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**LA THÈSE A ÉTÉ
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BOUNDARY INTEGRAL EQUATION
STRESS ANALYSIS OF CRACKED
THICK-WALLED CYLINDERS

Mong Lin Shim

A Thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the degree
of Master of Engineering

DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, ONTARIO
CANADA

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The undersigned recommend to the Faculty of
Graduate Studies and Research acceptance of the thesis;

BOUNDARY-INTEGRAL EQUATION
STRESS ANALYSIS OF CRACKED
THICK-WALLED CYLINDERS

submitted by Mong-Lin Shim in partial fulfillment of the
requirements for the degree of Master of Mechanical
Engineering.

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ABSTRACT

The Boundary Integral Equation (BIE) method has been developed into an economical and accurate numerical tool for solving complex engineering problems in elastostatics, especially when cracks are present. In this study, weight functions and polynomial influence coefficients for radial-edge cracks and semi-elliptical surface cracks, respectively, in thick-walled cylinders are obtained using the numerical BIE method. Utilizing these weight functions, which are obtained from two-dimensional stress analyses, stress intensity factors, K_I , for a wide range of radial-edge cracks in cylinders subjected to autofrettaged residual stresses are obtained using the linear superposition principle.

For semi-elliptical surface cracks in the cylinders, stress intensity factor solutions for internal pressure loading are presented. Both the case of when the internal pressure also acting on the crack faces, and the case when it does not, are investigated. K_I results quantifying the effects of autofrettaged residual stresses and thermal loads are also obtained. In these three-dimensional studies, the stress intensity factor solutions are obtained

from the polynomial influence coefficients using linear superposition. The results obtained should be useful in evaluating the structural integrity and/or safe-life of cracked cylindrical components.

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Cracks in pressure vessels or other cylindrical components are often found to be located at the internal bore of the cylinders. These cracks are well represented by semi-ellipses, thus requiring three-dimensional (3-D) treatment of the problem. In chapter 6, the weight/influence function coefficients for a wide range of crack geometries are obtained using the 3-D numerical BIE method. Employing the linear superposition technique, the influence function coefficients are used to evaluate K_I solutions for cracked cylinders subjected to a variety of loading conditions. These loading conditions include internal pressure with the case when the pressure acting on the crack faces as well as the case when it does not. Also the effects of autofrettaged residual stresses and thermal loads are investigated.

But before the results are presented and discussed, the basic theories of LEFM and BIE are reviewed, in the next two chapters respectively. This is then followed by a short discussion of the application of BIE to LEFM in chapter 4.

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NOMENCLATURE

- a - Characteristic crack length
- a - Semi-major axis of an elliptical surface crack (Chapter 6)
- a* - Relative crack-depth (Chapter 5);
 $a^* = a/(R_2 - R_1)$
- A(a*) - Constant coefficient of nodal weight function (Chapter 5)
- A_0, A_1, A_2, A_3 - Constant coefficients of polynomial influence function (Chapter 6)
- ΔA - Increase in crack area
- b - Semi-minor axis of an elliptical surface crack (Chapter 6)
- B(a*) - Constant coefficient of nodal weight function (Chapter 5)
- $C_{ij}(P)$ - Limiting value of the surface integral of $T_{ij}(P, Q)$
- C(a*) - Constant coefficient of nodal weight function (Chapter 5)
- d(b, c) - cth node of bth boundary element
- D(a*) - Constant coefficient of nodal weight function (Chapter 5)
- E - Young's modulus
- f_i - Body-force vector (Chapter 3)
- F - Non-dimensional compliance function; Eqn. (2.16)

$g_i(x)$	- Prescribed displacements
G	- Energy release rate; Eqn. (2.17)
G_c	- Critical value of G
$h_i(x)$	- Prescribed tractions
$h(a, r_s)$	- Nodal weight function (Chapter 5)
H	- Material modulus (Section 2.1)
$J(\xi)$	- Jacobian of transformation
k	- Cylinder radius ratio; $k=R_2/R_1$
K_I, K_{II}, K_{III}	- Stress intensity factors of mode I, II, III
K_{Ic}	- Critical value of K_I
K_I^*	- Value of K_I obtained from Eqns. (4.4) or (4.6), used to extrapolate K_I
K_I	- Weighted root-mean-square value of K_I
K_{Iref}	- Value of K_I at a reference point
δl	- Increment of crack length (Section 4.2)
M	- Total number of boundary elements
n	- Unit outward normal
n_i	- Components of n
$N(\xi)$	- Shape function
p	- An interior point
P	- A boundary surface point
P	- Internally applied pressure in thick-walled cylinder

- p^a - a^{th} nodal point on the boundary surface
 Q - General boundary surface point
 r - Polar coordinate defined in Figure 2.2
 r_{ref} - Distance of a reference point to the crack front
 $r(s)$ - Distance of a point on the crack-face to the crack-front
 $r^*(s)$ - $r^*(s) = r(s)/r_{\text{ref}}$
 r_s - Distance of a point from the crack-tip
 r^* - $r^* = r_s/a$
 R - Crack resistance (Eqn. (2.19))
 R_1, R_2 - Inner and outer radii of a cylinder
 nR_1 - Size of plastic deformation
 t_i - Traction vector
 ΔT - Difference between the temperature at the cylinder bore and at its outer circumference
 T_{ij} - Kelvin's point load solution for traction
 u_x, u_y, u_z - Displacements in the x, y, z directions (Chapter 2)
 U - Strain energy
 U_{ij} - Kelvin's point load solution for displacement
 v - Displacement in the y -direction

- v_{ref} - Displacement of a reference point
- $v(s)$ - Displacement of a point on the crack-face
- $v^*(s)$ - $v^*(s) = v(s) / v_{ref}$
- W - Surface energy (Chapter 2)
- W - Cylinder wall thickness, $W = R_2 - R_1$
- α - Coefficient of thermal expansion
- γ - Specific surface energy (Chapter 2)
- δ_{ij} - Kronecker delta
- ϵ_{ij} - Strain tensor
- θ - Polar coordinate as defined in Figure 2.2
- μ - Shear modulus
- ν - Poisson's ratio
- ν' - Equivalent Poisson's ratio, Eqn.(3.4a)
- ν^* - Equivalent Poisson's ratio, Eqn.(3.4b)
- ξ - Intrinsic coordinate
- σ_y - Yield stress
- $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}, \sigma_{yx}, \sigma_{yy}, \sigma_{yz}, \sigma_{zx}, \sigma_{zy}, \sigma_{zz}$ - Stress components at the vicinity of the crack-tip
- σ_{ij} - Stress tensor
- σ_o - External loading stress (Chapter 2)
- σ_θ - Hoop stress distribution in a cylinder
- $(\sigma_\theta)_e$ - Elastic hoop stress distribution
- $(\sigma_\theta)_p$ - Elasto-Plastic hoop stress distribution
- ϕ - Angle defined in Figure 6.1

CHAPTER 1

INTRODUCTION

Linear Elastic Fracture Mechanics (LEFM) analysis forms the basis for predicting the residual life and fracture strength of thick-walled cylinder components. According to the LEFM theory, the crack growth rate (da/dN) is a function of the cyclic change in the crack tip stress intensity factor (ΔK_I). Mathematically, the relationship may take the form of, for example, Paris law:

$$\frac{da}{dN} = c(\Delta K_I)^n \quad (1.1)$$

where a is the characteristic crack length; N is the number of load cycles; and c and n are the material constants. The stress intensity factor, K_I , is the characterizing parameter which embodies the effect of the stress field and crack geometry in the vicinity of the crack. For a given material, da/dN in a component is determined by K_I and its variations. Therefore, for a complete crack growth analysis a wide range of K_I solutions are essential.

Numerical K_I solutions for cracked thick-walled cylinders are available in the open literature (see, e.g., Bowie and Freese [1972], Heliot et al [1979], Atluri and Kathiresan [1979, 1980], Jan and Fenner [1979, 1980], and Kirkhope et al [1984]). However, these results are only applicable for the case of internally pressurized cylinders. K_I results quantifying the effects of autofrettaged residual stresses and thermal loads have not been extensively reported in the literature. This is particularly true for the case of surface cracks in cylinders. Related works only pertain to radial-edge cracks and are based on two-dimensional (2-D) analysis (see, e.g., Parker and Farrow [1981] and Pu and Hussain [1981]). Even when their solutions are only approximation in nature.

In order that repeated computer runs be avoided for a given cracked geometry subjected to different loading conditions, the weight function method (also commonly known as the influence or Green's function method) is employed. The basis of the weight function method is the principle of linear superposition. Once the weight functions for a cracked geometry have been determined, K_I can be evaluated for any crack size and shape from the "uncracked" stress field. In chapter 5, the two-dimensional numerical boundary integral equation (BIE) method is employed to compute the weight or

influence functions for the radial-edge cracks in thick-walled cylinders. Utilizing the results of these weight functions, numerical K_I values for a wide range of radial-edge cracks in cylinders subjected to autofrettaged residual stresses are obtained.

Cracks in pressure vessels or other cylindrical components are often found to be located at the internal bore of the cylinders. These cracks are well represented by semi-ellipses, thus requiring three-dimensional (3-D) treatment of the problem. In chapter 6, the weight/influence function coefficients for a wide range of crack geometries are obtained using the 3-D numerical BIE method. Employing the linear superposition technique, the influence function coefficients are used to evaluate K_I solutions for cracked cylinders subjected to a variety of loading conditions.

These loading conditions include internal pressure with the case when the pressure acting on the crack faces as well as the case when it does not. Also the effects of autofrettaged residual stresses and thermal loads are investigated.

But before the results are presented and discussed, the basic theories of LEFM and BIE are reviewed, in the next two chapters respectively. This is then followed by a short discussion of the application of BIE to LEFM in chapter 4.

CHAPTER 2

REVIEW OF LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

Many structures which have been designed based on the traditional concepts of yield or ultimate strength of the material, have failed prematurely even under "safe" loading conditions with disastrous consequences. These brittle failures have occurred in such diverse structures as suspension bridges, aircraft landing gears and pressure vessels. Careful examination of such failures usually produced a common feature: a small flaw or crack at the failure origin.

The key to control brittle fracture lies in the understanding of both the weakening effects of flaws or cracks in metals and those factors that influence these effects. To be of practical use, this understanding must be translated into the types of tests and structural mechanics familiar to the engineer. The branch of study concerning this type of failure is known as fracture mechanics.

2.1 Basic Fracture Parameters in LEFM

2.1.1 Stress Intensity Factors

The stress fields near crack tips can be categorized according to each of the three basic modes of deformation as shown in Figure 2.1. The opening mode, mode I, is associated with a local displacement in which the crack surfaces move directly apart. The edge-sliding mode, mode II, is characterized by displacements in which the crack surfaces slide over one another. In mode III, the tearing or shear mode, the crack surfaces slide with respect to one another parallel to the leading edge.

A crack problem may consist of any combination of these three basic modes of deformation under various loading conditions. In practice, mode I is generally being considered as the most important because brittle crack has the tendency to "propagate" in direction which minimizes the shear loading. Therefore, attention in this study will be confined to the opening mode (mode I) of failure.

Using the mathematical theory of linear elasticity and the Westergaard stress function (see, e.g., Irwin [1957] and Paris and Sih [1965]), it can be shown that the stress distributions in the vicinity of the

crack tip for the opening mode of deformation may be expressed in the following form:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{K_I \cos(\theta/2)}{\sqrt{2\pi r}} \begin{bmatrix} 1 - \sin(\theta/2)\sin(3\theta/2) \\ 1 + \sin(\theta/2)\sin(3\theta/2) \\ \sin(\theta/2)\cos(3\theta/2) \end{bmatrix} + O(r^0) \quad (2.1)$$

$$\sigma_{zz} = \nu' (\sigma_{xx} + \sigma_{yy}) \quad (2.2)$$

and

$$\sigma_{xz} = \sigma_{yz} = 0 \quad (2.3)$$

where K_I is the mode I stress intensity factor; r and θ are polar coordinates defined in Figure 2.2; $O(r^0)$ represents the higher order terms in r and are relatively insignificant to the crack-tip solution; ν is the Poisson's ratio; and

$$\nu' = \begin{cases} 0 & \text{(for plane stress)} \\ \nu & \text{(for plane strain)} \end{cases}$$

The corresponding displacement fields for the mode I deformation are

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \frac{K_I \sqrt{r/2\pi}(1 + \nu)}{2E'} \begin{bmatrix} (2\kappa - 1)\cos(\theta/2) - \cos(3\theta/2) \\ (2\kappa + 1)\sin(\theta/2) - \sin(3\theta/2) \end{bmatrix} + O(r^0) \quad (2.4)$$

$$u_z = -(\nu'' z/E)(\sigma_{xx} + \sigma_{yy}) \quad (2.5)$$

where E is the Young's modulus, and

$$\kappa = \frac{(3 - \nu)}{(1 + \nu)}, \quad \nu'' = \nu \quad (\text{for plane stress})$$

$$\kappa = (3 - 4\nu), \quad \nu'' = 0 \quad (\text{for plane strain})$$

The stress and displacement distributions near the crack tip for the mode II and mode III of deformation are:

mode II

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{K_{II}}{\sqrt{2\pi r}} \begin{bmatrix} -\sin(\theta/2)[2 + \cos(\theta/2)\cos(3\theta/2)] \\ \sin(\theta/2)\cos(\theta/2)\cos(3\theta/2) \\ \cos(\theta/2)[1 - \sin(\theta/2)\sin(3\theta/2)] \end{bmatrix} + O(r^0) \quad (2.6)$$

$$\sigma_{zz} = \nu' (\sigma_{xx} + \sigma_{yy}) \quad (2.7)$$

$$\sigma_{xz} = \sigma_{yz} = 0 \quad (2.8)$$

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \frac{K_{II} (1 + \nu)\sqrt{r/2\pi}}{2E} \begin{bmatrix} (2\kappa + 3)\sin(\theta/2) + \sin(3\theta/2) \\ -(2\kappa - 3)\cos(\theta/2) - \cos(3\theta/2) \end{bmatrix} + O(r^0) \quad (2.9)$$

$$u_z = -(\nu'' Z/E)(\sigma_{xx} + \sigma_{yy}) \quad (2.10)$$

mode III

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0 \quad (2.11)$$

$$\sigma_{xy} = 0 \quad (2.12)$$

$$\begin{bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \frac{K_{III}}{\sqrt{2\pi r}} \begin{bmatrix} -\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix} + O(r^0) \quad (2.13)$$

$$u_x = u_y = 0 \quad (2.14)$$

$$u_z = (K_{III} / 2E) \sqrt{r/2\pi} [2(1 + \nu)\sin(\theta/2)] \quad (2.15)$$

where K_{II} and K_{III} are the mode II and mode III stress intensity factors respectively. The solutions for mode II and mode III crack deformation are presented in the above for the sake of completeness. In the discussion hereafter, attention will be confined to the opening mode, mode I, of crack deformation.

It can be seen from Eqn. (2.1) that the stresses at the vicinity of the crack tip are singular, being inversely proportional to the square root of the distance from the crack tip. The constant of proportionality, or stress intensity factor, K_I , defines the strength of the singularity and is a function of applied load, crack size, and the geometry of the cracked component. K_I has the dimension of $\text{Pa}\sqrt{\text{m}}$ (i.e. stress $\cdot \sqrt{\text{length}}$). In general, K_I may be defined as:

$$K_I = F \sigma \sqrt{\pi a} \quad (2.16)$$

where σ = nominal applied stress,

F = Compliance function, and

a = crack length.

The basic assumption in LEFM is that unstable fracture occurs when K_I reaches a critical value, designated K_{IC} , commonly known as the fracture toughness. It is important here to distinguish the difference between K_I and K_{IC} . The stress intensity factor K_I is simply a quantifying parameter embodying the effects of the elastic stresses in the vicinity of the crack tip and of the geometry. Fracture toughness K_{IC} , on the other hand, is a particular value of K_I when reached will lead to unstable crack propagation. It is a material property and it reflects a material's ability to withstand a given stress at a crack tip.

2.1.2. Griffith's Energy Criterion

Another method commonly used for fracture prediction is the Griffith's energy criterion. Griffith [1920] has traditionally been considered the first to provide a fundamental understanding of fracture mechanics. According to Griffith, if the strain energy released (∂U) during an

Incremental crack extension (∂a) is equal to or greater than the corresponding increase in surface energy (∂W) required to create the new crack then the crack will grow.

The Griffith's energy release rate G can be defined as:

$$G = \frac{\partial U}{\partial a} \quad (2.17)$$

where U = strain energy and for a general three-dimensional loading system,

$$U = 1/2E \int \{ [\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - 2\nu(\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx})] + 2(1 - \nu)(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2) \} dV \quad (2.18)$$

The increase in surface energy that would result from an incremental crack extension is known as the crack resistance R ;

$$R = \frac{\partial W}{\partial a} = 2Y \quad (2.19)$$

where Y = specific surface energy.

It follows that the threshold condition for unstable crack growth is given by:

$$\frac{\partial U}{\partial a} = \frac{\partial W}{\partial a} \quad (2.20)$$

or

$$G = R = G_{\text{critical}} \quad (2.21)$$

where G_{critical} is a material property.

It can be shown by the principle of virtual work (Irwin [1957]) that

$$G = \frac{K_I^2}{H} \quad (2.22)$$

where H is the effective modulus and is given by

$$H = \begin{cases} E & \text{(for plane stress)} \\ \frac{E}{1-\nu^2} & \text{(for plane strain)} \end{cases} \quad (2.23)$$

Similarly,

$$G_{\text{critical}} = \frac{K_{IC}^2}{H} \quad (2.24)$$

It can be seen from Eqn. (2.22) that K_I can be computed from the change in strain energy when the crack is extended an infinitesimal amount.

2.2 Limitations of LEFM

From Eqn. (2.1), it is evident that the stresses at the crack tip are theoretically infinite as r approaches zero. In reality, these stresses are finite as plastic deformation will occur at the crack tip. Assuming elastic-perfectly plastic material the stress distribution at the vicinity of crack tip is shown in Figure 2.3.

The size of the plastic zone at the crack tip can be obtained as [Irwin (1960)]:

$$r_p = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_y} \right)^2 \quad (\text{for plane stress}) \quad (2.25)$$

$$r_p = \frac{1}{6\pi} \left(\frac{K_I}{\sigma_y} \right)^2 \quad (\text{for plane strain}) \quad (2.26)$$

The use of this Irwin plastic zone correction factor is adequate provided the plastic zone is small compared to the crack length. Further, it is not clear how Irwin's plastic correction factor can be applied to three-dimensional crack problems.

It should be emphasized here that if there are large amounts of plastic deformation, elastic-plastic fracture mechanics (EPFM) must be employed. Parameters have been proposed in EPFM to describe the non-linear plastic deformation behaviour at the crack tip; two such parameters are the J-integral and crack tip opening displacement, (COD) method. EPFM is beyond the scope of present study and hence, it will not be further discussed.

2.3 Determination of Stress Intensity Factor K_I

Although closed form solutions (e.g., Westergaard's and Muskhelishvili's stress functions) for the stress intensity factor K_I are available for some simple geometries, most practical problems are sufficiently complex to make the determination of K_I by analytical means an extremely difficult, if not an impossible, task. Recourse to experimental or numerical methods must therefore be sought. Experimental techniques for this purpose include such methods as photoelasticity and compliance method. While the finite element (FE) (see, e.g. Zienkiewicz [1977]) and boundary integral equation (BIE) methods (see, e.g. Cruse [1975], Tan and Fenner [1979]) are the most commonly used numerical techniques. In the present study, the BIE method is used.

The various techniques for evaluating K_I when using the BIE method will be discussed in chapter 4, after the review of the BIE method as a numerical tool in the next chapter.

CHAPTER 3

REVIEW OF THE BOUNDARY INTEGRAL EQUATION (BIE) METHOD

The application of the boundary integral equation (BIE) method to two-dimensional elastostatics was first formulated by Rizzo [1967]. His work was later extended by Cruse [1969, 1973] to three-dimensional applications. However, the advantages of the BIE method based on their formulation, when compared to the finite element (FE) method, were not clearly evident due to the relatively crude numerical schemes adopted. Since then, more sophisticated numerical formulations and implementation have been achieved by several researchers in the field, such as Lachat and Watson [1975, 1976], Rizzo and Shippy [1977] and Tan and Fenner [1978, 1979]. They have demonstrated the superiority of the BIE method over the FE method in solving many practical engineering problems.

The analytical basis of the BIE method is the transformation of the governing partial differential equation which is applicable throughout the domain into an integral equation which applies over its boundary. Thus a three-dimensional problem is being reduced numerically to a two-dimensional surface problem. This results in a smaller set of

resultant algebraic equations to be solved for the unknowns. Another advantage of the BIE method is the resolution and accuracy of the numerical results, particularly when rapidly varying stresses are involved. This is due in part to the lesser degree of approximations involved in the analysis, since the interior of the domain is not modelled.

The BIE method can be divided into two main categories: the direct and indirect methods. In the direct formulation, the physical tractions or displacements on the boundary are the unknowns which are directly solved for in the integral equation. On the other hand, the indirect formulation employs fictitious density functions in the integral equation. These fictitious density functions have no physical significance; but once the integral equation is solved, stresses and displacements on the boundary can be obtained in terms of these functions.

In the present study, only the direct formulation method will be used. The numerical strategy employed follows that originally proposed by Lachat and Watson [1976]. Cartesian tensor notations will be used throughout, where repeated suffices mean summation, and the comma suffix implies a partial differentiation with respect to the coordinates.

3.1 Formulation of the BIE

For a linear elastic solid domain R made of homogeneous, isotropic material in static equilibrium; the stresses are governed by:

$$\sigma_{ij,j} + f_i = 0 \quad \text{in } (R) \quad (3.1)$$

$$i, j = \begin{cases} 1, 2 & \text{in two-dimensions, (2-D)} \\ 1, 2, 3 & \text{in three-dimensions, (3-D)} \end{cases}$$

where σ_{ij} is the second-order stress tensor and f_i is the body force vector (force per unit volume). Also, the strain-displacement relationship is given by:

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (3.2)$$

where ϵ_{ij} is the strain tensor and u_i is the displacement component in the x_i direction.

Further, Hooke's law is given by:

$$\sigma_{ij} = 2\mu (\epsilon_{ij} + \frac{\nu'}{1-2\nu'} \epsilon_{kk} \delta_{ij}) \quad (3.3)$$

where μ is the shear modulus; δ_{ij} the Kronecker delta and ν' is defined as

$$\nu' = \begin{cases} \nu & \text{(for 3-D)} \\ \nu^* & \text{(for 2-D)} \end{cases} \quad (3.4a)$$

$$\text{where } \nu^* = \begin{cases} \frac{\nu}{(1+\nu)} & \text{(for generalized plane stress)} \\ \nu & \text{(for generalized plane strain)} \end{cases} \quad (3.4b)$$

and ν being the Poisson's ratio.

Substituting Eqn. (3.3) into Eqn. (3.1) and making use of Eqn. (3.2) results in the Cauchy-Navier equation:

$$\mu(u_{i,jj} + \frac{1}{1-2\nu^*} u_{j,ij}) + f_i = 0 \quad (3.5a)$$

Neglecting the body force vector f_i , this equation becomes

$$\mu(u_{i,jj} + \frac{1}{1-2\nu^*} u_{j,ij}) = 0 \quad (3.5b)$$

To solve Eqn. (3.5b) for a body R bounded by S (Figure 3.1), the appropriate boundary conditions given by Eqn. (3.6) below must be applied:

$$u_i(\chi) = g_i(\chi) \quad \text{on } S_u \quad (3.6a)$$

$$t_i(\chi) = \sigma_{ij} n_j = h_i(\chi) \quad \text{on } S_t \quad (3.6b)$$

where $u_i(\chi)$ and $t_i(\chi)$ are the displacement and traction components respectively on S ; $g_i(\chi)$ and $h_i(\chi)$ are prescribed functions, and n_j is the unit outward normal vector on S .

The fundamental singular solutions to Eqn. (3.5b) is the well known Kelvin's unit point load solution, in an infinite body. The displacements for this problem are given

by the following displacement tensors:

$$U_{ij}(p, Q) = \frac{1}{16\pi\mu(1-\nu)r} [(3-4\nu)\delta_{ij} + r_{,i}r_{,j}] \quad \text{for 3-D} \quad (3.7)$$

$$U_{ij}(p, Q) = \frac{1}{8\pi\mu(1-\nu^*)r} [(3-4\nu^*)\ln\left(\frac{1}{r}\right)\delta_{ij} + r_{,i}r_{,j}] \quad \text{for 2-D} \quad (3.8)$$

where $U_{ij}(p, Q)$ denotes the displacements in the x_i -direction at Q due to a unit point load in the x_j -direction at p . The tractions are given by the following traction tensors:

$$T_{ij}(p, Q) = \frac{-(1-2\nu)}{8\pi(1-\nu)r^2} \left[\frac{\partial r}{\partial n} (\delta_{ij} + \frac{3}{1-2\nu} r_{,i}r_{,j}) - n_j r_{,i} + n_i r_{,j} \right] \quad \text{for 3-D} \quad (3.9)$$

$$T_{ij}(p, Q) = \frac{-(1-2\nu^*)}{4\pi(1-\nu^*)r} \left[\frac{\partial r}{\partial n} (\delta_{ij} + \frac{2}{1-2\nu^*} r_{,i}r_{,j}) - n_j r_{,i} + n_i r_{,j} \right] \quad \text{for 2-D} \quad (3.10)$$

where $T_{ij}(p, Q)$ denotes the tractions in the x_i -direction at Q due to a unit point load in the x_j -direction at p . In the above equations, r is the distance from p to Q given by

$$r = \sqrt{(x_{Qi} - x_{pi})^2 + (y_{Qi} - y_{pi})^2} \quad (3.11)$$

$$r_{,i} = \frac{\partial r}{\partial x_{Qi}} = \frac{1}{r} (x_{Qi} - x_{pi}) \quad (3.12)$$

and

$$\frac{\partial r}{\partial n} = n_i \frac{\partial r}{\partial x_{Qi}} = \frac{n_i}{r} (x_{Qi} - x_{pi}) \quad (3.13)$$

Substituting the fundamental solution into the Betti's reciprocal work theorem (see, e.g., Sokolnikoff [1956]) and making use of the divergence theorem gives the Somigliana's identity:

$$u_i(p) = \int_S t_i(Q) U_{ij}(p, Q) ds - \int_S u_j(Q) T_{ij}(p, Q) ds \quad (3.14)$$

This identity gives the displacements $u_i(p)$ at an interior point p in terms of the boundary data. Taking the interior point p to a boundary point P by the usual limiting process results in the BIE, which is an integral constraint relating boundary tractions to boundary displacements:

$$\begin{aligned} C_{ij}(P) u_j(P) + \int_S T_{ij}(P, Q) u_j(Q) ds \\ = \int_S U_{ij}(P, Q) t_j(Q) ds \end{aligned} \quad (3.15)$$

where

$$C_{ij}(P) = \lim_{\epsilon \rightarrow 0} \int_{S_\epsilon} T_{ij}(P, Q) ds \quad (3.16)$$

In Eqn. (3.16) S_ϵ is the surface of R_ϵ which is the part of R contained within a sphere of radius ϵ centered at P . If the tangent plane at P is continuous, then

$$C_{ij}(P) = \frac{1}{2} \delta_{ij} \quad (3.17)$$

In most practical problems the BIE given in Eqn. (3.15) can only be solved by numerical means. Once it is solved,

the surface displacements and tractions will all be known everywhere on the boundary. From these data, the displacements at any desired interior point may be calculated using Eqn. (3.14). The corresponding stresses may be obtained from Eqn. (3.18) below, which is the Somigliana's identity for stresses:

$$\sigma_{ij}(p) = \int_S t_k(Q) D_{kij}(p, Q) ds - \int_S u_k(Q) S_{kij}(p, Q) ds \quad (3.18)$$

The tensors D_{kij} and S_{kij} in Eqn. (3.18) can be shown to be:

$$D_{kij}(p, Q) = \frac{1-2\nu}{8\pi(1-\nu)} \left(\frac{1}{r^2} \right) [\delta_{ki} r_{,j} + \delta_{kj} r_{,i} - \delta_{ij} r_{,k} + \frac{3}{1-2\nu} r_{,i} r_{,j} r_{,k}] \quad \text{for 3-D} \quad (3.19)$$

$$D_{kij}(p, Q) = \frac{1}{4\pi(1-\nu)r} [(1-2\nu)(\delta_{ki} r_{,j} + \delta_{kj} r_{,i} - \delta_{ij} r_{,k}) + 2r_{,i} r_{,j} r_{,k}] \quad \text{for 2-D} \quad (3.20)$$

and

$$\begin{aligned} S_{kij}(p, Q) = & \frac{E}{8\pi(1-\nu^2)} \left(\frac{1}{r^3} \right) \{ 3 \frac{\partial r}{\partial n} [(1-2\nu) \delta_{ij} r_{,k} + \nu (\delta_{ki} r_{,j} + \delta_{kj} r_{,i}) \\ & - 5 r_{,i} r_{,j} r_{,k}] \\ & + n_i (3 \nu r_{,j} r_{,k} + (1-2\nu) \delta_{kj}) \\ & + n_j (3 \nu r_{,i} r_{,k} + (1-2\nu) \delta_{ki}) \\ & + n_k [3(1-2\nu) r_{,i} r_{,j} - (1-4\nu) \delta_{ij}] \} \quad \text{for 3-D} \quad (3.21) \end{aligned}$$

$$\begin{aligned}
S_{kij}(p, Q) = \frac{E}{4\pi(1-\nu^2)r^2} \{ & 2 \frac{\partial r}{\partial n} [(1-2\nu)\delta_{ij}r_{,k} + (\delta_{ki}r_{,j} + \delta_{kj}r_{,i}) \\
& - 4r_{,i}r_{,j}r_{,k}] \\
& + 2\nu(n_{,i}r_{,j}r_{,k} + n_{,j}r_{,i}r_{,k}) \\
& + (1-2\nu)(2n_{,k}r_{,i}r_{,j} + n_{,j}r_{,ik} + n_{,i}r_{,jk}) \\
& + n_{,k}(1-4\nu)\delta_{ij}] \}
\end{aligned}$$

for 2-D (3.22)

3.2 Numerical Treatment of the BIE

Numerical solutions to the BIE are found by discretising the boundary into elements, and expressing the distributions of the unknowns over these elements in terms of suitable algebraic functions involving values at certain nodal points associated with the elements. In the present work, the numerical formulation used is that originally proposed by Lachat and Watson [1975, 1976] and Tan and Fenner [1978, 1979]. The surface of a 3-D solid body is represented by eight-node quadrilateral and/or six-node triangular isoparametric elements as shown in Figures (3.2) and (3.3) respectively. In the

case of a 2-D domain, the boundary is represented by three-node quadratic isoparametric line elements as illustrated in Figure 3.4. The representations of geometry and unknowns over each element are in terms of quadratic shape functions of intrinsic coordinates $N^c(\xi)$, similar to those used in the FE method (see, e.g., Zienkiewicz [1977]). Thus,

$$x_i(\xi) = N^c(\xi) x_i^c \quad (3.23a)$$

$$u_i(\xi) = N^c(\xi) u_i^c \quad (3.23b)$$

$$t_i(\xi) = N^c(\xi) t_i^c \quad (3.23c)$$

where

$$c = \begin{cases} 1, 2, 3, \dots, 8 & \text{for quadrilateral elements} \\ 1, 2, 3, \dots, 6 & \text{for triangular elements} \\ 1, 2, 3 & \text{for line elements} \end{cases}$$

The explicit expressions for $N^c(\xi)$ are given in Appendix I.

If the surface is represented by m surface (in 3-D) or line (in 2-D) elements and a total of q distinct nodes, Eqn. (3.15) then becomes

$$\begin{aligned} & C_{ij}(P^a) u_j(P^a) + \sum_{b=1}^m \sum_{c=1}^g u_j(P^{d(b,c)}) \int_S T_{ij}(P^a, Q(\xi)) N^c(\xi) J(\xi) d\xi \\ & = \sum_{b=1}^m \sum_{c=1}^g t_j(P^{d(b,c)}) \int_S U_{ij}(P^a, Q(\xi)) N^c(\xi) J(\xi) d\xi \end{aligned} \quad (3.24)$$

$$g_a = \begin{cases} 6 \text{ or } 8 & \text{in 3-D} \\ 3 & \text{in 2-D} \end{cases}$$

where p^a is the a^{th} node; $d(b,c)$ is the number of the c^{th} node of the b^{th} element and $J(\xi)$ is the Jacobian of transformation. For a 3-D problem, Eqn. (3.24) results in a system of $3q$ linear algebraic equations to be solved for the unknown boundary tractions or displacements. For a 2-D problem, the number of algebraic equations is $2q$. These equations can be solved by direct Gaussian elimination method.

The isoparametric representations can similarly be substituted into Eqns. (3.14) and (3.18), to obtain the desired interior point displacements and tractions, once Eqn. (3.24) has been solved.

CHAPTER 4

APPLICATION OF THE BIE METHOD TO LEFM

The most important characterising parameter in LEFM is the stress intensity factor K_I . A critical problem in the application of the BIE method to LEFM is the modelling of the stress singularity at the vicinity of the crack tip. Conventional isoparametric elements as described in the last chapter are generally considered to be inadequate to represent this stress singularity, unless they are used in large numbers. To introduce the stress singularity at the crack-tip, the special quarter-point node-shifting method, as is commonly used in finite element fracture mechanics studies (see, e.g., Barsoum [1974] and Henshall and Shaw [1975]), will be employed in this study.

The stress intensity factor, K_I , may be directly obtained from extrapolation techniques, making use of the stress or displacement results in the vicinity of the crack-tip as calculated from the BIE analysis. Alternatively, the energy method, or the weight-function method may be employed, as indirect means of determining K_I . All these techniques for determination of K_I will now be discussed in

the sections below.

4.1 Determination of K_I by Direct Extrapolation method

The direct extrapolation methods to determine the stress intensity factor K_I were first developed by Chan et al [1970] in their FE fracture study. When using these methods, the stresses and displacements at the vicinity of the crack tip, as obtained from the numerical analysis, are used to extract K_I .

The y-component displacement distribution near the crack tip is given by Eqn. (2.4);

$$u_y = \frac{K_I}{2E} (1+\nu) \sqrt{(r/2\pi)} [(2\kappa+1)\sin(\theta/2) - \sin(3\theta/2)] \quad (4.1)$$

Substituting $\theta = \pi$ into Eqn. (4.1), the crack-face displacements normal to the crack becomes

$$v = u_y = \frac{4K_I}{H} \sqrt{(r/2\pi)} \quad (4.2)$$

where H is the stiffness modulus as given in Eqn. (2.23).

Rearranging Eqn. (4.2) gives

$$K_I = \frac{Hv}{4} \sqrt{(2\pi/r)} \quad (4.3)$$

It is evident that Eqn. (4.3) cannot be employed directly to obtain the value of the stress intensity factor, since it is only defined at $r=0$. In order to compute the crack

tip K_I , it is necessary to take the limiting value as $r \rightarrow 0$:

$$K_I = \lim_{r \rightarrow 0} \frac{Hv}{4} \sqrt{(2\pi/r)} \quad (4.4)$$

The process of determining K_I according to Eqn. (4.4) is as follows. Using Eqn. (4.3), values of K_I , say, K_I^* , are calculated at a series of nodal points along a radial line on the crack face. By plotting K_I^* against r , the curve will consist of a region with constant slope over a span of r away from the crack tip in front. This region of constant slope will be nearer to $r=0$ the more refined the mesh is, around the crack-tip. By extrapolating the tangent to this constant slope portion of the curve to the K_I^* axis, the value of K_I may be obtained. Figure 4.1 shows a typical plot, which is for a radial-edge crack problem analysed in the present study. The computed value of K_I here is 2.744, which is about 0.94% lower than that obtained by Bowie and Freese [1972]. From this figure, it is clear that the K_I^* values for points very close to the crack tip has to be neglected in the extrapolation procedure because of the error introduced in the numerical modelling of the crack-tip singularity, unless the latter is implicitly modelled.

~~Instead of using the crack-face displacement~~

results, the stresses in the vicinity and ahead of the crack-tip may also be used for K_I determination. By letting $\theta=0$ in Eqn. (2.1), the direct stress component in the direction perpendicular to the crack-plane may be written as:

$$\sigma_{yy} = K_I / (\sqrt{2\pi r}) \quad (4.5)$$

Rewriting Eqn. (4.5) and remembering that K_I is defined for $r=0$,

$$K_I = \lim_{r \rightarrow 0} \sigma_{yy} \sqrt{2\pi r} \quad (4.6)$$

As in the displacement extrapolation method described earlier; Eqn. (4.6) is determined at a series of nodal points along the crack plane from the crack tip. These results are then used to obtain a plot as shown in Figure 4.2. The K_I values is then obtained by extrapolating the tangent of constant slope portion of the curve to $r=0$. The K_I value determined here is 2.483, which is about 10.4% lower than that obtained by Bowie and Freese [1972].

The displacement extrapolation method has generally been found to be more accurate than the stress extrapolation method in evaluating K_I . The reason for this is that in the BIE method, as in the FE method, displacements are the primary variables, whereas the stresses are the secondary results. The latter are hence, relatively

less accurate than the displacements.

4.2. Energy method to obtain K_I

The energy method (or indirect method) for the determination of K_I in three-dimensional BIE fracture analysis was first proposed by Cruse and Meyers [1977].

In this method, the crack-face nodal displacements, and the strain energy release rate G due to an infinitesimal crack increment are employed. For a cracked body, the strain energy release rate G is given by

$$G = \frac{\Delta U}{\Delta A} \quad (4.7)$$

where ΔU is the change in strain energy due to an increase of crack face area, ΔA . To compute ΔU , two BIE analyses would be required; these two analyses are identical in every respect except for a small perturbation of the crack size. The relationship between G and the weighted root-mean-square value of K_I , denoted by $\bar{K}_I$, is expressed as:

$$\bar{K}_I^2 = GH = \frac{1}{\Delta A} \int_{\Delta A} K_I^2(s) \delta l(s) ds \quad (4.8)$$

where δl is the distance between the initial and final perturbed crack fronts and s is the linear distance along the crack front, as illustrated in Figure 4.3.

The variation of K_I along the crack front $K_I(s)$

can be shown (see, e.g., Cruse and Meyers [1977]) to be given by

$$\frac{K_I(s)}{K_{I_{ref}}} = \frac{v(s)/v_{ref}}{\sqrt{r(s)/r_{ref}}} = \frac{v^*(s)}{\sqrt{r^*(s)}} \quad (4.9)$$

where $K_{I_{ref}}$ is the stress intensity factor at a reference point whose displacements and distance from the crack front are v_{ref} and r_{ref} respectively; $v(s)$ and $r(s)$ are the displacement and distance from the crack front for the particular point of interest.

Substituting Eqn. (4.9) into Eqn. (4.8);

$$GH = \frac{1}{\Delta A} \int_{\Delta A} (K_{I_{ref}})^2 [v^*(s)/\sqrt{r^*(s)}]^2 \delta l(s) ds \quad (4.10)$$

Rearranging Eqn. (4.10) becomes

$$K_{I_{ref}} = \left\{ \frac{1}{\Delta A} \int_{\Delta A} [v^*(s)/\sqrt{r^*(s)}]^2 \frac{\delta l(s)}{GH} ds \right\}^{-1/2} \quad (4.11)$$

Hence, $K_I(s)$ along the whole crack front can be computed from Eqn. (4.9) once $K_{I_{ref}}$ is known at a reference point.

The energy method is generally regarded to be more accurate than the displacement or stress extrapolation methods in evaluating K_I . However, the necessity to carry out two completely separate 3-D BIE analyses would make the energy method less economical even though a less refined mesh may now be used. But techniques have been developed to evaluate G without the need to perform two completely

separate numerical analyses (see, e.g. Tan and Fenner [1979])... Using these techniques, the second solution is obtained without re-establishing all the coefficients and reducing all the equations again. This is because only relatively few coefficients are changed when the crack front is extended by a small amount.

In two-dimensional fracture analysis, the computation of the strain energy release rate, G , using Eqn. (4.7) is relatively straight-forward. Although still requiring two analyses to obtain ΔU , the cost involved is generally very small. The stress intensity factor K_I can then be easily determined from Eqn. (2.22):

$$K_I = \sqrt{GH} \quad (4.12)$$

For the same radial-edge crack problem (i.e. $R_2=2.0$, $R_1=1.0$, $a/(R_2-R_1) = 0.3$ and $P=1.0$) analysed in the present study, K_I computed using the energy method is 2.757, which is only above 0.47% lower than that obtained by Bowie and Freese [1972].

4.2.1 Determination of K_I by Weight Function method

Many K_I solutions derived in the literature are only applicable to the loading conditions for which they were obtained. When a different load situation occurs, even though the cracked geometry remains the same, a new analysis would have to be performed to obtain the correct K_I values. An economical way of obtaining stress intensity factors for cracks under complex stress and geometry combinations is the use of the weight function method (also commonly known as the influence or Green's function method).

The basis of the weight function method in LEFM is the linear superposition principle. This is illustrated in Figure 4.4. The application of the linear superposition method essentially reduces the K_I solution of an arbitrary crack problem to the sum of sub-problem (1) the stress analysis of the uncracked component, and sub-problem (2) the crack component where only the crack face is subjected to loads. The loads acting on the crack face in sub-problem (2) are equal and opposite to the stresses ($\sigma(r_s)$ in Figure 4.4) that would otherwise exist across the prospective crack face in the absence of the crack. It can be shown (see, e.g., Bueckner [1970] and Rice [1972]) that for a

linear elastic solid containing a crack, if the crack-face displacements, $v(a, x, y)$, and the stress intensity factor, K_I^* , are known for a reference symmetrical loading system, then the stress intensity factor for any other symmetrical loading system on the same body may be obtained from:

$$K_I = \int_A \sigma(x, y) h(a, x, y) dA \quad (4.13)$$

In Eqn. (4.13) $h(a, x, y)$ is the weight function defined as:

$$h(a, x, y) = \frac{H}{K_I^*} \frac{\partial v(a, x, y)}{\partial a} \quad (4.14)$$

where $\partial v(a, x, y) / \partial a$ is the crack-face displacement derivative with respect to the crack extension under the known reference loading system. It can be seen from this equation that the reference problem provides sufficient data to enable the weight functions for the body to be evaluated. Thus the task of calculating K_I is reduced to evaluating the integral in Eqn. (4.13), once $h(a, x, y)$ and the uncracked stress field, $\sigma(x, y)$, in the prospective crack face are known.

The weight function $h(a, x, y)$ as defined in Eqn. (4.14) can be written to be valid for all points within the elastic solid body. However, for the purpose of evaluating the stress intensity factors at the crack front, only the weight functions along the prospective crack face are of

primary interest in the present work. In a two-dimensional linear elastic solid containing a crack of crack-length a , the crack face nodal weight function becomes;

$$h(a, r_s) = \frac{H}{K_I^*} \frac{\partial v(a, r_s)}{\partial a} \quad (4.15)$$

where r_s is the distance from the crack tip. The stress intensity factor is then obtained from:

$$K_I = \int_0^a \sigma(r_s) h(a, r_s) dr_s \quad (4.16)$$

In order to determine the weight function, $h(a, r_s)$, as given in Eqn. (4.15) by numerical means; consider a plane crack of crack-length a , under a known reference symmetrical loading system. The stress intensity factor K_I^* can be expressed in term of the strain energy U as

$$K_I^* = \left(H \frac{\partial U}{\partial a} \right)^{\frac{1}{2}} \quad (4.17)$$

Assuming unit thickness, the area of the crack is equal to a , and Eqn. (4.17) becomes:

$$K_I^* = \left(H \frac{\partial U}{\partial a} \right)^{\frac{1}{2}} \quad (4.18)$$

Substituting Eqn. (4.18) into Eqn. (4.15), the weight function reduces to

$$h(a, r_s) = \frac{1}{H} \left(\frac{\partial U}{\partial a} \right)^{-\frac{1}{2}} \frac{\partial v(a, r_s)}{\partial a} \quad (4.19)$$

To evaluate $h(a, r_s)$ using the incremental-difference method, two separate BIE elastic stress analyses of the cracked geometry would be required. As discussed in the previous section, these analyses would be identical in all respects except for a small perturbation of the crack-tip position, Δa . The corresponding incremental changes in strain energy, ΔU , and crack-face displacements at the nodal points, Δv , can also be obtained from the numerical analyses. These results would then be employed in the incremental form of Eqn. (4.19), i.e.

$$h(a, r_s) = \left(-\frac{1}{H} \frac{\Delta U}{\Delta a} \right)^{-\frac{1}{2}} \frac{\Delta v}{\Delta a} \quad (4.20)$$

Having obtained the discrete nodal weight function from Eqn. (4.20), they can be fitted into a mathematical function or polynomial to facilitate the numerical integration to obtain K_I solutions using Eqn. (4.16).

CHAPTER 5

• FRACTURE ANALYSIS OF THICK-WALLED CYLINDER WITH A SINGLE RADIAL-EDGE CRACK

Failure by fracture of components such as chemical reactor tubes, isostatic compaction chambers and gun barrels are likely to be catastrophic. It is therefore of practical importance that designers understand the fatigue and fracture behaviour of internally pressurized thick-walled cylinders. However, the stress analyses of cracked thick-walled cylinders are often complicated not only by the presence of internal pressure loads, but also by the presence of other stress fields such as thermal gradients and/or autofrettaged residual stresses. Autofrettage is a process commonly used to produce the favorable residual stresses in a thick-walled cylinder which enables the cylinder to increase its load carrying capacity and its fatigue life. In this chapter, the effects of autofrettaged residual stresses on stress intensity, K_I , solutions for single radial-edge cracks in thick-walled cylinders will be determined using the weight function method. Both elastic-perfectly plastic and strain-

hardening materials will be considered.

5.1 Review of Previous Work

Most cracks occurring in pressurized thick-walled cylinders are usually not straight-fronted; the problem is thus three dimensional in nature. However, as in many engineering problems, the two-dimensional assumption is often made to simplify the analysis and to provide upper-bound estimates of the solution.

Two-dimensional numerical K_I solutions are already available in the literature for a wide range of radial-edge cracks in internally pressurized thick-walled cylinder (see, e.g., Bowie and Freese [1972]). Emery and Segedin [1972] presented K_I solutions for external cracks in cylinders subjected to various loading conditions by employing a technique very similar to the finite difference approximations. K_I solutions for inner edge cracks in cylinders under different mechanical loading conditions were obtained by Labbens et al using the weight function method; the BIE method was employed in the computation of the weight function.

Parker and Farrow [1981] obtained K_I solutions for multiple radial edge cracks in an autofrettaged or

thermally stressed thick cylinder. However, these results are only approximations and their accuracies are questionable.

It is evident from the above review that the edge-cracked thick-walled cylinder problem has received a significant amount of attention; a fact attributed to its importance in engineering practice. However, it is surprising to note that all the stress intensity solutions available in the literature are restricted to non-autofrettaged internally pressurized cylinder. The effects of autofrettaged and thermal stresses on K_I have never previously been investigated in any great detail except for the approximate solutions by Parker and Farrow [1981]. These effects will now be considered here in this chapter.

5.2 Present Numerical Investigation

In order to obtain the stress intensity factors for an autofrettaged thick-walled cylinder containing a single radial-edge crack (Figure 5.1), it is assumed that the compressive residual stress field at the bore is developed first before the crack appears. Applying the principle of linear superposition, K_I for this problem thus becomes the sum of the solutions for the pressurized,

non-autofrettaged cylinder, $(K_I)_p$, and the solution for the unpressurized autofrettaged cylinder, $(K_I)_A$ (see Figure 5.2).

The weight functions for a cylinder containing a single radial-edge crack were determined by employing the two-dimensional BIE elastic stress analyses on the cracked component. Cylinder radius ratios $K = R_2/R_1 = 1.5, 2.0$ and 2.5 were considered, and for each of these ratios, relative crack-depth $a/(R_2 - R_1)$, ranging from 0.1 to 0.7 were treated. Poisson's ratio, ν , was taken to be 0.3 and plane strain end conditions was assumed throughout.

To determine the effects of the autofrettage residual stresses on the K_I solutions, numerical values of $(K_I)_A$ were computed for different degrees of overstrain using the weight function technique described in the last chapter. For use in these analyses, the respective residual stress distribution after each autofrettaged process was obtained by carrying out BIE elasto-plastic stress analysis on an uncracked cylinder. The material properties chosen for the present two-dimensional study correspond to those of an elastic-perfectly plastic material, Type 304 and A533B steels.

5.3 Determination of the Weight Functions, $h(a, r_s)$, using the BIE method

To determine the weight functions, $h(a, r_s)$, for a thick-walled cylinder containing a single radial-edge crack, the computational two-dimensional elastic BIE method was employed first to evaluate the nodal displacements $v(a, r_s)$. From these nodal displacement results, $h(a, r_s)$ at the nodal points were computed using the incremental-difference technique as described in the previous chapter.

A typical BIE mesh used for the evaluation of $h(a, r_s)$ is shown in Figure 5.3, where it can be seen that only one-half of the physical problem was modelled by virtue of the diametral plane of symmetry. The appropriate boundary conditions are as shown in Fig. 5.4. The accuracy of the nodal weight functions was enhanced by the introduction of singular boundary elements at the vicinity of the crack tip, where mid-point nodes adjacent to the crack-tip were shifted to quarter-points to introduce the $r^{-1/2}$ stress singularity there.

The requirement for two separate BIE elastic stress analyses for each crack geometry is not, in practical terms, a serious disadvantage. The additional effort required is rather insignificant; this arising from the fact that in

two-dimensional elastic BIE analysis, only the boundary of the numerical solutions domain is modelled. Further, the energy method is generally accepted to provide the most accurate K_I solutions.

5.3.1 Functional Representation of Nodal Weight Functions, $h(a, r_s)$

To facilitate the numerical integration to obtain K_I solutions using Eqn. (4.16), the discrete nodal weight functions obtained from the two-dimensional BIE elastic stress analyses could be expressed as mathematical functions or simple polynomials. Much effort (see, e.g., Besuner [1977], Labbens et al [1976], Petroski and Achenbach [1978] and Sha [1984]) have been devoted in recent years to finding suitable functions to represent the crack-face displacements, $v(a, r_s)$, in the reference problem from the discrete nodal values computed in the numerical stress analyses. The difficulty lies mainly in curve fitting of the nodal weight functions in the vicinity of the crack-tip, due to their singularity there. In the present work, the weight functions for a single radial-edge crack in a thick-walled cylinder are assumed to vary according to Eqn. (5.1) below:

$$h(a^*, r^*)\sqrt{W} = \frac{A(a^*)}{\sqrt{r^*}} + B(a^*) + C(a^*)\sqrt{r^*} + D(a^*)r^* \quad (5.1)$$

where $a^* = a/W$; $W = R_2 - R_1$ and $r^* = r_s/a^*$. The quantities a^* and r^* are dimensionless geometric variables being used here for generality; r_s being the distance from the crack-tip. Also A , B , C and D are constant coefficients for a given crack geometry. The reason for choosing the form in Eqn. (5.1) is because classical solutions have shown that the variation of the displacements in the vicinity of the crack-tip can be described in terms of a power series about the square root of the distance from the crack-tip. Further, it can be seen from Eqn. (4.15) that the weight functions are proportional to the first derivative of the displacement. Although more terms could be added to Eqn. (5.1), it was deemed unnecessary, as was later verified numerically. This was also reported by Sha [1984] who used a similar expression for the variation of weight functions in his finite element study.

Plots of some typical crack-face nodal weight functions obtained from the BIE elastic stress analyses for three relative crack lengths are presented in Figure 5.5. Note that the singularity nature of these variations at the vicinity of the crack-tip. The numerical values of constant coefficients A , B , C and D in Eqn. (5.1) are obtained

using least-squares curve fitting, and they are presented in Tables (5.1) to (5.3) for all the cylinder radius ratios and relative crack-length treated.

5.4 Determination of the Autofrettaged Residual

Stress Distributions

In order to evaluate the stress intensity factors for autofrettaged cracked thick-walled cylinder, $(K_I)_A$, using the weight function variations obtained in the earlier section; the magnitude of the autofrettaged compressive residual stresses must first be determined. Analytical (i.e. closed form) solutions for this elasto-plastic residual stresses are known only for non-strain hardening materials i.e. elasto-perfectly plastic material (see, e.g., Hill [1950]). However, for power-law strain hardening materials which obey the Prandtl-Reuss incremental stress-strain relations, such as Type 304 and A533B steels, computational methods must be used to determine the elasto-plastic residual stress distributions (see, e.g., Chu [1972] and Owen [1980]).

5.4.1 Elasto-Plastic Residual Stress Distributions for Cylinders made of Elastic-Perfectly Plastic Material

Assuming elastic-perfectly plastic material which obeys the Von Mises yielding criterion, the elasto-plastic residual hoop stress distributions in a partially yielded cylinder is given (see, e.g., Hill [1950]) by Eqns. (5.2) and (5.3) for plane strain end conditions:

$$(\sigma_{\theta})_p = \frac{2}{\sqrt{3}} \sigma_y \left\{ \frac{1}{k^2-1} \left(1 + \frac{R_2^2}{r^2} \right) \left(\frac{n^2-k^2}{2k^2} - \log_e n \right) + \left(\frac{n^2-k^2}{2k^2} - \log_e \frac{nR_1}{r} \right) \right\} \quad (5.2)$$

$$(\sigma_{\theta})_e = \frac{2}{\sqrt{3}} \sigma_y \left(1 + \frac{R_2^2}{r^2} \right) \left\{ \frac{n^2}{2k^2} + \frac{1}{k^2-1} \left(\frac{n^2-k^2}{2k^2} - \log_e n \right) \right\} \quad (5.3)$$

where σ_y is the yield stress of the material; nR_1 is the radius of elastic-plastic interface and k is the radius ratio (R_2/R_1). In Eqns. (5.2) and (5.3) linear elastic unloading was assumed when the autofrettage pressure was removed. In the present study, autofrettage conditions of up to 70% overstrain for cylinders of radius ratios $k=1.5$ and 2.0 were considered. For cylinders of $k=2.5$, up to 60% overstrain were considered. The percentage overstrain referred to here is the percentage of the cylinder wall

which has been plastically deformed during the autofrettage process. Overstrain conditions exceeding 60% were not considered for cylinders with $k=2.5$ because reverse yielding would occur if plastic deformation had developed to beyond that extent in the cylinder during the autofrettage process. Eqn. (5.2) would then be rendered invalid and computational elasto-plastic stress analysis must be carried out to determine the residual stress distribution in the cylinder. It should be noted here that reverse yielding would indeed occur if the cylinder of $k=2.5$ had been autofrettaged to 60% overstrain. However, it is restricted to only a very small region at the bore of the cylinder so that errors introduced to the results are unlikely to be very significant.

5.4.2 Residual Stress Distributions for Cylinders made of Type 304 and A533B Steels

The numerical BIE method for two-dimensional elasto-plasticity was employed to determine the autofrettaged residual stresses in uncracked thick-walled cylinders made of Type 304 and A533B steels. Detailed discussion on the use of the computer program employed is beyond the scope of the present work and will not be presented here.

A typical BIE mesh, used for the determination

of the elasto-plastic stresses is shown in Figure 5.6. As shown in the figure, only a 20° segment of the physical problem was modelled as the radial planes of symmetry was taken advantage to reduce the numerical size of the problem. The appropriate boundary conditions applied are as shown in Figure 5.7. In the elastic-plastic stress analysis, the stress-strain (σ - ϵ) relationship for Type 304 steel was taken to be;

$$\epsilon = \begin{cases} \sigma/E & (\sigma \leq \sigma_y) \\ \sigma/E + (\sigma/672.1)^{5.42} & (\sigma > \sigma_y) \end{cases} \quad (5.5)$$

where $E=207$ GPa and $\sigma_y=207$ mPa.

While for A533B steel, it was taken to be;

$$\epsilon = \begin{cases} \sigma/E & (\sigma \leq \sigma_y) \\ \sigma/E + (\sigma/827.3)^{10} & (\sigma > \sigma_y) \end{cases} \quad (5.6)$$

where $E=214.8$ GPa and $\sigma_y=275$ mPa.

The cylinder was assumed to be sufficiently long for plane strain end conditions to apply; and Von Mises yield criterion was assumed throughout.

Typical computed elasto-plastic hoop stress distributions in an uncracked, pressurized cylinder made of Type 304 steel and of radius ratio $k=2.0$ are presented in Figures 5.8 to 5.12, while for A533B steel, the computed elasto-plastic hoop stress distributions are shown in Figures

5.13 to 5.16. For convenience and ease of computing $(K_I)_A$ solutions using Eqn. (4.16), the elasto-plastic hoop stress distributions in the yielded region were least-squares curve-fitted into second-degree polynomials. The residual stress distributions in the yielded region are thus obtained from Eqn. (5.7a) below:

$$(\sigma_\theta)_p = (a_1 + b_1 r + c_1 r^2) - \frac{P}{k^2 - 1} \left(1 + \frac{R_2^2}{r^2}\right); \quad R_1 \leq r \leq nR_1 \quad (5.7a)$$

where P is the autofrettage pressure for plastic deformation up to nR_1 obtained from the elastic-plastic BIE analysis; and the constant coefficients a_1 , b_1 and c_1 are obtained from the least-squares curve-fit. In the unyielded

region, the residual stress distributions are approximated similarly by Eqn. (5.7b) below:

$$(\sigma_\theta)_e = (a_2 + b_2 r + c_2 r^2) - \frac{P}{k^2 - 1} \left(1 + \frac{R_2^2}{r^2}\right); \quad nR_1 \leq r \leq R_2 \quad (5.7b)$$

where a_2 , b_2 and c_2 are obtained from the least-squares curve-fit.

Similar to the elasto-perfectly plastic case, the percentage overstrain condition of up to 70% were considered for cylinders of radius ratios $k=1.5$ and 2.0 . However, for cylinder with $k=2.5$; it was found from the BIE elasto-plastic stress analysis that reverse yielding would occur if the cylinder has been autofrettaged to over 65%. Therefore, only overstrain conditions of up to 65% were considered for

cylinder with $k=2.5$.

5.5 Results and Discussion

5.5.1 K_I Solutions for Internally Pressurized, Cracked, Non-Autofrettaged Thick-Walled Cylinders.

In order to gain confidence and obtain an assessment of the accuracy of the weight functions obtained from the BIE elastic stress analyses, K_I solutions for internally pressurized, non-autofrettaged thick-walled cylinder containing a single radial-edge crack were evaluated. Cylinder radius ratios and relative crack-depth considered were defined as in section 5.2.

The K_I solutions were computed using Eqn. (4.16), i.e.

$$K_I = \int_0^a h(a, r_s) (P + \sigma_\theta(r_s)) dr_s \quad (5.8)$$

where $h(a, r_s)$ are the weight functions with constant coefficients A, B, C and D presented in Tables 5.1 to 5.3; P is the internally applied pressure and $\sigma_\theta(r_s)$ is the well known Lamé's hoop stress distribution given by Eqn. (5.9) below:

$$\sigma_\theta(r) = \frac{P}{k^2-1} \left(1 + \frac{R_2^2}{r^2} \right) \quad (5.9a)$$

or

$$\sigma_\theta(r_s) = \frac{P}{k^2-1} \left(1 + \frac{R_2^2}{(R_1+a-r_s)^2} \right) \quad (5.9b)$$

K_I solutions were obtained by evaluating the integral given in Eqn. (5.8) using the Gaussian integration scheme. The results presented in Figure 5.17. It can be seen from the figure that the computed K_I solutions and published K_I results (Bowie and Freese [1972]) are in excellent agreement indeed.

5.5.2 $(K_I)_A$ Solutions for Cracked, Autofrettaged Thick-Walled Cylinders

In the most general case of a cracked, autofrettaged thick-walled cylinder, the radial-edge crack may have grown into the unyielded region of the cylinder, so that two different residual hoop stress distributions may act on the same crack face as illustrated in Figure 5.18. From Eqn. (4.16), the residual stress intensity factors can be obtained, thus:

$$(K_I)_A = \int_0^{nR_1} h(a, r_s) (\sigma_\theta(r_s))_p dr_s + \int_{nR_1}^a h(a, r_s) \sigma_\theta(r_s) dr_s \quad \dots\dots\dots(5.10)$$

For the case where the crack lies within the yielded region of the cylinder, $(K_I)_A$ are determined from:

$$(K_I)_A = \int_0^a h(a, r_s) (\sigma_\theta(r_s))_p dr_s \quad (5.11)$$

The $(K_I)_A$ solutions for all the crack geometries and overstrain conditions treated were obtained by evaluating either Eqn.(5.10) or Eqn. (5.11) using Gaussian quadrature method. The normalized residual stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi a}\}$, for elastic-perfectly plastic material are listed in Tables 5.4 to 5.6; for Type 304 and A533B steels, they are presented in Figures 5.19 to 5.24.

It should perhaps be noted here that the qualitative features of the elasto-plastic hoop stress distributions for cylinders made of Type 304 and A533B steels presented in Figures 5.8 to 5.16 are as expected. However, it was found from the BIE elasto-plastic analyses that for a given radius ratio, and applied pressure, the computed normalized elasto-plastic hoop stresses, σ_θ / σ_y , for Type 304 and A533B steels were within less than 5% in value from each other. For practical purposes, the normalized residual stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi a}\}$, for cracked cylinders of the same geometry made of these two steels may therefore be taken as being equal in value.

It should be noted that all computed $(K_I)_A$ solutions are negative in values. A negative $(K_I)_A$ implies that the crack is being suppressed, i.e. the crack is closing, and fracture as a result of the residual stresses alone could not take place. However, the physical significance of

negative $(K_I)_A$ values become clear when the cracked, autofrettaged thick-walled cylinder is pressurized internally. In this case, the value of K_I is simply the algebraic sum of those for each of the independent loading cases as discussed in section 5.2, i.e.

$$K_I = (K_I)_A + (K_I)_p \quad (5.12)$$

Therefore, the net effect of a negative $(K_I)_A$ value is the reduction of the total value of the stress intensity factor.

The numerical accuracy of the $(K_I)_A$ solutions for cracked, autofrettaged cylinders is difficult to establish due to the fact that no previous work has been performed. However, the excellent agreement of the computed K_I solutions for internally pressurized, cracked and non-autofrettaged cylinders with known published analytical results indicate that the computed $(K_I)_A$ solutions should be within 3% of the true values.

5.6 Concluding Remark

Complex stress fields are often present in many engineering fracture problems, such as the autofrettaged residual stresses in the cracked thick-walled cylinder. Instead of carrying out costly, repeated and complete

numerical stress analyses, the weight function method reduces the problem to the determination of:

- (1) the weight functions, $h(a, \text{geometry})$, of the crack geometry, and
- (2) the stress distributions in the uncracked component

It is evident from the present study that the weight function method used in conjunction with the BIE method is a useful and economical tool for solving engineering fracture problems.

CHAPTER 6

FRACTURE ANALYSIS OF THICK-WALLED CYLINDER WITH AN INTERNAL SURFACE CRACK

Internal surface cracks which occur in pressurized thick-walled cylinders are rarely straight-fronted. Instead, they are usually part-through the cylinders wall thickness and are usually very well represented by semi-ellipses. To obtain accurate stress intensity factor, K_I , solutions for surface cracks in thick-walled cylinder, three-dimensional fracture analyses are required. In this chapter, the numerical three-dimensional BIE method will be employed to obtain K_I solutions for a wide range of semi-elliptical surface cracks in cylinders with radius ratios $K = 2.0$ and 2.5 . Several load conditions are considered; they include internal pressure as well as the situation when the cylinders are subject to thermal gradients and autofrettaged residual stress distributions.

6.1 Review of Previous Work

Recent advances in the application of computational techniques, such as the finite element method and the boundary integral equation method, in fracture analyses have made it possible to obtain accurate stress intensity factors for three-dimensional crack problems. The problem of semi-elliptical surface cracks in pressurized cylinders has been investigated by several researchers (see, e.g., Blackburn and Hellen [1977], Heliot et al [1979], Newman and Raju [1982], and McGowan and Raymund [1979]) using either the finite element or the boundary integral equation method of analysis in their three-dimensional fracture studies. However, many of these numerical K_I results reported in the open literature are only applicable to cylinders with radius ratios less than 1.5. Pressure vessels in high pressure engineering applications, on the other hand, usually have radius ratios greater than this value.

Numerical K_I results for surface cracks in thick-walled cylinders with radius ratios, $k = R_2/R_1$, greater than 1.5 have also been reported in the open literature (see, e.g., Atluri and Kathiresan [1979, 1980], Tan and Fenner [1979, 1980], Nadiri et al [1982], Kirkhope et al [1984], and Navid et al [1985]). However, the loading condition treated

in all of these investigations is that of internal pressurization of non-autofrettaged cylinders, with the internal pressure also acting on the crack faces. This loading condition probably represents the most severe case in many practical applications. For example, if the cylinder is subjected to relatively high frequency cyclic loading with a viscous pressurizing medium, the pressure on the crack faces can be expected to be significantly lower than the applied pressure at the cylinder bore. Or, in the case of internal liner being present, there will not be any pressure loads acting on the crack faces at all. Beside the pressure loads, the cylinder can often, in practice, also be subjected to thermal gradients and/or autofrettaged residual stresses. Surprisingly, the effects of these other loads on the K_I solutions for surface cracks in cylinders with k exceeding 1.5 have not been previously reported in the open literature. Related works on such thick-walled cylinders pertain to only edge cracks and are based on two-dimensional analysis (see, e.g., Pu and Hussain [1981] and Parker and Farrow [1981]). The effects of thermal gradients and autofrettaged residual stresses on the K_I solutions for surface cracks in cylinders with k greater than 1.5 will now be investigated in this chapter.

6.2 Present Numerical Investigation

The semi-elliptical surface crack in a plane through the axis of a thick-walled cylinder is shown in Figure 6.1. The lengths of the semi-major and semi-minor axes of the semi-ellipse representing the crack shape are defined by a and b respectively, while $W = (R_2 - R_1)$ is the cylinder wall thickness. In the present work, relative crack-depths b/W , ranging from 0.2 to 0.8 in thick-walled cylinders with radius ratios $k = R_2/R_1$ of 2.0 and 2.5 were analysed. However, only one crack shape, namely, $b/a = 0.8$, was considered throughout. This particular crack geometry was chosen because it is one commonly observed in high pressure Ni - Cr - Mo steel cylinders with k between 2.0 and 3.0 (see, e.g., Nishioka and Hidakawa [1977] and Crofton [1980]).

The numerical BIE method for three-dimensional elastostatics will be employed to obtain "Polynomial Influence Coefficients" or "crack-face pressure" mode I stress intensity factor solutions for all the crack geometries considered. More specifically, K_I solutions for the surface cracks with four different stress distributions, namely, uniform, linear, quadratic and cubic pressure variations, applied to the crack faces will be determined. Using the principle of linear superposition, these influence coefficients

from the four "crack-face pressure" solutions can be superposed to obtain stress intensity factor solutions for the cracked cylinder subjected to the following loading conditions:-

- (i) internal pressure: for both the case when the internal pressure is acting on the crack face and the case when it does not;
- (ii) autofrettaged residual stresses; and
- (iii) thermal loads in the form of steady-state radial temperature gradients.

6.3 Polynomial Influence Functions

The application of linear superposition principle to determine K_I solutions in linear elastic fracture mechanics has been discussed in detail in section 4.2.1 (see also Figure 4.4). The use of linear superposition method reduces the K_I solution of an arbitrary crack problem to the sum of sub-problem (1) the K_I solution for the crack-free problem, which is subjected to the same external loads, and sub-problem (2) the crack problem where only the crack face is subjected to loads. Thus,

$$K_I = K_I' + K_I'' \quad (6.1)$$

The loads acting on the crack face in sub-problem (2) are equal and opposite to the stresses that would otherwise

exist across the prospective crack face in the absence of the crack. Since $K_I' = 0$, the stress intensity factor for the original problem is essentially identical to that for the problem with the crack faces loaded according to the uncracked stress distribution, i.e.,

$$K_I = K_I'' \quad (6.2)$$

Suppose that the uncracked stress distribution across the prospective crack face due to arbitrary, external loads can be represented by a third-degree polynomial:

$$\sigma(r_s) = A_0 + A_1 r_s + A_2 r_s^2 + A_3 r_s^3 \quad (6.3)$$

where r_s is the non-dimensionalized distance measured from a reference origin and A_i ($i = 0, 3$) are the coefficients of the polynomial function which have units of stress. It is then possible to obtain general K_I solutions for the cracked problem in terms of polynomial influence coefficients $K_I^{(i)}$. These influence coefficients are the stress intensity factors, $K_I^{(0)}$, $K_I^{(1)}$, $K_I^{(2)}$, and $K_I^{(3)}$, respectively, for the problem with the crack faces subjected to the following direct stress distributions:

uniform ($\sigma = -A_0$), linear ($\sigma = -A_1 r_s$), quadratic ($\sigma = -A_2 r_s^2$), and cubic ($\sigma = -A_3 r_s^3$). The K_I solution for the cracked component subjected to the same arbitrary

external loads will be:

$$K_I = K_I^{(0)} + K_I^{(1)} + K_I^{(2)} + K_I^{(3)} \quad (6.4)$$

For a plane, uncracked thick-walled cylinder, the hoop stress distributions due to internal pressure, thermal gradient or autofrettage, are well known (see, e.g., Hill [1950]). These stress distributions can generally be very well fitted into a third-degree polynomial as given by Eqn. (6.3), with r_s conveniently defined as $(r - R_1)/(R_2 - R_1)$, r being the radial distance from the cylinder axis. Thus, if the influence coefficients $K_I^{(i)}$ are known for a given cracked cylinder geometry, the stress intensity factors for the cracked component subjected to the above-mentioned, or other, loading conditions can be determined with very little computational effort.

6.3.1 Polynomial Influence Coefficients

To obtain the polynomial influence coefficients or the crack-face pressure mode I stress intensity factor solutions, the numerical BIE method for three-dimensional elastostatics was employed. The computer program "BIE3D4" was used. The numerical strategy used in this computer program was based on the quadratic isoparametric formulation, whereby eight-noded quadrilateral and/or six-noded triangular isoparametric elements were used to model the

surface of the three-dimensional region of interest.

Figure 6.2 presents a typical BIE mesh used in the present work. Taking advantage of symmetry about both the plane of the crack and the cylinder cross-section containing its semi-minor axis, only one-quarter of the cylinder was modelled. The element configuration was varied to suit the different radius ratios and relative crack depths with up to 289 nodal points and 97 boundary elements (eight-node quadrilateral and six-node triangular elements) being used. Relatively small elements were employed in the vicinity of the crack front and mid-side nodes of elements immediately adjacent to the crack periphery were shifted to the quarter points nearest the front, in order to introduce the $r^{-1/2}$ stress singularity there. For each cylinder radius ratio, the length of the quarter-cylinder modelled was taken to be three times the wall thickness. Plane strain conditions were applied at the end of the cylinder remote from the crack and Poisson's ratio ν was taken to be 0.3 throughout. As the computer program BIE3D4 can handle multiple load cases, each computer run could treat simultaneously load cases corresponding to the uniform, linear, quadratic and cubic pressure variations on the crack face for each crack geometry considered. Values of $K_I^{(i)}$ were obtained from the crack face displace-

ment data using the displacement extrapolation method described in chapter 4. Plane strain conditions were assumed at all points along the crack front except at the internal surface of the cylinder, where plane stress was assumed.

The normalized influence coefficients, $K_I^{(i)} / \{A_i (b/w)^{1/2} \sqrt{\pi b}\}$, ($i = 0, 3$), corresponding to the four crack-face pressure loads (i.e. uniform ($\sigma = -A_0$), linear ($\sigma = -A_1 r_s$), quadratic ($\sigma = -A_2 r_s^2$) and cubic ($\sigma = -A_3 r_s^3$)) are listed in Tables 6.1 to 6.8 for the various crack geometries treated. The angle ϕ defines the position of points along the crack front, as shown in Figure 6.1; $\phi = 0^\circ$ corresponds to the point of maximum crack penetration while $\phi = 90^\circ$ corresponds to the internal surface of the cylinder. Tables 6.1 to 6.4 list the results for relative crack depths, $b/w = 0.2, 0.4, 0.6$ and 0.8 respectively, in a cylinder with radius ratio 2.0, while similar results for a cylinder with $k = 2.5$ are shown in Tables 6.5 to 6.8. Typical plots of these normalized influence coefficients for all four relative crack depths in a cylinder with $k = 2.0$ are also presented in Figures 6.3 to 6.6. It should perhaps be remarked here that the choice to include the term $(b/w)^{1/2}$ in the normalizing factor is a deliberate one. This is done so as to retain consistency in the number of significant figures for

the numerical values being presented. Thus, accuracy will not be sacrificed when they are used in the determination of the K_I solutions for practical load cases.

6.4 Results and Discussion

6.4.1 K_I solutions for Cracked, Non-autofrettaged Cylinder subjected to Internal Pressure Loads

In order to utilize the influence coefficients to obtain K_I solutions for cracked, non-autofrettaged cylinder subjected to internal pressure loads, these pressure loads must first be expressed as third-degree polynomial in the form of Eqn. (6.3). The hoop stress, σ_θ , distribution in an uncracked, non-autofrettaged, internally pressurized thick-walled cylinder is given by the well known Lamé's solution. It can be fitted into the form of Eqn. (6.3) by using the least-squares curve-fitting technique. For the cylinder with $k = 2.0$, the third-degree polynomial:

$$\sigma_\theta = P (1.6628 - 2.5018 r_s + 2.6718 r_s^2 - 1.2347 r_s^3) \quad \dots(6.5)$$

where P is the applied internal pressure, agrees to within 1% of the exact (i.e. Lamé's) solution for radial distance of up to $r_s = (r - R_1) / (R_2 - R_1) = 0.8$. For the cylinder

with $k = 2.5$, the corresponding expression, determined also to within 1% of the exact solution over the same relative radial depth, is:

$$\sigma_{\theta} = P (1.3702 - 3.0401 r_s + 3.7635 r_s^2 - 1.7545 r_s^3) \quad \dots(6.6)$$

To demonstrate how the stress intensity factors for the cracked, non-autofrettaged, internally pressurized cylinder are obtained using the influence coefficients; consider the cylinder with $k = 2.0$, containing the surface elliptical crack of relative depth $b/w = 0.2$. Suppose that the K_I at position $\phi = 0^\circ$ is to be determined. Using the influence coefficients listed in Table 6.1 and the third-order polynomial coefficients from Eqn. (6.5);

$$\begin{aligned} K_I^{(0)} &= 0.6700 A_0 \sqrt{\pi b} = 0.6700 (1.6628 P) \sqrt{\pi b} = 1.1141 P \sqrt{\pi b} \\ K_I^{(1)} &= 0.4582 A_1 \sqrt{\pi b} = 0.4582 (-2.5018 P) \sqrt{\pi b} = -0.2293 P \sqrt{\pi b} \\ K_I^{(2)} &= 0.3472 A_2 \sqrt{\pi b} = 0.3472 (2.6718 P) \sqrt{\pi b} = 0.0371 P \sqrt{\pi b} \\ K_I^{(3)} &= 0.2713 A_3 \sqrt{\pi b} = 0.2713 (-1.2347 P) \sqrt{\pi b} = -0.0027 P \sqrt{\pi b} \end{aligned}$$

The required K_I for the case when the internal pressure is not acting on the crack face is then obtained using Eqn.

(6.4):

$$K_I = \sum_{i=0}^3 K_I^{(i)} = [1.1141 - 0.2293 + 0.0371 - 0.0027] P\sqrt{\pi b}$$

$$= 0.9192 P\sqrt{\pi b}$$

In the case when the internal pressure also acts on the crack faces;

$$K_I = K_I^{(0)} + 0.9192 P\sqrt{\pi b} = 1.5892 P\sqrt{\pi b}$$

The set of results obtained by the above superposition procedure for the cracked, non-autofrettagged, internally pressurized thick-walled cylinder with $k = 2.0$, for the case when the pressure loads also acting on the crack faces, is presented in Table 6.9. Also presented in parentheses in Table 6.9 are corresponding results obtained by Tan and Fenner [1980] for the same problem. Tan and Fenner obtained their results directly using the numerical BIE method, instead of using the method of superposition employed in the present study. It can be seen from Table 6.9 that the two sets of results are in excellent agreement, with deviation of less than 2.5% at all, except two, of the points considered. These two points correspond to $\phi = 75^\circ$ and 90° when $b/w = 0.8$; even then the percentage deviations are less than 6%. The corresponding set of results for the cylinder with $k = 2.5$ is presented in Table 6.10. The sets of results in Tables 6.9 and 6.10 are also presented

graphically in Figures 6.7 to 6.10. As shown in these figures, the trends of the K_I variations around the respective crack points are very similar for both radius ratios.

The normalized stress intensity factors, $K_I/P\sqrt{\pi b}$, for the cracked, non-autofrettaged, internally pressurized cylinder for the case when the internal pressure is not acting on the crack faces are presented in Tables 6.11 to 6.12 for cylinders with $k = 2.0$ and 2.5 respectively. These sets of results are also presented graphically in Figures 6.11 to 6.14. Again, the trends of K_I variations around the respective crack points are very similar for both radius ratios. Comparing the results shown in Tables 6.9 and 6.10 with the corresponding values in Tables 6.11 to 6.12 respectively, it can be seen that the effect of the internal pressure acting on the crack faces in increasing the magnitudes of K_I is significant indeed. As to be expected, the increase in the K_I values, due to the pressure loads on the crack faces, becomes larger the greater the size of the crack. For the cylinder with $k = 2.0$, this increase in the magnitude of K_I can be as high as 103% and the corresponding increase for cylinder with $k = 2.5$ is 167%.

6.4.2 K_I solutions for Cracked, Autofrettaged Thick-Walled Cylinders

To obtain the stress intensity factors for a cracked, autofrettaged, internally pressurized cylinder, $(K_I)_{AP}$, it is assumed that the compressive autofrettaged residual stresses in the bore are developed first before the crack appears. $(K_I)_{AP}$ may be obtained from the sum of the solution for the pressurized, non-autofrettaged cylinder, say $(K_I)_p$, and the corresponding solution for the unpressurized autofrettaged cylinder, say $(K_I)_A$.

Numerical values of $(K_I)_p$ for the various crack geometries treated in the present work have been presented in the previous section; this section will present the determination of $(K_I)_A$ solutions.

Assuming elastic-perfectly-plastic material which obeys the Von Mises yielding criterion, the elastoplastic residual hoop stress distributions in a partially yielded cylinder for plane strain end conditions are given by Eqns. (5.2) and (5.3), they are:

$$(\sigma_\theta)_p = \frac{2}{\sqrt{3}} \sigma_y \left(\frac{1}{k^2 - 1} \left(1 + \frac{R_2^2}{r^2} \right) \left(\frac{n^2 - k^2}{2k^2} - \log_e n \right) + \left(\frac{n^2 + k^2}{2k^2} - \log_e \frac{nR_1}{r} \right) \right)$$

$$R_1 \leq r \leq nR_1 \quad (6.7)$$

$$(\sigma_{\theta})_e = \frac{2}{\sqrt{3}} \sigma_y \left(1 + \frac{R_2^2}{r^2}\right) \left\{ \frac{n^2}{2k^2} + \frac{1}{k^2 - 1} \left(\frac{n^2 - k^2}{2k^2} - \log_e n \right) \right\}$$

$$nR_1 \leq r \leq R_2 \quad (6.8)$$

where σ_y is the yield stress of the material; and nR_1 is the radius of elastic-plastic interface. In Eqns. (6.7) and (6.8) linear elastic unloading was assumed when the autofrettage pressure was removed. In the present study, autofrettage conditions ranging from 10% to 100% overstrain, at increments of 10% were considered for cylinder with $k = 2.0$. For cylinder with $k = 2.5$, the overstrain conditions considered were from 10% to 60% for reasons discussed earlier in Chapter 5.

In order to utilize the polynomial influence coefficients to evaluate $(K_I)_A$ for a given percentage overstrain, the autofrettaged residual hoop stresses as given by Eqns. (6.7) and (6.8) must be fitted into a third-degree polynomial in the form of Eqn. (6.3) so that the various polynomial constants, i.e. A_i , ($i = 0, 3$), can then be determined. When the size of the crack is such that the maximum crack-depth, b , lies within the plastic zone of the cylinder, i.e. b being less than nR_1 , it was found that, for both radius ratios considered, namely 2.0

and 2.5, the least-squares fitted third-degree polynomials obtained were indeed excellent representations of Eqn. (6.7). In the more general case when the crack has grown into the non-yielded region of the cylinder, i.e. b is greater than nR_1 , the crack faces will be subjected to the two different autofretted residual hoop stress distributions as given by Eqns. (6.7) and (6.8). This is illustrated schematically in Fig. 5.18. In this case, it is evident that a simple third-degree polynomial will not be adequate to represent the applied stress distributions across the crack faces, particularly near the elastic-plastic boundary. This was confirmed by least-squares curve fit computations. Therefore, the use of polynomial influence coefficients to evaluate $(K_I)_A$ for the cases when b is greater than nR_1 is precluded, and in the results which are presented below, the numerical values of $(K_I)_A$ for these cases have been obtained directly instead of by superposition procedures. This involves applying the appropriate tractions, as computed from Eqns. (6.7) and (6.8), on the crack faces in the numerical models.

The normalized stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi b}\}$, for cylinder with $k = 2.0$ are presented in Tables 6.13 to 6.16 for the crack sizes considered, namely $b/w = 0.2, 0.4, 0.6$ and 0.8 respectively. The corresponding

results for cylinder with $k = 2.5$ are presented in Tables 6.17 to 6.20. Some typical plots of $(K_I)_A / (\sigma_y \sqrt{\pi b})$ are presented in figures 6.15 to 6.20 for both radius ratios of 2.0 and 2.5 to present the variations of the stress intensity factors around the respective crack points under various overstrain conditions. From Tables 6.13 to 6.20, it can be seen that the numerical values of $(K_I)_A$ are generally negative for crack sizes with $b/w = 0.2$ and 0.4 . As discussed in the previous chapter, negative stress intensity factors per se have no physical meaning. The implication, however, is that the residual hoop stresses tend to close the crack. The net effect of a negative $(K_I)_A$ value is the reduction of the total value of the stress intensity factor for the cracked, internally pressurized, autofrettaged cylinder. It is interesting to note that for a given overstrain condition and crack size, the residual stresses do not always tend to close the crack everywhere near the crack periphery. This can be expected, in view of the fact that the residual hoop stresses are only compressive at and near the cylinder bore. At radial position close to and beyond the elastic-plastic interface, the residual stresses are tensile, as can be seen from Fig. 5.18. Indeed, as the crack grows deeper across the cylinder wall, the usefulness of the

residual hoop stresses in retarding the crack faces from opening near the crack front becomes increasingly questionable. In fact, for the cylinders considered, when the surface crack reaches the size where $b/w = 0.8$, the residual stresses tend to open rather than close the crack faces at most points there, although the crack opening due to these stresses is relatively small.

6.4.3 K_I solutions for a Cracked Cylinder subjected to Thermal Loads

Thick-walled cylinders can often, in practice, be subjected to thermal loads in the form of steady-state radial temperature gradients across the wall of the cylinder. The hoop stress distributions due to this steady-state temperature gradient are given by (see, e.g. Bolev and Weiner [1960]):

$$(\sigma_\theta)_T = \frac{E \alpha \Delta T}{2(1-\nu) \log_e k} \left(1 + \log_e \left(\frac{r}{R_2} \right) - \frac{1}{k^2 - 1} \left(1 + \frac{R_2^2}{r^2} \right) \log_e k \right) \quad (6.9)$$

where α is the coefficient of thermal expansion and ΔT is the difference between the temperature at the cylinder bore and at its outer circumference. It can be shown that Eqn. (6.9) takes the form of Eqn. (6.7). Substituting $n=k$, i.e. 100 percent overstrain condition, Eqn. (6.7)

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becomes

$$(\sigma_{\theta})_{A=100\%} = \frac{2}{\sqrt{3}} \sigma_y \left\{ \frac{1}{k^2-1} \left(1 + \frac{R_2^2}{r^2} \right) (-\log_e k) + \left(1 - \log_e \frac{R_2}{r} \right) \right\} \quad \dots\dots (6.10a)$$

or

$$(\sigma_{\theta})_{A=100\%} = \frac{2}{\sqrt{3}} \sigma_y \left\{ \left(1 + \log_e \frac{r}{R_2} \right) - \frac{1}{k^2-1} \left(1 + \frac{R_2^2}{r^2} \right) \log_e k \right\} \quad \dots\dots (6.10b)$$

Therefore, at a given point in a cylinder, the ratio of the elastic hoop stress due to the steady state radial temperature gradient, to the residual hoop stress in a 100 percent overstrained cylinder, is a constant factor involving the material properties and the cylinder radius ratio. Thus,

$$\frac{(\sigma_{\theta})_T}{(\sigma_{\theta})_{A=100\%}} = \frac{E \alpha \Delta T}{2(1-\nu) \log_e k} \cdot \frac{\sqrt{3}}{2\sigma_y} = \frac{\sqrt{3}}{4(1-\nu) \log_e k} \frac{E \alpha \Delta T}{\sigma_y} \quad \dots\dots (6.11)$$

This means that the stress intensity factors due to thermal loads, $(K_I)_T$, for the cracked cylinder with $k=2.0$ could be determined directly from the stress intensity factors $(K_I)_A$ due to the autofrettaged residual stresses corresponding to the 100% overstrain condition. This can be achieved by simply multiplying $(K_I)_A$ results as presented in Tables 6.13 to 6.16 by the constant factor given by Eqn. (6.11). The normalized values of the stress

intensity factors, $(K_I)_T / \{E\alpha \Delta T \sqrt{\pi b}\}$, for the cracked cylinder with $k = 2.0$ are listed in Table 6.21.

Alternatively, $(K_I)_T$ can be obtained from the polynomial influence coefficients using the method of superposition.

The corresponding third-degree polynomial, which could be used to represent the thermal hoop stresses in the uncracked cylinder, obtained by least-squares curve fitting technique to within 3% of the exact solution, is:

$$(\sigma_\theta)_T = E\alpha \Delta T (-0.8654 + 2.7026r_s - 2.0027r_s^2 + 0.7258r_s^3) \quad (6.12)$$

It should be remarked here that when the temperature at the cylinder bore is greater than that at the outer circumference (i.e., $\Delta T > 0$), the numerical values of $(K_I)_T$ are also generally negative. This has the same physical significance as the negative $(K_I)_A$ values discussed previously.

The relationship between the 100% overstrained residual hoop stresses and the elastic hoop stresses due to the steady-state thermal gradients for cylinder with $k = 2.0$ as established in Eqn. (6.11) does not hold true for cylinder with $k = 2.5$. This is because the autofrettaged residual hoop stress distribution given by Eqn. (6.7) is no longer valid for cylinder with $k = 2.5$ when it is 100% overstrained, as reverse yielding would have occurred.

To obtain the numerical values of $(K_I)_T$ for the cracked

cylinder with $k = 2.5$, the polynomial influence coefficients could be used. The thermal hoop stress distribution given by Eqn. (6.9) must first be expressed as a third-degree polynomial in the form of Eqn. (6.3). The corresponding polynomial obtained by the least-squares curve fitting technique to within 3% of the exact solution is:

$$(\sigma_{\theta})_T = E\alpha \Delta T(-0.8495 + 3.7468r_s - 3.3676r_s^2 + 1.3545r_s^3) \quad (6.13)$$

Using the superposition procedure as described previously, the normalized stress intensity factors, $(K_I)_A / (E\alpha \Delta T \sqrt{\pi b})$, for the cracked cylinder with $k = 2.5$ are presented in Table 6.22. The numerical values of $(K_I)_T$ as listed in Tables 6.21 and 6.22 are also presented graphically in figures 6.21 to 6.24 for the various relative crack-depth treated. As expected, the trends of the $(K_I)_T$ variations along the crack periphery for the two radius ratios are very similar.

6.5 Final Remarks

The accuracy of the numerical stress intensity factor solutions presented in this chapter is difficult to establish because no exact solutions are available. Further, no previous work to those carried out here has ever

been presented in the literature, except for the set of K_I results for the internally pressurized cracked cylinder with the pressure loads also acting on the crack faces when $k = 2.0$ (Table 6.9). A mesh refinement exercise could be performed to verify the convergence of the influence coefficients. But this can be very costly because of the costs involved in carrying out three-dimensional BIE stress analysis, even though it is still cheaper when compared to other numerical methods. However, experience with the use of the BIE numerical method has shown that the results are most likely to be within about 5% of their true values and are hence sufficiently accurate for engineering applications.

Finally, it should be noted that K_I solutions for other loading cases, such as those caused by thermal shock, could also be obtained by superposition of the influence coefficients presented in Tables 6.1 to 6.8. The accuracy of these solutions will of course depend upon the suitability and degree of accuracy to which the uncracked hoop stress distribution across the cylinder wall thickness is fitted into a simple third-degree polynomial.

CHAPTER 7

CONCLUSIONS

The basic principles of the theory of Linear Elastic Fracture Mechanics (LEFM) have been briefly reviewed. The application and limitation of LEFM were discussed, with attention confined to the opening mode, mode I, of deformation. The key mathematical equations of the Boundary Integral Equation (BIE) method for elastostatics, based on the direct formulation method, have also been presented for both two- and three-dimensional problems, with the numerical strategy employed briefly discussed.

In chapter 4, the various techniques to obtain the stress intensity factor K_I using the BIE method were discussed. Two of the most important schemes discussed were:

- (i) the displacement extrapolation method: this method utilises the crack face displacement data and offers an easy and accurate means of obtaining K_I directly at the crack tip front; and

- (ii) the weight or influence function method, which employs the principle of linear superposition to evaluate K_I indirectly.

The weight function method provides an economical and accurate means of determining K_I by eliminating repeated computer runs for a given cracked component subjected to several different loading conditions.

The 2-D numerical BIE method was successfully employed to obtain the weight functions for a wide range of radial-edge cracks in thick-walled cylinders. Using the principle of linear superposition, K_I results quantifying the effects of autofrettaged residual stresses for the various radial-edge cracks were evaluated.

Using the numerical 3-D BIE method, the polynomial influence coefficients for a wide range of semi-elliptical surface cracks were obtained. Numerical K_I results for the range of semi-elliptical surface cracks treated for cylinders subjected to the following loading conditions, were presented and discussed:

- (1) internal pressure loads, for the case with the internal pressure also acting on the crack faces and the case when it does not;

- (2) autofrettaged residual stresses; and
- (3) thermal loads

The 2-D and 3-D K_I numerical results presented in the present study are very useful for the structural assessment of cracked thick-walled cylinders.

APPENDIX 1

SHAPE FUNCTIONS

(1) 3-node quadratic boundary line element

$$N^1(\xi) = \frac{1}{2} \xi (\xi - 1)$$

$$N^2(\xi) = 1 - \xi^2$$

$$N^3(\xi) = \frac{1}{2} \xi (\xi + 1)$$

(2) 6-node triangular boundary surface element

$$N^1(\xi) = (1 - \xi_1 - \xi_2)(1 - 2\xi_1 - 2\xi_2)$$

$$N^2(\xi) = 4 \xi_1 (1 - \xi_1 - \xi_2)$$

$$N^3(\xi) = \xi_1 (2\xi_1 - 1)$$

$$N^4(\xi) = 4 \xi_1 \xi_2$$

$$N^5(\xi) = \xi_2 (2\xi_2 - 1)$$

$$N^6(\xi) = 4 \xi_2 (1 - \xi_1 - \xi_2)$$

(3) 8-node quadrilateral boundary surface element

$$N^1(\xi) = \frac{1}{4} (\xi_1 - 1)(1 - \xi_2)(1 + \xi_1 + \xi_2)$$

$$N^2(\xi) = \frac{1}{2} (1 - \xi_1^2)(1 - \xi_2)$$

$$N^3(\xi) = \frac{1}{4} (\xi_1 + 1)(\xi_2 - 1)(1 - \xi_1 + \xi_2)$$

$$N^4(\xi) = \frac{1}{2} (\xi_1 + 1)(1 - \xi_2^2)$$

$$N^5(\xi) = \frac{1}{4} (\xi_1 + 1)(\xi_2 + 1)(\xi_1 + \xi_2 - 1)$$

$$N^6(\xi) = \frac{1}{2} (\xi_2 + 1)(1 - \xi_1^2)$$

$$N^7(\xi) = \frac{1}{4} (\xi_1 - 1)(\xi_2 + 1)(1 + \xi_1 - \xi_2)$$

$$N^8(\xi) = \frac{1}{2} (1 - \xi_1)(1 - \xi_2^2)$$

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Table 5.1 Least-squares coefficients of Eqn. (5.1) for cylinder with a radial-edge crack; $R_2/R_1=1.5$.

$a^* = \frac{a}{R_2 - R_1}$	$A(a^*)$	$B(a^*)$	$C(a^*)$	$D(a^*)$
0.1	2.60233	1.07984	-2.28120	3.55051
0.2	1.85644	0.10390	0.67264	1.36625
0.3	1.51238	-0.07447	1.53625	1.01861
0.4	1.27754	0.09466	1.55447	1.34115
0.5	1.11496	0.26062	1.51593	1.76274
0.6	0.99319	0.39970	1.57295	2.07301
0.7	1.00285	0.00246	2.70847	1.58234

Table 5.2 Least-squares coefficients of Eqn. (5.1) for
cylinder with a radial-edge crack; $R_2/R_1=2.0$.

$a^* = \frac{a}{R_2 - R_1}$	$A(a^*)$	$B(a^*)$	$C(a^*)$	$D(a^*)$
0.1	2.64073	-0.21472	0.66194	1.31767
0.2	1.96743	-0.78961	1.96669	0.25321
0.3	1.58477	-0.61129	1.92255	0.24866
0.4	1.36033	-0.50153	1.95855	0.29871
0.5	1.27944	-1.16998	3.79468	-0.76131
0.6	1.09211	-0.35748	2.39452	0.15214
0.7	1.01132	-0.37214	3.14666	-0.39606

Table 5.3 Least-squares coefficients of Eqn. (5.1) for cylinder with a radial-edge crack; $R_2/R_1=2.5$.

$a^* = \frac{a}{R_2 - R_1}$	$A(a^*)$	$B(a^*)$	$C(a^*)$	$D(a^*)$
0.1	2.81908	-1.19584	2.19083	0.29485
0.2	1.93510	-0.76290	1.76896	0.06464
0.3	1.53986	-0.41897	1.28983	0.23857
0.4	1.33788	-0.47875	1.68200	-0.01470
0.5	1.19661	-0.47444	1.97442	-0.20475
0.6	1.10938	-0.45245	2.37584	-0.50680
0.7	1.01549	-0.35093	2.97393	-0.99961

Table 5.4 Normalized stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi a})$, due to residual stresses in an autofrettaged cylinder made of elastic-perfectly plastic material; $R_2/R_1 = 1.5$.

Percentage overstrain $= \frac{nR_1 - R_1}{R_2 - R_1} \times 100$	Relative crack-depth, $a/(R_2 - R_1)$						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
10	-0.0437	-0.0197	-0.0139	-0.0113	-0.0098	-0.0085	-0.0070
20	-0.1447	-0.0815	-0.0526	-0.0421	-0.0361	-0.0313	-0.0257
30	-0.2315	-0.1725	-0.1198	-0.0889	-0.0753	-0.0649	-0.0530
40	-0.3049	-0.2494	-0.2038	-0.1574	-0.1244	-0.1060	-0.0862
50	-0.3656	-0.3130	-0.2733	-0.2344	-0.1911	-0.1526	-0.1229
60	-0.4142	-0.3639	-0.3289	-0.2961	-0.2597	-0.2141	-0.1613
70	-0.4512	-0.4027	-0.3713	-0.3431	-0.3121	-0.2720	-0.2146

Table 5.5 Normalized stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi a}\}$, due to residual stresses in an autofrettaged cylinder made of elastic-perfectly plastic material; $R_2/R_1 = 2.0$.

Percentage overstrain $\frac{nR_1 - R_1}{R_2 - R_1} \times 100$	Relative crack-depth = $a/(R_2 - R_1)$						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
10	-0.0762	-0.0303	-0.0185	-0.0131	-0.0097	-0.0076	-0.0054
20	-0.2523	-0.1285	-0.0707	-0.0490	-0.0365	-0.0279	-0.0201
30	-0.3998	-0.2766	-0.1674	-0.1050	-0.0771	-0.0581	-0.0417
40	-0.5218	-0.3991	-0.2929	-0.1952	-0.1299	-0.0960	-0.0686
50	-0.6208	-0.4985	-0.3948	-0.3013	-0.2122	-0.1407	-0.0995
60	-0.6988	-0.5768	-0.4750	-0.3849	-0.3000	-0.2128	-0.1336
70	-0.7574	-0.6356	-0.5353	-0.4477	-0.3660	-0.2831	-0.1951

Table 5.6 Normalized stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi a}\}$, due to residual stresses in an autofrettaged cylinder made of elastic-perfectly plastic material; $R_2/R_1 = 2.5$.

Percentage overstrain $= \frac{nR_1 - R_1}{R_2 - R_1} \times 100$	Relative crack-depth = $a/(R_2 - R_1)$						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
10	-0.1025	-0.0359	-0.0195	-0.0124	-0.0083	-0.0054	-0.0032
20	-0.3419	-0.1569	-0.0756	-0.0471	-0.0312	-0.0203	-0.0123
30	-0.5376	-0.3429	-0.1866	-0.1029	-0.0668	-0.0432	-0.0263
40	-0.6964	-0.4938	-0.3354	-0.2018	-0.1153	-0.0736	-0.0448
50	-0.8233	-0.6144	-0.4543	-0.3222	-0.2018	-0.1120	-0.0676
60	-0.9220	-0.7083	-0.5468	-0.4158	-0.2980	-0.1872	-0.0954

Table 6.1 Normalized influence coefficients, $K_I^{(i)} / \{A_i (b/W)^{1/2}\}$, for internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.2$ and $r_s=(r-R_1)/W$

i	Type of Loading	ϕ (degree)						
		0	15	30	45	60	75	90
0	A_0	0.6700	0.6658	0.6592	0.6468	0.6357	0.6424	0.6770
1	$A_1(r_s)$	0.4582	0.4396	0.4160	0.3516	0.2772	0.1879	0.1094
2	$A_2(r_s)^2$	0.3472	0.3223	0.2874	0.2085	0.1369	0.0730	0.0407
3	$A_3(r_s)^3$	0.2713	0.2467	0.2068	0.1345	0.0741	0.0344	0.0195

Table 6.2 Normalized influence coefficients, $K_I^{(i)} / \{A_i (b/W)^i \sqrt{\pi b}\}$, for

internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1=2.0$; $b/a=0.8$, $b/W=0.4$ and $r_s=(r-R_1)/W$

i	Type of loading	ϕ (degree)							
		0	15	30	45	60	75	90	
0	A_0	0.6829	0.6823	0.6696	0.6602	0.6406	0.6483	0.6718	
1	$A_1(r_s)$	0.4632	0.4513	0.4186	0.3631	0.2798	0.1890	0.1116	
2	$A_2(r_s)^2$	0.3456	0.3370	0.3011	0.2164	0.1432	0.0766	0.0429	
3	$A_3(r_s)^3$	0.2673	0.2670	0.2247	0.1416	0.0789	0.0370	0.0212	

Table 6.3 Normalized influence coefficients, $K_I^{(i)} / \{A_i (b/W)^{1/2} b\}$, for

internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.6$ and $U_s = (r-R_1)/W$

i	Type of loading	ϕ (degree)						
		0	15	30	45	60	75	90
0	A_0	0.7147	0.7031	0.7016	0.6843	0.6720	0.6703	0.6969
1	$A_1(r_s)$	0.4733	0.4629	0.4296	0.3704	0.2894	0.2032	0.1256
2	$A_2(r_s)^2$	0.3623	0.3499	0.3092	0.2334	0.1489	0.0800	0.0499
3	$A_3(r_s)^3$	0.2815	0.2698	0.2329	0.1442	0.0827	0.0394	0.0249

Table 6.4 Normalized influence coefficients, $K_I^{(i)} / \{A_i (b/W)^{1/2}\}$, for

internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.8$ and $r_s=(r-R_1)/W$

i.	Type of loading	Φ (degree)							
		0	15	30	45	60	75	90	
0	A_0	0.8203	0.8064	0.7879	0.7541	0.7334	0.7266	0.7442	
1	$A_1(r_s)$	0.5186	0.4962	0.4657	0.3964	0.3104	0.2151	0.1482	
2	$A_2(r_s)^2$	0.3882	0.3686	0.3192	0.2378	0.1602	0.0894	0.0585	
3	$A_3(r_s)^3$	0.2984	0.2870	0.2467	0.1580	0.0901	0.0450	0.0313	

Table 6.5 Normalized influence coefficients, $K_I^{(i)} / \{A_i(b/W)^i \sqrt{\pi b}\}$, for internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1=2.5$, $b/a=0.8$, $b/W=0.2$ and $r_s=(r-R_1)/W$

i	Type of loading	ϕ (degree)						
		0	15	30	45	60	75	90
0	A_0	0.6656	0.6581	0.6536	0.6397	0.6258	0.6263	0.6582
1	$A_1(r_s)$	0.4519	0.4443	0.4103	0.3542	0.2719	0.1820	0.1067
2	$A_2(r_s)^2$	0.3333	0.3306	0.2941	0.2120	0.1385	0.0724	0.0394
3	$A_3(r_s)^3$	0.2896	0.2535	0.2139	0.1362	0.0743	0.0331	0.0178

Table 6.6 Normalized influence coefficients, $K_I^{(i)} / \{A_i (b/W)^{1/2} \sqrt{\pi b}\}$, for

internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1=2.5$, $b/a=0.8$, $b/W=0.4$ and $r_s=(r-R_1)/W$

i	Type of loading	ϕ (degree)							
		0	15	30	45	60	75	90	
0	A_0	0.6640	0.6627	0.6524	0.6387	0.6223	0.6166	0.6273	
1	$A_1(r_s)$	0.4568	0.4439	0.4165	0.3572	0.2765	0.1841	0.1022	
2	$A_2(r_s)^2$	0.3469	0.3247	0.2871	0.2068	0.1361	0.0698	0.0393	
3	$A_3(r_s)^3$	0.3157	0.2826	0.2340	0.1426	0.0784	0.0350	0.0192	

Table 6.7 Normalized influence coefficients, $K_I^{(i)} / \{A_i(b/W)^{1/2} \pi b\}$, for internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1 = 2.5$, $b/a = 0.8$, $b/W = 0.6$ and $r_s = (r - R_1)/W$

i	Type of loading	0	15	30	45	60	75	90
0	A_0	0.7230	0.7087	0.7065	0.6872	0.6692	0.6586	0.6532
1	$A_1(r_s)$	0.4758	0.4583	0.4321	0.3713	0.2888	0.1939	0.1171
2	$A_2(r_s)^2$	0.3512	0.3381	0.3067	0.2210	0.1473	0.0788	0.0478
3	$A_3(r_s)^3$	0.2966	0.2655	0.2271	0.1449	0.0914	0.0385	0.0246

Table 6.8 Normalized influence coefficients $K_I^{(i)} / \{ \lambda_{I1}(b/W)^i \sqrt{\pi b} \}$, for

internal semi-elliptical surface cracks in a cylinder:

$R_2/R_1 = 2.5$, $b/a = 0.8$, $b/W = 0.8$ and $r_s = (r - R_1)/W$

i	Type of loading	ϕ (degree)						
		0	15	30	45	60	75	90
0	A_0	0.8372	0.8155	0.7904	0.7418	0.7296	0.7118	0.7098
1	$A_1(r_s)$	0.5290	0.5163	0.4713	0.4037	0.3114	0.2183	0.1353
2	$A_2(r_s)^2$	0.4026	0.3674	0.3315	0.2550	0.1617	0.0897	0.0521
3	$A_3(r_s)^3$	0.3225	0.2872	0.2454	0.1557	0.0899	0.0449	0.0287

Table 6.9 Normalized stress intensity factors, $K_I/(P\sqrt{\pi b})$, for internal semi-elliptical surface cracks in a pressurized cylinder
(pressure also acting on the crack faces): $R_2/R_1=2.0$, $b/a=0.8$ and $\nu=0.3$

b/W	ϕ (degree)					
	0	15	30	45	60	75 90
0.2	1.584	1.585	1.576	1.567	1.568	1.624 1.752
	(1.617)	(1.604)	(1.588)	(1.565)	(1.565)	(1.604) (1.782)
0.4	1.481	1.488	1.476	1.476	1.481	1.567 1.694
	(1.492)	(1.473)	(1.469)	(1.455)	(1.481)	(1.573) (1.684)
0.6	1.466	1.442	1.459	1.452	1.476	1.546 1.709
	(1.490)	(1.480)	(1.460)	(1.463)	(1.473)	(1.582) (1.703)
0.8	1.621	1.603	1.555	1.522	1.549	1.629 1.765
	(1.623)	(1.607)	(1.582)	(1.526)	(1.520)	(1.536) (1.669)

Table 6.10 Normalized stress intensity factors, $K_I/(P\sqrt{\pi b})$, for internal semi-elliptical surface cracks in a pressurized cylinder
(pressure also acting on the crack faces): $R_2/R_1=2.5$, $b/a=0.8$
and $\nu=0.3$

b/W	0	15	30	45	60	75	90
0.2	1.349	1.336	1.341	1.331	1.338	1.384	1.501
0.4	1.192	1.195	1.186	1.188	1.212	1.276	1.384
0.6	1.210	1.201	1.216	1.196	1.229	1.300	1.390
0.8	1.378	1.304	1.305	1.251	1.281	1.332	1.453

Table 6.11 Normalized stress intensity factors, $K_I/(P\sqrt{\pi b})$, for internal

semi-elliptical surface cracks in a pressurized cylinder

(pressure not acting on the crack faces): $R_2/R_1=2.0$, $b/a=0.8$ and $\nu=0.3$

b/W	ϕ (degree)					
	0	15	30	45	60	75
0.2	0.919	0.919	0.917	0.921	0.932	0.982
0.4	0.799	0.806	0.806	0.816	0.840	0.919
0.6	0.754	0.738	0.757	0.768	0.804	0.876
0.8	0.801	0.797	0.768	0.767	0.815	0.902
						1.021
						1.012
						1.022
						1.075

Table 6.12 Normalized stress intensity factors, $K_I/(P\sqrt{\pi b})$, for internal semi-elliptical surface cracks in a pressurized cylinder (pressure not acting on the crack faces): $R_2/R_1=2.5$, $b/a=0.8$ and $\nu=0.3$

b/W	0	15	30	45	60	75	90
0.2	0.683	0.678	0.687	0.691	0.712	0.758	0.843
0.4	0.528	0.532	0.533	0.549	0.590	0.659	0.757
0.6	0.486	0.493	0.509	0.509	0.559	0.641	0.737
0.8	0.541	0.489	0.515	0.509	0.551	0.620	0.743

Table 6.13 Normalized stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi b}\}$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.2$ and $\nu=0.3$

% overstrain	0	15	30	ϕ (degree) 45	60	75	90
10	-0.005	-0.006	-0.006	-0.008	-0.020	-0.042	-0.096
20	-0.049	-0.055	-0.062	-0.085	-0.114	-0.158	-0.211
30	-0.139	-0.144	-0.151	-0.173	-0.202	-0.250	-0.309
40	-0.205	-0.211	-0.218	-0.241	-0.272	-0.324	-0.390
50	-0.273	-0.278	-0.284	-0.305	-0.334	-0.387	-0.457
60	-0.320	-0.326	-0.331	-0.352	-0.381	-0.435	-0.509
70	-0.356	-0.362	-0.367	-0.388	-0.417	-0.471	-0.548
80	-0.388	-0.387	-0.392	-0.413	-0.442	-0.497	-0.575
90	-0.396	-0.401	-0.407	-0.427	-0.456	-0.511	-0.590
100	-0.402	-0.407	-0.412	-0.432	-0.460	-0.511	-0.594

Table 6.14. Normalized stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.4$ and $\nu=0.3$

overstrain %	0	15	30	45	60	75	90
10	0.002	0.001	0.001	0.001	-0.001	-0.019	-0.065
20	0.007	0.005	0.001	-0.009	-0.017	-0.046	-0.164
30	-0.002	-0.009	-0.025	-0.049	-0.090	-0.150	-0.255
40	-0.037	-0.047	-0.065	-0.095	-0.154	-0.240	-0.333
50	-0.093	-0.103	-0.121	-0.151	-0.210	-0.299	-0.397
60	-0.137	-0.147	-0.164	-0.195	-0.254	-0.346	-0.448
70	-0.170	-0.180	-0.197	-0.228	-0.287	-0.382	-0.485
80	-0.193	-0.203	-0.220	-0.252	-0.311	-0.406	-0.512
90	-0.206	-0.217	-0.234	-0.265	-0.324	-0.420	-0.526
100	-0.211	-0.222	-0.238	-0.271	-0.330	-0.425	-0.531

Table 8.15 Normalized stress intensity factors, $(K_A)/(\sigma_y \sqrt{\pi b})$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.6$ and $\nu=0.3$

% overstrain	0	15	30	45	60	75	90
10	0.003	0.003	0.003	0.002	0.003	-0.013	-0.058
20	0.009	0.009	0.008	0.008	0.000	-0.033	-0.129
30	0.020	0.016	0.013	-0.001	-0.024	-0.085	-0.211
40	0.040	0.032	0.024	-0.006	-0.053	-0.122	-0.284
50	0.037	0.027	0.011	-0.024	-0.104	-0.195	-0.342
60	-0.006	-0.007	-0.038	-0.084	-0.158	-0.260	-0.399
70	-0.039	-0.039	-0.070	-0.115	-0.191	-0.295	-0.437
80	-0.061	-0.060	-0.092	-0.137	-0.213	-0.319	-0.464
90	-0.073	-0.072	-0.104	-0.149	-0.227	-0.372	-0.479
100	-0.076	-0.076	-0.109	-0.154	-0.232	-0.339	-0.484

Table 6.16 Normalized stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.0$, $b/a=0.8$, $b/W=0.8$ and $\nu=0.3$

% overstrain	0	15	30	45	60	75	90
10	0.004	0.004	0.004	0.004	0.004	-0.010	-0.054
20	0.013	0.012	0.011	0.013	0.013	-0.014	-0.106
30	0.041	0.043	0.041	0.029	0.007	-0.056	-0.184
40	0.042	0.049	0.033	0.015	0.006	-0.067	-0.259
50	0.054	0.053	0.050	0.007	-0.030	-0.124	-0.309
60	0.110	0.104	0.081	0.043	-0.038	-0.125	-0.337
70	0.063	0.063	0.046	-0.010	-0.094	-0.202	-0.397
80	0.045	0.028	0.028	-0.029	-0.134	-0.273	-0.413
90	0.032	0.014	0.014	-0.042	-0.147	-0.288	-0.429
100	0.028	0.010	0.009	-0.047	-0.152	-0.294	-0.434

Table 6.17 Normalized stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for an internal semi-elliptical surface crack in a cylinder with autofretage residual stresses: $R_2/R_1=2.5$, $b/a=0.8$, $b/W=0.2$ and $\nu=0.3$

% overstrain	ϕ (degree)						
	0	15	30	45	60	75	90
10	-0.005	-0.005	-0.005	-0.013	-0.021	-0.050	-0.134
20	-0.058	-0.065	-0.084	-0.107	-0.155	-0.217	-0.292
30	-0.176	-0.182	-0.201	-0.224	-0.272	-0.339	-0.424
40	-0.273	-0.277	-0.297	-0.319	-0.367	-0.438	-0.531
50	-0.351	-0.353	-0.374	-0.396	-0.444	-0.517	-0.617
60	-0.412	-0.414	-0.434	-0.456	-0.505	-0.579	-0.683

Table 6.18 Normalized stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.5$, $b/a=0.8$, $b/W=0.4$ and $\nu=0.3$.

% overstrain	0	15	30	ϕ (degree) 45	60	75	90
10	0.002	0.001	0.003	-0.002	-0.011	-0.031	-0.100
20	0.007	0.003	0.005	-0.001	-0.013	-0.057	-0.219
30	0.012	0.005	-0.011	-0.045	-0.098	-0.180	-0.334
40	-0.020	-0.030	-0.047	-0.095	-0.180	-0.295	-0.434
50	-0.090	-0.100	-0.117	-0.163	-0.250	-0.369	-0.513
60	-0.144	-0.154	-0.171	-0.217	-0.304	-0.427	-0.574

Table 6:19 Normalized stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi b}\}$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.5$, $b/a=0.8$, $b/W=0.6$ and $\nu=0.3$

% overstrain	$\phi(\text{degree})$						
	0	15	30	45	60	75	90
10	0.004	0.003	0.004	0.002	-0.001	-0.023	-0.089
20	0.011	0.010	0.010	0.009	0.006	-0.044	-0.180
30	0.022	0.019	0.014	0.011	-0.001	-0.089	-0.295
40	0.050	0.043	0.027	-0.013	-0.067	-0.174	-0.377
50	0.055	0.038	0.022	-0.025	-0.125	-0.240	-0.423
60	0.027	-0.002	-0.039	-0.077	-0.191	-0.346	-0.510

Table 6.20. Normalized stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for an internal semi-elliptical surface crack in a cylinder with autofrettage residual stresses: $R_2/R_1=2.5$, $b/a=0.8$, $b/W=0.8$ and $\nu=0.3$

% overstrain	ϕ (degree)					
	0	15	30	45	60	75 90
10	0.005	0.005	0.005	0.005	0.007	-0.014 -0.077
20	0.017	0.015	0.014	0.016	0.018	-0.037 -0.158
30	0.033	0.027	0.025	0.017	0.013	-0.022* -0.262
40	0.070	0.074	0.068	0.064	0.021	-0.095 -0.348
50	0.073	0.069	0.065	0.048	-0.019	-0.167 -0.381
60	0.128	0.110	0.094	0.043	-0.064	-0.221 -0.484

Table 6.21 Normalized stress intensity factors, $(K_I)_T / \{E\alpha \Delta T / \pi b\}$, for internal semi-elliptical surface cracks in a cylinder with a steady-state radial temperature gradient: $R_2/R_1=2.0$, $b/a=0.8$ and $\nu=0.3$

b/W	ϕ (degree)					
	0	15	30	45	60	75
0.2	-0.358	-0.363	-0.367	-0.386	-0.411	-0.456
0.4	-0.189	-0.198	-0.213	-0.242	-0.294	-0.380
0.6	-0.068	-0.068	-0.097	-0.137	-0.207	-0.302
0.8	0.025	0.009	0.008	-0.042	-0.136	-0.262
						-0.387

Table 6.22 Normalized stress intensity factors, $(K_I)_T / (E\alpha \Delta T \sqrt{\pi b})$, for

internal semi-elliptical surface cracks in a cylinder with

a steady-state radial temperature gradient: $R_2/R_1=2.5$, $b/a=0.8$ and $\nu=0.3$

b/W	ϕ (degree)						
	0	15	30	45	60	75	90
0.2	-0.269	-0.268	-0.285	-0.305	-0.346	-0.405	-0.484
0.4	-0.039	-0.048	-0.064	-0.106	-0.181	-0.282	-0.399
0.6	0.116	0.096	0.066	0.003	-0.074	-0.208	-0.342
0.8	0.230	0.262	0.197	0.198	0.027	-0.113	-0.290

Figure 2.1 Three displacement modes for crack surfaces.

Figure 2.2 Crack-tip stresses shown with the Cartesian co-ordinate system.

Figure 2.3 Stress distribution at the vicinity of the crack tip.

Figure 3.1 A boundary value problem in elastostatics.

Figure 3.2 An eight-node quadrilateral boundary surface element.

Figure 3.3 A six-node triangular boundary surface element.

Figure 3.4 A three-node quadratic boundary-line element.

Figure 4.1 Determination of stress intensity factor by displacements extrapolation method.

($R_2 = 2.0$, $R_1 = 1.0$, $a/(R_2 - R_1) = 0.3$ and $P = 1.0$)

Figure 4.2 Determination of stress intensity factor by stresses extrapolation method.

($R_2=2.0$, $R_1=1.0$, $a/(R_2-R_1)=0.3$ and $P=1.0$)

Figure 4.3 Perturbation of a crack front (Tan and Fenner [1979]).

Figure 4.4 Principle of the Weight Function method.

Figure 5.1 Thick-walled cylinder with a radial-edge crack.

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Figure 5.3 BIE mesh for cracked cylinder in elastic analysis ($R_2/R_1=2.0$, $a/(R_2-R_1)=0.5$ and 30 elements).

Figure 5.4 Boundary conditions for a thick-walled cylinder containing a single radial-edge crack, subjected to uniform pressure load on the crack-face.

Figure 5.5 Variation of crack-face nodal weight functions, $h\sqrt{W}$, with distance from the crack-tip, r^* ($=r_s/a$).

Figure 5.6 BIE mesh for uncracked cylinder in elastoplastic analysis.

Figure 5.7 Boundary conditions for uncracked cylinder, subjected to uniform pressure load on the bore surface.

Figure 5.8 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type 304 steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.607$.

Figure 5.9 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type 304 steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.651$.

Figure 5.10 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type 304 steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.694$.

Figure 5.11. Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type 304 steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.738$.

Figure 5.12 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type 304 steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.782$.

Figure 5.13 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type A533B steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.608$.

Figure 5.14 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type A533B steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.696$.

Figure 5.15 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type A533B steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.740$.

Figure 5.16 Elastic-plastic hoop stress distribution in a pressurized uncracked cylinder; Type A533B steel, $R_2/R_1=2.0$ and $P/\sigma_y=0.784$.

— Bowie and Freese [1972]

○ ○ ○ Weight Influence Function method

Figure 5.17 Comparison of stress intensity factors for unautofrettaged, pressurized cylinder with a single radial crack.

Figure 5.18. Typical residual hoop stress distribution in an autofrettaged cylinder.

Figure 5.19 Computed variation of the residual stress intensity factor, $(K_I)_A / (\sigma_y \sqrt{\pi a})$, with different degrees of overstrain; Type 304 steel and $R_2/R_1 = 1.5$.

Figure 5.20 Computed variation of the residual stress intensity factor, $(K_I)_A / \{\sigma_y \sqrt{\pi a}\}$, with different degrees of overstrain; Type 304 steel and $R_2/R_1 = 2.0$.

Figure 5.21 Computed variation of the residual stress intensity factor, $(K_I)_A / (\sigma_y \sqrt{\pi a})$, with different degrees of overstrain; Type 304 steel and $R_2/R_1 = 2.5$.

Figure 5.22 Computed variation of the residual stress intensity factor, $(K_I)_A / (\sigma_y \sqrt{\pi a})$, with different degrees of overstrain; Type A533B steel and $R_2/R_1 = 1.5$.

Figure 5.23 Computed variation of the residual stress intensity factor, $(K_I)_A / (\sigma_y \sqrt{\pi a})$, with different degrees of overstrain; Type A533B steel and $R_2/R_1 = 2.0$.

Figure 5.24 Computed variation of the residual stress intensity factor, $(K_I)_A / (\sigma_y \sqrt{\pi a})$, with different degrees of overstrain; Type A533B steel and $R_2/R_1 = 2.5$.

Figure 6.1 Semi-elliptical surface crack in a thick-walled cylinder.

Figure 6.2 BIE mesh for cylinder with $R_2/R_1=2.5$, $b/a=0.8$ and $b/W=0.4$ (97 boundary surface elements and 289 nodal points).

Figure 6.3 Influence function coefficients, $K_I^{(i)} \{A_i(b/w)^i \sqrt{\pi b}\}$, ($i=0,3$), for cylinder with $R_2/R_1=2.0$, $b/a=0.8$ and $b/w=0.2$.

Figure 6.4 Influence function coefficients, $K_I^{(i)} / \{A_I(b/W)^{1/2}\}$, ($i=0, 3$), for cylinder with $R_2/R_1=2.0$, $b/a=0.8$ and $b/W=0.4$.

Figure 6.5 Influence function coefficients, $K_I^{(i)} / (A_i (b/W)^i \sqrt{\pi b})$, ($i=0, 3$), for cylinder with $R_2/R_1=2.0$, $b/a=0.8$ and $b/W=0.6$.

Figure 6.6 Influence function coefficients, $K_I^{(i)} / (A_i (b/W)^i \sqrt{\pi b})$, $-(i=0,3)$, for cylinder with $R_2/R_1=2.0$, $b/a=0.8$ and $b/W=0.8$

Figure 6.7 Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders with pressure loads also acting on the crack faces. ($b/a=0.8$ and $b/W=0.2$)

Figure 6.8 Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders with pressure loads also acting on the crack faces. ($b/a=0.8$ and $b/W=0.4$)

Figure 6.9 Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders with pressure loads also acting on the crack faces. ($b/a=0.8$ and $b/W=0.6$)

Figure 6.10 Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders with pressure loads also acting on the crack faces. ($b/a=0.8$ and $b/W=0.8$)

Figure 6.11 Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders without pressure loads acting on the crack faces, ($b/a=0.8$ and $b/W=0.2$).

Figure 6.12. Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders without pressure loads acting on the crack faces. ($b/a=0.8$ and $b/W=0.4$)

Figure 6.13 Stress intensity factors, K_I ($P \sqrt{\pi b}$), for internally pressurized cylinders without pressure loads acting on the crack faces. ($b/a=0.8$ and $b/W=0.6$)

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Figure 6.14 Stress intensity factors, $K_I / (P \sqrt{\pi b})$, for internally pressurized cylinders without pressure loads acting on the crack faces. ($b/a=0.8$ and $b/W=0.8$)

Figure 6.15 Stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for cylinders with autofrettaged residual stresses.. ($R_2/R_1=2.0$, $b/a=0.8$ and overstrain=30%)

Figure 6.16 Stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for cylinders with autofrettaged residual stresses. ($R_2/R_1 = 2.0$, $b/a = 0.8$ and overstrain = 60%)

Figure 6.17 Stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi b}\}$, for cylinders with autofrettaged residual stresses. ($R_2/R_1 = 2.0$; $b/a = 0.8$ and overstrain = 80%)

Figure 6.18 Stress intensity factors, $(K_I)_A / \{\sigma_y \sqrt{\pi b}\}$, for cylinders with autofrettaged residual stresses. ($R_2/R_1=2.0$, $b/a=0.8$ and overstrain=100%)

Figure 6.19 Stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for cylinders with autofrettaged residual stresses: ($R_2/R_1=2.5$, $b/a=0.8$ and overstrain=40%)

Figure 6.20. Stress intensity factors, $(K_I)_A / (\sigma_y \sqrt{\pi b})$, for cylinders with autofrettaged residual stresses. ($R_2/R_1=2.5$, $b/a=0.8$ and overstrain=60%)

Figure 6.21 Stress intensity factors, $(K_I)_T / \{E\alpha \Delta T \sqrt{\pi b}\}$, for cylinders with steady-state radial temperature gradients. ($b/a=0.8$ and $b/W=0.2$).

Figure 6.22 Stress intensity factors, $(K_I)_T / (E \alpha \Delta T \sqrt{\pi b})$, for cylinders with steady-state radial temperature gradients. ($b/a=0.8$ and $b/W=0.4$)

Figure 6.23 Stress intensity factors, $(K_I)_T / \{E\alpha \Delta T \sqrt{\pi b}\}$, for cylinders with steady-state radial temperature gradients. ($b/a=0.8$ and $b/W=0.6$)

Figure 8.24 Stress intensity factors, $(K_I)_T / \{E\alpha \Delta T \sqrt{\pi b}\}$, for cylinders with steady-state radial temperature gradients. ($b/a=0.8$ and $b/W=0.8$)

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