MONOPOLY, COMPETITION, AND PRODUCTIVE EFFICIENCY

by

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Abstract

This thesis consists of three chapters that study the relationship between product market competition and productive efficiency. There is a long-standing belief among economists that a monopolist tends to slack off in his effort to control production costs while product market competition drives the manager of a firm to work harder. This thesis formally examines the validity of this belief in the context of three models where the manager chooses his effort level optimally.

Chapter 1 examines why monopoly pricing can cause productive inefficiency in an entrepreneurial firm where the owner of the firm manages it himself. The analysis demonstrates that output reduction due to monopoly pricing can actually lead to either the under-supply or the over-supply of effort. This distortion in effort, in turn, causes distortions in costs.

Chapter 2 sets up two principal-agent models to analyze productive inefficiency of monopoly in a managerial firm where ownership and management are separate. It is shown that under linear piece-rate incentive schemes, (i) the owner of monopoly firm chooses an incentive power different than that the owner of competitive firm does, and (ii) the manager of monopoly firm exerts less or more effort than the

manager of competitive firm. Furthermore, productive inefficiency of monopoly can be exacerbated or mitigated by the managerial incentive problem, depending on the output-effort relationship in the cost function.

Chapter 3 analyzes how competition increases productive efficiency through the effect of competition-provided information in a model of rank-order variable-prize tournaments. Agents choose effort that affects output level and a principal identifies a winner and a loser based on the agents' output rank: the winner is rewarded a linear piece-rate incentive scheme while the loser receives a fixed prize. First, the model is analyzed to determine the conditions under which the variable-prize tournament contracts dominate both fixed-prize tournaments and linear piece rates. Second, the model is applied to inter-firm competition in the case of a managerial duopoly or two managerial monopolies, in which the owners offer their managers yardstick managerial contracts of variable-prize tournament based on the rank of the two firms' realized marginal costs.

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CHAPTER 1

The Quiet Life of a Monopolist and the Efficiency Losses of Monopoly: The Case of an Entrepreneurial Firm

The best of all monopoly profits is a quiet life.

— Hicks (1935, p.8)

1.1. Introduction

The study of the efficiency losses of monopoly has a long tradition in economics. It is well known that monopoly pricing leads to output distortion, with the resulting deadweight loss “triangle”. However, the empirical significance of such output distortion was called into question by Harberger (1954), who reports that his estimated “triangle” deadweight welfare loss does not exceed 0.1 percent of the gross national product. In response economists have identified two other likely sources of efficiency losses, namely, rent seeking activities and the loss of productive efficiency. On the latter it has been suggested that a monopoly may slack off and may exert too little effort in reducing costs. However, despite its intuitive appeal, this idea has rarely been rigorously analyzed. To the contrary, many economists react to such an idea with distress and puzzlement (Farrell 1983). Why would a monopolist spare efforts to reduce costs when it stands to reap all the incremental profits arising from the

cost reduction (i.e. it does not have to worry about the incremental profits being competed away)?

There are, however, two analyses, namely Farrell (1983) and Nalebuff and Stiglitz (1983a), that do address this question. In both papers productive inefficiency of monopoly is caused by managerial slack arising from the separation of ownership and control. Using two parsimonious models these authors demonstrate that monopoly fails to minimize costs when it is operated by a manager who is not the owner. In their models, monopoly does not cause any loss of productive efficiency in an owner-managed firm. Consequently, the separation of ownership and control is a necessary condition for the loss of productive efficiency in these analyses.

The objective of the first two chapters is to conduct a more detailed analysis of the losses of productive efficiency caused by monopoly using a more general framework than those of Farrell (1983) and Nalebuff and Stiglitz (1983a). To be more specific, we study a model where a firm's total cost of production depends on output and the manager's effort. Total cost increases with output but decreases with effort.¹ We analyze two separate cases with respect to ownership and control: (1) the owner of the firm manages the firm (the "entrepreneurial firm"); and (2) the owner hires a manager to operate the firm (the "managerial firm").

¹A separate but related strand of literature studies the effects of product market competition on managerial slackness and agency costs. See, for example, Hart (1983), Scharfstein (1988), Willig (1987), Hermalin (1992), Martin (1993), Schmidt (1997), and Raith (2003). Another related strand of literature is that on the relationship between market power and the incentives to innovate. See, for example, Quirnbach (1993), Smulders and van de Klundert (1995), Symeonidis (1996), Bucci (1998), Vossen (1999), Pereira (2001), Kräkel (2004), Davidson and Ferrett (2007), and Moschini and Yerokhin (2008). Our analysis differs from these papers in the R&D literature in that the cost-reducing effort in our model has no spillover effect while R&D typically does.

The analysis demonstrates that even in the case where the owner manages his firm (Chapter 1), the level of effort exerted by an owner-manager monopolist generally cannot attain the *first-best* level exerted by a competitive firm's owner-manager. The direction of the bias depends on the relationship between output and effort in the cost function. The monopolist exerts too little effort if output and effort are complements in the cost function. The reverse is true if output and effort are substitutes. As a result, the monopolist does not necessarily enjoy a "quiet life".

An analogous result of the productive efficiency losses of monopoly is obtained in the case where the owner hires a manager to operate the firm (Chapter 2): the effort level exerted by the manager of the monopoly firm generally cannot attain the *second-best* level exerted by the manager of the competitive firm. It is shown that managerial slack exacerbates the monopolist's effort distortion if output and effort are complements in the cost function. On the other hand, if output and effort are substitutes, the separation of ownership and control offsets the over-supply of effort by the owner-manager monopolist. Moreover, monopoly power exacerbates managerial slack if there is cost complementarity between output and effort. On the other hand, if there is cost substitutability between output and effort, then monopoly power offsets managerial slack. This result provides an answer to the question addressed by Tirole (1988, p.75) about how market power on the product market affects "X-inefficiency" (Leibenstein 1966).

Furthermore, the effort distortion may lead to higher marginal costs of production in both the monopoly entrepreneurial firm and the monopoly managerial firm than in the competitive firm, which confirms the Tirole conjecture (1988, p.75-76)

in this regard. Using numerical examples, we demonstrate that the agency costs in a monopoly firm are not necessarily higher than in a competitive firm. Taking this into consideration, the total welfare losses caused by a managerial monopolist are not necessarily larger than those by an entrepreneurial monopolist, even in the case where cost and effort are complements.

As alluded to earlier, this chapter focuses on the case of an entrepreneurial firm. In Section 1.2, we present the model. Then we compare monopoly equilibrium with competitive equilibrium in Section 1.3. In Sections 1.4 we conduct an analysis of monopoly welfare losses. The analysis of a managerial firm is postponed to Chapter 2.

1.2. The Model

Consider a product market characterized by an inverse demand function $p(q)$, where q is the firm's output quantity and p is the output price. To study the efficiency losses of monopoly, we will compare two situations, one where the market is controlled by a monopolist and the other one where it is supplied by a representative competitive firm.² Each firm has access to a single-product technology represented by a stochastic cost function of production $C(q, e) - \varepsilon$, where e is the manager's cost-reducing effort (assumed to be one-dimensional), and ε is an idiosyncratic (firm-specific), random variable reflecting exogenous shocks to technology and giving rise

²In the analysis of the efficiency loss caused by monopoly, it is common to compare the monopoly equilibrium with that of a competitive equilibrium (represented by the intersection of the demand curve and industry marginal cost curve). Our use of a representative competitive firm is consistent with this common approach.

to stochastic fluctuations in the production cost. The random shock ε is i.i.d. (independently and identically distributed) across firms with the same support $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$ and variance σ_ε^2 , and continuous density function $f(\varepsilon)$ and cumulative distribution function $F(\varepsilon)$, which are commonly known to the firms. Assume that risk in the cost of production is nondiversifiable so that it cannot be controlled by firms and contracting parties.

The firm's cost function has the following properties:

(i) $C(q, e)$ is continuous and twice differentiable at all $q \geq 0$ and $e \geq 0$.

(ii) $C(\cdot, e)$ is a strictly increasing and convex function of q , i.e., $C_q(\cdot, e) > 0$, the cost is increasing for all $q \geq 0$. $C_{qq}(\cdot, e) \geq 0$, the marginal cost of output is nondecreasing for all $q \geq 0$.

(iii) $C(q, \cdot)$ is a strictly decreasing and convex function of e , i.e., $C_e(q, \cdot) < 0$ at all $e > 0$ (higher effort reduces the firm's cost of production). $C_{ee}(q, \cdot) \geq 0$ at all $e > 0$ (the marginal cost reduction of the manager's effort is nonincreasing).

(iv) $C_{qe}(q, e) \neq 0$, which implies that the marginal cost of production depends on effort. To be more specific, we will consider both the situation where $C_{qe}(q, e) < 0$ and the situation where $C_{qe}(q, e) > 0$. In the former situation, an increase in effort level is more effective at reducing the cost of production when the output increases, implying cost complementarity between output and effort. In the latter situation, an increase in effort level is less effective at reducing the cost of production when the output increases, implying cost substitutability between output and effort.³

³The cost complementarity and cost substitutability between output and effort can be traced back to the relationship between effort and other inputs in the production function. Complementarity between effort and other inputs in the production is sufficient for the complementarity in the cost

The inverse market demand function $p(q)$ is assumed to be twice continuously differentiable, strictly decreasing and concave at all $q \geq 0$, i.e., $p'(\cdot) < 0$, $p''(\cdot) \leq 0$. So both $p(q)$ and the marginal revenue curve, $MR(q)$, are strictly negatively sloped. To ensure that a positive quantity is produced and sold in equilibrium, we assume that $p(0) > C_q(0, e)$. For any given value of ε , the firm's realized profit is denoted by $\pi(q, e, \varepsilon)$: the profit of a monopoly firm can be written as $\pi(q, e, \varepsilon) = p(q)q - C(q, e) + \varepsilon$, while that of a competitive firm can be written as $\pi(q, e, \varepsilon) = pq - C(q, e) + \varepsilon$.

Assume that a manager incurs a disutility of effort (the cost of effort) when he exerts his cost-reducing effort in the production process. The disutility function of effort (or the cost function of effort) expressed in monetary terms is $\psi(e)$. It is twice continuously differentiable, strictly increasing and convex, i.e., $\psi'(\cdot) > 0$ (the cost-reducing effort is costly), $\psi''(\cdot) > 0$ and $\psi'''(\cdot) \geq 0$ (the marginal disutility of effort is convex). Moreover, $\psi(0) = \psi'(0) = 0$.

In this chapter, we consider the case where an owner manages the firm himself. We assume that the owner-manager is risk-neutral and his utility $V(q, e; \varepsilon)$ is equal to the firm's profit minus the cost of effort:

$$(1.1) \quad V(q, e; \varepsilon) = \pi(q, e, \varepsilon) - \psi(e).$$

We will compare the optimal choices of q and e in a monopoly firm and in a representative competitive firm. We will use subscripts with capital letters M and C to denote the monopoly firm and the representative competitive firm respectively,

function, while substitutability in the production function is necessary for the substitutability in the cost function. For a more detailed discussion, see Appendix A.

and will use superscripts with one asterisk to denote the optimal solutions in the cases of the entrepreneurial firm. For example, q_M^* denotes the optimal output of a monopoly entrepreneurial firm.

1.3. The Monopoly Efficiency Losses

The owner-manager monopolist chooses the levels of output q and effort e so as to maximize his expected utility $E[V_M(q, e; \varepsilon)]$ in (1.1).⁴ At an interior solution, his optimal levels of output q_M^* and effort e_M^* satisfy the following two first-order conditions:

$$(1.2) \quad p'(q_M^*)q_M^* + p(q_M^*) = C_q(q_M^*, e_M^*),$$

$$(1.3) \quad -C_e(q_M^*, e_M^*) = \psi'(e_M^*).$$

Condition (1.2) indicates that the marginal revenue is equal to the expected marginal cost of production. Condition (1.3) says that the expected marginal saving on production cost from the owner-manager's effort is equal to his marginal disutility of effort.

Now consider a competitive product market where the owner-manager of a representative competitive firm takes the market price as given to choose the levels of output q and effort e so as to maximize his expected utility $E[V_C(q, e; \varepsilon)]$ in (1.1). At an interior solution, his optimal levels of output q_C^* and effort e_C^* are characterized by

⁴In what follows, given a density function $f(x)$ of a random variable $x \in [\underline{x}, \bar{x}]$, the expected value of the function $g(x)$ over the possible realizations of x is given by $E[g(x)] = \int g(x)f(x)dx$. For notational simplicity, we do not explicitly write the limits of integration when the integral is over the entire range of possible realization values of x .

the following two first-order conditions:

$$(1.4) \quad p(q_C^*) = C_q(q_C^*, e_C^*),$$

$$(1.5) \quad -C_e(q_C^*, e_C^*) = \psi'(e_C^*).$$

Condition (1.4) states that the market price is equal to the firm's expected marginal cost of production, and condition (1.5) has the same form as (1.3).

The above two first-order conditions (1.2) and (1.3) (or (1.4) and (1.5)) characterize an output decision equation and an effort decision equation respectively: $q = q_i(e)$ and $e = e_i(q)$, $i \in \{M, C\}$. Using the comparative statics calculation with respect to these conditions, we have the following results:⁵

Lemma 1.1. (i) Both curves $q_i(e)$ and $e_i(q)$ are monotonically increasing (respectively, decreasing) in e in the (q, e) space as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$).

(ii) The curve $q_i(e)$ at every e has a less positive (respectively, negative) slope than the curve $e_i(q)$ as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$). Moreover, the two curves cross only once in the (q, e) space and a unique equilibrium $\{q_i^*, e_i^*\}$ exists.

A comparison of the optimization conditions under monopoly, (1.2)-(1.3), with those under competition, (1.4)-(1.5), reveals:

Lemma 1.2. (i) The output decision curve $q_M(e)$ of the monopoly firm at any $e \geq 0$ has less positive (respectively, negative) slope than $q_C(e)$ of the competitive firm in the (q, e) space as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$) and the curve $q_M(e)$ lies everywhere below the curve $q_C(e)$.

⁵The proofs of all lemmas and propositions in this chapter are presented in Appendix A.

(ii) Two effort decision curves $e_M(q)$ of the monopoly firm and $e_C(q)$ of the competitive firm coincide in the (q, e) space.

Comparing the owner-manager's optimal choices of the monopoly firm with those (the *first-best* solutions) of the competitive firm, we want to address whether the owner-manager monopolist would exert the *first-best* effort. We have the following proposition.

Proposition 1.1. (i) *The owner-manager monopolist expends lower effort than the owner-manager of the representative competitive firm, i.e., $e_M^* < e_C^*$ if $C_{qe}(q, e) < 0$. The opposite is true, i.e., $e_M^* > e_C^*$ if $C_{qe}(q, e) > 0$.*

(ii) *The optimal output of the monopoly firm is less than that of the competitive firm, i.e., $q_M^* < q_C^*$.*

In their simple parameterized model, Nalebuff and Stiglitz (1983a) show that managerial slack is the only source of the productive inefficiency of monopoly. Competitive firms always expend the *first-best* cost-reducing effort in both the case of the non-separation and the case of the separation of ownership and control. However, the owner-manager monopolist only expends the same *first-best* effort as competitive firms in the case of the non-separation of ownership and control. Because of more managerial slack arising in the moral hazard problem, the manager of the monopoly firm expends lower cost-reducing effort than the manager of a competitive firm in the case of the separation of ownership and control.

From Proposition 1.1 it can be seen that even though managerial slack is absent in the case of the non-separation of ownership and control, the equilibrium effort

expended by the owner-manager monopolist cannot attain the *first-best*, it can be either less or greater than that by the owner-manager of the representative competitive firm. But in either case the output distortion is exacerbated by the effort distortion. In other words, the monopolist would experience either a “quiet” life (exert lower effort) or a “hard” life (exert higher effort), and monopoly causes not only the output distortion, but also the effort distortion. Consequently, the monopoly firm is less efficient than the competitive firm in terms of both allocative efficiency and productive efficiency.

Figure 1.1. The output decision curve $q_M(e)$ and the effort-exerting curve $e_M(q)$ of the monopoly firm, and the output decision curve $q_C(e)$ and the effort-exerting curve $e_C(q)$ of the representative competitive firm when the technology exhibits *cost complementarity* between output and effort.

Figure 1.2. The output decision curve $q_M(e)$ and the effort-exerting curve $e_M(q)$ of the monopoly firm, and the output decision curve $q_C(e)$ and the effort-exerting curve $e_C(q)$ of the representative competitive firm when the technology exhibits *cost substitutability* between output and effort.

Lemmas 1.1, 1.2 and Proposition 1.1 are illustrated in Figures 1.1 and 1.2. In these two figures, curves $q_M(e)$ and $q_C(e)$ represent the relationships between q and e as defined by (1.2) and (1.4), respectively, and curve $e(q)$ represents the relationship determined by (1.3) or, equivalently, (1.5). Lemmas 1.1 and 1.2 suggest that curves $q_i(e)$ ($i = M, C$) and $e(q)$ are upward-sloping (respectively, downward-sloping) in the (q, e) space if $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$). In both cases, curve $e(q)$ is steeper than $q_i(e)$ ($i = M$, and C) and $q_C(e)$ always lies above $q_M(e)$. Thus, from Figure 1.1, which illustrates the case $C_{qe}(q, e) < 0$, we see that the owner-manager of a monopoly expends a lower level of effort. On the other hand, Figure 1.2 shows that the opposite is true in the case $C_{qe}(q, e) > 0$. From Figures 1.1 and 1.2 we also see

that the output level in a monopoly equilibrium is lower than that in a competitive equilibrium independent of the sign of $C_{qe}(q, e)$.

From Proposition 1.1 it is easy to show that inefficient effort supply by the owner-manager monopolist also causes cost distortions. This result is addressed in Proposition 1.2.

Proposition 1.2. *For any given output level q , the total production cost of a monopoly is higher than that of a competitive firm, i.e., $C(q, e_M^*) > C(q, e_C^*)$ in the case where $C_{qe}(q, e) < 0$, but $C(q, e_M^*) < C(q, e_C^*)$ in the case where $C_{qe}(q, e) > 0$. Furthermore, a monopoly produces a given output q at a higher marginal cost than the competitive firm, i.e., $C_q(q, e_M^*) > C_q(q, e_C^*)$.*

A more interesting question we also want to address is whether the effort distortion would reduce the owner-managers' utility. Even though we cannot compare directly the owner-managers' expected utilities in equilibrium, we can derive the following result based on Proposition 1.1:

Corollary 1.1. *The owner-manager monopolist's expected utility is greater than that of the representative competitive firm.*

Corollary 1.1 suggests that the owner-manager of a monopoly always enjoys a happier life, albeit not necessarily a quieter life, than that of a competitive firm.

1.4. The Monopoly Welfare Losses

In this section we will discuss the welfare consequences of the monopoly efficiency losses in the case of the entrepreneurial firm. We first give some definitions then

present the results of the monopoly welfare losses. We use Marshallian aggregate surplus to measure welfare. Expected total social welfare is defined as the sum of consumer surplus and producer surplus, or, equivalently, gross social welfare minus the expected production cost,

$$E[TSW(q, e; \varepsilon)] = \int_0^q p(x)dx - [C(q, e) - E(\varepsilon)].$$

To study the monopoly welfare losses caused by the deviation of the effort level from its *first-best*, we are interested in expressions of welfare losses as a function of the effort level. The following basic definition will be used in this section and Section 2.3 of Chapter 2.

Basic Definition 1.1. (i) *The deadweight welfare loss from the monopoly output distortion as a function of the effort level, $DWL_q(q_1(e), q_2(e), e)$, is defined as*

$$\begin{aligned} DWL_q(q_1(e), q_2(e), e) &= E[TSW(q_1(e), e; \varepsilon)] - E[TSW(q_2(e), e; \varepsilon)] \\ &= \int_{q_2(e)}^{q_1(e)} [p(x) - C_q(x, e)]dx. \end{aligned}$$

(ii) *For any two effort levels, $e_1 > e_2$, the deadweight welfare loss from the monopoly effort distortion as a function of the effort level, $DWL_e(q_1, q_2, e_1, e_2)$, is defined as the difference between the expected total social welfare at e_1 and at e_2 ,*

$$\begin{aligned} DWL_e(q_1(e_1), q_2(e_2), e_1, e_2) &= E[TSW(q_1(e_1), e_1; \varepsilon)] - E[TSW(q_2(e_2), e_2; \varepsilon)] \\ &= \int_0^{q_1(e_1)} [p(x) - C_q(x, e_1)]dx - \int_0^{q_2(e_2)} [p(x) - C_q(x, e_2)]dx. \end{aligned}$$

(iii) The saving on disutility of effort (cost of effort) is defined as

$$\Delta\psi(e_1, e_2) = \psi(e_1) - \psi(e_2).$$

It is well known that monopoly pricing results in a deadweight welfare loss (DWL) associated with the output distortion (known as allocative inefficiency produced by monopoly). In our model, moreover, monopoly causes additional DWL because of effort distortion. Here, we give two definitions of the DWL arising from both the monopoly output distortion and effort distortion, both of which are expressed in terms of the effort level.

Let $(q_i(e))_{i \in \{M, C\}}$ denote the output decision equation derived from the first-order conditions with respect to output for the owner-manager's expected utility maximizations of the representative competitive firm or the monopoly firm.

Definition 1.1. *Expected net social welfare of firm i in the case of the entrepreneurial firm as a function of effort is defined as the expected total social welfare minus the cost of the effort,*

$$\begin{aligned} E[NSW_1(q_i(e), e; \varepsilon)] &= E[TSW(q_i(e), e; \varepsilon)] - \psi(e) \\ (1.6) \quad &= \int_0^{q_i(e)} p(x)dx - C(q_i(e), e) + E(\varepsilon) - \psi(e), \quad i \in \{M, C\}. \end{aligned}$$

Definition 1.2. *The expected net social welfare loss created by the owner-manager monopolist (known as the cost of monopoly) in the case of the entrepreneurial firm as a function of effort is defined as the expected total deadweight welfare loss DWL^* minus the saving on cost of effort $\Delta\psi(e_C^*, e_M^*)$, which is equal to the difference between the*

expected net social welfare for the representative competitive firm and for the monopoly firm,

$$DWL^* - \Delta\psi(e_C^*, e_M^*) = E[NSW_1(q_C(e_C^*), e_C^*; \varepsilon)] - E[NSW_1(q_M(e_M^*), e_M^*; \varepsilon)],$$

where

$$DWL^* = DWL_q(q_C(e_M^*), q_M(e_M^*), e_M^*) + DWL_e(q_C(e_C^*), q_C(e_M^*), e_C^*, e_M^*).$$

For a given e , the first-order derivative of $E[NSW_1(q_i(e), e; \varepsilon)]$ with respect to output is

$$\left. \frac{\partial E[NSW_1(q_i(e), e; \varepsilon)]}{\partial q_i} \right|_{q_i(e)=q_C(e)} = p(q_C(e)) - C_q(q_C(e), e) = 0,$$

by the first-order condition with respect to output for the owner-manager's expected utility maximization of the competitive firm. The second-order derivative of $E[NSW_1(q_i(e), e; \varepsilon)]$ with respect to output is

$$\left. \frac{\partial^2 E[NSW_1(q_i(e), e; \varepsilon)]}{\partial q_i^2} \right|_{q_i(e)=q_C(e)} = p'(q_C(e)) - C_{qq}(q_C(e), e) < 0,$$

which implies that $E[NSW_1(\cdot, e; \varepsilon)]$ is strictly concave in output $q_i(e)$. When $q < q_C(e)$, $E[NSW_1(\cdot, e; \varepsilon)]$ is strictly increasing in output $q_i(e)$. Note that for any given $e \geq 0$, $q_M(e) < q_C(e)$. Hence,

$$E[NSW_1(q_C(e), e; \varepsilon)] - E[NSW_1(q_M(e), e; \varepsilon)] = DWL_q(q_C(e), q_M(e), e) > 0.$$

Besides the monopoly output distortion, there is an additional social welfare loss, $DWL_e(q_C(e_C^*), q_C(e_M^*), e_C^*, e_M^*) - \Delta\psi(e_C^*, e_M^*)$, which is related to the monopoly effort distortion. The expected net social welfare loss produced by the monopoly entrepreneurial firm is then equal to the sum of these two welfare losses,

$$\begin{aligned} & E[NSW_1(q_C^*, e_C^*; \varepsilon)] - E[NSW_1(q_M^*, e_M^*; \varepsilon)] \\ &= E[NSW_1(q_C(e_M^*), e_M^*; \varepsilon)] - E[NSW_1(q_M^*, e_M^*; \varepsilon)] \\ & \quad + E[NSW_1(q_C^*, e_C^*; \varepsilon)] - E[NSW_1(q_C(e_M^*), e_M^*; \varepsilon)], \end{aligned}$$

where $E[NSW_1(q_C(e_M^*), e_M^*; \varepsilon)] - E[NSW_1(q_M^*, e_M^*; \varepsilon)] = DWL_q(q_C(e_M^*), q_M(e_M^*), e_M^*)$,

$$E[NSW_1(q_C^*, e_C^*; \varepsilon)] - E[NSW_1(q_C(e_M^*), e_M^*; \varepsilon)] = DWL_e(q_C(e_C^*), q_C(e_M^*), e_C^*, e_M^*) - \Delta\psi(e_C^*, e_M^*).$$

Proposition 1.3. *The expected net social welfare loss from the monopoly effort distortion in the case of the entrepreneurial firm, $DWL_e(q_C(e_C^*), q_C(e_M^*), e_C^*, e_M^*) - \Delta\psi(e_C^*, e_M^*)$, is positive.*

Proposition 1.3 states that since the owner-manager monopolist does not expend the *first-best* effort, the effort distortion causes the extra expected net social welfare loss $DWL_e(q_C(e_C^*), q_C(e_M^*), e_C^*, e_M^*) - \Delta\psi(e_C^*, e_M^*)$, independent of whether $e_M^* < e_C^*$ or $e_M^* > e_C^*$. At the *first-best* effort $e = e_C^*$, $DWL_e(\cdot)$ reaches the minimum $DWL_e(\cdot) = 0$. While extremely simple and intuitive, this result has been missed in the existing analyses because of their (implicit) assumption $C_{qe} = 0$.

CHAPTER 2

The Quiet Life of a Monopolist and the Efficiency Losses of Monopoly: The Case of a Managerial Firm

In this chapter we examine the case of a managerial firm. To do so we extend the model in Chapter 1 to incorporate two versions of the principal-agent problem, where the principal is the owner and the agent is the manager of the firm. In the first version, the owner delegates all decisions of the firm to the manager (“complete delegation”). In the second version, the owner sets the output level but delegates the effort decision to the manager (“partial delegation”).

In terms of notations, we continue to use subscripts with capital letters M and C to denote the monopoly firm and the representative competitive firm, respectively. We will use superscripts with two asterisks to denote the optimal solutions in the cases of the managerial firm, and use a hat ($\hat{}$) to distinguish the case of partial delegation from that of complete delegation. For example, q_C^{**} is the optimal output of a competitive managerial firm under complete delegation, and $\hat{q}_M^{**}$ is the optimal output of a monopoly managerial firm under partial delegation.

This chapter is organized as follows. The principal-agent problem with complete delegation is analyzed in Section 2.1, and that with partial delegation is studied in Section 2.2. In Sections 2.3 we examine the welfare losses caused by monopoly. In

Section 2.4 we conduct numerical simulations of the general models in both Chapters 1 and 2. We conclude in Section 2.5.

2.1. The Principal-Agent Model with *Complete Delegation*

Suppose that the owner of each firm hires a manager to operate the firm. The firm's realized profit $\pi(q, e, \varepsilon)$ as a natural measure of its performance depends not only on output and effort decisions, but also on the realizations of the stochastic shocks to the production cost. Neither the owner nor the manager has *ex ante* information about the exogenous random shocks to the production cost except its probability density function and distribution function. Furthermore, the owner cannot even observe its *ex post* realization.

The asymmetric information between the owner and the manager is such that the manager's actual effort level is unobservable to the owner. In addition, the owner has the same information about market demand as the manager but he cannot control the output level, so he has to give the manager the *complete delegation* of operating the firm.¹ Although the owner can observe the realized output, costs and profits, he cannot infer the manager's effort level from the stochastic production cost function $C(q, e) - \varepsilon$, because the actual *ex post* realization of shocks is unobservable to him. Therefore, the contract cannot be based on the manager's actual effort level.

Assume that the owner does not monitor the manager's effort, because it is very costly for him to do so. Since the outcome of unobservable effort —the realized

¹If the owner can control the level of output, then he would choose an optimal output that is required for the manager to produce and offer the optimal incentive scheme to him. Given this output and incentive scheme, the manager only chooses his optimal effort. This setting is referred to as a partial delegation. We consider this case in Section 2.2.

profit— is assumed to be observable, verifiable and contractible, explicit incentive contracts are possible and feasible. To induce the manager to make desirable decisions, the owner can offer the manager an incentive scheme based on the firm's own realized profit $\pi(q, e, \varepsilon)$. For simplicity, we follow the approach suggested by Holmström and Milgrom (1987) and restrict our attention to a *linear* incentive scheme, denoted by $s(\pi(q, e, \varepsilon)) = \alpha\pi(q, e, \varepsilon) + \beta$, where α is the piece rate per unit of profit (the share of profit) and β is a fixed payment. In the literature, the piece rate α is known as the incentive power, or incentive strength or pay-performance sensitivity.

We follow the convention in the literature on principal-agent models and assume that the owner in a managerial firm is a risk-neutral expected utility maximizer, while the manager is a risk-averse expected utility maximizer. Furthermore, we assume that the manager has a utility function of the form,

$$U(q, e; \varepsilon) = u(s(\pi(q, e, \varepsilon)) - \psi(e)),$$

where $w \equiv s(\pi(q, e, \varepsilon)) - \psi(e)$ is the manager's net income, and $u(\cdot)$ is assumed to be continuously differentiable, strictly increasing and concave, i.e., $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

The production cost function, the owner's and the manager's preferences, and the cumulative distribution function $F(\varepsilon)$ of the random shocks ε and the associated probability density function $f(\varepsilon)$ are assumed to be common knowledge to both the owner and the manager in the sense of Aumann (1976).

In order to make a consistent comparison between the results obtained in the case of the managerial firm and those in the case of the entrepreneurial firm, we

adopt the state-space formulation suggested by Wilson (1969), Spence and Zeckhauser (1971) and Ross (1973) to express the principal-agent problem under asymmetric information. For simplicity, we will drop the variable subscripts M or C whenever our analysis does not involve comparisons between the monopoly firm and the competitive firm.

2.1.1. The Manager's Optimal Choices of Output and Effort and the Owner's Problem

The Manager's Problem

Knowing the incentive scheme $s(\pi(q, e, \varepsilon))$, the manager chooses output q and effort e to maximize his expected utility $E[U(q, e; \varepsilon)] = \int u[\alpha\pi(q, e, \varepsilon) + \beta - \psi(e)]f(\varepsilon)d\varepsilon$. An interior solution to the manager's expected utility maximization problem satisfies the following two first-order conditions:

$$(2.1) \quad \int u' \cdot \left[\alpha \frac{\partial \pi(q, e, \varepsilon)}{\partial q} \right] f(\varepsilon) d\varepsilon = 0,$$

$$(2.2) \quad \int u' \cdot \left[\alpha \frac{\partial \pi(q, e, \varepsilon)}{\partial e} - \psi'(e) \right] f(\varepsilon) d\varepsilon = 0.$$

Since $\partial \pi(q, e, \varepsilon)/\partial q$ and $\partial \pi(q, e, \varepsilon)/\partial e$ are independent of the random cost shock ε , the two first-order conditions (2.1) and (2.2) can be simplified as

$$(2.3) \quad \alpha \frac{\partial \pi(q, e, \varepsilon)}{\partial q} = 0,$$

$$(2.4) \quad \alpha \frac{\partial \pi(q, e, \varepsilon)}{\partial e} - \psi'(e) = 0.$$

Given any incentive power α of the linear incentive scheme, the manager's optimal choices of output and effort are functions of incentive power α and are denoted by $\{\tilde{q}_i(\alpha), \tilde{e}_i(\alpha)\}_{i \in \{M, C\}}$. Conducting comparative statics with respect to the conditions (2.3) and (2.4), we want to determine how the optimal output and effort chosen by the manager respond to changes in the incentive power α .²

Lemma 2.1. *(i) The optimal output chosen by the manager increases (respectively, decreases) with the incentive power if the production technology exhibits cost complementarity (respectively, substitutability) between output and effort, i.e., $d\tilde{q}_i(\alpha)/d\alpha \gtrless 0$ as $C_{qe}(q, e) \leq 0$.*

(ii) The optimal effort chosen by the manager always increases with the incentive power, i.e., $d\tilde{e}_i(\alpha)/d\alpha > 0$.

Lemma 2.1(ii) implies that a higher incentive power α chosen by the owner produces a strong incentive to elicit more effort from the manager.

The Owner's Problem

The owner offers the manager an optimal incentive scheme $s(\pi(q, e, \varepsilon))$ to induce him to choose output and effort that the owner desires. He chooses the incentive power α and the fixed payment β to maximize his expected utility $E[V(\alpha, \beta; \varepsilon)] = \int [\pi(q, e, \varepsilon) - s(\pi(q, e, \varepsilon))] f(\varepsilon) d\varepsilon$, taking into account the manager's optimal choices of output and effort to this incentive scheme.

Assume that the managers' market is perfectly competitive and all the managers in this market are identical, so the owner knows the managers' reservation wages and

²All the proofs of the lemmas, corollaries and propositions in this chapter are given in Appendix B.

their reservation utilities. The managers' participation constraint (or the individual rationality constraint) is that the manager gets an expected utility at least as great as his reservation utility u_0 , that is, $E[U(q, e; \varepsilon)] \geq u_0$. Following the first-order approach suggested by Mirrlees (1974) and Holmström (1979), we can replace the managers' incentive compatibility constraint by the two first-order conditions (2.3) and (2.4) of the manager's problem.³ Thus, the owner's problem is to maximize $E[V(\alpha, \beta; \varepsilon)]$ subject to the participation constraint and the two incentive compatibility constraints.

Substituting $\tilde{q}_i(\alpha)$ and $\tilde{e}_i(\alpha)$ into owner i 's expected utility function and manager i 's participation constraint we can write the owner's problem as

$$\max_{\{\alpha, \beta\}} E[V_i(\alpha, \beta; \varepsilon)] = (1 - \alpha)E\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) - \beta$$

subject to

$$(2.5) \quad Eu[\alpha\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) + \beta - \psi(\tilde{e}_i(\alpha))] \geq u_0.$$

Substituting $\beta = (1 - \alpha)E\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) - E(V_i)$ into the participation constraint (2.5), which must be binding at the optimum, yields

$$(2.6) \quad Eu[\alpha\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) + (1 - \alpha)E\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) - E(V_i) - \psi(\tilde{e}_i(\alpha))] = u_0.$$

³In principal-agent models, the distribution function needs to satisfy the monotone likelihood ratio property condition (MLRPC) and the convexity of distribution function condition (CDFC) such that the first-order condition approach is valid and effort $e(s(\pi))$ that satisfies the first-order condition is unique for any given $s(\pi)$. For more detailed discussions, see, for example, Mirrlees (1974), Grossman and Hart (1983), Rogerson (1985), Jewitt (1988), Alvi (1997), Araujo and Moreira (2001), Luporini (2006), and Ollier (2007). In our model there exists a unique solution to (2.3) and (2.4) for any given $s(\pi)$, so the first-order condition approach is valid.

Note that the maximization of $E[V_i(\alpha, \beta; \varepsilon)]$ is equivalent to the maximization of the left-hand side of (2.6) by choosing α . So we have

$$(2.7) \quad E \left\{ u' \cdot \left[\pi - E\pi + \alpha \frac{\partial \pi}{\partial q} \frac{\partial \tilde{q}_i(\alpha)}{\partial \alpha} + \left(\alpha \frac{\partial \pi}{\partial e} - \psi'(\tilde{e}_i(\alpha)) \right) \frac{\partial \tilde{e}_i(\alpha)}{\partial \alpha} + (1 - \alpha) \left(\frac{\partial E\pi}{\partial q} \frac{\partial \tilde{q}_i(\alpha)}{\partial \alpha} + \frac{\partial E\pi}{\partial e} \frac{\partial \tilde{e}_i(\alpha)}{\partial \alpha} \right) \right] \right\} = 0.$$

Using the first-order conditions (2.3) and (2.4) of the manager's problem, we simplify (2.7) as

$$(2.8) \quad \text{Cov}(u', \pi) + (1 - \alpha) \left(Eu' \cdot \frac{\partial E\pi}{\partial e} \right) \frac{\partial \tilde{e}_i(\alpha)}{\partial \alpha} = 0.$$

We want to determine what reasonable region the optimal incentive power α must lie in. From (2.8), we obtain the following standard result of principal-agent models:

Lemma 2.2. *The optimal incentive power α_i^{**} satisfies the condition $0 < \alpha_i^{**} < 1$.*

Lemma 2.2 implies that the profit-sharing scheme between the owner and the manager still holds in our model.

2.1.2. Managerial Slack in Managerial Firms

Before turning to the efficiency losses of monopoly under separation of ownership and control, we compare the effort and output levels in this case with those in the case where the owner also manages the firm. Given any incentive power, $\alpha \in (0, 1)$, now we can compare the two first-order conditions (2.3) and (2.4) for the manager's expected utility maximization in the case of the managerial firm with those (1.2)-(1.5)

in the case of the entrepreneurial firm. Observe that the output condition (2.3) is the same as those (1.2) and (1.4) in the case of the entrepreneurial firm in Section 1.3 of Chapter 1. However, the effort condition (2.4) would be the same as (1.3) and (1.5) only if $\alpha_i = 1$.

Thus, we obtain the dichotomy property of the pricing rule and incentive under asymmetric information: pricing rule is not affected by asymmetric information and it is the same as that in the case of the entrepreneurial firm, but the effort-exerting rule is affected by asymmetric information.⁴ Although the pricing rule is independent of the informational problems, this does not imply that the manager's production decision is independent of his effort decision: the reduction in cost-reducing effort affects output in the way described in Proposition 2.1.

Proposition 2.1. *For a given market structure (either monopoly or perfect competition) the optimal effort exerted by the manager in the managerial firm is less than that by the owner-manager in the entrepreneurial firm, i.e., $\tilde{e}_i(\alpha_i) < e_i^*$, $i \in \{M, C\}$. The optimal output chosen by the manager in the managerial firm is less (respectively, greater) than that by the owner-manager in the entrepreneurial firm if the production technology exhibits cost complementarity (respectively, substitutability) between output and effort, i.e., $\tilde{q}_i(\alpha_i) \leq q_i^*$ as $C_{qe}(q, e) \leq 0$, $i \in \{M, C\}$.*

Proposition 2.1 states that in the presence of the moral hazard problem, less effort on cost reduction is exerted in the case of the managerial firm than that in the case of the entrepreneurial firm. This implies that the managerial firms (both the monopoly

⁴For more detailed discussions about the incentive-pricing dichotomy under asymmetric information in the case of procurement and regulation, see Laffont and Tirole (1999).

managerial firm and the competitive managerial firm) engage in managerial slack, or X-inefficiency in the sense of Leibenstein (1966). This is another effort distortion caused by the moral hazard problem and is different from the effort distortion caused by the monopoly output distortion. Compared with the *first-best* solution of the competitive entrepreneurial firm, the monopoly managerial firm engages in two types of slacks: monopoly slack and managerial slack under cost complementarity between output and effort. Because of the managerial slack, the managerial firms always produce a given output at a higher marginal cost than the entrepreneurial firms. We will state this result formally at the end of Section 2.1.

2.1.3. Any Incentive Power $\alpha \in (0, 1)$ is Given

The first-order conditions (2.3) and (2.4) characterize an output decision equation and an effort decision equation respectively: $q = \tilde{q}_i(e)$ and $e = \tilde{e}_i(q; \alpha)$, $i \in \{M, C\}$. For any given incentive power $\alpha \in (0, 1)$, we have the following two lemmas that are similar to Lemmas 1.1 and 1.2 in Section 1.3 of Chapter 1:

Lemma 2.3. (i) Both curves $\tilde{q}_i(e)$ and $\tilde{e}_i(q; \alpha)$ are monotonically increasing (respectively, decreasing) in e in the (q, e) space as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$).

(ii) The curve $\tilde{q}_i(e)$ at every e has a less positive (respectively, negative) slope than the curve $\tilde{e}_i(q; \alpha)$ as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$). Moreover, the two curves cross only once in the (q, e) space and a unique equilibrium $\{\tilde{q}_i(\alpha), \tilde{e}_i(\alpha)\}$ exists.

(iii) Two output decision curves $\tilde{q}_i(e)$ in the case of the managerial firm and $q_i(e)$ in the case of the entrepreneurial firm coincide in the (q, e) space.

(iv) The effort decision curve $\tilde{e}_i(q; \alpha)$ in the case of the managerial firm has a more positive (respectively, negative) slope than $e_i(q)$ in the case of the entrepreneurial firm as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$) and the effort decision curve rotates counterclockwise.

A comparison of the manager's optimization conditions under monopoly and competition shows:

Lemma 2.4. (i) The output decision curve $\tilde{q}_M(e)$ of the monopoly firm at any $e \geq 0$ has less positive (respectively, negative) slope than $\tilde{q}_C(e)$ of the competitive firm in the case of the managerial firm in the (q, e) space as $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$) and the curve $\tilde{q}_M(e)$ lies everywhere below the curve $\tilde{q}_C(e)$.

(ii) Two effort decision curves $\tilde{e}_M(q; \alpha)$ of the monopoly firm and $\tilde{e}_C(q; \alpha)$ of the competitive firm coincide in the (q, e) space.

To analyze the efficiency losses of monopoly under asymmetric information, we now compare the manager's optimal choices of output and effort for the monopoly firm and the competitive firm.

Proposition 2.2. For any given incentive power $\alpha \in (0, 1)$, the optimal level of output chosen by the manager is lower, i.e., $\tilde{q}_M(\alpha) < \tilde{q}_C(\alpha)$, but the optimal level of effort can be either lower or higher in the monopoly equilibrium than in the competitive equilibrium. To be more specific, the optimal level of effort is lower (respectively,

higher) in the monopoly equilibrium than in the competitive equilibrium if the production technology exhibits cost complementarity (respectively, substitutability) between output and effort, i.e., $\tilde{e}_M(\alpha) \leq \tilde{e}_C(\alpha)$ as $C_{qe}(q, e) \leq 0$.

Proposition 2.2 says that given the same incentive power α chosen by both the owner of the monopoly firm and the owner of the competitive firm, the result on the output distortion and effort distortion of monopoly in the case of the managerial firm can be obtained, which is similar to Proposition 1.1.

From Lemma 2.1 and Proposition 2.2, we have Lemma 2.5.

Lemma 2.5. *For any incentive power $\alpha \in (0, 1)$,*

(i) $0 < d\tilde{e}_M(\alpha)/d\alpha < d\tilde{e}_C(\alpha)/d\alpha$ and $0 < d\tilde{q}_M(\alpha)/d\alpha < d\tilde{q}_C(\alpha)/d\alpha$ in the case where $C_{qe}(q, e) < 0$.

(ii) $d\tilde{e}_M(\alpha)/d\alpha > d\tilde{e}_C(\alpha)/d\alpha > 0$ and $0 > d\tilde{q}_M(\alpha)/d\alpha > d\tilde{q}_C(\alpha)/d\alpha$ in the case where $C_{qe}(q, e) > 0$.

Lemma 2.5 states that the same increase in the incentive power α can induce a larger increase (respectively, a smaller decrease) in output but a smaller (respectively, a larger) rise in effort in the monopoly firm than those in the competitive firm if the technology exhibits cost complementarity (respectively, substitutability) between output and effort. We have the following corollary in the case of the managerial firm, which is similar to Corollary 1.1 in the case of the entrepreneurial firm.

Corollary 2.1. *The manager's expected net income minus the fixed payment β in the monopoly firm is always greater than that in the representative competitive firm.*

2.1.4. The Owner's Optimal Choice of the Incentive Power α

Now we want to know how the owners of the two managerial firms choose the optimal incentive coefficients and whether these two incentive coefficients are same.

Proposition 2.3. *If the production technology exhibits cost complementarity (respectively, substitutability) between output and effort, then the incentive power α set by the owner of the monopoly firm is less (respectively, greater) than that by the owner of the competitive firm, i.e., $\alpha_M^{**} < \alpha_C^{**}$ as $C_{qe}(q, e) < 0$, and $\alpha_M^{**} > \alpha_C^{**}$ as $C_{qe}(q, e) > 0$.*

Proposition 2.3 states that whether the optimal incentive power α of the incentive scheme is stronger or weaker in the monopoly firm than in the competitive firm depends on the nature of production technology. The intuition behind this proposition is that when the technology exhibits cost complementarity between output and effort, the same increase in α has a smaller effect on the manager's effort in the monopoly firm than in the competitive firm. The owner of the monopoly firm responds to this by choosing a smaller incentive power than the owner of the competitive firm. The reverse is true if the technology exhibits cost substitutability between output and effort.

From Lemma 2.2 and Proposition 2.3 we have

$$0 < \alpha_M^{**} < \alpha_C^{**} < 1 \text{ as } C_{qe}(q, e) < 0,$$

$$1 > \alpha_M^{**} > \alpha_C^{**} > 0 \text{ as } C_{qe}(q, e) > 0.$$

2.1.5. The Monopoly Efficiency Losses

Comparing the manager's optimal choices of output and effort in the monopoly managerial firm with those (the *second-best* solution) in competitive managerial firm, we have the following proposition:

Proposition 2.4. *(i) If the technology exhibits cost complementarity between output and effort, the equilibrium effort and output chosen by the manager of the monopoly firm are less than those by the manager of the competitive firm, i.e., $e_M^{**} < e_C^{**}$ and $q_M^{**} < q_C^{**}$.*

(ii) If the technology exhibits cost substitutability between output and effort, the equilibrium effort exerted by the manager of the monopoly firm is greater than that by the manager of the competitive firm but the equilibrium output chosen by the manager of the monopoly firm is less than that by the manager of the competitive firm, i.e., $e_M^{**} > e_C^{**}$ and $q_M^{**} < q_C^{**}$.

Proposition 2.4 makes a comparison between the equilibrium solution of the monopoly managerial firm and the *second-best* solution of the competitive managerial firm. A comparison between Propositions 1.1 and 2.4 suggests that under certain conditions the effects of monopoly under the separation of ownership and control are qualitatively similar to those in the case where the owner manages the firm himself. That is, monopoly leads to lower effort if there is cost complementarity between output and effort; but the opposite is true if there is cost substitutability between output and effort.

Figure 2.1. The output decision curve $\tilde{q}_M(e)$ and the effort-exerting curve $\tilde{e}_M(q; \alpha_M)$ of the monopoly firm, and the output decision curve $\tilde{q}_C(e)$ and the effort-exerting curve $\tilde{e}_C(q; \alpha_C)$ of the representative competitive firm when the technology exhibits *cost complementarity* between output and effort and the incentive power satisfies $0 < \alpha_M < \alpha_C < 1$.

Lemmas 2.3, 2.4 and Proposition 2.4 are depicted in Figures 2.1 and 2.2, which are extensions of Figures 1.1 and 1.2 to the case of managerial firms. The output decision curve $\tilde{q}_i(e)$ and effort decision curve $\tilde{e}_i(q; \alpha_i)$ ($i = M, C$) in the case of managerial firms are determined by (2.3) and (2.4), respectively. Since $\alpha_i < 1$ in the managerial firms, their effort curves rotate inward relative to that of the entrepreneurial firm. In the case of cost complementarity, $\alpha_M < \alpha_C$ implies that the effort curve of the managerial monopoly rotates more than that of the managerial competitive firm. The opposite is true in the case of cost substitutability where $\alpha_M > \alpha_C$.

Figure 2.2. The output decision curve $\tilde{q}_M(e)$ and the effort-exerting curve $\tilde{e}_M(q; \alpha_M)$ of the monopoly firm, and the output decision curve $\tilde{q}_C(e)$ and the effort-exerting curve $\tilde{e}_C(q; \alpha_C)$ of the representative competitive firm when the technology exhibits *cost substitutability* between output and effort and the incentive power satisfies $1 > \alpha_M > \alpha_C > 0$.

Another way of considering the effects of monopoly under the separation of ownership and control is to use the competitive entrepreneurial firm (i.e. the first best equilibrium) as the benchmark. Comparing the optimal solutions of the monopoly managerial firm with the *first-best* solutions of the competitive entrepreneurial firm, we have the following proposition based on Propositions 1.1 and 2.1.

Proposition 2.5. (i) If $C_{qe}(q, e) < 0$, then $e_M^{**} < e_M^* < e_C^*$ and $q_M^{**} < q_M^* < q_C^*$. In this case, the managerial slack arising from the separation of ownership and control exacerbates monopoly efficiency losses.

(ii) If $C_{qe}(q, e) > 0$, then either $e_M^* > e_M^{**} > e_C^*$ or $e_M^{**} < e_C^* < e_M^*$, and $q_M^* < q_M^{**} < q_C^*$. In this case, the managerial slack offsets the over-supply of effort and the under-supply of output by the owner-manager monopolist.

Proposition 2.5 states the effects of managerial slack on the efficiency losses of monopoly when a monopoly *entrepreneurial* firm shifts to a monopoly *managerial* firm. Combining the insights from Propositions 1.1 and 2.1 we see that managerial slack arising from the separation of ownership and control exacerbates the monopoly efficiency losses if there is cost complementarity between output and effort. On the other hand, if there is cost substitutability between output and effort, managerial slack may mitigate or exacerbate the monopoly efficiency losses by offsetting the entrepreneurial monopolist's tendency to increase effort and reduce output. As a result, the effort exerted by the manager of the monopoly firm gets closer to or farther away from the *first-best* effort of the competitive entrepreneurial firm.

From Propositions 2.1 and 2.4, we obtain the following result on the effects of monopoly power on agency inefficiency.

Proposition 2.6. (i) If $C_{qe}(q, e) < 0$, then $e_M^{**} < e_C^{**} < e_C^*$ and $q_M^{**} < q_C^{**} < q_C^*$. In this case, monopoly power exacerbates agency inefficiency.

(ii) If $C_{qe}(q, e) > 0$, then either $e_C^{**} < e_M^{**} < e_C^*$ or $e_C^{**} < e_C^* < e_M^{**}$, and $q_M^{**} < q_C^* < q_C^{**}$. In this case, monopoly power offsets the under-supply of effort and the over-supply of output by the manager of the competitive firm.

Proposition 2.6 states that monopoly power exacerbates managerial slack when a *competitive* managerial firm shifts to a *monopoly* managerial firm in the case where

the technology exhibits cost complementarity between output and effort. Conversely, monopoly power offsets managerial slack if the technology exhibits cost substitutability between output and effort. This result provides an answer to Tirole's (1988, p.75) question concerning how market power on the product market affects "X-inefficiency" (Leibenstein 1966).

A proposition about cost distortions of monopoly in the case of the managerial firm (i.e. productive inefficiency of monopoly) can be directly obtained from Proposition 2.4, which is similar to Proposition 1.1 in the case of the entrepreneurial firm.

Proposition 2.7. *For any given output level q , the total production cost of a monopoly is higher than that of a competitive firm, i.e., $C(q, e_M^{**}) > C(q, e_C^{**})$ in the case where $C_{qe}(q, e) < 0$, but $C(q, e_M^{**}) < C(q, e_C^{**})$ in the case where $C_{qe}(q, e) > 0$. Furthermore, a monopoly produces a given output q at a higher marginal cost than the competitive firm, i.e., $C_q(q, e_M^{**}) > C_q(q, e_C^{**})$.*

From Proposition 1.2 we already know that in the case of entrepreneurial firms, the marginal cost curve of a monopoly is always above that of a competitive firm, and that the total cost curve of a monopoly is higher (respectively, lower) than that of a competitive firm if $C_{qe}(q, e) <$ (respectively, $>$) 0. The same pattern also holds in the case of managerial firms. Propositions 2.4 and 2.7 also suggest that the output distortion arising under monopoly leads to effort distortion, and the effort distortion causes the marginal cost of the monopoly firm to be higher than that of the competitive firm.

2.2. The Alternative Principal-Agent Model with *Partial Delegation*

In this section we assume that the owner can control the level of output. This allows the owner to delegate only partially the operation of the firm to the manager, under which the owner chooses an optimal output that the manager is required to produce. The other aspects of the model are the same as in the model with complete delegation. The only difference between the two models is that the manager chooses output under complete delegation, while the owner decides on output under partial delegation.

Given output and a *linear piece-rate* incentive scheme based on the firm's profit $s(\pi(q, e, \varepsilon)) = \alpha\pi(q, e, \varepsilon) + \beta$, the manager chooses effort e to maximize his expected utility $E[U(e; \alpha, q, \varepsilon)] = \int u[s(\pi(q, e, \varepsilon)) - \psi(e)]f(\varepsilon)d\varepsilon$.⁵

The first-order condition for the manager's expected utility maximization at an interior solution is

$$(2.9) \quad \int u' \cdot \left[\alpha \frac{\partial \pi(q, e, \varepsilon)}{\partial e} - \psi'(e) \right] f(\varepsilon) d\varepsilon = 0.$$

Since $\partial \pi(q, e, \varepsilon)/\partial e$ is independent of the random cost shock ε , the first-order conditions (2.9) can be simplified as

$$(2.10) \quad \alpha \frac{\partial E\pi(q, e, \varepsilon)}{\partial e} - \psi'(e) = -\alpha C_e(q, e) - \psi'(e) = 0.$$

⁵In this alternative model, since the owner is assumed to observe the market demand, the optimal incentive scheme based on the realization of the cost of production $C(q, e, \varepsilon)$ is equivalent to that based on the realization of the profit $\pi(q, e, \varepsilon)$.

From the first-order condition (2.10), the manager's optimal choice of effort e is a function of output q and the incentive power α and is denoted by $\widehat{e}_i(q, \alpha)$, $i \in \{M, C\}$. Conducting comparative statics with respect to the first-order condition (2.10), we determine how the optimal effort chosen by the manager responds to changes in q and α .

Lemma 2.6. (i) *The optimal effort $\widehat{e}_i(q, \alpha)$ chosen by the manager increases (respectively, decreases) with the output q if the production technology exhibits cost complementarity (respectively, substitutability) between output and effort, i.e., $\partial \widehat{e}_i(q, \alpha) / \partial q \gtrless 0$ as $C_{qe}(q, e) \lessgtr 0$.*

(ii) *The optimal effort $\widehat{e}_i(q, \alpha)$ chosen by the manager increases with incentive power α , i.e., $\partial \widehat{e}_i(q, \alpha) / \partial \alpha > 0$.*

The owner chooses α and β of the incentive scheme $s(\pi(q, e, \varepsilon))$ and output q to maximize his expected utility $E[V(q, \alpha, \beta; \varepsilon)] = \int [\pi(q, e, \varepsilon) - s(\pi(q, e, \varepsilon))] f(\varepsilon) d\varepsilon$ subject to the manager's participation constraint $E[U(e; \alpha, q, \varepsilon)] \geq u_0$ and incentive constraint (2.10). Substituting $\widehat{e}_i(q, \alpha)$ into owner i 's expected utility function and manager i 's participation constraint, we can write owner i 's problem as

$$\max_{\{\alpha, \beta, q\}} E[V_i(q, \alpha, \beta; \varepsilon)] = (1 - \alpha) E\pi(q_i, \widehat{e}_i(q, \alpha), \varepsilon) - \beta$$

subject to

$$(2.11) \quad Eu[\alpha\pi(q_i, \widehat{e}_i(q, \alpha), \varepsilon) + \beta - \psi(\widehat{e}_i(q, \alpha))] \geq u_0.$$

Substituting $\beta = (1 - \alpha)E\pi(q_i, \widehat{e}_i(q, \alpha), \varepsilon) - E(V_i)$ into the binding participation constraint (2.11) yields

$$(2.12) \quad Eu[\alpha\pi(q_i, \widehat{e}_i(q, \alpha), \varepsilon) + (1 - \alpha)E\pi(q_i, \widehat{e}_i(q, \alpha), \varepsilon) - E(V_i) - \psi(\widehat{e}_i(q, \alpha))] = u_0.$$

Note that the maximization of $E[V_i(q, \alpha, \beta; \varepsilon)]$ is equivalent to the maximization of the left-hand side of (2.12) by choosing α and q . So the first-order condition for maximizing the left-hand side of (2.12) with respect to α is

$$(2.13) \quad E \left\{ u' \cdot \left[\pi - E\pi + \left(\alpha \frac{\partial \pi}{\partial e} - \psi'(\widehat{e}_i(q, \alpha)) \right) \frac{\partial \widehat{e}_i(q, \alpha)}{\partial \alpha} + (1 - \alpha) \frac{\partial E\pi}{\partial e} \frac{\partial \widehat{e}_i(q, \alpha)}{\partial \alpha} \right] \right\} = 0.$$

Using the first-order condition (2.10), we simplify (2.13) as

$$(2.14) \quad \text{Cov}(u' \cdot \pi) + (1 - \alpha)Eu' \cdot \frac{\partial E\pi}{\partial e} \frac{\partial \widehat{e}_i(q, \alpha)}{\partial \alpha} = 0.$$

We want to determine what reasonable region the optimal incentive power α must lie in. From (2.14), we obtain the following standard result of principal-agent models:

Lemma 2.7. *The optimal incentive power $\widehat{\alpha}_i^{**}$ satisfies the condition $0 < \widehat{\alpha}_i^{**} < 1$.*

Lemma 2.7 implies that the profit-sharing scheme between the owner and the manager still holds in this alternative model.

Solving the incentive constraint (2.10) for α , we obtain $\alpha(q, e) = -\psi'(e)/C_e(q, e)$. Then substituting it into the owner's expected utility function, we rewrite the owner's

problem as

$$(2.15) \quad \max_{\{\beta, q, e\}} E[V(q, e, \beta; \varepsilon)] = (1 - \alpha(q, e))E\pi(q, e, \varepsilon) - \beta$$

subject to

$$(2.16) \quad Eu[\alpha(q, e)\pi(q, e, \varepsilon) + \beta - \psi(e)] \geq u_0.$$

The first-order conditions of the owner's problem (2.15)-(2.16) with respect to output and effort are respectively given by

$$(2.17) \quad -\frac{\partial \alpha(q, e)}{\partial q} E\pi(q, e, \varepsilon) + (1 - \alpha(q, e)) \frac{\partial E\pi(q, e, \varepsilon)}{\partial q} - \frac{\partial \beta(q, e; u_0)}{\partial q} = 0,$$

$$(2.18) \quad -\frac{\partial \alpha(q, e)}{\partial e} E\pi(q, e, \varepsilon) + (1 - \alpha(q, e)) \frac{\partial E\pi(q, e, \varepsilon)}{\partial e} - \frac{\partial \beta(q, e; u_0)}{\partial e} = 0.$$

The manager's participation constraint (2.16) is binding at the optimum. Inverting the function $Eu[\alpha(q, e)\pi(q, e, \varepsilon) + \beta - \psi(e)] = u_0$, we express β as a function of q , e and u_0 : $\beta = \beta(q, e; u_0)$. So we have

$$(2.19) \quad \frac{\partial \beta(q, e; u_0)}{\partial q} = -\frac{[\partial \alpha(q, e)/\partial q]E(u' \cdot \pi)}{Eu'} - \frac{\alpha(q, e)E(u' \cdot \partial \pi / \partial q)}{Eu'},$$

$$(2.20) \quad \frac{\partial \beta(q, e; u_0)}{\partial e} = -\frac{[\partial \alpha(q, e)/\partial e]E(u' \cdot \pi)}{Eu'} - \frac{E[u' \cdot (\alpha(q, e)(\partial \pi / \partial e) - \psi'(e))]}{Eu'}.$$

The manipulations of approximating the functions $E(u' \cdot \pi)$ and Eu' used in the proof of Proposition 2.3 can simplify the numerators of the first term on the right-hand side of (2.19) and (2.20) respectively. Moreover, the numerators of the second term on the right-hand side of (2.19) and (2.20) can be simplified since $\partial\pi/\partial q$ and $\partial\pi/\partial e$ are independent of the random cost shock ε . Thus, we have

$$(2.21) \quad \frac{\partial\beta(q, e; u_0)}{\partial q} \approx - \left\{ \frac{\partial\alpha(q, e)}{\partial q} [E\pi - r(\bar{w})\alpha(q, e)\text{Var}(\pi)] + \alpha(q, e) \frac{\partial E\pi}{\partial q} \right\},$$

$$(2.22) \quad \frac{\partial\beta(q, e; u_0)}{\partial e} \approx - \left\{ \frac{\partial\alpha(q, e)}{\partial e} [E\pi - r(\bar{w})\alpha(q, e)\text{Var}(\pi)] + \left[\alpha(q, e) \frac{\partial E\pi}{\partial e} - \psi'(e) \right] \right\}.$$

where $\bar{w} \equiv \alpha(q, e)E\pi(q, e, \varepsilon) + \beta - \psi(e)$ is the manager's expected net income, $r(\bar{w}) \equiv -u''(\bar{w})/u'(\bar{w})$ is the Arrow-Pratt coefficient of absolute risk aversion evaluated at $\bar{w}$ and $\text{Var}(\pi)$ is the variance of the firm's profit, which is equal to the variance of ε , i.e., $\text{Var}(\pi) = \sigma_\varepsilon^2$. In what follows, we denote $r(\bar{w})$ by the shorthand r .

Substituting (2.21) and (2.22) into these two first-order conditions (2.17) and (2.18) yields

$$(2.23) \quad \frac{\partial E[\pi(q, e, \varepsilon)]}{\partial q} - \frac{\partial RC(q, e)}{\partial q} = 0,$$

$$(2.24) \quad \frac{\partial E[\pi(q, e, \varepsilon)]}{\partial e} - \psi'(e) - \frac{\partial RC(q, e)}{\partial e} = 0,$$

where $RC(q, e) \equiv (r/2)[\alpha(q, e)]^2\sigma_\varepsilon^2$ is the premium paid by the owner to the manager for the risk he must bear and it is known as one of agency costs: the risk cost.

$\partial RC(q, e)/\partial q \equiv r\sigma_\varepsilon^2\alpha(q, e)[\partial\alpha(q, e)/\partial q]$ is the marginal risk cost of increasing output, and $\partial RC(q, e)/\partial e \equiv r\sigma_\varepsilon^2\alpha(q, e)[\partial\alpha(q, e)/\partial e]$ is the marginal risk cost of eliciting more effort from the manager.

Note that compared with the firm's pricing rule (2.3) under complete delegation, the pricing rule (2.23) under partial delegation is not separable from the managerial incentive problem. The effect of the marginal risk cost of increasing output on the pricing rule is captured by the second term on the left-hand side of condition (2.23), i.e., $\partial RC(q, e)/\partial q$. This effect can be negative (respectively, positive), depending on cost complementarity (respectively, cost substitutability) between output and effort. Moreover, the marginal risk cost of increasing effort also affects the manager's effort choice, which is captured by the second term on the left-hand side of condition (2.24), i.e., $\partial RC(q, e)/\partial e$, which is positive.

To ensure the existence and uniqueness of the optimal solution to the owner's problem (2.15)-(2.16), we need to impose the following additional restrictions on the cost function of production:

$$(i) \ C_{eqq}(q, e) \geq 0; \quad (ii) \ C_{eee}(q, e) \geq 0.$$

Lemma 2.8. *(i) $\alpha(q, e)$ is strictly decreasing (respectively, increasing) and convex in output q if the technology exhibits cost complementarity (respectively, cost substitutability) between output and effort.*

(ii) $\alpha(q, e)$ is strictly increasing and convex in effort e .

(iii) $\partial^2\alpha(q, e)/\partial q\partial e \leq 0$ if $C_{qe}(q, e) \leq 0$.

Corresponding to Propositions 2.1, 2.3 and 2.4 in the principal-agent model with *complete delegation*, three analogous results in the principal-agent model with *partial delegation* follow.

For any given incentive power $\alpha_i \in (0, 1)$, $i \in \{M, C\}$, comparing the optimal effort and output levels under partial delegation with those in the case where the owner also manages the firm, we have:

Proposition 2.8. *Assume that the marginal risk cost of eliciting more effort from the manager is large enough. For a given market structure (either monopoly or perfect competition), the equilibrium effort exerted by the manager in the managerial firm is less than that by the owner-manager of the entrepreneurial firm, i.e., $\widehat{e}_i(\alpha_i) < e_i^*$, $i \in \{M, C\}$. The equilibrium output chosen by the owner in the managerial firm is less (respectively, greater) than that by the owner-manager in the entrepreneurial firm if the technology exhibits cost complementarity (respectively, substitutability) between output and effort, i.e., $\widehat{q}_i(\alpha_i) \leq q_i^*$ as $C_{qe}(q, e) \leq 0$, $i \in \{M, C\}$.*

Note that if the marginal risk cost of eliciting more effort from the manager is small enough such that

$$-C_e(q_i^*, \widehat{e}(q_i^*, \alpha_i)) - \psi(\widehat{e}(q_i^*, \alpha_i)) - \frac{\partial RC(q_i^*, \widehat{e}(q_i^*, \alpha_i))}{\partial e} > 0,$$

then by similar reasoning, $\widehat{q}_i(\alpha_i) \geq q_i^*$ as $C_{qe}(q, e) \leq 0$, $i \in \{M, C\}$, but the relative magnitude between $\widehat{e}_i(\alpha_i)$ and e_i^* cannot be determined.

Comparing the monopoly equilibrium solution with the *second-best* solution of the representative competitive firm, we have the following two propositions similar to Propositions 2.3 and 2.4.

Proposition 2.9. (i) *If the technology exhibits cost complementarity (respectively, substitutability) between output and effort, then the optimal incentive power α set by the owner is lower (respectively, higher) in the monopoly firm than in the competitive firm, i.e., $\hat{\alpha}_M^{**} \leq \hat{\alpha}_C^{**}$.*

(ii) *The equilibrium output chosen by the owner of the monopoly firm is less than that by the owner of the competitive firm, i.e., $\hat{q}_M^{**} < \hat{q}_C^{**}$.*

Proposition 2.10. *If the technology exhibits cost complementarity (respectively, substitutability) between output and effort, then the equilibrium effort chosen by the manager of the monopoly firm is less (respectively, greater) than that by the manager of the competitive firm, i.e., $\hat{e}_M^{**} \leq \hat{e}_C^{**}$ as $C_{qe}(q, e) \leq 0$.*

Note that the results in Propositions 2.8, 2.9 and 2.10 are qualitatively the same as those in Propositions 2.1, 2.3 and 2.4. Hence, Propositions 2.5 to 2.7, which are based on Propositions 2.1, 2.3 and 2.4, still hold for this alternative principal-agent model. The welfare analyses in Section 2.3 below can be also applied to this model in the same way.

2.3. The Monopoly Welfare Losses

The monopoly welfare losses in the case of the separation of ownership and control have not been yet discussed in the traditional theory on monopoly. Here we make such a welfare analysis. To make a consistent comparison between the results in this section and Sections 1.4 of Chapter 1, we need to obtain the expressions of the monopoly welfare losses from both output distortion and effort distortion, which are

similar to those in Section 1.4. To do so, we rewrite the manager's expected utility function in terms of the risk premium (RP) or the certainty equivalent (CE) of the manager's net income $s(\pi(q, e, \varepsilon)) - \psi(e)$,

$$E[U(q, e; \varepsilon)] = Eu(s(\pi(q, e, \varepsilon)) - \psi(e)) \equiv u(Es(\pi(q, e, \varepsilon)) - \psi(e) - RP) \equiv u(CE).$$

For simplicity, we restrict the manager's expected utility function to a class of mean-variance functions.⁶ $Es(\pi(q, e, \varepsilon)) - \psi(e)$ in $u(\cdot)$ thus captures the mean of manager's net income, while RP captures its variance and may include other relevant constants such as the risk aversion coefficient. The variance of manager's net income is equal to $\alpha^2 \sigma_\varepsilon^2$. To be more specific, the risk premium is assumed to be a strictly increasing function of the variance σ_ε^2 of the random shock ε , the incentive power α and other constants denoted by a vector θ , i.e., $RP(\sigma_\varepsilon^2, \alpha, \theta)$. Assume that $RP(\sigma_\varepsilon^2, \cdot, \theta)$ is strictly convex in α and $RP(\sigma_\varepsilon^2, 0, \theta) = 0$. In addition, we maintain the assumption that the variance σ_ε^2 of ε is identical in both the monopoly firm and the competitive firm.

Since the manager's participation constraint is binding at the optimum, the certainty equivalent must be equal to his reservation wage, $CE = w_0$. Then the mean reward is given by $Es(\pi(q, e, \varepsilon)) = \psi(e) + RP(\sigma_\varepsilon^2, \alpha, \theta) + w_0$.

⁶When the expected utility function of a lottery only depends on the mean and variance of the outcomes, this utility is referred to as a mean-variance utility function. If a stochastic variable, which defines the outcomes, follows the normal distribution, then it can be completely characterized by the mean and variance, so the expected utility of a stochastic distribution of this variable can be expressed as a function of the mean and variance of the variable. Some examples of mean-variance utility function are given on pages 189-199 of Varian's (1992) textbook. A special mean-variance utility function is constant absolute risk aversion (CARA) utility function $u(w) = -\exp(-rw)$, where r is the Arrow-Pratt measure of absolute risk aversion, and w is normally distributed income. The CARA utility function is extensively used in capital asset pricing models and principal-agent models, for example.

The owner's expected utility is given by

$$\begin{aligned} E[V(q, e, \alpha; \varepsilon)] &= E\pi(q, e, \varepsilon) - Es(\pi(q, e, \varepsilon)) \\ &= E\pi(q, e, \varepsilon) - \psi(e) - RP(\sigma_\varepsilon^2, \alpha, \theta) - w_0. \end{aligned}$$

Note that compared with the owner-manager's expected utility in the case of the entrepreneurial firm, there are two extra terms included in the owner's expected utility in the case of the managerial firm: the risk premium and the manager's reservation wage. Their sum can be viewed as one of the agency costs, which the owner should pay to compensate the manager for the risk and the opportunity cost the manager must bear. In this sense for the owner the risk premium borne by him is just the risk cost. So we denote it by $RC(\sigma_\varepsilon^2, \alpha, \theta)$.

Then the manager's problem can be written as

$$\max_{\{q, e\}} E[U(q, e; \varepsilon)] = u(\alpha E\pi(q, e, \varepsilon) + \beta - \psi(e) - RC(\sigma_\varepsilon^2, \alpha, \theta)).$$

At an interior optimum, two simplified first-order conditions of the monopoly firm and of the competitive firm are respectively given by

$$\begin{aligned} p'(q)q + p(q) - C_q(q, e) &= 0, & p(q) - C_q(q, e) &= 0, \\ -\alpha C_e(q, e) - \psi'(e) &= 0. & -\alpha C_e(q, e) - \psi'(e) &= 0. \end{aligned}$$

For any given e , the above first-order conditions are actually a recursive system of equations for (q, α) . Hence, the optimal solutions of output and incentive power

$\{\tilde{q}_i(e), \alpha_i(e)\}_{i \in \{M, C\}}$ can be derived from these first-order conditions respectively. We obtain the optimal incentive power α set by the owner: $\alpha_i(e) = \frac{\psi'(e)}{-C_e(\tilde{q}_i(e), e)}$.

By Lemma 2.1, $d\alpha_i(e)/de > 0$. In addition, $d^2\alpha_i(e)/de^2 \geq 0$ by imposing an additional restrictions on the cost function of production: $C_{eee}(q, e) \geq 0$. Thus, $\alpha_i(\cdot)$ is a strictly increasing and convex function of e . When $e = 0$, the incentive power $\alpha_i = 0$ by assumption $\psi'(0) = 0$. So $\alpha_i(0) = \alpha'_i(0) = 0$, $i \in \{M, C\}$.

Let us give the following definitions for the case of the managerial firm, which are similar to those for the case of the entrepreneurial firm in Section 1.4 of Chapter 1.

Definition 2.1. *Expected net social welfare of firm i in the case of the managerial firm as a function of effort is defined as the expected total social welfare minus the cost of the effort and the risk cost,*

$$\begin{aligned} E[NSW_2(\tilde{q}_i(e), e; \varepsilon)] &= E[TSW(\tilde{q}_i(e), e; \varepsilon)] - \psi(e) - RC(\sigma_\varepsilon^2, \alpha_i(e), \theta) \\ (2.25) \quad &= \int_0^{\tilde{q}_i(e)} p(x)dx - C(\tilde{q}_i(e), e) + E(\varepsilon) - \psi(e) - RC(\sigma_\varepsilon^2, \alpha_i(e), \theta), i \in \{M, C\}. \end{aligned}$$

Definition 2.2. *The expected net social welfare loss created by the monopoly managerial firm (known as the cost of monopoly) in the case of the managerial firm as a function of effort is defined as the expected deadweight welfare loss DWL^{**} minus the savings on the cost of effort $\Delta\psi(e_C^{**}, e_M^{**})$ and on the risk cost $\Delta RC(e_C^{**}, e_M^{**})$, which is equal to the difference between the expected net social welfare for the representative competitive firm and for the monopoly firm,*

$$DWL^{**} - \Delta\psi(e_C^{**}, e_M^{**}) - \Delta RC(e_C^{**}, e_M^{**}) = E[NSW_2(q_C^*, e_C^*; \varepsilon)] - E[NSW_2(q_M^*, e_M^*; \varepsilon)],$$

where $DWL^{**} = DWL_q(\tilde{q}_C(e_M^{**}), \tilde{q}_M(e_M^{**}), e_M^{**}) + DWL_e(\tilde{q}_C(e_C^{**}), \tilde{q}_C(e_M^{**}), e_C^{**}, e_M^{**})$ and $\Delta RC(e_C^{**}, e_M^{**}) = RC(\sigma_\varepsilon^2, \alpha_C(e_C^{**}), \theta) - RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \theta)$.

The expected net social welfare loss under monopoly in the case of the managerial firm is then equal to the sum of these two welfare losses,

$$\begin{aligned} & E[NSW_2(q_C^{**}, e_C^{**}; \varepsilon)] - E[NSW_2(q_M^{**}, e_M^{**}; \varepsilon)] \\ &= E[NSW_2(\tilde{q}_C(e_M^{**}), e_M^{**}; \varepsilon)] - E[NSW_2(q_M^{**}, e_M^{**}; \varepsilon)] \\ & \quad + E[NSW_2(q_C^{**}, e_C^{**}; \varepsilon)] - E[NSW_2(\tilde{q}_C(e_M^{**}), e_M^{**}; \varepsilon)], \end{aligned}$$

where $E[NSW_2(\tilde{q}_C(e_M^{**}), e_M^{**}; \varepsilon)] - E[NSW_2(q_M^{**}, e_M^{**}; \varepsilon)] = DWL_q(\tilde{q}_C(e_M^{**}), \tilde{q}_M(e_M^{**}), e_M^{**}) - [RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \theta) - RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \theta)]$, and $E[NSW_2(q_C^{**}, e_C^{**}; \varepsilon)] - E[NSW_2(\tilde{q}_C(e_M^{**}), e_M^{**}; \varepsilon)] = DWL_e(\tilde{q}_C(e_C^{**}), \tilde{q}_C(e_M^{**}), e_C^{**}, e_M^{**}) - \Delta\psi(e_C^{**}, e_M^{**}) - \underbrace{[RC(\sigma_\varepsilon^2, \alpha_C(e_C^{**}), \theta) - RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \theta)]}_{\Delta RC_C}$.

Proposition 2.11. (i) *The expected net social welfare loss from the monopoly output distortion in the case of the managerial firm,*

$$(2.26) \quad DWL_q(\tilde{q}_C(e_M^{**}), \tilde{q}_M(e_M^{**}), e_M^{**}) - [RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \theta) - RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \theta)],$$

is positive.

(ii) *The expected net social welfare loss from the monopoly effort distortion in the case of the managerial firm,*

$$DWL_e(\tilde{q}_C(e_C^{**}), \tilde{q}_C(e_M^{**}), e_C^{**}, e_M^{**}) - \Delta\psi(e_C^{**}, e_M^{**}) - \Delta RC_C,$$

is positive.

Proposition 2.11 states that in the presence of agency costs, the expected net social welfare losses from the monopoly output distortion and effort distortion in the case of the managerial firm still exist. From Propositions 1.3 and 2.11, we directly have the following corollary:

Corollary 2.2. *Whatever the firm ownership structure (either entrepreneurial firm or managerial firm), the monopoly firms generally involve welfare losses from both allocative inefficiency and productive inefficiency.*

In Corollary 2.2 we verify the monopoly welfare losses from both allocative inefficiency and productive inefficiency in both the case of the entrepreneurial firm and the case of the managerial firm, but we do not know whether the monopoly welfare losses from the output distortion is greater than those from the effort distortion and whether the monopoly welfare losses in the case of the entrepreneurial firm is less than those in the case of the managerial firm. The numerical examples in Section 2.4 will give some insight of these aspects.

Agency Costs in the Managerial Firms

Finally, we consider the welfare losses from the moral hazard problem in both the monopoly and competitive managerial firms. Traditionally, they are called agency costs in principal-agent models, which are the social cost of the moral hazard problem measured by the extent to which the expected net social welfare falls short of its *first-best* exerted by the entrepreneurial firm's owner-manager.

Definition 2.3. *The agency costs of firm i are defined as the difference between the expected net social welfare of firm i in both the case of the entrepreneurial firm and the case of the managerial firm,*

$$\begin{aligned} AC(e_i^*, e_i^{**}) &= E[NSW_1(q_i(e_i^*), e_i^*; \varepsilon)] - E[NSW_2(\tilde{q}_i(e_i^{**}), e_i^{**}; \varepsilon)] \\ &= DWL_e(q_i(e_i^*), \tilde{q}_i(e_i^{**}), e_i^*, e_i^{**}) - \Delta\psi(e_i^*, e_i^{**}) + RC(\sigma_\varepsilon^2, \alpha_i(e_i^{**}), \theta), \quad i \in \{M, C\}, \end{aligned}$$

where $DWL_e(q_i(e_i^*), \tilde{q}_i(e_i^{**}), e_i^*, e_i^{**}) - \Delta\psi(e_i^*, e_i^{**})$ is the incentive cost (i.e. the cost of managerial slack) and $RC(\sigma_\varepsilon^2, \alpha_i(e_i^{**}), \theta)$ is the risk cost.

Note that in the literature of effort-based principal-agent models, the incentive cost is only measured by the net loss of firm's expected profit minus the saving on the cost of effort. The consumer's expected welfare loss is not taken into account. However, in our effort-based principal-agent model with output decision, the lower effort level exerted by the manager results in deadweight loss from the effort distortion, $DWL_e(q_i(e_i^*), \tilde{q}_i(e_i^{**}), e_i^*, e_i^{**})$, which measures the net expected loss of both firm i 's profit and consumer welfare when manager i 's effort falls short of its *first-best* level.

Proposition 2.12. (i) *The agency costs of the representative competitive firm are positive.*

(ii) *The agency costs of the monopoly firm are positive.*

Machlup (1967) suggests that managerial slack can only exist in imperfectly competitive product markets. Proposition 2.12 indicates that Machlup's (1967) suggestion is not complete; managerial slack can exist even if product markets are perfectly

competitive. His suggestion holds only in very special cases such as in Nalebuff and Stiglitz's (1983a) model where the cost function of production is assumed to exhibit constant returns to scale and the effort expended is assumed to be proportional to the level of output (i.e., $C_{qe} = 0$ in our notation).

2.4. The Parameterized Models and Numerical Simulations

To push further with our analysis and to verify the robustness of the results obtained in Chapters 1 and 2, in particular, to assess the quantitative significance of the qualitative results identified in the propositions and corollaries, we consider two parameterized models in which the cost functions are strictly convex in quantity and have the property of either cost complementarity ($C_{qe} < 0$) or cost substitutability ($C_{qe} > 0$) between output and effort: the specific cost functions are presented in Table 2.1,⁷ where $a > k > 0$. Suppose that the inverse market demand is linear, $p(q) = a - bq$, with $a, b > 0$, and the random shock ε is normally distributed with mean zero and variance σ_ε^2 .⁸ The disutility function of effort expressed in monetary terms is assumed to be quadratic,⁹ $\psi(e) = (R/2)e^2$, where $R > 0$ is a parameter characterizing disutility of effort.

⁷This is in contrast to the parameterized model of Nalebuff and Stiglitz (1983a) in which production technology exhibits constant returns to scale.

⁸The restriction on the normal distribution of the random variable is to pair it with exponential utility when we consider the case of the separation of ownership and control. Holmström and Milgrom (1987) suggested such a pair is convenient to solve some examples of the principal-agent model. For comparison, we impose the same restriction on the distribution of the random variable.

⁹This specification of the disutility function of effort is inspired from the example of Nalebuff and Stiglitz (1983a), the example of Holmström and Milgrom (1987) and Tirole's (1988) example that is taken from Parsons (1984).

Table 2.1. Cost Functions in Two Parameterized Models

	Cost Complementarity	Cost Substitutability
Parameterized Model 1	$C(q, e) = q^3 + (k - e)q$	$C(q, e) = q^3 + (q - k)e$
Parameterized Model 2	$C(q, e) = q^3 + (ke)/(q - k)$	$C(q, e) = q^3 + (k - e)/q$

As in the analysis of the general model, we consider both the case of an entrepreneurial firm and the case of a managerial firm (i.e., the separation of ownership and control). Here we present only the results from numerical simulations for Parameterized model 1 with cost complementarity. But the same numerical exercises can be done for Parameterized model 1 with cost substitutability as well as Parameterized model 2 with cost complementarity or cost substitutability.

2.4.1. The Case of Entrepreneurial Firms

Owner-manager i 's expected utility is

$$E[V_i(q, e; \varepsilon)] = E[\pi(q, e, \varepsilon)] - (R/2)e^2, \quad i \in \{M, C\}.$$

The first-order conditions of the owner-manager's expected utility maximization problem for the monopoly firm and the competitive firm are respectively given by

$$\begin{aligned} a - bq &= 3q^2 + (k - e), & a - 2bq &= 3q^2 + (k - e), \\ q &= eR. & q &= eR. \end{aligned}$$

Solving these first-order conditions for q and e , we obtain the optimal output and effort of the owner-manager monopolist and of the owner of the competitive firm

respectively

$$q_M^* = \frac{1 - 2bR + \sqrt{12(a - k)R^2 + (2bR - 1)^2}}{6R}$$

and $e_M^* = \frac{1 - 2bR + \sqrt{12(a - k)R^2 + (2bR - 1)^2}}{6R^2};$

$$q_C^* = \frac{1 - bR + \sqrt{12(a - k)R^2 + (bR - 1)^2}}{6R}$$

and $e_C^* = \frac{1 - bR + \sqrt{12(a - k)R^2 + (bR - 1)^2}}{6R^2}.$

2.4.2. The Case of Managerial Firms

Suppose that the manager has a constant absolute risk averse utility function,¹⁰ $u(w) = -\exp(-rw)$, where r is the Arrow-Pratt measure of absolute risk aversion, and $w = s(\pi(q, e, \varepsilon)) - \psi(e)$ is the manager's net income. Then his expected utility is given by

$$Eu(w) = - \int \exp(-rw) f(\varepsilon) d\varepsilon = u[E(w) - (r/2)\text{Var}(w)].$$

It follows that given the linear piece-rate incentive contract $s(\pi) = \alpha\pi(q, e, \varepsilon) + \beta$, the manager's certainty equivalent is

$$CE \equiv E(w) - (r/2)\text{Var}(w) = \alpha E[\pi(q, e, \varepsilon)] + \beta - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2,$$

¹⁰The constant absolute risk averse utility function is a special case of mean-variance utility functions and the most common specification in the principal-agent literature. It is taken from Lazear and Rosen (1981), and Holmström and Milgrom (1987).

where $(r/2)\alpha^2\sigma_\varepsilon^2$ is the risk cost borne by the manager. The manager's participation constraint is $CE \geq w_0$, where w_0 denotes his reservation wage. Since the manager's expected utility is increasing in his certainty equivalent, the maximization of his expected utility is equivalent to the maximization of his certainty equivalent.

The risk-neutral owner's expected utility is

$$E[V(\alpha; \varepsilon)] = E[\pi(q, e, \varepsilon) - s(\pi(q, e, \varepsilon))] = E[\pi(q, e, \varepsilon)] - \psi(e) - (r/2)\alpha^2\sigma_\varepsilon^2 - w_0.$$

2.4.2.1. The Principal-Agent Model with *Complete Delegation*. For any given incentive power $\alpha_i \in (0, 1)$, $i \in \{M, C\}$, the first-order conditions of the manager's certainty equivalent maximization problem for the monopoly firm and the competitive firm are respectively given by

$$\begin{aligned} a - bq &= 3q^2 + (k - e), & a - 2bq &= 3q^2 + (k - e), \\ \alpha_M q &= eR. & \alpha_C q &= eR. \end{aligned}$$

Solving these first-order conditions for q and e , we obtain the optimal output and effort of the monopoly firm and of the competitive firm respectively:

$$\begin{aligned} \tilde{q}_M(\alpha_M) &= \frac{\alpha_M - 2bR + \sqrt{12(a - k)R^2 + (2bR - \alpha_M)^2}}{6R}, \\ \tilde{e}_M(\alpha_M) &= \frac{\alpha_M(\alpha_M - 2bR + \sqrt{12(a - k)R^2 + (2bR - \alpha_M)^2})}{6R^2}, \\ \tilde{q}_C(\alpha_C) &= \frac{\alpha_C - bR + \sqrt{12(a - k)R^2 + (bR - \alpha_C)^2}}{6R}, \\ \tilde{e}_C(\alpha_C) &= \frac{\alpha_C(\alpha_C - bR + \sqrt{12(a - k)R^2 + (bR - \alpha_C)^2})}{6R^2}. \end{aligned}$$

From the Hessian matrix $\tilde{H}_M = \begin{bmatrix} \alpha_M(-2b - 6q_M) & \alpha_M \\ \alpha_M & -R \end{bmatrix}$ and $\tilde{H}_C = \begin{bmatrix} \alpha_C(-b - 6q_C) & \alpha_C \\ \alpha_C & -R \end{bmatrix}$, we obtain the elasticities of output and effort with respect to the incentive power respectively:

$$\begin{aligned} \eta_{\tilde{q}_M, \alpha_M} &= \frac{d\tilde{q}_M}{d\alpha_M} \frac{\alpha_M}{\tilde{q}_M} = \frac{(\alpha_M)^2}{\det(\tilde{H}_M)} < 1 \quad \text{and} \quad \eta_{\tilde{e}_M, \alpha_M} = \frac{d\tilde{e}_M}{d\alpha_M} \frac{\alpha_M}{\tilde{e}_M} = \frac{2\alpha_M(b + 3q_M)R}{\det(\tilde{H}_M)} > 1; \\ \eta_{\tilde{q}_C, \alpha_C} &= \frac{d\tilde{q}_C}{d\alpha_C} \frac{\alpha_C}{\tilde{q}_C} = \frac{(\alpha_C)^2}{\det(\tilde{H}_C)} < 1 \quad \text{and} \quad \eta_{\tilde{e}_C, \alpha_C} = \frac{d\tilde{e}_C}{d\alpha_C} \frac{\alpha_C}{\tilde{e}_C} = \frac{\alpha_C(b + 6q_C)R}{\det(\tilde{H}_C)} > 1. \end{aligned}$$

The first-order conditions of the owner's problem for the monopoly firm and the competitive firm are respectively:

$$-2(b + 3q_M)\mu_{M1} + \alpha_M\mu_{M2} = 0,$$

$$q_M - e_MR + \mu_{M1} - \mu_{M2}R = 0,$$

$$\mu_{M2}q_M - r\alpha_M\sigma_\varepsilon^2 = 0,$$

$$a - 2bq_M - 3q_M^2 - k + e_M = 0,$$

$$\alpha_Mq_M - e_MR = 0.$$

$$-(b + 6q_C)\mu_{C1} + \alpha_C\mu_{C2} = 0,$$

$$q_C - e_C R + \mu_{C1} - \mu_{C2} R = 0,$$

$$\mu_{C2}q_C - r\alpha_C\sigma_\varepsilon^2 = 0,$$

$$a - bq_C - 3q_C^2 - k + e_C = 0,$$

$$\alpha_C q_C - e_C R = 0.$$

From equations of the first-order condition with respect to output in both the case of the entrepreneurial firm and the case of the managerial firm, we obtain the output as a function of effort,

$$q_M(e) = \tilde{q}_M(e) = \frac{1}{3} \left(-b + \sqrt{3(a - k + e) + b^2} \right),$$

which is strictly increasing in effort.

In the case of the entrepreneurial firm, the output supposed to be produced by the owner-manager monopolist at the *first-best* level of effort e_C^* is

$$q_M(e_C^*) = \frac{1}{3} \left[-b + \sqrt{4(3(a - k) + b^2)R^2 + 2 \left(1 - bR + \sqrt{12(a - k)R^2 + (bR - 1)^2} \right)} \right] / (2R),$$

which is less than $q_C^* \equiv q_C(e_C^*)$ since $q_M(e) < q_C(e)$ for any $e \geq 0$.

In the case of the managerial firm, the *second-best* level of effort e_C^{**} is a solution to the nonlinear first-order conditions of the owner's problem of the representative competitive firm, so the output produced by the owner-manager monopolist at the

second-best level of effort e_C^{**} is

$$\tilde{q}_M(e_C^{**}) = \frac{1}{3} \left(-b + \sqrt{3(a - k + e_C^{**}) + b^2} \right),$$

which is less than $q_C^{**} \equiv \tilde{q}_C(e_C^{**})$ since $\tilde{q}_M(e) < \tilde{q}_C(e)$ for any $e \geq 0$.

2.4.2.2. The Principal-Agent Model with *Partial* Delegation. For any given incentive power $\alpha_i \in (0, 1)$, $i \in \{M, C\}$, the first-order conditions for the manager's certainty equivalent maximization problem are given by

$$(2.27) \quad \alpha_i q_i - e_i R = 0, \quad i \in \{M, C\}.$$

The first-order conditions of the owner's problem for the monopoly firm and the competitive firm are respectively:

$$a - 2bq_M - 3q_M^2 - k + e_M + \mu_M \alpha_M = 0,$$

$$q_M - e_M R - \mu_M R = 0,$$

$$\mu_M q_M - r \alpha_M \sigma_\varepsilon^2 = 0,$$

$$\alpha_M q_M - e_M R = 0.$$

$$a - bq_C - 3q_C^2 - k + e_C + \mu_C \alpha_C = 0,$$

$$q_C - e_C R - \mu_C R = 0,$$

$$\mu_C q_C - r \alpha_C \sigma_\varepsilon^2 = 0,$$

$$\alpha_C q_C - e_C R = 0.$$

Note that the first-order conditions of the owner's problem of the monopoly firm and of the representative competitive firm are systems of nonlinear equations, more precisely, systems of multivariate polynomial equations. Because of their complexity, we are not able to obtain analytical solutions for q , e and α . However, we can solve these systems of equations by numerical methods. This we turn to next.

2.4.3. The Numerical Simulations and Results

For numerical simulations of the models, *any* set of economically reasonable parameter values can be used because the theoretical results have been developed only on the basis of the general model with regular assumptions, not relying on the special assumptions about functional forms and parameters. In the base scenario, the following values for seven parameters are taken as an example:¹¹

$$a = 60, b = 10, k = 10, R = 1, \sigma_\epsilon^2 = 0.81, w_0 = 1, rw_0 = 9.$$

To facilitate numerical sensitivity analysis of the equilibrium values of the endogenous variables, of the changes in the welfare losses and of the changes in the agency costs, each time we change the value of one of the seven parameters to re-compute all the numerical results. Thus, we obtain eight sets of the optimal solutions for each of the two parameterized models with cost complementarity and cost substitutability and for each of the two cases, respectively. The different values of the seven parameters are listed in Table 2.2.

¹¹ rw_0 is average relative risk aversion and its value is taken from Caballero's (1990) numerical examples.

Table 2.2. Benchmark and new parameter values

Parameter	a	b	k	R	rw_0	σ_ε^2
Benchmark	60	10	10	1	9	0.81
New Value	80	20	20	3	3	0.36

The main quantitative results are shown in Tables 2.3 to 2.12. Tables 2.3 to 2.6 make comparisons between the monopoly equilibrium solution (in both the case of the entrepreneurial firm and the case of the managerial firm) and the *first-best* or *second-best* solution of the competitive firm, which are based on the numerical results with the benchmark parameters of Tables 2.7 and 2.10.

In Tables 2.7 to 2.12 we present samples of these numerical exercises. Note that the equilibria in Tables 2.7 to 2.12 are associated with seven sets of parameter values, the base scenario and sensitivity analysis with the value of one parameter being changed.

In Tables 2.7 and 2.10, equilibrium values of output, effort, where applicable, and the incentive power are presented. Various aspects of monopoly efficiency losses are in Tables 2.8 and 2.11. In Tables 2.9 and 2.12 we take a more detailed look at the agency costs.

Several interesting observations can be made from these tables.

Observation 1 All the results obtained from the general models in Chapters 1 and 2 are confirmed by the comparisons between the monopoly equilibrium solution (in both the case of the entrepreneurial firm and the case of the managerial firm) and the *first-best* or *second-best* solution of the representative competitive firm in Tables 2.7 and 2.10.

Observation 2 Compared with the traditional neoclassical models (under the strong assumption about the absence of monopoly slack), the welfare losses of monopoly can be larger or smaller in our parameterized models. As we can see from the last rows of Tables 2.8 and 2.11, compared with the traditional neoclassical models, the non-boldfaced numbers calculated from Parameterized models 1 and 2 with cost complementarity show that the welfare losses of monopoly are higher both in the entrepreneurial firm (“Case 1”) and in the managerial firm (“Case 2”). Moreover, the boldfaced numbers calculated from Parameterized models 1 and 2 with cost substitutability show that the welfare losses of monopoly are higher in “Case 1”. However, an exception from Parameterized models 1 and 2 with cost substitutability in “Case 2” is that the welfare losses of monopoly are lower.

Observation 3 In Tables 2.8 and 2.11, the expected net welfare loss from effort distortion is less than that from output distortion in both the case of the entrepreneurial firm and the case of the managerial firm for Parameterized models 1 and 2 with cost complementarity or cost substitutability. These numerical results do not support Leibenstein’s (1966) argument that the real costs of monopoly may derive more from organizational slack than price distortions.

Observation 4 The moral hazard problem arising from the separation of ownership and control does not necessarily exacerbate the welfare losses of monopoly. From the last row of Table 2.8, the non-boldfaced numbers with parameter value $b = 20$ and $R = 3$ calculated from Parameterized model 1 with cost complementarity show that the welfare losses arising from a monopoly run by a manager

are smaller than those from an entrepreneurial monopolist. The boldfaced numbers in the last row of Table 2.8 calculated from Parameterized model 1 with cost substitutability show that the moral hazard problem arising from the separation of ownership and control does mitigate the welfare losses of monopoly.

Observation 5 The agency costs are not always higher in a monopoly industry than in a competitive industry even though there is a higher level of managerial slack in monopoly firms than in competitive firms. In Table 2.9, the non-boldfaced numbers with parameter values $b = 20$ and $R = 3$ calculated from Parameterized model 1 with cost complementarity show that the agency costs in the monopoly industry are lower than those in the competitive industry. We find this quantitative evidence from the numerical simulation to support Jensen and Meckling's (1976) claim that agency costs are no less in competitive industries than in monopoly industries.

All the above observations based on the parameterized models and numerical examples suggest that the problem of the monopoly efficiency losses is quite complex.

Tables 2.3-2.6 are based on the numerical results of the parameterized models with the *benchmark parameters* in Tables 2.7 and 2.10.

Table 2.3. The Equilibrium Solution (q_M^*, e_M^*) of the Monopoly Firm in Case 1 versus the First-Best Solution (q_C^*, e_C^*)
The Efficiency Losses of Monopoly in Case 1

Cost Function	The Output determined by the Owner-Manager	
	Cost Complementarity	Cost Substitutability
Parameterized Model 1	$C(q, e) = q^3 + (k - e)q$ $q_M^* = 2 < q_C^* = 2.84933$ $e_M^* = 2 < e_C^* = 2.84933$	$C(q, e) = q^3 + (q - k)e$ $q_M^* = 2 < q_C^* = 2.84933$ $e_M^* = 8 > e_C^* = 7.15067$
Parameterized Model 2	$C(q, e) = q^3 + (ke)/(q - k)$ $q_M^* = 2.25082 < q_C^* = 3.11664$ $e_M^* = 1.29046 < e_C^* = 1.45278$	$C(q, e) = q^3 + (k - e)/q$ $q_M^* = 2.29825 < q_C^* = 3.14011$ $e_M^* = 0.435114 > e_C^* = 0.318461$

Table 2.4. The Equilibrium Solution (q_M^{}, e_M^{**}) of the Monopoly Firm in Case 2 versus the Second-Best Solution (q_C^{**}, e_C^{**})**
The Efficiency Losses of Monopoly in Case 2

Cost Function	The Output determined by the Manager		The Output determined by the Owner	
	Cost Complementarity	Cost Substitutability	Cost Complementarity	Cost Substitutability
Parameterized Model 1	$C(q, e) = q^3 + (k - e)q$ $q_M^{**} = 1.95859 < q_C^{**} = 2.79785$ $e_M^{**} = 0.680165 < e_C^{**} = 1.46247$ $\alpha_M = 0.347272 < \alpha_C = 0.522711$	$C(q, e) = q^3 + (q - k)e$ $q_M^{**} = 2.02573 < q_C^{**} = 2.88255$ $e_M^{**} = 7.17476 > e_C^{**} = 6.24734$ $\alpha_M = 0.899738 > \alpha_C = 0.877749$	$C(q, e) = q^3 + (k - e)q$ $q_M^{**} = 1.97288 < q_C^{**} = 2.82458$ $e_M^{**} = 0.686709 < e_C^{**} = 1.47596$ $\alpha_M = 0.348074 < \alpha_C = 0.522539$	$C(q, e) = q^3 + (q - k)e$ $q_M^{**} = 2.0027 < q_C^{**} = 2.8536$ $e_M^{**} = 7.17901 > e_C^{**} = 6.25373$ $\alpha_M = 0.89768 > \alpha_C = 0.875088$
Parameterized Model 2	$C(q, e) = q^3 + (ke)/(q - k)$ $q_M^{**} = 2.24559 < q_C^{**} = 3.10833$ $e_M^{**} = 0.239573 < e_C^{**} = 0.325256$ $\alpha_M = 0.185775 < \alpha_C = 0.224155$	$C(q, e) = q^3 + (k - e)/q$ $q_M^{**} = 2.30052 < q_C^{**} = 3.14119$ $e_M^{**} = 0.0109823 > e_C^{**} = 0.00436513$ $\alpha_M = 0.025265 > \alpha_C = 0.0137117$	$C(q, e) = q^3 + (ke)/(q - k)$ $q_M^{**} = 2.24656 < q_C^{**} = 3.11019$ $e_M^{**} = 0.239622 < e_C^{**} = 0.325329$ $\alpha_M = 0.185789 < \alpha_C = 0.224189$	$C(q, e) = q^3 + (k - e)/q$ $q_M^{**} = 2.30046 < q_C^{**} = 3.14117$ $e_M^{**} = 0.0109829 > e_C^{**} = 0.00436517$ $\alpha_M = 0.0252656 > \alpha_C = 0.0137118$

**Table 2.5. The Equilibrium Solution (q_M^*, e_M^*) of the Monopoly Firm in Case 2 versus the First-Best Solution (q_C^*, e_C^*)
The Effect of Agency Inefficiency on the Efficiency Losses of Monopoly**

Cost Function	The Output determined by the <i>Manager</i>		The Output determined by the <i>Owner</i>	
	Cost Complementarity	Cost Substitutability	Cost Complementarity	Cost Substitutability
	$C(q, e) = q^3 + (k - e)q$	$C(q, e) = q^3 + (q - k)e$	$C(q, e) = q^3 + (k - e)q$	$C(q, e) = q^3 + (q - k)e$
Parameterized Model 1	$q_M^* = 1.95859 < q_C^* = 2 < q_C^* = 2.84933$	$q_M^* = 2 < q_M^* = 2.02573 < q_C^* = 2.84933$	$q_M^* = 1.97288 < q_C^* = 2 < q_C^* = 2.84933$	$q_M^* = 2 < q_M^* = 2.0027 < q_C^* = 2.84933$
	$e_M^* = 0.680165 < e_C^* = 2 < e_C^* = 2.84933$	$e_M^* = 8 > e_M^* = 7.17476 > e_C^* = 7.15067$	$e_M^* = 0.686709 < e_C^* = 2 < e_C^* = 2.84933$	$e_M^* = 8 > e_M^* = 7.17901 > e_C^* = 7.15067$
Cost Function	$C(q, e) = q^3 + (ke)/(q - k)$	$C(q, e) = q^3 + (k - e)/q$	$C(q, e) = q^3 + (ke)/(q - k)$	$C(q, e) = q^3 + (k - e)/q$
Parameterized Model 2	$q_M^* = 2.24559 < q_C^* = 2.25082 < q_C^* = 3.11664$	$q_M^* = 2.29825 < q_M^* = 2.30052 < q_C^* = 3.14011$	$q_M^* = 2.24656 < q_C^* = 2.25082 < q_C^* = 3.11664$	$q_M^* = 2.29825 < q_M^* = 2.30046 < q_C^* = 3.14011$
	$e_M^* = 0.239573 < e_C^* = 1.29046 < e_C^* = 1.45278$	$e_M^* = 0.0109823 < e_C^* = 0.318461 < e_C^* = 0.435114$	$e_M^* = 0.239622 < e_C^* = 1.29046 < e_C^* = 1.45278$	$e_M^* = 0.0109829 < e_C^* = 0.318461 < e_C^* = 0.435114$

**Table 2.6. The Equilibrium Solution (q_M^*, e_M^*) of the Monopoly Firm in Case 2 versus the First-Best Solution (q_C^*, e_C^*)
The Effect of Monopoly Power on Agency Inefficiency**

Cost Function	The Output determined by the <i>Manager</i>		The Output determined by the <i>Owner</i>	
	Cost Complementarity	Cost Substitutability	Cost Complementarity	Cost Substitutability
	$C(q, e) = q^3 + (k - e)q$	$C(q, e) = q^3 + (q - k)e$	$C(q, e) = q^3 + (k - e)q$	$C(q, e) = q^3 + (q - k)e$
Parameterized Model 1	$q_M^* = 1.95859 < q_C^* = 2.79785 < q_C^* = 2.84933$	$q_M^* = 2.02573 < q_C^* = 2.84933 < q_C^* = 2.88255$	$q_M^* = 1.97288 < q_C^* = 2.82458 < q_C^* = 2.84933$	$q_M^* = 2.0027 < q_C^* = 2.84933 < q_C^* = 2.8536$
	$e_M^* = 0.680165 < e_C^* = 1.46247 < e_C^* = 2.84933$	$e_C^* = 6.24734 < e_C^* = 7.15067 < e_M^* = 7.17476$	$e_M^* = 0.686709 < e_C^* = 1.47596 < e_C^* = 2.84933$	$e_C^* = 6.25373 < e_C^* = 7.15067 < e_M^* = 7.17901$
Cost Function	$C(q, e) = q^3 + (ke)/(q - k)$	$C(q, e) = q^3 + (k - e)/q$	$C(q, e) = q^3 + (ke)/(q - k)$	$C(q, e) = q^3 + (k - e)/q$
Parameterized Model 2	$q_M^* = 2.24559 < q_C^* = 3.10833 < q_C^* = 3.11664$	$q_M^* = 2.30052 < q_C^* = 3.14011 < q_C^* = 3.14119$	$q_M^* = 2.24656 < q_C^* = 3.11019 < q_C^* = 3.11664$	$q_M^* = 2.30046 < q_C^* = 3.14011 < q_C^* = 3.14117$
	$e_M^* = 0.239573 < e_C^* = 0.325256 < e_C^* = 1.4527$	$e_C^* = 0.00436513 < e_M^* = 0.0109823 < e_C^* = 0.318461$	$e_M^* = 0.239622 < e_C^* = 0.325329 < e_C^* = 1.45278$	$e_C^* = 0.00436517 < e_M^* = 0.0109829 < e_C^* = 0.318461$

In Tables 2.7-2.12, the non-boldfaced numbers are calculated from the **parameterized models with cost complementarity** and the boldfaced numbers are calculated from the **parameterized models with cost substitutability**.

Table 2.7. The Equilibrium Solutions of Output, Effort and Incentive Power for different parameter values (Parameterized Model 1)

Types of Model	Traditional Model		Parameterized Model 1																	
Parameter Value	Benchmark		Benchmark						$a = 80$						$b = 20$					
Case	Case 1	Case 2	Case 1			Case 2			Case 1			Case 2			Case 1			Case 2		
Solution	$q(e^*)$	$q(e^{**})$	q^*	e^*	q^{**}	e^{**}	α^{**}	q^*	e^*	q^{**}	e^{**}	α^{**}	q^*	e^*	q^{**}	e^{**}	α^{**}			
Monopoly Firm	2.02648	1.98318	2.0	2.0	1.95859	0.68017	0.34727	2.60924	2.60924	2.57045	1.23081	0.47883	1.17572	1.17572	1.15451	0.178958	0.15501			
	2.02648	2.05449	2.0	8.0	2.02573	7.17476	0.89974	2.60924	7.39076	2.63381	6.51279	0.884147	1.17572	8.82428	1.19184	8.06506	0.915635			
Representative Competitive Firm	N/A	N/A	2.84933	2.84933	2.79785	1.46247	0.52271	3.5580	3.5580	3.51539	2.22793	0.63376	2.0	2.0	1.95859	0.680165	0.34727			
			2.84933	7.15067	2.88255	6.24734	0.877749	3.5580	6.4420	3.58899	5.46748	0.852827	2.0	8.0	2.02573	7.17476	0.899738			

Types of Model	Parameterized Model 1															
Parameter Value	$k = 20$					$R = 3$					$rw_0 = 3$			$\sigma_s^2 = 0.36$		
Case	Case 1		Case 2			Case 1		Case 2			Case 2			Case 2		
Solution	q^*	e^*	q^{**}	e^{**}	α^{**}	q^*	e^*	q^{**}	e^{**}	α^{**}	q^{**}	e^{**}	α^{**}	q^{**}	e^{**}	α^{**}
Monopoly Firm	1.66667	1.66667	1.62545	0.43527	0.26778	1.95773	0.65258	1.94014	0.095089	0.14704	1.97579	1.22693	0.62098	1.97128	1.08349	0.54964
	1.66667	18.33333	1.67965	17.9433	0.97942	2.16582	2.61139	2.18669	1.92126	0.737689	2.00917	7.70638	0.964403	2.01209	7.61283	0.953044
Representative Competitive Firm	2.44757	2.44757	2.39105	1.06181	0.44408	2.77778	0.925926	2.75185	0.236495	0.25782	2.82455	2.17967	0.77169	2.81863	2.01755	0.71582
	2.44757	17.5524	2.46402	17.1456	0.977736	3.02403	2.32532	3.04978	1.59879	0.690101	2.86134	6.82468	0.956017	2.86513	6.72187	0.942116

Table 2.8. The Welfare Losses of Monopoly for different parameter values (*Parameterized Model 1*)

Types of Model	Traditional Model		Parameterized Model 1					
Parameter Value	Benchmark		Benchmark		$a = 80$		$b = 20$	
Case	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
The Deadweight Welfare Loss from <i>Output Distortion</i> , DWL_q	8.61604	8.34859	8.45229	8.19757	12.3690	12.1114	9.64217	9.34564
	8.61604	8.79003	8.45229	8.6114	12.3690	12.5327	9.64217	9.86968
The Deadweight Welfare Loss from <i>Effort Distortion</i> , DWL_e	N/A	N/A	2.40668	2.17732	3.36129	3.48922	1.63793	0.977701
			-6.08662	-6.61667	-6.12627	-6.71887	-6.60488	-7.11187
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi$	N/A	N/A	-2.05934	-0.838096	-2.92560	-1.72438	-1.30884	-0.215299
			+6.43396	+6.22396	+6.56196	+6.26156	+6.93396	+6.78403
The Saving on the <i>Risk Cost</i> , $-\Delta RC$	N/A	N/A	N/A	-0.556333	N/A	-0.628312	N/A	-0.351999
				+0.142467		+0.198297		+0.10519
The Expected Net Social Welfare Loss, $\Delta E(NSW)$	8.61604	8.34859	8.79963	8.98047	12.805	13.2479	9.97125	9.75604
	8.61604	8.79003	8.79963	8.36116	12.805	12.2737	9.97125	9.64703

Types of Model	Parameterized Model 1					
Parameter Value	$k = 20$		$R = 3$		$rw_0 = 3$	$\sigma_s^2 = 0.36$
Case	Case 1	Case 2	Case 1	Case 2	Case 2	Case 2
The Deadweight Welfare Loss from <i>Output Distortion</i> , DWL_q	6.45313	6.21482	8.19224	8.08456	8.30313	8.27545
	6.45313	6.52865	9.48846	9.62065	8.50892	8.52695
The Deadweight Welfare Loss from <i>Effort Distortion</i> , DWL_e	1.89894	1.49001	0.757906	0.388751	2.67415	2.61642
	-13.7192	-14.0024	-1.99709	-2.24313	-6.30852	-6.37152
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi$	-1.60642	-0.46899	-0.647227	-0.070332	-1.62279	-1.44828
	+14.0117	+13.9962	+2.11839	+1.70271	+6.40605	+6.38583
The Saving on the <i>Risk Cost</i> , $-\Delta RC$	N/A	-0.457433	N/A	-0.163488	-0.255003	-0.340674
		+0.1012013		+0.247661	+0.0195662	+0.0335524
The Expected Net Social Welfare Loss, $\Delta E(NSW)$	6.746	6.7784	8.30292	8.23949	9.09949	9.10291
	6.746	6.53453	9.60976	9.32788	8.62601	8.57481

Table 2.9. The Agency Costs of the Monopoly Firm and the Representative Competitive Firm for different parameter values (*Parameterized Model I*)

Parameter Value	Benchmark		$a = 80$		$b = 20$	
Types of Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm
The Deadweight Welfare Loss from the <i>Effort Distortion</i> , $DWL_e(q(e^*), q(e^{**}), e^*, e^{**})$	3.43192 6.07347	3.91599 6.44442	4.57445 5.83387	4.70409 6.26277	1.65563 6.31183	2.61238 6.59131
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi(e^*, e^{**}) = -[\psi(e^*) - \psi(e^{**})]$	-1.76869 -6.26143	-2.98993 -6.05143	-2.64662 -6.10343	-3.84784 -5.80304	-0.675145 -6.41136	-1.76869 -6.26143
The Incentive Cost, $IC = DWL_e(q(e^*), q(e^{**}), e^*, e^{**}) - \Delta\psi(e^*, e^{**})$	1.66323 -0.187956	0.995912 0.392985	1.92784 -0.269565	0.856251 0.45973	0.980484 -0.0995259	0.843693 0.329889
The Risk Cost, $RC = (r/2)(\alpha(e^{**}))^2 \sigma_e^2$	0.439579 2.95073	0.995912 2.80826	0.835721 2.84935	1.46403 2.65106	0.0875806 3.05592	0.439579 2.95073
Total Agency Cost, $AC(e^*, e^{**}) = IC + RC + CE$	2.10281 2.76278	1.92198 3.20125	2.76356 2.57979	2.32028 3.11079	1.06807 2.9564	1.28327 3.28062

Parameter Value	$k = 20$		$R = 3$		$rw_0 = 3$		$\sigma_e^2 = 0.36$	
Types of Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm
The Deadweight Welfare Loss from the <i>Effort Distortion</i> , $DWL_e(q(e^*), q(e^{**}), e^*, e^{**})$	2.70543 6.93063	3.35268 7.13831	1.42932 4.94507	1.90616 5.05893	2.01812 2.16382	1.8998 2.3291	2.3901 2.85255	2.3572 3.06279
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi(e^*, e^{**}) = -[\psi(e^*) - \psi(e^{**})]$	-1.29416 -7.07435	-2.43159 -7.05888	-0.625219 -4.69219	-1.20211 -4.27651	-1.24732 -2.30584	-1.68387 -2.27793	-1.41302 -3.02237	-2.02408 -2.97425
The Incentive Cost, $IC = DWL_e(q(e^*), q(e^{**}), e^*, e^{**}) - \Delta\psi(e^*, e^{**})$	1.41127 -0.143715	0.921095 0.0794253	0.8041 0.252882	0.704043 0.782421	0.770799 -0.142014	0.215935 0.0511761	0.977081 -0.169827	0.333129 0.088544
The Risk Cost, $RC = (r/2)(\alpha(e^{**}))^2 \sigma_e^2$	0.261377 3.439651	0.71881 3.4845	0.078802 1.98356	0.24229 1.73589	0.46853 1.13004	0.723533 1.11047	0.489408 1.47143	0.830082 1.43788
Total Agency Cost, $AC(e^*, e^{**}) = IC + RC + CE$	1.67265 3.3528	1.63991 3.56393	0.8829 2.23644	0.94633 2.51832	1.23933 0.98802	0.93947 1.16165	1.46649 1.30161	1.16321 1.52643

Table 2.10. The Equilibrium Solutions of Output, Effort and Incentive Power for different parameter values (*Parameterized Model 2*)

Types of Model	Traditional Model		Parameterized Model 2														
Parameter Value	Benchmark		Benchmark						$a = 80$					$b = 20$			
Case	Case 1	Case 2	Case 1		Case 2			Case 1		Case 2			Case 1		Case 2		
Solution	$q(e^+)$	$q(e^{**})$	q^+	e^+	q^{**}	e^{**}	α^{**}	q^+	e^+	q^{**}	e^{**}	α^{**}	q^+	e^+	q^{**}	e^{**}	α^{**}
Monopoly Firm	2.25163	2.24602	2.25082	1.29046	2.24559	0.239573	0.185775	2.82035	1.39283	2.81456	0.292187	0.209949	1.36429	1.15798	1.36156	0.179763	0.155287
	2.29887	2.30055	2.29825	0.435114	2.30052	0.0109823	0.025265	2.84526	0.351461	2.84639	0.00584926	0.0166493	1.45221	0.688605	1.45777	0.0416043	0.06065
Representative Competitive Firm	N/A	N/A	3.11664	1.45278	3.10833	0.325256	0.224155	3.77231	1.60573	3.76291	0.418138	0.260797	2.25082	1.29046	2.24559	0.239573	0.185775
			3.14011	0.318461	3.14119	0.00436513	0.0137117	3.78049	0.264516	3.78104	0.00251357	0.0095039	2.29825	0.435114	2.30052	0.0109823	0.025265

Types of Model	Parameterized Model 2															
Parameter Value	$k = 20$					$R = 3$					$rw_0 = 3$			$\sigma_e^2 = 0.36$		
Case	Case 1		Case 2			Case 1		Case 2			Case 2			Case 2		
Solution	q^*	e^*	q^{**}	e^{**}	α^{**}	q^*	e^*	q^{**}	e^{**}	α^{**}	q^{**}	e^{**}	α^{**}	q^{**}	e^{**}	α^{**}
Monopoly Firm	2.24654	1.12654	2.24472	0.166993	0.148251	2.24654	0.429915	2.24455	0.0303666	0.070652	2.24701	0.524286	0.406478	2.24657	0.43762	0.339305
	2.34938	0.425642	2.3514	0.0102957	0.0242094	2.29982	0.14494	2.30057	0.00124105	0.0085654	2.30041	0.031368	0.072159	2.30045	0.023957	0.0551107
Representative Competitive Firm	3.10884	1.18405	3.10641	0.190913	0.16126	3.1095	0.483758	3.10625	0.0424411	0.087773	3.1109	0.674383	0.464589	3.11015	0.572076	0.394152
	3.17371	0.315089	3.17473	0.0042295	0.0134274	3.14084	0.106129	3.1412	0.0004895	0.0046127	3.14116	0.012746	0.040038	3.14117	0.0096563	0.0303319

Table 2.11. The Welfare Losses of Monopoly for different parameter values (*Parameterized Model 2*)

Types of Model	Traditional Model		Parameterized Model 2					
Parameter Value	Benchmark		Benchmark		$a = 80$		$b = 20$	
Case	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
The Deadweight Welfare Loss from <i>Output Distortion</i> , DWL_q	10.0604 9.91179	10.0041 9.9098	10.0522 9.91254	9.99984 9.90984	13.8265 13.6769	13.7564 13.6749	12.4325 12.3157	12.3775 12.3171
The Deadweight Welfare Loss from <i>Effort Distortion</i> , DWL_e	N/A	N/A	0.235793 -0.0371518	0.124322 -0.0021066	0.341823 -0.022999	0.201923 -0.000882	0.170949 -0.11033	0.077128 -0.0133114
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi$	N/A	N/A	-0.22264 +0.0439534	-0.024198 +0.00005078	-0.319205 +0.026778	-0.044733 +0.0000139	-0.162181 +0.142426	-0.0125402 +0.00080515
The Saving on the <i>Risk Cost</i> , $-\Delta RC$	N/A	N/A	N/A	-0.0573484 +0.00164138	N/A	-0.0872477 +0.0006812	N/A	-0.0379008 +0.011081
The Expected Net Social Welfare Loss, $\Delta E(NSW)$	10.0604 9.91179	10.0041 9.9098	10.0654 9.91934	10.0426 9.90943	13.8491 13.6806	13.8263 13.6747	12.4413 12.3478	12.4042 12.3157

Types of Model	Parameterized Model 2					
Parameter Value	$k = 20$		$R = 3$		$rw_0 = 3$	$\sigma_\varepsilon^2 = 0.36$
Case	Case 1	Case 2	Case 1	Case 2	Case 2	Case 2
The Deadweight Welfare Loss from <i>Output Distortion</i> , DWL_q	10.0047 9.86024	9.99043 9.85853	10.0093 9.91069	9.98945 9.90978	10.014 9.90997	10.0097 9.90992
The Deadweight Welfare Loss from <i>Effort Distortion</i> , DWL_e	0.068095 -0.0348368	0.028318 -0.0019108	0.0781375 -0.0123572	0.017515 -0.00023926	0.217859 -0.0059283	0.195138 -0.00455256
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi$	-0.0664416 +0.040946	-0.00428051 +0.000044057	-0.0737913 +0.0146164	-0.00131867 +0.000001951	-0.0899584 +0.00041074	-0.0678802 +0.00024034
The Saving on the <i>Risk Cost</i> , $-\Delta RC$	N/A	-0.0146771 +0.00147914	N/A	-0.00988695 +0.000189864	-0.0615013 +0.00437867	-0.0651693 +0.0034298
The Expected Net Social Welfare Loss, $\Delta E(NSW)$	10.0064 9.86635	9.99979 9.85814	10.0137 9.91295	9.99576 9.90973	10.0804 9.90883	10.0718 9.90904

Table 2.12. The Agency Costs of the Monopoly Firm and the Representative Competitive Firm for different parameter values (*Parameterized Model 2*)

Parameter Value	Benchmark		$a = 80$		$b = 20$	
Types of Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm
The Deadweight Welfare Loss from the <i>Effort Distortion</i> , $DWL_e(q(e^*), q(e^{**}), e^*, e^{**})$	1.47319 0.132356	1.63705 0.1001	1.69546 0.0894246	1.90552 0.0692988	1.20681 0.282931	1.35567 0.184454
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi(e^*, e^{**}) = -[\psi(e^*) - \psi(e^{**})]$	-0.803944 -0.0946016	-1.00239 -0.050699	-0.927295 -0.0617454	-1.20177 -0.0349812	-0.654304 -0.236223	-0.803944 -0.0946016
The Incentive Cost, $IC = DWL_e(q(e^*), q(e^{**}), e^*, e^{**}) - \Delta\psi(e^*, e^{**})$	0.669242 0.0377539	0.634666 0.0493108	0.768165 0.0276792	0.70375 0.0343176	0.552502 0.0467087	0.551723 0.0898527
The Risk Cost, $RC = (r/2)(\alpha(e^{**}))^2 \sigma_e^2$	0.125797 0.00232668	0.183145 0.0006853	0.160667 0.00101039	0.247914 0.00032923	0.0878962 0.0134077	0.125797 0.00232668
Total Agency Cost, $AC(e^*, e^{**}) = IC + RC + CE$	0.79504 0.04008	0.81781 0.05	0.92883 0.02869	0.95166 0.03465	0.6404 0.06012	0.67752 0.09218

Parameter Value	$k = 20$		$R = 3$		$rw_0 = 3$		$\sigma_e^2 = 0.36$	
Types of Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm	Monopoly Firm	Competitive Firm
The Deadweight Welfare Loss from the <i>Effort Distortion</i> , $DWL_e(q(e^*), q(e^{**}), e^*, e^{**})$	1.12176 0.129146	1.17584 0.0979326	0.559839 0.0448413	0.64032 0.0336322	1.07422 0.125994	1.13036 0.0973416	1.19566 0.128307	1.27886 0.0983253
The Saving on the <i>Cost of Effort</i> , $-\Delta\psi(e^*, e^{**}) = -[\psi(e^*) - \psi(e^{**})]$	-0.620604 -0.0905335	-0.682765 -0.0496316	-0.275858 -0.0315091	-0.34833 -0.0168946	-0.695204 -0.09417	-0.827886 -0.0506273	-0.736886 -0.094375	-0.891646 -0.0506619
The Incentive Cost, $IC = DWL_e(q(e^*), q(e^{**}), e^*, e^{**}) - \Delta\psi(e^*, e^{**})$	0.501159 0.0386124	0.493077 0.0483011	0.283981 0.0133322	0.29199 0.0167376	0.378995 0.0318243	0.302478 0.0467143	0.458774 0.033932	0.387215 0.0476634
The Risk Cost, $RC = (r/2)(\alpha(e^{**}))^2 \sigma_e^2$	0.0801108 0.00213632	0.0947879 0.00065717	0.0181947 0.00026741	0.0280817 0.00007755	0.200748 0.00632638	0.262249 0.00194771	0.186507 0.00492024	0.251677 0.00149044
Total Agency Cost, $AC(e^*, e^{**}) = IC + RC + CE$	0.58127 0.04075	0.58786 0.04896	0.30218 0.0136	0.32007 0.01682	0.57974 0.03815	0.56473 0.04866	0.64528 0.03885	0.63889 0.04915

2.5. Concluding Remarks

In Chapters 1 and 2 we have analyzed the productive inefficiency of monopoly using a model where a firm's total cost of production decreases with the effort exerted by the manager of the firm. Our analysis suggests that the assumption of output-independent marginal cost of production typically used in this literature is not without loss of generality. Our results show that when marginal cost varies with output level, monopoly causes productive inefficiency even if the owner manages the firm himself. Interestingly, the productive inefficiency of monopoly may be caused by too much effort rather than too little effort. In such a situation, moreover, the separation of ownership and control can mitigate the productive inefficiency of monopoly, thus raising the intriguing possibility that managerial slack can actually improve the efficiency of monopoly equilibrium. Therefore, to phrase our results in Hicks' (1935) terminology, a monopolist does not necessarily live a quiet life, and a quiet life is not necessarily a bad thing from the perspective of economic efficiency.

There are a couple of avenues for future research that we will explore. First, we will examine whether the adverse selection problem would increase the monopoly inefficiency. By introducing heterogeneity among owners and managers in their management ability into a model we can compare the efficiency loss of monopoly entrepreneurial firm with that of monopoly managerial firm. But this topic is beyond the scope of this thesis. Second, we will examine whether rank-order tournament contracts would alleviate managerial slack to increase the productive efficiency of

competitive firms. Given that rank-order tournament contracts would not be possible for a monopoly firm, the welfare loss of monopoly would be higher if rank-order tournament contracts can be used to reduce the agency inefficiency in a competitive industry. We return to this topic in Chapter 3.

CHAPTER 3

Incentives and Risks in Rank-Order Variable-Prize Tournaments

3.1. Introduction

It has long been recognized by economists that competition among firms (or agents) can provide information that allows the owner of a firm (or the principal) to improve productive efficiency by using incentive contracts based on the relative performance of the firm (or the agent). So far, two types of relative performance based incentive schemes have been studied in the literature. Under the first type of schemes, relative performance evaluation is based on cardinal performance information and the rewards to an agent are linked to the magnitude of relative performance. Studies of this type of incentive schemes include Holmström (1982), Nalebuff and Stiglitz (1983b, p.36-37), Carmichael (1983), and Marinakis and Tsoulouhas (2006). Under the second type of schemes, relative performance evaluation is based on ordinal performance information and the rewards to an agent are fixed and are based on the agent's rank alone. The second type schemes, often referred to as rank-order tournaments, has been analyzed by Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983b), and Nitzan (1991), just to name a few.

As is mentioned in Holmström and Tirole (1987) as well as Hart and Holmström (1987), relative performance evaluation enables the reduction in common environmental risk so it will reduce agency costs, but risk reduction does not necessarily reduce managerial slack. On the other hand, in the presence of common environmental risk, relative performance evaluation distorts firm's decision-making as a manager completely insulated from common environmental risk will ignore such risk when making investment or production decisions. In the absence of common environmental risk, according to the implications of sufficient statistic conditions suggested by Holmström (1982), rank-order tournaments perform worse than individualistic contracts. Therefore, designing an optimal scheme of relative performance evaluation that uses both ordinal and cardinal performance information seems desirable for eliminating these disadvantages.

In this chapter we analyze another incentive scheme that can be viewed as a hybrid of a linear piece rate scheme and the second type of relative performance schemes mentioned above. Specifically, we consider a variable-prize tournament under which the winner and the loser are identified by their performance rank but the winner is rewarded according to a linear piece-rate scheme based on his absolute performance and the loser receives a fixed payment. The model consists of one risk-neutral principal and two risk-averse agents. An agent's output depends on his effort, but it is also subject to agent-specific idiosyncratic shocks as well as shocks that are common to both agents. Using Taylor approximation, we characterize the optimal prize structure of this tournament and compare it with those of a fixed-prize tournament and of a linear

piece rate scheme. We derive conditions under which the variable-prize tournament dominates the fixed-prize tournament and linear piece rate scheme, respectively.

In comparison with the fixed-prize tournament, the variable-prize tournament ties the winner's income to his output level. This alleviates the effort distortion caused by the insulation of the agent's income from common shocks under the fixed-prize tournament. At the same time, by identifying the winner according to performance rank the variable-prize tournament preserves the feature of the fixed-prize tournament that reduces the impact of common shocks and, in doing so, reduces the agency costs. Indeed, it is shown that the variable-prize tournament dominates the fixed-prize tournament under the conditions that the degree of risk aversion, as measured by the coefficient of absolute risk aversion, is not too high and is nonincreasing in income.

Our analysis also shows that the variable-prize tournament dominates the linear piece rate scheme if the variance of the common shocks is relatively large. This condition is similar to the one for the dominance of the linear piece rate scheme by the fixed-prize tournament (Nalebuff and Stiglitz 1983b). However, the condition under the variable-prize tournament is less stringent than that under the fixed-prize tournament.

In addition, numerical simulations based on a parameterized model are conducted to analyze the relative efficiency of the variable-prize tournament. These simulations show, among other things, that the incentive power under the variable-prize tournament can be less than that under the linear piece rate.

This chapter is organized as follows. We begin in Section 3.2 by presenting a model of variable-prize tournaments and solve the model using Taylor approximation. Comparisons of variable-prize tournaments with fixed-prize tournaments and piece rates are made in Section 3.3. Results from numerical simulations of a parameterized model are presented in Section 3.4. We apply variable-prize tournaments to yardstick competition between managerial firms in Section 3.5. We then conclude in Section 3.6.

3.2. The Model

Consider a canonical model of tournaments, in which there are a principal (the firm) and two agents (managers) in a firm.¹ Following Lazear and Rosen (1981) as well as Holmström (1982), we will restrict our attention to a special output structure:

$$q_i = e_i + \varepsilon_i + \eta, \quad i = j, k,$$

where q_i is output produced by agent i , e_i is his effort, ε_i is an agent-specific idiosyncratic shock, and η is a common random shock affecting all of the agents. Both ε_i and η are independently distributed with zero mean.² The agent-specific shock ε_i is

¹In this paper we only focus on two-agent variable-prize tournaments. However, the model presented in this paper can be extended to analyze the case of multi-agent. Variable-prize tournament contracts can be designed as multinomial in the multi-agent setting. For example, multi-agent variable-prize tournaments can specify that the agent whose performance is ranked above the medium will be offered a linear incentive scheme while the agent whose performance is ranked below the medium will be given one of multiple different fixed prizes. Alternatively, the agent with top-rank performance will be offered a linear incentive scheme while other agents will be given one of different fixed prizes on the basis of their rank positions.

²The assumption of zero mean of ε_i and η is only for simplicity, and it is not necessary for the results of this paper to hold.

independently and identically distributed (i.i.d.) across agents, with a density function of $f(\varepsilon_i)$ over support $[\underline{\varepsilon}, \bar{\varepsilon}]$ and variance $\text{Var}(\varepsilon_i) = \sigma_\varepsilon^2$. The variance of common random shock η is denoted by $\text{Var}(\eta) = \sigma_\eta^2$. Because of the common random shock, the realized outputs produced by agents are correlated. Assume that the realizations of neither ε_i nor η is observable to the principal. Neither are the agents' effort levels. Furthermore, the agents do not observe ε or η until after they choose their effort levels. In other words, the agent's choice of effort cannot depend on the realizations of ε or η .³ Assume that production is additively separable across agents and there is a competitive output market with free entry. The price of output is denoted by V .

The contribution of this chapter is that we analyze a variable-prize tournament scheme. Specifically, suppose the principal offers the agents an incentive scheme as follows,

$$w = \begin{cases} w_1 \equiv \beta + \alpha q_i & \text{if } q_i > q_l \\ w_2 \equiv \gamma & \text{otherwise} \end{cases}, \quad i, l = j, k, \quad i \neq l,$$

where γ is a *fixed* prize to the loser if his realized output is less than that of the other agent and $\beta + \alpha q_i$ is a linear piece-rate scheme to the winner, in which β is a fixed component of the pay scheme and α is the piece rate per unit of output, and $\alpha \geq 0$.⁴

³In the literature on rank-order tournaments, there are different assumptions about the observability of common uncertainties. In Green and Stokey (1983), the agents observe only signals about η but not the realization of η itself before they choose their effort levels. Nalebuff and Stiglitz (1983b) assume that the workers decide on effort after observing η directly. In these models, the effort levels are contingent on either the signals of η or the realization of η . However, Holmström and Tirole (1987, p.90) suggest that choosing effort levels before and after the realization of η are conceptually equivalent.

⁴In footnote 11 of their paper, Lazear and Rosen (1981, p.857) suggest that combining piece rates in a contest against a fixed performance standard *must* be superior to using one alone. In this paper we will consider a similar incentive scheme in rank-order tournaments and characterize the conditions under which the similar results would hold in variable-prize tournaments.

In the literature the piece rate α has also been called the incentive power, incentive strength, or pay-performance sensitivity. To highlight the distinction with the fixed prize, we refer to αq_i as the *variable* prize.

It is easy to see that the incentive scheme specified above includes the fixed-prize tournament scheme as a special case. If we set $\alpha = 0$, this scheme degenerates into the standard fixed-prize tournament scheme that has first been studied by Lazear and Rosen (1981) and then extended by Green and Stokey (1983) and Nalebuff and Stiglitz (1983b). Therefore, the variable-prize tournament scheme with $\alpha > 0$ can be viewed as a mixture of linear piece rates and rank-order tournaments. While the fixed-prize scheme is based on ordinal performance information (performance rank) alone, the variable-prize scheme combines both ordinal and cardinal performance information (individual's absolute performance), so the latter can be viewed as a generalization of the former. Through this analysis, we want to establish the conditions under which the latter dominates the former. In what follows, we will use subscripts with capital letters PR, FT and VT to denote linear piece rates, fixed-prize tournaments and variable-prize tournament respectively, and will use superscripts with one asterisk to denote the optimal solution.

Since ε_i is i.i.d., the joint probability density function (p.d.f.) $f(\varepsilon_j, \varepsilon_k)$ of $(\varepsilon_j, \varepsilon_k)$ is simply the product of the density functions of ε_j and ε_k . Given the joint p.d.f. $f(\varepsilon_j, \varepsilon_k)$, the joint p.d.f. $f(\varepsilon_j, \varepsilon_k | q_j > q_k)$ of $(\varepsilon_j, \varepsilon_k)$ conditional on $q_j > q_k$ is given by $f(\varepsilon_j, \varepsilon_k | q_j > q_k) = f(\varepsilon_j)f(\varepsilon_k)/P$, where P is the probability that agent i wins (i.e.

his output exceeds that of his opponent),

$$P = \Pr(q_i > q_l) = \Pr(\varepsilon_l < e_i - e_l + \varepsilon_i) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_l + \varepsilon_i} f(\varepsilon_l) f(\varepsilon_i) d\varepsilon_l d\varepsilon_i.$$

Let E_ε and E_η denote the expectation operators with respect to the distributions of ε_i and η respectively. Given the variable-prize tournament scheme, agent i 's expected compensation Ew is given by

$$\begin{aligned} Ew &= E_\varepsilon[E_\eta(w_1)|q_i > q_l] \cdot \Pr(q_i > q_l) + w_2 \cdot \Pr(q_i \leq q_l) \\ &= P \cdot [Ew_1 + \alpha E_\varepsilon(\varepsilon_i | \varepsilon_l < e_i - e_l + \varepsilon_i)] + (1 - P) \cdot w_2, \end{aligned}$$

where $Ew_1 = \beta + \alpha e_i$ is the expected value of the variable prize w_1 in the state of winning and $E_\varepsilon(\varepsilon_i | \varepsilon_l < e_i - e_l + \varepsilon_i) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_l + \varepsilon_i} \varepsilon_i f(\varepsilon_l) f(\varepsilon_i) d\varepsilon_l d\varepsilon_i / P$ is the mean of ε_i conditional on $q_i > q_l$, which is positive and depends on the variance of ε_i . Thus, in contrast with fixed-prize tournaments, the idiosyncratic risks of output affect directly the agent's expected compensation Ew under variable-prize tournaments.

3.2.1. The Agent's Problem—*Optimal Choice of Effort*

Agents are assumed to be risk averse and identical ex ante. Each agent i has the utility function of the form $u(w - \psi(e_i))$,⁵ which is strictly increasing and concave in agent i 's net income $y_i = w - \psi(e_i)$, that is, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The cost of

⁵The assumption of *non-separability* between agent i 's income w_i and disutility of effort $\psi(e_i)$ in the utility function is only for simplicity, and it is not crucial for the following results to hold. In the literature of principal-agent models and rank-order fixed-prize tournaments, basic theoretic results hold true under the assumptions of both *non-separability* and *separability*. For example, under the assumption of *separability*, Green and Stokey (1983) and Nalenuff and Stiglitz (1983b) obtain results similar to those in Lazear and Rosen (1981) which are based on the assumption of *non-separability*.

effort $\psi(\cdot)$ is a strictly increasing and convex function of effort e_i , that is, $\psi'(\cdot) > 0$, $\psi''(\cdot) > 0$. Furthermore, we assume that $\psi'(0) = 0$. Thus, agent i 's expected utility is

$$(3.1) \quad EU_{VT}(w, e_i) = P \cdot E_\varepsilon \{ E_\eta [u(\beta + \alpha q_i - \psi(e_i))] | q_i > q_l \} + (1 - P) \cdot u(w_2 - \psi(e_i)),$$

where

$$\begin{aligned} & E_\varepsilon \{ E_\eta [u(\beta + \alpha q_i - \psi(e_i))] | q_i > q_l \} \\ &= \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_l + \varepsilon_i} E_\eta [u(\beta + \alpha(e_i + \varepsilon_i + \eta) - \psi(e_i))] f(\varepsilon_l) f(\varepsilon_i) d\varepsilon_l d\varepsilon_i \Big/ P. \end{aligned}$$

Agent i 's problem is to choose effort e_i to maximize his expected utility in a two-player tournament game. The first-order condition for an interior solution to agent i 's expected utility maximization problem (3.1) is given by

$$\begin{aligned} & \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} E_\eta [u(\beta + \alpha(e_i + \varepsilon_i + \eta) - \psi(e_i))] f(e_i - e_l + \varepsilon_i) f(\varepsilon_i) d\varepsilon_i \\ & + (\alpha - \psi'(e_i)) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_l + \varepsilon_i} E_\eta [u'(\beta + \alpha(e_i + \varepsilon_i + \eta) - \psi(e_i))] f(\varepsilon_l) f(\varepsilon_i) d\varepsilon_l d\varepsilon_i \\ (3.2) \quad & -u(w_2 - \psi(e_i)) \frac{\partial P}{\partial e_i} - (1 - P) u'(w_2 - \psi(e_i)) \psi'(e_i) = 0. \end{aligned}$$

In a symmetric Nash equilibrium, each agent i will supply the same level of effort, $e_i = e$, so the probabilities of winning and losing are equal, $P = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{\varepsilon_i} f(\varepsilon_l) f(\varepsilon_i) d\varepsilon_l d\varepsilon_i = 1/2$. The marginal winning probability of effort is then independent of the agent's effort level and only related to the distribution of ε_i , $\partial P / \partial e_i = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} f(\varepsilon_i) f(\varepsilon_i) d\varepsilon_i = E_\varepsilon[f(\varepsilon_i)]$. Thus, the agent's expected compensation becomes $EW = (1/2)[Ew_1 +$

$\alpha E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) + w_2]$, where the conditional mean of ε_i is

$$E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{\varepsilon_i} \varepsilon_i f(\varepsilon_l) f(\varepsilon_i) d\varepsilon_l d\varepsilon_i / (1/2).$$

To simplify notation, we will use $u(\tau)$ to denote $u(w_\tau - \psi(e))$ for $\tau = 1, 2$. The first-order condition (3.2) can then be written as

(3.3)

$$E_\varepsilon\{E_\eta[u(1) - u(2)]f(\varepsilon_i)\} + \frac{1}{2}\alpha E_\varepsilon\{E_\eta[u'(1)] | \varepsilon_i > \varepsilon_l\} - \frac{1}{2}\psi'(e)E_\varepsilon\{E_\eta[u'(1) + u'(2)] | \varepsilon_i > \varepsilon_l\} = 0.$$

The first two terms on the left-hand side of (3.3) capture the marginal benefits of supplying effort. The first term is the difference of utilities in two states of winning and losing, which is positive when the variable prize w_1 is greater than the fixed prize w_2 . The second term is the marginal gain from effort associated with the incentive power α . The last term is the marginal cost of supplying effort.

3.2.2. The Principal's Problem—*Optimal Variable-Prize Structure*

The principal is assumed to be risk neutral. In this chapter we focus on a situation where agents are scarce so there is competition for agents. Consequently, in equilibrium the principal earns zero expected profit, that is, the value of expected output $VE(q)$ must equal the expected compensation Ew . Then this zero expected profit condition can be written as,

$$(3.4) \quad VE(q) = Ve = Ew = \frac{1}{2}[\beta + \alpha(e + E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l)) + \gamma].$$

The principal's objective is to choose an optimal tournament contract, characterized by a triplet $\{\alpha, \beta, \gamma\}$, that maximizes the agent's expected utility:

$$(3.5) \quad EU_{VT}(w, e) = \frac{1}{2}E_{\varepsilon}\{E_{\eta}[u(\beta + \alpha(e + \varepsilon_i + \eta) - \psi(e)) | \varepsilon_i > \varepsilon_l] + \frac{1}{2}u(\gamma - \psi(e)),$$

subject to the agent's incentive constraint (3.3) and the zero expected profit constraint (3.4).

We assume that the distributions of ε_i and η satisfy the monotone likelihood ratio property condition (MLRPC) and the convexity of distribution function condition (CDFC) such that the first-order condition approach is valid.⁶

3.2.3. Solving the Model by Taylor Approximation

In order to compare the variable-prize tournaments with the fixed-prize tournaments and with the piece-rate scheme in the *second-best* optimum with risk averse agents and unobservability of effort, we need to know the properties of the optimal solutions to the general model presented above. Unfortunately, both the agent's effort supply function and the optimal variable-prize structure are nonlinear, which prevents us from directly comparing the solutions to the different general models. To get around this hurdle, we take a two-pronged approach. First, we obtain an approximate solution to this general model using the method of Taylor approximation and conduct an analysis based on the approximate solutions. Second, we derive the solutions to a model with specific functional forms and use numerical simulations to confirm our findings.

⁶For more detailed discussions, see, for example, Mirrlees (1974), Grossman and Hart (1983), Rogerson (1985), Jewitt (1988), Alvi (1997), Araujo and Moreira (2001), Luporini (2006), and Ollier (2007).

Following Lazear and Rosen (1981), we approximate the first-order conditions of the different general models by Taylor series expansions,⁷ then compare them to obtain qualitative results about the properties of the optimal solutions. Denote the agent's expected net income by $Ey = Ew - \psi(e)$. Taking a second-order Taylor series expansion at Ey to approximate $u(\tau)$ and a first-order Taylor series expansion for $u'(\tau)$ for $\tau = 1, 2$, we express the first-order condition for the agent's problem (3.3) as

$$(3.6) \quad Ew_1 + \alpha E_\epsilon(\epsilon_i | \epsilon_i > \epsilon_l) - w_2 \approx \frac{\psi'(e) + \chi(r, \alpha)}{\zeta(r, \alpha)},$$

⁷Loistl (1976) suggests that Taylor-series expansions for approximating expected utility functions, based on more terms of central moment, yield a better approximation result when the Taylor series converges. Pulley (1981), Kroll et al. (1984), and Ederington (1986) have also found that mean-variance Taylor-series expansions may be useful approximations to expected utility. According to the results in Loistl (1976), Hlawitschka (1994) and Cain and Peel (2004, p.152-156), given the chosen expansion point x_0 , the convergence intervals of Taylor series expansion for some of the most commonly used utility functions $u(x)$ in economics are as follows:

Utility Function	Convergence Region
Constant Relative Risk Aversion:	$x \in (0, 2x_0)$
logarithmic utility function ($u(x) = \log x, x \in (0, \infty)$)	
power utility function ($u(x) = x^a, x \in [0, \infty), a > 0$)	
Constant Absolute Risk Aversion:	$x \in (0, \infty)$
exponential utility function ($u(x) = -e^{-ax}, x \in [0, \infty), a > 0$)	
Markowitz: ($u(x) = 1 - e^{-ax} - axe^{-ax}, x \in [0, \infty), a > 0$)	$x \in (0, \infty)$

However, Hlawitschka (1994) presents empirical evidence to demonstrate that second-order expansions provide excellent approximations to expected utility when Taylor series converge or diverge; furthermore, including more terms worsens the approximation when Taylor series do converge.

where

$$\begin{aligned}
 \chi(r, \alpha) \equiv & \frac{\alpha^2}{2} r \{ -E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) (2E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)]) \\
 & + E_\varepsilon[\varepsilon_i^2 f(\varepsilon_i)] + E_\varepsilon[f(\varepsilon_i)] \sigma_\eta^2 \} \\
 (3.7) \quad & + \alpha \{ E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - \frac{1}{2} \},
 \end{aligned}$$

$$(3.8) \quad \zeta(r, \alpha) \equiv E_\varepsilon[f(\varepsilon_i)] + \frac{\alpha}{2} r \{ E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - \frac{1}{2} \}.$$

Let $r(Ey) \equiv -u''(Ey)/u'(Ey)$ denote the Arrow-Pratt measure of absolute risk aversion evaluated at the expected net income Ey . To simplify notation, we will denote $r(Ey)$ by the shorthand r .

Note that the value of the terms inside the braces in the last terms of (3.7) and (3.8), $E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - 1/2$, is less than zero if ε_i follows a standard distribution other than an exponential distribution. In the latter case it is always zero. For the purpose of analyzing variable-prize tournaments, we exclude the exponential distribution.

Substituting the zero expected profit constraint $Ve = Ew$ into Ey , we can express the agent's expected net income as a function of effort e : $Ey = Ve - \psi(e)$. Hence, r is also a function of effort e . Some further manipulation of the first-order condition (3.6) and the zero expected profit constraint (3.4) yields

$$(3.9) \quad Ew_1(e, \alpha) \approx Ve + \frac{\psi'(e) + \chi(r, \alpha)}{2\zeta(r, \alpha)} - \alpha E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l),$$

$$(3.10) \quad w_2(e, \alpha) \approx Ve - \frac{\psi'(e) + \chi(r, \alpha)}{2\zeta(r, \alpha)}.$$

Substituting (3.9) and (3.10) into the principal's problem (3.5), we have

(3.11)

$$\max_{\{e, \alpha\}} EU_{VT}(e, \alpha) = \frac{1}{2} E_\varepsilon \{ E_\eta [u(Ew_1(e, \alpha) + \alpha(\varepsilon_i + \eta) - \psi(e))] | \varepsilon_i > \varepsilon_l \} + \frac{1}{2} u(w_2(e, \alpha) - \psi(e))$$

The first-order conditions (FOCs) with respect to e and α are:

$$\begin{aligned} \frac{\partial EU_{VT}(e, \alpha)}{\partial e} &= u'(Ey) \left\{ V - \psi'(e) - r \frac{\psi'(e) + \chi(r, \alpha)}{4[\zeta(r, \alpha)]^2} \right. \\ &\quad \times \left[\psi''(e) + r'(Ey)(V - \psi'(e)) \left(\chi_r(r, \alpha) - \left(\frac{\psi'(e) + \chi(r, \alpha)}{\zeta(r, \alpha)} \right) \zeta_r(r, \alpha) \right) \right] \left. \right\} \approx 0, \\ \frac{\partial EU_{VT}(e, \alpha)}{\partial \alpha} &= \frac{1}{2} u''(Ey) \left\{ \frac{\psi'(e) + \chi(r, \alpha)}{[\zeta(r, \alpha)]^2} \right. \\ &\quad \times \frac{\chi_\alpha(r, \alpha) \zeta(r, \alpha) - [\psi'(e) + \chi(r, \alpha)] \zeta_\alpha(r, \alpha)}{2\zeta(r, \alpha)} + \alpha [\text{Var}(\varepsilon_i | \varepsilon_i > \varepsilon_l) + \sigma_\eta^2] \left. \right\} \approx 0. \end{aligned}$$

These two first-order conditions form the basis of our analysis below in Section

3.3.

3.3. Comparisons of Incentives and Income Risk

3.3.1. Comparison of the Optimal Effort Choices

3.3.1.1. Variable-Prize Tournaments versus Fixed-Prize Tournaments. Recall that the difference between a variable-prize tournament and a fixed-prize tournament can be expressed in terms of the value of α ; the former degenerates into the latter if $\alpha = 0$. Therefore, the first thing we need to determine is whether the solution to our model involves an interior solution with $\alpha > 0$. Given that we have excluded the exponential distribution, we have the following result.⁸

⁸It can be shown that in the case where ε_i follows the exponential distribution, the variable-prize tournament is always optimal at $\alpha = 0$, so it reduces to a fixed-prize tournament.

Lemma 3.1. *The optimal incentive power α_{VT}^* chosen by the principal under variable-prize tournaments is positive.*

The proofs of this lemma and all subsequent lemmas and propositions are relegated to Appendix C. Lemma 3.1 states that the optimal incentive power α_{VT}^* is not equal to zero under variable-prize tournaments. This implies that the prize of the winner is not fixed but depends on the realization of random output.

Proposition 3.1. *The solution $\{e_{VT}^*, \alpha_{VT}^*\}$ is a maximizer of the agents' expected utility function $EU_{VT}(e, \alpha)$ such that $EU_{VT}(e_{VT}^*, \alpha_{VT}^*) > EU_{VT}(e_{FT}^*, 0) \equiv EU_{FT}(e_{FT}^*)$.*

This proposition tells us that the agents' expected utility under variable-prize tournaments is greater than that under fixed-prize tournaments. But this does not necessarily mean that the agents supply a higher level of effort under variable-prize tournaments than under fixed-prize tournaments. Since $\alpha_{VT}^* > 0$, this would be true if effort e is an increasing function of the incentive power α . In what follows we characterize two conditions that are individually necessary but jointly sufficient for a positive relationship between the agents' expected marginal utility of effort, $\partial EU_{VT}(e, \alpha)/\partial e$, and the piece rate α as the tournament scheme switches from fixed-prize tournaments to variable-prize tournaments (i.e. as α is increased from 0).

Condition 1. $\sqrt{\frac{\psi''(e_{FT}^*)}{r_{FT}^*}} > V$, where $r_{FT}^* \equiv r(Ve_{FT}^* - \psi(e_{FT}^*))$.

The term $\psi''(e)$ in Condition 1 is related to the degree of disutility from exerting effort and agents are more averse to work when $\psi''(e)$ is large; r is the Arrow-Pratt coefficient of absolute risk aversion over income and a large r implies that agents are

more risk averse. Condition 1 requires that given the price of output V , agents are *less* averse to uncertainties in their net income than to work. Hence, agents prefer more risky variable-prize tournaments to less risky fixed-prize tournaments because if they win, then they receive more income proportional to their effort under variable-prize tournaments than under fixed-prize tournaments.

Condition 2. $r'(Ey) \leq 0$.

Condition 2 states that the utility function $u(\cdot)$ exhibits nonincreasing (constant or decreasing) absolute risk aversion.⁹ This condition implies that the utility cost of bearing a higher level of risk under variable-prize tournaments can be offset by exerting more effort and earning a higher expected income. Combining conditions 1 and 2 gives us the following result.

Lemma 3.2. *Provided that Conditions 1 and 2 are satisfied, the expected marginal utility of effort under variable-prize tournaments is increasing in α as α is increased from 0, i.e.,*

$$\frac{\partial}{\partial \alpha} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \Big|_{\alpha=0} > 0 \quad \text{for } e > 0.$$

This lemma states that under certain conditions the expected marginal utility of effort is indeed greater under variable-prize tournaments than under fixed-prize tournaments, implying that the effort supplied is greater under the former scheme than under the latter. This result is established in the following proposition.

⁹For example, the CRRA (constant relative risk aversion) utility functions such as logarithmic utility and power utility belong to the family of DARA (decreasing absolute risk aversion). The exponential utility function is CARA (constant absolute risk aversion).

Proposition 3.2. *Provided that Conditions 1 and 2 are satisfied, each agent exerts greater effort under variable-prize tournaments than under fixed-prize tournaments, i.e., $e_{VT}^* > e_{FT}^*$.*

This proposition states that under Conditions 1 and 2, the variable-prize tournament scheme provides more efficient incentives for an agent to exert effort than the fixed-prize tournament scheme so that the agent attains a higher level of expected utility under the former scheme than the latter.

Note that variable-prize tournaments dominate fixed-prize tournaments even though outputs of the agents are correlated due to the common shocks. This is because the common risks have no impact on agents' expected income under both tournaments. Consequently, neither Condition 1 nor Condition 2 is related to the common shocks.

3.3.1.2. Variable-Prize Tournaments versus Linear Piece Rates. In the literature comparisons of rank-order tournament scheme and linear piece rate scheme have produced mixed results. Using a numerical example of the utility function with constant relative but decreasing absolute risk aversion, Lazear and Rosen (1981) show that with risk aversion rank-order tournaments can dominate linear piece rates, and vice versa. But they are unable to characterize completely the conditions under which one scheme dominates the other. Based on a specific quadratic utility function (i.e. the utility function exhibits increasing absolute risk aversion), Nalebuff and Stiglitz (1983b, p.35-36) compare fixed-prize tournaments with linear piece rates and give the condition under which fixed-prize tournaments will always dominate linear piece rates.

In our model the inability to obtain analytic solutions means that we are not able to characterize the general conditions under which variable-prize tournaments dominate linear piece rates, or vice versa. What we are able to do, however, is to derive a result for the case where agents' degree of risk aversion is very low. It is easy to show that if the agents are risk neutral, both the variable-prize tournament scheme and the linear piece-rate scheme can achieve the *first-best* effort level under complete information. We will consider an infinitesimal increase in the Arrow-Pratt coefficient of absolute risk aversion from $r = 0$ and analyze the resulting difference between the *second-best* equilibrium under the variable-prize scheme and that under the linear piece-rate scheme.

We start by considering the change of the agent's expected marginal utility of effort under variable-prize tournaments, $\partial EU_{VT}(e, \alpha)/\partial e$, with respect to r as its value is increased from 0. It turns out that the following conditions are critical for determining the sign of this derivative.

Condition 3. $\frac{1}{4[\zeta(0, V)]^2} < \sigma_\varepsilon^2 + \sigma_\eta^2$.

Condition 4. $\frac{1}{4[\zeta(0, V)]^2} > \sigma_\varepsilon^2 + \sigma_\eta^2 > \frac{1+\chi_\alpha(0, V)}{4[\zeta(0, V)]^2}$.

It can be shown that for a variety of common probability distributions (e.g. normal, uniform, or logistic distributions), Conditions 3 and 4 essentially require that the variance of the common shock, σ_η^2 , be large relative to the variance of the agent-specific shock, σ_ε^2 . But Condition 4 is weaker than Condition 3 for variable-prize tournaments to dominate linear piece rates.

Lemma 3.3. *Assume that Condition 3 (or Condition 4) is satisfied. An infinitesimal increase in the value of r reduces the agents' expected marginal utility of effort, $\partial EU_{VT}(e, \alpha)/\partial e$. But the reduction is smaller under variable-prize tournaments than under piece rates.*

From Lemma 3.3 we have the following proposition similar to Propositions 3.1 and 3.2:

Proposition 3.3. *Assume that Condition 3 (or Condition 4) is satisfied. Then*

(i) *Each agent exerts greater effort under variable-prize tournaments than under linear piece rates, i.e., $e_{VT}^* > e_{FT}^* > e_{PR}^*$ (or $e_{VT}^* > e_{PR}^* > e_{FT}^*$);*

(ii) *Variable-prize tournaments dominate linear piece rates in terms of the agents' expected indirect utilities, i.e., $EU_{VT}(e_{VT}^*, \alpha_{VT}^*) > EU_{VT}(e_{FT}^*, 0) > EU_{PR}(e_{PR}^*)$ (or $EU_{VT}(e_{VT}^*, \alpha_{VT}^*) > EU_{PR}(e_{PR}^*) > EU_{VT}(e_{FT}^*, 0) \equiv EU_{FT}(e_{FT}^*)$).*

3.3.2. Comparison of the Means and the Variances of the Optimal Net Incomes and Agency Costs

A comparison of the agent's expected net incomes under variable-prize tournaments, fixed-prize tournaments and piece rates is quite straightforward when they all are expressed as the same function of effort e , $Ey_{VT} = Ey_{FT} = Ey_{PR} = Ve - \psi(e)$. So the agent's expected net income increases with effort e , that is, $V - \psi'(e) > 0$ for $e \in (0, e^{FB})$, where e^{FB} is the *first-best* effort level under complete information. The comparison of the expected net incomes under these three incentives schemes thus

reduces to the comparison of the effort levels. By Proposition 3.3, $e_{VT}^* > e_{FT}^* > e_{PR}^*$ (or $e_{VT}^* > e_{PR}^* > e_{FT}^*$), we have $Ey_{VT}^* > Ey_{FT}^* > Ey_{PR}^*$ (or $Ey_{VT}^* > Ey_{PR}^* > Ey_{FT}^*$).

The income risks borne by the agents are measured by the variances of the optimal net income y_{VT}^* . To obtain the variances of the optimal net income under variable-prize tournaments, we use the decomposition of variance suggested by Greene (2000, p.907), that is,

$$(3.12) \quad \text{Var}(y_{VT}^*) = E(\text{conditional variance of } y_{VT}^*) + \text{Var}(\text{conditional mean of } y_{VT}^*).$$

The conditional mean and conditional variance of the optimal net income y_{VT}^* are listed in Table 3.1.

Table 3.1. Conditional mean and conditional variance of the optimal net income in rank-order variable-prize tournaments

State	Conditional mean of y_{VT}^*	Conditional variance of y_{VT}^*
$q_i > q_l$	$E(y_{VT}^* q_i > q_l) = E(w_1^* q_i > q_l) - \psi(e_{VT}^*)$	$\text{Var}(y_{VT}^* q_i > q_l) = \text{Var}(w_1^* q_i > q_l) = (\alpha_{VT}^*)^2 [\text{Var}(\varepsilon_i \varepsilon_i > \varepsilon_l) + \text{Var}(\eta)]$
$q_i \leq q_l$	$E(y_{VT}^* q_i \leq q_l) = w_2^* - \psi(e_{VT}^*)$	0

From this table we can obtain the expectation of the conditional variance of y_{VT}^* and the variance of conditional mean of y_{VT}^* as follow:

$$\begin{aligned}
 E(\text{conditional variance of } y_{VT}^*) &= \text{Var}(y_{VT}^* | q_i > q_l) \cdot \Pr(q_i > q_l) \\
 &\quad + \text{Var}(y_{VT}^* | q_i \leq q_l) \cdot \Pr(q_i \leq q_l) \\
 (3.13) \quad &= \frac{1}{2} (\alpha_{VT}^*)^2 [\text{Var}(\varepsilon_i | \varepsilon_i > \varepsilon_l) + \sigma_\eta^2],
 \end{aligned}$$

$$\begin{aligned}
\text{Var}(\text{conditional mean of } y_{VT}^*) &= [E(y_{VT}^*|q_i > q_l) - Ey_{VT}^*]^2 \cdot \Pr(q_i > q_l) \\
&\quad + [E(y_{VT}^*|q_i \leq q_l) - Ey_{VT}^*]^2 \cdot \Pr(q_i \leq q_l) \\
&= \frac{1}{4} [Ew_1^* + \alpha_{VT}^* E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) - w_2^*]^2 \\
(3.14) \quad &\approx \frac{1}{4} \left[\frac{\psi'(e_{VT}^*) + \chi(r(Ey_{VT}^*), \alpha_{VT}^*)}{\zeta(r(Ey_{VT}^*), \alpha_{VT}^*)} \right]^2.
\end{aligned}$$

If $\alpha = 0$, then the variance of the optimal net income y_{FT}^* under fixed-prize tournaments is given by $\text{Var}(y_{FT}^*) \approx \frac{1}{4} \left[\frac{\psi'(e_{FT}^*) + \chi(r(Ey_{FT}^*), 0)}{\zeta(r(Ey_{FT}^*), 0)} \right]^2 = \frac{1}{4} \left[\frac{\psi'(e_{FT}^*)}{\zeta(r(Ey_{FT}^*), 0)} \right]^2$. If $\alpha > 0$, then the variance of the optimal net income y_{FT}^* under variable-prize tournaments is $\text{Var}(y_{VT}^*) \approx \frac{1}{2}(\alpha_{VT}^*)^2 [\text{Var}(\varepsilon_i | \varepsilon_i > \varepsilon_l) + \sigma_\eta^2] + \frac{1}{4} \left[\frac{\psi'(e_{VT}^*) + \chi(r(Ey_{VT}^*), \alpha_{VT}^*)}{\zeta(r(Ey_{VT}^*), \alpha_{VT}^*)} \right]^2$. Thus, compared with $\text{Var}(y_{FT}^*)$, an additional term is included in $\text{Var}(y_{VT}^*)$: the expectation of the conditional variance of y_{VT}^* .

The variances of conditional mean of the optimal net income y^* under the two tournaments are related to a different incentive power α . To compare them, we first find out how they change with α . Differentiating $\text{Var}(\text{conditional mean of } y^*)$ with respect to α and evaluating it at $\alpha = 0$, we have

$$\begin{aligned}
(3.15) \quad &\frac{d\text{Var}(\text{conditional mean of } y^*)}{d\alpha} \Big|_{\alpha=0} \approx \frac{\psi'(e_{FT}^*) \{ \psi''(e_{FT}^*) e'(0) \zeta(r_{FT}^*, 0) \}}{2[\zeta(r_{FT}^*, 0)]^3} \\
&+ \frac{\chi_\alpha(r_{FT}^*, 0) [\zeta(r_{FT}^*, 0) - (r_{FT}^*/2)\psi'(e_{FT}^*)]}{2[\zeta(r_{FT}^*, 0)]^3}.
\end{aligned}$$

The first term in the brace of the numerator of (3.15) includes the term $e'(0)$ which is associated with the second-order conditions for the principal's problem (3.11) evaluated at $\alpha = 0$ and is positive. So the first term is positive. In the second

term, $\chi_\alpha(r_{\text{FT}}^*, 0) < 0$, but the sign of the terms in the square brackets is uncertain. Therefore, we cannot determine the sign of (3.15). So the variances of the optimal net income under the two tournaments cannot be compared,

$$\text{Var}(y_{\text{VT}}^*) \geq \text{Var}(y_{\text{FT}}^*).$$

Neither can we compare the variance of the optimal net incomes under linear piece rates with that under fixed-prize tournaments, since the conditions used in Lemma 3.3 are neither sufficient nor necessary for comparing $\text{Var}(y_{\text{FT}}^*)$ with $\text{Var}(y_{\text{PR}}^*)$.

Agency costs include incentive costs and risk costs. To analyze the agency costs under variable-prize tournaments, we can express the agent's expected utility in terms of a certainty equivalent, which equals to the difference between the expected net income $Ey(e) = Ew - \psi(e) = Ve - \psi(e)$ and the risk premium RC ,

$$EU_{\text{VT}}(e) = u(Ey(e) - RC).$$

The risk premium RC is related to the risk attitude of the agent, the uncertainty of output and the incentive power α . In principal-agent models it is interpreted as one of agency costs that the principal should pay to compensate the agents for the risk and the opportunity cost the agents must bear. In this sense the risk premium is just the risk cost for the principal.

Incentive costs IC are the difference between the expected net income $Ey(e^{\text{FB}})$ at the *first-best* level e^{FB} and $Ey(e_{\text{VT}}^*)$ at the optimal effort level e_{VT}^* under variable-prize

tournaments,

$$IC = Ey(e^{\text{FB}}) - Ey(e_{\text{VT}}^*) = V(e^{\text{FB}} - e_{\text{VT}}^*) - [\psi(e^{\text{FB}}) - \psi(e_{\text{VT}}^*)],$$

where the first term on the right-hand side is the reduction of expected outputs and the second term is the saving of effort costs.

The expected utility is an increasing and concave function of the certainty equivalent, so the difference of the expected indirect utilities $EU_{\text{VT}}(e^{\text{FB}}) - EU_{\text{VT}}(e_{\text{VT}}^*)$ can be viewed an indirect expression of agency costs $AC = IC + RC$. Therefore, comparison between agency costs under different incentive schemes reduces to comparison between the expected indirect utilities. By Propositions 3.2 and 3.3, we have $EU_{\text{VT}}(e_{\text{VT}}^*, \alpha_{\text{VT}}^*) > EU_{\text{VT}}(e_{\text{VT}}^*, 0) > EU_{\text{PR}}(e_{\text{PR}}^*)$ (or $EU_{\text{VT}}(e_{\text{VT}}^*, \alpha_{\text{VT}}^*) > EU_{\text{PR}}(e_{\text{PR}}^*) > EU_{\text{VT}}(e_{\text{FT}}^*, 0)$). Consequently, we have the following ranking of agency costs: $AC_{\text{VT}} < AC_{\text{FT}} < AC_{\text{PR}}$ (or $AC_{\text{VT}} < AC_{\text{PR}} < AC_{\text{FT}}$).

3.4. The Parameterized Model and Numerical Results

In this section we construct and simulate a parameterized model to explore the properties of our model quantitatively. In this parameterized model, the production function is linear: $q_i = e_i + \varepsilon_i + \eta$, $i = j, k$. The output price V is normalized to one. We assume that each agent i has an exponential (CARA) utility function,

$$u(y_i) = -\exp(-r(y_i)),$$

where $y_i = w - \psi(e_i)$ is agent i 's net income and r is the constant absolute risk aversion coefficient.

The cost function of effort expressed in monetary terms is assumed to be quadratic, $\psi(e_i) = (R/2)e_i^2$, where $R > 0$ is a parameter of cost of effort.

The following are the parameter values used in the benchmark simulations:

$$R = \sigma_\eta^2 = \sigma_\varepsilon^2 = r = 1.$$

Sensitivity analyses are conducted using various other values for R , σ_η^2 and σ_ε^2 .¹⁰ We did numerical simulations using a number of probability distributions for the idiosyncratic shock ε_i and common shock η , including Gamma, Chi, Beta, uniform, normal and logistic distributions. The quantitative results from these simulations confirm the qualitative results presented in Sections 3.2 and 3.3.

In Table 3.2 and Figure 3.1, we present the details of the simulations using the logistic distribution. We chose the logistic distribution because it is similar to the normal distribution and a closed-form expression of its cumulative distribution function greatly reduces computational difficulties in finding numerical solutions.¹¹ The density function used in the simulations is given by $f(x) = \frac{\exp\left(-\frac{\pi x}{\sqrt{3}\sigma_x}\right) \frac{\pi}{\sqrt{3}\sigma_x}}{\left[1 + \exp\left(-\frac{\pi x}{\sqrt{3}\sigma_x}\right)\right]^2}$ with $x \in (-\infty, +\infty)$, which is obtained by using scale factor $\delta = \frac{\pi}{\sqrt{3}\sigma_x}$ in the original density function $f(x) = \frac{\delta \exp(-x\delta)}{(1 + \exp(-x\delta))^2}$ such that the rescaled distribution has zero mean and variance σ_x^2 .¹²

¹⁰We did not do numerical sensitivity analyses for parameter r because the effect of an increase in r is qualitatively the same as that of an increase in the idiosyncratic risk σ_ε^2 .

¹¹The conditional expectations of the utility and marginal utility are involved in the triple integral of a truncated density function, which cannot be evaluated numerically for the normal distribution because the normal cumulative distribution function has no closed-form expression.

¹²For a comparison of the normal and the rescaled logistic distributions, see Quandt (1988, p.9).

Table 3.2: A Comparison of the Optimal Solutions among Three Incentive Schemes

Optimal Solutions		PR	FT	VT	PR	FT	VT	PR	FT	VT
		Benchmark: $R = \sigma_\eta^2 = \sigma_\epsilon^2 = r = 1$			$\sigma_\eta^2 = \sigma_\epsilon^2 = r = 1, \mathbf{R} = 2$			$R = \sigma_\eta^2 = \sigma_\epsilon^2 = 1, \mathbf{r} = 2$		
e	(Effort Supply)	0.328458	0.236432	0.254705	0.0993649	0.0733793	0.083838	0.195036	0.129753	0.139227
α	(Piece Rate)	0.328458		0.191997	0.19873		0.12848	0.195036		0.104492
β	(High Fixed Prize)	0.220573	0.649477	0.484034	0.0796182	0.3216	0.235195	0.156997	0.359222	0.282322
γ	(Low Fixed Prize)		-0.176613	-0.129379		-0.174301	-0.149125		-0.0997166	-0.0760247
$E(y^*)$	(Mean Net Income)	0.274516	0.208482	0.222268	0.0894916	0.0679948	0.0768092	0.176016	0.121335	0.129535
$\text{Var}(y^*)$	(Income Risk)	0.215769	0.170606	0.178781	0.0789871	0.0613456	0.0682702	0.0760778	0.0526562	0.0555925
EU	(Expected Utility)	-0.847519	-0.882056	-0.869067	-0.951381	-0.963069	-0.956444	-0.820787	-0.868612	-0.853769
		$R = \sigma_\epsilon^2 = r = 1, \sigma_\eta^2 = 4$			$\sigma_\epsilon^2 = r = 1, \mathbf{R} = 2, \sigma_\eta^2 = 4$			$R = \sigma_\epsilon^2 = 1, \mathbf{r} = 2, \sigma_\eta^2 = 4$		
e		0.164087	0.236432	0.242443	0.0452238	0.0733793	0.0770245	0.0890971	0.129753	0.132826
α		0.164087		0.0668328	0.0904476		0.0459166	0.0890971		0.0358909
β		0.137163	0.649477	0.592457	0.0411334	0.32106	0.290588	0.0811588	0.359222	0.33289
γ			-0.176613	-0.160621		-0.174301	-0.165391		-0.0997166	-0.0917932
$E(y^*)$		0.150625	0.208482	0.213054	0.0431786	0.0679948	0.0710918	0.085128	0.121335	0.124005
$\text{Var}(y^*)$		0.134623	0.170606	0.172948	0.0409038	0.0613456	0.0637157	0.0396915	0.0526562	0.0534784
EU		-0.920641	-0.882056	-0.877496	-0.977586	-0.963069	-0.96069	-0.913935	-0.868612	-0.86346
		$R = r = 1, \sigma_\epsilon^2 = 4, \sigma_\eta^2 = 9$			$r = 1, \mathbf{R} = 2, \sigma_\epsilon^2 = 4, \sigma_\eta^2 = 9$			$R = 1, \mathbf{r} = 2, \sigma_\epsilon^2 = 4, \sigma_\eta^2 = 9$		
e		0.0709282	0.0785268	0.0848644	0.0184821	0.0214277	0.0241292	0.0367481	0.0406623	0.0439474
α		0.0709282		0.0389941	0.0369642		0.0232618	0.0367481		0.0201133
β		0.0658974	0.344383	0.29675	0.0177989	0.164154	0.137902	0.0353977	0.178565	0.155158
γ			-0.187329	-0.173328		-0.121298	-0.115855		-0.0972409	-0.0903253
$E(y^*)$		0.0684128	0.0754436	0.0812634	0.0181405	0.0209686	0.023547	0.0360729	0.0398356	0.0429817
$\text{Var}(y^*)$		0.0654006	0.0706794	0.0756223	0.0177626	0.0203708	0.0227838	0.0175555	0.0190173	0.0204128
EU		-0.965037	-0.960297	-0.956051	-0.990793	-0.989241	-0.987667	-0.96378	-0.958765	-0.954269
		$R = r = 1, \sigma_\epsilon^2 = 9, \sigma_\eta^2 = 16$			$r = 1, \mathbf{R} = 2, \sigma_\epsilon^2 = 9, \sigma_\eta^2 = 16$			$R = 1, \mathbf{r} = 2, \sigma_\epsilon^2 = 9, \sigma_\eta^2 = 16$		
e		0.0383148	0.0377151	0.0426189	0.00979397	0.00985844	0.0116113	0.0195286	0.0191952	0.0217089
α		0.0383148		0.0236009	0.0195879		0.0132584	0.0195286		0.0120065
β		0.0368468	0.227087	0.187076	0.00960213	0.108007	0.0861881	0.0191472	0.115619	0.0957268
γ			-0.151657	-0.14180		-0.0882899	-0.0850487		-0.0772286	-0.0724282
$E(y^*)$		0.0375808	0.0370038	0.0417107	0.00969805	0.00976125	0.0114765	0.0193379	0.019011	0.0214733
$\text{Var}(y^*)$		0.0367007	0.0358619	0.0402406	0.00959219	0.00963309	0.0113	0.00953416	0.00929756	0.0104665
EU		-0.980989	-0.981004	-0.977992	-0.995113	-0.99506	-0.994089	-0.980622	-0.980649	-0.97754

Note: **PR**, **FT** and **VT** denote linear piece rates, fixed-prize tournaments and variable-prize tournaments respectively. All the numerical results are computed under the assumption of **logistic distribution**: $\varepsilon_i \sim \text{Logistic}(0, \sigma_\epsilon^2)$ and $\eta \sim \text{Logistic}(0, \sigma_\eta^2)$. The bolds represent the parameter values different from the benchmark values.

Figure 3.1. The responses of e , $\text{Var}(y)$, EU , α and β to changes in R , σ_η^2 and σ_ε^2 under linear piece rates (PR), fixed-prize tournaments (FT) and variable-prize tournaments (VT).

Figure 3.1 illustrates how the agent's optimal effort supply e , the variance of the optimal net income $\text{Var}(y)$, the agent's expected utility EU , the optimal incentive power α and the fixed component of the pay scheme β respond to changes in parameters R , σ_η^2 and σ_ε^2 under the three incentive schemes.

Several interesting observations can be made from the numerical simulations illustrated in Figure 3.1.

Observation 1 When parameter R of the effort cost increases, e , $\text{Var}(y)$, EU , α and β decrease and their values for the three incentive schemes gradually converge. As R increases, the total and marginal costs of effort to an agent are higher at any given effort level. To induce the agent to supply the same effort, the principal has to pay the agent more and bear more agency costs. So the principal allows the agent to supply less effort to save agency costs. The optimal incentive power α_{VT}^* is less than α_{PR}^* but the fixed component of the pay β_{VT}^* is greater than β_{PR}^* , and their values for the three schemes gradually converge as R increases. This is because rank-order tournaments also provide incentives for increasing effort, so the principal gives lower incentive power and more insurance pay β to the agent than that under linear piece rates. Thus, α is less sensitive but β is more sensitive to the change in R under variable-prize tournaments than under linear piece rates.

Observation 2 When the common risk of output σ_η^2 increases, e , $\text{Var}(y)$ and EU do not change under fixed-prize tournaments because common uncertainty is filtered out by the tournaments. On the other hand, e , $\text{Var}(y)$ and EU decrease more under linear piece rates than variable-prize tournaments. When σ_η^2 is small,

that is, output is less risky, linear piece rates dominate the two tournaments. As σ_η^2 increases, variable-prize tournaments dominate linear piece rates but linear piece rates dominate fixed-prize tournaments. When σ_η^2 is sufficiently large, variable-prize tournaments dominate fixed-prize tournaments and the two tournaments dominate linear piece rates. As σ_η^2 increases, both α_{VT}^* and α_{PR}^* decrease and gradually converge but β_{VT}^* and β_{PR}^* change in the opposite direction. Put differently, as σ_η^2 becomes larger, α_{PR}^* and β_{PR}^* decrease together, while α_{VT}^* decreases but β_{VT}^* increases.

Observation 3 When the idiosyncratic risk of output σ_ε^2 increases, e , $\text{Var}(y)$ and EU decrease under the three incentive schemes. Starting from the benchmark equilibrium where the linear piece rates dominate the two tournaments, the values of e , $\text{Var}(y)$ and EU for the three incentive schemes first diverge then the gap among the three schemes remain relatively constant. Both α and β decrease under linear piece rates and variable-prize tournaments, and the values of α and β for the two schemes converge.

3.5. An Application of Variable-Prize Tournaments to Inter-Firm Competition

In this section we present a simple example to show how variable-prize tournament contracts can be used to reduce X-inefficiency when multiple firms face common cost shocks. In the analysis of monopoly efficiency losses in Chapter 2, we have shown that monopoly power on the product market affects the size of X-inefficiency under linear

piece rates.¹³ However, when there are other firms facing industry-wide uncertainties and market information about the performance rank of related firms is available, owners of these firms can use this information to design incentive contracts that are similar to variable-prize tournaments. Borrowing a term from the literature, we call the tournament scheme based on the rank of firms' performances the yardstick contract.¹⁴

A related literature has studied how product market competition affects internal firm inefficiency via the strategic effects arising from imperfect competition in the product market.¹⁵ These studies, however, ignore the informational role played by competition. Note that the intensity of the strategic effects may vary across different incentive schemes such as linear piece rates and rank-order tournaments. Thus, the strategic effects may offset or reinforce the information effect of competition on X-inefficiency.

Another related literature has focused on the information effect of competition on the type of incentive contracts. Farrell (1983) shows by a simple numerical example that there might be a significant welfare loss from X-inefficiency in monopoly, compared with a duopoly where tournament contracts (yardstick managerial contracts)

¹³See Proposition 2.6 in Section 2.1 of Chapter 2.

¹⁴Shleifer (1985) analyzes a regulatory scheme where a regulator uses the benchmark of other firms' costs to force regulated monopolies to compete with these identical or similar firms. This regulatory scheme is called yardstick competition. Since Shleifer, the term of yardstick competition has been used extensively in the literature on monopoly regulations. Related to our analysis here, Bertolotti and Poletti (1995, 1997) borrow this term to call managerial contracts of relative performance evaluations the yardstick contracts. Different from their model where the *absolute* performance of the other firms is used as a yardstick, the yardstick contracts in our model are on the firms' performance *rank*.

¹⁵See e.g. Martin (1993), Stenbacka (1993), Horn et al. (1994), Panunzi (1994), Alexander and Zhou (1995), Raith (2003), and Beiner et al. (2005).

force managers to reduce production costs by doing technology searches. Kobayashi and Yu (1984) study cost-reducing innovation made by identical risk-neutral firms in two similar schemes: tournament contracts (yardstick managerial contracts) designed by a social planner and competitive markets with price-cutting, in which both firms' rewards are based on relative performance. Bertolotti and Poletti (1995, 1997) analyze a duopoly affected by adverse selection on marginal costs in a one-stage structure (without strategic effects) and show how owners can use some market-provided information on the other firm's performance in writing yardstick managerial contracts to reduce the managers' information rents.

We will analyze how competition affects agency efficiency under yardstick managerial contracts in a moral hazard setting. To isolate the effect of competition on the choice among incentive schemes from the strategic effects, we consider a model of yardstick managerial contracts, in which the market structure is either oligopolies without strategic effects in the same market or monopolies producing the same product in separate markets (e.g. in different geographical markets). The former case is similar to the one considered by Bertolotti and Poletti (1995, 1997), while the latter is similar to that analyzed by Shleifer (1985).¹⁶

More specifically, we consider an industry with two firms: each firm produces a fixed output V of a homogeneous product with a constant price p . For example, these two firms could be companies that have won government contracts to supply a fixed quantity of a good or service at a fixed price.¹⁷

¹⁶Shleifer (1985) considers a model where cost yardstick-competition is used to regulate N identical risk-neutral monopoly firms that operate in separate markets but do not face an internal agency problem.

¹⁷For similar examples, see Laffont and Tirole (1999, Chapter 2).

The owner of each firm hires a manager to run the firm. The owners are risk neutral, while the managers are assumed to be risk averse and to have identical preferences. Each manager i can reduce the firm's marginal cost of production by exerting effort, denoted by e_i . A manager has the utility function $u(w_i - \psi(e_i))$, where w_i is his compensation and $\psi(\cdot)$ is his monetary disutility of effort. Assume that $u(\cdot)$ is strictly increasing and concave, that is, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. $\psi(\cdot)$ is a strictly increasing and convex function of effort e_i , that is, $\psi'(\cdot) > 0$, $\psi''(\cdot) > 0$. Furthermore, we assume that $\psi'(0) = 0$.

Each firm i ($i = 1, 2$) has the following average and marginal cost that is constant with respect to its output V :

$$c_i \equiv c(e_i, \varepsilon_i, \eta) = c_0 - e_i - \varepsilon_i - \eta,$$

where c_0 is a constant, ε_i is a firm i -specific shock, and η is an industry-wide shock affecting both firms. The common shock η implies that the costs of the two firms are correlated.

We maintain all the assumptions about ε_i and η made in Section 3.2. After choosing their effort levels, the managers observe the realizations of ε_i and η . But the owners cannot observe ε_i and η . An owner observes only the realized value of his own firm's marginal cost. Accordingly we assume that c_i is contractible. Furthermore, the ranking of the realized levels of the two firms' marginal costs is known to both owners and managers.

Given that managerial efforts affect marginal cost, managerial contracts can be made contingent on marginal costs to induce effort. Each owner i ($i = 1, 2$) offers his manager a variable-prize yardstick contract that bases manager i 's compensation w_i on a comparison of the two firms' realized marginal costs, given by

$$w_i = \begin{cases} \beta_i + \alpha_i(c_0 - c_i) & \text{if } c_i < c_j \\ \gamma_i & \text{otherwise} \end{cases}, \quad i, j = 1, 2, i \neq j,$$

where γ_i is a fixed prize to manager i if the realized level of his own firm's marginal cost is higher than or equal to that of the other firm and $\beta_i + \alpha_i(c_0 - c_i)$ is a linear piece-rate scheme to manager i otherwise. β_i is a fixed component of the pay scheme and α_i is the piece rate per unit of the observed cost reduction $c_0 - c_i$.

For simplicity, we assume that neither the owners nor the managers collude on decisions regarding contract and effort. To focus on the effect of competition on agency efficiency we need to exclude the effect of one firm's managerial contract on the other firm's behavior through strategic delegation.¹⁸ To do so, we assume that yardstick managerial contracts are not mutually observed by the owners and the managers of the two firms.

The timing of the game is as follows. First, each owner independently offers his manager a yardstick contract, and each manager independently decides to accept or

¹⁸In principal-agent models, an owner's choice of managerial incentives and a manager's effort choice are modeled as a two-stage game. If yardstick managerial contracts are mutually observed by the owners and the managers of the two firms, each owner will want to take into consideration the strategic effects of his action on the behavior of the other firm's manager. Thus, the owner's optimal choice of managerial incentives will depend on rival owners' choices. For discussions about strategic delegation in Cournot and Bertrand oligopoly games, see e.g. Fershtman and Judd (1987), Sklivas (1987), Fershtman et al. (1991), and Katz (1991). For strategic delegation in oligopolistic contests and tournaments, see Kräkel (2004, 2005).

refuse it. Then, the managers who have accepted the contracts simultaneously choose their levels of effort. Finally, each firm's marginal cost is realized and the owners pay the managers accordingly.

3.5.1. The Managers' Optimal Choices of Effort

Let P be the probability that the manager of firm i wins the tournament (i.e. the realized marginal cost of firm i is lower than that of the other firm),

$$P = \Pr(c_i < c_j) = \Pr(\varepsilon_j < e_i - e_j + \varepsilon_i) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_j + \varepsilon_i} f(\varepsilon_j) f(\varepsilon_i) d\varepsilon_j d\varepsilon_i.$$

Given the variable-prize yardstick contract, manager i 's expected compensation Ew_i is given by

$$\begin{aligned} Ew_i(e_i, e_j, \alpha_i, \beta_i, \gamma_i) &= E_\varepsilon[E_\eta(\beta_i + \alpha_i(c_0 - c_i)) | c_i < c_j] \cdot \Pr(c_i < c_j) + \gamma_i \cdot \Pr(c_i \geq c_j) \\ (3.16) \qquad \qquad \qquad &= P \cdot [\beta_i + \alpha_i e_i + \alpha_i E_\varepsilon(\varepsilon_i | \varepsilon_j < e_i - e_j + \varepsilon_i)] + (1 - P) \cdot \gamma_i, \end{aligned}$$

where $E_\varepsilon(\varepsilon_i | \varepsilon_j < e_i - e_j + \varepsilon_i) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_j + \varepsilon_i} \varepsilon_i f(\varepsilon_j) f(\varepsilon_i) d\varepsilon_j d\varepsilon_i / P$ is the mean of ε_i conditional on $c_i < c_j$, which is positive and depends on the variance of ε_i .

Then manager i 's expected utility is

$$\begin{aligned} (3.17) \qquad EU_{VT}(w_i, e_i) &= P \cdot E_\varepsilon\{E_\eta[u(\beta_i + \alpha_i(c_0 - c_i) - \psi(e_i))] | c_i < c_j\} + (1 - P) \cdot u(\gamma_i - \psi(e_i)), \end{aligned}$$

where

$$E_\varepsilon\{E_\eta[u(\beta_i + \alpha_i(c_0 - c_i) - \psi(e_i))] | c_i < c_j\}$$

$$= \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_j + \varepsilon_i} E_{\eta}[u(\beta_i + \alpha_i(e_i + \varepsilon_i + \eta) - \psi(e_i))] f(\varepsilon_j) f(\varepsilon_i) d\varepsilon_j d\varepsilon_i \Big/ P.$$

The managers simultaneously choose their effort levels. Each manager i chooses effort e_i to maximize his expected utility (3.17). An interior solution for manager i 's expected utility maximization problem satisfies the following first-order condition:

$$\begin{aligned} & \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} E_{\eta}[u(\beta_i + \alpha_i(e_i + \varepsilon_i + \eta) - \psi(e_i))] f(e_i - e_j + \varepsilon_i) f(\varepsilon_i) d\varepsilon_i \\ & + (\alpha_i - \psi'(e_i)) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{e_i - e_j + \varepsilon_i} E_{\eta}[u'(\beta_i + \alpha_i(e_i + \varepsilon_i + \eta) - \psi(e_i))] f(\varepsilon_j) f(\varepsilon_i) d\varepsilon_j d\varepsilon_i \\ (3.18) & - u(\gamma_i - \psi(e_i)) \frac{\partial P}{\partial e_i} - (1 - P) u'(\gamma_i - \psi(e_i)) \psi'(e_i) = 0. \end{aligned}$$

Given α_i, β_i and γ_i , equation (3.18) is manager i 's reaction function: his optimal choice of effort e_i is a function of manager j 's optimal choice of effort e_j . However, e_i is not a function of firm j 's contract $\{\alpha_j, \beta_j, \gamma_j\}$ since the contract of each firm is unobservable to the other firm's owner and manager. Thus, owner j 's choice of the managerial incentive structure $\{\alpha_j, \beta_j, \gamma_j\}$ has no strategic effect on manager i 's choice of effort e_i .

Assume that there is *ex ante* a competitive market of identical managers. Let w_0 be a manager's reservation wage. Then his participation constraint is

$$(3.19) \quad EU_{VT}(w_i, e_i) \geq u(w_0).$$

3.5.2. The Optimal Variable-Prize Structure

Owner i 's expected net profit $E\pi_{VT}(e_i, w_i)$ is equal to firm i 's expected profit net of manager i 's expected compensation payment $Ew_i(e_i, e_j, \alpha_i, \beta_i, \gamma_i)$ defined by (3.16), which is given by

$$(3.20) \quad E\pi_{VT}(e_i, w_i) = [p - Ec(e_i, \varepsilon_i, \eta)]V - Ew_i(e_i, e_j, \alpha_i, \beta_i, \gamma_i),$$

where $Ec(e_i, \varepsilon_i, \eta) \equiv E_\varepsilon[E_\eta(c_i)] = c_0 - e_i$ is firm i 's expected marginal cost of production.

Owner i independently chooses an optimal yardstick managerial contract, characterized by a triplet $\{\alpha_i, \beta_i, \gamma_i\}$ that maximizes his expected net profit (3.20) subject to manager i 's incentive constraint (3.18) and participation constraint (3.19).

We rewrite owner i 's problem in terms of his total costs, which are equal to the sum of his firm's production cost $c(e_i, \varepsilon_i, \eta)V$ and the managerial compensation payment w_i : $C(w_i, e_i, \varepsilon_i, \eta) = c(e_i, \varepsilon_i, \eta)V + w_i$. Given total revenue pV , owner i 's expected net profit maximization problem is equivalent to his expected total cost minimization problem:

$$(3.21) \quad \min_{\{\alpha_i, \beta_i, \gamma_i\}} EC(w_i, e_i, \varepsilon_i, \eta) = (c_0 - e_i)V + Ew_i(e_i, e_j, \alpha_i, \beta_i, \gamma_i)$$

subject to manager i 's incentive constraint (3.18) and participation constraint (3.19).

In a symmetric Nash equilibrium, the owners choose the same yardstick managerial contract $\{\alpha_i = \alpha, \beta_i = \beta, \gamma_i = \gamma\}$ and each manager i chooses the same level of effort $e_i = e$. So the probabilities of winning and losing are equal, $P =$

$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{\varepsilon_i} f(\varepsilon_j) f(\varepsilon_i) d\varepsilon_j d\varepsilon_i = 1/2$. The marginal winning probability of effort is then independent of manger i 's effort level and only related to the distribution of ε_i , $\partial P / \partial e_i = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} f(\varepsilon_i) f(\varepsilon_i) d\varepsilon_i = E_\varepsilon[f(\varepsilon_i)]$. Thus, manger i 's expected compensation in (3.16) is

$$Ew(e, \alpha, \beta, \gamma) = \frac{1}{2}[\beta + \alpha e + \alpha E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_j) + \gamma],$$

where the conditional mean of ε_i is $E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_j) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \int_{\underline{\varepsilon}}^{\varepsilon_i} \varepsilon_i f(\varepsilon_j) f(\varepsilon_i) d\varepsilon_j d\varepsilon_i / (1/2)$.

Let $u(1)$ and $u(2)$ denote the manger's utility in the winning and losing states respectively: $u(1) \equiv u(\beta + \alpha(e + \varepsilon_i + \eta) - \psi(e))$ and $u(2) \equiv u(\gamma - \psi(e))$. Then the manger's incentive constraint (3.18) and participation constraint (3.19) can be written respectively as

(3.22)

$$E_\varepsilon\{E_\eta[u(1) - u(2)]f(\varepsilon_i)\} + \frac{1}{2}\alpha E_\varepsilon\{E_\eta[u'(1)] | \varepsilon_i > \varepsilon_j\} - \frac{1}{2}\psi'(e)E_\varepsilon\{E_\eta[u'(1) + u'(2)] | \varepsilon_i > \varepsilon_j\} = 0,$$

(3.23)

$$EU_{VT}(w, e) = \frac{1}{2}E_\varepsilon\{E_\eta[u(1)] | \varepsilon_i > \varepsilon_j\} + \frac{1}{2}u(2) \geq u(w_0).$$

The owner's expected total cost minimization problem becomes

(3.24)

$$\min_{\{\alpha, \beta, \gamma\}} EC(w, e, \varepsilon_i, \eta) = (c_0 - e)V + Ew(e, \alpha, \beta, \gamma).$$

subject to (3.22) and (3.23). Therefore, the optimal solution to the model of yardstick managerial contracts is defined by (3.22), (3.23) and (3.24).

It is easy to see that the structure of yardstick managerial contracts defined by (3.22), (3.23) and (3.24) is the same as that of the model in Section 3.2. Thus, the

results in Sections 3.3 and 3.4 hold for the model in this section. Specifically, variable-prize tournaments improve agency efficiency, compared with fixed-prize tournaments and linear piece rates. Inter-firm competition can reduce X-inefficiency by making yardstick contracts feasible. Depending on market structure and the relative magnitude of industry-wide and firm-specific risks, the owners can use market-provided information about firms' performance correlation to make an optimal choice among available managerial incentive schemes.

3.6. Concluding Remarks

In this chapter we have analyzed a variable-prize tournament that has the combined features of relative performance evaluation and linear piece rates. We have shown that under certain conditions the variable-prize tournament dominates both the fixed-prize tournament and the linear piece rate scheme. Moreover, since it does not depend on information on the magnitude of relative performance, the variable-prize tournament requires less information than a scheme based on cardinal relative performance evaluation. These features of the variable-prize tournament make it a more appealing incentive scheme under some circumstances.

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APPENDIX A

Technical Details of Chapter 1

A.1. Cost Complementarity versus Cost Substitutability between Output and Effort with Some Examples

The cost complementarity or cost substitutability between output and effort directly comes from the nature of the production technology. To see this, consider a single-output, multi-input production technology which is represented by a production function $f(\mathbf{x}, e, \varepsilon)$, where $\mathbf{x} \equiv (x_i)_{i=1}^n$ is a non-negative vector of inputs with *constant* input prices denoted by the vector $\mathbf{w} \gg \mathbf{0}$, e is a one-dimensional effort, ε is an idiosyncratic, random variable. Assume that $f(\cdot)$ is a twice continuously differentiable concave function of inputs and effort and strictly increasing in ε . Let q denote the maximum quantity of output that is given by $f(\mathbf{x}, e, \varepsilon)$ and that can be produced using inputs $\mathbf{x} \equiv (x_i)_{i=1}^n$ and exerting effort e . For a given level of output and a given effort exerted by the firm's manager, the *cost minimization problem* (CMP) is

$$\begin{aligned} C(q, e, \varepsilon) &= \min_{\mathbf{x} \geq \mathbf{0}} \mathbf{w} \cdot \mathbf{x} \\ \text{s.t. } f(\mathbf{x}, e, \varepsilon) &\geq q. \end{aligned}$$

For constant input prices, the optimized value function of the CMP is given by the cost function $C(q, e, \varepsilon)$, which is a *strictly increasing* convex function of q , a

strictly decreasing convex function of e and a *strictly decreasing* function of ε .¹ The Lagrangian for the CMP is

$$(A.1) \quad \mathcal{L} = \mathbf{w} \cdot \mathbf{x} + \lambda [q - f(\mathbf{x}, e, \varepsilon)],$$

where λ is the Lagrange multiplier. For some $\lambda > 0$, the following first-order conditions of the Lagrangian (A.1) for interior solutions must hold for each of the inputs $x_i, i = 1, \dots, n$:

$$(A.2) \quad q - f(\mathbf{x}, e, \varepsilon) = 0,$$

$$(A.3) \quad w_i - \lambda \frac{\partial f(\mathbf{x}, e, \varepsilon)}{\partial x_i} = 0.$$

In matrix notation, the first-order conditions (A.2) and (A.3) can be expressed as

$$(A.4) \quad q - f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = 0,$$

$$(A.5) \quad \mathbf{w} - \lambda(q, e, \varepsilon) \mathbf{D}_{\mathbf{x}} f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \mathbf{0}.$$

Differentiating (A.4) and (A.5) with respect to e yields

$$(A.6) \quad -[\mathbf{D}_{\mathbf{x}} f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)]^{\top} \mathbf{D}_e \mathbf{x}(q, e, \varepsilon) - f_e(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = 0,$$

$$-\mathbf{D}_{\mathbf{x}} f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \lambda_e(q, e, \varepsilon) - \lambda \mathbf{D}_{\mathbf{x}}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \mathbf{D}_e \mathbf{x}(q, e, \varepsilon)$$

$$(A.7) \quad -\lambda \mathbf{D}_{\mathbf{x}e}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \mathbf{0}.$$

¹For more general results, see Mas-Colell et al. (1995), p. 140-141, Proposition 5.C.2.

Expressing (A.6) and (A.7) in partitioned matrix notation, we have

$$(A.8) = \begin{bmatrix} \mathbf{0} & -[\mathbf{D}_{\mathbf{x}}f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)]^\top \\ -\mathbf{D}_{\mathbf{x}}f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) & -\lambda \mathbf{D}_{\mathbf{x}}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \end{bmatrix} \begin{bmatrix} \lambda_e(q, e, \varepsilon) \\ \mathbf{D}_e \mathbf{x}(q, e, \varepsilon) \end{bmatrix}$$

$$= \begin{bmatrix} f_e(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \\ \lambda \mathbf{D}_{\mathbf{x}e}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \end{bmatrix},$$

$$\text{where } \mathbf{D}_{\mathbf{x}}f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \begin{bmatrix} \frac{\partial f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)}{\partial x_1} \\ \vdots \\ \frac{\partial f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)}{\partial x_n} \end{bmatrix}, \mathbf{D}_e \mathbf{x}(q, e, \varepsilon) = \begin{bmatrix} \frac{\partial x_1(q, e, \varepsilon)}{\partial e} \\ \vdots \\ \frac{\partial x_n(q, e, \varepsilon)}{\partial e} \end{bmatrix},$$

$$f_e(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \frac{\partial f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)}{\partial e},$$

$$\mathbf{D}_{\mathbf{x}}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = [f_{ij}], f_{ij} = \frac{\partial^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)}{\partial x_i \partial x_j}, i, j = 1, \dots, n,$$

$$\mathbf{D}_{\mathbf{x}e}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \begin{bmatrix} \frac{\partial^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)}{\partial x_1 \partial e} \\ \vdots \\ \frac{\partial^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)}{\partial x_n \partial e} \end{bmatrix}, \lambda_e(q, e, \varepsilon) = \frac{\partial \lambda(q, e, \varepsilon)}{\partial e}.$$

Let H be the first matrix on the left-hand side of the matrix expression (A.8), which is the bordered Hessian matrix of second derivatives of the Lagrangian for the cost minimization. The determinant of H is negative by the second-order conditions for minimization, i.e, $|H| < 0$. We rewrite the determinant of H in more concise notations as

$$(A.9) \quad |H| = \begin{vmatrix} \mathbf{0} & -\mathbf{F}_{\mathbf{x}} \\ -\mathbf{F}_{\mathbf{x}}^\top & -\lambda \mathbf{F}_{\mathbf{xx}} \end{vmatrix},$$

where $\mathbf{F}_\mathbf{x} \equiv [\mathbf{D}_\mathbf{x}f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)]^\top$ and $\mathbf{F}_{\mathbf{xx}} \equiv \mathbf{D}_\mathbf{x}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)$. By using the determinant expansion by cofactors formula, the determinant of H in (A.9) can be expanded as follows:

$$\begin{aligned}
 |H| &= \sum_{i=1}^n (-\mathbf{F}_\mathbf{x}^\top)_i \cdot (-1)^{(i+1)+1} \begin{vmatrix} -\mathbf{F}_\mathbf{x} \\ -\lambda (\mathbf{F}_{\mathbf{xx}})_i \end{vmatrix} \\
 &= -\sum_{i=1}^n (\mathbf{F}_\mathbf{x}^\top)_i \cdot \left[(-1)^{(i+1)+1} \lambda^{n-1} (-1)^n \begin{vmatrix} \mathbf{F}_\mathbf{x} \\ (\mathbf{F}_{\mathbf{xx}})_i \end{vmatrix} \right] \\
 (A.10) \quad &= -\lambda^{n-1} \sum_{i=1}^n (\mathbf{F}_\mathbf{x}^\top)_i \cdot \omega_i,
 \end{aligned}$$

where $\omega_i \equiv (-1)^{i+(n+2)} \sum_{j=1}^n (\mathbf{F}_\mathbf{x})_j \cdot (-1)^{1+j} |(\mathbf{F}_{\mathbf{xx}})_{ij}|$, $(\mathbf{F}_\mathbf{x}^\top)_i$ is the i th element in the vector $\mathbf{F}_\mathbf{x}^\top$, $(\mathbf{F}_\mathbf{x})_j$ is the j th element in the vector $\mathbf{F}_\mathbf{x}$, $(\mathbf{F}_{\mathbf{xx}})_i$ is the matrix obtained from the matrix $\mathbf{F}_{\mathbf{xx}}$ by deleting row i , and $(\mathbf{F}_{\mathbf{xx}})_{ij}$ is the matrix obtained from the matrix $\mathbf{F}_{\mathbf{xx}}$ by deleting row i and column j .

By the assumption that $f(\mathbf{x}, e, \varepsilon)$ is a twice continuously differentiable concave function of inputs $\mathbf{x} \equiv (x_i)_{i=1}^n$, $\mathbf{D}_\mathbf{x}f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \gg \mathbf{0}$, and $\mathbf{D}_\mathbf{x}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)$ is negative semidefinite for all $(\mathbf{x}, e)$, so $(\mathbf{F}_\mathbf{x}^\top)_i > 0$, $i = 1, \dots, n$. In ω_i , since $(\mathbf{F}_\mathbf{x})_j > 0$ and for each input x_i , $i = 1, \dots, n$, $|(\mathbf{F}_{\mathbf{xx}})_{ij}|$ are multiplied by the correct signs, $(-1)^{i+(n+2)}$ and $(-1)^{1+j}$, so that the number of the positive expansion terms in $(-1)^{i+(n+2)} (-1)^{1+j} |(\mathbf{F}_{\mathbf{xx}})_{ij}|$ are always much more than that of the negative terms.

Example A.1. When there are two inputs $\mathbf{x} \equiv (x_1, x_2)$ used in the production, for each input x_i , $i = 1, 2$, there are only two positive terms in

$$(-1)^{i+(2+2)} (-1)^{1+j} \left| (\mathbf{F}_{\mathbf{xx}})_{ij} \right|.$$

Example A.2. When there are three inputs $\mathbf{x} \equiv (x_1, x_2, x_3)$, for each input x_i , $i = 1, 2, 3$, there are five positive terms and one negative term in

$$(-1)^{i+(3+2)} (-1)^{1+j} \left| (\mathbf{F}_{\mathbf{xx}})_{ij} \right|.$$

Example A.3. When there are four inputs $\mathbf{x} \equiv (x_1, x_2, x_3, x_4)$, for each input x_i , $i = 1, 2, 3, 4$, there are sixteen positive terms and eight negative terms in

$$(-1)^{i+(4+2)} (-1)^{1+j} \left| (\mathbf{F}_{\mathbf{xx}})_{ij} \right|.$$

This ensures that the positive expansion terms in ω_i are always much more than the negative terms for $i = 1, \dots, n$. Moreover, $|H| < 0$ implies that $\sum_{i=1}^n (\mathbf{F}_{\mathbf{x}}^{\top})_i \cdot \omega_i > 0$ in (A.10). Hence, ω_i in $|H|$ must be positive for $i = 1, \dots, n$, i.e.,

$$(A.11) \quad \omega_i \equiv (-1)^{i+(n+2)} \sum_{j=1}^n (\mathbf{F}_{\mathbf{x}})_j \cdot (-1)^{1+j} \left| (\mathbf{F}_{\mathbf{xx}})_{ij} \right| > 0.$$

Solving for $\lambda_e(q, e, \varepsilon)$ in (A.8) by Cramer's rule, we obtain the effect of effort on the Lagrange multiplier λ ,

$$(A.12) \quad \lambda_e(q, e, \varepsilon) = \frac{\begin{vmatrix} f_e & -\mathbf{F}_x \\ \lambda \mathbf{F}_{xe} & -\lambda \mathbf{F}_{xx} \end{vmatrix}}{|H|},$$

where $f_e \equiv f_e(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)$ and $\mathbf{F}_{xe} \equiv \mathbf{D}_{xe}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)$. Let H^e be the partitioned matrix in the numerator of the fraction (A.12). The determinant of H^e in (A.12) is

$$(A.13) \quad |H^e| = \begin{vmatrix} f_e & -\mathbf{F}_x \\ \lambda \mathbf{F}_{xe} & -\lambda \mathbf{F}_{xx} \end{vmatrix}.$$

By using the determinant expansion by cofactors formula, the determinant of H^e in (A.13) can be expanded as follows:

$$\begin{aligned} |H^e| &= f_e \cdot (-1)^{1+1} |-\lambda \mathbf{F}_{xx}| + \sum_{i=1}^n \lambda (\mathbf{F}_{xe})_i \cdot (-1)^{(i+1)+1} \begin{vmatrix} -\mathbf{F}_x \\ -\lambda (\mathbf{F}_{xx})_i \end{vmatrix} \\ &= \lambda^n f_e \cdot (-1)^n |\mathbf{F}_{xx}| + \sum_{i=1}^n \lambda (\mathbf{F}_{xe})_i \cdot \left[(-1)^{(i+1)+1} \lambda^{n-1} (-1)^n \begin{vmatrix} \mathbf{F}_x \\ (\mathbf{F}_{xx})_i \end{vmatrix} \right] \\ (A.14) \quad &= \lambda^n \left[f_e \cdot (-1)^n |\mathbf{F}_{xx}| + \sum_{i=1}^n (\mathbf{F}_{xe})_i \cdot \omega_i \right], \end{aligned}$$

where $(\mathbf{F}_{xe})_i$ is the i th element in the vector $\mathbf{F}_{xe}$.

To make sure the sign of $|H^e|$, we only need to determine the sign and magnitude of each term of $|H^e|$ in (A.14). By the assumption that $f(\mathbf{x}, e, \varepsilon)$ is a twice continuously differentiable concave function of inputs and effort, $\mathbf{D}_{\mathbf{x}}f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \gg \mathbf{0}$, $f_e(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) > 0$ and $\mathbf{D}_{\mathbf{x}}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)$ is negative semidefinite for all $(\mathbf{x}, e)$, so $(-1)^n |\mathbf{F}_{\mathbf{xx}}| \geq 0$ and the first term in the bracket of $|H^e|$ is nonnegative. Therefore, the signs of the relevant terms of $|H^e|$ in (A.14) can be determined as follows:

$$(A.15) \quad |H^e| = \lambda^n \left[\underbrace{f_e}_{>0} \cdot \underbrace{(-1)^n |\mathbf{F}_{\mathbf{xx}}|}_{\geq 0} + \sum_{i=1}^n (\mathbf{F}_{\mathbf{x}e})_i \cdot \underbrace{\omega_i}_{>0} \right].$$

Let Γ denote the terms in the bracket of $|H^e|$ in (A.15). Thus, the sign of $|H^e|$ depends on the signs of $\mathbf{F}_{\mathbf{x}e} \equiv \mathbf{D}_{\mathbf{x}e}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon)$ and Γ . We obtain the following conditions for different signs of $|H^e|$:

If $\mathbf{D}_{\mathbf{x}e}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \gg \mathbf{0}$, then $|H^e| > 0$;

If $\mathbf{D}_{\mathbf{x}e}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \mathbf{0}$ and $(-1)^n |\mathbf{F}_{\mathbf{xx}}| > 0$, then $|H^e| > 0$;

If $\mathbf{D}_{\mathbf{x}e}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \mathbf{0}$ and $(-1)^n |\mathbf{F}_{\mathbf{xx}}| = 0$, then $|H^e| = 0$;

If $\mathbf{D}_{\mathbf{x}e}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \ll \mathbf{0}$, $(-1)^n |\mathbf{F}_{\mathbf{xx}}| > 0$ and $\Gamma \geq 0$, then $|H^e| \geq 0$;

If $\mathbf{D}_{\mathbf{x}e}^2f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \ll \mathbf{0}$ and $(-1)^n |\mathbf{F}_{\mathbf{xx}}| = 0$, then $|H^e| < 0$.

So we have

$$\begin{aligned}
 & \gg \\
 \lambda_e(q, e, \varepsilon) &= \frac{|H^e|}{|H|} < 0 \text{ if } \mathbf{D}_{\mathbf{x}e}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \mathbf{0} \text{ and } (-1)^n |\mathbf{F}_{\mathbf{xx}}| > 0; \\
 & \ll \text{ and } (-1)^n |\mathbf{F}_{\mathbf{xx}}| > 0 \text{ but } \Gamma > 0; \\
 \lambda_e(q, e, \varepsilon) &= \frac{|H^e|}{|H|} = 0 \text{ if } \mathbf{D}_{\mathbf{x}e}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) = \mathbf{0} \text{ and } (-1)^n |\mathbf{F}_{\mathbf{xx}}| = 0; \\
 \lambda_e(q, e, \varepsilon) &= \frac{|H^e|}{|H|} > 0 \text{ if } \mathbf{D}_{\mathbf{x}e}^2 f(\mathbf{x}(q, e, \varepsilon), e, \varepsilon) \ll \mathbf{0} \text{ and } (-1)^n |\mathbf{F}_{\mathbf{xx}}| > 0 \text{ but } \Gamma < 0; \\
 & \text{and } (-1)^n |\mathbf{F}_{\mathbf{xx}}| = 0.
 \end{aligned}$$

According to the envelope theorem, the marginal cost of output equals the Lagrange multiplier λ in the cost minimization problem, i.e.,

$$(A.16) \quad C_q(q, e, \varepsilon) \equiv \frac{\partial C(q, e, \varepsilon)}{\partial q} = \lambda(q, e, \varepsilon) > 0.$$

Hence, we have

$$(A.17) \quad C_{qe}(q, e, \varepsilon) \equiv \frac{\partial^2 C(q, e, \varepsilon)}{\partial q \partial e} = \lambda_e(q, e, \varepsilon) = \frac{|H^e|}{|H|}.$$

We can define the marginal technical complementarity (respectively, substitutability) between e and x_i as

$$(A.18) \quad \frac{\partial^2 f(\cdot)}{\partial x_i \partial e} > 0 \text{ (respectively, } < 0) \text{ for } i = 1, \dots, n,$$

which implies that effort and other input factors are complements (respectively, substitutes) in the production. Thus, the marginal technical complementarity between

effort and each of the inputs is a *sufficient* condition for cost complementarity between output and effort, but the marginal technical substitutability between effort and each of the inputs is only a *necessary* condition for cost substitutability between output and effort.

A more intuitive description of cost complementarity (respectively, substitutability) between output and effort is as follows. Consider a single-output production technology represented by a production function $f(x, e)$, where x is an index of all other inputs with the price $w > 0$ and e is the manager's effort. Assume that $f(\cdot)$ is a twice continuously differentiable concave function of x and effort e : $\partial f / \partial x > 0$, $\partial f / \partial e > 0$, $f_{xx} < 0$, and $f_{ee} < 0$.

Let q denote the maximum quantity of output that is given by $f(x, e)$. Differentiating $q = f(x, e)$ with respect to q and e yields

$$\frac{\partial x}{\partial q} = \frac{1}{f_x(x, e)} > 0 \quad \text{and} \quad \frac{\partial x}{\partial e} = -\frac{f_e(x, e)}{f_x(x, e)} < 0.$$

The cost function is $C(q, e) = wx(q, e)$. The marginal cost of output is given by

$$(A.19) \quad C_q(q, e) = w \frac{\partial x(q, e)}{\partial q} = w \frac{1}{f_x(x(q, e), e)}.$$

Differentiating $C_q(q, e)$ with respect to e yields

$$\begin{aligned}
 C_{qe}(q, e) &\equiv \frac{\partial C_q(q, e)}{\partial e} = -w \frac{1}{[f_x(\cdot)]^2} \left(f_{xx} \frac{\partial x}{\partial e} + f_{xe} \right) \\
 &= -\frac{w f_e}{[f_x(\cdot)]^2 x} \left(-\frac{f_{xx} \cdot x}{f_x} + \frac{f_{xe} \cdot x}{f_e} \right) \\
 (A.20) \quad &= -\frac{w f_e}{[f_x(\cdot)]^2 x} (\eta_{MPe} - \eta_{MPx}),
 \end{aligned}$$

where $\eta_{MPe} = \frac{\partial f_e}{\partial x} \frac{x}{f_e}$ is the *elasticity* of marginal product of effort e with respect to x and $\eta_{MPx} = \frac{\partial f_x}{\partial x} \frac{x}{f_x}$ is the *elasticity* of marginal product of x with respect to x .

If $f_{xe} > 0$, then $\eta_{MPe} > 0$, so $C_{qe}(q, e) < 0$.

If $f_{xe} < 0$, then $\eta_{MPe} < 0$, so $C_{qe}(q, e) \geq 0$ if $\eta_{MPe} \leq \eta_{MPx}$.

Thus, $f_{xe} > 0$ is a *sufficient* condition for cost complementarity between output and effort $C_{qe}(q, e) < 0$, but $f_{xe} < 0$ is only a *necessary* condition for cost substitutability between output and effort $C_{qe}(q, e) > 0$, so the sign of $C_{qe}(q, e)$ depends on which marginal product term is more sensitive to x .

It is easy to construct an example of the production function with the marginal technical complementarity between effort and each of the inputs. The typical Cobb-Douglas production function satisfies this property, which is as follows:

Example A.4. Assume that the production function is

$$f(L, K, e, \varepsilon) = L^\alpha K^\beta e^\gamma \varepsilon,$$

where L is labor, K is capital, $\alpha, \beta, \gamma \in (0, 1)$.

Then the production function satisfies the conventional properties:

$$\begin{aligned}\frac{\partial f(L, K, e, \varepsilon)}{\partial L} &= \alpha L^{\alpha-1} K^\beta e^\gamma \varepsilon > 0, \quad \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial L^2} = \alpha(\alpha-1) L^{\alpha-2} K^\beta e^\gamma \varepsilon < 0, \\ \frac{\partial f(L, K, e, \varepsilon)}{\partial K} &= \beta L^\alpha K^{\beta-1} e^\gamma \varepsilon > 0, \quad \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial K^2} = \beta(\beta-1) L^\alpha K^{\beta-2} e^\gamma \varepsilon < 0, \\ \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial L \partial K} &= \alpha \beta L^{\alpha-1} K^{\beta-1} e^\gamma \varepsilon > 0, \quad \frac{\partial f(L, K, e, \varepsilon)}{\partial e} = \gamma L^\alpha K^\beta e^{\gamma-1} \varepsilon > 0, \\ \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial e^2} &= \gamma(\gamma-1) L^\alpha K^\beta e^{\gamma-2} \varepsilon < 0.\end{aligned}$$

Moreover,

$$\frac{\partial^2 f(L, K, e, \varepsilon)}{\partial L \partial e} = \alpha \gamma L^{\alpha-1} K^\beta e^{\gamma-1} \varepsilon > 0, \quad \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial K \partial e} = \beta \gamma L^\alpha K^{\beta-1} e^{\gamma-1} \varepsilon > 0.$$

However, it is not easy to construct an example of the production function with the marginal technical substitutability between effort and each of the inputs. The following example of the production function is somewhat unusual:

Example A.5. Assume that the production function is

$$f(L, K, e, \varepsilon) = L^\alpha K^\beta + (L \cdot K - k)e^{-\gamma} \varepsilon,$$

where L is labor, K is capital, $\alpha, \beta, \gamma \in (0, 1)$ and $k > L \cdot K$.

Then the production function satisfies the conventional properties:

$$\begin{aligned}\frac{\partial f(L, K, e, \varepsilon)}{\partial L} &= \alpha L^{\alpha-1} K^\beta + K e^{-\gamma} \varepsilon > 0, \quad \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial L^2} = \alpha(\alpha-1) L^{\alpha-2} K^\beta < 0, \\ \frac{\partial f(L, K, e, \varepsilon)}{\partial K} &= \beta L^\alpha K^{\beta-1} + L e^{-\gamma} \varepsilon > 0, \quad \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial K^2} = \beta(\beta-1) L^\alpha K^{\beta-2} < 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 f(L, K, e, \varepsilon)}{\partial L \partial K} &= \alpha \beta L^{\alpha-1} K^{\beta-1} + e^{-\gamma} \varepsilon > 0, \quad \frac{\partial f(L, K, e, \varepsilon)}{\partial e} = -\gamma(L \cdot K - k)e^{-\gamma-1} > 0, \\ \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial e^2} &= \gamma(\gamma + 1)(L \cdot K - k)e^{-\gamma-2} < 0.\end{aligned}$$

Moreover,

$$\frac{\partial^2 f(L, K, e, \varepsilon)}{\partial L \partial e} = -\gamma K e^{\gamma-1} \varepsilon < 0, \quad \frac{\partial^2 f(L, K, e, \varepsilon)}{\partial K \partial e} = -\gamma L e^{\gamma-1} \varepsilon < 0.$$

The analysis above does not imply that every production technology falls into just one of the categories: cost complementarity or cost substitutability between output and effort, since each requires that the marginal cost of output responds to changes in effort in the same qualitative way, regardless of the level of output. More generally, production technologies exhibit cost complementarity, cost substitutability and even cost independence ($C_{qe}(q, e, \varepsilon) = 0$) between output and effort over only certain ranges of output. To have a clear understanding of the implications such technologies, we construct an example as follows:

Example A.6. Assume that the cost function is

$$C(q, e, \varepsilon) = q^3 + \left(k - e^{\frac{1}{2}}\right) \left(\bar{q}q - \frac{1}{2}q^2\right) - \varepsilon,$$

where $k > 0$ and $\bar{q} > q/2$. Then

$$\begin{aligned}C_q(q, e, \varepsilon) &= 3q^2 + \left(k - e^{\frac{1}{2}}\right) (\bar{q} - q), \quad C_{qq}(q, e, \varepsilon) = 6q - \left(k - e^{\frac{1}{2}}\right), \\ C_e(q, e, \varepsilon) &= -\frac{1}{2}e^{-\frac{1}{2}} \left(\bar{q}q - \frac{1}{2}q^2\right) < 0, \quad C_{ee}(q, e, \varepsilon) = \frac{1}{4}e^{-\frac{3}{2}} \left(\bar{q}q - \frac{1}{2}q^2\right) > 0, \\ C_{qe}(q, e, \varepsilon) &= -\frac{1}{2}e^{-\frac{1}{2}}(\bar{q} - q).\end{aligned}$$

For any $e > 0$, when $q \leq \bar{q}$, $C_{qe}(q, e, \varepsilon) \leq 0$, implying that when output $q \in (0, \bar{q})$, effort becomes more effective at reducing the marginal cost of output (cost complementarity between output and effort); when output $q = \bar{q}$, effort has no effect on reducing the marginal cost of output (cost independence between output and effort); when output $q \in (\bar{q}, \infty)$, effort becomes less effective at reducing the marginal cost of output (cost substitutability between output and effort).

A.2. Proofs of Lemmas, Corollaries and Propositions in Sections 1.3 and 1.4

Let $H_i \equiv [H_i^{ij}]$, $i, j = 1, 2$, $i \in \{M, C\}$ denote the Hessian matrix of second partial derivatives of owner-manager i 's expected utility function, where

$$\begin{aligned} H_M^{11} &= p''(q)q + 2p'(q) - C_{qq}(q, e) < 0, \\ H_C^{11} &= p'(q) - C_{qq}(q, e) < 0, \\ H_i^{12} &= H_i^{21} = -C_{qe}(q, e) \geq 0 \text{ as } C_{qe}(q, e) \leq 0, \\ H_i^{22} &= -C_{ee}(q, e) - \psi''(e) < 0. \end{aligned}$$

For firm $i \in \{M, C\}$, a second-order necessary condition for owner-manager i 's expected utility maximization problem is that at owner-manager i 's optimal choices $(q_i^*, e_i^*)_{i \in \{M, C\}}$ the Hessian matrix must be negative semidefinite. The second-order necessary condition is satisfied since the concavity of owner-manager i 's objective function ensures that the Hessian matrix is negative semidefinite for every (q_i, e_i) . In what follows, assume that the determinant of the Hessian matrix H_i is not equal

to zero for all (q_i, e_i) , so it must be positive, which implies the strict concavity of owner-manager i 's objective function. Hence, we have $\det(H_i) > 0$.

Proof of Lemma 1.1. (i) Conducting the comparative statics on conditions (1.2) and (1.3) (or conditions (1.4) and (1.5)), we have that the slope of curve $q_i(e)$ is $-H_i^{12}/H_i^{11} > 0$ and the slope of curve $e_i(q)$ is $-H_i^{22}/H_i^{21} > 0$ as $C_{qe}(q, e) < 0$, $i \in \{M, C\}$, implying both curves $q_i(e)$ and $e_i(q)$ are upward sloping. Similarly, both the slope of curve $q_i(e)$ and the slope of curve $e_i(q)$ is negative as $C_{qe}(q, e) > 0$, implying both curves $q_i(e)$ and $e_i(q)$ are downward sloping.

(ii) The difference between the slope of curve $q_i(e)$ and the slope of curve $e_i(q)$ is $\det(H_i)/(H_i^{11}H_i^{21}) < 0$ as $C_{qe}(q, e) < 0$ but $\det(H_i)/(H_i^{11}H_i^{21}) > 0$ as $C_{qe}(q, e) > 0$, $i \in \{M, C\}$. $\square$

Proof of Lemma 1.2. (i) In the case where $C_{qe}(q, e) < 0$, since $0 < -H_M^{12}/H_M^{11} < -H_C^{12}/H_C^{11}$, curve $q_M(e)$ has less positive slope than curve $q_C(e)$ for any $e \geq 0$. In the case where $C_{qe}(q, e) > 0$, since $0 > -H_M^{12}/H_M^{11} > -H_C^{12}/H_C^{11}$, curve $q_M(e)$ has less negative slope than curve $q_C(e)$ for any $e \geq 0$.

(ii) Since $-H_M^{22}/H_M^{21} = -H_C^{22}/H_C^{21}$, both curves $e_M(q)$ and $e_C(q)$ have the same slope. $\square$

Proof of Proposition 1.1. The proof is based on the first-order conditions of the owner-manager's expected utility maximization problem for the monopoly firm and for the representative competitive firm. Suppose that the owner-manager monopolist expends the *same* level effort e_C^* as the owner-manager of the competitive firm does. By assumption $p'(q) < 0$ for all $q \geq 0$, it follows from the owner-manager

monopolist's first-order condition with respect to output,

$$(A.21) \quad p'(q)q + p(q) = C_q(q, e_C^*),$$

that $p(q) > C_q(q, e_C^*)$.

Let $q_M(e_C^*)$ denote the solution to (A.21), which is the optimal level of output produced by the monopolist at e_C^* . Comparing the monopoly pricing rule (A.21) with the marginal cost pricing rule (A.22) of the representative competitive firm,

$$(A.22) \quad p(q_C^*) = C_q(q_C^*, e_C^*),$$

since $p'(q) < 0$ and $C_{qq}(q, e) \geq 0$ for all $q \geq 0$, we can conclude that $q_M(e_C^*) < q_C^*$.

Thus,

$$-C_e(q_M(e_C^*), e_C^*) \leq -C_e(q_C^*, e_C^*) \text{ as } C_{qe}(q, e) \leq 0,$$

it follows from the first-order condition (1.5) with respect to effort for the representative competitive firm that

$$(A.23) \quad -C_e(q_M(e_C^*), e_C^*) - \psi'(e_C^*) \leq 0 \text{ as } C_{qe}(q, e) \leq 0.$$

Differentiating the expression on the left-hand side of (A.23),

$$(A.24) \quad -C_e(q_M(e_C^*), e_C^*) - \psi'(e_C^*),$$

with respect to effort e , we have

$$\begin{aligned} & -C_{eq}(q_M(e_C^*), e_C^*) \frac{dq_M(e_C^*)}{de} - C_{ee}(q_M(e_C^*), e_C^*) - \psi''(e_C^*) \\ &= H_M^{21} \left[-\frac{H_M^{12}}{H_M^{11}} \right] + H_M^{22} = \frac{\det(H_M)}{H_M^{11}} < 0. \end{aligned}$$

So (A.24) is strictly decreasing in effort.

In the case $C_{qe}(q, e) < 0$, (A.24) is negative. The owner-manager monopolist must reduce his level of effort from e_C^* to e_M^* and correspondingly decrease the level of output from $q_M(e_C^*)$ to $q_M^* \equiv q_M(e_M^*)$ (since $dq_M(e)/de = -H_M^{12}/H_M^{11} > 0$ by Lemma 1.1) such that the monopolist's first-order condition (1.3) with respect to effort is satisfied. Thus, $e_M^* < e_C^*$, $q_M^* < q_C^*$.

Conversely, in the case $C_{qe}(q, e) > 0$, (A.24) is positive. By similar reasoning, the owner-manager monopolist must increase his level of effort from e_C^* to e_M^* and correspondingly decrease the level of output from $q_M(e_C^*)$ to $q_M(e_M^*)$ (since $dq_M(e)/de = -H_M^{12}/H_M^{11} < 0$ by Lemma 1.1) such that condition (1.3) is satisfied. Thus, $e_M^* > e_C^*$, $q_M^* < q_C^*$. $\square$

Proof of Proposition 1.2. First, consider the case $C_{qe}(q, e) < 0$. Proposition 1.1 states that $e_M^* < e_C^*$. Then $C_e(q, e) < 0$ implies that $C(q, e_M^*) > C(q, e_C^*)$, and $C_{qe}(q, e) < 0$ implies that $C_q(q, e_M^*) > C_q(q, e_C^*)$.

Similarly, we can show that $C(q, e_M^*) < C(q, e_C^*)$ and $C_q(q, e_M^*) > C_q(q, e_C^*)$ in the case $C_{qe}(q, e) > 0$. $\square$

Proof of Corollary 1.1. Let $(q_i(e))_{i \in \{M, C\}}$ denote owner-manager i 's optimal choice of output for given effort e , which is derived from the first-order condition

(1.2) or (1.4) with respect to output q . Then owner-manager i 's expected utility can be written as a functions of e :

$$E[V(q_i(e), e; \varepsilon)] = p(q_i(e))q_i(e) - C(q_i(e), e) + E(\varepsilon) - \psi(e).$$

Corresponding to the different effort levels e_M^* and e_C^* , the difference between owner-manager's expected utility of the monopoly firm and that of the competitive firm is given by $E[V(q_M(e_M^*), e_M^*; \varepsilon)] - E[V(q_C(e_C^*), e_C^*; \varepsilon)]$ which can be decomposed into

$$\begin{aligned} & E[V(q_M(e_M^*), e_M^*; \varepsilon)] - E[V(q_M(e_C^*), e_C^*; \varepsilon)] \\ (A.25) \quad & + E[V(q_M(e_C^*), e_C^*; \varepsilon)] - E[V(q_C(e_C^*), e_C^*; \varepsilon)]. \end{aligned}$$

First, check whether the first term on the right-hand side of (A.25) is positive. Differentiating $E[V(q_M(e), e; \varepsilon)]$ with respect to e yields

$$\begin{aligned} \frac{dE[V(q_M(e), e; \varepsilon)]}{de} &= [p'(q_M(e))q_M(e) + p(q_M(e)) - C_q(q_M(e), e)]q'_M(e) \\ &\quad - C_e(q_M(e), e) - \psi'(e) \\ &= -C_e(q_M(e), e) - \psi'(e). \end{aligned}$$

It follows from the the owner-manager monopolist's first-order condition (1.3) with respect to e that $dE[V(q_M(e_M^*), e_M^*; \varepsilon)]/de = 0$.

The second-order derivative of $E[V(q_M(e), e; \varepsilon)]$ with respect to e is

$$\begin{aligned} \frac{d^2 E[V(q_M(e), e; \varepsilon)]}{de^2} &= -[C_{qe}(q_M(e), e)] \frac{dq_M(e)}{de} - C_{ee}(q_M(e), e) - \psi''(e) \\ &= H_M^{21} \cdot \left[-\frac{H_M^{12}}{H_M^{11}} \right] + H_M^{22} = \frac{\det(H_M)}{H_M^{11}} < 0, \end{aligned}$$

which implies that $E[V(q_M(e), \cdot; \varepsilon)]$ is a strictly concave function of e . Hence, e_M^* is a global maximizer of $E[V(q_M(e), e; \varepsilon)]$. The concavity of $E[V(q_M(e), \cdot; \varepsilon)]$ in e and $e_M^* \leq e_C^*$ by Proposition 1.1 imply that

$$(A.26) \quad E[V(q_M(e_M^*), e_M^*; \varepsilon)] > E[V(q_M(e_C^*), e_C^*; \varepsilon)].$$

Then check whether the second term on the right-hand side of (A.25) is positive.

For given e_C^* , differentiating $E[V(q_i(e), e_C^*; \varepsilon)]$ with respect to q yields

$$\frac{\partial E[V(q_i(e), e_C^*; \varepsilon)]}{\partial q_i} = p'(q_i(e))q_i(e) + p(q_i(e)) - C_q(q_i(e), e_C^*).$$

It follows from the the owner-manager monopolist's first-order condition (1.2) with respect to q that

$$\left. \frac{\partial E[V(q_i(e), e_C^*; \varepsilon)]}{\partial q_i} \right|_{q_i(e)=q_M(e_M^*)} = p'(q_M(e_C^*))q_M(e_C^*) + p(q_M(e_C^*)) - C_q(q_M(e_C^*), e_C^*) = 0.$$

The second-order derivative of $E[V(q_i(e), e_C^*; \varepsilon)]$ with respect to q is

$$\frac{\partial^2 E[V(q_i(e), e_C^*; \varepsilon)]}{\partial q_i^2} = p''(q_i(e))q_i(e) + 2p'(q_i(e)) - C_{qq}(q_i(e), e_C^*) < 0,$$

which implies that $E[V(\cdot, e_C^*; \varepsilon)]$ is strictly concave in output $q_i(e)$. Hence, $q_M(e_C^*)$ is a global maximizer of $E[V(q_i(e), e_C^*; \varepsilon)]$. For given e_C^* , $q_M(e_C^*) < q_C(e_C^*)$. Hence, we must have

$$(A.27) \quad E[V(q_M(e_C^*), e_C^*; \varepsilon)] > E[V(q_C(e_C^*), e_C^*; \varepsilon)].$$

Thus, (A.26) and (A.27) imply that $E[V(q_M(e_C^*), e_C^*; \varepsilon)] > E[V(q_C(e_C^*), e_C^*; \varepsilon)]$. $\square$

Proof of Proposition 1.3. Differentiating the expected net social welfare function (1.6) of the competitive firm with respect to e yields

$$\begin{aligned} \frac{dE[NSW_1(q_C(e), e; \varepsilon)]}{de} &= [p(q_C(e)) - C_q(q_C(e), e)]q'_C(e) - C_e(q_C(e), e) - \psi'(e) \\ &= -C_e(q_C(e), e) - \psi'(e). \end{aligned}$$

It follows from the first-order condition (1.5) with respect to e , $-C_e(q_C(e_C^*), e_C^*) - \psi'(e_C^*) = 0$, that

$$\left. \frac{dE[NSW_1(q_C(e), e; \varepsilon)]}{de} \right|_{e=e_C^*} = 0.$$

The second-order derivative of $E[NSW_1(q_C(e), e; \varepsilon)]$ with respect to e is

$$\begin{aligned} \frac{d^2 E[NSW_1(q_C(e), e; \varepsilon)]}{de^2} &= -[C_{qe}(q_C(e), e)]q'_C(e) - C_{ee}(q_C(e), e) - \psi''(e) \\ &= H_C^{21} \cdot \left[-\frac{H_C^{12}}{H_C^{11}} \right] + H_C^{22} = \frac{\det(H_C)}{H_C^{11}} < 0, \end{aligned}$$

which implies that $E[NSW_1(q_C(e), \cdot; \varepsilon)]$ is a strictly concave function of e . Hence, e_C^* is a global maximizer of $E[NSW_1(q_C(e), e; \varepsilon)]$. The concavity of $E[NSW_1(q_C(e), \cdot; \varepsilon)]$

in e and $e_M^* \leq e_C^*$ by Proposition 1.1 imply that

$$\begin{aligned} & E[NSW_1(q_C^*, e_C^*; \varepsilon)] - E[NSW_1(q_C(e_M^*), e_M^*; \varepsilon)] \\ &= DWL_e(q_C(e_C^*), q_C(e_M^*), e_C^*, e_M^*) - \Delta\psi(e_C^*, e_M^*) > 0. \end{aligned}$$

□

APPENDIX B

Technical Details of Chapter 2

B.1. Proofs of Lemmas, Corollaries and Propositions in Section 2.1

Let $\tilde{H}_i \equiv [\tilde{H}_i^{jk}]$, $j, k = 1, 2$, $i \in \{M, C\}$ denote the Hessian matrix of second partial derivatives of manager i 's expected utility maximization problem, where

$$\begin{aligned}\tilde{H}_M^{11} &= \alpha [p''(q)q + 2p'(q) - C_{qq}(q, e)] < 0, \\ \tilde{H}_C^{11} &= \alpha [p'(q) - C_{qq}(q, e)] < 0, \\ \tilde{H}_i^{12} &= \tilde{H}_i^{21} = -\alpha C_{qe}(q, e) \geq 0 \text{ as } C_{qe}(q, e) \leq 0, \\ \tilde{H}_i^{22} &= -\alpha C_{ee}(q, e) - \psi''(e) < 0.\end{aligned}$$

For any incentive power α , a second-order necessary condition for manager i 's expected utility maximization problem is that at manager i 's optimal choices $(\tilde{q}(\alpha), \tilde{e}(\alpha))$ the Hessian matrix must be negative semidefinite, which is satisfied since the concavity of manager i 's objective function ensures that the Hessian matrix is negative semidefinite for every (q, e) . The assumption that the determinant of the Hessian matrix $\tilde{H}_i$ is not equal to zero for all (q, e) yields $\det(\tilde{H}_i) > 0$, which implies the strict concavity of the manager's objective function.

Proof of Lemma 2.1. Conducting the comparative statics on the first-order conditions (2.3) and (2.4), we obtain:

$$\begin{aligned}\frac{d\tilde{q}_i(\alpha)}{d\alpha} &= \frac{-C_e(q, e)}{\det(\tilde{H}_i)} \cdot \tilde{H}_i^{12} \geq 0 \text{ as } C_{qe}(q, e) \leq 0. \\ \frac{d\tilde{e}_i(\alpha)}{d\alpha} &= \frac{-C_e(q, e)}{\det(\tilde{H}_i)} \cdot (-\tilde{H}_i^{11}) > 0.\end{aligned}$$

□

Proof of Lemma 2.2. Suppose that $\alpha \leq 0$. Then the first term on the left-hand side, which is the covariance of u' and π , is non-negative since for $\alpha \leq 0$ the manager's net income ($w \equiv \alpha\pi(\tilde{q}(\alpha), \tilde{e}(\alpha), \varepsilon) + \beta - \psi(e)$) is nonincreasing in π , so that his marginal utility, which is a decreasing function of income by the assumption of concavity of u , is nondecreasing in π . The second term is strictly positive by assumption, $u'(\cdot) > 0$, $\partial\pi(q, e, \varepsilon)/\partial e = -C_e(q, e) > 0$, and $\partial\tilde{e}(\alpha)/\partial\alpha > 0$ by Lemma 2.1 (ii). Thus, the above equation cannot hold if $\alpha \leq 0$.

Similarly for $\alpha \geq 1$, the first term $\text{Cov}(u', \pi)$ is negative, but the second term is non-positive. This implies that the above equation cannot hold if $\alpha \geq 1$.

Hence, we infer that the optimal incentive power α_i^{**} must lie in a range from 0 to 1, i.e., $0 < \alpha_i^{**} < 1$. □

Proof of Proposition 2.1. The proof is based on the first-order conditions of the owner-manager's expected utility maximization problem in Section 1.3 and of the manager's expected utility maximization problem in Section 2.1. Suppose that the manager of each managerial firm i ($i \in \{M, C\}$) chooses the same level output q_i^* . Since $\alpha_i \in (0, 1)$ by Lemma 2.2, the first-order condition (2.4) with respect to effort

e implies that

$$(B.1) \quad -C_e(q_i^*, e_i) > \psi(e_i).$$

Let $\tilde{e}(q_i^*; \alpha_i)$ denote the optimal level of effort exerted by the manager at q_i^* , which is the solution to (2.4). It follows from the first-order conditions (1.3), (1.5) and the properties of $C(q, e)$ and $\psi(e)$ that $\tilde{e}(q_i^*; \alpha_i) < e_i^*$. Thus,

$$C_q(q_i^*, \tilde{e}(q_i^*; \alpha_i)) \geq C_q(q_i^*, e_i) \text{ as } C_{qe}(q, e) \leq 0.$$

It follows from the first-order condition (2.5) with respect to output q that

$$(B.2) \quad \begin{aligned} p'(q_M^*)q_M^* + p(q_M^*) - C_q(q_M^*, \tilde{e}(q_M^*; \alpha_i)) &\leq 0 \\ p(q_C^*) - C_q(q_C^*, \tilde{e}(q_C^*; \alpha_i)) &\leq 0 \end{aligned} \text{ as } C_{qe}(q, e) \leq 0,$$

Differentiating the expressions on the left-hand side of (B.2),

$$(B.3) \quad p'(q_M^*)q_M^* + p(q_M^*) - C_q(q_M^*, \tilde{e}(q_M^*; \alpha_i)),$$

$$(B.4) \quad p(q_C^*) - C_q(q_C^*, \tilde{e}(q_C^*; \alpha_i)),$$

with respect to q , we have

$$\begin{aligned}
& p''(q_M^*)q_M^* + p'(q_M^*) - C_{qq}(q_M^*, \tilde{e}(q_M^*; \alpha_i)) - C_{qe}(q_M^*, \tilde{e}(q_M^*; \alpha_i)) \frac{d\tilde{e}(q_M^*; \alpha_i)}{dq} \\
&= \tilde{H}_M^{11} + \tilde{H}_M^{12} \left[-\frac{\tilde{H}_M^{21}}{\tilde{H}_M^{22}} \right] = \frac{\det(\tilde{H}_M)}{\tilde{H}_M^{22}} < 0, \\
& p'(q_C^*) - C_{qq}(q_C^*, \tilde{e}(q_C^*; \alpha_i)) - C_{qe}(q_C^*, \tilde{e}(q_C^*; \alpha_i)) \frac{d\tilde{e}(q_C^*; \alpha_i)}{dq} \\
&= \tilde{H}_C^{11} + \tilde{H}_C^{12} \left[-\frac{\tilde{H}_C^{21}}{\tilde{H}_C^{22}} \right] = \frac{\det(\tilde{H}_C)}{\tilde{H}_C^{22}} < 0.
\end{aligned}$$

So both (B.3) and (B.4) are strictly decreasing in q .

In the case $C_{qe}(q, e) < 0$, (B.3) and (B.4) are negative. Manager i must decrease the level of output from q_i^* to $\tilde{q}_i(\alpha_i)$ and correspondingly reduce his level of effort from e_i^* to $\tilde{e}_i(\alpha_i) \equiv \tilde{e}(\tilde{q}_i(\alpha_i), \alpha_i)$ (since $d\tilde{e}(q; \alpha_i)/dq > 0$ by Lemma 2.3) such that the first-order condition (2.4) with respect to q is satisfied. Thus, $\tilde{e}_i(\alpha_i) < e_i^*$ and $\tilde{q}_i(\alpha_i) < q_i^*$.

Conversely, in the case $C_{qe}(q, e) > 0$, (B.3) and (B.4) are positive. Manager i must increase the level of output from q_i^* to $\tilde{q}_i(\alpha_i)$ and correspondingly reduce his level of effort from e_i^* to $\tilde{e}_i(\alpha_i) \equiv \tilde{e}(\tilde{q}_i(\alpha_i), \alpha_i)$ (since $d\tilde{e}(q; \alpha_i)/dq < 0$ by Lemma 2.3) such that condition (2.4) is satisfied. Thus, $\tilde{e}_i(\alpha_i) < e_i^*$ and $\tilde{q}_i(\alpha_i) > q_i^*$. $\square$

Proof of Lemma 2.3. (i) Conducting the comparative statics on conditions (2.3) and (2.4), we have that the slope of curve $\tilde{q}_i(e)$ is $-\tilde{H}_i^{12}/\tilde{H}_i^{11} > 0$ and the slope of curve $\tilde{e}_i(q; \alpha)$ is $-\tilde{H}_i^{22}/\tilde{H}_i^{21} > 0$ as $C_{qe}(q, e) < 0$, $i \in \{M, C\}$, implying both curves $\tilde{q}_i(e)$ and $\tilde{e}_i(q; \alpha)$ are upward sloping. Similarly, both the slope of curve $\tilde{q}_i(e)$ and the

slope of curve $\tilde{e}_i(q; \alpha)$ is negative as $C_{qe}(q, e) > 0$, implying both curves $\tilde{q}_i(e)$ and $\tilde{e}_i(q; \alpha)$ are downward sloping.

(ii) The difference between the slope of curve $\tilde{q}_i(e)$ and the slope of curve $\tilde{e}_i(q; \alpha)$ is $\det(\tilde{H}_i)/(\tilde{H}_i^{11}\tilde{H}_i^{21}) < 0$ as $C_{qe}(q, e) < 0$ but $\det(\tilde{H}_i)/(\tilde{H}_i^{11}\tilde{H}_i^{21}) > 0$ as $C_{qe}(q, e) > 0$, $i \in \{M, C\}$.

(iii) Since $-\tilde{H}_i^{12}/\tilde{H}_i^{11} = -H_i^{12}/H_i^{11}$, $i \in \{M, C\}$, both curves $e_M(q)$ and $e_C(q)$ have the same slope.

(iv) In the case where $C_{qe}(q, e) < 0$, since $-\tilde{H}_i^{22}/\tilde{H}_i^{21} > -H_i^{22}/H_i^{21} > 0$, $i \in \{M, C\}$, curve $\tilde{e}_i(q; \alpha)$ has more positive slope than curve $e_i(q)$. In the case where $C_{qe}(q, e) > 0$, since $-\tilde{H}_i^{22}/\tilde{H}_i^{21} < -H_i^{22}/H_i^{21} < 0$, $i \in \{M, C\}$, curve $\tilde{e}_i(q; \alpha)$ has more negative slope than curve $e_i(q)$. $\square$

Proof of Lemma 2.4. (i) In the case where $C_{qe}(q, e) < 0$, since $0 < -\tilde{H}_M^{12}/\tilde{H}_M^{11} < -\tilde{H}_C^{12}/\tilde{H}_C^{11}$, curve $\tilde{q}_M(e)$ has less positive slope than curve $\tilde{q}_C(e)$ for any $e \geq 0$. In the case where $C_{qe}(q, e) > 0$, since $0 > -\tilde{H}_M^{12}/\tilde{H}_M^{11} > -\tilde{H}_C^{12}/\tilde{H}_C^{11}$, curve $\tilde{q}_M(e)$ has less negative slope than curve $\tilde{q}_C(e)$ for any $e \geq 0$.

(ii) Since $-\tilde{H}_M^{22}/\tilde{H}_M^{21} = -\tilde{H}_C^{22}/\tilde{H}_C^{21}$, both curves $\tilde{e}_M(q; \alpha)$ and $\tilde{e}_C(q; \alpha)$ have the same slope. $\square$

Proof of Proposition 2.2. For any given incentive power $\alpha \in (0, 1)$, the two simplified first-order conditions (2.3) and (2.4) of the manager's problem are similar to the first-order conditions (1.2) and (1.3) (or conditions (1.2) and (1.3)) of the owner-manager's problem in the case of the entrepreneurial firm. Then compared the first-order conditions for the monopoly managerial firm with those for the competitive

managerial firm, the proof of this proposition is analogous to that of Proposition 1.1 and is omitted. $\square$

Proof Lemma 2.5. The result follows from Lemma 2.1 and Proposition 2.2. $\square$

Proof of Corollary 2.1. For any given incentive power, $\alpha \in (0, 1)$, let $\tilde{q}(\tilde{e}(\alpha))$ denote the optimal output at effort $\tilde{e}(\alpha)$, which is the solution to the simplified first-order condition (1.2). So the manager's net income is given by

$$w(\tilde{e}(\alpha); \varepsilon) = \alpha[p(\tilde{q}(\tilde{e}(\alpha))\tilde{q}(\tilde{e}(\alpha)) - C(\tilde{q}(\tilde{e}(\alpha)), \tilde{e}(\alpha)) + \varepsilon] + \beta - \psi(\tilde{e}(\alpha)).$$

Then the manager's expected utility can be written as a functions of effort e :

$$E[U(\tilde{e}(\alpha); \varepsilon)] = \int u[w(\tilde{e}(\alpha); \varepsilon)]f(\varepsilon)d\varepsilon.$$

The rest of the proof of this corollary is analogous to that of Corollary 1.1 by applying the first-order condition (1.3) and Proposition 2.2, so it is omitted. $\square$

Proof of Proposition 2.3. In equilibrium manager i 's participation constraint must be binding. Inverting the function $E u[\alpha\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) + \beta - \psi(\tilde{e}_i(\alpha))] = u_0$, we can express β as a function of α and u_0 , $\beta = \beta(\alpha; u_0)$. So

$$(B.5) \quad \frac{d\beta(\alpha; u_0)}{d\alpha} = -\frac{\partial E u / \partial \alpha}{\partial E u / \partial \beta} = -\frac{E(u' \cdot \pi_i)}{E(u')}.$$

Using the first-order condition (2.4), we can write owner i 's first-order condition with respect to α as

$$(B.6) \quad E[V'_i(\alpha; \varepsilon)] = (1 - \alpha) \frac{\partial E \pi_i}{\partial e} \frac{d\tilde{e}_i(\alpha)}{d\alpha} - E \pi_i - \frac{d\beta(\alpha; u_0)}{d\alpha} = 0,$$

where $E\pi_i \equiv \pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon)$. Taking a first-order Taylor series expansion, we approximate the function $u'(w_i)$ around manager i 's expected net income $\bar{w}_i = \alpha\pi(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha), \varepsilon) + \beta - \psi(\tilde{e}_i(\alpha))$ and obtain $u'(w_i) \approx u'(\bar{w}_i) + u''(\bar{w}_i)\alpha(\pi_i - E\pi_i)$. Hence, we have

$$(B.7) \quad E(u') \approx u'(\bar{w}_i) + u''(\bar{w}_i)\alpha E(\pi_i - E\pi_i) = u'(\bar{w}_i),$$

$$(B.8) \quad \begin{aligned} E(u' \cdot \pi_i) &\approx u'(\bar{w}_i)E\pi_i + u''(\bar{w}_i)\alpha E[(\pi_i - E\pi_i)\pi_i] \\ &= u'(\bar{w}_i)E\pi_i + u''(\bar{w}_i)\alpha \text{Var}(\pi_i), \end{aligned}$$

where $\text{Var}(\pi_i)$ is the variance of profit which is equal to the variance of ε , i.e., $\text{Var}(\pi) = \sigma_\varepsilon^2$.

Using (B.5), (B.7) and (B.8), we can approximate (B.6) as

$$(B.9) \quad E[V'_i(\alpha; \varepsilon)] \approx (1 - \alpha) [-C_\varepsilon(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha))] \frac{d\tilde{e}_i(\alpha)}{d\alpha} - r(\bar{w}_i)\alpha\sigma_\varepsilon^2 = 0,$$

where $r(\bar{w}_i) \equiv -u''(\bar{w}_i)/u'(\bar{w}_i)$ is the Arrow-Pratt coefficient of absolute risk aversion evaluated at $\bar{w}_i$.

By Proposition 2.2, $\tilde{e}_M(\alpha) \leq \tilde{e}_C(\alpha)$. Next, we compare the individual terms in (B.9) in the case $i = M$ with those in the case $i = C$. Proposition 2.2 and the convexity of $\psi(\cdot)$ imply that

$$\psi'(\tilde{e}_M(\alpha)) \leq \psi'(\tilde{e}_C(\alpha)) \quad \text{as } C_{qe}(q, e) \leq 0.$$

Hence, from manager i 's optimization condition with respect to effort, $-\alpha C_e(\tilde{q}_i(\alpha), \tilde{e}_i(\alpha)) = \psi'(\tilde{e}_i(\alpha))$, we obtain

$$(B.10) \quad -C_e(\tilde{q}_M(\alpha), \tilde{e}_M(\alpha)) \leq -C_e(\tilde{q}_C(\alpha), \tilde{e}_C(\alpha)) \text{ as } C_{qe}(q, e) \leq 0.$$

Furthermore, Lemma 2.5 states that for any $\alpha \in (0, 1)$,

$$(B.11) \quad \frac{d\tilde{e}_M(\alpha)}{d\alpha} \leq \frac{d\tilde{e}_C(\alpha)}{d\alpha} \text{ as } C_{qe}(q, e) \leq 0.$$

Finally, taking a second-order Taylor series expansion for terms in $u(w_i)$ around $\bar{w}_i$, we approximate the expected utility function $Eu(w_i)$:

$$Eu(w_i) \approx u(\bar{w}_i) + u'(\bar{w}_i)\alpha E(\pi_i - E\pi_i) + \frac{1}{2}u''(\bar{w}_i)\alpha^2 E(\pi_i - E\pi_i)^2 = u(\bar{w}_i) + \frac{1}{2}u''(\bar{w}_i)\alpha^2 \sigma_\varepsilon^2.$$

Since the manager participation constraint is binding for both $i = M$ and $i = C$, we have $\bar{w}_M = \bar{w}_C$. Thus,

$$(B.12) \quad r(\bar{w}_M) = r(\bar{w}_C).$$

On the basis of (B.9)-(B.12), we obtain

$$(B.13) \quad E[V'_M(\alpha; \varepsilon)] \leq E[V'_C(\alpha; \varepsilon)] \text{ as } C_{qe}(q, e) \leq 0$$

for any $\alpha \in (0, 1)$.

The optimal incentive power α_C^{**} chosen by the owner of the competitive firm satisfies the first-order condition: $E[V'_C(\alpha_C^{**}; \varepsilon)] = 0$. It follows from (B.13) that

$$(B.14) \quad E[V'_M(\alpha_C^{**}; \varepsilon)] \leq E[V'_C(\alpha_C^{**}; \varepsilon)] = 0 \quad \text{as } C_{qe}(q, e) \leq 0.$$

On the other hand, the optimal incentive power α_M^{**} chosen by the owner of the monopoly firm satisfies the first-order condition:

$$(B.15) \quad E[V'_M(\alpha_M^{**}; \varepsilon)] = 0.$$

Equations (B.14), B.15 and the concavity of $E[V_i(\cdot; \varepsilon)]$ in α implies that $\alpha_M^{**} \leq \alpha_C^{**}$ as $C_{qe}(q, e) \leq 0$. $\square$

Proof of Proposition 2.4. (i) By Proposition 2.2, for any α , $\tilde{e}_M(\alpha) < \tilde{e}_C(\alpha)$ as $C_{qe}(q, e) < 0$. So $\tilde{e}_M(\alpha_M^{**}) < \tilde{e}_C(\alpha_M^{**})$. Now $\alpha_M^{**} < \alpha_C^{**}$ by Proposition 2.3 and $0 < d\tilde{e}_M(\alpha)/d\alpha < d\tilde{e}_C(\alpha)/d\alpha$ by Lemma 2.5, we have $\tilde{e}_C(\alpha_M^{**}) < \tilde{e}_C(\alpha_C^{**})$. Hence,

$$e_M^{**} \equiv \tilde{e}_M(\alpha_M^{**}) < \tilde{e}_C(\alpha_C^{**}) \equiv e_C^{**}.$$

Similarly, for any α , $\tilde{q}_M(\alpha) < \tilde{q}_C(\alpha)$ as $C_{qe}(q, e) < 0$ by Proposition 2.2. So $\tilde{q}_M(\alpha_M^{**}) < \tilde{q}_C(\alpha_M^{**})$. Now $\alpha_M^{**} < \alpha_C^{**}$ by Proposition 2.3 and $0 < d\tilde{q}_M(\alpha)/d\alpha < d\tilde{q}_C(\alpha)/d\alpha$ by Lemma 2.5, we have $\tilde{q}_C(\alpha_M^{**}) < \tilde{q}_C(\alpha_C^{**})$. Hence,

$$q_M^{**} \equiv \tilde{q}_M(\alpha_M^{**}) < \tilde{q}_C(\alpha_C^{**}) \equiv q_C^{**}.$$

(ii) By similar reasoning, we can show that $e_M^{**} > e_C^{**}$ and $q_M^{**} < q_C^{**}$ in the case $C_{qe}(q, e) > 0$. $\square$

Proof of Proposition 2.5. (i) In the case $C_{qe}(q, e) < 0$, Proposition 1.1 states that $e_M^* < e_C^*$ and $q_M^* < q_C^*$; Proposition 2.1 implies that $e_M^{**} \equiv \tilde{e}_M(\alpha_M^{**}) < e_M^*$ and $q_M^{**} \equiv \tilde{q}_M(\alpha_M^{**}) < q_M^*$. This establishes the results in part (i).

Similarly, in the case $C_{qe}(q, e) > 0$, using Propositions 1.1 and 2.1, we can show the results in part (ii). $\square$

Proof of Proposition 2.6. In the case $C_{qe}(q, e) < 0$, Proposition 2.4 states that $e_M^{**} < e_C^{**}$ and $q_M^{**} < q_C^{**}$; Proposition 2.1 implies that $e_C^{**} \equiv \tilde{e}_C(\alpha_C^{**}) < e_C^*$ and $q_C^{**} \equiv \tilde{q}_C(\alpha_C^{**}) < q_C^*$. This establishes the results in part (i).

Similarly, in the case $C_{qe}(q, e) > 0$, using Propositions 2.4 and 2.1, we can show the results in part (ii). $\square$

Proof of Proposition 2.7. The proof follows from Proposition 2.4 and the properties of $C(q, e)$. $\square$

B.2. Proofs of Lemmas and Propositions in Section 2.2

Proof of Lemma 2.6. Conducting comparative statics on the first-order condition (2.10) of the manager's effort choice problem, we have

$$\begin{aligned} \frac{\partial \hat{e}(q, \alpha)}{\partial q} &= \frac{-\psi'(e)C_{eq}(q, e)}{\psi'(e)C_{ee}(q, e) - \psi''(e)C_e(q, e)} \geq 0 \text{ as } C_{qe}(q, e) \leq 0, \\ \frac{\partial \hat{e}(q, \alpha)}{\partial \alpha} &= \frac{[C_e(q, e)]^2}{\psi'(e)C_{ee}(q, e) - \psi''(e)C_e(q, e)} > 0. \end{aligned}$$

$\square$

Proof of Lemma 2.7. Suppose that $\alpha \leq 0$. Then the first term on the left-hand side of (2.14), $\text{Cov}(u', \pi)$, is non-negative since the manager's net income ($w \equiv$

$\alpha\pi(q, \widehat{e}(q, \alpha), \varepsilon) + \beta - \psi(\widehat{e}(q, \alpha))$ is nonincreasing in π for $\alpha \leq 0$, so that his marginal utility, which is a decreasing function of income by the assumption of the concavity of $u(\cdot)$, is nondecreasing in π . The second term is strictly positive because $u'(\cdot) > 0$, $\partial\pi(q, e, \varepsilon)/\partial e = -C_e(q, e) > 0$, $\partial\widehat{e}(q, \alpha)/\partial\alpha > 0$ by Lemma 2.6(ii). Thus, equation (2.14) cannot hold if $\alpha \leq 0$.

Similarly for $\alpha \geq 1$, the first term on the left-hand side of (2.14) is negative, but the second term is non-positive, suggesting that (2.14) cannot hold if $\alpha \geq 1$.

Hence, we infer that the incentive power $\widehat{\alpha}_i^{**}$ must satisfy $0 < \widehat{\alpha}_i^{**} < 1$. $\square$

Proof of Lemma 2.8. Conducting the comparative statics on the first-order condition of the manager's problem yields

$$\begin{aligned} \frac{\partial\alpha(q, e)}{\partial q} &= \frac{\psi'(e)C_{qe}(q, e)}{[C_e(q, e)]^2} \leq 0 \text{ as } C_{qe}(q, e) \leq 0, \\ \frac{\partial\alpha(q, e)}{\partial e} &= \frac{\psi'(e)C_{ee}(q, e) - \psi''(e)C_e(q, e)}{[C_e(q, e)]^2} > 0, \\ \frac{\partial^2\alpha(q, e)}{\partial q^2} &= \frac{\psi'(e)\{C_{eqq}(q, e)C_e(q, e) - 2[C_{qe}(q, e)]^2\}}{[C_e(q, e)]^3}, \\ \frac{\partial^2\alpha(q, e)}{\partial e^2} &= \frac{[\psi'(e)C_{eee}(q, e) - \psi'''(e)C_e(q, e)]C_e(q, e)}{[C_e(q, e)]^3} \\ &\quad - \frac{2[\psi'(e)C_{ee}(q, e) - \psi''(e)C_e(q, e)]C_{ee}(q, e)}{[C_e(q, e)]^3}, \\ \frac{\partial^2\alpha(q, e)}{\partial q\partial e} &= \frac{[\psi''(e)C_{qe}(q, e) + \psi'(e)C_{qee}(q, e)]C_e(q, e)}{[C_e(q, e)]^3} \\ &\quad - \frac{2\psi'(e)C_{qe}(q, e)C_{ee}(q, e)}{[C_e(q, e)]^3}. \end{aligned}$$

Under assumption (i), $\partial^2\alpha(q, e)/\partial q^2 > 0$, implying that $\alpha(q, e)$ is strictly decreasing (respectively, increasing) and convex in output if $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$). Under assumption (ii), $\partial^2\alpha(q, e)/\partial e^2 > 0$, implying that $\alpha(q, e)$ is strictly increasing and convex in effort. Furthermore, $\partial^2\alpha(q, e)/\partial q\partial e \leq 0$, implying that the marginal incentive power of effort is lower (respectively, higher) for a given effort level when output increases if $C_{qe}(q, e) < 0$ (respectively, $C_{qe}(q, e) > 0$), and the complementarity (respectively, substitutability) between output and effort in the incentive power $\alpha(q, e)$. $\square$

Let $\hat{H}_i \equiv [\hat{H}_i^{jk}]$, $j, k = 1, 2$, $i = \{M, C\}$ denote the Hessian matrix of the second partial derivatives of owner i 's expected utility maximization problem with respect to q and e , where

$$\begin{aligned}\hat{H}_M^{11} &= p''(q)q + 2p'(q) - \left[C_{qq}(q, e) + \frac{\partial^2 RC(q, e)}{\partial q^2} \right] < 0, \\ \hat{H}_C^{11} &= p'(q) - \left[C_{qq}(q, e) + \frac{\partial^2 RC(q, e)}{\partial q^2} \right] < 0, \\ \hat{H}_i^{12} &= \hat{H}_i^{21} = - \left[C_{qe}(q, e) + \frac{\partial^2 RC(q, e)}{\partial q\partial e} \right] \geq 0 \text{ as } C_{qe}(q, e) \leq 0, \\ \hat{H}_i^{22} &= - \left[C_{ee}(q, e) + \psi''(e) + \frac{\partial^2 RC(q, e)}{\partial e^2} \right] < 0,\end{aligned}$$

since

$$\begin{aligned}\frac{\partial^2 RC(q, e)}{\partial q^2} &= r\sigma_e^2 \left[\frac{\partial^2 \alpha(q, e)}{\partial q^2} \alpha(q, e) + \left(\frac{\partial \alpha(q, e)}{\partial q} \right)^2 \right] > 0, \\ \frac{\partial^2 RC(q, e)}{\partial e^2} &= r\sigma_e^2 \left[\frac{\partial^2 \alpha(q, e)}{\partial e^2} \alpha(q, e) + \left(\frac{\partial \alpha(q, e)}{\partial e} \right)^2 \right] > 0,\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 RC(q, e)}{\partial q \partial e} &= \frac{\partial^2 RC(q, e)}{\partial e \partial q} \\
&= r\sigma_\varepsilon^2 \left[\frac{\partial^2 \alpha(q, e)}{\partial q \partial e} \alpha(q, e) + \frac{\partial \alpha(q, e)}{\partial q} \frac{\partial \alpha(q, e)}{\partial e} \right] \leq 0 \text{ as } C_{qe}(q, e) \leq 0.
\end{aligned}$$

Let $(\hat{q}_i^{**}, \hat{e}_i^{**})_{i \in \{M, C\}}$ denote the equilibrium solution to owner i 's expected utility maximization problem. A sufficient second condition for owner i 's expected utility maximization problem is that the Hessian matrix is negative semidefinite at the optimal $(\hat{q}_i^{**}, \hat{e}_i^{**})$ and that the determinant of $\hat{H}_i$ is positive, $\det(\hat{H}_i) > 0$, $i \in \{M, C\}$.

Let $\hat{G}_M \equiv [\hat{G}_M^{jk}]$, $j, k = 1, 2$, be the Hessian matrix of the second partial derivatives of the owner's expected utility maximization problem with respect to q and α , where

$$\begin{aligned}
\hat{G}_M^{11} &= \hat{H}_M^{11} - \left[C_{qe}(q, \hat{e}(q, \alpha)) + \frac{\partial^2 RC(q, \hat{e}(q, \alpha))}{\partial q \partial e} \right] \frac{\partial \hat{e}(q, \alpha)}{\partial q} < 0, \\
\hat{G}_M^{12} &= \hat{H}_M^{21} \frac{\partial \hat{e}(q, \alpha)}{\partial \alpha} \geq 0 \text{ as } C_{qe}(q, e) \leq 0, \\
\hat{G}_M^{22} &= \hat{H}_M^{22} \frac{\partial \hat{e}(q, \alpha)}{\partial \alpha} < 0.
\end{aligned}$$

Conducting comparative statics on the first-order condition (2.23) with respect to e yields

$$\frac{dq_i(e)}{de} = -\frac{\hat{H}_i^{12}}{\hat{H}_i^{11}} \geq 0 \text{ as } C_{qe}(q, e) \leq 0, i \in \{M, C\}.$$

Substituting $\widehat{e}(q, \alpha)$ into the first-order conditions of the owner's problem, we convert it into the equivalent first-order conditions with respect to q and α ,

$$\begin{aligned} \frac{\partial E[\pi(q, \widehat{e}(q, \alpha), \varepsilon)]}{\partial q} - \frac{\partial RC(q, \widehat{e}(q, \alpha))}{\partial q} &= 0, \\ \frac{\partial E[\pi(q, \widehat{e}(q, \alpha), \varepsilon)]}{\partial e} - \psi'(\widehat{e}(q, \alpha)) - \frac{\partial RC(q, \widehat{e}(q, \alpha))}{\partial e} &= 0. \end{aligned}$$

Conducting comparative statics on the first-order condition (2.23) with respect to α yields

$$\frac{dq_i(\alpha)}{d\alpha} = -\frac{\widehat{G}_i^{12}}{\widehat{G}_i^{11}} \geq 0 \text{ as } C_{qe}(q, e) \leq 0, i \in \{M, C\}.$$

Proof of Proposition 2.8. The proof is based on the first-order conditions of the owner-manager's expected utility maximization problem in Section 1.3 and of the owner's expected utility maximization problem in Section 2.2. Suppose that the owner of each managerial firm i ($i \in \{M, C\}$) chooses the same level output q_i^* . Let $\widehat{e}(q_i^*, \alpha_i)$ denote the optimal level of effort exerted by manager i , which is the solution to the first-order condition (2.10) of the manager's problem. Since $\partial RC(q, e)/\partial q \leq 0$, the first-order condition (2.23) of the owner's problem implies that

$$\begin{aligned} (B.16) \quad p'(q_M^*)q_M^* + p(q_M^*) - C_q(q_M^*, \widehat{e}(q_M^*, \alpha_i)) &\leq 0 \\ p(q_C^*) - C_q(q_C^*, \widehat{e}(q_C^*, \alpha_i)) &\leq 0 \end{aligned} \text{ as } C_{qe}(q, e) \leq 0,$$

It follows from the first-order condition (2.23) and the properties of $p(q)$ and $C(q, e)$ that $\widehat{e}(q_i^*, \alpha_i) < e_i^*, i \in \{M, C\}$. The properties of $C(q, e)$ and $\psi(e)$ imply that

$$-C_e(q_i^*, \widehat{e}(q_i^*, \alpha_i)) > -C_e(q_i^*, e_i^*),$$

$$\psi'(\widehat{e}(q_i^*, \alpha_i)) < \psi'(e_i^*).$$

It follows from the first-order condition (1.3) or (1.5) of the owner-manager's problem that $-C_e(q_i^*, \widehat{e}(q_i^*, \alpha_i)) - \psi'(\widehat{e}(q_i^*, \alpha_i)) > 0$. $\partial RC(q, e)/\partial e > 0$ and the assumption that the marginal risk cost of eliciting more effort from the manager is large enough imply that

$$(B.17) \quad -C_e(q_i^*, \widehat{e}(q_i^*, \alpha_i)) - \psi'(\widehat{e}(q_i^*, \alpha_i)) - \frac{\partial RC(q_i^*, \widehat{e}(q_i^*, \alpha_i))}{\partial e} < 0.$$

Differentiating the left-hand side of (B.17),

$$(B.18) \quad -C_e(q_i^*, \widehat{e}(q_i^*, \alpha_i)) - \psi'(\widehat{e}(q_i^*, \alpha_i)) - \frac{\partial RC(q_i^*, \widehat{e}(q_i^*, \alpha_i))}{\partial e},$$

with respect to q , we have

$$\begin{aligned} & - \left[C_{eq}(q_i^*, \widehat{e}(q_i^*, \alpha_i)) + \frac{\partial^2 RC(q_i^*, \widehat{e}(q_i^*, \alpha_i))}{\partial e \partial q} \right] \\ & - \left[C_{ee}(q_i^*, \widehat{e}(q_i^*, \alpha_i)) + \psi''(\widehat{e}(q_i^*, \alpha_i)) + \frac{\partial^2 RC(q_i^*, \widehat{e}(q_i^*, \alpha_i))}{\partial e^2} \right] \frac{d\widehat{e}}{dq} \\ & = \widehat{H}_i^{21} + \widehat{H}_i^{22} \left[-\frac{\widehat{H}_i^{11}}{\widehat{H}_i^{12}} \right] = -\frac{\det(\widehat{H}_i)}{\widehat{H}_i^{12}} \leq 0 \text{ as } C_{qe}(q, e) \leq 0, i \in \{M, C\}. \end{aligned}$$

Let $\widehat{q}_i(\alpha_i)$ be the owner's optimal choice of output, which is the solution to the owner's optimization condition with respect to q .

In the case $C_{qe}(q, e) < 0$, (B.18) is strictly decreasing in q . So owner i must decrease the level of output from q_i^* to $\widehat{q}_i(\alpha_i)$, $i \in \{M, C\}$ and correspondingly manager i reduces his level of effort from e_i^* to $\widehat{e}_i(\alpha_i) \equiv \widehat{e}(\widehat{q}_i(\alpha_i), \alpha_i)$ (since $\partial \widehat{e}(q_i, \alpha_i) / \partial q_i > 0$) such that the first-order conditions (2.24) with respect to e of the owner's problem,

$$-C_e(\widehat{q}_i(\alpha_i), \widehat{e}_i(\alpha_i)) - \psi(\widehat{e}_i(\alpha_i)) - \frac{\partial RC(\widehat{q}_i(\alpha_i), \widehat{e}_i(\alpha_i))}{\partial e} = 0$$

are satisfied. Thus, $\widehat{e}_i(\alpha_i) < e_i^*$ and $\widehat{q}_i(\alpha_i) < q_i^*$ as $C_{qe}(q, e) < 0$.

Conversely, in the case $C_{qe}(q, e) > 0$, (B.18) is strictly increasing in q . So owner i must increase his level of output from q_i^* to $\widehat{q}_i(\alpha_i)$, $i \in \{M, C\}$ and correspondingly manager i decreases his level of effort from e_i^* to $\widehat{e}_i(\alpha_i) \equiv \widehat{e}(\widehat{q}_i(\alpha_i), \alpha_i)$ (since $\partial \widehat{e}(q_i, \alpha_i) / \partial q_i < 0$). Thus, $\widehat{e}_i(\alpha_i) < e_i^*$ and $\widehat{q}_i(\alpha_i) > q_i^*$ as $C_{qe}(q, e) > 0$. $\square$

Proof of Proposition 2.9. The proof is based on the first-order conditions of the owner's expected utility maximization problem for the monopoly firm and for the representative competitive firm. Suppose that the owner of the monopoly firm chooses the same output and incentive power $\widehat{q}_C^{**}$ and $\widehat{\alpha}_C^{**}$ as the owner of the competitive firm does. Given $\widehat{q}_C^{**}$ and $\widehat{\alpha}_C^{**}$, the manager of the monopoly firm must exert the same level of effort $\widehat{e}_C^{**} \equiv \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})$ as the manager of the competitive firm does. By assumption $p'(q) < 0$ for all $q \geq 0$, it follows from the first-order condition with respect to output for the monopoly firm,

$$(B.19) \quad p'(q)q + p(q) = C_q(q, \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})) + \frac{\partial RC(q, \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**}))}{\partial q},$$

that $p(q) > C_q(q, \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})) + \partial RC(q, \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**}))/\partial q$.

Let $q_M(e)$ denote the optimal level of output chosen by the owner of the monopoly firm at e , which is the solution to the owner's first-order condition (2.23) with respect to q . Comparing the monopoly pricing rule (B.19) with that (B.20) of the competitive firm,

$$(B.20) \quad p(\widehat{q}_C^{**}) = C_q(\widehat{q}_C^{**}, \widehat{e}_C^{**}) + \frac{\partial RC(\widehat{q}_C^{**}, \widehat{e}_C^{**})}{\partial q},$$

we can conclude that $q_M(\widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})) < \widehat{q}_C^{**}$ since $p'(q) < 0$, $C_{qq}(q, e) \geq 0$ for all $q \geq 0$ and $\partial^2 RC(q, e)/\partial q^2 > 0$.

Thus, $-C_e(q_M(\widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})), \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})) \leq -C_e(\widehat{q}_C^{**}, \widehat{e}_C^{**}) > 0$ and

$$-\frac{\partial RC(q_M(\widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})), \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**}))}{\partial e} \leq -\frac{\partial RC(\widehat{q}_C^{**}, \widehat{e}_C^{**})}{\partial e} < 0 \text{ as } C_{qe}(q, e) \leq 0.$$

It follows from the first-order condition (2.24) with respect to e for the competitive firm that

$$(B.21) \quad -C_e(q_M(\widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})), \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})) - \frac{\partial RC(q_M(\widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})), \widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**}))}{\partial e} \\ -\psi'(\widehat{e}(\widehat{q}_C^{**}, \widehat{\alpha}_C^{**})) \leq 0 \text{ as } C_{qe}(q, e) \leq 0.$$

Differentiating the expression on the left-hand side of (B.21) with respect to q , we have

$$\begin{aligned}
& \left[- \left(C_{eq}(q_M(\hat{e}_C^{**}), \hat{e}_C^{**}) + \frac{\partial^2 RC(q_M(\hat{e}_C^{**}), \hat{e}_C^{**})}{\partial e \partial q} \right) \frac{dq_M(\hat{e}_C^{**})}{de} \right. \\
& \quad \left. - \left(C_{ee}(q_M(\hat{e}_C^{**}), \hat{e}_C^{**}) + \psi''(\hat{e}_C^{**}) + \frac{\partial^2 RC(q_M(\hat{e}_C^{**}), \hat{e}_C^{**})}{\partial e^2} \right) \right] \frac{\partial \hat{e}(\hat{q}_C^{**}, \hat{\alpha}_C^{**})}{\partial q} \\
& = \left[\hat{H}_M^{21} \left(-\frac{\hat{H}_M^{12}}{\hat{H}_M^{11}} \right) + \hat{H}_M^{22} \right] \frac{\partial \hat{e}(\hat{q}_C^{**}, \hat{\alpha}_C^{**})}{\partial q} = \frac{\det(\hat{H}_M)}{\hat{H}_M^{11}} \frac{\partial \hat{e}(\hat{q}_C^{**}, \hat{\alpha}_C^{**})}{\partial q} \leq 0 \text{ as } C_{qe}(q, e) \leq 0.
\end{aligned}$$

Thus, the owner must decrease the level of output from $\hat{q}_C^{**}$ to $q_M(\hat{\alpha}_C^{**}) \equiv q_M(\hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**}))$ and correspondingly the manager decreases or increases his level of effort from $\hat{e}_C^{**} \equiv \hat{e}(\hat{q}_C^{**}, \hat{\alpha}_C^{**})$ to $\hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})$ (since $\partial \hat{e}(q, \alpha)/\partial q \geq 0$ as $C_{qe}(q, e) \leq 0$) such that the first-order condition (2.24) with respect to e is satisfied.

Furthermore, the owner of monopoly firm must decrease or increase the incentive power α and correspondingly decreases the output to reach his expected utility maximum. To see this, Differentiating the expression (B.22),

(B.22)

$$-C_e(q_M(\hat{\alpha}_C^{**}), \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})) - \frac{\partial RC(q_M(\hat{\alpha}_C^{**}), \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**}))}{\partial e} - \psi'(\hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})),$$

with respect to α , we have

$$\begin{aligned}
& - \left[C_{eq}(q_M(\hat{\alpha}_C^{**}), \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})) + \frac{\partial^2 RC(q_M(\hat{\alpha}_C^{**}), \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**}))}{\partial e \partial q} \right] \frac{dq_M(\hat{\alpha}_C^{**})}{d\alpha} \\
& - [C_{ee}(q_M(\hat{\alpha}_C^{**}), \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})) + \psi''(\hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})) \\
& + \frac{\partial^2 RC(q_M(\hat{\alpha}_C^{**}), \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**}))}{\partial e^2}] \cdot \left[\frac{\partial \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})}{\partial q} \frac{dq_M(\hat{\alpha}_C^{**})}{d\alpha} + \frac{\partial \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})}{\partial \alpha} \right] \\
& = \left[\hat{H}_M^{21} \left(-\frac{\hat{H}_M^{12}}{\hat{H}_M^{11}} \right) + \hat{H}_M^{22} \right] \cdot \left[\frac{\partial \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})}{\partial q} \frac{dq_M(\hat{\alpha}_C^{**})}{d\alpha} + \frac{\partial \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})}{\partial \alpha} \right] \\
& = \frac{\det(\hat{H}_M)}{\hat{H}_M^{11}} \cdot \left[\frac{\partial \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})}{\partial q} \frac{dq_M(\hat{\alpha}_C^{**})}{d\alpha} + \frac{\partial \hat{e}(q_M(\hat{\alpha}_C^{**}), \hat{\alpha}_C^{**})}{\partial \alpha} \right] < 0,
\end{aligned}$$

So (B.22) is strictly decreasing in α .

In the case $C_{qe}(q, e) < 0$, (B.22) is negative. The owner of the monopoly firm must decrease the incentive power from $\hat{\alpha}_C^{**}$ to $\hat{\alpha}_M^{**}$ and correspondingly decreases the level of output from $q_M(\hat{\alpha}_C^{**})$ to $\hat{q}_M^{**} \equiv q_M(\hat{\alpha}_M^{**})$ (since $q_M(\alpha)/d\alpha > 0$) such that the first-order condition of the owner's problem for the monopoly firm,

$$-C_e(q_M(\hat{\alpha}_M^{**}), \hat{e}(q_M(\hat{\alpha}_M^{**}), \hat{\alpha}_M^{**})) - \frac{\partial RC(q_M(\hat{\alpha}_M^{**}), \hat{e}(q_M(\hat{\alpha}_M^{**}), \hat{\alpha}_M^{**}))}{\partial e} - \psi'(\hat{e}(q_M(\hat{\alpha}_M^{**}), \hat{\alpha}_M^{**})) = 0$$

is satisfied. Thus, $\hat{\alpha}_M^{**} < \hat{\alpha}_C^{**}$ and $\hat{q}_M^{**} < \hat{q}_C^{**}$ as $C_{qe}(q, e) < 0$.

Conversely, In the case $C_{qe}(q, e) > 0$, (B.22) is positive. The owner of the monopoly firm must increase the incentive power from $\hat{\alpha}_C^{**}$ to $\hat{\alpha}_M^{**}$ and correspondingly decreases the level of output from $q_M(\hat{\alpha}_C^{**})$ to $\hat{q}_M^{**} \equiv q_M(\hat{\alpha}_M^{**})$ (since $q_M(\alpha)/d\alpha < 0$). Thus, $\hat{\alpha}_M^{**} > \hat{\alpha}_C^{**}$ and $\hat{q}_M^{**} < \hat{q}_C^{**}$ as $C_{qe}(q, e) < 0$. $\square$

Proof of Proposition 2.10. To compare the levels of effort exerted by the managers, we rewrite the left-hand side of the first-order condition for the manager's expected utility maximization with respect to effort e ,

$$E[U'(e; \alpha, q, \varepsilon)] = [-\alpha C_e(q, e) - \psi'(e)]Eu' = 0.$$

Differentiating $E[U'(e; \alpha, q, \varepsilon)]$ with respect to α and q yields

$$\begin{aligned} \frac{\partial E[U'(e; \alpha, q, \varepsilon)]}{\partial \alpha} &= -C_e(q, e)Eu' > 0, \\ \frac{\partial E[U'(e; \alpha, q, \varepsilon)]}{\partial q} &= -\alpha C_{eq}(q, e)Eu' \geq 0 \text{ as } C_{qe}(q, e) \leq 0. \end{aligned}$$

Proposition 2.9 states that $\hat{\alpha}_M^{**} \leq \hat{\alpha}_C^{**}$ and $\hat{q}_M^{**} < \hat{q}_C^{**}$ as $C_{qe}(q, e) \leq 0$. So for any $e > 0$, $E[U'(e; \hat{\alpha}_M^{**}, \hat{q}_M^{**}, \varepsilon)] \leq E[U'(e; \hat{\alpha}_C^{**}, \hat{q}_C^{**}, \varepsilon)]$ as $C_{qe}(q, e) \leq 0$.

The optimal level of effort $\hat{e}_C^{**}$ chosen by the manager of the competitive firm satisfies the first-order condition:

$$E[U'(\hat{e}_C^{**}; \hat{\alpha}_C^{**}, \hat{q}_C^{**}, \varepsilon)] = [-\hat{\alpha}_C^{**} C_e(\hat{q}_C^{**}, \hat{e}_C^{**}) - \psi'(\hat{e}_C^{**})]Eu' = 0.$$

Hence, we obtain

$$E[U'(\hat{e}_C^{**}; \hat{\alpha}_M^{**}, \hat{q}_M^{**}, \varepsilon)] \leq E[U'(\hat{e}_C^{**}; \hat{\alpha}_C^{**}, \hat{q}_C^{**}, \varepsilon)] = 0 \text{ as } C_{qe}(q, e) \leq 0.$$

On the other hand, the optimal level of effort $\hat{e}_M^{**}$ chosen by the manager of the monopoly firm also satisfies the first-order condition:

$$E[U'(\hat{e}_M^{**}; \hat{\alpha}_M^{**}, \hat{q}_M^{**}, \varepsilon)] = [-\hat{\alpha}_M^{**} C_e(\hat{q}_M^{**}, \hat{e}_M^{**}) - \psi'(\hat{e}_M^{**})]Eu' = 0.$$

The second-order condition for the manager's expected utility maximization with respect to effort,

$$\frac{dE[U'(e; \alpha, q, \varepsilon)]}{de} = [-\alpha C_{ee}(q, e) - \psi''(e)]Eu' + [-\alpha C_e(q, e) - \psi'(e)]^2 Eu'' < 0,$$

is satisfied, implying that $E[U'(e; \alpha, q, \varepsilon)]$ is strictly decreasing in effort e . This thus establishes the result: $\hat{e}_M^{**} \leq \hat{e}_C^{**}$ as $C_{qe}(q, e) \leq 0$. $\square$

B.3. Proofs of Propositions in Section 2.3

Conducting the comparative statics on the first-order conditions (2.3) and (2.4) with respect to e yields $d\tilde{q}_i(e)/de \geq 0$ as $C_{qe}(q, e) \leq 0$, $d\alpha_i(e)/de > 0$. For any $e > 0$, since $\tilde{q}_M(e) < \tilde{q}_C(e)$, thus $\alpha_M(e) \geq \alpha_C(e)$ as $C_{qe}(q, e) \leq 0$. So we have

$$\begin{aligned} 0 &\leq \frac{d\tilde{q}_M(e)}{de} \leq \frac{d\tilde{q}_C(e)}{de} \text{ as } C_{qe}(q, e) \leq 0, \\ \frac{d\alpha_M(e)}{de} &> \frac{d\alpha_C(e)}{de} > 0 \text{ as } C_{qe}(q, e) < 0 \\ \text{or } \frac{d\alpha_C(e)}{de} &> \frac{d\alpha_M(e)}{de} > 0 \text{ as } C_{qe}(q, e) > 0. \end{aligned}$$

The owner's expected utility can be rewritten as a function of the level of effort e : $E[V_i(\tilde{q}_i(e), e, \varepsilon)] = E\pi(\tilde{q}_i(e), e, \varepsilon) - \psi(e) - RC(\sigma_\varepsilon^2, \alpha_i(e), \theta) - w_0$, $i \in \{M, C\}$. The first-order conditions with respect to e of the owner's problem is

$$(B.23) \quad \frac{\partial E\pi(\tilde{q}_i(e), e, \varepsilon)}{\partial q} \tilde{q}'_i(e) + \frac{\partial E\pi(\tilde{q}_i(e), e, \varepsilon)}{\partial e} - \psi'(e) - \frac{\partial RC(\sigma_\varepsilon^2, \alpha_i(e), \theta)}{\partial \alpha_i(e)} \alpha'_i(e) = 0.$$

Using the first-order condition (2.3) with respect to q of the manager's problem, we can simplify it as

$$(B.24) \quad -C_e(\tilde{q}_i(e), e) - \psi'(e) - \frac{\partial RC(\sigma_\varepsilon^2, \alpha_i(e), \boldsymbol{\theta})}{\partial \alpha_i(e)} \alpha'_i(e) = 0.$$

Since $d^2\alpha_i(e)/de^2 \geq 0$, the second-order condition,

$$(B.25) \quad -C_{qe}(\tilde{q}_i(e), e) \tilde{q}'_i(e) - C_{ee}(\tilde{q}_i(e), e) - \psi''(e) - \frac{\partial^2 RC(\sigma_\varepsilon^2, \alpha_i(e), \boldsymbol{\theta})}{\partial \alpha_i^2(e)} [\alpha'_i(e)]^2 - \frac{\partial RC(\sigma_\varepsilon^2, \alpha_i(e), \boldsymbol{\theta})}{\partial \alpha_i(e)} \alpha''_i(e) < 0$$

must be satisfied.

Proof of Proposition 2.11. (i) The first term $DWL_q(\tilde{q}_C(e_M^{**}), \tilde{q}_M(e_M^{**}), e_M^{**})$ is positive. So we only need to check whether the second term of (2.26), $-[RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \boldsymbol{\theta}) - RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \boldsymbol{\theta})]$, is positive.

For any $e > 0$, $\alpha_M(e) \geq \alpha_C(e)$ as $C_{qe}(q, e) \leq 0$. By assumption, $RC(\sigma_\varepsilon^2, \cdot, \boldsymbol{\theta})$ is strictly increasing and strictly convex in α , so in the case $C_{qe}(q, e) < 0$,

$$RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \boldsymbol{\theta}) < RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \boldsymbol{\theta}).$$

Thus, the second term of (2.26) is thus positive.

Conversely, in the case $C_{qe}(q, e) > 0$, $RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \boldsymbol{\theta}) > RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \boldsymbol{\theta})$. The second term of (2.26) is thus negative. However, by Lemma 2.2, $\alpha_i(e_M^{**}) \in (0, 1)$, $i \in \{M, C\}$, so the effect of the incentive power α on $RC(\sigma_\varepsilon^2, \alpha, \boldsymbol{\theta})$ is trivial: the second term of (2.26) is less negative such that it must be outweighed by the positive

first term $DWL_q(\tilde{q}_C(e_M^{**}), \tilde{q}_M(e_M^{**}), e_M^{**})$. Hence, we have

$$\begin{aligned} & E[NSW_2(q_C(e_M^{**}), e_M^{**}; \varepsilon)] - E[NSW_2(q_M^{**}, e_M^{**}; \varepsilon)] \\ &= DWL_q(\tilde{q}_C(e_M^{**}), \tilde{q}_M(e_M^{**}), e_M^{**}) - [RC(\sigma_\varepsilon^2, \alpha_C(e_M^{**}), \theta) - RC(\sigma_\varepsilon^2, \alpha_M(e_M^{**}), \theta)] > 0. \end{aligned}$$

(ii) Differentiating the expected net social welfare function (2.25) of the competitive firm with respect to effort e yields

$$\begin{aligned} \frac{dE[NSW_2(\tilde{q}_C(e), e; \varepsilon)]}{de} &= [p(\tilde{q}_C(e)) - C_q(\tilde{q}_C(e), e)]\tilde{q}'_C(e) \\ &\quad - C_e(\tilde{q}_C(e), e) - \psi'(e) - \frac{\partial RC(\sigma_\varepsilon^2, \alpha_C(e), \theta)}{\partial \alpha_C} \alpha'_C(e) \\ &= -C_e(\tilde{q}_C(e), e) - \psi'(e) - \frac{\partial RC(\sigma_\varepsilon^2, \alpha_C(e), \theta)}{\partial \alpha_C} \alpha'_C(e). \end{aligned}$$

It follows from the owner's first-order condition (B.24) with respect to effort e of the competitive firm that

$$\left. \frac{dE[NSW_2(\tilde{q}_C(e), e; \varepsilon)]}{de} \right|_{e=e_C^{**}} = 0.$$

It follows from the owner's second-order condition (B.25) with respect to e of the competitive firm that the second-order derivative of $E[NSW_2(\tilde{q}_C(e), e; \varepsilon)]$ with respect to e is

$$\begin{aligned} \frac{d^2 E[NSW_2(\tilde{q}_C(e), e; \varepsilon)]}{de^2} &= -C_{qe}(\tilde{q}_C(e), e)\tilde{q}'_C(e) - C_{ee}(\tilde{q}_C(e), e) - \psi''(e) \\ &\quad - \frac{\partial^2 RC(\sigma_\varepsilon^2, \alpha_C(e), \theta)}{\partial \alpha_C^2} [\alpha'_C(e)]^2 - \frac{\partial RC(\sigma_\varepsilon^2, \alpha_C(e), \theta)}{\partial \alpha_C} \alpha''_C(e) < 0, \end{aligned}$$

which implies that $E[NSW_2(\tilde{q}_C(e), \cdot; \varepsilon)]$ is a strictly concave function of effort e . Hence, e_C^{**} maximizes the expected net social welfare function $E[NSW_2(\tilde{q}_C(e), e; \varepsilon)]$ of the competitive firm. The concavity of $E[NSW_2(\tilde{q}_C(e), \cdot; \varepsilon)]$ in e and $e_M^{**} \leq e_C^{**}$ by Proposition 2.4 imply that

$$\begin{aligned} & E[NSW_2(q_C^{**}, e_C^{**}; \varepsilon)] - E[NSW_2(\tilde{q}_C(e_M^{**}), e_M^{**}; \varepsilon)] \\ &= DWL_e(\tilde{q}_C(e_C^{**}), \tilde{q}_C(e_M^{**}), e_C^{**}, e_M^{**}) - \Delta\psi(e_C^{**}, e_M^{**}) - \Delta RC_C > 0. \end{aligned}$$

□

Proof of Proposition 2.12. We only need to prove that the incentive cost is positive in parts (i) and (ii). (i) The two first-order conditions with respect to output for the competitive firm in both the case of the entrepreneurial firm and the case of the managerial firm are same for any give e , so $q_C(e) = \tilde{q}_C(e)$. Using the expected net social welfare function (1.6) for the competitive firm in the case of the entrepreneurial firm, we can rewrite the incentive cost of the competitive managerial firm as

$$\begin{aligned} & DWL_e(q_C(e_C^*), \tilde{q}_C(e_C^{**}), e_C^*, e_C^{**}) - \Delta\psi(e_C^*, e_C^{**}) \\ &= E[NSW_1(q_C(e_C^*), e_C^*; \varepsilon)] - E[NSW_1(\tilde{q}_C(e_C^{**}), e_C^{**}; \varepsilon)] \\ &= E[NSW_1(q_C(e_C^*), e_C^*; \varepsilon)] - E[NSW_1(q_C(e_C^{**}), e_C^{**}; \varepsilon)]. \end{aligned}$$

Differentiating the expected net social welfare function (1.6) of the competitive firm in the case of the entrepreneurial firm with respect to effort e yields

$$\begin{aligned}\frac{dE[NSW_1(q_C(e), e; \varepsilon)]}{de} &= [p(q_C(e)) - C_q(q_C(e), e)]q'_C(e) - C_e(q_C(e), e) - \psi'(e) \\ &= -C_e(q_C(e), e) - \psi'(e).\end{aligned}$$

It follows from the the owner-manager's first-order condition (1.5) with respect to effort of the competitive firm that

$$\left. \frac{dE[NSW_1(q_C(e), e; \varepsilon)]}{de} \right|_{e=e_C^*} = 0.$$

The second-order derivative of $E[NSW_1(q_C(e), e; \varepsilon)]$ with respect to e is

$$\begin{aligned}\frac{d^2 E[NSW_1(q_C(e), e; \varepsilon)]}{de^2} &= -[C_{qe}(q_C(e), e)]q'_C(e) - C_{ee}(q_C(e), e) - \psi''(e) \\ &= H_C^{21} \cdot \left[-\frac{H_C^{12}}{H_C^{11}} \right] + H_C^{22} = \frac{\det(H_C)}{H_C^{11}} < 0,\end{aligned}$$

which implies that $E[NSW_1(q_C(e), \cdot; \varepsilon)]$ is a strictly concave function of effort e . Hence, e_C^* maximizes the expected net social welfare function $E[NSW_1(q_C(e), e; \varepsilon)]$ of the competitive firm. The concavity of $E[NSW_1(q_C(e), \cdot; \varepsilon)]$ in e and $e_C^{**} < e_C^*$ by Proposition 2.1 imply that

$$\begin{aligned}& E[NSW_1(q_C^*, e_C^*; \varepsilon)] - E[NSW_1(q_C^{**}, e_C^{**}; \varepsilon)] \\ &= DWL_e(q_C(e_C^*), \tilde{q}_C(e_C^{**}), e_C^*, e_C^{**}) - \Delta\psi(e_C^*, e_C^{**}) > 0.\end{aligned}$$

(ii) The two first-order conditions with respect to output for the monopoly firm in both the case of the entrepreneurial firm and the case of the managerial firm are the same for any given e , so $q_M(e) = \tilde{q}_M(e)$. Using the expected net social welfare function (1.6) for the monopoly firm in the case of the entrepreneurial firm, we can rewrite the incentive cost of the monopoly managerial firm as

$$\begin{aligned}
 & DWL_e(q_M(e_M^*), \tilde{q}_M(e_M^{**}), e_M^*, e_M^{**}) - \Delta\psi(e_M^*, e_M^{**}) \\
 &= E[NSW_1(q_M(e_M^*), e_M^*; \varepsilon)] - E[NSW_1(\tilde{q}_M(e_M^{**}), e_M^{**}; \varepsilon)] \\
 &= E[NSW_1(q_M(e_M^*), e_M^*; \varepsilon)] - E[NSW_1(q_M(e_M^{**}), e_M^{**}; \varepsilon)].
 \end{aligned}$$

Differentiating the expected net social welfare function (1.6) of the monopoly firm in the case of the entrepreneurial firm with respect to e , we obtain the first-order net welfare effect of the effort e :

$$(B.26) \quad \frac{dE[NSW_1(q_M(e), e; \varepsilon)]}{de} = -p'(q_M(e))q_M(e)q'_M(e) - C_e(q_M(e), e) - \psi'(e).$$

Set the initial level of effort $e = e_M^{**}$, the first-order net welfare effect (B.26) of an infinitesimal change in e is given by

$$\left. \frac{dE[NSW_1(q_M(e), e; \varepsilon)]}{de} \right|_{e=e_M^{**}} = -p'(\tilde{q}_M(e_M^{**}))\tilde{q}_M(e_M^{**})\tilde{q}'_M(e_M^{**}) - C_e(\tilde{q}_M(e_M^{**}), e_M^{**}) - \psi'(e_M^{**}) > 0,$$

it follows from the owner's first-order condition (B.24) with respect to e of the monopoly firm. Starting from e_M^{**} , the expected net welfare increases with the increase in e . When the level of effort increases from e_M^{**} to e_M^* , the level of output increases

or decreases from $q_M^{**} \equiv \tilde{q}_M(e_M^{**})$ to $q_M^* \equiv q_M(e_M^*)$ as $C_{qe}(q, e) \leq 0$, the first-order net welfare effect (B.26) of an infinitesimal change in e at the efficient level of effort e_M^* is given by

$$\left. \frac{dE[NSW_1(q_M(e), e; \varepsilon)]}{de} \right|_{e=e_M^*} = -p'(q_M(e_M^*))q_M(e_M^*)q'_M(e_M^*) - C_e(q_M(e_M^*), e_M^*) - \psi'(e_M^*) > 0,$$

which implies that the expected net social welfare does not reach the maximum at e_M^* . Hence, we obtain

$$\begin{aligned} & E[NSW_1(q_M^*, e_M^*; \varepsilon)] - E[NSW_1(q_M^{**}, e_M^{**}; \varepsilon)] \\ &= DWL_e(q_M(e_M^*), \tilde{q}_M(e_M^{**}), e_M^*, e_M^{**}) - \Delta\psi(e_M^*, e_M^{**}) > 0. \end{aligned}$$

□

APPENDIX C

Proofs of Lemmas and Propositions in Chapter 3

Proof of Lemma 3.1. Let $e(\alpha, \beta, \gamma)$ denote the agent's effort supply function derived from his expected utility maximization problem, given the variable-prize tournament contract $\{\alpha, \beta, \gamma\}$. Let λ be the Lagrange multiplier associated with the zero expected profit constraint. The first-order conditions (FOCs) for the Lagrangian of the principal's problem are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \alpha} &= \frac{1}{2} E_\varepsilon \{ E_\eta [u'(1) \cdot (e + \varepsilon_i + \eta)] | \varepsilon_i > \varepsilon_l \} \\ &\quad + \lambda \left\{ -\frac{1}{2} [e + E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l)] + \left(V - \frac{1}{2} \alpha \right) \frac{\partial e(\alpha, \beta, \gamma)}{\partial \alpha} \right\} = 0, \\ \frac{\partial \mathcal{L}}{\partial \beta} &= \frac{1}{2} E_\varepsilon \{ E_\eta [u'(1)] | \varepsilon_i > \varepsilon_l \} + \lambda \left\{ -\frac{1}{2} + \left(V - \frac{1}{2} \alpha \right) \frac{\partial e(\alpha, \beta, \gamma)}{\partial \beta} \right\} = 0, \\ \frac{\partial \mathcal{L}}{\partial \gamma} &= \frac{1}{2} u'(2) + \lambda \left\{ -\frac{1}{2} + \left(V - \frac{1}{2} \alpha \right) \frac{\partial e(\alpha, \beta, \gamma)}{\partial \gamma} \right\} = 0. \end{aligned}$$

To find a possible range of the optimal piece rate under variable-prize tournaments, we apply the method used by Stiglitz (1975, footnote 13) to our model.

From the first two FOCs, we obtain

$$\begin{aligned} \frac{\partial e}{\partial \beta} &= \frac{1}{2(V - \alpha/2)} + \frac{E_\varepsilon \{ E_\eta [u'(1)] | \varepsilon_i > \varepsilon_l \}}{(V - \alpha/2) E_\varepsilon \{ E_\eta [u'(1) \cdot (e + \varepsilon_i + \eta)] | \varepsilon_i > \varepsilon_l \}} \\ (C.1) \quad &\times \left\{ -\frac{1}{2} [e + E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l)] + (V - \alpha/2) \frac{\partial e}{\partial \alpha} \right\}. \end{aligned}$$

Setting the expected utility equal to constant level $\bar{U}$, we totally differentiate both sides of the expression with respect to α and β ,

$$dEU_{VT} = \frac{1}{2}E_\varepsilon\{E_\eta[u'(1) \cdot (e + \varepsilon_i + \eta)]|\varepsilon_i > \varepsilon_l\}d\alpha + \frac{1}{2}E_\varepsilon\{E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\}d\beta = 0.$$

So

$$(C.2) \quad \left. \frac{\partial \beta}{\partial \alpha} \right|_{\bar{U}} = - \frac{E_\varepsilon\{E_\eta[u'(1) \cdot (e + \varepsilon_i + \eta)]|\varepsilon_i > \varepsilon_l\}}{E_\varepsilon\{E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\}}.$$

Since

$$(C.3) \quad \frac{\partial e}{\partial \alpha} = \left. \frac{\partial e}{\partial \alpha} \right|_{\bar{U}} - \frac{\partial e}{\partial \beta} \left. \frac{\partial \beta}{\partial \alpha} \right|_{\bar{U}},$$

substituting (C.1) and (C.2) into (C.3) and rearranging, we have

$$(C.4) \quad -2(V - \alpha/2) \left. \frac{\partial e}{\partial \alpha} \right|_{\bar{U}} = \frac{E_\varepsilon\{E_\eta[u'(1) \cdot (e + \varepsilon_i + \eta)]|\varepsilon_i > \varepsilon_l\}}{E_\varepsilon\{E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\}} - [e + E_\varepsilon(\varepsilon_i|\varepsilon_i > \varepsilon_l)].$$

Let Cov_ε and Cov_η be the covariance operators with respect to the distributions of ε_i and η respectively. Expanding the first term on the right-hand side of (C.4) yields

$$\begin{aligned} & \frac{E_\varepsilon\{E_\eta[u'(1) \cdot (e + \varepsilon_i + \eta)]|\varepsilon_i > \varepsilon_l\}}{E_\varepsilon\{E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\}} \\ &= e + \frac{E_\varepsilon\{\varepsilon_i E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\} + E_\varepsilon\{E_\eta[\eta U'(1)]|\varepsilon_i > \varepsilon_l\}}{E_\varepsilon\{E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\}} \\ &= [e + E_\varepsilon(\varepsilon_i|\varepsilon_i > \varepsilon_l)] + \frac{\text{Cov}_\varepsilon\{\varepsilon_i, E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\} + \text{Cov}_\eta\{\eta, E_\varepsilon[u'(1)|\varepsilon_i > \varepsilon_l]\}}{E_\varepsilon\{E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\}}. \end{aligned}$$

Substituting it into (C.4), we have

$$(C.5) \quad e \left(V - \frac{\alpha}{2} \right) \xi_{e,\alpha} = \frac{\alpha}{2} \left\{ - \frac{\text{Cov}_\varepsilon\{\varepsilon_i, E_\eta[u'(1)]|\varepsilon_i > \varepsilon_l\} + \text{Cov}_\eta\{\eta, E_\varepsilon[u'(1)|\varepsilon_i > \varepsilon_l]\}}{E_\eta\{E_\varepsilon[u'(1)|\varepsilon_i > \varepsilon_l]\}} \right\},$$

where $\xi_{e,\alpha} \equiv \frac{\partial \ln e}{\partial \ln \alpha} \Big|_{\bar{U}}$ is the compensated supply *elasticity* of effort with respect to the piece rate α .

If $\alpha \leq 0$, then the left-hand side of (C.5) is positive but the right-hand side is non-positive because both covariances in the numerator are non-negative. If $\alpha \geq 2V$, then the left-hand side of (C.5) is non-positive but the right-hand side is positive because both covariances in the numerator are negative.

Hence, the optimal piece rate $\alpha_{VT}^* \in (0, 2V)$. $\square$

Proof of Proposition 3.1. Since $EU_{VT}(e, \alpha)$ is strictly concave, there exists a solution $\{e_{VT}^*, \alpha_{VT}^*\}$ which satisfies the first-order conditions such that $EU_{VT}(e_{VT}^*, \alpha_{VT}^*) > EU_{VT}(e, \alpha)$ for every $\{e, \alpha\} \in \mathbb{R}^2$. This establishes the result. $\square$

Proof of Lemma 3.2. We denote the derivatives of $\chi(r, \alpha)$ and $\zeta(r, \alpha)$ with respect to r and α by subscripts. Setting the initial piece rate $\alpha = 0$, we obtain that at $\alpha = 0$,

$$\chi(r, \alpha)|_{\alpha=0} \equiv \chi(r, 0) = 0,$$

$$\chi_\alpha(r, \alpha)|_{\alpha=0} \equiv \chi_\alpha(r, 0) = E_\varepsilon(\varepsilon_i|\varepsilon_i > \varepsilon_l)E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - \frac{1}{2} < 0,$$

$$\zeta(r, \alpha)|_{\alpha=0} \equiv \zeta(r, 0) = E_\varepsilon[f(\varepsilon_i)] > 0, \quad \zeta_\alpha(r, \alpha)|_{\alpha=0} \equiv \zeta_\alpha(r, 0) = \frac{\tau}{2}\chi_\alpha(r, 0) < 0;$$

$$\begin{aligned}\chi_r(r, \alpha)|_{\alpha=0} &\equiv \chi_r(r, 0) = 0, \quad \zeta_r(r, \alpha)|_{\alpha=0} \equiv \zeta_r(r, 0) = 0, \quad \chi_{r\alpha}(r, \alpha)|_{\alpha=0} \equiv \chi_{r\alpha}(r, 0) = 0, \\ \zeta_{r\alpha}(r, \alpha)|_{\alpha=0} &\equiv \zeta_{r\alpha}(r, 0) = E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - \frac{1}{2} = \chi_\alpha(r, 0) < 0.\end{aligned}$$

Define

$$\begin{aligned}\rho(e, r, \alpha) &\equiv \frac{\psi'(e) + \chi(r, \alpha)}{[\zeta(r, \alpha)]^2}, \\ \mathbf{v}(e, r, \alpha) &\equiv \psi''(e) + r'(Ey)[V - \psi'(e)] \left[\chi_r(r, \alpha) - \left(\frac{\psi'(e) + \chi(r, \alpha)}{\zeta(r, \alpha)} \right) \zeta_r(r, \alpha) \right].\end{aligned}$$

Thus, the expected marginal utility of effort can be rewritten as

$$\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \approx u'(Ey) \left[V - \psi'(e) - \frac{r}{4} \rho(e, r, \alpha) \cdot \mathbf{v}(e, r, \alpha) \right].$$

At $\alpha = 0$,

$$\begin{aligned}\rho(e, r, 0) &= \frac{\psi'(e) + \chi(r, 0)}{[\zeta(r, 0)]^2} = \frac{\psi'(e)}{[\zeta(r, 0)]^2} > 0, \\ \mathbf{v}(e, r, 0) &= \psi''(e) + r'(Ey)[V - \psi'(e)] \left[\chi_r(r, 0) - \left(\frac{\psi'(e) + \chi(r, 0)}{\zeta(r, 0)} \right) \zeta_r(r, 0) \right] \\ &= \psi''(e) > 0.\end{aligned}$$

Then we have

$$\begin{aligned}\frac{\partial EU_{VT}(e, 0)}{\partial e} &\approx u'(Ey) \left[V - \psi'(e) - \frac{r}{4} \rho(e, r, 0) \cdot \mathbf{v}(e, r, 0) \right] \\ &= u'(Ey) \left[V - \psi'(e) - \frac{r}{4} \frac{\psi'(e)}{[\zeta(r, 0)]^2} \psi''(e) \right].\end{aligned}$$

Note that the first-order condition with respect to effort at $\alpha = 0$ reduces to the first-order condition with respect to effort under fixed-prize tournaments,

$$\left. \frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right|_{\alpha=0} = 0 \implies u'(Ey) \left[V - \psi'(e) - \frac{r}{4} \frac{\psi'(e)}{[\zeta(r, 0)]^2} \psi''(e) \right] \approx 0.$$

Thus, the optimal effort level e_{FT}^* satisfies $\psi'(e_{FT}^*) \approx \frac{V}{1 + \frac{r_{FT}^* \psi''(e_{FT}^*)}{4[\zeta(r_{FT}^*, 0)]^2}}$, where $r_{FT}^* \equiv r(Ey_{FT}^*)$ and $Ey_{FT}^* = Ve_{FT}^* - \psi(e_{FT}^*)$ is the agent's expected net income at e_{FT}^* .

Differentiating the expected marginal utility of effort with respect to α and evaluating it at $\alpha = 0$, we get

$$\begin{aligned} \left. \frac{\partial}{\partial \alpha} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \right|_{\alpha=0} &\approx -\frac{r_{FT}^*}{4} u'(Ey_{FT}^*) \left[\rho(e_{FT}^*, r_{FT}^*, 0) \left. \frac{\partial v(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} \right. \\ &\quad \left. + v(e_{FT}^*, r_{FT}^*, 0) \left. \frac{\partial \rho(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} \right]. \end{aligned}$$

Differentiating $v(e, r, \alpha)$ with respect to α and evaluating it at $\alpha = 0$, we get

$$\begin{aligned} \left. \frac{\partial v(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} &= r'(Ey_{FT}^*) [V - \psi'(e_{FT}^*)] \left[\chi_{r\alpha}(r_{FT}^*, 0) - \left. \frac{\partial}{\partial \alpha} \left(\frac{\psi'(e) + \chi(r, \alpha)}{\zeta(r, \alpha)} \right) \right|_{\alpha=0} \zeta_r(r_{FT}^*, 0) \right. \\ &\quad \left. - \left(\frac{\psi'(e_{FT}^*) + \chi(r_{FT}^*, 0)}{\zeta(r_{FT}^*, 0)} \right) \zeta_{r\alpha}(r_{FT}^*, 0) \right] \\ &= r'(Ey) [V - \psi'(e_{FT}^*)] \left[-\frac{\psi'(e_{FT}^*)}{\zeta(r_{FT}^*, 0)} \chi_\alpha(r_{FT}^*, 0) \right] \leq 0. \end{aligned}$$

The last inequality follows from Condition 2.

Differentiating $\rho(e, r, \alpha)$ with respect to α and evaluating it at $\alpha = 0$, we get

$$\begin{aligned} \left. \frac{\partial \rho(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} &= \frac{\chi_\alpha(r_{\text{FT}}^*, 0) \zeta(r_{\text{FT}}^*, 0) - 2\zeta_\alpha(r_{\text{FT}}^*, 0) [\psi'(e_{\text{FT}}^*) + \chi(r_{\text{FT}}^*, 0)]}{[\zeta(r_{\text{FT}}^*, 0)]^3} \\ &= \frac{\chi_\alpha(r_{\text{FT}}^*, 0) \zeta(r_{\text{FT}}^*, 0) - r_{\text{FT}}^* \chi_\alpha(r_{\text{FT}}^*, 0) \psi'(e_{\text{FT}}^*)}{[\zeta(r_{\text{FT}}^*, 0)]^3} \\ &= \frac{\chi_\alpha(r_{\text{FT}}^*, 0)}{[\zeta(r_{\text{FT}}^*, 0)]^3} \cdot [\zeta(r_{\text{FT}}^*, 0) - r_{\text{FT}}^* \psi'(e_{\text{FT}}^*)]. \end{aligned}$$

$$\text{So sign} \left\{ -\left. \frac{\partial \rho(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} \right\} = \text{sign} \{ \zeta(r_{\text{FT}}^*, 0) - r_{\text{FT}}^* \psi'(e_{\text{FT}}^*) \}.$$

$$\begin{aligned} &\zeta(r_{\text{FT}}^*, 0) - r_{\text{FT}}^* \psi'(e_{\text{FT}}^*) \\ &= \zeta(r_{\text{FT}}^*, 0) - r_{\text{FT}}^* \frac{V}{1 + \frac{r_{\text{FT}}^* \psi''(e_{\text{FT}}^*)}{4[\zeta(r_{\text{FT}}^*, 0)]^2}} \\ &= \frac{\zeta(r_{\text{FT}}^*, 0)}{4[\zeta(r_{\text{FT}}^*, 0)]^2 + r_{\text{FT}}^* \psi''(e_{\text{FT}}^*)} \left\{ 4 \left[\zeta(r_{\text{FT}}^*, 0) - \frac{r_{\text{FT}}^* V}{2} \right]^2 + (r_{\text{FT}}^*)^2 \left[\frac{\psi''(e_{\text{FT}}^*)}{r_{\text{FT}}^*} - V^2 \right] \right\} > 0. \end{aligned}$$

The last inequality directly follows from Condition 1. Then

$$\left. \frac{\partial \rho(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} < 0.$$

Therefore, under Conditions 1 and 2 the expected marginal utility of effort is increasing in α as α is increased from 0,

$$\begin{aligned} \left. \frac{\partial}{\partial \alpha} \left(\frac{\partial EU_{\text{VT}}(e, \alpha)}{\partial e} \right) \right|_{\alpha=0} &\approx -\frac{r_{\text{FT}}^*}{4} u'(Ey_{\text{FT}}^*) \left[\rho(e_{\text{FT}}^*, r_{\text{FT}}^*, 0) \left. \frac{\partial \mathbf{v}(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} \right. \\ &\quad \left. + \mathbf{v}(e_{\text{FT}}^*, r_{\text{FT}}^*, 0) \left. \frac{\partial \rho(e, r, \alpha)}{\partial \alpha} \right|_{\alpha=0} \right] > 0. \end{aligned}$$

The strict concavity of $EU_{VT}(e, \alpha)$ implies that $\partial EU_{VT}(e, \alpha)/\partial e$ is monotonic in α . So $\partial EU_{VT}(e, \alpha)/\partial e$ is monotonically increasing in α for $\alpha > 0$. $\square$

Proof of Proposition 3.2. The proof of this proposition makes use of Lemma 3.2. Let $e(\alpha)$ be the optimal effort level chosen by the agent, which is derived from the first-order condition with respect to effort.

Applying the implicit function theorem to the first-order condition $\partial EU_{VT}(e, \alpha)/\partial e = 0$, we obtain the change in α with respect to a change in e ,

$$\frac{de(\alpha)}{d\alpha} = -\frac{\partial^2 EU_{VT}(e, \alpha)/\partial e \partial \alpha}{\partial^2 EU_{VT}(e, \alpha)/\partial e^2}.$$

As the denominator of this expression is negative due to the second-order conditions for maximization, we have $\text{sign}\{de(\alpha)/d\alpha\} = \text{sign}\{\partial^2 EU_{VT}(e, \alpha)/\partial e \partial \alpha\}$. By Lemma 3.2, $\left.\frac{de(\alpha)}{d\alpha}\right|_{\alpha=0} > 0$ and $e(\alpha)$ is monotonically increasing in α for $\alpha > 0$. Hence, we have

$$e_{VT}^* \equiv e(\alpha_{VT}^*) > e(0) \equiv e_{FT}^*.$$

$\square$

Proof of Lemma 3.3. We obtain that at $r = 0$,

$$\begin{aligned} \chi(r, \alpha)|_{r=0} &\equiv \chi(0, \alpha) = \alpha \{E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - \frac{1}{2}\} = \alpha \chi_\alpha(0, \alpha) < 0, \\ \chi_r(r, \alpha)|_{r=0} &\equiv \chi_r(0, \alpha) = \frac{\alpha^2}{2} \{-E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l)(2E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - E_\varepsilon(\varepsilon_i | \varepsilon_i > \varepsilon_l) E_\varepsilon[f(\varepsilon_i)]) \\ &\quad + E_\varepsilon[\varepsilon_i^2 f(\varepsilon_i)] + E_\varepsilon[f(\varepsilon_i)] \sigma_\eta^2\} > 0, \end{aligned}$$

$$\zeta(r, \alpha)|_{r=0} \equiv \zeta(0, \alpha) = E_\varepsilon[f(\varepsilon_i)] > 0,$$

$$\zeta_r(r, \alpha)|_{r=0} \equiv \zeta_r(0, \alpha) = \frac{\alpha}{2}\{E_\varepsilon(\varepsilon_i|\varepsilon_i > \varepsilon_l)E_\varepsilon[f(\varepsilon_i)] - E_\varepsilon[\varepsilon_i f(\varepsilon_i)] - \frac{1}{2}\} = \frac{\alpha}{2}\chi_\alpha(0, \alpha) < 0.$$

At $r = 0$, the agent's expected marginal utility of effort $\partial EU_{VT}(e, \alpha)/\partial e$ under variable-prize tournaments is

$$\begin{aligned} \left. \frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right|_{r=0} &\approx u'(Ey) \left[V - \psi'(e) - \frac{0}{4}\rho(e, r, \alpha) \cdot v(e, r, \alpha) \right] \\ &= u'(Ey)[V - \psi'(e)]. \end{aligned}$$

It follows from the FOC with respect to effort $\left. \frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right|_{r=0} = 0$ that $V - \psi'(e^{FB}) = 0$, where e^{FB} is the *first-best* effort level under complete information.

$$\begin{aligned} \rho(e, 0, \alpha) &\equiv \frac{\psi'(e) + \chi(0, \alpha)}{[\zeta(0, \alpha)]^2} = \frac{V + \alpha\chi_\alpha(0, \alpha)}{[\zeta(0, \alpha)]^2}, \\ v(e, 0, \alpha) &\equiv \psi''(e) + r'(Ey)[V - \psi'(e)] \left[\chi_r(0, \alpha) - \left(\frac{\psi'(e) + \chi(0, \alpha)}{\zeta(0, \alpha)} \right) \zeta_r(0, \alpha) \right] \\ &= \psi''(e). \end{aligned}$$

Differentiating the expected marginal utility of effort with respect to r and evaluating it at $r = 0$, we get

$$\left. \frac{d}{dr} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \right|_{r=0} = \left. \frac{\partial}{\partial r} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \right|_{r=0} + \left. \frac{\partial}{\partial \alpha} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \right|_{r=0} \cdot \left. \frac{d\alpha(r)}{dr} \right|_{r=0}$$

At the optimum, the piece rate under variable-prize tournaments is optimally chosen at $\alpha \neq V$. Hence, the effect of a change in r on α is zero, i.e., $\left. \frac{d\alpha(r)}{dr} \right|_{r=0} = 0$. Thus,

we have

$$\begin{aligned}
\left. \frac{d}{dr} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \right|_{r=0} &\approx \left. \frac{\partial}{\partial r} \left(\frac{\partial EU_{VT}(e, \alpha)}{\partial e} \right) \right|_{r=0} \\
&= -u'(Ey) \cdot \frac{1}{4} \rho(e, 0, \alpha) \cdot v(e, 0, \alpha) \\
&= -u'(Ey) \cdot \frac{1}{4} \frac{V + \alpha \chi_\alpha(0, \alpha)}{[\zeta(0, \alpha)]^2} \psi''(e) \\
&= -u'(Ey) V \psi''(e) \frac{1 + (\alpha/V) \chi_\alpha(0, \alpha)}{4 [\zeta(0, \alpha)]^2} < 0.
\end{aligned}$$

Under linear piece rates, the expected marginal utility of effort is given by

$$\frac{\partial EU_{PR}(e)}{\partial e} \approx u'(Ey) \{V - \psi'(e)[1 + r\psi''(e)(\sigma_\varepsilon^2 + \sigma_\eta^2)]\}.$$

So

$$\left. \frac{\partial EU_{PR}(e)}{\partial e} \right|_{r=0} = 0 \implies V - \psi'(e^{FB}) = 0.$$

$$\begin{aligned}
\left. \frac{d}{dr} \left(\frac{\partial EU_{PR}(e)}{\partial e} \right) \right|_{r=0} &\approx -u'(Ey) \psi'(e) \psi''(e) (\sigma_\varepsilon^2 + \sigma_\eta^2) \\
&= -u'(Ey) V C'''(e) (\sigma_\varepsilon^2 + \sigma_\eta^2) < 0.
\end{aligned}$$

It follows from Condition 3 that, if $\sigma_\varepsilon^2 + \sigma_\eta^2 > \frac{1}{4[\zeta(0, V)]^2}$,

$$\text{then } 0 > \left. \frac{d}{dr} \left(\frac{\partial EU_{VT}(e, V)}{\partial e} \right) \right|_{r=0} > \left. \frac{d}{dr} \left(\frac{\partial EU_{VT}(e, 0)}{\partial e} \right) \right|_{r=0} > \left. \frac{d}{dr} \left(\frac{\partial EU_{PR}(e)}{\partial e} \right) \right|_{r=0}.$$

From Condition 4, if $\frac{1 + \chi_\alpha(0, V)}{4[\zeta(0, V)]^2} < \sigma_\varepsilon^2 + \sigma_\eta^2 < \frac{1}{4[\zeta(0, V)]^2}$,

$$\text{then } 0 > \left. \frac{d}{dr} \left(\frac{\partial EU_{VT}(e, V)}{\partial e} \right) \right|_{r=0} > \left. \frac{d}{dr} \left(\frac{\partial EU_{PR}(e)}{\partial e} \right) \right|_{r=0} > \left. \frac{d}{dr} \left(\frac{\partial EU_{VT}(e, 0)}{\partial e} \right) \right|_{r=0}.$$

□

Proof of Proposition 3.3. By Lemma 3.3, under Condition 3, we obtain

$$\frac{\partial EU_{VT}(e, \alpha)}{\partial e} > \frac{\partial EU_{VT}(e, 0)}{\partial e} > \frac{dEU_{PR}(e)}{de} \geq 0 \quad \text{for } e > 0,$$

that is, for a given $\alpha > 0$, the expected utility curve $EU_{VT}(e, \alpha)$ has *greater* positive slope than curves $EU_{VT}(e, 0)$ and $EU_{PR}(e)$.

The optimal effort under linear piece rates, e_{PR}^* , satisfies the first-order condition with respect to effort: $dEU_{PR}(e_{PR}^*)/de = 0$. So $\partial EU_{VT}(e_{PR}^*, 0)/\partial e > 0$. The optimal effort under fixed-prize tournaments, e_{FT}^* , also satisfies the first-order condition with respect to effort: $\partial EU_{VT}(e_{FT}^*, 0)/\partial e = 0$.

The concavity of $EU_{VT}(\cdot, 0)$ in e implies that $e_{FT}^* > e_{PR}^*$. By similar reasoning, we can obtain $e_{VT}^* > e_{FT}^*$. Thus, we have $e_{VT}^* > e_{FT}^* > e_{PR}^*$.

In addition, for $e = 0$, $EU_{VT}(0, \alpha) = EU_{VT}(0, 0) = EU_{PR}(0) = 0$. Moreover, for all $e > 0$, $\frac{\partial EU_{VT}(e, \alpha)}{\partial e} > \frac{\partial EU_{VT}(e, 0)}{\partial e} > \frac{dEU_{PR}(e)}{de}$. It follows that

$$EU_{VT}(e_{VT}^*, \alpha_{VT}^*) > EU_{VT}(e_{FT}^*, 0) \equiv EU_{FT}(e_{FT}^*) > EU_{PR}(e_{PR}^*).$$

By similar reasoning, Lemma 3.3 under Condition 4 leads to $e_{VT}^* > e_{PR}^* > e_{FT}^*$ and $EU_{VT}(e_{VT}^*, \alpha_{VT}^*) > EU_{PR}(e_{PR}^*) > EU_{VT}(e_{FT}^*, 0) \equiv EU_{FT}(e_{FT}^*)$. □