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Dynamic Modelling and Measurement of a Structural Model with Thin Plate Elements

By

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A thesis submitted to
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the Faculty of Graduate Studies and Research
the acceptance of the thesis

Dynamic Modelling and Measurement of a
Structural Model with Thin Plate Elements

by

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in Partial Fulfilment of the Requirements
for the Degree of Master of Engineering

Thesis Supervisor

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Abstract

The direct boundary element method for thin plates has been shown to provide an accurate model for slabs supported on columns, and for use in the structural analysis of building frames. The method is extended for use in the dynamic analysis of non-symmetric flat plate building frames where storey twist displacements are an important consideration in the building design.

The direct boundary element method for thin plate bending analysis is used to model the flat plate slabs of a non-symmetric building frame. The model is reduced to 3 DOF per storey and used for dynamic analysis. The results are compared to those of a finite element model. A steel model is designed and fabricated and an experimental investigation is performed to verify the direct boundary element model.

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Table Of Contents

Acceptance Sheet	II
Abstract	III
Acknowledgments	IV
Table of Contents	V
Table of Figures and Tables	VII

1.	Introduction	1
1.1	General	1
1.2	The Direct Stiffness Matrix Method	3
1.3	Dynamic Analysis	4
1.4	Static and Dynamic Analysis of Models Incorporating Plate Elements	6
1.5	The Building Frame	9
1.6	Objective	10
2.	Building Frame Model	14
2.1	Introduction	14
2.2	Overview of the Direct Boundary Element Method	15
2.2.1	Discretization	15
2.2.2	Direct Boundary Element Method	16
2.2.3	Modelling Column Supports	18
2.2.4	Calculating the Stiffness Matrix	18
2.3	Intermediate Building Frame Model	19
2.3.1	Slab Elements	19
2.3.2	Column Elements	20
2.3.3	Global Stiffness Matrix	20
2.4	Compact Stiffness Model	24
3.	Analysis and Experimental Investigation	30
3.1	Introduction	30
3.2	Computer Model Investigation	30
3.2.1	Introduction	30
3.2.2	Dynamic Analysis	32
3.2.3	Boundary Element Model	33
3.2.4	BEM Dynamic Analysis Results	35
3.2.4.1	Finite Column-Slab Joint Model	35
3.2.4.2	Single Point Column Support Model	36
3.2.4.3	Discussion	36
3.2.5	Finite Element Model	37
3.2.6	FEM Dynamic Results	39
3.2.6.1	Dynamic Results by Full Finite Element Model	39

3.2.6.1.1	Finite Column-Slab Joint Model	39
3.2.6.1.2	Single-point Column Support Model	40
3.2.6.2	Dynamic Results by Compact Stiffness Matrix	40
3.2.6.2.1	Finite Column-Slab Joint Model	40
3.2.6.2.2	Single-point Column Supports	40
3.2.6.3	Discussion	41
3.3	Experimental Investigation	41
3.3.1	Introduction	41
3.3.2	Construction of the Test Model	42
3.3.3	Instrumentation	43
3.3.4	Method of Loading	43
3.3.5	Data Analysis	44
3.3.5.1	Fourier Transforms and Cross-spectral Density Functions	44
3.3.5.2	Extracting the Dynamic Characteristics of the Experimental Test Model	46
3.3.5.3	Example Calculation	48
3.3.6	Experimental Test Results	51
3.4	Discussion	52
3.4.1	Introduction	52
3.4.2	FEM Compact Stiffness Model and the Full FEM Model	53
3.4.3	Flexibility Matrices	53
3.4.4	Natural Frequencies of Vibration	54
3.4.4.1	BEM and Experimental Results	54
3.4.4.2	FEM and Experimental Results	55
3.4.5	Vibration Displacement Mode Shapes	58
3.4.6	Conclusion	57
4.	Conclusion	80
4.1	Summary	80
4.2	Conclusion	81
4.2	Future Work	82
	References	84
	Appendix A	87
	Appendix B	91

List of Figures and Tables

Figure 1.1	Shear Building	12
Figure 1.2	Building Frame Model of El-Kafrawy	12
Figure 1.3	Building Frame Model With 3 DOF Per Storey	13
Figure 2.1	Betti-Maxwell Reciprocal Theorem	26
Figure 2.2	States of Equilibrium Used in the BEM	26
Figure 2.3	Intermediate Building Frame Model	27
Figure 2.4	Column Element DOF	27
Figure 2.5 (a)	Example Structure	28
(b)	Column Displacements After Unit Twist	28
Figure 2.6	Geometry of Column Displacement	28
Figure 2.7	Column Element Displaced Shape Due to Unit Translation with DOF Forces	29
Figure 2.8	Example Structure for Static Lateral Load	29
Figure 3.1	Building Model General Configuration	59
Figure 3.2	Stiff Column Patch Used in the FEM Model	60
Figure 3.3	Boundary Element Mesh and Column Point Forces	61
Figure 3.4 (a)	Compact Flexibility Matrix for BEM Model With Finite Column-Slab Joints	61
Figure 3.4 (b)	Compact Flexibility Matrix for BEM Model Using 1-Point Column Supports	62
Figure 3.5	Lumped Mass Matrix	62
Table 3.1 (a)	Dynamic Results of BEM Model With Finite Column Supports	63
Table 3.1 (b)	Dynamic Results of BEM Model With Single Point Column Supports	63
Figure 3.6	Finite Element Mesh	64
Figure 3.7(a)	Compact Flexibility Matrix of FEM Model With Finite Column-Slab Joints	65
Figure 3.7 (b)	FEM Model Compact Flexibility Matrix of FEM Model With Single Point Column Supports	65
Table 3.2 (a)	Results of Dynamic Analysis of Full FEM Model with Finite Column-Slab Joints	66
Table 3.2 (b)	Results of Dynamic Analysis of Full FEM Model With Single Point Column Supports	66
Table 3.3 (a)	Results Dynamic Analysis of Compact FEM Model With Finite Column-Slab Joints	67
Table 3.3 (b)	Results of Dynamic Analysis of Compact FEM Model With Single Point Column Supports	67
Figure 3.8	Laboratory Model of Steel Construction	68
Figure 3.9 (a)	Model Base Mount	68
(b)	Base Clip Detail	68

Figure 3.10	Placement of Accelerometers on Model Plate	69
Figure 3.11	Shaking Table Secured to Floor	69
Figure 3.12	Plate Translating Laterally	70
Figure 3.13	Cosine Waves In Phase	70
Figure 3.14	Plate Rotating About Axis Perpendicular to Surface	71
Figure 3.15	Cosine Waves 180 Degrees Out of Phase	71
Figure 3.16	Triangular Accelerometer Configuration and unknown Displacements	72
Figure 3.17	Laboratory Model Acceleration vs Time Raw Data	73
Figure 3.18	Laboratory Model Acceleration vs Time Window Plot	74
Figure 3.19	Shaking Table Acceleration vs Time Raw Data	75
Figure 3.20	Fourier Spectrum of Test HAM1 Data	76
Figure 3.21	Cross-Spectra of Test HAM1 Data	77
Table 3.4	Experimental Dynamic Test Results	78
Figure 3.22	First Vibration Mode DOF Displacements	78
Figure 3.23	Second Vibration Mode DOF Displacements	79
Figure 3.24	Third Vibration Mode DOF Displacements	79
Plate A.1	Test Model and Shaking Table	87
Plate A.2	Model Base and Mounting Clamps	88
Plate A.3	I-Beam Anchoring Shaking Table	89
Plate A.4	Accelerometer	90

1. Introduction

1.1 General

This thesis addresses the analysis of frames comprised of one-dimensional beam elements and two-dimensional thin plate bending elements. A typical example of such a frame structure in civil engineering is the building frame of which the simplest form is the flat plate system. The flat plate building frame system is studied in detail in this thesis. although the methodology developed in the present study can be applied to the analysis of other complex structural systems as well.

It has been shown that an important consideration in the design of building structures is torsional effects due to lateral loads [12,14,18,24,33]. Torsion in a building occurs during earthquakes when the centre of mass of the building does not coincide with the building shear centre, resulting in a couple between the building inertial and stiffness forces. Torsional-lateral coupling in asymmetric buildings can lead to increased lateral storey drift during earthquakes. The distance between the shear centre and the centre of mass is the building eccentricity. For buildings whose translational natural frequency is close to that of the torsional natural frequency, the effective eccentricity of the building can be increased by as much as a factor of six [31]. It is therefore important to employ a 3-dimensional building model in the analysis to account for building twist in the design process.

Prior to the 1960's in some regions of North America, flat-plate slab buildings, mainly used for hotel, office and residential buildings, were designed to resist gravity loads only. A drawback of this particular type of building is that they are quite flexible and susceptible to large storey drift during earthquakes. Many existing buildings must therefore be re-evaluated using modern analysis techniques to determine their realistic strength capacities under earthquake loads and whether retrofitting is necessary [23]. Because of this reason and the need of an efficient tool for the analysis and design of new buildings, an efficient means for the dynamic analysis of a complex structure is needed by practising engineers.

The analysis of building frames under static and dynamic loadings is a complex undertaking. Although this task is attainable through the use of computers, for the practising engineer any technique which simplifies the problem while maintaining accuracy of results is an important tool. Simplifying the mathematical model of the physical structure generally leads to a reduction in the number of calculations performed in the analysis. Although model simplification is usually accompanied with loss of accuracy, sometimes appropriate simplifications based on engineering judgement with insignificant loss of accuracy can still be achieved.

In the present study, the vibrational behaviour of a building frame model comprised of column elements and slab super-elements modelled by the direct boundary element

method is studied. A simplified model of the building frame having only 3 DOF per storey is used in the dynamic analysis. Numerical results are compared with the results obtained from an experimental study of the building frame.

1.2 The Direct Stiffness Matrix Method

The direct stiffness matrix method involves the solution of the deformed shape of a given structure due to applied loads. In general, the given structure is discretized and displacements are found at selected discrete points, the degrees of freedom (DOF). Each DOF has associated stiffness values based on the physical properties of the problem. The displacements can be found by solving the equation

$$[K]\{d\} = \{f\} \quad (1)$$

in which $[K]$ is the stiffness matrix of the system, $\{d\}$ is a column matrix of DOF displacements, and $\{f\}$ is a column matrix of equivalent forces at the DOF due to applied loads.

When this method is applied to the analysis of framed structures, the local stiffness matrices of structural elements are formed and assembled into the global stiffness matrix of the entire structure. External loads on members are distributed to their associated DOF and inserted into the building global force matrix. The resulting system of equations

described by Equation (1) is then solved using the Gaussian forward and backward substitution procedure. The difficulty in the use of this method arises when non-standard structural elements are used. Slab stiffness coefficients, for example, are not as easily computed as those of simple frame elements and require advanced modelling methods such as the finite element method or the boundary element method. The most common usage of the direct stiffness method is in the analysis of building frames comprised of simple elements and in two dimensions.

1.3 Dynamic Analysis

The following is a brief overview of accepted methods of analysis used in earthquake response analysis of building frames.

1. Modal Superposition: The dynamic behaviour of a structural system is represented by its modal vibrational behaviour expressed in terms of natural frequencies and vibration displacement mode shapes, which can be found by the solution of the equation

$$\det([K] - \omega^2[M]) = 0 \quad (2)$$

where $[M]$ is the system mass matrix and ω is the natural vibration frequency. By the superposition of the response contribution from each vibration mode, an elastic analysis of a multi-degree-of-freedom structural system is simplified to the analysis of a series of

single-degree-of-freedom systems. Because of the reliance on the concept of superposition, the method is a linear analysis procedure and thus can not account for the effects of plastic deformations in the structure [9].

2. Equivalent Lateral Static Load: By using seismological characteristics of the site of the building, the dynamic properties of the building and the local geological and soil conditions of the site, the equivalent static lateral load method represents seismic load effects by lateral static design forces applied at the floor levels of the building, thus simplifying the analysis. The earthquake response of the structure is then obtained by static elastic analysis using the equivalent static lateral loads [1].

3. Inelastic Time-History Analysis: It is usually uneconomical and unnecessary to design a building to resist earthquake-induced forces in the elastic range. Under the strong shaking of a severe earthquake, buildings are expected to behave inelastically with plastic deformations at controlled locations and components of the structure. The direct time step integration technique in the time domain can be effectively used to determine the response of a building subjected to ground excitation, taking into account the effects of stiffness degradation and plastic deformations in the structure. This procedure involves the division of the loading time-history into a sequence of load steps. The structure is analysed under the load of each time step, using the results of the previous step as the initial conditions for the current time step. Within the analysis of each loading time step, the structure is assumed to behave linearly with the stiffness matrix remaining

constant, although the stiffness matrix may be altered between load steps to account for changes in stiffness due to plastic deformation within the structure. Inelastic earthquake response analysis by time-history analysis is perhaps the most detailed method of analysis, and also one of the most time consuming [9].

4. Inelastic Building Displacements From Elastic Analysis: It has been shown [27] that an upper bound estimate of building displacements can be obtained through the calculation of the elastic response by an elastic method of analysis, such as modal superposition. Therefore, although typical building structures are not designed to resist earthquake forces in the elastic range, an elastic analysis is still very useful as the results can be modified to predict the effects of plastic deformations in the structures.

1.4 Static and Dynamic Analysis of Models Incorporating Plate Elements

There exist several mathematical models which have been used for the static and dynamic analysis of building structures which incorporate plate elements. One of the simplest models used in the analysis of building frames is the shear building model shown in Figure 1.1, where floor slabs are assumed to be out-of-plane rigid, and slab-column joints do not rotate under load. For a 3-dimensional analysis, 2 translation and one twist degrees of freedom per storey are needed to describe building deformations. Corderoy and Thambiratnam [11], and Thambiratnam and Thevendran [28] have used the shear

building model along with energy methods to obtain stiffness and mass matrices for static and dynamic analyses. Although this method of analysis is easy to perform, it may not provide accurate results if slab elements are flexible, since slab-column joint rotations contribute to building lateral displacements.

In recent years, a class of FEM-based analysis techniques, called global-local methods, have been developed in order to reduce the number of DOF in building frame models. This methodology, exemplified by two procedures called the "displacement distribution factor method" by Leung and Wong [21], and the "finite storey method" by Pekau, Zielinski and Lin [26], generally employs a global analysis of a building structure to obtain lateral and twist displacements at each storey level, and then uses these displacements to find local deformations at areas of interest within each storey level through the use of interpolation functions which relate local deformations to global displacements. Using this technique, local areas of interest do not need to be defined until after the analysis is performed.

El-Kafrawy [13] has introduced a method of building frame static analysis which employs the finite element method. A 3-D frame model is formulated using two types of elements: the slab super-element and column element, with two translation and one twist DOF per slab element at the storey level, and two rotation and one axial DOF at slab-column joints, as shown in Figure 1.2. Finite elements are used to model the floor slab to generate the slab stiffness matrix, and the column stiffnesses are derived based on beam

theory. The stiffness matrix of the slab is then combined with that of the columns to form the building frame global stiffness matrix. Derivation of the slab stiffness matrix involves applying unit loads in succession to each slab-column joint DOF and using the finite element model to calculate the displacements corresponding to the DOF, which give the flexibility coefficients for each DOF. The resulting flexibility matrix is then inverted to obtain the slab stiffness matrix. Lateral stiffnesses of the building frame are derived from column stiffness coefficients.

Abdel-Akher [3] has used the direct boundary element method (DBEM) to model flat plate slabs supported directly on columns. By combining the stiffness matrix of the slab with that of the columns, similar to the work by El-Kafrawy, he has obtained a 3-dimensional building frame model which provides accurate results.

Hu [19] has extended Abdel-Akher's work by providing boundary element models for the bending analysis of plates with elastic edge beam supports.

It has been shown [17] through laboratory static and dynamic tests on a building model of reinforced concrete construction that the model flexibility matrix can be found by applying transverse and torsional static loads on the model while measuring storey displacements. Further, the natural frequencies and vibrational mode shapes calculated using the flexibility matrix were shown to have been in close agreement with measured findings.

As a further improvement, a model is developed in the present study which retains the simplicity of the shear building model in terms of the number of DOF per storey, but also has the accuracy of a more sophisticated model. A technique is developed which generates a building frame stiffness matrix with 3 DOF per storey in 3 dimensions and which accounts for the effects of building eccentricities and flexible slab-column joints.

A building frame is modelled using a sophisticated method. Unit loads are applied to each storey DOF: two lateral, and one twist, as shown in figure 1.3. The building flexibility matrix is derived from the resulting displacements. Inverting this matrix results in the stiffness matrix which may now be used for repeated static and dynamic analysis. This method produces a simple model (3 DOF per storey) and incorporates the accuracy of a more sophisticated model. Furthermore, the simplicity of the final model is constant regardless of the complexity of building geometry.

A requirement of the present analysis method is that the mass and inertial effects in the building frame are essentially concentrated at the slabs. This assumption is generally valid for most typical flat plate building frames [9].

1.5 The Building Frame

The present study deals specifically with building frames consisting of flat plate slabs

supported directly on columns in the absence of beams and girders. The method of analysis discussed here has been extended to the analysis of structural systems with other configurations [19]. Building material is assumed to be isotropic and linearly elastic. The floor slabs are modelled by thin plate Kirchhoff bending theory. Small deflections are assumed. Slabs are assumed to be rigid in-plane.

1.6 Objective

The use of the boundary element method (BEM) for analysing flat plate slabs supported on columns for displacement and stresses under gravity load has been developed and discussed in detail by Abdel-Akher [3] and Chen [8] in terms of accuracy and efficiency. Using the 3-dimensional building frame model developed by Abdel-Akher, Chen studied the performance of building frames under gravity and lateral loads.

Since the development of Chen's model, several problems have been noted. In Chen's model, a full 12 DOF frame element model for columns is incorporated directly with the slab super-element developed by Abdel-Akher. The numerical results obtained by Chen's model have been compared to the results from the experimental study of a plexiglass model that is significantly over stiff, and to a number of simple 2-dimensional systems. It is now known that her model does not correctly determine the stiffness terms related to the floor twist DOF. In addition to this, there are other problems relating to the algebraic signs of certain other stiffness coefficients as determined to be inconsistent with

the results of the present study using simple three-dimensional problems.

The author's contribution in part is a fundamental examination and formulation of the building frame model incorporating frame elements with the slab super-elements. To validate the analysis procedure, including the formulation of the DBEM for thin flat plate elements supported on columns, a flexible test-model is designed and fabricated to investigate the various dynamic behaviour patterns. The model is specially designed to bring out points raised earlier about torsional-lateral coupling. In a sense the model is an end in itself as a continuation of the work initiated in this thesis.

The frequency analysis performed in the present study includes the calculation of the building frame model natural frequencies and vibration displacement mode shapes only. The solution of these quantities represents the first step in a full dynamic analysis of a building frame under forced vibration loading where a method of analysis such as the modal superposition method is employed, and although a full dynamic analysis is not performed here, it is meant to demonstrate the possibility of using a reduced building frame model in a full dynamic analysis.

Figure 1.1 Shear Building

Figure 1.2 Building Frame Model of El-Kafrawy

Figure 1.3 Building Frame Model With 3 DOF Per Storey

2. Building Frame Model

2.1 Introduction

The simplified model of the building frame is obtained in three steps:

1. The slab is modelled using the direct boundary element method and the stiffness matrix of the slab is obtained using a method described in detail in [16]. The efficiency and accuracy of this numerical method of analysis is also described in [3]. The slab stiffness matrix uses 3 DOF per column-slab joint: 2 rotational about two orthogonal horizontal axes, and one axial along the vertical axis. The slab analysis is performed once for each slab of identical geometry used in the building frame.
2. The local stiffness matrix of each column and slab are assembled in the global stiffness matrix, referred to here as the 'intermediate model', consisting of slab super-elements, and column elements.
3. The compact stiffness matrix is generated from the intermediate model. Three DOF per storey are used in the compact stiffness matrix: two translational DOF and one twist DOF. A unit load is applied to the intermediate model at the location of each finite mass or inertial DOF of the compact stiffness model. The flexibility matrix is determined from the resulting displacements, from which the compact stiffness matrix is obtained by inversion.

In this chapter, a very brief description of the boundary element method is given. The

assembly of the stiffness matrix of the intermediate building frame model, described as step two above, is explained in more detail. Appendix B outlines the computer programs used in the modelling procedure as well as their proper usage.

2.2 Overview of the Direct Boundary Element Method

2.2.1 Discretization

In applying the direct boundary element method (DBEM) to analyse thin plates, the boundary is discretized into elements each having a specified number of nodes. At each node, there are two given boundary conditions and two unspecified quantities which are the basic unknowns of the problem. Values between nodes are approximated by applying interpolation functions to the nodal values in a manner similar to the finite element method.

At each boundary node, there are four variables: shear (V), moment (M), out-of-plane displacement (W) and rotation perpendicular to the edge (Θ). For each node, two of these variables are known, depending on the boundary conditions, as follows:

free edge: $V = 0$
 $M = 0$

fixed edge: $W = 0$
 $\Theta = 0$

pinned edge: $W = 0$
 $M = 0$

Therefore, for each boundary node there are two unknowns, and so two equations must be

produced per node to obtain a solution for unknown boundary variables.

2.2.2 Direct Boundary Element Method

The DBEM for plate bending uses the Betti-Maxwell reciprocal law to generate equations for the solution of the basic unknowns on the boundary. The Betti-Maxwell reciprocal law states that for two states of equilibrium a and b of a given elastic system, shown in Figure 2.1, the work done by the forces acting on state a moving through the deflections of state b equals the work done by the forces acting on state b moving through the deflections of state a .

Applying the Betti-Maxwell reciprocal theorem to plate bending, the given plate with loads and supports, as shown in Figure 2.2, is one state of equilibrium. The second can be any for which the solution is known. In the DBEM, this involves an infinite plate with either an applied concentrated point force or moment. The solution to this problem is known [29]. A portion of the infinite plate whose boundary shape is equivalent to that of the given plate is isolated, as shown in Figure 2.2, and is used as the second state of equilibrium in the Betti-Maxwell theorem, with the external forces and displacements along the boundary equal to internal forces and displacements of the infinite plate, which are known.

An equation is obtained by applying the Betti-Maxwell theorem and equating work terms.

Thus, any number of equations can be generated by applying a unit concentrated point force or moment force at different locations on the infinite plate. For $2N$ equations, N force locations, called source points, are needed, since two equations can be generated per source point. The equation resulting from a unit point force takes the form

$$1 \times w_p - \oint_s (WV' - \Theta M') ds - \sum_{i=1}^{N_c} W_i' C_i = \int \int_A p w' dA + \oint_s (W' V - \Theta' M) ds - \sum_{i=1}^{N_c} W_i' C_i - \sum_{i=1}^{N_r} w_i' R_i \quad (3)$$

where the first term is deleted if the source point is outside the boundary of the given plate; w_p is the plate vertical displacement at the point force; s is the plate boundary; A is the plate area; N_c is the number of corners; N_r is the number of internal reactions; C is a corner force reaction; and r is a reaction force. A unit concentrated moment produces the following equation:

$$1 \times \Theta_p' - \oint_s (WV^{m\bar{\xi}}) ds + \sum_{i=1}^{N_c} W_i C_i^{m\bar{\xi}} = \int \int_A p w^{m\bar{\xi}} dA + \oint_s W^{m\bar{\xi}} dA + \oint_s (W^{m\bar{\xi}} V + \Theta^{m\bar{\xi}} M) ds - \sum_{i=1}^{N_c} W_i^{m\bar{\xi}} C_i - \sum_{i=1}^{N_r} w_i^{m\bar{\xi}} R_i \quad (4)$$

Displacements at any location on the plate due to loads are found by placing a source point at that location, which will produce an equation having as a basic unknown the displacements at that point to be solved in addition to the various unspecified boundary

quantities. Internal forces and stresses are found by applying the equations of plate bending theory. The complete details describing this method can be found in [16].

2.2.3 Modelling Column Supports

Column-slab joints are modelled by placing a finite set of collocation points within the column connection area. The displacements of these points are compatible with the plane section assumption of the frame member and the point forces are statically equivalent to frame member forces at these locations. This method discretizes the pressure which arises between the column and slab due to loads. The pressure is thus represented as a small set of point forces. Since the column patch is assumed to displace rigidly, both vertically and rotationally, the displacements of each discrete point force can be calculated if the displacement of the patch is known. The sum of point sources must equal the total force due to pressure, and the moments at the column end are found by taking the sum of moments produced by each point force about the column centre lines.

2.2.4 Calculating the Stiffness Matrix

The stiffness matrix of the plate is generated using the following procedure [16]. The boundary of the slab is discretized into elements, and the column supports are modelled using the approach described in section 2.2.3 above. A unit displacement is given to a column DOF and a DBEM analysis of the slab is performed with the displacements of the point forces of that column patch known. The resulting system of equations is solved

allowing the column reactions to be calculated. Thus, one column of the stiffness matrix is found. This procedure is repeated for each column DOF.

2.3 Intermediate Building Frame Model

The model is an assemblage of two element types: column elements and slab super-elements. The intermediate building frame model has 3 DOF per column-slab joint and 3 DOF per storey, as shown in Figure 2.3. Joints have two rotational DOF about the horizontal axes, and one axial DOF along the vertical axis. Storeys have two in-plane translational DOF, and one in-plane twist DOF about the vertical axes, located at the slab centre of mass.

2.3.1 Slab Elements

The slab is modelled using the direct boundary element method, with column supports represented by a finite set of collocation points as described in section 2.2.3. The stiffness matrix of the slab is derived using the method described in section 2.2.4. Three DOF are given to each slab-column joint--two rotational DOF about horizontal axes, and one axial DOF along the vertical axis. Thus, a slab stiffness matrix is obtained having $3 \times (\text{no. columns})$ DOF resulting from the 3 DOF at each column end. The slab super-element of the intermediate model, as previously mentioned, is to have two translational and one twist DOF, and are located at the slab centre of mass. The slab is assumed to behave rigidly in-plane and therefore all columns framing into a particular slab experience the

same lateral displacements, so that the lateral displacements of all column ends framing into the slab may be described by a single degree of freedom. The stiffnesses associated with the slab super-element DOF are derived from the column stiffnesses. The method is explained in Section 2.3.3 below.

2.3.2 Column Elements

Column elements have local stiffness matrices as found in any structural analysis text book with 6 DOF per node, as shown in Figure 2.4 to be two rotational, two translational, one axial and one twist, and are calculated from column geometry and material properties [10]. The rotational and axial DOF of column ends correspond with the DOF of the slab stiffness matrix and are used for connectivity between the slab and columns when the building global stiffness matrix is assembled. Thus, column-slab joints have two rotational and one axial DOF.

2.3.3 Global Stiffness Matrix

Slab and column local stiffness matrices are assembled into the global stiffness matrix. The model global stiffness matrix has joint rotation and column axial DOF stiffness coefficients resulting from the direct insertion of those DOF stiffness coefficients of the column and slab elements.

Diagonal terms in the global stiffness matrix for the translational DOF of slab super-

elements are equal to the sum of the corresponding translational stiffness of all columns framing into the slab.

Deriving the stiffness coefficients relating to storey twist is more complicated. The spatial location of column end points must be introduced into the model in order to apply their twist stiffness to the entire storey and to take into account the off-diagonal stiffness connecting the storey twist with storey sway and joint rotations. The derivation of these stiffness coefficients is explained using the example structure shown in Figure 2.5 (a).

Figure 2.5 (b) is a plan view of the example structure showing the slab and column displacements after a unit twist is applied to the slab centre of mass. Figure 2.6 is a more detailed sketch of the displacement of one column and shows the geometry of the problem. Figure 2.7 is the deformed shape of a general column element given a unit translation at the column end with all other end DOF locked, and shows the resulting DOF forces required to maintain the deformed shape.

A slab unit twist causes the columns framing into it to displace laterally. The shear forces developed in the columns cause a moment about the twist origin. The sum of these moments equals the diagonal coefficient in the stiffness matrix for storey twist. The moments are calculated as follows:

As shown in Figure 2.6, the column is a distance R away from the twist origin, which has a horizontal component of length X and a vertical component of length Y . The column is

located at an angle Θ from the horizontal. Also shown is the column displacement vector S with horizontal and vertical components Δx and Δy respectively. The slab unit rotation is represented by the angle ϕ . The magnitudes of the shear force components in the x- and y-direction developed in the column due to the unit twist are proportional to the column shear forces caused by a column end unit translation shown in Figure 2.7, and are given by

$$\begin{aligned} F_x &= \frac{12EI_y}{l^3} \times \Delta x \\ F_y &= \frac{12EI_x}{l^3} \times \Delta y \end{aligned} \quad (5)$$

Corresponding moments about the twist origin are

$$M_t = F_x \times Y + F_y \times X \quad (6)$$

From the geometry of the system, shown in Figure 2.6, and a unit twist of $\phi=1$, we have

$$\begin{aligned} S &= R\phi = R \\ Y &= R\sin\theta & X &= R\cos\theta \\ \Delta x &= S\sin\theta & \Delta y &= S\cos\theta \end{aligned} \quad (7)$$

Substituting the values from equation (7) and equation (5) into equation (6) results in

$$M_t = \frac{12EI_y}{l^3} R^2 \sin^2\theta + \frac{12EI_x}{l^3} R^2 \cos^2\theta \quad (8)$$

The sum of moments about the twist origin for each column equals the diagonal stiffness matrix coefficient for storey twist. The slab unit twist also causes each column to twist axially. The column axial twist stiffness of each column is also added to the storey twist stiffness coefficient.

Off-diagonal coefficients in the global stiffness matrix relating storey twist and storey sway are defined as the shear force needed to maintain zero lateral displacement of the storey upon the application of the slab unit twist displacement. The magnitudes are equal to the sum of the shear forces developed in the columns due to the slab unit twist, given by equation (5). It is important to note that the direction of the column shear force is dependent on the column location. The twist-shear off diagonal coefficient in the stiffness matrix for x-translational DOF therefore equals the sum of F_x shear forces from every column, and for the y-translational DOF, the sum of the F_y shear forces.

The off-diagonal coefficients in the global stiffness matrix relating slab twist and column end rotations equals the moment needed to maintain zero column-slab joint rotations when the slab unit twist is applied. Figure 2.7 shows the deformed shape of a column element undergoing a unit translational displacement at the column end with all other DOF locked, and shows the column shear and moment forces required to maintain the deformed shape. The moment forces at the joint needed to maintain zero rotations due to the slab unit twist are proportional to those moment forces shown in Figure 2.7, and are scaled by the horizontal displacements of the column caused by the slab unit rotation.

The column end moment M_y about the y-axis is caused by the displacement Δx , which, from Equation (7) is equal to the distance Y , is given by the equation

$$M_y = \frac{6EI_y}{l^2} \times Y \quad (9)$$

Similarly, the column end moment about the x-axis, M_x , is caused by the displacement Δy of the column, and is given by

$$M_x = \frac{6EI_x}{l^2} \times X \quad (10)$$

It is important to note that the direction of the column end moments depends upon the column location with respect to the origin of twist.

2.4 Compact Stiffness Model

The intermediate building frame model described in the preceding section has $[3 \times (\text{no. columns}) + 3] \times [\text{no. storeys}]$ number of independent DOF. A two-storey building supported on six columns, for example, will have 42 DOF. The simplified model, derived from the application of unit loads and inverting the resulting flexibility matrix would have 6 DOF. Although this model uses a compact stiffness matrix with two lateral and one twist DOF per storey, the overall effect of column end rotations on building lateral displacements are included in the stiffness model.

A lateral load analysis was performed on the structure shown in figure 2.8 in order to compare the results of the two models to demonstrate that accuracy is preserved when using the compact stiffness matrix. The intermediate model was given a lateral load of 100 N on the second storey, resulting in first and second storey displacement in the direction of load of 0.2257 mm and 0.5537 mm respectively. The second model, which uses the compact stiffness matrix, was given the same loading and resulted in displacements of 0.2256 mm and 0.5535 mm for the first and second storey respectively. All results are within .04% of the intermediate model displacements which is an insignificant difference. The computer program uses variables of type double precision in the calculation of the building frame displacements.

Figure 2.1 Betti-Maxwell Reciprocal Theorem

Figure 2.2 States of Equilibrium Used in the BEM

Figure 2.3 Intermediate Building Frame Model

Figure 2.4 Column Element DOF

Figure 2.5 (a) Example Structure

(b) Column Displacements After Unit Twist

Figure 2.6 Geometry of Column Displacement

Figure 2.7 Column Element Displaced Shape Due to Unit Translation with DOF Forces

Figure 2.8 Example Structure for Static Lateral Load

3. Analysis and Experimental Investigation

3.1 Introduction

In this chapter, the results of a static and dynamic computer analysis of the structure shown in figure 3.1 are presented and compared to data collected through experimental investigation. The structure is a two-storey, asymmetrical (L-shaped) building frame of steel construction with clamped boundary conditions at the column base. The purpose of this investigation is to verify the boundary element model for flat plate slabs for the dynamic analysis of building frames, and to verify the validity of the compact stiffness matrix model.

3.2 Computer Model Investigation

3.2.1 Introduction

Two methods of computer analysis are employed: a boundary element method analysis and a finite element method analysis. In both cases, the structure shown in Figure 3.1 is modelled and loaded to obtain the compact stiffness matrix having x- and y-translational DOF and z-twist DOF at each storey centre of mass .

A dynamic analysis of the building frame is performed using both of the BEM and FEM compact stiffness models to obtain the natural frequencies and vibration displacement

mode shapes of the structure. Since the floor DOF are located at the slab centre of mass, a diagonal lumped-mass mass matrix is used. The dynamic analysis uses the inverse iteration method in a computer program developed by the author called Usesweep.c.

The lumped mass matrix used with the compact stiffness matrix for the dynamic analysis includes the column masses. A frequency analysis of the structure shown in Figure 3.1 using the compact stiffness model and the lumped mass matrix that includes column masses produces natural frequencies of vibration which are 8% lower than those produced by a similar analysis where column masses were not included in the lumped mass matrix. The frequencies of the model including the column mass lie closer to the results of the experimental investigation. Therefore, column masses are included in the model used for analysis. Also, the combined mass of the two slabs of the structure shown in Figure 3.1 is 60.2 kg, and the mass of all columns in the structure is 19.2 kg, which contributes significantly to the mass of the structure. It is therefore reasonable to include the column masses in the lumped mass matrix of the dynamic model.

The BEM model originally used 8 collocation points to model finite column-slab connections, and the FEM model used 2-dimensional columns elements where column-slab attachments were modelled by a single point. Better results are obtained with the FEM model when stiff plate elements are incorporated into the slab at the column-slab connections, as shown in Figure 3.2. The stiff plate elements, which cover an area equal to the cross-sectional area of the columns, models the effect of the column geometry on

the slab behaviour. For comparison, a BEM model is generated where column supports were modelled by single points. The results of all four models are given in the present study.

3.2.2 Dynamic Analysis

This section outlines the theory used by the program Usesweep.c to extract the dynamic characteristics of the numeric models. As mentioned in Chapter 1, the solution for the roots of the equation

$$\det([K] - \omega^2[M]) = 0$$

can be solved to obtain the natural frequencies and vibration displacement mode shapes of a structure. Direct solution of the eigenvalue problem is very time consuming and is possible only for small systems. In the present study, the eigenvalue problem is solved numerically by the inverse matrix iteration method [9]. Equating the system stiffness forces with the inertial forces gives the equation

$$[K]\{v_n\} = \omega_n^2[M]\{v_n\} \quad (12)$$

where ω_n is the nth natural frequency of vibration and $\{v_n\}$ is the nth mode of vibration displacement mode shape. The solution of equation (12) gives the mode shape which has the highest natural frequency of vibration of the system. Generally, the dynamic behaviour of a system is dominated by the contributions of a small number of mode

shapes which have the lowest frequencies of vibration of the system. It is therefore necessary to find only a number of the lowest modes of vibration, and Equation (12) is altered to do so in the following manner. Since the stiffness matrix is symmetric, it can be transformed to take the form

$$[K] = [L][U] \quad (13)$$

where [L] is a lower diagonal matrix and [U] is an upper diagonal matrix. This form of [K] is substituted into equation (12). An initial displacement vector is used in the left hand side of equation (12), and the equation is solved for a new {v} by employing two successive Gaussian forward and backward substitution procedures. This new {v} is used in the left hand side of equation (12) in the next cycle and the process is repeated until successive iterations result in a negligible difference in the displacement shape. The method converges toward the vibration mode shape of the system whose natural frequency is of the lowest value. Once a vibration mode is known, the condition of orthogonality between vibration modes can be used to modify the system resulting in the removal of the mode from the system, so that the iteration process will converge to the mode shape of the next highest frequency.

3.2.3 Boundary Element Model

The boundary element building frame model has floor slabs of 4.5 mm in thickness, and columns that were modelled after HSS 34x34x3.0. The columns have clamped end

conditions at their bases. All structural elements are assigned a Young's modulus of 200 000 MPa and a Poisson's ratio of 0.3, in accordance with the properties of structural steel.

Abdel-Akher's computer program, which applies the direct boundary element method to the analysis of elastic thin plates loaded out-of-plane, is used to model the building floor slabs supported on columns, as shown in Figure 3.3, to obtain the slab stiffness matrix.

As mentioned previously, two BEM models are generated: one with finite joints where column supports are modelled using 8 collocation points, and one where supports are modelled by only one point. This is done to determine the effect of column geometry on slab behaviour. For each BEM model, the global stiffness matrix of the building frame model is assembled using the slab and column local stiffness matrices, and is then used to obtain the compact flexibility matrix of the entire building, having 3 DOF per storey.

The compact flexibility matrices are shown in Figure 3.4 (a) and 3.4 (b).

A lumped-mass mass matrix is used for the dynamic analysis and is shown in Figure 3.5. Lumped floor masses and floor mass moments of inertia include slab masses plus one-half the length of all columns framing into the slab. Slab masses of the steel model were measured indirectly by subtracting the total mass of HSS used to build the model from the total mass of the model. A length of the HSS was weighed to find the mass per unit length.

3.2.4 BEM Dynamic Analysis Results

Usesweep.c is used to calculate the first six natural frequencies and mode shapes of the dynamic model comprised of the BEM compact flexibility matrix and the lumped mass matrix. The results for both BEM models are shown in Table 3.1 (a) and (b). The following three types of vibrational modes are found:

1. Translation and twist: Both translational and twist DOF undergo considerable displacements.
2. Pure translation: Only translational DOF undergo displacements, and twist DOF have zero or near-zero displacements.
3. Pure twist: Only twist DOF undergo displacements, and translational DOF have near-zero displacements.

3.2.4.1 Finite Column-Slab Joint Model

The BEM model which uses 8 collocation points to model column supports is found to have vibration mode shapes of Type 1 at 13.6 Hz and 74.5 Hz. Displacement magnitudes in the x- and y-direction within each storey are equal. Pure translation modes occur at 14.0 Hz and 77.5 Hz. Displacement magnitudes are again almost equal in the x- and y-directions for each storey. Pure twist occurs at 21.6 Hz and 103.3 Hz, where the twist DOF dominates motion, and translation in the x- and y-directions are minimal.

3.2.4.2 Single Point Column Support Model

The BEM model which uses 1 point to model column supports is found to have vibration mode shapes of Type 1 at 11.7 Hz and 73.0 Hz. Displacement magnitudes in the x- and y-directions are again equal. Pure translation mode occurs at 11.9 Hz and 75.8 Hz.

Displacement magnitudes are again almost equal in the x- and y-directions. Pure twist modes occur at 19.9 Hz and 101.4 Hz.

3.2.4.3 Discussion

The BEM model with finite column-slab joints has natural vibration frequencies which are approximately 2 Hz higher than those of the BEM model with single point column supports. This demonstrates that including the effects of column geometry stiffens the model. Since identical column stiffness and end constraints are used in both BEM models, it is seen that the discrepancy in the natural frequencies is due to the effect of column geometry on slab behaviour. Bending moments that are transferred from the wider column to the slab are more dispersed within the slab, allowing the slab to resist bending moments more easily and causing joint rotations to be of smaller magnitudes, resulting in a stiffer model.

Vibration displacement mode shapes of both models are close. The only significant discrepancy lies in the first mode shape, where the twist DOF displacement coefficients of the finite joint model are larger in magnitude than the twist DOF displacement

coefficients of the single point column support model. The mode shapes are normalized with respect to the DOF displacement of highest magnitude, which in this case is the storey lateral displacement, and so the discrepancy actually exists in the ratio of the storey twist to the storey translation of the two models. The flexibility matrices show that the finite joint model has lateral stiffness coefficients that are 18% more stiff than the single point column support model, and twist stiffness coefficients to be only 7% more stiff. The finite joint model therefore has a twist to lateral displacement ratio which is higher than that of the single point column support model, causing the discrepancy found in the first vibration displacement mode shape. The vibration displacement mode shapes of the other natural frequencies are not affected by the normalization procedure because they involve either only a translation motion, or only a twisting motion.

3.2.5 Finite Element Model

The same structure is modelled using the finite element analysis software package called Cosmos which was run on a Sun Sparc station. The building slabs are modelled using 3-node triangular thin plate elements, having 6 DOF per node, with isotropic material properties and a constant 4.5 mm thickness. The element node DOF include three translational DOF and three rotational DOF. The columns are modelled using 2-node elastic beam elements with the same 6 DOF per node as the plate elements, and having a square hollow cross-section, thus enabling the simulation of HSS 34x34x3.0 columns. Slab and column material properties are made equal to those used in the boundary

element model, and densities of the slab elements and column elements are calculated based on mass measurements made on the physical structure. The FEM model employs a total of 1452 elements, with each slab using 708 elements, and is shown in Figure 3.6. As mentioned previously, two FEM models were generated: one where stiff plate elements are incorporated into the slab at the column support to model a finite column-slab joint and to account for the effects of column geometry on the slab behaviour, and one where column supports were modelled by a single point. It is important to note that in the FEM model where stiff column patches are not included and the effect of column geometry on the slab behaviour is ignored, the model is in direct contrast to both the physical laboratory model and the DBEM model which uses 8 collocation points for columns supports.

Unit loads are applied to both FEM models and consequently their compact flexibility matrices are obtained and are shown in Figure 3.7 (a) and (b). Unit twist forces about the axis perpendicular to the slab are generated by applying a unit force couple centred about the slab centre of mass. This is done because the application of a unit twist force directly to the element node of the slab resulted in slab in-plane deformations which seemed unrealistically large. The application of the force couple produces more realistic slab deformations. The FEM software uses the Gaussian foreword elimination and backward substitution method when solving Equation (1) for load cases involving static loading.

3.2.6 FEM Dynamic Results

Two dynamic analyses were performed on both FEM models to obtain their first six natural frequencies and mode shapes:

1. The dynamic analysis of the full FEM model by standard finite element analysis
2. The dynamic analysis by using the compact stiffness matrix obtained from applying unit loads to the FEM model, and the same mass matrix used for the boundary element model dynamic analysis.

The results of the analyses are shown in Table 3.2 (a) and (b) and in Table 3.3 (a) and (b).

The same three types of vibration modes are found as in the BEM analysis: pure twist, pure translation, and twist and translation.

3.2.6.1 Dynamic Results by Full Finite Element Model

3.2.6.1.1 Finite Column-Slab Joint Model

The results of a frequency analysis of the full FEM model which modelled column connections by stiff plate elements found Type 1 vibrational mode shapes occurring at 12.6 Hz and 75.2 Hz. Type 2 vibrational mode shapes occur at 13.3 Hz and 76.7 Hz and Type 3 vibrational mode shapes occur at 17.6 Hz and 100.1 Hz.

3.2.6.1.2 Single-point Column Support Model

The dynamic analysis of the full FEM model which modelled column connections by a single point found Type 1 vibrational mode shapes, involving x- and y-translations and z-twist occurring at 12.1 Hz and 74.3 Hz. Type 2 vibrational mode shapes, which involves only lateral x- and y-displacements, occur at 12.7 Hz and 76.1 Hz. Type 3 vibrational mode shapes, involving z-twist displacements, occur at 16.8 Hz and 98.5 Hz.

3.2.6.2 Dynamic Results by Compact Stiffness Matrix

3.2.6.2.1 Finite Column-Slab Joint Model

The results of a frequency analysis of the model which takes into account the effect of finite column-slab joints and uses the compact stiffness matrix of the FEM model are presented in this section. Type 1 modes of vibration occur at the frequencies 13.2 Hz and 76.6 Hz. Type 2 modes of vibration occur at frequencies 14.0 Hz and 79.3 Hz, while type 3 modes of vibration occur at frequencies 18.3 Hz and 99.3 Hz.

3.2.6.2.2 Single-point Column Supports

The frequency analysis of the compact stiffness matrix of the FEM model where column supports are modelled by single points provides six vibrational mode shapes. Type 1 modes of vibration occur at the frequencies 12.7 Hz and 76.8 Hz. Type 2 modes of vibration occur at frequencies 13.3 Hz and 78.6 Hz, while type 3 modes of vibration

occur at frequencies 17.6 Hz and 100.6 Hz

3.2.6.3 Discussion

As with the BEM models, the FEM models that have column-slab joints of finite size are stiffer than the models with column supports modelled by a single point, and this is reflected in the natural frequencies of vibrations, where the FEM models with finite joints have natural frequencies of magnitude approximately 1 Hz higher than the single point column support models. The vibration displacement mode shapes, however, compare quite closely. It was shown in section 3.2.4.3 that some discrepancies exist in the vibration displacement mode shapes of the two BEM models because the inclusion of the finite joint in the model affects the lateral stiffness of the model to a higher degree than the twist stiffness. This discrepancy does not occur in mode shapes of the two FEM models, since they are equally affected by the inclusion of the finite joint. The FEM model with finite joints has lateral flexibility coefficients 7% lower than the single point column support model, and has twist flexibility coefficients 5% lower.

3.3 Experimental Investigation

3.3.1 Introduction

The laboratory investigation of the structure shown in Figure 3.1 was performed in order to verify the results of the computer model analyses. Testing was performed to measure

the building natural frequencies and vibration displacement mode shapes. Photographs of the experimental set-up can be found in Appendix A.

3.3.2 Construction of the Test Model

The experimental test model, shown in Figure 3.8, was fabricated in steel using welded construction. Slabs were 4.5 mm thick steel plates, and columns were HSS 34x34x3.0. Steel was the material chosen for use because of its isotropic elastic properties, and so the building would remain intact after testing for use in future research projects. The model was welded to a rigid base frame of HSS welded construction, as shown in Figure 3.9 (a) which was then bolted to the table using the clips shown in Figure 3.9 (b). This was done in order to ensure clamped end conditions of the columns, and also to allow freedom in choosing model geometry, so that column positions would not be strictly dictated by the position of bolt holes in the table.

It was noted in Section 1.1 that buildings that have lateral natural frequencies equal to their torsional lateral frequency can have an increased effective building eccentricity during earthquakes. The present study is partly concerned with validating the BEM method for thin plates used in building frames in a dynamic analysis, and therefore the model was designed to have differing lateral and torsional natural frequencies to ensure that the numerical and experimental results could be easily compared.

3.3.3 Instrumentation

Accelerations were measured using five single-DOF piezo-electric accelerometers which were channelled through a signal generator. Data was collected using a micro-computer at a frequency of 5000 data per second. Four accelerometers were located on the model slab as shown in Figure 3.10, with one pair at the slab centre of mass, and the other two near the slab extremities. Data was collected sequentially at each accelerometer.

However, any error which may arise due to the time lapse between the measured data points at each accelerometer is minimal because of the high frequency of data collection.

At the centre of mass, accelerometers were oriented in orthogonal directions, as were the two accelerometers at the slab extremities. One accelerometer was located on the shaking table to monitor the table motion during testing. With this set-up, the lateral motion of the model was measured directly, while storey twist could be calculated.

3.3.4 Method of Loading

The model was mounted on a single-DOF shaking table, which was fastened to the floor by spanning the surface of the table with two I-beams which were then secured to the floor using bar clamps, as shown in Figure 3.11. The actuator which would induce motion was not functional at the time that testing was performed, limiting this project's scope of study. In order to measure the vibrations of the model, excitation was provided by impulse loads using a pendulum. For every load applied, the drawback distance of the pendulum was measured in order to apply approximately the same load for each test.

Impulse loads were applied at three positions along the slab edge at both storey levels: at a point aligned with the centre of mass of the slab, and at points aligned with the columns at the slab corners, as shown in Figure 3.10. There were therefore a total of 6 impact load positions, and for each position 3 tests were performed. There were enough accelerometers to measure vibrations at only one storey at a time. Vibrations were therefore measured at one storey for impacts at each impulse load position, and were then measured at the other storey for all six impulse loads. In this way, information was gathered at both storeys under approximately the same load, and enough information was gathered to calculate the vibration displacement mode shapes for the structure.

3.3.5 Data Analysis

3.3.5.1 Fourier Transforms and Cross-spectral Density Functions

Using Fourier series, any periodic or non-periodic function can be approximately represented by the superposition of a series of sine and cosine waves. The function is transformed into the frequency domain from the time domain. The Fourier transform of the discrete function $p(t_m)$ and the inverse Fourier transform, which converts the function back to its original form in the time domain, are given by

$$p(t_m) = \sum_{n=-M}^M P_n e^{i t \frac{2\pi n m}{N}} \quad (14)$$

and

$$P_n = \frac{1}{N} \sum_{m=1}^{N-1} p(t_m) e^{-i t \frac{2\pi n m}{N}} \quad (15)$$

and are known as the Fourier transform pair, given here in their complex form, where P_n is the n th coefficient of the Fourier transform and N is the number of discrete points of the function. There exist simple and effective methods to calculate the discrete Fourier transform and its inverse numerically [5.6].

In dynamic analysis, the Fourier transform of the displacement-time history of a structure undergoing free vibration is used to obtain the natural frequencies of vibration of the structure.

The relation between two sets of statistical data can be analysed using the cross-correlation function, known as the cross-spectral density function when applied in the frequency domain. It is calculated using the equation

$$G_{xy}(f_k) = \frac{2}{N\Delta t} [X^*(f_k) Y(f_k)]$$

where $G_{xy}(f)$ is the cross-spectral density function. N is the number of discrete data points. Δt is the time between data points. f_k is the k th frequency. Y is the Fourier transform of the data set $y(t_n)$ and X^* is the complex conjugate of the Fourier transform of the data set $x(t_n)$. The value of G_{xy} is complex, which contains the information on its amplitude and phase angle.

The physical meaning of this function is illustrated by the example of a plate undergoing cyclical in-plane motion, where data which describe the motion of the plate are collected at two separate points on the surface of the plate. When the plate is translating laterally at a frequency of F Hz, as shown in Figure 3.12, the two points are moving synchronistically and their motions are similar to the two sinusoidal waves illustrated in Figure 3.13. The cross-spectral density function obtained from the two sets of data would show a spike at F Hz, and a phase angle of 0 degrees. When the plate is rotating about a fixed axis perpendicular to the surface of the plate, as shown in Figure 3.14, the two points move in opposite directions, and the cross-spectrum obtained would show a spike at F Hz and a phase angle of 180 degrees. Their motion may be described by the sinusoidal waves found in Figure 3.15.

3.5.5.2 Extracting the Dynamic Characteristics of the Experimental Test Model

The acceleration response of the test model due to impulse loading was measured and

used to extract the model natural frequencies and corresponding vibration displacement mode shapes. Each displacement mode shape is described by the storey displacements which consist of displacements in the x- and y-directions and the rotation about the z-axis at the centre of mass of the slab. The vibration response is assumed to be of a sinusoidal type. The dynamic response data is transformed into the frequency domain by Fourier transform. The relative direction of motion of each of the DOF are determined by cross-spectral analysis. Fourier analysis gives the natural frequencies of vibration, f_n , of the model, and the corresponding peak acceleration magnitudes, A_n . The Cross-spectra of the data from one accelerometer, selected as the reference basis, with the data from each of the other accelerometers are calculated to obtain their acceleration phase angles, ϕ_n . The phase angle indicates the relative direction of motion of each accelerometer. For the nth vibration mode, the acceleration of the ith accelerometer as a function of time can then be described by the equation

$$a_n'(t) = A_n' \cos(2\pi f_n t + \phi_n') \quad (17)$$

By integrating each equation twice, displacements are found for each accelerometer to be

$$d_n'(t) = -\frac{A_n'}{(2\pi f_n)^2} \cos(2\pi f_n t + \phi_n') \quad (18)$$

Using Equation (18), displacement vectors for each accelerometer over a time interval Δt can be calculated and used to determine the coefficients of the modal vibration displacement shapes. Because of the accelerometer layout, however, only the

displacement vector for the slab centre of mass is fully described, where both x- and y-displacement vector components are known. At the other two accelerometer locations, the x-displacement is known at one location, and the y-displacement is known at the other. However, since the original positions of the accelerometers are known (and form a triangular shape), the two unknown displacement vector components, shown in Figure 3.16, can be calculated assuming the triangular shape made by the accelerometer locations remains unchanged during excitation.

Vibration displacement mode shapes are found for each location where the impulse load was applied. The final results are an average of all test findings.

3.3.5.3 Example Calculation

The experimental data was analysed to find the natural frequencies and vibration displacement mode shapes of the laboratory model. The method used to analyse the experimental data is illustrated here by an example calculation of the first natural frequency and the corresponding vibration displacement mode shape. The data used is that of test HAM1, where accelerometers were located on the second storey of the test model in the manner shown in Figure 3.10, for an impact load applied to the edge of the second storey slab at a point aligned with the slab centre of mass.

A plot of the collected data of the model vibrations at accelerometer 2 due to the impact

load is shown in Figure 3.17, and a segment of the vibration plot is shown in Figure 3.18. Figure 3.19 shows a plot of the vibrations of the support table under the same load. It can be seen that the table vibrations attenuate to near zero values quickly after the load is applied. Collected data was truncated upon analysis so that only the data where there are no vibrations at the support were used. Also, the same window of data was used for each data set, beginning at the same time after impact, and having the same time span.

The results of the Fourier transforms of all four accelerometers are shown in Figure 3.20. Peaks occur at frequencies 12.2 Hz, 13.4 Hz, and 23.2 Hz. The peak magnitudes are equal to the maximum amplitudes of the oscillatory accelerations of each accelerometer and correspond to the modal frequencies of vibration of the model.

Cross-spectral density functions are calculated for accelerometer 2 with each of the other accelerometers subsequently. A portion of the spectra is shown in Figure 3.21 for the data of accelerometer 2 and all of the other accelerometers, showing the magnitude and phase angle plots. Peak magnitudes occur at the frequencies of 12.2 Hz, 13.4 Hz, and 23.2 Hz. From Figure 3.21, it is derived that accelerometer 2 is in phase with accelerometer 4 at the frequency of 12.2 Hz, and has a phase difference of 180 degrees with accelerometers 1 and 3. The relative directions of the accelerations of the accelerometers are now known.

Having obtained the acceleration frequencies, amplitudes, and the relative directions of

motion for each accelerometer, the displacement of each accelerometer can be described as a function of time using Equation (18). An arbitrary time interval can be used to calculate a corresponding displacement vector for each accelerometer.

At this stage, both the x- and y-displacement vector components are known at the slab centre of mass, while the slab rotation is still unknown. The x-displacement component only is known at one slab extremity, and the y-displacement component is known at the other extremity. The distances between accelerometers is also known, and remain constant, and are used to calculate the unknown vector displacement components of the points near the slab extremities shown in Figure 3.16. Accelerometer locations are plotted in the x-y plane before and after the displacement occurs. Changes in slope of the sides of the triangle formed by the accelerometers determine the slab rotation by the formula

$$\Delta\theta = \arctan(\Delta slope)$$

where $\Delta\theta$ is the slab rotation and $\Delta slope$ is the change of slope of the sides of the triangle.

The procedure described above is repeated for the data of test HAM19 where accelerometers are located on the first storey of the test model for an impact load applied at the same location as that of test HAM1.

Cross-spectra are found for the data of corresponding accelerometers of tests HAM1 and HAM19. That is, the data from accelerometer 2 of test HAM1 is crossed with the data from accelerometer 2 of HAM19 and so on. It is found that at each mode of vibration, the storeys are in phase.

At this point, the displacement values of the slab centres of mass are normalized by dividing each term by the displacement coefficient of the largest magnitude, giving the vibrational displacement mode shape for the frequency of 12.2 Hz shown in Table 3.4.

3.3.6 Experimental Test Results

Three natural frequencies and mode shapes were found for the laboratory model, and are shown in Table 3.4. The first vibrational mode shape occurs at 12.2 Hz, and involves translational and twist displacements. The second mode of vibration occurs at 13.4 Hz and involves translational displacements only. The third mode of vibration involves only twist displacements and occurs at 23.2 Hz. The amount of energy which was transmitted to the the model due to the impulse load was limited by the measuring capabilities of the accelerometers. Therefore, there may exist other modes of vibration which occur at higher frequencies that were not excited by the impulse load applied to the test model.

3.4 Discussion

3.4.1 Introduction

Several computer models of the frame structure are used in the dynamic analysis. Two versions of the BEM model and the FEM model are formulated: one version includes the effect of column geometry on slab behaviour at the column-slab attachments, and in the other version, column supports are represented by a single point. In this section, the results of these four computer models are discussed in relation to each other, and in relation to the results of the experimental investigation.

Flexibility matrices were produced from the static analysis of the BEM and FEM computer models by applying unit forces at the centres of mass of the slabs of the building frame. They are briefly discussed in this section.

The computer dynamic analyses for the BEM and FEM models and the laboratory tests produced vibration displacement mode shapes and frequencies. Variations in natural frequencies and of DOF displacement coefficients within each mode shape exist. Also, although the computer analyses are able to generate six natural frequencies and mode shapes, the laboratory experimental tests reveal only three modes of vibration. Therefore, the discussion pertains to these three modes of vibration. As mentioned earlier, three types of vibrational mode shapes are observed. Type 1 involves x- and y-translation displacements and twist displacements. Type 2 involves only x- and y-translations.

Type 3 involves only twisting displacements.

3.4.2 FEM Compact Stiffness Model and the Full FEM Model

The finite element model is used for two dynamic analyses: one using the full finite element model, and one using the compact stiffness model and the lumped mass matrix. The results of the two analyses are close. Frequencies from the compact stiffness matrix analysis are slightly higher than that of the full FEM model analysis, but all are within 5% of each other.

Upon comparison of the vibration displacement mode shapes found by the two computer analyses, it is seen that there are small variations in the DOF displacement coefficients, but overall the results are in agreement. The compact stiffness matrix along with the lumped mass matrix can therefore be assumed to be satisfactory as a dynamic model of the structure in comparison to the more detailed finite element model. Therefore, only the results obtained from the FEM and BEM compact stiffness and lumped-mass matrix models are discussed. They are compared with the experimental test results. In the following discussion, the FEM model signifies the FEM compact stiffness model with lumped-mass.

3.4.3 Flexibility Matrices

The compact flexibility matrices obtained from the BEM and FEM computer models are

shown in Figures 3.4 and 3.7 respectively. Upon comparing the BEM and FEM models where column geometry is included at the column-slab connection, the diagonal flexibility coefficients for the x- and y-translational DOF are within 5% of each other. The diagonal coefficients for the z-twist DOF in the BEM model are 25% lower than in the FEM model at the first storey and 32% lower at the second storey. The differences in the compact flexibility matrices obtained for the BEM and FEM models where column supports are modelled by a single point are more pronounced. Diagonal coefficients for the x- and y-translational DOF are 11% and 20% higher in the BEM model at stories one and two respectively, and diagonal coefficients for twist are 27% lower in the BEM model at both stories. The BEM model appears to be significantly less flexible in twist. When column geometry is taken into account in the model, the BEM and FEM models have equal lateral stiffness.

3.4.4 Natural Frequencies of Vibration

3.4.4.1 BEM and Experimental Results

The BEM model which uses finite column-slab joints has a first natural frequency 11% higher than the experimental result. The second frequency is 4% higher, and the third frequency is 7% lower. The BEM model using point supports for column-slab joints has a first natural frequency 4% lower than the experimental result, a second frequency 11% lower, and a third frequency 14% lower. The model using finite column-slab joints gives higher frequencies than the single point support model, indicating a stiffer model. The

inclusion of the finite joint has a significant influence on the results.

The single point support model provides a better result for the first frequency. The finite joint model provides better results for the second and the third frequency.

3.4.4.2 FEM and Experimental Results

The FEM model which incorporates the finite column-slab joint, taking into account the column geometry, has a first natural frequency 8% higher than the corresponding experimental frequency. The second frequency is 4% higher, and the third is 21% lower. The FEM model with single point column supports has a first frequency 4% higher than the experimental result, a second frequency which is 1% lower, and a third frequency which is 24% lower. The model with single point column supports has lower frequencies, indicating a less stiff model, with the first and second natural frequencies of vibration closer to the experimental results. Both models, however, provide results which are more or less in agreement with the experimental results for the first two natural frequencies. The finite column-slab joint model provides slightly better results for the third frequency, but here the results of both models are not in agreement with the experimental results.

3.4.5 Vibration Displacement Mode Shapes

Both computer analyses and the laboratory experimental test resulted in vibration displacement mode shapes which show that the first mode of vibration is of Type I where

both translation and twist displacements occur, the second mode of vibration is of Type 2 where mainly translation displacements occur, and the third is of Type 3 where twist displacements dominate.

The displacement shapes for the two BEM models, where one used finite column-slab joints, modelled by 8 collocation points, and the other used single point column supports, were generally in agreement with each other, as discussed previously in Section 3.2.4.3. The displacement shapes for the corresponding FEM models were also in agreement with each other, as discussed in Section 3.2.6.3. Therefore, only the models which included the effect of column geometry on slab behaviour are included in the discussion, since they more closely reflect the physical laboratory model.

A plot of the DOF displacement coefficients of the first, second and third vibration displacement mode shape is shown in Figure 3.22, 3.23 and 3.24 respectively for both of the computer and the experimental results. Discrepancies are found upon comparing the first two vibration displacement mode shapes. The FEM and BEM computer analyses give symmetric displacement shapes, where x- and y-translational displacements of a storey are equal within each mode shape. The experimental tests reveal mode shapes where the x- and y-translational displacements are not equal: x-displacements dominate in the first mode, and y-displacements dominate in the second mode. The frequencies of the first two vibration modes, which are found to be 12.2 Hz and 13.4 Hz, are however quite close to one another, so that they contribute almost equally to the dynamic

behaviour of the structure, in which case the unbalanced x- and y-displacements in each vibration mode may cancel each other out. The overall dynamic behaviour of the test structure may be symmetric.

For the first mode of vibration, the BEM twist displacement coefficients are adequate and the FEM model has twist DOF displacement coefficients which are much higher than the experimental results.. This indicates that the FEM model is too flexible torsionally.

For the second mode of vibration, the experimental results and both of the computer models have displacement shapes where twist displacements are small relative to the lateral displacements coefficients, so that in this respect they are in agreement with one another.

Although the largest discrepancy in natural frequency between the computer analyses and the test results occurred at the third vibration mode, the corresponding displacement mode shapes are quite close. It is also interesting to note that the BEM and FEM models and the laboratory tests all result in twist vibration mode shapes that are symmetric. That the displacement shapes for the twist mode are in agreement may not be surprising.

When the building frame twists, the magnitudes of the x- and y-displacement due to twist depend upon the location of the DOF with respect to the twist origin. Since both computer models and the laboratory test model have the same geometries, they would be expected to have a similar proportional relation between twist displacements and lateral

displacements.

3.4.6 Conclusion

Because the results of the dynamic analysis of the full FEM model and the results of the compact stiffness matrix model and the lumped mass matrix are comparable, it is seen that the compact stiffness matrix provides an adequate dynamic model.

The BEM and FEM model compact flexibility matrices show that these two computer models are equally flexible in translation, and that the BEM model is moderately stiffer than the FEM model in twist.

Including the effects of column geometry at the column-slab joint increases the natural frequencies of both BEM and FEM models, and therefore stiffens the model. While all models provide accurate results for the first two natural frequencies, only the BEM model which includes the effects of column geometry provides adequate results for the third frequency, which is associated with the twisting mode of vibration. The FEM models have twist frequencies which are too low, and the models are therefore too flexible in twist to adequately model the building frame.

The finite element model dynamic analysis, the boundary element model dynamic analysis, and the laboratory model investigation produce vibrational mode shapes of the

same nature. BEM and FEM model vibration displacement mode shapes are in agreement with each other and with the experimental results, with the exception of the FEM model having a larger displacement for the twist DOF in the first vibration mode. This is explained by the more torsionally flexible FEM model.

Figure 3.1 Building Model General Configuration

Figure 3.2 Stiff Column Patch Used in the FEM Model

Figure 3.3 Boundary Element Mesh and Column Point Forces

$0.4728e-6$	$0.7923e-8$	$-0.9646e-7$	$0.1084e-5$	$0.1830e-7$	$-0.2143e-6$
$0.7923e-8$	$0.4728e-6$	$-0.9645e-7$	$0.1830e-7$	$0.1084e-5$	$-0.2143e-6$
$-0.9646e-7$	$-0.9646e-7$	$0.1161e-5$	$-0.2136e-6$	$-0.2136e-6$	$0.2451e-5$
$0.1084e-5$	$0.1830e-7$	$-0.2136e-6$	$0.3343e-5$	$0.5774e-7$	$-0.6623e-6$
$0.1830e-7$	$0.1084e-5$	$-0.2136e-6$	$0.5774e-7$	$0.3343e-5$	$-0.6623e-6$
$-0.2144e-6$	$-0.2143e-6$	$0.2451e-5$	$-0.6623e-6$	$-0.6623e-6$	$0.7401e-5$

Figure 3.4 (a) Compact Flexibility Matrix for BEM Model With Finite Column-Slab Joints

$0.5769e-6$	$0.7278e-8$	$-0.9555e-7$	$0.1444e-5$	$0.1607e-7$	$-0.2110e-6$
$0.7278e-8$	$0.5769e-6$	$-0.9555e-7$	$0.1607e-7$	$0.1444e-5$	$-0.2110e-6$
$-0.9555e-7$	$-0.9555e-7$	$0.1254e-5$	$-0.2110e-6$	$-0.2110e-6$	$0.2770e-5$
$0.1444e-5$	$0.1607e-7$	$-0.2110e-6$	$0.4622e-5$	$0.4974e-7$	$-0.6530e-6$
$0.1607e-7$	$0.1444e-5$	$-0.2110e-6$	$0.4974e-7$	$0.4622e-5$	$-0.6530e-6$
$-0.2110e-6$	$-0.2110e-6$	$0.2770e-5$	$-0.6530e-6$	$-0.6530e-6$	$0.8573e-5$

Figure 3.4 (b) Compact Flexibility Matrix for BEM Model Using 1-Point Column Supports

39.8 kg	0	0	0	0	0
0	39.8 kg	0	0	0	0
0	0	8.38 kg-m^2	0	0	0
0	0	0	35.0 kg	0	0
0	0	0	0	35.0 kg	0
0	0	0	0	0	6.80 kg-m^2

Figure 3.5 Lumped Mass Matrix

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	13.6	.336	.336	-.223	1.0	1.0	-.653
2	14.0	-.337	.337	0.0	-1.0	1.0	0.0
3	21.6	.021	.021	.349	.064	.064	1.0
4	74.5	1.0	1.0	-.955	-.381	-.381	.413
5	77.5	-1.0	1.0	0.0	.382	-.382	.001
6	103.3	.102	.102	1.0	-.040	-.040	-.430

Table 3.1 (a) Dynamic Results of BEM Model With Finite Column Supports

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	11.7	.322	.322	-.146	1.0	1.0	-.432
2	11.9	-.321	.321	0.0	-1.0	1.0	0.0
3	19.9	.012	.012	.339	.043	.043	1.0
4	73.0	1.0	1.0	-.904	-.364	-.364	.394
5	75.8	-1.0	1.0	0.0	.364	-.364	0.0
6	101.4	.098	.098	1.0	-.036	-.036	-.416

Table 3.1 (b) Dynamic Results of BEM Model With Single Point Column Supports

Figure 3.6 Finite Element Mesh

$0.4804e-6$	$0.1034e-7$	$-0.1295e-6$	$0.1105e-5$	$0.2686e-7$	$-0.3270e-6$
$0.1026e-7$	$0.4802e-6$	$-0.1275e-6$	$0.2637e-7$	$0.1106e-5$	$-0.3227e-6$
$-0.1295e-6$	$-0.1292e-6$	$0.1527e-5$	$-0.3246e-6$	$-0.3204e-6$	$0.3586e-5$
$0.1105e-5$	$0.2669e-7$	$-0.3257e-6$	$0.3359e-5$	$0.7578e-7$	$-0.9650e-6$
$0.2621e-7$	$0.1106e-5$	$-0.3215e-6$	$0.7474e-7$	$0.3357e-5$	$-0.9468e-6$
$-0.3246e-6$	$-0.3246e-6$	$0.3581e-5$	$-0.9524e-6$	$-0.9524e-6$	$0.1096e-4$

Figure 3.7(a) Compact Flexibility Matrix of FEM Model With Finite Column-Slab Joints

$0.5110e-6$	$0.1123e-7$	$-0.1305e-6$	$0.1208e-5$	$0.2928e-7$	$-0.3230e-6$
$0.1118e-7$	$0.5106e-6$	$-0.1270e-6$	$0.2888e-7$	$0.1208e-5$	$-0.3182e-6$
$-0.1305e-6$	$-0.1287e-6$	$0.1592e-5$	$-0.3241e-6$	$-0.3153e-6$	$0.3806e-5$
$0.1208e-5$	$0.2969e-7$	$-0.3236e-6$	$0.3713e-5$	$0.8455e-7$	$-0.9567e-6$
$0.2930e-7$	$0.1208e-5$	$-0.3176e-6$	$0.8362e-7$	$0.3720e-5$	$-0.9211e-6$
$-0.3241e-6$	$-0.3182e-6$	$0.3806e-5$	$-0.9455e-6$	$-0.9262e-6$	$0.1173e-4$

Figure 3.7 (b) FEM Model Compact Flexibility Matrix of FEM Model With Single Point Column Supports

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	12.6	.280	.278	-.342	-.816	.812	-1.0
2	13.3	-.340	.342	.001	-.995	1.0	.003
3	17.6	.040	.040	.340	.118	.118	1.0
12	75.2	.981	1.0	-.652	-.423	-.430	.310
13	76.7	1.0	-.975	0.0	-.435	.425	0.0
18	100.1	.052	.049	1.0	-.023	-.023	-.471

Table 3.2 (a) Results of Dynamic Analysis of Full FEM Model with Finite Column-Slab Joints

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	12.1	.313	.311	-.342	.927	.919	-1.0
2	12.7	-.334	.337	.003	-.993	1.0	.009
3	16.8	.034	.034	.336	.105	.102	1.0
12	74.3	1.0	.994	-.689	-.422	-.416	.308
13	76.1	-.991	1.0	0.0	.0426	-.431	0.0
18	98.5	.057	.053	1.0	-.027	-.024	-.502

Table 3.2 (b) Results of Dynamic Analysis of Full FEM Model With Single Point Column Supports

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	13.2	.270	.272	-.341	.793	.795	-1.0
2	14.0	-.339	.341	.003	-.998	1.0	.004
3	18.3	.041	.041	.337	.123	.123	1.0
4	76.6	1.0	.958	-.549	-.392	-.340	.210
5	79.3	-.899	1.0	-.332	.348	-.385	.136
6	99.3	.081	.100	1.0	-.016	-.046	-.419

Table 3.3 (a) Results Dynamic Analysis of Compact FEM Model With Finite Column-Slab Joints

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	12.7	.306	.306	-.342	.907	.908	-1.0
2	13.3	-.335	.336	.005	-.996	1.0	.014
3	17.6	.036	.035	.334	.109	.107	1.0
4	76.8	.888	1.0	-.809	-.346	-.375	.313
5	78.6	1.0	-.902	-.055	-.388	.349	-.034
6	100.6	.093	.099	1.0	-.014	-.041	-.425

Table 3.3 (b) Results of Dynamic Analysis of Compact FEM Model With Single Point Column Supports

Figure 3.8 Laboratory Model of Steel Construction

Figure 3.9 (a) Model Base Mount

(b) Base Clip Detail

Figure 3.10 Placement of Accelerometers on Model Plate

Figure 3.11 Shaking Table Secured to Floor

Figure 3.12 Plate Translating Laterally

Figure 3.13 Cosine Waves In Phase

Figure 3.14 Plate Rotating About Axis Perpendicular to Surface

Figure 3.15 Cosine Waves 180 Degrees Out of Phase

Figure 3.16 Triangular Accelerometer Configuration and unknown Displacements

Figure 3.17 Laboratory Model Acceleration vs Time Raw Data

Figure 3.18 Laboratory Model Acceleration vs Time Window Plot

Figure 3.19 Shaking Table Acceleration vs Time Raw Data

Figure 3.20 Fourier Spectrum of Test HAM1 Data

Figure 3.21 Cross-Spectra of Test HAM1 Data

Mode #	Freq.	v1	v2	v3	v4	v5	v6
1	12.2	.470	.108	-.210	1.0	.242	-.422
2	13.4	-.177	.415	.046	-.355	1.0	.095
3	23.2	.025	.025	.499	.041	.035	1.0

Table 3.4 Experimental Dynamic Test Results

Figure 3.22 First Vibration Mode DOF Displacements

Figure 3.23 Second Vibration Mode DOF Displacements

Figure 3.24 Third Vibration Mode DOF Displacements

4. Conclusion

4.1 Summary

This thesis features the DBEM for elastic thin plates used in the modelling of flat slab building frames, where a finite set of collocation points placed within the column-slab connection area are used to model column supports, as developed in the work of Abdel-Akher. The method is applied to a structure and used in a frequency analysis and the results are verified by an experimental investigation.

Methods for static and dynamic analyses of building frames are reviewed in the first chapter. It is shown that elastic methods of analysis in dynamics are useful. A review of the literature has shown that an important consideration in the design of building structures is torsional effects due to lateral loads and that torsional-lateral coupling in asymmetric buildings can lead to increased lateral storey drift during earthquakes. A 3-dimensional building frame model is therefore important in the analysis of buildings.

The literature has also shown an interest in recent years in the simplification of the methods of analysis of building frames while using sophisticated modelling techniques. Chapter 2 features the intermediate building frame model which uses the direct boundary element method as developed by Abdel-Acker. A method which formulates the building frame stiffness coefficients for storey twist is developed, and the method of obtaining the

compact stiffness matrix model of a building frame is outlined. Finally, Chapter 5 presents the results of the dynamic analysis of the BEM and FEM models with both finite column-slab joints and single point columns supports, and of the laboratory experimental test results.

The contributions of this thesis may be summarized as follows:

1. Abdel-Acker's building frame model is modified by locating the floor slab DOF at the slab centre of mass.
2. A method for the formulation of the twist stiffness matrix coefficient for a building frame model is developed.
3. A study is done involving the dynamic analysis of BEM and FEM computer models, and the laboratory experimental investigation of a physical model.
4. The effect of including the finite column-slab joint in the numeric model of the building frame is studied.

4.2 Conclusion

The boundary element method for flat slabs incorporated into the building frame model, as developed in the work by Abdel-Acker, is an accurate tool for the dynamic modelling of flat plate building frames. The dynamic analysis results using the BEM model compared closely to the results of the experimental investigation of the physical model.

The method of calculating the building frame stiffness coefficients for storey twist presented in this study is adequate.

The compact stiffness matrix model of a building frame provides an accurate model for dynamic analysis. This is found as the standard finite element method dynamic analysis results compares closely with those of the finite element compact stiffness model.

The BEM model with finite column-slab joints, which more closely resembles the actual geometry of the physical model, provides a stiffer and more acceptable dynamic model than the BEM model which uses single point column supports.

4.2 Future Work

1. The physical model can be used for future studies. It would be useful to perform a static lateral load tests on the physical model to determine the model flexibility matrix coefficients to further verify the DBEM model.
2. It would be of interest to investigate the increase in building effective eccentricity during earthquakes due to equal lateral and torsional natural frequencies, mentioned in Section 1.1 of the present study. This effect could be achieved in the physical model by attaching point masses to the floor slabs at strategic locations to alter the model natural

frequencies.

3. The DBEM program for slab supported on columns of Abdel-Akher could be modified so that column-slab joint rotations slab bending moments due to storey sway can be found.

4. Forced-vibration testing of the laboratory model could be performed when the shaking-table actuator is functioning. It would be useful to study the use of the DBEM compact stiffness matrix computer model under forced-vibrations loading.

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Appendix A

Photographic Plates of Laboratory Test Set-up

Plate A.1 Test Model and Shaking Table

Plate A.2 Model Base and Mounting Clips

Plate A.3 I-Beam Anchoring Shaking Table

Plate A.4 Accelerometer

Appendix B

Instructions for the Use of the Computer Programs *Fslab.f*, *Frame_sdb.f* and *Usesweep.c*

The program *fslab.f* applies the DBEM to thin elastic plates supported directly on columns, and generates the stiffness matrix of the plate. It is used in the present study to generate the stiffness matrices of the slab elements of the intermediate building frame model. The program *frame_sdb.f* is used to assemble the slab element stiffness matrices and the column element stiffness matrices into the global stiffness matrix of the intermediate building frame model, and to analyse the building frame under static loads. *Usesweep.c* calculates the natural frequencies and vibration displacement mode shapes of a building frame.

The Usage of *Fslab.f*

Input data describing the plate boundary discretization and column supports is read from a file named *indata_1*. The format of the input data is outlined in *Appendix A* of [8]. The program creates a file called *ss* which contains the slab stiffness matrix.

The Usage of *Frame_sdb.f*

Input data for *frame_sdb.f* is read from a file called *indata_2*. The format of the input data is the following.

1. CX, CY
CX = x-coordinate of the building centre of mass
CY = y-coordinate of the building centre of mass
2. M, NS
M = number of columns
NS = number of storeys
3. E, PR, TH
E = Young's modulus
PR = Poisson's ratio
TH = slab thickness

4. I,X(I),Y(I),A(I),B(I),THETA
 I = column number
 X(I), Y(I) = x and y coordinate of the ith column
 A(I), B(I) = column x and y dimension
 THETA = angle of column orientation with respect to the global
 coordinate system

5. I1
 I2
 HH
 I1 = first floor number
 I2 = second floor number
 HH = column height

6. 0, 0.0
 0, 0.0

Usage of *Usesweep.c*

Usesweep.c requires the user to input the building frame flexibility matrix and mass matrix coefficients directly into the code of the program. Flexibility matrix coefficients are variables called *Flex[i][j]*, and mass matrix coefficients are found in *Mass[i]*, a column matrix having as values the coefficients of the diagonal mass matrix. The user must also indicate the dimension of the mass matrix using the variable *N*.

IMAGE EVALUATION TEST TARGET (QA-3)

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