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X-RAY SCATTER IMAGING IN MEDICINE: MODEL AND EXPERIMENTAL VALIDATION

by

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**X-RAY SCATTER IMAGING IN MEDICINE:
MODEL AND EXPERIMENTAL VALIDATION**

Submitted by

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in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

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22 Sept 2000

Abstract

The images produced in all radiological exams today are formed by capturing the undeviated (primary) x-ray photons. The scattered field, however, can offer different and often more useful information. I have devised a simple but useful model that considers the imaging via scattered x rays of a target object against a background material of the same dimensions when both are situated within a water phantom. The model predicts that there exist specific medical regimes where x-ray scatter imaging could offer higher contrast (C) and signal-to-noise ratio (SNR) than primary imaging. For example, the imaging of blood vessels versus white brain matter in a 15-cm-thick water phantom with the forward-scattered photons (2° to 12°) results in a maximum SNR, over all energies, greater than primary for essentially all vessel sizes ≤ 23 mm. Using an 80 kV beam would require dose increases of about 47% (scatter) and 205% (primary) relative to using a monoenergetic beam.

The model was validated experimentally with plastics placed at the centre of a 15-cm-diameter spherical water phantom. For example, to image a 2-cm-thick polymethyl methacrylate/polycarbonate combination using an 80 kV beam with the primary photons we obtain $C_{\text{expt}} = 0.01 \pm 0.02$, $C_{\text{pred}} = 0.008 \pm 0.002$, $\text{SNR}_{\text{expt}}/\sqrt{K_{\text{c}}^{\text{air}}} = 0.86 \pm 1.6 \text{ (mJ/kg)}^{-1/2}$ and $\text{SNR}_{\text{pred}}/\sqrt{K_{\text{c}}^{\text{air}}} = 0.51 \pm 0.14 \text{ (mJ/kg)}^{-1/2}$, where $K_{\text{c}}^{\text{air}}$ = air collision kerma [subscripts “expt” = experiment and “pred” = prediction]. The values obtained with the $\theta = 4^\circ$ scattered field were $C_{\text{expt}} = 0.26 \pm 0.06$, $C_{\text{pred}} = 0.19 \pm 0.01$, $\text{SNR}_{\text{expt}}/\sqrt{K_{\text{c}}^{\text{air}}} = 3.8 \pm 0.8 \text{ (mJ/kg)}^{-1/2}$ and $\text{SNR}_{\text{pred}}/\sqrt{K_{\text{c}}^{\text{air}}} = 3.2 \pm 0.3$

$(\text{mJ/kg})^{-1/2}$. The significant variation of C and SNR with different form factor data emphasizes the need for more accurate x-ray diffraction measurements. This work confirms the usefulness of scattered x rays and provides a model that can be used as a tool for designing and optimizing an x-ray scatter imaging system.

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I dedicate this thesis to “baby junior” who is on the way!

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Glossary

C	Contrast
d	Target and background objects' thickness
$\frac{d_{\kappa}\sigma}{d\Omega}$	Differential scatter cross section ($\text{cm}^{-2} \kappa^{-1} \text{sr}^{-1}$)
DQE	Detective quantum efficiency
$E_{\circ}$	Incident monoenergetic photon energy (keV)
F	Coherent form factor
HPGe	High-purity germanium (type of detector)
$K_{\text{c}}^{\text{air}}$	Air collision kerma (mJ kg^{-1})
μ	Total x-ray linear attenuation coefficient (cm^{-1})
N_{b}^{p}	Detected number of photons transmitted through the background
N_{b}^{s}	Detected number of photons singly scattered by the background
N_{t}^{p}	Detected number of photons transmitted through the target
N_{t}^{s}	Detected number of photons singly scattered by the target
$\Phi_{\circ}$	Incident photon fluence (photons cm^{-2})
POLYCA	Polycarbonate
POLYET	Polyethylene
PMMA	Polymethyl methacrylate
POLYST	Polystyrene
$\Psi_{\circ}$	Incident energy fluence (keV cm^{-2})
S	Incoherent scattering function
SNR	Signal-to-noise ratio
SNR_{max}	Maximum signal-to-noise ratio over all photon energies
θ	Scattering angle
X_{ion}	Exposure recorded by ion chamber (R)

CHAPTER 1

INTRODUCTION

1.1 Conventional X-Ray Imaging

In conventional x-ray imaging, which produces projection images of the patient, each image point or pixel value is determined by the integrated total linear x-ray attenuation coefficient, μ (cm^{-1}), along a specific line. In an ideal imaging system it would be the undeviated or “primary” photons that would be collected by the image receptor to record projection x-ray images of the patient. However a large number of scattered photons also reach the receptor. The general setup for conventional transmission (primary) imaging is shown in Fig. 1. Common x-ray imaging devices that acquire projection radiographs are screen-film combinations, video-based systems coupled to x-ray image intensifiers, and photo-stimulable phosphor devices.

At diagnostic energies (15-150 keV), the x rays interact with matter through three mechanisms: photoelectric absorption, incoherent (Compton) scattering and coherent scattering. For 40 keV photons in water, 69.6% of the interactions are Compton, 11% coherent, and the rest photoelectric.¹ The scattered x rays are generally regarded as a nuisance that reduces radiographic contrast (C) and signal-to-noise ratio (SNR). Since the scattered x rays can comprise up to 90% of the x rays reaching the image receptor² much effort has been devoted to eliminate them from the signal. The two most common techniques are to place an antiscatter grid prior to the

FIG. 1. Conventional x-ray imaging geometry.

detector and to increase the air gap between the patient and detector.

Motz and Danos³ wrote a landmark paper that quantified the information content of conventional primary images as a function of patient exposure. A schematic of their model is shown in Fig. 2. The task was to image a target object against a background material based upon the x-ray attenuation properties of the materials. The variables N_t and N_b are the number of x-ray photons at detector elements A and B on the image plane. They defined C and SNR as

$$C = 2 \frac{|N_t - N_b|}{N_t + N_b}, \quad (1)$$

$$\text{SNR} = \frac{|N_t - N_b|}{\sqrt{N_t + N_b}}, \quad (2)$$

and gave quantitative relationships between C , SNR, and patient exposure. The degrading effects of scattered radiation on image quality were assessed and estimates of the minimum exposures required for the detection of image signals associated with

FIG. 2. Schematic of the model of Motz and Danos³ to quantify the information content of conventional primary images.

particular radiological exams were given. Reliable detection was based upon the Rose⁴ criterion, namely that SNR must be ≥ 5 .

1.2 X-Ray Scatter Imaging

A completely different approach to x-ray imaging is possible by imaging the *scattered* radiation instead of the primary. Consider a detector element that subtends a solid angle $d\Omega$ at a specific point within the patient and that makes an angle θ with respect to the incident beam direction. The corresponding image pixel will represent a scatter signal characterized by the differential scatter cross section, $d\sigma/d\Omega$ ($\text{cm}^2 \text{electron}^{-1} \text{sr}^{-1}$) of the point of interest. The information obtained by detecting the scattered radiation is different and in some cases more useful than that using primary radiation. While the use of scattered x rays is still uncommon clinically, a number of investigators have devised special-purpose imaging systems that use the scattered field to form the image. Both projection images and cross-sectional scatter images have been generated. Basically, there are two types of x-ray scatter imaging devices: (1)

the forward-scatter devices that detect the low-angle scatter, and (2) the backscatter devices that detect the backscattered field. In Ref. 5, Speller and Horrocks gave a review of photon scattering and described the potential usefulness of scattered x rays to bring new information in medicine and biology.

X-ray diffraction computed tomography (CT), introduced by Harding *et al.*⁶⁻⁸ is a technique that generates a cross-sectional image of the object by detecting the forward scatter. Their imaging system, reproduced in Fig. 3, used a first generation CT geometry. A pencil beam of x rays was incident on the object and the low-angle scatter was collected by a set of detectors positioned at various angles from the primary beam. In this approach it is the integrated $d\sigma/d\Omega$ that is measured and used to reconstruct a tomogram. The images are characterized by the low-angle ($\leq 10^\circ$) x-ray diffraction properties of the object. These angular dependent images offer different and often more useful information than conventional radiological images.

Recently, Westmore *et al.*^{9,10} configured a coherent scatter CT system using a diagnostic x-ray image intensifier coupled to a charged-coupled device camera. Since many biological tissues have recognizable diffraction signatures, they foresee clinical applications of their system for bone mineral content measurements.

A table-top x-ray diffraction microtomography system was developed by Grant *et al.*¹¹ to obtain images of the internal structure of objects of low mass density at sub-millimetre resolution. Their phantom was composed of two cylindrical polyethylene sub-vessels which were inserted into a larger cylindrical-shaped polypropylene container. One inner vessel was filled with glycerol and the other with a lubrication oil. The space between the vessels and the outer container was filled with water. A 1.2 kW fixed molybdenum anode x-ray tube and a NaI scintillation photomultiplier detector were used. Since the mean energies of the spectra from the tube were low (≈ 17.5 keV), the coherent scatter was dominant over a larger range of angles

FIG. 3. Concept of x-ray diffraction computed tomography introduced by Harding *et al.*⁶⁻⁸

away from the forward direction. This was advantageous for their imaging system design. They demonstrated a major advantage of the diffraction CT technique over transmission CT to distinguish water from glycerol.

Luggar and Gilboy¹² have demonstrated that point scanned coherent scatter radiography can identify 2-mm-thick inclusions of plastics (e.g. nylon, polycarbonate, polyethylene) within a 10-mm-thick polymethyl methacrylate (PMMA) block with contrasts of up to 28% while point scanned transmission radiography was unable to detect them. The detector used was either a high-purity germanium (HPGe) or lithium-drifted silicon Si(Li) device. The samples were passed through the horizontal primary beam in 2.5 mm steps with acquisition times of 15 or 30 seconds at each position. The acquisition times were the same but the tube current was reduced in the transmission scans to prevent the detector overloading. They believe that their technique has applications in any system that currently uses conventional transmission radiography.

Breast tumors and microcalcifications are visualized on a mammogram due to differences in their x-ray attenuation properties. The properties of fibroglandular breast tissue and breast carcinomas are similar¹³ and this poses a challenging imaging task to conventional as well as to digital mammography, particularly when the tumor is surrounded by fibroglandular tissue. Kidaney *et al.*¹⁴ have shown differences between the low-angle x-ray scatter signatures of fibroglandular and cancerous breast tissues. They suggest that the tissue scatter information could greatly enhance the image contrast in breast imaging.

Compton or incoherently-scattered photons, whose cross section is approximately linear with electron density, have been used to produce: (i) two-dimensional cross-sectional images of the electron density distribution in the body¹⁵⁻¹⁷ and (ii) backscatter radiographs.¹⁸⁻²⁰ X-ray backscatter imaging, introduced by Jacobs *et al.*,^{18,19} is a technique that generates backscatter radiographs representing the integrated backwards scattering properties of the tissues along the incident beam direction. Figure 4 shows a schematic of their system for backscatter radiography. A pencil beam of x rays is raster scanned over the phantom and the backscattered field is collected by uncollimated large-area detectors adjacent to the incident beam. The diagram shows the changes of the backscatter intensity detected as the beam sweeps over air, bone, and fat structures. This approach is useful for the imaging of superficial structures, since the pathlength that the x rays must travel in the patient to reach the detector is small compared to primary imaging.

Subsequently, Herr *et al.*²¹ devised a Compton x-ray backscatter imaging system using a single detector and a mechanically rastered flying spot x-ray beam. The detector assembly consisted of a large area sodium iodide crystal, photomultiplier tube, and a line-focus collimator. It was mounted above the sweeping beam so that its focal line was parallel with the plane of the sweeping beam but perpendicular to the

FIG. 4. X-ray backscatter imaging concept introduced by Jacobs *et al.*^{18,19} for medical applications.

general direction of x-ray beam propagation. The 90° Compton scattered photons are used to form the scatter signals. At a given instant, the sweeping spot beam intersects the detector field of view, and defines an instantaneous sensitive volume. By sampling the scatter signal at fixed time intervals following the detection of each sweep start signal, the instantaneous sensitive volume is frozen at discrete locations. Their system can map the displacement and velocity patterns of the epicardial surface with a high degree of spatial and temporal precision.

In addition to these imaging technologies there have been studies which monitor the density variations in biological samples by detecting both types of scatter in a tightly collimated geometry. The characterization of tissue is accomplished by means of the coherent to Compton ratio²²⁻²⁵. The ratio is proportional to a power (1–5), dependent on the scattering angle, of the effective atomic number (Z_{eff}). Since Z_{eff}

FIG. 5. Systematic model for analyzing x-ray scatter imaging.

is larger for bone mineral than for soft tissue, changes in the ratio can be used to predict changes in bone mineral density.

1.3 Model

Although much experimental work has been done on the use of scattered x rays in medical imaging, a systematic approach, however, for analyzing their potential use has been lacking. Therefore, in analogy to the Motz and Danos³ model for primary imaging, I have devised a model for x-ray scatter imaging.²⁶ The model consists of a water phantom in which a target and a background object are situated as shown in Fig. 5. The goal is to distinguish these two objects based upon their scattering properties. Although the model seems simple at first, much information about the physics of scatter imaging can be gained.

The objective is to determine the regimes of medical interest in which x-ray scatter imaging could outperform conventional imaging in terms of C and SNR.

FIG. 6. Schematic top view of the scattering experiment for validation of model.

1.4 Outline

The thesis is structured as follows. In Chapter 2, an extract from our first publication,²⁶ I describe in detail various imaging geometries for the model and present the analytical/numerical results obtained for imaging various biological materials using monoenergetic beams for which the incident photon fluence Φ_0 was kept constant. This analysis allows us to make quantitative predictions of the performance of x-ray scatter imaging in medicine. Prior to publication, portions of this work were presented at the 1996-Vancouver²⁷ and 1997-Charlottetown²⁸ Canadian Organization of Medical Physicists (COMP) Annual Scientific Meetings, and at the 1997-Chicago Radiological Society of North America (RSNA) meeting.²⁹

In Appendix A, we show additional results obtained with monoenergetic beams for which the incident energy fluence, Ψ_0 , is kept constant. These results were presented at the 1999-San Diego Society of Photo-optical Instrumentation Engineers

(SPIE) Physics of Medical Imaging Conference.³⁰

Chapter 3 presents a straightforward extension of the model to study the effects of beam polychromaticity. Various energy fluence distributions were considered. To compare the results on a similar footing I matched their centroids and standard deviations. The method of matching is shown in Appendix B. This spectral blur analysis is based on our 1999 publication³¹ and some of the work was presented at the 1998-London (ON) COMP Conference,³² and at the 1999-SPIE Conference.³⁰

The validation of the model predictions was accomplished via a high-precision experimental apparatus. Figure 6 is a schematic top view of the apparatus. I chose to validate the model with plastics whose scattering signatures are relatively well known. The transmission and scatter signals from various plastics placed at the centre of the spherical water phantom were measured. Values of C and SNR were calculated for the task of imaging various combinations of plastics. This work is presented in Chapter 4 and has been submitted to the journal *Medical Physics*. In Appendices C (primary) and D (forward-scatter) we show additional results.

In Chapter 5, I give the conclusions of this work and also give directions for future work.

CHAPTER 2

A SEMIANALYTIC MODEL TO INVESTIGATE THE POTENTIAL APPLICATIONS OF X-RAY SCATTER IMAGING

In this chapter we describe the scattering cross-sections encountered in the diagnostic energy range, the models for radiological imaging with scattered versus primary photons and present some numerical results obtained with monoenergetic beams.

2.1 Scattering Cross Sections

X-ray scattering by atomic electrons in the diagnostic radiology energy range arises from coherent and incoherent interactions. Below, we briefly describe these phenomena, and give our sources for cross section data.

2.1.1 Coherent scatter

Coherent scattering is a cooperative phenomenon involving the electron distribution in the material. Upon scattering, the photons retain their original energies and no energy is transferred. Although coherent scatter constitutes only $\approx 10\%$ of the total linear attenuation coefficient in the energy range 15–50 keV, its angular distribution is strongly peaked in the forward direction. This behaviour is supported

both theoretically^{33,34} and experimentally.^{33,35} For example, the scattering of photons of energies < 20 keV from water at angles $\lesssim 10^\circ$ is dominated by the coherent process since the incoherent process is inhibited by electron binding. As the energy increases, the angular extent over which the coherent scatter dominates becomes narrower, i.e., tends towards smaller angles, and eventually becomes negligible at energies > 100 keV. The remarkable structure in the scatter intensity at small angles is the diffraction pattern of the material, which arises from interference between radiation scattered from different electrons. The atomic, molecular, and intermolecular electron distributions all affect the scatter distribution. The differential coherent scatter cross section per unit solid angle and per fundamental unit κ of the substance can be written as¹

$$\frac{d_\kappa \sigma_{\text{coh}}}{d\Omega} = \frac{d_e \sigma_o}{d\Omega} F^2[x, \rho_e(\mathbf{r})] \quad \left(\text{units are } \frac{\text{cm}^2}{\kappa \cdot \text{sr}} \right), \quad (3)$$

where $d_e \sigma_o/d\Omega$ is the Thomson differential cross section per free electron per steradian and F is the coherent form factor representing the collective effect of the electron distribution $\rho_e(\mathbf{r})$ within the scattering volume on the scattered wave amplitude. The variable $x = E_o/(hc) \sin(\theta/2)$ combines the dependence of F on the scattering angle θ and the photon energy E_o . There are no physical dimensions for F . Note however that F can be used to convert the Thomson differential cross section per free electron to a cross section per fundamental unit κ of the substance. The square of the form factor, F^2 , represents a ratio of the intensity scattered per κ to that which would be scattered by one isolated electron. For the scattering of x rays by a free atom, the fundamental unit is taken to be an atom ($\kappa=a$), whereas for scattering by a free molecule, the fundamental unit is a molecule ($\kappa=m$). For scattering by a group of molecules, it is most convenient to express F^2 as a ratio of the intensity scattered per bound electron to that scattered by a free electron ($\kappa=e$).

For our analysis, we extracted intermolecular form factors for various biological materials from the diffraction results of Kosanetzky *et al.*³⁶ for pig specimens. Their measured diffraction patterns covered the range $x \approx 0.25 - 4.3 \text{ nm}^{-1}$. In our work, we considered $0^\circ \leq \theta \leq 180^\circ$ and $10 \leq E_o \leq 200 \text{ keV}$, and hence the range $0 \leq x \leq 161 \text{ nm}^{-1}$ was required. The intermolecular form factor, F , was extended to zero at $x = 0$ linearly.

For higher values of x , F can be approximated by F_{free} , a free atom form factor which neglects the effects of interference between electrons of different molecules and between electrons of different atoms in a given molecule. As an example, according to the independent atomic scattering approximation,³⁷ for a water molecule (indicated by subscript w), the square of the form factor is

$$F_{\text{free, w}}^2 = 2 F_{\text{H}}^2 + F_{\text{O}}^2 \left(\frac{\text{free electron}}{\text{molecule}} \right), \quad (4)$$

where F_{H} and F_{O} are the atomic form factors of hydrogen and oxygen. This approximation is good for biological (amorphous) materials when x is large since the interference function, in the absence of long range order, tends to unity.^{37,38} We used atomic form factors from Hubbell *et al.*³⁹ The composition of each material was estimated in terms of fractional weights, w_i , taken from the literature (liver,³⁶ adipose tissue⁴⁰ (fat), brain matter,³⁶ blood,⁴¹ cortical bone,⁴² skeletal muscle⁴⁰). The calculation of F_{free} for water becomes

$$\begin{aligned} F_{\text{free, w}}^2 &= \frac{1}{Z} \sum_i m_i F_i^2 \\ &= \frac{1}{Z} \left[\frac{\frac{w_{\text{H}}}{A_{\text{H}}}}{\frac{w_{\text{H}}}{A_{\text{H}}} + \frac{w_{\text{O}}}{A_{\text{O}}}} F_{\text{H}}^2 + \frac{\frac{w_{\text{O}}}{A_{\text{O}}}}{\frac{w_{\text{H}}}{A_{\text{H}}} + \frac{w_{\text{O}}}{A_{\text{O}}}} F_{\text{O}}^2 \right] \left(\frac{\text{free electron}}{\text{bound electron}} \right), \end{aligned} \quad (5)$$

where A_i and w_i are respectively the atomic weights and fractional weights for $i=\text{O}$ (oxygen) and $i=\text{H}$ (hydrogen). The m_i coefficient is the fractional number of atoms of

FIG. 7. Free atom form factor, F_{free} , intermolecular form factor, F , and incoherent scattering function S for (a) white brain matter and (b) adipose tissue (fat). Note that F is extrapolated to zero at $x = 0$ and is matched with F_{free} towards higher values of x . [F was extracted from Kosanetzky *et al.*,³⁶ F_{free} and S were generated using the data of Refs. 39 and 43 respectively as described in the text]

species i . The division by the average atomic number, $\bar{Z}$, results in an F^2 expressed in units of free electron/bound electron.

As examples, Fig. 7 shows the resulting form factor for (a) white brain matter and (b) adipose tissue (fat). Note that F is well approximated by F_{free} for $1.5 \leq x \leq 4.2 \text{ nm}^{-1}$ but differs substantially for low x due to interference phenomena.

2.1.2 Incoherent scatter

In an incoherent scattering event, a photon interacts with an atomic electron and transfers some of its energy and momentum. The differential cross section per κ

per steradian can be written as

$$\frac{d_{\kappa}\sigma_{\text{inc}}}{d\Omega} = \frac{d_e\sigma_o}{d\Omega} F_{\text{KN}} S(x, Z) \left(\frac{\text{cm}^2}{\kappa \cdot \text{sr}} \right), \quad (6)$$

where F_{KN} is the Klein-Nishina factor¹ that modifies the Thomson cross section to give the probability of inelastic scattering from a free electron and $S(x, Z)$ is the incoherent scattering function which incorporates the effect of atomic binding on the probability of scattering. Similar to the coherent form factor, S has no physical dimensions.

Using tabulated values of S for single elements⁴³ (which range from 0 to Z), the scatter functions for the biological materials were calculated using the sum rule,

$$S_{\text{material}}(x) = \frac{1}{Z} \sum_i m_i S_i(x, Z_i) \left(\frac{\text{free electron}}{\text{bound electron}} \right). \quad (7)$$

Upon substituting Eq. 7 into Eq. 6, we obtain the differential incoherent scatter cross section per electron per steradian, $d_e\sigma_{\text{inc}}/d\Omega$. We note that $S(x)$ for white brain matter [Fig. 7(a)] and fat [Fig. 7(b)] increases from zero at $x = 0$ and approaches 1 as $x \rightarrow \infty$. The incoherent angular distribution for energies up to 100 keV is such that the backscattered field is comparable to the forward scattered intensity.

2.1.3 Attenuation coefficients

The total coherent scatter cross sections were generated by integrating the differential coherent scatter cross sections [Eq. 3] over 4π steradians. These cross sections, which included the effects of intermolecular interference, differed significantly from those calculated assuming only intra-atomic interference. For example, at 25 keV, the differences ranged from 3.0% for water to 24.4% for gray brain matter. Cross sections for incoherent scatter and the photoelectric effect were generated using the mixture rule from the data for elements of Plechaty *et al.*⁴⁴ Total linear attenuation

coefficients were obtained by multiplying the sum of the three preceding cross sections by the electron densities, and differed only slightly from those calculated assuming, for the coherent scattering process, independent contributions by each atom. For example, at 25 keV, the differences ranged from 0.4% for water to 3.2% for gray brain matter. This negligible difference is observed because the coherent scatter constitutes at most 12.7% (for water at 27 keV) of the total interaction cross section.

2.2 Models for Radiological Imaging With Scattered Versus Primary Photons

We have developed a simple but useful semianalytic approach which models imaging of a target object against a background material of the same dimensions when both are situated within a water phantom. The various imaging models chosen are: (i) for reference, a primary model representing conventional imaging, (ii) a total-scatter model detecting all photons singly scattered into 4π steradians, (iii) a forward-scatter model, and finally (iv) a backscatter model.

It is our purpose in this work to keep the analysis free from complications, and to extract quantitative information from the fundamental equations. In all four imaging models, the cross-sectional area A of the target and background objects is chosen sufficiently small that multiple scatter and self-attenuation within the objects can be neglected. Other assumptions are: the beam is non-divergent with a cross sectional area A , the scatter from the water itself does not reach the detector, and the detective quantum efficiency (DQE) of the detector is unity at all energies. Following Motz and Danos,³ we define C and SNR as given by Eqs. 1 and 2, where N_t and N_b represent the detected number of photons transmitted (for primary imaging) or singly scattered by the target and background objects respectively. We examine how C_{scatter} and $\text{SNR}_{\text{scatter}}$ vary as a function of primary photon energy E_0 and target thickness d . We compare

them with C_{primary} and $\text{SNR}_{\text{primary}}$.

In our analysis we chose a photon-counting detector but an energy integrating detector could have also been chosen. The only noise incorporated into our model is the Poisson statistics of the incident x-ray quanta. We know that the counts N_t and N_b are from two distinct targets and therefore are statistically independent Poisson variables. Thus the noise in the signal difference is simply $\sqrt{N_t + N_b}$. This model calculates the fundamental SNR accessible via x-ray scatter imaging. Such information is a good representation of the input signal to real x-ray scatter imaging systems for which $\text{DQE} < 1$.

2.2.1 Primary model

The primary imaging model, shown in Fig. 8, is that of Motz and Danos.³ Two objects are centred at an arbitrary distance from the surface of a slab-shaped water phantom of thickness L . Our analysis of primary imaging assumes the presence of an ideal grid which stops all the scatter from reaching the detector while transmitting all primary photons. The expressions for the number of primary photons transmitted through the target and background objects are:

$$N_t^p = N_o e^{-\mu_w(E_o)(L-d)} e^{-\mu_t(E_o)d} \quad \text{through the target,} \quad (8)$$

$$N_b^p = N_o e^{-\mu_w(E_o)(L-d)} e^{-\mu_b(E_o)d} \quad \text{through the background,} \quad (9)$$

where $N_o = \Phi_o A$ = incident number of photons on the phantom, Φ_o = primary photon fluence incident on the phantom, A = cross sectional area of target and background objects, E_o = primary photon energy, $\mu_i(E_o)$ = total linear attenuation coefficient evaluated at energy E_o , where the index i can be t (target), b (background), or w (water), L = thickness of water phantom, d = thickness of target and background objects.

FIG. 8. Geometry for the primary imaging model.

The contrast and signal-to-noise ratio due to the primary photons transmitted through the slab-shaped water phantom are given respectively by

$$C_{\text{primary}} = 2 \frac{|e^{-\mu_t(E_o)d} - e^{-\mu_b(E_o)d}|}{e^{-\mu_t(E_o)d} + e^{-\mu_b(E_o)d}}, \quad (10)$$

$$\text{SNR}_{\text{primary}} = C_{\text{primary}} \sqrt{\frac{N_o}{2} T_w^p T_o^p}, \quad (11)$$

where

$$T_w^p = e^{-\mu_w(E_o)(L-d)}, \quad (12)$$

$$T_o^p = \frac{1}{2} [e^{-\mu_t(E_o)d} + e^{-\mu_b(E_o)d}]. \quad (13)$$

Equations 10 to 13 reveal that $\text{SNR}_{\text{primary}}$ is equal to the product of C_{primary} with a factor $\sqrt{N_o/2}$ proportional to the square root of the number of incident photons, the square root of a transmission factor for primary T_w^p due to the slab-shaped water phantom, and the square root of a second transmission factor T_o^p which is the average

TABLE 1. Asymptotic expressions for C_{primary} and $\text{SNR}_{\text{primary}}$ obtained from Eqs. 10–13. The total linear attenuation coefficients μ_w , μ_t and μ_b are evaluated at the primary photon energy, E_o .

Region	C_{primary}	$\text{SNR}_{\text{primary}}$
$ \mu_t - \mu_b d \ll 1$	$ \mu_b - \mu_t d$	$\sqrt{\frac{\Phi_o A}{2}} e^{-\mu_w L} e^{-(\mu_b - \mu_w)d} \mu_b - \mu_t d$
$ \mu_t - \mu_b d \gg 1$	$2 \frac{ 1 - e^{(\mu_t - \mu_b)d} }{1 + e^{(\mu_t - \mu_b)d}} \rightarrow 2$	$\sqrt{\Phi_o A} e^{-\mu_w L} e^{-(\mu_b - \mu_w)d}$

proportion of photons transmitted through the target and background objects. This relationship between $\text{SNR}_{\text{primary}}$ and C_{primary} will help us to interpret the graphical results. Since a wide range of primary photon energies and object thicknesses are considered, it seems appropriate to list several asymptotic expressions for the two measures. In Table 1, the asymptotic expressions indicate that for $|\mu_t - \mu_b|d \ll 1$, $C_{\text{primary}} \propto d$ and $\text{SNR}_{\text{primary}} \propto d$, whereas for $|\mu_t - \mu_b|d \gg 1$, $C_{\text{primary}} \rightarrow 2$ and $\text{SNR}_{\text{primary}}$ is only weakly dependent on d . With reference to these asymptotic expressions, we study the transition region between the two extremes.

2.2.2 Total-scatter model

Our first x-ray scatter imaging model shown in Fig. 9 consists of a spherical water phantom of radius R , in which the target and background objects are centred. To determine the fundamental limits of scatter imaging, a 4π steradian detector is assumed. All the single scattered photons from both objects which manage to exit the phantom are collected. Since our model assumes scattering from the centre of the target a reasonable estimate of the total photon pathlength through water from entering the phantom to escaping as a scattered photon is $2R - d/2$. The number

FIG. 9. Geometry for the total-scatter imaging model. Detection over 4π steradians is accomplished via a spherical detector. A total photon pathlength through water of $2R - d/2$ is assumed.

of x-ray photons scattered from object element dl at l , reaching the detector, can be written as:

$$dN_i^s = N_o e^{-\mu_w(E_o)(R-d/2)} e^{-\mu_i(E_o)l} \rho_i^e \int_{\theta=0}^{\theta=\pi} \left[\frac{d_e \sigma_i^{\text{coh}}(\theta, E_o)}{d\Omega} + \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} \right] \times e^{-\mu_w(E')R} 2\pi \sin \theta d\theta dl, \quad (14)$$

where ρ_i^e is the electron density (number per unit volume) for object i , E' is the energy of the scattered photon, and $d\Omega/d\theta = 2\pi \sin \theta$ is the solid angle per unit angular increment. The factor $\exp(-\mu_w(E')R)$ can be taken outside of the integral for coherent scatter ($E' = E_o$) and must remain inside for incoherent scatter since E' is a function of θ . The total number of scattered x rays from object i , reaching the detector, is obtained by integrating Eq. 14 over the thickness d , giving

$$N_i^s = N_o e^{-\mu_w(E_o)(R-d/2)} (1 - e^{-\mu_i(E_o)d}) \frac{\rho_i^e}{\mu_i(E_o)} \left\{ \sigma_i^{\text{coh}}(E_o) e^{-\mu_w(E_o)R} + \int_{\theta=0}^{\theta=\pi} \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} e^{-\mu_w(E')R} 2\pi \sin \theta d\theta \right\}, \quad (15)$$

where $\sigma_i^{\text{coh}}(E_o)$ is the total cross section per electron for coherent scattering of material i evaluated at E_o .

If we set the energy E' of the Compton scattered photons equal to that of the monochromatic primary beam, then numerical integration is avoided. The total numbers of detected single-scattered photons from our objects are given by

$$N_i^s = N_o e^{-\mu_w(E_o)(2R-d/2)} \frac{\rho_i^e [\sigma_i^{\text{coh}}(E_o) + \sigma_i^{\text{inc}}(E_o)]}{\mu_i(E_o)} (1 - e^{-\mu_i(E_o)d}) , \quad (16)$$

$$i = \begin{cases} t & \text{for target} \\ b & \text{for background} \end{cases}$$

where $\sigma_i^{\text{inc}}(E_o)$ is the total cross section for incoherent scattering of material i . For convenience, we define $\mu_i^{\text{coh}} = \rho_i^e \sigma_i^{\text{coh}}$ and $\mu_i^{\text{inc}} = \rho_i^e \sigma_i^{\text{inc}}$ as the partial linear attenuation coefficients respectively for coherent and incoherent processes. Noting that all coefficients are evaluated at E_o , the expressions for C_{scatter} and $\text{SNR}_{\text{scatter}}$ can according to Eqs. 1, 2, and 16 be written as

$$C_{\text{scatter}} = 2 \frac{\left| \frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} (1 - e^{-\mu_t d}) - \frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b} (1 - e^{-\mu_b d}) \right|}{\frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} (1 - e^{-\mu_t d}) + \frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b} (1 - e^{-\mu_b d})} , \quad (17)$$

$$\text{SNR}_{\text{scatter}} = C_{\text{scatter}} \sqrt{\frac{N_o}{2} T_w^{s1} T_w^{s2} I_o^s} , \quad (18)$$

where

$$T_w^{s1} = e^{-\mu_w(R-d/2)} , \quad (19)$$

$$T_w^{s2} = e^{-\mu_w R} , \quad (20)$$

$$I_o^s = \frac{1}{2} \left[\frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} (1 - e^{-\mu_t d}) + \frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b} (1 - e^{-\mu_b d}) \right] . \quad (21)$$

Similar to the primary model, $\text{SNR}_{\text{scatter}}$ is given by the product of C_{scatter} with $\sqrt{N_o}/2$ and the square root of three other factors: two transmission factors T_w^{s1} (before scattering) and T_w^{s2} (after scattering) due to the spherical phantom, and a factor I_o^s which is a measure of the average scatter production in the target and background objects. The two terms in I_o^s are the fractions of photons incident on the target and background which scatter, at any angle. The asymptotic expressions for C_{scatter} and $\text{SNR}_{\text{scatter}}$ shown in Table 2 indicate that for radiologically thin objects (i.e. $\mu_{i=t,b}d \ll 1$), the linear attenuation coefficients for scatter are of importance, while for thick objects (i.e. $\mu_{i=t,b}d \gg 1$), the important quantities are the ratios of the scatter to total cross sections (or the ratios of linear attenuation coefficients). Note that for $\mu_i d \ll 1$, C_{scatter} is independent of d , $\text{SNR}_{\text{scatter}} \propto \sqrt{d}$, whereas for $\mu_i d \gg 1$, C_{scatter} saturates independent of d and $\text{SNR}_{\text{scatter}}$ is only weakly dependent on d .

To approach the conditions practical for a clinical device, we next consider two geometries which utilize only the scattered photons within a certain angular range.

2.2.3 Forward-scatter model

The forward-scatter imaging model also consists of a spherical water phantom of radius R in which we have centred our target and background materials (Fig. 10). For this model, however, an annular detector subtends a smaller solid angle and receives only those photons scattered in the forward direction within $\theta_{\min} \leq \theta \leq \theta_{\max}$. The total photon pathlength in water remains $2R - d/2$. The expression for the number of x-ray quanta scattered from object element dl at l and reaching the detector is

$$dN_i^s = N_o e^{-\mu_w(E_o)(R-d/2)} e^{-\mu_i(E_o)l} \rho_i^e \int_{\theta_{\min}}^{\theta_{\max}} \left[\frac{d_e \sigma_i^{\text{coh}}(\theta, E_o)}{d\Omega} + \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} \right] \times e^{-\mu_w(E')R} 2\pi \sin \theta d\theta dl . \quad (22)$$

TABLE 2. Asymptotic expressions for C_{scatter} and $\text{SNR}_{\text{scatter}}$ for the total-scatter model, obtained from Eqs. 17 to 21. The coefficients $\mu_{i=t,b,w}$, μ_t^{coh} , μ_t^{inc} , μ_b^{coh} , and μ_b^{inc} are evaluated at the primary photon energy, E_o .

Region	Asymptotic Expressions
$\mu_i d \ll 1,$ $i=t,b$	$C_{\text{scatter}} = 2 \frac{ (\mu_t^{\text{coh}} + \mu_t^{\text{inc}}) - (\mu_b^{\text{coh}} + \mu_b^{\text{inc}}) }{(\mu_t^{\text{coh}} + \mu_t^{\text{inc}} + \mu_b^{\text{coh}} + \mu_b^{\text{inc}})}$ $\text{SNR}_{\text{scatter}} = \sqrt{\Phi_o A e^{-\mu_w(2R-d/2)}} d \frac{ (\mu_t^{\text{coh}} + \mu_t^{\text{inc}}) - (\mu_b^{\text{coh}} + \mu_b^{\text{inc}}) }{\sqrt{\mu_t^{\text{coh}} + \mu_t^{\text{inc}} + \mu_b^{\text{coh}} + \mu_b^{\text{inc}}}}$
$\mu_i d \gg 1,$ $i=t,b$	$C_{\text{scatter}} = 2 \frac{\left \left(\frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} \right) - \left(\frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b} \right) \right }{\left(\frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} \right) + \left(\frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b} \right)}$ $\text{SNR}_{\text{scatter}} = \sqrt{\Phi_o A e^{-\mu_w(2R-d/2)}} \frac{\left \frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} - \frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b} \right }{\sqrt{\frac{\mu_t^{\text{coh}} + \mu_t^{\text{inc}}}{\mu_t} + \frac{\mu_b^{\text{coh}} + \mu_b^{\text{inc}}}{\mu_b}}}$

By confining our angular range to small angles ($\theta_{\text{max}} = 12^\circ$ in our work), the detection of coherently scattered photons is favoured and a valid approximation is to set $E' = E_o$. In our analysis the largest Compton energy loss is ≈ 2 keV corresponding to $\theta = 12^\circ$ and $E_o = 200$ keV. The total number of single-scattered photons from object i reaching the detector becomes

$$N_i^s = N_o e^{-\mu_w(E_o)(2R-d/2)} \frac{U_i(E_o, \theta_{\text{min}}, \theta_{\text{max}})}{\mu_i(E_o)} (1 - e^{-\mu_i(E_o)d}), \quad (23)$$

FIG. 10. Geometry for the forward-scatter imaging model. The energy of the incoherently scattered photons is set equal to that of the monochromatic primary beam.

where

$$U_i(E_o, \theta_{\min}, \theta_{\max}) = \rho_i^e \int_{\theta_{\min}}^{\theta_{\max}} \left[\frac{d_e \sigma_i^{\text{coh}}(\theta, E_o)}{d\Omega} + \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} \right] 2\pi \sin \theta d\theta \quad (24)$$

is the probability per unit object thickness of scattering into the solid angle $\Omega = \int_{\theta_{\min}}^{\theta_{\max}} 2\pi \sin \theta d\theta$ for object i . In other words, U_i is the component of the total linear attenuation coefficient μ_i due to scattering in the range $\theta_{\min} \leq \theta \leq \theta_{\max}$. The C_{scatter} and $\text{SNR}_{\text{scatter}}$ expressions become

$$C_{\text{scatter}} = 2 \frac{\left| \frac{U_t(E_o, \theta_{\min}, \theta_{\max})}{\mu_t(E_o)} (1 - e^{-\mu_t d}) - \frac{U_b(E_o, \theta_{\min}, \theta_{\max})}{\mu_b(E_o)} (1 - e^{-\mu_b d}) \right|}{\frac{U_t(E_o, \theta_{\min}, \theta_{\max})}{\mu_t(E_o)} (1 - e^{-\mu_t d}) + \frac{U_b(E_o, \theta_{\min}, \theta_{\max})}{\mu_b(E_o)} (1 - e^{-\mu_b d})}, \quad (25)$$

$$\text{SNR}_{\text{scatter}} = C_{\text{scatter}} \sqrt{\frac{N_o}{2} T_w^{s1} T_w^{s2} I_o^s}, \quad (26)$$

where

$$I_o^s = \frac{1}{2} \left[\frac{U_t(E_o, \theta_{\min}, \theta_{\max})}{\mu_t(E_o)} (1 - e^{-\mu_t d}) + \frac{U_b(E_o, \theta_{\min}, \theta_{\max})}{\mu_b(E_o)} (1 - e^{-\mu_b d}) \right]. \quad (27)$$

The transmission factors T_w^{s1} and T_w^{s2} are the same as for the total-scatter model (i.e. are given by Eqs. 19 and 20 respectively). Table 3 shows the asymptotic expressions obtained from these results. The behaviour parallels that for the total-scatter analysis.

TABLE 3. Asymptotic expressions for C_{scatter} and SNR_{scatter} for the forward-scatter model, obtained from Eqs. 25–27. $U_{i=t,b}$ corresponds to the probability per unit object thickness of scattering into the angular range $\theta_{\min} \leq \theta \leq \theta_{\max}$ (Eq. 24).

Region	C_{scatter}	SNR_{scatter}
$\mu_i d \ll 1$ $i=t,b$	$2 \frac{ U_t - U_b }{U_t + U_b}$	$\sqrt{\Phi_o A e^{-\mu_w(2R-d/2)}} d \frac{ U_t - U_b }{\sqrt{U_t + U_b}}$
$\mu_i d \gg 1$ $i=t,b$	$2 \frac{\left \frac{U_t}{\mu_t} - \frac{U_b}{\mu_b} \right }{\frac{U_t}{\mu_t} + \frac{U_b}{\mu_b}}$	$\sqrt{\Phi_o A e^{-\mu_w(2R-d/2)}} \frac{\left \frac{U_t}{\mu_t} - \frac{U_b}{\mu_b} \right }{\sqrt{\frac{U_t}{\mu_t} + \frac{U_b}{\mu_b}}}$

2.2.4 Backscatter model

For the task of imaging superficial structures with the backscattered field, we refer to Fig. 11, where both objects are at a distance z from the surface of a slab-shaped water phantom having thickness L . An annular detector surrounds the x-ray source and subtends a solid angle Ω defined by an angular range $\theta_{\min} \leq \theta \leq \theta_{\max}$ with respect to the scattering site. To simplify the numerical calculations, all detected backscattered photons are assumed to have travelled a distance $k = z/|\cos \bar{\theta}|$ after scattering in the water phantom. The lateral extent of the phantom is assumed large enough such that the pathlength k is entirely within water. Similar to the forward scatter formulation, the expression for the number of x-ray quanta scattered from

FIG. 11. Geometry for the backscatter imaging model. All detected backscattered photons are assumed to have traveled a distance $k = z / |\cos \bar{\theta}|$ in water after scattering.

object element dl at l , reaching the detector, is

$$dN_i^s = N_o e^{-\mu_w(E_o)(z-d/2)} e^{-\mu_i(E_o)l} \rho_i^e \int_{\theta_{\min}}^{\theta_{\max}} \left[\frac{d_e \sigma_i^{\text{coh}}(\theta, E_o)}{d\Omega} + \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} \right] \times e^{-\mu_w(E')k} 2\pi \sin \theta d\theta dl. \quad (28)$$

The incoherent scattering process must be treated realistically (i.e., $E' < E_o$ since the detection of the backscattered field involves large scattering angles). The total single scatter from object i reaching the detector becomes

$$N_i^s = N_o e^{-\mu_w(E_o)(z-d/2)} \frac{W_i(E_o, \theta_{\min}, \theta_{\max})}{\mu_i(E_o)} (1 - e^{-\mu_i(E_o)d}), \quad (29)$$

where

$$W_i(E_o, \theta_{\min}, \theta_{\max}) = \rho_i^e \int_{\theta_{\min}}^{\theta_{\max}} \left[\frac{d_e \sigma_i^{\text{coh}}(\theta, E_o)}{d\Omega} e^{-\mu_w(E_o)k} + \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} e^{-\mu_w(E')k} \right] 2\pi \sin \theta d\theta \quad (30)$$

is the probability per unit object thickness of backscattering within $\theta_{\min} \leq \theta \leq \theta_{\max}$ and reaching the detector. Note that W_i differs from U_i in that it includes not only

the probability of scattering, but also the transmission factor for scatter to reach the detector. The C_{scatter} and $\text{SNR}_{\text{scatter}}$ expressions become

$$C_{\text{scatter}} = 2 \frac{\left| \frac{W_t(E_o, \theta_{\min}, \theta_{\max})}{\mu_t(E_o)} (1 - e^{-\mu_t d}) - \frac{W_b(E_o, \theta_{\min}, \theta_{\max})}{\mu_b(E_o)} (1 - e^{-\mu_b d}) \right|}{\frac{W_t(E_o, \theta_{\min}, \theta_{\max})}{\mu_t(E_o)} (1 - e^{-\mu_t d}) + \frac{W_b(E_o, \theta_{\min}, \theta_{\max})}{\mu_b(E_o)} (1 - e^{-\mu_b d})}, \quad (31)$$

$$\text{SNR}_{\text{scatter}} = C_{\text{scatter}} \sqrt{\frac{N_o}{2} T_w^{s1} T_w^{s2} I_o^s}, \quad (32)$$

where

$$T_w^{s1} = e^{-\mu_w(E_o)(z-d/2)}, \quad (33)$$

and the product of the final two factors under the root is

$$T_w^{s2} I_o^s = \frac{1}{2} \left[\frac{W_t(E_o, \theta_{\min}, \theta_{\max})}{\mu_t(E_o)} (1 - e^{-\mu_t d}) + \frac{W_b(E_o, \theta_{\min}, \theta_{\max})}{\mu_b(E_o)} (1 - e^{-\mu_b d}) \right]. \quad (34)$$

The resulting asymptotic expressions are given in Table 4. The behaviour is as in the previous two models.

2.2.5 Integration procedures

Due to the forward-peaked nature of coherent scatter for high photon energies, some care was required in the numerical integration. For example, in the forward-scatter model $x \approx 2 \text{ nm}^{-1}$ for $\theta = 2^\circ$ and $x \approx 10 \text{ nm}^{-1}$ for $\theta = 12^\circ$ when $E_o = 120 \text{ keV}$. Since the region $x = 0-5 \text{ nm}^{-1}$ is of great importance due to intermolecular interference effects, more emphasis should be given in evaluating the integrals in this region. We followed the integration procedure used by Hubbell *et al.*⁴³ and chose the integration variable as $1 - \cos \theta$, from which the values of $x = (1/\lambda) \sin(\theta/2) = (1/\lambda) \sqrt{(1 - \cos \theta)/2}$ can easily be computed. The integration range was from $1 - \cos \theta_{\min}$ to $1 - \cos \theta_{\max}$, divided into intervals equally spaced in the logarithm of $1 - \cos \theta$.

TABLE 4. Asymptotic expressions for C_{scatter} and $\text{SNR}_{\text{scatter}}$ for the backscatter model, obtained from Eqs. 31 to 34. $W_{\text{i=t,b}}$ corresponds to the probability per unit object thickness of scattering into the angular range $\theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}$ and reaching the detector (Eq. 30).

Region	C_{scatter}	$\text{SNR}_{\text{scatter}}$
$\mu_i d \ll 1$ $i=t,b$	$2 \frac{ W_t - W_b }{W_t + W_b}$	$\sqrt{\Phi_o A} e^{-\mu_w(z-d/2)} d \frac{ W_t - W_b }{\sqrt{W_t + W_b}}$
$\mu_i d \gg 1$ $i=t,b$	$2 \frac{\left \frac{W_t}{\mu_t} - \frac{W_b}{\mu_b} \right }{\frac{W_t}{\mu_t} + \frac{W_b}{\mu_b}}$	$\sqrt{\Phi_o A} e^{-\mu_w(z-d/2)} \frac{\left \frac{W_t}{\mu_t} - \frac{W_b}{\mu_b} \right }{\sqrt{\frac{W_t}{\mu_t} + \frac{W_b}{\mu_b}}}$

Values of F and S at the integration mesh points were obtained by second-order log-log interpolation. To perform the integrations, we used Simpson's rule with 1001 integration points (1000 intervals).

2.3 Results

Our results for C and SNR as a function of object thickness, d , and primary photon energy, E_o , are displayed via contour plots. The selected ranges $d = 0.01 - 40$ mm and $E_o = 10 - 200$ keV generally allow both verification of the asymptotic expressions and investigation of the intermediate transition regions. In each imaging model, the primary photon fluence was held constant at $\Phi_o = 3.126 \times 10^9 \text{ cm}^{-2}$, corresponding to an entrance exposure for 60 keV photons of 100 mR.

The circular cross-sectional areas of the target and background objects were fixed at $A = \pi r^2 = 10^{-2} \text{ cm}^2$, where r is the radius of the cylindrical objects. The small area permits us to omit the multiple scatter occurring within the target. For

example, a 60 keV incident photon that scatters coherently at the centre of a liver target has only a 1% chance of interacting again within the target, but a 92% chance of interacting within the water before escaping a spherical water phantom of radius $R = 12.5$ cm. For a 90° scattered photon, the assumption of a distance R through water instead of r through liver plus $R - r$ through water makes a difference of $< 0.04\%$ in the number of photons transmitted. Multiple scatter effects in the water are neglected but in the discussion section we address what would happen if they were taken into account.

The target and background objects were chosen from the following list of biological materials: liver, fat, brain matter, bone, muscle, and blood. The task of imaging liver versus fat was used as a basis for comparing the four imaging models. Several other target/background combinations were also investigated.

2.3.1 Primary Model

Figure 12 shows C_{primary} and $\text{SNR}_{\text{primary}}$ for the tasks of imaging liver versus fat [(a) and (b)] and bone versus muscle [(c) and (d)]. The thickness of the slab-shaped water phantom is $L = 25$ cm for both imaging tasks since this value is representative of the abdomen thickness for the average person.

The results for contrast in Figs. 12(a) and 12(c) show that, as E_o increases, the contrast decreases monotonically for fixed thickness d . As d increases, the contrast increases for fixed E_o . For high-contrast regions (i.e. $|\mu_t - \mu_b| d \gg 1$, upper left corners of both contrast plots), $C_{\text{primary}} \rightarrow 2$ at a smaller value of d for the bone/muscle combination since $|\mu_{\text{bone}} - \mu_{\text{muscle}}| \gg |\mu_{\text{liver}} - \mu_{\text{fat}}|$ at low energies. The higher contrast seen for the task of imaging bone versus muscle is a direct consequence of the stronger photoelectric effect for bone. In both imaging tasks, $C_{\text{primary}} \rightarrow |\mu_t - \mu_b| d$ for $|\mu_t - \mu_b| d \ll 1$ (bottom right corners of the plots).

FIG. 12. Results from primary imaging model. (a) C_{primary} and (b) $\text{SNR}_{\text{primary}}$ for the task of imaging liver versus fat. (c) C_{primary} and (d) $\text{SNR}_{\text{primary}}$ for the task of imaging bone versus muscle.

For low energies, $T_w^p \rightarrow 0$ and hence $\text{SNR}_{\text{primary}} \rightarrow 0$. As the transmission factors T_w^p and T_o^p increase with increasing energy, a distinct difference between the two imaging tasks is observed. For the imaging task of liver versus fat, the SNR increases monotonically with energy. For the higher contrast task of bone versus muscle, however, the SNR has an optimum energy for all d . This behaviour is a direct consequence of the competition between the increase in the number of photons reaching the detector and the decrease in contrast as E_o increases. For $E_o > 100$ keV, the SNR reaches a plateau where the small increase in the number of photons

is counterbalanced by the small decrease in contrast.

2.3.2 Total-Scatter Model

In implementing the total-scatter model, we assumed that $E' = E_o$. Calculations (not shown here) over the range of diagnostic energies and object thicknesses d show that this approximation has a negligible effect on C and slightly overestimates SNR. Despite small differences, the variation of C_{scatter} and $\text{SNR}_{\text{scatter}}$ as a function of E_o and d both remain similar.

In Fig. 13 are the results for the tasks of imaging liver versus fat [(a) and (b)] and white versus gray brain matter [(c) and (d)]. The target and background tissues are centred in a spherical water phantom of radius $R = 12.5$ cm for the former task, and $R = 7.5$ cm for the latter. The smaller water phantom is representative of the human head.

Figure 13(a) shows that for $\mu_{i=t,b} d \ll 1$ (bottom quarter of plot), C_{scatter} is independent of d as predicted by Table 2. Upon leaving the $\mu_i d \ll 1$ region, C_{scatter} initially decreases with an increase in d . For specific combinations of d and E_o (upper left corner), C_{scatter} is zero. As indicated in Eq. 17 and Table 2, C_{scatter} may be zero if the difference between the target and background's scattering properties are identical. It is surprising that there exists an extensive zero contrast locus which extends on the figure from $d = 3.8$ mm, $E_o = 10$ keV through $d = 40$ mm, $E_o = 28.3$ keV. For large values of $\mu_i d$, C_{scatter} increases and finally saturates for $\mu_i d \gg 1$ (occurs beyond the range of d shown on the plots).

In Fig. 13(c), the smaller linear attenuation coefficients of white and gray brain matter enlarge the $\mu_i d \ll 1$ region in terms of d , i.e. the contrast at a given E_o remains constant over a larger range of d . Their similar $\bar{Z}$ and electron densities result in a lower C_{scatter} as compared to the liver/fat combination. Coherent scattering

FIG. 13. Results from total-scatter imaging model. (a) C_{scatter} and (b) $\text{SNR}_{\text{scatter}}$ for the task of imaging liver versus fat. (c) C_{scatter} and (d) $\text{SNR}_{\text{scatter}}$ for the task of imaging white brain matter versus gray matter.

is primarily responsible for the higher contrast at low energies, whereas incoherent scattering gives lower contrast which extends to high energies.

Figures 13(b) and 13(d) show the SNR results. The general shape of the SNR plot for the liver versus fat combination is similar to that obtained with the primary imaging model. In both models, the SNR exhibits no optimum energy for all d . In the scatter imaging models, it is the product of the transmission factors T_w^{s1} , T_w^{s2} , and the scatter production factor I_o^s which dictates the number of detected scattered photons. Although the I_o^s factors are similar for both imaging tasks, the smaller water

FIG. 14. Maximum SNR over all photon energies versus object thickness d for the total-scatter and primary imaging models. The imaging tasks consist of (a) liver versus fat and (b) white versus gray brain matter.

phantom in the latter causes an increase in the transmission of low energy photons. This increase combined with the higher C at lower energies results in an SNR which has an optimum energy for all d .

Figure 14 shows the maximum signal-to-noise ratio (SNR_{max}) over all photon energies as a function of d for the primary and total-scatter imaging models. For the liver/fat combination, $\text{SNR}_{\text{max}}^{\text{scatter}} > \text{SNR}_{\text{max}}^{\text{primary}}$ for all d [Fig. 14(a)], whereas for white versus gray brain matter, this occurs for $d \leq 12$ mm [Fig. 14(b)].

2.3.3 Forward-Scatter Model

For our analysis of forward-scatter imaging, an angular range of $2^\circ \leq \theta \leq 12^\circ$ (≈ 0.13 steradian) was chosen. The approximation of setting $E' = E_0$ was applied. Figures 15(a) and 15(b) show the results of C_{scatter} and $\text{SNR}_{\text{scatter}}$ for the liver/fat

FIG. 15. Results from forward-scatter imaging model. (a) C_{scatter} and (b) $\text{SNR}_{\text{scatter}}$ for the task of imaging liver versus fat. (c) C_{scatter} and (d) $\text{SNR}_{\text{scatter}}$ for the task of imaging white brain matter versus blood.

combination, while Figs. 15(c) and 15(d) are those for white brain matter/blood. The spherical water phantom has a radius $R = 12.5$ cm for the former and 7.5 cm for the latter.

The structure and magnitude of C_{scatter} is essentially governed by the coherent scatter, while the incoherent scatter behaves as an underlying structureless background. For sufficiently small d , where $\mu_i d \ll 1$ [lower portion of Figs. 15(a) and 15(c)], C_{scatter} is essentially proportional to $F_{\text{target}}^2 - F_{\text{background}}^2$ for both imaging tasks [Eqs. (1), (24), and (25)]. As was shown in Table 3, for $\mu_i d \gg 1$, C_{scatter} is proportional to the

FIG. 16. Maximum SNR over all photon energies versus object thickness d for the forward-scatter and primary imaging models. The imaging tasks consist of (a) liver versus fat and (b) white brain matter versus blood.

difference in the target and background's relative probabilities of producing forward scatter. In this region, C_{scatter} passes through a significant maximum for the liver/fat combination [upper left corner of Fig. 15(a)]. The broad maximum in the middle of both C_{scatter} plots occurs because the intermolecular interference effects are similar for the corresponding range in x .

In Figs. 15(b) and 15(d), the E_o for which SNR is maximized is almost independent of d . These maxima occur at ≈ 79 keV and ≈ 51 keV for, respectively, the liver/fat and white brain matter/blood combinations. A significant optimum is obtained for the latter.

Figure 16(a) indicates that $SNR_{\max}^{\text{scatter}} > SNR_{\max}^{\text{primary}}$ for $d \leq 8$ mm when imaging liver versus fat. Figure 16(b) shows that for $d \leq 23$ mm, $SNR_{\max}^{\text{scatter}} > SNR_{\max}^{\text{primary}}$ for white brain matter versus blood.

2.3.4 Backscatter Model

An angular range of $158^\circ \leq \theta \leq 178^\circ$ (≈ 0.45 steradian) was chosen to investigate the potential applications of x-ray scatter in imaging superficial structures. The target and background objects are located at depth $z = 3$ cm from the surface of a slab-shaped water phantom. For comparison, primary-model calculations were done using a thickness of $L = 25$ cm. The object combinations consisted of liver/fat and bone/muscle.

The behaviours of C and SNR are similar for both imaging tasks. Again, the plots of C [Figs. 17(a) and 17(c)] exhibit zero contrast for specific combinations of E_o and d . The significant increase in SNR [Figs. 17(b) and 17(d)] in the region $\mu_i d \gg 1$ (slightly to the right of the upper left corner) is due to the high contrast and the large number of incident photons reaching the objects. These two factors compensate for the lower I_o^s scatter production factor. The SNR in both cases passes through a shallow maximum for all d .

Figures 18(a) and 18(b) reveal that $\text{SNR}_{\text{max}}^{\text{scatter}} > \text{SNR}_{\text{max}}^{\text{primary}}$ for $d \leq 22$ mm when imaging liver versus fat, and for $d \leq 6$ mm for bone versus muscle. For the latter, the slope of SNR_{max} changes at $d \approx 23$ mm since the maximum jumps from high energies to low energies [Fig. 17(d)].

2.4 Discussion

A method to compare quantitatively the information content (C and SNR) of x-ray scatter images versus primary photon images has been described.

The results obtained for the task of imaging liver versus fat in a 25 cm thick water phantom, via the four imaging models, indicate that for targets of thickness $d \leq 10$ mm and energies $E_o \geq 60$ keV, x-ray scattered photons offer a higher C than

FIG. 17. Results from backscatter imaging model. (a) C_{scatter} and (b) $\text{SNR}_{\text{scatter}}$ for the task of imaging liver versus fat. (c) C_{scatter} and (d) $\text{SNR}_{\text{scatter}}$ for the task of imaging bone versus muscle.

primary photons (e.g. for $d = 1$ mm and $E_o = 60$ keV, $C_{\text{forward-scatter}}/C_{\text{primary}} \approx 133$). Although significant advantages in C are obtained, it is the magnitude of the SNR which is more important. Conceptually, the highest SNR in x-ray scatter images would be realized if all the single scatter leaving the patient were detected. Figure 14(a) indicates that imaging liver versus fat deep within the body with all the single scatter would provide a higher SNR than primary photons for all object thicknesses up to and including the maximum thickness studied of 40 mm.

Calculations were performed to assess the clinical potential of using restricted

FIG. 18. Maximum SNR over all photon energies versus object thickness d for the backscatter and primary imaging models. The imaging tasks consist of (a) liver versus fat and (b) bone versus muscle.

solid angles. Although forming the signal with the forward scattered photons from 2° to 12° corresponds to using only 1% of the possible detector solid angle, the resulting SNR_{max} ranges from 34.1% ($d = 0.01$ mm) to 40.2% ($d = 40$ mm) of the value for a 4π sr detector. The maximum object thickness for which x-ray scatter imaging is advantageous is reduced from 40 mm to 8 mm. The imaging of blood versus white brain matter in a 15 cm thick water phantom with the forward scattered photons (2° to 12°), results in an SNR greater than primary for essentially all vessel sizes ≤ 23 mm. Backscattered photons from 158° to 178° (0.45 sr, $\approx 3.6\%$ of 4π sr) could distinguish liver from fat, 3 cm below the surface of a 25 cm thick water phantom, better than primary photons for liver structures with $d \leq 22$ mm. These findings suggest that a clinical imaging system capturing the scattered field could detect small lesions where standard transmission imaging might not.

In our work, several assumptions were made in order to keep the analysis simple. The amount of multiple scatter and attenuation in the water phantom is much larger than in the target objects, so that the latter can be neglected. A non-divergent beam was assumed for convenience of modelling, but a divergent beam, as in real devices, would not change the results significantly.

If the multiple scatter generated in the water phantom is taken into account, a reduction in both C and SNR would occur. To estimate the amount of image degradation, we refer to the Monte Carlo (MC) simulation study of Neitzel *et al.*⁴⁵ In their analysis, they assumed a pencil beam of monoenergetic 60 keV x rays normally incident upon a slab shaped water phantom of thickness 20 cm. Their object to detector distance was 4 cm. Their scatter profiles indicate that the multiple scatter component: (1) is practically independent of scatter angle, and (2), in the angular range 2° to 12° , constitutes $\approx 20\%$ of the signal. In such cases where the unwanted scatter represents a nearly constant background, the contrast reduction can be compensated for by varying the image display, whereas the reduction in the SNR is inevitable. If we assume that the multiple scatter provides a uniform contribution, M , to both N_t and N_b , then the added unwanted scatter causes the noise to increase from⁴⁶ $\sqrt{N_t + N_b}$ to $\sqrt{N_t + N_b + 2M}$, and the SNR to be multiplied by a factor $1/\sqrt{1 + M/\bar{N}}$ (where $\bar{N} = (N_t + N_b)/2$). Hence, having a 20% multiple scatter component in a forward-scatter imaging device would reduce the SNR by $\approx 8.7\%$. For backscatter, the multiple scatter percentage is greater, and would cause a larger reduction in the SNR.

For an image receptor where the DQE < 1 , the SNR would decrease (e.g. for DQE = 0.6, SNR = $\sqrt{0.6}$ SNR_{DQE=1}). Although the DQE of the detector could vary as a function of energy, the Compton energy shifts are small (≈ 6 keV for $E_o = 60$ keV, $\theta = 90^\circ$) so that the DQE could still be approximated as constant.

CHAPTER 3

ANALYSIS OF SPECTRAL BLUR EFFECTS IN X-RAY SCATTER IMAGING

In practice, polyenergetic beams are almost always used due to their higher photon fluence rate. In this chapter we extend our model to study the effects of beam polychromaticity.

3.1 Incident Energy Fluence

The preceding chapter (based on Ref. 26) presented results obtained with monoenergetic beams for which the incident photon fluence Φ_o was fixed. Here we keep the incident energy fluence Ψ_o constant so that, to a first approximation, similar doses are delivered by the various photon beams. Figure 19 shows the effect of changing from constant Φ_o to constant Ψ_o in our model. Shown are contour lines of SNR as a function of incident monoenergetic photon energy E_o and of d for the task of imaging liver versus fat with the total single scatter from both objects. The water phantom is a sphere of diameter 25 cm. Both Φ_o and Ψ_o were chosen such that for 60 keV photons the entrance air kerma is $8.73 \times 10^{-4} \text{ J kg}^{-1}$ (exposure of 100 mR), which is the order of magnitude used for conventional primary imaging in the clinic. The smaller photon fluence at higher energies for the fixed Ψ_o spectra decreases SNR such that it passes through a shallow maximum between 80–163 keV [Fig. 19(b)], whereas

FIG. 19. Contour lines of SNR as a function of E_o and d for the task of imaging liver/fat with the total single scatter. (a) Fixed $\Phi_o = 3.13 \times 10^9 \text{ cm}^{-2}$, (b) fixed $\Psi_o = 1.88 \times 10^{11} \text{ keV} \cdot \text{cm}^{-2}$.

for the spectra with fixed Φ_o there is no optimum energy below 200 keV [Fig. 19(a)]. Other examples obtained with monoenergetic beams for which Ψ_o is fixed are shown in Appendix A [taken from Ref. 30]. The general conclusions of Chapter 2 do not change if Ψ_o is fixed instead of Φ_o . X-ray scatter images are still predicted, in many cases, to offer more diagnostic information than conventional radiological images.

3.2 Spectral Blur

The four photon beams characterized by the energy fluence distributions $d\Psi/dE$ shown in Fig. 20 were used to analyze the effects of spectral blur on x-ray scatter imaging. The first corresponds to an ideal monoenergetic beam with photon energy E_o (e.g., from a synchrotron source), the second to a beam with two separate mo-

FIG. 20. Energy fluence distributions of four photon beams used to analyze the effects of spectral blur on x-ray scatter imaging.

monoenergetic peaks of energies E_1 and E_2 (e.g., the $K\alpha$ and $K\beta$ lines of a fluorescence foil), the third to a clinical x-ray beam, and the fourth to a polyenergetic beam with uniform energy fluence between energies $E_{\min}$ and $E_{\max}$. We assumed that the latter represents an upper limit to spectral broadening. In order to compare the results obtained with these spectral distributions, the centroids $\langle E \rangle$ and spectral widths (as given by the standard deviation, σ) of the dual peak and rectangular spectra were matched to those of the clinical x-ray spectrum, as described in Appendix B. The implementation of beam polychromaticity is a straightforward extension of the equations of Chapter 2 to include integration over the incident spectrum. The SNR was evaluated via Eq. 2 in terms of number of photons.

3.3 Results

We study the effects of spectral blur on x-ray scatter imaging by first increasing the spectral width and then by considering large spectral widths including clinical

TABLE 5. Centroids $\langle E \rangle_i$ and spectral widths σ_i of the energy fluence distributions shown in Fig. 21.

index (i)	$\langle E \rangle_i$ (keV)	σ_i (keV)
1	57.3	5.8
2	56.2	10.8
3	56.9	16.8

x-ray spectra. For various imaging tasks, values of SNR are calculated as a function of object thickness $d = 0.01 - 40$ mm and incident photon spectrum. For each of the four beams the total energy fluence incident on the phantom is $\Psi_o = 1.88 \times 10^{11}$ keV cm^{-2} corresponding to an entrance air kerma of 8.73×10^{-4} J kg^{-1} (100 mR) for 60 keV photons.

3.3.1 Increasing the Spectral Width

The effect of spectral width is studied for imaging 1-cm-thick liver/fat structures with the forward-scattered photons ($2^\circ - 12^\circ$). Both objects are at the centre of a spherical water phantom of diameter 25 cm. A 100 kV spectrum⁴⁷ was bandpass filtered symmetrically in 1 keV steps about the optimum monoenergetic energy E_o . The corresponding dual peak and rectangular spectra were generated to have the same $\langle E \rangle$ and σ as the filtered 100 kV spectra. Three energy fluence distributions with spectral widths σ_1 , σ_2 , and σ_3 are shown in Fig. 21 for each type of polyenergetic spectrum. Their $\langle E \rangle$ and σ are shown in Table 5. At $\sigma = \sigma_3$ the entire 100 kV spectrum is considered and at $\sigma = 0$ the optimum energy for monoenergetic imaging is chosen.

In Fig. 22 the SNR is seen to decrease smoothly as σ increases. The arrows indicate the locations corresponding to the spectral widths σ_1 , σ_2 , and σ_3 . At $\sigma = \sigma_3$ the SNR has decreased to 84.4% for the dual peak beam, to 86.3% for the rectangular

FIG. 21. Three energy fluence distributions characterized by σ_1 , σ_2 , and σ_3 for each of the three types of polyenergetic spectra.

beam, and to 87.5% for the 100 kV beam. Following Rose,⁴ this specific imaging task does not require monoenergetic beams since $\text{SNR} > 5$ for all spectra.

3.3.2 Large Spectral Widths

In Fig. 23 are shown ratios of SNR for polyenergetic scatter imaging to that for optimum monoenergetic scatter imaging (SNR_{opt}) as a function of object size d within the phantom. Note that SNR_{opt} is a function of both energy and d .

We consider the task of imaging liver/fat combinations ranging from 0.01 to 40 mm thick via the total-scatter [Fig. 23(a)], forward-scatter [Fig. 23(b)], and backscatter [Fig. 23(c)] models. Both objects are centred inside a 25-cm-diam spherical water phantom for the total-scatter and forward-scatter models. For the backscatter model,

FIG. 22. SNR as a function of σ for imaging liver versus fat structures 1 cm thick with the forward-scattered photons ($2^\circ - 12^\circ$).

they are 3 cm deep in a slab of water. The angular range of detection is $2^\circ - 12^\circ$ for the forward-scatter model and is $158^\circ - 178^\circ$ for the backscatter model.

The $\langle E \rangle$ and σ of the dual peak and rectangular beams were matched to the following spectra: for Fig. 23(a), a 140 kV beam (total scatter), for Fig. 23(b), a 100 kV beam (forward scatter), and for Fig. 23(c), an 80 kV beam (backscatter). The values of $\langle E \rangle$ and σ are shown in Table 6. These constant potential spectra⁴⁷ were chosen based upon the optimum monoenergetic energies which were between 80 – 163 keV (total scatter), 57 – 59 keV (forward scatter) and 21 – 67 keV (backscatter).

A common feature for all three scatter models and spectra is that the decrease in SNR is nearly independent of object size d for $d < 2$ mm. For larger values of d , SNR decreases and at $d = 40$ mm is 82% (140 kV, total scatter), 87% (100 kV, forward scatter) and 44% (80 kV, backscatter) of SNR_{opt} . The reason for the decrease in SNR for $d \geq 2$ mm is due to lower C regions being blurred into the signal. Refer

FIG. 23. Ratios of SNR for polyenergetic imaging to SNR for optimum monoenergetic imaging (SNR_{opt}) for (a)-(e) scatter models and (f) primary model.

TABLE 6. Centroids $\langle E \rangle$ and standard deviations σ of the clinical x-ray spectra.

kV	$\langle E \rangle$ (keV)	σ (keV)
80	47.7	13.4
100	56.9	16.8
140	70.5	25.4

to Figs. 13(a), 15(a), and 17(a) in Chapter 2 which show contour plots of C as a function of d and incident monoenergetic energy E_0 . The range in SNR by choosing one spectral distribution versus another was less than 3.0% of SNR_{opt} (total-scatter model), 3.5% (forward-scatter model), and 3.7% (backscatter model).

Figures 23(d) and 23(e) show ratios of $\text{SNR}/\text{SNR}_{\text{opt}}$ for, respectively, imaging white brain matter versus blood with the forward scatter ($2^\circ - 12^\circ$) and imaging bone versus muscle with the backscatter ($158^\circ - 178^\circ$) using an 80 kV beam. The former combination is centred inside a 15-cm-diam spherical water phantom and the latter is 3 cm deep in a slab of water. The decrease in SNR for the white matter/blood imaging task is similar to that for the task of imaging liver/fat, 100 kV, forward-scatter model [Fig. 23(b)]. For imaging bone/muscle structures with the backscattered photons, the SNR is ≈ 0 for $d = 19.1$ mm. The lower energy photons have blurred a $C = 0$ locus into the signal. The existence of the $C = 0$ locus was shown in Chapter 2 [see Fig. 17(c) (bone/muscle, backscatter)].

3.3.3 Primary Imaging

Finally, for comparison with our results for scatter imaging, Fig. 23(f) shows ratios of SNR for polyenergetic primary imaging to SNR for optimum monoenergetic primary imaging (SNR_{opt}) as a function of d . The slab-shaped water phantom is 15 cm thick for the imaging task involving white brain matter and is otherwise 25 cm

thick. For liver/fat and white matter/blood, the ratio is nearly independent of d . For the bone/muscle imaging task an increase occurs for $d > 2$ mm due to beam hardening. The SNR ranges from 57.1% (white matter/blood) to 85.5% (liver/fat) of SNR_{opt} .

3.4 Discussion

A quantitative description of spectral blur effects in x-ray scatter imaging has been presented. The incident energy fluence was kept constant so that to a first approximation similar doses are delivered by the monoenergetic, dual peak, rectangular, and clinical x-ray beams. For the task of imaging liver versus fat via the three scatter models, the range in SNR by choosing one spectrum versus another was less than 3.5% of SNR_{opt} .

Initially we thought that the rectangular beams would represent the upper limit to spectral broadening. However, Fig. 23(b) (liver versus fat, forward-scatter model) shows a slightly worse SNR with the dual peak beam. This behavior, surprising at first, is related to the forward-peaked nature of the coherent scatter that causes the energy for which SNR is maximized to be ≈ 57 keV for all d . The energies of the dual peak beam, $E_1 = 40$ keV and $E_2 = 74$ keV, are further away from this optimum energy and thus produce a smaller SNR.

The SNR obtained with the optimum monoenergetic beam (SNR_{opt}) can also be obtained with the polyenergetic beams at the expense of an increase in dose. Each entry in Table 7 is the fractional increase in dose required to achieve a SNR equal to that for a monoenergetic beam optimized in energy for that specific thickness d and type of imaging (primary or one of the three scatter models). The cost in dose as a result of spectral blur is generally moderate for scatter imaging and there is a similar cost in primary imaging. For example, imaging liver/fat combinations 10 mm

TABLE 7. % dose increase required such that SNR obtained with the clinical x-ray spectra equals SNR_{opt} for $d = 1, 10$, and 40 mm.

Target vs. Background	Imaging Model and kV	% dose increase		
		$d = 1$ mm	$d = 10$ mm	$d = 40$ mm
Liver vs. Fat	Primary 80 kV	37	37	37
	Total Scatter 140 kV	19	23	49
	Forward Scatter 100 kV	30	31	32
	Backscatter 80 kV	15	20	408
White Matter vs. Blood	Primary 80 kV	206	206	204
	Forward Scatter 80 kV	45	46	49
Bone vs. Muscle	Primary 80 kV	105	83	55
	Backscatter 80 kV	18	259	588

thick with the forward scattered photons using a 100 kV beam would require a 31% increase in dose whereas primary imaging with an 80 kV beam would require a 37% increase. There are some x-ray scatter imaging tasks [e.g., bone/muscle, $d = 19.1$ mm, backscatter model, Fig. 23(e)] for which a $C = 0$ locus exists and the spectrum should be chosen to avoid it.

As discussed in Chapter 2, a reduction in SNR will occur due to the multiple scatter in the water phantom. There will probably not be much difference between monoenergetic and polyenergetic beams. For example, for the forward-scatter geometry, the Monte Carlo simulation study of Neitzel *et al.*⁴⁵ found only minor differences between the spatial distributions of multiple scatter at the image plane for 140 kVp versus 60 keV pencil beams incident on a 10 cm water phantom. A 20% multiple

scatter component would reduce the SNR by $\approx 8.7\%$.

CHAPTER 4

EXPERIMENTAL VALIDATION: PRIMARY AND FORWARD-SCATTER MODELS

In this chapter, we present an experimental validation of the primary and the forward-scatter x-ray imaging models when polyenergetic beams are used. Plastic samples are used because their coherent scatter form factors are relatively well known as compared to those for biological tissues. The reason for this is that the preparation of biological samples is much more difficult. For our model predictions we used the form factors that were extracted from the x-ray diffraction measurements of Kosanetzky *et al.*³⁶. This was the only source of diffraction data available at the beginning of the project. Such data were essential for the forward-scatter calculations since these form factors incorporate the important intermolecular interference effects that arise at scattering angles $\theta < 12^\circ$. Although there now exist more recent sources of diffraction data, there is not yet consensus in the literature. Some preliminary results obtained with biological tissues are presented.

4.1 Method

4.1.1 Apparatus

The main components of the experiment were an optical bench, an x-ray tube, a pinhole collimator, a water phantom, a target positioning assembly, a slit assembly,

and a HPGe detector. Figure 24 shows top views of the primary and forward-scatter geometries. Polyenergetic photon beams were generated by a conventional rotating anode x-ray tube [Machlett Dynamax 62R, large focal spot (1.2 mm nominal) used] driven by a single-phase generator. The beam was collimated with a pinhole of diameter d_{pin} to pencil beams prior to entering a spherical water phantom of radius R . The amount of off-focal radiation reaching the phantom was reduced by placing a 3-mm-diam aperture at the exit window of the x-ray tube. An ionization chamber was placed on the optical bench at a distance L_{ion} from the focal spot and recorded the exposure X_{ion} . The phantom, centred on the rotating stage at a distance L_c from the focal spot, was used for both primary and forward-scatter measurements. The target samples were placed at the centre of the phantom via three orthogonal adjustable sliding stages. A slit assembly mounted on the rotating stage allowed us to restrict the scattering volume seen by the HPGe detector. An aperture in front of the HPGe detector defined the active diameter of the detector.

Measurements were also performed with the water phantom removed. There were four different types of measurements: (1) in-air primary, (2) in-water primary, (3) in-air scatter and (4) in-water scatter.

The HPGe detector (Ortec model GLP 16195/10-P) had a crystal diameter of 16 mm and thickness of 10 mm. A bias of -1500 V was provided by an Ortec 456 power supply. The detector was housed in a mobile rack and positioned accurately to detect the scattered photons at angle θ as follows. A laser was mounted on the mobile rack directly below the central axis of the detector. The rotating stage carried two plates each with a pinhole. The detector at a distance L_{td} from the target centre was correctly positioned for a given angle θ once the laser beam passed through both pinholes.

The pulse-height analysis system consisted of a Canberra 2024 amplifier, Can-

berra 8077 ADC, and Aptec PC MCA acquisition card and software. Unipolar pulses were used with a shaping time of $2 \mu\text{s}$. Pileup rejection was used. The bin size used by the MCA software on acquisition was 0.058 keV; subsequently the data were rebinned to 1 keV for analysis.

4.1.2 Materials

The plastics were nylon, polycarbonate (POLYCA), polyethelene (POLYET), polymethyl methacrylate (PMMA) and polystyrene (POLYST). These samples were machined to have a square cross section of area $A = 1 \text{ cm}^2$ and thicknesses $d = 0.5, 0.75, 1, 1.5, 2, 2.5, 3, 3.5,$ and 4 cm . The biological materials were beef specimens of liver and muscle. A sample of water was also used. The water and biological samples were placed within a plastic container whose inner width was 1.01 cm and whose entrance and exit windows were milled down to a negligible thickness of 0.1 mm.

The intermolecular coherent scattering form factors (F), used as input parameters to the imaging models, were extracted from the x-ray diffraction measurements of Kosanetzky *et al.*³⁶

X-ray scattering in the diagnostic radiology energy range also arises from incoherent interactions. Using tabulations of the incoherent scattering function S for single elements,⁴³ S values for the composite materials were calculated using the sum rule (Eq. 7).

A 15-cm-diam spherical water phantom was used for in-water measurements. The PMMA spherical wall of the phantom was incorporated into the overall diameter.

4.1.3 Measurements of Primary and Scatter

For all of our measurements we operated the x-ray tube in fluoroscopic mode with a tube current ranging from 0.2 mA to 3 mA, a potential between 70 and 120

(a)

(b)

FIG. 24. Schematic top views of the (a) primary and (b) forward-scatter experiments. (Not to scale).

kV and 2.5 mm Al filtration. The numbers of photons transmitted $N_{\text{expt}}^{\text{primary}}$ and scattered $N_{\text{expt}}^{\text{scatter}}$ by a plastic target sample placed at the centre of the phantom were measured. These measurements were repeated for the different plastics and results of C and $\text{SNR}/\sqrt{K_{\text{e}}^{\text{air}}}$ [units are $(\text{mJ/kg})^{-1/2}$] were calculated as a function of target thickness d , scattering angle θ , and kV. We have divided SNR by $\sqrt{K_{\text{e}}^{\text{air}}}$, the square root of the air collision kerma, to normalize out variations of x-ray tube output and measurement time. Both in-air and in-water measurements were performed for the plastic samples whereas only in-air measurements were done for the biological materials.

Note that all of our spectra contain the Compton profile and the increase in counts toward lower energies due to the escape of secondary electrons, bremsstrahlung, and characteristic x rays from the Ge crystal. In this analysis we do not correct for the detector's response function. This error would be small in comparison to the other sources of error described later.

4.1.4 Ion Chamber Measurements

A quantitative validation of our models requires an estimate of the number of incident photons, N_0 , at different locations corresponding to the four different types of measurements. For in-air primary and scatter measurements we estimated N_0 at the target entrance and for in-water measurements at the phantom entrance. We determined these from ion chamber measurements as follows.

The ionization chamber volume was partially irradiated during our measurements because of the 3-mm-diam off-focal radiation aperture at the x-ray tube exit window. To correct for this we recorded for a given kV and mAs, an ion chamber reading Q_{partial} . We then removed the 3-mm-diam aperture and recorded for the same kV and mAs an ion chamber reading Q_{entire} . Then for all ion chamber readings Q

made with the same kV, the exposure X_{ion} is given in Roentgens by

$$X_{\text{ion}} = \frac{Q_{\text{entire}}}{Q_{\text{partial}}} C_X Q, \quad (35)$$

where $C_X = (2.18 \pm 0.15) \times 10^8 \text{ R C}^{-1}$ is the ion chamber calibration factor quoted by the manufacturer. The corresponding air collision kerma¹ is

$$K_c^{\text{air}} = 8.73 X_{\text{ion}} \text{ mJ kg}^{-1}. \quad (36)$$

4.1.5 Primary model

We now describe the specific details concerning the primary measurements, when using the plastic targets and an 80 kV beam. The detector was placed on the axis of the beam, at a distance $L_{\text{td}} = 230.0 \pm 0.2 \text{ cm}$ from the target object centre. The geometry is shown in Fig. 24(a). The distances from the focal spot to the ionization chamber center and to the phantom center were $L_{\text{ion}} = 50.6 \pm 0.3 \text{ cm}$, $L_c = 83.7 \pm 0.3 \text{ cm}$. The ratio $Q_{\text{entire}}/Q_{\text{partial}}$ was 1.71 ± 0.07 . A small circular aperture with $d_{\text{ap}}^p = 1.0 \pm 0.1 \text{ mm}$ was positioned in front of the Ge entrance surface at a distance of $1.2 \pm 0.2 \text{ cm}$. The distance between the 1-mm-diam aperture and the focal spot was $L_{\text{ap}} = 312.5 \pm 0.3 \text{ cm}$. This small aperture reduced the beam area so that in-air primary measurements could be done with relatively low pulse counting electronics dead times. The theoretical minimum pinhole diameter and slit opening required were $d_{\text{pin}}^{\text{min}} = 0.244 \text{ mm}$ and $\Delta w_{\text{slit}}^{\text{min}} = 0.294 \text{ mm}$. We chose $d_{\text{pin}} = (1.5 \pm 0.1) \text{ mm}$ and $\Delta w_{\text{slit}} = 1.15 \text{ mm}$ to allow for small misalignments. A small rectangular opening was formed between the phantom and detector by combining the slit shown in Fig. 24 with an additional orthogonal slit with a 3-mm-opening.

For both types of primary measurements, the expression for the number of incident photons N_0^{primary} per K_c^{air} is

$$\frac{N_0^{\text{primary}}}{K_c^{\text{air}}} = \frac{\Phi}{X} \frac{\pi}{4} (d_{\text{ap}}^p)^2 \left(\frac{L_{\text{ion}}}{L_{\text{ap}}} \right)^2 T_{\text{air}}(L_{\text{air}}) \frac{1}{8.73} [\text{mJ}^{-1} \text{ kg}]. \quad (37)$$

TABLE 8. Number of incident photons N_o per K_c^{air} for in-air and in-water measurements at 80 kV. In air: $L_{\text{air}} = L_{\text{td}} - d + L_c - L_{\text{ion}}$. In water: $L_{\text{air}} = L_{\text{td}} - 2R + L_c - L_{\text{ion}}$. The value shown for in-air is for a 4-cm-thick target but the same value was used for other d .

	Primary		Scatter	
	In Air	In Water	In Air	In Water
L_{air} (cm)	259.1 ± 0.5	240.9 ± 0.5	62.6 ± 0.4	51.6 ± 0.4
T_{air}	0.912	0.918	0.978	0.982
$\frac{N_o}{K_c^{\text{air}}} \text{ (kg mJ}^{-1}\text{)}$	$(3.8 \pm 0.8) \times 10^5$	$(3.7 \pm 0.7) \times 10^5$	$(1.7 \pm 0.2) \times 10^7$	$(1.70 \pm 0.2) \times 10^7$

From the Catalogue of Spectral Data for Diagnostic X-rays⁴⁷ we obtain, for an 80 kV beam, the photon fluence per unit exposure $\Phi/X = 1.69 \times 10^{11} \text{ cm}^{-2} \text{ R}^{-1}$. The factor $T_{\text{air}}(L_{\text{air}})$ takes into account that there is attenuation of the transmitted photons through air before reaching the detector. The distance L_{air} is the distance through air between ion chamber and detector. The transmission factor is

$$T_{\text{air}}(L_{\text{air}}) = \sum_{i=1}^N \rho(E_i) e^{-\mu_{\text{air}}(E_i)(L_{\text{air}})}, \quad (38)$$

where $\rho(E)$ represents the normalized distribution of the incident photon spectrum, N is the number of 1 keV bins, and μ_{air} is the linear attenuation coefficient for air.

The values of N_o^{primary} represent the number of photons that would be detected if there was no attenuation in the target and water phantom. The values are shown in Table 8.

4.1.6 Forward-scatter model

For the scatter measurements we moved the x-ray tube, pinhole, and phantom stage along the optical bench to be closer to the detector in order to compensate for the lower number of photons detected during scatter measurements. With this

new geometry, the ratio $Q_{\text{entire}}/Q_{\text{partial}}$ was 2.37 ± 0.12 . We sampled the scattered field in the horizontal plane defined by the x-ray beam, the target sample, and the detector. The sampling was from $\theta = 2^\circ$ to 12° in 1° ($\pm 0.1^\circ$) increments. For each θ , we aligned the detector such that L_{td} did not vary significantly (less than 3 mm). In our calculations, however, the specific value of L_{td} was used and the average value was $\bar{L}_{\text{td}} = 36.19 \pm 0.03$ cm. The ion chamber and phantom were at $L_{\text{ion}} = 51.9 \pm 0.3$ cm and $L_{\text{c}} = 82.3 \pm 0.3$ cm from the focal spot. A pinhole with $d_{\text{pin}} = 1.5 \pm 0.1$ mm was placed on the optical bench at a distance $L_{\text{pin}} = (74.8 \pm 0.3)$ cm from the focal spot. The incident number of photons $N_{\text{o}}^{\text{scatter}}$ per $K_{\text{c}}^{\text{air}}$ is

$$\frac{N_{\text{o}}^{\text{scatter}}}{K_{\text{c}}^{\text{air}}} = \frac{\Phi}{X} \frac{\pi}{4} (d_{\text{pin}})^2 \left(\frac{L_{\text{ion}}}{L_{\text{pin}}} \right)^2 T_{\text{air}}(L_{\text{air}}) \frac{1}{8.73} [\text{mJ}^{-1} \text{ kg}]. \quad (39)$$

and the values are shown in Table 8.

A larger aperture with $d_{\text{ap}}^{\text{s}} = (5.00 \pm 0.1)$ mm replaced the 1-mm-diam aperture in front of the HPGe detector. This larger aperture increased the solid angle of detection. Figure 24(b) shows a top view of the geometry. The active diameter of the detector was $d_{\text{detector}} \approx 5.17 \pm 0.06$ mm. The detector subtended an angle θ_{A} at the target center given on average by

$$\theta_{\text{A}} = 2 \tan^{-1} \left[\frac{d_{\text{detector}}}{2(\bar{L}_{\text{td}})} \right] = 0.82^\circ \pm 0.01^\circ, \quad (40)$$

and a solid angle given by

$$\Omega_{\text{detector}} = \frac{\pi d_{\text{detector}}^2}{4(\bar{L}_{\text{td}})^2} = (1.60 \pm 0.03) \times 10^{-4} \text{ sr}. \quad (41)$$

For a scattering angle θ , the detector sees the photons within the angular range $\theta_{\text{min}} = \theta - \theta_{\text{A}}/2$ to $\theta_{\text{max}} = \theta + \theta_{\text{A}}/2$. For the model calculations we assumed an annular detector that subtended a solid angle given by

$$\Omega_{\text{annular}} = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} 2\pi \sin \theta d\theta. \quad (42)$$

Since x-ray scattering from amorphous materials is circularly symmetric about the primary beam direction, we multiplied $N_{\text{expt}}^{\text{scatter}}$ by $\Omega_{\text{annular}}/\Omega_{\text{detector}}$. Results of C and $\text{SNR}/\sqrt{K_{\text{c}}^{\text{air}}}$ were calculated as a function of θ . We also generated results by taking the total number of scattered photons from 2° to 12° to be the signal. This would be accomplished by summing the counts obtained in each annular ring.

4.1.7 Beam Alignment

Before proceeding to do measurements for all target thicknesses in air and in water, we had to make sure that our beam alignment was good.

The alignment of the focal spot, pinhole, and detector for our primary measurements was verified as follows. A 4-cm-thick polycarbonate target was placed in air. The incident number of photons N_0 was calculated as described earlier. We then calculated the ratio of the number of photons detected to the predicted number. The calculation was repeated for different alignments of the x-ray tube. The best x-ray tube position yielded a ratio of ≈ 0.73 . We also scanned the target horizontally and vertically across the beam in order to make sure that our beam was hitting the target centrally.

Measurements in air and in water were then done for all other plastics. The ratio of detected to predicted was calculated for these measurements and the average values were 0.71 ± 0.01 for in air and 0.72 ± 0.01 for in water. This indicated an overall systematic misalignment of our system. Although C values are unaffected, we corrected the experimental SNR values by dividing them by $\sqrt{0.71}$.

For our scatter measurement geometry, the beam alignment was verified again via transmission measurements with a polycarbonate target 11 cm thick. A much thicker target was chosen because the distance between the focal spot and the detector was much smaller. Using a thinner target would have paralyzed the pulse height

electronics. This alignment assured us that the incident photons were hitting the center of the targets. Although a similar systematic misalignment occurred as for the primary geometry, we did not correct the SNR values since the beam alignment after the point of scattering is not crucial.

4.1.8 Correction for Photons Scattered by Water

The intersection of the conical primary beam and the conical volume seen by the detector defined a complex shaped scattering volume. Besides the scatter from the target there is unwanted single scatter from the water which we must correct for.

According to our analytic expressions [refer to Eqs. 8, 23, and 24], for a target of thickness d at the center of a spherical water phantom, the ratio of the fraction of incident photons that are scattered to the fraction that are transmitted is

$$\kappa = \frac{\frac{N_{\text{scatter}}}{N_{\text{o}}^{\text{scatter}}}}{\frac{N_{\text{primary}}}{N_{\text{o}}^{\text{primary}}}} = \frac{e^{-\mu_w(E_o)(d/2)} \frac{U_i(E_o, \theta_{\min}, \theta_{\max})}{\mu_i(E_o)} (1 - e^{-\mu_i(E_o)d})}{e^{-\mu_i(E_o)d}}, \quad (43)$$

where E_o is the photon energy, μ_w and μ_i are the linear attenuation coefficients for water and object i , and

$$U_i(E_o, \theta_{\min}, \theta_{\max}) = \rho_i^e \int_{\theta_{\min}}^{\theta_{\max}} \left[\frac{d_e \sigma_i^{\text{coh}}(\theta, E_o)}{d\Omega} + \frac{d_e \sigma_i^{\text{inc}}(\theta, E_o)}{d\Omega} \right] 2\pi \sin \theta d\theta \quad (44)$$

is the probability per unit object thickness of scattering into the solid angle $\Omega = \int_{\theta_{\min}}^{\theta_{\max}} 2\pi \sin \theta d\theta$ for object i . The electron density ρ_i^e is expressed in units of electrons cm^{-3} . The μ values were obtained using the mixture rule from the data for elements of Plechaty *et al.*⁴⁴ In Eq. 43, the ratio $\frac{U_i(E_o, \theta_{\min}, \theta_{\max})}{\mu}$ represents the fraction of interactions that are scatterings into the solid angle Ω . The remaining factors are due to transmission and for a 2-cm-thick polyethylene target their combined average value over the 2° to 12° range was 0.38 ± 0.03 . The experimental value for the scatter-to-primary ratio κ of Eq. 43 is obtained by dividing the scatter spectrum by the primary

spectrum bin by bin. Prior to dividing, we multiplied the bins in the scatter spectra by $\Omega_{\text{annular}}/\Omega_{\text{detector}}$ and divided the bins in both spectra by the calculated values of N_o determined from X_{ion} . Note that in Eq. 43 we assume that the Compton scattered photon energy is equal to that of the incident photon. This is a valid approximation since the largest Compton energy loss is ≈ 0.27 keV, corresponding to $\theta = 12^\circ$ and $E = 80$ keV.

Consider a 2-cm-thick polyethylene target in air. The spectrum obtained via transmission measurements using an 80 kV beam is shown in Fig. 25(a). Note that we have normalized by N_o^{-1} . A bin size of 1 keV was chosen. The collection time was 1 min and the dead time was 6.13%. The total number of counts was 236477. The spectrum consists of continuous bremsstrahlung plus peaks at 59 keV (tungsten $K\alpha$ radiation) and at 67 keV ($K\beta$). Figure 25(b) shows the spectrum obtained when one detects instead the scattered field at $\theta = 3^\circ$. The collection time was 1.5 minutes and the dead time was 0.45%. The total number of counts was 10484. The center of the broad maximum occurs at $E = 57$ keV and corresponds to $x = 1.2 \text{ nm}^{-1}$ where

$$x = \frac{E}{hc} \sin\left(\frac{\theta}{2}\right) \quad (45)$$

is a parameter that combines the dependence of scattering on photon energy and scattering angle. The peak occurs due to the polymer structure of polyethylene. Figure 25(c) shows the spectrum obtained at $\theta = 6^\circ$. The peak corresponding to $x = 1.2 \text{ nm}^{-1}$ has shifted toward lower energies because of the tradeoff between E and θ in the parameter x . The use of such material-specific spectra has been demonstrated by Harding et al.⁴⁸ with an energy-dispersive x-ray diffraction tomographic imaging system.

The scatter-to-primary ratio obtained experimentally and through prediction for a 2-cm-thick polyethylene target in air as a function of energy is shown in Fig. 26

FIG. 25. Spectra for a 2-cm-thick polyethylene sample measured in air with an 80 kV beam. (a) transmission, (b) $\theta = 3^\circ$, and (c) $\theta = 6^\circ$.

FIG. 26. Scatter-to-primary ratio for a 2-cm-thick polyethylene target as a function of θ and energy, measured in air. The distributions of dots are experimental results while the solid lines are predictions (Eq. 43).

for several angles. Note that the attenuation factor in Eq. 43 due to a water phantom does not apply in air. Excellent agreement is obtained at $\theta = 4^\circ$, 5° , and 6° . While good agreement is obtained for the locations of peaks at other angles, the magnitude differs significantly.

Figure 27 shows the results obtained in water for the 2-cm-thick polyethylene target. The magnitude of the ratio has increased with respect to the in-air measurements. This increase in scatter signal is due to water contamination.

Figure 28 shows the scatter-to-primary ratio as a function of θ and energy, obtained when the 2-cm-thick polyethylene target was removed from the water phantom. This measurement can be thought of as measuring the scatter signal of a 2-cm-thick “water target” in water. The coherent scattering form factors for water used to calculate predictions were those measured at 20°C from Ref. 49, which we take to be the

FIG. 27. Scatter-to-primary ratio for a 2-cm-thick polyethylene target as a function of θ and energy, measured in water.

gold standard for water. The general features as a function of θ in the experimental results are similar to those predicted. At the majority of energies, experiment yields higher interaction scatter fractions. Dividing on a bin-by-bin basis the experimental results by the theoretical ones yields the panels in Fig. 29. If there was no water contamination, these values should all be centered around unity. The fraction of water contamination at each angle was given by

$$\beta = \frac{\bar{\eta} - 1}{\bar{\eta}}, \quad (46)$$

where

$$\eta = \frac{\kappa_{\text{expt}}}{\kappa_{\text{predicted}}}. \quad (47)$$

We have chosen the energy range from 30 to 70 keV to calculate the values of β . Below 30 keV the number of counts in the primary spectra is overestimated while above 70 keV the number of counts in the scatter spectra is small. The fraction

FIG. 28. Scatter-to-primary ratio for a 2-cm-thick “water target” as a function of θ and energy, measured in water.

FIG. 29. Water contamination component for 2-cm-thick targets as a function of θ and energy.

of scatter due to water contamination ranged from $\beta = 0.26$ at $\theta = 10^\circ$ to 0.75 at $\theta = 2^\circ$. These estimates obtained with the 2-cm-thick “water target” measurements also corresponded to those for the 2-cm-thick plastic targets. Similar measurements and calculations were performed for the other target thicknesses.

For a specific scattering angle θ and plastic target thickness d , the number of scattered x rays due to water contamination is given by

$$N_{\text{wc}}(\theta, d) = \beta(\theta, d)N_{\text{water}}(\theta, d), \quad (48)$$

where $N_{\text{water}}(\theta, d)$ corresponds to the total number of scattered x rays detected when the plastic target was removed from the phantom (i.e. “water target” measurement). Note that the same geometry is used. The corrected experimental values of C and SNR become

$$C = 2 \frac{|N_t - N_b|}{N_t + N_b - 2N_{\text{wc}}(\theta, d)} , \quad (49)$$

and

$$\text{SNR} = \frac{|N_t - N_b|}{\sqrt{N_t + N_b - 2N_{\text{wc}}(\theta, d)}} . \quad (50)$$

4.2 Results

In Section 4.2.1 we present results of C and SNR obtained for imaging combinations of plastic targets and combinations of plastic/water targets. This analysis allows us to test our model predictions. In Section 4.2.2 we present, for interest, some preliminary results obtained in air for various combinations of biological tissues. Unless otherwise stated an 80 kV beam with 2.5 mm Al filtration was used.

The experimental error bars for C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ obtained via the primary measurements were calculated using Poisson statistics, the 1% uncertainty in our ion

chamber readings Q and the 7% systematic uncertainty in the ion chamber calibration factor C_x . These error bars are much larger than those obtained for the predicted values. These latter ones, based on the uncertainties in d (0.5 – 4%), R (< 1%) and μ (2%, which was in the range of 1 – 4% quoted by Ref. 50) are not shown. As mentioned, the SNR values for our primary measurements were scaled up by a factor $1/\sqrt{0.71}$ in order to correct for the beam misalignment.

For the scatter measurements, we included the uncertainty in the solid angle subtended by the detector and also propagated the error for the water contamination component. Recall that no beam alignment correction was applied to the scatter measurements.

Although the uncertainties in the form factor data were not taken into account, we have dedicated a section that shows the variation of the predicted values when one uses a different source of form factor data.

4.2.1 Plastic Targets

A. Primary

Figure 30 shows C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ results obtained for the tasks of imaging different 2-cm-thick plastic target combinations with the transmitted photons. Panels (a) and (b) are for in-air measurements while (c) and (d) correspond to in-water measurements. For the majority of combinations, the agreement between experiment and prediction is good.

Figure 31 shows results for imaging PMMA versus polycarbonate as a function of target thickness d . The model predicts well the variation of C and SNR as a function of d for both in-air (a),(b) and in-water (c),(d) measurements. Refer to Appendix C for other examples.

FIG. 30. C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ for primary measurements as a function of target/background combinations 2-cm-thick. (a),(b) in air, and (c),(d) in water. The open circles are predictions while the solid circles with error bars are from experiment.

B. Scatter

For the scatter results we have not taken the absolute value of the signal difference $N_t - N_b$ in Eqs. 1, 2, 49, and 50. This allows us to demonstrate that contrast reversal as a function of θ can occur.

Figure 32 shows C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ results obtained for the tasks of imaging a PMMA (target) versus polycarbonate (background) 2-cm-thick combination as a function of θ . Panels (a) and (b) are for in-air measurements while (c) and (d) correspond to in-water measurements. At $\theta = 2^\circ$ PMMA scatters more than polycarbonate

FIG. 31. C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ for (a),(b) in-air and (c),(d) in-water primary measurements as a function of d . Target = PMMA, and background = polycarbonate. The model predictions are shown as solid lines.

whereas at $\theta = 4^\circ$ the opposite occurs. The agreement between experiment and theory is moderate. Note the high values of C and SNR obtained at a scattering angle of 2° . For higher values of θ where intermolecular interference effects are reduced, C and SNR are generally smaller. Both experiment and prediction demonstrate contrast reversal as a function of θ . Refer to Appendix D for other examples.

Figure 33 shows C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ results obtained for the tasks of imaging different 2-cm-thick plastic combinations with the total scatter from 2° to 12° . These results are a superposition of the type of results shown in Fig. 32. Panels (a) and (b) are for in-air measurements while (c) and (d) correspond to in-water measurements. The variation of C and SNR obtained experimentally as a function

FIG. 32. C and $SNR/\sqrt{K_c^{air}}$ for in-air (a),(b) and for in-water (c),(d) forward scatter measurements as a function of the scattering angle θ . Target = PMMA, background = polycarbonate, and $d = 2$ cm. Open circles are predictions and solid circles are from experiment.

of target/background combination follows a similar pattern to the prediction. Their absolute magnitudes, however, differ substantially for most imaging tasks. Later in Section 4.3, we show scatter results for 2° to 12° versus target thickness d for PMMA/polycarbonate combinations in water.

C. Effects of kV on Scatter Imaging

In Fig. 34 are shown the spectra obtained at $\theta = 5^\circ$ in water for “water” and polyethylene 4-cm-thick targets using 80 and 120 kV beams. The tungsten $K\alpha$ and $K\beta$ peaks are clearly seen in the 120 kV spectra.

Table 9 shows the effect of using 120 versus 80 kV to image 4-cm-thick plastic

FIG. 34. Spectra at $\theta = 5^\circ$ for “water” and polyethylene targets 4-cm-thick measured in water using 80 and 120 kV beams.

targets against water at $\theta = 5^\circ$. No correction for water contamination was applied to the experimental data. Predicted and experimental results indicate that changing the kV from 80 to 120 kV generally decreases C and increases SNR. The 80 kV spectrum has a narrower energy distribution and therefore C is usually higher because the spectral blurring is smaller. The SNR obtained with the 120 kV spectrum is, however, higher because of beam hardening.

4.2.2 Biological Tissues

For the primary measurements we used a 70 kV beam whereas for the forward-scatter measurements 80 kV was chosen. A pinhole with $d_{\text{pin}} = (1.00 \pm 0.05)$ mm was used to collimate the incident beam in both cases. The experimental setups were similar to those for the plastic targets and we calculated $N_0^{\text{primary}}/K_c^{\text{air}} = (7.5 \pm 0.8) \times$

TABLE 9. The effect of kV on C and SNR for imaging 4-cm-thick plastic targets against water at $\theta = 5^\circ$.

Target	Background	$\frac{C(120 \text{ kV})}{C(80 \text{ kV})}$		$\frac{\text{SNR}(120 \text{ kV})}{\text{SNR}(80 \text{ kV})}$	
		Expt	Pred	Expt	Pred
PMMA	Water	0.86 ± 0.10	0.91 ± 0.06	1.04 ± 0.14	1.12 ± 0.12
Polycarbonate		0.99 ± 0.11	0.97 ± 0.06	1.18 ± 0.15	1.20 ± 0.12
Polystyrene		0.99 ± 0.11	1.01 ± 0.06	1.19 ± 0.15	1.24 ± 0.13
Polyethylene		1.11 ± 0.13	1.13 ± 0.07	1.31 ± 0.18	1.37 ± 0.14
Nylon		0.90 ± 0.11	0.95 ± 0.06	1.09 ± 0.15	1.17 ± 0.12

10^5 kg mJ^{-1} and $N_0^{\text{scatter}}/K_c^{\text{air}} = (7.4 \pm 0.8) \times 10^6 \text{ kg mJ}^{-1}$. Only in-air measurements were performed. The scattered field was sampled from 2° to 10° in 1° increments.

Figure 35 shows C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ for the task of imaging (a),(b) water/beef liver, (c),(d) water/beef muscle, and (e),(f) beef liver/beef muscle via the primary and forward-scatter models. The abscissa indicates which type of imaging geometry is used. The first corresponds to primary imaging, the next nine correspond to scatter imaging with annular detectors at specific θ , and the final to scatter imaging with all scatter between 2° to 10° . The C obtained with scatter especially at low angles $\theta \leq 4^\circ$ is higher than what is obtained with primary imaging for all three combinations.

For water/beef liver good agreement between experiment and prediction for C [Fig. 35(a)] occurs for most imaging geometries. Figure 35(b) shows corresponding SNR values and moderate agreement is obtained.

The variation of C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ as a function of scattering angle is similar between prediction and experiment for water versus muscle [Figs. 35(c) and 35(d)]. For liver versus muscle [Figs. 35(e) and 35(f)], we obtain agreement for angles $\geq 6^\circ$. The preparation of the samples was quick and likely, small amounts of other tissue types were present in a given sample. This would affect C and SNR.

FIG. 35. C and $\text{SNR} / \sqrt{K_c^{\text{air}}}$ for imaging the following combinations in air: (a),(b) water/liver, (c),(d) water/muscle, and (e),(f) liver/muscle.

FIG. 36. Coherent form factors as a function of x for PMMA. These curves were extracted from various x-ray diffraction studies. (Kosanetzky *et al.*,³⁶ Tartari *et al.*,⁵¹ Peplow and Verghese,⁵² and our laboratory.⁵³)

4.3 Effects of New Data for Form Factors

One of the reasons for disagreement between experiment and prediction can be attributed to the form factor data which were used as input to the forward-scatter calculations. As mentioned previously, the form factors F of the plastic and biological tissues used in this work were extracted from the diffraction measurements of Kosanetzky *et al.*³⁶ Although there exist some other sources of data,^{51,52} there is not yet consensus in the literature. Figure 36 shows the form factor data as a function of x for PMMA measured by three different authors^{36,51,52} plus some preliminary work in our lab.⁵³ Although all four curves have been made to coincide at large x where atoms scatter independently, they differ substantially at lower values of x .

Figure 37 shows results of C and $\text{SNR}/\sqrt{K_{\text{c}}^{\text{air}}}$ for the task of imaging PMMA versus polycarbonate in water as a function of target thickness d . An angular range

FIG. 37. C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ for in-water forward-scatter measurements as a function of d . Target = PMMA, background = polycarbonate. Solid lines are predictions using: #1 = our form factor data,⁵³ #2 = Peplow and Verghese⁵² data, and #3 = Kosanetzky *et al.*³⁶ data. Solid circles are experiment.

of detection from 2° to 12° was used. The predictions were calculated by using three sources of form factor data: our laboratory,⁵³ Peplow and Verghese,⁵² and Kosanetzky *et al.*³⁶ We see that using different sources of diffraction data has a significant effect on C and SNR.

4.4 Discussion

We have configured an experiment to validate our primary and forward-scatter x-ray imaging models. Overall good agreement between prediction and experiment

was obtained.

In the analytical formulation of the models we assumed that our scattering volume contained only the target sample. However, with our measurements we showed that a significant contribution of scatter from the water phantom also reaches the detector. With the aid of our seminanalytic expressions we were able to quantify and correct for the water contamination component.

The reproducibility of our measurements was assessed by measuring, for the 2-cm-thick plastic targets, the $\theta = 5^\circ$ scattered field on two separate days. Between these two sets of measurements the detector, spherical phantom, and target samples were moved although the incident beam alignment was not changed. The width of the slit was also changed between measurements. The detector to target center distance L_{td} was approximately the same for both sets. For the different possible target/background combinations, the root-mean-square deviations in C and SNR were 0.01 and 0.9 for in-air, and 0.03 and 0.6 for in-water measurements.

The effect of using different form factor data showed variations in the predicted values of C and SNR. Although different authors show the same structure for the form factors, there is still doubt about their magnitude. Work in parallel in our laboratory, therefore, consists of obtaining accurate form factors for plastic and biological tissues via x-ray diffraction measurements.

CHAPTER 5

SUMMARY

5.1 Conclusions

I have developed a simple but useful model to quantify the information content of x-ray scatter images. The analytic expressions for C and SNR provided quantitative descriptions of the unique relationship that exists between them, which in turn, helped in interpreting the results. My model predicts that there exist specific medical regimes where x-ray scatter imaging could offer higher C and SNR than conventional x-ray imaging whether monoenergetic or polyenergetic beams are used.

The experiment that was configured to validate the model proved to be a crucial component. Plastics were chosen since their scattering properties were thought to be relatively well known. However I showed that the model predictions varied significantly with different available form factor data.

In our formulation of the forward-scatter model we neglected the fact that the intersection of the conical primary beam and the conical volume seen by the detector defined a complex shaped scattering volume. Within the volume one finds the plastic of interest as well as a volume of unwanted water. Combining our analytical expressions for the numbers of transmitted and scattered x rays with our experimental measurements we were able to determine the amount of water contamination and correct for it.

Overall, good agreement between prediction and experiment was obtained. We are confident that our model can be used as a tool for designing and optimizing an x-ray scatter imaging system.

5.2 Future Work

Suggestions for future work are divided into short term and long term projects. The items are listed in decreasing order of importance.

5.2.1 Short term

1. In Section 4.3 we showed that using different sources of form factor data has significant effects on the predicted C and SNR. Our analysis suggests that more accurate form factor data for the plastics are needed. A complete set of diffraction data for biological tissues including both healthy and cancerous samples would help advance the technology towards clinical applications.
2. In-water measurements with biological tissues should also be done. A new holder for biological tissues should be fabricated in order to allow these measurements. The preparation of biological samples should be done much more precisely. Perhaps collaboration with the Departments of Chemistry and Biology would be advantageous.
3. The validation of the backscatter model with a slab-shaped water phantom should be done. The experimental apparatus was designed in order to allow measurements of the backscattered field.
4. Similar measurements with a 25-cm-diam phantom (available) should be done and compared to the results obtained in this work. Such measurements could help demonstrate that multiple scatter effects are negligible in our system.

Imaging plastic combinations in either phantom should yield the same C when a monoenergetic beam is used.

5. The systematic misalignment should be investigated. Measurements with no target and no phantom should be done with different sized apertures prior to the detector entrance. These different sized apertures must be placed at the same distance from the detector. A new end-cap holder for the apertures should be fabricated. The ratio of the number of detected photons to the predicted number should be plotted as a function of aperture size. The curve should have a slope of zero. The detector's response function can also be characterized as a function of aperture size. For alignment purposes, the x-ray tube stand should be modified in order to allow small movements in all directions including rotation capabilities.
6. Measurements with several monoenergetic beams could validate our monoenergetic calculations and also provide more insight into the forward-scatter measurements. A monoenergetic beam would narrow the momentum transfer range and allow one to investigate specific regions of F .
7. Although we believe that using the spectra in Ref. 47 for our model prediction calculations is sufficient, it should be verified by measuring the actual spectra and correcting for the detector's response function.

5.2.2 Long term

1. Our analysis suggests that more work will be needed in order to determine the optimum angular range of detection. Our results obtained with the total scatter between 2° to 12° are generally smaller than what is obtained at a given small angle. Perhaps a combination of smaller angles will offer the

best signal. Also measurements done at ultra-low angles ($< 1^\circ$) could be beneficial.⁵⁴

2. As was shown in Appendix A there could be possible applications of using the forward-scattered x rays as a new source of information in neuroradiology. In support of this, the model should be extended to analyse x-ray diffraction CT.

APPENDIX A

Model Predictions: Fixed Energy Fluence

In Chapter 2 we gave results obtained with monoenergetic beams for which the incident photon fluence Φ_o was constant. In Chapter 3, for our analysis of spectral blur effects, we kept the incident energy fluence Ψ_o constant. Figure 19 showed that using constant Ψ_o versus constant Φ_o had small effects on C and SNR. Here we give several other examples obtained with monoenergetic beams for which $\Psi_o = 1.88 \times 10^{11} \text{ keV} \cdot \text{cm}^{-2}$ corresponding to an entrance exposure of 100 mR for 60 keV photons.

An angular range of $2^\circ \leq \theta \leq 12^\circ$ was chosen for our analysis of forward-scatter imaging and $158^\circ \leq \theta \leq 178^\circ$ was chosen for the backscatter model. For the backscatter model, the target objects were located at depth $z = 3 \text{ cm}$. As before, the cross-sectional areas of the target and background objects were chosen as $A = 1.0 \text{ mm}^2$, sufficiently small to validate the omission of multiple scatter and self-attenuation effects.

The SNR was evaluated numerically as the target object thickness $d = 0.01 - 40 \text{ mm}$ and incident monoenergetic photon energy $E_o = 10 - 200 \text{ keV}$ were systematically varied. We show the maximum SNR, denoted by SNR_{max} , over all photon energies versus object thickness d for the imaging of bone/muscle [Fig. 38(a)], fat/muscle [Fig. 38(b)], water/gray matter [Fig. 38(c)] and white/gray matter [Fig. 38(d)] via

FIG. 38. The maximum SNR over all photon energies versus object thickness d within the water phantom for the imaging of (a) bone/muscle, (b) fat/muscle, (c) water/gray matter, and (d) white/gray matter combinations via the four models.

all four imaging models. The thicknesses of the spherical and slab-shaped water phantoms are 25 cm for the imaging of the first two combinations and 15 cm for the second two. Figures 38(a) and 38(b) demonstrate the potential advantages of an x-ray backscatter imaging device to distinguish superficial structures. For the bone/muscle combination, $SNR_{\max}^{\text{backscatter}} > SNR_{\max}^{\text{primary}}$ for $d \leq 3.6$ mm, whereas for fat/muscle this occurs for $d \leq 17$ mm. A similar result holds for fat/liver (not shown), for which $SNR_{\max}^{\text{backscatter}} > SNR_{\max}^{\text{primary}}$ for $d \leq 22$ mm. The potential benefits of using a forward-scatter device in neuroradiology are shown in Figs. 38(c) and 38(d). In both cases, $SNR_{\max}^{\text{forward-scatter}} > SNR_{\max}^{\text{primary}}$ for all d considered.

An interesting feature in Figs. 38(c) and 38(d) is that the SNR obtained via the total scatter model is smaller than what is obtained with an annular detector defined by $\theta_{\min} = 2^\circ$ and $\theta_{\max} = 12^\circ$. The higher C (not shown) obtained with the scattered x rays between 2° to 12° compensates for the smaller solid angle of detection.

APPENDIX B

Centroid and Standard Deviation Matching

In Section 3.2, we explained that our analysis of spectral blur involved three different types of polyenergetic beams with constant Ψ_0 : (1) a dual peak beam, (2) a rectangular beam, and (3) a filtered kV bremsstrahlung beam. In order to compare C and SNR obtained with these different beams we matched the $\langle E \rangle$ and σ of the dual and rectangular spectra to those of the filtered kV spectra. As shown below, the method of matching $\langle E \rangle$ and σ is straightforward.

Suppose we apply a bandpass filter to some kV spectrum. Depending on the filter's lower and upper limits, we either obtain a hypothetical bandpass filtered kV spectrum or an actual clinical x-ray spectrum. We denote their mean and standard deviation as $\langle E \rangle_{\text{kV}}$ and σ_{kV} . One can easily show that a dual peak beam with energies E_1 and E_2 given by

$$\begin{aligned} E_1 &= \langle E \rangle_{\text{kV}} - \sigma_{\text{kV}}, \\ E_2 &= \langle E \rangle_{\text{kV}} + \sigma_{\text{kV}}, \end{aligned} \tag{A1}$$

and a rectangular beam with lower and upper limits $E_{\min}$, $E_{\max}$ given by

$$\begin{aligned} E_{\min} &= \langle E \rangle_{\text{kV}} - \sqrt{3} \sigma_{\text{kV}}, \\ E_{\max} &= \langle E \rangle_{\text{kV}} + \sqrt{3} \sigma_{\text{kV}}, \end{aligned} \tag{A2}$$

both have $\langle E \rangle = \langle E \rangle_{\text{kV}}$ and $\sigma = \sigma_{\text{kV}}$.

APPENDIX C

Primary Results

In Section 4.2.1 we showed results of C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ obtained with the primary geometry for imaging PMMA versus polycarbonate targets d -cm-thick using 80 kV beams and 2.5 mm Al. Both in-air and in water results were shown [Fig. 31]. In this section we show additional results obtained using an 80 kV beam for different combinations of target and background objects. Recall that the SNR curves have been scaled up by a factor $1/\sqrt{0.71}$ to correct for the systematic misalignment of our system. The model predictions are the solid lines.

Primary results obtained in air are shown in Fig. 39. Although C and SNR obtained for the imaging tasks involving PMMA appear to be low for $d \geq 2$ cm, the agreements between experiment and prediction for other combinations are excellent.

Primary results obtained in water are shown in Fig. 40. Good agreement is obtained for all combinations involving two plastics. The agreement for the tasks of imaging a plastic against water however is moderate. To image PMMA, polycarbonate, and nylon targets against water, experiment is higher than prediction, whereas to image polystyrene and polyethylene against water prediction is higher.

A probable explanation for these differences is that for our measurements with plastics there is also the phenomenon of x-ray refraction that takes place between the interfaces of the plastics and water. For imaging plastic combinations these

effects cancel out but for imaging a plastic versus water they do not. A variety of imaging techniques have been devised to produce x-ray refraction contrast images with laboratory x-ray sources^{55,56} and synchrotron sources.^{57,58}

FIG. 39. C and $\text{SNR} / \sqrt{K_c^{\text{air}}}$ for in-air primary measurements as a function of d .

FIG. 39 cont'd

FIG. 39 cont'd

FIG. 40. C and $SNR/\sqrt{K_c^{air}}$ for in-water primary measurements as a function of d .

FIG. 40 cont'd

FIG. 40 cont'd

FIG. 40 cont'd

FIG. 40 cont'd

APPENDIX D

Forward-Scatter Results

In Section 4.2.1 we showed results of C and $\text{SNR}/\sqrt{K_c^{\text{air}}}$ obtained as a function of θ for the task of imaging PMMA versus polycarbonate 2-cm-thick combinations in air and in water using 80 kV beams and 2.5 mm Al [Fig. 32]. In this section we show in Fig. 41 additional results as a function of θ and d for several combinations of target and background objects using an 80 kV beam. Only in-water results are shown: (a) PMMA versus polycarbonate, (b) PMMA versus polystyrene, (c) PMMA against water, (d) polycarbonate versus polystyrene, (e) polycarbonate against water, and (f) polystyrene against water. The error bars for predictions are shown in order to demonstrate that these uncertainties are small in comparison to experimental error bars. Experimental results are closed circles and model predictions are open circles.

The agreements obtained for imaging PMMA [Fig. 41(c)] and polycarbonate [Fig. 41(e)] against water are good and moderate agreement is obtained for PMMA versus polycarbonate [Fig. 41(a)] and polystyrene against water [Fig. 41(f)]. However for PMMA versus polystyrene [Fig. 41(b)] and polycarbonate versus polystyrene [Fig. 41(d)] the differences are significant. As explained in Chapter 4, more accurate coherent form factor data for the plastics could explain the differences.

PMMA VERSUS POLYCARBONATE

FIG. 41. C and $\text{SNR} / \sqrt{K_c^{\text{air}}}$ obtained as a function of θ and d for imaging in water (a) PMMA versus polycarbonate.

PMMA VERSUS POLYSTYRENE

FIG. 41 cont'd (b) PMMA versus polystyrene.

PMMA VERSUS WATER

FIG. 41 cont'd (c) PMMA against water.

POLYCARBONATE VERSUS POLYSTYRENE

FIG. 41 cont'd (d) polycarbonate versus polystyrene.

POLYCARBONATE VERSUS WATER

FIG. 41 cont'd (e) polycarbonate against water.

POLYSTYRENE VERSUS WATER

FIG. 41 cont'd (f) polystyrene against water.

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