INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI®

Prices and Pricing in Imperfectly Competitive Markets

by

Ying Kong

B.S.(Honours) Peking University, M.A. Carleton University

A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfilment of the requirement for the degree of
Doctor of Philosophy
in the
Department of Economics

CARLETON UNIVERSITY

Ottawa, Ontario

September 2000

Copyright © 2000, Ying Kong

All rights reserved. This work may not be reproduced
in whole or in part, by photocopying or other means without
permission of the author

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-57613-2

Canada

The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis,

“Prices and Pricing in Imperfectly Competitive Markets”

submitted by

Ying Kong, B.S., M.A.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Economics

Thesis Supervisor

External Examiner

Carleton University
September 2000

Abstract

In this thesis I study price competition in imperfectly competitive markets. Three basic models are proposed to analyse price competition using game theory. The purpose of my thesis is to extend competition analysis with new assumptions.

Chapter 1 studies equilibrium price dispersion with congestion costs. In the literature, explanations of price dispersion invariably depend on the assumptions of imperfect information and/or some form of heterogeneity among either consumers or firms. The objective of this chapter is to demonstrate that price dispersion is possible even in a world of perfect information and identical consumers and firms. The driving force of the model in this chapter is service capacity and congestion cost. Each firm chooses a service capacity (e.g., the size of a store or the number of telephones operators taking customers orders). Customers of a firm bear a congestion cost that, for a fixed service capacity, is an increasing function of the number of customers served by this firm. We demonstrate that even though customers have perfect information about prices and both consumers and firms are *ex ante* identical in all aspects, there exist equilibriums where firms choose different service capacities and charge different prices for the same product. Under certain conditions, the total profits of the two firms and total surplus are higher in the price-dispersion equilibriums than that in the one-price equilibrium.

Chapter 2 examines the strategic pricing behaviour of brand name drugs in response to generic entry in the pharmaceutical market. The objective of this chapter is to provide a theoretical model that

explains the “Generic Competition Paradox” which states that the brand-name drug price would rise in response to generic entry. Without relying on the assumption that one set of “loyal” consumers consume only brand-name drugs, the model demonstrates that the determining factor is the size of marginal cost of production of the generic firms relative to certain characteristics of market demand. Under the assumption that all consumers will consume both brand-name and generic drugs, the model shows that the price of brand-name drug will rise following generic entry if the marginal cost of production for generic firm is relatively large. More specifically, the condition for a brand-name price increase can be met when the market share of price-insensitive consumers is relatively large and the factor of cross substitutability for these same consumers is relatively large. Similarly, the same condition can be met if the cross substitute factor for price-insensitive consumers is relatively small and the relative market share of price-insensitive consumers is small.

Chapter 3 presents a welfare analysis of dumping in an oligopolistic domestic good market. The existing literature on dumping focuses in large part on the analysis of anti-dumping duties based on the implicit assumption that the domestic import-competing industry is perfectly competitive or monopoly. Taking dumping as a given in an oligopolistic home market, the model tries to determine when dumping can negatively affect the home country’s total welfare in the absence of any anti-dumping duties. The key variables of interest in this model are the number of domestic firms and the degree of substitutability among home produced products. The model demonstrates the important ways in which the value of the cross elasticity of demand between home produced goods and imported goods will affect the change in the total welfare that follows dumping, especially for the case of differentiated home goods. With a linear marginal cost function, dumping cannot reduce the

total welfare in an oligopolistic home market if its product is homogenous. However, total welfare in the oligopolistic home market can be reduced if the products are sufficiently differentiated.

Acknowledgements

I would like to express my deep appreciation to my supervisor Professor Zhiqi Chen. He has given me invaluable input and guidance extending far beyond the vital steps required to complete this thesis. Working with Professor Chen over the years has been a truly learning and rewarding experience for me.

I also want to express my sincere thanks to Professor Richard A. Brecher, the Ph.D Student Supervisor, Professor Nicolas Rowe, Chair in the Department of Economics at Carleton University and Professor Rose Anne Devlin, the director of Doctoral Studies of the Joint Ph.D Program with the University of Ottawa. During the past several years, they have given me endless support in many aspects in my thesis studies. Without their understanding and help, I could not fulfil my Ph.D studies in Economics successfully.

A very special thank-you to Professor Steven Ferris, the most warm-hearted professor I have ever known. I am grateful for his timely assistance and his forever support and encouragement in both my studies and personal life. Working with Professor Ferris has provided me with the building blocks needed to develop and apply my analytical abilities with confidence.

I would like to express my appreciation for all the kindness and help from Professor Keith Acheson, Professor Donald G. McFetridge, Professor E. U. Choudri and Professor Kier Armstrong. My thesis has benefited greatly by following their advices on the initiation of the topics as well as the

development of the models.

Further, I would like to extend my thanks to Ms. Judy Poole, Ms. Ginette Lafleur and Ms. Claudia Van Decker for providing an excellent supportive environment for the students. Studying in the Department of Economics at Carleton University is a rewarding experience that I will remember for my entire life.

At last, thanks to my dear wife Diana, for her understanding and patience. I would like to dedicate this thesis to her for her invaluable love and support.

Table of Contents

Acceptance Sheet.....	ii
Abstract.....	iii
Acknowledgements.....	vi
Table of Contents.....	viii
List of Tables.....	x
List of Figures.....	xi
 Chapter 1. Capacity Utilization, Congestion Cost and Price Dispersion.....	 1
I. Introduction.....	1
II. The Basic Model.....	3
III. The Case of Linear Congestion Cost.....	11
IV. Comparison of the Two Types of Equilibria.....	22
V. A Special Example.....	36
VI. Concluding Remarks.....	42
Appendix A.....	43
Appendix B	46
Reference.....	58
 Chapter 2. Pricing Name-Brand Products in the Presence of Generic Entry - A Study Of The "Generic Competition Paradox".....	 60
I. Introduction.....	60
II. The Basic Model.....	66
III. An Extension: The Brand-Name Firm Produces A Generic Product in Response to Generic	

Entry.....	87
IV. Concluding Remarks.....	94
Appendix.....	96
Reference.....	102
 Chapter 3. Dumping in Oligopoly Market: A Welfare Analysis.....	105
I. Introduction.....	105
II. The Basic Model.....	109
III. The Linear Demand with Homogeneous Home Produced Goods.....	116
IV. The Linear Demand with Differentiated Home Produced Goods.....	130
V. Concluding Remarks.....	139
Appendix.....	141
Reference.....	151

List of Tables

Table 1.1 A Comparison between Symmetric and Asymmetric Equilibria.....	39
Table 1.2 The Role of θ	41

List of Figures

Figure 1.1 The Price Setting with $y_1=y_2$	14
Figure 1.2 The Price Setting with $y_1 \neq y_2$	14
Figure 1.3 The Capacity Levels in Asymmetric Equilibrium - Case 1.....	20
Figure 1.4 The Capacity Level in Asymmetric Equilibrium - Case 2.....	21
Figure 2.1 The Demand Curve in the Brand-name Market.....	73
Figure 2.2 Brand-Name Drug Price Increases with Generic Entry when $c_b > (1/\gamma)(\alpha-\beta c_g)$	85
Figure 2.3 Brand-Name Drug Price Increases with Generic Entry when $c_b < (1/\gamma)(\alpha-\beta c_g)$	86
Figure 3.1 Changes in Consumer Surplus in the Imported Good Market.....	122
Figure 3.2 Changes in Total Surplus in the Home Produced Good Market.....	123
Figure 3.3 The Change in Total Welfare in the Home Good Market As n Increases.....	125
Figure 3.4 The Change in Total Welfare Change As a Function of the Number of the Home Firms.....	126
Figure 3.5 Changes in Total Welfare in Home Produced Good Market When b Changes.....	129
Figure 3.6 Changes of the Total Welfare in the Case of $\underline{\theta} < \theta < \theta^*$ given n	137
Figure 3.7 Changes of Total welfare when θ Decreases in the Domestic Good Market.....	138
Figure 3.8 Proof of the Changes in the Total Welfare in the Imported Good Market	144
Figure 3.9 Proof of the Changes in the Total Welfare in the Domestic Good Market.....	145

CHAPTER 1

CAPACITY UTILIZATION, CONGESTION COST AND PRICE DISPERSION

I. INTRODUCTION

It is well-known by now that price competition among firms does not always lead to a single equilibrium price. In the existing literature there are many theoretical models that generate equilibrium price dispersion (see chapter 16 in Shy (1995) for a lucid discussion of various price dispersion theories). Following Stigler (1961), all these models share a common assumption which is (at least some) consumers have imperfect information about the prices offered by firms. Consumers learn the price of a firm either through search (e.g. Salop and Stiglitz 1982, Carlson and McAfee 1983, Burdett and Judd 1983) or through advertisements (Butters 1977, Bester and Petrakis 1995) or both (Robert and Stahl II 1993). Furthermore, it is common in these models to assume that either consumers or firms are heterogenous (e.g. Reinganum 1979, Wilde and Schwartz 1979, Luski 1976, and Reitman 1991).

The objective of this chapter is to demonstrate that equilibrium price dispersion is possible even if consumers have perfect information about prices and if consumers and firms are homogeneous. Our model extends the framework of Luski 1976 and Reitman 1991. Based on the premise that a

consumer has to incur a congestion cost when a large number of consumers buy from the same firm and, the congestion cost rises when the number of customers served by the firm increases. For example, suppose a consumer orders a good through the telephone. Given that the number of telephone operators working for a firm is fixed, a consumer who calls will be put on hold longer when there are more consumers calling the same firm. Alternatively, suppose that the firm in our question is a retail store. Given a fixed shop space and a fixed number of cash registers, the store becomes more crowded and the queues at the check-out counters are longer when the number of customers increases. Our model is more general than Luski (1976) and Reitman (1991) in the sense that the level of service capacity is endogenous - firms can choose the number of telephone operators or the size of shop space and the number of check-out lines.

To be more specific, we consider a two-stage model of two firms that have identical cost structure and produce homogeneous goods with N identical consumers. At stage one, the firms simultaneously choose the levels of service capacity. At stage two, the firms set prices, and each consumer who observes the prices perfectly decides from which firm he/she will buy the product. We demonstrate that even though customers have perfect information about prices and both consumers and firms are *ex ante* identical in all aspects, there exist equilibria where firms choose different service capacities and charge different prices for the same product. Under certain conditions, the total profits of the two firms are higher in the price-dispersion equilibria than that in the one-price equilibrium. More interestingly, under certain conditions the price dispersion equilibria produce higher total surplus than the single price equilibrium.

Discussions are organized as follows. Section II discusses the two-stage model. Section III considers the case of linear congestion cost. In section IV we compare the price dispersion equilibriums and the one-price equilibrium. Section V analyzes a specific example and conducts numerical simulations. Concluding remarks are given in section VI.

II. THE BASIC MODEL

1. *A Two-stage Game Model*

We can formulate the strategic behavior of both firms and consumers in a two-stage game. To add specificity, we suppose that our example covers a new product or a new service in the market where two firms have to decide their service capacities. The firms then undergo a price competition corresponding to the consumers' demand. In the first stage of this game, firms compete in capacity while in the second stage of the game, firms compete in price. The best way of solving the sub-game equilibrium of a dynamic finite game is to solve it backwards. Therefore, we begin with the second stage by asking what prices firms will choose in a Nash-equilibrium one-shot price game where the service capacity is taken as given from a prior first-stage choice. We then analyze the first stage to find a sub-game equilibrium in capacity levels. Here, the second-stage equilibrium market prices solved for the first stage are taken into considerations by the firms.

Consider a market where two firms that are *ex ante* identical in all aspects sell an identical product. The marginal cost of production is constant and is normalized to zero. We consider a two-stage game between these two firms. At stage 1, firms choose the service capacity levels (for example, the size of the store or the number of telephone operators); and at stage 2, firms choose prices taking as given the capacity levels. Thus, the strategic choices of each firm are the price P_i ($i=1,2$) and the service capacity level y_i with capacity cost $K(y_i)$. It is reasonable to assume that capacity is an increasing function of y_i . Furthermore, we assume that the marginal capacity cost rises when capacity level increases, i.e.

$$K'(y_i) > 0, \quad K''(y_i) > 0, \quad K'(0) = 0, \quad K'(\infty) = \infty.$$

On the demand side, we assume that there are N *ex ante* identical consumers with a common reservation price of V . Each consumer has perfect information about firms in terms of the price and the service capacity. Also, each consumer will buy at most one unit of the product. In other words, the total demand in the market for this particular product is equal to N when the full product price is less than or equal to V .

In addition to the price P_i , we assume that a consumer incurs a congestion cost when buying the product from firm i . This congestion cost increases with the number of consumers who buy from the same firm, n_i , and decreases with the capacity level, y_i . We denote this congestion cost as $C(n_i, y_i)$. The congestion cost has the following properties:

$$\frac{\partial C(n_i, y_i)}{\partial n_i} > 0, \quad \frac{\partial^2 C(n_i, y_i)}{\partial n_i^2} \geq 0, \quad \frac{\partial C(n_i, y_i)}{\partial y_i} < 0.$$

Above assumption implies that the marginal congestion cost increases as the number of consumer increase. The surplus a consumer obtains from buying a unit of the product from firm i is $V - P_i - C(n_i, y_i)$. The consumer will choose a purchasing strategy that yields him/her the highest surplus. Given that consumers are *ex ante* identical, all of them will buy from the same firm as long as

$$V - P_i - C(n_i, y_i) \neq V - P_j - C(n_j, y_j) \quad i \neq j$$

Therefore, the following equation must hold in equilibrium,

$$V - P_i - C(n_i, y_i) = V - P_j - C(n_j, y_j) \quad i \neq j \quad (1.1)$$

where n_i , n_j are the numbers of consumers who buy from firm i and firm j .

Since $N = n_i + n_j$, we can rewrite (1.1) as

$$P_j - P_i = C(n_i, y_i) - C[(N - n_i), y_j] \quad (1.2)$$

Let $X = P_j - P_i$ and $D = C(n_i, y_i) - C[(N - n_i), y_j]$. Then, equation (1.2) can be written as

$$X = D(n_i, y_i, y_j) \quad (1.3)$$

Since

$$\frac{\partial D}{\partial n_i} = C_1(n_i, y_i) + C_1(N - n_i, y_j) > 0 \quad (1.4)$$

where $C_1(\cdot)$ is the first order derivative with respect to the first variable of the function, the function of X is monotonically increasing in n_i . Therefore, we can solve (1.3) for n_i as a function of X . Let $n_i = G(X, y_i, y_j)$, (1.4) implies that $\partial G / \partial X > 0$ ¹.

---

¹Note:

$$\frac{\partial X}{\partial n_i} = \frac{\partial D}{\partial n_i} > 0 \quad \text{then} \quad \frac{\partial G}{\partial X} = \frac{\partial n_i}{\partial X} > 0$$

By symmetry,

$$n_j = G(P_i, -P_j, y_j, y_j) = G(-X, y_j, y_j)$$

2. Stage Two Equilibrium

Firm i 's profit can be written as

$$\pi_i(X, y_i, y_j) = P_i \cdot n_i = P_i \cdot G(X, y_i, y_j) \quad (1.5)$$

At stage 2, firm i chooses P_i taking as given P_j . The solution to P_i is

$$G(X, y_i, y_j) - P_i \cdot G_1(X, y_i, y_j) = 0$$

where $G_1(\cdot)$ represents the derivative of function $G(\cdot)$ with respect to its first variable, X .

Then we can solve the price level of firm i as,

$$P_i = \frac{G(X, y_i, y_j)}{G_1(X, y_i, y_j)} \quad (1.6)$$

By symmetry, the following condition can be obtained from firm j 's F.O.C.

$$P_j = \frac{G(P_i - P_j, y_j, y_j)}{G_1(P_i - P_j, y_j, y_j)} = \frac{G(-X, y_j, y_j)}{G_1(-X, y_j, y_j)} \quad (1.7)$$

Equation (1.6) and (1.7) imply

$$\begin{aligned}
 X=P_j-P_i &= \frac{G(-X,y_i,y_j)}{G_1(-X,y_i,y_j)} - \frac{G(X,y_i,y_j)}{G_1(X,y_i,y_j)} = \frac{N-G(X,y_i,y_j)}{\partial[N-G(X,y_i,y_j)]/\partial(-X)} - \frac{G(X,y_i,y_j)}{G_1(X,y_i,y_j)} \\
 &= \frac{N-G(X,y_i,y_j)}{G_1(X,y_i,y_j)} - \frac{G(X,y_i,y_j)}{G_1(X,y_i,y_j)} = \frac{N-2G(X,y_i,y_j)}{G_1(X,y_i,y_j)}
 \end{aligned} \tag{1.8}$$

Define the function $F(X,y_i,y_j)$ as

$$F(X,y_i,y_j) = \frac{N-2G(X,y_i,y_j)}{G_1(X,y_i,y_j)} \tag{1.9}$$

Equation (1.8) is equivalent to

$$X=F(X,y_i,y_j) \tag{1.10}$$

We now discuss equation (1.10) in two different situations.

(1) Suppose $y_i=y_j$, then $X=0$ is a solution to (1.8)

When $X=0$,

$$F(0,y_i,y_j) = \frac{N-2G(0,y_i,y_j)}{G_1(0,y_i,y_j)} \tag{1.11}$$

$G(0,y_i,y_j)=N/2$ is true if and only if $y_i=y_j$, because the number of consumers in firm i will be the same

as the number of consumers in firm j as $P_i = P_j$ and $y_i = y_j = y$ from equations (1.2), (1.3) and (1.4). The result of $G(0, y, y) = N/2$ implies that $F(0, y, y) = 0$. Therefore, $y_i = y_j = y$ is the sufficient condition for $X = 0$ as a fixed point of $F(X, y, y)$.

PROPOSITION 1.1

When two firms choose the same capacity level, there exists an equilibrium in which the two firms charge the same price, i.e. $P_1 = P_2$.

(2) Suppose $y_i \neq y_j$, then $X = 0$ is not a solution to (1.8) and, $P_i > P_j$ if and only if $y_i > y_j$

Suppose $X = 0$ and $y_i \neq y_j$. Equation (1.2) implies that $n_i \neq n_j$ or $n_i \neq N/2$. Thus, the numerator of the right hand of (1.8) is

$$N - 2n_i < 0 \quad (1.12)$$

and the left hand of (1.8) is

$$X = 0 \quad (1.13)$$

So, $X = 0$ is not a solution to (1.8).

Therefore $X \neq 0$, or $P_i \neq P_j$ if $y_i \neq y_j$. Furthermore, equation (1.2) also implies that given $y_i > y_j$, Equation (1.10) can be satisfied only when $X < 0$ or $P_i > P_j$. The discussions above bring to us

Proposition 1.2

PROPOSITION 1.2

When two firms choose different capacities, there exists an equilibrium in which the two firms charge different prices and the firm with the relatively larger capacity level will charge higher price while the firm with relatively smaller capacity level will charge a lower price.

3. Stage One Equilibrium

In the first stage, the two firms set capacities y_i and y_j , respectively. The profit of firm i is

$$\pi_i = P_i \cdot n_i - K(y_i) = P_i(y_i, y_j) \cdot G(X(y_i, y_j), y_i, y_j) - K(y_i) \quad (1.14)$$

Substituting (1.6) into (1.14),

$$\pi_i = \frac{[G(X(y_i, y_j), y_i, y_j)]^2}{G_1(X(y_i, y_j), y_i, y_j)} - K(y_i) \quad (1.15)$$

Firm i 's choice of y_i is determined by the F.O.C.

$$\frac{2G(\cdot) \cdot G_1(\cdot) \left(\frac{\partial G}{\partial X} \cdot \frac{\partial X}{\partial y_i} + \frac{\partial G}{\partial y_i} \right) - \left[\frac{\partial^2 G}{\partial X^2} \cdot \frac{\partial X}{\partial y_i} + \frac{\partial^2 G}{\partial X \partial y_i} \right] \cdot [G(\cdot)]^2}{[G_1(\cdot)]^2} - K'(y_i) = 0 \quad (1.16)$$

By symmetry, the solution to y_j can be represented as the solution to

$$\frac{2\bar{G}(\cdot) \cdot \bar{G}_1(\cdot) \left(\frac{\partial \bar{G}}{\partial X} \cdot \frac{\partial X}{\partial y_j} + \frac{\partial \bar{G}}{\partial y_j} \right) - \left[\frac{\partial^2 \bar{G}}{\partial X^2} \cdot \frac{\partial X}{\partial y_j} + \frac{\partial^2 \bar{G}}{\partial X \partial y_j} \right] \cdot [\bar{G}(\cdot)]^2}{[\bar{G}_1(\cdot)]^2} - K'(y_j) = 0 \quad (1.17)$$

where

$$\bar{G}(\cdot) = G(X(y_j, y_i), y_j, y_i)$$

Since (1.16) and (1.17) are symmetric equations, there exists at least a pair of symmetric roots which are $y_i = y_j = y^s$. What we are interested in, however, is whether (1.16) and (1.17) have a pair of asymmetric roots, i.e., $y_i \neq y_j$, in which case, according to Proposition 2, equilibrium price dispersion would be observed. Since (1.16) and (1.17) are in general nonlinear functions, it is possible that they have roots of $y_i \neq y_j$. To demonstrate this possibility, we will make a further assumption on the shape of the congestion cost function in the next section.

III. THE CASE OF LINEAR CONGESTION COST

Suppose the congestion cost function is a linear function given by

$$C_i(n_i, y_i) = c_i + an_i - by_i \quad a > 0, b > 0, \quad i=1,2 \quad (1.18)$$

where, c_i is a fixed congestion cost. Equation (1.18) implies that congestion cost is positive even when the number of customs is small if a firm chooses minimum capacity level (i.e. $y_i = 0$). Obviously, the congestion cost increases in n_i and decreases in y_i ,

$$\frac{\partial C(n_i, y_i)}{\partial n_i} > 0, \quad \frac{\partial^2 C(n_i, y_i)}{\partial n_i^2} = 0, \quad \frac{\partial C(n_i, y_i)}{\partial y_i} < 0, \quad \frac{\partial^2 C(n_i, y_i)}{\partial y_i^2} = 0.$$

Since the surplus a consumer obtains from buying a unit of the product from firm i is $V - P_i - C(n_i, y_i)$, the price difference between the two firms is

$$P_2 - P_1 = C_1 - C_2 = an_1 - by_1 - a(N - n_1) + by_2$$

The demand function for firm 1 is

$$n_1 = \frac{1}{2a} [P_2 - P_1 + aN - b(y_2 - y_1)] \quad (1.19)$$

Repeating the analysis for firm 2, we find

$$n_2 = \frac{1}{2a} [P_1 - P_2 + aN - b(y_1 - y_2)] \quad (1.20)$$

1. The Game at Stage Two

We define the profit of firm i as π_i . Solving firm i 's profit-maximization problem at stage 2.

$$\text{Max}_{P_i} \pi_i = P_i \cdot n_i = \frac{P_i}{2a} [P_j - P_i + aN - b(y_j - y_i)] \quad i=1,2; j=1,2; i \neq j$$

Given the capacity choices of the two firms y_1 and y_2 , we obtain,

$$P_1 = \frac{P_2}{2} + \frac{aN}{2} - \frac{b}{2}(y_2 - y_1) \quad (1.21)$$

$$P_2 = \frac{P_1}{2} + \frac{aN}{2} - \frac{b}{2}(y_1 - y_2) \quad (1.22)$$

Equation (1.21) and (1.22) imply

$$P_2 - P_1 = \frac{2b}{3}(y_2 - y_1) \quad (1.23)$$

Equation (1.23) indicates that the existence of price dispersion in this linear congestion cost case relies on firms' choice of capacity as we have stated in Proposition 1 and 2. If $y_1 = y_2$, then $P_1 - P_2 = 0$ or $P_1 = P_2$. If $y_1 \neq y_2$, then $P_1 \neq P_2$. If $y_2 > y_1$, then $P_2 > P_1$.

We can also illustrate these properties in Figures 1 and 2 for the two separate cases where the intersection of the reaction functions determines the equilibrium prices. In Figure 1.1, $y_1=y_2$ so that $P_1=P_2=aN$. In Figure 1.2, $y_1 \neq y_2$ and P_1 and P_2 can be solved by

$$P_1 = aN - \frac{b}{3}(y_2 - y_1) \quad (1.24)$$

$$P_2 = aN - \frac{b}{3}(y_1 - y_2) \quad (1.25)$$

The second order condition for firms' stage 1 profit-maximization problem is provided in the Appendix.

FIGURE 1.1. The Price Setting with $y_1 = y_2$

FIGURE 1.2. The Price Setting with $y_1 \neq y_2$

Substituting (1.23) into (1.19) and (1.20), we have the demand functions for the two firms.

$$n_1 = \frac{1}{2a} \left[aN - \frac{b}{3} (y_2 - y_1) \right] \quad (1.26)$$

$$n_2 = \frac{1}{2a} \left[aN - \frac{b}{3} (y_1 - y_2) \right] \quad (1.27)$$

The above result highlights the importance of firms' capacity choices. If they choose the same capacity level in the first stage, there will be a one-price equilibrium in the second stage and the number of consumers will be even distributed between two firms. On the other hand, if they choose different capacity levels in the first stage, there will be price-dispersion equilibrium in the second stage and there are more consumers in one firm than that in the other firm.

2. The Game at Stage One

Now, we can formulate the firm i 's profit maximization problem in stage 1 as

$$\text{Max}_{y_i} \pi_i = P_i \cdot n_i - K(y_i) = \frac{1}{2a} \left[aN - \frac{b}{3} (y_j - y_i) \right]^2 - K(y_i) \quad i=1,2; \quad j=1,2; \quad i \neq j \quad (1.28)$$

Solving this profit maximization problem, we get the firm 1's reaction function.

$$\frac{b}{3} N - \frac{b^2}{9a} y_2 + \frac{b^2}{9a} y_1 = K'(y_1) \quad (1.29)$$

As shown in the Appendix, the second order condition for this maximum is

$$K''(y_1) > \frac{b^2}{9a} \quad (1.30)$$

Completely differentiating Equation (1.29) with respect to the two endogenous variables, y_1 and y_2 , we obtain the slope of firm 1's reaction curve:

$$\frac{dy_2}{dy_1} = \frac{b^2/9a - K''(y_1)}{b^2/9a} < 0 \quad (1.31)$$

where the sign follows from the second order condition (1.30).

By symmetry, firm 2's reaction function and its slope are

$$\frac{b}{3}N - \frac{b^2}{9a}y_1 + \frac{b^2}{9a}y_2 = K'(y_2) \quad (1.32)$$

and

$$\frac{dy_1}{dy_2} = \frac{b^2/9a - K''(y_2)}{b^2/9a} < 0 \quad (1.33)$$

where the second order condition is symmetric to (1.30), i.e.,

$$K''(y_2) > \frac{b^2}{9a} \quad (1.34)$$

Suppose $y_1=y_2=y^*$. Either equation (1.29) or (1.32) yields

$$K'(y^*) = \frac{b}{3}N \quad (1.35)$$

Given the assumptions that $K'(0)=0$, $K'(\infty)=\infty$ and $K''(y) < 0$, (1.35) has a unique solution. In other words, there exists a symmetric equilibrium where $y_1=y_2=y^*$.

Consider the absolute value of the slope of two firms' reaction curves at $y_1=y_2=y^*$, we have

$$\left| \frac{dy_2}{dy_1} \right|_1 = \frac{K''(y^*) - b^2/9a}{b^2/9a} \quad (1.36)$$

$$\left| \frac{dy_2}{dy_1} \right|_2 = \frac{b^2/9a}{K''(y^*) - b^2/9a} \quad (1.37)$$

Then at $y_1=y_2=y^*$, firm 1's reaction curve is flatter than firm 2's reaction curve if and only if:

$$\frac{K''(y^*) - b^2/9a}{b^2/9a} < \frac{b^2/9a}{K''(y^*) - b^2/9a}$$

or,

$$K''(y^*) < \frac{2b^2}{9a} \quad (1.38)$$

This situation is illustrated in Figure 1.3.

The necessary and sufficient condition for a price-dispersion equilibrium is that there is at least one crossing of the two reaction curves between the point of (y^*, y^*) and y_2 axis. Given (1.38), this will happen if firm 1's reaction curve intersects y_2 axis at a higher value of y_2 than the point where firm 2's reaction intersects. Consider the intersections of the two reaction curves at the vertical axis, and letting $y_1=0$ in both firms' reaction functions (1.29) and (1.32), we solve

$$y_2|_1 = \frac{3a}{b}N \quad (1.39)$$

and,

$$y_2|_2 = \frac{9a}{b^2}K'(y_2|_2) - \frac{3a}{b}N \quad (1.40)$$

As y_2 values where the two reaction functions intersect on y_2 axis, we let $\hat{y}$ is as the solution to equation (1.40). If $\hat{y}$ is less than $3aN/b$, from Figure 1.3 we know that there exists at least one equilibrium with $y_1 \neq y_2$. We state this formally as Proposition 1.3.

PROPOSITION 1.3

Given the linear congestion cost functions, there exists at least one asymmetric equilibrium where $y_1 \neq y_2$ if:

1) $\hat{y} < 3aN/b$, and

2) $b^2/9a < K''(y^*) < 2b^2/9a$.

Considering an alternative case where the slope of firm 1's reaction curve is steeper than that of firm 2's reaction curve at $y_1=y_2=y^*$, an equilibrium with $y_1 \neq y_2$ also exists if $\hat{y} > 3aN/b$ (see Figure 1.4). This brings to us Proposition 1.4.

PROPOSITION 1.4

Given the linear congestion cost functions, there exists at least one asymmetric equilibrium where $y_1 \neq y_2$ if:

- 1) $\hat{y} > 3aN/b$, and
- 2) $K''(y^*) < -2b^2/9a$.

FIGURE 1.3. The Capacity Levels in Asymmetric Equilibrium - Case 1

FIGURE 1.4. The Capacity Levels in Asymmetric Equilibrium - Case 2

IV. COMPARISON OF THE TWO TYPES OF EQUILIBRIA

From Sections II and III , we have proved the existence of symmetric and asymmetric capacity levels of two firms that lead to one-price or price-dispersion equilibria. The interesting results may not only come from the asymmetric capacity levels of two firms but also from the differences in terms of firms' profits, consumer surplus and total surplus between the two firms². Such comparison will shed light on which of the types of equilibria is more likely to occur as the actual outcome of the game and which is more desirable from the society's perspective. For example, if the price-dispersion equilibrium can bring a larger profits than that in one-price equilibrium, the price-dispersion equilibrium is more likely to be the outcome as long as some form of side payments are possible. In what follows, we compare the asymmetric equilibrium with the symmetric equilibrium derived in the case of linear congestion cost function.

1. The Profit of Each Firm

According to Equation (1.28), firm i 's profit π_i can be written as

$$\pi_i = \frac{1}{2\alpha} [aN - \frac{b}{3}(y_j - y_i)]^2 - K(y_i) \quad (1.41)$$

It is satisfied when $y_i = y_j$, $\pi_i = \pi_j$. Hence, restricting our attention to those cases where $y \neq y_j$, we can

---

² One issue that we do not study here is the stability property of these equilibria. In our model, firms choose capacity rather than output. Once chosen, the capacity of a firm can not be changed without incurring significant adjustment cost. An analysis of such adjustment process is beyond the scope of this chapter.

differentiate (1.41) completely with y_i . We find

$$\frac{\partial \pi_i}{\partial y_i} = -\frac{b}{3a} \left[aN - \frac{b}{3}(y_j - y_i) \right] \frac{dy_j}{dy_i} + \left[\frac{b}{3}N - \frac{b^2}{9a}y_j + \frac{b^2}{9a}y_i - K'(y_i) \right] \quad (1.42)$$

Consider the sign of (1.42) along firm i 's reaction curve, the second term of (1.42) is zero because of the first order condition in (1.29). The first term of (1.42) is the result that the demand multiplies $(-b/3a)$ and the slope of reaction curves. Since

$$\frac{b}{3a} \left[aN - \frac{b}{3}(y_j - y_i) \right] = \frac{2}{3}n_i > 0, \quad \frac{dy_j}{dy_i} < 0$$

we have

$$\frac{\partial \pi_i}{\partial y_i} = -\frac{b}{3a} \left[aN - \frac{b}{3}(y_j - y_i) \right] \frac{dy_j}{dy_i} > 0 \quad (1.43)$$

Equation (1.43) implies that a firm's profit will go up when its capacity level increases along its reaction curve. We state this result as Proposition 1.5.

PROPOSITION 1.5

In an asymmetric equilibrium, a firm with a larger capacity level earns relatively higher profits than a firm with a smaller capacity level.

2. The Total Profits of the Two Firms

The total profits of the two firms are defined as

$$\begin{aligned}\Pi &= \pi_1 + \pi_2 = \frac{1}{2a} \left[aN - \frac{b}{3}(y_2 - y_1) \right]^2 + \frac{1}{2a} \left[aN - \frac{b}{3}(y_1 - y_2) \right]^2 - K(y_1) - K(y_2) \\ &= aN^2 + \frac{b^2}{9a} (y_2 - y_1)^2 - K(y_1) - K(y_2)\end{aligned}$$

Considering the changes in Π along with firm 1's reaction curve, we have

$$\frac{\partial \Pi}{\partial y_1} = \frac{2b^2}{9a} \left(1 - \frac{dy_2}{dy_1} \right) (y_1 - y_2) - K'(y_1) - K'(y_2) \frac{dy_2}{dy_1} \quad (1.44)$$

Substitute (1.31) into (1.44),

$$\begin{aligned}\frac{\partial \Pi}{\partial y_1} &= \frac{2b^2}{9a} \cdot \frac{K''(y_1)}{b^2/9a} (y_1 - y_2) - K'(y_1) - K'(y_2) \left[1 - \frac{K''(y_1)}{b^2/9a} \right] \\ &= 2K''(y_1)(y_1 - y_2) - K'(y_1) - K'(y_2) \left[1 - \frac{K''(y_1)}{b^2/9a} \right]\end{aligned} \quad (1.45)$$

When $y_1 = y_2 = y^*$,

$$\begin{aligned}
\frac{\partial \Pi}{\partial y_1} \Big|_{y_1=y_2=y^*} &= -K'(y^*) \left[2 - \frac{K''(y^*)}{b^2/9a} \right] \\
&= -K'(y^*) \left[\frac{2b^2/9a - K''(y^*)}{b^2/9a} \right] \quad \begin{cases} > 0 & \text{if } K''(y^*) < 2b^2/9a \\ < 0 & \text{if } K''(y^*) > 2b^2/9a \end{cases} \quad (1.46)
\end{aligned}$$

The first term of equation (1.44) is positive if y_1 is greater than y_2 and, it is negative if y_1 is less than y_2 . For the second and third terms of equation (1.44), we take the derivative with respect to y_1 .

$$\begin{aligned}
\frac{\partial [-K'(y_1) - K'(y_2) \frac{dy_2}{dy_1}]}{\partial y_1} &= -K''(y_1) - K''(y_2) \left(\frac{dy_2}{dy_1} \right)^2 - K'(y_2) \frac{d^2 y_2}{dy_1^2} \\
&= -K''(y_1) - K''(y_2) \left[1 - \frac{K''(y_1)}{b^2/9a} \right]^2 + K'(y_2) \frac{K'''(y_1)}{b^2/9a} \quad \begin{cases} < 0 & \text{if } K'''(y_1) < 0 \\ > 0 & \text{only if } K'''(y_1) > 0 \end{cases} \quad (1.47)
\end{aligned}$$

The third degree derivative of $K'''(y_1)$ represents the changes in the rate of marginal cost increase. A positive $K'''(y_1)$ implies that the rate of marginal cost increase will be rising as the capacity level increases. By symmetry, we have the same argument from firm 2's point view. Let (y_a, y_b) be the capacity levels at asymmetric equilibrium and $y_a > y_b$. According to (1.45)-(1.47), we have Proposition 1.6.

PROPOSITION 1.6

Suppose $K'''(y_i)$ is positive and is so large, that

$$\frac{\partial[-K'(y_i)-K'(y_j)\frac{dy_j}{dy_i}]}{\partial y_i} > 0$$

along firm i 's reaction curve for y_i in the range of $[y_a, y_b]$, the total profits of the two firms at asymmetric equilibrium are higher than that in symmetric equilibrium.

Proof:

The prove of Proposition 1.6 is based on two situations along firm 1's reaction curve.

$$(1) \quad K''(y^*) > 2b^2/9a$$

In this case, we have

$$\frac{\partial \Pi}{\partial y_1} \Big|_{y_1=y_2=y^*} > 0, \quad \frac{\partial[-K'(y_1)-K'(y_2)\frac{dy_2}{dy_1}]}{\partial y_1} > 0$$

The above inequalities imply that starting from the point (y^*, y^*) , as y_1 increases and y_2 decreases, the first term in (1.45) becomes positive, and the second and third terms in (1.45) remain positive.

Therefore, we have

$$\frac{\partial \Pi}{\partial y_1} > 0 \quad \forall \quad y_1 > y^s \quad (1.48)$$

Equation (1.48) demonstrates that the total profits from asymmetric equilibrium are higher than that from symmetric equilibrium. For $y_1 < y^s < y_2$, we can generate the same results in firm 2's reaction curve.

$$(2) \quad b^2/9a < K''(y^s) < 2b^2/9a$$

In this case, we have

$$\frac{\partial \Pi}{\partial y_1} \Big|_{y_1=y_2=y^s} < 0, \quad \frac{\partial [-K'(y_1) - K'(y_2) \frac{dy_2}{dy_1}]}{\partial y_1} > 0$$

According to the above inequalities, starting from the point (y^s, y^s) , as y_1 decreases and y_2 increases, the first term in (1.45) becomes negative and the second term in (1.45) is also negative. Therefore, we have

$$\frac{\partial \Pi}{\partial y_1} > 0 \quad \forall \quad y_1 < y^s \quad (1.49)$$

Equation (1.49) demonstrates that the total profits from asymmetric equilibrium are higher than that from symmetric equilibrium.

Q.E.D.

3. Consumer Surplus

Using (1.1), (1.21) and (1.22), the consumer surplus (CS) of N consumers from buying one product from firm 1 or firm 2 is represented as

$$\begin{aligned} CS &= n_1[V - P_1 - C(n_1, y_1)] + n_2[V - P_2 - C(n_2, y_2)] = N[V - (P_1 + ay_1 - by_1)] \\ &= VN - \frac{3aN^2}{2} + \frac{bN}{2}(y_1 + y_2) \end{aligned} \quad (1.50)$$

Equation (1.50) demonstrates that CS is an increasing function of total capacity $y_1 + y_2$.

The change of CS along firm i 's reaction curve is given by

$$\frac{\partial CS}{\partial y_i} = \frac{bN}{2} \left(1 + \frac{dy_j}{dy_i}\right) = \frac{bN}{2} \left[2 - \frac{K''(y_i)}{b^2/9a}\right] \quad (1.51)$$

At symmetric equilibrium

$$\frac{\partial CS}{\partial y_i} \Big|_{y_i=y_j=y^*} = \frac{bN}{2} \left(2 - \frac{K''(y^*)}{b^2/9a}\right) \quad \begin{cases} > 0 & \text{if } K''(y^*) < 2b^2/9a \\ < 0 & \text{if } K''(y^*) > 2b^2/9a \end{cases} \quad (1.52)$$

Taking derivative from (1.51) with respect to y_i , we have

$$\frac{\partial^2 CS}{\partial y_i^2} = -\frac{9aN}{2b} K'''(y_i) \quad \begin{cases} < 0 & \text{iff } K'''(y_i) > 0 \\ > 0 & \text{iff } K'''(y_i) < 0 \end{cases} \quad (1.53)$$

We have the following proposition.

PROPOSITION 1.7

The consumer surplus from asymmetric equilibrium is lower than that from symmetric equilibrium if $K'''(y) > 0$. The consumer surplus from asymmetric equilibrium is higher than that from symmetric equilibrium if $K'''(y) < 0$.

Proof:

We prove this proposition for four different cases along firm 1's reaction curve.

(1) The case of $K'''(y) > 0$ and $b^2/9a < K'''(y^s) < 2b^2/9a$

$$\frac{\partial CS}{\partial y_1} \Big|_{y_1=y_2=y^s} > 0, \quad \frac{\partial^2 CS}{\partial y_1^2} < 0$$

Starting from the point (y^s, y^s) , as y_1 decreases and y_2 increases, $\partial CS / \partial y_1$ increases. Then,

$$\frac{\partial CS}{\partial y_1} > 0, \quad \forall y_1 < y^s$$

Therefore, in this case consumer surplus in the asymmetric equilibrium is lower than that in the symmetric equilibrium.

(2) The case of $K'''(y_i) > 0$ and $K''(y^s) > 2b^2/9a$

$$\frac{\partial CS}{\partial y_1} \Big|_{y_1=y_2=y^s} < 0, \quad \frac{\partial^2 CS}{\partial y_1^2} < 0$$

Starting from the point (y^s, y^s) , as y_1 increases and y_2 decreases, $\partial CS / \partial y_1$ decreases. Then,

$$\frac{\partial CS}{\partial y_1} < 0, \quad \forall y_1 > y^s$$

Therefore, in this case consumer surplus in the asymmetric equilibrium is lower than that in the symmetric equilibrium.

(3) The case of $K'''(y_i) < 0$ and $b^2/9a < K''(y^s) < 2b^2/9a$

$$\frac{\partial CS}{\partial y_1} \Big|_{y_1=y_2=y^s} > 0, \quad \frac{\partial^2 CS}{\partial y_1^2} > 0$$

Starting from the point (y^s, y^s) , as y_1 increases and y_2 decreases, $\partial CS / \partial y_1$ increases. Then,

$$\frac{\partial CS}{\partial y_1} > 0, \quad \forall y_1 > y^s$$

Therefore, in this case consumer surplus in the asymmetric equilibrium is higher than that in the symmetric equilibrium.

(4) The case of $K'''(y) < 0$ and $K''(y^s) > 2b^2/9a$

$$\frac{\partial CS}{\partial y_1} \Big|_{y_1=y_2=y^s} < 0, \quad \frac{\partial^2 CS}{\partial y_1^2} > 0$$

Starting from the point (y^s, y^s) , as y_1 decreases and y_2 increases, $\partial CS / \partial y_1$ decreases. Then,

$$\frac{\partial CS}{\partial y_1} < 0, \quad \forall y_1 < y^s$$

Therefore, in this case, consumer surplus in the asymmetric equilibrium is higher than that in the symmetric equilibrium.

Q.E.D.

4. The Total Welfare

The total welfare in this model is defined as W .

$$W = \pi_1 + \pi_2 + CS \equiv aN^2 + \frac{b^2}{9a}(y_2 - y_1)^2 - K(y_1) - K(y_2) + VN - \frac{3aN^2}{2} + \frac{bN}{2}(y_1 + y_2) \quad (1.54)$$

Considering the changes in W along with firm 1's reaction curve, we have

$$\frac{\partial W}{\partial y_1} = \frac{2b^2}{9a}(y_2 - y_1)\left(\frac{dy_2}{dy_1} - 1\right) - K'(y_1) - K'(y_2)\frac{dy_2}{dy_1} + \frac{bN}{2}\left(1 + \frac{dy_2}{dy_1}\right)$$

$$= \frac{dy_2}{dy_1} \left[\frac{2b^2}{9a} (y_2 - y_1) - K'(y_2) + \frac{bN}{2} \right] + \frac{bN}{2} - \frac{2b^2}{9a} (y_2 - y_1) - K'(y_1) \quad (1.55)$$

When $y_1 = y_2 = y^*$, (1.55) becomes

$$\frac{\partial W}{\partial y_i} \Big|_{y_1=y_2=y^*} = \left[\frac{bN}{2} - K'(y^*) \right] \left(\frac{dy_2}{dy_1} + 1 \right) = \left[\frac{bN}{2} - K'(y^*) \right] \left[2 - \frac{K''(y^*)}{b^2/9a} \right]$$

From (1.35), we have $K'(y^*) = bN/3$. Then,

$$\frac{\partial W}{\partial y_i} \Big|_{y_1=y_2=y^*} = \frac{bN}{6} \left[2 - \frac{K''(y^*)}{b^2/9a} \right] \quad \begin{cases} > 0 \text{ if } K''(y^*) < 2b^2/9a \\ < 0 \text{ if } K''(y^*) > 2b^2/9a \end{cases} \quad (1.56)$$

Combining (1.31) and (1.55),

$$\begin{aligned} \frac{\partial W}{\partial y_1} &= \left[1 - \frac{9a}{b^2} K''(y_1) \right] \left[\frac{2b^2}{9a} (y_2 - y_1) - K'(y_2) + \frac{bN}{2} \right] + \frac{bN}{2} - \frac{2b^2}{9a} (y_2 - y_1) - K'(y_1) \\ &= bN - K'(y_1) - K'(y_2) - K''(y_1) [2(y_2 - y_1) - \frac{9a}{b^2} K'(y_2)] - \frac{9aN}{2b} K''(y_1) \end{aligned} \quad (1.57)$$

From (1.29) and (1.32), we have

$$K'(y_1) + K'(y_2) = \frac{2bN}{3}$$

and from (1.32) , we have

$$2(y_2 - y_1) = \frac{18a}{b^2} K'(y_2) - \frac{6aN}{b}$$

Then (1.57) becomes

$$\frac{\partial W}{\partial y_1} = \frac{bN}{3} - K''(y_1) \left[\frac{9a}{b^2} K'(y_2) - \frac{6aN}{b} \right] - \frac{9aN}{2b} K''(y_1) \quad (1.58)$$

From (1.58),

$$\begin{aligned} \frac{\partial^2 W}{\partial y_1^2} &= -K'''(y_1) \left[\frac{9a}{b^2} K'(y_2) - \frac{6aN}{b} \right] - \frac{9a}{b^2} K''(y_1) K''(y_2) \frac{dy_2}{dy_1} - \frac{9aN}{2b} K'''(y_1) \\ &= -K'''(y_1) \left[\frac{9a}{b^2} K'(y_2) - \frac{3aN}{2b} \right] - \frac{9a}{b^2} K''(y_1) K''(y_2) \frac{dy_2}{dy_1} \end{aligned} \quad (1.59)$$

Since

$$K''(y_1) > 0, \quad K''(y_2) > 0, \quad \frac{dy_2}{dy_1} < 0$$

the second term in (1.59) is negative. The sign of $\partial^2 W / \partial y_1^2$ then depends on the signs of $K'''(y_1)$ and

$$[9aK'(y_2)/b^2 - 3aN/2b]:$$

$$\frac{\partial^2 W}{\partial y_1^2} > 0 \quad \begin{cases} \text{if i) } K'''(y_1) > 0 \text{ and } K'(y_2) > bN/6 \\ \text{or ii) } K'''(y_1) < 0 \text{ and } K'(y_2) > bN/6 \end{cases}$$

Combining (1.56) and (1.59), we have Proposition 1.8 .

PROPOSITION 1.8

The total welfare from asymmetric equilibrium is higher than that from symmetric equilibrium if either $K'''(y) > 0$ and $K'(y) > bN/6$ or $K'''(y) < 0$ and $K'(y) > bN/6$.

Proof:

We can prove Proposition 8 in two cases along the firm 1's reaction curve. The conditions in the proposition ensure that $\partial^2 W / \partial y_1^2 > 0$.

(1) The case of $K''(y^*) \leq 2b^2/9a$.

In this case, we have

$$\frac{\partial W}{\partial y_1} \Big|_{y_1=y_2=y^*} \geq 0$$

Starting from the point (y^*, y^*) , as y_1 increases and y_2 decreases, $\partial W / \partial y_1$ increases.

Then,

$$\frac{\partial W}{\partial y_1} > 0, \quad \forall y_1 > y^s$$

For $y_1 > y^s$, we have a higher total welfare from asymmetric equilibrium than that from symmetric equilibrium.

(2) The case of $K''(y^s) > 2b^2/9a$.

In this case, we have

$$\frac{\partial W}{\partial y_1} \Big|_{y_1=y_2=y^s} < 0$$

Starting from the point (y^s, y^s) , as y_1 decreases and y_2 increases, $\partial W / \partial y_1$ increases.

Then,

$$\frac{\partial W}{\partial y_1} < 0, \quad \forall y_1 < y^s$$

For $y_1 < y^s$, we have a higher total welfare from asymmetric equilibrium than that from symmetric equilibrium.

Q.E.D.

V. A SPECIFIC EXAMPLE

In this section, we further illustrate our results by considering an example where the capacity cost takes a particular functional form. We will first derive the analytical solution to the model, and then conduct numerical simulation. To be more specific, we suppose

$$K(y_i) = F + \theta y_i^3 \quad \text{where, } \theta > 0 \text{ and } i=1,2.$$

The above cost function implies that a firm has to incur a cost of F if it chooses the minimum capacity ($y_i=0$). Capacity cost rises with y_i . It is easy to verify that

$$K'(y_i) = 3\theta y_i^2 > 0, K''(y_i) = 6\theta y_i > 0, K'''(y_i) = 6\theta > 0 \text{ and } K'(0) = 0.$$

Equations (1.29) and (1.32) imply that the reaction functions at stage 1 are

$$3abN - b^2y_2 + b^2y_1 - 27a\theta y_1^2 = 0 \quad (1.60)$$

$$3abN - b^2y_1 + b^2y_2 - 27a\theta y_2^2 = 0 \quad (1.61)$$

By solving equations (1.60) and (1.61), we have three pairs of equilibrium capacity levels:

(1) A Pair of Symmetric Equilibrium Capacity Levels

$$y_1 = y_2 = y^s = \left(\frac{bN}{9\theta}\right)^{1/2} \quad (1.62)$$

(2) Two Pairs of Asymmetric Equilibrium Capacity Levels

$$y_1 = \frac{b^2 + (81\alpha^2 b \theta N - b^4)^{1/2}}{27\alpha\theta}, \quad y_2 = \frac{b^2 - (81\alpha^2 b \theta N - b^4)^{1/2}}{27\alpha\theta} \quad (1.63)$$

and,

$$y_1 = \frac{b^2 - (81\alpha^2 b \theta N - b^4)^{1/2}}{27\alpha\theta}, \quad y_2 = \frac{b^2 + (81\alpha^2 b \theta N - b^4)^{1/2}}{27\alpha\theta} \quad (1.64)$$

To ensure that the values of y_1 and y_2 are real and positive, the following condition must hold.

$$b^2 - (81\alpha^2 b \theta N - b^4)^{1/2} > 0, \quad 81\alpha^2 b \theta N - b^4 > 0$$

or,

$$\frac{b^3}{81\alpha^2 N} < \theta < \frac{2b^3}{81\alpha^2 N}$$

As shown in Appendix, second order condition requires

$$\theta < \frac{5b^3}{324a^2N}$$

Then, for the asymmetric equilibrium to exist the value of θ must be in the range of

$$\frac{b^3}{81a^2N} < \theta < \frac{5b^3}{324a^2N} \quad (1.65)$$

In Table 1.1, we present a summary of prices, capacities, profits, consumer surplus and the total welfare in the symmetric (column 1) and asymmetric (column 2) equilibria of this specific example. Details of derivation are given in Appendix.

In the third column in Table 1.1 we present the differences in equilibrium values between the symmetric and asymmetric equilibrium. In particular, notice that the total profit and welfare in asymmetric equilibrium is greater than that of the symmetric equilibrium.

TABLE 1.1 A Comparison between Symmetric and Asymmetric Equilibriums

Symmetric Equilibrium ($y_1=y_2=y^s$)	Asymmetric Equilibrium ³ (y_1, y_2)	Comparison
$P_1=P_2=P^s=aN$	$P_1=aN+\frac{2bJ}{3}, P_2=aN-\frac{2bJ}{3}$	$P_1>P^s$ $P_2<P^s$
$n_1=n_2=n^s=\frac{N}{2}$	$n_1=\frac{N}{2}+\frac{bJ}{3a}, n_2=\frac{N}{2}-\frac{bJ}{3a}$	$n_1>n^s$ $n_2<n^s$
$\pi_1=\pi_2=\pi^s=\frac{aN^2}{2}-\theta(\frac{bN}{9\theta})^{3/2}-F$	$\pi_1=\frac{1}{2a}(aN+\frac{2bJ}{3})^2-\theta(\frac{b^2}{27a\theta}+J)^3-F$ $\pi_2=\frac{1}{2a}(aN-\frac{2bJ}{3})^2-\theta(\frac{b^2}{27a\theta}-J)^3-F$	$\pi_1>\pi^s$ $\pi_2<\pi^s$
$\Pi^s=2\pi^s=aN^2-2\theta(\frac{bN}{9\theta})^{3/2}-2F$	$\Pi=\pi_1+\pi_2=aN^2+\frac{2b^2J^2}{9a}-\frac{2b^6}{27^3a^3\theta^2}-2F$	$\Pi>\Pi^s$
$CS^s=N(V-C)-\frac{3aN^2}{2}+\frac{(bN)^{3/2}}{(9\theta)^{1/2}}$	$CS=N(V-C)-\frac{3aN^2}{2}+\frac{b^3N}{27a\theta}$	$CS<CS^s$
$W^s=N(V-C)+\frac{7bN}{27}(\frac{bN}{\theta})^{1/2}-\frac{aN^2}{2}-2F$	$W=N(V-C)+\frac{5b^3N}{81a\theta}-\frac{8b^6}{27^3a^3\theta^2}-\frac{aN^2}{2}-2F$	$W>W^s$

³ J is a positive parameter combination whose value is provided in the Appendix.

One important parameter in this specific example is θ which measures the slope of marginal cost of capacity. In what follows we illustrate the role of θ through numerical simulation. The parameters are chose as: $N=110$, $\alpha=1$, $b=5.4$, $c=310$, $V=500$, $F=9$, the value of θ is limited within $[0.0177, 0.022]$ which satisfies the constraint of (49). We set different values to θ and find out how the equilibrium changes as the value of θ changes. From the results demonstrated in Table 1.2, several comments can be made.

- (1) Firms' equilibrium capacity levels will decrease with the increase of θ , because the capacity cost increases with θ .
- (2) As θ increases, the total congestion costs in both symmetric equilibrium and asymmetric equilibrium will increase because the firms' capacity levels decrease.
- (3) The consumer surplus in both equilibriums will decrease as θ increases because, the capacity levels decrease and the congestion costs increase.
- (4) The total profits of the two firms increase in both symmetric and asymmetric equilibriums when θ increases. However, the total profits of the two firms in asymmetric equilibrium increases by larger proportion than that in symmetric equilibrium.
- (5) The difference in welfare levels between the asymmetric equilibrium and symmetric equilibrium increase with θ increases because the change in consumer surplus is offset by the change in profits.

TABLE 1.2 The Role of θ

	$\theta=0.0177$	$\theta=0.0180$	$\theta=0.0190$	$\theta=0.0195$
y^s	61.06	60.55	58.94	58.19
y_i	63.41	68.16	72.42	73.19
y_j	58.62	51.84	41.26	37.58
P^s	110	110	110	110
P_i	118.63	139.39	166.08	174.11
P_j	101.37	80.61	53.92	45.89
n^s	55	55	55	55
n_i	59	70	83	87
n_j	51	40	27	23
C^s	35.25	38.01	46.73	50.84
C_i	26.88	11.61	1.97	1.81
C_j	44.14	70.39	114.13	130.04
$K(y^s)$	4039.23	4005.5	3898.91	3848.71
$K(y_i)$	4522.64	5710.07	7225.43	7655.34
$K(y_j)$	3574.42	2515.93	1344.01	1043.56
CS^s	39022.02	38718.49	37759.15	37307.4
CS	38994.07	38390	36514.21	35648.46
π^s	2010.78	2044.5	2151.09	2201.29
π_i	2513.79	4005.26	6565.7	7502.21
π_j	1563.61	732.74	109.72	9.28
$\Pi^s=2\pi^s$	4021.55	4089	4302.19	4402.58
$\Pi=\pi_i+\pi_j$	4077.4	4738	6675.42	7511.49
$W^s=\Pi^s+CS^s$	43043.57	42807.49	42061.34	41709.98
$W=\Pi+CS$	43071.47	43128	43189.63	43159.95
$W-W^s$	27.9	320.51	1128.29	1449.97

VI. CONCLUDING REMARKS

In this chapter we contribute to the literature on price dispersion by demonstrating that price dispersion can prevail even in a world where consumers have perfect information and both consumers and firms are *ex ante* identical. The results of our model are consistent with the phenomenon that large, spacious stores charging relatively higher prices co-exist with small, crowded stores charging relatively lower prices for the same products.

In our model the situation where two firms choosing the same capacity and charging the same price also exists as an equilibrium. However under the conditions identified in the paper, this one-price equilibrium involves lower total industry profits than the price dispersion equilibriums. In such instances we expect that, as long as some form of explicit or implicit side payments between firms is possible, a price dispersion equilibrium will prevail as the actual outcome. This, from the society's perspective, may also be the preferred outcome since the total surplus may be higher.

APPENDIX A

The Proof of the Second Order Condition

(1) *The Second Order Condition in the linear Congestion Cost Model*

(i) *Firm 1 in the Second Stage of the Game*

F.O.C.

$$\frac{\partial \pi_1}{\partial P_1} = \frac{1}{2a} [P_2 - 2P_1 + aN - b(y_2 - y_1)]$$

S.O.C.

$$\frac{\partial^2 \pi_1}{\partial P_1^2} = -\frac{1}{a} < 0$$

Here, the second order condition is satisfied.

(ii) *Firm 1 in the First Stage of the Game*

$$\pi_1 = P_1 \cdot n_1 - d \cdot y_1^2 = \frac{1}{2a} [aN - \frac{b}{3}(y_2 - y_1)]^2 - K(y_1)$$

F.O.C.

$$\frac{\partial \pi_1}{\partial y_1} = \frac{bN}{3} - \frac{b^2}{9a} (y_2 - y_1) - K'(y_1)$$

S.O.C.

$$\frac{\partial^2 \pi_1}{\partial y_1^2} = \frac{b^2}{9a} - K''(y_1)$$

Here, the second order condition is satisfied if the function $K(y)$ satisfies

$$K''(y_1) > \frac{b^2}{9a}$$

We can follow the same procedure to prove the second order condition for firm 2 according to the symmetric situation.

Q.E.D.

(2) The Second Order Condition in the Specific Example

Substituting $K''(y) = 6\theta y$, into the second order condition which we already proved above,

$$K''(y_i) = 6\theta y_i > \frac{b^2}{9a} \Rightarrow \theta > \frac{b^2}{54ay_i}$$

Since y_i has two possible values,

$$y_i = \frac{b^2 \pm (81a^2b\theta N - b^4)^{1/2}}{27a\theta}$$

The second order condition becomes

$$2b^2 \pm 2(81a^2b\theta N - b^4)^{1/2} > b^2 \Rightarrow b^2 > \mp 2(81a^2b\theta N - b^4)^{1/2}$$

Therefore, the second order condition is satisfied if

$$\theta < \frac{5b^3}{324a^2N}$$

Q.E.D.

APPENDIX B

A Comparison between Symmetric and Asymmetric Equilibria in the Specific Case

(1) *The Prices, the Demands and the Capacity Costs*

In symmetric equilibrium,

$$P_1 = P_2 = P^s = aN, \quad n_1 = n_2 = n^s = \frac{N}{2}, \quad y_1 = y_2 = y^s = \left(\frac{bN}{9\theta}\right)^{1/2}, \quad C^s = C + an^s - by^s = c + \frac{aN}{2} - b\left(\frac{bN}{9\theta}\right)^{1/2}$$

In asymmetric equilibrium, we consider the case of $y_1 \neq y_2$ and let,

$$J = \frac{(81a^2b\theta N - b^4)^{1/2}}{27a\theta}$$

Then,

$$y_1 = \frac{b^2}{27a\theta} + J, \quad y_2 = \frac{b^2}{27a\theta} - J$$

$$P_1 = aN + \frac{2bJ}{3}, \quad P_2 = aN - \frac{2bJ}{3}$$

$$n_1 = \frac{N}{2} + \frac{bJ}{3a} = \frac{1}{2a}\left(aN + \frac{2bJ}{3}\right), \quad n_2 = \frac{N}{2} - \frac{bJ}{3a} = \frac{1}{2a}\left(aN - \frac{2bJ}{3}\right)$$

$$C_1 = c + \frac{1}{2}\left(aN + \frac{2bJ}{3}\right) - b\left(\frac{b^2}{27a\theta} + J\right), \quad C_2 = c + \frac{1}{2}\left(aN - \frac{2bJ}{3}\right) - b\left(\frac{b^2}{27a\theta} - J\right)$$

(2) Firm's Profits

In symmetric equilibrium,

$$\pi_i = \pi_j = \pi^s = P^s \cdot n^s - K(y^s) = \frac{aN^2}{2} - \theta \left(\frac{bN}{9\theta} \right)^{3/2} - F$$

In asymmetric equilibrium,

$$\pi_1 = P_1 \cdot n_1 - K(y_1) = \frac{1}{2a} \left(aN + \frac{2bJ}{3} \right)^2 - \theta \left(\frac{b^2}{27a\theta} + J \right)^3 - F$$

$$\pi_2 = P_2 \cdot n_2 - K(y_2) = \frac{1}{2a} \left(aN - \frac{2bJ}{3} \right)^2 - \theta \left(\frac{b^2}{27a\theta} - J \right)^3 - F$$

$$\pi_i - \pi^s = \frac{1}{2a} \left(aN \pm \frac{2bJ}{3} \right)^2 - \theta \left(\frac{b^2}{27a\theta} \pm J \right)^3 - \frac{aN^2}{2} + \theta \left(\frac{bN}{9\theta} \right)^{3/2}, \quad i = 1, 2$$

$$= \frac{1}{2a} (aN)^2 \left(1 \pm \frac{4bJ}{3aN} \right) - \theta \left(\frac{b^2}{27a\theta} \right)^3 \left[1 \pm \frac{3J(27a\theta)}{b^2} \right] - \frac{aN^2}{2} + \theta \left(\frac{bN}{9\theta} \right)^{3/2}$$

$$= \pm \frac{2bNJ}{3} + \theta \left(\frac{bN}{9\theta} \right)^{3/2} - \theta \left(\frac{b^2}{27a\theta} \right)^3 \mp 3\theta J \left(\frac{b^2}{27a\theta} \right)^2$$

Then we have

$$\pi_1 - \pi^s = \frac{2bNJ}{3} + \theta \left(\frac{bN}{9\theta} \right)^{3/2} - \theta \left(\frac{b^2}{27a\theta} \right)^3 - 3\theta J \left(\frac{b^2}{27a\theta} \right)^2$$

$$\pi_2 - \pi^s = -\frac{2bNJ}{3} + \theta \left(\frac{bN}{9\theta} \right)^{3/2} - \theta \left(\frac{b^2}{27a\theta} \right)^3 + 3\theta J \left(\frac{b^2}{27a\theta} \right)^2 \quad (B1)$$

Since

$$\theta > \frac{b^3}{81a^2N}$$

we have

$$\frac{2bNJ}{3} > 3\theta J \left(\frac{b^2}{27a\theta}\right)^2, \quad \theta \left(\frac{bN}{9\theta}\right)^{3/2} > \theta \left(\frac{b^2}{27a\theta}\right)^3 \quad (\text{B2})$$

Thus,

$$\pi_1 - \pi^* > 0$$

From (B1) , we let

$$\left(\frac{b^2}{27a\theta}\right)^3 > 3J \left(\frac{b^2}{27a\theta}\right)^2 \Rightarrow \left(\frac{b^2}{27a\theta}\right)^6 > 9J^2 \left(\frac{b^2}{27a\theta}\right)^4 \Rightarrow \theta < \frac{10b^3}{27^2a^2N}$$

$$\frac{2bNJ}{3} > \theta \left(\frac{bN}{9\theta}\right)^{3/2} \Rightarrow 4\left(\frac{bN}{9\theta} - \frac{b^4}{27^2a^2\theta^2}\right) > \frac{bN}{81\theta} \Rightarrow \theta > \frac{b^3}{315a^2N}$$

Since

$$\frac{10b^3}{27^2a^2N} > \frac{b^3}{81a^2N}, \quad \frac{b^3}{315a^2N} < \frac{b^3}{81a^2N}$$

we have

$$\pi_2 - \pi^s < 0$$

(3) The Total Profits

In symmetric equilibrium,

$$\Pi^s = 2\pi^s = aN^2 - 2\theta\left(\frac{bN}{9\theta}\right)^{3/2} - 2F = aN^2 - \frac{2bN}{27}\left(\frac{bN}{\theta}\right)^{1/2} - 2F$$

$$\frac{\partial \Pi^s}{\partial \theta} = \frac{1}{27}\left(\frac{bN}{\theta}\right)^{3/2} > 0$$

The above equation implies that the total profits in symmetric equilibrium increase as θ increases.

In asymmetric equilibrium,

$$\begin{aligned} \Pi = \pi_1 + \pi_2 &= \frac{1}{2a} \left[\left(aN + \frac{2bJ}{3} \right)^2 + \left(aN - \frac{2bJ}{3} \right)^2 \right] - \theta \left[\left(\frac{b^2}{27a\theta} + J \right)^3 + \left(\frac{b^2}{27a\theta} - J \right)^3 \right] - 2F \\ &= \frac{1}{2a} \left[2a^2N^2 + 2\left(\frac{2bJ}{3}\right)^2 \right] - 2\theta \left[\left(\frac{b^2}{27a\theta} \right)^3 + \frac{b^2J^2}{9a\theta} \right] - 2F = aN^2 + \frac{2b^2J^2}{9a} - \frac{2b^6}{(27)^3a^3\theta^2} - 2F \\ &= aN^2 + \frac{2b^3N}{81a\theta} - \frac{8b^6}{(27)^3a^3\theta^2} - 2F \end{aligned}$$

$$\frac{\partial \Pi}{\partial \theta} = -\frac{2b^3N}{81a\theta^2} + \frac{16b^6}{(27a\theta)^3}$$

Letting $\partial \Pi / \partial \theta > 0$, we have

$$\theta < \frac{8b^3}{243a^2N}$$

Since

$$\frac{b^3}{81a^2N} < \frac{8b^3}{243a^2N}$$

then, we proved that the total profits in asymmetric equilibrium increase as θ increases.

$$\begin{aligned}\Delta \Pi &= \Pi - \Pi^s = \frac{2b^2J^2}{9a} - \frac{2b^6}{(27)^3a^3\theta^2} + \frac{2bN}{27} \left(\frac{bN}{\theta} \right)^{1/2} \\ &= \frac{2b^3N}{81a\theta} - \frac{2b^6}{(81)^2a^3\theta^2} - \frac{2b^6}{(27)^3a^3\theta^2} + \frac{2bN}{27} \left(\frac{bN}{\theta} \right)^{1/2} \\ &= \frac{2b^3N}{81a\theta} - \frac{8b^6}{(27)^3a^3\theta^2} + \frac{2bN}{27} \left(\frac{bN}{\theta} \right)^{1/2}\end{aligned}$$

Since at the point of

$$\theta = \frac{b^3}{81a^2N}$$

$$\frac{2b^6}{(27)^3a^3\theta^2} = \frac{2b^6}{(27)^3a^3} \cdot \frac{(81)^2a^4N^2}{b^6} = \frac{2}{3}aN^2$$

$$\frac{2bN}{27} \left(\frac{bN}{\theta} \right)^{1/2} = \frac{2bN}{27} \cdot \left(bN \cdot \frac{81a^2N}{b^3} \right)^{1/2} = \frac{2}{3} aN^2$$

$$J^2 = 3abN - \frac{b^4}{27a} \cdot \frac{81a^2N}{b^3} = 3abN - 3abN = 0$$

Therefore,

$$\Delta \Pi \Big|_{\theta = \frac{b^3}{81a^2N}} = 0$$

Next, we look at the total profit change with the change of θ .

$$\frac{\partial \Delta \Pi}{\partial \theta} = -\frac{2b^3N}{81a\theta^2} + \frac{4b^6}{(81)^2a^3\theta^3} + \frac{4b^6}{(27)^3a^3\theta^3} - \frac{1}{27} \left(\frac{bN}{\theta} \right)^{3/2}$$

If

$$-\frac{2b^3N}{81a\theta^2} + \frac{4b^6}{(81)^2a^3\theta^3} > 0 \quad (B3)$$

and

$$\frac{4b^6}{(27)^3a^3\theta^3} - \frac{1}{27} \left(\frac{bN}{\theta} \right)^{3/2} > 0 \quad (B4)$$

then, $\partial\Pi/\partial\theta>0$.

From (B3) and (B4) we have

$$\theta < \frac{2b^3}{81a^2N}, \quad \theta < \frac{(16)^{1/3}b^3}{81a^2N}$$

Since

$$\frac{b^3}{81a^2N} < \frac{5b^3}{324a^2N} < \frac{2b^3}{81a^2N}, \quad \frac{b^3}{81a^2N} < \frac{5b^3}{324a^2N} < \frac{(16)^{1/3}b^3}{81a^2N}$$

therefore in the range of (see condition of (63))

$$\frac{b^3}{81a^2N} < \theta < \frac{5b^3}{324a^2N}$$

Inequalities (B3) and (B4) are sufficient conditions so that $\partial\Pi/\partial\theta>0$. Therefore, we have proved

$$\Pi - \Pi^s > 0$$

(4) *Consumer Surplus*

In symmetric equilibrium,

$$CS^s = N[V - P^s - C(n^s, y^s)] = N(V - C) - \frac{3aN^2}{2} + \frac{(bN)^{3/2}}{(9\theta)^{1/2}}$$

In asymmetric equilibrium,

$$CS_1 = n_1(V - P_1 - c_1) = \left(\frac{N}{2} + \frac{bJ}{3a}\right) \left[V - aN - \frac{2bJ}{3} - C - \frac{aN}{2} - \frac{bJ}{3} + b\left(\frac{b^2}{27a\theta} + J\right)\right]$$

$$= \left(\frac{N}{2} + \frac{bJ}{3a}\right) \left(V - C - \frac{3aN}{2} + \frac{b^3}{27a\theta}\right)$$

$$CS_2 = n_2(V - P_2 - c_2) = \left(\frac{N}{2} - \frac{bJ}{3a}\right) \left[V - aN + \frac{2bJ}{3} - C - \frac{aN}{2} + \frac{bJ}{3} + b\left(\frac{b^2}{27a\theta} - J\right)\right]$$

$$= \left(\frac{N}{2} - \frac{bJ}{3a}\right) \left(V - C - \frac{3aN}{2} + \frac{b^3}{27a\theta}\right)$$

then

$$CS = CS_1 + CS_2 = \left(\frac{N}{2} + \frac{bJ}{3a}\right) \left(V - C - \frac{3aN}{2} + \frac{b^3}{27a\theta}\right) + \left(\frac{N}{2} - \frac{bJ}{3a}\right) \left(V - C - \frac{3aN}{2} + \frac{b^3}{27a\theta}\right)$$

$$= N(V - C) - \frac{3aN^2}{2} + \frac{b^3N}{27a\theta}$$

$$\frac{\partial CS^s}{\partial \theta} < 0, \quad \frac{\partial CS}{\partial \theta} < 0$$

$$\Delta CS = CS - CS^s = \frac{b^3N}{27a\theta} - \frac{(bN)^{3/2}}{(9\theta)^{1/2}}$$

Let $\Delta CS < 0$, we have

$$\frac{Nb^3}{27a\theta} < \frac{(bN)^{3/2}}{(9\theta)^{1/2}} \Rightarrow \theta > \frac{b^3}{81a^2N}$$

Since above inequality is the lower limit of the value of θ , we proved that $CS - CS^s > 0$.

(5) The Total Welfare

In symmetric equilibrium,

$$\begin{aligned} W^s &= \Pi^s + CS^s = aN^2 - 2\theta \left(\frac{bN}{9\theta} \right)^{3/2} - 2F + N(V-C) - \frac{3aN^2}{2} + \frac{(bN)^{3/2}}{(9\theta)^{1/2}} \\ &= N(V-C) + \frac{7}{27} \frac{(bN)^{3/2}}{\theta^{1/2}} - \frac{aN^2}{2} - 2F \end{aligned} \quad (B5)$$

$$\frac{\partial W^s}{\partial \theta} < 0$$

In asymmetric equilibrium,

$$\begin{aligned} W &= \Pi + CS = aN^2 + \frac{2b^2J^2}{9a} - \frac{2b^6}{(27)^3 a^3 \theta^2} - 2F + N(V-C) - \frac{3aN^2}{2} + \frac{b^3N}{27a\theta} \\ &= aN^2 + \frac{2b^3N}{81a\theta} - \frac{8b^6}{(27)^3 a^3 \theta^2} - 2F + N(V-C) - \frac{3aN^2}{2} + \frac{b^3N}{27a\theta} \\ &= N(V-C) + \frac{5b^3N}{81a\theta} - \frac{8b^6}{(27)^3 a^3 \theta^2} - \frac{aN^2}{2} - 2F \end{aligned} \quad (B6)$$

$$\frac{\partial W}{\partial \theta} = -\frac{5b^3N}{81a\theta^2} + \frac{16b^6}{27^3a^3\theta^3}$$

Let $\partial W / \partial \theta = 0$, we have

$$\theta < \frac{16b^3}{1215a^2N}$$

Since

$$\frac{b^3}{81a^2N} < \frac{16b^3}{1215a^2N} < \frac{5b^3}{324a^2N}$$

we have

$$\frac{\partial W}{\partial \theta} > 0 \quad \text{when} \quad \frac{b^3}{81a^2N} < \theta < \frac{16b^3}{1215a^2N}$$

$$\frac{\partial W}{\partial \theta} < 0 \quad \text{when} \quad \frac{16b^3}{1215a^2N} < \theta < \frac{5b^3}{324a^2N}$$

From (B5) and (B6),

$$\begin{aligned} \Delta W &= N(V-C) + \frac{5b^3N}{81a\theta} - \frac{8b^6}{(27)^3a^3\theta^2} - \frac{aN^2}{2} - 2F - [N(V-C) + \frac{7}{27} \frac{(bN)^{3/2}}{\theta^{1/2}} - \frac{aN^2}{2} - 2F] \\ &= \frac{5b^3N}{81a\theta} - \frac{8b^6}{(27)^3a^3\theta^2} - \frac{7}{27} \frac{(bN)^{3/2}}{\theta^{1/2}} \end{aligned} \quad (B7)$$

At the lower limit point of θ ,

$$\Delta W|_{\frac{b^3}{81a^2N}} = 5aN^2 - \frac{8}{3}aN^2 - \frac{63}{27}aN^2 = 5aN^2 - 5aN^2 = 0 \quad (\text{B8})$$

The change of ΔW when θ increasing,

$$\frac{\partial \Delta W}{\partial \theta} = -\frac{5b^3N}{81a\theta^2} + \frac{16b^6}{(27)^3a^3\theta^3} + \frac{7}{54}\left(\frac{bN}{\theta}\right)^{3/2}$$

From (B2) , we know

$$\left(\frac{bN}{9\theta}\right)^{3/2} > \left(\frac{b^2}{27a\theta}\right)^3 \Rightarrow \left(\frac{bN}{\theta}\right)^{3/2} > \frac{b^6}{(27)^2a^3\theta^3}$$

If we can prove that

$$-\frac{5b^3N}{81a\theta^2} + \frac{16b^6}{(27)^3a^3\theta^3} + \frac{7}{54}\frac{b^6}{(27)^2a^3\theta^3} > 0 \quad (\text{B9})$$

we will have

$$\frac{\partial \Delta W}{\partial \theta} > 0$$

From (B9), we have

$$-\frac{5b^3N}{81a\theta^2} + \frac{16b^6}{(27)^3a^3\theta^3} + \frac{7}{2} \frac{b^6}{(27)^3a^3\theta^3} > 0 \Rightarrow \left[\frac{16+3.5}{(27)^3} \right] \frac{b^3}{a^2\theta} > \frac{5N}{81} \Rightarrow \theta < (0.016) \frac{b^3}{a^2N}$$

Since the up limit if θ is

$$\theta < \frac{5b^3}{324a^2N} = (0.015) \frac{b^3}{a^2N}$$

Therefore, inequity (B9) is satisfied and, in turn, we have that $\partial W / \partial \theta > 0$. Thus, we proved $W^* > 0$ when θ is chosen in the relevant range.

REFERENCE

- Abbott, A. (1994), "Observed Price Dispersion: Product Heterogeneity, Regional Market, or Local Market Power?" *Journal of Economics and Business* 46: 21-37.
- Burdett, K and Judd, K.L (1983), "Equilibrium Price Dispersion," *Econometrica* 51: 955-969
- Carleson, J.A. and McAfee, R.P. (1983), "Discrete Equilibrium Price Dispersion," *Journal of Political Economy*, 91: 480-493
- Dahlby, B. and D. S. West (1986), "Price Dispersion in an Automobile Insurance Market," *Journal of Political Economy*, 94: 418-437
- Diamond, P. (1971), "A Model of Price Adjustment," *Journal of Economic Theory* 3.
- Lippman, S., and J. McCall (1976), "The Economics of Job Search: A Survey," *Economic Inquiry* 14: 347-368
- Luski, I. (1976), "On Partial Equilibrium in a Queuing System with Two servers," *Review of Economic Studies*, 43: 519-525
- McAfee, R.P. (1995), "Multiproduct Equilibrium Price Dispersion," *Journal of Economic Theory* 67:83-105
- McCall, J. (1970), "Economics of Information and Job Search," *Quarterly Journal of Economics* 84: 113-126
- Nelson, P. (1970), "Information and Consumer Behavior," *Journal of Political Economy* 78: 311-329
- Pen, I.P.L. and D. Reitman (1994), "Service time competition," *The Rand Journal of Economics*, 25: 619-634,
- Reinganum, J. (1979), "A simple Model of Equilibrium Price Dispersion," *Journal of Political Economy* 87: 851-858
- Reitman, D. (1991), "Endogenous Quality Differentiation in Congested Markets," *Journal of Industrial Economics*, 39: 621-648
- Rosthschild, M. (1974), "Search for the Lowest Price When the Distribution of Price is Unknown," *Journal of Political Economy* 82:689-711

- Salp, S. (1977), "The Noisy Monopolist: Imperfect Information, Price Dispersion, and Price Discrimination," *Review of Economic Studies* 44: 393-406
- Salop, S., and J. Stiglitz. (1977), "Bargains and Ripoffs ,," *Review of Economic Studies* 44: 493-510
- Shilony, Y. (1977), "Mixed Pricing in Oligopoly," *Journal of Economic Theory* 14:373-388
- Shy, O. (1995), *Industrial Organization*, Cambridge, Mass. MIT Press 1995
- Stigler, G. (1961), "The Economics of Information," *Journal of Political Economy* 69; 213-225
- Varian, H. (1980), " A Model of Sales," *American Economic Review* 70: 651-659
- Wilde, L., and A. Schwartz. (1979), "Equilibrium Comparison Shopping," *Review of Economic Studies* 46: 543-553

CHAPTER 2

PRICING NAME-BRAND PRODUCTS IN THE PRESENCE OF GENERIC ENTRY

— A STUDY OF THE "GENERIC COMPETITION PARADOX"

I. INTRODUCTION

Since the 1970's the easing of entry barriers by legislation and the expiration of many brand-name patents has increased dramatically the number of generic products in the pharmaceutical market and stimulated strong competition between brand-name drugs and their generic counterpart. The traditional market model argues that the entry of more products and the availability of a greater selection of substitutes will result in a fall in the price of brand-name products. Indeed, Wiggings and Maness (1994) find price responses to generic entry that are consistent with this prediction. In their analysis of anti-infective drugs, brand-name prices declined as entry expanded generic choice. On the other hand, several economists find that brand-name drug prices have sometimes risen when generic competition materializes (Frank and Salkever 1992, Grabowski and Vernon 1992). These observations challenge what is expected of traditional analysis and has led to this problem being called

the "Generic Competition Paradox"¹.

How general is the paradox? Caves, Whinston and Hurwitz (1991) undertook an exploratory analysis over the 1976-1987 time period of the price effect that followed patent expiration and the subsequent entry of generic drugs. They found that brand-name drug prices were depressed by only 4.5 percent following entry into previously uncontested markets. However, their test also showed that this decline was insignificantly different from zero. On the other hand, Wagner and Duffy (1988) also examined price changes for the top selling generics and name brands. Their research showed that substantial price increases in the brand-name drugs were associated with large reductions in generic prices as entry occurred. Similarly, Grabowski and Vernon (1992) examined data on 18 major orally-administered drug products that were subject to generic competition between 1983 and 1987. They found a 7 percent average rise in brand-name drug prices one year subsequent to generic entry. A more common finding of researchers, then is that brand-name firms either maintain or raise their prices in response to generic entry.

To explain the "Generic Competition Paradox," Frank and Salkever (1992, 1995) built a simple market segmentation model based on the persistency of physicians' prescription patterns. They argue that some physicians are more likely to prescribe brand-name drugs while others will more often prescribe generics. Accordingly, they divide the world of drug buyers into two groups: price-sensitive consumers and price-insensitive consumers.

---

¹. F.M. Scherer, "Pricing, Profits, and Technological Progress in the Pharmaceutical Markets", *Journal of Economic Perspectives*, Vol. 7, 1993

The existence of two consumer groups with distinctly different price sensitivities can be rationalized in the following way. Patients who are treated by physicians in HMOs (Health Maintenance Organizations) are more likely to be prescribed generics. Hospitals and other health maintenance organizations can surmount information hurdles by convening committees to recommend the most cost-effective drug for their organization's drug formulary. Hence consumers within this group tend to be more price-elastic, switching to generic drugs when brand-name prices rise. Frank and Salkever argue that this group of consumers will not consume any brand-name drugs when cheaper generic substitutes become available.

On the other hand, individual physicians are typically more risk-averse, less sensitive to cost, and tend to be creatures of habit. Such physicians often continue to prescribe brand-name drugs even when less expensive generic substitutes exist. Reinforcing this tendency, consumers who arrive at retail pharmacies normally lack the knowledge needed to evaluate the risks of substituting away from prescribed brand-name drugs. Individuals who purchase prescription drugs in a retail outlet are then more likely to form a group of price-insensitive "loyal" consumers.

Frank and Salkever argue that the increase in brand-name prices can be explained by the optimizing behaviour of brand-name producers. The key in their model is the existence of "loyal" consumers whose demands for brand-name products are totally independent of generic drug prices. In other words, even if the price of a generic drug is much lower than a branded drug, these "loyal" consumers

will continue to consume only the brand-name product ². The demand by “loyal” consumers then allows brand-name drug producer to maintain its high price level in the face of entry by lower-priced generic drugs. In some cases, it can be more profitable for the brand-name firm to abandon its price-sensitive market by increasing price rather than try to retain its price-sensitive consumers by lowering price.

The critical assumption in Frank and Salkever’s model is that “loyal” consumers consume only brand-name drugs. However, the empirical analysis of Hellerstein (1994) indicates that “almost all physicians prescribe both types of drugs to their patients.”³ Therefore, this assumption is not consistent with the empirical evidence. More generally, consumers do look for a lower price in the market. Even price-insensitive consumers will switch to generic products if their price becomes low enough, while some price-sensitive consumers will continue to consume some brand-name drugs even when less expensive generic substitutes are available.

The objective of our study is to provide a theoretical model that can explain the “Generic Competition Paradox” with more realistic assumptions. In particular, we assume that both price-sensitive and price-insensitive consumers consume both brand-name and generic drugs. In other words, we attempt to explain the “Generic Competition Paradox” without the restrictive assumption that “loyal”

---

². In their model, the brand name producer’s demand function is $Q_b = D_L(P_b) + D_S(P_b, P_g)$ where Q_b is the quantity demand of brand-name drug, P_b and P_g are the brand name and generic prices respectively, and D_L and D_S represent the loyal and cross-price sensitive segments of the brand name firm’s demand function.

³ Judith K. Hellerstein, “The Demand for Post-patent Prescription Pharmaceuticals”, *NBER WORKING PAPERS SERIES*, NO. 4981, December 1994, page 32.

consumers consume only brand-name drugs. The driving force in our model is the market share of price-sensitive consumers in combination with cross substitutability between the two types of drugs for price-insensitive consumers. As long as there exists a relatively large share of price-insensitive consumers in the market when cross substitutability is relatively large, or a relatively small share of price-insensitive consumers in the market when the cross substitutability is relatively small, the brand-name drug's price is likely to increase. In contrast to Frank and Salkever's model where prices will always rise in the face of generic entry, our approach shows that if the assumption of brand loyalty is replaced with the more realistic assumption that all consumers in the drug market consume both brand-name and generic drugs, the price of the brand-name drug will rise only under certain conditions. In this sense the "Generic Competition Paradox" becomes more fragile. We also extend the analysis to consider the case where the brand-name firm chooses to produce its own generic product in response to generic entry.

A related strand of literature are the analysis of competition among brand-name drugs. For example, Lu and Comanor (1998) examines the pricing strategies of newly patented Pharmaceuticals. Because a patent is given for a drug's novel chemical composition rather than its therapeutic properties, many pharmaceutical firms can receive a patent despite their drug having a therapeutic function similar to that of an existing drug. Based on their empirical evidence, Lu and Comanor find that for drugs which represent important therapeutic gain, the launch price can be two or three times higher than those of existing drugs used for the same purposes. Drugs that largely copy the actions of currently available products are typically priced at comparable level. With the passing of time, the two pricing strategies become quite different. The first practice typically sets a high initial price and then lowers

it over time. The second practice launches the new product at a low price and then gradually raises that price. Our analysis is different in that we focus on brand-name firm's pricing strategy after (as opposed to before) the expiration of patent.

The remainder of this chapter is organized into three sections. Section II presents the basic model; Section III extends that model to allow the brand-name firm to produce a generic product; and Section IV presents our concluding remarks.

II. THE BASIC MODEL

1. *Assumptions*

(1) *The Demand Side*

We assume that there are two groups of consumers that consume both brand-name and generic drugs, *Group 1* and *Group 2*. The consumers in *Group 1* are assumed to be relatively insensitive to price changes between the two drug types and thus have a lower price elasticity of demand than those in *Group 2*. We view *Group 1* as those consumers treated by office-based physicians and *Group 2* as those treated in hospitals.

This characterization of the pharmaceutical market generalizes the “loyal” consumers model advanced by Frank and Salkever (1992). Hence following that analysis, the demand side of the market for prescription drugs has two distinct segments. First, consumers treated by hospitals and health maintenance organizations are price-sensitive because their prescriptions come from institutions where the large volume of purchases makes it economic to compare the effectiveness of brand-name and generic drugs and thus choose the most cost-effective drugs at relatively low administrative cost. The second segment consists of individuals whose prescriptions are presented at retail pharmacies. These individuals are price-insensitive for two reasons. First, individual prescribing physicians tend to be risk-averse and insensitive to cost. Hence they continue to prescribe brand-name drugs even though generic substitutes may be equally effectively but less expensive. Secondly, the physicians’ prescription is compounded by the fact that consumers who purchase drugs at a retail pharmacy normally lack sufficient knowledge to evaluate the alternatives and risks of substituting away from

a prescribed brand-name drug. Therefore, it is reasonable to assume that when generic substitutes exist, a drug buyer will typically belong to one of two quite different groups: those who are price sensitive and those who are not.

Suppose then that there are two goods in one particular drug market: a brand-name drug X_b and a generic drug substitute X_g . In contrast to Frank and Salkever's assumption of a group of purely "loyal" consumers, we assume that both groups of consumers treat the brand-name and the generic drugs as imperfect substitutes and hence consume some quantity of both. The preferences of both groups of consumers can then be represented by different parameters within the same quadratic utility function:

$$U_i = X_0 + \frac{\alpha_i}{\beta_i} \sum_j X_{ji} - \frac{1}{2\beta_i} \sum_j X_{ji}^2 - \frac{\gamma_i}{\beta_i} \sum_j X_{ji} \quad (2.1)$$

where $i=1, 2$ represents the two groups of consumers and where $j=b, g$ represents respectively the brand-name product (b) and the generic product (g). X_0 represents units of a competitively produced numeraire good. To emphasize the substitution effect of prices, the income effects of price changes are ignored.

The demand function for each product can be derived by solving⁴ :

---

⁴ See the Appendix at the end of the chapter for the proof.

$$\frac{\partial U_i}{\partial X_{bi}} = MU_{bi} = p_b, \quad \frac{\partial U_i}{\partial X_{gi}} = MU_{gi} = p_g \quad (2.2)$$

Solved simultaneously, they yield the inverse demand functions⁵

$$p_b = \frac{1}{\beta_i}(\alpha_i - X_{bi} - \gamma_i X_{gi}), \quad p_g = \frac{1}{\beta_i}(\alpha_i - X_{gi} - \gamma_i X_{bi})$$

or the two demand curves

$$X_{bi} = \frac{1}{1-\gamma_i^2}[\alpha_i(1-\gamma_i) - \beta_i p_b + \beta_i \gamma_i p_g], \quad X_{gi} = \frac{1}{1-\gamma_i^2}[\alpha_i(1-\gamma_i) - \beta_i p_g + \beta_i \gamma_i p_b] \quad (2.3)$$

The parameters β_i and γ_i represent factors determining the responsiveness of product demand to changes in the own and the substitute good price. Factor β_i measures the change in the slope of the demand curve by the two groups for the drugs. γ_i measures the substitutability between the brand name and generic product. When $\gamma_i = 0$, the two are completely different in the eyes of group i 's consumers. Each producer is then a monopolist in its own brand. As $\gamma_i \rightarrow 1$, the products are increasing

---

⁵ We assume that there is no price discrimination by the brand-name firm. This assumption is reasonable because drug sales to hospital are largely channelled through the same wholesalers that sell to the pharmacy market and their ability to arbitrage across markets prevents the producer from forcing price discrimination. In addition the practice of price discrimination may violate anti-trust law.

viewed as being homogeneous. In the limit, group i 's consumers view the two products as being identical. The discussion that follows assumes that $0 < \gamma_i < 1$. This means we are assuming that the brand-name and the generic product are imperfect substitutes. In addition, we assume that $\alpha_i < \beta_i < \gamma_i$, which means that the own substitute effect is greater than the cross substitute effect. These parameter sizes then translate our desired assumptions of the model into explicit form. From the demand functions in (2.3), we have

$$\frac{\partial X_{bi}}{\partial p_b} = -\frac{\beta_i}{1-\gamma_i^2}, \quad \frac{\partial X_{bi}}{\partial p_g} = \frac{\beta_i \gamma_i}{1-\gamma_i^2}$$

Since we assume that the demand of *Group 2*'s consumers is more price sensitive than *Group 1*'s consumers, we have

$$\frac{\beta_1}{1-\gamma_1^2} < \frac{\beta_2}{1-\gamma_2^2}, \quad \frac{\beta_1 \gamma_1}{1-\gamma_1^2} < \frac{\beta_2 \gamma_2}{1-\gamma_2^2} \quad (2.4)$$

A sufficient condition for (2.4) to hold is $\beta_1 < \beta_2$ and $\gamma_1 < \gamma_2$. Therefore, we assume that $\beta_1 < \beta_2, \gamma_1 < \gamma_2$.

Across the two groups, the aggregated demand for X_b and X_g are:

$$X_b = \theta X_{b1} + (1-\theta) X_{b2}$$

$$X_g = \theta X_{g1} + (1-\theta) X_{g2}$$

or

$$X_b = \frac{\theta\alpha_1}{1+\gamma_1} + \frac{(1-\theta)\alpha_2}{1+\gamma_2} - \left[\frac{\theta\beta_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2}{1-\gamma_2^2} \right] p_b + \left[\frac{\theta\beta_1\gamma_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2\gamma_2}{1-\gamma_2^2} \right] p_g \quad (2.5)$$

$$X_g = \frac{\theta\alpha_1}{1+\gamma_1} + \frac{(1-\theta)\alpha_2}{1+\gamma_2} - \left[\frac{\theta\beta_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2}{1-\gamma_2^2} \right] p_g + \left[\frac{\theta\beta_1\gamma_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2\gamma_2}{1-\gamma_2^2} \right] p_b \quad (2.6)$$

where θ represents the share of *Group 1*'s demand. If we let

$$\alpha = \frac{\theta\alpha_1}{1+\gamma_1} + \frac{(1-\theta)\alpha_2}{1+\gamma_2}, \quad \beta = \frac{\theta\beta_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2}{1-\gamma_2^2}, \quad \gamma = \frac{\theta\beta_1\gamma_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2\gamma_2}{1-\gamma_2^2}$$

then equations (2.5) and (2.6) can be rewritten as market demand curves

$$X_b = \alpha - \beta p_b + \gamma p_g, \quad X_g = \alpha - \beta p_g + \gamma p_b \quad (2.7)$$

or as inverse demand functions,

$$p_b = \frac{1}{\beta}(\alpha + \gamma p_g - X_b), \quad p_g = \frac{1}{\beta}(\alpha + \gamma p_b - X_g). \quad (2.8)$$

Figure 2.1 illustrates the key features of the brand-name demand for all price possibilities. The

highest price that the brand-name firm can charge before it prices itself out of the market is illustrated by point D on the price axis. This point is calculated from (2.8) by setting $X_b=0$ and solving for D as

$$D = \frac{1}{\beta}(\alpha + \gamma p_g) \quad (2.9)$$

Given the price of the generic substitute, when p_b falls the market demand for the brand-name drug will increase along Dd . At the point d , the price of brand-name product has fallen sufficiently such that the generic firm's output has fallen to zero, i.e. $X_g=0$. By setting $X_g=0$ and solving for p_b , (2.7) yields

$$d = \frac{1}{\gamma}(\beta p_g - \alpha). \quad (2.10)$$

where $d \geq 0$ requires $P_g \geq \alpha/\beta$.

Even though the generic firm's demand has fallen to zero at $P_b = d$, the brand-name firm could continue to cut price if it so wished. It would not increase customers at the same rate as before since the generic firm no longer has customers to lose. The second branch of the brand-name demand curve, dF , is steeper and can be calculated from (2.7) and (2.8). Solving for p_g when X_g equals zero from (2.7) and substituting into (2.8). The inverse demand of p_b in terms of X_b becomes

$$p_b = \frac{1}{\beta^2 - \gamma^2}[\alpha(\beta + \gamma) - \beta X_b] = \frac{\alpha}{\beta - \gamma} - \frac{\beta}{\beta^2 - \gamma^2} X_b \quad (2.11)$$

The demand curve for the brand name drug can then be written as

$$X_b = \frac{\alpha(\beta + \gamma)}{\beta} - \frac{\beta^2 - \gamma^2}{\beta} p_b = a - b p_b \quad (2.12)$$

where

$$a = \frac{\alpha(\beta + \gamma)}{\beta}, \quad b = \frac{\beta^2 - \gamma^2}{\beta}. \quad (2.13)$$

FIGURE 2.1. The Demand Curve in the Brand-name Market

(2) The Supply Side

We assume that there exists one brand-name firm and that n ($n \geq 2$) generic firms enter the market after the brand-name firm's patent expires. The brand-name producer is treated as a dominant firm that determines its own price in response to generic entry. The n generic firms are all identical and play a Cournot game in setting quantities. The generic price is then determined from the Cournot quantity outcome given the brand-name price. The brand-name firm sets its price taking the number of current and future generic producers as exogenous. Therefore, the brand-name and the generic firms are assumed to play a Stackelberg price setting game. In doing so, the marginal costs of the brand-name and generic firms producing drugs are constant and assumed to be represented by c_b and c_g respectively.

2. Brand-name Price Setting in a Two-stage Model

There are two stages in which the brand-name firm and the generic firms compete with each other. First, the brand-name firm is a Stackelberg leader that determines its market price, given the generic price function. Then, the n generic firms play the Cournot quantity setting game to decide the total generic quantity taking as given the price of brand-name product from the first stage of the game. From this we can determine the generic price as a function of the price of the brand-name product. In order to solve this model, we start from the second stage of the game.

(1) The Second Stage of the Model: Generic Firms' Cournot Competition

In the generic drugs market, n generic profit maximizing firms compete in quantities, taking the price of the brand-name product as given. From the inverse demand from (2.8), generic firm i 's reaction

function is derived as

$$r_i(x_{-g_i}) = \argmax(p_g - c_g) \cdot x_{g_i} = \argmax\left\{\frac{1}{\beta}[\alpha + \gamma p_b - (x_{g_1} + x_{g_2} + \dots + x_{g_i} + \dots + x_{g_n})]x_{g_i} - c_g x_{g_i}\right\}. \quad (2.14)$$

Because the n generic firms are assumed identical, i.e. $x_{g_1} = x_{g_2} = x_{g_3} = \dots = x_{g_n} = x_g$. Equation (2.14) can be solved as

$$x_g = \frac{1}{n+1}(\alpha + \gamma p_b - \beta c_g) \quad (2.15)$$

The market supply forthcoming from the generics is then

$$X_g = nx_g = \frac{n}{n+1}(\alpha + \gamma p_b - \beta c_g) \quad (2.16)$$

Combining (2.16) with (2.8), we can solve for the generic price as a function of the price of brand-name product and the marginal cost of generic production. This is

$$p_g = \frac{\alpha + \gamma p_b}{(n+1)\beta} + \frac{n}{n+1}c_g. \quad (2.17)$$

(2) The First Stage of the Model: Brand Name Firm's Price Setting

Substituting the generic price function in (2.17) back into (2.7) yields the demand function for brand-

name producer facing n generic entrants with marginal cost c_g . This is found as.

$$X_b = \frac{\alpha[(n+1)\beta + \gamma] + n\beta\gamma c_g}{(n+1)\beta} - \frac{(n+1)\beta^2 - \gamma^2}{(n+1)\beta} p_b \quad (2.18)$$

Letting

$$\bar{a} = \frac{\alpha[(n+1)\beta + \gamma] + n\beta\gamma c_g}{(n+1)\beta} \quad \text{and} \quad \bar{b} = \frac{(n+1)\beta^2 - \gamma^2}{(n+1)\beta}$$

The demand function of the brand-name product can be written more concisely as

$$X_b = \bar{a} - \bar{b}p_b \quad (2.19)$$

The brand-name producer sets the price that maximizes profits. Hence the maximization problem

$$\text{Max}_{p_b} \pi_b = (p_b - c_b)X_b = (p_b - c_b)(\bar{a} - \bar{b}p_b) \quad (2.20)$$

yields the optimal price

$$p_b^* = \frac{\bar{a}}{2\bar{b}} + \frac{1}{2}c_b = \frac{\alpha[(n+1)\beta + \gamma] + n\beta\gamma c_g}{2[(n+1)\beta^2 - \gamma^2]} + \frac{c_b}{2} \quad (2.21)$$

This implies,

$$\frac{\partial P_b^*}{\partial n} = \frac{2(\alpha\beta + \beta\gamma c_g)[(n+1)\beta^2 - \gamma^2] - 2\beta^2\{\beta[\alpha(n+1) + n\gamma c_g] + \alpha\gamma\}}{2[(n+1)\beta^2 - \gamma^2]^2} \quad (2.22)$$

As shown in the Appendix, $\partial P_b^*/\partial n > 0$ if

$$c_g > \frac{\alpha}{\beta + \gamma} \quad (2.23)$$

The result in (2.23) is interesting. It implies that the price response by the brand-name product to the generic entry depends on the cost of the generic product instead of the cost of the brand-name product. From (2.23), we conclude that if the marginal cost of the generic product is relatively large, the brand-name drug price will increase with generic entry. This conclusion can be summarized as Proposition 2.1.

PROPOSITION 2.1.

If the marginal cost of the generic drug is relatively large, i.e.

$$c_g > \frac{\alpha}{\beta + \gamma}$$

the brand-name drug price will increase with generic entry.

Using the definitions of α , β and γ , we can rewrite (2.23) as

$$c_g > \left[\left(\frac{\theta}{1-\theta} \right) \frac{\alpha_1}{1+\gamma_1} + \frac{\alpha_2}{1+\gamma_2} \right] / \left[\left(\frac{\theta}{1-\theta} \right) \frac{\beta_1(1+\gamma_1)}{1-\gamma_1^2} + \frac{\beta_2(1+\gamma_2)}{1-\gamma_2^2} \right]$$

Given the value of c_g the inequality above is also equivalent to

$$\left(\frac{\theta}{1-\theta} \right) \frac{\beta_1 c_g (1+\gamma_1) - \alpha_1 (1-\gamma_1)}{1-\gamma_1^2} > \frac{\alpha_2 (1-\gamma_2) - \beta_2 c_g (1+\gamma_2)}{1-\gamma_2^2} \quad (2.24)$$

The detailed proofs of (2.23) and (2.24) are provided in the Appendix.

The restrictions implied by (2.24) depend on the sign of both side of the inequality. In order to simplify the analysis we make a further assumption as

$$\varphi_1 = \frac{\alpha_1 - \beta_1 c_g}{\alpha_1 + \beta_1 c_g} > 0, \quad \varphi_2 = \frac{\alpha_2 - \beta_2 c_g}{\alpha_2 + \beta_2 c_g} > 0, \quad \varphi_2 > \varphi_1, \quad \alpha_2 > \alpha_1$$

Then, we can discuss (2.24) based on three possible cases:

(i) $\gamma_1 > \varphi_1$ and $\gamma_2 < \varphi_2$

In this case, the values on both sides of inequality (2.24) are positive. Then, we have

$$\frac{\theta}{1-\theta} > \frac{\alpha_2(1-\gamma_2)-\beta_2c_g(1+\gamma_2)}{\beta_1c_g(1+\gamma_1)-\alpha_1(1-\gamma_1)} \cdot \left(\frac{1-\gamma_1^2}{1-\gamma_2^2}\right).$$

(ii) $\gamma_1 > \varphi_1$ and $\gamma_2 > \varphi_2$

In this case, the left side of the inequality (2.24) is always positive while the right side of the inequality is negative. Hence even if $\theta \rightarrow 0$, the cross elasticity in the price sensitive market is sufficiently large to have the brand name price rise following generic entry.

(iii) $\gamma_1 < \varphi_1$ and $\gamma_2 > \varphi_2$

In this case, the values on both sides of inequality (2.24) are negative. Then, we have

$$\frac{\theta}{1-\theta} < \frac{\beta_2c_g(1+\gamma_2)-\alpha_2(1-\gamma_2)}{\alpha_1(1-\gamma_1)-\beta_1c_g(1+\gamma_1)} \cdot \left(\frac{1-\gamma_1^2}{1-\gamma_2^2}\right).$$

Three conclusions can be drawn from the above discussion. First, if the cross substitute factor for *Group 1* consumers, γ_1 , is relatively large while the cross substitute factor for *Group 2* consumers, γ_2 , is relatively small, the brand-name price will increase only when the market share of *Group 1* consumers is relatively large. Second, if the cross substitute factors are both relatively large, the brand-name price will increase regardless of the relative share of *Group 1* consumers. Third, if the cross substitutability factor for *Group 1* consumers is relatively small, an increase in the brand-name price requires the relative share of *Group 1* consumers to be relatively large. These results give us

the following proposition.

PROPOSITION 2.2.

Given the marginal cost of generic firms, the brand-name drug price will increase with generic entry if

- (i) the relative share of the demand by Group 1 consumers is large and the cross substitute factor of Group 1 consumers is relative large when the cross substitute factor of Group 2 consumer is relative small, i.e. $\gamma_1 > \varphi_1$ and $\gamma_2 < \varphi_2$ or,*
- (ii) the relative share of Group 1 consumers is small and the cross substitute factor of Group 1 consumers is relatively small when then cross substitute factor of Group 2 consumer is relative large , i.e. $\gamma_1 < \varphi_1$ and $\gamma_2 > \varphi_2$ or,*
- (iii) the cross substitute factors of Group 1 and Group 2 consumers are relative large, i.e. $\gamma_1 > \varphi_1$ and $\gamma_2 > \varphi_2$.*

A key to understanding Proposition 2.1 and 2.2 is the movement of the demand curve in the brand-name market. On the one hand, comparing (2.8) and (2.19), we find that the slope of the demand for brand-name drugs becomes steeper with generic entry because,

$$\frac{1}{b} = \frac{(n+1)\beta}{(n+1)\beta^2 - \gamma^2} = \frac{1}{\beta - [\gamma^2/(n+1)\beta]} > \frac{1}{\beta}$$

On the other hand, as we let $X_g=0$ in (2.16), we find that the point d in Figure 2.1 moves down to the point d' in Figure 2.2 and 2.3, i.e.

$$d' = \frac{1}{\gamma}(\beta c_g - \alpha) < d = \frac{1}{\gamma}(\beta p_g - \alpha)$$

Furthermore, the intercept of the demand curve on the vertical axis moves up to the point D' in Figure 2.2 and 2.3, i.e. ⁶

$$\frac{\bar{a}}{\bar{b}} > \frac{\alpha}{\beta}$$

Therefore Proposition 2.2 is easily understood. Before generic entry, the brand-name products serve both *Group 1* and *Group 2* consumers. At this stage, the demand for the brand-name products is relatively elastic. After generic entry, the lower generic prices attract mostly *Group 2* consumers' demand thus, *Group 2* consumers consume fewer brand-name products in the brand-name market. The aggregated demand for the brand-name product is then relatively inelastic. At this stage, the brand-name producer tends to increase its price. Furthermore, the change in the slope of demand for brand-name products also depends on the cross substitute factor of *Group 1* consumers, γ_1 . If γ_1 is relatively large, *Group 1* consumers will also easily switch some of their consumption to generic products. Therefore, a large share of *Group 1* consumers is a sufficient condition to increase brand-name price. If γ_1 is relatively small, it means *Group 1* consumers are extremely price insensitive to difference in drug prices so that only a very limited demand are switched from brand-name products

---

⁶. See the Appendix at the end of the chapter for the proof.

to generic products in the presence of generic entry. On the other hand, if *Group 1* consumers occupy a large share of demand in the market, it means that the brand-name market is already inelastic before generic entry. Following generic entry, only a very small amount of demand by *Group 2* consumers will switch to generic drugs thus, the elasticity of brand-name drug market will not change significantly, i.e. the price of brand-name products will not change significantly either. Therefore, a small share of *Group 1* consumers is sufficient to increase brand-name price.

Substituting (2.21) in to (2.18) we have the quantity of brand-name products,

$$X_b = \frac{1}{2(n+1)\beta} \left\{ (n+1)\alpha\beta + n\beta\gamma c_g + \alpha\gamma + [(n+1)\beta^2 - \gamma^2]c_b \right\} \quad (2.25)$$

Equation (2.25) implies that the change in total quantity of the brand-name drug with generic entry depends, among other things, on the marginal cost of the brand-name firm. Then, we have Proposition 2.3.

PROPOSITION 2.3.

Given the marginal cost of generic drugs, the output level of the brand-name drug decreases with generic entry when the marginal cost of the brand-name drug is relatively small. On the other hand, the output level of the brand-name drug products can increase with generic entry if the marginal cost of brand-name drugs is sufficiently large.

Proof:

From (2.25), we have

$$\frac{\partial X_b}{\partial n} = \frac{\gamma}{2\beta(n+1)^2} (\beta c_g - \alpha + \gamma c_b) \quad \begin{cases} < 0, & \text{if } c_b < (1/\gamma)(\alpha - \beta c_g) \\ > 0, & \text{if } c_b > (1/\gamma)(\alpha - \beta c_g) \end{cases} \quad (2.26)$$

Q.E.D.

Proposition 2.3 is illustrated in figure 2.2 and 2.3. When the marginal cost of brand-name products is relatively small, i.e. $c_b < (1/\gamma)(\alpha - \beta c_g)$, the equilibrium output of brand-name products decreases with generic entry because revenue falls in the neighbourhood of c_b . On the other hand, the equilibrium output of the brand-name product will increase with generic entry when the marginal cost of brand-name products is relatively large i.e. $c_b > (1/\gamma)(\alpha - \beta c_g)$. In this case, marginal revenue rises in the neighbourhood of c_b .

Now let us go back to the generic drug market to look at the equilibrium generic price and quantity. Given the price of brand-name product shown by equation (2.21), the equilibrium generic drug price can be derived from (2.17).

$$\begin{aligned} p_g^* &= \frac{\alpha}{(n+1)\beta} + \frac{n}{n+1} c_g + \frac{\gamma}{(n+1)\beta} p_b^* \\ &= \frac{\alpha}{(n+1)\beta} + \frac{n}{n+1} c_g + \frac{\alpha\gamma[(n+1)\beta + \gamma] + n\beta\gamma^2 c_g}{2(n+1)\beta[(n+1)\beta^2 - \gamma^2]} + \frac{\gamma c_b}{2(n+1)\beta} \end{aligned}$$

$$= \frac{\alpha}{(n+1)\beta} + \frac{n}{n+1}c_g + \frac{\alpha\gamma[\beta+\gamma/(n+1)]+n\beta\gamma^2c_g/(n+1)}{2\beta[(n+1)\beta^2-\gamma^2]} + \frac{\gamma c_b}{2(n+1)\beta} \quad (2.27)$$

From (2.27), we conclude that p_g^* will decrease as n increases. This is summarized as Proposition 2.4.

PROPOSITION 2.4

In equilibrium, the generic price will decrease with generic entry.

FIGURE 2.2. Brand-name Drug Price Increases with Generic Entry when $c_b < (1/\gamma)(\alpha - \beta c_g)$

FIGURE 2. 3. Brand-name Drug Price Increases with Generic Entry when $c_b < (1/\gamma)(\alpha - \beta c_g)$

III. AN EXTENSION: THE BRAND-NAME FIRM PRODUCES A GENERIC PRODUCT IN RESPONSE TO GENERIC ENTRY

We now discuss the extended case where the brand-name firm responds to generic entry by producing a generic product. Although this case is not often observed, several such cases have arisen in the pharmaceutical market. For example, industry leader Merck introduced a separate generic drug division in 1992 (based in Rahway, New Jersey). Similarly West Point Pharma, made a generic version of Merck's analgesic Dolobid, whose patent expired in 1989. At the same time, the company also announced that it would market generic formulation of all its products.⁷ To analyse this case, we use a model similar to the one in the previous section except that the brand-name firm and n generic firms now play a Stackelberg-quantity-setting game in the second stage of the game.

Since the profit loss that the brand-name firm experiences after generic entry comes from its loss of market share as price-sensitive consumers switch to generic products, it is reasonable for the brand-name firm to consider joining the production of one or more generic products in order to make up or minimizing its loss. Relative to generics, the brand-name firm considering production of a generic product has access to better technology, a better reputation for product quality, economies of scale etc. Therefore, the strategy of producing both brand-name and a generic product may maximize the brand-name firm's profit potential when faced with generic entry.

---

⁷Milt Fredenheim, " Merck Sets Generic Drug Sales." *New York Times*, Sept. 8, 1992.

1. The Second Stage of the Model: Stackelberg Quantity Competition in the Generic Drug Market

In the second stage, the brand-name firm is assumed to be the leader in quantity-setting game. We suppose that the brand-name firm produces one generic product, x_{bg} , where the marginal cost of producing that generic drug is assumed to be a constant, c_{bg} . The total supply of generic products from both the brand-name firm and the n generic firms is now

$$X_g = x_{bg} + (x_{g_1} + x_{g_2} + \dots + x_{g_n}) = x_{bg} + nx_g.$$

Substituting this into (2.8), the inverse demand facing generic producers becomes

$$p_g = \frac{1}{\beta} [\alpha + \gamma p_b - (x_{bg} + nx_g)] \quad (2.28)$$

We can write the i th generic firm's profit π_i as function

$$\pi_i = (p_g - c_g)x_{g_i} = \frac{1}{\beta} [\alpha + \gamma p_b - (x_{bg} + x_{g_1} + \dots + x_{g_i} + \dots + x_{g_n}) - \beta c_g] x_{g_i}. \quad (2.29)$$

Using the first order condition of (2.29) and assuming that all generic firms are identical, the reaction function of the i th generic firm and the aggregate for all generic firms can be found as

$$x_g = \frac{1}{n+1}(\alpha + \gamma p_b - x_{bg} - \beta c_g); \quad nx_g = \frac{n}{n+1}(\alpha + \gamma p_b - x_{bg} - \beta c_g) \quad (2.30)$$

The profit function facing the brand-name firm producing one generic product is defined by π_{bg} . Hence using Equation(2.30), π_{bg} can be presented as

$$\begin{aligned} \pi_{bg} &= (p_g - c_{bg})x_{bg} = \frac{1}{\beta} \left[\alpha + \gamma p_b - x_{bg} - \frac{n}{n+1}(\alpha + \gamma p_b - x_{bg} - \beta c_g) - \beta c_{bg} \right] x_{bg} \\ &= \frac{1}{(n+1)\beta} [\alpha + \gamma p_b + n\beta c_g - (n+1)\beta c_{bg} - x_{bg}] x_{bg}. \end{aligned} \quad (2.31)$$

It follows from the first order condition of (2.31) that the optimal generic output choice for the brand-name firm is

$$x_{bg} = \frac{1}{2} [\alpha + \gamma p_b + n\beta c_g - (n+1)\beta c_{bg}] \quad (2.32)$$

Substituting (2.32) back into (2.30), we can now determine the supply of the non brand name generic firms and the total supply output by both brand-name and generic firms as

$$nx_g = \frac{n}{n+1}(\alpha + \gamma p_b - \beta c_g) - \frac{n}{n+1}x_{bg} = \frac{n}{2(n+1)} [\alpha + \gamma p_b + (n+1)\beta c_{bg} - (n+2)\beta c_g]$$

and

$$X_g = x_{bg} + nx_g = \frac{1}{2(n+1)} [(2n+1)(\alpha + \gamma p_b) - n\beta c_g - (n+1)\beta c_{bg}] \quad (2.33)$$

Substituting (2.33) back into (2.28), we can determine the generic inverse demand function given the brand-name price.

$$p_g = \frac{1}{2\beta(n+1)} [\alpha + \gamma p_b + (n+1)\beta c_{bg} + n\beta c_g] \quad (2.34)$$

Substituting this price into (2.7), the demand function in the brand name market is found as

$$X_b = \frac{2\alpha\beta(n+1) + \alpha\gamma + n\beta\gamma c_g + (n+1)\beta\gamma c_{bg}}{2\beta(n+1)} - \frac{2(n+1)\beta^2 - \gamma^2}{2\beta(n+1)} p_b = \overline{a'} - \overline{b'} p_b \quad (2.35)$$

where

$$\overline{a'} = \frac{\alpha[2\beta(n+1) + \gamma] + \beta\gamma[nc_g + (n+1)c_{bg}]}{2\beta(n+1)} \quad \text{and} \quad \overline{b'} = \frac{2(n+1)\beta^2 - \gamma^2}{2\beta(n+1)}$$

2. The First Stage of the Model: Price Setting in Brand-name Drug Market

Given the demand function in (2.35), optimal price setting for the profit maximizing brand-name firm can be found as the solution to the following problem.

$$\begin{aligned}
Max_{p_b} \pi_b &= (p_b - c_b)X_b + (p_g - c_{bg})x_{bg} = (p_b - c_b)(\bar{a}' - \bar{b}'p_b) + (p_g - c_{bg})x_{bg} \\
&= (p_b - c_b)(\bar{a}' - \bar{b}'p_b) + \frac{1}{4\beta(n+1)}[\alpha + \gamma p_b + n\beta c_g - (n+1)\beta c_{bg}]^2
\end{aligned}$$

The first order condition for an interval maximum yields

$$p_b^{*'} = \frac{2[\alpha\beta(n+1) + \alpha\gamma + n\beta\gamma c_g] + [2(n+1)\beta^2 - \gamma^2]c_b}{4(n+1)\beta^2 - 3\gamma^2} \quad (2.36)$$

We can now determine the brand name producer's price response to entry by taking the derivative of (2.36) with respect to n , $\partial p_b^{*'} / \partial n$. This yields

$$\frac{\partial p_b^{*'}}{\partial n} > 0 \quad \text{if} \quad c_g > \frac{\alpha(4\beta + 3\gamma) - 2\beta\gamma c_b}{4\beta^2 - 3\gamma^2} \quad (2.37)$$

The detailed solution and sign proofs of (2.36) and (2.37) are given in the Appendix.

We can now compare the case where the brand-name firm produces only the brand-name drug with the case where the brand-name firm produces a generic drug as well. From (2.35) and (2.37), we see that the difference between these two cases is that in the second case the brand-name drug's price depends not only on the cost of the generic drug but also on the cost of brand-name drug. If the cost of producing the brand-name drug is relatively large, the price of brand-name drug is likely to rise with generic entry regardless of the relative share of *Group 1* consumers. Comparing (2.37) to

(2.22), we find that the price of brand-name drug is more likely to rise in the second case if the following condition holds⁸, i.e.

$$c_b > \frac{\alpha(7\beta+6\gamma)}{2\beta(\beta+\gamma)} \quad (2.38)$$

These results lead to the following proposition.

PROPOSITION 2.5.

With generic entry, the brand-name drug price is more likely to increase when the brand-name firm also produces a generic product than in the case where the brand-name firm produces only a brand-name product as long as the marginal cost of brand-name drug is large enough, i.e. if

$$c_b > \frac{\alpha(7\beta+6\gamma)}{2\beta(\beta+\gamma)}$$

The explanation of this proposition is straight-forward. If the brand-name firm produces only the brand-name products, some of its price-sensitive consumers will switch to consume more generic products after the patent expires. To avoid this profit loss due to the loss of market share of demand,

---

⁸.See the Appendix at the end of the chapter for the proof.

the brand-name producer is reluctant to raise its price especially when there is a larger number of price-sensitive consumers in the market. However, if the brand-name firm joins the production of one or more generic products with generic entry, it is not as necessary for the brand-name firm to worry about the loss of profits because the brand-name firm is also a leader in the generic drug market. By producing in both markets, the brand name firm continues to attract both price-sensitive and price-insensitive consumers. The higher is the cost of producing the brand-name drug, the more likely it is for the brand-name firm to increase its brand-name price.

IV. CONCLUDING REMARKS

This chapter represents the first attempt to explain the "Generic Competition Paradox" without relying on the assumption that one set of "loyal" consumers consume only brand-name drugs. We have demonstrated that the determining factor in the model is the size of marginal cost of production of the generic firms relative to certain characteristics of market demand. Under the assumption that all consumers will consume both brand-name and generic drugs, we have shown that the price of the brand-name drug will rise following generic entry if the marginal cost of production for generic firms is relatively large.

Another key difference between our model and the existing literature lies on the relative market share of price-insensitive consumers and price-sensitive consumers. We have shown that the condition for a brand-name price increase can be met when the market share of price-insensitive consumers is relatively large and the factor indicating cross substitutability for these same consumers is relatively large. Similarly, the same condition can be met if the cross substitute factor for price-insensitive consumers is relatively small and if the relative market share of price-insensitive consumers is small.

One worthwhile extension of the basic model is to the case where a brand-name firm produces one or more generic products after its own patent expires. Our model provides an explanation to why the brand-name price will rise when entry leads both the brand-name and generic firms increase their supply of generic products. In this case, the price of the brand-name drug depends explicitly on both the cost of production in the generic firm and the cost of producing the brand-name product.

Compared with the case where the brand-name firm produces only the brand-name product, our model demonstrates that the price of the brand-name product is more likely to rise when the cost of its product is relatively high.

Our analysis has implications for the debate over the length of patent protection granted to brand-name drugs. In 1989, amendment to the Canadian Patent Act gave extended patent protection to brand-name firms by extending its duration to twenty years. Critics of this amendment argue that the lengthening of patent protection will allow brand-name producers to keep their prices high for longer periods and will have the effects of driving up the overall costs of drugs. However, the analysis in our model suggests that this is not always true. Under certain condition, generic competition after the expiration of patent may drive brand-name's price up so that overall the general drug price level may stay the same or even rise after patent expiration. In this case, the lengthening of patent protection granted to brand-name drugs may not lead to higher overall drug costs.

APPENDIX

1. The Proof of Demand Functions

The demand functions for the two drugs by the two groups of consumers come from the following utility maximization problem:

$$Max_{X_0, X_{ji}} U_i = X_0 + \frac{\alpha_i}{\beta_i} \sum_j X_{ji} - \frac{1}{2\beta_i} \sum_j X_{ji}^2 - \frac{\gamma_i}{\beta_i} \Pi_j X_{ji}$$

Subject to the budget constraint:

$$p_0 X_0 + \sum_j p_j X_{ji} = Y$$

where Y is income.

Since the price of the numeraire good is 1, i.e. $p_0 = 1$, $X_0 = Y - \sum_j p_j X_{ji}$. Substituting X_0 back into the objective function, we have

$$Max_{X_{ji}} U_i = \frac{\alpha_i}{\beta_i} \sum_j X_{ji} - \frac{1}{2\beta_i} \sum_j X_{ji}^2 - \frac{\gamma_i}{\beta_i} \Pi_j X_{ji} + Y - \sum_j p_j X_{ji}$$

The first order condition then yields,

$$\frac{\partial U_i}{\partial X_{bi}} = MU_{bi} = p_b, \quad \frac{\partial U_i}{\partial X_{gi}} = MU_{gi} = p_g$$

Q.E.D.

2. The Proof of Equation (2.22) and Inequalities (2.23), (2.24)

$$\frac{\partial P_b^*}{\partial n} = \frac{2(\alpha\beta + \beta\gamma c_g)[(n+1)\beta^2 - \gamma^2] - 2\beta^2\{\beta[\alpha(n+1) + n\gamma c_g] + \alpha\gamma\}}{2[(n+1)\beta^2 - \gamma^2]^2}$$

Requiring $\partial P_b^* / \partial n > 0$ implies

$$2(\alpha\beta + \beta\gamma c_g)[(n+1)\beta^2 - \gamma^2] > 2\beta^2\{\beta[\alpha(n+1) + n\gamma c_g] + \alpha\gamma\}$$

or,

$$(\alpha + \gamma c_g)[(n+1)\beta^2 - \gamma^2] > \beta[\alpha\beta(n+1) + \alpha\gamma + n\beta\gamma c_g]$$

$$\Rightarrow c_g(\beta^2 - \gamma^2) > \alpha(\beta - \gamma) \Rightarrow c_g > \frac{\alpha(\beta - \gamma)}{\beta^2 - \gamma^2} = \frac{\alpha}{\beta + \gamma}$$

Alternatively, the definitions of β , γ and α can be used to express the condition as

$$\left[\frac{\theta\beta_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2}{1-\gamma_2^2} + \frac{\theta\beta_1\gamma_1}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2\gamma_2}{1-\gamma_2^2} \right] c_g > \frac{\theta\alpha_1}{1+\gamma_1} + \frac{(1-\theta)\alpha_2}{1+\gamma_2}$$

$$\Rightarrow \left[\frac{\theta\beta_1(1+\gamma_1)}{1-\gamma_1^2} + \frac{(1-\theta)\beta_2(1+\gamma_2)}{1-\gamma_2^2} \right] c_g > \frac{\theta\alpha_1}{1+\gamma_1} + \frac{(1-\theta)\alpha_2}{1+\gamma_2}$$

$$\begin{aligned}
& \rightarrow \left[\frac{\theta\beta_1}{1-\gamma_1} + \frac{(1-\theta)\beta_2}{1-\gamma_2} \right] c_g > \frac{\theta\alpha_1}{1+\gamma_1} + \frac{(1-\theta)\alpha_2}{1+\gamma_2} \\
& \rightarrow \left(\frac{\theta}{1-\theta} \right) \frac{\beta_1 c_g (1+\gamma_1) - \alpha_1 (1-\gamma_1)}{1-\gamma_1^2} > \frac{\alpha_2 (1-\gamma_2) - \beta_2 c_g (1+\gamma_2)}{1-\gamma_2^2} \\
& \rightarrow \left[\left(\frac{\theta}{1-\theta} \right) \frac{\beta_1}{1-\gamma_1} + \frac{\beta_2}{1-\gamma_2} \right] c_g > \left(\frac{\theta}{1-\theta} \right) \frac{\alpha_1}{1+\gamma_1} + \frac{\alpha_2}{1+\gamma_2}
\end{aligned}$$

3. The Proof of Equation (2.36) and Inequality (2.37)

$$Max_{P_b} \pi_b = (P_b - c_b)X_b + (P_g - c_{bg})x_{bg} = (P_b - c_b)(\overline{a'} - \overline{b'}P_b) + (P_g - c_{bg})x_{bg}$$

From the second stage of the game, we have

$$\begin{aligned}
P_g - c_{bg} &= \frac{1}{2\beta(n+1)} [\alpha + \gamma P_b + n\beta c_g + (n+1)\beta c_{bg}] - c_{bg} \\
&= \frac{1}{2\beta(n+1)} [\alpha + \gamma P_b + n\beta c_g + \beta(n+1)c_{bg} - 2\beta(n+1)c_{bg}] \\
&= \frac{1}{2\beta(n+1)} [\alpha + \gamma P_b + n\beta c_g - \beta(n+1)c_{bg}]
\end{aligned}$$

and

$$x_{bg} = \frac{1}{2}[\alpha + \gamma p_b + n\beta c_g - (n+1)\beta c_{bg}]$$

Then, the profit maximization problem becomes

$$\begin{aligned} \text{Max}_{p_b} \pi_b &= (P_b - c_b)X_b + (P_g - c_{bg})x_{bg} \\ &= (p_b - c_b)(\bar{a}' - \bar{b}'p_b) + \frac{1}{4\beta(n+1)}[\alpha + \gamma p_b + n\beta c_g - (n+1)\beta c_{bg}]^2 \end{aligned}$$

F.O.C.

$$\begin{aligned} \frac{\partial \pi_b}{\partial P_b} &= \bar{a}' - \bar{b}'P_b - \bar{b}'(P_b - c_b) + \frac{\gamma}{2\beta(n+1)}[\alpha + \gamma P_b + n\beta c_g - (n+1)\beta c_{bg}] \\ &= \frac{\alpha[2\beta(n+1) + \gamma] + \beta\gamma[nc_g + (n+1)c_{bg}]}{2\beta(n+1)} - \frac{2(n+1)\beta^2 - \gamma^2}{2\beta(n+1)}(2P_b - c_b) \\ &\quad + \frac{\gamma}{2\beta(n+1)}[\alpha + \gamma P_b + n\beta c_g - (n+1)\beta c_{bg}] = 0 \\ \rightarrow P_b^{*'} &= \frac{2[\alpha\beta(n+1) + \alpha\gamma + n\beta\gamma c_g] + [2(n+1)\beta^2 - \gamma^2]c_b}{4(n+1)\beta^2 - 3\gamma^2} \end{aligned}$$

Taking the derivative of (2.36) with respect to n yields

$$\frac{\partial p_b^{*'}}{\partial n} = \frac{2(\alpha\beta + \beta\gamma c_g + \beta^2 c_b)[4(n+1)\beta^2 - 3\gamma^2] - 4\beta^2\{2[\alpha\beta(n+1) + \alpha\gamma + n\beta\gamma c_g] + [2\beta^2(n+1) - \gamma^2]c_b\}}{[4(n+1)\beta^2 - 3\gamma^2]^2}$$

Since the denominator is positive, the sign of the above equation is determined by the sign of numerator. Hence we need look only at the sign of the numerator. Letting N represent the numerator of above equation, we have

$$\begin{aligned} N &= 8\alpha\beta^3(n+1) - 6\alpha\beta\gamma^2 + 8\beta^3\gamma(n+1)c_g - 6\beta\gamma^3c_g \\ &+ 8\beta^4(n+1)c_b - 8\alpha\beta^3(n+1) - 8\alpha\beta^2\gamma - 8n\beta^3\gamma c_g - 8\beta^4(n+1)c_b + 4\beta^2\gamma^2c_b \\ &= 2\beta\gamma c_g(4\beta^2 - 3\gamma^2) + 4\beta^2\gamma^2c_b - 2\alpha\beta\gamma(3\gamma + 4\beta) \end{aligned}$$

Now, requiring $N > 0$ produces the sufficient condition for $\partial p_b^{*'} / \partial n > 0$. This implies

$$\frac{\partial p_b^{*'}}{\partial n} > 0 \quad \text{if} \quad c_g > \frac{\alpha(4\beta + 3\gamma) - 2\beta\gamma c_b}{4\beta^2 - 3\gamma^2}$$

Q.E.D.

4. The Proof of the Inequality Condition of (2.38)

From (2.22) and (2.37), we let,

$$\frac{\alpha(4\beta+3\gamma)-2\beta\gamma c_b}{4\beta^2-3\gamma^2} < \frac{\alpha}{\beta+\gamma}$$

or,

$$\alpha(4\beta+3\gamma)(\beta+\gamma)-2\beta\gamma c_b(\beta+\gamma) < \alpha(4\beta^2-3\gamma^2) \quad \Rightarrow \quad 2\beta(\beta+\gamma)c_b > \alpha(7\beta+6\gamma)$$

$$\Rightarrow c_b > \frac{\alpha(7\beta+6\gamma)}{2\beta(\beta+\gamma)}$$

Q.E.D.

REFERENCE

- Anis H. Aslam (1992), "Pharmaceutical Prices with Insurance Coverage and Formularies", *Canadian Journal of Economics*, 25 (2).
- Anis H. Aslam and Quan Wen (1998), " Price regulation of Pharmaceuticals in Canada," *Journal of Health Economics* 17: 21-38.
- Berndt, E.R., Cockburn I.M., Cocks D.L., Estein A and Z. Griliches (1997), " Is Price Inflation Different for the Elderly? An Empirical Analysis of Prescription Drugs", *NBER Working Paper* (6182), Sept. 1997.
- Bobula Joel D.(1996), "A New Era in Pharmaceutical Pricing", *Journal of Research in Pharmaceutical Economics*, 7(1-2).
- Caves R.E., Whinston M.D., and Hurwitz M.A. (1991), "Patent Expiration, Entry and Competition in the U.S. Pharmaceutical Industry: An Exploratory Analysis," *Brookings Papers on Economic Activity*.
- Charles E. Phelps (1997), *Health Economics*, addition-Wesley Educational Publishers Inc.
- Comanor W.S.(1986), "The Political Economy of the Pharmaceutical Industry", *Journal of Economic Literature*.
- Eastman Harry C. (1994), " Pharmaceutical Price Review in Canada", *Pharmaco Economics*, 5 (4).
- Elzinga Kenneth G. And Mills David E. (1997), " The Distribution and Pricing of Prescription Drugs", *International Journal of the Economics of Business*, 4(3).
- Frank R.G., and Salkever D.S. (1992), " Pricing Patent Loss and the Market for Pharmaceutical", *Southern Economic Journal*.
- Frank R.G., and Salkever D.S. (1995), "Generic Entry and the Pricing of Pharmaceutical", *NBER Working Paper* (5306), Oct. 1995.
- Grabowski H. and Vernon J. (1996), " Brand Loyalty, Entry and Price Competition in Pharmaceutical After the 1984 Drug Act", *Journal of Law and Economics*.
- Hellerstein J. K. (1994), "The Demand for Post-patent Prescription Pharmaceutical", *NBER Working Paper* (4981), Dec. 1994.
- Hudson John (1992), "Pricing Dynamics in the Pharmaceutical Industry", *Applied Economics*, 24(1).

IMS of Canada, Ltd., *Canadian Pharmaceutical Market: Drug Store and Hospital Purchases*, December 1997.

Jambulingam Thanigavelan and Kreling David H.(1995),” Relationship between the Number of Firms and Generic Pricing Competition”, *Journal of Research in Pharmaceutical economics*, 6(3).

Jesse J. Friedman (1981), *Pharmaceutical Prices in Canada: guiding principles for government policy*, Jesse J Friedman & Associates Economic Consultants, Washington, D.C.

Lu, Z. John, and William S. Comanor (1998), “Strategic Pricing of New Pharmaceutical”, *The Review of Economics and Statistics*, Vol. LXXX.

Martin Shubik and Richard Levitan (1980), *Market Structure and Behaviour*, Harvard University Press, Cambridge, Massachusetts.

Mullins C. Daniel (1995), “Toward an Understanding of Pharmaceutical Pricing Strategies through the Use of Simple Game Theoretic Models”, *Journal of Research in Pharmaceutical Economics*, 6(3).

Office of Technology Assessment (OTA), *Pharmaceutical R&D Costs, Risks and Rewards*, U.S.G.P.O. 1993.

Patented Medicines Prices Review Board, 1988. Bull.: the first issue.

Patented Medicines Prices Review Board, 1993. International Price Comparison Studies.

Patented Medicines Prices Review Board, 1988. Bull.: Issue No. 12.

Peter Zweifel and Friedrich Breyer (1997), *Health Economics*, Oxford University Press.

Philip Jacobs (1997), *The Economics of Health and Medical Care*, Aspen Publishers, Inc.

Scherer F. M. (1993), “Pricing, Profits, and Technological Progress in the Pharmaceutical Industry”, *Journal of Economic Perspectives*, 7(3).

Schweitzer Stuart O. (1997), *Pharmaceutical Economics and Policy*, Oxford University Press.

Shapiro Daniel M. And Switzer Lorne N.(1993), “ The Stock Market Response to Changing Drug Patent Legislation: The Case of Compulsory Licensing in Canada”, *Managerial and Decision Economics*, 14(3).

Shulman Sheila R. (1994), “ the Canadian Patented Medicine Prices Review Board: New Rules and New Status” , *Pharmaco Economics*, 6 (0).

Wagner, J. L. and B. M. Duffy (1988), "Containing the Costs of Prescription Drugs: The U.S. Experience", paper presented at the Symposium: Controlling Costs While Maintaining Health. Bonn.

Wiggins S.N., and Maness R. (1994), "Price Competition in Pharmaceutical Markets", *Working Paper*, Department of Economics, Texas A&M University.

CHAPTER 3

DUMPING IN OLIGOPOLY MARKETS: A WELFARE ANALYSIS

I. INTRODUCTION

In Article VI of the *General Agreements on Tariffs and Trade (GATT)*, “dumping” is defined as the offer of a product for sale in an export market at a price below its ‘normal’ or ‘fair’ market value, where this value is usually defined as the price charged by the firm in its home market. In the absence of a domestic price (a home market), either the highest comparable price charged in a third market or the exporting firm’s estimated cost of production may be used to determine the normal value. There are basically two types of dumping in international trade: price-based dumping and cost-based dumping. In the case of the price-based dumping, dumping margins are calculated by comparing the export price of a product with the price for the same good when consumed in the home market. In the case of the cost based dumping, the export price is compared with a fair-value standard calculated by estimating its cost of production, to which a markup for profits and administration expenses is added. When the foreign industry has excess capacity, the actual price of both domestic sales and exports sold to the importing country might be below full cost. Hence, dumping can arise even when the overseas and the import price are the same.¹ Importing countries tend to accept the self-serving

---

¹ For a theoretical analysis of this version of dumping, see Wilfred Ethier, ‘Dumping,’ *Journal of Political Economy*, 90 (June 1982): 487-506.

perception that dumping is either 'unfair' to them and/or is a predatory move designed to drive them out of business. Tivig and Walz (2000) provide a more thorough discussion of definitions of dumping. In order to offset or prevent dumping and protect its domestic industries, *GATT* allows a country to levy an anti-dumping duty on any dumped product. The implication is that dumping imposes costs on the recipient country.

Economists, in general, are suspicious of the economic benefits of such a duty. A consensus opinion among economists appears to be that dumping improves (rather than reduces) social welfare in the home country by allowing domestic consumers to buy foreign goods at lower prices. As long as the exporter finds that overseas markets are more competitive than the protected domestic market, the export price will be set lower than the price charged to customers in the exporter's home country and even lower than it would be in the absence of discrimination. The importing country therefore benefits from a lower price. If this were the whole story, however, the importing country should welcome dumping. The puzzle for economists is why anti-dumping laws persist when it would appear that dumping improves the home country's welfare. Such a theory, however, requires the domestic industry in importing countries be perfectly competitive.

In the past twenty years, economists have developed analyses of the effects of dumping on an imperfectly competitive domestic industry that focus almost entirely on anti-dumping actions (e.g., tariffs or subsidies). The welfare analysis of dumping itself is only implicit. Webb (1992), for example, built a duopoly model where each country has a firm that sells in both the domestic and the foreign market. He concludes that under Cournot behaviour, anti-dumping duties will have an ambiguous

price and welfare effect on the home country. Dixit's (1987) oligopoly model, where both domestic and foreign firms sell in the home market, examines whether imperfect competition could justify on the aggregate normative grounds the policies of anti-dumping and countervailing duties. When home goods are homogenous, his model implies that the home government can alter equilibrium through two policy instruments — a production subsidy to the domestic firms or a tariff on imports (or both). Day and Mohanty (1987) studied the effect on the welfare of the exporting (dumping) country. They found that dumping is a form of price discrimination that causes a domestic consumption distortion in the exporting country. Thus here is an economic justification for consumers in the dumping country to be concerned about the act of dumping.

The most recent study of dumping's welfare effects is that of Anderson, Schmitt and Thisse (1995). They construct a duopoly model with one domestic and one foreign firm. Under both Bertrand and Cournot competition, they find that any government whose objective was to maximize total domestic surplus would prefer to have no legislation, regardless of the action of the other government. In their model, the enforcement of anti-dumping legislation reduces total welfare because it increases in domestic price. This reduces the sum of consumer surplus and domestic profit in the home country.

In our review of the existing literature on the welfare effects of dumping, we have found that most authors base their analysis on the assumption that there is only one domestic firm. Even though Dixit has an oligopoly model, his study focusses on anti-dumping actions rather than on the relationship between market structure and changes of welfare. Our model differs from the existing literature by explicitly analysing dumping's welfare effects and by assuming that the domestic import-competing

industry is a homogeneous or differentiated oligopoly. Our goal is to study dumping's net benefits or net loss for the home country.

Instead of focusing on the analysis of anti-dumping duties, our interest is to determine when dumping can negatively affect the home country's social welfare in the absence of any anti-dumping duties. Therefore, we take dumping as a given and analyse its welfare effects when the import-competing industry is oligopolistic. We demonstrate that (i) the number of domestic firms and, (ii) the level of differentiation of the domestic products are crucial in determining the sign of the change in the home country's welfare once dumping occurs. When the home market is oligopolistic, the home firm competes with all other firms along with participating in the competition between home goods and imported rivals, since the profits made by the domestic industry, and thus the total welfare, will vary with the level of competition among home firms. Furthermore, in the case of differentiated home produced goods, the domestic firms' competitive behaviour depends on the degree of substitutability among home goods.

The analysis of this chapter is organized as follows. Section II considers a basic model. Section III focuses on the welfare effects when the domestic industry produces a homogenous product. Section IV modifies that model to consider differentiated products. Section V provides concluding remarks.

II. THE BASIC MODEL

We assume that consumers in the home country are identical and consume three goods. One good is produced in a foreign country and exported to the home country. Its quantity is denoted as Q_f . The other two goods are produced domestically and we denote their respective quantities as Q_h and Q_o . The good Q_h is an imperfect substitute for Q_f and the good Q_o is taken to be the numeraire good. We assume that labour is the only factor of production in the home country and can be purchased in a competitive market at a constant wage rate. All home country producers of Q_h are assumed to have the same cost function. Finally the market in which the numeraire good Q_o is produced is assumed to be perfectly competitive.

1. *The Model with Homogeneous Home Produced Goods*

We consider a model where the utility function of a representative consumer is given by

$$U = Q_o + u(Q_h, Q_f)$$

The demand functions for the product Q_h and product Q_f are determined by ²:

$$\frac{\partial u(Q_h, Q_f)}{\partial Q_h} = P_h, \quad \frac{\partial u(Q_h, Q_f)}{\partial Q_f} = P_f \quad (3.1)$$

---

² See the Appendix at the end of the chapter for the proof.

where, P_h and P_f represent the price of the home produced good Q_h and the price of the imported good Q_f respectively.

Home country welfare is defined as the sum of the social value of consumption and income minus the amount paid to the foreign producer and the domestic cost of production. Welfare can be written as:

$$W = U - P_f \cdot Q_f - C(Q_h) + y \quad (3.2)$$

where all citizens possess identical tastes and wage income y , and $C(Q_h)$ is the total cost function of industry h in the home country ³.

Since, according to *GATT* dumping occurs when "a product is to be considered as being introduced into the commerce of an importing country at less than its normal value", dumping is reflected in our framework as a fall in price of the imported goods in a certain time period. Hence, the change in the home country's welfare depends on the sign of the derivative of W with regard to P_f . According to (3.2), we obtain

$$\frac{\partial W}{\partial P_f} = \frac{\partial u}{\partial Q_h} \cdot \frac{\partial Q_h}{\partial P_f} + \frac{\partial u}{\partial Q_f} \cdot \frac{\partial Q_f}{\partial P_f} - Q_f - P_f \frac{\partial Q_f}{\partial P_f} - \frac{\partial C(Q_h)}{\partial Q_h} \cdot \frac{\partial Q_h}{\partial P_f} = -Q_f + (P_h - C') \frac{\partial Q_h}{\partial P_f} \quad (3.3)$$

where C' is the marginal cost for industry h .

---

³. See the Appendix at end of the chapter for the proof.

When the home produced good market is a competitive market, the equilibrium price is equal to the marginal cost. Then, the change in social welfare becomes:

$$\frac{\partial W}{\partial P_f} = -Q_f < 0 \quad (3.4)$$

Equation (3.4) brings the well-known proposition,

PROPOSITION 3.1

The total welfare of the home country will always increase with dumping when the home good market is perfectly competitive.

The first term of Equation (3.3) reflects the change in consumer surplus in imported good market which always increases when price P_f decreases. The second term of Equation (3.3) reflects the change in the total welfare in the home market which decreases with the fall in Q_h as long as P_h is greater than marginal cost C' . Then, the explanation of Proposition 3.1 is obvious. Since there is no welfare loss in the competitive home market when all firms have marginal cost equal to price, the change of home country's total welfare depends only on consumer surplus in the imported good market. As foreign goods become cheaper, domestic consumers are made better off.

When the home market is imperfectly competitive, the change in the home country's total welfare depends on the home firms' price setting strategy and the change in the quantity of home production associated with the price change of imported good. If these changes cause the second term of

Equation (3.3) to be greater than the first, then total welfare falls with a fall in P_f . This implies the following proposition for the welfare effects following dumping when the home market is imperfectly competitive.⁴

PROPOSITION 3.2

The effect of dumping on the social welfare of the home country depends on the cross elasticity of demand for the home good with respect to the price of imported good when the home produced good market is imperfectly competitive. To be more specific, dumping will increase the home country's welfare if and only if the cross elasticity of demand is less than the ratio of the expenditure shares on the imported good and the home produced good relative to the Lerner Index, i.e.,

$$\frac{\partial W}{\partial P_f} < 0 \quad \text{iff} \quad \epsilon_{hf} < \frac{P_f Q_f / P_h Q_h}{(P_h - C_h) / P_h}$$

According to Proposition 3.2, the direction of the change in social welfare (in both the imported goods market and the home produced goods market) depends on the cross elasticity of demand for the home produced goods with respect to the imported good's price. When P_f drops, we can compare the welfare changes in two segment markets: the home produced good market and the imported good market. Generally, there is a welfare loss in the home produced good market due to the reduction in

---

⁴See the Appendix at end of the chapter for the proof.

the home good production while there is always a consumer surplus gain in the imported good market. If the cross elasticity of demand for the home produced good is small, the loss in the home produced good market is smaller than the gain in the imported good market. In this case, there is a welfare gain; otherwise, there is a welfare loss.

2. The Model with Differentiated Home Produced Goods

Suppose now that consumers view the different home goods produced as differentiated from each other in the market. In order to simplify, we also assume that domestic firms face a constant marginal cost. The aggregate utility function now becomes

$$U = Q_0 + u(q_1, \dots, q_n, Q_f) \quad (3.5)$$

where q_i represents the output level of firm i in home produced goods market.

Equation (3.5) generates the following inverse demand functions.

$$P_f = \frac{\partial u(\cdot)}{\partial Q_f}, \quad P_{h_i} = \frac{\partial u(\cdot)}{\partial q_i} \quad (3.6)$$

where P_{h_i} is the price of product q_i .

Using the same methodology as in the homogeneous case, social welfare can be written as the following.

$$W = U - P_f Q_f - \sum_{i=1}^n C(q_i) + y \quad (3.7)$$

Then, the change in social welfare resulting from a decrease in the imported good price is shown by Equation (3.8).

$$\begin{aligned} \frac{\partial W}{\partial P_f} &= \sum_{i=1}^n \frac{\partial u(\cdot)}{\partial q_i} \cdot \frac{\partial q_i}{\partial P_f} + \frac{\partial u(\cdot)}{\partial Q_f} \cdot \frac{\partial Q_f}{\partial P_f} - P_f \frac{\partial Q_f}{\partial P_f} - Q_f - \sum_{i=1}^n \frac{\partial C(q_i)}{\partial P_f} \\ &= -Q_f + \sum_{i=1}^n (P_{h_i} - C_i') \frac{\partial q_i}{\partial P_f} \end{aligned} \quad (3.8)$$

The first term in Equation (3.8) again represents the gain in consumer surplus from the imported good market that follows the decrease in P_f . The second term represents the total surplus change in the home good market which generally is negative for a decrease in P_f . Once again, if the first term in Equation (3.8) is less than the second term in the equation, there will be a welfare loss under dumping. This produces the following proposition.

PROPOSITION 3.3

In the case of differentiated home products, dumping can reduce the home country's social welfare if the sum of the product of each home firm's Lerner Index, its sales, and its cross elasticity of demand exceeds total expenditures on the foreign good, i.e.,

$$\frac{\partial W}{\partial P_f} > 0 \quad \text{if} \quad \sum_{i=1}^n \frac{P_{h_i} - C'_i}{P_{h_i}} (P_{h_i} \cdot q_i) \varepsilon_{h,f} > P_f Q_f$$

Otherwise, dumping will increase the home country's welfare.

The proof follows the proof of Proposition 3.2.

Both Proposition 3.2 and 3.3 show the general condition for the effect of dumping on total welfare in the home country. To derive more specific conditions for when welfare will increase or decrease with a fall in the foreign price, P_f , we need to specify a particular utility function and the type of competitive behaviour engaged in by home producers. This is done in the next two sections.

III. THE LINEAR DEMAND WITH HOMOGENEOUS HOME PRODUCED GOODS

In this section, we take a close look at the relationship between the number of the domestic firms and changes in the total welfare when the price of the imported products decreases. We assume that the consumer's utility function is given as:

$$U = Q_0 + \frac{1}{b^2 - d^2} [a(b+d)(Q_h + Q_f) - \frac{b}{2}(Q_h^2 + Q_f^2) - dQ_h Q_f] \quad (3.9)$$

The utility function is chosen as quadratic in Q_h , Q_f and linear in Q_0 so that there are linear demand functions with no income effects. The demand functions for good Q_h and good Q_f are:

$$Q_h = a - bP_h + dP_f \quad Q_f = a - bP_f + dP_h \quad (3.10)$$

Here, b and d can be viewed as factors of own- and cross-price elasticity of demand. If $d=0$, the market demands for the two goods are independent. As d rises, the products become closer substitutes and the total demand becomes more elastic. We then impose $a > b > d > 0$ to assure that the products Q_h and Q_f are imperfect substitutes for each other and the own-price effect is larger than the cross-price effect.

On the supply side, we assume that there are n home firms with the same increasing marginal cost

competing in quantity in the home produced goods market, i.e. they play a Cournot competition game taking the price of the imported goods P_f as given. The total cost function of each home firm is as:

$$C = \frac{1}{2}cq_i^2 \quad \text{where } c \geq 0 \text{ and } i=1,2,3,\dots,n$$

Then, the firm's marginal cost is $C' = cq_i$.

The reaction function of firm i comes from

$$r_i(q_{-i}) = \operatorname{argmax}[P_h \cdot q_i - \frac{1}{2}cq_i^2] = \operatorname{argmax}[\frac{1}{b}(a + dP_f - Q_h) \cdot q_i - \frac{1}{2}cq_i^2] \quad (3.11)$$

where, q_i is the output of firm i . Since all home firms have the same cost function, Equation (3.11) generates the symmetric output level in *Nash equilibrium*, i.e., $q_i = q^c = Q_h^c / n$. Then,

$$q^c = \frac{a + dP_f}{n+1+bc},$$

and

$$Q_h^c = nq^c = \frac{n(a + dP_f)}{n+1+bc} \quad (3.12)$$

From (3.12), we have

$$\varepsilon_{hf} = \frac{\partial Q_h^c}{\partial P_f} \cdot \frac{P_f}{Q_h^c} = \frac{dP_f}{a + dP_f} \quad (3.13)$$

and

$$P_f = \frac{1}{nd} [na - (n+1+bc)Q_h^c] \quad (3.14)$$

From the demand function (3.10), we have

$$P_f = \frac{1}{d} (Q_h^c - a + bP_h^c) \quad (3.15)$$

Combining (3.13) and (3.15), we have the "Cournot Expansion Path (CEP)" :

$$P_h^c = \frac{(1+bc)}{nb} \cdot Q_h^c = \frac{1+bc}{b(n+1+bc)} (a + dP_f) \quad (3.16)$$

The Cournot Expansion Path gives the relationship between the Cournot equilibrium price P_h^c and the imported goods price P_f , holding the number of home firms output. Equation (3.16) illustrates clearly how the price of home produced goods changes under dumping when the domestic market is oligopolistic.

Since the firm's marginal cost can be written as⁵:

$$C' = cq^c = c \cdot \frac{Q_h^c}{n}$$

Equation (3.16) can be written as:

$$P_h^c - C' = \frac{Q_h^c}{nb} = \frac{a + dP_f}{b(n+1+bc)} \quad (3.17)$$

where the right hand equality uses (3.10). Substituting (3.16) into (3.10), we have

$$Q_f = a - bP_f + dP_h^c = a - bP_f + \frac{d(1+bc)(a+dP_f)}{b(n+1+bc)} \quad (3.18)$$

Taking (3.13), (3.17) and (3.18), Proposition 3.2 implies that $\Delta W / \Delta P_f > 0$ if

$$Q_f > \frac{nd}{n+1+bc} (P_h^c - cq) = \frac{nd(a+dP_f)}{b(n+1+bc)^2} \quad (3.19)$$

or

$$a - bP_f + \frac{d(1+bc)(a+dP_f)}{b(n+1+bc)} > \frac{nd(a+dP_f)}{b(n+1+bc)^2}$$

---

⁵ Since domestic firms all have the same cost function, the marginal cost of the industry is equal to the firm's marginal cost.

$$\begin{aligned}
&\Rightarrow b(n+1+bc)^2(a-bP_f)+(a+dP_f)[d(1+bc)(n+1+bc)-nd]>0 \\
&\Rightarrow b(n+1+bc)^2(a-bP_f)+(a+dP_f)[d(1+bc)+bdc(n+1+bc)]>0
\end{aligned} \tag{3.20}$$

Since $a > bP_f$, (3.20) always holds given any value of n . This situation implies that the home country's welfare always increases when P_f decreases. This result also can be extended to the case of constant marginal cost if we let c in (3.20) to be equal to zero which implies

$$b(n+1)^2(a-bP_f)+d(a+dP_f)>0 \tag{3.21}$$

The inequalities (3.20) and (3.21) produce the following corollary.

COROLLARY 3.1

When home firms produce a homogenous product under either increasing or constant marginal cost, dumping always increases the home country's social welfare whether or not the home market is imperfectly competitive.

The result for competitive markets was illustrated earlier. The case for imperfectly competitive markets is illustrated in Figure 3.1 that illustrates the change in consumer surplus in the imported good market. When price P_f drops from P_n to P_p , the consumption level of consumers will increase from Q_n to Q_p . This results a relative price effect along the same demand curve along with an

offsetting change in quantity demanded appearing as a shift in the demand curve because Q_f and Q_h are imperfectly substitutable for each other. The increase in consumer surplus is represented by the area $P_f ABCP_{f2}$. Figure 3.2 illustrates the change in total surplus in the home produced good market. The home good price and production level fall from P_{h1} to P_{h2} and from Q_{h1} to Q_{h2} when the foreign price P_f decreases. The reduction of total surplus in the home market is shown in the area $DFGH$ in Figure 2. Hence, the changes of the total welfare in both markets are equivalent to the difference between the value of area $P_f ABCP_{f2}$ and the value of area $DFGH$, i.e. $\Delta W = P_f ABCP_{f2} - DFGH$. It can be shown that the value of welfare represented by area $P_f ABCP_{f2}$ is always greater than that represented by area $DFGH$. Hence, social welfare always increases⁶.

---

⁶. See the Appendix at end of the chapter for the proof.

FIGURE 3.1 Changes in Consumer Surplus in the Imported Good Market

FIGURE 3.2 Changes in Total Surplus in Home Produced Good Market

In addition, the area $DFGH$ in the home good market will depend on the number of domestic firms. Given the price of imported goods, the area $DFGH$ increases when n increases. Then, to determine how changes in scale change welfare, we take the derivative of the change in welfare with respect to P_f with respect to the changes in n ⁷. That is from

$$\frac{\partial W}{\partial P_f} = Q_f + (P_d - C_f) \frac{\partial Q_d}{\partial P_f} = - \left[a - bP_f + \frac{d(1+bc)(a+dP_f)}{b(n+1+bc)} \right] + \frac{nd(a+dP_f)}{b(n+1+bc)^2} < 0$$

we find,

$$\frac{\partial^2 W}{\partial P_f \partial n} = \frac{d(a+dP_f)[2+(3+n+bc)bc]}{b(n+1+bc)^3} > 0 \quad (3.22)$$

Inequality (3.22) implies that the total welfare will increase in a large scale when the number of firms n decreases, and the total welfare will increase in a small scale when the number of firms n increases. This is shown in Figure 3.3 and 3.4 and brings to us another corollary.

COROLLARY 3.2

Given the price of P_f , the increase in home country's total welfare resulted from dumping will be relatively large when the home produced good market is relatively concentrated.

---

⁷See the Appendix at end of the chapter for the proof.

FIGURE 3.3 The Change in the Total Welfare in the Home Good Market As n Increases

FIGURE 3.4 The Change in Total Welfare As a Function of the Number of the Home Firms

It is interesting to note how the roles of own price elasticity and the cross elasticity of demand for the home produced goods affect the results. Since any change in elasticity factor b or cross elasticity factor d will cause a change in the shape of demand curves (either a slope or an intercept), the surplus in both markets will be different with different b 's and d 's. The change in welfare with the changes of b and d is shown as⁸ :

$$\frac{\partial^2 W}{\partial P_f \partial b} = P_f + \frac{bcd[nbc + (d+bc)(bc+2)] + (n+1)d}{b^2(n+1+bc)^3} > 0 \quad (3.23)$$

$$\frac{\partial^2 W}{\partial P_f \partial d} = - \frac{(a+2dP_f)[nbc + (1+bc)^2]}{b(n+1+bc)^2} < 0 \quad (3.24)$$

Inequalities (3.23) and (3.24) gives us Corollary 3.3 as follows.

COROLLARY 3.3

The increase in the home country's total welfare resulted from dumping will be relatively large:

- (1) the more inelastic is the demand for home produced goods, or*
- (2) the larger is the cross elasticity of demand between the home produced goods and the imported goods.*

The increase in b can be seen from Figure 3.5. A large elasticity of home produced goods implies that the

---

⁸See the Appendix at the end of the chapter for the proof.

slopes of both CEP and the demand curve will be relatively flat. Therefore, if we compare the total welfare loss due to dumping, we will find the area $DFGH$ for the elastic demand curve is larger than area $D'F'G'H'$, representing the case of inelastic markets. Thus, the welfare loss in the home good market will be smaller. A similar analysis can be done in the imported market where there is a relatively large increase in consumer surplus. On the other hand, when d increases, the demand curve in both the home good and the imported good markets will shift further inward. Therefore, there is a relatively large increase in the consumer surplus in the imported good market and a relatively small decrease in the total surplus in the home produced good market.

FIGURE 3.5 Changes in the Total Welfare in the Home Produced Good Market When b Changes

IV. THE LINEAR DEMAND WITH DIFFERENTIATED HOME PRODUCED GOODS

Since products can be differentiated in many dimensions, we assume that consumers' preferences can be represented by a quadratic utility function of n domestic goods $q_1, \dots, q_n$, one imported good Q_f and one numeraire good Q_0 .

$$U = \alpha(q_1 + \dots + q_n + Q_f) - \frac{1}{2}[\beta(Q_f^2 + \sum_{i=1}^n q_i^2 + 2\theta \sum_{i=1}^n q_i Q_{-hi}) + 2\gamma Q_f \sum_{i=1}^n q_i] + Q_0 \quad (3.25)$$

where $\alpha > \beta > \gamma$ and

$$Q_{-hi} = \sum_{j \neq i} q_j$$

Equation (3.25) yields the following demand functions.

$$P_f = \alpha - \beta Q_f - \gamma \sum_{i=1}^n q_i, \quad P_{hi} = \alpha - \beta(q_i + \theta Q_{-hi}) - \gamma Q_f \quad (3.26)$$

where P_{hi} is the price of good q_i . Then we have

$$Q_f = \frac{1}{\beta}(\alpha - P_f - \gamma \sum_{i=1}^n q_i)$$

Substitute the above equation about Q_i in the demand function of P_{hi} , it yields

$$P_{hi} = \alpha - \beta(q_i + \theta Q_{-hi}) - \frac{\gamma}{\beta}(\alpha - P_f - \gamma \sum_{i=1}^n q_i) = \alpha - \frac{\gamma}{\beta}(\alpha - P_f) - \beta(q_i + \theta Q_{-hi}) + \frac{\gamma^2}{\beta} \sum_{i=1}^n q_i$$

The parameter θ measures the degree of product differentiation for the home produced goods. If $\theta=0$, demands are independent. Each home producer is then a monopolist of its own brand within the home produced good market. If $\theta=1$, and the products are homogeneous, we are back to the cases discussed in the previous section. Generally, we assume $0 < \theta < 1$ which means that home produced goods are imperfectly substitutable for each other.

Now, we still assume that n home goods producers play a Cournot game and take price P_f as given. In order to simplify the case, we assume that domestic firms have a constant marginal cost, c . Then, the reaction function of firm i comes from

$$r_i(q_{-i}) = \argmax [P_{hi} \cdot q_i - cq_i] = \argmax [\alpha - \frac{\gamma}{\beta}(\alpha - P_f) - \beta(q_i + \theta Q_{-hi}) + \frac{\gamma^2}{\beta} \sum_{i=1}^n q_i - c] q_i$$

It generates

$$q_i = \frac{1}{2(\beta^2 - \gamma^2)} [\alpha\beta - \gamma(\alpha - P_f) - \beta^2\theta Q_h + \gamma^2 \sum_{j \neq i} q_j - \beta c] \quad (3.27)$$

Equation (3.27) is a linear function. There is only one equilibrium, namely the symmetric single-period Nash equilibrium in which $q_i = q^c$ and $P_h = P_h^c$. In this case, we solve n reaction functions and have

$$q^c = \frac{\beta(\alpha - c) - \gamma(\alpha - P_f)}{\rho} \quad (3.28)$$

where

$$\rho = \beta^2[2 + (n-1)\theta] - (n+1)\gamma^2$$

If θ is not so small, then the value of ρ is greater than zero, i.e. $\rho > 0$ if $\theta > \underline{\theta}$, where

$$\underline{\theta} = \max \left\{ 0, \left(\frac{n+1}{n-1} \right) \frac{\gamma^2}{\beta^2} - \frac{2}{n-1} \right\}$$

The total output of the home produced goods is

$$Q_h^c = nq^c = \frac{n[\beta(\alpha - c) - \gamma(\alpha - P_f)]}{\rho} \quad (3.29)$$

Solving P_f from (3.29)

$$P_f = \frac{\rho Q_h^c}{n\gamma} - \frac{\beta}{\gamma}(\alpha - c) + \alpha \quad (3.30)$$

From (3.25) , we have

$$P_f = \alpha - \frac{\beta}{\gamma}(\alpha - P_h) + \frac{Q_h^c}{n\gamma}[\beta^2 + (n-1)\theta\beta^2 - n\gamma^2] \quad (3.31)$$

Combining (3.30) and (3.31), we have the Cournot Expansion Path.

$$P_h^c = c + \frac{Q_h^c}{n\beta}(\beta^2 - \gamma^2) = c + \frac{[\beta(\alpha - c) - \gamma(\alpha - P_f)](\beta^2 - \gamma^2)}{\beta\rho} \quad (3.32)$$

From (3.25) and (3.29),

$$Q_f = \frac{1}{\beta}(\alpha - P_f - \gamma Q_h^c) = \frac{(\alpha - P_f)\rho - n\gamma[\beta(\alpha - c) - \gamma(\alpha - P_f)]}{\beta\rho} \quad (3.33)$$

$$\frac{\partial Q_h^c}{\partial P_f} = \frac{n\gamma}{\beta^2[2 + (n-1)\theta] - (n+1)\gamma^2} = \frac{n\gamma}{\rho} \quad (3.34)$$

Then, we can apply Proposition 3.3 to (3.32), (3.33) and (3.34), i.e. $\partial W / \partial P_f > 0$ if⁹,

---

⁹See the Appendix at end of the chapter for proof.

$$P_f > \varphi \quad (3.35)$$

where

$$\varphi = \alpha - \frac{m\gamma\beta(\alpha - c)(\beta^2 - \gamma^2 + \rho)}{\rho(\rho + m\gamma^2) + m\gamma^2(\beta^2 - \gamma^2)}$$

Inequality (3.35) brings us a corollary as follows.

COROLLARY 3.4

When home produced goods are differentiated, dumping will reduce the home country's social welfare as long as the foreign price is relatively high, i.e. $P_f > \varphi$.

The explanation of Corollary 3.4 is straight forward. When reduction of P_f is small, the consumer surplus gain is relatively small in the imported good market. In the home produced good market, the reduction in P_h is also small due to P_f decrease. Therefore, the total welfare of the home country decreases.

Inequality (3.35) can be rewritten as

$$(\alpha - P_f)\rho^2 - m\gamma\varphi\rho < m\gamma\varphi(\beta^2 - \gamma^2) \quad (3.36)$$

where $\varphi = \beta(\alpha - c) - \gamma(\alpha - P_f)$. The sufficient condition for the existence of Inequality (3.36) is $\varphi > 0$. It implies

$$P_f > \alpha - \frac{\beta(\alpha - c)}{\gamma} \quad (3.37)$$

From Inequality (3.36), we have

$$\rho < \frac{n\gamma\varphi + [(n\gamma\varphi)^2 + 4n\gamma\varphi(\beta^2 - \gamma^2)(\alpha - P_f)]^{1/2}}{2(\alpha - P_f)} \quad (3.38)$$

Taken the definition of ρ , (3.38), it implies that $\theta < \theta^*$ where

$$\begin{aligned} \theta^* &= \frac{1}{(n-1)\beta^2} \left\{ \frac{n\gamma\varphi + [(n\gamma\varphi)^2 + 4n\gamma\varphi(\beta^2 - \gamma^2)(\alpha - P_f)]^{1/2}}{2(\alpha - P_f)} + (n+1)\gamma^2 - 2\beta^2 \right\} \\ &= \frac{1}{(n-1)\beta^2} \left\{ \frac{n\gamma\varphi + [(n\gamma\varphi)^2 + 4n\gamma\varphi(\beta^2 - \gamma^2)(\alpha - P_f)]^{1/2}}{2(\alpha - P_f)} \right\} + \underline{\theta} \end{aligned} \quad (3.39)$$

The discussion above brings us another corollary.

COROLLARY 3.5

Given the number of the domestic firms, dumping will reduce the total welfare if the degree of substitution among the home produced goods is relatively low in the sense that $\underline{\theta} < \theta < \theta^*$.

Next, we discuss the relationship between the magnitude of the changes in welfare and the degree of substitution. From Proposition 3.3, we have the equation that explains the welfare change in response to the changes in the foreign price P_f and substitution factor θ ¹⁰,

$$\frac{\partial W}{\partial P_f \partial \theta} = - \frac{(n-1)\beta n \gamma \phi}{\rho^2} \left[\frac{2(\beta^2 - \gamma^2)}{\rho} + 1 \right] < 0 \quad (3.40)$$

Inequality (3.40) implies that the total welfare decreases in a smaller scale when θ increases. These situations can be summarized in Figure 3.6 and Corollary 3.6

COROLLARY 3.6

In the case where $\underline{\theta} < \theta < \theta^*$, the lower the degree of substitution among the home produced goods, the greater the reduction in welfare from dumping.

Corollary 3.6 can be understood by considering the changes of welfare in both the imported good market and the home produced good market. In the above case, the welfare gain in the imported good market (the area $P_f ABC P_f$ in Figure 3.1) is less than the welfare loss in the domestic good market when θ is small. From Figure 3.7, we understand that the changes of substitution factor θ will affect CEP . As θ decreases, the equilibrium price of P_h will be relatively high. Then, the welfare loss in the domestic good market is large.

---

¹⁰See the Appendix at end of the chapter for proof.

FIGURE 3.6 Changes of the Total Welfare in the Case of $\underline{\theta} \cdot \theta \cdot \theta^*$ given n

FIGURE 3.7 Changes of the Total Welfare when θ Decreases in the Domestic Good Market

V. CONCLUDING REMARKS

This chapter demonstrates the important way in which the value of the cross elasticity of demand between home produced goods and imported goods will affect the change in the total welfare that follows dumping, especially for the case of differentiated home goods. With a linear marginal cost function, dumping cannot reduce the total welfare in an oligopolistic home market if its product is homogenous. However, total welfare in the oligopolistic home market can be reduced if the products are sufficiently differentiated.

Two other findings of this chapter are of interest:

(1) The degree of substitution among home produced goods plays a key role in determining the size of the change in total welfare. If the domestic goods are less substitutable for each other, the price set for home goods will be relatively high. Then the welfare loss in the home market can be larger than the gain in consumer surplus in the imported good market.

(2) In the case of a homogenous home market, the increase in the home country's welfare generated by the reduction in the foreign price will be larger if the home market is the more concentrated.

To summarize, this chapter analyses the effects on total welfare of dumping activities when the home country's market is imperfectly competitive. The model provides a more general treatment of dumping than the existing analysis of the two extreme forms of market structure: monopoly or competitive markets. The

results highlight the importance of the market structure in determining dumping's welfare effects and provide a reference point for policy makers to evaluate dumping's effects on the domestic economy.

It should be noted that in this analysis, dumping is assumed to have taken place and so arises for reasons exogenous to the analysis. The reason for doing this is to simplify the analysis. We recognize that in general the welfare effects of dumping may be related to a firm's decision to engage in dumping both can depend on the demand function and the market structure within the home country. For example, suppose we follow the tradition of Viner (1966) and view dumping as a form of price discrimination in international trade. As such, it occurs whenever the conditions for separated markets and imperfect competition are met. Monopolist generally set prices above marginal costs and price differently on markets with different demand elasticities: if domestic demand is less elastic than world demand because a domestic monopolist meets competition of other producers in foreign markets, domestic prices will be higher than world prices, provided that markets are separated. From this point of view, dumping is less likely to be observed when the cross elasticity of demand is small. Notice that these conditions that make dumping likely are also those under which dumping reduces welfare in our model. Therefore, putting the dumping decision as an endogenous variable in our model will likely generate more interesting results. This is a project worthy of future research.

APPENDIX

1. The Proof of Demand Functions

The demand function for the domestic consumers comes from the following utility maximization problem:

$$\text{Max}_{Q_h, Q_f, Q_o} U = Q_o + u(Q_h, Q_f)$$

Subject to the budget constraint: $P_o Q_o + P_f Q_f + P_h Q_h = y$, where y is income. Since the price of the numeraire good is normalized to 1, i.e. $P_o = 1$, we have $Q_o = y - P_f Q_f - P_h Q_h$, by substituting Q_o back into the objective function, we have

$$\text{Max}_{Q_h, Q_f} U = y - P_f Q_f - P_h Q_h + u(Q_h, Q_f)$$

The first order condition then yields,

$$\frac{\partial u}{\partial Q_f} = P_f \quad \frac{\partial u}{\partial Q_h} = P_h$$

Q.E.D.

2. The Proof of the Welfare Function

Given consumer's utility function $U(Q_h, Q_f, Q_o)$, the total welfare function can be written as

$$W = U(Q_h, Q_f, Q_o) + Y - P_h Q_h - P_f Q_f$$

where, Y represents income, which is the sum of profits and wage. Further more, we assume wage is constant as y .

Since the total profits of domestic firms can be written as

$$P_h Q_h - nC(q_i)$$

The total welfare function becomes

$$W = U(Q_h, Q_f, Q_o) + P_h Q_h - nC(q_i) + y - P_h Q_h - P_f Q_f = U(Q_h, Q_f, Q_o) - P_f Q_f - nC(q_i) + y$$

Q.E.D.

3. The Proof of the Changes in Welfare when P_f Decreases

We compare two equilibria : $(P_{f1}, P_{h1}, Q_{f1}, Q_{h1})$ and $(P_{f2}, P_{h2}, Q_{f2}, Q_{h2})$.

Suppose $P_{f1} > P_{f2}$, $Q_{f1} < Q_{f2}$; $P_{h1} > P_{h2}$, $Q_{h1} < Q_{h2}$ with foreign price dumping and the domestic firms have a constant marginal cost c .

The total welfare functions for the two equilibria are:

$$W_1 = U(Q_f, Q_{h1}, Q_o) - P_f Q_f - c Q_{h1} + y$$

$$W_2 = U(Q_{f2}, Q_{h2}, Q_o) - P_{f2} Q_{f2} - c Q_{h2} + y$$

Then, the changes in the total welfare under foreign price dumping can be written as

$$\begin{aligned} \Delta W &= W_2 - W_1 = U(Q_{f2}, Q_{h2}, Q_o) - U(Q_f, Q_{h1}, Q_o) + P_f Q_f - P_{f2} Q_{f2} + c Q_{h1} - c Q_{h2} \\ &= U(Q_{f2}, Q_{h2}, Q_o) - U(Q_{f2}, Q_{h1}, Q_o) + U(Q_{f2}, Q_{h1}, Q_o) - U(Q_f, Q_{h1}, Q_o) \\ &\quad + P_f Q_f - P_{f2} Q_{f2} + P_{f2} Q_f - P_{f2} Q_{f2} + c(Q_{h1} - Q_{h2}) \\ &= [U(Q_{f2}, Q_{h1}, Q_o) - U(Q_f, Q_{h1}, Q_o) - P_{f2}(Q_{f2} - Q_f)] \\ &\quad + (P_f - P_{f2})Q_f + [U(Q_{f2}, Q_{h2}, Q_o) - U(Q_{f2}, Q_{h1}, Q_o) + c(Q_{h1} - Q_{h2})] \\ &= [U(Q_{f2}, Q_{h1}, Q_o) - U(Q_f, Q_{h1}, Q_o) - P_{f2}(Q_{f2} - Q_f)] \\ &\quad + (P_f - P_{f2})Q_f - [U(Q_{f2}, Q_{h1}, Q_o) - U(Q_{f2}, Q_{h2}, Q_o) - c(Q_{h1} - Q_{h2})] \end{aligned}$$

The following diagrams of the imported good market and the domestic good market illustrate clearly the changes in welfare under dumping.

FIGURE 3.8 Proof of the Changes in the Total Welfare in the Imported Good Market

FIGURE 3.9 Proof of the Changes in the Total Welfare in the Domestic Good Market.

4. The Proof for Proposition 3.2

Proof :

Let $\partial W / \partial P_f > 0$ in Equation (3.3), then

$$-Q_f + (P_h - C') \frac{\partial Q_h}{\partial P_f} > 0$$

and

$$\frac{\partial Q_h}{\partial P_f} \frac{P_f}{Q_h} > \frac{P_f Q_f}{(P_h - C') Q_h}$$

then

$$\epsilon_{hf} > \frac{P_f Q_f}{(P_h - C') Q_h}$$

or

$$\frac{P_h - C'}{P_h} \cdot \epsilon_{hf} > \frac{P_f Q_f}{P_h Q_h} \Rightarrow \epsilon_{hf} > \frac{P_f Q_f / P_h Q_h}{(P_h - C') / P_h}$$

Q.E.D.

5. The Proof for Proposition 3.3

Let $\partial W / \partial P_f > 0$, it implies

$$\sum_{i=1}^n (P_{h_i} - C'_i) \frac{\partial q_i}{\partial P_f} > Q_f$$

or,

$$\sum_{i=1}^n (P_{h_i} - C'_i) q_i \frac{\partial q_i}{\partial P_f} \frac{P_f}{q_i} > P_f Q_f$$

and

$$\sum_{i=1}^n \frac{P_{h_i} - C'_i}{P_{h_i}} (P_{h_i} q_i) \epsilon_{h_i, f} > P_f Q_f$$

Q.E.D.

6. The Proofs for Welfare Changes in the Case of Homogenous Goods

From (3.13), (3.17) and (3.18)

$$\frac{\partial W}{\partial P_f} = -Q_f + (P_d - C') \frac{\partial Q_h}{\partial P_f} = -a + bP_f - \frac{d(1+bc)(a+dP_f)}{b(n+1+bc)} + \frac{nd(a+dP_f)}{b(n+1+bc)^2}$$

we have

$$\frac{\partial^2 W}{\partial P_f \partial n} = \frac{d(1+bc)(a+dP_f)}{b(n+1+bc)^2} + \frac{db(a+dP_f)(n+1+bc)^2 - 2nbd(n+1+bc)(a+dP_f)}{b^2(n+1+bc)^4}$$

$$= \frac{d(1+bc)(a+dP_f)(n+1+bc) + d(a+dP_f)(n+1+bc) - 2nd(a+dP_f)}{b(n+1+bc)^3}$$

$$= \frac{d(a+dP_f)[2+(3+n+bc)bc]}{b(n+1+bc)^3} > 0$$

Then

$$\frac{\partial^2 W}{\partial P_f \partial b} = P_f - \frac{bcd(a+dP_f)(n+1+bc) - (n+1+2bc)d(1+bc)(a+dP_f)}{b^2(n+1+bc)^2} - \frac{nd(a+dP_f)(n+1+3bc)}{b^2(n+1+bc)^3}$$

$$P_f + \frac{(a+dP_f)[bcd(n+1+bc) - (d+bcd)(n+1+2bc)]}{b^2(n+1+bc)^2} - \frac{nd(a+dP_f)(n+1+3bc)}{b^2(n+1+bc)^3}$$

$$= P_f + \frac{bcd[nbc + (d+bc)(bc+2)] + (n+1)d}{b^2(n+1+bc)^3} > 0$$

and

$$\begin{aligned} \frac{\partial^2 W}{\partial P_f \partial d} &= -\frac{(1+bc)(a+dP_f) + d(1+bc)P_f}{b(n+1+bc)} + \frac{n(a+dP_f) + ndP_f}{b(n+1+bc)^2} \\ &= -\frac{(1+bc)(a+2dP_f)}{b(n+1+bc)} + \frac{na + 2ndP_f}{b(n+1+bc)^2} = -\frac{(a+2dP_f)[nbc + (1+bc)^2]}{b(n+1+bc)^2} < 0 \end{aligned}$$

7. The Proofs for Welfare Changes in the Case of Differentiated Goods

From (3.32), (3.33) and (3.34), the Proposition 3.3 implies

$$\frac{\partial W}{\partial P_f} = -Q_f + (P_h^c - c) \frac{\partial Q_h^c}{\partial P_f} = -\frac{(\alpha - P_f)\rho - n\gamma[\beta(\alpha - c) - \gamma(\alpha - P_f)]}{\beta\rho} + \frac{n\gamma(\beta^2 - \gamma^2)[\beta(\alpha - c) - \gamma(\alpha - \beta P_f)]}{\beta\rho^2} > 0$$

$$\Rightarrow \frac{n\gamma(\beta^2 - \gamma^2)[\beta(\alpha - c) - \gamma(\alpha P_f)]}{\rho} > (\alpha - P_f)\rho - n\gamma[\beta(\alpha - c) - \gamma(\alpha P_f)]$$

$$\Rightarrow n\gamma(\beta^2 - \gamma^2)[\beta(\alpha - c) - \gamma(\alpha P_f)] > (\alpha - P_f)\rho^2 - n\gamma\rho[\beta(\alpha - c) - \gamma(\alpha P_f)]$$

$$\Rightarrow \alpha - P_f < \frac{n\beta\gamma(\alpha - c)(\beta^2 - \gamma^2 + \rho)}{\rho(\rho + n\gamma^2) + n\gamma^2(\beta^2 - \gamma^2)}$$

$$\Rightarrow P_f > \frac{\alpha[\rho(\rho + n\gamma^2) + n\gamma^2(\beta^2 - \gamma^2)] - n\beta\gamma(\alpha - c)(\beta^2 - \gamma^2 + \rho)}{\rho(\rho + n\gamma^2) + n\gamma^2(\beta^2 - \gamma^2)} = \alpha - \frac{n\beta\gamma(\alpha - c)(\beta^2 - \gamma^2 + \rho)}{\rho(\rho + n\gamma^2) + n\gamma^2(\beta^2 - \gamma^2)}$$

and

$$\frac{\partial W}{\partial P_f} = -Q_f + (P_h^c - c) \frac{\partial Q_h^c}{\partial P_f} = -\frac{(\alpha - P_f)\rho - n\gamma[\beta(\alpha - c) - \gamma(\alpha - P_f)]}{\beta\rho} + \frac{n\gamma(\beta^2 - \gamma^2)[\beta(\alpha - c) - \gamma(\alpha - \beta P_f)]}{\beta\rho^2}$$

$$= -\frac{(\alpha - P)\rho - n\gamma\varphi}{\beta\rho} + \frac{n\gamma\varphi(\beta^2 - \gamma^2)}{\beta\rho^2}$$

Then

$$\frac{\partial^2 W}{\partial P \partial \theta} = -\left[\frac{n\gamma\varphi}{\beta\rho^2} + \frac{2n\gamma\varphi(\beta^2 - \gamma^2)}{\beta\rho^3}\right] \frac{\partial \rho}{\partial \theta} = -\frac{n(n-1)\beta\gamma\varphi}{\rho^2} \left[\frac{2(\beta^2 - \gamma^2)}{\rho} + 1\right] < 0$$

REFERENCE

- Anderson, Simon P., Nicolas Schmitt and Jacques-Francois Thesis (1995), "Who Benefits from Anti-dumping Legislation? " *Journal of International Economics* 38 , 3212-337.
- Blackorby, Charles , Daniel Primont and R. Robert Russell (1978) *Duality, Separability, and Functional Structure: Theory and Economic Applications*, North-Holland, Inc.
- Boltuck, Richard 1991, "Assessing the Effects on the Domestic Industry of Price Dumping," *Policy Implications of Anti-dumping Measures*, North-Holland.
- Brander , James and Paul Krugman (1983), "A 'Reciprocal Dumping' Model of International Trade," *Journal of International Economics* 15.
- Day , Satya P. and Adwait K. Mohanty (1987), "Welfare of the Dumping Country: A Comprehensive Ranking of Policies", *Journal of Quantitative Economics*, Vol.3 NO.1 , 13-34
- Dixit , Avinash (1987), "Anti-Dumping and Countervailing Duties under Oligopoly," *European Economic Review* 32, 355-68
- Eaton, Jonathan and Gene M. Grossman (1986), "Optimal Trade and Industrial Policy Under Oligopoly," *The Quarterly Journal of Economics*.
- Ethier, Wilfred (1982) , " Dumping," *Journal of Political Economy*, 90, 487-506
- Glance, Simon (1994), "*COMPAS Model Documentation*," Canadian International Trade Tribunal.
- Helpman, Elhanan and Paul R. Krugman (1986), *Market Structure and Foreign Trade*, Cambridge, Mass.: MIT Press.
- Lahiri , Sajal and Yoshiyasu Ono (1997), " Asymmetric Oligopoly, International Trade, and Welfare: a Synthesis," *Journal of Economics*.
- Leidy, Michael P. and Bernard M. Hoekman (1990), " Production effects of price- and cost-based anti-dumping laws under flexible exchange rates," *Canadian Journal of Economics*, No. 4. 873-895
- Martin , Shubik (1980), *Market Structure and Behaviour*, Cambridge, MA: Harvard University Press.
- Martin , Stephen (1996), *Advanced Industrial Economics*. Blackwell Publishers Inc.
- Morkre, Morris E. (1993), " The Effect of Subsidized Imports on Domestic Industry: A Comparison of

Market Structure," *Journal of Policy Modelling* 15(1), 49-61

Palsson, Halldor P. (1997), " The Economic Impact of Dumping on the Canadian Refined Sugar Market," *Working Paper of Competition Bureau, Industry Canada*.

Sugden, Robert and Allan Williams (1978), *The Principles of Practical Cost-benefit Analysis*, Oxford University Press.

Tirole, J. 1988, *The Theory of Industrial Organization*. Cambridge, Mass.: MIT Press.

Tivig, Thusnelda and Uwe Walz (2000), " Market share, cost-based dumping, and anti-dumping policy ," *Canadian Journal of Economics*, Vol. 33, No.1

Viner, Jacob (1966), *Dumping: A Problem in International Trade*, New York: Kelley

Webb, Michael (1992) " The Ambiguous Consequences of Anti-Dumping Laws, " *Economics Inquiry*, Vol. XXX, 437-448