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Development of an Infrared Absorption Tomography Instrument for Temporally and Spatially Resolved Fuel Concentration Measurements

by

Mihaela Jekic, B. Eng.

A thesis submitted to the Faculty of Graduate Studies and Research

in partial fulfillment of the requirements for the degree of

Master of Applied Science

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering

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submitted by

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in partial fulfillment of the requirements for the degree of

Master of Applied Science

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To Nera and Mico

Abstract

Fuel-to-air ratio non-uniformities play a key role in the noise characteristics and nitrogen oxide emissions performance of lean premixed combustors. An instrument was developed to measure spatially and temporally resolved fuel concentration fields through line-of-sight hydrocarbon absorption of $3.39\text{ }\mu\text{m}$ laser radiation. An optical system was designed to deliver a collimated infrared laser sheet into the test section and the attenuated laser sheet power is recorded by a custom-built infrared detector array. Absorption data acquired at multiple radial and angular positions is combined using the principles of computed tomography to reconstruct the time-resolved concentration profiles. A tomography algorithm called the constrained conjugate gradient method was implemented and tested with an artificially generated Gaussian function that models the concentration distribution of a fuel jet. An attempt was made to experimentally validate the algorithm through measurements of known concentration fluctuations induced by a modulated axisymmetric methane jet in co-flowing air.

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Chapter 1

Introduction

Over the last decade, there has been a resurgence of research interest in gas turbine combustion noise, including the fundamental causes, passive and active noise control, and computational models to help develop control strategies and improved combustor designs. This interest has in part stemmed from environmental regulations imposing increasing constraints on the emission levels of pollutants such as nitrogen oxides (NO_x) and unburned hydrocarbons (UHC). The continual drive toward lower emissions has led to the development of lean premixed (LP) combustors that have a propensity to vibrations. Unlike diffusion combustors, in which fuel and air are mixed by turbulent diffusion in the flame zone, LP combustors contain a separate region called a premixer where the fuel and air are mixed prior to entering the flame zone. The term “lean” comes from the fact that LP combustors

operate at lean equivalence ratios of ~ 0.5 (actual fuel-to-air ratio normalized by the fuel-to-air ratio required for complete combustion) to keep the flame temperatures below ~ 1900 K and thus mitigate the formation of thermal NO. LP combustors can be gaseous-fuelled, as in most stationary gas turbines, or liquid-fuelled, in which case known as as lean premixed prevaporized (LPP).

A well known instability mechanism in LP combustors is referred to as “acoustic feed-system coupling” or “convective time delay”. Perturbations to the fuel and air supply lines produce ϕ fluctuations that are convected into the flame front, and lead to oscillations in flame temperature, location, and surface area. The resulting heat release fluctuations generate pressure waves that propagate upstream to the supply lines. Depending on the phase relations between the heat release fluctuations and the dynamic pressure oscillations in the combustor, this feedback mechanism can amplify or damp the instability. In LP combustors, a combination of localized regions of extinction and re-ignition resulting from operating very close to the lean stability limit, and relatively low acoustic losses due to the absence of liner holes and wall-cooling slots, can potentially result in significant dynamic pressure fluctuations.

Based on the above instability mechanism, ϕ non-uniformities are expected to be strongly linked to the noise characteristics and NOx emissions performance of LP combustors. To mitigate noise generation and emissions, not only spatial but also temporal uniformity is required. There is recent evidence that the cross-correlations between time-resolved ϕ and dynamic pressure measurements can predict the level of combustion instability. The ability to instantaneously capture ϕ fluctuations over the entire combustor inlet plane, coupled with simultaneously acquired dynamic pressure measurements, could provide great benefits in understanding combustor noise.

1.1 Objective and Scope

The overall aim of this program is to design a non-intrusive, low-cost, spatially and temporally resolved absorption tomography instrument for the measurement of 2D fuel concentration profiles. Absorption tomography is a technique popular in the medical field that involves line-of-sight absorption measurements of radiation after traversing a medium. This data is acquired at multiple angles or “projections” and is subsequently numerically combined

to identify 2D species concentrations. The scope of this thesis was to design and characterize the required optical system and select the associated electronics necessary to obtain the tomography data, and to select, implement and perform preliminary testing of the tomographic reconstruction algorithm. In addition, preliminary validation of the instrument was attempted employing an enclosed axisymmetric methane jet in co-flowing air.

It is important to note that a crucial difference exists between this program and the research described in literature in that both the experimental setup and the tomographic reconstruction algorithm are designed for time-resolved capture of non-axisymmetric fuel concentration fields. The reason why an axisymmetric jet is used in preliminary validation attempts is to limit the number of projections to one before additional funds are invested in the lasers, optics and detectors for the remaining projections. No attempt has been made yet to apply the technique to a combustion situation.

1.2 Organization

Following the literature survey in Chapter 2, the design and characterization of the optical and electronic components will be presented in Chapter 3. Chapter 4 will outline the theory behind tomographic reconstruction, discuss the software developed to reconstruct the fuel concentration, and present the preliminary results obtained with artificial data. Flow characterization through hot wire measurements, and validation attempts utilizing the axisymmetric methane jet, will be presented in Chapter 5. Finally, conclusions and recommendations will be discussed in Chapter 6.

Chapter 2

Literature Review

2.1 Introduction to Absorption Tomography

The word tomography comes from the Greek words *tomos* (a slice or cross section) and *graphein* (to write), and it means a technique of creating cross-sectional images. The history of the tomographic reconstruction techniques originated in 1917 when the Austrian mathematician J. Radon proved that an object can be reconstructed uniquely from the infinite set of all its projections or “shadows” (Gordon et al., 1975). In contrast to the infinite number of exact projections required by Radon’s theorem, when the reconstruction is applied to a finite amount of real data accompanied by noise, it yields reconstructions that are at best only estimates of the object’s actual structure (Gordon et al., 1975). However, through the ongoing development of sophisticated tomographic reconstruction algorithms, it is possible to obtain

a reasonable estimate of the actual object.

Tomography first found applications in medicine by passing X-rays at different angles through a cross-section of the human body, and using special sensors to measure the amount of radiation absorbed by different biological tissues such as bone, fat, muscle, skin, or tumors. This is a significant improvement over conventional X-rays, which project a single image of the tissues onto a sheet of film and where the overlapping structures are often difficult or impossible to distinguish (Herman and Lent, 1976). While CT (Computed Tomography) commercial instruments have been extensively used in hospitals around the world during the last thirty years, tomography has also found increasing application in various other fields including fluid mechanics and combustion (e.g. Emmerman et al., 1980, Santoro et al., 1981, Goulard and Ray, 1985, Kawazoe et al., 1997, Timmerman, 1997, Hindle et al., 2001 and Girard et al., 2002). Unlike CT scans, where the body composition does not change rapidly and therefore a relatively small number of X-ray source-detector pairs can be rotated sequentially around the human body, in fluid mechanics the problem is compounded by the rapidly varying fluid flows and the consequent requirement for simultaneous data acquisition

at each angle. This greatly increases the number of detection units and the complexity of the data acquisition system. This complexity has hindered advances in the absorption technique beyond measuring fuel concentration at a single point, at steady state conditions, or for axisymmetric flows. To the author's knowledge only Hindle et al. (2001) have obtained some preliminary data with coarsely resolved full 2D tomography. Several approaches to absorption and tomography studies by various researchers, including Hindle, will be presented in this section.

In order to utilize tomography for hydrocarbon concentration measurements, the first step is finding a quantitative link between the measured laser beam absorption and the fuel concentration. This relationship is given by the Beer-Lambert Law:

$$\frac{I}{I_o} = e^{-\alpha PLX_f} \quad (2.1)$$

where I is the attenuated laser beam intensity after traversing a distance L through the medium, I_o the unattenuated intensity in the absence of the absorbing species, α the absorption coefficient, P the pressure inside the test

section, and X_f the fuel mole fraction. By measuring I and I_o at the same time as the test section pressure and temperature on which the absorption coefficient is dependent, the path-averaged fuel concentration can be extracted for a given path line through the test section. If this line-of-sight absorption measurement is captured simultaneously over a number of radial and angular positions, the data can be combined through a tomographic reconstruction algorithm to yield 2D or 3D profiles of hydrocarbon distribution. This provides the vital link between absorption and tomography for determining fuel concentration profiles.

2.2 Fluid Mechanics and Combustion Applications

2.2.1 Alternatives to Tomography for Concentration Measurements

Several imaging techniques currently exist to measure 2D fuel concentration fields, the most notable of which is planar laser-induced fluorescence (PLIF). Numerous studies have applied PLIF to determine species concentrations at atmospheric pressure, and recently some researchers such as Fink et al. (2002) have extended it to LPP combustors at elevated pressure conditions

using n-heptane as fuel. However, the instrumentation which includes a tunable Excimer laser and a photometric digital camera is very expensive, and in most cases fluorescent dopants such as acetone must be added and corrected for quenching. Most importantly, data is limited to the repetition rate of the pulsed laser (typically <100 Hz), and only measurements parallel to the combustor axis can be made without severe modifications to the engine. This not only makes the application non-trivial and expensive to implement but also precludes the determination of the fuel-to-air ratio field perpendicular to the combustor axis. Instead of direct imaging techniques, line-of-sight methods such as tomography that permit the detection instruments to be positioned on the same plane as the light source and perpendicular to the flow direction appear to be better suited for fuel concentration measurements in real combustors.

2.2.2 1D Tomography

Spatial and temporal non-uniformities in the fuel-to-air ratio are thought to play a major role in the noise characteristics and NO_x emissions performance of LP combustors. As a result, there have been recent efforts to develop a non-intrusive instrument capable of measuring fuel-to-air ratio fluctuations

and to subsequently relate them to other measured parameters. Systems utilizing 1D tomography will be presented first, followed by extensions to 2D fields.

One such instrument has been designed by Quang-Viet Nguyen of NASA Glenn Research Centre and presented in Nguyen (2002). This instrument captures real-time fuel-to-air ratio fluctuations at an LPP combustor inlet, which are acquired synchronously with the combustor dynamic pressure, fuel injector pressure, flame luminosity and NO_x level measurements. The technique utilizes line-of-sight absorption by the C-H vibrational stretching mode at 3.39 μm of Jet A fuel that is decomposed into smaller hydrocarbons at high temperatures. Nguyen (2002), who states that to his knowledge this type of real-time data had never been described in literature prior to the publication of his work, has found that the cross correlation between the fuel-to-air ratio and dynamic pressure predicts the level of combustion instability.

This work has significant limitations. Real-time fuel-to-air ratio measurements can only be obtained at a single point, while 2D mapping is only valid for steady-state flows. However, it is not possible to acquire data that

is both spatially and temporally resolved. The ability to instantaneously capture the entire fuel-to-air ratio field at the premixer outlet has the potential to provide great benefits in studying the fundamental behaviour of combustion noise and its relationship to emissions, which are not yet fully understood.

A temporally resolved point measurement system has recently been designed, constructed and validated by Rolls-Royce Canada in collaboration with the National Research Council of Canada. This instrument is based on hydrocarbon absorption measurements at $3.39\text{ }\mu\text{m}$ and contains a calibrated gas cell into which fuel is drawn through a sampling probe with a typical spatial resolution of 1 mm^2 . More details regarding the system validation and the application to the study of the effects of turbulent mixing on the spatial dispersion of equivalence ratio fluctuations downstream of the fuel injection can be found in Yimer and Bourque (2005). Although useful in observing concentration variation at a single point, this technique cannot be extended to two dimensions while remaining time-resolved.

2.2.3 2D Tomography

One of the first attempts at capturing 2D fuel concentration profiles through absorption tomography came as early as 1980 in a paper presented by Emmerman et al. and a similar paper by Santoro et al. (1981). These researchers succeeded in capturing the steady-state concentration distribution of an off-axis methane-argon jet in a quiescent air, and furthermore investigated the effect of the number of projections on the accuracy of frequency domain tomographic reconstructions. Several important results emerged from this work. The authors observed a significant degradation in reconstruction quality close to the jet inlet, including deviation from expected peak magnitude and reconstruction artifacts away from the jet. These effects were attributed to aliasing resulting from insufficient data points to resolve the steep gradients (high frequencies) and were similar to the aliasing artifacts observed when the number of projections was reduced further downstream. In addition, the authors noted that due to the fairly smooth concentration maps typically encountered in combustion applications, the number of viewing angles could potentially be reduced. A few years later, this work evolved into the measurement of temperature and OH concentration distributions in a premixed methane flame within 5 ms using ultraviolet beam scanning (for

further information, refer to Goulard and Ray (1985)).

A 2D technique called Statistical Absorption Tomography has been developed by Sivathanu et al. (2001), which considers the spatial correlation of local transmittances in order to achieve very high resolution up to $\frac{1}{50}th$ of the integral length scale of turbulent flows. Based on this technique, the authors have designed a six-axis optical spray patternator that has since become a commercial product (Lim and Sivathanu, 2002). However, this method cannot capture instantaneous concentration fields.

Another application of 2D tomography has been the work of Girard, Dibble and Chen of the University of California, who have utilized infrared light emitting diodes (LEDs) to detect the time-averaged and RMS axisymmetric radial profile of methane concentration in air by employing a genetic algorithm. Girard et al. (2002) have applied Large Eddy Simulations (LES) to generate a data set to synthetically test the genetic algorithm. The authors have found that the LES results show a reasonable agreement with experimental data reconstructed with the genetic algorithm. Although the above researchers indicated that this new application of tomography to reproduce

both the time-averaged and RMS concentration profiles could potentially be extended to non-axisymmetric flow fields, the author has not found subsequent updates to this work.

Several researchers have applied tomography to measure fuel concentration fields in internal combustion engines. These systems are typically fiber-optic based, but differ in complexity and reconstruction technique employed. In order to analyze the fuel-air mixture formation process in automotive engines, Kawazoe et al. (1997) have developed a tomographic system based on laser light absorption at $3.39\text{ }\mu\text{m}$ by the C-H vibrational stretching mode. Quantitative gaseous fuel distribution is obtained using the convolution back-projection algorithm, which falls under the category of frequency domain tomographic reconstruction techniques that require parallel, equally spaced beams inside the measurement section. The first part of the experiment involved an open diffusion flame with an axisymmetric methane jet in air and was performed by translating and rotating a single infrared source-detector pair. The second part of the experiment involved measurement of steady-state fuel-air mixing within the cylinder of a model engine and resulted in significant complications due to light refraction by the quartz cylinder wall.

The authors resolved this problem through accurate calculation of the positions and angles of the transmitter-receiver fiber optic pair and subsequent painstaking alignment to generate parallel equidistant beams required by the algorithm. Whereas spatially resolved, this system is not temporally resolved over a 2D plane due to the limitation of a single transmitter-receiver pair.

Carey et al. (1999) have conducted the preliminary design of a similar but more advanced absorption tomography technique for the measurement of 2D gasoline vapour distribution within the cylinder of an internal combustion engine. Through the application of algebraic reconstruction techniques (ART), instead of frequency domain techniques employed by Kawazoe et al. (1997) to generate the 2D concentration profile, non-parallel and irregularly spaced beams can be utilized. Taking advantage of the flexibility in the positioning of transmitter-receiver pairs gained by fiber optic based transmission and signal collection, these irregularly spaced beams can be distributed in desired configurations. However, in the subsequent update and prototype construction, Hindle et al. (2001) retained the use of ART but, like Kawazoe (1997), appear to have carefully adjusted the fiber positions to generate parallel, equally spaced beams. They utilized 4 projections spaced at 45°

intervals, each containing 8 transmitter-receiver pairs for a total of 32 simultaneously sampled beams. The resulting concentration profiles are obtained with a coarse resolution on the order of $\frac{1}{5}^{th}$ of the test section diameter and 1.2 ms (due to signal averaging). To the author's knowledge, these are the first and only published absorption tomograms showing 2D spatially and temporally resolved, non-axisymmetric species imaging. This work, conducted at the University of Manchester Institute of Science and Technology (UMIST), appears to have terminated in 2001.

Unlike most absorption studies, Carey and Hindle have exploited the weak absorption of long-chain hydrocarbons at $1.7\text{ }\mu\text{m}$ in the near infrared, which is the second harmonic of the fundamental C-H vibrational stretching mode. The potential advantage of $1.7\text{ }\mu\text{m}$ over $3.39\text{ }\mu\text{m}$ is access to off-the-shelf optoelectronic components designed for telecommunications. However, the diode laser is not readily available and the above researchers have employed a customized laser manufactured specifically for the application, resulting in higher cost.

2.3 Tomographic Reconstruction Techniques

2.3.1 Frequency Domain vs. Algebraic Techniques

Among the numerous tomography algorithms that have emerged over the past three decades, the two classes of algorithms that have found the most widespread use have been frequency domain and algebraic reconstruction techniques. Both types of techniques can be illustrated by Figure 2.1 in the spatial domain, where a sequence of beams traversing and interacting with the field of interest generates a set of path-averaged measurements termed a projection. Whereas algebraic techniques can utilize a variety of beam orientations, the frequency domain methods require parallel, equidistant beams.

Variants of the frequency domain techniques are based on the “Fourier Slice Theorem” that will be briefly introduced here. For a more rigorous mathematical derivation, the reader is referred to Kak and Slaney (1982). The Fourier Slice Theorem states that the Fourier Transform (FT) of a projection at an angle θ in the physical space gives a slice of the FT of the original object at the equivalent angle in the frequency plane. Adding a series of projections at various angles generates an image representation in the frequency domain. Therefore, it should theoretically be possible to retrieve the im-

Figure 2.1: Projection Illustration in the Spatial Domain (Adapted from Williams and Beck, 1995)

age in the spatial domain by performing a 2D inverse FT. However, in most practical cases where the computational load and speed of reconstruction are important, filtered back-projection is preferred to direct inversion. This algorithm involves applying a weighting function to the FT of a projection, and smearing or back-projecting the 2D inverse FT over the image plane. The addition of each subsequent projection to the reconstruction generates improved estimates of the image. Only a finite number of projections can be acquired, leading to greater error in the high frequency than the low frequency components. This results in image degradation manifested as streaks and Moire patterns (Williams and Beck, 1995). Although the image quality can be improved by interpolating between the points, for a small number of projections the artifacts would be excessive.

Unlike the frequency domain methods, ART permits reasonable image reconstruction with only a few viewing angles. This is beneficial from the point of view of hardware since greater gains are achieved by minimizing the number of projections (detector arrays) than the number of sampling points per projection (individual detector elements). ART offers the ability to constrain the intermediate function values to lower and upper bounds or

set negative concentrations to zero, define known flow region boundaries, or introduce “equal support” such that the sum of all paths per projection covering the same region is the same for all projections (Medoff et al., 1982). In addition to constraints, a priori information such as the average fuel concentration known from mass conservation can serve as the initial image estimate. ART is iterative in nature, and presents a considerable challenge in establishing the termination criteria for the iterative process. Furthermore, algebraic approaches are very sensitive to perturbations in the measured data, such as detector electronic noise and misalignment (Timmerman, 1997). Watt and Conery (1993) have found that angular misalignment leads to blurring whereas the more critical lateral misalignment leads to severe line artifacts. A combination of the two effects results in extremely corrupted images, with largest errors in the regions of steepest gradients. Another challenge with ART is the fact that it is essential to have accurate knowledge of the ray paths connecting the transmitter and receiver (Kak and Slaney, 1982).

2.3.2 Axisymmetric Reconstruction

Axisymmetric flow fields, such as a round fuel jet injected into co-flowing air, require measurements at only a single arbitrary angle. This type of flow is

commonly reconstructed utilizing 1D tomographic reconstruction techniques such as onion peeling, Abel inversion, or filtered backprojection reduced to one dimension. Among these, onion peeling is by far the most common. The details regarding these techniques, including the implementation algorithms and performance comparison, are presented by Dasch (1992). However, all three methods require equally spaced narrow beams through the measurement plane, which is not the case in the presence of wall refraction. In fact, the general form of the deconvolution expresses the function distribution as (Dasch, 1992):

$$f(r_i) = \frac{1}{\Delta r} \sum_{j=0}^{\infty} D_{ij} P(r_j) \quad (2.2)$$

where Δr is the spacing between the rays, $r_i = i\Delta r$ is the distance from the centre of the object, and D_{ij} are linear operator coefficients specific to each technique. It can be observed that the requirement of equally spaced intervals is built into the fundamental equation. Therefore, although relatively simple to implement, these techniques cannot be utilized for reconstructing axisymmetric concentration fields where test chamber refraction creates unevenly spaced divergent beams.

2.3.3 Limited Data Tomography

In practice it may not always be possible to have full 360° optical access or projections evenly distributed over 180° . As a result, some authors have investigated methods of reducing the severe streak artifacts induced by this limited-angle case with application to CT scanners. The missing projections must be consistent with the measured data and are estimated by utilizing information not contained in the measurements, such as constraints, in order to produce a unique image. A similar limitation may occur in fluid mechanics due to, for example, the presence of a central body in the middle of the test section or opaque test section supports. An interesting method of dealing with limited data by modifying existing iterative reconstruction algorithms is presented by Medoff et al. (1982). These authors formalize the conditions that must be satisfied by a consistent estimate of the missing data into a single functional relationship. Another method that involves the integration of the probabilistic “Bayesian” approach into iterative reconstruction techniques to reduce the limited data artifacts has been proposed by Hanson and Wecksung (1983). A very different approach suggested by Buonocore et al. (1981) utilizes the “natural” pixel decomposition of the x-y plane into a set

of discrete overlapping pixels uniquely defined by the beam paths and intersections. Based on this representation, a framework is established for filtered back projection algorithms with limited data.

A line-of-sight, non-intrusive commercial instrument capable of measuring time-resolved, 2D fuel concentration profiles does not currently exist, but could provide great benefits in understanding combustor noise. As a result, the development of such an instrument, which combines absorption of infrared radiation by hydrocarbon fuel and the principles of computed tomography, was undertaken. The design of the optical and electronic components required for the instrument operation will be discussed in the following chapter.

Chapter 3

Component Design and Characterization

In this chapter the design, selection, and characterization of the optical and electronic components required for the tomography system are discussed. These include the optical access to the flow, detector arrays and data acquisition system, laser, and beam transforming optics. In addition, the power budget estimate and reflection loss profiles are presented.

3.1 Optical Access Design

3.1.1 Geometry

Several designs were considered for the optical access geometry. The first was a square tube, which has the benefit that the rays remain parallel in the

absence of wall curvature refraction effects. This in turn broadens the range of applicable tomographic reconstruction techniques, including frequency domain methods that require uniformly spaced parallel rays. However, upon further analysis, a number of limitations prevented this design from being implemented. Unless only two orthogonal projections are utilized, which is typically insufficient to provide reasonable accuracy, the window supports would block portions of the laser sheets and result in loss of data and severe image distortion. In order to maintain the same number of detectors (i.e., data points) with two projections as with three, 1.5 times as many detectors per projection would be required and the test section size would consequently increase. This is a serious problem because the longer path length through the medium would result in significantly lower signal levels as attenuation increases exponentially with path length. In addition to the above limitations, the secondary recirculation zones produced at the four corners would preclude generating axisymmetric flows useful for validation of the instrument.

Another design considered was a hexagon-shaped tube suited for three projections. This geometry would result in ray paths being altered within the test section due to wall refraction but emerging collinear with the inci-

dent rays. Similar to the previous design, the joints between the hexagon sides would be opaque to the laser sheet and would result in loss of data. In addition, this shape is not representative of a real pre-mixer geometry, has not been well studied, and it would not submit to simple analytical expressions for jet propagation, if so required for validation purposes.

The chosen design is a circular tube, which was initially rejected due to complex beam propagation resulting from the curved refracting walls. Each ray within the test section makes a different angle with the centreline and diverges further upon exiting the test section. This effect is clearly illustrated in the ray diagram produced with OSLO optical design software (educational edition) in Figure 3.1. However, the disadvantages are outweighed by the numerous advantages. These advantages include full 360° optical access, the flexibility to increase the number of projections with no support blocking, the ability to generate axisymmetric flow fields for validation, and the fact that this geometry has been extensively studied and is closer to a real pre-mixer geometry. In addition, the spreading of the laser sheet due to refraction can be utilized in a beneficial way to allow more detectors per given test section diameter. This in turn improves the resolution while minimizing the

Figure 3.1: Ray Trace through the Sapphire Tube

attenuation. Another advantage of this design is the reflection loss profile, with reflection losses highest in the centre of the tube where the laser sheet intensity is also highest due to a Gaussian intensity profile.

3.1.2 Optical Material

The principal criteria for selecting the test section material are good transmission at $3.39\text{ }\mu\text{m}$, low reflection losses, and the ability to withstand compressive stresses in order to support the remainder of the test section mounted on top. Sapphire is the material that appears to best meet these requirements. It has a broad transmission range from 180 nm to $5\text{ }\mu\text{m}$ that is not temperature-sensitive, a relatively low index of refraction ($n = 1.7$) which results in reasonably low reflection losses, excellent mechanical strength that enables the use of thinner walls, and surface hardness which makes it highly scratch resistant. IR grade fused quartz or silica are less expensive; however, $3.39\text{ }\mu\text{m}$ is very close to their spectral cutoff in transmittance, and their material strength properties are inferior to those of sapphire.

3.1.3 Optical Access Size

The test section diameter was initially selected as 50.8 mm to match an existing contraction on the test rig. However, since the power input to the detectors was calculated to be extremely low as a result of significant methane absorption, the diameter was reduced to 23.8 mm . Although the signal could

be electronically amplified, the noise would be as well, and the quality of the results would be degraded due to a low signal-to-noise ratio (S/N). Other advantages of a 23.8 mm diameter cross section include a smaller laser sheet span that further increases the available power, and cost reduction since 23.8 mm falls within the standard size range offered by most sapphire tube manufacturers. The spatial resolution would improve, while the difference between adjacent detector readings would diminish. However, the instrument cannot rely on significant differences between adjacent detector readings since it must be able to capture fairly smooth concentration profiles expected at the premixer outlet.

An optically polished sapphire tube with a wall thickness of 1.6 mm was selected, and the test section was designed to minimize the loads imposed on this slender tube. The thickness was specified with a tolerance of ± 0.05 mm, which is expected to have only minor effects on beam propagation; furthermore, tighter tolerances would have led to a significant cost increment. The detector casing size dictated the minimum distance from the tube to avoid physical interference between neighbouring detector arrays. Although the details such as the specific distances will be discussed in subsequent sections,

Figure 3.2: Ray Trace of Three Intersecting Laser Sheets Traversing the Sapphire Tube

the portion of the cross section captured by three arrays is illustrated as the inner hexagon in Figure 3.2.

3.2 Light Source Selection

IR light emitting diodes (LEDs) initially appeared to be a low-cost, simple solution for the $3.39\text{ }\mu\text{m}$ source. However, in order to gain temporal and spatial resolution, multiple LEDs would have to operate simultaneously and the attenuated signal would have to be captured by dedicated detectors for each LED. A number of problems were evident with this arrangement. Resolution was limited by the fact that LEDs would have to be encased in protective packaging (minimum size approximately 1 cm), thus severely limiting the number of data points per projection. The broad spectral shapes of LEDs exceed the CH_4 absorption peak bandwidth, leading to adjacent wavelengths and insufficient attenuated signal to be detected. Power output would be excessively low (μW range), which would be further exacerbated by removing a portion of the beams to monitor power fluctuations. LEDs are only partially collimated, which means that a collimating lens would have to be placed in front of each LED, leading to high cost and complexity. Finally, potential for interference would exist between adjacent LEDs. On the other hand, the $3.39\text{ }\mu\text{m}$ Helium-Neon (HeNe) laser is near diffraction-limited and can be collimated with much lower divergence than an LED source, while generating significantly higher power. The linewidth of the HeNe laser line is less than

that of the methane absorption line, thus ensuring that only the signal that has been attenuated by methane absorption is received by the detectors. In addition, a single laser per projection can be utilized by expanding the laser beam into a sheet, which means that detector spacing rather than source spacing would limit the resolution.

3.3 Detection System Selection

3.3.1 Detectors

The choice of detector material was straightforward because lead selenide (PbSe) has excellent responsivity at $3.39\ \mu\text{m}$ and is commercially available at relatively low cost. However, a number of configurations were possible with these detectors. Transmitting and receiving the signal through fiber optics would offer greater flexibility in beam placement, but at the expense of increased data acquisition system complexity, as well as power reduction due to division of laser output among fiber optic lines. Charge coupled device (CCD) line arrays would be the optimal choice because of high resolution that can furthermore be adjusted by grouping the pixels into “bins”. Unfortunately, the CCD-type arrays suited for $3.39\ \mu\text{m}$ possessed insufficient span

and were very expensive. An alternative to CCDs is the bolometer arrays that alter resistance in response to a temperature change resulting from incident IR radiation. However, these highly sensitive devices are primarily used for the far infrared, are very expensive, and have excessively long response times (typically greater than 50 ms).

Another type of sensing units considered were photodetectors, which are sensitive to fluctuations in the number of incident photons at specific optimized wavelengths. These fast-response detectors can be operated in a photovoltaic or photoconductive mode. Detectors offered commercially in the photoconductive mode are significantly less expensive, but they require a bias voltage up to 100 V, and cannot be utilized at low frequencies due to excess “1/f” or “flicker” noise. However, to remove the effects of the slowly varying background temperature, an optical chopper would be employed that would also mitigate the low frequency detector noise. Higher frequencies can be achieved by increasing the number of slots on the optical chopper without introducing unwanted vibration resulting from more revolutions per second of the motor. As a result of the lower cost (particularly for multiple detectors) and the ability to overcome the disadvantages, photoconductive-mode

Figure 3.3: Single Detector Circuit

detectors were selected. An illustration of a circuit corresponding to a single detector element is presented in Figure 3.3, with optional electronic amplification and filtering.

Individual detectors are typically offered in standard packaging that limits the spacing between adjacent elements and therefore the resolution. As a result, the decision was made to purchase unpackaged elements in the form of plate cells and construct the detector array and circuit. A thermoelectric cooler mounted on a heat sink was utilized to lower thermal noise. In addition, amplifiers located at the back of the circuit board were intended to reduce noise by minimizing the transmission distance. However, due to problems encountered in mounting the tiny amplifiers, no amplification was

included in the preliminary setup.

The next issue was the choice of the detector active area, which was narrowed down to 1 x 1 mm or 2 x 2 mm. After careful consideration, it was evident that 1 x 1 mm active area offered several advantages. Due to a reduction in plate cell size, it is possible to fit more detectors into a smaller area, thus increasing the spatial resolution. The correspondingly narrower laser sheet span increases the available power and decreases the risk of interference between adjacent projections. While the active area dimension specifies the beam width for the purpose of tomographic reconstruction, rays of zero thickness would be ideal. Since the finite width acts as a low pass spatial filter by averaging the signal over the beam dimension, it is advantageous to utilize narrower beams that more closely approximate rays. Finally, the responsivity ($\frac{V}{W}$) increases by a factor of two for the smaller active area.

3.3.2 Data Acquisition Boards

IR photodetectors have continuous output and cannot hold information, which precludes the use of a multiplexer and necessitates a dedicated analog-to-digital converter (ADC) for each detector. It was found that the best

performance-to-cost ratio was obtained with National Instruments 8-channel, 16-bit data acquisition (DAQ) boards synchronized in a master/slave arrangement. Some of the properties are an acquisition rate of 31 kHz/channel (sufficient), analog input signal range of ± 5 V, and no built-in filtering. The cost of the DAQ boards limits the number of detectors and projections. This criterion, combined with the minimum plate cell spacing, the known detector distance from the test section, and the ray trace in Figure 3.1, resulted in the selection of 2 DAQ boards or 16 detectors per projection. It should be noted that an additional DAQ board must be utilized for the acquisition of experimental parameters such as static pressure, dynamic pressure, and methane and air flow rates. The cost of DAQ boards and synchronization concerns limited the tomography system to 3 projections (60° apart), or 7 DAQ boards.

3.4 Power Budget and Reflection Losses

Following the component selection, a power budget for each projection based on worst-case scenario was estimated as shown in Table 3.1. The data is based on methane absorption at the equivalence ratio of one, and for sim-

Table 3.1: Worst-Case Power Budget Estimate

Laser power	3	mW
Laser sheet span at detector location	50	mm
Laser sheet thickness at detector location	1	mm
Reflection losses through the sapphire tube	25	%
Absorption coefficient at equivalence ratio of 1	7.805	$\frac{1}{atmcm}$
Fuel mole fraction	0.095	
Chamber pressure	1	atm
Longest path length through the sapphire tube	2.5	cm
Minimum transmission through CH_4	15.7	%
Detector active area width	1	mm
Detector active area height	1	mm
Other optical losses (beamsplitter, lenses, ...)	50	%
Detector responsivity	10000	$\frac{V}{W}$
Detector input power	3.5	μW
Detector output voltage	35	mV

plicity assumes a uniform average laser sheet power rather than the actual Gaussian distribution. Although the power is very low, the high detector responsivity generates a reasonable voltage output.

The reflection losses induced by the sapphire tube depend strongly on the polarization of the laser. Output of linearly polarized lasers may be either p or s polarized with respect to a given surface, which greatly influences the surface reflectance. P polarization is desired since it produces significantly lower losses. At the Brewster angle, p polarized light suffers no reflection loss. To achieve p polarization, a germanium window at the Brewster angle

was positioned in front of the laser, and the laser was rotated until maximum power output was detected.

The reflectance is dependent on the angle formed by the laser sheet and the surface normal. For p polarization, the reflectance for a single surface is expressed as (Born and Wolf, 1980):

$$R_p = \left(\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \right)^2 \quad (3.1)$$

where subscript i denotes incident angle and t transmitted angle. Combining the reflection data for the two air-sapphire and two sapphire-air interfaces yields the reflection loss profile across the tube displayed in Figure 3.4. At the tube centreline, both the fuel absorption and the reflection losses are highest. However, this is in part mitigated by the laser sheet Gaussian intensity distribution with the peak at the centreline. Nonetheless, the significant degree of attenuation limits the methane concentration that can be captured by the instrument.

Figure 3.4: Total Reflection Loss Across the Sapphire Tube

3.5 Optics Design

Numerous challenges were encountered with designing the optics suited for $3.39\ \mu\text{m}$. The majority of the components were not available “off the shelf”, thus necessitating custom fabrication. Furthermore, the optics alignment was a significant challenge for many reasons. These include the invisibility of IR radiation to the human eye, lack of mid-infrared beam characterization systems, and low power. In addition, visible and IR beams differ in their propagation characteristics and their response to beam transforming optics, which precludes utilizing visible lasers for quantitative characterization of IR

systems.

The above challenges necessitated a significant reliance on theory in designing the custom optics. To understand the design, it is important to understand that real laser beams do not behave as a collection of parallel rays. This may only be true at the beam waist where the wavefront is flat (infinite radius of curvature). However, beyond this point the beams diverge according to the following relation (Sasnet, 1989):

$$w(z) = w_o \sqrt{1 + \left(\frac{\lambda M^2 z}{\pi w_o^2} \right)^2} \quad (3.2)$$

where w_o is the $\frac{1}{e^2}$ Gaussian beam waist radius, w is the beam radius at distance z from the waist and λ is the wavelength. M^2 is a parameter quantifying the deviation of the laser beam from the diffraction limited single-mode Gaussian distribution, and is expressed in terms of the far field divergence angle θ as follows (Sasnet, 1989):

$$M^2 = \frac{\pi w_o \theta}{2\lambda} \quad (3.3)$$

M^2 values greater than 1 are associated with increased beam divergence and focussed spot sizes. Equation (3.2) was widely employed in the design of the present system, starting from the laser sheet thickness specification discussed in the next section.

3.5.1 Laser Sheet Dimensions

Prior to proceeding with the optical design, the $\frac{1}{e^2}$ horizontal and vertical dimensions of the laser sheet entering the sapphire tube had to be defined. The focus or beam waist was set to be at the centre of the test section (CTS) in Figure 3.8 to ensure a symmetric sheet thickness distribution inside the test section. For ease of alignment and to capture maximum power, the laser sheet would ideally underfill the 1 x 1 mm detectors in the vertical plane. Due to the difficulty of focussing the laser sheet to a very small beam waist radius (w_{img}), the detectors should be positioned as close as possible to the CTS. Physical interference among adjacent detector arrays placed a lower limit on this distance as 60 mm for 3 projections, 60° apart. Therefore, solving Equation (3.2) for $z = 60$ mm and $w(z) \leq 0.4$ mm yields a range of suitable w_{img} values. Although the M^2 parameter was not provided by

the laser manufacturer, for HeNe lasers this value is typically very close to 1.0. Therefore, two curves were plotted in Figure 3.5 for the range of w_{img} corresponding to $M^2 = 1.0$ and $M^2 = 1.1$ in order to ascertain that the chosen value met both criteria. The valid range extended from 0.18 to 0.36 mm for $M^2 = 1.0$, and from 0.21 to 0.34 mm for $M^2 = 1.1$. A lower-end $w_{img} = 0.2$ mm was selected because the real waist would likely be larger than that predicted by theory. This value deviates less than 5% inside the test section. In terms of the collimated sheet span (CS) entering the sapphire tube, a value of 21 mm appeared reasonable to capture a sufficient portion of the cross section while avoiding photons originating from one projection being detected by another (Figure 3.2).

3.5.2 Laser Characterization

The laser had to be experimentally characterized to determine the location (ZL) and size (w_o) of the beam waist inside the laser cavity, illustrated in Figure 3.8. The principle used in the measurements was the relationship between the transmission (T) through an aperture of known diameter (D) and the beam waist (w), derived from Eckbreth, 1996:

Figure 3.5: Beam Radius at the Detector Plane as a Function of the Image Waist

$$w = D \sqrt{\frac{-1}{2 \ln(1 - T)}} \quad (3.4)$$

At least two measured beam radii at known distances from the laser were required in order to simultaneously solve two instances of Equation (3.2) with two unknowns, ZL and w_o . The transmittance was determined by taking the ratio of power transmitted through the aperture and total unob-

structed power. The power meter was operating at the lowest limit of its dynamic range, hence each measurement was repeated four times to obtain the average transmittance. At each location, the beam radius was measured for three different apertures, and averaged to get the final radius. The results were $ZL = 0.3$ m from the laser outlet and $w_o = 0.45$ mm. Despite the crude experimental technique used to define the laser output, the optics characterization showed that these values enabled reasonably accurate lens design.

3.5.3 Lens Configuration

Two types of lens configurations were considered to transform the beam into a collimated sheet. On one hand, the Keplerian system of Figure 3.6 consists of two spherical plano-convex (PCX) lenses with a common internal focus. The first lens focusses the beam, which subsequently expands and is collimated by the second lens. An additional cylindrical PCX lens can be added to focus the beam in one plane. The advantage of this configuration is that a spatial filter in the form of a small aperture can be introduced at the common focus to block scattered light and produce a very smooth irradiance profile. Another benefit of expanding the entire beam prior to focussing it

Figure 3.6: Keplerian System (Adapted from Melles Griot Optics Guide, 2005)

with the cylindrical lens is that divergence, which is inversely proportional to beam diameter, is minimized. This means that the optical path lengths for different projections would not have to be precisely matched, and more flexibility in the optical setup would be allowed. However, the beam would have to be focussed from 21 mm to 0.2 mm, leading to a magnification factor of approximately $\frac{1}{100}$. This extremely high demagnification would not only be difficult to achieve, but would lead to undesirable steep gradients or extremely long transmission distances.

On the other hand, the Galilean system of Figure 3.7 is composed of a

Figure 3.7: Galilean System (Adapted from Melles Griot Optics Guide, 2005)

plano-concave (PCV) and a PCX lens. Utilizing a cylindrical rather than spherical PCV lens, a fan could be formed in one plane while leaving the orthogonal plane unaffected. Through appropriate matching of the focal lengths f_1 and f_2 and lens distances, a single spherical PCX lens could both collimate the fan and focus it in the orthogonal direction. The shorter distances, smoother gradients, and cost and alignment benefits of fewer lenses led to the selection of the Galilean system.

3.5.4 Lens Material

Material selection for the beam transforming optics was a challenging task since standard, low-cost materials do not transmit radiation at $3.39\text{ }\mu\text{m}$. For IR-grade fused quartz, $3.39\text{ }\mu\text{m}$ is extremely close to the cutoff wavelength. Materials such as germanium ($n = 4$), silicon ($n = 3.43$) and zinc selenide ($n = 2.44$) suffer from significant reflection losses due to high indices of refraction, as shown by the following relationship for each surface (Born and Wolf, 1980):

$$r = \left(\frac{n - 1}{n + 1} \right)^2 \quad (3.5)$$

Sapphire is too expensive as a lens material, while some fluorides such as barium and lithium fluoride can be sensitive to atmospheric moisture. Finally, calcium fluoride (CaF_2) was selected due to its excellent transmittance, low n , and the off-the-shelf availability of CaF_2 spherical PCX lenses. Unfortunately, cylindrical PCV lenses would have to be custom built. Low n of 1.41 leads to low reflection losses, which means that the lenses can be utilized without anti-reflection coating (cost reduction).

3.5.5 Theory

A combination of Gaussian beam optics and geometrical optics was utilized to specify the focal lengths and positions of the lenses. The assumption that the lens focal length is equal to the distance between the lens and the image, as predicted by the simple lens equation, is not valid for Gaussian beams. Therefore, an alternative formulation developed by Self (1983) was employed. Imaging of Gaussian beams by simple lenses is described by a modified lens equation that treats the input beam waist as the object and the output beam waist as the image. This equation is expressed in terms of the Rayleigh range, which is defined as the distance over which the beam waist radius spreads by a factor of $\sqrt{2}$. The input beam Rayleigh range, Ray_{obj} , is expressed as (Sasnet, 1989):

$$Ray_{obj} = \frac{\pi w_o^2}{\lambda M^2} \quad (3.6)$$

In the Galilean system, the lens separation is given as $f_2 - f_1$. Therefore, the object distance (S_{obj}) between the beam waist inside the laser cavity and

the PCX lens can be expressed in terms of the distance from the laser to the PCV lens (Z_{pcv}), as follows:

$$S_{obj} = ZL + Z_{pcv} + f_2 - f_1 \quad (3.7)$$

Once the object distance is known, both the image distance (S_{img}) between the PCX lens and the CTS and w_{img} can be computed as follows (Self, 1983):

$$S_{img} = -f_2 \frac{-S_{obj}^2 + S_{obj}f_2 - Ray_{obj}^2}{Ray_{obj}^2 - 2S_{obj}f_2 + S_{obj}^2 + f_2^2} \quad (3.8)$$

$$w_{img} = w_o \frac{f_2}{\sqrt{(f_2 - S_{obj})^2 + Ray_{obj}^2}} \quad (3.9)$$

Geometrical optics is employed to create a link among several parameters, utilizing the thin-lens approximation. Referring to Figure 3.7, magnification can be defined in two ways through similar triangles. The first is a ratio of focal lengths, and the second a ratio of the collimated sheet span and diameter of the beam incident on the PCV lens. Both expressions are presented in the following equation:

$$M = \frac{f_2}{f_1} = \frac{CS}{2w_{pcv}} \quad (3.10)$$

The equations discussed in this section will next be applied to quantitatively define the optical parameters.

3.5.6 Optical Parameter Specification

The optical parameters were computed for a range of possible distances between the laser and the PCV lens to determine which set of parameters best satisfied the requirements. From Equation (3.2), w_{pcv} was calculated and subsequently combined with known CS to find the magnification from Equation (3.10). Off-the-shelf availability of 250 mm or 500 mm focal length spherical PCX lenses constrained f_2 to two possibilities. For a given f_2 and M , f_1 was calculated from Equation (3.10). This provided sufficient information to determine the remaining parameters from Equations (3.7), (3.8) and (3.9).

The criterion for selecting the desired set of parameters was the resulting value of w_{img} , which was previously chosen as 0.2 mm. Both f_2 values gen-

erated a solution that met this requirement. Although the 250 mm design was much more compact (distance between the laser and *CTS* approximately 50% shorter), it had other disadvantages. The PCV lens focal length would be very small (32 mm), which would be difficult and costly to manufacture. In addition, the proximity to the laser may corrupt the results due to visible discharge or the beam not having settled into a fully Gaussian mode. Therefore, the 500 mm PCX lens was selected (in fact, the actual focal length at 3.39 μm was measured to be 490 mm). The following table is a summary of the above parameters, illustrated in Figure 3.8:

Table 3.2: Summary of Optical Parameters

f_1	f_2	M	S_{obj}	S_{img}	CS	w_o	w_{pcv}	w_{img}
0.13 m	0.49 m	3.77	1.5 m	0.72 m	21 mm	0.45 mm	2.8 mm	0.21 mm

3.5.7 Lens Size

The lens aperture was selected to minimize the effect of diffraction by the aperture. For a Gaussian beam profile, an aperture diameter D should be greater than $4.6w$. A smaller aperture results in diffraction of the laser beam, which distorts the Gaussian profile, resulting in a set of circular diffraction rings whose radial intensity profile is described by the Airy pattern (Self, 1983). Therefore, the spherical PCX lens diameter was chosen to be 50.8

mm while the cylindrical PCV lens dimensions were 17.8 x 17.8 mm. Furthermore, these PCV lens dimensions enable the use of a standard 25.4 mm diameter stock.

3.5.8 Number of Projections

In order to minimize the cost of the system, the initial design attempted to utilize a single laser beam split into three equal portions rather than three independent lasers. This system was extremely difficult to design as a result of the complex optical setup and the requirement to match the optical path lengths of all projections. Power would be significantly lower due to splitting into multiple sheets as well as the optical losses introduced by interaction with numerous lenses, mirrors and beamsplitters. Alignment of the entire system would be an enormous task. Path length matching was critical due to beam expansion as a function of distance, requiring different magnification for different distances from the laser. This meant that without path length equalization, different custom lenses would be necessary for each projection, thus greatly adding to the cost. In addition, the gradient in the laser sheet thickness would differ among the projections. Although path length matching would resolve these difficulties, it presented a number of problems. First,

the total path lengths had to be significantly larger than with three lasers due to the necessity of directing two beams at 60° to the original. This is schematically illustrated in Figure 3.9, with the portion to be matched shown in bold. Another important problem was the fact that adjustments due to disagreement with theory would be difficult to implement for all three projections simultaneously as altering any parameter would change all the others. Beamsplitters suited for $3.39\text{ }\mu\text{m}$ that maintain the p polarization would have to be custom built and may degrade the beam quality. In addition, this configuration was not suited for introducing more than three projections if required.

This system is displayed in Figure 3.10. As a result of the complexities discussed above, a simpler setup with dedicated lasers for each projection and parameters from Table 3.2 was selected instead. Nonetheless, the single-laser, three-projection setup is a valuable reference and may be useful at a future date.

3.5.9 Optics Characterization

In order to generate the desired laser sheet, lens alignment was carried out with a visible HeNe laser by superimposing the reflections created by the front and back surfaces of each lens. Adjustment was made easier through sliding rails and kinematic mounts that together enabled 6 degree-of-freedom lens motion. Beam characterization was subsequently performed with a liquid nitrogen cooled mercury-cadmium-telluride detector with a centered pinhole in front. The output was recorded in phase with the optical chopper using a phase-sensitive instrument. Traversing the detector with a manual translating stage in the horizontal and vertical directions and recording the output voltage produced the laser beam intensity profiles. Near the focal plane (77 cm from the PCX lens), a well-defined Gaussian profile was observed except for a bright spot on one side (Figure 3.11). To determine whether this resulted from lens effects or the laser beam itself, the lenses were bypassed with two gold mirrors and the profiles were recorded at 2.19 m from the laser. Figure 3.12 displays the horizontal profile while Figure 3.13 displays the vertical. The same bright spot as in Figure 3.11 was observed in the horizontal plane, which indicated that the laser intensity profile was not perfectly Gaussian. This meant that $M^2 > 1.0$, and a larger-than-expected w_{img} was likely to

occur. However, the bright spot would not affect the final results since it would be included in both I and I_o measurements.

In addition to the intensity profile, the beam size was estimated at 2.19 m from the laser by estimating the positions corresponding to $\frac{1}{e^2}$ of the peak intensity. In the horizontal plane, the size was estimated as 5.4 mm and in the vertical as 5.6 mm. On the other hand, Equation (3.2) based on experimentally determined ZL and w_o predicted 6.0 mm in both planes. Given the crude size estimation method and the laser power fluctuations during the measurements, the 10% difference was acceptable.

Other measurements included moving the detector along the beam propagation direction to estimate the location of the focus. While theory predicted $S_{img} = 0.72$ m, the actual focal plane was found to be at $S_{img} = 0.77$ m. In the horizontal plane, the $\frac{1}{e^2}$ laser sheet span was estimated to be 18 mm at 8 cm behind the PCX lens, 17 mm at 77 cm and 19 mm at 112 cm. Rather than remaining collimated, the sheet appeared to contract and then expand again, while the span was less than the predicted 21 mm. Collimation was realized by adjusting the PCX lens position, and checked by measuring the

beam profile approximately 20 cm in front of and 10 cm behind the CTS. At both locations, the $\frac{1}{e^2}$ span was estimated to be 19 mm. Shifting both lenses away from the laser while maintaining the inter-lens separation could increase this size to 21 mm, but was not undertaken due to physical space constraints.

Following the beam characterization, it was evident that, despite the measurement errors and uncertainties, a reasonable overall agreement with theoretical predictions was obtained. This is an important conclusion due to the reliance on theory in the design of mid-infrared optical systems.

3.6 Removal of Power Fluctuation Effects

Another aspect critical to system performance was an independent power measurement of a fraction of the laser output to monitor fluctuations with time. Normalizing both I and I_o by this value would remove the effect of laser power fluctuations. Rather than utilizing a costly beamsplitter designed for $3.39\ \mu m$, a simple, low-cost, 25.4 mm diameter sapphire window was employed. The natural surface reflectance of sapphire provided sufficient input to the detector, while the very small 0.5 mm thickness ensured negligible

beam displacement.

Figure 3.8: Illustration of Optical Parameters

Figure 3.9: Optical Path Matching

SCALE 0.100

Figure 3.10: Optical Setup with a Single Laser and Three Projections

Figure 3.11: Horizontal Laser Intensity Profile near the Focal Plane

Figure 3.12: Horizontal Intensity Profile 2.19 m from the Laser

Figure 3.13: Vertical Intensity Profile 2.19 m from the Laser

Chapter 4

Tomographic Reconstruction Techniques

As outlined in Section 2.3.1, frequency domain and algebraic reconstruction techniques each present a number of advantages and disadvantages, which were evaluated in selecting the suitable algorithm class. In the present case, refraction due to curved sapphire walls precludes the use of parallel, equidistant beams. The restriction in the cost and complexity of the data acquisition system limits the number of projections or viewing angles. Furthermore, this limitation requires the ability to compensate for incomplete data sets through constraints and a priori information. As a result, in spite of important drawbacks, the algebraic reconstruction techniques (ART) were the clear choice. This chapter presents a generic introduction to ART, followed by the implementation of the selected algorithm.

In algebraic methods, the image is represented as a group of basis functions $\mathbf{f}$ whose coefficients define the values of different spatial segments of the image. The simplest basis functions are square “pixels” or “cells” superimposed over the region of interest. It is assumed that in each cell the fuel concentration is constant and is defined by the basis function f_i , where the subscript i represents the cell index ranging from 1 to N in the sequence displayed in Figure 4.1. For the purpose of tomography, a ray can be defined as a “fat” line equal to the detector active area at the array location, which would have uniform width if no refracting walls were present (such as in Figure 4.1).

A series of algebraic equations are generated that express the measured projection data R_m in terms of the weighted unknowns f_i , as follows:

$$R_m = \sum H_{m,i} f_i \quad \text{or} \quad \mathbf{R} = \mathbf{H}\mathbf{f} \quad (4.1)$$

where $H_{m,i}$ represents the contribution of the i^{th} cell to the m^{th} beam, defined by their area of intersection illustrated in Figure 4.1. Most of $H_{m,i}$ matrix values are zero since a given beam only intersects a relatively small number

Figure 4.1: Square Pixel Discretization

of cells. In order to reduce the complexity and the computation time, $H_{m,i}$ values are sometimes considered to be only a function of the perpendicular distance between the centre of the m^{th} ray and the centre of the i^{th} cell, or simply replaced by 1 or 0 depending on whether the centre of the image cell is within the ray. Although easy to implement, these approximations often

lead to artifacts and “salt and pepper” noise. In the presence of refraction induced by the circular test section walls, each segment of the collimated sheet corresponding to a tomography ray expands to a different degree and forms a distinct angle with the centreline. This makes the area of intersection calculations exceedingly complex. A suitable compromise may be the assumption of infinitesimally thin beams that originate within the collimated sheet and terminate at the centre of a particular detector element. According to this approach, the intersection length rather than area forms the coefficients of $H_{m,i}$.

The total number of rays in all projections (M) represents the number of equations available to reconstruct the field whereas the number of square pixels in the grid (N) represents the number of unknowns to be solved. When noise is present in the measurement data and when $M < N$, as is typically the case, direct matrix inversion is not possible and some least squares iterative method has to be employed. Utilizing 3 projections with 16 detectors per projection to reconstruct a 10 x 10 grid would result in 48 equations and 100 unknowns.

The aim of iterative techniques is to minimize the difference between the measured projection data ($\mathbf{R}$) and the estimated line integrals referred to as “ray sums” ($\mathbf{R}'$). The general iterative reconstruction algorithm has the form:

$$\mathbf{f}^{k+1} = \mathbf{f}^k + \alpha^k \mathbf{v}^k \quad (4.2)$$

where $\mathbf{v}^k$ is a vector representing the direction in which to move from the current image estimate $\mathbf{f}^k$, and α^k is a scalar step size (Medoff et al., 1982). Different choices of $\mathbf{v}^k$ and α^k result in different iterative schemes, which are divided into three categories: sequential updating, simultaneous updating, and multiplicative algorithms. These algorithms will be introduced here, whereas for a more in-depth treatment the reader is referred to Watt and Conery (1993), Kak and Slaney (1982), Medoff et al. (1982), and Timmerman (1997).

Sequential algorithms update the pixel values after each equation representing a particular path through the test section is evaluated, and are expressed as:

$$f_i^{k+1} = f_i^k + \alpha H_{m,i} \frac{\Delta R_m}{\left(\sum_{j=1}^N H_{m,j}\right)^2} \quad (4.3)$$

In simultaneous methods, all equations are evaluated simultaneously at each iteration, and the entire image estimate is updated by back-projecting a static projection error $\mathbf{R} - \mathbf{R}'$ as follows:

$$\mathbf{f}^{k+1} = \mathbf{f}^k + \beta \mathbf{H}^T (\mathbf{R} - \mathbf{R}')^k \quad (4.4)$$

Finally, the multiplicative algorithms rescale the image value based on the the ratio of the projection estimate and the actual projection, as shown in Equation 4.5. These algorithms are inherently non-negative.

$$f_i^{k+1} = f_i^k \left(\frac{R_m^k}{R_m} \right) H_{m,i} \quad (4.5)$$

Kak and Slaney (1982) suggest that a combination of sequential and simultaneous methods called the Simultaneous Algebraic Reconstruction Technique (SART) yields reconstructions of good quality and numerical accuracy

in only one iteration by utilizing bilinear elements instead of pixel basis functions. The pyramid-shaped bilinear elements are the simplest higher-order basis functions, the values of which are determined based on the four neighbouring points of the square pixel sampling lattice. An advantage of this method is that a circular domain can be reconstructed more accurately since, unlike square pixels, the bilinear elements can be set to originate and terminate at the domain boundaries. Under pixel discretization, only several corner cell values can be set to zero, leading to, for example, a reduction of 12 unknowns on a 10x10 grid. On the other hand, the bilinear elements result in a reduction of about 25 unknowns since the reconstruction circle covers approximately 75% of the domain. However, square pixel discretization is a necessary first step in implementing bilinear elements as it establishes the groundwork for this more complex method.

Although the sequential and multiplicative algorithms have much faster convergence, simultaneous methods generally perform better in the presence of excessive noise. Therefore, for low signal levels due to significant absorption, resulting in low signal-to-noise ratio, the simultaneous category is preferred. A variant of the simultaneous approach, which improves con-

vergence by making the step direction vector of Equation 4.2 orthogonal to all previous steps, is called the conjugate gradient technique (CGT) and is displayed in Equations 4.6 to 4.10. When a non-negativity constraint is introduced between Equations 4.6 and 4.7, the method is referred to as constrained conjugate gradient technique (CCGT). CGT updates the parameter β of Equation 4.4 based on the previous search direction utilizing two vectors $\mathbf{p}$ and $\mathbf{r}$. Typically, the algorithm initializes $\mathbf{f}$ to the zero image, and $\mathbf{p}^0 = \mathbf{r}^0 = \mathbf{H}^T \mathbf{R}$. For finite dimensional problems, CGT is guaranteed to converge in a finite number of steps (Timmerman, 1997).

$$\beta^k = \left(\frac{|r^k|}{|Hp^k|} \right)^2 \quad (4.6)$$

$$f^{k+1} = f^k + \beta^k p^k \quad (4.7)$$

$$r^{k+1} = H^T (R - Hf^{k+1}) \quad (4.8)$$

$$\alpha^k = \left(\frac{|r^{k+1}|}{|r^k|} \right)^2 \quad (4.9)$$

$$p^{k+1} = r^{k+1} + \alpha^k p^k \quad (4.10)$$

4.1 Discretization and Weighting Coefficient Determination

Utilizing the lowest order square pixel basis functions, the circular test section was discretized into a grid with equal number of pixels on each side ($P = \sqrt{N}$). The grid resolution was chosen as 10x10 in order to capture the integral length scale of the turbulent flow imaged. Based on the assumption that the size of the largest eddies is on the order of the test section diameter (D), as a first approximation the integral length scale was roughly estimated as $\frac{1}{10}D$. The length of the intersection of each ray with a particular pixel within the grid determines the coefficients of $\mathbf{H}$, representing the contribution of each pixel to a given ray sum. Generating these weighting coefficients will be the focus of this section.

In order to define a common reference for the rays propagating through the tube, the coordinate system illustrated in Figure 4.1 is utilized as the reference frame for all three projections. The first step in computing the coefficients of $\mathbf{H}$ is to work backward to determine at what distance from the centreline the rays corresponding to the known detector positions are located. This is performed utilizing Snell's Law and the known sapphire

Figure 4.2: Single Ray Path through the Sapphire Tube

tube geometry and index of refraction. As a result, the coordinates of the intersection points between the rays and the test section inner wall can be determined. To accomplish this task, the nomenclature displayed in Figure 4.2 is utilized. A large number of closely spaced parallel rays spanning the tube inner radius are numerically “sent” through the test section, and each of their positions (h_3) at the receiver plane is determined. For each detector, its known position is compared to the calculated h_3 values, and the ray resulting in minimum difference is taken to be the one for which the line integration (ray sum) is performed. The coordinates of the rays in the other half of the test section have the same magnitude but different sign due to symmetry about the centreline.

Next, the row and column number of the pixels that contain the coordinates $(w1_1, h1_1)$ and $(w2_1, h2_1)$, which represent the intersection between a given ray and the inner tube wall, and the corresponding pixel index from 1 to N , are found. The horizontal and vertical distances between these coordinates and the nearest pixel edges to the right and above, respectively, are calculated and utilized to determine the angle (ε) from the intersection coordinate to the upper right corner of the pixel (Figure 4.3). This angle serves as the condition for whether a particular ray passes through the next pixel on the right (case 1), the pixel one column to the right and one row above (case 2), or directly through the pixel above (case 3). If the angle β , which the ray makes with the horizontal as it passes through the test section, is less than the calculated angle ($\beta < \varepsilon$), case 1 occurs. Similarly, if $\beta = \varepsilon$, case 2 occurs and if $\beta > \varepsilon$, case 3 occurs as illustrated in Figure 4.4. Each ray or detector element traverses the test section at a distinct angle β with respect to the horizontal.

Depending upon which of the three possibilities takes place, the horizontal and vertical increments (Δw and Δh , respectively) from $(w1_1, h1_1)$ to the point where the ray exits the cell, are calculated differently. From this, the

Figure 4.3: Condition for Defining Ray-Cell Intersection

Figure 4.4: Ray Propagation Possibilities through the First Cell Intersected within the Test Section

ray path length through the cell is defined as:

$$l = \sqrt{(\Delta w)^2 + (\Delta h)^2} \quad (4.11)$$

This intersection length had to be normalized by some parameter in order to generate unitless weighting coefficients. The total path length through the test section was not suitable since it is specific to each ray. Instead, the pixel diagonal, which is the maximum possible path length through a cell,

was chosen to normalize the coefficients to a scale of 0 to 1.

Following the exit from the first cell, a ray can traverse the subsequent pixels in several ways that differ in how Δw and Δh are calculated. Depending on whether a ray enters or leaves a cell through a vertical edge, a corner, or a horizontal edge, designations 1, 2 or 3 are assigned, respectively. The example in Figure 4.5 illustrates a ray entering a cell through the left border and exiting through the right (designation 1-1), and subsequently entering the neighbouring cell across the left border and leaving through the upper edge (designation 1-3). The sequence of 9 possible classifications (1-1, 1-2, 1-3, 2-1, 2-2, 2-3, 3-1, 3-2, 3-3) defines 21 distinct ways to calculate Δw and Δh . For the first cell in Figure 4.5, ΔX of Figure 4.3 is equal to the cell width, which means that for known β , Δh can be obtained through simple trigonometry. For the second cell, Δh is the difference between ΔY and Δh of the previous pixel. Again, for known β , Δw and l are easily computed through trigonometry. The last pixel overlapping the test section is corrected for the fact that the ray terminates at the tube wall rather than at the edge of the pixel. For the first and last pixels, the actual intersection is used in computation while the reconstructed function is only resolved down to a

Figure 4.5: Example of Path-Based Classification

pixel. Although this yields the appropriate weighting coefficient, the circular test section boundaries are not clearly resolved but will appear jagged.

In order to apply the same procedure to the remaining two projections, a coordinate transformation is performed by applying a rotation matrix about the z axis to coordinates $(w1_1, h1_1)$ and $(w2_1, h2_1)$ as follows:

$$\begin{bmatrix} \cos(\psi) & \sin(\psi) \\ -\sin(\psi) & \cos(\psi) \end{bmatrix}$$

For the second projection, $\psi = \frac{\pi}{3}$, while β is replaced by $\frac{\pi}{3} + \beta$ for rays above the second projection centreline and $\frac{\pi}{3} - \beta$ for the rays below the centreline. For the third projection, the angle $\frac{\pi}{3}$ becomes $\frac{2\pi}{3}$. The procedure for determining Δw and Δh , and thus the intersection length l , is similar to the first projection. The accuracy of the computed intersection path lengths has

been checked by discretizing the domain more coarsely into, for example, 3x3 cells and manually checking the results. In addition, the accuracy has been checked qualitatively by displaying $\mathbf{H}$ on a 10x10 grid for each individual ray, and confirming that the ray distribution follows the projection angle ψ .

4.2 Software Validation with Artificial Data

Preliminary testing of the CCGT algorithm was performed by utilizing a Gaussian function to simulate the fuel jet concentration profile, decreasing as a function of the distance from the centreline. Since the laser sheet undergoes non-uniform stretching due to refraction, it was extremely difficult to artificially generate the “measured” data at the detector plane in place of the real projections. As a result, refraction was ignored by setting $n = 1$ for the sapphire tube, thus synthetically emulating an open jet. While this simulated parallel rays, it constrained the number of detector elements in each array to 10 instead of 16 since the full detector span is greater than the test section diameter. Under the assumption of axisymmetry, these simulated projection measurements were applied to all three projections.

Figure 4.6: Gaussian Function on a 1000 x 1000 Grid

As the first step, a Gaussian surface was generated by the point-by-point multiplication of two Gaussian curves on a 1000x1000 grid, each created by the function $e^{-\frac{r^2}{2}}$ over a range of $-4 < r < 4$ (Figure 4.6). For fixed sapphire tube dimensions, decreasing r reduces the gradients to be reconstructed and vice versa. The Gaussian surface was subsequently averaged over a 10x10 grid for direct comparison with the reconstructed function (Figure 4.7). When determining the ray sums, the rows of the 1000x1000 grid most closely corresponding to each detector position were utilized, and their elements added.

Two means of monitoring the iteration convergence are the normalized

Figure 4.7: Gaussian Function on a 10 x 10 Grid

residual projection error $\frac{\|\delta \mathbf{R}\|}{\|\mathbf{R}\|}$, where $\|\delta \mathbf{R}\| = \sqrt{(R_1 - R'_1)^2 + \dots + (R_M - R'_M)^2}$, and the normalized residual image error $\frac{\|\delta \mathbf{f}\|}{\|\mathbf{f}\|}$, which is unknown since we do not know the actual concentration distribution. This means that the projection error rather than the image error must be utilized as the termination condition. Unfortunately, there is no simple way to relate the projection error to the quality of the image and thus no obvious way to stop the iterations before the image becomes degraded. Watt and Conery (1993) have numerically tested the convergence of several algebraic algorithms and found that all except CCGT converge to a minimum image error as the projection error continues to converge, leading to image degradation. It is therefore desirable

to have slow divergence from the minimum error solution. Although CCGT with square pixel basis functions demonstrated rapid divergence from the minimum image error, the image and projection error appeared to reach a minimum at approximately the same iteration, providing a convenient termination method (Watt and Conery, 1993).

In the present simulation, utilizing CCGT with square pixel basis functions, the iterations were carried out until the residual projection error between adjacent iterative steps no longer decreased. It is clear that the line integrals cannot be obtained exactly due to inaccuracies that manifest themselves as noise in the data. As a result, through trial and error, the reconstruction was set to terminate if the projection residual decreased below 10^{-4} . Based on the above, the Gaussian function was reconstructed in 28 iterations, and the reconstruction displayed in Figure 4.8. The function shape corresponds well with the original, and there are few edge artifacts. However, the magnitude is over-predicted, the reason behind which is currently unclear.

The image residual and the projection residual are displayed in Figure 4.9, with the minimum projection and image errors labelled on the graphs. The

Figure 4.8: Reconstructed Function (28 Iterations)

author did not observe the correspondence between the minimum image and projection error presented by Watt and Conery (1993), nor the expected rapid divergence from the minimum error solution. Instead, the error remained stable over a large number of iterations. One possibility that may explain this discrepancy with the results of Watt and Conery (1993) is the fact that noise was not included in the simulation. The image begins to exhibit significant edge artifacts far beyond the termination point, such as after 100 iterations, as illustrated in Figure 4.10. Although this test was simplified by assuming parallel rays and no measurement noise, it utilized only 10 instead of 16 detectors or “equations” per projection. Therefore, the artificial validation

Figure 4.9: Projection and Image Error

provides positive indications that the reconstruction will be effective in the actual measurement situation.

Figure 4.10: Reconstructed Function (100 Iterations)

Chapter 5

Experimental Results

Prior to proceeding to the experimental stage, the methane jet and the surrounding airflow properties had to be defined. The starting parameters were the average jet inlet velocity of $20 \frac{m}{s}$ and the fuel-air momentum ratio of 0.1 (momentum is defined as the product of mass and velocity), while the jet radius at the measurement plane was expected to be 80% of the test section radius. These values resulted in the remaining flow specifications displayed in Table 5.1. Figure 5.1 presents a schematic drawing of the test setup and illustrates the elements that will be referred to in subsequent sections.

Figure 5.1: Test Setup

5.1 Hot Wire Measurements

A series of constant temperature anemometry experiments were carried out in order to characterize the flow uniformity and the response of the fuel jet to forced pulsations induced by the shaker. For this purpose, a single DISA Type 55 wire positioned orthogonal to the flow was employed to capture the streamwise velocity component. Throughout the experimental phase, the hot wire was calibrated daily to correct for changes in wire and cable resistance and to obtain the coefficients A , B , and n relating the voltage output (E) and the flow velocity (V) according to Equation 5.1. Good curve fits were obtained, with the coefficient of determination (R^2) ranging from 0.9994 to 0.9998.

$$E^2 = A + BV^n \quad (5.1)$$

5.1.1 Flow Uniformity at the Measurement Plane

The existing contraction with a 50.8 mm outlet diameter had to be adapted to match the 23.8 mm inner diameter of the sapphire tube. As a result, a 76.2 mm long extension was designed. To ensure the compatibility of the two contractions, the flow uniformity at the measurement plane was checked

without the central fuel pipe. The hot wire measurements were acquired by traversing the flow in 1.78 mm increments over a rectangular region enclosing the tube, resulting in a 14 x 14 grid. The mean velocity was determined as the time-average of the data, and the turbulence intensity (TI) as:

$$TI = \frac{\sqrt{\frac{1}{N} \sum (u - \bar{u})^2}}{\bar{u}} \quad (5.2)$$

where u is the total velocity, including both the mean and the fluctuating component, and $\bar{u}$ is the mean velocity. In order to decide the acquisition frequency and the number of samples at each location while minimizing the data storage requirements, the variation in mean velocity and turbulence intensity was investigated. Comparing the results acquired over 5 seconds at 10 kHz to those acquired over 3 seconds at 1 kHz, approximately 1% deviation was observed. Therefore, the latter case satisfied the requirements.

The mean velocity and turbulence intensity profiles across the measurement plane centreline are displayed in Figure 5.2. The mean velocity of 9.5 $\frac{m}{s}$ was very uniform, leading to confidence in the contraction performance. The turbulence intensity away from the edges was approximately 3%, and

Figure 5.2: Mean Velocity and Turbulence Intensity Profile at Contraction Outlet

was also uniform.

5.1.2 Steady State Velocity Profile across Contraction Outlet and Jet Inlet

The steady-state flow uniformity and axisymmetry in the airflow surrounding the fuel pipe was checked at the contraction outlet plane. A single line of measurements was acquired in four orthogonal directions between the outer wall of the fuel pipe and the edge of the contraction outlet. The mean velocity in the four directions, referred to as North, South, East and West, is displayed in Figure 5.3, whereas the turbulence intensity profile is shown in

Figure 5.3: Mean Air Velocity outside of Fuel Pipe at Contraction Outlet

Figure 5.4. Due to physical constraints in the probe holder size, the x-axis “zero” displayed on the charts is approximately 7 mm from the coordinate system of Figure 5.1. The mean velocity was found to be $11.5 \frac{m}{s}$, and is highly uniform and axisymmetric. Turbulence intensity exhibits uniformity and axisymmetry in all four directions, with a mean value of 2.5% inside the main flow field. As expected, turbulence intensity grows toward the contraction edges where there is a steep mean velocity gradient.

The second portion of this experiment involved measuring the steady-

Figure 5.4: Turbulence Intensity outside of Fuel Pipe at Contraction Outlet

state profile across the jet inlet (at the exit of the fuel pipe). Air was utilized instead of methane and the density difference compensated for when determining the required mass flow. The results are displayed in Figure 5.5 for two orthogonal “sweeps” across the jet labelled West-East (WE) and North-South (NS). A high degree of symmetry is observed, as well as a good correspondence with the predicted mean velocity of $20 \frac{m}{s}$.

Figure 5.5: Mean Velocity and Turbulence Intensity across the Jet Inlet

5.1.3 Frequency Characterization and Velocity-Dynamic Pressure Correlation

Previous acoustic studies in a similar setup indicated that a standing wave was produced between the two open ends of the fuel pipe. Dynamic pressure measurements acquired at a single point 70 mm from one end were sufficient to describe the pressure oscillations in the tube. First, the effect of varying the frequency of flow pulsations induced by the shaker on the amplitude of dynamic pressure oscillations was investigated. It was found that the peak fluctuation amplitude occurred at approximately 20 Hz, gradually decreased by up to 30% until around 155 Hz, and fell significantly beyond that point.

Therefore, it was decided to perform the validation experiment at 20 and 155 Hz.

The next step was to correlate the dynamic pressure and hot wire velocity measurements in order to be able to derive the magnitude of velocity fluctuations based solely on dynamic pressure measurements. This study was performed at the selected frequencies of 20 Hz and 155 Hz by progressively increasing the amplitude of oscillations and simultaneously recording the dynamic pressure and velocity data (at the centre of the pipe). The results are displayed in Figures 5.6 and 5.7, which show a reasonable goodness of fit.

5.2 Time Resolved Absorption Measurements

Time-resolved concentration fluctuations, produced by modulating a methane jet at 20 Hz and 155 Hz in co-flowing air, were measured with the 16-element detector array. Data, with a sample size of 4096, were acquired at a sampling rate of 10 kHz. The infrared laser beam was optically chopped at 808 Hz, since the photoconductive detectors perform well only at high frequencies. The chosen frequency also had to be sufficiently removed from the physical

Figure 5.6: Peak-to-Peak Velocity and Dynamic Pressure Correlation at 20 Hz

Figure 5.7: Peak-to-Peak Velocity and Dynamic Pressure Correlation at 155 Hz

process of interest (20 and 155 Hz). At both modulation frequencies, the amplitude of oscillations was adjusted to produce the same peak-to-peak velocity fluctuation of $9 \frac{m}{s}$ (see Figures 5.6 and 5.7). In order to eliminate the 60 Hz noise component introduced by the power line, a 4th order Butterworth bandpass filter was implemented in the data acquisition software.

Since the methane jet is pulsed and the light source (laser) itself is optically chopped, all at different frequencies, data analysis is best performed in the frequency domain. Applying Fast Fourier Transform (FFT) to results measured by detector 8 yields Figures 5.8 and 5.9, which display a strong peak at the chopping frequency and the second largest peak at 20 Hz and 155 Hz, respectively. In addition, although the original time series does not appear to contain a 20 Hz component, a simple moving average of 66 points that crudely acts as a low pass filter at 150 Hz reveals the 20 Hz signal displayed in Figure 5.10.

In order to generate the ratio of signal strength with and without methane, corrected for the laser power drift, it is assumed that the ratio of voltages $\frac{V/V_{ref}}{V_o/V_{o_{ref}}}$ is equivalent to $\frac{I/I_{ref}}{I_o/I_{o_{ref}}}$. All the corrections and scaling are performed

Figure 5.8: FFT for Jet Modulation at 20 Hz

Figure 5.9: FFT for Jet Modulation at 155 Hz

Figure 5.10: 20 Hz Signal Revealed through Moving Average of Time Series

at the 808 Hz chopping frequency utilizing voltage amplitudes generated by the FFT. For example, the output at 20 Hz (V_{20}) is scaled by the following constant:

$$V_{20} \frac{\frac{V_{808}}{V_{808,ref}}}{\frac{V_{0808}}{V_{0808,ref}}} \quad (5.3)$$

It is assumed that each detector signal is a sine wave at the frequency of interest, such as 20 Hz, with the amplitude obtained through FFT and corrected as above. Simultaneous data acquisition by the 16 detectors should therefore produce waveforms at the same frequency and phase, but varying

amplitude depending on the extent of absorption. The expected profile is roughly inverted Gaussian, with the middle detectors recording the highest attenuation due to peak concentration at the jet centreline and the longest path length through the sapphire tube. On the other hand, the edge detectors should record similar values with and without methane because the expected jet radius at the measurement plane is $\sim 80\%$ of the test section radius. Although it is anticipated that responsivity would vary to some extent among the detector elements, this effect should in theory be eliminated through the ratio $\frac{V}{V_o}$ since the responsivity is constant for measurements taken with and without methane.

Individual detector readings were sequentially acquired at the same position near the middle of the test section in the absence of the absorbing gas, methane. Ideally, a plot of the voltage amplitudes at the chopping frequency should yield a nearly straight line. However, severe irregularities in the profile and enormous differences among adjacent detectors are observed, such as a magnitude difference of 87 times between detectors 8 and 9 (Figure 5.11). The voltage amplitudes at 20 Hz in the presence of the modulated methane jet are plotted in Figure 5.12, where it is observed that the discrepancy in

Figure 5.11: FFT Voltage Amplitude without Absorbing Species

detector responsivity is not removed by the correction factor in Equation 5.3. In addition, the lower end of the chart displays noise amplitude at 808 Hz taken with no input signal. For detectors 1, 4, 6, 7, 9, 15, and 16 the noise amplitude is comparable or even greater than the signal amplitude.

One possibility for the problems in detector performance is effects of atmospheric moisture on the detectors. The detectors are currently exposed to atmosphere rather than sealed in protective infrared-transmitting windows. An attempt to compensate for this was made by reversing the Pelletier element initially used for cooling in order to heat the detectors and remove

Figure 5.12: FFT Voltage Amplitude of Modulated Jet at 20Hz and Dark Noise

any potential moisture absorption. However, this did not improve the performance. Although the bias voltage supplied by the DC-to-DC converter is expected to be relatively uniform across each detector, significant differences in bias voltage were measured that correspond to the observed differences in detector responsivity.

A very likely cause of measurement noise not directly related to responsivity is the absence of amplifiers between the detector output and the data acquisition boards. As mentioned in Chapter 3, mounting difficulties prevented the small amplifiers from being installed at the back of the circuit

board, which would minimize the transmission distance and improve S/N. Sixteen high-quality external amplifiers may be utilized instead, but are currently not available. Therefore, the detector output is sent directly to a ± 5 V differential data acquisition board with no amplification. For output of less than 10 mV recorded by several detectors, and 10 V range of the data acquisition board, only $\frac{1}{1000}th$ of the dynamic range is utilized.

Serious problems with the detector circuit board prevented testing the tomography algorithm with experimental data at the present time. Prior to reaching that stage in the instrument validation, a number of changes in the circuit board design and construction will be necessary.

Table 5.1: Specification of Flow Parameters

GEOMETRY		
Fuel pipe ID	6.35	<i>mm</i>
Fuel pipe OD	9.53	<i>mm</i>
Sapphire tube ID	23.8	<i>mm</i>
METHANE/AIR FLOW RELATION		
Fuel-air momentum ratio	0.1	
METHANE JET FLOW PARAMETERS		
Inlet velocity	20	$\frac{m}{s}$
Mass flow rate	0.000431	$\frac{kg}{s}$
Momentum	0.008614	$\frac{kgm}{s^2}$
AIR FLOW PARAMETERS		
Momentum	0.0861	$\frac{kgm}{s^2}$
Inlet velocity	13.86	$\frac{m}{s}$
Mass flow rate	0.00621	$\frac{kg}{s}$
FLOW GEOMETRY		
Jet radius at the measurement plane as a fraction of the sapphire tube inner radius	0.80	R_i
Jet radius at the measurement plane	9.52	<i>mm</i>
Distance of jet inlet from virtual jet origin	49	<i>mm</i>
Distance in jet diameters between jet inlet and measurement plane	11.73	<i>J.D.</i>
ADDITIONAL PARAMETERS		
Fuel-air ratio	0.0693	
Equivalence ratio	1.21	
Entrainment rate	0.000984	$\frac{kg}{s}$

Chapter 6

Conclusions and Recommendations

Measurement of time-resolved equivalence ratio profiles across the combustor inlet could provide great benefits in understanding combustion noise and NO_x emissions performance of lean premixed combustors. As a result, an attempt was made to develop a non-intrusive, temporally and spatially resolved instrument for measuring hydrocarbon concentration fields. To achieve this, hydrocarbon absorption of laser radiation at 3.39 μm and the principles of computed tomography were utilized. An optical system employing an infrared laser was designed to deliver a light sheet into the measurement zone, collimated in one plane and focussed to the required beam waist in the orthogonal plane. A tomography algorithm based on the constrained conjugate gradient method was implemented, tested with artificial data, and found to

perform well in the absence of experimental noise.

A 16-element infrared detector array was constructed and tested with known concentration oscillation frequencies of 20 Hz and 155 Hz. Although FFT results indicated that the detectors sensed the above concentration fluctuations, severe hardware performance problems prevented experimental data reconstruction with the tomography algorithm. Some fundamental changes in the data acquisition hardware may be necessary. The detector circuit should be re-designed to accommodate the amplifiers on the back of the board. Potential electronic interference among closely spaced adjacent elements should be checked. The detector elements must be enclosed in a chamber with an infrared-transmitting window and a continuous supply of dry air. This would prevent condensation and absorption of atmospheric moisture by the detector surfaces. Furthermore, dry air would enable more significant cooling than that provided by the Pelletier element. For example, a liquid nitrogen chamber could be installed, which would improve responsivity and S/N.

A single-ended rather than differential data acquisition board may be

considered to avoid the grounding problems encountered in acquiring the output from single-ended detectors. Selectable-range boards may be useful because the signal levels with and without methane absorption are quite different. Relying entirely on selectable hardware gain to bring the signal to the desired level amplifies the noise as well. However, even though the data acquisition boards are not optimal, they are workable.

Many of the data acquisition difficulties would be avoided if sufficiently large commercial detectors arrays, similar to charge-coupled devices (CCDs) but suited for the mid-infrared spectrum, currently existed. There are indications that such devices are under development and may become available in a few years.

Once confidence in detector circuit performance is established, calibration of concentration as a function of voltage will be required for each detector since the absorption coefficient is dependent on concentration. Based on Equation 2.1, concentration can be expressed as:

$$X = -\frac{1}{\alpha(X)} \frac{1}{PL} \ln \left(\frac{V/V_{ref}}{V_o/V_{o_{ref}}} \right) \quad (6.1)$$

For a range of known concentrations at steady state, and the corresponding measured static pressure and voltage outputs, the expression for α in terms of X for each detector can be determined. It is important to note that the logarithmic expression should be evaluated in the time domain, followed by FFT to determine the concentration magnitude at the chopping frequency for steady-state conditions. Because of non-linearity, this process is not equivalent to directly applying FFT to voltages and subsequently evaluating the logarithm.

Once the system validation has been performed, the effect of reducing the distance between the laser and the test section by shortening the lens focal lengths could be investigated. While spherical PCX lenses with $f_2 = 250$ mm are commercially available, the reduced focal length of the custom cylindrical PCV lenses would result in manufacturing difficulties due to high curvature, and potential degradation in the laser sheet intensity profile. However, if these effects are shown to be insignificant, the size of the system could be significantly reduced. With improved data acquisition hardware, the power reduction due to a single laser for all three projections may become accept-

able. Another potential consideration is the C-H absorption peak at $1.7\text{ }\mu\text{m}$. If a suitable diode laser can be obtained at a reasonable cost, it may be easier to find commercial detectors developed for telecommunications at this wavelength.

The unknown that remains in this system is the performance of the tomography algorithm with experimental data in the presence of noise. The quality of reconstruction should be investigated for known 20 Hz and 155 Hz concentration fluctuations at a given number of jet diameters downstream of the inlet. Comparing the projection residual and the image residual, the suitability of the minimum projection error as the termination criterion can be examined. In addition, the implementation of bilinear elements rather than square pixels as the basis functions should be explored.

The author believes that this temporally and spatially resolved absorption tomography system is feasible with improvements in data acquisition hardware, and could become a valuable tool in combustion diagnostics.

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