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**A MICROSTRIP ANTENNA
WITH AN INSET MATCHING NETWORK**

by

Lise Quesnel

**A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of**

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Carleton University
Ottawa, Ontario**

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**A MICROSTRIP ANTENNA
WITH AN INSET MATCHING NETWORK**

submitted by

Lise Quesnel B. App. Sc.

in partial fulfilment of the requirements for the degree of
Master of Engineering

Thesis Supervisor

Chair, Department of Electronics

Carleton University
September, 1995

Abstract

This thesis investigates the merits of an unconventional matching network applied to a microstrip antenna. The antenna consists of a single element mounted on an air dielectric substrate, operating at L band. It is capacitively coupled to a coaxial connector, positioned on the underside of the ground plane, by an inset matching network etched on its surface. For the chosen substrate thicknesses and dielectric constant ϵ_r , the design provides a linear polarization and yields bandwidths in the range of 5 to 10 % for a return loss of -15dB, while providing adequate radiation patterns. The analysis of the antenna system demonstrates the flexibility and the usefulness of such a matching network.

The design is taken one step further by adding a second port to the single element antenna to obtain radiation in two orthogonal polarizations. Experimental results demonstrate the feasibility of the dual-polarized microstrip antenna with inset matching networks.

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*One of the greatest gifts granted to people
Is an insatiable thirst for learning.
The satisfaction of mastering a concept or a skill
Be it academic or physical,
Virtues a soul like no material can.*

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List of Symbols and Abbreviations

α	attenuation factor [11]
a	radius of the circular antenna [12]
A	area
β	propagation constant [11]
b	radius of the aperture in the ground plane in the C_g expression
bmw	beamwidth
B	susceptance representing the fringing fields of the microstrip antenna
BW	bandwidth
Γ_m	reflection coefficient
C	general capacitance
C_g	coaxial connector to ground plane capacitance
C_{ij}	mutual capacitance
ΔL	line extension representing the fringing fields of the microstrip antenna
d	probe radius
$d\Omega$	unit solid angle
D	directivity of an antenna
ϵ_0	free space permittivity
ϵ_r	dielectric constant of the substrate
ϵ_{eff}	effective dielectric constant of the microstrip structure
ϵ	general permittivity
ϵ'	real part of ϵ
ϵ''	imaginary of ϵ
E	electric field
f_c	center frequency
f_{op}	center frequency of operation
$F(\theta, \phi)$	normalized electric far field
F_v	electric vector potential
GHz	gigahertz
η	free space characteristic impedance
h	height of the microstrip antenna
H	magnetic field
k_0	free space propagation constant
k	propagation constant
λ	general wavelength
λ_0	free space wavelength
λ_{eff}	effective wavelength of the microstrip structure
L	length of the microstrip antenna
L_v	simplified integral of F_v
μ	magnetic permeability
M	magnetic current
MHz	megahertz

ρ	charge density
P	power
P_c	power dissipated in conductors
P_d	power dissipated in dielectrics
P_r	radiated power
$P_{r\ end}$	power radiated from the end fringing fields
$P_{r\ side}$	power radiated from the side fringing fields
Q	quality factor of a resonator
Q_c	charge in coulomb
r_o	radius of the probe [11]
r	distance vector
R	general resistance
R	distance vector [appendix]
R_c	resistivity of conductor
R_r	radiation resistance
σ	conductivity
σ	unknown constant charge density [appendix]
σ_c	conductor conductivity
S	surface
$\tan \delta$	loss tangent
TEM	transverse electromagnetic
v	volume
V	voltage
VSWR	voltage standing wave ratio
ω	angular frequency
W	width of the microstrip antenna
W_e	average electric energy
W_m	average magnetic energy
x	Cartesian axis
X_l	reactance of the short-circuited transmission line [12]
$X_{l.}$	inductive reactance of the coaxial probe [11]
y	Cartesian axis
Y	length in the C_g expression
z	Cartesian axis
Z_{eff}	effective characteristic impedance of the microstrip structure
Z_{in}	input impedance

Chapter one

Introduction

Indoor wireless communications is an important topic in the world of research and development. The Systems and Computer Engineering department at Carleton University is heavily involved in many aspects of this field. One of their areas of research is the investigation of diversity schemes used to maintain the signal integrity between transmitters and receivers.

One of their experiments was designed to study the polarization diversity scheme. For the study, a wideband dual-polarized antenna was required. An additional requirement was that the antenna design be relatively inexpensive since many units would be needed for an indoor system. Consequently, the goal of the research herein was to design a wideband, dual-polarized antenna which could be built at a relatively low cost.

The microstrip antenna was chosen for the study since it can easily support two polarizations and is relatively inexpensive. In addition, the wideband requirement can be implemented in a variety of ways.

The simplest way to improve the bandwidth of a microstrip antenna is to increase the height of its substrate, and add a feed network coupled with a matching network to obtain an acceptable level of power transfer between the antenna and the coaxial connector. This can be implemented in several ways. One type of structure consists of adding a matching network of stubs and quarter-wave transformers at the input of a probe-fed antenna. A second structure consists of superimposing two radiating conductors over the ground plane where each is separated from the other by a dielectric, and using

a feed aperture located underneath the ground plane. This is referred to as a stacked configuration. A third structure consists of correcting the inductive input impedance of a probe-fed microstrip antenna by adding a capacitive contributor inside the antenna instead of at its input port. Other types of structures also exist.

Early in my research I discovered the matching capabilities of cutting a rectangular ring on the top conductor of the probe-fed microstrip antenna to create a capacitive contributor to the system. It was a simple design, which provided the required matching and had adequate radiation patterns. I later found references where the authors used the same design applied to a circular microstrip antenna. The advantages of this design over the ones mentioned in the previous paragraph is that it captures the wide bandwidth of a thicker substrate, and makes it available to a 50 ohm coaxial connector without adding any components to the structure. Therefore, it maintains a low cost in addition to allowing a second port to be inserted without a problem. Consequently, the microstrip antenna with an inset matching network, i.e. the rectangular ring, fulfills all the requirements for the study of the polarization diversity scheme.

Considering the success of the inset matching network, the following objective was chosen for this thesis.

1.1 Thesis Objective

The objective of this thesis is to investigate the design of a microstrip antenna with an inset matching network. The emphasis of the investigation is placed on understanding how variations in the physical parameters affect the performance parameters. Transmission line theory is used as the basic tool of this analysis. The input impedance and the radiation patterns of the antenna are the main topics of the thesis.

1.2 Thesis Outline

In chapter two, selected topics provide the necessary background information required to analyze the design under review. The topics covered are as follows: microstrip parameters, transmission line model, radiation model, bandwidth enhancement techniques and matching networks.

The third chapter contains the theoretical analysis of the design under review. The chapter is divided into two sections: the input impedance analysis and the radiation pattern analysis. The input impedance section begins by presenting additional microstrip parameters. Then, the different phenomena occurring in the structure are analyzed using transmission line theory. With the transmission line representation of the microstrip antenna, the bandwidth enhancement technique applied in the design is examined. Following this, the inherent problem of using a feed probe in a thick substrate is presented and discussed. Next, the inset matching network, which provides the solution to the problem, is analyzed. Finally, the design procedure for the input impedance of the complete antenna is discussed. The radiation analysis section develops a composite radiation pattern to examine the different sources of radiation present in the structure.

In the fourth chapter, input impedance and radiation pattern results from the theoretical analysis are compared with the experimental results. In addition, theoretical input impedance results, simulated from the commercial software package *Ensemble* (based on a full-wave analysis) are also included in the above comparison. Further experimental results are then presented to complete the demonstration of the design under review.

The fifth chapter introduces the two port version of the microstrip antenna with an inset matching network. Experimental results are presented.

Conclusions and recommendations for future work are presented in the final chapter.

Chapter Two

Literature Survey

2.1 The Microstrip Antenna

A general microstrip antenna consists of two conductors separated by a dielectric layer. The larger conductor serves as the ground plane of the system while the top conductor is cut so as to excite the required mode of operation. Shown in Figure 2.1 are

Figure 2.1 The general structure of the microstrip antenna.

the four basic physical degrees of freedom of the simple microstrip antenna : the length, **L**, the width, **W**, the substrate height, **h**, and substrate dielectric permittivity, ϵ_r . These physical parameters influence the input impedance which in turn determines the following performance parameters: the frequency of operation and the bandwidth. In addition, the physical parameters affect the radiation pattern. The links between both sets of parameters will be addressed in the appropriate sections throughout the thesis.

2.1.1 Microstrip Parameters

The microstrip antenna is simply a radiating microstrip line. Therefore, part of its behavior can be explained with the same closed-formed equations devised for the microstrip line.

Researchers have developed a set of equations representing the microstrip structure which can be used in the standard transmission line equations. One of the earliest developments originated from the work of Harold A. Wheeler. He derived an expression for the characteristic impedance of the microstrip structure [2,3]. The authors of [4] extended this work and derived an effective dielectric constant equation, ϵ_{eff} in (2-1), in addition to providing an alternative equation for the characteristic impedance, Z_{eff} in (2-2).

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + 12 \frac{h}{W}\right)^{-0.5} \quad (2-1)$$

$$Z_{eff} = \frac{120\pi/\sqrt{\epsilon_{eff}}}{\frac{W}{h} + 1.393 + 0.667 \ln \left[\frac{W}{h} + 1.444 \right]} \quad (2-2)$$

Subsequently, with a single effective dielectric constant, the wavelength perceived in the structure is now:

$$\lambda_{eff} = \frac{\lambda_o}{\sqrt{\epsilon_{eff}}} \quad (2-3)$$

When the microstrip line is truncated to build an open circuit termination, the fringing fields created at the end must also be included in the impedance analysis. The open end discontinuity is represented by an excess capacitance. An alternative microstrip

parameter for the capacitance, derived by Hammerstad [5] and modified by [6], is the line extension. Researchers have derived the closed-formed equation of the line extension as an equivalent representation of the discontinuity. The line extension ΔL , (2-4), is added at the ends of the antenna's length. This equation can then be used to obtain the difficult-to-calculate capacitance so that the open end discontinuity can be modelled as a susceptance, B , in (2-5). The line extension or the susceptance can be used in the analysis. The variable k_o represents the free space propagation constant.

$$\Delta L = h \cdot 0.412 \frac{(\epsilon_{eff} + 0.3)(\frac{W}{h} + 0.264)}{(\epsilon_{eff} - 0.258)(\frac{W}{h} + 0.8)} \quad (2-4)$$

$$B = \frac{k_o \Delta L \sqrt{\epsilon_{eff}}}{Z_{eff}} \quad (2-5)$$

Consequently, the effective permittivity, the effective characteristic impedance and the line extension can be used to represent the microstrip antenna as a standard transmission line.

2.1.2 Transmission Line Model

Although, the transmission line model cannot offer the accuracy of the numerical models, it provides a good interpretation of the behavior of the dominant mode of operation in a microstrip antenna, the **TEM** mode.

In reference [10], the author presents the microstrip antenna as a transmission line weighted at each end by a radiation load. The load consists of the open end susceptance,

B, and of the radiation resistance, **R_r**, representing the power dissipated as radiation. The load farthest from the feed point is transformed across the transmission line and added in parallel to the second load to produce the input impedance of the antenna, **Z_{in}**. This is shown in Figure 2.2.

Figure 2.2 The basic transmission line representation of the microstrip antenna.

Reference [11] goes one step further and includes the effect of the probe used to feed the antenna. The author uses an expression referred to as an 'inductive reactance of the coaxial probe', which he then places between the source and the transmission line representation of the antenna:

$$X_L = \frac{\beta \alpha W}{2\pi} \ln \frac{2}{1.78107 \beta r_o} \quad (2-6)$$

The r_o variable is the radius of the probe, β the effective propagation constant, α the attenuation factor and W the width of the antenna. Another author, [12], who discusses a probe-fed circular microstrip antenna, represents the probe as a short circuited transmission line:

$$X_l = \omega L = \frac{Z}{\pi} \tan\left(\frac{2\pi h}{\lambda_{eff}}\right) \quad (2-7)$$

where

$$Z = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{a}{d}\right) \quad (2-8)$$

and where the effective wavelength is used. The variable d is the probe radius and a the radius of the circular antenna under consideration.

A more involved model of the feed probe is the tapered transmission line model found in [13]. In this conference paper, the author establishes the field line configuration

Figure 2.3 Probe's approximate field distribution (from ref. 13).

between the probe and the ground plane as shown in Figure 2.3, and then he uses the mid-value equipotential line of the field distribution as a conductor. This conductor becomes the outer conductor of a tapered transmission line whose inner conductor is the probe. The shape of the outer conductor is assumed to be sinusoidal in form. The tip of the outer conductor is forced to diverge from the equipotential path to end on the top conductor as a short circuit. The resulting reactance value is doubled to compensate for the use of the mid-value equipotential line. The author's theoretical results agree well with the experimental ones.

2.1.3 Radiation Model

Since the input impedance is being represented with the transmission line approach, a radiation resistance is required to model the power released by the fringing fields at the ends of the antenna.

Most radiation models found in the literature incorporate approximations applicable to a thin substrate. [14 - 17] These approximations lead to a variety of models with fundamental differences but all providing an adequate description of the microstrip antenna's dominant radiation characteristics. Consequently, those models can also describe the general behaviour of a microstrip antenna with a slightly thicker substrate in which the **TEM** mode remains dominant.

The radiation model developed in [16] is adopted in this thesis. The fringing fields occurring at the ends of the antenna are represented respectively by radiating slots placed perpendicular to the plane of the conductors as shown in Figure 2.4.

Figure 2.4 The equivalent slot representation of the end fringing fields of the microstrip antenna.

Since the microstrip antenna is being represented by a transmission line model, the electric field is assumed to be uniform in the slot. With this information, the far field expression of each slot can be derived. The derivation of these fields is taken from chapter eleven of [16] and is included in Appendix A.

From these far field expressions, the resistance representing the radiated power is calculated in the standard way. First, the average radiated power, P_r , is derived with the Poynting vector using the far field expressions, then, with circuit theory the radiation resistance, R_r , becomes the following, where the voltage, V , is the one across the slot

$$R_r = \frac{|V|^2}{2 P_r} \quad (2-9)$$

2.2 Bandwidth Enhancement Techniques and their Matching Networks

The microstrip antenna has traditionally been chosen for its inconspicuous profile. The low profile produces a high Q resonant behaviour, where the variable Q denotes the quality factor of a resonator. Consequently, the traditional microstrip antenna is associated with a narrow bandwidth.

However, this does not need to be the case if the specifications can tolerate a potentially more complex profile. The bandwidth of the microstrip antenna can be increased by the addition of a matching network or by using a bandwidth enhancement technique coupled with a matching network.

Before continuing along this line, an important concept needs to be introduced. A paper by Fano [18], demonstrates that the improvement of the bandwidth of an arbitrary impedance, through the use of a broadband matching network, is limited by the Q of this impedance and by the physical realizability of the matching network. An example of his results is illustrated with the following simple case shown in Figure 2.5. This example is found in [8]. For the parallel combination of an arbitrary resistance, R , and arbitrary capacitance, C ,

Figure 2.5 An arbitrary RC parallel configuration to be matched.

a matching network needs to be devised to provide the reflection coefficient, Γ_m , over the maximum bandwidth. Equation (2-10) states the theoretical limit of a general case, while equation (2-11) is the more specific case of the ideal reflection coefficient: the value of one outside the band and the minimum value of Γ_m inside the band.

$$\int_0^w \ln \frac{1}{|\Gamma(\omega)|} d\omega \leq \frac{\pi}{RC} \quad (2-10)$$

$$\Delta\omega \ln \frac{1}{\Gamma_m} \leq \frac{\pi}{RC} \quad (2-11)$$

The ω variable denotes the angular frequency in the equations. With equation (2-11), it is obvious that for a fixed load, a better match, that is a lower Γ_m , can only be obtained at the expense of the bandwidth and vice versa. More importantly, as R or C increases, that is the Q increases, either the bandwidth must decrease or the reflection coefficient must worsen. This observation indicates that a higher Q impedance load has poorer matching capabilities than a lower Q impedance.

This conclusion is directly applicable to microstrip antennas. A high Q microstrip antenna is difficult to match over a wide bandwidth. Consequently, bandwidth enhancement techniques are used in conjunction with matching networks.

A wider bandwidth can be obtained with two types of techniques. In the first

case, the microstrip antenna is modified so as to excite multiple resonances in the response, thereby artificially widening the bandwidth. In the second case, the Q of the microstrip antenna is lowered before any matching is attempted. With these modifications, according to Fano's results, a better match is achievable over a wider bandwidth.

By definition, the Q of a resonator is:

$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss per second}} \quad (2-12)$$

Therefore, the second technique consists of decreasing the ratio of the stored energy to power loss. This can be achieved by adding a dissipative load to affect strictly the power loss, or by altering the physical structure to modify both quantities. An appropriate matching network is then applied.

2.2.1 Wideband Microstrip Antennas

The authors of reference [19] present a series of results to demonstrate the necessity of using bandwidth enhancement techniques to obtain wideband microstrip antennas. They first show that for the rectangular microstrip antenna without any bandwidth enhancement technique, a significant increase in bandwidth proves impossible to achieve with a multi-stub matching network. By increasing the antenna width to approximately two effective wavelengths, the structure excites a second harmonic in the direction of the width, thus a bandwidth enhancement technique has been incorporated. With the multiple resonance, a two stub matching network is then successful in increasing the bandwidth by a factor of 4 over the unmatched antenna. Next, the authors demonstrate the bandwidth enhancement technique consisting of increasing the power loss to reduce the Q . A matching load, i.e. a lossy load, attached to the initial rectangular antenna, enables a two stub matching network to provide a voltage standing wave ratio $VSWR < 2$ over an 18% bandwidth around the center frequency of 3.1 GHz. It should

be noted that the absolute bandwidth is 18% of the center frequency. The addition of a matching load to the antenna allows it to be designated as a travelling-wave antenna. Unfortunately, this design inherently has a poor radiation efficiency. However, according to the authors, by applying the concept to a travelling-wave array, an improved radiation efficiency can be obtained.

Reference, [20], demonstrates an alternative method of increasing the bandwidth by lowering the Q . The design consists of an aperture-coupled, stacked microstrip antenna structure. The structure, shown in Figure 2.6, provides an antenna with a similar bandwidth as the previous example, greater than 20% for a $VSWR < 2$ and a center frequency of 4.45 GHz. The bandwidth ranges from 3.98 GHz to 4.92 GHz.

Figure 2.6 Configuration of an aperture-coupled stacked microstrip patch antenna (from ref. 20).

2.2.2 Capacitively-Coupled, Probe-Fed Microstrip Antennas

The Q of the microstrip antenna can also be lowered by increasing the thickness of the substrate relative to the wavelength. In reference [21], the authors catalogue a series of experimental results taken from a variety of thick substrate antennas. The

expected rise in bandwidth for an increasing height-to-wavelength ratio occurs along with a decrease in the resonant frequency. Unfortunately, when these antennas are probe-fed, the input impedance suffers an increase in inductance. The increase is significant enough to shift the reactance curve completely above the zero point. Therefore, such an input impedance requires a reactive matching network for it to be compatible with a 50 ohm coaxial cable.

The presence of the inductive contribution in the microstrip antenna's input impedance is a problem for which viable solutions exist. The basic concept behind the correction is simply tuning out the unwanted inductance by adding capacitance to the system. The two methods presented here consist of capacitively coupling the probe to the antenna.

In reference [22], the authors study a circular antenna capacitively coupled to its feed probe through an annular ring cut around the probe tip as shown in Figure 2.7. The presence of the ring not only neutralizes the inductance, but it also places the antenna's input impedance near the 50 ohm point. Consequently, the antenna can be directly connected to the coaxial connector. In addition, the authors give a model of the antenna which consists of representing the resonator by its lumped equivalent model, the capacitive gap by an empirically modified expression of a capacitive reactance and the

Figure 2.7 Patch geometry (from ref. 22).

probe by a tapered transmission line.

The reference [23] also presents a capacitively coupled feed probe design shown in Figure 2.8. The design is reminiscent of a stacked patch configuration. However, as the authors point out, since the middle conductor is kept small to avoid radiation, the analysis of the design differs greatly from the stacked configuration of two radiators. In this design, the smaller conductor produces the required capacitance. In the reference, the flexibility of the design is well demonstrated through numerous theoretical results and

Figure 2.8 The capacitively fed microstrip antenna element (from ref. 23).

supported with experimental prototypes.

In summary, this chapter has discussed the following topics: the microstrip antenna, the transmission line representation of the antenna, bandwidth enhancement techniques and matching networks.

Chapter three

Theoretical Analysis of the Microstrip Antenna with an Inset Matching Network

This chapter contains the analysis of the microstrip antenna with an inset matching network. The analysis is based on the transmission line representation of the microstrip antenna. This type of approach is chosen because with it, the phenomena at work in the structure can be considered separately. Therefore it provides a good understanding of the design under review.

In this analysis, the physical attributes being addressed are shown in figure 3.1. The physical components are the top conductor, the double layered substrate, the feed probe and the inset position of the probe, while the physical parameters are the width W and length L of the top conductor, and the height h and dielectric constant ϵ_r of the substrate. The inset matching network consists of the rectangular ring cut on the top conductor.

Figure 3.1 The physical attributes of the microstrip antenna under review.

The performance parameters being addressed in relation to the physical attributes are the input impedance, the frequencies of operation, the bandwidth and the radiation patterns.

3.1 Input Impedance Analysis

3.1.1 Analysis Overview

Before launching into the technicalities of the analysis, an overview of the design at hand is necessary. The analysis is divided into two subjects to better understand the antenna behavior: the unmatched antenna and the matched antenna.

The unmatched microstrip antenna can be segmented as shown in Figure 3.2a).

Figure 3.2a) Schematic representation of the unmatched microstrip antenna.

The resonator and the radiation loads represent the behavior of the simple microstrip antenna and are determined using the standard microstrip design equations. As the substrate is thickened, the impact of the probe goes beyond a simple inductance and it is regarded as creating an impedance transformation. With the geometry involved, the probe causes the input impedance of the resonator to be shifted higher in inductance as well as in resistance.

To compensate for the increase in inductance, the resonator needs to be capacitively coupled to the feed probe. The inset matching network shown in Figure 3.1 introduces the necessary capacitance to cancel the unwanted inductance. In addition, the

combination of the probe and the matching network transforms the resonator response in such a way as to provide frequencies with impedances of approximately of $50 + j0$ ohms. Consequently, no additional components are required besides the inset matching network. Schematically, the matching network is placed between the resonator and the impedance transformer as in Figure 3.2b).

Since a cascade type transmission line analysis is used to predict the behavior of an integrated design, limitations on the accuracy of the prediction are expected.

Figure 3.2b) Schematic representation of the matched microstrip antenna.

3.1.2 Microstrip Parameters

In this section, the behavior of the resonator and the properties of the radiator pertaining to the input impedance analysis are presented. Then, with the equations formulated, the impact of the physical attributes on the performance parameters is investigated.

3.1.2.1 Resonance

The square microstrip antenna falls in the category of standing-wave antennas due to its resonant configuration. The resonance is established with a half-wavelength long, open circuit microstrip line. Ideally, with this configuration, a standing wave with a minimum occurring halfway down the line is created within the structure. Such a

behavior results in a parallel-type resonance [8]. The strategic location of the minimum ultimately will allow a second feed probe to be placed in the orthogonal direction with a degree of isolation from the first probe.

At the input of the ideal lossless parallel resonator, the existing phenomenon is actually one of anti-resonance i.e. the impedance approaches infinity at a certain frequency. However, when losses are included in the analysis, the reactance becomes finite and reaches a value of zero at that frequency. The general resonator input impedance curve is shown Figure 3.3 and is obtained with the software package *Libra*.

Figure 3.3 The general input reactance and resistance curves of a resonator.

It's important to note the expression "center frequency" is taken from Figure 3.3. It represents the frequency occurring at the value of peak resistance. It is used throughout the thesis as a pointer to indicate changes in the input impedance response. However, the term "center frequency of operation" is used literally and is not associated with the previous definition.

Since the software package *Libra* is designed to work with microstrip lines, it provides the simulation medium required for the transmission line representation. The transmission line representation of the resonator consists of a microstrip line to represent

the physical parameters of the antenna and two sets of radiation loads to represent the impact of the radiating characteristics on the input impedance. *Libra* inherently factors in the effective wavelength and dielectric constant, and lumped elements can be easily added to represent the radiation loads.

With *Libra*, the structure shown in Figure 3.4, is used to analyze the impact of

Figure 3.4 The transmission line representation of the resonator and radiator properties of the microstrip antenna.

the resonator properties on the input impedance of the microstrip antenna.

Effective Wavelength

In chapter one, the microstrip parameters were given. They were developed to represent the heterogeneous medium consisting of the interface between the air and the substrate of permittivity ϵ_r of the microstrip structure with an effective homogeneous medium. Since the presence of a dielectric forces the phase velocity to be smaller than the free space value, the wavelength perceived inside the microstrip structure has to be corrected accordingly. The required correction is simply the free-space wavelength, λ_o , divided by the root of the effective dielectric constant:

$$\lambda_{eff} = \frac{\lambda_o}{\sqrt{\epsilon_{eff}}} \quad (3-1)$$

3.1.2.2 Dielectric Constant of the Double Layered Substrate

With the structure shown in Figure 3.1, the dielectric constant value ϵ_r needs to be calculated to represent the following double layered substrate. The thick microstrip antenna is supported by a layer of *FR-4* fibre glass to add mechanical support; this fibre glass is then placed on the substrate consisting of the honeycomb structure (*Hexcel*). Both of these structures have different dielectric permittivities. To continue the use of the **TEM** transmission line approach, these two values must be combined into one. The usual method of achieving this consists of considering the inhomogeneous substrate as two capacitances in series. Each capacitance, C , is calculated as:

$$C = \frac{\epsilon A}{h} \quad (3-2)$$

where ϵ is the dielectric permittivity, A , the area of the conductors and h the height of the dielectric layer. The overall capacitance is then calculated and the new dielectric constant is extracted from this value. It should be noted that the area does not influence the value of the new dielectric constant.

The impact of the dielectric constant on the frequency behavior can be explained as follows. An input frequency of 1.9 GHz has a free-space wavelength of λ_0 . Due to the presence of the dielectric, that λ_0 is perceived as $\lambda_{eff} = \lambda_0 / \sqrt{\epsilon_{eff}}$, or a higher frequency of 2.002 GHz, for an effective dielectric constant of 1.11. Therefore, the behavior of 2.002 GHz has now been pushed down to the 1.9 GHz frequency. In another context, if for example a center frequency of 2.5 GHz is required, the nominal half-wavelength dimension of 6 cm, would have to be shortened according to $\lambda_{eff} = \lambda_0 / \sqrt{\epsilon_{eff}}$, therefore to 5.7 cm.

3.1.2.3 Loss Tangent of the Double Layered Substrate

A second issue regarding the dielectric permittivity, ϵ , concerns its imaginary component, ϵ'' :

$$\epsilon = \epsilon' - j\epsilon'' \quad (3-3)$$

The imaginary component in the dielectric permittivity indicates the presence of losses originating from the damping of the vibrating dipole moments. Often these losses are lumped with the conductor loss of this dielectric since, as shown in reference [8], the dielectric loss $\omega\epsilon''$ cannot be differentiated from the conductor loss σ . The combined loss is accounted for in the analysis through the loss tangent:

$$\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon' \epsilon_0} \quad (3-4)$$

Subsequently, the dielectric substrate is often represented by its permittivity taken as equal to its real component, and its loss tangent.

In the analysis herein, an effective loss tangent of the double layered substrate is needed for the *Libra* algorithm. Therefore, the properties of the individual dielectrics are combined to define the properties of one effective medium. To achieve this goal, I developed the following sequence of equations.

The power dissipated in the two dielectrics provides the necessary link between the individual loss tangents of the fibre glass and the honeycomb, and the required effective loss tangent. Initially, the power in each dielectric is calculated using the respective conductivities.

$$P_{d_i} = h_i \int_s \sigma_i |E_i|^2 dS_i \quad (3-5 \text{ a})$$

$$P_{d_i} = h_i W E_o^2 \frac{1}{2} L \sigma_i \quad (3-5 \text{ b})$$

The variable **S** represents the surface of the dielectric; the subscript **i** represents both dielectrics, 1 and 2; the variable **h** is the height, **W** the width, **L** the length, **E** the electric field, and the σ the conductivity.

To evaluate the equation, an electric field representation is required for the two layers. An arbitrary electric field is assigned to the top dielectric, while the lower electric field stems from the former, using boundary conditions for adjacent dielectrics:

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n} \quad (3-6)$$

The power dissipated in both volumes is then summed:

$$P_{d\ 12} = h_1 W E_o^2 \frac{1}{2} L \sigma_1 + h_2 W E_o^2 \left(\frac{\epsilon_{r\ 1}}{\epsilon_{r\ 2}} \right)^2 \frac{1}{2} L \sigma_2 \quad (3-7)$$

To subsequently use this summation to obtain an effective conductivity, the electric field for the single effective dielectric has to be linked to the two previous individual fields. This link is established using the definition of the potential difference. It has to be the

same value across the dual layer as that across the single effective layer. Therefore, the maximum potential expression, $V_{\max}$, of the dual layer is calculated, in addition to that of the single layer:

$$V_{\max} = \int_0^{h_2} (E_o \frac{\epsilon_1}{\epsilon_2})_{2n} dz + \int_{h_2}^{h_2+h_1} (E_o)_{1n} dz \quad (3-8 a)$$

$$V_{\max} = \int_0^h E_m dz \quad (3-8 b)$$

The maximum electric field value of the top layer, E_o , is arbitrarily assigned, while the maximum electric field value, E_m , of the single effective layer is the unknown being sought. By comparing the two expressions, the unknown is calculated:

$$E_m = \frac{h_2 E_o \left(\frac{\epsilon_1}{\epsilon_2} \right) + h_1 E_o}{h_1 + h_2} \quad (3-9)$$

and subsequently used, in conjunction with the summation of dissipated power, P_{d12} , to extract the effective conductivity:

$$\sigma_{eff} = \frac{P_{d12}}{h \frac{1}{2} W L E_m^2} \quad (3-10)$$

Equation (3-10) is of the same form as (3-5b). As was previously shown, the link

between the loss tangent and the conductivity is:

$$\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'\epsilon_0} \quad (3-11)$$

and it becomes:

$$\tan\delta_{eff} = \frac{\sigma_{eff}}{\omega\epsilon'\epsilon_0} \quad (3-12)$$

since the losses are lumped into one conductivity. With this definition, the effective loss tangent becomes frequency independent. The previous procedure can be summarized into a single expression and used to calculate the effective loss tangent:

$$\tan\delta_{eff} = \frac{h_1 \tan\delta_1 \epsilon_{r1} + h_2 \left(\frac{\epsilon_{r1}}{\epsilon_{r2}} \right)^2 \tan\delta_2 \epsilon_{r2}}{h \epsilon' \left[\frac{h_2 \left(\frac{\epsilon_1}{\epsilon_2} \right) + h_1}{h} \right]^2} \quad (3-13)$$

The conductivities of the two initial dielectrics have been replaced by their equivalent form taken from equation (3-12).

With these definitions, the double layered dielectric has been converted to a single effective medium with an effective dielectric constant and loss tangent.

3.1.2.4 Line Extension

In chapter two, the modelling of the microstrip antenna's open circuit was discussed. Essentially, the reactance of the open circuit load was mapped into a line extension. As was pointed out, this extension was simply added to both of the open ends of the antenna, as shown in Figure 3.5. Therefore, the impact of the open circuit is to lower the center resonant frequency.

Figure 3.5 The line extension locations.

However, it is more convenient to use the susceptance, B , associated with the open end line which can be calculated from the line extension as follows:

$$B = \frac{k_o \Delta L \sqrt{\epsilon_{eff}}}{Z_{eff}} \quad (3-14)$$

The susceptance is placed at each end of the transmission line in parallel with the radiation resistance to represent both open circuit conditions.

3.1.2.5 Inset Position of the Feed Point

A method for modifying the input impedance of the microstrip antenna is to feed the resonating structure at a point inset from the edge. This characteristic is shown in

Figure 3.1. Theoretically, this modification does not change the center frequency, but it does lower the resistance and compress the reactance of the adjacent frequencies. This trend is shown in Figure 3.6. The bandwidth is affected slightly, but more importantly, the lower impedance at the input facilitates the task of matching the antenna. In the analysis, the inset translates to joining two transmission lines, each with a radiation load, where they are added in parallel at the feed probe.

Figure 3.6 The impact of the inset position of the feed point on the input impedance.

3.1.2.6 Radiation Resistance

For all structures supporting electromagnetic waves, any radiation, wanted or unwanted, represents a loss of power. For the purpose of the impedance analysis, any power loss contributes a resistance. Three radiation loads are included in the model, one for the fringing fields, one for the probe and the last one for the inset matching network.

Fringing Fields

The radiation resistance described in chapter one is used to represent the radiation emanating from the ends of the antenna. The equation for the resistance (3-16) is developed in Appendix A. One resistance is located at each of the extremities of the transmission line and is placed in parallel with the susceptance representing the reactive contribution from the fringing fields, **B**.

$$R_r = \frac{|hE_o|^2 \eta}{\left[\frac{E_o khW}{2\pi}\right]^2 \int_0^\pi \int_0^\pi (\sin(\theta))^3 \left[\text{sinc}\left(\frac{kh}{2}\sin(\theta)\cos(\phi)\right) \text{sinc}\left(\frac{kW}{2}\cos(\theta)\right)\right]^2 d\theta d\phi}$$

(3-16)

In addition to these fields, the power radiated by the side fringing fields is also taken into account. The derivation of this radiation resistance is the same as the previous one except that the internal electric field has a cosine distribution. To include the effects of the side radiation into the analysis, two approximations must be adopted. These are based on the diagram of Figure 3.7.

Figure 3.7 Diagram for the calculation of the radiation resistance of the side slots.

First, even though the radiated power expression can be derived for a slot whose voltage varies along its length, the definition of the radiation resistance is applicable only to a slot with a constant voltage across it. To counter this problem, the significant portion of the radiated power is assumed to exit from the extremities of the side slots, as shown in Figure 3.7, where a constant voltage is present. With this allowance, the resistance can now be calculated on the assumption that the radiated power exits from this constant voltage. Secondly, since the voltage existing at the extremities of the side slots is the same as that of the end slots, due to the junction of the end slots and the side slots, the power radiated from the sides can be lumped with the end radiation resulting in a parallel combination of resistances. Each end slot receives half the contribution from each side slot. Therefore, the radiation resistance from one slot, representing half of the power from both side slots, is added to one end radiation resistance. The equation for the resistance, similar to (3-16), is included in Appendix A.

Probe Radiation

The feed probe also contributes radiation to the overall radiation patterns since a current exists along its length. The explanation of the modelling of the probe is left to the section 3.2.3. It is sufficient to say that the probe is assumed to be a top-loaded monopole over a ground plane. Its radiation resistance is calculated from the far field

pattern. The results indicate that the probe resistance is much smaller than the other contributors, and thus it is not included in the transmission line representation.

3.1.2.7 Impact of the Width on the Input Impedance

The impact of the length and the dielectric permittivity have been established and that of the height will be determined in the next section. With the radiation resistance in place, the impact of the width on the input impedance response can now be examined.

If in certain designs a rectangular geometry is allowable, the width can be used to alter the input impedance. With a microstrip implementation, the width to height ratio affects the effective dielectric constant and the line extension, therefore it indirectly influences the center frequency. In addition, the width can be used to modify the radiation load, an important aspect of the matching process. Table 3.1 provides numbers representing the impact of the changing width on the microstrip constants, and Figure 3.8 illustrates the variations in the input resistance and reactance for those widths. The results are based on an antenna of a length of 6.0 cm and a height of 1.3 cm.

Table 3.1 Impact of the width on the microstrip constants and on the center frequency, f_c

widths	ϵ_{eff}	line extension	f_c
5.0 cm	1.0821	7.95 mm	1.885 GHz
6.0 cm	1.084	8.09 mm	1.875 GHz
7.0 cm	1.0856	8.19 mm	1.867 GHz

Figure 3.8 The impact of the width on the input impedance.

As the width is increased, the trends are similar to those associated with an inset increase: the resistance is lowered and the reactance is compressed. However, in addition, the center frequency is also lowered.

This completes the analysis of the behavior of the microstrip antenna as a simple resonator. The next section deals with the impact of the height of the substrate on the bandwidth of the resonator.

3.1.3 Bandwidth Enhancement Technique

The bandwidth of a microstrip antenna may be enhanced by increasing the height of the substrate. With this design, the height of the honeycomb layer shown in Figure 3.1 is increased. In a transmission line resonator, increasing the distance between the conductors lowers the ratio of the average stored energy over the power loss. Consequently, the quality factor, Q , of the resonator decreases, indicating an increase in the bandwidth. The definition of the Q of a resonator is:

$$Q = \omega \frac{\text{average stored energy per second}}{\text{average power loss per second}} \quad (3-17)$$

Therefore the trend of the bandwidth can be demonstrated by calculating the energy and the power loss associated with the **TEM** mode, then by examining the impact of increasing the substrate height. In this case, the Q becomes:

$$Q = \omega \frac{W_e + W_m}{P_d + P_c + P_{r\text{ end}} + P_{r\text{ side}}}$$

The radiated power expressions ($P_{r\text{ end}}$ and $P_{r\text{ side}}$) are taken from the radiation resistances' algorithms. The power loss expressions associated with the dielectric and the conductor, taken from [28] are the following:

$$P_d = h \int_s \sigma_{\text{eff}} |E|^2 ds \quad (3-18)$$

$$P_c = 2 \int_s R |H|^2 ds \quad (3-19)$$

where the effective dielectric conductivity, σ_{eff} , is calculated using the effective loss

tangent. The R_c variable, (3-20) denotes the resistivity of the conductor and the copper conductivity is used in the resistivity equation:

$$R_c = \sqrt{\frac{2\pi f \mu_o}{2\sigma_c}} \quad (3-20)$$

The definitions of the average stored energy are the following:

$$W_e = \frac{\epsilon}{4} \int_v E \cdot E^* dv \quad (3-21)$$

$$W_m = \frac{\mu}{4} \int_v H \cdot H^* dv \quad (3-22)$$

where the first expression is the electric energy and the second one the magnetic energy, with μ representing the magnetic permeability. The equations are taken from [8]. The integrals are evaluated over the volume of the antenna structure. The field distributions inside the volume encircling the resonator are, for the same coordinate system as the radiation resistances:

$$E_x = E_o \sin\left(\frac{\pi y}{L}\right) \quad (3-23)$$

$$H_z = \frac{E_x}{\eta} \quad (3-24)$$

where η represents the characteristic impedance of the surrounding medium. The ensuing derivation of the energy expressions are left to the Appendix B. In the final equation the Q becomes:

$$Q = \omega \frac{\frac{1}{8} h L (\epsilon + \frac{\mu}{\eta^2})}{\frac{W h^2 k^2}{8 \pi^2 \eta} [..] + h \sigma_{eff} \frac{L}{2} + 2 R \frac{L}{2 \eta^2}} \quad (3-25)$$

where the square parentheses indicate the integrals of the radiated power to be evaluated.

Since the contribution of the conductor loss is small, the fact that it does not contain the height variable has very little impact in the following observations. In the final Q expression, the height variable exerts a dual influence on the value of the Q . The first influence of the height is its presence in the sinc functions of the integrals. In that portion of the equation, an increase in height entails a decrease in the value of the integrals, therefore an increase in Q . The second influence of the height is as a multiplicative factor in the denominator. In this case, the height is inversely proportional to the Q . Even though both influences produce opposite trends, the impact of the second one dominates. Therefore, it is clear that the Q of the resonator decreases with increasing height, thereby increasing its bandwidth. Table 3.2 contains a few values of height and Q to support this conclusion.

Table 3.2 Q values for different substrate heights

height	Q factor
1 mm	118
1 cm	12
1.3 cm	9

A second point concerning the height of the substrate is the influence it exerts on the center frequency. As the height is increased, the line extension representing the fringing fields' reactive contribution is also increased. For example, for a height of 3 mm, the line extension is 2 mm, while that of a height of 1 cm is 6.4 mm. With this factor, the center frequency, for a square antenna with a length of 6 cm, drops by 160 MHz.

Having established the bandwidth enhancement technique, the impact of the chosen feed configuration is now dealt with in the next section.

3.1.4 A Probe-fed, Thick Substrate Microstrip Antenna

The two previous sections described the behavior of a microstrip antenna without any feed network. The next step therefore consists of analyzing the impact of the feed network on the input impedance. In the design under review, the antenna is fed by a coaxial connector placed underneath the ground plane. Its inner conductor is extended by a probe and the latter is connected to the top conductor of the antenna. This configuration is chosen for its simplicity and its compactness since it does not add any extra volume to the overall structure.

In the traditional thin substrate antenna, the presence of the probe can be represented as an inductance and added in series to the input impedance of the resonator. Several examples were given in chapter one.

However, the impact of the probe strays from the behavior of a simple inductance as it increases in length inside the thicker substrate. Another representation of the probe is required to model its impact. In keeping with the transmission line approach, the probe impact is analyzed with the concept of a tapered transmission line. In chapter one, such a case was discussed [13]. The method adopted here tackles the problem from a different angle.

To describe the behavior of the probe impact with a tapered line implies that an impedance transformation is occurring. To demonstrate how this transformation comes

Figure 3.9 The approximate transverse representation of the field lines around the probe.

about. Figure 3.9 displays an approximate transverse representation of the field lines around the probe. The approximation is taken from Figure 2.3, itself from reference [13].

Along the resonator, the two conductors are separated by a fixed distance h and the field lines are constant. However, along the probe, the distance between the probe and the ground plane decreases. The field lines gradually cover less and less distance. Consequently, an impedance transformation occurs along the probe, and it resembles that of a tapered line. By removing the ground plane and inserting the equivalent image of the probe, a two transmission line system emerges, Figure 3.10, which is then used to establish the presence of an impedance transformation.

Figure 3.10 The probe and its image.

Geometrically, the probe and its image represent the extreme case of a tapered transmission line where the angle between the conductors is 180 degrees as shown in Figure 3.11.

Figure 3.11 The probe representing the extreme case of a tapered transmission line. The transverse fields between both sets of conductors are shown.

Unfortunately, this condition imposes a significant limit on the success of the tapered line approach. In a normal tapered line, the field lines surrounding the circumference of both conductors, in a transverse cut, are not constant as shown in the lower right corner of Figure 3.11, while those of the vertical probe and its image, around their circumferences, are constant. Consequently, the definitions of the distributed capacitance, inductance and conductance calculated for the standard transmission line geometry will carry an error when used to model the probe impact. The error occurs in the distributed elements because they are calculated from the transverse field distribution [25]. Without knowing the true nature of the field distribution around the probe, it is difficult to predict the error. For the purpose of this analysis, an empirical correction is applied to the theoretical results to provide an interpretation of the source of the error in lieu of a full scale analysis. Before tackling this issue, the tapered line approach is presented.

With the change along the probe being gradual, reference [9] indicates that the input impedance of a tapered line can be calculated by segmenting the line and treating each step as a standard transmission line. Subsequently, the input impedance of one step becomes the load of the previous step, Figure 3.12.

Figure 3.12 The segmentation of the tapered line.

To accommodate the vertical position of the conductors the distance between each pair of steps is chosen as the height existing between their middle points as shown in Figure 3.13. Then, in keeping with the definition of two-wire lines, the distance is augmented by twice the radius of the probe.

Figure 3.13 The distance, D , between each pair of transmission lines.

For each step, the characteristic impedance can now be calculated using the distributed elements of a two-wire line. The resonator input impedance, previously defined, provides the initial load condition required to lead to the calculation of the final input impedance.

3.1.4.1 Coaxial Connector to Ground Plane Capacitance

An extra correction factor to the center frequency arises because of the finite thickness of the ground plane, surrounding the inner conductor of the coaxial connector. Since the coaxial connector is affixed to the underside of the ground plane, its inner conductor and the circular aperture cut in the ground plane form a small capacitive cylinder. This is shown in Figure 3.14a) .

Figure 3.14a) The physical components creating the capacitance C_g .

This geometry produces a capacitance in parallel with the input impedance of the antenna. The value of this capacitance, C_g is taken from static field theory:

$$C_g = \frac{2\pi\epsilon Y}{\ln\left(\frac{b}{d}\right)} \quad (3-15)$$

where the d is the probe radius, the b the radius of the aperture in the ground plane and Y the length in question. In addition, the connector's dielectric juts halfway up into the circular aperture. Consequently, two capacitances exist in series: one with a Teflon dielectric and the other an air dielectric.

The Figure 3.14 b) demonstrates that for a low impedance, the impact of this capacitance is very slight. However, for the same capacitance and a high impedance, the impact worsens as shown in 3.14 c).

Figure 3.14 b) The impact of the combination of the aperture in the ground and the probe for a low impedance.

Figure 3.14 c) The impact of the combination of the aperture in the ground and the probe for a high impedance.

When the experiments were conducted for this thesis, the dimensions of the aperture in the ground plane and the thickness of the ground plane varied from case to case and the individual numbers were not logged. Therefore, for the purpose of the transmission line analysis, an average set of dimensions is adopted for every case. Although the equation is only an approximation of the situation, it provides a good indication of a potential source of distortion.

Finally, the representation of the input impedance of the unmatched, thick substrate, probe-fed, microstrip antenna is simply the impedance at the input of the tapered line, in parallel with the coaxial connector to ground plane capacitance. This is shown in Figure 3.15.

Figure 3.15 The transmission line representation of the unmatched antenna.

Using this representation, in conjunction with experimental results, the impact of the probe on the input impedance is first examined, then the success and limitation of this representation are discussed.

In Figure 3.16 a), b) and c), the experimental results, the theoretical resonator

Figure 3.16 a) Input resistance of the experimental results, the theoretical resonator and the theoretical probe fed resonator.

input impedance and the theoretical probe-fed resonator input impedance are compared. The first observation concerns the credibility of the tapered line approach.

Figure 3.16 b) Input reactance of the experimental results, the theoretical resonator and the theoretical probe fed resonator.

In comparing the theoretical resonator-only curves to the experimental one, the latter's reactance is seen to be more inductive and its resistance to be higher in value. The subsequent representation of the probe by a simple inductance would not be adequate in this case since it could not explain the increase in the resistance. In contrast, the tapered line approach, in Figure 3.16, predicts the reactance and the resistance's behavioral trends. Consequently, the impact of the probe can be represented by an impedance transformation. This phenomenon is clearer when observed on the Smith Chart as shown in Figure 3.16 c).

It is important to note that the impact of the parallel capacitance representing the

thickness of the ground plane is significant for this impedance level, and that it provides only an approximation of the actual variation. In this example, it represents a shift of approximately 25 ohms for the peak of the resistance and the positive peak of the reactance.

Figure 3.16 c) Smith Chart of the input impedance's experimental results, $\square$, the theoretical resonator, $+$, and the theoretical probe fed resonator, $\diamond$. (frequencies: 1.6 to 2.1 GHz)

The results of this representation indicate that the probe's geometry coupled with its location under the resonator very likely generate the unwanted shifts in the input impedance of the resonator.

The results depicted in Figure 3.16 also demonstrate the limitation of the tapered line approach: a significant overestimation of the reactance and the resistance's contributions. As discussed earlier, it is assumed that the source of error resides in the definition of the distributed elements of the tapered line.

An empirical correction is discussed to shed some light on the limitation of the

tapered line approach adopted here. With a two-wire transmission line, the inductance and the conductance are derived from the capacitance of the lines [25]. The resistance of the lines does not change since the conductor area remains the same. Since the present definitions of the elements overestimate the inductance, the obvious correction lies in increasing the capacitance. Such a modification also forces a decrease in the inductance contribution since it is inversely proportional to the capacitance. Therefore, to obtain a simple empirical correction, I varied the capacitance by increments while comparing the theoretical representation to the experimental results. An empirical constant of $0.3\text{e-}11$ farads added to the existing capacitance provides a demonstration of the improvement between the correlation of the resonator impedance and the experimental results as shown in Figure 3.17 a) and b).

Figure 3.17 a) Input resistance of the experimental results, the uncorrected transmission line results and the empirically corrected transmission line results.

Figure 3.17 b) Input reactance of the experimental results, the uncorrected transmission line results and the empirically corrected transmission line results.

The nature of the correction indicates that the field between the probe and its image would be stronger than those between the standard two-wire lines, since the distributed capacitance is higher. To obtain a solution with a higher accuracy for the impact of the probe, the field analysis would have to take into account the vertical position of the probe, the conductor discontinuity occurring at the junction, where the flat conductor meets the cylindrical one, and the change in the local charge density occurring on the flat conductor at that junction. Such a scenario, however, is not pursued in this thesis.

Even though the representation of the probe fed microstrip antenna is limited in accuracy, it does explain the source of the trends in the input impedance of the antenna. By knowing the pros and cons of the feed probe and its impedance transformation nature, an appropriate matching network can be developed.

A last trend needs to be demonstrated. Previously, the modification of the inset position of the feed probe affected both the resistance and the reactance, but did not alter the center frequency. However, with the probe's impact now accounted for, a new load appears for each new inset at the probe's extremity thereby modifying the center frequency as well. This is illustrated in Figure 3.18.

Figure 3.18 The impact on the input impedance for a variation in the inset position of the probe.

The end of this section concludes the analysis of the unmatched microstrip antenna. The next section presents the matched antenna. Therefore, the inset matching network is presented and discussed.

3.1.5 Inset Matching Network

The original design of a thick substrate microstrip antenna fed with a probe was chosen for its simplicity and compactness. The required matching network should be designed along the same lines, as well as provide the necessary capacitance to counteract the unwanted inductance of the probe fed configuration. The inset matching network is a suitable network which capacitively couples the resonator to the probe, while adding zero volume to the structure. This is achieved by cutting a rectangular ring on the top conductor, around the probe tip, Figure 3.19.

Figure 3.19 The microstrip antenna with an inset matching network.

With this design, the smaller conductor is referred to as the inner conductor and the larger one as the outer conductor. In addition to reducing the input reactance, the inset ring also reduces the input resistance. Therefore, the microstrip antenna with its inset matching network can be directly connected to the coaxial connector without the need of additional matching components. In the following paragraphs, the matching network is discussed and its behavior is analyzed with the empirically corrected version of the tapered line approach.

Discussion on the Inset Matching Network

The first point to discuss is the use of the term 'matching network'. In the modified design, the rectangular ring cut on the antenna, technically, cannot be solely referred to as the matching network. There exist several contributors to the matching mechanism. The role of the ring is strictly to introduce a capacitive contribution by creating a two-conductor system between the probe and the resonator. An important effect of the rectangular ring is the introduction of a second contributor to the matching process. The inner conductor of the inset matching network acts as a transmission line thereby significantly transforming the combination of the resonator response and the capacitance prior to the probe impedance transformation. Then, a very minor modification transpires from the radiation resistance associated with the ring. Lastly, the tapered transmission line properties of the probe system, transforms its new load to an input impedance centered around the $50 + j0$ ohm point, where the adjacent frequencies are designated as the operating frequencies. Consequently, the rectangular ring, the inner conductor, the radiation resistance of the slots and the probe system constitute the matching network under consideration. This is shown in Figure 3.20.

Figure 3.20 The transmission line representation of the matched antenna.

Nevertheless, the rectangular ring and the inner conductor provide the flexibility

of the matching network, hence the use of the term 'inset matching network'. By changing the ring area and the area of the inner conductor, the resulting different loads are transformed to a variety of input impedances. The reason behind creating disparate loads lies in choosing the range of the operating frequencies. In all cases, the resonator behavior essentially remains centered near the same center frequency. With each new load, the input impedance is basically shifted up or down in resistance and reactance, subsequently putting a new range of operating frequencies around the 50 ohm point. Unfortunately, the choice of frequencies is restricted to a discrete set of values since both the resistance and the reactance changes are interdependent due to the inset property of the network.

Since a choice of range of operating frequencies exists, a range of bandwidths is also available. However, there is no direct control over the range of bandwidth. The input impedance of the frequencies adjacent to the chosen frequency of operation determines the bandwidth of operation.

3.1.5.1 The Theoretical Representation of the Rectangular Ring

The ring is represented by a static capacitance placed as shown previously in Figure 3.20. The capacitance can be assumed to be static since the dimensions of the ring are much smaller than the wavelengths involved. The value of this capacitance is based on the actual three dimensional structure. It represents the mutual capacitance between the inner conductor and the outer conductor forming the ring.

The basic procedure behind calculating a capacitance is the following. In the static domain, the capacitance representing a two conductor system is simply:

$$C = \frac{Q_s}{V} \quad (3-26)$$

The charge densities existing on the surfaces are calculated using the numerical Method

of Moments technique. Then, the total charge, Q_i , on each conductor is calculated from this charge density. The capacitance is calculated using the potential voltage difference, V , between the surfaces.

For a multi-conductor system, mutual capacitances need to be included to represent the influence of the voltage of a two-conductor sub-system, on a third conductor. The relationships between the variables for a three conductor system are then:

$$Q_{s1} = V_1 C_{11} + V_2 C_{12} \quad (3-27)$$

$$Q_{s2} = V_1 C_{12} + V_2 C_{22} \quad (3-28)$$

where the two voltages are referenced to ground, the third conductor. The mutual capacitance, C_{ij} , is sought to represent the capacitive contribution created by the two conductors forming the rectangular ring. The other capacitances C_{ii} are not needed in this case. Therefore, the basic procedure for calculating a single capacitance is simply extended to consider two two-dimensional charge densities and two independent voltage variables. The mutual capacitances C_{12} and C_{21} are equal since the system is passive, therefore either voltages can be used to calculate C_{ij} while the second voltage is forced to zero. The algorithm implemented to execute those calculations is explained in Appendix A. In Table 3.3, the capacitances associated with examples of the dimension of the ring and the inner conductor's dimensions are given. The nomenclature of the dimensions are tied to the position of the antenna in Figure 3.21.

Figure 3.21 The nomenclature of the dimensions of the inset matching network.

Table 3.3 Capacitances of the inset matching networks

cases	1c	1d	1s	7m	7c
top gap	1.0 mm	2.0 mm	1.0 mm	2.0 mm	1.0 mm
bottom gap	1.0 mm	2.0 mm	1.0 mm	2.0 mm	1.0 mm
right gap	1.0 mm	1.0 mm	1.0 mm	2.5 mm	1.0 mm
left gap	1.0 mm	1.0 mm	1.0 mm	2.0 mm	1.0 mm
top edge	4.0 mm	3.0 mm	3.0 mm	3.5 mm	7.0 mm
inner W,L	1cm x 1cm	1cm x 1cm	14cmx12cm	12cmx13cm	1cm x 1cm
conductor	inset 1.0cm	inset 1.0cm	inset 1.0cm	inset 1.3cm	inset 1.3cm
C_{ij}	4.341e-13 F	4.116e-13 F	5.751e-13 F	4.881e-13 F	4.366e-13 F

The behavior of the inset ring is parallel to that of a normal capacitor. As the distance between the two conductor increases, the capacitance is reduced, 1c to 1d, and as the inner conductor is increased, the capacitance follows suit, 1c to 1s.

The radiation resistance of the ring is calculated by breaking it down into four

slots. Each slot's radiation resistance is calculated according to the dimensions shown in Figure 3.22, then they are added in parallel since they are all joined by the same two conductors. The resulting value is then placed in parallel with the overall capacitance of the ring. The calculation of these resistances is the same one used with the slots supporting a uniform electric field give in Appendix A.

Figure 3.22 The four-slot representation of the rectangular ring.

3.1.5.2 The Theoretical Representation of the Inner Conductor of the Inset Matching Network

The representation of the impedance transformation properties of the inner conductor poses a problem since the design is of an integrated nature. However, a reasonable approximation of the situation can be achieved by considering the following points. By definition, the calculated mutual capacitance represents the total capacitance contribution of the rectangular ring. This capacitance value is distributed around the inner conductor. Then, each differential element of the capacitance is transformed halfway across the inner conductor to the probe. To approximate this distributed behavior within the context of the transmission line approach, the known total capacitance value is placed in series at the junction of the two transmission lines representing the antenna with an inset feed position, then the resulting combination is transformed down a transmission line of width equal to the inner conductor's and half the value of its length. It's important that the two transmission lines be joined in the model at AA' , despite the presence of the ring, to represent the areas on each side of the rectangular ring which are joined at the inset

position. The orientation of the transmission line representation of the inner conductor is chosen to conform with the orientation of the other transmission lines in the model as was shown in Figure 3.20.

Discussion of the Impact of the Inset Matching Network on the Input Impedance of the Antenna

The rectangular ring and the inner conductor dimensions establish the range of operating frequencies of the antenna. The dramatic improvement brought by this matching network over the unmatched antenna is surprisingly easy to achieve. The simplicity of this matching network is its most attractive feature. There exists three types of variations with the rectangular ring and the inner conductor: the size of the area of the ring, the size of the area of the inner conductor and the inset position of the probe. Each type of variation is addressed by comparing results from the empirically corrected transmission line representation.

3.1.5.2 Impact of the Inset Position of the Feed Probe

The inset position of the probe is actually a dimension which has to be decided upon in the first steps of designing the antenna since it has been shown to alter the center frequency. The operating frequency is also modified, in a different manner, by the changing inset value. Upon observing the experimental results, the frequency of operation usually sits in the band of frequencies located in the lower peak of the reactance curve. As depicted in Figure 3.23, a higher value of inset reduces the peaks of the resistance and the reactance thereby pushing the 50 ohm point to a lower range of operating frequencies.

Figure 3.23 Impact on the input impedance for an increase of the inset position of the feed probe.

In addition, as the curves become more compact, the potential bandwidth is increased. This modification is quite important since it offers an added degree of freedom. Another consequence of this degree of freedom is the modification of the operating frequency to antenna dimensions ratio. The lower the frequency for a given antenna, the larger the beamwidth of the radiation pattern.

3.1.5.4 Impact of the Inner Conductor of the Inset Matching Network

The size of the area of the inner conductor also has an effect on the input impedance. As the area is increased, the capacitance increases, therefore the capacitive reactive contribution is reduced. Ultimately, the input reactance increases as shown in Figure 3.24. By doing so, the zero reactance point can be pushed to lower frequencies. Consequently the size of the inner conductor provides another limited degree of freedom

in the choice of the operating values. However, the example in Figure 3.24 demonstrates the limit of such a degree of freedom; the inner conductor can be too large and the zero reactance point missed entirely. The resistance curve is affected mostly at its peak value.

Figure 3.24 Impact on the input impedance for an increase in the dimensions of the inner conductor.

Both these physical modifications have a limited range of values due to the restricted space on the top conductor of the antenna. The range of dimensions depends on the length and width chosen.

3.1.5.5 Impact of the Rectangular Ring

The size of the area of the rectangular ring affects the overall input impedance in a systematic way. The modifications in the reactance curves and in the resistance curves are shown in Figure 3.25, where two cases have the same inner conductor dimensions but different ring areas.

Figure 3.25 Impact on the input impedance for an increase in the ring area.

In the Table 3.4, the static capacitances for the two-conductor system are given. The larger ring area gives the smallest capacitance, as expected, which in turn pushes the input reactance in the negative area.

Table 3.4 Static capacitance of the rectangular ring

Antenna Dimensions		Inset Matching Network Dimensions	
length	6.0 cm	inner conductor	gap widths
inset	1.0 cm	1.0cm x 1.0cm	1cm x 1cm x 1cm x 1cm
height	1.3 cm	1.0cm x 1.0cm	1cm x 1cm x 2cm x 2cm
static capacitance		4.341e-13 F	4.1161e-13 F

Consequently, the ring area is used to move the input impedance curve to the vicinity of

the 50 ohm point once the operating frequency has been obtained. The Smith Chart shown in Figure 3.26 illustrates the movement of the impedance response near the 50 ohm point, which is located in its middle.

Figure 3.26 The Smith Chart for the impact on the input impedance for an increase of the ring area. An increase of the ring area, from $\perp$ to $+$, rotates the response down in the capacitive portion of the Smith Chart.

With the knowledge of the impact of the physical parameters, the construction of the microstrip antenna with an inset matching network becomes a relatively easy task. In the next section, a design procedure is elaborated to facilitate the completion of a prototype.

3.1.6 Design Procedure for the Microstrip Antenna with an Inset Matching Network

The antenna's physical dimensions are chosen to accommodate three principal performance parameters: the center frequency of operation, the bandwidth and the beamwidths of the radiation patterns. Since the antenna is an integrated design, the performance parameters are unfortunately all interdependent. Consequently, the number of different available performance parameters are limited. Nevertheless, if the following sequence of choices is adopted, the flexibility of the design emerges.

The physical parameters should be chosen in the following order: the height of the substrate, the dimensions of the top conductor and the inset, then the size and position of the rectangular ring.

The center frequency is the fundamental component of the whole design. Once all the corrections are accounted for, it varies slightly with the choice of the rectangular ring. However, this center frequency is chosen in the second step since it is highly dependent on the substrate height.

The height of the substrate roughly decides the bandwidth capacity of the antenna and thus is chosen first. Next, a trial set of dimensions for the length, width and height of the unmatched antenna are chosen and the center frequency noted. Knowing, from the experimental results, that the operating frequency is located above the center frequency and sits in the lower peak of the reactance, and that the rectangular ring will increase the center frequency slightly, the dimensions of the unmatched antenna are adjusted until the operating frequency is in the right location of the reactance curve. Then, from the trends previously demonstrated, the general type of rectangular ring and inner conductor is chosen and applied. Next, the position of the operating frequency range on the reactance curve is verified. It can be further adjusted by changing the length and width or the inset position. After a few iterative steps, a prototype design is obtained.

3.1.7 Commercial Software *Ensemble*

The commercial software *Ensemble* provides the second source of theoretical results used to validate the design under review. The theoretical calculations are based on a full-wave approach using the mixed-potential integral equation formulation along with the Methods of Moments.

The results were obtained from version 3.0 of the software. Although this version accommodates two substrate layers, it cannot simulate a probe reaching up to the top substrate. Consequently, the dielectric permittivity, calculated to represent the dual dielectric substrate devised for the transmission line representation, must also be used with this model.

In addition, the analysis assumes a uniform current on the probe. The manual indicates that the results may start to deteriorate when the height of the substrate is beyond 0.05λ and that they can become unacceptable past 0.1λ . The substrate heights considered herein fall between the above two limits, consequently, the success of the results will be limited. Despite the loss of accuracy, the results have proven to be adequate in validating the behavior of the design.

3.2 Radiation Pattern Analysis

The analysis of the radiation pattern of the microstrip antenna with an inset matching network is the next topic of discussion. Three sources of radiation were identified in the impedance analysis: the fringing fields of the top conductor, the fringing fields of the rectangular ring, and the feed probe. The radiation patterns of the sources are examined separately. Then, a composite pattern is presented and the results are discussed.

3.2.1 Dominant Fringing Fields

The dominant fields used in the impedance analysis were developed using the method presented in [16]. To facilitate the eventual construction of the composite pattern, a different but equivalent approach, [15], is used for the development of the radiation patterns. In this paper, the slots are parallel to the top conductor and placed around its perimeter. The coordinate system is positioned such that the x axis is along the width of the antenna, the y axis along the length and the z axis is pointing above it as shown in Figure 3.27. Since the theta angle originates at the z axis, the range of interest is from -90 to $+90$ degrees. The phi angle originates at the x axis and is restricted to the xy plane, therefore the principal cuts are at 0 and 180 degrees, and at 90 and 270 degrees. The origin is placed in the middle of the top conductor. The results of the far fields expressions are identical to those of [16] when the following considerations are adopted. In keeping with the impedance analysis, the length of the antenna remains the physical length plus the double line extension contribution. This effective length extends from the middle of one end slot to the middle of the other. Then, the width of the slot is taken as the height of the substrate, as is the case in [16]. For the side slots, the slot width is again the height of the substrate, while the distance between the middle points is the physical width of the antenna. With these dimensions, both methods produce the same beamwidths for the principal cuts. The advantage of the technique in [15] lies in the clear separation of the co-polarized fields and the cross-polarized fields.

The far field expressions are calculated using the Fourier Transform of the Cartesian coordinates of the excitation's electric field inside the slots. The slots at each end of the antenna produce the y coordinate of the Fourier Transform and in the final expression will represent the co-polarized signal. The side slots produce the x coordinate of the Fourier Transform and they will represent the cross-polarized signal.

Figure 3.27 The representation of the fringing fields of the microstrip antenna by four slots placed around the perimeter of the top conductor.

Within the context of the Fourier transform, the phases, normally attributed to the array factor, are added to the expressions. Finally, the fields are converted to the spherical coordinates and into the far field expressions, in the same way as in [16].

The definition of the co- and cross-polarization is the third definition of Ludwig [7]. Therefore, the linear polarizations herein are those that can be measured using the standard transmitter-receiver set-up. The transmitter is fixed in azimuth and elevation, and can rotate around its horizontal axis when it is placed vertically, while the receiver is fixed in elevation and the signal is recorded as it rotates in the azimuth. It also can be rotated around its horizontal axis to provide a direction parallel or perpendicular to the transmitter. Consequently, according to the above coordinate system, the azimuth of the receiver is always the range of θ from $+90$ to 0 degrees. The E plane pattern is the signal recorded in the plane created by the z axis and $\phi=90$ and 270 degrees, and the H plane pattern is measured in the plane where $\phi=0$ and 180 degrees. Subsequently, based on (3-29), within the E plane pattern, the co-polarized signal is the θ component

of the electric far field and the cross-polarized signal is the phi component. Within the H plane pattern, the co-polarized signal is the phi component, while the theta component provides the cross-polarized signal. The generic far field expression is the following, the E_x and the E_y are the rectangular far field coordinates.

$$E(r) = \frac{jke^{-jkr}}{2\pi r} i_\theta [E_x \cos(\phi) + E_y \sin(\phi)] + i_\phi [E_x \sin(\phi) + E_y \cos(\phi) \cos(\theta)]$$

(3-29)

In looking at the equation, the co-polarized H plane cut is the E_y of the phi component. Since it is multiplied by a cosine of the theta angle, the ideal pattern cut will always go to zero at the ends. Consequently, the H plane pattern is always expected to produce a single lobe.

A different scenario exists for the E plane due to the array of the two slots. Since the effective length is often longer than the half-wavelength value of the operating frequency, the creation of sidelobes is now possible. Whether the presence of sidelobes or nulls in the pattern becomes problematic is very much application dependent. Its co-polarized signal is the E_y of the theta component.

In Figure 3.28, the E plane cut of the dominant contributor to the antenna pattern is shown superimposed with the H plane cut. In other words, the E plane represents the elevation cut of the antenna when it is placed lengthwise vertically. The H plane then represents the azimuth cut, width-wise.

Figure 3.28 The principal cuts of the dominant fields of the antenna radiation pattern.

The cross-polarized signals for the H plane and the E plane are respectively the E_x term of the theta component and that of the phi component. In the theoretical patterns, the cross-polarized signals of the principal cuts are zero. In the E plane, this zero occurs since the magnetic currents of the side slots are out of phase and thus cancel along the length of the antenna, as directed by the phase factor. The H plane's zero value results from the reversal of phase occurring in the cosine distribution along the side slot which provide a cancellation along the width of the antenna. However, the nulls are often not visible due to various reasons. These will be discussed in the composite pattern section.

3.2.2 Rectangular Ring Field

Since the rectangular ring is an aperture by nature, it inevitably becomes a radiator. To provide a first approximation of its radiation contributions, the ring is

expression to bring the response to the middle of the antenna.

The radiation resistance of the probe can be calculated using the far field expressions. With the present dimensions, the radiation resistance is of the order of 10^{-2} , therefore its contribution is negligible in the presence of the other contributions in the impedance analysis.

3.2.4 Composite Radiation Pattern

The composite radiation pattern is built by adding each of the three sources of radiation with its appropriate phase. The coordination of all the phases requires some extra calculations and one subsequent modification. These are shown along with the calculation of the directivity of the theoretical pattern.

Within the context of the far field expressions, the phases associated with the array factors were included. At this point, the phase of the signal at each of the sources must be calculated. This is accomplished by placing an arbitrary voltage at a certain point and calculating the phases at each source point. The transmission line representation is used to obtain the required phases. Since the transmission line representation is broken into two parts, the independent arbitrary voltage is not assigned at the input of the antenna, but at the junction between the two parts, Figure 3.29. It has a magnitude of one and a phase of 0 degrees.

Figure 3.29 The arbitrary voltage of magnitude of one and phase of 0 applied at the port BB'.

The current required to produce this arbitrary voltage at the tip of the probe is calculated using the tapered transmission line representation. Then, using *Libra's* harmonic balance analysis, the phases of the voltages occurring at the ends of the antenna, as well as that across the static capacitance are calculated. In addition, the voltages' maximum magnitude can be recorded and used to calculate the maximum electric field magnitude present in each aperture.

Since the transmission line representation lumps the rectangular ring into one parallel RC sub-circuit, each of the radiating four slots are then given the same voltage magnitude and phase calculated with the *Libra* algorithm.

The required modification previously mentioned concerns the relationship between the two end slots. In the *Libra* algorithm used to calculate the voltages' phase and magnitude, the phase reversal due to the approximate half-wavelength configuration occurring between the ends of the antenna is inherently included in the results. Consequently, it must be removed from the derivation of the far field expressions.

In the derivation of the far field expressions, the phase reversal of the excitation fields of each slot forced their magnetic current to be in phase. Therefore, the fields of the second slot were simply added to the first slot. To remove the impact of the phase reversal, the contribution from the second slot is now subtracted from the first one. By doing this, the individual phases derived for each slot in *Libra* can be used.

The directivity to be used in the theoretical representation is calculated from its definition. According to [24], the directivity is the ratio of the radiation intensity to the average radiation intensity:

$$D = \frac{\frac{1}{2} \Re(E \times H^*) r^2 \bar{r}}{\frac{1}{2\pi} \iint \frac{1}{2} \Re(E \times H^*) r^2 d\Omega} \quad (3-30)$$

With several manipulations of the above quantities (given in [24]), the directivity becomes:

$$D = \frac{|F(\theta, \phi)|^2}{\iint |F(\theta, \phi)|^2 d\Omega} \frac{1}{2\pi} \quad (3-31)$$

The variable F denotes the normalized electric far field, the r , the distance vector and the $d\Omega$ the unit solid angle. Since the microstrip antenna radiates over a hemisphere, the usual 4π is replaced by half its value. The algorithms for the radiation resistances are used to evaluate the denominator. This value is then incorporated in the composite pattern algorithm.

3.2.4.1 Results of the Composite Radiation Pattern

The co-polarized signals of the composite radiation pattern are first examined, then the cross-polarized signals are discussed.

Co-polarized Signal

The E plane cut from the dominant co-polarized fringing fields is shown in Figure 3.30. In this example, the end slots have a different voltage and the phase difference is different than 180 degrees. Superimposed along with it is the ideal E plane cut of the same antenna, with only a 180 degree phase difference and even voltages. The extra phase difference between the two sources forces a limited amount of beam scanning to occur. Subsequently, the side lobe's impact is worsened and the direction of

Figure 3.30 The E plane cut of the dominant fringing fields with uneven voltages and offset phase superimposed with the cut of even voltages and a 180 degree phase difference.

the maximum directivity is pushed away from the center. The reason behind the extra phase stems from the difference between the center frequency, which corresponds to the effective half-wavelength dimension of the antenna, and the operating frequency in question. In most of the results gathered, the operating frequencies are higher than the center frequency, therefore the antenna is always longer than the operating half-wavelength, and the beam scanning is always present. In a few experimental prototypes, the operating frequencies were lower than the center frequency and the sidelobe was not present. Whether beam scanning is a problem or not, is application dependent.

In the H plane, the beam scanning is not present because the pattern of the array factor is omnidirectional in this plane. Therefore, the H plane pattern remains a single lobe.

The impact of the co-polarized contribution of the rectangular ring added to the dominant radiation pattern is shown in Figure 3.31 a) curve B. Included also is the impact of strictly the dominant pattern, curve A.

Figure 3.31 a) The E plane cuts of the composite pattern. The A curve denotes the pattern of the dominant source, the B curve the pattern of the dominant source and the ring and the C curve the pattern of the three sources.

Figure 3.31 b) The H plane cut of the composite pattern.

The power radiated from the two slots generating the co-polarized signal of the ring worsens the beam scanning in the E plane. In the H plane, the contributions from the slots cancel each other due to their alignment in this direction. Therefore, no change occurs in the single lobe as shown in Figure 3.31 b).

The curve denoted C shown in the Figure 3.31 a), is the complete pattern grouping the dominant fields, the rectangular ring fields and the probe field. Since the probe has only a theta component, it does not affect the H plane's co-polarized field. In the E plane, if the probe was indeed radiating like a top-loaded monopole above a ground plane, the extra radiated power would smother the null and increase the beamwidth of the main lobe.

Cross-polarized Signal

The composite pattern of the cross-polarized signal is more difficult to predict because there is not one dominant field contribution. Therefore, with the combination of three sets of approximations for three sources of low power levels, the accuracy of the predicted patterns is more questionable. For the purpose of this thesis, a theoretical pattern is not pursued for these reasons. However, a discussion on each source of cross-polarization is presented.

The dominant far field expressions are derived based on the fact that the electric field is uniform in the end slots and that a cosine exists in the side slots. This response forces a null along both principal planes. However, knowing from the harmonic balance analysis of *Libra* that the voltages in each end slot are unequal and that the phase difference is not 180 degrees, the non-ideal response will most likely not be zero. This is the case for the null in the H plane; when the voltage is uneven at each end of the side slots, the expected cancellation along the width of the antenna will not happen since both sides of the cosine will not be equal. Along the E plane, to suppress the null, the voltage distribution in one end slot would have to vary along its length thereby forcing the distribution in each side slot to be different. However, since the antenna is symmetric relative to the y axis, it is assumed that both side slots will most likely have similar voltage distributions, ensuring a significant null in the E plane direction. This null would

also be visible in the middle of the H plane.

The cross-polarization contribution of the rectangular ring is simply the inverse of its co-polarized counterparts. The two signals cancel in the E plane and do not contribute to the overall cross-polarized signal, while they contribute some power to the H plane. A null would exist in the middle of the H plane pattern due to the E plane's cancellation.

The probe contributes only to the cross-polarized signal of the H plane. It has the same doughnut shape regardless of the direction. Its pattern therefore has the same shape as the other contributors.

Assembling all the observations concerning the cross-polarized signals, the composite pattern would most likely have a very small contribution of power in the E plane and a more significant one in the H plane resembling the transverse cut of the doughnut shape.

Chapter four

Theoretical and Experimental Results

This chapter presents a comparison between the theoretical and experimental results for the microstrip antenna with an inset matching network. In addition, several more sets of experimental data are presented to complete the characterization of the design under review.

4.1 Comparison between Theoretical and Experimental Results

4.1.1 Input Impedance Responses

The comparison between results is broken down into two sections. First, the results from the transmission line analysis and the experimental results are examined. Then, comparisons are carried out between the *Ensemble* results and the experimental ones.

4.1.1.1 Transmission Line versus Experimental

In chapter two, a general case comparing the two sets of input impedance results was presented in order to demonstrate the limitation of the analytical representation. This issue is revisited briefly, then, the validity of the trends predicted by the transmission line analysis is demonstrated.

The limitation of the transmission line approach resides in the representation of the field surrounding the probe. As was mentioned before, to obtain a solution with a higher accuracy, the field analysis would need to take into account the vertical position of the probe, the conductor discontinuity occurring at the junction where the flat conductor meets the cylindrical one, and the change in the local charge density occurring on the flat conductor at that junction. Furthermore, any changes within the inset matching network, as well as the position of the probe along the inner conductor, modifies the field distribution and also would need to be accounted for in the analysis. Consequently, the empirical correction provided earlier is useful for demonstrating where the tapered line approach is limited, but if it was to replace a rigorous analysis, it would have to be converted to an empirical equation with the physical dimensions as the independent variables.

Figure 4.1 a) Variation of the resistance for an increase in the ring area for the experimental results and the theoretical probe fed resonator. (from 1c to 1d)

Since the error in the tapered line approach is systematic in nature, it allows the

transmission line analysis to predict the correct trends in the performance parameters according to the modifications in the physical parameters. To demonstrate this, the following comparisons are made using the uncorrected versions of the transmission line analysis and the corresponding experimental results.

In Figure 4.1 a) and b), the reduction in the resistance and the reactance, for the increase in the ring area, (1c to 1d), is visible in the experimental results as well as the theoretical ones.

Figure 4.1 b) Variation of the reactance for an increase in the ring area for the experimental results and the theoretical probe fed resonator. (from 1c to 1d)

In figure 4.2 a) and b), the greater inset value is reflected in the compressed response of the reactance and the resistance (1a to 7a).

Figure 4.2 a) Variation of the reactance for an increase in the inset position of the probe for the experimental results and the theoretical probe fed resonator. (from 1a to 7a)

Figure 4.2 b) Variation of the resistance for an increase in the inset position of the probe for the experimental results and the theoretical probe fed resonator. (from 1a to 7a)

In figure 4.3 a) and b), the impact of the inset modification is visible in the compressed curves, (9s to 7m), but in addition, the larger inner conductor of the smaller

Figure 4.3 a) Variation of the resistance for an increase of the inset value and a decrease of the inner conductor for the experimental and the theoretical probe fed results. (9s to 7m)

inset, 9s, pushes its reactance higher in value. This distinction is visible since the two reactance curves do not cross in their upper part as they do in figure 4.2 b).

It is important to note that in the graphs such as these, the markers, $\square$, $+$, $\diamond$ and $\times$ are used to differentiate the curves. They do not represent specific values.

Figure 4.3 b) Variation of the reactance for an increase of the inset value and a decrease of the inner conductor for the experimental and the theoretical probe fed results. (9s to 7m)

The microstrip antenna with an inset matching network is easy to build once all the design trends are known. The transmission line analysis has provided an important understanding of these design trends.

4.1.1.2 Ensemble versus Experimental

The commercial software *Ensemble* also has a limitation where the probe impact is concerned. An interpretation of this limitation is provided using the results of the analysis of the transmission line representation and the experimental results. Despite this problem, the simulation results from *Ensemble* are useful in confirming the behavior of the inset rectangular ring. This is possible since the authors do not indicate any limitations on the modelling of a thick substrate microstrip structure.

In other words, the interpretation of the limitation of the probe modelling is based on the fact that the resonator behavior is being correctly modelled while the impact of the probe is not being sufficiently modelled. Therefore, using the terminology of the transmission line analysis, the limitation is interpreted as the lack of the impedance transformation attributed to the probe. To demonstrate the issue, the transmission line representation of the resonator behavior is compared to that of the uncorrected probe-fed resonator. Then, the *Ensemble* results are compared to the experimental ones and the conclusions are drawn.

Figure 4.4 a) Reactance of the transmission line resonator and the transmission line probe fed resonator for the unmatched antenna.

of/de

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Figure 4.4 b) Resistance of the transmission line resonator and the transmission line probe fed resonator for the unmatched antenna.

To begin, in Figure 4.4 a) and b), the transmission line resonator impedance and the transmission line probe-fed resonator impedance of an unmatched antenna are depicted. In this specific case, the impact of the probe increases both the reactance and the resistance.

In Figure 4.5 a) and b), the corresponding *Ensemble* results and experimental results are shown. The lower resistance in the *Ensemble* result is the first indication that the impedance transformation is missing. The fact that the reactances are similar however could result from the presence of the simplified probe model and therefore it does not provide useful information.

Figure 4.5 a) Resistance of the experimental results and the *Ensemble* results for the unmatched antenna.

Figure 4.5 b) Reactance of the experimental results and the *Ensemble* results for the unmatched antenna.

With a matched antenna, the transmission line results indicate that the impact of

the probe is the same as that of the unmatched antenna, that is, an increase in both the reactance and the resistance. However, it's important to notice that with the presence of the inset ring, the resonator response is now much lower in reactance before the impact of the probe is added as shown in Figure 4.6.

In the corresponding *Ensemble* and experimental results of Figure 4.7 a) and b), the lower theoretical resistance is again an indication of the missing impedance transformation, and this time, the difference in the reactances can also be attributed to it. Since the *Ensemble* reactance is higher than the resonator's, Figure 4.6, and lower than the experimental value, it could be interpreted as being the inductance of the simplified probe model, which is not sufficient to represent the impedance transformation of the capacitive resonator response. However, since the reactance is significantly more capacitive than it was in the unmatched antenna, this indicates that the inset rectangular ring is functioning as required.

Figure 4.6 The reactance of the transmission line resonator and the probe fed capacitively coupled resonator.

Figure 4.7 a) Resistance of the experimental results and the *Ensemble* results for the capacitively coupled antenna.

Figure 4.7 b) Reactance of the experimental results and the *Ensemble* results for the capacitively coupled antenna.

The last indication that the *Ensemble* results are lacking an impedance transformation is present in the variation of the resistance in response to a change in the inner conductor's dimension. In the Figure 4.8, the variation in resistance for a change in the inner conductor, depicted for the transmission line resonator result and the transmission line probe fed resonator result, indicates that the probe's impact reversed the trends in resistance of the initial resonator impedance. A larger inner conductor, denoted as 1s, produces a drop in resistance for the resonator case (res.1c to res.1s) and an increase in resistance for the probe fed resonator (pro.1c to pro.1s).

Figure 4.8 Variation in resistance for a change in the inner conductor's dimensions for the transmission line resonator and the probe fed resonator.

In the corresponding *Ensemble* results, the resistance trend follows that of the resonator, not the probe fed resonator as shown in Figure 4.9.

Figure 4.9 Variation in resistance for an increase of the inner conductor for the *Ensemble* results.

With all these indications, the limitation of the *Ensemble* results most likely can be interpreted as a missing impedance transformation. Nevertheless, the results support the trends of the inset matching network demonstrated in chapter two with the transmission line analysis. They predict the rise in inductance for an increase in the inner conductor, the lowering of the reactance as the slots are enlarged as shown in Figure 4.10 a) and b), and the compression of the impedance curve as the inset is increased as shown in Figure 4.11 a) and b). The Figures 4.10 and 4.11 compare the experimental trend to the *Ensemble* trend.

Figure 4.10 a) Variation of the resistance for an increase in the ring area for the experimental results and the *Ensemble* results. (from 1c to 1d)

Figure 4.10 b) Variation of the reactance for an increase in the ring area for the experimental results and the *Ensemble* results. (from 1c to 1d)

Figure 4.11 a) Variation of the resistance for an increase in the inset position of the probe for the experimental results and the *Ensemble* results. (from 1a to 7a)

Figure 4.11 b) Variation of the reactance for an increase in the inset position of the probe for the experimental results and the *Ensemble* results. (from 1a to 7a)

4.1.2 Radiation Patterns

The radiation patterns of the experimental prototype 7m are now compared to their theoretical representation. The composite radiation pattern consists of the contributions from the antenna and from the rectangular ring. In Figure 4.12 a), the E plane co-polarized signals of both cases are shown, while those of the H plane are in 4.12 b) for a frequency of 1.9 GHz. In addition, the measured cross-polarized signal of both planes are added to the figures.

Figure 4.12 a) The theoretical results (_) and the experimental results (...) for the E plane of the prototype 7m.

Figure 4.12 b) The theoretical results (_) and the experimental results (...) for the H plane of the prototype 7m.

The beam scanning predicted by the composite pattern consisting of the dominant field and the rectangular ring field is visible in the co-polarized experimental results. Since in the theoretical case, the contribution of the probe smothered the nulls of the sidelobe, this discrepancy would lead one to believe that the probe's radiation is not reaching the overall far field. Confirmation of this observation would require a series of new measurements designed to isolate the radiation of the probe. This issue will not be pursued here.

The cross-polarized results support the hypotheses presented earlier. The cross-polarized signal in the E plane is low while that of the H plane has the double peak of the general doughnut shape.

The theoretical and experimental maximum gains and beamwidths (bmw) are included in Table 4.1.

Table 4.1 Performance parameters of the radiation pattern of the prototype 7m

	Maximum Gain	- 3dB E bmw	-3dB H bmw
<i>theoretical</i>	8.0 dBi	60 °	76 °
<i>experimental</i>	7.9 dBi	72 °	76 °

4.2 Experimental Results

4.2.1 Input Impedance Measurements

A series of prototype results are presented below to demonstrate the bandwidth ranges for operating frequencies between 1.85 GHz and 1.95 GHz, for an antenna with a top conductor of dimensions of 6.0 cm x 6.0 cm.

In Table 4.2, the dimensions, the center frequency of operation and the bandwidth for a return loss of -15dB and -10dB are given for four prototypes. Figure 4.13 contains the return loss for each of the cases. The center frequency of operation is denoted by f_{op} . The prototype's nomenclature is the same as that from chapter two, Figure 3.21.

Table 4.2 Physical and performance parameters of four experimental prototypes

prototype	1b	9s	7m	10x
inset	1.0 cm	1.0 cm	1.3 cm	1.0 cm
top edge	4.5 mm	3.0 mm	3.5 mm	2.5 mm
inner cond.	1.0cm x 1.0cm	1.4cm x 1.2cm	1.2cm x 1.3cm	1.0cm x 0.95cm
top gap	0.5 mm	2.0 mm	2.0 mm	3.0 mm
bottom gap	0.5 mm	2.0 mm	2.0 mm	3.0 mm
right gap	0.5 mm	2.0 mm	2.5 mm	2.0 mm
left gap	0.5 mm	2.0 mm	2.0 mm	3.0 mm
f_{op}	1.990 GHz	1.945 GHz	1.905 GHz	1.945 GHz
BW (-15 dB)	130 MHz	137 MHz	155 MHz	205 MHz
BW (-10 dB)	235 MHz	270 MHz	300 MHz	500 MHz

The first prototype, 1b, has the smallest inner conductor and the smaller of the two inset values. In addition, it also has the smallest bandwidth. In the second prototype, 9s, by increasing the inner conductor and adjusting the slots accordingly, the bandwidth is improved slightly and the frequency of operation is lowered. This change occurs since

Figure 4.13 Return loss for each of the prototypes in Table 4.1.

the portion of the reactance curve where it is more compressed. Since an increase in the inset position of the curve compresses the whole reactance curve, it also improves the bandwidth as well as lowers the operating frequency. Such is the case with the third prototype, 7m. Finally, the last prototype, 10x, shows a significant improvement in bandwidth over the other results because its height is increased.

A second set of experimental results are presented to demonstrate the transition between the unmatched antenna and a set of the matched ones. Illustrated in figure 4.14 a) and b)

Figure 4.14 a) The reactance of the unmatched antenna and the unmatched antennas of Table 4.3.

Figure 4.14 b) The resistance of the unmatched antenna and the matched antennas of Table 4.3.

is the fact that the center frequency remains closely aligned in the transition from the unmatched antenna to the matched antennas. Therefore, it can be chosen before the inset ring is cut. Table 4.3 gives the dimensions, center operating frequency and the bandwidths of the prototypes involved. Within these three cases, only the inner conductor and the slots were varied. The length and width are 6.0 cm each, the height is 1.3 cm and the inset is 1.3 cm. The Smith Chart representation of the results given in figure 4.14 c) demonstrates how the different dimensions of the inset matching network move the impedance response around the 50 ohm point.

Figure 4.14 c) The Smith Chart representation of the cases from Table 4.3.

Table 4.3 Physical and performance parameters of the matched prototypes

prototype	7j	7m	7k
inner cond.	1.0cmx1.0cm	1.2cmx1.3cm	1.0cmx1.1cm
top edge	6.0 mm	3.5 mm	3.5 mm
top gap	2.0 mm	2.0 mm	1.9 mm
bottom gap	2.0 mm	2.0 mm	2.0 mm
right gap	0.5 mm	2.5 mm	1.0 mm
left gap	0.5 mm	2.0 mm	1.0 mm
f_{op}	1.94 GHz	1.905 GHz	1.935 GHz
BW -15dB	150 MHz	155 MHz	162 MHz
BW -10 dB	315 MHz	300 MHz	330 MHz

4.2.2 Radiation Patterns

The radiation patterns of the prototype 10x with a substrate height of 1.6 cm are now shown. In Figure 4.15 a), the E plane co- and cross-polarized signals are depicted, while the H plane is in Figure 4.15 b). The frequency is 1.852 GHz.

Figure 4.15 a) The E plane cut of the prototype 10x.

Figure 4.15 b) The H plane cut of the prototype 10x.

In the examples given in Table 4.1, with an increase in height of approximately 3 mm, or one eighth of an inch, the bandwidth increases from 8% to 10% for the -15dB return loss. In the H plane pattern, the cross-polarization of this case is worse than that of the lower substrate height for approximately the same frequency.

The maximum gain and the -3dB beamwidths (bmw) are given in Table 4.4.

Table 4.4 Performance parameters of the radiation pattern of the prototype 10x

Maximum Gain	-3dB E bmw	-3dB H bmw
7.4 dBi	70 °	78 °

Chapter five

Dual-Polarized Microstrip Antenna with Inset Matching Networks

The microstrip antenna, with its square geometry, can easily support two orthogonal modes of propagation. To excite the second mode, a feed probe is simply inserted in the direction orthogonal to the first probe. By choosing the half-wavelength configuration, the ideal voltage standing wave has a null along this orthogonal direction. Since a current path ideally does not exist in this direction, the second probe is effectively isolated from the first one. Practically, the isolation is finite, but it is sufficient to demarcate two independent ports.

This chapter presents the experimental results of the dual-polarized microstrip antenna with inset matching networks.

5.1 Design Procedure and Experimental Results

The one port version of the microstrip antenna has been sufficiently characterized to facilitate its design procedure. The jump from a one port to a two port antenna is easy since both ports are relatively independent of each other due to their orthogonality. The design procedure for building a dual port antenna is the following. First, a one port antenna is designed at an operating frequency slightly above the required one. This

difference is needed since experimental results show that the addition of a second rectangular ring in the vicinity of the first one pushes the frequency response to a slightly lower frequency range. Then, the one port is roughly matched to the 50 ohm point. The dimensions of the inset matching network serve as a starting point for the two port. The best way to adjust the matching network is to initially cut both rectangular rings according to the one port's dimensions, then work on one of them to optimized it. This method is possible since the behavior of one port is not as sensitive to modifications in the second ring as it is to modifications to its own ring. A prototype two port antenna was built from a one port version. The dual port microstrip antenna is shown in Figure 5.1.

Figure 5.1 The dual-port microstrip antenna with inset matching networks.

The impedance curves are shown in Figure 5.2 a) and b) for the one port prototype and the two port prototype. Table 5.1 gives the dimensions, the center operating frequency and the bandwidths of the two ports.

Table 5.1 Physical and performance parameters of the single port and dual-port antennas

ports	port A	port B	single port antenna
inset position	1.3 cm	1.3 cm	1.0 cm
inner conductor	1.4cmx1.12cm	1.4cmx1.2cm	1.4cmx1.2cm
top edge	3.0 mm	3.8 cm	3.0 mm
top gap	2.5 mm	2.0 mm	2.0 mm
bottom gap	2.5 mm	2.0 mm	2.0 mm
right gap	2.0 mm	2.0 mm	2.0 mm
left gap	2.5 mm	2.0 mm	2.0 mm
f_{op}	1.880 GHz	1.870 GHz	1.945 GHz
BW (-15dB)	121 MHz	133 MHz	137 MHz
BW (-10dB)	240 MHz	250 MHz	270 MHz

Figure 5.2 a) The resistance of the single port and dual port antennas from Table 5.1.

Figure 5.2 b) The reactance of the single port and dual port antennas from Table 5.1.

The lower operating frequency of the two port prototype is visible, while the similarity in the impedances indicates that the transition is possible from a one port to a two port. Since the matching networks are cut by hand, the two ports of the same antenna cannot be identical and this fact is reflected in the slight difference between the responses.

One representative example of the radiation patterns is shown in Figure 5.3 a) and b). The first cut is the E plane cut while the second is the H plane cut for a frequency of 1.9 GHz.

Figure 5.3 a) The H plane cut of one of the ports of the dual-polarized antenna.

Figure 5.3 b) The E plane cut of one of the ports of the dual-polarized antenna.

All the trends discussed for the one port antenna are visible in the results of the two port antenna.

The S_{21} parameter measured between the two ports is used as an indication of the isolation of those ports. For the -10dB return loss, it ranges from -22dB to -15 dB, and for the -15dB return loss from -21dB to -16dB.

Table 5.2 contains the gain and the beamwidths of the dual port antenna.

Table 5.2 Performance parameters of the dual-port antenna

Maximum Gain	-3db E bmw	-3dB H bmw
7.6 dBi	70 degrees	83 degrees

Chapter six

Conclusions

The investigation of the inset matching network through the theoretical analysis has provided the link between the physical parameters and the performance parameters. Therefore, the objective of this thesis has been reached. The trends in the input impedance due to variations in the dimensions of the antenna and the dimensions of the inset matching network have been demonstrated. They have provided a design procedure which enables one to quickly obtain the desired performance parameters.

The dual-polarized version of the microstrip antenna with inset matching networks was also investigated experimentally, and its performance was determined to be of the same nature as the single port antenna. Consequently, the goal of this research, the design of a wideband, dual-polarized microstrip antenna of relatively low cost, was attained.

Subsequently, the Systems and Computer Engineering department used the antenna to study certain aspects of a polarization diversity scheme applicable to indoor wireless communications. The results indicated the feasibility of such a communication scheme and also established the candidacy of the microstrip antenna with inset matching networks for this application [26].

6.1 Future research

An important advantage of the inset matching network is that it can be built relatively quickly using the results from the transmission line representation. However,

the use of this design would become more widespread if a rigorous theoretical model was developed for it. With such a model, the suitability of the performance parameters would be established theoretically without the need for experimental results. With this information, designers would be more willing to include this matching network in their systems. Consequently, future research on the inset matching network should be expended towards developing a rigorous theoretical model.

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APPENDIX A

Derivation of the Radiation Resistance Equation and the Rectangular Ring's Static Capacitance

Radiation Resistance Equation

There are two radiation resistances to calculate: one for the end radiation and one for the side radiation. The development of the electric far field for the end slot is found in [16], chapter eleven. It is based on the following coordinate system.

Figure A-1 The coordinate system for the end slots.

The field distribution inside the slot is:

$$\vec{E}_a = \vec{a}_x E_o \quad (\text{A-1})$$

and the resulting magnetic current according to the equivalence principle and Maxwell's equations is:

$$\vec{M} = -2 \hat{n} \times \vec{E}_a = -2 \vec{a}_y \times \vec{a}_x E_o \quad (\text{A-2})$$

$$\vec{M}_z = \hat{a}_z 2 E_o \quad (\text{A-3})$$

The next step requires the calculation of the electric vector potential F_v . In the reference, the author presents a short cut to this procedure by processing the information necessary only for the calculation of the far field. The derivation of the calculation of the far field is then reduced to a two-step procedure using the magnetic current as the input. First, the simplified integral of the electric vector potential is designated as L_v and calculated according to the direction of the magnetic current.

$$\vec{F}_v = \frac{\epsilon e^{-jkr}}{4\pi r} \vec{L}_v \quad (\text{A-4})$$

Then, the far fields expressions are constructed using those L_v terms. For the z directed magnetic current, the theta component of the L_v terms comes into play as:

$$\vec{L}_\theta = \iint_s -M_z \sin(\theta) e^{jk(x \sin(\theta)\cos(\phi) + z \cos(\theta))} dx dy \quad (\text{A-5})$$

$$\vec{L}_\theta = -2E_o \sin(\theta) \iint_s e^{jkx \sin(\theta)\cos(\phi)} e^{j kz \cos(\theta)} dx dy \quad (\text{A-6})$$

$$\bar{L}_\theta = 2E_o hW \sin(\theta) \operatorname{sinc} \left(\frac{kh \sin(\theta) \cos(\phi)}{2} \right) \operatorname{sinc} \left(\frac{Wk \cos(\theta)}{2} \right) \quad (\text{A-7})$$

where the integrals are evaluated over the surface of the slot. The primed coordinates indicate the surface of the source, which is the slot in this case. Then the E field is simply:

$$\bar{E}_\phi = \frac{jk e^{-jkr}}{4\pi r} \bar{L}_\theta \quad (\text{A-8})$$

$$\bar{E}_\phi = \frac{jkhWE_o e^{-jkr}}{2\pi r} \sin(\theta) \operatorname{sinc} \left(\frac{kh}{2} \sin(\theta) \cos(\phi) \right) \operatorname{sinc} \left(\frac{kW}{2} \cos(\theta) \right) \quad (\text{A-9})$$

The radiated power is the average Poynting vector of the far field integrated over the hemisphere above the microstrip antenna. The magnetic field is related to the electric field by the following linear relationship for the far field case.

$$\bar{H}_\theta = \frac{\bar{E}_\phi}{\eta} \quad (\text{A-10})$$

Therefore,

$$P = \iint_s \frac{1}{2} \Re (\bar{E} \times \bar{H}^*) \cdot \hat{a}_r r^2 \sin(\theta) d\theta d\phi \quad (\text{A-11})$$

$$P = \frac{1}{2} \Re \frac{1}{\eta} \iint_s |\bar{E}_\phi|^2 r^2 \sin(\theta) d\theta d\phi \quad (\text{A-12})$$

For the purpose of calculating the radiation resistance, these integrals are evaluated using the trapezoidal function in the commercial software package *Matlab*. Then, the radiation resistance is calculated according to the following circuit theory concept.

$$R_r = \frac{|V|^2}{2 P_r} \quad (\text{A-13})$$

$$V = h E_o \quad (\text{A-14})$$

Therefore, the expression for the radiation resistance is the following:

$$R_r = \frac{|hE_o|^2 \eta}{\left[\frac{E_o khW}{2\pi} \right]^2 \int_0^\pi \int_0^\pi (\sin(\theta))^3 \left[\text{sinc} \left(\frac{kh}{2} \sin(\theta) \cos(\phi) \right) \text{sinc} \left(\frac{kW}{2} \cos(\theta) \right) \right]^2 d\theta d\phi} \quad (\text{A-15})$$

Using the coordinate system as in A-1 with a 90 degree rotation in the zy plane, the derivation of the far field for the slot with a cosine field distribution becomes identical to the previous one except for the added term of the cosine distribution. The subsequent changes are the following:

$$\vec{E}_x = E_o \sin\left(\frac{\pi z}{L}\right) \quad (\text{A-16})$$

$$\vec{M}_z = 2E_o \sin\left(\frac{\pi z}{L}\right) \quad (\text{A-17})$$

$$\vec{L}_\theta = -2E_o \sin(\theta) \iint_s \sin\left(\frac{\pi z}{L}\right) e^{jkx \sin(\theta) \cos(\phi)} e^{jkz \cos(\theta)} dx dz \quad (\text{A-18})$$

$$\vec{L}_\theta = 2E_o \sin(\theta) \iint_s \left[\frac{e^{\frac{j\pi z}{L}}}{2j} - \frac{e^{-\frac{j\pi z}{L}}}{2j} \right] e^{jkx \sin(\theta) \cos(\phi)} e^{jkz \cos(\theta)} dx dz \quad (\text{A-19})$$

$$\vec{L}_\theta = 2E_o \sin(\theta) h L \text{sinc}\left(\frac{kh}{2} \sin(\theta) \cos(\phi)\right) \frac{j}{2} \left[\text{sinc}\left(\frac{kL}{2} \cos(\theta) + \frac{\pi}{2}\right) - \text{sinc}\left(\frac{kL}{2} \cos(\theta) - \frac{\pi}{2}\right) \right] \quad (\text{A-20})$$

$$\vec{E}_\phi \approx \frac{jk e^{-jkr} \vec{L}_\theta}{4\pi r} \quad (\text{A-21})$$

$$\vec{E}_\phi \approx \frac{ke^{-jkr}}{4\pi r} E_o h L \sin(\theta) L \text{sinc}\left(\frac{kh}{2} \sin(\theta) \cos(\phi)\right) [\text{sinc}(\dots) - \text{sinc}(\dots)] \quad (\text{A-22})$$

The radiation resistance is calculated with the same method as before.

$$R_r = \frac{|hE_o|^2 \eta}{\left[\frac{E_o khL}{4\pi} \right]^2 \int_0^\pi \int_0^\pi (\sin(\theta))^3 [\text{sinc}(\dots)] [\text{sinc}(\dots) \text{sinc}(\dots)]^2 d\theta d\phi} \quad (\text{A-23})$$

Rectangular Ring's Static Capacitance

The transmission line representation of the inset rectangular ring consists of two elements: a static capacitance and a transmission line. The derivation of the algorithm for the calculation of the static capacitance is given here.

The first step in calculating the mutual capacitance between the inner and outer conductor consists in setting up the coordinate system. In figure A-2, the conductors are shown to lie in the x-y plane. They are located at a distance h in the z direction, while their images are at $-h$.

Figure A-2 The location of the conductors for the Moment Method analysis.

Also depicted in the figure are the distance variables from the sources to the observation point P. Four such variables, one for each conductor and their image, of the following general form come into play in the equation, where the primed coordinates indicate the source and the unprimed coordinates indicate the observation point:

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \quad (\text{A-24})$$

The following equation relates the potential to the neighborhood surface charge densities ρ .

$$V = \iint_s \frac{\rho}{4\pi\epsilon R} dS. \quad (\text{A-25})$$

By expanding the charge densities to represent the four sources, and combining the related pairs, the equation becomes:

$$V = \iint_s \frac{\rho_1}{4\pi\epsilon} \left[\frac{1}{R_1} - \frac{1}{R_{-1}} \right] dS + \iint_s \frac{\rho_2}{4\pi\epsilon} \left[\frac{1}{R_2} - \frac{1}{R_{-2}} \right] dS \quad (\text{A-26})$$

Next, the charge densities are each subdivided and the sub-areas assigned an unknown two-dimensional constant charge density σ . The two conductors are divided up as shown in the Figure A-3.

Figure A-3 The subdivision of the conductors for the Moment Method analysis.

To follow the protocol of the Methods of Moments, the σ 's are the unknown coefficients of the charge density:

$$\rho = \sum \sigma_{ij} P_{ij} \quad (\text{A-27})$$

where the P terms equal one over the sub-area in question. The potential equation becomes the following:

$$V = \sum \frac{\sigma_{1ij}}{4\pi\epsilon} \iint_{s_1} \left[\frac{1}{R_1} - \frac{1}{R_{-1}} \right] dS + \sum \frac{\sigma_{2ij}}{4\pi\epsilon} \iint_{s_2} \left[\frac{1}{R_2} - \frac{1}{R_{-2}} \right] dS \quad (\text{A-28})$$

The following test function is used to convert the equation into a matrix:

$$\iint_s \delta(x - x_m) \delta(y - y_m) V(x,y) dx dy = V(x_m, y_m) \quad (\text{A-29})$$

It is evaluated over the area containing the charge densities. The coordinate point (x_m, y_m) is the middle point of each sub-area of the conductors. By applying the test function to both sides of the potential equation, the only modification in the previous potential equation is replacing the observation point coordinates (x,y) by the test function coordinates (x_m, y_m) . Subsequently, a square matrix is developed where

$$[A] [\sigma] = [V] \quad (\text{A-30})$$

and the A terms are the evaluation of the double integral. Therefore, for each set of coordinates (x_m, y_m) the double integral must be evaluated over both areas supporting a charge density. This procedure builds a square matrix with each row associated with a set of (x_m, y_m) coordinates and each column associated with an unknown charge density of one sub-area (x', y') . The voltage column represents the voltage associated with the area represented by the (x_m, y_m) coordinate.

The matrix is solved using the Gauss's method taken from reference [27].

Appendix B

Quality Factor for the TEM mode

This appendix contains the derivation of the quality factor, **Q**, for a microstrip antenna with a **TEM** mode. The coordinate system is placed in the middle of the antenna with the y axis along the length of the antenna, the z axis along its width and the x axis along the height of the substrate.

By definition, the **Q** is the ratio of the average energy over the average power loss:

$$Q = \omega \frac{(W_e + W_m)}{P_d + P_c + P_{r\ end} + P_{r\ side}} \quad (\text{B-1})$$

The following electric field is assumed to exist in the antenna structure:

$$\vec{E}_x = E_o \sin\left(\frac{y\pi}{L}\right) \quad (\text{B-2})$$

where the magnetic field is:

$$\vec{H}_z = \frac{\vec{E}_x}{\eta} \quad (\text{B-3})$$

The average electric energy is [8]:

$$W_e = \frac{\epsilon}{4} \int_V \vec{E} \vec{E}^* dv \quad (\text{B-4})$$

and it becomes:

$$W_e = \frac{\epsilon}{8} E_o^2 h W L \quad (\text{B-5})$$

The average magnetic energy is [8]:

$$W_m = \frac{\mu}{4} \int_V \left(\frac{\vec{E}}{\eta} \right)^2 dv \quad (\text{B-6})$$

and it becomes:

$$W_m = \frac{\mu}{8} \frac{1}{\eta^2} E_o^2 h W L \quad (\text{B-7})$$

The average power dissipated in the dielectric is [28]:

$$P_d = h \int_s \sigma_d |E|^2 ds \quad (\text{B-8})$$

$$P_d = h W \sigma_d E_o^2 \frac{L}{2} \quad (\text{B-9})$$

and the average power loss in the conductor is [28]:

$$P_c = 2 \int_s R |H|^2 ds \quad (\text{B-10})$$

$$P_c = 2 R \frac{1}{\eta^2} W \frac{L}{2} E_o^2 \quad (\text{B-11})$$

The radiated power is taken from the algorithms for the radiation resistances. They are combined and abbreviated in the next equation:

$$P_{r, \text{end} + \text{side}} = \frac{1}{2} \frac{1}{\eta} \frac{W^2 h^2 E_o^2 k^2}{4\pi^2} [\text{integral for } P_{r, \text{end}} + \text{integral for } P_{r, \text{side}}] \quad (\text{B-12})$$

Combining all the equations, the Q expression becomes:

$$Q = \omega \frac{\frac{E_o^2}{8} h W L (\epsilon + \frac{\mu}{\eta^2})}{\frac{W^2 h^2 E_o^2 k^2}{8 \pi^2 \eta} [\dots] + h W \sigma_{\text{eff}} E_o^2 \frac{L}{2} + 2 R W L \frac{E_o^2}{2 \eta^2}} \quad (\text{B-13})$$

and is reduced to:

$$Q = \omega \frac{\frac{1}{8} h L (\epsilon + \frac{\mu}{\eta^2})}{\frac{W h^2 k^2}{8 \pi^2 \eta} [\dots] + h \sigma_{\text{eff}} \frac{L}{2} + 2 R \frac{L}{2 \eta^2}} \quad (\text{B-14})$$

END

1 1-0 6-9 6

FIN