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SAVINGS INCENTIVES AND
THE RETIREMENT INCOME SYSTEM

by Keith Horner, B.A., M.A.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Department of Economics

Carleton University

OTTAWA, Ontario

October, 1983

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ABSTRACT

Keith Horner, Ph.D. Carleton University, October 1983

SAVINGS INCENTIVES AND THE RETIREMENT INCOME SYSTEM

Thesis Supervisor: W. Irwin Gillespie

This thesis investigates the possible roles of tax incentives and public pensions as components of government policy in support of retirement incomes. The encouragement of individual saving clearly provides an alternative to public pensions as a means of supporting retirement incomes. It has also been argued that measures are needed to offset the displacement of private saving by unfunded public pensions.

The thesis begins by reviewing the theoretical and empirical literature dealing with the taxation of capital income, the economic rationale for public pensions, and the effect of the public pensions on private saving levels. The complex set of tax provisions and income transfer programs which support retirement income in Canada is then described and analysed using a two period lifecycle model of consumption and savings. The major sections of the thesis are devoted to the development, estimation and analysis of a multi-period lifecycle savings model which embodies several realistic features including uncertain longevity, a wealth constraint, and provision for variation in the size of the consuming unit over its lifetime. This model is used to examine the effects of uncertainty -- regarding longevity, interest rates, and future earnings -- on consumer behavior and the

consequent demand for income insurance services which may be provided by public or private pensions. An illustrative policy analysis is provided which compares the effects on consumption patterns and consumer welfare of a cut in taxes on capital income versus an increase in the level of public pension income. Survey data on the spending patterns of individual households is used to estimate the lifecycle consumption function in a manner which retains the exact form of the model solution. The estimation results provide further evidence regarding the degree of consumer risk aversion and the interest elasticity of savings.

The main findings of the thesis are as follows. (1) As private markets which provide insurance against inadequate retirement income are hampered by moral hazard and adverse selection problems, public pensions have a potential role in increasing the retirement income insurance available to consumers. To the extent that public pensions do fulfil this role, they are not perfectly substitutable for private saving. (2) The estimation of the savings model developed in the paper provides some evidence that the level of consumer risk aversion is greater than has often been assumed in the literature. This adds support to the idea that consumer demand for income insurance is strong. (3) Both the theoretical and empirical evidence available in the literature suggest that the original estimates by Feldstein of the savings displacement effect of public pensions are overstated. Nevertheless, it remains very probable that some degree of displacement does occur. (4) Because they may provide valuable insurance services, an increase in public pensions (from present levels in Canada) may

raises the net return on private saving for retirement. On the other hand, the savings incentive measure is likely to be more effective in raising retirement consumption levels. For this reason, and because an increase in public pensions tends to increase the interest elasticity of savings, public pensions and savings incentives should be considered as complementary policy measures. (5) Finally, an analysis of the effects of the tax deduction for registered pension and savings plan contributions indicates that while it clearly stimulates savings, it has a doubtful influence on the allocation of consumption over consumers' lifetimes. Other forms of tax incentive are likely to be more effective in encouraging the deferral of consumption.

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A special purpose Family Expenditure Survey micro-data file was created for the project by the Consumer Income and Expenditure Division of Statistics Canada. Because the micro-data is secret, Statistics Canada also produced tabulations and regressions from this file and converted a complex set of instructions into a working, variable assignment program. My thanks to Gail Oja, Harry Champion and Roger Love for guidance on the development of the file and to Ulysse Nevrumont and Robin Bregg for their considerable efforts in creating the file and programming the estimation model.

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CHAPTER 1

INTRODUCTION

In most industrial countries, retirement income levels are supported both by public pensions and by tax incentives designed to encourage individual saving for retirement. The appropriate degree of reliance on collectively versus privately provided retirement income is a subject of recurring debate in these countries. In Canada, this question has recently been addressed in several important studies: The Retirement Income System in Canada: Problems and Alternative Policies for Reform, the report of a federal government task force (Lazar Report, 1980); the Report of the Royal Commission on the Status of Pensions in Ontario, (Haley Report, 1981); and the federal government green paper, Better Pensions for Canadians (Government of Canada, 1982c). A related question, which has received considerable attention by economists in the past decade, is whether the introduction or enrichment of unfunded public pensions is likely to lead to a serious drop in national saving and hence in the productive capacity of the country. Thus, policies to encourage private saving can be seen as an alternative to public pensions as an instrument for supporting retirement incomes. Additionally, they can be viewed as a means of offsetting possible adverse consequences of a public pension regime.

A curious gap exists between the analysis of these issues provided in the policy studies and that provided in the economic literature. The policy studies embody a common view that the present retirement income system leaves a substantial proportion of the retired population with inadequate income, judged either in relation to poverty lines or to pre-retirement income levels. Various remedies are proposed for this situation including: the expansion of public pensions, a voluntary supplement to the Canada and Quebec Pension Plans (C/QPP), and measures aimed at expanding coverage and improving benefit levels of private pensions. None of the studies pays much attention to the level of household savings apart from pension contributions. Nor is there much consideration of the response of household saving (and wealth at retirement) or the average age of retirement to changes in tax or pension provisions.

In contrast, the economic journal literature relating to retirement income is sharply focused on just those questions which are skimmed over in the public policy studies: the response of total household savings and retirement age to the expansion of public (and private) pensions and the implication of these responses for economic efficiency and social welfare. Little explicit consideration is given to questions of income inadequacy among the retired population. Moreover, much of the literature employs an analytical framework which provides no convincing rationale for the existence of public or private pension plans.

Accordingly, this thesis attempts to provide a bridge between these two approaches by comparing the effectiveness of public pensions and savings incentives in raising retirement incomes in the context of a model of household savings which can reflect the savings responses with which the economic journal literature is concerned.^{1,2} As the model depends on utility maximization by consumers, it also provides a basis for examining the welfare implications of alternative policy approaches. While a number of interesting results are obtained in the thesis, much of it is devoted to developing the household savings model, investigating its characteristics, and examining how well it performs in explaining the consumption and savings patterns of Canadian households. The possible application of the model to simulate the effects of specific policy changes on families with different age and income characteristics is suggested in the paper by the inclusion of a few illustrative simulation results.

The thesis is organized as follows. Chapter 2 reviews the theoretical literature on the tax treatment of investment income. In so doing, it shows how the optimal level of capital taxation depends on the interest elasticity of saving. This result is of particular interest because the interest elasticity of saving is later seen to

1. The response of labour supply to tax or pension system changes will be briefly considered but it can not be explicitly examined with the household savings model developed here.
2. A few studies do exist which provide comments on both the pension adequacy and savings-response questions. These include Pesando and Rea (1977), the Economic Council of Canada study, One in Three-- Pensions for Canadians to 2030 (1979), and Daley (1980).

be affected by the level of public pensions. Thus evidence is provided of the interdependency of appropriate tax and public pension policies.

Chapter 3 reviews the possible economic roles that have been proposed for public pensions, that is, the ways in which they might increase social welfare. It also provides a brief review of the controversy surrounding the effects of public pensions on national saving and income levels. An important conclusion of the first part of this chapter is that, since public pensions may possibly play an important role in providing consumers with insurance against income inadequacy in retirement, models designed to examine the effects of public pensions should be capable of representing the effects of uncertainty on the behavior of individuals and on the levels of welfare they can attain. Although uncertainties connected with real interest rates, future earnings and longevity are all important, the latter has the greatest consequence for the design of the household savings model.

Chapter 4 reports on some empirical estimates obtained in the literature. First, some summary comments are provided regarding the many estimates that have been made of the response of saving to public pension increases. Then a review is made of estimates that have been obtained for two important parameters, the interest elasticity of saving and the coefficient of relative risk aversion, a utility function parameter which relates both to the interest elasticity of saving and to the consumer's demand for income insurance.

While analytical models very often represent the tax system by a single tax rate and public pensions by a single income level, these policy instruments are far more complex in reality. Chapter 5 describes the detailed tax and income-transfer provisions which affect retirement incomes and household saving in Canada.

Chapter 6 begins by developing the basic lifecycle consumption (or household savings) model in a simple two-period form with no allowance for uncertain longevity. In the model, the consumer maximizes his utility subject to given resource endowments and a given interest rate by choosing consumption levels in the two periods. No consumption-leisure choice is considered, nor is any bequest motive. In order to permit the effects of tax or public pension policies to be observed more clearly, the utility function is assumed to take a particular form (iso-elastic and additively separable in C_0 and C_1). Development of the model in this simple form permits the nature of the solution and its relationship to parameters such as the interest elasticity of savings to be examined more easily than in the multi-period model of Chapter 7. It also permits the examination of the effects of detailed tax and pension policies for which the multi-period model would prove too cumbersome. These include the Guaranteed Income Supplement (GIS) program and the deduction for contributions to registered pension plans (RPPs) and registered retirement savings plans (RRSPs).

In Chapter 7, the lifecycle consumption model of Chapter 6 is further developed and then employed to examine a number of questions. Its development is guided by two purposes. First, the model should

permit the analysis of questions relating to uncertain lifetime, the provision of annuities, and uncertainties in the interest rate or in future earnings levels. Second, the model should be sufficiently realistic to permit its estimation using consumption and saving information for a cross-section of Canadian households. This implies, for example, that the effects of varying family size over the lifetime of the household must be taken into account. The model developed is one where individuals in a household maximize their collective expected utility by choosing current year consumption levels (and planned future consumption profiles) based on their resource constraints. The household is assumed to know the survival probabilities by age of the (adult) family members and to maximize expected utility levels subject to those probabilities. This model results in a realistic, humped profile of consumption levels by age which can be altered by introducing public pensions or private annuities. The solution is also affected by the requirement that net wealth levels remain non-negative. Again, no bequests are assumed in the model but the effect of including them is discussed. After providing analyses of the effects of the uncertainties associated with longevity, the interest rate and future income, the chapter concludes by providing an illustrative analysis of the implications for consumption levels pre- and post-retirement and for welfare levels of the policy options of (a) reducing the tax rate on investment income and (b) raising public pension levels.

Chapter 8 describes how the model was estimated using data on the spending patterns of individual households from the 1978 Family Expenditure Survey, and it describes the results of this estimation. Estimation of the model requires that proxy variables be generated where resource constraint information is unavailable on the survey. Chief among these proxy variables are the future earnings profiles of the household head and spouse. These variables are developed based on an analysis of earnings patterns by age, sex, education and occupation. With proxy variables assigned to households, the equation predicting household consumption levels in the current year is estimated in two ways. First, it is estimated non-stochastically, which has the effect of forcing all the coefficients of the explanatory variables to take their theoretically predicted values. Second, it is estimated using the ordinary least squares method, which permits the coefficients of the explanatory variables to vary so as to minimize the standard error of estimate. In each case, the model is estimated based on model solutions embodying different estimates of the risk aversion parameter. Thus, the model estimation provides evidence of the degree of risk aversion exhibited by Canadian households and the consequent interest elasticity of savings. Finally, the estimation results are compared with those obtained from a more traditional consumption function embodying age and family size dummies but no variables reflecting current wealth or expected future earnings.

Chapter 9 concludes the thesis by providing a summary of findings.

CHAPTER 2

ISSUES RELATING TO CAPITAL TAXATION

Nearly all recent analysis of capital taxation and public pension issues has been based on the lifecycle theory of personal savings. Under this theory, the primary purpose of personal savings is to translate an uneven stream of income over the lifetime into a smoother preferred stream of consumption expenditures. As recognized by Fisher (1930), the optimally chosen consumption levels will depend not only on income receipts but on the after-tax rate of return available on accumulated capital. Questions concerning the tax treatment of capital income are thus closely linked to questions relating to the provision of retirement income. This link is also reflected in government policies like the Registered Pension Plan (RPP) and Registered Retirement Savings Plan (RRSP) tax deductions which are designed to raise retirement incomes by increasing the net rate of return on saving. The issues relating to capital taxation are, of course, broader than the question of its effect on income and consumption in retirement. Nevertheless, the importance of the capital taxation-retirement income link and the fact that the literature on the welfare implications of capital taxation is more fully developed than that on the public provision of retirement income suggest that a review of capital taxation questions provides an appropriate starting point for the analysis of this thesis.

It is a long standing conclusion of public finance theory that reducing the tax rates on investment income to levels below those applied to labour income would lead to gains in economic efficiency, although at a cost in terms of horizontal and vertical equity. (See, for example, Musgrave and Musgrave, 1973, p. 449.) The acceptance of the first part of this proposition has been reflected in recurring proposals that the general income tax be replaced by taxes on labour income and/or consumer expenditures. While the conclusion appears to remain valid, the logic underlying it has been subject to both elaboration and qualification in recent years. (See especially Feldstein, 1978a, King, 1980, and the review article by McLure, 1980.) Some of the points raised in the discussion of this proposition also have implications for the evaluation of the welfare effects of public pensions. The main elements of the discussion are reviewed below under the headings: (1) static efficiency, (2) optimal taxation results, (3) dynamic efficiency, and (4) equity.

2.1 STATIC EFFICIENCY

Consider the effects of an increase in the net rate of return on capital resulting from a reduction in the personal income tax rate applicable to investment income. This increase in the net interest rate is equivalent to a fall in the price of future consumption, C_1 , relative to that of current consumption, C_0 , and so encourages the consumer to defer consumption to the future. The gain in economic efficiency produced by the tax cut comes about because the tax imposes

a divergence between the marginal rate at which present resources can be transformed into future resources through production and the rate at which consumers can exchange present and future consumption.

Under competitive assumptions, the intertemporal rate of transformation in production is $1+i$ where the pre-tax interest rate, i , is also the marginal product of capital. The corresponding substitution rate for a tax-paying consumer is $1+(1-m)i$ where m is the consumer's marginal tax rate on investment income. A reduction in the tax rate brings the intertemporal rates of substitution closer to equality, reduces the distortion in the time profile of consumption, and thus raises the consumer's level of lifetime consumption and utility.

Several points are worth noting about the relation of this efficiency effect to taxes and saving. First, it is important to recognize that a reduction in the tax rate on investment income need not lead to a decline in current consumption. The reason for this is that an uncompensated decline in taxes on interest income produces an increase in the consumer's lifetime consumption possibilities quite apart from its effect on the relative prices of C_0 and C_1 . In response to this positive income effect, the consumer will tend to increase both C_0 and C_1 , and it is quite possible that the increase in C_0 will outweigh the decline in C_0 resulting from the price incentive to substitute C_1 for C_0 . Thus, in the general case, a decline in taxes on investment income, if unaccompanied by any compensating tax increase or benefit reduction, will increase future consumption levels but

will have an ambiguous effect on current consumption.¹ An important implication of this point is that efficiency gains or losses can not be inferred directly from current consumption responses to change in capital income taxation. Since the efficiency cost of the tax arises solely from the distortion between the intertemporal rates of substitution in consumption and production, the income effects of a tax reduction must be factored out to arrive at an assessment of the efficiency gains. This can be accomplished by use of the Slutsky equation which, expressed in elasticity form is

$$\xi_{C_0 h} = \epsilon_{C_0 h} + \alpha_1 \eta_0 \quad (2.1)$$

where ξ and ϵ represent compensated and uncompensated elasticities of demand for current consumption with respect to the price of future consumption, $h = (1+(1-m)i)^{-1}$ is the price of future consumption, α_1 is the proportion of lifetime income spent on future consumption and η_0 is the income elasticity of current consumption. Since it is uncompensated elasticities which are usually estimated in econometric analyses, it is important that these be translated into compensated elasticities if the purpose is to assess the welfare effects of tax changes.

1. There is a case where a reduction in taxes on interest income could lead to a reduction in future consumption. For this to occur it is necessary that the consumer's current resources, $W + Y_0$, be small relative to expected future non-capital income, Y_1 , so he is a net borrower, and that the tax reduction result in an increase in borrowing as well as lending rates (i.e., that interest costs be deductible from taxable income). In this case the tax reduction would have a negative income effect which could outweigh the substitution effect favouring C_1 .

A second point which reinforces the previous one concerns the response of savings to an interest rate change. Feldstein (1978a, 1978c) has pointed out that, in a model with no bequests, current saving is equivalent to expenditure on future consumption ($S = hC_1$). Accordingly, the familiar quantity - expenditure relationship implies $\epsilon_{Sh} = 1 + \epsilon_{C_1h}$ (and $\xi_{Sh} = 1 + \xi_{C_1h}$) so that a zero elasticity of savings with respect to the interest rate does not mean that an interest rate change has no allocative effect.

A third point to note about this efficiency argument is that it ignores the taxation of capital in the hands of corporations. Feldstein (1974a) mentions evidence in Brittain (1966) and Feldstein (1970) that firms have adjusted their earnings retention rates in response to changes in personal income tax rates. King (1977) also demonstrates that adjustments to the corporate tax treatment of interest payments and depreciation can produce a combined personal and corporate tax system in which intertemporal distortions related to corporate assets are eliminated.

Fourth, it should also be noted that in reality different classes of investment income are subject to widely different tax treatment. In Canada, for example: interest payments, dividends and capital gains are all taxed differently; incomes from incorporated and unincorporated businesses are unequally taxed; and imputed rent and capital gains on owner-occupied housing escape taxation completely (although housing is subject to property taxes). There is some evidence (see, for example, St. Hilaire and Whalley, 1981) that

inter-industry resource mis-allocation costs resulting from such differences in tax treatment may be considerably more important than the efficiency costs of distortions in the intertemporal allocation of consumption.

2.2 OPTIMAL TAXATION RESULTS

The simple model in which a welfare gain results from any reduction in the tax rate on capital income involves an implicit assumption that the capital tax can be replaced by some non-distortionary tax. However, all taxes of any importance distort economic activity in one way or another.² Non-universal sales taxes distort the pattern of consumption across commodities, labour incomes taxes distort the choice between leisure and the goods and services purchased from labour income, and so on.

Literature on choice in "second-best" situations, where the impossibility of achieving grand optimality conditions is admitted, suggests that if not all distortions can be eliminated then none should be eliminated. Welfare maximization in these situations requires a balancing of distortions rather than their elimination. The attempt to derive tax policy prescriptions in a second-best world

---

2. Poll taxes and random taxes are two conceivable "lump-sum" or non-distortionary taxes. Such taxes may indeed have a role to play. For example, there is a random-tax element in the use of lotteries to generate government revenues. However these taxes fail to meet equity goals and have other drawbacks sufficient to restrict them to minor roles in the tax system.

has been pursued in a growing literature on "optimal taxation". This form of analysis typically proceeds by the maximization of a social welfare function (or representative utility function) through the choice of a set of tax rates to meet a government revenue constraint. The model employed in the analysis also contains some form of production function describing the link between the supply of labour and capital and the income available for consumption. The welfare maximizing set of tax rates is generally found to depend on the nature of the representative utility function and in particular on the values of the own- and cross-price elasticities of the demand functions derived from it.

The model developed and results obtained by King (1980) will serve to illustrate this analysis. King's analysis is based on a steady state growth model where the capital/labour ratio may be either taken as fixed by other policies (eg. debt policy that fixes interest rate i) or dependent upon the tax rates imposed on capital and labour income. In either case the government acts to maximize steady state utility, where utility for each member of the society (identical except for generation) depends on labour supplied in period 0 and consumption in periods 0 and 1 (pre- and post-retirement). Thus $U=U(L, C_0, C_1)$. The government's choice of tax rates on capital and labour incomes, t_i and t_w , results in relative prices $h = (1+(1-t_i)i)^{-1}$ and $l = (1-t_w)w$ of retirement consumption and leisure $(1-L)$ respectively. When the government can use other policy instruments to set the capital/labour ratio at its optimal level, the optimal second-best tax rate on capital income will satisfy

$$t_i = \left(\frac{1+i}{i} \right) \left(\frac{t_w}{1-t_w} \right) \left(\frac{\xi_{Ll} - \xi_{C_1 l}}{\xi_{Lh} - \xi_{C_1 h}} \right) \quad (2.2)$$

where the ξ_{ij} are compensated elasticities of L and C_1 with respect to the prices l and h . This condition, first derived by Corlett and Hague (1953), shows that a zero tax rate on capital income will be optimal only if $\xi_{Ll} = \xi_{C_1 l}$. From the elasticity definitions it can be shown that this would also imply $\xi_{C_0 l} = \xi_{C_1 l}$. That is, optimality of $t_i = 0$ would require that a compensated change in the wage rate lead to equi-proportionate changes in C_0 , C_1 and L . This in turn would require that the ratio C_1/C_0 be independent of the wage rate; i.e., that C_0 and C_1 be equally strong substitutes for leisure. Optimality of $t_i > 0$, on the other hand, requires that a rise in the net wage be allocated in higher proportion to C_0 than C_1 . This seems plausible since the increase in L tends to reduce first period utility. As would be expected, (2.2) also indicates that, for given t_w , optimal t_i will be higher the greater the own-price elasticity of labour supply (ξ_{Ll}) and the smaller the elasticity of savings to the net rate of interest.³ The important point here is that the optimal

3. Since the uncompensated elasticities of savings to the interest rate, i , are most commonly discussed and estimated, it is useful to express (2.2) in terms of this elasticity. Based on the Slutsky relations, and the relationship between C_0 and $S_0 = W_0 + Y_0 - C_0$, one can write

$$-\xi_{lh} = \left(\frac{\alpha_0}{1-\alpha_0} \right) \left(\frac{1+i}{i} \right) \left(\frac{S_0}{C_0} \right)^* \varepsilon_{Si} + \alpha_0 \eta_0$$

or $-\xi_{lh} = k \varepsilon_{Si} + \alpha_0 \eta_0$.

Accordingly, (2.2) can be written as

$$t_i = \left(\frac{1+i}{i} \right) \left(\frac{t_w}{1-t_w} \right) \left(\frac{\xi_{Ll} - \xi_{C_1 l}}{\xi_{Lh} + k \varepsilon_{Si} + \alpha_0 \eta_0} \right)$$

In this formulation it is clear that optimal t_i will vary inversely with the interest elasticity of savings, ε_{Si} .

tax rate on investment income depends not only upon the compensated elasticity of savings to the rate of interest but on the compensated elasticities of labour supply to the wage rate and the interest rate. Without information regarding these elasticities, there can be no presumption that efficiency would be improved by the elimination of taxes on capital income. King observed that the potential importance of labour supply responses is neglected in three recent estimations of the welfare cost of capital income taxation. Boskin (1978) and Summers (1981) both assume that labour supply is fixed which implies $\epsilon_{LL} = \epsilon_{CL} = 0$ and optimal $t_1 = 0$. Feldstein (1978a) assumes that the uncompensated own- and cross-price elasticities of labour supply are zero. Again, this is found to imply that optimal $t_1 = 0$. Thus all three estimates start from assumptions which lead to the polar case of $t_1 = 0$ and so guarantee high welfare cost estimates.

King also provides two extensions to the result of (2.2) which are of interest. First, he employs his model to reach the intuitive conclusion that tax rates on nominal capital income should be reduced in the face of inflation if the capital tax system is unindexed. He shows that the optimal tax rate t_1' under inflation may be obtained from a relation which differs from (2.2) only in the replacement of $\frac{1+i}{i}$ by $\frac{1+i+P+iP}{i+P+iP}$ where P is the inflation rate. It is evident from this relation that t_1' may be considerably lower than t_1 if P is large relative to i .

Second, he adjusts his model to reflect the existence of a second period resource endowment (eg. public pension benefits). The result of this is to increase the interest elasticity of savings and so reduce the optimal level of capital income taxation. This is an important result in the context of this study for it suggests that an increase in public pension promises should be complemented by a reduction in the rate of taxation on income from savings. The effect of expected future income on the interest elasticity of savings is further examined in sections 6.1 and 7.5 based on more fully elaborated lifecycle models.

2.3 DYNAMIC EFFICIENCY

Those who argue for the reduction in taxes on capital often do so not because of the distortion of labour-leisure and intertemporal consumption choices but because capital taxes are presumed to depress savings and output. In contrast, King comes to the counter-intuitive conclusion that the optimal level of capital income taxes will be higher in a world where general equilibrium effects must be taken into account than in one where "other policies" are available to ensure that the economy remains on its optimal growth path. To begin with, King shows that the efficiency condition (2.2) is a special case of a more general condition

$$t_1 = \left(\frac{1+i}{i} \right) \left(\frac{t_w}{1-t_w} \right) \left(\frac{\xi_{L_2} - \xi_{C_1 l}}{\xi_{L_h} - \xi_{C_1 h}} \right) + \frac{i-n}{n} \left(\frac{1}{\xi_{L_h} - \xi_{C_h 1}} \right) \quad (2.3)$$

with (2.2) being obtained by assuming that debt policy can be used to maintain $i=n$. To see the significance of this, note that $i=n$ is the "golden rule" condition for optimal capital intensity in a steady state growth model where n is the rate of population growth and technological change is ignored. If $i > n$, capital intensity and income per worker are below their optimal levels. If $i < n$, the capital/labour ratio and income per worker is too high in the sense that a decline in the rate of savings would result in a decline in capital per worker and income per worker but an increase in steady state consumption per worker. When no other policies exist to set $i=n$, the optimal t_i and t_w must be chosen to minimize (subject to the revenue constraint) the welfare cost arising from the failure to achieve optimal capital intensity as well as from the labour-leisure and consumption-timing distortions. The case which is generally considered to be realistic is the inadequate savings rate case where $i > n$. Here the equilibrium condition (2.3) leads paradoxically to the finding that t_i should be higher than if the economy was on its optimal growth path ($i=n$). King provides two explanations for this result. First, income is lower in the non-optimal growth case than when $i=n$; thus higher tax rates are required to meet an unchanged government revenue requirement. Second, there is an additional reason to favour capital over labour taxes which is that, since capital taxes are paid in retirement rather than in period 0, savings must increase simply to finance period 1 taxes. King then shows that if debt policy is available (and costless) so that the government can lend or borrow at will, then the timing of tax payments ceases to be of consequence and the relevant efficiency condition reverts to (2.2).

These findings regarding the dynamic efficiency considerations relating to capital income taxation can not be simply applied in the case of a small open economy where i can be taken as fixed. In this case, the level of t_1 affects the level of domestic ownership of world capital rather than the level of capital and output in the economy.

Taking both static and dynamic efficiency arguments into consideration, King departs from Boskin, Summers and Feldstein in concluding that "it is difficult to argue strongly for either an income or an expenditure tax on efficiency grounds". He reaches this conclusion based on the lack of sound knowledge regarding the elasticities in the efficiency conditions (2.2) or (2.3).

Another efficiency consideration, obscured in the preceding models, is that preferential taxation of capital income may bias the allocation of resources away from investment in human capital, the returns on which are fully taxed.

2.4 EQUITY

In addition to its implications for economic efficiency, capital income taxation has implications for horizontal and vertical equity and for the distribution of income between generations. The principle of horizontal equity is that individuals with the same "ability to pay" should be equally taxed regardless of the source of their incomes. In the context of the Haig-Simons definition of

income as any increase in wealth occurring over the course of the taxation period (a year), this principle implies that all forms of investment income should be taxed at the same rate as is labour income. Recently, some analysts, eg. Feldstein (1978a) and Davies (1981) have criticized this formulation of the horizontal equity principle on the grounds that "ability to pay" should be defined in lifetime terms since ability to pay is a proxy for utility and since individuals act to maximize lifetime rather than annual utility. If ability to pay is defined on a lifetime basis, it is appropriately measured by either the present value of non-capital income plus gifts and inheritances or the present value of consumption expenditures plus bequests. This has two implications for equitable taxation: first, tax rates should be proportional unless complete income averaging is permitted, and second, only labour income plus gifts and inheritances, but not capital income should be taxed. Only if these two conditions are met will individuals with equal lifetime incomes, but differing patterns of income over their lifetimes, be equally taxed. Note that the assumptions upon which these conclusions depend are strong and controversial. In particular, Simons explicitly rejected the equation of ability to pay with utility. (See Hettich, 1979.)

The fact that investment income is taxed may actually have more to do with vertical equity, and monitoring and enforcement costs, than with horizontal equity. Since investment income is quite highly concentrated among those with high total incomes, its taxation contributes to the progressivity of the system. This point is reinforced by the fact that alternative taxes, especially the sales

tax, are much less progressive than the income tax. Also, taxing investment income provides a proxy for the taxation of gifts and inheritances which are largely untaxed in Canada and for which monitoring and enforcement costs are high.

To summarize, there are horizontal and vertical equity arguments, albeit contentious ones, which support the continuation of capital income taxation. In the context of a shift from capital taxation to wage or consumption taxation, an inter-generational equity question may also arise. Summers (1981) and Auerbach and Kotlikoff (1981a) have noted that shifting from one tax base to another may result in transfers to or from cohorts that are advanced in age at the time of the change. For example, if most investment income is earned by those in retirement, then the replacement of capital taxation by increases in the tax rates on either wages or consumption will produce a transfer to the retired cohorts at the expense of younger cohorts.

2.5 SUMMARY

The efficiency and equity considerations outlined above cast serious doubt upon the proposition that capital income taxation should be eliminated while leaving open the question of whether taxes on capital income should be decreased from their present levels. It does seem clear that the high and uncertain tax rates that can result from the lack of inflation indexing of the income base of capital taxes (see section 5.1) must impose efficiency costs. However,

improved indexing of the capital income tax base would appear to be a more appropriate response to this problem than a general reduction in the rate at which capital income is taxed.

The question of whether efficiency gains could be achieved by reducing tax rates on capital and raising them on consumption or labour income (even in a zero inflation environment) depends upon the interest elasticity of consumption (or savings) and also upon the own-price and cross-elasticities of consumption and leisure.

To the extent that an increase in a second-period resource endowment (such as a public pension) increases the interest-elasticity of saving, it implies a reduction in the optimal rate of capital taxation.

CHAPTER 3

PUBLIC PENSIONS, SAVINGS, AND WELFARE

In the small but rapidly growing literature on the economics of public pensions, two main questions have been addressed. The first concerns the role of public pensions, while the second concerns the effects of public pensions on private saving and the supply of labour. Although the first question is logically prior, the second has received considerably more attention in the literature, at least until quite recently. It is also interesting to note that in analyses of the interaction between public pensions and private savings, the two have usually been assumed to be perfect substitutes. In effect, public pensions have been characterized as playing no economic role. In this review of the literature, possible rationales for the existence of public pensions are considered first; their effect on savings and labour supply is then briefly examined.

3.1 THE ROLE OF PUBLIC PENSIONS

Some discussions of the role of public pensions focus on earnings-related public pensions such as the Canada/Quebec Pension Plans (C/QPP) or the U.S. Social Security program. Here, a broader notion of public pensions is considered which encompasses as well flat-rate demogrant programs, like Canada's Old Age Security program (OAS), and income-tested retirement benefits like the Guaranteed Income Supplement program (GIS). Obviously, a successful explanation

for the existence of public pensions must also explain the existence of different types of public pension program. Four main roles for public pensions that have been suggested in the literature are considered below under the headings: (a) income redistribution, (b) forced savings, (c) gains from intergenerational transfers, and (d) incomplete private insurance markets.

3.1.a Income Redistribution

The simplest rationale for public pensions is to provide welfare gains through income transfers to lower-income, retired individuals in the community. This model of public pensions could be based upon utility interdependence among the individuals alive at any point in time. Alternately, it could be based on the existence of external costs of poverty without the existence of utility interdependence. The model need not involve any notion of public pensions as a continuing series of intergenerational transfers.

This simple model is consistent with the existence of income-tested transfers such as GIS. It is less satisfactory as an explanation of a universal demogrant for the elderly such as OAS. However, if OAS, for example, were eliminated and GIS increased to maintain the same minimum income guarantee, the work and savings disincentives associated with GIS would be increased because an increase would be required in either the GIS tax-back rate or the income range over which the tax-back rate applies. With this added consideration,

income redistribution may supply a role for both income-tested and demogrant pensions. On the other hand, intra-cohort income redistribution provides no motivation for earnings-related pension plans like the C/QPP.

3.1.b Forced Saving

Given the existence of redistributive pensions, Buchanan (1968) and Browning (1973, 1975) see a role for a contributory, C/QPP-type program in coercing all those individuals who can do so to make reasonable provision for their own income support during retirement. The collectivity of individuals has an interest in reducing the moral hazard costs attending the provisions of income-tested retirement pensions. The moral hazard costs of an income-tested transfer program are the increased transfer costs and lost output associated with reductions in saving and labour supply which result from the high marginal tax rates implicit in the income-testing of benefits under the transfer program.

The fact that the high tax rates embodied in an income-tested public pension only extend to a certain level of retirement income (see the analysis of section 6.2.c), suggests that a compulsory, contributory public pension plan need only provide a base level of benefits in order to eliminate most of the moral hazard costs associated with redistributive public pension plans. This is an important point to Browning particularly as he emphasizes the potential welfare costs of public pensions. Browning argues that a public pension can reduce individual welfare levels through the restrictions it imposes on the

level and pattern of household saving. The importance of this argument is weakened by the fact that most individuals save in other ways than by contributing to public pension plans (e.g. by paying off the mortgage on a home) and so can alter the level and timing of their saving to some extent even with an inflexible schedule of social security contributions.

A forced saving role for public pensions is sometimes based on the premise that individuals are myopic and systematically undersave. For example, Diamond (1977) suggests that public pensions may be motivated by paternalism on the part of government and provides evidence that a sizeable fraction (30%) of the three middle-income quintiles of the United States population would have "irrationally low" wealth levels at age 60 in the absence of Social Security. This rationale for public pensions is unappealing since it either rejects the premise of rational utility maximization upon which most economic analysis is based or it raises and leaves unanswered the question of why individuals should systematically save at "irrationally low" rates. The Buchanan and Browning model provides at least a partial answer to the question.

An important question concerning forced saving as a rationale for public pensions is why the goal is not sought simply by legislating a requirement for all individuals to purchase adequate pension coverage from private firms. (Indeed, arrangements exist in the U.K. for individuals to opt out of a tier of the public pension system upon proof of equivalent employer-sponsored pension benefits.) Buchanan finds the answer to this in the argument outlined below that welfare gains can be achieved through a system of intergenerational transfers.

3.1.c Gains from Intergenerational Transfers

Another possible role for public pensions, which has received considerable attention in the literature, derives from the intergenerational transfers implicit in a public pension which is financed on a pay-as-you-go (PAYGO) basis rather than on a fully funded basis. With PAYGO financing, public pension benefits in any year are financed out of general governmental revenues which come predominantly from taxes on the incomes of pre-retirement age cohorts. Thus an intergenerational income transfer is involved. With a fully funded plan on the other hand, a portion of the taxes paid by the working cohorts goes into a fund which is later used to pay their own pensions, so no intergenerational transfer occurs. Extending a pioneering analysis by Samuelson (1958), Aaron (1966) showed that, with steady population growth at rate n and real wage growth at rate g , the welfare of all generations in a population can be increased by the institution of a sequence of intergenerational transfers so long as $n+g$ exceeds i , the real rate of interest. His result is based simply on the fact, when $n+g$ exceeds i , a given tax rate will yield higher benefits to a representative taxpayer under a PAYGO system where all tax proceeds are distributed to retired cohorts than if the tax proceeds are invested at i and the accumulated capital returned to the taxpayer as a retirement pension. Thus, this finding provides a role for public pensions insofar as they are financed on a PAYGO basis.

A number of comments and criticisms have been made in respect of this finding which greatly weaken its appeal as a rationale for public pensions. Feldstein (1974b) rejected the relevance of the result by asserting that $i > n + g$ is the realistic case. Based on this premise, he argued that welfare would be increased not only by fully funding public pension benefits but by raising (U.S.) Social Security taxes even further to accumulate an endowment fund to pay the public pensions of future generations.

Kurz (1977) and Asimakopulos (1976, 1980) note that Aaron's result is simply an application of the golden rule of accumulation introduced in Chapter 2. If the economy is on a steady-state growth path with $i < n + g$, then the rate of savings is higher than the optimum. A PAYGO public pension is not necessary in order to achieve the available welfare gain; any reduction in the level of savings will do.

Kurz and Asimakopulos also point out that, whereas the excessive savings rate associated with $i < n + g$ is inefficient in that everyone can be made better off by an increase in consumption levels, the less-than-optimal saving rate associated with $i > n + g$ is not inefficient. Present day consumers would be required to sacrifice consumption and utility to improve the welfare of future consumers who would benefit from the higher steady state capital/labour ratio. Thus, the movement to a long-run optimum saving rate would not be a Pareto improvement. While this observation attacks the welfare basis

of Feldstein's endowment fund prescription, it also lends credence to his assertion that $i > n + g$, for the Pareto improvement available if $i < n + g$ should make any equilibrium in that range unstable.

An equally important attack on the Samuelson-Aaron result was made by Lerner in his criticism of Samuelson's original formulation of the proposition (1959 and "Rejoinder" to Samuelson's "Reply"). Samuelson analyzed a polar case of Aaron's model in which capital is not storable between periods so $i = 0$. Lerner objected to Samuelson's optimal allocation on the grounds that it has the characteristics of a chain letter. A decline in the rate of population growth (or wage growth) would result in a situation where the sequence of transfers could not be maintained, with the result that some generations of consumers would have sacrificed consumption when young for little or no return when retired. He asserted that, since future conditions are essentially unknowable, the only legitimate social contract is one between the individuals alive in a given period without reference to future periods. He would have his government maximize not the representative individual's multi-period utility function, eg. $U_i(C_{1i}, C_{2i}, C_{3i})$, but the social welfare function $W_t(C_{w1}, C_{w2}, C_r)$ where the arguments are the consumption levels of all generations alive in period t . That successive governments will choose compromise positions between the conservative Lerner optimum and the risky Samuelson optimum which promises current generations greater welfare so long as future generations can and will maintain the sequence of intergenerational transfers, and that these compromise positions will

shift according to forecasts of future economic and demographic trends, is a central theme of the later analysis of public pensions provided by Asimakopulos and Weldon (1968, 1970, 1976, 1980).

Together, these three criticisms make the Samuelson-Aaron result inadequate to explain the existence of public pensions as a permanent institution.

3.1.d Incomplete Private Insurance Markets

The final role which has been suggested for public pensions is to provide a substitute for insurance services which private markets are unable to supply. This possible role was foreshadowed, in Samuelson's 1958 analysis, by the substitution of intergenerational transfers for absent capital markets; however, it was really only developed in Diamond (1977) where uncertainty was first introduced into the analysis. Diamond considered that individuals might wish to insure themselves against uncertainties relating to: (i) the rate of return on saving, (ii) longevity, and (iii) contingencies such as ill health or unemployment which might shorten their working lives. Uncomfortably low retirement incomes could be produced by unanticipated outcomes in each of these areas. To establish a role for public pensions in providing insurance against risks such as these, two propositions must be demonstrated. The first is that private markets for insurance services in these areas will be less than complete. The second is that collective provision of insurance through public pensions will provide a net welfare gain after taking into account

the welfare costs associated with the public pensions. These would include the costs associated with compulsory purchase of collectively determined levels of retirement income insurance and the moral hazard costs generated as individuals alter their consumption behaviour (eg. plan to retire earlier) in response to changed prices and endowments. Diamond provided an informal analysis of the first proposition but did not address the other in his 1977 article. Moreover, with the exception of the analysis of government provision of longevity insurance by Eckstein, Eichenbaum and Peled (1983), no further advances have been made in elaborating and testing the propositions.

With respect to the uncertainty affecting the rate of return on retirement saving, Diamond judged that there is an absence of safe investment opportunities which could guarantee a positive real rate of return. Privately offered instruments which pool the risks associated with individual assets do not appear to provide a safe real return since mutual funds, for example, are subject to large value changes in a short time. The main lack would appear to be a vehicle for pooling widespread risks, particularly the risk of unanticipated inflation, across different asset holding periods. Diamond suggests that, given the fact that wide variation in inflation rates has been a recent experience, the absence of a private sector vehicle for insuring this risk may only indicate a lag in its development. On the other hand, Friedman and others have noted that governments receive windfall gains from rising inflation, in the form of reductions in the real value of government debt, and also that governments have the power to create inflation. In this situation, it may be unrealistic

to conclude that private markets will ever be able to provide instruments with guaranteed positive real rates of return. One suggested solution to this problem is for the government to issue "index bonds" which pay a rate of return specified in real terms. This suggestion is in line with the proposals by Buchanan and Browning which involve the correction of externalities in ways which involve the minimum degree of intervention in private markets and household decision making. Here the question is whether government provision of such an instrument could substitute for the presumably more cumbersome and costly superstructure of a Social Security plan. The evidence of capital market failures not related to unexpected inflation suggests that index bonds may not provide the whole answer to inadequate retirement income levels. In addition, experience with index bonds in several other countries shows that their issue can have undesirable consequences particularly for open economies which are price takers in export markets (see van Tonnigen et al, Appendix 12, Lazar Report, 1980).

The second market failure suggested by Diamond concerns the provision of fair, real annuities. In addition to sharing the problems associated with insuring the risk of unanticipated inflation, it appears that the annuities market is limited by adverse selection as well. Adverse selection problems in the annuities market derive from the situation in which potential purchasers of annuities have more information about their life expectancies than potential sellers can obtain (at a cost which would make the transaction worthwhile). In this situation, annuity prices based on total population mortality

rates will tend to be attractive to those with above-average life expectancies and unattractive to others. Only those with above-average life expectancies will purchase annuities and, assuming competition in supply, losses by suppliers will drive prices up to levels where annuity purchase is only attractive to a relatively small number of (high life-expectancy) individuals in the population. Evidence that this situation is a realistic one is given by the observation that insurance companies quote standard annuity prices unvaried by factors other than age and sex despite evidence of systematic variation in life expectancies as represented by the finding of Wigle and Mao (1980) that life expectancies are strongly correlated with income. They found, for example, a six year difference in the life expectancies of males in high-income and low-income urban census tracts. Further evidence of potential adverse selection problems is provided by the fact that nearly all private sector annuities take the form of employer-sponsored pension plans in which membership is often a condition of employment. Insurance which is compulsory for members of a group is a logical response to the problem of adverse selection. Documentation of the extent of private sector annuity coverage, through group and individual plans, is provided in section 7.8 of Chapter 7.

Recently, Eckstein, Eichenbaum and Peled (1983) have applied the earlier models of Rothschild and Stiglitz (1976) and Wilson (1977) to provide a more formal theoretical analysis of the adverse selection problem in the private annuities market. They have also examined the operation of the private annuities market in the presence

of a universal public pension. Their analysis assumes perfect asymmetry of information: individuals know their survival probabilities while annuity sellers know the distribution of survival probabilities without knowing where any potential annuity purchaser is in the distribution. Annuity sellers operate in a competitive market but one in which all contracts specify both price and quantity and no seller will provide a consumer with a second contract. Equilibrium allocations are defined where no contracts make expected losses and all profitable contracts are exploited. In this model, all possible equilibrium allocations involve less than full longevity insurance for some or all of the population. When the operation of the market is examined in the presence of a public pension, an interesting result is obtained. The nature of the equilibrium allocation is the same as when no public pension exists, but the shortfall from complete insurance provision is reduced. The public pension does not merely substitute for private annuity contracts but actually makes the private market more efficient by reducing the scale of the adverse selection problem. In rough terms, the existence of the public pension permits sellers to identify customers demanding additional longevity insurance as being predominantly high-life expectancy individuals. A correctly set public pension provides an unambiguous welfare gain in this model.

The third and probably most serious failure of private capital markets concerns the lack of adequate insurance against the risk of a shorter than planned working life due to ill health or unemployment. As Diamond notes, this contingency can have a dramatic

than expected retirement reduces income and savings (in the years just prior to normal retirement when households tend to exhibit their highest savings rates) and simultaneously increases the expected number of years of retirement income which must be provided out of accumulated savings. The failure of private markets to provide anything like full insurance against this risk can be explained by the serious moral hazard and adverse selection problems its provision would encounter. The moral hazard problem arises because full income insurance would provide many individuals with a strong incentive to exaggerate problems of ill health or unemployability in order to retire early. Since the conditions of ill health and "inability to obtain suitable employment" are difficult and costly to monitor, the moral hazard problem can not be easily overcome.¹ Moreover, since those who have the greatest doubts concerning their future employability have the greatest incentive to purchase earnings insurance, an adverse selection process tends to drive up the price of such insurance in the manner detailed for the case of annuities. The responses found to moral hazard problems include monitoring arrangements, up to the point where they become too costly, and co-insurance (i.e. partial insurance). The latter is seen in private pension plans which provide actuarially-reduced or slightly better than actuarially-reduced pension

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1. In much of the literature on Social Security, retirement age is treated as purely a "choice" variable, and the uncertainty associated with it is ignored. In this connection a finding reported in Kurz (1981) is of some interest. The longitudinal "Retirement History Survey" in the U.S. permitted a comparison of the expected date of retirement of men aged 58 to 63 in 1969 and the actual age at which they later declared themselves as "retired". The correlation between these two variables was

benefits to those who retire prior to the normal retirement age. As noted above, compulsory insurance, including that provided by a universal public pension, has the potential to provide welfare gains by reducing the impact of adverse selection problems.

The findings of this section may be summarized as follows: Income distribution ~~effects~~ can provide an explanation for income-tested pensions such as GIS and, to a lesser extent, for taxable demogrant's such as OAS. The effect of a forced savings plan in offsetting the disincentive effects of income-tested pensions can provide a role for a base level public pension plan, such as C/QPP, which is not redistributive in nature. The possible gains from intergenerational transfers when earnings growth exceeds the interest rate were not found to provide convincing rationale for the institution of a PAYGO public pension plan. Finally, potential welfare gains resulting from the substitution for, or complementing of, imperfect private insurance markets were identified. Based on the preliminary analysis of these potential gains that has been accomplished so far, their existence seems probable enough to warrant taking them seriously. Thus analyses of the welfare costs or optimal size of public pensions which ignore them should be considered to be seriously flawed. On the other hand, the existence and scope of a welfare-increasing insurance role for public pensions must also depend on the welfare costs of these plans which are manifest in the distortions they create in the patterns of consumption, savings, and labour supply over individuals' lifetimes. These latter effects are considered in the following section.

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3.2 THE EFFECT OF PUBLIC PENSIONS ON PRIVATE SAVINGS AND LABOUR SUPPLY

A lively discussion of the effect of unfunded public pensions on savings, labour supply, and output has occurred in the literature following a provocative article by Feldstein (1974a). In that and succeeding articles (1974b, 1976a, 1976b, 1977, 1978b, 1979a, 1979b, 1982; Feldstein and Pellechio, 1978), Feldstein has argued that the U.S. Social Security system has strongly negative effects on savings, capital formation and output. He has based his argument on a lifecycle model of household consumption and savings in which anticipated Social Security benefits are counted by consumers as a component of their wealth and so substitute for the assets obtained through private saving. Since future Social Security benefits in a PAYGO system will be financed by future taxes, no public investment in real assets replaces the foregone private capital accumulation. The real wealth resulting from private saving is thus replaced by "fiat" wealth. The result is a permanent decline in the level of savings and the corresponding levels of capital and national income. In 1974a, Feldstein estimated that, in the absence of Social Security, private savings and the level of capital in the U.S. would be 60% higher, and national income 15% higher. In his savings displacement estimates Feldstein used either a gross or net "Social Security Wealth" variable. These differed according to whether anticipated future Social Security payroll taxes were deducted in estimating household wealth.

Feldstein has also pointed out that the "replacement effect" of Social Security on household saving may be disguised by an offsetting "retirement effect". Families will allocate part of their expanded household wealth to increased leisure, generally in the form of earlier retirement, and this may result in increased personal savings rates over the now shorter span of working years. The strength of this retirement effect was exaggerated (at the time of Feldstein's writing) by the fact that U.S. Social Security benefits were earnings-tested. This restriction on the receipt of retirement benefits has since been virtually eliminated. Feldstein has also suggested that accumulation to finance induced earlier retirement may provide a more orthodox explanation than Katona's "goal gradient hypothesis" or Cagan's "recognition effect" of their findings that the rates of discretionary saving by private pension contributors were higher than those of non-contributors indicating that private pensions did not substitute for discretionary saving (Katona, 1964; Cagan, 1965).

In Feldstein's initial paper he made no precise statements regarding the consequences of Social Security for economic efficiency or social welfare, although an implication of reduced welfare is clear from his emphatically stated conclusions regarding the resulting reduction in real income levels. Nor did he give any consideration to the economic rationale for the program. There are several points to consider regarding the welfare implications of the Feldstein savings and labour supply effects. To begin with, one should note

that no changes are claimed to occur in the rate of return on saving or the price of labour; thus no distortion of lifetime consumption or labour supply patterns is claimed. Welfare losses arise in Feldstein's model because a spurious income effect leads to a reduction in capital accumulation when the capital/labour ratio is at or below its golden rule steady state level. In 1974b, Feldstein made explicit this steady state utility foundation of his argument and its relationship to the Samuelson-Aaron rule for PAYGO public pensions. The fact that an increase in the savings rate to bring it closer to the golden rule level makes some cohorts worse off and so is not a Pareto improvement was noted in the previous section.

A major criticism of Feldstein's model has come from Barro (1974), who proposed an alternative model of intergenerational transfers in which an increase in unfunded Social Security wealth has no effect on private consumption or investment. The basis of Barro's model is as follows: the existence of intergenerational transfers, from young to old in the form of support for retired parents by their children and from old to young in the form of bequests, implies that the welfare levels of individuals in different generations are arguments in each other's utility functions. Moreover, this utility interdependence extends indefinitely into time. If generation two values the welfare of generation three, and generation one values the welfare of generation two, then generation one also values the welfare of generation three. The implication of this model is that where private intergenerational

transfers are "operational", an externally imposed transfer -- e.g. an increase in unfunded pension benefits which implies a future increase in tax rates on workers -- will be fully offset by an adjustment in the level of the private transfers. For example, a pensioner who has selected a positive bequest already has the option of shifting resources from his heirs to himself; he should thus be expected to react to an increased unfunded pension which will burden his heirs with increased taxes by increasing the level of his bequest.

To the extent that this model is borne out in reality, it has two important implications for public pension questions. The first and most obvious is that attempts to raise retirement consumption levels by raising public pensions will be offset either by reduced family support of pensioners or by increased bequests. According to this view, the growth of public pensions can be seen largely as a "formalization" of the support of the elderly via the public sector. The second implication of the model is that, as adjustments to private transfers offset the transfers implicit in unfunded public pensions (or public debt in general), the "fiat wealth" of unfunded pension benefits has no effect on consumption and savings levels.

The welfare implications of fully-funded versus PAYGO public pensions when generations are linked by inter-dependent utility functions (ie. the Barro model) have also been investigated by Sheshinski and Weiss (1981) under the assumption that the public

pensions provide annuity services unavailable through private markets. While the public pensions provide clear welfare gains in this model, the extent of these gains is unaffected by the funding choice.

Barro's model has been critized (e.g. by Feldstein, 1976a) on the grounds that "the subtlety of the required anticipations (of future tax burdens) makes it unlikely that households do respond as Barro suggests". It must also be assumed that some pensioners are not linked to the younger generation through operational transfers. Thus the extent to which Barro effects on private intergenerational transfers offset public intergenerational transfers, and thus reduce the effects of public pensions on saving, is a matter for empirical testing.

Another criticism of Feldstein's analysis is that, in paying no attention to the possible role of public pensions, it implicitly assumes that public pensions are perfectly substitutable for private saving. It is obvious, of course, that they are not. For example, public pension rights can not be used to support consumption in the event of pre-retirement contingencies (eg. unemployment) whereas private savings often can. The implications of the imperfect substitutability of public and private savings are not clear, however. When public pensions provide longevity insurance services for instance, a public pension dollar may be capable of replacing more than a dollar of private saving. This situation is modeled in sections 7.8 and 7.9. In contrast, the Eckstein, Eichenbaum and Peled analysis of

public pensions and private annuities provides examples where the provision of public pensions and the consequent diminution of adverse selection problems can actually lead to an increase in private saving.

Thus, while serious concerns have been raised by Feldstein and others about the possible effects of public pensions on saving and labour supply, subsequent analyses have tended to allay these concerns. They have showed that the Feldstein model is based on extreme assumptions regarding utility non-interdependence and the substitutability of public pensions for private saving. Feldstein's analysis also provides a poor basis for making judgements about the welfare consequences of public pensions since his model admits no role for public pensions and therefore no associated welfare gains. In this context, it should be stressed that the observation of trends to earlier retirement and increased pre-retirement consumption in response to the introduction of public pensions does not permit one to infer welfare costs; these trends would also be expected to occur in response to any efficiency gains generated by the introduction of the public pension plans.

Notwithstanding these criticisms and qualifications of Feldstein's analysis, there remains the possibility that savings could be depressed to some degree by the introduction or expansion of unfunded public pensions and that this could impose welfare costs on future generations.

CHAPTER 4

EMPIRICAL ESTIMATES

Whether or not unfunded public pensions lead to a significant decline in national savings is "perhaps the most widely debated policy question in economics today". (Auerbach and Kotlikoff, 1981b). The argument of the previous two chapters suggests that two other empirical questions are also highly relevant to the policy issues raised. They are the interest elasticity of saving and the level of risk aversion exhibited by consumers. The first of these is key to the issue of the optimal level of capital taxation while the second is important in determining the costliness of uncertainty to consumers and the consequent demand for the insurance services provided by public and private pensions. As shown in Chapter 6, it is also an important determinant of the interest elasticity of savings. The empirical findings to date on the savings displacement effect and the two parameters are briefly outlined below.

4.1 SAVINGS DISPLACEMENT BY PUBLIC PENSIONS

The controversy regarding the effect of unfunded public pensions on national savings and long-run income levels was set off by Feldstein (1974a). Feldstein based his calculations on the time series equation

$$\hat{C} = 228 + .530 YD + .120 YD_{-1} + .356 RE + .014 W_{-1} + .021 SSW \quad (4.1)$$

where C is current period consumption, YD is disposable income, RE is corporate retained income (a proxy for capital gains), W is household wealth, and SSW is the present value of pension promises under the U.S. Social Security program. From this equation, Feldstein estimated that, by 1971, U.S. Social Security had reduced aggregate personal saving by 50% and thus reduced total private saving and the private capital stock by 38%. Based on a Cobb-Douglas production function, Feldstein found this to imply a reduction of 13% in GNP in the long-run.

Munnell (1974) estimated a similar equation but included a variable to measure explicitly the earlier retirement induced by the social security wealth (which was predicted but not estimated by Feldstein). Her results were consistent with Feldstein's model in finding a significant effect of social security wealth on retirement duration which offset part of a gross wealth effect of SSW on saving.

In the years following 1974, savings displacement effects were repeatedly estimated by Feldstein and others, not only in time-series form but in household cross-section and cross-national models. Much of this work is summarized in Feldstein (1979a and 1979b). The empirical estimates, particularly in time series, were also criticized by Upton (1975), Esposito (1978), and Barro (1978). The critiques noted that removing the pre-WWII years or adding an unemployment variable to the time-series equations reduced the level and significance of the SSW coefficient. Thus, SSW may have been acting as a dummy variable for pre- and post-war differences in consumption patterns. Barro also added a variable representing the size of the government

surplus to reflect his hypothesis that private saving would respond to changes in the level of burden on future generations implied by changes in the levels of government debt. Barro found that the inclusion of his government surplus variable reduced the coefficient on SSW almost to zero.

Canadian estimates, such as Boyle and Murray (1979), Wrage (1980), and Daley and Wrage (1981) have tended to reject the hypothesis of savings displacement although the latter authors found some effect on savings associated with the OAS program.

The savings displacement debate took another turn when Leimer and Lesnoy (1982) attempted to replicate Feldstein's original result and found a programming error which produced a large (up to 37%) progressive overstatement in SSW. Estimating Feldstein's equations with corrected data produced an insignificant positive coefficient on SSW for the 1930-1974 sample period and a significant negative coefficient for the post war period.

After all this empirical work, the importance of a savings displacement rate remains highly uncertain. Moreover, doubt has been cast upon the ability of standard time-series or cross-section regression methods to distinguish between competing hypotheses (eg. the Feldstein and Barro models). For example, Auerbach and Kotlikoff (1981b) use synthetically produced observations to demonstrate that, even if a strong savings displacement effect exists, regression

coefficients are likely to be unstable and may not indicate that the effect exists. Some support for this finding is found in the cross-section estimates presented in Chapter 8.

4.2 THE INTEREST ELASTICITY OF SAVINGS

In the estimation of Keynesian consumption functions in the decades following WWII, the interest rate was often ignored as an explanatory variable. When included, usually in pre-tax nominal form, its coefficient was often found to have the wrong sign (ie. positive) or to be insignificantly different from zero. One exception to this was a study by Wright (1969) which employed a post-tax interest rate and calculated an interest elasticity of savings of 0.2.

The importance of interest elasticities in optimal taxation results led to a renewed interest in this parameter, and, in 1978, Boskin produced new estimates which depart from most earlier estimates and are now widely quoted and applied. Boskin estimated the equation

$$\ln \hat{C} = -38 + .56 \ln YD + .18 \ln YD_{-1} + .28 \ln W_{-1} - .003 \ln U - 1.07R \quad (4.2)$$

where C is per capita private consumption, YD and W are disposable income and household wealth, U is the unemployment rate, and R is the real, post-tax, average return on capital. This produced an estimate of the interest elasticity of saving of about 0.25, slightly higher than that obtained by Wright. To remove simultaneity bias, Boskin then re-estimated equation (4.2) with the values of YD , W , and R

replaced by values estimated using the principal components of exogenous variables from a macro-economic model. This approach resulted in an increase in the estimated elasticity to 0.4.

The idea that the interest elasticity is of substantial size has become even more widespread in the wake of an analysis by Summers (1981). Summers obtained savings elasticity values from synthetic data generated by a stylized lifecycle model similar in some respects to the multi-period model developed in this thesis. Based on different assumed interest rates and levels of consumer risk aversion, he obtained values of up to 3.7 for the interest elasticity of saving. Apart from assuming certain longevity, his model differs from the model of this paper by embodying a general equilibrium solution for consumption, savings and the interest rate in the context of steady state growth. Thus, it is not clear how his generated values should compare with other estimates.

The model developed in Chapter 7 of this thesis is used to estimate an aggregate interest elasticity of savings for Canadian households which can be compared to these other recent estimates.

4.3 THE COEFFICIENT OF RELATIVE RISK AVERSION

The risk aversion parameter (which is v in the notation of this thesis) characterizes a dimension of consumer tastes which is critical to the argument of this thesis. As detailed in Chapter 6, the iso-elastic utility function adopted here (and widely throughout

the recent economic literature) takes the form $U_t = (1/1-v) C_t^{1-v}$ for $v > 1$ and $U_t = \ln C_t$ for $v = 1$. Minus v is the elasticity of marginal utility, and v can also be shown to be Pratt's (1964) coefficient of relative risk aversion and the inverse of the intertemporal elasticity of substitution in consumption. In other terms, v can be seen to measure the degree of concavity or curvature of the utility function and also to indicate how sensitive the allocation of consumption between periods is to the interest rate.

In analyses employing the iso-elastic utility function, economists have often assumed $v \leq 1$ or, for mathematical convenience, $v = 1$ (for example, Tobin, (1967)). Summers (1981) refers to $v=1$ as a plausible value. Results will be obtained in Chapter 7, however, which depend critically on whether $v > 1$. As outlined below, the values obtained for v in the empirical literature have varied quite widely but have been consistently greater than one.

Given the several implications of the utility function parameter, it is not surprising that it has been estimated in a number of contexts. The earliest attempt to estimate v appears to have been by Fisher (1927). Maital (1973) documents three more recent estimates by Fellner (1967), Powell, Moa and Wilson (1968), and Mera (1969) each of which found v to be 1.5. Fellner's estimate followed the lead of Fisher in determining v by manipulating the first order conditions of a consumer demand problem and assuming that cross-elasticities were zero. His estimate was derived from a cross-section estimate of food expenditures in the U.S. Powell et al

obtained v in a similar context and arrived at its value by estimating a linear expenditure system with time-series data. Mera obtained his estimate in quite a different context by assuming that income tax progressivity embodies an equal absolute utility sacrifice principle (Musgrave and Musgrave, 1973, p. 199) and thus estimating v from the schedules of average tax rates applying in the U.S. in several different years.

Estimates obtained in the 1970's have suggested even higher values for v . Friend and Blume (1975) focused on the risk aversion interpretation of v and estimated the coefficient using data on individual portfolios of risky and non-risky assets. They concluded that v was likely to be greater than 2. Farber (1975) also focused on risk aversion in studying the choices made in collective bargaining in the coal industry. He obtained estimates of 2.5 and 3.7. Finally, Ghez and Becker (1975) estimated v using a model of the allocation of consumption and leisure over the lifecycle. Their estimate implied $v \geq 3.6$.

The findings on these empirical questions may be summarized by noting that little consensus has developed on the savings displacement issue while recent estimates of both the interest elasticity of savings and the coefficient of relative risk aversion have tended to provide higher values for these parameters than was earlier assumed.

CHAPTER 5

DESCRIPTION OF TAX-TRANSFER PROVISIONS
WHICH AFFECT HOUSEHOLD SAVINGS

For the purposes of the theoretical analysis of household savings, the tax-transfer system is often represented as having two features: a single tax rate applying to capital income and the right to some level of public pension income. The tax-transfer system as it affects household savings is, of course, rather more complicated than this. This chapter provides a description of the tax rates, income tax deductions and public pension programs which directly affect household saving in Canada.

5.1 TAX RATES

— The simple analytical model with a single tax rate on income from capital is immediately complicated in at least four ways. First, different forms of investment income are taxed at different rates. Second, the progressive structure of the income tax system means that marginal tax rates rise with the level of taxable income, even in the absence of deductions. Third, corporate-source income is taxed twice: once as corporate profits and again as the personal income of shareholders. This aspect of the tax system is considered beyond the scope of this study so no further consideration is given to the effects of possible changes in corporate tax provisions.

Fourth, the effective tax rate on investment income, and thus the post-tax real rate of return on saving, is affected by a change in the rate of inflation. These points are elaborated below.

Four main forms of investment income should be considered: interest income, dividend income, taxable capital gains, and income arising from homeownership. Another quantitatively important form of household investment income is income from equity in farms, professional practices and unincorporated businesses. This income is concentrated in relatively few hands, however, and the wealth stocks and income flows are subject to particularly serious problems of measurement and classification. For these reasons, household saving in this form is not considered here.

Interest income can arise from deposits in banks, credit unions, etc., from holdings of bonds, mortgages, and Guaranteed Investment Certificates, and from private loans. Along with most forms of dividend income and taxable capital gains, interest income has been eligible, since 1974, for the \$1000 Interest and Dividend deduction as detailed in the next section. Interest income in excess of this deduction is taxable at the same rates as apply to wage and salary income. These tax rates are applied to the nominal amount of investment income without allowance for any loss in the value of the invested capital due to inflation; as a result, the effective marginal tax rate rises with the rate of inflation as is shown in Figure 5.1 on page 54.

Dividends from Canadian corporations are subject to "gross-up" and tax credit provisions which have the effect of lowering the marginal tax rates applying to them, particularly for taxpayers with low levels of taxable income. One purpose of this treatment of dividends is to reflect the fact of their prior taxation as corporate profits. The formula for the marginal tax rate on dividends, and which has applied since 1978, is provided below. It assumes an Ontario resident subject to provincial taxes at 46% of the Basic Federal Tax. If f is the taxpayer's federal marginal tax rate,

$$m_d = 1.46 (1.5f - .375).^1$$

The taxation of capital gains on stocks, bonds, real estate (apart from principal residence) and personal property was introduced as part of the major tax reform which took effect in 1972. Since taxable capital gains are defined as one-half of reported capital gains, the effective (nominal) marginal tax rate is one-half that on wage and salary income. As with interest and dividend income, the effective marginal tax rate on real capital gains rises with the rate of inflation. Up to certain limits, net capital losses may be

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1 A minor change in the tax treatment of dividends was proposed in the budget of November 12, 1981. The modified formula is $m_d = 1.46 (1.5f - .34)$.

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deducted from taxable income.² When flows of taxable income are related to household wealth portfolios, it will be noted that stocks give rise to a blend of capital gains and dividend income. Bonds may also yield capital gains as well as interest income.

Homeownership gives rise to capital gains income upon the sale of the home and, more immediately, to imputed rental income, i.e. the rental costs avoided by the fact of homeownership. Models of the rental vs. ownership choice, such as that outlined by Fallis (1980), suggest that the level of a family's imputed rental income can be approximated by taking the product of the mortgage interest rate and the amount of the family's equity in the home. There is no tax on capital gains or imputed rental income arising from ownership of one's principal residence.

The two graphs in Figure 5.1 illustrate the relationships between the marginal tax rates on wages and salaries and various forms of investment income. Graph (a) shows how the marginal tax rates on interest, dividends and capital gains income compare to those on wages and salaries at various levels of taxable income. In this graph a zero rate of inflation is assumed, so the tax rates on

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2. For most assets, one-half of a net capital loss may be deducted from income up to a limit of \$2000 in the current tax year. Net loss amounts in excess of \$2000 may be carried back one year and forward until fully applied. Losses on the sale of listed personal property are not deductible.

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nominal and real income are identical. Graph (b) suggests how the effective tax rate on interest income depends on the rate of inflation. Here the marginal tax rates are expressed as a percentage of real, inflation-adjusted investment income.³

FIGURE 5.1

MARGINAL TAX RATES ON VARIOUS INCOME SOURCES

3. The inflation rate is assumed to be fully embodied in the nominal rate of return. A constant real rate of return of 3% is assumed. For the case where the inflation rate is 10% and the investor's 1978 taxable income level is between \$16731 and \$21294 (combined federal and provincial marginal tax rate of 41%), the real marginal tax rate is derived as follows:

$$\text{nominal return} = (1.03)(1.10) - 1 = .133$$

$$\text{net real return} = \frac{1 + .133(1 - .41)}{1.10} - 1 = \frac{1.0785}{1.10} - 1 = -.0196$$

$$\text{real marginal tax rate} = \frac{.03 - (-.0196)}{.03} \times 100\% = 165\%$$

From these graphs, it is evident that, even with the tax preferences applicable to dividend and capital gains income, high inflation rates can result in effective tax rates that are substantially higher than corresponding tax rates on wage and salary income and, indeed, greater than 100%.

5.2 EXEMPTIONS AND DEDUCTIONS

The tax treatment of investment income set out above is substantially modified by the presence of several exemptions or deductions. Six of these are described here: (i) the Age Exemption, (ii) the Interest and Dividend Deduction, (iii) the Pension Income Deduction, (iv) the deduction for Registered Pension Plan Contributions and Registered Retirement Savings Plan Premiums (RPP/RRSP deduction), (v) the Registered Homeownership Savings Plan Deduction (RHOSP) and (vi) the deduction of interest costs on money borrowed to earn investment income.

(i) Age Exemption. This personal exemption may be claimed by any taxpayer aged 65 or over. It amounted to \$1520 in 1978 and is indexed to changes in the Consumer Price Index as are other personal exemptions and the rate structure of the income tax system.⁴ The estimated revenue cost to the federal government in 1978 was \$160 million (Department of Finance, 1980) which implies a combined federal and provincial revenue cost of about \$220 million.

4. The federal budget of June 28, 1982 retracted the extent of tax structure indexing to 6% and 5% for the tax years 1983 and 1984.

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(ii) Interest and Dividend Deduction. This deduction was introduced in 1974. One of its main purposes is to compensate, in a rough way, for the lack of full inflation indexing in the taxation of investment income. It applies to interest, dividend and capital gains income and provides a maximum deduction of \$1000. This maximum is not indexed and has remained fixed since 1974. The federal revenue cost for this deduction was estimated by the Department of Finance to be \$515 million in 1978 so the corresponding federal and provincial revenue cost would be about \$720 million. The Interest and Dividend Deduction appears to have been effective in eliminating the taxation of investment income for a majority of families. Evidence in support of this is presented in Tables 5.1 and 5.2 below. Table 5.1 provides estimates of the median level of net worth and total assets of families and unattached individuals in different income classes. From this is derived an estimate of the median level of those assets which yield taxable income, i.e. deposits, bonds, stocks, miscellaneous financial assets (mainly mortgages) and real estate other than the home.

These estimates of taxable assets are based on the assumption that the median wealth holder in each income class has the same portfolio as the average wealth holder in the class, after adjustment to remove equity in businesses, farms and professions from that portfolio. On account of the skewness of wealth holdings, the median rather than average level of wealth holdings is considered to be more representative of the position of family units in each income class.

TABLE 5.1

TAXABLE ASSET HOLDINGS BY INCOME LEVEL, 1977

<u>Income Group</u>	<u>% of Family Units</u>	<u>Median Net Wealth</u>	<u>Corresponding Total Assets</u>	<u>Corresponding Taxable Assets</u>
(\$000)				
F 3	8.2	\$ 1,065	\$ 1,159	\$ 309
3-7	17.5	5,680	6,101	2,034
7-11	14.8	9,506	10,633	3,481
11-15	14.2	14,140	17,016	5,109
15-20	16.4	25,086	31,476	8,594
20-25	12.0	33,449	43,611	10,247
25-35	11.4	49,223	59,592	17,672
35+	5.5	98,200	108,869	53,814
	100.0			

Source: Statistics Canada Cat. 13-570, The Distribution of Income and Wealth in Canada, 1977, Tables 5 and 13, and calculations by author.

Based on an assumed nominal return of ten percent, these estimates indicate that only in the top three income classes, accounting for 28.9% of the population, is the median wealth holder likely to have a sufficient level of taxable assets for the income derived from those assets to exceed \$1000. More direct information on the proportion of taxpayers with taxable incomes greater than \$1000 is provided in Table 5.2. This table shows the proportion of family units with investment income (from taxable sources) greater than \$1000 and greater than \$2000. (In a two-taxfiler family, two Interest and Dividend Deductions are available).

Table 5.2

TAXABLE INVESTMENT INCOME LEVELS BY AGE OF HEAD, 1977

Age of Head	\$1000	\$2000
25	2.9%	1.7%
25-34	6.3	2.5
35-44	13.0	6.6
45-54	18.7	10.6
55-64	25.9	16.2
65-69	34.0	22.5
70+	30.0	20.2
All	16.1	9.5

Source: Statistics Canada, 1978 Survey of Consumer Finances microdata tape, "Economic Families, 1977 Income."

For all age groups, only 16.1% of family units had investment income exceeding \$1000 and only 9.5%, investment income exceeding \$2000. Moreover, the proportion of families with investment income over \$1000 or \$2000 was highest in the 65 and over age groups where the Age Deduction is available to complement the Interest and Dividend Deduction. On the other hand, a downward bias is imparted to these proportions by the underreporting of investment income, by about 40%, in the Survey of Consumer Finances. On balance it seems unlikely that more than 20% of family units, concentrated in higher income and older age groups, earns sufficient investment income to exceed the Interest and Dividend Deduction and other deductions available to them.

(iii) Pension Income Deduction. This deduction applies to income, to a maximum of \$1000, received: a) by a taxfiler of any age as a lump sum or as annuity payments from a pension fund, or b) by a taxfiler over age 65 (or widowed spouse regardless of age), as annuity payments from an RRSP, a RRIF⁵, or a deferred profit-sharing plan or as the taxable portion of other annuities. It does not apply to income from public pensions such as Old Age Security or the Canada Pension Plan. This deduction first took effect in 1975 and is not indexed. Its 1978 revenue cost is estimated at \$95 million to the federal government or about \$130 million to the federal and provincial governments together.

(iv) RPP/RRSP Deduction. This tax provision permits the taxfiler to deduct from taxable income the sum of his (employee) contribution to a Registered Pension Plan plus the amount paid into a Registered Retirement Savings Plan up to a joint maximum. Since 1976, the maximum has been the lesser of 20% of earned income or a dollar amount, which is \$3500 for those who are eligible for Registered Pension Plans fully or partly funded by employer contributions and \$5500 for those not eligible for such plans. These maximum dollar

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5. A RRIF, or Registered Retirement Income Fund, is an alternative, non-annuity vehicle for receiving the capital value of an RRSP over a number of years.

amounts are not indexed but were raised from previous levels of \$1500/\$2500 in 1965 to 1971 and \$2500/\$4000 in 1972 to 1975. As a result of these irregular increases the maximums have varied between 18% and 32% of the average assessed income of taxpayers and are currently at the low end of this range.

The interest earned by pension funds and savings plans is also excluded from income until such time as pension benefits are received or the taxpayers' equity in the pension or savings plan is liquidated. If the accumulated equity is paid out as an annuity (or converted into one), the annuity payments are fully taxable. (In the case of annuities purchased with capital which has already been taxed when received as income, only the interest portion of the annuity is taxable). If the accumulated equity is withdrawn in a lump sum, then the full sum must be included in taxable income. Thus the RPP/RRSP deduction provides a facility for reducing lifetime tax liabilities in face of a progressive tax system by smoothing out the time profile of taxable income and thus reducing the taxpayer's lifetime average tax rate. In addition, it provides tax deferral advantages which in most cases are generous enough to permit the real return on the investment to escape taxation completely.

The federal revenue cost of the RPP/RRSP deduction is estimated to have been \$1650 million in 1978 or about \$2310 million for the federal and provincial treasuries combined. The total of gross contributions (contributions without allowance for withdrawals) to

RPP's and RRSP's has risen steadily from 1.4% of personal disposable income in 1966 to 3.3% in 1978. The personal savings rate has also risen sharply with the result that the ratio of RPP/RRSP contributions to personal savings has risen less steadily and less significantly though it is higher now than in the mid-1960's. Over the same period the RRSP share of total RPP/RRSP contributions has risen from 30% to over 50%.

The pattern of utilization of the RPP/RRSP deduction by taxfilers is set out in Table 5.3. The first half of the table shows what proportions of taxfilers in various age/income categories reported some contribution to RPPs or RRSPs in 1978. The second half shows the corresponding percentages of taxfilers which reported contributions equal to the RPP/RRSP limit (i.e. \$3500 or \$5500 or 20% of earned income).

TABLE 5.3

RPP/RRSP CONTRIBUTORS, 1978

A. Percent of Taxfilers Who Are RPP and/or RRSP Contributors:

Age	<25	25-44	45-64	65+	All
Income (\$000)					
<5	3.4	4.0	3.6	.8	3.1
5-10	18.1	25.9	28.3	13.0	23.2
10-15	35.6	47.7	58.2	26.0	48.1
15-20	40.6	62.4	68.9	47.3	62.5
20-25	40.5	64.6	75.5	56.6	67.2
25-50	33.7	74.8	84.9	59.0	78.6
50+	55.2	86.2	88.4	53.9	85.9
Total	14.2	34.4	40.0	4.7	28.4

B. Percent of Taxfilers Who Contribute to Limit of RPP/RRSP Deduction:

Age	<25	25-44	45-64	65+	All
Income (\$000)					
<5	.1	.5	1.3	.4	.4
5-10	.1	2.1	5.6	4.2	2.4
10-15	.3	1.6	7.0	8.4	3.1
15-20	.7	2.7	8.9	15.3	4.7
20-25	1.7	4.1	13.3	18.6	7.2
25-50	9.5	18.2	31.6	35.8	24.3
50+	20.7	61.5	64.5	41.9	62.2
Total	.2	2.8	8.0	1.8	3.5

Source: Revenue Canada, Taxation. Special tabulation from the Tax Model (1% sample of tax returns of individuals).

Overall, 28.4% of taxfilers reported RPP/RRSP contributions. (The split between RPP's and RRSP's was: RPP only, 17.4%; RRSP only, 6.6%; both, 4.4%). The proportion contributing was substantially higher (over two-thirds) for taxfilers of age 25-64 and income over \$15,000.

The proportion of taxfilers contributing to the RPP/RRSP limit was only 3.5%. Only in the highest income groups (\$25,000+) and older age groups (45+) were proportions as high as 30% found. Among contributors, only 12.2% contributed to the limit, and among contributors with incomes over \$20,000, the proportion contributing to the limit was still only 19.5%. Thus the limit to the RPP/RRSP deduction was binding only for a small proportion of the highest income taxfilers.

(v) RHOSP Deduction. This provision, introduced in 1974, permits taxpayers to escape taxation on contributions of up to \$1000 per year for a maximum of ten years if these amounts are invested in a savings plan to be used in the purchase of a home. An RHOSP contributor may not already own a home or have purchased a home with RHOSP savings in the past. The accumulated capital is not taxable upon withdrawal at the time of home purchase. In 1978, the measure had a revenue cost of \$92 million to the federal government or about \$130 million to the federal and provincial governments together..

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(vi) Deduction of Interest costs. The cost of interest on monies borrowed to finance the purchase of assets (apart from principal residences and, as a result of changes proposed in the November, 1981 federal budget, RRSPs) is deductible from taxable income. This lowers the cost of borrowing and raises the net return on the investment of borrowed capital relative to the investment of saved capital. It can also result in low tax rates on investment income for tax filers who borrow to earn capital gains or invest in tax preferred vehicles like MURBs or drilling funds.

5.3 PUBLIC PENSION PROGRAMS

When all the provincial income supplements, tax credits and property tax relief programs are included, there is a large number of government programs which provide income transfers to those over 65. In addition, there are both universal (e.g. medicare) and age-categorized (e.g. pharmacare in several provinces) programs which provide significant in-kind income transfers to those over 65. With this caveat, only the three major public pension plans will be described here: namely, Old Age Security, the Guaranteed Income Supplement (and Spouse's Allowance), and the Canada/Quebec Pension Plans.

Old Age Security (OAS). The federal OAS program provides a flat rate benefit to all individuals over 65 subject to some length-of-residency restrictions. The 1978 annual benefit level averaged \$1904 for a total cost of \$4131 million (1978/79 fiscal year). In its current demogrant form, OAS benefits were first paid in 1952.

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The age of eligibility was ~~at 70~~ but was progressively lowered to 65 between 1966 and 1970. ~~The benefit level~~ level was revised upwards irregularly between 1952 and 1968, was more regularly increased between 1968 and 1972, and has been fully indexed to the Consumer Price Index (CPI) since 1973.⁶ From 1967 to 1980 it grew at an annual average rate of 7.5%, slightly faster than the CPI rate of increase of 7.1%.

The Guaranteed Income Supplement (GIS) and Spouse's Allowance (SPA). The federal GIS program provides income supplements to individuals over 65 who have little or no income apart from OAS. The benefit reduction rate in the face of additional income is 50%. The maximum benefits in 1978 were \$1335 for a single pensioner and \$2372 for a two-pensioner family; full or partial benefits were paid to about 55% of OAS recipients; the total cost of the GIS program in that fiscal year was about \$1234 million. The GIS program was introduced in 1967, and was originally intended as a transitional program to supplement incomes while the Canada/Quebec Pension Plans were being phased in. The maximum benefit levels were adjusted upwards by 2% per annum from 1967 to 1971, increased more substantially in 1971 and 1972, and indexed to CPI growth beginning in 1972. Additional increases in benefit levels, over and above normal indexing were made in 1979

6. The budget of June 28, 1982 restricted the indexation of OAS benefits to 6% and 5% for 1983 and 1984. However, full indexation to the CPI is retained for pensioners qualifying for GIS benefits.

and 1980. Over the period 1967 to 1980, GIS maximum benefit levels have grown at a rate of 15.4% per annum, more than twice the average rate of inflation. Over the same period, the maximum benefits under the OAS and GIS programs combined grew at a rate of 12.3% per annum, considerably faster than the 7.1% average inflation rate and also faster than the 9.1% growth rate in average weekly wages and salaries. Accordingly, in the simulations and estimations presented in this thesis, minimum public pension benefits are assumed to grow as fast (in real or nominal terms) as wages.

The SPA program is an extension of the GIS program to cover individuals 60 to 64 years of age who are spouses of OAS pensioners. Its maximum benefit rate is the same as for a two-pensioner GIS. The allowance is reduced at a 75% rate in the face of other income down to the maximum single GIS benefit level, and at 25% beyond that. The total cost of the program in fiscal year 1978 was \$126 million.

The Canada/Quebec Pension Plans (C/QPP). These plans provide pensions to those over 65 according to a formula which relates benefits to contribution and earnings levels throughout the pensioner's working life. Since the inception of the plan in 1966, taxes to finance C/QPP benefits have been levied equally on employees and employers at a combined rate of 3.6% of "pensionable" earnings (i.e. earnings between defined annual floor and ceiling levels). The maximum pensionable earnings level currently stands at about 80% of the average level of wages and salaries so a large fraction of the

work force contributes at the maximum. Maximum benefits are payable to those who have contributed at the maximum rate throughout their careers, and proportionately lower pensions to those who have contributed at lower rates for all or a significant part of their careers. Retirement benefits are paid to those over 65 who have applied for them, regardless of whether or not the beneficiary continues to receive earned income. The maximum benefit level is 25% of an average of the earnings ceiling in the year of retirement and the two-preceding years. This earnings ceiling is currently increased by 12.5% per annum and will be indexed to average wage and salary levels when it catches up to that average. In the post-retirement period, pensions are indexed to the CPI. The spouse of a deceased pensioner receives a survivor's pension equal to 60% of the contributor's retirement pension. The C/QPP plans also provide disability benefits and lump sum death benefits.

The C/QPP plans had a ten year phase-in period during which partial retirement benefits were paid. This phase-in period ended in 1975. The maximum retirement pension in 1978 was \$2333 and the average payment to new beneficiaries was about one-half that amount. Total expenditures under the plans were \$1782 million. The impact of the introduction of the C/QPP plans on retirement income levels was offset to some extent by the fact that the contribution rates and benefit levels of most of the larger private pension plans were correspondingly reduced.

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CHAPTER 6

ANALYSIS OF TAX-TRANSFER PROVISIONS IN A TWO-PERIOD MODEL

In this chapter, simple two-period lifecycle models are developed and employed to provide an initial analysis of the effect of the various tax rates, deductions and public pension promises on household saving. Restricting the analysis to two periods makes it easier to model complex tax provisions. Also, slight modifications to the model permit one to focus on particular features of various tax provisions such as tax deferral, inflation neutrality, and income averaging in a progressive tax system. A straightforward analysis of the effect of pension rights on household saving is provided as well; however, consideration of the insurance aspect of public pensions is deferred to Chapter 7.

Section 6.1 develops the two-period lifecycle model which forms the basis of the analysis. In Section 6.2, this model is used to provide the basic analysis of changes in deductions, pension rights, and post-tax rates of return. Section 6.3 focuses on the RPP/RRSP deduction and examines its effects in several contexts (e.g. with and without inflation).

6.1 THE TWO-PERIOD LIFECYCLE MODEL

The lifecycle model consumption and savings is based on the concept, proposed by Fisher (1930) and developed by Modigliani and Brumberg (1954), that the main purpose of personal saving is to increase utility by converting an uneven income stream into a smoother lifetime profile of consumption. Fisher also concluded that the chosen pattern of consumption over the lifetime would reflect the consumer's opportunity to raise his level of lifetime consumption by earning interest on deferred consumption.

6.1.a Form of the Model

In the simplest, two-period form of the model the consumer maximizes a utility function

$$U = U(C_0, C_1) \quad (6.1)$$

subject to a constraint set

$$C_0 = Y_0 - Z_0 - B \quad (6.2)$$

$$C_1 = Y_1 + rB - Z_1$$

where C_0, C_1 = consumption levels in periods 0 and 1,

Z_0, Z_1 = taxes in periods 0 and 1,

r = one plus the interest rate,

B = period 0 savings in the single available asset (bonds).

1. The constraint could equivalently be written to express the identity of the present values of lifetime wealth and consumption; e.g.

$$C_0 + C_1 r^{-1} = Y_0 - Z_0 + (Y_1 - Z_1)r^{-1}.$$

Some important simplifying features of this model should be noted. First, by its omission of leisure from the utility function the model ignores all labour supply considerations including the possibility that a tax or pension policy change might result in induced retirement with associated effects on optimal savings levels. Second, the model ignores bequests; consideration for the welfare of heirs is not reflected in the utility function. Third, the marginal utility schedules for C_0 and C_1 and the elements of the constraint (r , Y_t , Z_t) are all assumed to be known with certainty. This rules out the precautionary motive for saving: that is, saving to provide self-insurance against adverse contingencies such as unforeseen medical expenses, unexpected longevity, or income shortfalls. The effects of relaxing this important assumption are examined in Chapter 7. Fourth, the "consumer" or "household" is treated as a single utility-maximizing individual. When the lifecycle model is employed to examine the savings levels of actual Canadian households in Chapter 8, this assumption is modified to recognize variations in size and composition among households. Fifth, all saving is assumed to occur via purchases of a single asset (B for bonds) in the initial formulation of the model. In section 6.3, a second asset (R for RPP or RRSP contributions) is included in the model. Additional observations regarding the effects of including other assets are provided in Appendix 6.A following this chapter. Finally, note that Z_1 is assumed to be independent of B; that is, investment income is non-taxable. This assumption is relaxed in sections 6.2 and 6.3.

To facilitate the comparison of model solutions under different tax and public pension regimes, a specific form of the utility function is adopted. Following a widespread example (Atkinson (1970), Nordhaus (1973), Blinder (1974), Levhari and Mirman (1977), and Davies (1979, 1980)), an additively separable, iso-elastic utility function is employed. It is written

$$U = \frac{C_0^{1-v}}{1-v} + a \frac{C_1^{1-v}}{1-v} \quad (6.3)$$

where a is the consumer's subjective time preference discount factor ($a \leq 1$). Others, including Tobin (1967), have studied consumption behaviour with models based on $U_t = \log C_t$, which is in fact a special case of the iso-elastic utility function in which $v = 1$.

Additive separability of the utility function means that the marginal utilities of C_0 and C_1 are independent, $\frac{\partial^2 U}{\partial C_0 \partial C_1} = 0$. Intuitively plausible phenomena, such as habit persistence, cast doubt on the realism of this assumption, but it is essential for the model to be analytically tractable.

The iso-elasticity of the utility function provides several advantages in the derivation and interpretation of analytical and empirical results. The parameter v determines the direction and degree of curvature of the utility function. The elasticity of U_t with respect to C_t is $(1-v)$; the elasticity of marginal utility is $-v$. The assumption of diminishing marginal utility implies a concave

utility function ($U_t' > 0$ and $U_t'' < 0$) and this requires $v > 0$. The degree of concavity of the utility function also defines the degree of risk aversion shown by the consumer. The more concave the utility function, the less willing a consumer would be to give up a certain level of consumption of, say, \$20,000 for an even chance of \$10,000 or \$30,000. Pratt's (1964) measure of relative risk aversion ($= C_t U_t''/U_t'$) is equal to v , a feature which will find application in Chapter 7. Another expression of the concavity of the utility function is in terms of the elasticity of the intertemporal substitution in consumption, $\sigma_{C_0 C_1}$, which is $1/v$ for this utility function.² This coefficient measures the sensitivity of the allocation of consumption between periods to the rate of interest, and it makes intuitive sense that it should be inversely related to the degree of risk aversion.

6.1.b Model Solution

The consumer maximizes utility (6.3) subject to the constraint (6.2) by choosing the level of bond purchases, B , such that

$$U_B = U_0 \frac{dC_0}{dB} + U_1 \frac{dC_1}{dB} = 0$$

2. The elasticity of intertemporal substitution in consumption can be expressed as the proportional change (in the limit) in C_0/C_1 occasioned by a proportional change in the relative price or ratio of marginal utilities of the two commodities. Thus, if $U_i = \partial U / \partial C_i$,

$$\begin{aligned} \sigma_{C_0 C_1} &= d(C_0/C_1)/(C_0/C_1) \div d(U_1/U_0)/(U_1/U_0) \\ &= (U_1/U_0)/(C_0/C_1) \div d(U_1/U_0)/d(C_0/C_1) \\ &= a(C_0/C_1)^v/(C_0/C_1) \div av(C_0/C_1)^{v-1} \\ &= 1/v \end{aligned}$$

$$\text{since } U_0 = C_0^{-v} \text{ and } U_1 = aC_1^{-v}.$$

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That is $C_0^{-v} (-1) + a C_1^{-v} (r) = 0$

or $C_1 = (ar)^{1/v} C_0$ (6.4)

Substituting this first-order condition into (6.2) provides the optimal level of C_0 . Thus,

$$\begin{aligned} C_1 &= (ar)^{1/v} C_0 = Y_1 + rB - Z_1 \\ &= Y_1 + r(Y_0 - Z_0 - C_0) - Z_1 \\ \text{and } C_0 &= \frac{(Y_0 - Z_0) + (Y_1 - Z_1)r^{-1}}{1 + (ar^{1-v})^{1/v}} \end{aligned} \quad (6.5)$$

The second order condition for the solution to provide maximum utility is

$$\frac{d^2 U}{dC_0^2} = U_{11} U_2^2 - 2 U_{12} U_1 U_2 + U_{22} U_1^2 < 0$$

or

$$\frac{d^2 U}{dC_0^2} = -v C_0^{-v-1} a^2 C_1^{-2v} + 0 - v a C_1^{-v-1} C_0^{-2v} < 0$$

which is satisfied so long as $v > 0$.

From (6.4) it is evident that, with $v > 0$, optimally chosen consumption will be greater in period 1 than period 0 as long as $ar > 1$. In other words, optimal consumption will rise over the consumer's lifetime providing that the interest rate exceeds his subjective time preference rate. For a given value of ar , the ratio C_1/C_0 depends inversely on v . The less risk averse a consumer, the more uneven a pattern of consumption he will tolerate in order to increase his total lifetime consumption.

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Another point to note is that optimal C_1/C_0 is independent of the scale of consumption. This derives from the homotheticity of the utility function. It has the implication that the existence of differing levels of lifetime resources is not sufficient by itself to generate savings rates which vary systematically with income.

A final point of interest concerning this model solution is the response of the optimal level of C_0 to a change in the interest rate. From (6.5), the elasticity of C_0 with respect to the interest factor can be shown to be

$$e_{C_0 r} = \left(\frac{v-1}{v} \right) \frac{(ar^{1-v})^{1/v}}{1+(ar^{1-v})^{1/v}} - \frac{(Y_1 - Z_1)r^{-1}}{(Y_0 - Z_0) + (Y_1 - Z_1)r^{-1}} \quad (6.6)$$

With $v < 1$, $e_{C_0 r}$ is unambiguously negative, and a rise in the interest factor depresses current consumption and stimulates saving. With $v > 1$, an interest rate rise only stimulates saving if the ratio of discounted future income, $(Y_1 - Z_1)r^{-1}$, to lifetime income is sufficiently greater than the corresponding ratio of future consumption to lifetime consumption. In other words, savings are more likely to be increased by an interest rate rise for a consumer who is a net borrower in period 0 than for one who is a net saver. The reason is that an interest rate has a positive income effect for a net saver but a negative income effect for a net borrower. The final effect on C_0 depends on the direction and strength of the income effect in relation to the substitution effect. The latter occurs because a rise in the interest rate is equivalent to a fall in the price of C_1 relative to

the price of C_0 . Thus, for a net borrower the income and substitution effects are reinforcing, and an interest rate rise must lead to increased saving. For a net saver, though, the positive income effect, which permits increased consumption in both periods, may outweigh the substitution effect and produce an increase in C_0 . Whether the income effect will more than offset the substitution effect depends on both the strength of the income effect and the consumer's degree of risk aversion; for a consumer with little risk aversion ($\nu < 1$), the substitution effect will always dominate the income effect.

6.2 ANALYSIS OF TAX AND PUBLIC PENSION CHANGES

6.2.a The Taxation of Investment Income

In the model of section 6.1, the tax functions, Z_0 and Z_1 , were left unspecified except that Z_1 was assumed to be independent of B . A more explicit tax function which also makes investment income taxable is now considered. The second condition in (6.2) is replaced by

$$C_1 = Y_1 + rB - \alpha(Y_1 + (r-1)B - A_1)^\beta \quad (6.7)$$

where the final term is period 1 taxes, Z_1 . With the bracketed expression representing taxable income (income less deductions, A_1), this functional form provides a reasonably accurate continuous form representation of Canada's progressive income tax schedule when $\beta = 1.27$.

Introducing this tax regime affects the model solution via the term dC_1/dB which changes from r to

$$\begin{aligned} \frac{dC_1}{dB} &= r - \alpha\beta (Y_1 + (r-1)B - A_1)^{\beta-1}(r-1) \\ &= r - m_1(r-1) \equiv \rho, \end{aligned} \quad (6.8)$$

where m_1 is the period 1 marginal tax rate and ρ represents the post-tax rate of return on investment. The first order condition (6.4) now becomes

$$C_1 = (\alpha\rho)^{1/\gamma} C_0 \quad (6.9)$$

and, since $\rho < r$, the consumption ratio C_1/C_0 is seen to be reduced by the taxation of investment income. An explicit general solution for C_0 , corresponding to (6.5) is not obtainable because, when B is eliminated from an expanded (6.9), the resulting expression in C_0 can not be simplified.³ Whether taxing investment income reduces saving can not be determined in the general case. As detailed in section 6.1, the effect on the level of C_0 (and savings) depends on the relative strength of opposing income and substitution effects.

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3. If a simplified tax system (which can still be progressive) is considered which has a constant marginal tax rate, m_1 , then an explicit solution for C_0 can be obtained. It differs from (6.5) only by the replacement of r with ρ .

6.2.b Exemptions and Deductions

Since the Age Exemption, the Interest and Dividend Deduction and the Pension Income Deduction reduce the taxes of those in retirement, they are often presumed to encourage greater saving prior to retirement. Application of the lifecycle savings model to this question does not provide an unambiguous result that can be generalized to individuals of different income levels and profiles. However, it is sufficient to indicate that these tax preferences, assuming they are uncompensated by other tax increases or government spending cuts, are more likely to discourage than encourage saving.

To begin with, all three tax provisions reduce the consumer's tax liabilities. Since the consumer allocates this increase in lifetime income to consumption in both periods, there is a tendency for pre-retirement (period 0) savings to decline. Whether or not this income effect is offset, or perhaps outweighed, by a substitution effect caused by the tax provisions, depends on the particular circumstances of the consumer. Three general cases may be considered. Case one: the Interest and Dividend Deduction or the Pension Income Deduction exceed the consumer's investment income so that this income is non-taxable. In this case, an increase in either of these two deductions, or in the Age Exemption, will have no substitution effect as it will not affect the net rate of return on saving. Case two: the consumer's investment income just brings him to the limit of the Interest and Dividend Deduction or the Pension Income Deduction so he faces a net return on saving at the margin of $\rho < r$. Here an increase

in either of these two deductions, but not in the Age Exemption, will raise the net rate of return on saving and so produce an increase in the optimal C_1/C_0 ratio. Case three: the consumer's investment income exceeds the Interest and Dividend deduction or the Pension Income Deduction even after an increase in their levels. Here the situation is even more complex. If the consumer's marginal tax rate were effectively constant, his net rate of return on saving would be unchanged by the deduction increases, at $\rho = r - m_1(r-1)$. However, with a progressive rate structure, the increase in any period 1 deduction (eg. the Age Exemption) could lower m_1 and so reduce ρ and produce an increase in optimal C_1/C_0 . In practice, this effect will be quite small. Thus, the substitution effects leading to an increase in saving will be generally small or zero except for those consumers with investment income levels just at the deductibility limits prior to an increase in those limits. In consequence, the income effects of deduction increases, which tend to depress savings, are likely to outweigh the pro-saving, substitution effects for most consumers.⁴ Finally, if one supposes instead that the tax reductions are financed through compensating revenue increases which leave each consumer's lifetime income level unaffected, then the result would be reversed. With all income effects equal to zero, the deductions would unambiguously increase savings, although only to a minor extent, since the induced changes in the rate of return would generally be small.

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4. However, the analysis of Appendix 6.A, where investment in several assets is considered, suggests that investment income levels close to the deductibility limits are likely to be observed for a substantial number of investors.

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6.2.c Public Pension Programs

In the context of a two-period, life-cycle savings model, the effect of two of the three basic types of public pension program on savings can be stated quite simply. For both OAS and the C/QPP programs, an increase in benefit levels (not offset by increased taxes) will result in an increase in both C_0 and C_1 and hence a decline in saving. In the case where the increase in benefits is just financed by an increase in period 0 taxes, consumption levels will not be increased but personal savings will still decline. This case may be described as a replacement of private by public saving. These two cases may be illustrated by means of a simple two-period indifference curve analysis.

FIGURE 6.1

EFFECT OF CAS OR C/QPP INCREASES ON SAVING

Part (a) of Figure 6.1 illustrates the case where benefits are increased without any increase in taxes. Both C_0 and C_1 increase to C'_0 and C'_1 and savings falls from $YD_0 - C_0$ to $YD_0 - C'_0$. Part (b) illustrates the case where the increase in benefits is fully financed by an increase in period 0 taxes. The consumer is left on his original budget line but his savings level falls from $YD_0 - C_0$ to $YD'_0 - C_0$. The main point in both of these cases is that the net rate of return on savings is unaffected by these benefit level changes; thus they have income effects but no substitution effects.⁵ Note that, while C/QPP benefits are conditioned on past earnings, they are not related to levels of private investment income and so do not affect the net rate of return on saving.

As noted earlier, this simple lifecycle analysis neglects the insurance aspect of public pensions. When this is considered in Chapter 7, a different conclusion will be reached regarding the effects of public pensions on saving.

The effect of the GIS program on saving incentives is somewhat more complicated because the program is not universal and because its expansion can alter the terms of trade between period 0 and period 1 consumption for some consumers. Figure 6.2 illustrates the effect of a GIS increase for an individual whose period 1 income is slightly in excess of the break-even prior to the increase in benefits.

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5. Any small changes in the tax rate on investment income arising from the increased benefit levels in period 1 are neglected here.

FIGURE 6.2

EFFECT OF A GIS INCREASE ON SAVING

The effect of the GIS increase is to alter the consumer's budget line from abcd to aefd. The reduced slope of bc and ef relative to ad reflects the benefit reduction rate of 50% which lowers the net rate of return for consumers who become eligible for GIS benefits. In the example portrayed, the consumer's optimal consumption pair shifts from C_0, C_1 to C'_0, C'_1 , and his savings level falls from $YD_0 - C_0$ to $YD_0 - C'_0$. The income effect of the GIS increase is reflected in the increased present value of C'_0, C'_1 . The substitution effect resulting from the reduction in the net rate of return on savings is reflected (given homothetic indifference curves) in a shift in the allocation of consumption in favour of period 0 ($C'_0/C'_1 > C_0/C_1$). Note that the aggregate effect of increased GIS benefits on the consumption choices

of beneficiaries depends on how many consumers become GIS recipients as a result of the increase; the net return on savings is unchanged for those who were already beneficiaries before the increase.

6.3 THE RPP/RRSP DEDUCTION

For savings in RPPs and RRSPs, deductibility of plan contributions results in the deferral of taxation on both principal and interest until the accumulated savings are removed from the plans for purposes of consumption. This special tax treatment is most easily modeled, following Hood (1982) and Daly (1981), by considering RPP/RRSP savings as a separate asset, R, and having the consumer maximize utility by the simultaneous choice of R and B in period 0. In the discussion which follows, several cases of such a model are analysed including cases with and without taxability of the returns on B and cases with inflation and multi-period tax deferral taken into account.

6.3.a The RPP/RRSP Deduction with Bond Interest Non-Taxable

To begin with, consider an artificially simple case where interest on B is non-taxable and the marginal tax rates, m_0 and m_1 , are fixed. Also, no limits are assumed on deductible RPP/RRSP contributions. The constraint set (6.2) is replaced by

$$\begin{aligned} C_0 &= Y_0 - Z_0 (Y_0 - R) - R - B \\ C_1 &= Y_1 + r(R + B) - Z_1 (Y_1 + rR) \end{aligned} \quad (6.10)$$

with $Z_0 = m_0$, $Z_1 = m_1$

Here $dC_0/dB = -1$ and $dC_1/dB = r$ while $dC_0/dR = -(1-m_0)$ and $dC_1/dR = r(1-m_1)$. Thus the net rates of return on investment in the two assets, dC_1/dC_0 , are $-r$ and $-r \frac{(1-m_1)}{(1-m_0)}$ for B and R respectively. Since m_1 may be expected to be less than m_0 (because of a decline in income after retirement and because of the special tax deductions which apply in retirement), this analysis suggests that all saving will be in the form of R, the asset with the higher rate of return.

The above case, while instructive, is unrealistic since it ignores the dependency of m_0 and m_1 on the level of R. An increase in R will tend to lower m_0 and raise m_1 , thus bringing the net rate of return on R closer to r . With the marginal tax rates treated as endogenous, a more realistic solution can be obtained. Here (6.10) is replaced by

$$\begin{aligned} C_0 &= Y_0 - \alpha(Y_0 - R)^\beta - R - B \\ C_1 &= Y_1 + r(R+B) - \alpha(Y_1 + rR)^\beta \end{aligned} \quad (6.11)$$

The resulting first order conditions for the optimum are as follows:

$$\begin{aligned} U_B = 0 &\Rightarrow C_0^{-v} = ar C_1^{-v} \\ C_1 &= (ar)^{1/v} C_0 \end{aligned} \quad (6.12a)$$

$$\begin{aligned} \text{and } U_R = 0 &\Rightarrow C_0^{-v} (1-m_0) = ar(1-m_1) C_1^{-v} \\ C_1 &= \left[ar \frac{(1-m_1)}{(1-m_0)} \right]^{1/v} C_0 \end{aligned} \quad (6.12b)$$

Since both these conditions must be met,

$$\frac{1-m_1}{1-m_0} = 1$$

or $m_0 = m_1$ at the optimum. (6.12)

This implies that R will be increased to the point where lifetime taxation is minimized by complete income averaging. As the net rate of return on both R and B at this point is r , a second result is that the introduction of R has no effect on C_1/C_0 or the allocation of consumption between periods. By expanding (6.12), the optimal level of R can be obtained. Thus,

$$\begin{aligned} \alpha\beta(Y_0 - R)^{\beta-1} &= \alpha\beta(Y_1 + rR)^{\beta-1} \\ \text{so } Y_0 - R &= Y_1 + rR \\ \text{and } R &= \frac{Y_0 - Y_1}{r + 1} \end{aligned} \quad (6.13)$$

Further, with the common average tax rate defined as

$$\begin{aligned} \bar{f} &= \alpha(Y_0 - R)^{\beta-1} \\ &= \alpha\left(Y_0 - \frac{Y_0 - Y_1}{r + 1}\right)^{\beta-1} \\ &= \alpha\left(\frac{rY_0 + Y_1}{r + 1}\right)^{\beta-1}, \end{aligned}$$

the equilibrium levels of C_0 and B can be easily shown to be

$$C_0 = \frac{(1-\bar{f})(Y_0 + Y_1 r^{-1})}{1 + (ar^{1-v})^{1/v}} \quad (6.14)$$

and
$$B = (1-\bar{f})(Y_0 + Y_1 r^{-1}) \left[\frac{r}{r+1} - \frac{1}{1 + (ar^{1-v})^{1/v}} \right] \quad (6.15)$$

As Hood concluded, the consumer will use R to maximize his lifetime post-tax income and, from that point, will use B to allocate consumption between periods. This solution is illustrated in Figure 6.3.

FIGURE 6.3

CONSUMPTION ALLOCATION WITH RRSPs AND BONDS

Starting at post-tax endowment point a (assuming $R=0$), the consumer achieves a lifetime income increase by investing in R to attain point a' . From there he reaches his optimal consumption allocation b' by choosing a level of B . In the case illustrated, the consumer borrows or chooses $B < 0$. Since the consumer's budget lines, L and L' through a and a' , both have slope $-r$, and the utility function is homothetic,

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the consumer remains on the same allocation ray with and without access to R (i.e. b' versus b). Note that where the consumer can contribute to RPP/RRSPs up to some limit, he will attain a consumption pair on the segment bb'; the limit on RPP/RRSP contributions serves only to reduce the income gain. Another point of interest is that the return $r \left(\frac{1-m_1}{1-m_0} \right)$ is not much greater than r for a significant range of R values near a'. This fact, considered together with transactions costs in the market for R and real-world uncertainty regarding future marginal tax rates, may explain in good part why more taxpayers are not observed to be constrained by the RPP/RRSP deductibility limits.

6.3.b The RPP/RRSP Deduction with Bond Interest Taxable

First consider the case where interest on B is taxable over the whole range of B. That is, ignore the \$1000 Interest and Dividend Deduction and assume that, for net borrowers in B, the interest payments can be deducted from taxable income. The result of this change in the lending-borrowing constraint is that the first order condition $U_R = 0$ is unchanged from (6.12b), but the condition, $U_B = 0$, becomes

$$\begin{aligned} C_1 &= (ap)^{1/v} C_0 \\ &= a(r-m_1(r-1))^{1/v} C_0 \end{aligned}$$

as in (6.8) and (6.9). Since both $U_R = 0$ and $U_B = 0$ must be met, it follows that

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$$r - m_1(r-1) \equiv \rho \Rightarrow r \left(\frac{1-m_1}{1-m_0} \right)$$

which implies

$$\frac{m_1}{1-m_1} = r \left(\frac{m_0}{1-m_0} \right) \quad (6.16)$$

From (6.16), it is clear that $m_1 > m_0$ at equilibrium, which contrasts with the result $m_1 = m_0$ when B was non-taxable. As was the case with B taxable but R absent, an explicit algebraic solution for C_0 is not obtainable. Nevertheless, the qualitative implications of this solution are clear enough. Compared to the preceding case, the consumer will invest more heavily in R since his opportunity cost has fallen from r to $r - m_1(r-1)$. This result is also apparent from the condition $m_1 > m_0$ which implies a higher level of period 1 taxable income than in the case of non-taxable bond interest. Despite the relative increase in period 1 taxable income, the allocation of lifetime consumption shifts in favor of the present. This can be observed from the condition $U_B = 0$ which gives $C_1/C_0 = (ap)^{1/v}$, down from $C_1/C_0 = (ar)^{1/v}$.

Perhaps a more important question is the effect on consumption and savings of introducing or increasing R when bond interest is taxable. With bond interest non-taxable, it was seen that introducing R increased C_0 and C_1 but left C_1/C_0 unchanged. With bond interest taxable (and debt interest tax deductible), introducing R again raises C_0 and C_1 (assuming no compensatory tax increases) but now produces a decline in C_1/C_0 . That is, with bond interest taxable, the introduction or increase in RPP/RRSP deductibility actually

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discourages consumption deferral. The reason is that, by deferring taxation on rR to period 1, the investment in R raises m_1 and thus lowers ρ , the price at which the consumer can exchange C_0 for C_1 . This is an interesting and perhaps counter-intuitive conclusion given the common acceptance of the notion that tax assistance to retirement saving encourages consumption deferral.⁶

This result also draws attention to two other tax provisions: the \$1000 Interest and Dividend Deduction and the tax deductibility (or not) of interest paid on loans obtained to finance RRSP contributions. If interest costs are not deductible, then, for any consumer who invests in R to the extent that optimal B is negative, the intertemporal rate of exchange is r not ρ . For such a consumer, the introduction of R does increase C_1/C_0 . Moreover, the range of $(r-1)B$ over which this intertemporal exchange rate holds is extended to +\$1000 by the Interest and Dividend deduction. As negative B is a quite plausible result for many consumers, the removal of interest deductibility from borrowing to finance RRSP purchases in the November 1981 federal budget (Government of Canada, 1981) may well have a significant effect in encouraging retirement consumption.

Despite its questionable effect on the time pattern of consumption, the RPP/RRSP provision has a substantial effect on the personal saving rate. The reason for this apparently contradictory

6. This result is available but not remarked in Hood (1982) and Daly (1981).

result is that the utilization of the RPP/RRSP deduction leads to the deferral of taxes to a later period. As a result of this tax deferral, current period disposable income rises. To maintain an unchanged time pattern of consumption, savings must rise as well. The situation might be summarized by noting that nearly all of the increased savings are required to pay off the deferred tax liabilities. In Appendix 6.B, a very simple two-period numerical example is provided to illustrate this result. With an initial endowment of $Y_0 = \$20,000$, $Y_1 = \$10,000$, the result of introducing an RPP/RRSP provision is a small increase in lifetime income of \$174, consequent increases of about \$88 in both C_0 and C_1 , and a substantial increase in the period 0 rate of saving out of disposable income from .230 to .325. The increase in the dollar value of savings is approximately equal to the level of tax savings produced by the deduction. Accordingly it is reasonable to estimate that the aggregate effect of the RPP/RRSP deduction on personal savings is similar in magnitude to the total revenue cost to government of the deduction. In 1978, this amounted to \$2310 million or about 1.5% of personal disposable income. Thus the effect of the RPP/RRSP provision on the timing of tax collections may have contributed up to 1.5 percentage points to the 10.8% personal saving rate in 1978 -- without having an important effect on consumption levels in retirement.

In light of this result, the question immediately arises as to whether the RPP/RRSP deduction serves any useful purpose. One argument in support of the provision is that it may play an informational role, encouraging savings and the deferral of consumption by calling the attention of taxfilers to the question of whether their retirement

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incomes are adequate. This is a weak argument both because the strength of any such effect is doubtful and because more direct and less costly means of providing such information are surely available. A second argument for the deduction is that it reduces the distortion of intertemporal consumption choice inherent in the taxation of investment income in an income tax system. Thus, it may be claimed to be a move toward a more efficient, consumption tax system. In Chapter 2, some questions were raised regarding the superior efficiency claimed for the consumption tax system. Putting this issue aside, it has been demonstrated in this section that the RPP/RRSP deduction will not reduce the intertemporal distortion of consumption in the general case but only in the special case where optimal B is shifted from a taxable range to a non-taxable range. A third justification for the deduction is that it corrects a horizontal inequity inherent in a progressive income tax system: namely, its bias against those with uneven time profiles of income. This justification also invites the response that there may be alternative ways of correcting this horizontal inequity which are more efficient and/or are more desirable from a vertical equity stand-point. Considered apart from other provisions of the tax system, the RPP/RRSP deduction clearly results in a transfer from low- to high-income families no matter whether income is measured on an annual or lifetime basis. The rationale for the RPP/RRSP deduction can not be properly evaluated, however, without considering two other aspects of it.

6.3.c Tax Deferral and Inflation Protection via the RPP/RRSP Deduction

The rate of return formula derived from (6.10) shows that permitting the deferral of taxation on both the principle and interest associated with an RPP/RRSP contribution is equivalent to taxing the contribution but completely exempting the interest income from taxation. That is, for $m_0 = m_1$, $-dC_1/dC_0$ (via R) = $r(1-m_1)/(1-m_0) = r$.

An important result of this tax treatment is that the return on R is largely unaffected by the rate of inflation. If P is one plus the inflation rate, and $m_0 = m_1$, then the real return on R is

$$-dC_0/dC_1 = P^{-1} [Pr (1-m_1)/(1-m_0)] = r$$

as before. (However, RRSP investments are still subject to some inflation risk in the sense that the expected or long run average real return, r , may not be realized during periods of rising inflation as capital losses are incurred on longer term debt instruments in the portfolio. The same holds for the returns realized on RPP funds.) In contrast, the effective rate of taxation of real interest on bonds rises with inflation as was demonstrated in the previous chapter. The real return on B under inflation is $-dC_1/dC_0 = P^{-1} [Pr - m_1(Pr-1)]$. For $r=1.03$, $m_1=.4$ and zero inflation ($P=1$), this is 1.018 (i.e. a 1.8% real rate of return). With 8% inflation ($P=1.08$), $-dC_1/dC_0$ falls to .988 for a net real rate of return of -1.2%. This analysis suggests that investments in RPPs and RRSPs are likely to increase in importance in household wealth portfolios in times of high inflation.

The existence of this instrument with its low inflation risk may also help to explain why Canadian savings rates have not fallen in the face of the high inflation rates of the past decade.⁷

Thus, the RPP/RRSP deduction has been shown to have several interesting characteristics. It is a vehicle for tax reduction through income averaging. It is a relatively safe investment in inflationary periods. Because of this and because so much tax deferral is involved in a RPP/RRSP contribution, the expanded level and utilization of the deduction over the 1970's has without doubt contributed materially to the increase in Canada's personal saving rate. Finally, and notwithstanding all the foregoing, the deduction was shown to provide little or no incentive for consumption deferral (a result which is unchanged when an inflationary environment is assumed).

In the light of this strong result, it seems appropriate to review the effect of various other provisions of the tax system on the intertemporal allocation of consumption. To begin with, the taxation of investment income, at any positive rate, has been shown to distort the pattern of consumption in favour of the present. The interaction of inflation with the tax system can produce a dramatic increase in tax rates and so amplify this effect. This situation is

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7. As Jump (1980a,b,c) has demonstrated, a measurement bias during inflationary periods may also have contributed to Canada's recent high rates of personal savings. Since a portion of interest income serves only to offset the erosion of capital values by inflation, both income and savings are overstated in inflationary periods.

offset to a considerable extent, however, by the reduced tax rates applying to some forms of investment income. Thus, dividends and certain capital gains are taxed at substantially reduced rates and the income from homeownership completely escapes taxation.⁸

Deductions relating to investment or retirement income may also influence the intertemporal allocation of consumption. For small investors, these deductions (the Age Deduction, the \$1000 Interest and Dividend Deduction, and the \$1000 Pension Income Deduction) may be sufficient to permit their investments to escape taxation. (For individuals with low lifetime income, however, the 50% reduction rate on GIS benefits may effectively tax investment income.) At higher levels of investment or pension income, these deductions were shown to have only a minor, second order impact on the investors' marginal tax rates on income from saving.

Given the very limited effectiveness of the various deductions, including the RPP/RRSP deduction in promoting consumption deferral, it is reasonable to ask what other provisions (short of the elimination of all taxation on investment income) might be more effective. A

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8. The taxation of capital gains when realized rather than when accrued also provides a small reduction in the marginal tax rate. For an N year holding period, the net-of-tax real rate of return is

$[r^N - m(r^N - 1)]^{1/N}$ for taxation on realization and $r - m(r - 1)$ for accrual taxation. For $N=10$, $m=.4$ and $r=1.03$, the respective net-of-tax rates of return are 1.89% and 1.80%.

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variety of such provisions could be considered, but only two will be noted here.⁹ Both provisions act to encourage saving by insulating after-tax rates of return from the effects of inflation. The first provision is the Registered Shareholder Investment Plan (or RSIP) which was proposed in the federal budget of June 28, 1982 and has since been promised by the present Minister of Finance in his Economic Statement of October 27, 1982 (Government of Canada, 1982a, 1982b). The RSIP eliminates taxation on the inflation component of capital gains. The reduced tax rate on capital gains is maintained, but gains are taxed upon accrual (possibly subject to some amortization process) rather than upon realization. The effect is to limit taxation to a maximum rate of about 25% of the real return on capital. The proposed application of the RSIP is limited to common stocks in Canadian companies.

The second provision is designed as an alternative to the RPP/RRSP deduction, and is based on the notion of a level of deductions against retirement income which is dependent upon the consumer's savings behaviour. The provision would work as follows. A contribution to a RPP or RRSP would generate no tax deduction when made; however, the amount of the contribution would be added to a notional account called a Retirement Income Deduction Account (or RIDA) by Revenue

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9. The question is analogous to questions of how to design incentive structures for ends such as charitable giving. Options considered in such analyses include deductions at rates over 100%, tax credits, and matching grants. See for example Hood et. al. (1977), Rosen (1979).

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Canada. The balance of this account for each contributor would be updated annually by the income tax indexation factor. In retirement, both the principle and interest components of the consumer's resulting pension income would be taxable (as with RPP/RRSP income now); however, the accumulated balance in the RIDA would be drawn down to provide annual deductions against the pension income. Using the same notation as before, the real post-tax rate of return per annum for an N year holding period would be

$$\begin{aligned} & [P^{-N}(P^N r^N (1-m_1) + m_1 P^N)]^{1/N} \\ &= r^N (1-m_1) + m_1 \\ &= r^{N-m_1} (r^N - 1). \end{aligned}$$

For $m_1 = .4$, $r = 1.03$ and $N = 10$, the net real return would be 1.89% for an effective tax rate on the real gain of about 37%. For longer holding periods, the effective tax rate would be slightly lower owing to the deferral of taxation on the investment income. As with the RSIP, the effective tax rate is unaffected by the rate of inflation.

These two provisions suggest ways in which the bias of the current system against consumption deferral could be somewhat reduced without rejecting the personal income tax system in favour of consumption or wage tax systems.

CAPITAL INFLUENCE

APPENDIX 6.A

TAX RATES AND DEDUCTIONS IN A MULTIPLE ASSET PORTFOLIO

It was observed in the text that, despite the Dividend Tax Credit and the 50% exemption of capital gains income, the tax rates on the real returns from capital can be much higher than corresponding rates on wage and salary income. With inflation rates of 10% or more the effective tax rates on income from common stocks can exceed 100%, and the tax rates on real interest income would be higher yet. The effect of these high tax rates on household saving is substantially offset, however, by the effects of the Interest and Dividend Deduction and the non-taxation of the income derived from homeownership. The importance of the latter tax preference is suggested by the fact that, in 1977, equity in owner-occupied homes accounted for 44% of the aggregate net worth of family units. This tax preference, together with the Interest and Dividend Deduction, is sufficient to produce a situation where, even with historically high savings rates, only a small fraction of families is exposed to the high marginal tax rates on (real) interest and dividends, and capital gains income.

A corollary of this is that a change in the tax rate on, say, interest income may have less effect on the household savings rate than on the division of household wealth portfolios between interest-bearing and other assets. This point can be elaborated with the help of a two-asset model (with no uncertainty). Such a model is

developed below. It is then applied to examine the effects of a tax rate change and a change in the level of a deduction. The analysis parallels that of a price-discriminating monopolist. Consider an individual whose wealth is divided between a financial asset, F , the income from which is taxable subject to a deduction, and investment in a home, H , the returns from which are non-taxable. The marginal return on investments in F is i_b up to the point where the deduction is exhausted and i_a beyond that. The (implicit) marginal return on investment in a principal residence is assumed to decline continuously with the level of investment reflecting diminishing marginal utility in the consumption of housing services and/or increasing costs in the maintenance of the home.

FIGURE 6.A.1

EFFECT OF A TAX RATE CHANGE IN A TWO-ASSET MODEL

Under these circumstances the individual will apportion his wealth first to H until the implicit return on H drops to i_b , then to F until the deduction is exhausted, then to H until the return on H drops to i_a and then to F. This combined marginal return schedule is shown in the third part of Figure 6.A.1. The total level of assets invested will be determined as the level of W at which the marginal return curve, R, intersects a supply curve, S, which may be considered as deriving from an interest-sensitive consumption function.

If the effective tax rate on F (beyond the deduction) is high, this analysis immediately suggests that at a single level, F_d , of investment in financial assets may be consistent with a considerable range of total wealth positions, W_b to W_a . The extent of this range depends on the degree to which the marginal utility of housing consumption diminishes with extra units of consumption.¹

Effect of a Tax Rate Change

Figure 6.A.1 also portrays the response of the wealth level and portfolio composition to an increase in i_a to i'_a occasioned, for example, by a decline in the tax rate on interest income or a rise in the Dividend Tax Credit. Assuming an initial wealth level W_1 (with investment in F in excess of F_d), the result of the increase in i_a is an increase in total wealth from W_1 to W_2 which corresponds to a

1. This model does not provide any representation of the increase in home equity by mortgage repayment. Such an option could be represented by a horizontal section of R_H at i_b .

larger increase in F and an offsetting decline in H. Three simplifications made by this analysis should be noted. First, the equilibrium positions portrayed will only emerge over time; housing assets are less liquid than financial ones. Second, no value is recognized for the liquidity services provided by the financial asset; the individual may be expected to derive an implicit return on the initial units of investment in F which is greater than i_0 . Third, no explicit consideration is given to the degree of substitutability of F and H. In general, the more closely substitutable are F and H, the greater will be the change in the holdings of one in response to a change in the rate of return on the other. Nevertheless, the example demonstrates that the change in the holding of an asset in response to a change in its rate of return will overstate the change in total saving because the quantity of other assets held will change in an offsetting way. In some situations, the rate of return change may result in a substantial portfolio shift but very little change in the level of total saving.

Effect of a Change in the Level of a Deduction

In Figure 6.A.2, the simple portfolio choice model is used to examine the effects of a change in the level of the tax deduction on income earned on the financial asset. In the case shown, the increase in the deduction results in a change in the marginal rate of return on total assets earned by the investor.² As in the previous

2. This would not be true if the investor's initial holding of F was less than F_0 or greater than F_2 . In the former of these cases, the increase in the deduction would have no effect on i or W . In the latter case, the increase in the deduction would have a pure income effect; the marginal rate of return i and the wealth holding W would be unaffected, but the total return from that wealth holding would rise:

FIGURE 6.A.2

EFFECT OF A CHANGE IN A TAX DEDUCTION
IN A TWO-ASSET MODEL

example, a new equilibrium at i'_a and W_2 is established and the increase in total assets results from an increase in F and a partly offsetting decrease in H . The important point here is that, even though the income from F is now fully covered by the deduction, the marginal rate of return facing the investor has risen only from i_a to i'_a , a considerably smaller change than $i_b - i_a$. In summary, the existence of a portfolio of assets whose returns are subject to varying tax treatment means that the effect on total savings of a change in a tax rate or deduction must be expected to be smaller than would be the case if there were only one asset.

APPENDIX 6.B

TABLE 6.B.1

EXAMPLE OF THE EFFECT OF THE RPP/RRSP
DEDUCTION ON CONSUMPTION AND SAVINGS

Basic Data

$$Y_0 = \$20,000$$

$$Y_1 = \$10,000$$

$$r = 1.035$$

$$a = 1$$

$$v = 2.5$$

$$(ar)^{1/v} = 1.013856$$

$$D = 1 + (ar^{1-v})^{1/v} = 1.97957$$

NO DEDUCTION

Tax Rates

$$f_0 = .023(Y_0)^{.27} = .3334$$

$$f_1 = .023(Y_1)^{.27} = .2765$$

RRP/RRSP Investment

$$0$$

Lifetime Income

$$\begin{aligned} L_0 &= (1-f_0)Y_0 + (1-f_1)Y_1r^{-1} \\ &= \$20,322 \end{aligned}$$

Consumption

$$C_0 = L_0/D = \$10,266$$

$$C_1 = (ar)^{1/v} C_0 = \$10,408$$

Period 0 Disposable Income

$$YD_0 = (1-f_0)Y_0 = \$13,332$$

WITH DEDUCTION

$$\bar{f} = .023 \frac{rY_0 + Y_1}{r+1}^{.27} = .3090$$

$$R = \frac{Y_0 - Y_1}{r+1} = \$4914$$

$$\begin{aligned} L_0 &= (1-\bar{f})(Y_0 + Y_1r^{-1}) \\ &= \$20,496 \end{aligned}$$

$$C_0 = L_0/D = \$10,354$$

$$C_1 = (ar)^{(1/v)} C_0 = \$10,497$$

$$YD_0 = R + (1-\bar{f})(Y_0 - R) = \$15,338$$

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CHAPTER 7

DEVELOPMENT AND ANALYSIS OF
A MULTI-PERIOD LIFECYCLE SAVINGS MODEL

The two-period lifecycle model employed in the last chapter, which is the basic model underlying the econometric analyses of Feldstein (1974, etc.) and others, has two serious limitations when applied to policy questions relating to the effects of taxes or public pensions on savings. First, a two-period model does not have the flexibility to permit the assessment of the differential impacts of policy measures on consumers of different age levels or income profiles. Thus it provides a poor basis for looking at the income transfer or equity effects of the policy measures. Second, and more important, by abstracting from uncertainty about future income, the rate of return on saving, and longevity, it ignores the insurance role of saving. It also ignores a major point of difference between private wealth and public pension rights since the value of the latter must reflect the fact that the benefits are paid out as a life annuity and are indexed to the rate of inflation. As a result of these limitations, an analysis based on the simple two-period model is probably not able to admit the possibility that the optimal retirement income system could include a mix of income tax measures and public pension programs.

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The aim of this chapter is to develop and analyse a multi-period lifecycle model which overcomes these limitations. Section 7.1 begins by expanding the two-period model of Chapter 6 to T periods without dropping the assumption of certain longevity. This provides a reference case against which the solution of the model with uncertain lifetime can be compared. The uncertain longevity model is the core model of the thesis. It is developed in section 7.2, and its mathematical solution is outlined in Appendix 7.A. In section 7.3, the model is elaborated to consider a multi-person consuming unit and the effects of expected changes in the size of the household over its lifetime. This is necessary to permit the model to be estimated (in Chapter 8) using data on household consumption patterns. Section 7.4 provides a note on how the model solution would be affected by the inclusion of a bequest motive. In section 7.5, the analysis of the model of section 7.2 begins with an examination of the interest elasticity of consumption. Section 7.6 employs these results in considering the effect on consumption levels of uncertainty about the rate of return on savings. Similarly, section 7.7 examines the effect on consumption and savings of uncertainty regarding expected future streams of earnings or transfer income. In section 7.8, an alternative form of the model is examined in which the consumer is assumed to have the option of investing in actuarially fair annuities. The analysis of the model concludes in section 7.9 with an illustrative comparison of two broad types of policy measure designed to increase the level of household consumption in retirement. The two measures are an increase in the after-tax rate of return on saving and an increase in public pension income. The findings of the chapter are summarized in section 7.10.

ADDITIONAL INFORMATION

7.1 THE MULTI-PERIOD MODEL WITH CERTAIN LIFETIME

The model of equations (6.2) and (6.3) can be quite simply extended to a multi-period context. In making such an extension, all the assumptions of that model are preserved: the form of the utility function (no leisure or bequest terms, additive separability, iso-elasticity), the consuming unit as a single utility-maximizing individual, the single asset with a certain rate of return available to savers, and the perfect knowledge of longevity and future income flows. In a multi-period context, utility function (6.3) becomes

$$U = \sum_{t=0}^{\hat{T}} a^t C_t^{1-v} \quad (7.1)$$

where: C_t = consumption in year t ;

$\hat{T}$ = years of life remaining; death is taken to occur at the end of year $\hat{T}$;

a = the subjective time preference factor;

v = the measure of the consumer's degree of risk aversion.

Note that in this formulation the marginal utility of consumption in any period, $a^t C_t^{-v}$, depends only on a^t and C_t . Possible differences in the marginal utility of a unit of consumption pre- and post-retirement, for example, are ignored.

Utility is constrained in this model by the requirement that the present value of the consumption stream be equal to lifetime wealth, the sum of current (non-human) wealth and the present value of future earnings and transfers. That is,

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$$L_0 = \sum_{t=0}^{\hat{T}} C_t r^{-t} = W_0 + \sum_{t=0}^{\hat{T}} Y_t r^{-t} \quad (7.2)$$

where: L_0 = lifetime wealth;

r = the interest factor (i.e. one plus the real, net-of-tax rate of return on assets);

W_0 = net worth available for consumption in period 0 (includes interest earned in period 0);

Y_t = net-of-tax, non-capital income in period t (earnings plus money and in-kind transfers less taxes).

The solution to this simple form of the model is easily obtained by setting $J_{C_t} = 0$ (for all t) where

$$J = \sum_{t=0}^{\hat{T}} a^t C_t^{1-v} + \lambda (L_0 - \sum_{t=0}^{\hat{T}} C_t r^{-t})$$

This results in the first order conditions

$$C_t = (ar)^{t/v} \lambda^{-1/v} = (ar)^{t/v} C_0. \quad (7.3)$$

Substituting the optimal C_t into the constraint yields

$$C_0 = \frac{W_0 + \sum_{t=0}^{\hat{T}} Y_t r^{-t}}{\sum_{t=0}^{\hat{T}} (ar^{1-v})^{t/v}} = \frac{L_0}{\sum_{t=0}^{\hat{T}} x^t} \quad (7.4)$$

where $x = (ar^{1-v})^{1/v}$. Some characteristics of the solution should be noted. First, C_t grows or declines monotonically depending on whether $ar > 1$. If $ar = 1$, then $C_t = C_{t-1} = C_0 = \frac{L_0}{\hat{T}+1}$; that is, lifetime

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wealth is merely divided into equal parts. If instead we assume that the consumer wants to maintain his relative position in the distribution of consumption levels when incomes are subject to positive secular growth, then we would expect $C_t/C_{t-1} = (ar)^{1/v} > 1$. If secular growth is represented by a factor $g > 1$ and we assume that the consumer wishes to maintain $C_t/C_{t-1} = g$, then we can deduce values of a for given g, r and v from $g = (ar)^{1/v}$. For example, $g = 1.02$, $r = 1.035$ and $v = 1.73$ is consistent with $a = 1$ (subjective discount rate = 0). For lower r or higher g , a lower level of relative risk aversion, v , is required to preserve positive or zero time preference. Another point to note is that optimal savings rates and wealth levels can be positive or negative depending chiefly on the size of anticipated future income in relation to base period income and wealth. In this simple model, negative net worth positions are constrained only by the size of the stream of future earnings and transfers.

7.2 UNCERTAIN LIFETIME

Once uncertainty regarding longevity, income, or the interest rate is admitted, the consumer can only attempt to maximize expected utility. Except in special cases, the consumption choice which maximizes expected utility will be found to differ from that which maximizes utility when the random variables take their expected values. The development of a lifecycle consumption model with uncertain lifetime is due to Yaari (1964 and 1965). The case of uncertain lifetime was also analyzed by Tobin (1967) but, for a reason described below, his solution is incorrect. The derivation presented below

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follows Yaari in its essential aspects. However, as the model is developed in discrete time instead of continuous time, the mathematical procedure used to derive the solution is dynamic programming rather than optimal control theory.

Let d_t = the probability of death at the end of period t , with $\sum_{t=0}^T d_t = 1$; then $s_t = \sum_{j=t}^T d_j$ is the probability of survival to t . Here T is the consumer's planning horizon, the final year for which positive s_t and d_t are defined. Expected years of life remaining from any age k is given by $\hat{T} = \sum_{t=k}^T \frac{s_t}{s_k}$. The individual maximizes

$$E(U) = \sum_{t=0}^T d_t \left[\sum_{k=0}^t a^k \frac{C_k^{1-v}}{1-v} \right],$$

that is, the expected value of a series of "lifetime utilities" each corresponding to the event of death in some year t . The order of summation of the C_k terms can be reversed giving

$$\begin{aligned} E(U) &= \sum_{t=0}^T \left[\sum_{k=t}^T d_k \right] a^t \frac{C_t^{1-v}}{1-v} \\ E(U) &= \sum_{t=0}^T s_t a^t \frac{C_t^{1-v}}{1-v} \end{aligned} \quad (7.5)$$

The global constraint (equation 7.2) which requires discounted lifetime resources to equal discounted lifetime consumption is no longer operational because future C_t and Y_t can now be defined only in probabilistic terms. Instead, a constraint such as

$$W_t = r(W_{t-1} + Y_{t-1} - C_{t-1}) \geq 0 \quad (7.6)$$

must hold for all t in the planning period $0, \dots, T$. (Failure to recognize this change in the nature of the constraint lead Tobin to his incorrect solution). A completely realistic model would reflect the fact that negative net worth positions are available to some, younger consumers as documented in table 7.1 below. However, the incidence of negative net worth is limited, and observed levels of negative net worth are generally low. Therefore, $W_t \geq 0$ for all t would seem to be a reasonable approximation. This is the constraint employed in the models of Yaari (1965) and Davies (1980).

TABLE 7.1
FREQUENCY OF NEGATIVE WEALTH LEVELS
FOR CANADIAN FAMILY UNITS, 1977

Percent of Family Units with Net Worth Less Than:			
Age of Head	<u>\$0</u>	<u>-\$1000</u>	<u>-\$5000</u>
15-24	27.5	13.8	1.5
25-34	14.6	9.9	2.2
35+	<u>4.3</u>	<u>2.4</u>	<u>0.6</u>
All Ages	9.3	5.4	1.1

Source: Statistics Canada, 1977 Survey of Consumer Finances micro data tape, "Economic Families, 1976 Income, 1977 Assets and Debts".

ADDITIONAL INFORMATION

In examining the solution to the model with uncertain longevity, it is convenient to begin by considering the case where the constraint $W_t \geq 0$ is not binding until period T . This case would occur when $W_0 + Y_0$ is relatively high and future levels of Y_t are low or zero. The solution to the model in this case is derived in Appendix 7.A. The result of introducing uncertain longevity into the model is that the first order conditions (7.3), which define the optimal time path, become

$$C_t = s_t^{1/v} (ar)^{t/v} C_0 \quad (7.7)$$

and the equation for base period consumption (7.4) becomes

$$C_0 = \frac{W_0 + \sum_{t=0}^T Y_t r^{-t}}{\sum_{t=0}^T s_t^{1/v} (ar^{1-v})^{t/v}} \quad (7.8)$$

The effect of uncertain lifetime on consumer behaviour can be examined with reference to these equations. First, since $0 \leq s_t \leq 1$, the ratio C_t/C_0 will generally be smaller with uncertainty than without. As s_t falls further below 1 in later years of life, C_t/C_{t-1} will become less than 1 at some point and continue to decline thereafter, even with $ar > 1$. This means that, rather than rising monotonically, the level of household consumption may be expected to rise for a period, when the consumer is young and s_t is close to 1, and then fall as the consumer becomes older. These effects are illustrated in Figure 7.1. Part (a) of the figure shows the average survival probabilities for males and females in Canada in the mid-1970's.

FIGURE 7.1

SURVIVAL PROBABILITIES AND CONSUMPTION PLANS

(a) Survival Probabilities

(b) Consumption Plans with Uncertain Lifetime

($L_0 = \$1,000,000$)

Source for (a): Statistics Canada, Life Tables, Canada and the Provinces, 1975-77, Catalogue 85-532.

Part (b) shows the lifetime consumption profiles (with $L_0 = \$1,000,000$) that arise from this model for a single male with $a=1$, $r=1.035$ and two values of the risk aversion parameter, $v=1$ and $v=2.5$.

Note that the effect of a declining probability of survival is considerably greater when $v < 1$ than when $v=2.5$. That is, planned consumption by the less risk averse consumer falls off to much lower levels than does that of the more risk averse consumer. The low risk aversion consumer is seen to be relatively unwilling to give up earlier consumption to provide for the contingency of unexpected longevity. At the same time, he is more sensitive to the opportunity provided by a positive interest rate to increase expected lifetime consumption by deferring consumption while young.

Suppose now that the consumer were to die at age 80. His wealth at death can be represented by the area under his planned consumption curve to the right of age 80 (discounted at 3.5% per annum). This wealth level would be an unintended bequest (since there is no bequest argument in the consumer's utility function), which would result solely from the consumer's desire to provide for the contingency of unexpected longevity. Accordingly the unintended bequest must be seen as a cost arising from the uncertainty regarding longevity. It is clear from Figure 7.1 that, for both $v=1$ and $v=2.5$, the size of the unintended bequest is considerable.

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Another indication of the cost of uncertain longevity may be obtained by comparing the equations (7.4) and (7.8) for base-period consumption. By writing the numerators as L_0 , and again letting $x = (ar^{1-v})^{1/v}$, the equations can be rewritten as

$$C_0 = L_0 \div \sum_{t=0}^T x^t \quad (7.4')$$

and
$$C_0 = L_0 \div \sum_{t=0}^T s_t^{1/v} x^t \quad (7.8')$$

Note that $\sum_{t=0}^T s_t = T+1$ and also that, as v increases, $s_t^{1/v}$ increases as well. From these observations it may be deduced that for the usual case of $ar \geq 1$ ($x^t \geq 1$), uncertain longevity will result in a reduction in C_0 for any consumer with $v > 1$. Moreover, an increase in the level of uncertainty regarding longevity will further reduce C_0 so long as $v < 1$. These results were demonstrated by Levhari and Mirman (1977) and add greater precision to the conclusion reached both by Fisher and Yaari that, in the absence of a bequest motive, uncertainty of lifetime will increase the rate of preference for present over future consumption. Thus, it is true that C_t/C_{t+1} will be higher under uncertainty than in the certain lifetime case, but initial consumption, C_0 , will be reduced by uncertainty so long as the consumer's level of relative risk aversion is sufficiently high ($v > 1$). For the risk averse consumer, the decline in C_0 (and thus in all the C_t) relative to the no-uncertainty case provides a basis for measuring the cost of uncertainty to the consumer. This point will be further developed in the section on annuities.

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While the solution just discussed is instructive, it is not representative of the more common circumstance in which a consumer is prevented from borrowing on future earnings or pension income by the constraint that $W_t \geq 0$. Once the wealth constraint is introduced, the solution can only be described with reference to time intervals over which the constraint is or is not effective. Davies has christened these "blocked" and "free" intervals. For blocked intervals,

$$C_t = W_t + Y_t \quad (7.9)$$

with W_t usually equal to zero. Now consider a free interval from $t=g$ to $t=h$. Inside this interval, consumption grows or diminishes according to

$$C_t = \frac{s_t}{s_g} \left(ar \right)^{\frac{1}{v} \frac{(t-g)}{v}} C_g \quad (7.10)$$

and initial consumption in the interval is defined by

$$C_g = \frac{W_g + \sum_{t=g}^h Y_t r^{-(t-g)}}{\sum_{t=g}^h \frac{s_t}{s_g} \left(ar \right)^{\frac{1-v}{v} \frac{t-g}{v}}} \quad (7.11)$$

Finally, continuity of consumption between blocked and free intervals demands (unless $g=0$)

$$C_g = \left(\frac{s_g}{s_{g-1}} \right)^{1/v} (ar)^{1/v} C_{g-1} \quad (7.12)$$

and (unless $h=T$)

$$C_{h+1} = \left(\frac{s_{h+1}}{s_h} \right)^{1/v} (ar)^{1/v} C_h \quad (7.13)$$

The solution of the consumer's allocation problem using these four relationships involves the determination of the free interval boundaries (e.g. g and h) simultaneously with the C_t . This is probably done most conveniently if the model is cast in continuous time. However, this requires that each family's income profile be represented by a continuous function (some kind of polynomial) and so would make the simulation of policy changes (e.g. an increase in OAS) much more cumbersome and approximate. Accordingly, the discrete form of the model has been retained here, and the solution is based on an algorithm which first applies the unconstrained model and then, when some W_t falls below zero, searches for the interval boundaries which best satisfy conditions (7.9) to (7.13).

Figure 7.2 shows the effect of the wealth constraint on planned consumption for a realistic income profile in which earnings are concentrated in the 30-64 age span and a significant level of public and private pension income is assumed. Both earnings and pension income are expressed in constant dollars and are assumed to reflect a secular income growth of 2% per annum. With initial wealth at age 15 equal to zero, constrained consumption (dotted line) may be seen to be limited to current income for the first dozen or so years and also for the years from about age 90 to $T(=104)$. Unconstrained consumption, which would result if all Y_t were equal to zero and

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lifetime resources concentrated instead in W_0 , is higher than constrained consumption in the early years but markedly lower in the later years of life.

FIGURE 7.2

CONSUMPTION PROFILES
WITH AND WITHOUT A WEALTH CONSTRAINT

Because the wealth constraint renders the model solution analytically more cumbersome, it will be treated as if not binding in some subsequent sections of this chapter. Thus the reference solution will be (7.7) and (7.8) rather than (7.9) to (7.13). However, in the model simulations to derive interest elasticities and policy responses and in the estimation of the model in Chapter 8, the wealth constraint will be respected.

A final point which should be raised under the heading of lifetime uncertainty is the existence of systematic variation in life expectancy among consumers. The gap between female and male life expectancies, which has emerged over the past several decades and now stands at about seven years, provides one important example. A second is provided by the well-documented (Kitagawa and Hauser (1973), Billette (1976), and Wigle and Mao (1980)) relationship of longevity to socio-economic status or family income. For example, the last-mentioned study found that, in Canadian urban areas in 1971, the difference between life expectancies in the highest- and lowest-income groupings (quintiles) of census tracts was 6.2 years for males and 2.9 years for females.* Since within-tract variation in incomes and life expectancies could not be identified in the study, the positive relationship between income and longevity is probably understated in these estimates.

The effect of, say, a decrease in life expectancy is naturally to increase planned consumption in early years and decrease it in later years. This also results in a reduction in planned wealth accumulation. To provide one indication of the magnitude of the effect, a shift from female to male survival probabilities for a single consumer having the income profile shown in Figure 5.2 was found to reduce the peak wealth holding (at age 65) by about 20%.

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7.3 VARIATION IN HOUSEHOLD SIZE

To this point, the model has treated the consuming unit as a single individual. However, in applying a model of consumption to household data, the differing consumption demands of large versus small families must be recognized as well as the pooling of items of expenditure by family or household members. In a model which involves the allocation of expected consumption levels across future years, the prospect of varying family size over the lifecycle must also be taken into account. The simplest way to reflect these considerations is to assume that the collectivity of a family or household is the utility-maximizing unit, and to further assume that future consumption demands are assessed in relation to the expected future size of the consuming unit. This is the approach adopted here. It does not permit the consideration of intergenerational transfers occurring within the family, but has the advantage of being capable of application with existing household survey data,

The application of this approach first requires that the notion of the survival of the spending unit be defined. Second, the way in which the evaluation of consumption reflects family size must be specified. With regard to the first of these questions, the lifespan of the spending unit is taken here to be equivalent to the remaining life of the longest surviving spouse of a couple. The survival probability of the couple, s_t , can then be represented as the sum of the probabilities of one spouse surviving alone, s_{1t} , and both spouses surviving, s_{2t} , where s_{1t} and s_{2t} are defined in terms of the survival probabilities of the husband and wife.

$$\begin{aligned} s_{2t} &= s_{Ht} s_{Wt} \\ s_{1t} &= s_{Ht} + s_{Wt} - 2s_{Ht} s_{Wt} \end{aligned} \quad (7.14)$$

No allowance is made in estimating the model for the possibility of marriage, divorce or separation occurring in future years. When family size varies, two possible expressions of utility that have been considered (e.g. by Arrow and Kurz, (1970) and Irvine (1979) and (1980)) are:

$$U = U\left(\frac{C}{F}\right) \quad (7.15)$$

$$\text{and } U = F U\left(\frac{C}{F}\right) \quad (7.16)$$

where F is either family size or a family size equivalence factor which takes into account economies of scale in consumption. Consider family consumption in two periods which differ only by the sizes of the family, F_1 and F_2 , with $F_2 > F_1$. With the iso-elastic utility function, maximizing utility by equating the marginal utilities of family consumption in the two periods leads to the following conditions:

$$\frac{C_2}{C_1} = \left(\frac{F_2}{F_1}\right)^{\frac{v-1}{v}} \quad (7.15')$$

and

$$\frac{C_2}{C_1} = \frac{F_2}{F_1} \quad (7.16')$$

In the model of (7.16), the ratio of optimal family consumption levels in the two periods is proportional to (equivalence scale-adjusted) family size. In the model of (7.15) on the other hand,

optimal C_2/C_1 depends on v and, with $v < 1$, has the unfortunate property that $C_2/C_1 < 1$ for $F_2 > F_1$. Accordingly, utility expression (7.16) has been adopted in the model estimation. (A close variant of it is also employed by Davies (1982).) Now let F_1 and F_2 be the family size factors, corresponding to the events of one or both of the spouses surviving to a given year. With s_{1t} and s_{2t} as defined in (7.14), utility function (7.5) can then be expressed as

$$\begin{aligned} E[U] &= \sum_{t=0}^T \left[\frac{s_{1t} F_1 a^t \left(\frac{C_t}{F_1} \right)^{1-v}}{1-v} + \frac{s_{2t} F_2 a^t \left(\frac{C_t}{F_2} \right)^{1-v}}{1-v} \right] \\ &= \sum_{t=0}^T [s_{1t} F_1^v + s_{2t} F_2^v] \frac{a^t}{1-v} C_t^{1-v} \\ &= \sum_{t=0}^T \tilde{s}_t \frac{a^t C_t^{1-v}}{1-v} \end{aligned} \quad (7.17)$$

Thus, the solution to the consumer choice problem can be obtained as before simply by replacing s_t by $\tilde{s}_t$ which depends upon v , upon both male and female survival probabilities, and upon the age of the two spouses in the base year. This procedure can be further amended by increasing the F for a period of years starting with the base year to reflect the presence of children or other family members.

An example of the effect of introducing multi-person households as the utility maximizing unit is provided in Figure 7.3. In this example, the income pattern of Figure 7.2 is retained¹, but the

1. This ignores the fact that the presence of a spouse will result in an increase in the income available for consumption; at the minimum a second stream of OAS benefits will be received.

consuming unit is taken to include both a husband and wife, subject to their survival probabilities, plus two children who are assumed to influence household consumption between parents' ages 30 and 54 for the first child and 35 and 59 for the second. The parents are assumed to be of equal age.

FIGURE 7.3

EFFECT OF ADDITIONAL FAMILY MEMBERS
ON THE HOUSEHOLD'S LIFETIME CONSUMPTION PROFILE

The effect of adding family members is to concentrate the household's lifetime consumption in the age span 30-60 when family size is greatest. The extra consumption in these years is largely at the expense of consumption in later years of the parents' lives. In addition to altering the lifetime consumption profile, the addition of family members also delays the accumulation of wealth and reduces the peak wealth holding (at age 65) by nearly one-half.

7.4 A NOTE ON THE INCLUSION OF A BEQUEST MOTIVE

The model of section 7.2, which is used to derive interest elasticity and policy response estimates below and is estimated in Chapter 8, includes no provision for intended bequests. The present section is restricted, therefore, to providing some comments on the subject of extending the model to include bequests.

Some idea of the effect of including a bequest motive can be obtained by considering the certain lifetime model of section 7.1 and modifying the utility function to include a bequest term. Thus, consider

$$U = \sum_{t=0}^{\hat{T}} a^t \frac{C_t^{1-v}}{1-v} + b^w \frac{a^{\hat{T}} B^{1-w}}{1-w} \quad (7.18)$$

By means of some simplifying assumptions, the effect of changing the model in this way can be investigated. First, take $w=v$ so the elasticity of marginal utility is the same for bequests as for consumption. Next, let the strength of bequest parameter, b , equal one.² With these assumptions, the bequest argument in the utility function has the same effect as an additional year of life. Consumption in each period from 0 to $\hat{T}$ is reduced slightly as lifetime resources are effectively apportioned over $\hat{T} + 1$ periods. Changing the value of b

2. Davies (1979) found that the observed level of inheritances was consistent with an average value of b of 0.93. However, this evidence is likely to be misleading since his model did not take account of the level of unintended bequests that results from uncertain lifetime.

would change the proportion of resources devoted to B. A value of $w \neq v$ would mean that the proportion of lifetime resources devoted to bequests would vary according to the level of lifetime wealth.

It is not so simple to apply this bequest motive formulation to a model in which lifetime is uncertain and a fair annuities market is assumed not to exist. (When $W_0 > 0$ but all $Y_t = 0$, a complex expression for C_0 was derived which has much the same properties as the solution of (7.18); when $Y_t > 0$, no analytical expression for C_0 could be obtained.) An interesting result of the uncertain lifetime model, however, is that the expected level of unintended bequests may be substantially higher than the level of planned bequests. For example, with the income profile of Figure 7.2, which assumes a modest level of annuity-form income, the expected level of the unintended bequest generated by the wealth-constrained model is about \$90,000. This compares with a planned bequest level of about \$24,000 when $w=v$ and $b=1$ in (7.18) and the level of lifetime resources is the same. This suggests that the desire to bequeath is likely to add only modestly to savings rates and planned wealth levels when annuity markets are imperfect. The most noticeable effect of including bequests in the model would be an increase in wealth levels among those at advanced ages (over 80), and even this might not be a very realistic result given the likely ages and wealth levels of the heirs by that time. These results cast some doubt on the Kotlikoff and Summers (1981) finding, based on a model with certain lifetime, that bequests explain much of the household saving.

A related consequence of considering uncertain lifetime in conjunction with an imperfect annuities market is that the economic risks of uncertain longevity are seen to be shared, unavoidably, by the consumer and his heirs. Unexpected longevity results simultaneously in low-consumption years for the consumer and a reduction in the level of bequests.

The model of equation (7.18) does not provide any representation of the determinants of bequests or other intergenerational transfers. Indeed, Kurz (1981) claims that such models amount to "meaningless formalism" on this account. Thus, in order to permit consideration of the Barro effects of Chapter 3, where implicit intergeneration transfers deriving from unfunded pension promises result in compensatory changes in bequest levels, the model would have to be elaborated to include explicitly the utility levels of heirs. The development of such a model in a form which could be estimated is beyond the scope of this thesis.

7.5 THE INTEREST ELASTICITY OF CONSUMPTION AND SAVINGS

In the analysis of the two period model in section 6.1.b, it was shown that the level and sign of the interest elasticity of consumption depended upon v and upon whether the consumer was a net saver or a net borrower. The elasticity was more likely to be strongly negative for a net borrower with low level of risk aversion (low v). This result holds in general in the multi-period model although it is altered in some instances by the wealth constraint. Since the consumer's

ratio of current net worth to expected future earnings and transfers varies with age, the interest elasticity of consumption also varies with age.

For a consumer in a free interval, the interest elasticity of consumption, $\epsilon_{C_0, r}$, can be obtained by differentiating equation (7.11) and multiplying the result by r/C_0 . This yields

$$\epsilon_{C_0, r} = \frac{(v-1)}{v} \frac{\sum_{t=g}^h (t-g) \left(\frac{s_t}{s_g}\right)^{1/v} (ar^{1-v})^{\frac{t-g}{v}}}{\sum_{t=g}^h \left(\frac{s_t}{s_g}\right)^{1/v} (ar^{1-v})^{\frac{t-g}{v}}} - \frac{\sum_{t=g}^h (t-g) Y_t r^{-(t-g)}}{W_g + \sum_{t=g}^h Y_t r^{-(t-g)}} \quad (7.19)$$

The expression is very similar to (6.6); the only differences are the presence of the survival probabilities and the factors $(t-g)$ in the numerator. In the two-period case, of course, $(t-g)$ is either zero or one. Thus, the dependence of $\epsilon_{C_0, r}$ on v and on the importance of expected future income relative to total lifetime resources is closely analogous to that found in the two-period case. Here it is important that "lifetime" be interpreted as the free interval ending in year h rather than the consuming unit's total potential lifetime.

For a consumer for whom C_0 is constrained by $W_0 + Y_0$, (7.19) does not apply. For such a consumer,

$$\epsilon_{C_0, r} = 0. \quad (7.20)$$

If one accepts the central assumption of the lifecycle model, namely, that the consumer bases his consumption on a rational estimate of his future earnings stream as well as upon his current resources, (7.19) and (7.20) can be applied to estimate typical interest elasticities of consumption. One set of such elasticities is presented in Table 7.2 below. The elasticities are derived for families (couples with no children) with middle incomes (for 1978) of about \$15000 in the base year. The future income stream employed in the estimates includes income from both public and private pensions. The elasticities are shown for different ages of the husband (the wife is two years younger) and three different values of v . In all these cases, $a = .99$ and $r = 1.03$ are assumed. (Evans (1983) has demonstrated that the elasticities are sensitive to a and r ; however, these are plausible values.) Base period wealth levels are assumed to be \$10,000, \$20,000, \$50,000; \$60,000, \$70,000, and \$70,000 at ages 20 to 70 respectively. With these wealth levels, the wealth constraint is never binding, so $^E C_{0r}$ is always non-zero. In elasticity estimations for young, lower-income families, the constraint was binding in some cases.

TABLE 7.2
INTEREST ELASTICITIES OF CONSUMPTION, $^E C_{0r}$

Husband's Age	Risk Aversion Parameter, v		
	.75	1.5	2.5
20	-66.69	-48.86	-42.94
30	-55.23	-40.01	-34.74
40	-43.32	-31.22	-26.67
50	-32.27	-22.47	-18.74
60	-20.83	-13.77	-10.96
70	-7.68	-3.37	-1.58

The wide range of the elasticities is immediately apparent and is caused both by differences in assumed risk aversion levels and variation in the ratio of wealth to future earnings over the lifecycle.³ This wide range in interest-elasticities, also noted by Summers (1981), provides reason to believe that aggregate interest elasticities as estimated by Boskin (1978) and others could be unstable over time in the face of demographic changes.

While the interest elasticity of base period consumption falls most directly from the lifecycle model, the elasticity of savings to the interest rate is a more commonly discussed and estimated parameter. The latter parameter is easily derived from the former. Let $i=r-1$ be the interest rate. Then

$$\epsilon_{ri} = \frac{\partial r}{\partial i} \frac{i}{r} = \frac{i}{r}$$

Since base-period savings, S , equals $W_0 + Y_0 - C_0$,

$$\epsilon_{SC_0} = \frac{\partial S}{\partial C_0} \cdot \frac{C_0}{S_0} = - \frac{C_0}{W_0 + Y_0 - C_0}$$

---

3. The range of these $\epsilon_{C,r}$ does not extend to positive values. Previous estimates, in which less generous pension income streams were assumed, did include positive values.

The interest elasticity of savings can now be expressed as

$$\begin{aligned}\epsilon_{S_i} &= \epsilon_{SC_o} \cdot \epsilon_{C_o r} \cdot \epsilon_{ri} \\ &= (\epsilon_{ri} \cdot \epsilon_{SC_o}) \epsilon_{C_o r} \\ &= - \frac{i}{r} \left(\frac{C_o}{W_o + Y_o - C_o} \right) \epsilon_{C_o r} \quad (7.21)\end{aligned}$$

Table 7.3 provides estimates of ϵ_{S_i} which correspond to the $\epsilon_{C_o r}$ estimates of Table 7.2.

TABLE 7.3

INTEREST ELASTICITIES OF SAVINGS, ϵ_{S_i}

Husband's Age	Risk Aversion Parameter, v		
	<u>.75</u>	<u>1.5</u>	<u>2.5</u>
20	2.80	2.05	1.80
30	1.18	.86	.74
40	.40	.29	.25
50	.24	.16	.14
60	.12	.08	.07
70	.05	.02	.01

Again a wide range is evident in the elasticities over age groups and values of the risk aversion parameter. Note that the higher elasticities for younger age groups where future income is large relative to current wealth supports King's conclusion, reported in Chapter 2, that an increase in public pensions would increase the elasticity of savings and thus reduce the optimal rate of capital taxation.

As a final exercise in this section, (7.19), (7.20) and (7.21) are applied to estimate interest elasticities of consumption and savings at the aggregate level for the population of Canadian households. The ideal method of making such an estimate would be to apply the above mentioned formulae at a microdata level using reported or assigned levels of C_o , W_o , Y_o , Y_t . A more approximate method has been employed here. The population was first disaggregated into 18 age/income groups (6 age groups times 3 income groups) based on income data from the 1978 Survey of Consumer Finances (1977 incomes). Representative wealth levels were then assigned to each age/income group. These were chosen to balance to the average wealth levels reported by age group in the 1977 Assets and Debts Survey. Average consumption levels were derived from consumption to income ratios calculated for age/income groups in the 1978 Family Expenditure Survey.⁴ The low income groups were assigned the survival probability vector of single males; the middle and high-income groups were assigned the survival probability vector of a couple without children. After calculating the $^e C_o$ and $^e S_i$ for individual age/income groups, the corresponding aggregate elasticities were calculated by taking appropriate weighted averages. For example, if S is aggregate savings and equals $\sum_{k=1}^N S_k$ where S_k is the savings of the k^{th} individual, then

4. The three data sources are:

- (1) Statistics Canada, Income Distributions by Size in Canada, 1977 (Catalogue 13-207, annual).
- (2) Statistics Canada, The Distribution of Income and Wealth in Canada, 1977 (Catalogue 13-570).
- (3) Statistics Canada, special purpose microdata file from the 1978 Family Expenditure Survey.

$$\epsilon_{Si} = \frac{\partial S}{\partial i} \frac{i}{S}$$

$$= \sum_{k=1}^n = \frac{\partial S_k}{\partial i} \frac{i}{S}$$

$$= \sum_{k=1}^n = \frac{\partial S_k}{\partial i} \frac{i S_k}{S_k S}$$

$$= \sum_{k=1}^n = \alpha_k \epsilon_k$$

(7.22)

where $\alpha_k = S_k/S$ and $\epsilon_k = \frac{\partial S_k}{\partial i} \frac{i}{S_k}$. The aggregate elasticities obtained through these procedures are displayed in Table 7.4 below.

TABLE 7.4

AGGREGATE INTEREST ELASTICITIES OF CONSUMPTION
AND SAVINGS FOR CANADIAN HOUSEHOLDS IN 1978

Risk Aversion Parameter, v	$\epsilon_{C_{or}}$	ϵ_{Si}
.75	-28.25	.24
1.5	-17.82	.15
2.5	-13.98	.12

These estimates (of ϵ_{Si}) are of similar order, but somewhat lower than the estimates obtained by Beatty and Davies. They are substantially lower than the values calculated by Summers using stylized data on a multi-cohort population.

7.6 UNCERTAIN RATE OF RETURN

A fact of life not reflected in the model so far is uncertainty regarding the inflation-adjusted rate of return on savings. This uncertainty is considered by some analysts (e.g. Diamond, 1977) to contribute to a failure of capital markets sufficient to provide a rationale for the existence of public pensions. If policies can be designed which are capable of reducing rate of return uncertainty, they may be worth considering as possible lower-cost alternatives to expansion of the public pension system. Proposals for the provision of indexed bonds or for government action which would lead financial institutions to offer price-indexed pensions and annuities provide an example of this sort of policy.

The effect of uncertainty regarding the rate of return on savings has been examined by Phelps (1962) and Merton (1969) who obtained parallel results to those derived below. The approach taken here does not follow closely either of these prior papers.

Rate of return uncertainty is most conveniently introduced into the lifecycle consumption model in the case where all income derives from non-human capital (i.e. $L_0 = W_0, Y_t = 0$). For simplicity, it can also be assumed that the consumer's duration of life, T , is known and that the random interest factor r_t , once drawn from its probability distribution, is constant over time. The model is again solved by dynamic programming since the consumer's wealth at any time depends on the random interest rate. For n possible interest rate states, $\sum_{i=1}^n p_i = 1$,

$$E[W_t] = \sum_i p_i [r_i (W_{t-1} - C_{t-1})] \quad (7.23)$$

The solution for C_0 can be quickly sketched as follows:

$$E[Z_T] = \max_{C_T} a \frac{C_T^{1-v}}{1-v} = a \frac{E[W_T]^{1-v}}{1-v}$$

$$E[Z_{T-1}] = \max_{C_{T-1}} \frac{a^{T-1} C_{T-1}^{1-v}}{1-v} + E[Z_T]$$

$$\begin{aligned} \frac{\partial Z_{T-1}}{\partial C_{T-1}} = 0 &\rightarrow a^{T-1} C_{T-1}^{-v} = a^T \sum_{i=1}^n [p_i [r_i (W_{T-1} - C_{T-1})]^{-v} r_i] \\ C_{T-1}^{-v} &= a (\sum_{i=1}^n p_i r_i^{1-v}) (W_{T-1} - C_{T-1})^{-v} \end{aligned} \quad (7.24)$$

since $(W_{T-1} - C_{T-1})$ is constant in this expression.

$$\begin{aligned} \text{Thus, } C_{T-1} &= (a \sum_{i=1}^n p_i r_i^{1-v})^{-1/v} (W_{T-1} - C_{T-1}) \\ C_{T-1} &= \frac{W_{T-1}}{1 + (a \sum_{i=1}^n p_i r_i^{1-v})^{1/v}} \end{aligned}$$

and, by extension,

$$C_0 = \frac{W_0}{\sum_{t=0}^{T-1} (a \sum_{i=1}^n p_i r_i^{1-v})^{t/v}} \quad (7.25)$$

In the case of certain r , the expression corresponding to 7.24 is

$$C_{T-1}^{-v} = (ar^{1-v}) (W_{T-1} - C_{T-1})^{-v} \quad (7.26)$$

The "certainty equivalent" interest factor $\bar{r}$, can be defined as that certain interest factor which would result in the same consumption stream as results when r is uncertain. That is, when $r = \bar{r}$ in 7.26, equations 7.24 and 7.26 will yield the same C_{T-1} . Accordingly, $\bar{r}$ can be expressed as

$$\bar{r} = (\sum p_i r_i^{1-v})^{1/(1-v)} \quad (7.27)$$

Except when $v=1$, $\bar{r}$ will be less than the expected interest factor, $\bar{r} = \sum p_i r_i$. This is demonstrated in Figure 7.4 for the two cases $v > 1$ and $v < 1$. The effect of rate of return uncertainty on consumption can

FIGURE 7.4

CERTAINTY EQUIVALENT AND EXPECTED INTEREST RATES

now be analysed by examining the effect of a reduction in the interest factor (by the "risk premium", $\bar{r} - \bar{r}$) on C_0 . This is simply accomplished by examining the elasticity of C_0 with respect to r as developed in

$$\epsilon_{C_0, r} = \frac{(v-1)}{v} \frac{\sum_{t=0}^T t(ar^{1-v})^{t/v}}{\sum_{t=0}^T (ar^{1-v})^{t/v}} \quad (7.28)$$

Thus, $\epsilon > 0$ as $v > 1$ and $\epsilon = 0$ when $v=1$. This means that interest rate uncertainty leads to a decline in C_0 when $v > 1$ but an increase in C_0 when $v < 1$. The dependence of the effect of interest rate uncertainty upon v can be explained in terms of income and substitution effects. The effective decline in r produced by uncertainty reduces the lifetime consumption possibilities associated with a given W_0 and thus tends to reduce C_0 . However the same decline in r has an offsetting substitution effect which favours present over future consumption. For relatively risk averse consumers (with $v > 1$), the income effect dominates so C_0 declines. For less risk averse consumers (with $v < 1$) for whom the intertemporal elasticity of substitution ($1/v$) is relatively high, the substitution effect dominates, and interest rate uncertainty tends to stimulate current consumption. This is illustrated by a simple indifference curve analysis in Figure 7.5 where current consumption declines from x to y for the more risk-averse consumer while rising from x to z for the less risk-averse consumer.

4

FIGURE 7.5

INTEREST RATE UNCERTAINTY AND CONSUMPTION
WHEN $L_0 = W_0$

When the model is generalized to include non-capital income ($Y_t > 0$), the effect of rate of return uncertainty becomes more complicated. No analytical expression can now be derived for C_0 although Phelps (1962) obtained a solution for the special case where Y_t is constant. His conclusions were essentially the same as those presented here.

The relationship between $\hat{r}$ and $\bar{r}$ is unchanged as is the substitution effect of a change in r . However, a reduction in the effective interest rate now has two offsetting income effects. The consumer's set of attainable annual consumption levels now depends on

both the consumption possibilities obtainable from W_0 and those obtainable from $\sum_{t=0}^T Y_t r^{-t}$. The former possibilities are reduced by a drop in r while the latter are expanded. Compared to the case of $L_0 = W_0$, it is now more likely that current consumption will rise as a result of the effective interest rate decline attributable to uncertainty.

To complete the discussion of the effect of rate of return uncertainty on current consumption, it remains to obtain some idea of the degree of uncertainty actually faced by Canadian consumers. This is obviously a very difficult magnitude to determine. A starting point is provided by estimates of the annual average real rates of return on various financial instruments prepared by Carty and Pressman (Lazar Report, 1980, vol. II, Appendix 10). These estimates cover the period 1952 to 1978 and include capital gains and losses (including those due to inflation) as well as nominal interest or dividend yields. They make no allowance for taxes though, for they were developed principally to derive an estimated average real rate of return on private pension plan portfolios. (The average rate so derived was $r=1.035$). For a selection of the instruments, Table 7.5 shows the range of the annual rates of return, the average annual rate of return, $\bar{r}$, and the standard deviation of the annual rates, σ_r . In addition, the table provides values for the certainty equivalent rates of return $\hat{r}$, for $v=0.5$ and $v=2.5$, as well as the corresponding values of the risk premium, $\bar{r}-\hat{r}$.

TABLE 7.5

VARIABILITY OF REAL RATES OF RETURN
FOR SELECTED FINANCIAL ASSETS, 1952-1978

	Financial Instruments				
	3-Month Treasury Bills	Conventional Mortgages	Long-Term Government of Canada Bonds	MacLeod Young Weir 10 Indus- trials	Stocks
Range of r					
- low r	.9710	.9580	.8744	.8511	.7850
- high r	1.0376	1.0868	1.1325	1.0845	1.3202
- range	.0666	.1288	.2581	.2334	.5352
Average, $\bar{r}^{(a)}$	1.0091	1.0352	1.0011	1.0075	1.0529
Standard Deviation, σ_r	.0161	.0251	.0512	.0611	.1201
Certainty Equivalent, $\hat{r}$					
$v=0.5$	1.0091	1.0350	1.0004	1.0066	1.0494
$v=2.5$	1.0088	1.0344	.9978	1.0027	1.0355
Risk Premium, $\bar{r} - \hat{r}$					
$v=0.5$	.0000	.0000	.0000	.0009	.0035
$v=2.5$	.0003	.0008	.0033	.0048	.0174

Note (a) arithmetic average

Source: Calculated from estimates in Carty and Pressman, Appendix 10, Lazar Report (1980), vol. II, Table 1, p. 10-5.

The table illustrates several things. To begin with, the range of annual returns varies from a low of 6.7% for Treasury Bills to a high of 53.5% for stocks. Negative real rates of return were recorded by all instruments in at least one year. The standard deviations of the annual rates ranged from 1.61% for Treasury Bills to 12.0% for stocks. The average annual rates of return also varied significantly, from 0.11% for long-term Government of Canada bonds to 5.29% for stocks. There was a correlation between risk (σ_r) and return (r) when debt and equity instruments were compared but not within the class of debt instruments. The degree of variation in annual returns is reflected in the risk premiums which the model predicts would be reflected in the savings behaviour of consumers with different levels of risk aversion. As one would expect, the less risk averse consumer has considerably lower risk premiums than his more risk averse counterpart. The risk premiums for the risk averse consumer are negligible for Treasury Bills and Mortgages but rise to 0.33% for long-term government bonds, 0.48% for corporate bonds, and 1.74% for stocks. To derive some idea of the appropriate risk premium for a typical consumer (given v), one must consider the effects of several complicating factors. First, the annual rates estimated by Carty and Pressman are averages of portfolios of assets within each class of instrument. Thus an individual purchasing a small portfolio of stocks or bonds, may be faced with a much higher level of uncertainty regarding the annual rate of return for any type of instrument than is evident here. On the other hand, the averaging of interest rates over multi-year holding periods will lead to a substantial reduction in uncertainty. The accumulation of a portfolio of several instruments will also reduce uncertainty although considerable correlation between the annual returns

for different instruments is evident in the Carty and Pressman estimates.⁵ It must also be recognized that no estimate of rate of return variability is provided here for equity in the home, the most important asset in household wealth portfolios. Taking all these factors into account, one can only conclude that the average rate of 3.5% employed in the Lazar Report is likely to overstate the certainty equivalent rate appropriate for household wealth portfolios. For a risk averse consumer ($v=2.5$), a reasoned guess at an average risk premium might be one-half of one per cent.

7.7 UNCERTAIN FUTURE INCOME

Uncertainty regarding future income (earnings plus transfer income) is another potentially important determinant of household savings behaviour. Diamond (1977) suggests that this uncertainty (especially the risk of earlier than planned retirement because of ill-health or unemployment) could provide an important rationale for public pensions. Unexpectedly low income at age 55-64, say, can have

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5. It is natural to assume that low income, low wealth holders will face greater rate of return uncertainty than those with bigger, presumably more diverse asset portfolios. However, this appears to be offset by the tendency of low wealth holders to concentrate their financial asset holdings on highly liquid (low return) assets such as bank deposits and Canada Savings Bonds. The proportion of financial assets held in such liquid assets declines monotonically over income and wealth levels from about 90% to less than 40%. (Source: Statistics Canada, catalogue 13-570, The Distribution of Income and Wealth in Canada, 1977, Tables 10 and 13.) At the same time, this portfolio composition information is probably consistent with the hypothesis of a positive correlation between certainty-equivalent pre-tax rates of return and income or wealth levels.

a very substantial effect on the level of resources that an individual is able to accumulate to finance consumption in his later years of life. Private insurance against the risks of such an income loss exists but only in a very limited form because of the moral hazard problems associated with this kind of insurance (i.e. the difficulty of objectively assessing the inability to work or find employment).

The effect of future income uncertainty on current consumption and saving is most conveniently examined using a two-period model, where C_0 and C_1 are optimally chosen subject to given $W_0 + Y_0$ and uncertain Y_1 . The first order condition, analogous to equation (7.24) in the case of rate of return uncertainty, is

$$C_0^{-v} = ar \sum_{i=1}^n p_i [W_1 + Y_{1i}]^{-v} \quad (7.29)$$

or, more compactly,

$$C_0^{-v} = ar \sum_{i=1}^n p_i R_i^{-v} \quad (7.30)$$

where period 1 resources, $R_i = W_1 + Y_{1i}$ with

$$\bar{R} = \sum_{i=1}^n p_i (W_1 + Y_{1i}).$$

Again a certainty-equivalent level of period 1 resources, $\hat{R}$, can be defined as the certain level of R in

$$C_0^{-v} = ar \hat{R}^{-v} \quad (7.31)$$

which would lead to the same choice of C_0 as in the uncertain case (7.30). The resulting expression for $\hat{R}$ is:

$$\hat{R} = \left[\sum_{i=1}^m p_i (W_1 + Y_{1i})^{-v} \right]^{-1/v} \quad (7.32)$$

An examination of $\hat{R}$ and $\bar{R}$, parallel to that made of $\hat{r}$ and $\bar{r}$ in the preceding section, shows that $\hat{R} < \bar{R}$ so that again uncertainty can be described in terms of a risk premium which must be deducted from observed values of R to obtain certainty equivalent levels. Note that the size of the risk premium depends not only on the $p_i Y_{1i}$ but on the level of W_0 and Y_0 (via their influence on W_1) as well. The effect of this reduction in the "effective" level of Y_1 on current consumption is much simpler than in the case of rate of return uncertainty. Intertemporal substitution possibilities are unaffected and there is an unambiguous income effect. A reduction in effective future income results in a reduction in current consumption and an increase in savings. In the slightly more general case of a multi-period model where only one of the future Y_t is subject to change or uncertainty, the elasticity of C_0 with respect to the Y_t is given by:

$$\varepsilon_{C_0, Y_t} = \frac{Y_t r^{-t}}{W_0 + \sum_{t=0}^T Y_t r^{-t}} \quad (7.33)$$

Thus the elasticity is unambiguously positive and its magnitude depends directly on the size of discounted Y_t as a fraction of total lifetime resources.

The case where all Y_t ($t=0, \dots, T$) are drawn from a known and constant probability distribution is analytically much more complex. However, Miller (1974) has demonstrated that the C_0 obtained when all the Y_t take their expected values will be an upper bound, or, in other words, that the income uncertainty will depress current consumption except in a highly improbable boundary case. In practice we would expect the two period case to provide a rough guide to the effects of uncertainty in several future periods since we would expect that, in many instances, prediction errors for a particular individual ($Y_t - \bar{Y}_t$; $t=0, \dots, T$) would be positively correlated. For example, the effects of unemployment, disability or an unexpected opportunity on income levels will often persist for a number of years.

To provide some basis for judging the quantitative significance of uncertainty regarding future income, an analysis was made of the variability of ~~non~~ investment income (i.e. earnings plus government transfers) for working age men and women disaggregated into age, family size, education and occupation classes. The disaggregation by age, education and occupation classes reflects some at least of the reduction in income uncertainty which results from individuals' knowledge of their human capital levels. No attempt was made to examine the variability of the total income of multi-earner husband-wife families. Within-class income variances were calculated from Survey of Consumer Finance microdata on 1977 family incomes.⁶ For males the

6. Statistics Canada, 1978 Survey of Consumer Finances microdata tape, "Economic Families, 1977 Income".

structure of disaggregation was as follows: (1) family size - one person, two or more persons; (2) education - university degree, post-secondary with certificate or diploma, grades 11-13 and incomplete post-secondary, grades 9-10, less than grade 9;

(3) occupation - managerial, professional/technical, sales, clerical and service, blue collar plus farming and fishing; (4) age - <25, 25-34, 35-44, 45-54, 55-59, 60-64. For women, the family size variable was further split by a "presence of children under the age of twelve" variable.

The sub-group of greatest numerical and income importance is males in multi-person families. For this group the average coefficient of variation (CV) across education, occupation and age groups was 43%, resulting from an average standard deviation of \$7463 on an overall mean income of \$17160. The CV's were about 42% for the under 55 age groups and over 50% for the 55-64 age group.⁷ Within this group of males, there was no evidence that the severity of income uncertainty varies with income level. (When the CV's were regressed on class income means for the 65 most populated education/occupation/age classes, both the $\bar{R}^2$ and the coefficient of the independent variable were approximately zero). For men in single-person families, mean incomes were about 25% lower than for men in multi-person families

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7. The estimated level of income uncertainty is understated for the 55-64 group, because the within-occupation variances do not reflect the shift from working to retired status in this age group.

and the CV's were substantially higher, averaging about 60%. For women, income variation was generally greater than for men with CV's of about 60% for those without children and CV's of over 90% for those with children. Income variation reflects both choice and chance, and it seems likely that choice plays a bigger role in the variation in married women's income than it does in the incomes of men.

With this information on income variation as background, consider the implications for the current savings and consumption of an individual who makes the following assessment of his post-65 resources:

<u>annuity level</u>	<u>probability</u>
\$ 6,000	.3
\$12,000	.7

The \$6000 level of public/private pension income might reflect, for example, the contingency of forced early retirement because of ill-health. This distribution of annuity levels has a mean of \$10,200 and a standard deviation of \$2750 for a CV of 27%. For a risk aversion level $v=2.5$, the risk premium $\bar{R}-\bar{R}$, derived from (7.32) would be \$1741 or 17% of the expected annuity level. This example suggests that, even with considerably less income uncertainty than is indicated by the variability of male and female earnings (e.g. CV=27% vs. CV>40%), uncertainty regarding future income levels could have a substantial effect on consumption and savings levels.

7.8 AVAILABILITY OF ANNUITIES

In this section, an alternative version of the uncertain lifetime model is examined in which the consumer is assumed to have the option of investing in actuarially fair annuities. An annuity is a stream of income which continues as long as the recipient survives. In this broad sense, public pensions and many private pensions are annuities. Thus annuity income can be represented in the model simply as a stream of income which extends to year T (but which cannot be borrowed against in the absence of other wealth). A private annuity can also be considered as a tradeable asset which can be distinguished from other assets by its rate of return and by the fact that its capital value becomes zero upon the death of the holder. The rate of return will be higher than that earned by other assets because of the fact that the income stream derived from the asset (the annuity payments) exhausts the principal as well as reflecting the regular interest earned on it. If the perfect capital market assumption is extended to include "fair" annuities, then the interest factor for an annuity will be $\frac{s_{t-1}}{s_t} r$ which depends on the age of the holder but always exceeds r . The solution of the lifecycle consumption model when both regular assets and fair annuities are available to the consumer is again due to Yaari (1965).

For the case with no bequest motive, the consumer's utility is maximized by a policy of investing all wealth in annuities. The logic of the result may be appreciated by noting that only through this policy can unintended bequests, which are valueless to the

consumer, be eliminated. The dynamic programming derivation of the solution for the discrete time version of Yaari's model is provided in Appendix 7.B. The optimal consumption path is given by

$$C_t = (ar)^{t/v} C_0 \quad (7.34)$$

and base period consumption is defined by the equation

$$C_0 = \frac{W_0 + \sum_{t=0}^T Y_t s_t r^{-t}}{\sum_{t=0}^T s_t (ar^{1-v})^{t/v}} \quad (7.35)$$

Note that (7.34) is identical to (7.3) and that (7.35) is similar to (7.4).⁸ Thus, the presence of a complete annuity market permits the consumer to attain almost the same consumption profile as in the case where longevity is certain.

For the case with a bequest motive, the utility maximizing consumer saves for future consumption entirely by purchasing annuities. Saving for bequests, though, is necessarily accomplished by the purchase of non-annuity assets. Again the presence of a complete market in fair annuities permits the costs of uncertain longevity to be eliminated.

8. The denominators of (7.4) and (7.35) differ slightly except if $s=r=1$ which results in $\sum_{t=0}^T s_t (ar^{1-v})^{t/v} = T+1$. The numerators also differ, except when $L=W_0$, since future income is discounted at the actuarial rate of interest in (7.35).

FIGURE 7.6

POST-65 CONSUMPTION PROFILES
WITH AND WITHOUT ANNUITIES

In the "no bequest motive" case, the existence of a fair annuity market permits a substantial increase in consumption in the later years of life (and a corresponding reduction in unintended bequests). To give some idea of the extent of this effect, Figure 7.6

shows the optimal consumption profiles for a consumer in the presence or absence of an annuities market. The consumer is assumed to have no rights to public or private pension income ($Y_t=0$ for all t) but to have sufficient wealth to purchase an annuity, graduated according to $(ar)^{t/v}$, where $a \geq 1$ and $r=1.035$, which pays \$5000 at age 65. Two cases are considered: that of a consumer with medium-high risk aversion ($v=2.5$) and that of a low risk aversion consumer ($v=1$). For the case where $v=2.5$, the cost of the annuity is \$60,695 payable at age 65. With this annuity the individual's planned level of consumption rises from \$5000 at age 65 to over \$8500 at age 107. If the consumer had the same wealth level but no access to fair annuities, his optimal planned consumption would start at \$3545, rise slightly to \$3559 at age 68, and then decline continuously to the end of the planning horizon. For the less risk-averse consumer with the same wealth at 65, consumption at age 65 would be \$4200 with or without annuities, but would diverge more sharply in later years than in the case of the more risk-averse consumer.

These model solutions can be used to determine the value of the fair annuity market to the consumer. First, the utility levels reached, with and without annuities, can be calculated by evaluating the expression for expected lifetime utility when all the C_t take their optimal values. The utility levels reached with annuities exceed those reached in the no-annuity cases. Next, one can calculate the wealth that would be required to reach the annuity-case utility levels in the absence of annuities. As demonstrated by Kotlikoff and Spivak (1981), this can be most easily accomplished by the evaluation

of indirect utility functions. An indirect utility function is an expression for utility in terms of the given parameters in the optimization problem (i.e., W_0, Y_t, r, a , and v). It can be obtained by substituting the optimal consumption levels (the C_t) into the expression for expected utility. For the iso-elastic utility function ($v \neq 1$), with and without annuities, the indirect utility functions are:

$$Z_0^A = \frac{L_0^{1-v}}{1-v} \left[\sum_{t=0}^T s_t (ar^{1-v})^{t/v} \right]^v \quad (7.36)$$

$$Z_0 = \frac{L_0^{1-v}}{1-v} \left[\sum_{t=0}^T s_t^{1/v} (ar^{1-v})^{t/v} \right]^v \quad (7.37)$$

Now let kL_0 be the wealth required in the absence of annuities to achieve the same utility as achieved with L_0 when annuities are available. That is, $Z_0(kL_0) = Z_0^A(L_0)$. For given s_t, a, r and v , the value of k can be determined as

$$k = \left(\frac{\theta^A}{\theta} \right)^{1/1-v} \quad (7.38)$$

where θ^A and θ are the bracketed expressions in (7.36) and (7.37) respectively. Note that k is independent of L_0 ; with the iso-elastic utility function, the value of access to annuities does not depend on the consumer's wealth level. For the case of $v=2.5$, $k=1.774$, so \$177,740 in non-annuity wealth would be required to replace \$100,000 worth of annuities. For $v=1$, $k=1.661$, so, for the less risk averse consumer, a smaller amount of \$166,100 in non-annuity wealth would replace \$100,000 in annuities.

An interesting extension to this analysis is provided in the article by Kotlikoff and Spivak. They note that the pooling of consumption expenditure among family members (of the same generation) results in a pooling of the risks of unexpected longevity. Their findings indicate that (for $v=2.5$) the risk-pooling inherent in marriage has a dollar value at age 65 equal to about thirty per cent of the value of a complete annuities market.

The main prediction of this version of the model, that all wealth not accumulated to finance bequests will be held in annuity form, provides a basis upon which to examine both the realism of the model of section 7.2 and the common assertion that annuity markets are very incomplete. Before some information on the importance of annuity vs. non-annuity income is examined, three points deserve mention. First, no information exists on desired bequest levels. Moreover, desired bequests cannot be inferred from actual bequests except by making assumptions concerning the completeness of the annuities market for, as demonstrated in the previous section, unintended bequests are a result of an incomplete annuities market. Second, if other savings motives (e.g. speculative or various precautionary motives) are admitted, then consumers can be expected to hold non-annuity assets even in the absence of a bequest motive. Finally, there are reasons to expect incompleteness of annuity markets. For one thing, the existence of transactions costs and uncertainty regarding future rates of return means that all capital markets are imperfect. In addition, the fact that annuity buyers may have an advantage over sellers in predicting their own longevity means that

annuity markets will be shaped and limited by the effects of adverse selection (i.e. the relatively strong incentive for those with above-average expected longevity to purchase annuities).

TABLE 7.6
ANNUITY AND NON-ANNUITY INCOME
OF THOSE OVER AGE 65, 1978

<u>Income Component</u>	<u>Amount</u> (\$millions)	(%)
<u>Public Pensions</u>		
<u>OAS/GIS/SPA</u>		
- reported by RCT	2560	
- unreported (a)	2820	
<u>C/QPP</u>		
- reported by RCT	978	
- unreported (a)	634	
Total	7092	40
<u>Private Annuities</u>		
Private pensions	1586	
<u>Annuities</u>	125	
Total	1711	10
Other Investment Income	4614	26
Earnings	1953	11
Imputed Rent (b)	2428	13
<u>Total Income</u>		100

Sources: All figures from Revenue Canada, Taxation, Taxation Statistics, 1980 (1978 Incomes), Table 4, except:

- (a) unreported OAS/GIS/SPA and C/QPP income estimated from unpublished program data. Unreported income from other sources is ignored in this table.
- (b) calculated as 11% of total home equity as estimated from data reported in Statistics Canada (catalogue 13-570) The Distribution of Income and Wealth in Canada, 1977.

Some information on the importance of annuity-form income for those over 65 in 1978 is provided in Table 7.6. These estimates suggest that total annuity-form income amounted to about 50% of the group's total income. Imputed rental income from home-ownership accounted for 13% of the total; earnings 11% and other non-annuity investment income, 26%. Over 80% of the annuity-form income came from public pension programs. The balance of the annuity income consisted mainly of private pension income with individual annuity contracts providing less than 2% of total annuity-form income.⁹ Several inferences may be drawn from this information. First, while annuity income forms a substantial part of the total income of those over 65, it does not dominate other income sources to the extent that the model of this section predicts. That is, imputed rent and other non-annuity investment income appear too important to be explained purely by bequests and speculative or precautionary savings motives in the presence of an option to purchase fair annuities. Second, the conclusion that private annuity markets are highly imperfect is supported by two observations:

- the relative importance of public pension income compared to private annuity income;
- the unimportance of private pension and annuity income compared to investment income from homeownership and other assets.

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9. The rapid growth in RRSP contributions, together with the requirement that RRSP wealth be converted to some form of annuity before the holder reaches age 72, suggests that individual-contract annuities will become more important in the future.

The first observation suggests that governments have moved to overcome a perceived market failure in private annuity markets. Given the history of private pension contributions, the second observation suggests that contributors may often choose when they can to obtain their pension rights in capital amounts for personal investment rather than in annuity form. It thus adds to the evidence of annuity market imperfections. Another interesting observation is that, in private annuity markets, individual annuity contracts are very unimportant relative to the group annuities provided through private pensions. This suggests that the adverse selection problem is a real one, and also indicates that privately organized, compulsory group annuities (pensions) provide at least a partial solution to it.

When combined with the large wealth gains that were shown to result from the provision of a complete annuities market, these observations suggest that considerable scope probably remains for improving social welfare via measures which either provide greater public pensions or permit improvements in the operation of private markets for individual and group annuities. Measures aimed at encouraging a reverse mortgage market, which permits the annuitization of equity in the home without loss of title prior to death, have been proposed as one way of meeting this need.

With regard to the development of a model for policy simulation purposes, the evidence provided above suggests that the model with the availability of a fair and complete annuities market assumed is less realistic than the model of section 7.2, in which uninsurable

uncertainty regarding longevity is a cause of wealth accumulation and unintended bequests. In applying that model, the rights to public and private pension income can be reflected explicitly in the Y_t .

7.9 COMPARATIVE EFFECTS OF TWO POLICY MEASURES AIMED AT INCREASING CONSUMPTION IN RETIREMENT

One strength of the model developed in this chapter is that it can be used to simulate the effects of tax or public pension provisions on the consumption behaviour and welfare levels of individual households differentiated by age, income, wealth and family size. Thus the model provides a tool which can be used to examine both the efficiency and distributional effects of possible policy changes. Time constraints have prevented any full-scale application of the model in this fashion as part of this thesis. However, the present section provides an illustrative policy analysis for a single representative household. The case is that of a married worker of age 45 whose income profile is that portrayed in Figure 7.2. This income profile represents that of a blue collar worker with high school graduation whose wife does not work.

The model employed includes public and private pensions in the anticipated income stream, Y_t , and assumes that the household does not have the option of converting ordinary assets into fair annuities. No bequest motive is included. The model also includes the wealth constraint of section 7.2. In the context of the policy

simulations, this means that where the constraint is effective (eg. at age 90), an increase in Y_t is reflected in an equal increase in C_t . No part of this income increase can be consumed in other years.

Two policy comparisons are made. In the first, the effects of three transfers to the household are compared without allowance for the tax increases that would be necessary to fund the transfers. The three transfers, which are designed to have identical present value revenue costs, are as follows:

- ΔW_{45} - a simple increase in the household's wealth at age 45;
- Δr - an increase in the net-of-tax rate of return by 0.5%, from 3.5% to 4.0%;
- ΔY_p - an increase in public pension benefits.

The increase in public pension benefits is spread over the post-65 ages in accordance with the assumption that all public pensions are indexed at a rate of 2% per annum in real terms to maintain their relativity to wages. The comparison is made assuming two different values, 0.75 and 2.5, for the risk aversion parameter, v . When $v=0.75$ each of the transfers has a present value cost of \$11,867; when $v=2.5$ the cost is \$10,022. For the public pension alternative this is an expected revenue cost; the realized cost will depend on the longevity of the husband and wife. Several sets of simulation results are set out below.

First, the changes in expected levels of pre- and post-65 consumption are compared. Also a measure of the target-efficiency of the public pension increase is presented. This measure is the ratio of the present value of the increase in post-65 consumption levels to the present value of the corresponding pension income increases.

	Risk Aversion Parameter, v	
	<u>0.75</u>	<u>2.5</u>
Change in pre-65 consumption (%)		
ΔW_{45}	2.15	1.76
Δr	-1.97	0.58
ΔY_p	3.57	3.51
Change in post-65 consumption (%)		
ΔW_{45}	2.09	1.75
Δr	9.02	4.20
ΔY_p	4.17	3.66
Target-efficiency of ΔY_p	.419	.395

Several observations may be made on these results. First, the interest rate subsidy (or investment income tax cut) results in the greatest increase in retirement consumption. Also, its advantage over the other policy options in this respect is substantially greater when $v=0.75$ than when $v=2.5$. Second, the wealth increases occurring when wealth at age 45 or public pensions are increased is allocated both to both pre-65 and post-65 consumption roughly in proportion to existing consumption levels. In the case of ΔY_p , this means that the pension income increase is only about 40% effective in raising consum-

ption in retirement. In the case of Δr , there is a substitution of future for present consumption and, when $v=0.75$, this outweighs the positive income effect so pre-65 consumption actually drops.

Second, it is interesting to compare the changes in the household's utility level that result from the three policy options. Note that the utility changes assuming $v=0.75$ can't be compared with those based on $v=2.5$.

$v =$	<u>0.75</u>	<u>2.5</u>
<u>Change in Utility level</u>		($\times 10^{-6}$)
ΔW_{45}	7.7	.131
Δr	8.9	.157
ΔY_p	15.2	.280

The utility increases obtained from ΔW and Δr are similar and substantially lower than those resulting from ΔY_p . One reason for the difference in utility gains may be seen by examining the changes in the levels of unintended bequests which, it will be recalled, are valueless to the household. For $v = 0.75$, the changes in bequest levels are: for ΔW_{45} , + \$4810; for Δr , + \$18,326; and for ΔY_p , - \$10,783. (These values weren't calculated for $v = 2.5$.) Thus by increasing the fraction of lifetime resources which accrues in

annuity form, the increase in public pensions permits a decline in valueless bequests and an corresponding increase in utility.¹⁰

A second type of policy analysis considers revenue-neutral changes. As a simple proxy for this type of comparison, the changes Δr and ΔY_p were offset by a reduction in wealth at age 45. Only the case of $v = 2.5$ was simulated. The results are as follows:

	<u>Δr</u>	<u>ΔY_p</u>
Change in pre-65 consumption (%)	-1.29	1.74
Change in post-65 consumption (%)	2.28	1.90
Change in utility level ($\times 10^{-6}$)	.016	.154
Change in bequest (\$)	\$3832	-18,557

Again the interest rate subsidy is found to be most efficient in increasing post-retirement consumption while the increase in public pensions provides a significantly higher utility gain to the household.

7.10 SUMMARY OF FINDINGS

In this chapter, a multi-period lifecycle model of consumption has been developed which incorporates uncertain lifetime and a non-negative wealth constraint. An elaboration of the model to permit

10. Clearly the welfare gain from ΔY_p would be lower if the utility function employed here were replaced by one which included the welfare of heirs as an argument. In such a model, the risk of unexpected longevity is pooled between the consumer and his heirs, bequests are no longer valueless, and a smaller welfare gain would derive from the reduction of unintended bequests.

the consumption of multi-person consuming units to be considered has also been provided. Models with certain lifetime, bequests, and fair annuities have been examined for purposes of comparison. The model has been employed to investigate: (a) interest elasticities of consumption and savings, (b) uncertainty regarding longevity, the rate of interest, and the level of future income, and (c) alternative policy measures aimed at increasing retirement consumption. The main findings are summarized below.

Uncertain longevity tends to depress planned consumption in later years when survival probabilities are relatively low. In contrast, the opportunity to raise lifetime consumption by deferring consumption when the interest rate exceeds the consumer's subjective time preference rate tends to produce an increasing age profile of consumption. When these two tendencies are combined, a "humped" age profile of consumption emerges. The concentration of consumption in the peak consumption years close to retirement age is greatest for consumers with low levels of risk aversion. These consumers are most sensitive to lifetime consumption-increasing opportunities and least sensitive to the risk of low consumption levels resulting from unexpected longevity. The fact that the consumer, in the absence of a perfect private annuity market or the complete replacement of one by public pensions, must self-insure against unexpected longevity means that a substantial level of bequests will be observed even in the absence of any bequest motive.

When the model is modified to represent the consumption of a family with children at home up to the parent's age of 55 or 60, the result is an advance in peak consumption to a younger age and a considerable decline in the level of wealth at retirement.

Elasticities of savings to the rate of interest varied from 2.8 to 0.1 for middle income families depending on their ages and risk aversion levels. The elasticities tended to be greatest for young families and families with low risk aversion levels. Aggregate interest elasticities of savings of .12 to .24 (depending on v) were estimated for the Canadian household sector in 1978. These elasticities are somewhat lower than those estimated by Boskin, Summers, and Davies.

Uncertainty regarding the interest rate and future income can be represented by certainty equivalent values of these variables which are lower than their expected values. Uncertainty regarding future income has only an income effect and produces an unambiguous decline in current consumption. An increase in interest rate uncertainty leads to an increase in current consumption (decline in savings) in most but not all cases; the strength of the consumption response depends on the income and substitution effects embodied in the interest elasticity of consumption. Evidence regarding the variability of annual real rates of return on financial assets suggests that the difference between the expected and certainty equivalent interest rates might be as high as one-half of one per cent for a consumer with $v=2.5$.

Following Yaari, it was demonstrated that a consumer with no bequest motive, who can invest in actuarially fair annuities, will invest only in annuities. The existence of a perfect annuities market would permit the consumer to completely avoid the cost of uncertain longevity and attain the same lifetime consumption profile as a consumer with certain longevity. The value of a perfect annuities market was determined by comparing indirect utility functions with and without annuities. Depending on the level of risk aversion, the introduction of a perfect annuities market would be as valuable as a 66% to 78% increase in non-annuity wealth for a 65 year old consumer with no claims to annuity-form income. Evidence on the importance of public and private annuity-form income in Canada suggests that private annuity markets are not well developed. Private annuity income accounts for only ten per cent of the incomes of Canadians over 65 years of age. Based on this and other observations, the assumption of an incomplete annuities market is accepted as a basis for modeling and estimating household consumption.

Finally, a comparison was made of two policy measures designed to increase retirement consumption: an increase in the after-tax interest rate and an increase in public pension income. The former measure is found to be the more effective in raising retirement consumption; however, the latter provides the consumer with a greater welfare gain.

APPENDIX 7.A

SOLUTION OF THE LIFECYCLE CONSUMPTION

MODEL WITH UNCERTAIN LIFETIME

The notation is as defined in the body of Chapter 7.

Expected lifetime utility,

$$E(U) = \sum_{t=0}^T s_t a^t \frac{C_t^{1-v}}{1-v} \quad (7.5)$$

is maximized subject to the condition

$$W_t = r(W_{t-1} + Y_{t-1} - C_{t-1}) \geq 0 \quad (7.6)$$

which must hold for all t . In this derivation of the model solution, the wealth constraint is assumed not to be binding until $\bar{t}=T$. The dynamic programming solution depends on Bellman's (1956) principle of optimality which states that the consumer can choose an optimal consumption level in period t by equating the marginal utility of C_t to the expected marginal valuation of wealth over the $T-t$ periods remaining in the planning horizon, given that the $C_{t+1}, C_{t+2} \dots C_T$ are themselves optimally chosen. Accordingly, the derivation proceeds in the following manner. The optimal consumption level, C_T , in the last period of the planning horizon is (trivially) chosen as a function of W_T . The indirect utility function, $Z_T = \max U(W_T)$, is then derived. Next, the optimal choice of C_{T-1} given W_{T-1} and Z_T is made, and from this is derived Z_{T-1} . By comparing the optimal C_T and C_{T-1} , the time path of the C_t can be determined. From the expressions for C_{T-1} or C_{T-2} , the expression for C_t and C_0 (base year consumption) can be obtained by extension.

$$Z_T = \max \left\{ s_T a^T \frac{C_T}{1-v} \right\} = s_T a^T \frac{C_T}{1-v} (W_T + Y_T)^{1-v}$$

$$\begin{aligned} Z_{T-1} &= \max \left\{ s_{T-1} a^{T-1} \frac{C_{T-1}}{1-v} + s_T a^T (W_T + Y_T)^{1-v} \right\} \\ &= \max \left\{ s_{T-1} a^{T-1} \frac{C_{T-1}}{1-v} + s_T a^T [r(W_{T-1} + Y_{T-1} - C_{T-1}) + Y_T]^{1-v} \right\} \end{aligned}$$

$$\frac{\partial Z_{T-1}}{\partial C_{T-1}} = 0 \Rightarrow s_{T-1} a^{T-1} C_{T-1}^{-v} = s_T a^T C_T^{-v} r$$

$$C_T = \left(\frac{s_T}{s_{T-1}} \right)^{1/v} (ar)^{1/v} C_{T-1} \quad (7.7)$$

But, $C_T = r(W_{T-1} + Y_{T-1} - C_{T-1}) + Y_T$

Thus, $\left(\frac{s_T}{s_{T-1}} \right)^{1/v} (ar)^{1/v} C_{T-1} = r(W_{T-1} + Y_{T-1} - C_{T-1}) + Y_T$

$$\left[r + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} (ar)^{1/v} \right] C_{T-1} = rW_{T-1} + rY_{T-1} + Y_T$$

(÷r+simplify)

$$C_{T-1} = \frac{W_{T-1} + Y_{T-1} + Y_T r^{-1}}{1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} (ar^{1-v})^{1/v}}$$

$$\begin{aligned}
 \text{Thus, } Z_{T-1} &= s_{T-1} a^{T-1} \frac{C_{T-1}^{1-v}}{1-v} + s_T a^T \left[\left(\frac{s_T}{s_{T-1}} \right)^{1/v} (ar)^{1/v} C_{T-1} \right]^{1-v} \\
 &= s_{T-1} a^{T-1} \frac{C_{T-1}^{1-v}}{1-v} \left[1 + \left(\frac{s_T}{s_{T-1}} \right) a \left(\frac{s_T}{s_{T-1}} \right)^{\frac{1-v}{v}} (ar)^{\frac{1-v}{v}} \right] \\
 &= s_{T-1} a^{T-1} \frac{C_{T-1}^{1-v}}{1-v} \left[1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} (ar^{1-v})^{1/v} \right]
 \end{aligned}$$

Let $x = (ar^{1-v})^{1/v}$

$$\begin{aligned}
 \text{Then } Z_{T-1} &= \left(1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} x \right) s_{T-1} a^{T-1} \frac{C_{T-1}^{1-v}}{1-v} \\
 &= \left(1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} x \right) s_{T-1} \frac{a^{T-1}}{1-v} \left[\frac{W_{T-1} + Y_{T-1} + Y_T r^{-1}}{1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} x} \right]^{1-v} \\
 &= \left(1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} x \right)^v s_{T-1} \frac{a^{T-1}}{1-v} [W_{T-1} + Y_{T-1} + Y_T r^{-1}]^{1-v}
 \end{aligned}$$

Next consider

$$\begin{aligned}
 Z_{T-2} &= \max \left\{ s_{T-2} \frac{a^{T-2}}{1-v} C_{T-2}^{1-v} + Z_{T-1} \right\} \\
 &= \max \left\{ s_{T-2} \frac{a^{T-2}}{1-v} C_{T-2}^{1-v} + \right. \\
 &\quad \left. \left(1 + \left(\frac{s_T}{s_{T-1}} \right)^{1/v} x \right)^v s_{T-1} \frac{a^{T-1}}{1-v} [W_{T-1} + Y_{T-1} + Y_T r^{-1}]^{1-v} \right\}
 \end{aligned}$$

Recall

$$W_{T-1} = r(W_{T-2} + Y_{T-2} - C_{T-2})$$

Thus,

$$\frac{\partial Z}{\partial C_{T-2}} = 0 \implies$$

$$s_{T-2} a^{T-2} C_{T-2}^{-v} = \left(1 + \left(\frac{s_T}{s_{T-1}}\right)^{1/v} x\right)^v s_{T-1} a^{T-1} r [W_{T-1} + Y_{T-1} + Y_T r^{-1}]^{-v}$$

$$W_{T-1} + Y_{T-1} + Y_T r^{-1} = \left(1 + \left(\frac{s_T}{s_{T-1}}\right)^{1/v} x\right) \left(\frac{s_{T-1}}{s_{T-2}}\right)^{1/v} (ar)^{1/v} C_{T-2} \equiv \theta C_{T-2}$$

$$\theta C_{T-2} = r(W_{T-2} + Y_{T-2} - C_{T-2}) + Y_{T-1} + Y_T r^{-1}$$

$$C_{T-2} = \frac{W_{T-2} + Y_{T-2} + Y_{T-1} r^{-1} + Y_T r^{-2}}{1 + \frac{\theta}{r}}$$

$$= \frac{W_{T-2} + \sum_{t=T-2}^T Y_t r^{T-2-t}}{\left(\frac{s_T}{s_{T-1}}\right)^{1/v} x \left(\frac{s_{T-1}}{s_{T-2}}\right)^{1/v} x}$$

$$= \frac{W_{T-2} + \sum_{t=T-2}^T Y_t r^{T-2-t}}{1 + \left(\frac{s_{T-1}}{s_{T-2}}\right)^{1/v} x + \left(\frac{s_T}{s_{T-2}}\right)^{1/v} x^2}$$

Finally, by extension and noting that $s_0 = 1$,

$$C_0 = \frac{W_0 + \sum_{t=0}^T Y_t r^{-t}}{\sum_{t=0}^T s_t^{1/v} (ar^{1-v})^{t/v}} \quad (7.8)$$

APPENDIX 7.B

SOLUTION OF THE LIFECYCLE CONSUMPTION MODEL WITH UNCERTAIN LIFETIME WHEN ANNUITIES ARE AVAILABLE

The solution proceeds as in Appendix 7.A except that the constraint (7.6) is replaced by

$$W_t = \frac{s_{t-1}}{s_t} r (W_{t-1} + Y_{t-1} - C_{t-1}) \geq 0 \quad (7.B.1)$$

Since no bequest motive is assumed and since $\frac{s_{t-1}}{s_t} r > r$,

all wealth will be held in the form of annuities.

$$Z_t = \max \left\{ s_T a^T \frac{C_T}{1-v} \right\} = s_T a^T (W_T + Y_T)^{1-v}$$

$$Z_{T-1} = \max \left\{ s_{T-1} a^{T-1} \frac{C_{T-1}}{1-v} + s_T a^T \left[\frac{s_{T-1}}{s_T} r (W_{T-1} + Y_{T-1} - C_{T-1}) + Y_T \right]^{1-v} \right\}$$

$$\frac{\partial Z_{T-1}}{\partial C_{T-1}} = 0 \Rightarrow s_{T-1} a^{T-1} C_{T-1}^{-v} = s_T a^T \frac{s_{T-1}}{s_T} r [C_T]^{-v}$$

$$C_T = (ar)^{1/v} C_{T-1} \quad (7.34)$$

$$\text{Thus, } (ar)^{1/v} C_{T-1} = \frac{s_{T-1}}{s_T} r (W_{T-1} + Y_{T-1} - C_{T-1}) + Y_T$$

$$\left[s_{T-1} r + s_T (ar)^{1/v} \right] C_{T-1} = s_{T-1} r (W_{T-1} + Y_{T-1}) + s_T Y_T$$

$$C_{T-1} = \frac{W_{T-1} + Y_{T-1} + \frac{s_T}{s_{T-1}} r^{-1} Y_T}{1 + \left(\frac{s_T}{s_{T-1}} \right) (ar^{1-v})^{1/v}}$$

$$\text{By extension, } C_0 = \frac{W_0 + \sum_{t=0}^T s_t r^{-t} Y_t}{\sum_{t=0}^T s_t (ar^{1-v})^{t/v}} \quad (7.35)$$

CHAPTER 8

ESTIMATION OF THE MODEL USING DATA ON EXPENDITURES BY CANADIAN HOUSEHOLDS

This chapter investigates the ability of the lifecycle savings model to explain the pattern of consumption among Canadian households. The analysis is based on the model of sections 7.2 and 7.3; that is:

$$C_o = \min \left\{ \begin{array}{l} \frac{W_o + \sum_{t=0}^H Y_t r^{-t}}{\sum_{t=0}^H \tilde{s}_t^{1/v} (ar^{1-v})^{t/v}} \\ W_o + Y_o \end{array} \right. \quad (8.1)$$

where H is the last year of the first free interval encountered in or after the base year and $\tilde{s}$ is the survival probability of the household, treated as a collective. Two approaches are taken to estimating the model. The first and most demanding is to obtain estimates of the several components of the right hand side of (8.1) and use these to compute C_o directly. This is repeated for several values of v in order to provide an indication of the degree of risk aversion exhibited, on average, by Canadian households. This approach is demanding since it yields exact estimates of C_o rather than allowing the parameters of the model to be estimated by means of a least square error procedure. Its strength is that it avoids the problems of multi-collinearity and spurious correlation which often render regression estimates suspect.

Its weakness is that it depends directly on proxies for unobservable variables such as expected future earnings and on assumptions regarding the parameters α and r . For comparison purposes, the model is also estimated by a more traditional approach in which the components of the right-hand-side of (8.1) are treated as independent variables in a regression equation. Finally, the results of both these approaches are compared to those obtained by regressing C_0 on current income, family size, and the age of the household head.

The chapter is organized as follows. Section 8.1 provides an overview of the estimation procedure. Sections 8.2 through 8.7 describe the detailed procedures used in estimating and assigning the variables of equation (8.1). Section 8.8 describes the sample of households employed in the estimation. The results obtained are examined in Section 8.9.

8.1 OVERVIEW OF THE ESTIMATION PROCEDURE

For purposes of estimation, (8.1) can be re-expressed as

$$C = \min \begin{cases} Q[WH + WF + Y + YF + P + G] \\ WH + WF + Y \end{cases} \quad (8.2)$$

where:

C = base year consumption expenditure

Y = base year income

WH = equity in the home

WF = other components of net worth

YF = the present value of future non-pension income

P = the present value of future private pension income

G = the present value of future public pension income

$$Q = \left[\sum_{t=0}^H \frac{s_t}{s_0} (ar^{1-v})^{t/v} \right]^{-1}$$

This estimating model has the advantage of incorporating considerations of future incomes, family size, life expectancies and wealth constraints directly and explicitly, in contrast to most models found in the literature which rely on age and family size dummies and very crude proxies of permanent income. The model is applied to income and expenditure data for individual Canadian "spending units" (i.e. groups of individuals who share a dwelling unit and pool major items of expenditure) obtained from the 1978 Family Expenditure Survey (FAMEX). (See Statistics Canada catalogue 62-551 for published data). To derive values of the proxy variables WF and YF, additional information is used which is obtained from a microdata file of records from the 1977 Assets and Debts Survey (Statistics Canada catalogues 13-570 and 13-572) and from the microdata file of the 1978 Survey of Consumer Finances (1977 incomes) (Statistics Canada catalogue 13-207 annual). Proxy variables P and G are based partly upon household information in the FAMEX records and partly upon external information relating to the provisions and past performance of private and public pension plans. Variable Q incorporates vectors of male and female survival probabilities as well as the assigned values for a, r and v. The assignment of H presents a problem since P, G and Q depend upon H, and H, in turn, depends upon the general parameters, a, r and v, upon the vector of survival probabilities $\tilde{s}_t$ which incorporates family size considerations, upon

the level of current resources, $WH + WF + Y$, and upon the time profile of future income receipts embodied in YF , P , and G . The problem is to reach an acceptable compromise between the increased precision that could be obtained with a large number of options for H and the need to limit the cost and complexity of the variable assignment procedure. Based on a large number of model simulation runs, a set of six cases is assigned for a given value of v (with a and r also set at given values).

With the variables thus defined, the model is estimated in two different ways. First, estimates denoted C_i^* for the i^{th} spending unit are obtained directly from (8.2) and the errors $e_i^* = C_i - C_i^*$ examined. Second, the equation

$$\begin{aligned} \hat{C}_i = \hat{a}_0 + \hat{a}_1 (Q.WH) + \hat{a}_2 (Q.WF) + \hat{a}_3 (Q.Y) + \hat{a}_4 (Q.YF) \\ + \hat{a}_5 (Q.P) + \hat{a}_6 (Q.G.) \end{aligned} \quad (8.3)$$

is estimated using the whole sample and again after dropping the observations for which $C_i^* = WH + WF + Y$. A much higher error must be expected in C_i^* than $\hat{C}_i$, of course, because the former estimating approach is equivalent to forcing $a_0 = 0$ and $a_1 = a_2 = \dots = a_6 = 1$ in the latter. The second, weaker test of the model is applied in order to permit comparison with traditional cross-section consumption function estimates and in order to assess the explanatory power of the proxy variables representing the several components of lifetime resources.

The estimating procedure suffers from several important limitations. To begin with, the usual errors-in-variables problem resulting from sampling and non-sampling error is greatly exaggerated by the need to introduce proxy variables which are both based on information from outside data sources and required to represent consumers' expectations about their future incomes and life expectancies. Second, considerable error may be introduced by the adoption, without testing, of given values for a and r and by the assumption that a , r , and v are invariant across households.¹ Cost and time constraints made the testing of different values of a or r impractical. A third and similar limitation is the restriction of the survival probability vectors (the $\tilde{s}_t$) to four cases (based on the sex of head, presence of spouse and presence of children) in the face of the much greater actual variety in family composition and the fact, noted earlier, that life expectancy varies systematically with income. Fourth, the restriction to six values of H is a potentially important source of error, especially for households with atypical age-income and age-wealth profiles. Ideally the optimization model would be solved for each household separately, yielding an H_i^* along with C_i^* . Cost considerations prohibited this approach. However, tests showed that C^* was not very sensitive to changes in H of one or two years, the reason being that when H is increased, both the numerator and denominator of (8.1) are increased. In a typical example, the error in C^* was found to rise from 0.014 to 0.092 to 0.234 of a percentage point as H was reduced one, two and three years below its optimally-

1. For example, Evans (1983) has shown that the interest elasticity of savings is quite sensitive to the assumed value of a .

chosen value. These limitations mean that the evidence regarding the likely level of v must be taken as merely indicative. A more serious attempt to estimate v would require varying v to minimize errors in the C_i^* while using several different sets of assumptions regarding a , r , and the $\tilde{s}_t$, and the levels of expected future income. Despite these limitations, however, the present estimation process should provide a reasonable idea of the applicability of the lifecycle consumption model with uncertainty to the savings behaviour of Canadian households.

The detailed assignment procedures for the variables in (8.2) are described below in the following order: C, Y, WH and WF in section 8.2, YF in 8.3, P in 8.4, G in 8.5, Q in 8.6 and H in 8.7.

8.2 CURRENT CONSUMPTION, INCOME AND WEALTH

The dependent variable, base year consumption is obtained from FAMEX records as follows:

C = total consumption expenditures
+ gifts given
+ charitable contributions
+ imputed rent on owner-occupied home (= 0.11 X equity in home)
+ 0.5 x account balancing difference

Gifts are thus treated as consumption rather than as investments upon which an eventual return is expected. Following the model set out in Fallis (1980), the current consumption of home equity is estimated as the product of equity in the home and the current rate of interest on mortgages. The account balancing difference is the discrepancy between the income and other money receipts reported by a spending unit and the sum of its consumption expenditures and its increase in net assets. Half of this discrepancy is allocated to consumption expenditures and half to saving. No attempt is made to replace expenditures on consumer durables by an estimate of the consumption of services from the spending unit's stock of durables.

Base year disposable income is defined in the following manner.

Y = total income of the spending unit

- interest income
- dividend income
- other investment income
- income taxes paid
- Unemployment Insurance premiums
- C/QPP contributions
- contributions to employer pensions or savings plans
- government health premiums
- + 0.11 (equity in home)

Current and future investment receipts are excluded from income because they are capitalized in the estimate of base year net worth. As discussed in section 8.4, contributions to employer pension plans are treated as given to the consumer in the same way as are the taxes levied to finance future C/QPP benefits.

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Housing wealth, WH, is obtained directly from FAMEX records as the difference between the "expected selling price of an owned home" and the "balance of principle outstanding on all mortgages".

The balance of current net worth, (financial wealth) WF, is assigned to each spending unit according to the following equation:

$$\begin{aligned} WF = & \text{other money receipts} \\ & + \text{gifts received} \\ & + 10 \text{ (interest income)} \\ & + 17 \text{ (dividend income)} \\ & + 25 \text{ (other investment income)} \\ & - 10 \text{ (interest paid on non-mortgage debt)} \\ & + \text{FRSP (net increase in assets held in RHOSP's and RRSP's)} \end{aligned}$$

where FRSP takes the value 1 when the savings plan asset change is negative or zero, and the values 2, 4, 6, 8, 10 when the asset change is positive and the age of the spending unit head is under 25, 25-34, 35-44, 45-54 and 55 and over.

"Other money receipts" includes money gifts from persons outside the spending unit, inheritances, lump sum insurance settlements, windfall gains and income tax refunds. Gifts received refers to non-money gifts from outside the spending unit.

The multipliers used to estimate asset stocks from the interest, dividend and other investment income amounts reported in FAMEX were derived from a regression equation wherein realized amounts of "investment income" reported (for 1976) in the 1977 Assets and Debts Survey were related to the amounts of the three classes of asset held at the time of interview (May, 1977). The regressions were carried out at a microdata level; that is, using data on the assets and incomes of individual family units. The simplest regression equation had an $\bar{R}^2$ of 0.365 or 0.423 depending on whether weighted or unweighted observations were used. Some increase in explanatory power was obtained by including non-linear terms and age and earned-income dummies but this increase was not felt to be sufficient to justify the additional complexity that would be implied for the assignment process.

The multiplier used to estimate outstanding debt was calculated as a rough average of the rates faced by debtors on bank and installment credit in 1978.

The FRSP coefficients were derived by examining average RRSP contribution levels by income (from 1978 FAMEX records) on one hand and average RRSP holdings by age and income class (from 1977

Assets and Debts survey data) on the other. As would be expected, it was found that both contribution levels and asset holdings rose with income, and that accumulated asset holdings also rose with age. It is clear that this facet of the wealth assignment procedure is a very crude one so the resulting wealth assignments will be subject to considerable error. In its defense, however, is the fact that RRSP wealth makes up only 1.8% of the total asset portfolio of Canadian households. (Statistics Canada, catalogue 13-570, Table 13, p. 84).

No estimate was made of net assets in unincorporated businesses; instead spending units with over \$1,000 in self-employment income were eliminated from the FAMEX sample used in the estimation. This course was followed because of the additional difficulty in defining the incomes, assets and savings rates of business owners and the likelihood that their savings patterns differ from those in the balance of the population.

8.3 FUTURE NON-PENSION INCOME

The variable YF is a proxy variable for the present value of the expected future disposable income of the spending unit up to the year when the head is age 64. All heads and spouses are assumed to plan to retire at age 65 and no earned income is assigned in respect of ages 65 and over. For those over 65 in the base year who do report earnings, this component of income is assumed to fall to zero in the following year. The several steps involved in the creation and assignment of YF can be summarized as follows:

- choose a limited number of cross-sectional age-earnings profiles based on the sex, education and occupation of the earner;
- determine and apply a tax function to convert to post-tax earnings;
- convert the cross-sectional profile to a longitudinal one assuming 2% real growth in after-tax earnings; then using a 3% discount rate, calculate a matrix of present values of future income with columns representing the selected sex, education, occupation, characteristics and rows representing the age of the earner in 1978;
- finally, in the assignment procedure, adjust the selected present value by a factor relating reported base year net earnings to the predicted net earnings embodied in the present value; sum the adjusted present values of husband and wife where appropriate.

The following paragraphs document this procedure in more detail.

8.3.a Selection of Age-Earnings Profiles

The basis of the selection of a small set of representative age-earnings profiles was a pair of cross-tabulations of incomes for males and females by age, education, occupation and family type.

These tabulations were obtained from Statistics Canada's public use micro-data file of the 1978 Survey of Consumer Finances (Economic Families - 1977 Income). The income variable included earnings plus non-investment income including government transfers since the YF variable is meant to be inclusive of benefits such as Social Assistance and Unemployment Insurance. The incomes were averaged over ten year age groups from 15-24 to 55-64. Five education categories were employed: (a) less than grade 9, (b) grades 9, 10, (c) grade 11 up to some post-secondary, (d) post-secondary certificate or diploma, and (e) university degree. Six occupation classes were used (based on previous, more detailed tabulations): (a) managerial, administration, (b) professional, technical, (c) sales, (d) clerical, service, (3) blue collar (farming, etc., mining, fabrication, construction, transport), and (f) never worked before or last worked over five years ago. For males, two family types were considered: single-person and multi-person family units. For females, the mutli-person family category was further split according to the presence or absence of children under age twelve.

The selection of a limited number of age-earnings profiles resulted from a series of experiments with estimating equations and was shaped by consideration of the way in which the earnings information was to be adjusted and employed in the consumption estimation model. Two principles in particular were followed. First, separate profiles were only defined for education/occupation classes including substantial fractions of the population of earners. Second, where age-income profiles differed mainly by level rather than "age-shape" this was

taken into consideration by the adjustment of present values by a constant term rather than by the definition of separate profiles. For example, a regression of the ratio of earnings for unattached males to the earnings of males in multi-person families (controlling for age/education/occupation group) showed that single males earn about 75% as much as males in larger family units. The coefficients of age, education and occupation dummy variables included in the equation to explain the variation in the ratio were generally insignificant and showed no coherent patterns (e.g. the coefficients for the lowest and highest education classes were negative; the other education class coefficients were positive). Accordingly, age-earnings profiles were derived for males in multi-person families and the resulting present values multiplied by 0.75 when applied to male unattached individuals.

For males, the result of the procedures was the representation of cross-sectional earnings profiles by five representative profiles as shown in Figure 8.1.

The representation of female income profiles is complicated by several factors, including:

- greater income variation due to smaller sample sizes (of women reporting an occupation) and the effect of part-time work);

- historically lower labour force participation rates of married women as compared to single women;
- the effect of young children on employment and earnings levels;
- the effect of past interruptions of employment on the current earnings of older women;
- the likelihood that female earnings profiles will be much less affected by marital status and child raising in the future than to date.

FIGURE 8.1

REPRESENTATIVE CROSS-SECTION AGE-EARNINGS PROFILES

Classes

Male	1. degree; managerial, professional
	2. (no degree; managerial, professional) + (grade 11+; sales)
	3. grade 11+; clerical, blue collar
	4. under grade 11; sales, clerical, blue collar
	5. non-working
Female	6. grade 11 to some post-secondary; clerical/service

Source: Derived from tabulations from Statistics Canada's public use microdata tape from the 1978 Survey of Consumer Finances (Economic Families-1977 Incomes).

On the other hand, the 1977 data showed women to be concentrated in a few education, occupation classes: 50% were in the high school graduation level education class; among occupations, 54% were in the clerical and service group and a further 20% were in the professional/technical group. Several observations emerged from the analysis of the earnings tabulations. First, education and occupation differences were reflected in earnings levels as with the males, but differences in the age profile of earnings by education or occupation were obscured by the effects of family status differences. Second, the effect of family status and presence of children was quite complex. Below age 45, women with young children tended to have lower incomes (given their education/occupation class) than either single women or married women without children. Among women over age 45, however, a similar earnings gap existed between married women, with or without children, and single women. Given the doubt about whether these effects of family status on earnings would persist in the future and also the likelihood of a change in family status over the working life of an individual, the decision was made to make no explicit adjustment to future earnings based on family status. (An implicit adjustment is made, though, when the present value of future earnings is adjusted by a factor related to reported current net earnings). The earnings profile assignment procedure adopted was to assign all women the earnings profile of a high school graduate (grade 11 up to some post-secondary) in a clerical or service occupation. This is shown as profile 6 in figure 8.1. Later in the assignment procedure the present values of future earnings were then adjusted by the factor 0.65 for those with less than grade 11 education, and not in a mana-

gerial or professional occupation, or by the factor 1.5 for those with a post-secondary diploma or university degree who also were in a managerial or professional occupation. Finally, the present values were further adjusted in light of reported base year earnings as detailed in section 8.3.d. Perhaps the most important thing to note about this assignment procedure is that it involves the assignment of some future earnings even to women not currently in the labour force.

Before the adjustment for tax liabilities, these predicted earnings levels derived from Survey of Consumer Finance data for 1977 were multiplied by the factor 1.062 to bring them up to 1978 levels. This factor was calculated from the growth in the Industrial Composite Index of Average Weekly Wages and Salaries between 1977 and 1978.

8.3.b Adjustment for Tax Liabilities

The second step was to adjust the income estimates for tax liabilities. This was done on the basis of income tax data (Revenue Canada, Taxation, Taxation Statistics, 1980 Edition (1978 tax year), Supply and Services, Canada, Table 2) since the microdata available on the Survey of Consumer Finances public use tape did not permit the allocation of income taxes to heads and spouses. Tax liabilities were estimated by regressing an average tax rate on the independent variable "assessed income" using data grouped by level of assessed income for taxfilers who were taxed as single with no dependents. Parallel equations were estimated for taxfilers with dependents and taxfilers taxed as married but were discarded because the tax rate

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MicroD

differences were quite small and because taxfilers taxes as single with no dependents accounted for over two-thirds of all taxfilers. In calculating the average tax rates, C/QPP and UI contributions were included as well as federal and provincial income tax liabilities; the variation of exemption levels with income was also automatically accounted for.

Average tax rates rise rapidly from zero at zero assessed income and then slowly approach an asymptote at the maximum combined federal/provincial marginal tax rate. The average tax rate function which best fits this pattern is a logarithmic reciprocal transformation. For 1978, the specification was

$$atr = \exp \left(-1.27 - \frac{10171}{AY} \right)$$

where atr= average tax rate and AY = assessed income. The errors at different income levels may be seen by comparing the actual and estimated tax rates in Figure 8.2.

FIGURE 8.2

ESTIMATION OF AVERAGE TAX RATES

Source: Estimated from data in National Revenue Taxation, Taxation Statistics, 1980 Edition (1978 incomes), Table 2.

Two important assumptions were made in applying the tax rates to derive net-of-tax earnings levels. First, the effect of investment income on the rate of tax on earnings was neglected. This can be justified on the grounds that investment income has less effect on average than on marginal tax rates since the bulk of investment income (e.g. imputed rent, first \$1000 of interest, dividends, or capital gains) escapes taxation, and that the tax rates are slightly overestimated for higher income groups where taxable investment

income is likely to be found. The second assumption is that the tax rates are constant over time in the face of real as well as purely nominal income increases. To assume otherwise would imply an improbable increase over time in the share of national income which passes through government accounts.

8.3.c Calculation of Present Values of Future Net Earnings

After multiplying the cross-section earnings estimates by $(1-atr)$, the thirty numbers (six profiles times five, ten-year age groups) were expanded to form six, fifty-year (cross-section) age-income profiles now in single years of age. This was done simply by the ten-fold replication of each estimate rather than by any interpolation or smoothing procedure. Next, the initial year's estimate (age 15) was dropped to reflect the fact that the present value desired is that of income starting in base year plus one. Finally, present values were calculated of future earnings to age 64 for each age of earner within each earnings profile. The discount rate employed, $1.02/1.03=.99029$, embodies two considerations. It incorporates the assumption of 2% real wage growth over time (i.e. the conversion of the cross-sectional income profiles into longitudinal age-income profiles). It also embodies the assumption of a risk-adjusted, net-of-tax real rate of return on savings of 3%. The choice of the latter discount rate was based on information and analysis presented in Chapters 6 and 7 which suggests that effective marginal tax rates on investment income are low or zero for most households, that an

average pre-tax rate of return of 3.5% is earned on a typical pension and savings plan portfolio,² and that a realistic risk premium might be as high as one-half of one percent.

8.3.d Adjustment to Reflect Reported Earnings Levels

Two kinds of adjustment were made to the present values obtained in the previous step. The first kind was explained in section 8.3.a and included the adjustment by a factor of 0.75 of the earnings variable for single males and the adjustment by factors of 0.65 or 1.5 of the earnings of women in certain education/occupation groups. The second form of adjustment involved the multiplication of the earnings present values by the factor

$$k = 0.5 \left(1 + \frac{RY}{PY(I,J)} \right)$$

where RY is the reported earnings (in FAMEX) of the spending unit head or spouse (=earnings + U.I. benefits - income taxes paid) and PY(I,J) is the base year predicted income level for the appropriate age/education/occupation class as used in the derivation of the present values. The effect of this adjustment is to raise or lower the estimate of expected future earnings by one-half the percentage difference between reported earnings and predicted earnings for the earner's age/education/occupation class. For example, if $RY/PY(I,J)$

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2. These estimates were taken from Carty and Pressman (Lazar Report, 1980, vol. II, Appendix 10).

were 1.6, k would be 1.3; if $RY/PY(I,J)$ were 0.6, k would be 0.8.

The purpose of this adjustment is to take account of the finding of Lillard (1977, reported in Irvine 1979 and 1980) that non-transitory influences, not due to observable factors such as age and education, explained a significant proportion of the variation in earnings for a longitudinal sample with two to five observations included for each individual. More precisely, Lilliard found that, while age, education, IQ and family background variables explained 30% of the variation in earnings levels, 57% of the remaining variation was associated with a permanent deviation of each individual's earnings from the values predicted by the earnings function.

The assignment of YF for each spending unit was completed by summing these adjusted present values for head and spouse where applicable.

8.4 PRIVATE PENSION INCOME

In considering how to treat the assets, savings and expected benefits associated with private pension plans, a basic choice between two approaches must be made at the outset. The first is to treat past (employer and employee) contributions to pension plans as a component of current assets to be added to WH and WF and to include current contributions as a component of saving. An alternative approach is to treat private pensions analogously to public pensions, including anticipated pension benefits as part of expected future income and including pension contributions with taxes rather than

savings. As indicated earlier, the latter approach has been adopted here. Its choice may be justified on the grounds that membership in a pension plan is usually a condition of employment rather than a deliberate choice by the contributor. Also, several features of existing pension plans including vesting and portability limitations, lack of indexing of deferred pensions, and the popularity of final earnings benefit formulae mean that accumulated pension contributions are likely to be a poor predictor of future pension benefits.

The Lazar Report (p. 39 and ff.) provides a convenient review of information on employer-sponsored pension plans. Some points from that review which were taken into account in assigning expected private pension benefits (P) are as follows:

- in 1976, 48% of paid workers were plan members though the figure rises to 62% if part-time workers and those under 25 and over 64 are excluded;
- about 20% of covered employees are in non-contributory plans; the employee share of total contributions is 26% in private-sector plans and 41% in public-sector plans;
- the vast majority of plan members are in defined benefit plans where benefits are determined by a formula rather than by the level of accumulated contributions. The split among various types of plans according to benefit definition is: final or best average, 56%; career average 18%; flat benefit, 20%; defined contribution and other, 6%;

- most defined benefit plans provide benefit levels equal to a percentage of (final or average) earnings times the number of eligible years of service. A rate of 2% or more per year of service applied to 80% of members in final average and 62% of those in career average plans; for a further 8% and 22% of those in final and career average plans, the benefit rate was between 1.5% and 2%;
- while private-sector plans generally do not provide inflation-indexing of pensions-in-pay, a survey of private sector plans including 21% of covered employees resulted in the finding that almost 80% of the sampled plans provided some form of ad hoc inflation adjustment to pensions-in-pay. Over the period 1971-1975, these adjustments averaged 66% of the change in the Consumer Price Index.

In-light of this picture of private pension plans, the basic method chosen was to assign future pension benefits only to those who reported pension contributions (31% of spending units) and to base the benefit levels on the predicted earnings level of the spending unit head at age 64. For males, the pension at age 65 was taken to be 35% of this earnings level; for females, a lower rate of 25% was assumed in reflection of the likelihood of a lower number of eligible years of service. These expected benefit levels are based on the popular 2% benefit rate and assume integration of the plans with the Canada/Quebec Pension Plans (i.e. combined C/QPP and private pension benefits amount to 2% of final earnings per year of service).

This procedure may over-estimate expected pension benefits since no account is taken of any perceived probability of loss of pension credits through unemployment or job change. For spending units headed by someone over age 65, this procedure was replaced by one of extrapolating reported pension income.

To determine a present value of P, two discounting processes were applied. First, the benefits from age 65 to age H, the consumption planning horizon, were discounted by the factor 0.9434 and summed. This discount factor assumes a decline of about 3% per year in the real value of private pension benefits due to inflation. Thus, it is consistent with ad hoc inflation adjustments of the order of 6% per annum with a 9% inflation rate. Second, the present value of pension benefits at age 65, thus calculated, was discounted to the head's age in the base year using the discount factor of 0.99029 which embodies the 2% real growth in the earnings base as well as the 3% rate of return on savings. For those over 65, benefits from the current year to the year when the head's age becomes H were simply accumulated at the discount factor of 0.9434. The actual formulae employed to accomplish these conversions are shown below for the cases of a working and retired individual.

$$(\text{working}) P = (\text{CON}) (P_{65}) (17.667) (1 - 0.9434^{H-65}) (0.99029^{65-HAGE})$$

$$(\text{retired}) P = (P_{\text{reported}}) (16.667) (1 - 0.9434^{H-HAGE})$$

where CON takes values 1 or 0 depending on the presence or absence of reported pension contributions and HAGE is the age of the spending unit head in 1978.

8.5 PUBLIC PENSION INCOME

A full-fledged simulation of public pension entitlements would require: (a) the simulation of Canada and Quebec Pension Plan (C/QPP) entitlements based on the earnings history (past and future) of the head and spouse, (b) the trivial assignment of Old Age Security (OAS) benefits, and (c) the simulation of Guaranteed Income Supplement (GIS) benefit levels based on the estimates of other components of retirement income. For the purpose of reflecting expected future public pension levels in base year consumption-savings decisions, a short-hand approach was considered to be adequate. Five representative cases were constructed, based on family status and the level of C/QPP contributions. As with the assignment of private pension income, these cases were used to define values of the level of benefits at age 65 (G_{65}), and then this value was extrapolated to age H at an appropriate discount rate and summed to obtain a present value. Again a modified procedure was used to assign benefits to those over age 65 in 1978. The five cases are set out in Table 6.1 below.

TABLE 8.1

ASSIGNMENT OF PUBLIC PENSION BENEFITS

Family Status	C/QPP Contribution in 1978	Description of Benefit	1978 Benefit Level (G ₆₅)
Single	<\$150	OAS+max GIS	\$3239
Single	≥\$150	OAS+max CPP + partial GIS	\$4405
Couple	<\$150	2.0AS+max GIS	\$6180
Couple	\$150-299	2.0AS+1 max CPP + postal GIS	\$7346
Couple	≥\$300	2.0AS+2.max CPP +partial GIS	\$8513

In constructing these cases, the basic assumption made was that the retiree has little income aside from public pensions. Thus a single pensioner with no C/QPP income would receive the maximum GIS benefit of \$1335 per year (at 1978 levels) while a pensioner with the maximum CPP pension of \$2333 would still receive a partial GIS benefit of \$168 per year. It was the fact that changes in C/QPP benefits, right up to the maximum level, are offset at a 50% rate by changes in GIS benefits, which suggested that an abbreviated method of modeling public pension benefits would be adequate. A second important assumption was that GIS benefits will be increased over time so that the combined OAS/GIS maximum benefit grows at the same rate as average wages. This assumption is consistent with the history of maximum GIS benefit levels over the period since the plan was introduced in 1967. If, on the other hand, GIS benefits were assumed to remain fixed in

real terms, then GIS benefit levels would fall relative to C/QPP benefits over the long run (since C/QPP benefits are effectively indexed to wages up to the contributor's year of retirement) and the public pensions of those largely dependent on OAS/GIS would fall behind those with substantial C/QPP pensions. In defining the cases, the cases with C/QPP contributions at or above \$150 (\$300 for a couple) represent the situation of those who contribute at or close to the 1978 maximum rate of \$169.20. Those with lower contribution rates are also expected to exhibit more interruptions (e.g. due to unemployment) in their contribution history and so are expected to be eligible for substantially lower C/QPP pensions than those who contribute at the maximum.

In calculating the present value of the stream of public pension benefits extending to the consumption planning horizon, age H, the benefit levels from Table 8.1 were extended over the retirement period and discounted by the factor $1.02/1.03 = 0.99029$, reflecting (as with YF) 2% real growth and a 3% discount rate. Since OAS benefits and C/QPP benefits-in-pay do not in fact grow over time in real terms, it may be objected that a higher discount rate should be applied in the cases where OAS and C/QPP benefits are large relative to GIS benefits. When such cases were simulated, however, the resulting present values were found not to differ by much from those where the discount factor .99029 was used. (This is again because of the partial substitution of GIS benefits for the more slowly growing OAS and C/QPP benefits.)

For those over 65 in 1978, the variable G_{65} is replaced by the variable G_{BASE} which is defined for singles as $\max [\$3239, \text{reported OAS/GIS} + C/QPP]$ and for couples as $\max [\$6180, \text{reported OAS/GIS} + C/QPP]$.

The actual formulae used to obtain the present values are as follows:

$$(\text{working}) G = (102) (G_{65}) (1 - .99029^{H-65}) (.99029^{65-HAGE})$$

$$(\text{retired}) G = (102) (G_{BASE}) (1 - .99029^{H-HAGE})$$

where HAGE again is the age of the spending unit head in 1978.

8.6 THE VARIABLE Q

The denominator of the expression for optimal consumption is $Q^{-1} = \sum_{t=0}^H \tilde{s}_t \frac{1}{v} (ar^{1-v})^{t/v}$. The value of Q^{-1} also depends on the value of H and the age of the spending unit head in the base year.

In the estimation procedure, values of Q are assigned to spending units from a matrix of pre-calculated Q's. The matrix has 60 rows, representing head's ages 15-74, and six columns representing six cases defined by family type and the value of H.

Among the six columns of Q values, four family types are represented: (a) single males, (b) single females, (c) couples without children under age 16, and (d) couples with children. For family types (a) and (b), the $\tilde{s}_t$ are simply the male and female survival probabilities taken from Statistics Canada's Life Tables, Canada and the Provinces, 1975-1977 (Catalogue 84-532). For couples without children, $\tilde{s}_t$ is derived from:

$$\begin{aligned}\tilde{s}_t &= F(1)^V s_{1t} + F(2)^V s_{2t} \\ &= [s_{Ht} + s_{Wt} - 2s_{Ht} \cdot s_{Wt}] + (1.262)^V s_{Ht} s_{Wt}\end{aligned}$$

where s_{1t} and s_{2t} are the probabilities of one and both spouses, respectively surviving to t , s_{Ht} and s_{Wt} are the survival probabilities of the husband wife, and $F(1) = 1.336$ is the family size equivalence scale employed. In applying this formula, the wife's reported age was ignored; instead, the wife was assumed to be two years younger than her spouse. (An analysis of the ages of husbands and wives based on the ages reported in the 1978 Survey of Consumer Finances indicated that 15% of husbands were younger than their spouses, 48% were from zero to three years older, 21% were from four to six years older, and 16% were more than six years older).

In the case which represents couples with children, the above formula was altered by setting $F(1) = 2.336$ for head's ages 30-34 and 55-59, and $F(1) = 3.336$ for head's ages 35-54. $F(2)$ was set at 3.336 and 4.336 for the same periods. Thus the effect of children on lifetime consumption patterns is represented, in the estimation procedure, by the case of a two-child family where the children's influence on consumption levels extends from head's age 30 to age 59 and is strongest in the age span 35-54.

An important feature of the estimation procedure is its reliance on single assumed values of a and r , particularly as it has been shown that parameters such as the interest elasticity of saving depend significantly on these assumed values. While it would be

interesting to observe whether, for example, the minimum error value of v is sensitive to the choice of a or r , cost and time considerations precluded this analysis. As noted earlier, the estimate $r = 1.03$ is based on the average real rate of return of 3.5% estimated for pension fund assets by Carty and Pressman (Lazar Report, vol. II, Appendix 10). The assumed value, $a = 0.99$, is based on simulation results for typical households. This value of the subjective time preference factor, in conjunction with $r = 1.03$, gave the most realistic looking age profiles of consumption.

8.7 THE CONSUMPTION PLANNING HORIZON, H

The need to assign a value of H to all spending units in the estimation sample derives, first, from the fact that the existence of universal public pensions virtually guarantees that the wealth constraint will become operative at some point in the consumer's potential lifespan and, second, from the fact that the option of solving the optimization problem separately for each sample point was too costly to be feasible. Instead, a large number of different solutions to the optimization problem were examined in order to derive a workable number of representative cases. In the course of this examination, it was noted that the value of H depends chiefly on two factors. To begin with, H varies positively with the weight of the survival probabilities $\tilde{s}_t$ in later years of life as compared to earlier years. The value of H is greater, therefore, for women than for men and for childless couples than for couples with children. In addition, H varies positively with the importance of base year wealth

and pre-retirement income relative to the level of expected pension income. As noted in the overview, it was also found that the optimal consumption level C^* was fairly insensitive to departures of one or two years in H from its optimally determined value.

The analysis of solutions to the optimization problem resulted in the choice of six values of H for use in the definition of the Q vectors and in the assignment of public and private pension present values. The six cases are described in Table 8.2 below.

TABLE 8.2

CONSUMPTION PLANNING HORIZON, H :
DESCRIPTION OF SIX CASES

Case	Description	$v =$	H (age of head)			
			0.75	1.5	2.5	3.5
1.	Single male		80	83	86	89
2.	Single female		86	87	90	92
3.	Couple: low YF, W^*		84	83	82	81
4.	Couple with children: low YF, W^*		83	80	79	79
5.	Couple: high YF, W^*		87	88	89	90
6.	Couple with children: high YF, W^*		85	86	87	88

* high YF, W denotes a spending unit with the head in the highest YF group (university degree, manager or professional) and with $WH+WF \geq \$50,000$; low YF, W denotes all other husband-wife spending units.

8.8 SAMPLE SELECTION

The model estimation employed a sample of income and expenditure records for individual Canadian spending units taken from the working files of the 1978 Family Expenditure Survey. The working files covered urban and rural households and included a total of 9370 spending units. Several classes of spending units were excluded from this sample prior to estimation. First, 829 spending units which were not "economic families" were excluded. As noted earlier, a "spending unit" is a group of people who share a dwelling unit and pool major items of expenditure. An economic family is a group of people who share a dwelling unit and are related by blood, marriage or adoption. The rationale for excluding non-Economic Family spending units was that they are often unlikely to be arrangements of long duration and so the consumption behaviour exhibited by these groups is unlikely to be explicable in the context of a lifecycle model. Second, 1064 spending units were excluded because they had self-employment income in excess of \$1000. The measurement of self-employment income, assets and saving flows pose special problems. Also, self-employed persons may have atypical savings patterns. Thus it seemed preferable to eliminate self-employed individuals from the estimation sample. Third, 561 spending units, where the head's age exceeded 74, were excluded. Some upper limit to the head's age had to be introduced to prevent cases where the head's age exceeded the consumption planning horizon, H . A further reason for restricting HAGE is that the error introduced by choosing H from a limited number of cases is likely to be particularly

significant when HAGE is large. For example, the true H for an elderly single male with a small pension entitlement but a high level of wealth (WH+WF) is likely to be considerably greater than the assigned value of H (i.e. 86). Moreover, when H-HAGE is small, an error in H can have a big effect on predicted consumption levels. Finally, 2 spending units were excluded because their reported base year income was negative or zero. The estimation sample included 6932 observations. No allowance was made for varying sample weights in the estimation.

8.9 ESTIMATION RESULTS

The model was estimated for four values of v (.75, 1.5, 2.5 and 3.5). The estimation results are set out in Tables 8.3, 8.4 and 8.5 below. Table 8.3 summarizes the results obtained when the C_i^* were estimated directly from equation (8.2). In Table 8.4, averages of (unsquared) errors $C_i - C_i^*$ are provided for eighteen age-income groups to indicate where the model systematically under-or over-estimates consumption. Table 8.5 provides the results of estimating regression equation (8.3) for both the full and "restricted" (i.e. only observations for which wealth constraint not binding) samples. It also provides the results of estimating a simple linear consumption function with current income, family size and age dummies as the independent variables.

It is perhaps surprising that an estimating equation which does not result from an error minimization process can explain up to 46% of the variance in a cross-section of household consumption levels. On the other hand, the standard errors are not small in

relation to the mean level of consumption, ranging up to 39.7% of it.³ The differences in the performance of the model based on the assumption of different levels of consumer risk aversion seem fairly small when considered in the light of the widely differing age-consumption profiles shown in Figure 7.2 of Chapter 7. The standard errors do vary with v , however, and the minimum standard error is found at $v=1.5$.

TABLE 8.3

SUMMARY OF RESULTS OF ESTIMATING C^*

	$v=$	<u>.75</u>	<u>1.5</u>	<u>2.5</u>	<u>3.5</u>
1. Standard Error of Estimate (SEE)(\$)		6978	6568	6753	6900
2. Coefficient of Variation (SEE $\div \bar{C}$) ^a		.397	.374	.384	.392
3. $\bar{R}^2$		.392	.462	.431	.406
4. % of observations for which wealth constraint is binding		19	26	28	29
5. $\bar{C}^* / \bar{C}$		.820	.935	.980	.999
$n=6932$ $\bar{C}=\\$17580$ $SD(C) = \\$8950$					

3. Note that the calculated standard errors assume a simple random sample and take no account of sample design effects. See Statistics Canada, Consumer Income and Expenditure Division, "An Investigation of Variance Estimation in Family Expenditure Surveys" February, 1980.

It is interesting to note that the percentage of observations for which the constraint, $C^* \leq WH + WF + Y$, is binding rises substantially as v increases. This result is consistent with the consumption patterns of Figure 7.1. (Moreover, more detailed results confirm that the observations for which C^* is wealth constrained are concentrated in the younger age groups). The rise in the number of wealth-constrained consumers, as v increases, is paralleled by an increase in the predicted mean consumption level. For $v=.75$, the aggregate household consumption of the sample is underestimated by 18%; for $v=3.5$ it is estimated almost exactly. The implications of the error in estimating $\bar{C}$ deserve consideration.

The under-estimate of $\bar{C}$ for $v = .75$ relative to higher values of v is consistent with Figure 7.2, particularly when it is noted that 55% of the sampled spending units are headed by someone under age 45. However, an under-estimate of $\bar{C}$ could possibly be explained instead by a systematic under-estimation of consumers' anticipated lifetime resource levels. In particular, the assigned value of YF and the values of P and G for those below age 65 in the base year could be subject to systematic error. While this possibility cannot be dismissed, there are reasons to suspect that the estimates of expected future earnings and pension incomes are more likely to be over-estimated than under-estimated. For one thing, the 2% real growth assumed for the average wage rates and for public pension benefit levels may exceed consumers' expectations in 1978 given the low income and productivity increases in the post-OPEC period. Second, the estimates of YF , P , and G have not been discounted for the uncertainty that

consumers must attach to them. If it is true that some upward bias exists in the assigned estimates of expected future income, then the estimate $v = 1.5$ obtained from the model may be biased downwards rather than upwards.

In considering the average errors by age and income groups in Table 8.4, first note that a positive error signifies that the model under-estimates consumption for that group.

TABLE 8.4

AVERAGE ERRORS, $\frac{1}{n_k} \sum (C_i - C_i^*)$, BY AGE AND INCOME GROUP (\$)

Age	Income	<\$10,000	\$10,000 -24,999	\$25,000+
<25	$v = .75$	1884	3325	9687
	1.5	1398	1283	4439
	2.5	1271	662	2479
	3.5	1223	433	1716
25-34	$v = .75$	2341	2561	6581
	1.5	1677	-680	524
	2.5	1451	-1913	-2027
	3.5	1361	-2427	-3099
35-44	$v = .75$	1349	2414	7085
	1.5	735	-559	2064
	2.5	501	-1762	-34
	3.5	384	-2259	-912
45-54	$v = .75$	1591	2644	10132
	1.5	1110	766	7058
	2.5	902	12	5797
	3.5	808	-287	5280
55-64	$v = .75$	161	2801	9289
	1.5	62	1999	7665
	2.5	24	1666	7058
	3.5	7	1517	6752
65+	$v = .75$	-707	2586	11728
	1.5	-381	2778	11884
	2.5	-220	2921	12118
	3.5	-180	2888	12124

With two exceptions, the cell sample sizes should be sufficient to provide reasonable estimates of group mean consumption levels. The exceptions are the youngest and oldest groups within the \$25000+ income category, with 10 and 33 observations respectively. The groups with the largest frequencies are the 25-34 and 35-44 age groups in the middle income range; together they account for over 30% of the total sample.

A first observation from these errors is that the model consistently (i.e. for all the values of v) underestimates consumption by the under 25 age group. This may be taken as an indication that the non-negative wealth constraint is more severe than that which actually faces young consumers. Evidence on this point was presented in Chapter 7 where it was seen that about one quarter of under-25 family units, and about 14% of age 25-34 family units had negative net worth positions. Further evidence is found in the observation from data not reported here that, for young households for which the constraint was not binding, consumption was over-estimated for all cases except $v = .75$. A second observation is that consumption tends to be systematically under-estimated for the high income groups (and, to a lesser extent, the middle income groups) in the over-45 age categories. For those over 65, this almost certainly results from the crude assumption, made in the variable assignment process, that reported earnings decline to zero in the year following the base year. For the 45-64 age group, the systematic error could well arise from the assignment of future earnings and private pension benefit levels which are below the true expectations of the higher income

earners in the group. One cause of this would be the failure to assign private pension benefits to those in non-contributory pension plans. About 20% of covered employees are in non-contributory plans. Another source of systematic error could be a downward bias in assigned wealth levels due to the under-reporting of wealth in the Asset and Debt Survey. This was documented by Davies (1979 a,b). It would affect mainly the older, higher-income groups.

TABLE 8.5

REGRESSION ESTIMATES

A. EQUATION (8.3): FULL SAMPLE

$v \pm$	<u>.75</u>	<u>1.5</u>	<u>2.5</u>	<u>3.5</u>
SEE (\$)	4948	4721	4654	4708
$\bar{R}^2$	.694	.722	.730	.723
Constant	2299 (10.9)*	3962 (18.6)	4883 (21.9)	5388 (22.7)
QWH	.474 (7.7)	.428 (7.7)	.434 (8.0)	.423 (7.8)
QWF	.457 (17.5)	.461 (19.3)	.472 (19.8)	.462 (19.4)
QY	12.72 (50.6)	13.34 (58.9)	13.40 (60.9)	13.0 (59.5)
QYF	.862 (59.9)	.500 (45.3)	.380 (36.7)	.347 (33.6)
QP	-.226 (-2.4)	.068 (0.8)	.114 (1.4)	.202 (2.6)
QG	-.329 (-6.0)	-.760 (-13.2)	-.862 (-14.8)	-.919 (-14.8)

n = 6932

* t values in brackets

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TABLE 8.5 (Continued)

REGRESSION ESTIMATES

B. EQUATION (8.3): RESTRICTED SAMPLE**

** wealth constraint not binding

v =	.75	1.5	2.5	3.5
SEE(\$)	5160	5018	4985	5045
$\bar{R}^2$	.673	.699	.705	.699
n	5603	5139	4974	4907
Constant	3110 (12.2)*	5153 (18.4)	6162 (20.5)	6815 (21.8)
QWH	.430 (6.6)	.386 (6.4)	.383 (6.3)	.360 (6.0)
QWF	.452 (16.6)	.459 (18.0)	.472 (18.4)	.462 (18.1)
QY	12.17 (45.2)	12.66 (50.5)	12.70 (51.3)	12.26 (49.9)
QYF	.864 (52.4)	.509 (37.5)	.390 (29.7)	.360 (27.3)
QP	-.090 (-.9)	.201 (2.1)	.241 (2.5)	.325 (3.4)
QG	-.389 (-6.1)	-.873 (-12.5)	-.979 (13.5)	-1.05 (-13.6)

* t values in brackets

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TABLE 8.5 (Continued)

REGRESSION EQUATIONS

C. STANDARD CONSUMPTION FUNCTION: FULL SAMPLE

SEE (\$)		4109
$\bar{R}^2$		.789
n		6932
<u>variable</u>	<u>coefficient</u>	<u>t value</u>
constant	2215	(14.1)
Income*	.794	(134.)
Family Size	5.48	(14.4)
D<25	426	(2.1)
D 25-34	-	-
D 35-44	-19	(.0)
D 45-54	-121	(-.8)
D 55-64	-728	(-12.9)
D 65+	-1025	(-6.0)

* Disposable income including investment income and imputed rent.

Turning to the estimates of regression equation (8.3) it is immediately apparent that this equation explains considerably more of the variance in C than does equation (8.2). This comes as no surprise since (8.2) is equivalent to (8.3) with a_0 constrained to equal zero and $a_1, \dots, a_6$ constrained to equal one. The $\bar{R}^2$'s range from .694 to .730 for the full sample and from .673 to .705 for the restricted sample (observations for which wealth constraint is not binding).

Part C of Table 8.5 shows that the explanatory power of the model

with unconstrained coefficients still falls short of that of a standard consumption function with current disposable income, family size and age dummies as independent variables. The $\bar{R}^2$ for the standard model is .789. Whether due to the undoubtedly substantial errors in assigning proxy variables for lifetime resources or to inadequacy of the underlying theoretical model, the model as developed in equations (8.2) and (8.3) does not improve upon the predictive power of the simple consumption function.

The poorest-fitting version of equation (8.3) is again that with $v=.75$, but now the least error case is $v=2.5$ instead of $v=1.5$. These results hold for both the full and restricted samples which provide remarkably similar results.

Predictably perhaps, some of the coefficients are found to stray quite far from the values appropriate to equation (8.2). The constant term is always well above its supposed value of zero. Most of the coefficients of the components of lifetime resources are well below 1.0 with the coefficient of QP being generally close to zero and the coefficient of QG being invariably negative. The coefficient of QY, on the other hand, ranges from 12.7 to 13.40. Part of the explanation for these unexpected coefficient values and the variation in some of them for different samples and values of v lies in collinearity among the independent variables. For example, positive correlation is found between all pairs of QWH, QY, QP and QG with r^2 values ranging from .39 to .68. In addition, QWH and QG are negatively correlated with YF. One result of this is the negative coefficient

of QG which occurs despite the fact that the simple correlation between C and QG is positive. In the context of the lifecycle model or any other model of consumption, some of these cross-sectional correlations are spurious. An increase in P or G, for example, must be expected to increase C regardless of the pattern of P and G levels across income groups. Thus it seems reasonable to argue that more attention should be paid to the constrained coefficient version of the model, equation (8.2), despite the higher standard errors associated with the model in that form.

The findings of this chapter may be summarized as follows:

1. Even when estimated directly, rather than using a error minimization procedure, the model can explain close to one-half the variation in household consumption.

2. The proportion of variance explained rises to 73% when the model is estimated as a simple regression equation. It is higher yet, at 79% for a simple regression equation in which consumption is explained only by current income, the size of the household and the age of its head. While the inability of the lifecycle model to improve on the explanatory power of the standard consumption function is disappointing, it is not really unexpected. The reasons for this are, first, that many serious errors are undoubtedly introduced into the lifecycle estimates through the use of rough proxy variables for most of the components of lifetime income, and, second, that the systematic variation of the average propensity to consume by age,

which is a key feature of the lifecycle model, is also captured in the age dummies of the standard consumption function. Moreover, the standard consumption function includes age and family size as explanatory variables without restricting the way in which they are permitted to influence consumption.

3. There are noticeable differences in the explanatory power ($\bar{R}^2$'s) of the model when estimated with different values of the risk aversion parameter, v . However, these differences are not great in comparison to the change in $\bar{R}^2$ which results from moving from deterministic to stochastic estimation of the model (i.e. from equation (8.2) to equation 8.3)). In addition, the minimum error value of v is not the same for both the deterministic and stochastic estimation methods. For the former, $v = 1.5$ provides the best estimate; for the latter, $v = 2.5$ does. Under both methods, the low risk aversion case of $v = 0.75$ ranks last in explanatory power. The inability of these estimates to show either of $v = 1.5$ or $v = 2.5$ as the clearly preferred estimate of v may help to explain why both values have been obtained in previous estimates of this parameter in the literature. Clearly, the difference between average values of v of 1.5 and 2.5 does not give rise to striking differences in the observed pattern of consumption across households.

4. According to the estimates provided in Chapter 7, a value of the risk aversion parameter in the 1.5 to 2.5 range implies an aggregate interest elasticity of savings of about 0.13, considerably lower than recent estimates in the literature. This level of risk

aversion also implies that improvements in the market for annuities (assuming it to be not well developed to begin with) could provide substantial welfare gains to consumers.

CHAPTER 9

SUMMARY OF FINDINGS

This thesis has investigated the comparative advantages of public pensions and tax incentives for saving, two ways in which governments can assist or encourage individuals to obtain higher incomes in retirement. It has also examined the interactions between and welfare implications of the two types of policy. The main approach taken in the investigation has been to develop and analyse a lifecycle model of household consumption and savings which is based on the hypothesis of utility maximization and elaborated in such a way as to permit important features of the retirement income system to be taken into consideration. For example, the model permits explicit consideration of the response of savings to the post-tax interest rate. Second, a multi-period framework is employed so that the variation in circumstances and savings behaviour of households at different stages in the lifecycle can be examined. This approach also permits age-related policies to be faithfully represented (e.g. public pensions payable from age 65). Third, uncertain longevity is explicitly incorporated in the model so that the consumer's response to this uncertainty can be analysed. Another reason for employing a model which is realistic in many respects is to enable the model to be estimated using survey data on the expenditure patterns of Canadian households. The main disadvantage of employing a model of this level of detail and realism is that it is too cumbersome to permit the nature of some responses to be conveniently illustrated or to permit

some policy questions to be examined (for example, variations in the tax treatment of different assets). For this reason, a number of questions were investigated with the aid of a simpler, two-period version of the model. The following paragraphs summarize the findings derived from the review of the literature in Chapters 2, 3, and 4 and the analysis and estimation of the lifecycle models in Chapters 6, 7, and 8.

In the simplest, partial equilibrium models, taxes on income from saving reduce economic efficiency and social welfare by creating a divergence between the rates at which goods can be exchanged over time in consumption and in production. When the existence of the distortions created by other taxes is recognized, the optimal level of capital taxation is found to depend on the interest elasticity of savings, elasticity of labour supply and other cross-elasticities. There is no longer a presumption that income from capital should be untaxed. As investment income is concentrated among high-income groups, this result is supported by vertical equity considerations which are absent from most optimal taxation models used to compare the efficiency of different tax regimes. In addition, it may be supported by the administrative costliness of alternative taxes such as taxes on gifts and inheritances. If tax rates are applied to nominal investment income, a reduction in those tax rates may be justified when inflation rises. However, the indexation of capital taxes would appear to be a preferred solution to this problem. The optimal level of capital taxation varies inversely with the interest elasticity of savings. Therefore, one can argue that any measures

which significantly raise the interest elasticity of saving should be complemented by a decrease in the marginal tax rates on investment income.

In the context of economic models of social welfare, four possible roles for public pensions have been identified. A redistributive rôle, based on utility interdependence or the external costs of poverty, could explain the existence of public pension benefits which are concentrated among lower-income groups in the retired population. A second role is found in the use of contributory public pension programs to force middle- and higher-income earners to attain a base level of retirement income. This could reduce the work- and savings-disincentive costs arising from high tax-back rates on redistributive public pensions. Such a "forced savings" role could explain the existence of universal contributory public pensions (like C/QPP) covering a base level of earnings from employment. The introduction of unfunded public pensions has also been proposed as a means of exploiting the gains available through a system of transfers from younger to older generations when the rate of growth of the payroll tax base exceeds the interest rate. This role for public pensions has little appeal since, even if such gains are available now (which is doubtful), they are likely to be replaced by losses for some age cohorts during periods when a decline is experienced in the rate of growth of total wages.

A fourth possible role for public pensions is to provide insurance services which private markets can not provide. Retirement income levels are affected by uncertainties associated with the rate of return on saving, longevity, and the level of earnings in the years prior to the normal age of retirement. In each of these cases, reasons may be found to suppose that private insurance markets will be incomplete. No insurance is available at the present time to protect investments from losses due to the effects of inflation. The market for annuities is restricted by the effects of adverse selection. The provision of insurance against the loss of earnings in late career is hampered both by moral hazard and adverse selection problems. The risk pooling and compulsion inherent in the provision of a universal public pension can overcome most of the problems which beset private insurance markets. Furthermore, one analysis has indicated that, by reducing the scale of adverse selection effects, the provision of a public pension may actually lead to an increase in the private provision of annuities (rather than merely substituting for less efficient private markets).

It seems clear, therefore, that a potential insurance role exists for public pensions. However, moral hazard costs and other inefficiencies also attend the provision of public pension income, so it can not be demonstrated at this level of analysis that a net welfare gain is available from the provision of insurance services through public pensions. This caveat is important, for much of the analysis of the later chapters of the thesis proceeds under the assumption that private insurance markets are incomplete and welfare gains are available

through the provision or expansion of public pensions. The premise of the thesis is that the likelihood that such potential welfare gains do exist is at least strong enough to make it worthwhile to examine retirement income policy in a model which assumes incomplete private insurance markets.

Feldstein has argued that the provision of unfunded public pension benefits leads to a decline in national saving and productive capacity. Private saving will fall as a result of the income effect of the promised pensions; with an unfunded plan there will be no offsetting increase in public saving. On the basis of the considerable theoretical and empirical analysis that has followed Feldstein's article, it is likely that these savings displacement effects are relatively small. Much of the theoretical basis for Feldstein's result has been removed by Barro's argument that, since individuals value the welfare of their parents and children (as demonstrated by family support of the elderly and by bequests and inter vivos transfers to children), they will react to measures which burden other generations by making offsetting adjustments to the level of their private intergenerational transfers. The imperfect substitutability of public pension promises and private savings should further diminish the degree to which public pensions displace private savings. While Feldstein has found strong displacement effects empirically, his empirical models have been criticized, and most other researchers have failed to find significant effects.

With regard to two other empirical questions relating to pensions and saving, recent research on both the interest elasticity of saving and the coefficient of relative risk aversion has supported higher estimates than were previously accepted.

The two period model with no uncertainty was used to examine the effects on consumption and savings of changes in tax rates on investment income, tax deductions, different public pensions, and the tax deduction on RPP/RRSP contributions. A change in the marginal rate of tax on investment income implies a change in the net rate of return on saving. For a consumer with no second period endowment (eg. no public pension), the response of current consumption to a change in the rate of return depends on the relative strengths of offsetting income and substitution effects. The increased interest rate favours consumption deferral while the positive income effect favours an increase in consumption in both periods. For consumers with low risk aversion ($v < 1$), the substitution effect always dominates, and an increase in interest rates always leads to a decline in current consumption. For more risk averse consumers ($v > 1$) the converse is true. The case of a consumer with a second period endowment is complicated by the fact that a rise in the interest rate reduces the present value of the future endowment. This reduces the positive income effect of the increased rate of return with the result that current consumption can be reduced even for risk averse consumers with $v > 1$. These results may be restated in terms of the interest elasticity of saving: (1) the elasticity tends to be greatest for low-risk aversion consumers with $v < 1$, and (2) public pension promises

or other second period resource endowments tend to offset the positive income effect of an interest rate increase and so increase the interest elasticity of saving.

A tax deduction like the age deduction has a negligible effect on the marginal rates of return of most consumers, but (if uncompensated) has a positive income effect; thus, it tends to depress savings. Deductions on income from savings, such as the \$1000 interest, dividend and capital gains deduction and the \$1000 pension income deduction, do raise net interest rates for a significant proportion of taxpayers and so are more likely to stimulate savings. For taxpayers with investment or pension incomes over \$1000, though, they have no substitution effect and provide no encouragement of savings. When changes in tax rates and deductions are considered in the context of a multi-asset model, the effects of the measures on total saving are found to be diminished by the tendency of consumers to respond to the tax changes by adjusting their asset portfolios.

Pension benefits (like OAS and C/QPP) which are unrelated to levels of investment or private pension income in retirement are found to affect saving only through their income effects. Income-tested pensions like GIS, however, can provide powerful disincentives to save. Even quite generous savings incentives are likely to have little effect on individuals at lower-income levels who expect to receive some level of GIS benefits in retirement.

The tax deductibility of RPP and RRSP contributions is a major government program when measured by its current-year revenue costs. It was found to have a strong impact on savings merely by virtue of the obligation it imposes to build up sufficient capital to pay the deferred taxes. Based on recent contribution levels, this tax provision may have contributed up to 1.5 percentage points to the personal savings rate in Canada. The effect of the RPP/RRSP deduction on post-tax income and consumption levels in retirement is much less clear. Following Hood's analysis, it was shown that the deduction (whether or not subject to limits) would only produce an increase in the rate of return on savings, at the margin, under certain circumstances; in other cases it could actually lead to an increase in the ratio of present to future consumption. The deduction would only lead to consumption deferral if it had the effect of moving the consumer from a situation of having taxable investment income to one where his investment income was under \$1000 (or negative) and so non-taxable. This result suggests that the removal of tax deductibility on monies borrowed to purchase RRSPs may have a significant effect in encouraging savings. Finally, it was noted that saving and consumption deferral could be encouraged by other tax provisions, such as inflation indexing of capital gains and a retirement income deduction which is earned by making non-deductible pension or savings plan contributions.

A number of results were obtained by examining the multi-period form of the model with uncertain lifetime and wealth constrained to be non-negative. To begin with, the model was found to produce a

humped consumption profile over the lifetime as has been observed in cross-section expenditure data. The shape of the profile was affected by the assumed values of the time preference rate and the interest rate and, even more strongly, by the level of consumer risk aversion. To facilitate estimation of the model with expenditure survey data, a method was developed to enable variation in the expected size of the family over its lifetime to be represented by modifications in the survival probabilities applied. The resulting consumption profiles varied appropriately with family composition.

Estimates were made of the interest elasticity of savings, assuming different levels of risk aversion, for households of different ages and for the population as a whole. For individual households, the elasticities varied markedly with age, suggesting that demographic shifts could produce variations in the aggregate elasticity. The elasticity tended to fall with age, reflecting a growing ratio of current wealth to expected future endowments (i.e. future earnings and public pensions). The age differences in the elasticity were substantially greater than its variation over the assumed levels of risk aversion. For the population as a whole, the elasticity varied from 0.24 with $v=.5$ to 0.12 with $v=2.5$. These values are somewhat lower than those obtained from time series data by Boskin.

The effect of uncertainty regarding the net rate of return on saving can be represented by a certainty equivalent rate of interest which is lower than the expected rate. Based on annual data on real rates of return on pension fund assets over the last two decades, a

tentative estimate was made which suggested a risk premium of about one-half of one percent (on an expected return of 3.5%). The effects of rate of return uncertainty on consumption and savings depend on the relative strengths of the income and substitution effects embodied in the interest elasticity of savings.

A similar analysis of the effect of uncertainty regarding future non-capital income led to the conclusion that, since no offsetting substitution effects are involved, income uncertainty should have a strong effect in stimulating savings.

The introduction of uncertain longevity into the model (without changing expected longevity) was found to lead to an acceleration of consumption, ie. an increase in the ratio of current to future consumption. Again the balance of income and substitution effects produced the result that current consumption would only increase in absolute terms for low-risk aversion consumers with $v < 1$. In the absence of annuity markets or public pension income, a consumer could expect to retain a substantial level of accumulated savings at the time of death, and this result is obtained with no bequest motive assumed. The level of unintended bequest, which measures the extent to which a consumer will self-insure the contingency of unexpected longevity, naturally rises with the assumed level of risk aversion. When annuity income is relatively low, the level of unintended bequests can be substantial; thus, it is found that the introduction of a bequest motive may not have a very great effect on observed wealth levels in retirement.

When an actuarially fair market in annuities is introduced into the model, the consumer with no bequest motive will place all his savings in annuities. The effective rate of return on savings is increased, and, as unintended bequests can be reduced to zero, a substantial positive income effect occurs. Once again, current consumption levels will rise or fall according to whether the consumer's coefficient of risk aversion, v , is greater than or less than one. As annuities permit the consumer to economize on self-insurance against unexpected longevity (or to reduce the expected level of unintended bequests), a dollar of annuity wealth or public pension income was found to substitute for more than a dollar of non-annuity wealth. As expected, the ratio was found to rise with v , from 1.66 for $v=1$ to 1.77 for $v=2.5$. An examination of the composition of the retirement income of Canadians over age 65 in 1978 showed that annuity-form income accounts for about 60% of total income apart from earnings and that public pension income accounts for over 75% of annuity-form income. Although the strength of the bequest motive is unknown, this information may provide some support for the notion that welfare gains could be achieved either by improvements in the efficiency of private annuity markets or by an increase in the level of public pensions.

As a policy analysis illustration, the effects of an increase in the post-tax rate of return on savings (via tax rate cuts) were compared with those of an increase in the level of public pension benefits. The interest rate increase was found to be more effective

in raising post-retirement consumption levels while the public pension increase was found to provide the greater improvement in consumer welfare.

When the model was estimated deterministically using expenditure information for a sample of Canadian households, it was found to explain a surprisingly high proportion (46%) of the variance in current year consumption levels. The proportion of variance explained rose substantially, to 73% when the equation was re-estimated using least squares. However, the equation suffered from multi-collinearity, and some coefficients on the explanatory variables (eg. on public pensions) were found to have inappropriate signs. A simple Keynesian model, with age and family size dummies added, was found to explain an even higher proportion of variance (79%) perhaps simply because the least restrictions were placed on the coefficients. The equations were estimated based on four different assumed variables for the risk aversion parameter, v . While the deterministic and least squares methods did not favour the same value of v , the two results placed v at either 1.5 or 2.5. (Note that the impact of changes in v on spending patterns declines sharply as v increases; thus the choice between $v=1.0$ and $v=1.5$ is much more significant than the choice between $v=2.0$ and $v=2.5$.) A level of risk aversion parameter of 1.5 to 2.5 was found to be consistent with an estimated interest elasticity of savings of about 0.13.

The set of findings obtained in this thesis can be summarized in six propositions.

1. There is evidence that public pensions can and do play a significant role in raising the level of retirement income insurance available to individuals. This evidence includes theoretically predicted problems (particularly the adverse selection problem) with private insurance markets, evidence of a relatively high ($v \geq 1.5$) level of risk aversion among consumers, and the observation that public pension income is currently much greater than private annuity income.

2. Some (less than dollar-for-dollar) reduction in private saving in response to an increase in public pension benefits is very likely.

3. Government policies which stimulate savings would likely be more effective than public pension increases in raising the retirement consumption levels of those now in the labour force; however, public pension increases are the more likely to raise welfare levels.

4. An increase in public pension promises tends to increase the interest elasticity of savings.

5. The preceding four propositions imply that savings incentives and public pension improvements should be considered as complementary policy approaches rather than merely as substitutes for one another.

6. Given the large income effects but limited substitution effects associated with the deductibility of RPP and RRSP contributions, consideration should be given to other possible forms of savings incentive which would have more certain effects on the post-tax rate of return on savings at the margin.

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