are in thermal equilibrium with the photon bath and treat them as relativistic with the appropriate degrees of freedom. Plots of the DM number per comoving volume vs $x = m/T$ are shown in Figs. 5.1 and 5.2, where we take m to

Figure 5.1: DM number densities as a function of $x = m/T$ for three DM species. The solid curves represent the equilibrium density; the dashed curves represent the scenario with only self-annihilation to SM particles, which we label as the base; the curves with both dots and dashes represent scenarios with the base interactions plus a single additional interaction as labeled in the plot; and the dotted curve represent the result including all the additional interactions of the previous curves. The green, blue, and red curves represent the number density of ϕ_1 , ϕ_2 , and ϕ_3 with masses of 100, 120, and 150 GeV, respectively. Note that for the green and blue curves, the various curves sit very closely on top of each other.

be the mass of the heaviest stable DM particle.

In all cases, the solid curves represent the equilibrium number densities; the dashed lines represent the scenarios with only self-annihilation to SM particles, which we label as the “base”; the lines with a combination of dashes and dots include the base interactions and a single additional interaction, ignoring the contributions from crossing symmetry; and finally the dotted lines show the results when all the additional interactions are included. We label the species as ϕ_i with the index i increasing with increasing species mass.

Figure 5.2: DM number densities as a function of $x = m/T$ for four DM species. The line labeling is as in Fig. 5.1. The green, blue, red, and purple curves represent the number density of ϕ_1 , ϕ_2 , ϕ_3 , and ϕ_4 with masses of 100, 120, 150, and 180 GeV, respectively. Note that for the green, blue, and red curves the various curves sit very closely on top of each other.

In Fig. 5.1, we consider three DM species, ϕ_1 (green), ϕ_2 (blue), and ϕ_3 (red), with masses of 100, 120, and 150 GeV, respectively. We will start with the most general application of Eq. (5.15) that is applicable to this case and examine each of the contributions in turn. For this case, Eq. (5.15) reduces to the following set of three coupled equations:

$$\frac{dY_1}{dx} = -\frac{xs}{H(m)} \left\{ 2\langle\sigma_{11\rightarrow SM+SM}v\rangle(Y_1^2 - (Y_1^{eq})^2) - 2\langle\sigma_{23\rightarrow 11}v\rangle \left(Y_2Y_3 - Y_1^2 \frac{Y_2^{eq}Y_3^{eq}}{(Y_1^{eq})^2} \right) \right\}, \quad (5.16)$$

$$\begin{aligned} \frac{dY_2}{dx} = & -\frac{xs}{H(m)} \left\{ 2\langle\sigma_{22\rightarrow SM+SM}v\rangle(Y_2^2 - (Y_2^{eq})^2) \right. \\ & + \langle\sigma_{23\rightarrow 11}v\rangle \left(Y_2Y_3 - Y_1^2 \frac{Y_2^{eq}Y_3^{eq}}{(Y_1^{eq})^2} \right) \\ & \left. - \langle\sigma_{13\rightarrow 12}v\rangle \left(Y_3 - Y_2 \frac{Y_3^{eq}}{Y_2^{eq}} \right) Y_1 \right\}, \end{aligned} \quad (5.17)$$

$$\begin{aligned} \frac{dY_3}{dx} = & -\frac{xs}{H(m)} \left\{ 2\langle\sigma_{33\rightarrow SM+SM}v\rangle(Y_3^2 - (Y_3^{eq})^2) \right. \\ & + \langle\sigma_{23\rightarrow 11}v\rangle \left(Y_2Y_3 - Y_1^2 \frac{Y_2^{eq}Y_3^{eq}}{(Y_1^{eq})^2} \right) \\ & \left. + \langle\sigma_{13\rightarrow 12}v\rangle \left(Y_3 - Y_2 \frac{Y_3^{eq}}{Y_2^{eq}} \right) Y_1 \right\}. \end{aligned} \quad (5.18)$$

From this set of equations, we can examine which factors influence each of the DM species' number densities.

Starting with ϕ_1 , the only additional term that can affect the ϕ_1 number density is $\phi_2 + \phi_3 \leftrightarrow \phi_1 + \phi_1$ because ϕ_1 only acts as a spectator in the $\phi_1 + \phi_3 \leftrightarrow \phi_1 + \phi_2$ interaction, leaving the ϕ_1 number density unchanged. The interesting temperature is when ϕ_1 freezes out when the actual ϕ_1 number density departs from the equilibrium number density. This occurs when $Y_1 = cY_1^{eq}$ for $c \gtrsim 1$ [24]. Because ϕ_1 is lighter than

the other DM species, ϕ_2 and ϕ_3 have already departed from chemical equilibrium so that $Y_2 \gg Y_2^{eq}$ and $Y_3 \gg Y_3^{eq}$. This gives

$$\begin{aligned} Y_1^2 - (Y_1^{eq})^2 &\approx (1 - c^{-2})Y_1^2, \\ Y_2Y_3 - Y_1^2 \frac{Y_2^{eq}Y_3^{eq}}{(Y_1^{eq})^2} &\approx Y_2Y_3. \end{aligned} \tag{5.19}$$

Since $Y_1 \gg Y_2$ and $Y_1 \gg Y_3$ at the temperature where $Y_1 = cY_1^{eq}$, the second line in Eq. (5.19) is negligible and can safely be ignored during a numerical scan. This explains why the two additional terms in Fig. 5.1 have no significant effect on the ϕ_1 relic density.

We next examine the temperature region around where ϕ_2 freezes out; $Y_2 = cY_2^{eq}$. In this case, $Y_1 \approx Y_1^{eq}$ and $Y_3 \gg Y_3^{eq}$. This leads to

$$\begin{aligned} Y_2^2 - (Y_2^{eq})^2 &\approx (1 - c^{-2})Y_2^2, \\ Y_2Y_3 - Y_1^2 \frac{Y_2^{eq}Y_3^{eq}}{(Y_1^{eq})^2} &\approx Y_2Y_3, \\ \left(Y_3 - Y_2 \frac{Y_3^{eq}}{Y_2^{eq}} \right) Y_1 &\approx Y_1Y_3. \end{aligned} \tag{5.20}$$

Since $Y_2^2 \gg Y_2Y_3$, the second line in Eq. (5.19) will be negligible. The third line could potentially be important since $Y_1 \gg Y_2 \gg Y_3$. However, it turns out that for our choice of masses the $\phi_1 + \phi_3 \leftrightarrow \phi_1 + \phi_2$ interaction causes $Y_2^2 \gg Y_1Y_3$, so this term also has a negligible contribution compared to the first line in Eq. (5.20). The net result is that the interactions added to the base case do not significantly alter the ϕ_2 relic abundance.

Finally, we consider the temperature where ϕ_3 freezes out; $Y_3 = cY_3^{eq}$. In this

case, $Y_1 \approx Y_1^{eq}$ and $Y_2 \approx Y_2^{eq}$. This gives:

$$\begin{aligned}
Y_3^2 - (Y_3^{eq})^2 &\approx (1 - c^{-2})Y_3^2, \\
Y_2Y_3 - Y_1^2 \frac{Y_2^{eq}Y_3^{eq}}{(Y_1^{eq})^2} &\approx (1 - c^{-1})Y_2Y_3, \\
\left(Y_3 - Y_2 \frac{Y_3^{eq}}{Y_2^{eq}}\right) Y_1 &\approx (1 - c^{-1})Y_1Y_3.
\end{aligned} \tag{5.21}$$

In this case, the largest reaction rate is for $\phi_1 + \phi_3 \leftrightarrow \phi_1 + \phi_2$, corresponding to the third line of Eq. (5.21), followed by $\phi_2 + \phi_3 \leftrightarrow \phi_1 + \phi_1$, corresponding to the second line of Eq. (5.21), because of the hierarchy of the number densities, $Y_1 \gg Y_2 \gg Y_3$. This is what we see in Fig. 5.1, where the curves with the additional interactions stay in thermal equilibrium longer, resulting in a smaller relic density for ϕ_3 . So, for the case of annihilation of ϕ_3 with ϕ_1 , there is a small amount of ϕ_3 which sees a large amount of ϕ_1 so that the interaction $\phi_1 + \phi_3 \leftrightarrow \phi_1 + \phi_2$ depletes ϕ_3 significantly but barely affects the ϕ_1 density, which is what is seen in Fig. 5.1. Because ϕ_2 has a smaller relic density than ϕ_1 , it does not deplete ϕ_3 as much, which is also observed in Fig. 5.1.

In Fig. 5.2, we repeat the exercise for four DM species, ϕ_1 (green), ϕ_2 (blue), ϕ_3 (red), and ϕ_4 (purple), with masses of 100, 120, 150, and 180 GeV, respectively. For this case, Eq. (5.15) leads to a set of four coupled equations. ϕ_1 , ϕ_2 , and ϕ_3 behave similarly to the previous case, so to avoid repetition, we will focus on the behavior of the ϕ_4 number density. The differential equation for Y_4 is given by:

$$\begin{aligned}
\frac{dY_4}{dx} = & -\frac{xs}{H(m)} \left\{ 2\langle\sigma_{44 \rightarrow SM+SM}v\rangle(Y_4^2 - (Y_4^{eq})^2) \right. \\
& \left. + \langle\sigma_{34 \rightarrow 12}v\rangle \left(Y_3Y_4 - Y_1Y_2 \frac{Y_3^{eq}Y_4^{eq}}{Y_1^{eq}Y_2^{eq}} \right) \right\}
\end{aligned} \tag{5.22}$$

$$\begin{aligned}
& + \langle \sigma_{24 \rightarrow 13} v \rangle \left(Y_2 Y_4 - Y_1 Y_3 \frac{Y_2^{eq} Y_4^{eq}}{Y_1^{eq} Y_3^{eq}} \right) \\
& + \langle \sigma_{14 \rightarrow 23} v \rangle \left(Y_1 Y_4 - Y_2 Y_3 \frac{Y_1^{eq} Y_4^{eq}}{Y_2^{eq} Y_3^{eq}} \right) \Bigg\}.
\end{aligned}$$

To obtain some insight into the behavior of the ϕ_4 number density, we start by considering the temperature where ϕ_4 freezes out, $Y_4 = c Y_4^{eq}$ with $c \gtrsim 1$, and assume that in this region $Y_1 \approx Y_1^{eq}$, $Y_2 \approx Y_2^{eq}$, and $Y_3 \approx Y_3^{eq}$:

$$\begin{aligned}
Y_4^2 - (Y_4^{eq})^2 & \approx (1 - c^{-2}) Y_4^2, \\
Y_3 Y_4 - Y_1 Y_2 \frac{Y_3^{eq} Y_4^{eq}}{Y_1^{eq} Y_2^{eq}} & \approx (1 - c^{-1}) Y_3 Y_4, \\
Y_2 Y_4 - Y_1 Y_3 \frac{Y_2^{eq} Y_4^{eq}}{Y_1^{eq} Y_3^{eq}} & \approx (1 - c^{-1}) Y_2 Y_4, \\
Y_1 Y_4 - Y_2 Y_3 \frac{Y_1^{eq} Y_4^{eq}}{Y_2^{eq} Y_3^{eq}} & \approx (1 - c^{-1}) Y_1 Y_4.
\end{aligned} \tag{5.23}$$

These expressions can explain why the $\phi_4 + \phi_3 \leftrightarrow \phi_1 + \phi_2$ (dot-dot-dashed) curve sits above the $\phi_4 + \phi_2 \leftrightarrow \phi_1 + \phi_3$ (dot-dashed-dashed) and $\phi_4 + \phi_1 \leftrightarrow \phi_2 + \phi_3$ (dot-dot-dashed-dashed) curves but fails to explain other details of this plot. As pointed out above, the number density of a DM species depends on its mass so that there is less of ϕ_3 to annihilate with than ϕ_1 and ϕ_2 so that ϕ_4 is depleted less by its interactions with ϕ_3 than with the other DM species.

Next, we consider the behavior of the ϕ_4 number density for the curves when the $\phi_4 + \phi_2 \leftrightarrow \phi_1 + \phi_3$ and $\phi_4 + \phi_1 \leftrightarrow \phi_2 + \phi_3$ interactions are present in the temperature region around ϕ_3 and ϕ_4 freeze-out where $Y_4 = c_1 Y_4^{eq}$ and $Y_3 = c_2 Y_3^{eq}$ with $c_1 \simeq c_2 \gtrsim 1$:

$$\begin{aligned}
Y_4^2 - (Y_4^{eq})^2 & \approx (1 - c_1^{-2}) Y_4^2, \\
Y_3 Y_4 - Y_1 Y_2 \frac{Y_3^{eq} Y_4^{eq}}{Y_1^{eq} Y_2^{eq}} & \approx (1 - c_1^{-1} c_2^{-1}) Y_3 Y_4,
\end{aligned} \tag{5.24}$$

$$\begin{aligned}
Y_2 Y_4 - Y_1 Y_3 \frac{Y_2^{eq} Y_4^{eq}}{Y_1^{eq} Y_3^{eq}} &\approx (1 - c_2 c_1^{-1}) Y_2 Y_4, \\
Y_1 Y_4 - Y_2 Y_3 \frac{Y_1^{eq} Y_4^{eq}}{Y_2^{eq} Y_3^{eq}} &\approx (1 - c_2 c_1^{-1}) Y_1 Y_4.
\end{aligned}$$

In this temperature region, the factors, $(1 - c_2 c_1^{-1})$, in the third and fourth equations in Eq. (5.24), are small, so the ϕ_4 number density is dominated by the first equation in Eq. (5.24). The second equation is not relevant here as it applies to the $\phi_4 + \phi_3 \leftrightarrow \phi_1 + \phi_2$ case, which we are not considering here. The result is that the curves for $\phi_2 + \phi_4 \leftrightarrow \phi_1 + \phi_3$ and $\phi_1 + \phi_4 \leftrightarrow \phi_2 + \phi_3$ have similar values of Y_4 shortly after ϕ_4 departs from the equilibrium curve.

Lastly, we examine the region where ϕ_2 freezes out, corresponding to the temperature where $Y_2 = c Y_2^{eq}$ with $c \gtrsim 1$ as before. We assume that $Y_1 \approx Y_1^{eq}$, $Y_3 \gg Y_3^{eq}$, and $Y_4 \gg Y_4^{eq}$ so that

$$\begin{aligned}
Y_4^2 - (Y_4^{eq})^2 &\approx (1 - c^{-2}) Y_4^2, \\
Y_3 Y_4 - Y_1 Y_2 \frac{Y_3^{eq} Y_4^{eq}}{Y_1^{eq} Y_2^{eq}} &\approx Y_3 Y_4, \\
Y_2 Y_4 - Y_1 Y_3 \frac{Y_2^{eq} Y_4^{eq}}{Y_1^{eq} Y_3^{eq}} &\approx Y_2 \left(Y_4 - c^{-1} Y_3 \frac{Y_4^{eq}}{Y_3^{eq}} \right), \\
Y_1 Y_4 - Y_2 Y_3 \frac{Y_1^{eq} Y_4^{eq}}{Y_2^{eq} Y_3^{eq}} &\approx Y_1 \left(Y_4 - c Y_3 \frac{Y_4^{eq}}{Y_3^{eq}} \right).
\end{aligned} \tag{5.25}$$

It turns out that Y_4 is numerically comparable to $Y_3 \frac{Y_4^{eq}}{Y_3^{eq}}$ at the temperature range we are considering. As the temperature decreases, c , which is a measure of the difference between Y and Y^{eq} , increases so that $\left(Y_4 - c Y_3 \frac{Y_4^{eq}}{Y_3^{eq}} \right)$ will become negative and the reverse reaction will dominate. At the same time, $\left(Y_4 - c^{-1} Y_3 \frac{Y_4^{eq}}{Y_3^{eq}} \right)$ remains positive and continues to deplete ϕ_4 . This is why in Fig. 5.2 the $\phi_4 + \phi_1 \leftrightarrow \phi_2 + \phi_3$ (dot-dot-dashed-dashed) curve increases while the $\phi_4 + \phi_2 \leftrightarrow \phi_1 + \phi_3$ (dot-dashed-dashed) curve decreases.

curve decreases. The dotted line shows the result when all processes are included. It can be seen that the reverse reaction is important up until ϕ_1 starts to freeze out, at which point the $\phi_1 + \phi_4 \leftrightarrow \phi_2 + \phi_3$ interaction becomes dominant and starts depleting ϕ_4 .

These simple examples show that even for a simple multispecies DM sector the behavior of the relic abundance can be quite complicated and the resulting relic abundances can vary substantially depending on the details of the underlying model. It is not always *a priori* obvious how the details of the model will work out and one needs to understand how each of the interactions contribute to the final result. All things being equal, heavier DM species will be more affected by their interactions with lighter DM species than vice versa. The interactions between multispecies SM can lead to experimental consequences. Say, for example, that a model has many DM species but only one interacts with a portal to the visible Universe. One could imagine a scenario where the model could account for the measured DM relic density, but the component that interacts with the visible Universe has a very small relic density, making it unlikely that it could be observed in direct or indirect detection experiments.

In the next sections, we will explore in detail a specific multispecies DM model, the hidden gauged SU(3) model. Because the parameter space is large, we will restrict ourselves to several representative benchmark points with interesting phenomenology.

5.4 Hidden Gauged SU(3) Model

In the remainder of this chapter, we study the phenomenology of the hidden gauged SU(3) model of Arcadi *et al.* [112], which consists of spin-1 and spin-0 states. In their paper, they examined two representative limiting cases that made their numerical

analysis tractable. We extend their studies to consider different points in the parameter space that leads to more complex DM scenarios. We mention that others have also studied hidden SU(3) models [124, 131, 143].

5.4.1 Details of the model

For completeness and clarity, we start by reproducing the details of the hidden gauged SU(3) model of Arcadi *et al.* [112]. The model consists of a gauged SU(3) which is fully broken by two complex scalar triplets, Φ_1 and Φ_2 , so that all the new gauge bosons acquire a mass. These new scalars are not charged under the SM gauge groups and can only interact with the SM through the SM Higgs doublet. To simplify the Lagrangian and insure additional stable states, a $\mathbb{Z}_2$ symmetry is also imposed such that

$$\begin{aligned}\Phi_1 &\rightarrow -\Phi_1, \\ \Phi_2 &\rightarrow \Phi_2,\end{aligned}\tag{5.26}$$

which has the effect of only including even powers of the scalar triplets in the Lagrangian. Following Ref. [112], and imposing this additional $\mathbb{Z}_2$ symmetry, we write the Lagrangian as

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{portal}} + \mathcal{L}_{\text{hidden}},\tag{5.27}$$

where

$$-\mathcal{L}_{\text{SM}} \supset V_{\text{SM}} = m_H^2 |H|^2 + \frac{\lambda_H}{2} |H|^4,\tag{5.28}$$

$$-\mathcal{L}_{\text{portal}} = V_{\text{portal}} = \lambda_{H11} |H|^2 |\Phi_1|^2 + \lambda_{H22} |H|^2 |\Phi_2|^2,\tag{5.29}$$

$$-\mathcal{L}_{\text{hidden}} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} + |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 - V_{\text{hidden}}, \quad (5.30)$$

$$V_{\text{hidden}} = m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 \quad (5.31)$$

$$+ \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right]. \quad (5.32)$$

Here, $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \tilde{g} f^{abc} A_\mu^b A_\nu^c$ is the field strength tensor for the hidden SU(3), where $\tilde{g}$ is the gauge coupling and f^{abc} are the SU(3) structure constants. The covariant derivative is given by $D_\mu = \partial_\mu - i\tilde{g} A_\mu^a t^a$, where t_a are the Gell-Mann matrices (see, for example, Ref. [149], where the $t_a = \frac{1}{2}\lambda_a$).

In general, the scalar doublet and triplets will have the form

$$\begin{aligned} H &= \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{1,r}^H + i\phi_{1,i}^H \\ v + \phi_{2,r}^H + i\phi_{2,i}^H \end{pmatrix}, \\ \Phi_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{1,r}^1 + i\phi_{1,i}^1 \\ \phi_{2,r}^1 + i\phi_{2,i}^1 \\ v_1 + \phi_{3,r}^1 + i\phi_{3,i}^1 \end{pmatrix}, \\ \Phi_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{1,r}^2 + i\phi_{1,i}^2 \\ v_2 + \phi_{2,r}^2 + i\phi_{2,i}^2 \\ v_3 + \phi_{3,r}^2 + i\phi_{3,i}^2 \end{pmatrix}, \end{aligned} \quad (5.33)$$

where the v_i are the vacuum expectation values (vevs), the superscript indicates to which of the Φ_i the field belongs, and the subscripts indicate the position in the triplet and whether it is the real or imaginary component. When all three new vevs are taken to be nonzero, we have some freedom in defining the Goldstone bosons of

the hidden SU(3). One convenient way to define them is

$$\begin{aligned}
G^1 &= \phi_{1,i}^2, \quad G^2 = \phi_{1,r}^2, \quad G^3 = \phi_{2,i}^2, \quad G^4 = \phi_{1,i}^1, \quad G^5 = \phi_{1,r}^1, \\
G^6 &= \frac{v_1 \phi_{2,i}^1 + v_2 \phi_{3,i}^2}{\sqrt{v_1^2 + v_2^2}}, \quad G^7 = \frac{v_1 \phi_{2,r}^1 + v_3 \phi_{2,r}^2 + v_2 \phi_{3,r}^2}{\sqrt{v_1^2 + v_2^2 + v_3^2}}, \\
G^8 &= \frac{v_2 v_3 \phi_{2,i}^1 + (v_1^2 + v_2^2) \phi_{3,i}^1 + v_1 v_3 \phi_{3,i}^2}{\sqrt{(v_1^2 + v_2^2)(v_1^2 + v_2^2 + v_3^2)}},
\end{aligned} \tag{5.34}$$

which leaves the physical states

$$\varphi_1 = \frac{v_2 \phi_{2,i}^1 - v_3 \phi_{3,i}^1 + v_1 \phi_{3,i}^2}{\sqrt{v_1^2 + v_2^2 + v_3^2}}, \tag{5.35}$$

$$\varphi_2 = \frac{v_3 \phi_{2,r}^1 + v_1 \phi_{2,r}^2}{\sqrt{v_1^2 + v_3^2}}, \tag{5.36}$$

$$\varphi_3 = \frac{v_1 v_2 \phi_{2,r}^1 + v_2 v_3 \phi_{2,r}^2 + (v_1^2 + v_3^2) \phi_{3,r}^2}{\sqrt{(v_1^2 + v_2^2)(v_1^2 + v_2^2 + v_3^2)}}, \tag{5.37}$$

$$\varphi_4 = \phi_{3,r}^1. \tag{5.38}$$

We will see in Sec. 5.4.2.3 that in order to have a consistent vacuum state we must have $v_1 v_3 (\lambda_4 + \lambda_5) = 0$. Since we require $v_1 \neq 0$ to fully break the SU(3), we will study the cases where we take $v_3 = 0$ and allow $\lambda_4 \neq -\lambda_5$. For completeness, we give the details of the $\lambda_4 = -\lambda_5$ case in App. B for the interested reader who may wish to study this possibility.

In the $v_3 = 0$ case, the resulting Goldstone bosons are given by

$$\begin{aligned}
G^1 &= \phi_{1,i}^2, \quad G^2 = \phi_{1,r}^2, \quad G^3 = \phi_{2,i}^2, \quad G^4 = \phi_{1,i}^1, \quad G^5 = \phi_{1,r}^1, \\
G^8 &= \phi_{3,i}^1, \quad G^6 = s_\beta \phi_{2,i}^1 + c_\beta \phi_{3,i}^2, \quad G^7 = s_\beta \phi_{2,r}^1 - c_\beta \phi_{3,r}^2,
\end{aligned} \tag{5.39}$$

where we define $s_\beta = \sin \beta = \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$ and $c_\beta = \cos \beta = \frac{v_2}{\sqrt{v_1^2 + v_2^2}}$. This leads to the

convenient definition for the physical states given by

$$\begin{aligned}\varphi_1 &= \phi_{3,r}^1, \quad \varphi_2 = \phi_{2,r}^2, \quad \varphi_3 = c_\beta \phi_{2,r}^1 + s_\beta \phi_{3,r}^2, \\ \varphi_4 &= c_\beta \phi_{2,i}^1 - s_\beta \phi_{3,i}^2,\end{aligned}\tag{5.40}$$

and the resulting form of the triplets in the unitary gauge is given by

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ c_\beta(\varphi_3 + i\varphi_4) \\ v_1 + \varphi_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 + \varphi_2 \\ s_\beta(\varphi_3 - i\varphi_4) \end{pmatrix}.\tag{5.41}$$

Spontaneous symmetry breaking in the SM happens in the usual way so that the Higgs doublet in the unitary gauge is given by

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix},\tag{5.42}$$

where v is the SM vev.

In the unitary gauge, we write down the scalar mass matrix in the form $\mathcal{L} \supset \frac{1}{2} \Phi^T m_{\text{scalar}}^2 \Phi$, where $\Phi^T = (h, \varphi_1, \varphi_2, \varphi_3, \varphi_4)$ are the scalars in the model and where

$$m_{\text{scalar}}^2 = \begin{pmatrix} \lambda_H v^2 & \lambda_{H11} v v_1 & \lambda_{H22} v v_2 & 0 & 0 \\ \lambda_{H11} v v_1 & \lambda_1 v_1^2 & \lambda_3 v_1 v_2 & 0 & 0 \\ \lambda_{H22} v v_2 & \lambda_3 v_1 v_2 & \lambda_2 v_2^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{(\lambda_4 + \lambda_5)}{2} (v_1^2 + v_2^2) & 0 \\ 0 & 0 & 0 & 0 & \frac{(\lambda_4 - \lambda_5)}{2} (v_1^2 + v_2^2) \end{pmatrix}. \quad (5.43)$$

We see that, because the second row of Φ_1 and the third row of Φ_2 do not have a vev, φ_3 and φ_4 cannot mix with any of the other scalars. In addition, because we do not have CP violation in the scalar sector, φ_3 and φ_4 cannot mix with each other, so both are mass eigenstates. To differentiate the mass states from the gauge states, we relabel the mass eigenstates as $\varphi_3 = \mathcal{H}$ and $\varphi_4 = \chi$. We label the remaining mass eigenstates as h_1 , h_2 , and h_3 and parametrize the mixing between them using three angles θ_1 , θ_2 , and θ_3 as follows:

$$\begin{pmatrix} m_{h_1}^2 & 0 & 0 \\ 0 & m_{h_2}^2 & 0 \\ 0 & 0 & m_{h_3}^2 \end{pmatrix} = U^T \begin{pmatrix} \lambda_H v^2 & \lambda_{H11} v v_1 & \lambda_{H22} v v_2 \\ \lambda_{H11} v v_1 & \lambda_1 v_1^2 & \lambda_3 v_1 v_2 \\ \lambda_{H22} v v_2 & \lambda_3 v_1 v_2 & \lambda_2 v_2^2 \end{pmatrix} U, \quad (5.44)$$

where

$$U = Y_1 X_2 Y_3, \quad Y_1 = \begin{pmatrix} c_1 & 0 & -s_1 \\ 0 & 1 & 0 \\ s_1 & 0 & c_1 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix},$$

$$Y_3 = \begin{pmatrix} c_3 & 0 & -s_3 \\ 0 & 1 & 0 \\ s_3 & 0 & c_3 \end{pmatrix}. \quad (5.45)$$

Here, m_{h_i} is the mass of h_i , $s_i = \sin \theta_i$, $c_i = \cos \theta_i$, for $i = 1, 2, 3$. The advantage of this parametrization is that, in the limit of small θ_2 , we can define an effective mixing angle $\theta = \theta_1 + \theta_3$, which mixes h and φ_2 . This is convenient since the mixing between the SM Higgs and the dark scalars is constrained to be small to be consistent with precision electroweak measurements. References [112, 150] find that $\sin \theta \lesssim 0.3 - 0.4$ depending on the mass of the second scalar. For all our benchmark points, we will take $\sin \theta = 0.1$.

In the vector boson sector, there is only mixing between A^3 and A^8 through the following mass matrix:

$$\mathcal{L} \supset \frac{\tilde{g}}{8} \begin{pmatrix} A_\mu^3 & A_\mu^8 \end{pmatrix} \begin{pmatrix} v_2^2 & -\frac{v_2^2}{\sqrt{3}} \\ -\frac{v_2^2}{\sqrt{3}} & \frac{1}{3}(4v_1^2 + v_2^2) \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ A_\mu^8 \end{pmatrix}. \quad (5.46)$$

We define the mass eigenstates as

$$A_\mu^{3'} = c_\alpha A_\mu^3 + A_\mu^8 s_\alpha, \quad (5.47)$$

$$A_\mu^{8'} = -s_\alpha A_\mu^3 + A_\mu^8 c_\alpha, \quad (5.48)$$

where $c_\alpha = \cos \alpha$, $s_\alpha = \sin \alpha$, and

$$\alpha = \begin{cases} \frac{1}{2} \arctan \left(\frac{\sqrt{3}v_2^2}{2v_1^2 - v_2^2} \right) & \text{for } v_2^2 \leq 2v_1^2 \\ \frac{1}{2} \arctan \left(\frac{\sqrt{3}v_2^2}{2v_1^2 - v_2^2} \right) + \frac{\pi}{2} & \text{for } v_2^2 > 2v_1^2 \end{cases}. \quad (5.49)$$

We note that $\alpha \in (0, \frac{\pi}{3})$ and that $t_\alpha = \tan \alpha > 0$. We label all the mass eigenstates with a prime to distinguish between the mass and gauge eigenstates. The resulting vector boson masses are given by

$$\begin{aligned} m_{A^{1'}} &= m_{A^{2'}} = \frac{\tilde{g}}{2} v_2, \quad m_{A^{4'}} = m_{A^{5'}} = \frac{\tilde{g}}{2} v_1, \\ m_{A^{6'}} &= m_{A^{7'}} = \frac{\tilde{g}}{2} \sqrt{v_1^2 + v_2^2} \\ m_{A^{3'}} &= \frac{\tilde{g} v_2}{2} \left(1 - \frac{t_\alpha}{\sqrt{3}}\right)^{1/2}, \quad m_{A^{8'}} = \frac{\tilde{g} v_1}{2} \left(1 - \frac{t_\alpha}{\sqrt{3}}\right)^{-1/2}. \end{aligned} \quad (5.50)$$

We can also write $m_{A^{3'}}$ and $m_{A^{8'}}$ explicitly in terms of the vevs as

$$m_{A^{3'}}^2 = \frac{\tilde{g}^2}{6} \left(v_1^2 + v_2^2 - \sqrt{v_1^4 - v_1^2 v_2^2 + v_2^4} \right), \quad (5.51)$$

$$m_{A^{8'}}^2 = \frac{\tilde{g}^2}{6} \left(v_1^2 + v_2^2 + \sqrt{v_1^4 - v_1^2 v_2^2 + v_2^4} \right). \quad (5.52)$$

From this form, we can see that $A_\mu^{3'}$ is the lightest vector boson while $A_\mu^{8'}$ is the heaviest, or one of the heaviest. When $v_1 = v_2$, $A_\mu^{8'}$ becomes degenerate with $A_\mu^{6'}$ and $A_\mu^{7'}$.

This model has a global $\mathbb{Z}_2 \times \mathbb{Z}_2' \times \mathbb{Z}_2''$ symmetry. Two of these $\mathbb{Z}_2$'s are remaining symmetries from the breaking of the SU(3) as described in Ref. [113], and the third $\mathbb{Z}_2$ being a consequence of the additional $\mathbb{Z}_2$ symmetry imposed on the scalar triplets which was not in the model studied in Ref. [112].

The first $\mathbb{Z}_2$ symmetry is a subgroup of two remaining U(1) symmetries, one for the vectors and one for the scalars. The U(1) for the vectors is characterized by the matrix

$$U = e^{i\delta_1/3} \text{diag}(e^{-i\delta_1}, 1, 1). \quad (5.53)$$

Under the transformation $A_\mu^a t^a \rightarrow U A_\mu^a t^a U^\dagger$, setting $\delta_1 = \pi$ takes $A_\mu^{1,2,4,5} \rightarrow -A_\mu^{1,2,4,5}$ and does not change the other vector fields. The U(1) for the scalars is described by taking $\Phi_{1,2} \rightarrow e^{i\delta_2} \Phi_{1,2}$. The $\mathbb{Z}_2$ subgroup corresponds to choosing $\delta_1 = \pi$ and $\delta_2 = \pi$. The $\mathbb{Z}_2'$ symmetry is associated with complex conjugation, which is an outer automorphism of SU(3) [113]. The $\mathbb{Z}_2''$ symmetry is one that does not appear in Ref. [112]. This symmetry is a result of having added the extra $\mathbb{Z}_2$ in Eq. (5.26). This added symmetry results in the scalar potential being invariant under rephrasing ϕ_3 and ϕ_4 . Independently, a U(1) subgroup of SU(3) characterized by the matrix

$$U' = e^{i\delta_3/3} \text{diag}(1, e^{-i\delta_3}, 1) \quad (5.54)$$

leaves the the field strength tensor invariant. In order for the scalar kinetic terms to be invariant, we must make a specific choice for the rephasing of ϕ_3 and ϕ_4 , as well as for δ_3 . This results in the following transformations:

$$\begin{aligned} \phi_{3,4} &\rightarrow -\phi_{3,4} \\ A_\mu^{1,2,6,7} &\rightarrow -A_\mu^{1,2,6,7} \end{aligned} \quad (5.55)$$

We summarize the charges of the physical states under the $\mathbb{Z}_2 \times \mathbb{Z}_2' \times \mathbb{Z}_2''$ symmetries in Table 5.1. A minus sign in the table represents multiplying the field by -1 under the symmetry.

From these charges, we can identify the expected DM species in this model. From the charges in Table 5.1, we expect a minimum of three stable DM species and a maximum of seven DM species. Because the mass degeneracies, $m_{A^{1'}} = m_{A^{2'}}$ and $m_{A^{4'}} = m_{A^{5'}}$, put restrictions on which decays are kinematically allowed, the actual minimum number of stable DM species is four.

Table 5.1: Charges of the physical states under the $\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2$ symmetry.

Physical states	$\mathbb{Z}_2 \times \mathbb{Z}'_2 \times \mathbb{Z}''_2$
h_1, h_2, h_3	$(+, +, +)$
$\mathcal{H}, A_\mu^{7'}$	$(+, +, -)$
$A_\mu^{3'}, A_\mu^{8'}$	$(+, -, +)$
$A_\mu^{5'}$	$(-, +, +)$
$A_\mu^{4'}$	$(-, -, +)$
$A_\mu^{2'}$	$(-, +, -)$
$\chi, A_\mu^{6'}$	$(+, -, -)$
$A_\mu^{1'}$	$(-, -, -)$

5.4.2 Theoretical constraints on Lagrangian parameters

The hidden gauged SU(3) model has numerous parameters. Much of the parameter space can be excluded by theoretical consistency of the scalar potential and by experimental measurements. In the following subsections, we lay out the details of the theoretical constraints.

5.4.2.1 Unitarity

We work in the high energy limit where only four-point diagrams contribute to $2 \rightarrow 2$ scalar scattering since all diagrams involving scalar propagators are suppressed by the square of the collision energy. Thus, the dimensionful couplings m_H^2 , m_{11} , and m_{22} are not constrained directly by perturbative unitarity. In the high energy limit, it is valid to use the Goldstone bosons as the physical degrees of freedom instead of the longitudinally polarized vector bosons. We neglect scattering processes involving transversely polarized gauge bosons and fermions.

To obtain a minimal list of constraints, we must ensure that all the eigenvalues of the coupled-channel scattering matrix $\mathcal{M}$, which includes each possible combination of two scalar fields in the initial and final states, satisfy $|\text{Re } a_0| < 1/2$. We include

Table 5.2: Charges of the scalar gauge eigenstates under the nine global $\mathbb{Z}_2$ symmetries of the quartic part of the scalar potential.

Fields	$\mathbb{Z}_2$ Charges								
$\phi_{1,r}^H$	−	+	+	+	+	+	+	+	+
$\phi_{1,i}^H$	+	−	+	+	+	+	+	+	−
$\phi_{2,r}^H$	+	+	−	+	+	+	+	+	+
$\phi_{2,i}^H$	+	+	+	−	+	+	+	+	−
$\phi_{1,r}^1$	+	+	+	+	−	+	+	−	+
$\phi_{1,i}^1$	+	+	+	+	−	+	+	−	−
$\phi_{2,r}^1$	+	+	+	+	+	−	+	−	+
$\phi_{2,i}^1$	+	+	+	+	+	−	+	−	−
$\phi_{3,r}^1$	+	+	+	+	+	+	−	−	+
$\phi_{3,i}^1$	+	+	+	+	+	+	−	−	−
$\phi_{1,r}^2$	+	+	+	+	−	+	+	+	+
$\phi_{1,i}^2$	+	+	+	+	−	+	+	+	−
$\phi_{2,r}^2$	+	+	+	+	+	−	+	+	+
$\phi_{2,i}^2$	+	+	+	+	+	−	+	+	−
$\phi_{3,r}^2$	+	+	+	+	+	+	−	+	+
$\phi_{3,i}^2$	+	+	+	+	+	+	−	+	−

a symmetry factor of $1/\sqrt{2}$ for each pair of identical particles in the initial or final states.

We use the fields in the form used in Eq. (5.33). This leads to a large 120×120 dimension scattering matrix. However, the part of the scalar potential that leads to the quartic interactions obeys nine $\mathbb{Z}_2$ discrete symmetries which bring the matrix $\mathcal{M}$ into block diagonal form, which greatly simplifies the problem. In a $2 \rightarrow 2$ scattering process, the initial and final states must have the same charges under these $\mathbb{Z}_2$ symmetries. The charges we assign for the scalar fields that leave the potential invariant under these symmetries are given in Table 5.2. We can understand the origins of these charges by examining the various terms in the potential. The first four symmetries arise because the doublet, H , only appears as $|H|^2 = \frac{1}{2}((\phi_{1,r}^H)^2 + (\phi_{1,i}^H)^2 + (\phi_{2,r}^H)^2 + (\phi_{2,i}^H)^2 + (\phi_{3,r}^H)^2 + (\phi_{3,i}^H)^2)$, so changing the sign of any one of the fields does not change

the term. The fifth through seventh symmetries can be understood by looking at the quadratic terms involving the triplets which appear in the potential, namely, $|\Phi_1|^2$, $|\Phi_2|^2$, and $\Phi_1^\dagger \Phi_2$. From this, we see that there are never any terms which multiply fields from different scalar triplet rows. The eighth symmetry is related to the extra $\mathbb{Z}_2$ symmetry given in Eq. (5.26) that we imposed on the potential. Finally, the last symmetry comes from taking the complex conjugate of the potential. The resulting constraints are given by

$$|\lambda_H| \leq 8\pi, \quad (5.56)$$

$$|\lambda_{H11}| \leq 8\pi, \quad (5.57)$$

$$|\lambda_{H22}| \leq 8\pi, \quad (5.58)$$

$$|\lambda_3 - \lambda_4| \leq 8\pi, \quad (5.59)$$

$$|\lambda_3 + \lambda_4| \leq 8\pi, \quad (5.60)$$

$$|\lambda_3 - \lambda_5| \leq 8\pi, \quad (5.61)$$

$$|\lambda_3 + \lambda_5| \leq 8\pi, \quad (5.62)$$

$$|\lambda_3 + 3\lambda_4 - 4\lambda_5| \leq 8\pi, \quad (5.63)$$

$$|\lambda_3 + 3\lambda_4 + 4\lambda_5| \leq 8\pi, \quad (5.64)$$

$$|\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}| \leq 4\pi, \quad (5.65)$$

$$|\lambda_1 + \lambda_2 - \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}| \leq 4\pi, \quad (5.66)$$

$$|\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}| \leq 4\pi, \quad (5.67)$$

$$|\lambda_1 + \lambda_2 - \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}| \leq 4\pi, \quad (5.68)$$

$$|\text{Roots}(P(z))| \leq 8\pi, \quad (5.69)$$

where

$$\begin{aligned}
P(z) = & z^3 - 2z^2(4\lambda_1 + 4\lambda_2 + 3\lambda_H) \\
& - 4z \left((3\lambda_3 + \lambda_4)^2 - 12(\lambda_1 + \lambda_2)\lambda_H \right. \\
& \left. - 16\lambda_1\lambda_2 + 6\lambda_{H11}^2 + 6\lambda_{H22}^2 \right) \\
& - 24 \left(- (3\lambda_3 + \lambda_4)^2\lambda_H + 16\lambda_1\lambda_2\lambda_H - 8\lambda_2\lambda_{H11}^2 \right. \\
& \left. - 8\lambda_1\lambda_{H22}^2 + 4(3\lambda_3 + \lambda_4)\lambda_{H11}\lambda_{H22} \right). \tag{5.70}
\end{aligned}$$

This last polynomial comes from finding the eigenvalues of the 3 by 3 matrix:

$$P(z) = \det \begin{pmatrix} 8\lambda_1 - z & 2(3\lambda_3 + \lambda_4) & \sqrt{24}\lambda_{H11} \\ 2(3\lambda_3 + \lambda_4) & 8\lambda_2 - z & \sqrt{24}\lambda_{H22} \\ \sqrt{24}\lambda_{H11} & \sqrt{24}\lambda_{H22} & 6\lambda_H - z \end{pmatrix}.$$

The bounds from Eq. (5.69) can be translated into the following bounds using the same technique used in Ref. [4]:

$$\begin{aligned}
4\pi & \geq |2\lambda_1 + 2\lambda_2 - \sqrt{4(\lambda_1 - \lambda_2)^2 + (3\lambda_3 + \lambda_4)^2}|, \\
4\pi & \geq |2\lambda_1 + 2\lambda_2 + \sqrt{4(\lambda_1 - \lambda_2)^2 + (3\lambda_3 + \lambda_4)^2}|, \\
\lambda_H & < \frac{4\pi}{3} + \frac{4((3\lambda_1 + \lambda_4)\lambda_{H11}\lambda_{H22} + 2\lambda_{H11}^2(\pi - \lambda_2) + 2\lambda_{H22}^2(\pi - \lambda_1))}{(3\lambda_1 + \lambda_4)^2 - 16(\pi - \lambda_1)(\pi - \lambda_2)}, \\
\lambda_H & > -\frac{4\pi}{3} + \frac{4((3\lambda_1 + \lambda_4)\lambda_{H11}\lambda_{H22} + 2\lambda_{H11}^2(-\pi - \lambda_2) + 2\lambda_{H22}^2(-\pi - \lambda_1))}{(3\lambda_1 + \lambda_4)^2 - 16(-\pi - \lambda_1)(-\pi - \lambda_2)}.
\end{aligned}$$

5.4.2.2 Scalar potential bounded from below

We next study the constraints on the scalar couplings by requiring the potential to be bounded from below. We follow the approach of Refs. [4] and [151]. First, we make the following definitions:

$$r = \sqrt{|H|^2 + |\Phi_1|^2 + |\Phi_2|^2}, \quad (5.71)$$

$$r^2 \cos^2 \gamma_1 = |H|^2, \quad (5.72)$$

$$r^2 \cos^2 \gamma_2 \sin^2 \gamma_1 = |\Phi_1|^2, \quad (5.73)$$

$$r^2 \sin^2 \gamma_2 \sin^2 \gamma_1 = |\Phi_2|^2, \quad (5.74)$$

$$\zeta = \frac{|\Phi_1^\dagger \Phi_2|^2}{|\Phi_1|^2 |\Phi_2|^2}, \quad (5.75)$$

$$\omega = \frac{(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2}{2|\Phi_1|^2 |\Phi_2|^2}. \quad (5.76)$$

The parameters ζ and ω are bounded by

$$\zeta \in [0, 1], \quad (5.77)$$

$$\omega \in [-1, 1]. \quad (5.78)$$

Making these substitutions, we can write the quartic part of the potential as

$$V_4 = \frac{r^4}{(1 + \tan^2 \gamma_2)^2 (1 + \tan^2 \gamma_1)^2} \mathbf{x}^T A \mathbf{y}, \quad (5.79)$$

where

$$\mathbf{x} = \begin{pmatrix} 1 \\ \tan^2 \gamma_2 \\ \tan^4 \gamma_2 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 1 \\ \tan^2 \gamma_1 \\ \tan^4 \gamma_1 \end{pmatrix}, \quad (5.80)$$

and

$$A = \frac{1}{2} \begin{pmatrix} \lambda_H & \lambda_{H11} & \lambda_1 \\ 2\lambda_H & \lambda_{H11} + \lambda_{H22} & 2(\lambda_3 + \zeta\lambda_4 + \omega\lambda_5) \\ \lambda_H & \lambda_{H22} & \lambda_2 \end{pmatrix}. \quad (5.81)$$

The fraction in Eq. (5.79) is always positive and grows with the overall field excursion r . The $\mathbf{x}^T A \mathbf{y}$ term in Eq. (5.79) can be positive or negative; we require it to be positive to ensure that the potential is bounded from below. This term can be expressed as a biquadratic in $\tan \gamma_2$ with coefficients being other biquadratics in $\tan \gamma_1$. A biquadratic of the form $a + bz^2 + cz^4$ will be positive for all values of z if the following conditions are satisfied:

$$a > 0, \quad c > 0, \quad \text{and} \quad b + 2\sqrt{ac} > 0. \quad (5.82)$$

This provides us with the following constraints:

$$\lambda_H > 0, \quad (5.83)$$

$$\lambda_1 > 0, \quad (5.84)$$

$$\lambda_2 > 0, \quad (5.85)$$

$$\lambda_{H11} + 2\sqrt{\lambda_1 \lambda_H} > 0, \quad (5.86)$$

Table 5.3: Input parameters for the benchmark points. m_χ , $m_{\mathcal{H}}$, and $m_{A^{1'}}$ were allowed to vary for benchmark point A, B, and C, respectively, along with the gauge coupling, which was varied to obtain the correct relic density. In all cases, we take the mixing in the scalar sector to only be between h_1 and h_3 with a mixing angle of $\sin \theta = 0.1$. We also set m_{h_1} equal to the SM Higgs mass.

Parameters	Scenarios A	Scenarios B	Scenarios C
$m_{A^{1'}}$	300 GeV	300 GeV	200 – 500 GeV
m_{h_2}	2500 GeV	2500 GeV	600 GeV
m_{h_3}	650 GeV	650 GeV	225 GeV
$m_{\mathcal{H}}$	1000 GeV	400 – 600 GeV	250 GeV
m_χ	50 – 200 GeV	1000 GeV	250 GeV
v_1/v_2	10	10	1.2
DM	$\chi, A^{1'}, A^{2'}, A^{3'}$	$\mathcal{H}, A^{1'}, A^{2'}, A^{3'}$	$\mathcal{H}, \chi, A^{1'}, A^{2'}, A^{3'}, A^{4'}, A^{5'}$

$$\lambda_{H22} + 2\sqrt{\lambda_2\lambda_H} > 0, \quad (5.87)$$

$$\lambda_3 + \zeta\lambda_4 + \omega\lambda_5 + \sqrt{\lambda_1\lambda_2} > 0. \quad (5.88)$$

This last bound can be split up into the different cases

$$\lambda_3 + \sqrt{\lambda_1\lambda_2} > \begin{cases} \lambda_5 & \text{for } \lambda_4 > 0, \lambda_5 > 0, \\ -\lambda_4 + \lambda_5 & \text{for } \lambda_4 < 0, \lambda_5 > 0, \\ -\lambda_5 & \text{for } \lambda_4 > 0, \lambda_5 < 0, \\ -\lambda_4 - \lambda_5 & \text{for } \lambda_4 < 0, \lambda_5 < 0. \end{cases} \quad (5.89)$$

5.4.2.3 Avoiding alternative minima

In this section, we discuss the structure of the vacuum of the model to confirm that the choice of vevs corresponds to the absolute minimum of the potential. First, we

write the potential in terms of the vevs:

$$\begin{aligned}
V(v, v_1, v_2, v_3) = & \frac{1}{2} (m_H^2 v^2 + m_{11}^2 v_1^2 + m_{22}^2 (v_2^2 + v_3^2)) \\
& + \frac{1}{8} (\lambda_H v^4 + \lambda_1 v_1^4 + \lambda_2 (v_2^4 + v_3^4)) \\
& + \frac{1}{4} (\lambda_{H11} v_1^2 + \lambda_{H22} (v_2^2 + v_3^2)) v^2 \\
& + \frac{1}{4} (\lambda_3 v_1^2 v_2^2 + \lambda_2 v_2^2 v_3^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 v_3^2) .
\end{aligned} \tag{5.90}$$

Minimizing the potential leads to the following constraints:

$$\frac{\partial V}{\partial v} = 0 = v (2m_H^2 + \lambda_H v^2 + \lambda_{H11} v_1^2 + \lambda_{H22} (v_2^2 + v_3^2)) , \tag{5.91}$$

$$\frac{\partial V}{\partial v_1} = 0 = v_1 (2m_{11}^2 + \lambda_{H11} v^2 + \lambda_1 v_1^2 + \lambda_3 v_2^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_3^2) , \tag{5.92}$$

$$\frac{\partial V}{\partial v_2} = 0 = v_2 (2m_{22}^2 + \lambda_{H22} v^2 + \lambda_3 v_1^2 + \lambda_2 (v_2^2 + v_3^2)) , \tag{5.93}$$

$$\frac{\partial V}{\partial v_3} = 0 = v_3 (2m_{22}^2 + \lambda_{H22} v^2 + (\lambda_3 + \lambda_4 + \lambda_5) v_1^2 + \lambda_2 (v_2^2 + v_3^2)) . \tag{5.94}$$

Note that $v_2 \frac{\partial V}{\partial v_3} = v_3 \frac{\partial V}{\partial v_2} + (\lambda_4 + \lambda_5) v_1^2 v_3$. From this, we conclude that to have a consistent vacuum configuration requires that $(\lambda_4 + \lambda_5) v_1^2 v_3 = 0$. Since we need $v_1 \neq 0$ to fully break the hidden SU(3) and setting $\lambda_4 = -\lambda_5$ is an unmotivated tuned choice, we will assume that $v_3 = 0$. Given a collection of input parameters, we wish to ensure that we are in the deepest minima. From the inputs, we can obtain values for the scalar couplings, the vevs, and $\tilde{g}$, but not m_H^2 , m_{11}^2 , and m_{22}^2 . We use Eq. (5.91) to (5.94) to solve for these parameters, which ensures that our choice of parameters is one of the extrema of the potential. We then find all possible values for (v, v_1, v_2, v_3) which satisfy Eq. (5.91) to (5.94) and check to see which values lead to the smallest value of the potential. If the values of the vevs with which we started do not lead to

the deepest minima, we must reject that point.

5.5 Results and Discussion for the Hidden Gauged SU(3) Model

The hidden gauged SU(3) model has too many parameters to make a full scan of the parameter space practical. Therefore, we will explore the implications of three representative benchmark points. The parameter values for benchmark point A were chosen to explore the region where one of the DM species could annihilate to SM particles through a Higgs resonance. Benchmark point B was chosen to explore a region where no such resonant effects would occur. Finally, benchmark point C was chosen to explore the region where v_1 and v_2 have similar values and where there are many more stable DM particles. All benchmark points satisfy all the theoretical constraints discussed so far.

We will vary the mass of one of the DM species and find the gauge coupling $\tilde{g}$ which results in the correct relic density. We vary the mass of χ , $\mathcal{H}$, and $A^{1'}$ for benchmark points A, B, and C, respectively. Some of the hidden sector particles are stable because symmetry respecting decays are kinematically forbidden. Thus, benchmark points A and B have four stable DM species, while benchmark point C has seven DM species. Our purpose is to study some of the features that arise from having many stable DM species. The benchmark points we consider are defined in Table 5.3. For all the benchmark points, we take $\theta_2 = 0$ and combine θ_1 and θ_3 into a single angle θ , which we set to $\sin \theta = 0.1$, which is consistent with experimental constraints coming from the SM Higgs couplings [112, 150]. We also ignore any unstable dark-sector species by assuming that they would have decayed out long before freeze-out occurred. Finally, we include all tree level diagrams involving stable DM species and

the SM species in the initial and final states. We make sure to include the widths in the propagators of unstable species in s-channel diagrams.

5.5.1 DM relic density from the hidden gauged SU(3) model

When varying the mass of one of the DM species for a benchmark point, we used a binary search to find a value of the gauge coupling which would result in the relic density lying in the measured range of $\Omega_{CDM}h^2 = 0.1186 \pm 0.002$ [152]. Figure 5.3 shows the resulting values for the coupling as a function of the DM mass for each benchmark point. If the required value of the coupling fell outside the unitarity bound of $\sqrt{4\pi}$, we did not consider it. This was the case for benchmark point A for a range of m_χ masses above the threshold for annihilation via the Higgs boson resonance as can be seen by the lack of points between half the Higgs mass and about 90 GeV.

Another insightful variable is the ratio of the relic density of vector particles to the total relic density. This is defined by

$$f_A = \frac{\sum_i \Omega_{A^{i'}}}{\Omega_{\text{tot}}}. \quad (5.95)$$

This variable shows how the composition of the DM changes as we vary parameters such as the gauge coupling. It is very useful when looking at direct and indirect detection since the scalar and vector particles interact very differently with detectors. Figure 5.3 shows how f_A varies for each benchmark point.

5.5.2 DM direct-detection

We now examine the constraints obtained from direct detection measurements using the latest results from the XENON1T experiment [153, 154]. Because there are multi-

Figure 5.3: Constraints on the allowed SU(3) gauge coupling values and the resulting ratio of the vector relic density to total relic density as a function of the DM mass. The left column shows values of the gauge coupling required so that the relic density is consistent with $\Omega_{CDM}h^2 = 0.1186 \pm 0.002$, while the right column shows the fraction of DM that consists of vector particles. The dashed lines in the left column show the unitarity bound for $\tilde{g}$ which is $\sqrt{4\pi}$, and the dashed lines in the right column show the maximum value that f_A can take, which is 1. Plots (a) and (b) show the results for scenario A, plots (c) and (d) show the results for scenario B, and plots (e) and (f) show the results for scenario C.

ple DM species that can each interact differently with the detector, we cannot simply calculate a cross section and compare it with the reported experimental bound. Instead, we must find the predicted number of events that the experiment would have seen and compare that to what the experiment observed. The theoretical rate of events is given by

$$\frac{dR}{dE_R} = \sum_i f_i \left(\frac{dR}{dE_R} \right)_i, \quad (5.96)$$

where $f_i = \frac{\Omega_i}{\Omega_{\text{tot}}}$ and [117]

$$\left(\frac{dR}{dE_R} \right)_i = \frac{\sigma_{iN}^0 \rho^{\text{loc}}}{2\mu_{iN}^2 m_i} F_i^2(E_R) I_i(E_R). \quad (5.97)$$

Here, E_R is the nuclear recoil energy in the detector; σ_{iN}^0 is the DM nucleus cross section at zero momentum transfer; ρ^{loc} is the local DM density for the i th species; $\mu_{iN} = m_i m_N / (m_i + m_N)$ is the reduced mass of the nucleus DM system; $F_i(E_R)$ is a nuclear form factor; and $I_i(E_R)$ is the mean of the inverse of the speed, $\langle v_i^{-1} \rangle$, of the i th species in the DM halo for a given E_R . Explicit formulas for $I_i(E_R)$ and $F_i(E_R)$ can be found in Ref. [117]. Once the differential rate is calculated, we integrate it over the energy range appropriate for the experiment and multiply the result by an efficiency factor which could in principle be different for each species. In the case for XENON1T, the energy range for single nuclear recoil events is 4.9 to 40.9 keV, and we assume a detection efficiency of 89%, which for simplicity we took to be the same for all DM species and recoil energies.

For the parameter values to be allowed, the calculated theoretical rate should fall below the experimental limit. At a confidence level of $1 - \beta$, a background rate ν_b , and a total of n_{obs} observed events, we can find the upper limit on the signal rate

ν_s^{up} by numerically solving

$$\beta = \frac{e^{-(\nu_s^{\text{up}} + \nu_b)} \sum_{n=0}^{n_{\text{obs}}} \frac{(\nu_s^{\text{up}} + \nu_b)^n}{n!}}{e^{-\nu_b} \sum_{n=0}^{n_{\text{obs}}} \frac{\nu_b^n}{n!}}. \quad (5.98)$$

For XENON1T [153, 154], we use $\nu_b = 735$ and $n_{\text{obs}} = 739$. To compare the experimental rate to the theoretical prediction, the experimental rate needs to be given in terms of events per unit time per unit mass, which is achieved by dividing the experimental values by the exposure. For XENON1T, this is $1.0 \text{ t} \times \text{yr}$. At the 95% confidence level, the resulting limit on the rate is $1.58 \times 10^{-4} \text{ kg}^{-1} \text{ day}^{-1}$ [154].

For the cross section, our benchmark points have two mediating particles between the dark and visible sectors, namely, h_1 and h_3 . The cross section for DM species i and a nucleon is thus given by

$$\sigma_{in}^0 = \frac{f_N^2}{4\pi} \left(\frac{g_{iih_1} c_\theta}{m_{h_1}^2} + \frac{g_{iih_3} s_\theta}{m_{h_3}^2} \right)^2 \frac{m_n^4}{(m_n + m_i)^2 v^2}, \quad (5.99)$$

where c_θ and s_θ are $\cos \theta$ and $\sin \theta$, respectively; f_N is the SM Higgs effective coupling to nucleons, which is approximately 0.30 ± 0.03 [82]; m_n is the nucleon mass which we take to be 0.93895 GeV ; m_i is the mass of the DM species in question; v is the SM vev; and g_{iih_1} and g_{iih_3} are the couplings between the DM and h_1 and h_3 which can be found in Appendix C, respectively. In the case of scalar DM, we take the Feynman rule to be of the form $-ig$, and in the case of vector DM, the Feynman rule is of the form $-igg_{\mu\nu}$. To transform Eq. (5.99) to a cross section with the entire nucleus, we use [117]

$$\sigma_{iN}^0 = \sigma_{in}^0 \frac{A^2 \mu_{iN}^2}{\mu_{in}^2}, \quad (5.100)$$

where A is the atomic number of the nucleus, μ_{iN} is the reduced mass between the nucleus and the DM particle, and μ_{in} is the reduced mass between the nucleon and the DM particle. The scalar DM particles (χ and $\mathcal{H}$) interact weakly with the nuclear target. This is because the scalar- h_1 coupling is weak due to our choice of $\lambda_{H11} = 0$ and λ_{H22} to be small and h_3 interacts with nucleons via its SM Higgs component which is small (recall $\sin \theta = 0.1$). As a consequence, when DM is mainly scalar, it is weakly constrained by direct detection. The results for each benchmark point are shown in Fig 5.4. Again, the gap between half the Higgs mass and roughly 90 GeV for benchmark point A is because the value of the coupling that gives the correct relic abundance is not allowed because it violates the unitarity constraint. In Fig 5.4(a), we see that there is an allowed region at high mass and in the Higgs resonance region. In Fig 5.4(b), the gauge coupling is quite small and has a large fraction of scalar DM, so direct detection does not impose any significant constraints on benchmark point B. In Fig 5.4(c), we see that direct detection excludes most of the mass region due to the large vector fraction and large gauge coupling. There is, however, the tip of a resonance which allows for efficient annihilation of vectors around 340 GeV, which remains allowed. This corresponds to when the mass of $A^{3'}$ is just below 300 GeV and where there is significant annihilation of $A^{3'}$ through the 600 GeV h_2 .

We see that f_A is nontrivial in each case; that is to say, it is not close to being exactly 1 or exactly 0. This nontrivial behavior tells us that it is important to consider all the relevant particles when doing the freeze-out calculation, not just the lightest or most weakly coupled. In the cases where a resonance effect is present, it is particularly important to be precise, as these regions require quite different couplings constants to obtain the observed relic density.

Figure 5.4: The direct-detection event rate as a function of the DM mass. Plots (a), (b), and (c) show the results for benchmark points A, B and C, respectively. The allowed points are below the dashed line at $1.58 \times 10^{-4} \text{ kg}^{-1}\text{day}^{-1}$. In all the cases, the regions where the coupling constant $\tilde{g}$ was large to give the allowed relic density are ruled out by these direct detection constraints. These are also the regions where a significant portion of the DM consists of scalars.

5.5.3 DM indirect detection

Dwarf spheroidal satellite galaxies (dSphs) are typically DM dominated and so are a good place to study DM [155, 156]. The Fermi Collaboration [18] has acquired six years of data from observing 15 dSphs and has released bounds for WIMP DM annihilation based on the observed gamma ray flux. They considered the following representative final states for the DM annihilation: e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$, $u\bar{u}$, $b\bar{b}$, and W^+W^- [18].

We translate these bounds into constraints on our model by considering the cross sections of pairs of identical DM particles into the above final states scaled by their fraction squared and their branching ratio. Explicitly, we will compare

$$\sigma_{\text{scaled}} = \left(\frac{\Omega_i}{\Omega_{DM}} \right)^2 \frac{(\sigma_{ii \rightarrow SM+SM})^2}{\sigma_{ii}^{tot}}, \quad (5.101)$$

where $\sigma_{ii \rightarrow SM+SM}$ is the cross section from two identical DM particles to one of the SM final states considered by the Fermi Collaboration and σ_{ii}^{tot} is the sum of the cross sections to all possible final states. Of all the final states considered by Fermi, only the W^+W^- final state has the potential to provide useful constraints as the scaled cross sections to the other final states are orders of magnitude below the Fermi constraints. This is because the cross sections to these fermions are proportional to the mass squared of the fermion, which are small compared to the W mass squared. Figure 5.5 shows the results for each of the benchmark points. In all cases, the scalar DM ends up having a significantly larger scaled cross section to W^+W^- than any vector DM because the vector DM annihilates preferentially to other DM particles. We see that, although the constraint is close, indirect detection does not impose any constraints. For benchmark point A, the experimental limits are quite close to

the theoretical predictions, while for benchmark points B and C, the experimental limits are about an order of magnitude above the theoretical predictions. Thus, in all three cases, there is the potential to rule out large regions of parameter space with moderate improvements to the experimental bounds. It should also be noted that a more sophisticated analysis that considers the resulting spectrum from multiple DM species coannihilating to the same final state could result in more stringent constraints.

These results lead to the interesting observation that if the DM is dominated by vector species then it will be more easily detected in direct-detection experiments, while if the DM is dominated by scalar species then it would more likely to be detected by indirect detection experiments. This could lead to an observation in one type of experiment, but no signal in the other, and emphasizes the importance of the complementarity of multiple types of searches. Arcadi *et al.* [112] and many others have also pointed this out, e.g., Refs. [13–17, 117, 135, 139].

To end this section, we note that the constraints used are conservative. We only considered constraints from the annihilation of one species at a time. This bound could be improved with a more detailed study of the gamma-ray spectrum [157, 158] or cosmic-ray spectra [159] resulting from multiple species annihilating or coannihilating and comparing this to the observed spectrum from dSphs [120].

5.6 Summary and Conclusions

This chapter reported a study of multispecies DM, in particular, the hidden gauged SU(3) model. We calculated the DM relic density to constrain the parameters of the model. Using the resulting allowed values of the parameters, we studied the prospects for observing multispecies DM in direct-detection and indirect detection

Figure 5.5: Indirect detection constraints for the W^+W^- final state. Figures (a), (b), and (c) show the results for benchmark points A, B, and C, respectively. The line colors are as follows: red for $\chi\chi \rightarrow W^+W^-$; blue for $\mathcal{H}\mathcal{H} \rightarrow W^+W^-$; green for $A^{1'}A^{1'} \rightarrow W^+W^-$ or $A^{2'}A^{2'} \rightarrow W^+W^-$, which have identical results; purple for $A^{3'}A^{3'} \rightarrow W^+W^-$; and orange for $A^{4'}A^{4'} \rightarrow W^+W^-$ or $A^{5'}A^{5'} \rightarrow W^+W^-$, which have identical results. The dashed lines are the constraints from Fermi [18], while the solid lines are the predictions of the model. Note that the purple and green dashed lines are almost superimposed because they have very similar masses leading to similar constraints. We also note that the solid orange line does not appear because it is too small.

measurements. Before examining the hidden gauged $SU(3)$ model, we studied the relic abundance of simplified scenarios to gain some insight into how adding different types of interactions between the DM species would affect the relic abundance. We then examined a specific model, the hidden gauged $SU(3)$ model of Arcadi *et al.* [112]. With many DM particles, it was not possible to implement a detailed scan of the parameter space. Instead, we studied three representative benchmark points. In all cases, we fixed the gauge coupling to give the observed relic density. When the relic density was dominated by vector particles, we found that it could more easily be constrained by direct-detection experiments, while if the relic abundance were dominated by scalars, indirect detection would have the potential to be more constraining in the future, even with conservative bounds. This reinforces the need to search for DM with a broad experimental program. An important result was that for the different benchmark points the required value for the gauge coupling could vary quite significantly based on the choice of parameter values. This can lead to significantly different relic abundances among particle species, which in turn changes the detection prospects of a particular parameter point. We also saw that for each benchmark point the ratios of vectors to scalars were never dominated by either scalars or vectors and that the DM consisted of some nontrivial mixture of both. This shows the importance of not only including the lightest stable particle in the freeze-out calculation but including all relevant particles.

Chapter 6

Real singlet scalar DM extension of the Georgi-Machacek model

6.1 Introduction

Since the discovery of a Standard Model (SM)-like 125 GeV Higgs boson at the CERN Large Hadron Collider (LHC) [5], the determination of the Higgs boson's couplings to other particles has become a top priority. At the LHC, these couplings are extracted from signal rates in various resonant Higgs production and decay channels, which can be written in the narrow width approximation as

$$\text{Rate}_{ij} = \sigma_i \frac{\Gamma_j}{\Gamma_{\text{tot}}} = \kappa_i^2 \sigma_i^{\text{SM}} \frac{\kappa_j^2 \Gamma_j^{\text{SM}}}{\sum_k \kappa_k^2 \Gamma_k^{\text{SM}} + \Gamma_{\text{new}}}. \quad (6.1)$$

Here σ_i is the Higgs production cross section in production mode i , Γ_j is the Higgs decay partial width into the final state j , Γ_{tot} is the total width of the Higgs boson, the corresponding quantities in the SM are denoted with a superscript, and Γ_{new} represents the partial width of the Higgs boson into any new, non-SM final states.

The coupling modification factors κ_i parametrize the deviations of the Higgs couplings from their SM values [160].

The extraction of the Higgs couplings κ_i from these LHC rate measurements is plagued by a well-known “flat direction” [161] that appears when new decay modes are present. For example, one can imagine a scenario in which all the coupling modification factors have a common value $\kappa_i^2 \equiv \kappa^2 > 1$ and there is a new, unobserved contribution to the Higgs total width, $\Gamma_{\text{new}} > 0$. In this case the Higgs production and decay rates measurable at the LHC are given by

$$\text{Rate}_{ij} = \frac{\kappa^4 \sigma_i^{\text{SM}} \Gamma_j^{\text{SM}}}{\kappa^2 \Gamma_{\text{tot}}^{\text{SM}} + \Gamma_{\text{new}}}. \quad (6.2)$$

All measured Higgs production and decay rates will be equal to their SM values if

$$\Gamma_{\text{new}} = \kappa^2(\kappa^2 - 1)\Gamma_{\text{tot}}^{\text{SM}} \geq 0. \quad (6.3)$$

In particular, a simultaneous enhancement $\kappa^2 > 1$ of all the Higgs couplings to SM particles can mask, and be masked by, the presence of new decay modes of the Higgs that are not (yet) directly detected at the LHC.¹

Our goal in this chapter is to study an explicit benchmark model in which this scenario could be realized. We focus on models with extended Higgs sectors. Our first requirement is a model in which the Higgs couplings to W and Z bosons and to fermions can be enhanced relative to those in the SM. To achieve $\kappa_W, \kappa_Z > 1$ in an extended Higgs model, we need scalars in isospin representations larger than doublets that carry non-negligible vacuum expectation values (vev’s). Only a few

¹Measuring such an enhancement in the Higgs couplings would be straightforward at a lepton-collider Higgs factory such as the International Linear Collider, where a direct measurement of the total Higgs production cross section in $e^+e^- \rightarrow Zh$ can be made with no reference to the Higgs decay branching ratios by using the recoil mass method (see, e.g., Ref. [162]).

such models exist that preserve the ρ parameter at tree level: the Georgi-Machacek (GM) model with isospin triplets [163, 164], generalizations of the GM model to higher isospin [165–169], and an extension of the Higgs sector by an isospin septet with appropriately chosen hypercharge [170–172]. In this chapter we choose the GM model as the simplest extension suitable for our purposes. Its phenomenology has been extensively studied [168, 171, 173–194]. It has also been incorporated into the scalar sectors of little Higgs [195, 196] and supersymmetric [197, 198] models, and an extension with an additional isospin doublet [199] has been considered.

Our second requirement is a new decay mode for the 125 GeV Higgs boson. A particularly attractive prospect is to link Higgs physics to the mystery of DM in the Universe (for a recent pedagogical review see Ref. [15]) by allowing the Higgs to decay into pairs of DM particles. To this end we extend the GM model through the addition of a real isospin-singlet scalar field S , upon which we impose a Z_2 symmetry $S \rightarrow -S$. We will require that S accounts for the observed DM relic abundance in the Universe via the standard thermal freeze-out mechanism. Real singlet scalar extensions of the SM [2, 65–84] and of two-Higgs-doublet models [2, 88–96, 96–98] have been extensively studied in the literature. These models tend to be tightly constrained by the combination of relic density, DM direct-detection limits, and limits on the indirect detection of DM annihilation by-products from nearby dwarf galaxies.

We will find that the situation is rather similar in the singlet scalar DM extension of the GM model. The two strongest constraints are the requirement of the correct DM relic abundance from thermal freeze-out [24] and the direct-detection cross section limit from the LUX experiment [200]. These constraints restrict the allowed range of singlet scalar masses to lie just below the 125 GeV Higgs pole for resonant annihilation (57–62 GeV) or above the Z boson mass. The constraint from

125 GeV Higgs boson invisible decays is currently weaker than that from direct-detection. Constraints coming from Higgs properties and signals also significantly constrain this model. They do however allow for some interesting deviations from the Standard Model that the GM model without the singlet does not allow.

One important difference compared to the singlet scalar extension of the SM is the prospect for future DM direct-detection experiments to probe the model at heavier singlet masses. While an absence of signal at the planned XENON 1T experiment would exclude singlet scalar masses up to 4.5 TeV in the singlet scalar extension of the SM [82], in the singlet scalar extension of the GM model a large swath of parameter space with singlet scalar masses as light as 125 GeV remains beyond the reach of XENON 1T. In fact, there is some allowed parameter space with singlet scalar masses near the 125 GeV Higgs pole for resonant annihilation (60–62 GeV) and some with singlet scalar masses above about 150 GeV which have a direct-detection cross section that lies below the neutrino floor. This is mainly due to the contribution of the additional scalars in the GM model to the production of the correct relic density, while not contributing strongly to the direct-detection cross section.

This chapter is organized as follows. In Sec. 6.2 we begin with a description of the singlet scalar extension of the GM model. In Sec. 6.3 we extend the theoretical constraints on the GM model to include the singlet scalar extension. In Sec. 6.4 we describe the details of the thermal freeze-out and imposing the relic abundance constraints on the model parameters while in Sec. 6.5 we describe the numerical scan procedure used to map out the allowed parameter space. In Sec. 6.6 we briefly summarize the direct- and indirect search constraints on the additional scalars in the GM model. In Sec. 6.7 we compute the DM relic abundance and direct and indirect detection cross sections and display the impact of the observational constraints

on the allowed parameter space. In Sec. 6.8 we consider the constraints from the 125 GeV Higgs boson invisible decays and signal strengths in visible channels. Finally in Sec. 6.9 we summarize our conclusions. Feynman rules for couplings involving the singlet scalar are collected in App. D.

6.2 The Georgi-Machacek model extended by a real singlet scalar

The scalar sector of the GM model [163, 164] consists of the usual complex doublet (ϕ^+, ϕ^0) with hypercharge² $Y = 1$, a real triplet (ξ^+, ξ^0, ξ^-) with $Y = 0$, and a complex triplet $(\chi^{++}, \chi^+, \chi^0)$ with $Y = 2$. The doublet is responsible for the fermion masses as in the SM. In order to preserve the custodial $SU(2)$ symmetry and avoid large tree-level contributions to the electroweak ρ parameter, the scalar potential is constructed to preserve a global $SU(2)_L \times SU(2)_R$ symmetry, which breaks down to the diagonal subgroup [known as the custodial $SU(2)$ symmetry] upon electroweak symmetry breaking. To make the global $SU(2)_L \times SU(2)_R$ symmetry explicit, we write the doublet in the form of a bidoublet Φ and combine the triplets to form a bitriplet X :

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, \quad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}. \quad (6.4)$$

The vev's are defined by $\langle \Phi \rangle = \frac{v_\phi}{\sqrt{2}} I_{2 \times 2}$ and $\langle X \rangle = v_\chi I_{3 \times 3}$, where I is the unit matrix and the Fermi constant G_F constrains

$$v_\phi^2 + 8v_\chi^2 \equiv v^2 = \frac{1}{\sqrt{2}G_F} \approx (246 \text{ GeV})^2. \quad (6.5)$$

²We use $Q = T^3 + Y/2$.

The most general gauge-invariant scalar potential involving these fields and the real singlet S , while conserving the global $SU(2)_L \times SU(2)_R$ and the Z_2 symmetry $S \rightarrow -S$, is given by

$$\begin{aligned}
V(\Phi, X) = & \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\
& + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b) \\
& - M_1 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t^a X t^b) (UXU^\dagger)_{ab} \\
& + \frac{\mu_S^2}{2} S^2 + \lambda_a S^2 \text{Tr}(\Phi^\dagger \Phi) + \lambda_b S^2 \text{Tr}(X^\dagger X) + \lambda_S S^4.
\end{aligned} \tag{6.6}$$

The first three lines of this potential are identical to that given, e.g., in Ref. [188].³ The last line contains the new terms involving the singlet scalar S . Here the $SU(2)_L$ generators for the doublet representation are $\tau^a = \sigma^a/2$ with σ^a being the Pauli matrices, the generators for the triplet representation are

$$\begin{aligned}
t^1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & t^2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \\
t^3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},
\end{aligned} \tag{6.7}$$

³A translation table to other parametrizations of the GM model scalar potential has been given in the appendix of Ref. [188].

and the matrix U , which rotates X into the Cartesian basis, is given by [177]

$$U = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}. \quad (6.8)$$

We will work in the vacuum in which S does not get a vev, so that the Z_2 symmetry remains unbroken and S is stable. The presence of S then has no effect on the mass spectrum or potential-minimization conditions of the GM sector of the model, which can be taken from Ref. [188]. We summarize the physical spectrum here.

The physical fields can be organized by their transformation properties under the custodial $SU(2)$ symmetry into a custodial fiveplet, a custodial triplet, and three custodial singlets, one of which is S . The custodial-fiveplet and -triplet states are given by

$$\begin{aligned} H_5^{++} &= \chi^{++}, & H_5^+ &= \frac{(\chi^+ - \xi^+)}{\sqrt{2}}, \\ H_5^0 &= \sqrt{\frac{2}{3}}\xi^0 - \sqrt{\frac{1}{3}}\chi^{0,r}, \\ H_3^+ &= -s_H\phi^+ + c_H\frac{(\chi^+ + \xi^+)}{\sqrt{2}}, \\ H_3^0 &= -s_H\phi^{0,i} + c_H\chi^{0,i}, \end{aligned} \quad (6.9)$$

and their complex conjugates, where the vev's are parametrized by

$$c_H \equiv \cos \theta_H = \frac{v_\phi}{v}, \quad s_H \equiv \sin \theta_H = \frac{2\sqrt{2}v_\chi}{v}, \quad (6.10)$$

and we have decomposed the neutral fields into real and imaginary parts according

to

$$\begin{aligned}\phi^0 &\rightarrow \frac{v_\phi}{\sqrt{2}} + \frac{\phi^{0,r} + i\phi^{0,i}}{\sqrt{2}}, & \chi^0 &\rightarrow v_\chi + \frac{\chi^{0,r} + i\chi^{0,i}}{\sqrt{2}}, \\ \xi^0 &\rightarrow v_\chi + \xi^0.\end{aligned}\tag{6.11}$$

The masses within each custodial multiplet are degenerate at tree level and can be written (after eliminating μ_2^2 and μ_3^2 in favor of the vev's) as⁴

$$\begin{aligned}m_5^2 &= \frac{M_1}{4v_\chi}v_\phi^2 + 12M_2v_\chi + \frac{3}{2}\lambda_5v_\phi^2 + 8\lambda_3v_\chi^2, \\ m_3^2 &= \frac{M_1}{4v_\chi}(v_\phi^2 + 8v_\chi^2) + \frac{\lambda_5}{2}(v_\phi^2 + 8v_\chi^2) = \left(\frac{M_1}{4v_\chi} + \frac{\lambda_5}{2}\right)v^2.\end{aligned}\tag{6.13}$$

The gauge singlet S remains a mass eigenstate, with the physical mass squared given by

$$m_S^2 = \mu_S^2 + 2\lambda_a v_\phi^2 + 6\lambda_b v_\chi^2,\tag{6.14}$$

which we require to be positive to avoid breaking the Z_2 symmetry.

The other two custodial $SU(2)$ -singlet mass eigenstates are given by

$$\begin{aligned}h &= \cos\alpha\phi^{0,r} - \sin\alpha H_1^{0r}, \\ H &= \sin\alpha\phi^{0,r} + \cos\alpha H_1^{0r},\end{aligned}\tag{6.15}$$

where

$$H_1^{0r} = \sqrt{\frac{1}{3}}\xi^0 + \sqrt{\frac{2}{3}}\chi^{0,r}.\tag{6.16}$$

⁴Note that the ratio M_1/v_χ is finite in the limit $v_\chi \rightarrow 0$,

$$\frac{M_1}{v_\chi} = \frac{4}{v_\phi^2} [\mu_3^2 + (2\lambda_2 - \lambda_5)v_\phi^2 + 4(\lambda_3 + 3\lambda_4)v_\chi^2 - 6M_2v_\chi],\tag{6.12}$$

which follows from the minimization condition $\partial V/\partial v_\chi = 0$ [188].

The mixing angle and masses are given by

$$\begin{aligned}\sin 2\alpha &= \frac{2\mathcal{M}_{12}^2}{m_H^2 - m_h^2}, & \cos 2\alpha &= \frac{\mathcal{M}_{22}^2 - \mathcal{M}_{11}^2}{m_H^2 - m_h^2}, \\ m_{h,H}^2 &= \frac{1}{2} \left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \mp \sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2} \right],\end{aligned}\tag{6.17}$$

where we choose $m_h < m_H$, and

$$\begin{aligned}\mathcal{M}_{11}^2 &= 8\lambda_1 v_\phi^2, \\ \mathcal{M}_{12}^2 &= \frac{\sqrt{3}}{2} v_\phi [-M_1 + 4(2\lambda_2 - \lambda_5) v_\chi], \\ \mathcal{M}_{22}^2 &= \frac{M_1 v_\phi^2}{4v_\chi} - 6M_2 v_\chi + 8(\lambda_3 + 3\lambda_4) v_\chi^2.\end{aligned}\tag{6.18}$$

6.3 Theoretical Constraints on Lagrangian Parameters

The singlet scalar DM extension of the GM model has 13 free parameters, two of which can be fixed by G_F and the 125 GeV Higgs mass. Before scanning over the remaining parameters, we first study the relevant theoretical and experimental constraints. The theoretical constraints come from (1) perturbative unitarity imposed on $2 \rightarrow 2$ scalar scattering amplitudes, (2) the requirement that the scalar potential be bounded from below, and (3) that the custodial SU(2)-preserving minimum is the true global minimum of the potential.

6.3.1 Perturbative unitarity of $2 \rightarrow 2$ scattering amplitudes

The quartic scalar couplings in Eq. (6.6) can be bounded by perturbative unitarity of the $2 \rightarrow 2$ scalar field scattering amplitudes. We apply the condition $|\text{Re } a_0| < 1/2$ to

each eigenvalue of the coupled-channel scattering matrix $\mathcal{M}$ including each possible combination of two scalar fields in the initial and final states. Because the scalar potential is invariant under $\text{SU}(2)_L \times \text{U}(1)_Y$, the scattering processes preserve electric charge and hypercharge and can be conveniently classified by the total electric charge and hypercharge of the incoming and outgoing states. We include a symmetry factor of $1/\sqrt{2}$ for each pair of identical particles in the initial and final states.

The basis states for $Q = Y = 0$ are

$$\begin{aligned} &\chi^{++*}\chi^{++}, \chi^{+*}\chi^+, \xi^{+*}\xi^+, \phi^{+*}\phi^+, \\ &\frac{\xi^0\xi^0}{\sqrt{2}}, \chi^{0*}\chi^0, \phi^{0*}\phi^0, \frac{S^2}{\sqrt{2}}, S\xi^0. \end{aligned} \quad (6.19)$$

Scattering amplitudes involving these states yield eight distinct eigenvalues of $\mathcal{M}$ given by

$$\begin{aligned} x_2^\pm &= 4\lambda_1 - 2\lambda_3 + 4\lambda_4 \pm \sqrt{(4\lambda_1 + 2\lambda_3 - 4\lambda_4)^2 + 4\lambda_5^2}, \\ y_1 &= 16\lambda_3 + 8\lambda_4, \\ y_2 &= 4\lambda_3 + 8\lambda_4, \\ z_b &= 4\lambda_b, \\ z_{1,2,3} &= \text{Roots}(P(z)), \end{aligned} \quad (6.20)$$

where z_1 , z_2 , and z_3 are the roots of the polynomial

$$P(z) = \det \begin{pmatrix} 24\lambda_1 - z & 12\lambda_2 & 4\lambda_a \\ 12\lambda_2 & 28\lambda_3 + 44\lambda_4 - z & 6\lambda_b \\ 4\lambda_a & 6\lambda_b & 12\lambda_S - z \end{pmatrix}. \quad (6.21)$$

We have followed the notation of Refs. [177, 188] where possible. Note that the pair of eigenvalues $x_1^\pm$ of Refs. [177, 188] is recovered by taking $\lambda_a = \lambda_b = \lambda_S = 0$ in $P(z)$.

The basis states for $Q = 0$ and $Y = 1$ are

$$\phi^+\xi^{+*}, \phi^0\xi^0, \chi^+\phi^{+*}, \chi^0\phi^{0*}, S\phi^0. \quad (6.22)$$

Scattering amplitudes involving these states yield four additional distinct eigenvalues of $\mathcal{M}$ are given by

$$\begin{aligned} y_3 &= 4\lambda_2 - \lambda_5, \\ y_4 &= 4\lambda_2 + 2\lambda_5, \\ y_5 &= 4\lambda_2 + 4\lambda_5, \\ z_a &= 4\lambda_a. \end{aligned} \quad (6.23)$$

Scattering amplitudes involving basis states with other values of Q and Y only repeat eigenvalues that have already been found. Note that by adding the real singlet scalar S we have replaced the two eigenvalues $x_1^\pm$ of Refs. [177, 188] with five new eigenvalues $z_{1,2,3,a,b}$. We obtain the unitarity bounds by requiring that the absolute value of each of the eigenvalues in Eqs. (6.20) and (6.23) be less than 8π .

The three unitarity constraints $|z_{1,2,3}| < 8\pi$ can be made more algebraically tractable by replacing them with three equivalent conditions as follows. First, since Eq. (6.21) is linear in λ_S , we can solve the equation $P(z) = 0$ for λ_S as a function of the root z ,

$$\lambda_S(z) = \frac{1}{6} \left(\frac{z}{2} + \frac{2\lambda_a^2(7\lambda_3 + 11\lambda_4 - \frac{1}{4}z) + 9\lambda_b^2(3\lambda_1 - \frac{1}{8}z) - 18\lambda_2\lambda_a\lambda_b}{2(7\lambda_3 + 11\lambda_4 - \frac{1}{4}z)(3\lambda_1 - \frac{1}{8}z) - 9\lambda_2^2} \right). \quad (6.24)$$

This function has two poles, across which $\lambda_S(z)$ changes sign. There are thus three values of z that yield the same value of λ_S , corresponding to the three roots $z_{1,2,3}$ of the polynomial $P(z)$. We now require that all three of these roots satisfy $|z_{1,2,3}| < 8\pi$. For this to be possible, the two poles in $\lambda_S(z)$ must also lie at z values between -8π and 8π . The positions of these two poles are given by $z = x_1^\pm$, where

$$x_1^\pm = 12\lambda_1 + 14\lambda_3 + 22\lambda_4 \pm \sqrt{(12\lambda_1 - 14\lambda_3 - 22\lambda_4)^2 + 144\lambda_2^2}. \quad (6.25)$$

Therefore we require $|x_1^\pm| < 8\pi$, reproducing two of the unitarity constraints from the original GM model [177, 188]. The third condition restricts λ_S to lie in the range for which the three roots of $P(z)$ all lie within $(-8\pi, 8\pi)$,

$$\lambda_S^{\min} < \lambda_S < \lambda_S^{\max}, \quad (6.26)$$

where $\lambda_S^{\min} = \lambda_S(z = -8\pi)$ and $\lambda_S^{\max} = \lambda_S(z = 8\pi)$ from Eq. (6.24).

To summarize, we will require that the following constraints from perturbative unitarity be satisfied:

$$\begin{aligned} 8\pi &> \left| 12\lambda_1 + 14\lambda_3 + 22\lambda_4 \pm \sqrt{(12\lambda_1 - 14\lambda_3 - 22\lambda_4)^2 + 144\lambda_2^2} \right| = |x_1^\pm|, \\ 8\pi &> \left| 4\lambda_1 - 2\lambda_3 + 4\lambda_4 \pm \sqrt{(4\lambda_1 + 2\lambda_3 - 4\lambda_4)^2 + 4\lambda_5^2} \right| = |x_2^\pm|, \\ 8\pi &> |16\lambda_3 + 8\lambda_4| = |y_1|, \\ 8\pi &> |4\lambda_3 + 8\lambda_4| = |y_2|, \\ 8\pi &> |4\lambda_2 - \lambda_5| = |y_3|, \end{aligned}$$

$$\begin{aligned}
8\pi &> |4\lambda_2 + 2\lambda_5| = |y_4|, \\
8\pi &> |4\lambda_2 + 4\lambda_5| = |y_5|, \\
8\pi &> |4\lambda_a| = |z_a|, \\
8\pi &> |4\lambda_b| = |z_b|, \\
\lambda_S &< \frac{1}{6} \left(4\pi + \frac{2\lambda_a^2(7\lambda_3 + 11\lambda_4 - 2\pi) + 9\lambda_b^2(3\lambda_1 - \pi) - 18\lambda_2\lambda_a\lambda_b}{2(7\lambda_3 + 11\lambda_4 - 2\pi)(3\lambda_1 - \pi) - 9\lambda_2^2} \right), \\
\lambda_S &> \frac{1}{6} \left(-4\pi + \frac{2\lambda_a^2(7\lambda_3 + 11\lambda_4 + 2\pi) + 9\lambda_b^2(3\lambda_1 + \pi) - 18\lambda_2\lambda_a\lambda_b}{2(7\lambda_3 + 11\lambda_4 + 2\pi)(3\lambda_1 + \pi) - 9\lambda_2^2} \right). \tag{6.27}
\end{aligned}$$

6.3.2 Requirement that the scalar potential be bounded from below

We next examine the constraints on the scalar couplings imposed by requiring that the scalar potential be bounded from below. Following the approach of Ref. [151], we parametrize the potential using the following definitions:

$$\begin{aligned}
r &= \sqrt{\text{Tr}(\Phi^\dagger\Phi) + \text{Tr}(X^\dagger X) + S^2}, \\
r^2 \cos^2 \gamma \sin^2 \beta &= \text{Tr}(\Phi^\dagger\Phi), \\
r^2 \sin^2 \gamma \sin^2 \beta &= \text{Tr}(X^\dagger X), \\
r^2 \cos^2 \beta &= S^2, \\
\zeta &= \frac{\text{Tr}(X^\dagger X X^\dagger X)}{(\text{Tr}(X^\dagger X))^2}, \\
\omega &= \frac{\text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(X^\dagger t^a X t^b)}{\text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X)}. \tag{6.28}
\end{aligned}$$

Making these substitutions, we can write the quartic part of the potential as

$$V_4 = \frac{r^4}{(1 + \tan^2 \gamma)^2 (1 + \tan^2 \beta)^2} \mathbf{x}^T A \mathbf{y}, \tag{6.29}$$

where

$$\mathbf{x} = \begin{pmatrix} 1 \\ \tan^2 \beta \\ \tan^4 \beta \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 1 \\ \tan^2 \gamma \\ \tan^4 \gamma \end{pmatrix}, \quad (6.30)$$

and

$$A = \begin{pmatrix} \lambda_S & 2\lambda_S & \lambda_S \\ \lambda_a & \lambda_a + \lambda_b & \lambda_b \\ \lambda_1 & \lambda_2 - \lambda_5 \omega & \lambda_3 \zeta + \lambda_4 \end{pmatrix}. \quad (6.31)$$

The first fraction in Eq. (6.29) is always positive, and grows with the overall field excursion r . The $\mathbf{x}^T A \mathbf{y}$ term in Eq. (6.29) can be positive or negative; we require it to be positive to ensure that the potential is bounded from below. This term can be expressed as a biquadratic in $\tan \gamma$ with coefficients being other biquadratics in $\tan \beta$. A biquadratic of the form $a + bz^2 + cz^4$ will be positive for all values of z if the following conditions are satisfied:

$$a > 0, \quad c > 0, \quad \text{and} \quad b + 2\sqrt{ac} > 0. \quad (6.32)$$

In our case this leads to the following constraints on the elements of the matrix A in Eq. (6.31):

$$\begin{aligned} 0 < A_{11} &= \lambda_S, \\ 0 < A_{33} &= \zeta \lambda_3 + \lambda_4, \\ 0 < A_{13} &= \lambda_S, \\ 0 < A_{31} &= \lambda_1, \\ 0 < A_{12} + 2\sqrt{A_{11}A_{13}} &= 4\lambda_S, \end{aligned}$$

$$\begin{aligned}
0 < A_{32} + 2\sqrt{A_{31}A_{33}} &= \lambda_2 - \omega\lambda_5 + 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)}, \\
0 < A_{21} + 2\sqrt{A_{11}A_{31}} &= \lambda_a + 2\sqrt{\lambda_1\lambda_S}, \\
0 < A_{23} + 2\sqrt{A_{13}A_{33}} &= \lambda_b + 2\sqrt{(\zeta\lambda_3 + \lambda_4)\lambda_S}, \\
0 < x_i A_{i2} + 2\sqrt{x_j A_{j1} x_k A_{k3}} \\
&= \mathbf{x}^T \mathbf{A} \mathbf{e}_2 + 2\sqrt{(\mathbf{x}^T \mathbf{A} \mathbf{e}_1)(\mathbf{x}^T \mathbf{A} \mathbf{e}_3)}, \\
0 < A_{2i} y_i + 2\sqrt{A_{1j} y_j A_{3k} y_k} \\
&= \mathbf{e}_2^T \mathbf{A} \mathbf{y} + 2\sqrt{(\mathbf{e}_1^T \mathbf{A} \mathbf{y})(\mathbf{e}_3^T \mathbf{A} \mathbf{y})},
\end{aligned} \tag{6.33}$$

where it should be understood that repeated indices are summed over, and $\mathbf{e}_i$ is a unit vector with a 1 in the i th component and zeros everywhere else. The last two conditions do not provide any new information as they are always satisfied when the others are, but we list them for completeness.

The ranges of the parameters ζ and ω are given, as in the original GM model [188], by

$$\zeta \in \left[\frac{1}{3}, 1\right], \quad \omega \in \left[-\frac{1}{4}, \frac{1}{2}\right]. \tag{6.34}$$

For a given value of ζ , we can write $\omega \in [\omega_-, \omega_+]$, where [188]

$$\omega_{\pm}(\zeta) = \frac{1}{6}(1 - B) \pm \frac{\sqrt{2}}{3} \left[(1 - B) \left(\frac{1}{2} + B \right) \right]^{\frac{1}{2}}, \tag{6.35}$$

with

$$B \equiv \sqrt{\frac{3}{2} \left(\zeta - \frac{1}{3} \right)} \in [0, 1]. \tag{6.36}$$

Therefore, we can write our constraints as follows:

$$\lambda_1 > 0,$$

$$\begin{aligned}
\lambda_4 &> \begin{cases} -\frac{1}{3}\lambda_3 & \text{for } \lambda_3 \geq 0, \\ -\lambda_3 & \text{for } \lambda_3 < 0, \end{cases} \\
\lambda_2 &> \begin{cases} \frac{1}{2}\lambda_5 - 2\sqrt{\lambda_1 \left(\frac{1}{3}\lambda_3 + \lambda_4\right)} & \text{for } \lambda_5 \geq 0, \lambda_3 \geq 0, \\ \omega_+(\zeta)\lambda_5 - 2\sqrt{\lambda_1 (\zeta\lambda_3 + \lambda_4)} & \text{for } \lambda_5 \geq 0, \lambda_3 < 0, \\ \omega_-(\zeta)\lambda_5 - 2\sqrt{\lambda_1 (\zeta\lambda_3 + \lambda_4)} & \text{for } \lambda_5 < 0, \end{cases} \\
\lambda_a &> -2\sqrt{\lambda_1 \lambda_S}, \\
\lambda_b &> \begin{cases} -2\sqrt{\left(\frac{1}{3}\lambda_3 + \lambda_4\right) \lambda_S} & \text{for } \lambda_3 \geq 0, \\ -2\sqrt{(\lambda_3 + \lambda_4) \lambda_S} & \text{for } \lambda_3 < 0, \end{cases} \\
\lambda_S &> 0.
\end{aligned} \tag{6.37}$$

The first three of these constraints are identical to those in the original GM model, while the last three are new.

We note that the full parameter space of the quartic scalar couplings as allowed by perturbative unitarity and the requirement that the scalar potential be bounded from below can be covered by scanning over the following ranges. For the couplings λ_1 – λ_5 , the ranges are the same as in the original GM model [188],

$$\begin{aligned}
\lambda_1 &\in \left(0, \frac{\pi}{3}\right), \quad \lambda_2 \in \left(-\frac{2\pi}{3}, \frac{2\pi}{3}\right), \quad \lambda_3 \in \left(-\frac{\pi}{2}, \frac{3\pi}{5}\right), \\
\lambda_4 &\in \left(-\frac{\pi}{5}, \frac{\pi}{2}\right), \quad \lambda_5 \in \left(-\frac{8\pi}{3}, \frac{8\pi}{3}\right).
\end{aligned} \tag{6.38}$$

For the new couplings λ_a , λ_b , and λ_S in the singlet scalar DM extension of the GM

model, the ranges are,⁵

$$\begin{aligned}\lambda_a &\in \left(-\frac{2\pi(3\sqrt{2}-2)}{7}, 2\pi\right), & \lambda_b &\in \left(-\frac{4\pi}{\sqrt{33}}, \frac{4\pi}{3}\right), \\ \lambda_S &\in \left(0, \frac{2\pi}{3}\right).\end{aligned}\tag{6.39}$$

Within these ranges, the conditions in Eqs. (6.27) and (6.37) must still be applied and any points in violation discarded.

6.3.3 Conditions to avoid alternative minima

Finally we check that the scalar potential does not contain any deeper minima that spontaneously break the custodial symmetry or that give the singlet a vev. The constraints on the parameters required to ensure that the desired electroweak-breaking and custodial SU(2)-preserving minimum is the true global minimum were studied for the original GM model in Ref. [188]. These continue to apply in the singlet extension that we study here and we implement them as follows. Using ζ and ω from Eq. (6.28) and introducing the additional parameters

$$\begin{aligned}\sigma &= \frac{\text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b)(UXU^\dagger)_{ab}}{\text{Tr}(\Phi^\dagger \Phi)[\text{Tr}(X^\dagger X)]^{\frac{1}{2}}}, \\ \rho &= \frac{\text{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}}{[\text{Tr}(X^\dagger X)]^{\frac{3}{2}}},\end{aligned}$$

⁵The upper limits of these ranges come from the unitarity constraints in Eq. (6.27). The upper limit on λ_a comes directly from $|z_a| < 4\pi$. The upper limit on λ_b comes from the upper and lower bounds on λ_S : for large enough λ_b these two bounds meet each other, and the least stringent bound on λ_b comes from taking all other quartic couplings equal to zero in these expressions. The upper limit on λ_S comes directly from the expression in Eq. (6.27), which is least stringent when all other quartic couplings are set to zero.

The lower limit on λ_a comes from an interplay of the bounded-from-below constraint $\lambda_a > -2\sqrt{\lambda_1 \lambda_S}$ in Eq. (6.37) and the upper bound on λ_S from Eq. (6.27) when $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_b = 0$ and $\lambda_S = 2\lambda_1$. The lower limit on λ_b comes from an interplay of the constraint in Eq. (6.37) and the bound on λ_3 and λ_4 from $|x_1^\pm| < 8\pi$ in Eq. (6.27). The least stringent limit occurs when $\lambda_1 = \lambda_2 = \lambda_3 = 0$. The lower limit on λ_S comes trivially from Eq. (6.37).

$$\begin{aligned}
x^2 &= \text{Tr}(\Phi^\dagger \Phi), \\
y^2 &= \text{Tr}(X^\dagger X), \\
z^2 &= S^2,
\end{aligned} \tag{6.40}$$

the scalar potential can be written as

$$\begin{aligned}
V &= \frac{\mu_2^2}{2}x^2 + \frac{\mu_3^2}{2}y^2 + \frac{\mu_S^2}{2}z^2 + (\lambda_2 - \lambda_5\omega)x^2y^2 \\
&\quad + \lambda_ax^2z^2 + \lambda_by^2z^2 + \lambda_1x^4 + (\lambda_3\zeta + \lambda_4)y^4 \\
&\quad + \lambda_Sz^4 - M_1\sigma x^2y - M_2\rho y^3.
\end{aligned} \tag{6.41}$$

The parameters ζ , ω , σ , and ρ capture the dependence on which component(s) of X obtain a vev. The correct custodial $\text{SU}(2)$ -preserving vacuum corresponds to $\zeta = 1/3$, $\omega = 1/2$, $\sigma = \sqrt{3}/4$, and $\rho = 2/\sqrt{3}$ [188]. For a given set of Lagrangian parameters, we check that these values yield the lowest value of the potential V by using the convenient parametrization [188]

$$\begin{aligned}
\zeta &= \frac{1}{2}\sin^4\theta + \cos^4\theta, \\
\omega &= \frac{1}{4}\sin^2\theta + \frac{1}{\sqrt{2}}\sin\theta\cos\theta, \\
\sigma &= \frac{1}{2\sqrt{2}}\sin\theta + \frac{1}{4}\cos\theta, \\
\rho &= 3\sin^2\theta\cos\theta,
\end{aligned} \tag{6.42}$$

and scanning over $\theta \in [0, 2\pi)$.

We then check that the potential does not have any deeper minima in which S gets a vev. If μ_S^2 , λ_a and λ_b are all positive, then S cannot get a vev. We only have to worry about this possibility if one or two of these parameters are negative (all

three cannot be negative because we require $m_S^2 = \mu_S^2 + 2\lambda_a v_\phi^2 + 6\lambda_b v_\chi^2 > 0$). Taking $\partial V/\partial z = 0$ yields two possible extrema,

$$\begin{aligned} z &= 0, \\ z^2 &= -\frac{1}{4\lambda_S}(\mu_S^2 + 2\lambda_a x^2 + 2\lambda_b y^2). \end{aligned} \tag{6.43}$$

We then take $\partial V/\partial x = 0$ and $\partial V/\partial y = 0$, plug in each of the two solutions for z from Eq. (6.43), solve for the possible values of x and y in each case, and then plug these back into V to obtain the depth of the potential at each extremum. Points are discarded if a minimum with $z \neq 0$ is deeper than the desired one with $z = 0$.

6.4 Thermal relic density

We now turn to constraints from the DM relic abundance. We assume that the scalar DM candidate S constitutes all of the DM. We will use the observed relic density to fix a combination of λ_a and λ_b . We show that the direct-detection constraints restrict the DM mass to be near half the Higgs mass around 62 GeV or above approximately 120 GeV. All Feynman rules involving particle species from the original GM model can be found in the appendix of Ref. [188].

6.4.1 Thermal freeze-out

The relic density of S through thermal freeze-out in the early Universe is determined by the annihilation cross section for $SS \rightarrow \text{anything}$. We calculate the thermally averaged cross section as a function of temperature using [145, 147]

$$\langle \sigma_{12 \rightarrow 34} v_{\text{rel}} \rangle = \frac{g_1 g_2 T}{32\pi^4 n_1^{\text{eq}} n_2^{\text{eq}}} \tag{6.44}$$

$$\times \int_{4m_S^2}^{\infty} \sigma_{12 \rightarrow 34} [s - 4m_S^2]^2 \sqrt{s} K_1 \left(\frac{\sqrt{s}}{T} \right) ds,$$

where $s = (p_1 + p_2)^2$ is the usual Mandelstam variable, $g_1 = g_2 \equiv g_S = 1$ is the number of internal degrees of freedom of S , v_{rel} is the relative velocity of particles 1 and 2, and K_1 is the modified Bessel function of the second kind of order 1. We use the thermally averaged total annihilation cross section as input for the usual Boltzmann equation [24, 145, 147]:

$$\frac{dn_S}{dt} + 3Hn_S = -\langle \sigma v_{\text{rel}} \rangle \left[n_S^2 - (n_S^{\text{eq}})^2 \right], \quad (6.45)$$

where H is the Hubble parameter and n_S is the number density of S . Here n_S^{eq} is the equilibrium number density of S and is given by [145, 147]

$$\begin{aligned} n_S^{\text{eq}} &= \frac{g_S}{(2\pi^3)} \int e^{-E_S/T} d^3 p_i \\ &= \frac{g_S m_S^2 T}{2\pi^2} K_2(m_S/T) \end{aligned} \quad (6.46)$$

$$\approx g_S \left(\frac{m_S T}{2\pi} \right)^{3/2} e^{-m_S/T}, \quad (6.47)$$

where K_2 is the modified Bessel function of the second kind of order 2 and where the approximation holds for when $m_S \gg T$. We then solve this equation numerically to obtain the number density today which translates to a value for the relic abundance.

We need to include all final states arising from SS annihilation into SM particles and various other scalar final states appearing in the model. We group them by final state.

6.4.1.1 $f\bar{f}$, VV , and H_3V final states

We begin with the final states for which the SS annihilation proceeds by the s -channel exchange of h and H bosons only (the first diagram in Fig. 6.1). We write this annihilation cross section by incorporating the expression for the SM Higgs decay width, setting the Higgs mass to the center of mass energy. For decays to $AB = f\bar{f}$ or VV , the resulting expression is:

$$\sigma v_{\text{rel}} = \frac{2}{\sqrt{s}} \left| \frac{g_{SSh}g_{hAB}}{s - m_h^2 - im_h\Gamma_h} + \frac{g_{SSH}g_{HAB}}{s - m_H^2 - im_H\Gamma_H} \right|^2 \frac{1}{g_{h_{SM}AB}^2} \times \Gamma(m_{h_{SM}} = \sqrt{s}, h_{SM} \rightarrow AB), \quad (6.48)$$

where Γ is the decay width of a SM Higgs boson with a mass of $\sqrt{s}$ into the final state AB . This decay width is calculated using the usual SM formulas. Here also, Γ_h is the width of the Higgs boson, and Γ_H is the width of H .

We must also include the final state with one H_3 scalar and one vector boson. The cross section of this process when $\sqrt{s} > m_3 + m_V$ is

$$\sigma v_{\text{rel}} = \frac{(s^2 - 2s(m_3^2 + m_V^2) + (m_3^2 - m_V^2)^2)^{\frac{3}{2}}}{8\pi s^2 m_W^2} \times \left| \frac{g_{SSh}g_{hH_3V}}{s - m_h^2 - im_h\Gamma_h} + \frac{g_{SSH}g_{HH_3V}}{s - m_H^2 - im_H\Gamma_H} \right|^2. \quad (6.49)$$

When $\sqrt{s} < m_3 + m_V$, we include the off-shell process $SS \rightarrow V^*H_3$, whose cross section is given by

$$\sigma v_{\text{rel}} = \left| \frac{g_{SSh}g_{hH_3V}}{s - m_h^2 - im_h\Gamma_h} + \frac{g_{SSH}g_{HH_3V}}{s - m_H^2 - im_H\Gamma_H} \right|^2 \delta_V \frac{3m_V^2}{8\pi^3 v^2} G_{ij}, \quad (6.50)$$

where [201–203]

Figure 6.1: Feynman diagrams for SS annihilation. In the first, the final state can be any allowed pair of SM particles or GM scalars. However, in the second and third diagrams, only scalar final states can appear. Crossed diagrams (not shown) are also included in the calculation.

$$G_{ij} = \frac{1}{4} \left[2(-1 + k_j - k_i) \sqrt{\lambda_{ij}} \left(\frac{\pi}{2} + \arctan \left(\frac{k_j(1 - k_j + k_i) - \lambda_{ij}}{(1 - k_i) \sqrt{\lambda_{ij}}} \right) \right) + (\lambda_{ij} - 2k_i) \log k_i + \frac{1}{3}(1 - k_i) \left(5(1 + k_i) - 4k_j + \frac{2\lambda_{ij}}{k_j} \right) \right], \quad (6.51)$$

with

$$\delta_W = \frac{3}{2}, \quad \delta_Z = 3 \left(\frac{7}{12} - \frac{10}{9} s_W^2 + \frac{40}{27} s_W^4 \right). \quad (6.52)$$

Here $k_i = m_3^2/s$, $k_j = m_V^2/s$, s_W is the sine of the weak mixing angle, and $\lambda_{ij} = -1 + 2k_i + 2k_j - (k_i - k_j)^2$.

6.4.1.2 Final states involving H_3 or H_5 pairs

We now compute the SS annihilation cross sections into final states that consist of neutral or charged H_3 and H_5 scalars. The possible final states considered here are

$$(H_3^0, H_3^0), (H_3^+, H_3^-), (H_5^0, H_5^0), (H_5^+, H_5^-), (H_5^{++}, H_5^{--}). \quad (6.53)$$

Since the two particles in each of these final states have the same mass, we will label it as m_i . Annihilation into these final states proceeds via s -channel and four-point diagrams (the first two diagrams in Fig. 6.1). The cross section for final-state particles h_1 and h_2 is given by

$$\sigma v_{\text{rel}} = \delta_{12} \frac{(s - 4m_{h_1}^2)^{\frac{1}{2}}}{16\pi s^{\frac{3}{2}}} \times \left| g_{SSH_1 h_2} + \frac{g_{SSH} g_{hh_1 h_2}}{s - m_h^2 - im_h \Gamma_h} + \frac{g_{SSH} g_{Hh_1 h_2}}{s - m_H^2 - im_h \Gamma_h} \right|^2, \quad (6.54)$$

where $\delta_{12} = 1/2$ for identical final-state particles $h_1 = h_2$ and $\delta_{12} = 1$ for nonidentical final-state particles.

6.4.1.3 (h, H) , (h, h) , and (H, H) final states

The cross sections with (h, H) , (h, h) , or (H, H) in the final state proceed via s -channel, four-point, and t - and u -channel diagrams (all the diagrams in Fig. 6.1 plus the crossed diagram). For nonidentical h_i, h_j in the final state, we obtain

$$\sigma v_{\text{rel}} = \frac{\sqrt{s^2 + (m_{h_j}^2 - m_{h_i}^2)^2 - 2s(m_{h_j}^2 + m_{h_i}^2)}}{16\pi s^2} g(s), \quad (6.55)$$

and for identical particles in the final state, we obtain

$$\sigma v_{\text{rel}} = \frac{\sqrt{s - 4m_{h_i}^2}}{32\pi s^{3/2}} g(s), \quad (6.56)$$

where g , a , b , c , and d are defined as follows:

$$g(s) = 2a^2 + \frac{b^2}{c^2(c^2 - d^2)} + \frac{b(b - 4ac^2)}{c^3 d} \tanh^{-1} \left(\frac{d}{c} \right), \quad (6.57)$$

$$a = g_{SSH_i h_j} + \sum_k \frac{g_{SSH_k} g_{h_k h_i h_j}}{s - m_{h_k}^2 - i m_{h_k} \Gamma_{h_k}}, \quad (6.58)$$

$$b = 4g_{SSH_i} g_{SSH_j} (s - (m_{h_i}^2 + m_{h_j}^2)), \quad (6.59)$$

$$c^2 = (s - m_{h_j}^2 - m_{h_i}^2)^2, \quad (6.60)$$

$$d^2 = \left(1 - \frac{4m_S^2}{s}\right) \left((s - m_{h_j}^2 + m_{h_i}^2)^2 - 4sm_{h_i}^2\right), \quad (6.61)$$

where the sum in a runs over $h_k = h, H$.

6.4.2 Imposing relic density

Here we give details on how we impose the relic density as a constraint on λ_a and λ_b . We first note that the thermally averaged cross section is a strictly increasing function of $|\lambda_a|$ and $|\lambda_b|$. In our numerical scans, we would like to be able to randomly select a particular linear combination of the couplings λ_a and λ_b and then scale them both until the correct relic density is obtained. After generating a scan point in the original GM model (see next section for details), we select the values of λ_a , λ_b , and μ_S^2 as follows:

- First, generate a random angle $\theta_\lambda \in [-\frac{\pi}{2}, \frac{\pi}{2}]$;
- Randomly select either the positive or negative solution for λ_a ;
- Set $\lambda_b = \lambda_a \tan \theta_\lambda$;
- Generate a random mass $m_S > 0$ GeV;
- Find a value of λ_a that yields a relic density of $0.1064 \leq \Omega_{DM} h^2 \leq 0.1176$ [152];
- Once the value of λ_a is found we can find λ_b and, in turn, μ_S^2 using Eq. (6.14).

The first three steps allow us to select a particular linear combination of λ_a and λ_b . Generating m_S directly lets us avoid unphysical negative m_S^2 values regardless of the actual values of λ_a and λ_b . Finally, numerically searching for the correct value of λ_a is straightforward because σv_{rel} is an increasing function of λ_a and $\sigma v_{\text{rel}} \rightarrow \infty$ when $\lambda_a \rightarrow \infty$ and $\sigma v_{\text{rel}} \rightarrow 0$ when $\lambda_a \rightarrow 0$.

6.5 Numerical Scan Procedure

To map out the allowed parameter space, we perform numerical scans. In these scans, we start by imposing the theoretical constraints and the relic density constraint. We then check whether the points pass the remaining experimental constraints. In the plots that follow, points that pass the experimental constraints will be blue, while points that fail at least one constraint will be red.

For the dimensionful parameters, the ranges we scan over are:

- $-4 \times 10^4 \text{ GeV}^2 \leq \mu_3^2 \leq 10^6 \text{ GeV}^2$;
- $0 \leq M_1 \leq \max(3500 \text{ GeV}, 3.5\sqrt{|\mu_3^2|})$;
- $|M_2| \leq \max(250 \text{ GeV}, 1.3\sqrt{|\mu_3^2|})$, with either sign allowed;
- $0 \leq m_S \leq 1000 \text{ GeV}$, $0 \leq m_S \leq 125 \text{ GeV}$, or $56 \leq m_S \leq 63 \text{ GeV}$ (see text below for explanation of these three regions).

The ranges for M_1 and M_2 are chosen to minimize the number of points generated which fail the theory constraints while still scanning the whole parameter space. The mass parameters μ_3^2 and m_S do not have upper bounds so we impose arbitrary bounds for the purpose of the scan. We perform a scan with $0 \leq m_S \leq 1000 \text{ GeV}$ in order to obtain a general picture of the parameter space, one with $0 \leq m_S \leq 125 \text{ GeV}$ in

order to obtain higher statistics in the interesting low- m_S region, and finally a smaller dedicated scan with $56 \leq m_S \leq 63$ GeV to further investigate the Higgs pole region.

From these values, we calculate λ_1 , μ_2^2 , and μ_S^2 , and all the masses and couplings, and then use the relic density to fix a random linear combination of λ_a and λ_b .

6.6 Direct and indirect collider constraints on the GM model

Very low masses for $H_3^{0,\pm}$ and $H_5^{0,\pm,\pm\pm}$ can have a substantial effect on the DM relic abundance through annihilations into pairs of these scalars. We constrain these masses using direct experimental search limits as follows. LHC limits on anomalous like-sign dimuon production [204] set a lower bound on the mass of a doubly charged scalar decaying to like-sign W boson pairs. This was studied in Ref. [205] for the Higgs triplet model [206–210] and recast into the GM model in Ref. [169]. This yields a lower bound $m_5 \geq 76$ GeV, so long as H_3 is heavier than H_5 so that the decays $H_5^{\pm\pm} \rightarrow W^\pm H_3^\pm$ do not compete with the decays into like-sign W pairs. Searches for a charged Higgs boson at the CERN Large Electron-Positron collider [211] exclude charged Higgs masses below 78 GeV, assuming that the charged Higgs decays entirely into a combination of $\tau\nu$ and cs final states. This limit can be applied to $H_3^\pm$ so long as decays $H_3^\pm \rightarrow W^\pm H_5^0, ZH_5^\pm$ do not compete with the decays to fermions. This holds when H_5 is heavier than H_3 . We therefore impose the lower bounds

$$m_3 \geq 76 \text{ GeV}, \quad m_5 \geq 76 \text{ GeV}. \quad (6.62)$$

Low $H_3^\pm$ masses can also be constrained from their effect on the loop-induced decay of $b \rightarrow s\gamma$. We use the “loose” constraint determined for the GM model

Figure 6.2: Feynman diagram for direct-detection via t -channel Higgs exchange in $SN \rightarrow SN$ scattering. There is a second diagram in which h is replaced by H .

in Ref. [192], which is based on an experimental average from the Heavy Flavour Averaging Group [212, 213] and a theoretical prediction from the public code SuperIso v3.3 [214]. The $b \rightarrow s\gamma$ constraint sets a maximum value of v_χ as a function of m_3 . Although this could potentially be constraining, all points in our numerical scan satisfied this constraint.

6.7 Constraints from DM

6.7.1 DM direct-detection

When a DM particle is in close proximity to a nucleon, there may be a scattering via the t -exchange of a Higgs boson. This transfer of momentum can be detected from the nucleon recoil so that experimental limits can be used to constrain our model. In our model, this process proceeds via exchange of a virtual h or H as shown in Fig. 6.2.

The spin-independent cross section for the scattering of a scalar DM particle S off of a single nucleon in this model is given by

$$\sigma = \left(\frac{g_{SSh}c_\alpha}{m_h^2 c_H} + \frac{g_{SSH}c_\alpha}{m_H^2 c_H} \right)^2 \frac{f_N^2 m_N^4}{4\pi(m_N + m_S)^2 v^2}, \quad (6.63)$$

where we neglect the momentum transfer relative to the h or H mass, c_H was defined in Eq. (6.10), $c_\alpha \equiv \cos \alpha$, $s_\alpha \equiv \sin \alpha$, and f_N is the nucleon vertex factor [82],

$$f_N = \sum_q f_q = \sum_q \frac{m_q}{m_N} \langle N | \bar{q}q | N \rangle, \quad (6.64)$$

where the sum is over all quark flavours, and the Feynman rule for the Higgs-nucleon vertex is $-if_N m_N/v$. We follow Ref. [82] in using $f_N = 0.30 \pm 0.03$ and $m_N = (m_n + m_p)/2 = 938.95$ MeV.

In Fig. 6.3 we illustrate the effect of the direct-detection constraints on our model. The scan points shown are those that satisfy the theoretical constraints and yield the correct DM relic abundance. The blue points satisfy the constraints from the DM direct-detection experiments as well as limits from indirect detection (see next subsection), while the red points fail those constraints. The current most stringent direct-detection cross section limit comes from the LUX experiment [200] and is shown as the solid magenta line in Fig. 6.3. As can be seen, this constraint is responsible for excluding the great majority of the red (excluded) points in our scan, except for a small collection of points on the lower-mass side of the Higgs pole at $m_h/2 = 62.5$ GeV. We also show the projected limits from DEAP-3600 [215] (dotted green) and XENON1T [216] (dotted brown), as well as the “neutrino floor” (the yellow shaded region) below which coherent neutrino scattering becomes an irreducible background to the DM direct-detection experiments [217].

6.7.2 DM indirect detection

Dwarf spheroidal satellite galaxies (dSphs) are typically DM dominated so are a good place to study DM. The Fermi Collaboration has acquired 6 yr worth of data

Figure 6.3: Direct-detection cross section as a function of m_S . The top plot is an enlargement of the low- m_S region while the bottom plot shows the full m_S range scanned. The red (grey) points are eliminated by direct or indirect DM detection measurements and the blue (black) points are the remaining ones that are allowed. On the right side of the figures, starting at the top are the exclusion limits from LUX (solid magenta) [200], as well as projected sensitivities of DEAP-3600 (dashed green) [215], and XENON1T (dashed brown) [216]. The yellow shaded region below the lowest line is the coherent neutrino scattering background (the neutrino floor) [217]. Data files were taken from the DMTools website [218].

observing 15 dSphs and have released bounds for WIMP DM annihilation based on their gamma ray flux. They considered the following representative final states for the DM annihilation: e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$, $u\bar{u}$, $b\bar{b}$, and W^+W^- [18].

We can translate the Fermi bounds into constraints on our model by considering the branching ratio of the singlet annihilation to these final states. Although all of the final states are considered in our analysis, the strongest constraint comes from the $b\bar{b}$ final state for singlet scalar masses just below half the h mass. Figure 6.4 shows the results of applying the $b\bar{b}$ constraint to the scan points. As can be seen, there is a sharp dip in the cross section followed by a sharp peak. The dip can be understood as coming from having to lower the values of λ_a and λ_b near the Higgs resonance, therefore lowering the g_{SSh} coupling, in order to obtain the correct relic density. As the singlet mass approaches the Higgs pole, the thermal distribution during freeze-out pushes the center of mass energy above the pole. This results in increased values of λ_a and λ_b to obtain the correct relic density. However, since the temperature of DM is much lower today (we use the approximation that $v_{\text{rel}} = 0$), the increased coupling appears at a center of mass energy closer to the Higgs resonance and creates this peak. The indirect detection constraint thereby excludes a small collection of points on the heavier side of the h pole dip in Fig. 6.3 that are not yet excluded by direct detection.

Figure 6.4: Present-day annihilation cross section for $SS \rightarrow b\bar{b}$ as a function of m_S . The blue (black) points are allowed while the red (grey) points are excluded by direct-detection or dwarf spheroidal galaxy constraints. Points above the blue line are excluded by the dSphs $b\bar{b}$ constraint from the Fermi satellite [18]. Fermi gives the thermally averaged cross section while we use the low velocity approximation and take $s = 4m_S^2$.

6.8 Constraints from Higgs boson properties

6.8.1 Higgs invisible width

When $m_S < m_{h,H}/2$, the decay of the Higgs boson to two DM candidates is kinematically accessible. For convenience we define:

$$\begin{aligned}\kappa_f^h &= \frac{g_{hff}}{g_{hff}^{SM}} = \frac{c_\alpha}{c_H}, \\ \kappa_V^h &= \frac{g_{hVV}}{g_{hVV}^{SM}} = c_\alpha s_H - \sqrt{\frac{8}{3}} s_\alpha s_H, \\ \kappa_\gamma^h &= \left[\frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma^{SM}(h \rightarrow \gamma\gamma)} \right]^{1/2}.\end{aligned}\tag{6.65}$$

Note that κ_f^h is the same for all fermions and κ_V^h is the same for $V = Z$ and $W^\pm$. $\Gamma(h \rightarrow \gamma\gamma)$ receives contributions from H_3^+ , H_5^+ , and H_5^{++} in addition to the modified $ht\bar{t}$ and hWW couplings. The expression for the width the Higgs decaying to two DM particles is

$$\Gamma(h \rightarrow \text{inv}) = \frac{g_{SSh}^2}{32m_h^2\pi} \sqrt{m_h^2 - 4m_S^2}.\tag{6.66}$$

The most stringent LHC constraint on invisible Higgs decay comes from Higgs production in vector boson fusion (VBF). To compare with the experiment, we therefore consider the ratio [219]

$$\frac{\sigma_{VBF} \text{BR}_{\text{inv}}}{\sigma_{VBF}^{SM}} = \frac{(\kappa_V^h)^2 \Gamma(h \rightarrow \text{inv})}{\Gamma_{\text{tot}}} < 0.29,\tag{6.67}$$

written in terms of the VBF production cross section and the invisible branching ratio. In the total width Γ_{tot} of h we include decays to $b\bar{b}$, $c\bar{c}$, $\tau\tau$, gg , WW^* , ZZ^* , $\gamma\gamma$, and SS as computed above.

Figure 6.5: Cross section times branching ratio for h production in vector boson fusion followed by invisible decays to SS , normalized to the SM cross section, as a function of m_S . The blue (black) points are allowed while the red (grey) points are excluded by direct detection or dwarf spheroidal galaxy constraints. Points above the blue line are excluded by the ATLAS limit on $\sigma_{VBF}BR_{\text{inv}}/\sigma_{VBF}^{SM}$ [219] .

The ratio in Eq. (6.67) is shown in Fig. 6.5, plotted against m_S in the kinematically allowed region. The experimental constraint of $\sigma_{VBF}BR_{\text{inv}}/\sigma_{VBF}^{SM} < 0.29$ [219] is shown as the horizontal blue line. The bound from invisible Higgs decays is currently not as strong as the constraints from direct detection of DM.

6.8.2 Higgs couplings and signal strength

We finally apply the latest combined measurements of Higgs couplings from CMS and ATLAS from run 1 of the LHC [220] to our model. In this section we discard the points that are excluded by DM direct-detection or indirect detection constraints. We will find that the Higgs coupling measurements exclude a significant fraction of the remaining points, in particular those for which the h coupling to fermion or vector boson pairs is sufficiently different from the SM.

We write the cross sections and branching ratios in terms of the appropriate SM values and the κ factors defined in Eq. (6.65) as follows:

$$\begin{aligned}\sigma(AB \rightarrow h) &= (\kappa_{AB}^h)^2 \sigma^{SM}(AB \rightarrow h), \\ \text{BR}(h \rightarrow AB) &= (\kappa_{AB}^h)^2 \frac{\Gamma_{\text{tot}}^{SM}}{\Gamma_{\text{tot}}} \text{BR}^{SM}(h \rightarrow AB),\end{aligned}\tag{6.68}$$

where $\kappa_g^h = \kappa_f^h$.

We compute a χ^2 using the ATLAS+CMS combined results for the LHC run 1 Higgs properties from Table 9 and the correlation matrix from Fig. 28 of Ref. [220]. The inputs we use in this analysis are: $\sigma(gg \rightarrow h \rightarrow ZZ)$, $\sigma_{\text{VBF}}/\sigma_{ggF}$, σ_{Wh}/σ_{ggF} , σ_{Zh}/σ_{ggF} , $\sigma_{tth}/\sigma_{ggF}$, $\text{BR}^{WW}/\text{BR}^{ZZ}$, $\text{BR}^{\gamma\gamma}/\text{BR}^{ZZ}$, $\text{BR}^{\tau\tau}/\text{BR}^{ZZ}$, and $\text{BR}^{bb}/\text{BR}^{ZZ}$. We start by symmetrizing the uncertainties for a given observable by taking the root mean square of the asymmetric uncertainties. We then construct the variance matrix V from these symmetrized uncertainties and the correlation matrix and define:

$$\chi^2 = (\mathbf{x} - \mathbf{y})^T V^{-1} (\mathbf{x} - \mathbf{y}),\tag{6.69}$$

where $\mathbf{x}$ is a vector of the experimental best fit values and $\mathbf{y}$ is a vector of calculated values using the kappas and the SM predictions from Table 9 of Ref. [220]. Using the SM predictions for $\mathbf{y}$ we get:

$$\chi_{SM}^2 = 30.0448.\tag{6.70}$$

We will consider a point in our model to be consistent with the experimental measurements of these observables if:

$$\Delta\chi^2 \equiv |\chi^2 - \chi_{SM}^2| < 4.$$

Figure 6.6: Couplings of h to fermions (κ_f^h) and W and Z bosons (κ_V^h) normalized to their SM values, for only the points allowed by the DM direct- and indirect detection constraints. The dark blue (black) points satisfy the $\Delta\chi^2 < 4$ condition for the Higgs coupling measurements from the LHC run 1 data [220], while the red (grey) points fail this condition.

We apply the constraints from the Higgs coupling fit only to the points that have passed all previous DM constraints. Figure 6.6 shows the points that pass (blue/black) and those that fail (red/grey) the χ^2 constraint from Higgs couplings in the κ_V^h - κ_f^h plane. The Higgs coupling measurements exclude points for which κ_V^h or κ_f^h deviate too much from their SM value of 1.

Of particular interest for Higgs phenomenology is the case where the singlet is lighter than half the Higgs mass. This allows the Higgs to decay to a pair of singlets which would then escape the detector. Figure 6.7 shows the branching ratio of $h \rightarrow SS$ as a function of κ_V^h for the points that passed all previous constraints and have a singlet mass less than half the Higgs mass.

The only observable in the χ^2 analysis that is sensitive to the total decay width of the Higgs boson is the $\sigma(gg \rightarrow h \rightarrow ZZ)$ cross section, because the total width

Figure 6.7: Branching ratio of $h \rightarrow SS$ as a function of κ_V^h , for only the points allowed by the DM direct- and indirect detection constraints and for which $m_S < m_h/2$. The dark blue (black) points satisfy the $\Delta\chi^2 < 4$ condition for the Higgs coupling measurements from the LHC run 1 data [220], while the red (grey) points fail this condition.

cancels out in all the other inputs. In particular, we have

$$\frac{\sigma(gg \rightarrow h \rightarrow ZZ)}{\sigma^{SM}(gg \rightarrow h \rightarrow ZZ)} = (\kappa_f^h \kappa_V^h)^2 \frac{\Gamma_{\text{tot}}^{SM}}{\Gamma_{\text{tot}}}. \quad (6.71)$$

This observable allows us to potentially distinguish between our model and the original GM model without the scalar singlet DM candidate. In Fig. 6.8 we plot this observable versus κ_V^h for the points that survive the $\Delta\chi^2 < 4$ constraint and for which the mass of the singlet is less than half the Higgs mass, so that $h \rightarrow SS$ is kinematically allowed. These are the blue (black) points. We then take the same points, set $\mu_s = \lambda_a = \lambda_b = \lambda_S = 0$ while keeping the other Lagrangian parameters fixed, and remove the singlet from the theory. These points are plotted in red (gray). For these points the couplings κ_f^h , κ_V^h , and κ_γ^h are the same as in the full model, but $\text{BR}(h \rightarrow SS)$ (and its contribution to the Higgs total width) is eliminated.

Figure 6.8: Prediction for the cross section $\sigma(gg \rightarrow h \rightarrow ZZ)$ normalized to its SM value as a function of κ_V^f . The dark blue (black) points are the points that pass the DM direct- and indirect detection constraints and satisfy the $\Delta\chi^2 < 4$ constraint from the Higgs coupling measurements. The red (gray) points are the same parameter points but with the $h \rightarrow SS$ decays switched off (see the text for details) –these correspond to the predictions in the original GM model without the singlet scalar.

As can be seen, for the original GM model without the singlet, the red points fall roughly along a line due to the correlation between κ_V^h and $\text{BR}(h \rightarrow ZZ)$ after the rest of the Higgs coupling measurements are imposed. For the full GM model with the singlet scalar DM candidate, we see that the contribution from the competing $h \rightarrow SS$ decay mode is not large enough to significantly affect $\text{BR}(h \rightarrow ZZ)$ and so we see similar values of $\sigma(gg \rightarrow h \rightarrow ZZ)/\text{SM}$.

6.9 Conclusions

In this chapter we studied the addition of a scalar DM candidate to the Georgi-Machacek model. This provides a concrete implementation of a scenario in which the Higgs couplings to vector boson and fermion pairs can be enhanced while a new, non-SM decay mode is also present, thereby allowing an exploration of the interplay of Higgs production and decay constraints. We showed that the DM candidate in this model can be made to respect all current constraints. However, for the region below half the Higgs mass, we do not see a significant branching ratio from the Higgs to the DM.

The model consists of the Georgi-Machacek model with the addition of a real singlet which has a Z_2 symmetry to make it stable. We first studied the theoretical constraints on the new parameters by imposing tree-level unitarity in $2 \rightarrow 2$ scalar scattering amplitudes, requiring that the potential is bounded from below, and requiring that deeper custodial $\text{SU}(2)$ -violating minima are absent. We found that we could translate all the constraints from the original GM model to our extended model and simply add a few new constraints on the new Lagrangian parameters.

We performed a numerical scan over the Lagrangian parameters, imposing the theoretical constraints and requiring that the singlet scalar accounts for all of the

DM in the Universe through thermal freeze-out. We identified the parameter regions that satisfy the constraints from DM direct-detection searches as well as the indirect constraints from gamma ray measurements of dwarf spheroidal galaxies. This constrained the DM mass to be either near the Higgs pole for resonant annihilation (57–62 GeV) or sparsely distributed above about 120 GeV. We also saw that for parameter values where the DM mass was near the Higgs pole, we could not attain a sizeable branching ratio for $h \rightarrow SS$. We also saw that the current limit on the Higgs invisible decays does not further constrain the model.

We finally studied the constraints from the LHC run 1 Higgs coupling measurements. While these measurements further constrain the parameter space, they do so mostly by constraining the $h f \bar{f}$, $h V V$, and $h \gamma \gamma$ couplings. The Higgs coupling measurements are not yet precise enough to be sensitive to the modification of signal rates by the presence of the $h \rightarrow SS$ decay mode, so that the constraints from Higgs measurements are the same so far as they would be in the original GM model without the singlet scalar.

The allowed region of parameter space that we identified can be further probed in the future by the next generation of DM direct-detection experiments.

Chapter 7

Conclusion

As physicists, it is our job to model the Universe as best we can. Not only would we want to understand what will happen given a scenario, but be able to have a deeper understanding of why the laws of physics are what they are. This could eventually come with a grand unified theory, but we are not there yet and our current models only show us some of the deep secrets of the Universe. The existence of DM suggests additional particles, which if understood could hint towards these more complete models of physics.

In this thesis, we considered many different ways in which DM physics could manifest. We started by taking a look at how modifying the expansion rate of the Universe while DM was undergoing freeze-out could lead to parametrically different values for the cross-section or the mass. These types of studies are important because it is typical in physics to try to solve two problems at once. For example, someone may try to solve a problem in early Universe cosmology and simultaneously explain DM through a freeze-out mechanism. We found that adding additional contributions to the energy density slowed the expansion rate of the Universe, giving more time to the DM to annihilate as the temperature cooled slower. This implies that either a

smaller cross-section is required or a larger mass.

In the following chapter, we considered effects from DM self-interactions. There are many astronomical observations of colliding clusters from which we can study the effects of large clumps of DM passing through each other. From these, we can put limits on the interaction cross-section based on the deformation of the DM halos which are not accounted for by gravity. This translates quite naturally to particle physics models. We considered a real singlet scalar extension of the SM as the simplest extension and showed that the model is viable if the mass of the scalar was less than about 0.1 GeV and underwent *freeze-in*. We were able to salvage the freeze-out scenario by extending the scalar sector by an additional 2HDM as long as flavour conservation was achieved by Yukawa alignment, although the result was still quite fine-tuned.

Next, we discussed the possibility that DM could be composed of multiple DM species. Having more than one species significantly changes the experimental bounds since having more species allows us to "hide" some or all of the DM into species that interact very little with the SM. We considered an additional SU(3) gauge group with two complex scalar triplets and found that the scalars are more easily detectable using indirect detection, while vectors were more easily found using direct-detection. In this model, we could have between four and seven stable DM species. We found that although in most cases, the lightest DM species will be the dominant contribution to the relic density, this is not always the case and a full calculation involving the densities of all the species provides a much more accurate answer.

Finally, we looked at the possibility that the Higgs boson could be hiding the DM. We considered an extension of the SM where the couplings of the Higgs boson to vector bosons was enhanced, which would result in a larger production rate through

VBF at the LHC. We added a real singlet scalar as a DM candidate and looked at different mass ranges. In particular, if the Higgs boson could decay invisibly to a pair of these scalars, then the enhanced production rate would not be detected. We found that the only place this could happen is if we tune the mass of the scalar to be very close to half the Higgs boson mass because this is the only region where we require a small coupling to obtain the correct relic density. At values of the DM mass larger than about 200 GeV, there were no constraints.

Throughout this thesis, we have seen the Higgs boson play a significant role in how the SM talks to the DM. This is well motivated by the fact that the Higgs boson is responsible for giving mass to the SM particles, with the exception of neutrinos, so it is not outrageous to think that the Higgs boson could play a part in giving mass to the DM. If this were the case, then the Higgs boson would also interact with the DM. Such a portal would lead to a fundamental connection between Higgs physics and DM physics. This kind of connection can lead us down a rabbit hole where the Higgs boson or DM introduces us to a wide range of new and exciting physics and phenomenology.

Appendix A

Derivation of Friedmann equations

In this appendix, we derive the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. First, let $g_{\mu\nu}$ be the metric. We define the Christoffel symbols by:

$$\Gamma^\sigma_{\mu\alpha} = \frac{1}{2}g^{\sigma\beta}[g_{\mu\beta,\alpha} + g_{\beta\alpha,\mu} - g_{\mu\alpha,\beta}] \quad (\text{A.1})$$

These come in in the definition of the covariant derivative: they take into account the change in a quantity due to the curvature of the background space. Next, we define the Riemann curvature tensor, the Ricci curvature tensor, and the scalar curvature:

$$R^\mu_{\lambda\nu\beta} = \Gamma^\mu_{\lambda\beta,\nu} - \Gamma^\mu_{\lambda\nu,\beta} + \Gamma^\mu_{\sigma\nu}\Gamma^\sigma_{\lambda\beta} - \Gamma^\mu_{\sigma\beta}\Gamma^\sigma_{\lambda\nu}, \quad (\text{A.2})$$

$$R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}, \quad (\text{A.3})$$

$$R = g^{\mu\nu}R_{\mu\nu}. \quad (\text{A.4})$$

From the general theory of relativity, we can relate these to the stress energy tensor:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}. \quad (\text{A.5})$$

Where G is Newton's constant and Λ is the cosmological constant.

When deriving the FLRW metric, we start by making some simplifying assumptions about the Universe:

- The Universe is homogeneous on large scales.
- The Universe is isotropic on large scales.

These conditions imply that the spatial part of the metric be spherically symmetric.

It also implies that $g_{i0} = g_{0i} = 0$. This gives us the metric:

$$ds^2 = dt^2 - a^2 \left(\frac{dr^2}{1 - \frac{r^2}{k^2}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right), \quad (\text{A.6})$$

where a is an arbitrary time dependent function, and k describes the curvature of the Universe: $k^2 \rightarrow \infty$ describes a flat Universe, $k^2 > 1$ describes a positively curved Universe, and $k^2 < 0$ describes a negatively curved Universe. With coordinates $\xi^\mu = (t, r, \theta, \phi)$, the metric tensor is written as:

$$g_{\mu\nu} = \text{diag} \left(1, -\frac{a^2}{1 - \frac{r^2}{k^2}}, -a^2 r^2, -a^2 r^2 \sin^2 \theta \right). \quad (\text{A.7})$$

From this, we can calculate the Christoffel symbols and the various curvature tensors.

The non-zero elements of the Christoffel symbols are given by:

$$\Gamma^0_{11} = \frac{a\dot{a}}{1 - \frac{r^2}{k^2}} \quad (\text{A.8})$$

$$\Gamma^0_{22} = r^2 a \dot{a} \quad (\text{A.9})$$

$$\Gamma^0_{33} = r^2 a \dot{a} \sin^2 \theta \quad (\text{A.10})$$

$$\Gamma^1_{01} = \Gamma^1_{10} = \Gamma^2_{02} = \Gamma^2_{20} = \Gamma^3_{03} = \Gamma^3_{30} = \frac{\dot{a}}{a} \quad (\text{A.11})$$

$$\Gamma^1_{11} = \frac{r}{k^2 \left(1 - \frac{r^2}{k^2}\right)} \quad (\text{A.12})$$

$$\Gamma^1_{22} = -r \left(1 - \frac{r^2}{k^2}\right) \quad (\text{A.13})$$

$$\Gamma^1_{33} = -r \left(1 - \frac{r^2}{k^2}\right) \sin^2 \theta \quad (\text{A.14})$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \quad (\text{A.15})$$

$$\Gamma^2_{33} = -\cos \theta \sin \theta \quad (\text{A.16})$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot \theta \quad (\text{A.17})$$

The non-zero components of the Riemann tensor are given by:

$$R^1_{010} = R^2_{020} = R^3_{030} = -\frac{\ddot{a}}{a}, \quad (\text{A.18})$$

$$R^0_{101} = \frac{a\ddot{a}}{1 - \frac{r^2}{k^2}}, \quad (\text{A.19})$$

$$R^0_{202} = r^2 a\ddot{a}, \quad (\text{A.20})$$

$$R^0_{303} = r^2 a\ddot{a} \sin^2 \theta, \quad (\text{A.21})$$

$$R^1_{212} = \frac{r^2}{k^2} + r^2 \dot{a}^2, \quad (\text{A.22})$$

$$R^1_{313} = \frac{r^2}{k^2} \sin^2 \theta + r^2 \sin^2 \theta \dot{a}^2, \quad (\text{A.23})$$

$$R^2_{121} = R^3_{131} = \frac{1}{k^2 \left(1 - \frac{r^2}{k^2}\right)} + \frac{\dot{a}^2}{1 - \frac{r^2}{k^2}}, \quad (\text{A.24})$$

$$R^2_{323} = \frac{r^2}{k^2} \sin^2 \theta + r^2 \dot{a}^2 \sin^2 \theta, \quad (\text{A.25})$$

$$R^3_{232} = \frac{r^2}{k^2} + r^2 \dot{a}^2, \quad (\text{A.26})$$

$$R^\mu_{\nu\alpha\beta} = -R^\mu_{\nu\beta\alpha}. \quad (\text{A.27})$$

The Ricci tensor is given by:

$$R_{\mu\nu} = \text{diag} \left(-3\frac{\ddot{a}}{a}, \frac{1}{1 - \frac{r^2}{k^2}} \left[a\ddot{a} + \frac{1}{k^2} + \dot{a}^2 \right], \right. \quad (\text{A.28})$$

$$\left. r^2 \left[2a\ddot{a} + \frac{2}{k^2} + \dot{a}^2 \right], r^2 \sin^2 \theta \left[a\ddot{a} + \frac{2}{k^2} + 2\dot{a}^2 \right] \right), \quad (\text{A.29})$$

and the scalar curvature is given by:

$$R = -6 \left(\frac{1}{k^2 a^2} + \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right). \quad (\text{A.30})$$

Finally, the left hand side of Eq. [A.5](#) is given by:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = \text{diag} \left(-\Lambda + \frac{3}{k^2 a^2} + 3\frac{\dot{a}^2}{a^2}, -\frac{1}{1 - \frac{r^2}{k^2}} \left[\frac{1}{k^2} - \Lambda a^2 + \dot{a}^2 + 2a\ddot{a} \right] \right. \quad (\text{A.31})$$

$$\left. -r^2 \left[\frac{1}{k^2} - \Lambda a^2 + \dot{a}^2 + 2a\ddot{a} \right], -r^2 \sin^2 \theta \left[\frac{1}{k^2} - \Lambda a^2 + \dot{a}^2 + 2a\ddot{a} \right] \right). \quad (\text{A.32})$$

Note that this is none zero as long as $a > 0$. This means that the Universe expanding can be seen as a curvature of the Universe. This can be seen intuitively by imagining two photons moving parallel to each other. As the Universe between them expands, they seem to veer off away from each other.

We will now model the stuff in the Universe as a perfect fluid with a pressure p and energy density ρ . The resulting stress-energy tensor is:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \quad (\text{A.33})$$

where u_μ is the four velocity of the fluid. Because we are assuming the Universe is

homogeneous and isotropic, we can go to the frame where the fluid is at rest. This gives:

$$T_{\mu\nu} = \text{diag} \left(\rho, p \frac{a^2}{1 - \frac{r^2}{k^2}}, pa^2 r^2, pa^2 r^2 \sin^2 \theta \right).$$

Using Eq. A.5 and looking at the individual components, we find the two independent equations:

$$\frac{3}{k^2 a^2} + 3 \frac{\dot{a}^2}{a^2} - \Lambda = 8\pi G \rho, \quad (\text{A.34})$$

$$\frac{1}{k^2} + \dot{a}^2 + 2a\ddot{a} - \Lambda a^2 = -8\pi G p a^2. \quad (\text{A.35})$$

Now we define the Hubble parameter as:

$$H = \left(\frac{\dot{a}}{a} \right)^2, \quad (\text{A.36})$$

and rearranging the previous equations, we get what are referred to as the Friedmann equations:

$$H = \frac{8\pi G}{3} \rho - \frac{1}{k^2 a^2} + \frac{\Lambda}{3}, \quad (\text{A.37})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}. \quad (\text{A.38})$$

Appendix B

Details of the $\lambda_4 = -\lambda_5$ case in the hidden SU(3) model

In this appendix, we give the details of the $\lambda_4 = -\lambda_5$ case for the hidden SU(3) model.

In this case, the unitary gauge is given by:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v \end{pmatrix},$$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \frac{v_1 v_2 \varphi_3}{\sqrt{(v_1^2 + v_2^2)(v_1^2 + v_2^2 + v_3^2)}} - \frac{v_3 \varphi_2}{\sqrt{v_1^2 + v_3^2}} - i \frac{v_2 \varphi_1}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \\ v_1 + \varphi_4 - i \frac{v_3 \varphi_1}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \end{pmatrix}, \quad (\text{B.1})$$

$$\Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 + \frac{v_1 \varphi_2}{\sqrt{v_1^2 + v_3^2}} - \frac{v_2 v_3 \varphi_3}{\sqrt{(v_1^2 + v_2^2)(v_1^2 + v_2^2 + v_3^2)}} \\ v_3 + \sqrt{\frac{v_1^2 + v_3^2}{v_1^2 + v_2^2 + v_3^2}} \varphi_3 + i \frac{v_1 \varphi_1}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \end{pmatrix}. \quad (\text{B.2})$$

We write down the scalar mass matrix with $\lambda_4 = -\lambda_5$ in the form $\mathcal{L} \supset \frac{1}{2} \Phi^T m_{\text{scalar}}^2 \Phi$, where $\Phi^T = (h, \varphi_4, \varphi_2, \varphi_3)$ are the scalars in the model, and where

$$m_{\text{scalar}}^2 = \begin{pmatrix} \lambda_H v^2 & \lambda_{H11} v v_1 & \frac{\lambda_{H22} v v_1 v_2}{\sqrt{v_1^2 + v_3^2}} & \frac{\lambda_{H22} v v_3 (v_1^2 - v_2^2 + v_3^2)}{\sqrt{(v_1^2 + v_3^2)(v_1^2 + v_2^2 + v_3^2)}} \\ \lambda_{H11} v v_1 & \lambda_1 v_1^2 & \frac{\lambda_3 v_1^2 v_2}{\sqrt{v_1^2 + v_3^2}} & \frac{\lambda_3 v_1 v_3 (v_1^2 - v_2^2 + v_3^2)}{\sqrt{(v_1^2 + v_3^2)(v_1^2 + v_2^2 + v_3^2)}} \\ \frac{\lambda_{H22} v v_1 v_2}{\sqrt{v_1^2 + v_3^2}} & \frac{\lambda_3 v_1^2 v_2}{\sqrt{v_1^2 + v_3^2}} & \frac{\lambda_2 v_1^2 v_2^2}{v_1^2 + v_3^2} & \frac{\lambda_2 v_1 v_2 v_3 (v_1^2 - v_2^2 + v_3^2)}{(v_1^2 + v_3^2) \sqrt{(v_1^2 + v_2^2 + v_3^2)}} \\ \frac{\lambda_{H22} v v_3 (v_1^2 - v_2^2 + v_3^2)}{\sqrt{(v_1^2 + v_3^2)(v_1^2 + v_2^2 + v_3^2)}} & \frac{\lambda_3 v_1 v_3 (v_1^2 - v_2^2 + v_3^2)}{\sqrt{(v_1^2 + v_3^2)(v_1^2 + v_2^2 + v_3^2)}} & \frac{\lambda_2 v_1 v_2 v_3 (v_1^2 - v_2^2 + v_3^2)}{(v_1^2 + v_3^2) \sqrt{(v_1^2 + v_2^2 + v_3^2)}} & \frac{\lambda_2 v_3^2 (v_1^2 - v_2^2 + v_3^2)^2}{(v_1^2 + v_3^2)(v_1^2 + v_2^2 + v_3^2)} \end{pmatrix}. \quad (\text{B.3})$$

The pseudo-scalar φ_1 does not mix with the others and has a mass given by:

$$m_{\varphi_1}^2 = \lambda_4 (v_1^2 + v_2^2 + v_3^2). \quad (\text{B.4})$$

This case also results in more mixing between the gauge bosons. They are described by the following mass terms:

$$\begin{aligned}
\mathcal{L} \supset & \frac{\tilde{g}^2}{8} \begin{pmatrix} A_\mu^1 \\ A_\mu^4 \end{pmatrix}^T \begin{pmatrix} v_2^2 & v_2 v_3 \\ v_2 v_3 & v_1^2 + v_3^2 \end{pmatrix} \begin{pmatrix} A^{1\mu} \\ A^{4\mu} \end{pmatrix} \\
& + \frac{\tilde{g}^2}{8} \begin{pmatrix} A_\mu^2 \\ A_\mu^5 \end{pmatrix}^T \begin{pmatrix} v_2^2 & v_2 v_3 \\ v_2 v_3 & v_1^2 + v_3^2 \end{pmatrix} \begin{pmatrix} A^{2\mu} \\ A^{5\mu} \end{pmatrix} \\
& + \frac{\tilde{g}^2}{8} \begin{pmatrix} A_\mu^3 \\ A_\mu^6 \\ A_\mu^8 \end{pmatrix}^T \begin{pmatrix} v_2^2 & -v_2 v_3 & \frac{-v_2^2}{\sqrt{3}} \\ -v_2 v_3 & v_1^2 + v_2^2 + v_3^2 & \frac{-v_2 v_3}{\sqrt{3}} \\ \frac{-v_2^2}{\sqrt{3}} & \frac{-v_2 v_3}{\sqrt{3}} & \frac{4v_1^2 + v_2^2 + 4v_3^2}{3} \end{pmatrix} \begin{pmatrix} A^{3\mu} \\ A^{6\mu} \\ A^{8\mu} \end{pmatrix} \\
& + \frac{\tilde{g}^2}{8} (v_1^2 + v_2^2 + v_3^2) A_\mu^7 A^{7\mu}. \tag{B.5}
\end{aligned}$$

We note that the mixing between $A_\mu^1 - A_\mu^4$ and $A_\mu^2 - A_\mu^5$ pairs is the same and vanishes when $v_3 = 0$. Similarly, A_μ^6 becomes an eigenstate degenerate with A_μ^7 when $v_3 = 0$; however, there is still mixing between A_μ^3 and A_μ^8 .

Appendix C

Feynman rules involved in Direct-Detection for the hidden SU(3) model

C.1 Feynman rules involved in Direct-Detection

In this appendix, we give the Feynman rules relevant for direct-detection. We give these in the gauge basis, not the mass basis, for simplicity. To convert a rule to the mass basis, multiply by the relevant mixing factors described in Equations [5.44](#), [5.45](#), [5.48](#), [5.49](#). In this appendix, we will use $s_\beta \equiv \sin \beta = \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$ and $c_\beta \equiv \cos \beta = \frac{v_2}{\sqrt{v_1^2 + v_2^2}}$. When describing the form of the coupling, S will be a placeholder for a scalar and A will be a placeholder for a vector.

C.1.1 Couplings involving a single h

The Feynman rules for couplings involving a single h . These couplings will have the form $-ig_{SSH}$ with all particles incoming. The couplings are defined as follows:

$$g_{\phi_3\phi_3h} = g_{\phi_4\phi_4h} = v(\lambda_{H22}s_\beta^2 + \lambda_{H11}c_\beta^2). \quad (\text{C.1})$$

The SM Higgs does not directly interact with the gauge bosons in this model, only through mixing.

C.1.2 Couplings involving a single ϕ_1

The Feynman rules for couplings involving a single ϕ_1 . This particle can mix with the SM Higgs and contribute to direct-detection, although the mixing was set to zero in the benchmark points studied in the main chapter. Starting with the couplings involving only scalars, these couplings will have the form $-ig_{SS\phi_1}$ with all particles incoming. The couplings are defined as follows:

$$g_{\phi_1\phi_3\phi_3} = v_1(\lambda_3s_\beta^2 + \lambda_1c_\beta^2 + \lambda_4 + \lambda_5), \quad (\text{C.2})$$

$$g_{\phi_1\phi_4\phi_4} = v_1(\lambda_3s_\beta^2 + \lambda_1c_\beta^2 + \lambda_4 - \lambda_5). \quad (\text{C.3})$$

There are also couplings which involve two scalars and a vector which have the form $ig_{\phi_1SA}(p_1 - p_2)_\mu$ where p_1 and p_2 are the momenta of the scalar going into the vertex. These are given by:

$$g_{\phi_1\phi_3A^7} = g_{\phi_1\phi_4A^6} = \frac{i}{2}\tilde{g}c_\beta. \quad (\text{C.4})$$

Finally, there are couplings involving two vectors. These will have the form $ig_{\phi_1 AA}g^{\mu\nu}$ with all particles incoming. The couplings are defined as follows:

$$g_{\phi_1 A^4 A^4} = g_{\phi_1 A^5 A^5} = g_{\phi_1 A^6 A^6} = g_{\phi_1 A^7 A^7} = -\frac{1}{2}\tilde{g}^2 v_1, \quad (\text{C.5})$$

$$g_{\phi_1 A^8 A^8} = -\frac{2}{3}\tilde{g}^2 v_1. \quad (\text{C.6})$$

C.1.3 Couplings involving a single ϕ_2

The Feynman rules for couplings involving a single ϕ_2 . This particle can mix with the SM Higgs and contribute to direct-detection. These couplings will have the form $-ig_{\phi_2 SS}$ with all particles incoming. The couplings are defined as follows:

$$g_{\phi_2 \phi_3 \phi_3} = v_2(\lambda_2 s_\beta^2 + \lambda_3 c_\beta^2 + \lambda_4 + \lambda_5), \quad (\text{C.7})$$

$$g_{\phi_2 \phi_4 \phi_4} = v_2(\lambda_2 s_\beta^2 + \lambda_3 c_\beta^2 + \lambda_4 - \lambda_5). \quad (\text{C.8})$$

There are also couplings which involve two scalars and a vector which have the form $ig_{\phi_2 SA}(p_1 - p_2)_\mu$ where p_1 and p_2 are the momenta of the scalar going into the vertex. These are given by:

$$g_{\phi_2 \phi_3 A^7} = g_{\phi_2 \phi_4 A^6} = \frac{i}{2}\tilde{g}s_\beta. \quad (\text{C.9})$$

Finally, there are couplings involving two vectors. These will have the form $ig_{\phi_2 AA}g^{\mu\nu}$ with all particles incoming. The couplings are defined as follows:

$$g_{\phi_2 A^1 A^1} = g_{\phi_2 A^2 A^2} = g_{\phi_2 A^3 A^3} = g_{\phi_2 A^6 A^6} = g_{\phi_2 A^7 A^7} = -\frac{1}{2}\tilde{g}^2 v_2, \quad (\text{C.10})$$

$$g_{\phi_2 A^3 A^8} = \frac{1}{2\sqrt{3}}\tilde{g}^2 v_2, \quad (\text{C.11})$$

$$g_{\phi_2 A^s A^s} = \frac{1}{6} \tilde{g}^2 v_2. \quad (\text{C.12})$$

Appendix D

Feynman rules involving the DM S , in the GM-model

D.1 Feynman rules for couplings involving S

In this appendix, we give the new Feynman rules involving the additional scalar S .

D.1.1 Triple scalar couplings

The Feynman rules for couplings to S are given by $-ig_{SSH_i}$ with all particles incoming and the couplings defined as follows:

$$\begin{aligned} g_{SSH} &= -4 \left(\lambda_a s_\alpha v_\phi - \sqrt{3} \lambda_b c_\alpha v_\chi \right), \\ g_{SSH} &= -4 \left(\lambda_a c_\alpha v_\phi + \sqrt{3} \lambda_b s_\alpha v_\chi \right), \end{aligned} \tag{D.1}$$

where we use the notation $s_\alpha \equiv \sin \alpha$ and $c_\alpha \equiv \cos \alpha$.

D.1.2 Quartic scalar couplings

The Feynman rules for couplings to S are given by $-ig_{SSs_1s_2}$ with all particles incoming and the couplings defined as follows:

$$\begin{aligned}
g_{SShh} &= -4 \left(\lambda_a c_\alpha^2 + \lambda_b s_\alpha^2 \right), \\
g_{SSHH} &= -4 \left(\lambda_a s_\alpha^2 + \lambda_b c_\alpha^2 \right), \\
g_{SShH} &= -4 s_\alpha c_\alpha (\lambda_a - \lambda_b), \\
g_{SSG^0 G^0} &= g_{SSG^+ G^{+*}} = -4 \left(\lambda_a c_H^2 + \lambda_b s_H^2 \right), \\
g_{SSH_3^0 H_3^0} &= g_{SSH_3^+ H_3^{+*}} = -4 \left(\lambda_a s_H^2 + \lambda_b c_H^2 \right), \\
g_{SSG^0 H_3^0} &= g_{SSG^+ H_3^{+*}} = g_{SSH_3^+ G^{+*}} = -4 s_H c_H (\lambda_b - \lambda_a), \\
g_{SSH_5^0 H_5^0} &= g_{SSH_5^+ H_5^{+*}} = g_{SSH_5^{++} H_5^{++*}} = -4 \lambda_b,
\end{aligned} \tag{D.2}$$

where we use the notation $s_H \equiv \sin \theta_H$ and $c_H \equiv \cos \theta_H$, and G^0 and $G^\pm$ are the Goldstone bosons.

All other Feynman rules are identical to those in the original GM model and can be found in Appendix A of Ref. [188]

References

- [1] A. Poulin, *Dark Matter freezeout in modified cosmological scenarios*, *Phys. Rev. D* **D100** (2019), no. 8 043022, [[arXiv:1905.03126](#)].
- [2] R. Campbell, S. Godfrey, H. E. Logan, A. D. Peterson, and A. Poulin, *Implications of the observation of dark matter self-interactions for singlet scalar dark matter*, *Phys. Rev. D* **D92** (2015), no. 5 055031, [[arXiv:1505.01793](#)].
- [3] A. Poulin and S. Godfrey, *Multicomponent dark matter from a hidden gauged $SU(3)$* , *Phys. Rev. D* **D99** (2019), no. 7 076008, [[arXiv:1808.04901](#)].
- [4] R. Campbell, S. Godfrey, H. E. Logan, and A. Poulin, *Real singlet scalar dark matter extension of the Georgi-Machacek model*, *Phys. Rev. D* **D95** (2017), no. 1 016005, [[arXiv:1610.08097](#)].
- [5] **ATLAS**, G. Aad et al., *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett. B* **B716** (2012) 1–29, [[arXiv:1207.7214](#)].
- [6] **CMS**, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, *Phys. Lett. B* **B716** (2012) 30–61, [[arXiv:1207.7235](#)].
- [7] J. H. Oort, *The force exerted by the stellar system in the direction perpendicular to the galactic plane and some related problems*, *Bulletin of the Astronomical Institutes of the Netherlands* **6** (Aug., 1932) 249.
- [8] F. Zwicky, *Die Rotverschiebung von extragalaktischen Nebeln*, *Helv. Phys. Acta* **6** (1933) 110–127. [Gen. Rel. Grav.41,207(2009)].
- [9] F. Zwicky, *On the Masses of Nebulae and of Clusters of Nebulae*, *Astrophys. J.* **86** (1937) 217–246.
- [10] V. C. Rubin and W. K. Ford, Jr., *Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions*, *Astrophys. J.* **159** (1970) 379–403.

- [11] V. C. Rubin, N. Thonnard, and W. K. Ford, Jr., *Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4605 /R = 4kpc/ to UGC 2885 /R = 122 kpc/, Astrophys. J.* **238** (1980) 471.
- [12] M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, L. David, W. Forman, C. Jones, S. Murray, and W. Tucker, *Direct constraints on the dark matter self-interaction cross-section from the merging galaxy cluster 1E0657-56, Astrophys. J.* **606** (2004) 819–824, [[astro-ph/0309303](#)].
- [13] G. Bertone, D. Hooper, and J. Silk, *Particle dark matter: Evidence, candidates and constraints, Phys. Rept.* **405** (2005) 279–390, [[hep-ph/0404175](#)].
- [14] J. L. Feng, *Dark Matter Candidates from Particle Physics and Methods of Detection, Ann. Rev. Astron. Astrophys.* **48** (2010) 495–545, [[arXiv:1003.0904](#)].
- [15] G. B. Gelmini, *The Hunt for Dark Matter, in Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: Journeys Through the Precision Frontier: Amplitudes for Colliders (TASI 2014): Boulder, Colorado, June 2-27, 2014*, pp. 559–616, 2015. [arXiv:1502.01320](#).
- [16] L. Roszkowski, E. M. Sessolo, and S. Trojanowski, *WIMP dark matter candidates and searches—current status and future prospects, Rept. Prog. Phys.* **81** (2018), no. 6 066201, [[arXiv:1707.06277](#)].
- [17] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, *The waning of the WIMP? A review of models, searches, and constraints, Eur. Phys. J.* **C78** (2018), no. 3 203, [[arXiv:1703.07364](#)].
- [18] **Fermi-LAT**, M. Ackermann et al., *Searching for Dark Matter Annihilation from Milky Way Dwarf Spheroidal Galaxies with Six Years of Fermi Large Area Telescope Data, Phys. Rev. Lett.* **115** (2015), no. 23 231301, [[arXiv:1503.02641](#)].
- [19] Y. Ema, F. Sala, and R. Sato, *Light Dark Matter at Neutrino Experiments, Phys. Rev. Lett.* **122** (2019), no. 18 181802, [[arXiv:1811.00520](#)].
- [20] **CMS**, A. M. Sirunyan et al., *Search for invisible decays of a Higgs boson produced through vector boson fusion in proton-proton collisions at $\sqrt{s} = 13$ TeV, arXiv:1809.05937*.
- [21] **ATLAS**, M. Aaboud et al., *Combination of searches for invisible Higgs boson decays with the ATLAS experiment, Submitted to: Phys. Rev. Lett.* (2019) [[arXiv:1904.05105](#)].

- [22] **Planck**, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, [arXiv:1807.06209](#).
- [23] M. Battaglieri et al., *US Cosmic Visions: New Ideas in Dark Matter 2017: Community Report*, in *U.S. Cosmic Visions: New Ideas in Dark Matter* College Park, MD, USA, March 23-25, 2017, 2017. [arXiv:1707.04591](#).
- [24] E. W. Kolb and M. S. Turner, *The Early Universe*, *Front. Phys.* **69** (1990) 1–547.
- [25] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*. Addison-Wesley, Reading, USA, 1995.
- [26] H. E. Logan, *TASI 2013 lectures on Higgs physics within and beyond the Standard Model*, [arXiv:1406.1786](#).
- [27] H. Georgi, *Lie algebras in particle physics*, *Front. Phys.* **54** (1999) 1–320.
- [28] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*. W. H. Freeman, San Francisco, 1973.
- [29] P. Gondolo and J. Edsjo, *Neutralino relic density including coannihilations*, *Phys. Atom. Nucl.* **61** (1998) 1081–1097. [*Yad. Fiz.*61,1181(1998)].
- [30] G. Steigman, B. Dasgupta, and J. F. Beacom, *Precise Relic WIMP Abundance and its Impact on Searches for Dark Matter Annihilation*, *Phys. Rev.* **D86** (2012) 023506, [[arXiv:1204.3622](#)].
- [31] E. V. Linder, *Exploring the expansion history of the universe*, *Phys. Rev. Lett.* **90** (2003) 091301, [[astro-ph/0208512](#)].
- [32] R. von Marttens, L. Casarini, D. F. Mota, and W. Zimdahl, *Cosmological constraints on parametrized interacting dark energy*, *Phys. Dark Univ.* **23** (2019) 100248, [[arXiv:1807.11380](#)].
- [33] J. D. Barrow, *MASSIVE PARTICLES AS A PROBE OF THE EARLY UNIVERSE*, *Nucl. Phys.* **B208** (1982) 501–508.
- [34] M. Kamionkowski and M. S. Turner, *THERMAL RELICS: DO WE KNOW THEIR ABUNDANCES?*, *Phys. Rev.* **D42** (1990) 3310–3320.
- [35] H. Stoica, S. H. H. Tye, and I. Wasserman, *Cosmology in the Randall-Sundrum brane world scenario*, *Phys. Lett.* **B482** (2000) 205–212, [[hep-th/0004126](#)].
- [36] N. Okada and O. Seto, *Relic density of dark matter in brane world cosmology*, *Phys. Rev.* **D70** (2004) 083531, [[hep-ph/0407092](#)].

- [37] E. Abou El Dahab and S. Khalil, *Cold dark matter in brane cosmology scenario*, *JHEP* **09** (2006) 042, [[hep-ph/0607180](#)].
- [38] W.-L. Guo and X. Zhang, *Constraints on Dark Matter Annihilation Cross Section in Scenarios of Brane-World and Quintessence*, *Phys. Rev.* **D79** (2009) 115023, [[arXiv:0904.2451](#)].
- [39] H. Abdusattar and H. Iminniyaz, *Abundance of Asymmetric Dark Matter in Brane World Cosmology*, *Commun. Theor. Phys.* **66** (2016), no. 3 363–368, [[arXiv:1505.03716](#)].
- [40] A. Arbey and F. Mahmoudi, *SUSY constraints from relic density: High sensitivity to pre-BBN expansion rate*, *Phys. Lett.* **B669** (2008) 46–51, [[arXiv:0803.0741](#)].
- [41] A. Arbey, A. Deandrea, and A. Tarhini, *Anomaly mediated SUSY breaking scenarios in the light of cosmology and in the dark (matter)*, *JHEP* **05** (2011) 078, [[arXiv:1103.3244](#)].
- [42] M. Bastero-Gil, R. Cerezo, and J. G. Rosa, *Inflaton dark matter from incomplete decay*, *Phys. Rev.* **D93** (2016), no. 10 103531, [[arXiv:1501.05539](#)].
- [43] P. Salati, *Quintessence and the relic density of neutralinos*, *Phys. Lett.* **B571** (2003) 121–131, [[astro-ph/0207396](#)].
- [44] F. Rosati, *Quintessence and the dark matter abundance*, in *4th International Conference on Where Cosmology and Fundamental Physics Meet Marseille, France, June 23-26, 2003*, 2003. [hep-ph/0309124](#).
- [45] S. Profumo and P. Ullio, *SUSY dark matter and quintessence*, *JCAP* **0311** (2003) 006, [[hep-ph/0309220](#)].
- [46] C. Pallis, *Quintessential kination and cold dark matter abundance*, *JCAP* **0510** (2005) 015, [[hep-ph/0503080](#)].
- [47] H. Iminniyaz and X. Chen, *Relic Abundance of Asymmetric Dark Matter in Quintessence*, *Astropart. Phys.* **54** (2014) 125–131, [[arXiv:1308.0353](#)].
- [48] R. Catena, N. Fornengo, A. Masiero, M. Pietroni, and F. Rosati, *Dark matter relic abundance and scalar - tensor dark energy*, *Phys. Rev.* **D70** (2004) 063519, [[astro-ph/0403614](#)].
- [49] R. Catena, M. Pietroni, and L. Scarabello, *Einstein and Jordan reconciled: a frame-invariant approach to scalar-tensor cosmology*, *Phys. Rev.* **D76** (2007) 084039, [[astro-ph/0604492](#)].

- [50] N. Bernal, C. Cosme, and T. Tenkanen, *Phenomenology of Self-Interacting Dark Matter in a Matter-Dominated Universe*, *Eur. Phys. J.* **C79** (2019), no. 2 99, [[arXiv:1803.08064](#)].
- [51] N. Bernal, C. Cosme, T. Tenkanen, and V. Vaskonen, *Scalar singlet dark matter in non-standard cosmologies*, *Eur. Phys. J.* **C79** (2019), no. 1 30, [[arXiv:1806.11122](#)].
- [52] S. Hamdan and J. Unwin, *Dark Matter Freeze-out During Matter Domination*, *Mod. Phys. Lett.* **A33** (2018), no. 29 1850181, [[arXiv:1710.03758](#)].
- [53] E. Hardy, *Higgs portal dark matter in non-standard cosmological histories*, *JHEP* **06** (2018) 043, [[arXiv:1804.06783](#)].
- [54] F. D’Eramo, N. Fernandez, and S. Profumo, *When the Universe Expands Too Fast: Relentless Dark Matter*, *JCAP* **1705** (2017), no. 05 012, [[arXiv:1703.04793](#)].
- [55] F. D’Eramo, N. Fernandez, and S. Profumo, *Dark Matter Freeze-in Production in Fast-Expanding Universes*, *JCAP* **1802** (2018), no. 02 046, [[arXiv:1712.07453](#)].
- [56] C. Maldonado and J. Unwin, *Establishing the Dark Matter Relic Density in an Era of Particle Decays*, [arXiv:1902.10746](#).
- [57] H. Iminniyaz, B. Salai, and G. Lv, *Relic Density of Asymmetric Dark Matter in Modified Cosmological Scenarios*, *Commun. Theor. Phys.* **70** (2018), no. 5 602, [[arXiv:1804.07256](#)].
- [58] K. Dienes, F. Huang, and B. Thomas Unpublished paper.
- [59] R. Massey et al., *The behaviour of dark matter associated with four bright cluster galaxies in the 10 kpc core of Abell 3827*, *Mon. Not. Roy. Astron. Soc.* **449** (2015), no. 4 3393–3406, [[arXiv:1504.03388](#)].
- [60] F. Kahlhoefer, K. Schmidt-Hoberg, J. Kummer, and S. Sarkar, *On the interpretation of dark matter self-interactions in Abell 3827*, *Mon. Not. Roy. Astron. Soc.* **452** (2015), no. 1 L54–L58, [[arXiv:1504.06576](#)].
- [61] A. H. G. Peter, M. Rocha, J. S. Bullock, and M. Kaplinghat, *Cosmological Simulations with Self-Interacting Dark Matter II: Halo Shapes vs. Observations*, *Mon. Not. Roy. Astron. Soc.* **430** (2013) 105, [[arXiv:1208.3026](#)].

- [62] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez, and M. Bradac, *Constraints on the Self-Interaction Cross-Section of Dark Matter from Numerical Simulations of the Merging Galaxy Cluster 1E 0657-56*, *Astrophys. J.* **679** (2008) 1173–1180, [[arXiv:0704.0261](#)].
- [63] M. Rocha, A. H. G. Peter, J. S. Bullock, M. Kaplinghat, S. Garrison-Kimmel, J. Onorbe, and L. A. Moustakas, *Cosmological Simulations with Self-Interacting Dark Matter I: Constant Density Cores and Substructure*, *Mon. Not. Roy. Astron. Soc.* **430** (2013) 81–104, [[arXiv:1208.3025](#)].
- [64] D. Harvey, R. Massey, T. Kitching, A. Taylor, and E. Tittley, *The non-gravitational interactions of dark matter in colliding galaxy clusters*, *Science* **347** (2015) 1462–1465, [[arXiv:1503.07675](#)].
- [65] M. J. G. Veltman and F. J. Yndurain, *RADIATIVE CORRECTIONS TO W W SCATTERING*, *Nucl. Phys.* **B325** (1989) 1–17.
- [66] V. Silveira and A. Zee, *SCALAR PHANTOMS*, *Phys. Lett.* **161B** (1985) 136–140.
- [67] J. McDonald, *Gauge singlet scalars as cold dark matter*, *Phys. Rev.* **D50** (1994) 3637–3649, [[hep-ph/0702143](#)].
- [68] C. P. Burgess, M. Pospelov, and T. ter Veldhuis, *The Minimal model of nonbaryonic dark matter: A Singlet scalar*, *Nucl. Phys.* **B619** (2001) 709–728, [[hep-ph/0011335](#)].
- [69] J. McDonald, *Thermally generated gauge singlet scalars as selfinteracting dark matter*, *Phys. Rev. Lett.* **88** (2002) 091304, [[hep-ph/0106249](#)].
- [70] V. Barger, P. Langacker, M. McCaskey, M. J. Ramsey-Musolf, and G. Shaughnessy, *LHC Phenomenology of an Extended Standard Model with a Real Scalar Singlet*, *Phys. Rev.* **D77** (2008) 035005, [[arXiv:0706.4311](#)].
- [71] A. Goudelis, Y. Mambrini, and C. Yaguna, *Antimatter signals of singlet scalar dark matter*, *JCAP* **0912** (2009) 008, [[arXiv:0909.2799](#)].
- [72] M. Gonderinger, Y. Li, H. Patel, and M. J. Ramsey-Musolf, *Vacuum Stability, Perturbativity, and Scalar Singlet Dark Matter*, *JHEP* **01** (2010) 053, [[arXiv:0910.3167](#)].
- [73] X.-G. He, T. Li, X.-Q. Li, J. Tandean, and H.-C. Tsai, *The Simplest Dark-Matter Model, CDMS II Results, and Higgs Detection at LHC*, *Phys. Lett.* **B688** (2010) 332–336, [[arXiv:0912.4722](#)].

- [74] S. Profumo, L. Ubaldi, and C. Wainwright, *Singlet Scalar Dark Matter: monochromatic gamma rays and metastable vacua*, *Phys. Rev.* **D82** (2010) 123514, [[arXiv:1009.5377](#)].
- [75] C. E. Yaguna, *The Singlet Scalar as FIMP Dark Matter*, *JHEP* **08** (2011) 060, [[arXiv:1105.1654](#)].
- [76] A. Drozd, B. Grzadkowski, and J. Wudka, *Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson*, *JHEP* **04** (2012) 006, [[arXiv:1112.2582](#)]. [Erratum: JHEP11,130(2014)].
- [77] A. Djouadi, O. Lebedev, Y. Mambrini, and J. Quevillon, *Implications of LHC searches for Higgs-portal dark matter*, *Phys. Lett.* **B709** (2012) 65–69, [[arXiv:1112.3299](#)].
- [78] M. Kadastik, K. Kannike, A. Racioppi, and M. Raidal, *Implications of the 125 GeV Higgs boson for scalar dark matter and for the CMSSM phenomenology*, *JHEP* **05** (2012) 061, [[arXiv:1112.3647](#)].
- [79] A. Djouadi, A. Falkowski, Y. Mambrini, and J. Quevillon, *Direct Detection of Higgs-Portal Dark Matter at the LHC*, *Eur. Phys. J.* **C73** (2013), no. 6 2455, [[arXiv:1205.3169](#)].
- [80] K. Cheung, Y.-L. S. Tsai, P.-Y. Tseng, T.-C. Yuan, and A. Zee, *Global Study of the Simplest Scalar Phantom Dark Matter Model*, *JCAP* **1210** (2012) 042, [[arXiv:1207.4930](#)].
- [81] P. H. Damgaard, D. O’Connell, T. C. Petersen, and A. Tranberg, *Constraints on New Physics from Baryogenesis and Large Hadron Collider Data*, *Phys. Rev. Lett.* **111** (2013), no. 22 221804, [[arXiv:1305.4362](#)].
- [82] J. M. Cline, K. Kainulainen, P. Scott, and C. Weniger, *Update on scalar singlet dark matter*, *Phys. Rev.* **D88** (2013) 055025, [[arXiv:1306.4710](#)]. [Erratum: Phys. Rev.D92,no.3,039906(2015)].
- [83] S. Baek, P. Ko, and W.-I. Park, *Invisible Higgs Decay Width vs. Dark Matter Direct Detection Cross Section in Higgs Portal Dark Matter Models*, *Phys. Rev.* **D90** (2014), no. 5 055014, [[arXiv:1405.3530](#)].
- [84] L. Feng, S. Profumo, and L. Ubaldi, *Closing in on singlet scalar dark matter: LUX, invisible Higgs decays and gamma-ray lines*, *JHEP* **03** (2015) 045, [[arXiv:1412.1105](#)].
- [85] **ATLAS**, T. A. collaboration, *Search for an Invisibly Decaying Higgs Boson Produced via Vector Boson Fusion in pp Collisions at $\sqrt{s} = 8$ TeV using the ATLAS Detector at the LHC*, .

- [86] **CMS**, C. Collaboration, *Search for invisible decays of Higgs bosons in the vector boson fusion production mode*, .
- [87] L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, *Freeze-In Production of FIMP Dark Matter*, *JHEP* **03** (2010) 080, [[arXiv:0911.1120](#)].
- [88] X.-G. He, T. Li, X.-Q. Li, J. Tandean, and H.-C. Tsai, *Constraints on Scalar Dark Matter from Direct Experimental Searches*, *Phys. Rev.* **D79** (2009) 023521, [[arXiv:0811.0658](#)].
- [89] B. Grzadkowski and P. Osland, *Tempered Two-Higgs-Doublet Model*, *Phys. Rev.* **D82** (2010) 125026, [[arXiv:0910.4068](#)].
- [90] H. E. Logan, *Dark matter annihilation through a lepton-specific Higgs boson*, *Phys. Rev.* **D83** (2011) 035022, [[arXiv:1010.4214](#)].
- [91] M. S. Boucenna and S. Profumo, *Direct and Indirect Singlet Scalar Dark Matter Detection in the Lepton-Specific two-Higgs-doublet Model*, *Phys. Rev.* **D84** (2011) 055011, [[arXiv:1106.3368](#)].
- [92] X.-G. He, B. Ren, and J. Tandean, *Hints of Standard Model Higgs Boson at the LHC and Light Dark Matter Searches*, *Phys. Rev.* **D85** (2012) 093019, [[arXiv:1112.6364](#)].
- [93] Y. Bai, V. Barger, L. L. Everett, and G. Shaughnessy, *Two-Higgs-doublet-portal dark-matter model: LHC data and Fermi-LAT 135 GeV line*, *Phys. Rev.* **D88** (2013) 015008, [[arXiv:1212.5604](#)].
- [94] X.-G. He and J. Tandean, *Low-Mass Dark-Matter Hint from CDMS II, Higgs Boson at the LHC, and Darkon Models*, *Phys. Rev.* **D88** (2013) 013020, [[arXiv:1304.6058](#)].
- [95] Y. Cai and T. Li, *Singlet dark matter in a type II two Higgs doublet model*, *Phys. Rev.* **D88** (2013), no. 11 115004, [[arXiv:1308.5346](#)].
- [96] L. Wang and X.-F. Han, *A simplified 2HDM with a scalar dark matter and the galactic center gamma-ray excess*, *Phys. Lett.* **B739** (2014) 416–420, [[arXiv:1406.3598](#)].
- [97] C.-Y. Chen, M. Freid, and M. Sher, *Next-to-minimal two Higgs doublet model*, *Phys. Rev.* **D89** (2014), no. 7 075009, [[arXiv:1312.3949](#)].
- [98] A. Drozd, B. Grzadkowski, J. F. Gunion, and Y. Jiang, *Extending two-Higgs-doublet models by a singlet scalar field - the Case for Dark Matter*, *JHEP* **11** (2014) 105, [[arXiv:1408.2106](#)].

- [99] **ATLAS, CMS**, G. Aad et al., *Combined Measurement of the Higgs Boson Mass in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments*, *Phys. Rev. Lett.* **114** (2015) 191803, [[arXiv:1503.07589](#)].
- [100] **Particle Data Group**, K. A. Olive et al., *Review of Particle Physics*, *Chin. Phys.* **C38** (2014) 090001.
- [101] **LHC Higgs Cross Section Working Group**, J. R. Andersen et al., *Handbook of LHC Higgs Cross Sections: 3. Higgs Properties*, [arXiv:1307.1347](#).
- [102] **ATLAS**, T. A. collaboration, *Measurements of the Higgs boson production and decay rates and coupling strengths using pp collision data at $\sqrt{s} = 7$ and 8 TeV in the ATLAS experiment*, .
- [103] **CMS**, V. Khachatryan et al., *Precise determination of the mass of the Higgs boson and tests of compatibility of its couplings with the standard model predictions using proton collisions at 7 and 8 TeV*, *Eur. Phys. J.* **C75** (2015), no. 5 212, [[arXiv:1412.8662](#)].
- [104] L. Lopez-Honorez, O. Mena, S. Palomares-Ruiz, and A. C. Vincent, *Constraints on dark matter annihilation from CMB observations before Planck*, *JCAP* **1307** (2013) 046, [[arXiv:1303.5094](#)].
- [105] **Planck**, P. A. R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13, [[arXiv:1502.01589](#)].
- [106] Z. Kang, *Upgrading sterile neutrino dark matter to FIMP using scale invariance*, *Eur. Phys. J.* **C75** (2015), no. 10 471, [[arXiv:1411.2773](#)].
- [107] S. L. Glashow and S. Weinberg, *Natural Conservation Laws for Neutral Currents*, *Phys. Rev.* **D15** (1977) 1958.
- [108] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, *Theory and phenomenology of two-Higgs-doublet models*, *Phys. Rept.* **516** (2012) 1–102, [[arXiv:1106.0034](#)].
- [109] N. Craig, J. Galloway, and S. Thomas, *Searching for Signs of the Second Higgs Doublet*, [arXiv:1305.2424](#).
- [110] V. D. Barger, J. L. Hewett, and R. J. N. Phillips, *New Constraints on the Charged Higgs Sector in Two Higgs Doublet Models*, *Phys. Rev.* **D41** (1990) 3421–3441.
- [111] A. Pich and P. Tuzon, *Yukawa Alignment in the Two-Higgs-Doublet Model*, *Phys. Rev.* **D80** (2009) 091702, [[arXiv:0908.1554](#)].

- [112] G. Arcadi, C. Gross, O. Lebedev, Y. Mambrini, S. Pokorski, and T. Toma, *Multicomponent Dark Matter from Gauge Symmetry*, *JHEP* **12** (2016) 081, [[arXiv:1611.00365](#)].
- [113] C. Gross, O. Lebedev, and Y. Mambrini, *Non-Abelian gauge fields as dark matter*, *JHEP* **08** (2015) 158, [[arXiv:1505.07480](#)].
- [114] K. R. Dienes and B. Thomas, *Dynamical Dark Matter: I. Theoretical Overview*, *Phys. Rev.* **D85** (2012) 083523, [[arXiv:1106.4546](#)].
- [115] H. Baer, A. Lessa, and W. Sreethawong, *Coupled Boltzmann calculation of mixed axion/neutralino cold dark matter production in the early universe*, *JCAP* **1201** (2012) 036, [[arXiv:1110.2491](#)].
- [116] D. Chialva, P. S. B. Dev, and A. Mazumdar, *Multiple dark matter scenarios from ubiquitous stringy throats*, *Phys. Rev.* **D87** (2013), no. 6 063522, [[arXiv:1211.0250](#)].
- [117] K. R. Dienes, J. Kumar, and B. Thomas, *Direct Detection of Dynamical Dark Matter*, *Phys. Rev.* **D86** (2012) 055016, [[arXiv:1208.0336](#)].
- [118] K. M. Zurek, *Multi-Component Dark Matter*, *Phys. Rev.* **D79** (2009) 115002, [[arXiv:0811.4429](#)].
- [119] S. Esch, M. Klasen, and C. E. Yaguna, *A minimal model for two-component dark matter*, *JHEP* **09** (2014) 108, [[arXiv:1406.0617](#)].
- [120] G. Elor, N. L. Rodd, T. R. Slatyer, and W. Xue, *Model-Independent Indirect Detection Constraints on Hidden Sector Dark Matter*, *JCAP* **1606** (2016), no. 06 024, [[arXiv:1511.08787](#)].
- [121] K. K. Boddy, J. L. Feng, M. Kaplinghat, and T. M. P. Tait, *Self-Interacting Dark Matter from a Non-Abelian Hidden Sector*, *Phys. Rev.* **D89** (2014), no. 11 115017, [[arXiv:1402.3629](#)].
- [122] A. DiFranzo and G. Mohlabeng, *Multi-component Dark Matter through a Radiative Higgs Portal*, *JHEP* **01** (2017) 080, [[arXiv:1610.07606](#)].
- [123] S. Bhattacharya, P. Poulose, and P. Ghosh, *Multipartite Interacting Scalar Dark Matter in the light of updated LUX data*, *JCAP* **1704** (2017), no. 04 043, [[arXiv:1607.08461](#)].
- [124] P. Ko and Y. Tang, *Residual Non-Abelian Dark Matter and Dark Radiation*, *Phys. Lett.* **B768** (2017) 12–17, [[arXiv:1609.02307](#)].

- [125] D. Feldman, Z. Liu, P. Nath, and G. Peim, *Multicomponent Dark Matter in Supersymmetric Hidden Sector Extensions*, *Phys. Rev.* **D81** (2010) 095017, [[arXiv:1004.0649](#)].
- [126] F. D'Eramo and J. Thaler, *Semi-annihilation of Dark Matter*, *JHEP* **06** (2010) 109, [[arXiv:1003.5912](#)].
- [127] M. Aoki, M. Duerr, J. Kubo, and H. Takano, *Multi-Component Dark Matter Systems and Their Observation Prospects*, *Phys. Rev.* **D86** (2012) 076015, [[arXiv:1207.3318](#)].
- [128] K. R. Dienes, F. Huang, S. Su, and B. Thomas, *Dynamical Dark Matter from Strongly-Coupled Dark Sectors*, *Phys. Rev.* **D95** (2017), no. 4 043526, [[arXiv:1610.04112](#)].
- [129] K. R. Dienes, J. Fennick, J. Kumar, and B. Thomas, *Randomness in the Dark Sector: Emergent Mass Spectra and Dynamical Dark Matter Ensembles*, *Phys. Rev.* **D93** (2016), no. 8 083506, [[arXiv:1601.05094](#)].
- [130] L. Forestell, D. E. Morrissey, and K. Sigurdson, *Non-Abelian Dark Forces and the Relic Densities of Dark Glueballs*, *Phys. Rev.* **D95** (2017), no. 1 015032, [[arXiv:1605.08048](#)].
- [131] A. Karam and K. Tamvakis, *Dark Matter from a Classically Scale-Invariant $SU(3)_X$* , *Phys. Rev.* **D94** (2016), no. 5 055004, [[arXiv:1607.01001](#)].
- [132] K. J. Bae, H. Baer, A. Lessa, and H. Serce, *Coupled Boltzmann computation of mixed axion neutralino dark matter in the SUSY DFSZ axion model*, *JCAP* **1410** (2014), no. 10 082, [[arXiv:1406.4138](#)].
- [133] T. Hur, H.-S. Lee, and S. Nasri, *A Supersymmetric $U(1)$ -prime model with multiple dark matters*, *Phys. Rev.* **D77** (2008) 015008, [[arXiv:0710.2653](#)].
- [134] H. S. Cheon, S. K. Kang, and C. S. Kim, *Doubly Coexisting Dark Matter Candidates in an Extended Seesaw Model*, *Phys. Lett.* **B675** (2009) 203–209, [[arXiv:0807.0981](#)]. [Erratum: *Phys. Lett.* **B698**, 324(2011)].
- [135] K. R. Dienes, J. Kumar, B. Thomas, and D. Yaylali, *Dark-Matter Decay as a Complementary Probe of Multicomponent Dark Sectors*, *Phys. Rev. Lett.* **114** (2015), no. 5 051301, [[arXiv:1406.4868](#)].
- [136] M. Aoki, J. Kubo, and H. Takano, *Multicomponent Dark Matter in Radiative Seesaw Model and Monochromatic Neutrino Flux*, *Phys. Rev.* **D90** (2014), no. 7 076011, [[arXiv:1408.1853](#)].
- [137] K. R. Dienes and B. Thomas, *Dynamical Dark Matter: II. An Explicit Model*, *Phys. Rev.* **D85** (2012) 083524, [[arXiv:1107.0721](#)].

- [138] M. Aoki and T. Toma, *Implications of Two-component Dark Matter Induced by Forbidden Channels and Thermal Freeze-out*, *JCAP* **1701** (2017), no. 01 042, [[arXiv:1611.06746](#)].
- [139] K. R. Dienes, J. Kumar, B. Thomas, and D. Yaylali, *Off-diagonal dark-matter phenomenology: Exploring enhanced complementarity relations in nonminimal dark sectors*, *Phys. Rev.* **D96** (2017), no. 11 115009, [[arXiv:1708.09698](#)].
- [140] K. R. Dienes, J. Fennick, J. Kumar, and B. Thomas, *Dynamical Dark Matter from Thermal Freeze-Out*, *Phys. Rev.* **D97** (2018), no. 6 063522, [[arXiv:1712.09919](#)].
- [141] S. Bhattacharya, P. Ghosh, T. N. Maity, and T. S. Ray, *Mitigating Direct Detection Bounds in Non-minimal Higgs Portal Scalar Dark Matter Models*, *JHEP* **10** (2017) 088, [[arXiv:1706.04699](#)].
- [142] A. Ahmed, M. Duch, B. Grzadkowski, and M. Iglicki, *Multi-Component Dark Matter: the vector and fermion case*, *Eur. Phys. J.* **C78** (2018), no. 11 905, [[arXiv:1710.01853](#)].
- [143] G. Arcadi, F. S. Queiroz, and C. Siqueira, *The Semi-Hooperon: Gamma-ray and anti-proton excesses in the Galactic Center*, *Phys. Lett.* **B775** (2017) 196–205, [[arXiv:1706.02336](#)].
- [144] M. Aoki and T. Toma, *Boosted Self-interacting Dark Matter in a Multi-component Dark Matter Model*, *JCAP* **1810** (2018), no. 10 020, [[arXiv:1806.09154](#)].
- [145] J. Edsjo and P. Gondolo, *Neutralino relic density including coannihilations*, *Phys. Rev.* **D56** (1997) 1879–1894, [[hep-ph/9704361](#)].
- [146] G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov, and B. Zaldivar, *micrOMEGAs5.0 : Freeze-in*, *Comput. Phys. Commun.* **231** (2018) 173–186, [[arXiv:1801.03509](#)].
- [147] P. Gondolo and G. Gelmini, *Cosmic abundances of stable particles: Improved analysis*, *Nucl. Phys.* **B360** (1991) 145–179.
- [148] K. Griest and D. Seckel, *Three exceptions in the calculation of relic abundances*, *Phys. Rev.* **D43** (1991) 3191–3203.
- [149] **Particle Data Group**, C. Patrignani et al., *Review of Particle Physics*, *Chin. Phys.* **C40** (2016), no. 10 100001.
- [150] A. Falkowski, C. Gross, and O. Lebedev, *A second Higgs from the Higgs portal*, *JHEP* **05** (2015) 057, [[arXiv:1502.01361](#)].

- [151] A. Arhrib, R. Benbrik, M. Chabab, G. Moulhaka, M. C. Peyranere, L. Rahili, and J. Ramadan, *The Higgs Potential in the Type II Seesaw Model*, *Phys. Rev.* **D84** (2011) 095005, [[arXiv:1105.1925](#)].
- [152] **WMAP**, E. Komatsu et al., *Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation*, *Astrophys. J. Suppl.* **192** (2011) 18, [[arXiv:1001.4538](#)].
- [153] **XENON**, E. Aprile et al., *First Dark Matter Search Results from the XENON1T Experiment*, *Phys. Rev. Lett.* **119** (2017), no. 18 181301, [[arXiv:1705.06655](#)].
- [154] **XENON**, E. Aprile et al., *Dark Matter Search Results from a One Ton-Year Exposure of XENON1T*, *Phys. Rev. Lett.* **121** (2018), no. 11 111302, [[arXiv:1805.12562](#)].
- [155] T. R. Slatyer, *Indirect Detection of Dark Matter*, in *Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics : Anticipating the Next Discoveries in Particle Physics (TASI 2016): Boulder, CO, USA, June 6-July 1, 2016*, pp. 297–353, 2018. [arXiv:1710.05137](#).
- [156] J. Conrad, J. Cohen-Tanugi, and L. E. Strigari, *WIMP searches with gamma rays in the Fermi era: challenges, methods and results*, *J. Exp. Theor. Phys.* **121** (2015), no. 6 1104–1135, [[arXiv:1503.06348](#)]. [*Zh. Eksp. Teor. Fiz.*148,no.6,1257(2015)].
- [157] K. K. Boddy, K. R. Dienes, D. Kim, J. Kumar, J.-C. Park, and B. Thomas, *Lines and Boxes: Unmasking Dynamical Dark Matter through Correlations in the MeV Gamma-Ray Spectrum*, *Phys. Rev.* **D94** (2016), no. 9 095027, [[arXiv:1606.07440](#)].
- [158] K. K. Boddy, K. R. Dienes, D. Kim, J. Kumar, J.-C. Park, and B. Thomas, *Boxes, Boosts, and Energy Duality: Understanding the Galactic-Center Gamma-Ray Excess through Dynamical Dark Matter*, *Phys. Rev.* **D95** (2017), no. 5 055024, [[arXiv:1609.09104](#)].
- [159] K. R. Dienes, J. Kumar, and B. Thomas, *Dynamical Dark Matter and the positron excess in light of AMS results*, *Phys. Rev.* **D88** (2013), no. 10 103509, [[arXiv:1306.2959](#)].
- [160] **LHC Higgs Cross Section Working Group**, A. David, A. Denner, M. Duehrssen, M. Grazzini, C. Grojean, G. Passarino, M. Schumacher, M. Spira, G. Weiglein, and M. Zanetti, *LHC HXSWG interim recommendations to explore the coupling structure of a Higgs-like particle*, [arXiv:1209.0040](#).

- [161] D. Zeppenfeld, R. Kinnunen, A. Nikitenko, and E. Richter-Was, *Measuring Higgs boson couplings at the CERN LHC*, *Phys. Rev.* **D62** (2000) 013009, [[hep-ph/0002036](#)].
- [162] H. Baer, T. Barklow, K. Fujii, Y. Gao, A. Hoang, S. Kanemura, J. List, H. E. Logan, A. Nomerotski, M. Perelstein, et al., *The International Linear Collider Technical Design Report - Volume 2: Physics*, [arXiv:1306.6352](#).
- [163] H. Georgi and M. Machacek, *DOUBLY CHARGED HIGGS BOSONS*, *Nucl. Phys.* **B262** (1985) 463–477.
- [164] M. S. Chanowitz and M. Golden, *Higgs Boson Triplets With $M(W) = M(Z) \cos \theta_W$* , *Phys. Lett.* **165B** (1985) 105–108.
- [165] P. Galison, *Large Weak Isospin and the W Mass*, *Nucl. Phys.* **B232** (1984) 26–60.
- [166] R. W. Robinett, *EXTENDED STRONGLY INTERACTING HIGGS THEORIES*, *Phys. Rev.* **D32** (1985) 1780.
- [167] H. E. Logan, *Radiative corrections to the $Z b$ anti- b vertex and constraints on extended Higgs sectors*. PhD thesis, UC, Santa Cruz, 1999. [hep-ph/9906332](#).
- [168] S. Chang, C. A. Newby, N. Raj, and C. Wanotayaroj, *Revisiting Theories with Enhanced Higgs Couplings to Weak Gauge Bosons*, *Phys. Rev.* **D86** (2012) 095015, [[arXiv:1207.0493](#)].
- [169] H. E. Logan and V. Rentala, *All the generalized Georgi-Machacek models*, *Phys. Rev.* **D92** (2015), no. 7 075011, [[arXiv:1502.01275](#)].
- [170] J. Hisano and K. Tsumura, *Higgs boson mixes with an $SU(2)$ septet representation*, *Phys. Rev.* **D87** (2013) 053004, [[arXiv:1301.6455](#)].
- [171] S. Kanemura, M. Kikuchi, and K. Yagyu, *Probing exotic Higgs sectors from the precise measurement of Higgs boson couplings*, *Phys. Rev.* **D88** (2013) 015020, [[arXiv:1301.7303](#)].
- [172] C. Alvarado, L. Lehman, and B. Ostdiek, *Surveying the Scope of the $SU(2)_L$ Scalar Septet Sector*, *JHEP* **05** (2014) 150, [[arXiv:1404.3208](#)].
- [173] J. F. Gunion, R. Vega, and J. Wudka, *Higgs triplets in the standard model*, *Phys. Rev.* **D42** (1990) 1673–1691.
- [174] J. F. Gunion, R. Vega, and J. Wudka, *Naturalness problems for $\rho = 1$ and other large one loop effects for a standard model Higgs sector containing triplet fields*, *Phys. Rev.* **D43** (1991) 2322–2336.

- [175] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide*, *Front. Phys.* **80** (2000) 1–404.
- [176] H. E. Haber and H. E. Logan, *Radiative corrections to the $Z b$ anti- b vertex and constraints on extended Higgs sectors*, *Phys. Rev.* **D62** (2000) 015011, [[hep-ph/9909335](#)].
- [177] M. Aoki and S. Kanemura, *Unitarity bounds in the Higgs model including triplet fields with custodial symmetry*, *Phys. Rev.* **D77** (2008), no. 9 095009, [[arXiv:0712.4053](#)]. [Erratum: *Phys. Rev.* **D89**, no. 5, 059902 (2014)].
- [178] S. Godfrey and K. Moats, *Exploring Higgs Triplet Models via Vector Boson Scattering at the LHC*, *Phys. Rev.* **D81** (2010) 075026, [[arXiv:1003.3033](#)].
- [179] I. Low and J. Lykken, *Revealing the electroweak properties of a new scalar resonance*, *JHEP* **10** (2010) 053, [[arXiv:1005.0872](#)].
- [180] H. E. Logan and M.-A. Roy, *Higgs couplings in a model with triplets*, *Phys. Rev.* **D82** (2010) 115011, [[arXiv:1008.4869](#)].
- [181] A. Falkowski, S. Rychkov, and A. Urbano, *What if the Higgs couplings to W and Z bosons are larger than in the Standard Model?*, *JHEP* **04** (2012) 073, [[arXiv:1202.1532](#)].
- [182] C.-W. Chiang and K. Yagyu, *Testing the custodial symmetry in the Higgs sector of the Georgi-Machacek model*, *JHEP* **01** (2013) 026, [[arXiv:1211.2658](#)].
- [183] C.-W. Chiang, A.-L. Kuo, and K. Yagyu, *Enhancements of weak gauge boson scattering processes at the CERN LHC*, *JHEP* **10** (2013) 072, [[arXiv:1307.7526](#)].
- [184] C. Englert, E. Re, and M. Spannowsky, *Triplet Higgs boson collider phenomenology after the LHC*, *Phys. Rev.* **D87** (2013), no. 9 095014, [[arXiv:1302.6505](#)].
- [185] R. Killick, K. Kumar, and H. E. Logan, *Learning what the Higgs boson is mixed with*, *Phys. Rev.* **D88** (2013) 033015, [[arXiv:1305.7236](#)].
- [186] C. Englert, E. Re, and M. Spannowsky, *Pinning down Higgs triplets at the LHC*, *Phys. Rev.* **D88** (2013) 035024, [[arXiv:1306.6228](#)].
- [187] A. Efrati and Y. Nir, *What if $\lambda_{hhh} \neq 3m_h^2/v$* , [[arXiv:1401.0935](#)].
- [188] K. Hartling, K. Kumar, and H. E. Logan, *The decoupling limit in the Georgi-Machacek model*, *Phys. Rev.* **D90** (2014), no. 1 015007, [[arXiv:1404.2640](#)].

- [189] C.-W. Chiang and T. Yamada, *Electroweak phase transition in Georgi–Machacek model*, *Phys. Lett.* **B735** (2014) 295–300, [[arXiv:1404.5182](#)].
- [190] C.-W. Chiang, S. Kanemura, and K. Yagyu, *Novel constraint on the parameter space of the Georgi–Machacek model with current LHC data*, *Phys. Rev.* **D90** (2014), no. 11 115025, [[arXiv:1407.5053](#)].
- [191] S. I. Godunov, M. I. Vysotsky, and E. V. Zhemchugov, *Double Higgs production at LHC, see-saw type II and Georgi–Machacek model*, *J. Exp. Theor. Phys.* **120** (2015), no. 3 369–375, [[arXiv:1408.0184](#)].
- [192] K. Hartling, K. Kumar, and H. E. Logan, *Indirect constraints on the Georgi–Machacek model and implications for Higgs boson couplings*, *Phys. Rev.* **D91** (2015), no. 1 015013, [[arXiv:1410.5538](#)].
- [193] C.-W. Chiang and K. Tsumura, *Properties and searches of the exotic neutral Higgs bosons in the Georgi–Machacek model*, *JHEP* **04** (2015) 113, [[arXiv:1501.04257](#)].
- [194] S. I. Godunov, M. I. Vysotsky, and E. V. Zhemchugov, *Suppression of $H \leftarrow VV$ decay channels in the Georgi–Machacek model*, *Phys. Lett.* **B751** (2015) 505–507, [[arXiv:1505.05039](#)].
- [195] S. Chang and J. G. Wacker, *Little Higgs and custodial $SU(2)$* , *Phys. Rev.* **D69** (2004) 035002, [[hep-ph/0303001](#)].
- [196] S. Chang, *A ‘Littlest Higgs’ model with custodial $SU(2)$ symmetry*, *JHEP* **12** (2003) 057, [[hep-ph/0306034](#)].
- [197] L. Cort, M. Garcia, and M. Quiros, *Supersymmetric Custodial Triplets*, *Phys. Rev.* **D88** (2013), no. 7 075010, [[arXiv:1308.4025](#)].
- [198] M. Garcia-Pepin, S. Gori, M. Quiros, R. Vega, R. Vega-Morales, and T.-T. Yu, *Supersymmetric Custodial Higgs Triplets and the Breaking of Universality*, *Phys. Rev.* **D91** (2015), no. 1 015016, [[arXiv:1409.5737](#)].
- [199] S. El Hedri, P. J. Fox, and J. G. Wacker, *Exploring the Dark Side of the Top Yukawa*, [arXiv:1311.6488](#).
- [200] LUX, D. S. Akerib et al., *Results from a search for dark matter in the complete LUX exposure*, *Phys. Rev. Lett.* **118** (2017), no. 2 021303, [[arXiv:1608.07648](#)].
- [201] K. Hartling, K. Kumar, and H. E. Logan, *GMCALC: a calculator for the Georgi–Machacek model*, [arXiv:1412.7387](#).

- [202] A. Djouadi, J. Kalinowski, and P. M. Zerwas, *Two and three-body decay modes of SUSY Higgs particles*, *Z. Phys.* **C70** (1996) 435–448, [[hep-ph/9511342](#)].
- [203] A. G. Akeroyd, *Three body decays of Higgs bosons at LEP-2 and application to a hidden fermiophobic Higgs*, *Nucl. Phys.* **B544** (1999) 557–575, [[hep-ph/9806337](#)].
- [204] **ATLAS**, G. Aad et al., *Search for anomalous production of prompt same-sign lepton pairs and pair-produced doubly charged Higgs bosons with $\sqrt{s} = 8$ TeV pp collisions using the ATLAS detector*, *JHEP* **03** (2015) 041, [[arXiv:1412.0237](#)].
- [205] S. Kanemura, M. Kikuchi, H. Yokoya, and K. Yagyu, *LHC Run-I constraint on the mass of doubly charged Higgs bosons in the same-sign diboson decay scenario*, *PTEP* **2015** (2015) 051B02, [[arXiv:1412.7603](#)].
- [206] J. Schechter and J. W. F. Valle, *Neutrino Masses in $SU(2) \times U(1)$ Theories*, *Phys. Rev.* **D22** (1980) 2227.
- [207] T. P. Cheng and L.-F. Li, *Neutrino Masses, Mixings and Oscillations in $SU(2) \times U(1)$ Models of Electroweak Interactions*, *Phys. Rev.* **D22** (1980) 2860.
- [208] M. Magg and C. Wetterich, *Neutrino Mass Problem and Gauge Hierarchy*, *Phys. Lett.* **94B** (1980) 61–64.
- [209] G. Lazarides, Q. Shafi, and C. Wetterich, *Proton Lifetime and Fermion Masses in an $SO(10)$ Model*, *Nucl. Phys.* **B181** (1981) 287–300.
- [210] R. N. Mohapatra and G. Senjanovic, *Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation*, *Phys. Rev.* **D23** (1981) 165.
- [211] **LEP Higgs Working Group for Higgs boson searches**, **ALEPH**, **DELPHI**, **L3**, **OPAL**, *Search for charged Higgs bosons: Preliminary combined results using LEP data collected at energies up to 209-GeV, in Lepton and photon interactions at high energies. Proceedings, 20th International Symposium, LP 2001, Rome, Italy, July 23-28, 2001*, 2001. [[hep-ex/0107031](#)].
- [212] **Particle Data Group**, J. Beringer et al., *Review of Particle Physics (RPP)*, *Phys. Rev.* **D86** (2012) 010001.
- [213] **Heavy Flavor Averaging Group**, D. Asner et al., *Averages of b -hadron, c -hadron, and τ -lepton properties*, [[arXiv:1010.1589](#)].
- [214] F. Mahmoudi, *SuperIso: A Program for calculating the isospin asymmetry of $B \rightarrow \bar{\ell} K^* \gamma$ in the MSSM*, *Comput. Phys. Commun.* **178** (2008) 745–754, [[arXiv:0710.2067](#)].

- [215] **DEAP**, P. A. Amaudruz et al., *DEAP-3600 Dark Matter Search*, *Nucl. Part. Phys. Proc.* **273-275** (2016) 340–346, [[arXiv:1410.7673](#)].
- [216] P. Cushman et al., *Working Group Report: WIMP Dark Matter Direct Detection*, in *Proceedings, 2013 Community Summer Study on the Future of U.S. Particle Physics: Snowmass on the Mississippi (CSS2013): Minneapolis, MN, USA, July 29-August 6, 2013*, 2013. [arXiv:1310.8327](#).
- [217] J. Billard, L. Strigari, and E. Figueroa-Feliciano, *Implication of neutrino backgrounds on the reach of next generation dark matter direct detection experiments*, *Phys. Rev.* **D89** (2014), no. 2 023524, [[arXiv:1307.5458](#)].
- [218] A. Desai and A. Moskowitz, “DMTools Limit Plot Generator.” <http://dmtools.brown.edu>. Accessed: 2019-04-26.
- [219] **ATLAS**, G. Aad et al., *Constraints on new phenomena via Higgs boson couplings and invisible decays with the ATLAS detector*, *JHEP* **11** (2015) 206, [[arXiv:1509.00672](#)].
- [220] **ATLAS, CMS**, G. Aad et al., *Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV*, *JHEP* **08** (2016) 045, [[arXiv:1606.02266](#)].