EXPLORATORY STUDIES IN A NEW THREE-DIMENSIONAL
PHOTOTHERMOELASTIC TECHNIQUE

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A thesis submitted to the Faculty of Graduate Studies in
partial fulfilment of the requirements for the degree of
Doctor of Philosophy.

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Carleton University

Ottawa, Ontario

May, 1972

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The undersigned recommend to the Faculty of Graduate
Studies acceptance of the thesis

"Exploratory Studies in a New Three-
Dimensional Photothermoelastic Technique"
submitted by Satish Kumar Bhan, in partial fulfilment of the
requirements for the degree of Doctor of Philosophy.

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Mechanics and Structural Engineering

Carleton University

May 8, 1972

ABSTRACT

A new technique of freezing thermal strains due to a steady temperature gradient has been investigated. It consists of casting and curing a photoelastic model under the desired temperature profile. Thus the casting solidifies and cures stress-free and still supports a temperature profile. If this temperature profile is now removed above the critical temperature, thermal stresses related to the temperature profile and the material constants are developed. These stresses can be frozen by cooling the model to room-temperature. Internal stresses caused by any mechanical constraints or loads during the casting process can be annealed by removing the model from the mold and heating it to the critical temperature. In doing so, these stresses will be released due to the absence of all restraints whereas the thermal strains will remain as long as the temperature is uniform in the model. These thermal strains will be refrozen on cooling.

The method has been evaluated by hollow thin disks under steady-state temperature profile in the radial direction. The isochromatic fringe order in the vicinity of the outer and the inner radius has seriously been affected by rapidly developing time-edge stresses which are tensile in nature. A model similar to the main model has been cast under a uniform temperature. The time-edge effect fringe order distribution in the latter model has

been superimposed on the distribution in the main model to obtain the thermal stress fringe pattern. Comparison of the isochromatic fringe order obtained in this manner has shown close agreement with theory.

The experimental details along with the calibration techniques have been described. Recommendations to discover and employ materials with lesser time-edge effects have been made.

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to his advisor, Dr. G. W. Bigg, for his guidance during the author's stay at Carleton University and for his guidance throughout this investigation. Mr. R. C. Sampson provided the author with information about the epoxy material and made other suggestions; the author is very grateful to Mr. Sampson for his help. The author also wishes to thank Mr. John Herler and his staff in the workshop for their co-operation during the fabrication of the experimental equipment. Thanks are also due to Messrs. J. Patry and S. M. Sathikh for their suggestions during the progress of this investigation.

This project was funded in part by the National Research Council Grant A3294 and the financial assistance is gratefully acknowledged.

Finally, the author is grateful for the patient understanding shown by his wife, Linda, during the preparation of this thesis.

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NOMENCLATURE

b	Width of tensile calibration specimen
c_1, c_2	Arbitrary constants
D	Diameter of calibration disk
E	Modulus of Elasticity, psi
f	Material fringe value, psi.in.per fringe
k	Coefficient of thermal conductivity, BTU per foot. $^{\circ}\text{F} \cdot \text{hour}$
ℓ	Axial length of the model
L	Gage length on calibration specimens
n	Isochromatic fringe order
N_0	Isochromatic fringe order at the center of the calibration disk
P	Applied Load, pounds
Q	Rate of heat conduction, BTU per hour
Q_{α}	Photothermoelastic figure of merit
r	Radius
t	Thickness of slice
T	Temperature, $^{\circ}\text{F}$
ΔT	Temperature difference between the outer and the inner radius of the cylinder, $^{\circ}\text{F}$
u	Radial displacement
x,y,z	Rectangular coordinates
X	Ratio (r/r_i) for hollow disk and ratio $(2x/D)$ for Calibration disk

Y	Ratio $(2y/D)$ for calibration disk
Z	Ratio (axial distance from one end/axial length of hollow disk)
α	Coefficient of thermal expansion, inch per inch, $^{\circ}F$
ϵ	Normal strain
β	Ratio of the outer to the inner radius
δ	Change in length
λ	Variance
ν	Poisson's ratio
σ	Normal stress
γ	Percentage curing shrinkage in material
o, i	Subscripts representing the outer and inner surface of a hollow cylinder, respectively
x, y, z, r, θ	Subscripts representing the three rectangular, radial and tangential directions, respectively.
long., lat.	Subscripts representing the longitudinal and the lateral directions, respectively.

I. INTRODUCTION

A. PRELIMINARY

The efficiency of high speed turbomachines depends on the permissible level of stresses in the rotating parts and on the character of the load. In particular, performance of rotors in gas turbines is influenced by thermal stresses which occur due to the temperature gradients between the center and the rim during starting, stopping and change of load¹(1). During load changes, the temperature equilibrium in the rim is reached about five to six times faster than in the center of the rotor. This enhances the temperature difference and hence the thermal stresses in the rotor disk. The turbine blades in some gas turbines are cooled by a coolant passing through a radial hole in the blade while hot gases pass on the outside. Thermal stresses generated by these temperature differences within the blade can exceed the material yield point which then leads to a decrease in the strength of the blades and in some cases rupturing them completely⁽²⁾. It is extremely difficult to make theoretical and direct experimental studies of stress distributions in the working elements of these turbines.

¹ Number in parenthesis refers to the number of Reference in the REFERENCES.

David⁽³⁾ reported incidences of case cracking in steam turbine casings during the daily load variations in British thermal power plants. The present recommended 'start up' and 'operational' procedures being based upon operational experience rather than accurate thermal stress analysis. High-temperature strain gage techniques have been used to evaluate the 'surface thermal strains' but the overall thermal stress distribution within the body has not been determined.

Thermal stresses arise in heat exchanger plants and nuclear reactor plants during load variations. The very high temperatures in the cores compared with the cooler outside in nuclear reactors results in thermal stresses; a full knowledge of their distribution is essential to sound design.

Failure of diesel engine pistons is caused by effects of repeated combustion pressure and the steady temperature gradient through the piston crown. The fatiguing action of repeated combustion pressures combined with the tensile stress due to temperature gradient usually causes cracking on the coolant side of the piston at the junction of the crown and wall⁽⁴⁾.

Analytical results show that aerodynamic heating causes significant thermal stresses in aircraft structures⁽⁵⁾; these calculations have been, so far, based on idealized representations of wing structures. Accurate theoretical solution of the

thermal stress problem is not available.

A rocket propellant-grain is usually a thick-walled hollow cylindrical grain bonded to the motor shell. The inner boundary of the grain cross-section is usually star shaped with sharp fillets. The coefficient of thermal expansion of the propellant is several times larger than the coefficient of thermal expansion of the case. Hence, with a uniform change in temperature differential thermal expansion or contraction causes thermal stresses in both the grain and the case. A change in temperature of the motor shell results in temperature gradient through the grain thus, again, causing thermal stresses with concentrations in the fillets⁽⁶⁾.

A low temperature fluid flowing at various levels in a pipe acts as a heat sink and thus introduces a temperature gradient through the pipe⁽⁷⁾. The resulting thermal stresses deform the pipe shape and may result in large reaction forces and moments at the supports which may thereby cause high local stresses⁽⁸⁾.

Besides the few examples listed above there are numerous structural components which are subject to thermal stresses. Theoretical analysis of these stresses in three dimensional bodies is relatively complex and available exact solutions are limited to a few simple configurations and loading conditions⁽⁹⁾. Recently, the finite element method has been developed and it has found application to some thermal stress problems (10,11). This numerical

method has two serious limitations: 1. Maximum stresses in a structure commonly occur at sudden changes in geometry like notches, grooves and holes. The number of nodes necessary to handle these geometries is large. However, the number of nodes used is limited by the capacity of the computers. 2. The results of a numerical analysis need to be verified experimentally.

Consequently, considerable interest centers upon the experimental techniques of determining thermal stresses.

B. EXPERIMENTAL STRESS ANALYSIS TECHNIQUES.

Having illustrated the need for experimental analysis of thermal stress problems it is advisable to discuss the various tools of experimental stress analysis available. The most commonly used is the 'strain-gage' technique. This method is very simple, cheap and least time consuming. The use of strain-gage techniques is somewhat involved at high temperatures and generally entails large scale metal models with extensive heating facilities for thermal-strain measurements. Recently, the use of strain gages has become more popular with the introduction of high-temperature strain-gage techniques. The need for expensive heating facilities has been eliminated by employing plastic models which have a much higher coefficient of thermal expansion than their metal prototypes⁽⁴⁾. Plastics being materials with a low coefficient of

thermal conductivity, exhibit unusual effects. The gage current forms a hot spot under the gage due to the poor dissipation of heat generated by the current. This results in an increase in the gage temperature. There is an apparent strain because of a change in gage resistance with temperature and the differential thermal expansion between the gage and the plastic base. Finally, there is a reinforcing effect of the gage installation on the model. Lawton⁽⁴⁾ has fully studied these effects and investigated the contribution of each to the error in the measured strain.

Brittle coating and stress coating techniques⁽¹²⁾ have limited application and cannot be employed with much accuracy to structural components with complex geometries. The Moire-⁽¹³⁾ and the Grid-techniques have found a wide application in many engineering problems but they are mostly restricted to simpler geometries. Holography has been recently applied to some simpler thermoelastic problems⁽¹⁴⁾.

In all these methods only the surface strains can be measured. In April 1970 Epelle and Meyer⁽¹⁵⁾ reported the measurement of thermal strains by strain gages embedded in epoxy model. The validity of the results has been questioned by many discussers. Although it is true that maximum stresses frequently occur at structural boundaries, a complete stress analysis through the body of the structure is essential for sound design.

Three dimensional photoelasticity appears to be a promising technique for the complete thermoelastic stress analysis of complex structure. 'Scattered light' technique^(16,17) can be employed to analyse special thermoelastic problems. However, fringe patterns obtained in this method are faint and difficult to evaluate. Another problem in the application of such a method is in the application of thermal loading without interfering with the already complex optical instrumentation.

C. PHOTOTHERMOELASTICITY

When a transparent homogeneous isotropic elastic material is in an unstressed state it exhibits a uniform index of refraction at all points and in all planes in the material. However, when the material is stressed it becomes double refracting with the fast and the slow axes coinciding with the axes of principal stress in the plane perpendicular to the axis of propagation of light². In 1853, Maxwell noted that changes in the indices of refraction were linearly proportional to the principal stresses². This optical phenomenon is called the photoelastic effect and its application to elastic stress problems is called photoelasticity.

Photothermoelasticity is a branch of photoelasticity that

² PP. 166, Ref. (12)

deals with thermal stresses in the elastic range. Thermal stresses can result from one or a combination of the following:

1. When a structural component is made out of more than one material with different coefficients of thermal expansion, any change in the temperature of the component will cause differential thermal expansion which results in generating thermal stresses even in the absence of external constraints.

2. A temperature difference between the various parts of a body will generate mutual constraints within the body and result in thermal stresses.

3. A sudden change in temperature in any part of the body will result in transient thermal stresses.

Photothermoelasticity is strictly a modelling technique. Interpretation of test results on a model, in terms of stresses in the prototype require special considerations (19, 20). At this stage, photothermoelasticity, while useful, is limited only to certain types of applications listed later.

D. PURPOSE AND SCOPE OF INVESTIGATION

With the development of better photoelastic materials and refinement of the photoelastic techniques three dimensional photoelasticity has been widely accepted as a refined and accurate tool of experimental stress analysis. Although inability to

match Poisson's ratio of the photoelastic model (≈ 0.5) to that of the prototype ($\approx 0.3 - 0.4$) diminishes to some degree the value of the frequently used 'freezing technique', practice has shown that information from such tests is frequently of invaluable assistance in engineering design (21).

Three dimensional photothermoelasticity has so far found application to only a certain class of problems. Some techniques developed either determine the surface strains only (22) or the stresses in a preselected plane of the three dimensional model (23). In some special class of problems, mechanical analogies have been used to simulate thermal loading (21, 24). A photo-thermoelastic method to determine the complete stress distribution in a three-dimensional body under a temperature gradient is yet to be developed. Many problems in engineering design encounter transient thermal stresses. Amongst these a large number of problems have a relatively slow change in temperature as in the case of a rocket-propellant grain and an aircraft structure in changing altitude. In the above cases the inertia effect can be assumed to be relatively small. With this assumption, the thermal stresses can be determined as in the case of a steady temperature gradient by using a gradient of temperature that corresponds to a particular time (24).

It is the purpose of this investigation to develop a

new three-dimensional photothermoelastic method for application to a general thermoelastic problem which involves a steady temperature gradient. 'Stress Freezing' is by far the most versatile photoelastic technique. When applied in its conventional form to models with temperature gradients, the resulting fringe pattern requires interpretation in terms of highly variable mechanical and physical material properties (23). This fringe pattern is a result of both frozen and unfrozen stresses (22). Any slicing of such a model disturbs the unfrozen stress distribution and makes it impossible to interpret the fringe pattern in the slice.

In this investigation a modified form of the 'freezing technique' has been devised to freeze only the thermal stresses in the model. This frozen stress model can be analysed for stresses in the conventional form of the 'frozen stress' technique of three-dimensional photoelasticity. The new method has been applied to a two-dimensional problem with a known exact theoretical solution. The experimental results have been compared with theory to assess the accuracy of this technique.

E. RESUME OF PREVIOUS WORK

Although Weibel (25) was the first to apply photothermoelasticity to the solution of thermal stress problems, it was Gerard and Gilbert (5) who established it as a reliable method.

Initially, the method was applied to two-dimensional problems and considerable effort went into developing the validity of the photothermoelastic method by comparing analytical solutions of well-known thermoelastic problems with the experimental results.

After successful application of photothermoelasticity to two-dimensional problems (26-28), Gerard and Trampusch (23) extended photothermoelasticity to three-dimensional problems by devising the 'embedded polariscope method', also called the 'cemented polariscope method'. In this method a polariscope, consisting of two thin sheets of polarizing material, is cemented into the model with a preselected layer of material between the polarizers. Stress distribution in the specific layer of the model can be interpreted from the photoelastic fringe pattern. This method has limitations in that the fringe pattern is confined to a preselected region of the model and only one set of isoclinics within the region are visible, which makes the separation of stresses difficult. The latter limitation has been modified recently (29, 30). Strength of cemented joints in such models has been found to be high. These joints are stress free and do not disturb the continuity of stress distribution. Because of its non-destructive nature and lower Poisson's ratio, this method may be preferred to the conventional 'stress freezing' method in some applications. This method has found a wide application to thermoelastic problems involving both the steady temperature gradients as well as the

transient thermal gradients (7, 8, 27, 28, 31-39). Photovisco-elastic behavior in three-dimensional bodies has also been studied by employing this method (40).

In 1961, Gerard and Trampusch (23) disqualified the 'stress freezing' technique for three-dimensional photothermoelasticity because of the reportedly 'rapidly changing material properties in the range of the critical temperature and the difficult interpretation of the resulting fringe pattern in terms of highly variable physical and mechanical properties'.

In 1969, Lambie (41) reported a methyl methacrylate composition of exactly zero birefringence and this gave birth to a new method known as the 'sandwich technique'. In this method a model is constructed out of the zero birefringence material (commercially known as "Perspex") into which a birefringent layer of Araldite is cemented in the plane under investigation. Stress distribution in an extrusion container has been studied by using the sandwich technique (42). Again, this method has the following limitations:

1. The stress distribution can be determined only in a pre-selected plane of the model.
2. Complete matching of the mechanical properties between the birefringent and the non-birefringent components of the model is difficult.

The design of a Rocket Propellant-grain has been the subject of research for a long time (6, 24, 43-49). Following are the various loading conditions on the grain:

1. Thermal loading: the three types of thermal loading are:

(a) Differential thermal expansion: The coefficient of thermal expansion for the grain is much higher than that for the casing. With a uniform change in temperature of the motor, thermal stresses result from differential thermal expansion or contraction of the grain and the casing. Durelli (24) simulated this type of loading with restrained curing shrinkage. Sampson (47) utilized the property of homogenous and isotropic contraction accompanying the slow polymerization of epoxy resins to produce a three dimensional photoelastic model of the same size and shape, at the elevated temperature, as the container it was cast in. Reducing the temperature of the bonded model-container composite structure through the transition temperature of the epoxy material resulted in photoelastic stress patterns which corresponded to differential thermal contraction stresses between the model and the container. These patterns were analysed by using the conventional three-dimensional photoelastic techniques. Durelli and Parks (49) came up with a two-dimensional simulation of the problem. Internal pressure in the model was assumed to simulate, as a first approximation, the stress distribution produced by restrained shrinkage.

In a second method, a steel ring with a urethane-rubber model of the grain cross-section was subjected to cold temperatures and the photoelastic isochromatic patterns were recorded for various temperatures. In a third approach, the mechanical equivalent of thermal loading involved the manufacturing of a urethane-rubber grain model. This model was oversized by about one percent relative to the rigid model of the case into which it was forced. The resulting photoelastic fringe pattern was recorded.

(b) Steady temperature gradients: The steady temperature difference between the inside of the grain and the outside of the casing introduces thermal stresses in both components of the structure. This problem has been solved by Durelli (24) with the help of a mechanical analogy developed by Biot (50). It consists in cutting the multiply connected cylinder as many times as necessary to make it simply connected. Mechanical distortions (radial, tangential and rotational) are applied to the lips of these cuts. These displacements are calculated from analogy between the Laplace equations for plane steady-heat flow and plane voltage distribution. These distortions are applied to the cuts by means of a deformer (25, 43, 44).

(c) Transient thermal loading: Transient thermal stresses result from sudden changes in temperature of either of the motor boundaries. Bonded polariscope techniques have been used to study these stresses (46).

2. Internal pressure: this loading has been studied and reported in Ref. (48).

In early applications of photothermoelasticity the model materials used suffered large changes in their mechanical and physical properties at temperatures not too far above the room temperature. Thus thermal loading was introduced by cooling; in most cases dry ice was used. With the development of better photoelastic materials (51) this practice has been replaced by using higher temperatures (47).

In another application of photothermoelasticity to a special class of problems, Slot (20) simulated thermal stresses by mechanical prestraining. This method consists of building a photoelastic model out of components in which mechanical strains are frozen in such a way that with the rise in temperature these mechanical prestrains are released and result in model stresses that correspond to the thermal stresses of interest. Applicability of this method, however, depends on the ability to obtain properly pre-strained components. The thermal stresses in a circular disk with a temperature step across a diametral plane was simulated by cementing a prestrained semidisk (tension along the diametral plane) to an unstrained semidisk. When the model disk was heated beyond the critical temperature of the model material, the frozen prestrains in the disk were released and the associated expansion

produced thermal stresses that corresponded to the temperature field of interest. These stresses in the model were frozen on subsequent cooling to room temperature. Leven and Sampson (52) used a prestrained tension strip in a two dimensional model of a nuclear fuel plate assembly to simulate differential thermal expansion between hot fuel bearing interior plates and a cooler non-fuel bearing end plate. Slot (20) used the prestraining method to investigate the thermal stresses in the divider-lane region of a U-tube type steam generator.

Photoelastic materials used in the 'freezing technique' have a Poisson's ratio of nearly 0.5. The particular value of Poisson's ratio imposes a constant-volume condition on the prestraining method. Thermal strains are always accompanied by a volume change; hence, they cannot be simulated properly. However, when the thermal strain field of interest varies in one direction only, the thermal stresses can be simulated fairly accurately. In many cases, the error in stresses due to imperfect prestraining is of localized nature only. The scope of application of the method has been widened recently by the application of 'unit thermal fields' which permits the modelling of thermoelastic problems by a quasistationary approach (53).

With the introduction of high-temperature strain-gage techniques, attempts were made, notably by Dennis and Radford (54), to measure steady thermal stresses in piston heads of diesel

engines. However, various difficulties involved in application of these techniques included the problems of compensating the bridges, gage instability at temperatures above 750° F, hot spots forming under the gage and the high cost of heating and cooling equipment for high temperature rigs. In 1968, Lawton (4) applied the strain gage technique to plastic models of diesel engine piston heads. Thermal strains are in proportion to the product ($\alpha\Delta T$). Plastics have a coefficient of thermal expansion much greater than that for the metal prototypes and thus large thermal strains can be obtained by using a much lower range of temperatures. In this particular application calculations were made to account for the errors caused in strain reading due to hot spots under the strain gages. Bush (55) employed the bonded strain gage technique to measure thermal stresses in composite cylinders over the range - 47° F to 255° F. The inner cylinder was made of cured epoxy - impregnated glass tape and the outer cylinder was made of flexible epoxy casting resins. Considerable effort has been made at determining apparent strain corrections. Epelle and Meyer (15) embedded strain gages in epoxy models to measure thermal strains. The discussion of Ref. 15 by other authors involved in similar work brings forward various problems involved in such an application.

The scattered light method has been used to study three-dimensional stresses in shrink fit problems (56). Measurement of

thermal strain in various other practical problems is discussed in references (2, 57-63).

Jessop (64) has reported that the thermal stresses in photoelastic materials constitute a difficult problem for the investigators. He suggested that this sensitivity of the material may be used at any rate to give some qualitative information regarding thermal stresses in structures and machines. Interpretation of the fringe pattern was described as impossible even in the case of a simple three-dimensional model.

The earlier attempts at freezing thermal strains due to a temperature gradient were made by Hosp (65) in 1961 and by Smith and Haines (66) in 1965. Hosp reported that the fringe pattern in his uniformly cold disk was the same as would have been expected if the thermal strains had been frozen in but the introduction of notches into his disk disturbed the pattern. No explanation of this observation was offered. Smith and Haines reported that the stresses induced by cooling a cylinder of stress freezing photoelastic plastic which supported a given temperature profile were of two types: frozen stresses and unfrozen stresses. A variation of the scattered light technique was employed to analyse thermal strains without slicing the model. In 1969, Haines and Wright (22) analysed the frozen and the unfrozen stresses reported in the previous work. The removal of the temperature profile at room

temperature has been suggested as the cause of the unfrozen stresses developed along with the frozen stresses. The disturbance of these unfrozen stresses by notching is most likely the cause of the phenomenon noticed by Hosp (described earlier). A detailed discussion of reference (22) is given in Appendix A. A new strain-gage technique has been devised to measure the surface thermal strains in such models. A strain gage rosette was bonded to the surface of the model (with combined frozen and unfrozen strains). The portion of the model, to which the strain gage rosette was bonded, was removed from the rest of the model thus releasing all unfrozen strains. The change in the resistance of the strain gage was observed as a measure of the unfrozen surface thermal strains.

Similarity relationships for the uncoupled thermo-elastic problems have been derived for thermal displacements, strains and stresses between model and prototype by Hovanesian and Kowalski (19). Favre and Schumann (67) have proposed a method to obtain sufficient information photoelectrically in order to determine separately thermal stresses on the surface of two dimensional models.

II. THEORY OF THE NEW PHOTOTHERMOELASTIC TECHNIQUE

A. STRESS FREEZING:

Stress freezing is frequently used in three-dimensional photoelasticity and the various available techniques are:

1. Conventional Stress Freezing. The conventional stress freezing method was first developed in 1936 by G.U. Oppel and consists of locking in the model deformations due to applied load. It is based on a diphase model of the behavior of many polymeric materials when they are heated (68). Some of the long chain hydrocarbon molecules of polymeric materials are well bonded into a three-dimensional network of primary bonds. However, a large number of molecules are less solidly bonded together into shorter secondary chains. When the polymer is at room temperature, both sets of bonds, the primary and the secondary, act to resist the deformation due to applied load. As the temperature of the material increases these secondary chains break up continuously until a temperature, known as the 'Critical temperature' of the material is reached. At this temperature all the secondary chains are broken and the primary bonds, in effect, carry the entire applied load. The critical temperature is different for each material. Since the primary bonds constitute a very small portion of the polymer, the deformations in the primary bond structure are quite large yet elastic in character. If the temperature of the model

is lowered while the load is maintained constant, the secondary bonds reform between the highly deformed primary bonds and thus serve to lock these deformations. At room temperature when the load is removed, the secondary bonds allow only a small relaxation of the deformations in the primary bonds, which are now permanently locked on a molecular level. Thus these deformations and the associated birefringence are maintained in any small section cut from the original model. In these polymers the process of breaking down of the secondary bonds is a function of time and temperature. At temperatures above the critical temperature, the maximum fringe order is obtained instantly and for temperatures lower than the critical temperature the variation of fringe order with time is shown in Fig. 1, taken from Ref. (12). Ordinarily, the load on the model is maintained for a couple of hours before cooling to the room temperature. This eliminates the creep effects which are usually present during the first few minutes.

In the three-dimensional photoelastic method, sections of interest are cut out of the stress frozen models and the locked-in birefringence analysed. Standard methods have been developed to calculate the three-dimensional stress distribution from the photoelastic data (69, 70).

The conventional stress freezing method is not applicable to freezing thermal strains caused by a temperature gradient. When

a model is subjected to a temperature profile (above the critical temperature), thermal strains are produced in the primary phase of the material. On cooling the model through the critical (or freezing) temperature, as each element of the secondary phase freezes around the already deformed primary phase, it becomes stress free but still supports a temperature profile. Thus, when the room temperature conditions are reached and the temperature profile is still supported the secondary phase is stress free and has the deformations of the primary phase locked-in. When this temperature profile is removed, a stress system is induced in the secondary phase and is related to the temperature profile and the post-freezing constants of the material. Therefore, the birefringence observed in the model at room temperature consists of two parts: frozen and unfrozen (22). The separation of these two quantities is very complex and involves a large number of unknown variables. Any sectioning of the model will disturb the unfrozen stresses and thus make the interpretation of the photoelastic data more complex. However, this method can be used along with strain-gage techniques to obtain surface thermal strains as proposed by Haines and Wright (22).

2. The Creep Method (71, 72)

The creep method is also known as the extended frozen-stress method and is based upon the fact that many plastics which

creep exhibit birefringence which remains directly proportional to the load even after considerable creep. A model (usually manufactured from a phenolformaldehyde type of plastic) is allowed to creep under load at room temperature for a long time (up to 1 month in some cases). The deformation and the birefringence increase with time. When the load is removed, there is an instantaneous recovery. However, a portion of the birefringence, which is proportional to the elastic stress distribution remains. The recovery of this residual birefringence is slow enough to allow examination of one or more slices removed from simple models before the recovery is complete. This method can be improved by employing epoxy materials and loading at slightly elevated temperatures ($\approx 140^{\circ}\text{F}$ - not high enough to break the secondary bonds). The creep rate of the material is greatly accelerated at this temperature and the desired response is obtained in a much shorter time. If the model is cooled to room temperature before unloading, the creep rate governing the recovery of the model is greatly reduced. This recovery may extend over a period of several months thus allowing the investigator to slice the model and examine the slices at his own convenience. In the temperature range of this method, the Poisson's ratio of the material is low and close to that of most engineering materials. In comparison with the conventional freezing method, the modulus of elasticity and the material fringe constant are high for the materials used in the creep method. Thus, the creep method is preferred to the

conventional stress freezing method for problems in which the following factors are involved:

(a) the stress distribution is appreciably effected by the magnitude of Poisson's ratio (certain bending problems for plates).

(b) only small deformations of the elastic body can be tolerated.

(c) thin members buckle prematurely at the critical temperature.

The creep method is not applicable to thermal stress problems with temperature gradients for the following reasons:

(a) The creep rate is a non-linear function of temperature. Thus the different regions of the model would creep to a different amount. It is highly unlikely that the resulting birefringence will be directly proportional to the thermal stress distribution.

(b) The removal of the temperature profile will introduce residual stresses such that the resultant birefringence will include the effect of both the thermal as well as the residual stress distributions. Separation of these two stresses would involve too many unknown variables.

The author has failed to find any application of this method to a thermal stress problem in the available literature and no such attempt has been made in this investigation.

3. The Curing Method (73)

The stage-curing phenomena typical of thermosetting resins is used to lock deformations in the curing method. When an epoxy-resin model is cast, the liquid mixture gels after some time into a mass which has the consistency of a soft rubber eraser. At this stage the polymerization process is well in progress and some molecular chains have been formed. The model is loaded (mechanically) at this stage, thus causing deformation in the already formed molecular chains. By maintaining the load on the model until the polymerization is completed, new strain-free molecular bonds are formed around the originally deformed bonds which are thus locked in their deformed state. When the load is removed, the locked-in birefringence remains and the model can be sliced and examined to determine the three-dimensional elastic stress distribution.

To the author's knowledge, no mention has been made of any application of the curing method to photothermoelastic problems in the available literature.

Replacing the mechanical load by a thermal load due to temperature gradient in the curing method involves the same problems as in the conventional stress freezing method. To illustrate, consider an epoxy-resin model in its gel-stage and apply a temperature gradient. This will result in deformation

of the already formed bonds. If the temperature gradient is maintained until the polymerization is completed, new molecular bonds will be formed around the originally deformed bonds and thus lock them in their deformed state. However, it is important to realize that the molecular bonds that are formed after the temperature gradient is applied, are stress free and still support the original temperature profile. When the polymerization is completed and the temperature gradient removed, a stress system is introduced into the molecular bonds which are formed after the gel stage. The associated birefringence is related to the temperature gradient. The resultant birefringence consists of a frozen part and an unfrozen part. The separation of these two components is difficult. Any slicing of the model will disturb the unfrozen stresses thus making the interpretation of the photoelastic data more complex.

It is, therefore, concluded that none of the three methods discussed above have a direct application in freezing thermal stresses caused by a temperature profile.

B. MULTIPHASE THEORY OF PLASTICS: (74, 75)

The properties observed with the plastics used as model materials for stress freezing can be very well explained by the multiphase theory. These materials are assumed to be composed of an elastic phase and a plastic phase (like a sponge soaked with

grease). The modulus of elasticity of the elastic phase is nearly independent of temperature and it follows Hooke's law. This phase corresponds to the primary bonds in the two phase theory illustrated earlier.

The plastic phase has a multitude of components of different softening temperatures. This phase is hard at low temperatures and grows softer with rising temperatures until it is completely soft at a temperature known as the critical temperature. These assumptions have been derived from the creep behavior of these plastics. Creeping occurs in plastic stressed at any temperature but the rate of deformation is higher with higher stress and higher temperature. At the critical temperature full deformation at a certain stress will be reached in a few seconds and will be completely relieved within a short time on removal of the stress. The velocity of deformation or redeformation decreases after loading or unloading respectively. If this velocity were proportional to the difference between the instantaneous deformation and the definite one, it would lead to the assumption that the plastic phase consists of a uniform material. Actually, the velocity of deformation decreases faster than the difference of deformations which means some components of the plastic phase deform faster than others.

In photoelastic models at room temperature, the load is

partially carried by the elastic phase and partially by the plastic phase. The stress remains low in the elastic phase because its modulus of elasticity is much lower than that determined from the deformations at low temperature. At temperatures above the critical temperature, when the plastic components are soft or liquid, the entire load is carried by the elastic part. Thus, the deformations are much higher at the high temperature³ than at the low temperature. In the freezing method, the deformations in the elastic phase are locked in by the plastic phase which, at high temperature, has changed its shape to conform to the deformation of the elastic phase.

It has been found by Kuske (75) that the double refraction is proportional to stress in the elastic phase as well as in the plastic phase (it is not to be confused with plastic deformation in the elastic phase). The photoelastic constant is different in the two phases and is opposite in sign for some photoelastic materials like polyester. This explains the decreasing photoelastic effect with increasing deformation in some materials. With increasing deformation or temperature, stress increases in the elastic phase and thus the resulting double

³ High temperature refers to temperature above the critical temperature.

refraction is influenced more and more by this phase. If the sign of the photoelastic constant is different in the two phases, double refraction will decrease and become zero at a certain deformation. With further increasing deformation at constant load, the double refraction will change its sign and increase asymptotically to a certain amount. All materials showing increased birefringence with increased deformation under constant load have the same sign of the photoelastic constant for the elastic as well as the multi-component plastic phase.

Chemists question the multiphase nature of plastics and explain the mechanical behavior of plastics by a decreasing inner friction at increasing temperatures. The change of the sign of photoelastic constants with temperature, however, cannot be explained in this way.

Javornicky (76) has questioned the validity of Kuske's multiphase theory on the basis of its disagreement with the chemical and physical nature of macromolecular materials. The physical-chemical investigation of the reason for birefringence in plastic leads to the presumption of a double nature; stress birefringence and orientation birefringence. The polarization of macromolecules is assumed to be the result of the internal rotation of the individual links of the molecular chains. This molecular motion is influenced both by the internal forces between the links

and by the stretching and shifting of the whole chains. Javornicky has concluded that the law of final birefringence for one-dimensional state of stress beyond the elasticity limit, ϵ_e , is

$$\delta = [c\sigma + K(\epsilon - \epsilon_e)]t$$

where δ is the final birefringence, c is the photoelastic constant, ϵ is the strain due to conventional stress σ , K is the proportionality constant of the orientation birefringence, ϵ_e is the strain corresponding to the elasticity limit and t is the thickness of the model.

It has been noted by Pindera (77) that the interpretation of 'stress freezing' effects on the basis of the so-called "bi-phase" or "multi-phase" theory is purely phenomenological and not capable of predicting the behavior of material on the basis of its structure, and is contradictory to known facts. The particular photoviscoelastic behavior of high polymers in the process of creep deformation can be explained in a consistent manner by the influence of orientation birefringence, as explained by Javornicky. According to Pindera, since these orientation effects are not always linear functions of strain, it is not correct to assume that the photoelastic effects at the temperature above the critical temperature are linearly related to stress in the whole range of deformation, as it is commonly assumed in the freezing techniques. Alfrey⁴ holds that the different retarded elastic

⁴ Reference 78, pp.164.

elements do not refer to different kinds of materials but to different mechanisms of response of a single homogeneous material. For this reason, a multiphase structure should not be assumed merely to account for the existence of complex photoelastic properties in a material. The fundamental origin of such properties lies in the possibility of different modes of response, rather than the presence of different types of material, each possessing simple mechanical properties.

From the above discussion it is clear that although there is not a complete agreement between the various authors about the exact nature of the photoelastic materials, they have a common goal in explaining the photoelastic properties of these materials. Thus, for explaining the phenomenon of 'stress freezing', the 'bi-phase theory' and the 'multi-phase' theory are acceptable within the elastic limit of stress above the critical temperature. The present investigation deals with stresses within the elastic limit and the creep deformation is assumed to be negligible in the temperature range above the critical temperature. With this assumption the 'bi-phase' model of the material satisfactorily explains the phenomenon of 'stress freezing'. The author feels that although the 'bi-phase' theory does not correctly explain the total behavior of the photoelastic plastics, within the limits laid above, it is an accurate model of the material behavior. It must also be noted that it is the deformations or strains and

not the stresses that are locked in. However, within the elastic limit they are linearly related and the term 'stress freezing' can be used safely.

C. THERMOELASTIC STRESS FREEZING

In the proposed technique of thermoelastic stress freezing studied in the present investigation, the model is cast out of a liquid epoxy at room temperature. After the initiation of polymerization within a few hours of the pouring, the desired temperature profile is applied to the model (which is in the form of a very thick viscous liquid at that time). Polymerization is an exothermic reaction and increases in intensity with temperature. After the initial stages of polymerizations are over, the temperature level of the whole model can be safely raised without the danger of excessive exothermic reaction. Polymerization is accompanied by shrinkage which is roughly balanced by thermal expansion. This is achieved by gradually raising the temperature level of the model to above the critical temperature of the material, at a predetermined rate. The temperature profile is maintained constant throughout this time. The model is then retained at this level of temperatures for a couple of days to complete the process of polymerization. As each element of the material solidified, it is stress free but retains the temperature profile. At the end of this period the model is still stress free under the temperature

profile. This temperature profile is now removed and the whole model is maintained at a constant temperature (above the critical temperature); thermal stresses are developed in the model which are related to the temperature profile and the material constants in the range of the temperature profile. Thermal stresses are a function of the product $E\alpha$, the temperature gradient and the geometry of the model. If the product $E\alpha$ and the material fringe constant are constant over the range of the temperature profile, the birefringence resulting from the thermal stresses can be very easily interpreted. It is important to realise that for this model which has been cast stress free under a certain temperature profile, a uniform temperature is equivalent to a thermal loading condition on a model machined out of material cast at uniform temperature. The thermal load for such an equivalent model is a temperature profile which is the reverse of the one used in the real model. To illustrate, consider two geometrically similar uniform beams of rectangular cross-section with free ends (Fig. 2). Model A is cast in the manner described earlier with a steady state temperature profile as shown in Fig. 2a. At this stage, the model is stress free. Model B is machined out of a stress free casting which has been cast at uniform temperature. In Fig. 2c, model B is shown with uniform temperature and therefore stress free. When the temperature profile is removed from Model A, thermal stresses, as shown in Fig. 2b, are

developed. Similar thermal stresses are developed in Model B, if the temperature profile (as shown in Fig. 2d) is applied to it. It may be observed that the temperature profile in Fig. 2d is the reverse of the temperature profile under which Model A was cast. Thus Model B can be assumed to be the equivalent of Model A in the corresponding states shown in Fig. 2.

For a free body, a uniform change in temperature does not result in any thermal stresses. Hence, for a model cast under a temperature profile, a uniform state of temperature at any temperature above the critical temperature results in thermal stresses which are related to the temperature profile under which the model has been cast. These thermal stresses result in deformation of the primary bonds which can be locked in by slowly cooling the model to room temperature. On cooling, the secondary bonds form around the already deformed primary bonds. This locking-in process takes place on a molecular level and the deformations in the primary phase are retained even after slicing the model.

During the cooling process, stresses resulting from differential thermal expansion between the mold components and the model; and stresses arising from restrained shrinkage may be frozen-in along with the thermal stresses. In order to remove these undesirable effects, the model is removed from the mold and

slowly heated to a temperature above the critical temperature. It is retained at this temperature for a few days until the locked-in stresses due to differential thermal expansion and restrained curing shrinkage are annealed. It should be recalled here that since the temperature of the model is uniform (and above the critical temperature) the thermal stresses related to the temperature profile (although not locked-in anymore) are always present. During the same annealing cycle calibration models are loaded. After annealing for a couple of days only the desired thermal stresses are present. Deformations resulting from these stresses are locked in by slowly cooling the model to room temperature. This model can be sliced and the thermal stress distribution determined by using the well established techniques of three-dimensional photoelastic analysis.

In a minor variation of the above method, if the model geometry is changed in any form such as by slicing and a part or portion of the model is annealed at the high temperature, the thermal stress distribution will change. This can be explained by recalling that at a temperature above the critical temperature, the thermal stresses are not locked-in. In such a state, the equivalent model (Fig. 2) will have the new geometry (although the same temperature profile) and therefore a new thermal stress distribution. This will result in a redistribution of thermal

stresses and on cooling the deformations resulting from the new stress distribution will be locked-in. Therefore, the model should be cured before slicing.

III EXPERIMENTAL VERIFICATION

A. EXPLORATORY EXPERIMENT

1. Purpose

In order to assess the accuracy of the proposed photo-thermoelastic method it was essential to apply it to a thermoelastic problem with an accurately known theoretical solution. There are only a few exact theoretical solutions to thermal stress problems available (9) and out of these, a thin disk with a concentric hole under a steady-state temperature profile in the radial direction only, was chosen. This exploratory experiment was designed to:

(a) Choose an appropriate epoxy material; evaluate its performance in the application of the technique; establish a standard casting procedure and a thermal cycle;

(b) Find out if the temperature profile can be maintained constant without creating an excessively large exothermic reaction.

(c) Evaluate the effect of thermocouples embedded in the model on the isochromatic fringe order distribution and establish the accuracy of the temperature measuring devices.

(d) Find out if the model is stress free at the end of the curing cycle above the critical temperature, as proposed in the theory of the new technique. Also, to observe the variation of the isochromatic fringe order in the model during

the thermal cycle.

(e) Determine the resulting isochromatic fringe order distribution in the model and compare it qualitatively with the theoretical solution.

(f) Determine the material properties and establish standard calibration techniques.

2. Material

In photothermoelasticity, the thermal stresses are related to the product $E \alpha$ and the fringe order is related to $(E \alpha / f)$. Unlike mechanical loading where the load may usually be increased to obtain a sufficiently high fringe order for the desired accuracy, thermal loading is usually limited by the available temperature difference that can be established experimentally. Most epoxy resins deteriorate at a temperature not too far above their critical temperature, thus limiting the available temperature difference. Therefore, it is desirable to use materials of sufficiently high sensitivity to achieve some accuracy in the photothermoelastic investigation. This sensitivity is conveniently judged by use of the photothermoelastic figure of merit, Q_α , defined in Ref. (5) as

$$Q_\alpha = \alpha E / f .$$

It is well known from the available information that epoxy resins have a relatively high Q_α .

The main requirement for an epoxy material to be used in the method is the constancy of the photothermoelastic figure of merit in the range of the temperature profile above the critical temperature. Many castable epoxy resins commonly used in three-dimensional photoelastic stress freezing meet this requirement to some extent. This technique requires a material to be castable under a temperature profile. A number of desirable qualities to assess the superiority of one material over the other are listed below, in the order of preference:

- (a) Constancy of the photothermoelastic figure of merit in the range of temperatures above the critical temperature.
- (b) High photothermoelastic figure of merit.
- (c) Mild exothermic reaction during polymerization.
- (d) Low creep at temperatures above the critical temperature.
- (e) Homogeneity of polymerization shrinkage within a casting.
- (f) Low polymerization shrinkage to avoid excessive shrinkage stresses.
- (g) Moderately low critical temperature.
- (h) Large temperature difference between the deterioration and the critical temperature.

(i) Low time-edge effects.

(j) Other desirable qualities as in conventional stress freezing materials such as good machineability, high optical transparency and high strength above the critical temperature.

Due to the limited time available, the author did not make any effort to develop a new epoxy resin composition most suitable for this application. Out of the available materials, the epoxy composition developed by Sampson (47) was chosen. It consists of 50 parts by weight (pbw) Bakelite ERL-2774 resin, 50 pbw Bakelite ERL-2795 and 25 pbw amine hardener Bakelite ZZL-0803. The same material has been used by Marloff and Daniel (60).

3. Design of Model

(a) Theoretical Calculations of Thermal Stresses in a Thin Hollow Circular Disk (9):

When the temperature, T does not vary over the thickness of the disk, it is assumed that the displacement and stress due to thermal gradient also do not vary over the thickness. In polar coordinates, the shear stress, $\tau_{r\theta} = 0$ due to the symmetry about the center. The stresses, σ_r and σ_θ satisfy the equation of equilibrium:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1)$$

If ϵ_r represents the actual radial strain, $\epsilon_r - \alpha T$ represents the part due to stress, where T is the temperature change from the uniform temperature state of the disk. Thus, we have

$$\epsilon_r - \alpha T = \frac{1}{E} (\sigma_r - \nu \sigma_\theta) \quad (2)$$

and similarly

$$\epsilon_\theta - \alpha T = \frac{1}{E} (\sigma_\theta - \nu \sigma_r) \quad (3)$$

solving equations (2) and (3) for σ_r , σ_θ we find

$$\sigma_r = \frac{E}{1-\nu} [\epsilon_r + \nu \epsilon_\theta - (1+\nu)\alpha T] \quad (4)$$

and

$$\sigma_\theta = \frac{E}{1-\nu} [\epsilon_\theta + \nu \epsilon_r - (1+\nu)\alpha T] \quad (5)$$

Substituting from the above two equations in eq.(1) we have,

$$r \frac{d}{dr} (\epsilon_r + \nu \epsilon_\theta) + (1-\nu)(\epsilon_r - \epsilon_\theta) = (1+\nu)\alpha r \frac{dT}{dr} \quad (6)$$

If u denotes the radial displacement, the radial and tangential strains are given by

$$\epsilon_r = \frac{du}{dr}, \quad \epsilon_\theta = \frac{u}{r} \quad (7)$$

Substituting these in eq. (16), we obtain

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = (1 + \nu) \alpha \frac{dT}{dr}$$

or

(8)

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d(ru)}{dr} \right] = (1 + \nu) \alpha \frac{dT}{dr}$$

Integrating both sides, we find

$$u = (1 + \nu) \alpha \frac{1}{r} \int_{r_i}^r T r \, dr + c_1 r + \frac{c_2}{r} \quad (9)$$

where r_i is the arbitrary lower limit in the case of a circular disk with a hole of radius r_i .

The stress components are now found by using the solution from eq. (9) in eqs. (4) and (5). Then,

$$\sigma_r = -\alpha E \cdot \frac{1}{r^2} \int_{r_i}^r T r \, dr + \frac{E}{1 - \nu} [c_1(1 + \nu) - c_2(1 - \nu) \frac{1}{r^2}] \quad (10)$$

$$\sigma_\theta = \alpha E \cdot \frac{1}{r^2} \int_{r_i}^r T r \, dr - \alpha E T + \frac{E}{1 - \nu} [c_1(1 + \nu) + c_2(1 - \nu) \frac{1}{r^2}] \quad (11)$$

The constants c_1 and c_2 are determined from the boundary conditions $\sigma_r = 0$, at $r = r_i$ and $\sigma_r = 0$, at $r = r_0$. Substituting these boundary conditions in eq. (10), we have

$$0 = 0 + \frac{E}{1-\nu} \left[c_1(1+\nu) - c_2(1-\nu) \cdot \frac{1}{r_i} \right] \quad (12)$$

and

$$0 = -\alpha E \cdot \frac{1}{r_0} \int_{r_i}^{r_0} Tr dr + \frac{E}{1-\nu} \left[c_1(1+\nu) - c_2(1-\nu) \cdot \frac{1}{r_0} \right] \quad (13)$$

Subtracting eq. (13) from eq. (12)

$$c_2 = \frac{\alpha(1+\nu)}{\left(\frac{r_0}{r_i} - 1\right)} \int_{r_i}^{r_0} Tr dr \quad (14)$$

and

$$c_1 = \frac{(1-\nu)}{(1+\nu)} \frac{c_2}{r_i} \quad (15)$$

The radial steady state temperature profile in a hollow circular cylinder is given in Ref. (90) as

$$T(r) - T_i = - \frac{(T_i - T_0)}{\ln(r_0/r_i)} \cdot \ln(r/r_i) \quad (16)$$

where $T(r)$ is the temperature at a radius r and T_i and T_0 are the inner and outer surface temperatures, respectively.

If the temperature, T_i is the reference temperature at which no stress conditions existed and $\Delta T = T_i - T_0$ is the temperature difference between the inner and outer radius, the above equation can be rewritten as

$$T = - \frac{\Delta T}{\ln \beta} \cdot \ln \left(\frac{r}{r_i} \right) \quad (17)$$

where $\beta = r_0/r_i$.

Evaluating the integral

$$\begin{aligned} \int_{r_i}^r T r dr &= \int_{r_i}^r - \frac{\Delta T}{\ln \beta} \ln \left(\frac{r}{r_i} \right) r dr \\ &= - \frac{\Delta T}{\ln \beta} \left[\frac{r^2}{2} \ln \left(\frac{r}{r_i} \right) - \frac{r^2}{4} + \frac{r_i^2}{4} \right] \end{aligned} \quad (18)$$

Therefore

$$\int_{r_i}^{r_0} T r dr = - \frac{\Delta T}{\ln \beta} \left[\frac{r_0^2}{2} \ln \beta - \frac{1}{4}(r_0^2 - r_i^2) \right] \quad (19)$$

Substituting from eq. (19) into eqs. (14) and (15), we have

$$c_2 = - \frac{\alpha(1+\nu)}{(\beta^2 - 1)} \frac{\Delta T}{\ln \beta} \left[\frac{r_0^2}{2} \ln \beta - \frac{1}{4}(r_0^2 - r_i^2) \right] \quad (20)$$

and

$$c_1 = - \frac{\alpha(1-\nu)}{(\beta^2 - 1)} \frac{\Delta T}{\ln \beta} \left[\frac{\beta^2}{2} \ln \beta - \frac{1}{4}(\beta^2 - 1) \right] \quad (21)$$

Substituting from eqs. (14-21) in eqs. (10), (11) we have

$$\sigma_r = - \frac{\alpha E}{r^2} \int_{r_i}^r T r dr + \frac{E}{(1+\nu)} c_2 \frac{(X^2 - 1)}{r^2}$$

and

$$\sigma_{\theta} = \frac{\alpha E}{r} \int_{r_i}^r T r dr + \alpha E \Delta T \frac{\ln X}{\ln \beta} + \frac{E}{(1+\nu)} c_2 \frac{(X^2 + 1)}{r^2} \quad (22)$$

or

$$\sigma_r = \frac{\alpha E \Delta T}{2} \left[- \frac{\beta^2}{(\beta^2 - 1)} \cdot \frac{(X^2 - 1)}{X^2} + \frac{\ln X}{\ln \beta} \right] \quad (22)$$

and

$$\sigma_{\theta} = \frac{\alpha E \Delta T}{2} \left[- \frac{\beta^2}{(\beta^2 - 1)} \cdot \frac{(X^2 + 1)}{X^2} + \frac{\ln X}{\ln \beta} + \frac{1}{\ln \beta} \right] \quad (23)$$

Substituting these values in the stress-optic law:

$$\sigma_{\theta} - \sigma_r = \frac{fn}{t}$$

where n is the isochromatic fringe order, t is the thickness of the model and f is the stress-optical constant (also known as the material fringe value). Therefore,

$$\frac{\alpha E \Delta T}{2} \left(\frac{2\beta^2}{\beta^2 - 1} \cdot \frac{1}{X^2} + \frac{1}{\ln \beta} \right) = \frac{fn}{t}$$

or

$$n = - \frac{\alpha E t \Delta T}{2f} \left(\frac{2\beta^2}{\beta^2 - 1} \cdot \frac{1}{X^2} - \frac{1}{\ln \beta} \right) \quad (24)$$

where

$$\Delta T = (T_i - T_0) \quad .$$

The fringe order distribution in a long hollow circular cylinder under similar boundary conditions can be calculated in a similar manner to be (9)

$$n' = - \frac{\alpha E t}{2(1-\nu)} \frac{\Delta T}{f} \left(\frac{2\beta^2}{\beta^2 - 1} \cdot \frac{1}{X^2} - \frac{1}{\ln \beta} \right) \quad (25)$$

A comparison of the theoretical stress distribution in the two cases is shown in Fig. 3.

The radial displacement can be calculated by substituting the values in eq. (9), we have

$$\begin{aligned} u &= - (1+\nu) \frac{\alpha}{r} \frac{\Delta T}{\ln \beta} \left[\frac{r^2}{2} \ln X - \frac{1}{4}(r^2 - r_i^2) \right] \\ &\quad - \frac{\alpha(1+\nu)}{(\beta^2 - 1)} \frac{\Delta T}{\ln \beta} \cdot r \left[\frac{r_0^2}{2} \ln \beta - \frac{1}{4}(r_0^2 - r_i^2) \right] \left[\frac{(1-\nu)}{(1+\nu)} \frac{1}{r_i^2} + \frac{1}{r^2} \right] \\ &= - \frac{(1+\nu) \alpha \Delta T}{4 \ln \beta} \cdot r_i \left[2X \ln X - \frac{(X^2 - 1)}{X} \right] \\ &\quad + \frac{1}{(\beta^2 - 1)} \{ \beta^2 (2 \ln \beta - 1) + 1 \} \left\{ \frac{(1-\nu)}{(1+\nu)} X + \frac{1}{X} \right\} \end{aligned} \quad (26)$$

(b) Dimensions of the Model

The model employed in this experiment was a thick walled hollow cylinder exposed to a steady state heat flow in the radial direction with constant temperature distribution along the axis.

The cylindrical model was used so that a thin disk machined out of the model in the midlength region would have no temperature variation through the thickness. This thin disk when cured would represent a thin disk model for which the theoretical thermoelastic solution has been determined earlier. The overall size of the model was limited by the casting facilities available. The ratio of the outer to inner radius was chosen as 8 , in order to get sufficient isochromatic fringes in a fairly large field. The dimensions of the proposed model (without accounting for curing shrinkage) are as follows:

outer diameter = 6 in.

inner diameter = $3/4$ in.

length of the cylinder = $2\frac{1}{4}$ in.

The critical temperature of the material is in the range of 170-175°F (47). The material deteriorates in a few hours at temperatures above 215°F (80). This limits the working temperature range for the material to less than 40°F. A temperature differential of 35°F between the higher temperature at the inner radius and the temperature at the outer radius was chosen for this experiment.

The theoretical solution of the thermoelastic problem has been shown earlier. Calculations of the isochromatic fringe order at the inner and outer radius (assuming the values of the

material constants from the average of the values given by Sampson in Ref. (47)) showed that the isochromatic fringe order was high enough to be recorded accurately with the available equipment (Fig. 4). Thin circular disks removed from the model when cured at the high temperature would be examined for photoelastic data and the results correlated with the thin disk (plane stress case) solution.

4. Design of Equipment

(a) Mold

Figure 5 shows the sketch outline of the mold in this exploratory experiment. The mold was fabricated from aluminum cylinders. Aluminum is a very good conductor of heat and thus efficient in removing the exothermic heat of polymerization from the model to the outside. The mold was supported such that the axis of the cylindrical model remained horizontal.

The two vertical faces of the model were made out of 1/4" thick pyrex glass plates. Three quarters of one inch diameter concentric holes were drilled in the circular glass plates which were placed in thin brass rings with threads on the outside. Rings cut out from 1/32 inch thick teflon sheets were used as sealants between the brass rings and the glass plates. The inside of the main aluminum mold was threaded on both sides to receive the brass rings. Teflon rings were used as sealants

along with a liberal use of high vacuum grease. A 5 degree taper was made on the inner radius of the glass disks to receive the tapered core tube. A thin layer of aluminum was deposited on the outer face of one of the glass plates by using the process of vacuum deposition. This surface was faced outward from the main model thus acting as a mirror for observation of the model through the other glass plate. A reflection - polariscope arrangement utilizing this mirror surface was designed to obtain photoelastic data from the model during the casting and curing processes. The glass faces of the mold were insulated on the outside by 3 3/4 inch long cylindrical Malenite insulation blocks. Four circular holes 1 1/2 inches in diameter each were bored out in one insulation block at a staggered central distance from the main axis such that the entire radial field of the cylindrical model could be observed through these holes. The holes in the block were fitted with 4 loose-fit Malenite plugs.

Aluminum pipes were used to transfer the heating oil to and from the core-tube. The ends of the tubes were tapered to seal the connection between the core-tube, the glass plate and the tubes themselves. Calculations were made (assuming standard material constants) for the relative thermal expansion of the brass ring and the aluminum cylindrical mold. The clearance between these two increases at higher temperatures by a negligible amount.

cores have been recommended by many authors for use in three-dimensional photoelastic castings (51). It is flexible enough to take most of the curing shrinkage of epoxy material without resulting in excessive stresses.

A hollow RTV rubber core-tube, 3/4" outside dia., was cast in a brass mold. The wall thickness of the tube was limited to 1/16" to reduce the temperature drop across the wall thickness. Simple heat transfer calculations indicated that the temperature differential across the wall thickness of the core tube did not exceed 8°F at the upper working temperature range of the experiment. The small wall thickness resulted in some problems in the casting of the rubber core. The rubber was too thick to flow down the 1/16" wide passage in the mold. Excessive amounts of thinner resulted in a highly flexible tube. After a few trials the problem was solved by leaving the liquid rubber mixture in a reservoir on top of the mold and allowing it to flow down by placing the mold in a vacuum chamber at 29 inches of mercury vacuum. The correct length of the rubber tube was cut and the ends were tapered on a grinder to allow its smooth entry into the tapered holes in the glass plates.

(b) Temperature Control

A temperature differential between the inner and the outer radius of the model was introduced by raising the temperature

at the inner radius above that at the outer radius. Paraffin oil (0-120, Fisher Scientific Co.) was passed through the core-tube to control the temperature at the inner radius while the temperature at the outer radius was controlled by the oven temperature. The temperatures indicated by the thermocouples, embedded at the inner and the outer radius of the model, were used to measure the temperature differential on the model. The oil was heated outside the oven in a 'Lab-Line Constant Temperature Bath' provided with a coil heater, mechanical stirrer and thermostatic temperature control. A small centrifugal pump was used to pump the oil through 1/4 in. diam. flexible copper tubing (insulated with figerglass wool) into the core of the model and back into the oil bath. The temperature at the inner radius was raised in steps by changing the temperature setting on the oil bath every hour. This manual temperature control was found to result in a maximum variation of 4^oF from the desired temperature differential. The mold was placed in a 'hotpack' oven recommended for use in three-dimensional photoelastic analysis which thus controlled the temperature at the outer radius of the model. The temperature cycle of the oven was programmed on a cam which automatically controlled the oven temperature.

(c) Temperature Measurement

A thick walled cylinder with constant temperature distribution along its axis when exposed to steady-state heat flow

from the inside to the outside has an axisymmetrical radial temperature distribution. The radial temperature distribution in the model was monitored by twelve copper/constantan, 30 gage, glass braded thermocouples connected to a twelve point Bristol Multipoint Recorder. The recorder recorded the output of all the thermocouples within one minute. Preliminary experiments were conducted to observe the effect of thermocouples embedded in epoxy on the stress distribution in the epoxy model. Various cylindrical castings were made with the combined thermocouple wire and each lead separately embedded in the epoxy. In all cases high stresses were generated which affected a large area of the models. Since the thermocouples could not be removed from the models, these residual stresses were not annealed by curing at temperatures above the critical temperature. Thermocouple wires impregnated with variable thicknesses of RTV rubber were tried; this resulted in a decrease in the intensity of stresses but still effected a large area of the model around the wires. Finally, bare thermocouple leads were embedded in the epoxy and the resulting stresses were found to be small and very localized in nature. The epoxy acts as a good electrical insulation. The thermocouple grid was made out of bare conductors of copper and constantan. The location of the thermocouples is shown in Fig. 6(a). Thermocouple junctions were soldered by using a soft silver solder. The thermocouple grid output was checked against the thermometer reading in a constant temperature bath; a maximum variation in the recorded temperature equal to one percent of the true temperature was

noted. Insulation was retained on the thermocouple leads near the outer radius of the mold. Epoxy cement (CIBA 6020-100 pbw and hardner CIBA HN951-10 pbw) was used to cement the thermocouple grid to the aluminum mold.

The thermocouple output in the temperature range of the experiment exceeded the 1 millivolt range of the recorder. Figure 6(b) shows the potentiometer circuit used to feed only a part of the output to the recorder. The rheostat readings were calibrated to determine the amount of reduction in the thermocouple output. Appendix B shows the calculations for the design of the circuit. An ice-bath was used as the reference point.

Crushed ice was allowed to stabilize at room temperature before pouring into a thermos-flask. One quarter of the volume was filled with cold water. The cold junction of the thermocouple was placed centrally at two thirds depth from the level of the ice, as recommended in Ref. 79. The ice in the cold bath was changed every 24 hours.

The rheostat in the potentiometer circuit had to be adjusted every 4 hours in order to keep the output of all thermocouples within the range of the recorder during a changing temperature cycle.

The output of the thermocouple junction adjacent to the

core tube in the model was used as a guide to control the temperature of the oil bath. The difference of the temperature output of the thermocouple junctions at the inner and the outer radius of the mold was monitored constant by the oil temperature in the core tube. Thus, as the oven temperature was raised at a uniform rate, the temperature of the oil in the core tube was adjusted every hour such that the above mentioned thermocouples indicated a constant temperature differential on the model.

5. Experimental Procedure

In the early stages of the experiment the casting procedure for the material was unknown. The first casting was made by pouring the hardener ZZL-0803 into a mixture of the epoxies ERL-2774 and ERL-2795 at 170°F. This resulted in an instantaneous and highly exothermic reaction. The mixture started hardening up into a beehive type structure which started expanding at an alarming rate and had to be discarded. The material components were mixed successfully at room temperature. Further exploratory tests were not needed as most of the introductory information about the material was received from its developer (80).

(a) Mold Preparation

The aluminum mold was cleaned with acetone before assembly. A thin coating of Dow Corning-7 mold release compound was applied on the inner surface of the mold. A very fine layer

of the mold release compound was applied to the RTV rubber core and the thermocouple leads. Pallet (81) has recommended a procedure for applying a very thin permanent film of mold release on aluminum molds for repeated use. This procedure was used to coat the glass plates. It was found at the end of the first experiment that the epoxy had adhered to glass plates and thus resulted in excessive stresses due to restrained shrinkage and differential thermal expansion. In the next attempt at casting the model, the glass plates were coated with a liberal use of the mold release compound. The mold release, although fairly transparent at room temperature, turns into a whitish opaque substance on contact with epoxy at high temperatures. Thus the glass plates were not transparent anymore and the photoelastic observations in the model during the casting and curing processes could not be made. The mold was assembled and the thermocouple grid was placed in the designed location. The relative position of the various thermocouple junctions was located from the inner surface of the mold. A standard hollow cylindrical mold and a plate mold were also cleaned and coated with the mold release compound for casting the calibration disk and plate models, respectively.

(b) Casting

Fifty parts by weight (pbw) of epoxy ERL-2774 and fifty pbw of epoxy ERL-2795 were mixed at room temperature and heated to 140°F. The mixture was relatively thin at this temperature and

permitted complete mixing. The mixture was cooled to room temperature and 25 pbw of the amine hardener ZZL-0803 was added to it. The mixture was continuously stirred with a glass rod for five minutes after which it was degassed under 29 inches of mercury vacuum until the large mass of bubbles disappeared. This did not take more than fifteen minutes. The epoxy mixture was fairly thin and was poured into the molds. In two hours the level of the epoxy in the main mold had gone down due to polymerization shrinkage. Epoxy (much thicker at the moment) was poured into the empty mold space continually for the next three hours. A few air bubbles were entrapped in the epoxy during pouring, but most of these bubbles rose to the surface soon after pouring.

(c) Thermal Cycle

The temperature distribution in the model was recorded from the time the epoxy was poured into the mold. Two hours after the pouring of the casting, the temperature of the oil was raised such that the difference in the temperatures indicated by the thermocouples near the inner and the outer radius of the model was 35°F. The oven was retained at room temperature. It took about 5 hours for the temperature distribution in the model to reach a steady state. This state of temperature was maintained until 24 hours after the pouring. The oven temperature was then increased uniformly at the rate of 1°F per hour. The temperature of the oil in the core tube was raised every hour such that the temperature

difference between the inner and the outer radius of the model was maintained at 35°F . After the temperature at the outer radius of the model had reached 170°F and that at the inner radius equal to 205°F , further heating was stopped. This temperature profile was sustained for three days. A temporary disturbance in the temperature profile was caused by a power failure on the last day and it took about four hours to stabilize the original temperature profile. At the end of the three day period, the oil supply was cut off and the model was allowed to reach a uniform temperature of 170°F . It took two hours for the model to achieve a uniform temperature (within 2°F of the oven temperature). After allowing the model temperature to stabilize for three more hours the model was cooled to room temperature at the rate of 1.35°F per hour. Figure 7 shows the designed temperature cycle represented by the variation in the inner and the outer temperatures of the model. Figure 8 shows the actual temperature profile at different intervals of time. The temperature at the inner radius has been extrapolated. The average temperature profile has been compared with the theoretical temperature profile in Figure 9.

(d) Observations

At room temperature the model was removed from the mold and examined in a polariscope. The isochromatic fringe order and the the isoclinic distribution were recorded. Figure 10 shows the

distribution of the isochromatic fringe order on the lower vertical axis of the model. Figure 25 shows the dark and light field isochromatic fringe patterns for the full thickness of the model.

The model was then sliced into four cross-sectional slices of varying thickness on a band saw. Blade speed of 4000 feet per minute and a low feed were used. The slices were then faced on a lathe with a carbide tipped tool. Figure 11 shows the slicing plan for the model. These slices were examined in the polariscope and the isochromatic fringe order distribution was recorded. In order to anneal the undesirable gravitational and other non-thermal stresses these slices were laid flat on a horizontal aluminum plate coated with mold release compound and placed in the oven. The temperature of the slices was raised to 185°F at the rate of 2.25°F per hour, retained at that temperature for 5 days and cooled to room temperature at the rate of 1.35°F per hour. The isochromatic fringe order distribution in each of these slices was again noted. Figures 26, 27 show the comparison of the isochromatic fringe order in the slices, before and after the curing cycle. Figure 11 shows the observed isochromatic fringe order distribution as compared with the theoretical distribution. One of the slices was very carefully cut in the radial direction. This did not result in any noticeable change in the isochromatic fringe pattern existing before the cut was made. This proved the frozen character of the observed birefringence. The fringe order

seemed to be changing with time near the boundaries which was presumed to result from the time-edge effect. However, the observations were made immediately after the slices were removed from the oven. The axisymmetric distribution of stresses resulted in isochromatic fringes as concentric circles and isoclinics running in the radial direction. Tardy's method of compensation was used to measure the fractional fringe order. The thermocouple positions were measured from slice no.3. The experimental temperature distribution was plotted according to these measurements.

6. Calibration

Reference (47) gives the range of material properties which shows a large batch to batch variation. Therefore, it is important to measure the material constants for each batch of castings.

(a) Material Fringe Constant, f

The stress optical law in the elastic limit of stress can be written as

$$\sigma_1 - \sigma_2 = f \cdot \frac{n}{d}$$

where $(\sigma_1 - \sigma_2)$ is the principal stress difference, n is the isochromatic fringe order, d is the thickness of the model through which the polarized light travels and f is the material

fringe constant; f is dependent on the light wave length used.

The standard calibration technique of a circular disk under a diametral concentrated load was employed. A circular disk (2 in. diameter and 1/4" thickness) was machined out of the calibration cylinder cast along with the main model. The disk was raised to 185°F during the annealing cycle. A 5 pound diametral concentrated load was applied to the disk and after five days at the high temperature, the calibration model was cooled to the room temperature at the rate of 1.35°F per hour. The isochromatic fringe order distribution along the horizontal diameter was recorded. The theoretical fringe order distribution has been derived in Appendix C as

$$n = \frac{8P}{\pi f D} \left\{ \frac{1 - X^2}{(1 + X^2)^2} \right\}$$

where P is the diametral load

f is the material fringe constant

D is the diameter of the disk

X is the ratio of the distance from the centre along the horizontal diameter to the radius of the disk,

and n is the fringe order at position X on the horizontal diameter.

The value of $f = 1.4$ psi. in./fringe was calculated from the observed data.

The value of the material fringe constant, f was also calculated from the tensile calibration tests described later. The calculated values from the two specimen are:

$$f_1 = 1.38 \text{ psi. in. per fringe}$$

$$f_2 = 1.43 \text{ psi. in. per fringe}$$

The average value of

$$f = 1.4 \text{ psi. in. per fringe}$$

has been used in calculating the theoretical fringe order distribution.

(b) Modulus of Elasticity and Poisson's Ratio

The measurements of strains in calibration models by a cathetometer and strain gages were found to result in a wide scatter in the values of the constants E and ν .

A crude form of an optical comparator using an optical micrometer was finally employed to measure the strains in the calibration models. An optical micrometer is a very simple optical measuring device consisting of a microscope with a traveling micrometer eye-piece. Variable sensitivity can be obtained by using the different objectives as shown below:

Objective Number	Total Magnification (X)	Sensitivity (inches) $\times 10^{-6}$	Range (inches)
1	312.5	6.36	.00787
2	125	15.75	.0197
3	43.7	43.3	.0551

In the application of an optical micrometer to the measurement of strains, a double cross slide is mounted on the base of the microscope. The specimen is mounted on the cross slide and the gage lines are alternately brought in the field of the microscope by a standard inside micrometer controlled displacement of the slide.

Standard gage lengths are marked on a plastic specimen with the help of a co-ordinatograph. The change in gage length between two gage marks (G1 and G2) is measured in the following manner.

Gage mark G1 is aligned with the zero reading on the micrometer eye-piece scale. The inside-micrometer is adjusted to a length 'd' (very nearly equal to gage length between G1 and G2). The cross-slide is slowly displaced by a distance 'd' by fitting in the fixed inside-micrometer. The reading ' d_1 ' of the gage mark G2 is noted on the micrometer eye-piece scale. Thus the initial gage length equals $(d \pm d_1)$ (the sign depending upon

the side of the zero to which the gage mark G2 falls). The final gage length $(d \pm d_2)$ is measured in a similar manner. Therefore the change in gage length is given by $[(d \pm d_2) - (d \pm d_1)]$ or $\pm (d_2 - d_1)$. Inside-micrometers are dependable gages and thus, at room temperature, distance 'd' is assumed to remain constant between the two readings. This has been proved by accurate reproduction of results.

Initially, the gage lengths were marked with a steel marker. The sides of the gage lines appeared very rough under the microscope and hence, the alignment of the cross-wire with the gage line was not very accurate. Gage lines were replaced by small gage points and the cross-wire in the eye-piece was aligned tangent to the circumference of the gage point (which appears as a circle under the microscope). This arrangement was found to give a much higher accuracy than the gage lines.

Longitudinal and lateral gage lengths of 1 inch were marked on two tensile specimens (one of which is shown in Fig.12). The initial gage lengths were measured in the manner explained above. Two different tensile loads were applied to the two specimens at the critical temperature during the second thermal cycle. On cooling to room temperature the deformations due to the tensile loads were frozen in. The new gage lengths were measured at room temperature. The values of the change in length thus measured,

the applied load and the dimensions of the model were substituted in eqs. C.2, C.3 (Appendix C) to calculate the values of the material constants E and ν . The average values of these constants were calculated as $E = 1850$ psi. and $\nu = 0.49$.

The value of α was assumed constant in the range of temperatures above the critical temperature on the basis of the Sampson's curves in Fig. 13. A gage length of 1 in. was marked on a 1/8 in. thick piece of the plastic. The specimen was placed in an insulated box with glass windows. Hot oil at controlled temperature was passed through the box, which was placed in the field of the microscope. The change in the gage length between 175°F and 185°F was measured with the optical micrometer. The value of α was calculated from the change in length per unit length per °F to be equal to .000098 in. per in. per °F. The variation of α with temperature was not investigated in this experiment and will be discussed in the second experiment.

7. Evaluation of the Experimental Data

(a) Temperature

The electrical output of the thermocouples on the recorder chart was converted into temperatures by using the standard conversion tables. The thermocouple grid had been slightly displaced during the pouring of the epoxy. This displacement was accounted for in plotting the temperature profile. The

leads be placed in the axial direction before joining together.

(iv) The temperature indicated by a thermocouple is the average of the temperatures through the bead. This average may not indicate the true temperature of the epoxy at the apparent location.

(v) Error in the recording instrument.

(b) Photoelastic Data

Figure 25 shows the isochromatic fringe pattern in the model at the end of the first thermal cycle. The concentration of fringes at the bottom of the cylindrical model shows the presence of an external load at that point. This, along with the appearance of the fringe order distribution in the lower half of the model, suggests the presence of gravitational stresses. Reference (84)⁵ gives an equation for calculation of the gravitational stresses in a solid disk. These stresses, superimposed on thermal stresses caused by the temperature profile give an isochromatic fringe pattern close to that observed in the model. The two fringe order distributions are compared in Figure 10. Apparently, the shrinkage of the model on curing resulted in the diameter of the model being less than that of the mold and hence a concentrated reaction on the model at the bottom edge. The reaction at this edge combined with the self-weight resulted in the undesirable non-thermal stresses.

The above observation suggested that the cylindrical model be cast with its axis vertical.

The self weight stresses are, however, annealed in the next thermal cycle as observed in Figs. 26, 27. Figure 11 shows a comparison of the experimental fringe order distribution with the theoretically calculated one. The axisymmetric distribution of stresses was shown by all isoclinics appearing as radial lines. The theoretical distribution has been calculated by using the plane stress formula (eq. 24) derived in the theory. Since the slices are of finite thickness (varying from 0.34 inches to 1.00 inches) the assumption of plane stress is not completely true but only a close approximation. The concentration of rings of high order isochromatic fringes near the boundaries suggested the presence of a tensile time-edge effect. The slices were laid flat on an aluminum plate coated with mold release compound and heated at 110⁰F for four days. The time-edge effect stresses appeared to have considerably reduced at the end of this thermal cycle. Fig. 11 shows a close correlation between the theoretical and experimental distribution except in the region very close to the boundaries. At this stage of the investigation time-edge effect appeared to be the only cause of high fringe order near the inner radius and a reduced fringe order near the outer radius of the model. Other possible sources of error were:

(i) The temperature control system of the oven failed during the time when the polymerization of the epoxy was in progress at the high temperature. It took almost five hours before the original temperature profile could be reinstated.

(ii) Defective fabrication of the glass walls in the mold resulted in the central hole being offset by 1/4 in. from the true center of the mold.

(iii) The temperature at the inner surface of the model was controlled manually and therefore the rise in temperature was not as gradual as that of the outer surface temperature which was controlled by the automatic control on the oven.

(iv) The thermocouple grid had been designed to measure the axial variations of the temperature in the model. Dislocation of the grid at the time of pouring of the epoxy did not permit it to do so. There might have been an axial variation in the temperature of the model.

In spite of the variation between the experimental and the theoretical fringe order near the inner and the outer radius and the exploratory nature of the experiment, it had proved to be a qualitative support to the theory behind the new photothermo-elastic technique. Further application of the technique was desired to evaluate its scope.

In this experiment the temperature profile has been maintained constant without causing excessively large exothermic reactions. The thermocouples have affected the stress distribution in a small region around them. The stress distribution in the model above the critical temperature could not be measured due to experimental difficulties listed earlier.

8. Rebored Disks

In the theory of the new photothermoelastic technique it has been postulated that if a model is cast and cured under a temperature profile, a uniform temperature in the model is equivalent to a reversed temperature profile on an equivalent model. Thermal stresses are thus developed in the model at the critical temperature when the temperature is uniform. These stresses are related to the geometry of the model, the temperature profile it supported during casting and curing and the material constants at the critical temperature. Thermal stresses are frozen in the model at room temperature. If a section of material is now removed from the model and raised to a uniform temperature above the critical temperature, a new thermal stress distribution will be developed. This stress distribution is related to the new geometry and the temperature profile supported by the model during casting. These thermal stresses can be locked-in on cooling the model to the room temperature. To verify this theory two slices from the previous experiment were rebored to 1 in. dia. and 2 in. dia., respectively. The slices were then heated to 185°F^6 (above critical temperature) and retained at that temperature for four hours, followed by cooling to room temperature at the rate of 1.35°F per hour. The -----

⁶ The temperature equal to 185°F was chosen as an arbitrary average of the temperature profile under which the model had been cast.

slices were examined in the polariscope for isochromatic fringe order distribution. The temperature profiles for the new geometries were read from Fig. 9. The theoretical fringe order distributions for the new geometries were calculated from eq.(24). The experimental and the theoretical fringe order distributions for the two new geometries have been plotted in Figs. 14, 15. Close correlation between theory and experiment is observed through most of the model with the exceptions of regions in the vicinity of the inner and the outer surfaces, where the experimental fringe order is much higher than the theoretical one as in the case of the original model. Tensile time-edge stresses were suspected at both the inside and the outside radius, but it was not yet known if the error could be attributed completely to the time-edge effect. More accurate evaluation of the technique was, therefore, necessary.

B. THE REVISED EXPERIMENT

1. Purpose

The purpose of the second experiment was to conduct an accurate application of the new photothermoelastic technique and thus check the results of the previous experiment.[#] Valuable information had been gained from the previous experiment in the application of the method. Since no definite cause of the variation between theoretical and experimental values of isochromatic

fringe order near the boundaries had been found, it was the purpose of this experiment to ascertain the reason for the existence of such errors.

2. Design of Model

The new method was again applied to a thin hollow disk under a steady state temperature profile in the radial direction. In order to achieve uniform axial temperature in the disk, it was machined from the midlength of a thick walled cylinder cast under a radial temperature profile. An increase in the fringe order at the outer radius was achieved by choosing the ratio of the outer radius to the inner radius equal to two (Fig. 4). The inner radius was limited by the size of the core tube available. The length of the cylinder was chosen to be 3 inches. The final dimensions of the model were:

inner radius = 1 in.

outer radius = 2 in.

length of cylinder = 3 in.

temperature difference between the inner radius and the
outer radius = 30°F .

The length of the cylindrical model was designed to be equal to 3 in. so that the central 1 in. length used for machining thin disks would not be effected by the temperature variations at the ends of the cylinder. Thus it was assumed that axial temperature variation in the central length of the cylindrical model would be minimum. The validity of this assumption will be evaluated in a later discussion.

3. Design of Equipment

(a) Mold

Figure 16 shows the sketch outline of the mold. (see also Fig. 28). The outer shell of the mold was manufactured from an aluminum pipe. The top and bottom faces of the mold cavity were made of aluminum disks insulated from the outside by 2 inches thick Malenite insulation blocks. The core tube was cast from RTV silastic rubber as described in the previous experiment. The wall thickness of 2 inches dia. core tube was $5/64$ in. Theoretical calculations showed that the tube was safe against buckling under pressure caused by the curing shrinkage of the epoxy material. The mold was supported with its axis vertical. The core tube was supported concentrically between the two horizontal plates by two aluminum pipes through tapered end plates. These pipes also served as the inlet and the outlet for the heating oil. The nuts on the outside were tightened to pull the aluminum pipes against the end plates and thus seal the junctions between the core tube and the flat plates. It was learned from the previous experiment that the epoxy at room temperature was viscous enough to achieve sealing of the connections between various component parts of the mold with a liberal use of the mold release compound only. A $3/8$ in. dia. hole in the upper insulation block was used for pouring the epoxy into the mold.

(b) Temperature Control

Unlike the previous experiment, the temperature at the inner radius of the model in this experiment was controlled automatically. This was achieved by procuring an automatic temperature controlling instrument similar to the one mounted on the oven (Fig. 30). The thermal cycle was programmed on identical cams to give a constant temperature difference between the inner and outer radius of the model. The thermocouple junction of the new temperature controller was placed centrally in the core tube of the model. The temperature controller thus controlled the temperature of the oil inside the core, which indirectly controlled the temperature at the inner radius of the model. Equation (16) shows that the temperature profile remains constant for a constant temperature difference between the inner and outer radius of a hollow disk. It had been assumed that a constant temperature difference between the oven temperature and the oil temperature inside the core would result in a constant temperature profile through the model. The temperature outputs of the thermocouples embedded in the model were checked at various intervals of time. The maximum variation of the temperature differential across the model, thus measured, was less than 1°F in 6 hours. The temperature of the oil was adjusted to give a constant temperature differential of 30°F across the model, once in every six hour interval. Figure 18(a) shows the temperature profile on the model recorded at

various time intervals. The effect of the temperature control system response time on the temperature profile was negligible; this was attributed to the very slow rates of heating (1°F per hour) and cooling (1.35°F per hour) used in the experiment.

(c) Temperature Measurement

The temperature in the model was monitored by twelve copper/constantan thermocouples which fed into a Bristol Multipoint Recorder through a potentiometer circuit described in the previous experiment. Figure 17(a) shows the distribution of the thermocouple junctions through the model. The thermocouple grid was fabricated as in the previous experiment. The leads were placed axially in a uniform temperature zone before forming the thermocouple junction. A hollow rectangular cage ($2 \frac{15}{16}$ inches long, 1 inch wide and $\frac{1}{32}$ inch wall thickness) was sliced out of a $\frac{1}{8}$ inch thick epoxy plate cast in the previous experiment. The thermocouple grid was cemented to this plastic support, the bottom side of which was supported on the lower face of the model. The support was manufactured out of the plastic to reduce stresses due to differential thermal expansion around it. The position of all the thermocouple junctions was located with respect to the bottom outer corner of the plastic support. The assembly was then placed radially in the space between the core tube and the outer wall of the mold. The outer wall of the plastic support was in close contact with the vertical wall of the mold. The leads from the

thermocouples at the bottom were taken out through a 1/8 inch dia. hole in the side of the mold. This hole was sealed with silastic RTV rubber. The leads from the thermocouples at the top were passed through the pouring hole (Fig. 28) . All the thermocouple leads were soldered to corresponding thermocouple wires from the recorder. The rest of the temperature recording arrangement was the same as in the previous experiment. A continuous record of the thermocouple output was maintained through the experiment. The temperature profile at different intervals of time has been plotted in Fig. 18(a). The location of the thermocouples was measured from slices machined at the end of the first thermal cycle. The results from the temperature records will be discussed later.

4. Experimental Procedure

The new model was cast in the same manner as in the previous experiment. The exothermic heat of polymerization was removed in the first two hours after pouring by pumping oil at room temperature into the core of the mold. Axisymmetric temperature distribution was ensured by pumping in oil at the bottom of the core tube, thus keeping the core tube filled with oil at all times. Two hours after pouring, the temperature at the inner radius was raised by 30°F above the temperature at the outer radius by heating the oil. The remaining thermal cycle of the model was the same as in the previous experiment with the exception that the temperature

at the outer radius of the model was 177°F in the high temperature range. This temperature was well above the range of the critical temperature for this material specified in Ref. (47).

At the end of the first thermal cycle, the model was removed from the mold and observed in circularly polarized light (Fig. 32). Cross-sectional slices were removed from the model according to the slice plan shown in Fig. 17(b). These disks were then faced on a lathe. Two tensile specimens were machined out of the calibration plate and a circular disk was machined out of the calibration cylinder. The slices from the main model were laid flat on an aluminum plate coated with mold release compound. The slices along with the calibration molds were heated to 185°F ⁷ at the rate of 5°F per hour. The calibration specimens were loaded and after three days, the temperature was lowered to room temperature at the rate of 1.35°F per hour. At room temperature, the slices were examined in a polariscope. In all observations made the polariscope was provided with a profile projector of magnification 10X. The slices were mounted on the carriage of the polariscope which had longitudinal and transverse micrometer feeds. The location of the different fringe orders was thus measured to the nearest 0.001 in.

⁷ The curing temperature equal to 185°F is an arbitrary value, well above the critical temperature.

5. Calibration

Two tensile specimens and a circular disk under diametral load were used to find the value of the material constants E , ν and f at temperatures above the critical temperature. The dimensions of the calibration specimen were the same as those used in the previous experiment (Fig. 12). One inch gage lengths were marked as described in the previous experiment on

- (i) the two tensile specimens in the longitudinal and transverse directions,
- (ii) the vertical and the horizontal diameters of the calibration disk.

The initial gage lengths were measured with the optical micrometer. The calibration models were loaded during the annealing cycle of the main model (after the casting cycle). The gage lengths were again measured at the end of the stress freezing cycle. The stress frozen calibration models were examined in circularly polarized light.

Following is the data from the calibration tests used in calculating the constants E , ν and f . The formulas used have been derived in Appendix C

Calibration disk:

Load, $P = 5.00$ pounds

Diameter, $D = 2.000$ in.

Thickness, $t = 0.250$ in.

Horizontal and vertical gage lengths = 1.000 in.

Change in gage length along the horizontal diameter,

$$\delta x \begin{matrix} +D/4 \\ -D/4 \end{matrix} = 0.00586 \pm .000075 \text{ in.}$$

Change in gage length along the vertical diameter,

$$\delta y \begin{matrix} +D/4 \\ -D/4 \end{matrix} = 0.01020 \pm .000075 \text{ in.}$$

Fringe order at the center of the disk,

$$N_o = 4.25 \text{ (Green Light)}$$

Substitution of the above values in Eqs. (C.13), (C.17) and (C.21) , gives

$$E = 2,100 \pm 65 \text{ psi}$$

$$\nu = 0.49 \pm 0.02$$

$$f = 1.50 \pm 0.04 \text{ psi.in. per fringe [Green Light]}$$

Tensile Specimens:

All gage lengths = 1.000 in.

Data	Tensile Specimen	
	No. 1	No. 2
Width, b (in.)	1.1100	1.1100
Thickness, t (in.)	0.1205	0.1195
Load, P (pounds)	2.445	3.530
Stress, $\sigma = P/bt$ (psi)	18.26	26.85
Long. strain, $\epsilon_{\text{Long.}}$	0.00869	0.0128
Lat. strain, $\epsilon_{\text{Lat.}}$	-0.00414	-0.00635
Isochromatic fringe order in the uniform section,		
n (green light)	1.48	2.16
E , psi (eq.C.2)	$2,095 \pm 70$	$2,085 \pm 70$
ν (eq.C.3)	0.47 ± 0.025	0.49 ± 0.025
f , psi.in. per fringe, (eq.C.4, green light)	1.480 ± 0.104	1.475 ± 0.104

The average value of the material constants used in the calculation of the theoretical solution for the thermoelastic problem are:

$$E = 2100 \pm 100 \text{ psi}$$

$$\nu = 0.49 \pm 0.025$$

$$f = 1.50 \pm 0.13 \text{ psi. in. per fringe.}$$

The value of α was calculated for various temperatures in the same manner as in the first experiment. Figure 13 shows the

comparison of the measured value of α with the values supplied by Sampson (80). The value of α has been found to fluctuate about a constant value in the range of temperatures above the critical temperature. Thus, a maximum scatter of ten percent from the average value can be noticed. This scatter is within the experimental error. The distribution of the experimental points appears to indicate a partial verification of the values shown by Sampson's curve. A more accurate means of measurement would be required to ascertain the variation of α with temperature in the high temperature range. In this investigation, the value of α has been assumed to be constant without any further verification. Average value of $\alpha = .000100 \pm .000006$ in. per in. per $^{\circ}\text{F}$.

Sampson's curve for E/f (Fig. 13) shows a constant value in the high temperature range. Fessler (82) has reported similar observations for several other stress freezing materials. This observation has not been verified in the present investigation. The calculations have been based on the assumption that the value of E/f is constant in the range of temperatures above the critical temperature.

6. Evaluation of Experimental Data

(a) Temperature

Figure 18(a) shows the temperature profile on the model at various time intervals. The temperature profile, two hours after pouring shows that the exothermic heat of the polymerization

reaction has been efficiently transferred out of the model by the cooling fluid. The temperature profile along the radial lines A-A and B-B, placed symmetrically along the axis of the cylindrical model, show a constant difference of 2 to 3⁰F in the later stages of the experiment. The temperature recorded at the point C and the corresponding temperatures along A-A and B-B show a variation of temperature along the axis of the cylindrical model. This variation in temperature reaches a maximum of 2.5⁰F per inch of axial length between the point C and the corresponding point on the radial line A-A. The thickness of the two thin disks machined out of the cylindrical model is equal to 0.1528 in. and 0.1706 in. respectively. Thus the maximum variation in temperature profile through the thickness of such a disk is less than 0.5⁰F. This axial temperature difference is very small compared to the temperature difference of 30⁰F along the radius of the disk and is within the experimental error of temperature measurement. The third disk machined out of the middle section of the model has a thickness of 0.9690 in. and the small variations of temperature along the axis do not appear to have any noticeable effect on the stress distribution, as discussed later.

The average experimental and theoretical temperature profiles have been plotted in Fig. 18(b). The theoretical temperature distribution has been calculated from eq.(16). There is a close correlation between the two distributions and the maximum

difference between the two is less than 5 percent (within experimental error). The main source of this variation appears to be the influence of the temperature variation along the axis of the cylindrical model. The heat transfer in the model has not been truly radial as has been assumed in the calculation of the theoretical temperature profile. Some of the other possible sources have been listed in the previous experiment. The calculations for the theoretical fringe order distribution have been based on the theoretical steady-state temperature profile.

(b) Photoelastic Data

The fringe order distributions in the thin slices (no. 2 and no. 4) and the central, thick slice (no.3) at the end of a curing cycle of individual slices separately, have been plotted in Figs.19, 21, 20, respectively. Theoretical fringe order distributions have been calculated on the assumption of plane stress conditions in these disks (eq. 24). These distributions correlate fairly well with the experimental fringe order distributions except near the inner and the outer radius. The variation in fringe order distribution near the inner and the outer radius was recorded every day for a couple of days. The increasing fringe order at both boundaries indicated the existence of tensile time-edge effect fringes. The tensile nature of the time-edge effect was more obvious at the outer radius where the expected

fringe order had a very small fringe gradient. The superimposition of a positive time-edge effect on the existing negative fringe pattern at the outer radius resulted in the appearance of a zero order fringe at the outer radius within half an hour of the removal of the slice from the oven. This zero order fringe was slowly moving inwards with time and higher order tensile fringes were formed at both the outer and the inner radius. These slices were heated at 115°F for three days to remove time-edge effect and observed in a polariscope immediately after removal from the oven. The appearance of the zero fringe order near the outer boundary and its inward motion within a few minutes of the first observation lead to the conclusion that the plastic material is seriously affected by time-edge stresses.

7. Model Without Temperature Gradient

In the evaluations of the test results from the previous two experiments it has been suggested that the discrepancy between the experimental and theoretical values of fringe order may be the result of the time-edge effect. It has been suggested in the literature (12, 51, 82, 84, 86) that time-edge effect caused by absorption of moisture can be reduced by heating the slices at a slightly elevated temperature ($110^{\circ} - 150^{\circ}\text{F}$) for a few days. The slices from both the experiments were again maintained at 115°F for three days and cooled to room temperature at the rate of

1.35°F per hour. The isochromatic fringe order distributions in the slices were recorded immediately after their removal from the oven. The distribution of the fringe order was again observed to be changing rapidly with time and by the end of one set of observations the time-edge effect had remarkably affected the distribution near the boundaries.

In order to account for these non-thermal stresses a hollow cylindrical model was cast in the mold from the second experiment, with no temperature profile. The model was cast and cured in exactly the same procedure and thermal cycle as the second experiment except that a uniform temperature was maintained in the model at all times. At the end of the first curing cycle, slices (0.15 in. and 1.0 in. thickness) from the new model were machined on a band-saw and faced on a lathe. These slices along with the slices from the previous two experiments were heated at 185°F for three days. A circular disk machined out of the new batch of casting was also loaded for the purposes of calibrating the new material. The calibration procedure was the same as used in the previous two experiments. During the curing period at the high temperature, the slices were examined in a reflection polariscope arrangement through the glass window in the oven. Isochromatic fringes were present in all slices except in those from the model cast under a uniform temperature. All slices were cooled to the room temperature at the rate of 2.25°F per hour. The isochromatic

fringe order distribution in all the slices was recorded. Figures 33, 34 show the isochromatic fringe pattern in a slice (No.3) with the thermal gradient and the slice without a thermal gradient, respectively. The fringe order distribution in the slices from the second experiment and in those from the model without a temperature profile have been plotted in Figs. (19-21). The fringe order distribution due to thermal stresses has been calculated as the difference between the two fringe order distributions plotted and compared with the values calculated from the theoretical solution (thin disk formula). Figure 21 also shows the comparison of the experimental fringe order distribution from the three slices with the theoretical distribution. The material constants for the new batch of castings have been calculated to be within 1.0 percent of those for the main model. The value of material constants calculated for the model without thermal stresses are:

$$E = 2120 \pm 100 \text{ psi}$$

$$\nu = 0.480 \pm .025$$

$$f = 1.49 \pm 0.13 \text{ psi. in. per fringe.}$$

In another attempt to remove the time-edge affect, all slices were placed in an oil⁸ bath and the oil was heated to the critical temperature. After three days, the oil bath was cooled to

⁸ IMF-163 oil from Photoelastic Inc., Malvern, Pa.

the room temperature at the rate of 2.25°F per hour. Isochromatic fringes were observed near the edges of the slices (from the model with no temperature gradient) and the distribution was similar to the one due to time-edge effect. The exact origin of these fringes is not well known but it is believed that the diffusion of moisture between the epoxy material and the oil (some oils have slight solubility in water) may have caused the edge stresses. Similar edge effects in plastics immersed in various liquids have been reported by Heywood (86). Serious time-edge effects in amine-cured epoxies have also been reported by Leven (51).

Axial slices were removed from the main model with temperature gradient and the model without the temperature gradient. Figures 22, 23 show the isochromatic fringe order distribution in these slices in the radial and the axial directions, respectively. The isochromatics in these slices represent the stress difference $(\sigma_z - \sigma_r)$. The isochromatic fringe order due to thermal stress has been calculated by superimposing the fringe order distribution of the two slices, one upon the other. The axial stress, σ_z has been calculated from the formula

$$(\sigma_z - \sigma_r) = f \frac{n}{t}$$

where σ_r is given by Eq. (22), and is plotted in Fig. 22. The axial stress, σ_z is indicated by the difference of the axial

fringe order distributions in Fig. 23. This difference between the fringe order in the slice with a temperature profile and the slice without a temperature profile is very small and constant along the axial line C-C. The radial stress, σ_r can be assumed to be constant along the axial line. The value of σ_r for $X = 1.4$ has been taken from Fig. 22 and simple calculations have shown that the axial stress $\sigma_z = 0$, along the line C-C.

Figure 3 shows that the magnitude of the fringe order due to thermal stresses ($\sigma_z - \sigma_r$) in an axial slice is almost as high as that in a cross-sectional slice ($\sigma_\theta - \sigma_r$). The low fringe order due to thermal stress in the axial slice shows that the disk (slice #3), from which the axial slices have been removed, has a thermal stress distribution close to that given by the thin disk solution.

The two axial slices were cured at 185°F for three days. The isochromatic fringe pattern in both slices remained unaltered. This further shows that the fringe pattern in the axial slice from the model with a temperature gradient was mainly due to the time-edge effect. This slice, shown in Fig. 22, when annealed at the high temperature would develop thermal stresses corresponding to the new geometrical shape. For simplification, assuming the temperature variation between the outer and the inner edges to be

linear, it is found that no thermal stresses would be developed⁹ for such a geometry and thermal loading conditions. From Fig. 18(b) it can be seen that the temperature profile can be approximated by a straight line.

Time-edge stresses in photoelastic models follow their edges¹⁰. Figures 33, 34 show the effect of a cut on the slices from models with and without a temperature gradient. The fringes in the former slice are disturbed to a much lesser degree than those in the latter. The thermal stresses frozen in the former slice were not disturbed by a cut, whereas the time-edge stresses were disturbed. In the latter slice, the stresses (only time-edge effect) are not frozen-in and hence are greatly effected by the cut. The new fringes run nearly parallel to the cut (Fig. 34). These photographs were taken immediately after the cut was machined.

⁹ pp. 403, Ref. 9.

¹⁰ pp. 132, Ref. 86.

IV DISCUSSION OF RESULTS

There is a close correlation between the theoretical and experimental temperature distribution in both experiments (Figs. 9, 18(b)). Hence, the theoretical thermal stress calculations have been based on the theoretical steady-state temperature profile (eq. 16).

In discussing the isochromatic fringe order distributions the time-edge effect has resulted in large variations of the experimental fringe order. "Time-edge effect" refers to fringes that develop with time at the edges of a newly machined slice or a model after removal from the oven at the end of a thermal cycle. These fringes are not related to the stresses and strains produced by the applied loads and deformations. There are two types of time-edge stresses. One type of time-edge stress is believed to result from the diffusion of water vapour from the air into the plastic or from the plastic into the air. This type of edge-effect can be temporarily removed by driving off the moisture by heating the plastic. Depending on the moisture content and temperature, the sign of apparent stresses corresponding to the edge-effect can be positive or negative (84)¹¹. Leven (51) found that the time-edge effect was appreciably less for the epoxy resins cured with an anhydride type of hardening agent, instead of an amine hardener. Anhydride cured resins show an increasing time-edge effect in the

¹¹ pp. 157

first few days and as stability is reached within a couple of months, the time-edge effect vanishes completely. All amine cured epoxies show continual increase in time-edge stresses for a prolonged period of time before any stability in its value is noticed.

The second type of time-edge stress is believed to result from the oxidation of the edges at higher temperatures. This type of time-edge effect is caused during annealing at the high temperature and cannot be removed unless the edge is machined off. This effect is also known as the 'rind effect' and is found predominantly in polystyrene type of resins (86)¹². The rind effect has been reduced in some plastics by curing in an inert atmosphere or any mineral oil; other methods used for the same purpose are listed in the above reference.

In the present investigation the time-edge effect is believed to be a combined effect of water vapour diffusion and oxidation. Time-edge stresses have been noticed in the model cast without a temperature profile immediately after removal from the oven at the end of the second thermal cycle, thus suggesting the existence of the rind effect. The time-edge stresses in these models have been noticed to increase rapidly thereafter. This increase in the time-edge stresses at room temperature is believed to be caused by water vapour diffusion between the plastic and the

atmosphere. The initially rapidly increasing time-edge effect has been observed to slow down within a few days. The slices have been put through a thermal cycle (recommended by Leven (51)) to remove these time-edge stresses. The examination of these slices immediately after removal from the oven has shown that the time stresses were introduced very rapidly either during cooling of the oven (in the lower temperature ranges) or during the removal of the slices from the oven. These rapidly introduced stresses were also noticed in the stress-frozen calibration disk model immediately after removal from the oven. The zero order fringe which normally exists at the outer boundary of the horizontal diameter in the calibration disk was found on the inside of the boundary. A bending-beam compensation device was used to determine the nature of the fringe order between the zero order fringe and the outer boundary. A tensile time stress had developed at the edge which was increasing in magnitude and thus moving the zero order fringe farther away from the boundary.

The tensile nature of the birefringence resulting from the time-edge effect was also ascertained from the slices removed from the model cast under a uniform temperature. Figure 34 shows the time-edge effect in the slices (without thermal stresses). The slice was slotted and the change in fringe order distribution is also shown in the same figure. The fringes closely follow the

contour of the cut and thus prove the non-frozen character of the stresses in these slices. As mentioned before, these slices were examined in a reflection polariscope arrangement at different temperatures. Fringes were not observed from the critical temperature to temperatures close to the room temperature (110°F). Since the slices were machined out of a model cast under a uniform temperature and they were free of any external loads or constraints during the curing cycle, the isochromatic fringes observed at room temperature (which are not locked-in) are assumed to be the result of the time-edge effect.

The frozen character of the fringes observed in slices from the first two experiments has been proved by carefully machining a radial slot through a slice (Fig. 33). A very slight disturbance in the continuity of fringes in the interior portion of the slice has been noticed. This small disturbance in the fringe pattern appears to have resulted from the release of time-edge stresses near the cut.

The material constants were calculated from the calibration tests for the model in the second experiment (with temperature profile) and the model without a temperature profile. The values of these constants for the two batches of castings have been found to match within one percent; hence, for the purpose of correlation, these values are assumed to be the same.

The fringe order at the center of the calibration disk has not shown relaxation over a period of one month. It is therefore assumed that stress distribution in slices from the second experiment is the result of time-edge effect stresses superimposed on the frozen thermal stresses.

There are only time-edge stresses in the slices from the model without a temperature profile. These two models have the same geometry and material constants; hence, the thermal stress distribution can be found by superimposing the two distributions. The isochromatic fringe order distribution due to thermal stresses has been plotted in Figs. 19-21 for the second experiment. The time-edge stress distribution varies slightly with the thickness of slice; therefore, correspondingly equal thickness slices were used in the above calculations. Figures 19 and 21 show the fringe order distributions in the thin slices (No. 2 and No. 4, 0.1528 in. and 0.1706 in. thick, respectively). The experimental values of fringe order due to thermal stresses correlate very well with the calculated values from the thin disk formula. The experimental values plotted in Fig. 20 are from a thick slice (No. 3, .9691 in. thick). Although this slice is too thick to be considered as a thin disk, the experimental values of the fringe order correlates well with those from the thin disk formula. Figure 21 also shows a comparison of the experimental fringe pattern from the three slices with the theoretical distribution.

The orientation of the molecular chains in plastics under a high stress is known to effect the value of the material constants (87). The Poisson's ratio for most plastics is 0.5 at the critical temperature which results in the constancy of volume condition. This holds true for small deformations only and the value of the Poisson's ratio decreases with increasingly large deformations. However, the stress level and the deformations in this experiment have been so small that the material constants were not a function of the stress level. Therefore, the distribution of the time-edge stresses was not altered by the thermal stresses. Hence, the fringe pattern representing the thermal stress component only can be determined by a direct superposition of the fringe patterns in the slice cast under a temperature profile and the slice cast under a uniform temperature.

The fringe order distributions in the slices from the first experiment have not been corrected for the time-edge effect. A good correlation exists between the experimental and theoretical distributions except near the boundaries (Fig. 11). Figures 14 and 15 show a close comparison between the theory and the experiment for the rebores slices except near the boundaries where compensations should be made for the time-edge effect. These slices were machined from the original model (first experiment) by reborring the slices and they did not show any change in the fringe order distribution. When these rebores slices were heated above the

critical temperature, the frozen strains were released. Since the slices (which had been cast stress-free under a temperature profile) were at a uniform temperature, thermal stresses related to the temperature profile, the material constants and the new geometry were developed. Since the geometry was different, the new stress distribution was different from that in the original model. This is supported by the close correlation between theory and experiment (Figs. 14, 15).

The calibration constants calculated from the calibration tests have shown good repeatability within the experimental error. The assumption of constant values for the product $(E \alpha / f)$ is based on the curves in Fig. 13 and needs to be verified in more detail. These curves have been supplied by Sampson (80) who developed the material. Experimentally determined values of α compare closely to those of Sampson and so does the one value of E/f (from stress freezing tests).

Polymerization is known to be a function of temperature in thermosetting resins like the one used in this analysis (88,89). The initial rate of polymerization in the amine cured epoxies is very high and rapidly slows down thereafter. In this technique, the initially vigorous polymerization reaction has considerably slowed down within two hours after mixing the material components.

The introduction of the temperature gradient two hours

after mixing is assumed to vary only slightly the rate of polymerization of the material through the model. By maintaining this temperature profile as each molecular chain is formed, it is strain free and supports a temperature profile. As the temperature level of the model is raised, mutual strains are developed between the already formed chains because the value of the material constants varies over the range of the temperature profile (Fig.13). The same holds true for the molecular chains formed during the process of changing temperature from the room temperature level to the critical temperature level. These strains keep on changing with temperature and when the model reaches the high temperatures (above critical temperature) each element or molecular chain has gone through an integrated change in length represented by

$$\int_{T_0}^{T_f} \alpha \, dT$$

where $\alpha \, dT$ is the change in a unit length for a temperature increase of dT .

T_0 is the initial temperature at which the chain is formed strain free and T_f is the final temperature (above the critical temperature).

This integrated change in length is not the same for each chain. However, the material is in a soft gelly form until

it reaches the critical temperature. The birefringence due to mutual restraints in the small molecular chains formed before the material reaches the critical temperature is therefore assumed to be small in comparison with the birefringence due to thermal stress. The material cures under a temperature profile for three days. Most of the long molecular bonds are formed in this period. Since no temperature changes occur during the three day period, no mutual strains are developed in the newly formed chains.

Some mechanical strains may be produced by external constraints like differential thermal expansion between the model and the core, restrained curing shrinkage and the self weight. These strains, when frozen-in with the thermal strains during curing, can be annealed by removing the model from the mold and curing it again at the critical temperature.

It is surprising to notice that the severe time-edge effects associated with the material have not been reported in either of the earlier works that used this material (47, 60). However, Sampson has acknowledged high time-edge stresses in slices thicker than $1/8$ in., through a recent communication (80). It is for this reason that time edge stresses were not taken into account while choosing this material. The maximum fringe order due to time-edge effect has been measured to be 6.5 fringes per inch. In regions of high fringe order this time-edge effect is

relatively small and could be easily overlooked. Marloff and Daniel (60) have determined the stress distribution in the fibre-reinforced composite models by using the principle of superposition to separate the effects of shrinkage from those of combined shrinkage and loading. However, time-edge effects may have existed in their models and were accounted for by superposition without their knowledge. There is ample evidence of the presence of time-edge effect stresses in slices available from the photographs of the isochromatic fringe pattern, in Ref. 60. The inward bending of the fringes near the edges of the model subjected to external loading and matrix shrinkage, suggests an apparent tensile stress at the edge. The presence of time-edge effect is not very obvious from the fringe photographs shown in Ref. 47.

In this analysis, the value of the maximum fringe order has been limited by the small range of temperature profile available. The model shape chosen was very simple with an accurate theoretical solution available for comparison. Within these limitations the maximum fringe order due to the time-edge stresses was almost twice that due to thermal stresses. However, the use of the principle of superposition to separate the effects of time-edge stresses from those of combined time-edge and thermal stresses has shown good results (Fig. 21). The maximum variation of the experimental fringe order (Fig. 21) from the theoretical value is 10 percent, which is less than the experimental error of 13 percent, estimated in the error analysis (Appendix D).

V CONCLUSION

A. GENERAL

On the basis of this exploratory study, it appears that the primary purpose of the investigation, namely, to develop a general technique of three-dimensional photothermoelasticity for constant temperature gradient problems has been partly fulfilled. A new technique of freezing thermal strains in a general three-dimensional body has been proposed and successfully applied to a two-dimensional thermoelastic problem. Although, the presence of unavoidable stresses (assumed to be time-edge stresses) have altered the all important fringe order near the boundary, this problem has been overcome by the principle of superposition which was described earlier. The fairly close correlation between experiment and theory (Figs. 19-21) has given a foundation to the general principles of the new technique.

The choice of the material for its high instant time-edge effects has been poor. Most of the methods recommended in literature for removal of the time-edge effect have failed. Although these effects have been compensated for by casting a similar model under no load conditions, the extra effort could be avoided by using a material with a better time-edge property. The most commonly used epoxies with less time-edge effect are cured with anhydrides and have to be allowed to gel within a very small range

of temperature to avoid the precipitation of the anhydrides (91). Any material chosen to replace the material used in this investigation should have the merits listed earlier. It is therefore concluded that until a suitable material is chosen to replace the present material, compensation models under no thermal load should be cast along with the main model.

This technique has all the merits of the frequently used three-dimensional stress-freezing technique. Although the thermal stresses due to a constant temperature profile only can be frozen, transient thermal stress problems with low inertia effects can be simulated by employing the instantaneous temperature profile. These temperature profiles may be applied by using heat absorbent paints locally. The methods of analysis of photoelastic data from stress-frozen model has been well developed and the same can be employed in analysing thermal stresses frozen by this technique. The property of this technique to maintain thermal stresses in the model even at the critical temperature, allows the superposition and freezing of other types of stresses along with the thermal stresses. This can be applied to problems with various types of stresses as in an impeller blade where centrifugal and hydrostatic pressure stresses can be superimposed on the thermal stress due to a temperature gradient.

This technique may be used to obtain a qualitative picture

of the thermal stress distribution due to steady temperature gradients. Quantitative results can only be relied upon after verification of the method by using a different model material with much lower time-edge effects. The reproducibility and accuracy of the test results using this technique have not yet been firmly established. However, a limited amount of evidence has been obtained to recommend further work in assessing the technique. Some of the limitations of the new technique are:

1. The application and maintainance of a constant temperature profile that simulates the actual prototype conditions for complex models could be an impossible task.
2. The magnitude of the stresses frozen is limited by the range of temperature profile available (between the critical and the degradation temperature of the material).
3. This technique can only be applied to steady-state and not all transient thermal gradient problems.

B. RECOMMENDATION

It is recommended that a new epoxy material with very small time-edge effects be ~~sought~~ for applying the technique. An increase in the working temperature (between the critical and the deterioration temperature) of the new epoxy material would be desirable. The rate at which the temperature profile is raised

from the room temperature level to that above the critical temperature should be much faster than that used in this investigation. Thus a much larger portion of the molecular chains will be formed above the critical temperature. This would overshadow the effect of any mutual restraints between chain lengths formed at lower temperatures, as discussed earlier.

The new material should be employed for assessing the technique. The constancy of the value of the photothermoelastic figure of merit, $Q_{\alpha} = (E \alpha / f)$ in the range of the temperature profile above the critical temperature, is essential for the application of this technique. This condition should be completely verified for the new material.

Until such a material is discovered, the present material may be used in the application of the technique, along with the compensation model for time-edge effect.

The calibration techniques have shown good repeatability and are recommended for standardization of the calibration procedure.

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Fig. 1. DEFLECTION AS FUNCTION OF TIME AFTER LOAD
APPLICATION FOR TEMPERATURES BELOW THE
CRITICAL TEMPERATURES, T_{cr}
(pp.256,Ref.12)

Fig. 2. EQUIVALENT MODEL OF A MODEL CAST UNDER A
TEMPERATURE PROFILE

Thermal Stresses In A Long Cylinder

Thermal Stresses In A Thin Disk

Fig. 3. THERMAL STRESSES* THEORETICAL SOLUTION OF A THIN DISK
AND A LONG CYLINDER UNDER A STEADY STATE
TEMPERATURE PROFILE

Fig. 4. VARIATION OF ISOCROMATIC FRINGE ORDER AT THE OUTER RADIUS AND THE INNER RADIUS, POSITION OF 'ZERO ORDER FRINGE' WITH β , IN A THIN DISK UNDER A STEADY STATE TEMPERATURE PROFILE

Sectional View

Side View

Scale $-\frac{1}{2}$ Full Size
All Dimensions In In.

Fig. 5. MOLD (experiment no. 1)

Fig. 7. TEMPERATURE CYCLE
(experiment no. 1)

Fig. 8. TEMPERATURE PROFILE ON THE MODEL AT VARIOUS TIME INTERVALS
(experiment no. 1)

Fig. 9. AVERAGE TEMPERATURE DISTRIBUTION IN THE MODEL
(experiment no. 1)

Fig. 10. ISOCHROMATIC FRINGE ORDER DISTRIBUTION IN THE MODEL
 AT THE END OF THE FIRST THERMAL CYCLE
 (experiment no. 1)

SLICING PLAN

All dimensions in in.

Fig. 11. RADIAL DISTRIBUTION OF ISOCHROMATIC FRINGES IN SLICES (experiment no. 1)

Fig.12. CALIBRATION SPECIMENS

(All dimensions in in.)

Fig. 13. VARIATION OF MATERIAL CONSTANTS WITH TEMPERATURE

Fig. 14. REDISTRIBUTION OF ISOCHROMATIC FRINGES IN REBORED DISK
 (slice no. 2, experiment no. 1)

Fig. 15. REDISTRIBUTION OF ISOCHROMATIC FRINGES IN REBORED DISK
(slice no. 4, experiment no.1)

Scale- Half full size
All Dimensions In in.

Fig. 16. MOLD (experiment no. 2)

Fig. 17(a). THERMOCOUPLE GRID

Scale- Full Size
All Dimensions In In.

Fig. 17(b). SLICE PLAN FOR MODEL(experiment no. 2)
All Dimensions In In.

Fig. 18(a). TEMPERATURE PROFILE ON THE MODEL AT VARIOUS TIME INTERVALS (experiment no. 2)

Fig. 18(b). AVERAGE TEMPERATURE DISTRIBUTION IN THE MODEL
 (experiment no. 2)

SUPERPOSITION OF FRINGE PATTERNS FOR SLICE NO. 2 (thickness = 0.1528 in.)

Fig. 19. THERMALLY INDUCED FRINGE PATTERN IN SLICE NO. 2 (experiment no. 2)

SUPERPOSITION OF FRINGE PATTERNS FOR SLICE NO. 3 (thickness = 0.9690 in.)

Fig. 20. THERMALLY INDUCED FRINGE PATTERN IN SLICE NO. 3
(experiment no. 2)

Fig. 21. THERMALLY INDUCED FRINGE PATTERN IN SLICES
(experiment no. 2)

Fig. 22. THERMAL STRESS DISTRIBUTION ALONG RADIAL LINE A-A
IN AXIAL SLICES

Fig. 25. ISOCHROMATIC FRINGE ORDER DISTRIBUTION IN AXIAL SLICES
ALONG THE AXIAL DIRECTION

Fig. 24. CASTING MOLD IN THE OVEN (experiment no. 1)

Fig. 24. CASTING MOLD IN THE OVEN (experiment no. 1)

Fig. 25. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERNS IN
THE FULL MODEL AT THE END OF THE FIRST THERMAL CYCLE
(experiment no. 1)

Fig. 25. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERNS IN THE FULL MODEL AT THE END OF THE FIRST THERMAL CYCLE (experiment no. 1)

Fig. 26. DARK FIELD ISOCHROMATIC FRINGE PATTERN IN SLICE NO. 3
BEFORE AND AFTER THE SECOND CURING CYCLE (experiment no. 1)

Fig. 26. DARK FIELD ISOCHROMATIC FRINGE PATTERN IN SLICE NO. 3
BEFORE AND AFTER THE SECOND CURING CYCLE (experiment no. 1)

Fig. 27. DARK FIELD ISOCHROMATIC FRINGE PATTERN IN SLICE NO. 4
BEFORE AND AFTER THE SECOND CURING CYCLE (experiment no. 1)

Fig. 27. DARK FIELD ISOCHROMATIC FRINGE PATTERN IN SLICE NO. 4
BEFORE AND AFTER THE SECOND CURING CYCLE (experiment no. 1)

Fig. 28 MOLD FOR EXPERIMENT NO. 2

Fig. 28 MOLD FOR EXPERIMENT NO. 2

Fig. 29. MAIN MOLD AND THE CALIBRATION MOLDS IN THE OVEN
DURING THE FIRST THERMAL CYCLE (experiment no. 2)

Fig. 29. MAIN MOLD AND THE CALIBRATION MOLDS IN THE OVEN
DURING THE FIRST THERMAL CYCLE (experiment no. 2)

Fig. 30. TEMPERATURE CONTROL AND RECORDING EQUIPMENT (experiment no. 2)

Fig. 30. TEMPERATURE CONTROL AND RECORDING EQUIPMENT (experiment no. 2)

Fig. 31. CYLINDRICAL MODEL AND THE CALIBRATION MODELS IN THE
OVEN DURING THE SECOND THERMAL CYCLE (experiment no. 2)

Fig. 31. CYLINDRICAL MODEL AND THE CALIBRATION MODELS IN THE
OVEN DURING THE SECOND THERMAL CYCLE (experiment no. 2)

Fig. 32. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERN IN
THE FULL MODEL AFTER THE FIRST THERMAL CYCLE
(experiment no. 2, light - Laser)

Fig. 32. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERN IN
THE FULL MODEL AFTER THE FIRST THERMAL CYCLE
(experiment no. 2, light - Laser)

Fig. 33. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERN IN
SLICE NO. 3 BEFORE AND AFTER MAKING A RADIAL CUT
(experiment no. 2, light - Laser)

Fig. 33. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERN IN
SLICE NO. 3 BEFORE AND AFTER MAKING A RADIAL CUT
(experiment no. 2, light - Laser)

Fig. 34. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERN IN
SLICE FROM THE MODEL WITHOUT A TEMPERATURE PROFILE"
BEFORE AND AFTER MAKING A RADIAL CUT (light - Laser)

Fig. 34. DARK AND LIGHT FIELD ISOCHROMATIC FRINGE PATTERN IN
SLICE FROM THE MODEL WITHOUT A TEMPERATURE PROFILE"
BEFORE AND AFTER MAKING A RADIAL CUT (light - Laser)

VIII APPENDIX

APPENDIX A

DISCUSSION OF REFERENCE (22)

A copy of the discussion of the paper sent to SESA for publication is attached. The discussion was not accepted by the Society For Experimental Stress Analysis (SESA) because the six-month post-publication period for the paper during which discussions are accepted, had expired.

'An Experimental Method of Determining Thermal Strains'

by Haines, D. J. and Wright, G. P.
'EXPERIMENTAL MECHANICS' v.9n.7 July 1969, p.327-331

Discussion

by Satish Bhan
Graduate Teaching Assistant
Faculty of Engineering
Carleton University, Ottawa, Canada.

The authors have made an important contribution towards the application of photoelasticity to thermal stress problems. Although the proposed method can be used only to determine surface thermal strains, it has opened the way for further application of the conventional 'freezing' technique to thermoelastic problems.

In the theory section of the paper, the authors have given a comprehensive breakdown of the frozen and the unfrozen stress systems. The discussor, however, wonders about the accuracy of the numerical values of these components due to the following reasons:

1. The experimental temperature distribution in a hollow cylinder cooling at the outer surface has been shown in Fig. 2 of the paper. The temperature distribution can be reasonably assumed to be in steady state. The theoretical calculations of this temperature distribution are fairly simple (1) and shows a curvature convex downwards; whereas, on the contrary, fig. 2 shows the temperature distribution convex upwards. The discussor wonders about the origin of this discrepancy.

2. The authors, in their 'Introduction', have rightly explained that the 95% of the unfrozen stresses arise from the removal of the temperature gradient. In Figs. 1 and 3(a), the authors have rightly shown that the value of the frozen fringe order is negative at the inner boundary and positive at the outer. This frozen birefringence is caused by the application of the temperature profile and is thereafter frozen in transition of the model through the 'freezing' temperature. As each element of the material freezes, it becomes stress free but still supports a temperature profile so that when the temperature profile is later removed, a stress system is introduced which is related to the temperature profile and the post freezing constants of the material. The discussor contends that this unfrozen stress system will have a sign opposite to that of the frozen stress system and in this case the former will give a positive fringe order at the inner boundary and a negative fringe order at the outer boundary. The authors have, however, shown the frozen and the unfrozen fringe order of the same sign (fig. 1 and 3a).

3. The authors have mentioned (p. 328) that parallel and tapered disks of material have been extracted from the frozen cylinder and the total stress system (frozen and unfrozen) present in the disks, measured in units of relative retardation. The discussor feels that although the frozen stress system is not disturbed by change in geometry (as in slicing), the unfrozen stress system will certainly be disturbed. Thus, in a parallel

disk sliced from a frozen cylinder, the frozen stress system will be given by the long circular cylinder formula (2) (plain strain) and the unfrozen stress system by the thin disk formula (3) (plain stress). In the case of a tapered disk, the unfrozen stress system will be more complex than in the parallel disk. The discussor wonders if the authors have taken these facts into consideration while separating the frozen and the unfrozen birefringence.

Lastly, the discussor wants to draw attention to a typographical error in the line before the sub-title 'The Thermal-strain Measuring Technique'. Instead of ' $\alpha_2/\alpha_1 \approx 3$ ' it should read ' $\alpha_2/\alpha_1 \approx 1/3$ '.

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APPENDIX B

DESIGN OF POTENTIOMETER FOR TEMPERATURE RECORDER

The range of the Bristol Multipoint recorder used is from 0 to 1.0 millivolt. The output from the copper-constantan thermocouples in the range of temperatures used in the technique varies from 0.8 to 4.25 millivolts. In order to record all the temperatures on the recorder a bucking voltage (variable millivolts) was provided by a potentiometer circuit shown in Fig. (6b). Thus the input to the recorder was adjusted to be always less than 1 millivolt. The rheostat readings were calibrated to give the bucking voltage and this voltage was added to the recorder output to calculate the true thermocouple output.

The potentiometer circuit provided a bucking voltage ranging from zero mv ($\frac{1.5}{200 + .5} \times 0$) to 3.74 mv ($\frac{1.5}{200 + .5} \times 0.5$).

Thus the maximum output that could be measured with this arrangement was $= 3.74 + 1.0 = 4.74$ mv, which corresponded to a temperature $= 230^{\circ}\text{F}$. The maximum temperature that could be used without degrading the material was only 215°F .

The millivolt output from the thermocouples has been converted to temperature by using the standard conversion tables.

APPENDIX C

EVALUATION OF MATERIAL CONSTANTS

1. Tensile Specimen

The stress in a rectangular cross-sectional tensile specimen of width b and thickness t is given by

$$\sigma = \frac{P}{bt} \quad (C.1)$$

where, P is the tensile load. The longitudinal strain is given by

$$\epsilon_{\text{Long.}} = \frac{P}{bt E}$$

or

$$E = \frac{P}{bt \epsilon_{\text{Long.}}} \quad (C.2)$$

and

$$\nu = - \frac{\epsilon_{\text{Lat.}}}{\epsilon_{\text{Long.}}} \quad (C.3)$$

The longitudinal and the lateral strains were measured by an optical micrometer.

If n is the isochromatic fringe order in the uniform section, using stress-optic law

$$\frac{P}{bt} = f \cdot \frac{n}{t}$$

or

$$f = P/bn \quad (C.4)$$

2. Calibration Disk

The stresses on the horizontal diameter of a circular disk of diameter, 'D' and thickness, 't' under a concentrated load, P acting at the ends of the vertical diameter are given by (85)

$$\sigma_x = \frac{2P}{\pi t D} \left[\frac{D^2 - 4x^2}{D^2 + 4x^2} \right]^2 \quad (C.5)$$

$$\sigma_y = - \frac{2P}{\pi t D} \left[\frac{4D^4}{(D^2 + 4x^2)^2} - 1 \right] \quad (C.6)$$

The stresses on the vertical axis are given by

$$\sigma_x = \frac{2P}{\pi t D} \quad (C.7)$$

$$\sigma_y = - \frac{2P}{\pi t} \left[\frac{2}{D - 2y} + \frac{2}{D + 2y} - \frac{1}{D} \right] \quad (C.8)$$

Strain in any small element of length, dx along the horizontal diameter and distance, x from the center is

$$\epsilon_x = \frac{1}{E} [\sigma_x - \nu \sigma_y] \quad (C.9)$$

Substituting in the above equation from (C.7) and (C.8), we have

$$\epsilon_x = \frac{2P}{\pi t D E} \left[\left(\frac{D^2 - 4x^2}{D^2 + 4x^2} \right)^2 + \nu \left(\frac{4D^4}{(D^2 + 4x^2)^2} - 1 \right) \right] \quad (C.10)$$

Therefore, the change in length of this small element is

$$\delta x = \epsilon_x \cdot dx$$

and change in length of a gage length from $-x_1$ to $+x_2$ is

$$\delta x)_{\text{gage}} = \int_{-x_1}^{+x_2} \epsilon_x \cdot dx \quad (\text{C.11})$$

For a horizontal gage length, ℓ placed symmetrically about the center,

$$x_1 = x_2 = \ell/2 \quad \text{and}$$

$$\delta x)_{\text{gage}} = \frac{2P}{\pi t D E} \left[D \left\{ -2 \arctan \left(\frac{\ell}{D} \right) + \frac{2\ell D}{\ell^2 + D^2} + \frac{\ell}{D} \right\} + \nu D \left\{ 2 \arctan \left(\frac{\ell}{D} \right) + \frac{2\ell D}{\ell^2 + D^2} \right\} \right] \quad (\text{C.12})$$

For a gage length $\ell = \frac{D}{2}$

$$\delta x \begin{matrix} + \frac{D}{4} \\ - \frac{D}{4} \end{matrix} = \frac{2P}{\pi t E} (.374 + 1.226 \nu) \quad (\text{C.13})$$

Similarly, strain in any small element of length dy along the vertical diameter and distance y from the center of the disk is

$$\epsilon_y = \frac{1}{E} [\sigma_y - \nu \sigma_x] \quad (\text{C.14})$$

and change in length of this small element is

$$\delta y = \epsilon_y \cdot dy$$

For a gage length on the vertical diameter from y_1 to y_2

$$\delta y)_{\text{gage}} = \int_{y_1}^{y_2} \epsilon_y \cdot dy \quad (\text{C.15})$$

Substituting (C.5), (C.6) and (C.14) in (C.15)

$$\begin{aligned} \delta y)_{\text{gage}} &= \int_{y_1}^{y_2} - \frac{2P}{\pi t E} \left[\frac{2}{D-2y} + \frac{2}{D+2y} - \frac{(1-\nu)}{D} \right] dy \\ &= - \frac{4P}{\pi t E} \left[\ln \left(\frac{1 + 2 \frac{y}{D}}{1 - 2 \frac{y}{D}} \right) - \frac{2(1-\nu)}{D} y \right]_{y_1}^{y_2} \quad (\text{C.16}) \end{aligned}$$

If the vertical gage length $= \frac{D}{2}$ is placed symmetrically about the center

$$\begin{aligned} y_1/D &= -1/4 \quad \text{and} \quad y_2/D = 1/4 \\ &+ D/4 \\ \text{and} \quad \delta y &= - \frac{2P}{\pi t E} (1.2 + \nu) \cdot \quad (\text{C.17}) \\ &- D/4 \end{aligned}$$

The isochromatic fringe order 'n' on the horizontal diameter is given by the stress-optic law as

$$\sigma_x - \sigma_y = f \cdot \frac{n}{t} \quad (\text{C.18})$$

Substituting (C.5) and (C.6) in (C.18), we have

$$\frac{8PD}{\pi t} \frac{(D^2 - 4x^2)}{(D^2 + 4x^2)^2} = f \cdot \frac{n}{t}$$

or

$$f = \frac{8P}{\pi n} \frac{(D^2 - 4x^2)}{(D^2 + 4x^2)^2} \quad (C.19)$$

Substituting $X = 2x/D$ we obtain

$$f = \frac{8P}{\pi n D} \left[\frac{1 - X^2}{(1 + X^2)^2} \right] \quad (C.20)$$

If $n = N_o$, at the center of the disk

$$f = \frac{8P}{\pi N_o D} \quad (C.21)$$

Equations (C.13), (C.17), and (C.21) have been used to calculate the values of the material constants E , ν and f from the measurements of the changes in the diametral gage lengths and the fringe order at the center of the disk, N_o .

Linear dispersion of material fringe value, f with wave length can be noticed from the following calculations:

$$\begin{aligned} \text{Wave length ratio} &= \frac{\lambda_{\text{laser}}}{\lambda_{\text{green light}}} \\ &= \frac{6328}{5460} \\ &= 1.16 \end{aligned}$$

$$\begin{aligned} \text{Fringe value ratio} &= \frac{f_{\text{laser}}}{f_{\text{green light}}} \\ &= \frac{1.75}{1.5} \\ &= 1.166 \end{aligned}$$

The two ratios are nearly equal in magnitude.

APPENDIX D

ERROR ANALYSIS

If the final result y is related to components x_i by the relation

$$y = F(x_1, x_2, \dots, x_n)$$

where F is a known functional form then the probable error λ in y is given by (92, 93)

$$\lambda^2 = \sum_{i=1}^n \left(\frac{\partial F}{\partial x_i} \right)^2 \lambda_i^2 \quad (D.1)$$

where λ_i is the probable error in component x_i and is calculated from a set of observations of each component.

For the tensile calibration specimen, from eqs. (C.2), (C.3), and (C.4)

$$\begin{aligned} E &= \frac{P}{bt \epsilon_{\text{Long.}}} = \frac{PL}{bt(L_2 - L_1)_{\text{Long.}}} \\ \nu &= - \frac{\epsilon_{\text{Lat.}}}{\epsilon_{\text{Long.}}} = - \frac{(L_2' - L_1')_{\text{Lat.}}}{(L_2 - L_1)_{\text{Long.}}} \end{aligned} \quad (D.2)$$

$$f = \frac{P}{bn}$$

Using $P, b, t, L, L_1, L_2, L_1', L_2', n$ as the components in eq.(D.1) for functional forms in eq.(D.2), the p.e.'s in E, ν , and f can be calculated.

¹³ Probable errors (hereafter referred to as p.e.), λ_i used in this analysis is equal to the maximum difference between values in a set of six measurements of quantity x_i and is hence conservative.

The data used are

$$\begin{aligned}
 P &= 2.44 \text{ pounds} & \lambda_P &= 0.05 \text{ pounds} \\
 b &= 1.110 \text{ in.} & \lambda_b &= 0.001 \text{ in.} \\
 t &= 0.120 \text{ in.} & \lambda_t &= 0.001 \text{ in.} \\
 L &= 1.000 \text{ in.} & \lambda_L &= .001 \text{ in.} \\
 L_2 - L_1 &= 0.0087 \text{ in.} & \lambda_{L_1} = \lambda_{L_2} = \lambda_{L_1'} = \lambda_{L_2'} &= .0002 \text{ in.} \\
 L_2' - L_1' &= - 0.0041 \text{ in.} \\
 n &= 1.48 \\
 E &= 2090 \text{ psi} \\
 \nu &= 0.470 \\
 f &= 1.48 \text{ psi. in. per fringe}
 \end{aligned}$$

The value of λ_L takes into account the thermal expansion that may be caused by a change in the room temperature (2° maximum).

These values have been substituted in equation (D.2) to obtain the values of the p.e.'s as:

$$\begin{aligned}
 \lambda_E &= 70 \text{ psi} & &= 3.4 \text{ percent} \\
 \lambda_\nu &= 0.027 & &= 5.6 \text{ percent} & (D.3) \\
 \lambda_f &= 0.1035 \text{ psi.in. per fringe} & &= 6.9 \text{ percent}
 \end{aligned}$$

Similarly, for the calibration disk, the equations (C.13), (C.17), and (C.21) can be combined to give the functional forms for the calibration constants as:

$$E = \frac{2P}{\pi t(Y_2 - Y_1)} \left[1.2 + \frac{0.8}{1.226 - \frac{X_2 - X_1}{Y_2 - Y_1}} \right]$$

$$\nu = \frac{0.86}{1.226 - \frac{X_2 - X_1}{Y_2 - Y_1}} \quad (D.4)$$

$$f = \frac{8P}{\pi N_o D}$$

where X_1, X_2, Y_1, Y_2 are the initial and the final gage length along the horizontal and vertical axis, respectively. Thus the functional forms can be written as:

$$E = E(P, t, X_1, X_2, Y_1, Y_2)$$

$$\nu = \nu(X_1, X_2, Y_1, Y_2) \quad (D.5)$$

$$f = f(P, N_o, D)$$

The p.e.'s λ_E, λ_ν , and λ_f have been calculated from equations (D.1), (D.4), (D.5).

The data used are

$P = 5.00$ pounds	$\lambda_P = 0.05$ pounds
$D = 2.000$ in.	$\lambda_D = 0.001$ in.
$t = 0.250$ in.	$\lambda_t = 0.001$ in.
$X_2 - X_1 = 0.0058$ in.	$\lambda_{X_1} = \lambda_{X_2} = \lambda_{Y_1} = \lambda_{Y_2} = 0.0002$ in.
$Y_2 - Y_1 = -0.0102$ in.	
$N_o = 4.2$	$\lambda_N = 0.1$

$$E = 2110 \text{ psi}$$

$$\nu = .490$$

$$f = 1.5 \text{ psi. in. per fringe}$$

The calculated values of the p.e.'s are

$$\lambda_E = 64 \text{ psi} \quad \approx 3.0 \text{ percent}$$

$$\lambda_\nu = 0.024 \quad \approx 5.0 \text{ percent} \quad (D.6)$$

$$\lambda_f = 0.0354 \text{ psi.in.per fringe} \approx 2.4 \text{ percent}$$

The p.e. for the coefficient of thermal expansion can be calculated from

$$\alpha = \frac{X_2 - X_1}{X \cdot (T_2 - T_1)} \quad (D.7)$$

or $\alpha = \alpha[X, X_1, X_2, T_1, T_2] .$

Substituting in equation (D.1) and (D.7) the data

$$\text{Gage length } X = 1.000 \text{ in.} \quad \lambda_X = .001 \text{ in.}$$

$$X_2 - X_1 = 0.000525 \text{ in.} \quad \lambda_{X_1} = \lambda_{X_2} = 0.000016 \text{ in.}$$

$$T_2 - T_1 = 5.0^\circ\text{F} \quad \lambda_T = 0.1^\circ\text{F}$$

$$\alpha = .000100 \text{ in.per in. per } ^\circ\text{F}$$

The calculated value of the p.e. is

$$\lambda_\alpha = .000006 \text{ in. per in. per } ^\circ\text{F} = 6 \text{ percent} \quad (D.8)$$

From eq. (24), the expression for the theoretical

isochromatic fringe order in a thin disk is given by

$$n = \frac{\alpha E t \Delta T}{2f} \left(\frac{2\beta^2}{\beta^2 - 1} \cdot \frac{1}{X^2} - \frac{1}{\ln \beta} \right)$$

The maximum fringe order is given at $X = 1$ by

$$N = \frac{\alpha E \Delta T}{2f} \left(\frac{2\beta^2}{\beta^2 - 1} - \frac{1}{\ln \beta} \right) \quad (D.9)$$

where $N = n/t$

$$N = N(\alpha, E, T_1, T_2, f, \beta) .$$

The p.e. in β has been calculated from

$$\beta = \frac{r_0}{r_i} \quad \text{or} \quad \beta = \beta(r_i, r_0)$$

and using $\lambda_{r_0} = \lambda_{r_i} = .001$ in. in eq. (D.1); the calculated value of $\lambda_\beta = 0.0024$ (D.10)

The probable error in the maximum fringe order per inch thickness has been calculated from eqs. (D.1) and (D.9). There is a small recovery in the frozen strain of the calibration model when the load is removed at the room temperature. The effect of this recovery is automatically taken into account by calibration in ordinary photoelastic analysis because the recovery takes place in both the main model and the calibration model. However, in the new technique of freezing thermal strains under investigation, the thermal load remains on the model even at room temperature. The

exact mode of recovery of strains, if any, is not exactly known. Any error due to this recovery in the main mold is assumed negligible and only the error due to recovery in the calibration mold is taken into account. A rough estimate of the latter error is the ratio (E/E_r) where E and E_r represent the modulus of elasticity at the critical and room temperature, respectively. For most stress freezing materials, this ratio is less than 1.5 percent (51). This error has been taken into account in the following calculations.

$$\begin{aligned}
 E &= 2100 \text{ psi} & \lambda_E &= 105 \text{ psi (5 percent)} \\
 \alpha &= 100 \times 10^{-6} \text{ per } ^\circ\text{F} & \lambda_\alpha &= 6 \times 10^{-6} \text{ per } ^\circ\text{F} \\
 f &= 1.5 \text{ psi.in.per fringe} & \lambda_f &= 0.13 \text{ psi.in per fringe} \\
 T_1 - T_2 &= 30^\circ\text{F} & & (8.5 \text{ percent}) \quad (D.11) \\
 & & \lambda_{T_1} &= 2.0^\circ\text{F} \\
 & & \lambda_{T_2} &= 2.0^\circ\text{F} \\
 \beta &= 1.95 & \lambda_\beta &= 0.0024 \\
 N_{\max} &= 2.6 \text{ fringe per inch}
 \end{aligned}$$

The calculated value of the p.e. in the maximum fringe order is

$$\begin{aligned}
 \lambda_N &= 0.34 \text{ fringes} \\
 &= 13 \text{ percent} \quad (D.12)
 \end{aligned}$$