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**LA THÈSE A ÉTÉ
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PROPERTIES OF
RAKING RATIO ESTIMATORS

by

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APRIL 1977

6 Michael Douglas Bankier 1977

The undersigned recommend to the Faculty
of Graduate Studies acceptance of the thesis

"Properties of
Raking Ratio Estimators"

submitted by Michael Douglas Bankier, B.Sc.,
in partial fulfilment of the requirements for
the degree of Master of Science.

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May 9, 1977

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ABSTRACT

Assume a census is done which provides population values for certain basic characteristics while other characteristics are measured on only a sample basis. Raking ratio estimation is an iterative procedure that provides estimates of the population values for the sample characteristics. It utilizes the known population values of the basic characteristics. The use of this supplementary information makes possible the improvement of the efficiency of the estimators. Also in a cross-tabulation of a sample characteristic against a basic characteristic, estimates of the basic characteristic produced by this method can be made equal to the known population values.

In this thesis the large-sample variance and bias of the raking ratio estimators up to the fourth iteration are given under the assumptions of simple random sampling and cluster sampling. The asymptotic variance of the maximum likelihood estimator for the total of a sample characteristic is derived. An empirical study of the raking ratio estimators under simple random sampling and cluster sampling is done. The large-sample efficiency of the raking ratio estimators versus the maximum likelihood estimators under simple random sampling is calculated. The design effect for cluster sampling versus simple random sampling is computed.

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CHAPTER 1

INTRODUCTION

Assume we have a two-way $r \times s$ cross-classification table which classifies the N units in a population into rs categories. Let N_{ij} be the number of these units which fall in the (i,j) th cell of the table. The corresponding number in a sample of n units is denoted by n_{ij} . Suppose the population marginals

$$N_{i.} = \sum_j N_{ij} \quad i = 1, 2, \dots, r$$

$$N_{.j} = \sum_i N_{ij} \quad j = 1, 2, \dots, s$$

are known.¹ The problem now is to find estimates $\hat{N}_{ij}$ of the cell frequencies N_{ij} subject to the constraints

$$\sum_j \hat{N}_{ij} = N_{i.} \quad i = 1, 2, \dots, r$$

$$\sum_i \hat{N}_{ij} = N_{.j} \quad j = 1, 2, \dots, s-1 \quad \left(\sum_{i=1}^r N_{i.} = \sum_{j=1}^s N_{.j} = N \right) \quad (1.1)$$

Several iterative procedures have been proposed. One well known method is called the raking ratio estimation procedure (RREP). The p th iteration raking ratio estimator (RRE) for

1. Throughout this thesis we will denote summation over a subscript by replacing that subscript by a dot (.).

2.

N_{ij} is given by

$$\hat{N}_{ij}^{(p)} = w_{ij}^{(p)} n_{ij}, \quad p = 0, 1, 2, \dots \quad (1.2)$$

where

$$w_{ij}^{(0)} = \frac{N}{n} \text{ for all } i, j$$

and for $p \geq 1$

$$w_{ij}^{(p)} = \begin{cases} w_{ij}^{(p-1)} \frac{N_{.j}}{\sum_a n_{aj} w_{aj}^{(p-1)}} & \text{if } p \text{ is even} \\ w_{ij}^{(p-1)} \frac{N_{i.}}{\sum_b n_{ib} w_{ib}^{(p-1)}} & \text{if } p \text{ is odd.} \end{cases} \quad (1.3)$$

It is readily seen that for p odd the $\hat{N}_{ij}^{(p)}$ satisfy the row constraints exactly and similarly for p even the column constraints are satisfied exactly, i.e.,

$$\begin{aligned} \sum_j \hat{N}_{ij}^{(2p-1)} &= N_{i.} \\ \sum_i \hat{N}_{ij}^{(2p)} &= N_{.j} \end{aligned} \quad (1.4)$$

At Statistics Canada the RREP was applied to a somewhat different problem. In both the 1971 and 1976 censuses a basic questionnaire called the 2A form was distributed to two thirds of the households in each enumeration area (EA). Every third household received the more extensive 2B form which included the 2A questions. This procedure was carried out throughout Canada with two exceptions: all collective dwellings (i.e. institutions, lodging - houses) plus all households in difficult to enumerate

areas such as the Yukon or North West Territories were enumerated on the 2B form.

The cells of the cross-classification table correspond to 2A-items. Thus the N_{ij} values are known unlike the usual applications of the RREP. The weights $w_{ij}^{(p)}$, however, are calculated as before using only the known population marginals and the sample values n_{ij} obtained from the 2A portion of the 2B forms. Denote by Y the total of a 2B-category of interest (for example the category Employed). Then the p th iteration RRE of Y is given by

$$\hat{Y}^{(p)} = \sum_i \sum_j w_{ij}^{(p)} y_{ij} \quad (1.5)$$

where y_{ij} is the total for the 2B-category of those sample units that fall in the (i,j) th cell of the table.

The EA's are formed into groups called weighting areas (WA's). The RREP is carried out separately in each WA. Thus within a weighting area the sampling scheme is a stratified systematic sample for household characteristics. For characteristics of household members, it is a stratified systematic sample of clusters.

The RREP is used for a number of reasons. First the weights are easy to compute. Also the use of known values of $N_{i.}$ and $N_{.j}$ in the calculation of the weights may improve the efficiency of the estimators. Another point is that in cross-tabulations of 2B-categories with 2A-categories, estimates of the 2A-categories

appear. Using the RREP, these estimates can be made to agree with the known population values. This is important in that lack of agreement between the known population values and the estimates from the sample can cause concern to the general user of census data. Finally, it should be noted that the RREP uses one set of weights for all 2B-categories. This is an advantage from a computational point of view and also helps insure consistency between the estimates in different tabulations.

This thesis will cover the following topics. In Chapter 2 a review of the literature is given. Large-sample approximations for the variance and bias of the RRE up to the fourth iteration are presented. These formulae are developed under the assumptions of simple random sampling (s.r.s.) and cluster sampling. The effect of the actual samples being systematic (within EA's) is assumed to be negligible. The fact that in reality the sampling was stratified (EA's within WA's) is ignored. For most 2B-categories such as Employed, the gains in efficiency from stratification on the EA level are thought to be minimal. The variance of the estimators for certain 2A-categories such as Rented Apartments however might be significantly reduced by stratification. Further study is required, however, to confirm these conjectures.

In Chapter 3, we consider as a special case estimators of the number of people that belong to a 2B-category. Let N_{ijk} be the number of people that are associated with the k th 2B-category of interest and fall in the (i,j) th cell of the

cross-classification table. The asymptotic variance-covariance matrix of the maximum likelihood estimators (m.l.e.) π_{ijk}^* of $\pi_{ijk} = \frac{N_{ijk}}{N}$ is found where the m.l.e. are subject to the constraints

$$\sum_j \sum_k \pi_{ijk} = \pi_{i..} \quad i = 1, \dots, r-1$$

$$\sum_i \sum_k \pi_{ijk} = \pi_{.j.} \quad j = 1, \dots, s-1 \quad (1.6)$$

$$\sum_i \sum_j \sum_k \pi_{ijk} = 1.$$

In the above $\pi_{i..} = \frac{N_{i..}}{N}$ and $\pi_{.j.} = \frac{N_{.j.}}{N}$ are the known

marginal proportions. These results are derived under the assumptions of s.r.s. and sampling with replacement (w.r.). Efficiency comparisons of the RRE and the m.l.e. (which are asymptotically most efficient) are then possible.

Empirical results on the estimated standard error and bias for the RRE under s.r.s. are given in Chapter 4. The efficiency of the RRE relative to the m.l.e. is also calculated in this chapter.

Chapter 5 gives, for various categories, the estimated standard error and bias of the RRE under cluster sampling. The estimated standard error of $\hat{y}^{(0)}$ under s.r.s. is also calculated for these categories. From this a "design effect" is found. It is hoped that the knowledge of a design effect will permit estimates of the standard error of $\hat{y}^{(p)}$ under cluster sampling to be found using the computationally simpler formula

for the estimated standard error of $\hat{Y}^{(0)}$ under s.r.s.

In Chapter 6 is given a summary of the most important results and conclusions.

CHAPTER 2

REVIEW OF THE LITERATURE

2.1: Initial Papers on the RREP

In 1940 Deming and Stephan [5] made the first study of the RREP. They proposed to find the values of m_{ij} that minimize

$$S = \sum_i \sum_j \frac{(m_{ij} - n_{ij})^2}{n_{ij}} \quad (2.1)$$

subject to the constraints

$$\sum_j m_{ij} = m_{i.} \quad i = 1, 2, \dots, r \quad (2.2)$$

$$\sum_i m_{ij} = m_{.j} \quad j = 1, 2, \dots, s-1 \quad (\sum_i m_{i.} = \sum_j m_{.j} = n)$$

where

$$m_{i.} = N_{i.} \frac{n}{N} \quad (2.3)$$

$$m_{.j} = N_{.j} \frac{n}{N}$$

are known and S is a χ^2 -type measure of discrepancy. They provided an exact solution to the problem which involved solving $s-1$ normal equations in $s-1$ unknowns. The RREP was proposed as an iterative process for finding the solution of this problem. They also discussed an extension to cross-tabulations in three dimensions.

Stephan [16] pointed out that in the 1940 paper they were wrong in believing that the RREP converged to the solution that minimized S in (2.1) subject to the constraints (2.2). The solution from the RREP satisfies the marginal constraints but it

does not minimize S . Stephan developed an alternative iterative procedure which does lead to the correct solution.

Smith [15], El-Badry and Stephan [6], and Friedlander [8] proposed other procedures for getting estimates of the population cell frequencies which satisfy the constraints (1.1).

Ireland and Kullback [10] minimized the "discrimination information".

$$I(\pi/p) = \sum_{i=1}^r \sum_{j=1}^s \pi_{ij} \ln (\pi_{ij}/p_{ij}) \quad (2.4)$$

subject to the marginal restrictions

$$\begin{aligned} \pi_{i.} &= \sum_{j=1}^s \pi_{ij} \\ \pi_{.j} &= \sum_{i=1}^r \pi_{ij} \end{aligned} \quad (2.5)$$

where $\pi_{i.}$ and $\pi_{.j}$ are known. They defined $\pi_{ij} = \frac{N_{ij}}{N}$ and

$p_{ij} = \frac{n_{ij}}{n}$. The solution π_{ij}^* to this problem is given by the

RREP provided that $n_{ij} > 0$ for all i, j . The estimators π_{ij}^* are asymptotically normal and asymptotically most efficient.

Fienberg [7] gave a geometrical proof that the RREP converges to values which satisfy the marginal constraints (2.5) if $n_{ij} > 0$ for all i, j . Bishop and Fienberg [3] and Goodman [9] also examined the convergence properties of the RREP.

2.2 Large-Sample Bias and Variance Formulae for the RREP

Under S.R.S. Without Replacement

Rao [11] derived the large-sample variances of the estimators $\hat{Y}^{(0)}$ and $\hat{Y}^{(2)}$ for the 2B-items.

Noting that, conditionally for a fixed $n_{i.}$, a sample of size $n_{i.}$ from the $N_{i.}$ units of the i th row is a s.r.s., a simplified derivation of the large-sample variance formulae was given for $\hat{Y}^{(1)}$ and $\hat{Y}^{(2)}$ by Rao [12]. Also variance estimators valid for $n_{i.} \geq 2$ were obtained.

Assuming s.r.s.w.r., the asymptotic variance-covariance matrix of the m.l.e. of the π_{ij} 's was obtained by Rao [13]. The calculation of this covariance matrix involves only the inversion of a $(c-1) \times (c-1)$ matrix where c is the minimum of r and s . Also the large-sample bias of $\hat{Y}^{(2)}$ and an estimator of this bias were found.

Arora and Brackstone [1] derived the large-sample bias and variance of the RRE for 2B-items up to the fourth iteration.

The method used in deriving the large-sample formulae uses a Taylor Series expansion of the estimator and will now be demonstrated for the second iteration. Two large-sample approximations applied in these derivations are

$$B(X \frac{w}{x}) \doteq B(w) - R B(x) - \frac{1}{E(x)} C(x, w - Rx) \quad (2.6)$$

and

$$C\left(\sum_i E(x_i) \frac{w_i}{x_i}, \sum_k E(z_k) \frac{q_k}{z_k}\right) \\ = C\left(\sum_i (w_i - R_i x_i), \sum_k (q_k - R_k^* z_k)\right) \quad (2.7)$$

where $B(X \frac{W}{X})$ is the bias of $X \frac{W}{X}$ in estimating W and C is the covariance operator. In the above w and x are the estimators of W and X . Also

$$R = \frac{E(w)}{E(x)}, R_i = \frac{E(w_i)}{E(x_i)}, \text{ and } R_k^* = \frac{E(q_k)}{E(z_k)}.$$

Let y_{ijk} be the observed value of a certain 2B-item for the k th sample unit in the (i,j) th cell. Let Y_{ijk} be the corresponding value for the k th population unit. Then we have

$$y_{ij} = \sum_{k=1}^{n_{ij}} y_{ijk} \text{ and } Y_{ij} = \sum_{k=1}^{N_{ij}} Y_{ijk}. \text{ Other notation which will}$$

be used is the following:

$$\hat{N}_{ij}^{(p)} = w_{ij}^{(p)} n_{ij}, \quad N_{ij}^{(p)} = E(\hat{N}_{ij}^{(p)})$$

$$\hat{Y}_{ij}^{(p)} = w_{ij}^{(p)} y_{ij}, \quad Y_{ij}^{(p)} = E(\hat{Y}_{ij}^{(p)})$$

$$R_{i.} = \frac{y_{i.}}{N_{i.}}, \hat{R}_{i.}^{(p)} = \frac{\hat{Y}_{i.}^{(p)}}{\hat{N}_{i.}^{(p)}}, \quad R_{i.}^{(p)} = \frac{E(\hat{Y}_{i.}^{(p)})}{E(\hat{N}_{i.}^{(p)})}$$

$$R_{.j} = \frac{y_{.j}}{N_{.j}}, \hat{R}_{.j}^{(p)} = \frac{\hat{Y}_{.j}^{(p)}}{\hat{N}_{.j}^{(p)}}, \quad R_{.j}^{(p)} = \frac{E(\hat{Y}_{.j}^{(p)})}{E(\hat{N}_{.j}^{(p)})}$$

$$\rho_{ij} = \frac{N_{ij}}{N_{i.}}, \hat{\rho}_{ij}^{(p)} = \frac{\hat{N}_{ij}^{(p)}}{\hat{N}_{i.}^{(p)}}, \quad \rho_{ij}^{(p)} = \frac{E(\hat{N}_{ij}^{(p)})}{E(\hat{N}_{i.}^{(p)})}$$

$$\kappa_{ij} = \frac{N_{ij}}{N_{.j}}, \hat{\kappa}_{ij}^{(p)} = \frac{\hat{N}_{ij}^{(p)}}{\hat{N}_{.j}^{(p)}}, \quad \kappa_{ij}^{(p)} = \frac{E(\hat{N}_{ij}^{(p)})}{E(\hat{N}_{.j}^{(p)})}$$

$$\gamma_{ia} = \sum_j \kappa_{ij} \rho_{aj}, \quad \alpha_i = N_{i.}^2 \left(\frac{1}{n_{i.}} - \frac{1}{N_{i.}} \right).$$

The weights $w_{ij}^{(p)}$ are given by equations (1.3). It can be seen

that $\hat{\rho}_{ij}^{(0)} = \hat{\rho}_{ij}^{(1)} = \frac{n_{ij}}{n_{i.}}$ and will be written as $\hat{\rho}_{ij}$.

Similarly $\hat{\kappa}_{ij}^{(1)}$ will be denoted by $\hat{\kappa}_{ij}$. We will let

$$\hat{\gamma}_{ia} = \sum_j \hat{\kappa}_{ij} \hat{\rho}_{aj}.$$

For any variables V, W taking the values V_{iu}, W_{iu} for the u th unit of the population in the i th row let

$$S^2(V_{iu}) = \frac{1}{N_{i.}-1} \sum_{u=1}^{N_{i.}} (V_{iu} - \bar{V}_i)^2$$

$$S(V_{iu}, W_{iu}) = \frac{1}{N_{i.}-1} \sum_{u=1}^{N_{i.}} (V_{iu} - \bar{V}_i)(W_{iu} - \bar{W}_i) \text{ for the}$$

$$\text{population, and } s^2(v_{iu}) = \frac{1}{n_{i.}-1} \sum_{u=1}^{n_{i.}} (v_{iu} - \bar{v}_i)^2$$

$$s(v_{iu}, w_{iu}) = \frac{1}{n_{i.}-1} \sum_{u=1}^{n_{i.}} (v_{iu} - \bar{v}_i)(w_{iu} - \bar{w}_i) \quad \text{for the}$$

sample, where $\bar{v}_i, \bar{w}_i$ are respectively the population and sample means of V in the i th row.

Also, let us define

$j^{a_{iu}} = 1$ if the u th unit of the sample in the i th row belongs to the j th column.

$= 0$ otherwise

and z_{iu} = observed value of the 2B-item for the u th unit of the sample in the i th row.

We write $j^{A_{iu}}$ and Z_{iu} when referring to the units in the population rather than the sample.

Finally, a subscript 2 on an expectation or covariance indicates that it is conditional on the set of $n_{i.}$'s while a subscript 1 implies the expectation is over the $n_{i.}$'s.

The large-sample bias of $\hat{Y}^{(2)}$ is derived first:

$$\begin{aligned} B(\hat{Y}^{(2)}) &= \sum_j B(N_{.j}) \frac{\hat{Y}_{.j}^{(1)}}{\hat{N}_{.j}^{(1)}} \\ &= - \sum_j \frac{1}{N_{.j}^{(1)}} C(N_{.j}^{(1)}, \hat{Y}_{.j}^{(1)} - R_{.j}^{(1)} \hat{N}_{.j}^{(1)}) \\ &= - \sum_j \frac{1}{N_{.j}^{(1)}} C \left(\sum_i \frac{N_{i.}}{n_{i.}} \sum_{u=1}^{n_{i.}} j^{a_{iu}}, \sum_i \frac{N_{i.}}{n_{i.}} \sum_{u=1}^{n_{i.}} j^{a_{iu}} (z_{iu} - R_{.j}^{(1)}) \right) \end{aligned}$$

$$\begin{aligned}
&= - \sum_j \frac{1}{N_{.j}^{(1)}} E_1 \sum_i N_i^2 C_2 \left(\frac{1}{n_{i.}} \sum_{u=1}^{n_{i.}} j^{a_{iu}}, \frac{1}{n_{i.}} \sum_{u=1}^{n_{i.}} j^{a_{iu}} (z_{iu} - R_{.j}^{(1)}) \right) \\
&= - \sum_j \frac{1}{N_{.j}^{(1)}} \sum_i E_1 [\alpha_i s(j^{a_{iu}}, j^{a_{iu}} (z_{iu} - R_{.j}^{(1)}))] \quad (2.8) \\
&= - \sum_j \frac{1}{N_{.j}^{(1)}} \sum_i E_1 \left[\frac{\alpha_i}{N_{i.}-1} \left\{ \sum_{u=1}^{N_{i.}} j^{a_{iu}} [j^{a_{iu}} (z_{iu} - R_{.j}^{(1)})] \right. \right. \\
&\quad \left. \left. - \frac{(\sum_{u=1}^{N_{i.}} j^{a_{iu}}) [\sum_{u=1}^{N_{i.}} j^{a_{iu}} (z_{iu} - R_{.j}^{(1)})]}{N_{i.}} \right\} \right]
\end{aligned}$$

$$= - \sum_j \frac{1}{N_{.j}^{(1)}} \sum_i E_1 \left[\alpha_i \frac{N_{i.}}{N_{i.}-1} \left(\frac{y_{ij}}{N_{i.}} - \rho_{ij} R_{.j}^{(1)} \right) (1 - \rho_{ij}) \right]. \quad (2.9)$$

An estimator of $B(\hat{Y}^{(2)})$ can be found by substituting the sample covariance $s(j^{a_{iu}}, j^{a_{iu}} (z_{iu} - \hat{R}_{.j}^{(1)}))$ in (2.8).

This gives

$$b(\hat{Y}^{(2)}) = - \sum_j \frac{1}{N_{.j}^{(1)}} \sum_i \alpha_i \frac{N_{i.}}{N_{i.}-1} \left(\frac{y_{ij}}{N_{i.}} - \hat{\rho}_{ij} \hat{R}_{.j}^{(1)} \right) (1 - \hat{\rho}_{ij}). \quad (2.10)$$

Next the large-sample variance of $\hat{Y}^{(2)}$ is found. Compared to terms of order $\frac{1}{n}$, $V_1 E_2(\hat{Y}^{(2)})$ is negligible. Thus

$$V(\hat{Y}^{(2)}) \doteq E_1 V_2 \left(\sum_j N_{.j}^{(1)} \frac{\hat{Y}_{.j}^{(1)}}{N_{.j}^{(1)}} \right)$$

$$= E_1 V_2 \left(\sum_j \hat{Y}_j^{(1)} - R_j^{(1)} \hat{N}_j^{(1)} \right)$$

$$= E_1 V_2 \left(\sum_i \frac{N_i}{n_i} \left(\sum_{u=1}^{n_i} z_{iu} - \sum_j R_j^{(1)} \sum_{u=1}^{n_i} j^{a_{iu}} \right) \right)$$

$$= E_1 \sum_i \alpha_i S^2 (z_{iu} - \sum_j j^{a_{iu}} R_j^{(1)})$$

$$= E_1 \sum_i \frac{\alpha_i}{N_i - 1} \left[\sum_{u=1}^{N_i} (z_{iu} - \sum_j j^{a_{iu}} R_j^{(1)})^2 \right.$$

$$\left. - \frac{1}{N_i} \left(\sum_{u=1}^{N_i} (z_{iu} - \sum_j j^{a_{iu}} R_j^{(1)}) \right)^2 \right]$$

$$= E_1 \sum_i \frac{\alpha_i}{N_i - 1} \left[\sum_{u=1}^{N_i} z_{iu}^2 - 2 \sum_j y_{ij} R_j^{(1)} + \sum_{u=1}^{N_i} \sum_j j^{a_{iu}} R_j^{(1)2} \right.$$

$$\left. - \frac{1}{N_i} (y_{i.} - \sum_j N_{ij} R_j^{(1)})^2 \right]$$

$$= E_1 \sum_i \frac{\alpha_i}{N_i - 1} \left[\left(\sum_j \sum_{k=1}^{N_{ij}} y_{ijk}^2 - \frac{y_{i.}^2}{N_i} \right) - 2 \sum_j y_{ij} R_j^{(1)} \right.$$

$$\left. + \sum_j N_{ij} R_j^{(1)2} + 2 R_{i.} \sum_j N_{ij} R_j^{(1)} - \frac{1}{N_i} \left(\sum_j N_{ij} R_j^{(1)} \right)^2 \right] \quad (2.11)$$

$$= V(\hat{Y}^{(1)}) + E_1 \sum_i \alpha_i \frac{N_i}{N_i - 1} \left[\sum_j \rho_{ij} R_j^{(1)2} \right.$$

$$\left. - \frac{2}{N_i} \sum_j R_j^{(1)} (y_{ij} - \rho_{ij} y_{i.}) - \left(\sum_j \rho_{ij} R_j^{(1)} \right)^2 \right] \quad (2.11)$$

An estimator of $V(Y^{(2)})$ is given by

$$v(\hat{Y}^{(2)}) = v(\hat{Y}^{(1)}) + \sum_i \alpha_i \frac{n_{i.}}{n_{i.}-1} \left[\sum_j \hat{\rho}_{ij} \hat{R}_{.j}^{(1)2} - \frac{2}{n_{i.}} \sum_j \hat{R}_{.j}^{(1)} (y_{ij} - \hat{\rho}_{ij} y_{i.}) - \left(\sum_j \hat{\rho}_{ij} \hat{R}_{.j}^{(1)} \right)^2 \right]. \quad (2.12)$$

The large-sample formulae for the other iterations are as follows.

$$B(\hat{Y}^{(0)}) = 0 \quad (2.13)$$

$$V(\hat{Y}^{(0)}) = N^2 \left(\frac{1}{n} - \frac{1}{N} \right) \frac{1}{N-1} \sum_i \sum_j \sum_k (y_{ijk} - \bar{y})^2. \quad (2.14)$$

$$B(\hat{Y}^{(1)}) = 0 \quad (2.15)$$

$$V(\hat{Y}^{(1)}) = E_1 \sum_i \frac{\alpha_i}{N_{i.}-1} \left(\sum_{j,k=1}^{N_{ij}} y_{ijk}^2 - \frac{y_{i.}^2}{N_{i.}} \right) \quad (2.16)$$

$$B(\hat{Y}^{(3)}) = B(\hat{Y}^{(2)}) + E_1 \sum_i R_{i.}^{(2)} \sum_j \frac{1}{N_{.j}^{(1)}} \left\{ \alpha_i \frac{N_{i.}}{N_{i.}-1} \rho_{ij} (1 - \rho_{ij}) \right.$$

$$\left. - \kappa_{ij}^{(1)} \sum_a \alpha_a \frac{N_{a.}}{N_{a.}-1} \rho_{aj} (1 - \rho_{aj}) \right\} + E_1 \sum_i \frac{1}{N_{i.}^{(2)}}$$

$$\left\{ \alpha_i \frac{N_{i.}}{N_{i.}-1} \left[\sum_j \kappa_{ij}^{(1)} \frac{y_{ij}}{N_{i.}} - y_{i1} R_{i.} \right] - \sum_a \alpha_a \frac{N_{a.}}{N_{a.}-1} \right.$$

$$\left. \left[\sum_j \frac{y_{ij}^{(1)}}{N_{.j}^{(1)}} \rho_{aj} (\kappa_{ij}^{(1)} - y_{ia}) - R_{i.}^{(2)} \left(\sum_j \kappa_{ij}^{(1)2} \rho_{aj} - y_{ia}^2 \right) \right] \right\}$$

(2.17)

$$\begin{aligned}
V(\hat{Y}^{(3)}) &= V(\hat{Y}^{(2)}) + E_1 \left[\sum_i \alpha_i \frac{N_{i.}}{N_{i.}-1} \left\{ \sum_j \rho_{ij} \left(\sum_a R_{a.}^{(2)} \kappa_{aj}^{(1)} \right)^2 \right. \right. \\
&\quad - \left(\sum_a R_{a.}^{(2)} \gamma_{ai} \right)^2 + 2 \sum_j \left(\frac{Y_{ij}}{N_{i.}} - \rho_{ij} R_{.j}^{(1)} \right) \left(\sum_a R_{a.}^{(2)} \kappa_{aj}^{(1)} \right) \\
&\quad \left. \left. - 2 \left(R_{i.} - \sum_j \rho_{ij} R_{.j}^{(1)} \right) \left(\sum_a R_{a.}^{(2)} \gamma_{ai} \right) \right\} \right] \quad (2.18)
\end{aligned}$$

$$\begin{aligned}
B(\hat{Y}^{(4)}) &= B(\hat{Y}^{(3)}) - E_1 \sum_i \sum_j R_{.j}^{(3)} \rho_{ij}^{(2)} \left\{ \frac{1}{N_{.b}^{(1)}} \left(\alpha_i \frac{N_{i.}}{N_{i.}-1} \rho_{ib} (1 - \rho_{ib}) \right. \right. \\
&\quad \left. \left. - \kappa_{ib}^{(1)} \sum_a \alpha_a \frac{N_{a.}}{N_{a.}-1} \rho_{ab} (1 - \rho_{ab}) \right) + E_1 \sum_j R_{.j}^{(3)} \sum_i \frac{1}{N_{i.}^{(2)}} \right. \\
&\quad \left. \left\{ -\alpha_i \frac{N_{i.}}{N_{i.}-1} \rho_{ij} (\kappa_{ij}^{(1)} - \gamma_{ii}) + \sum_a \alpha_a \frac{N_{a.}}{N_{a.}-1} \right. \right. \\
&\quad \left. \left. \left[\rho_{aj} \kappa_{ij}^{(1)} (\kappa_{ij}^{(1)} - \gamma_{ia}) - \rho_{ij}^{(2)} \left(\sum_b \kappa_{ib}^{(1)} \rho_{aj} - \gamma_{ia}^2 \right) \right] \right\} \right. \\
&\quad \left. - E_1 \sum_j \frac{1}{N_{.j}^{(3)}} \sum_a \alpha_a \frac{N_{a.}}{N_{a.}-1} \sum_{a'} \rho_{a'j}^{(2)} \left[\left(\frac{Y_{aj}}{N_{a.}} - R_{.j}^{(1)} \rho_{aj} \right) (\kappa_{a'j}^{(1)} - \gamma_{a'a}) \right. \right. \\
&\quad \left. \left. + \sum_i \left(\frac{Y_{ij}}{N_{i.}^{(2)}} - R_{.j}^{(3)} \rho_{ij}^{(2)} \right) \left(\sum_b \kappa_{ib}^{(1)} \kappa_{a'b}^{(1)} \rho_{ab} - \gamma_{ib} \gamma_{a'a} \right) \right] \right] \quad (2.19)
\end{aligned}$$

$$\begin{aligned}
V(\hat{Y}^{(4)}) &= V(\hat{Y}^{(3)}) + E_1 \left[\sum_i \alpha_i \frac{N_{i.}}{N_{i.}-1} \left\{ \sum_j \rho_{ij} \left(\sum_{a,b} \kappa_{aj}^{(1)} \rho_{ab}^{(2)} R_{.b}^{(3)} \right)^2 \right. \right. \\
&\quad \left. \left. - \left(\sum_{a,b} R_{.b}^{(3)} \rho_{ab}^{(2)} \gamma_{ai} \right)^2 - 2 \sum_j \left(\frac{Y_{ij}}{N_{i.}} - \rho_{ij} \left(R_{.j}^{(1)} - \sum_a R_{a.}^{(2)} \kappa_{aj}^{(1)} \right) \right) \right. \right. \\
&\quad \left. \left. \left(\sum_{a,b} \kappa_{aj}^{(1)} \rho_{ab}^{(2)} R_{.b}^{(3)} \right) + 2 \left(R_{i.} - \sum_j \rho_{ij} \left(R_{.j}^{(1)} - \sum_a R_{a.}^{(2)} \kappa_{aj}^{(1)} \right) \right) \right. \right. \\
&\quad \left. \left. \left(\sum_{a,b} R_{.b}^{(3)} \rho_{ab}^{(2)} \gamma_{ai} \right) \right\} \right] \quad (2.20)
\end{aligned}$$

Estimators for the above formulae can be derived in a similar fashion to those given for the second iteration.

2.3 Large-Sample Bias and Variance Formulae for the RREP Under Cluster Sampling Without Replacement

In [2] Arora and Brackstone derived the large-sample bias and variance of the RRE for 2B-items up to the fourth iteration under cluster sampling. The bias and variance for the second iteration estimator will be derived to demonstrate the technique.

In this situation it is assumed that the population consists of A clusters (i.e. households). Let N_{hij} be the number of elements (i.e. members) in the h th cluster that fall in the (i,j) th cell of the cross-classification table. For the 2B-item Y that we are interested in, Y_{hij} is the total of the variable Y for those N_{hij} units. A s.r.s. of a of these clusters is drawn. The sample quantities corresponding to N_{hij} and Y_{hij}

are n_{hij} and y_{hij} . We will write $N_{.ij} = \sum_{h=1}^A n_{hij}$

and $Y_{.ij} = \sum_{h=1}^A y_{hij}$ as N_{ij} and Y_{ij} respectively to

simplify the notation. Similarly n_{ij} and y_{ij} will represent

$n_{.ij}$ and $y_{.ij}$. The p th iteration RRE is given by eq. (1.5).

The weights are calculated by formulae (1.3) as under the s.r.s.

case. The notation in section 2.2 is used with the exception of

the following terms:

$$\alpha = A^2 \left(\frac{1}{a} - \frac{1}{A} \right)$$

$$S^2(v_h) = \frac{1}{A-1} \sum_{h=1}^A (v_h - \bar{v})^2, \quad s(v_h, w_h) = \frac{1}{A-1} \sum_{h=1}^A (v_h - \bar{v})(w_h - \bar{w})$$

$$s^2(v_h) = \frac{1}{a-1} \sum_{h=1}^a (v_h - \bar{v})^2, \quad s(v_h, w_h) = \frac{1}{a-1} \sum_{h=1}^a (v_h - \bar{v})(w_h - \bar{w})$$

where for example the variable v takes the value v_h in the

h th household of the population. Also $\bar{v} = \frac{1}{A} \sum_{h=1}^A v_h$ and

$$\bar{v} = \frac{1}{a} \sum_{h=1}^a v_h$$

The derivation of the large-sample bias of $\hat{Y}^{(2)}$ is done

first.

$$B(\hat{Y}^{(2)}) = \sum_j B(N_{.j} \frac{\hat{Y}_{.j}^{(1)}}{N_{.j}^{(1)}})$$

$$= B(\hat{Y}^{(1)}) - \sum_j R_{.j}^{(1)} B(N_{.j}^{(1)}) - \sum_j \frac{1}{N_{.j}^{(1)}} C(N_{.j}^{(1)}, \hat{Y}_{.j}^{(1)} - R_{.j}^{(1)} \hat{N}_{.j}^{(1)})$$

$$= B(\hat{Y}^{(1)}) - \sum_j R_j^{(1)} \sum_i B(N_i, \frac{\frac{A}{a} n_{ij}}{\frac{A}{a} n_{i.}}) - \sum_j \frac{1}{N_j^{(1)}} C(\sum_i N_i, \frac{\frac{A}{a} n_{ij}}{\frac{A}{a} n_{i.}})$$

$$\sum_i N_i, \frac{\frac{A}{a} y_{ij}}{\frac{A}{a} n_{i.}} - R_j^{(1)} (\sum_i N_i, \frac{\frac{A}{a} n_{ij}}{\frac{A}{a} n_{i.}})$$

$$= B(\hat{Y}^{(1)}) + \sum_j R_j^{(1)} \sum_i \frac{1}{N_i} C[\frac{A}{a} n_{i.}, \frac{A}{a} (n_{ij} - \frac{N_{ij}}{N_i} n_{i.})]$$

$$- \sum_j \frac{1}{N_j^{(1)}} C[\sum_i \frac{A}{a} (n_{ij} - \rho_{ij} n_{i.}), \sum_i \frac{A}{a} (y_{ij} - \frac{y_{ij}}{N_i} n_{i.})]$$

$$- R_j^{(1)} \sum_i \frac{A}{a} (n_{ij} - \rho_{ij} n_{i.})]$$

$$= B(\hat{Y}^{(1)}) + \sum_j R_j^{(1)} \frac{A^2}{N_i} C[\frac{1}{a} \sum_{h=1}^a n_{hi.}, \frac{1}{a} \sum_{h=1}^a (n_{hij} - \rho_{ij} n_{hi.})]$$

$$- \sum_j \frac{A^2}{N_j^{(1)}} C[\frac{1}{a} \sum_{h=1}^a (n_{h.j} - \sum_i \rho_{ij} n_{hi.}), \frac{1}{a} \sum_{h=1}^a$$

$$(y_{h.j} - \sum_i \frac{y_{ij}}{N_i} n_{hi.} - R_j^{(1)} (n_{h.j} - \sum_i \rho_{ij} n_{hi.}))]$$

$$= B(\hat{Y}^{(1)}) + \sum_j R_j^{(1)} \sum_i \frac{\alpha}{N_i} S(N_{hi.}, N_{hij} - \rho_{ij} N_{hi.})$$

$$- \sum_j \frac{\alpha}{N_{.j}^{(1)}} s(n_{h.j} - \sum_i \rho_{ij} n_{hi.}, y_{h.j} - \sum_i \frac{y_{ij}}{N_{i.}} n_{hi.} - R_{.j}^{(1)})$$

$$(n_{h.j} - \sum_i \rho_{ij} n_{hi.}) \quad (2.21)$$

$$= B(Y^{(1)}) + \frac{\alpha}{A-1} \sum_j R_{.j}^{(1)} \sum_i \frac{1}{N_{i.}} \sum_{h=1}^A n_{hi.} (n_{hi.j} - \rho_{ij} n_{hi.})$$

$$- \frac{\alpha}{A-1} \sum_j \frac{1}{N_{.j}^{(1)}} \sum_{h=1}^A (n_{h.j} - \sum_i \rho_{ij} n_{hi.}) (y_{h.j} - \sum_i \frac{y_{ij}}{N_{i.}} n_{hi.} - R_{.j}^{(1)})$$

$$(n_{h.j} - \sum_i \rho_{ij} n_{hi.}) \quad (2.22)$$

An estimator of $B(Y^{(2)})$ is obtained by substituting

$s(n_{hi.}, n_{hi.j} - \rho_{ij} n_{hi.})$ and $s(n_{h.j} - \sum_i \hat{\rho}_{ij} n_{hi.},$

$y_{h.j} - \sum_i \frac{y_{ij}}{n_{i.}} n_{hi.} - \hat{R}_{.j}^{(1)} (n_{h.j} - \sum_i \hat{\rho}_{ij} n_{hi.}))$ in

(2.21) which gives

$$b(Y^{(2)}) = b(Y^{(1)}) + \frac{\alpha}{a-1} \sum_j \hat{R}_{.j}^{(1)} \sum_i \frac{1}{\frac{A}{a} n_{i.}} \sum_{h=1}^a n_{hi.} (n_{hi.j} - \rho_{ij} n_{hi.})$$

$$= \frac{\alpha}{a-1} \sum_j \frac{1}{N_{.j}^{(1)}} \sum_{h=1}^a (n_{h.j} - \sum_i \hat{\rho}_{ij} n_{hi.}) (y_{h.j} - \sum_i \frac{y_{ij}}{n_{i.}} n_{hi.} - \hat{R}_{.j}^{(1)})$$

$$(n_{h.j} - \sum_i \hat{\rho}_{ij} n_{hi.}) \quad (2.23)$$

The large-sample variance of $\hat{Y}^{(2)}$ is found as follows.

$$V(\hat{Y}^{(2)}) = V\left(\sum_j N_{.j} \frac{\hat{Y}_{.j}^{(1)}}{\hat{N}_{.j}^{(1)}}\right)$$

$$\doteq V(\hat{Y}^{(1)} - \sum_j R_{.j}^{(1)} \frac{\hat{N}_{.j}^{(1)}}{N_{.j}})$$

$$= V\left(\sum_i N_{i.} \frac{\frac{A}{a} y_{i.}}{\frac{A}{a} n_{i.}} - \sum_j R_{.j}^{(1)} \sum_i N_{i.} \frac{\frac{A}{a} n_{ij}}{\frac{A}{a} n_{i.}}\right)$$

$$\doteq V\left(\sum_i \frac{A}{a} (y_{i.} - R_{i.} n_{i.}) - \sum_j R_{.j}^{(1)} \sum_i \frac{A}{a} (n_{ij} - \rho_{ij} n_{i.})\right)$$

$$= A^2 V\left[\frac{1}{a} \sum_{h=1}^a (y_{h..} - \sum_i R_{i.} n_{hi.} - \sum_j R_{.j}^{(1)} (n_{h.j} - \sum_i \rho_{ij} n_{hi.}))\right]$$

$$= \alpha S^2 \left[(y_{h..} - \sum_i R_{i.} n_{hi.}) - \sum_j R_{.j}^{(1)} (n_{h.j} - \sum_i \rho_{ij} n_{hi.}) \right]$$

$$= \frac{\alpha}{A-1} \sum_{h=1}^A ((y_{h..} - \sum_i R_{i.} n_{hi.}) - \sum_j R_{.j}^{(1)} (n_{h.j} - \sum_i \rho_{ij} n_{hi.}))^2$$

$$(2.24)$$

An estimator of $V(\hat{Y}^{(2)})$ is given by

$$v(\hat{Y}^{(2)}) = \frac{\alpha}{a-1} \sum_{h=1}^a ((y_{h..} - \sum_i \hat{R}_{i.}^{(1)} n_{hi.}) - \sum_j \hat{R}_{.j}^{(1)} (n_{h.j} - \sum_i \rho_{ij} n_{hi.}))^2 \quad (2.25)$$

The large-sample bias and variance formulae for the other iterations are now given.

$$B(\hat{Y}^{(0)}) \doteq - \frac{\alpha}{A-1} \frac{1}{N} \sum_{h=1}^A (N_{h..} y_{h..} - \frac{Y}{N} N_{h..}^2) \quad (2.26)$$

$$V(\hat{Y}^{(0)}) \doteq \frac{\alpha}{A-1} \sum_{h=1}^A (y_{h..} - \frac{Y}{N} N_{h..})^2 \quad (2.27)$$

$$B(\hat{Y}^{(1)}) \doteq - \frac{\alpha}{A-1} \sum_i \frac{1}{N_{i.}} \sum_{h=1}^A (N_{hi.} y_{hi.} - R_{i.} N_{hi.}^2) \quad (2.28)$$

$$V(\hat{Y}^{(1)}) \doteq \frac{\alpha}{A-1} \sum_{h=1}^A (y_{h..} - \sum_i R_{i.} n_{hi.})^2 \quad (2.29)$$

$$\dot{B}(\hat{Y}^{(3)}) \doteq B(\hat{Y}^{(2)}) - \frac{\alpha}{A-1} \sum_i \sum_j R_{i.}^{(2)} \kappa_{ij}^{(1)} \sum_a \frac{1}{N_{a.}} \sum_{h=1}^A$$

$$[N_{ha.} N_{haj} - \rho_{aj} N_{ha.}^2] + \frac{\alpha}{A-1} \sum_i \sum_j R_{i.}^{(2)} \frac{1}{N_{.j}^{(1)}} \sum_{h=1}^A$$

$$[(N_{h.j} - \sum_a \rho_{aj} N_{ha.}) (N_{hi.} - \rho_{ij} N_{hi.}) - \kappa_{ij}^{(1)}]$$

$$(N_{h.j} - \sum_a \rho_{aj} N_{ha.})] + \frac{\alpha}{A-1} \sum_i \frac{1}{N_{i.}^{(2)}} \sum_{h=1}^A \sum_j [\{\sum_i \kappa_{ij}^{(1)}]$$

$$(N_{h.j} - \sum_a \rho_{aj} N_{ha.}) \{ (Y_{hi.} - R_{i.} N_{hi.}) - \sum_j \frac{Y_{ij}^{(1)}}{N_{.j}^{(1)}} \}$$

$$(N_{h.j} - \sum_a \rho_{aj} N_{ha.}) + R_{i.}^{(2)} \sum_j \kappa_{ij}^{(1)} (N_{h.j} - \sum_a \rho_{aj} N_{ha.}) \} \} \quad (2.30)$$

$$\begin{aligned} V(\hat{Y}^{(3)}) &= \frac{\alpha}{A-1} \sum_{h=1}^A [(Y_{h..} - \sum_i R_{i.} N_{hi.}) - \sum_j R_{.j}^{(1)} (N_{h.j} - \sum_i \rho_{ij} N_{hi.}) \\ &+ \sum_i \sum_j R_{i.}^{(2)} \kappa_{ij}^{(1)} (N_{h.j} - \sum_a \rho_{aj} N_{ha.})]^2 \end{aligned} \quad (2.31)$$

$$B(\hat{Y}^{(4)}) = B(\hat{Y}^{(3)}) + \frac{\alpha}{A-1} \sum_i \sum_j R_{.j}^{(3)} \rho_{ij}^{(2)} \sum_{h=1}^A [\sum_a \sum_b \kappa_{ib}^{(1)} \frac{1}{N_{a.}}]$$

$$(N_{ha.} N_{hab} - \rho_{ab} N_{ha.}^2) - \sum_b \frac{1}{N_{.b}^{(1)}} (N_{h.b} - \sum_a \rho_{ab} N_{ha.})$$

$$\{ (N_{hib} - \rho_{ib} N_{hi.}) - \kappa_{ib}^{(1)} (N_{h.b} - \sum_a \rho_{ab} N_{ha.}) \} \}$$

$$- \frac{\alpha}{A-1} \sum_i \sum_j R_{.j}^{(3)} \frac{1}{N_{i.}^{(2)}} \sum_{h=1}^A [\{ \sum_a \kappa_{ib}^{(1)} (N_{h.b} - \sum_a \rho_{ab} N_{ha.}) \}]$$

$$\{ (N_{hij} - \rho_{ij} N_{hi.}) - \kappa_{ij}^{(1)} (N_{h.j} - \sum_a \rho_{aj} N_{ha.}) + \rho_{ij}^{(2)} \sum_b \kappa_{ib}^{(1)}$$

$$(N_{hib} - \sum_a \rho_{ab} N_{ha.}) \} \} - \frac{\alpha}{A-1} \sum_j \frac{1}{N_{.j}^{(3)}} \sum_{h=1}^A [\{ \sum_i \sum_b \rho_{ij}^{(2)} \kappa_{ib}^{(1)}$$

$$(N_{h.b} - \sum_a \rho_{ab} N_{ha.}) \} \{ (Y_{h.j} - \sum_a \frac{Y_{aj}}{N_{a.}} N_{ha.})$$

$$\begin{aligned}
& - R_{.j}^{(1)} (N_{h.j} - \sum_a \rho_{aj} N_{ha.}) + \sum_i \sum_b \frac{Y_{ij}^{(2)}}{N_i^{(2)}} \kappa_{ib}^{(1)} (N_{h.b} - \sum_a \rho_{ab} N_{ha.}) \\
& - R_{.j}^{(3)} \sum_i \sum_b \rho_{ij}^{(2)} \kappa_{ib}^{(1)} (N_{h.b} - \sum_a \rho_{ab} N_{ha.}) \} \quad (2.32)
\end{aligned}$$

$$\begin{aligned}
V(\hat{Y}^{(4)}) &= \frac{\alpha}{A-1} \sum_{h=1}^A [(Y_{h..} - \sum_i R_i N_{hi.}) - \sum_j \hat{R}_{.j}^{(1)} (N_{h.j} - \sum_i \rho_{ij} N_{hi.}) \\
& + \sum_i \sum_j R_i^{(2)} \kappa_{ij}^{(1)} (N_{h.j} - \sum_a \rho_{aj} N_{ha.}) \\
& - \sum_i \sum_j \sum_b R_{.j}^{(3)} \rho_{ij}^{(2)} \kappa_{ib}^{(1)} (N_{h.b} - \sum_a \rho_{ab} N_{ha.})]^2 \quad (2.33)
\end{aligned}$$

Estimators for the above formulae can be derived like those for the second iteration were. When the clusters are of size one (i.e. s.r.s.) it can be shown that the above formulae for the bias and variance are approximately equal to the formulae under s.r.s. given in section 2.2. The formulae are only approximately equal because a conditional approach was used in section 2.2 while an unconditional approach is used in this section.

CHAPTER 3
ASYMPTOTIC VARIANCE OF THE MAXIMUM LIKELIHOOD ESTIMATOR
FOR A 2B-ITEM

3.1 Introduction

Suppose there are t 2B-categories and let

$$\begin{aligned} N_{ijk} & \quad i = 1, 2, \dots, r \\ & \quad j = 1, 2, \dots, s \\ & \quad k = 1, 2, \dots, t \end{aligned}$$

be the number of individuals in the population that belong to the k th 2B-category and fall in the (i,j) th cell of the 2A-category

cross-classification table. Define $\pi_{ijk} = \frac{N_{ijk}}{N}$ so that

$$\begin{aligned} \sum_j \sum_k \pi_{ijk} &= \pi_{i..} & i &= 1, \dots, r-1 \\ \sum_i \sum_k \pi_{ijk} &= \pi_{.j.} & j &= 1, \dots, s-1 \\ \sum_i \sum_j \sum_k \pi_{ijk} &= 1 \end{aligned} \quad (3.1)$$

where $\pi_{i..} = \frac{N_{i..}}{N}$ and $\pi_{.j.} = \frac{N_{.j.}}{N}$ are the known marginal proportions.

In this chapter the asymptotic variance - covariance (V-C) matrix of the m.l.e. π_{ijk}^* of π_{ijk} is found subject to the constraints (3.1). This leads to the asymptotic variance of $\pi_{..k}^*$:

$$V(\pi_{..k}^*) = \sum_i \sum_j \sum_{i'j'} C(\pi_{ijk}^*, \pi_{i'j'k}^*). \quad (3.2)$$

Since the m.l.e. $\pi_{\dots k}^*$ are asymptotically most efficient, we can compute, using (3.2), the loss in efficiency of the RRE relative to the m.l.e.

3.2 Derivation

Let us assume that we have taken a s.r.s.w.r. Then the likelihood of the sample frequencies n_{ijk} 's is given by

$$L \propto \prod_{i,j,k} (\pi_{ijk})^{n_{ijk}}. \quad (3.3)$$

We maximize

$$\ln L = \text{constant} + \sum_i \sum_j \sum_k n_{ijk} \ln \pi_{ijk} \quad (3.4)$$

subject to the constraints (3.1) to find the m.l.e. π_{ijk}^* . To do this we would have to solve a system of nonlinear equations iteratively.

Silvey [14] has given a general method for finding the asymptotic V-C matrix of the m.l.e. Let

$$B(\pi) = \frac{1}{n} (E \left(- \frac{\partial^2 \ln L}{(\partial \pi_{ijk})(\partial \pi_{i'j'k'})} \right)) \quad (3.5)$$

denote the $(rst) \times (rst)$ information matrix. Let $H(\pi)$ be the $(rst) \times (r + s - 1)$ matrix of derivatives of (3.1) with respect to the π_{ijk} 's. The derivatives are in the order that the π_{ijk} 's fall in π where

$$\pi = (\pi'_{1..} | \pi'_{2..} | \dots | \pi'_{r-1..} | \pi'_{..s} | \pi'_{r..} | \pi'_{rs..}) \quad (3.6)$$

$$\pi'_{i..} = (\pi'_{i1..} | \pi'_{i2..} | \dots | \pi'_{i,s-1..}) \quad (3.7)$$

$$\pi'_{..j} = (\pi'_{1j..} | \pi'_{2j..} | \dots | \pi'_{r-1,j..}) \quad (3.8)$$

$$\text{and } \pi'_{ij.} = (\pi_{ij1}, \pi_{ij2}, \dots, \pi_{ijt}) \quad (3.9)$$

Then the asymptotic V-C matrix of the π^*_{ijk} is given by

$\frac{1}{n} D(\pi)$ where

$$\begin{bmatrix} B(\pi) & H(\pi) \\ H'(\pi) & 0_{(r+s-1), (r+s-1)} \end{bmatrix}^{-1} = \begin{bmatrix} D(\pi) & Q(\pi) \\ Q'(\pi) & R(\pi) \end{bmatrix} \quad (3.10)$$

and $0_{(r+s-1), (r+s-1)}$ is a matrix of zeroes of order $(r+s-1) \times (r+s-1)$.

Using the formula for the inverse of a partitioned matrix we find

$$\begin{aligned} D(\pi) &= B^{-1}(\pi) - B^{-1}(\pi) H(\pi) [H'(\pi) B^{-1}(\pi) H(\pi)]^{-1} H'(\pi) B^{-1}(\pi) \\ &= B^{-1}(\pi) - A F^{-1} A' \end{aligned} \quad (3.11)$$

where $A = B^{-1}(\pi) H(\pi)$ and $F = H'(\pi) B^{-1}(\pi) H(\pi)$. A method

to calculate the inverse of F , an $(r+s-1) \times (r+s-1)$ order matrix, by inverting only a $s-1 \times s-1$ order matrix will be given (without loss of generality we assume $s = \min(r, s)$).

Let us define the following notation:

$$\text{diag } (\delta') = \text{diag } (\delta_1, \delta_2, \dots, \delta_p) \quad (3.12)$$

where $\delta' = (\delta_1, \dots, \delta_p)$ and

$$\text{diag } (\delta_1 | \delta_2 | \dots | \delta_m)$$

$$= \left[\begin{array}{c|c|c|c|c|c|c} \text{m columns} & & & & & & \\ \hline \delta_1 & 0_{-p} & 0_{-p} & \dots & 0_{-p} & 0_{-p} & 0_{-p} \\ \hline 0_{-p} & \delta_2 & 0_{-p} & \dots & & 0_{-p} & 0_{-p} \\ \hline 0_{-p} & 0_{-p} & & & & 0_{-p} & 0_{-p} \\ \hline \vdots & \vdots & & & & \vdots & \vdots \\ \hline 0_{-p} & 0_{-p} & & & & 0_{-p} & 0_{-p} \\ \hline 0_{-p} & 0_{-p} & 0_{-p} & \dots & 0_{-p} & \delta_{m-1} & 0_{-p} \\ \hline 0_{-p} & 0_{-p} & 0_{-p} & \dots & 0_{-p} & 0_{-p} & \delta_m \end{array} \right] \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} m \\ \text{submatrices} \end{array} \quad (3.13)$$

where $\delta'_i = (\delta_{1i}, \delta_{2i}, \dots, \delta_{pi})$. In the above 0_{-p} is a vector of

zeroes of order $p \times 1$. Let

$$E_m^p = \text{diag } (\delta_1 | \delta_2 | \dots | \delta_m) \quad (3.14)$$

where $\delta_1 = \frac{1}{-p}$ and $\frac{1}{-p}$ is a vector of ones of order $p \times 1$.

Also let

$$\left(\begin{matrix} E \\ \vdots \\ E \\ \vdots \\ E \end{matrix} \right)_m^p = \left\{ \begin{matrix} E_m^p \\ E_m^p \\ \vdots \\ E_m^p \end{matrix} \right\} \quad q \text{ submatrices.} \quad (3.15)$$

Finally define

$$\left(\begin{matrix} e \\ \vdots \\ e \end{matrix} \right)_m^p = \left\{ \begin{matrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{matrix} \right\} \quad m \text{ rows} \quad (3.16)$$

where the one is in the p th row.

Using this notation, we can write

$$H(\pi) = \begin{bmatrix} E_{r-1}^{t(s-1)} & (E_{s-1}^t)_{r-1} & l_{t(s-1)(r-1)} \\ E_{r-1}^t & 0_{t(r-1),s-1} & l_{t(r-1)} \\ 0_{t(s-1),r-1} & E_{s-1}^t & l_{t(s-1)} \\ 0_{t,r-1} & 0_{t,s-1} & l_t \end{bmatrix} \quad (3.17)$$

Also

$$-\frac{\partial^2 \ln L}{(\partial \pi_{ijk})(\partial \pi_{i'j'k'})} = \begin{cases} \frac{n_{ijk}}{\pi_{ijk}^2} & \text{if } i = i' \\ & j = j' \\ & k = k' \\ 0 & \text{otherwise} \end{cases} \quad (3.18)$$

so that

$$E \left[-\frac{\partial^2 \ln L}{(\partial \pi_{ijk})(\partial \pi_{i'j'k'})} \right] = \begin{cases} \frac{n}{\pi_{ijk}} & \text{if } i = i' \\ & j = j' \\ & k = k' \\ 0 & \text{otherwise.} \end{cases} \quad (3.19)$$

Therefore

$$B(\pi) = \text{diag}(\alpha') \quad (3.20)$$

where $\alpha_{ijk} = \frac{1}{\pi_{ijk}}$ and α is the vector of the rst elements

α_{ijk} , arranged in the same order as in π . From the above it

$$\text{follows that } B^{-1}(\pi) = \text{diag}(\pi') \quad (3.21)$$

and $A =$

$\text{diag}(\pi_{1..} \pi_{2..} \dots \pi_{r-1, \dots})$ $t(s-1)$	$\text{diag}(\pi_{11.} \pi_{12.} \dots \pi_{1, s-1, \dots})$ t	$\pi_{1..}$
	$\text{diag}(\pi_{21.} \pi_{22.} \dots \pi_{2, s-1, \dots})$ t	$\pi_{2..}$
	$\vdots$	
	$\text{diag}(\pi_{r-1, 1.} \pi_{r-1, 2.} \dots \pi_{r-1, s-1, \dots})$ t	$\pi_{r-1, \dots}$
$\text{diag}(\pi_{1s.} \pi_{2s.} \dots \pi_{r-1, s.})$ t	$0_{t(r-1), s-1}$	$\pi_{..s.}$
$0_{t(s-1); r-1}$	$\text{diag}(\pi_{r1.} \pi_{r2.} \dots \pi_{r, s-1, \dots})$ t	$\pi_{r..}$
$0_{t, r-1}$	$0_{t, s-1}$	$\pi_{rs.}$

(3.22)

From (3.17) and (3.22) we get

$$F = \begin{bmatrix} F_{11} & f_{12} \\ f'_{12} & 1 \end{bmatrix} \quad (3.23)$$

where

$$f'_{12} = (\pi_{1..}, \pi_{2..}, \dots, \pi_{r-1..}, \pi_{.1.}, \pi_{.2.}, \dots, \pi_{.,s-1.}) \quad (3.24)$$

$$E_{11} = \begin{bmatrix} E_1 & E_2 \\ E'_2 & E_3 \end{bmatrix} \quad (3.25)$$

$$E_1 = \text{diag} (\pi_{1..}, \pi_{2..}, \dots, \pi_{r-1..}) \quad (3.26)$$

$$E_3 = \text{diag} (\pi_{.1.}, \pi_{.2.}, \dots, \pi_{.,s-1.}) \quad (3.27)$$

$$E_2 = \begin{bmatrix} \pi_{11.} & \pi_{12.} & \dots & \pi_{1,s-1,.} \\ \pi_{21.} & \pi_{22.} & \dots & \pi_{2,s-1,.} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{r-1,1.} & \pi_{r-1,2.} & \dots & \pi_{r-1,s-1,.} \end{bmatrix} \quad (3.28)$$

$$\pi_{ij.} = \sum_k \pi_{ijk}$$

$$\pi_{i..} = \sum_j \sum_k \pi_{ijk} \quad (3.29)$$

$$\text{and } \pi_{.j.} = \sum_i \sum_k \pi_{ijk}$$

The inversion of F can now be reduced to the inversion of a $(s-1) \times (s-1)$ matrix by using repeatedly the formula for the

inverse of a partitioned matrix. Let

$$\tilde{F}^{-1} = \begin{bmatrix} \tilde{F}^{11} & \tilde{f}^{12} \\ \tilde{f}^{12'} & \tilde{f}^{22} \end{bmatrix} \quad (3.30)$$

where

$$\tilde{f}^{22} = (1 - \tilde{f}'_{12} \tilde{F}^{-1}_{11} \tilde{f}_{12})^{-1} \quad (3.31)$$

$$\tilde{f}^{12} = -(\tilde{F}^{-1}_{11} \tilde{f}_{12}) \tilde{f}^{22} \quad (3.32)$$

$$\tilde{F}^{11} = \tilde{F}^{-1}_{11} [I + (\tilde{f}_{12} \tilde{f}'_{12}) \tilde{F}^{-1}_{11} \tilde{f}^{22}] \quad (3.33)$$

and

$$\tilde{F}^{-1}_{11} = \begin{bmatrix} \tilde{E}^1 & \tilde{E}^2 \\ (\tilde{E}^2)' & \tilde{E}^3 \end{bmatrix} \quad (3.34)$$

$$\tilde{E}^3 = (\tilde{E}_3 - \tilde{E}'_2 \tilde{E}^{-1}_1 \tilde{E}_2)^{-1} \quad (3.35)$$

$$\tilde{E}^{-1}_1 = \text{diag} \left(\frac{1}{\pi_{1..}}, \frac{1}{\pi_{2..}}, \dots, \frac{1}{\pi_{r-1..}} \right) \quad (3.36)$$

$$\tilde{E}^2 = -\tilde{E}^{-1}_1 \tilde{E}_2 \tilde{E}^3 \quad (3.37)$$

$$\tilde{E}^1 = \tilde{E}^{-1}_1 [I + \tilde{E}_2 \tilde{E}^3 \tilde{E}'_2 \tilde{E}^{-1}_1] \quad (3.38)$$

Thus we see that the inverse of $\tilde{F}$ can be found by calculating the inverse of $\tilde{E}_3 - \tilde{E}'_2 \tilde{E}^{-1}_1 \tilde{E}_2$ which is a $(s-1) \times (s-1)$ matrix.

Therefore the asymptotic V-C matrix $\frac{D(\pi)}{n}$ of the m.l.e.

π_{ijk}^* is obtained by substituting (3.21), (3.22), and (3.30) in

(3.11).

Finally we find explicitly the expression for the asymptotic variance of $\pi^*_{..k}$. Defining

$$c' = \underbrace{[(e_t^k)' | (e_t^k)' | \dots | (e_t^k)']}_{rs \text{ submatrices}} \quad (3.39)$$

we can write (3.2) as

$$\begin{aligned} V(\pi^*_{..k}) &= \frac{c' (\text{diag}(\pi') - AF^{-1}A') c}{n} \\ &= \frac{1}{n} (\pi_{..k} - a' F^{-1} a) \end{aligned} \quad (3.40)$$

$$\text{where } a' = (\pi_{1.k}, \pi_{2.k}, \dots, \pi_{r-1,k}, \pi_{1.k}, \pi_{2.k}, \dots, \pi_{s-1,k}, \pi_{..k}) \quad (3.41)$$

This final result is important because it demonstrates that we only have to calculate F^{-1} rather than the C-V matrix $\frac{D(\pi)}{n}$

to find the asymptotic variance of the m.l.e. for a 2B-item. This is a considerable computational savings. It also demonstrates that, as would be expected, the value of the asymptotic variance does not depend on t , the number of 2B-items.

CHAPTER 4

Empirical Study of the Bias and Variance Under Simple Random Sampling

4.1 Categories Examined and Notation

To get a better idea of the properties of the variance and bias of the RRE under s.r.s., we have made an empirical study using the data from the 1974 test census. An electoral district (ED) with 15 WA's was chosen for this purpose. The 2A-items in the test census for which information was collected were: relation to head of household, sex, age, marital status, mother tongue, type of dwelling, and tenure (owned or rented). The 2B-item values collected were: school attendance, years of schooling, post-secondary education, academic qualifications, labour force status, and address five years ago.

For each WA the 2A information was cross-classified in the household table given in Appendix I by Table 1. A strategy which involved collapsing together rows and/or columns was adopted to ensure that certain requirements involving $N_{i.}$, $n_{i.}$, $N_{.j}$ and $n_{.j}$ were met (e.g. $n_{i.}$ and $n_{.j}$ were greater than 0). After collapsing, the largest household cross-classification (weighting) table for a WA had 18 rows and 4 columns while the smallest one had 7 rows and 3 columns (compared to 20 rows and 4 columns in the original table).

The RREP can begin with either the rows or columns of the household table. In the theory given in Chapter 2, the weights

were such that $\hat{N}_{.j}^{(4)} = N_{.j}$. For the household table used in

this study, it was considered more important that $\hat{N}_{i.}^{(4)} = N_{i.}$.

Thus the RREP started with the columns (i.e. $\hat{N}_{.j}^{(1)} = N_{.j}$).

Under the assumption of s.r.s. we can deal only with 2B-items pertaining to households. Thus the categories considered in this study are related to tenure, type of dwelling and characteristics of the head of the household. Since there are many possible categories, a representative cross-section of 15 categories was selected for the empirical study. The categories picked fall into 3 basic classes.

- A. Categories defined by 2B-items. Only those 15 years or older were counted in these categories.
- B. Categories defined by 2A-items that are not directly related to any row or column of the weighting table.
- C. Categories defined by 2A-items that correspond directly to some rows or columns of the weighting table.

Though the main interest is in the 2B-categories, the 2A-categories are also included for two reasons. First, they give examples of categories that are correlated in some fashion with the rows or columns of the weighting table. Secondly, in cross-tabulations of 2B-categories against 2A-categories, estimates of the 2A-categories appear. Unless these 2A-categories correspond to some row or column of the collapsed weighting table, there is no guarantee that the estimates will equal the known population

values. Thus the variance of these estimators of 2A-categories is of some interest.

The actual categories discussed at the WA level are:

- Class A (2B-Items):
- A1 Households with Employed Heads
 - A2 Households with Unemployed Heads
 - A3 Households with Heads Not in Labour Force
 - A4 Households with Heads Not Moved in
5 Years
 - A5 Households with Heads Moved in Last
5 Years in Same Municipality
 - A6 Highest Grade of Head is 1 to 10
 - A7 Heads with Bachelor Degree or Higher
- Class B (2A-Items):
- B1 Households with 3 or 4 Persons
 - B2 Age of Head is Less Than 25
 - B3 Age of Head is 25 to 34
 - B4 Heads Who are Widowed, Divorced, or
Separated
- Class C (2A-Items):
- C1 Households with 2 or Fewer Persons
 - C2 Age of Head is 65 or More
 - C3 Owned Dwellings
 - C4 Rented Apartments.

The raking weights were calculated at the WA level to provide estimates for the WA. However, the WA level weights were also used to provide estimates at the EA level for the 3 EA's from WA 2. This was done for the categories A1, A4, A6, B1 and B3.

In Appendix II the estimates at the WA level are given in Tables 1 to 15 for the 15 categories listed above and for five WA's arranged from the largest (WA 8) to the smallest (WA 12). Results for the remaining WA's are similar.

Tables 1 to 15 list for each iteration (p = 0, 1, 2, 3, 4) estimates of the population total ($\hat{Y}^{(p)}$), the standard error (SE_p), the coefficient of variation (CV_p), and the "ratio of errors" (RE_p) which is defined as SE_p expressed as a percentage of SE_0 . Estimates at the ED level are also given which serve as a summary. Also a measure of convergence of the $\hat{Y}^{(p)}$ between iterations is given in these tables. It is defined as

$$C(k-p) = \sqrt{\frac{\sum_{m=1}^{15} (\hat{Y}_m^{(p)} - \hat{Y}_m^{(k)})^2}{\sum_{m=1}^{15} (\hat{Y}_m^{(k)})^2}} \times 100\% \quad (4.1)$$

where $\hat{Y}_m^{(p)}$ is the population estimate at the pth iteration for weighting area m. $C(k-p)$ can be regarded as the average of the percentage change in the population estimates between the kth and pth iteration. Similar results at the EA level for the five chosen categories are given by Tables 16 to 20 in Appendix II.

We also use another measure of convergence denoted by

$$M_p = \left[\sqrt{\frac{\sum_i (\hat{N}_{i.}^{(p)} - N_{i.})^2}{\sum_i N_{i.}^2}} \times 100\% \text{ for } p = 1, 3 \right. \\ \left. \sqrt{\frac{\sum_j (\hat{N}_{.j}^{(p)} - N_{.j})^2}{\sum_j N_{.j}^2}} \times 100\% \text{ for } p = 2, 4 \right] \quad (4.2)$$

It is clear from (4.2) that M_p measure the agreement between $\hat{N}_{i.}^{(p)}$ and $N_{i.}$ for $p = 1, 3$ and between $\hat{N}_{.j}^{(p)}$ and $N_{.j}$ for $p = 2, 4$.

In Chapter 3 the asymptotic variance of the m.l.e., $\pi_{..k}^*$ of $\pi_{..k}$ was derived under s.r.s.w.r. To allow efficiency comparisons with the RRE, the large-sample variances of the RRE without the f.p.c. are used. Two estimates of the asymptotic standard error of $N\pi_{..k}^*$ are calculated. One estimator SE_0^* uses zero iteration raking ratio estimates $\hat{\pi}_{ijk}^{(0)} = \frac{w_{ij}^{(0)} n_{ijk}}{N}$ for the π_{ijk} 's in (3.40). The other estimator SE_4^* uses fourth iteration raking ratio estimates of the π_{ijk} 's. SE_u^* ($u = 0, 4$) expressed as a percentage of SE_p (the estimated p th iteration RRE standard error under s.r.s.w.r.) will be called RE_p^u . Tables 21 to 25 in Appendix II give SE_p , RE_p^0 , RE_p^4 ($p = 0, 1, 2, 3, 4$), SE_0^* , and SE_4^* for the categories A1, A4,

A6, B1, and C4, in five WA's. Results at the ED level are also given. Tables for the other 10 categories were calculated, but are not given in Appendix II. because the results are similar.

4.2 Analysis of the Results

Bias

For the 15 categories of interest and three representative WA's, the estimated bias of the RRE as a percentage of the population estimate never exceeds .24% in absolute value at the WA level. At the EA level, this ratio never exceeds .39%. These upper limits are small compared to the corresponding coefficients of variation. Thus the bias of the RRE may be considered as negligible even at the EA level.

Standard Error

The standard errors for categories at the ED level (obtained from the standard errors at the WA level) are examined initially. Looking at the A class categories first, it is clear from Figure 4.1 that categories which have larger population estimates tend to have greater gains in efficiency. This seems reasonable in that categories which make up a substantial portion of the elements in certain rows or columns of the household table should have large gains in efficiency. Some small categories however (e.g. A3) can have most of the households in a few rows or columns belong to them and thus have large reductions in variance result.

Fig. 4.1. The relationship between RE_4 and $\hat{Y}^{(4)}$ for the A class categories under s.r.s. The points in the figure are labelled to indicate the category they correspond to.

With the exception of the migration categories A4 and A5, the greatest gains in efficiency are realized at the second iteration rather than the first. This indicates there is a correlation between the A class categories and the rows of the weighting table. In contrast, there is a considerable reduction in the variance at both the first and second iteration for the migration

categories. This implies a correlation of these categories with both the rows and columns of the weighting table. For all the A class categories, the difference in relative efficiencies $RE_2 - RE_4$ lies between 0% and 1.1% and the measure of convergence $C(2-4)$ does not exceed .41%. This indicates that the gain in efficiency of the fourth iteration RRE relative to the second iteration RRE is very small and that the population estimates are stable. It might therefore be concluded that there is not much benefit going beyond the second iteration. However, Figure 4.2 demonstrates that the agreement between $\hat{N}_{.j}^{(p)}$ and $N_{.j}$ is much closer for $p = 4$ than for $p = 2$. This close agreement is important in the census situation as mentioned in Chapter 1. For this reason, use of the fourth iteration estimates seems advisable.

Fig. 4.2 Measure of how closely marginal constraints are satisfied for three WA's (See eq. (4.2) for definition of M_p).

WA	M_1	M_2	M_3	M_4
8	9.75	2.17	1.02	.54
11	6.82	1.18	.46	.15
12	7.17	1.86	.37	.12

For the B class categories, almost all the gains in efficiency are made at the second iteration. Also the population estimates remain stable after the second iteration. These results are not

surprising since there are obvious relationships between most of the B class categories and certain rows of the weighting table.

Examining the C class categories, there should be zero variances for C3 and C4 at the first and third iterations (since the estimates agree with the known population values for all samples) if no collapsing of the weighting table has taken place. Category C3 estimators do in fact have zero variances at these iterations since columns 1 and 2 of the weighting table are never collapsed. For category C4 in WA 12, columns 3 and 4 of the household table are collapsed together and instead of a zero variance we get $RE_3 = 96.0\%$. This shows the important effect that collapsing can have on gains in efficiency.

Next the efficiency of the RRE is compared to the m.l.e. Since the 0 iteration estimator of π_{ijk} is unbiased, it is felt that SE_0^* might be a better estimator of the asymptotic standard error of the m.l.e. $N\pi^* \dots k$ than SE_4^* ; Thus we will use RE_p^0 rather than RE_p^4 in this discussion. RE_p^0 never exceeds 100%. For categories whose estimators for all iterations have nonzero variances, $RE_4^0 \geq 99.8\%$ except for C4 which instead has $RE_3^0 = 99.9\%$. These results are strictly valid for large n and s.r.s.w.r. but they do indicate that the RRE is likely to be very efficient under s.r.s.w.o.r.

Finally, we examine the five categories at the EA level.

Since the WA level weights are used to produce these estimates, the gains in efficiency at the EA level are not as large as those at the WA level.

CHAPTER 5

Empirical Study of the Bias and Variance Under Cluster Sampling

5.1 Categories Examined

In this chapter we apply the cluster sampling formulae derived in Chapter 2 to categories which deal with characteristics of the individual rather than those of the household. The notation in this chapter is identical to that used in Chapter 4. Also the same ED is used for the empirical study. The persons for each WA are cross-classified according to the population table (Table 2) given in Appendix I. The population table has 34 rows and 30 columns. After collapsing, however, the largest table has 28 rows and 26 columns while the smallest has 21 rows and 14 columns. As with the household table in Chapter 4, the RREP starts with the columns of the population table so that

$$\hat{N}_{i.}^{(4)} = N_{i.}$$

Results for fifteen categories at the WA level are given here. These are split into three classes A, B, and C which are defined in Chapter 4. The categories are:

- Class A (2B-Items):
- A1 Employed
 - A2 Unemployed
 - A3 Not in the Labour Force
 - A4 Not Moved in Last 5 Years
 - A5 Moved in Last 5 Years in Same Municipality
 - A6 Highest Grade is 1 to 8

- A7 Highest Grade is 9 to 10
- A8 Attending School Full Time
- A9 Bachelor Degree or Higher

Class B (2A-Items): B1 Widowed

B2 Mother Tongue German

B3 Mother Tongue Italian

Class C (2A-Items): C1 Single. (Never Married)

C2 Mother Tongue English

C3 Mother Tongue French.

Tables 1 to 15 in Appendix III give the results for the 15 categories listed above. Estimates are also calculated at the EA level using WA level weights for the categories A1, A4, A6, B2, and C1. The results for these 5 categories are presented in Tables 16 to 20 of Appendix III.

5.2 Analysis of the Results

Bias

The bias is calculated only up to the second iteration for one average sized WA because of high computational costs. For the 15 categories considered at the WA level, the estimated bias of the RRE as a percentage of the population estimate never exceeds .13% in absolute value. Thus the bias of the RRE may be considered as negligible at least for the first two iterations.

Standard Error

Standard errors at the ED level are examined initially.

Turning first to the A class categories, Figure 5.1 demonstrates a tendency for categories with larger population estimates to have greater gains in efficiency.

Fig. 5.1 The relationship between RE_4 and $\hat{Y}^{(4)}$ for the A class categories under cluster sampling. The points in the figure are labelled to indicate the category they correspond to.

For all A class categories there are important gains in efficiency at both the first and second iteration. This implies that the A categories are correlated with both the rows and columns of the population table. This dual correlation is probably the result of a similarity between the variables of the rows and columns. All this is in contrast to the results of Chapter 4 where, with the exception of the migration categories, most large gains in efficiency were realized at the second iteration. It can be seen that after the second iteration there are almost no further gains in efficiency and that the population estimates change little. But as mentioned in the last chapter, it is advisable to use fourth iteration estimators because of the closer agreement between $\tilde{N}_{.j}^{(p)}$ and $N_{.j}$ for $p = 4$ than for $p = 2$.

Looking next at the B categories, it can be seen again that those categories with the larger population estimates lead to greater gains in efficiency. For the category B1, the gain in efficiency is substantial at both first and second iterations because the rows, and to a certain extent the columns, of the population table are associated with marital status. Moreover, after the second iteration small additional gains in efficiency are realized, unlike the A class categories. The major gains in efficiency for the categories B2 and B3 are realized at the first iteration because the columns of the population table classify individuals by mother tongue.

Examining the C class categories, we know that $SE_2 = SE_4 = 0$ for C1, and $SE_1 = SE_3 = 0$ for C2 and C3 before collapsing. For most WA's, collapsing of rows or columns together causes the variances of the estimators to be nonzero. However, large gains in efficiency are still made for all three categories.

For the EA categories, the gains in efficiency are not as large as for the corresponding categories at the WA level.

5.3 The Design Effect

On a large scale in a census situation, the standard errors of the RRE's under cluster sampling are probably too complex to compute. If a sample of v WA's is randomly picked, then we can calculate, for the u th WA in the sample, the "design effect" D_P^u which is defined as the ratio of the estimated standard error under cluster sampling for $\hat{Y}^{(p)}$ to the estimated standard error (SE_0^s) under s.r.s. for $\hat{Y}^{(0)}$. For estimators of counts of individuals that belong to a particular category, the formula for SE_0^s reduces to a computationally simple form. Thus for any WA outside the sample of WA's, an estimate of the standard error of $\hat{Y}^{(p)}$ under cluster sampling can be calculated by multiplying the SE_0^s for that WA by

$$C_P = \frac{1}{v} \sum_{u=1}^v D_P^u \quad (5.1)$$

where C_p is the average of the design effects D_p^u for the sample of WA's.

Design effects D_p^u for the six categories A1, A4, A6, A9, B2, and C1 in an average size WA are given in Table 21 of Appendix III. It should be noted that for the categories A4 and B2, D_0^u is substantially greater than 1 indicating, as would be expected, a clustering effect for the variables Not Moved in Last 5 Years, and Mother Tongue German. A similar procedure could be applied to household categories to calculate an estimate of the standard error for $Y^{(p)}$ under s.r.s. from SE_0^s .

CHAPTER 6

CONCLUSIONS

The empirical results of Chapters 4 and 5 may be summarized as follows:

- (a) The bias of the RRE is negligible for all practical purposes.
- (b) Not only do larger categories start with smaller coefficients of variation, but they also tend to have greater gains in efficiency.
- (c) After the second iteration, the population estimates change little and the additional gains in efficiency are generally small or negligible. However, assuring close agreement of N_j and its estimated value implies that fourth iteration estimates should be used.
- (d) Care must be taken in the choice of collapsing strategies as it has been shown that substantial losses in efficiency can result from unfavourable collapsing.
- (e) Categories at the EA level using WA level weights tend to have smaller gains in efficiency than the corresponding WA level categories.
- (f) Under s.r.s. the third or fourth iteration RRE is practically as efficient as the m.l.e. This indicates that the RRE are likely to be very efficient under s.r.s.

One drawback to the RREP is that with census data there is no guarantee that the procedure will converge to estimates that

satisfy the constraints (1.1) unless $n_{ij} > 0$ for all i, j .

However in practice these constraints seem to have been satisfied reasonably well by the fourth iteration even when some of the n_{ij} 's are zero.

The next area of research that should be examined is the extension of the large-sample bias and variance formulae to stratified cluster sampling. By taking stratification into account, a more accurate estimate of the variance of the RRE could then be calculated.

APPENDIX I

Cross-Classification Tables

TABLE 1
Cross-Classification Table For
Housing and Household Weights

Household Type	Sex of Head	Age of Head	Number of Persons	Row No.	OWNED		RENTED	
					Single Detached	Other	Apartment	Other
					Column 1	Column 2	Column 3	Column 4
ONE OR MORE FAMILY HOUSEHOLDS	MALE	15-34	LE 2	1				
			GT 2	2				
		35-64	LE 2	3				
			GT 2	4				
		GE 65	LE 2	5				
			GT 2	6				
	FEMALE	15-34	LE 2	7				
			GT 2	8				
		35-64	LE 2	9				
			GT 2	10				
		GE 65	LE 2	11				
			GT 2	12				
ONE PERSON NON-FAMILY HOUSEHOLDS	MALE	15-34		13				
		35-64		14				
		GE 65		15				
	FEMALE	15-34		16				
		35-64		17				
		GE 65		18				
TWO-OR-MORE PERSON NON-FAMILY HOUSEHOLDS	MALE			19				
	FEMALE			20				

TABLE 2. SEXES- CLASSIFICATION FOR ANALYSIS BY RACE AND MARITAL STATUS	FAMILY MEMBERS										NON-FAMILY MEMBERS																			
	HEAD IN A ONE SPINSE IN A MARRIAGE					CHILDREN IN THE FAMILIES					HEAD OF HOUSEHOLD					RELATED MEMBER OF HOUSEHOLD					UNRELATED MEMBER OF HOUSEHOLD									
	WITHIN FAMILY		MOTHER TONGUE		OTHER		MOTHER TONGUE		OTHER		MOTHER TONGUE		OTHER		MOTHER TONGUE		OTHER		MOTHER TONGUE		OTHER		MOTHER TONGUE		OTHER		MOTHER TONGUE		OTHER	
	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE	MOTHER TONGUE
AGE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
SEX	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
EVER MARRIED	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
NEVER MARRIED	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
MALE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
FEMALE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

APPENDIX II

Tables of Results for Categories Under S.R.S.

Tables 1 to 15 give results for the RRE of 15 categories under s.r.s.w.o.r. at the WA level. For each iteration, the estimated population total, standard error, coefficient of variation, and "ratio of errors" are given by columns labelled Pop. Est., S.E., C.V., and R.E. respectively. These estimates are given for 5 WA's and also at the ED level. Tables 16 to 20 present similar results at the EA level for 5 categories and the three EA's from WA.2. Tables 21 to 25 give a measure of the efficiency of the RRE versus the m.l.e. under s.r.s.w.r. for 6 categories. For each iteration SE_p , RE_p^0 , RE_p^4 , SE_0^* , and SE_4^* are given at both the WA and ED level.

CATEGORICAL AT WA LEVEL UNDER SIMPLE RANDOM SAMPLING

TABLE 1

A1

HOUSEHOLDS WITH EMPLOYED HEADS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
0.27 1.46 0.16 0.14 1.40 0.03

UA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.
8	1,209.1	30.5	0.025	1,207.6	30.4	0.025	1,191.3	22.8	0.019	1,191.4	22.8	0.019	1,191.1	22.8	0.019
9	1,141.4	24.8	0.022	1,142.2	24.7	0.022	1,140.4	19.4	0.017	1,141.0	19.4	0.017	1,140.6	19.4	0.017
11	1,111.0	18.0	0.016	1,111.5	18.0	0.016	1,107.5	15.1	0.014	1,107.5	15.1	0.014	1,107.8	15.1	0.014
12	920.0	18.1	0.020	918.7	18.2	0.020	921.5	13.9	0.015	922.1	13.9	0.015	922.2	13.9	0.015
ED	5,575.7	11.8	0.022	5,455.6	11.8	0.022	5,375.5	8.9	0.018	5,381.1	9.9	0.019	5,377.7	9.9	0.019
ED	13,873.7	84.0	0.006	13,878.3	83.6	0.006	13,844.3	65.7	0.005	13,852.6	65.8	0.005	13,845.9	65.7	0.005

TABLE 2

A2

HOUSEHOLDS WITH UNEMPLOYED HEADS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.71 2.41 0.54 0.26 2.58 0.41

UA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.
8	75.2	12.1	0.160	76.3	12.3	0.161	76.2	12.1	0.159	76.0	12.1	0.159	76.2	12.1	0.159
9	57.2	10.5	0.184	56.9	10.5	0.185	57.2	10.4	0.183	57.2	10.4	0.183	57.2	10.4	0.183
11	17.7	5.8	0.331	17.7	5.8	0.330	17.8	5.9	0.331	17.8	5.9	0.331	17.8	5.9	0.331
12	18.1	6.0	0.331	17.9	5.9	0.331	17.8	5.9	0.331	17.8	5.9	0.331	17.8	5.9	0.331
ED	542.0	32.4	0.068	543.4	32.6	0.066	543.4	32.4	0.059	541.7	32.3	0.060	542.5	32.3	0.059

TABLE 3

A3

HOUSEHOLDS WITH HEADS NOT IN LABOUR FORCE

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
0.71 4.31 0.43 0.37 4.15 0.09

UA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.
8	658.7	24.6	0.045	658.9	24.7	0.045	676.0	21.8	0.032	676.2	21.8	0.032	676.3	21.8	0.032
9	355.4	23.5	0.066	354.8	23.4	0.066	356.3	17.6	0.049	355.5	17.6	0.049	355.9	17.6	0.049
11	177.3	17.3	0.098	176.9	17.3	0.098	181.2	14.2	0.078	181.3	14.2	0.078	181.1	14.2	0.078
12	178.0	17.4	0.098	179.4	17.5	0.098	176.3	13.3	0.075	176.1	13.2	0.075	176.0	13.2	0.075
ED	4,502.5	80.9	0.018	4,496.4	80.6	0.018	4,531.2	61.4	0.014	4,524.2	61.3	0.014	4,530.3	61.3	0.014

TABLE 4

A4

HOUSEHOLDS WITH HEADS NOT MOVED IN 5 YEARS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.63 1.90 0.62 0.27 1.51 0.37

UA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.
8	890.3	31.2	0.035	879.5	26.5	0.030	884.5	24.9	0.028	886.3	24.4	0.028	887.1	24.4	0.027
9	806.1	28.0	0.035	802.5	24.4	0.030	809.6	23.1	0.029	808.3	22.7	0.029	807.9	22.7	0.028
11	650.1	25.3	0.035	652.6	23.8	0.036	674.9	21.8	0.032	670.8	21.8	0.033	672.2	21.8	0.032
12	304.6	21.2	0.069	305.3	19.7	0.065	304.7	18.9	0.062	306.1	18.8	0.061	305.6	18.8	0.061
ED	4,528.1	94.7	0.011	4,488.0	85.5	0.010	4,470.0	80.5	0.009	4,492.7	79.4	0.009	4,482.4	79.4	0.009

CATEGORIES AT VA LEVEL UNDER SIMPLE RANDOM SAMPLING

TABLE 3

A3

HOUSEHOLDS WITH HEADS MOVED IN LAST 5 YEARS IN SAME MUNICIPALITY

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)

1.27 1.82 0.49 0.24 1.50 0.27

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	R.E.
8	571.5	28.5	0.050	577.3	27.1	0.047	569.4	26.7	0.047	568.0	26.5	0.047	569.0	26.5	0.047
9	370.4	23.8	0.064	372.5	23.1	0.062	376.5	22.3	0.059	378.4	22.2	0.059	377.5	22.2	0.059
11	339.8	22.2	0.065	339.8	22.0	0.065	333.7	21.3	0.064	335.3	21.3	0.063	334.7	21.3	0.064
12	310.7	21.3	0.069	310.6	21.4	0.069	312.8	21.1	0.068	312.5	21.1	0.068	312.5	21.1	0.068
ED	94.4	12.5	0.133	93.5	12.5	0.133	94.5	12.2	0.129	94.3	12.2	0.129	94.3	12.2	0.129
ED	4,911.0	84.6	0.017	4,930.9	83.1	0.017	4,934.3	80.9	0.016	4,942.3	80.7	0.016	4,948.6	80.7	0.016

TABLE 6

A6

HIGHEST GRADE OF HEAD IS 1 to 10

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)

0.57 0.92 0.23 0.17 0.75 0.10

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	R.E.
8	1,091.8	31.0	0.028	1,090.2	30.9	0.028	1,087.5	29.8	0.027	1,090.0	29.8	0.027	1,088.4	29.8	0.027
9	632.4	27.5	0.044	630.0	27.4	0.043	634.8	26.7	0.042	634.0	26.7	0.042	634.5	26.7	0.042
11	460.8	24.2	0.052	459.2	24.0	0.052	465.4	23.1	0.050	465.4	23.1	0.050	466.1	23.1	0.050
12	365.0	22.3	0.061	364.6	22.3	0.061	364.1	21.0	0.058	363.9	21.0	0.058	363.6	21.0	0.058
ED	309.6	17.5	0.057	310.2	17.1	0.055	315.2	16.3	0.052	313.8	16.3	0.052	314.1	16.3	0.052
ED	9,346.6	93.9	0.010	9,355.4	93.4	0.010	9,356.8	89.7	0.010	9,362.0	89.7	0.010	9,357.6	89.7	0.010

TABLE 7

A7

HEADS WITH BACHELOR DEGREE OR HIGHER

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)

0.68 2.69 0.31 0.25 2.60 0.30

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	R.E.
8	66.2	11.3	0.171	66.7	11.5	0.172	68.3	11.4	0.166	68.3	11.4	0.166	68.5	11.4	0.166
9	158.6	17.0	0.106	160.1	17.1	0.107	158.9	16.7	0.105	158.8	16.7	0.105	158.7	16.7	0.105
11	263.9	18.4	0.090	263.8	18.4	0.090	263.8	18.2	0.091	263.8	18.2	0.091	263.8	18.2	0.091
12	102.6	13.7	0.134	102.3	13.7	0.134	100.5	13.5	0.135	100.4	13.5	0.135	100.5	13.5	0.135
ED	45.4	8.1	0.228	45.2	8.0	0.227	45.2	7.9	0.228	45.2	7.9	0.228	45.2	7.9	0.228
ED	1,364.3	48.6	0.037	1,366.0	48.6	0.037	1,316.8	47.7	0.036	1,317.2	47.7	0.036	1,318.4	47.7	0.036

TABLE 8

B1

HOUSEHOLDS WITH 3 OR 4 PERSONS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)

0.51 2.76 0.35 0.33 2.52 0.11

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	C.V.	Pop. Est.	S.E.	R.E.
8	577.5	28.6	0.049	577.2	28.4	0.049	577.4	28.4	0.049	577.4	28.4	0.049	577.4	28.4	0.049
9	557.2	26.8	0.048	555.9	26.8	0.048	555.3	26.8	0.048	555.3	26.8	0.048	555.3	26.8	0.048
11	517.1	24.7	0.048	515.0	24.6	0.048	513.4	24.6	0.048	513.4	24.6	0.048	513.4	24.6	0.048
12	476.6	23.5	0.049	475.4	23.6	0.050	473.4	23.6	0.050	473.4	23.6	0.050	473.4	23.6	0.050
ED	2,340.1	74.4	0.062	2,340.1	74.4	0.062	2,340.1	74.4	0.062	2,340.1	74.4	0.062	2,340.1	74.4	0.062
ED	6,983.4	93.4	0.013	6,977.1	92.8	0.013	6,910.7	92.8	0.013	6,910.7	92.8	0.013	6,910.7	92.8	0.013

CATEGORIES AT VA LEVEL UNDER SIMPLE RANDOM SAMPLING

N7

AGE OF HEAD IS LESS THAN 25

TABLE 9

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
3.76 6.13 1.65 1.01 5.12 0.68

VA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.
8	255.7	21.1	0.083	280.4	20.3	0.078	96.3	760.9	17.0	0.065	80.3	257.4	17.0	0.066	80.7	259.9	16.9	0.065	80.2	
9	102.4	13.9	0.136	104.7	13.3	0.127	95.4	105.6	12.4	0.118	89.3	107.0	12.4	0.116	89.3	106.2	12.3	0.115	89.0	
11	94.3	9.2	0.207	95.3	8.6	0.190	94.0	42.1	8.3	0.197	90.5	43.2	8.2	0.130	89.4	42.8	9.2	0.150	83.4	
2	96.5	13.3	0.138	96.5	12.8	0.133	96.1	97.9	11.8	0.120	88.0	97.0	11.8	0.121	88.2	97.4	11.7	0.120	87.9	
12	32.4	7.6	0.240	32.7	7.7	0.235	99.1	30.6	7.3	0.239	94.2	31.1	7.3	0.235	94.1	31.0	7.3	0.235	94.1	
ED	1,351.4	49.9	0.037	1,375.4	48.5	0.035	97.2	1,403.3	43.2	0.031	86.5	1,390.5	43.2	0.031	86.5	1,398.6	43.0	0.031	86.2	

TABLE 10

AGE OF HEAD IS 25 TO 34

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.36 6.86 0.67 0.61 6.60 0.20

	C(0-1)				C(1-3)				C(2-4)				C(3-4)				C(4-5)								
	1.36		6.86		0.47		0.61		6.60		0.20		0.53		68.9		306.9		15.9		0.052		58.0		
8	324.8	23.3	0.072	327.9	23.2	0.071	99.3	306.7	15.9	0.052	68.1	305.2	16.1	0.053	69.9	306.9	15.9	0.052	68.0	15.9	0.052	68.0	15.9	0.052	58.0
9	334.3	23.0	0.069	334.7	22.2	0.066	96.3	326.8	12.4	0.038	54.6	328.4	12.9	0.039	56.0	326.5	12.4	0.038	53.9	12.4	0.038	53.9	12.4	0.038	53.9
11	322.1	21.8	0.068	320.7	21.6	0.067	99.1	278.9	8.3	0.030	38.1	280.4	8.4	0.030	38.8	278.4	8.3	0.030	37.8	8.3	0.030	37.8	8.3	0.030	37.8
12	307.7	21.2	0.069	303.8	20.3	0.067	95.8	306.2	11.3	0.037	53.0	305.6	11.5	0.038	54.3	305.5	11.2	0.037	52.0	11.2	0.037	52.0	11.2	0.037	52.0
ED	4,163.1	80.0	0.019	4,181.8	79.2	0.019	99.0	4,123.3	42.0	0.010	52.5	4,120.8	42.7	0.010	53.4	4,127.0	41.9	0.010	52.4	41.9	0.010	52.4	41.9	0.010	52.4

TABLE 11

HEADS WHO ARE WIDOWED, DIVORCED OR SEPARATED

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.40 4.75 0.64 0.57 4.45 0.12

	C(0-1)				C(1-2)				C(2-3)				C(3-4)				C(2-4)				C(1-3)				C(1-4)			
	1.40				4.75				9.04				0.57				4.65				-0.12							
	8	9	11	12	ED	8	9	11	12	ED	8	9	11	12	ED	8	9	11	12	ED	8	9	11	12	ED			
Pop. Est.	553.4	334.3	322.1	307.7	188.7	556.3	336.7	320.7	303.8	187.9	570.7	323.6	314.7	278.9	274.0	570.9	323.4	314.5	278.4	273.5	326.8	322.8	314.6	278.4	273.5			
S.E.	28.2	23.0	21.8	21.2	16.1	28.2	23.0	21.6	20.3	16.0	16.3	15.4	14.7	11.9	11.3	16.3	15.4	14.7	11.9	11.3	16.3	15.4	14.7	11.9	11.3			
C.V.	0.051	0.069	0.068	0.069	0.085	0.051	0.066	0.067	0.067	0.085	0.031	0.038	0.039	0.037	0.040	0.032	0.039	0.040	0.037	0.038	0.032	0.039	0.040	0.037	0.038			
R.E.	100.0	96.3	99.1	95.8	99.5	100.0	96.3	99.1	95.8	99.5	100.0	96.3	99.1	95.8	99.5	100.0	96.3	99.1	95.8	99.5	100.0	96.3	99.1	95.8	99.5			
Pop. Est.	569.0	328.4	320.4	305.6	174.0	569.0	328.4	320.4	305.6	174.0	570.7	323.6	314.7	278.9	274.0	570.9	323.4	314.5	278.4	273.5	326.8	322.8	314.6	278.4	273.5			
S.E.	17.9	12.4	11.3	11.5	6.9	17.9	12.9	11.5	11.2	6.9	17.9	12.4	11.3	11.9	6.9	17.9	12.9	11.5	11.2	6.9	17.9	12.4	11.3	11.9	6.9			
C.V.	0.031	0.038	0.037	0.037	0.040	0.031	0.039	0.038	0.038	0.040	0.032	0.039	0.040	0.037	0.040	0.032	0.039	0.040	0.037	0.038	0.032	0.039	0.040	0.037	0.038			
R.E.	63.5	54.6	53.0	53.0	43.1	63.5	54.6	53.0	53.0	43.1	63.5	54.6	53.0	53.0	43.1	63.5	54.6	53.0	53.0	43.1	63.5	54.6	53.0	53.0	43.1			
Pop. Est.	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5	3266.5			
S.E.	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3	49.3			
C.V.	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015			
R.E.	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4	68.4			

TABLE 12

HOUSEHOLDS WITH 2 OR FEWER PEOPLE

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.36 4.45 0.66 0.60 4.05 0.07

	C(0-1)	C(1-2)	C(2-3)	C(3-4)	C(1-3)	C(2-4)
	1.36	4.45	0.66	0.60	4.05	0.07
8	1,037.7	31.2	0.030	1,042.2	29.6	0.028
9	634.4	27.4	0.045	617.2	26.4	0.043
11	384.2	23.0	0.060	388.3	22.0	0.057
2	365.0	22.3	0.061	366.8	22.0	0.060
12	165.1	15.4	0.093	163.5	15.3	0.094
ED	6,835.2	91.8	0.013	6,856.8	87.5	0.013
	94.8	1,091.1	10.7	0.010	34.2	1,084.8
	96.4	613.6	10.5	0.017	38.3	615.0
	95.4	392.2	8.4	0.021	36.3	393.6
	98.9	375.7	9.0	0.024	40.3	374.9
	99.4	169.8	8.2	0.048	57.9	168.4
	95.3	6,952.2	37.7	0.003	41.1	6,924.6
	94.8	1,091.1	10.7	0.010	34.2	1,084.8
	36.5	1,090.7	10.6	0.010	34.1	1,090.7
	39.8	613.7	10.5	0.017	38.2	613.7
	41.0	392.2	8.3	0.021	38.0	392.2
	40.4	170.1	8.2	0.024	53.1	170.1
	52.9	6,949.9	37.7	0.005	42.9	6,949.9
	41.0	1,090.7	10.6	0.010	34.1	1,090.7

CATEGORIES AT VA LEVEL UNDER SIMPLE RANDOM SAMPLING

TABLE 13

C2
AGE OF HEAD IS 65 OR MORE

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.35 7.85 0.65 0.61 7.81 0.09

VA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.
8	472.2	26.8	0.057	98.7	489.8	26.5	0.058	98.7	486.5	6.2	0.013	23.2	487.3	6.4	0.013	23.8	485.8	6.2	0.013	23.2
9	259.0	20.9	0.081	98.5	257.5	20.5	0.080	98.5	266.1	4.7	0.018	22.3	265.2	4.7	0.018	22.7	265.7	4.5	0.018	22.3
11	109.3	14.0	0.128	100.4	109.5	14.1	0.128	100.4	110.3	7.2	0.065	51.5	110.1	7.2	0.065	51.4	110.0	7.2	0.065	51.2
2	111.6	14.3	0.128	101.4	113.5	14.4	0.127	101.4	103.2	2.4	0.023	16.6	103.1	2.4	0.023	16.8	103.2	2.4	0.023	16.5
12	32.4	7.8	0.240	98.6	32.8	7.8	0.238	100.6	42.6	4.4	0.096	53.0	42.1	4.1	0.098	53.1	42.4	4.1	0.097	53.3
ED	2,760.0	67.6	0.024	98.8	2,743.6	66.8	0.024	98.8	2,766.0	17.1	0.006	25.3	2,760.7	17.6	0.006	26.1	2,765.6	17.1	0.006	25.3

TABLE 14

C3
OWNED DWELLINGS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
2.79 0.96 0.97 0.26 0.00 0.72

VA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.
8	944.4	31.3	0.033	922.0	922.0	0.0	0.000	0.0	908.9	14.3	0.016	45.6	922.0	0.0	0.000	0.0	917.4	4.3	0.005	13.9
9	1,078.2	25.8	0.024	1,068.0	1,068.0	0.0	0.000	0.0	1,075.3	10.9	0.010	42.1	1,068.0	0.0	0.000	0.0	1,059.7	3.1	0.003	11.5
11	1,078.5	19.2	0.018	1,079.0	1,079.0	0.0	0.000	0.0	1,084.5	6.0	0.006	31.3	1,079.0	0.0	0.000	0.0	1,082.0	1.4	0.001	7.5
2	728.9	22.6	0.031	724.0	724.0	0.0	0.000	0.0	719.1	7.9	0.011	35.1	724.0	0.0	0.000	0.0	723.3	1.5	0.002	6.4
12	327.3	17.5	0.054	326.0	326.0	0.0	0.000	0.0	331.6	4.5	0.014	25.7	326.0	0.0	0.000	0.0	326.4	0.3	0.001	1.9
ED	12,624.0	89.3	0.007	12,477.0	12,477.0	0.0	0.000	0.0	12,419.8	34.7	0.003	38.9	12,422.0	0.0	0.000	0.0	12,459.2	9.3	0.001	10.4

TABLE 15

C4
RENTED APARTMENTS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
5.46 3.27 3.20 0.85 0.15 2.41

VA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.	Pop. Est.	S.E.	C.V.	R.E.
8	649.7	29.5	0.045	661.0	661.0	0.0	0.000	0.0	680.8	11.7	0.017	39.7	661.0	0.0	0.000	0.0	665.2	3.4	0.005	11.6
9	286.1	21.7	0.076	299.0	299.0	0.0	0.000	0.0	294.1	8.7	0.030	40.1	299.0	0.0	0.000	0.0	297.7	2.4	0.008	11.2
11	82.7	12.3	0.149	90.0	90.0	0.0	0.000	0.0	87.7	5.6	0.064	45.5	90.0	0.0	0.000	0.0	89.2	1.4	0.016	11.8
2	84.5	12.6	0.149	95.0	95.0	0.0	0.000	0.0	95.1	3.3	0.035	26.4	95.0	0.0	0.000	0.0	95.2	0.4	0.005	3.5
12	20.8	6.3	0.305	20.7	20.7	6.2	0.299	98.9	20.0	6.0	0.300	96.0	20.4	6.0	0.294	96.0	22.4	6.0	0.294	96.3
ED	2,642.3	65.4	0.025	2,615.1	2,615.1	11.8	0.004	18.1	2,769.4	28.7	0.010	43.8	2,715.9	11.4	0.004	17.5	2,731.5	13.8	0.005	21.2

CATEGORIES AT EA LEVEL UNDER SIMPLE RANDOM SAMPLING

TABLE 16

A1

HOUSEHOLDS WITH EMPLOYED HEADS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.34 0.45 0.08 0.05 0.43 0.11

EA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop. Est	S.E.	C.V.	Pop. Est	S.E.	C.V.	Pop. Est	R.E.	Pop. Est	S.E.	C.V.	R.E.	Pop. Est	S.E.	C.V.	R.E.	Pop. Est	S.E.	C.V.	R.E.
2	428.3	23.1	0.054	423.4	21.9	0.052	94.9	94.9	424.0	21.3	0.050	92.1	424.1	21.2	0.050	91.9	424.3	21.2	0.050	91.9
1	362.0	22.2	0.061	360.0	21.4	0.059	96.3	96.3	362.5	20.9	0.058	93.8	362.3	20.8	0.058	93.7	362.1	20.8	0.058	93.7
3	179.7	13.2	0.117	135.3	14.7	0.108	96.3	96.3	135.3	14.2	0.105	93.2	135.7	14.1	0.104	92.6	135.7	14.1	0.104	92.6

TABLE 17

A4

HOUSEHOLDS WITH HEADS NOT MOVED IN 5 YEARS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
0.24 2.16 0.48 0.20 1.99 0.28

2	117.6	14.6	0.124	118.0	14.6	0.123	99.8	99.8	121.0	14.2	0.118	97.6	121.4	14.2	0.117	97.4	121.3	14.2	0.117	97.5
1	171.9	17.1	0.100	172.2	16.3	0.095	95.3	95.3	168.9	15.7	0.093	91.6	169.8	15.6	0.092	91.1	169.4	15.6	0.092	91.1
3	15.1	5.5	0.363	15.2	45.5	0.362	100.4	100.4	14.8	5.5	0.370	100.0	14.9	5.5	0.368	100.0	14.9	5.5	0.368	100.0

TABLE 18

A6

HIGHEST GRADE OF HEAD IS 1 TO 10

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.22 0.86 0.13 0.09 0.89 0.21

2	144.8	16.0	0.110	143.4	15.8	0.110	98.9	98.9	142.2	15.4	0.108	96.2	142.3	15.4	0.108	96.2	142.2	15.3	0.108	96.2
1	196.1	18.1	0.092	194.8	17.6	0.090	97.4	97.4	194.0	17.2	0.089	95.1	193.7	17.2	0.089	95.0	193.5	17.2	0.089	95.0
3	24.1	6.9	0.287	26.4	7.2	0.273	104.3	104.3	27.9	7.0	0.252	101.9	27.9	7.0	0.251	101.4	27.9	7.0	0.251	101.4

TABLE 19

B1

HOUSEHOLDS WITH 2 OR 4 PERSONS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.21 1.23 0.12 0.08 1.13 0.18

2	193.0	18.0	0.093	191.2	17.5	0.092	97.6	97.6	189.3	16.4	0.087	91.5	189.3	16.4	0.087	91.3	189.2	16.4	0.087	91.3
1	220.2	18.9	0.086	218.3	18.4	0.084	97.6	97.6	220.5	17.6	0.080	93.2	220.2	17.6	0.080	93.2	220.0	17.6	0.080	93.1
3	63.3	11.0	0.174	65.8	11.2	0.171	102.1	102.1	63.6	10.9	0.171	99.2	63.8	10.9	0.171	99.0	63.8	10.9	0.171	99.0

TABLE 20

B3

AGE OF HEAD IS 25 TO 34

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
2.30 1.69 0.43 0.32 1.32 0.37

2	156.8	16.5	0.105	153.7	15.1	0.108	91.6	91.6	151.9	12.8	0.084	87.3	152.0	12.8	0.084	87.3	152.4	12.7	0.084	87.1
1	96.5	13.3	0.138	95.1	12.7	0.134	95.3	95.3	97.7	11.6	0.119	87.0	96.9	11.6	0.120	87.2	97.3	11.6	0.119	87.0
3	54.3	10.7	0.188	57.1	10.4	0.183	102.1	102.1	56.6	9.6	0.170	94.1	56.7	9.6	0.169	93.8	56.9	9.6	0.168	93.6

ESTIMATED EFFICIENCY OF THE RRE VERSUS THE M.L.E. UNDER S.R.S.W.R.

TABLE 21

AI

HOUSEHOLDS WITH EMPLOYED HEADS

WA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4				M.L.E.
	SE ₀	RE ₀ ⁰	RE ₀ ⁴	SE ₁	RE ₁ ⁰	RE ₁ ⁴	SE ₂	RE ₂ ⁰	RE ₂ ⁴	SE ₃	RE ₃ ⁰	RE ₃ ⁴	SE ₄	RE ₄ ⁰	RE ₄ ⁴	SE ₄	RE ₄ ⁰	RE ₄ ⁴	SE*	SE*	
8	37.1	74.6	75.5	37.1	74.7	75.6	27.7	99.9	101.2	27.7	99.9	101.2	27.7	99.9	101.2	27.7	99.9	101.2	27.7	28.0	
9	30.3	78.0	78.2	30.2	78.4	78.5	23.7	99.9	100.1	23.7	99.9	100.0	23.7	99.9	100.1	23.7	99.9	100.1	23.7	23.7	
11	22.2	83.4	84.3	22.1	83.6	84.5	18.5	99.6	100.7	18.5	99.8	100.8	18.5	99.9	100.9	18.5	99.9	100.9	18.5	18.7	
2	22.1	77.3	77.9	22.0	77.7	78.3	17.1	99.8	100.5	17.1	99.9	100.6	17.1	99.9	100.6	17.1	99.9	100.6	17.1	17.2	
12	14.6	83.7	84.8	14.4	84.3	85.4	12.2	99.8	101.1	12.2	99.8	101.1	12.2	99.8	101.1	12.2	99.8	101.1	12.2	12.3	
20	102.9	77.4	77.7	101.7	78.3	78.6	79.8	99.8	100.2	79.9	99.8	100.1	79.7	99.9	100.2	79.7	99.9	100.2	79.7	79.9	

TABLE 22

ESTIMATED EFFICIENCY OF THE HRE VERSUS THE M.L.E. UNDER S.R.S.W.R.

A4

HOUSEHOLDS WITH HEADS NOT MOVED IN 5 YEARS

WA	Iteration 0		Iteration 1		Iteration 2		Iteration 3		Iteration 4		M.L.E.				
	SE ₀	RE ₀ ⁰	SE ₀ ⁴	RE ₁ ⁰	SE ₁	RE ₂ ⁰	SE ₂	RE ₃ ⁰	SE ₃	RE ₄ ⁰	SE ₄	RE ₄ ⁴	SE ₄ [*]	SE ₄ [*]	
8	38.1	78.1	78.0	32.4	91.8	91.6	30.5	97.5	97.3	29.9	99.7	99.5	29.9	99.8	29.7
9	34.2	81.0	81.2	29.9	92.9	93.1	28.3	97.9	98.1	27.8	99.7	100.0	27.8	99.7	28.6
11	31.1	85.3	85.9	29.0	91.2	91.9	26.6	99.5	100.3	26.6	99.8	100.5	26.5	99.9	26.7
2	25.9	88.0	88.2	24.0	94.8	95.0	23.0	99.1	99.3	22.8	99.8	100.0	22.8	99.9	22.8
12	21.0	79.0	78.1	17.4	95.5	94.3	16.8	98.6	97.5	16.6	99.8	98.6	16.6	99.8	16.4
ED	116.0	83.3	83.2	104.4	92.6	92.5	98.0	98.6	98.5	96.8	99.8	99.7	96.8	99.8	96.5

TABLE 23

A6

ESTIMATED EFFICIENCY OF THE RRE VERSUS THE M.L.E. UNDER S.R.S.W.R.

HIGHEST GRADE OF HEAD IS 1 TO 10

WA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4			M.L.E.	
	SE ₀	RE ₀ ⁰	RE ₀ ⁴	SE ₁	RE ₁ ⁰	RE ₁ ⁴	SE ₂	RE ₂ ⁰	RE ₂ ⁴	SE ₃	RE ₃ ⁰	RE ₃ ⁴	SE ₄	RE ₄ ⁰	RE ₄ ⁴	SE ₀ [*]	SE ₀ ⁴
8	38.0	95.6	95.5	37.7	96.3	96.2	36.3	99.9	99.8	36.4	99.9	99.7	36.3	99.9	99.8	36.3	36.3
9	33.7	96.9	97.0	33.4	97.5	97.6	32.6	99.9	100.0	32.6	99.9	100.0	32.6	99.9	100.1	32.6	32.6
11	29.7	95.3	95.6	29.4	96.2	96.5	28.3	99.9	100.2	28.3	99.9	100.2	28.3	99.9	100.2	28.3	28.4
2	27.2	94.1	93.8	27.1	94.5	94.2	25.7	99.9	99.6	25.7	99.8	99.5	25.6	99.9	99.6	25.6	25.5
12	21.5	92.3	92.0	20.9	95.0	94.7	19.9	99.7	99.4	19.9	99.8	99.5	19.9	99.8	99.5	19.9	19.8
ED	115.1	94.9	94.7	113.9	95.9	95.7	109.4	99.8	99.7	109.4	99.8	99.7	109.3	99.9	99.7	109.2	109.0

ESTIMATED EFFICIENCY OF THE RRE VERSUS THE M.L.E. UNDER S.R.S.W.R.

TABLE 24

B1

HOUSEHOLDS WITH 3 OR 4 PERSONS

WA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4				M.L.E.
	SE ₀ ⁰	RE ₀ ⁰	SE ₀ ⁴	RE ₀ ⁴	SE ₁ ⁰	RE ₁ ⁰	SE ₁ ⁴	RE ₁ ⁴	SE ₂ ⁰	RE ₂ ⁰	SE ₂ ⁴	RE ₂ ⁴	SE ₃ ⁰	RE ₃ ⁰	SE ₃ ⁴	RE ₃ ⁴	SE ₄ ⁰	RE ₄ ⁰	SE ₄ ⁴	RE ₄ ⁴	
8	35.0	74.6	72.3	34.7	75.2	72.9	26.1	99.8	96.7	26.3	99.3	96.2	26.1	99.9	96.8	26.1	25.3				
9	32.8	83.5	84.0	32.7	83.8	84.2	27.5	99.9	100.4	27.5	99.8	100.3	27.4	99.9	100.5	27.4	27.6				
11	30.4	85.7	86.0	30.2	86.3	86.6	26.1	99.9	100.2	26.1	99.8	100.0	26.1	99.9	100.2	26.1	26.1				
2	28.7	83.0	83.1	28.7	83.1	83.2	23.9	99.8	99.9	23.9	99.8	99.9	23.9	99.9	100.0	23.8	23.9				
12	21.4	88.1	87.5	21.2	88.9	88.3	18.9	99.8	99.1	18.9	99.8	99.1	18.9	99.8	99.2	18.9	18.7				
ED	114.4	84.1	84.0	113.2	85.1	84.9	96.4	99.9	99.7	96.6	99.6	99.5	96.4	99.9	99.7	96.3	96.1				

ESTIMATED EFFICIENCY OF THE RRE VERSUS THE M.L.E. UNDER S.R.S.W.R.

TABLE 25

C4

RENTED APARTMENTS

[illegible]

APPENDIX III

Tables of Results for Categories Under Cluster Sampling

Tables 1 to 15 give results for the RRE of 15 categories under cluster sampling at the WA level. For each iteration, the estimated population total, standard error, coefficient of variation, and "ratio of errors" are given by the columns labelled Pop. Est., S.E., C.V., and R.E. respectively. These estimates are given for 5 WA's and also at the ED level. Tables 16 to 20 give similar results at the EA level for 5 categories and the three EA's from WA 2. Table 21 presents the design effect D_p^u , SE_0^s , and SE_p^c for 6 categories and one average sized weighting area.

CATEGORIES AT WA LEVEL UNDER CLUSTER SAMPLING

A1
EMPLOYEDC(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.43 0.70 0.36 0.11 0.40 0.12

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.
8	2,144.7	56.7	0.026	2,120.1	50.6	0.024	89.2	2,118.9	44.3	0.021	78.2	2,109.8	44.7	0.021	78.8
9	2,052.3	55.0	0.027	2,089.8	45.1	0.022	82.2	2,087.4	41.2	0.020	75.0	2,087.7	41.7	0.020	75.9
11	1,995.6	48.4	0.024	2,017.0	37.8	0.019	78.0	2,020.2	34.7	0.017	71.7	2,027.1	35.0	0.017	72.3
12	1,562.2	43.6	0.028	1,580.1	35.1	0.022	80.5	1,592.8	31.0	0.019	71.1	1,591.2	31.9	0.020	73.1
ED	24,464.4	180.9	0.038	24,511.7	152.4	0.031	80.4	24,485.7	137.7	0.026	76.1	24,478.0	139.0	0.026	76.8

TABLE 2

A2
UNEMPLOYEDC(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
2.74 1.50 0.71 0.64 1.13 0.29

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.
8	238.8	23.4	0.080	232.3	22.5	0.089	96.1	247.9	22.3	0.090	95.6	249.3	22.3	0.089	95.3
9	237.8	22.3	0.084	238.7	21.8	0.091	98.1	240.5	21.7	0.090	97.6	239.4	21.8	0.091	97.7
11	138.5	18.3	0.132	145.8	17.7	0.127	97.8	145.0	17.4	0.120	95.2	145.0	17.5	0.121	95.5
12	102.6	14.8	0.143	104.0	14.4	0.138	94.8	108.7	14.0	0.134	94.8	103.8	14.1	0.135	94.9
ED	2,234.5	71.1	0.032	2,223.3	69.9	0.031	96.8	2,219.7	68.0	0.031	95.7	2,223.0	68.1	0.031	95.7

TABLE 3

NOT IN THE LABOUR FORCE

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.69 1.02 0.39 0.45 0.89 0.15

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.
8	1,841.3	54.7	0.030	1,826.8	50.2	0.028	51.8	1,834.3	42.9	0.023	78.4	1,839.5	43.2	0.023	78.9
9	1,468.3	50.1	0.034	1,490.4	43.9	0.029	87.5	1,497.7	40.5	0.027	80.8	1,496.1	39.9	0.027	79.7
11	1,332.9	41.8	0.037	1,339.1	33.9	0.031	85.8	1,339.8	33.1	0.029	79.3	1,339.1	33.0	0.029	78.9
12	878.7	37.6	0.043	892.5	34.3	0.038	91.3	898.5	30.4	0.034	81.0	891.6	30.9	0.035	82.2
ED	5,814.5	276.8	0.009	5,749.3	257.7	0.044	51.1	5,784.3	23.7	0.041	84.2	5,807.7	23.8	0.041	84.4

TABLE 4

NOT MOVED IN LAST 5 YEARS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.70 1.32 0.32 0.33 1.12 0.17

VA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.
8	2,109.0	81.3	0.039	2,064.3	70.8	0.034	87.1	2,074.4	65.1	0.031	83.1	2,071.7	64.8	0.031	79.7
9	2,190.2	85.4	0.039	2,225.9	69.4	0.031	81.2	2,231.3	64.1	0.029	75.0	2,224.0	64.4	0.029	75.1
11	1,765.1	78.2	0.044	1,812.5	66.5	0.037	85.1	1,822.9	61.7	0.033	78.9	1,837.9	62.1	0.034	79.4
12	772.2	59.8	0.078	787.0	54.4	0.069	90.8	793.9	51.1	0.064	85.4	788.1	51.2	0.065	85.5
ED	22,565.9	280.9	0.012	22,574.1	242.4	0.011	86.3	22,527.3	227.7	0.010	81.1	22,534.3	227.9	0.010	81.2

CATEGORIES AT VA LEVEL UNDER CLUSTER SAMPLING

AS

MOVED IN LAST 5 YEARS IN SAME MUNICIPALITY

TABLE 5

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.25 1.16 0.44 0.31 1.07 0.22

VA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4				
	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	
8	1,121.4	61.7	0.055	97.9	1,113.7	60.4	0.054	97.9	1,113.2	59.1	0.053	95.7	95.7	1,112.2	58.9	0.053	95.5	1,114.5	59.0	0.053	95.5
9	840.8	57.7	0.049	97.0	852.1	56.0	0.046	97.0	840.3	55.6	0.044	92.9	92.9	847.3	53.9	0.044	93.4	838.8	53.5	0.054	92.9
11	790.0	56.8	0.072	98.6	797.3	56.0	0.070	98.6	787.1	54.5	0.069	96.0	96.0	785.3	54.4	0.070	96.1	785.3	54.4	0.069	95.8
2	710.1	51.7	0.073	98.0	725.0	50.7	0.070	98.0	728.0	50.5	0.069	97.7	97.7	725.2	50.4	0.070	97.5	727.5	50.4	0.069	97.5
12	210.0	28.2	0.134	96.2	205.0	27.2	0.133	96.2	201.7	26.6	0.132	94.3	94.3	200.4	26.6	0.133	94.4	201.0	26.5	0.132	94.1
ED	10,829.9	207.3	0.019	97.0	10,943.0	201.0	0.018	97.0	10,940.3	196.2	0.018	94.7	94.7	10,940.6	196.2	0.018	94.7	10,941.3	196.0	0.018	94.6

TABLE 6

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.92 1.61 0.75 0.71 1.17 0.18

8	1,228.5	49.2	0.040	1,222.3	44.7	0.037	90.9	90.9	1,235.9	41.6	0.034	84.6	84.6	1,230.1	41.9	0.034	85.2	1,236.4	41.7	0.034	84.7
9	609.2	37.2	0.061	625.4	35.3	0.056	94.9	94.9	629.2	33.8	0.054	90.7	90.7	630.3	33.7	0.054	90.6	630.4	33.7	0.054	90.6
11	427.4	32.9	0.077	436.2	29.5	0.068	90.2	90.2	440.5	28.0	0.064	85.3	85.3	439.3	28.1	0.064	85.4	440.6	28.0	0.063	85.1
2	378.7	32.1	0.085	377.2	30.1	0.080	90.9	90.9	379.7	27.9	0.074	87.0	87.0	378.4	27.9	0.074	87.0	380.8	27.9	0.073	86.9
12	298.7	26.5	0.089	286.9	23.7	0.083	89.6	89.6	293.6	21.3	0.073	80.5	80.5	292.4	21.6	0.074	81.7	294.7	22.3	0.072	80.3
ED	10,460.2	151.8	0.014	10,681.0	133.5	0.012	87.9	87.9	10,456.9	122.5	0.011	80.7	80.7	10,688.8	123.6	0.012	81.4	10,688.1	122.4	0.011	80.6

HIGHEST GRADE IS 1 TO 8

TABLE 7

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
2.12 0.89 0.30 0.30 0.85 0.10

8	913.2	40.5	0.044	898.7	39.7	0.044	98.1	98.1	897.6	38.7	0.043	95.5	95.5	898.6	39.0	0.043	96.2	898.4	38.7	0.043	95.5
9	813.3	38.6	0.048	822.2	38.1	0.046	98.6	98.6	827.7	36.6	0.044	94.6	94.6	826.4	36.7	0.044	95.0	827.1	36.6	0.044	94.6
11	680.9	37.2	0.055	694.7	34.9	0.050	93.9	93.9	706.3	34.4	0.049	92.4	92.4	705.5	34.3	0.049	92.3	706.6	34.3	0.049	92.4
2	473.4	32.3	0.068	484.2	31.0	0.064	95.9	95.9	490.1	30.2	0.062	93.5	93.5	487.9	30.5	0.062	94.3	490.2	30.3	0.062	93.6
12	445.8	27.9	0.082	434.6	26.9	0.082	96.6	96.6	424.8	26.5	0.083	95.3	95.3	426.4	26.4	0.082	94.7	424.8	26.5	0.082	95.0
ED	10,165.5	137.4	0.014	10,137.3	131.9	0.013	96.0	96.0	10,141.0	128.0	0.013	93.1	93.1	10,145.4	128.4	0.013	93.4	10,139.3	127.9	0.013	93.1

TABLE 8

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
4.48 2.35 1.61 1.55 1.32 0.16

8	432.1	33.3	0.074	436.5	29.7	0.068	99.2	427.3	27.6	0.065	93.4	431.6	28.2	0.065	94.6	427.8	27.7	0.065	93.1
9	391.0	31.5	0.070	395.8	29.3	0.069	99.0	401.2	27.7	0.066	96.8	393.2	28.3	0.066	98.3	402.2	27.7	0.066	98.8
11	474.6	35.3	0.074	497.8	24.4	0.069	99.0	493.7	24.1	0.069	98.3	494.7	24.1	0.069	98.3	493.4	24.0	0.069	98.8
2	289.5	28.5	0.102	302.8	22.4	0.074	95.9	306.5	20.9	0.068	90.7	304.1	20.9	0.068	90.8	306.2	20.8	0.068	90.6
12	195.2	22.3	0.114	199.9	16.1	0.076	87.8	189.5	14.9	0.079	87.0	192.0	14.7	0.077	86.2	189.6	14.9	0.079	86.9
ED	5,383.0	121.3	0.022	5,390.9	92.6	0.017	96.3	5,406.8	88.2	0.016	92.7	5,413.7	89.0	0.016	93.4	5,404.5	88.1	0.016	92.6

HIGHEST GRADE IS 9 TO 10

AS

ATTENDING SCHOOL FULL TIME

CATEGORIES AT WA LEVEL UNDER CLUSTER SAMPLING

TABLE 2

A9
BACHULOR DEGREE OR HIGHERC(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.31 1.86 0.61 0.46 1.75 0.33

WA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	
8	106.1	15.6	0.150	102.3	15.2	0.148	97.5	101.3	14.9	0.147	95.7	101.3	14.9	0.147	95.6	101.3	14.9	0.147	95.6	
9	237.6	23.8	0.100	238.3	23.5	0.099	98.5	241.6	23.0	0.095	96.3	241.6	23.1	0.096	96.9	241.6	23.0	0.095	96.4	
11	283.0	25.4	0.090	280.8	24.8	0.088	97.5	279.2	24.3	0.087	95.6	278.8	24.3	0.087	95.4	278.7	24.3	0.087	95.5	
12	145.0	18.4	0.127	147.4	18.1	0.123	98.0	145.1	17.8	0.123	96.8	145.0	17.9	0.123	96.8	144.7	17.8	0.123	96.7	
ED	1,898.3	68.4	0.036	1,903.8	66.6	0.035	97.6	1,896.8	65.5	0.035	95.7	1,897.1	65.5	0.035	95.8	1,897.3	65.4	0.034	95.6	

TABLE 10

B1

MIXED

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
6.40 3.79 2.58 1.64 2.45 1.44

	8	9	11	12	ED
Pop. Est.	431.3	262.0	129.7	108.5	2,629.3
S.E.	30.5	23.7	16.3	15.8	75.8
C.V.	0.071	0.090	0.126	0.149	0.029
Pop. Est.	460.3	248.5	139.8	101.1	2,687.7
S.E.	23.4	17.7	12.1	13.1	57.0
C.V.	0.050	0.071	0.087	0.129	0.021
R.E.	76.6	74.7	74.2	82.3	75.2
Pop. Est.	459.3	251.6	136.4	96.8	2,667.4
S.E.	18.3	15.9	11.5	11.0	51.0
C.V.	0.040	0.063	0.082	0.114	0.019
R.E.	60.0	67.4	70.4	69.2	67.4
Pop. Est.	447.0	248.1	138.6	99.3	2,708.5
S.E.	17.3	15.2	10.5	10.9	49.1
C.V.	0.037	0.061	0.076	0.109	0.018
R.E.	56.7	64.1	64.6	68.3	64.8
Pop. Est.	444.9	236.8	139.2	99.7	2,689.5
S.E.	16.7	14.9	10.6	10.6	48.1
C.V.	0.035	0.059	0.076	0.107	0.018
R.E.	54.9	62.8	64.9	67.0	63.5

TABLE 11

B2

MOTHER TONGUE GERMAN

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
10.61 3.16 1.88 0.65 2.83 1.63

	8	9	10	11	12	ED
Pop. Est.	8.9	30.5	61.9	44.4	70.7	431.7
S.E.	5.5	9.8	17.4	14.9	20.1	42.6
C.V.	0.616	0.323	0.280	0.336	0.284	0.099
Pop. Est.	10.9	33.4	66.7	44.9	70.1	425.4
S.E.	5.2	9.2	16.9	12.7	18.2	38.0
C.V.	0.473	0.276	0.224	0.283	0.260	0.089
Pop. Est.	94.1	93.8	88.0	85.4	84.8	89.3
S.E.	11.3	34.4	68.1	45.1	44.3	37.6
C.V.	5.2	9.2	14.8	12.6	12.6	0.087
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44.3	44.5	429.1
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.6	84.8	84.5	84.6	88.6
S.E.	10.9	34.3	66.2	44.5	44.5	429.7
C.V.	5.2	9.2	14.7	12.6	12.6	0.088
Pop. Est.	93.9	93.7	85.0	84.7	84.5	88.8
S.E.	10.8	34.4	65.9	44		

TABLE 12

B3

MOTHER TONGUE ITALIAN

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
10.83 1.66 1.11 0.38 0.73 0.79

	8	9	11	12	ED
Pop. Est.	441.1	35.0	441.1	441.1	407.9
S.E.	39.6	14.1	37.2	37.2	34.0
C.V.	8.9	39.6	8.4	8.4	7.6
R.E.	5.9	17.6	5.6	5.6	5.6
Pop. Est.	3.0	2.5	3.1	3.1	3.1
S.E.	0.078	0.119	0.083	0.083	0.083
C.V.	25.3	37.2	25.3	25.3	25.3
R.E.	72.8	89.3	72.8	72.8	72.8
Pop. Est.	1,059.8	65.0	1,067.1	1,067.1	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	61.5
Pop. Est.	410.8	33.8	410.8	410.8	410.8
S.E.	12.4	12.4	12.4	12.4	12.4
C.V.	2.9	36.4	2.9	2.9	2.9
R.E.	64.2	88.0	64.2	64.2	64.2
Pop. Est.	1,059.8	64.3	1,059.8	1,059.8	1,059.8
S.E.	64.3	12.4	64.3	64.3	64.3
C.V.	6.1	18.8	6.1	6.1	6.1
R.E.	72.1	89.2	72.1	72.1	72.1
Pop. Est.	409.4	33.8	409.4	409.4	409.4
S.E.	33.8	12.4	33.8	33.8	33.8
C.V.	8.1	36.4	8.1	8.1	8.1
R.E.	61.5	88.0	61.5	61.5	6

CATEGORIES AT WA LEVEL UNDER CLUSTER SAMPLING

13

(CERTAINLY NEVER MARRIED)

MA	Iteration 0				Iteration 1				Iteration 2				Iteration 3				Iteration 4			
	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.	Pop.Est.	S.E.	C.V.	R.E.
C(0-1)	2,632.6	49.9	0.019	46.8	2,646.7	23.3	0.009	46.8	2,635.4	6.1	0.002	12.3	2,635.9	16.0	0.006	32.0	2,617.2	6.1	0.002	12.2
C(1-2)	2,644.1	46.5	0.018	35.9	2,608.6	17.4	0.007	35.9	2,628.0	6.7	0.003	13.7	2,610.6	13.8	0.005	28.5	2,627.4	6.5	0.003	13.6
C(2-3)	2,494.9	53.0	0.017	2,499.0	12.1	0.005	28.2	2,490.0	5.5	0.002	12.7	2,489.4	9.7	0.004	22.4	2,489.9	5.4	0.002	12.6	
C(3-4)	38-2	38-2	0.021	36.6	1,787.0	14.0	0.008	36.6	1,787.5	4.7	0.003	12.4	1,795.5	12.5	0.007	32.7	1,787.2	4.7	0.003	12.3
C(4-5)	2,215.5	32.2	0.027	26.7	1,235.5	8.6	0.007	26.7	1,233.0	0.0	0.000	0.0	1,235.7	7.0	0.006	21.6	1,233.0	0.0	0.000	0.0
ED	2,046.0	143.3	0.093	33.0	3,073.4	57.2	0.002	35.0	3,015.4	22.5	0.001	13.7	3,034.8	45.4	0.001	27.8	3,018.2	22.3	0.001	13.7

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MOTHER TONGUE ENGLISH

[illegible]

ST. PETERS

5

NOTRE TONGUE FRENCH

	C(0-1)	C(1-2)	C(2-3)	C(3-4)	C(1-3)	C(2-4)
	3.81	0.35	0.33	0.05	0.10	0.29
8	1,377.3	80.3	0.058	1,382.0	0.0	0.000
9	813.8	77.1	0.082	984.7	0.0	0.010
10	483.5	59.3	0.123	519.8	9.5	0.020
11	279.3	32.1	0.169	246.2	12.5	0.028
12	141.0	15.1	0.126	141.0	12.5	0.028
13	66.8	6.6	0.055	66.8	9.5	0.010
14	33.4	3.3	0.022	33.4	4.7	0.005
15	16.7	1.7	0.010	16.7	2.4	0.002
16	8.3	0.8	0.005	8.3	1.2	0.001
17	4.2	0.4	0.002	4.2	0.6	0.000
18	2.1	0.2	0.001	2.1	0.3	0.000
19	1.0	0.1	0.000	1.0	0.1	0.000
20	0.5	0.0	0.000	0.5	0.0	0.000

CATEGORIES AT EA LEVEL UNDER CLUSTER SAMPLING

A1
EMPHOTEDC(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.78 0.37 0.20 0.13 0.47 0.10

WA	Iteration 0			Iteration 1			Iteration 2			Iteration 3			Iteration 4		
	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.	Pop.Est.	S.E.	C.V.
2	721.9	44.5	0.062	732.4	42.6	0.058	737.8	41.6	0.056	735.4	41.9	0.057	737.3	41.4	0.056
1	639.1	44.7	0.070	652.6	41.8	0.064	653.9	40.8	0.062	652.1	40.7	0.062	652.9	40.7	0.062
3	201.2	26.0	0.129	205.1	25.2	0.123	201.9	24.4	0.121	202.7	24.5	0.121	202.1	24.4	0.121

TABLE 17

A4

NOT MOVED IN LAST 5 YEARS

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.89 0.96 0.57 0.66 0.44 0.14

2	281.1	38.7	0.138	285.1	38.0	0.132	287.9	37.2	0.129	286.0	37.3	0.130	287.8	37.2	0.129
1	443.8	49.7	0.112	452.7	46.7	0.103	457.0	44.7	0.098	454.7	44.6	0.098	457.7	44.4	0.097
3	47.3	18.6	0.392	49.2	18.3	0.372	49.0	18.2	0.371	48.4	18.2	0.376	48.7	18.2	0.373

TABLE 18

A5

HIGHEST GRADE IS 1 TO 8

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.43 1.65 0.38 0.61 1.36 0.30

2	121.3	18.8	0.155	120.0	17.8	0.149	123.9	17.1	0.138	123.1	17.1	0.139	124.1	17.1	0.138
1	213.0	26.0	0.122	215.3	25.8	0.120	214.6	25.0	0.116	214.2	24.9	0.116	215.3	24.9	0.116
3	44.4	12.2	0.275	42.0	11.6	0.277	41.1	11.4	0.277	41.1	11.4	0.277	41.8	11.4	0.275

TABLE 19

B2

MOTHER TONGUE GERMAN

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.60 1.72 2.43 0.46 1.52 2.23

2	29.8	12.5	0.424	29.6	11.1	0.375	30.4	11.0	0.361	29.5	11.0	0.371	29.7	11.0	0.370
1	14.8	8.7	0.562	15.3	7.8	0.506	14.7	7.7	0.523	14.8	7.7	0.519	14.9	7.7	0.517
3	0.0	0.0	-	0.0	0.0	-	0.0	0.0	-	0.0	0.0	-	0.0	0.0	-

TABLE 20

G1

SINGLE (NEVER MARRIED)

C(0-1) C(1-2) C(2-3) C(3-4) C(1-3) C(2-4)
1.28 1.18 0.48 0.46 0.94 0.03

2	792.8	54.6	0.069	779.8	51.9	0.067	776.2	51.5	0.066	779.7	51.5	0.066	776.1	51.5	0.066
1	772.2	55.4	0.072	769.7	52.3	0.068	758.6	50.9	0.067	762.4	51.4	0.067	758.9	50.9	0.067
3	251.5	37.0	0.147	245.7	35.7	0.139	252.2	35.1	0.138	253.4	35.1	0.138	252.2	35.1	0.139

TABLE 21. The design effect $D_p^u = \frac{SE_p^C}{SE_0^S}$ for 6 categories in an average size WA. SE_p^S and SE_p^C are

estimates of the standard errors of $\hat{y}(p)$ under s.r.s. and cluster sampling respectively.

Categories	Iteration 0			Iteration 1		Iteration 2		Iteration 3		Iteration 4	
	SE ₀ ^s	SE ₀ ^c	D ₀ ^u	SE ₁ ^c	D ₁ ^u	SE ₂ ^c	D ₂ ^u	SE ₃ ^c	D ₃ ^u	SE ₄ ^c	D ₄ ^u
A1 Employed	48.0	48.4	1.008	37.8	.788	34.7	.723	35.0	.729	34.7	.723
A4 Not Moved in Last 5 Years	47.1	78.2	1.660	66.5	1.412	61.7	1.310	62.1	1.318	61.6	1.308
A6 Highest Grade is 1 to 8	27.6	32.9	1.192	29.6	1.072	28.0	1.014	28.1	1.018	28.0	1.014
A9 Bachelor Degree or Higher	22.8	25.4	1.114	24.8	1.088	24.3	1.066	24.3	1.066	24.3	1.066
B2 Mother Tongue German	10.9	17.4	1.596	14.9	1.367	14.8	1.358	14.7	1.349	14.7	1.349
C1 Single (Never Married)	48.9	43.0	.879	12.1	.247	5.5	.112	9.7	.198	5.4	.110

BIBLIOGRAPHY

- [1] Arora, H.R., and Brackstone, G.J. (1977). An investigation of the properties of raking ratio estimators with simple random sampling. To appear in Survey Methodology.
- [2] Arora, H.R., and Brackstone, G.J. (1977). An investigation of the properties of raking ratio estimators with cluster sampling (unpublished manuscript).
- [3] Bishop, Y.M.M., and Fienberg, S.E. (1969). Incomplete two-dimensional contingency tables. Biometrics 25 119-128.
- [4] Brackstone, G.J., and Rao, J.N.K. (1976). Raking ratio estimators. Survey Methodology 2 63-69.
- [5] Deming, W.E., and Stephan, F.F. (1940). On a least squares adjustment of a sampled frequency table when the expected marginal totals are known. Ann. Math. Statist. 11 427-444.
- [6] El-Badry, N.A., and Stephan, F.F. (1955). On adjusting sample tabulations to census counts. J. Amer. Statist. Assoc. 50 738-762.
- [7] Fienberg, S.E. (1970). An iterative procedure for estimation in contingency tables. Ann. Math. Statist. 41 907-917.
- [8] Friedlander, D. (1961). A technique for estimating a contingency table, given the marginal totals and some supplementary data. J. Roy. Statist. Soc., Ser. A 124 412-420.

- [9] Goodman, L.A. (1968). The analysis of cross-classified data: independence, quasi-independence, and interactions in contingency tables with or without missing entries. J. Amer. Statist. Assoc. 63 1091-1131.
- [10] Ireland, C.T., and Kullback, S. (1968). Contingency tables with given marginals. Biometrika 55 179-188.
- [11] Rao, J.N.K. (1974). Raking ratio estimators. Technical Report, Statistics Canada.
- [12] Rao, J.N.K. (1974). Raking ratio estimators, II. Technical Report, Statistics Canada.
- [13] Rao, J.N.K. (1974). Raking ratio estimators, III. Technical Report, Statistics Canada.
- [14] Silvey, S.D. (1959). The lagrangian multiplier test. Ann. Math. Statist. 30 389-407.
- [15] Smith, J.H. (1947). Estimation of linear functions of cell proportions. Ann. Math. Statist. 18 231-254.
- [16] Stephan, F.F. (1942). Iterative method of adjusting sampling frequency tables when expected marginal totals are known. Ann. Math. Statist. 13 166-178.

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FIN