1

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Service

Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

**Modelling Regeneration and Pest Control Alternatives
for a Forest System in the presence of Fire Risk**

by

Richard H.H. Moll, B.Sc., M.Sc., M.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Department of Systems and Computer Engineering,

Carleton University,
Ottawa, Ontario, Canada

January 14, 1991

© copyright
1991, Richard H.H. Moll

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Service Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

The author has granted an irrevocable non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of his/her thesis by any means and in any form or format, making this thesis available to interested persons.

The author retains ownership of the copyright in his/her thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without his/her permission.

L'auteur a accordé une licence irrévocable et non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de sa thèse de quelque manière et sous quelque forme que ce soit pour mettre des exemplaires de cette thèse à la disposition des personnes intéressées.

L'auteur conserve la propriété du droit d'auteur qui protège sa thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

ISBN 0-315-68854-8

Canada

The undersigned hereby recommend to the Faculty of Graduate Studies and Research acceptance of the thesis

*Modelling Regeneration and Pest Control Alternatives
for a Forest System in the presence of Fire Risk.*

submitted by Richard H.H. Moll, B.Sc., M.Sc., M.Eng., in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Chairman,
Department of Systems and
Computer Engineering

Thesis Supervisor

External Examiner

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University
January 14, 1991

Abstract

This thesis attempts to provide solutions for the fire and pest protection forest management *modelling* and *model representation* problems. The modelling problem seeks the optimal regeneration harvest and pest protection schedules by age and species taking into account fire risk, that produces a desired wood quantity over time at maximum discounted net profit. An LP model is developed to economically evaluate alternative regimes for protection spraying of susceptible forest species against insect infestations and alternative harvesting strategies which include conversion of susceptible species to non-susceptible, by planting. These strategies are evaluated subject to catastrophic loss due to fire. The model representation problem, which involves issues of understandability, was addressed by using a high level model development system and a processing network representation of the underlying algebraic description. An iterative simulation-optimization approach is developed which tests how well the deterministic model holds in a simulated stochastic environment. This validation procedure involves solving the optimization problem deterministically using average values for the fire and infestation proportions and also at each time period updating the system state by simulating the state transition for the next time period with the simulation model using randomly generated proportions and re-solving using the updated state as the new initial condition. An optimal wood supply trajectory in a simulated stochastic environment is therefore constructed. The results from the iterative stochastic solution provided a confidence measure for the deterministic solution. The effectiveness of the deterministic model was evaluated in terms of how flexible its structure is in distinguishing alternative policy options.

Acknowledgements

I am especially indebted to my research adviser Dr. J.W. Chinneck who has been a source of constant help, encouragement and guidance during the development of this dissertation. The years of association and the many fruitful conversations with him have been invaluable to me in reaching a focussed direction for this research.

I would like to thank Dr. C.M. Woodside for acting as my program adviser during the early stages of my study and for providing constructive criticism in the formation of an appropriate program which involved the interrelated areas of modelling, simulation, optimization and control of biological systems. I am also grateful to Drs. N.U Ahmed and E. Petriu (Univ. of Ottawa, Elect. Eng.) and Dr. G.R. Carmody (Carleton Univ., Biology) for their committee involvement. I extend special thanks to Dr. D.L. Martell (Univ. of Toronto, Faculty of Forestry) for serving as my external examiner.

I gratefully acknowledge the support of Statistics Canada which enabled me to pursue this degree part-time. In particular I would like to thank Mr. J.S. Wells, Assistant Chief Statistician, Mr. T. Gigantes, Director General and Mr. K.E. Hamilton, Manager, Environment and Natural Resources for their support and understanding during this research. I hope that the outcome of this research effort may be fruitfully applied to the program of strengthening the national statistical base on the environment.

I cannot adequately express my appreciation to my wife, Marita, for enduring the various moods I underwent, from elation to dejection, during the course of this research. In particular I must thank her for bearing the lioness's share of responsibility associated with the nurturing of our two children, Rachel and Christopher, during these important years of their life. I sincerely hope that my graduate study was well worth the hardships endured by them.

Finally, I thank Mr. P. Prins of Arbex, Ottawa for providing funds to purchase PC-GAMS without which this research could not have been implemented.

This thesis is dedicated to my father, the late Dr. H.H. Moll., M.D., F.R.C.P.

Contents

1	Introduction	1
1.1	A Need for Mathematical Modelling in the Forest Industry	1
1.2	Thesis Objective	3
1.3	A Research Overview	5
1.4	Thesis Outline	10
2	Background	12
2.1	The Classical "Rotation" Problem	12
2.2	The General Economic Forest Management Problem	14
2.3	Forest Harvest Scheduling	16
2.4	Traditional Methods	18
2.4.1	Area Control	19
2.4.2	Volume Control	19
2.5	Processing Networks and Linear Programming	20
2.6	Model Development Systems	24
2.6.1	GAMS and the Solution Method	24
2.6.2	The GAMS/MINOS Solver	26
3	State-of-the-Art Forest Harvest Scheduling	28
3.1	Mathematical Programming Models	28
3.2	Model I Linear Programming	29
3.3	Model II Linear Programming	33

3.4	Model II and the Age-Structured Model III	35
3.5	Discussion of Models I, II and III	41
3.6	Reed and Errico Model III	43
3.7	Stochastic Fire Hazard Model	49
3.8	Stochastic Fire and Pest Hazard Model	53
3.9	The Pest Control Model of Boychuk	59
4	The Fire and Pest Protection Forest Modelling Problem.....	61
4.1	A Statement of the Problem	61
4.1.1	The Fire and Pest Protection Modelling Problem.....	61
4.1.2	The Model Representation Problem	63
4.2	Why solve these problems ?.....	65
4.3	Have these problems been solved before ?	66
5	Network Representation	67
5.1	Network Representation of the Fire and Pest Model with no Protection Spraying or Conversion Harvests	68
5.2	Network Representation of the Fire and Pest Model with Conversion Harvests	73
5.3	Network Representation of the Fire and Pest Model with Protection Spraying	76
5.4	Network Representation of the Fire and Pest Model with Protection and Conversion Harvests	79
6	Algebraic Representation	84
6.1	The Fire and Pest Model with no Protection Spraying or Conversion Harvests	84
6.2	The Fire and Pest Model with Conversion Harvests	93
6.3	The Fire and Pest model with Protection Spraying	96
6.4	The Fire and Pest Model with Protection Spraying and Conversion	

	Harvests	100
6.5	Conclusions	103
7	Model Validation	104
7.1	Stochastic Validation	104
7.1.1	Stochastic LP Simulator	106
7.1.2	Results and Conclusions from the Stochastic Simulation	110
7.1.3	Stochastic Growth	113
7.2	Sensitivity Analysis	113
7.2.1	Discount Rate Sensitivity	114
7.2.2	Fire Rate Sensitivity	115
7.2.3	Sequential Flow Constraint Sensitivity	116
7.2.4	Conclusions from the Sensitivity Analysis	118
8	Model Effectiveness: Distinguishing Policy Options	119
8.1	A Comparative Scenario Analysis	119
8.2	Baserun Scenario	121
8.3	Conversion Harvest Scenario	125
8.4	Protection Scenario	126
8.5	The Conversion Harvest and Protection Scenario	128
8.6	Effect of Fire Loss on all Scenarios	129
8.7	Conclusions	133
9	Conclusions	134
9.1	Summary of Results	134
9.2	Summary of Contributions	136
9.3	Suggestions for Further Work	137
	References	140

Appendix A Calibration of the Fire and Pest Protection Model	149
A.1 Kamloops TSA Inventory Data	149
A.2 Kamloops Fire Data	156
A.3 Western Spruce Budworm Infestation Data	159
Appendix B GAMS: Fire and Pest Protection Model.....	161
Appendix C GAMS: Stochastic LP Simulator.....	178
Appendix D GAMS: Kamloops Data Aggregation.....	184
Appendix E The Inverse Transformation Method.....	191

List of Figures

2.1	A Processing Node with r Inflow and $t-r$ Outflow Terminals	23
3.1	Graphical representation of the Age-Structured Model III.....	38
3.2	Graphical representation of the Fire Pest Model of Reed and Errico [1987].....	58
5.1	The Fire and Pest Model with no Protection or Conversion Harvests.....	70
5.2	Oldest Age-Class Non-Susceptible.....	71
5.3	Non-Susceptible Intermediate Periods and Age-Classes.....	72
5.4	Susceptible Uninfested Node: Age-Class 1.....	73
5.5	Susceptible Uninfested Intermediate Periods and Age-Classes.....	74
5.6	The Fire and Pest Model with Conversion Harvests.....	75
5.7	Susceptible Infested Unprotected.....	76
5.8	Susceptible Infested Protected.....	77
5.9	The Fire and Pest Model with Protection Spraying.....	78
5.10	Susceptible Infested Unprotected.....	79
5.11	Susceptible Infested Protected.....	80
5.12	Regeneration Age-Class Non-Susceptible; A Regular Node.....	81
5.13	Regeneration Age-Class Susceptible; A Regular Node.....	82
5.14	The Fire and Pest Protection Model with Protection Spraying and Conversion Harvests.....	83
7.1	Feedback application of the Deterministic Control in a Stochastic Environment : An Iterative Optimization Procedure.....	109
7.2	An Example of the application of the Deterministic Control using	

	Randomly generated Fire Probabilities; one sample path st4.....	111
7.3	Optimal Feedback application of the Deterministic Control using Randomly generated Fire and Infestation Probabilities; eight Sample Paths (st1 to st8) plus an overlay chart of the Optimal Harvest solution using .005 average annual fire loss and average period infestation rate of .2 (the deterministic solution)	112
7.4	Effect of the Discount Rate on the Timber Supply	114
7.5	Effect of the Discount Rate on the N.P.V.	115
7.6	Effect of changing the Fire Rate on Timber Supply	116
7.7	A Comparison of the Timber Supply associated with Maximum Proportional Changes in Sequential Pulp Flow corresponding to a ten-year period.	117
8.1	Initial Age-Class Distribution for the <i>Baserun+Infestation</i> Scenario.....	122
8.2	Optimal Harvest Level Solutions for the <i>Baserun</i> and <i>Baserun+Infestation</i> Scenario.....	123
8.3	Non-Susceptible area harvested for the <i>Baserun+Infestation</i> Scenario.....	123
8.4	Susceptible area harvested for the <i>Baserun+Infestation</i> Scenario.....	124
8.5	Susceptible Infested area harvested for the <i>Baserun+Infestation</i> Scenario.....	124
8.6	Period-10 Age-Class Distribution for the <i>Baserun+Infestation</i> Scenario.....	125
8.7	Period-10 Age-Class Distribution for the <i>Conversion Harvest</i> Scenario.....	126
8.8	Optimal Harvest Levels for the <i>Baserun+Infestation</i> and the <i>Protection</i> Scenarios.....	127
8.9	Area Protected by Spraying for the <i>Protection</i> Scenario.....	127
8.10	Period-10 Age-Class Distribution for the <i>Protection</i> Scenario.....	128
8.11	Period-10 Age-Class Distribution for the <i>Conversion and Protection</i> Scenario..	129
8.12	Optimal Harvest Levels using average period Fire Loss for the <i>Baserun</i> Scenario.....	130
8.13	Optimal Harvest Levels using average period Fire Loss for the <i>Baserun+Infested</i> Scenario.....	131
8.14	Optimal Harvest Levels using average period Fire Loss for the <i>Protection</i> Scenario.....	131
8.15	A Comparison of the effect of Fire on all Optimal Harvest Level Scenarios....	132

A.1	Aggregate Inventory for the Kamloops TSA.....	155
A.2	Aggregate Volume-at-Age Curves for the Kamloops TSA.....	155
A.3	Annual Area Loss Due to Fire in the Kamloops TSA (1950-1989).....	156
E-1	The Inverse Transformation Method to generate Random Samples.....	192

List of Tables

- 3.1 Possible regeneration harvest sequences over six periods for an age-class that contains merchantable volume in period one and where there must be at least two periods between regeneration harvests. Each H represents a possible regeneration harvest. (from Johnson and Scheurman [1977])..... 32
- A.1 Cumulative Frequency Distribution of annual rates of Fire for the last 38 years in the Kamloops TSA..... 158

Glossary of Acronyms and Symbols

FPM	-----	Forest Management Problem
GAMS	-----	Generalized Algebraic Modelling System
LP	-----	Linear Programming
NPV	-----	Net Present Value
PC	-----	Personal Computer
FMU	-----	Forest Management Unit
TSA	-----	Timber Supply Area
m.s.y	-----	maximum sustainable yield
MUSYC	-----	Multiple Use Sustained Yield Calculation
TimberRAM	-----	Timber Resources Allocation Method
FORPLAN	-----	Forest Planning Model
TREES	-----	Timber Resources Economic Estimation
NSR	-----	Not Sufficiently Restocked
OR	-----	Operations Research
DSS	-----	Decision Support Systems
MS	-----	Management Science
$V(t)$	-----	volume-at-age t
p	-----	price per unit volume
c	-----	cost per unit volume
r	-----	instantaneous interest rate
$NPV(t)$	-----	net present value of an infinite series of rotations t
e^{rt}	-----	continuous discount factor
$x(t,i)$	-----	area of forest at time t in aged i

$c(t,i)$	-----	area of forest in aged i cut at time t
P_i	-----	processing ratio figure for terminal i
X_j	-----	flow variable for edge j
x_{hq}	-----	area of forest management unit h assigned to regeneration harvest sequence q
D_{hq}	-----	net present value of unit h under management regime q
A_i	-----	hectares of cutting unit i present at the beginning of period 1.
Y_{ij}	-----	hectares of initial stratum i that are harvested in cutting period j
Z_{jk}	-----	hectares regenerated in period j and harvested in cutting period k .
W_i	-----	hectares of initial stratum i that are left unharvested through the planning
	-----	period
U_j	-----	hectares regenerated in period j and subsequently left unharvested until the
	-----	end of the planning period
D_{ij}	-----	per hectare present value of costs and incomes associated with stands from
	-----	initial stratum i that are harvested in period j
E_{jk}	-----	per hectare present value of costs and incomes associated with stands that are
	-----	regenerated in period j and harvested in period k
T_i	-----	per hectare value of leaving a stand from initial stratum i unharvested at the
	-----	end of the planning period.
Q_j	-----	per hectare present value of leaving a stand that was regenerated in cutting
	-----	period j unharvested at the end of the planning period
$x_i(t)$	-----	area of forest with even-aged stands in age-class i at the start of period t
$h_i(t)$	-----	area of forest clearcut harvested in age-class i during period t .
v_i	-----	yield per hectare.
a	-----	upper bound on period cut
b	-----	lower bound on period cut
μ	-----	maximum increase or decrease in volume cut between periods.
$p_i(t)$	-----	proportion of forest in age-class i destroyed by fire during period t
$y_i(t)$	-----	area of uninfested susceptible timber in age i during period t
$z_i(t)$	-----	area of infested susceptible timber in age i during period t .
$r_i(t)$	-----	proportion of forest area not infested in period t which becomes infested.
$g_i(t)$	-----	area of (un-)infested timber in age-class i clearcut harvested during period t .
$q_i(t)$	-----	proportion of infested timber in age-class i destroyed by fire during period t .
MUP	-----	planting cost per hectare

MUC	-----	harvesting cost per hectare
MUS	-----	spraying cost per hectare
β	-----	percent volume reduction in infested protected stands
PD	-----	sales price of pulplogs per cubic metre
VNS_i	-----	volume-at-age i of non-susceptible forest
VS_i	-----	volume-at-age i of susceptible forest
$p(t)$	-----	proportion of non-susceptible timber burned during period t .
$q(t)$	-----	proportion of susceptible uninfested timber burned during period t
$ru(t)$	-----	proportion of susceptible infested burned in period t
$s_i(t)$	-----	proportion of susceptible uninfested timber in age-class i that becomes
	-----	infested in period t and remains in age-class i in period $t+1$
$f_i(t)$	-----	area of infested timber harvested in age-class i during period t .
$gc_i(t)$	-----	area of uninfested harvested in period t and converted to non-susceptible
$fc_i(t)$	-----	area of infested harvested in period t and converted to non-susceptible.
$fcp_i(t)$	-----	area of infested protected susceptible harvested in period t and converted to
	-----	non-susceptible
$zp_i(t)$	-----	area of protected infested timber in age i at the start of period t
$rp(t)$	-----	proportion of susceptible protected infested burned per period
$pr_i(t)$	-----	area of infested protected by spraying in period t
$fp_i(t)$	-----	area of infested protected timber in age-class i harvested in period t

Chapter 1

Introduction

1.1 A Need for Mathematical Modelling in the Forest Industry

In an address to the International Forestry Congress in Quebec, Pearce [1984] clearly outlined that the forest sector in Canada is in transition. In particular he asserted that "*. . . the forest industry if it is to survive at all, must shift its source of raw materials from the depleting stocks of original timber to forests that are grown and managed.*"

This commitment to *intensive forest management* has been reinforced by representatives of provincial and federal governments, the forest industry and universities for some time (e.g. Armson [1976]; Baskerville [1983 and 1985]; Reed [1978 and 1985]; and Sadler [1983]). The need for the forest sector to respond to this changing resource base will present a number of adjustment problems for the industry. Managed or second growth timber has different technical qualities requiring different harvesting technologies and is suitable for different products. Also these forests are of different species and are located in different areas than the frontier virgin timber which has been harvested for over a century. As a result of this position, effort in the development of technologies to assist in understanding the complex inter-relationships of biophysical and socio-economic factors in intensive forestry, has been accelerated.

Problems in forest management may be characterized by a responsive, evolving natural resource system which requires adaptive management strategies that influence, and are influenced by, systems dynamics. As reviewed by Williams [1989], this is evidenced by a growing number of applications of *dynamic optimization methods* in renewable natural resource management. In particular, the methods of *systems engineering*, especially *mathematical modelling* and *computer simulation* have been fruitfully applied to the analysis of renewable natural resource systems. These methods address the central problem of optimal utilization of a biotic resource whilst minimizing adverse ecological impact.

As pointed out by Williams [1989], applications of dynamic optimization procedures in natural resources management can be classified in terms of six approaches:

- (1) variational mathematics;
- (2) dynamic programming;
- (3) quasilinearization and linear programming;
- (4) non-linear programming;
- (5) simulation-optimization; and
- (6) classical optimization methods.

These methods have been reviewed by Jacquette [1972], Walters and Hilborn [1978], Goh [1980], Shoemaker [1981], Reed [1986], Cohen [1987] and Williams [1989]. All the methods are characterized by:

- (1) a mathematical model to describe the natural resource system;
- (2) a set of feasible control strategies; and
- (3) an objective functional to measure the costs and benefits of the strategies.

1.2 Thesis Objective

The objective of this thesis is to develop an *optimization modelling framework* to provide solutions for the *general forest management problem* (FMP) of determining optimal policies for long term forest management. In particular, the fire and pest protection FMP is addressed which solves for: *the optimal timber harvest and pest protection schedule subject to fire risk that produces a desired wood supply over time at maximum discounted net profit*. This objective is guided by reference to the research goals outlined below.

(1) ---- *The suggested model framework should address policy issues pertaining to either a forest products company or a government agency which must develop a regional forest management plan.*

The model should be conceived as a "strategic" (c.f. Barros and Weintraub [1982] and Weintraub et al [1986]) as opposed to a "tactical" forest planning tool. Tactical planning occurs at the stand level such that the coordination over time of silvicultural activities such as cutting, thinning and planting are achieved in the best way. For tactical models these silvicultural activities are defined for stands homogeneous in terms of species type, age, location, site-class etc. The tactical model is capable of measuring the impact of local investment decisions associated with the aforementioned silvicultural decisions.

On the other hand, the strategic forest planning model is more aggregate and is concerned with macro-level investment decisions such as plant/mill expansion and construction, whether to spray or not and road building. The strategic model is more spatially aggregated; stands are grouped into macro-stands which are homogeneous with respect to the characteristics relevant to the problem. In short, the strategic model need not be concerned with the level of detail required for tactical decision making. The relationship between the two levels of decision making is critical. The higher level strategic model should be capable of integration with the disaggregated tactical model. The strategic model would lead to investment recommendations which the tactical model should follow.

(2) ---- *The framework should be flexible, amenable to evolutionary improvements and "transparent" to the user.*

Extant forest planning models such as TimberRAM (Navon [1971]), MUSYC (Johnson and Jones [1977]) and TREES (Tedder et al. [1980]) are in the form of complex, fixed structure FORTRAN based programs. Users find these systems complex, difficult to understand and time consuming to work with. We suggest a modelling framework where field data from a particular forest should be easily aggregated to provide a model for solution which experienced foresters may use to judge the feasibility of a harvesting schedule in the light of constraints which may not be easily mathematized.

(3) ---- *A simulation-optimization or simulation-gaming solution paradigm is sought.*

We recognize that we are not aiming for a single "optimal" solution to the FMP. Rather, we are attempting to understand, by altering the structure of the problem, the impact of alternative silvicultural strategies and environmental uncertainty on the form of the optimal solution. This method is a collection of approaches that are more exploratory and less directed at genuine optima. The *simulation-optimization approach* is more suited to complex large scale problems which defy an exact solution.

(4) ---- *A validation procedure should be developed to measure the worth of the prescriptive model.*

Traditional model validation (of descriptive models) involves designing tests such that the model, if false, would fail to pass. We suggest a validation procedure for the prescriptive fire and pest protection model which: a) tests how confident we are to use the deterministic policy when faced with obvious uncertainties such as fire and infestation losses; and b)

provides a sensitivity analysis to examine the model's response to parametric and structural changes. Once validated we show that the deterministic model is useful for distinguishing alternative silvicultural policies

1.3 A Research Overview

In response to the thesis objective, an overview of the major research accomplishments is listed in order of importance:

- (1) the development of a flexible mathematical programming modelling system to address the *fire and pest protection* forest management problems.
- (2) a concise *model representation method* using GAMS, a generalized algebraic modelling system and a *processing network* description of the forest management problem which motivates the underlying algebraic structure as well as suggesting the use of faster *network flow* solution algorithms;
- (3) validation of the model by: a) providing an approximate solution to the stochastic fire and pest protection forest management control problem using an *iterative simulation-optimization procedure* ; and b) testing how sensitive the model is to changes in discount, fire and inter-period flow rates.
- (4) a *comparative scenario analysis* to economically evaluate alternative deterministic infestation protection policies which demonstrates the usefulness of the model in distinguishing policy options.

These areas of research are outlined below.

1. The Fire and Pest Protection Model

The fire and pest protection model provides us with a generalized mathematical programming system which allows the flexible integration of all the following concepts in a single framework:

- (1) species diversity;
- (2) infestation of susceptible species;
- (3) natural regeneration and planting;
- (4) conversion of susceptible to non-susceptible species;
- (5) pest protection by spraying;
- (6) age-specific harvesting; and
- (7) catastrophic losses due to fire.

The design of this framework was influenced by the four research goals outlined in the previous section:

- the model is capable of answering strategic questions such as: does it pay to protect infested stands by spraying or by conversion harvests or both in the long run? (*goal 1*);
- the fire and pest protection problem was concisely and efficiently modelled (*goal 2*);
- model results were concerned with changes in the form of the optimal timber supply rather than exact solutions (*goal 3*); and
- catastrophic losses due to fire were simulated in an iterative simulation-optimization state updating procedure (*goal 4*);

The fire and pest protection model builds on the work of Reed and Errico [1986], [1987] and Boychuk [1988]. Reed and Errico [1986], [1987] represented the notions of catastrophic loss due to fire and infestation of susceptible species in separate model structures and did not address the concept of protection by spraying.

Boychuk articulated protection of susceptible species in a problem representation that required efficient modelling. The model framework in this thesis integrates all of these concepts in an efficient and evolutionary manner. That is, the concepts of conversion harvests and protection by spraying are introduced in a way that shows how the base model's structure is augmented. Each model is solved to show, by a comparative scenario analysis, the effect of including structural changes which reflect alternative policy options.

Furthermore, Reed and Errico [1986] and [1987] transformed the generic so-called Model II (Johnson and Scheurman [1977]) into a "standard form" LP using matrix algebra so that the matrix generation (i.e. matrix A and the right hand side vector b) could be constructed with SAS PROC MATRIX (SAS Institute Inc., 1982). This mathematical approach loses the intuitive appeal of a natural algebraic "scripting" of the various problem structures which are *directly* implementable using a *high level modelling language*.

2. Model Representation Method

Acceptance of complex mathematical programming systems by forest managers is an important consideration. This is achieved only if the managers understand the *problem statement*. This *model representation problem* which involves the successive development of a problem statement includes a formal model, model instance, algorithm output and final presentation was addressed by using a high level modelling language called GAMS (Brooke, Kendrick and Meeraus [1988]). Since model representations are tools to help conceptualize and communicate the problem or decision situation a premise underlying this thesis is that, throughout the problem solving process, the corresponding representations should make it as easy as possible to use the solution technology. This idea is considered

to be one of the principal contributions of the *decision support* system methodology which lays more emphasis on infrastructure (i.e. the representations surrounding the problem solving process) than on algorithms. We feel that the *model representation problem* was effectively solved through the use of the *GAMS language* and the *processing network* representation of the forest management problems.

The inter-temporal dynamics of growth, harvesting, insect infestation, protection by spraying and catastrophic losses due to fire were graphically represented by a network structure. This network contains "processing nodes" as described by Chinneck [1990] and so is amenable to faster LP solution algorithms. We take advantage of this special processing network structure to illustrate the mechanisms of growth, regeneration harvests, conversion harvests, protection of infested stands and catastrophic loss due to fire. The *flow conservation rule* at a node together with the *ratio of flow rule* motivates the algebraic development of the alternative model structures.

3. Stochastic LP Simulator

An iterative simulation-optimization procedure was developed to examine how well the deterministic model using average per period fire-loss and infestations would hold in a stochastic environment. Questions which are asked include: *how confident are we to use the deterministic policy when faced with obvious uncertainties such as infestations and fire losses?* and *how accurate are the deterministic timber supply projections?* The "stochastic LP simulator" involved:

- (1) solving the deterministic fire and pest protection problem based on the current system state using average period infestation and fire rates; and
- (2) updating the system state based on the optimal harvests and protections obtained from the first period in (1), and simulating the state transition at the next period with the dynamical area balance equations (the simulation model)

incorporating random infestation and fire proportions drawn from an empirical distribution.

A sample path of the approximate optimal solution to the *stochastic control problem* was obtained. Several such sample paths were constructed and compared with the deterministic solution. It was found that the deterministic solution follows closely the approximate stochastic solutions. From this stochastic simulation we concluded that the deterministic fire and pest protection model is valid for evaluating alternative policy options.

4. Comparative Scenario Analysis

This analysis, believed to be the first attempt to economically evaluate the effect of introducing management controls for Western Spruce Budworm directly in the forest management problem structure, involves solving the fire and pest protection model with various controls articulated. The solution obtained from solving the baserun fire and pest model with no fire and no protection for infestation by spraying or conversion harvests which rehabilitate the susceptible uninfested and infested stands was compared with a solution from a model which allowed for either:

- (1) conversion harvests; or
- (2) protection spraying; or
- (3) conversion harvests *and* protection spraying.

The results indicated that it does not pay to rehabilitate the susceptible uninfested or infested stands by conversion harvests. The optimal solution does select some conversion harvests over harvesting the susceptible and replanting with susceptible but there is no change in the total discounted net present value (NPV) or the optimal timber supply trajectory.

On the other hand, inclusion of protection of the infested stands by spraying does have a significant impact on the long run maximum sustained yield. The annual harvest level is increased by 23 percent. The total NPV for the protection solution is $\$4.838 \times 10^9$ as compared with $\$3.948 \times 10^9$ for the baserun-infestation scenario. It clearly pays to protect the stands by spraying.

From a modelling point of view the fire and pest protection model successfully discriminates between policy options which demonstrates that the model is useful. We have proposed that a suitable criterion of worth for the prescriptive model is flexibility in evaluating alternative futures. The successful discrimination of alternative policies is demonstrated in the comparative scenario analysis.

1.4 Thesis Outline

In the following chapter we provide some background for modelling the general economic forest management problem. In chapter three state-of-the-art mathematical programming abstractions of this problem are discussed. This identifies a need for models that are capable of representing the growth and harvesting process over a long time horizon, with sufficient harvest zones so as to reasonably represent a region, and permit the optimal allocation of wood produced from harvesting as well as recognizing the effect of catastrophic losses from insects, diseases and fire on the long run supply of timber.

In chapter four we provide a formal statement of the proposed forest *modelling* and *model representation* problem: *to develop a concise, easy to understand mathematical programming model to determine the optimal timber regeneration harvest and pest protection schedule that produces a specified wood quantity in the presence of fire and insect infestation risk at maximum net profit.*

In chapter five the alternative problem structures are represented graphically. This reveals an underlying *processing network structure* which motivates the algebraic description of the models and suggests more efficient solution methods. Chapter six

provides an algebraic description of the fire and pest protection models, showing how protection of infested stands by spraying and conversion harvests are integrated into the model structure.

Model validation is addressed in chapter seven by testing the deterministic policies in a simulated stochastic fire and infestation environment by using the optimal deterministic solution in a feedback procedure. We extend this validation by performing a sensitivity analysis to examine the timber supply response to changes in key parameters. In chapter eight the usefulness of the "validated" deterministic model is examined in terms of how flexible the design approach to mathematical programming is, in comparing alternative policy options. Finally, chapter nine presents conclusions and suggests areas of further research.

Chapter 2

Background

In this chapter we provide some background for modelling forest management problems. This consists of three topics: first, an historical perspective of forest harvest scheduling systems (sections (§) 2.1 to § 2.4) ; second, an introduction to embedded network problems in linear programming solutions to the forest management problems which are identified as processing networks (§ 2.5) ; and finally a discussion of high level modelling systems used to implement these problems (§ 2.6).

2.1 The Classical "Rotation" Problem

The classical forest economic stand management problem (the so-called "rotation" problem) chooses the age at which trees ought to be cut so as to maximize the present value of the flow of profits. This problem was first solved by Faustman [1849]. The basic idea is that the volume of wood on a land unit (hectare) over time may be described by a function $V(t)$, $t \geq 0$. The net revenue $NR(t)$ for cutting a forest aged t years is $NR(t) = pV(t) - c$ where p is the price per unit volume and c represents costs of harvesting and regeneration. The problem is to decide on the *rotation period* t which maximizes the net present value $NPV(t)$ of an infinite series of rotations under continuous (instantaneous) interest rate r . If we start with a forest at age zero then we solve:

$$\text{Max}_t \{ \text{NPV}(t); t \geq 0 \} \quad (2.1)$$

$$\begin{aligned} \text{Where }^1 \quad \text{NPV}(t) &= (pV(t)-c)e^{-rt}[1+e^{-rt}+(e^{-rt})^2+\dots] \\ &= \sum_{r=1}^{\infty} e^{-nrt} (pV(t)-c) \\ &= \frac{pV(t)-c}{e^{rt}-1} \end{aligned} \quad (2.2)$$

The Faustman solution is that the optimal rotation t^* must satisfy :

$$\frac{dV(t^*)}{dt} \frac{1}{(V(t^*)-c/p)} = \frac{r}{(1-e^{-rt^*})} \quad (2.3)$$

This follows from determining the optimum t^* for $\frac{d\text{NPV}(t)}{dt} = 0$

The important point to recognize is that this problem is focussed on the growth process for a unit area of land (i.e. a hectare). As pointed out by Nautiyal and Fowler [1980] the use of the Faustman formula implies a perfectly competitive stumpage market and an "atomistically small landowner" supplying wood to a market whose price cannot be

¹where e^{-rt} is the continuous discount factor and is related to the discrete discount factor $(1+i)^{-t}$ where i is the annual interest rate (compounded annually) by $e^r=1+i$. This reasoning by Johannson and Lofgren [1985 p 17] is reiterated as follows. Consider a period of one year, which is divided into a large number n of elementary periods. If r is the continuous interest rate, then r/n is the interest earned on one dollar during the short period $1/n$. At the end of the year the dollar will be worth $v=(1+r/n)^n$. If the period is made infinitely small v will converge to e^r .

That is $\lim_{n \rightarrow \infty} v = e^r$ where e is the base of natural logarithms. Recall from calculus the

limit : $\lim_{n \rightarrow \infty} [1+(1/n)]^n = e$.

affected by the amount of wood supplied. For an industry characterized by large landowners, as in Canada, these assumptions are not valid. Instead we must consider the sector requirements of a forest industry. We will contrast these simple dynamic economic models which are appropriate for the small land owner with the long-term mathematical programming models which address the needs of a forest industry operating on a large region. First, the general forest management problem (FMP) will be defined.

2.2 The General Economic Forest Management Problem

The economic FMP is a generalization of the classical "rotation" problem. The more general FMP takes into account the temporal evolution of the age-structure of many stands on the future supply of timber as well as allowing for variation in timber prices. We consider a timber firm with a given amount of land X , to manage, where the trees are of different ages. A *continuous time model*, borrowed from Berck [1976] will be used. Let $x(t,i)$ be the number of hectares at time t of timber in age-class i . Let Ω be the set of all possible ages.

$$\text{Then } X = \int_{w \in \Omega} x(t,w) dw \quad (2.4)$$

We define timber cutting as $c(t,i)$, the number of hectares with trees of age i which is cut at time t .

The growth process is defined by:

$$x(t,i) = x(t-s,i-s) - \int_{w=0}^s c(t-w,i-w)dw \text{ for all } s. \quad (2.5)$$

The area where timber of age-class i grows at time t equals the area that at time $t-s$, was occupied by age-class $i-s$ minus what was cut in the interval $[0,t-(t-s)] = [0,s]$. Finally the area occupied by age-class 0 at time t is given by :

$$x(t,0) = \int_{w \in \Omega} c(t,w) dw \quad (2.6)$$

Equations (2.4)-(2.6) define the forest's technology. For an *economic management problem* we must define the volume of timber of age-class w at time t and its unit price. Let $g(w)$ be the amount of wood (cubic metres) on a hectare of land in age-class w . Also, we assume that the price, $p(t)$, of any age-class depends only on time and not on age-class. The forest owners investment criterion is to maximize the present value of the income stream of the cutting.

Hence the general economic FMP is:

$$\text{Max}_{c(t,i)} \int_{t=0}^{\infty} [p(t)e^{-rt} * \int_{w \in \Omega} c(t,w)g(w)dw]dt \quad (2.7)$$

subject to the aforementioned technology (i.e. (2.4)-(2.6)), some initial conditions on the state of the forest and the time stream of wood product demand at the mill. Here $p(t)$ is the price per cubic metre of wood and r is the prevailing continuous interest rate in a perfect capital market. The general FMP, as defined above, is an *infinite horizon, continuous time, investment decision problem*, where the profit function may be nonlinear, constrained

by a nonlinear forest technology. The complexity of this, as yet unsolved, generalization of the FMP in the theory of forest economics was identified by Johannsson and Lofgren [1985].

2.3 Forest Harvest Scheduling

Operational management of the forest over time is called *forest harvest scheduling*; in its most general sense it can be viewed as the *what, where* and *when* of the management of a forest. Forests are classified as being composed of either *even-aged* or *uneven-aged* stands depending on the way in which they are managed. An even-aged stand is a group of trees whose individual trees originated at about the same time, either artificially or naturally. It grows, is cared for, may undergo various intermediate cuttings (either precommercial or commercial thinnings) during its development and is eventually removed in one or more major harvest cuts after which a new stand is established. The interval of time between these two points is the *rotation*. This establishment may occur naturally or be the result of site preparation (*scarification*) and planting (*basic regeneration*). On the other hand, *uneven* or many aged stands have no beginning (except when the stand is first established) or ending; they may be managed in a continuing series of partial cuttings. The period between these harvests is called the *cutting cycle*. In North America most forests are managed on an even-aged basis.

How the forest is harvested and regenerated (e.g. whether by clear-cutting and planting leaving seed trees or some form of partial cutting) is called the *silvicultural system*. The schedules of activities undertaken by management (i.e. the sequence in time of thinnings and final cuttings) are called *management regimes*. According to the model classification system of Shugart [1984] the "*mixed-aged, monospecies, non spatial tree*" model structures (e.g. Susuki and Umemura [1974] and Susuki [1984]) are relevant within the scope of this problem. The forest managers main problem in formulating a suitable harvesting policy is to generate a sustained yield of wood products while aiming toward a

targeted desirable forest structure (the *silvicultural goal*) sometime during or at the end of the planning horizon.

What is meant by this statement? The concept of *sustainability*, as its name implies, means to produce a forest structure (i.e. a distribution of area by age-class) such that each year the amount which is removed from the forest is less than or equal to the increment which grows during that year. Foresters have understood this to mean that a structure which consists of equal area in each age-class. This so-called *balanced* or *normal* or *fully regulated* forest produces a harvest which is sustainable. The range of age-classes in the forest determines how fast the forest will "rotate" through these age-classes. Various regulatory schemes have been developed which convert a present structure to the desirable target structure over time. These will be discussed in the next section.

Within this transition, the forest manager is concerned with determining the annual allowable cut of timber: the *maximum sustainable yield (m.s.y)* from period to period which will permit reaching this target forest. Forest harvest scheduling is the determination of this cut; it is the means by which a forest is converted from an initial structure to a desirable one. In Canada, the provincial governments are responsible for regulating how much timber should be cut from the forests each year. The so-called *annual allowable cut* or *maximum allowable depletion* is specified for each allowable cut administration unit (*Forest Management Unit*: [FMU] or *Timber Supply Area* [TSA]) over a long time horizon. Commercial timber companies are then responsible for managing these FMU's under a long term forest management agreement with the provincial Crown. These companies frequently ask the question:

- *What are the minimum production costs associated with forest management regimes designed to attain various sustainable harvest levels over time for the FMU?*

That is, for each sustainable harvest level (the annual allowable cut):

- *What are the associated minimum cost timber management strategies or what is the potential for this FMU to respond over time to various intensities of management?*

Management intensity is taken to mean the costs of accessing, tending, harvesting and transporting the products from various stands within a forest over time. To answer these questions it is necessary to characterize the forest such that its component stands, subject to different management activities, and the areas cut-over can be quantitatively projected. Also management must estimate the costs and effects of harvesting and transporting components of stands at different ages and various silvicultural treatments such as scarification and planting, precommercial and commercial thinning.

2.4 Traditional Methods

As pointed out by Davis [1966], over the years of forest history, a great many methods for *determining the cut* or so-called *harvest scheduling* have been developed. Most of these methods originated in Europe and only a few have survived. Those that have, may be classified into two general approaches:

- (1) area control; and
- (2) volume control.

With both, the answer is always in the volume of timber to be cut, and as rightly asserted by Davis [1966], neither can be applied exclusively in giving a complete answer. Since timber volume is necessarily cut in whole trees that occupy area the relationship of volume with area is inescapable. When trees are cut they release area - land - which is the fundamental production base for the forest industry. With clear-cutting the relationship is simple and is absorbed in the volume per unit area yield curves. Consequently, although

area and volume control may be viewed as separate methods the end result sought is always the volume to be cut from areas and neither approach can give the complete answer.

2.4.1 Area Control

The simplest approach to harvest scheduling is *area control* which defines the annual harvest in terms of land area. A special case is when this area is defined equal to the total forest area, A , divided by the "rotation age", R , (that age at which the mean annual increment of the stand's wood volume is maximum). If this annual rate of cutting is effected over the entire forest for R years a fully regulated forest will be achieved in R years. This regulated forest will sustain an annual cut equal to $A \cdot v(R)/R$, (in theory), in perpetuity where $v(R)$, the so called yield curve, is the yield per hectare at age R . These curves normally reach a maximum and then decline to reflect the loss in yield due to mortality in the stand. The problem with area control is that if the forest's age structure departs from normality the volume obtained will be uneven over time. For example, if the forest is *over-aged* and the oldest age cut rule is used, harvest volumes generally start low and then increase during the transition period toward the long run sustainable allowable cut. This is unacceptable to the mill owner who demands a steady flow of timber for production.

2.4.2 Volume Control

In contrast to area control is *volume control* in which the annual harvest is specified in terms of wood volume and the cut over area follows from this. Recognition of the impact of these annual harvests on the areal distribution of the stands for the whole forest must be made. Historically, volume control has been applied by simple formulae methods (e.g.

Hanzlik's² or Von Mantel's³ formulae). For example, if the forest contains much over-age timber the allowable cut is calculated using Hanzlik's method as that which liquidates the over-age timber in R (the rotation age) years plus an allowance for increment in the younger stands. These formulae methods do not explicitly take into consideration the time evolution of the forest's age structure.

2.5 Processing Networks and Linear Programming

More recently, mathematical programming models have been used to solve the general forest management problem. They utilize two simplifying assumptions. First, a *finite planning horizon* is assumed. This assumption implies that at the end of the planning horizon the forest has no value. Since the age distribution of the forest in any given future period has implications for the future supply, this assumption is not valid. However, it can be shown that if prices are bounded, the loss in present value from stopping after a finite number of periods may be made arbitrarily small. The second simplifying assumption required is that time is *discrete* and not *continuous*. With a *linear technology* and *linear objective* the problem becomes a standard linear programming model [LP] for which a solution may be obtained using the *simplex method* of Dantzig [1963].

In Canada, Beck [1985] describes the transition of the western provinces from using simple formula based methods (e.g. Hanzlik's or Von Mantel's formulae) to calculate annual allowable cuts to data intensive mathematical programming harvest scheduling systems which have been developed in the United States. These linear programming models which include TimberRAM (Navon [1971], MUSYC (Johnson and Jones [1977]) and TREES (Tedder et al [1980]) have been adapted to Canadian conditions such that management alternatives not represented in the original model structure could be included.

² Hanzlik's formula (Hanzlik [1922] was developed to initiate management on unregulated Douglas Fir stands of the Pacific Northwest. The assumed cut = $V/R + I$, where V =volume of mature merchantable timber above rotation age R and I is an estimate of the increment.

³ Von Mantel's formula is based on the assumption that the growing stock, G , may be represented by the area of a right triangle. The assumed cut is $2G/R$. c.f. Davis [1966] p110.

The LP formulations are based on the flow of areas between time periods, species types and age-classes and can be conveniently described as a *network flow problem* with side constraints as we will see in chapter five. These network problems which have proportional flow restrictions on the arcs entering or leaving some nodes are called *processing network problems*. Processing network terminology was introduced by Koene [1982] and a survey of applications at that time is given in the same paper.

The forestry problems consist of two types of constraints: those supplied by the user in the problem formulation, and the "structural" constraints which enforce inter-temporal conservation of area. It is these structural constraints which may be portrayed by a processing network.

As pointed out by Chinneck [1990] the distinguishing feature of a processing network is the presence of one or more processing nodes as shown in Figure 2.1. A strictly positive number, p_i , called the "processing ratio figure", is associated with each terminal i of the processing node. In Figure 2.1 we label the inflow terminals from 1 to r and the outflow terminals $r+1$ to t then for a processing node we ensure conservation of flow across the node by maintaining the identity:

$$\sum_{i=1}^r p_i = \sum_{i=r+1}^t p_i \quad (2.8)$$

The processing ratio figures are used to construct two-way flow ratio equations of the following form:

$$x_i / x_j = p_i / p_j \quad (2.9)$$

where x_i represents the edge flow at terminal i . In an LP formulation the following processing ratio equation is used :

$$p_j x_i - p_i x_j = 0 \quad (2.10)$$

If the processing node has t terminals then it may be represented by $t-1$ independent equations of the form (2.10). Most processing network problems consist of both "processing" nodes denoted by squares and "regular" nodes denoted by circles. Pure processing network problems are *minimum cost flow* problems. Each edge in the processing network may have an associated cost per unit of flow. The problem is to assign values to edge flows which minimize the total cost.

We utilize the processing network representation to motivate the underlying algebraic description of the "structural" constraints which represent the temporal interdependence of forest areas in each age-class under harvesting, insect infestation, protection by spraying, intensive silviculture (i.e. planting) and catastrophic losses due to fire. According to Bixby and Fourer [1988], embedded networks in linear programming problems are of interest for two reasons. First, as we have alluded to, they suggest a useful and transparent interpretation of the linear program. Second, the presence of network constraints can allow the LP to be solved faster provided its structure is exploited by specialized fast network algorithms such as those described by Chen and Engquist [1986], Koene [1984] and Glover and Klingman [1981].

P_i : processing ratio figure for terminal i

X_j : flow variable for edge j

Figure 2.1: A Processing Node with r Inflow and $t-r$ Outflow Terminals

2.6 Model Development Systems

In a recent survey, Sharda [1988] identifies two high level mathematical programming languages GAMS (Bisschop and Meeraus [1982]) and XPRESS-LP (Dash Associates, UK). The basic idea behind these *model development systems* is that most mathematical programming problems have only a few generic types of constraints. By using a modelling language, it is necessary to enter those equations once only. Similar constraints are automatically generated using exogenously defined data. Both systems provide conditional rows, column generation, and general algebraic expressions for the coefficients. GAMS requires the data to be included in the problem specification whereas XPRESS-LP can read data from external files.

We have chosen a high level model development system since, as recognized by Arden [1980], the limitations of mathematical programming do not lie in the computational algorithms but rather in the amount of work required to prepare large problems and analyze the results. That is, we are attempting to address the *model representation problem* which requires a concise, easy to understand, statement of the mathematical programming problem.

The purpose of the naming convention in a high level modelling system is to make the statement of the problem as transparent as possible. Since these representations are written in straightforward algebra, few of the operators require explanation. The models become self-documenting and understandable.

2.6.1 GAMS and the Solution Method

The forest management problem (FMP) proposed in chapter four can be solved using linear programming. Since the model structure involves concepts which have not been articulated in existing timber harvest scheduling systems such as a Model I TimberRAM by Navon [1971] or a Model II FORPLAN by Johnson [1986] a generalized mathematical programming (modelling) environment was used for the implementation of the FMP. This

involved several phases of effort that required verification of their utility and validity. As reviewed by Sharda [1988] the phenomenal growth of microcomputers has resulted in a proliferation of management science (MS) and operations research (OR) software, including LP software. Since we are concerned with a *model representation problem* a high level mathematical modelling language was sought for solving the fire and pest protection models.

We chose GAMS/MINOS because it provided a simple, clear statement of a complex problem. GAMS/MINOS is a combination of two mainframe programs, GAMS (Bisschop and Meeraus [1982]) and MINOS (Murtagh and Saunders [1982]). By using a high level modelling language such as GAMS it was possible to develop concise, syntactically correct, representations of the alternative forest fire and pest protection models so that the results could be interpreted and validated more easily. High level modelling languages base their representation style on the commonly used algebraic notation for mathematical programs. Model development systems concentrate on easy to use representations of the mathematical programming problem rather than insightful representations of the algorithm output. They are designed to make the construction and solution of large and complex mathematical programming models more straightforward for programmers and more comprehensible to users of models from other disciplines e.g. economists. It can substantially improve the productivity of modellers and greatly expand the extent and usefulness of mathematical programming applications in policy analysis and decision making.

The PC version of GAMS (Brooke et al. [1988]) allows easy development of customized harvest scheduling models using a *design approach* to model building. That is a library of examples is catalogued so that further models may be constructed building on the ideas of previous models. Automatic model documentation is achieved by having a language that is easily read by people and computers. This assures that the model the analyst conceives is in fact being solved by the computer. Also the readability of the high level modelling programs means the analyst can explain the model to others with ease and benefit from their comments and suggestions.

2.6.2 The GAMS/MINOS Solver

The problems defined in the next sections are all linear programming problems and are solved using a specially adapted version of the MINOS optimizer called GAMS/MINOS. GAMS/MINOS is a FORTRAN-based system designed to solve large-scale optimization problems expressed in the following form:

$$\begin{array}{ll} \text{Minimize } \mathbf{c}^t \mathbf{x} & (2.11) \\ \mathbf{x} \end{array}$$

$$\text{subject to } \mathbf{Ax} \leq, \geq \text{ or } = \mathbf{b} \quad (2.12)$$

$$\mathbf{l} \leq \{ \mathbf{x} \} \leq \mathbf{u} \quad (2.13)$$

where the vectors $\mathbf{c}$, $\mathbf{b}$, $\mathbf{l}$, $\mathbf{u}$ and the matrix $\mathbf{A}$ are constant. The components of $\mathbf{x}$ are called the linear variables and the equations in (2.12) are the linear constraints. Let m and n denote the total number of constraints and variables. Thus, $\mathbf{A}$ has m rows and n columns. The constraints (2.13) specify upper and lower bounds on all variables. The vector $\mathbf{b}$ is called the right-hand side.

Linear Programming

Problems defined in § 6.1 to § 6.4 are linear programming problems. GAMS/MINOS converts all constraints into equalities, and the only remaining inequalities are simple bounds on variables. The linear program is written as :

$$\begin{array}{ll} \text{Minimize } \mathbf{c}^t \mathbf{x} & \text{subject to } \mathbf{Ax} + \mathbf{Is} = \mathbf{0}, \mathbf{l} \leq \{ \mathbf{x}, \mathbf{s} \} \leq \mathbf{u} \\ \mathbf{x}, \mathbf{s} \end{array}$$

where the elements of x are GAMS variables, and s is a set of *slack variables*: one for each general constraint.

GAMS/MINOS solves linear programs using a reliable implementation of the *primal simplex method* (Dantzig [1963]), in which the constraints $Ax + Is = 0$ are partitioned into the form $Bx_b + Nx_n = 0$ where the basis matrix B is square and nonsingular. The elements of x_b and x_n are called the basic and nonbasic variables respectively.

GAMS/MINOS maintains a sparse LU factorisation of the basis matrix B , using a Markowitz ordering scheme and Bartels-Golub updates, as implemented in the Fortran package LUSOL (Gill, et al [1987]). (See Bartels and Golub [1969]; Bartels [1971]; Reid [1976] and [1982]). The basis factorisation is central to the efficient handling of sparse linear and nonlinear constraints.

Chapter 3

State-of-the-Art Forest Harvest Scheduling

In this chapter a review of the current mathematical programming abstractions of the general economic forest management problems is given. A detailed description of the two most widely used linear programming formulations: Model I and II forms (Johnson and Scheurman [1977]) is given in sections 3.1 to 3.3. The general dynamic linear system age-structured model of Reed and Errico [1986] is introduced in section 3.4 and its relationship to the Model II form is discussed in section 3.5. In section 3.6 the Reed and Errico Model III is formulated as a linear deterministic control problem which can be solved using standard linear programming. The general linear system age-structured model is augmented in section 3.7 by Reed and Errico [1986] to reflect catastrophic fire losses and shown to be a stochastic control problem for which approximation solution methods are suggested. Finally the best control model of Boychuk [1988] is described.

3.1 Mathematical Programming Models

To date the most widely used technique for scheduling timber harvest and investment on an area of timberland under even-aged management is either a Model I or Model II LP formulation as classified by Johnson and Scheurman [1977]. The difference between Model I and II stems from the definition of the activity. In Model I, an activity refers to a

complete set of actions that occur on a particular site class or management unit (whose initial classification is maintained over the entire planning horizon). Model I was originally suggested by Kidd et al [1966] and Ware and Clutter [1971] and implemented by Navon [1971] in the Timber Resources Allocation Method TimberRAM. Alternatively, in Model II each age-class containing hectares in the first period forms a management unit until those hectares are harvested. Then these harvested hectares form a new age-class (the youngest) and management unit until harvested again. Model II form was implemented by Johnson and Jones [1979] in the Multiple Use Sustained Yield Calculation known as MUSYC. This model has been calibrated for Canadian conditions by the British Columbia Ministry of Forests to develop a Timber Supply Analysis for the Kamloops Timber Supply Area.

Both of the Model forms I and II initially assume that:

- (1) the forest has one site class which is divided into a number of age-classes;
- (2) the forestland area is fixed for the period of analysis;
- (3) the number of years between age-classes equals the length of time periods;
- (4) clear-cutting is used as the regeneration harvest;
- (5) regeneration occurs in the same period as the regeneration harvest; and
- (6) all losses from fire, storms, insects and disease are incorporated into the yield.

3.2 Model I Linear Programming

To motivate understanding of Model I consider the problem where a forest manager wishes to schedule harvests over the next six (10-year) periods for a forest that now has two 10-hectare cutting units¹ (age-classes), one four periods old and one eight periods old. It takes a minimum of two periods for a stand to reach merchantability. To use Model I, for

¹ A cutting unit is a mutually exclusive geographically distinct area of the forest. Cutting units may be created by (1) identification of a spatially separate stand; (2) consolidating geographically contiguous stands that are managed, after the initial harvest, as one stand; (3) pooling similar but non-contiguous stands that are close enough geographically to be managed as a single unit; or by (4) combining similar stands over the entire ownership (i.e. by aggregating all stands of a certain age-class).

each sequence of regeneration harvests that can occur over the next six periods in each age-class an activity must be defined. Table 3.1 shows that there are 21 such sequences (activities or management regimes) including the option of not harvesting at all over the next six periods. Each sequence differs from the others in the number of harvests during the time horizon and (or) in the age at time of harvest. Each alternative will produce a different pattern of wood production over time.

Also, as pointed out by Gunn and Rai [1987], these alternatives will produce different wood commodities (e.g. sawlogs and pulpwood). In the following model structures it is assumed that all harvests occur at the mid point of the cutting periods.

Model I

If the objective of management is to maximize the utility of the forest for the owner (as suggested by Ware and Clutter [1971]; that is to maximize the total discounted net revenue from timber harvests over the planning horizon plus the value of the inventory left at the end of the planning horizon, the optimization problem becomes a Model I formulation:

$$\text{Max}_x \sum_{h=1}^2 \sum_{q=1}^{21} D_{hq} x_{hq} \quad (3.1)$$

subject to the following area constraints:

$$\sum_{q=1}^{21} x_{1q} = 10 \quad \text{and} \quad \sum_{q=1}^{21} x_{2q} = 10 \quad (3.2)$$

where x_{hq} = hectares of management unit h assigned to regeneration harvest sequence q

and D_{hq} = discounted net value per hectare of management unit h over the planning horizon if management regime q is used.

Here a management unit is an age-class defined at the beginning of the projection period and the harvest sequence q , (the activity) absorbs the temporal aspect of the problem. The use of alternative harvest sequences avoids the mathematical representation of the time process associated with each alternative activity both in the stand growth dynamics and the objective function. Such mathematical simplicity is obtained at the expense of defining an enormous number of decision variables for a typical realistic problem.

For example, Johnson and Scheurman [1977], identify 265,665 alternative harvest sequences for a problem with 5-year periods and age-classes, a 100-year time horizon and a minimum of two periods between regeneration harvests. However, as pointed out by Gunn and Rai [1987], this poses no problem for a Model I solution, since it decomposes by management unit and the solution to each management unit (age-class) is to choose the harvest sequence q^* which maximizes $\{D_{hq}\}$. In fact the real problem is to compute D_{hq} since this requires a full discounted cash flow analysis for each harvest sequence q .

harvest sequence number q	*	period number					
	*	1	2	3	4	5	6

1	*	no harvest					
2	*						H
3	*					H	
4	*				H		H
5	*				H		
6	*			H		H	
7	*			H			H
8	*			H			
9	*		H		H		
10	*		H		H		H
11	*		H			H	
12	*		H				H
13	*		H				
14	*	H		H		H	
15	*	H		H			H
16	*	H		H			
17	*	H			H		
18	*	H			H		H
19	*	H				H	
20	*	H					
21	*	H					H

Table 3.1: Possible regeneration harvest sequences over six periods for an age-class that contains merchantable volume in period one and where there must be at least two periods between regeneration harvests. Each H represents a possible regeneration harvest. (from Johnson and Scheurman [1977])

3.3 Model II Linear Programming

The principal difference between the Model I and Model II formulations lies in the definition of the activity variables. With a Model I, each activity variable is associated with a particular pattern of forest operations that spans the planning period. In the Model II formulation, each harvesting opportunity is represented by a separate activity variable. Consider management regime 6 given in the previous section (see Table 3.1). With a Model I formulation, the pattern of clearcut and planting in period 3, clearcut and plant in period 5 and ownership of a 15 year old plantation at the end of the planning period is associated with a single variable. In a Model II formulation, three activity variables would be required to describe this pattern; one to provide for harvest of the initial existing stand in period 3, another to provide for harvest of the second stand in period 5, and a third to define the existence of the 15-year-old stand at the end of the planning period. In the Model I the initial subdivisions of land (in our case the initial age-classes) are referred to as *cutting units* (unit 1 is the stand 4 periods old at the beginning of the planning period etc.) With a Model II formulation, the initial subdivisions of the forest do not retain their identity through the planning period. The two initial forest types are referred to as *initial strata*. (In most Model II formulations, the initial strata are the various age-classes in existence at the beginning of the planning period). A Model II formulation requires the definition of the following activity variables:

A_i = hectares of cutting unit i present at the beginning of period 1.

Y_{ij} = hectares of initial stratum i that are harvested in cutting period j ($i=1, 2$; $j=1, \dots, 6$)

Z_{jk} = hectares regenerated in cutting period j and harvested in cutting period k .

($j=2,3,4,5$; $k=j+1, \dots, 6$)

We assume that regenerated stands must grow at least one period before being harvested which explains the $j+1$.

W_i = hectares of initial stratum i that are left unharvested through periods i ($i=1, 2$)

U_j = hectares regenerated in period j and subsequently left unharvested until the end of the planning period ($j=2, \dots, 6$)

The following economic inputs are required:

D_{ij} = per hectare present value of costs and incomes associated with stands from initial stratum i that are harvested in period j ($i=1, 2 ; j=1, \dots, 6$)

E_{jk} = per hectare present value of costs and incomes associated with stands that are regenerated in period j and harvested in period k ($j=2, 3, 4 ; k=j+1, \dots, 5$)

T_i = per hectare value of leaving a stand from initial stratum i unharvested at the end of the planning period ($i=1, 2$)

Q_j = per hectare present value of leaving a stand that was regenerated in cutting period j unharvested at the end of the planning period ($j=2, 3, 4$)

The Model II objective is:

$$\text{Max } \phi = \sum_i \sum_j D_{ij} Y_{ij} + \sum_{j=2} \sum_{k=j+1} E_{jk} Z_{jk} + \sum_i T_i W_i + \sum_j Q_j U_j \quad (3.3)$$

$$\text{s.t. } \sum_j Y_{ij} = A_i - W_i \quad (i=1,2) \quad (3.4)$$

$$\text{and} \quad \sum_{k=j+1}^6 Z_{jk} + U_j - \sum_{i=1}^2 Y_{ij} = 0 \quad (j=2,3,4) \quad (3.5)$$

The first constraint requires that the assignment hectares for each initial stratum sum to the number of hectares A_i originally present in that initial stratum i . That is the area harvested equals the difference between the initial area of stratum i (A_i) and the surviving area at the end of the time horizon of initial stratum i (W_i). The j th constraint of the second set requires hectares of stands originating in cutting period j to equal hectares of stands assigned for harvest in period j .

3.4 Model II and the Age-Structured Model III

In order to clarify the interpretation of the way time is incorporated into the decision variables, reference to the age-structured Model II as developed by Reed and Errico [1986] will be made. Model II or what we call Model III requires the definition of the following variables. Let $x_i(t)$ for ($i=1, \dots, k-1$) be the area of the forest with even-aged stands in age-class i (i.e. with stands of age between $(i-1)$ periods and i periods at the beginning of period t , and $x_k(t)$ is the area of the forest with stands in oldest age-class k (i.e. with stands of age $(k-1)$ periods or older). Let $h_i(t)$ be the area clearcut harvested in age-class i during period t . We shall assume that when an area is cut it returns to age-class 1 and that it is reforested (or regenerates naturally) and at the start of period $t+1$ will be in age-class 2.

The dynamics of growth and alternative harvests may be understood more clearly by a graphical representation. Let us consider the previously discussed problem where there is forested area in two age-classes, one aged 4-periods and the other aged 8-periods at the beginning of the 6-period time horizon. The age-class width is 20 years as well as the simulation time step. The harvest, growth and regeneration possibilities are given in the graph of Figure 3.1. For the variable x each node may be referenced by (i,t) the age-class i

and the beginning of the time period t . At each node the sum of inflowing edges is equal to the state variable $x_i(t)$. We may construct the dynamics from the graph. Starting at nodes $(4,1)$ and $(8,1)$ we trace the evolution of harvest, growth and regeneration activities. For example the edge from $(8,1)$ to $(1,2)$ represents the area $h_g(1)$ harvested in time period 1 from age-class 8 and regenerates as age-class 1 at the start of time period 2. The arc from $(8,1)$ to $(8,2)$ represents the surviving forest area, $x_8(1)-h_g(1)$ from age-class 8 which "grows" into age-class 8 in the next time period 2. Since age-class 8 contains all forest area aged 8 and older there is not a jump in the age-class. At each node there is conservation of land area. For example at node $(1,2)$, $x_1(2) = h_4(1)+h_g(1)$. That is inflowing area equals outflowing area. The complete dynamics may be constructed for each time period. At the start of time period 1 we have the initial areas $x_4(1)$ and $x_8(1)$, then for periods $t=2, 6$ the following area balance equations hold; the first equation reads: the areas $h_4(1)$ and $h_g(1)$ harvested in period 1 from age-classes 4 and 8 become, at the start of period 2, age-class 1.

$$\begin{array}{ll}
 t=2 & \begin{aligned}
 x_1(2) &= h_4(1)+h_g(1) \\
 x_5(2) &= x_4(1)-h_4(1) \\
 x_8(2) &= x_8(1)-h_g(1)
 \end{aligned} \\
 \\
 t=3 & \begin{aligned}
 x_1(3) &= h_5(2)+h_g(2) \\
 x_2(3) &= x_1(2) \\
 x_6(3) &= x_5(2)-h_5(2) \\
 x_8(3) &= x_8(2)-h_g(2)
 \end{aligned} \\
 \\
 t=4 & \begin{aligned}
 x_1(4) &= h_2(3)+h_6(3)+h_g(3) \\
 x_2(4) &= x_1(3) \\
 x_3(4) &= x_2(3)-h_2(3) \\
 x_7(4) &= x_6(3)-h_6(3) \\
 x_8(4) &= x_8(3)-h_g(3)
 \end{aligned} \\
 \\
 t=5 & \begin{aligned}
 x_1(5) &= h_2(4)+h_3(4)+h_7(4)+h_g(4) \\
 x_2(5) &= x_1(4) \\
 x_3(5) &= x_2(4)-h_2(4) \\
 x_4(5) &= x_3(4)-h_3(4) \\
 x_8(5) &= x_7(4)-h_7(4)+x_8(4)-h_g(4)
 \end{aligned}
 \end{array}$$

$$\begin{aligned}t=6 \quad & x_1(6) = h_2(5) + h_3(5) + h_4(5) + h_8(5) \\& x_2(6) = x_1(5) \\& x_3(6) = x_2(5) - h_2(5) \\& x_4(6) = x_3(5) - h_3(5) \\& x_5(6) = x_4(5) - h_4(5) \\& x_8(6) = x_8(5) - h_8(5)\end{aligned}$$

Figure 3.1: Graphical Representation of the Age-Structured Model III.

It is easy to see from above that the general dynamics may be represented by :

$$x_1(t+1) = h_1(t) + h_2(t) + h_3(t) + \dots + h_k(t)$$

$$x_i(t+1) = x_{i-1}(t) - h_{i-1}(t) \quad \text{for } i = 2, 3, \dots, k-1 \quad (3.6)$$

$$x_k(t+1) = x_{k-1}(t) - h_{k-1}(t) + x_k(t) - h_k(t)$$

How do these variables relate to those used in the Model II formulation? We see that the variables $h_i(t)$ associated with the original forest area defined at the beginning of the planning period correspond with the Y_{ij} of the model II formulation. In period 1 if we define stratum $i=1$ to be the age-class 4 and $i=2$ to be age-class 8 then the following correspondence occurs: $Z_{11}=h_4(1)$, $Z_{12}=h_5(2)$, $Z_{13}=h_6(3)$, $Z_{14}=h_7(4)$. However, for $t=5$ since in the age-class model area surviving from age-class 7 becomes age-class 8 it is no longer possible to distinguish whether $h_8(5)$ consists of originally stratum 1 or 2 (i.e age-class 4 or 8 at the beginning of the planning period). The age-class model is different and will be called Model III.

Similarly we can make a correspondence between the Z_{jk} of Model II and the $h_i(t)$ of the age-class or Model III formulation. For example we have $Z_{23}=h_2(3)$, $Z_{24}=h_3(4)$ and $Z_{25}=h_4(5)$.

For the unharvested area at the end of the planning period we can only say $W_1 + W_2 = x_8(6)$ since we cannot distinguish whether $x_8(6)$ consists of original stratum 1 or 2.

For the U_j 's in this example we see that :

$$U_2 = x_5(6), U_3 = x_4(6), U_4 = x_3(6) \text{ and } U_5 = x_2(6).$$

We can verify these correspondences by examining the constraint :

$$\sum_{k=j+1}^5 Z_{jk} + U_j - \sum_{i=1}^2 Y_{ij} = 0$$

for $j=2$ we have:

$$Z_{23} + Z_{24} + Z_{25} + U_2 - Y_{12} - Y_{22} = 0$$

We want to prove that the corresponding variables:

$$h_2(3) + h_3(4) + h_4(5) + x_5(6) - h_4(1) - h_8(1) = 0$$

Using (2.5) we eliminate the variables $x_5(6)$, $x_4(5)$, $x_3(4)$ etc. recursively and obtain:

$$\begin{aligned} h_2(3) + h_3(4) + x_4(5) - h_4(1) - h_8(1) &= h_2(3) + x_3(4) - h_4(1) - h_8(1) \\ &= x_2(3) - h_4(1) - h_8(1) \\ &= x_1(2) - h_4(1) - h_8(1) \\ &= h_4(1) + h_8(1) - h_4(1) - h_8(1) \\ &= 0 \end{aligned}$$

3.5 Discussion of Models I, II and III

The Model II and III forms have several advantages over Model I. First, they require far fewer decision variables and secondly their mathematical structures are more amenable to systems analysis techniques and parameterization of fire, insect and other loss agents within the structure of the model. However, a loss in usefulness will result from not being able to identify the original stand structure over the planning horizon in this model form. It is argued by Gunn and Rai [1987] that for the purposes of long range planning this is not an important issue. At the strategic level of policy analysis with which we are concerned this loss in accuracy is less important. Two other concerns have been raised by Barros and Weintraub [1982] regarding the accuracy of the Model II form. First, intermediate or partial cuts are not considered and second, since the stand identification is lost, location specific information such as transportation costs are difficult to represent.

So far we have discussed only simple Model I, II and III forms and, as Gunn and Rai [1987] point out, these model forms, when applied to a single stand, are no different to the classical "rotation" problem. However, Models I, II or III, in practice, are never applied to a single stand. Rather, they are applied to a forest region with many stands and there are always area and volume constraints on the inter-period harvests. The inclusion of these area/volume constraints requires the use of a full scale LP. Implementation of Model I has resulted in the development of TimberRAM by Navon [1971] and of Model II FORPLAN by Johnson [1986] or MUSYC by Johnson and Jones [1979].

Even though the addition of inter-period volume and/or area constraints transforms the simple stand problem into the *forest level scheduling problem*, Models I, II and III disregard an important qualitative aspect to the timber supply problem. That is, the consideration of the type of timber that is produced, not just the volume. For example, as Gunn and Rai [1987] point out, a 30cm. diameter log will have alternative uses (sawlogs, pulpwood), whereas, a 12cm log can only be used for pulpwood. There are also different species which must be taken into account and the sawmill by-products could be modelled.

There is a need for more detailed articulation of the industrial requirements for different wood products within the FMP framework.

In summary, we have contrasted the simple dynamic economic models which are appropriate for the small land owner with the long-term mathematical programming models which address the needs of a forest industry operating on a large region. The LP models, to date, are capable of ensuring an adequate volume of timber from period to period by using two types of constraints. The early models (e.g Ware and Clutter [1971]) used production constraints for each period. Here, total production of a given wood product type (sawlogs) must lie within prespecified bounds each period. The second type, called *harvest flow constraints* by Johnson and Scheurman [1977], require that the production of a given wood type in one period must not exceed that produced in a previous period. Both of these constraint types ensure an adequate supply of timber.

However, they are *supply driven* in the sense that they are based solely on the biological capacity of the forest. A more reasonable approach, as suggested by Barros and Weintraub [1982], is to recognize that the industry's requirements for wood products may be satisfied in different ways. For example, a pulpmill's fibre requirement may be satisfied through small diameter trees, large diameter trees or chips produced as by-products from the sawmill. Using a Model I form to consider the supply from 300 stands, they represented a forest industry system which included:

- (1) allocation of different wood commodities to the sawmill, pulpmill, and export sectors;
- (2) chip by-production at the sawmill and its use by the pulpmills; and
- (3) the material balance between the amount of each commodity produced in each period through harvesting and the allocation of that wood to the mills.

For each stand, only three to five regeneration harvest sequences, were used to keep the number of decision variables less than 2500. This proliferation of activity variables, as mentioned earlier, is an artifact of the Model I formulation. Gunn and Rai have consequently suggested an integrated forest industry approach based on the Model II form

which has the advantage of requiring far fewer decision variables to represent more management alternatives. Also, as we shall see in the next section, the size of the Model II LP may be further reduced by eliminating some of the variables by taking advantage of its special structure.

The structures of Models II and III are also amenable to other methods of solution and more easily allow for the introduction of stochastic effects such as fire, insect and disease. These stochastic effects are relevant within the Canadian context. To motivate understanding of the Model II or III formulation, the general dynamic linear systems model structure of forest dynamics, as developed by Reed and Errico [1986], will be described in the next section.

3.6 Reed and Errico Model III

In section 3.4 the motivation for Model III as depicted by the system of equations (3.6) was given. In matrix notation we have a linear discrete-time dynamical system:

$$\mathbf{x}(t+1) = \mathbf{R}\mathbf{x}(t) - \mathbf{S}\mathbf{h}(t) \quad (3.7)$$

$$\text{where } \mathbf{R} = \begin{bmatrix} 0 & . & . & . & . & 0 \\ 1 & . & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & 1 \end{bmatrix} \quad (3.8)$$

$$\text{and } \mathbf{S} = \begin{bmatrix} -1 & -1 & . & . & . & -1 \\ 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & 1 \end{bmatrix} \quad (3.9)$$

The idea is simple. During period t all the area in age-class 1 would move to age-class 2 at the beginning of period $(t+1)$; the area in age-class 2 would move up to age-class 3 etc.; and the area in age-class k would stay in that same age-class. We shall suppose that when an area is harvested, the yield per hectare is v_i ($i=1, \dots, k$). Note that we are assuming that all areas in the oldest age-class k have the same unit value. This may be justified by assuming that the volume-at-age curve reaches an asymptote or some constant level. Where this is not the case, the number of age-classes, k , should be sufficiently large to allow a full description of volume yields by age through the values v_i . This treatment is suggested by Reid and Errico [1986] and differs from the Model II form 1 model of Johnson and Sheurman [1977]. If the harvest in period t is given by the vector $h(t)$, the total volume cut in period t , $H(t)$ is:

$$H(t) = \mathbf{v}' \mathbf{h}(t) = \sum_{i=1}^k v_i h_i(t) \quad (3.10)$$

where $\mathbf{v}'$ is the volume-at-age vector

$$\mathbf{v}' = (v_1, v_2, v_3, \dots, v_k) \quad (3.11)$$

In order to ensure a controlled flow of timber from the forest we would want to impose some constraints on the sequence of volumes $H(t)$. These harvest flow constraints could be of several forms. Three possible forms are given.

1. Upper and lower bound constraints

If the total period cut is constrained to lie between specified lower and upper limits a and b , say, we would have :

$$a \leq H(t) \leq b \quad t = 1, 2, \dots, N \quad (3.12)$$

2. Step constraints

It may be desired that the change in volume cut from period to period not exceed certain specified limits. In this case:

$$\mu_1 \leq H(t) - H(t-1) \leq \mu_2 \quad t = 2, 3, \dots \quad (3.13)$$

where μ_1 is the specified maximum decrease in volume cut and μ_2 is the specified maximum increase in volume cut.

3. Sequential flow constraints

Instead of specifying a maximum absolute increase or decrease in volumes cut from period to period it may be preferable to specify maximum percentage, or proportional increase or decrease. If this is the case the harvest flow constraints would be of the form:

$$(1 - \beta_1)H(t-1) \leq H(t) \leq (1 + \beta_2)H(t-1) \quad \text{for } t = 2, 3, \dots \quad (3.14)$$

where β_1 is the maximum proportional decrease ($0 \leq \beta_1 \leq 1$) and β_2 is the maximum proportional increase ($0 \leq \beta_2 \leq 1$). Of course it would be possible to have combinations of the three types of constraints. Subject to the harvest flow constraints we wish to maximize some long run measure of yield from the forest. Without these forest level constraints the problem would be reduced to simply optimizing activities at the stand level. *These forest level constraints define the forest management problem.* One way of defining an objective is to specify a discount factor ∂ which will discount future harvests to a present value. The discount factor, ∂ , is related to the annual interest rate i : $\partial = [1/(1+i)]^T$ where T is the period length in years. The total discounted yield over an infinite time horizon is given by :

$$J = \sum_{t=1}^{\infty} \partial^t H(t) \quad (3.15)$$

The full maximization problem then can be specified as :

Choose a sequence of harvests $\{ h(t) \}$ to

$$\text{maximize} \quad J = \sum_{t=1}^{\infty} \partial^t v' h(t) \quad (3.16)$$

subject to:

$$h(t) \leq x(t) \quad (3.17)$$

$$x(t+1) = Rx(t) - Sh(t) \quad (3.18)$$

and subject to the specified harvest flow constraints and given the initial state of the forest $x(1)$. This is a *deterministic control problem* and since it is linear in the variables $x(t)$ and $h(t)$ both in the objective and constraints it can be solved by standard linear programming methods.

As shown by Reed and Errico [1986] the size of the LP can be substantially reduced by eliminating the variables $x(t)$ ($t = 2, \dots, N$) from the problem by using (3.7) recursively.

From (3.7) we have

$$x(2) = Rx(1) - Sh(1)$$

$$x(3) = Rx(2) - Sh(2)$$

$$\begin{aligned}
&= \mathbf{R}(\mathbf{R}\mathbf{x}(1)-\mathbf{S}\mathbf{h}(1))-\mathbf{S}\mathbf{h}(2) \\
&= \mathbf{R}^2\mathbf{x}(1)-(\mathbf{R}\mathbf{S}\mathbf{h}(1)+\mathbf{S}\mathbf{h}(2))
\end{aligned}$$

and by induction we obtain

$$\mathbf{x}(t) = \mathbf{R}^{t-1}\mathbf{x}(1)-(\mathbf{R}^{t-2}\mathbf{S}\mathbf{h}(1)+\mathbf{R}^{t-3}\mathbf{S}\mathbf{h}(2)+\dots+\mathbf{S}\mathbf{h}(t-1)) \quad (3.19)$$

Using this we can express the constraints (3.17) as:

$$\mathbf{R}^{t-2}\mathbf{S}\mathbf{h}(1)+\mathbf{R}^{t-3}\mathbf{S}\mathbf{h}(2)+\dots+\mathbf{S}\mathbf{h}(t-1)+\mathbf{h}(t) \leq \mathbf{R}^{t-1}\mathbf{x}(1) \quad \text{for } t = 1, \dots, N$$

Expressed as an LP in standard form the maximization problem becomes:

$$\text{maximize} \quad \mathbf{c}' \mathbf{h} \quad (3.20)$$

$$\text{subject to} \quad \mathbf{A} \mathbf{h} \leq \mathbf{b} \quad (3.21)$$

$$\text{and} \quad \mathbf{h} \geq \mathbf{0} \quad (3.22)$$

where $\mathbf{h}$ is the Nk -dimensional vector of activities

$$\mathbf{h} = [\mathbf{h}(1), \mathbf{h}(2), \dots, \mathbf{h}(N)]'$$

formed by concatenating vertically the vectors $\mathbf{h}(1), \mathbf{h}(2), \dots, \mathbf{h}(N)$ $\mathbf{c}'$ is the (Nk) -dimensional row vector given by: $\mathbf{c}' = (\partial \mathbf{v}', \partial^2 \mathbf{v}', \dots, \partial^N \mathbf{v}')$ formed by horizontal concatenation and $\mathbf{A}$ is the matrix of dimension $(k+2)N \times kN$ formed by concatenating blocks of dimension $(k \times k)$ and $(2k)$ in the following way:

2-dimensional vector $[b-a]'$ k times. The lower part of b are the upper and lower bound constraints on the volume cut in period t as defined in (2.19) i.e. $a \leq v'h(t) \leq b$ for $t=1, 2, \dots, N$. This LP can be solved using the *Revised Simplex Algorithm* (Dantzig [1963]).

3.7 Stochastic Fire Hazard Model

The general model introduced in section 3.5 to represent the deterministic forest dynamics was modified by Reed and Errico [1986] to incorporate random losses due to fire or other agents. The following additional notation was required. Let $p_1(t), p_2(t), \dots, p_k(t)$ represent the random proportions of age-classes $1, \dots, k$ destroyed by fire during period t and subsequent to reforestation (or natural regeneration) end up at the beginning of period $t+1$ in age-class 1.

The tree growth dynamics including fire will be:

$$x_1(t+1) = \sum_{i=1}^k h_i(t) + \sum_{i=1}^k p_i(t) [x_i(t) - h_i(t)]$$

$$x_i(t+1) = \{1 - p_{i-1}(t)\} \{x_{i-1}(t) - h_{i-1}(t)\} \quad \text{for } i = 2 \text{ to } k-1 \quad (3.23)$$

$$x_k(t+1) = \{1 - p_{k-1}(t)\} \{x_{k-1}(t) - h_{k-1}(t)\} + \{1 - p_k(t)\} \{x_k(t) - h_k(t)\}$$

In matrix notation we have:

$$x(t+1) = R(t)x(t) - S(t)h(t) \quad (3.24)$$

$$R(t) = \begin{bmatrix} p_1(t) & p_2(t) & . & . & p_k(t) \\ (1-p_1(t)) & 0 & . & . & 0 \\ 0 & (1-p_2(t)) & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & (1-p_{k-1}(t)) & (1-p_k(t)) \end{bmatrix}$$

$$\text{and } S(t) = \begin{bmatrix} (-1+p_1(t)) & (-1+p_2(t)) & . & . & (-1+p_k(t)) \\ (1-p_1(t)) & 0 & . & . & 0 \\ 0 & (1-p_2(t)) & 0 & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & (1-p_{k-1}(t)) & (1-p_k(t)) \end{bmatrix}$$

We note that the basic dynamic equation (3.24) is a *stochastic difference equation* since the elements $p_i(t)$ in $R(t)$ and $S(t)$ are random variables. Subject to the harvest flow constraints which we defined for the deterministic case we wish to maximize some long run measure of yield from the forest.

For the stochastic problem the objective function is the expected discounted yield from period t over an infinite time horizon.

$$\text{i.e. } J = E \left\{ \sum_{t=1}^{\infty} \partial^t v^t \cdot h(t) \right\}.$$

The *stochastic control problem* then becomes:

Choose a sequence of harvests $\{h(t)\}$ to

$$\text{maximize } J = E \left\{ \sum_{t=1}^{\infty} \partial^t v^t h(t) \right\} \quad (3.25)$$

subject to :

$$h(t) \leq x(t)$$

$$x(t+1) = R(t)x(t) - S(t)h(t)$$

$$x(t), h(t) \geq 0$$

and subject to the specified *harvest flow constraint* and given the *initial state* of the forest $x(1)$. As pointed out by Reed and Errico [1986] the solution will be a *feedback policy* where the optimal harvest $h^*(t)$ in period t will depend on the current state $x(t)$ and possibly on previous states and harvests. In general, solutions are difficult to obtain.

In the special case when there are no harvest flow constraints Reed and Errico have shown that the optimal solution to this problem is the same as the optimal solution to the simpler deterministic control problem. This result is of little practical significance since with no harvest flow constraints the problem can be broken down into a set of mutually exclusive stand optimizations for which the solutions are readily available. However, approaching the problem deterministically led Reed and Errico to an approximation method for the stochastic control problem which may be adequate. The idea is to use the optimal solution to the deterministic control problem with harvest flow constraints in a feedback fashion to obtain an approximate solution to the stochastic control problem.

$$\text{i.e. } \max \left\{ J = \sum_{t=1}^{\infty} \partial^t v^t h(t) \right\} \quad (3.2)$$

$$\text{s.t.} \quad h(t) \leq x(t)$$

$$\text{and} \quad x(t+1) = R x(t) - S h(t)$$

and the harvest flow constraints where $\mathbf{R}$ and $\mathbf{S}$ are the expected values of the random matrices $\mathbf{R}(t)$ and $\mathbf{S}(t)$. The feedback method would first observe the first period harvest $h^*(1)$ given the initial state $\mathbf{x}(1)$ optimal for the *deterministic control problem* which uses average fire rates. Then one would observe what actually happened with respect to fires (i.e the particular realization of the random variables $\mathbf{p}(t)$) and then simulate the new state variable $\mathbf{x}(2)$. The whole process would now be re-run using $\mathbf{x}(2)$ as the initial state vector to determine the optimal harvest $h^*(2)$. In current whole forest scheduling models a similar feedback approach is used to update the state vector with observed catastrophic losses. However, these models do not specify whether the destruction occurs in mature or immature stands or whether average per annum fire is incorporated in the model structure for the subsequent periods of the time horizon. Reed and Errico have included this possibility. The approximation method may be interpreted as being equivalent to the deterministic present value if fixed rather than random proportions were destroyed by fire.

Reed and Errico [1986] cast this approximation method in the linear programming framework and used the revised simplex method to solve for the optimal cut at each iteration. To summarize: the optimal exploitation of a forest composed of stands of different ages, and subject to random catastrophic losses (i.e fire) has been considered. The problem has been treated by Reed and Errico [1986] as a stochastic control problem. An approximation method was proposed by using the optimal policy associated with the deterministic control problem in a feedback fashion. The deterministic formulation assumes that fixed rather than random proportions of the areas in each age-class, will be destroyed by fire in each period. It may be shown that the closeness of the deterministic optimal policy to the stochastic optimal policy depends on the variance of the random variables which are the proportions burnt in each age-class. If the forest consists of a large area, the proportions burned will have small variance and the deterministic solution should provide a reasonable approximation to the stochastic control problem. Because of this randomness of losses the time series of actual harvests, using the deterministic optimal

policy in a feedback fashion, will exhibit some fluctuation i.e. it will be a particular realization of a stochastic process.

Preliminary results from Reed and Errico [1986] showed that even comparatively low rates of fire could result in considerably lower harvest projections than those obtained assuming no catastrophic losses. They found that the size of the reductions in harvest are in close agreement with those obtained from their stand level analyses (Reed and Errico [1985]) and those obtained by simulation analysis (Van Wagner [1983]). For example for the particular growth curve used a one percent per annum loss rate through fire translates into a forty-percent loss in maximum sustainable yield. The harvest trajectory required to approach m.s.y is also affected. The implication of these results for forest management is that, given the existence of catastrophic losses, current timber supply projections based on whole forest models which ignore these losses are too high. Also the concept of a "normal" forest must be revised. When age-independent fire-risk is included a "normal" forest will have, not a uniform age distribution, but one exhibiting a geometric decline. Where fire risk is age-dependent then the age distribution will show a less regular decline.

3.8 Stochastic Fire and Pest Hazard Model

The previous section examined how it is possible to include the effect of fire risk in the long run supply of timber. We now consider losses due to insect infestations. In Ontario, the Boreal forest experiences large periodic outbreaks of spruce budworm infestations. These outbreaks have resulted in an annual estimated loss of approximately 23 million cubic metres which exceeds the annual softwood harvest volume of 16 million cubic metres (Howse [1986]). In the face of increasing wood supply costs and possible shortages Ontario has increased protection of trees from insect caused mortality by large scale spraying programs. It is believed that a component of mature or over-mature balsam fir is necessary for an outbreak to occur and that white spruce is also susceptible to attack. Since spruce budworm feed on progressively older needles, black spruce is less susceptible,

since it carries more years of needles and can withstand more years of attack before being killed. Younger trees which grow vigorously are also less susceptible. The characteristics of budworm damage must be reflected in the model structure.

Recent work in this area by Reed and Errico [1987] have attempted to analyze the forest harvest scheduling problem subject to the risk of fire *and* insect damage. They describe the state of the forest at any time by a vector $[x(t), y(t)]'$ where $x(t)' = (x_1(t), x_2(t), \dots, x_k(t))$ represents the area of uninfested timber in the various age-classes $1, 2, \dots, k$ at the beginning of period t and $y(t)' = (y_1(t), y_{i+1}(t), \dots, y_k(t))$ where $y_i(t)$ is the area of timber which was infested at age i and is still standing at the start of period t . The age $i (\geq 1)$ is the youngest age at which infestation can occur. In their model we note that the age-classes for infested timber correspond to the age at which infestation occurred, and not the current (period t) age of the timber as for the non-infested. This reflects the situation where no growth occurs after the infestation has happened, i.e. catastrophic infestation where the stands are completely killed and the residual component stagnates. We could have growth after infestation or a combination of the two cases. This will be described in more detail later. Reed and Errico [1987] define the random proportions $r_i(t)$, $r_{i+1}(t), \dots, r_k(t)$ of the areas of previously non-infested timber struck by pests in period t and move into the infested categories. They also allowed for the possibility of catastrophic fires by assuming that random proportions $p_1(t)$, $p_2(t), \dots, p_k(t)$ of the areas in various age-classes of the non-infested timber and random proportions $q_i(t)$, $q_{i+1}(t), \dots, q_k(t)$ of the areas of infested timber are destroyed by fire in period t and are returned to the first age-class of non-infested timber at the start of time period $t+1$; that is the fire performs a rehabilitation role. We might expect higher rates of fire in the infested timber than among non-infested timber due to the higher level of fuel from dead trees. With these assumptions the following *stochastic difference equation* of Reed and Errico [1987] describes the forest.

$$x_1(t+1) = h_1(t) + \dots + h_k(t) + p_1(t)\{x_1(t) - h_1(t)\} + \dots + p_k(t)\{x_k(t) - h_k(t)\} + g_1(t) + \dots + g_k(t) \\ + q_1(t)\{y_1(t) - g_1(t)\} + \dots + q_k(t)\{y_k(t) - g_k(t)\}$$

$$x_2(t+1) = \{1 - p_1(t)\}\{x_1(t) - h_1(t)\}$$

$$x_i(t+1) = \{1 - p_{i-1}(t) - r_{i-1}(t)\}\{x_{i-1}(t) - h_{i-1}(t)\} \quad \text{for } i=2, 3, \dots, k-1$$

$$x_k(t+1) = \{1 - p_{k-1}(t) - r_{k-1}(t)\}\{x_{k-1}(t) - h_{k-1}(t)\} + \{1 - p_k(t) - r_k(t)\}\{x_k(t) - h_k(t)\}$$

(3.26)

$$y_i(t+1) = r_i(t)\{x_i(t) - h_i(t)\} + \{1 - q_i(t)\}\{y_i(t) - g_i(t)\}$$

$$y_k(t+1) = r_k(t)\{x_k(t) - h_k(t)\} + \{1 - q_k(t)\}\{y_k(t) - g_k(t)\}$$

$$\text{where } h' = \{h_1(t), h_2(t), \dots, h_k(t)\} \quad \text{and } g' = \{g_1(t), \dots, g_k(t)\}$$

represent the areas in the various age-classes of infested and uninfested timber respectively that are harvested in period t .

Graphical representation

This model may also be represented by a graph as shown in Figure 3.2. For each variable x and y a node may be referenced by (i, t) , the age-class i and the beginning of time period t . The state variables $x_i(t)$ and $y_i(t)$ represent the sum of inflowing area. For example at the start of the first time period, $x_i(1)$ is the initial area in age-class i and is considered input to node $(i, 1)$. At each node there is conservation of area. That is the sum of flow lines into a node equals the sum leaving. The outputs from nodes $(i, 1)$ are the area harvested during period 1, $h_i(1)$, the proportion of the remaining forest $x_i(1) - h_i(1)$ that burns (i.e

$p_i(1)(x_i(1)-h_i(1))$), the proportion that becomes infested $r_i(1)[x_i(1)-h_i(1)]$ and the surviving area which grows into the next age-class $\{1-p_i(1)-r_i(1)\}[x_i(1)-h_i(1)]$.

The edges represent flows of area from node to node. For example, $h_4(1)$ represents the area harvested from age-class 4 in time period 1 and regenerates as age-class 1 at the start of time period 2. It is connected from node (4,1) to (1,2), which means that there is a transfer of area from age-class 4 in period 1 which will be regenerated in age-class 1 at the start of time period 2. The transfer of area from uninfested age-class 4 in period 1 to infested in period 2 and moves into age-class 5 $\{y_5(2)\}$ is $r_4(1)\{x_4(1)-h_4(1)\}$. Burning and harvesting of infested area {i.e. $y_i(t)$ } regenerates as uninfested area in age-class 1 in the next period.

Equations (3.26) may be written in vector notation form as:

$$X(t+1) = R(t)X(t) - S(t)h(t) \quad (3.27)$$

where $X(t)' = \{x(t)', y(t)'\}$ and $R(t)$ and $S(t)$ are $(2k-a+1) \times (2k-a+1)$ square matrices.

With this dynamic equation we seek to maximize the expected present value J of the flow of harvests from the resource subject to the harvest flow constraints and other forest level constraints that may be imposed.

$$\text{i.e.} \quad \text{Max}_t J = E \left\{ \sum_t \frac{1}{(1+i)^t} [v'h(t) + w'g(t)] \right\}$$

where $1/(1+i)$ is the per annum discount factor and $v' = \{v_1, v_2, \dots, v_k\}$ and $w' = \{w_1, w_{1+1}, \dots, w_k\}$ are vectors representing the net stumpage values per hectare of timber in the various infested and uninfested classes. An approximation solution to this stochastic optimization problem can be obtained using the feedback technique described in section 3.6. In each period the deterministic optimization problem is solved by replacing the random variables by their expected values. This model structure assumes that,

following harvest of both infested or uninfested area, natural regeneration of the same species occurs in the next time period. There is no provision for controlling the development of an infestation by spraying. Also, it would be reasonable to represent planting of harvested area with a species which is either less susceptible or not susceptible to an insect infestation. Some of these ideas have been addressed by Boychuk [1988] who recently developed a *pest control model* for part of the Ontario Boreal Forest. In the next section a description of the Boychuk model and suggestions for a more *efficient* model structure will attempt to relate concepts implemented in the Boychuk model to the generic dynamic linear systems structure of Reed and Errico.

Figure 3.2: Graphical Representation of the Fire Pest Model of Reed and Errico [1987]

3.9 The Pest Control Model of Boychuk

The model developed by Boychuk [1988] has an underlying structure similar to a Model III (Reed and Errico [1986, 1987]). It was designed to evaluate protection spraying of susceptible white spruce/balsam fir stands against spruce budworm in the production of a continuous supply of black spruce, white spruce and balsam fir wood fibre from Ontario's boreal forest. In this model multiple outbreaks of budworm were assumed to occur over the time horizon of the optimization. Two types of protection spraying were evaluated for each age-class. The first protected the trees for harvest in the current period and the second protected the trees so that they could be harvested in a subsequent period. The forest was stratified into susceptible and non-susceptible species so that susceptible stands of white spruce/balsam fir could be converted to non-susceptible black spruce after harvest.

The Boychuk model differs from the Reed and Errico model in that pest control decisions were built into the problem structure as decision variables. These controlled the fraction of the attacked forest that is killed. Both the Boychuk and Reed and Errico models assume perfect foresight on future outbreaks; that is a deterministic approximation of a stochastic control problem is made. A "*not sufficiently restocked*" (NSR) land class was also recognized and two regeneration classes were considered:

- (1) *natural regeneration* where nothing was done other than zero cost activities at the time of harvesting; and
- (2) *basic regeneration* which consisted of planting or seeding as appropriate for the site together with the necessary tending.

Even-aged stands were assumed and six 20-year age-classes used. Seven 20-year time periods constituted the time horizon and silvicultural activities were assumed to occur in the

middle of the periods. *Clearcut harvesting* was used and for black spruce a *strip cutting cycle* was completed in each period. The following silvicultural activities were included: after a stand is harvested or killed by spruce budworm, natural or basic regeneration or conversion from susceptible to non susceptible forest is carried out. Each regeneration or conversion activity may be successful or not with a certain probability.

It was assumed each 20-year period 20 percent of NSR regenerated naturally to susceptible forest type. The remainder could be rehabilitated to non-susceptible forest type using basic regeneration. Pest outbreaks were characterized by *time* (period of occurrence), *extent* (proportion of area attacked) and *severity* (proportion of attacked killed). Mortality of attacked susceptible stands is a function of age-class and regeneration type.

Two modes of protection spraying were represented:

- (1) keeping the trees alive for harvest in the current period; and
- (2) keeping the trees alive for harvest beyond the current period.

Protection spraying is also age and regeneration class specific. The first protection scheme was assumed to require three applications and save 80% of the sprayed area and the second required six applications and saved 60% of the sprayed area. Survived attacked stands are assumed to have reduced volume due to growth loss during the attack. This model structure does not keep track of attacked stands so that an adjustment was made to reflect the loss in volume by holding back attacked stands which were not protected and survived one age-class; thus reflecting the loss in volume.

Chapter 4

The Fire and Pest Protection Forest Modelling Problem

In this chapter a *statement of the problem* suggests both a fire and pest protection *modelling* and a *model representation* problem.

4.1 A Statement of the Problem

The *modelling problem* involves the integration of concepts that have been addressed separately in extant planning models as well as the mathematization of new protection controls for the infested areas and intensive silvicultural harvesting strategies. This approach permits a scenario analysis so that alternative policy options may be compared for economic efficiency. The *model representation problem* involves issues of understandability. The approach taken is to develop a "transparent" or "white box" model to promote understandability and avoid ambiguity. A discussion of the importance of these problems for forest management in Canada is given.

4.1.1 The Fire and Pest Protection *Modelling* Problem

The difficulty with recent forest planning models (Reed and Errico [1986], [1987] and Boychuk [1988]) is that each cover only part of the forest fire and pest management

problem. A generalized mathematical programming model is sought which allows for the flexible integration of all the following concepts so that the fire and pest protection forest management problem can be addressed in a single mathematical programming framework:

- (1) *species diversity;*
- (2) *infestation of susceptible species;*
- (3) *natural regeneration and planting;*
- (4) *conversion of susceptible uninfested, infested and protected stands to non-susceptible species;*
- (5) *pest protection ;*
- (6) *minimum age for harvesting; and*
- (7) *catastrophic losses due to fire;*

This *design framework* is concerned with the development of a optimization model to economically evaluate:

- (1) *alternative protection spraying of susceptible species against insect infestations; and*
- (2) *intensive silvicultural strategies which include conversion of susceptible species to non-susceptible species by planting.*

Both (1) and (2) are evaluated subject to the risk of catastrophic losses due to fire. The forest modelling problem then becomes: *develop a model to determine the optimal timber regeneration¹ harvest and pest protection schedule by age in time period t subject to fire risk that produces a desired sustainable wood supply over time at maximum discounted net profit.*

¹ either natural or basic regeneration.

We recognize the need to model :

- (1) protection spraying of infested stands ;
- (2) conversion harvests of susceptible species; and
- (3) minimum age for harvesting.

We seek an augmented state-space "Reed and Errico" model structure which includes protection and intensive silviculture conversion harvests (i.e (1) and (2) above).

In order to analyze the economic efficiency of the alternate policies an optimization modelling approach is sought whereby a baserun model structure is solved for comparison with solutions from augmented structures that include the policy options separately. This approach should be implemented using a high level algebraic model development system.

4.1.2 The *Model Representation* Problem

There is also a problem in forest management of the *understandability* of the mathematical programming systems available. Current technology for harvest scheduling is in the form of complex fixed structure FORTRAN based programs. Users of these systems frequently find that the computer representation of the models is complicated and time consuming to understand and work with. This problem is not confined to the forestry community but to the users of any large scale mathematical programming system. As pointed out by Lucas and Mitra [1988] our ability to construct models and our understanding of the many facets of these models are less developed than the technology used to solve the problems.

According to Geoffrion [1985] "*Modelling as done today is a much lower productivity process than it ought to be. It takes too long to build, verify and document models. It is too hard to maintain and make evolutionary improvements .*" Krabek et al [1980] pointed out that the steps of managing data, building the model and reporting and analysing results are much more expensive than that of optimization. Furthermore, these

steps prove to be a barrier to the effective use of the LP. Also, Greenberg [1983] observed that " *comprehension is the present bottleneck in using large scale models.*"

Another objective of this research is to *provide a flexible, easy to understand, mathematical programming (modelling) environment* such that field data from a particular forest may be rapidly transferred to the computer for solution so experienced foresters are able to judge the feasibility of a harvesting schedule in the light of constraints which may not be easily mathematized. Acceptance of complex mathematical programming systems by forest managers will only be achieved if they are able to understand the model abstraction of the problem. An attempt will be made to develop a useful model through the process of producing more and more precise representations of the generalized forest management problem.

This process is recognized by the Operations Research (OR) community as the *model representation problem* which involves successively developing a problem statement which includes formal model, model instance, algorithm output, and final presentation. We adopt the view of Jones [1988] that the field of OR has traditionally concentrated on developing the models and algorithms. The representations used for the other project phases have usually been ad hoc. When well done, (e.g. graphical representations) they can both significantly aid the decision maker and expand the use of OR techniques.

It is now accepted that one of the principal contributions of the research in *Decision Support Systems (DSS)* has been its emphasis on tools to support the development of these representations. As pointed out by Jones [1988] " *OR has both a challenge and an opportunity to move from the art of representation to a science of representation*". Issues of representation have been identified by Sprague and Carlson [1982] who present a framework for designing DSS that explicitly includes representation. The small amount of research in OR that addresses issues of representation has generally involved developing software for particular solution paradigms.

For example, the development of high level modelling languages enables the modelier to implement large and complex problems in a concise and reliable manner. Recent experience by Paules and Floudas [1989] suggest that efforts to provide a model

that reflects a real world system may result in a problem that is difficult or impossible to solve with extant general purpose codes.

4.2 Why solve these problems?

The question of timber supply in Canada has been receiving increasing attention over the last few years. Conclusions drawn from a Timber Demand/Supply Analysis in Canada-Workshop, May 3-5, 1988 were distributed in a memorandum from J.C. Nautiyal [1988], the Associate Dean of the Faculty of Forestry, University of Toronto, entitled *"Timber Supply and Management of Insect Damage: Networks of Excellence"*. Two main conclusions were identified:

- (1) Regional supply studies need to be integrated into a national supply model which then could face the national and international demand scenario; and
- (2) Damage due to insects, diseases and fire should be incorporated into supply studies.

In particular, the memorandum stated " *the extent of damage to growing stock from pests and fires is substantial in Canada and the alternative of increasing economic supplies by incurring protection expenditures needs to be compared with costs of increasing output by more intensive management* ". It continues, " *any supply analysis is incomplete if it excludes the possibilities of losses due pests and fires. Though the manner in which insects, diseases and fires affect short and long run supplies is different, they also may have certain common features. Dealing with all three sources of loss in detail may be too daunting a problem. . . .* "

For example, in Ontario annual losses of 23 million cubic metres exceeded the 1986 harvest volume of 16 million cubic metres. Also, preliminary results from Reed and Errico [1986] suggested that even low rates of fire can result in considerably lower harvest

projections than those obtained assuming no catastrophic losses. It is expected that the combined effect of loss due to insect infestations and fire will further reduce the sustainable harvest level. The nature of the harvest trajectory required to approach *maximum sustained yield* (m.s.y) may also change. Implication of these concepts for forest management is that, given the existence of catastrophic losses, current timber supply projections based on forest models which ignore these losses may well be too high.

In order to investigate the policy consequences of such catastrophic events it would be necessary to conduct sensitivity analyses on the impact of variations in fire and insect parameters on the time path of the long run sustained yield.

4.3 Have these problems been solved before?

Separate aspects of the forest fire and pest protection management problem have been modelled (Reed and Errico [1986] , [1987] and Boychuk [1988]). Reed and Errico [1987] suggested a model structure which represented the impact of the timing and intensity of an infestation but they did not model protection spraying. Boychuk [1988] allowed for protection spraying and conversion harvests in a mathematical programming system which had many redundant constraints and variables. The Boychuk model had approximately 1770 variables and 1070 constraints for a problem with six twenty-year age-classes, seven time periods and five state variables. We seek a parsimonious state-space description of the pest protection problem. Also the Boychuk model did not include a state variable to represent the protected stands nor did it include catastrophic loss due to fire. Neither of these recent developments recognized a minimum age for harvesting.

The model representation problem has not been addressed before. Extant software for operational and prototype harvest scheduling systems (Johnson and Jones [1979], Reed and Errico [1986], [1987]) are in the form of complex, difficult to understand FORTRAN programs.

Chapter 5

Network Representation

In this chapter, we present graphical descriptions of the fire and pest protection models. We introduced, in chapter three, the network flow description of the inter-temporal relationships of growth, harvesting, infestation, protection and catastrophic losses due to fire. This network representation is shown to have proportional flow restrictions on the arcs entering or leaving some nodes which reveals a *processing network* structure (see section 2.5). We take advantage of the network flow description to motivate understanding of the fire and pest protection algebraic structures which are detailed in chapter six.

We are concerned with "proof of concept" and will keep the model structure as simple as possible yet sufficiently rich to be able to examine the required concept. We need to be able to represent the evolution of a forest with several species. Since we are also concerned with insect infestation it is reasonable to first partition the forest into *susceptible* and *non-susceptible* species. For the susceptible species we further subdivide the areas into those *infested* and those that are *uninfested*. Furthermore, for the infested areas we need to account for areas which are *protected* through spraying programs.

First, we consider one type of spraying: to protect the stands for harvest in the current period. It will become evident that detail in the model structure may be added quite easily. We are attempting to achieve *understandability* of the way in which these models structures may be augmented to reflect alternative silvicultural activities. Since we are addressing strategic problems associated with a large region it may be sufficient to treat the losses due to fire and insect infestations deterministically. The regeneration options (i.e.

natural versus planting) will be associated with the definition of the harvesting variables. For example the definition of the harvesting variables will assume either natural regeneration or regeneration by planting.

5.1 Network Representation of the Fire and Pest Model with no Protection Spraying or Conversion Harvests.

We describe the state of the forest at the start of period t by three state variables $x_i(t)$, $y_i(t)$ and $z_i(t)$ which represent the area of timber at the start of time period t in age-class i which is non-susceptible, susceptible uninfested and susceptible infested respectively. Each state variable (a node in the network) may be referenced by (i,t) the age-class i and the beginning of a time period

The network of Figure 5.1 is an overview of the basic fire and pest model which is described algebraically in section 6.2. Here we consider only four time periods and four age-classes. The *ordinal* value of the sets defined in section 6.2 indexes the age-class and time period. There are three sets of nodes corresponding to the state variables, $x_i(t)$, $y_i(t)$ and $z_i(t)$. If we consider node $(4,2)$, the area of non-susceptible species in age-class 4 at the beginning of time period 2 is $x_4(2)$ and equals the flow from node $(4,1)$ plus the flow from node $(3,1)$. The arcs or edges represent flows of area from node to node. At each node there is conservation of area: i.e. the sum of inflowing area equals the sum of outflowing area.

It is easy to identify from Figure 5.1 the principal activities which occur at each node. For each variable we may classify activities that occur at the oldest age-class, intermediate age-classes and the first or regeneration class. Since there is no protection of infested area by spraying or conversion harvests the area transitions are simple. For the susceptible and non-susceptible species we notice that for any age-class other than the first, area flows to the regeneration class at the start of the next period as a result of regeneration

harvesting or fire loss. In this simple network the minimum age for harvesting is age-class 3.

The "susceptible uninfested" states may become "infested" by spruce budworm and so an edge flow to the state "susceptible infested" would occur. For the "infested" states the "grow" lines are horizontal indicating that there is no jump in age-class between periods. This reflects an arresting of growth due to the infestation. For example, loss in volume in the forest due to a budworm infestation may be represented in two ways :

- (1) by reducing the volume-at-age curve by a certain percentage; or
- (2) by not allowing the area of the forest to shift up an age-class over one time period.

Figure 5.1: The Fire and Pest Model with no Protection or Conversion Harvests

Processing Network Representation

By reference to the system of equations associated with the nodes (i,t) for $x_i(t)$, $y_i(t)$, $z_i(t)$ (described in chapter six) we notice that there are proportional flow restrictions at the nodes. This means that processing nodes must exist in the network. The network in Figure 5.1 may in fact be represented by a *processing network*. Each node (i,t) associated with the variables x , y , and z may be represented by a composite node consisting of regular node which we denote by circles and processing nodes represented by rectangles. Conservation of flow exists at both types of nodes. In the network of Figure 5.1 we expand the node $(4,1)$ for the non-susceptible species into one *regular node* and one *processing node* as shown in Figure 5.2. The regular node is considered to be a source node with input area $x_4(1)$ (the initial condition). We have inflow to the regular node of $x_4(1)$, the area in age-class 4 at the start of period 1. The harvest during period 1 from age-class 4 is $h_4(1)$. The unharvested area, $x_4(1)-h_4(1)$, inflows to a processing node.

Figure 5.2: Oldest Age-Class Non-Susceptible

A fixed proportion, $p_4(1)$, of the unharvested area, $\{x_4(1)-h_4(1)\}$, may burn and regenerate at the start of period 2 as age-class 1 (i.e. $p_4(1)\{x_4(1)-h_4(1)\}$) and the remaining

area, $[1-p_4(1)]\{x_4(1)-h_4(1)\}$, "grows" into the next age-class. In this example there is no age-class jump because 4 is the oldest age-class.

Each node may be examined in this fashion. We note that for intermediate periods and age-classes the sum of the inflowing areas is renamed at each composite node by the state variable. For example, composite node (4,2) may be represented by Figure 5.3 where $x_4(2) = [1-p(1)]\{x_4(1)-h_4(1)+x_3(1)-h_3(1)\}$, the sum of outflows from nodes (4,1) and (3,1) respectively. The area that burns is $p(2)\{x_4(2)-h_4(2)\}$ and the surviving area that grows into the next age-class is $[1-p(2)]\{x_4(2)-h_4(2)\}$.

Figure 5.3: Non-Susceptible Intermediate Periods and Age-Classes

5.2 Network Representation of the Fire and Pest Model with Conversion Harvests

The network in Figure 5.6 shows the inclusion of conversion harvests from susceptible to non-susceptible as described algebraically in section (§) 6.2. The processing nodes associated with susceptible variables y and z represent conversion harvests, and infestation processes as well as fire losses and natural regeneration harvests.

Susceptible Uninfested Node

We consider in Figure 5.4 a composite source node (3,1) (expanded from Figure 5.6) for the "susceptible uninfested" state variable y .

Figure 5.4: Susceptible Uninfested Node: Age-Class 1.

As before the input to the regular node is $y_3(1)$. The outflows are the harvested area $g_3(1)$ which naturally regenerates and the harvested area which is converted to non-susceptible, $gc_3(1)$, and the surviving area subject to fire and insect infestation $y_3(1) - g_3(1) - gc_3(1)$ is

input to the processing node. Outputs from the processing node are the area that burns $q(1)\{y_3(1)-gc_3(1)-g_3(1)\}$ and the infested area $s_3(1)\{y_3(1)-gc_3(1)-g_3(1)\}$. The surviving area that grows into the next age-class is $[1-q(1)-s_3(1)]\{y_3(1)-gc_3(1)-g_3(1)\}$. For the next time period (2) the composite node (3,2) would consist of two regular nodes and a processing node as shown in Figure 5.5.

Figure 5.5: Susceptible Uninfested Intermediate Periods and Age-Classes

The output of the the first regular node (input to the processing node) is the state, $y_3(2)$, and is equal to the area that grows from nodes (3,1) and (2,1) respectively:

$$y_3(2) = [1-q(1)-s_3(1)]\{y_3(1)-gc_3(1)-g_3(1)+y_2(1)-gc_2(1)-g_2(1)\}.$$

Figure 5.6: The Fire and Pest Model with Conversion Harvests

5.3 Network Representation of the Fire and Pest Model with Protection Spraying

In Figure 5.9 the network of Figure 5.1 is augmented to include protection spraying of the stands infested with spruce budworm. The "susceptible infested" and "protected" states, $z_{pi}(t)$, are added to the network showing how a transition from the "infested unprotected" to "protected" state may occur.

Susceptible Infested Unprotected

Consider, in Figure 5.9, the node (4,2) for the "susceptible infested unprotected" state variable z expanded as two regular and one processing nodes in Figure 5.7.

Figure 5.7: Susceptible Infested Unprotected

For a typical interior composite nodes we have incoming area from the "uninfested" state which has become infested during the previous period. The output from the first regular node $z_4(2)$ would be the infested area $s_4(1)\{y_4(1)-g_4(1)\}$ plus the area that survived from the same age-class and previous period $[1-ru(1)]\{z_4(1)-f_4(1)-pr_4(1)\}$ where $ru(1)$ is the proportion of unprotected burned during period 1, $f_4(1)$ is the area regeneration harvested

and $pr_4(1)$ is the area in age-class 4 protected by spraying during period 1. Again there are no age-class jumps since after infestation the forest is presumed not to grow. We may harvest $f_4(2)$ or protect amount $pr_4(2)$ by spraying. A proportion $ru(2)$ of the remaining area $z_4(2)-f_4(2)-pr_4(2)$ may burn and the surviving proportion $[1-ru(2)]$ of the remaining area $\{z_4(2)-f_4(2)-pr_4(2)\}$ "grows" to the same age-class at start of the next period. We note that protection by spraying also "protects" stands for one period from catastrophic loss due to fire. However, once these stands become protected they are then subject to fire hazard. This "bonus fire protection" is a model artifact associated with aggregating age-classes and time periods. To avoid this, more temporal resolution is needed, at the expense of an increase in computational burden as mentioned in section 9.3.

Susceptible Infested Protected

In the "protected" states, for the oldest age-class i.e. node (4,2) in Figure 5.9 we have two regular and one processing node as shown in Figure 5.8. Inputs to the first regular node include growth from nodes (4,1) and (3,1) and the protected area during period 1. The "protected" state $zp_4(2)$ is input to the second regular node whose outflows include the area regeneration harvested $fp_4(2)$ and the surviving area which is subject to a proportion $rp(1)$ fire loss is given by: $zp_4(2)=pr_3(1)+pr_4(1)+[1-rp(1)]\{zp_4(1)-fp_4(1)+zp_3(1)-fp_3(1)\}$.

Figure 5.8: Susceptible Infested Protected

Figure 5.9: The Fire and Pest Model with Protection Spraying

5.4 Network Representation of the Fire and Pest Model with Protection and Conversion Harvests.

We now include conversion harvests from all allowable "susceptible" states. These are shown as harvest flow lines from the "susceptible" states to the regeneration age-class of the "non-susceptible" state at the beginning of the subsequent period. The complete fire and pest protection model as shown in the network of Figure 5.14 may be viewed as a superposition of Figures 5.6 and 5.9.

Susceptible Infested Unprotected

Consider node (4,2) of network (Figure) 5.14 for the "susceptible infested unprotected" state z .

Figure 5.10: Susceptible Infested Unprotected

As shown by Figure 5.10 for typical interior composite nodes we have incoming area from the "uninfested" state which has become infested during the previous period. The output from the first regular node $z_4(2)$ would be the infested area $s_4(1)\{y_4(1)-g_4(1)-gc_4(1)\}$ plus

the area that survived $[1 - ru(1)]\{z_4(1) - f_4(1) - fc_4(1) - pr_4(1)\}$ from the same age-class and previous period. Again there are no age-class jumps since after infestation the forest is presumed not to grow. We may harvest $f_4(2)$ and/or harvest and convert, $fc_4(2)$, and the remaining area is subject to fire hazard, $ru(2)$, or may be protected from the insect infestation and transfers to the protected state. The proportion $[1 - ru(2)]$ of the surviving area $\{z_4(2) - f_4(2) - fc_4(2) - pr_4(1)\}$ "grows" to the same age-class in the next period.

Susceptible Infested Protected

Similarly for the oldest age-class node e.g. node (4,2) in network 5.14 there are two regular nodes and one processing node as shown in Figure 5.11.

Figure 5.11: Susceptible Infested Protected

Inputs to the first regular node include growth from nodes (4,1) and (3,1) and the protected area during period 1. The state $zp_4(2)$, input to the second regular node whose outflows include the area regeneration harvested, $fp_4(2)$, and conversion harvested, $fc_4(2)$, and the surviving area which is subject to a fire loss proportion $rp(2)$, is given by:

$$zp_4(2) = pr_3(1) + pr_4(1) + [1 - rp(1)]\{zp_4(1) - fp_4(1) - fcp_4(1) + zp_3(1) - fp_3(1) - fcp_3(1)\}.$$

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Regeneration: Non-Susceptible

In the processing network representation the regeneration nodes or nodes associated with age-class 1 in all periods are regular nodes since they accumulate harvests or fire/insect infestation losses for the period. For example the node (1,2) for the non-susceptible species, as shown in Figure 5.12, would be a regular node with inputs of :

- (1) natural regeneration harvests from all harvestable age-classes i during period 1;
- (2) burned area ;
- (3) conversion harvests from "susceptible infested unprotected" and "protected" states; and
- (4) burned area during period 1 from all age-classes.

The output , $x_1(2)$, grows into the next age-class at the start of period 2.

$$x_1(2) = \sum_i (h_i(1) + fc_i(1) + fcp_i(1) + p_i[x_i(1) - h_i(1)])$$

Figure 5.12: Regeneration Age-Class Non-Susceptible; A Regular Node.

Regeneration: Susceptible

Regeneration, represented by a single node in figure 5.13 has inputs:

- (1) harvests from susceptible infested protected in period 1: $\sum g_i(1)$;
- (2) harvests from susceptible infested unprotected period 1: $\sum f_i(1)$;
- (3) harvests from susceptible uninfested in period 1: $\sum fp_i(1)$;
- (4) burned area from susceptible uninfested in period 1: $\sum q_i[y_i(1)-g_i(1)-gc_i(1)]$;
- (5) burned area from infested unprotected in period 1: $\sum ru_i[z_i(1)-f_i(1)-fc_i(1)]$; and
- (6) burned area from infested protected in period 1: $\sum rp_i(1)[zp_i(1)-fp_i(1)-fcp_i(1)]$.

The output from this node is $y_1(2) = (1)+(2)+(3)+(4)+(5)+(6)$ is the area regenerated at the start of period 2.

Figure 5.13: Regeneration Age-Class Susceptible; A Regular Node.

Figure 5.14: The Fire and Pest Protection Model with Protection Spraying and Conversion Harvests.

Chapter 6

Algebraic Representation

In this chapter the model structures introduced graphically in chapter five are documented algebraically using a modelling style similar to the *model development system* GAMS (Brooke, Kendrick and Meeraus [1988]). First, the basic fire and pest model without protection spraying or conversion harvests is developed. Next, the model is augmented to show how protection by spraying of the infested stands and conversion harvests are separately included. Finally, the complete fire and pest protection model with protection spraying and conversion harvests is presented. The sections 6.1 to 6.4 are the algebraic representations of the associated network descriptions of sections 5.1 to 5.4 respectively.

6.1 The Fire and Pest Model with no Protection Spraying or Conversion Harvests.

In the documentation of the following mathematical programming problems we will adopt a notation similar to that used in GAMS. This promotes a consistent, unambiguous modelling style. First, all entities of the model are identified and grouped by type (e.g. indices, decision variables etc.). Second, the ordering of entities is chosen so that no symbol is referred to until it is defined. Third, the units of all entities are specified, and, fourth, the units are chosen to a scale such that the numerical values to be encountered by the optimizer have relatively small absolute orders of magnitude. Terminology varies

amongst modellers. For example, economists use the terms "exogenous variable" and "endogenous variable" for "given data" and "decision variable", respectively. In GAMS, the terminology adopted is as follows: indices are called **SETS**, given data are called **PARAMETERS**, decision variables are called **VARIABLES**, and constraints and the objective function are called **EQUATIONS**.

SETS

Sets are fundamental building blocks and allow a model to be succinctly stated. These describe the level of disaggregation while providing a notation for indexing. The elements in a set are the labels.

$i \in I = \{i-20, i-40, i-60, i-80, i-100, i-120, i-140, i-160, i-180\}$
nine 20-year age-classes

$t \in T^2 = \{p-1*p-10\}$
ten 20-year time periods

$k \in K = \{k-1*k-20\}$
twenty 1-year time periods

SUBSETS

A minimum age for harvesting is imposed by segregating the age-classes into those from which cutting is and is not allowed.

$I_{cns} = \{i-140, i-160, i-180\}$
cutting age-classes non-susceptible

² The asterisk "*" is used to alleviate the writing of a sequence of elements for a set. Here we have ten twenty year time periods. Instead of typing all ten, GAMS builds up the label list by looking at the difference between the two labels. If the only characters that differ are digits, and if the number (say L) formed by these digits in the left one is less than that for the right one (R), then a label is constructed from every integer in the sequence L to R.

I_{ns}	= {i-20, i-40, i-60, i-80, i-100, i-120}	no cutting age-classes non-susceptible
I_s	= {i-20, i-40, i-60, i-80, i-100}	no cutting age-classes susceptible
I_{cs}	= { i-120, i-140, i-160, i-180}	cutting age-classes susceptible
I_{inf}	= {i-60, i-80, i-100, i-120, i-160, i-180}	infestable classes

DATA³

The data are generally of two types, parameters and variables. A scalar is a parameter of dimension one.

SCALARS

MUP	planting cost	(\$ per hectare)
MUC	cutting cost	(\$ per hectare)
PD	sales price of pulplogs	(\$ per cubic metre)
i	annual discount rate	
μ	percent volume reduction of infested stands	
δ	maximum proportional change in sequential pulp flow for a ten year period	

³ The fire and pest models are calibrated using inventory and volume-at-age data obtained for the Kamloops Timber Supply Area in British Columbia. These data were aggregated according to procedures explained in Appendix A section A1. The associated GAMS code which implemented the massaging of the data is given in Appendix D

PARAMETERS

VNS_i	volume of non-susceptible forest	(cubic metres per ha)
VS_i	volume of susceptible forest	(cubic metres per ha)
$p(t)$	average proportion of <i>non-susceptible</i> timber burned in period t and naturally regenerates at the start of period $t+1$	
$q(t)$	average proportion of <i>susceptible uninfested</i> timber burned in period t	
$ru(t)$	proportion of <i>susceptible infested</i> burned in period t	
$s_i(t)$	proportion of <i>susceptible uninfested</i> timber in age-class i that becomes <i>infested</i> in period t and remains in age-class i in period $t+1$.	

VARIABLES (subject to non-negativity constraints)

Variables are endogenous or the decision variables.

$h_i(t)$	area of <i>non-susceptible</i> timber harvested in age-class i in period t and regenerates as age-class $i-20$ at the start of period $t+1$		(ha per period)
$g_i(t)$	area of <i>uninfested susceptible</i> timber harvested in age-class i in period t and regenerates as age-class $i-20$ at the start of period $t+1$		(ha per period)
$f_i(t)$	area of <i>infested susceptible</i> species harvested in age-class i and regenerates as age-class $i-20$ at the start of period t		(ha per period)

OTHER VARIABLES (subject to non-negativity constraints)

We describe the state of the forest at the start of period t by three state variables $x_i(t)$, $y_i(t)$ and $z_i(t)$ which represent the area of timber at the start of time period t in age-class i which is non-susceptible, susceptible uninfested and susceptible infested respectively. These variables are related inter-temporally by fire, infestation, harvesting and growth processes which are described algebraically in the area balance equations or graphically in the network description in the previous chapter.

$x_i(t)$ area of *non-susceptible* timber in age-class i at the start of time period t (ha)

$y_i(t)$ area of *susceptible uninfested* timber in age-class i at the start of period t (ha)

$z_i(t)$ area of *susceptible infested* timber in age-class i at the start of period t (ha)

pulp(t) final shipments of pulp (cubic metres per period)

phip(t) planting cost (\$ per period)

phil(t) cutting cost (\$ per period)

phix(t) sales revenue (\$ per period)

EQUATIONS

Together with the objective function the constraints are considered as equations.

The fire pest model with no protection or conversion harvests:

$$\text{MAX } \phi = \sum_t \frac{1}{(1+i)^{20(t-1)}} [\phi_{ix}(t) - \phi_{ip}(t) - \phi_{il}(t)] \quad (6.5)$$

$$\text{where, } \phi_{ix}(t) = PD * \text{pulp}(t) \quad (6.6)$$

In order to reflect the loss of harvest volume due to the infestation the merchantable volume recovered from the infested stands is reduced by μ percent.

The final shipments of pulp (cubic metres per period) are therefore:

$$\text{pulp}(t) \leq \sum_{i \in I_{cns}} VNS_i * h_i(t) + \sum_{i \in I_{cs}} VS_i [g_i(t) + (1-\mu)f_i(t)] \quad (6.7)$$

The logging costs :

$$\phi_{il}(t) = MUC \left\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t) + f_i(t)] \right\} \quad (6.8)$$

and the planting⁴ costs :

$$\phi_{ip}(t) = MUP \left\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t) + f_i(t)] \right\} \quad (6.9)$$

⁴ we have assumed every hectare harvested will regenerate by planting.

The area balance constraints are:

NON SUSCEPTIBLE

for the youngest age-class :

$$x_{i-20}(t+1) = \sum_{i \in I_{cns}} [h_i(t) + p(t)(x_i(t) - h_i(t))] + \sum_{i \in I_{ns}} p(t) x_i(t) \quad (6.10)$$

the no-cutting classes:

$$x_i(t+1) = [1-p(t)] x_{i-1}(t) \quad \text{for } i \in I_{ns} \text{ not including "i-20"} \quad (6.11)$$

the cutting classes

$$x_i(t+1) = [1-p(t)] x_{i-1}(t) \quad \text{for } i = \text{"i-140"} \quad (6.12)$$

$$x_i(t+1) = [1-p(t)] * \{x_{i-1}(t) - h_{i-1}(t)\} \quad \text{for } i = \text{"i-160"} \quad (6.13)$$

$$x_i(t+1) = [1-p(t)] * \{x_{i-1}(t) - h_{i-1}(t) + x_i(t) - h_i(t)\} \quad \text{for } i = \text{"i-180"} \text{ the oldest age-class} \quad (6.14)$$

SUSCEPTIBLE UNINFESTED

the youngest age-class :

$$y_{i-20}(t+1) = \sum_{i \in I_{cs}} \{ g_i(t) + f_i(t) + q(t)[y_i(t) - g_i(t)] + ru(t)[z_i(t) - f_i(t)] \} \\ + \sum_{i \in I_s} \{ ru(t) * z_i(t) + q(t) * y_i(t) \} \quad (6.15)$$

the no-cutting classes:

$$y_i(t+1) = \{1-q(t)-s_{i-1}(t)\} * y_{i-1}(t) \quad \text{for } i \in I_s \text{ not including "i-20"} \quad (6.16)$$

for the cutting classes :

$$y_i(t+1) = \{1-q(t)-s_{i-1}(t)\} * y_{i-1}(t) \quad \text{for } i = \text{"i-120"} \quad (6.17)$$

$$y_i(t+1) = \{1-q(t)-s_{i-1}(t)\} * [y_i(t)-g_i(t)] \quad \text{for } i \in \{i-140, i-160\} \quad (6.18)$$

$$y_i(t+1) = \{1-q(t)-s_{i-1}(t)\} * [y_i(t)-g_i(t)+y_{i-1}(t)-g_{i-1}(t)] \quad \text{for } i = \text{"i-180"} \quad (6.19)$$

SUSCEPTIBLE INFESTED

We notice that in the infested equations there is no inter-temporal jump in the age-class .

For the no-cutting classes:

$$z_i(t+1) = s_i(t) * y_i(t) + [1-ru(t)] * z_i(t) \quad \text{for } i \in I_s \quad (6.20)$$

and for the cutting classes

$$z_i(t+1) = s_i(t) * \{y_i(t)-g_i(t) + [1-ru(t)] * \{z_i(t)-f_i(t)\} \} \quad \text{for } i \in I_{cs} \quad (6.21)$$

We must observe the restrictions on the area cut :

$$h_i(t) \leq x_i(t) \quad \text{for } i \in I_{cns} \quad \text{non-susceptible} \quad (6.22)$$

$$g_i(t) \leq y_i(t) \quad \text{for } i \in I_{cs} \quad \text{susceptible uninfested} \quad (6.23)$$

$$\text{and } f_i(t) \leq z_i(t) \text{ for } i \in I_{CS} \quad \text{susceptible infested} \quad (6.24)$$

To ensure a steady flow of wood the following sequential flow constraints are used:

$$\text{pulp}(t) \geq (1-\delta)^2 * \text{pulp}(t-1) \quad (6.25)$$

$$\text{pulp}(t) \leq (1+\delta)^2 * \text{pulp}(t-1) \quad (6.26)$$

$(1-\delta)$ is squared because δ is specified for a ten-year period and $(1-\delta)^2$ is the twenty year period proportion. This model will be run to provide a *baselrun scenario* so that the effect of including policy controls such as *protection* of the infested stands by spraying and *converting* the harvested susceptible area by planting with non-susceptible species may be compared for economic efficiency.

6.2 The Fire and Pest Model with Conversion Harvests

In this section the "baserun" model described in § 6.1 is augmented to include the possibility of converting timberland from susceptible to non-susceptible after harvest. The following conversion harvest variables are included for infested and uninfested susceptible stands. The new variables are shown as **bold**.

VARIABLES (subject to non-negativity constraints)

$gc_i(t)$ area of *uninfested susceptible* harvested in period t and is converted to *non-susceptible* age-class "i-20" at the start of period $t+1$ (ha per period)

$fc_i(t)$ area of *infested susceptible* harvested in period t and is converted to *non-susceptible* age-class "i-20" at the start of period $t+1$. (ha per period)

EQUATIONS

The fire pest model with conversion harvests :

$$\text{MAX } \phi_i = \sum_t \frac{1}{(1+i)^{20(t-1)}} [\phi_{ix}(t) - \phi_{ip}(t) - \phi_{il}(t)] \quad (6.27)$$

where conversion harvests would be included in :

the final shipments of pulp,

$$\text{pulp}(t) \leq \sum_{i \in I_{\text{cns}}} \text{VNS}_i * h_i(t)$$

$$+\sum_{i \in I_{cs}} VS_i * \{g_i(t)+gc_i(t)+(1-\mu)[f_i(t)+fc_i(t)] \} \quad (6.28)$$

the logging costs

$$phil(t) = MUC\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t)+gc_i(t)+f_i(t)+fc_i(t)] \} \quad (6.29)$$

and the planting costs

$$phip(t) = MUP\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t)+gc_i(t)+f_i(t)+fc_i(t)] \} \quad (6.30)$$

The area balance constraints are modified to reflect the conversion harvests:

NON SUSCEPTIBLE

for the youngest age-class :

$$\begin{aligned} x_{i-20}(t+1) = & \sum_{i \in I_{cns}} [h_i(t) + p(t)(x_i(t) - h_i(t))] + \sum_{i \in I_{ns}} p(t) x_i(t) \\ & + \sum_{i \in I_{cs}} [gc_i(t) + fc_i(t)] \end{aligned} \quad (6.31)$$

SUSCEPTIBLE UNINFESTED

$$\begin{aligned}
 y_{i-20}''(t+1) = & \sum_{i \in I_{CS}} \{ g_i(t) + f_i(t) + q(t) [y_i(t) - g_i(t) - gc_i(t)] + ru(t) [z_i(t) - f_i(t) - fc_i(t)] \} \\
 & + \sum_{i \in I_S} \{ ru(t) * z_i(t) + q(t) * y_i(t) \}
 \end{aligned} \tag{6.32}$$

for the cutting classes :

$$y_i(t+1) = \{ 1 - q(t) - s_{i-1}(t) \} * [y_i(t) - g_i(t) - gc_i(t)] \quad \text{for } i \in \{ i-140, i-160 \} \tag{6.33}$$

$$\begin{aligned}
 y_i(t+1) = & \{ 1 - q(t) - s_{i-1}(t) \} * [y_i(t) - g_i(t) - gc_i(t) + y_{i-1}(t) - g_{i-1}(t) - gc_{i-1}(t)] \\
 & \text{for } i = "i-180" \text{ the oldest age-class}
 \end{aligned} \tag{6.34}$$

SUSCEPTIBLE INFESTED

for the cutting classes

$$z_i(t+1) = s_i(t) * \{ y_i(t) - g_i(t) - gc_i(t) \} + (1 - ru(t)) * \{ z_i(t) - f_i(t) - fc_i(t) \} \quad \text{for } i \in I_{CS} \tag{6.35}$$

We must modify the restrictions on the area cut for the susceptible stands:

$$g_i(t) + gc_i(t) \leq y_i(t) \quad \text{for } i \in I_{CS} \quad \text{susceptible uninfested} \tag{6.36}$$

$$\text{and} \quad f_i(t) + fc_i(t) \leq z_i(t) \quad \text{for } i \in I_{CS} \quad \text{susceptible infested} \tag{6.37}$$

6.3 The Fire and Pest Model with Protection Spraying

In this section we show how the "baserun" model described in § 6.1 is augmented to include the **protection** of infested stands by spraying as decision variables. We need to add a state variable: $zp_i(t)$, the area of susceptible forest, infested at age i , which is still surviving at the start of period t and was protected by a spraying program in period $t-1$. Again the new variables are shown as **bold**.

SCALARS

MUS spraying cost (\$ per hectare)
 β percent volume reduction in infested protected stands

PARAMETERS

$rp(t)$ average proportion of *susceptible protected infested* burned per period

VARIABLES (subject to non-negativity constraints)

$pr_i(t)$ area of infested protected by spraying in period t (ha per period)

$fp_i(t)$ area of infested protected timber in age-class i harvested in period t and
 regenerates as age-class "i-20" at the start of period $t+1$. (ha per period)

OTHER VARIABLES (subject to non-negativity constraints)

$zp_i(t)$ area of *protected infested* timber in age i at the start of period t (ha)

phis(t) spraying cost

(\$ per period)

EQUATIONS

The fire pest model with protection spraying:

$$\text{MAX } \phi = \sum_t \frac{1}{(1+i)^{20(t-1)}} [\phi_{ix}(t) - \phi_{ip}(t) - \phi_{il}(t) - \phi_{is}(t)] \quad (6.38)$$

where the final shipments of pulp include harvesting the protected stands:

$$\text{pulp}(t) \leq \sum_{i \in I_{cns}} VNS_i * h_i(t) + \sum_{i \in I_{cs}} VS_i [g_i(t) + (1-\mu)f_i(t) + (1-\beta)fp_i(t)] \quad (6.39)$$

The cost of spraying is :

$$\phi_{is}(t) = \text{MUS} \sum_{i \in I_{inf}} pr_i(t) \quad (6.40)$$

The logging and planting costs will include the protected area harvested:

$$\text{Logging } \phi_{il}(t) = \text{MUC} \left\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t) + f_i(t) + fp_i(t)] \right\} \quad (6.41)$$

$$\text{Planting } \phi_{ip}(t) = \text{MUP} \left\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t) + f_i(t) + fp_i(t)] \right\} \quad (6.42)$$

The susceptible area balance constraints need to reflect the protection of stands by spraying.

SUSCEPTIBLE UNINFESTED

the youngest age-class :

$$\begin{aligned}
 y_{i-20}(t+1) = & \sum_{i \in I_{cs}} \{ g_i(t) + f_i(t) + fp_i(t) + q(t)[y_i(t) - g_i(t)] + ru(t)[z_i(t) - f_i(t)] \\
 & + rp(t)[zp_i(t) - fp_i(t)] \} \\
 & + \sum_{i \in I_s} \{ ru(t)[z_i(t) - pr_i(t)] + q(t)*y_i(t) + rp(t)*zp_i(t) \} \quad (6.43)
 \end{aligned}$$

SUSCEPTIBLE INFESTED NOT PROTECTED

For the no-cutting classes:

$$z_i(t+1) = s_i(t)*y_i(t) + (1 - ru(t))*[z_i(t) - pr_i(t)] \quad \text{for } i \in I_s \quad (6.44)$$

and for the cutting classes

$$z_i(t+1) = s_i(t)*[y_i(t) - g_i(t)] + (1 - ru(t))*[z_i(t) - f_i(t) - pr_i(t)] \quad \text{for } i \in I_{cs} \quad (6.45)$$

SUSCEPTIBLE INFESTED PROTECTED

The no-cutting classes

$$zp_i(t+1) = pr_{i-1}(t) + [1 - rp(t)] zp_{i-1}(t) \quad \text{for } i \in I_s \quad (6.46)$$

and the cutting classes :

$$zp_i(t+1) = pr_{i-1}(t) + [1-rp(t)] zp_{i-1}(t) \quad \text{for } i = "i-120" \quad (6.47)$$

$$zp_i(t+1) = pr_{i-1}(t) + [1-rp(t)] * \{zp_{i-1}(t) - fp_{i-1}(t)\} \quad \text{for } i \in \{i-140, i-160\} \quad (6.48)$$

$$zp_i(t+1) = pr_i(t) + pr_{i-1}(t) + [1-rp(t)] * \{zp_i(t) - fp_i(t) + zp_{i-1}(t) - fp_{i-1}(t)\} \\ \text{for } i = "i-180" \quad (6.49)$$

The restrictions on the areas cut and protected must be modified to include the protected area:

$$f_i(t) + pr_i(t) \leq z_i(t) \quad \text{for } i \in I_{CS} \quad \text{infested unprotected} \quad (6.50)$$

$$pr_i(t) \leq z_i(t) \quad \text{for } i \in I_S \quad \text{infested unprotected} \quad (6.51)$$

$$fp_i(t) \leq zp_i(t) \quad \text{for } i \in I_{CS} \quad \text{infested protected} \quad (6.52)$$

6.4 The Fire and Pest Model with Protection Spraying and Conversion Harvests.

In this section the fully articulated fire and pest protection model ⁵ is described. We need to include a conversion harvest variable for the "infested protected" stands. This model is the combination of the conversion harvest model § 6.2 and the protection model § 6.3. New variables are again shown as bold .

VARIABLES (subject to non-negativity constraints)

fcp_i(t) area of *infested protected susceptible* harvested in period t and is converted to *non-susceptible* age-class "i-20" at the start of period t+1 (ha per period)

EQUATIONS

The fire pest model with protection spraying and conversion harvests :

$$\text{MAX } \phi_i = \sum_t \frac{1}{(1+i)^{20(t-1)}} [\phi_{ix}(t) - \phi_{ip}(t) - \phi_{il}(t) - \phi_{is}(t)] \quad (6.53)$$

subject to the following constraints including conversion harvests for the protected stands:

the final shipments of pulp,

$$\text{pulp}(t) \leq \sum_{i \in I_{\text{cns}}} \text{VNS}_i * h_i(t)$$

⁵ In Appendix B the GAMS model code and solution run is provided for this particular model.

$$+\sum_{i \in I_{cs}} VS_i \{ g_i(t)+gc_i(t)+(1-\mu)[f_i(t)+fc_i(t)]+(1-\beta)[fp_i(t)+fcp_i(t)] \} \quad (6.54)$$

the logging costs

$$phil(t) = MUC\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t)+gc_i(t)+f_i(t)+fc_i(t)+fp_i(t)+fcp_i(t)] \} \quad (6.55)$$

and the planting costs

$$phip(t) = MUP\{ \sum_{i \in I_{cns}} h_i(t) + \sum_{i \in I_{cs}} [g_i(t)+gc_i(t)+f_i(t)+fc_i(t)+fp_i(t)+fcp_i(t)] \} \quad (6.56)$$

The area balance constraints are modified to reflect the conversion harvests:

NON SUSCEPTIBLE

for the youngest age-class :

$$\begin{aligned} x_{i-20}(t+1) = & \sum_{i \in I_{cns}} [h_i(t) + p(t)(x_i(t) - h_i(t))] + \sum_{i \in I_{ns}} p(t) * x_i(t) \\ & + \sum_{i \in I_{cs}} [gc_i(t) + fc_i(t) + fcp_i(t)] \end{aligned} \quad (6.57)$$

SUSCEPTIBLE UNINFESTED

$$y_{i-20}(t+1) = \sum_{i \in I_{cs}} \{ g_i(t) + f_i(t) + fp_i(t) + q(t)[y_i(t) - g_i(t) - gci(t)] + ru(t)[z_i(t) - f_i(t)] + rp(t)[zp_i(t) - fp_i(t) - fcp_i(t)] \}$$

$$+ \sum_{i \in I_S} \{ ru(t)[z_i(t)-pr_i(t)]+q(t)*y_i(t)+rp(t)*zp_i(t) \} \quad (6.58)$$

SUSCEPTIBLE INFESTED NOT PROTECTED

for the cutting classes

$$z_i(t+1) = s_i(t)*\{y_i(t)-g_i(t)-gc_i(t)\}+(1-ru(t))*\{z_i(t)-f_i(t)-fc_i(t)-pr_i(t)\} \quad \text{for } i \in I_{CS} \quad (6.59)$$

SUSCEPTIBLE INFESTED PROTECTED

For the cutting classes :

$$zp_i(t+1) = pr_{i-1}(t)+[1-rp(t)]*\{zp_{i-1}(t)-fp_{i-1}(t)-fcp_i(t)\} \quad \text{for } i \in \{i-140, i-160\} \quad (6.60)$$

$$zp_i(t+1) = pr_i(t)+pr_{i-1}(t)+[1-rp(t)]*\{zp_i(t)-fp_i(t)-fcp_i(t)+zp_{i-1}(t)-fp_{i-1}(t)-fcp_{i-1}(t)\}$$

$$\text{for } i = "i-180" \text{ the oldest age-class} \quad (6.61)$$

The restrictions on the conversion harvests must be included:

$$f_i(t)+fc_i(t)+pr_i(t) \leq z_i(t) \quad \text{for } i \in I_{CS} \quad \text{infested unprotected} \quad (6.62)$$

$$fp_i(t)+fcp_i(t) \leq zp_i(t) \quad \text{for } i \in I_{CS} \quad \text{infested protected} \quad (6.63)$$

6.5 Conclusions

The fully articulated model structures presented in this chapter have 771 variables and 593 constraints. This efficient, parsimonious state-space model structure enabled the problems to be solved on a personal computer. Since computational time for solution of linear programs by the Simplex Algorithm is more dependent on the number of constraints we obtained considerable improvement in solution time, as compared with the Boychuk [1988] model (1770 variables and 1070 constraints), due to the reduction in the number of constraints.

The model structures presented were *generic*. That is to say, the complexity of the model is managed by making the data operations *set driven*. This way, it is not necessary to write down separate equations for, say, each age-class but rather a set of age-classes were created in a single equation defined over the set. Therefore, large models which are a hallmark of the FMP may be documented with models which include only a small number of equation types. The generic equations hold over all elements within a set. Exceptions may be made by using subsets and operators that exclude certain elements of the set.

Also, the generic structure allows easy evolutionary changes to be made. For example, the inclusion of protection and intensive forest management controls by augmenting the basic model enabled a *comparative scenario analysis* (chapter 8) so the relative merits of each policy could be judged.

Chapter 7

Model Validation

In this chapter we develop confidence in the model by testing how well the deterministic policy holds in a simulated stochastic environment as well as providing a sensitivity analysis to examine the model's response to parametric and structural changes. In the first section we attempt to validate the assumptions of using deterministic fire and infestation hazard rates. Due to the long term nature of the problem it is not possible to validate the fire and pest protection model in the real world; we cannot wait a hundred years! However, we propose a "validation procedure" which tests how well the model works in a simulated stochastic environment. In the second section the impact on the "optimal" timber supply of changing the discount rate, annual fire rates and the maximum proportional increase or decrease in the sequential flow constraints is provided in a sensitivity analysis.

7.1 Simulation of a Stochastic Solution

Events such as forest fires and insect infestations occur randomly. Therefore, we need to know how well the deterministic policies hold in a simulated stochastic environment. We can then test the performance of the deterministic model in a simulated "real" world. The solution to the simpler deterministic problem may be sufficient to provide an estimate of the long run sustainable yield subject to fixed rather than random losses due to fire and insect

infestation. In order to test the validity of the deterministic policy we seek an answer to the question :

Q1. How confident are we to use the deterministic policy when faced with obvious uncertainties such as fire and infestation losses ?

A procedure was developed to answer this question by attempting to solve the stochastic problem (equation 3.25). This is a problem in *stochastic control theory* since the dynamical system describing the forest's stand age-class structure consists of stochastic difference equations. The solution will be a "feedback policy" where the optimal harvest will depend on the current state and possibly on previous states and harvests. Since a solution to the true stochastic problem is difficult to obtain an approximation of the *stochastic control problem* is suggested.

This solution may then be compared with the simpler deterministic policy. The closeness of these two policies will provide a measure of confidence for the simpler policy. The idea is to use the optimal solution to the deterministic problem in a *feedback fashion* to obtain an approximately optimal solution to the stochastic control problem. An iterative simulation-linear programming approach is used to solve the fire and pest protection problem when fire proportions were allowed to be random variables with empirically based distributions. The *iterative simulation-LP approach* involves:

- (1) solving the deterministic problem at each time period based on the current system state and average values for the fire and infestation proportions; and
- (2) updating the the system state based on the optimal harvests and protection controls, and then simulating the state transition at the next period with the simulation model (the dynamical area balance equations), using random fire infestation proportions. This procedure is shown in Figure 7.1.

The *approximate optimal harvest strategy* obtained in this fashion is compared with the *deterministic solution*. Problems in which parameters are unknown or uncertain rarely have exact solutions. Heuristic and approximation methods are needed. One heuristic commonly used is the *certainty equivalence* approach (see Chow [1975]). In this method the expected value of the random variables are used. Then the stochastic control problem is converted to a deterministic control problem.

7.1.1 Stochastic LP Simulator

To answer question Q1 of the validation procedure an iterative simulation-optimization approach ¹ allows the fire and infestation proportions to be entered randomly into a state updating scheme. First, an optimal deterministic solution is obtained using an average per annum fire rate of .005 and a period infestation rate of .2. We record this solution (call it LP1) to compare with the *sample paths of the approximate stochastic solution* which will be constructed in the following iterative optimization procedure. First, we define some sets in the usual modelling style.

SETS

$y \in Y$	$=\{y-1*y-20\}$	twenty single years
$t \in T$	$=\{P-1*P-10\}$	ten twenty year periods
$i \in I18$	$=\{I-1*I-18\}$	index of discrete fire rates

PARAMETERS

q_i	random annual fire rate
$F(q_i)$	empirical cumulative probability distribution function for fire occurrence

¹ The GAMS program for the simulator is documented in Appendix C.

$R_y(t)$	random number in the interval [0,1]
$QSAMP_y(t)$	random sample annual fire rates
$P(t)$	random period fire rate
$S(t)$	random period infestation rate

Twenty random annual fire rates for the ten twenty-year simulation time periods are drawn from the empirical distribution using the following procedure. First, we set the seed for the pseudo-random number generator.

Seed = 6502

- * the inverse transformation method is used to draw the random fires from the
- * distribution, using linear interpolation.

Loop (t = 1 to 10

 Loop (y = 1 to 20

$R_y(t) = \text{UNIFORM}(0,1)$;

 Loop (i = 1, 18

 If ($R_y(t) > F(q_i)$) then

$$QSAMP_y(t) = q_i + (R_y(t) - F(q_i)) * (q_{i+1} - q_i) / (F(q_{i+1}) - F(q_i))$$

)

The twenty year period random fire rate is then calculated for the ten time periods.

$$p^*(t) = 1 - \prod_{y=1}^{20} (1 - QSAMP_y(t)) \quad \text{for } t = 1, 2, \dots, 10.$$

The random fire rates $q^*(t)$, $ru^*(t)$ and $rp^*(t)$ are set equal to $p^*(t)$.

We assume that the random annual infestation rate is uniformly distributed between zero and .02. The random sample annual infestation rates $(R_y(t) \cdot .02)$ will be obtained by linear interpolation using the inverse transformation method where the cumulative distribution function is a ramp whose (x,y) coordinates are $(0,0)$ and $(.02, 1)$. The period infestation rates are thus obtained:

$$s^*(t) = 1 - \prod_{y=1}^{20} (1 - R_y(t) \cdot .02) \quad \text{for } t = 1, 2, \dots, 10.$$

Iterative Optimization Procedure

- (1) From LP1 record the optimal first period ("P-1") timber supply $PULP("P-1")$
- (2) Using simulated random fire and infestation rates: $p^*(\text{"P-1"})$, $q^*(\text{"P-1"})$, $ru^*(\text{"P-1"})$, $rp^*(\text{"P-1"})$ and $s^*(t)$ together with the optimal harvests and areas protected obtained from LP1: $h^*(\text{"P-1"})$, $g^*(\text{"P-1"})$, $gc^*(\text{"P-1"})$, $f^*(\text{"P-1"})$, $fc^*(\text{"P-1"})$, $fp^*(\text{"P-1"})$, $fcp^*(\text{"P-1"})$ and $pr^*(\text{"P-1"})$ we update the state vectors: $X(\text{"P-2"})$, $Y(\text{"P-2"})$, $Z(\text{"P-2"})$ and $ZP(\text{"P-2"})$ by using the dynamical equations (simulation system) described in chapter six.
- (3) The deterministic problem is re-solved (LP2) using these updated states as the new initial state vector to determine the optimal second period harvests and areas protected $h^*(\text{"P-2"})$, $g^*(\text{"P-2"})$, $gc^*(\text{"P-2"})$, $f^*(\text{"P-2"})$, $fc^*(\text{"P-2"})$, $fp^*(\text{"P-2"})$, $fcp^*(\text{"P-2"})$ and $pr^*(\text{"P-1"})$. We note that average annual fire rates of .005 are used in all future periods as well as an average period infestation rates of .2 From LP2 we record the optimal second period timber supply: $PULP(\text{"P-2"})$.

- (4) This procedure is repeated to produce solutions LP3 to LP10 from which the rest of the optimal period timber supplies are obtained for periods P-3 to P-10 (i.e. PULP("P-3") to PULP("P-10")).

This optimal sample path constructed may be viewed as an approximate solution to the stochastic control problem. The iterative *simulation-linear programming procedure* reminds us that the major purpose in using a timber management scheduling model is only to consider the long run impact of management actions in developing a schedule for the immediate period. Decisions concerning the use of limited resources in future periods should be delayed until more information becomes available.

The key to this process is that at any point (period) in time one is solving for the *immediate actions* only. Other actions are considered only to help measure the long-term impacts of the immediate actions. Uncertainty makes it difficult to evaluate the long-term impacts.

Figure 7.1: Feedback application of the Deterministic Control in a Stochastic Environment : An Iterative Optimization Procedure.

7.1.2 Results and Conclusions from the Stochastic Simulation

Application of the iterative "feedback" procedure using randomly generated fire probabilities was efficiently implemented using a SAVE and RESTART feature of GAMS. The deterministic fire pest model, as described in § 6.3, was solved (*the deterministic solution*) and saved using the GAMS SAVE command which saves an entire problem setup and values of all defined parameters during execution. A complete description of this model and its solution is given in appendix B. The subsequent solutions LP2 to LP10 are obtained from another GAMS file, the "stochastic LP simulator" which was described in § 7.2 by calling upon the saved file which contained *the deterministic solution* using the GAMS RESTART command. The GAMS program for this simulator is given in appendix C. The stochastic simulator is providing, for each re-run LP2-LP10, new initial conditions on the states for the originally defined problem.

This enables all the stochastic solutions to be referenced to the same deterministic run. Furthermore, we recall the fire and pest model has a *network flow structure*. One characteristic of network problems is that an initial feasible solution is difficult to obtain. This is evidenced by the large number of iterations required to obtain the initial optimal deterministic solution. However, once a feasible solution is found subsequent solves, as required in the creation of the sample path (i.e. solutions LP2-LP10), were obtained much faster.

An example of a sample path is given in Figure 7.2. For the sample path (st4), each point on the graph (i.e. at each time period) is obtained from one LP solution as explained in the above iterative optimization procedure. We obtained eight sample paths by resetting the seed of the pseudo random number generator for each one.

Figure 7.2: An example of the application of the Deterministic Control using Randomly generated Fire Probabilities; one sample path st4

We compare the approximate solutions to the stochastic control problem with the deterministic policy which assumed fixed fire and infestation rates. In Figure 7.3 eight sample paths (labelled st1 to st8) obtained by simulating fire destruction and insect infestation rates for eight independent ten-period time horizons are shown together with an overlay of the deterministic solution obtained using an average period infestation rate of .2 and average period fire rate corresponding to a rate of .005 per annum which is traced by the solid line. We observe that the deterministic solution follows closely the approximation to the stochastic solutions for the first six periods and then for the last four periods the randomness increases. The timber supply trajectories obtained from this iterative optimization procedure provide a region of confidence for the deterministic solution. This region widens towards the end of the time horizon indicating that we cannot be as sure of the timber supply in the long term when faced with uncertainties such as insect infestations or fire loss.

Figure 7.3: Optimal Feedback application of the Deterministic Control using Randomly generated Fire and Infestation Probabilities; eight sample paths (st1 to st8) plus an overlay chart of the Optimal Harvest solution using .005 average annual fire loss and average period infestation rate of .2 (the Deterministic Solution).

This increase in the variability of the optimal timber supply obtained from the state updating procedure may be explained by the restrictions imposed on cutting younger stands. During the latter part of the time horizon all the original older stands have been harvested and a large amount of regenerated timber is available for harvest in period 7. Specifically, the initial age-class distribution is uneven and has considerable over mature timber in age-classes i-120 and i-180 with little timber in i-140 and i-160 as can be seen in Figure 8.1. Since we restrict cutting to at least age-class i-120 (non-susceptible) and i-100 (susceptible), by period 6 most of the original over-mature stands have been harvested and by period 7 much regenerated timber is available for harvest. The effect of fire in the younger stands appears to increase the variability in the optimal harvest. This is an important problem since, in our ideal model, we restrict the cutting of younger age-classes,

yet in reality younger stands are often chosen for harvest for reasons of accessibility. If this suggestion is correct then the variability in the optimal supply would be higher and the problem compounded.

From these results we conclude that, since the optimal solution using average per period fire and infestation rates is sufficiently close to the approximate stochastic solutions obtained using the feedback application of the deterministic control using randomly generated fire and infestation probabilities, the deterministic fire and pest protection model is valid for evaluating policies for the present while taking the long term effects into consideration.

7.1.3 Stochastic Growth

We have not included in the stochastic simulation the possibility of stochastic growth as a result of environmental changes in growing conditions. Variability in growth would be reflected in the volume-at-age curves. However, we have absorbed, in our aggregate model, considerable variability in growing conditions and species diversity in the calibration of the fire and pest protection model as described in appendix A. This aggregation was based on inventory data defined for seven species, sixteen twenty year age-classes and two site classes. These data were used in a Multiple Use Sustained Yield Calculation (MUSYC) for the Kamloops TSA area. The aggregate model contained nine twenty year age-classes and two species (non-susceptible and susceptible). Without the representation of fire and infestation the optimal timber supply from the aggregate model compared favourably with a disaggregate MUSYC run. We, therefore, are confident that the aggregation procedure captured sufficient variability in a parsimonious model structure.

7.2 Sensitivity Analysis

We have "validated" the stochastic components of the model. We extend this validation by providing answers to the following question in a sensitivity analysis:

Q2. How sensitive is the optimal timber supply to variations in key model parameters such as discount rate, fire and inter-period flow constraints?

7.2.1 Discount Rate Sensitivity

We solve the fully articulated fire and pest protection model using average period fire rates corresponding to a fixed per annum probability of .005 and period discount rates corresponding to 0, 1, 2, 3, 4, 5, and 6 percent real interest rates per annum.

Figure 7.4 Effect of the Discount Rate on Timber Supply.

We see from Figure 7.4 that the optimal timber supply trajectory is relatively insensitive to changes in the discount rate. However, the total discounted net present value obtained from these solutions is affected as shown in Figure 7.5.

Figure 7.5 Effect of the Discount Rate on the total NPV.

7.2.2 Fire Rate Sensitivity

We show in Figure 7.6 the optimal timber supply subject to changes in the fire rate. The solutions corresponding to annual fire destruction rates of 0, .001, .0025, .005, .0075, .01, and .0125 are compared. The trajectory using a fire destruction rate of .005 is highlighted since we used this rate in the scenario analysis described in chapter eight.

Historical data for the Kamloops TSA suggest a mean annual fire rate of approximately 1/10%. We have solved for fire rates which are 2.5, 5, 7.5, 10, and 12.5 times as large. These rates would not necessarily be expected to happen and are used to demonstrate that the model solution is sensitive to changes in the average fire rate. For example a 12.5 times increase in the average fire rate from .001 to .0125 produces a 30% drop in the optimal harvest level for the first period.

Figure 7.6: Effect of changing the Fire Rate on Timber Supply.

7.2.3 Sequential Flow Constraint Sensitivity

In Figure 7.7 we show the effect on the "optimal" timber supply of changing the maximum proportional inter-period change in sequential pulp flow for a ten year period. We see how important this constraint is in determining the shape of the optimal harvest level. We have overlaid the solution obtained using a non-declining yield constraint. This constraint is equivalent to using a sequential flow constraint with zero maximum proportional decrease inter-period flow (i.e. $pulp(t) \geq pulp(t-1)$). This solution provides a sustainable optimal harvest level of 1.53×10^6 cubic metres per year. By increasing the maximum proportional decrease/increase for a ten-year period from 0 to .14 (i.e. 0% to 14%) the optimal first period supply increases 100%. This demonstrates that the supply is very sensitive to these inter-period flow constraints. We also observe in Figure 7.7 that increasing the maximum proportional decrease/increase in inter-period flow produces a supply characterised by a

higher initial harvest which decreases to a minimum after 120 years and then increases towards the end of the planning horizon. We have identified the extreme trajectories for $\beta = 0, .14$ (as defined in equation (3.14), p45) as well as that corresponding to $\beta = .1$ which was used in the stochastic validation and the scenario analysis of the next chapter.

We have shown that the harvest flow constraints determine the nature of the "optimal" timber supply trajectory. The choice of these constraints will reflect government objectives as they relate to the forest industry. A non-declining yield, for example, could be used if the objective was to maintain stability in employment for a region. We have selected, in the scenario analysis of the next chapter, an inter-temporal flow restriction (corresponding to a ten-year period) of ten percent. This was chosen arbitrarily without any specific policy objectives in mind.

Figure 7.7: A comparison of the Optimal Timber Supply due to changes in the maximum proportional increase/decrease in sequential pulp flow corresponding to a ten-year period.

7.2.4 Conclusions from the Sensitivity Analysis

We have examined how sensitive the fire and pest protection model optimal harvest trajectories are to changes in the discount rate, fire hazard rate and inter-temporal maximum proportional increase and decrease in sequential pulp flow. From this analysis we may conclude that model solutions are highly sensitive to changes in the fire and inter-period flow rates yet relatively insensitive to changes in the discount rate. We further conclude that since the optimal timber supply is so sensitive to changes in inter-temporal flow restrictions it supports the idea that the *forest-level harvest scheduling problem* is largely characterised by the form and magnitude of these constraints.

Chapter 8

Model Effectiveness: Distinguishing Policy Options

In the traditional sense, validation of a *descriptive* model involves the designing of tests such that the "model", if false, would fail to pass. We would then reject the truth of the model. A model which attempts to describe natural phenomena is accepted if it explains all the facts. If it does not, then we either reject the model or improve it, and test it again.

When the model is *prescriptive* rather than descriptive i.e. the forest management problems, it attempts to predict what *should* be done rather than what *is* actually done, the ability to explain facts is no longer a suitable criterion of worth. How then do we measure the worth of a prescriptive model? How effective is this model? In the next sections we explore the worthiness of the prescriptive model in terms of how *flexible* its structure is in distinguishing alternative policy options.

8.1 A Comparative Scenario Analysis

We assess the usefulness of the fire and pest protection models by conducting experiments to answer the following question:

Q3. How well does the model evaluate alternative management policies as compared with other model structures? In particular, does it pay to protect

infested stands by spraying or by conversion harvests or both in the long run ?

This question may be answered by performing a *comparative scenario analysis*. Since the fire and pest model represents management control actions such as conversion harvests and spraying directly in the model structure, alternative policies may be compared for economic efficiency by running the model with and without those controls articulated. For example the, baserun model, as described in § 6.1 with no protection for pest infestation or conversion harvests would be compared with results obtained from the solution of a structure which allows for either:

- (1) conversion harvests (§ 6.2); or
- (2) protection spraying (§ 6.3); or
- (3) conversion harvests and protection spraying (§ 6.4).

Indicators of model performance would include:

- (1) changes in the net present value (NPV) of timber production; and
- (2) changes in the time path of the optimal timber supply in its approach to the equilibrium maximum sustained yield (m.s.y).

A decrease in total NPV would indicate a higher cost policy. The equilibrium m.s.y. is the goal of forest management in the long run. Controlling for an infestation may result in an elevated m.s.y. The analysis would compare the effect of different fire/infestation scenarios on the wood supply trajectories for different control actions.

In the next sections we economically evaluate *protection* for Western Spruce Budworm (WSB) infestations by spraying and compare with intensive forest management

strategies such as *conversion harvests*. Five scenarios are obtained by solving the different problem statements defined algebraically in sections 6.1 to 6.4:

- (1) baserun (§ 6.1);
- (2) baserun+infestation (§ 6.1);
- (3) conversion harvest (§ 6.2);
- (4) protection (§ 6.3); and
- (5) conversion+protection (§ 6.4).

Each involved the solution of different model structures to show the impact on the long run sustained yield of the alternative control strategies. A final scenario showing the effect of an average period fire loss is also given.

8.2 Baserun Scenario

First, the *baserun scenario* was obtained by solving the model as described in § 6.1 with all fire and infestation parameters set to zero. Then, the same model was solved with a WSB infestation¹. We call this the *baserun+infestation scenario*. It was assumed that all area in infestable ages of the susceptible species were infested at the start of period 1. A 20% period infestation rate was imposed for all future periods on the infestable age-classes (i.e. i-60 through i-180) for the susceptible timber. The initial age-class distribution of the forest is shown in Figure 8.1. In both the *non-susceptible* (x) and *susceptible infested* (z) stands there is considerable over-aged timber (i.e older than 180 years).

¹ A description of the infestation data obtained for the Kamloops TSA is given in Appendix A, section A3.

Figure 8.1: Initial Age-Class distribution for the *Baserun+Infestation* Scenario

We compare the optimal harvest level solutions for the baserun and baserun+infestation scenarios in Figure 8.2. We recall that a ten percent (for ten-year period) upper and lower bound sequential flow constraint on the inter-period timber supply was used. This constraint completely determines the supply trajectory. After only ten periods the optimal solution had not reached a steady state but exhibits the expected cyclical shape consistent with these sequential flow constraints.

The nature of the *baserun+infestation* schedule (i.e. the pattern of harvesting) for the three states (x, y and z) are given in Figures 8.3 to 8.5 respectively. For the non-susceptible species the optimal solution selects the oldest aged timber (highest volume) first for harvesting.

For the susceptible timber, only infested timber is harvested in the early periods. Harvesting uninfested timber occurs later in the time horizon when the immature uninfested has reached a harvestable age.

Figure 8.2: Optimal Harvest level solutions for the *Baserun* and *Baserun+Infestation* Scenarios

Figure 8.3: Non-Susceptible area harvested for the *Baserun+Infestation* Scenario

Figure 8.4: Susceptible area harvested for the *Baserun+Infestation* Scenario

Figure 8.5: Susceptible Infested area harvested for the *Baserun+Infestation* Scenario

8.3 Conversion Harvest Scenario

We now explore the effect of including conversion harvest decision variables as described in the model of § 6.2. The results from the *baserun+infestation scenario* showed that there was no change in the total NPV or the optimal harvest trajectory. However, the optimal controls do select *conversion harvests* over harvesting the susceptible and planting with susceptible. The results indicate that conversion harvests do not have, in this case, an impact on the m.s.y. This may be explained as follows:

- (1) conversion harvests produce the same volume as non-conversion harvests; and
- (2) the effect of a conversion harvest is reflected only in the evolution of the forest's age-class distribution.

We show the final (period-10) age-class distribution for the two scenarios in Figures 8.6 and 8.7. In the *conversion harvest scenario* the increase in mature non-susceptible is beginning to show. However, a longer time horizon would be needed to show an impact on the m.s.y.

Figure 8.6: Period-10 Age-Class distribution for the *Baserun+Infestation Scenario*

Figure 8.7 Period-10 Age-Class distribution for the *Conversion Harvest* Scenario

8.4 Protection Scenario.

To evaluate the impact of including protection by spraying we solved the model as defined in § 6.3 using the same starting distributions as in the *baserun+infestation* scenario. Again there is no fire and the infestation parameters were the same. The comparison of the *baserun+infestation* with the *protection scenario* optimal timber supply is given in Figure 8.8.

An expected elevation of the supply occurs with protection. The total net present value of the protection solution is $\$4.838 \times 10^9$ as compared with $\$3.948 \times 10^9$ for the *baserun+infestation*. It clearly pays to protect the infested stands by spraying. The pattern of protection as calculated in the optimal solution is given in Figure 8.9. It pays to protect as soon as possible.

Figure 8.8: Optimal harvest levels for the *Baserun+Infestation* and the *Protection* Scenarios.

Figure 8.9 Area Protected by Spraying for the *Protection* Scenario

8.5 The Conversion Harvest and Protection Scenario

We finally solve the model as described in § 6.3 which includes both protection and conversion harvest variables. The optimal harvest level is the same as obtained in the protection scenario since we discovered that the conversion harvests do not have an impact on the supply within the selected ten period time horizon. The conversion harvests selected in the optimal solution are different to those obtained in the conversion only scenario. This is because conversion harvests occur also in the protected infested stands. We compare the last period age-class distribution for the protection scenario (Figure 8.10) with that for the *conversion and protection scenario* (Figure 8.11). These two distributions are quite similar except for an increase in some of the non-susceptible age-classes due to conversion harvests of protected stands.

Figure 8.10: Period-10 Age-Class distribution for the *Protection* Scenario

Figure 8.11: Period-10 Age-Class distribution for the *Conversion and Protection* Scenario

8.6 Effect of Fire on all Scenarios

We compare the impact of an average per period fire loss on the *baserun*, *baserun+infestation* and *protection* scenarios. The mean proportion burned per year is .00096. This was calculated in § A.2 from the historical annual losses for the Kamloops TSA. The optimal deterministic harvest trajectories corresponding to an annual fire rate of .005 were then used for comparison purposes.

The twenty year period fire rate p was calculated in the following way. Given an annual fire rate $q(t)$ for year t , then the proportion not burned after the first year t_1 is $1-q(t_1)$. The proportion not burned after the second year t_2 is $(1-q(t_2))(1-q(t_1))$. Therefore, the proportion not burned after 20 years is: $(1-q(t_{20}))(1-q(t_{19})) \dots (1-q(t_1))$ or the proportion burned for a twenty year period p is calculated as:

$$p = 1 - \prod_{t=1}^{20} (1-q(t))$$

In the deterministic solutions we use a fixed annual fire rate of $q = .005$ which corresponds to a period fire rate of $p = 1 - (1 - 0.005)^{20} = .095$. This rate was used for all the species. In Figures 8.12 to 8.14 the effect of a per annum fire rate of .005 on the *baserun*, *baserun+infestation* and *protection* scenarios is shown. It can be seen from these figures that even a .005 per annum fire rate can reduce the wood supply significantly from the *no fire* scenarios. The optimal harvest solutions have not reached equilibrium due to the relatively short time horizon used. However, we observe that the first period harvests are reduced as a result of fire. As was pointed out by Reed and Errico, the implication of this for forest planners is that, in the presence of fire risk, the m.s.y will be lower and that if a smooth descent to m.s.y is required, initial harvest levels must be reduced.

Figure 8.12: Optimal Harvest Levels using average period Fire Loss for the *Baserun* Scenario

Figure 8.13: Optimal Harvest Levels using average period Fire Loss for the *Baserun+Infested* Scenario

Figure 8.14: Optimal Harvest Levels using average period Fire Loss for the *Protection* Scenario.

In the next chart (Figure 8.15) we show how all the scenarios compare. The baserun (base) shows the optimal timber supply curve when no infestation, protection or fire present. This is an optimistic supply curve. The next highest supply trajectory is obtained in the *protection* scenario which consists of the *baserun+infestation+protection*. The next three curves are indistinguishable. We may say that these scenarios are equivalent. That is: *baserun+fire = baserun+infestation +protection+fire =baserun+infestation*. From this we may conclude that for these infestation and fire rates the effect of fire and infestation are equivalent and that protection cancels out the effect of an infestation in the presence of fire. Finally, the lowest supply trajectory is obtained when both fire and infestation are present with no protection.

Figure 8.15: A Comparison of the effect of Fire on all Optimal Harvest Level Scenarios.

8.7 Conclusions

We have suggested that a suitable criterion of worth may be measured by the flexibility of the model to evaluate alternative policy options. In the previous sections we demonstrated that the fire and pest protection model is capable of discriminating alternative policy options. Specifically, we were able to provide answers to the questions identified in section 8.1. Since the policy options were articulated directly in the structure of the model it was possible to evaluate each separately. We may conclude that the *fire and pest protection model* is capable of distinguishing policy options. Existing harvest scheduling systems such as MUSYC or FORPLAN do not model fire and infestation hazards or allow for protection of infested stands by spraying. Specifically, we conclude that it *pays to protect* infested stands by spraying. However, the results indicate that intensive silviculture, such as conversion harvests, is not cost effective. The optimal solution does select some conversion harvests but there is no increase in the total discounted net present value of the return streams.

The impact of an average annual fire risk reduces the first period and subsequent period cuts indicating that if fire risk is not accounted for we are overestimating the annual timber supply using conventional scheduling systems which do not take into account potential average losses in present *and* future periods in the solution method. We emphasize that all prescriptive models have descriptive elements such as the random process of catastrophic loss due to fire. These descriptive uncertain elements are also untestable; we cannot wait for the future to happen. We know these components of the decision making model should recognize an uncertain future. We showed in the previous chapter how well the deterministic model performed in a simulated "real" world.

Finally, this analysis demonstrated that the alternative forest management problems were efficiently modelled on a personal computer and that the high level modelling system used enabled easy transparent structural changes to be efficiently made.

Chapter 9

Conclusions

9.1 Summary of Results

In this thesis, the primary conclusion is that we have successfully addressed the fire and pest protection forest *modelling* problem by developing a flexible optimization *modelling framework* to determine the optimal timber harvest and pest protection schedule subject to fire and infestation risk that produces a desired wood supply over time at maximum discounted net profit. We have developed an *efficient, generic*, parsimonious state-space fire and pest protection forest management model which allows the integration of all the following concepts in a single optimization framework:

- (1) species diversity;
- (2) infestation of susceptible species;
- (3) natural regeneration and planting;
- (4) conversion of susceptible to non-susceptible species;
- (5) pest protection by spraying;
- (6) age-specific harvesting; and
- (7) catastrophic losses due to fire.

We have shown that the *model representation problem* was effectively solved by using a high level model development system such as GAMS. Algebraic modelling systems are powerful tools to help conceptualize and communicate the problem or decision situation. By using a high level language, model documentation is automatic and the model is easily verified. Also, the readability of these programs means the model can be explained more easily to others who might provide useful comments and suggestions.

We have also shown that the inter-temporal dynamics of growth, harvesting, insect infestation, protection by spraying and catastrophic losses due to fire may be represented by a processing network structure. We take advantage of this special network to motivate the understanding of the algebraic development of the alternative model structures. Network flow problems, in particular processing network problems, are amenable to faster solution algorithms.

We "validated" the deterministic model by examining how well the deterministic policy of using average per period fire and infestation loss holds in a stochastic environment. It was found that the deterministic solution follows closely the approximate stochastic solution for the first six periods. However, in the last four periods the randomness in the stochastic solution increases. We concluded that the simpler deterministic policy was a good enough approximation to the solution obtained from the application of the deterministic control in a feedback procedure using randomly generated fire and infestation probabilities. We therefore concluded that we cannot be as sure of the timber supply in the long term when faced with uncertainties such as fire and infestation losses. This suggests how these models should be used operationally. The optimization problem should be solved in each period after updating the forest inventory to take account of fire and infestation that has occurred.

We explored the worthiness of the deterministic model by testing how well it evaluates alternative management policies compared with other models? The results indicated that it *does not pay* to rehabilitate the susceptible uninfested or infested stands by conversion harvests. The optimal solution does select some conversion harvests over harvesting the susceptible and replanting with susceptible but there is no change in the total

discounted net present value (NPV) or the optimal timber supply trajectory. On the other hand, inclusion of protection of the infested stands by spraying does have a significant impact on the long run maximum sustained yield. The annual harvest level is increased. The total NPV for the protection solution is $\$4.838 \times 10^9$ as compared with $\$3.948 \times 10^9$ for the baserun-infestation scenario. It clearly pays to protect the stands by spraying. We conclude that the model is capable of distinguishing policy options.

9.2 Summary of Contributions

The primary research contribution of this thesis is the development of a mathematical programming modelling system to address the *general economic forest management problem*; however, within this broad accomplishment there are several specific achievements.

- (1) The development of a parsimonious state-space representation of the *Fire and Pest Protection Forest Management Problem* to determine the optimal timber regeneration harvest by age and species in each time period and pest protection schedule that produces a desired wood supply over time at maximum discounted net profit.
- (2) An efficient, low cost implementation of a concise, easily understood *model representation* of the fire and pest protection model using a high-level algebraic modelling language.
- (3) A new *processing network description* of the fire and pest protection model which motivates the algebra of the problem and suggests faster solution methods.

- (4) *A stochastic validation* which tests how well the deterministic model holds in simulated stochastic environment by using the deterministic solution in an iterative feedback procedure.
- (5) *A comparative scenario analysis* of alternative silvicultural strategies for the Western Spruce Budworm infestation problem.

9.3 Suggestions for Further Work

Assessment of Engineering Value

An assessment of the engineering value of our forest management system is provided by answering the following question: "*What developments to this research analysis system are required so that it could be used for operational forest management ?*" The answers are delineated below.

1) Data Aggregation.

This research model is a simplified representation of reality. It assumes that timber in only two aggregate species, susceptible and non-susceptible, grows along their respective volume-at-age curves. To be used as an operational whole-forest timber supply model we need to represent more species and provide their associated volume-at-age curves. Timber grows on different site classes and this variety in the quality should be represented. The dimensionality of the problem would increase and would have to be solved on a workstation which is not restricted by the limits of DOS 640K RAM.

2) Temporal Resolution

The temporal resolution of the model is defined by the age-class width of the inventoried stand data. Since forest management is planned on a yearly basis the model's stand age-class distribution and time periods could be disaggregated into single years. This magnifies the model size twenty times and would also raise issues of software and hardware requirements. Models developed with these characteristics would involve several thousand variables and constraints. There is a need for research into specialized algorithms which can efficiently solve these multi-period scheduling problems. For example, the special staircase structure of a multi-period scheduling problem could be exploited as alluded to by Chvatal [1980]. Dantzig [1955] observed that these problems required more iterations of the simplex method than other LP problems of comparable size. This observation is explained by the fact that, such problems are characterized by an activity such as a stock or inventory which is maintained at a positive level during one period, will also remain positive in a subsequent period. The inter-period structure of the LP problem is manifest by blocks rather than haphazard points. Some modifications to the *simplex method* have been attempted (Perold and Dantzig [1978]) to exploit this staircase structure. Another approach by Ho and Manne [1974] used an iterative application of the Dantzig-Wolfe decomposition algorithm. There is much scope for research developments in faster more efficient algorithms for these types of problems.

3) Scenario Management

If we adopt a simulation-optimization solution paradigm the numerous resultant scenarios require management so that they are comprehensible. A *scenario management system* is required which "models" the resultant scenarios by keeping track of the combination of inputs that produce the individual scenario outputs so that we may create:

- (1) a catalogue of individual results; and

(2) the output may be obtained again by re-running the model.

Finally, complex, large-scale modelling problems require appropriate data management and graphic tools for their comprehension. There is a need for a graphical interface to model development systems. It should be possible to interactively portray both the input and output structure of a large-scale model graphically.

References

Arden, B.W (1980) *What Can Be Automated? The Computer Science and Engineering Research Study*, MIT Press, Cambridge, Massachusetts.

Armson, K.A (1976) *Forest Management in Ontario*. Toronto: Ministry of Natural Resources. V+ 172pp.

Barros, O, and A. Weintraub (1982) Planning for a vertically integrated forest industry. *Oper. Res.* 30: 1168-1183.

Bartels, R.H (1971) A stabilization of the simplex method, *Numerische Mathematik* 16: 414-434.

Bartels, R.H and G.H. Golub (1969) The simplex method of linear programming using LU decomposition, *Communications of the ACM* 12: 266-268.

Baskerville, G.L (1983) *Good Forest Management: a Commitment to action*
Fredericton: New Brunswick Department of Natural Resources, 13pp.

Baskerville, G.L (1985) *Facing the future "pie in the sky" versus " what you see is what you get "* ppE121-125. In Canadian Pulp and Paper Association Preprint book of papers presented at 66th. annual meeting. Montreal Woodlands section.

Can. Pulp and Paper Association 146p.

Bell, P.C (1985) Visual Interactive Modeling as an Operations Research Technique.
Interfaces 15: 26-33

Beck, J.A (1984) Harvest scheduling models in use in Western Canada. pp 69-73 in
Nagumo H. et al (ed) Forest Planning for Improved Management Research and
Managerial Economics. Tokyo Dept. of Forestry, University of Tokyo, ix + 812pp

Berck, P (1976) *Natural Resources in a Competitive Economy*, MIT, Ph.D.
dissertation.

Boychuk, D (1988) *An LP model to evaluate spruce budworm control alternatives.*
Paper presented at TIMS/ORSA Joint National Meeting. Washington, D.C
April 25-27.

Bisschop, J and A. Meeraus (1982) On the Development of a General Algebraic
Modeling System in a Strategic Planning Environment. *Math. Prog. Study* 20, 1-
29.

Bixby, R.E and R. Fourer (1988) Finding Embedded Network Rows in Linear
Programs I Extraction Heuristics. *Management Science* Vol 34. No 3 pp342-349.

Brooke, A, A. Drud, and A. Meeraus (1985) *Modeling Systems and Nonlinear
Programming in a Research Environment.* Computers in Engineering Vol 3 ed. R.
Raghavan and S.M. Rhode A.S.M.E.

Brooke, A, D. Kendrick, and A. Meeraus (1988) *GAMS: User's Guide*
Scientific Press 651 Gateway Blvd., Suite 1100, South San Francisco, California

94080-7014.

Chinneck, J.W (1990) Formulating processing network models: viability theory.
Naval Research Logistics, 35: 245-261.

Chow, G. (1975), *Analysis and Control of Dynamic Economic Systems*, J. Wiley and Sons, New York

Chvatal, V (1983) *Linear Programming* . W.H. Freeman and Company, New York., N.Y.

Chen, C-H. J and M. Engquist (1986) A Primal Simplex Approach to Pure Processing Networks. *Manag. Sci.* Vol 32 No. 12, 1582-1587.

Cohen, Y. (1987) , A review of harvest theory and applications of optimal control theory in fisheries management. *Can. J. Fish. Aquat. Sci.* 44, 75-83.

Dantzig, G.B (1955) Upper bounds, secondary constraints, and block triangularity in linear programming. *Econometrica* 23: 174-183.

Dantzig, G.B (1963) *Linear programming and extensions*. Princeton University Press, Princeton N.J.

Davis, K.P (1966) *Forest Management: Regulation and Valuation*. Mcgraw-Hill Book Co. New York.

Faustman, M (1849) Berechnung des Werthes, welchen Waldboden sowie nach nicht haubare Holzbestände für die Waldwirtschaft besitzen. (Calculation of values for forest land and for immature stands used for timber production) *Allg Forst*

Jagd Z 25: 441-445

Fisher, M.L (1986) Interactive Optimization. *Ann. Opns. Res.*, 5, 541-556.

Geoffrion, A.M (1985) *Structured Modeling*. Western Management Science Institute, Graduate School of Management, University of California, Los Angeles, CA 90024.

Gill, P.E, W. Murray, M.A. Saunders and M.H. Wright (1987) Maintaining LU factors of a general sparse matrix, *Linear Algebra and its Applications* 88/89: pp 239-270.

Goh, B.S (1980), *Management and Analysis of Biological Systems*, Elsevier Scientific Publ., New York.

Glover, F. and D. Klingman (1981) The Simplex SON Algorithm for LP/Embedded Network Problems. *Math. Prog. Study* 15 : pp148-176.

Greenberg, H.J (1983) A functional description of ANALYZE: a Computer-assisted modelling and analysis system for linear programming models. *ACM Trans. Math. Soft.* 9 (1): 18-56.

Gunn, E.A and A.K. Rai (1987) Modelling and decomposition for planning long term forest harvesting in an integrated industry structure. *Can. J. For. Res.* 17: 1507-1517.

Hanzlik, E.J., (1922), Determination of the annual cut on a sustained yield basis for virgin American Forests. *J. Forestry*, 20: 611-625.

- Hillier, F.S and G.J. Lieberman (1986)** *Introduction to Operations Research*. fourth edition, Holden-Day, Oakland, California.
- Ho, J.K, and A.S. Manne (1974)** Nested decomposition for dynamic models, *Mathematical Programming* 6: 121-140.
- House G.M (1986)** Lecture delivered to the Faculty of Forestry, Univ. of Toronto,
- Jacquette, D.L (1972)** Mathematical models for controlling biological populations; a survey, *Oper. Res.* 20, 1142-1152.
- Johansson, P-O and K-G. Lofgren (1985)** *The Economics of Forestry and Natural Resources*. Basil Blackwell Ltd, New York.
- Johnson, K.N. (1986)** *FORPLAN version 2. An overview*. Land Management Planning Systems Section. USDA Forest Service, Washington, D.C.
- Johnson, K.N and D.B. Jones (1979)** *A User's Guide to Multiple Use - Sustained Yield Resource Calculation (MUSYC) Timber Management*, USDA Forest Service, Fort Collins, CO.
- Johnson, K.N and H.L. Scheurman (1977)** Techniques for Prescribing Optimal Timber Harvest and Investment under Different Objectives - discussion and synthesis. *For. Sci. Monogr.* No 18
- Jones, C.V (1988)** The Three-Dimensional Gantt Chart . *Operations Research* Vol. 36, No. 6, November-December.
- Kidd, W.E, E.F. Thompson, and P.H. Hoepner (1966)** Forest Regulation by

Linear Programming: a Case Study. *J. For.* 64: 611-613

Koene, J. (1984) Minimal Cost Flow in Processing Networks; A Primal Approach.
CWI Tract 4.

Krabek, C.B, R.J. Sjoquist and D.C. Sommer (1980) The Apex Systems:
past and future. *SIGMAP Bulletin* 29, 3-23.

Lembersky, M.R, and U.H. Chi (1984) Decision Simulators Speed
Implementation and Improve Operations. *Interfaces* 14, 4 1-15.

Lucas, C. and G. Mitra (1988) Computer-Assisted Mathematical Programming
(Modelling) System: CAMPS. *The Computer Journal* . Vol. 31, No 4, 1988.

Murtagh, B.A and M.A. Saunders (1982) A projected Lagrangean algorithm and
its implementation for sparse nonlinear constraints, *Mathematical Programming
Study* 16: 84-117.

Nautiyal, J.C (1988) Memorandum, Faculty of Forestry, University of Toronto.

Nautiyal, J.C and K.S. Fowler (1980) Optimal forest rotation in an imperfect
stumpage market. *Land Econ.* 56(2): 213-226.

Navon, D.I (1971) *TimberRAM, a long range planning method for commercial
Timberlands under multiple use management* . USDA For. Serv, Res. Pap. PSW-
70

Paules, G.E and C.A. Floudas (1989) APROS: Algorithmic Development
Methodology For Discrete-Continuous Optimization Problems.

Oper. Res. Vol 37(6) pp 902-918.

Pearse, P.H (1984) *Obstacles and incentives for silviculture in Canadian forest policies..* prepared for the International Forest Congress, Quebec. Aug 5-9.

Perold, A.F and G.B. Dantzig (1978) A basis factorization method for block triangular linear programs in *proceedings of the Symposium on Sparse Matrix Computations*, I.S. Duff and G.W. Stewart, eds. Knoxville, Tennessee, 283-312.

Pontryagin, L.S, V.G. Boltyanskii, R.V. Gamkrelidze, and E.F. Mishenko (1962) *The Mathematical Theory of Optimal Processes*, John Wiley and Sons, New York.

Pritsker, A.A.B and C.D. Pegden (1979) *Introduction to Simulation and SLAM*, A Halsted Press Book, John Wiley and Sons, Inc., New York.

Reed, F.L.C (1978) *Forest management in Canada*. Vol 1 Can. For. Serv. Forest Manage. Inst. Info. Rep. FMR-X-102 vii+155p.

Reed, F.L.C (1985) Reshaping forest policy in British Columbia *J. Bus.Admin.* 15: 111-137.

Reed, W.J. (1986) Optimal harvesting models in forest management- a survey, *Nat. Res. Model.* 1, 55-75.

Reed, W.J and D. Errico (1985) Assessing the long-run yield of a forest stand subject to the risk of fire. *Can. J. For. Res.*, 15: 680-687

Reed, W.J and D. Errico (1986) Optimal harvest scheduling at the forest level in

the presence of the risk of fire. *Can J. For. Res.* 16: 266-278.

Reed, W.J and D. Errico (1987) Techniques for assessing the effect of pest hazards on the long run timber supply FEPA report 86-6, Univ. of B.C.

Reid, J.K (1976) *Fortran subroutines for handling sparse linear programming bases*, Report R8269, Atomic Energy Research Establishment, Harwell, England.

Reid, J.K (1982) A sparse-exploiting variant of the Bartels-Golub decomposition for linear bases, *Mathematical Programming* . 24: 55-69.

Sadler, B (ed) (1983) *Canada's forests : transition to management* . Banff . Alberta : The Banff Centre, viii +112pp.

Sharda, R (1988) The State of the Art Linear Programming on Personal Computers. *Interfaces* 18: 49-58.

Shoemaker, C.A. (1981) Application of dynamic programming and other optimization methods in pest management. *IEEE Trans. Automatic Control* AC-26, 1125-1132.

Shugart, H.H (1984) *A Theory of Forest Dynamics*. Springer - Verlag, New York Inc.

Sprague, R.H, and E.D. Carlson (1982) *Building Effective Decision Support Systems*. Prentice-Hall, Englewood Cliffs, N.J.

Susuki, T and T. Umemura. (1974) *Forest transition as a stochastic process*. 11 pp327-352 in J Fries (ed) *Growth models for trees and stand simulation*. Dept.

Forest Yield Research Royal College of Forestry, Stockholm . Res. Notes 30

Susuki, T (1984) *The Gentan Probability. A model for the improvement of the normal forest concept and forest planning.* In proceedings IUFRO Symposium on Forest Management Planning and Management Economics. Oct15/19 University of Tokyo pp12-24.

Tedder, P.L, J.S. Schmidt, and J. Gourley (1980) TREES: Timber Resource Economic Estimation System Volume I, a users manual for forest management and harvest scheduling. Oreg. State Univ., Corvallis *For. Res. Lab. Res. Pap.* 31a

Van Wagner, C.E (1983) Simulating the effect of forest fire on long-term annual timber supply. *Can. J. For. Res.* 13: 451-457.

Walters, C.J and R. Hilborn (1978), Ecological optimization and adaptive management, *Ann. rev. Ecol. Syst.* 9, 157-188.

Wagner, H (1975) *Principles of Operations Research.* 2nd Edition Prentice Hall, Englewood Cliffs, NJ.

Ware, G.O, and J.L. Clutter (1971) A mathematical programming system for the management of industrial forests. *For. Sci.* 17: 428-445

Weintraub, A, S. Guitart and V. Kohn (1986) Strategic planning in forest Industries. *J. of Oper. Res.* 24: 152-162.

Williams, B.K (1989) Review of dynamic optimization methods in renewable resource management, *Nat. Res. Model.* 3 No 2, 137-216.

Appendix A

Calibration of the Fire and Pest Protection Model

This appendix discusses how data obtained for the Kamloops Timber Supply Area (TSA) in British Columbia were aggregated to calibrate a tractable fire and pest protection forest management model described in detail in chapters 4 and 5.

A.1 Kamloops TSA Inventory Data

Inventory data for used for a Multiple Use Sustained Yield Calculation, MUSYC¹, (Johnson and Jones [1979]) were obtained from the B.C. Ministry of Forests Timber Management Group. Forest stands are aggregated into twenty year age-classes, working groups (species) and land class types (site-classes) We model these data by first defining sets which correspond to the stratification of the inventory and volume-at-age curves.

SETS

I	twenty-year age-classes	{ i-20, i-40, i-60, i-80, i-100, i-120, i-140, i-160, i-180, i-200, i-220, i-240, i-260, i-280, i-300, i-320 }
W	working groups	{ Fir-wet, Fir-dry, Cedar, Hemlock, Spruce, Balsam, Pine }

¹ defined in chapter 2.

L land class type

{ good-medium, poor-low }

The notation "i-20" indexes all stands aged zero to twenty years old. The Inventory Branch of the B.C Ministry of Natural Resources collects single-year age-class data. MUSYC asks for the average age of an aggregate age-class. For twenty-year age-classes "i-20" would correspond to the age class whose average age is 10 years old. We have 16 twenty-year age-classes so that the oldest timber is 320+ years old.

The Kamloops TSA is comprised of 824,140 hectares of commercially available forest land. This area is a *net land base* figure and results from a complex "*netting down*" procedure conducted by the B.C. Inventory Branch which takes into account reductions due to:

- (1) non-crown land;
- (2) non-forest land;
- (3) non-stocked land;
- (4) unclassified roads, trails and landings;
- (5) environmentally sensitive areas;
- (6) low productivity sites;
- (7) deciduous forest land; and
- (8) problem forest types.

From a total land base of 2,110,893 hectares there are 1,435,715 hectares of productive crown forest which is "netted down" to 824,140 hectares of commercially available timber.

The *land classes* are an aggregation of four *site classes*, good, medium, poor and low. Site class is an indicator of site productivity. Working groups refer to the leading or predominant species in the stand. Only Douglas Fir leading stands are considered susceptible to Western Spruce Budworm (WSB). We need to segregate the forest into

PARAMETERS

$X_{i,w,l}$	initial inventory	(hectares)
$V_{i,w,l}$	volume at age for initial inventory	(cubic metres per hectare)
$AGX_{i,s,l}$	aggregate forest inventory	(hectares)
$AGXV_{i,s,l}$	aggregate total forest volume	(cubic metres)
$AGV_{i,s,l}$	aggregate yield curve	(cubic metres per hectare)
$AGGXV_{i,s}$	aggregate total forest volume	(cubic metres)
$AGGV_{i,s}$	aggregate yield curve	(cubic metres per hectare)
$AGGX_{i,s}$	aggregate forest inventory	(hectares)
$AGOLDXV_s$	total volume old age classes	(cubic metres)
$AGOLDX_s$	area old age classes	(hectares)
$AGOLDV_s$	aggregate volume old age-classes	(cubic metres per hectare)

The aggregate total volume is obtained for the susceptible and non-susceptible species:

$$AGXV_{i,"susc",l} = \sum_{w \in SW} X_{i,w,l} * V_{i,w,l} \quad (a.1)$$

$$AGXV_{i,"non-susc",l} = \sum_{w \in NSW} X_{i,w,l} * V_{i,w,l} \quad (a.2)$$

The area of susceptible and non-susceptible forest land are created:

$$AGX_{i,"susc",l} = \sum_{w \in SW} X_{i,w,l} \quad (a.3)$$

$$AGX_{i, \text{"non-susc"}, l} = \sum_{w \in NSW} X_{i, w, l} \quad (a.4)$$

and the aggregate per hectare yield calculated for the aggregated species $s \in S$.

$$AGV_{i, s, l} = AGXV_{i, s, l} / AGX_{i, s, l} \quad (a.5)$$

The same procedure is applied to "aggregate out" or "squeeze out" the land class type l .

First we create the total forest volume :

$$AGGXV_{i, s} = \sum_l AGV_{i, s, l} * AGX_{i, s, l} \quad (a.6)$$

and area for the targetted aggregation (i, s)

$$AGGX_{i, s} = \sum_l AGX_{i, s, l} \quad (a.7)$$

to obtain the unit yield:

$$AGGV_{i, s} = AGGXV_{i, s} / AGGX_{i, s} \quad (a.8)$$

Since the fire and pest model as defined in chapter five has its oldest age-class "i-180" which includes all stands older than 160 years we need to create an aggregate unit yield for this age-class for the susceptible and non-susceptible species. In the same manner as (a.6) the aggregate total volume for those age classes older than 160 years is:

$$AGOLDV_s = \sum_{i \in I_{old}} AGGV_{i, s} * AGGX_{i, s} \quad (a.9)$$

and the unit yield for stands older than 160 years:

$$AGOLD_s = AGOLDV_s / \sum_{i \in I_{old}} AGGX_{i,s} \quad (a.10)$$

These aggregation procedures produce two main parameters for the fire and pest model:

- (1) the starting inventory (Figure A.1); and
- (2) the per hectare volume-at-age curves (Figure A.2).

Figure A.1: Aggregate Inventory for the Kamloops TSA.

Figure A.2: Aggregate Volume-at-Age Curves for the Kamloops TSA

A.2 Kamloops Fire Data

In this section annual losses due to fire for the Kamloops TSA region are analysed. An historical time series from 1950 to 1989 of areal losses were obtained from the Protection Branch of the B.C Ministry of Forestry. The *Fire Reporting System* of the Protection Branch summarizes fire losses in forests for 1959 to 1989 in terms of area of forest fires > 4 ha, other fires which include non forested area and a total which includes forest fires ≤ 4 ha. We have included the forest fires which were ≤ 4 ha in our analysis by subtracting the other from the total. This is important since, in general, there are many small fires of ≤ 4 ha each year which account for as much area as the few large fires. The historical time series of area loss due to fire from 1950 to 1989 is given in Figure a.3.

Figure A.3: Annual Area Loss Due to Fire in the Kamloops TSA (1950-1989).

We see from Figure A.3 considerable variability in annual losses. A probability distribution of the fire rate is constructed from the data as follows.

SETS

$k \in K$ years {1950*1989}

PARAMETERS

QA(k) annual fire loss for Kamloops TSA (ha per year)
 Q(k) annual proportion burned
 q fire rate

The proportion burned each year k is calculated :

$$Q(k) = QA(k) / \text{total area}$$

and these fire rates were ranked noting their frequency as seen in table a.1.

$$\text{The mean proportion burned} = \sum_q q * f(q) = .00096$$

The actual mean proportion was not used in the deterministic optimizations. Instead a mean annual fire rate of .005 was used. Consequently the fire rates q were scaled up by a factor $.005/.00096 = 5.21$ so that comparisons with the optimal deterministic harvest trajectory corresponding to a per annum fire rate of .005 or 1/2 percent per year could be made. The cumulative distribution function F(q) was used to generate a random sample PA(k) (k=1, 2, 3 . . . 20) for each year in a twenty year time frame, by using the *inverse transformation method*³ (Pritsker, A.A.B., and C.D. Pegden [1979]).

³ described in detail in Appendix D

annual fire rate q	frequency R	Probability $f_Q(q) = \text{Prob}[Q=q]$ R/n	Cumulative Frequency $F_Q(q) = \text{Prob}[Q \leq q]$
$\leq .0002$	19	.487	.487
.00022	1	.026	.513
.0003	1	.026	.539
.00034	1	.026	.565
.00035	3	.077	.642
.00036	1	.026	.668
.00042	1	.026	.694
.00048	1	.026	.720
.00053	1	.026	.746
.00065	1	.026	.772
.00074	1	.026	.798
.00079	2	.0513	.849
.00103	1	.026	.8753
.00149	1	.026	.9013
.00165	1	.026	.927
.00447	1	.026	.953?
.00812	1	.026	.9793
.0099	<u>1</u>	.026	1.00
	n = 38		

Table A.1: Cumulative Frequency Distribution of Annual Rates of Fire for the last 38 years in the Kamloops TSA.

A.3 Western Spruce Budworm Infestation Data

Data on the spread of WSB for the Kamloops TSA were obtained from the Protection Branch of the B.C Ministry of Forests. Only Douglas Fir leading stands are considered susceptible to WSB. This area is 39% of the total net commercial land base. Typical infestation scenarios were summarized by the Protection Branch for the Kamloops TSA:

- (1) 50% of the susceptible area will be infested with WSB at any given infestation;
- (2) it was assumed that all infested areas would experience the same infestation cycle. It was also recognized that the total susceptible area would likely be infested at some time during an infestation. However, each area would experience different infestation intensities and durations. It was assumed that only 50% of the total susceptible area would be infested; and
- (3) A typical infestation cycle would be.

Year	Infestation Intensity
1	Light
2	Light
3	Moderate
4	Moderate
5	Severe
6	Moderate
7	Moderate
8	Light
9	Light

Given this pattern of infestation, data indicated that the volume of harvest loss for an uneven aged stand would be 30% and would occur immediately. Similar data for even

aged stands indicated a 23% loss in harvest volume assuming the average age of infested stands was 50 years.

Protection involved three sprayings applied for three consecutive years, one per year each at a cost of \$53 per hectare. Since the time resolution of our model is twenty years it was assumed only one infestation cycle could occur per period. The period protection cost for infested stands would be $3 \times 53 = \$159$ / hectare/period.

Appendix B

GAMS: Fire and Pest Protection Model

SETS

I	age-classes	/ I-20, I-40, I-60, I-80, I-100, I-120, I-160, I-180/
ICUTNS(I)	cutting non-susceptible	/I-140, I-160, I-180/
INCUTNS(I)	no cutting non-susceptible	/ I-20, I-40, I-60, I-80, I-100, I-120/
ICUT(I)	cutting classes susceptible	/I-120, I-140, I-160, I-180/
INCUT(I)	no cutting classes susceptible	/I-20, I-40, I-60, I-80, I-100/
T	time periods	/P-1*P-10/
K	twenty years	/K-1*K-20/

;

SCALARS

MUP	planting cost	(\$ per hectare)	/600/
MUC	cutting cost	(\$ per hectare)	/1600/
MUS	spraying cost	(\$ per hectare)	/159/
PD	sales price of pulplogs	(\$ per cubic metre)	/65/
II	real annual interest rate		/.05/
MU	percent volume reduction infested unprotected		/.3/
BETA	percent volume reduction protected		/.03/
GAMMA	ten year upper and lower bound on supply		/.1/

;

PARAMETERS

VNS(I)	volume of non-susceptible forest	(m**3 per ha)
	/I-20 1, I-40 45, I-60 67, I-80 162, I-100 208, I-120 278, I-140 302, I-160 281, I-180 332/	
VS(I)	volume of susceptible forest	(m**3 per ha)
	/I-20 .26, I-40 6, I-60 72, I-80 152, I-100 199, I-120 219, I-140 268, I-160 263, I-180 234/	
X0(I)	initial area of non susceptible forest	(ha)
	/ I-20 35525, I-40 16524, I-60 35614, I-80 64964, I-100 87660, I-120 94121, I-140 35512, I-160 7116, I-180 126152/	
Y0(I)	initial area of susceptible timber	(ha)
	/I-20 12455, I-40 4774, I-60 12943, I-80 36621, I-100 87660, I-120 94121, I-140 26961, I-160 13563, I-180 81327/	
Z0(I)	initial area of infested susceptible not protected	(ha)
PA(K,T)	annual fire rate non-susceptible forest	
P(T)	average proportion of non-susceptible forest burned per period	
Q(T)	average proportion of susceptible uninfested burned per period	
RA(K,T)	annual fire rate for infested species	
S(I,T)	average proportion infested per period	
RU(T)	average proportion of unprotected infested burned per period	
RP(T)	proportion of protected infested burned per period	
DELT(T)	period discount factor	
SUPPLY(T,*)	annual timber supply per period;	
*		
*	all initial susceptible is infested	
*		
Z0(INFEST)	= Y0(INFEST);	
Y0(INFEST)	= 0.0;	
ZP0(I)	= 0.0;	
*		

* the period infestation rates are set

*

$S(I,T) = .2;$

$S("I-20",T) = 0.0;$

$S("I-40",T) = 0.0;$

*

* the annual fire rates are translated into period fire rates for the non-susceptible

*

$PA(K,T) = .005;$

$P(T) = 1 - \text{PROD}(Y, (1 - PA(K,T)));$

$Q(T) = P(T);$

*

* and for the susceptible species

*

$RA(K,T) = .005;$

$RU(T) = 1 - \text{PROD}(K, (1 - RA(K,T)));$

$RP(T) = RU(T);$

*

* the period discount factor

*

$\text{DELT}(T) = (1 + I) ** (-20 * (\text{ORD}(T) - 1));$

POSITIVE VARIABLES

$H(\text{ICUTNS},T)$	area of non-susceptible timber regeneration harvested	(ha)
$G(\text{ICUT},T)$	area of uninfested regeneration harvested	(ha)
$GC(\text{ICUT},T)$	area of uninfested conversion harvested	(ha)
$F(\text{ICUT},T)$	area of infested regeneration harvested	(ha)
$FC(\text{ICUT},T)$	area of infested conversion harvested	(ha)
$PR(I,T)$	area of susceptible infested protected	(ha)
$FP(\text{ICUT},T)$	area of infested protected regeneration harvested	(ha)
$FCP(\text{ICUT},T)$	area of infested protected regeneration harvested	(ha)
$X(I,T)$	area of non-susceptible forest	(ha)

Y(I,T)	area susceptible uninfested forest	(ha)
Z(I,T)	area of susceptible forest infested at age I	(ha)
ZP(I,T)	area of infested forest protected	(ha)

FREE VARIABLES

PHI	benefit	(\$)
PHIP(T)	planting cost	(\$ per period)
PHIL(T)	cutting cost	(\$ per period)
PHIX(T)	sales revenue	(\$ per period)
PHIS(T)	spraying cost	(\$ per period)

EQUATIONS

PBAL(T)	pulplogs balance	(m**3 per period)
ASALES(T)	sales revenue	(\$ per period)
ACUTC(T)	cutting cost	(\$ per period)
ASPRAYC(T)	spraying cost	(\$ per period)
APLANT(T)	planting cost	(\$ per period)
BENEFIT	total benefit	(\$ per period)
CUTX(ICUTNS,T)	area cut restriction non-susceptible	(ha)
CUTY(ICUT,T)	area cut restriction susceptible-uninfested	(ha)
NCUTZ(INCUT,T)	area cut restriction infested no cut ages	(ha)
CUTZ(ICUT,T)	area cut restriction infested cut ages	(ha)
CUTZP(ICUT,T)	area cut restriction infested protected	(ha)
BALX1(T)	area balance non-susceptible no cut age-classes	(ha)
BALX2(INCUTNS,T)	area balance non-susceptible no cut age-classes	(ha)
BALX3(T)	area balance non-susceptible boundary no cut-cut ages	(ha)
BALX4(ICUTNS,T)	area balance non susceptible cutting age-classes	(ha)
BALX5(T)	area balance non-susceptible oldest age class	(ha)
BALY1(T)	area balance uninfested age-class 1	(ha)
BALY2(INCUT,T)	area balance uninfested no cut age-classes	(ha)
BALY3(T)	area balance uninfested boundary no cut-cut ages	(ha)

BALY4(ICUT,T)	area balance uninfested cutting age classes	(ha)
BALY5(T)	area balance uninfested oldest age classes	(ha)
BALZ2(INCUT,T)	area balance infested unprotected no cut ages	(ha)
BALZ4(ICUT,T)	area balance infested unprotected no cut ages	(ha)
BALZP2(INCUT,T)	area balance protected cutting age classes	(ha)
BALZP3(T)	area balance protected boundary no cut-cut age classes	(ha)
BALZP4(ICUT,T)	area balance protected cutting age-classes	(ha)
BALZP5(T)	area balance oldest age-class	(ha)
SY1(T)	max declining yield	(m**3 per period)
SY2(T)	max increasing yield	(m**3 per period)
P1G	max increasing yield stochastic 1	(m**3 per period)
P1L	max declining yield stochastic 1	(m**3 per period)
P2G	max increasing yield stochastic 2	(m**3 per period)
P2L	max declining yield stochastic 2	(m**3 per period)
P3G	max increasing yield stochastic 3	(m**3 per period)
P3L	max declining yield stochastic 3	(m**3 per period)
P4G	max increasing yield stochastic 4	(m**3 per period)
P4L	max declining yield stochastic 4	(m**3 per period)
P5G	max increasing yield stochastic 5	(m**3 per period)
P5L	max declining yield stochastic 5	(m**3 per period)
P6G	max increasing yield stochastic 6	(m**3 per period)
P6L	max declining yield stochastic 6	(m**3 per period)
P7G	max increasing yield stochastic 7	(m**3 per period)
P7L	max declining yield stochastic 7	(m**3 per period)
P8G	max increasing yield stochastic 8	(m**3 per period)
P8L	max declining yield stochastic 8	(m**3 per period)
P9G	max increasing yield stochastic 9	(m**3 per period)
P9L	max declining yield stochastic 9	(m**3 per period)

;

$$\begin{aligned}
 \text{PBAL(T)..} \quad \text{PULP(T)} = & \text{L= SUM(ICUTNS,VNS(ICUTNS)*H(ICUTNS,T)} \\
 & + \text{SUM(ICUT,VS(ICUT)*(GC(ICUT,T)} \\
 & + (1-\text{BETA})*(\text{FP(ICUT,T)+FCP(ICUT,T))} \\
 & + (1-\text{MU})*(\text{FC(ICUT,T)+F(ICUT,T)+G(ICUT,T)}));
 \end{aligned}$$

ASALES.. $PHIX(T) = E = PD * PULP(T);$
 ACUTC(T).. $PHIL(T) = E = MUC * (SUM(ICUTNS, H(ICUTNS, T))$
 $+ SUM(ICUT, G(ICUT, T) + F(ICUT, T) + FP(ICUT, T)$
 $+ GC(ICUT, T) + FC(ICUT, T) + FCP(ICUT, T)));$
 ASPRAYC(T).. $PHIS(T) = E = MUS * SUM(I, PR(I, T));$
 APLANT(T).. $PHIP(T) = E = MUP * (SUM(ICUTNS, H(ICUTNS, T))$
 $+ SUM(ICUT, G(ICUT, T) + F(ICUT, T) + FP(ICUT, T)$
 $+ GC(ICUT, T) + FC(ICUT, T) + FCP(ICUT, T)));$
 SY1(T-1).. $PULP(T) = G = ((1 - GAMMA) ** 2) * PULP(T-1);$
 SY2(T-1).. $PULP(T) = L = ((1 + GAMMA) ** 2) * PULP(T-1);$
 BALX1(T+1).. $X("I-20", T+1) = E =$
 $SUM(ICUTNS, H(ICUTNS, T) + P(T) * (X(ICUTNS, T) - H(ICUTNS, T)))$
 $+ SUM(ICUT, GC(ICUT, T) + FCP(ICUT, T) + FC(ICUT, T))$
 $+ SUM(INCUTNS, P(T) * (X(INCUTNS, T)));$
 BALX2(INCUTNS, T+1) \$ (ORD(INCUTNS) NE 1)..
 $X(INCUTNS, T+1) = E = (1 - P(T)) * X(INCUTNS - 1, T);$
 BALX3(T+1)..
 $X("I-140", T+1) = E = (1 - P(T)) * X("I-120", T);$
 BALX4(ICUTNS, T+1)
 $$ ((ORD(ICUTNS) NE 1) * (ORD(ICUTNS) NE CARD(ICUTNS)))..$
 $X(ICUTNS, T+1) = E = (1 - P(T)) * (X(ICUTNS - 1, T) - H(ICUTNS - 1, T));$
 BALX5(T+1)..

$$X("I-180",T+1) = E = (1-P(T)) * (X("I-160",T) - H("I-160",T) + X("I-180",T) - H("I-180",T));$$

BALY1(T+1)..

$$\begin{aligned} Y("I-20",T+1) = E = \\ \text{SUM}(\text{ICUT}, G(\text{ICUT},T) + Q(T) * (Y(\text{ICUT},T) - G(\text{ICUT},T) - G(\text{ICUT},T)) \\ + \text{FP}(\text{ICUT},T) + F(\text{ICUT},T) + \text{RP}(T) * (Z(\text{INCUT},T) - \text{FP}(\text{ICUT},T) - \text{FCP}(\text{ICUT},T)) \\ + \text{RU}(T) * (Z(\text{INCUT},T) - \text{PR}(\text{INCUT},T)) \\ + \text{SUM}(\text{INCUT}, \text{RU}(T) * (Z(\text{INCUT},T) - \text{PR}(\text{INCUT},T)) \\ + Q(T) * Y(\text{INCUT},T) + \text{RP}(T) * Z(\text{INCUT},T)); \end{aligned}$$

BALY2(INCUT,T+1) \$ (ORD(INCUT) NE 1)..

$$Y(\text{INCUT},T+1) = E = (1-Q(T) - S(\text{INCUT}-1,T)) * Y(\text{INCUT}-1,T);$$

BALY3(T+1)..

$$Y("I-120",T+1) = E = (1-Q(T) - S("I-100",T)) * Y("I-100",T);$$

BALY4(ICUT,T+1) \$ ((ORD(ICUT) NE 1) * (ORD(ICUT) NE CARD(ICUT)))..

$$Y(\text{ICUT},T+1) = E = (1-Q(T) - S(\text{ICUT}-1)) * (Y(\text{ICUT}-1,T) - G(\text{ICUT}-1,T) - G(\text{ICUT}-1,T));$$

BALY5(T+1)..

$$\begin{aligned} Y("I-180",T+1) = E = \\ (1-Q(T) - S("I-160",T)) * (Y("I-160",T) - G("I-160",T) - G(\text{"I-160",T}) + Y("I-180",T) - G("I-180",T) - G(\text{"I-180",T})); \end{aligned}$$

BALZ2(INCUT,T+1)..

$$\begin{aligned} Z(\text{INCUT},T+1) = E = \\ S(\text{INCUT},T) * (Y(\text{INCUT},T)) + (1-\text{RU}(T)) * (Z(\text{INCUT},T) - \text{PR}(\text{INCUT},T)); \end{aligned}$$

BALZ4(ICUT,T+1)..

$$Z(\text{ICUT},T+1) = E =$$

$$S(ICUT,T)*(Y(ICUT,T)-G(ICUT,T)-GC(ICUT,T)) \\ +(1-RU(T))*(Z(ICUT,T)-F(ICUT,T)-FC(ICUT,T)-PR(ICUT,T));$$

BALZP2(INCUT,T+1)..

$$ZP(INCUT,T+1) = E = PR(INCUT-1,T) + (1-RP(T))*ZP(INCUT-1,T);$$

BALZP3(T+1)..

$$ZP("I-120",T+1) = E = PR("I-100",T) + (1-RP(T))*ZP("I-100",T);$$

BALZP4(ICUT,T+1) \$ ((ORD(ICUT) NE 1)*(ORD(ICUT) NE CARD(ICUT)))..

$$ZP(ICUT,T+1) = E =$$

$$PR(ICUT-1,T) + (1-RP(T))*(ZP(ICUT-1,T) - FP(ICUT-1,T) - FCP(ICUT-1,T));$$

BALZP5(T+1)..

$$ZP("I-180",T+1) = E =$$

$$PR("I-180",T) + PR("I-160",T) + (1-RP(T))*(ZP("I-160",T) - FP("I-160",T) - \\ FCP("I-160",T) + ZP("I-180",T) - FP("I-180",T) - FCP("I-180",T));$$

CUTX(ICUTNS,T)..

$$H(ICUTNS,T) = L = X(ICUTNS,T) ;$$

CUTY(ICUT,T)..

$$G(ICUT,T) + GC(ICUT,T) = L = Y(ICUT,T);$$

CUTZ(ICUT,T)..

$$F(ICUT,T) + FC(ICUT,T) + PR(ICUT,T) = L = Z(ICUT,T);$$

NCUTZ(INCUT,T)..

$$PR(INCUT,T) = L = Z(INCUT,T);$$

CUTZP(ICUT,T)..

$$FP(ICUT,T) + FCP(ICUT,T) = L = ZP(ICUT,T);$$

```

P1G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","DETERM");
P1L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","DETERM");
P2G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-2");
P2L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-2");
P3G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-3");
P3L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-3");
P4G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-4");
P4L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-4");
P5G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-5");
P5L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-5");
P6G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-6");
P6L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-6");
P7G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-7");
P7L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-7");
P8G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-8");
P8L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-8");
P9G.. PULP("P-1") =G= ((1-GAMMA)**2)*20*SUPPLY("P-1","P-9");
P9L.. PULP("P-1") =L= ((1+GAMMA)**2)*20*SUPPLY("P-1","P-9");

```

BENEFIT..

```

      PHI =E= SUM(T,DELT(T)*(PHIX(T)-PHIP(T)-PHIL(T)-PHIS(T)));

```

*

*****initialize the state variables*****

*

```

X.FX(I,"P-1")   =  X0(I);
Y.FX(I,"P-1")   =  Y0(I);
Z.FX(I,"P-1")   =  Z0(I);
ZP.FX(I,"P-1")  =  ZP0(I);

```

*

*****select all parameters for a model run*****

*

MODEL FPM

```

      /PBAL.ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,

```

CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY\$,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2 /;

MODEL FPM1

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P1G,P1L /;

MODEL FPM2

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P2G,P2L /;

MODEL FPM3

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P3G,P3L /;

MODEL FPM4

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P4G,P4L /;

MODEL FPM5

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P5G,P5L /;

MODEL FPM6

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P6G,P6L /;

MODEL FPM7

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P7G,P7L /;

MODEL FPM8

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P8G,P8L /;

MODEL FPM9

/PBAL,ASALES,ACUTC,ASPRAYC,APLANT,BENEFIT,CUTX,
CUTY,CUTZ,NCUTZ,CUTZP,BALX1,BALX2,BALX3,BALX4,
BALX5,BALY1,BALY2,BALY3,BALY4,BALY5,BALZ2,BALZ4,
BALZP2,BALZP3,BALZP4,BALZP5,SY1,SY2,P9G,P9L /;

*

SOLVE FPM USING LP MAXIMIZING PHI;

*

*****select from solution the annual timber supply*****

*

SUPPLY(T, " DETERM") = PULP.L(T)/20;

*

*****display output variables*****

*

DISPLAY SUPPLY, H.L, G.L, GC.L, PR.L, F.I., FC.L, FP.L, FCP.L;

COMPILATION TIME = 0.412 MINUTES

MODEL STATISTICS

BLOCKS OF EQUATIONS 29 SINGLE EQUATIONS 593

BLOCKS OF VARIABLES 18 SINGLE VARIABLES 771

NON ZERO ELEMENTS 2938

GENERATION TIME = 3.992 MINUTES

EXECUTION TIME = 4.315 MINUTES

SOLVE SUMMARY

MODEL FPM	OBJECTIVE PHI
TYPE LP	DIRECTION MAXIMIZE
SOLVER MINOS5	FROM LINE 215

**** SOLVER STATUS 1 NORMAL COMPLETION

**** MODEL STATUS 1 OPTIMAL

**** OBJECTIVE VALUE 4838442830.5000

RESOURCE USAGE, LIMIT 17.553 1000.000

ITERATION COUNT, LIMIT 453 1500

MINOS 5.2 (Mar 1988)

B. A. Murtagh, University of New South Wales

and

P. E. Gill, W. Murray, M. A. Saunders and M. H. Wright

Systems Optimization Laboratory, Stanford University.

=====

MINOS 5.2 (Mar 1988)

=====

WORK SPACE NEEDED (ESTIMATE) -- 30871 WORDS.

WORK SPACE AVAILABLE -- 31992 WORDS.

EXIT -- OPTIMAL SOLUTION FOUND

PARAMETER SUPPLY ANNUAL TIMBER SUPPLY M**3 PER PERIOD

DETERM

P-1 2947539.903
P-2 2387507.321
P-3 1933880.930
P-4 1566443.553
P-5 1268819.278
P-6 1027743.615
P-7 832472.329
P-8 1007291.517
P-9 1218822.736
P-10 1474775.511

**VARIABLE H.L AREA OF NON-SUSCEPTIBLE TIMBER REGENERATION
HARVESTED (HA)**

	P-1	P-2	P-3	P-4	P-5	P-6
I-140	35512.000	49493.321	47219.407	23153.928	28428.630	16524.000
I-160	7116.000					
I-180	126152.000			44627.679	40440.593	41810.072
+	P-7	P-8	P-9	P-10		
I-140	35525.000	66708.048	66608.021			
I-160			13230.054			
I-180	7185.370			88841.898		

VARIABLE G.L. AREA OF UNINFESTED REGENERATION HARVESTED (HA)

P-6 P-7

I-120 6227.500 5852.325
I-140 1193.500

VARIABLE GC.L AREA OF UNINFESTED CONVERSION HARVESTED (HA)

P-9

I-140 2026.817

VARIABLE PR.L AREA OF SUSCEPTIBLE INFESTED PROTECTED (HA)

P-1 P-2 P-3 P-5

I-60 12943.000 2387.000
I-80 36621.000 6227.500
I-100 64451.000 1193.500
I-120 67857.000
I-140 17114.701
I-180 47567.744 33759.256

VARIABLE F.L AREA OF INFESTED REGENERATION HARVESTED (HA)

P-1

I-140 9846.299
I-160 13563.000

VARIABLE FC.L AREA OF INFESTED CONVERSION HARVESTED (HA)

ALL 0.000

**VARIABLE FP.L AREA OF INFESTED PROTECTED REGENERATION
HARVESTED (HA)**

	P-2	P-3	P-4	P-7
I-140	67857.000	64451.000	36621.000	3580.500
I-180	47567.744	33759.256		

**VARIABLE FCP.L AREA OF INFESTED PROTECTED CONVERSION
HARVESTED (HA)**

	P-2	P-5	P-7
I-120			6227.500
I-140		12943.000	
I-160	17114.701		

VARIABLE X.L AREA NON-SUSCEPTIBLE FOREST (HA)

	P-1	P-2	P-3	P-4	P-5	P-6
I-20	35525.000	168780.000	66608.021	47219.407	67781.608	81812.223
I-40	16524.000	35525.000	168780.000	66608.021	47219.407	67781.608
I-60	35614.000	16524.000	35525.000	168780.000	66608.021	47219.407
I-80	64964.000	35614.000	16524.000	35525.000	168780.000	66608.021
I-100	87660.000	64964.000	35614.000	16524.000	35525.000	168780.000
I-120	94121.000	87660.000	64964.000	35614.000	16524.000	35525.000
I-140	35512.000	94121.000	87660.000	64964.000	35614.000	16524.000
I-160	7116.000		44627.679	40440.593	41810.072	7185 370
I-180	126152.000			44627.679	40440.593	41810.072
	P-7	P-8	P-9	P-10		

I-20	58334.072	48937.870	66708.048	81864.893
I-40	67781.608	81812.223	58334.072	48937.870
I-80	47219.407	67781.608	81812.223	58334.072
I-100	66608.021	47219.407	67781.608	81812.223
I-120	168780.000	66608.021	47219.407	67781.608
I-140	35525.000	168780.000	66608.021	47219.407
I-160			102071.952	
I-180	7185.370		88841.898	

VARIABLE Y.L AREA OF SUSCEPTIBLE UNINFESTED FOREST (HA)

	P-1	P-2	P-3	P-4	P-5	P-6
I-20	12455.000	23409.299	115424.744	98210.256	36621.000	
I-40	4774.000	12455.000	23409.299	115424.744	98210.256	36621.000
I-60		4774.000	12455.000	23409.299	115424.744	98210.256
I-80			2387.000	12455.000	11704.650	115424.744
I-100				2387.000	6227.500	11704.650
I-120					1193.500	6227.500
I-140						1193.500

+	P-7	P-8	P-9	P-10
I-20	7421.000	9432.825		
I-40		7421.000	9432.825	
I-60	36621.000		7421.000	9432.825
I-80	49105.128	36621.000		7421.000
I-100	57712.372	49105.128	36621.000	
I-120	5852.325	57712.372	49105.128	36621.000
I-140			57712.372	49105.128
I-160				55685.555

VARIABLE Z.L AREA OF SUSCEPTIBLE FOREST INFESTED AT AGE I (HA)

	P-1	P-2	P-3	P-5	P-6	P-7
I-60	12943.000		2387.000	11704.650	11704.650	60809.778
I-80	36621.000			6227.500		57712.372
I-100	64451.000			1193.500		5852.325
I-120	67857.000					
I-140	26961.000					
I-160	13563.000					
I-180	81327.000	33759.256				

	+	P-8	P-9	P-10
I-60		60809.778	60809.778	60809.778
I-80		57712.372	57712.372	57712.372
I-100		5852.325	5852.325	5852.325

VARIABLE ZP.L AREA OF INFESTED FOREST PROTECTED (HA)

	P-2	P-3	P-4	P-5	P-6	P-7
I-60	12943.000		2387.000			
I-100	36621.000	12943.000		2387.000	6227.500	
I-120	64451.000	36621.000	12943.000		3580.500	6227.500
I-140	67857.000	64451.000	36621.000	12943.000		3580.500
I-160	17114.701					
I-180	47567.744	33759.256

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Appendix C

GAMS: Stochastic LP Simulator

\$TITLE the stochastic fire pest regeneration model using Kamloops fire data

*

\$OFFSYMXREF OFFSYMLIST OFFUELLIST OFFUELXREF

OPTIONS LIMROW=0,LIMCOL=0,LP=MINOS5,SOLPRINT=OFF,DECIMALS=5;

*

*

SETS

I18	index of fire rate	/1*18/
K	years	/K-1*K-20/
T	periods	/P-1*P-10/;

SCALARS

PP	random fire rate per period non-susceptible
QQ	random fire rate per period susceptible
SS	random infestation rate
RRU	random fire rate infested
RRP	random fire rate protected

;

PARAMETERS

PAA(I18) random annual fire rate non-susceptible

/1 0, 2 .00022, 3 .0003, 4 .00034, 5 .00035, 6 .00036, 7 .00042, 8 .00048, 9 .00053, 10
.00065, 11 .00074, 12 .00079, 13 .00103, 14 .00149, 15 .00165, 16 .00447, 17 .00812,
18 .0099/

PAASC(I18) scaled fire rate

FFP(I18) empirical cumulative distribution function non-susceptible

/1 .487, 2 .513, 3 .539, 4 .565, 5 .642, 6 .668, 7 .694, 8 .72, 9 .746, 10 .772, 11 .798,
12 .849, 13 .8753, 14 .9013, 15 .927, 16 .9533, 17 .9793, 18 1.0/

QAA(I18) random annual fire rate susceptible

YY(K,T) a random number between 0 and 1

QSAMP(K,T) random sample drawn

PPP(T) random fire rate

SSS(T) random infestation rate

ACUTEST(T) estimated annual cut using average fire rates

ACUT(T) annual cut under simulated feedback;

SCALARS

mean	mean fire rate	/0.00096/
scale	scale factor for .01 fire rate;	

scale = .005/mean;

paasc(i18) = scale*paa(i18);

option seed = 3124

;

*

QQ = PP;

RRP = PP;

RRU = PP;

*

* calculate random period infestation rate

*

SSS(T,=1-PROD(I20,(YY(I20,T)*.02));

*

* update states using randomly generated fires

*

X0("I-20") =

SUM(ICUTNS,H.L(ICUTNS,"P-1")

+PP*(X.L(ICUTNS,"P-1")-H.L(ICUTNS,"P-1"))

+SUM(ICUT,GC.L(ICUT,"P-1")+FCP.L(ICUT,"P-1")+FC.L(ICUT,"P-1"))

+SUM(INCUTNS, PP*(X.L(INCUTNS,"P-1")));

X0(INCUTNS) \$ (ORD(INCUTNS) NE 1) = (1-PP)*X.L(INCUTNS-1,"P-1") ;

X0("I-140") = (1-PP)*X.L("I-120","P-1");

X0(ICUTNS) \$ ((ORD(ICUTNS) NE 1)*(ORD(ICUTNS) NE CARD(ICUTNS)))

= (1-PP)*(X.L(ICUTNS-1,"P-1")-H.L(ICUTNS-1,"P-1"));

X0("I-180") = (1-PP)*(X.L("I-160","P-1")-H.L("I-160","P-1")

+X.L("I-180","P-1")-H.L("I-180","P-1"));

Y0("I-20") = SUM(ICUT,G.L(ICUT,"P-1")

+QQ*(Y.L(ICUT,"P-1")-GC.L(ICUT,"P-1")-G.L(ICUT,"P-1"))

+FP.L(ICUT,"P-1")+F.L(ICUT,"P-1")

+RRP*(ZP.L(ICUT,"P-1")-FP.L(ICUT,"P-1")-FCP.L(ICUT,"P-1"))
 +RRU*(Z.L(ICUT,"P-1")-F.L(ICUT,"P-1")-FC.L(ICUT,"P-1")
 -PR.L(ICUT,"P-1"))
 +SUM(INCUT, RRU*(Z.L(INCUT,"P-1")-PR.L(INCUT,"P-1"))
 +QQ*Y.L(INCUT,"P-1")+RRP*ZP.L(INCUT,"P-1"));

Y0(INCUT) \$ (ORD(INCUT) NE 1)=(1-QQ-SS)*Y.L(INCUT-1,"P-1");

Y0("I-120") = (1-QQ-SS(T))*Y.L("I-100","P-1");

Y0(ICUT) \$ ((ORD(ICUT) NE 1)*(ORD(ICUT) NE CARD(ICUT)))
 =(1-QQ-SS(T))*(Y.L(ICUT-1,"P-1")-G.L(ICUT-1,"P-1")-GC.L(ICUT-1,"P-1"));

Y0("I-180") = (1-QQ-SS(T))*(Y.L("I-160","P-1")-G.L("I-160","P-1")
 -GC.L("I-160","P-1")+Y.L("I-180","P-1")-G.L("I-180","P-1")-GC.L("I-180","P-1"));

Z0(INCUT) =
 SS(T)*(Y.L(INCUT,"P-1"))+(1-RRU)*(Z.L(INCUT,"P-1")
 -PR.L(INCUT,"P-1"));

Z0(ICUT) =
 SS(T)*(Y.L(ICUT,"P-1")-G.L(ICUT,"P-1")-GC.L(ICUT,"P-1"))
 +(1-RRU)*(Z.L(ICUT,"P-1")-F.L(ICUT,"P-1")-FC.L(ICUT,"P-1")
 -PR.L(ICUT,"P-1"));

ZP0(INCUT) = PR.L(INCUT-1,"P-1")+(1-RRP)*ZP.L(INCUT-1,"P-1");

ZP0("I-120") = PR.L("I-100","P-1")+(1-RRP)*ZP.L("I-100","P-1");

```

ZP0(ICUT) $ ((ORD(ICUT) NE 1 )*(ORD(ICUT) NE CARD(ICUT)))
      = PR.L(ICUT-1,"P-1")
      +(1-RRP)*(ZP.L(ICUT-1,"P-1")-FP.L(ICUT-1,"P-1")-FCP.L(ICUT-1,"P-1"));

```

```

ZP0("I-180") =
      PR.L("I-160","P-1")+PR.L("I-180","P-1")
      +(1-RRP)*(ZP.L("I-160","P-1")-FP.L("I-160","P-1")-FCP.L("I-160","P-1")
      +ZP.L("I-180","P-1")-FP.L("I-180","P-1")-FCP.L("I-180","P-1"));

```

```

*
```

```

***** INITIALIZE THE STATE VARIABLES *****

```

```

*
```

```

X.FX(I,"P-1")  =  X0(I);
Y.FX(I,"P-1")  =  Y0(I);
Z.FX(I,"P-1")  =  Z0(I);
ZP.FX(I,"P-1") =  ZP0(I);

```

```

*
```

```

***** RE-SOLVE *****

```

```

*
```

```

SOLVE FPM(T) USING LP MAXIMIZING PHI;

```

```

*
```

```

* TAKE THE FIRST PERIOD CUT

```

```

*
```

```

SUPPLY(T+1)=PULP.L("P-1")/20;          );

```

```

*****
***** END OF T LOOP *****
*****

```

```

DISPLAY PPP, SSS, QSAMP, SUPPLY;

```

Appendix D

GAMS: Kamloops Data Aggregation

SETS

I	ten year age-classes	/I-1*I-31/
I20	twenty year age classes/I-20, I-40, I-60, I-80, I-100, I-120, I-140, I-160, I-180, I-200, I-220, I-240, I-260, I-280, I-300, I-320/	
IOLD(I20)	age-classes greater than 160 /I-180, I-200, I-220, I-240, I-260, I-280, I-300, I-320/	
W	working group	/fir-wet, fir-dry, cedar, hemlock, spruce, balsam, pine/
SW(W)		/fir-wet, fir-dry/
NSW(W)		/cedar, hemlock, spruce, balsam, pine/
L	land-class	/good-med, poor-low/
S	susceptible non-susceptible	/susc, non-susc/;

TABLE X(I20,W, L) initial inventory (thousand hectares)

	fir-wet.g-m	fir-wet.p-l	fir-dry.g-m	fir-dry.p-l	
I-20	5.391	1.949	2.404	2.711	
I-40	0.654	0.746	1.044	2.330	
I-60	4.949	3.797	0.550	3.647	
I-80	19.595	11.112	1.276	4.638	
I-100	39.940	4.772	3.590	16.149	
I-120	37.542	4.180	9.641	16.494	
I-140	19.444	1.368	3.562	2.587	
I-160	8.133	0.380	3.166	1.884	
I-180	8.507	0.329	8.269	5.812	
I-200	5.287	0.204	5.948	3.251	
I-220	5.529	0.100	11.292	6.980	
I-240	4.625	0.063	2.314	1.893	
I-260	0.991	0.007	1.782	1.133	
I-280	1.134	0.008	1.097	0.626	
I-300	1.027		0.651	0.303	
I-320	0.701		0.964	0.500	
+ cedar.g-m cedar.p-l hemlock.g-m hemlock.p-l spruce.g-m					
I-20	0.596	.399	.441	.190	11.753
I-40	.263	.631	.262	.464	.570
I-60	.479	.200	.694	.259	3.521
I-80	.598	.086	.737	.127	7.552
I-100	.729	.311	.358	.329	7.273
I-120	.088	.444	.496	.493	18.341
I-140	.264	.513	.420	.232	8.506
I-160	.148	.307	.360	.284	1.292
I-180	.164	.376	.091	.093	5.124
I-200	.226	.170	.216	.194	4.283
I-220	.425	1.206	.288	.380	4.387
I-240	1.179	1.924	.194	1.646	8.811
I-260	.912	.716	.181	.636	2.761
I-280	2.106	1.659	.338	.097	3.896
I-300	8.478	2.392	.513	.572	2.796
I-320	10.537	2.968	1.372	1.427	1.285
+ spruce.p-l balsam.g-m balsam.p-l pine.g-m pine.p-l					
I-20	1.810	5.787	2.625	7.872	4.052
I-40	0.469	2.433	3.960	1.603	5.869
I-60	1.739	2.319	3.418	12.663	10.322
I-80	.008	5.013	2.252	45.771	2.820
I-100	1.105	2.488	2.437	54.026	18.604

I-120	0.714	4.116	2.489	66.940	
I-140	1.672	2.553	0.725	20.625	.002
I-160	0.544	0.275		1.653	2.253
I-180	1.560	0.158		1.496	1.488
I-200	1.343	0.095		1.360	1.530
I-220	5.300	0.159		0.558	0.655
I-240	5.668	0.399		0.048	0.191
I-260	3.946	0.038		0.062	0.103
I-280	3.078	0.016			
I-300	12.285	0.406		0.075	
I-320	6.672	0.423		0.021	

TABLE V(I,W,L) existing volume tables

	fir-wet.g-m	fir-wet.p-l	fir-dry.g-m	fir-dry.p-l	cedar.g-m
I-1	.0006				.0009
I-2	.0023	.0001	.0002		.0058
I-3	.0281	.0032	.0061		.0158
I-4	.0759	.0249	.0305	.0064	.0308
I-5	.1253	.0507	.0698	.0213	.0502
I-6	.1702	.0751	.1106	.0391	.0729
I-7	.2067	.0955	.1437	.0550	.0971
I-8	.2357	.1118	.1679	.0676	.1224
I-9	.2580	.1246	.1845	.0770	.1478
I-10	.2751	.1344	.1957	.0838	.1729
I-11	.2892	.1426	.2041	.0891	.1958
I-12	.3002	.1490	.2101	.0932	.2171
I-13	.3089	.1542	.2144	.0963	.2366
I-14	.3157	.1583	.2177	.0988	.2541
I-15	.3212	.1617	.2203	.1008	.2694
I-16	.3256	.1645	.2223	.1025	.2818
I-17	.3292	.1669	.2240	.1039	.2792
I-18	.3321	.1689	.2254	.1051	.2745
I-19	.3346	.1705	.2266	.1062	.2699
I-20	.3366	.1720	.2276	.1071	.2653
I-21	.3383	.1732	.2285	.1080	.2607
I-22	.3402	.1745	.2295	.1088	.2607
I-23	.3418	.1757	.2304	.1096	.2607
I-24	.3433	.1767	.2312	.1103	.2607
I-25	.3446	.1777	.2320	.1109	.2607
I-26	.3459	.1786	.2327	.1115	.2607
I-27	.3470	.1794	.2333	.1120	.2607
I-28	.3480	.1801	.2339	.1125	.2607
I-29	.3489	.1808	.2345	.1130	.2607
I-30	.3498	.1814	.2350	.1135	.2607
I-31	.3498	.1814	.2350	.1135	.2607

+

	cedar.p-l	hemlock.g-m	hemlock.p-l	spruce.g-m	spruce.p-l	balsam.g-m
I-1	.0004	.0003	.0001			.0054
I-2	.0027	.0046	.0016	.0004		.0220
I-3	.0081	.0173	.0067	.0101		.0464
I-4	.0171	.0401	.0167	.0374	.0084	.0752
I-5	.0293	.0716	.0311	.0711	.0253	.1054
I-6	.0444	.1091	.0489	.1083	.0460	.1352
I-7	.0612	.1485	.0681	.1458	.0688	.1625
I-8	.0791	.1878	.0876	.1822	.0929	.1873
I-9	.0977	.2252	.1065	.2163	.1171	.2093
I-10	.1164	.2595	.1242	.2475	.1407	.2286
I-11	.1343	.2899	.1400	.2766	.1635	.2452
I-12	.1515	.3163	.1540	.3028	.1850	.2593
I-13	.1675	.3389	.1662	.3263	.2049	.2712
I-14	.1824	.3579	.1766	.3474	.2232	.2813
I-15	.1961	.3737	.1854	.3661	.2398	.2897
I-16	.2083	.3866	.1928	.3827	.2548	.2967
I-17	.2193	.3970	.1988	.3972	.2684	.3024
I-18	.2247	.4053	.2037	.4088	.2804	.3071
I-19	.2220	.4117	.2077	.4186	.2906	.3110
I-20	.2192	.4165	.2108	.4272	.2991	.3140
I-21	.2163	.4200	.2132	.4348	.3065	.3165
I-22	.2163	.4267	.2172	.4424	.3138	.3203
I-23	.2163	.4325	.2206	.4492	.3204	.3237
I-24	.2163	.4377	.2237	.4554	.3264	.3268
I-25	.2163	.4422	.2265	.4609	.3318	.3296
I-26	.2163	.4462	.2290	.4659	.3367	.3321
I-27	.2163	.4498	.2312	.4704	.3411	.3343
I-28	.2163	.4530	.2332	.4745	.3451	.3364
I-29	.2163	.4559	.2350	.4781	.3487	.3383
I-30	.2163	.4585	.2367	.4814	.3520	.3401
I-31	.2163	.4585	.2367	.4814	.3520	.3401

+	balsam.p-l	pine.g-m	pine.p-l
I-1	.0022	.0002	0
I-2	.0095	.0044	.0001
I-3	.0206	.3243	.0016
I-4	.0341	.0564	.0137
I-5	.0487	.0962	.0356
I-6	.0634	.1366	.0604
I-7	.0773	.1757	.0861
I-8	.0902	.2108	.1109
I-9	.1019	.2413	.1337
I-10	.1123	.2671	.1541
I-11	.1215	.2897	.1728

I-12	.1295	.3046	.1881
I-13	.1364	.3145	.2007
I-14	.1424	.3217	.2115
I-15	.1475	.3241	.2203
I-16	.1519	.3243	.2270
I-17	.1556	.3243	.2291
I-18	.1587	.3243	.2293
I-19	.1614	.3243	.2293
I-20	.1636	.3243	.2293
I-21	.1655	.3243	.2293
I-22	.1681	.3243	.2293
I-23	.1705	.3243	.2293
I-24	.1726	.3243	.2293
I-25	.1746	.3243	.2293
I-26	.1764	.3243	.2293
I-27	.1781	.3243	.2293
I-28	.1797	.3243	.2293
I-29	.1811	.3243	.2293
I-30	.1824	.3243	.2293
I-31	.1824	.3243	.2293

PARAMETERS

AGX(I20,S,L)	aggregate forest inventory	(ha)
AGXV(I20,S,L)	aggregate total forest yield	(m**3)
AGV(I20,S,L)	aggregate unit yield	(m**3 per hA)
AGGXV(I20,S)	aggregate total forest yield over L	(m**3)
AGGV(I20,S)	aggregate unit yield	(m**3 per ha)
AGGX(I20,S)	aggregate forest inventory over L	(ha)
X20(I20,S)	forest inventory	(ha)
V20(I20,S)	volume-at-age	(m**3 per ha)
V20WL(I20,W,L)	volume-at-age 20 year age-classes	
AGOLDXV(S)	total volume old age-classes	
AGOLDX(S)	area old age-classes	
AGOLDV(S)	aggregate volume old age class;	

V20WL("I-20",W,L) = V("I-1",W,L);

V20WL("I-40",W,L) = V("I-3",W,L);

V20WL("I-60",W,L) = V("I-5",W,L);


```

V20WL("I-80",W,L) = V("I-7",W,L);
V20WL("I-100",W,L) = V("I-9",W,L);
V20WL("I-120",W,L) = V("I-11",W,L);
V20WL("I-140",W,L) = V("I-13",W,L);
V20WL("I-160",W,L) = V("I-15",W,L);
V20WL("I-180",W,L) = V("I-17",W,L);
V20WL("I-200",W,L) = V("I-19",W,L);
V20WL("I-220",W,L) = V("I-21",W,L);
V20WL("I-240",W,L) = V("I-23",W,L);
V20WL("I-260",W,L) = V("I-25",W,L);
V20WL("I-280",W,L) = V("I-27",W,L);
V20WL("I-300",W,L) = V("I-29",W,L);
V20WL("I-320",W,L) = V("I-31",W,L);
AGXV(I20,"susc",L) = SUM(SW,X(I20,SW,L)*V20WL(I20,SW,L));
AGXV(I20,"non-susc",L) = SUM(NSW,X(I20,NSW,L)*V20WL(I20,NSW,L));
AGX(I20,"susc",L) = SUM(SW,X(I20,SW,L));
AGX(I20,"non-susc",L) = SUM(NSW,X(I20,NSW,L));
AGV(I20,S,L) = AGXV(I20,S,L)/AGX(I20,S,L);
AGGXV(I20,S) = SUM(L,AGV(I20,S,L)*AGX(I20,S,L));
AGGX(I20,S) = SUM(L,AGX(I20,S,L));
AGGV(I20,S) = AGGXV(I20,S)/AGGX(I20,S);
AGOLDXV(S) = SUM(IOLD,AGGV(IOLD,S)*AGGX(IOLD,S));
AGOLDX(S) = SUM(IOLD,AGGX(IOLD,S));
AGOLDV(S) = AGOLDXV(S)/AGOLDX(S);
XTOTT = SUM(S,XTOT(S));

```

DISPLAY AGGX,AGGV,AGOLDX,AGOLDV;

PARAMETER AGGX AGGREGATE FOREST INVENTORY OVER L

	SUSC	NON-SUSC
I-20	12.455	35.525
I-40	4.774	16.524
I-60	12.943	35.614

I-80	36.621	64.964
I-100	64.451	87.660
I-120	67.857	94.121
I-140	26.961	35.512
I-150	13.563	7.116
I-180	22.917	10.550
I-200	14.690	9.417
I-220	23.901	13.358
I-240	8.895	20.060
I-260	3.913	9.355
I-280	2.865	11.190
I-300	1.981	27.517
I-320	2.165	24.705

PARAMETER AGGV AGGREGATE UNIT YIELD (M**3 PER HA)

SUSC NON-SUSC

I-20		0.001
I-40	0.006	0.045
I-60	0.072	0.067
I-80	0.152	0.162
I-100	0.199	0.208
I-120	0.219	0.278
I-140	0.268	0.302
I-160	0.263	0.281
I-180	0.232	0.333
I-200	0.238	0.343
I-220	0.218	0.336
I-240	0.262	0.356
I-260	0.225	0.348
I-280	0.252	0.355
I-300	0.275	0.322
I-320	0.244	0.303

PARAMETER AGOLDX AREA OLD AGE-CLASSES

SUSC 81.327, NON-SUSC 126.152

PARAMETER AGOLDV AGGREGATE VOLUME OLD AGE CLASS

SUSC 0.234, NON-SUSC 0.332

EXECUTION TIME = 0.213 MINUTES

Appendix E

The Inverse Transformation Method

Pseudo random numbers are used as the basis for generating random samples from an empirical distribution function. The simplest method is the *inverse transformation method* which uses the information that the random variable R is uniformly distributed on the unit interval $[0,1]$. Thus to generate a random sample from $F(q)$, we generate a random number r and solve the equation $r=F(q)$ for the corresponding value of $q=F^{-1}(r)$. For example, consider the cumulative distribution function obtained from the annual fire data as depicted in Figure E-1:

To obtain a sample, a random number is generated in the range 0 to 1 and the graph is entered at this ordinate value. The resultant random sample is then obtained by tracing across the graph to the cumulative curve and then down to the q -axis. Since the data supports a cumulative distribution function for discrete values of the fire rates we need to obtain the random samples for any ordinate value on the y axis. One method is to use linear interpolation. For example if the random number generated is .6 the random sample fire rate would be :

$$q_L + \frac{(.6 - L)}{(U - L)} [q_U - q_L]$$

where U and L are the upper and lower bounds of the cumulative distribution function and q_U and q_L are the fire rates associated with these bounds. This is shown schematically in Figure E-1.

Figure E-1 The Inverse Transformation Method to generate Random Samples

END

02.03.92

FIN