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An Adaptive Kalman Filter for Tactical Missiles Guidance Systems

by

Jingming Li

B.Eng., Beijing University of
Aeronautics and Aerospace 1985

A thesis submitted to the faculty of
Graduate Studies and Research in partial
fulfillment of the requirements for the Degree of

Master of Engineering

**Department of
Mechanical and Aerospace Engineering**

**Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering**

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Ottawa, Ontario, Canada

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submitted by Jingming Li, B.Eng.
in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Supervisor

Chair, Department of Mechanical and Aerospace Engineering

Carleton University
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ABSTRACT

The Kalman filter can be used to upgrade Inertial Navigation Systems numerically without upgrading the hardware, which is costly. In this thesis, current methods of integrating Kalman filter into Inertial Navigation Systems are investigated; a standard Kalman filter algorithm is implemented in the INS for a glide weapon system in order to investigate the performance of the Kalman filter; an adaptive Kalman filter is implemented to show the performance improvement. The Kalman filter estimates the navigation error in real time with the aid of other external signal sources. The Inertial Navigation System receives these signals and makes appropriate compensations. The estimation results from the standard Kalman filter may diverge from the actual error signal either because of the changing dynamics or inaccurate parameters. The adaptive Kalman filter could tune itself to work within the new environment. A Kalman filter convergent criterion and an on-line noise covariance estimation algorithm are described in this thesis. An approach that combines the S Kalman filter and on-line variable identifications is suggested in this thesis. The simulation results for the standard and adaptive Kalman filters are discussed and compared.

Dedication

*To my wife Qian Wang, my parents
and aunt A. Tan*

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CHAPTER I

INTRODUCTION

Guided missiles first appeared in Germany near the ending of the Second World War. The V-2 ground to ground missile was the first pilot stand-off guided weapon system with an Inertial Navigation System. It was actually the first real application of the Inertial Navigation System. The Inertial Navigation System or INS has the advantage of being self-dependent. It is the only navigation system which can provide the position and speed of a vehicle with high accuracy without the real need for other external signals such as radio waves, laser, etc. Therefore, it is unlikely to be interrupted by external interference. The military advantage of the Inertial Navigation System is obvious.

Since the World War II, many countries like United States, USSR, Britain, France, China and India have been doing research and development extensively on Inertial Navigation Systems. Along with the rapid progress in computing technology, electronics and instrumentation technology, the Inertial Navigation technology has been rapidly progressing. Now, nearly all middle to long range ground attack missiles, such as cruise missiles, large commercial airplanes, military aircraft, helicopters, large commercial and warships and submarines, are all equipped with Inertial Navigation Systems. It is one of the most important navigation tools in modern transportation and weapon delivery systems.

The Inertial Navigation System consists of gyroscopes and accelerometers. It makes

use of gravity and inertial characteristics of gyros and accelerometers to provide all the navigation parameters.

The characteristics of gyros and accelerometers have been known for quite a long time, but the real INS was not possible until the middle of this century for several reasons:

- The large scale real-time complex numerical computations are impossible without using advanced computing technology.
- The inertial devices such as gyros and accelerometers used for INS must have very high precision. That requires very sophisticated instrumentation techniques and the cost of gyros was very high. It discouraged the large scale use of INS.

The revolution occurred in late 1950's and 1960's as the theory for INS became more advanced and computers became more and more available and suitable for INS applications. The large-scale, high speed computer made INS possible for the use of real time navigation. In early 1960's Kalman filtering theory was introduced by R.E Kalman [20]. The Kalman filter provides a mathematical solution to upgrade the INS without using high accuracy gyros, thus substantially reduces the cost of INS. The Kalman filter actively provides estimations of errors in the navigation parameters provided by INS. These errors can subsequently be subtracted from those parameters. Many engineering applications have proved that Kalman filter can upgrade the INS precision to a higher class. Now, most of the commercial Inertial Navigation Systems are using middle grade precision gyros and utilize the Kalman filter to provide high precision navigation results.

Fig. 1-1 shows the benefit of using a Kalman filter. The error on position information provided by the Inertial Navigation System provided is reduced substantially,

Fig. 1-1 RMS Error Estimation on Position
(From Proc.of IEEE Conference on National Aerospace Electronics, 1986, Vol.1)

However, many systems using the Kalman filter show that the estimated results are quite different from their actual results. This may be because the system model on which the Kalman filter is based does not describe the system accurately or because there are some uncertainties in Kalman filter system parameters.

Another restriction to the Kalman filter applications is the data processing time for the Kalman filter. First, if the Kalman filter is directly an integrated part of the system, then each processing step has to be completed in less than the data sampling time of the discrete system. That sampling time could be very short for a high speed aircraft. But, in fact, the Kalman filter data processing programs are always running at a lower priority, since the central on board computer is also needed for more critical programs such as control and stability algorithms[1]. That could cause a long processing time. It is necessary either to use more powerful computers or to chose simpler models for Kalman filtering.

There are two trends in current Kalman filter applications. One is adaptive Kalman filtering and the other is parallel Kalman filter processing. Adaptive Kalman filter [1] [3] [5] [7] [10] can tune the system's parameters to adjust itself to the changing work conditions. In this way, it works better than the standard Kalman filter does in many cases.

There are many research works in this area. One of them can be found in [7] , where, Paul W. Mcburney of Stanford Telecom, describes approaches to building a reliable, robust and adaptive filter. The Schmidt-Kalman filter is to allow certain errors which are normally ignored to be included in the measurement process. An adaptive process scheme based on measurement residual monitoring was present.

J.Z. Sasiadek of Carleton University and P.J. Wojcik of Alberta Research Council [10] presented an adaptive filter algorithm and its simulation results. This algorithm works in cases where there is not much information on the system noise. Instead of determining

Kalman gain in each step which depends on the noise information. this algorithm adaptively predicts the next step Kalman gain in advance. The performance analysis can be found in [10]. It is helpful for design of Inertial Navigation Kalman filters in cases where there is not much information on system or measurement noise available.

All adaptive Kalman filters are sub-optimal filters. Their errors on estimations will be greater than those of Kalman filters. But adaptive Kalman filters can provide useful results even though they may be downgraded.

With rapid progress in computer technology, the costs for computer hardware are getting lower, and the computing powers are much greater. Parallel Kalman filter processing makes use of an array of processors [4] [21] [22], typically four to six processors in a transputer system. One processor serves as the master processor, the others act as the slave processors. Each processor is dedicated to a particular part of a large data processing task. These processors communicate with each other, so that the final results can be sent back to the central computer. H.R. Hashemipour [4] described several algorithms for the distributed computing system.

Fig. 1-2 Parallel Processing Structure

The object of this thesis is to investigate various ways to integrate Kalman filters into Inertial Navigation Systems, and to implement a standard and an adaptive Kalman filter into an INS which was proposed for use in a pilot stand-off glide weapon system for the American Air-Force in the late 1970's. Performance examinations of these Kalman filters are carried out to find out potential problems and ways to overcome them and improve the performance. Some figures in this paper are from other published materials as indicated. The author has the written permission from the original authors. The thesis organization is as follows:

- **Chapter 2** The basic theory of Inertial Navigation Systems. The fundamental inertial navigation equations, reference frames, a local-level terrestrial INS and

error considerations are described.

- **Chapter 3** The introduction to the Kalman filter. Kalman filter theory and algorithms are described.
- **Chapter 4** Application of Kalman filters into Inertial Navigation Systems. This chapter describes the investigations on various ways to integrate Kalman filter into INS, their benefits and restrictions, as well as the general design approaches.
- **Chapter 5** Implementation of a standard Kalman Filter into a strapdown INS/RAC guidance system for the glide weapon systems. It include system modelling Kalman filter design, and computer simulations.
- **Chapter 6** The adaptive Kalman filter. This chapter describes ways to improve the Kalman filter performance. An Adaptive Kalman filter algorithm is suggested for use in the INS for the glide weapon system. Simulation results are given to prove the suggestions.
- **Chapter 7** Conclusions and Discussions.

CHAPTER 2

THE INERTIAL NAVIGATION SYSTEM

2.1 Introduction to Inertial Navigation System

Navigation is the determination of a physical body's position and velocity relative to the reference coordinate frames. Generally terrestrial navigation involves the determination of a vehicle's position and velocity relative to the earth. An inertial navigation system utilizes the inertial properties of the sensors mounted on board the vehicle to execute the navigation function. The system accomplishes this task through appropriately processing the data obtained from force and inertial angular velocity measurements.

Thus a good Inertial Navigation System is capable of continuous determination of the velocity and position of a vehicle without the need for external radiation or optical information. While the INS has obvious advantages for military applications, there are also many INS applications in non-military industries.

All Inertial Navigation Systems must perform the following functions :

- instrument a reference frame;
- measure specific force;
- time integrate the specific force data to obtain velocity and position information.

The first function is accomplished by use of gyroscopic instruments. Gyroscopes are devices that display strong angular momentum characteristics. Since inertial referred time

rate of change of angular momentum is proportional to the applied torque. gyroscopic devices can maintain a known spatial direction through appropriate torque control. Three such devices are capable of instrumenting a three-dimensional cartesian coordinate frame. Each is typically used as the sensing element in a closed loop servo system which operates to maintain the gyro's spatial direction.

Thus if three un-torqued gyros are mounted on a gimbaled platform whose gimbals are driven to maintain the gyro's orientation, an inertial non-rotation coordinate frame will be instrumented by the platform.

The inertial platform can also be instrumented to the rotation reference coordinator, such as the local geographic coordinator.

Another typical configuration of INS is referred to as the "strapdown" INS. In this kind of configuration, the gyros are structurally mounted on the vehicle, each gyro is used as the sensing element in a closed loop servo system. In this system the torque applied to the gyro is proportional to the gyro's inertial referenced angular rotation. While the gyro no longer remains non-rotating to the reference space, the applied torquing signal which is proportional to the inertial referred angular velocity, can be used to analytically calculate the relative angular orientation between the gyro's initial and present spatial direction. If three structurally mounted gyros are used, the relative orientation between the initial and present vehicle coordinates can be determined. In this case, a reference frame is analytically instrumented.

The second function, the specific force measurement is accomplished by using the devices called accelerometers. Although there are many ways of making force

measurements, most of the devices are sophisticated variations of the simple pendulum. The motion of the pendulous element is related to motion of the platform or structural element upon which the accelerometer is mounted. However, it is impossible to distinguish between effects of inertial acceleration and gravitational fields. Thus, in order for the navigation system to correct the motion of the pendulous element with inertial acceleration, detailed knowledge of the local gravitational field is necessary.

Once a measurement of the inertial acceleration is obtained, it follows that the first integration with time will yield velocity information, and the second integration with time will yield position information. This data processing function is accomplished by an on board digital computer in modern systems.

A typical gimbaled inertial measurement unit is shown on Fig. 2-1

Fig. 2-1 Basic INS Structure

In fig. 2-1, the out-most gimbal is attached to the vehicle which can undergo any change in angular orientation. The platform maintains a fixed orientation in relation to the desired coordinate frame. Gyros on the platform sense the angular rate of the platform with respect to the inertial space, and their outputs are sent electronically to the torque motors on the gimbal structure, commanding them to maintain a desired platform orientation regardless of the orientations of the gimbals.

The platform remains aligned with a known reference coordinator frame. Accelerometers on the platform then provide vehicle acceleration with respect to this reference coordinate frame. The acceleration will then be integrated to generate the vehicle's position and velocity.

2.2 Types of Inertial Navigation Systems

Inertial Navigation Systems can be classified into three basic categories: geometric, semi-analytic and analytic. Because these systems utilize similar inertial sensors, the similarities among the various types of INS are much more apparent than the differences.

The geometric system was the first practical inertial navigation system. The navigation information was available in analog fashion directly from the gimbals' angles. Two reference frames, the inertial non-rotating frame and the local navigational frame are physically instrumented. This type of system is mechanically complex and requires minimum computations. It is the early stage of INS development.

The semi-analytic systems physically have only one reference frame, either an inertial non-rotating frame or a local navigational frame. Three gimbals are necessary to effect

this coordinate frame. Position information is provided by the on board computer. This type of configuration was quite popular in the late 1960's and 1970's, and still can be found in use today.

An analytic Inertial Navigation System does not physically instrument any reference frame. It uses gyro outputs to calculate the relative orientation between the system's initial and present state. The lack of gimbal structure in a strap-down system affords a reduction in system size, weight, fuel consumption, and cost. The complexity of calculation is increased. All the modern INS systems are strapdown INS systems.

2.3 The Fundamental Inertial Navigation Equation

The Inertial Navigation System's function is , in the simplest term, the integration of the acceleration measured by the accelerometer to determine the velocity and position of the vehicle. The position is conventionally presented by latitude and longitude (L and l) and the distance to the earth (height) in cases of above the surface of earth. In theoretical analysis, the vector r is usually used to present the position.

The following equation is called the fundamental inertial navigation equation:

$$\frac{d^2 r}{dt^2} = f_s + g^*(r)$$

Where : $r = [r_x, r_y, r_z]$, the position vector

$f_s = [f_x, f_y, f_z]$ the specific force

$\mathbf{g}^* = [g_x, g_y, g_z]$ the gravitational field acceleration at the vector in system's location.

The specific force $\mathbf{f}_s$ is defined as $\mathbf{F}_a/m$. Where $\mathbf{F}_a$ is the acceleration force at the accelerometer, m is the mass. The $\mathbf{f}_s$ represents the accelerometer indicated acceleration (m/s^2). The integration of the acceleration of the vehicle must be carried out with respect to a specific reference frame. The calculation results may be transformed among several different frames.

2.4 Reference Frames

Reference frames are important for understanding the functions of Inertial Navigation System.

1. Inertial Frame (i frame, x y z)

In practice, a coordinate frame that is non-rotating relative to the stars and has its origin at the earth's center can be regarded as the inertial frame. The axes has been chosen such that the x, y inertial axes lie in the equatorial plane and the z axis is coincident with Earth's angular velocity vector.

2. Geographic Frame (n frame; N, E, D axes)

The geographic frame is a local navigational frame which has its origin at the system's location and its axes aligned with north, east, and down, (N, E, D). Down is defined to be normal to the reference ellipsoid, an analytically defined surface. The north axis, N, is in the direction of the projection of the earth's inertial angular velocity vector into the local horizontal plane. The east axis, E, completes the right hand orthogonal set.

3. Earth Frame

The Earth frame is a coordinate frame which has its origin at the earth's mass center and has axes fixed on the earth. The axes are arranged so that the earth and inertial frames are coincident at starting time.

The following relationships apply:

$$l = l_0 + \lambda - \omega_{ie}t$$

Where:

l is terrestrial longitude from Greenwich

λ is celestial longitude

l_0 is initial terrestrial longitude

ω_{ie} is earth's inertial angular velocity

t is time.

4. Geocentric Frame (C frame)

The local geocentric frame has its origin at the system's location, coincident with the origin of the geographic frame. The Z_c axis is in the direction opposite to the geocentric position vector. The Y_c is in the direction of the east, and the X_c axis, which is in the local meridian plane, completes the right handed orthogonal axis set.

Fig. 2-2 shows the above reference frames.

Fig. 2-2 Coordinate frame geometry. (N,E,D)-geographic: (x,y,z)-inertial: (x_e, y_e, z_e)-earth: (x_e, y_e, z_e)-geocentric. (From K.R.Britting, Inertial Navigation Systems Analysis. MIT, 1971)

5. Body Frame (b frame)

The body frame constitutes the vehicle axes of roll, pitch, and yaw. See fig 2-3

Fig. 2-3 Body Frame

6. Tangent Frame (t frame)

This is an earth fixed frame, which is aligned with a geographic frame at a fixed location on earth. This location is usually the landing site, guidance radar, or some other convenient point of reference.

2.5 The Local-Level Terrestrial Navigator

One typical gimballed INS is the local level terrestrial navigator. It physically

instruments the local geographic coordinate frame. The platform axes are commanded to align with the local north-east-down coordinate system. Three accelerometers are present to measure the geographically referred specific force. In many practical applications, the vertical navigation (i.e. altitude) is not desired. Only two accelerometers are required in most of the local-level terrestrial navigators. The navigation quantities are the vehicle's position, which can be determined by the latitude (L) , longitude (l), and height (h), and velocity (v).

Fig 2-4 is the functional block diagram for the local level of terrestrial navigator.

Fig 2-4 Data flow of local-level inertial guidance system

The basic navigation equations from the [12] are presented in the followings:

1. Platform Commands

The central on board computer must send commands to the torque motors to force the platform to align with the local geographic frame. The platform would be ideally commanded at the angular velocity of the geographic frame with respect to the inertial frame.

$$\omega_{in}^n = \begin{pmatrix} \dot{\lambda} \\ -\dot{L} \\ -\dot{\lambda} \sin L \end{pmatrix}$$

where : ω_{in}^n is the angular velocity of the geographic frame with respect to the inertial frame

L is the latitude of the vehicle, and

$$\dot{\lambda} = \dot{l} + \omega_{ie}$$

where: ω_{ie} is Earth self rotation angular velocity.

l is the longitude of the vehicle.

2. Specific Force

Since both the gyro and accelerometer are directly mounted on the platform, the specific force with respect to the local geographic frame is directly available.

$$\mathbf{f}^n = \mathbf{f}^a$$

3. Earth -Referenced Velocity

The earth referenced velocity is the result of the integration of the appropriately compensated specific force data. The velocity is obtained by :

$$\dot{\mathbf{v}}^n = \begin{pmatrix} \dot{v}_N \\ \dot{v}_E \\ \dot{v}_D \end{pmatrix} = \begin{pmatrix} f_N - v_E(l + 2\omega_{ie})\sin L + \dot{L}v_D \\ f_E + v_N(l + 2\omega_{ie})\cos L \\ f_D - v_E(l + 2\omega_{ie})\cos L - \dot{L}v_N + g \end{pmatrix}$$

The computed velocity is given by:

$$\mathbf{v}^n = \int \dot{\mathbf{v}}^n dt + \mathbf{v}^n(0)$$

where : v_N is the vehicle Earth referenced velocity component in the north direction.

v_E is the vehicle Earth referenced velocity component in the east direction.

v_D is the vehicle Earth referenced velocity component in the down direction.

f is the specific force.

4. Gravity Calculation

The vertical channel requires an explicit computation of the magnitude of the earth's gravity field. Its value depends on the system's location. However, in some cases, a constant value can be assumed.

5. Latitude and Longitude

The further integration of velocity generates position information, the Geographic Latitude, L and the Terrestrial Longitude, l . See fig 2-2 for their definition. Usually, the altitude information is given by some other external source, since the inertia navigation altitude channel is theoretically unstable from the control point of view.

$$\dot{L} = \frac{v_N}{(r_0 + h)}$$

Where:

r_0 is the earth's equivalent radius.

h is the vehicle's height.

and

$$l = \frac{v_E}{(r_0 + h) \cos L}$$

$$L = \int \dot{L} dt + L(0)$$

$$l = \int \dot{l} dt + l(0)$$

The system equations are graphically represented in Fig. 2-5. Where the inertial system is also used to calculate the system's altitude. The altimeter is present to provide additional altitude measurement.

Fig. 2-5 Local-Level INS Computation Chart
(From Kenneth R. Britting "Inertial Navigation Systems Analysis")

2.6 INS Error Considerations

Many factors can affect the accuracy of the INS. The following are some of the known major error sources :

- Gyro Drift Error.

Ideally, the gyro can maintain a fixed direction relative to inertial space without any external torque or maintain the desired direction through accurate torque control. But because of the changes of temperature, friction, uneven mass , etc, the gyro can drift away from its desired position. This drift sometimes is unrepeatable and unpredictable. The platform will drift away from the local geographic frame due to the gyro drifting in local level INS configurations.

- Accelerometer Error.

The inaccuracy of the accelerometer will cause measurement errors in specific forces.

- Accelerometer Alignment Error.

Ideally, the accelerometer coordinates should be an orthogonal axis set. But the accelerometers can not be mounted in a perfect orthogonal set mechanically. The sensitivity axes of the accelerometers are not in the ideal directions .

- System Alignment Error.

System alignment error is defined as initial misalignment of the platform to the geographic frame in local-level INS configuration.

- Attitude Error.

Attitude error is defined as the orthogonal transformation error between platform and

geographic axes, presented in geographic axes. In local-level INS, this error occurs when the platform is not aligned with local geographic frame due to non-precision commands or motor torque errors.

- Altimeter Error.

This is the error contained in height measurement or calculations.

- Gravity Field Error.

The gravity field is usually calculated under certain assumptions of the earth's deflections of the vertical and gravity anomalies are not compensated. Gravity field error is the error contained in this assumption.

The above are major sources of error in the INS. One way to improve the navigation accuracy is to physically eliminate these sources. A lot of sophisticated technologies have been developed for this purpose, but they are very costly. A better alternative is to mathematically improve accuracy with the use of the Kalman filter.

CHAPTER 3

KALMAN FILTER THEORY

3.1 Introduction

Kalman filtering theory was first introduced by R.E Kalman in a paper " A New Approach to Linear Filtering and Prediction Problems " published in **Journal of Basic Engineering** (ASME) in March,1960. The Kalman filter is a linear, unbiased, and minimum error variance, recursive algorithm which can optimally estimate the unknown state of a dynamic system from noisy data taken at discrete real-time. It has been widely used in many industrial applications, such as video and laser tracking systems, vehicle and weapon navigation systems, missile trajectory estimations and radar and fire control systems. With recent developments in high speed computers and real-time operating systems, the Kalman filter has become even more useful for very complicated real-time applications.

Kalman filtering is to estimate state vector of a dynamic system based upon all the data measurements. Optimal estimation is an estimator that minimizes the estimation error in a well-defined statistical sense. The Kalman filter needs the following information to perform its functions:

- Knowledge of system and measurement dynamics.

- Assumed statistics of system noise and measurement noise.
- Initial condition information.

3.2 Kalman Filter Algorithm

Assume a system state space model is:

$$\mathbf{x}(t+1) = \mathbf{A}_{t+1/t} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{G} \mathbf{w}(t) \quad (3 - 1)$$

$$\mathbf{z}(t+1) = \mathbf{H}_{t+1} \mathbf{x}(t+1) + \mathbf{v}(t+1) \quad (3 - 2)$$

Where :

$\mathbf{A}_{t+1/t}$ is a (m,n) matrix, called the state space transit matrix

$\mathbf{B}$ is a (n,1) matrix, called the system control matrix

$\mathbf{H}$ is a (n,1) matrix, called the observation matrix

$\mathbf{x}$ is an n dimension vector representing system states

$\mathbf{u}$ is the input control vector

$\mathbf{z}$ is the system state measurement vector

$\mathbf{G}$ is the coefficient matrix for state noise

$\mathbf{w}$ is Gaussian random zero bias noise for state vector

$\mathbf{v}$ is Gaussian random zero bias noise, measurement error

Kalman filtering is a process to determine $\mathbf{x}^*$ such that the sum of the deviations, $\mathbf{z} - \mathbf{z}^*$ is minimized. i.e. the quantity:

$$\mathbf{J} = (\mathbf{z} - \mathbf{H}\mathbf{x}^*)^T (\mathbf{z} - \mathbf{H}\mathbf{x}^*) \quad (3-3)$$

where $\mathbf{x}^*$ stands for the estimation of state vector $\mathbf{x}$.

Assume all the processes are Gaussian processes, then the state estimation can be regarded as the linear combinations of past state measurements:

$$\hat{x}^- = \sum A_i * z_i$$

The error of this estimation will be:

$$\Delta x = x - \hat{x}^-$$

The best estimation of x will be yielded when Δx is at minimum value. Fig. 3-1 geometrically shows the meaning of optimal estimation. Δx is at minimum when $\hat{x}^-$ is the projection of x .

Fig. 3-1 Measurement Plane and State Estimation

Let P to be the estimation error covariance, $P = E[\Delta x \Delta x^T]$. At time t , assume we know:

$\{ x(0), x(1), x(2), \dots, x(t) \}$

$\{ P(0), P(1), P(2), \dots, P(t) \}$

and

$$\mathbf{R} = \mathbf{E} \{ \mathbf{w}(t)\mathbf{w}(t)^T \}$$

$$\mathbf{Q} = \mathbf{E} \{ \mathbf{v}(t)\mathbf{v}(t)^T \}$$

where $\mathbf{R}$ is the covariance of the state noises; $\mathbf{Q}$ is the covariance of state measurement noises. They are diagnostic matrixes for Gaussian white noise.

The best estimates of the states in the next time step $t+1$ is:

$$\mathbf{x}_{t+1}^- = \mathbf{A}\mathbf{x}_t + \mathbf{B}u \quad (3-4)$$

The estimation error covariance is:

$$\mathbf{P}_{t+1/t} = \mathbf{E} (\Delta \mathbf{x} \Delta \mathbf{x}^T) = \mathbf{E} ([\mathbf{x}_{t+1} - \mathbf{x}_{t+1/t}^-][\mathbf{x}_{t+1} - \mathbf{x}_{t+1/t}^-]^T) \quad (3-5)$$

The estimation error is:

$$\begin{aligned} \Delta \mathbf{x}_{t+1/t} &= \mathbf{x}_{t+1} - \mathbf{x}_{t+1/t}^- \\ &= \mathbf{A}_{t+1/t} \mathbf{x}_t + \mathbf{B}u + \mathbf{w}_t - \mathbf{A}_{t+1/t} \mathbf{x}_t^- - \mathbf{B}u \\ &= \mathbf{A}_{t+1/t} [\mathbf{x}_t - \mathbf{x}_t^-] + \mathbf{w}_t \\ &= \mathbf{A}_{t+1/t} \Delta \mathbf{x}_t + \mathbf{w}_t \end{aligned} \quad (3-6)$$

Substitute (3-6) into (3-5) gives :

$$\begin{aligned}
P_{t+1/t} &= E[\Delta x_{t+1/t} \Delta x_{t+1/t}^T] \\
&= A_{t+1/t} E[\Delta x_t \Delta x_t^T] A_{t+1/t}^T \\
&\quad + E[w_t w_t^T] + E[w_t \Delta x_t A_{t+1/t}^T] \\
&\quad + E[A_{t+1/t} \Delta x_t w_t^T]
\end{aligned}
\tag{3-7}$$

The last two items in (3-7) are zero because all the noises are Gaussian. Apply the noise statistics previously defined; the estimation error covariance can now be written as:

$$P_{t+1/t} = A_{t+1/t} P_t A_{t+1/t}^T + Q_t \tag{3-8}$$

At time $t+1$, the measurement signal $z(t+1)$ is obtained .

It is apparent that state estimation of $x_{t+1/t}$ can be corrected by this information:

$$\begin{aligned}
\bar{x}_{t+1} &= \bar{x}_{t+1/t} + K_{t+1} [z(t+1) - z^-(t+1)] \\
&= \bar{x}_{t+1/t} + K_{t+1} [z(t+1) - H_{t+1} \bar{x}_{t+1/t}]
\end{aligned}
\tag{3-9}$$

The correction coefficient K_{t+1} , or Kalman gain, is a vector which can be determined by system information. It is called Kalman gain¹.

¹ Kalman gain is derived from the partial derivative of final estimation error covariance P_{t+1} with respect to K_{t+1} [6]

$$K_{t+1} = P_{t+1/t} H_{t+1}^T [H_{t+1} P_{t+1/t} H_{t+1}^T + R_{t+1}]^{-1} \quad (3-10)$$

The estimation error covariance is :

$$\begin{aligned} P_{t+1} &= E[\Delta x_{t+1} \Delta x_{t+1}^T] \\ &= E[(I - K_{t+1} H_{t+1}) \Delta x_{t+1/t} - K_{t+1} v_{t+1}] \\ &\quad * [(I - K_{t+1} H_{t+1}) \Delta x_{t+1/t} - K_{t+1} v_{t+1}]^T \\ &= P_{k+1/k} - K_{t+1} H_{t+1} P_{t+1/t} \end{aligned} \quad (3-11)$$

Equation (3-4), (3-8), (3-9), (3-10) and (3-11) are a complete set of Kalman filter algorithms. The Kalman filter works in an iterative process. So the initial state values and initial covariance matrix are needed.

Kalman filter can also be used for states estimation on a non-linear system and colored non-Gaussian process. There is not a unique solution for the non-linear system or non-white process. Additional restriction or information must be provided with respect to specified problems .

Fig. 3-2 is a simple structure diagram of a discrete system with a discrete Kalman filter. Fig. 3-3 is the data flow diagram of a Kalman filter.

Fig. 3-2 Functional Chart of System Model and Kalman Filter
(From A. Gelb "Applied Optimal Estimation" The MIT Press, 1974)

Fig. 3-3 Data Flow Chart of Kalman Filter Algorithm

CHAPTER 4

INTERFACING KALMAN FILTERS WITH INS

4.1 The Benefits of Using Kalman Filters

The Inertial Navigation System, or INS, can provide very good information due to tight loop control, computer commanding, etc. However, the system drifts at a slow rate and could shift quite away from its ideal position over a long time period because of the characteristics of the gyro. A typical INS position error can be up to one nautical mile in an hour.

Some other navigation tools, such as doppler radar and radio navigation aid, work in just the opposite way to INS. The data they provide is good on average, but subject to considerable high frequency noise, due to instrument noise, atmospheric effects , antenna oscillation and other interference.

It would be natural to think of using a Kalman filter in order to combine information from both INS and external data sources in an optimal manner to provide very accurate navigation parameters. The Kalman filter uses statistical characteristics of errors in both the INS and the external data source to determine this optimal combination. Essentially, Kalman filters use good low frequency data from external sources to compensate the slowly growing errors inherent in the INS. In this way, the costly high grade precision

gyro is not necessary. The cost for the INS will be decrease drastically decreased.

The external source could be a radar, a radio navigator, or the signal from satellites - GPS (Global position system).

The substantial benefit of using the Kalman filter to aid INS can be gained so long as an adequate system model can be developed.

Fig. 4-1 Position errors from INS. a), Pure INS b).INS with Doppler aide. (From P.Maybeck, Stochastic models, estimation, and control, Vol.1, 1982, with permission from the author.)

From Fig.4-1, it can be seen that an INS at an accuracy of 1.5 n mile per hour is upgraded to the accuracy of 1/500 n mile per hour after using a Kalman filter with Doppler radar aid.

4.2 Systems Configurations of INS with a Kalman Filter

There are two types of configuration to interface Kalman filters with Inertial Navigation systems. One is the total state space versus error state space formulation, or direct Kalman filtering. The other is feed-forward versus feedback mechanization, or, indirect Kalman filtering.

In the direct Kalman filtering configuration, total system states such as position, velocity, attitude, etc are among the state variables in a Kalman filter model. The INS accelerometer outputs and external signal source are directly used as measurements.

Fig. 4-2 Direct Kalman filtering
(From P.Maybeck, Stochastic models, estimation, and control, Vol.1, 1982)

Fig. 4-2 shows the configuration of a direct Kalman filter. The Kalman filter is directly in the INS control loop. The INS accelerometer data and external source data are

both fed into the filter. The Kalman filter will not only provide desired navigation information, but also send appropriate commands to the torque motors mounted on the gimbals to force the platform to align with the chosen coordinate frame.

The benefits of such a configuration are that the information is utilized optimally in real time. It can improve the performance of an INS in the best possible way.

However, there are several serious drawbacks to this configuration. Being in the INS control loop, the filter would have to maintain explicit, accurate awareness of vehicle angular motion, as well as suppress noisy and erroneous data. The sampling rate of the Kalman filter must be at least twice the highest frequency of data to be sampled. The common engineering practice is to choose the sampling rate of the filter to be five to six times of the data frequency. Consider the INS used in aircraft and missiles: the roll rate capability is in the order of 400 deg/sec, the Kalman filter has to be running at least a sampling time of 1/800 second.

This is a very difficult situation for the on board central computer. The central computer allocates the most portion of CPU time and memory on more critical control and stability algorithms; only a very small portion of that can be used for the Kalman filter algorithm. The filtering algorithm can only be scheduled in low priority. The computing time for the Kalman filter may be quite long. The conflict between system requirement and computer capacity is obvious.

Another drawback is that if the Kalman filter fails for some reason, the entire navigation system fails, because the Kalman filter is an integrated part of the system and the INS can not operate without the filter. From a reliability standpoint, it would be

desirable to provide an emergency degraded performance mode in case of such failure.

So, the direct configuration for the Kalman filter to the INS is restricted to laboratory testing, ground alignment and calibration, and submarines while static or low frequency data is dealt with.

In the indirect formula, the errors in INS indicated positions and velocities are among the filter state variables. Each measurement datum sent to the Kalman filter is the difference between the INS and the external source data.

Fig. 4-3 is the system structure diagram. The data from the INS and the external source is first processed to form the error information. This information can then be fed into the Kalman filter as the measurement signal. The Kalman filter estimates the navigation errors. These errors are then subtracted from the INS navigational output.

Fig. 4.3 Indirect Feed-Forward Kalman Filter

(From P.Maybeck, Stochastic models, estimation, and control, Vol.1, 1982)

In indirect Kalman filter configuration, the filter is based on a set of inertial system error propagation equations which are relatively well developed. The errors are low frequency. Because the filter is out of the INS control loop, there are no strict requirements for the Kalman filter sampling rate. This sampling rate can be much lower than that of a direct filter. The computing requirements can be met. Further, if the Kalman filter fails, the INS still works independently. For these reasons, the indirect Kalman filter configuration is used in almost all terrestrial Inertial Navigation Systems.

There are actually two types of configurations of indirect Kalman filters. Fig 4.3 shows the structure of an indirect feed-forward Kalman filter, the filter compares the two sets of data and uses the results to estimate the errors in the inertial system. By subtracting these estimated errors from the INS output, the on board computer maintains the optimal estimates of position, velocity, and attitude. The Inertial Navigation System operates as if there were no Kalman filter. It is unaware of the existence of the filter or external data. So if either of them should fail, the unaltered INS information would still be available. But since Kalman filter performance depends on adequacy of a linear dynamic model, it is necessary that errors in the INS remain at small magnitudes. The INS has to have its own means to bound the errors.

Minimum computation is required for the central computer in this configuration. Fig. 4-4 is the structure diagram for the indirect feedback Kalman filter. The Kalman filter not only generates estimation errors of navigation parameters, but also errors of the INS system's parameters such as the gyro's drift rate. These estimations are then fed back to the INS so that they can be compensated. In this way, the INS errors are checked

regularly and eliminated; the adequacy of the linear model is enhanced. There is a further advantage to this configuration. Since the INS is corrected after each measurement sample, many of the predicted error states at the next sample time will be zero, and thus these components of state vector need not be computed explicitly. The computation is relatively larger than that of feed-forward configuration, but provides better performance. The INS for large commercial aircraft, helicopters which utilize a powerful on board computer mostly use this kind of configuration for Kalman filter application.

Fig. 4-4 Indirect feedback Kalman filter

(From P.Maybeck, Stochastic models, estimation, and control, Vol.1, 1982)

4.3 External Data Sources

Typical external data sources used are position data, velocity data, and altitude data:

Position data source include :

- 1) Radar, on board or ground based
- 2) Radio navigation aids: TACAN, OMEGA, etc
- 3) Global Position System (GPS) navigation system
- 4) Position fixes, star sightings, landmarks
- 5) Radiometric area correlation
- 6) Laser ranging.

Velocity data source include:

- 1) Doppler radar
- 2) Indicated airspeed from the air data system.

Altitude data source include:

- 1) Barometric altimeter
- 2) Radar altimeter
- 3) Laser altimeter.

Additional altitude data is needed for the INS since the INS itself cannot provide good altitude data.

Radar is the most commonly used reliable navigation tool, and is a very good external aid to the INS. The Kalman filter optimally combines both information from radar and from INS to achieve high accuracy of navigation. INS/Doppler-radar is the most common combination for aircraft navigation.

In the recent developments of space program, one navigation tool is becoming more and more popular -- the Global Positioning System, or GPS.

The Global Positioning System is a system that consists of 24 navigation satellites. Eight satellites in each of three different orbits. Each satellite contains a transmitter and a receiver to send and receive signals and a quartz crystal oscillator "clock". Periodically, a ground tracking network measures and updates the ephemeris and satellite clock phase and frequency in order to maintain synchronization of all satellite clocks. The satellites in turn continually transmit this information together with an identification code so that the user can distinguish satellites from each other. The system user with a signal receiver can thus compare the time shift between satellite time and user un-synchronized time.

The signal is transmitted in the speed of light, the distance between user and each satellite will be known. The user's position can be determined by the distances to three satellites, each in a different orbit. This distance R can be determined by:

$$R_{j\text{-sat}} = C t_u - C t_{j\text{-sat}}$$

where:

$R_{j\text{-sat}}$ is distance from j-th satellite to user

t_u is time of the user

$t_{j\text{-sat}}$ is time of the j-th satellite

C is speed of light.

If a vehicle is equipped with INS and GPS receivers, the difference between the GPS indicated position and the INS indicated position can be used as input for the Kalman filter to estimate navigation parameters and those of the INS .

A distance divergence measurement can be written in terms of INS indicated range R_{j-ins} and satellite indicated distance R_{j-sat} , true distance R_{j-true} , and associated errors δR_{j-sat} , δR_{j-ins} .

$$\begin{aligned} z_j(t_i) &= R_{j-INS}(t_i) - R_{j-sat}(t_i) \\ &= [R_{j-true}(t_i) + \delta R_{j-ins}(t_i)] - [R_{j-true}(t_i) + \delta R_{j-sat}(t_i)] \\ &= \delta R_{j-INS}(t_i) - \delta R_{j-sat}(t_i) \end{aligned}$$

Since INS indicated range is derived from INS indicated position of the user vehicle and satellite-provided ephemeris data on its own position, range error is due to errors from both these sources. But since satellite orbital parameters are very precisely updated, the errors from this source are negligible.

The range measurement error model, validated through empirical data, is :

$$\delta R_{j-sat}(t_i) = c\delta t_u(t_i) - c\delta t_{sj}(t_i) + b_{rj}(t_i) - v_j(t_i)$$

where:

δt is clock error of user and/or j-th satellite.

b_{rj} is a small range bias term accounting for tropospheric delay and uncertainty in the speed of light.

$v_j(t_i)$ is white gaussian measurement noise which models the high

frequency corruption of satellite signal and quantization error.

4.4 A Radar Aided INS--LITTON LR-80 AHRS ^[14]

LITTON LR-80 AHRS is a good example of Kalman filter application in INS system. LR-80 is a commercial inertial navigation product designed for use in the AH-64A Apache helicopter, AHIP OH-58D scout helicopter, and some others. LR-80 has been developed by Litton Guidance and Control Systems Ltd, one of the most important players in this field. This system uses a Kalman filter and Radar navigation aid to perform its functions.

1. LR-80 AHRS System Configuration

The LR-80 is a strapdown Attitude and Reference System with its design based on helicopter requirements. It utilizes two two-degree-of-freedom dry tuned gyros and three accelerometers to sense inertial motions.

These sensed data are then used to compute the attitude, position, velocity, etc. The system has been designed primarily to be interfaced with a doppler radar; the velocity difference between doppler radar indicated velocity and INS indicated velocity can be fed into the Kalman filter.

2. Kalman Filter Design

The Kalman filter was implemented to perform initial in-ground alignment, and in flight navigation error estimation. The Kalman filter state vector consists of:

- 1) Position error states in navigation frame X,Y direction, written as

directions X_N , Y_N . These errors directly affect navigation accuracy.

- 2) Velocity error states in X, Y direction. The velocity errors significantly affect accuracy and can be well measured through doppler radar.
- 3) Gyro drift error states in X,Y,Z direction.
- 4) Gyro scale factor error state, which reflects uncertainties in gyro bias.
- 5) Accelerometer bias error states, which reflects accelerometer error in X,Y,Z direction.
- 6) Doppler scale factor error state and doppler bias error state.

Table 4-1 is a list of state selection of LR-80.

Table 4-1 Kalman Filter States in LR_80

State				
1	$\bar{P}_x$	Position error in X_N direction (ft)		
2	$\bar{P}_y$	Position error in Y_N direction (ft)		
3	$\bar{V}_x$	Velocity error in X_N direction (ft/s)		
4	$\bar{V}_y$	Velocity error in Y_N direction (ft/s)		
5	$\bar{T}_x$	Rotation about X_N to level Y_N (Rad)		
6	$\bar{T}_y$	Rotation about Y_N to level X_N (Rad)		
7	$-B_{Gx}$	-X gyro bias drift rate (Rad/s)		
8	$-B_{Gy}$	-Y gyro bias drift rate (Rad/s)		
9	$-B_{Gz}$	-Z gyro bias drift rate (Rad/s)		
10	B_{Ax}	X accei bias (ft/s ²)	$-S_{Gz}$	-Z gyro scale factor errors (per unit)
11	B_{Ay}	Y accei bias (ft/s ²)	$\Delta\alpha$	Computed alpha - True alpha (Rad)
12	B_{Az}	Z accei bias (ft/s ²)	S_{Dx}	Doppler scale factor error in X_B (per unit)
13	$-\Omega_x$	-Earth rate about X_N (Rad/s)	B_{Dx}	Doppler bias error in X_B (ft/s)
14	$-\Omega_y$	-Earth rate about Y_N (Rad/s)	B_{Dy}	Doppler bias error in Y_B (ft/s)

The initial state values and the state driving noises are determined through observation of day-to-day and long term bias and scale factor changes of gyros and accelerometers. Still, there may be some other system errors that have not been modeled, and the real covariance will be larger than it is modeled. The covariance must be modeled so that it can reflect the increased uncertainty.

The doppler radar provides velocity information in vehicle body frame, whereas the INS propagates velocity in the navigation frame. In order to calculate the velocity error, first the INS transforms the INS velocity into body frame velocity, then it is modified to simulate the tracking lags to the doppler; then the Doppler velocity is extrapolated to synchronize it with INS data; these two data are then differentiated to form a velocity error, and subsequently transformed back to the navigation frame to update Kalman states.

The Kalman filter here monitors all important known error sources. It uses the velocity error input to estimate current error states and feeds back the error estimations to the INS. The INS will make appropriate adjustments to eliminate its internal errors such as gyro drift. The positions, velocities, and attitude information will also be corrected.

Fig. 4.5 shows a simple functional data flow of LR-80.

Fig. 4-5 LR-80 Functional Data Flow

(From D. Lottman, Kalman filter in Litton LR-80 AHRS, IEEE, Proc. National Aerospace and Electronics, 1984)

4.5 General Kalman Filter Design Approaches

Although establishing a Kalman filter that can be used in real flight applications is a difficult test-analysis-test iterative process, some general rules still can be followed.

- 1) Develop a system model that can well describe the system.
- 2) Generate a Kalman filter based on this model and analyze its capacity.
- 3) Simplify the system model and reduce the Kalman filter order depending on system requirements and center computer loading situation.
- 4) Conduct noise analysis to determine covariance matrices. Validate them with lab tests.

- 5) Conduct Kalman filter performance analysis off line and on line.
- 6) Perform final tuning.

CHAPTER 5

KALMAN FILTER DESIGN SIMULATION AND ANALYSIS

5.1 Background

In the late 1970's, the American Air Force was developing a tactical weapon system that would not need a pilot stand on during ordinance delivery. Such a weapon system is a middle course missile equipped with a Strapdown Inertial Navigation System. A radiometric area correlator is used as the navigation aid for INS.

For the radiometric area correlator, the computer pre-stores the radio wave picture of the desired area that the vehicle is going to fly over. During the course, the radiometric sends the radio picture immediately below the vehicle to the computer. By comparing this picture with the restored picture, the position measurement can be generated.

Since this kind of weapon system can be used only once, (once it hit the target, it explodes) the navigation and computer system's hardware should be chosen at the lowest cost, so long as they can meet the precision requirements. The most important subject in missile navigation is the position precision, i.e. whether the weapon can hit the target.

A Kalman filter has been designed to combine the information from the INS and the Radio Area Correlator (RAC) to estimate the errors generated by INS and RAC. These signals are to be fed back to the central computer to correct the INS provided signals.

Because of the limited computer memory, a very simple Kalman filter has been

chosen. Only the basic navigation parameters such as position, velocity, and attitude are among the Kalman filter states.

5.2 System Error Model for Kalman Filter

We define:

δx_e = position error of INS in the east

δx_n = position error of INS in the north

δv_e = velocity error of INS in the east

δv_n = velocity error of INS in the north

ϕ_e = attitude error in the east

ϕ_n = attitude error in the north.

It is apparent that :

$$\delta \dot{x}_e = \delta v_e$$

$$\delta \dot{x}_n = \delta v_n$$

Attitude errors account for a small misalignment of actual sensing frame to the local north-east frame. Ideally, the sensing axes of the accelerometers in the north and east positions would be orthogonal to gravity field. But in the practice, these axes are almost impossible to orthogonalize perfectly to the gravity field. The gravity field would have effects on north and east accelerometers. These will be in amounts of: $g \cdot \sin(\phi_e)$ and

$g \sin(\phi_n)$ in north and east respectively. See fig. 5-1, the sensing frame $\{e, n, z\}$ has a small angle rotation along axis e to $\{e, n', z'\}$, the earth gravity g will have an effect of $g \sin(\phi_e)$. Same to the north direction.

Fig. 5-1 a) Attitude Error caused acceleration in North
b) Attitude Error caused acceleration in East
c) Earth coordinate

There are many other factors that can cause errors in accelerometers in north and east such as scale factors, bias, etc. Since we do not model them into the state space equation, we have to use a Gaussian Noise to reflect those uncertain quantities causing errors in accelerometers.

We can now have the equation :

$$\delta \dot{v}_e = -g * \phi_n + w_{e1}$$

and

$$\delta \dot{v}_n = g * \phi_e + w_{n1}$$

where: w_{e1} , w_{n1} are the Gaussian noises associated with acceleration error in the east and north.

g is the gravitational acceleration of the earth.

Assume the vehicle travels at a speed of v in the north, the attitude error in the east will cause an error in v , i.e. δv_n . The time rate of the attitude error is a angular velocity. It is in an amount of $\delta v_n / (R_0 + h)$. R_0 is the earth radius. Same applies to the east. There are many other factors affect the attitude error, the most significant of which is the gyro drift rate in that specific direction.

Gaussian noise is used to reflect the effects of un-modeled factors. Write the above in the mathematical equations, we have:

$$\dot{\phi}_n = \delta v_e / (R_0 + h) + w_{e2}$$

and

$$\dot{\phi}_e = -\delta v_n / (R_0 + h) + w_{n2}$$

where: w_{e2} , w_{n2} are the Gaussian noises associated with errors in attitude angular rate in east and north

R_0 is the equatorial radius of the earth

h is the vehicle flight height.

These equations are written in a state space form :

$$\begin{pmatrix} \delta \dot{x}_e \\ \delta \dot{v}_e \\ \dot{\phi}_n \\ \delta \dot{x}_n \\ \delta \dot{v}_n \\ \dot{\phi}_e \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 1/R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & g \\ 0 & 0 & 0 & 0 & -1/R & 0 \end{pmatrix} * \begin{pmatrix} \delta x_e \\ \delta v_e \\ \phi_n \\ \delta x_n \\ \delta v_n \\ \phi_e \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} w_{e1} \\ w_{e2} \\ w_{n1} \\ w_{n2} \end{pmatrix}$$

(5-1)

This is the system error model in continuous time. The Kalman filter is based on this

model. The position measurement can be generated by the difference between the INS indicated position and the RAC indicated position.

$$x_{e-INS} = x_{e-true} + \delta x_e$$

$$x_{n-INS} = x_{n-true} + \delta x_n$$

and

$$x_{e-RAC} = x_{e-true} - v_1$$

$$x_{n-INS} = x_{n-true} - v_2$$

where:

x_{e-RAC} is the position error from Radio Area Correlator

v_1 , v_2 are measurement noise.

Write the measurement in the form of matrix:

$$\begin{aligned}
\begin{pmatrix} z_e \\ z_n \end{pmatrix} &= \begin{pmatrix} x_{e-INS} - x_{e-RAC} \\ x_{n-INS} - x_{n-RAC} \end{pmatrix} \\
&= \begin{pmatrix} \delta x_e + v_1 \\ \delta x_n + v_2 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} * \begin{pmatrix} \delta x_e \\ \delta v_e \\ \phi_n \\ \delta x_n \\ \delta v_n \\ \phi_e \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}
\end{aligned}$$

(5-2)

So, the system model in the form of :

$$\dot{x}(t) = F * x(t) + G * w(t)$$

(5-3)

$$z(t) = H * x(t) + v(t)$$

where :

$x(t)$ is the system state vector

$$x(t) = \begin{pmatrix} \delta \dot{x}_e & \delta \dot{v}_e & \dot{\phi}_n & \delta \dot{x}_n & \delta \dot{v}_n & \dot{\phi}_e \end{pmatrix}^T$$

F is the system dynamic model

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 1/R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & g \\ 0 & 0 & 0 & 0 & -1/R & 0 \end{pmatrix}$$

G is the coefficient matrix for the noise

$$G = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and H is the measurement matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

z is the measurement vector

$$z = [x_e, x_n]^T$$

5.3 Kalman Filter Design

The above model is a theoretical model applicable only in continuous time. Since we are not taking measurement continuously but receive information at intervals of T_s seconds,-- T_s is the sampling time, we need to discrete the system model.

a). Discrete System Model

The relationship between the transition matrix A and system dynamics matrix F is :

$$A(t) = L^{-1}([sI - F]^{-1})$$

Where I is the identity matrix, and L^{-1} is the inverse Laplace transform.

For discrete system, we can say that discrete transition matrix is the continuous transition matrix evaluated at sampling time T_s [8]

$$A_k = A(T_s)$$

We can have the discrete transition matrix A_k

$$A_k =$$

$$\begin{pmatrix} 1 & \sin(\omega T_s)/\omega & R(1-\cos(\omega T_s)) & 0 & 0 & 0 \\ 0 & \cos(\omega T_s) & g\sin(\omega T_s)/\omega & 0 & 0 & 0 \\ 0 & -\sin(\omega T_s)/(\omega R) & \cos(\omega T_s) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sin(\omega T_s)/\omega & R(1-\cos(\omega T_s)) \\ 0 & 0 & 0 & 0 & \cos(\omega T_s) & -g\sin(\omega T_s)/\omega \\ 0 & 0 & 0 & 0 & \sin(\omega T_s)/(\omega R) & \cos(\omega T_s) \end{pmatrix}$$

where: ω is the Schuller frequency, and defined as :

$$\omega^2 = g/R$$

R is the radius of earth, g is the gravitation acceleration of earth. It's value is at about 1 out of 84 minuets.

The noise coefficient can be obtained from

$$G_k = \int_0^k A*(\tau)*Gd\tau$$

$$G_A =$$

$$\begin{pmatrix} (1 - \cos(\omega T_s))/\omega^2 & R(T_s - \sin(\omega T_s))/\omega^2 & 0 & 0 \\ \sin(\omega T_s)/\omega & g(1 - \cos(\omega T_s))/\omega^2 & 0 & 0 \\ -(1 - \cos(\omega T_s))/(R\omega^2) & \sin(\omega T_s)/\omega & 0 & 0 \\ 0 & 0 & (1 - \cos(\omega T_s))/\omega^2 & R(T_s - \sin(\omega T_s))/\omega^2 \\ 0 & 0 & \sin(\omega T_s)/\omega & -g(1 - \cos(\omega T_s))/\omega^2 \\ 0 & 0 & (1 - \cos(\omega T_s))/(R\omega^2) & \sin(\omega T_s)/\omega \end{pmatrix}$$

We now have a discrete recursive model:

$$x_k = A_k * x_{k-1} + G_k * w_k$$

$$z_k = H * x_k + v_k$$

(5-4)

Kalman filter algorithms can be executed based on this model. Properties of the system's noise, i.e. the covariance matrices of the noise and the error state vector initial value have to be known.

b) Kalman Filter Equations

The discrete Kalman filter can now be written as:

$$x_k = A_k * x_{k-1} + K_k * (z_k - H * A_k * x_{k-1})$$

(5-5)

$$P_{k|k-1} = A_k * P_k * A_k^T + G * Q_k * G^T$$

(5-6)

$$K_k = P_{k|k-1} H^T [H * P_{k|k-1} * H^T + R]^{-1}$$

(5-7)

$$P_k = (I - K_k * H) * P_{k/k-1}$$

(5- 8)

and :

$$P_0 = P_0 ; x_0 = x_0$$

$$Q_k = E[w(t_k) * w(t_k)^T]$$

$$R_k = E[v(t_k) * v(t_k)^T]$$

c) Initial Values

Initial values , x_0 and P_0 , theoretically will not affect the final Kalman filter estimation results. But they do affect how fast the Kalman filter will reach its stable value. For real applications, initial covariance come from day to day observations of the gyro's drift. Initial position error can be considered to be error because of the precise ground alignment processes.

d) Covariance Analysis

To get a satisfactory Kalman filter performance, the covariance matrices must be carefully obtained through many analyses and experimental procedures. In order to simplify the Kalman filter model, many known error sources are not included in the system. Their effects on the system will be included in the noise. There are also some other error sources which are not known that may also have

effects on the performance and also have to be included in the noise considerations.

In this particular configuration of RAC-INS, the following factors must be considered in the noise

1) Accelerometer Errors

These errors are described as day to day non-repeatable bias, scale factor errors, input axis misalignment, etc.

2) Uncertainty in the Knowledge of Gravity

The errors are the differences between real earth geoid and assumed ellipsoid for INS computation.

3) Gyro Errors

These errors are gyro drift rate bias, scale factor errors, input axis misalignment, gyro drift rate, g sensitive drift, and g^2 sensitive drift.

4) RAC errors

The errors in RAC data are modelled as a corruptive white Gaussian noise. The strength of such noise can be denoted as:

$$R = [\sigma * h(t_i)]^2$$

5) Altimeter Errors

Because of the instability of the INS vertical channel, the altitude information is derived from the external altimeter. The altimeter errors have important effects on navigation accuracy. These errors are defined as altimeter bias, and scale factors.

P. Maybeck in "Analysis of a Kalman filter for a strapdown inertial/radiometer area correlator guidance system" ([1]) listed all the major error quantities . The most important error information is listed in the following table.

TABLE 5-1 ERROR SOURCES

ERROR SOURCE	P_0 term	Q term
Accelerometer biases (non repeatable)	$(200 \mu g)^2$	0
Accelerometer scale factors errors	$(405.6 \text{ ppm})^2$	0
Accelerometer input axis misalignment	$(30 \text{ arc sec})^2$	0
Accel. Meter bias	$(600 \mu g)^2$	$2P_0/60$
Gravity deflection	$(26 \mu g)^2$ (east) $(17 \mu g)^2$ (north)	$2P_0/10$
Gravity anomaly	$(35 \mu g)^2$	$2P_0/60$
Gyro drift rate biases	$(2.0 \text{ deg/hr})^2$ (roll axis)	0
Gyro scale factor errors	$(500 \text{ ppm})^2$	0
Gyro input axis misalignment	$(30 \text{ arc sec})^2$	0
Gyro drift rate	$(0.4 \text{ deg/hr})^2$	$2P_0/60$
RAC bias	P_{ba}	0
Altimeter bias	$(500 \text{ ft})^2$	$2P_0/250$
Altimeter Scale factor	$(0.03)^2$	0

When the Q term is zero, this does not necessarily mean that the particular error source does not have any effect on the system. It may indicate that, for that particular error source, the characteristics of the strength is not yet known.

In the design of this system, the above error information must be combined and fine tuned to find the Q, R to get the most satisfactory results for position and velocity, attitude error estimations.

5.4 Kalman Filter Simulations

In order to examine the Kalman filter performance, computer simulations of the Kalman filter are needed. As designed above, only the position error, the velocity error and the attitude error are among the Kalman filter states. For missiles system, the most important issue is accurate position control, while speed and attitude are of secondary importance. Only position measurement is available through RAC external data source. Simulations are against three different scenarios :

- 1) Slowly increased position error with stationary noise process.
- 2) Drastically changing dynamics with stationary process.
- 3) Uncertain parameters.

The input data for simulations (i.e. the Kalman filter input data) is synthesized according to the position error patterns being described in several published papers. ([13], [24])

The measurement noise error could be coming from either the INS or the RAC system,

or both. If the noise is coming from the RAC, the Kalman filter should be able to filter out this noise. The correct estimation of INS error states should be established. If the noise is contained in the INS, the Kalman filter should be able to estimate this noise in real time, thus the corrections in the central computer can be made.

5.4.1 Slowly Increased Position Error

The INS position error is a slowly increasing error due the slow drift of the gyros. The measurement data that inputs to the Kalman filter is as shown in Fig. 5-2. This data is generated by a mathematical function shown in Fig. 5-3 and corrupted by a computer generated random white Gaussian noise with a strength of $\sigma=3$.

1) Initial Values

$$\mathbf{x}_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$$

$$P_0 = \begin{pmatrix} 2.25 \times 10^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & .25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.25 \times 10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 \end{pmatrix}$$

The initial errors can be considered as zero because of accurate ground lab processes.

The initial error covariance is from [2].

2) RAC Measurement Noise.

$R = 9$, assuming the noise is from the RAC.

$R = 0$, assuming the noise is from the INS.

3) INS System Noise

$Q = [0]$, assuming the noise is from the RAC

$$Q_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.16 & 0 & 0 & 0 & 0 \\ 0 & 0 & .10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.10 \end{pmatrix}$$

assuming the noise is from the INS.

4) Sampling Time

A sampling time of $T_s = 9$ seconds is assumed.

5) Simulation Results and Analysis

Fig. 5-4 gives the Kalman filter result for position error estimation, assuming noise is from the RAC. It can be seen that the Kalman filter estimation is very close to the actual signal .

Fig. 5-5 , Fig. 5-6, and Fig. 5-7 are the Kalman filter estimation results for position error, velocity error and attitude error, assuming the noise is from the INS itself. Kalman filter correctly provides the noise information of the states. It can be seen that Kalman

filter can provide state estimations.

5.4.2 Changing Data Dynamics Process

The changing data process is the most common data pattern in INS errors. Because of changing working conditions such as temperature, moist, etc, the gyro may drift at quite different rates. The measurement data is shown in Fig. 5-8. This data is generated by a mathematical function as shown in Fig. 5-9, and corrupted by a Gaussian random noise process with the strength $\sigma=3$.

1) Initial Values:

Initial values are the same as in case of slow increased position error.

2) RAC Measurement Noise:

$R = 9$, assuming the measurement noise is coming from RAC.

3) INS System Noise:

$$Q = 0.$$

4) Sampling Time:

A sampling time $T_s = 9$ second is assumed.

5) Simulations Results and Analysis:

Fig. 5-10, Fig. 5-11, fig. 5-12 show the position error, velocity error and attitude error estimated by the Kalman filter. The actual position error signal is shown in fig. 5-9. Fig.5-13 is the comparison of actual position error and estimated position error. It can be seen that the estimated error is quite different from the actual data.

5.4.3 Parameter Uncertainties

The main concern here is that the predefined Q , R values may be quite different from

the actual error covariance in the noise sequence. Three experiments have been done to examine the effects.

- 1) Measurement noise processes with covariance of $R = 0.5, 1.0, 9, 13$ are assigned to the Kalman filter to do the computations. The actual noise strength in the measurement data is 9 as shown in Fig. 5-1. Fig.5-14 is the comparison of the Kalman filtering results with different R . It can be seen that the stable states are the same. Only the transitional states are different from each other.
- 2) INS system noise processes contained in velocity error is assigned with different noise strength $Q(2,2)=1$ and $Q(2,2)=0.1$ at each time . The measurement data for the Kalman filter is the same as shown in Fig. 5-1. The results are shown in Fig.5-15. The state values are different because of different noise strength assumptions.
- 3) The measurement noise strengths are slowly growing. Fig.5-16 shows a measurement data process, where σ grows from 3 to 4 to 5. The Kalman filter is assigned with the strength of $R=9$. The result is shown in Fig. 5-17 and compared with $\sigma=3$ case. It can be seen that the estimation for this case is just slightly shifted away from the result of constant noise.

Fig. 5-2 Measurement Data

Fig. 5-3 Real Error Data

Fig. 5-4 Position Error

Fig. 5-5 Position Error of INS

Fig. 5-6 Velocity Error of INS

Fig. 5-7 Attitude Error of INS

Fig. 5-8 Measurement Data of Changing Dynamic

Fig. 5-9 Real Error Data

Fig. 5-10 Position Error From Kalman Filter

Fig. 5-11 Velocity Error From Kalman Filter

Fig. 5-12 Attitude Error from Kalman Filter

Fig. 5-13 Comparison Estimated Data with Real Data

Fig. 5-14 Comparison Position Error with Different R

Fig. 5-15 Comparison of Position Error With Different Q

Fig. 5-16 Measurement Data with Varying Noise

Fig. 5-17 Comparison Position Error of Constant Noise and Varing Noise

5.5 Summary

The Following table is the summary of simulation experiments.

Table 5-1 Simulations Summary

Experiment No.	Fig. No.	Remarks	Parameters
1	5-3, 5-4	Slowly growing error	$R=9, Q=0$
2	5-5 5-6 5-7	Slowly growing error	$R=0$ $Q(2,2)=0.16$ $(3,3)=0.10$ $Q(5,5)=0.16$ $Q(6,6)=0.10$
3	5-8 5-9, 5-10 5-13	Changing error dynamics. Comparison of data	$Q=0$ $R=9$
4	5-14	Different pre-assigned R	$R=0.5; 1; 9; 13$ $\sigma=3, Q=0$
5	5-15	Different pre-assigned Q	$Q(2,2)=1; 0.1$ $R=0$
6	5-16, 5-17	Varying σ	$R=3, Q=0$ $\sigma=3; 4; 5$

From the above simulations, it can be seen that this Kalman filter works well with slowly growing errors. The error in the RAC noise strength R does not affect the final results, as indicated by experiment No. 4. The system noise Q will certainly affect the error state estimation.

This filter fails in the case of changing data dynamics. As the data estimated from Kalman filter is quite different from its actual data. The reasons for this will be discussed in next chapter.

CHAPTER 6

ADAPTIVE KALMAN FILTER

The key to success for a real time Kalman filtering application is to generate a system model that can adequately predict the actual system's performance. Ideally the filter should have a wide range of convergence so that it can work in a wide range of dynamic environments. It is even better if the filter can detect and rectify situations of sensor failure and divergence. The adaptive Kalman filter has some of these features.

6.1 The Reason for Filter Divergence

The Kalman filter can provide the most accurate computation results only within certain predefined conditions. The system model and dynamics need to be well known. But in real applications, the systems usually are not well known enough to establish the perfect system model. There are still some computation limitations for the size of the system model.

For example, in the Litton-80 INS (Chapter 4) for helicopters, a fairly large model can be built to accurately describe the system performance. An indirect feedback Kalman filter is used to monitor all potential error sources. A standard Kalman filter may be the best choice.

In the missile applications, only a simple model can be built because of computing

limitations. All other error sources will be considered as Gaussian random white noise. There is no direct estimation or monitoring inside the model to actively check the possible errors. The system model is only valid when the error is small. But there is no mechanism inside the system to make sure the error is small. It is possible that this system model is not adequate.

The Kalman filter generates the estimation of the state at time step t_i based wholly on the measurement data prior to that time and current measurement data. Any error in those data will cause the errors in current state and later state estimation.

As the Kalman computation approaches stability, the Kalman gain is gradually decreased to zero, as shown in Fig. 6-1. Fig. 6-1 is the Kalman gain in case of drastically changed data in chapter 5. Although the input data has changed, the Kalman gain is near the zero. The input data in this stage will have little effect on the Kalman filter estimation.

This is a problem since the previous data may not contain any dynamic information on the later measurement data, and the later data dynamics do not have much influence on the current computing process because the Kalman Gain is close to zero.

This is a typical problem for the Kalman filter. The Kalman filter gets so used to the dynamics of the early stage of data that it can not be aware that the process has changed its course. The computed results will definitely be wrong .

This explains what happened in experiment No.3, (Fig. 5-13). The position error data has three different parts during the 1000 sampling times. At about 200 sampling times, the Kalman filter enters into stable state. The Kalman filter algorithm uses information

on first part of data to estimate the second part, and third part. Data in second and third parts does not have much effect on the their state estimations.

In general, there are two main reasons for the Kalman filter failure :

- 1) The discrepancy between the mathematical model that the Kalman filter is based on and the actual physical process.
- 2) Lack of knowledge of the system. This will cause the uncertainties in system parameters.

Fig. 6-1 The Kalman Gain

6.2 Possible Modifications to the Standard Kalman Filters

There are many ways to overcome the drawbacks of the standard Kalman filter. But there is no general formula. The way to improve the Kalman filter varies from one application to another.

a) Extended Kalman Filter

In many cases, some error states are not modeled into the system --either for the purpose of simplifying the model or because of lack of the knowledge of the system. This may be the reason for the Kalman filter divergence. It is obvious that we should put back those error states into the system model.

Let "f" to indicate those additional states. Since f can affect the estimation of x, the original state vector, there must be some kind of relation between them. We use U to indicate this relation. Now, the new discrete model is:

$$x_{k+1} = A_{k+1,k} * x_k + U_{k+1,k} * f_k + w$$

and f can be described as :

$$f_{k+1} = \phi_{k+1,k} f_k + w_1$$

w , w₁ are white noise associated with states. ϕ is the transition matrix for f

Nothing is changed in the system measurement model. The new state vector for

the Kalman filter will be:

$$r_{k+1} = \begin{pmatrix} x_{k+1} \\ f_{k+1} \end{pmatrix}$$

The new transit matrix will be :

$$\Phi_{k+1,k} = \begin{pmatrix} A_{k+1,k} & U_{k+1,k} \\ 0 & \Phi_{k+1,k} \end{pmatrix}$$

The measurement will be :

$$z_{k+1} = (H_{k+1} \quad 0) * r_{k+1} + v_k$$

The algorithm will be the same as that of the standard Kalman filter.

Although this algorithm can provide satisfactory results, it is not practical in many applications. First, it is hard to find the missing factors in the system that cause the problem. It may be even more difficult to determine the relation (**U**) of these states to the original states (**x**).

The most serious drawback for this application is that it will considerably increase the work load of the computer for the Kalman filter computing since the size of state vector has been increased. This is very undesirable for many applications.

Paul W. McBurny in "A Robust Approach to Reliable Real-Time Kalman Filtering" ([7]) suggests a "SCHMIDT-KALMAN FILTER". This allows some missing

states to be considered in the gain and covariance updating stage, but not being actually estimated. It certainly will help to reduce the computing load.

b) Finite Memory Filtering

The standard Kalman filter estimates the current state(s) x_{i+1} based on the initial condition and all measurements prior to the time $i+1$. As it progresses, the filter 'remembers' an ever growing number of measurements. However, the previous data may not mean anything if the dynamics of the data have changed. The finite memory filtering provides state estimations based upon the most recent N measurements.

$$E[x(t_i)/z_{i-N+1}, z_{i-N+2}, z_{i-N+3}, \dots, z_i]$$

Where N is the fixed integer predefined upon a particular application. This kind of filter always be implemented by maintaining storage of the most recent N measurement values and performing an N step recursive processing. It is usually in the general form of :

$$x_N(t_i^+) = C_1(t_i) * x_N(t_{i-1}^+) + C_2(t_i) * z_i - C_3(t_i) * z_{i-N}$$

In the step of $i-1$ to i , the information z_i is added to propagate the state estimation, and the information of z_{i-N} is subtracted from it. Although it looks simple, the exact equation for the general case has not yet been established. The calculations of C_1 C_2 , C_3 for particular cases are very complex. In addition, this kind of algorithm may generate

extensive computation load and may have instability problems.

c) Overweighing Most Recent Data

The reason for using a finite memory Kalman filter is that the system model is not adequate over long period of time, and old data may not be meaningful anymore. One alternative to finite memory filtering is to overweigh the most recent data, so that it will have a greater influence on current data estimation.

One such method is called "exponential age weighing of data". The strengths of measurement corruption noises for previous measurements are artificially increased before their influences are brought to bear on current state estimation.

Fig.6-2 portrays how the $\mathbf{R}$ varies in the case of stationary measurement noise, where $\mathbf{R}_i = \mathbf{R}$.

FIG 6-2 Measurement Error Covariance

There is an adaptive approach for such methods. This will be discussed in more detail later.

d) Frozen Kalman Gain

We know that as the Kalman filtering approaches a stable state, the Kalman gain K approaches zero. The information from measurement data has little or no effect on the current estimation. Technically, we can freeze the K at a certain level so that the current measurement data can be brought into consideration. This method follows the same idea as the overweighing of the most recent data.

All the above methods to modify the Kalman filter are usually considered as compensation methods. They need pre-defined detailed information such as additional states, affective memory length N , or weighing factor, etc. There is no mechanism that can make the Kalman filter smart enough to identify the system's performance and tuning by itself so as to meet the requirements of system.

6.3 Adaptive Approaches to Kalman Filtering

For many applications, various techniques can be used to produce an adequate system description, or system model, which is driven by known Gaussian white noises, and with which it is possible to develop an optimal state estimator and/or controller. However, such an optimal estimation is dependent upon complete knowledge of the system.

In any real time practical application, some quantities are only known with some

uncertainty. The unpredicted parameter changes could have severe effects on system performance.

In the last section, some techniques that can improve the system performance were discussed. But somehow these are in a pre-defined manner. The adaptive Kalman filter will have the ability to self-tune according to the system's variations, thus providing a better result than a divergent Kalman filter.

There are many adaptive Kalman filter algorithms depending on particular applications and hardware availability. One popular algorithm can provide estimation results even though the noise information is totally unknown (black box). Only the system transit matrix and measurement matrix are known.

6.3.1 Self-Tuning Gain - An Adaptive Kalman Filter

The system model is the same as we described before :

$$x_{k+1} = A * x_k + B * w_k$$

$$z_k = C * x_k + v_k$$

If all parameters are known, i.e, A, B, C, Q, R, the estimation equations can be written as:

$$x_{k+1} = Ax_k + K_{k+1}(z_{k+1} - H_{k+1}A_kx_k)$$

$$K_{k+1} = P_{k+1/k}H_{k+1}^T[H_{k+1}P_{k+1/k}H_{k+1}^T + R]^{-1}$$

and $P_{k+1/k}$ can be obtained by :

$$P_{k+1/k} = A_{k+1}P_kA_{k+1}^T + GQG^T$$

If only A, C are known, the Kalman gain has to be estimated on the basis of the current measurement data. In this sense, it has the ability to adjust itself according to the data received. The current K_i can be estimated by last time step K_{i-1}

$$K_i = K_{i-1} + \Phi^{-1} \begin{pmatrix} \Gamma_{i-1}(1) \\ \cdot \\ \cdot \\ \cdot \\ \Gamma_{i-1}(n) \end{pmatrix} * [\Gamma_{i-1}(0)]^{-1}$$

Where:

$$\Gamma(k) = \frac{1}{N} * \sum_{i=k+1}^N u_i * u_{i-k}^T$$

and

$$u_k = z_k - H * x_{k|k-1}$$

and Φ^{-1} is the pseudoinverse of matrix Φ which is defined as :

$$\Phi = \begin{pmatrix} H * A \\ H * A * (1 - K * H) * A \\ \cdot \\ \cdot \\ \cdot \\ H * [A * (1 - K * H)]^{n-1} * A \end{pmatrix}$$

The performance of this adaptive Kalman filter has been analyzed by Dr. J.z. Sasiadek and P. J. Wojcik ([10]). The result is published in Journal of Dynamics Systems, Measurement and Control. Simulations of the noise signal with frequency from 0 up to certain values were made . Their results are helpful in designing an adaptive Kalman filter for a specific application.

6.4 The Use of S Kalman Filter in the INS-RAC Guidance System for Glide Weapon System

IN this specific missile application, a major consideration is the computer CPU time and memory allocated for Kalman filter. The Kalman filter is always in supporting role and running in the background in a lower priority. Due to the fact that this system is only being used once and the number of weapons could be numerous, the hardware was to be obtained at the lowest possible cost. Only very limited CPU time and computer memory can be contributed to the Kalman filter since the central on board computer must perform more critical task such as dynamic stability control, attitude control and inertia navigation computations. For this reason the first proposed Kalman filter structure was very simple.

The ideal Kalman filter will be relatively simple and would not take up too much capacity of the central computer and would still meet the system's requirements.

The S-Kalman filter is an adaptive Kalman filter which is actually a modified overweighing Kalman filter and very close to the standard Kalman filter. It works such that when the estimation error is increasing, the Kalman gain is also gets larger. This causes the current measurement signal to have more influence on data estimation, thus providing more accurate results.

The system model is as before :

$$x_{k+1} = A * x_k + G * w$$

$$z_{k+1} = H * x_k + v$$

We artificially define the measurement noise as the following :

$$E[v_n v_n^T / t=t_k] = R_{n/k} = S^{k-n} * R_n$$

where : $k \geq n$

R_n is the actual measurement noise strength in time n

S is the artificially assumed coefficient larger than 1.

The actual meaning of the above equation is that when the filter processes data at step n , the measurement noise strength prior to that step will be increased purposely . The earlier the time step, the higher its measurement covariance. This is equivalent to making the earlier measurement data to have lower influence on the current estimation. That, in turn, is saying the current measurement will have a greater influence on the current estimation.

The S Kalman filter algorithm from the above model is almost identical to the standard Kalman filter.

$$x_{k+1} = Ax_k + K_{k+1}(z_{k+1} - H_{k+1}A_k x_k)$$

(6-4-4)

$$P_{k+1/k} = S * A_{k+1} * P_k * A_{k+1}^T + G * Q_k * G^T$$

(6-4-5)

$$K_{k+1} = P_{k+1/k} H_{k+1}^T [H_{k+1} P_{k+1/k} H_{k+1}^T + R]^{-1}$$

(6-4-6)

$$P_{k+1} = (I - K_{k+1} H_{k+1}) P_{k+1/k}$$

(6-4-7)

In reference [3], the above algorithm has been proved.

Up to this point, this algorithm is no different than an overweighing Kalman filter if a predefined value S is used in the calculation. It becomes a standard Kalman filter if $S = 1$.

This algorithm can become an adaptive filter if it can adjust S itself according to the system measurements rather than remaining a fixed, predefined value.

This algorithm will work in this way :

- 1) Standard Kalman filter, $S = 1$ if no divergence is detected.
- 2) If divergence detected, switch to S Kalman filter assign S with some value that is greater than 1.
- 3) If divergence disappears, go back to Standard Kalman filter, $S = 1$.

- **How to detect system divergence ?**

As in the "adaptive gain" Kalman filter, the update sequence is used to judge the system performance.

Let u_k be denoted as:

$$u_k = z_k - H * x_{k/k-1}$$

If the following equation is met, it indicates that the Kalman filter is convergent.

$$u_k^T * u_k \leq \gamma * \text{tr}[E(u_k * u_k^T)]$$

where: γ is a scale factor larger than 1.

tr is the trace, the sum of diagonal elements of a matrix .

If the above equation is not met, it is possible that Kalman filter is divergent:

$$\begin{aligned} E[u_k * u_k^T] &= E([z_k - H_k * x_{k/k-1}] * [z_k - H_k * x_{k/k-1}]^T) \\ &= E[z_k * z_k^T] + H_k * E[x_{k/k-1} * x_{k/k-1}^T] * H_k^T \\ &= H_k * P_{k/k-1} * H_k^T + R_k \end{aligned}$$

The criterion for convergence can be written as :

$$u_k^T * u_k \leq \gamma * tr[H_k * P_{k|k-1} * H_k^T]$$

(6-4-8)

- How to determine S ?

When system divergence is detected, S should be able to automatically pick up some value. S can be determined interactively using system parameters.

Equation (6-4-8) is the criterion for divergence; the most strict condition is that actually $\gamma = 1$.

Let $\gamma = 1$:

$$u_k^T * u_k = tr(E[u_k * u_k^T])$$

The above equation can be obtained from :

$$\begin{aligned} u_k * u_k^T &= E[u_k * u_k^T] \\ &= H_k * P_{k|k-1} * H_k^T + R_k \end{aligned}$$

(6-4-9)

Combining (6-4-5) and (6-4-9) we get :

$$u_k * u_k^T = S * H_k * A_k * P_k * A_k^T * H_k^T + H_k * Q_{k-1} * H_k^T + R_k$$

(6-4-10)

From (6-4-10) we have :

$$S_k * I = [u_k * u_k^T - H_k * Q_{k-1} * H_k^T - R_k] * [H_k * A_k * P_{k-1} * A_k^T * H_k^T]^{-1}$$

(6-4-11)

Write the above equation in a form :

$$S_k = \frac{1}{m} \text{tr}([u_k * u_k^T - H_k * Q_{k-1} * H_k^T - R_k] * [H_k * A_k * P_{k-1} * A_k^T * H_k^T]^{-1})$$

(6-4-12)

In a system that has strict limits on computing capacity, (6-4-12) is not ideal since it involves large amount of matrix manipulations.

An other version of S equation can be obtained by rewriting (6-4-10) as follows:

$$S_k * H_k * A_k * P_{k-1} * A_k^T * H_k^T = u_k * u_k^T - H_k * Q_{k-1} * H_k^T - R_k$$

The traces of left and right matrix should be equal

$$S_k * \text{tr}[H_k * A_k * P_{k-1} * A_k^T * H_k^T] = \text{tr}[u_k * u_k^T - H_k * Q_{k-1} * H_k^T - R_k]$$

or :

$$S_k = \frac{\text{tr}[u_k u_k^T - H_k G Q G^T H_k^T - R]}{\text{tr}[H_k A_k P_k A_k^T H_k^T]} \quad (6-4-13)$$

The use of the above equation is recommended since it reduces the inverse matrix calculations.

6.5 Using On-Line System Identifications to Correct Uncertainties of Q,R

By using an S-Kalman filter, we can overcome the problem that standard Kalman filter can not deal with. But S-Kalman filter alone can not overcome the problem caused by inaccurate description of the system noise during the modeling procedure. It would be ideal to have some means to check and correct them from time to time during the real-time filtering procedure.

P. Maybeck in his book "Stochastic Models, Estimation, and Control" ([1]) describes a method using maximum likelihood equation to perform on-line estimations of the variations of the concerned parameters. The procedures are quite tedious. Only results are given here.

If a set of data which contains a number of N data, including measurement and estimation is available in time j , the maximum likelihood estimation of measurement noise would be :

$$R(t_i) = \frac{1}{N} * \sum_{j=i-N+1}^i \left([z_j - H(t_j)x(t_j^+)] [z_j - H(t_j)x(t_j^+)]^T + H(t_j)P(t_j^+)H^T(t_j) \right)$$

(6-14)

The system noise estimation in an explicit format is more difficult. A general format does not exist yet [1]. If we assume the system noise is stationary noise over most recent N steps, the system noise strength estimation can be found as :

$$Q(t_i) = \frac{1}{N} * \sum_{j=i-N+1}^i \left(G^{-1}(t_{j-1}) * [\Delta x_j * \Delta x_j^T + P(t_j) - A * P(t_{j-1}) * A^T] * G^{-1}(t_{j-1}) \right)$$

(6-4-15)

where: $\Delta x_j = x_j^+ - x_j^-$

x^+ is the estimation after receiving the current measurement data

x^- is the estimation before receiving the current measurement data.

It is apparent that these computations will take up a lot of computing capacity and consume many CPU time. But for most of applications there is no need for step by step Q,R estimation during recursive processes. Off-line noise analysis during the modelling stage is most important. Periodically checking the difference between estimated Q, R and

pre-defined Q , R can help prevent severe errors.

6.6 A Comprehensive Engineering Approach to Better Kalman Filtering

A practical engineering approach to the INS/RAC navigation system for this weapon system can now be established by combining standard Kalman filtering, S Kalman filtering, and on-line noise strength estimation.

The S Kalman filter is suitable for this application simply because its algorithm is relatively simple, and the system is a grey box of which we already know some of the noise statistics.

The new approach can be described as follows :

- 1) At beginning of the process, perform standard Kalman filtering. Ground lab modelling and testing can usually determine the initial covariance values and state value, and state dynamics quite well, so the kalman filter can provide best filtering results.
- 2) After some time steps, during each recursive cycle, check if the convergent condition is still being met, if it is, continue to use the standard Kalman filter. If that condition is not being met, then calculate S and start S Kalman filtering.
- 3) At each of several pre-set points in time steps, calculate estimated Q , R . If the differences between estimated value and pre-defined value are significant, use estimated value instead of old value.

Fig. 6-3 shows the schematic of this approach.

Fig. 6-3 S-Kalman Filter Algorithm Data Flow

6.7 Simulations and Results

The purpose of computer simulations of the S Kalman filter and Q, R identification is to show how the adaptive approach can provide better estimation results.

The measurement data is the same as that used in simulation of the standard Kalman filter, so there is a base for comparison.

1) Changing Data Process

All the parameters and measurement data are the same as those in experiment No.3 for standard Kalman filter.

Fig. 6-4 is the S-Kalman filter effect on the Kalman gain K . The data dynamic change will automatically increase Kalman gain, so the new data will have effects on final estimation.

Fig. 6-5 is the position data from S Kalman filter. Fig. 6-6 compares the data from standard and adaptive Kalman filters and the actual signal. It can be seen that the performance improvement is quite significant.

Fig. 6-7, Fig 6-8 are the velocity and attitude data from the S-Kalman filter.

2) Uncertainties in Noise

Fig. 6-9 shows the measurement data for the Kalman filter. This data has increased noise strength σ from 3 to 5 then to 7 with the changing dynamics.

The noise is assumed to be committed by the measurement noise. The R is set to a value of 9 ($\sigma=3$).

The result is shown in Fig. 6-10. When the noise strength increases, the estimated position data become more and more noisy. The solid line shows the result when there is no increase in strength in measurement data.

3) Combining On-line Noise Estimation with S Kalman Filtering

S Kalman filter is sensitive to the R selection. It is useful to combine the on line noise estimation algorithm described as equation (6-4-14). N is set at 50. Checking points are from 190 till 250 and 300 till 450. If the estimated R is different from the pre-set R, the estimated value will replace the pre-set value.

Fig. 6-11 is the estimated R (solid line) and the comparison to noise covariance, σ^2 (i.e. R). The estimated noise strength can be close to the actual noise strength.

Fig. 6-12 is the position error from the Kalman filter and comparison of data from the S Kalman filter combined with R identification, S Kalman filter alone and the actual signal. It can be seen that the combined methods provide much better results.

Table 6-1 is the summary of these experiments.

Table 6-1 Summary of Simulations on Adaptive Kalman Filter

Experiments	Figure	Remarks	Parameters
7	Fig.6-4, 6-5, 6-6, 6-7, 6-8	Using S Kalman Filter in changing dynamics.	$Q = 0$, $R=9$, $\sigma=3$
8	Fig.6-9, 6-10	Using S Kalman Filter in varying noise strength.	$Q = 0$, $R=3$ $\sigma = 3$ if $t < 450$ $\sigma = 5$ if $t < 600$ $\sigma = 7$ if $t < 1000$
9	Fig.6-11,6-12	Using S-Kalman filter combined with on-line noise strength estimation.	$Q = 0$ $R_0 = 0$

Fig. 6-4 Comparison of Gain of S-Kalman Filter and Kalman Filter

Fig. 6-5 Position Errors From S-Kalman Filter

Fig. 6-6 Comparison of Position Errors

Fig. 6-7 Velocity Error from S-Kalman Filter

Fig. 6-8 Attitude Error From S-Kalman Filter

Fig. 6-9 Measurement Data

Fig. 6-10 Position Error with Varing Noise

Fig. 6-11 Measurement Noise Covariance Estimation

Fig. 6-12 Position Error from S-Kalman Filter with On-line R Estimation

From the simulation results, it can be seen that using the new approach can bring the system up to better performance levels. The algorithm is adaptive in the sense that it can automatically adjust the Kalman gain through a system determined factor S . Although this filter is designed for the model of INS-RAC for a glide weapon system, it should be applicable to any real-time system where the computation capacity is restricted.

CHAPTER 7

CONCLUSIONS AND DISCUSSIONS

7.1 Conclusions

Due to many restrictions, the original design of the Kalman filter presented by P. Maybeck in reference [2] is very simple. Many important factors, such as gyro drift rate, were left out of the Kalman filter state vector. In chapter 5, simulation results of the Kalman filter against different type of input data show that the Kalman filter could fail in some cases. An adaptive Kalman filter, S Kalman filter, combined with on line covariance matrices estimation is introduced in this thesis. Simulation results in chapter 6 show the major improvements of the Kalman filter performance.

The Inertial Navigation System is one of the major navigation tools used in many military aircraft and other space vehicles. The key components of the Inertial Navigation System are gyroscopes and accelerometers.

Gyroscopes can provide a measurement platform for the accelerometers, which provide specific force measurements. The inertial navigation process is the integration process of the measured specific force with respect to certain defined frame. The accuracy of the INS depends mainly on the quality of the gyroscope and system alignment. A poor gyro

will drift away from its reference direction and any measurement made with respect to that gyro will contain considerable errors. Very sophisticated instrumentation technology has to be used to achieve satisfactory results.

Kalman filtering is an optimal estimation technology which can extract useful information from noise corrupted data with minimum estimation error.

Kalman filtering provides a way to numerically upgrade the Inertial Navigation System. There are two ways to integrate the Kalman filter into INS; direct or indirect applications. The direct Kalman filter is rarely used due to computer system's requirements. Indirect Kalman filtering can work in lower frequencies which estimate navigation error with the aid of external measurement data.

One major concern in Kalman filter application in INS for spacecraft is the CPU time and memory allocated for the Kalman filter process. The Kalman filter is always running in the background and only very limited computing capacity can be allocated to it. This may cause the Kalman filter system model to be over simplified and cause Kalman filter failure.

An adaptive Kalman filtering approach which combines adaptive S Kalman filter and maximum likelihood on-line measurement noise and system noise estimation is introduced. . The S Kalman filter through automatic S factor justification can indirectly fine tune the Kalman gain during the filtering process. Such an algorithm works as it would normally work as a standard Kalman filter, but after un-convergent signals are received, it will switch to the S Kalman filter. Covariance verifications will be done periodically.

Simulations of the new adaptive Kalman filter in the pilot stand-off missile system has been done. Simulation results prove that adaptive Kalman filtering can provide better estimation results in case of drastically changed data dynamics and parameter uncertainties.

7.2. Further Research Direction

Throughout the thesis, the computing restriction on the Kalman filter application is an important concern. Since the central computer is engaged with other more critical processes such as vehicle control and stability algorithms.

As the cost of processors is reduced drastically, the local distributed processor network, or transputer system will commonly be available. Computing languages such as Parallel C can support such a hardware configuration. Using such hardware configuration, the central processor that is used to do all Kalman filtering computation tasks will only assume the function of initiating communications with all other local processors, and making summaries of the local processor's computation results, while the local processors will actually be dedicated to a sub-Kalman filtering, i.e. a reduced size Kalman filter. In this way, the high speed Kalman filter will be obtained. There is a great potential for the parallel Kalman filter processing.

For the S Kalman filter, a better method for on line estimation of noise strength is still needed. Simulation with real INS data is suggested.

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