BUS REAL-TIME ARRIVAL PREDICTION USING STATISTICAL PATTERN RECOGNITION TECHNIQUE

By

Nam Hoai Vu, M.Sc., (2000)
Hanoi University of Civil Engineering, Vietnam

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Department of Civil and Environmental Engineering
Carleton University
Ottawa, Ontario, Canada

© **December 2006**

Nam Hoai Vu

The Doctor of Philosophy in Civil Engineering is a joint program with the University of
Ottawa, administrated by the Ottawa-Carleton Institute for Civil Engineering

Library and
Archives Canada

Bibliothèque et
Archives Canada

Published Heritage
Branch

Direction du
Patrimoine de l'édition

395 Wellington Street
Ottawa ON K1A 0N4
Canada

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

ISBN: 978-0-494-23303-0

Our file Notre référence

ISBN: 978-0-494-23303-0

NOTICE:

The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

AVIS:

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protègent cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.

Canada

Abstract

Given the realization that real-time bus arrival information is viewed positively by passengers of public transit, many bus transit agencies of various sizes are developing Real-time Bus Arrival Information System (RETBAIS) following the implementation of Automatic Vehicle Location (AVL) and Automatic Passenger Counter (APC) systems. This research focuses on one important element of the RETBAIS, the real-time prediction model. Data required for the research were retrieved from the APC and AVL systems of the City of Ottawa/OC Transpo. The developed model has two main modules: Running Time Prediction Module (RTM) and Dwell Time Prediction Module (DTM).

The RTM is based on the statistical pattern recognition methodology. Given a pattern defining bus running time being predicted, the trained RTM automatically searches through the historical patterns which are the most similar to the new pattern and based on that, the prediction of a bus running time is made. The RTM was tested with different data sets of various bus running time situations. It was found that it worked well as indicated by the average relative prediction error of as low as 5% for the Transitway route and about 7% for the mixed-traffic bus route. Moreover, this module performed in a consistent manner even when unusual bus operational scenarios were used.

The DTM has four sub-modules. The first two sub-modules are also based on a recognition technique for predicting separately the number of passengers boarding and alighting. The third sub-module is used to examine the relationship between actual dwell times and various explanatory variables. The last one is based on the fact that passengers

choose the most convenient door for boarding and alighting. Having tested with datasets, the DTM proved that it can predict passenger activities with satisfactory accuracy without any specific prior assumptions on the complicated relationship between dwell time and the influencing factors.

When the constituent modules are integrated, the whole model can predict bus arrival times at every downstream stop. The prediction accuracy increased with new data availability. The average relative prediction error varied from 3 to 8%.

In order to provide bus dispatchers with tools for managing bus fleet, two methods to detect bus on-time performance and bus bunching were developed. By using these tools, a bus dispatcher can easily know ahead of time if the bus is on-time, late, early, or bunching is likely to occur.

By offering fast, accurate and reliable predictions, it is contended that the developed real-time prediction model will enhance the bus arrival information system and therefore will be a contribution to public transportation operation.

Acknowledgments

I would like to express my deepest gratitude to Prof. Ata. M. Khan for encouragement, patience, support and invaluable scientific guidance in supervising this thesis.

I am grateful to the staffs of the OC Transpo (Ottawa, Canada) for several discussions, comments and data provisions on the various aspects of this thesis. Special thanks to Mr. Joe Koffman, Mr. Brian Barclay and Ms. Sylvie Paquette. I would like to say thank you to Mr. Kean Lew and Mrs. Stephen Hotard (PTV America Inc.) for helping me to use the VISSIM software.

I am greatly thankful to Prof. Yasser Hassan, Prof. William Johnson, and Prof. Steven Prus for valuable suggestions on this thesis research.

Financial support by the Vietnamese Government is gratefully acknowledged.

I want to thank my friends; Mr. Phung Viet Anh who was always willing to help me during difficulties; Mr. Jarbar Siddique for interesting conversations in the common favorite area of bus transit; Ms. Sandra Majkik who shared data with me.

I am so indebted to my wife Mrs. Huyen Vu and to my son Hieu Vu for continuous encouragements, patience, sacrifice and their love. I love you both.

Four years of living and studying in this country tattooed in my mind about a beautiful country with clement people. Thank you Canada!

Nam Hoai Vu

TABLE OF CONTENTS

ABSTRACT.....	i
ACKNOWLEDGMENTS.....	iii
TABLE OF CONTENTS.....	iv
LIST OF TABLES.....	xi
LIST OF FIGURES.....	xv
ABBREVIATIONS.....	xviii

CHAPTER 1: INTRODUCTION

1.1	Overview.....	1
1.2	Background.....	4
1.3	Problem Statement.....	6
1.4	Goals and Objectives.....	7
1.5	Study Methodology.....	8
1.6	Thesis Document and Organization.....	11

CHAPTER 2: LITERATURE REVIEW

2.1	Introduction.....	14
2.2	Real-Time Bus Arrival Information System: Current State of Development...	14
2.2.1	AVL System and APC System.....	15
2.2.1.1	Automatic Vehicle Location System	15
2.2.1.2	Automatic Passenger Counting System	19
2.2.1.3	Uses of Retrieved AVL-APC data in RETBAIS.....	21
2.2.2	Bus Running Time Prediction Algorithms.....	25
2.2.2.1	Blacksburg (Virginia) Prediction Algorithms.....	25
2.2.2.2	Portland (Oregon) and King County Metro, Seattle (Washington) Prediction Algorithm.....	26

2.2.2.3	Texas Transportation Institute Algorithms.....	28
2.2.2.4	New Jersey University Algorithm.....	28
2.2.2.5	University of Toronto Algorithm.....	29
2.2.2.6	Artificial Neural Networks (ANN)–Based Algorithms.....	30
2.2.2.7	Classical Statistical Regression Models.....	31
2.2.2.8	Other Prediction Models.....	33
2.2.3	Bus Dwell Time Prediction Models.....	35
2.2.4	Media and Communication System.....	38
2.2.4.1	Information Dissemination Media.....	39
2.2.4.2	Communication System.....	40
2.3	Summary.....	40

CHAPTER 3: MECHANISM OF THE PROPOSED MODEL AND AVL -APC DATA COLLECTION

3.1	Introduction.....	42
3.2	Component of Bus Trip Time and Influencing Factors.....	42
3.2.1	Actual Moving Time.....	44
3.2.2	Dwell Time.....	45
3.2.3	Traffic Signal Delay.....	46
3.2.4	General Delay.....	46
3.2.5	Recovery Time.....	48
3.3	Proposed Structures and Components.....	48
3.3.1	General Discussion.....	48
3.3.2	Assumptions.....	50
3.3.3	Structure and the Building Modules of the Proposed Model.....	51
3.3.4	Mechanism of the Proposed Model.....	53
3.4	APC and AVL Data Collection.....	59
3.4.1	Bus Route Selection.....	60
3.4.2	Data Sets.....	62

3.4.3	APC and AVL Data Description.....	64
3.4.4	Data Characteristics	66
3.4.4.1	Arrival Time.....	66
3.4.4.2	Dwell Time and Passenger Activities.....	76
3.5	Summary.....	82

CHAPTER 4: BUS RUNNING TIME PREDICTION MODULE

4.1	Introduction.....	83
4.2	Problem Statement.....	84
4.3	Nonparametric Regression Methods in Statistical Pattern Recognition.....	85
4.3.1	LOWESS Estimation Method.....	87
4.3.1.1	General Functions.....	87
4.3.1.2	Bandwidth Selection.....	89
4.3.1.3	Weighting Kernel Function Selection.....	93
4.3.1.4	Degree of Polynomial Regression.....	93
4.4	Modelling Bus Running Time Prediction by using LOWESS Method and APC -AVL Data.....	94
4.4.1	APC and AVL data.....	94
4.4.2	Parameter Selection.....	94
4.4.3	Pattern Selection.....	95
4.4.4	Pattern Recognition.....	96
4.4.4.1	Euclidean Distance Calculation.....	96
4.4.4.2	Optimal Bandwidth by Leave-one-out Method.....	96
4.4.4.3	Recognition of the Neighbours... ..	100
4.4.5	Prediction.....	100
4.4.6	Update Prediction.....	102
4.5	Summary.....	103

CHAPTER 5: BUS DWELL TIME PREDICTION MODULE

5.1	Introduction.....	104
5.2	Dwell Time Prediction Module	105
5.2.1	Discussions on Previous Works.....	105
5.2.2	Dwell Time Prediction Module (DTM).....	108
5.2.3	Real-Time Boarding Passenger Prediction Sub-module.....	110
5.2.3.1	Parameter Selection.....	110
5.2.3.2	Pattern Selection.....	110
5.2.3.3	Pattern Recognition.....	114
5.2.3.4	Prediction.....	114
5.2.3.5	Update Prediction.....	115
5.2.4	Real-time Alighting Passenger Prediction Sub-module.....	115
5.2.4.1	Parameter Selections.....	115
5.2.4.2	Pattern Selection.....	116
5.2.4.3	Pattern Recognition.....	116
5.2.4.4	Prediction.....	117
5.2.4.5	Update Prediction.....	117
5.2.5	Regression Sub-Module	118
5.2.5.1	Variables Selection and Preparation.....	118
5.2.5.2	Regression Functions.....	120
5.2.6	Busiest Door Prediction Sub-Module.....	127
5.2.6.1	Rigid-Body Bus.....	128
5.2.6.2	Articulated Bus.....	136
5.2.7	Method A vs. Method B and the Selection.....	142
5.2.7.1	Method A.....	143
5.2.7.2	Method B.....	143
5.2.7.3	Rigid-body Bus.....	144
5.2.7.4	Articulated Bus.....	144
5.2.7.5	Accuracy Performance of the Two Methods.....	146
5.3	Summary.....	147

CHAPTER 6: MODEL PERFORMANCE

6.1	Introduction.....	149
6.2	Computer Program Development.....	150
6.3	Simulation of Bus Operation Scenarios.....	150
6.3.1	The VISSIM Simulator.....	151
6.3.2	Transit Route Coding in VISSIM.....	153
6.3.3	Calibration and Validation.....	155
6.3.4	Bus Operation Scenarios, Micro-simulation Runs, and the VISSIM Outputs.....	157
6.4	Model Testing and Comparison.....	162
6.4.1	Evaluation Criteria for Prediction Performance.....	162
6.4.2	Reference Predictors.....	163
6.4.2.1	The Naïve Model.....	164
6.4.2.2	The Kalman Filter- Based Model.....	164
6.4.3	Testing the Developed Model and the Reference Predictors.....	166
6.4.3.1	Data Issues.....	166
6.4.3.2	Running Time Prediction Performance.....	167
6.4.3.3	Boarding Passenger Prediction Performance.....	176
6.4.3.4	Alighting Passenger Prediction Performance.....	178
6.4.3.5	Prediction Performance with Actual Data.....	180
6.4.3.6	Tukey Test for Performance Comparison.....	184
6.4.3.7	Bus Arrival Time Prediction Performance.....	186
6.5	Summary.....	188

CHAPTER 7: REAL-TIME PREDICTION INTERVAL, ON-LINE SCHEDULE ADHERENCE EVALUATION AND BUS BUNCHING DETECTION

7.1	Introduction.....	190
7.2	Real-time Prediction Interval.....	190
7.3	On-line Schedule Adherence Evaluation.....	194

7.3.1 Methodology.....	194
7.3.2 A Numerical Example.....	196
7.3.3 Discussion.....	201
7.4 Real-time Bus Bunching Detection Method.....	203
7.4.1 Methodology.....	203
7.4.2 A Numerical Example.....	206
7.5 Further Possible Applications of the Developed Model.....	207
7.6 Real-time Bus Arrival Broadcasting.....	208
7.7 Level of Spatial and Temporal Detail of Data for the Developed Model.....	209
7.8 Summary.....	210

CHAPTER 8: CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions.....	211
8.2 Recommendations and Future Research.....	216

REFERENCES.....	219
------------------------	------------

APPENDICES.....	234
------------------------	------------

Appendix A.....	235
Appendix B1.....	237
Appendix B2.....	243
Appendix B3.....	246
Appendix C.....	249
Appendix D1.....	258
Appendix D2.....	267
Appendix D3.....	281
Appendix D4.....	285
Appendix D5.....	317
Appendix D6.....	330
Appendix D7.....	341
Appendix D8.....	382

Appendix D9.....	384
------------------	-----

LIST OF TABLES

Table 2.1	Transit Agencies with Real-time Bus Arrival Information System.....	23
Table 3.1	Factors that influence Bus Running Time and Delay.....	43
Table 3.2	Distance between stops BLAIR and LEBRETON. Bus Route 95 (Orleans-Nepean South).....	61
Table 3.3	Distance between stops GREENBORO and RIDEAU. Bus Route 1 (South Keys - Ottawa Rockliffe).....	62
Table 3.4	Data Sets for Bus Route 95 and Bus Route 1	63
Table 3.5	Data Collectable by APC and AVL Systems.....	65
Table 3.6	Means and Standard Deviations of Bus on-time Performance in the Winter Bus Route 1 (Greensboro- Rideau).....	67
Table 3.7	Means and Standard Deviations of Bus on-time Performance in the Summer Bus Route 1(Greensboro-Rideau)	68
Table 3.8	Means and Standard Deviations of Bus on-time Performance in the Winter. Bus Route 95(Blair- Lebreton).....	68
Table 3.9	Means and Standard Deviations of Bus on-time Performance in the Summer . Bus Route 95(Blair-Lebreton).....	69
Table 3.10	Two-sample t statistics for Winter and Summer Time. Route 95- Morning period (8 a.m. to 10 a.m.).....	70
Table 3.11	Two-sample t statistics for Winter and Summer Time. Route 95- Noon period (12 a.m. to 2 p.m.).....	70
Table 3.12	Two-sample t statistics for Winter and Summer Time Route 95- Afternoon period (4 p.m. to 6 p.m.).....	71
Table 3.13	Two-sample t statistics for Winter and Summer Time Route 1- Morning period (8 a.m. to 10 a.m.).....	72
Table 3.14	Two-sample t statistics for Winter and Summer Time Route 1- Noon period (12 a.m. to 2 p.m.).....	72

Table 3.15	Two-sample t statistics for winter and summer time Route 1- Afternoon period (4 p.m. to 6 p.m.).....	72
Table 3.16	Two-sample t statistics for Different Time Periods of Day. Route 95- the winter time (the WINTER data set).....	73
Table 3.17	Two-sample t statistics for Different Time Periods of Day. Route 95- the summer time (the SUMMER data set).....	74
Table 3.18	Two-sample t statistics for Different Time Periods of Day. Route 1- the winter time (the WINTER data set).....	74
Table 3.19	Two-sample t statistics for Different Time Periods of Day. Route 1- the summer time (the SUMMER data set).....	75
Table 3.20	Descriptive Statistics of Passenger Activities. Route 1- time period (8 a.m. to 10 a.m.).....	79
Table 3.21	Descriptive Statistics of Passenger Activities. Route 1- time period (12 a.m. to 2 p.m.).....	80
Table 3.22	Descriptive Statistics of Passenger Activities Route 1- time period (4 p.m. to 6 p.m.).....	80
Table 3.23	Descriptive Statistics of Passenger Activities Route 95- time period (8 a.m. to 10 a.m.).....	80
Table 3.24	Descriptive Statistics of Passenger Activities. Route 95- time period (12 a.m. to 2 a.m.).....	81
Table 3.25	Descriptive Statistics of Passenger Activities. Route 95- time period (4 p.m. to 6 p.m.).....	81
Table 4.1	Common Kernel Functions for Univariate Data.....	93
Table 5.1	Variable Selections for RESM.....	119
Table 5.2	Coefficients of the Best Regression.....	123
Table 5.3	Model Summary for the Best Regression.....	124
Table 5.4	Proposed Non-linear Regression Models.....	126
Table 5.5	Validated Dataset for Logistic Regressions.....	132
Table 5.6	Best Binary Logistic Regression Equations.....	133

Table 5.7	Model Summary of Type B-L.1.....	133
Table 5.8	Prediction Performance of Type BL-1.....	134
Table 5.9	Coefficients of Type BL-1.....	135
Table 5.10	The Best Multinomial Logistic Regression Equations.....	139
Table 5.11	Likelihood Ratios of Type B-ML.3.....	140
Table 5.12	Parameters of Type B-ML.3.....	141
Table 5.13	Measures of Accuracy.....	143
Table 5.14	Average Service Time Selection(s/p).....	144
Table 5.15	Example Output of Dwell time Prediction of Method A and Method B Applied for Rigid-body Bus.....	145
Table 5.16	Example Output of Dwell time Prediction of Method A and Method B Applied for Articulated Bus.....	146
Table 5.17	Mean Absolute Prediction Error of Method A vs. Method B.....	147
Table 6.1	Differences between VISSIM(version 4.1) and CORSIM (TSIS 5.1) for public transit simulation.....	152
Table 6.2	Kinetic Characteristics of the OC bus.....	153
Table 6.3	VISSIM Calibrated Parameters.....	157
Table 6.4	Dimensionality and Required Sample Size.....	159
Table 6.5	Scenario Design of Simulation.....	160
Table 6.6	Accuracy Criteria for Model Testing.....	163
Table 6.7	Cross-validation MAPE, MRE, and RMSE.....	169
Table 6.8	Route 1– Running Time Prediction with Overall Situations: MAPE, MRE, and RMSE	171
Table 6.9	Studentized Range of Test Sets.....	185
Table 6.10	Result of Tukey’s Procedure at Significance Level =0.05.....	185

Table 6.11	Holmwood Stop: Bus Arrival Time Prediction Performance of the 3 predictors.....	186
Table 6.12	Gladstone Stop: Bus Arrival Time Prediction Performance of the 3 predictors.....	187
Table 6.13	Rideau Stop: Bus Arrival time prediction performance of the 3 predictors.....	187
Table 6.14	Numerical Examples of Real-time Bus Arrival Prediction Using Developed Models for 3 full trips (seconds).....	188
Table 7.1	Real-time Prediction Interval for Bus Running Time of Route 1 (Time in second).....	194
Table 7.2	Goodness-of-fit test Summary ($\alpha = 0.05$).....	198
Table 7.3	Parameters of the Two Consecutive Buses.....	206
Table 7.4	Level of Spatial and Temporal Details for Data Capture.....	209

LIST OF FIGURES

Figure 1.1	Research Framework.....	9
Figure 2.1	Signpost-based Automatic Vehicle Location.....	18
Figure 2.2	GPS-base Automatic Vehicle Location.....	18
Figure 2.3	The APC System Mounted on a Bus.....	19
Figure 2.4	AVL and APC Systems on a Bus.....	21
Figure 3.1	Bus Trip Time Components.....	43
Figure 3.2	A Simplified Sketch of Bus Operation.....	51
Figure 3.3	Process of Real-time Prediction.....	54
Figure 3.4A	Bus Route 95 (Blair- Lebereton)....	61
Figure 3.4B	Bus Route 1 (South Keys-Rideau).....	62
Figure 3.5	Structures of APC Data at pointcheck.....	64
Figure 3.6	Structure of APC Data on Passenger Activity.....	66
Figure 3.7	Dwell time Histogram. Route 1- Time period (8 a.m. to 10 a.m.).....	76
Figure 3.8	Dwell time Histogram. Route 1- Time period (12 a.m. to 2 p.m.).....	77
Figure 3.9	Dwell time Histogram. Route 1- Time period (4 p.m. to 6 p.m.).....	77
Figure 3.10	Dwell time Histogram. Route 1- Time period (4 p. m. to 6 p.m.).....	77
Figure 3.11	Dwell time Histogram. Route 95 - Time period (8 a.m. to 10 a.m.).....	78
Figure 3.12	Dwell time Histogram. Route 95 - Time period (12 a.m. to 2 p.m.).....	78
Figure 4.1	Bandwidth Selection and NP Regression.....	90
Figure 4.2	Constant Bandwidth Problem.....	90
Figure 4.3	N-Nearest Type of Bandwidth.....	91
Figure 4.4	Automatic Process of Searching Optimal Number of Neighbours.....	97

Figure 5.1	Dwell time Prediction Module. Method A.....	109
Figure 5.2	Dwell time Prediction Module. Method B.....	109
Figure 5.3	Boxplot of Dwell time. Bus route 1 (All seasons and all time dataset)	121
Figure 5.4	Boxplot of validated dwell time. Bus route 95 (All seasons and all time dataset).....	121
Figure 6.1	Structure and Constituent Functions of the Computer Program.....	150
Figure 6.2	Bus Routes 95 and 1 coded in VISSIM 4.10.....	154
Figure 6.3	Vehicle Record Configuration Tool in the VISSIM.....	155
Figure 6.4	Route 95- Running Time Prediction: MAPE and MRE.....	169
Figure 6.5	Route 95- Running Time Prediction: RMSE.....	170
Figure 6.6	Route 1- Running Time Prediction with Overall Situation: MAPE and MRE.....	171
Figure 6.7	Route 1- Running Time Prediction with Overall Situation: RMSE.....	171
Figure 6.8	Route1-The Differences of MAPE Between Overall and Slowdown zone Situations.....	173
Figure 6.9	Route1-The difference of RME Between Overall and Slowdown zone Situations.....	173
Figure 6.10	Route1-The Difference of RMSE Between Overall and Slowdown zone Situations.....	173
Figure 6.11	Nearest Neighbour and Cross-Validation Least-square Error (CR) for Bus Running Time Prediction. Case 43-2200, Route 1.....	175
Figure 6.12	Nearest Neighbour and Cross-Validation Least-square Error (CR) for Bus Running Time Prediction. Case 54-2200, Route 95.....	175
Figure 6.13	Route 95- Boarding Passenger Prediction: MAPE and MRE.....	177
Figure 6.14	Route 95- Boarding Passenger Prediction: RMSE.....	177
Figure 6.15	Nearest Neighbour and Cross-Validation Least-square Error (CR)	

	for boarding passenger prediction. Case 24-1200, Route 95.....	178
Figure 6.16	Route 95- Alighting Passenger Prediction: MAPE and MRE.....	179
Figure 6.17	Route 95- Alighting Passenger Prediction: RMSE.....	179
Figure 6.18	Nearest Neighbour and Cross-Validation Least-square Error (CR) for Alighting passenger Prediction. Case 20-1200, Route 95.....	180
Figure 6.19	Route 95-Running Time Prediction with Actual Data: MAPE and MRE.....	181
Figure 6.20	Route 95-Running Time Prediction with Actual data: RMSE.....	181
Figure 6.21	Route 1- Running Time Prediction with Actual Data: MAPE and MRE.....	181
Figure 6.22	Route 1-Running Time Prediction with Actual Data: RMSE.....	182
Figure 6.23	Route 95- Boarding Passenger Prediction with Actual Data MAPE and MRE.....	182
Figure 6.24	Route 95- Boarding Passenger Prediction with Actual Data: RMSE....	183
Figure 6.25	Route 95- Alighting Passenger Prediction with Actual Data MAPE and MRE.....	183
Figure 6.26	Route 95- Alighting Passenger Prediction with Actual Data: RMSE....	183
Figure 7.1	Fitted Density for OAP at Gladstone Stop-Route1.....	200
Figure 7.2	On-time Limits and Predictions Interval.....	201
Figure 7.3	Tendency of Bus Bunching $P > 0$	204
Figure 7.4	Tendency of Bus Bunching $P = 0$	204

ABBREVIATIONS

ANN	<i>Artificial Neural Networks</i>
APC	<i>Automatic Passenger Counter</i>
APTS	<i>Advanced Public Transportation System</i>
AVL	<i>Automatic Vehicle Location</i>
ATIS	<i>Advanced Traveler Information System</i>
BDSM	<i>Busiest- door Prediction Sub-Module</i>
CBD	<i>Central Business District</i>
CORSIM	<i>Corridor Simulation</i>
CV	<i>Cross-Validation</i>
CR	<i>Cross-validation Least-square Error</i>
DGPS	<i>Differential Global Positioning System</i>
DMS	<i>Dynamic Message Sign</i>
DR	<i>Dead Reckoning</i>
DTM	<i>Dwell Time Prediction Module</i>
FTA	<i>Federal Transit Administration</i>
GPS	<i>Global Positioning System</i>
GIS	<i>Geographic Information System</i>
OAP	<i>Off-line Adherence Performance</i>
OC Transpo	<i>Ottawa-Carleton Regional Transit Commission</i>
OLS	<i>Ordinary Least Square</i>
PBSM	<i>Punctuality and Bunching Prediction Sub-Module</i>
PDA	<i>Personal Digital Assistant</i>

ITS	<i>Intelligent Transportation System</i>
LORAN-C	<i>C configuration of Long Range Navigation</i>
LOWESS	<i>Locally Weighted Scatter Plot Smoothing</i>
MAPE	<i>Mean Absolute Prediction Error</i>
NP	<i>Non-Parametric Regression</i>
RASM	<i>Real-time Alighting Passenger Prediction Sub-Module</i>
RAP	<i>Real-time Adherence Performance</i>
RBSM	<i>Real-time Boarding Passenger Prediction Sub-Module</i>
RMSE	<i>Root Mean Square Error</i>
RETBAIS	<i>Real-Time Bus Arrival Information System</i>
RESM	<i>Regression Sub- Module</i>
RTM	<i>Running time Prediction Module</i>
SPS	<i>Standard Positioning System</i>
SO	<i>Signpost and Odometer</i>
SP Recognition	<i>Statistical Pattern Recognition</i>
Tri-Met	<i>Tri-County Metropolitan Transportation District of Oregon</i>
VISSIM	<i>Verkehr In Stadten-SIMulation</i>

Chapter 1

INTRODUCTION

1.1 Overview

Many cities around the world are struggling with transportation problems. Managing congestion, reducing air and noise pollution, minimizing fuel consumption, maintaining adequate supply to serve an increasing demand for travel, and preserving community quality of life, are always the challenges for the cities to face. New or widened highways are no longer the preferred solutions of today due to fiscal constraints, conservation of land and petroleum fuels, and environmental quality objectives.

To solve the transportation problems, the development of a convenient and modern public transportation network in the city is usually considered as a first priority solution by the city authorities. Enhancing customer satisfaction by improving service quality will be a key to the success in retaining existing riders and diverting users of private automobile to the public transportation system. In America, if all people who are riding transit to work drove instead, their cars would be as long as 23,000 miles line of traffic, long enough to circle the earth (FTA 1996 Report).

Innovations in communication networks, navigation systems, Global Positioning System (GPS), Geographic Information System (GIS), sensor technologies, the Internet, and computer software, are contributing to the enhancement of the public transportation

system. At present, the Intelligent Transportation Systems (ITS) in various countries are the vivid examples of the successes in applying new technologies in the transportation field.

The advanced public transportation system (APTS) is one of the main components in the ITS architecture in many countries. The APTS implemented by GPS, GIS, Automatic Vehicle Location System (AVL), and Automatic Passenger Counter System (APC), enables transit agencies to capture the real-time information including vehicle location, speed, operational status, and passenger occupancy. Real-time information is a valuable and reliable input for transit passenger information systems to assist transit providers in planning, operation and management.

Although various transit agencies have implemented new technologies to collect real-time information about their en-route bus fleets, they have been facing difficulties in disseminating reliable bus arrival information to the customer. Traffic conditions reflect stochastic processes. Therefore, they influence a bus adherence to schedule in a random way. It is a challenge for transit agencies to know the arrival time ahead of time. Only the real-time predictions based on real-time information are capable of attracting customers to the public transit and the use of the predicted arrival information can potentially influence the daily activities of travellers.

The APC systems have been deployed in some transit agencies in North America (the United States and Canada) since the 1980s. The time when the first AVL system was launched was even earlier. In 1969, Chicago (USA), implemented an AVL system and Toronto, (Canada) installed one in 1972. The names of AVL and APC systems explain their functions. The AVL system provides dynamically the location, speed, and

operational status of a remote bus while an APC system is able to transmit the information about passenger boarding and alighting at stops, running time between stops, dwell times, and several service characteristics to the control center. The AVL and APC systems provide much data which can potentially serve as inputs for the development of bus arrival prediction models.

When the first AVL system was initiated in Chicago, a 10-year vision of using AVL systems to improve the schedule performance, was described optimistically by Klopfenstein, (1986, p.3). Since that time, indeed, many transit agencies still do not operate AVL and APC systems. As of 2000, only 88 out of 548 transit agencies in the United States were operating AVL system and 145 others were planning AVL systems (Schweiger, 2003). A small fraction of the 88 agencies are providing real-time arrival information to the customer via Advanced Traveller Information System (ATIS) accessed by passengers at home, work places, and bus stations via the World Wide Web (WWW) Internet system or wireless devices such as internet capable cell phones, Palm Pilots, and Personal Digital Assistants (PDAs). The reason behind the delay is not only that an AVL system is still a significant investment but also the agencies are waiting for the most preferable real-time information system that is capable of providing bus real-time arrival information, workable with the existing technologies and infrastructure, and suitable for their fiscal conditions.

The number of transit agencies using APC systems was even fewer. A survey conducted by Daniel (1998) showed that out of the 27 US transit agencies in the survey, only five agencies use the APC system. Almost all of these agencies used pencil and manual check for different purposes. The Ottawa-Carleton Regional Transit Commission

(OC Transpo), an 827-bus system serving the capital city of Ottawa (Canada), is a pioneer and long-time user of the APC system. Over the years, data collected from the APC system and the signpost-based AVL system are serving the OC Transpo in terms of increasing the quality of service, monitoring the average loads and passenger activities, and maintaining schedule adherence. However, like many other transit agencies, the OC Transpo has not yet deployed a real-time bus arrival information system or RETBAIS for short.

Clearly, there is a need to develop models that can utilize the huge and valuable AVL-APC database for predicting bus arrival times in order to improve the quality of the arrival information disseminated to passengers.

1.2 Background

Transit agencies provide bus information to passengers in different ways. These vary from traditional methods such as printed schedules to modern methods such as arrival times on an electronic sign, also known as Dynamic Message Sign (DMS) located on a bus stop. There are three kinds of information that a transit passenger wants to know: the pre-trip information, the en-route information and the in-vehicle information.

Pre-trip information is static information including schedules, maps, destinations, fares, and policies. This information helps passengers in planning trips. It is updated periodically for informing passengers about every change in schedule time of bus routes, the new routes, or the new policies and fares, except the actual bus operational situations such as delay, late or early buses. To know the scheduled arrivals, a passenger can use a free brochure showing the bus map and schedule of the bus. Also, the passengers can

access the transit agency's website to find the schedules if they are provided. Calling the transit representatives is also a method for passengers to know bus arrivals. By simply providing the bus station number, a passenger can get the scheduled times via telephone.

From the passenger's point of view, there is no certainty that the expected bus will arrive on schedule. Hence, if transit agencies cannot provide passengers actual arrivals or in other words, the en-route information, the waiting passengers can only depend on their own waiting time experiences. Therefore, they will tend to arrive to the stop a few minutes earlier than the schedule to avoid missing the bus. If the bus arrives late, this increases the passengers' waiting time. Even if the bus arrives on time, the waiting time could be perceived to be long. On the other hand, when the bus comes to the stop earlier than scheduled, passengers can miss the bus even though they come to the stop on time. In this case, the waiting time is now not a few minutes but as long as an headway. Moreover, if passengers do not know that they missed the last bus, they may experience anxiety. For on-board passengers, in-vehicle arrival information to the destinations can help them to have smooth connections with other inter-modal transit services.

Obviously, the uncertainty of bus arrival time increases passenger's waiting time and anxiety, deteriorates schedule adherence, and reduces the smooth transition among inter-modal transfer. Thus, passenger dissatisfaction is unavoidable if transit agencies still provide their customers only the static information. To satisfy customers, transit agencies should provide their passengers not only static information (e.g., pre-trip information) but also en-route and in-vehicle arrival information. Interestingly, some transit agencies that provide real-time information to passengers recognized that diversion to private

automobile was low even when bus arrivals varied but passengers knew this information ahead of time (Schweiger *et al.*, 2003). Therefore, providing passengers real-time arrival information is necessary and this will encourage passengers to use the transit system.

1.3 Problem Statement

Surveys carried out by some transit agencies revealed that the most important factor influencing bus riders' decision to ride a bus, surprisingly, is not the bus on-time performance but the real-time information about the bus of interest. On-time performance only comes as the second important factor. This shows that real-time information is playing an important role in customer's perception of transit service quality (Zhong *et al.*, 2003).

Passengers' demand for real-time information on bus arrivals is pressing transit agencies to meet that demand in order to increase or at least, to keep their customers. Having recognized the importance of this task, various agencies in European countries, in North America (i.e., the United States, Canada), and Asian countries (e.g., Japan, Taiwan, and Korea) have been installing AVL-APC systems to assist their bus fleets as well as to develop a suitable RETBAIS. However, most of these agencies are using AVL-APC data mainly for off-line performance studies. This practice continues in spite of the fact that much more is to be gained in on-line applications such as predicting bus arrivals, predicting passengers boarding or alighting at stops, and proactive dispatch programs.

The use of APC data and AVL data is also different. While APC data downloaded daily or weekly from on-board microcomputers have been used successfully for years in the off-line analyses, for instance, passenger loads, on-time performances, etc., the AVL

data have not yet been used adequately for neither off-line nor on-line services, except for incident response by many agencies. The reason behind the failure in using useful AVL data is that the transit agencies did not recognize the potential benefits achievable from useful AVL data. Therefore, their AVL systems were not used to match and track vehicles, and AVL and the APC systems have been used as stand-alone systems. This practice limits the on-line applications based on archived AVL-APC data (Furth *et al.*, 2003, p.6).

The purpose of this research is to develop a methodological framework and the constituent model for advancing dynamic predictions and thus to support the real-time bus arrival information system. By using the AVL-APC data intensively, the model will be suitable for transit agencies whose bus fleets are equipped with AVL and APC systems.

1.4 Goals and Objectives

This research is intended to enhance the interests of transit passengers as well as transit service providers. The study aims at increasing the quality of actual arrival information (e.g., en-route and in-vehicle information) to be provided to bus transit passengers. Also, this research is intended to support bus transit providers in term of real-time control and management. For achieving the first goal, a real-time prediction model for bus arrivals was developed. To attain the second goal, the applications of the model were further developed for bus bunching and punctuality prediction in order to help dispatcher in managing the fleet.

Specific objectives of this research are:

1. To define a research framework and constituent modules for the development of methods to support bus real-time arrival prediction.
2. To develop modules and integrate the modules to create a dynamic model that can predict real-time bus arrivals and dwell times at every stop.
3. To evaluate the accuracy of each module and the integrated set of modules.
4. To develop bunching and punctuality prediction algorithms that can help bus dispatchers in controlling the fleet under real-time situations.
5. To propose the architecture of a-real-time bus arrival system based on the developed model.

1.5 Study Methodology

The overall research framework is shown in Figure 1.1.

The problem definition step covered the general lack of real-time information on bus arrivals and dwell time. As noted in the previous section, the users of bus transit are placing much importance on this type of information. However, for a number of reasons, a very high proportion of public transit agencies do not provide such information. Although a number of transit agencies have taken steps to implement AVL and APC systems, they have not taken the next step to make use of the data to develop a RETBAIS. Also in this step, the issues of the existing bus arrival prediction algorithms were noted. Consequently, the problem definition step dealt with the characterization of the problem to be solved. Here, broad research goals and specific objectives were defined.

Figure 1.1: Research Framework

In order to fulfill the research goals and objectives, further steps were accomplished. Two next steps were the data collection and the data generation. These are carried out side-by-side.

In the data collection step, the APC and AVL data collected from the OC Transpo, accompanied with other data of the transit network, were used for two purposes. On one hand, these data were mined directly for the model development and testing steps. On the other hand, they were used as the inputs for the data generation step.

The purpose of data generation step was to generate data, given the assumptions of different bus operational scenarios which are difficult to study in the form of field experiments (e.g., congestion, incidents, etc.). The VISSIM 4.10 traffic software was used for these simulation purposes. In order to make the simulated bus operations in the transit network as similar to the actual operations as possible, a calibration procedure was developed. After the calibration procedure was completed, the generated data were then used for testing the model under each scenario.

The next step involved model development and model testing procedures where the variables were defined and theoretical aspects were researched. Based on the AVL and APC data and the use of statistical pattern recognition technique and nonlinear regression methodologies, a model consisting of three modules and four sub-modules was developed. This developed model has the capability to make real-time bus arrival time and dwell time predictions. After the model was tested in terms of accuracy and reliability by using simulated data obtained from computer simulations of several bus operational scenarios, the model was also tested with real-world data. At the end of this

step, the developed model's prediction performance on bus running time, bus dwell time and passenger activities were assessed.

Finally, for bunching detection, punctuality study, and dispatcher help, the developed model was expanded by using predicted bus running times and dwell times.

1.6 Thesis Document and Organization

The dissertation consists of eight chapters.

Chapter 1 provides an introduction to the subject of this research, research objectives, and the research study framework.

Chapter 2 is devoted to the literature review including current applications and the potential prediction algorithms for bus running time and dwell time.

Chapter 3 presents a broad view of real-time bus arrival prediction model developed in this thesis research. This model includes three linked modules: Running Time Prediction Module (RTM), Dwell Time Prediction Module (DTM), Punctuality and Bunching Prediction Module (PBM). The DTM is composed of four sub-modules, including: Real-Time Boarding Passenger Prediction (RBSM), Real-time Alighting Passenger Prediction (RASM), Regression sub-module (RESM), and Busiest Door Prediction (BDSM).

Chapter 4 covers the first module, namely the RTM. In this chapter, a pattern recognition process based on a non-parametric statistical technique is presented which can predict running times on every downstream link of an en-route bus. The module's refined procedures are also presented in this chapter.

Chapter 5 commences with some discussions on previous dwell time prediction algorithms, which have been described in the literature. Next, a framework for dwell time module and its four sub-modules are presented, followed by the solutions and procedures for the first two sub-modules. The text describes how several deficiencies of previous dwell time prediction models reported by various studies are overcome. This chapter also covers data collected by the OC Transpo on dwell time and the number of boarders, alighters and other variables such as on-time performance, bus location, bus types, and time of day. Several statistical assumptions on their relationships are examined in order to find the best statistical model for estimating dwell time.

Chapter 6 presents the method for the estimation of the developed model. First, the procedure to simulate several bus operational scenarios by using VISSIM software is introduced. Second, for comparison purposes, two reference models for bus arrival time prediction are described. One of these models can be currently considered as an advanced model and the other as the simplest model. Third, the developed model and the two reference models are compared in term of bus running time, passenger activities, and bus dwell time predictions given different bus operational scenarios and a variety of data. The comparisons are based on the Tukey's test procedure in order to provide sound conclusions. By using the evaluation system, models' accuracy and reliability are assessed.

Chapter 7 covers the methodologies for the on-line detection of bus on-time performance and bus bunching. For this purpose, a method to estimate real-time prediction interval is developed given a user-defined level of significance. On the basis of probability theory and the historical bus arrival time distribution, two methods are

developed, one for on-line bus on-time performance evaluation and the other for on-line bus bunching detection. These provide the probability that such events may occur. Finally, the discussions on announcing of bus arrival times as well as spatial aspect and details of the database for the developed model are discussed.

Chapter 8 presents conclusions and recommendations. The contributions of this thesis research are highlighted. Recommendations are provided that include future research on different aspects of data, mathematical algorithms, and some limitations of the developed model.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

The application of AVL and APC systems in bus transit is becoming popular in North America and Europe. This enables the agencies to develop a RETBAIS. Based on the review of a number of publications (i.e., syntheses, reports, and articles), this chapter examines the current practices, the experiences, the benefits and problems. Also, the prediction algorithms of bus arrival time and dwell time used by transit agencies around the world are reviewed.

2.2 Real-Time Bus Arrival Information System: Current State of Development

The deployment of the AVL and APC systems in many transit agencies has a long history dating back to the early 1970s. However, this became widespread through the United States and Europe only since the late 1980s and 1990s for the purpose of increasing operational efficiency. After years of using AVL and APC systems, transit agencies realized that these systems not only can increase the operational efficiency but also provide passenger real-time information which is bringing substantial benefits to the transit providers.

The fundamental elements for deploying a RETBAIS are: the technical characteristics of the underlying AVL system, the real-time prediction models, type of media to distribute the real-time information, related costs of the system, customer and the media reaction to the system, and institutional issues with the system (Schweiger *et al.*, 2003, p.2). In the field of interest, the literature review focuses only on the first three elements. Each element is discussed in a corresponding part, namely: the AVL and APC systems, arrival and dwell time prediction algorithms, and media and communication systems.

2.2.1 AVL System and APC System

2.2.1.1 Automatic Vehicle Location System

Automatic Vehicle Location (AVL) system is the backbone technology for a RETBAIS. The AVL system tracks the dynamic location of a remote bus and transmits the data to the control center. The transmitted locations of the bus along with other data such as speed, acceleration, or historical data collected from APC systems and other diagnostic systems on the bus, are used to predict the new location of the bus. As a result, the arrival time at a selected stop is predicted. Different agencies are using different technologies that belong to the four AVL categories. These, in the increasing level of advanced information technology, are presented below.

- **Dead-Reckoning (DR)-based AVL system:** This is the oldest and simplest navigation technology. The devices frequently used are odometers and

compasses. Data recorded from instruments must be transmitted by radio to the central control for processing, analysis, and display. In a DR-based AVL system, a bus is equipped with a heading sensor known as an adaptation of a magnetic compass, wheel odometers, an information processor, and radio equipment. The readings of heading sensor and the readings of wheel odometers are processed by the on-board information processor and converted into modulated waves that are compatible with the bandwidth requirements, workable with both receivers and antennas, and transmittable to the control center. At the control center, the waves received from a bus then will be decoded for tracking and displaying the location of the bus.

- **Radio Navigation or LORAN C-based AVL system:** The C configuration of the Long Range Navigation (LORAN-C) system is the triangulation with three or more land-based transmitting stations. The instruments of a bus using LORAN-C are: a LORAN-C receiver, antenna, a data processor, and land mobile transceivers. Location is delivered based on the distances from the bus to a pair of fixed radio stations' locations. No new LORAN C-based AVL system is anticipated in the future (Okunieff, 1997).

- **Signpost and Odometer (SO)-based AVL system:** The system, known as radio frequency signposts, or signpost for short (both active and passive) uses radio beacon mounted on the top of the facilities. In a passive signpost system, each beacon mounted on the pole or structure above the height of the bus has a unique ID. The beacon sends a low power coded signal in order to save

electricity. When a bus passes a signpost, the specialized receivers attached on the bus receive and decode the transmission from the signpost to recognize the signpost's ID. The location of the bus then is determined based on the last beacon ID the bus detected and the distances travelled. Figure 2.1 represents a passive SO-based AVL system. In contrast with passive system, each bus in the active system has a unique ID on a radio channel designated solely for automatic vehicle locating use. The vehicle transmission is continuous and at very low power. Therefore, when the bus crosses a signpost pole equipped with a receiver, its location is relayed and transmitted by that signpost to the control center. This alternative transmits information via underground wire system so the bus does not require a wireless transfer of data to the control center. Details on SO-AVL system can be found in Skomal (1981) and Okunieff (1997).

- **Global Positioning System (GPS) - based AVL system:** Development of work on GPS commenced in 1973 for the sake of supporting the positioning requirements of military operations. As of 2000, the full constellation of 24 satellites was launched to ensure that there will always be at least four satellites at all sites on the globe. Each GPS satellite transmits a unique navigation signal on two L-band frequencies, the L1 frequency 1575.42 MHz and the L2 frequency 1227.60 MHz. Both bands carry the navigation message and Standard Positioning System (SPS). A bus equipped with GPS antenna and GPS receiver connected to an on-board microcomputer receives GPS signals, determines its location and sends the location data to the control center via wireless communication system. Figure 2.2 depicts a GPS-based AVL system for bus transit

Figure 2.1: Signpost- based Automatic Vehicle Location
 (Source: [http:// www. calccit.org/ itsdecision](http://www.calccit.org/itsdecision))

Notes:

- 1- The location antenna mounted on the bus receives the signpost signals and transmits them to the on-board Vehicle Location Unit (VLU). The VLU then recognizes the signpost's ID.
- 2- The location of the bus based on the signpost's ID then is transmitted to the control center via radio system.

Figure 2.2: GPS-based Automatic Vehicle Location
 (Source: [http:// www. calccit.org/ itsdecision](http://www.calccit.org/itsdecision))

- **Differential GPS (DGPS)-based AVL system:** In order to enhance the accuracy, DGPS requires that one of the receivers to be located as a base location. From the known location of the base, the bias errors due to the estimated position of the orbiting satellites are calculated. A bus on the DGPS based AVL system, like other GPS-based AVL systems, has a GPS antenna, GPS receivers, and an on-board microcomputer. The required additional unit is a radio link that receives correction from differential reference station.

2.2.1.2 Automatic Passenger Counting System

Automatic Passenger Counting System or APC system is an automatic means for counting boarding and alighting passengers as well as the time that the bus is at a specific location. Figure 2.3 depicts the components of an APC system mounted on a bus.

Figure 2.3: The APC system mounted on a bus (Source: <http://www.infodev.ca>)

The heart of an APC system is the on-board microcomputer connected to the counting sensors attached on the door areas of a bus. Each time a passenger gets on or off the bus, the sensors will record the passenger activities and store data in the on-board microcomputer. There are two counting technologies applied in the APC system: infrared beam sensors attached on the door areas, and treadle mats mounted at the step levels. Based on the order of the broken beam in the case of infrared beam sensor or the order of stepping on the mat in the case of treadle mats, the APC system can differentiate if a passenger is getting off or alighting on the bus. The data stored in the onboard computer can be transmitted manually with data collector. To transfer data automatically, infrared or radio modems are needed. In case of using infrared devices, when the bus stops (usually at the garage or bus stop), two infrared links, one on the bus and the second on the bus stop or garage area, download data from onboard computer automatically. If the bus is equipped with a radio modem, data stored on the on-board computer can be copied automatically at the distances of up to 500 ft between the radio transceivers. Time to download a full day data is only about 20-30 seconds (<http://www.infodev.ca>).

The APC system can work as a stand-alone system using one of the four AVL categories mentioned above to match passenger activities to predetermined bus stops. Figure 2.4 illustrates the integration of the APC and AVL systems. Such system can measure over 20 variables including passenger activities (boarding and alighting), bus locations (GPS coordinates, direction, bus stop location), and operational activities (speed, running time, arrival and departure time, dwell time, stop distance, access time, idle time, passenger load, etc). Moreover, an APC system integrated with the AVL system and the radio systems can provide real-time data for real-time dispatching and

arrival data dissemination purposes (John, 1999). Therefore, the APC system has great potential as valuable source of data for either on-line or off-line applications in RETBAIS.

Figure 2.4: AVL and APC Systems on a Bus (Source: Furth *et al.*, 2003)

2.2.1.3 Uses of Retrieved AVL-APC data in RETBAIS

Transit agencies use collected AVL and APC data for two main purposes: off-line performance analyses and on-line applications. While many agencies have used the collected data for off-line studies, a few agencies use these data for on-line applications, such as providing passengers en-route and in-vehicle information. The degree of using

these data differs from one agency to another. The APC data were usually used in off-line estimates while the AVL data were used for incident response, safety, and recently for real-time location tracking. However, many agencies failed in using the AVL system for both on-line and off-line estimations (Furth *et al.*, 2003). Very few transit agencies (for instant: Calgary Transit, Canada) considered using AVL data for off-line analysis as an objective. While the APC system can collect and store many valuable variables useful for on-line analysis, they have not been used for the real-time applications because they were not integrated with the AVL and radio systems. In contrast, the AVL system, which is considered a more “advanced” technology in real-time application, is not programmed to store data. Consequently, it has been used only as a means for incident response (Furth *et al.*, 2003). Ironically, many agencies did have AVL system but still relied on manual check to measure running time and on-time performance whereas other agencies that had APC systems but did not integrate them with AVL systems and radio system still could not provide passengers real-time arrival information.

Some successes in North America, European countries, and Asian countries in implementing real-time bus arrival system are listed in Table 2.1. Some typical applications are detailed in terms of technology and scope.

- ***NextBus System, United States***

The Nextbus System is well-known, serving 28 agencies over 13 states in the United States. Based on the GPS-based AVL system, satellites and wireless communication, the Nextbus System is capable of predicting real-time arrivals for buses at all bus stops. This information is updated every minute and provided to passengers in

various ways. At bus stops, real-time arrivals are announced via dynamic message signs (DMSs).

Table 2.1: Transit Agencies with Real-time Bus Arrival Information System

Agency Location	AVL system	Description	Website address
<i>Nextbus</i>	GPS	Real-time arrivals via www, palm pilot, DMS, and internet capable cell phone.	www.nextbus.com
Cape cod, RTA, Dennis, MA	GPS	Real-time bus's location via Internet	www.capcodtransit.org
Los Angeles, LADOT, LACMTA	Signpost	Real-time arrivals via DMS at Metro rapid bus stops.	www.ladottransit.com
San Louis Obispo Transit, CA	GPS	Real-time bus arrivals at major bus stop	
Denver RTD	GPS	Real-time bus arrival via telephone and mobile applications	www.rtd-denver.com
Seattle, King County Metro	Signpost	<i>Busview</i> and <i>Mybus</i> programs to provide real-time information and others via Internet and mobile phone	http://mybus.org
Ohio State University	GPS	Real-time arrivals via website and message sign at stops	
Tri-Met (Portland, OR)	GPS	Real-time arrivals via DMSs and real-time schedule via website.	http://www.trimet.org
Metro transit, Halifax, Canada	GPS	Real-time arrival via telephone	www.halifax.ca/metrotransit/gotime.html
TranLink, Vancouver, Canada	GPS	Real-time arrival on 98-B lines.	www.tranlink.bc.ca
London, United Kingdom	Signpost	<i>Countdown</i> system provides real-time bus arrivals at stops via message signs	http://www.tfl.gov.uk/buses
Helsinki, Finland	DGPS and Signpost	<i>PROMISE</i> , provides real-time arrival via internet and wireless terminals	
Magdeburg, Germany	GPS	<i>PIEPSE</i> , provides passengers the delay information that may occur.	
Brussels, Belgium	GPS	<i>Phoebus</i> : Real-time arrivals at stop and Internet	www.stib.irisnet.be/FR/3600F.html
Paris, France	GPS	<i>SNCF</i> Real-time arrival via media, internet.	
Kaohsiung, Taiwan	GPS	Real-time arrival via Internet	www.mybus.com.tw
Taichung, Taiwan	GPS	Real-time arrival via Internet	www.mybus.com.tw

The arrival information is also posted on the internet at [http:// www.nextbus.com](http://www.nextbus.com).

Passengers can access the website, select the bus route, direction and stop in order to

know the dynamic arrival information. Passengers with Palm Pilot and internet capable cell phones can also get bus arrival information at anytime and anywhere.

- **Countdown System, London, United Kingdom:**

Transport for London or London Bus services Limited has a very large number of buses, a total of 5700 buses. The information system, called Countdown, was first piloted in 1992 in an endeavour to provide real-time arrivals for passengers. In 1996, a full scaled AVL system and Countdown Program were approved with an approximately 27 million dollars of budget. To date, 2000 countdown signs have been installed, and 2000 others are expected to be installed by the following years, covering 25 percent of the stops and serving over 60 percent of all passenger journeys. All 5700 buses in the system were equipped with signpost-based AVL system. The buses are localized by signposts in the 5000 signpost-network covering the service area. Each bus location is polled every 30 seconds. The predicted arrivals are disseminated to passengers via internet and DMSs. The information in the forms of audio and visual are in the plan. The Countdown system is well received by passengers (Source: <http://www.tfl.gov.uk/buses>)

- **Tri- County Metropolitan Transportation District of Oregon (Tri-Met)**

Tri-Met can be considered as the pioneer among over 30 agencies in the United States for using collected data from APC and AVL systems for both on-line arrival predictions and off-line performance studies. One of the keys to the success of Tri-Met after years of using the systems was the development of the Bus Dispatching System

(BDS). Another key was that the AVL-APC interface provided the local referencing of the APC data. Hence, it increased the utilization of data resources for different purposes (Furth *et al.*, 2003). At present, Tri-Met has a real-time bus arrival system called Tri-Met Transit Tracker system that provides real-time arrival information for selected bus routes. Passengers can access the website at <http://trimet.org> to get bus arrival information. The Transit Tracker project is aimed at deploying initially 250 bus stops that announce real-time arrivals to passengers. The expected rate of implementation is 50 stops per year.

2.2.2 Bus Running time Prediction Algorithms

Different arrival prediction models and algorithms have been developed by various transit agencies and universities. The scale and the scope of these models vary, depending upon data needs. While some models solely use data collected from the GPS-based AVL system, others use data collected from the APC systems. Several models require more data than others and additional technologies are used to collect such data (e.g., road conditions, traffic conditions). This subsection describes non-proprietary algorithms and models that have been documented and published.

2.2.2.1 Blacksburg (Virginia) Prediction Algorithms

Four algorithms were developed by Lin and Zen, (1999) to predict bus arrivals. According to the authors, first, the location of the buses and stops are tracked and relayed by satellites and GPS-based AVL system installed on the buses. To match the locations of a bus to the locations of the stops, especially on the loop routes where some stops have

the same coordinates (e.g., longitude and latitude) but in different distances, a matching module was proposed. Second, four different algorithms were developed based on different data needs: GPS bus location data, GPS bus location data + bus schedule table, GPS bus location + bus schedule table + delay, and GPS bus location + bus schedule table + delay + time check point. In the first two algorithms, the running time of a bus is predicted based on a simple interpolation from the bus schedule table, the scheduled travel time between two major stops, the ratio of the distance between two major stops and the distance between any link pair between these two stops (e.g., from the tracked GPS bus location to a stop or link). Having observed that dwell time at time check stop was the major contributor to schedule adherence, the authors developed the last two algorithms to take delays and dwell time into consideration. Delays in these algorithms are calculated given the assumption that the lateness or earliness at an upstream stop is perceived by bus drivers. These delays may be diminished because drivers tend to adjust the speed to maintain on-time arrivals at check stops.

2.2.2.2 Portland (Oregon) and King County Metro, Seattle (Washington) Prediction Algorithms

The models created by Dailey *et al.*, (2001) and Cathey *et al.*, (2003) have been applied on Mybus System in King County Metro (Seattle), and Tri-Met (Portland). The model includes three components: the Tracker, the Filter, and the Predictor. According to Cathey *et al.*, (2003), the mechanism behind this model can be summarized as follows.

- **Tracker:** When a request is received regarding the arrival time of a bus at a bus stop, the actual location of that bus is relayed by GPS-based AVL system. To match this location to a particular trip of a bus route, an

elaborate tracking methodology is used. The core of this methodology is that the original GPS data on bus locations (e.g., longitude, latitude, and time) are converted into distances from these locations to an underlying pattern, called *distance into pattern*. Therefore, the bus locations are stored and updated easily. This eliminates the ambiguities of differentiating bus directions when a vehicle is operating back and forth on a block over the same roadway.

- **Filter:** A Kalman filter is applied to estimate the bus dynamic state. The state vector X includes three elements: distance into pattern, speed, and acceleration. At time step $(k-1)$, before the AVL system on the bus sends the new data, the prior bus state at time step k is projected. This estimation is based on the current state at time step $(k-1)$ and a transition matrix. When new AVL data are available at time step k , the optimal estimation of the new state (or the posterior projected state) of the bus is calculated based on the prior state and a correction. The correction is an optimal weighting of values done by multiplying a Kalman gain with the difference between the actual measured value of *distance into pattern* and the best prediction of its value before it is actually measured. These values then will be used to predict the next state of the bus at time step $k+1$ and the procedure is applied until the bus actually arrives at the stop.
- **Predictor:** Locations of the bus are updated frequently at a predetermined time interval such as at every time point or constant time intervals.

The arrival is predicted synchronously at each time interval when the new AVL data of the bus become available.

2.2.2.3 Texas Transportation Institute Algorithms

The TransLink Lab at the Texas Transportation Institute developed two algorithms to predict bus arrival: 1-minute zone algorithm, and distance-based model. In the former algorithm, the bus route map is divided into small 1-minute zones based on the historical travel time data. When a current location of bus is detected, the travel time from that location to the predetermined destination is then calculated by counting the number of zones between these pairs. In the later algorithm, the variation in running speeds in various time period of a day (e.g., the class time), and the distance to destination are modelled to reflex the bus travel time (Chien *et al.*, 2004).

2.2.2.4 New Jersey University Algorithm

Chen *et al.*, (2004) created a model to predict bus arrival on the New Jersey Transit bus route 39. Unlike the previous models discussed above, this model uses data collected from APC system intensively. Having argued that the artificial neural network (ANN) cannot be practically used in real-time bus arrival prediction due to long training time, the authors developed a combination of an ANN and a Kalman filter. First, data collected from APC data and other data sources (e.g., historical running times, distances between stops, weather conditions) were used as the inputs to train the ANN. The training back propagation algorithm and several hidden layers were examined. The well-trained

ANN then was used for initial predictions of running times between stops, given the road conditions and traffic conditions. To capture the real-time positions of the bus, a Kalman filter-based method was also developed. The values predicted from ANN are used as the initial states of the bus at the first stop. Each time the bus arrives at the next stop, the new location, the actual running time, and the actual arrival time are relayed by AVL and APC systems, and then a Kalman gain loop uses these data (e.g., the most updated data) for updating the new states of the bus. Therefore, the predictions are adjusted with new state and updated data.

2.2.2.5 University of Toronto Algorithms

Farhan (2002), Shalaby and Farhan (2004), developed models to predict running times and dwell times for bus route 5 in Toronto (Canada). Two Kalman filter procedures, which were first proposed by Reihoundt *et al.*, (1997) for predicting running time and dwell time, have been developed in detail (Shalaby *et al.* 2004, p. 48). The predictions are made at every timepoint. To predict running time of a current bus on a link, the average historical running time (e.g., 3 days in the past) and the current running time of the previous bus on that link, are used as the initial state of the current bus. Each time the bus comes to a next stop, the state is updated by a Kalman gain and the predictions are adjusted with the most updated data collected from the AVL and APC systems. The same procedure is applied for predicting arrival rate of boarding passengers at every stop. The predicted arrival rate is assumed to be a uniform distribution, so dwell time is the product of that rate and the headway. The model outperformed in arrival

prediction in comparison with other models (e.g., the ANN-based model and the average based- model).

2.2.2.6 Artificial Neural Network (ANN)-based Algorithms

Several ANN- based models applied in the bus transit field can be found in Chien *et al.*, (2002) and Kalaputapu *et al.*, (1995). Unlike the prediction models discussed so far, which require only AVL data or combined AVL-APC data, these models need different data sources collected from field measurements or simulated from simulators.

Chien *et al.*, (2002) proposed two ANN-based models to predict arrivals: link-based model, and stop-based model. In the first model, a segment bounded by two bus stops was considered as a series of traversed links. One bus route may have several segments. On each link, the following data were required: volume, speed, delay, average link queue time, passenger demand at stops, and bus travel distance on a link. The ANNs with the variable listed above were trained by the back propagation algorithm. During the training step, several parameters have been examined such as number of hidden layers, learning rate. If there are M links on a segment, ANN will be applied M times to predict bus running time on each link. Therefore, the arrival time at stop $i+1$ is the sum of the departure time at stop i and the total predicted travel times of all M links of that segment. Unlike the first model, the second model integrated the traversed links into one link. Therefore, each segment between a pair of two stops is a link. The data that were used to train the ANN are integrated as the means and standard deviations of speeds, volumes, and delays. This model is simpler than the first one because the trained ANN is used only one time to predict running time on the link. However, it also required more variables

than that of the link-based model to train the ANNs. There were 9 variables in this model: distance between stops, mean and standard deviation of volumes, mean and standard deviation of link speeds, mean and standard deviation of delays, number of intersections traversed between a pair of stops, passenger demand at stop. On the basis of mean absolute predictor error, both models showed good predictions when they were applied to bus route 39, New Jersey Transit.

2.2.2.7 Classical Statistical Regression Models

These models based on the classical statistic regressions vary from simple multiple linear regressions to non-linear regressions. Besides the data that can be collected by the APC or AVL systems (for example, the stop distances, average running times on the links, the number of passengers getting on or off the bus, the average speeds of bus on links), more data were required such as average delay on the link, traffic volume, densities, the number of left turning vehicles, type of vehicle, traffic signal timing, etc. The models treated bus running time or bus delay as dependent variable and other variables as independent ones.

Alfa *et al.*, (1988) proposed three statistical models to investigate the bus running time for the transit network in the city of Winnipeg, Canada. Many variables were collected and 11 of them were carefully selected in order to get the most influencing variables. After removing the variables which had small correlations with the response variable, only four variables were retained: number of bus stops, number of stop signs, number of traffic lights, and length of the segment. These variables then were used to developed three models, one is multiple linear regression and two others are non-linear

regression models. The second model was suitable for extrapolation of bus running time while the first model was preferable for interpolation.

Abdelfattah and Khan, (1998) developed six statistical models to capture the variations in bus transit delays under several bus operational scenarios: normal road condition (model 1), bus route in business district area (model 2), bus route including high proportion of heavy vehicles (models 3, 4), and one-lane blocked route (models 5 & 6). The data for building these models were measured in the field (the OC Transpo bus route 2, Ottawa, Canada) and the simulated data obtained from TRAF-NETSIM software. Thirteen variables were considered as independent variables and bus delay was used as the dependent variable. To retain the most influencing variable, several stepwise regression procedures in the SPSS software were applied. The variables with the highest influence on bus delays were: densities of left turn and through vehicles, link length, bus efficiency ratio, number of stations, number of stops, and heavy vehicle traffic density per link. The first model was linear multiple regression type and the others were all non-linear models.

Patnaik *et al.*, (2004) proposed a statistical regression based model using APC data. The APC database was split into two parts: 80 percent database was used for developing multiple regression models and the remaining for testing the developed models. Eleven variables collected from the APC system were used as the inputs and two models were developed using the stepwise regression method.

2.2.2.8 Other Prediction Models

Other prediction models applied for bus arrival were found in the literature and discussed briefly as follows:

- ***Simulation model:*** Chien *et al.*, (2000) proposed a model to predict bus arrival based on simulation. Although the enhancements developed by the authors for CORSIM (CORridor SIMulation) software were intended to increase the capability of CORSIM to capture the waiting passenger, these also were used to estimate stop-to-stop travel time. Based on the proposed model, bus schedule of each bus route was specified by users and used as input. Then the simulator generated the buses on the network based on input schedule. Therefore, given simulated road conditions, passengers, and actual bus schedule, the enhanced CORSIM could be used to predict arrival times.
- ***Non-real-time AVL data model:*** Having recognized that historical AVL data are capable of providing estimated bus running time, Horbury (1999) proposed some models to estimate moving time, bus speed and dwell time. The signpost-based AVL system data collected from route 18 in London (England) were decomposed into moving time and dwell time at stop. Moving time for a trip from the origin to the destination was calculated by subtracting the total travel time and the dwell times at stops. To calculate the dwell time, boarding and alighting time models were constructed from the survey data. These models have simple linear regression forms. As a result, the moving times can be obtained. Because

the models only used the AVL historical data, it is suitable for off-line estimations.

- ***Bus stop-to-stop line model:*** Son *et al.*, (2004) developed a real-time prediction model to project both the travel times from a bus stop to a stop line at an intersection, and from stop line to a downstream bus stop. The arrival time at the first stop and the location of the bus were recorded by the AVL system mounted on the bus ITS devices. Two Kalman filters were used to predict these travel times. The predicted arrival time at the second stop is the sum of the two predicted travel times plus the waiting times at the signal.
- ***GPS map-based model:*** Weigang *et al.*, (2004) developed models for estimating bus arrival time and bus speeds based on the GIS map. The mechanism behind the models is that the bus route is divided into a number of short, straight line sub-routes. The location of bus relayed by GPS system then is matched to find the bus position in the GIS map and an algorithm was developed to make an accurate prediction for the distance from the tracked bus position. If bus speed information is available, then the running time is simply projected by dividing the distance by the speed.

2.2.3 Bus Dwell time Prediction Models

Most of the models developed in the past considered dwell time as a delay and explored the influencing factors on dwell time. Prior to the applications of the APC system to bus transit, because of high cost and labor-power in data collection, studies at that time focused mostly on the relationship between dwell time and the number of passengers boarding and alighting which were counted manually at point checks.

Back in 1974, Kraft and Berger found that the time for serving one boarding passenger is about 4.5 seconds and dwell time for boarding is 2 seconds plus time for serving M boarding passengers (Kraft *et al.*, 1974).

Having studied dwell time and passenger activities in Lafayette, Indiana (USA), Guenther *et al.*, (1983) generated some conclusions about dwell time:

- The number of passengers boarding and alighting at each stop follows Poisson distribution only when they are relatively low.
 - Negative binomial distribution is acceptable as a descriptor of boarding and alighting passengers under all volume conditions.
 - The assumption that dwell time per passenger is independent of the number of passengers boarding and alighting may be erroneous.
 - The relationship between average dwell time per passenger and the total number of boarding and alighting passengers at a bus stop was found as
- $$\varepsilon = 5.0 - 1.2[\ln(z)] \quad (2.1)$$

Where: z is the number of boarders and alighters at a stop

Another model developed for bus dwell time estimation can be found in the works of Senevirate (1988). The author found that the Gamma distribution fits data of boarding and alighting time distribution better than the Normal and the log-normal distributions did.

In the early 1990s, the equations relating dwell time to the number of passengers as well as other data collected by the APC and AVL systems such as, fare box type, number of vehicle's doors, lifting operations, bus schedule adherence performances, bus locations, and bus speed were first developed.

Marshall *et al.*, (1990) developed six regression models (four linear and two exponential regressions) to relate dwell time to the number of passengers, the fare collection methods (coins, bill) and the bus induced-delay. The authors concluded that exponential regressions were suitable and did not create negative dwell time when no boarding and alighting passenger are recorded.

Levine *et al.*, (1994) considered dwell time to consist of five components (boarding and alighting time, waiting time for other passenger activities, waiting time by buses that advanced ahead of schedule, waiting time for re-entry to traffic stream, and time for attending equipment). The Levine's models were mainly devoted to low-floor-bus boarding time. The findings showed that a low- floor bus saves on the average 17.5 sec per full bus per stop, 0.16 sec per passenger with unimpeded movement and 5.78 sec per passenger using mobility aids.

The Highway Capacity Manual (HCM) 2000 suggested a formula for the calculation of dwell time as follows

$$D = (P_a)(t_a) + (P_b)(t_b) + t_{op} \quad (2.2)$$

where: D = dwell time in seconds
 P_a = Number of alighting passengers through the busiest door
 t_a = Passenger alighting time per passenger
 P_b = Number of boarding passengers through the busiest door
 t_b = Passenger boarding time per passenger
 t_{op} = Door opening and closing time

Rajbhandari *et al.*, (2003) used statistical regressions to generate four models for dwell time as a function of the number of boarding and alighting passengers, and the number of standees. Like other authors, Rajbhandari found that the dwell time has a non-linear relation with the number of passengers as shown below

$$DT = a + b (Total) + c (Total) (S) \quad (2.3)$$

where: a, b, c = are regression parameters

Total = Number of boarding and alighting passengers

S = Number of standees

DT= Dwell time.

Dueker *et al.*, (2004) collected data by using the APC system in TriMet transit system for developing multiple regression models. The determinants of dwell time include passenger activity, lift operation and other effects.

Bertini and Geneidy, (2004) also used data from TriMet to model the dwell time. The data were 452 stop samples of 14 trips of bus route 14. The authors found that dwell time and passenger activities are related as follows:

$$\text{Dwell time (s)} = 5.8 + 0.85 N_a + 3.6 N_b \quad (2.4)$$

Where: N_a , N_b is the number of boarding and alighting passengers, respectively

Shalaby *et al.*, (2004) proposed a Kalman filter to predict the passenger arrival rate based on the historical data of the number of passengers arriving at a stop. Once the passenger arrival rate has been predicted, it is assumed to be constant during the headway. Thus, the predicted number of boarding passengers is the product of the headway and the predicted passenger arrival rate. Dwell time is then calculated by multiplying the predicted boarding passengers and an average serving time per boarder.

Fang and Min (2005) generated a model to estimate the dwell time. First, the author surveyed the distribution of passengers boarding and alighting times via front door and the rear doors. Second, they developed several probabilistic distributions (e.g., log-normal, Gamma, and Weibull) to fit the frequency distribution. The probability of the service time duration for both alighting and boarding is predicted by using Monte-Carlo simulation.

2.2.4 Media and Communication System

Media and communication system play an important role in the success of a RETBAIS. Real-time arrivals will be useless if they cannot be disseminated immediately to passengers. A bus fleet well-equipped with APC and AVL systems will not work efficiently if the data collected cannot be transmitted sufficiently and dynamically to the control center. This subject is discussed in two parts: information dissemination media for transit users and communication system for transit providers.

2.2.4.1 Information Dissemination Media

Printed medium is the simplest way transit agencies provide information to passengers. Besides this traditional method, passengers can access the telephone system to get traveller information. The innovations in information technologies now enable transit agencies to have various media to distribute information to passengers. Currently, passengers can get transit information from cell phones, PDAs, pagers, Palm Pilots, internet capable mobile phones, kiosks, video monitors, DMSs, and the Internet.

The DMSs and video monitors are prevalent at bus stops. The DMSs and Video monitors are non-interactive displays where passengers can get only en-route or in-vehicle information if such devices are installed in the buses. The information shown on the electronic signs usually includes: time and date, route number and destination, waiting time (countdown or time range), and service interruption. Kiosk, internet capable cell phones, and the Internet allow passengers to get information actively. Therefore, passengers can get all kind of information they need (pre-trip information, en-route and in-vehicle information).

Providing information via Palm pilots, pagers, PDAs and cell phones is a preferable method that transit agencies aim at because such devices are personal and wireless. Therefore, they enable passengers to get real-time information anytime and anywhere at low cost.

Some current deployments of dissemination media are listed in Table 2.1. (Further details can be seen in TCRP Report 92, 2003).

2.2.4.2 Communication System

Most of agencies use radio system to transmit data. As the demand for real-time information is increasing, the requirements for data transmission are on the rise. This can cause “overcrowding of spectrum” placed on the available radio channels dedicated for the AVL system. Therefore, the agencies are looking for the alternative solutions such as using “spread spectrum”, narrowing radio communication band, renting commercial radio service, using infrared or high capacity frequency link (John, 1999).

2.3 Summary

In many transit agencies, AVL and APC have long been used for increasing operational efficiency. However, it is in recent years that the agencies have recognized the benefits of applying AVL, APC and other advanced technologies for the provision of passenger real-time information.

The use of collected APC data for off-line analysis has been the practice in transit agencies, but the AVL system was not considered as an objective. Many agencies installed AVL systems for their bus fleet, but cannot provide real-time arrival information to passengers. They use the AVL system only for incident response while they use manual check for on-time performance estimation and running time analysis. The APC systems have been used for off-line studies only in spite of the fact that these can provide valuable data for on-line applications.

Bus arrival prediction models reported in literature very seldom use the collected APC data for real-time prediction while it is known that an integration of APC and AVL system with radio systems can meet the real-time information requirements. Almost all of the prediction models are based on the GPS-based AVL data and other diverse sources of data.

None of the arrival prediction models based on AVL data predict dwell times and include them in the total predicted trip time. This may cause inaccurate predictions of arrival for downstream stops when the bus serves crowded upstream stops.

Chapter 3

MECHANISM OF THE PROPOSED MODEL AND AVL-APC DATA COLLECTION

3.1 Introduction

Following the examination of the previous works involved in real-time bus running time and dwell predictions, this chapter presents the framework of the developed model for real-time bus arrival predictions and its building blocks. This chapter is organized as follows: First, the components of a bus trip time are investigated in order to emphasize the roles of running time and dwell time in bus trip time analyses and predictions. Second, various modules are described and their integration into the complete model is covered. Last, the study design including the process of APC and AVL data collection and data descriptions are discussed.

3.2 The Components of Bus Trip Time and Influencing Factors

To date, many models and algorithms have been developed to investigate the factors that influence bus running time and delay. The most influencing factors found in studies are shown in Table 3.1. The identified factors influence different components of the bus trip time. Therefore, in analyzing and estimating accurately the components of the

bus trip time, causal factors should be studied, given that their effects will ultimately influence actual bus operation on the route.

Table 3.1: Factors that influence Bus Running Time and Delay

Studies	Factors
Afdail <i>et al.</i> , (1988) Abdelfattah and Khan (1998)	<ul style="list-style-type: none"> - Number of bus stops - Link length - Number of traffic lights - Densities, left turning and through vehicles - Proportion of heavy vehicles in mixed-traffic stream
Abkowitz and Engelstein (1984)	<ul style="list-style-type: none"> - The number of passengers boarding or alighting at stops.
Senevirante (1988) Patnaik <i>et al.</i> , (2004)	<ul style="list-style-type: none"> - Dwell time - Service characteristics
McKnight <i>et al.</i> , (2003)	<ul style="list-style-type: none"> - Traffic congestion

According to Maloney and Boyle (1999), a bus trip time includes the following components: (1) actual moving times between stops; (2) dwell time at stops; (3) traffic signal delay; (4) general traffic delays, and (5) recovery time at the end of each trip.

Figure 3.1: Bus Trip Time Components
(Modified from Manoley *et al.*, 1999)

Maloney and Boyle (1999) showed that actual moving time between stops is the primary component accounting for overall 59 percent of total trip time, followed by recovery time (13%), deadhead time (9%), time maneuvering to access and leave the stop (7%), dwell time (7%), and signal delay (5%) (see Figure 3.1). The following paragraphs briefly discuss each component of the bus trip time.

3.2.1 Actual Moving Time

Actual moving time of a bus is the time that the bus is in motion. It is heavily influenced by three main factors: schedule, distance between stops, and speed of the bus.

A short schedule will force bus drivers to speed up whenever possible and rush to the stops. Therefore, a short schedule may shorten the moving time of the bus in some cases, but may raise safety concerns on the road and put pressure on the driver and the dispatcher to keep the schedule adherence. In contrast, a long schedule may lead the bus to come to the stops earlier than scheduled time. If the arrivals are consistently early, the moving times may be increased intentionally by operators (i.e., to slow down the bus). As a result, passengers will spend more time in underutilized buses (Senevirante, 1990).

The spaces between bus stops also influence moving time. A short distance does not permit the attainable cruise speed of buses since the available distance between stops is used for deceleration and acceleration operations. On the other hand, Levison (1983) pointed out that the fewer the bus stops, the greater the number of passengers waiting at downstream stop. Therefore, elimination of some stops will enhance average speed but can increase access time for passengers. However, Levison found that on the basis of

statistical analysis, reducing the number of stops did not influence the average speed to a high degree given the fact of traffic signals.

The speed of the bus is an important factor that influences the bus movement time. However, speed is affected by many factors. Traffic densities and congestions are the major causes of low speeds of buses running in mixed traffic bus routes in the downtown area. For instant, the kilometers travelled by a bus in one hour decreases significantly from 22.5 km/h in suburban area to 16 km/h in the city, and 8 km/h in the central business district (CBD) (Levinson, 1983). The speed of bus during the rush hour periods was 1.4 to 1.6 times as slow as that of the car (Levinson, 1983) or equal to 42-59 percent of car's speeds (McKnight *et al.*, 1997). In a manner similar to the effect of the number of bus stops, the number of traffic signals also reduces the speed of the buses when they approach and leave the traffic signals.

3.2.2 Dwell Time

Dwell time is the time required for the bus at stop to serve at the busiest door plus time for closing and opening door (HCM 2000). Major factors that affect dwell time found by many studies are the number of boarding and alighting passengers, bus stop spacing, fare collection method, type of bus, and type of on-board circulation. However, the relationship between dwell time and the number of boarding and alighting passengers was considered by many researchers to be sufficient for their purposes. Many models have been developed to investigate this relationship. They vary from simple descriptive statistical models (Bertini *et al.*, 2004; Rajat *et al.*, 2003; Lin and Wilson, 1992), to probabilistic models such as Poisson model (Chien *et al.*, 2000), Gamma model

(Senevirante, 1988), natural logarithm model (Guenthner *et al.*, 1983), and exponential model (Leo *et al.*, 1990). The previous works on dwell time shows another trend, namely to project dwell time based on the relationships between dwell time and the bus service characteristics. The models belonging to this trend can be found in following publications (Kraft *et al.*, 1974; Kostopolous *et al.*, 1985; Dueker *et al.*, 2004). These models are the multiple regressions where dwell time is the dependent variable and types of fare collection, number of doors, and on time performance are used as independent variables.

3.2.3 Traffic Signal Delay

Traffic signal delay accounts for a large proportion (i.e., up to 32- 45 percent) of total delay (Levison, 1986). Most of this delay is due to waiting and queuing at signalized intersections. In the Highway Capacity Manual 2000 (TRB 2000), a bus in a traffic flow is converted into equivalent passenger car units so that an average delay per queuing vehicle is calculated. This value can be considered as the delay of a bus at a signalized intersection.

3.2.4 General Delay

General delay refers to delays caused by traffic congestion, crowded bus berths, and lane changing maneuvers. It is fair to say that bus travel time delay is not only caused by ordinary vehicles sharing the lane with buses but also by the buses themselves. Slow and large buses in heavy traffic stream, like other heavy vehicles, cause car drivers to avoid travelling in the same lane and consequently result in intricate lane changing

maneuvers. Hence, this characteristic of traffic flow increases delay. According to Ubitran *et al.*, (1986), traffic congestion has a severe adverse impact on bus travel time. So far, very few studies have been found in the literature that examined the direct impacts of congestion on bus travel time. Currently, MacKnigh *et al.*, (2003) developed models to quantify the impacts of congestion on bus travel time in their studies of 39 bus routes in Northern New Jersey (USA). The following is such a model:

$$dBTT_{r_ff} = 0.73(TTT_{r_a} - TTT_{r_ff}) \quad (3.1)$$

Where:

$dBTT_{r_ff}$ = Change in travel time rate (minutes per mile) due to congestion for buses on route

TTT_{r_a} = Travel time rate for traffic under existing conditions

TTT_{r_ff} = Travel time rate for traffic under free flow conditions

0.73 = Statistical parameter resulted from calibration study

To quickly estimate the actual travel time rate, travel time rate indices (TRIs), which are the extra amounts of time (in terms of minutes/mile) for travel over a link, were used.

$$TRI_s = TTT_{r_a} / TTT_{r_ff} \text{ or } TTT_{r_a} = TRI_s \cdot TTT_{r_ff} \quad (3.2)$$

From Equations 3.1 & 3.2, bus travel time increment due to congestion is as below.

$$dBTT_{r_ff} = 0.73TTT_{r_ff}(TRI_s - 1) \quad (3.3)$$

2

3.2.5 Recovery Time

Recovery time is the required time for layover of a bus driver at the end of each trip. The length of recovery time is based on the labor agreement with bus drivers and it is usually equal to at least 10 percent of a round trip time (Pile *et al.*, 1998).

Obviously, if buses are running on the exclusive bus route or on the Transitway, many factors influencing the travel time components under mixed operations are eliminated. In contrast, if buses are running in the mixed-traffic bus lanes where other vehicles share the same lane with buses, the buses are vulnerable to delays caused by the many frictions such as traffic volume, incidents, curb parking operations, etc. For this reason, predicting the influences on the components of a bus trip in mixed traffic bus route will continue to be a challenge. For information, it should be noted that mixed-traffic bus operation accounts for almost 99 percent of total bus route distances in North America (Kittelson and Associates Inc., 2003).

3.3 Proposed Model Structures and Components

3.3.1 General Discussion

As discussed above, bus travel time is complex and is influenced by many stochastic factors. Hence, to predict bus travel time, one should consider all bus trip time components. The recovery time component is an exception which does not influence the accuracy of bus arrival time predictions. Other identified components play a role in the success of a bus arrival prediction model. If any of these components is not adequately

modelled, one may get poor predictions of bus arrivals, especially when the bus operates in the mixed traffic route during rush hours.

The models reported in literature can be divided into three groups. In the first group are the models which aggregate all components (e.g., actual moving time, general delays, traffic signal delays, and dwell times) into one variable called running time and predict its value. These models can be found in the works of Chen *et al.* (2004), Chien *et al.* (2002), Abdelfattah *et al.* (1998), and Kalaputapu *et al.* (1995). The second group includes the models that integrate moving time, general delays and either traffic signal or dwell time into the running time. The bus arrival prediction is the sum of that running time and estimated traffic signal delays or dwell time at stops. Please refer to the works of Shalaby *et al.* (2004), Son *et al.* (2004), Lin and Zen (1999), and Horbury (1999) for further details of these models. The last group includes the models that take into account only one main component of bus travel time, the actual moving time. These models are based only on the data of the actual location of the bus provided by the GPS-based AVL system and make the prediction of a new location of the bus in the next time step. Some of these models can be found in the publications of Dailey *et al.* (2001), Cathey *et al.* (2003), and Weigang *et al.* (2004).

Each group has drawbacks and advantages. The models in the first group do not require as elaborate data as required by the second group. However, because of the aggregation, these models cannot capture the effects of one component on another. For example, a long moving time can cause a crowded downstream stop(s) and this, in turn, results in a long dwell time contributing to a late arrival at the following stops. On the contrary, the models in the second group which separate dwell time as an independent

component can differentiate the effects. Consequently, they are more flexible than the first ones. Because of their flexibility, such models may be applicable for both arrival time prediction and dynamic bus control actions. The models in the last group were advanced for tracking the dynamic bus locations. However, based only on the locations of the bus and ignoring dwell time and other components of a bus trip time, the accuracy of these models is good for an arrival prediction of one stop ahead but it may deteriorate at the downstream stops after that stop.

The model proposed in this study takes into account the advantages of different models aforementioned. In order to use data collectable by APC and AVL systems, moving times, traffic signal delays and general delays are integrated into running times while dwell time is treated separately. The proposed model has three modules. The first module is for predicting the running time, the second module is for predicting dwell time, and the last module is for model application. By separating dwell time and running time prediction, the proposed model can capture the effects of running time on dwell time and vice versa. Also, basing on the latest updates of the dynamic locations of the bus, each module will adjust its predictions at every time step. Therefore, the whole model can be applied for real-time predictions.

3.3.2 Assumptions

The following assumptions are made in this research study:

- A bus route under consideration is composed of links and bus stops/time-points where a bus link is a route section whose two ends are two bus stops or timepoints.
- All buses in the bus route are equipped with APC, AVL and radio systems.

- Each time a bus arrives at and departs from a stop, the following data are recorded and transmitted dynamically to the control center: location of the bus at this stop, the bus trip ID, the times the bus arrives and departs the stop, the number of passengers boarding and alighting at the stop, and the running time the bus travelled on the previous link.

Figure 3.2 presents a simplified sketch of bus operation on a route.

Figure 3.2: A Simplified Sketch of Bus Operation on a route

3.3.3 Structure and the Building Modules of the Proposed Model

As mentioned above, to take advantages of the previous models, the proposed model for bus arrival time prediction in this study is the combination of the three modules. The first module predicts running times and is therefore called Running Time

prediction Module (RTM). The second module predicts dwell times, and it is called Dwell Time prediction Module (DTM). The last module predicts bus punctuality and bunching, and it is called Punctuality and Bunching prediction Module (PBM).

Running times in a bus link are predicted by the first module (RTM) in the following manner. Given a current running time situation, the historical data of running times of each link are used as a basis to predict running times. Each time the bus comes to a stop or when the data collected by APC and AVL become available, the most updated data are used to predict new bus running times. These data are also stored to update historical data. The running time prediction module is described in details in chapter 4 and 6.

Dwell time and passenger occupancy prediction module (DTM) is based on the passenger activities recorded at the stops (i.e., both historical data and the current data). The proposed dwell time prediction module is presented in chapter 5 and 6.

The punctuality and bunching prediction module (PBM) is described in chapter 7. In this module, a method to estimate prediction interval is to be examined. Based on this method and by using probability theory, two algorithms are developed for bus bunching and bus on-time detection.

In general, to predict bus arrival/departure time at stop $i+1$, given the recorded departure time of the bus at previous stop (i) denoted as $Dp_{actual, i}$, the first two modules (i.e. RTM and DTM) are connected together by two simple general equations:

$$Ar_{i+1} = Dp_{actual, i} + r_{i_i+1} \quad (3.4)$$

$$Dp_{i+1} = Ar_{i+1} + dwell_{i+1} \quad (3.5)$$

Where:

Ar_{i+1} is the predicted arrival time at stop $i+1$

Dp_{i+1} is the predicted departure time at stop $i+1$

$Dp_{actual,i}$ is the actual departure time of the bus at stop i

r_{i_i+1} = predicted running time the bus will spend on the link between two stops, obtained by RTM.

$dwell_{i+1}$ = predicted dwell time of the bus at stop $i+1$, obtained by DTM

Once the arrival prediction has been made for a bus stop, this value will be compared to the scheduled arrival time that the bus is supposed to be at that stop. This predicted time difference will indicate if the bus is likely to be early, on-time, or late. Based on this information and other predicted variables, the PBM will provide the best suggestions to bus dispatcher in support of decision making.

3.3.4 The Mechanism of The Proposed Model

Figure 3.3 shows the process of real-time prediction of the proposed model. For detailed descriptions, let us assume that the bus route under consideration has n bus stops denoted as $STOP_i$ (where $i=1$ to n) and bus k has just arrived at bus stop $STOP_i$. We call the actual time that bus arrived at stop i as $Ar_{actual,i}$ and the actual time the bus left the stop as $Dp_{actual,i}$. It should be noted that the time $Ar_{actual,i}$ may be different from the scheduled time Sc_i of bus k at stop i due to travel time variations. Let us denote Ar_j and Dp_j as the predicted arrival time and departure time of bus k at a downstream stop j (where $j= i+1$ to n).

Figure 3.3: Process of Real-time Prediction

To predict the arrival times of bus k at downstream stops (e.g., $i+1$, $i+2 \dots n$), the following steps are applied.

1. Initial Arrival Predictions:

- Running time module will predict running times of bus k on all downstream links (i.e., $LINK_{i_{i+1}}$, $LINK_{i+1_{i+2}} \dots LINK_{n-1_n}$). The predicted running times are coded as $r_{i_{i+1}}$, $r_{i+1_{i+2}} \dots r_{n-1_n}$.
- Dwell time module will predict dwell times of bus k at all downstream stops (i.e., $STOP_{i+1}$, $STOP_{i+2} \dots STOP_n$). The predicted dwell times are denoted as: $dwell_{i+1}$, $dwell_{i+2} \dots dwell_n$.
- Because the actual departure time at stop i was recorded ($Dp_{actual, i}$), the predicted arrival of bus k at the next stop ($i+1$) is

$$Ar_{i+1} = Dp_{actual, i} + r_{i_{i+1}} \quad (3.6)$$

- The predicted arrival time of bus k at stop $i+2$ then will be calculated

$$Ar_{i+2} = Dp_{i+1} + r_{i+1_{i+2}}. \quad (3.7)$$

$$Dp_{i+1} = Ar_{i+1} + dwell_{i+1}. \quad (3.8)$$

Substitute Dp_{i+1} of Equation 3.8 into Equation 3.7, so we have:

$$Ar_{i+2} = Ar_{i+1} + dwell_{i+1} + r_{i+1_{i+2}} \quad (3.9)$$

Substitute Ar_{i+1} of Equation 3.6 into Equation 3.9, so we have:

$$Ar_{i+2} = Dp_{actual, i} + r_{i_{i+1}} + dwell_{i+1} + r_{i+1_{i+2}} \quad (3.10)$$

- The same mathematical operation is applied for all other stops. For example, the predicted arrival time of bus k at stop $i+3$ will be calculated as:

$$Ar_{i+3} = Dp_{i+2} + r_{i+2_i+3} \quad (3.11)$$

$$Dp_{i+2} = Ar_{i+2} + dwell_{i+2} \quad (3.12)$$

Substitute Dp_{i+2} of Equation 3.12 into Equation 3.11. We have:

$$Ar_{i+3} = Dp_{actual, i} + r_{i_i+1} + dwell_{i+1} + r_{i+1_i+2} + dwell_{i+2} + r_{i+2_i+3} \quad (3.13)$$

Hence, we have initial predicted arrivals of bus k at all downstream stops even when bus k is currently at an upstream stop.

2. Update Arrival Predictions

The predicted travel time to each down stream stop will be updated when the most recent bus arrival information becomes available at every h seconds. For example, as the bus departs from stop 1, the running time prediction module, together with dwell time prediction module, predicts arrival times from first stop to all downstream stops in the fashion as mentioned above. By the time the bus arrives at stop 2, new updated arrival information, running times and passenger activities of the buses on the bus route have already been collected by the AVL and APC systems. These data then will be used by the two modules to make new predictions of running times and dwell times. As a result, the arrival predictions will be updated.

In general, when bus k comes to the next stop (e.g., stop $i+1$), the new actual arrival time and departure time were recorded as $Ar_{actual, i+1}$ and $Dp_{actual, i+1}$, respectively. New updated arrival predictions then will be calculated at time step $Dp_{actual, i+1}$ as follows:

- Running time module will use updated information to predict new running times of bus k on all downstream links after stop $i+1$ (e.g., $LINK_{i+1_i+2}, LINK_{i+2_i+3} \dots, LINK_{n-1_n}$). The new predicted running times are coded as $r^{(1)}_{i+1_i+2}, r^{(1)}_{i+2_i+3} \dots, r^{(1)}_{n-1_n}$. The superscripts attached with each variables indicates the number of times the predicted running times have been updated. In this case, the running times have been updated for the first time.
- Dwell time module will use the most updated information to predict dwell times of bus k at all downstream stops (e.g., $STOP_{i+2}, STOP_{i+3} \dots, STOP_n$). The new predicted dwell times are denoted as: $dwell^{(1)}_{i+2}, \dots, dwell^{(1)}_n$. Like running time variables, the superscripts indicate the number of times that dwell times have been updated.
- New updated arrival prediction at stop $(i+2)$ will be calculated as follows

$$Ar^{(1)}_{i+2} = Dp_{actual,i+1} + r^{(1)}_{i+1_i+2} \quad (3.14)$$

The same mathematical operation will be applied to update the arrival predictions for all downstream stops. For example, the updated arrival time of bus k at stop $i+3$ will be as follows:

$$Ar^{(1)}_{i+3} = Dp^{(1)}_{i+2} + r^{(1)}_{i+2_i+3} \quad (3.15)$$

$$Dp^{(1)}_{i+2} = Ar^{(1)}_{i+2} + dwell^{(1)}_{i+2} \quad (3.16)$$

Substituting Equations 3.14 and 3.16 into Equation 3.15 gives

$$Ar^{(1)}_{i+3} = Dp_{actual,i+1} + r^{(1)}_{i+1_i+2} + dwell^{(1)}_{i+2} + r^{(1)}_{i+2_i+3} \quad (3.17)$$

- The same procedures will be applied every time the bus actually comes to a downstream stop (e.g., stop $i+2$, $i+3 \dots, n$). For example, the prediction on arrival time of bus k at stop $(i+3)$ is updated when bus k arrives at next stop (i.e. stop $i+2$) as shown below.

$$Ar^{(2)}_{i+3} = Dp_{actual, i+2} + r^{(2)}_{i+2_i+3} \quad (3.18)$$

If we compare the first updated value of predicted arrival time at stop $(i+3)$ (Eq. 3.17), and that of the initial prediction (Eq.3.13), we can see that the new value is updated based on the two most recent updated information. First, it is based on the actual departure time recorded at new stop $Dp_{actual, i+1}$. Therefore, the model can capture the adverse effects of an incident that may cause a late departure at stop $(i+1)$ when the bus was running in the $LINK_{i_i+1}$. Second, it is based on the first time updated running times (i.e. $r^{(1)}_{i+1_i+2}$, $r^{(1)}_{i+2_i+3}$) and dwell time $dwell^{(1)}_{i+2}$. These variables are updated by the RTM and DTM, given the newest data collected from the APC and AVL systems. Once again, if we compare the first updated arrival time at stop $(i+3)$ in Equation 3.17 and the second updated one in Equation 3.18, we can see the second updated arrival based on the new actual departure time at stop $Dp_{actual, i+2}$ and the second updated running time $r^{(2)}_{i+2_i+3}$. Hence, the model can capture the dynamic changes in bus travel times.

3. Interpolate Arrival times at Skipped Stops

Following the mechanism presented above we can see that, at each time step, or in other words, at each time the bus left an upstream stop, say $Dp_{actual, i}$, the predictions of arrivals of this bus at all following stops will be made based on the actual recorded time

$Dp_{actual,i}$, the predicted running times r_j and the predicted dwell times $dwell_j$ ($j = i+1$ to n). A question can be asked as to how the proposed model predicts the arrivals at downstream stops when the bus comes to an upstream stop without stopping due to serving requirements? In practice, if the bus does not stop at a bus stop, neither location of the bus stop nor passenger activities at this stop are recorded. In this case, dwell time is definitely zero but the arrival time is not known. In case of a bus equipped with GPS-based AVL system, the interpolated time that the bus approaches the stop can be considered as the GPS time recorded when the bus was closest to this stop. However, if the bus did not have a GPS-based AVL system, (e.g., the bus has another AVL technology such as S/O-based AVL or DR-based AVL), the arrival time of the bus can be estimated as follows:

- Assume three bus stops A, B, and C and that bus k passed stop B without stopping.
- The previous buses had the running times in $LINK_{A_B}$ are $r(k-d)$ (where $d = 1, 2 \dots$). Approximately, the interpolated arrival of bus k will be

$$Ar(k)_{interpolated, B} = Dp(k)_{actual, A} + r(k)_{Average A_B} \quad (3.19)$$

Where: $r(k)_{Average A_B}$ is the average of the same-day historical running times of the previous buses running on the $LINK_{A_B}$ in one hour period.

3.4 APC and AVL Data Collection

Based on how the proposed model works (i.e. its mechanism) and its building elements, a study design containing the processes of data collection and data reduction was developed. It includes: data collection, data description, and data reduction.

3.4.1 Bus Route Selection

In order to develop a model capable of predicting real-time bus arrival under different operational scenarios such as buses operating during off-peak and peak-hour, on mixed traffic route or on exclusive bus lane, the selected bus routes have to cover these situations. In this study, data retrieved from APC and AVL systems of the OC Transpo were used. Two bus routes were selected as the study subjects; route 95 (Orleans-Fallow Field) and bus route 1 (South Keys-Ottawa Rockcliffe). The reasons behind the selections are:

- Bus route 95 is one of the three major bus routes in Ottawa (i.e. route 95, route 96 and route 97) and it operates 22 hours a day from 4:00 a.m. to 2:30 a.m. Almost all of the buses (95%) operating on this route are articulated buses.
- Almost all of the segments of route 95 are on the Transitway and the others are designated as exclusive bus lanes located on highway 417, Albert and Salter streets.
- Route 1 is a mixed-traffic bus route located almost entirely on Bank Street, an important radial road in the city of Ottawa.
- Both bus routes cross the central business district (CBD) of Ottawa.

Bus route 95 has 63 stops. It connects Orleans to South Nepean and is 31.4 kilometers in length. In this study, the route segment shown in Figure 3.4A from Blair to Lebreton station was selected because it contains both Transitway and exclusive bus lanes. This segment is 10.26 km long; it includes 26 stops and the average distance between two stops is 0.39 km. Out of 26 stops in this segment, seven stops were chosen and their data are collected and tabulated in Table 3.2. The schedule headways during

weekday vary from 3 to 5 minutes in peak-hour to 15 minutes after 10 p.m., and 30 minutes after midnight.

Figure 3.4A: Bus Route 95 (Blair-Lebreton)

Table 3.2: Distance between stops BLAIR and LEBRETON. Bus Route 95 (Orleans-Nepean South)

Stop ID	STOP DESCRIPTION	Distance between stops (km)	Accumulated Distances (km)
EE915	BLAIR 2B	0.00	0.00
EB 905	ST LAURENT 2B	2.79	2.79
AE900	TRAIN 2A	1.19	3.98
CE940	LEES 2B	2.16	6.14
CD910	MACKEZINE KING 2A*	1.87	8.01
CA920	ALBERT-KENT*	1.19	9.2
CJ 900	LEBRETON*	1.06	10.26

Note: * bus stop is in CBD area. Please see **Appendix A** for the map of route 95

Bus route 1 has 65 bus stops lying between Ottawa Rockcliffe and South Keys. It is a mixed-traffic bus route because it is designated mainly on Bank, Wellington, Rideau, St. Patrick and Beachwood streets.

Figure 3.4B: Bus route 1 (South Keys-Rideau)

Table 3.3: Distance between stops GREENSBORO and RIDEAU. Bus Route 1 (South Keys - Ottawa Rockliffe)

Stop ID	STOP DESCRIPTION	Distance between stops (km)	Accumulated Distances (km)
RF900	GREENSBORO 1A	0.00	0.00
RF075	BANK-JOHNSTON NS	3.19	3.19
RA190	BANK- WALKLEY	1.19	4.38
RA945	BILLINGS BRIDGE 4C	2.70	7.08
CH080	BANK -HOLMWOOD*	2.12	9.20
CF630	BANK- GLADSTONE*	1.55	10.75
CD 920	RIDEAU 3A*	2.1	12.85

Note: * bus stop is in CBD area. Please see **Appendix A** for the map of route 1

In this study, the segment connecting Greensboro stop and Rideau stop as shown in Figure 3.4B, is selected and seven bus stops were considered as timepoints (see Table 3.3). The schedule of bus route 1 varies depending upon the time of day, from 12 minutes in peak-hour period to 30 minutes after 7 p.m.

3.4.2 Data Sets

Data from the APC-AVL systems available from the OC Transpo for each selected stop were clustered by time periods in a day and seasons in a year. In this study,

only weekday data were of interest. Weekday peak period data are considered as the ones that record bus arrivals at stops from 8 a.m. to 10 a.m., 12:00 noon to 2 p.m., from 4 p.m. to 6 p.m. in the afternoon. Weekday evening is considered as the time of bus arrivals after 7 p.m. However, this study did not consider weekday evening.

Data were also clustered by season in order to capture the various passenger activities and on-time bus performance in different seasons. Four seasonal data sets, namely SEASON1 (January 1st to April 30th), SEASON2 (May 1st to 31st Jun), SEASON3 (July 1st to August 30th), and SEASON4 (September 1st to December 30th) were also clustered. The data for bus route 95 and bus route 1 were collected for one year, covering from Season 4 of 2004 to Season 4 of 2005. The number of observations during peak-periods in each sample is shown in Table 3.4. Each observation noted in Table 3.4 contains a set of information, including stop ID, route and route direction, time period and date, actual and schedule arrival, the number of passengers boarding and alighting at a stop, the number of passengers on-board when the bus leaves the stop or passes it. These data are synthesized in the tabular form called APC point check as shown in Figure 3.5.

Table 3.4: Data Sets for Bus Route 95 and Bus Route 1

Route	SEASON 1			SEASON 2			SEASON 3			SEASON 4		
	8-10 a.m.	12-2 p.m.	4- 6p.m.	8-10 a.m.	12-2 p.m.	4- 6p.m.	8-10 a.m.	12-2 p.m.	4- 6p.m.	8-10 a.m.	12-2 p.m.	4- 6p.m.
95	472	998	1220	890	744	931	849	494	700	592	450	600
1	513	335	189	291	203	113	297	210	130	389	312	150

Table 3.5: Data Collectable by APC and AVL systems

Collectable Data	Description
SCHED DEP TIME	<i>Scheduled departure time at specified section</i>
DATE	<i>The date on which the observation occurs</i>
TIME OF DAY	<i>Time period of bus operation</i>
DOOR OPENING TIME	<i>Bus door opening time</i>
DOOR CLOSING TIME	<i>Bus door closing time</i>
EXCESS TIME	<i>The portion of time spent at a stop other than servicing passengers; that is, the stop time at a stop excluding the dwell time</i>
ATC DEP TIME	<i>Actual departure time</i>
MOVING TIME	<i>Time spent Moving Between Stops; The proportion of time between departing the first stop and the arrival at the second stop-excluding any idle times, stop times, and stop- and- go times.</i>
STOPGO TIME	<i>The portion of the running time spent in a stop and go situation (stop time less than 10 seconds).</i>
IDLE TIME	<i>The portion of the running time spent in an idle situation.</i>
DWELL TIME	<i>The portion of running time spent in servicing passengers</i>
EXPECTED TIME	<i>The scheduled time at the stop, as obtained from the itinerary</i>
ACTUAL TIME	<i>The actual time that observed trip passed the stop.</i>
ONS 1, ONS 2, ONS 3	<i>The number of boarding passengers at the bus stop used front door, the first rear door, and the second rear door.</i>
OFFS 1, OFFS 2, OFF3	<i>The number of alighting passengers the bus stop at the stop used front door, the first rear door, and the second rear door.</i>
LOAD AT DEPARTURE	<i>The number of passengers on the bus as the bus leaves or passes the stop</i>
STOP DISTANCE	<i>Travel distance between two consecutive stops</i>
STATTIME	<i>Stationary time: The time the bus not in motion; vary from 10 to 45 seconds</i>
TRIP ID	<i>Unique index associated with a trip</i>
STOP ID	<i>Unique index associated with a bus stop</i>
TRIP STATUS	<i>Start or end a trip</i>
MATCH	<i>Unique index associated with a pattern in each pick data</i>
LAT	<i>GPS Latitude* (As of August 2005)</i>
LON	<i>GPS Longitude*(As of August 2005)</i>

Note: * Not available

Another type of data that records the passenger activity at bus doors is also collected by the OC Transpo. Figure 3.6 presents an example of such data. There are 6780 and 2800 recorded observations for bus route 95 and bus route 1 for all selected stops, respectively.

BOOKING	TRIP_ID	DATE	STOP	LOSTYPE	MATCH	ACTTIME	EXPTIME	STATIME	DWELLTIME	RTENS	LOADEP	ON1	OFF1	ON2	OFF2	ON3	OFF3
APR05	1000	5/02/2015 0:00	EEB15	1	E	00010	00040	21	18	178	23	8	1	1	1	2	0
APR05	2000	5/04/2015 0:00	EEB15	1	E	50005	57000	50	53	178	42	26	0	1	3	0	14
APR05	3000	5/17/2015 0:00	EEB15	1	E	57000	57000	02	54	177	46	15	3	9	4	6	1
APR05	2000	0/25/2015 0:00	EEB15	1	E	57005	57000	42	33	175	38	13	0	5	7	6	5
APR05	2000	4/27/2015 0:00	EEB15	1	E	05075	05100	43	35	175	54	8	1	7	5	2	3
APR05	2000	5/5/2015 0:00	EEB15	1	E	05100	05100	10	11	1	20	3	0	0	3	0	3
APR05	2042	4/28/2015 0:00	EEB15	1	E	02370	02340	31	29	175	32	12	0	2	2	4	0
APR05	2042	5/4/2015 0:00	EEB15	1	E	02205	02340	30	15	177	19	7	0	4	3	0	0
APR05	2053	5/30/2015 0:00	EEB15	1	E	01050	01020	46	30	2	57	13	1	7	0	9	2
APR05	2007	4/25/2015 0:00	EEB15	1	E	50025	50400	30	26	179	54	8	2	10	4	5	3
APR05	2000	0/17/2015 0:00	EEB15	1	E	03015	03000	18	9	177	18	4	2	2	1	2	1
APR05	2000	5/17/2015 0:00	EEB15	1	E	03480	03000	38	61	279	15	5	0	0	0	0	0
APR05	2000	5/10/2015 0:00	EEB15	1	E	03000	03000	38	34	174	07	14	0	8	2	4	8
APR05	2000	5/02/2015 0:00	EEB15	1	E	03535	03000	38	13	279	23	5	0	0	1	0	2
APR05	2075	5/11/2015 0:00	EEB15	1	E	04410	04300	24	20	178	14	13	0	0	0	0	0

Figure 3.6: Structure of APC Data on Passenger Activity

3.4.4 Data Characteristics

Before developing the model, it is necessary to analyze the acquired data, so that we can obtain the general characteristics of arrival times and dwell times at each bus stop in different period of time. In order to do that, descriptive statistics were applied.

3.4.4.1 Arrival time

Arrival time was analyzed in term of bus on-time performance, measuring the differences between the schedule and actual arrival. The data were collected at each bus stop and clustered by time of day and seasonal time (e.g., SEASON 1, SEASON 2, SEASON 3, and SEASON 4). A positive value of difference indicates the bus was behind the schedule, or lateness, while a negative one represents earliness.

Two periods of time in the year 2005 were chosen to consider the effects of weather on bus on-time performance. The WINTER data set (Jan 3rd to Apr 23rd, 2005) represents winter conditions and the SUMMER data set (Jun 26th to September 4th, 2005) accounts for summer conditions. The means and standard deviations of lateness and earliness of buses on bus route 1 in the two seasons are shown in Tables 3.6 and 3.7 while that of bus route 95 are shown in Tables 3.8 and 3.9.

Table 3.6: Means and Standard Deviations of Bus on-time Performance in the winter. Bus Route 1 (Greensboro – Rideau)

Time Period	Stop Name	Mean (minute)	Standard Deviation (Minute)	Number of observations
8 a.m.-10 a.m. (WINTER)	GREENSBORO 1A	-0.02	1.38	51
	BANK-JOHNSTON NS	-0.88	1.48	49
	BANK- WALKLEY	-1.59	1.47	56
	BILLINGS BRIDGE 4C	0.06	1.90	67
	BANK –HOLMWOOD	2.02	2.40	78
	BANK- GLADSTONE	4.71	2.55	81
	RIDEAU 3A	3.17	3.43	53
12 noon -2 p.m. (WINTER)	GREENSBORO 1A	1.07	2.15	45
	BANK-JOHNSTON NS	-0.63	2.66	43
	BANK- WALKLEY	-0.64	3.02	42
	BILLINGS BRIDGE 4C	1.05	2.71	42
	BANK –HOLMWOOD	1.63	3.11	40
	BANK- GLADSTONE	4.65	3.72	40
	RIDEAU 3A	4.87	3.84	49
4 p.m. - 6 p.m. (WINTER)	GREENSBORO 1A	0.67	1.39	20
	BANK-JOHNSTON NS	-0.87	3.03	23
	BANK- WALKLEY	-0.83	2.93	24
	BILLINGS BRIDGE 4C	0.54	3.16	24
	BANK –HOLMWOOD	2.42	3.73	24
	BANK- GLADSTONE	4.52	3.80	23
	RIDEAU 3A	4.17	4.61	28

Table 3.7: Means and Standard Deviations of Bus on-time Performance in the Summer. Bus Route 1 (Greensboro – Rideau)

Time Period	Stop Name	Mean (minute)	Standard Deviation (Minute)	Number of observations
8 a.m.-10 a.m. (SUMMER)	GREENSBORO 1A	0.66	1.21	32
	BANK-JOHNSTON NS	-0.91	1.81	33
	BANK- WALKLEY	-1.88	1.95	34
	BILLINGS BRIDGE 4C	-0.27	1.99	49
	BANK –HOLMWOOD	2.17	2.73	48
	BANK- GLADSTONE	5.23	3.39	47
	RIDEAU 3A	3.55	3.73	45
12 noon -2p.m. (SUMMER)	GREENSBORO 1A	0.96	2.33	28
	BANK-JOHNSTON NS	-0.41	3.07	27
	BANK- WALKLEY	-0.11	3.39	27
	BILLINGS BRIDGE 4C	1.26	3.38	27
	BANK –HOLMWOOD	1.48	2.67	27
	BANK- GLADSTONE	5.30	3.39	27
	RIDEAU 3A	3.55	4.82	27
4 p.m.-6p.m. (SUMMER)	GREENSBORO 1A			
	BANK-JOHNSTON NS	-0.60	1.82	21
	BANK- WALKLEY	-0.5	1.93	20
	BILLINGS BRIDGE 4C	0.95	1.68	19
	BANK –HOLMWOOD	1.29	2.03	18
	BANK- GLADSTONE	3.65	2.17	18
	RIDEAU 3A	0.94	2.01	33

Table 3.8: Means and Standard Deviations of Bus on-time Performance in the Winter. Bus Route 95 (Blair – Lebreton)

Time period	Stop Name	Mean (minute)	Standard Deviation (minute)	Number of Observations
8a.m.-10 a.m. (WINTER)	BLAIR 2B	0.37	3.02	119
	ST LAURENT 2B	0.55	2.65	126
	TRAIN 2A	0.79	2.34	122
	LEES 2B	1.10	2.31	126
	MACKEZINE KING 2A	2.10	2.64	128
	ALBERT -KENT	2.36	2.78	129
	LEBRETON	0.95	3.04	134
12 noon-2 p.m. (WINTER)	BLAIR 2B	-0.59	1.66	111
	ST LAURENT 2B	-0.27	1.70	111
	TRAIN 2A	-0.03	2.63	107
	LEES 2B	-0.18	1.73	119
	MACKEZINE KING 2A	0.68	1.80	115
	ALBERT -KENT	0.99	1.98	138
	LEBRETON	-0.40	2.12	110
4 p.m.-6 p.m. (WINTER)	BLAIR 2B	-0.11	1.72	138
	ST LAURENT 2B	0.40	1.81	138
	TRAIN 2A	0.94	1.94	134
	LEES 2B	0.97	2.04	143
	MACKEZINE KING 2A	2.76	2.88	138
	ALBERT -KENT	3.20	2.50	143
	LEBRETON	0.59	2.66	143

Table 3.9: Means and Standard Deviations of Bus on-time Performance in the Summer. Bus route 95 (Blair – Lebreton)

Time period	Stop Name	Mean (Minute)	Standard Deviation (Minute)	Number of Observations
8 a.m.-10 a.m. (WINTER)	BLAIR 2B	0.17	0.162	98
	ST LAURENT 2B	0.47	1.68	95
	TRAIN 2A	0.78	1.67	98
	LEES 2B	0.85	1.80	99
	MACKEZINE KING 2A	1.71	1.87	103
	ALBERT -KENT	2.28	2.16	102
	LEBRETON	0.61	2.43	103
12 noon -2p.m. (WINTER)	BLAIR 2B	-0.19	1.64	59
	ST LAURENT 2B	0.43	1.70	61
	TRAIN 2A	0.97	1.67	61
	LEES 2B	0.62	1.55	63
	MACKEZINE KING 2A	1.43	1.65	61
	ALBERT -KENT	2.08	2.11	64
	LEBRETON	0.63	2.52	60
4 p.m-6 p.m. (WINTER)	BLAIR 2B	0.67	1.97	80
	ST LAURENT 2B	1.27	1.96	83
	TRAIN 2A	1.78	2.09	81
	LEES 2B	1.88	2.27	79
	MACKEZINE KING 2A	3.26	2.57	79
	ALBERT -KENT	3.63	2.88	80
	LEBRETON	0.90	2.87	84

From the tables, we can see that, in the same time period, the lateness of the stops located in the CBD area are always longer than that of the stops located outside the CBD. For route 1, the means of times decrease in the afternoon both in the winter and summer, implying that the adherence to the schedule is better in the afternoon. In contrast, for route 95, the better adherence to the schedule is found in the middle of the day.

Further analyses were carried out in the form of two-sample-t test technique. The means of bus on-time performances at the same bus stop were compared in terms of different time periods (e.g., 8 a.m. to 10 a.m., 12 noon to 2 p.m., and 4 p.m. to 6 p.m.) and various seasons (e.g., the winter and the summer). The results of the tests for bus

route 95 are shown in the Tables 3.10 to 3.12, and those of bus route 1 are presented in Tables 3.13 to 3.15.

**Table 3.10: Two-sample t statistics for Winter and Summer Time
Route 95- Morning period (8 a.m. to 10 a.m.)**

<i>H₀: m₁=m₂</i>			
<i>H_a: m₁≠ m₂</i>			
STOP	Two-sample t values (2 tail-test)		
	<i>t_l</i>	<i>df_l</i>	<i>T_l</i>
BLAIR 2B	<u>0.721</u>	98	1.984
ST LAURENT 2B	<u>0.268</u>	95	1.984
TRAIN 2A	<u>0.037</u>	98	1.984
LEES 2B	<u>0.912</u>	99	1.984
MACKEZINE KING	<u>1.312</u>	103	1.984
ALBERT -KENT	<u>0.307</u>	102	1.984
LEBRETON	<u>0.957</u>	103	1.984

Notes for tables (3.10) to (3.12)

t_l = t values resulted from the comparisons the means measured in the winter time to the mean measured in the summer time , given the same time period of day.

df_l = Degree of freedom corresponding with *t_l*

T_l = Critical T values with 0.05 level of significance corresponding with *df_l*.

Underlined values: Not significant at 0.05 level or cannot reject *H₀*. (*t* < *T*)

**Table 3.11: Two-sample t statistics for winter and summer time
Route 95- Noon period (12 a.m. to 2 p.m.)**

<i>H₀: m₁=m₂</i>			
<i>H_a: m₁≠ m₂</i>			
STOP	Two-sample t values (2 tail-test)		
	<i>t_l</i>	<i>df_l</i>	<i>T_l</i>
BLAIR 2B	<u>1.507</u>	59	2.001
ST LAURENT 2B	2.684	61	2.000
TRAIN 2A	3.726	63	2.000
LEES 2B	<u>1.749</u>	61	2.000
MACKEZINE KING	2.779	63	2.000
ALBERT -KENT	3.482	61	2.000
LEBRETON	2.689	64	2.000

**Table 3.12: Two-sample t statistics for Winter and Summer Time
Route 95- Afternoon period (4 p.m. to 6 p.m.)**

<i>H₀: $m_1 = m_2$</i>			
<i>H_a: $m_1 \neq m_2$</i>			
STOP	Two-sample t values (2 tail-test)		
	t_I	df_I	T_I
BLAIR 2B	2.949	80	1.990
ST LAURENT 2B	3.287	83	1.990
TRAIN 2A	2.933	81	1.990
LEES 2B	2.963	79	1.990
MACKEZINE KING	1.319	79	1.990
ALBERT -KENT	1.120	80	1.990
LEBRETON	0.816	84	1.990

For route 95 (i.e. the Transitway bus route), as shown in Tables 3.10 to 3.12, almost all of the means based on collected data at bus stops during the noon period (12 noon to 2 p.m.) and during the afternoon period (4 p.m. to 6 p.m.) are statistically significant in terms of difference between winter and summer at 0.05 level of significance. This means that the buses of route 95 adhered to the schedule differently depending upon seasons during such time periods. However, for the morning period (8 a.m. to 10 a.m.), such differences were not found. In other words, during the morning period, there is no difference in bus on-time performance between the winter and the summer.

Also, for bus route 1 (i.e. mixed traffic bus route), the test results in Tables 3.13 to 3.15 show that at 0.05 significance level, the null hypothesis (i.e., the means measured in the winter time are the same as the means recorded in the summer time) cannot be rejected, for any time period of day. These imply that there are no statistical differences in bus adherence to schedule between the winter and the summer time for the mixed-bus route.

**Table 3.13: Two-sample t statistics for Winter and Summer Time
Route 1- Morning period (from 8 a.m. to 10 a.m.)**

<i>H₀: m₁=m₂</i>			
<i>H_a: m₁≠ m₂</i>			
STOP	Two-sample t values (2 tail-test)		
	<i>t₁</i>	<i>df₁</i>	<i>T₁</i>
GREENSBORO 1A	2.359	32	2.038
BANK-JOHNSTON NS	0.079	33	2.036
BANK- WALKLEY	0.748	34	2.034
BILLINGS BRIDGE	1.978	49	2.009
BANK –HOLMWOOD	1.327	48	2.011
BANK- GLADSTONE	1.813	47	2.013
RIDEAU 3A	0.521	45	2.015

Notes for tables (3.13) to (3.15)

t₁ = t value that is resulted from the comparisons the means measured in the winter time to the mean measured in the summer time, given the same time period of day.

df₁ = Degree of freedom corresponding with *t₁*

T₁ = Critical T values with 0.05 level of significance corresponding with *df₁*.

Underlined values: Not significant at 0.05 level or cannot reject *H₀*. (*t* < *T*)

**Table 3.14: Two-sample t statistics for winter and summer time
Route 1- Noon period (12 a.m. to 2 p.m.)**

<i>H₀: m₁=m₂</i>			
<i>H_a: m₁≠ m₂</i>			
STOP	Two-sample t values (2 tail-test)		
	<i>t₁</i>	<i>df₁</i>	<i>T₁</i>
GREENSBORO 1A	0.202	28	2.048
BANK-JOHNSTON NS	0.307	27	2.052
BANK- WALKLEY	0.661	27	2.052
BILLINGS BRIDGE	0.272	27	2.052
BANK –HOLMWOOD	0.211	27	2.052
BANK- GLADSTONE	0.740	27	2.052
RIDEAU 3A	1.225	27	2.052

**Table 3.15: Two-sample t statistics for winter and summer time
Route 1- Afternoon period (4 p.m. to 6 p.m.)**

<i>H₀: m₁=m₂</i>			
<i>H_a: m₁≠ m₂</i>			
STOP	Two-sample t values (2 tail-test)		
	<i>t₁</i>	<i>df₁</i>	<i>T₁</i>
GREENSBORO 1A			
BANK-JOHNSTON NS	0.356	21	2.080
BANK- WALKLEY	0.444	20	2.086
BILLINGS BRIDGE	0.542	19	2.093
BANK –HOLMWOOD	1.257	18	2.101
BANK- GLADSTONE	0.929	18	2.101
RIDEAU 3A	3.440	28	2.048

In order to test if the on-time performance of the buses varies depending upon the different time periods of the same day for a given season, the means of on-time performance collected at bus stop during each time period were also compared by the same technique described above. The test results are shown in Tables 3.16 and 3.17 for bus route 95, and Tables 3.18 and 3.19 for bus route 1.

**Table 3.16: Two-sample t statistics in Different Time Periods of Day
Route 95- The winter time (the WINTER data set)**

<i>Ho: $m_1=m_2$, $H_a: m_1 \neq m_2$; $Ho': m_1=m_3$, $H_a': m_1 \neq m_3$</i>						
STOP	Two-sample t values (2 tail-test)					
	t_2	df_2	T_2	t_3	df_3	T_3
BLAIR 2B	3.014	110	1.984	<u>1.533</u>	119	1.984
ST LAURENT 2B	2.867	110	1.984	<u>0.532</u>	126	1.984
TRAIN 2A	2.477	107	1.984	<u>0.555</u>	122	1.984
LEES 2B	4.926	119	1.984	<u>0.486</u>	126	1.984
MACKEZINE	4.628	115	1.984	<u>1.950</u>	128	1.984
ALBERT -KENT	4.941	138	1.984	2.609	129	1.984
LEBRETON	4.074	110	1.984	<u>1.046</u>	134	1.984

Notes for tables (3.16) & (3.17):

Underlined values: Not significant at 0.05 level or cannot reject H_o or H_o' . ($t < T$)

t_2, t_3 = Values that are resulted from the comparisons between the means measured the morning period (8 A.M. to 10 A.M.), the means measured in noon period (12 A. M. to 2 P.M.), and those of afternoon period (4 P.M. to 6 P.M.), respectively.

df_2, df_3 = Degree of freedom corresponding with t_2, t_3

T_2, T_3 = Critical T values with 95 percent degree of confidence corresponding with df_2, df_3 .

For bus route 95, the test results show clear differences between the means of on-time performance measured in different time periods of day. As shown in Table 3.16, during the winter time, there are statistically significant differences of bus on-time performance between morning and noon time period. For the summer time, the statistical differences of the means are found between morning and afternoon period (Table 3.17). For bus route 1, the differences between the means are not as clear as those of bus route

**Table 3.19: Two-sample t statistics in Different Time Periods of Day
Route 1- The summer time (the SUMMER data set)**

<i>Ho: $m_1=m_2$, $H_a: m_1 \neq m_2$; Ho': $m_1=m_3$, $H_a: m_1 \neq m_3$</i>						
STOP	Two-sample t values (2 tail-test)					
	t_2	df_2	T_2	t_3	df_3	T_3
GREENSBORO 1A	0.613	28	2.048			
BANK-JOHNSTON NS	0.747	27	2.052	0.611	21	2.080
BANK- WALKLEY	2.414	27	2.052	2.528	20	2.086
BILLINGS BRIDGE	2.155	27	2.052	2.547	19	2.093
BANK -HOLMWOOD	1.066	27	2.052	1.419	18	2.101
BANK- GLADSTONE	0.086	27	2.052	2.209	18	2.101
RIDEAU 3A	0.00	27	2.052	2.633	28	2.048

In this subsection, the data on arrival time of the bus were investigated thoroughly. Based on the results of two-sample-t tests, some conclusions on the bus arrival times of the two bus routes can be drawn.

- For mixed traffic bus route (i.e. bus route 1), the statistical differences of bus on time performance recorded at stops during winter and summer time are not found for all three time periods of the day (i.e. morning, noon, and afternoon). This implies that the winter does not have much influence on punctuality of the buses on a mixed bus route. They were always behind schedule. However, for Transitway (i.e. bus route 95), such differences were found for middle of the day and the afternoon.

- For the same season and for the same day, bus on-time performance was different, depending upon time periods and type of bus routes. For bus route 95, these differences were very clear. At almost all the selected stops of this bus route, the differences can be found between morning time and noon time during the winter time and between morning and afternoon time in the summer time. In the winter time, buses tend to be on-time in the noon time. During summer time, buses tend to arrive to stop later in

the afternoon. For bus route 1, the differences were not clear. On the average, buses were behind schedule by the same amount.

3.4.4.2 Dwell Time and Passenger Activities

According to the OC Transpo, dwell time is the time counted from door opening time to the moment when the last passenger activity is finished. In order to eliminate the abnormal dwell time that could have bad effects on the results of the analysis, the study deleted dwell times of over 180 seconds (3 minutes). According to OC Transpo, these are regarded as “unusual events” and therefore these data should not be included in the analysis. Figures 3.7 to 3.12 present the histograms of dwell times based on one year’s observations of each time period of day for each bus route.

Figure 3.7: Dwell time Histogram for Route 1-Time period (8 a.m. to 10 a.m.)

Figure 3.8: Dwell time Histogram for Route 1- Time period (12 a.m. to 2 p.m.)

Figure 3.9: Dwell time Histogram for Route 1- Time period (4 p.m. to 6 p.m.)

Figure 3.10: Dwell time Histogram for Route 95- Time period (8 a.m. to 10 a.m.)

Figure 3.11: Dwell time Histogram for Route 95- Time period (12 a.m. to 2 p.m.)

As shown in the histograms, route 95 has smaller means and standard deviations than that of route 1. As described previously, these differences reflect type of bus used and route characteristics. This means that its dwell time per stop are smaller and more stable than that of bus route 1. The average dwell time per stop tends to be longer in the noon and afternoon compared to that of morning in both types of bus routes. It varies in the range of 12 to 17 seconds for route 95 and 19 to 21 seconds for route 1.

Figure 3.12: Dwell time Histogram for Route 95-Time period (4p.m. to 6 p.m.)

Passenger activity is composed of the passenger movements through the bus doors. Most of the buses in route 95 are low-floor articulated buses with 3 doors while the buses running on route 1 are mostly rigid-body buses. Boarding passengers are required to board at front door while alighting passengers are allowed to get off the bus at all doors. However, boarding passengers can also get on an articulated bus at the rear doors in the peak-hours.

Tables 3.20 to 3.25 present the descriptive statistics of passenger activities on each bus route. As shown in the tables, the number of passengers using front door for boarding is about 2.5 times larger than that of using rear doors on average. By comparing the variables *OFF 1* (the number of alighting passengers using front door) in route 1 and route 95, we can see that the ratios between the number of passengers using rear doors and the front door for alighting are about 1.2 to 1.5 for route 1, but about 5.8 to 6.3 for route 95. Obviously, passengers in rigid body buses tend to use front door for alighting more frequently compared to that of articulated buses.

**Table 3.20: Descriptive Statistics of Passenger Activities.
Route 1- Time period: (8 a.m. to 10 a.m.)**

Variable	N	Minimum	Maximum	Mean	Std. Deviation
ON_1	1056	.00	54.00	4.3163	6.62927
OFF_1	1056	.00	19.00	.9489	1.89468
OFF_2	1056	.00	19.00	.1913	1.12758
OFF_3	1056	.00	37.00	1.5426	3.72237
TOTAL_ON	1056	.00	54.00	4.3920	6.72462
TOTAL_OFF	1056	.00	55.00	3.0123	5.66329
LOAD_ARR	1056	.00	71.00	19.2983	16.29168
TOTAL_PAS	1056	.00	57.00	7.5616	9.02708
LOAD_DEP	1056	.00	65.00	19.5019	16.34428

Note: Variables have been defined in Table 3.5.

**Table 3.21: Descriptive Statistics of Passenger Activities.
Route 1- Time period: (12 a.m. to 2 p.m.)**

Variable	N	Minimum	Maximum	Mean	Std. Deviation
ON_1	1072	.00	29.00	4.0606	5.31589
OFF_1	1072	.00	19.00	1.3302	2.48977
OFF_2	1072	.00	16.00	.2407	1.41945
OFF_3	1072	.00	27.00	1.7043	3.69738
TOTAL_ON	1072	.00	29.00	4.0886	5.31760
TOTAL_OFF	1072	.00	37.00	3.2752	5.99532
LOAD_ARR	1072	.00	56.00	15.3330	13.56143
LOAD_DEP	1072	.00	56.00	16.1465	13.76907
TOTAL_PAS	1072	.00	44.00	7.3638	8.80063

**Table 3.22: Descriptive Statistics of Passenger Activities.
Route 1- Time period: (4 p.m. to 6 p.m.)**

Variable	Observations	Minimum	Maximum	Mean	Std. Deviation
ON_1	648	.00	33.00	4.5077	6.21833
OFF_1	648	.00	14.00	1.4105	2.46672
OFF_2	648	.00	17.00	.1867	1.35414
OFF_3	648	.00	19.00	1.7160	3.27176
TOTAL_ON	648	.00	33.00	4.5556	6.22082
TOTAL_OFF	648	.00	29.00	3.3133	5.48954
LOAD_ARR	648	.00	59.00	14.7469	13.32245
TOTAL_PAS	648	.00	50.00	7.8688	9.29774
LOAD_DEP	648	.00	58.00	15.9892	13.47623

**Table 3.23: Descriptive Statistics of Passenger Activities.
Route 95- Time period (8 a.m. to 10 a.m.)**

Variables	Observations	Minimum	Maximum	Mean	Std. Deviation
ON_1	2039	.00	21.00	2.7533	2.94677
ON_2	2039	.00	15.00	.8524	1.74539
ON_3	2039	.00	15.00	.8818	1.76050
OFF_1	2039	.00	21.00	.7396	1.59007
OFF_2	2039	.00	27.00	2.1918	2.98886
OFF_3	2039	.00	40.00	2.4998	3.38369
TOTAL_ON	2039	.00	37.00	4.4875	5.33277
TOTAL_OFF	2039	.00	79.00	5.4311	7.14610
LOAD_DEP	2039	.00	97.00	30.2325	20.35251
LOAD_ARR	2039	.00	96.00	31.1761	17.77509

**Table 3.24: Descriptive Statistics of Passenger Activities.
Route 95- Time period (12 a.m. to 2 p.m.)**

Variables	N	Minimum	Maximum	Mean	Std. Deviation
ON_1	1843	.00	23.00	3.6316	3.76717
ON_2	1843	.00	17.00	1.3820	2.37809
ON_3	1843	.00	15.00	1.3006	2.32106
OFF_1	1843	.00	15.00	.9078	1.58890
OFF_2	1843	.00	23.00	2.6088	3.33336
OFF_3	1843	.00	22.00	3.0222	3.96141
TOTAL_ON	1843	.00	41.00	6.3142	7.11558
TOTAL_OFF	1843	.00	48.00	6.5388	7.96463
LOAD_ARR	1843	.00	88.00	29.2702	14.47510
LOAD_DEP	1843	.00	98.00	29.0456	16.98444

**Table 3.25: Descriptive Statistics of Passenger Activities.
Route 95- Time period (4 p.m. to 6 p.m.)**

Variables	N	Minimum	Maximum	Mean	Std. Deviation
ON_1	2839	.00	33.00	4.0775	4.18456
ON_2	2839	.00	19.00	1.7263	2.66858
ON_3	2839	.00	26.00	1.8859	2.93571
OFF_1	2839	.00	17.00	1.0398	1.93940
OFF_2	2839	.00	26.00	2.9362	4.11590
OFF_3	2839	.00	36.00	3.6404	5.09293
TOTAL_ON	2839	.00	52.00	7.6897	8.05438
TOTAL_OFF	2839	.00	68.00	7.6164	10.34543
LOAD_ARR	2839	.00	104.00	39.6840	19.96143
LOAD_DEP	2839	.00	106.00	39.7573	21.18925

The average load factors (the number of passengers in a transit vehicle prior to stopping divided by vehicle capacity) of both bus routes are quite low at about 35% and 52% for route 1 and route 95, respectively. On the average, there are 4 boarding passengers and 3 alighting passengers for each stop of route 1. The corresponding numbers for route 95 are 6 and 7 for each stop.

3.5 Summary

In this chapter, based on the observations of a bus travel time and its components, the framework of the proposed model for bus arrival prediction and its modules were presented. Because moving time accounts for about 60 percent of total travel time and it is vulnerable to delays caused by many unforeseen stochastic traffic-related factors, it is difficult to examine the effect of each factor on moving time separately. Therefore, the proposed model aggregated moving time and relevant delays into running time. By doing so, running time can be obtained from the departure time and the arrival time recorded by the APC and AVL systems between two consecutive bus stops. Dwell time is predicted separately from running time so that the model can capture the effect of long running times to long dwell time at bus stops, which may cause the bus to be behind the schedule at the stops thereafter.

The mechanism of the whole model with its three constituent modules was presented in order to explain how the model predicts real-time bus arrivals. Before going to develop the prediction model, the APC and AVL data were collected from the OC Transpo and processed. To understand the preliminary operational characteristics of the two bus routes, namely route 95 and 1, which were selected for modelling purposes, several descriptive statistics were discussed. In the next chapter, the first module of the model will be described.

Chapter 4

BUS RUNNING TIME PREDICTION MODULE

4.1 Introduction

Observations on bus travel time components presented in the previous chapter show that running time is the key component in overall terms. Hence, if a model makes poor predictions of bus running time, this will cause the failure of the overall model itself. For this reason, almost all bus arrival prediction models and algorithms reported in literature paid most attention to running time prediction. Also, almost all of these models tried to incorporate most influencing factors in their predictions. In theory, on the basis of all influencing factors, it should be possible to make better predictions. In practice, however, this has been a difficult task because of the data availability and technical constraints.

Data availability is of most concern when a researcher attempts to develop a prediction model, given that the quality and quantity of data influences model development in a major way. Data can be divided into static data and dynamic data. The static data such as number of stops, number of intersections, link length, curve radius and so on, of course, are the simplest data that can be collected by common measurements. However dynamic data, whose characteristics vary over time such as densities, volumes and speeds of traffic stream, proportion of left turn and heavy vehicles, are not always

available and collectable. Although, from a technical perspective, sensors embedded on the bus routes or the traffic cameras can capture these dynamic data; practically, not all the bus routes have sensors and cameras or even if this was the case, not all these data can be transmitted dynamically. This eliminates the applications of the models based on the above data to make real-time predictions where up-to-minute input data are the vital requirements.

To eliminate the difficulties of data availability, an innovative algorithm for bus running time prediction is advanced in this research in order to develop a real-time bus arrival prediction model that is based on the available APC and AVL data. The algorithm is based on a statistical pattern recognition technique where the bus operational state in the past that is most similar to a current bus state reported by the APC-AVL systems is recognized and used to predict the running time of the bus. The methodology to apply the technique to bus running time predictions and the detailed procedure are presented in the following sections.

4.2 Problem Statement

Running time is a stochastic process influenced by many stochastic factors. Hence, capturing the distribution of running times with these factors by using specific probabilistic formulations is difficult. No publication reporting an examination of such distributions has been found in the published literature.

The duration of time that a bus has just spent from one stop to another, by itself, reflects the sum of influences including traffic conditions. For example, if the bus was unusually behind the schedule, it could be due to an incident while the bus was running

on the link. If the bus was late in the usual manner, this could be due to traffic surges during rush hours. If the bus was earlier than schedule time, it could be due to travel in light traffic. Intuitively, the buses running on the same link and in short headways under similar traffic conditions will tend to have similar running situations on that link. In other words, running times of buses observed around a specific time in the past, say t , may contain information about the running time a bus will spend at time t in the future. This standpoint stimulates the applications of pattern recognition to bus running time predictions.

A pattern can be seen as a set of measurements or observations represented in a vector or matrix notation (Schalkoff, 1992, p.6). According to this definition, running times of the buses, together with other traffic data, around a specific departure time of a bus under consideration, can be considered as a pattern for the feature of interest (i.e. running time) of this bus at the time in question. In this research, the statistical pattern recognition technique was chosen to develop a model for predicting bus running times. Specifically, a nonparametric regression technique is proposed for the recognition of the bus running situations in the past that exhibit the closest similarity to the current bus running situation. These, the local neighbours, then will be used to estimate the running times of the buses.

4.3 Nonparametric Regression Method in Statistical Pattern Recognition

Nonparametric regression (NP) methods, or kernel-based methods, are the new developments in statistical pattern (SP) recognition (Schalkoff, 1992). They stem from curve smoothing techniques and non-linear time series methods defined in the 1970s.

The term nonparametric was used to differentiate these methods from the parameter regression ones where the data were used to build a series of equations with parameters resulting from parameter fittings. Various prior relations among data variables have been defined such as linear, logistic, log-linear, and so on, depending upon the data.

The nonparametric regression methods, in contrast, do not need any specific prior relation among data. Hence, there are no parameter estimates in NP regression. Instead, to forecast, the NP regression retains the data and searches through them for past similar cases. This strength makes NP regression to be a powerful method due to its flexible adaptations in a wide variety of situations.

Another advantage of the NP regression over the traditional parametric regression is that because of no parameter estimates, NP regression can adapt easily with new data added to the model while parameter regression method needs expensive parameter re-computations. This is an especially important characteristic of NP for real-time applications to predictions where new data are added to the model continuously.

Like artificial neural network (ANN) models, the NP regression models can be trained from actual data; but unlike an ANN model which is considered as a “black-box” method, the NP regression provides statistical inference and confidence interval. There are three popular types of nonparametric regression (Andersen, 2004).

- Locally Weighted Scatter plot Smoothing (LOWESS): Model fits data by local polynomial regressions and join them together. This type is most widely used in many application fields.
- Smoothing Splines: Models partition the data and fit separate piecewise regression to each section, smoothing them together where they join.

- Generalized additive models: The extensions of the types above to specific situations such multiple and multivariate regression, and to non-quantitative dependent variables.

In this study, the use of LOWESS method is proposed as the basis for model development. The choice of this method over other methods such as spline regression is based on the task at hand. Our interest is with pointwise estimates and not with the continuous regression curve. The details of LOWESS method are discussed below.

4.3.1 LOWESS Estimation Method

4.3.1.1 The General Functions

LOWESS method was first introduced by Cleveland (1979) and further developed for multivariate models (Cleveland *et al.*, 1988). In the 1979 publication, Cleveland proposed the model for bivariate data $(X_1, Y_1), \dots, (X_n, Y_n)$ which are the independent and identically distributed (i.i.d) samples drawn from population (X, Y) . Given a value defined by x_0 , the estimated value of $Y|X = x_0$ is calculated by the regression function $m(x)$. Cleveland supposed that function $m(x)$ can be expanded by Taylor series expansion in a *neighbourhood* of x_0 as follows:

$$m(x) \approx m(x_0) + m'(x_0)(x - x_0) + \frac{m''}{2!}(x - x_0)^2 + \dots + \frac{m^{(p)}}{p!}(x - x_0)^p \quad (4.1)$$

This polynomial is fitted locally by a weighted least squares regression minimization (Fan *et al.*, 1996).

$$\sum_{i=1}^n \{Y_i - \beta_0 - \sum_{j=0}^p \beta_j (X_i - x_0)^j\}^2 K_h(X_i - x) \quad (4.2)$$

$$K_h(.) = K(. / h) / h \quad (4.3)$$

Where: h = bandwidth controlling the size of the local neighbourhood

$K_h(.)$ is a Kernel function assigning weights to each datum point.

The solution of Equation 4.2 gives a series of $\hat{\beta}_j, j=0, \dots, p$.

Obviously, from Taylor expansion, we have

$$\hat{\beta}_j = \frac{\hat{m}_j(x_0)}{j!} \quad (4.4)$$

where: $\hat{m}_j(x_0)$ is an estimator of $m^{(j)}(x_0)$

As $\hat{m}_j(x_0)$ is defined from Equation 4.4, $m^{(j)}(x_0)$ is also defined to estimate the entire function $m^{(j)}(.)$.

In the matrix notation, Equation 4.2 can be denoted as

$$\text{Min}_{\beta} (y - X\beta)^T W (y - X\beta) \quad (4.5)$$

where:

$$X = \begin{bmatrix} 1 & X_1 - x_0 & \dots & (X_1 - x_0)^p \\ 1 & X_2 - x_0 & \dots & (X_2 - x_0)^p \\ \vdots & \vdots & \ddots & \vdots \\ 1 & X_n - x_0 & \dots & (X_n - x_0)^p \end{bmatrix}; \quad y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix}; \quad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix} \quad \text{and} \quad \hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_p \end{pmatrix};$$

W is the $n \times n$ diagonal matrix of weights

$$W = \text{diag}\{K_h(X_i - x_0)\} \quad (4.6)$$

The solution of Equation 4.5 can be written as:

$$\hat{\beta} = (X^T W X)^{-1} X^T W y \quad (4.7)$$

where: X^T is the transpose matrix of matrix X

Once $\hat{\beta}$ is defined, the estimation of Y given $X=x_0$ is easily defined, or a prediction is made.

From the general equations, we can see that there is a series of issues that need to be discussed. They are: the bandwidth parameter (or smoothing parameter), the type of Kernel function, the degree of polynomial of the regression function.

4.3.1.2 Bandwidth Selection

Bandwidth plays an important role in the accuracy of the estimation made by LOEWSS. Bandwidth selection is the process of recognizing suitable neighbours around x_0 , or the *focal point* (Andersen, 2004). In ST Recognition, the recognition of a point belonging to a neighbourhood of a focal point is usually based on the *Euclidian distance* from the focal point to that point, and a selected bandwidth. In essence, to find an optimal bandwidth is to balance the trade-off between variance and bias (Fan *et al.*, 1996). A too large bandwidth can create a large modelling bias. In contrast, a very small bandwidth can cause a noisy estimate or large variance. Figure 4.1 depicts the effects of bandwidth on NP regression.

Several methods have been developed to find an optimal bandwidth given a dataset. Generally speaking, there are two kinds of bandwidth selection: constant bandwidth and variable bandwidth. Constant bandwidth regulates a constant range of x given a focal point while variable bandwidth changes upon the location of the focal point. Constant bandwidth is simple to apply but has a drawback of no neighbour recognition.

Figure 4.1: Bandwidth Selection and NP Regression

(Source: Modified from Fan et al., 1996, p.8)

Note: (1) Too small bandwidth ($h=0$) causes noisy estimation and large modelling variance, (2) Too large bandwidth ($h=\infty$) causes modelling bias, (3) As h runs from 0 to ∞ , the resulting estimate runs from the most complicated curve to the simplest curve.

Figure 4.2: Constant bandwidth Problem

Note: If constant bandwidth is 4 units, no neighbour is found around focal A whereas focal B has 2 neighbours

Figure 4.3: N-Nearest type of bandwidth

Note: If N nearest neighbours are 3, focal A always has 3 neighbours and so does the focal B

Therefore, the N -nearest neighbour type of bandwidth, where N neighbours nearest to a given focal are always recognized, is more common. The difference between the two methods is shown in Figures 4.2 & 4.3.

In a study published in 1979, Cleveland proposed to use the distance from the r^{th} nearest neighbour of x_0 as the defined bandwidth. However, the author did not discuss the value of r that should be chosen. Instead, Cleveland suggested that “*Instead of thinking in terms of q , the number of points in the neighbourhood, we think in terms of $f=q/n$, the fraction of points in the neighbourhood*”. Because the f values were not suggested, so f was chosen without a certainty that the bandwidth is optimal.

Another method to find optimal bandwidth is *cross-validation* (Hardle, 1990). The main feature of this method is the use of two samples drawn from the same

population. The first sample, called the screening sample contains n_c data points used for building a regression model. The model then is applied to the predictors for the second sample called the validation sample with n_v data points. There are different ways to split n_c and n_v from population of n to built up different models. Out of those models, there will be several models offering the best average predictive ability. When sample size is small and the time for computing bandwidth is limited, the simplest *cross-validation* with $n_v=1$ is preferable. This method can also be called the *leave-one-out* method.

In the leave-one-out method presented by Hardle (1990), for each given (x_i, y_i) , $(n-1)$ remaining points after removing (x_i, y_i) from the sample are used to build regression function $m(x)$ and to predict value of $y_i = \hat{m}_{h,i}(x_i)$, given x_i and a bandwidth h . The values of h then are changed to find the optimal bandwidth $\hat{h}$ satisfying the equations (Hardle, 1990, pp.148-153)

$$\hat{h} = \arg \min_h [CV(h)] \quad (4.8)$$

Where $CV(h)$ is *cross-validation* function

$$CV(h) = n^{-1} \sum_{j=1}^n \left[y_j - y^{(h)}_j \right]^2 \quad (4.9)$$

Other methods to find the optimal bandwidth are: plug-in-out method (details can be seen in Hardle 1990; Fan *et al.*, 1996; and Loader, 1999), automatic bandwidth and order selection (William 1995; Zhang 2000). These methods are based on the trade-off between variance and bias to find optimal bandwidth automatically. However, these methods are very complicated and difficult to apply in practice.

4.3.1.3 Weighting Kernel Function Selection

The goal of Kernel function is to assign suitable weights in response to each neighbour x_i of a focal x_0 . The kernel function is a continuous and symmetric function which integrates to 1.

$$\int K(u)du = 1 \quad (4.10)$$

There are several possible kernel functions that can be applied for estimation. According to Loader (1999), the selection of Kernel function has small influence on the trade-off between variance and bias of the estimation.

Table 4.1: Common Kernel Functions for Univariate data

Kernel Function	Analytic Form, K(x)
<i>Rectangular</i>	$\frac{1}{2} \text{ for } u \leq 1, 0 \text{ otherwise}$
<i>Triangular</i>	$1 - u \text{ for } u < 1, 0 \text{ otherwise}$
<i>Biweight</i>	$\frac{15}{16}(1 - u^2)^2 \text{ for } u < 1, 0 \text{ otherwise}$
<i>Tri-cube weight</i>	$(1 - u ^3)^3 \text{ for } u < 1, 0 \text{ otherwise}$
<i>Normal</i>	$\frac{1}{\sqrt{2\pi}} \exp(-u^2 / 2)$
<i>Epanechnikov</i>	$\frac{3}{4}(1 - u^2 / 5) / \sqrt{5} \text{ for } u < \sqrt{5}, 0 \text{ otherwise}$

Note: $u = \frac{x_i - x_0}{h}$; $h = \text{bandwidth}$. Source: Webb Andrew (2002), p.109.

4.3.1.4 Degree of Polynomial Regression

The selection of degree of polynomial is the balance between computational ease and the need to reproduce patterns in data. Cleveland (1979) suggested using linear regression for adequate smoothing and computational ease.

4.4 Modelling Bus Running Time Prediction by Using LOWESS Method and APC - AVL Data

Before applying LOWESS method and AVL and APC data for modelling bus running time prediction, it is necessary to determine the parameters for the LOWESS method and the APC-AVL data that will be used in the model.

4.4.1 APC and AVL Data

As the purpose of this research is to develop a model capable of predicting running time by using APC and AVL data, it is necessary to discuss the data being used in the model. Data collected by the APC and the AVL systems were shown in Table 3.5. Actual running times from one stop to another stop can be obtained in two ways: subtracting the arrival time of the bus at the stop and the actual departure time of that bus at the stop right before; or summation of the moving time, the stationary and the stop-and-go time of the bus. These running times are used in modelling.

4.4.2 Parameter Selection

The role of each parameter in LOWESS method has been presented in the previous parts of this chapter. To apply this method for modelling real-time bus running time predictions, the following parameters and assumptions are used.

- Bandwidth selection method: The *leave-one-out* method is chosen because of its ease in computation while it can maintain an automatic optimal bandwidth. Hence, it is suitable for real-time estimations.
- Kernel function selection: The *Tri-cube* weight function is selected.
- Degree of polynomial regression function: The linear regression is proposed.

4.4.3 Pattern Selection

Assume that bus k leaves stop i at time $Dp(k)_{actual,i}$. We have to predict running time the bus will spend to arrive at the next stop $i+1$, $r_0(k)$. By that time $Dp(k)_{actual,i}$, the running times of the previous buses (e.g., $k-1$, $k-2$, ..., $k-d$) in the link bounded by stop i and stop $(i+1)$ have been collected. Let us denote these running times as $r_0(k-1)$, $r_0(k-2)$, ..., $r_0(k-d)$. Now, the running time being predicted $r_0(k)$ is presented by *pattern* $\mathbf{R}_0$

$$\mathbf{R}_0 = [r_0(k-1), r_0(k-2), \dots, r_0(k-d)]^T \quad (4.11. A)$$

Where:

d = the number of previous buses that should be taken into account in making the running prediction of the current bus.

Also in the past, each running time experienced by bus k in the link (i_i+1) is also presented by its own pattern whose elements are defined similarly above. So we have $\mathbf{R}_1$, $\mathbf{R}_2$, ..., $\mathbf{R}_m$ patterns, representing the corresponding running times of bus k as recorded in the past $r_1(k)$, $r_2(k)$, ..., $r_m(k)$.

$$\mathbf{R}_j = [r_j(k-1), r_j(k-2), \dots, r_j(k-d)]^T \quad (4.11.B)$$

(m = number of observations collected in the past, $j= 1$ to m).

To predict running time of bus k at current time with a given pattern R_0 , the following steps are developed to recognize the running times of bus k in the past $r_j(k)$ which have the most similar patterns to pattern R_0 .

4.4.4 Pattern Recognition

The common way that is applied in SP recognition so as to recognize and classify features in the same group, is based on the *Euclidian Distances* and a selected bandwidth. Therefore, this step includes two sub-steps: calculate the Euclidian distances from pattern R_0 to the other patterns $R_1, R_2, \dots, R_m$, and recognize the neighbourhood.

4.4.4.1 Euclidian Distance Calculation

The Euclidian distance between patterns R_0 and R_j ($j=1$ to m) can be calculated as follows:

$$u_j = \sqrt{[r_j(k-1) - r_0(k-1)]^2 + [r_j(k-2) - r_0(k-2)]^2 + \dots + [r_j(k-d) - r_0(k-d)]^2} \quad (4.12)$$

4.4.4.2 Finding Optimal Bandwidth by Leave-One-Out Method

A running time recorded in the past $r_j(k)$ will be recognized as the neighbour of the one being predicted $r_0(k)$, called the focal point, when its Euclidian distance to the focal point is smaller than a regulated bandwidth.

Figure 4.4: Automatic Process of Searching the Optimal Number of Neighbours.

The distances have been defined by Equation 4.12 for all running times. Now we define the bandwidth by using *leave-one-out* technique presented by Hardle (1990).

We have m observations of running times $r_j(k)$ from the past and the corresponding pattern for each, $\mathbf{R}_j(k)$. Following the algorithm suggested by Hardle, and applying *Nadaraya-Watson* kernel (shown on the next page) estimation for the sake of shortening the time to find the optimal bandwidth, the following steps shown in Figure 4.4 are pursued.

- Regulate the initial *N-nearest* neighbours as $0.1m$ (for example, if $m=100$ observations, so $N=10$, meaning that 10 neighbours will be around a focal point) and the increment for each step is $0.1m$.
- **FOR $N=0.1m, N=N+0.1m$ TO $M=0.8m$ DO (LOOP1)**

FOR $i=1$ to M do (LOOP 2)

1. Consider the running time $r_i(k)$ as the focal point, ($i=1$ to m). It should be noted that it is represented by a pattern namely $\mathbf{R}_i(k) = [r_i(k-1), r_i(k-2), \dots, r_i(k-d)]^T$.
2. Use $(m-1)$ remaining running times to build a *N-nearest* neighbour regression to predict the value of $r_i(k)$, as follows:

- Calculate the Euclidean distances from the remaining running times to the focal $r_i(k)$ as:

- Calculate the Euclidian distance

$$dis_j = \sqrt{[r_j(k-1) - r_i(k-1)]^2 + [r_j(k-2) - r_i(k-2)]^2 + \dots + [r_j(k-d) - r_i(k-d)]^2}$$

with $(i \neq j)$ (4.13)

- Arrange the dis_j in the increasing order

- Select the N^{th} smallest of the arrangement above. The bandwidth is the distance from the focal to the N^{th} nearest neighbour. Let us call that value as $h(N)$.

- Calculate the weight by applying *tri-cube* weight Kernel function:

$$W(j) = \frac{70}{81} \left(1 - \left| \frac{dis_j}{h(N)} \right|^3\right)^3 \cdot I_{[0,1]} \quad (4.14)$$

Where: $I_{[0,1]} = 1$ if $0 \leq \left| dis_j / h(N) \right| < 1$ otherwise $I_{[0,1]} = 0$

- For the sake of computational ease, the predicted value of $r_i(k)$ made at $h(N)$ is computed by the *Nadaraya-Watson* kernel estimation.

$$\hat{r}_i(k) = \frac{\sum_{j=1, j \neq i}^{m-1} W_j \cdot r_j(k)}{\sum_{j=1, j \neq i}^{m-1} W_j} \quad (4.15)$$

- Calculate the prediction error made at $h(N)$ as :

$$Err_i(k) = [\hat{r}_i(k) - r_i(k)]^2 \quad (4.16)$$

END LOOP 2

3. Calculate the Cross-validation function

$$CV(h) = \frac{\sum_{i=1}^m Err_i(k)}{m} \quad (4.17)$$

- **END LOOP 1**

- Determine the optimal number of neighbours

$$OpNei = \arg \min_h \{CV(h)\} \quad (4.18 A)$$

- Determine the optimal bandwidth

$$H = dis_{OpNei} \quad (4.18 B)$$

Where: dis_{OpNei} is the *Euclidian* distance from the *focal point* to the smallest $Opnei^{th}$ neighbour.

4.4.4.3 Recognition of the Neighbours

The *Euclidian distances* (Eq. 4.12) and the optimal band width H (Eq. 4.18B) have been defined. Hence, given focal point $r_0(k)$ and its pattern R_0 (see Eq.4.11A), a point $r_j(k)$ will be recognized as the neighbour of $r_0(k)$ by applying *Tri-cube weight kernel function* as follows:

$$- \text{ Calculate the weight } K_j = \frac{70}{81} (1 - |\frac{u_j}{H}|^3)^3 I_{[0, 1]} \quad (4.19)$$

Where: $I_{[0, 1]} = 1$ if $0 \leq |u_j / H| < 1$ otherwise $I_{[0, 1]} = 0$; ($j=1$ to m)

- From Equation 4.19, we can see that if $r_j(k)$ has a distance larger than the bandwidth H , the weight assigned for that point is zero, meaning that $r_j(k)$ does not belong to the neighbourhood of focal $r_0(k)$. Otherwise, it is a neighbour of $r_0(k)$. The influence of this neighbour on the focal point is taken into account by a kernel weight K_j depending upon its location.

4.4.5 Prediction

When well defined, the weighted neighbours will be used as independent variables of a polynomial regression in order to estimate the dependent variable which is

being predicted. In our model, the running time of bus k being predicted is the dependent variable (i.e., $r_0(k)$). The weighted neighbours of this dependent variable are the historical running times recorded in the past (i.e., $r_j(k)$) and recognized by the steps mentioned above. Although a higher degree of polynomial regression results in better predictions, the polynomial regression in this study is proposed to be linear regression for the sake of balancing computational ease and adequate prediction (Cleveland et al., 1979).

Equations from 4.2 to 4.7 present the procedure for predicting a univariate variable by using weighted least-square regression minimization. In our case, the prediction is univariate response but its pattern is a multi-dimensional vector. Therefore, the procedure above has been further developed by applying the works of Fan *et al.*, (1996) for multivariate regression as follows.

- Weighted least square regression function has similar form as for Equation 4.5

$$\min (y - X\beta)^T W (y - X\beta) \quad (4.20)$$

$$\text{Where: } X = \begin{bmatrix} 1 & r_1(k-1) - r_0(k-1) & r_1(k-2) - r_0(k-2) & \dots & r_1(k-d) - r_0(k-d) \\ 1 & r_2(k-1) - r_0(k-1) & r_2(k-2) - r_0(k-2) & \dots & r_2(k-d) - r_0(k-d) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_m(k-1) - r_0(k-1) & r_m(k-2) - r_0(k-2) & \dots & r_m(k-d) - r_0(k-d) \end{bmatrix}$$

$$y = \begin{pmatrix} r_1(k) \\ r_2(k) \\ \vdots \\ r_m(k) \end{pmatrix}; \quad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_d \end{pmatrix} \quad \text{and the estimated values of beta } \hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_d \end{pmatrix};$$

W is the matrix of weights defined as below

$$W = \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_2 & 0 & . & . & . \\ . & 0 & K_3 & 0 & . & . \\ . & . & 0 & . & 0 & . \\ . & . & . & 0 & . & 0 \\ 0 & . & . & . & 0 & K_m \end{bmatrix}$$

Where: K_s ' values are computed in Equation (4.19).

- The solution of Equation 4.20 is

$$\hat{\beta} = (X^T W X)^{-1} X^T W y \quad (4.21)$$

The ***predicted value*** of $r_0(k)$ or the running time that bus k will likely spend in the $LINK_{i_{i+1}}$ is

$$r_0(k) = \hat{\beta}_0 + \sum_{j=1}^d \hat{\beta}_j r_0(k-j) \quad (4.22)$$

4.4.6 Update Prediction

The mechanism of how the bus arrival time prediction model updates its predictions was discussed in the previous chapter. As described earlier, when the bus arrives at the next stop, new data are collected and transmitted to the control center. These data then will be used by this module to predict running times and for all downstream links. The updating process follows the same steps above as noted from pattern recognition to prediction. Each time the bus arrives at a stop, if there are M links between the current stop (i.e. the stop the bus has just arrived) and the last stop, the RTM will be applied M times for predicting the running time that the bus will likely spend on each link.

4.5 Summary

In this chapter, an innovative algorithm to predict running times of bus transit is developed and its details are noted. The methodology applied in this algorithm is based on the LOWESS method, a method in the family of Statistical Pattern Recognition. The reasons for choosing this method are as follows: (1) it is a powerful method because it is not based on any specific assumptions on the relationship of variables. (2) The predictions made by LOWESS method are based on the local historical database (both past and current data base) which was recorded around the time the prediction is being made, so the method can capture the surges in traffic stream during peak hours as well as the changes due to traffic incidents.

To predict running time by applying LOWESS-based method, four steps were proposed: (1) pattern selection, (2) pattern recognition, (3) prediction, and (4) updating of prediction. Of the four steps, the second step is the most elaborate. This includes a procedure from defining Euclidian distances, finding optimal bandwidth by using leave-one-out method, and recognizing the neighbours. After recognition, the weighted neighbours are used in predicting running times by applying multivariate linear regression.

The prediction performance of this module will be presented in chapter 6.

Chapter 5

BUS DWELL TIME PREDICTION MODULE

5.1 Introduction

From the transit passenger point of view, one would argue that the predictions of dwell times, departure times and the number of passengers are not what a passenger wants to know. This standpoint is true; however, these predictions contribute to the accuracy of announced arrivals that passengers are interested in. Although dwell time accounts for a smaller portion of total delay (12-26%) compared to that of running time (48-75%), this cannot be neglected, especially when the buses serve high density population areas (Levinson, 1983).

As opposed to the above point of view, bus transit providers have considered that data on passenger activities at each stop, and the number of on-board passengers are extremely important for analyzing ridership and the relationship between passenger loading, running times and on-time performance (Pile *et al.*, 1998). In terms of real-time control, the predicted number of passengers boarding and alighting at a stop can help bus dispatchers to have suitable pro-active solutions once a crowded bus stop is detected in advance or the knowledge that bunching is likely to occur.

Dwell time is influenced by many factors belonging to two groups. The first group relates to passenger activities such as a number of passengers boarding and

alighting, types of passengers (e.g., age, physical health, sex). The second group relates to bus service activities such as types of fare collection, number of doors, seat-capacity, type of bus (e.g., rigid-body bus, articulated bus, low-floor bus or high-deck bus), and service frequencies. Before the invention of the APC system, dwell time analysis was limited due to time consuming and highly labor-intensive manual counting. The APC systems, at present, are creating great potential for studies on dwell time by providing much data not only in quantity but also in quality. This also enables the applications of real-time prediction method for predicting bus passenger activities and analyzing delays.

The proposed real-time dwell time prediction module described in this chapter is one part of the entire proposed model developed for predicting bus arrivals. Specifically, predicted dwell times are the chains to link a pair of predicted running times detailed in chapter 4. Thus, this module enables the entire model to predict arrival times and departure times of buses at all stops. This chapter is organized as follows. First, some drawbacks of previous dwell time prediction models are discussed. Second, four proposed sub-modules are then described in detail and integrated in a complete bus dwell time prediction module. Last, the summary closes the chapter.

5.2 Bus Dwell Time Prediction Module

5.2.1 Discussions on the Previous Works

A number of problems uncovered in the previous works on dwell time prediction require further discussions:

- As shown in the literature review, it was recognized that the number of boarding and alighting passengers is the vital key to determine the dwell time. However, no discussion on how to predict these numbers is found except in the works of Farhan (2002) and Shalaby *et al.*, (2004). To predict dwell time, the previous models require the values of the number of passenger boarding and alighting at stops. Unfortunately, these values were not available and need to be predicted as well. Therefore, it is reasonable to say that the number of predicted alighters and boarders at a stop is the root of dwell time prediction if one want to know dwell time in advance. Once the number boarding and alighting passengers has been predicted, all the previously noted models can be used to predict dwell time. Otherwise, they can be used only for off-line estimation purposes when the information on passenger activities of buses is available.
- The assumption that a long headway or a long service frequency can result in a long dwell time at downstream stops was widely accepted and used by many authors (Koutsopoulos *et al.*, 1985; Lin *et al.*, 1999; Ding *et al.*, 2000; Chien *et al.*, 2000; Fu and Liu, 2003; Shalaby and Farhan, 2004). However, the relationship between dwell time and headway was still underestimated. Instead, this assumption is quantified by a simple equation shown below:

$$dwell = sf. \text{apar. } t_b \quad (5.1)$$

Where:

$dwell$ = Dwell time (sec)

sf = Service frequency or headway (sec)

λ = Average passenger arrival rate (passenger/sec) where arrival rate distribution is usually assumed as a Poisson distribution or uniform distribution;

t_b = Average boarding time per passenger (sec/pas).

This assumption has some drawbacks:

- First, as shown in Equation 5.1, these models assume that arrival passenger flow is uniform. This assumption may be erroneous in case of long headways where passengers tend to arrive at stops around the announced arrival times (Abkowitz *et al.*, 1986).
- Second, based on the assumption above, one could argue that if no passenger comes to the stop during a headway time, the value of average passenger arrival rate will be zero. This will result in a zero value of dwell time, meaning that the bus will not stop at that stop because there is no boarding demand. However, there may be an alighting demand and a stop may be made with a positive dwell time.

Hence, the above assumption should be modified to capture this case.

- It is the fact that boarding passengers at the front door usually wait for the last alighting passenger before getting on the bus. If the bus has two doors and some passengers get off at the front door, dwell time may be lengthened. Therefore, the passenger decision on taking front door for getting off the bus will play an important role in predicting dwell time. If the bus has more than 2 doors (e.g., articulated bus), where passengers can board in rear doors, the dwell time will be the time for serving the busiest door. However, which door

will be the busiest door in the above cases and what is the role the passengers play in determining dwell time are still open questions.

5.2.2 The Dwell time Prediction Module (DTM)

The DTM proposed in this study is dynamic in a manner that it is based on the real-time predicted boarding and alighting passengers at a stop. In order to do this, two of its sub-modules, namely Real-time Boarding Passenger Prediction (RPSM) and Real-time Alighting Passenger Prediction (RASM), have been developed separately. These predictions then are used as the inputs for two other sub-modules called Regression (RESM) and Busiest door (BDSM). The RESM is based on the collected data from AVL and APC systems in the OC Transpo. In this sub-module, the relationship between dwell time and a series of variables (e.g. the number of passengers, bus type, on time performance, and so on) is examined in the form of several statistical regressions. The BDSM is based on the equation suggested by HCM 2000 (Eq. 2.2) with modifications in order to find the busiest door.

By combining the sub-modules, two methods (A&B) are proposed in this research for predicting dwell time. Method A is the combination of BPSM, APSM, and RESM. Method B includes BPSM, APSM, and BDSM. Figures 5.1 and 5.2 depict the frameworks of the two methods.

Figure 5.1: Dwell time Prediction Module. Method A

Figure 5.2: Dwell time Prediction Module. Method B

The details of each sub-module are presented separately in the following sections and the better prediction method is identified.

5.2.3 Real-time Boarding Passenger Prediction Sub-Module

The LOWESS-based method developed in chapter 4 is used here to predict boarding passengers at a stop. The steps to obtain a prediction on the number of passengers boarding bus (k) at stop ($i+1$) are presented below.

5.2.3.1 Parameter Selection

The method applied to select parameters in chapter 4 is used in this chapter as follows: *leave-one-out* method for optimal bandwidth selection; *Tri-cube weight* function to recognize the neighbours, and *Nadaraya-Watson* kernel function for prediction.

5.2.3.2 Pattern Selection

As discussed earlier, there is an erroneous assumption on the distribution of passenger arrival flow at a stop. In this research, the author intended to capture the relationship between headway and the number of boarding passengers at a downstream stop without any prior mathematical assumption about this relationship.

Assume that bus k leaves stop i at time $Dp(k)_{actual,i}$. We have to predict the number of passengers boarding on bus k when it arrives at the next stop $i+1$. Let us denote this variable as $b_0(k)$. At time step $Dp(k)_{actual,i}$, data on the number of boarding passengers on previous buses at stop ($i+1$) have been collected. We denote these numbers as $b_0(k-1)$, $b_0(k-2), \dots, b_0(k-d)$.

Also at that time step, running times of the previous buses (e.g., $r_0(k-1)$, $r_0(k-2), \dots, r_0(k-d)$) in the link bounded by stop i and stop $(i+1)$ have been available. We already have the predicted running time that bus k on the $LINK_{i+1}$. This value is $r_0(k)$ in Equation 4.22 (Chapter 4).

In order to take the relationship between the bus headway and the number of boarding passengers without any prior assumption on it, we consider the number of boarding passengers on the previous bus at stop $i+1$ as well as the recorded headway of this bus as the elements of a pattern defining the number of boarding passengers being predicted when bus k arrives at stop $i+1$. Because the running time of bus k , which has been predicted, may have influence on the number of boarders being predicted, it is also included as an element of the pattern above.

Based on the above assumption, we can say that the boarding passengers being predicted $b_0(k)$ is presented by a *pattern*, namely (B_0) , whose three elements are the number of boarding passengers recorded from the previous bus, the running time of that bus, and the predicted running time bus k will spend to come to stop $i+1$.

$$B_0 = [b_0(k-1), r_0(k-1), r_0(k)]^T \quad (5.2)$$

Also in the past, the number of boarding passengers on bus k at stop $(i+1)$ is also presented by its own pattern whose elements are defined similarly above. So we have $B_1, B_2, \dots, B_m$ patterns, representing the corresponding number of boarders of bus k recorded in the past $b_1(k), b_2(k), \dots, b_m(k)$.

$$B_j = [b_j(k-1), r_j(k-1), r_j(k)]^T \quad (5.3)$$

(m = number of observations collected in the past, $j=1$ to m).

Because the patterns contain elements measured on different scales, it is necessary to standardize them. We have:

$$\mathbf{SB}_j = [sb_j(k-1), sr_j(k-1), sr_j(k)]^T \quad (5.4)$$

Where:

- $\mathbf{SB}_j$ ($j=1$ to m) = standardized pattern of B_j
- $sb_j(k-1)$ = standardized values of $b_j(k-1)$; $j=1$ to m

$$sb_j(k-1) = \frac{b_j(k-1) - \bar{b}(k-1)}{Sb(k-1)} \quad (5.5)$$

- $\bar{b}(k-1)$ = The mean of the number of boarding passengers on bus ($k-1$)

$$\bar{b}(k-1) = \frac{\sum_{j=1}^m b_j(k-1)}{m} \quad (5.6)$$

- $Sb(k-1)$ = Standard deviation of boarding passengers on bus ($k-1$)

$$Sb(k-1) = \sqrt{\frac{\sum_{j=1}^m [b_j(k-1) - \bar{b}(k-1)]^2}{m-1}} \quad (5.7)$$

- $sr_0(k-1)$ = Standardized value of running time $r_0(k-1)$

$$sr_0(k-1) = \frac{r_0(k-1) - \bar{r}(k-1)}{Sr(k-1)} \quad (5.8)$$

- $sr_j(k-1)$ = Standardized values of running times $r_j(k-1)$; ($j=1$ to m)

$$sr_j(k-1) = \frac{r_j(k-1) - \bar{r}(k-1)}{Sr(k-1)} \quad (5.9)$$

Where:

- $\bar{r}(k-1)$ = Mean of running times of bus $k-1$

$$\bar{r}(k-1) = \frac{\sum_{j=1}^m r_j(k-1)}{m} \quad (5.10)$$

- $Sr(k-1)$ = Standard deviation of running times of bus $k-1$

$$Sr(k-1) = \sqrt{\frac{\sum_{j=1}^m [r_j(k-1) - \bar{r}(k-1)]^2}{m-1}} \quad (5.11)$$

- $sr_0(k)$ = Standardized value of running time $r_0(k)$
- $sr_j(k)$ = Standardized values of running times $r_j(k)$

$$sr_0(k) = \frac{r_0(k) - \bar{r}(k)}{Sr(k)} \quad (5.12)$$

$$sr_j(k) = \frac{r_j(k) - \bar{r}(k)}{Sr(k)} \quad (5.13)$$

Where:

- $\bar{r}(k)$ = Mean of running times of bus k

$$\bar{r}(k) = \frac{\sum_{j=1}^m r_j(k)}{m} \quad (5.14)$$

- $Sr(k)$ = Standard deviation of running times of bus k

$$Sr(k) = \sqrt{\frac{\sum_{j=1}^m [r_j(k) - \bar{r}(k)]^2}{m-1}} \quad (5.15)$$

5.2.3.3 Pattern Recognition

Following standardization, the patterns are used to recognize the neighbours of $b_0(k)$ represented by pattern B_0 (Eq.5.2). The automatic procedure to recognize the neighbours around $b_0(k)$ is similar to the one presented in Section 4.4.4 of which we replace the $r_0(k)$ and $r_j(k)$ with their patterns by the focal $b_0(k)$ and $b_j(k)$ with the corresponding patterns. For the sake of reducing a lengthy expression, this procedure is not shown here.

5.2.3.4 Prediction

To predict the standardized value of boarders $sb_0(k)$, we use the *Nadaraya-Watson* kernel estimation function as follows.

$$\hat{b}_0(k) = \frac{\sum_{j=1, j \neq i}^{OpNei} K_j \cdot b_j(k)}{\sum_{j=1, j \neq i}^{OpNei} K_j} \quad (5.16)$$

Where

$OpNei$ is the optimal number of neighbours around $b_0(k)$. Please see equations 4.18A and 4.18 B for the definition of $OpNei$.

K_j is the weight assigned for the j^{th} neighbour of the defined neighbourhood. Please refer to equation 4.19 for the definition of K_j

$\hat{b}_0$ = The predicted value of $b_0(k)$

5.2.3.5 Update Prediction

The updating process follows the same steps above as noted from pattern recognition to prediction. If there are M stops between the current stop (i.e. the stop the bus has just arrived) and the last stop, the DTM will be applied M times for predicting the number of boarders on the bus when it arrives at each stop.

5.2.4 Real-Time Alighting Passenger Prediction Sub-module

LOWESS-based method is also applied to predict the number of alighting passengers at a stop. The procedure is as shown below.

5.2.4.1 Parameter Selection

The following methods are applied to select parameters in modelling alighting passenger predictions: leave-one-out method for optimal bandwidth selection; *Tri-cube weight* function to recognize the neighbours, and *Nadaraya-Watson* kernel function for prediction.

5.2.4.2 Pattern Selection

The number of alighting passengers at a stop does not depend on the service frequency. Therefore, the pattern for the number of alighting passengers of bus k that are being predicted at stop $i+1$, written as $a_0(k)$, is selected as the composition of only the number of alighting passengers of the two previous buses denoted as $a_0(k-1)$ and $a_0(k-2)$

$$A_0 = [a_0(k-1), a_0(k-2)]^T \quad (5.17)$$

Where:

$a_0(k)$ = the being predicted number of passengers alighting from bus k at stop $i+1$.

$a_0(k-1), a_0(k-2)$ = the number of alighting passengers on the two previous buses $(k-1)$ and $(k-2)$ at stop $i+1$

Also in the past, each number of alighting passengers on bus k at stop $(i+1)$ is presented by its own pattern with its elements defined in a manner similar to the one above. So we have the patterns $A_1, A_2 \dots A_m$

$$A_j = [a_j(k-1), a_j(k-2)]^T \quad (5.18)$$

Where: $j=1$ to m ; m = the number of observations in the past

5.2.4.3 Pattern Recognition

The procedure to recognize the neighbours around $a_0(k)$ is similar to the one presented in Section 4.4.4.

5.2.4.4 Prediction

Again, we use *Nadaraya-Watson* kernel estimation function as follows

$$\hat{a}_0(k) = \frac{\sum_{j=1, j \neq i}^{OpNei} K_j \cdot a_j(k)}{\sum_{j=1, j \neq i}^{OpNei} K_j} \quad (5.19)$$

Where

$OpNei$ is the optimal number of neighbours around $a_0(k)$. Please see equations 4.18A and 4.18 B for definition of $OpNei$

K_j is the weight assigned for the j^{th} neighbour of the defined neighbourhood.

Please refer to equation 4.19 for the definition of K_j

$\hat{a}_0$ = The predicted value of $a_0(k)$

5.2.4.5 Update Prediction

The updating process follows the same steps as noted above from pattern recognition to prediction. Each time the bus arrives at a stop, if there are M stops between the current stop (i.e. the stop the bus has just arrived) and the last stop, the RASM will be applied M times for predicting the number of de-boarders on the bus when it arrives at each stop.

5.2.5 Regression Sub-Module

This sub-module is developed in order to examine the relationships between dwell time and other variables influencing it. It is a component of method A mentioned in Section 5.2.2. Data collected from APC and AVL systems of the OC Transpo are used to find these relationships.

5.2.5.1 Variable Selection and Preparation

The variables selected in the regression models are shown in Table 5.1. Because some variables were not collected by the APC-AVL systems but can be transformed from other variables, it is necessary to prepare data before going to the regression analyses.

The variables were transformed and calculated as shown below. The explanations of variables are in Tables 3.5 and 5.1.

$$LOAD_ARR: = (LOAD_DEP) + (TOTAL_OFFS) - (TOTAL_ONS) \quad (5.20)$$

$$TOTAL_ONS = (ON_1) + (ON_2) + (ON_3) \quad (5.21)$$

$$TOTAL_OFFS = (OFF_1) + (OFF_2) + (OFF_3) \quad (5.22)$$

$$PUNT = (ACT_TIME) - (EXPEC_TIME) \quad (5.23)$$

$$TOTAL_PASS = (TOTAL_ONS) + (TOTAL_OFFS) \quad (5.24)$$

$$LF = (LOAD_ARR) / BUS_CAPA \quad (5.25)$$

where: BUS_CAPA is the number of designated seats of transit bus; BUS_CAPA = 40 for rigid bus; = 65 for articulated bus.

Table 5.1: Variable Selections for RESM

Variable	Type of Variable	Code	Description of Variable	Reason for Selection
Boarding passengers	Numeric	<i>TOTAL_ONS</i>	The total number of passengers boarding at a stop	It has a major influence on dwell time.
Alighting passengers	Numeric	<i>TOTAL_OFFS</i>	The total number of passengers alighting at a stop	It has an influence on dwell time
Boarding and alighting	Numeric	<i>TOTAL_PASS</i>	Total passengers getting on and off at a bus stop	It has a major influence on dwell time.
On- board passengers prior stop	Numeric	<i>LOAD_ARR</i>	Number of passengers on the bus before it arrives or passes the stop.	This effects the circulation in the bus. Therefore it will influence dwell time
Loading factor	Numeric	<i>LF</i>	Ratio between the number of on-board passengers prior stop and the capacity of the bus	A value of LF close to 1 means that the bus is likely full. Hence, this may cause a long dwell time.
Alighting passengers at front door	Numeric	<i>OFF_1</i>	Number of alighting passengers using front door	If passengers alight at front door, it will increase dwell time.
Punctuality	Numeric	<i>PUNT</i>	Measured by lateness and earliness of the bus.	A long lateness can cause a crowded downstream stop resulting in a long dwell time.
Bus Type	Dummy	<i>BUS_TYPE</i>	Articulated bus or rigid bus (all are low-floor buses). 1=Articulated Bus; 0 = Rigid bus	Dwell time depends on bus type
Time	Dummy	<i>TIME</i>	Time of day (Morning, noon, after noon). 1= Morning; 2= Noon; 3= afternoon	Dwell time depends on time of day
Season	Dummy	<i>SEASON</i>	0= Winter; 1 = Spring; 2=Summer;3= Fall	Dwell time may be different for summer and winter
Number of Doors	Numeric	<i>DOORS</i>	Self explanatory	Increasing number of doors reduces dwell time
Stop location	Dummy	<i>STOP_LOCA</i>	1= Stop located at CBD 0= Otherwise	The location of bus stop may effect dwell time

5.2.5.2 Regression Functions

Before going to regression analyses, dwell times being used in the regression functions are processed to delete unreliable records. One of these records is the zero values, meaning that the bus does not stop at the bus stops. Such records are not included in the model and being deleted. The other records that should be removed from the dataset are the extreme values of dwell times and outliers. However, deleting these records cannot be done without any consideration. In this study, the records of dwell time over 180 seconds are deleted without any concern because such time durations are usually the layovers. After removing these records, the remaining data set was examined to find additional records that should be deleted.

Examinations of the data show that the mean of dwell time recorded for all seasons and for all times of day for bus 95 and 1 are 15.86 and 20.40 seconds, respectively. The records, which are suggested by the SPSS package as the extreme cases and outliers, are the ones with durations of over 44 seconds in bus route 95 and 57 seconds in bus route 1. The *boxplots* shown in Figures 5.3 and 5.4 present the outliers and extreme cases which are signified by small circles and asterisks¹, respectively. According to the SPSS, the cases with values between 1.5 and 3 box lengths from the upper or lower edge of the box are considered as outliers. The cases with values higher than 3 box lengths from both edges of the box are considered as extreme cases. The box length is the interquartile range.

The outliers and extreme cases as mentioned above (i.e. cases with recorded dwell time larger than 57 seconds for buses of route 1 and 44 seconds for bus 95) should be

¹ The number beside the asterisk (or circle) is the case number

erased in order to improve the prediction capability of the regression models. In this study, the dwell times lower than 57 seconds are kept for both routes. The sample size of the database for regression analysis reduces from 9983 cases to 8685 cases after outlier and zero value deletions².

Figure 5.3: Boxplot of Dwell time. Bus route 1

Figure 5.4: Boxplot of Dwell time. Bus route 95

² It was found that when using all data without any outlier and extreme case deletion for a pilot multiple-regression, the R^2 of this equation was rather low. Please see Appendix B1 for this regression.

Type A-1: Simple Linear Regression

The general regression equation is shown below.

$$\begin{aligned} dwell = & \beta_0 + \beta_1.(TOTAL_ON) + \beta_2.(TOTAL_OFF) + \beta_3.(TOTAL_PASS) + \\ & \beta_4.(LOAD_ARR) + \beta_5.(LF) + \beta_6.(PUNT) + \beta_7.(BUS_TYPE) + \beta_8.(TIME) + \\ & \beta_9.(SEASON) + \beta_{10}.(DOORS) + \beta_{11}.(STOP_LOCA) \end{aligned} \quad (5.26)$$

The backward stepwise regression method is selected in order to track the effect of each variable on dwell time. As a part of this method, variables with significance values of more than 0.15 (i.e., they are statistically not significant) are not included in the model. From this perspective, one may argue that a significance level of 0.05 should be better. Of course, a selection of a low value of level of significance (i.e. high confidence level) will reduce the risk of taking an unimportant variable in the regression model. However, 0.05 is too low and often results in the deletion of the important variables from the model. Therefore, the values of $\alpha = 0.15-0.2$ are suggested (Scott Menard, 2001, p. 64).

The SPSS (Version 13.0) package was used to develop the regression model. Eighty percent of the total cases were drawn randomly. The remaining cases were used for testing the models. The models showing the largest *R-square*, along with significant *t-test* for parameters, were selected. The results of the three runs are shown in Tables 5.2 and 5.3.

Table 5.2: Coefficients of the Best Regression

Coefficients ^a					
Model		Unstandardized Coefficients		Standardized Coefficients	Sig.
		B	Std. Error	Beta	
1	(Constant)	15.750	1.154		.000
	TOTAL_ONS	1.220	.016	.673	.000
	TOTAL_OFFS	.438	.016	.246	.000
	LOAD_ARR	.072	.029	.112	.013
	LF	-4.935	1.649	-.125	.003
	TIME	.113	.119	.008	.341
	PUNT	.003	.001	.047	.000
	SEASON	.097	.088	.009	.272
	STOP_LOCATE	2.125	.217	.089	.000
2	(Constant)	16.020	1.119		.000
	TOTAL_ONS	1.222	.016	.674	.000
	TOTAL_OFFS	.439	.016	.246	.000
	LOAD_ARR	.075	.028	.117	.009
	LF	-5.086	1.642	-.129	.002
	PUNT	.003	.001	.047	.000
	SEASON	.093	.088	.009	.291
	STOP_LOCATE	2.127	.217	.089	.000
	DOORS	-4.385	.419	-.160	.000
3	(Constant)	16.175	1.109		.000
	TOTAL_ONS	1.222	.016	.674	.000
	TOTAL_OFFS	.438	.016	.246	.000
	LOAD_ARR	.074	.028	.116	.009
	LF	-5.064	1.641	-.128	.002
	PUNT	.003	.001	.047	.000
	STOP_LOCATE	2.126	.217	.089	.000
	DOORS	-4.397	.418	-.161	.000

a. Dependent Variable: DWELL_3

$$\begin{aligned}
 dwell &= 16.175 + 2.126 (STOP_LOCATE) + 0.438 (TOTAL_OFFS) + \\
 R^2 &= 0.581 \quad t=14.583 \quad t=9.812 \quad t=27.163 \\
 &+ 1.222 (TOTAL_ONS) - 4.397 (DOORS) + 0.074 (LOAD_ARR) \\
 &- 5.064 (LF) + 0.003 (PUNT) \\
 &\quad t=75.856 \quad t=-10.511 \quad t=2.605 \quad t=-3.085 \quad t=5.116
 \end{aligned}
 \tag{5.27}$$

After 3 runs, the stepwise regression method returned the best regression as shown in Table 5.2. Two out of 9 variables were removed from the model (i.e. *TIME* and *SEASON*). All parameters are statistically significant. With *R-square* of 0.581, the model is satisfactory.

Following Equation 5.27, if we control other variables, dwell time will increase by 2.13 seconds if bus stop is located in the CBD area compared to that of a stop outside this

area. The number of doors has a strong influence on dwell time. If the bus has 3 doors instead of 2 doors, dwell time can decrease up to 4.34 seconds when other variables are controlled. Dwell time will increase if the bus has more boarding or alighting passengers.

Table 5.3: Model Summary for the Best Regression

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.763 ^a	.582	.581	7.66728
2	.763 ^b	.582	.581	7.66722
3	.763 ^c	.581	.581	7.66729

a. Predictors: (Constant), DOORS, SEASON, STOP_LOCATE, TIME, TOTAL_OFFS, LF, TOTAL_ONS, PUNT, LOAD_ARR

b. Predictors: (Constant), DOORS, SEASON, STOP_LOCATE, TOTAL_OFFS, LF, TOTAL_ONS, PUNT, LOAD_ARR

c. Predictors: (Constant), DOORS, STOP_LOCATE, TOTAL_OFFS, LF, TOTAL_ONS, PUNT, LOAD_ARR

Type A-2: Non- Linear Regression

Besides the multiple linear regression developed above, a series of non-linear regression types of function are also examined.

- Type A-2.1:

$$dwell = \beta_0 + \beta_1.(TOTAL_ON) + \beta_2.(TOTAL_ON)^2 + \beta_3.(TOTAL_OFF) + \beta_4.(TOTAL_PASS) + \beta_5.(LOAD_ARR) + \beta_6.(LF) + \beta_7.(PUNT) + \beta_8.(BUS_TYPE) + \beta_9.(TIME) + \beta_{10}.(SEASON) + \beta_{11}.(DOORS) + \beta_{12}.(STOP_LOCA) \quad (5.28)$$

- Type A-2.2

$$dwell = \beta_0 + \beta_1.(TOTAL_ON) + \beta_2.(TOTAL_ON)^2 + \beta_3.(TOTAL_ON)^3 + \beta_4.(TOTAL_OFF) + \beta_5.(TOTAL_PASS) + \beta_6.(LOAD_ARR) + \beta_7.(LF) + \beta_8.(PUNT) + \beta_9.(BUS_TYPE) + \beta_{10}.(TIME) + \beta_{11}.(SEASON) + \beta_{12}.(DOORS) + \beta_{13}.(STOP_LOCA) \quad (5.29)$$

- Type A-2.3

$$\begin{aligned}
\text{dwell} = & \beta_0 + \beta_1(TOTAL_ON) + \beta_2(TOTAL_ON)^2 + \beta_3(TOTAL_OFFS)^2 + \\
& \beta_4(TOTAL_OFF)(TOTAL_OFF) + \beta_5(TOTAL_PASS) + \beta_6(LOAD_ARR) + \beta_7(LF) + \\
& \beta_8(PUNT) + \beta_9(BUS_TYPE) + \beta_{10}(TIME) + \beta_{11}(SEASON) + \beta_{12}(DOORS) + \beta_{13}(STOP_LOCA)
\end{aligned}
\tag{5.30}$$

- Type A-2.4 (Gauss Regression)

$$\text{dwell} = b_1.(1 - b_3 \exp(-b_2.(TOTAL_PASS)^2)) \tag{5.42}$$

- Type A-2.5 (Weibull Regression)

$$\text{dwell} = b_1 - b_2.\exp(-b_3.(TOTAL_PASS)^{b_4}) \tag{5.31}$$

- Type A-2.6 (Morgan-Mercer-Florin Regression)

$$\text{dwell} = (b_1.b_2 + b_3.TOTAL_PASS^{b_4})/(b_2 + TOTAL_PASS^{b_4}) \tag{5.32}$$

The types from A-2.1 to A-2.3 are polynomial regressions in term of variables *TOTAL_ONS* and *TOTAL_OFF* whereas types from A-2.4 to A-2.6 are the common non-linear regressions where the relationship between dwell time and *TOTAL_PASS* (e.g., total number of passengers getting on and off the bus) is examined. To find the best regression equation for each type, backward regression method was applied for the types from A-2.1 to A-2.3 and the Gauss-Newton method was applied for the rest. By using the SPSS version 13, we can find the best regression equations. Their *R-square* values and parameters are presented in Table 5.4. Details can be seen in **Appendix B1**.

Out of the types from A-1 to A2-6, type A-2.3 has the highest *R-square*, meaning that it is the most accurate type as compared with others. As a result, the use of type A-2.3 is proposed in this research to predict dwell time.

Although type A-2.6 is less accurate than type A-2.3, it is a compact one requiring only the total of passenger activities through bus doors recorded or predicted. Therefore, it should be used in case of lack of data on some variables.

Table 5.4: Proposed Non-linear Regression Models

Type	The best regression equations	R-square
A-2.1	$dwel\ell = 13.177 + 1.928(TOTAL_ONS) - 0.030(TOTAL_ONS)^2 - 0.438(TOTAL_OFF) - 0.830(LF) - 3.922(DOORS) + 1.957(STOP_LOCA) + 0.003(PUNT)$ <small> $t=21.024$ $t=51.635$ $t=-20.636$ $t=-28.834$ $t=-2.414$ $t=-17.342$ $t=9.664$ $t=5.456$ </small>	0.610
A-2.2	$dwel\ell = 12.843 + 2.351(TOTAL_ON) - 0.071(TOTAL_ON)^2 + 0.001(TOTAL_ONS)^3 + 0.429(TOTAL_OFF) + 0.03(PUNT) - 0.954(LF) - 0.153(SEASON) - 4.073(DOORS) + 1.808(STOP_LOCA)$ <small> $t=19.804$ $t=33.657$ $t=-11.873$ $t=7.050$ $t=-28.173$ $t=5.490$ $t=-2.771$ $t=1.821$ $t=-17.953$ $t=-8.987$ </small>	0.618
A-2.3	$dwel\ell = 14.434 + 2.014(TOTAL_ON) - 0.026(TOTAL_ON)^2 + 0.516(TOTAL_OFFS) + 0.004(TOTAL_OFFS)^2 - 0.022(TOTAL_ONS)(TOTAL_OFFS) + 0.078(Load_ARR) - 5.047(LF) + 0.003(PUNT) - 4.748(DOORS) + 2.087(STOP_LOCA) + 0.196(SEASON)$ <small> $t=13.921$ $t=54.071$ $t=-17.944$ $t=15.364$ $t=3.387$ $t=-11.548$ $t=2.982$ $t=-3.349$ $t=5.492$ $t=-12.295$ $t=10.384$ $t=2.419$ </small>	0.634
A-2.4	$dwel\ell = 31.08(1 - 0.795 \exp(-0.346(10^{-1}TOTAL_PASS)^2))$	0.502
A-2.5	$dwel\ell = 1184.802 - 91.856 \exp(-2.557(10^{-1}TOTAL_PASS)^{-0.003})$	0.477
A-2.6	$dwel\ell = (-146.259 + 470.907(TOTAL_PASS)^{0.561}) / (97.767 + (10^{-1}TOTAL_PASS)^{0.561})$	0.528

To predict dwell time, the number of boarding passengers ($TOTAL_ONS$) and the number of alighting passengers ($TOTAL_OFFS$), which can be predicted from previous sub-modules (i.e., BPSM and APSM), is used in regression A-2.3. Other related variables (i.e., $LOAD_ARR$, $PUNT$, and LF) can also be determined indirectly by using Equations 5.20, 5.23 and 5.25. It should be noted that data on all variables that are used in this type are all obtainable from the APC and AVL systems mounted on the buses.

5.2.6 Busiest Door Prediction Sub-Module

If we assume that all passengers board at the front door and alight at the rear-door, the dwell time can be easily predicted as the maximum of the two service times, one for boarding passengers at front door and one for alighting passengers at the rear door. However, data collected from the APC system by the OC Transpo shows the fact that the on-board passengers alight at all doors while waiting passengers can get on the bus at the front door of a 2 door-bus and from all doors of the 3-door bus. This implies that the dwell time is actually the service time at the busiest door. This can be seen in the suggestions of the Highway Capacity Manual 2000 (Equation 2.2). However, how to find the busiest door was not discussed in literature. In this sub-module, the equation provided by the HCM (2000) is used with changes in finding the most crowded door. It should be recalled that the sub-module is a part of method B, another method proposed in this study for predicting dwell time in real-time, given the predicted number of boarding and alighting passengers. Please refer to Fig.5.2 for details. In this research, dwell time is calculated by using Equation 5.33.

$$dwell = a.t_a + b.t_b + t_{op} \quad (5.33)$$

Where: $dwell$ = Predicted dwell time in seconds

a = Predicted number of alighting passengers through the busiest door

t_a = Passenger alighting time per passenger (follows HCM 2000)

b = Predicted number of boarding passengers through the busiest door

t_b = Passenger boarding time per passenger (follows HCM 2000)

t_{op} = Door opening and closing time (4 seconds)

In order to find the busiest door for each type of bus, data are clustered upon rigid body bus and articulated bus. The methodologies to find the busiest door for each type are presented below.

5.2.6.1 Rigid-Body Bus

A rigid-body bus has two doors, one in front and the other in the rear. At a bus stop, passengers can only get on the bus from the front door while they can get off the bus at any door convenient for them. Before the bus completely opens the doors, alighting passengers tend to move to the doors for ease of exiting. Alighting passengers have priority to get off the bus first. If some passengers get off via the front door, on-ground passengers have to wait until the last alighter leaves the bus before getting on. This will lengthen dwell time compared to the case when all alighting passengers use the rear door and all boarding passengers use the front door. For example, assume that there are 8 alighting passengers and 3 boarding passengers. Serving time for each alighter is assumed to be 2.5 seconds and 5 seconds for each boarder. Hence, servicing time to board all 3 passengers is 15 seconds at the front door, and 20 seconds are required for 8 alighting passengers, provided that all alighters use the rear door and all boarders use the front door. The busier door is the rear door (i.e., 20 seconds). However, if 4 out of 8 alighters use the front door, the servicing time at this door will be 25 seconds and for the rear door it is only 10 seconds. The busier door now is the front door instead of the rear door as found in the first case and the extra service time is 5 seconds. Obviously, if

passengers get off at the front door, the dwell time is increased. If we can find the busier door, we will have a better prediction of dwell time of the bus.

On-board passengers are likely to choose the most convenient door for alighting. Of course, passengers near a door will usually use that door for alighting because of a short distance. Based on the distance and the ease to move to the doors, passengers in the middle of the bus will choose a door for alighting. In general, passengers will use the door that offers them the highest convenience. Intuitively, the convenience of a door can be presented by several variables such as the number of boarding and alighting passengers, the number of standees on the bus, the stop location, time of day, season, and so on. In other words, the likelihood (or the probability) that a door is the busier door depends on an unobserved variable (i.e. the convenience that a door offers passengers at each bus stop) whose values can be quantified by a series of data retrieved from APC and AVL systems. Let us call U as “convenience” function. Assume that we have to relate probability that the front door is the busier door (P_{front_door}) with convenience value that the front door offers alighting passengers at bus stops.

Obviously, we cannot model this probability directly as $P_{front_door} = U$, because the values of U may vary outside the range 0 and 1 while the probability must lie between this range. To solve the problem, a commonly used approach in logistic regression is to relate the probability with the *odds*.

Let us assume that the $odds_{front_door}$ describes the ratio of the probability that the front door is the busier door to the probability that it is not so. It is easy to convert back and forth between *odds* and probability by using Equations 5.34 and 5.35. As shown in the equations, probabilities range from 0 to 1, whereas *odds* range from 0 to infinity. An

odds equal to 1 indicates equal probabilities that the front door is the busier door and it is not so (i.e., 50%). For *odds* greater than 1, that the front door is the busier door is more likely and vice versa.

$$odds_{front_door} = \frac{P_{front_door}}{1 - P_{front_door}} \quad (5.34)$$

$$P_{front_door} = \frac{odds_{front_door}}{1 + odds_{front_door}} \quad (5.35)$$

Where:

P_{front_door} = the probability the front door is the busier door

The natural logarithm of the odds that the front door is the busier door is called the *Logit (front_door)*.

$$Logit (front_door) = \ln (odds_{front_door}) = \ln \left(\frac{P_{front_door}}{1 - P_{front_door}} \right) \quad (5.36)$$

Obviously, the variable *Logit (front_door)* varies from negative infinity to positive infinity. Therefore, if we model *Logit (front_door)* = *U*, we will have a form of logistic regression.

$$Logit (front_door) = U \quad (5.37)$$

We can convert *Logit (front_door)* back to the odds by exponentiation. From Equations 5.37 and 5.38 we have

$$Odds_{front_door} = e^U \quad (5.38)$$

Substitute the $odds_{front_door}$ in Equation 5.38 into Equation 5.35, so the probability that the front door is the busier door can be calculated as below.

$$P_{front_door} = P_1 = \frac{e^U}{1 + e^U} \quad (5.39)$$

The probability that the rear door is the busier door (P_{rear_door}) is as shown below

$$P_{rear_door} = P_2 = \frac{1}{1 + e^U} \quad (5.40)$$

Several types of U equations are proposed in the Equations from 5.41 to 5.43. The regression coefficients of each equation are estimated through an interactive maximum likelihood method.

Type B-L.1

$$U = \beta_0 + \beta_1.(TOTAL_ONS) + \beta_2.(TOTAL_OFFS) + \beta_3.(LOAD_ARR) + \beta_4.(LF) + \beta_5.(PUNT) + \beta_6.(TIME) + \beta_7.(SEASON) + \beta_8.(STOP_LOCA) \quad (5.41)$$

Type B-L.2

$$U = \beta_0 + \beta_1.(TOTAL_ON) + \beta_2.(TOTAL_ON)^2 + \beta_3.(TOTAL_ON)^3 + \beta_4.(LOAD_ARR) + \beta_5.(LF) + \beta_6.(PUNT) + \beta_7.(TIME) + \beta_8.(SEASON) + \beta_9.(STOP_LOCA) \quad (5.42)$$

Type B-L.3

$$U = \beta_0 + \beta_1.(TOTAL_ON) + \beta_2.(TOTAL_ON)^2 + \beta_3.(TOTAL_OFFS)^2 + \beta_4.(TOTAL_ONS)(TOTAL_OFFS) + \beta_5.(LOAD_ARR) + \beta_6.(LF) + \beta_7.(PUNT) + \beta_8.(TIME) + \beta_9.(SEASON) + \beta_{10}.(STOP_LOCA) \quad (5.43)$$

Now, we can apply binary logistic regression. APC data of 7 bus stops of route 1 were used to develop the model. At each stop, the sums of passengers through the front door and the rear door were calculated for each door and compared to each other for every bus trip. If the front door is the busier door then it is coded as 1, otherwise as 0. In this research, the dwell times longer than 180 seconds or equal to zero are not included in

the model. Also, the cases that recorded boarding passengers at the rear door are considered as “unusual events” and erased.

The validated dataset is shown in Table 5.5. Out of the 2642 cases, there are 1740 cases recording that the busier door is the front door, accounting for 68.3 percent of the total.

The backward stepwise method was used in this research in order to delete the variables with significance level larger than 0.1, and as the sequence, the best regression was being found. To do this, the SPSS 13 package was used to develop the model for each type of U presented in Equations 5.41, 5.42 and 5.43. The overall significance was tested with *Model Chi-squared* as called by SPSS, which is derived from the likelihood of observing the actual data under assumption that the model that has been fitted is accurate.

The best regression function of each type from B-L.1 to B-L.3 after the backward stepwise procedure were found and tabulated in Table 5.6. Details can be seen in **Appendix B2**.

Table 5.5: Validated Dataset for Logistic Regressions

VALIDATED_BUSIER_DOOR					
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	.00	808	30.6	31.7	31.7
	1.00	1740	65.9	68.3	100.0
	Total	2548	96.4	100.0	
Missing	System	94	3.6		
Total		2642	100.0		

Table 5.6: Best Binary Logistic Regression Equations

Type	The Best Logistic Regression Equation	-2LL	Nagelkerke <i>R</i> square	Overall Accuracy
B-L.1	$U = -1.023 + 1.205(TOTAL_ONS) - 0.314(TOTAL_OFFS) +$ $0.017(Load_ARR) - 0.001(PUNT) + 0.704(STOP_LOCATE(1))$	1804.39	0.586	86.3 %
B-L.2	$U = 0.510 + 1.091(TOTAL_ON) - 0.018(TOTAL_ON)^2$ $- 0.028(Load_ARR) - 0.821(STOP_LOCA)$	948.378	0.469	89%
B-L.3	$U = 1.227 + 1.460(TOTAL_ON) - 0.665(TOTAL_OFF) -$ $0.024(TOTAL_ON)^2 + 0.018(TOTAL_OFFS)^2 -$ $0.022(TOTAL_ONS)(TOTAL_OFFS) - 0.042(Load_ARR)$	861.847	0.529	89%

As can be seen in Table 5.6, all types are quite powerful in term of prediction. To obtain further information for selection, we use the R^2_L , which was suggested by Menard (2001, p.27). According to him “ R^2_L is the most appropriate for logistic regression because it is conceptually closest to the Ordinary Least Square”.

$$R^2_L = \frac{\chi^2}{\chi^2 - 2LL} \quad (5.44)$$

Where: χ^2 = the *Chi-square* of the logistic regression model.

Following Equation 5.44, the R^2_L for types B-L.1, B-L.2 and B-L.3 are 0.433, 0.372 and 0.430, respectively. Type B-L.1 shows the highest R^2_L . As a result, it is used for this research.

Table 5.7: Model Summary of Type B-L1

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	1804.229(a)	.418	.586
2	1804.777(a)	.418	.586

a Estimation terminated at iteration number 8 because parameter estimates changed by less than .001.

Table 5.8: Prediction Performance of Type B-L.1

Classification Table ^a					
Observed			Predicted		
			BUSIER DOOR		Percentage Correct
			Busier door is rear door	Busier Door is Front door	
Step 1	BUSIER_DOOR	Busier door is rear door	647	161	80.1
		Busier Door is Front door	189	1551	89.1
	Overall Percentage				86.3
Step 2	BUSIER_DOOR	Busier door is rear door	647	161	80.1
		Busier Door is Front door	188	1552	89.2
	Overall Percentage				86.3

a. The cut value is .500

As shown in Table 5.7, it has a *pseudo R²* statistics of 0.586 (*Nagelkerke R²*) and 0.418 (*Cox & Snell R²*), which indicate a satisfactory model.

Table 5.8 presents prediction performance of type BL-1. The overall correct prediction of the model is up to 86.3 percent. Out of the 1740 cases recorded showing that the busier door is the front door, this model predicted correctly 1552 cases. This results in an accuracy of up to 89.1 percent. Also, out of the observed 708 cases where the rear door is the busier door, the model predicted correctly 647 cases, resulting in a good prediction up to 80.1% of accuracy.

Table 5.9 tabulates the coefficients of type B-L.1. The *odds ratio* associated with each coefficient is presented as $Exp(B)$ shown in the last column of this table. The odds ratio is the number by which we would multiply the *odds_{front_door}* (the probability divided by 1 minus the probability, as shown in Equation 5.34) for each one-unit increase in the dependent variables. An *odds ratio* greater than 1 indicates that the *odds_{front_door}*

increases when the independent variable increases and an odds ratio smaller than 1 indicate that the $odds_{front_door}$ decrease when the independent variable increases.

Table 5.9: Coefficients of Type BL-1

		<i>B</i>	<i>S.E.</i>	<i>Wald</i>	<i>df</i>	<i>Sig.</i>	<i>Exp(B)</i>
Step 1(a)	TOTAL_ONS	1.203	.059	419.818	1	.000	3.331
	TOTAL_OFFS	-.314	.021	217.103	1	.000	.730
	LOAD_ARR	.017	.006	9.586	1	.002	1.018
	PUNT	-.001	.000	3.190	1	.074	.999
	SEASON			7.004	3	.072	
	SEASON(1)	-.183	.154	1.416	1	.234	.833
	SEASON(2)	-.088	.172	.260	1	.610	.916
	SEASON(3)	.238	.170	1.954	1	.162	1.268
	STOP_LOCATE(1)	.704	.177	15.838	1	.000	2.023
	TIME			.548	2	.760	
	TIME(1)	.089	.156	.328	1	.567	1.093
	TIME(2)	.112	.154	.526	1	.468	1.118
	Constant	-1.105	.250	19.548	1	.000	.331
Step 2(a)	TOTAL_ONS	1.205	.059	420.852	1	.000	3.337
	TOTAL_OFFS	-.314	.021	218.743	1	.000	.730
	LOAD_ARR	.017	.006	9.620	1	.002	1.018
	PUNT	-.001	.000	3.213	1	.073	.999
	SEASON			6.913	3	.075	
	SEASON(1)	-.186	.154	1.462	1	.227	.830
	SEASON(2)	-.096	.171	.317	1	.573	.908
	SEASON(3)	.230	.170	1.840	1	.175	1.259
	STOP_LOCATE(1)	.704	.177	15.844	1	.000	2.023
	Constant	-1.023	.223	21.020	1	.000	.359

a Variable(s) entered on step 1: TOTAL_ONS, TOTAL_OFFS, LOAD_ARR, PUNT, SEASON, STOP_LOCATE, TIME.

Because the *odd* ratios of the independent variables *TOTAL_ON*, *LOAD_ARR* and *STOP_LOCATE(1)* are all greater than 1, the $odds_{front_door}$ will increase when these variables increase. In contrast, the $odds_{front_door}$ will decrease with the increments of the variables *TOTAL_OFF* and *PUNT*.

Specifically, while controlling other variables, if the bus has one more boarding passenger, the $odds_{front_door}$ will increase 3.33 times. This is obvious because all

passengers have to board at the front door. Similarly, the $odds_{front_door}$ will increase 1.02 with one unit loading factor increment. This result is expected because on-board passengers tend to alight via front door because of the difficulties in circulation. This $odds$ will be doubled if the bus serves in CBD area. In contrast, if the bus has one more alighting passenger, the $odds_{front_door}$ will decrease 0.83 times. Also, the $odds_{front_door}$ will reduce if the bus arrives at stop later.

Once the busier door has been defined, dwell time will be determined by Equations 5.45, 5.46 and 5.47.

$$dwell = P_1(TOTAL_OFFS)t_a + (TOTAL_ONS)t_b + t_{op} \quad \text{if } P_1 \geq P_2 \quad (5.45)$$

$$dwell = \text{Max} \{ P_1(TOTAL_OFFS)t_a + (TOTAL_ONS)t_b; P_2(TOTAL_OFFS)t_a \} + t_{op} \quad \text{if } P_1 < P_2 \quad (5.46)$$

$$P_2 = 1 - P_1 \quad (5.47)$$

Where:

$TOTAL_ONS$ and $TOTAL_OFFS$ = Total the number of boarding and alighting passengers.

t_a, t_b = Average service time per passenger for alighting and boarding, respectively.

t_{op} = Door opening and closing time (4 seconds)

5.2.6.2 Articulated Bus

Unlike a rigid bus, an articulated bus has three doors and passengers can board via rear doors. To predict the busiest door out of the three doors of the bus, multinomial *Logit* regression is applied.

Mathematically, the extension of the binary logistic regression, which was applied for rigid-body bus, to multinomial logistic regression is straightforward. Let us assume the second rear door is the reference category (i.e., the probability that this door is the busiest door). We call the natural logarithm of the *odd* (*front door*) (the ratio of the probability that the front door is the busiest door to the probability that the second rear door is the busiest door) is the *Logit* of front door, written as *Logit* (*front_door*). Similarly, we denote *Logit* (*first_rear_door*) as the natural logarithm of the ratio of the probability that the first rear door is the busiest door to the probability that second rear door is the busiest door). In mathematical notations, we have:

$$\text{Logit}(\text{front_door}) = \ln \frac{P_1}{P_3} \quad (5.48)$$

$$\text{Logit}(\text{first_rear_door}) = \ln \frac{P_2}{P_3} \quad (5.49)$$

$$P_1 + P_2 + P_3 = 1 \quad (5.50)$$

Where:

P_1 , P_2 , and P_3 = the probabilities that the front door, the first rear door and the last rear door is the busiest door, respectively.

$\ln$ = natural logarithm or logarithm base e

Similar to the explanations in the case of two-door bus, we use the convenience functions namely U_1 and U_2 whose variables are collectable from APC and AVL systems, one for each door (i.e., the front door and the first rear door) relative to the second rear door (i.e., the reference category), to describe the relationship between the probabilities and the collected data. In other words, we model $\text{Logit}(\text{front_door}) = U_1$ and Logit

(*first_rear_door*) = U_2 . Therefore, based on Equations 5.48, 5.49, and 5.50 with several mathematical operations, we have the probability that the front door is the busiest door is:

$$P_1 = \frac{e^{U_1}}{e^{U_1} + e^{U_2} + 1} \quad (5.51)$$

Similarly, the probability that the first rear door is the busiest door is:

$$P_2 = \frac{e^{U_2}}{e^{U_1} + e^{U_2} + 1} \quad (5.52)$$

And, the probability that the second rear door is the busiest door is:

$$P_3 = \frac{1}{e^{U_1} + e^{U_2} + 1} \quad (5.53)$$

Some types of U_1 and U_2 equations are examined in order to find the best multinomial *Logit* regressions. The types of U 's function are formulated as follows

Type B-ML.1

$$U_1 = \beta_0 + \beta_{1-1} \cdot (TOTAL_ONS) + \beta_{2-1} \cdot (TOTAL_OFFS) + \beta_{3-1} \cdot (LOAD_ARR) + \beta_{4-1} \cdot (LF) + \beta_{5-1} \cdot (PUNT) + \beta_{6-1} \cdot (TIME) + \beta_{7-1} \cdot (SEASON) + \beta_{8-1} \cdot (STOP_LOCA) \quad (5.54)$$

$$U_2 = \beta_0 + \beta_{1-2} \cdot (TOTAL_ONS) + \beta_{2-2} \cdot (TOTAL_OFFS) + \beta_{3-2} \cdot (LOAD_ARR) + \beta_{4-2} \cdot (LF) + \beta_{5-2} \cdot (PUNT) + \beta_{6-2} \cdot (TIME) + \beta_{7-2} \cdot (SEASON) + \beta_{8-2} \cdot (STOP_LOCA) \quad (5.55)$$

Type B- ML.2

$$U_1 = \beta_0 + \beta_{1-1} \cdot (TOTAL_ONS) + \beta_{2-1} \cdot (TOTAL_OFFS) + \beta_{3-1} \cdot (TOTAL_ON)^2 + \beta_{4-1} \cdot (LOAD_ARR) + \beta_{5-1} \cdot (LF) + \beta_{6-1} \cdot (PUNT) + \beta_{7-1} \cdot (TIME) + \beta_{8-1} \cdot (SEASON) + \beta_{9-1} \cdot (STOP_LOCA) \quad (5.56)$$

$$U_2 = \beta_0 + \beta_{1-2} \cdot (TOTAL_ONS) + \beta_{2-2} \cdot (TOTAL_OFFS) + \beta_{3-2} \cdot (TOTAL_ON)^2 + \beta_{4-2} \cdot (LOAD_ARR) + \beta_{5-2} \cdot (LF) + \beta_{6-2} \cdot (PUNT) + \beta_{7-2} \cdot (TIME) + \beta_{8-2} \cdot (SEASON) + \beta_{9-2} \cdot (STOP_LOCA) \quad (5.57)$$

Type B- ML.3

$$U_1 = \beta_0 + \beta_{1-1} \cdot (TOTAL_ONS) + \beta_{2-1} \cdot (TOTAL_OFFS) + \beta_{3-1} \cdot (TOTAL_ON)^2 + \beta_{4-1} \cdot (TOTAL_OFF)^2 + \beta_{5-1} \cdot (TOTAL_ONS) \cdot (TOTAL_OFFS) + \beta_{6-1} \cdot (LOAD_ARR) + \beta_{7-1} \cdot (LF) + \beta_{8-1} \cdot (PUNT) + \beta_{9-1} \cdot (TIME) + \beta_{10-1} \cdot (SEASON) + \beta_{11-1} \cdot (STOP_LOCA) \quad (5.58)$$

In order to find the coefficients of the regression types above, APC data from 7 stops on the bus route 95 were used. Total number of boarding and alighting passengers at each door of the bus at each bus stop were calculated and compared with those of the other doors. If a door is the busiest door, it is coded as 1 and the remaining doors are coded as zeros. By using the multinomial logistic regression function of the SPSS 13.0 software for each type of U s named as B-ML.1, B-ML.2 and B-ML.3, we have the best Multinomial Logistic Regression for each. The results of these procedures are tabulated in Table 5.10. Please see **Appendix B3** for the detail of each type.

Table 5.10: The Best Multinomial Logistic Regression Equations

Type	The Best Logistic Regression Equation	χ^2 - deviance	McFaden <i>R</i> square	Overall Accuracy
B-ML.1	$U_1 = 1.302 + 0.098 * TOTAL_ONS - 0.308 * TOTAL_OFFS - 6.10^{-3} * LOAD_ARR - 0.105 * TIME + 6.7.10^{-3} * SEASON + 0.549 * STOP_LOCA$ $U_2 = -0.133 - 0.001 * TOTAL_ONS - 0.023 * TOTAL_OFFS + 0.004 * LOAD_ARR - 0.114 * TIME - 0.022 * SEASON + 0.057 * STOP_LOCA$	10360.49	0.193	62.3
B-ML.2	$U_1 = 1.602 + 1.50 * TOTAL_ONS - 0.30 * TOTAL_OFFS - 0.003 * TOTAL_ON^2 - 0.013 * LOAD_ARR - 0.116 * TIME + 0.122 * SEASON + 0.659 * STOP_LOCA$ $U_2 = -0.228 + 0.016(TOTAL_ONS) - 0.023(TOTAL_OFFS) + 0.003(LOAD_ARR) - 0.104(TIME) - 0.011(SEASON) + 0.034(STOP_LOCA)$	8352.10	0.194	65.2
B-ML.3	$U_1 = 2.230 + 0.166 * TOTAL_ONS - 0.513 * TOTAL_OFFS - 0.006 * TOTAL_ON^2 + 0.006 * TOTAL_OFF^2 + 0.01 * TOTAL_ONS * TOTAL_OFFS - 0.016 * LOAD_ARR - 0.172 * TIME + 0.133 * SEASON + 0.601 * STOP_LOCA$ $U_2 = -0.178 + 0.19 * TOTAL_ONS - 0.032 * TOTAL_OFFS - 0.001 * TOTAL_ON^2 + 0.003 * LOAD_ARR - 1.05 * TIME - 0.01 * SEASON + 0.033 * STOP_LOCA$	861.85	0.224	65.9

As shown in Table 5.10, type ML-3 provides the largest *McFadden* R^2_L of 0.224 and the overall accuracy of 65.9 percent. Therefore, type ML-3 is suggested for busiest door prediction of articulated bus. Also, it works fairly well, as indicated by χ^2 of 861.847 and *McFadden* R^2_L of 0.224.

The overall Likelihood Ratio tests of this type are presented in Table 5.11 on the basis of SPSS application. The results show that all parameters are statistically significant, except the variable *PUNT*.

Table 5.11: Likelihood Ratios of type B-ML.3

Effect	Model Fitting Criteria			Likelihood Ratio Tests		
	AIC of Reduced Model	BIC of Reduced Model	-2 Log Likelihood of Reduced Model	Chi-Square	df	Sig.
Intercept	8372.224	8503.287	8332.224	285.443	2	.000
LOAD_ARR	8090.781	8234.950	8046.781(a)	.000	0	.
LF	8090.781	8234.950	8046.781(a)	.000	0	.
PUNT	8086.832	8217.895	8046.832	.051	2	.975
SEASON	8104.913	8235.976	8064.913	18.132	2	.000
STOP_LOCATE	8143.819	8274.882	8103.819	57.038	2	.000
TIME	8099.926	8230.989	8059.926	13.145	2	.001
TOTAL_ONS	8213.854	8344.917	8173.854	127.073	2	.000
TOTAL_OFFS	9061.769	9192.832	9021.769	974.988	2	.000
Total_On_squared	8205.349	8336.412	8165.349	118.568	2	.000
TotalOffPw	8125.214	8256.277	8085.214	38.433	2	.000
TOTALon_and_OFF	8191.669	8322.732	8151.669	104.888	2	.000

The chi-square statistic is the difference in -2 log-likelihoods between the final model and a reduced model. The reduced model is formed by omitting an effect from the final model. The null hypothesis is that all parameters of that effect are 0.

a This reduced model is equivalent to the final model because omitting the effect does not increase the degrees of freedom.

As shown in Table 5.12, the probabilities of front door and the first rear door are both reduced when more passengers get off the buses, but the front door has a larger reduction. In contrast, these probabilities increase when more passengers get on the bus

and the influence on the front door has a larger increment. Interestingly, location of bus stop has a quite strong influence on passenger's door choice. This can be seen shown in *Logit (door_1_busiest)* and *Logit (door_2_busiest)* of 1.824 and 1.033, if the stop is located in the CBD area.

Table 5.12: Parameters of Type B-ML.3

Parameter Estimates								
Busiest door ^a		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)
								Lower Bound Upper Bound
door 2 busiest	Intercept	-.178	.179	.993	1	.319		
	LOAD_ARR	.003	.003	1.300	1	.254	1.003	.998 1.008
	LF	0 ^b	.	.	0	.	.	.
	PUNT	.000	.000	.008	1	.930	1.000	.999 1.001
	SEASON	-.010	.039	.061	1	.806	.990	.918 1.069
	STOP_LOCATE	.033	.092	.127	1	.721	1.033	.863 1.237
	TIME	-.105	.051	4.232	1	.040	.901	.815 .995
	TOTAL_ONS	.019	.017	1.276	1	.259	1.019	.986 1.054
	TOTAL_OFFS	-.032	.014	5.113	1	.024	.969	.942 .996
	Total_On_squared	-.001	.001	1.115	1	.291	.999	.998 1.000
	TotalOffPw	.000	.000	.621	1	.431	1.000	1.000 1.001
	TOTALon_and_OF	.000	.001	.112	1	.738	1.000	.998 1.001
Door 1 busiest	Intercept	2.230	.159	195.865	1	.000		
	LOAD_ARR	-.016	.002	43.579	1	.000	.984	.979 .989
	LF	0 ^b	.	.	0	.	.	.
	PUNT	.000	.000	.022	1	.882	1.000	.999 1.001
	SEASON	.133	.036	13.534	1	.000	1.142	1.064 1.226
	STOP_LOCATE	.601	.088	47.085	1	.000	1.824	1.536 2.166
	TIME	-.172	.048	12.841	1	.000	.842	.767 .925
	TOTAL_ONS	.166	.016	102.289	1	.000	1.181	1.143 1.219
	TOTAL_OFFS	-.513	.018	851.036	1	.000	.599	.578 .620
	Total_On_squared	-.006	.001	92.083	1	.000	.994	.993 .995
	TotalOffPw	.006	.001	53.559	1	.000	1.006	1.004 1.007
	TOTALon_and_OF	.010	.001	72.065	1	.000	1.010	1.007 1.012

a. The reference category is: door 3 busiest.

b. This parameter is set to zero because it is redundant.

Following the best multinomial logistic regression type, we can estimate the dwell time at the busiest door, as shown in Equations 5.60 and 5.61.

$$dwell = P_{Busiest_door}(TOTAL_ONS)t_a + P_{Busiest_door}(TOTAL_OFFS)t_b + t_{op} \quad (5.60)$$

$$P_{Busiest_door} = \text{Max} (P_1, P_2, \text{ and } P_3) \quad (5.61)$$

P_1 , P_2 , P_3 are the probabilities that the busiest door is the front door, the first rear door and the second rear door defined in Equations 5.51 to 5.53 and type B-ML.3, respectively.

5.2.7 Method A vs. Method B and the Selection

Two methods (A& B) and their components have been developed to predict dwell time in the previous sections. Method A is based on the best dwell time regression function (i.e., type A-2.3) to examine the relationship between dwell time and the number of boarders and alighters as well as other variables. Method B is based on the revisions on busiest door selection method. Once the busiest door is selected on the basis of probability, dwell time is predicted by using the predicted number of boarders and alighters at that door as shown in Equations 5.45 and 5.46 for rigid-body buses or Equations 5.60 and 5.61 for articulated buses. In order to find enhanced methodology for use in this research, two methods are compared, based on their prediction performance in terms of the overall accuracy.

Overall accuracy can be determined by calculating one of the measures presented in Table 5.13. In this study, the mean absolute percentage error (MAPE) is used as a yardstick to select the better of the two methods.

Table 5.13: Measures of Accuracy

Measure	Equation
MAE Mean Absolute	$MAE = \frac{1}{N} \sum x_i - x_i^{\Delta} \quad (5.62)$
MAPE Mean Absolute Percentage Error	$MAPE = \left(\frac{1}{N} \sum_{i=1}^N \frac{ x_i - x_i^{\Delta} }{x_i} \right) \cdot 100 \quad (5.63)$
MRE Maximum Relative Error	$MRE = \max_i \frac{ x_i - x_i^{\Delta} }{x_i} \quad (5.64)$
RMSE Root Mean Square Error	$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - x_i^{\Delta})^2} \quad (5.65)$

5.2.7.1 Method A

In method A, as shown in type A-2.3, dwell times are predicted as below:

$$\begin{aligned}
 dwell = & 14.434 + \underset{t=13.921}{2.014}(TOTAL_ON) - \underset{t=-17.944}{0.026} \cdot (TOTAL_ON)^2 + \underset{t=15.364}{0.516}(TOTAL_OFFS) + \\
 & \underset{t=3.387}{0.004}(TOTAL_OFFS)^2 - \underset{t=-11.548}{0.022}(TOTAL_ONS)(TOTAL_OFFS) + \underset{2.982}{0.078} \cdot (LOAD_ARR) \\
 & - \underset{-3.349}{5.047}(LF) + \underset{t=5.492}{0.003}(PUNT) - \underset{t=-12.295}{4.748}(DOORS) + \underset{t=10.384}{2.087}(STOP_LOCA) + \underset{t=2.419}{0.196}(SEASON)
 \end{aligned} \quad (5.66)$$

5.2.7.2 Method B

While method A predicts dwell time without clustering data, method B clusters data into two samples, one for rigid-body bus and the other for articulated bus.

The values t_a , t_b , are in accordance with the HCM (2000) suggestions (Exhibit 27-9, p.27-10) as shown in Table 5.14.

Table 5.14: Average Service Time Selection(s/p)

Bus Type	Average dwell time per boarding passenger through		Average dwell time per alighting passenger through	
	Front door	Rear door	Front Door	Rear Door
Rigid-Body Bus	1.2	NA	0.9	0.9
Articulated Bus	0.9	0.6	0.6	0.6

5.2.7.3 Rigid-Body Bus

In the case of rigid-body bus, dwell times are predicted by using Equations 5.45 to 5.47 and the logistic regression type namely B-L.1. Table 5.15 shows a fraction extracted from the outputs of two dwell time prediction methods applied for rigid-body buses running on bus route 1, the OC Transpo, Canada.

5.2.7.4 Articulated Bus

In case of articulated bus, dwell times are predicted by using Equations 5.60 and 5.61 and the logistic regression type namely B-ML3 whose multinomial logistic regression functions are:

$$U_1 = 2.230 + 0.166 * TOTAL_ONS - 0.513 * TOTAL_OFFS - 0.006 * TOTAL_ON^2 + 0.006 * TOTAL_OFF^2 + 0.01 * TOTAL_ONS * TOTAL_OFFS - 0.016 * LOAD_ARR - 0.172 * TIME + 0.133 * SEASON + 0.601 * STOP_LOCA \quad (5.67)$$

$$U_2 = -0.178 + 0.19 * TOTAL_ONS - 0.032 * TOTAL_OFFS - 0.001 * TOTAL_ON^2 + 0.003 * LOAD_ARR - 1.05 * TIME - 0.01 * SEASON + 0.033 * STOP_LOCA \quad (5.68)$$

Table 5.15: Example Output of Dwell time Prediction of Method A and Method B Applied for Rigid-body-Bus

Bus Stop	Actual_Arr_Time	Expec_Time	Date	Actual_Dwell_Time (s)	Predicted_dwell_Method A (s)	Predicted_dwell_Method B(s)
CH080	33105	32880	9/14/2004	7	11.88	6.07
CF630	32700	32580	9/14/2004	6	.	.
CD920	33840	33600	9/16/2004	19	27.84	24.03
RA190	30375	30540	9/24/2004	2	4.39	2.65
CC190	29220	29280	10/5/2004	42	33.67	41.6
RF075	31500	31500	10/6/2004	5	7.29	2.46
CC190	33570	32880	10/8/2004	7	10.04	6.35
CC190	30510	30480	10/8/2004	28	18.49	22.69
CD920	33240	32400	10/8/2004	15	26.3	18.29
CF630	29085	28980	10/8/2004	4	10.07	4.21
CH080	29520	29280	10/8/2004	9	8.34	4.8
CH080	32490	31680	10/8/2004	12	15.26	7.59
CD920	30330	30000	10/14/200	4	9.37	4.2
CH080	29595	29280	10/14/2004	9	8.95	3.99
CH080	29460	29280	10/18/2004	10	13.61	8.63
CD920	32850	32400	10/20/2004	30	32.03	23
RA190	31680	31740	10/20/2004	4	5.48	3.46
RA945	31140	31080	10/20/2004	12	8.71	3.95
RA945	31110	31080	10/20/2004	10	16.14	9.19
CH080	31980	31680	10/20/2004	11	15.43	9.42
RA190	30480	30540	10/25/2004	10	11.73	7.59
RA945	31125	31080	10/25/2004	31	32.88	25.4
CF630	31605	31380	10/25/2004	7	10.62	4.94
RA945	34635	34680	10/27/2004	5	6.42	2.71
RA945	34560	34680	10/27/2004	25	23.97	16.1
CF630	35025	35100	10/27/2004	2	8.08	2.93
CD920	33690	33600	11/1/2004	13	17.52	10.74
CH080	33105	32880	11/1/2004	7	10.87	4.69

Table 5.16 shows another fraction of the outputs of the two dwell time prediction methods applied for articulated buses running on bus route 95, the OC Transpo, Canada.

Table 5.16: Example Output of Dwell time Prediction of Method A and Method B Applied for Articulated Bus

Bus Stop	Actual_Arr_Time	Expec_Time	Date	Actual_Dwell_Time (s)	Predicted_dwell_Method A (s)	Predicted_dwell_Method B (s)
EE915	29475	29400	9/7/2004	19	23.41	7.73
CJ900	30975	30720	9/7/2004	9	9.91	4.9
CA920	30780	30420	9/7/2004	7	14.67	6.64
CA920	31620	31440	9/8/2004	9	10.61	5.38
CJ900	31830	31740	9/8/2004	16	7.07	3.65
NI910	32415	32340	9/8/2004	9	5.04	3.56
EB905	30615	30600	9/8/2004	16	20.05	6.95
CD910	31245	31140	9/8/2004	3	6.7	3.86
CJ900	33345	33300	9/9/2004	4	4.6	3.23
EB905	32250	32160	9/9/2004	8	15.51	8.23
CD910	32775	32700	9/9/2004	24	18.24	7.31
EE915	35730	35820	9/10/2004	26	16.07	5.86
EE915	31575	31500	9/10/2004	21	11.69	5.89
EB905	31785	31680	9/10/2004	7	7.83	4.62
CA920	32730	32520	9/10/2004	8	11.41	7.12
CA920	30480	30240	9/10/2004	11	11.02	4.41
NI910	32505	32340	9/10/2004	2	3.27	2.8
NI910	32490	32340	9/10/2004	3	2.28	2.86
NI910	31410	31080	9/10/2004	3	4.26	3.25
CE940	32100	31980	9/10/2004	3	3.14	2.74
EB905	30585	30600	9/10/2004	5	8.71	5.81
NI910	35940	35940	9/14/2004	2	2.92	3.1
CJ900	35400	35340	9/14/2004	1	3.57	2.52
EE915	31575	31500	9/15/2004	17	28.93	11.17
EB905	31770	31680	9/15/2004	8	13.07	4.73
CE940	32115	31980	9/15/2004	12	15.65	8.11
CJ900	32100	32100	9/17/2004	25	9.14	4.93
NI910	32655	32700	9/17/2004	6	5.53	4.27
CD910	31575	31500	9/17/2004	21	20.94	5.94
CD910	33255	32880	9/17/2004	11	13.09	8.34
CJ900	33585	33420	9/20/2004	2	3.88	2.53

5.2.7.5 Accuracy Performance of the Two Methods

Obviously, the higher the value of MAPE, the poorer the prediction is. We can see from Table 5.17 that while method A is quite poor in predictions for rigid bus (i.e.

MAPE up to 59.80%) it works well for predicting dwell time for articulated buses with MAPE as low as 35.79%. In contrast, method B outperforms method A in case of rigid-body bus (MAPE= 32.42%) as compared to that of articulated bus (MAPE=45.72%).

Table 5.17: Mean Absolute Prediction Error of Method A vs. Method B

Bus Type	MAPE (%)		20% sample size
	Method A	Method B	
Rigid-body-Bus	59.80	32.42	473 cases
Articulated Bus	35.79	45.72	1846 cases

As a result, method A is suggested for predicting dwell time for an articulated bus while method B should be used for a rigid-body bus.

5.3 Summary

A comprehensive research methodology and proof of concept analyses on dwell time predictions are reported in this chapter.

Dwell time in previous works found in the literature was usually considered as delay and its values were defined by multiple regressions where the most influencing variables are total alighters and boarders. When using the models developed in the past to predict dwell time at a stop, the analyst is asked to enter the number of passengers boarding and alighting at that stop. Unfortunately, these numbers are not available because the bus has yet come to the stop. The dwell time prediction model developed in this chapter is a dynamic model because it can predict the number of boarding and alighting passengers. Based on the statistical pattern recognition technique, the historical

data and the up-to-minute data transmission from the APC and AVL systems, the predictions of passengers' activities are predicted and updated dynamically by the two sub-modules, which were called the RBSM and the RASM.

Method A and method B were developed to examine the fact that dwell time has a complicated relationship with many characteristics of passenger activities and service activities. A series of regression types varying from multiple linear regression, namely non-linear regression, binary *Logit* regression, and multinomial *Logistic* regressions were applied to explore this relationship. Data collected by the APC system mounted on the OC Transpo buses were used as the inputs to these methods. The comparisons among them show that method A (i.e. non-linear regression model) outperformed method B (i.e., Multinomial *Logit* regression) in predicting dwell time for the articulated bus. On the other hand, method B (i.e. binary *Logit*) is suitable for the rigid-body bus.

The performance of boarding passenger prediction and alighting passenger prediction sub-modules is presented in the next chapter.

Chapter 6

MODEL PERFORMANCE

6.1 Introduction

In the previous chapters, the study framework, the methodologies, the mathematical algorithms, and the detailed procedures for each constituent element of the developed model have been presented. Prior to application, it should be evaluated in terms of accuracy and reliability. From the accuracy perspective, the model's predictions on bus arrival time should be evaluated on the basis of several measurements of goodness-of-fit. Furthermore, these predictions would also be compared with predictions resulting from the use of other predictors, given the same input data. In terms of reliability, the developed model should be tested with different bus running scenarios in order to assess if it is a versatile model and if its predictions are reliable under any scenario.

This chapter presents information on the conversion of the developed mathematical prediction algorithms into computer programs, simulation of several bus operational scenarios, and evaluation of the developed model's performance based on simulated as well as real-world data.

6.2 Computer Program Development

The developed model was coded in Matlab computer language, version 7.0. The computer program consists of two main parts, called “functions” by Matlab. Each function contains several sub-functions in order to make the whole program work efficiently. The structure of the computer program, its functions, and sub-functions are shown in Figure 6.1. The source codes for each can be seen in **Appendix C** of this document.

Figure 6.1: Structure and Constituent Functions of the Computer Program

6.3 Simulations of Bus Operation Scenarios

A number of bus operation scenarios that are difficult to carry out as field experiments were simulated. The simulated bus arrival times, dwell times, and passenger activities were used to test the developed model in term of its accuracy and reliability.

Three bus operation scenarios were set up. These are: bus running under normal traffic condition, bus running under incident condition, and bus running under high passenger demand at some busy stops. It should be noted that the purpose of these simulations is to experience randomness of real-life bus operations generated by simulation tools. This approach goes beyond the analysis of a single operation case.

In order to obtain simulated data, transit routes were coded in suitable traffic software which was calibrated before being used for simulation purposes.

6.3.1 VISSIM Simulator

VISSIM is a popular microscopic traffic simulation software, *Verkehr In Städten-SIMulation* (PTV 2004). Like other traffic microsimulators such as CORSIM, SIMtraffic, etc., a key feature of VISSIM is the capability of simulating the randomness of traffic flow. The vehicle-following, lane-changing, and route selection components of VISSIM are recognized to mimic the real-world traffic operations. Furthermore, the flexibility and convenience of modelling interactions such as stopping, yielding, queuing, and passing, as well as the plentiful user-defined parameters, all make VISSIM to be a very useful traffic simulation tool.

VISSIM was chosen for this study due to not only attributes noted earlier but also owing to capability to simulate transit. Other simulators have limitations in this aspect. With VISSIM, a public transit planner can define different bus routes for the same road; each with corresponding time-based schedule or headway-based schedule. At each bus stop, passenger activities such as dwell time, number of boarders and de-boarders can be defined separately following either normal distribution or empirical distributions for

every single bus route sharing the same bus stop. Furthermore, VISSIM allows the users to define various types of acceleration and deceleration characteristics of the buses under study as well as bus driver behaviour. These enable transit planners and operators to predict effects of locations of transit facilities and passenger activities, generation of the real-time traveller information, and the study of operational alternatives before actual implementation. Table 6.1 compares some transit-related features of VISSIM versus CORSIM, the other commonly used micro-simulation software.

Table 6.1: Comparison of VISSIM (version 4.10) and CORSIM (TSIS 5.1) for public transit simulation

Module	VISSIM	CORSIM
Dispatching buses	- Headway-based schedule ¹ - Time-based schedule ²	- Headway-based schedule - N/A ⁶
Distribution of dwell time	- Normal Distribution - Empirical distribution ³	-Mean dwell time and distribution tables.
Estimate dwell time	- Additive method ⁴ - Maximum method ⁵	- Additive method - N/A
Estimate number of boarders and de-boarders	- Number of boarders and de-boarders	- N/A
Estimate number of waiting passenger	- Capability exists	- N/A
Calculate passenger waiting time	- Capability exists	- N/A
Number of dedicated bus lanes	- 2 or more	- only one

Notes

1. Headway-based schedule: Users can define headway (or service frequency) and start trip number.
2. Time-based schedule: Users can define the time the bus is dispatched at all bus terminals.
3. Empirical distribution: Users can define empirical distribution of dwell time at any bus stop for any bus route based on actual dwell time.
4. Additive method: Dwell time is calculated by additional boarding and alighting times.
5. Maximum method: Dwell time is calculated based on the maximum service time for boarding or alighting passengers, otherwise sum of both. This is suitable for one directional door.
6. N/A : Not applicable

6.3.2 Transit Route Coding in VISSIM

The two bus route segments of interest in this research, namely route 95 and route 1, were coded in VISSIM. Traffic and geometric data for each were provided by the City of Ottawa while the data on bus operations were obtained from APC systems of the OC Transpo. Several field surveys were also made at sites for which data were not available. A bitmap file of the routes' layout was imported into the simulator and the network geometry was built based on the background map using links and connectors. The traffic network coding required much effort because VISSIM specifies elaborate coding procedures for every single facility of the network. Other supplemental features such as number of lanes, speed limits, stop signs, right-turn-on-red, and actuated traffic signals, were also carefully coded. Furthermore, traffic and signal timings for intersections for the 2-hour peak period in the morning (8:00 a.m.-10:00 a.m.) were coded.

Since operation of the buses on selected routes was of much interest in this research, the kinetic characteristics of the actual buses (i.e., acceleration and deceleration rates) running on the OC Transpo network reported by Zargari (1997) and Zargari and Khan (2003) were used (see Table 6.2) instead of using VISSIM default parameters.

Table 6.2: Kinetic Characteristics of the OC bus

Parameter	Maximum Acceleration (m/s²)	Maximum Deceleration (m/s²)
VISSIM defaults	1.52	4.58
Zargari and Khan (2003)	1.72	4.02

In order to make the simulated bus operation to be more realistic, besides the selected bus stops for which the actual APC data on passenger activities (e.g., the number

of boarders and alighters) were used as the VISSIM inputs, other supplemental bus stops were also included in the routes. For these stops, empirical distributions of dwell time at bus stops were used (e.g., mean, standard deviation). Also, real-world bus dispatching schedules were imported for each route. The two bus routes in the coded form are shown in Figure 6.2.

Figure 6.2: Bus Routes 95 and 1 coded in VISSIM 4.1

VISSIM can automatically record the history of every single bus running on the bus route at time steps as small as 0.1 second interval and tabulates the data into the built-in table, namely the Vehicle Record. The software can provide over 75 fields in a such a table varying from kinetic information of the vehicles to emissions, fuel consumption,

passenger activities of public transit (i.e., dwell time, the number of boarders and de-boarders, passenger waiting time, bus arrival time and departure time, etc.), vehicle delay (e.g., queuing, number of stops, etc.), and GPS data (e.g., X, Y, Z coordinates). Figure 6.3 shows the user-defined process in VISSIM for selecting transit bus parameters of the simulated network.

Figure 6.3 Vehicle Record-Configuration Tool in VISSIM

6.3.3 Calibration and Validation

Micro-simulation is useful but dangerous if it is not carried out properly (Fox, 2000). Although the purpose of simulations in this study is to take advantage of randomness of transportation data generated by VISSIM for the developed model testing, in order to enhance its value, the calibration is needed. The calibration process applied in this study follows the instructions provided by Dowling *et al.*, (2004).

The process started with on-screen animation so that coding errors could be easily detected. Each time an observed vehicle's behaviour appeared to be unrealistic such as unusual stop on the route, left turning without priority rule, unrealistic collision, or unrealistic congestion, the potential causes were analyzed and the relevant parameters were adjusted. This process was repeated several times in order to ensure that the simulated network is consistent, the coding is accurate, and the simulation is a reality.

Once the on-screen error checking was completed, the calibration process progressed to the next step. In doing so, the two most influencing parameters which have a major influence on safety distance between vehicles and thus affect the saturation flow rate were chosen for calibration. These parameters are *BX_ADD* and *BX_MULT*, where *BX_ADD* is the additive part and *BX_MULT* is the multiplicity part of desired safety distance (VISSIM 4.1 User Manual, p.5-26). Please refer to Wiedemann *et al.*, (1994) for further details.

Based on these parameters, the calibration targets were set up. Bus running times between a pair of stops (i.e., the links) generated by VISSIM were compared with actual running times of the OC Transpo buses. According to the calibration targets, the differences should be less than 15 percent for each link, and the number of such links should account for more than 85 percent of the total. These are suggested by Dowling *et al.*, (2002, p.64). Details of the calibration process and results are presented in **Appendix D1**. Table 6.3 shows the calibrated values that were used for the follow-up steps.

Table 6.3: VISSIM Calibrated Parameters

Parameters	Default values	Calibrated values
Average stand still distance (m)	2.0	1.6
BX_ADD (m)	3.0	2.0
BX_MULT (m)	3.5	3.0
Gap acceptance (s)	5	7-10 ¹

Note:

¹ The calibrated value was based on the on-screen error checking

6.3. 4 Bus Operation Scenarios, Micro-simulation Runs, and the VISSIM Outputs

The simulation model for the two bus routes and part of traffic network was coded and calibrated in VISSIM. This enabled the use of VISSIM to generate data on bus operations and passenger activities that correspond with transit and traffic conditions. Three scenarios were simulated. The first scenario related to routine traffic, and the hourly and daily traffic variations were simulated. The second scenario involved an incident on the bus route. The incident could be a one-lane closure or a slowdown zone for road maintenance, or it may be due to traffic accident. The last scenario was devoted to the simulations of passenger activities at some busy stops that were caused by special events such as Boxing Days, or a football match.

The number of simulation runs was also carefully considered. In VISSIM, each simulation run requires a random seed number, which is used by VISSIM to generate traffic volumes and vehicle arrival rates. Consequently, multiple runs in VISSIM with different random seeds provide variations in results. The necessary number of simulation runs (or sample size) can be established on the basis of confidence interval, sample

standard deviation and the allowed error range of a parameter being statistically analyzed.

For example, Zong *et al.*, (2002) suggested the following equation:

$$n = \left(\frac{z_{\alpha/2} \sigma}{E} \right)^2 \quad (6.1)$$

Where: n = required number of simulation runs

σ = sample standard deviation

$\alpha/2$ = threshold value for significance level

E = Allowed error range.

However, the main purpose of simulation runs in this research is to generate random data for model testing rather than using these for statistical and analytical purposes. Therefore, the above suggested approach was not applied.

If we refer to the mathematical models developed in previous chapters, we can see that they are all based on a statistical pattern recognition technique where the recognition of a neighbour is based on the similarities of its several features with that of a given focal point. In mathematical terms, each neighbour is a multi-dimensional point near the focal point. The dimensionality, the required sample size, and the recognition of neighbourhood are the difficulties to cope with for those who are applying statistical pattern recognition in practice. The difficulties are that at high dimensions, a point may be no longer the neighbour of a given focal point although it is so at low dimensions. This was referred to as “dimensionality curse” in the literature (Ernest, 1961, pp.94; Cleveland *et al.*, 1988). For a better illustration, it should be interesting to read the following:

“Regions of high density may contain few samples, even for moderate sample sizes. For example, in the 10-dimensional unit multivariate normal distribution (Silverman, 1986), 99% of the mass of the distribution

is at points at a distance greater than 1.6, whereas in one dimension, 90% of the distribution lies between ± 1.6 " (Webb Andrew, 2002, pp.116).

This means that if one wants to avoid non-recognition in a high dimensional space, the required sample size has to be increased. Table 6.4 presents suggestions by Silverman (1986) on required sample size based on dimensionality¹.

Table 6.4: Dimensionality and Required Sample size

Dimensionality	Required Sample size
1	4
2	19
3	67
4	223
5	768
6	2790
7	10700
8	43700
9	187000
10	842000

(Source: Silverman, 1986, pp. 94)

For the developed model, the maximum dimensionality is three. Therefore, according to Silverman (1986), the required sample size should be more than or equal to 67 simulation runs. In this study, 100 and 70 simulation runs were considered for route 1 and route 95, respectively. The number of runs for each scenario is shown in Table 6.5.

¹ Silverman (1986) suggested it for non-parametric density estimation in high dimensional space. Although our purpose is not the same, this reference would be valuable under the circumstance that information about required sample size for non-parametric regression in high dimensionality is difficult to find.

Table 6.5: Scenario Design for Simulation

Scenario	Situation	Description	Number of simulation runs	
			Route 95	Route 1
1000	1100	Simulate traffic networks with volumes 15% less than normal traffic	N/A	15
	1200	Simulate network with normal traffic volumes	40	20
	1300	Simulate network with volumes 15% more than normal traffic	N/A	15
2000	2100	Simulate traffic network with one-lane closed and normal traffic volumes	N/A	15
	2200	Simulate traffic network with slow-down zone with normal traffic volumes	15	20
3000	3000	Simulate traffic network with busy stops and normal traffic volumes.	15	15
Total			70	100

Notes:

N/A: Not applicable because the buses run on the dedicated bus lanes. As a consequence, ordinary traffic variations would have not influence on bus transit.

Scenarios 1000 and 2000 were further divided into situations coded as 1100, 1200, 1300, 2100, and 2200. In the situations 1100 and 1300, daily traffic variations were simulated by increasing and decreasing normal traffic by 15 percent, respectively. Hourly traffic variations are based on the actual traffic counts and assumed to be the same for everyday.

Situation 2100 is for one-lane closure due to road construction zone on the segment of Bank St. from Holmwood to Gladstone stops. This situation does not apply to bus route 95.

Situation 2200 simulates slowdown zone. For bus route 1, the speed limit is as low as 10-12 km/h between Holmwood and Gladstone stops. For bus route 95, this zone is in between Hurdman and Campus stations where speed limit is 20-25 km/h. These slowdown zones were effective for 2 hours and removed after that time.

Scenario 3000 simulates busy bus stops by increasing the number of boarding passengers by threefold. For bus route 1, these stops are Billings Bridge and Homlwood. For route 95, such stops are St. Laurent and Hudman.

For each simulation run, 15 minutes (i.e., 900 seconds) were added to the simulation time as a *warm-up* time, the time to fill-up the empty traffic network with vehicles so that meaningless outputs are excluded. The state of the bus is set up to be updated at every 0.5 simulation second. This can keep the necessary accuracy while reducing simulation time and the magnitude of each output file². The output file for each simulation was recorded by VISSIM in a table, namely OC Transpo_VEH_RECORD in the form of Microsoft Access database.

Because the database is huge but only bus running times and passenger activities at stops are needed for the study, such information were filtered and re-tabulated. These tables are listed in **Appendix D2**.

If we consider each VISSIM output after each simulation run as a record of a service day of the buses, then we have 100-day database for route 1 and 70 days of records for route 95. For these days, the situations mentioned in Table 6.5 may happen

² Each simulation run by Intel Celeron Microprocessor 2.8 MHz, 1 GB RAM, and Window Home XP environment, is about 3600s for 8100s actual time with 0.1sec-update set, and about 1400 seconds with 0.5 sec-update set. As a consequence, the magnitude of the file reduces from 32 MB to approximate 14-17 MB for each record, correspondingly.

randomly, more or less frequently, due to the stochastic nature of traffic. In order to reflect these phenomena, the records were mixed up randomly. Also, by mixing the data, we could test the developed prediction model in term of its capability to recognize the similar bus running situations given a specified scenario.

6.4 Model Testing and Comparison

Like several models discussed in literature review, the developed model in this study is devoted to real-time bus arrival and related predictions. In order to study the relative merits of the developed model, it is necessary to compare it with other models in term of prediction accuracy and reliability. However, this comparison does not mean that the developed model is more or less advanced than others. Actually, it is aimed at informing the readers about the strengths and weakness of the compared models in case these are to be implemented in the field.

6.4.1 Evaluation Criteria for Prediction Performance

Three accuracy criteria shown in Table 6.6 are used to test the developed model as well as selected other predictors.

Table 6.6: Accuracy Criteria for Model Testing

Accuracy Criteria	Equation	Measurement
MAPE Mean Absolute Percentage Error	$MAPE = \left(\frac{1}{N} \sum_{i=1}^N \frac{ x_i - \hat{x}_i }{x_i} \right) \cdot 100$	MAPE provides unit-free measurement of the performance. It can be used to compare predictors across different test sets.
MRE Maximum Relative Error	$MRE = \max_i \frac{ x_i - \hat{x}_i }{x_i}$	MRE distinguishes the magnitude of the error from a predictor
RMSE Root Mean Square Error	$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \hat{x}_i)^2}$	RMSE measures the goodness-of-fit of the predictor.

Notes: x - actual data; $\hat{x}$ - predicted data; N - sample size

6.4.2 Reference Predictors

As discussed earlier, for reference purposes, the developed model should be compared with some other predictors, given the same input data. Two predictors were selected. The first one can be considered as the simplest model, namely the naïve model. The second reference model, which can be seen as one of the most advanced models in term of bus real-time arrival, was first developed by Reinhoudt *et al.* (1997), and further applied by Farhan (2002), and Shalaby *et al.* (2004). It is a Kalman filter-based model.

The reason to choose the first reference model is that it is a simple model without any computational difficulties. The estimation of a bus operation (i.e., running time or passenger activity) between a pair of stops at current time is simply calculated by taking historical data of the bus itself and that of the same-day data just before the prediction time.

The Kalman filter-based model aforementioned was chosen because it was successfully applied with both actual and simulated data for bus route 5 in downtown Toronto, Canada. Moreover, this model did outperform other models in a comparative

assessment (Shalaby *et al.*, 2004), especially, the ANN-based model – which is a very powerful prediction tool. For the sake of illustration, brief presentations about these predictors are made below. The details can be seen in **Appendix D3**.

Obviously, any prediction model that does not show consistent and more accurate forecast than the naïve model would have little value.

6.4.2.1 The Naïve Model

This predictor is rather simple in concept. Naïve predictions were calculated in a manner that take into account both current condition and historical bus running times or passenger activities as follows

$$r(k) = \left(\frac{r(k-1)}{r_{his}(k-1)} \right) r_{his}(k) \quad (6.1)$$

Where:

$r(k)$ and $r_{his}(k)$: running time or passenger activity of the bus for bus k at current time and one-day before.

$r(k-1)$ and $r_{his}(k-1)$: running time or passenger activity of the bus $(k-1)$ at current time and one-day before.

6.4.2.2 The Kalman Filter-based Model

The Kalman filter technique involves the state-space estimation. That is, the state of a subject in question at current time step is optimized (filtered) on the basis of the information about its historical states at previous discrete time steps and the latest

information. Reinhoudt *et al.* (1997), Farhan (2002), and Shalaby *et al.* (2004), used this technique in their research where the authors considered the new state being filtered is the bus running time between a pair of stops; the newest received information at the current time step is the running time of the same-day previous bus, which has just finished the run right before the bus in question. The historical data are the means and variances of running times recorded for the buses on several days before.

According to Farhan (2002), the Kalman algorithm for bus running time prediction and passenger arrival rate is based on the system of four main equations as shown below.

$$g(k+1) = \frac{e(k) + VAR[local_{data}]_{k+1}}{e(k) + 2VAR[local_{data}]_{k+1}} \quad (6.2)$$

$$e(k+1) = VAR[local_{data}]_{k+1} \cdot g(k+1) \quad (6.3)$$

$$a(k+1) = 1 - g(k+1) \quad (6.4)$$

$$P(k+1) = a(k+1).art(k) + g(k+1).art_1(k+1) \quad (6.5)$$

Where

g: filter-gain

a: loop-gain

e: filter error

P: prediction

$art(k)$: actual running time or passenger arrival rate of previous bus at instant (k)

$art_1(k+1)$: actual running time or actual passenger arrival rate of the previous day at instant $(k+1)$

$VAR [data_{local}]_{k+1}$: average variance of the last three days' data at time step $k+1$.

If we take a closer look at this system, we can see that the determination of *filter-gain* (e.g., $g(k+1)$) plays the key role to update the prediction in Equations 6.4 and 6.5. To determine $g(k+1)$, we need to know $e(k)$ (Eq. 6.2), which in turn, requires $g(k)$ to be defined (Eq.6.3). However, $g(k)$ then needs $e(k-1)$ to be estimated and so on. In other words, this is a recursive procedure where the bus running time being predicted at time step k requires an update, prediction, and correction (filter) procedure beginning from the very first trip running on that link (e.g. time step 1, or initial estimation). In his study, Farhan used 3 day-historical data for updating the variable $VAR [data_{local}]_{k+1}$. In this research, 3-day historical data were also used. An Excel code was developed to apply this algorithm for bus running time prediction.

6.4.3 Testing the Developed Model and Reference Predictors

6.4.3.1 Data issues

The *cross-validation* technique is used in order to assess the accuracy of estimates. In this technique, the original sample data are mutually split into testing sample and learning sample. This technique is intended to avoid overly optimistic estimates when one uses the same data to construct the parameters for a prediction model and assess its performance. Although no parameters in our models are to be estimated (i.e., non-

parametric model³), this technique is still applied in order to enhance the evaluations of model's performance.

There are various ways of performing data separation depending upon the magnitude of the original data. However, in a nutshell for the relatively small sample size that we have in hand, it is not practical to divide it in the way discussed above since it reduces the effective sample size. Instead, we consider the way based on a modified version of *cross-validation* technique, namely the *leave-one-out* cross validation (Hardle, 1990; Kwon *et al.*, 2000). We treat one day as a test set and the remaining as historical data. By doing so, we can calculate the *cross-validation* evaluation criteria (e.g., MAPE, MRE, and RMSE) for the developed model and the reference predictors. In this study, 20 test sets, each for one time, were drawn randomly for the purpose of model testing.

As the developed model combines the running time module and dwell time module, the performance of each module, as well as the whole model for bus arrival, was estimated separately and reported in the following sections.

6.4.3.2 Running Time Prediction Performance

During the simulation time (i.e., 8100 seconds), each VISSIM record contained 9 trips^{4, 5} for bus route 1 and 35 trips for bus route 95. Consequently, we have 180 simulated trips for route 1 and 700 simulated trips for route 95, given 20 test sets for each route. Moreover, each trip belonging to route 1 has 3 running times (i.e., the time spent

³ Please refer to chapter 4, section 4.3 for discussions on parametric and non-parametric models.

⁴ Under this circumstance, each trip is defined when a bus departs at the first stop and arrives at the last stop of the studied segment. For bus route 1, a trip starts when a simulated bus arrives at Billings Bridges stop and ends when the bus arrives at Rideau stop. For route 95, the two end stops are St.Laurent and Lebreton stops.

⁵ VISSIM uses the term "course" instead of "trip".

by each trip between a pair of understudied stops). Also, bus route 95 has 5 running times. In theory, if we apply *cross-validation* technique regarding every running time between all consecutive stops as well as every trip, we will have to cope with 540 and 3500 running time predictions for route 1 and route 95, respectively. These are actually large numbers and beyond the scope of this thesis research.

In order to reduce the work load while we can maintain the generality in estimating the predictors (i.e., the developed model, the KF and the naïve model), only specified trips and running times of each route are selected for model testing. For route 1, only the trips that start at 8:59 a.m. everyday at Billings Bridge station (i.e., trip 6) and only running times between Billings Bridge and Gladstone stops were considered. For bus route 95, only the trips that start routinely at 8:59 a.m. at St. Laurent station (i.e., trip 24) and only the running times between St. Laurent and Mackenzie stations were studied. By doing so, the number of predictions reduces to 20 for each route.

To apply the running time module described in chapter 4, each 8:59 a.m. bus trip of route 1 was drawn from the 100-record database and considered as a test set. Running time of the 8:59 a.m. bus (i.e., trip 6) that is being predicted was defined by its pattern matrix containing three previous running times of 8:47 a.m. bus (trip 5), 8:37 a.m. bus (trip 4), and 8:27 a.m. bus (trip 3). The remaining data (i.e., 99 records) were treated as the historical pattern matrices. The differences between the actual running times and predicted running times in the test sets were calculated. The same procedure was also applied for the 8:59 a.m. bus trips at St. Laurent station of route 95.

For the Kalman-filter based algorithm (KF), the record of 8:59 a.m. bus (trip 6) was drawn from the 100-record database and also considered as the test set. Based on the

running time between Billings Bridges and Gladstone stops for trips 5, 4, 3, 2, 1 and the historical running times experienced by 8:59 a.m. buses (trip 6) of 3 days ago, the running time was predicted and compared with the actual running time. For route 95, the same steps were applied for 8:59 a.m. bus trips running between St.Laurent and Mackenzie stations. In the case the Naïve model, predictions were obtained from the use of Equation 6.1. The details for the use of data for testing the predictors are presented in **Appendix D4**.

Figures 6.4, 6.5 and Table 6.7 present the results of *cross-validation* MAPE, MRE, and RMSE of the three predictors applied to route 95.

Table 6.7: Cross-validation MAPE, MRE, and RMSE

Cross-validation criteria	MAPE (%)	MRE (%)	RMSE (sec)
Developed Model	3.94	10.46	39.31
KF model	9.64	24.76	82.50
Naïve model	15.49	90.53	181.73

Figure 6.4: Route 95- Running Time Prediction: MAPE and MRE

Figure 6.5: Route 95- Running Time Prediction: RMSE

From Figures 6.4 and 6.5, we can see that the developed model is superior to the two reference predictors when they were all applied to the Transitway route (i.e., Route 95). The error measurements for the developed model are only a half and a quarter compared to those of the Kalman Filter and the naïve predictor, respectively. This could be due to the fact that many types of impedance on bus running time were eliminated on the Transitway route so that the differences between situations are quite clear. Hence, the developed model recognized and classified the situations quite well.

For route 1, a mixed-traffic bus route, due to many frictions that influence the bus running time, different bus running situations may be alike (i.e., they may have quite similar pattern matrices). Therefore, it makes it difficult for the developed model to cope with these cases when recognizing and classifying similar situations. This observation is supported by Figures 6.6, 6.7 and Table 6.8 when the developed model and the reference predictors were applied for route 1.

**Table 6.8: Route 1- Running Time Prediction with Overall Situations:
MAPE, MRE, and RMSE**

Cross-validation criteria	MAPE (%)	MRE (%)	RMSE (sec)
Developed Model	9.66	20.39	118.07
KF model	10.32	43.10	154.29
Naïve Model	13.59	35.37	168.90

**Figure 6.6: Route 1- Running Time Prediction with Overall Situations:
MAPE and MRE**

Figure 6.7 Route1- Running Time Prediction with Overall Situations: RMSE

Results shown in figures 6.6 and 6.7 suggest that the developed model for bus running time still outperforms the reference predictors since all indicators are favorable. However, the differences were not as remarkable as in the case of the Transitway route.

In order to further explore the developed model, more comparisons were carried out. Fifteen running situations coded as 2200 (i.e., slowdown zone situation), each for one time, were extracted from database of route 1. It should be noted that these situations were mixed up randomly with other situations in order to reflect the variable nature of traffic as well as to test the developed model's capability to recognize the similar situations in a mixed data.

For each predictor, by comparing the prediction results of MAPE, MRE and RMSE regarding these slowdown zone situations with those of 20 overall situations mentioned previously, we can see that the accuracy of the KF predictor changed considerably. Its MAPE increases from 10.32% to 19.03%, the RMSE goes up from 154s to 291s (i.e., up to 88%). In contrast, the prediction results of the developed model are still stable. The MAPE decreases from 9.66% to 8.76%. Furthermore, the MRE and RMSE are almost the same as they were. Figures 6.8, 6.9, and 6.10 present these discrepancies. The comparisons point out that the developed model is more reliable than other predictors when dealing with abnormal traffic situations.

Figure 6.8: Route 1-The Differences of MAPE between the Overall and Slowdown zone Situations

Figure 6.9: Route 1- The Differences of MRE between the Overall and Slowdown zone Situations

Figure 6.10: Route 1- The Differences of RMSE between the Overall and Slowdown zone Situations

The poorer prediction performances of the reference predictors are explainable. In order to predict a current bus running time, the KF and the naïve predictors were based on the historical data of several days before that would have no running situations similar to the present one. For instance, to predict running time of a situation that has Dcode = 43 and Scode = 2200 in table D4.1 (see Appendix D4), the KF model was based on the running times of 3 previous days which have different situations with Dcode-Scode as 78-3000, 29-1300, and 87-1100, respectively. As a consequence, the increase of prediction errors of these models is understandable.

Unlike the KF and the naïve models, the developed model searched through the whole database, recognized the most similar situations, and gathered them for estimations. For example, with the same case above (i.e., the situation coded as 43-2200), the automatic neighbour searching procedure⁶ searched through from 10 to 70 nearest neighbours around this case in order to find the optimal number of similar neighbours which provides the smallest cross-validation least-square prediction error (i.e. CR for short). It was found that 20 nearest neighbours offered smallest prediction error in this case, as shown in Figure 6.11. The figure was returned from the Matlab program coded for the developed model. Figure 6.12 depicts another example of such a neighbour searching for route 95, case 54-2200.

⁶ Please refer to chapter 4, section 4.4 for the details of this procedure.

Figure 6.11: Nearest Neighbour and Cross-Validation Least-square Prediction Error (CR) for Bus Running Time Prediction. Case 43-2200, Route 1.

Figure 6.12: Nearest Neighbour and Cross-Validation Least-square Error (CR) for Bus Running Time Prediction. Case 54-2200, Route 95.

6.4.3.3 Boarding Passenger Prediction Performance

As the number of boarders and de-boarders at the stops of route 1 are relatively small compared to that of route 95, only boarding passenger data of route 95 were used to test the predictors⁷. Also, for the sake of reducing computational work, only the number of boarding passengers at Mackenzie station of the buses that begin their trips at 8:59 a.m. (trip 24) at St. Laurent station was predicted. The boarding passenger prediction sub-module presented in chapter 5 (section 5.2.3) was applied for this purpose where the pattern matrix was defined by that of the previous bus (trip 20) served at Mackenzie station, the running time experienced by this bus, and the predicted running time of the bus itself (i.e., trip 24). The historical data of boarding passengers and running times of such buses were used as the historical pattern matrix.

The KF model developed by Farhan (2000) was applied to estimate the passenger arrival rate at a stop. The number of boarders for trip 24 then was predicted by multiplying this rate and the predicted running time. To apply the naïve predictor, Equation 6.1 was used.

The details of each predictor are presented in **Appendix D5**.

Figures 6.13 and 6.14 depict the *cross-validation* MAPE, MRE, and RMSE of the three predictors applied to forecast the number of boarders for 20 test sets. As shown in the figures, the developed model is the best, followed by the Kalman Filter and the naïve predictor.

⁷ The average boarding passenger at stops of route 1 is only about 10-15 pas/ h /stop, or it equals to 3-4 pas/ bus/ stop. Therefore, even the difference between predicted and actual data of only 1 passenger/stop can cause a prediction error of up to 30% or more. This may lead to misjudgments on the predictors.

Figure 6.13: Route 95 - Boarding Passenger Prediction: MAPE and MRE

Figure 6.14: Route 95 - Boarding Passenger Prediction: RMSE

Figure 6.15 shows an example of a neighbour searching result for a boarding passenger prediction. It should be noted that while the KF predictor needs the controversial assumption of a uniform passenger arrival rate, there is no such assumption for the developed model.

Figure 6.15: Nearest Neighbour and Cross-Validation Least-square Error (CR) for Boarding Passenger Prediction. Case 24-1200, Route 95.

6.4.3.4 Alighting Passenger Prediction Performance

The number of alighting passengers of the buses at Mackenzie station that start at 8:59 a.m. (trip 24) from St.Laurent station was used for testing the predictors. In doing so, the alighting passenger prediction sub-module in chapter 5 was used. Following the sub-module, alighting passengers of the two previous buses (i.e., trips 20 and 19) that served at Mackenzie stop were used as the elements of the current pattern matrix defining the de-boarders of trip 24 that are being predicted. The historical data of alighting passengers of these buses was used as the historical pattern matrix for neighbour searching and prediction purposes.

The Kalman Filter algorithm developed by Farhan *et al.*, (2002) was not originally developed by the above authors for alighting passenger prediction. In this study, this method was modified with minor changes so that it can be used for alighting passenger prediction. For the naïve model, the predictions were based on Equation 6.1.

The details of each predictor are presented in **Appendix D6**.

Figures 6.16 and 6.17 depict the MAPE, MRE, and RMSE of the 3 predictors applied to forecast the number of de-boarders.

Figure 6.16: Route 95- Alighting Passenger Prediction: MAPE and MRE

Figure 6.17: Route 95- Alighting Passenger Prediction: RMSE

Figure 6.18: Nearest Neighbour and Cross-Validation Least-square Error (CR) for Alighting Passenger Prediction. Case 20-1200, Route 95.

The developed model outperformed the comparative predictors in term of alighting passenger prediction as shown in Figures 6.16 and 6.17. Figure 6.18 shows the result of an example of a neighbour searching.

6.4.3.5 Prediction Performance with Actual Data

Running time data and the information of the number of boarding and alighting passengers retrieved from the APC and AVL systems embedded on the buses of route 1 and route 95 were used to test the predictors. Figures from 6.19 to 6.26 present the MAPE, MPE, and RMSE of the 3 predictors applied to the selected segments and stops of route 95 and route 1 with different test sets. Please see Appendix D7 for details.

Figure 6.19: Route 95 - Running Time Prediction with Actual Data: MAPE and MRE

Figure 6.20: Route 95 -Running Time Prediction with Actual Data: RMSE

Figure 6.21: Route 1-Running time Prediction with Actual Data: MAPE and MRE

Figure 6.22: Route 1-Running Time Prediction with Actual Data: RMSE

Figure 6.23: Route 95 - Boarding Passenger Prediction with Actual Data: MAPE and MRE

Figure 6.24: Route 95 - Boarding Passenger Prediction with Actual Data: RMSE

Figure 6.25: Route 95 - Alighting Passenger Prediction with Actual Data: MAPE and MRE

Figure 6.26: Route 95 - Alighting Passenger Prediction with Actual Data: RMSE

The figures reveal that the developed model has the best prediction performance compared to the reference predictors. For running time prediction, the average MAPE is only about 6% for both routes. The MRE varies between 10% and 11%, and the RMSE is in the range from 45 to 65 seconds. For boarding passenger predictions, the developed model's MAPE and MRE are as low as 20% and 46%, respectively. For alighting passenger predictions, its MAPE and MRE are 8% and 23%. All indicators reflect a favorable prediction model.

6.4.3.6 Tukey Test for Performance Comparison

The prediction performances of the three predictors were evaluated with different test sets. Although the charts presented earlier visually depict that the developed model outperformed the reference models, the comparisons based on the sound statistical analysis are the basis for reliable conclusions. In this part, Tukey's test was used to compare the MAPE values of the predictors. The details of Tukey procedure are presented in **Appendix D8**. Tables 6.9 and 6.10 present the results.

The results of the Tukey's procedure prove that the developed model has smallest MAPE compared to that of Kalman Filter and the naïve models in most of the test sets at 95% confidence level. Therefore, the developed model can be considered as the preferred prediction model.

Table 6.9: Studentized Range of the Test Sets

<i>Test sets</i>	<i>q</i> _{0.05, 3, 57}	<i>MSE</i>	<i>w</i>
Route 95: Simulated data of running times	3.42	15.1	2.97
Route 1: Simulated data of running times	3.42	8.72	2.25
Route 1: Simulated data of running times for slowdown-zone scenario	3.43*	7.78	2.47
Route 95: Simulated data of boarding passengers	3.42	79.74	6.82
Route 95: Simulated data of alighting passengers	3.42	23.67	3.72
Route 1: Actual data of running times	3.43*	6.55	2.26
Route 95: Actual data of running times	3.43*	13.79	3.29
Route 95: Actual data of boarding passengers	3.43*	114.46	9.47
Route 95: Actual data of alighting passengers	3.43*	16.91	3.64

Notes: MSE: Mean Square Error within groups; q: Studentized range distribution value (Netter, 1985); w: range

* Resulted from *q* _{0.05, 3, 42}

Table 6.10: Results of Tukey's Procedure at Significance Level = 0.05

TETS SETS	MAPE (%)		
	Developed Model	KF.Model	Naïve Model
Route 95: Simulated data of running times	3.94	9.64	15.49
<i>Pairwise Comparison</i>		5.70	5.85
Route 1: Simulated data of running times	9.66	10.32	13.59
<i>Pairwise Comparison</i>		0.66	3.27
Route1: Simulated data of running times for slowdown zone scenario	8.76	19.03	23.62
<i>Pairwise Comparison</i>		6.94	3.28
Route 95: Simulated data of boarding passengers	26.11	34.00	42.96
<i>Pairwise Comparison</i>		7.89	8.96
Route 95: Simulated data of alighting passengers	5.73	24.65	14.59
<i>Pairwise Comparison</i>		8.86	10.06
Route 95: Actual data of running times	6.6	10.26	26.72
<i>Pairwise Comparison</i>		3.66	16.46
Route 1: Actual data of running times	6.02	8.38	15.78
<i>Pairwise Comparison</i>		2.36	7.40
Route 95: Actual data of boarding passengers	20.18	37.39	69.14
<i>Pairwise Comparison</i>		17.21	31.75
Route95: Actual data of alighting passengers	8.28	12.37	20
<i>Pairwise Comparison</i>		4.09	7.63

6.4.3.7 Bus Arrival Time Prediction Performance

Each component of the developed model was statistically tested and the performance results are encouraging. Therefore, the author is confident that the integrated model will outperform the reference predictors when applied to bus arrival time predictions.

In order to estimate performance of the integrated model for bus arrival time predictions, 15 *full* trips of route 1 that start at Billings Bridge station at 8:59 a.m. were selected randomly. For each trip, running times, dwell times, the number of boarding and alighting passengers, departure times, and arrival times of the buses at four selected stops (i.e., Billings Bridge, Holmwood, Gladstone, and Rideau stops) were predicted.

To predict bus arrival times at the stops, besides the use of the developed modules and sub-modules for predicting and updating bus running times on the links and dwell times at the stops, the updating predictions procedure presented in chapter 3 was applied in order to update arrival and departure time information of the buses at stops. The details of these predictions can be seen in **Appendix D9**.

Tables 6.11, 6.12 and 6.13 display the performance of the three predictors in term of bus arrival time at Holmwood, Gladstone and Rideau stops.

Table 6.11. Holmwood Stop: Bus Arrival Time Prediction Performance of the 3 Predictors

Cross-validation criteria	MAPE (%)	MRE (%)	RMSE (sec)
Developed Model	8.60	20.14	62.31
KF model	12.43	43.10	81.77
Naïve Model	12.63	57.42	118.13

Table 6.12. Gladstone Stop: Bus Arrival Time Prediction Performance of the 3 Predictors

Cross-validation Criteria	MAPE (%)		MRE (%)		RMSE(sec)	
	<i>1st update</i>	<i>2nd update</i>	<i>1st update</i>	<i>2nd update</i>	<i>1st update</i>	<i>2nd update</i>
Developed Model	5.58	2.93	16.82	7.95	85.18	40.69
KF. model	8.93	3.85	17.24	8.79	97.86	50.95
Naïve Model	10.53	7.46	42.64	23.41	160.53	100.90

Table 6.13. Rideau Stop: Bus Arrival Time Prediction Performance of the 3 Predictors

Cross-validation Criteria	MAPE (%)			MRE (%)			RMSE (sec)		
	<i>1st update</i>	<i>2nd update</i>	<i>Last update</i>	<i>1st update</i>	<i>2nd update</i>	<i>Last update</i>	<i>1st update</i>	<i>2nd update</i>	<i>Last update</i>
Developed Model	5.25	3.32	2.58	12.75	11.87	4.17	97.16	70.43	47.19
KF. model	6.27	4.52	3.99	15.04	9.41	8.68	120.80	68.61	60.25
Naïve Model	10.22	6.17	6.87	27.11	12.33	12.55	196.63	123.64	127.33

The results presented in the tables suggest that the developed model is the most preferable model, followed by the KF and the naïve model. Also, the predicted arrival times at the stops become more accurate when the most updated bus information became available. For example, at Rideau stop, the average of MAPE of the 15 full-trips decreased from 5.25% to 3.32% and 2.58% corresponding to the first, the second and the last update. Table 6.14 shows a part of real-time prediction results for bus arrival times at the stops.

Table 6.14: Numerical Examples of Real-time Bus Arrival Prediction Using Developed Model for 3 full trips (seconds)

<i>Predicted Arrival Time At Stop</i>					<i>dcode</i>
<i>Holmwood</i>	<i>Gladstone</i>	<i>Rideau</i>	<i>Updated</i>		
544.37	970.24	1583.09	1st		28
	953.34	1581.21	2nd		
		1590.66	last		
<i>Actual Arrival Time at Stop</i>	561	971	1591		
<i>Holmwood</i>	<i>Gladstone</i>	<i>Rideau</i>	<i>Updated</i>		<i>dcode</i>
579.64	1274.42	1891.46	1st		49
	1336.06	2033.75	2nd		
		2017.58	last		
<i>Actual Arrival Time at Stop</i>	661	1410	2001		
<i>Holmwood</i>	<i>Gladstone</i>	<i>Rideau</i>	<i>Updated</i>		<i>dcode</i>
548.47	1191.79	1793.04	1st		56
	1193.45	1783.68	2nd		
		1870.02	last		
<i>Actual Arrival Time at Stop</i>	572	1247	1848		

6.5 Summary

Different test sets were used to test different constituent modules as well as the integrated developed model in term of prediction capability. In order to compare the developed model with other predictors, the Kalman filter-based model selected from the literature and the naïve model were also used to predict bus running times and passenger activities. It was found that all modules of the developed model were statistically superior to the reference predictors, given the same input data.

When applied for obtaining running time predictions, the running time prediction module worked satisfactorily, given its MAPE of only nearly 5% and 7% on average when using both actual and simulated data for route 95 and route 1, respectively.

The real-time boarding passenger and alighting passenger sub-modules also worked quite well and all outperformed the reference predictors. The MAPE for boarding passenger prediction was from 20% to 26% and from 6% to 8% for alighting passenger estimation. It should be recalled that while other prediction models mentioned in the literature review need a controversial assumption on passenger arrival rate, the developed model does not require this assumption.

While the reference models showed instability when dealing with unusual event occurring in bus service activity, the developed model showed only small changes in its prediction performance. That is, it worked consistently.

For real-time bus arrival time prediction, the integrated model produced good results as can be appreciated with average MAPE as low as 3%. The updating method helps to improve the accuracy of the model when new information becomes available and fed into the model.

Chapter 7

REAL-TIME PREDICTION INTERVAL, ON-LINE ADHERANCE EVALUATION AND BUS BUNCHING DETECTION

7.1 Introduction

In this chapter, methods to define real-time prediction interval, bus on-time performance, and bus bunching detection are reported. In addition, further applications of the developed model are discussed. Finally, the spatial aspect and the level of detail for database are mentioned in order to facilitate model efficiency.

7.2 Real-time Prediction Interval

Prediction is a challenge and will never be a fixed value. Therefore, a prediction interval is important under any circumstances. In this study, prediction interval for bus running time carries the evidences of non-parametric regression and thus, it is complicated and time consuming.

Several methods have been developed to determine the prediction interval for non-parametric regression family. These methods can be seen in the books of Hardle (1990), Fan *et al.*, (1996) or Loader *et al.*, (1999). However, only the methods that are theoretically sound and can be applied easily are of interest in this research. In this

section, an automatic computer-intensive procedure was applied to define prediction interval dynamically with each prediction.

According to Fan *et al.*, (1996) the prediction intervals for pointwise locally weighted regression can be expressed as follows.

$$\hat{m}_j(x_0) - j! \hat{b}_{j,p}(x_0) \pm z_{1-\alpha/2} \cdot j! \left\{ \hat{V}_{j,p}(x_0) \right\}^{1/2} \quad (7.1)$$

Where:

$\hat{m}_j(x_0)$ and j have been explained in chapter 4

$z_{1-\alpha/2}$ denotes the $(1-\alpha/2)^{\text{th}}$ quantile of the standard Gaussian distribution

$\hat{b}_{j,p}(x_0)$, $\hat{V}_{j,p}(x_0)$ the estimated conditional bias and variance

The estimates of bias and variance are obtained by calculating expectation and variance from equation defining β as follows (Please see Equation 4.7, chapter 4)

$$E(\beta(x_0) | X_1, X_2, \dots, X_n) = (X^T W X)^{-1} X^T W m = \beta + (X^T W X)^{-1} X^T W r \quad (7.2)$$

$$Var(\beta(x_0) | X_1, X_2, \dots, X_n) = (X^T W X)^{-1} (X^T \Pi X) (X^T W X)^{-1} \quad (7.3)$$

Where: $m = (m(X_1), m(X_2), \dots, m(X_n))^t$, $\beta = (m(x_0), \dots, m^{(p)}(x_0)/p!)^t$ and $r = m - X\beta$, the residual of the local polynomial, and Π is square covariance diagonal matrix with

$\Pi_{i,i} = K^2 \left(\frac{X_i - x_0}{h} \right) \sigma^2(X_i)$; Please refer to chapter 4 for the definitions of X and W .

The bias in equation 7.2 and the variance in equation 7.3 cannot be directly usable because of unknown quantities r and Π . Therefore, they need to be approximated through $\hat{b}_{j,p}(x_0)$, $\hat{V}_{j,p}(x_0)$ which are respectively the estimated conditional bias and

variance. These are the decompositions of the estimator of mean squared error ($\hat{MSE}$) of

$\hat{\beta}_j = \hat{m}_j(x_0) / j!$ as shown below

$$\hat{MSE}_{j,p}(x_0, h) = \hat{b}_{j,p}^2(x_0) + \hat{V}_{j,p}(x_0) \quad (7.4)$$

According to Loader *et al.* (1999), the estimated bias and variance can be defined by the following equations.

$$\hat{V}_{j,p}(x_0) \approx \hat{\sigma}^2(x_0) \quad (7.5)$$

Where:

$$\hat{\sigma}^2(x_0) = \frac{1}{tr(W) - tr((X'WX)^{-1}X'W^2X)} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 . K\left(\frac{X_i - x_0}{h}\right) \quad (7.6)$$

$$\hat{b}_{j,p}(x_0) = (X'WX)^{-1} X'W\tau \quad (7.7)$$

Where: τ is a row vector which replaces the residual vector in equation 7.2 in a higher order of polynomial.

While the estimator of variance can be approximated easily, it was found that the estimator of bias shown in equation 7.7 is unrealizable as it is based on a larger order of regression function with different pilot bandwidths (Fan *et al.*, 1996). Therefore, it is an expensive and complicated computation.

In chapter 4, we applied *leave-one-out* method to define an optimal bandwidth, given a focal point. Based on that bandwidth, the estimator of mean square error $\hat{MSE}$ was defined as a by-product of the bandwidth computation. As thus, we can estimate the estimator of bias by using the system of equations 7.4, 7.5 and 7.6. Once the estimators

of bias and variance have been determined, the prediction interval can be defined. The steps to determine a prediction interval are shown below:

Step 1: Determine the optimal bandwidth by *leave-one-out* method presented in chapter 4.

Step 2: Based on the optimal bandwidth and the recognition of the neighbours around the given focal, determine the local regression function.

Step 3: Based on the local regression function, calculate mean squared error ($\hat{MSE}$) of the local neighbours in the domain of the optimal bandwidth.

Step 4: Determine the estimator of variance based on equations 7.6.

Step 5: Determine the estimator of bias based on $\hat{MSE}$ and the estimator of variance.

Step 6: Estimate prediction interval based on equation 7.1 with z statistic replaced by t statistic due to the approximation of variance with the degree of freedom shown below (Fan *et al.*, 1996).

$$d_n(x_0) = \left\{ \sum_{i=1}^n K_h(X_i - x_0) \right\}^2 / \sum_{i=1}^n K_h^2(X_i - x_0) \quad (7.8)$$

The overall procedure was coded in Matlab. Please see **Appendix C**.

Table 7.1 presents pointwise prediction intervals for running time test sets of route 1 with two levels of significance at 0.05 and 0.01. As seen in the table, the predicted values were not the center of corresponding prediction intervals due to bias. The predicted values tend to be closer to the upper bounds of their prediction intervals.

**Table 7.1: Real-time Prediction Interval for Bus Running Time of Route 1
(Time in second)**

RCODE*	Predicted Running time	$\alpha=0.05$		$\alpha=0.01$	
		lower bound	upper bound	lower bound	upper bound
1	946	764	1003	707	1060
7	962	735	1025	656	1095
12	923	740	984	682	1041
17	925	672	1014	591	1094
18	949	755	1015	695	1075
23	935	699	1007	629	1073
28	859	606	950	525	1025
33	912	718	975	659	1033
37	951	751	1015	690	1076
40	828	638	895	578	955
46	896	655	975	579	1050
50	909	742	967	688	1020
57	899	596	1006	499	1103
61	904	733	959	680	1012
68	954	724	1033	651	1070

* RCODE: a unique code set by the author for the purpose of computer programming

With a real-time prediction interval, a bus transit planner can capture the variation domain of the predicted bus running time, given a certain confidence level. Therefore, the provision of real-time prediction interval enables other applications of the developed model in the bus transit field. The following sections present the applications of this method for defining probabilities of the bus being on-time or in pair form.

7.3 On-line Schedule Adherence Evaluation

7.3.1 Methodology

As discussed in previous chapters, bus service providers and passengers need to know ahead of time whether the bus will be on-schedule or not. The model developed in

this thesis research allows transit providers to know about bus arrival time as well as the prediction interval in real-time at any stop. However, the methodology to determine the probability of bus being on-time has yet to be developed.

Data for every 3 monthly periods that were retrieved from the AVL-APC systems in the OC Transpo were used to estimate bus on-time performance. Comparisons were made of the retrieved bus arrival times at bus stops with the scheduled times and then the differences were statistically analyzed. Such a difference is called the *off-line adherence performance (OAP)* of the bus. The OC Transpo as well as other bus transit agencies throughout North America consider that the bus is on-time if its *OAP* falls between 1 minute of earliness and 3 minutes of lateness (Stramathan *et al.*, 1999; Shara, 2002). We call these values as the *on-time limits* and the time between the limits is *on-time standard range*.

To forecast the probability of the bus being on time in real-time, it is easy to apply the above *on-time standard range*, given a real-time bus arrival prediction. Now we call the difference between a scheduled arrival time and a real-time arrival time as *Real-time Adherence Performance (RAP)*. Is the bus on-time given a predicted *RAP*? A simple answer is that if this *RAP* is within the *on-time standard range*, then the bus can be considered as on-time. However, in order to provide bus dispatchers more information about on-line arrival performance of the bus, further steps should be taken as follows

1. At each stop, the history of bus arrival times is collected and compared with scheduled arrival time.
2. Examine the density distribution of *OAP* by assuming that it follows one of the following distributions: the Gaussian or Lognormal distributions.

3. Find the best fit distribution using χ^2 (*Chi-square*) test or *Kolmogorov-Smirnov* test (K-S test).
4. Determine the *RAP* given a predicted bus arrival time and its prediction interval
5. Calculate the probability of the bus being on-time and other possibilities, given the best-fit distribution, the *RAP*, and the prediction interval.

7.3.2 A Numerical Example

For the sake of illustration, we select arrival times recorded at Gladstone stop of the buses that are scheduled to depart routinely at 8:59 a.m. (i.e., at the 1st second) at Billings Bridge station and to arrive at Gladstone stop at 9:10 a.m. (i.e., at the 660th second). The data were retrieved from the APC systems for one year period from 2004 to 2005.

1. The differences between historical arrival times and scheduled arrival times (*OAP*) were calculated and arranged in the increasing order.
2. To fit the *OAP* distribution, the Gaussian and Lognormal probability distributions were applied. Equations 7.8 and 7.9 present these distributions, respectively.

Lognormal Distribution

$$\begin{cases} f(x) = \frac{\Delta h}{(x - \min)\sqrt{2\pi\theta^2}} \exp\left(-\frac{[\ln(x - \min) - \zeta]^2}{2\theta^2}\right) & x > \min \\ f(x) = 0 & \text{if } x \leq \min \end{cases} \quad (7.8)$$

Gaussian distribution

$$f(x) = \frac{\Delta h}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right) \quad (7.9)$$

Where:

$\min$ = minimum x

Δh = interval length; $\Delta h = (\text{maximum } x - \text{minimum } x) / \text{total classes}$

ζ = Location parameter of a lognormal distribution

θ = shape parameter

σ = standard deviation

μ = mean

3. A *null hypothesis* (H_0) that the *Gaussian* or the *Lognormal* probability functions provide good fits for the distribution of *OAP* was made and tested. For the chi-square test, let's call χ^2_{eti} as the chi-square value estimated from the goodness-of-fit procedure. If χ^2_{eti} is greater than $\chi^2_{critical}$ at $\alpha = 0.05$, then H_0 is rejected, meaning a poor fit. Otherwise, if χ^2_{eti} is smaller than $\chi^2_{critical}$, H_0 cannot be rejected (DO NOT REJECT), or a good fit is recognized. For the K-S test, the K-S estimated will be compared with K-S critical at $\alpha = 0.05$. If the K-S estimated is smaller than K-S critical, then H_0 cannot be rejected (DO NOT REJECT).

4. For each type of distribution, the theoretical frequencies were calculated and compared to the observed frequencies of *OAP*. These discrepancies were then used to estimate χ^2 and *Kolmogorov-Smirnov* test. Table 7.2 presents the results of the fitting procedures. These are the products of the StaFit 2.0 software.

Table 7.2: Goodness-of-fit Test Summary ($\alpha=0.05$)

data points	29	
estimates	moment estimates	
accuracy of fit	3.e-004	
level of significance	5.e-002	
Summary		
distribution	Chi Squared	Kolmogorov - Smirnov
Lognormal	0.855 (3)	0.149
Normal	0.746 (3)	0.176
Detail		
Lognormal Distribution		
minimum	=	105. [fixed]
mu	=	4.63621
sigma	=	0.615371
Chi Squared		
total classes	15	
interval type	equal length	
net bins	4	
chi**2	0.855	
degrees of freedom	3	
alpha	5.e-002	
chi**2(3,5.e-002)	7.81	
p-value	0.836	
result	DO NOT REJECT	

Kolmogorov-Smirnov

data points	29
K-S stat	0.149
alpha	5.e-002
K-S stat(29,5.e-002)	0.246
p-value	0.495
result	DO NOT REJECT

Gaussian distribution

mean	=	229.655
sigma	=	83.107

Chi Squared

total classes	15
interval type	equal length
net bins	4
chi**2	0.746
degrees of freedom	3
alpha	5.e-002
chi**2(3,5.e-002)	7.81
p-value	0.862
result	DO NOT REJECT

Kolmogorov-Smirnov

data points	29
K-S stat	0.176
alpha	5.e-002
K-S stat(29,5.e-002)	0.246
p-value	0.296
result	DO NOT REJECT

5. The results show that both distributions were successfully in fitting the actual data at $\alpha=0.05$ (see Table 7.2 and Fig.7.1) but the Gaussian distribution gave a better fit. Therefore, the best-fit probability function is:

$$f(x) = \frac{29}{83.107\sqrt{2\pi}} \exp\left(-\frac{(x - 229.655)^2}{2(83.107)^2}\right) \quad (7.10)$$

Figure 7.1: Fitted Density for OAP at Gladstone Stop- Route 1

Assume that a real-time prediction of bus arrival time, together with its prediction interval, has just been made as 946^{th} second at Gladstone stop (e.g., the case with $RCODE=1$, $\alpha=0.05$, Table 7.1). The scheduled arrival time for this bus is at 660^{th} second. We calculate the differences between the lower prediction bound (764^{th} second), the predicted value (946^{th} second), and the upper prediction bound (1003^{rd} second), with the scheduled arrival time. These are $X_1=104s$, $X_2=286s$, and $X_3=343s$, respectively. As

discussed earlier, the bus is considered to be on time if the predicted arrival time compared to schedule times is 60 seconds (1 minute) of earliness or 180 seconds (3 minutes) of lateness. We denote the *on-time limits* as O_1 (lower limit) and O_2 (upper limit). By applying Equation 7.10 with the values of X and O , we can have the probabilities that the bus will be on-time or not. Figure 7.2 depicts the locations of those points.

Figure 7.2: On-time Limits and Prediction Interval

In Figure 7.2, the *lower limit* O_1 is not presented since the lowest value of OAP is 100 seconds of lateness. Therefore, there is no chance that the bus is on-time at the stop during the time range of $600^{th} \sim 760^{th}$ second.

Because the predicted *RAP* (i.e., $X_2=286$ seconds) is larger than the on-time *upper limit* ($O_2=180$ seconds) the bus is very likely not on-time. However, as the prediction lower bound ($X_1=104$ seconds) is smaller than the upper limit, there is still a chance the bus is on-time. The probability that the bus is on-time and its *RAP* falls between the range X_1O_2 (i.e., $104\sim180$ seconds) is exactly the area bounded by the distribution curve (Equation 7.10) and part of the horizontal axis X_1O_2 as found below:

$$P(104 \leq RAP \leq 180) = \int_{104}^{180} \frac{29}{83.107\sqrt{2\pi}} \exp\left(-\frac{(x - 229.655)^2}{2(83.107)^2}\right) dx \approx 21\%$$

In other words, the probability that the bus is on-time is 21%, with the bus arrival time is in between $764^{th} \sim 840^{th}$ second.

Similarly, the probability that the bus is behind the schedule and the lateness is within 180 to 343 seconds (O_2X_3) is as follows

$$P(180 \leq RAP \leq 343) = \int_{180}^{343} \frac{29}{83.107\sqrt{2\pi}} \exp\left(-\frac{(x - 229.655)^2}{2(83.107)^2}\right) dx \approx 62\%$$

If we convert the *RAP* into the arrival time, the probability that the bus is late is 62% with the bus arrival time in between $841^{st} \sim 1003^{rd}$.

In general term, the probability that the bus is not on-time is 79%. That is high enough to conclude that the bus is not on-time.

7.3.3 Discussion

1. Unlike the simple answer that the bus is certainly not on-time (e.g., probability that the bus is not on-time is equal to 1), if a predicted bus arrival time is out of *the on-time standard range*, the above example showed that there is still a chance the bus is on-

schedule. This happens when an overlap between prediction interval and standard range exists. If the probability that the bus is not on-time is higher than that of the bus is on-time, there is potential that the bus is not-on time and vice versa.

2. If there is no overlap and the prediction upper bound X_3 is smaller than the on-time lower limit (O_1), then the probability that the bus is not on-time and early is 1. Similarly, if prediction lower bound X_1 is larger than the on-time upper limit (O_2), then the probability that the bus is not on-time and is late is 1.

3. By combining the method to predict real-time prediction interval in previous section and the methodology developed in this section, bus dispatchers can quantitatively analyze all possibilities of bus adherence to the schedule. This would help them to make better decisions in managing their bus fleet.

7.4 Real-time Bus Bunching Detection Method

7.4.1 Methodology

Once bus arrival times at every stop can be predicted in real time, the tendency of buses to form in pairs (or bus bunching) can be detected. Knowledge in advance of a bus bunching will help bus dispatchers to make a proactive solution to prevent it. The decision should be based on the probability that bunching may take place and the experience of the bus dispatchers. In this section, a method to provide that probability is developed.

Given the *OAP* distributions of bus k and $k+1$ at a specified stop, the predicted bus arrival times of bus k and bus $k+1$, and their prediction intervals, the probability of a

bus bunching at the stop is positive if the upper prediction bound ($AR_{3,k}$) of bus k is larger than the lower prediction bound ($AR_{1,k+1}$) of bus $k+1$ as shown in Figure 7.3. Otherwise, this probability is equal to zero as illustrated in Figure 7.4.

Figure 7.3: Tendency of Bus Bunching, $P>0$

Figure 7.4: Tendency of bus Bunching, $P=0$

In Figure 7.3, the probability of bus bunching can be derived as follows:

$$P_b = \int_{AR_{1,k+1}-Sc_k}^{AR_{3,k}-Sc_k} f_k(x)dx \cdot \int_{AR_{1,k+1}-Sc_{k+1}}^{AR_{3,k}-Sc_{k+1}} f_{k+1}(x)dx \quad (7.11)$$

Where:

$AR_{1,j}$ and $AR_{3,j}$ ($j=k, k+1$) = the lower and the upper prediction bounds of a prediction interval of bus j

Sc_k and Sc_{k+1} = scheduled arrival times for bus k and $k+1$

$f_k(x)$ and $f_{k+1}(x)$ = probability functions of *OAP* of bus k and $k+1$ at the stop

Obviously, the larger the P_b , the closer the bus dispatchers should pay attention to this phenomenon. The developed method for bus bunching detection is summarized as follows

1. Determine a pair of buses needed to be examined for bunching at a certain stop (e.g., bus k and bus $k+1$).
2. Use the developed model to predict bus arrival times and the corresponding prediction intervals for each bus.
3. Determine the goodness-of-fit functions of *OAP* for bus k and $k+1$ at the stop (e.g., $f_k(OAP)$ and $f_{k+1}(OAP)$).
4. Compare the upper bound prediction of the preceding bus ($AR_{3,k}$) with the lower prediction bound of the following bus ($AR_{1,k+1}$).

If $AR_{1,k+1} \geq AR_{3,k}$ then $P_b = 0$ or there is no chance the two buses can form a pair.

If $AR_{1,k+1} < AR_{3,k}$ then P_b is found from Equation 7.11

7.4.2 A Numerical Example

To illustrate the methodology described in the previous section, a pair of 8:56 a.m. and 8:59 a.m. buses departing from St.Laurent station of route 95 was selected. The scheduled arrival times of these buses at Mackenzie stop are 9:05 a.m. and 9:08 a.m., respectively. Historical data of bus arrival times recorded by the APC systems were used to examine the *OAP* distributions of the two buses at Mackenzie stop. The real-time arrival times and the prediction intervals of these buses were predicted by using the developed model. We have to predict if the buses are forming into a pair or not. The parameters of the two buses are presented in Table 7.3.

Table 7.3: Parameters of the Two Consecutive Buses

Parameter	8:56 a.m. bus trip	8:59 a.m. bus trip
Off-line Adherence Performance function at Mackenzie station	$f_k(x) = \frac{36}{0.7598(x-30)\sqrt{2\pi}} \exp\left(-\frac{[\ln(x-30) - 4.2439]^2}{2(0.7598)^2}\right)$	$f_{k+1}(x) = \frac{50.5}{0.5827(x+135)\sqrt{2\pi}} \exp\left(-\frac{[\ln(x+135) - 5.1567]^2}{2(0.5827)^2}\right)$
Scheduled Arrival Time	$Sc_k = 540^{th}$ second	$Sc_{k+1} = 720^{th}$ second
On-time limits	$-60 \sim +180$ seconds (equivalent to arrival time in between $480^{th} \sim 720^{th}$ second)	$-60 \sim +180$ seconds (equivalent to arrival time in between $660^{th} \sim 900^{th}$ second)
Real-time Prediction of bus arrival time	Lower bound $AR_{1,k} = 611^{th}$ sec Predicted arrival time = 748^{th} sec Upper bound $AR_{3,k} = 794^{th}$ sec	Lower bound $AR_{1,k+1} = 724^{th}$ sec Predicted arrival time = 879^{th} sec Upper bound $AR_{3,k+1} = 898^{th}$ sec

As shown in Table 7.3, $AR_{3,k}$ is larger than $AR_{1,k+1}$ so there is a chance that the two buses will form a pair at Mackenzie stop. The probability of this incidence is calculated as follows

$$P_b = \int_{AR_{1,k+1}-Sc_k}^{AR_{3,k}-Sc_k} f_k(x)dx \cdot \int_{AR_{1,k+1}-Sc_{k+1}}^{AR_{3,k}-Sc_{k+1}} f_{k+1}(x)dx = \int_{724-540}^{794-540} f_k(x)dx \cdot \int_{724-720}^{794-720} f_{k+1}(x)dx =$$

$$P_b = 0.067 * 0.235 = 0.0154 \text{ or } 1.54 \%$$

The probability is too small to be concerned about bus bunching at Mackenzie stop.

7.5 Further Possible Applications by Using the Developed Model

This section focuses on further possible applications of the developed model in the field of bus transit control. Instead of detailing the control methods currently used in transit agencies since it may lead to another field of bus transit and therefore divert the focus of the thesis out of its targets, this section aims at listing some real-time control models whose critical bases are the availability of real-time bus arrival and dwell time information at bus stops.

The bus control strategies are moving towards real-time responses to the sporadic service problems (Strathman *et al.*, 2000). In the real-time bus control strategies, vehicle holding, short-turning, stop skipping, and speed modification are the tools to maintain the headway and service reliability given the knowledge of bus service problem in advance such as lateness, earliness or bunching. Having assumed that real-time bus running times and dwell times are available, many researchers did develop models for real-time bus

control strategies for different purposes. The readers should refer to the work of Strathma *et al.*, (2000) for automatic bus dispatching model, or the work of Ding *et al.*, (2002); and Fu *et al.*, (2003, 2004) for optimizing headway variance. Those who are interested in speed modification can read the work of Shalaby *et al.*, (2004). For service reliability or passenger utility, the works of Turnquist (1982) and Rabi *et al.*, (2000) are suggested.

7.6 Real-time Bus Arrival Information Broadcasting

When arriving at a bus stop, passengers need arrival information of the next bus (buses) that is supposed to be broadcasted. Because broadcasted time changes as real-time arrivals are updated, passengers may get one or more than one bus arrival announcements.

In the developed model, the predicted bus arrival time of the current bus is based on the running times of previous buses. Therefore, if the headway is long, the prediction may be inaccurate. The author suggests that if the headway between buses is longer than 30 minutes, the real-time prediction should not be made and the announced arrival should be “no real-time information”.

For the incoming stop (i.e., the stop closest to the bus), besides the real-time bus arrival time being announced, bus transit agencies should consider broadcasting the information about bus on-time possibility. The bus should be announced as “on-time” when the probability that the bus is on-time is larger than it is not so¹. Otherwise, it should be broadcasted as “not on-time”. For instance, in the numerical example in section 7.3.2, the probability that the bus is on-time is 21%, while the probability that it is not so

¹ The method to estimate this probability was developed in section 7.3

is 79%. Hence, the announcement at Gladstone stop should be as “Next bus is not on-time and will arrive in 4 minutes²”. This would help passengers to know clearly that the expected bus is coming or it has already passed the stop.

7.7 Level of Spatial and Temporal Detail of Data for the Developed Model

A sound and well organized database is needed in order to facilitate the applications of the developed model. This section discusses a key aspect of a database for the model, the requirement of data capture and database design.

Table 7.4: Level of Spatial and Temporal Details for Data Capture

Level	Description	Event-Independent records	Event Records	Between-stop Performance Data
A	AVL data without real-time Tracking	Infrequency (60-120 sec)		
B	AVL data with real-time Tracking	Infrequency (60-120 sec)	each timepoint	
C	APC and event recorder		each stop	
D	Event recorder with between stop summaries		Each stop and in between stop	Recorded events and summaries
E	Event recorder/trip recorder	Very frequent	all types	All events, full speed profiles

Furth *et al.*, (2003) concluded that there are five levels of spatial and temporal detail, as illustrated in Table 7.4. Referring to the table and the data contents that the

² Number 4 is only an example. In fact, the announced time is the difference between the predicted arrival time and the current clock- time when the arrival time is announced.

developed model requires, it is suggested that the level of spatial and temporal detail of data for the model should be level C and up. However, an integration of the APC system with the AVL technologies and a radio system is a requirement for the applications of the developed model for real-time bus arrival prediction. The reason is that on one hand, the on-board micro computer can match the vehicle to the route, and on the other hand the stop record can be sent over the air to the control center.

7.8 Summary

Chapter 7 is devoted to the methodologies that can predict the likelihood that the bus is on-time, late, early or bunching. These resulted from the conjunction of the real-time prediction interval presented in section 7.2 and the historical distribution of bus on-time performance at each bus stop. By knowing in advance the probabilities that the bus is not on-time, bus dispatcher can make a good decision to apply suitable control methods in order to keep service adherence.

The broadcasting of real-time bus arrival was also mentioned. As illustrated previously, even if a real-time bus arrival prediction falls within an on-time standard range, there is still a chance that the bus is not on-time and vice-versa since a prediction, in essence, is a probabilistic estimation. The announcement of if the bus is on-time or not should be broadcasted with the consideration of such probabilities.

For bus bunching detection, a method which is simple in application was developed. By applying this method, bus dispatchers can easily quantify a likelihood of bus bunching. As a consequence, they will have suitable proactive control methods to prevent it.

Chapter 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The growing urban traffic congestion in medium and large size cities is forcing the city authorities to search for efficient solutions in order to reduce the adverse effects. It is widely recognized that among possible solutions, an advanced urban public transit system in the urban areas is potentially the most effective solution.

In a modern society where travellers value their time highly, the provision of bus real-time information is instrumental in improving bus transit's ridership. Therefore, it is important to both public transit providers and the passengers. For transit providers, the development of a real-time bus arrival information system (RETBAIS) is an important objective that they have to meet. For passengers, real-time bus arrival information is an important sign in their perception of a reliable, attractive, and convenient public transit system. Over the last twenty years, many transit providers have been equipping their bus fleets with the AVL and APC systems which are considered to be the technology backbone for the development of a RETBAIS. However, these expensive systems are underused. Moreover, the vital role of real-time bus arrival information has been adequately recognized by the transit providers only in the last few years.

This thesis research can be considered as a contribution to this new field of bus transit. All thesis research goals and objectives were addressed comprehensively in terms

of methodologies that provide up-to-minute information of bus arrivals as well as to help bus dispatcher the on-line decision support tools for managing the bus fleet.

Urban bus transit has a major challenge that it has to strictly adhere to schedule but at the same time it is vulnerable to delays which are very complicated to manage in the urban context. In order to overcome the difficulties of the study of influencing factors on bus arrival time and also to use intensively the available data recoded in the AVL and APC systems, the research separated bus arrival time into two important parts, namely bus running time in a link and bus dwell time at a stop. These philosophies lead to the development of two modules.

Following the development of the running time prediction module, based on the basis of a statistical pattern recognition technique, namely the LOWESS method, and the similarities of bus running times experienced in the same period of time in the past, and in the same bus link, a prediction algorithm was formulated. In order to enhance the algorithm's prediction capability as well as to shorten the prediction time, the author applied leave-one-out cross validation technique and Nadaraya-Watson kernel regression in searching the similarities among variables. The entire algorithm is an automatic computer-intensive procedure for searching, matching and predicting bus running time whenever the AVL-APC data are available and updated. Tests made with simulated and actual data for various bus operation scenarios show that this module performed well, with the average mean relative error were just about 5% for Transitway route and 7% for mixed-traffic bus route. In this type of research, such relative error levels or even higher than these, would be acceptable to the consumer. Moreover, the module worked reliably when dealing with unusual events of bus operation. This characteristic was not found in

the Kalman filter model, a reference model used in this study. It is useful to note that the Kalman filter model outperformed several other comparative models including the Artificial Neural Network-based model in term of bus running time prediction.

Following the development of bus dwell time prediction module, based on the logic that the number of boarding passengers and alighting passengers are the determinants for dwell time prediction, the research moved on to the development of a real-time passenger activity prediction algorithm. Four sub-modules were developed separately for predicting boarders, de-boarders and bus dwell time. The first two sub-modules were also based on the same basis of statistical pattern recognition which was applied in the running time prediction module. In order to avoid using a controversial assumption on the distribution of passenger arrival rate, the algorithm developed for boarding passenger prediction used bus headway and boarding historical data as the elements for a pattern being recognized. It was found that both sub-modules outperformed the reference models and the results were acceptable with the average prediction error of about 22% for boarding prediction, and 6% for alighting prediction.

Unlike the first two sub-modules where the predictions were driven by the real-time data, the last two sub-modules were developed in the form of pre-defined functions by relating dwell time with the number of boarders, de-boarders, and other selected variables. The first sub-module of these two, namely regression sub-module, was based on the classical functions regressing dwell time to several variables whose data were retrieved from the APC systems. The author found that the regression type A-2.3 is the most suitable with its *R-square* of up to 63%. In case of data shortage, the type A-2.6 is suggested. The last sub-module, namely the busiest door prediction sub-module, was

based on the fact that alighting and boarding passengers' door choice influence dwell time and therefore the determination of the busiest door is important. Several types of binary and multinomial logistic regressions functions, which stem from the discrete choice theory, were examined with the retrieved APC data. The author suggested that binary logistic regression type B-L.1, together with Equations from 5.45 to 5.47, is suitable for rigid-body bus while multiple regression (i.e., type A-2.3) should be used for articulated bus. This developed algorithm also solved the problem of bus stop skipping prediction.

When the two modules were integrated for predicting bus arrival times at every bus stop of the full-trips, the entire model worked satisfactorily. The average relative prediction error increased with the new data availability and varied from 3% to 8%.

As extensions of the developed model, two algorithms were formulated, one for bus on-time performance evaluation and the other for real-time bus bunching detection. In order to use these algorithms, the author developed a method to delineate real-time bus arrival prediction interval. It is worth noting that most existing bus arrival time prediction models found in the literature could forecast only the mean value of arrival time. Therefore, how certain the prediction is and what the prediction interval is, were not provided by these models. The author of this thesis can quantitatively analyze all measures of bus on-time performance, which again were not found in previous models devoted to bus arrival time prediction.

On the basis of the nature of real-time bus arrival predictions, a method to broadcast bus arrivals to passengers was discussed. Bus arrival time broadcasting should be based on the sound probabilities that the bus will be on-time or not. Also, suggestions

were made on the spatial characteristics and details of the database for the developed model where the integration of the APC system with AVL and a radio system is a prerequisite.

This research can be considered as a contribution to the development of real-time bus arrival information system. In this research, the term “*real-time*” was illustrated clearly. Here, a real-time prediction model meant to not only to be capable of producing prediction with real-time data but the predictive characteristics of the method itself. Specifically, one important feature for a real-time prediction model is accurate and rapid responsiveness. Therefore, any real-time model should be a computer-intensive one with fast calculation rate. The developed model met this requirement. It was found that a prediction was made by the developed model in only about 15 seconds. All prediction procedures of optimal searching, recognizing, and predicting were done automatically. Hence, this research can provide bus transit providers an accurate, fast, and reliable prediction model.

In the case of dwell time prediction, the term “real-time passenger activity prediction” was first introduced by the author and a methodology to predict it was comprehensively developed. The model can help bus transit planner to deal with highly busy bus stops as well as to predict bus dwell time in real-time that should be spent by the buses to serve such stops or to estimate the load factor for each bus, the waiting time of passengers at stop, and so on. Moreover, the algorithm developed in this study can solve a number of existing issues in term of dwell time estimation.

By using the methods developed by the author for analyzing bus on-time performance, the bus dispatcher can easily detect the possibilities of “not on-time bus

arrivals” and “bus bunching”. These capabilities would help transit agencies to provide passengers more accurate and reliable information as well as to manage their bus fleet economically and efficiently.

The entire model was developed in a manner that it solely used the AVL-APC data. No additional traffic data such as traffic volume, speed, density or so on, are needed. Moreover, because the data are updated at stops or timepoints (i.e., *time-at-location*¹), tracking is not required (Furth *et al*, 2003). Therefore, the developed model is suitable even if a transit agency is still using the APC systems with the old AVL technologies (i.e., non GPS-based AVL technology).

In conclusion, the author believes that the real-time bus arrival time prediction model developed in this thesis will enhance the bus arrival information system and will be a contribution to public transit operation.

8.2 Recommendations and Future Research

The model developed in this thesis research based on the statistical pattern recognition technique is innovative and produces results that are clearly more accurate than obtainable from other models. As is the case with other methods, there are limitations of the developed model that should be understood prior to application. However, on the balance, the bus arrival prediction process would be considerably enhanced by adopting the model developed in this thesis. Based on the findings and the

¹ *Time-at-location*: a record of when a bus passed a predetermined point such as a stop or a timepoint. This data collection method is opposite to the *location-at-time* method - i.e., where a bus was at a given time. *Time-at-location* is strongly preferred to *location-at-time* data for management used of achieved AVL-APC data (Furth *et al.*, 2003, p.42).

difficulties experienced during the course of carrying out this research, the following recommendations are made.

The results found in chapter 3 raise the question “why the buses running on route 1 are mostly behind the schedule?” and “what should be done to solve that problem?” Based on the distribution of bus on time performance, it was found that the lateness of the buses have been caused not only by the traffic congestion but also by the schedule design. For example, the statistical mean of actual bus running times between Billings Bridge and Gladstone stops is about 15 minutes whereas the running time scheduled for the buses of this segment is only 11 minutes. With that set up, the probability for the bus “on-time” was only about 43%. Therefore, the OC transpo should adjust the scheduled running times between stops closer to the actual means. Such an adjustment would be appreciated by their customers.

In chapter 4, the author used Euclidian distance to calculate the similarity among patterns. Mathematically speaking, the author actually treated the elements in a pattern matrix equally with the same weights. However, running times of the buses closer to the understudied bus may get more weights because they run under the most similar road and traffic condition with the bus of interest. Further study should be carried on with this standpoint in order to improve the developed model.

Due to constraints in data collection and simulation, a number of elements taken into a pattern in this study were eliminated in order to ensure a sound statistical inference. However, whenever the database is large, more elements should be added into the patterns so that it can improve the prediction results of the developed model.

The elements in the form of categorical variables should be considered. For example, for a pattern of boarding passenger prediction, besides the elements which were presented, we can add categorical elements such as bus type, seating plan, and so on. The method to recognize such patterns should be further explored in order to make the developed model more versatile.

It is possible that matrix $B = (X^T W X)$ in equation 4.7 will become a singular matrix so that no prediction can be made. In order to solve the problem, further study to convert the linear regression shown in chapter 4 into the form of ridge regression is needed.

Finally, data organization for the retrieved APC-AVL data as well as the data provisions for each module of the developed model requires further studies.

REFERENCES

Abdelfattah, AliM., and Khan Ata M., (1998). Models for Predicting Bus Delays. *Transportation Research Record* 1623, pp. 8-15.

Abkowitz, Mark, Eiger Amir, and Engestein Israel, (1986). Optimal Control of Headway Variation on Transit Routes. *Journal of Advanced Transportation*, Vol.20 (1), pp.73-88.

Abkowitz, Mark, and Lepofsky, (1988). Implementing Headway Based Reliability Control on Transit Routes. *Journal of Transportation Engineering*, Vol.116, No.1, pp. 49- 63.

AC Transit. Rider Information. <http://www.actransit.org>

Akcelik & Associates Pty Ltd. Micro-Simulation and Analytical Methods for Modeling Urban Traffic. October 2001.

Akcelic, R., and Besly M., (2001). Acceleration and Deceleration Models. *The 23rd Conference of Australian Institutes of Transport Research (CAITR 2001)*. Monash University, Melbourne, Australia, December 2001.

Alfa, Attahiru Sule, William B.Menzies, James Purcha, and R.McPherson, (1988). A regression Models for Bus Running Times in Suburban Areas of Winnipeg. *Journal of Advanced Transportation*, Vol. 21, pp. 227-237.

Andersen, Bob, (2005). *Introducing Nonparametric Regression*. Lecture notes. McGill University.

- Ben, Akiva, and Steven Lermark**, (1985). *Discrete Choice Analysis*. The MIT Press
- Bertini, R.L., and Ahmed M., Geneidy El.**, (2004). Modeling Transit Time Using Archived Bus Dispatch System Data. *Journal of Transportation Engineering*, ASCE Vol. 130, No. 1, January 1.
- Berry, W., Feldman S.**, (1985). *Multiple Regressions in Practice*. Sage Publications.
- Bodin, L.D., Ball M.O., Duguid R., and Mitchell M.**, (1985). The Vehicle Scheduling Problem with Interlining. *Computer scheduling of Public Transport 2*, Edited by Jean-Marc Rousseau, Elsevier Science Publishers B.V, pp.427-450.
- Chang, Hyeon, Theo Arentze, and Harry Timmermans**, (2001). Pattern Recognition in Complex Activity Travel Patterns; Comparison of Euclidian Distance, Signal-Processing, Theoretical and Multidimensional Sequence Alignment Methods. *Transportation Research Record* 1752, pp. 16-22.
- Chapman, R.A., and Michel J.F.**, (1978). Modeling the Tendency of Buses to Form Pair. *Transportation Science*, Vol. 12, No. 2, pp.165-175. *Transportation Research record* 1752, pp. 16-22.
- Cathey, F.W., and D.J. Dailey**, (2003). A Prescription for Transit Arrival/ Departure Prediction Using Automatic Vehicle Location Data. *Transportation Research Part C*, Vol.11, pp.241-264.
- Chen, Mei, Liu Xiaobo, Xia Jingxin, and Chien Steven**, (2004). A Dynamic Bus-Arrival Time Prediction Model Based on APC Data. *Computer- Aided Civil and Infrastructure Engineering*, Vol. 19, pp. 364-376.

- Chien, Steven I., Shoai M. Chowdhury, Kyriacos C., Mouskos, and Yuqing Ding,** (2000). Enhancements of CORSIM Model in Simulating Transit Operations. *Journal of Transportation Engineering*, Vol. 126, No. 5, September 1, pp. 396-404.
- Chien, Steven I., Yuqing Ding, and Chienhung Wei.,** (2002). Dynamic Bus Arrival Time Prediction with Artificial Neural Networks. *Journal of Transportation Engineering*, Vol. 128, No.5, September 1, pp. 429-438.
- Cleveland, William S.,** (1979). Robust Locally Weighted Regression and Smoothing Scatter Plots. *Journal of the American Statistical Association*, Vol.74, No.368, pp.829-836.
- Cleveland, William S.,** (1988). Locally Weighted Regression: An Approach by Local Fitting. *Journal of the American Statistical Association*, Vol.88, pp.596-610.
- Conover, W.J.,** (1971). *Practical Nonparametric Statistics*. John Wiley & Sons Inc.
- Dailey, D.J, S.D Maclean, F.W. Cathey, and Z.R. Wall,** (2001). Transit Vehicle Arrival Prediction, Algorithm and Large-Scale Implementation. *Transportation Research Record*, 1771, pp. 46-51.
- Daniel, L. Gerlough and Frank C. Barnes.,** (1971). *Poisson and Other Distributions in Traffic*. Eno Foundation for Transportation. Saugaruck, Conecticut.
- Daniel, Wayne W.,** (1978). *Applied Nonparametric Statistics*. Houghton Mifflin Company.
- Daniel, K. Boley,** (1998). *Passenger Counting Technologies and Procedures*, TCRP synthesis 29, TRB, Washington, DC.

- Davis, Gary A., Nancy L. Nihan, Mohammad M. Hamed, and Lesli N. Jacobson,** (1990). Adaptive Forecasting of Freeway Traffic Congestion. *Transportation Research Record*, 1287, pp. 29-33.
- Devroye, Luc, Laszlo Györfi, and Lugosi,** (1996). *A probabilistic Theory of Pattern Recognition*. Springer Publication.
- Dowling, R., Skabardonis A., and Alexiadis V.,** (2004). *Traffic Analysis Toolbox- Volume 3: Guidelines for Applying Traffic Simulation Software*. Report FHWA-HRT-04-040.
- Ding, Yuquing, and Chien Steven,** (2002). Improving Transit Quality and Headway Regularity with Real-Time Control. *Transportation Research Record*, 1760, pp. 161-170.
- Dueker, Kenneth J., Thomas J. Kompel, and James G. Strathman,** (2004). Determinants of Bus Dwell Time. *Journal of Public Transportation*, Vol.7, No.1, pp. 21-38. Website <http://www.nctr.usf.edu/jpt/journalarticles.html>
- Ernest, Bellman R.,** (1961). *Adaptive Control Process*. Princeton, N.J, Princeton University Press.
- Fan and Gijbels I.,** (1996). *Local Polynomial Modeling and Its Applications*. Chapman & Hall Publisher.
- Fang, Zhao and Min-Tang-Li.,** (2005). Calibration of Highway / Transit Speed relation For Improved Transit network Modeling in FSUTMS. BD-015-07, Final Report. Lehman Center for Transportation Research - Florida International University.

- Fang, Zhao and Nokil Park**, (2004). Using Geographically Weighted Regression Models to Estimate Annual Average Daily Traffic. *Transportation Research Record*, 1879, pp. 99-107.
- Farhan, Ali**, (2002). *Bus Arrival Time Prediction for Dynamic Operations Control and Passengers Information Systems*. Thesis Dissertation of Masters of Applied Science, University of Toronto, Canada.
- Fox, John**, (1991). *Regression Diagnostics*. Series in Quantitative Applications in Social Sciences, Sage University Paper, Vol.79.
- Fox, K.**, (2000). SMARTTEST-new tools for evaluating ITS. *Traffic Engineering and Control*, Vol.41 (1), pp.20-22.
- Fukotomi, Akihira**, (2004). *Simulation Approach to Study of Driving Behaviour, Fuel Consumption and Emissions*. Master Thesis Dissertation, Carleton University, Ottawa-Canada.
- Fu, Liping and Liu Qing**, (2004). Real-time Optimization Model for Dynamic Scheduling of Transit Operations. *Transportation Research Record*, 1857, pp. 48-55.
- Fu, Liping and Yang X.**, (2003). Design and Implementation of Bus-Holding Control Strategies with Real-time Information. *Transportation Research Record*, 1791, pp. 6-12.
- Furth, G. Peter**, (2003). *Use of Archived AVL-APC Data to Improve Transit Performance and Management: Review and Potential*. Transit Cooperative Research Program Web Document 23 (Project H-28), TRB, Washington, DC.

- Gibbons, J. Dickinson.,** (1984). *Nonparametric Methods for Quantitative Analysis*. Second Edition. American Sciences Press, Inc.
- Guether Richard P., and Kumares C. Sinha,** (1983). Modeling Bus Delays due to Passenger Boarding and Alighting. *Transportation Research Record*, 915, pp. 7-13.
- Gyorfi Lazlo, Hardle Wolfgang, Sarda Pascal, and Vieu Philippe,** (1989). *Nonparametric Curve Estimation from Time Series*. Springer-Verlag.
- Harris, Richard,** (2000). *A Primer of Multivariate Statistics*. Third Edition, IEA Publisher
- Hardle, Wolfgang,** (1990). *Applied Nonparametric Regression*. Cambridge University Press.
- Hay, William and Winkler Robert,** (1971). *Statistics, Probability, Inference, and Decision*. Holt, Rinehart and Winston, Inc.
- Helali, K.N and Khan A.M.,** (1993). Regression Based Urban Arterial Road Delay Models. *Canadian Journal of Civil Engineering*, Vol. 20, No.1, Feb.1993, pp.37-49.
- Higham, Desmond and Higham Nicholas,** (2000). *MATLAB Guide*. SIAM Philadelphia.
- Horbury, Antoneta X.,** (1999). Using non-real-time Automatic Vehicle Location Data to Improve bus Services. *Transportation research Part B*. V33. pp. 559-579.
- Ishak, S, C., and Alecsandru,** (2004). Optimizing Traffic Prediction Performance of Neural Networks under Various Topological, Input, and Traffic Condition Settings. *Journal of Transportation Engineering*, Vol. 130, No.4, July 1, pp. 452-465.

Iversen, G. R., and Norpoth Helmut, (1976). *Analysis of Variance*. Sage Publications.

Kalaputapu, R., and Demetsky, (1985). Application of Artificial Neural Networks and Automatic Vehicle Location Data for Bus Transit schedule behavior Modeling. *Transportation Research Record 1497*, pp.44-52.

Kimpe, J. Thomas., James G. Strathman, David Griffin, Steven Callas, and Richard L. Gehart, (2003). Automatic Passenger Counter Evaluation Implication for National Transit Database Reporting. *Transportation Research Record 1835*, pp. 93-99.

Kittelson, and Associates, Inc., (2003). *Transit Capacity and Quality of Transit Manual*. The Second Edition. TRB, Washington D.C

Klopfenstein, R., (1986). “*Bus Communication System*”. Transportation Research Board, National Research Council. Washington D.C.

Koutsopoulos, H.N, A.R. Odoni, and N.H.M Wilson, (1985). Determination of Headway as Function Time varying Characteristics on a Transit Network. *Computer scheduling of Public Transport 2*, Edited by Jean-Marc Rousseau, Elsevier Science Publishers B.V., 1985. pp. 391-413.

Kraft, W., and TH. Deutschman, (1974). Bus Passenger Service-time Distribution. *Transportation Research Record*, 625, pp.37-43.

Kwon, Coifman B., and Bickel P., (2000). Day to Day Travel Time Trends and Travel Time Prediction from Loop Detector Data. *Transportation Research Record*, 1717, pp. 120-128.

Leuzbatch, Wilhelm, (1988). *Introduction to the Theory of Traffic Flow*. Springer

- Levine, Jonathan C., and Gwo-Wei Torng**, (1994). Dwell-time Effects of Low-Floor Bus Design. *Journal of Transportation Engineering*, Vol. 120, No.6, November, pp. 914-929.
- Levinson, Herbert**, (1983). Analyzing Transit Travel Time Performance. *Transportation Research Record*, 915, pp. 1-7.
- Li, Weigang, Marlon W. Koendjiharie, Ricardo C.deM.JuCa', Yaeko Yamashita, and Andrew Maciver**, (?). Algorithms for Estimating Bus Arrival Times Using GPS Data. Retrieved from the Website
- Linda, B., and Virginia C.**, (1992). *Processing data-The survey Example*. Series Quantitative Applications in Social Sciences, Sage University Paper 85.
- Lewis P.A, and Orav. E.J.**,(1980). *Simulation Methodology for Statistic, Operation and Analysts, Engineers –Volume 1*. Wadsworth & Brooks/code Statistics/ Probability Series.
- Lin, T., and N.H. Wilson**, (1992). Dwell time Relationships for Light Rail Systems. *Transportation Research Record*, 1361, pp. 287-295.
- Lin ,Wei Hua and Bertini Rober L.**, (2004). Modeling Schedule Recovery Process in Transit Operations for Bus Arrival Time Prediction. *Journal of Advanced Transportation*, Vol. 38, No.3, pp. 347-365.
- Lin, Wei-Hua and Zeng Jian**, (1999). Experimental Study of Real- Time Bus Arrival Time Prediction with GPS Data. *Transportation Research Record*, 1666, pp. 101-109.
- Loader, Clive**, (1999). *Local Regression and Likelihood*. Statistics and Computing, Springer.

- Lyshevski, Sergey**, (2003). *Engineering and Scientific Computations Using MATLAB*, John Wiley and Sons.
- Maloney, Mark and Boyle Daniel**, (1999). Components of Travel Time on the Glendale Beeline Bus Network. *Transportation Research Record 1666*, pp. 23-27.
- Marshall, Leo F., Herbert S. Levison, Lawrence C. Lennon, and Jerry Cheng**, (1990). Bus Service Times and Capacities in Manhattan. *Transportation Research Record*, 1266, pp. 189-196.
- Maybeck, P.S.**, (1979). *Stochastic Models, Estimation, and Control*. Volume 1. Academic Press.
- McKnight, Claire, Herbert Levison, Kaan Ozbay, Cammile Kamga, and Rober E. Passwell**, (2003). Impact of Congestion on Bus Operations and Costs. FHWA-NJ-2003-008, Final Report.
- McKnight C., and Passwell R.E.**, (1997). Impact of Congestion on New York Bus Service. *UTRC for MTA NewYork City Transit*.
- Mei, Chen and Chien Steven I. J.**, (2001), Dynamic Freeway Travel-Time Prediction with Probe Vehicle Data. *Transportation Research Record*, 1768, pp. 157-161.
- Menard, Scott**, (2001). *Applied Logistic Regression Analysis*. 2^{sd} Edition, Sage University Paper. Series: Quantitative Applications in the Social Science.
- Mishalani, Rabi G., Sungjoon, Lee and Mark, R. McCord**, (2000). Evaluating Real-Time Bus Arrival Information Systems. *Transportation Research Record*, 1731, pp. 81-87.

- Morgan, D.,** (2002). *A Microscopic Simulation Laboratory for Advanced Public Transportation System Evaluation*. Master Thesis Dissertation, MIT.
- Neter J., Wasserman W., and Kutner M.,** (1985). *Applied Linear Statistical Models*. Second Edition, Richard D. Irwin, Inc.
- Normand R., Draper and Harry, Smith,** (1998). *Applied Regression Analysis*. Wiley series in Probability and Statistics. Wiley Inter-science Publishers. Third Edition.
- Okunieff, Paula E.,** (1997). AVL Systems for bus Transit. *Synthesis of Transit Practice 24, TRR*, National Academy Press, Washington D.C
- Patnaik, J., Steven Chien, and Athanassios, Bladikas,** (2004). Estimation of Bus Arrival Times Using APC Data. *Journal of Public Transportation*, Vol.7, No.1, pp. 1-19. Website: <http://www.nctr.usf.edu/jpt/journalarticles.html>
- Pile, Randall, Jame Niemayer, and Rusell Chrishom,** (1998). *Transit Scheduling: Basic and Advanced Manuals*. TCRP Report 30, TRB, Washington, DC.
- Pedhazur, Elazar,** (1997). *Multiple Regression in Behavioral Research, Explanation and Prediction*. Harcourt College Publishers, Third Edition.
- PTV America,** (2004). *VISSIM User Manual*. PTV Planung Transport Verkehr AG
- PTV America,** (2005). *NEMA Editor Manual (Version 3.3)*. PTV Planung Transport Verkehr AG
- Rabi, Mishalani, Sungjoon Lee, and Mark R. McCord.,** (2000). Evaluating Real-time Bus Arrival Information Systems. *Transportation Research Record* 1731, pp. 81-87.

- Rajbhandari Rajat, Chien Steven I., and Daniel Janice R.**, (2003). Estimation of Bus Dwell Times with Automatic Passenger Counter Information. *Transportation Research Record*, 1841, pp. 120-127.
- Reinhoundt, Edwin M., and Veslatin S.A.**, (1997). A Dynamic Predicting Algorithm for Estimating Bus Arrival Time. Presents of the 8th IFAC/IFIP/IFORS Symposium, Crete, Greece, June 1997. Vol3. pp. 1225-8.
- Richard.P., and Dennis C.**,(1984). Cross-validation of Regression Models. *Journal of the American Statistical Association*, Vol.79, No.387, pp.575-583.
- Roess, Roger, Prassas Elena, and McShane William.**, (2004). *Traffic Engineering*, 3rd Edition, Prentice Hall.
- Ruihong, Huang and Zhong-Ren Peng.**, (2002). Schedule-Based Path-Finding Algorithms for Transit Trip-Planning Systems. *Transportation Research Record*, 1783, pp. 142-148.
- Schalkoff, Robert**, (1992). *Pattern Recognition, statistical, structural and neural Approaches*. John Wiley & Sons, Inc.
- Schiavon, John**, (1999). *Understanding and Applying Advanced on Board Bus Electronics*. TCRP Report 43. *Transportation Research Board*.
- Schuldze, Thomas, and Fliees Thomas.**, (1997). Urban Traffic Simulation with Psycho-physical Vehicle-Following Models. *Proceedings of the 1997 Winter Simulation Conference*.
- Schweiger, L.**, (2003). Real-time Bus Arrival Information Systems, a Synthesis of Transit Practice. *Transportation Research Board*. Washington D.C

- Senevirante, P.N.**, (1988). Simulation of Fixed Bus Travel Time. *Journal of Advanced Transportation*, Vol. 22, pp. 39-53.
- Senevirate, Prianka**, (1990). Analysis of On-time Performance of Bus Services Using Simulation. *Journal of Transportation Engineering*, Vol.116, No.4, pp. 517-531.
- Shalaby, A., Farhan Ali.**, (2004). Prediction Models of Bus Arrival and Departure Times Using AVL and APC Data. *Journal of Public Transportation*, Vol.7, No.1, pp. 41-60. Website: <http://www.nctr.usf.edu/jpt/journalarticles.html>
- Shara R.J.**, (2002). *Development of a Public Transit Information System Using GIS and ITS Technologies*. Master Thesis Dissertation, Carleton University, Ottawa-Canada.
- Silverman, B.W.**, (1986). *Density Estimation for Statistic and Data Analysis*. Monographs on Statistics and Applied Probability.
- Skomal, Edward**, (1981). *Automatic Vehicle Locating System*. Van Nostrand Reihold Company.
- Smith, Brian L., and Michael J., Demetsky**, (1994). Short-term Traffic Flow Prediction: Neural Network Approach. *Transportation Research Record*, 1453, pp. 98-104.
- Son, Bongsoon, Kim Hyung Jin, Shin Chi huyn, and Lee sang keon**, (2004). Intelligent Multimedia Solusion and security for the next generation Mobile Network-Bus Arrival Time Prediction Method for ITS Application. *Lecture Notes in Computer Science*. V.3215, pp.88-94.

- Stephan, R.S., Kenth A.B, and David W.S.,** (1994). Cross-validation of Multivariate Densities. *Journal of the American Statistical Association*, Vol.89, No.427, pp.807-817.
- Strathman, James G., Thomas J.Kimple, and Kentheth J. Dueker,** (2000). Bus Transit Operations Control: Review and Experiment Involving Tri-Met's Automated Bus Dispatching System.
- Sun, HongYu, Liu Henry, Xiao Heng, He Rachel, and Ran Bin,** (2003). Use of Local Linear Regression Model for Short-term Traffic Forecasting. *Transportation Research Record*, 1836, pp. 143-150.
- Transportation research Board.** *Highway Capacity Manual. Special Report 209.* TRB, National Research Council, Washington, D.C, 2000.
- Transit Cooperative Research Program,** (2001). *Transit Capacity and Quality of Service Manual*, Second Edition, TRB, Washington, DC.
- Transit Cooperative Research Program,** (2000). *Strategies for Improved Traveler Information*, TCRP Report 92, TRB, Washington, DC.
- Transit Cooperative Research Program,** (1998). *Understanding and Applying Advanced On-Board Bus Electronics*, Report 43, TRB, Washington, DC.
- Turnquist, MA.,** (1982). *Strategies for Improving Bud Transit Service Reliability.* Evanston, IL: Northwestern University. (NTIS Report No. DOT/RSPA/DPB-50-81-27).USDOT, Research and Special Programs Administration.
- Ubitran and Levison,** (1986). *Bus Priority Improvement Study: Analysis Of bus Priority Proposal.* November 1986

- Versteegrt, H.H, and Temprere C., MJ.,** (2003). PredicTime, a Stage of the Art and Functional Architecture. TNO- Inro Report 2003-07. Institute for Traffic and Transport, Logistic and Spatial Development, Nederland.
- Vuchic, VuKan R.,** (2004). *Urban Transit Operation, Planning, and Economics*. John Willey and Sons, Inc.
- Webb, Andrew,** (2002). *Statistical Pattern Recognition*. John Wiley and Sons Ltd. Second Edition.
- Wei, Hua Lin, Qingying Lu, and Joy Dahgren,** (2002). Dynamic Procedure for Short-Term Prediction of Traffic Conditions. *Transportation Research Record*, 1783, pp. 149-157.
- Wei, Hua Lin, Amit Kulkarni, and Pitu Mirchandani,** (2004). Short-term Arterial Travel Time Prediction for Advanced Traveler Information Systems. *Intelligent Transportation System*, Vol.8, pp. 143-154.
- Weidemann, Reiter U.,** (1994). *Micro-traffic simulation, the simulation system mission: Background and Actual State*. Publication (no information).
- Weigang, Li, Koendjiharie Marlon, Ricardo C.de M.JuCa, Yaeko Yamashita, and Andrew Macive,** (2004). *Algorithm for Estimating Bus Arrival Times using GPS data*. Retrieved from Website.
- William, R. Schucany,** (1995). Adaptive Bandwidth Choice for Kernel Regression. *Journal of American Statistical Association*, Vol.90, No.430, pp.535-540.
- Zargari, S.,** (1997). *Optimization of Integrated Multimodal Urban Transportation Corridors*. Ph.D Thesis Dissertation, Carleton University, Ottawa, Canada.

- Zargari S., and Khan A.M.,** (2003). Optimization of Travel in Bus Rapid Transit–Based Multimodal Corridors. *Journal of Public Transportation*, Vol.6, No.2, pp.87-104.
- Zhang, Wenyang and Lee Sik-Yum,** (2000). Variable Bandwidth Selection in Varying-Coefficient Models. *Journal of Multivariate Analysis*, Vol. 74, pp.116-134.
- Zhong, Ren Peng, Danlin Yu, and Edward Beimborn.,** (2002). Transit User Perceptions of the Benefits of Automatic Vehicle Location. *Transportation Research Record*, 1791, pp. 127-133.
- Zong, Z., Tian, Thomas U., Roelof E., and Kevin B.,** (2002). Variations in Capacity and Delay Estimates from Microscopic Traffic Simulation Models. *Transportation Research Record*, 1802, pp. 23-31.
- Yoon, Paul and Hwang, Ching-lai,** (1995). *Multiple Attribute Decision Making*. An Introduction series Quantitative Applications in Social Sciences, Sage University Paper 104.

APPENDICIES

Appendix A

1. General

This appendix shows the maps of bus route 95 and route 1. The maps were retrieved from the Website www.octranspo.com

2. Bus Route Maps

Figure A.1: Bus route 95

Figure A.2: Bus route 1

Appendix B1

1. General

This Appendix covers the results of six regression types described in chapter 5.

2. SPSS results for Classical Regression Types

Table B1:1 Model Summary for Type A-2.1

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.781 ^a	.610	.609	7.35328	1.800
2	.781 ^b	.610	.609	7.35280	
3	.781 ^c	.610	.609	7.35339	

a. Predictors: (Constant), DOORS, SEASON, PUNT, TotalOnPw, TOTAL_OFFS, STOP_LOCATE, LF, TOTAL_ONS, LOAD_ARR

b. Predictors: (Constant), DOORS, SEASON, PUNT, TotalOnPw, TOTAL_OFFS, STOP_LOCATE, LF, TOTAL_ONS

c. Predictors: (Constant), DOORS, PUNT, TotalOnPw, TOTAL_OFFS, STOP_LOCATE, LF, TOTAL_ONS

d. Dependent Variable: DWELL_3

Table B1.2: Parameters of Type A-2.1

Coefficients ^a								
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	13.346	1.070		12.469	.000		
	TOTAL_OFFS	.438	.015	.250	28.700	.000	.843	1.186
	TotalOnPw	-.030	.001	-.413	-20.587	.000	.159	6.287
	TOTAL_ONS	1.928	.037	1.054	51.417	.000	.152	6.558
	LOAD_ARR	.012	.027	.019	.450	.653	.034	29.251
	LF	-1.532	1.575	-.039	-.973	.331	.040	25.161
	PUNT	.003	.001	.048	5.441	.000	.840	1.191
	SEASON	.118	.084	.011	1.414	.158	.990	1.010
	STOP_LOCATE	1.975	.206	.083	9.573	.000	.846	1.182
DOORS	-4.050	.399	-.150	-10.152	.000	.295	3.386	
2	(Constant)	12.961	.645		20.095	.000		
	TOTAL_OFFS	.439	.015	.251	28.871	.000	.851	1.176
	TotalOnPw	-.030	.001	-.414	-20.673	.000	.160	6.255
	TOTAL_ONS	1.929	.037	1.055	51.657	.000	.154	6.508
	LF	-.840	.344	-.021	-2.443	.015	.833	1.200
	PUNT	.003	.001	.047	5.437	.000	.840	1.191
	SEASON	.118	.084	.011	1.410	.158	.990	1.010
	STOP_LOCATE	1.956	.202	.083	9.682	.000	.882	1.134
	DOORS	-3.902	.227	-.144	-17.219	.000	.915	1.093
3	(Constant)	13.177	.627		21.024	.000		
	TOTAL_OFFS	.438	.015	.250	28.834	.000	.853	1.173
	TotalOnPw	-.030	.001	-.413	-20.636	.000	.160	6.248
	TOTAL_ONS	1.928	.037	1.054	51.635	.000	.154	6.500
	LF	-.830	.344	-.021	-2.414	.016	.834	1.199
	PUNT	.003	.001	.048	5.456	.000	.840	1.191
	STOP_LOCATE	1.957	.202	.083	9.684	.000	.882	1.134
	DOORS	-3.922	.226	-.145	-17.342	.000	.919	1.089

a. Dependent Variable: DWELL_3

Table B1.3: Model Summary for Type A-2.2

Model Summary ^a					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.786 ^a	.618	.618	7.26409	
2	.786 ^b	.618	.618	7.26403	1.843

a. Predictors: (Constant), TotalOnCubic, PUNT, SEASON, DOORS, TOTAL_OFFS, STOP_LOCATE, LF, TOTAL_ONS, LOAD_ARR, TotalOnPw

b. Predictors: (Constant), TotalOnCubic, PUNT, SEASON, DOORS, TOTAL_OFFS, STOP_LOCATE, LF, TOTAL_ONS, TotalOnPw

c. Dependent Variable: DWELL_3

Table B1.4: Parameters of Type A-2.2

Coefficients ^a								
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	13.664	1.075		12.706	.000		
	TOTAL_OFFS	.427	.015	.243	27.949	.000	.834	1.200
	TotalOnPw	-.071	.006	-.964	-11.837	.000	.010	104.957
	TOTAL_ONS	2.347	.070	1.288	33.545	.000	.043	23.319
	LOAD_ARR	.026	.027	.041	.956	.339	.034	29.035
	LF	-2.415	1.566	-.061	-1.542	.123	.040	25.093
	PUNT	.003	.001	.048	5.485	.000	.831	1.203
	SEASON	.153	.084	.015	1.824	.068	.985	1.015
	STOP_LOCATE	1.848	.205	.078	8.995	.000	.839	1.192
	DOORS	-4.387	.399	-.161	-10.989	.000	.295	3.392
	TotalOnCubic	.001	.000	.361	7.034	.000	.024	41.712
2	(Constant)	12.843	.649		19.804	.000		
	TOTAL_OFFS	.429	.015	.244	28.173	.000	.841	1.189
	TotalOnPw	-.071	.006	-.966	-11.873	.000	.010	104.851
	TOTAL_ONS	2.351	.070	1.290	33.657	.000	.043	23.241
	LF	-.954	.344	-.024	-2.771	.006	.823	1.214
	PUNT	.003	.001	.048	5.490	.000	.831	1.203
	SEASON	.153	.084	.015	1.821	.069	.985	1.015
	STOP_LOCATE	1.808	.201	.076	8.987	.000	.874	1.144
	DOORS	-4.073	.227	-.149	-17.953	.000	.913	1.095
	TotalOnCubic	.001	.000	.362	7.050	.000	.024	41.700

^a. Dependent Variable: DWELL_3

Table B1.5: Parameters of Type A-2.4

Parameter Estimates				
Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
b1	31.287	.384	30.534	32.039
b2	.346	.014	.319	.374
b3	.795	.005	.784	.805

Table B1.6: Model Summary for Type A-2.4

ANOVA ^a			
Source	Sum of Squares	df	Mean Squares
Regression	1627451	3	542483.7
Residual	405012.0	6032	67.144
Uncorrected Total	2032463	6035	
Corrected Total	813235.7	6034	

Dependent variable: DWELL_3

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .502.

Table B1.7: Parameters of Type A-2.5

Parameter Estimates				
Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
b1	1184.802	1268661	-2485838.131	2488207.734
b2	91.856	998524.5	-1957368.933	1957552.645
b3	-2.557	9811.205	-19235.985	19230.870
b4	-.003	7.131	-13.981	13.976

Table B1.8: Model Summary for Type A-2.5

ANOVA ^a			
Source	Sum of Squares	df	Mean Squares
Regression	1643479	4	410869.8
Residual	441527.8	6095	72.441
Uncorrected Total	2085007	6099	
Corrected Total	843898.3	6098	

Dependent variable: DWELL_3

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .477.

Table B1.9: Parameters of Type A-2.6

Parameter Estimates				
Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
b1	-1.496	1.613	-4.658	1.666
b2	97.767	234.859	-362.640	558.175
b3	470.907	1253.074	-1985.570	2927.383
b4	.561	.117	.332	.790

Table B1.10: Model Summary for Type A-2.6

ANOVA ^a			
Source	Sum of Squares	df	Mean Squares
Regression	1643120	4	410779.9
Residual	387654.5	5996	64.652
Uncorrected Total	2030774	6000	
Corrected Total	819684.1	5999	

Dependent variable: DWELL_3

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .527.

Table B1.11: Model Summary for Pilot Regression without Deletion of Outlier and Extreme values

Model Summary ^d										
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics					Durbin-Watson
					R Square Change	F Change	df1	df2	Sig. F Change	
1	.608 ^a	.370	.364	15.99027	.370	65.929	8	898	.000	2.041
2	.608 ^b	.370	.365	15.98183	.000	.052	1	898	.820	
3	.607 ^c	.369	.365	15.98773	-.001	1.664	1	899	.197	

a. Predictors: (Constant), BUS_TYPE, STOP_LOCATE, SEASON, TOTAL_ONS, PUNT, LF, TOTAL_OFFS, LOAD_DEP

b. Predictors: (Constant), BUS_TYPE, STOP_LOCATE, SEASON, TOTAL_ONS, LF, TOTAL_OFFS, LOAD_DEP

c. Predictors: (Constant), BUS_TYPE, STOP_LOCATE, TOTAL_ONS, LF, TOTAL_OFFS, LOAD_DEP

d. Dependent Variable: DWELL_1

Table B1.12: Coefficients for the Pilot Regression

Coefficients ^a										
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B		Collinearity Statistics	
		B	Std. Error	Beta			Lower Bound	Upper Bound	Tolerance	VIF
1	(Constant)	17.323	1.958		8.846	.000	13.480	21.166		
	STOP_LOCATE	1.819	1.182	.044	1.539	.124	-.501	4.139	.843	1.187
	SEASON	.605	.468	.034	1.292	.197	-.314	1.524	.989	1.011
	PUNT	.001	.003	.007	.228	.820	-.006	.007	.835	1.198
	LF	-27.184	9.753	-.409	-2.787	.005	-46.325	-8.044	.033	30.758
	TOTAL_OFFS	.829	.178	.280	4.643	.000	.478	1.179	.193	5.176
	LOAD_DEP	.408	.165	.392	2.467	.014	.083	.733	.028	35.896
	TOTAL_ONS	1.124	.189	.381	5.936	.000	.753	1.496	.170	5.876
BUS_TYPE	-15.769	2.268	-.328	-6.952	.000	-20.221	-11.317	.315	3.170	
2	(Constant)	17.314	1.957		8.848	.000	13.473	21.154		
	STOP_LOCATE	1.879	1.151	.046	1.632	.103	-.380	4.139	.887	1.127
	SEASON	.604	.468	.034	1.290	.197	-.315	1.522	.989	1.011
	LF	-27.081	9.737	-.408	-2.781	.006	-46.191	-7.971	.033	30.692
	TOTAL_OFFS	.831	.178	.280	4.661	.000	.481	1.180	.194	5.166
	LOAD_DEP	.408	.165	.392	2.470	.014	.084	.733	.028	35.895
	TOTAL_ONS	1.125	.189	.381	5.945	.000	.754	1.497	.170	5.874
	BUS_TYPE	-15.810	2.260	-.329	-6.996	.000	-20.245	-11.374	.317	3.151
3	(Constant)	18.216	1.828		9.963	.000	14.627	21.804		
	STOP_LOCATE	1.923	1.151	.047	1.671	.095	-.336	4.183	.888	1.126
	LF	-27.223	9.740	-.410	-2.795	.005	-46.338	-8.107	.033	30.688
	TOTAL_OFFS	.829	.178	.280	4.649	.000	.479	1.179	.194	5.165
	LOAD_DEP	.410	.165	.393	2.479	.013	.085	.734	.028	35.893
	TOTAL_ONS	1.128	.189	.382	5.957	.000	.756	1.500	.170	5.873
	BUS_TYPE	-15.965	2.258	-.332	-7.072	.000	-20.396	-11.535	.318	3.142

a. Dependent Variable: DWELL_1

a. Dependent Variable: DWELL_1

Appendix B2

1. General

This appendix presents the outputs of SPSS 13 applied for three types of binary logistic regressions. Each type has three tables recording the model summary, the omnibus tests, and model parameters.

2. SPSS Results for Binary Logistic Regression

Table B2.1: Model Summary for Type B-L.1

Model Summary			
Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	1804.229 ^a	.418	.586
2	1804.777 ^a	.418	.586

a. Estimation terminated at iteration number 8 because parameter estimates changed by less than .001.

Table B2.2: Omnibus Tests of Model Coefficients of Type B-L.1

Omnibus Tests of Model Coefficients				
		Chi-square	df	Sig.
Step 1	Step	1379.104	10	.000
	Block	1379.104	10	.000
	Model	1379.104	10	.000
Step 2 ^a	Step	-.548	2	.760
	Block	1378.557	8	.000
	Model	1378.557	8	.000

a. A negative Chi-squares value indicates that the Chi-squares value has decreased from the previous step.

Table B2.3: Model Parameters of Type B-L.1

Classification Table ^a					
Observed			Predicted		
			BUSIER DOOR		Percentage Correct
			Busier door is rear door	Busier Door is Front door	
Step 1	BUSIER_DOOR	Busier door is rear door	647	161	80.1
		Busier Door is Front door	189	1551	89.1
	Overall Percentage				86.3
Step 2	BUSIER_DOOR	Busier door is rear door	647	161	80.1
		Busier Door is Front door	188	1552	89.2
	Overall Percentage				86.3

a. The cut value is .500

Table B2.4: Model Summary for Type B-L.2

Model Summary			
Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	942.654 ^a	.262	.473
2	942.743 ^a	.262	.473
3	944.813 ^b	.261	.471
4	948.127 ^c	.260	.469
5	948.378 ^d	.260	.469

a. Estimation terminated at iteration number 15 because parameter estimates changed by less than .001.

b. Estimation terminated at iteration number 16 because parameter estimates changed by less than .001.

c. Estimation terminated at iteration number 10 because parameter estimates changed by less than .001.

d. Estimation terminated at iteration number 8 because parameter estimates changed by less than .001.

Table B2.5: Model Parameters of Type B-L.2

Classification Table ^a					
Observed			Predicted		
			BUSIER DOOR		Percentage Correct
			Busier door is rear door	Busier Door is Front door	
Step 1	BUSIER_DOOR	Busier door is rear door	102	159	39.1
		Busier Door is Front door	41	1566	97.4
	Overall Percentage				89.3
Step 2	BUSIER_DOOR	Busier door is rear door	100	161	38.3
		Busier Door is Front door	41	1566	97.4
	Overall Percentage				89.2
Step 3	BUSIER_DOOR	Busier door is rear door	96	165	36.8
		Busier Door is Front door	42	1565	97.4
	Overall Percentage				88.9
Step 4	BUSIER_DOOR	Busier door is rear door	98	163	37.5
		Busier Door is Front door	43	1564	97.3
	Overall Percentage				89.0
Step 5	BUSIER_DOOR	Busier door is rear door	95	166	36.4
		Busier Door is Front door	40	1567	97.5
	Overall Percentage				89.0

a. The cut value is .500

Table B2.6: Omnibus Tests of Model Coefficients of Type B-L.2

Omnibus Tests of Model Coefficients				
		Chi-square	df	Sig.
Step 1	Step	568.401	12	.000
	Block	568.401	12	.000
	Model	568.401	12	.000
Step 2 ^a	Step	-.090	1	.765
	Block	568.311	11	.000
	Model	568.311	11	.000
Step 3 ^a	Step	-2.070	2	.355
	Block	566.241	9	.000
	Model	566.241	9	.000
Step 4 ^a	Step	-3.314	3	.346
	Block	562.928	6	.000
	Model	562.928	6	.000
Step 5 ^a	Step	-.251	1	.617
	Block	562.677	5	.000
	Model	562.677	5	.000

a. A negative Chi-squares value indicates that the Chi-squares value has decreased from the previous step.

Appendix B3

1. General

This appendix illustrate the outputs of the SPSS applied for Multinomial Logistic Regression models proposed in chapter 5, section 5.3.6.2

2. SPPSS Results for Multinomial Logistic Regression

Table B3.1: Goodness-of-Fit for Type B-ML.1

Goodness-of-Fit			
	Chi-Square	df	Sig.
Pearson	21001.411	12222	.000
Deviance	10360.485	12222	1.000

Table B3.2: Pseudo R-square for Type B-ML.1

Pseudo R-Square	
Cox and Snell	.333
Nagelkerke	.379
McFadden	.193

Table B3.3: Model Parameters for Type B-ML.1

Parameter Estimates									
Busiest door ^a		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
door 2 busiest	Intercept	-.133	.123	1.172	1	.279			
	LOAD_ARR	.004	.002	2.911	1	.088	1.004	.999	1.008
	LF	0 ^b			0				
	PUNT	.000	.000	.005	1	.945	1.000	1.000	1.000
	SEASON	-.022	.033	.416	1	.519	.979	.917	1.045
	STOP_LOCATE	.057	.077	.549	1	.459	1.058	.911	1.230
	TIME	-.114	.044	6.803	1	.009	.892	.819	.972
	TOTAL_ONS	-.001	.005	.025	1	.874	.999	.988	1.010
	TOTAL_OFFS	-.023	.004	36.024	1	.000	.978	.970	.985
Door 1 busiest	Intercept	1.302	.121	115.269	1	.000			
	LOAD_ARR	-.006	.002	7.351	1	.007	.994	.990	.998
	LF	0 ^b			0				
	PUNT	.000	.000	.502	1	.479	1.000	1.000	1.001
	SEASON	.067	.032	4.414	1	.036	1.069	1.005	1.139
	STOP_LOCATE	.549	.075	53.817	1	.000	1.732	1.496	2.006
	TIME	-.105	.042	6.284	1	.012	.900	.829	.977
	TOTAL_ONS	.098	.006	269.578	1	.000	1.103	1.090	1.116
	TOTAL_OFFS	-.308	.010	1013.492	1	.000	.735	.721	.749

a. The reference category is: door 3 busiest.

b. This parameter is set to zero because it is redundant.

Table B3.1: Goodness-of-Fit for Type B-ML.2

Goodness-of-Fit			
	Chi-Square	df	Sig.
Pearson	23844.277	10336	.000
Deviance	8352.103	10336	1.000

Table B3.2: Pseudo R-square for Type B-ML.2

Pseudo R-Square	
Cox and Snell	.321
Nagelkerke	.372
McFadden	.194

Table B3.3: Model Parameters of Type B-ML.2

Parameter Estimates									
Busiest door ^a		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
door 2 busiest	Intercept	-.228	.161	2.020	1	.155			
	LOAD_ARR	.003	.003	1.510	1	.219	1.003	.998	1.008
	LF	0 ^b	.	.	0	.	.	.	.
	PUNT	.000	.000	.003	1	.957	1.000	.999	1.001
	SEASON	-.011	.039	.082	1	.775	.989	.916	1.067
	STOP_LOCATE	.034	.090	.139	1	.710	1.034	.867	1.234
	TIME	-.104	.051	4.142	1	.042	.902	.816	.996
	TOTAL_ONS	.016	.016	1.004	1	.316	1.016	.985	1.048
	TOTAL_OFFS	-.023	.006	14.870	1	.000	.978	.967	.989
	Total_On_squared	.000	.001	.863	1	.353	1.000	.999	1.001
Door 1 busiest	Intercept	1.602	.146	121.095	1	.000			
	LOAD_ARR	-.013	.002	30.584	1	.000	.987	.983	.992
	LF	0 ^b	.	.	0	.	.	.	.
	PUNT	.000	.000	.004	1	.947	1.000	.999	1.001
	SEASON	.122	.035	12.121	1	.000	1.130	1.055	1.210
	STOP_LOCATE	.659	.084	61.675	1	.000	1.933	1.640	2.279
	TIME	-.116	.046	6.455	1	.011	.890	.814	.974
	TOTAL_ONS	.150	.015	95.282	1	.000	1.161	1.127	1.197
	TOTAL_OFFS	-.300	.010	869.260	1	.000	.741	.726	.756
	Total_On_squared	-.003	.001	38.672	1	.000	.997	.996	.998

a. The reference category is: door 3 busiest.

b. This parameter is set to zero because it is redundant.

Appendix C

1. General

This Appendix presents the MatLab source codes for the developed model. It includes the source code for bus running time, and boarding and de-boarding passenger predictions.

2. Bus running time function

```
name: running.m

function runpre = running(X,Y,Xreal)

%Performs bus real-time prediction using modified LOWESS method.
%LOWESS was first introduced by Cleveland 1978 and Cleveland et al., 1988
%Input variables:
%
% X = data set in form of matrix ( m by d)
%     each element of this matrix plays running time of the
%     previous buse recored in the past
%     there may be d previous buses taken into the pattern and
%     there are m observations( records)
%Xreal= the pattern vector ( 1 by d) whose each element
%       is the record of the running time of the previous
%       bus right before. There are d previous buses.
%Y = data set in form of vector (m by 1) whose each element
%   is the record of running time of the bus in question
%   in the past. There are m observations.
%Outputs:
%
% runpre      = real-time prediction of bus running time, it is a scalar.
%
%.....
% WRITTEN BY: NAM VU, PH.D CANDIDATE, CARLETON UNIVERSITY
% START DATE: Jan 15,2006
% FINISH DATE: NO INFORMATION
% THIS PROGRAM IS FOR ACADEMIC USES ONLY.
%.....
% the next lines is to check if the real-time predictions should be made or
% not
%f=input('Please enter the bus service frequency in second f= ');
```

```

% first check for the prediction horizon (if d smaller than 2, there is no
% real-time prediction)
%if f >1800
%  warning('the bus service frequency is too long');
%  fprintf('\n! No real-time estimation being made');
%  dum= input('\nType any key to proceed');
%  end
%  clear f;

% NOW HERE IS THE MAIN PROGRAM.....

%load Rpastpattern.dat;
% enter history data based on bus ID and service time
%load Rpastdata.dat;
% enter history running times of the bus
%load The Rcurrenpattern.dat

% the following lines is to find the optimal bandwidth H by applying
% leave-one-out method (see Wofgang 1996) and Nadaraya-Watson estimation
%
%clear
[m,k]=size(X);
hmin= round(0.1*m);
hmax=round(0.8*m);
s= round(0.1*m);
for h=hmin:s:hmax
    for i=1:1:m
        Y_test=Y(i);
        Y_train=Y;
        Y_train(i)=[];
        test_1=X(i,:);
        test_2=X;
        test_2(i,:)=[];% the training set after removing the ith row
        for n=1:size(test_2,1)
            d(n,1)= sqrt(sum((test_2(n,:)-test_1(1,:)).^2));
        end
        disp(' The test set is')
        disp(Y_train);
        disp(' the Edist to the first  second... neighbor')
        disp(d');
        q=sort(d);
        bandwidth=q(h)% Select the h-nearest neighbours
        K_weigh=d./bandwidth;
        for p=1:length(d)
            if K_weigh(p,1)<1
                w(p,1)=(70/81)*(1-(abs(K_weigh(p,1))^3)^3);
            else
                w(p,1)=0;
            end
        end
        w;
        if w==0
            warning('No neighbour has been found');
        end
        y_hat =(Y_train*w)/sum(w);
    end
end

```

```

        error= (y_hat-Y_test)^2;
        E(i)=error;
    end
    disp('The matrix of Error is')
    disp(E);
    Cv =sum(E)/m
    j=1+(h-hmin)/s;
    CR(j)=Cv;
    nnear(j)=hmin+(j-1)*s;
end
disp('The CROSS_VALIDATION factors')
disp(CR)
[miCV, pointer]=min(CR);
OpNei=round(hmin+s*(pointer-1))
%.....PLOTING.....
%
    plot(nnear,CR);
    xlabel('# of nearest neighbours'),ylabel('CR');
    title('h and CR Relationship');
%
%.....PREDICTION STEP.....
for i=1:m
    eudis0(i,1)=sqrt(sum((X(i,:)-Xreal(1,:)).^2));
    end
    eudis0;
    B=sort(eudis0);
    OpBand=B(OpNei);
    k0=eudis0./OpBand;
    for i=1:m
        if k0(i,1)<1
            w0(i,1)=(70/81)*(1-(abs(k0(i,1))^3)^3);
        else
            w0(i,1)=0;
        end
    end
    w0;
    BanMa=diag(w0);
    EstiMa=[ones(m,1),X];
    IXma=inv(EstiMa*BanMa*EstiMa)
    Beta=IXma*EstiMa*BanMa*Y;
    slope=Beta;
    slope(1,:)=[];
    runpre=Beta(1)+ Xreal*slope
    %runpre=(Y*w0)/sum(w0);
    fprintf('THE PREDCTED BUS RUNNING TIME IS %4.2f SECONDS.',runpre);
%.....
%.....CONFIDENCE INTERVAL ESTIMATION.....
de=0;
for i=1:m
    if w0(i,1)~=0
        PRE(i,1)=Beta(1)+ (X(i,:) *slope(:,1));
        ERRO(i,1)=(Y(i,1)-PRE(i,1))^2;
        weigERRO(i,1)=ERRO(i,1)*w0(i,1);
        de=de+w0(i,1)^2;
    end
end
end

```

```

de % sum of the squared weights
PRE;% Matrix of predicted values of neighbors around x0
weigERRO; % Matrix of weighted errors
MSE=sum(ERRO) % Matrix of prediction error
ns=IXma*EstiMa*BanMa*BanMa*EstiMa
us=EstiMa*BanMa*BanMa*EstiMa
ms=1/(trace(BanMa)-trace(ns))
sigma=sum(weigERRO)*ms
bias= (MSE-sigma)/OpNei
freDegree=(sum(w0))^2/de
fprintf('the degree of freedom is %4.0f',freDegree)
critical=input('Please enter the confidence limit critical= ');
lower=runpre-abs(sqrt(bias))-critical*abs(sqrt(sigma))
upper=runpre+sqrt(bias)+critical*sqrt(sigma)
fprintf('THE LOWER LIMIT IS %4.2f seconds at 95 percent confidence level',lower);
fprintf('THE UPPER LIMIT IS %4.2f seconds at 95 percent confidence level',upper);
fprintf('HAVE A NICE DAY')

```

3. Bus boarder function

File Name: boarder.m

```

function boarder = nuboarder(X,Y,Xreal)
%Performs real-time prediction of number of boarders on the bus using
%modified LOWESS method.
%LOWESS was first introduced by Cleveland 1978 and Cleveland et al., 1988
%Input variables:
%
% X = Data set in form of matrix ( m by 4)
%      each element of this matrix plays BOADERS (2
%      observations)of the previous buses in same day.
%      and HEADWAYS (2 observations) of the previous bus and
%      the bus itself in the past at time k.
%      There may be m events observed(recorded)
% Y = The pattern vector ( 1 by 4) whose elements of this
%      vector are the records of the boarders in the bus k in
%      history
%      and the boarders of the 2 previous buses.
% Xreal = Data set in form of vector (m by 1) whose each element
%      is the predicted runninf time of the bus, the
%      running time of the bus right before it, the number
%      of boarders of the 2 previous buses
%
%Output(s):
%
% boarder = Predicted number of boarders. It is a scalar.
%
%.....

```

```

% WRITTEN BY NAM VU, PH.D CANDIDATE, CARLETON UNIVERSITY
% START DATE: MARCH 1,2006
% FINISH DATE: NO INFORMATION
% THIS PROGRAM IS FOR ACADEMIC USES ONLY.
%.....
% the next lines is to check if the real-time predictions should be made or
% not
f=input('Please enter the bus service frequency in second f= ');
% first check for the prediction horizon (if d smaller than 2, there is no
% real-time prediction)
if f > 1800
    warning('the bus service frequency is too long');
    fprintf('\n! No real-time estimation being made');
    dum= input('\nType any key to proceed');
end
clear f;

% .....NOW HERE IS THE MAIN PROGRAM.....

%load ('Bpastpattern.dat');
% enter history data based on bus ID and service time
%load ('Bpastdata.dat');
% enter history # of boarders of the bus
%load ('Bcurrenpattern.dat');

% We should first STANDARDIZE all variables

[m,k]=size(X);
for j=1:k % normalize data of past pattern Matrix
    mX(1,j)=mean(X(:,j));
    sX(1,j)=std(X(:,j));
    X(:,j)=(X(:,j)-mX(1,j))/sX(1,j);
    Xreal(1,j)=(Xreal(1,j)-mX(1,j))/sX(1,j); % Current Pattern is also normalized
end
% mY=mean(Y);
% sY=std(Y);
% Y=(Y-mY)/sY;
% Exhibit the inputs
mX
Y
disp(Xreal)
X
hmin= round(0.1*m);
hmax=round(0.7*m);
s= round(0.1*m);
for h=hmin:s:hmax
    for i=1:1:m
        Y_test=Y(i);
        Y_train=Y;
        Y_train(i)=[];
        test_1=X(i,:);
        test_2=X;
        test_2(i,:)=[]; % the training set after removing the ith row
        for n=1:size(test_2,1)

```

```

        d(n,1)= sqrt(sum((test_2(n,:)-test_1(1,:)).^2));
    end
    disp(' The test set is')
    disp(Y_train)
    disp('the Edist to the first second... neighbor')
    disp(d');
    q=sort(d);
    bandwidth=q(h)% Select the h-nearest neighbours
    K_weigh=d./bandwidth;
    for p=1:length(d)
        if K_weigh(p,1)<1
            w(p,1)=(70/81)*(1-(abs(K_weigh(p,1))^3)^3);
        else
            w(p,1)=0;
        end
    end
    w;
    if w==0
        warning('No neighbour has been found');
    end
    y_hat =(Y_train*w)/sum(w);
    error= (y_hat-Y_test)^2;
    E(i)=error;
end
    disp('The matrix of Error is')
    disp(E);
    Cv =sum(E)/m
    j=1+(h-hmin)/s;
    CR(j)=Cv;
    nnear(j)=hmin+(j-1)*s;
end
disp('The CROSS_VALIDATION factors')
disp(CR)
[miCV, pointer]=min(CR);
OpNei=round(hmin+s*(pointer-1))
%.....PLOTING.....
%
    plot(nnear,CR);
    xlabel('# of nearest neighbours'),ylabel('CR');
    title('h and CR Relationship');
%.....PREDICTION STEP.....
for i=1:m
    eudis0(i,1)=sqrt(sum((X(i,:)-Xreal(1,:)).^2));
end
eudis0;
B=sort(eudis0);
OpBand=B(OpNei);
k0=eudis0./OpBand;
for i=1:m
    if k0(i,1)<1
        w0(i,1)=(70/81)*(1-(abs(k0(i,1))^3)^3);
    else
        w0(i,1)=0;
    end
end
w0;

```

```

BanMa=diag(w0);
EstiMa=[ones(m,1),X];
IXma=inv(EstiMa**BanMa*EstiMa);
disp(EstiMa');
Beta=IXma*EstiMa**BanMa*Y;
disp(Beta);
boarder=round(Y*w0/sum(w0))
fprintf(2,'THE REAL-TIME PREDCTED # of PASS OF THE BUS IS %4.2f PAS.',boarder);
disp('Program complete.HAVE A NICE DAY ');

```

4. Bus de-boarder function

File name: deboarder.m

```

function deboarder = deboarder(X,Y,Xreal)
%Performs real-time prediction of number of boarders on the bus using
%modified LOWESS method.
%LOWESS was first introduced by Cleveland 1978 and Cleveland et al., 1988
%Input variables:
%
% X = Data set in form of matrix ( m by 4)
%      each element of this matrix plays DEBOARDERS (2
%      observations)of the previous buses in the past.
% Y = The pattern vector ( 1 by 4) whose elements of this
%      vector are the records of the deboarders in the bus k in
%      history
%      and the boarders of the 2 previous buses.
% Xreal = Data set in form of vector (m by 1) whose each element
%      is the the number of deboarder of the 2 previous
%buses
%
%Output(s):
%
% deboarder = Predicted number of deboarders. It is a scalar.
%
%.....
% WRITTEN BY NAM VU, PH.D CANDIATE, CARLETON UNIVERSITY
% START DATE: MARCH 30,2006
% FINISH DATE: NO INFORMATION
% THIS PROGRAM IS FOR ACADEMIC USES ONLY.
%.....
% the next lines is to check if the real-time predictions should be made or
% not
%f=input('Please enter the bus service frequency in second f= ');
% first check for the prediction horizon (if d smaller than 2, there is no
% real-time prediction)
%if f >1800
% warning('the bus service frequency is too long');

```



```

% fprintf('\n! No real-time estimation being made');
% dum= input('\nType any key to proceed');
%end
%clear f;

% .....NOW HERE IS THE MAIN PROGRAM.....

%load ('Bpastpattern.dat');
% enter history data based on bus ID and service time
%load ('Bpastdata.dat');
% enter history # of boarders of the bus
%load ('Bcurrenpattern.dat');
% We should first STANDARDIZE all variables
[m,k]=size(X);
for j=1:k % normalize data of past pattern Matrix
    mX(1,j)=mean(X(:,j));
    sX(1,j)=std(X(:,j));
    X(:,j)=(X(:,j)-mX(1,j))/sX(1,j);
    Xreal(1,j)=(Xreal(1,j)-mX(1,j))/sX(1,j); % Current Pattern is also normalized
end
% mY=mean(Y);
% sY=std(Y);
% Y=(Y-mY)/sY;
% Exhibit the inputs
mX
Y
disp(Xreal)
X
hmin= round(0.05*m);
hmax=round(0.7*m);
s= round(0.1*m);
for h=hmin:s:hmax
    for i=1:1:m
        Y_test=Y(i);
        Y_train=Y;
        Y_train(i)=[];
        test_1=X(i,:);
        test_2=X;
        test_2(i,:)=[]; % the training set after removing the ith row
        for n=1:size(test_2,1)
            d(n,1)= sqrt(sum((test_2(n,:)-test_1(1,:)).^2));
        end
        disp(' The test set is')
        disp(Y_train)
        disp(' the Edist to the first second... neighbor')
        disp(d');
        q=sort(d);
        bandwidth=q(h)% Select the h-nearest neighbours
        K_weigh=d./bandwidth;
        for p=1:length(d)
            if K_weigh(p,1)<1
                w(p,1)=(70/81)*(1-(abs(K_weigh(p,1)))^3)^3;
            else
                w(p,1)=0;
            end
        end
    end
end

```

```

end
w;
if w==0
    warning('No neighbour has been found');
end
y_hat =(Y_train*w)/sum(w);
error= (y_hat-Y_test)^2;
E(i)=error;
end
disp('The matrix of Error is')
disp(E);
Cv =sum(E)/m
j=1+(h-hmin)/s;
CR(j)=Cv;
nnear(j)=hmin+(j-1)*s;
end
disp('The CROSS_VALIDATION factors')
disp(CR)
[miCV, pointer]=min(CR);
OpNei=round(hmin+s*(pointer-1))
%.....PLOTING.....
%
plot(nnear,CR);
xlabel('# of nearest neighbours'),ylabel('CR');
title('h and CR Relationship');
%.....PREDICTION STEP.....
for i=1:m
    eudis0(i,1)=sqrt(sum((X(i,:)-Xreal(1,:)).^2));
end
eudis0;
B=sort(eudis0);
OpBand=B(OpNei);
k0=eudis0./OpBand;
for i=1:m
    if k0(i,1)<1
        w0(i,1)=(70/81)*(1-(abs(k0(i,1)))^3)^3;
    else
        w0(i,1)=0;
    end
end
w0;
BanMa=diag(w0);
EstiMa=[ones(m,1),X];
IXma=inv(EstiMa'*BanMa*EstiMa);
disp(EstiMa');
Beta=IXma*EstiMa'*BanMa*Y;
disp(Beta);
deboarder=round(Y*w0/sum(w0))
fprintf(2,'THE REAL-TIME PREDCTED # of PASS OF THE BUS IS %4.2f PAS.',deboarder);
disp('Program complete.HAVE A NICE DAY ');

```

Appendix D1

1. General

This appendix presents the calibration procedure described in chapter 6. First, two runs were made with VISSIM default parameters (see Table 6.3). By holding the same random seed numbers for each run and the same traffic volume, two other runs were also made with VISSIM but with calibrated values (see Table 6.3). The simulated results on bus travel times then were compared with corresponding actual bus travel times of each bus route provided by the OC Transpo at 95% confidence level.

2. Travel time Measurement Configuration in VISSIM 4.1

Simulated bus travel times were measured by a VISSIM's tool namely Travel time Measurement Configuration which records travel time of all buses running between stops. A travel time includes actual travel time, stop and go time, dwell time and all delays at intersections. Figure D1.1 shows this tool of VISSIM.

Figure D1.1: Travel time measurement tool in VISSIM

3. Calibration Procedure

Tables D1.1 to D1.6 present the comparisons between simulated bus travel times and the actual ones provided by the OC Transpo. As shown in the tables, the calibrated parameters provided closer bus running times to the actual data than the use of VISSIM default parameters did. However, if we use the default values suggested by VISSIM, the model still meets calibration targets. Therefore, no further calibration efforts were made after the first adjustment. The parameters used in this study are reasonable.

Tables D1.7- D1.10 tabulate VISSIM outputs during calibration process

4. APC-AVL data from OC Transpo

Tables D1.11-D1.16 present actual bus travel time data provided by the OC Transpo

Table D1.1: VISSIM Calibration: Billings-Holmwood Link

Parameters (1)	Difference. % (2)	Significance Level (3)	Simulated data			OC transpo data		
			# of Simulation runs (4)	Mean (5)	Std (6)	# of APC records (7)	Mean (8)	Standard Dev. (9)
BX_ADD=2.0 BX_MULT=3.0	6.1	0.05	16	464.16 s	31.99	37	435.6	48
BX_ADD=3.0 BX_MULT=3.5	7.8	0.05	16	469.98	28.38			

Table D1.2: VISSIM Calibration: Holmwood- Gladstone Link

Parameters (1)	Diff. % (2)	Significance Level (3)	Simulated data			OC transpo data		
			# of simulation runs (4)	Mean (5)	Standard Dev. (6)	# of APC records (7)	Mean (8)	Standard Dev. (9)
BX_ADD=2.0 BX_MULT=3.0	9.49	0.05	16	395.85	36.21	38	437.4	47.4 s
BX_ADD=3.0 BX_MULT=3.5	12.50	0.05	16	382.73	32.68			

Table D1.3: VISSIM Calibration: Gladstone- Rideau Link

Parameters (1)	Difference (2)	Significance Level (3)	Simulated data			OC transpo data		
			# of simulation runs (4)	Mean (5)	Standard Dev. (6)	# of APC records (7)	Mean (8)	Standard Dev. (9)
BX_ADD=2.0 BX_MULT=3.0	5	0.05	15	595.88	67.52	35	567.6 s	55.8 s
BX_ADD=3.0 BX_MULT=3.5	4.24	0.05	15	591.65	63.37			

Table D1.4: VISSIM Calibration: Laurent- Mackenzie Link

Parameters (1)	Difference (2)	Significance Level (3)	Simulated data			OC transpo data		
			# of simulation runs (4)	Mean (5)	Standard Dev. (6)	# of APC records (7)	Mean (8)	Standard Dev. (9)
BX_ADD=2.0 BX_MULT=3.0	7.6	0.05	72	611.5	28.7	104	568.2 s	50.4
BX_ADD=3.0 BX_MULT=3.5	8.9	0.05	72	618.8	29.1			

Table D1.5: VISSIM Calibration: Mackenzie- Kent Link

Parameters (1)	Difference (2)	Significance Level (3)	Simulated data			OC transpo data		
			# of simulation runs (4)	Mean (5)	Standard Dev. (9) (6)	# of APC records (7)	Mean (8)	Standard Dev. (9)
BX_ADD=2.0 BX_MULT=3.0	10.12	0.05	70	269.1	10.9	74	299.4 s	30.6 s
BX_ADD=3.0 BX_MULT=3.5	15.34	0.05	70	253.5	12.2			

Table D1.6: VISSIM Calibration: Kent- Lebreton Link

Parameters (1)	Difference (2)	Significance Level (3)	Simulated data			OC transpo data		
			# of Simulation runs (4)	Mean (5)	Standard Dev. (6)	# of APC records (7)	Mean (8)	Standard Dev. (9)
BX_ADD=2.0 BX_MULT=3.0	4.1	0.05	69	153.6	16.1	74	160.2	19.2
BX_ADD=3.0 BX_MULT=3.5	2.4	0.05	69	156.2	16.7			

**Table D1.7: Simulated bus travel time data with VISSIM default values,
Random seed =1111**

Table of Travel Times												
File: c:\myottawa\octranspo2.inp												
Comment: OC Transpo bus routes 1 and 95												
Date: wednesday, May 10, 2006 11:17:59 AM												
No.	1	(Billings-Holmewood):	from link	344	at	128.7 m	to link	127	at	620.7 m,		
Distance	2119.0 m											
No.	2	(Holmwood-Gladstone):	from link	127	at	635.7 m	to link	10133	at	4.7 m,		
Distance	1534.7 m											
No.	3	(Gladstone-Rideau):	from link	116	at	11.9 m	to link	417	at	72.2 m,		
Distance	2148.8 m											
No.	5	(Makezine-Kent):	from link	352	at	106.1 m	to link	39	at	56.3 m,		
Distance	1099.6 m											
No.	6	(Kent-Lebreton):	from link	39	at	110.2 m	to link	11	at	142.5 m,		
Distance	1049.0 m											
No.	7	(Laurent-Mackezine):	from link	350	at	208.2 m	to link	20	at	71.6 m,		
Distance	5227.6 m											
Time; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh;												
VehC; Rigid bus;; Rigid bus;; Rigid bus;; Articulated bus;; Articulated bus;;												
Articulated bus;;												
No.;	1;	1;	2;	2;	3;	3;	5;	5;	6;	6;	7;	7;
Name;Billings-Holmewood;Billings-Holmewood;Holmwood-Gladstone;Holmwood-Gladstone;Gladstone-Rideau;Gladstone-Rideau;Makezine-Kent;Makezine-Kent;Kent-Lebreton;Kent-Lebreton;												
Laurent-Mackezine;Laurent-Mackezine;												
900;	0.0;	0;	0.0;	0;	0.0;	0;	254.9;	1;	0.0;	0;	538.8;	2;
1800;	437.1;	1;	0.0;	0;	0.0;	0;	224.0;	5;	147.0;	4;	604.8;	5;
2700;	472.4;	1;	404.2;	1;	0.0;	0;	205.1;	4;	159.6;	5;	640.6;	5;
3600;	469.3;	1;	439.3;	2;	590.4;	2;	220.3;	5;	159.3;	5;	653.2;	5;
4500;	468.8;	2;	344.4;	1;	569.2;	2;	204.2;	5;	199.7;	5;	625.3;	4;
5400;	464.8;	1;	403.6;	2;	737.3;	1;	216.7;	4;	139.3;	3;	658.3;	4;
6300;	476.7;	1;	0.0;	0;	629.6;	1;	201.7;	3;	150.3;	5;	632.4;	4;
7200;	462.9;	1;	364.4;	1;	498.8;	1;	194.4;	4;	152.9;	3;	630.4;	3;
8100;	0.0;	0;	330.3;	1;	0.0;	0;	208.7;	4;	137.8;	4;	628.7;	4;

**Table D1.8: Simulated bus travel time data with VISSIM defaults values,
Random seed =9999**

Table of Travel Times									
File: c:\myottawa\octranspo2.inp									
Comment: OC Transpo bus routes 1 and 95									
Date: Wednesday, May 10, 2006 12:10:47 PM									
No.	1 (Billings-Holmewood):	from link	344	at	128.7 m	to link	127	at	620.7 m,
Distance	2119.0 m								
No.	2 (Holmwood-Gladstone):	from link	127	at	635.7 m	to link	10133	at	4.7 m,
Distance	1534.7 m								
No.	3 (Gladstone-Rideau):	from link	116	at	11.9 m	to link	417	at	72.2 m,
Distance	2148.8 m								
No.	5 (Makezine-Kent):	from link	352	at	106.1 m	to link	39	at	56.3 m,
Distance	1099.6 m								
No.	6 (Kent-Lebreton):	from link	39	at	110.2 m	to link	11	at	142.5 m,
Distance	1049.0 m								
No.	7 (Laurent-Mackezine):	from link	350	at	208.2 m	to link	20	at	71.6 m,
Distance	5227.6 m								
Time; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh;									
VehC; Rigid bus;; Rigid bus;; Rigid bus;; Articulated bus;;									
Articulated bus;; Articulated bus;;									
No.;;	1;	1;	2;	2;	3;	3;	5;	5;	6;
Name;	Billings-Holmewood;	Billings-Holmewood;	Holmwood-Gladstone;	Holmwood-Gladstone;	Gladstone-Rideau;	Gladstone-Rideau;	Makezine-Kent;	Makezine-Kent;	Kent-Lebreton;
Laurent-Mackezine;									
900;	0.0;	0;	0.0;	0;	0.0;	0;	187.9;	1;	0.0;
1800;	0.0;	0;	0.0;	0;	0.0;	0;	225.6;	4;	166.9;
2700;	477.2;	2;	401.3;	1;	0.0;	0;	228.2;	5;	154.6;
3600;	467.9;	2;	369.9;	2;	529.0;	2;	201.4;	6;	152.7;
4500;	565.1;	1;	391.9;	1;	551.0;	1;	195.5;	4;	163.3;
5400;	455.1;	1;	369.2;	2;	625.7;	2;	202.9;	4;	156.2;
6300;	435.2;	1;	0.0;	0;	555.5;	1;	217.9;	3;	174.1;
7200;	453.3;	1;	391.0;	1;	694.9;	1;	201.4;	4;	133.8;
8100;	0.0;	0;	332.3;	1;	579.0;	1;	209.3;	4;	132.4;

**Table D1.9: Simulated bus travel time data with VISSIM Calibrated values,
Random seed =1111**

Table of Travel Times									
File: c:\myottawa\octranspo2.inp									
Comment: OC Transpo bus routes 1 and 95									
Date: Wednesday, May 10, 2006 3:05:00 PM									
No. 1	(Billings-Holmewood):	from link	344	at	128.7 m	to link	127	at	620.7 m,
Distance		2119.0 m							
No. 2	(Holmwood-Gladstone):	from link	127	at	635.7 m	to link	10133	at	4.7 m,
Distance		1534.7 m							
No. 3	(Gladstone-Rideau):	from link	116	at	11.9 m	to link	417	at	72.2 m,
Distance		2148.8 m							
No. 5	(Makezine-Kent):	from link	352	at	106.1 m	to link	39	at	56.3 m,
Distance		1099.6 m							
No. 6	(Kent-Lebreton):	from link	39	at	110.2 m	to link	11	at	142.5 m,
Distance		1049.0 m							
No. 7	(Laurent-Mackezine):	from link	350	at	208.2 m	to link	20	at	71.6 m,
Distance		5227.6 m							
Time; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh;									
VehC; Rigid bus;; Rigid bus;; Rigid bus;; Articulated bus;; Articulated									
bus;; Articulated bus;;									
No.;;	1;	1;	2;	2;	3;	3;	5;	5;	6;
Name;Billings-Holmewood;Billings-Holmewood;Holmwood-Gladstone;Holmwood-									
Gladstone;Gladstone-Rideau;Gladstone-Rideau;Makezine-Kent;Makezine-Kent;Kent-									
Lebreton;Kent-Lebreton;Laurent-Mackezine;Laurent-Mackezine;									
900;	0.0;	0;	0.0;	0;	0.0;	0;	232.4;	1;	0.0;
1800;	434.5;	1;	0.0;	0;	0.0;	0;	221.5;	5;	146.3;
2700;	471.9;	1;	414.9;	1;	0.0;	0;	209.6;	4;	156.6;
3600;	468.2;	2;	440.1;	2;	621.6;	2;	208.4;	6;	153.8;
4500;	460.4;	1;	354.1;	1;	566.4;	2;	209.3;	4;	196.7;
5400;	462.8;	1;	442.3;	2;	667.0;	1;	202.9;	4;	142.6;
6300;	468.2;	1;	0.0;	0;	631.2;	1;	213.8;	3;	139.3;
7200;	464.9;	1;	374.8;	1;	496.7;	1;	188.5;	4;	170.5;
8100;	0.0;	0;	434.3;	1;	0.0;	0;	203.7;	4;	131.9;
								4;	628.7;

**Table D1.10: Simulated Bus Travel Time data with VISSIM Calibrated Values,
Random seed =9999**

Table of Travel Times											
File: c:\myottawa\octranspo2.inp											
Comment: OC Transpo bus routes 1 and 95											
Date: wednesday, May 10, 2006 2:16:14 PM											
No.	1	(Billings-Holmewood):	from link	344	at	128.7 m	to link	127	at	620.7 m,	
Distance	2119.0 m										
No.	2	(Holmwood-Gladstone):	from link	127	at	635.7 m	to link	10133	at	4.7 m,	
Distance	1534.7 m										
No.	3	(Gladstone-Rideau):	from link	116	at	11.9 m	to link	417	at	72.2 m,	
Distance	2148.8 m										
No.	5	(Makezine-Kent):	from link	352	at	106.1 m	to link	39	at	56.3 m,	
Distance	1099.6 m										
No.	6	(Kent-Lebreton):	from link	39	at	110.2 m	to link	11	at	142.5 m,	
Distance	1049.0 m										
No.	7	(Laurent-Mackezine):	from link	350	at	208.2 m	to link	20	at	71.6 m,	
Distance	5227.6 m										
Time; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh; Trav;#Veh;											
VehC; Rigid bus;; Rigid bus;; Rigid bus;; Articulated bus;;											
Articulated bus;; Articulated bus;;											
No.;	1;	1;	2;	2;	3;	3;	5;	5;	6;	6;	7;
Name;Billings-Holmewood;Billings-Holmewood;Holmwood-Gladstone;Holmwood-Gladstone;Gladstone-Rideau;Gladstone-Rideau;Makezine-Kent;Makezine-Kent;Kent-Lebreton;Kent-Lebreton;Laurent-Mackezine;Laurent-Mackezine;											
900;	0.0;	0;	0.0;	0;	0.0;	0;	191.3;	1;	0.0;	0;	542.5;
1800;	0.0;	0;	0.0;	0;	0.0;	0;	225.9;	4;	158.3;	5;	588.5;
2700;	477.3;	2;	367.1;	1;	0.0;	0;	228.2;	5;	144.7;	4;	593.4;
3600;	455.3;	2;	374.9;	2;	530.3;	2;	201.5;	6;	146.4;	5;	628.8;
4500;	564.0;	1;	377.6;	1;	549.9;	1;	194.7;	4;	159.8;	5;	617.7;
5400;	410.2;	1;	391.3;	2;	621.4;	2;	203.1;	4;	156.4;	4;	628.1;
6300;	435.3;	1;	0.0;	0;	554.1;	1;	217.1;	3;	170.2;	3;	652.6;
7200;	452.8;	1;	390.7;	1;	771.4;	1;	201.4;	4;	150.2;	5;	588.2;
8100;	0.0;	0;	323.0;	1;	588.5;	1;	209.3;	4;	130.0;	4;	598.6;

Table D1. 11: Actual Bus travel time: Billings- Holmwood stops

PROCESSED: 2006-05-09 13:11:35			
AUTOMATIC PASSENGER COUNTING SYSTEM			
REVENUE TIME UTILIZATION			
WEEKDAY SERVICE			
BOOKING: APR05			
FROM STOP: RA945 BILLINGS BRIDGE 4C		PERIOD: 2005-04-24 TO 2005-06-25	
TO STOP: CF630 BANK HOLMWOOD FS		DAY TYPE: WEEKDAY	
ROUTE: 1 Bank - - Rockcliffe		TIME: 08:00:00 TO 10:00:00	
	Average Time per Trip Minutes	Std Dev	% of Time
Moving between stops	4.85	0.86	66.73
Stop and go time	0.05	0.26	0.74
Idle time	0.48	0.67	6.56
Dwell time	0.83	0.35	11.40
Excess time	1.06	0.49	14.56
TOTAL	7.26	0.80	100%
Layover time	2.58		
Average sched time per trip	5.32		
Total distance (KM)	2.12		
Average moving speed (KM/HR)	26.24		
Average total speed (KM/HR)	17.51		
Total trips captured:	37		

Table D1. 12: Actual Bus travel time: Holmwood- Gladstone stops

PROCESSED: 2006-05-09 13:11:55			
AUTOMATIC PASSENGER COUNTING SYSTEM REVENUE TIME UTILIZATION WEEKDAY SERVICE			
BOOKING: APR05		PERIOD: 2005-04-24 TO 2005-06-25	
FROM STOP: CF630 BANK HOLMWOOD FS		DAY TYPE: WEEKDAY	
TO STOP: CH080 BANK GLADSTONE NS		TIME: 08:00:00 TO 10:00:00	
ROUTE: 1 Bank - - Rockcliffe			
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	3.90	1.13	53.57
Stop and go time	0.28	0.61	3.79
Idle time	0.04	0.19	0.60
Dwell time	1.27	0.38	17.46
Excess time	1.79	0.53	24.57
	-----	-----	-----
TOTAL	7.29	0.79	100%
Layover time	0.00		
Average sched time per trip	5.00		
Total distance (KM)	1.55		
Average moving speed (KM/HR)	23.83		
Average total speed (KM/HR)	12.77		
Total trips captured:	38		

Table D1. 13: Actual Bus travel time: Gladstone-Rideau

PROCESSED: 2006-05-09 13:12:16			
AUTOMATIC PASSENGER COUNTING SYSTEM REVENUE TIME UTILIZATION WEEKDAY SERVICE			
BOOKING: APR05		PERIOD: 2005-04-24 TO 2005-06-25	
FROM STOP: CH080 BANK GLADSTONE NS		DAY TYPE: WEEKDAY	
TO STOP: CD920 RIDEAU 3A		TIME: 08:00:00 TO 10:00:00	
ROUTE: 1 Bank - - Rockcliffe			
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	6.43	2.05	67.93
Stop and go time	0.58	1.46	6.12
Idle time	0.30	0.54	3.12
Dwell time	1.05	0.32	11.05
Excess time	1.11	0.60	11.78
	-----	-----	-----
TOTAL	9.46	0.93	100%
Layover time	0.00		
Average sched time per trip	13.00		
Total distance (KM)	2.10		
Average moving speed (KM/HR)	19.60		
Average total speed (KM/HR)	13.32		
Total trips captured:	35		

Table D1. 14: Actual Bus travel time: St.Laurent-Mackenzie stops

PROCESSED: 2006-05-09 13:11:11			
AUTOMATIC PASSENGER COUNTING SYSTEM REVENUE TIME UTILIZATION WEEKDAY SERVICE			
BOOKING: APR05			
FROM STOP: EB905 ST LAURENT 2B	PERIOD: 2005-04-24 TO 2005-06-25		
TO STOP: CD910 MACKENZIE KING 2A	DAY TYPE: WEEKDAY		
ROUTE: 95 Transitway	TIME: 08:00:00 TO 10:00:00		
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	8.17	0.75	86.29
Stop and go time	0.03	0.19	0.33
Idle time	0.13	0.32	1.35
Dwell time	0.69	0.34	7.24
Excess time	0.45	0.17	4.79
	-----	---	----
TOTAL	9.47	0.84	100%
Layover time	0.00		
Average sched time per trip	8.69		
Total distance (KM)	5.22		
Average moving speed (KM/HR)	38.32		
Average total speed (KM/HR)	33.07		
Total trips captured:	104		

Table D1. 15: Actual Bus travel time: Mackenzie- Kent stops

PROCESSED: 2006-05-09 13:11:10			
AUTOMATIC PASSENGER COUNTING SYSTEM REVENUE TIME UTILIZATION WEEKDAY SERVICE			
BOOKING: APR05			
FROM STOP: CD910 MACKENZIE KING 2A	PERIOD: 2005-04-24 TO 2005-06-25		
TO STOP: CA920 ALBERT KENT	DAY TYPE: WEEKDAY		
ROUTE: 95 Transitway	TIME: 08:00:00 TO 10:00:00		
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	3.55	0.53	71.09
Stop and go time	0.01	0.06	0.14
Idle time	0.19	0.39	3.86
Dwell time	0.64	0.31	12.75
Excess time	0.61	0.29	12.16
	-----	---	----
TOTAL	4.99	0.51	100%
Layover time	0.00		
Average sched time per trip	4.72		
Total distance (KM)	1.19		
Average moving speed (KM/HR)	20.12		
Average total speed (KM/HR)	14.30		
Total trips captured:	74		

Table D1. 16: Actual Bus travel time: Kent – Lebreton stops

PROCESSED: 2006-05-11 11:40:57			
AUTOMATIC PASSENGER COUNTING SYSTEM			
REVENUE TIME UTILIZATION			
WEEKDAY SERVICE			
BOOKING: APR05			
FROM STOP: CA920 ALBERT KENT			
TO STOP: CJ900 LEBRETON 1A			
ROUTE: 95 Transitway			
PERIOD: 2005-04-24 TO 2005-06-25			
DAY TYPE: WEEKDAY			
TIME: 08:00:00 TO 10:00:00			
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	2.45	0.36	91.56
Stop and go time	0.03	0.19	1.14
Idle time	0.00	0.00	0.00
Dwell time	0.09	0.08	3.37
Excess time	0.10	0.08	3.93
	-----	----	----
TOTAL	2.67	0.32	100%
Layover time	0.00		
Average sched time per trip	5.00		
Total distance (KM)	1.06		
Average moving speed (KM/HR)	26.01		
Average total speed (KM/HR)	23.82		
Total trips captured:	74		

Appendix D2

1. General

This Appendix presents the VISSIM's outputs following simulation and processing. There were 170 tables for the two bus routes. These record the operation information of the buses. Because the data are large¹, only samples are printed out for the sake of illustration.

2. Legend

t= Arrival Time at stop (second)

ToD= Time of Day;

STime =Start Time of the course (trip) at the first stop (second)

Line= Bus line

Stp= Stop Number

StpBd=Number of boarding Passengers

StpAlt= Number of Alighting Passengers

StpDwl= Stop Dwell Time (second)

StpSvcT= Stop Service Time (second)

StpWP= Waiting Passenger

Bus Station Name	Code (Stp)	Line
St Laurent 2A	51	95
Train 2A	16	95
Hurdman	20	95
Lees	21	95
Campus	50	95
Laurier	25	95
Mackenzie King	23	95
Billings Bridge	1	1
Bank-Holmwood	54	1
Bank-Gladstone	3	1
Rideau Center	53	1

¹ It could be about 1200 pages if all of the above tables are printed out.

Table D2.1: Route 95- Situation 1200

	ToD	Stp	Course	Stp	Stp	Stp	Stp	Stp	Stp
26	07:45:26.0	0.5	95	1	51	0	8	27	27
126.5	07:47:06.5	0.5	95	1	16	0	8	15.4080185674615	15.4080185674615
206	07:48:26.0	180.5	95	2	51	10	8	52	52
229	07:48:49.0	0.5	95	1	20	14	9	64.5	64.5
331.5	07:50:31.5	180.5	95	2	16	10	8	16.6408717933666	16.6408717933666
355.5	07:50:55.5	0.5	95	1	21	14	9	20.7682394096475	20.7682394096475
386.5	07:51:26.5	360.5	95	3	51	11	8	54.5	54.5
450.5	07:52:30.5	0.5	95	1	50	14	9	41.1177085856324	41.1177085856324
437.5	07:52:17.5	180.5	95	2	20	12	12	67	67
521.5	07:53:41.5	0.5	95	1	25	14	9	11.8510497799656	11.8510497799656
568.5	07:54:28.5	180.5	95	2	21	12	12	17.6706932648818	17.6706932648818
516	07:53:36.0	360.5	95	3	16	11	8	13.0275490869938	13.0275490869938
566	07:54:26.0	540.5	95	4	51	7	8	44.5	44.5
619.5	07:55:19.5	360.5	95	3	20	10	12	62	62
652	07:55:52.0	0.5	95	1	23	18	18	97	97
661	07:56:01.0	180.5	95	2	50	12	12	20.1698061319809	20.1698061319809
711.5	07:56:51.5	180.5	95	2	25	12	12	9.41338555159671	9.41338555159671
683	07:56:23.0	540.5	95	4	16	7	8	20.6037198994942	20.6037198994942
734.5	07:57:14.5	360.5	95	3	21	0	0	0	0
747	07:57:27.0	720.5	95	5	51	6	8	42	42
792.5	07:58:12.5	540.5	95	4	20	11	11	62	62
809.5	07:58:29.5	360.5	95	3	50	0	0	19.1968338518466	19.1968338518466
820	07:58:40.0	0.5	95	1	42	18	18	29.2409836316985	29.2409836316985
773	07:57:53.0	180.5	95	2	23	4	21	69.5	69.5
865.5	07:59:25.5	720.5	95	5	16	6	8	30.6395278375851	30.6395278375851
883.5	07:59:43.5	0.5	95	1	24	18	18	32.0886676006806	32.0886676006806
859.5	07:59:19.5	360.5	95	3	25	0	0	25.8094241290089	25.8094241290089
918	08:00:18.0	540.5	95	4	21	11	11	23.7811306750596	23.7811306750596
926	08:00:26.0	900.5	95	6	51	4	8	37	37
941	08:00:41.0	180.5	95	2	42	4	21	9.55396359766676	9.55396359766676
953	08:00:53.0	0.5	95	1	7	13	25	102	102
989	08:01:29.0	720.5	95	5	20	18	11	79.5	79.5
993.5	08:01:33.5	180.5	95	2	24	4	21	26.2339346047374	26.2339346047374
1013.5	08:01:53.5	360.5	95	3	23	6	20	72	72
1016.5	08:01:56.5	540.5	95	4	50	11	11	25.8639488739813	25.8639488739813
1038	08:02:18.0	900.5	95	6	16	4	8	19.4601644732678	19.4601644732678
1072	08:02:52.0	540.5	95	4	25	11	11	29.635358426345	29.635358426345
1074	08:02:54.0	180.5	95	2	7	2	17	54.5	54.5
1085.5	08:03:05.5	0.5	95	1	15	13	25	34.943301091224	34.943301091224
1105	08:03:25.0	1080.5	95	7	51	19	8	74.5	74.5

1135	08:03:55.0	720.5	95	5	21	18	11	14.6498056895333	14.6498056895333
1141	08:04:01.0	540.5	95	4	23	6	19	69.5	69.5
1148.5	08:04:08.5	900.5	95	6	20	8	10	52	52
1182.5	08:04:42.5	180.5	95	2	15	2	17	29.3198984205231	29.3198984205231
1180.5	08:04:40.5	360.5	95	3	42	6	20	7.28735577834652	7.28735577834652
1234	08:05:34.0	360.5	95	3	24	6	20	11.268640117897	11.268640117897
1251	08:05:51.0	1080.5	95	7	16	19	8	9.91379574376476	9.91379574376476
1258	08:05:58.0	0.5	95	1	52	13	25	7.94756682398519	7.94756682398519
1228.5	08:05:28.5	720.5	95	5	50	18	11	19.6868938284895	19.6868938284895
1266.5	08:06:06.5	900.5	95	6	21	8	10	11.1554981443269	11.1554981443269
1279.5	08:06:19.5	720.5	95	5	25	18	11	40.5561034458096	40.5561034458096
1286.5	08:06:26.5	1260.5	95	8	51	12	8	57	57
1300.5	08:06:40.5	540.5	95	4	42	6	19	35.219331599476	35.219331599476
1314	08:06:54.0	360.5	95	3	7	2	18	57	57
1319	08:06:59.0	180.5	95	2	52	2	17	33.9811784389407	33.9811784389407
1349.5	08:07:29.5	1080.5	95	7	20	6	15	59.5	59.5
1354	08:07:34.0	900.5	95	6	50	8	10	25.3888255677701	25.3888255677701
1369	08:07:49.0	540.5	95	4	24	6	19	5.49294724121229	5.49294724121229
1375.5	08:07:55.5	720.5	95	5	23	5	22	74.5	74.5
1410.5	08:08:30.5	900.5	95	6	25	8	10	21.915413922672	21.915413922672
1417	08:08:37.0	1260.5	95	8	16	12	8	19.9467728345976	19.9467728345976
1422.5	08:08:42.5	360.5	95	3	15	2	18	13.9579757504817	13.9579757504817
1433.5	08:08:53.5	540.5	95	4	7	3	18	59.5	59.5
1472	08:09:32.0	1080.5	95	7	21	6	15	22.2812288908245	22.2812288908245
1493.5	08:09:53.5	900.5	95	6	23	5	17	62	62
1467	08:09:27.0	1440.5	95	9	51	8	8	47	47
1525.5	08:10:25.5	1260.5	95	8	20	5	13	52	52
1542	08:10:42.0	720.5	95	5	42	5	22	27.7101882253563	27.7101882253563
1542	08:10:42.0	540.5	95	4	15	3	18	14.9754120586199	14.9754120586199
1559	08:10:59.0	360.5	95	3	52	2	18	18.2594138743781	18.2594138743781
1591.5	08:11:31.5	1440.5	95	9	16	8	8	21.7979516728509	21.7979516728509
1566.5	08:11:06.5	1080.5	95	7	50	6	15	28.223327691404	28.223327691404
1605.5	08:11:45.5	720.5	95	5	24	5	22	2.03845422638412	2.03845422638412
1624	08:12:04.0	1080.5	95	7	25	6	15	29.4021079828607	29.4021079828607
1641	08:12:21.0	1260.5	95	8	21	5	13	11.4381248583569	11.4381248583569
1646	08:12:26.0	1620.5	95	10	51	16	8	67	67
1661	08:12:41.0	900.5	95	6	42	5	17	21.4709414145211	21.4709414145211
1674	08:12:54.0	720.5	95	5	7	2	19	59.5	59.5
1678.5	08:12:58.5	540.5	95	4	52	3	18	24.2338642213828	24.2338642213828
1716.5	08:13:36.5	900.5	95	6	24	5	17	28.1892212176801	28.1892212176801
1707.5	08:13:27.5	1440.5	95	9	20	10	12	62	62
1727	08:13:47.0	1260.5	95	8	50	5	13	11.8356045866382	11.8356045866382

1731.5	08:13:51.5	1080.5	95	7	23	7	21	77	77
1768.5	08:14:28.5	1260.5	95	8	25	5	13	52.8966901428382	52.8966901428382
1783.5	08:14:43.5	720.5	95	5	15	2	19	25.2340207650103	25.2340207650103
1785.5	08:14:45.5	1620.5	95	10	16	16	8	22.960157967385	22.960157967385
1826	08:15:26.0	1800.5	95	11	51	15	8	64.5	64.5
1794.5	08:14:54.5	900.5	95	6	7	0	15	44.5	44.5
1837	08:15:37.0	1440.5	95	9	21	10	12	52.824036320219	52.824036320219
1861	08:16:01.0	1260.5	95	8	23	4	18	62	62
1897	08:16:37.0	1620.5	95	10	20	8	14	62	62
1901.5	08:16:41.5	1080.5	95	7	42	7	21	13.7412054252605	13.7412054252605
1906	08:16:46.0	900.5	95	6	15	0	15	24.8785079302955	24.8785079302955
1920.5	08:17:00.5	720.5	95	5	52	2	19	0	0
1952.5	08:17:32.5	1080.5	95	7	24	7	21	29.4062008271227	29.4062008271227
1965	08:17:45.0	1800.5	95	11	16	15	8	3.4983609929123	3.4983609929123
1969	08:17:49.0	1440.5	95	9	50	10	12	1.6792013983905	1.6792013983905
2003	08:18:23.0	1440.5	95	9	25	10	12	20.219276874692	20.219276874692
2019	08:18:39.0	1260.5	95	8	42	4	18	25.3860101450702	25.3860101450702
2022	08:18:42.0	1620.5	95	10	21	8	14	17.1548377015565	17.1548377015565
2006.5	08:18:26.5	1980.5	95	12	51	24	8	87	87
2034.5	08:18:54.5	1080.5	95	7	7	7	19	72	72
2039.5	08:18:59.5	900.5	95	6	52	0	15	19.2632181295951	19.2632181295951
2077.5	08:19:37.5	1260.5	95	8	24	4	18	21.8194859816282	21.8194859816282
2059	08:19:19.0	1800.5	95	11	20	8	14	62	62
2095.5	08:19:55.5	1440.5	95	9	23	14	19	89.5	89.5
2113	08:20:13.0	1620.5	95	10	50	8	14	20.908244199681	20.908244199681
2138.5	08:20:38.5	1080.5	95	7	15	7	19	14.7975736539457	14.7975736539457
2153.5	08:20:53.5	1260.5	95	8	7	3	15	52	52
2163	08:21:03.0	1620.5	95	10	25	8	14	35.3233184129663	35.3233184129663
2167.5	08:21:07.5	1980.5	95	12	16	24	8	20.5656558967728	20.5656558967728
2185	08:21:25.0	2160.5	95	13	51	11	8	54.5	54.5
2186	08:21:26.0	1800.5	95	11	21	8	14	17.2122399097436	17.2122399097436
2235.5	08:22:15.5	1080.5	95	7	52	7	19	34.7580535812084	34.7580535812084
2263.5	08:22:43.5	1440.5	95	9	42	14	19	19.1026094806307	19.1026094806307
2265	08:22:45.0	1260.5	95	8	15	3	15	27.0040373374394	27.0040373374394
2278	08:22:58.0	1980.5	95	12	20	9	16	69.5	69.5
2279.5	08:22:59.5	1800.5	95	11	50	8	14	26.5560844710106	26.5560844710106
2319	08:23:39.0	1440.5	95	9	24	14	19	22.707752223182	22.707752223182
2312	08:23:32.0	2160.5	95	13	16	11	8	25.1572047241585	25.1572047241585
2332	08:23:52.0	1620.5	95	10	23	11	21	87	87
2337	08:23:57.0	1800.5	95	11	25	8	14	18.3323028581116	18.3323028581116
2366.5	08:24:26.5	2340.5	95	14	51	22	8	82	82
2395	08:24:55.0	1440.5	95	9	7	4	23	74.5	74.5
2398.5	08:24:58.5	1260.5	95	8	52	3	15	11.7219697401678	11.7219697401678

2425	08:25:25.0	2160.5	95	13	20	7	12	54.5	54.5
2412	08:25:12.0	1980.5	95	12	21	9	16	24.2663476350826	24.2663476350826
2453.5	08:25:53.5	1800.5	95	11	23	8	20	77	77
2502.5	08:26:42.5	1440.5	95	9	15	4	23	35.7978008883888	35.7978008883888
2512.5	08:26:52.5	1980.5	95	12	50	9	16	34.9375145429651	34.9375145429651
2499.5	08:26:39.5	1620.5	95	10	42	11	21	6.28853073269999	6.28853073269999
2521	08:27:01.0	2340.5	95	14	16	22	8	20.2395481154767	20.2395481154767
2545.5	08:27:25.5	2520.5	95	15	51	21	8	79.5	79.5
2542.5	08:27:22.5	2160.5	95	13	21	7	12	27.474141306356	27.474141306356
2552.5	08:27:32.5	1620.5	95	10	24	11	21	20.4017029958996	20.4017029958996
2577.5	08:27:57.5	1980.5	95	12	25	9	16	23.2010066579892	23.2010066579892
2621	08:28:41.0	1800.5	95	11	42	8	20	16.2928305636922	16.2928305636922
2628	08:28:48.0	2340.5	95	14	20	12	16	77	77
2634	08:28:54.0	1620.5	95	10	7	5	22	74.5	74.5
2642	08:29:02.0	2160.5	95	13	50	7	12	4.9348794933683	4.9348794933683
2674	08:29:34.0	1800.5	95	11	24	8	20	22.9099954820845	22.9099954820845
2676	08:29:36.0	2160.5	95	13	25	7	12	23.468804485881	23.468804485881
2693	08:29:53.0	1980.5	95	12	23	16	24	107	107
2698	08:29:58.0	2520.5	95	15	16	21	8	22.6049955504088	22.6049955504088
2700.5	08:30:00.5	1440.5	95	9	52	4	23	0	0
2726.5	08:30:26.5	2700.5	95	16	51	18	8	72	72
2740	08:30:40.0	1620.5	95	10	15	5	22	21.1668147437831	21.1668147437831
2767	08:31:07.0	2340.5	95	14	21	12	16	36.6406023740624	36.6406023740624
2754	08:30:54.0	1800.5	95	11	7	2	20	62	62
2812.5	08:31:52.5	2160.5	95	13	23	1	19	57	57
2809	08:31:49.0	2520.5	95	15	20	13	15	77	77
2873	08:32:53.0	2700.5	95	16	16	18	8	25.8976918188092	25.8976918188092
2878	08:32:58.0	1620.5	95	10	52	5	22	32.1622888171239	32.1622888171239
2878	08:32:58.0	2340.5	95	14	50	12	16	5.67213631849137	5.67213631849137
2862.5	08:32:42.5	1980.5	95	12	42	16	24	9.55724702847379	9.55724702847379
2867.5	08:32:47.5	1800.5	95	11	15	2	20	6.78266611184534	6.78266611184534
2906.5	08:33:26.5	2880.5	95	17	51	21	8	79.5	79.5
2913	08:33:33.0	2340.5	95	14	25	12	16	25.7887515919864	25.7887515919864
2913.5	08:33:33.5	1980.5	95	12	24	16	24	7.00857075452838	7.00857075452838
2949	08:34:09.0	2520.5	95	15	21	13	15	22.5148356183951	22.5148356183951
2957.5	08:34:17.5	1800.5	95	11	52	2	20	17.7503283888678	17.7503283888678
2980.5	08:34:40.5	2160.5	95	13	42	1	19	39.1189068557031	39.1189068557031
2989.5	08:34:49.5	2700.5	95	16	20	13	15	77	77
2993.5	08:34:53.5	1980.5	95	12	7	3	28	84.5	84.5
3045.5	08:35:45.5	2520.5	95	15	50	13	15	26.1703266971289	26.1703266971289
3060.5	08:36:00.5	2880.5	95	17	16	21	8	12.9772107090858	12.9772107090858
3052.5	08:35:52.5	2340.5	95	14	23	7	25	87	87

3086	08:36:26.0	3060.5	95	18	51	28	8	97	97
3101.5	08:36:41.5	2520.5	95	15	25	13	15	24.9566980658624	24.9566980658624
3092.5	08:36:32.5	2160.5	95	13	24	1	19	19.5688205404824	19.5688205404824
3119	08:36:59.0	1980.5	95	12	15	3	28	19.2069067504596	19.2069067504596
3131.5	08:37:11.5	2700.5	95	16	21	13	15	28.5398107125153	28.5398107125153
3163.5	08:37:43.5	2880.5	95	17	20	9	15	67	67
3172.5	08:37:52.5	2520.5	95	15	23	3	25	77	77
3173.5	08:37:53.5	2160.5	95	13	7	1	14	44.5	44.5
3218.5	08:38:38.5	2340.5	95	14	42	7	25	43.3106773439273	43.3106773439273
3239	08:38:59.0	1980.5	95	12	52	3	28	7.28084833846673	7.28084833846673
3236.5	08:38:56.5	2700.5	95	16	50	13	15	33.0503709327271	33.0503709327271
3267	08:39:27.0	3240.5	95	19	51	19	9	77	77
3256.5	08:39:16.5	3060.5	95	18	16	28	8	11.7016795737678	11.7016795737678
3283.5	08:39:43.5	2160.5	95	13	15	1	14	38.4944297014503	38.4944297014503
3294.5	08:39:54.5	2880.5	95	17	21	9	15	45.2037699409313	45.2037699409313
3300.5	08:40:00.5	2700.5	95	16	25	13	15	26.6685695871849	26.6685695871849
3339	08:40:39.0	2520.5	95	15	42	3	25	0.143383196888053	0.143383196888053
3332.5	08:40:32.5	2340.5	95	14	24	7	25	18.2476665302477	18.2476665302477
3357	08:40:57.0	3060.5	95	18	20	5	18	64.5	64.5
3393	08:41:33.0	2520.5	95	15	24	3	25	17.5281466034446	17.5281466034446
3412.5	08:41:52.5	2340.5	95	14	7	4	22	72	72
3413.5	08:41:53.5	2700.5	95	16	23	7	24	84.5	84.5
3416	08:41:56.0	2880.5	95	17	50	9	15	0	0
3420	08:42:00.0	3240.5	95	19	16	19	9	35.5914647586737	35.5914647586737
3447	08:42:27.0	2880.5	95	17	25	9	15	35.7159251832657	35.7159251832657
3436	08:42:16.0	2160.5	95	13	52	1	14	42.2990516638717	42.2990516638717
3472	08:42:52.0	2520.5	95	15	7	0	20	57	57
3485	08:43:05.0	3060.5	95	18	21	5	18	55.1546256119374	55.1546256119374
3505.5	08:43:25.5	3480.5	95	20	51	29	8	99.5	99.5
3525	08:43:45.0	2340.5	95	14	15	4	22	49.2666795560478	49.2666795560478
3533.5	08:43:53.5	2880.5	95	17	23	0	23	64.5	64.5
3547	08:44:07.0	3240.5	95	19	20	11	16	74.5	74.5
3581	08:44:41.0	2700.5	95	16	42	7	24	28.303029537133	28.303029537133
3589	08:44:49.0	2520.5	95	15	15	0	20	14.7797399089687	14.7797399089687
3615	08:45:15.0	3060.5	95	18	50	5	18	24.6661945810962	24.6661945810962
3643	08:45:43.0	2700.5	95	16	24	7	24	31.0556529278476	31.0556529278476
3670	08:46:10.0	3060.5	95	18	25	5	18	10.8625354505967	10.8625354505967
3676.5	08:46:16.5	3480.5	95	20	16	29	8	12.1156619534737	12.1156619534737
3687	08:46:27.0	3240.5	95	19	21	11	16	20.293115927708	20.293115927708
3701.5	08:46:41.5	2880.5	95	17	42	0	23	22.8139938790113	22.8139938790113
3679.5	08:46:19.5	2340.5	95	14	52	4	22	27.6033627495755	27.6033627495755

3713	08:46:53.0	2700.5	95	16	7	1	21	62	62
3719	08:46:59.0	2520.5	95	15	52	0	20	15.1548953894126	15.1548953894126
3746	08:47:26.0	3720.5	95	21	51	21	8	79.5	79.5
3772.5	08:47:52.5	3060.5	95	18	23	8	23	84.5	84.5
3775.5	08:47:55.5	3480.5	95	20	20	14	19	89.5	89.5
3758.5	08:47:38.5	2880.5	95	17	24	0	23	15.6802729934569	15.6802729934569
3784.5	08:48:04.5	3240.5	95	19	50	11	16	29.2068057900778	29.2068057900778
3822.5	08:48:42.5	2700.5	95	16	15	1	21	21.189326216139	21.189326216139
3845	08:49:05.0	3240.5	95	19	25	11	16	0	0
3834	08:48:54.0	2880.5	95	17	7	2	16	52	52
3894.5	08:49:54.5	3240.5	95	19	23	3	25	77	77
3899.5	08:49:59.5	3720.5	95	21	16	21	8	37.0796820626057	37.0796820626057
3926	08:50:26.0	3480.5	95	20	21	14	19	12.2899039012735	12.2899039012735
3939.5	08:50:39.5	3060.5	95	18	42	8	23	34.7271146053383	34.7271146053383
3945.5	08:50:45.5	2880.5	95	17	15	2	16	26.6992100535395	26.6992100535395
3959	08:50:59.0	2700.5	95	16	52	1	21	31.5222598313149	31.5222598313149
4010.5	08:51:50.5	3480.5	95	20	50	14	19	17.5260660874827	17.5260660874827
3986	08:51:26.0	3960.5	95	22	51	17	8	69.5	69.5
4008	08:51:48.0	3060.5	95	18	24	8	23	32.2580614143162	32.2580614143162
4026.5	08:52:06.5	3720.5	95	21	20	15	16	84.5	84.5
4057	08:52:37.0	3480.5	95	20	25	14	19	20.1878627712418	20.1878627712418
4079	08:52:59.0	2880.5	95	17	52	2	16	13.5816157059634	13.5816157059634
4062	08:52:42.0	3240.5	95	19	42	3	25	26.625886904377	26.625886904377
4072	08:52:52.0	3060.5	95	18	7	1	22	64.5	64.5
4131	08:53:51.0	3480.5	95	20	23	5	29	92	92
4123.5	08:53:43.5	3240.5	95	19	24	3	25	25.9107241235012	25.9107241235012
4128	08:53:48.0	3960.5	95	22	16	17	8	23.8041358846617	23.8041358846617
4175	08:54:35.0	3720.5	95	21	21	15	16	21.6590086922245	21.6590086922245
4185.5	08:54:45.5	3060.5	95	18	15	1	22	53.3741020803076	53.3741020803076
4194.5	08:54:54.5	3240.5	95	19	7	1	19	57	57
4226.5	08:55:26.5	4200.5	95	23	51	12	8	57	57
4272	08:56:12.0	3720.5	95	21	50	15	16	9.47203578202862	9.47203578202862
4241	08:55:41.0	3960.5	95	22	20	16	15	84.5	84.5
4298.5	08:56:38.5	3480.5	95	20	42	5	29	16.5083276733241	16.5083276733241
4306	08:56:46.0	3240.5	95	19	15	1	19	19.2484204738052	19.2484204738052
4311.5	08:56:51.5	3720.5	95	21	25	15	16	25.0472432560491	25.0472432560491
4340	08:57:20.0	3060.5	95	18	52	1	22	36.2744676634914	36.2744676634914
4352	08:57:32.0	3480.5	95	20	24	5	29	3.74275448956048	3.74275448956048
4361	08:57:41.0	4200.5	95	23	16	12	8	23.6964912551724	23.6964912551724
4375.5	08:57:55.5	3720.5	95	21	23	11	27	102	102
4389.5	08:58:09.5	3960.5	95	22	21	16	15	12.9703745153732	12.9703745153732
4433.5	08:58:53.5	3480.5	95	20	7	4	24	77	77

4439.5	08:58:59.5	3240.5	95	19	52	1	19	26.1875991525283	26.1875991525283
4477	08:59:37.0	3960.5	95	22	50	16	15	36.3476847942022	36.3476847942022
4465	08:59:25.0	4440.5	95	24	51	21	8	79.5	79.5
4479	08:59:39.0	4200.5	95	23	20	13	14	74.5	74.5
4539	09:00:39.0	3720.5	95	21	42	11	27	30.372170229074	30.372170229074
4543	09:00:43.0	3960.5	95	22	25	16	15	14.0515556003056	14.0515556003056
4544.5	09:00:44.5	3480.5	95	20	15	4	24	32.0448772867643	32.0448772867643
4603	09:01:43.0	3720.5	95	21	24	11	27	31.1350461911719	31.1350461911719
4613	09:01:53.0	3960.5	95	22	23	3	26	79.5	79.5
4620.5	09:02:00.5	4440.5	95	24	16	21	8	15.7389234860793	15.7389234860793
4621.5	09:02:01.5	4200.5	95	23	21	13	14	12.2836130248426	12.2836130248426
4673.5	09:02:53.5	3720.5	95	21	7	1	27	77	77
4677.5	09:02:57.5	3480.5	95	20	52	4	24	39.279605069313	39.279605069313
4706	09:03:26.0	4680.5	95	25	51	20	8	77	77
4712	09:03:32.0	4200.5	95	23	50	13	14	4.15574188593224	4.15574188593224
4730.5	09:03:50.5	4440.5	95	24	20	12	16	77	77
4748.5	09:04:08.5	4200.5	95	23	25	13	14	5.06061638903156	5.06061638903156
4785	09:04:45.0	3720.5	95	21	15	1	27	17.0526883141438	17.0526883141438
4781	09:04:41.0	3960.5	95	22	42	3	26	8.0156513865455	8.0156513865455
4833.5	09:05:33.5	3960.5	95	22	24	3	26	17.1927505673281	17.1927505673281
4855.5	09:05:55.5	4200.5	95	23	23	7	23	82	82
4856.5	09:05:56.5	4680.5	95	25	16	20	8	5.29336716021017	5.29336716021017
4876	09:06:16.0	4440.5	95	24	21	12	16	16.2282314832981	16.2282314832981
4919	09:06:59.0	3720.5	95	21	52	1	27	22.3055422753444	22.3055422753444
4914	09:06:54.0	3960.5	95	22	7	2	21	64.5	64.5
4951	09:07:31.0	4680.5	95	25	20	11	16	74.5	74.5
4968.5	09:07:48.5	4440.5	95	24	50	12	16	0	0
4947.5	09:07:27.5	4920.5	95	26	51	16	8	67	67
5001.5	09:08:21.5	4440.5	95	24	25	12	16	20.9663978509401	20.9663978509401
5021.5	09:08:41.5	4200.5	95	23	42	7	23	0	0
5030	09:08:50.0	3960.5	95	22	15	2	21	12.3463063881264	12.3463063881264
5076	09:09:36.0	4200.5	95	23	24	7	23	12.6588642997845	12.6588642997845
5089	09:09:49.0	4680.5	95	25	21	11	16	29.6445976483914	29.6445976483914
5090.5	09:09:50.5	4920.5	95	26	16	16	8	20.4897853294716	20.4897853294716
5094.5	09:09:54.5	4440.5	95	24	23	5	26	84.5	84.5
5158.5	09:10:58.5	3960.5	95	22	52	2	21	3.77946960685088	3.77946960685088
5155	09:10:55.0	4200.5	95	23	7	1	21	62	62
5185	09:11:25.0	5160.5	95	27	51	20	8	77	77
5193.5	09:11:33.5	4680.5	95	25	50	11	16	19.6499393782459	19.6499393782459
5202.5	09:11:42.5	4920.5	95	26	20	15	15	82	82
5244	09:12:24.0	4680.5	95	25	25	11	16	16.5429576075664	16.5429576075664
5264.5	09:12:44.5	4440.5	95	24	42	5	26	20.9501583037169	20.9501583037169

5270	09:12:50.0	4200.5	95	23	15	1	21	25.3010242162322	25.3010242162322
5320.5	09:13:40.5	4440.5	95	24	24	5	26	24.0670833095815	24.0670833095815
5333	09:13:53.0	4680.5	95	25	23	5	25	82	82
5335	09:13:55.0	5160.5	95	27	16	20	8	8.99609205785535	8.99609205785535
5349.5	09:14:09.5	4920.5	95	26	21	15	15	32.580970474208	32.580970474208
5395.5	09:14:55.5	4440.5	95	24	7	1	21	62	62
5400.5	09:15:00.5	4200.5	95	23	52	1	21	13.61664373854	13.61664373854
5433	09:15:33.0	5160.5	95	27	20	14	16	82	82
5426.5	09:15:26.5	5400.5	95	28	51	23	8	84.5	84.5
5460.5	09:16:00.5	4920.5	95	26	50	15	15	8.74029392445983	8.74029392445983
5500.5	09:16:40.5	4680.5	95	25	42	5	25	9.18784937144202	9.18784937144202
5500.5	09:16:40.5	4920.5	95	26	25	15	15	25.3067173453278	25.3067173453278
5510	09:16:50.0	4440.5	95	24	15	1	21	19.488750675755	19.488750675755
5588	09:18:08.0	5400.5	95	28	16	23	8	0.10553875567286	0.10553875567286
5554	09:17:34.0	4680.5	95	25	24	5	25	11.3271640958308	11.3271640958308
5574.5	09:17:54.5	4920.5	95	26	23	5	25	82	82
5578.5	09:17:58.5	5160.5	95	27	21	14	16	14.5694630291844	14.5694630291844
5634	09:18:54.0	4680.5	95	25	7	1	21	62	62
5641	09:19:01.0	4440.5	95	24	52	1	21	28.9065577999884	28.9065577999884
5666	09:19:26.0	5640.5	95	29	51	19	8	74.5	74.5
5666.5	09:19:26.5	5160.5	95	27	50	14	16	0	0
5682.5	09:19:42.5	5400.5	95	28	20	17	17	92	92
5741	09:20:41.0	4920.5	95	26	42	5	25	12.8237554008622	12.8237554008622
5697	09:19:57.0	5160.5	95	27	25	14	16	6.39417429101949	6.39417429101949
5745.5	09:20:45.5	4680.5	95	25	15	1	21	14.9165469343161	14.9165469343161
5795.5	09:21:35.5	4920.5	95	26	24	5	25	22.9048427862369	22.9048427862369
5804.5	09:21:44.5	5640.5	95	29	16	0	0	0	0
5812	09:21:52.0	5160.5	95	27	23	8	26	92	92
5843	09:22:23.0	5400.5	95	28	21	17	17	3.2860347695154	3.2860347695154
5879	09:22:59.0	4680.5	95	25	52	1	21	37.8215949947311	37.8215949947311
5874	09:22:54.0	4920.5	95	26	7	0	21	59.5	59.5
5891.5	09:23:11.5	5640.5	95	29	20	8	16	67	67
5906	09:23:26.0	5880.5	95	30	51	12	8	57	57
5924.5	09:23:44.5	5400.5	95	28	50	17	17	18.4728386632098	18.4728386632098
5985	09:24:45.0	4920.5	95	26	15	0	21	23.7798581361778	23.7798581361778
5975	09:24:35.0	5400.5	95	28	25	17	17	26.1032905974223	26.1032905974223
5980.5	09:24:40.5	5160.5	95	27	42	8	26	3.44662979712462	3.44662979712462
6022.5	09:25:22.5	5640.5	95	29	21	8	16	4.42756581463529	4.42756581463529
6034	09:25:34.0	5160.5	95	27	24	8	26	27.1119473786949	27.1119473786949
6035.5	09:25:35.5	5880.5	95	30	16	12	8	13.9237116884215	13.9237116884215
6055.5	09:25:55.5	5400.5	95	28	23	7	29	97	97
6102	09:26:42.0	5640.5	95	29	50	8	16	23.9712883692326	23.9712883692326
6114	09:26:54.0	5160.5	95	27	7	3	24	74.5	74.5

6120	09:27:00.0	4920.5	95	26	52	0	21	19.6718139861789	19.6718139861789
6138	09:27:18.0	5880.5	95	30	20	7	14	59.5	59.5
6156	09:27:36.0	5640.5	95	29	25	8	16	38.0878146071646	38.0878146071646
6146.5	09:27:26.5	6120.5	95	31	51	5	8	39.5	39.5
6219.5	09:28:39.5	5160.5	95	27	15	3	24	6.66599889247703	6.66599889247703
6222	09:28:42.0	5400.5	95	28	42	7	29	10.7026956547843	10.7026956547843
6250	09:29:10.0	5880.5	95	30	21	0	0	0	0
6276	09:29:36.0	5400.5	95	28	24	7	29	13.4475356103968	13.4475356103968
6264	09:29:24.0	6120.5	95	31	16	5	8	18.2622089357901	18.2622089357901
6293.5	09:29:53.5	5640.5	95	29	23	8	23	84.5	84.5
6326	09:30:26.0	5160.5	95	27	52	3	24	31.940801226161	31.940801226161
6323.5	09:30:23.5	5880.5	95	30	50	0	0	13.2403125421837	13.2403125421837
6355	09:30:55.0	5400.5	95	28	7	3	25	77	77
6366.5	09:31:06.5	5880.5	95	30	25	0	0	4.59490935936899	4.59490935936899
6378.5	09:31:18.5	6120.5	95	31	20	10	12	62	62
6387.5	09:31:27.5	6360.5	95	32	51	8	8	47	47
6412.5	09:31:52.5	5880.5	95	30	23	3	20	64.5	64.5
6465	09:32:45.0	5400.5	95	28	15	3	25	31.1792059609331	31.1792059609331
6459.5	09:32:39.5	5640.5	95	29	42	8	23	10.5236023096192	10.5236023096192
6510.5	09:33:30.5	6120.5	95	31	21	10	12	17.7938943048758	17.7938943048758
6512.5	09:33:32.5	6360.5	95	32	16	8	8	16.9861651926325	16.9861651926325
6513.5	09:33:33.5	5640.5	95	29	24	8	23	3.92747389807214	3.92747389807214
6579	09:34:39.0	5880.5	95	30	42	3	20	15.4358496228558	15.4358496228558
6601	09:35:01.0	5400.5	95	28	52	3	25	21.8043619229966	21.8043619229966
6607.5	09:35:07.5	6120.5	95	31	50	10	12	33.125921396223	33.125921396223
6621.5	09:35:21.5	6360.5	95	32	20	9	12	59.5	59.5
6593.5	09:34:53.5	5640.5	95	29	7	4	21	69.5	69.5
6626	09:35:26.0	6600.5	95	33	51	9	8	49.5	49.5
6633	09:35:33.0	5880.5	95	30	24	3	20	23.2027227829131	23.2027227829131
6673.5	09:36:13.5	6120.5	95	31	25	10	12	0.142594699579	0.142594699579
6705	09:36:45.0	5640.5	95	29	15	4	21	23.5039948765551	23.5039948765551
6713.5	09:36:53.5	5880.5	95	30	7	0	16	47	47
6750.5	09:37:30.5	6600.5	95	33	16	9	8	9.2634927830178	9.2634927830178
6776.5	09:37:56.5	6120.5	95	31	23	16	19	94.5	94.5
6747.5	09:37:27.5	6360.5	95	32	21	9	12	18.2914109014894	18.2914109014894
6822	09:38:42.0	5880.5	95	30	15	0	16	28.8570505891686	28.8570505891686
6839	09:38:59.0	5640.5	95	29	52	4	21	32.1118785517802	32.1118785517802
6844.5	09:39:04.5	6360.5	95	32	50	9	12	40.5978623003054	40.5978623003054
6865.5	09:39:25.5	6840.5	95	34	51	5	8	39.5	39.5
6851	09:39:11.0	6600.5	95	33	20	9	13	62	62
6917.5	09:40:17.5	6360.5	95	32	25	9	12	25.1264076828181	25.1264076828181
6958.5	09:40:58.5	5880.5	95	30	52	0	16	40.5808391462845	40.5808391462845
6947	09:40:47.0	6120.5	95	31	42	16	19	9.528538795536	9.528538795536

6976.5	09:41:16.5	6840.5	95	34	16	5	8	36.5970839685049	36.5970839685049
6996	09:41:36.0	6120.5	95	31	24	16	19	0	0
7015.5	09:41:55.5	6360.5	95	32	23	4	20	67	67
6978.5	09:41:18.5	6600.5	95	33	21	9	13	33.2707220892744	33.2707220892744
7075.5	09:42:55.5	6120.5	95	31	7	5	24	79.5	79.5
7087	09:43:07.0	6600.5	95	33	50	9	13	15.3258228142802	15.3258228142802
7100	09:43:20.0	6840.5	95	34	20	11	12	64.5	64.5
7106	09:43:26.0	7080.5	95	35	51	7	8	44.5	44.5
7133	09:43:53.0	6600.5	95	33	25	9	13	35.6970750506626	35.6970750506626
7182	09:44:42.0	6360.5	95	32	42	4	20	28.2816946303335	28.2816946303335
7195	09:44:55.0	6120.5	95	31	15	5	24	4.44997661544161	4.44997661544161
7213	09:45:13.0	7080.5	95	35	16	0	0	0	0
7226	09:45:26.0	6840.5	95	34	21	11	12	20.0434564677048	20.0434564677048
7246	09:45:46.0	6360.5	95	32	24	4	20	19.5910203146401	19.5910203146401
7253.5	09:45:53.5	6600.5	95	33	23	9	20	79.5	79.5
7300	09:46:40.0	7080.5	95	35	20	10	12	62	62
7314	09:46:54.0	6360.5	95	32	7	2	16	52	52
7321.5	09:47:01.5	6120.5	95	31	52	5	24	19.0624231850498	19.0624231850498
7351	09:47:31.0	6840.5	95	34	25	11	12	17.5248523435433	17.5248523435433
7318	09:46:58.0	6840.5	95	34	50	11	12	3.58440917993988	3.58440917993988
7421.5	09:48:41.5	6600.5	95	33	42	9	20	2.43932507561824	2.43932507561824
7425	09:48:45.0	7080.5	95	35	21	10	12	3.00675717633991	3.00675717633991
7425.5	09:48:45.5	6360.5	95	32	15	2	16	35.1387273006741	35.1387273006741
7474.5	09:49:34.5	6600.5	95	33	24	9	20	32.9090432907553	32.9090432907553
7492	09:49:52.0	6840.5	95	34	23	6	19	69.5	69.5
7502.5	09:50:02.5	7080.5	95	35	50	10	12	19.7813045612462	19.7813045612462
7552	09:50:52.0	7080.5	95	35	25	10	12	8.84483974387476	8.84483974387476
7554.5	09:50:54.5	6600.5	95	33	7	1	20	59.5	59.5
7612.5	09:51:52.5	7080.5	95	35	23	6	20	72	72
7620.5	09:52:00.5	6360.5	95	32	52	2	16	13.1094724464612	13.1094724464612
7660.5	09:52:40.5	6840.5	95	34	42	6	19	16.0723240200001	16.0723240200001
7663.5	09:52:43.5	6600.5	95	33	15	1	20	12.1277774792325	12.1277774792325
7712.5	09:53:32.5	6840.5	95	34	24	6	19	34.7761975694417	34.7761975694417
7793.5	09:54:53.5	6840.5	95	34	7	5	18	64.5	64.5
7779	09:54:39.0	7080.5	95	35	42	6	20	36.944007232616	36.944007232616
7799.5	09:54:59.5	6600.5	95	33	52	1	20	7.50651086312656	7.50651086312656
7848.5	09:55:48.5	7080.5	95	35	24	6	20	43.3738742069109	43.3738742069109
7973.5	09:57:53.5	7080.5	95	35	7	0	18	52	52

Table D2.2: Route1 -Situation 1200

		Stm				StpBd	StpAl		StpS	StpWP
1335	08:07:15.0	1320.5	1	1	1	32	1	87	87	32
1466	08:09:26.0	1320.5	1	1	39	32	1	5.4576016	5.4576016	32
1512.5	08:10:12.5	1320.5	1	1	38	32	1	29.279263	29.279263	32
1583	08:11:23.0	1320.5	1	1	37	32	1	30.155894	30.155894	32
1641.5	08:12:21.5	1320.5	1	1	36	32	1	27.893874	27.893874	32
1706.5	08:13:26.5	1320.5	1	1	35	32	1	22.669224	22.669224	32
1777.5	08:14:37.5	1320.5	1	1	34	32	1	41.966357	41.966357	32
1837	08:15:37.0	1320.5	1	1	33	32	1	26.49125	26.49125	32
1935.5	08:17:15.5	1920.5	1	2	1	17	5	49.5	49.5	17
1912	08:16:52.0	1320.5	1	1	32	32	1	21.308117	21.308117	32
1959.5	08:17:39.5	1320.5	1	1	54	3	1	14.5	14.5	12
2027	08:18:47.0	1320.5	1	1	43	3	1	8.0508578	8.0508578	12
2054.5	08:19:14.5	1920.5	1	2	39	17	5	25.892111	25.892111	17
2086	08:19:46.0	1320.5	1	1	44	3	1	2.1300374	2.1300374	12
2140	08:20:40.0	1320.5	1	1	45	3	1	19.025988	19.025988	12
2189.5	08:21:29.5	1320.5	1	1	27	3	1	14.825459	14.825459	12
2166.5	08:21:06.5	1920.5	1	2	38	17	5	17.856813	17.856813	17
2257.5	08:22:37.5	1320.5	1	1	26	3	1	32.318241	32.318241	12
2244.5	08:22:24.5	1920.5	1	2	37	17	5	24.447085	24.447085	17
2296	08:23:16.0	1920.5	1	2	36	17	5	1.262845	1.262845	17
2321.5	08:23:41.5	1920.5	1	2	35	17	5	25.467909	25.467909	17
2342	08:24:02.0	1320.5	1	1	28	3	1	0.238119	0.238119	12
2363	08:24:23.0	1320.5	1	1	3	0	0	7	7	12
2390.5	08:24:50.5	1920.5	1	2	34	17	5	54.593859	54.593859	17
2423	08:25:23.0	1320.5	1	1	40	0	0	8.3886843	8.3886843	12
2463	08:26:03.0	1920.5	1	2	33	17	5	9.4538681	9.4538681	17
2502	08:26:42.0	1320.5	1	1	29	0	0	15.033406	15.033406	12
2513.5	08:26:53.5	1920.5	1	2	32	17	5	7.141502	7.141502	17
2535	08:27:15.0	2520.5	1	3	1	9	3	29.5	29.5	9
2547	08:27:27.0	1920.5	1	2	54	2	1	12	12	9
2568	08:27:48.0	1320.5	1	1	46	0	0	18.982776	18.982776	12
2599	08:28:19.0	1920.5	1	2	43	2	1	3.2858575	3.2858575	9
2608.5	08:28:28.5	2520.5	1	3	39	9	3	7.4919369	7.4919369	9
2650.5	08:29:10.5	1320.5	1	1	30	0	0	14.60863	14.60863	12
2654	08:29:14.0	2520.5	1	3	38	9	3	30.689474	30.689474	9
2653	08:29:13.0	1920.5	1	2	44	2	1	40.306499	40.306499	9
2724	08:30:24.0	2520.5	1	3	37	9	3	4.1422931	4.1422931	9
2742	08:30:42.0	1920.5	1	2	45	2	1	33.266526	33.266526	9
2743	08:30:43.0	1320.5	1	1	41	0	0	13.267352	13.267352	12
2775.5	08:31:15.5	2520.5	1	3	36	9	3	46.905667	46.905667	9
2806	08:31:46.0	1920.5	1	2	27	2	1	17.962784	17.962784	9
2827	08:32:07.0	1320.5	1	1	31	0	0	0	0	12
2842	08:32:22.0	2520.5	1	3	35	9	3	45.471238	45.471238	9
2864.5	08:32:44.5	1320.5	1	1	47	0	0	30.329177	30.329177	12
2901	08:33:21.0	1920.5	1	2	26	2	1	38.642742	38.642742	9
2913.5	08:33:33.5	2520.5	1	3	34	9	3	20.252189	20.252189	9
2951	08:34:11.0	2520.5	1	3	33	9	3	0.6925741	0.6925741	9
2956.5	08:34:16.5	1320.5	1	1	53	0	0	23.75157	23.75157	12
2991	08:34:51.0	2520.5	1	3	32	9	3	35.313859	35.313859	9
3001.5	08:35:01.5	1920.5	1	2	28	2	1	20.597245	20.597245	9
3042.5	08:35:42.5	1920.5	1	2	3	0	0	7	7	17
3085.5	08:36:25.5	1920.5	1	2	40	0	0	22.666966	22.666966	17
3051.5	08:35:51.5	2520.5	1	3	54	10	0	32	32	10
3112.5	08:36:52.5	2520.5	1	3	43	10	0	18.541121	18.541121	10
3135	08:37:15.0	3120.5	1	4	1	12	1	37	37	12
3139.5	08:37:19.5	1920.5	1	2	29	0	0	30.632116	30.632116	17
3192	08:38:12.0	2520.5	1	3	44	10	0	31.79721	31.79721	10
3226.5	08:38:46.5	1920.5	1	2	46	0	0	17.940633	17.940633	17
3254.5	08:39:14.5	3120.5	1	4	39	12	1	46.623965	46.623965	12
3284.5	08:39:44.5	2520.5	1	3	45	10	0	30.344633	30.344633	10
3315	08:40:15.0	1920.5	1	2	30	0	0	3.7213291	3.7213291	17
3345	08:40:45.0	2520.5	1	3	27	10	0	29.535785	29.535785	10
3374.5	08:41:14.5	3120.5	1	4	38	12	1	47.244925	47.244925	12
3406.5	08:41:46.5	1920.5	1	2	41	0	0	33.994919	33.994919	17
3428.5	08:42:08.5	2520.5	1	3	26	10	0	0	0	10
3445.5	08:42:25.5	3120.5	1	4	37	12	1	35.577628	35.577628	12
3450.5	08:42:30.5	2520.5	1	3	28	10	0	26.298122	26.298122	10
3470.5	08:42:50.5	1920.5	1	2	31	0	0	22.255488	22.255488	17
3500	08:43:20.0	2520.5	1	3	3	7	0	24.5	24.5	21
3529.5	08:43:49.5	1920.5	1	2	47	0	0	31.603019	31.603019	17
3545.5	08:44:05.5	3120.5	1	4	36	12	1	21.006394	21.006394	12
3587	08:44:47.0	3120.5	1	4	35	12	1	27.860643	27.860643	12
3562.5	08:44:22.5	2520.5	1	3	40	7	0	31.212293	31.212293	21

3625.5	08:45:25.5	2520.5	1	3	29	7	0	10.208356	10.208356	21
3645.5	08:45:45.5	3120.5	1	4	34	12	1	39.903886	39.903886	12
3658.5	08:45:58.5	1920.5	1	2	53	0	0	60.452303	60.452303	17
3703	08:46:43.0	3120.5	1	4	33	12	1	0	0	12
3706	08:46:46.0	2520.5	1	3	46	7	0	25.125668	25.125668	21
3729.5	08:47:09.5	3120.5	1	4	32	12	1	42.42492	42.42492	12
3735	08:47:15.0	3720.5	1	5	1	14	3	42	42	14
3790.5	08:48:10.5	2520.5	1	3	30	7	0	46.219653	46.219653	21
3797	08:48:17.0	3120.5	1	4	54	4	0	17	17	4
3843	08:49:03.0	3120.5	1	4	43	4	0	17.47007	17.47007	4
3854.5	08:49:14.5	3720.5	1	5	39	14	3	21.080692	21.080692	14
3882.5	08:49:42.5	2520.5	1	3	41	7	0	44.69781	44.69781	21
3885.5	08:49:45.5	3120.5	1	4	44	4	0	7.3980621	7.3980621	4
3928	08:50:28.0	3720.5	1	5	38	14	3	20.545554	20.545554	14
3941	08:50:41.0	3120.5	1	4	45	4	0	41.351083	41.351083	4
3949	08:50:49.0	2520.5	1	3	31	7	0	24.846459	24.846459	21
3983	08:51:23.0	3720.5	1	5	37	14	3	47.41648	47.41648	14
4012	08:51:52.0	3120.5	1	4	27	4	0	44.485596	44.485596	4
4065	08:52:45.0	2520.5	1	3	47	7	0	12.064501	12.064501	21
4074	08:52:54.0	3720.5	1	5	36	14	3	10.094787	10.094787	14
4117.5	08:53:37.5	3120.5	1	4	26	4	0	16.390518	16.390518	4
4104	08:53:24.0	3720.5	1	5	35	14	3	34.430367	34.430367	14
4157.5	08:54:17.5	2520.5	1	3	53	7	0	19.801833	19.801833	21
4212	08:55:12.0	3120.5	1	4	28	4	0	20.504793	20.504793	4
4176	08:54:36.0	3720.5	1	5	34	14	3	42.966766	42.966766	14
4236.5	08:55:36.5	3720.5	1	5	33	14	3	39.467628	39.467628	14
4269.5	08:56:09.5	3120.5	1	4	3	20	0	57	57	20
4311	08:56:51.0	3720.5	1	5	32	14	3	16.461351	16.461351	14
4348.5	08:57:28.5	3120.5	1	4	40	20	0	13.757	13.757	20
4352.5	08:57:32.5	3720.5	1	5	54	4	1	17	17	4
4397	08:58:17.0	3120.5	1	4	29	20	0	0	0	20
4424	08:58:44.0	3720.5	1	5	43	4	1	43.594182	43.594182	4
4437	08:58:57.0	3120.5	1	4	46	20	0	40.25191	40.25191	20
4455.5	08:59:15.5	4440.5	1	6	1	9	4	29.5	29.5	9
4489.5	08:59:49.5	3720.5	1	5	44	4	1	42.469772	42.469772	4
4512	09:00:12.0	3120.5	1	4	30	20	0	50.600424	50.600424	20
4530	09:00:30.0	4440.5	1	6	39	9	4	28.765089	28.765089	9
4601.5	09:01:41.5	3720.5	1	5	45	4	1	8.7762561	8.7762561	4
4598	09:01:38.0	4440.5	1	6	38	9	4	9.4041155	9.4041155	9
4603	09:01:43.0	3120.5	1	4	41	20	0	19.39257	19.39257	20
4655.5	09:02:35.5	4440.5	1	6	37	9	4	58.193015	58.193015	9
4641	09:02:21.0	3720.5	1	5	27	4	1	18.189548	18.189548	4
4692.5	09:03:12.5	3720.5	1	5	26	4	1	21.816198	21.816198	4
4670	09:02:50.0	3120.5	1	4	31	20	0	34.651951	34.651951	20
4739	09:03:59.0	3120.5	1	4	47	20	0	10.047024	10.047024	20
4756.5	09:04:16.5	4440.5	1	6	36	9	4	19.992113	19.992113	9
4762.5	09:04:22.5	3720.5	1	5	28	4	1	27.582663	27.582663	4
4833	09:05:33.0	3120.5	1	4	53	20	0	38.131512	38.131512	20
4797	09:04:57.0	4440.5	1	6	35	9	4	27.719453	27.719453	9
4812.5	09:05:12.5	3720.5	1	5	3	0	0	7	7	0
4850.5	09:05:50.5	4440.5	1	6	34	9	4	25.145266	25.145266	9
4842	09:05:42.0	3720.5	1	5	40	0	0	45.733809	45.733809	0
4896	09:06:36.0	4440.5	1	6	33	9	4	23.289467	23.289467	9
4947	09:07:27.0	3720.5	1	5	29	0	0	23.412439	23.412439	0
4972	09:07:52.0	4440.5	1	6	32	9	4	30.372853	30.372853	9
5026	09:08:46.0	3720.5	1	5	46	0	0	16.464009	16.464009	0
5028.5	09:08:48.5	4440.5	1	6	54	1	0	9.5	9.5	1
5088.5	09:09:48.5	4440.5	1	6	43	1	0	5.0112064	5.0112064	1
5111	09:10:11.0	3720.5	1	5	30	0	0	26.937431	26.937431	0
5147.5	09:10:47.5	4440.5	1	6	44	1	0	19.51663	19.51663	1
5211.5	09:11:51.5	4440.5	1	6	45	1	0	16.170945	16.170945	1
5203	09:11:43.0	3720.5	1	5	41	0	0	8.7140979	8.7140979	0
5238.5	09:12:18.5	3720.5	1	5	31	0	0	30.991937	30.991937	0
5259	09:12:39.0	4440.5	1	6	27	1	0	36.61299	36.61299	1
5320	09:13:40.0	3720.5	1	5	47	0	0	39.178277	39.178277	0
5331	09:13:51.0	4440.5	1	6	26	1	0	10.976648	10.976648	1
5364.5	09:14:24.5	4440.5	1	6	28	1	0	24.464754	24.464754	1
5411.5	09:15:11.5	4440.5	1	6	3	2	0	12	12	2
5446.5	09:15:46.5	4440.5	1	6	40	2	0	19.664625	19.664625	2
5459	09:15:59.0	3720.5	1	5	53	0	0	4.7647244	4.7647244	0
5514.5	09:16:54.5	4440.5	1	6	29	2	0	25.633836	25.633836	2
5535.5	09:17:15.5	5520.5	1	7	1	15	4	44.5	44.5	15
5575.5	09:17:55.5	4440.5	1	6	46	2	0	25.972412	25.972412	2
5654	09:19:14.0	5520.5	1	7	39	15	4	12.325186	12.325186	15
5655	09:19:15.0	4440.5	1	6	30	2	0	39.763651	39.763651	2
5714.5	09:20:14.5	5520.5	1	7	38	15	4	40.036219	40.036219	15
5745	09:20:45.0	4440.5	1	6	41	2	0	41.726113	41.726113	2
5783	09:21:23.0	5520.5	1	7	37	15	4	18.74583	18.74583	15

5809.5	09:21:49.5	4440.5	1	6	31	2	0	20.321632	20.321632	2
5829	09:22:09.0	5520.5	1	7	36	15	4	30.83984	30.83984	15
5866	09:22:46.0	4440.5	1	6	47	2	0	15.730392	15.730392	2
5879.5	09:22:59.5	5520.5	1	7	35	15	4	22.113042	22.113042	15
5927.5	09:23:47.5	5520.5	1	7	34	15	4	14.155327	14.155327	15
5957.5	09:24:17.5	4440.5	1	6	53	2	0	37.11808	37.11808	2
5974	09:24:34.0	5520.5	1	7	33	15	4	25.927102	25.927102	15
6051	09:25:51.0	5520.5	1	7	32	15	4	19.132478	19.132478	15
6095.5	09:26:35.5	5520.5	1	7	54	3	1	14.5	14.5	3
6164.5	09:27:44.5	5520.5	1	7	43	3	1	52.390536	52.390536	3
6238.5	09:28:58.5	5520.5	1	7	44	3	1	0	0	3
6281.5	09:29:41.5	5520.5	1	7	45	3	1	6.4722438	6.4722438	3
6318	09:30:18.0	5520.5	1	7	27	3	1	18.891125	18.891125	3
6385	09:31:25.0	5520.5	1	7	26	3	1	8.2249188	8.2249188	3
6415	09:31:55.0	5520.5	1	7	28	3	1	27.883884	27.883884	3
6462.5	09:32:42.5	5520.5	1	7	3	4	0	17	17	4
6502.5	09:33:22.5	5520.5	1	7	40	4	0	26.902577	26.902577	4
6569	09:34:29.0	5520.5	1	7	29	4	0	8.6644114	8.6644114	4
6645.5	09:35:45.5	5520.5	1	7	46	4	0	15.449596	15.449596	4
6675	09:36:15.0	6660.5	1	8	1	16	2	47	47	16
6735.5	09:37:15.5	5520.5	1	7	30	4	0	49.522318	49.522318	4
6795	09:38:15.0	6660.5	1	8	39	16	2	25.28876	25.28876	16
6826.5	09:38:46.5	5520.5	1	7	41	4	0	21.891848	21.891848	4
6897	09:39:57.0	6660.5	1	8	38	16	2	13.235967	13.235967	16
6907	09:40:07.0	5520.5	1	7	31	4	0	17.445411	17.445411	4
6960	09:41:00.0	5520.5	1	7	47	4	0	12.745853	12.745853	4
6980.5	09:41:20.5	6660.5	1	8	36	16	2	25.938058	25.938058	16
6940.5	09:40:40.5	6660.5	1	8	37	16	2	8.755851	8.755851	16
7027	09:42:07.0	6660.5	1	8	35	16	2	30.233526	30.233526	16
7080.5	09:43:00.5	5520.5	1	7	53	4	0	37.665976	37.665976	4
7089	09:43:09.0	6660.5	1	8	34	16	2	24.391749	24.391749	16
7131.5	09:43:51.5	6660.5	1	8	33	16	2	1.7780939	1.7780939	16
7160.5	09:44:20.5	6660.5	1	8	32	16	2	29.271663	29.271663	16
7216	09:45:16.0	6660.5	1	8	54	8	0	27	27	8
7273	09:46:13.0	6660.5	1	8	43	8	0	5.1971143	5.1971143	8
7306.5	09:46:46.5	6660.5	1	8	44	8	0	25.060252	25.060252	8
7381	09:48:01.0	6660.5	1	8	45	8	0	11.438539	11.438539	8
7423	09:48:43.0	6660.5	1	8	27	8	0	11.162477	11.162477	8
7462.5	09:49:22.5	6660.5	1	8	26	8	0	25.012257	25.012257	8
7514.5	09:50:14.5	6660.5	1	8	28	8	0	11.078448	11.078448	8
7549	09:50:49.0	6660.5	1	8	3	3	0	14.5	14.5	3
7586.5	09:51:26.5	6660.5	1	8	40	3	0	34.01282	34.01282	3
7680	09:53:00.0	6660.5	1	8	29	3	0	29.961759	29.961759	3
7738	09:53:58.0	6660.5	1	8	46	3	0	18.006331	18.006331	3
7811	09:55:11.0	6660.5	1	8	30	3	0	20.333498	20.333498	3
7815	09:55:15.0	7800.5	1	9	1	11	2	34.5	34.5	11
7903	09:56:43.0	6660.5	1	8	41	3	0	31.899362	31.899362	3
7934.5	09:57:14.5	7800.5	1	9	39	11	2	13.838277	13.838277	11
7971	09:57:51.0	6660.5	1	8	31	3	0	29.420716	29.420716	3
7994	09:58:14.0	7800.5	1	9	38	11	2	26.232242	26.232242	11
8035.5	09:58:55.5	6660.5	1	8	47	3	0	35.974127	35.974127	3
8062.5	09:59:22.5	7800.5	1	9	37	11	2	40.146187	40.146187	11

Appendix D3

1. General

This Appendix presents the basic concepts of the Kalman filter (KF) technique, and the applications of this technique to bus running time prediction developed by Reihoundt (1997), Farhan (2002) and Sahalaby *et al.* (2004).

2. Fundamental Concepts of Kalman Filter Technique

To illustrate how the Kalman filter works, Maybeck (1979, pp 8-15) presented a very clear example, as follows.

Suppose that a measurement of any one-dimensional location of a moving point is taken to be z_I at time t_I and the standard deviation sd_{zI} . Thus, the conditional probability of $x(t_I)$, the position at time t_I conditioned on the observed value of the measurement being z_I is normally distributed with mean z_I and standard deviation of sd_{zI} .

Based on the conditional probability density, the best estimate of the point location

$$\hat{x}(t_1) = z_1 \quad (\text{D3.1})$$

And the variance of the error in estimate is

$$sd_x^2(t_1) = sd_{z1}^2 \quad (\text{D3.2})$$

Suppose that there is another more accurate measurement of the point location z_2 of the true position but taken at time $t_2 \equiv t_I$ with new standard variation sd_{z2} .

With two measurements available for the same point location, the problem is how to give the best prediction $x(t_2)$, given $t_2=t_1$ and already known z_1 and z_2 . The Kalman filter assumes that the conditional probability density of the new position at time t_2 is a Gaussian density with mean μ and standard deviation sd where

$$\mu = [sd_{z_2}^2 / (sd_{z_1}^2 + sd_{z_2}^2)]z_1 + [sd_{z_1}^2 / (sd_{z_1}^2 + sd_{z_2}^2)]z_2 \quad (D3.4)$$

$$1/sd^2 = (1/sd_{z_1}^2) + (1/sd_{z_2}^2) \quad (D3.5)$$

The best estimation is, of course, when $\hat{x}(t_2) = \mu$ or

$$\hat{x}(t_2) = [sd_{z_2}^2 / (sd_{z_1}^2 + sd_{z_2}^2)]z_1 + [sd_{z_1}^2 / (sd_{z_1}^2 + sd_{z_2}^2)]z_2 = [sd_{z_1}^2 / (sd_{z_1}^2 + sd_{z_2}^2)](z_2 - z_1)$$

Therefore,

$$\hat{x}(t_2) = \hat{x}(t_1) + K(t_2)[z_2 - \hat{x}(t_1)] \quad (D3.6)$$

where: K is a Kalman Gain

$$K(t_2) = [sd_{z_1}^2 / (sd_{z_1}^2 + sd_{z_2}^2)] \quad (D3.7)$$

Now the new variance is updated as follows

$$sd_x^2(t_2) = sd_x^2(t_1) - K(t_2)sd_x^2(t_1) \quad (D3.8)$$

In conclusion, the system of equations says that the optimal estimate of new state at time t_2 , $\hat{x}(t_2)$, is equal to the best prediction of its value before z_2 is actually taken, $\hat{x}(t_1)$, plus a correction term of an optimal weighing value multiplied by the difference between z_2 and $\hat{x}(t_1)$.

Figure D3.1: Kalman Filter Technique- Basic Concepts (Source: Maybeck 1979)

3. The KF Model for Bus running time Prediction

As mentioned in chapter 6, the Kalman Filter algorithm for bus link running time is the system of 4 main equations

$$g(k+1) = \frac{e(k) + VAR[data_{out}]_{k+1}}{e(k) + VAR[data_{out}]_{k+1} + VAR[data_{in}]_{k+1}} \quad (D3.9)$$

$$e(k+1) = VAR[data_{in}]_{k+1} \cdot g(k+1) \quad (D3.10)$$

$$a(k+1) = 1 - g(k+1) \quad (D3.11)$$

$$P(k+1) = a(k+1) \cdot art(k) + g(k+1) \cdot art_1(k+1) \quad (D3.12)$$

Where

g: filter-gain

a: loop-gain

e: filter error

P: prediction

$art(k)$: actual running time of previous bus at instant(k)

$art_i(k+1)$: actual running time of the previous day at instant(k+1)

$VAR[data_{in}]_{k+1}$ = Variance of the last three days data at time step k+1.

$VAR[data_{out}]_{k+1}$ = Variance of Prediction

One difficulty we have to cope with is that the estimation of prediction variance. This has not been available both in the predicted form or in the form of actual observation. Farhan (2002) assumed as follows

$$VAR[data_{out}] = VAR[data_{in}] = VAR[local_{data}]$$

Substitute $VAR[local_{data}]$ into Equations D3.9 and D3.11, then we have Equations 6.1 and 6.2 in chapter 6.

To calculate $VAR[data_{in}]$ the following equation is applied

$$VAR[data_{in}] = E \{ [art_i(k+1) - E[art_i(k+1)]]^2 \} \quad (D3.13)$$

Where i = number of observed day at time step $(k+1)$

Appendix D4

1. General

This Appendix presents database for bus routes 1 and 95 as well as the predictions which are obtained from the developed model, the KF and the naive predictors.

2. Database for Route1

Table D4.1: Database for Route 1, segment Billings-Gladstone

DCODE	SCODE	R1	R2	R3	R4
71	3000	950	902.5	938	813
72	3000	747.5	833.5	959	788.5
90	1100	905	975	1104	925.5
52	2200	1403	1318	1323	1451
38	2200	1252	1407	1367	1401
23	1300	905	977.5	904.5	1038
22	1300	806.5	1056	1052	914.5
25	1300	912.5	880.5	977.5	886.5
39	2200	1273	1356	1239	1287
5	1200	912.5	1049	971	865.5
11	1200	986.5	1055	886	996.5
28	1300	1002	961.5	889	889.5
50	2200	1355	1424	1167	1385
53	2200	1229	1332	1357	1214
73	3000	1009	824.5	961.5	830.5
54	2200	1362	1421	1490	1460
74	3000	851.5	782	805	789
61	2100	921	974	1078	924
6	1200	915.5	841	865.5	917
70	2100	946.5	1036	1083	934.5
26	1300	863.5	812.5	995.5	956
68	2100	990	1047	998	832.5
21	1300	998	981.5	983	932.5
59	2100	1110	834	905	830
8	1200	1023	1041	997.5	910.5

92	1100	880	949	879.5	918.5
94	1100	862	941	972	914.5
57	2100	1065	851.5	999	941.5
4	1200	968.5	859.5	866	912.5
3	1200	852.5	912.5	792	833
16	1200	892.5	927	985.5	1018
75	3000	716.5	774.5	827.5	821
40	2200	1396	1219	1358	1281
55	2200	1166	1178	1471	1438
96	1100	1101	841	1025	934
76	3000	730.5	773.5	827.5	821
37	2200	1358	1176	1254	1276
10	1200	864.5	953.5	896	818.5
30	1300	975	856.5	1093	933.5
45	2200	1228	1366	1488	1421
35	1300	964	933	1099	1021
12	1200	861.5	848	1073	959.5
9	1200	967.5	860	925.5	1011
65	2100	749	953	806	936
58	2100	989	856.5	907.5	935.5
34	1300	860.5	944.5	761.5	1003
77	3000	1022	787.5	832	888.5
95	1100	1104	1018	980.5	1042
69	2100	771.5	848.5	869	1045
17	1200	1019	968	1016	953.5
97	1100	868	1039	995	1030
93	1100	843	1041	1089	1027
60	2100	880.5	956	962.5	1050
87	1100	842	918	907	933
29	1300	1095	946	875	952.5
78	3000	953	837	798.5	884
43	2200	1141	1231	1346	1193
46	2200	1394	1400	1250	1344
24	1300	1111	1044	1083	945.5
19	1200	930.5	896	1005	930
51	2200	1124	1358	1239	1307
62	2100	911.5	860.5	1094	833
44	2200	1362	1343	1473	1168
36	2200	1275	1397	1417	1411
32	1300	956	949	1005	1015
47	2200	1404	1223	1132	1269
66	2100	1041	1155	1103	927.5
56	2100	877.5	816.5	892.5	848.5
79	3000	660.5	775.5	857	874.5
27	1300	1023	1001	1106	1026
88	1100	995	851.5	913.5	912
74	3000	1040	988.5	1136	973.5

1	1200	926.5	1036	1098	935.5
80	3000	962	1008	979	899
81	3000	831.5	781.5	823.5	876
89	1100	992.5	1055	1067	969
42	2200	1359	1416	1263	1318
82	3000	961.5	889	928	820.5
49	2200	1236	1338	1352	1345
2	1200	864.5	1032	1047	1053
98	1100	933.5	918.5	964.5	1055
67	2100	970.5	967	863	815.5
99	1100	962.5	956.5	860	922.5
64	2100	1104	1160	1122	931.5
63	2100	982.5	861	985.5	906.5
33	1300	988.5	1043	985	1005
18	1200	897	940	1108	949
83	3000	952	853.5	1035	819.5
48	2200	1471	1400	1270	1290
20	1200	860	845	1005	920
31	1300	926	982.5	983.5	1040
91	1100	965	942.5	860	929.5
100	1100	906.5	1029	1043	1049
84	3000	963	832	802.5	774
13	1200	798	977.5	871.5	917
14	1200	999.5	959.5	811	1056
101	1100	906.5	1029	1043	1049
15	1200	887	837.5	881.5	918
85	3000	1005	945	889	836
7	1200	916.5	883	788.5	938.5

Note:

1. Dcode= Date code; Scode= Situation code (Please refer to Table 6.5 for the codes); R1, R2, R3, and R4 = running time of trip 6, 5, 4, and 3.

2. Predictions Resulted from the Proposed Model and Reference Predictors for Route1

The results of the proposed model were returned from the Matlab program coded for the bus running time prediction module (Please see Appendix C). In this section, the example of Matlab outputs is partially presented for the sake of illustration. The predictions obtained from the developed model as well as those of reference predictors are tabulated in Tables D4.38 and D4.39.

the Edist to the first second... neighbor

1.0e+003 *

Columns 1 through 15

0.1962	0.2324	0.8580	0.9067	0.1797	0.3161	0.1960	0.7398	0.2573	0.2056	0.1366	0.7964
0.7745	0.2122	1.0262									

Columns 16 through 30

0.1812	0.3033	0.0903	0.3319	0.2194	0.2864	0.2181	0.1666	0.2632	0.1142	0.1939	0.2129
0.0851	0.1096	0.2168									

Columns 31 through 45

0.1646	0.7442	0.8952	0.2402	0.1653	0.6451	0.1759	0.3052	0.9766	0.3251	0.2869	0.1565
0.0722	0.1220	0.0928									

Columns 46 through 60

0.1162	0.2561	0.1375	0.2429	0.2743	0.3509	0.2189	0.1237	0.1079	0.0720	0.7039	0.8024
0.3353	0.2171	0.7504									

Columns 61 through 75

0.3240	0.8554	0.9388	0.2390	0.5855	0.4152	0.1528	0.1426	0.3497	0.1316	0.3648	0.3446
0.2312	0.1242	0.3281									

Columns 76 through 90

0.8079	0.1828	0.8300	0.3196	0.2140	0.1665	0.1038	0.4333	0.2008	0.2617	0.3242	0.2753
0.7888	0.2206	0.2411									

Columns 91 through 98

0.0935	0.3129	0.1728	0.1276	0.1420	0.3129	0.1055	0.1564
--------	--------	--------	--------	--------	--------	--------	--------

bandwidth =

325.1407

The matrix of Error is

1.0e+005 *

Columns 1 through 15

0.0062	0.3230	1.0369	0.2701	0.0120	0.2047	0.0032	0.3904	0.0122	0.0205	0.0527	0.7882
0.2316	0.0734	0.7597									

Columns 16 through 30

```

0.0526 0.0065 0.0008 0.0000 0.0457 0.0195 0.0302 0.3620 0.0571 0.0250 0.0641 0.1678
0.0202 0.0533 0.0253
Columns 31 through 45
0.4410 1.0650 0.0757 0.2722 0.3832 0.8301 0.0395 0.0128 0.2075 0.0032 0.0625 0.0149
0.3201 0.0430 0.0420
Columns 46 through 60
0.0986 0.2597 0.2452 0.0565 0.0618 0.1297 0.0380 0.0784 0.2796 0.0087 0.0470 1.0164
0.2781 0.0009 0.0205
Columns 61 through 75
0.0078 0.8448 0.3544 0.0016 1.1412 0.0690 0.0213 0.7150 0.0561 0.0518 0.0900 0.0043
0.0026 0.0878 0.0211
Columns 76 through 90
0.8060 0.0147 0.2323 0.0766 0.0004 0.0205 0.0126 0.2059 0.0277 0.0190 0.0239 0.0037
1.5853 0.0574 0.0034
Columns 91 through 99
0.0155 0.0190 0.0164 0.1868 0.0497 0.0190 0.0143 0.0662 0.0006
Cv =
1.8679e+004
The CROSS_VALIDATION factors
1.0e+004 *
1.0100 0.9455 1.0059 1.2357 1.5169 1.7389 1.8679
OpNei = 20
Beta =
736.0225
0.0223
-0.1607
0.3956
THE PREDCTED BUS RUNNING TIME IS 946.57 SECONDS.Program complete

```

Figure D4.1: An Example of the Matlab Outputs (Case 90-1100, R1)

3. Kalman Filter model's prediction results for Route1

Table D4.3: Kalman Filter for test case 90-1100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	938.5	813	788.5	846.667	8433.36	1133.44	3383.36	4783.4	0.5	2391.7	0.5	896.75	925.5
8:37	4	788.5	889	833.5	837	2352.25	2704	12.25	2528.13	0.66056	1669.98	0.33944	864.728	1104
8:47	5	883	902.5	833.5	873	100	870.25	1560.25	485.125	0.81626	395.986	0.18374	883.203	975
8:59	6	916.5	950	747.5	871.333	2040.03	6188.44	15334.7	4114.24	0.52296	2151.57	0.47704	856	905

Table D4.4: Kalman Filter for test case 50-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	865.5	996.5	889.5	917.167	2669.44	6293.78	765.444	4481.61	0.5	2240.81	0.5	947.25	1423.5
8:37	4	971	886	889	915.333	3098.78	860.444	693.444	1979.61	0.68071	1347.54	0.31929	1059.66	1167
8:47	5	1049	1054.5	961.5	1021.67	747.111	1078.03	3620.03	912.569	0.71237	650.084	0.28763	1020.61	1384.5
8:59	6	912.5	986.5	1002	967	2970.25	380.25	1225	1675.25	0.58125	973.737	0.41875	1162	1354.5

Table D4.5: Kalman Filter for test case 6-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1459.5	789	924	1057.5	161604	72092.3	17822.3	116848	0.5	58424.1	0.5	964.5	917
8:37	4	1490	805	1077.5	1124.17	133834	101867	2177.78	117851	0.59932	70630.1	0.40068	1013.19	865.5
8:47	5	1421	782	974	1059	131044	76729	7225	103887	0.62685	65121.1	0.37315	933.513	841
8:59	6	1361.5	851.5	921	1044.67	100383	37313.4	15293.4	68848.4	0.66054	45477.2	0.33946	893.8	915.5

Table D4.6: Kalman Filter for test case 26-1300

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	924	917	934.5	925.167	1.36111	66.6944	87.1111	34.0278	0.5	17.0139	0.5	969.75	956
8:37	4	1077.5	865.5	1083	1008.67	4738.03	20496.7	5525.44	12617.4	0.50034	6312.93	0.49966	1019.54	995.5
8:47	5	974	841	1036	950.333	560.111	11953.8	7338.78	6256.94	0.66766	4177.5	0.33234	1022.54	812.5
8:59	6	921	915.5	946.5	927.667	44.4444	148.028	354.694	96.2361	0.97798	94.1168	0.02202	943.5	863.5

Table D4.7: Kalman Filter for test case 21-1300

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	934.5	956	832.5	907.667	720.028	2336.11	5650.03	1528.07	0.5	764.035	0.5	918.75	932.5
8:37	4	1083	995.5	998	1025.5	3306.25	900	756.25	2103.13	0.57686	1213.21	0.42314	970.284	983
8:47	5	1036	812.5	1047	965.167	5017.36	23307.1	6696.69	14162.2	0.52054	7371.96	0.47946	1016.31	981
8:59	6	946.5	863.5	990	933.333	173.361	4876.69	3211.11	2525.03	0.79673	2011.76	0.20327	988.2	998

Table D4.8: Kalman Filter for test case 8-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	832.5	932	830	864.833	1045.44	4511.36	1213.36	2778.4	0.5	1389.2	0.5	917.5	910.5
8:37	4	998	983	905	962	1296	441	3249	868.5	0.72219	627.219	0.27781	906.528	997.5
8:47	5	1047	981	834	954	8649	729	14400	4689	0.53134	2491.47	0.46866	910.625	1040.5
8:59	6	990	998	1109.5	1032.5	1806.25	1190.25	5929	1498.25	0.72699	1089.22	0.27301	1091	1023

Table D4.9: Kalman Filter for test case 57-2100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	910.5	918.5	914.5	914.5	16	16	0	16	0.5	8	0.5	959.75	941.5
8:37	4	997.5	879.5	972	949.667	2288.03	4923.36	498.778	3605.69	0.50055	1804.85	0.49945	956.767	999
8:47	5	1040	949	941	976.667	4011.11	765.444	1272.11	2388.28	0.63712	1521.61	0.36288	962.047	851.5
8:59	6	1022.5	880	862	921.5	10201	1722.25	3540.25	5961.63	0.55659	3318.16	0.44341	857.3	1065

Table D4.10: Kalman Filter for test case 3-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	914.5	941.5	912.5	922.833	69.4444	348.444	106.778	208.944	0.5	104.472	0.5	958.75	833
8:37	4	972	999	866	945.667	693.444	2844.44	6346.78	1768.94	0.51434	909.841	0.48566	849.973	792
8:47	5	941	851.5	859.5	884	3249	1056.25	600.25	2152.63	0.58723	1264.09	0.41277	831.638	912.5
8:59	6	862	1065	968.5	965.167	10643.4	9966.69	11.1111	10305	0.52889	5450.27	0.47111	942.1	852.5

Table D4.11: Kalman Filter for test case 69-2100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1002.5	888.5	1041.5	977.5	625	7921	4096	4273	0.5	2136.5	0.5	1023.25	1044.5
8:37	4	761.5	832	980.5	858	9312.25	676	15006.3	4994.13	0.5881	2937.07	0.4119	1006.86	869
8:47	5	860.5	1021.5	1103.5	995.167	18135.1	693.444	11736.1	9414.28	0.56747	5342.32	0.43253	1002.07	848.5
8:59	6	860.5	1021.5	1103.5	995.167	18135.1	693.444	11736.1	9414.28	0.61051	5747.53	0.38949	1004	771.5

Table D4.12: Kalman Filter for test case 87-1100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1029.5	1027	1049.5	1035.33	34.0278	69.4444	200.694	51.7361	0.5	25.8681	0.5	1027.25	933
8:37	4	995	1089	962.5	1015.5	420.25	5402.25	2809	2911.25	0.50221	1462.06	0.49779	947.815	907
8:47	5	968	1039	1041	1016	2304	529	625	1416.5	0.6702	949.342	0.3298	996.807	918
8:59	6	868	843	880.5	863.833	17.3611	434.028	277.778	225.694	0.83887	189.329	0.16113	886.5	842

Table D4.13: Kalman Filter for test case 78-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1049.5	933	952.5	978.333	5064.69	2055.11	667.361	3559.9	0.5	1779.95	0.5	978.75	884
8:37	4	962.5	907	875	914.833	2272.11	61.3611	1586.69	1166.74	0.71636	835.802	0.28364	877.553	798.5
8:47	5	956	918	946	940	256	484	36	370	0.7652	283.124	0.2348	911.367	837
8:59	6	880.5	842	1095	939.167	3441.78	9441.36	24284	6441.57	0.51075	3290.04	0.48925	968.8	953

Table D4.14: Kalman Filter for test case 43-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	933	952.5	884	923.167	96.6944	860.444	1534.03	478.569	0.5	239.285	0.5	944.5	1192.5
8:37	4	907	875	798.5	860.167	2193.36	220.028	3802.78	1206.69	0.5451	657.772	0.4549	977.73	1345.5
8:47	5	918	946	837	900.333	312.111	2085.44	4011.11	1198.78	0.60764	728.429	0.39236	1036.51	1230.5
8:59	6	842	1095	953	963.333	14721.8	17336.1	106.778	16028.9	0.51111	8192.53	0.48889	1089	1141

Table D4.15: Kalman Filter for test case 24-1300

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	884	1192.5	1343.5	1140	65536	2756.25	41412.3	34146.1	0.5	17073.1	0.5	1174.25	945.5
8:37	4	798.5	1345.5	1250	1131.33	110778	45867.4	14081.8	78322.7	0.54914	43010.1	0.45086	1112.71	1082.5
8:47	5	837	1230.5	1399.5	1155.67	101548	5600.03	59454.7	53574.2	0.64322	34459.8	0.35678	1286.4	1044
8:59	6	953	1141	1394	1162.67	43960.1	469.444	53515.1	22214.8	0.71841	15959.2	0.28159	1295	1111

Table D4.16: Kalman Filter for test case 51-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1343.5	945.5	930	1073	73170.3	16256.3	20449	44713.3	0.5	22356.6	0.5	967.5	1306.5
8:37	4	1250	1082.5	1005	1112.5	18906.3	900	11556.3	9903.13	0.76512	7577.1	0.23488	1075.82	1238.5
8:47	5	1399.5	1044	896	1113.17	81986.8	4784.03	47161.4	43385.4	0.54016	23434.8	0.45984	1053.5	1357.5
8:59	6	1141	1394	930.5	1155.17	200.694	57041.4	50475.1	28621	0.64524	18467.4	0.35476	1082	1124

Table D4.17: Kalman Filter for test case 56-2100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1015.2	1269	927.5	1070.57	3065.47	39375.8	20468.1	21220.6	0.5	10610.3	0.5	966.25	848.5
8:37	4	1005	1132	1102.5	1079.83	5600.03	2721.36	513.778	4160.69	0.78023	3246.28	0.21977	1046.68	892.5
8:47	5	949	1223	1154.5	1108.83	25546.7	13034	2085.44	19290.4	0.53881	10393.8	0.46119	1033.67	816.5
8:59	6	956	1404	1041	1133.67	31565.4	73080.1	8587.11	52322.8	0.54517	28525.1	0.45483	938.9	877.5

Table D4.18: Kalman Filter for test case 33-1300

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	922.5	931.5	906.5	920.167	5.44444	128.444	186.778	66.9444	0.5	33.4722	0.5	955.75	1005
8:37	4	860	1122	985.5	989.167	16684	17644.7	13.4444	17164.4	0.50049	8590.54	0.49951	995.241	985
8:47	5	956.5	1159.5	861	992.333	1284.03	27944.7	17248.4	14614.4	0.61357	8966.99	0.38643	908.917	1042.5
8:59	6	962.5	1104	982.5	1016.33	2898.03	7685.44	1144.69	5291.74	0.72933	3859.42	0.27067	998.7	988.5

Table D4.19: Kalman Filter for test case 27-1300

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	927.5	848.5	874.5	883.5	1936	1225	81	1580.5	0.5	790.25	0.5	939.75	1025.5
8:37	4	1102.5	892.5	857	950.667	23053.4	3383.36	8773.44	13218.4	0.51451	6801.01	0.48549	938.805	1106
8:47	5	1154.5	816.5	775.5	915.5	57121	9801	19800	33461	0.54613	18273.9	0.45387	925.506	1001
8:59	6	1041	877.5	660.5	859.667	32881.8	318.028	39667.4	16599.9	0.67751	11246.6	0.32249	770.3	1023

Table D4.20: Kalman Filter for test case 74-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	890	792100	792100	792100	792100	0.5	0	0.5		1005
8:27	3	874.5	1025.5	912	937.333	3948.03	7773.36	641.778	5860.69	0.5	2930.35	0.5	958.5	973.5
8:37	4	857	1106	913.5	958.833	10370	21658	2055.11	16014	0.54191	8678.19	0.45809	940.985	1136
8:47	5	775.5	1001	851.5	876	10100.3	15625	600.25	12862.6	0.62612	8053.6	0.37388	957.868	988.5
8:59	6	660.5	1023	995	892.833	53978.8	16943.4	10438	35461.1	0.55099	19538.6	0.44901	992.1	1040

Table D4.21: Kalman Filter for test case 99-1100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	890	792100	792100	792100	792100	0.5	0	0.5		1005
8:27	3	1053	1055	815.5	974.5	6162.25	6480.25	25281	6321.25	0.5	3160.63	0.5	910.25	922.5
8:37	4	1047	964.5	863	958.167	7891.36	40.1111	9056.69	3965.74	0.64247	2547.87	0.35753	884.273	860
8:47	5	1032	918.5	967	972.5	3540.25	2916	30.25	3228.13	0.64148	2070.79	0.35852	928.639	956.5
8:59	6	864.5	933.5	970.5	922.833	3402.78	113.778	2272.11	1758.28	0.68531	1204.97	0.31469	966.1	962.5

Table D4.22: Kalman Filter for test case 91-1100

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	890	792100	792100	792100	792100	0.5	0	0.5		1005
8:27	3	1290	920	1039.5	1083.17	42780	26623.4	1906.78	34701.7	0.5	17350.8	0.5	1022.25	929.5
8:37	4	1269.5	1005	983.5	1086	33672.3	6561	10506.3	20116.6	0.65066	13089	0.34934	964.635	860
8:47	5	1400	845	982.5	1075.83	105084	53284	8711.11	79184	0.53817	42614.5	0.46183	925.926	942.5
8:59	6	1471	860	926	1085.67	148482	50925.4	25493.4	99703.6	0.58804	58629.6	0.41196	932.8	965

Table D4.23: Kalman Filter for test case 52-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	813	788.5	925.5	842.333	860.444	2898.03	6916.69	1879.24	0.5	939.618	0.5	965.25	1450
8:37	4	938	959	1104	1000.33	3885.44	1708.44	10746.8	2796.94	0.57191	1599.59	0.42809	1252.12	1322.5
8:47	5	902.5	833.5	975	903.667	1.36111	4923.36	5088.44	2462.36	0.62259	1533.03	0.37741	1106.15	1317.5
8:59	6	950	747.5	905	867.5	6806.25	14400	1406.25	10603.1	0.53371	5658.98	0.46629	1097	1421

Table D4.24: Kalman Filter for test case 38-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	788.5	925.5	1450.5	1054.83	70933.4	16727.1	156552	43830.3	0.5	21915.1	0.5	1227.75	1400
8:37	4	959	1104	1322.5	1128.5	28730.3	600.25	37636	14665.3	0.71382	10468.4	0.28618	1344.68	1367
8:47	5	833.5	975	1317.5	1042	43472.3	4489	75900.3	23980.6	0.58958	14138.5	0.41042	1337.82	1406.5
8:59	6	747.5	905	1402.5	1018.33	73350.7	12844.4	147584	43097.6	0.57046	24585.3	0.42954	1404	1252

Table D4.25: Kalman Filter for test case 39-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1038	914.5	886.5	946.333	8402.78	1013.36	3580.03	4708.07	0.5	2354.03	0.5	945.75	1286.5
8:37	4	904.5	1052	977.5	978	5402.25	5476	0.25	5439.13	0.58895	3203.37	0.41105	1104.51	1238.5
8:47	5	977.5	1056	880.5	971.333	38.0278	7168.44	8250.69	3603.24	0.65386	2356.02	0.34614	1004.42	1356
8:59	6	905	806.5	912.5	874.667	920.111	4646.69	1431.36	2783.4	0.64869	1805.55	0.35131	1068	1273

Table D4.26: Kalman Filter for test case 54-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1384.5	1213.5	830.5	1142.83	58402.8	4993.78	97552.1	31698.3	0.5	15849.1	0.5	917.75	1459.5
8:37	4	1167	1356.5	961.5	1161.67	28.4444	37960	40066.7	18994.2	0.64719	12293	0.35281	1137.2	1490
8:47	5	1423.5	1332	824.5	1193.33	52976.7	19228.4	136038	36102.6	0.57274	20677.4	0.42726	1108.84	1421
8:59	6	1354.5	1228.5	1008.5	1197.17	24753.8	981.778	35595.1	12867.8	0.72275	9300.25	0.27725	1123	1362

Table D4.27: Kalman Filter for test case 40-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	833	1017.5	821	890.5	3306.25	16129	4830.25	9717.63	0.5	4858.81	0.5	913	1280.5
8:37	4	792	985.5	827.5	868.333	5826.78	13728	1667.36	9777.4	0.59951	5861.65	0.40049	1008.92	1358
8:47	5	912.5	927	774.5	871.333	1694.69	3098.78	9376.69	2396.74	0.77506	1857.62	0.22494	905.751	1218.5
8:59	6	852.5	892.5	716.5	820.5	1024	5184	10816	3104	0.61516	1909.45	0.38484	909.7	1396

Table D4.28: Kalman Filter for test case 55-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1017.5	821	1280.5	1039.67	491.361	47815.1	58000.7	24153.2	0.5	12076.6	0.5	1142.75	1438
8:37	4	985.5	827.5	1358	1057	5112.25	52670.3	90601	28891.3	0.58644	16942.9	0.41356	1391.09	1470.5
8:47	5	927	774.5	1218.5	973.333	2146.78	39534.7	60106.7	20840.7	0.6445	13431.9	0.3555	1308.09	1177.5
8:59	6	892.5	716.5	1395.5	1001.5	11881	81225	155236	46553	0.56304	26211.1	0.43696	1300	1166

Table D4.29: Kalman Filter for test case 37-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1438	934	821	1064.33	139627	16986.8	59211.1	78306.8	0.5	39153.4	0.5	913	1275.5
8:37	4	1470.5	1025	827.5	1107.67	131648	6833.78	78493.4	69240.9	0.61021	42251.3	0.38979	1002.13	1254
8:47	5	1177.5	841	773.5	930.667	60926.7	8040.11	24701.4	34483.4	0.68995	23791.8	0.31005	922.48	1176
8:59	6	1166	1100.5	730.5	999	27889	10302.3	72092.3	19095.6	0.69192	13212.7	0.30808	867.7	1358

Table D4.30: Kalman Filter for test case 45-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1275.5	818.5	933.5	1009.17	70933.4	36353.8	5725.44	53643.6	0.5	26821.8	0.5	969.25	1420.5
8:37	4	1254	896	1092.5	1080.83	29986.7	34163.4	136.111	32075	0.64742	20766	0.35258	1208.15	1487.5
8:47	5	1176	953.5	856.5	995.333	32640.4	1750.03	19274.7	17195.2	0.68825	11834.6	0.31175	1053.22	1365.5
8:59	6	1357.5	86.5	975	806.333	303785	518160	28448.4	410972	0.5071	208403	0.4929	1167	1228

Table D4.31: Kalman Filter for test case 43-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	933	952.5	884	923.167	96.6944	860.444	1534.03	478.569	0.5	239.285	0.5	944.5	1192.5
8:37	4	907	875	798.5	860.167	2193.36	220.028	3802.78	1206.69	0.5451	657.772	0.4549	977.73	1345.5
8:47	5	918	946	837	900.333	312.111	2085.44	4011.11	1198.78	0.60764	728.429	0.39236	1036.51	1230.5
8:59	6	842	1095	953	963.333	14721.8	17336.1	106.778	16028.9	0.51111	8192.53	0.48889	1089	1141

Table D4.32: Kalman Filter for test case 46-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	952.5	884	1192.5	1009.67	3268.03	15792.1	33428	9530.07	0.5	4765.03	0.5	1098.75	1343.5
8:37	4	875	798.5	1345.5	1006.33	17248.4	43194.7	115034	30221.6	0.53654	16215	0.46346	1344.57	1250
8:47	5	946	837	1230.5	1004.5	3422.25	28056.3	51076	15739.3	0.66999	10545.2	0.33001	1236.94	1399.5
8:59	6	1095	953	1141	1063	1024	12100	6084	6562	0.72276	4742.76	0.27724	1213	1394

Table D4.33: Kalman Filter for test case 51-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1343.5	945.5	930	1073	73170.3	16256.3	20449	44713.3	0.5	22356.6	0.5	967.5	1306.5
8:37	4	1250	1082.5	1005	1112.5	18906.3	900	11556.3	9903.13	0.76512	7577.1	0.23488	1075.82	1238.5
8:47	5	1399.5	1044	896	1113.17	81986.8	4784.03	47161.4	43385.4	0.54016	23434.8	0.45984	1053.5	1357.5
8:59	6	1141	1394	930.5	1155.17	200.694	57041.4	50475.1	28621	0.64524	18467.4	0.35476	1082	1124

Table D4.34: Kalman Filter for test case 44-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	930	1306.5	833	1023.17	8680.03	80277.8	36163.4	44478.9	0.5	22239.5	0.5	919	1168
8:37	4	1005	1238.5	1094	1112.5	11556.3	15876	342.25	13716.1	0.72386	9928.61	0.27614	1114.43	1472.5
8:47	5	896	1357.5	860.5	1038	20164	102080	31506.3	61122.1	0.53756	32856.8	0.46244	1143.51	1342.5
8:59	6	930.5	1124	911.5	988.667	3383.36	18315.1	5954.69	10849.2	0.80113	8691.68	0.19887	997.2	1362

Table D4.35: Kalman Filter for test case 47-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1168	1410.5	1015.2	1197.9	894.01	45198.8	33379.3	23046.4	0.5	11523.2	0.5	1010.1	1269
8:37	4	1472.5	1416.5	1005	1298	30450.3	14042.3	85849	22246.3	0.60286	13411.3	0.39714	1109.85	1132
8:47	5	1342.5	1397	949	1229.5	12769	28056.3	78680.3	20412.6	0.62364	12730.1	0.37636	1017.87	1223
8:59	6	1362	1275	956	1197.67	27005.4	5980.44	58402.8	16492.9	0.63923	10542.8	0.36077	1052	1404

Table D4.36: Kalman Filter for test case 42-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	899	876	969	914.667	245.444	1495.11	2952.11	870.278	0.5	435.139	0.5	987	1318
8:37	4	781.5	823.5	1066.5	890.5	11881	4489	30976	8185	0.51295	4198.47	0.48705	1188.99	1263
8:47	5	1008	781.5	1054.5	948	3600	27722.3	11342.3	15661.1	0.5591	8756.12	0.4409	1146.43	1415.5
8:59	6	962	831.5	992.5	928.667	1111.11	9441.36	4074.69	5276.24	0.72674	3834.46	0.27326	1108	1359

Table D4.37: Kalman Filter for test case 48-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	2	0	0	0	1005	1E+06	1E+06	1E+06	1E+06	0.5	0	0.5		1005
8:27	3	1005	949	819.5	924.5	6480.25	600.25	11025	3540.25	0.5	1770.13	0.5	912.25	1290
8:37	4	985	1107.5	1035	1042.5	3306.25	4225	56.25	3765.63	0.59515	2241.13	0.40485	1138.24	1269.5
8:47	5	1042.5	940	853.5	945.333	9441.36	28.4444	8433.36	4734.9	0.59569	2820.51	0.40431	1021.69	1400
8:59	6	988.5	897	952	945.833	1820.44	2384.69	38.0278	2102.57	0.70073	1473.33	0.29927	1086	1471

Table D4.38: Prediction Error of the Predictors (Overall situation)

Dcode	Scode	Actual	Developed Model	KF. Model	Naïve Model	d1	d2	d3
90	1100	905	946	856	874.4	0.045303867	0.05414	0.03381
50	2200	1354.5	1377.6	1068.3	1484	0.017054264	0.2113	0.09561
6	1200	915.5	917.61	893.84	795.25	0.002304752	0.02366	0.13135
26	1300	863.5	1009.84	943.55	742.31	0.169473075	0.0927	0.14035
21	1300	998	931.29	988.17	928.06	0.066843687	0.00985	0.07008
8	1200	1022.5	952.37	1090	1384.2	0.068586797	0.06601	0.35374
57	2100	1065	941.96	857	780.01	0.115530516	0.19531	0.2676
3	1200	852.5	889.85	942	1028.22	0.043812317	0.10499	0.20612
69	2100	771.5	928.82	1104	920.21	0.203914452	0.43098	0.19275
87	1100	842	945.54	886	845.5	0.122969121	0.05226	0.00416
78	3000	953	822.44	968	968.83	0.136998951	0.01574	0.01661
43	2200	1141	1296	1088	1401.03	0.135845749	0.04645	0.2279
24	1300	1111	943.17	1212.7	1039	0.151062106	0.09154	0.06481
51	2200	1124	1358.5	1082	1409.7	0.203291815	0.03737	0.25418
56	2100	877.5	874.29	938.89	736.22	0.00365812	0.06996	0.161
33	1300	988.5	950.32	998.74	1189.6	0.038624178	0.01036	0.20344
99	1100	962.5	896	966.09	959.56	0.069090909	0.00373	0.00305
91	1100	965	902.38	932.8	888.3	0.064891192	0.03337	0.07948
44	2200	1362	1132.69	997	1422.5	0.164236417	0.26799	0.04442
27	1300	1023	911.27	770.31	852.56	0.109217986	0.24701	0.16661

Note: d1,d2, d3: The absolute relative error (%) of the Developed Model, the Kalman Filter and the Naïve predictor, respectively.

Table D4.39: Prediction Error of the Predictors (situation coded as 2200)

Dcode	Scode	Actual	Developed Model	KF Model	NAIVE	d1	d2	d3
52	2200	1402.5	1290.21	1097	1222.91	0.080064	0.217825	0.12805
38	2200	1251.5	1306.05	1404.02	1497.24	0.038895	0.108749	0.175216
39	2200	1272.5	1331.72	1068.3	795.25	0.042225	0.145597	0.340285
50	2200	1354.5	1377.61	1162.2	1483.46	0.016478	0.137112	0.09195
40	2200	1395.5	1227.24	909.69	1127.25	0.119971	0.346389	0.191266
37	2200	1357.5	1211	867.75	1384.2	0.104456	0.349198	0.019037
57	2200	1065	941.96	857	780.01	0.087729	0.148307	0.203201
54	2200	1361.5	1259.5	1122.9	1028.22	0.072727	0.170125	0.237633
46	2200	1394	1332	1212.7	920.21	0.044207	0.129269	0.337818
44	2200	1362	1132.69	997.21	1422.5	0.163501	0.2601	0.043137
47	2200	1404	1187	1052	1233.32	0.154724	0.25098	0.121697
42	2200	1359	1332	1108.1	1332.27	0.019251	0.178895	0.019059
48	2200	1471	1306.9	1086.1	1562	0.117005	0.274439	0.064884
51	2200	1124	1358.5	1082	1409.7	0.167201	0.029947	0.203708

Note: d1,d2, d3: The absolute relative error (%) of the Developed Model, the Kalman Filter and the Naïve Models, respectively.

4. Database for Route 95

**Table D4.40: Database for Route 95, segment Laurent-Mackenzie
(70 simulation runs)**

Dcode	Scode	R1	R2	R3	R4
54	2200	794.5	681.5	677	747
58	2200	681.5	798	662	741.5
33	1200	571	537.5	561.5	605.5
42	1200	867	747	746	686.5
47	1200	747	866.5	747	686.5
62	2200	699	672.5	673	642.5
48	1200	746	866.5	747	808.5
36	1200	746.5	755	748	686
16	3000	632	632.5	648.5	592
52	2200	781.5	785	686.5	740
3	3000	803.5	647.5	795	731.5
2	3000	673.5	666	687	633.5
7	3000	698.5	677	666.5	725
21	1200	563	638	548.5	526
28	1200	562	657	561.5	511.5
19	1200	553.5	547.5	546	613
4	3000	782	792	680.5	619
11	3000	685	753.5	797	641.5
45	1200	747	773	747.5	686
39	1200	746	746.5	643.5	698.5
5	3000	690	770.5	679.5	735.5
55	2200	685	795	677.5	737.5
38	1200	746.5	747	763	688
13	3000	668	792.5	677	728.5
64	2200	694	830.5	755	752
14	3000	689	674	683	627
10	3000	672.5	661.5	667	615.5
29	1200	559	548	563	614.5
35	1200	548	521.5	559.5	499.5
53	2200	820	694.5	671.5	743.5
17	1200	550	526	550.5	589.5
6	3000	720	810	750	700
12	3000	689.5	795.5	680	717
18	1200	550	641.5	571.5	525
31	1200	558	558.5	557	509
56	2200	805.5	699	672.5	728
8	3000	681	647.5	691.5	743
49	1200	748.5	747.5	746	686.5

9	3000	679.5	804.5	682	745
59	2200	778.5	781	687	740
41	1200	748	754.5	638.5	809.5
46	1200	747	766.5	746.5	807
57	2200	671	691.5	683	734
51	2200	694.5	777.5	686	734.5
1	3000	702	676.5	676.5	737
22	1200	556	540.5	554	502
40	1200	747.5	746.5	746	708
63	2200	798	796.5	675	732
50	1200	820	788	777	790
61	2200	683	763.5	688	833.5
43	1200	748	746.5	644.5	811.5
24	1200	557	542	564	580
25	1200	566	535.5	554.5	612.5
37	1200	746	869	747.5	707.5
65	2200	685	772	686	712
44	1200	646	746.5	748.5	686.5
60	2200	686	678	681	722.5
20	1200	554	538.5	561.5	509
34	1200	553.5	657.5	554.5	514.5
26	1200	553	640	555.5	496.5
23	1200	569.5	548.5	567	609
30	1200	565.5	552.5	568.5	631
32	1200	572.5	569.5	675.5	513
27	1200	573.5	553.5	551.5	513
15	3000	655	673	648	632
66	1200	755.5	866.5	633.5	685.5
67	1200	747	746.5	747.5	809.5
68	1200	756.5	866.5	755.5	931.5
69	1200	747	746	747.5	806.5
70	1200	746.5	748.5	748	696

5. Route 95- Running time Predictions of Developed Model and the Reference Predictors

The results of the developed model were returned from the Matlab program (Please see Appendix C). The predictions of the developed model as well as that of reference predictors are presented in the Table D4.62.

the Edist to the first second... neighbor

Columns 1 through 12

109.1627 295.7930 9.8234 118.3860 119.4288 163.0376 11.9269 184.8574 83.9673 116.9198 120.1395

112.2297

Columns 13 through 24

284.4477 277.8394 296.8063 111.2542 73.4592 26.4670 104.5490 82.0762 94.0997 17.0660 89.6284

99.5440

Columns 25 through 36

120.5664 143.5627 284.7236 354.5046 104.9976 315.9980 61.6624 85.2877 268.0340 327.9482 95.7836

124.9090

Columns 37 through 48

9.7596 99.4636 81.9344 157.8243 112.4600 94.4352 78.5318 109.4406 344.2906 12.3288 94.4934

106.0059

Columns 49 through 60

150.7689 155.1016 299.9237 299.6389 121.0485 68.2074 9.7211 100.8043 337.4185 280.4737 297.7058

283.4255

Columns 61 through 68

273.6078 266.0569 331.8528 140.6991 164.7559 113.5187 263.5157 110.5294

bandwidth =

263.5157

The matrix of Error is

1.0e+004 *

Columns 1 through 12

0.2264 0.4125 2.0691 0.0331 0.0002 0.0279 0.0433 0.0211 0.2991 0.6475 0.0509 0.0459

```

Columns 13 through 24
    0.4526    0.4624    0.6318    0.3921    0.1759    0.0451    0.0442    0.1399    0.1937    0.0433    0.3629    0.1302

Columns 25 through 36
    0.0057    0.0012    0.5612    0.8049    0.9362    0.7362    0.0066    0.1432    0.7324    0.6111    0.6731    0.1487

Columns 37 through 48
    0.0523    0.2523    0.2698    0.0367    0.0328    0.3069    0.1098    0.0450    0.6474    0.0473    0.5196    0.8569

Columns 49 through 60
    0.2245    0.0367    0.6500    0.4379    0.0288    0.1767    0.6687    0.1408    0.6969    0.5965    0.5925    0.4422

Columns 61 through 69
    0.4560    0.5661    0.3782    0.0712    0.0915    0.0331    0.0728    0.0332    0.0433

Cv =
    3.1777e+003

The CROSS_VALIDATION factors
    1.0e+003 *
    2.0559    1.7628    1.7629    2.0098    2.5940    2.8404    3.1777

OpNei =
    14

Beta =
    1.0e+003 *
    1.6125
    0.0000
   -0.0017
    0.0003

THE PREDCTED BUS RUNNING TIME IS 721.66 SECONDS.Program complete

```

Figure D4.2: An Example of the Matlab Outputs (Case 54-2200, R95)

Table D4.42: Kalman Filter for case 54-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	513	513	632	552.667	1573.44	1573.44	6293.78	1573.44	0.5	786.722	0.5	631	747
8:39	19	675.5	551.5	648	625	2550.25	5402.25	529	3976.25	0.54501	2167.1	0.45499	693.044	677
8:43	20	552.5	569.5	553.5	558.5	36	121	25	78.5	0.96622	75.8485	0.03378	557.671	681.5
8:59	24	573.5	573.5	655	600.667	738.028	738.028	2952.11	738.028	0.52444	387.049	0.47556	667.602	794.5

Table D4.43: Kalman Filter for case 47-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	605.5	686.5	686.5	659.5	2916	729	729	1822.5	0.5	911.25	0.5	658.25	686.5
8:39	19	662	561.5	746	656.5	30.25	9025	8010.25	4527.63	0.54572	2470.8	0.45428	718.97	746
8:43	20	789	537.5	747	691.167	9571.36	23613.4	3117.36	16592.4	0.53465	8871.1	0.46535	746.535	866.5
8:59	24	681.5	571	867	706.5	625	18360.3	25760.3	9492.63	0.65923	6257.82	0.34077	866.83	747

Table D4.44: Kalman Filter for case 48-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	1	686.5	686.5	642.5	671.833	215.111	215.111	860.444	215.111	0.5	107.556	0.5	636.25	808.5
8:39	19	746	747	673	722	576	625	2401	600.5	0.5411	324.929	0.4589	735.181	747
8:43	20	747	866.5	672.5	762	225	10920.3	8010.25	5572.63	0.51416	2865.24	0.48584	708.695	866.5
8:59	24	867	747	699	771	9216	576	5184	4896	0.61319	3002.16	0.38681	763.791	746

Table D4.45: Kalman Filter for case 36-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	686.5	642.5	808.5	712.5	676	4900	9216	2788	0.5	1394	0.5	719.25	686
8:39	19	747	673	747	722.333	608.444	2433.78	608.444	1521.1111	0.65712	999.546	0.34288	726.084	748
8:43	20	866.5	672.5	866.5	801.833	4181.78	16727.1	4181.78	10454.444	0.52281	5465.71	0.47719	809.953	755
8:59	24	747	699	746	730.667	266.778	1002.78	235.111	634.77778	0.90575	574.952	0.09425	746.848	746.5

Table D4.46: Kalman Filter for case 16-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	642.5	808.5	686	712.3333	4876.69	9248.03	693.444	7062.3611	0.5	3531.18	0.5	658	592
8:39	19	673	747	748	722.6667	2466.78	592.111	641.778	1529.4444	0.76792	1174.49	0.23208	711.795	648.5
8:43	20	672.5	866.5	755	764.6667	8494.69	10370	93.4444	9432.3611	0.5293	4992.59	0.4707	704.871	632.5
8:59	24	699	746	746.5	730.5	992.25	240.25	256	616.25	0.90101	555.245	0.09899	735.215	632

Table D4.47: Kalman Filter for case 21-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	731.5	633.5	725	696.667	1213.36	3990.03	802.778	2601.6944	0.5	1300.85	0.5	677.5	526
8:39	19	795	687	666.5	716.167	6214.69	850.694	2466.78	3532.6944	0.57774	2040.99	0.42226	607.173	548.5
8:43	20	647.5	666	677	663.5	256	6.25	182.25	131.125	0.94307	123.66	0.05693	669.684	638
8:59	24	647.5	666	677	663.5	256	6.25	182.25	131.125	0.66022	86.5712	0.33978	663.749	563

Table D4.48: Kalman Filter for case 19-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	725	526	511.5	587.5	18906.3	3782.25	5776	11344.25	0.5	5672.13	0.5	570.75	613
8:39	19	666.5	548.5	561.5	592.1667	5525.44	1906.78	940.444	3716.1111	0.71642	2662.3	0.28358	576.104	546
8:43	20	677	638	657	657.3333	386.778	373.778	0.11111	380.27778	0.8889	338.029	0.1111	644.668	547.5
8:59	24	698.5	563	562	607.8333	8220.44	2010.03	2100.69	5115.2361	0.51599	2639.42	0.48401	554.982	553.5

Table D4.49: Kalman Filter for case 45-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	613	619	641.5	624.5	132.25	30.25	289	81.25	0.5	40.625	0.5	635.75	686
8:39	19	546	680.5	797	674.5	16512.3	36	15006.3	8274.125	0.50122	4147.19	0.49878	741.636	747.5
8:43	20	547.5	792	753.5	697.667	22550	8898.78	3117.36	15724.403	0.55825	8778.21	0.44175	750.85	773
8:59	24	553.5	782	685	673.5	14400	11772.3	132.25	13086.125	0.62558	8186.43	0.37442	717.949	747

Table D4.50: Kalman Filter for case 39-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	619	641.5	686	648.833	890.028	53.7778	1381.36	471.903	0.5	235.951	0.5	658	698.5
8:39	19	680.5	797	747.5	741.667	3741.36	3061.78	34.0278	3401.57	0.51676	1757.8	0.48324	723.821	643.5
8:43	20	792	753.5	773	772.833	367.361	373.778	0.02778	370.569	0.85171	315.617	0.14829	753.796	746.5
8:59	24	782	685	747	738	1936	2809	81	2372.5	0.53118	1260.23	0.46882	746.766	746

Table D4.51: Kalman Filter for case 38-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	735.5	735.5	737.5	736.167	0.44444	0.44444	1.77778	0.44444	0.5	0.22222	0.5	683.75	688
8:39	19	643.5	679.5	677.5	666.833	544.444	160.444	113.778	352.444	0.50016	176.278	0.49984	682.748	763
8:43	20	746.5	770.5	795	770.667	584.028	0.02778	592.111	292.028	0.61592	179.866	0.38408	782.709	747
8:59	24	746	690	685	707	1521	289	484	905	0.5452	493.402	0.4548	713.198	746.5

Table D4.52: Kalman Filter for case 64-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	737.5	688	728.5	718	380.25	900	110.25	640.125	0.5	320.063	0.5	679.25	752
8:39	19	677.5	763	677	705.833	802.778	3268.03	831.361	2035.4	0.53645	1091.88	0.46355	711.767	755
8:43	20	795	747	792.5	778.167	283.361	971.361	205.444	627.361	0.73265	459.637	0.26735	782.474	830.5
8:59	24	685	746.5	668	699.833	220.028	2177.78	1013.36	1198.9	0.58043	695.877	0.41957	736.18	694

Table D4.53: Kalman Filter for case 29-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	752	627	615.5	664.833	7598.03	1431.36	2433.78	4514.69	0.5	2257.35	0.5	622.75	614.5
8:39	19	755	683	667	701.667	2844.44	348.444	1201.78	1596.44	0.70709	1128.83	0.29291	651.622	563
8:43	20	830.5	674	61.5	522	95172.3	23104	212060	59138.1	0.50473	29848.6	0.49527	309.879	548
8:59	24	694	689	672.5	685.167	78.0278	14.6944	160.444	46.3611	0.99845	46.2893	0.00155	672.307	559

Table D4.54: Kalman Filter for case 6-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	499.5	743.5	589.5	610.833	12395.1	17600.4	455.111	14997.8	0.5	7498.89	0.5	609.75	700
8:39	19	559.5	671.5	550.5	593.833	1178.78	6032.11	1877.78	3605.44	0.75489	2721.73	0.24511	587.143	750
8:43	20	521.5	694.5	526	580.667	3500.69	12958	2988.44	8229.36	0.57095	4698.56	0.42905	622.107	810
8:59	24	548	820	550	639.333	8341.78	32640.4	7980.44	20491.1	0.55143	11299.4	0.44857	666.629	720

Table D4.55: Kalman Filter for case 56-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	717	525	509	583.667	17777.8	3441.78	5575.11	10609.8	0.5	5304.89	0.5	569.5	728
8:39	19	680	571.5	557	602.833	5954.69	981.778	2100.69	3468.24	0.71668	2485.61	0.28332	605.448	672.5
8:43	20	795.5	641.5	558.5	665.167	16986.8	560.111	11377.8	8773.44	0.56204	4931.02	0.43796	608.427	699
8:59	24	689.5	550	558	599.167	8160.11	2417.36	1694.69	5288.74	0.65898	3485.16	0.34102	606.084	805.5

Table D4.56: Kalman Filter for case 8-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	525	509	728	587.333	3885.44	6136.11	19787.1	5010.78	0.5	2505.39	0.5	679	743
8:39	19	571.5	557	672.5	600.333	831.361	1877.78	5208.03	1354.57	0.74023	1002.7	0.25977	690.814	691.5
8:43	20	641.5	558.5	699	633	72.25	5550.25	4356	2811.25	0.57567	1618.36	0.42433	695.818	647.5
8:59	24	550	558	805.5	637.833	7714.69	6373.36	28112.1	7044.03	0.55152	3884.92	0.44848	734.64	681

Table D4.57: Kalman Filter for case 49-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	509	728	743	660	22801	4624	6889	13712.5	0.5	6856.25	0.5	686.5	686.5
8:39	19	557	672.5	691.5	640.333	6944.44	1034.69	2618.03	3989.57	0.73108	2916.68	0.26892	690.155	746
8:43	20	558.5	699	647.5	635	5852.25	4096	156.25	4974.13	0.61336	3050.92	0.38664	685.584	747.5
8:59	24	558	805.5	681	681.5	15252.3	15376	0.25	15314.1	0.54529	8350.7	0.45471	711.238	748.5

Table D4.58: Kalman Filter for case 59-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	743	686.5	745	724.833	330.028	1469.44	406.694	899.736	0.5	449.868	0.5	687.5	740
8:39	19	691.5	746	682	706.5	225	1560.25	600.25	892.625	0.60064	536.143	0.39936	705.163	687
8:43	20	647.5	747.5	804.5	733.167	7338.78	205.444	5088.44	3772.11	0.53318	2011.2	0.46682	749.648	781
8:59	24	681	748.5	679.5	703	484	2070.25	552.25	1277.13	0.72026	919.866	0.27974	707.893	778.5

Table D4.59: Kalman Filter for case 24-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	790	833.5	811.5	811.667	469.444	476.694	0.02778	473.069	0.5	236.535	0.5	720.75	580
8:39	19	777	688	644.5	703.167	5451.36	230.028	3441.78	2840.69	0.51998	1477.12	0.48002	613.539	564
8:43	20	788	763.5	746.5	766	484	6.25	380.25	245.125	0.8754	214.584	0.1246	723.761	542
8:59	24	820	683	748	750.333	4853.44	4533.78	5.44444	4693.61	0.51117	2399.25	0.48883	647.302	557

Table D4.50: Kalman Filter for case 25-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	833.5	811.5	580	741.667	8433.36	4876.69	26136.1	6655.028	0.5	3327.51	0.5	605	612.5
8:39	19	688	644.5	564	632.167	3117.36	152.111	4646.69	1634.736	0.7522	1229.65	0.2478	576.018	554.5
8:43	20	763.5	746.5	542	684	6320.25	3906.25	20164	5113.25	0.55367	2831.04	0.44633	547.579	535.5
8:59	24	683	748	557	662.667	413.444	7281.78	11165.4	3847.611	0.63448	2441.21	0.36552	549.141	566

Table D4.61: Kalman Filter for case 20-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	712	686.5	722.5	707	25	420.25	240.25	222.625	0.5	111.313	0.5	676.25	509
8:39	19	686	748.5	681	705.167	367.361	1877.78	584.028	1122.57	0.52362	587.798	0.47638	599.062	561.5
8:43	20	772	746.5	678	732.167	1586.69	205.444	2934.03	896.069	0.62349	558.691	0.37651	634.137	538.5
8:59	24	685	646	686	672.333	160.444	693.444	186.778	426.944	0.69776	297.903	0.30224	641.419	554

Table D4.62: Prediction Error of the Predictors

Dcode	Scode	Actual	Developed Model	KF Model	Naïve Model	d1	d2	d3
54	2200	794.5	721.66	667.6	663.32	0.0917	0.1597	0.165110132
47	1200	747	745.72	866.83	1005.69	0.0017	0.1604	0.346305221
48	1200	746	695.64	763.79	900.06	0.0675	0.0238	0.206514745
36	1200	746.5	731.6	746.85	649.93	0.02	0.0005	0.129363697
16	3000	632	630.23	735.21	529.45	0.0028	0.1633	0.162262658
21	1200	563	548.75	663.75	658.26	0.0253	0.179	0.16920071
19	1200	553.5	563.92	554.98	468.33	0.0188	0.0027	0.153875339
45	1200	747	716.24	717.47	70.72	0.0412	0.0395	0.905327979
39	1200	746	696.85	746.17	721.39	0.0659	0.0002	0.032989276
38	1200	746.5	746.24	713.2	643.64	0.0003	0.0446	0.137789685
64	2200	694	760.47	736.18	700	0.0958	0.0608	0.008645533
29	1200	559	566.51	672.31	557.11	0.0134	0.2027	0.003381038
6	3000	720	739.9	666.63	846.95	0.0276	0.0741	0.176319444
56	2200	805.5	721.24	606.08	698.37	0.1046	0.2476	0.132998138
8	3000	681	731.69	734.64	746.15	0.0744	0.0788	0.095668135
49	1200	748.5	730.5	711.24	786.17	0.024	0.0498	0.050327321
59	2200	778.5	713.85	707.89	659.65	0.083	0.0907	0.152665382
24	1200	557	562.93	647.3	543.08	0.0106	0.1621	0.024991023
25	1200	566	557.38	549.14	550.32	0.0152	0.0298	0.02770318
20	1200	554	555.73	641.42	544.85	0.0031	0.1578	0.016516245

Note: d1, d2, d3: The absolute relative error (%) of the Developed model, the Kalman Filter and the Naïve models, respectively

Appendix D5

1. General

This Appendix presents the database, the prediction results of the developed model, the Kalman Filter and the naïve models applied for boarding passenger prediction.

2. Database for the Developed Model and Kalman Filter Model

Table D5.1: Database of Boarding Passengers at Mackenzie Station (Route 95)

Day	Service	IB	OB	Model	R2
54	2200	3	5	794.5 (721.16)	681.5
58	2200	2	8	681.5	798
33	1200	7	5	571	537.5
42	1200	12	6	867	747
47	1200	9	7	747 (745.72)	866.5
62	2200	1	6	699	672.5
48	1200	11	10	746 (695.64)	866.5
36	1200	6	8	746.5 (730.63)	755
16	3000	9	9	632(630.23)	632.5
52	2200	8	9	781.5	785
3	3000	12	3	803.5	647.5
2	3000	7	9	673.5	666
7	3000	3	6	698.5	677
21	1200	7	11	563(548.75)	638
28	1200	8	11	562	657
19	1200	7	7	553.5 (563.92)	547.5
4	3000	5	16	782	792
11	3000	1	8	685	753.5
45	1200	10	8	747(716.24)	773
39	1200	7	10	746(696.85)	746.5
5	3000	6	6	690	770.5
55	2200	6	9	685	795
38	1200	9	9	746.5(746.24)	747
13	3000	7	9	668	792.5
64	2200	12	9	694(760.47)	830.5
14	3000	7	4	689	674
10	3000	4	0	672.5	661.5
29	1200	7	7	559(566.51)	548
35	1200	6	6	548	521.5

53	2200	11	7	820	694.5
17	1200	5	5	550	526
6	3000	6	6	720(739.39)	810
12	3000	9	6	689.5	795.5
18	1200	9	11	550	641.5
31	1200	7	5	558	558.5
56	2200	7	4	805.5 (721.24)	699
8	3000	8	6	681(731.69)	647.5
49	1200	6	4	748.5(730.5)	747.5
9	3000	7	8	679.5	804.5
59	2200	12	7	778.5(713.85)	781
41	1200	1	12	748	754.5
46	1200	10	4	747	766.5
57	2200	2	5	671	691.5
51	2200	9	10	694.5	777.5
1	3000	8	10	702	676.5
22	1200	8	7	556	540.5
40	1200	10	5	747.5	746.5
63	2200	14	10	798	796.5
50	1200	9	5	820	788
61	2200	9	10	683	763.5
43	1200	5	12	748	746.5
24	1200	8	13	557(562.93)	542
25	1200	7	2	566(557.38)	535.5
37	1200	1	5	746	869
65	2200	9	9	685	772
44	1200	6	5	646	746.5
60	2200	3	4	686	678
20	1200	8	10	554(555.73)	538.5
34	1200	10	8	553.5	657.5
26	1200	8	5	553	640
23	1200	6	7	569.5	548.5
30	1200	5	7	565.5	552.5
32	1200	12	3	572.5	569.5
27	1200	6	7	573.5	553.5
15	3000	11	9	655	673
66	1200	5	12	746.5	748.5
67	1200	2	10	755.5	866.5
68	1200	4	10	747	765.5
69	1200	10	11	756.5	866.5
70	1200	4	8	747	746

Note:

1. R1, R2, B1, and B2 = Running time and Boarding passengers of trip 24 and trip 20 respectively.
2. Numbers in brackets are the predicted running times of the bus in question. These, along with B1, B2, and R2, are the elements of the pattern matrix defining the boarding passenger being predicted.

Table D5.2: Database for Kalman Filter Predictor

Index	Arrival Time	Arrival Time	Arrival Time	Arrival Time
54	2200	0.003776	0.007337	0.001477
58	2200	0.0029347	0.010025	0.006042
33	1200	0.0122592	0.009302	0.005343
42	1200	0.0138408	0.008032	0.002681
47	1200	0.0120482	0.008078	0.005355
62	2200	0.0014306	0.008922	0.007429
48	1200	0.0147453	0.011541	0.005355
36	1200	0.0133958	0.010596	0.010695
16	3000	0.0174051	0.014229	0.020046
52	2200	0.0102367	0.011465	0.005827
3	3000	0.0149347	0.004633	0.011321
2	3000	0.0103935	0.013514	0.004367
7	3000	0.0042949	0.008863	0.0015
21	1200	0.0124334	0.017241	0.009116
28	1200	0.0142349	0.016743	0.014248
19	1200	0.0126468	0.012785	0.003663
4	3000	0.0063939	0.020202	0.005878
11	3000	0.0014599	0.010617	0.015056
45	1200	0.0133869	0.010349	0.016054
39	1200	0.0093834	0.013396	0.004662
5	3000	0.0086957	0.007787	0.007358
55	2200	0.0087591	0.011321	0.008856
38	1200	0.0120563	0.012048	0.010485
13	3000	0.010479	0.011356	0.002954
64	2200	0.0172911	0.010837	0.005298
14	3000	0.0101597	0.005935	0.004392
10	3000	0.005948	0	0.005997
29	1200	0.0125224	0.012774	0.001776
35	1200	0.0109489	0.011505	0.025022
53	2200	0.0134146	0.010079	0.004468
17	1200	0.0090909	0.009506	0.00545
6	3000	0.0083333	0.007407	0.006667
12	3000	0.0130529	0.007542	0.005882
18	1200	0.0163636	0.017147	0.008749
31	1200	0.0125448	0.008953	0.012567
56	2200	0.0086903	0.005722	0.004461
8	3000	0.0117474	0.009266	0.013015
49	1200	0.008016	0.005351	0.004021
9	3000	0.0103017	0.009944	0.005865
59	2200	0.0154143	0.008963	0.008734
41	1200	0.0013369	0.015905	0
46	1200	0.0133869	0.005219	0.002679
57	2200	0.0029806	0.007231	0.008785
51	2200	0.012959	0.012862	0.004373

1	3000	0.011396	0.014782	0.007391	0.006784
22	1200	0.0143885	0.012951	0.021661	0.003984
40	1200	0.0133779	0.006698	0.010724	0.004237
63	2200	0.0175439	0.012555	0.001481	0.009563
50	1200	0.0109756	0.006345	0.005148	0.006329
61	2200	0.0131772	0.013098	0	0.011998
43	1200	0.0066845	0.016075	0.001552	0.011091
24	1200	0.0071813	0.023985	0.001773	0.006897
25	1200	0.0123675	0.003735	0.00541	0.006531
37	1200	0.0013405	0.005754	0.009365	0.00424
65	2200	0.0131387	0.011658	0.004373	0.005618
44	1200	0.0092879	0.006698	0.005344	0.005827
60	2200	0.0043732	0.0059	0.007342	0.005536
20	1200	0.0108303	0.01857	0.003562	0.011788
34	1200	0.0180668	0.012167	0.007214	0.007775
26	1200	0.0144665	0.007813	0.010801	0.008056
23	1200	0.0105356	0.012762	0	0.00821
30	1200	0.0088417	0.01267	0.008795	0.012678
32	1200	0.0209607	0.005268	0.013323	0.003899
27	1200	0.0104621	0.012647	0.012693	0.001949
15	3000	0.0167939	0.013373	0.018519	0.004747

3. Prediction Results of the Developed Model

The following table shows part of an output returned from Matlab source code for the sake of illustration. All predictions are tabulated in Table D5.23

The CROSS_VALIDATION factors

11.6279 9.3710 9.6050 9.4747 9.5225 9.6709 9.7343

OpNei = 10

Columns 1 through 12

1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
-0.8663 0.2075 -0.8663 -0.5083 -0.1504 -0.5083 0.9233 0.2075 0.5654 0.5654 -1.5821 0.5654
1.2755 -0.0305 -1.3076 2.1134 0.7265 0.1718 0.7150 0.7207 -0.6026 1.1252 1.3795 -0.1229
-0.2401 0.9386 -1.6971 0.4226 1.6317 -0.3312 1.6317 0.5036 -0.7359 0.8071 -0.5841 -0.3970

Columns 13 through 24

```

1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
-0.5083 1.2812 1.2812 -0.1504 3.0708 0.2075 0.2075 0.9233 -0.5083 0.5654 0.5654 0.5654
0.1660 -1.4000 -1.4116 -1.5098 1.1310 0.0100 0.7265 0.7150 0.0678 0.0100 0.7207 -0.1865
-0.2857 -0.6803 -0.4880 -1.5959 0.8779 0.4884 0.6857 0.4176 0.6604 0.9083 0.4226 0.8830
Columns 25 through 36
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
0.5654 -1.2242 -2.6559 -0.1504 -0.5083 -0.1504 -0.8663 -0.5083 -0.5083 1.2812 -0.8663 -1.2242
0.1140 0.0562 -0.1345 -1.4462 -1.5734 1.5702 -1.5503 0.4145 0.0620 -1.5503 -1.4578 1.4026
1.2675 -0.3160 -0.4425 -1.5909 -1.8590 -0.1086 -1.8135 1.0601 0.9133 -0.6448 -1.4846 -0.0631
Columns 37 through 48
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
-0.5083 -1.2242 0.2075 -0.1504 1.6392 -1.2242 -0.8663 0.9233 0.9233 -0.1504 -0.8663 0.9233
-0.0363 0.7438 -0.0536 1.0906 0.7381 0.7265 -0.1518 0.1198 0.2064 -1.4809 0.7323 1.3159
-0.5841 0.4277 1.0044 0.7666 0.4985 0.6199 -0.1389 0.7312 -0.2907 -1.6668 0.4176 0.9235
Columns 49 through 60
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
-0.8663 0.9233 1.6392 -1.9400 -0.8663 0.5654 -0.8663 -1.2242 0.9233 0.2075 -0.8663 -0.1504
1.5702 -0.0131 0.7381 -1.3653 0.7150 0.0100 -0.4408 0.0215 -1.5040 -1.5098 -1.5156 -1.3249
0.8375 0.5896 0.4176 -1.7174 1.6570 0.6756 0.4176 -0.2755 -1.6870 -0.4830 -0.6600 -1.5858
Columns 61 through 69
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
-0.1504 -1.5821 -0.1504 0.5654 1.6392 0.9233 0.9233 1.2812 0.2075
-1.3711 -1.2902 -1.2787 -0.3367 0.7207 0.8247 0.7265 0.8363 0.7265
-1.5454 -1.3733 -1.5352 -0.3261 0.4378 1.6317 0.6098 1.6317 0.4125
boarder = 8
THE REAL-TIME PREDCTED # of PASS OF THE BUS IS 8.00 PAS.Program complete.HAVE A NICE DAY

```

Figure D5.1: An Example of the Matlab outputs for Boarding Passenger Prediction (Case 24-1200)

Table D5.3: Kalman Filter for test case 54-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	8
8:36	18	0.0039	0.00195	0.00475	0.00353	1.3E-07	2.5E-06	1.5E-06	1.3E-06	0.5	6.6E-07	0.5	0.01487	0.00669	
8:39	19	0.01332	0.01269	0.01852	0.01485	2.3E-06	4.6E-06	1.3E-05	3.5E-06	0.54337	1.9E-06	0.45663	0.01312	0.00148	
8:43	20	0.00527	0.01265	0.01337	0.01043	2.7E-05	4.9E-06	8.7E-06	1.6E-05	0.52822	8.3E-06	0.47178	0.00776	0.00734	
8:59	24	0.02096	0.01046	0.01679	0.01607	2.4E-05	3.1E-05	5.2E-07	2.8E-05	0.56542	1.6E-05	0.43458	0.013	0.004	

Table D5.4: Kalman Filter for test case 47-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	10
8:36	18	0.00135	0.01321	0.00291	0.00582	2E-05	5.5E-05	8.5E-06	3.7E-05	0.5	1.9E-05	0.5	0.01396	0.00437	
8:39	19	0.00604	0.00534	0.00268	0.00469	1.8E-06	4.3E-07	4E-06	1.1E-06	0.946	1.1E-06	0.054	0.00277	0.00536	
8:43	20	0.01003	0.0093	0.00803	0.00912	8.2E-07	3.3E-08	1.2E-06	4.3E-07	0.77802	3.3E-07	0.22198	0.00744	0.00808	
8:59	24	0.00293	0.01226	0.01384	0.00968	4.5E-05	6.7E-06	1.7E-05	2.6E-05	0.50316	1.3E-05	0.49684	0.011	0.012	

Table D5.5: Kalman Filter for test case 48-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	5
8:36	18	0.00291	0.00437	0.00467	0.00398	1.1E-06	1.5E-07	4.7E-07	6.5E-07	0.5	3.2E-07	0.5	0.01483	0.00618	
8:39	19	0.00604	0.00743	0.00536	0.00628	5.4E-08	1.3E-06	8.5E-07	6.9E-07	0.59477	4.1E-07	0.40523	0.00569	0.00536	
8:43	20	0.00803	0.00808	0.00892	0.00834	9.7E-08	7.1E-08	3.3E-07	8.4E-08	0.85511	7.2E-08	0.14489	0.00841	0.01154	
8:59	24	0.01384	0.01205	0.00143	0.00911	2.2E-05	8.7E-06	5.9E-05	1.6E-05	0.50115	7.8E-06	0.49885	0.006	0.015	

Table D5.6: Kalman Filter for test case 36-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	9
8:36	18	0.00437	0.00469	0.00618	0.00508	5.1E-07	1.5E-07	1.2E-06	3.3E-07	0.5	1.6E-07	0.5	0.01559	0.00437	
8:39	19	0.00536	0.00743	0.00536	0.00605	4.8E-07	1.9E-06	4.8E-07	1.2E-06	0.53226	6.4E-07	0.46774	0.0049	0.0107	
8:43	20	0.00808	0.00892	0.01154	0.00951	2.1E-06	3.5E-07	4.1E-06	1.2E-06	0.60435	7.3E-07	0.39565	0.01121	0.0106	
8:59	24	0.01205	0.00143	0.01475	0.00941	7E-06	6.4E-05	2.8E-05	3.5E-05	0.50511	1.8E-05	0.49489	0.013	0.013	

Table D5.7: Kalman Filter for test case 16-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	10
8:36	18	0.00467	0.00618	0.00437	0.00508	1.7E-07	1.2E-06	4.9E-07	7E-07	0.5	3.5E-07	0.5	0.01469	0.00338	
8:39	19	0.00743	0.00536	0.0107	0.00783	1.6E-07	6.1E-06	8.2E-06	3.1E-06	0.52635	1.6E-06	0.47365	0.00723	0.02005	
8:43	20	0.00892	0.01154	0.0126	0.01102	4.4E-06	2.7E-07	2.5E-06	2.3E-06	0.63043	1.5E-06	0.36957	0.01535	0.01423	
8:59	24	0.00143	0.01475	0.0134	0.00986	7.1E-05	2.4E-05	1.3E-05	4.7E-05	0.50764	2.4E-05	0.49236	0.014	0.017	

Table D5.8: Kalman Filter for test case 21-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	6
8:36	18	0.00685	0.01263	0.01517	0.01155	2.2E-05	1.2E-06	1.3E-05	1.2E-05	0.5	5.8E-06	0.5	0.02009	0.00761	
8:39	19	0.01132	0.00437	0.0015	0.00573	3.1E-05	1.9E-06	1.8E-05	1.7E-05	0.57461	9.5E-06	0.42539	0.0041	0.00912	
8:43	20	0.00463	0.01354	0.00886	0.00901	1.9E-05	2.1E-05	2.2E-08	2E-05	0.5967	1.2E-05	0.4033	0.00897	0.01724	
8:59	24	0.00143	0.01029	0.00429	0.00534	1.5E-05	2.5E-05	1.1E-06	2E-05	0.61458	1.2E-05	0.38542	0.009	0.012	

Table D5.9: Kalman Filter for test case 19-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	8
8:36	18	0.01517	0.00761	0.00978	0.01085	1.9E-05	1.1E-05	1.2E-06	1.5E-05	0.5	7.3E-06	0.5	0.01739	0.00761	
8:39	19	0.0015	0.00912	0.01425	0.00829	4.6E-05	6.9E-07	3.6E-05	2.3E-05	0.56753	1.3E-05	0.43247	0.01138	0.00912	
8:43	20	0.00886	0.01724	0.01674	0.01428	2.9E-05	8.8E-06	6.1E-06	1.9E-05	0.6291	1.2E-05	0.3709	0.01391	0.01279	
8:59	24	0.00429	0.01243	0.01423	0.01032	3.6E-05	4.5E-06	1.5E-05	2E-05	0.61362	1.3E-05	0.38638	0.014	0.013	

Table D5.10: Kalman Filter for test case 45-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	4
8:36	18	0.00437	0.00469	0.00618	0.00508	5.1E-07	1.5E-07	1.2E-06	3.3E-07	0.5	1.6E-07	0.5	0.01559	0.00761	
8:39	19	0.00536	0.00743	0.00536	0.00605	4.8E-07	1.9E-06	4.8E-07	1.2E-06	0.53226	6.4E-07	0.46774	0.00641	0.00912	
8:43	20	0.00808	0.00892	0.00886	0.00862	2.9E-07	9.1E-08	5.9E-08	1.9E-07	0.81132	1.6E-07	0.18868	0.00891	0.01035	
8:59	24	0.00143	0.01029	0.00146	0.00439	8.8E-06	3.5E-05	8.6E-06	2.2E-05	0.50179	1.1E-05	0.49821	0.006	0.013	

Table D5.11: Kalman Filter for test case 39-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	10
8:36	18	0.00969	0.00624	0.00875	0.00822	2.2E-06	4E-06	2.7E-07	3.1E-06	0.5	1.5E-06	0.5	0.01687	0.00761	
8:39	19	0.00588	0.01506	0.01605	0.01233	4.2E-05	7.4E-06	1.4E-05	2.5E-05	0.51511	1.3E-05	0.48489	0.01196	0.00912	
8:43	20	0.0202	0.01062	0.01035	0.01372	4.2E-05	9.6E-06	1.1E-05	2.6E-05	0.5983	1.5E-05	0.4017	0.00985	0.0134	
8:59	24	0.00639	0.00146	0.01339	0.00708	4.7E-07	3.2E-05	4E-05	1.6E-05	0.66256	1.1E-05	0.33744	0.013	0.009	

Table D5.12: Kalman Filter for test case 38-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	7
8:36	18	0.00143	0.00816	0.01356	0.00772	3.9E-05	2E-07	3.4E-05	2E-05	0.5	9.9E-06	0.5	0.01928	0.00761	
8:39	19	0.00446	0.00736	0.00886	0.00689	5.9E-06	2.2E-07	3.9E-06	3.1E-06	0.80921	2.5E-06	0.19079	0.00862	0.00912	
8:43	20	0.014	0.00779	0.01132	0.01103	8.8E-06	1.1E-05	8.2E-08	9.7E-06	0.55683	5.4E-06	0.44317	0.01034	0.01204	
8:59	24	0.00134	0.00939	0.00876	0.0065	2.7E-05	8.4E-06	5.1E-06	1.7E-05	0.56666	9.9E-06	0.43334	0.01	0.013	

Table D5.13: Kalman Filter for test case 64-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	8
8:36	18	0.00437	0.00469	0.00618	0.00508	5.1E-07	1.5E-07	1.2E-06	3.3E-07	0.5	1.6E-07	0.5	0.01559	0.00761	
8:39	19	0.00536	0.00743	0.00536	0.00605	4.8E-07	1.9E-06	4.8E-07	1.2E-06	0.53226	6.4E-07	0.46774	0.00641	0.00912	
8:43	20	0.00808	0.00892	0.00886	0.00862	2.9E-07	9.1E-08	5.9E-08	1.9E-07	0.81132	1.6E-07	0.18868	0.00891	0.0108	
8:59	24	0.00143	0.01029	0.01048	0.0074	3.6E-05	8.4E-06	9.5E-06	2.2E-05	0.50177	1.1E-05	0.49823	0.011	0.017	

Table D5.14: Kalman Filter for test case 29-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	5
8:36	18	0.00437	0.00469	0.00618	0.00508	5.1E-07	1.5E-07	1.2E-06	3.3E-07	0.5	1.6E-07	0.5	0.01559	0.00326	
8:39	19	0.00536	0.00743	0.00536	0.00605	4.8E-07	1.9E-06	4.8E-07	1.2E-06	0.53226	6.4E-07	0.46774	0.00437	0.00178	
8:43	20	0.00808	0.00892	0.00886	0.00862	2.9E-07	9.1E-08	5.9E-08	1.9E-07	0.81132	1.6E-07	0.18868	0.00753	0.01277	
8:59	24	0.01729	0.01016	0.00595	0.01113	3.8E-05	9.5E-07	2.7E-05	1.9E-05	0.502	9.8E-06	0.498	0.009	0.013	

Table D5.15: Kalman Filter for test case 6-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	5
8:36	18	0.00437	0.00469	0.00618	0.00508	5.1E-07	1.5E-07	1.2E-06	3.3E-07	0.5	1.6E-07	0.5	0.01559	0.00326	
8:39	19	0.00536	0.00743	0.00536	0.00605	4.8E-07	1.9E-06	4.8E-07	1.2E-06	0.53226	6.4E-07	0.46774	0.00437	0.00178	
8:43	20	0.00808	0.00892	0.00886	0.00862	2.9E-07	9.1E-08	5.9E-08	1.9E-07	0.81132	1.6E-07	0.18868	0.00753	0.0074	
8:59	24	0.01729	0.01016	0.00909	0.01218	2.6E-05	4.1E-06	9.5E-06	1.5E-05	0.50258	7.6E-06	0.49742	0.008	0.008	

Table D5.16: Kalman Filter for test case 56-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	7
8:36	18	0.00697	0.00952	0.00982	0.00877	3.2E-06	5.6E-07	1.1E-06	1.9E-06	0.5	9.5E-07	0.5	0.01741	0.01099	
8:39	19	0.00588	0.00875	0.01257	0.00907	1E-05	1E-07	1.2E-05	5.1E-06	0.54247	2.8E-06	0.45753	0.01185	0.00446	
8:43	20	0.00754	0.01715	0.00895	0.01121	1.3E-05	3.5E-05	5.1E-06	2.4E-05	0.52698	1.3E-05	0.47302	0.00683	0.00572	
8:59	24	0.01305	0.01636	0.01255	0.01399	8.7E-07	5.6E-06	2.1E-06	3.3E-06	0.83155	2.7E-06	0.16845	0.011	0.009	

Table D5.17: Kalman Filter for test case 8-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	6
8:36	18	0.00952	0.00982	0.01099	0.01011	3.5E-07	8.4E-08	7.7E-07	2.1E-07	0.5	1.1E-07	0.5	0.01799	0.00942	
8:39	19	0.00875	0.01257	0.00446	0.00859	2.5E-08	1.6E-05	1.7E-05	7.9E-06	0.50337	4E-06	0.49663	0.00692	0.0013	
8:43	20	0.01715	0.00895	0.00572	0.01061	4.3E-05	2.7E-06	2.4E-05	2.3E-05	0.54024	1.2E-05	0.45976	0.00369	0.00923	
8:59	24	0.01636	0.01254	0.00869	0.01253	1.5E-05	1.4E-10	1.5E-05	7.3E-06	0.72791	5.3E-06	0.27209	0.009	0.012	

Table D5.18: Kalman Filter for test case 49-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	6
8:36	18	0.01099	0.00942	0.0094	0.00994	1.1E-06	2.6E-07	2.9E-07	6.9E-07	0.5	3.4E-07	0.5	0.0172	0.00942	
8:39	19	0.01302	0.0044	0.00587	0.00776	2.8E-05	1.1E-05	3.6E-06	1.9E-05	0.50438	9.8E-06	0.49562	0.00763	0.0013	
8:43	20	0.00927	0.00535	0.00994	0.00819	1.2E-06	8E-06	3.1E-06	4.6E-06	0.75795	3.5E-06	0.24205	0.00785	0.00535	
8:59	24	0.01175	0.00802	0.0103	0.01002	3E-06	4E-06	7.8E-08	3.5E-06	0.66626	2.3E-06	0.33374	0.009	0.009	

Table D5.19: Kalman Filter for test case 59-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	7
8:36	18	0.00942	0.00583	0.0094	0.00821	1.5E-06	5.7E-06	1.4E-06	3.6E-06	0.5	1.8E-06	0.5	0.0172	0.00942	
8:39	19	0.01302	0.00402	0.00587	0.00763	2.9E-05	1.3E-05	3.1E-06	2.1E-05	0.52042	1.1E-05	0.47958	0.00757	0.0013	
8:43	20	0.00927	0.00535	0.00994	0.00819	1.2E-06	8E-06	3.1E-06	4.6E-06	0.77141	3.6E-06	0.22859	0.00797	0.00896	
8:59	24	0.01175	0.00802	0.0103	0.01002	3E-06	4E-06	7.8E-08	3.5E-06	0.66827	2.3E-06	0.33173	0.01	0.015	

Table D5.20: Kalman Filter for test case 24-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	7
8:36	18	0.00633	0.012	0.01109	0.00981	1.2E-05	4.8E-06	1.7E-06	8.4E-06	0.5	4.2E-06	0.5	0.01805	0.0069	
8:39	19	0.00515	0	0.0015	0.00222	8.6E-06	4.9E-06	5.1E-07	6.8E-06	0.61911	4.2E-06	0.38089	0.00356	0.00177	
8:43	20	0.00635	0.0131	0.01608	0.01184	3E-05	1.6E-06	1.8E-05	1.6E-05	0.55814	8.9E-06	0.44186	0.00976	0.02399	
8:59	24	0.01098	0.01318	0.00668	0.01028	4.9E-07	8.4E-06	1.3E-05	4.4E-06	0.74975	3.3E-06	0.25025	0.011	0.007	

Table D5.21: Kalman Filter for test case 25-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	
8:36	18	0.012	0.01191	0.0069	0.01027	3E-06	2.7E-06	1.1E-05	2.8E-06	0.5	1.4E-06	0.5	0.01595	0.0069	
8:39	19	0	0.00155	0.00177	0.00111	1.2E-06	2E-07	4.4E-07	7.1E-07	0.74969	5.3E-07	0.25031	0.00306	0.00177	
8:43	20	0.0131	0.01608	0.02399	0.01772	2.1E-05	2.7E-06	3.9E-05	1.2E-05	0.51086	6.1E-06	0.48914	0.01312	0.00374	
8:59	24	0.01318	0.00668	0.00718	0.00901	1.7E-05	5.4E-06	3.4E-06	1.1E-05	0.60632	6.9E-06	0.39368	0.006	0.012	3

Table D5.22: Kalman Filter for test case 20-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.0006	0.0006	0.0006	0.0006	0.5	0	0.5		0.025	
8:36	18	0.00562	0.00587	0.00554	0.00568	3.3E-09	3.9E-08	1.9E-08	2.1E-08	0.5	1E-08	0.5	0.01527	0.0069	
8:39	19	0.00437	0.00534	0.00734	0.00569	1.7E-06	1.2E-07	2.7E-06	9.2E-07	0.50283	4.6E-07	0.49717	0.00712	0.00177	
8:43	20	0.01166	0.0067	0.0059	0.00809	1.3E-05	1.9E-06	4.8E-06	7.3E-06	0.51528	3.8E-06	0.48472	0.0039	0.01857	
8:59	24	0.01314	0.00929	0.00437	0.00893	1.8E-05	1.3E-07	2.1E-05	8.9E-06	0.58762	5.2E-06	0.41238	0.01	0.011	7

Table D5. 23: Prediction Error of the 3 Predictors

ID	Age	Actual	Developed Model	KF Model	Naïve Model	d1	d2	d3
54	2200	3	5	8	6	0.667	1.66667	1
47	1200	9	7	10	14	0.222	0.11111	0.555556
48	1200	11	7	5	2	0.364	0.54545	0.818182
36	1200	6	7	9	9	0.167	0.5	0.5
16	3000	9	7	10	11	0.222	0.11111	0.222222
21	1200	7	9	6	6	0.286	0.14286	0.142857
19	1200	7	6	8	5	0.143	0.14286	0.285714
45	1200	10	6	4	1	0.4	0.6	0.9
39	1200	7	7	10	10	0	0.42857	0.428571
38	1200	9	7	8	6	0.222	0.11111	0.333333
64	2200	12	7	8	7	0.417	0.33333	0.416667
29	1200	7	6	5	4	0.143	0.28571	0.428571
6	3000	6	8	5	6	0.333	0.16667	0
56	2200	7	5	7	5	0.286	0	0.285714
8	3000	8	4	6	11	0.5	0.25	0.375
49	1200	6	7	7	5	0.167	0.16667	0.166667
59	2200	12	7	7	6	0.417	0.41667	0.5
24	1200	8	8	7	5	0	0.125	0.375
25	1200	7	8	3	1	0.143	0.57143	0.857143
20	1200	8	7	7	8	0.125	0.125	0

Note: d1.d2, d3= Absolute Relative Error the for Developed Model, KF model, and the naïve model, respectively

Appendix D6

1. General

This Appendix presents the database, the prediction results of the developed model, the Kalman Filter, and the Naive model applied for alighting passenger prediction.

2. Database for the Developed Model and Kalman Filter Model

Table D6.1: Database of Alighting Passengers at Mackenzie Station (Route 95)

Day	Time	Actual	Predicted	As
54	2200	23	27	23
58	2200	21	26	23
33	1200	25	28	21
42	1200	12	13	13
47	1200	11	14	13
62	2200	21	23	27
48	1200	14	17	13
36	1200	13	14	10
16	3000	17	17	16
52	2200	28	28	23
3	3000	17	17	17
2	3000	17	16	17
7	3000	17	16	17
21	1200	22	33	25
28	1200	22	31	25
19	1200	23	33	23
4	3000	17	17	15
11	3000	17	17	17
45	1200	14	16	12
39	1200	13	14	11
5	3000	17	17	15
55	2200	23	31	20
38	1200	12	14	12
13	3000	17	17	17
64	2200	22	21	22
14	3000	17	17	16

10	3000	17	17	16
29	1200	25	28	20
35	1200	25	27	22
53	2200	28	21	26
17	1200	25	35	22
6	3000	17	17	15
12	3000	17	17	17
18	1200	29	27	23
31	1200	25	28	28
56	2200	22	22	20
8	3000	17	17	17
49	1200	11	14	13
9	3000	17	17	17
59	2200	27	29	25
41	1200	13	17	9
46	1200	12	17	12
57	2200	24	24	19
51	2200	26	29	22
1	3000	17	17	16
22	1200	27	26	23
40	1200	12	12	13
63	2200	25	25	22
50	1200	12	14	11
61	2200	20	30	22
43	1200	12	14	11
24	1200	23	29	26
25	1200	26	29	25
37	1200	15	16	13
65	2200	20	22	21
44	1200	11	14	13
60	2200	25	23	23
20	1200	26	26	23
34	1200	27	26	27
26	1200	24	29	26
23	1200	25	26	19
30	1200	21	27	22
32	1200	20	22	23
27	1200	19	26	23
15	3000	17	17	16

Note:

A1, A2, and A3 = Alighting Passengers of trip 24th, trip 20th, and trip 19th, respectively.

3. Prediction Results of the Developed Model

The following table shows part of an output returned from Matlab source code for the sake of illustration. All predictions are tabulated in Table D6.22

```

The CROSS_VALIDATION factors
NaN 4.2983 4.0432 4.2601 5.0041 6.0432 7.1430 8.0114

OpNei = 15

Columns 31 through 45

1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000 1.0000
2.1420 -0.7306 -0.7306 0.8653 1.0249 0.0673 -0.7306 -1.2094 -0.7306 1.1845 -0.7306 -0.7306
0.3865 1.1845 -0.7306
0.6395 -0.7446 -0.3491 0.8372 1.8258 0.2441 -0.3491 -1.1400 -0.3491 1.2327 -1.9309 -1.3377
0.0463 0.6395 -0.5468

Columns 46 through 60

1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000 1.0000
0.7057 -1.5286 0.5461 -1.2094 1.3440 -1.2094 1.1845 1.1845 -0.8902 0.0673 -1.2094 0.2269
0.7057 1.1845 0.7057
0.8372 -1.1400 0.6395 -1.5354 0.6395 -1.5354 1.4304 1.2327 -1.1400 0.4418 -1.1400 0.8372
1.6281 1.4304 0.0463

Columns 61 through 64

1.0000 1.0000 1.0000 1.0000
0.8653 0.0673 0.7057 -0.7306
0.6395 0.8372 0.8372 -0.5468

20.6431
2.8937
2.0531

deboarder = 25

THE REAL-TIME PREDCTED # of PASS OF THE BUS IS 25.00 PAS.Program complete.HAVE A NICE DAY

```

Figure D6.1: An Example of Matlab output for de-boarding Prediction (Case 20-1200)

Table D6.2: Kalman Filter for test set 54-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	23	23	16	20	9	9	16	9	0.5	4.5	0.5	9	23
8:43	20	22	26	17	21.6667	0.1111	18.778	21.778	9.44444	0.5962	5.63077	0.4038	19.4228	27
8:59	24	20	19	17	18.6667	1.7778	0.1111	2.7778	0.94444	0.8744	0.82583	0.1256	18	23

Table D6.3: Kalman Filter for test set 47-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	23	23	21	20	9	9	1	9	0.5	4.5	0.5	11.5	13
8:43	20	27	26	28	27	0	1	1	0.5	0.90909	0.45455	0.09091	26.6364	14
8:59	24	23	21	25	23	0	4	4	2	0.55102	1.10204	0.44898	20	11

Table D6.4: Kalman Filter for test set 48-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	13	13	27	20	49	49	49	49	0.5	24.5	0.5	14.5	13
8:43	20	13	14	23	16.6667	13.444	7.1111	40.111	10.2778	0.77189	7.93328	0.22811	20.7189	17
8:59	24	12	11	21	14.6667	7.1111	13.444	40.111	10.2778	0.63923	6.56991	0.36077	20	14

Table D6.5: Kalman Filter for test set 36-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	13	27	13	20	49	49	49	49	0.5	24.5	0.5	7.5	10
8:43	20	14	23	17	18	16	25	1	20.5	0.68702	14.084	0.31298	14.8092	14
8:59	24	11	21	14	15.3333	18.778	32.111	1.7778	25.4444	0.60838	15.48	0.39162	14	13

Table D6.6: Kalman Filter for test set 16-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	27	13	10	20	49	49	100	49	0.5	24.5	0.5	6	16
8:43	20	23	17	14	18	25	1	16	13	0.74257	9.65347	0.25743	14.5149	17
8:59	24	21	14	13	16	25	4	9	14.5	0.62487	9.06064	0.37513	15	17

Table D6.7: Kalman Filter for test set 21-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	17	17	17	20	9	9	9	9	0.5	4.5	0.5	9.5	25
8:43	20	17	16	16	16.3333	0.4444	0.1111	0.1111	0.27778	0.94505	0.26252	0.05495	16.4945	33
8:59	24	17	17	17	17	0	0	0	0	1	0	0	17	22

Table D6.8: Kalman Filter for test set 19-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	17	25	25	20	9	25	25	17	0.5	8.5	0.5	13.5	23
8:43	20	16	33	31	26.6667	113.78	40.111	18.778	76.9444	0.52617	40.486	0.47383	27.2094	33
8:59	24	17	22	22	20.3333	11.111	2.7778	2.7778	6.94444	0.87229	6.05754	0.12771	23	23

Table D6.9: Kalman Filter for test set 45-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	23	15	17	20	9	25	9	17	0.5	8.5	0.5	9.5	12
8:43	20	33	17	17	22.3333	113.78	28.444	28.444	71.1111	0.5282	37.5607	0.4718	14.641	16
8:59	24	23	17	17	19	16	4	4	10	0.82627	8.2627	0.17373	17	14

Table D6.10: Kalman Filter for test set 39-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	15	17	12	20	25	9	64	17	0.5	8.5	0.5	7	11
8:43	20	17	17	16	16.6667	0.1111	0.1111	0.4444	0.11111	0.98726	0.1097	0.01274	15.9363	14
8:59	24	17	14	14	15	4	1	1	2.5	0.51073	1.27684	0.48927	14	13

Table D6.11: Kalman Filter for test set 38-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	11	15	20	20	81	25	0	53	0.5	26.5	0.5	11	12
8:43	20	14	17	31	20.6667	44.444	13.444	106.78	28.9444	0.65701	19.0168	0.34299	24.4832	14
8:59	24	13	17	23	17.6667	21.778	0.4444	28.444	11.1111	0.73057	8.11742	0.26943	21	12

Table D6.12: Kalman Filter for test set 64-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	20	12	17	20	0	64	9	32	0.5	16	0.5	9.5	22
8:43	20	31	14	17	20.6667	106.78	44.444	13.444	75.6111	0.54784	41.4228	0.45216	19.2608	21
8:59	24	23	12	17	17.3333	32.111	28.444	0.1111	30.2778	0.7031	21.2882	0.2969	18	22

Table D6.13: Kalman Filter for test set 29-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	22	16	16	20	4	16	16	10	0.5	5	0.5	9	20
8:43	20	21	17	17	18.3333	7.1111	1.7778	1.7778	4.44444	0.68	3.02222	0.32	17.96	28
8:59	24	22	17	17	18.6667	11.111	2.7778	2.7778	6.94444	0.58936	4.09275	0.41064	22	25

Table D6.14: Kalman Filter for test set 6-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	22	26	22	20	4	36	4	20	0.5	10	0.5	12	15
8:43	20	17	21	35	24.3333	53.778	11.111	113.78	32.4444	0.56677	18.3884	0.43323	26.3353	17
8:59	24	25	28	25	26	1	4	1	2.5	0.89311	2.23277	0.10689	24	17

Table D6.15: Kalman Filter for test set 56-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	17	23	28	20	9	9	64	9	0.5	4.5	0.5	15	20
8:43	20	17	27	28	24	49	9	16	29	0.536	15.544	0.464	24.288	22
8:59	24	17	29	25	23.6667	44.444	28.444	1.7778	36.4444	0.58789	21.4252	0.41211	24	22

Table D6.16: Kalman Filter for test set 8-3000

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	23	28	20	20	9	64	0	36.5	0.5	18.25	0.5	11	17
8:43	20	27	28	22	25.6667	1.7778	5.4444	13.444	3.61111	0.85823	3.09918	0.14177	21.2912	17
8:59	24	29	25	22	25.3333	13.444	0.1111	11.111	6.77778	0.59304	4.01951	0.40696	20	17

Table D6.17: Kalman Filter for test set 49-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	20	17	13	20	0	9	49	4.5	0.5	2.25	0.5	7.5	13
8:43	20	17	14	17	16	1	4	1	2.5	0.65517	1.63793	0.34483	15.6207	14
8:59	24	25	22	17	21.3333	13.444	0.4444	18.778	6.94444	0.55275	3.83851	0.44725	16	11

Table D6.18: Kalman Filter for test set 59-2200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	17	13	17	20	9	49	9	29	0.5	14.5	0.5	9.5	25
8:43	20	17	14	17	16	1	4	1	2.5	0.87179	2.17949	0.12821	18.0256	29
8:59	24	17	11	17	15	4	16	4	10	0.54913	5.49133	0.45087	22	27

Table D6.19: Kalman Filter for test set 24-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	11	22	11	20	81	4	81	42.5	0.5	21.25	0.5	6.5	26
8:43	20	14	30	14	19.3333	28.444	113.78	28.444	71.1111	0.565	40.1775	0.435	19.2201	29
8:59	24	12	20	12	14.6667	7.1111	28.444	7.1111	17.7778	0.76526	13.6046	0.23474	16	23

Table D6.20: Kalman Filter for test set 25-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	22	11	26	20	4	81	36	42.5	0.5	21.25	0.5	14	25
8:43	20	30	14	29	24.3333	32.111	106.78	21.778	69.4444	0.56635	39.3298	0.43365	27.2654	29
8:59	24	20	12	23	18.3333	2.7778	40.111	21.778	21.4444	0.73918	15.8513	0.26082	25	26

Table D6.21: Kalman Filter for test set 20-1200

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:36	17	0	0	0	20	400	400	400	400	0.5	0	0.5		2
8:39	19	21	13	23	20	1	49	9	25	0.5	12.5	0.5	12.5	23
8:43	20	23	14	23	20	9	36	9	22.5	0.6087	13.6957	0.3913	23	26
8:59	24	20	11	25	18.6667	1.7778	58.778	40.111	30.2778	0.59223	17.9313	0.40777	25	26

**Table D6.22: Prediction Error of the 3 Predictors for
Alighting Passenger Prediction**

Dcode	Scode	Actual	Developed Model	KF. Model	Naïve Model	d1	d2	d3
54	2200	23	25	18	30	0.087	0.217	0.304348
47	1200	11	12	20	13	0.091	0.818	0.181818
48	1200	14	14	20	16	0	0.429	0.142857
36	1200	13	12	14	12	0.077	0.077	0.076923
16	3000	17	17	15	16	0	0.118	0.058824
21	1200	22	24	17	35	0.091	0.227	0.590909
19	1200	23	24	23	23	0.043	0	0
45	1200	14	12	17	16	0.143	0.214	0.142857
39	1200	13	12	14	12	0.077	0.077	0.076923
38	1200	12	12	21	10	0	0.75	0.166667
64	2200	22	23	18	21	0.045	0.182	0.045455
29	1200	25	23	22	28	0.08	0.12	0.12
6	3000	17	17	24	12	0	0.412	0.294118
56	2200	22	23	24	20	0.045	0.091	0.090909
8	3000	17	17	20	17	0	0.176	0
49	1200	11	12	16	14	0.091	0.455	0.272727
59	2200	27	24	22	29	0.111	0.185	0.074074
24	1200	23	25	16	25	0.087	0.304	0.086957
25	1200	26	25	25	23	0.038	0.038	0.115385
20	1200	26	25	25	28	0.038	0.038	0.076923

Appendix D7

1. General

This appendix includes the databases for the two bus routes and the prediction results of the developed model and the reference predictors.

2. Database for Route 1 and Route 95

The databases were retrieved from the APC-AVL system embedded on the buses running on the two bus routes.

Table D7.1 Database for route 1 (seconds)

RCODE	8:59 trip	8:47 trip	8:37 trip	8:27 trip
1	877	992	940	818
2	762	914	900	713
3	929	903	1003	789
4	945	911	1080	922
5	810	800	923	850
6	1029	917	900	880
7	1030	950	946	915
8	914	968	1016	726
9	948	1074	890	817
10	934	901	873	814
11	900	997	956	794
12	986	1013	960	846
13	863	1062	1003	951
14	1122	846	920	855
15	905	958	954	933
16	928	961	932	814
17	1017	844	959	858
18	923	869	1083	809
19	909	1040	919	893
20	840	926	862	842
21	945	975	1001	887
22	793	947	896	895
23	984	879	968	852

24	1070	886	917	912
25	853	980	1006	1000
26	998	850	1008	889
27	1035	1012	950	856
28	768	992	807	913
29	783	862	975	861
30	971	914	1088	789
31	880	827	1043	887
32	982	732	844	857
33	994	1031	959	840
34	984	890	913	936
35	1038	969	887	972
36	980	960	870	825
37	1029	940	987	863
38	805	844	932	807
39	916	850	970	899
40	903	903	937	711
41	767	827	930	918
42	946	840	994	871
43	819	1076	1021	873
44	1050	944	1127	932
45	842	824	861	840
46	967	935	887	849
47	938	833	1045	779
48	831	812	935	754
49	714	914	859	814
50	912	1028	989	874
51	823	950	1085	824
52	975	1000	1020	775
53	965	829	1116	923
54	1076	895	1004	813
55	973	914	1108	856
56	857	825	939	857
57	953	815	878	842
58	823	1026	949	882
59	916	939	1057	841
60	913	878	880	754
61	918	1014	1014	793
62	888	960	919	872
63	1020	846	1008	779
64	742	1064	881	834
65	958	966	972	799
66	1001	1077	881	784
67	986	1045	927	794
68	972	874	1001	832
69	921	817	941	868
70	981	932	1060	850

Table D7.2: Database for route 95

8:59	8:43	8:40	8:37	B1	B2	A1	A2	A3
632	611	702	618	6	11	11	13	11
639	699	594	529	3	22	10	11	12
571	333	631	553	9	9	9	10	11
653	465	730	622	6	9	8	7	9
487	557	563	720	4	16	14	11	10
667	592	637	567	8	7	11	12	9
608	484	606	766	8	2	10	10	11
521	569	528	547	1	3	11	14	8
542	417	411	687	4	12	10	11	11
619	520	422	499	6	3	10	12	9
620	509	562	541	10	17	9	12	11
554	635	544	579	5	9	12	13	10
584	486	558	646	7	8	10	14	10
684	449	488	804	9	10	9	12	11
583	450	602	635	3	6	12	14	11
610	567	601	585	1	3	17	23	15
651	465	536	674	13	12	19	18	15
656	700	638	510	6	16	15	22	15
616	597	473	615	11	18	17	17	15
521	571	513	668	13	9	15	22	17
519	524	569	596	8	13	18	17	15
631	571	497	609	9	8	17	17	13
568	674	671	642	5	16	15	19	17
639	316	689	665	5	7	17	19	17
533	669	602	786	2	8	16	19	17
633	502	489	816	10	13	13	17	15
642	540	632	692	10	8	15	21	17
629	628	410	494	11	8	17	19	13
553	598	549	660	4	1	14	18	15
589	378	807	570	6	4	17	19	19
632	389	616	524	8	11	13	15	15
522	597	507	692	3	4	17	19	14
570	448	647	671	6	4	18	17	18
550	376	567	606	7	8	17	18	15
662	503	658	588	7	4	9	9	7
701	500	561	540	5	3	10	11	9
646	423	679	448	11	13	8	9	8
568	428	656	468	9	1	9	9	7
620	568	601	512	4	5	8	8	9
596	348	566	585	9	10	9	11	6
562	671	604	642	6	5	8	9	9
627	606	573	647	3	7	8	9	7
614	512	581	506	12	11	7	9	9
547	537	634	692	6	8	9	11	8

571	449	531	666	8	25	8	11	8
593	435	531	764	1	2	7	9	9
643	577	586	576	6	14	9	11	9
541	630	536	624	12	12	7	9	9
576	667	483	574	8	7	8	9	7
582	527	646	694	4	3	17	19	15
661	601	568	729	8	7	19	19	15
703	735	478	673	5	7	19	14	17
685	766	559	891	3	6	15	18	15
657	502	576	609	14	4	15	21	13
626	763	425	666	8	19	15	15	13
612	523	492	436	7	4	16	16	13
527	469	461	572	7	8	14	17	15
585	596	663	443	12	9	18	19	17
607	534	576	614	9	13	17	15	15
620	656	663	498	11	11	13	20	15
554	605	625	560	3	9	14	15	18
572	384	619	538	6	3	17	17	15
638	236	539	610	8	14	15	14	15
544	593	606	716	9	18	13	15	14
566	438	494	774	8	3	14	6	5
623	630	536	666	12	4	16	22	19
691	698	540	571	6	6	18	26	21
551	441	600	642	16	8	21	23	17
642	640	462	551	3	1	18	23	18
565	555	490	534	17	3	21	23	23
591	481	600	490	5	7	17	18	19
576	697	688	442	6	2	21	23	18
676	488	611	618	6	12	23	22	23
654	662	556	689	2	7	21	23	18
494	698	583	645	4	8	11	12	8
589	330	592	624	8	18	13	13	11
638	834	721	627	9	2	10	12	10
524	675	581	571	13	18	11	12	9
597	644	651	806	7	14	10	10	11
585	463	581	604	8	6	11	14	8
635	514	531	489	6	9	10	11	11
561	402	608	596	15	17	10	12	9
612	413	659	570	4	9	9	12	11
580	450	471	613	6	10	12	13	10
532	598	511	587	9	10	10	14	10
587	724	573	544	5	6	9	12	11
583	674	618	458	10	16	12	14	11
592	490	522	582	9	14	9	12	11
592	322	541	476	10	19	10	12	9
522	563	629	616	7	4	22	24	18
667	466	494	712	12	11	23	23	19

680	361	658	852	4	12	23	18	22
601	584	646	675	9	9	19	23	19
568	597	542	664	14	16	19	26	17
620	625	632	726	8	11	18	18	17
586	513	675	666	7	6	20	20	16
612	498	608	749	1	4	18	22	19
513	519	645	607	10	6	23	24	21
699	522	595	637	8	9	21	19	19
626	561	587	562	2	3	17	25	18

For each trip, APC data of the whole year of observations were processed so that the mean and standard deviation can be used. The following tables provided by the OC Transpo are shown for the sake of illustration.

Table D7.3 OC Transpo data for bus trips to Gladstone during 8:30-9:00 AM

PROCESSED: 2006-06-20 11:12:12			
AUTOMATIC PASSENGER COUNTING SYSTEM			
REVENUE TIME UTILIZATION			
WEEKDAY SERVICE			
FROM STOP: RA945 BILLINGS BRIDGE 4C		BOOKING: JAN05	
TO STOP: CH080 BANK GLADSTONE NS		PERIOD: 2005-01-03 TO 2005-04-23	
ROUTE: 1 SOUTH KEYS - OTTAWA ROCKLIFFE		DAY TYPE: WEEKDAY	
		TIME: 08:30:00 TO 08:45:00	
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	7.03	3.17	43.88
Stop and go time	1.46	2.07	9.11
Idle time	1.02	0.98	6.40
Dwell time	2.98	0.85	18.63
Excess time	3.52	0.80	21.99
	-----	-----	-----
TOTAL	16.02	1.28	100%
Layover time	0.91		
Average sched time per trip	10.00		
Total distance (KM)	3.67		
Average moving speed (KM/HR)	31.33		
Average total speed (KM/HR)	13.75		
Total trips captured:	6		

PROCESSED: 2006-06-20 11:24:24

AUTOMATIC PASSENGER COUNTING SYSTEM

REVENUE TIME UTILIZATION

WEEKDAY SERVICE

BOOKING: SEP05

PERIOD: 2005-09-05 TO 2005-12-17

DAY TYPE: WEEKDAY

TIME: 08:30:00 TO 08:45:00

FROM STOP: RA945 BILLINGS BRIDGE 4C

TO STOP: CH080 BANK GLADSTONE NS

ROUTE: 1 Bank - - Rockcliffe

	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	10.52	1.02	65.78
Stop and go time	0.00	0.00	0.00
Idle time	0.22	0.43	1.36
Dwell time	2.45	0.79	15.35
Excess time	2.80	0.69	17.51
	-----	-----	-----
TOTAL	15.99	1.66	100%

Layover time	0.00
Average sched time per trip	10.00
Total distance (KM)	3.67
Average moving speed (KM/HR)	20.94
Average total speed (KM/HR)	13.77

Total trips captured: 4

PROCESSED: 2006-06-20 11:12:32

AUTOMATIC PASSENGER COUNTING SYSTEM

REVENUE TIME UTILIZATION

WEEKDAY SERVICE

BOOKING: JAN05

PERIOD: 2005-01-03 TO 2005-04-23

DAY TYPE: WEEKDAY

TIME: 08:45:00 TO 09:00:00

FROM STOP: RA945 BILLINGS BRIDGE 4C

TO STOP: CH080 BANK GLADSTONE NS

ROUTE: 1 SOUTH KEYS - OTTAWA ROCKLIFFE

	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	10.21	1.85	66.28
Stop and go time	0.29	1.21	1.91
Idle time	0.45	0.60	2.94
Dwell time	2.06	0.54	13.37
Excess time	2.39	0.86	15.49
	-----	-----	-----
TOTAL	15.41	1.56	100%

Layover time	7.20
Average sched time per trip	10.00
Total distance (KM)	3.67
Average moving speed (KM/HR)	21.56
Average total speed (KM/HR)	14.29

Total trips captured: 17

PROCESSED: 2006-06-20 11:15:15			
AUTOMATIC PASSENGER COUNTING SYSTEM			
REVENUE TIME UTILIZATION			
WEEKDAY SERVICE			
FROM STOP: RA945 BILLINGS BRIDGE 4C		BOOKING: SEP05	
TO STOP: CH080 BANK GLADSTONE NS		PERIOD: 2005-09-05 TO 2005-12-17	
ROUTE: 1 Bank - - Rockcliffe		DAY TYPE: WEEKDAY	
		TIME: 08:45:00 TO 09:00:00	
	Average Time per Trip Minutes	Std Dev	% of Time
	-----	---	----
Moving between stops	9.08	1.73	59.26
Stop and go time	0.65	1.17	4.22
Idle time	0.83	0.71	5.45
Dwell time	1.85	0.60	12.10
Excess time	2.91	0.87	18.97
	-----	-----	-----
TOTAL	15.32	1.54	100%
Layover time	2.17		
Average sched time per trip	10.00		
Total distance (KM)	3.67		
Average moving speed (KM/HR)	24.26		
Average total speed (KM/HR)	14.38		
Total trips captured:	12		

Table D7.4 OC Transpo data for bus trips to Mackenzie during 8:00 -10:00 AM

PROCESSED: 2005-09-14 10:22:01								
AUTOMATIC PASSENGER COUNTING SYSTEM								
POINT CHECK								
WEEKDAY SERVICE								
STOP: CD910 MACKENZIE KING 2A			BOOKING: APR05					
ROUTE: 95 Transitway			PERIOD: 2005-04-24 TO 2005-06-25					
DIRECTION: 1 WBND			TIME: 08:00:00 TO 10:00:00					
			DAY OF THE WEEK: MON TUE WED THU FRI					
T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95020A	2	08:01	08:03	2	5	10	42	2005-05-10 TUE
			08:04	3	11	11	54	2005-05-30 MON
			AVERAGE	3	8	11	48 (2)	
95024A	3	08:04	08:06	2	11	4	47	2005-04-25 MON
			08:06	2	3	8	50	2005-05-05 THU
			08:06	2	4	9	31	2005-06-15 WED
			AVERAGE	2	6	7	43 (3)	
95052A	2	08:08	08:11	3	3	9	28	2005-05-19 THU
95054A	3	08:11	08:11	0	5	3	21	2005-05-04 WED
			08:12	1	8	10	40	2005-05-30 MON
			08:14	3	6	9	61	2005-05-31 TUE
			08:12	1	7	11	69	2005-06-07 TUE
			08:17	6	8	10	52	2005-06-17 FRI
			08:14	3	7	11	42	2005-06-21 TUE
			AVERAGE	2	7	9	48 (6)	
95010A	2	08:14	08:19	5	10	12	82	2005-05-17 TUE
			08:18	4	9	11	46	2005-06-06 MON
			08:19	5	3	11	53	2005-06-08 WED
			AVERAGE	5	7	11	60 (3)	
95003A	3	08:17	08:20	3	5	12	53	2005-04-25 MON
			08:21	4	1	9	49	2005-04-27 WED
			08:17	0	2	5	39	2005-05-03 TUE
			08:19	2	9	7	59	2005-05-12 THU
			08:21	4	4	9	21	2005-05-17 TUE
			AVERAGE	3	4	8	44 (5)	
95056A	2	08:20	08:23	3	10	7	60	2005-05-18 WED

			08:23	3	17	9	53	2005-06-02 THU
			AVERAGE	3	14	8	57 (2)	
95012A	3	08:24	08:26	2	17	5	48	2005-05-10 TUE
PROCESSED: 2005-09-14 10:22:02								
AUTOMATIC PASSENGER COUNTING SYSTEM								
POINT CHECK								
WEEKDAY SERVICE								
STOP:	CD910 MACKENZIE KING 2A	BOOKING: APR05						
ROUTE:	95 Transitway	PERIOD: 2005-04-24 TO 2005-06-25						
DIRECTION:	1 WBND	TIME: 08:00:00 TO 10:00:00						
DAY OF THE WEEK: MON TUE WED THU FR								
T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
679A	14	08:26	08:30	4	4	5	33	2005-05-18 WED
95055A	2	08:27	08:28	1	8	8	35	2005-05-17 TUE
			08:28	1	0	3	26	2005-06-03 FRI
			AVERAGE	1	4	6	31 (2)	
95057A	3	08:30	08:32	2	6	16	41	2005-05-03 TUE
			08:32	2	11	16	58	2005-05-06 FRI
			08:36	6	9	14	48	2005-05-10 TUE
			08:32	2	7	10	56	2005-06-15 WED
			AVERAGE	3	8	14	51 (4)	
95026A	2	08:33	08:36	3	12	8	69	2005-05-18 WED
			08:36	3	8	7	51	2005-05-20 FRI
			08:37	4	9	8	66	2005-06-16 THU
			AVERAGE	3	10	8	62 (3)	
95005A	3	08:36	08:38	2	3	8	26	2005-05-06 FRI
			08:38	2	2	9	31	2005-05-17 TUE
			08:39	3	5	16	58	2005-05-19 THU
			AVERAGE	2	3	11	38 (3)	
680A	14	08:38	08:44	6	1	1	26	2005-05-09 MON
95009A	3	08:40	08:43	3	8	6	69	2005-04-26 TUE
			08:41	1	0	9	36	2005-04-27 WED
			08:43	3	8	16	66	2005-05-02 MON
			08:43	3	3	11	50	2005-05-03 TUE
			08:42	2	9	13	62	2005-05-04 WED
			AVERAGE	2	6	11	57 (5)	
95060A	2	08:43	08:45	2	11	4	45	2005-04-26 TUE
			08:46	3	4	15	56	2005-05-10 TUE
			08:50	7	1	22	45	2005-05-25 WED
			08:43	0	6	11	55	2005-06-09 THU
			08:44	1	2	10	43	2005-06-20 MON
PROCESSED: 2005-09-14 10:22:02								
AUTOMATIC PASSENGER COUNTING SYSTEM								
POINT CHECK								
WEEKDAY SERVICE								
STOP:	CD910 MACKENZIE KING 2A	BOOKING: APR05						
ROUTE:	95 Transitway	PERIOD: 2005-04-24 TO 2005-06-25						
DIRECTION:	1 WBND	TIME: 08:00:00 TO 10:00:00						
DAY OF THE WEEK: MON TUE WED THU FRI								
T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95060A	2	08:43	AVERAGE	3	5	12	49 (5)	
95001A	3	08:46	08:47	1	4	13	22	2005-04-28 THU
659A	14	08:47	08:51	4	5	7	30	2005-05-17 TUE
			08:46	-1	1	5	22	2005-06-24 FRI
			AVERAGE	2	3	6	26 (2)	
95008A	3	08:49	08:51	2	4	13	55	2005-04-29 FRI
			08:52	3	3	8	31	2005-05-24 TUE
			AVERAGE	3	4	11	43 (2)	
95019A	2	08:53	08:54	1	5	10	41	2005-04-26 TUE
			08:53	0	8	12	53	2005-05-03 TUE
			08:55	2	7	8	61	2005-05-04 WED
			08:50	-3	3	8	29	2005-05-31 TUE
			AVERAGE	0	6	10	46 (4)	
95014A	3	08:57	08:59	2	6	18	41	2005-05-06 FRI
			08:57	0	3	10	24	2005-05-09 MON

			08:56	-1	8	8	30	2005-05-17 TUE
			AVERAGE	0	6	12	32 (3)	
95053A	4	08:59	09:04	5	6	14	45	2005-04-27 WED
			09:06	7	4	12	37	2005-04-29 FRI
			09:03	4	0	7	21	2005-05-13 FRI
			08:58	-1	0	5	45	2005-06-02 THU
			08:54	-5	1	2	28	2005-06-06 MON
			AVERAGE	2	2	8	35 (5)	
95015A	3	09:01	09:01	0	4	10	51	2005-05-09 MON
			09:01	0	6	8	36	2005-05-10 TUE
			AVERAGE	0	5	9	44 (2)	
95023A	2	09:05	09:05	0	5	4	47	2005-05-09 MON
			09:04	-1	8	8	34	2005-05-10 TUE
PROCESSED: 2005-09-14 10:22:02								
AUTOMATIC PASSENGER COUNTING SYSTEM								
POINT CHECK								
WEEKDAY SERVICE								
STOP:	CD910 MACKENZIE KING 2A				BOOKING: APR05			
ROUTE:	95 Transitway				PERIOD: 2005-04-24 TO 2005-06-25			
DIRECTION:	1 WBND				TIME: 08:00:00 TO 10:00:00			
				DAY OF THE WEEK: MON TUE WED THU FRI				
T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95023A	2	09:05	09:08	3	23	9	59	2005-05-17 TUE
			AVERAGE	1	12	7	47 (3)	
95050A	3	09:09	09:10	1	2	8	25	2005-05-18 WED
			09:10	1	5	13	41	2005-05-19 THU
			09:11	2	7	20	41	2005-05-20 FRI
			09:11	2	2	6	33	2005-05-26 THU
			AVERAGE	2	4	12	35 (4)	
95025A	3	09:13	09:16	3	2	15	37	2005-05-25 WED
			09:15	2	9	6	32	2005-05-26 THU
			09:13	0	6	7	25	2005-06-02 THU
			09:18	5	12	17	74	2005-06-13 MON
			AVERAGE	3	7	11	42 (4)	
95018A	10	09:16	09:21	5	10	20	47	2005-04-25 MON
			09:18	2	11	7	32	2005-04-26 TUE
			09:19	3	1	11	47	2005-04-27 WED
			09:16	0	1	10	25	2005-04-28 THU
			09:16	0	3	7	19	2005-05-05 THU
			AVERAGE	2	5	11	34 (5)	
95021A	2	09:20	09:22	2	10	15	40	2005-05-30 MON
95004A	3	09:24	09:26	2	5	17	35	2005-04-26 TUE
			09:26	2	6	3	20	2005-04-27 WED
			09:23	-1	6	2	24	2005-04-28 THU
			09:28	4	6	18	51	2005-05-05 THU
			AVERAGE	2	6	10	33 (4)	
95022A	3	09:28	09:31	3	0	5	16	2005-05-11 WED
			09:31	3	5	9	26	2005-05-12 THU
			AVERAGE	3	3	7	21 (2)	
95007A	2	09:32	09:33	1	9	13	44	2005-05-20 FRI
PROCESSED: 2005-09-14 10:22:03								
AUTOMATIC PASSENGER COUNTING SYSTEM								
POINT CHECK								
WEEKDAY SERVICE								
STOP:	CD910 MACKENZIE KING 2A				BOOKING: APR05			
ROUTE:	95 Transitway				PERIOD: 2005-04-24 TO 2005-06-25			
DIRECTION:	1 WBND				TIME: 08:00:00 TO 10:00:00			
				DAY OF THE WEEK: MON TUE WED THU FRI				
T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95028A	3	09:36	09:39	3	6	12	36	2005-05-26 THU
95013A	3	09:40	09:44	4	5	15	21	2005-05-03 TUE
			09:43	3	1	4	15	2005-05-04 WED
			AVERAGE	4	3	10	18 (2)	
95011A	9	09:44	09:45	1	8	5	19	2005-04-25 MON
			09:49	5	9	17	40	2005-04-27 WED
			AVERAGE	3	9	11	30 (2)	
95017A	3	09:48	09:48	0	6	11	37	2005-05-06 FRI
			09:47	-1	5	15	18	2005-05-09 MON
			09:50	2	10	16	34	2005-05-10 TUE
			09:47	-1	6	9	29	2005-06-24 FRI
			AVERAGE	0	7	13	30 (4)	

95027A	3	09:52	09:52	0	5	13	34	2005-05-09 MON
			09:53	1	6	10	30	2005-06-16 THU
			AVERAGE	1	6	12	32 (2)	
95024A	2	09:56	09:59	3	1	6	12	2005-04-25 MON
			09:58	2	10	12	41	2005-05-05 THU
			09:58	2	9	8	23	2005-06-15 WED
			AVERAGE	2	7	9	25 (3)	
95016A	3	10:00	09:59	-1	5	8	15	2005-05-04 WED
			10:02	2	10	6	19	2005-05-09 MON
			10:00	0	7	10	24	2005-05-17 TUE
			10:02	2	3	26	18	2005-06-02 THU
			09:59	-1	3	10	23	2005-06-15 WED
			AVERAGE	0	6	12	20 (5)	

PROCESSED: 2005-09-14 10:25:42

AUTOMATIC PASSENGER COUNTING SYSTEM
POINT CHECK
WEEKDAY SERVICE

STOP: CD910 MACKENZIE KING 2A
ROUTE: 95 ORLEANS - NEPEAN SOUTH
DIRECTION: 1 WBND

BOOKING: SEP04
PERIOD: 2004-09-05 TO 2004-12-18
TIME: 08:00:00 TO 10:00:00
DAY OF THE WEEK: MON TUE WED THU FRI

T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95015A	3	08:00	08:05	5	4	11	55	2004-09-23 THU
			07:59	-1	4	3	15	2004-11-12 FRI
			AVERAGE	2	4	7	35 (2)	
95005A	2	08:03	08:04	1	5	6	28	2004-10-08 FRI
95064A	3	08:06	08:13	7	9	9	64	2004-10-18 MON
			08:13	7	10	7	71	2004-10-19 TUE
			08:07	1	8	4	38	2004-10-26 TUE
			08:09	3	8	8	56	2004-10-27 WED
			08:14	8	8	9	41	2004-11-01 MON
			AVERAGE	5	9	7	54 (5)	
95063A	2	08:09	08:13	4	14	9	54	2004-10-18 MON
95062A	3	08:12	08:19	7	11	14	63	2004-09-17 FRI
			08:18	6	7	4	46	2004-10-21 THU
			AVERAGE	7	9	9	55 (2)	
95007A	2	08:15	08:22	7	16	7	60	2004-09-29 WED
			08:20	5	5	6	39	2004-10-29 FRI
			08:20	5	14	10	77	2004-11-09 TUE
			AVERAGE	6	12	8	59 (3)	
95009A	3	08:19	08:22	3	1	2	33	2004-09-10 FRI
			08:22	3	8	7	42	2004-11-15 MON
			AVERAGE	3	5	5	38 (2)	
95020A	2	08:22	08:28	6	6	12	58	2004-09-07 TUE
			08:25	3	9	10	58	2004-11-19 FRI
			AVERAGE	5	8	11	58 (2)	
95019A	3	08:25	N/A					
437A	4	08:27	08:33	6	0	8	40	2004-10-27 WED

PROCESSED: 2005-09-14 10:25:42

AUTOMATIC PASSENGER COUNTING SYSTEM
POINT CHECK
WEEKDAY SERVICE

STOP: CD910 MACKENZIE KING 2A
ROUTE: 95 ORLEANS - NEPEAN SOUTH
DIRECTION: 1 WBND

BOOKING: SEP04
PERIOD: 2004-09-05 TO 2004-12-18
TIME: 08:00:00 TO 10:00:00
DAY OF THE WEEK: MON TUE WED THU FRI

T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95056A	2	08:29	08:32	3	7	4	43	2004-10-13 WED
			08:28	-1	6	11	39	2004-11-29 MON
			AVERAGE	1	7	8	41 (2)	
95025A	3	08:32	08:34	2	7	11	47	2004-11-03 WED
			08:40	8	5	10	54	2004-11-05 FRI

			08:36	4	3	9	36	2004-11-08 MON
			AVERAGE	5	5	10	46 (3)	
95008A	2	08:35	08:42	7	7	15	28	2004-11-05 FRI
634A	4	08:37	08:38	1	0	5	21	2004-10-01 FRI
95026A	3	08:39	08:41	2	4	3	42	2004-09-08 WED
			08:42	3	1	7	27	2004-09-10 FRI
			08:42	3	5	6	28	2004-09-23 THU
			08:40	1	3	6	36	2004-09-24 FRI
			08:41	2	2	9	34	2004-12-01 WED
			08:41	2	8	6	40	2004-12-06 MON
			AVERAGE	2	4	6	35 (6)	
95018A	3	08:42	N/A					
95022A	2	08:45	08:46	1	8	9	50	2004-09-17 FRI
			08:46	1	11	6	48	2004-09-21 TUE
			08:48	3	4	15	57	2004-12-06 MON
			AVERAGE	2	8	10	52 (3)	
699A	4	08:47	08:47	0	0	0	15	2004-09-21 TUE
			08:50	3	1	4	34	2004-09-22 WED
			08:53	6	0	7	23	2004-09-24 FRI
			AVERAGE	3	0	4	24 (3)	
95002A	3	08:49	08:50	1	3	11	34	2004-09-14 TUE

PROCESSED: 2005-09-14 10:25:43

AUTOMATIC PASSENGER COUNTING SYSTEM
POINT CHECK
WEEKDAY SERVICE

STOP: CD910 MACKENZIE KING 2A
ROUTE: 95 ORLEANS - NEPEAN SOUTH
DIRECTION: 1 WBND

BOOKING: SEP04
PERIOD: 2004-09-05 TO 2004-12-18
TIME: 08:00:00 TO 10:00:00
DAY OF THE WEEK: MON TUE WED THU FRI

T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95014A	3	08:53	N/A					
95016A	2	08:57	08:59	2	3	5	41	2004-09-10 FRI
			09:00	3	9	7	54	2004-09-15 WED
			09:00	3	11	10	60	2004-09-28 TUE
			AVERAGE	3	8	7	52 (3)	
423A	4	08:59	09:02	3	1	3	10	2004-11-05 FRI
95052A	3	09:01	09:01	0	3	4	32	2004-10-15 FRI
			09:01	0	1	8	40	2004-10-19 TUE
			09:01	0	12	11	43	2004-11-10 WED
			09:01	0	4	7	29	2004-11-16 TUE
			AVERAGE	0	5	8	36 (4)	
95027A	3	09:05	09:06	1	7	6	41	2004-09-09 THU
			09:03	-2	2	3	40	2004-10-28 THU
			AVERAGE	-1	5	5	41 (2)	
95013A	2	09:08	N/A					
95023A	3	09:13	N/A					
95050A	10	09:16	09:18	2	4	4	36	2004-10-05 TUE
			09:17	1	2	8	25	2004-10-28 THU
			09:17	1	4	8	23	2004-10-29 FRI
			09:24	8	5	18	51	2004-11-05 FRI
			09:18	2	3	2	32	2004-11-10 WED
			AVERAGE	3	4	8	33 (5)	
95004A	2	09:18	N/A					
95055A	2	09:20	09:20	0	4	9	23	2004-11-09 TUE
95010A	3	09:24	09:28	4	13	9	47	2004-09-21 TUE
			09:26	2	2	12	38	2004-09-30 THU

PROCESSED: 2005-09-14 10:25:43

AUTOMATIC PASSENGER COUNTING SYSTEM
POINT CHECK
WEEKDAY SERVICE

STOP: CD910 MACKENZIE KING 2A
ROUTE: 95 ORLEANS - NEPEAN SOUTH
DIRECTION: 1 WBND

BOOKING: SEP04
PERIOD: 2004-09-05 TO 2004-12-18
TIME: 08:00:00 TO 10:00:00
DAY OF THE WEEK: MON TUE WED THU FRI

T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95010A	3	09:24	09:25	1	11	5	32	2004-10-20 WED
			AVERAGE	2	9	9	39 (3)	
95024A	3	09:28	N/A					
95001A	2	09:32	N/A					
435A	3	09:34	09:34	0	1	5	15	2004-09-17 FRI
			09:36	2	3	5	12	2004-09-21 TUE
			09:37	3	2	7	20	2004-10-13 WED
			09:37	3	1	10	18	2004-10-14 THU
			09:39	5	4	9	36	2004-10-26 TUE
			AVERAGE	3	2	7	20 (5)	
95057A	3	09:36	09:40	4	10	6	49	2004-10-29 FRI
			09:38	2	5	15	43	2004-11-04 THU
			09:39	3	3	17	24	2004-11-18 THU
			AVERAGE	3	6	13	39 (3)	
95006A	3	09:40	09:43	3	5	12	33	2004-09-14 TUE
95017A	9	09:44	N/A					
95058A	3	09:46	09:48	2	1	10	19	2004-10-01 FRI
			09:44	-2	3	17	27	2004-10-07 THU
			09:49	3	9	10	44	2004-11-01 MON
			09:49	3	10	8	35	2004-11-03 WED
			09:49	3	11	14	23	2004-11-08 MON
			AVERAGE	2	7	12	30 (5)	
95012A	3	09:48	09:50	2	1	9	19	2004-10-28 THU
			09:49	1	8	21	25	2004-10-29 FRI
			AVERAGE	2	5	15	22 (2)	
95051A	3	09:50	09:56	6	8	21	30	2004-10-12 TUE
			09:55	5	4	7	27	2004-11-05 FRI

PROCESSED: 2005-09-14 10:25:43

AUTOMATIC PASSENGER COUNTING SYSTEM
POINT CHECK
WEEKDAY SERVICE

STOP: CD910 MACKENZIE KING 2A
ROUTE: 95 ORLEANS - NEPEAN SOUTH
DIRECTION: 1 WBND

BOOKING: SEP04
PERIOD: 2004-09-05 TO 2004-12-18
TIME: 08:00:00 TO 10:00:00
DAY OF THE WEEK: MON TUE WED THU FRI

T I M E								
RUN	PAT	EXPECTED	ACTUAL	DIFFERENCE	ONS	OFFS	LOAD AT DEPARTURE	DATE
95051A	3	09:50	AVERAGE	6	6	14	29 (2)	
95021A	3	09:52	N/A					
95015A	2	09:56	09:56	0	2	8	21	2004-09-23 THU
			09:56	0	8	8	13	2004-11-12 FRI
			AVERAGE	0	5	8	17 (2)	
95011A	3	10:00	N/A					

3. Running Time Prediction Results of the Developed Model :Route 1

The running time predictions obtained from the developed model are tabulated in Table D7.19 of this appendix. For illustration, the following table depicts the output of the model applied for test case RCODE=18.

Table D7.5: An Example of Developed Model's Output

....
the Edist to the first second... neighbor
Columns 1 through 12
138.2730 210.8571 87.9496 77.7618 189.6905 163.5890 132.6671 136.3413
224.3460 193.2873 134.9488 129.1932
Columns 13 through 24
175.2300 164.9713 137.6980 136.4368 133.9282 183.4395 198.7468 82.0132
171.2472 106.2629 162.6202 166.4866
Columns 25 through 36
105.1114 136.7360 267.5160 110.4786 69.3883 112.0025 294.2738 141.9854
175.7238 215.7770 193.6065 74.5923
Columns 37 through 48
160.6042 131.7126 187.8599 180.5488 115.3895 151.7052 106.8601 226.4662
173.5288 122.7170 197.4734 205.1065
Columns 49 through 60
121.8729 40.4245 109.3997 137.6448 76.3830 51.7569 161.5609 216.7234
149.0179 12.5770 211.3152 109.9486
Columns 61 through 68
145.7678 122.9194 223.5827 107.5712 239.6461 183.4306 84.3911 166.3349

bandwidth =

180.5488

The matrix of Error is

1.0e+004 *

Columns 1 through 12

0.2593	2.8210	0.0043	0.0005	1.4051	1.0797	0.9566	0.0297	0.0692			
0.0196	0.0739	0.3250									

Columns 13 through 24

0.5662	3.9312	0.0856	0.0017	0.8072	0.0462	0.6723	0.0004	1.8555			
0.2964	2.1052	0.7821									

Columns 25 through 36

0.3783	1.1622	2.4488	2.3185	0.0933	0.3824	0.3668	0.4822	0.3481			
1.4091	0.3011	0.7268									

Columns 37 through 48

1.5127	0.0346	0.0641	2.8624	0.0193	1.4681	1.2478	0.6955	0.2016			
0.0056	0.8899	4.3129									

Columns 49 through 60

0.0596	1.5289	0.1847	0.0511	1.8846	0.0905	0.5140	0.0841	1.1825			
0.0911	0.0183	0.0172									

Columns 61 through 69

0.2168	0.9249	3.4738	0.0822	0.6648	0.4521	0.1124	0.0061	0.1337			
--------	--------	--------	--------	--------	--------	--------	--------	--------	--	--	--

Cv =

7.7825e+003

The CROSS_VALIDATION factors

1.0e+003 *

9.6041	8.1329	7.6980	7.9301	7.8487	7.7976	7.7825					
--------	--------	--------	--------	--------	--------	--------	--	--	--	--	--

OpNei =

21

Beta =

1.0e+003 *

1.3238

-0.0000

-0.0002

-0.0002

THE PREDCTED BUS RUNNING TIME IS 948.96 SECONDS.Program complete

ans =

948.9596

Note: Only the most important part of the output is printed out.

4. Running Time Prediction Results of the KF Predictor: Route 1

The predictions of Kalman Filter predictors are tabulated in Tables D7.6-D7.20.

Table D7.6. Kalman Filter for test case RCODE=1

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	832	867	849	849.333	300.444	312.111	0.11111	306.278	0.5	153.139	0.5	874.5	818
8:37	19	1007	940.5	1060.2	1002.57	19.6544	3852.27	3321.6	1935.96	0.51902	1004.81	0.48098	943.707	939.8
8:47	20	874.3	817.4	931.5	874.4	0.01	3249	3260.41	1624.51	0.61811	1004.12	0.38189	934.67	991.6
8:57	24	986	971.6	921.3	959.633	695.201	143.201	1469.44	419.201	0.77248	323.827	0.22752	937.294	877.7

Table D7.7 Kalman Filter for test case RCODE=7

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	922.2	850	880.4	884.2	1444	1169.64	14.44	1306.82	0.5	653.41	0.5	890.2	915.4
8:37	19	1079.5	923.2	900.2	967.633	12514.2	1974.32	4547.25	7244.24	0.52158	3778.42	0.47842	907.472	946.3
8:47	20	910.7	800.3	916.5	875.833	1215.68	5705.28	1653.78	3460.48	0.67657	2341.27	0.32343	926.138	949.5
8:57	24	944.5	810.2	1029	927.9	275.56	13853.3	10221.2	7064.42	0.57108	4034.32	0.42892	994.901	1029

Table D7.8. Kalman Filter for test case RCODE=12

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	817.2	813.5	794.1	808.267	79.8044	27.3878	200.694	53.5961	0.5	26.7981	0.5	847.05	845.8
8:37	19	889.6	872.9	856	906.167	274.454	1106.67	2483.36	690.563	0.50952	351.853	0.49048	901.949	960.2
8:47	20	1073.5	900.5	996.7	990.233	6933.34	8052.07	41.8178	7492.7	0.51147	3832.3	0.48853	978.869	1013.2
8:57	24	948.2	933.9	900.1	927.4	432.64	42.25	745.29	237.445	0.94487	224.355	0.05513	906.335	1013.2

Table D7.9: Kalman Filter for test case RCODE=17

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	854.6	932.9	814	867.167	157.921	4320.87	2826.69	2239.4	0.5	1119.7	0.5	857	931
8:37	19	919.7	953.7	931.8	935.067	236.134	347.201	10.6711	291.668	0.82874	241.716	0.17126	931.663	959
8:47	20	845	948	960.6	917.867	5309.55	908.018	1826.14	3108.78	0.51871	1612.56	0.48129	959.83	844
8:57	24	1121.9	904.8	928.2	984.967	18750.7	6426.69	3222.45	12588.7	0.5301	6673.23	0.4699	888.634	1017

Table D7.10: Kalman Filter for test case RCODE=18

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	932.9	814	857.5	868.133	4194.72	2930.42	113.068	3562.57	0.5	1781.28	0.5	878.75	808.6
8:37	19	953.7	931.8	959.1	948.2	30.25	268.96	118.81	149.605	0.92809	138.847	0.07191	948.278	1083.1
8:47	20	958.2	960.6	844	920.933	1388.8	1573.44	5918.74	1481.12	0.52239	773.72	0.47761	958.197	868.9
8:57	24	904.8	928.2	1017.4	950.133	2055.11	481.071	4524.8	1268.09	0.61688	782.259	0.38312	960.507	923

Table D7.11. Kalman Filter for test case RCODE=23

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	841.5	886.7	895.3	874.5	1089	148.84	432.64	618.92	0.5	309.46	0.5	897.65	852.2
8:37	19	861.7	1001.2	895.8	919.567	3348.55	6664	564.854	5006.28	0.51499	2578.18	0.48501	874.654	967.9
8:47	20	926.1	974.9	946.6	949.2	533.61	660.49	6.76	597.05	0.84173	502.553	0.15827	949.971	878.9
8:57	24	839.6	945.1	792.5	859.067	378.951	7401.73	4431.12	3890.34	0.53034	2063.19	0.46966	833.079	984.4

Table D7.12. Kalman Filter for test case RCODE=28

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	999.7	889.4	856.3	915.133	7151.52	662.204	3461.36	3906.86	0.5	1953.43	0.5	878.15	913.4
8:37	19	1006.3	1007.7	949.5	987.833	341.018	394.684	1469.44	367.851	0.86321	317.532	0.13679	944.562	807.4
8:47	20	979.9	849.5	1011.5	946.967	1084.6	9499.75	4164.55	5292.18	0.51456	2723.16	0.48544	912.422	991.6
8:57	24	852.8	997.9	1034.9	961.867	11895.5	1298.4	5333.87	6596.97	0.58554	3862.8	0.41446	1016.95	768.4

Table D7.13. Kalman Filter for test case RCODE=33

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	788.6	887	856.7	844.1	3080.25	1840.41	158.76	2460.33	0.5	1230.17	0.5	878.35	839.9
8:37	19	1087.5	1043.4	843.8	991.567	9203.2	2686.69	21835	5944.95	0.54688	3251.18	0.45312	842.033	959.1
8:47	20	913.5	827.2	732.2	824.3	7956.64	8.41	8482.41	3982.52	0.64493	2568.46	0.35507	812.765	1030.7
8:57	24	971	880.1	982.1	944.4	707.56	4134.49	1421.29	2421.03	0.6733	1630.07	0.3267	997.978	993.9

Table D7.14. Kalman Filter for test case RCODE=37

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	863.4	807.1	899.1	856.533	47.1511	2443.65	1811.92	1245.4	0.5	622.701	0.5	899.55	710.9
8:37	19	987.4	932.2	969.5	963.033	593.734	950.694	41.8178	772.214	0.64367	497.051	0.35633	877.353	936.9
8:47	20	940.4	844.4	849.7	878.167	3872.99	1140.19	810.351	2506.59	0.5451	1366.35	0.4549	889.367	903.3
8:57	24	1029	805	915.7	916.567	12641.3	12447.1	0.75111	12544.2	0.52582	6596.04	0.47418	909.82	902.9

Table D7.15. Kalman Filter for test case RCODE = 40

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	873	932	840	881.667	75.1111	2533.44	1736.11	1304.28	0.5	652.139	0.5	870	848.8
8:37	19	1021.1	1127.4	860.9	1003.13	322.801	15442.2	20230.3	7882.5	0.51986	4097.81	0.48014	855.09	886.7
8:47	20	1076.2	943.6	824.4	948.067	16418.2	19.9511	15293.4	8219.05	0.59977	4929.56	0.40023	849.334	934.5
8:57	24	818.7	1050	842.2	903.633	7213.67	21423.2	3774.05	14318.4	0.57343	8210.62	0.42657	881.572	967

Table D7.16. Kalman Filter for test case RCODE = 46

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	778.5	754.2	813.5	782.067	12.7211	776.551	988.054	394.636	0.5	197.318	0.5	856.75	874.4
8:37	19	1044.9	935.3	859.1	946.433	9695.68	123.951	7627.11	4909.82	0.50985	2503.27	0.49015	866.599	989.4
8:47	20	832.8	812.1	913.9	852.933	405.351	1667.36	3716.93	1036.36	0.77352	801.645	0.22648	930.999	1027.6
8:57	24	938.1	831.3	714.4	827.933	12136.7	11.3344	12889.8	6074.01	0.53095	3225.01	0.46905	861.306	912

Table D7.17. Kalman Filter for test case RCODE = 50

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	813	856	857	842	841	196	225	518.5	0.5	259.25	0.5	878.5	841.6
8:37	19	1003.9	1108.4	939.1	1017.13	175.121	8329.6	6089.2	4252.36	0.51479	2189.08	0.48521	891.792	877.7
8:47	20	895.3	913.8	824.8	877.967	300.444	1284.03	2826.69	792.236	0.79006	625.91	0.20994	835.906	814.9
8:57	24	1076	972.9	857.3	968.733	11506.1	17.3611	12417.4	5761.75	0.52576	3029.29	0.47424	837.192	953.1

Table D7.18. Kalman Filter for test case RCODE = 57

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	881	840	754	825	3136	225	5041	1680.5	0.5	840.25	0.5	827	793.4
8:37	19	949	1056.6	879.5	961.7	161.29	9006.01	6756.84	4583.65	0.54198	2484.25	0.45802	840.065	1014
8:47	20	1025.5	939.4	878	947.633	6063.22	67.7878	4848.8	3065.5	0.64418	1974.73	0.35582	926.392	1013
8:57	24	823	915	913	883.667	3680.44	981.778	860.444	2331.11	0.64877	1512.35	0.35123	948.123	917.6

Table D7.19. Kalman Filter for test case RCODE = 61

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	798.8	784.4	793.9	792.367	41.3878	63.4678	2.35111	52.4278	0.5	26.2139	0.5	846.95	832.2
8:37	19	971.8	881.2	927.3	926.767	2028	2076.32	0.28444	2052.16	0.50317	1032.59	0.49683	880.052	1000.7
8:47	20	965.5	1076.8	1044.9	1029.07	4040.72	2278.47	250.694	3159.6	0.57023	1801.69	0.42977	1025.9	874.3
8:57	24	957.6	1001.4	986	981.667	579.204	389.404	18.7778	484.304	0.82518	399.638	0.17482	966.473	971.6

Table D7.20. Kalman Filter for test case RCODE = 68

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:17	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		900
8:27	18	798.8	784.4	793.9	792.367	41.3878	63.4678	2.35111	52.4278	0.5	26.2139	0.5	846.95	832.2
8:37	19	971.8	881.2	927.3	926.767	2028	2076.32	0.28444	2052.16	0.50317	1032.59	0.49683	880.052	1000.7
8:47	20	965.5	1076.8	1044.9	1029.07	4040.72	2278.47	250.694	3159.6	0.57023	1801.69	0.42977	1025.9	874.3
8:57	24	957.6	1001.4	986	981.667	579.204	389.404	18.7778	484.304	0.82518	399.638	0.17482	966.473	971.6

Table D7.21: Prediction Error of the Predictors

Rcode	Actual	Developed Model	KF Model	Naïve Model	Devlp. Error	KF Error	Naïve Error	d1	d2	d3
1	877.1	946	984	1117.82	-68.90	-106.90	-240.72	0.0786	0.1130	0.2446
7	1029.8	962.11	994.9	1066.05	67.69	34.90	-36.25	0.0657	0.0363	0.0364
12	985.5	923.07	906.33	915	62.43	79.17	70.50	0.0633	0.0858	0.0778
17	1017.4	924.92	888.63	815.53	92.48	128.77	201.87	0.0909	0.1392	0.2272
18	923.1	948.96	960.5	1047.42	-25.86	-37.40	-124.32	0.0280	0.0394	0.1294
23	984.4	935.03	833.08	735.82	49.37	151.32	248.58	0.0502	0.1618	0.2984
28	768.4	858.62	1016.95	1014.54	-88.19	-248.55	-246.14	0.1148	0.2902	0.2420
33	993.9	911.56	997.97	1382.48	82.34	-4.07	-388.58	0.0828	0.0045	0.3894
37	1029	950.59	967.12	959.79	78.41	61.88	69.21	0.0762	0.0651	0.0716
40	902.9	828.25	909.82	973.46	74.65	-6.92	-70.56	0.0827	0.0084	0.0776
46	967	896.47	881.57	954.68	70.53	85.43	12.32	0.0729	0.0953	0.0140
50	912	908.77	861.31	803.28	3.23	50.69	108.72	0.0035	0.0558	0.1262
57	953.1	899.4	837.19	847	53.70	115.91	106.10	0.0563	0.1289	0.1267
61	917.6	903.53	948.12	1054.21	14.07	-30.52	-136.61	0.0153	0.0338	0.1441
68	971.6	953.53	966.47	825.02	18.07	5.13	146.58	0.0186	0.0054	0.1517

Note: d1,d2,d3: The absolute relative error (%) of the Proposed Model, the Kalman Filter and the Naïve predictor

5. Running Time Prediction Results of the Developed Model: Route 95

The running time prediction results of the developed model are tabulated in Table D7.36 of this appendix.

6. Running Time Prediction Results of the KF Model: Route 95

Tables from D7.22-D7.35 present the running time prediction results of KF model applied to route 95.

7. Boarding Passenger Prediction Results of the Developed Model: Route 95

The boarding passenger prediction results of the developed model are tabulated in Table D7.51 in this appendix.

8. Boarding Passenger Prediction Results of the KF Model: Route 95

Tables from D7.37-D7.50 show the predictions results of the KF model applied for boarding passenger predictions.

9. Alighting Passenger Prediction Results of the Developed Model: Route 95

The alighting passenger prediction results of the developed model are tabulated in Table D7.65 of this appendix.

10. Alighting Passenger Prediction Results of the KF Model: Route 95

Tables from D7.52 -D7.64 show the prediction results of the KF model applied for alighting passenger predictions.

Table D7.22. Kalman Filter for test case RCODE = 6

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	552.97	621.76	719.6	631.443	6158.06	93.7669	7771.6	3125.92	0.5	1562.96	0.5	674.8	567.21
8:39	19	631.05	729.68	562.75	641.16	102.212	7835.79	6148.13	3969	0.58225	2310.96	0.41775	564.613	636.53
8:43	20	332.88	464.9	556.97	451.583	14090.5	177.334	11106.3	7133.91	0.5697	4064.16	0.4303	591.205	592.14
8:59	24	570.61	653.09	487.24	570.313	0.08801	6851.98	6901.18	3426.03	0.68615	2350.78	0.31385	520.163	667.33

Table D7.23. Kalman Filter for test case RCODE = 11

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	546.59	686.64	498.97	577.4	949.256	11933.4	6151.26	6441.32	0.5	3220.66	0.5	564.485	541.44
8:39	19	527.57	411.32	422.01	453.633	5466.63	1790.42	1000.04	3628.52	0.65369	2371.93	0.34631	463.37	562.26
8:43	20	569.18	416.69	520.39	502.087	4501.52	7292.59	335.012	5897.05	0.58372	3442.22	0.41628	537.82	509.18
8:59	24	520.88	541.71	619.26	560.617	1579	357.462	3439.04	968.232	0.81999	793.938	0.18001	599.444	619.99

Table D7.24. Kalman Filter for test case RCODE = 17

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	804.36	635.28	585.23	674.957	16745.2	1574.24	8050.87	9159.73	0.5	4579.87	0.5	607.615	673.8
8:39	19	488.48	601.83	600.5	563.603	5643.52	1461.28	1361.36	3552.4	0.69598	2472.39	0.30402	622.785	536.49
8:43	20	449.25	450.1	567.23	488.86	1568.95	1502.34	6141.86	1535.64	0.72299	1110.26	0.27701	558.715	464.78
8:59	24	683.63	583.12	609.83	625.527	3376	1798.33	246.385	2587.16	0.58833	1522.11	0.41167	550.118	651.2

Table D7.25. Kalman Filter for test case RCODE = 20

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	673.8	509.6	615.47	599.623	5502.18	8104.2	251.117	6803.19	0.5	3401.59	0.5	622.735	667.52
8:39	19	536.49	638.44	472.78	549.237	162.478	7957.23	5845.62	4059.86	0.64762	2629.25	0.35238	541.402	513.23
8:43	20	464.78	699.71	596.78	587.09	14959.7	12683.3	93.8961	13821.5	0.54343	7510.97	0.45657	558.633	571.42
8:59	24	651.2	656.49	615.51	641.067	102.684	237.879	653.143	170.282	0.97831	166.589	0.02169	614.554	520.63

Table D7.26. Kalman Filter for test case RCODE = 26

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	641.95	665.15	785.78	697.627	3099.89	1054.73	7771.01	2077.31	0.5	1038.66	0.5	707.89	816.42
8:39	19	671.25	689.29	602.43	654.323	286.512	1222.67	2692.92	754.59	0.70383	531.104	0.29617	665.807	489.25
8:43	20	673.73	315.57	668.65	552.65	14660.4	56206.9	13456	35433.6	0.50372	17848.6	0.49628	579.617	501.51
8:59	24	568.25	638.57	533.32	580.047	139.161	3424.98	2183.38	1782.07	0.91678	1633.76	0.08322	530.673	632.92

Table D7.27. Kalman Filter for test case RCODE = 37

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	$g(k+1)$	$e(k+1)$	$a(k+1)$	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	606.12	588.43	539.63	578.06	787.364	107.537	1476.86	447.45	0.5	223.725	0.5	584.815	448
8:39	19	566.6	657.59	560.98	595.057	809.782	3910.42	1161.22	2360.1	0.52263	1233.45	0.47737	507.046	678.75
8:43	20	376.24	503.44	499.59	459.757	6975.03	1908.23	1586.69	4441.63	0.56096	2491.58	0.43904	578.248	423.23
8:59	24	549.87	662.18	701.45	637.833	7737.55	592.76	4047.08	4165.15	0.61512	2562.06	0.38488	594.368	646.35

Table D7.28. Kalman Filter for test case RCODE = 41

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	468.41	512.04	584.84	521.763	2846.58	94.5432	3978.67	1470.56	0.5	735.28	0.5	607.42	641.53
8:39	19	655.77	600.91	565.51	607.397	2339.98	42.0768	1754.49	1191.03	0.61793	735.977	0.38207	594.555	603.81
8:43	20	428.38	568.42	348.34	448.38	400	14409.6	10008	7404.8	0.52367	3877.68	0.47633	470.028	670.83
8:59	24	567.68	620.42	595.99	594.697	729.9	661.69	1.67271	695.795	0.86795	603.917	0.13205	605.872	561.98

Table D7.29. Kalman Filter for test case RCODE = 44

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	468.62	512.04	584.84	521.833	2831.66	95.9094	3969.84	1463.78	0.5	731.892	0.5	607.42	641.53
8:39	19	603.81	572.99	580.7	585.833	323.161	164.951	26.3511	244.056	0.79995	195.234	0.20005	592.869	603.81
8:43	20	670.83	605.89	511.84	596.187	5571.63	94.1547	7114.36	2832.89	0.51866	1463.63	0.48334	556.293	537.06
8:59	24	561.98	626.73	613.99	600.9	1514.77	667.189	171.348	1090.98	0.70074	764.492	0.29926	590.968	546.88

Table D7.30. Kalman Filter for test case RCODE = 44

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	764.38	575.74	624.08	654.733	12022.4	6239.95	939.627	9131.17	0.5	4565.58	0.5	627.04	574.27
8:39	19	530.86	585.63	536	550.83	398.801	1211.04	219.929	804.92	0.86966	700.005	0.13034	540.988	483.28
8:43	20	434.99	576.71	629.51	547.07	12561.9	878.53	6796.35	6720.23	0.52475	3526.45	0.47525	560.014	667.21
8:59	24	593.06	643.42	540.65	592.377	0.46694	2605.42	2675.65	1302.94	0.78753	1026.11	0.21247	567.54	575.79

Table D7.31. Kalman Filter for test case RCODE = 49

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	614.36	498.37	560.15	557.627	3218.67	3511.35	6.36721	3365.01	0.5	1682.51	0.5	595.07	538.09
8:39	19	576.35	662.93	624.75	621.343	2024.4	1729.45	11.6054	1876.93	0.65475	1228.91	0.345254	594.83	619.16
8:43	20	534.33	655.71	605.24	598.427	4108.38	3281.38	46.4215	3694.88	0.57129	2110.86	0.428707	611.20	384.35
8:59	24	606.93	620.4	553.62	593.65	176.358	715.563	1602.4	445.96	0.85148	379.728	0.148516	528.48	572.17

Table D7.32. Kalman Filter for test case RCODE = 62

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	550.66	534.16	489.59	524.803	668.567	87.5472	1239.98	378.057	0.5	189.029	0.5	559.80	441.73
8:39	19	462.31	490.41	599.8	517.507	3046.67	734.229	6772.19	1890.45	0.52381	990.232	0.476192	524.53	687.54
8:43	20	640.15	555.21	480.67	558.677	6637.9	12.0178	6085.04	3324.96	0.5648	1877.95	0.435196	570.70	696.77
8:59	24	641.84	564.99	591.14	599.323	1807.67	1178.78	66.9669	1493.22	0.69303	1034.85	0.30697	623.57	575.55

Table D7.33. Kalman Filter for test case RCODE = 80

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	626.64	571.11	805.78	667.843	1697.71	9357.34	19026.5	5527.53	0.5	2763.76	0.5	717.89	603.65
8:39	19	721.25	581.13	650.93	651.103	4920.55	4896.27	0.03004	4908.41	0.60984	2993.36	0.390158	632.48	580.66
8:43	20	834.31	674.97	644.36	717.88	13555.9	1841.27	5405.19	7698.61	0.58138	4475.84	0.418617	617.69	462.64
8:59	24	637.95	524.02	596.9	586.29	2668.76	3877.55	112.572	3273.15	0.70304	2301.15	0.296962	557.03	585.08

Table D7.34. Kalman Filter for test case RCODE = 86

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	570.04	613.06	587.27	590.123	403.34	526.091	8.14151	464.715	0.5	232.358	0.5	608.64	543.67
8:39	19	658.73	471.33	510.89	546.983	12487.3	5723.43	1302.73	9105.37	0.5063	4610.04	0.493701	527.07	572.92
8:43	20	834.31	449.86	598.27	627.48	42778.6	31548.9	853.224	37163.8	0.5292	19667.1	0.470799	586.34	723.62
8:59	24	612.37	579.89	531.71	574.657	1422.3	27.3878	1844.42	724.842	0.96567	699.961	0.034325	538.30	587.31

Table D7.35. Kalman Filter for test case RCODE = 91

time	trip	art3	art2	art1	Average	$\Delta 3$	$\Delta 2$	$\Delta 1$	Var[data] _n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	630	396900	396900	396900	396900	0.5	0	0.5		630
8:36	18	581.56	476.12	615.52	557.733	567.71	6660.74	3339.3	3614.22	0.5	1807.11	0.5	622.76	711.86
8:39	19	522.39	541.33	629.14	564.287	1755.33	527.009	4205.95	1141.17	0.72095	822.724	0.279052	652.22	494.04
8:43	20	490.23	321.6	563.06	458.297	1019.74	18686	10975.4	9852.86	0.52004	5123.87	0.479961	529.93	465.82
8:59	24	592.32	591.79	522.28	568.797	553.347	528.693	2163.8	541.02	0.91282	493.855	0.087178	517.36	667.16

Table D7.36: Prediction Error of the Predictors

R CODE	Actual Running time	Developed Model	KF Model	Naïve Model	d1	d2	d3
6	667	600.44	520.16	518.0104	0.100247	0.220546	0.223767
11	620	597.99	599.44	605.9335	0.035495	0.033156	0.022683
17	651	590.09	550.12	499.6859	0.093843	0.155222	0.232669
20	521	594.16	614.55	589.355	0.141228	0.180392	0.131999
26	633	610.78	530.67	400.0117	0.034981	0.161553	0.367991
37	646	607.15	594.37	594.2325	0.060654	0.080426	0.080639
41	562	595.24	605.87	1147.79	0.059166	0.078081	1.04237
44	547	598.06	590.97	644.2334	0.093582	0.080617	0.178012
49	576	600.62	567.54	573.0316	0.043107	0.014343	0.004806
62	572	606.61	528.48	351.5722	0.06018	0.076369	0.385553
72	576	615.58	623.57	856.9015	0.069557	0.083439	0.488848
80	585	593.81	557.03	428.5672	0.014912	0.047951	0.267513
86	587	606.63	538.3	643.1271	0.032888	0.083455	0.095031
91	667	587.2	517.36	432.0833	0.119848	0.224531	0.352352
96	586	604.26	574.94	508.2925	0.030391	0.019606	0.133254

Note: d1, d2, d3: The absolute relative error (%) of the Developed Model, the Kalman Filter and the Naïve predictors, respectively

Table D7.37. Kalman Filter for test case RCODE = 6

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predicted	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	6
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.011776	0.005482	0.010662	0.009307	1.04E-05	0.99998	1.04E-05	1.97E-05	0.010662	0.018852	
8:43	20	0.027036	0.019355	0.028726	0.025039	1.81E-05	0.611056	1.11E-05	0.388944	0.024886	0.011821	
8:59	24	0.015773	0.009187	0.008209	0.011056	1.29E-05	0.650571	8.37E-06	0.349429	0.009472	0.011988	

Table D7.38. Kalman Filter for test case RCODE = 11

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predicted	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	7
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.003791	0.003791	0.009478	0.005687	3.59E-06	0.999993	3.59E-06	6.84E-06	0.009478	0.010671	
8:43	20	0.005271	0.028798	0.005765	0.013278	0.000152	0.505824	7.71E-05	0.494176	0.008189	0.033386	
8:59	24	0.00192	0.007384	0.009689	0.006331	1.03E-05	0.894747	9.2E-06	0.105253	0.012183	0.016129	

Table D7.39. Kalman Filter for test case RCODE = 17

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	6
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.005376	0.011631	0.004996	0.007334	1.11E-05	0.999979	1.11E-05	2.12E-05	0.004996	0.020503	
8:43	20	0.022259	0.01333	0.005289	0.013626	3.73E-05	0.564989	2.11E-05	0.435011	0.011907	0.025819	
8:59	24	0.013165	0.005145	0.00164	0.00665	2.24E-05	0.660199	1.48E-05	0.339801	0.009856	0.019963	

Table D7.40. Kalman Filter for test case RCODE = 20

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	10
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.020503	0.014097	0.021151	0.018584	1.19E-05	0.999977	1.19E-05	2.27E-05	0.021151	0.021433	
8:43	20	0.025819	0.022867	0.030161	0.026282	5.94E-06	0.750292	4.46E-06	0.249708	0.027982	0.01575	
8:59	24	0.019963	0.009139	0.017871	0.015658	3.05E-05	0.534033	1.63E-05	0.465967	0.016883	0.02497	

Table D7.41. Kalman Filter for test case RCODE = 26

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	5
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.005959	0.007254	0.011619	0.008277	3.21E-06	0.999994	3.21E-06	6.11E-06	0.011619	0.016351	
8:43	20	0.023748	0.022182	0.011964	0.019298	1.41E-05	0.551251	7.75E-06	0.448749	0.013933	0.025921	
8:59	24	0.008799	0.00783	0.00375	0.006793	2.55E-06	0.801586	2.04E-06	0.198414	0.008149	0.0158	

Table D7.42. Kalman Filter for test case RCODE = 37

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	6
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.012354	0.010645	0.010696	0.011232	8.02E-07	0.999998	8.02E-07	1.53E-06	0.010696	0.007366	
8:43	20	0.021263	0.007945	0.006005	0.011738	5.26E-05	0.503788	2.65E-05	0.496212	0.00668	0.030716	
8:59	24	0.01273	0.010571	0.007128	0.010143	3.44E-06	0.896926	3.08E-06	0.103074	0.009559	0.017019	

Table D7.43. Kalman Filter for test case RCODE = 41

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	8
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.012199	0.006657	0.012378	0.010411	8.65E-06	0.999984	8.65E-06	1.65E-05	0.012378	0.008281	
8:43	20	0.002334	0.008796	0.028708	0.013279	6.99E-05	0.529109	3.7E-05	0.470891	0.019089	0.007453	
8:59	24	0.015854	0.006447	0.015101	0.012467	2.39E-05	0.718421	1.71E-05	0.281579	0.012947	0.010676	

Table D7.44. Kalman Filter for test case RCODE = 44

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	11
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.008281	0.006981	0.017221	0.010827	1.06E-05	0.99998	1.06E-05	2.03E-05	0.01722	0.014186	
8:43	20	0.007453	0.011553	0.021491	0.013499	2.02E-05	0.604362	1.22E-05	0.395638	0.018601	0.014896	
8:59	24	0.010676	0.004787	0.019544	0.011669	2.42E-05	0.600668	1.45E-05	0.399332	0.017688	0.010971	

Table D7.45. Kalman Filter for test case RCODE = 49

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	11
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.005651	0.011953	0.020522	0.012709	2.52E-05	0.999952	2.52E-05	4.79E-05	0.020521	0.006208	
8:43	20	0.004598	0.024276	0.019062	0.015979	9.92E-05	0.556339	5.52E-05	0.443661	0.013359	0.010491	
8:59	24	0.001686	0.009325	0.022195	0.011069	4.55E-05	0.688638	3.14E-05	0.311362	0.018551	0.013894	

Table D7.46. Kalman Filter for test case RCODE = 62

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	4
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.01388	0.016593	0.009604	0.013359	5.36E-06	0.99999	5.36E-06	1.02E-05	0.009604	0.012921	
8:43	20	0.024329	0.016775	0.01487	0.018658	1.79E-05	0.565307	1.01E-05	0.434693	0.014023	0.007805	
8:59	24	0.014829	0.01773	0.005419	0.012659	1.52E-05	0.624555	9.5E-06	0.375445	0.006315	0.010486	

Table D7.47. Kalman Filter for test case RCODE = 72

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	4
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.012978	0.010196	0.01167	0.011615	1.94E-06	0.999996	1.94E-06	3.69E-06	0.01167	0.005818	
8:43	20	0.001562	0.005403	0.014563	0.007176	1.73E-05	0.526458	9.12E-06	0.473542	0.010422	0.00287	
8:59	24	0.004674	0.030089	0.008458	0.014407	0.00017	0.513042	8.74E-05	0.486958	0.005737	0.010425	

Table D7.48. Kalman Filter for test case RCODE = 86

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	7
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.008319	0.024091	0.018435	0.016948	6.27E-05	0.999881	6.27E-05	0.000119	0.018433	0.013777	
8:43	20	0.002397	0.026668	0.021727	0.016931	0.000153	0.58506	8.95E-05	0.41494	0.018428	0.012969	
8:59	24	0.014108	0.024808	0.011727	0.016881	3.53E-05	0.779663	2.75E-05	0.220337	0.012001	0.013673	

Table D7.349. Kalman Filter for test case RCODE = 91

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	8
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.010626	0.016973	0.015659	0.01442	1.05E-05	0.99998	1.05E-05	1.99E-05	0.015659	0.006982	
8:43	20	0.021813	0.022229	0.016715	0.020252	3.17E-06	0.811191	2.57E-06	0.188809	0.014877	0.008292	
8:59	24	0.006532	0.010347	0.016926	0.011268	1.16E-05	0.549753	6.4E-06	0.450247	0.013039	0.008513	

Table D7.50. Kalman Filter for test case RCODE = 96

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed	Boarders
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025	8
8:36	18	0	0	0	1.025	1.050625	0.5	0.525313	0.5	0.0125	0	
8:39	19	0.010626	0.022168	0.007947	0.01358	4.12E-05	0.999922	4.12E-05	7.85E-05	0.007947	0.018217	
8:43	20	0.028558	0.059079	0.007104	0.03158	0.000383	0.525559	0.000201	0.474441	0.012377	0.023614	
8:59	24	0.015194	0.016898	0.013403	0.015165	1.5E-06	0.992643	1.49E-06	0.007357	0.013478	0.017987	

Table D7.51: Boarding Passenger Prediction Results of the 3 Predictors

RCODE	Actual Boarding Passengers	Developed Model	KF Model	Naïve Model	d1	d2	d3
6	8	7	6	2	0.1250	0.2500	0.7500
11	10	9	7	34	0.1000	0.3000	2.4000
17	13	8	6	4	0.3846	0.5385	0.6923
20	13	7	10	6	0.4615	0.2308	0.5385
26	10	8	5	3	0.2000	0.5000	0.7000
37	11	8	6	21	0.2727	0.4545	0.9091
41	6	7	8	5	0.1667	0.3333	0.1667
44	6	7	11	9	0.1667	0.8333	0.5000
49	8	7	11	7	0.1250	0.3750	0.1250
62	6	7	4	1	0.1667	0.3333	0.8333
72	6	6	4	1	0.0000	0.3333	0.8333
80	8	7	10	3	0.1250	0.2500	0.6250
86	5	7	7	5	0.4000	0.4000	0.0000
91	12	8	8	19	0.3333	0.3333	0.5833
96	7	7	8	2	0.0000	0.1429	0.7143

Note: d1, d2, d3: The absolute relative error (%) of the Developed Model, the Kalman Filter and the Naïve predictors, respectively

Table D7.52. Kalman Filter for test case RCODE = 1

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	15	17	15	15.66667	1.111111	0.506934	0.56326	0.493066	12.0416	15
8:43	20	17	19	15	17	2	0.561717	1.123434	0.438283	15	20
8:59	24	14	18	17	16.33333	4.111111	0.560105	2.302652	0.439895	18	13

Table D7.53. Kalman Filter for test case RCODE = 6

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	11	9	10	10	1	0.507692	0.507692	0.492308	9.507692	9
8:43	20	10	7	11	9.333333	2.944444	0.539685	1.589072	0.460315	10.07937	12
8:59	24	9	8	14	10.33333	3.611111	0.590172	2.131178	0.409828	13	11

Table D7.54. Kalman Filter for test case RCODE = 11

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	8	11	9	9.333333	2.277778	0.503407	1.146648	0.496593	9	11
8:43	20	14	11	12	12.33333	2.277778	0.600544	1.367906	0.399456	11.60054	12
8:59	24	11	10	10	10.33333	0.277778	0.855584	0.237662	0.144416	11	9

Table D7.55. Kalman Filter for test case RCODE = 14

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	11	10	10	10.33333	0.277778	0.526627	0.146285	0.473373	9.526627	11
8:43	20	12	13	14	13	0.5	0.563808	0.281904	0.436192	12.69143	12
8:59	24	9	12	10	10.33333	2.277778	0.529138	1.205258	0.470862	11	9

Table D7.56. Kalman Filter for test case RCODE = 17

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	8
8:39	19	11	11	15	12.33333	1.777778	0.504356	0.896633	0.495644	11.53049	15
8:43	20	12	14	23	16.33333	12.11111	0.517848	6.271712	0.482152	19.14278	18
8:59	24	9	12	17	12.66667	6.944444	0.655544	4.552387	0.344456	17	19

Table D7.57. Kalman Filter for test case RCODE = 23

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	8
8:39	19	15	15	15	15	0	1	0	0	15	17
8:43	20	18	22	17	19	5	0.5	2.5	0.5	17	22
8:59	24	19	15	17	17	4	0.619048	2.47619	0.380952	19	15

Table D7.58. Kalman Filter for test case RCODE = 26

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	8
8:39	19	17	17	17	17	0	1	0	0	17	17
8:43	20	19	19	18	18.66667	0.111111	0.5	0.055556	0.5	17.5	15
8:59	24	15	17	16	16	1	0.513514	0.513514	0.486486	15	13

Table D7.59. Kalman Filter for test case RCODE = 30

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	8
8:39	19	17	13	15	15	4	0.501946	2.007782	0.498054	11.51362	19
8:43	20	21	19	18	19.33333	1.444444	0.705015	1.018355	0.294985	18.29498	19
8:59	24	15	17	14	15.33333	1.444444	0.630316	0.910457	0.369684	16	17

Table D7.60. Kalman Filter for test case RCODE = 37

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	8
8:39	19	15	7	9	10.33333	16.44444	0.500475	8.230027	0.499525	8.500475	8
8:43	20	18	9	11	12.66667	20.94444	0.582105	12.19187	0.417895	9.746315	9
8:59	24	17	9	10	12	17	0.63197	10.74349	0.36803	10	8

Table D7.61. Kalman Filter for test case RCODE = 41

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	8
8:39	19	7	9	6	7.333333	1.444444	0.505351	0.729951	0.494649	6.989298	9
8:43	20	9	8	11	9.333333	0.944444	0.639365	0.603845	0.360635	10.27873	9
8:59	24	9	8	9	8.666667	0.277778	0.760413	0.211226	0.239587	9	8

Table D7.62. Kalman Filter for test case RCODE = 44

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	9	9	9	9	0	1	0	0	9	7
8:43	20	9	11	9	9.666667	1.111111	0.5	0.555556	0.5	8	9
8:59	24	7	9	7	7.666667	1.111111	0.6	0.666667	0.4	9	8

Table D7.63. Kalman Filter for test case RCODE = 55

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	17	15	13	15	2	0.503876	1.007752	0.496124	11.0155	13
8:43	20	14	18	21	17.66667	6.777778	0.534599	3.623393	0.465401	17.27679	15
8:59	24	19	15	15	16.33333	4.444444	0.644793	2.865749	0.355207	15	15

Table D7.64. Kalman Filter for test case RCODE = 59

time	trip	art3	art2	art1	Average	Var[data]n	g(k+1)	e(k+1)	a(k+1)	Predict	Observed
8:33	17	0	0	0	0.025	0.000625	0.5	0	0.5		0.025
8:36	18	0	0	0	0.25	0.0625	0.5	0.03125	0.5	0.0125	9
8:39	19	13	15	17	15	2	0.503876	1.007752	0.496124	13.03101	15
8:43	20	16	17	19	17.333333	0.944444	0.673952	0.63651	0.326048	17.69581	15
8:59	24	16	14	18	16	2	0.568641	1.137282	0.431359	17	17

Table D7.65: Boarding Passenger Prediction Results of the 3 Predictors

RCODE	Actual number of De-boarders	Developed Model	KF Model	Naïve Model	d1	d2	d3
1	11	11	18	17	0.0000	0.3846	0.5455
6	11	10	13	15	0.0909	0.0833	0.6667
11	9	11	10	10	0.2222	0.1667	0.0909
14	9	11	11	9	0.2222	0.1667	0.1818
17	19	16	17	13	0.1579	0.0556	0.1333
20	15	16	19	22	0.0667	0.1364	0.2941
23	15	17	18	19	0.1333	0.0526	0.1176
26	13	16	15	13	0.2308	0.1176	0.1333
30	17	16	16	15	0.0588	0.1579	0.2105
37	8	8	10	8	0.0000	0.1111	0.0000
41	8	8	9	7	0.0000	0.0000	0.2222
44	9	9	9	9	0.0000	0.1818	0.1250
49	8	8	8	7	0.0000	0.1111	0.0000
55	15	15	15	11	0.0000	0.0000	0.1538
59	17	16	17	14	0.0588	0.1333	0.0667

Note: d1, d2, d3: The absolute relative error (%) of the Developed Model, the Kalman Filter and the Naïve predictors, respectively

Appendix D8

1. General

This appendix covers the mathematical concepts of the Tukey test used for the purpose of evaluating the models' performance. As discussed in chapter 6, section 6.4.3.6 and section 6.4.1, MAPE is the most suitable criterion to be compared so that the performance of the models can be ranked.

2. Tukey Test Procedure

The results presented in chapter 6 were obtained by following the steps suggested by Netter *et al.*, (1985).

Step 1: Statistical hypothesis

$$H_0: \text{MAPE}_N = \text{MAPE}_K = \text{MAPE}_P$$

$$H_a: \text{MAPE}_N \geq \text{MAPE}_K \geq \text{MAPE}_P \text{ or } \text{MAPE}_K \geq \text{MAPE}_P \geq \text{MAPE}_N$$

Where N = Naïve model, K= Kalman Filter Model, P = Developed model

Let i = The number of models being compared; $i=3$

j = the number of elements of the sample test

Select level of significance $\alpha = 0.05$, then select suitable $Q_{\alpha, i, i(j-1)}$ from the studentized range distribution.

Step 2: Determine range $w = Q_{\alpha} \sqrt{MSE / j}$ where MSE = mean square error within group.

Step 3: List the sample's MAPE in the increasing order and make pairwise comparisons. If the difference is smaller than the predetermined range, the null hypothesis cannot be rejected, or there is no statically significant difference between the pairwise models at $\alpha = 0.05$. Such a difference is underlined.

Appendix D9

1. General

This appendix covers the predicted arrival times of 15 full trips at three selected stops. These are Holmwood, Gladstone, and Rideau stops.

2. Prediction Results

The results are tabulated in Tables D9.1-D9.45. The left hand side part of the tables shows the predicted running times between stops and dwell times at the stops by applying developed running time and dwell time prediction modules, respectively. The right hand side part of the tables presents the predicted arrival time obtained from the general equations 3.4 and 3.5 (chapter 3). The updated predictions followed the procedure presented in chapter 3, section 3.3.4.

The following legends are used in Tables D9.1 to D9.45

T6-R1-54: Running time from stop 1 to stop 54, trip 6;

T6-R54-3: Running time from stop 54 to stop 3, trip 6

Boar: Number of boarding Passenger at a stop

Ali: Number of alighting passenger at a stop

Dwell: Dwell time

Arv.Holwood: Arival time at Holmwood stop

(number): The number showing the times the prediction is updated.

Error: Prediction Error, the difference between the predicted arrival time at last stop to that of schedule time.

%. Relative Prediction Error

Numbers in the colored boxes are the predicted values

Numbers in non-colored boxes are the actual values

**Table D9.1- D9.15. Real-time Bus Arrival Prediction at Stops: Holmwood, Gladstone, and Rideau.
Route 1-Direction: Downtown 8:59 a.m. bus trips. The Developed Model**

Table D9.1

dcode	Update	t6-R1-54	boar54	all.54	dwell 54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
17	1st	556.61	11		34.5	368.75	7		24.5	603.94	556.61	959.86	1588.30
		630.5	13		39.5		10		32				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					354.55			7	583.95		1024.55	1615.50
										T6-R3-53(3)			
	last									585.62			1255.62
										569.5	630.5	670	1239.50

Table D9.2

dcode	Update	t6-R1-54	boar 54	ali. 54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
12	1st	513.47	2		12	457.22	2		12	605.42	513.47	982.69	1600.11
		604.5	2		12		3		14.5				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					407.63	4		17	585.36		1024.13	1626.49
						397.5	3		14.5	T6-R3-53(3)			
	last									614.92			1643.42
										673	604.5	1014	1701.50

Table D9.3

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
1	1st	572.75	3		14.5	329.57	3		14.5	592.52	572.75	916.82	1523.84
		558.5	1	0	9.5		2	0	12				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					409.4	3		14.5	672.18		977.40	1664.08
						373.5	2		12	T6-R3-53(3)			
	last									594.12			1547.62
										534	558.5	941.5	1487.50

Table D9.4

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
2	1st	591.08	3		14.5	370.02	3		14.5	611	591.08	975.60	1601.70
		492	3		14.5		3						
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					406.58	4		17	632.22		913.08	1562.30
						372.5	3		14.5	T6-R3-53(3)			
	last									602.23			1495.73
										568	492	879	1461.50

Table D9.5

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
3	1st	591.08	3		14.5	385.46	3		14.5	583.16	591.08	991.04	1588.70
		509.87	1		9.5		2		12				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					372.39	4		17	581.17		891.76	1489.93
						391.5	2		12	T6-R3-53(3)			
	last									607			1529.87
										578	509.87	910.87	1500.87

Table D9.6

dcode	Update	t6-R1-54	boar54	all54	dwel54	T6-R54-3(1)	boar3	all3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
4	1st	516.99	3		14.5	362.67	3		14.5	590.08	516.99	894.16	1498.74
		538.5	1		9.5		0		7				
	2nd					T6-R54-3(2)				T6-R3-53(2)			
						388.54	4		17	579.94		936.54	1533.48
						435.5	0		7	T6-R3-53(3)			
	last									592.16			1582.66
										659	538.5	983.5	1649.50

Table D9.7

dcode	Update	t6-R1-54	boar54	all54	dwel54	T6-R54-3(1)	boar3	all3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
5	1st	544.5	4		17	404.1	4		17	606.53	544.50	965.60	1589.13
		590	2		12		3		14.5				
	2nd					T6-R54-3(2)				T6-R3-53(2)			
						433.25	5		19.5	801.87		1035.25	1656.62
						395.5	3		14.5	T6-R3-53(3)			
	last									644.45			1656.45
										614.5	590	997.5	1626.50

Table D9.8

dcode	Update	t6-R1-54	boar54	all54	dwel54	T6-R54-3(1)	boar3	all3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
20	1st	577.86	8		27	402	4		17	573.81	577.86	1006.86	1597.67
		616	9		29.5		3		14.5				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					451.13	4		17	584.43		1096.63	1698.06
						480	3		14.5	T6-R3-53(3)			
	last									607.06			1747.06
										540	616	1125.5	1680.00

Table D9.9

dcode	Update	t6-R1-54	boar54	all54	dwel54	T6-R54-3(1)	boar3	all3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
24	1st	552.65	10	32	412.23	412.23	2		12	573.81	552.65	996.88	1582.69
		622	16	47			2		12				
	2nd					T6-R54-3(2)				T6-R3-53(2)			
						316.58	4		17	811.22		985.58	1613.80
						321	2		12	T6-R3-53(3)			
	last									595.45			1597.45
										674	622	990	1676.00

Table D9.10

dcode	Update	t6-R1-54	boar54	all54	dwel54	T6-R54-3(1)	boar3	all3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
28	1st	544.37	12	37	388.87	388.87	2		12	600.85	544.37	970.24	1583.09
		561	14	42			6		22				
	2nd					T6-R54-3(2)				T6-R3-53(2)			
						350.34	5		19.5	608.37		953.34	1581.21
						368	6		22	T6-R3-53(3)			
	last									597.66			1590.66
										597.5	561	971	1590.50

Table D9.11

dcode	Update	t6-R1-54	boar54	all54	dwel54	T6-R54-3(1)	boar3	all3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
49	1st	579.64	3	14.5	680.28	680.28	3		14.5	802.54	579.64	1274.42	1891.46
		660.5	4	17			1		9.5				
	2nd					T6-R54-3(2)				T6-R3-53(2)			
						658.56	5		19.5	678.19		1336.06	2033.75
						732.5	1		9.5	T6-R3-53(3)			
	last									598.08			2017.58
										586	660.5	1410	2005.50

Table D9.12

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
52	1st	585.88	3		14.5	704.84	3		14.5	606.4	585.88	1305.22	1926.12
		499.95	3		14.5		4		17				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					733.99	3		14.5	602.43		1248.44	1865.37
						642	4		17	T6-R3-53(3)			
	last									588.5			1771.95
										649.5	499.95	1156.45	1822.95

Table D9.13

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
56	1st	548.47	2		12	631.32	4		17	584.25	548.47	1191.79	1793.04
		572	3		14.5		6		22				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					606.95	3		14.5	575.73		1193.45	1783.68
						660	6		22	T6-R3-53(3)			
	last									601.52			1870.02
										579.5	572	1246.5	1848.00

Table D9.14

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau
65	1st	558.67	3		14.5	472.99	3		14.5	613.75	558.67	1046.16	1674.41
		493	1		9.5		4		17				
						T6-R54-3(2)				T6-R3-53(2)			
	2nd					398.17	3		14.5	588.96		901.67	1505.13
						393	4		17	T6-R3-53(3)			
	last									610.95			1523.45
										572.5	493	895.5	1485.00

Table D9.15

dcode	Update	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. . Rideau
70	1st	394.37	3		14.5	334.58	5		19.5	609.38	394.37	743.45	1372.33
		390.5	1		9.5		3		14.5				
	2nd					T6-R54-3(2)	4		17	609.85		741.27	1368.12
	last					324	3		14.5	T6-R3-53(3)			1338.98
										600.18			1397.00
										658.5	390.5	724	

**Table D9.16 -D9.30. Real-time Bus Arrival Prediction at Stops: Holmwood, Gladstone, and Rideau.
Route 1-Direction: Downtown 8:59 a.m. bus trips. Kalman Filter-based model**

Table D9.16

dcode	t6-R1-54	boar54	ali.54	dwell 54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
17	508.01	11		34.5	412.6	7		24.5	547.52	508.01	955.11	1527.13	1st
	630.5	13		39.5		10		32					
					T6-R54-3(2)				T6-R3-53(2)				
					406.79			7	543.44		1076.79	1627.23	2st
					342.5				T6-R3-53(3)				
									544.06			1556.56	last
									569.5	630.5	1012.5	1582.00	

Table D9.17

dcode	t6-R1-54	boar 54	ali. 54	dwell54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Updated
12	480.39	2		12	367.15	2		12	574.08	480.39	859.54	1445.62	1st
	604.5	2		12		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					393.51	4		17	577.56		1010.01	1604.57	2st
						3		14.5	T6-R3-53(3)				
									583.33			1214.33	last
									673	604.5	616.5	1304.00	

Table D9.18

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
1	544.4	3		14.5	453.9	3		14.5	536.39	544.40	1012.80	1663.69	1st
	558.5	1	0	9.5		2	0	12					
					T6-R54-3(2)				T6-R3-53(2)				
					448.01	3		14.5	596.97		1016.01	1627.48	2st
					373.5	2		12	T6-R3-53(3)				
									663.09			1616.59	last
									534	558.5	941.5	1487.50	

Table D9.19

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
2	570.63	3		14.5	400.1	3		14.5	545.46	570.63	985.23	1545.19	1st
	492	3		14.5		3							
					T6-R54-3(2)				T6-R3-53(2)				
					407.23	4		17	587.84		913.73	1518.57	2st
					372.5	3		14.5	T6-R3-53(3)				
									549.06			1442.56	last
									568	492	879	1461.50	

Table D9.20

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
3	492.37	3		14.5	351.49	3		14.5	552.77	492.37	858.36	1425.63	1st
	509.87	1		9.5		2		12					
					T6-R54-3(2)				T6-R3-53(2)				
					392.97	4		17	578.51		912.34	1507.85	2st
					391.5	2		12	T6-R3-53(3)				
									620.06			1542.93	last
									578	509.87	910.87	1500.87	

Table D9.21

code	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
4	490.25	3		14.5	391.63	3		14.5	590.11	490.25	896.38	1500.99	1st
	538.5	1		9.5		0		7					
					T6-R54-3(2)				T6-R3-53(2)				
					375.72	4		17	585.06		923.72	1525.78	2st
					435.5	0		7	T6-R3-53(3)				
									571.49			1561.99	last
									659	538.5	983.5	1649.50	

Table D9.22

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
5	544.25	4		17	438.25	4		17	669.27	544.25	999.50	1685.77	1st
	590	2		12		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					445.95	5		19.5	620.73		1047.95	1688.18	2st
					395.5	3		14.5	T6-R3-53(3)				
									672.81			1684.81	last
									614.5	590	997.5	1626.50	

Table D9.23

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
20	588.21	8		27	385.84	4		17	586.86	588.21	971.08	1574.73	1st
	616	9		29.5		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					381.12	4		17	605.18		1026.62	1648.80	2st
					480	3		14.5	T6-R3-53(3)				
									578.38			1718.38	last
									540	616	1125.5	1680.00	

Table D9.24

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
24	608.9	10		32	401.6	2		12	623.93	608.90	1042.50	1678.43	1st
	622	16		47		2		12					
					T6-R54-3(2)				T6-R3-53(2)				
					348.41	4		17	667.29		1017.41	1701.70	2st
					321	2		12	T6-R3-53(3)				
									650.24			1652.24	last
									674	622	990	1676.00	

Table D9.25

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
28	480.3	12		37	436.61	2		12	556.66	480.30	953.91	1522.57	1st
	561	14		42		6		22					
					T6-R54-3(2)				T6-R3-53(2)				
					400.67	5		19.5	547.88		1003.67	1571.05	2st
					368	6		22	T6-R3-53(3)				
									531.34			1524.34	last
									597.5	561	971	1590.50	

Table D9.26

dcode	t6-R1-54	boar54	ali54	dwell54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
49	595.59	3		14.5	681	3		14.5	585.51	595.59	1291.09	1891.10	1st
	660.5	4		17		1		9.5					
					T6-R54-3(2)				T6-R3-53(2)				
					677.25	5		19.5	588.08		1354.75	1982.33	2st
					732.5	1		9.5	T6-R3-53(3)				
									607.54			2027.04	last
									586	660.5	1410	2005.50	

Table D9.27

dcode	t6-R1-54	boar54	ali54	dwell54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
52	612.75	3		14.5	722.12	3		14.5	586.87	612.75	1348.37	1950.74	1st
	499.95	3		14.5		4		17					
					T6-R54-3(2)				T6-R3-53(2)				
					721.09	3		14.5	629.59		1235.54	1879.73	2st
					642	4		17	T6-R3-53(3)				
									643.98			1817.43	last
									649.5	499.95	1156.45	1822.95	

Table D9.28

dcode	t6-R1-54	boar54	ali54	dwell54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
56	671.3	2		12	615.33	4		17	593.68	671.30	1298.63	1909.37	1st
	572	3		14.5		6		22					
					T6-R54-3(2)				T6-R3-53(2)				
					647.37	3		14.5	588.66		1233.87	1837.03	2st
					660	6		22	T6-R3-53(3)				
									574.71			1843.21	last
									579.5	572	1246.5	1848.00	

Table D9.29

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
65	613.16	3		14.5	422.19	3		14.5	597.06	613.16	1049.85	1661.41	1st
	493	1		9.5		4		17					
					T6-R54-3(2)				T6-R3-53(2)				
					420.55	3		14.5	642.29		923.05	1579.84	2st
					393	4		17	T6-R3-53(3)				
									571.64			1484.14	last
									572.5	493	895.5	1485.00	

Table D9.30

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
70	477.29	3		14.5	326.3	5		19.5	576.85	477.29	818.09	1414.44	1st
	390.5	1		9.5		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					334.87	4		17	605.83		734.87	1357.70	2st
					324	3		14.5	T6-R3-53(3)				
									563.7			1302.20	last
									658.5	390.5	724	1397.00	

**Table D9.31 -D9.45. Real-time Bus Arrival Prediction at Stops: Holmwood, Gladstone, and Rideau.
Route 1-Direction: Downtown 8:59 a.m. bus trips. The Naïve model**

Table D9.31

dcode	t6-R1-54	boar54	ali. 54	dwell 54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
17	465.59	11		34.5	328.23	7		24.5	519.84	465.59	828.32	1372.66	1st
	630.5	13		39.5		10		32					
					T6-R54-3(2)				T6-R3-53(2)				
					439.5			7	460.5		1109.50	1577.00	2st
					342.5				T6-R3-53(3)				
									620			1632.50	last
									569.5	630.5	1012.5	1582.00	

Table D9.32

dcode	t6-R1-54	boar 54	ali. 54	dwell54	T6-R54-3(1)	boar3	ali3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Updated
12	436.09	2		12	353.7	2		12	558.89	436.09	801.79	1372.68	1st
	604.5	2		12		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					350.94	4		17	529.49		967.44	1513.93	2st
					397.5	3		14.5	T6-R3-53(3)				
									550.56			1579.06	last
									673	604.5	1014	1701.50	

Table D9.33

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
1	606.29	3	0	14.5	387.5	3		14.5	577.33	606.29	1008.29	1600.72	1st
	558.5	1		9.5		2	0	12					
					T6-R54-3(2)				T6-R3-53(2)				
					474.8	3		14.5	452.9		1042.80	1510.20	2st
					373.5	2		12	T6-R3-53(3)				
									681.57			1635.07	last
									534	558.5	941.5	1487.50	

Table D9.34

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
2	622.22	3		14.5	353.41	3		14.5	660.85	622.22	990.13	1665.48	1st
	492	3		14.5		3							
					T6-R54-3(2)				T6-R3-53(2)				
					373.5	4		17	679.49		880.00	1576.49	2st
					372.5	3		14.5	T6-R3-53(3)				
									473.54			1369.04	last
									568	492	879	1461.50	

Table D9.35

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
3	411.6	3		14.5	277.86	3		14.5	546.09	411.60	703.96	1264.55	1st
	509.87	1		9.5		2		12					
					T6-R54-3(2)				T6-R3-53(2)				
					351.05	4		17	522.17		870.42	1409.59	2st
					391.5	2		12	T6-R3-53(3)				
									688.18			1611.05	last
									578	509.87	910.87	1500.87	

Table D9.36

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
4	498.01	3		14.5	481.6	3		14.5	687.3	498.01	994.11	1695.97	1st
	538.5	1		9.5		0		7					
					T6-R54-3(2)				T6-R3-53(2)				
					311.3	4		17	626.77		859.30	1503.07	2st
					435.5	0		7	T6-R3-53(3)			1442.52	last
									452.02				
									659	538.5	983.5	1649.50	

Table D9.37

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
5	578.3	4		17	499.36	4		17	711	578.30	1094.66	1822.66	1st
	590	2		12		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					629	5		19.5	574.89		1231.00	1825.39	2st
					395.5	3		14.5	T6-R3-53(3)				
									862			1874.00	last
									614.5	590	997.5	1626.50	

Table D9.38

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
20	616.12	8		27	395.5	4		17	515.73	616.12	1038.62	1571.35	1st
	616	9		29.5		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					349.5	4		17	604		995.00	1616.00	2st
					480	3		14.5	T6-R3-53(3)				
									631.03			1771.03	last
									540	616	1125.5	1680.00	

Table D9.39

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
24	684.8	10		32	509.52	2		12	619.61	684.80	1226.32	1857.93	1st
	622	16		47		2		12					
					T6-R54-3(2)				T6-R3-53(2)				
					293.92	4		17	696.23		962.92	1676.15	2st
					321	2		12	T6-R3-53(3)				
									613.8			1615.80	last
									674	622	990	1676.00	

Table D9.40

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
28	455.05	12		37	709.84	2		12	615.73	455.05	1201.89	1829.62	1st
	561	14		42		6		22					
					T6-R54-3(2)				T6-R3-53(2)				
					384.63	5		19.5	495.38		987.63	1502.51	2st
					368	6		22	T6-R3-53(3)				
									578.7			1571.70	last
									597.5	561	971	1590.50	

Table D9.41

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
49	570.06	3		14.5	768.45	3		14.5	612	570.06	1353.01	1979.51	1st
	660.5	4		17		1		9.5					
					T6-R54-3(2)				T6-R3-53(2)				
					570.8	5		19.5	566		1246.30	1833.80	2st
					732.5	1		9.5	T6-R3-53(3)				
									636.17			2055.67	last
									586	660.5	1410	2005.50	

Table D9.42

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv.	Arv.	Gladstone	Arv.	Rideau	Update
52	527.5	3		14.5	714.73	3		14.5	559.38	527.50		1256.73		1830.61	1st
	499.95	3		14.5		4		17							
					T6-R54-3(2)				T6-R3-53(2)						
					661.05	3		14.5	702.05			1175.50		1892.05	2st
					642	4		17	T6-R3-53(3)						
									844.4					2017.85	last
									649.5	499.95		1156.45		1822.95	

Table D9.43

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv.	Arv.	Gladstone	Arv.	Rideau	Update
56	637.23	2		12	557.52	4		17	569.62	637.23		1206.75		1793.37	1st
	572	3		14.5		6		22							
					T6-R54-3(2)				T6-R3-53(2)						
					578.25	3		14.5	526.27			1164.75		1705.52	2st
					660	6		22	T6-R3-53(3)						
									487.41					1755.91	last
									579.5	572		1246.5		1848.00	

Table D9.44

dcode	t6-R1-54	boar54	all54	dwell54	T6-R54-3(1)	boar3	all3	dwell3	T6-R3-53(1)	Arv.	Arv.	Gladstone	Arv.	Rideau	Update
65	776.07	3		14.5	494.29	3		14.5	603.25	776.07		1284.86		1902.61	1st
	493	1		9.5		4		17							
					T6-R54-3(2)				T6-R3-53(2)						
					493.88	3		14.5	664.76			996.38		1675.64	2st
					393	4		17	T6-R3-53(3)						
									650.82					1563.32	last
									572.5	493		895.5		1485.00	

Table D9.45

dcode	t6-R1-54	boar54	ali54	dwel54	T6-R54-3(1)	boar3	ali3	dwel3	T6-R3-53(1)	Arv. Holmwood	Arv. Gladstone	Arv. Rideau	Update
70	511.35	3		14.5	335.67	5		19.5	597.76	511.35	861.52	1478.78	1st
	390.5	1		9.5		3		14.5					
					T6-R54-3(2)				T6-R3-53(2)				
					332	4		17	658		732.00	1407.00	2st
					324	3		14.5	T6-R3-53(3)				
									550.27			1288.77	last
									658.5	390.5	724	1397.00	