Stress Intensity Factors and Weight Functions for Cracks in Fastener Holes

By

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Master of Applied Science (Engineering)

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of
Mechanical and Aerospace Engineering
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Abstract

General solutions of stress intensity factors for through-the-thickness cracks including single and double symmetric cracks and a corner crack at a fastener hole under arbitrary loading conditions are derived using weight function methods. The derived weight function solutions are obtained from the assumed approximated general weight functions and two reference stress intensity factors. For through cracks, the stress intensity factor weight function solutions are valid over the range of crack length over hole radius ($0.2 \leq c/R \leq 1.5$) and are verified against the existing results for quadratic and cubic stress distributions on the crack face as well as finite element data for remote tension loading. For a corner crack, the derived weight function solutions are valid for the range of relative crack depths (a/t) between 0.2 and 0.8, crack aspect ratio (a/c) between 0.5 and 1.5 and R/t (the ratio of hole radius over plate thickness) of 0.5. The weight functions were verified using existing results for quadratic and cubic stress distribution on the crack face and remote tension loading.

Applications of the derived weight function solutions are performed to predict the fatigue crack growth behavior under remote tension with and without cold working for a cracked fastener hole. The solutions derived in this thesis are suitable for the fracture assessment and fatigue crack analysis for through and corner cracked fastener holes.

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Nomenclature

I'	integral contour
$\Delta\alpha, \Delta l$	virtual crack extension
δ_{ij}	Kronecker symbol
$\delta l(s)$	local crack front advance at point s
δU	change in the total potential energy
$\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$	strain components
θ	angle of polar coordinates
ν	poisson's ratio
a_0, a_1, b_0, b_1, b_2	constants
c_0, c_1, d_0, d_1, d_2	constants
ξ, η	local coordinates for isoparametric element
$\sigma(x)$	local stress distribution normal to the prospective crack plane
σ_0	characteristic stress or nominal stress
Φ	parametric angle of a corner crack
A	crack depth for a corner crack
A	hole surface point of a corner crack
$A_{10}, A_{20}, A_{30}, A_{40}$	parameters for F_{A0}
$A_{11}, A_{21}, A_{31}, A_{41}$	parameters for F_{A1}
$[B], [B]_I$	B matrix of finite elements
B	front surface point of a corner crack

$B_{10}, B_{20}, B_{30}, B_{40}$	parameters for F_{B0}
$B_{11}, B_{21}, B_{31}, B_{41}$	parameters for F_{B1}
C	crack length for through cracks and a corner crack
$[D]$	stress-strain matrix
$Det J $	determinant of Jacobin matrix
E	modulus of elasticity
F_{A0}	reference boundary correction factor by curve fitting for point A
F_{A1}	reference boundary correction factor by curve fitting for point A
F_{B0}	reference boundary correction factor by curve fitting for point B
F_{B1}	reference boundary correction factor by curve fitting for point B
F_n	boundary correction factors
F_{P0}	reference boundary correction factor by curve fitting for local point P
F_{P1}	reference boundary correction factor by curve fitting for local point P
G	strain energy release rate, or a function depending on geometry in weight function
H	general elastic modulus
J, J_N	integral, J-integral values at node N
$[J]$	Jacobian matrix
K	stress intensity factor
K_{r0}, K_{1r}	reference stress intensity factors
$L_{i0}, K_{i0}, H_{i0}, Q_{i0}, P_{i0}$	parameters for F_{P0}
$L_{i1}, K_{i1}, H_{i1}, Q_{i1}, P_{i1}$	parameters for F_{P1}
$m(x, c), m_A(x, c),$	weight function

$m_B(x, c)$	
M_1, M_2, M_3	curve fitting parameters from Newman
M_{1l}, M_{12}, M_{13}	weight function parameters for through cracks
M_{1A}, M_{2A}, M_{3A}	weight function parameters for hole surface point A of a corner crack
M_{1B}, M_{2B}, M_{3B}	weight function parameters for front surface point B of a corner crack
M_{1P}, M_{2P}, M_{3P}	weight function parameters for local point P of a corner crack
n_i, m_i, l_i	outer normal
N_i, N_I	shape function corresponding to element's i or I node
P	local point of a corner crack
q, q_k	smooth function in domain integral
r	radius from crack tip
R	hole radius
SIF	stress intensity factor
t	plate thickness
T	stress work
T_i	components of traction vector
U	potential energy
u_i	displacement vector components or nodal displacement in x-direction for node i
U_r	reference crack opening displacement field
v_i	nodal displacement in y-direction for node i
w	strain energy density, weight in Gauss integration
W, L	plate width, pipe width

w_i	weights in Gauss integration
x_i	x, y, z axis or nodal x-direction coordinate
y_i	nodal y-direction coordinate
Y_0, Y_I	reference stress intensity factors for double through cracks

Chapter 1

Introduction

In linear elastic fracture mechanics, the stress intensity factor, K , characterizes the stress field in the vicinity of a crack tip in a linear elastic material. Because the stress intensity factor plays an important role in fracture mechanics, the criterion for failure due to unstable crack growth in a brittle material is dependent on this parameter and fatigue growth rate can be predicted in ductile materials using this parameter. In addition, many ductile fracture assessment procedures presented in terms of a failure assessment diagram requires the calculation of the elastic stress intensity factor. Therefore, the most important aspect in fracture and fatigue analysis is to find the solution of stress intensity factor for the cracked geometry under consideration.

In practice, many failures in engineering structures are due to fatigue cracks initiating and developing from fastener holes at which there are large stress concentrations. The United States Air Force, for example, has adopted fracture mechanics based design criteria that require new aircraft to resist failure from pre-existent cracks assumed present at fastener holes [1]. Such analyses require accurate stress intensity factor (K_I) solution for the reliable prediction of fracture and fatigue growth. The present research will focus on finding stress intensity factor solutions for through-the-thickness cracks and quarter elliptical cracks at a fastener hole in a finite thickness plate as shown in Figure 1.1 and Figure 1.2.

Stress intensity factor for through cracks and corner cracks at a fastener hole depend on the crack geometry and the applied loading type. The geometrical parameters for through cracks at a hole in a plate that affect the stress intensity factor solutions are the ratio, c/R , where c is crack depth, R is the radius of the hole. Also the parameters for corner cracks at a hole in a fastener hole in a finite thickness plate are the aspect ratio, a/c , and the relative crack depth, a/t , where a is the crack length, and t is the thickness. The details and parameters of geometry for the through and corner cracks are shown in

Figure 1.3 and Figure 1.4. The applied loads may be idealized as an arbitrary crack face pressure in a manner which allows the results to be applied to many other practical loading conditions. The goal of this thesis is to generate general stress intensity factor solutions using weight function method for a wide range of geometrical parameters and arbitrary nonuniform loading conditions.

Due to the complexity of the problem, exact solutions are not available, and approximate methods must be used. Although there are many approximate methods which can be used to solve this problem, the most accepted stress intensity factor solutions for the present problem were obtained using the finite element method. Grandt and Kullgren [2] presented the stress intensity factor solution for through-the-thickness, corner cracks flawed hole. The accuracy of their solutions were verified by other results using finite element-alternation method [3].

For through cracks at a fastener hole in a plate, Grandt and Kullgren [2] calculated the stress intensity factor for a single and double cracked hole loaded with a crack face pressure with the ratios, c/R , of 0.2 to 0.5. For corner cracks at a fastener hole in a plate, Grandt and Kullgren [4] calculated the stress intensity factor single corner cracked hole loaded with crack face pressure with aspect ratios, a/c , of 0.5, 0.9, 1.1, 1.5 and relative depths, a/t , of 0.2, 0.4, 0.5, 0.6, 0.8 and c/R of 0.5.

Because these results of stress intensity factors were presented for one geometry at a time, for the purpose of present research, interpolation of these data is required to obtain a general solution for the entire region of a/c between 0.5 and 1.5 and a/t between 0.2 and 0.8 for corner cracks with R/t ratio of 0.5 and the entire region of c/R between 0.2 and 1.5 for through cracks. The curve fitting is used to obtain the general expressions.

Stress intensity factors also depend on loading conditions, and the finite element method can only solve one loading condition at a time. The results from finite element methods are therefore limited to a few simple loading conditions. Although superposition from Grandt and Kullgren [2] [4] results can be used when the loading condition can be

decomposed into the sum of simple loading conditions whose stress intensity factor solutions are available, the most efficient method to calculate stress intensity factor for complex stress distributions is the weight function method.

The weight function method was developed for the calculation of stress intensity factors by Bueckner [5] and Rice [6] in the early 1970's. Mathematically, the weight function is the Green function for the stress intensity factor problem, which can be interpreted as the stress intensity factor corresponding to a unit point load applied to the crack surface. The weight function is only dependent on crack geometry. The stress intensity factor can be calculated by the integration of the product of the weight function and the stress distribution acted on the prospective crack plane. Once the weight function is known for a particular geometry, the stress intensity factor from a variety of loading conditions can be calculated.

There are several ways to obtain the weight function. Rice [6] found that the weight function could be derived from a reference stress intensity factor solution for a specific geometry and the corresponding crack opening displacement fields. Although appropriate reference stress intensity factors can be found for much geometry, the crack opening displacement field is more difficult to obtain because it is a field function changing along crack length and is seldom published together with stress intensity factor solutions. To overcome this difficulty, many authors proposed approximate weight function forms from which the weight function can be obtained from reference stress intensity factor solutions alone. Recently, Shen and Glinka [7], Wang and Lambert [8] found general forms that can be used to approximate weight functions for a variety of two and three-dimensional cracks. Based on these general forms, the weight function for a given geometry can be obtained from two reference stress intensity solutions corresponding to two different crack face stress distributions and one boundary condition for each weight function considered. This approach has been successfully applied to derive weight functions for semi-elliptical surface cracks [8], quarter elliptical corner cracks in a plate [9]. In the current research, it is used to obtain weight functions for through cracks and corner cracks at a fastener hole.

1.1 Outline of Thesis

The objective of this thesis is using weight function method to derive general solutions of stress intensity factors for through-the-thickness cracks and quarter elliptical cracks at a fastener hole in a finite thickness plate with wide range of a/c , a/t , and c/R ratios for arbitrary loading conditions.

The weight function solutions for single and double symmetric through cracks at a fastener hole in a plate were provided. The results are validated by two dimensional finite element analyses of the same crack geometry under remote tension loading conditions as well as the existing results under parabolic and cubic loading on the crack face with c/R of 0.2, 0.5, 1.0 and 1.5.

For a corner crack at a fastener hole in a finite thickness plate, weight function solutions are obtained for any point along the crack front by combining the weight function approach both for surface points with $\phi = 0$, $\phi = \pi/2$ and local points with $0 < \phi < \pi/2$. The derived weight functions are verified by the existing results with parabolic and cubic stress distribution on the crack face and Newman data under remote tension loading. These weight functions are valid over a range of aspect ratio, $0.5 \leq a/c \leq 1.5$, and relative crack depth, $0.2 \leq a/t \leq 0.8$ with $R/t = 0.5$.

As the application of the developed weight function solutions, for single through crack and a corner crack at a fastener hole, a fatigue crack propagation analysis is performed to predict the crack growth behavior under remote tension loading with and without introduced cold working process.

There are seven chapters included in this thesis. In chapter 2, an overview of the calculation of stress intensity factors using the weight function method is presented.

Chapter 3 presents the weight function solutions for single and double through cracks at a fastener hole. Chapter 4 and 5 give the weight function solutions for the surface and local points of a corner crack at a fastener hole. Chapter 6 shows the applications of the weight function solutions for the fatigue analysis of a through cracked and corner cracked fastener hole in a plate. Conclusions are made in Chapter 7 along with some recommendations.

Figure 1.1 Through cracks at a fastener hole (3-D view).

Figure 1.2 One corner crack at a fastener hole in a finite thickness plate (3-D view).

Figure 1.3 Single or double symmetric through cracks at a fastener hole.

Figure 1.4 Single or double quarter elliptical cracks at a fastener hole in a finite thickness plate.

Chapter 2

Calculation of Stress Intensity Factors

In this chapter, an overview on the calculation of stress intensity factors using the finite element method and the weight function approach is presented.

2.1 Stress Intensity Factors

There are three types of displacement models in a loaded and cracked body shown in Fig. 2.1. Mode I is called the opening mode caused by tensile loading normal to crack face plane and consists of the crack faces simply moving apart. Mode II, the sliding mode, the crack face slides relative to one another in a direction normal to the leading edge of the crack. Mode III, the tearing mode, also involves relative sliding of the crack faces with the moving direction parallel to the leading edge. Most cracking problems of engineering interest involve primarily Mode I and are due to normal stress. In this thesis, only Mode I will be discussed.

2.1.1 Stress Intensity Factor

The stress intensity factor K characterizes the magnitude (intensity) of the stress in the vicinity of an ideally sharp crack tip in a linear elastic, isotropic material. A coordinate system for describing the stresses in the vicinity of a crack is shown in Figure 2.2. The polar coordinates r and θ lie in the x - y plane, which is normal to the plane of the crack, and the z -direction is parallel to the leading edge of the crack. For any case of Mode I loading, the stresses near the crack tip depend on r and θ as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \begin{bmatrix} 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \\ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \\ \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \end{bmatrix} + \text{other terms} \quad (2.1)$$

The stresses in the third direction are given by

$$\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0 \quad (\text{plane stress})$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \quad , \quad \tau_{xz} = \tau_{yz} = 0 \quad (\text{plane strain}) \quad (2.2)$$

As r approaches zero, the first term in Eq. (2.1) dominates, while the other terms approach zero or constants. Mathematically the stresses are thought to be a square root singularity. The parameter K_I is the stress intensity factor. Since the stress intensity factor characterizes the stress field for the cases where crack tip plasticity is small, it may be used in the determination of residual strength (for brittle materials) and fatigue crack growth.

In the present research, the main objective is to calculate the stress intensity factors for through cracks and corner cracks at a fastener hole in a finite thickness plate as shown in figures 1.3 and 1.4.

Stress intensity factors for through cracks under tension stress field can be determined by the general form used by most previous researches as follows:

$$K_I = \sigma_0 \sqrt{\pi c F} \quad (2.3)$$

Stress intensity factors vary along the crack front for corner cracks. Stress intensity factors for corner cracks under tension stress field are always normalized as follows:

$$K_I = \sigma_0 \sqrt{\pi c/Q} F \quad (2.4)$$

Where F is called the boundary correction factor, c is the crack depth shown in Figure 1.3 and 1.4. σ_0 is nominal stress and Q is the shape factor of a ellipse and is given by the square of the complete elliptic integral of second kind. The following empirical equation for Q is used [10]:

$$Q = 1.0 + 1.464 \left(\frac{a}{c} \right)^{1.65} \quad (a/c \leq 1) \quad (2.5)$$

$$Q = 1.0 + 1.464 \left(\frac{a}{c} \right)^{-1.65} \quad (a/c > 1) \quad (2.6)$$

One thing should be emphasized that the $r^{-1/2}$ singularity which appears in equation (2.1) vanishes at the intersection of three free surfaces [11] such as points A and B shown in Figure 1.4. That is the “ $r^{-1/2}$ ” occurs only near crack front points embedded entirely in the material. However, it was shown by Sih and Lee [11], for engineering materials with a Poisson ratio, $\nu=0.3$, the dominant singularity near the surface point of a surface crack was $r^{-0.4523}$, which in practical terms does not represent a dramatic departure from the $r^{-1/2}$ singularity.

The stress intensity factor can be determined directly from the local crack tip stress field, Eq. (2.1), or the corresponding strain and displacement field. It can also be obtained indirectly through other fracture parameters such as the strain energy release rate, G , or the J -integral, which are uniquely related to the stress intensity factor for linear elastic materials.

2.1.2 Strain Energy Release Rate and J -integral

The strain energy release rate G and the J -integral are energy based parameters in fracture mechanics. For a linear elastic material, they are directly related to the stress intensity factor K . If one of these three parameters can be obtained, the other two can be known from the relationship between them.

The strain energy release rate, G , proposed by Griffith [12], which is defined as the rate of change in potential energy of a cracked body with an increase in crack area for linear elastic materials shown in Figure 2.3, can be expressed as

$$G = -\frac{dU}{dA} \quad (2.7)$$

Where U is the potential energy stored in the member, and A is the cracked area.

The change in crack area dA , is $t da$, where a and t are the crack length and specimen thickness, respectively. Thus, G characterizes the energy per unit crack area available to extend the crack. The relationship between K and G is:

$$G = \frac{K^2}{H} \quad (2.8)$$

where H is the general elastic modulus:

$$H=E \quad (\text{plane stress})$$

$$H=E/(1-\nu^2) \quad (\text{plane strain})$$

with E , modulus of elasticity, and ν , Poisson's ratio.

The J -integral was proposed by Rice [13] to characterize the crack tip stress field in non-linear elastic material. Consider an arbitrary counter-clockwise path (Γ) around the tip for a crack, as illustrated in figure 2.4. The J -integral is given by

$$J = \int_{\Gamma} (w dy - T_i \frac{\partial u_i}{\partial x} ds) \quad (2.9)$$

Where w is the strain energy density, T_i are components of the traction vector, u_i are the displacement vector components, and ds is increments of length s along the contour Γ . Rice showed that the value of the J -integral is independent of the path of integration around the crack. Thus J is a path independent integral. Also this integral can be considered as an energy release rate for nonlinear elastic materials. Equation (2.7) defines the energy release rate for linear materials. The same definition holds for nonlinear elastic materials, except that G is replaced by J :

$$J = -\frac{dU}{dA} \quad (2.10)$$

If the material considered is linear elastic, the J -integral is equal to the strain energy release rate, G :

$$J=G \quad (\text{for linear elastic materials}) \quad (2.11)$$

Thus, the J -integral is related to the stress intensity factor by Eq. (2.8)

2.1.3 Methods of Calculation Stress Intensity Factors

Over the past forty years, many methods of obtaining stress intensity factor solutions have been developed. These can be divided into three categories depending on their degree of sophistication and the time required to obtain a solution (Aliabadi and Rooke [14]):

Level one: Handbooks

Level two: Superposition, Stress Concentration, Stress Distribution, Green's Function, Approximate Weight Function and Compounding.

Level three: Collocation (Mapping), Integral Transformation, Body Force Method, Alternating Technique, Boundary Element Method, Line Spring Method, Finite Element Method.

The stress intensity factor depends on the geometrical configuration of a cracked body and the loading condition. For simple geometrical configurations and loading conditions, it is possible to use handbooks ([15], [16], [17]). Where a solution cannot be obtained directly from a handbook then, one of the relatively simple methods in level two may be adequate. They seldom require more than a few hours to obtain stress intensity factor values. The accuracy of the methods in level two depends on the accuracy of the method itself and the accuracy of the reference stress intensity factors used. However, for complicated problems, where very accurate stress intensity factors are important, numerical methods in level three are needed. Among them, the finite element method is most commonly applied.

The combination of finite element calculations and the weight function approach is a good compromise. Normally the weight function solution of stress intensity factor can be derived from the reference stress intensity factors calculated by the finite element method for simple loading conditions. In the current research, the combination of weight function and finite element results are used for through cracks at a fastener hole in a finite thickness plate.

In the next two sections, the application of the finite element method and the weight function approach are discussed.

2.2 Finite Element Method

The finite element method (FEM) has emerged over the past two decades as one of the most powerful numerical tools for the solution of crack problems in fracture mechanics. Special techniques are employed to treat fracture mechanics problems. In general, the behavior of the displacement field in common finite elements is at most quadratic or cubic, so that the stress field is at most linear or quadratic [18]. On the other hand, the stresses in the neighborhood of a crack tip in a linear elastic material are singular. Therefore, in order to properly model the stresses in the region adjacent to the crack top, singular quarter-point, isoparametric elements were employed in the present research. The stress intensity factor must be extracted from either the stress or displacement field or by energy based method. In the present research, K is calculated from the J -integral, which is obtained using the domain integral method.

2.2.1 Modeling the Crack Tip Stress Field

Stresses within regular finite elements are generally represented by functions containing at most quadratic terms ([19] and [18]). These elements are unable to adequately describe the singular behavior of stresses in the neighborhood of a crack tip. Because singular elements are much more efficient as mentioned above, for two dimensional through cracks in present research, eight noded singular elements (collapsed rectangular) were employed at the crack tip.

Two dimensional eight noded (collapsed) singular elements

The information of the isoparametric elements is well documented ([18]). For a two dimensional eight node isoparametric element as shown in figure 2.5, the geometry of the element is mapped into the normalized square space (ξ, η) , $(-1 \leq \xi \leq 1, -1 \leq \eta \leq 1)$ through the following transformation

$$\begin{aligned}
x &= \sum_{i=1}^8 N_i(\xi, \eta) x_i \\
y &= \sum_{i=1}^8 N_i(\xi, \eta) y_i
\end{aligned} \tag{2.12}$$

where N_i is the shape function corresponding to node i , whose co-ordinates are (x_i, y_i) in the x - y coordinate system and (ξ, η) in the transformed ξ - η system. $(\xi, \eta = \pm 1)$ for corner points and zero for mid-side node:

$$\begin{aligned}
N_i = & [(1+\xi \xi_i)(1+\eta \eta_i)-(1+\eta \eta_i)(1-\xi^2)] \xi_i^2 \eta_i^2 / 4 + \\
& (1-\xi^2)(1+\eta \eta_i)(1-\xi^2) \eta_i^2 / 2 + (1-\eta^2)(1+\xi \xi_i)(1-\eta^2) \xi_i^2 / 2
\end{aligned} \tag{2.13}$$

The displacement are interpolated of follows:

$$\begin{aligned}
u &= \sum_{i=1}^8 N_i(\xi, \eta) u_i \\
v &= \sum_{i=1}^8 N_i(\xi, \eta) v_i
\end{aligned} \tag{2.14}$$

The strain in the element are determined from

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = [B] \begin{Bmatrix} u_1 \\ v_1 \\ . \\ . \\ u_8 \\ v_8 \end{Bmatrix} \tag{2.15}$$

$$\text{where } [B] = [[B]_1, [B]_2, \dots, [B]_8] \tag{2.16}$$

with

$$[B] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad (2.17)$$

and

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \quad (2.18)$$

J is the Jacobian matrix and is given by

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (2.19)$$

The strain in Eq. (2.15) can be rewritten in a form by combining Eq. (2.15) , (2.16), and (2.18) as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [J]^{-1} [B(\xi, \eta)] \begin{Bmatrix} u_1 \\ v_1 \\ \cdot \\ \cdot \\ \cdot \\ u_8 \\ v_8 \end{Bmatrix} \quad (2.20)$$

The stress is given by:

$$\{\sigma\} = [D]\{\varepsilon\} \quad (2.21)$$

where $[D]$ is the stress-strain matrix, i.e., Hook's law for elastic materials.

The $N_i(\xi, \eta)$ are polynomials and hence, from Eq. (2.15), $\partial N_i / \partial \xi$ and $\partial N_i / \partial \eta$ are non-singular. For normal elements, the determinant of Jacobian $[J]$, $\det [J]$ is not zero anywhere in the element, and the stress and strain fields are linear. Therefore, this element cannot be used directly to model the stress (strain) singularity at the crack tip. However, Barsoum [20] and Henshell and Shaw [21] independently observed that by moving the mid-side node for an eight noded isoparametric quadrilateral element to a quarter point position, the determinant of Jacobian, $\det [J]$ becomes zero at crack tip, and the desired $1/\sqrt{r}$ variation in strains and stresses can be achieved. Barsoum however, showed that the $1/\sqrt{r}$ variation for strains is only achieved along the boundary of the element for such elements. To achieve the $1/\sqrt{r}$ singularity along any ray that emanates from the node at the crack tip in the interior of the element, he collapsed the quadrilateral element into a triangular element by coalescing nodes along one side and constraining them to have the same displacement, as shown in Figure 2.6.

2.2.2 Extracting Stress Intensity Factors

Two methods were used to determine the stress intensity factor once having the stress and displacement fields in a cracked body, the direct and indirect methods. The most commonly used direct method is displacement extrapolation [23]. Strain energy rate, G , or the J -integral from energy change is employed in indirect method. In present research, J -integral calculation is used to extract the stress intensity factors.

For linear elastic materials, the stress intensity factor K can be obtained by using following relationship between K and J ,

$$K = \sqrt{\frac{JE}{(1-\nu^2)}} \quad (\text{plane strain}) \quad (2.22)$$

$$K = \sqrt{JE} \quad (\text{plane stress}) \quad (2.23)$$

In present calculation of stress intensity factor using finite element method for through crack, equation (2.22) is used.

2D Domain Integral Formulation

The J -integral numerical analysis was found by Shih [22] using the energy domain integral methodology which can be applied to both quasi-static and dynamic problems with elastic or elastic-plastic material and accommodate crack face traction and thermal loading. Moreover, the domain integral formulation [22] is relatively simple to implement numerically, and it is very efficient.

Equation (2.9) is limited to quasi-static conditions, requires the absence of body forces and crack faces free of traction. In order to remove these limitations, Shih

proposed the generalized expression of J -integral which the contour surrounding the crack tip, Γ_0 , be vanishingly small:

$$J = \lim_{\Gamma_0 \rightarrow 0} \int_{\Gamma_0} \left[(w + T)n_1 - \sigma_{ij}n_i \frac{\partial u_j}{\partial x_1} \right] ds \quad (2.24)$$

where w and T are the stress work and kinetic energy densities, respectively, σ_{ij} and u_i are the components of the stress and displacement, n_i are the unit vectors normal to Γ_0 , and ds is the arc length as shown in Figure 2.7.

Under quasi-static conditions ($T=0$), Eq. (2.22) becomes

$$J = \lim_{\Gamma_0 \rightarrow 0} \int_{\Gamma_0} \left[w\delta_{li} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right] n_i ds \quad (2.25)$$

where $i=1,3$ represent the directions of axis x, y and z , while L represent the nodal point in each element, this convention is slightly different with the one used in section 2.2.1.

Equation (2.25) can be used in a material with the plastic deformation and its body forces, thermal strains and crack face tractions being ignored. This equation is path independent which means its value does not depend on the crack tip contour Γ_0 and Eq. (2.25) becomes Eq.(2.9). If any of these conditions is not satisfied, Eq. (2.25) must be used.

Because the form of Eq.(2.25) is not suitable for engineering calculation, following numerical analysis is developed to evaluate the stresses and strains along a vanishingly small contour.

Referring to figure 2.7, considering a closed contour Γ^* connecting inner Γ_0 and outer Γ_l contour, the outer contour, Γ_l is finite, while Γ_0 is vanishingly small. Γ_- and Γ_+ are the lower and upper crack faces, so that

$$\Gamma^* = \Gamma_I + \Gamma_+ + \Gamma_- + \Gamma_0$$

therefore

$$\Gamma_0 = -\Gamma^* + \Gamma_I + \Gamma_+ + \Gamma_-$$

For linear and nonlinear elastic materials, Li [24] rewrite Eq.(2.25) in the form:

$$J = \lim_{\Gamma_0 \rightarrow 0} \int_{\Gamma_0} \left[w \delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x_1} \right] m_i q ds - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j} \frac{\partial u_j}{\partial x_1} q ds \quad (2.26)$$

where q is an arbitrary but smooth function that is equal to unity on Γ_0 and zero on Γ_I .

Note that $m_i = -n_i$ on Γ_0 and $m_j = 0$ and $m_2 = \pm 1$ on Γ_+ and Γ_- . The second term in (2.26) is the contribution due to crack surface tractions.

Applying the divergence theorem to the closed contour integral in Eq.(2.26) :

$$J = \int_{A^*} \frac{\partial}{\partial x_i} \left\{ (w \delta_{1i} - \sigma_{ij} \frac{\partial u_j}{\partial x_1}) q \right\} dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j} \frac{\partial u_j}{\partial x_1} q ds \quad (2.27)$$

where A^* is the annular area enclosed by Γ^* . Invoking equilibrium for a linear elastic material under quasi-static conditions in the absence of thermal strain and body force, Eq.(2.27) becomes the domain integral expression for the energy release rate,

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w \delta_{1i} \right] \frac{\partial q}{\partial x_i} dA - \int_{\Gamma_+ + \Gamma_-} \sigma_{2j} \frac{\partial u_j}{\partial x_1} q ds \quad (2.28)$$

Equation (2.28) is employed in finite element calculation to obtain the J -integral in two dimensions.

The inner contour, Γ_0 is taken as the crack tip, in which A^* corresponds to the area inside of Γ_I . The boundary, Γ_I should coincide with element boundaries. The q function must be specified at all nodes within the area. The shape of the q function is arbitrary, as long as q has the correct values on the domain boundaries. In a plane stress or plane strain problem, $q=1$ at Γ_0 (crack tip) and $q=0$ at the outer boundary. The function q can be obtained from nodal virtual crack extension [25] [26]. Banks-Sills [27] showed that the q function can be interpreted as a normalized virtual displacement: $q(x,y)=\Delta u(x,y)/\Delta a$, although such an interpretation is not required in the derivation of the domain integral formulation. Here Δa is the crack tip virtual extension, and $\Delta u(x,y)$ is the displacement within the area A^* in x_I direction caused by this virtual crack extension. Figure 2.8 shows two common examples of q functions, with their corresponding virtual nodal displacements. The pyramid function (Figure 2.8(a)) is equal to 1 at the crack tip but varies linearly to zero in all directions, while the plateau function (Figure 2.8(b)) equals 1 in all region except the outer ring of elements. Shih have shown that the computed value of J is insensitive to the assumed shape of the q function.

The value of q within an element can be interpolated as :

$$q(x_i) = \sum_{I=1}^L N_I(\xi, \eta) q_I \quad (2.29)$$

where L is the number of nodes per element, q_I are the nodal values of q and N_I are the element shape functions. Then the derivative of q is given by

$$\frac{\partial q}{\partial x_j} = \sum_{I=1}^L \left(\frac{\partial N_I}{\partial \xi} \frac{\partial \xi}{\partial x_j} + \frac{\partial N_I}{\partial \eta} \frac{\partial \eta}{\partial x_j} \right) q_I \quad (2.30)$$

and the discretized form of the domain integrals as follows:

$$J = \sum_{\substack{\text{all elements} \\ \text{within } A}} \sum_{p=1}^{NG} \left\{ \left[\sigma_{ij} \frac{\partial u_j}{\partial x_i} - w \delta_{ij} \right] \frac{\partial q}{\partial x_i} \right\}_p w_p - \sum_{\substack{\text{all edges on} \\ \Gamma_+ + \Gamma_-}} \left\{ t_j \frac{\partial u_j}{\partial x_i} q \right\} w \quad (2.31)$$

The quantities within $\{ \}_p$ are evaluated at the Gauss integration points $p=1, NG$, and w_p and w are the respective weights at these points. The second term in (2.31) is the contribution due to crack face traction t_j . Eq.(2.31) is the numerical formulation for J used in the two dimensional finite element calculation.

2.3 Weight Function Method

In the present research, the weight function method was used to determine stress intensity factor solutions under arbitrary loading systems. The weight functions were determined using reference stress intensity factor solutions obtained using from [2], [4].

2.3.1 Theoretical Background

Bueckner [5] first demonstrated that for a cracked body as shown in Figure 2.9(a), loaded by a stress field S , the stress intensity factor is the same as the cracked body loaded by crack surface pressure, $\sigma(x)$ is the stress distribution acting on the prospective crack plane for the uncracked geometry. Therefore, the calculation of the stress intensity factor for a given crack geometry under any applied load is equivalent to the calculation of the stress intensity factor for the same crack geometry with the corresponding crack pressure $\sigma(x)$ acting on the crack surface. The stress intensity factor for a cracked body with loading on the crack surface can be calculated by integrating the product of the weight function $m(x,a)$ and the stress distribution $\sigma(x)$ in the crack plane:

$$K = \int_0^a \sigma(x) m(x, a) dx \quad (2.32)$$

The weight function $m(x, a)$ depends on the geometry of the cracked body. Once the weight function has been determined, the stress intensity factor for this geometry can be obtained from Eq.(2.32) for any stress distribution $\sigma(x)$.

Mathematically the weight function $m(x, a)$ is a generalization of Green's function for the present stress intensity factor problem. Green's function was defined by Green in 1828 as the response of a system to a impulse. The weight function is the stress intensity factor for a pair of unit point loads on the crack surface as shown in Figure 2.9 (c).

Bueckner [4] found that the weight functions could be interpreted as normalized displacement fields as well, which satisfy the equations of linear elasticity, but have a singularity at the crack tip which normally would not be admissible as a physical field. Bueckner refers to this as the “fundamental field”. Based on the “fundamental field”, the direct determination of the weight function involves a complex elastic analysis of the cracked body. However, the determination of the weight functions, $m(x, a)$, can be simplified by using the following relationship derived by Rice [6]:

$$m(x, a) = \frac{H}{K} \frac{\partial u_r(x, a)}{\partial a} \quad (2.33)$$

where H is the generalized elastic modulus, which equals E for plane stress or $E/(1-\nu^2)$ for plane strain. In order to determine weight function from equation (2.33), a reference stress intensity factor, K_r , has to be known for a specific geometry and applied loading system in addition to the corresponding crack opening displacement field, $u_r(x, a)$.

For a given problem, an appropriate reference stress intensity factor can be found either in the literature or by numerical calculation. However, it is difficult to obtain the corresponding analytical expression for the crack opening displacement function, $u_r(x, a)$. To overcome this difficulty, some authors have proposed approximate expressions for the

displacement, $u_r(x, a)$ and the weight function, $m(x, a)$, which can be evaluated when combined with only reference stress intensity factors.

2.3.2 Approximate Weight Functions

Petroski-Achenbach Method

In Eq.(2.33), Petroski [28] gave an approximate expression of $u_r(x, a)$ for plane problem:

$$u_r = \frac{\sigma_0}{H\sqrt{2}} \left[4F\sqrt{a(a-x)} + \frac{G(a-x)^{\frac{3}{2}}}{\sqrt{a}} \right] \quad (2.34)$$

where σ_0 is the nominal stress for the specific problem defined using, $\sigma_r(x) = \sigma_0 p(x)$ and $p(x)$ is the nondimensional reference stress distribution, and F is the geometry correction factor for the reference stress intensity factor:

$$F = \frac{K_r}{\sigma_0 \sqrt{\pi a}} \quad (2.35)$$

Thus, with the known K_r , the function G in equation (2.34) that depends only on the geometrical parameters was found from the self-consistency of equation (2.32) by [28].

Setting $K=K_r$, $\sigma(x) = \sigma_r(x)$ and substituting from (2.33) for $m(x, a)$, K_r can be expressed by:

$$K_r = \frac{H}{K_r} \int_0^a \sigma_r(x) \frac{\partial u_r(x, a)}{\partial a} dx \quad (2.36)$$

Substituting Eq.(2.34) into (2.36), the function G can be solved. After G is determined and substituted into Eq.(2.34) and then Eq.(2.33), this leads to the following weight function:

$$m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a}\right) + M_2 \left(1 - \frac{x}{a}\right)^2 \right] \quad (2.37)$$

where,

$$M_1 = \frac{8 \frac{\partial F}{\partial a} + 4F + 3G}{4F}$$

and

$$M_2 = \frac{2 \frac{\partial G}{\partial a} a - 3G}{4F}$$

Petroski approach requires only one reference stress intensity factor solution and shows the ideas to develop the general weight function expression. This expression has been found to be sufficient to approximate many weight functions for 2D and 3D cracked problems ([29], [30] and [31]). However, by checking the accuracy of the resultant weight function from [28], it is found that this crack opening displacements form does not accurately describe the crack opening displacement for some cracked bodies and therefore the resultant weight function is not accurate for equation (2.34) ([32], [33] and [34]).

Shen-Glinka Method

In order to get more accurate weight functions, several authors proposed other expressions to approximate weight functions. Glinka and Shen [35] proposed accurately approximated general expression for a variety of existing weight functions:

$$m(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_1 \left(1 - \frac{x}{a}\right)^{\frac{1}{2}} + M_2 \left(1 - \frac{x}{a}\right) + M_3 \left(1 - \frac{x}{a}\right)^{\frac{3}{2}} \right] \quad (2.38)$$

where M_i ($i=1, 2, 3$) are parameters which depend only on the geometrical configuration of the cracked body. The determination of these three parameters requires three reference stress intensity factor solutions.

These parameters M_i can be determined from Eq.(2.32) providing that three stress intensity factor expressions, K_{ri} ($i=1, 2, 3$), in the case of three different stress distribution σ_{ri} ($i=1, 2, 3$) known by solving the simultaneous equations after substituting these stress distribution and weight function of (2.38) into Eq.(2.32).

However, the number of reference stress intensity factors can be reduced by using appropriate boundary conditions. For example, for the double edge crack in a finite thickness plate as shown in Figure 2.10, it has been shown that the curvature of the crack surface at the crack mouth (free corner) is zero (Fett, [32]),

$$\frac{\partial^2 u_r(x, a)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.39)$$

This is due to the fact that the surfaces near the crack mouth area stress free. Subsequently, applying the relationship between $u_r(x, a)$ and $m(x, a)$ in Eq.(2.33), the second derivative of the weight function for such a crack should be zero at $x=0$:

$$\frac{\partial^2 m(x, a)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.40)$$

This equation can be used as an additional condition needed for the derivation of the weight function.

In the present research, Shen and Glinka approach for the weight function solution for surface cracks can be used both for the through cracks and the hole surface point A and the front surface point B at a fastener hole in a plate with a slightly difference crack geometry parameters name notations as shown in Figure 1.3 and 1.4.

Using Eq. (2.32), the stress intensity factor K for through cracks and corner crack in current research was written as:

$$K = \int \sigma(x)m(x,c)dx \quad (2.41)$$

where $m(x, c)$ is weight function for specified crack geometry and $\sigma(x)$ is the stress distribution on the potential crack plane. In the present research, the stress distribution only depends on x , and there is no change for stress distribution in other direction.

For through cracks, Shen and Glinka found that the general expression of the weight function could be written as follows:

$$m(x,c) = \frac{2}{\sqrt{2\pi(x-c)}} \left[1 + M_1 \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_2 \left(1 - \frac{x}{c}\right) + M_3 \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \quad (2.42)$$

For the additional condition applied to solve these three parameters M_i , Shen and Glinka suggest the following equation should be used:

$$\frac{\partial^2 m(x,c)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.43)$$

In present research, equation (2.42) is used as the general form of weight function for through cracks at a fastener hole in a plate, the detail of the derivation will be discussed in Chapter 3.

In [9], Shen and Glinka also gave the general expression of the weight function for the surface and deepest point of a semi-elliptical crack in a finite thickness plate shown in figure 2.11.

From figure 2.11, these weight functions can be expressed as:

$$m_A(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left[1 + M_{1A} \left(1 - \frac{x}{a}\right)^{\frac{1}{2}} + M_{2A} \left(1 - \frac{x}{a}\right) + M_{3A} \left(1 - \frac{x}{a}\right)^{\frac{3}{2}} \right] \quad (2.44)$$

$$m_B(x, a) = \frac{2}{\sqrt{\pi x}} \left[1 + M_{1B} \left(\frac{x}{a}\right)^{\frac{1}{2}} + M_{2B} \left(\frac{x}{a}\right) + M_{3B} \left(\frac{x}{a}\right)^{\frac{3}{2}} \right] \quad (2.45)$$

Based on the weight functions in Eqs. (2.44) and (2.45), the weight functions for the corner crack two surface points shown in figure 2.12 were written as follows according to the stress distribution direction:

$$m_A(x, c) = \frac{2}{\sqrt{\pi x}} \left[1 + M_{1A} \left(\frac{x}{c}\right)^{\frac{1}{2}} + M_{2A} \left(\frac{x}{c}\right) + M_{3A} \left(\frac{x}{c}\right)^{\frac{3}{2}} \right] \quad (2.46)$$

$$m_B(x, c) = \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{1B} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{2B} \left(1 - \frac{x}{c}\right) + M_{3B} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \quad (2.47)$$

According to equation (2.43), the second derivative of the weight function for point *B* at $x=0$ must be zero. Thus, one additional boundary condition can be written by:

$$\frac{\partial^2 m_B(x, c)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.48)$$

Because the weight function must vanish at $x=c$, to satisfy this condition, for point A , we have:

$$m_A(x, c)|_{x=c}=0 \quad (2.49)$$

By knowing these boundary conditions for a given crack geometry, only two reference stress intensity factor solutions are needed, and a system of three simultaneous equations can be written to find the three unknown coefficients M_i .

Wang-Lambert method

The weight function form from Shen and Glinka is used to solve the stress intensity factor for the deepest point and surface point for semi-elliptical surface crack. For any local point, $0 \leq \Phi \leq \pi/2$, along a semi-elliptical surface crack in a finite thickness plate shown in Figure 2.11, Wang and Lambert [8] derived the weight function form from an approximate general weight function using two reference stress intensity factor solutions as following:

For $0 \leq x \leq a \sin \Phi$,

$$m_p(x, a) = \frac{\sqrt{\sin \Phi + 1}}{\sqrt{\pi(a \sin \Phi - x)}} \left[1 + M_{1p} \left(1 - \frac{x}{a \sin \Phi} \right) + M_{2p} \left(1 - \frac{x}{a \sin \Phi} \right)^2 \right] \quad (2.50)$$

For $a \sin \Phi \leq x \leq a$,

$$m_p(x, a) = \frac{\sqrt{1 - \sin \Phi}}{\sqrt{\pi(x - a \sin \Phi)}} \left[1 + M_{3p} \left(\frac{x}{a \sin \Phi} - 1 \right)^{\frac{1}{2}} + M_{4p} \left(\frac{x}{a \sin \Phi} - 1 \right) \right] \quad (2.51)$$

As shown in figure 2.12 for present research, using the derivation of weight function for local point at a semi-elliptical surface crack from Wang, the general form of weight function is applied for corner cracks at a fastener hole in a plate as follows.

For $0 \leq x \leq a \sin \Phi$,

$$m_p(x, c) = \frac{\sqrt{\sin \Phi + 1}}{\sqrt{\pi(c \sin \Phi - x)}} \left[1 + M_{1p} \left(1 - \frac{x}{c \sin \Phi} \right) + M_{2p} \left(1 - \frac{x}{c \sin \Phi} \right)^2 \right] \quad (2.52)$$

$$\frac{\partial^2 m_p(x, c)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.53)$$

For $a \sin \Phi \leq x \leq c$,

$$m_p(x, c) = \frac{\sqrt{1 - \sin \Phi}}{\sqrt{\pi(x - c \sin \Phi)}} \left[1 + M_{3p} \left(\frac{x}{c \sin \Phi} - 1 \right)^{\frac{1}{2}} + M_{4p} \left(\frac{x}{c \sin \Phi} - 1 \right) \right] \quad (2.54)$$

$$m_p(x, c) \Big|_{x=c} = 0 \quad (2.55)$$

Equations (2.53) and (2.55) come from the boundary conditions described in equations (2.48) and (2.49).

All these detail derivation of weight function for the present research will be described and discussed in later chapters.

Figure 2.1 Three basic modes of crack surface displacement.

Figure 2.2 Three dimensional coordinate system for the region at the crack tip.

Figure 2.3 Cracked member under a Mode I load P .

Figure 2.4 Arbitrary contour around the tip of a crack.

a)

b)

Figure 2.5 Eight noded isoparametric element.

a). global co-ordinates

b). local co-ordinates

Figure 2.6 Eight noded collapsed singularity element.

Figure 2.7 Inner and outer contours

(a) The pyramid function.

(b) The plateau function.

Figure 2.8 Examples of q functions in two dimensions, with the corresponding virtual nodal displacements.

Figure 2.9 Weight function notation for cracks under mode I loading.

- a) Applied stress system S and the corresponding local stress field $\sigma(x)$;
- b) Applied local stress field $\sigma(x)$;
- c) Physical meaning of weight function.

Figure 2.11 Semi-elliptical surface crack in a finite thickness plate.

Figure 2.12 Surface points A and B, local point P of a corner crack at a fastener hole in a finite thickness plate.

Chapter 3

Through Cracks at a Fastener Hole in a Plate

For cracks at a fastener hole in a plate, there are several configurations including through-the-thickness, corner and surface cracked hole geometries. For two dimensional crack analysis, single and symmetric double through cracks at a fastener hole in a plate were chosen for which weight functions were developed. The weight function results are presented for cracks over the range of c/R ratio between 0.2 and 1.5.

In this chapter, available stress intensity factors are reviewed first, weight functions are derived based on two reference stress intensity factor solutions with uniform and linear stress distributions on the crack face, the derived weight functions are then verified using other stress distributions.

3.1 Stress Intensity Factor Results

In present research, the existing stress intensity factor solutions obtained by Grandt [2] are used as reference stress intensity factors to derive the weight function solution for the concerned geometry. The specified crack geometry with the parameters of c , crack depth and R , hole radius is shown in Figure 3.1.

3.1.1 Results for Stress Distributions on the Crack Face

In order to employ the general weight function form specified in previous chapter, the conversion of existing results is conducted from Grandt solution under individual crack face pressure to the standard loading system used in Shen [9] and Wang [8].

For single and two symmetric through cracks at a fastener hole in a plate as shown in Figure 3.1, Grandt [2] gave the stress intensity factor solution with c/R ratio

restricted to the range between 0.2 and 1.5. The through cracked hole is loaded with a crack face pressure $\sigma_n (x/R)^n$ as n varied in the range $n=0, 1, 2, 3$. The stress intensity factor results were summarized in the dimensionless form $M_n = K_n / (\sigma_0 \sqrt{\pi c})$ in [2], where M_n is the boundary correction factor, σ_0 is the nominal stress and c is the crack length.

Conversion of Grandt's results

The stress intensity factor obtained from Grandt result is under the loading condition of $\sigma_0 (x/R)^n$ (Figure 3.1). However, the widely used general form of weight function for through cracks is related to the loading system in which the crack face pressure is expressed as $\sigma_0 (1-x/c)^n$ (Figure 3.2). To facilitate the derivation of weight functions, the conversion of results using superposition method between these two kinds of loading is conducted in the present research.

Referring to figure 3.1, the loading system for through cracks in Grandt approach is :

$$\sigma_g(x) = \sigma_0 \left(\frac{x}{R}\right)^n \quad (3.1)$$

from figure 3.2 the standard loading condition on the crack face is :

$$\sigma_s(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n \quad (3.2)$$

where σ_0 is nominal stress applied on the crack face, R is the hole radius and c is the crack depth, $n=0, 1, 2, 3$.

The stress intensity factors corresponding to these two loading systems are:

$$K_g |_{\sigma_g(x)} = \sigma_0 \sqrt{\pi c} M_n \quad (3.3)$$

$$K_s |_{\sigma_s(x)} = \sigma_0 \sqrt{\pi c} F_n \quad (3.4)$$

where M_n is the dimensionless stress intensity factor obtained from Gandt's approach, F_n is the dimensionless stress intensity factor under the standard stress distribution Eq.(3.2) applied on the crack face.

By expanding equation (3.2) to be polynomial form, $\sigma_s(x)$ can be treated as the sum of some single terms which is function of $\frac{x}{c}$ item.

$$\sigma_s(x) = \begin{cases} \sigma_0, & n = 0 \\ \left[1 - \left(\frac{x}{c} \right) \right] \sigma_0, & n = 1 \\ \left[1 - 2 \left(\frac{x}{c} \right) + \left(\frac{x}{c} \right)^2 \right] \sigma_0, & n = 2 \\ \left[1 - 3 \left(\frac{x}{c} \right) + 3 \left(\frac{x}{c} \right)^2 - \left(\frac{x}{c} \right)^3 \right] \sigma_0, & n = 3 \end{cases} \quad (3.5)$$

Suppose for any single term loading condition:

$$\sigma_{sn}(x) = \sigma_0 \left(\frac{x}{c} \right)^n \quad (3.6)$$

The stress intensity factor K_{sn} can be expressed :

$$K_{sn} |_{\sigma_{sn}(x)} = \sigma_0 \sqrt{\pi c} X_n \quad (3.7)$$

where X_n is boundary correction factor with loading condition expressed in Eq. (3.6).

Also $\sigma_g(x)$ can be expressed as:

$$\sigma_g(x) = \sigma_0 \left(\frac{x}{R}\right)^n = \sigma_0 \left(\frac{c}{R}\right)^n \left(\frac{x}{c}\right)^n = \sigma_{sn}(x) \left(\frac{c}{R}\right)^n \quad (3.8)$$

therefore, the relation between K_g and K_{sn} can be found:

$$K_g |_{\sigma_g(x)} = \left(\frac{c}{R}\right)^n K_{sn} |_{\sigma_{sn}(x)} \quad (3.9)$$

X_n can now be expressed in terms of M_n :

$$X_n = \left(\frac{c}{R}\right)^{-n} M_n \quad (3.10)$$

Once knowing X_n , from Eq.(3.5), F_n , the final boundary correction factor under standard loading system can be obtained by writing F_n individually in terms of some X_n items in the sequence of increasing n :

$$\begin{aligned} F_0 &= X_0 & n=0 \\ F_1 &= X_0 - X_1 & n=1 \\ F_2 &= X_0 - 2X_1 + X_2 & n=2 \\ F_3 &= X_0 - 3X_1 + 3X_2 - X_3 & n=3 \end{aligned} \quad (3.11)$$

in terms of M_n equation (3.11) can be written as:

$$\begin{aligned} F_0 &= M_0 & n=0 \\ F_1 &= M_0 - \left(\frac{c}{R}\right)^{-1} M_1 & n=1 \end{aligned}$$

(3.12)

$$F_2 = M_0 - 2\left(\frac{c}{R}\right)^{-1} M_1 + \left(\frac{c}{R}\right)^{-2} M_2 \quad n=2$$

$$F_3 = M_0 - 3\left(\frac{c}{R}\right)^{-1} M_1 + 3\left(\frac{c}{R}\right)^{-2} M_2 - \left(\frac{c}{R}\right)^{-3} M_3 \quad n=3$$

Results of single cracked hole loaded with a crack face pressure for $n=0, 1, 2, 3$ are presented in table 3.1. Results of a double cracked hole loaded with a crack face pressure for $n=0, 1, 2, 3$ are listed in table 3.2.

3.1.2 Results for Remote Tension Loading

For the purpose of validation for the derived weight function results (derived later), two dimensional finite element calculation of stress intensity factor for double symmetric through cracks at a fastener hole was performed. The geometry and coordinate system are shown in Figure 3.3.

Finite Element Model

ABAQUS version 6.2 programming was used in the finite element analyses with 8-noded isoparametric 2D solid elements and reduced integration. In order to model the square root singularity at the crack tip, two dimensional prism elements with one mid-side node at the quarter point and coalescing nodes along one side such that they have same displacement as discussed in chapter 2, section 2.2.1.

Using the plane strain relationship between J and K which can be found as:

$$K = \sqrt{\frac{JE}{(1-\nu^2)}} \quad (3.13)$$

the stress intensity factor, K , can be calculated from the J -integral which was solved using the domain integral method (chapter 2, section 2.2.2). The analyses were made with a linear elastic material mode with a Young's modulus, E , of 207 GPa and Poisson's ratio, ν of 0.25.

Double symmetric through cracks at a fastener hole with ratio of c/R , 0.2, 0.5, 1 and 1.5 were analyzed by finite element method. Uniform tensile loading is applied on the remote side of the plate normal to the crack face as shown in figure 3.3. Because the whole plate with cracked hole is symmetric, only a quarter of the plate was meshed and plotted in figure 3.4. In order to eliminate the influence of the plated width and height, sufficient elements were added in the width and height direction with $W/c \geq 15$ and $H/c \geq 15$.

Verification of FEM Model

In order to verify the finite element model, the stress intensity factors with the constant loads applied to the crack surface as shown in figure 3.5 were obtained by the finite element model, the results is listed in table 3.3. Comparisons were made between the stress intensity factors calculated from the finite element analysis and from the Grandt' paper [2] for double symmetric cracks at a fastener hole with corresponding to the ratio of c/R , 0.2, 0.5, 1 and 1.5. The results of comparison were listed in table 3.4. The maximum difference is within 6%.

The stress intensity factors, K , from Grandt is in terms of dimensionless boundary correction factors M . In table 3.4, M results from Grandt is given directly by using the following equation:

$$M = \frac{K_g}{\sigma_0 \sqrt{\pi c}} \quad (3.14)$$

where σ_0 is the constant loads applied on the crack face.

The inspection of table 3.4 indicates that the present finite element models were suitable for the analysis of through cracked hole.

Results of Finite Element Model for Remote Tension

The finite element mesh as shown in figure 3.4 can be used to obtain the stress intensity factors for double symmetric through cracks at a fastener hole under remote tension directly. For one single through crack, Shah [36] developed a conversion factor to express the relationship between one and two corner cracks:

$$K^I = \sqrt{\frac{\frac{4}{\pi} + \frac{ac}{2tR}}{\frac{4}{\pi} + \frac{ac}{tR}}} K^{II}$$

where K^I is stress intensity factor for one corner crack, K^{II} is stress intensity factor for two symmetric corner cracks, t is plate thickness, a is the crack depth of a corner crack and c is the crack length of a corner crack.

Using this equation given by Shah [36], the stress intensity factors for single through crack can be obtained from the finite element results for double symmetric through cracks, setting the ratio of a/t equal to one, the relationship of stress intensity factors for single and double symmetric through cracks can be written as:

$$K^I = \sqrt{\frac{\frac{4}{\pi} + \frac{c}{2R}}{\frac{4}{\pi} + \frac{c}{R}}} K^{II} \quad (3.15)$$

where K^I is stress intensity factor for one single through crack, K^{II} is stress intensity factor for two symmetric through cracks.

For single and double through cracks, the results with the constant tension acted on the remote side of the plate as shown in figure 3.3 is listed in table 3.5.

3.2 Weight Function for One Single Through Crack

The Shen-Glinka approach discussed in chapter 2 was employed to derive the weight functions on the range of $0.2 \leq c/R \leq 1.5$.

3.2.1 Reference Stress Intensity Factors

At the start of the derivation of weight functions, the converted results performed in section 3.1.1 are used for the reference stress intensity factor solutions under the standard loading system with corresponding to a constant and linearly decreasing stress field acting on the crack surface.

The stress intensity factors for uniform tension on the crack face:

$$\sigma(x) = \sigma_0 \quad (3.16)$$

are:

$$K_{r0} = \sigma_0 \sqrt{\pi c} F_0 \quad (3.17)$$

The stress intensity factors for linearly decreasing crack face pressure:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right) \quad (3.18)$$

are:

$$K_{r1} = \sigma_0 \sqrt{\pi c} F_1 \quad (3.19)$$

where F_0 and F_1 is the boundary correction factor for one single through crack.

For the ease of computation, F_0 and F_1 can be expressed in the form of equations. Curve fitting for the results listed in table 3.1 with c/R ratio between 0.2 and 1.5 and in the case $n=0$ and $n=1$ are performed to obtain the expression for the reference stress intensity factor solution after converting the existing results.

The dimensionless boundary correction factor F_0 and F_1 can be expressed as the function of c/R ratio as follows by curve fitting method:

For $n=0$

$$F_0 = 1.0548 - 0.3905\left(\frac{c}{R}\right) + 0.2236\left(\frac{c}{R}\right)^2 - 0.0405\left(\frac{c}{R}\right)^3 - 0.001\left(\frac{c}{R}\right)^4 \quad (3.20)$$

For $n=1$

$$F_1 = 0.4037 - 0.2647\left(\frac{c}{R}\right) + 0.1573\left(\frac{c}{R}\right)^2 - 0.0311\left(\frac{c}{R}\right)^3 - 0.0001\left(\frac{c}{R}\right)^4 \quad (3.21)$$

The curve fitting results with an accuracy of 1% or better are shown in Figure 3.6.

3.2.2 Weight Functions

The general expression of stress intensity factors in term of weight function parameters can be chosen the form of Eq.(2.32). For one single crack geometry (Figure 3.2, for one crack), it is adjusted as:

$$K = \int_0^c \sigma(x) m(x, c) dx \quad (3.22)$$

The following weight function form is from Shen-Glinka [35] as discussed in chapter 2:

$$m(x, c) = \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{i1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{i2} \left(1 - \frac{x}{c}\right) + M_{i3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \quad (2.42)$$

Applying the additional boundary condition represented by Eq.(2.43), which can be written as :

$$\frac{\partial^2 m(x, c)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.43)$$

Substituting Eqs.(3.16) ~ (3.21) into (3.22) and combined by Eqs.(2.43) and (2.42) leads to following system equations obtained:

$$K_{r0} = \sigma_0 \sqrt{\pi c} F_0 = \int_0^c \sigma_0 \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{i1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{i2} \left(1 - \frac{x}{c}\right) + M_{i3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (3.23a)$$

$$K_{r1} = \sigma_1 \sqrt{\pi c} F_1 = \int_0^c \sigma_1 \left(1 - \frac{x}{c}\right) \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{i1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{i2} \left(1 - \frac{x}{c}\right) + M_{i3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (3.23b)$$

$$\frac{\partial^2}{\partial x^2} \left\{ \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{i1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{i2} \left(1 - \frac{x}{c}\right) + M_{i3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \right\} \Big|_{x=0} = 0 \quad (3.23c)$$

After integration and differentiation, a set of three algebraic equations was solved by determining the parameters of M_{I1} , M_{I2} and M_{I3} :

$$M_{I1} = \frac{\pi}{\sqrt{2}} (4F_0 - 6F_1) - \frac{24}{5} \quad (3.24)$$

$$M_{I2} = 3 \quad (3.25)$$

$$M_{I3} = 2 \left(\frac{\pi}{\sqrt{2}} F_1 - M_{I1} - 4 \right) \quad (3.26)$$

The weight function for through cracks can be determined in the form of Eq.(2.42) once knowing these three parameters M_{I1} , M_{I2} and M_{I3} .

3.2.3 Validation of Weight Functions

For any given loading systems, the stress intensity factor can be obtained by substituting the weight function parameters M_{I1} , M_{I2} and M_{I3} given in Eqs. (3.24), (3.25) and (3.26) as well as Eq.(2.42) into Eq.(3.22), the results is valid for through cracks with the ratio of c/R between 0.2 and 1.5.

Two comparisons were made to verify the results of weight function approach presented in this chapter.

Comparisons with the converted results

The first was performed between the predictions using weight function method and the converted results based on the Grandt data [2].

Under the loading situation expressed by equation (3.2), that is:

$$\sigma_s(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n \quad (3.2)$$

The weight function method can be used to calculate the stress intensity factors for the following stress fields:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2 \quad (3.27)$$

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3 \quad (3.28)$$

Substituting the stress distributions on the crack face expressed in equations (3.27) and (3.28) with the derived weight function expressed in Eq. (2.42) into equation (3.22), the stress intensity factors for single through crack can be obtained.

Table 3.6 and 3.7 show the comparisons of stress intensity factors in terms of boundary factors F for the quadratic and cubic stress distribution using weight function method and the converted Grandt results with corresponding to stress distribution expressed in Eqs.(3.27) and (3.28).

For the region of c/R from 0.2 to 1.5, the maximum difference of the stress intensity factors between these two approaches is less than 7.5 percent. The comparison is also plotted in Figure 3.7.

Comparison with the finite element data

The second comparison is made between the predictions using weight function approach and the finite element data in table 3.5.

With uniform tensile loading applied on the far end side of plate, Petroski [28] also gave the expression of stress distribution on the crack face, based on the present crack geometry shown in figure 3.3, it can be written as:

$$\sigma(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] \quad (3.29)$$

Where σ_0 is the nominal stress, R is the hole radius shown in figures 3.3.

Substituting the stress distribution on the crack face into equation (3.22), the results of stress intensity factors using weight function method and the data obtained from finite element method were listed in table 3.8 and plotted in Figure 3.8. The comparison shows the accuracy is less than 7.5 percent based on the finite element data for the same region of c/R specified above.

3.3 Weight Function for Double Symmetric Through Cracks

The Shen-Glinka approach discussed in chapter 2 was employed to derive the weight functions for double symmetric through cracks at a fastener hole on the range of $0.2 \leq c/R \leq 1.5$.

3.3.1 Reference Stress Intensity Factors

Following the same procedure of obtaining the reference stress intensity factors with section 3.2, the stress intensity factors for constant crack face pressure:

$$\sigma(x) = \sigma_0 \quad (3.16)$$

are:

$$K_{r0} = \sigma_0 \sqrt{\pi c} Y_0 \quad (3.30)$$

the stress intensity factors for linearly decreasing crack face pressure:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right) \quad (3.18)$$

are:

$$K_{r1} = \sigma_0 \sqrt{\pi c} Y_1 \quad (3.31)$$

where Y_0 and Y_1 is the boundary correction factor for double symmetric through cracks.

For the ease of computation, Y_0 and Y_1 can be expressed in the form of equations. Curve fitting for the results listed in table 3.2 with c/R ratio between 0.2 and 1.75 and in the case $n=0$ and $n=1$ are performed to obtain the expression for the reference stress intensity factor solution after converting the existing results.

The dimensionless boundary correction factor Y_0 and Y_1 can be expressed as the function of c/R ratio as follows by curve fitting method:

For $n=0$

$$Y_0 = 1.007 - 0.1881\left(\frac{c}{R}\right) + 0.1676\left(\frac{c}{R}\right)^2 - 0.028\left(\frac{c}{R}\right)^3 - 0.005\left(\frac{c}{R}\right)^4 \quad (3.32)$$

For $n=1$

$$Y_1 = 0.3658 - 0.1052\left(\frac{c}{R}\right) + 0.0728\left(\frac{c}{R}\right)^2 + 0.0093\left(\frac{c}{R}\right)^3 - 0.0103\left(\frac{c}{R}\right)^4 \quad (3.33)$$

The curve fitting results with an accuracy of 1% or better are shown in Figure 3.9.

3.3.2 Weight Functions

Using the general expression of stress intensity factors K as discussed in section 3.2 in this chapter,

$$K = \int_0^c \sigma(x) m(x, c) dx \quad (3.22)$$

the following weight function form is from Shen-Glinka [33] as discussed in chapter 2:

$$m(x, c) = \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{y1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{y2} \left(1 - \frac{x}{c}\right) + M_{y3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \quad (3.34)$$

Applying the additional boundary condition represented by Eq.(2.43), which can be written as :

$$\frac{\partial^2 m(x, c)}{\partial x^2} \Big|_{x=0} = 0 \quad (2.43)$$

Substituting Eqs.(3.16), (3.18), (3.30) and (3.31) into (3.22) and combined by Eqs.(3.34) and (2.43) leads to following system equations:

$$K_{r0} = \sigma_0 \sqrt{\pi c} Y_0 = \int_0^c \sigma_0 \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{y1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{y2} \left(1 - \frac{x}{c}\right) + M_{y3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (3.35a)$$

$$K_{r1} = \sigma_1 \sqrt{\pi c} Y_1 = \int \sigma_1 \left(1 - \frac{x}{c}\right) \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{y1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{y2} \left(1 - \frac{x}{c}\right) + M_{y3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}}\right] dx \quad (3.35b)$$

$$\frac{\partial^2}{\partial x^2} \left\{ \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{y1} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{y2} \left(1 - \frac{x}{c}\right) + M_{y3} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}}\right] \right\} \Big|_{x=0} = 0 \quad (3.35c)$$

After integration and differentiation, a set of three algebraic equations was solved by determining the weight function parameters of M_{y1} , M_{y2} and M_{y3} :

$$M_{y1} = \frac{\pi}{\sqrt{2}} (4Y_0 - 6Y_1) - \frac{24}{5} \quad (3.36)$$

$$M_{y2} = 3 \quad (3.37)$$

$$M_{y3} = 2 \left(\frac{\pi}{\sqrt{2}} Y_1 - M_{y1} - 4 \right) \quad (3.38)$$

The weight function for double symmetric through cracks can be determined in the form of Eq.(3.34) once knowing these three parameters M_{y1} , M_{y2} and M_{y3} .

3.3.3 Validation of Weight Functions

Two comparisons were made to verify weight function results presented in this chapter.

Comparisons with the converted results

The first was performed between the predictions using weight function method and the converted results in table 3.2.

The weight function method can be used to calculate the stress intensity factors for the following stress fields applied on the crack face:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2 \quad (3.39)$$

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3 \quad (3.40)$$

Tables 3.9 and 3.10 show the results of stress intensity factors in terms of boundary factors F for the quadratic and cubic stress distribution using weight function method and the converted Grandt results with corresponding to stress distribution expressed in Eqs.(3.39) and (3.40).

For the region of c/R from 0.2 to 1.5, the difference of the stress intensity factors between these two approaches is less than six percent. The comparison is also plotted in Figure 3.10.

Comparison with the finite element data

The second comparison is made between the predictions using weight function approach and the finite element data from the present calculations for double through cracks at a fastener hole under uniform remote tension loading as shown in Figure 3.3.

With uniform tensile loading applied on the far end side of plate, as discussed in section 3.2, the crack face stress distribution can be written as:

$$\sigma(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] \quad (3.29)$$

Where σ_0 is the nominal stress, R is the hole radius shown in figures 3.3.

Substituting the stress distribution on the crack face into equation (3.22), the results of stress intensity factors using weight function method and the data obtained from finite element method were listed in table 3.11 and plotted in Figure 3.11 with the accuracy less than ten percent based on the finite element data for the same region of c/R specified above.

3.4 Discussion

The weight function solutions for single through crack and double symmetric through crack at a fastener hole in a plate were performed.

Both for single and double through cracks, comparisons of the derived weight function based SIF to the converted results of Grandt (table 3.1 and 3.2) were made. For quadratic and cubic stress distributions, the difference between the weight functions and converted Grandt's results is less than 7.5 percent.

Another comparison was made between the weight functions and finite element data for far end tension loading. The results are presented in tables 3.6 and 3.9 as well as Figures 3.8 and 3.11. The predictions from the weight functions were within ten percent of the finite element data.

On the range of ratios, $0.2 \leq c/R \leq 1.5$, the stress intensity factors for through cracks at a fastener hole in a flat plate can be obtained by integration of the product of the weight function and the stress distribution on the potential plane. The derived weight functions are suitable for fatigue and fracture analysis for fastener holes with through crack under arbitrary stress distributions.

c/R	F0(n=0)	F1(n=1)	F2(n=2)	F3(n=3)
0.2	0.9817	0.3542	0.2217	0.1631
0.3	0.9592	0.3395	0.2105	0.1537
0.4	0.9351	0.3234	0.1982	0.1438
0.5	0.9119	0.3079	0.1867	0.1349
0.6	0.8914	0.2942	0.1762	0.1262
0.8	0.861	0.2744	0.1612	0.1134
1	0.8442	0.2637	0.1534	0.1073
1.3	0.8351	0.258	0.1492	0.104
1.5	0.8337	0.2578	0.1494	0.1041

Table 3.1 The converted results of boundary correction factor F_n for single through crack under the standard crack face stress distribution $\sigma_s(x) = \sigma_0(1 - \frac{x}{c})^n$,

$$F_n = \frac{K|_{\sigma_s(x)}}{\sigma_0 \sqrt{\pi c}} .$$

c/R	F0(n=0)	F1(n=1)	F2(n=2)	F3(n=3)
0.2	0.9752	0.3467	0.2144	0.1559
0.3	0.9651	0.3418	0.2117	0.1544
0.4	0.9582	0.3372	0.208	0.1514
0.5	0.9529	0.3335	0.2053	0.1495
0.6	0.9435	0.325	0.1973	0.1418
0.8	0.9484	0.3298	0.2022	0.1469
1	0.9542	0.3332	0.2046	0.1485
1.3	0.967	0.3412	0.2105	0.1532
1.5	0.9816	0.3505	0.2176	0.1595

Table 3.2 The converted results of boundary correction factor F_n for double symmetric through crack under the standard crack face stress distribution $\sigma_s(x) =$

$$\sigma_0 \left(1 - \frac{x}{c}\right)^n, F_n = \frac{K|_{\sigma_s(x)}}{\sigma_0 \sqrt{\pi c}}.$$

c/R	0.2	0.5	1	1.5
F (double)	1.03	0.9941	0.9806	0.9795

Table 3.3 Boundary correction factors F for double through cracks at a fastener hole using finite element method with constant stress distribution on the crack face.

$\sigma(x) = \sigma_0$	Boundary Correction Factors F		
c/R	FEA	Grandt	Accuracy
0.2	1.03	0.9752	6%
0.5	0.9941	0.9529	3.30%
1	0.9806	0.9542	2.80%
1.5	0.9795	0.9816	0.20%

Table 3.4 Comparison of boundary correction factors F for double through cracks using finite element method (FEA) and Grandt [2] with constant stress distribution applied on crack face.

c/R	0.2	0.5	1	1.5
F (single)	2.4596	1.7754	1.3397	1.1579
F (double)	2.5476	1.9156	1.5169	1.3556

Table 3.5 Boundary correction factors F for single and double through cracks at a fastener hole using finite element method with constant remote tension.

$\sigma(x)=\sigma_0(1-x/c)^2$	Boundary Correction Factors F		
c/R	WF	Converted	Accuracy
0.2	0.2183	0.2217	1.51%
0.5	0.1808	0.1867	3.15%
1	0.1498	0.1534	2.32%
1.5	0.1427	0.1494	4.51%

Table 3.6 Comparison of boundary correction factors F for single through crack using weight function (WF) and converted Grandt [2] results with quadratic loading on the crack face.

$\sigma(x)=\sigma_0(1-x/c)^3$	Boundary Correction Factors F		
c/R	WF	Converted	Accuracy
0.2	0.1567	0.1631	3.94%
0.5	0.1267	0.1349	6.11%
1	0.102	0.1073	4.96%
1.5	0.0963	0.1041	7.49%

Table 3.7 Comparison of boundary correction factors F for single through crack using weight function (WF) and converted Grandt [2] results with cubic loading on the crack face.

Tension	Boundary Correction Factors F		
c/R	WF	FEA	Accuracy
0.2	2.3186	2.4596	5.73%
0.5	1.6775	1.7754	5.52%
1	1.2404	1.3397	7.41%
1.5	1.0858	1.1579	6.22%

Table 3.8 Comparison of boundary correction factors F for single through crack using weight function (WF) and finite element (FEA) results with uniform remote tension.

$\sigma(x)=\sigma_0(1-x/c)^2$	Boundary Correction Factor F		
c/R	WF	Converted	Accuracy
0.2	0.2109	0.2144	1.70%
0.5	0.1991	0.2053	3.00%
1	0.1994	0.2046	2.50%
1.5	0.2133	0.2176	2.00%

Table 3.9 Comparison of boundary correction factors F for double symmetric through cracks using weight function (WF) and converted Grandt [2] results with quadratic loading on the crack face.

$\sigma(x)=\sigma_0(1-x/c)^3$	Boundary Correction Factor F		
c/R	WF	Converted	Accuracy
0.2	0.1505	0.1559	3.50%
0.5	0.1411	0.1495	5.70%
1	0.1412	0.1485	3.90%
1.5	0.1523	0.1595	3.50%

Table 3.10 Comparison of boundary correction factors F for double symmetric through cracks using weight function (WF) and converted Grandt [2] results with cubic loading on the crack face.

Tension	Boundary Correction Factors F		
c/R	WF	FEA	Accuracy
0.2	2.2913	2.5476	10.00%
0.5	1.7668	1.9156	7.70%
1	1.4414	1.5169	5.00%
1.5	1.3427	1.3556	1.00%

Table 3.11 Comparison of boundary correction factors F for double symmetric through cracks using weight function (WF) and finite element (FEA) results with uniform remote tension.

Figure 3.1 Single or double through cracks at a fastener hole with crack face loading, $\sigma(x) = \sigma_0 \left(\frac{x}{R}\right)^n$ from Grandt [2].

Figure 3.2 Single or double through cracks at a fastener hole with standard crack face loading, $\sigma(x) = \sigma_n \left(1 - \frac{x}{c}\right)^n$.

Figure 3.3 Symmetric through cracks at a fastener hole in a plate with uniform remote tension.

Figure 3.4 Finite element mesh for through cracks at a fastener hole in a plate.

Figure 3.5 Symmetric through cracks at a fastener hole in a plate with uniform crack face pressure.

Figure 3.6 The comparison of stress intensity factor results from curve fitting and converted Grandt's results for single through crack at a fastener hole with constant and linearly decreasing stress field.

Figure 3.7 Comparison of weight function results and the predictions of converted Gandt's results [2] with quadratic and cubic stress distribution for single through crack at a fastener hole.

Figure 3.8 Comparison of weight function results and the results of finite element calculation of with uniform remote tension stress distribution for single through crack at a fastener hole.

Figure 3.9 The comparison of stress intensity factor results from curve fitting and converted Grandt's results for double symmetric through cracks at a fastener hole with constant and linearly decreasing stress field.

Figure 3.10 Comparison of weight function results and the predictions of converted Gandt's results [2] with quadratic and cubic stress distribution for double through cracks at a fastener hole.

Figure 3.11 Comparison of weight function results and the results of finite element calculation of with uniform remote tension stress distribution for double symmetric through cracks at a fastener hole.

Chapter 4

Weight Function for Two Surface Points in a Corner Crack

The power of the weight function method can be seen from the derivation of stress intensity factors for through-the-thickness cracks at a fastener hole in a plate in chapter 3. Once the weight function for a particular cracked body has been determined, the stress intensity factor for any loading system applied to that body could be calculated by simple integration. In practice, corner cracks at a fastener hole occur naturally in many structural components and may result in a premature failure. Accurate stress intensity factor solutions for such cracks are needed for reliable fracture assessment and prediction of fatigue crack growth. In this chapter, the weight function solutions for the two surface points A and B of a corner crack at a fastener hole as shown in figure 4.1 are derived by employing the universal expression of weight functions for such cracked geometry found by Shen-Glinka [35]. The solution of weight functions for the local points along the corner crack front other than the two surface points will be discussed in Chapter 5.

The derived weight functions for these surface points are valid for the range of relative crack depths, a/t between 0.2 and 0.8, crack depth ratio, a/c between 0.5 and 1.5 and an R/t ratio of 0.5. The weight functions were derived using reference SIF results from converted Grandt's results with constant and linear stress distributions on the crack face and verified using converted Grandt's results [4] for parabolic and cubic stress distribution on the crack face and the Newman and Raju's curve fitting results [10] for uniform remote tension loading.

4.1 Available Stress Intensity Factors for Corner Cracks

4.1.1 Available Results for Polynomial Stress Distributions

The stress intensity factor solution for corner cracks at a fastener hole in a plate given by Grandt is summarized in [4]. The dimensionless stress intensity factor M is presented as a function of crack shape a/c , size a/t position along the crack front angle parameter Φ with R/t ratio of 0.5 as shown in figure 4.1.

4.1.2 Conversion of Grandt's Result

Following the same procedure described in chapter 3, the converted results can be found by following process:

Referring to figure 4.1, the loading system perpendicular to crack face for corner cracks in Grandt approach is :

$$\sigma_g(x) = \sigma_0 \left(\frac{x}{R}\right)^n \quad (3.1)$$

using the same standard loading condition on the crack face expressed by Eq.(3.2) is :

$$\sigma_s(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n \quad (3.2)$$

Where σ_0 is nominal stress applied on the crack face, R is the hole radius and c is the crack depth, $n=0, 1, 2, 3$.

The stress intensity factors in Grandt's result under the loading condition by Eq.(3.1) for corner crack is:

$$K_g |_{\sigma_g(x)} = \sigma_0 \sqrt{\pi a} M_n \quad (4.1)$$

The following stress intensity factor expression is adopted corresponding to the standard stress distribution showed in Eq.(3.2), that is:

$$K_s |_{\sigma_s(x)} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_n \quad (4.2)$$

where M_n in equation (4.1) is the dimensionless stress intensity factor obtained from Grandt's approach, F_n in equation (4.2) is the dimensionless stress intensity factor under the standard stress distribution Eq.(3.4) applied on the crack face and Q is the shape factor for an ellipse and is given by the square of the complete elliptic integral of the second kind [37].

Very useful empirical expressions for Q have been developed by Rawe [38]. The expressions are:

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65} \quad \text{for } \left(\frac{a}{c}\right) \leq 1 \quad (4.3a)$$

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{-1.65} \quad \text{for } \left(\frac{a}{c}\right) > 1 \quad (4.3b)$$

$\sigma_s(x)$ can be expressed as the sum of some single term which is function of $\frac{x}{c}$ item as follows:

$$\sigma_s(x) = \begin{cases} \sigma_0, & n = 0 \\ \left[1 - \frac{x}{c}\right] \sigma_0, & n = 1 \\ \left[1 - 2\frac{x}{c} + \left(\frac{x}{c}\right)^2\right] \sigma_0, & n = 2 \\ \left[1 - 3\left(\frac{x}{c}\right) + 3\left(\frac{x}{c}\right)^2 - \left(\frac{x}{c}\right)^3\right] \sigma_0, & n = 3 \end{cases} \quad (3.5)$$

for any single term loading condition, using equation (3.6):

$$\sigma_{sn}(x) = \sigma_0 \left(\frac{x}{c}\right)^n \quad (3.6)$$

the stress intensity factor K_{sn} with the loading condition by Eq.(3.6) can be expressed :

$$K_{sn} |_{\sigma_{sn}(x)} = \sigma_0 \sqrt{\frac{\pi c}{Q}} X_n \quad (4.4)$$

where X_n is the boundary correction factor for loading condition Eq. (3.6).

Also $\sigma_g(x)$ can be expressed in terms of $\sigma_{sn}(x)$ as:

$$\sigma_g(x) = \sigma_0 \left(\frac{x}{R}\right)^n = \sigma_0 \left[\left(\frac{R}{t}\right)^{-1} \left(\frac{a}{t}\right) \left(\frac{a}{c}\right)^{-1}\right]^n \left(\frac{x}{c}\right)^n = \left[\left(\frac{R}{t}\right)^{-1} \left(\frac{a}{t}\right) \left(\frac{a}{c}\right)^{-1}\right]^n \sigma_{sn}(x) \quad (4.5)$$

Therefore, the relation between K_g and K_{sn} can be found:

$$K_g |_{\sigma_g(x)} = \left[\left(\frac{R}{t}\right)^{-1} \left(\frac{a}{t}\right) \left(\frac{a}{c}\right)^{-1}\right]^n K_{sn} |_{\sigma_{sn}(x)} \quad (4.6)$$

Substituting Eq.(4.1) and Eq.(4.4) into Eq.(4.6), X_n can be expressed in terms of M_n :

$$X_n = \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-n} M_n \quad (4.7)$$

Analogous to Eq.(3.11), F_n , the final boundary correction factor under standard loading system can be established by writing F_n individually in terms of some X_n items in the sequence of increasing n :

$$\begin{aligned} F_0 &= X_0 & n=0 \\ F_1 &= X_0 - X_1 & n=1 \\ F_2 &= X_0 - 2X_1 + X_2 & n=2 \\ F_3 &= X_0 - 3X_1 + 3X_2 - X_3 & n=3 \end{aligned} \quad (3.11)$$

in terms of M_n equation (3.11) can be written as:

$$\begin{aligned} F_0 &= \sqrt{\frac{a}{c}} \sqrt{Q} M_0 & n=0 \\ F_1 &= \sqrt{\frac{a}{c}} \sqrt{Q} M_0 - \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-1} M_1 & n=1 \\ F_2 &= \sqrt{\frac{a}{c}} \sqrt{Q} M_0 - 2 \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-1} M_1 + \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-2} M_2 & n=2 \end{aligned}$$

$$\begin{aligned}
F_3 = & \sqrt{\frac{a}{c}} \sqrt{Q} M_{0-3} \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-1} M_{I+3} \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-2} M_2 \\
& - \sqrt{\frac{a}{c}} \sqrt{Q} \left[\left(\frac{R}{t} \right)^{-1} \left(\frac{a}{c} \right)^{-1} \left(\frac{a}{t} \right) \right]^{-3} M_3 \quad n=3
\end{aligned} \tag{4.8}$$

The converted results from equation (4.8) were listed in tables 4.1~4.4 which n is from zero to three corresponding to the Grandt's data. The results for points A and B together with points C ($\Phi=\pi/3$) and D ($\Phi=\pi/6$) in the middle portion of the crack front as shown in figure 4.1 are also presented in these tables.

4.1.3 Available Results from Newman

Another set of stress intensity factor solutions for corner crack at a fastener hole was obtained using the finite element method by Newman and Raju [10]. Corresponding to the present crack geometry, Newman provided empirical stress intensity factor equations by curve fitting method using finite element analysis results for quarter elliptical corner cracks at a hole in finite plate subjected to remote tensile loading.

These empirical equations are function of parametric angle, crack depth and length, plate thickness and hole radius. These equations also consider the effect of plate width and height. A wide range of configuration parameters was included in these equations. The aspect ratio, a/c , ranged from 0.2 to 2, the relative crack depth ratio, a/t , ranged from 0 to 1 and the ratio of hole radius, R , to plate thickness t ranged from 0.5 to 2.

For two symmetric corner cracks with $0.2 \leq \left(\frac{a}{c}\right) \leq 2$, $0 \leq \frac{a}{t} \leq 1$, $0.5 \leq \frac{R}{t} \leq 1$, $0 \leq \Phi \leq \pi/2$, the curve fitting equation of stress intensity factor is:

$$K_I|_{\sigma_s(x)} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_n\left(\frac{a}{c}, \frac{a}{t}, \Phi\right) \quad (4.9)$$

The function F_n was found to be:

$$F_n = \left[M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right] g_1 g_2 g_3 f_\Phi f_w \quad (4.10)$$

for $a/c \leq 1$

$$M_1 = 1.13 - 0.09 \left(\frac{a}{c} \right) \quad (4.11)$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + \frac{a}{c}} \quad (4.12)$$

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14 \left(1 - \frac{a}{c} \right)^{24} \quad (4.13)$$

$$g_1 = 1 + \left[0.1 + 0.35 \left(\frac{a}{t} \right)^2 \right] (1 - \sin \Phi)^2 \quad (4.14)$$

$$g_2 = \frac{1 + 0.358\lambda + 1.425\lambda^2 - 1.578\lambda^3 + 2.156\lambda^4}{1 + 0.13\lambda^2} \quad (4.15)$$

where

$$\lambda = \frac{1}{1 + \frac{c}{R} \cos(0.85\Phi)} \quad (4.16)$$

The function g_3 is given by :

$$g_3 = \left(1 + 0.04 \frac{a}{c}\right) \left[1 + 0.1(1 - \cos \Phi)^2 \left[0.85 + 0.15 \left(\frac{a}{t}\right)^{\frac{1}{4}}\right]\right] \quad (4.17)$$

The parametric angle f_Φ function is given:

$$f_\Phi = \left[\left(\frac{a}{c}\right)^2 \cos^2 \Phi + \sin^2 \Phi\right]^{\frac{1}{4}} \quad (4.18)$$

For $a/c > 1$

$$M_1 = \sqrt{\frac{c}{a}} \left(1 + 0.04 \frac{c}{a}\right) \quad (4.19)$$

$$M_2 = 0.2 \left(\frac{a}{c}\right)^{-4} \quad (4.20)$$

$$M_3 = -0.11 \left(\frac{a}{c}\right)^{-4} \quad (4.21)$$

$$g_1 = 1 + \left[0.1 + 0.35 \left(\frac{a}{c}\right)^{-1} \left(\frac{a}{t}\right)^2\right] (1 - \sin \Phi)^2 \quad (4.22)$$

Functions g_2 and λ are given by Eqs (4.15) and (4.16).

$$g_3 = \left[1.13 - 0.09 \left(\frac{a}{c} \right)^{-1} \right] \left[1 - 0.1(1 - \cos \Phi)^2 \right] \left[0.85 + 0.15 \left(\frac{a}{t} \right)^{\frac{1}{4}} \right] \quad (4.23)$$

Because the present research focus on a fastener hole in a finite thickness plate, the ratio of crack depth c and plate width w can be assumed to be infinite, the function f_w considering the plate width effect is set to 1 in the present study for both cases, that is:

$$f_w = 1 \quad (4.24)$$

The dimensionless stress intensity factors using these empirical equations are obtained by Matlab programming. The results for two surface points A and B in figure 4.1 were listed in tables 4.5 and 4.6.

4.2 Weight Functions

Based on the stress intensity factor results converted from Grandt [4] for uniform and linear stress distributions, weight functions at the hole surface point and the front surface point for a corner crack at a fastener hole were derived for the range of $0.5 \leq \frac{a}{c} \leq 1.5$, $0.2 \leq \frac{a}{t} \leq 0.8$ and the ratio, $\frac{R}{t} = 0.5$.

The Shen-Glinka approach discussed in section 2.3.2 was employed to derive the weight functions.

4.2.1 Weight Function for the Hole Surface Point A of a Corner Crack

Two reference stress intensity factors were used with respect to constant and linearly decreasing stress fields acting on the potential crack plane. The crack geometry is shown in figure 4.1.

Reference Stress Intensity Factors

The converted results performed in section 4.1.2 are used as the reference stress intensity factor solutions under the standard loading system with corresponding to a constant and linearly decreasing stress field acting on the crack surface.

The stress intensity factors for constant pressure:

$$\sigma(x) = \sigma_0 \quad (4.25)$$

are:

$$K_{r0} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{A0} \quad (4.26)$$

The stress intensity factors for linearly decreasing crack face pressure:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right) \quad (4.27)$$

are:

$$K_{r1} = \sigma_1 \sqrt{\frac{\pi c}{Q}} F_{A1} \quad (4.28)$$

where F_{A0} and F_{A1} are boundary correction factors for hole surface point A with corresponding to standard loading situations, $n=0$ and $n=1$ in equation (3.2).

Based on the results in tables 4.1 to 4.4, the empirical formulas of the boundary correction factors for hole surface point A were fitted with accuracy of 1% or better. The expressions are valid for $0.2 \leq \frac{a}{t} \leq 0.8$, $0.5 \leq \left(\frac{a}{c}\right) \leq 1.5$ and $\frac{R}{t} = 0.5$.

Using the curve fitting results shown in figures 4.2 and 4.3, the boundary correction factors F_{A0} and F_{A1} for point A can be expressed as:

$$F_{A0} = A_{10} + A_{20}\left(\frac{a}{t}\right)^2 + A_{30}\left(\frac{a}{t}\right)^4 + A_{40}\left(\frac{a}{t}\right)^6 \quad (4.29)$$

where

$$A_{10} = -0.1353 + 2.2882\left(\frac{a}{c}\right) - 1.4998\left(\frac{a}{c}\right)^2 + 0.2646\left(\frac{a}{c}\right)^3$$

$$A_{20} = 1.6974 - 5.1522\left(\frac{a}{c}\right) + 5.1577\left(\frac{a}{c}\right)^2 - 1.6504\left(\frac{a}{c}\right)^3$$

$$A_{30} = -8.4548 + 29.4903\left(\frac{a}{c}\right) - 31.7494\left(\frac{a}{c}\right)^2 + 10.4671\left(\frac{a}{c}\right)^3$$

$$F_{A1} = A_{11} + A_{21}\left(\frac{a}{t}\right)^2 + A_{31}\left(\frac{a}{t}\right)^4 + A_{41}\left(\frac{a}{t}\right)^6 \quad (4.30)$$

$$A_{11} = -0.0421 + 1.6954\left(\frac{a}{c}\right) - 1.0235\left(\frac{a}{c}\right)^2 + 0.1479\left(\frac{a}{c}\right)^3$$

$$A_{21} = -0.6833 + 3.0473\left(\frac{a}{c}\right) - 3.7048\left(\frac{a}{c}\right)^2 + 1.2717\left(\frac{a}{c}\right)^3$$

$$A_{31} = -2.031 + 6.5666\left(\frac{a}{c}\right) - 6.8213\left(\frac{a}{c}\right)^2 + 32.2450\left(\frac{a}{c}\right)^3$$

$$A_{41} = 4.0382 - 15.1709\left(\frac{a}{c}\right) + 17.0867\left(\frac{a}{c}\right)^2 - 5.7950\left(\frac{a}{c}\right)^3$$

Weight Functions

Since that two reference stress intensity factor solutions K_{r0} and K_{r1} in Eqs. (4.26) and (4.28) are known, a set of two equations can be written by substituting Eqs (4.25)~(4.28) and (2.46) into Eq. (2.41):

$$K_{r0} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{A0} = \int_0^c \sigma_0 \frac{2}{\sqrt{\pi x}} \left[1 + M_{1A} \left(\frac{x}{c}\right)^{\frac{1}{2}} + M_{2A} \left(\frac{x}{c}\right) + M_{3A} \left(\frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (4.31)$$

$$K_{r1} = \sigma_1 \sqrt{\frac{\pi c}{Q}} F_{A1} = \int_0^c \sigma_1 \left(1 - \frac{x}{c}\right) \frac{2}{\sqrt{\pi x}} \left[1 + M_{1A} \left(\frac{x}{c}\right)^{\frac{1}{2}} + M_{2A} \left(\frac{x}{c}\right) + M_{3A} \left(\frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (4.32)$$

The third condition necessary to determine parameters M_{1A} , M_{2A} and M_{3A} was derived by satisfying the condition that the weight function in Eq. (2.49) must vanish for $x=c$, that is:

$$m_A(x, c) \Big|_{x=c} = 0 \quad (4.33)$$

The three unknown parameters M_{1A} , M_{2A} and M_{3A} can be determined by simultaneously solving Eqs. (4.31), (4.32) and (4.33):

$$M_{1A} = 30 \frac{\pi}{2\sqrt{Q}} F_{A1} - 18 \frac{\pi}{2\sqrt{Q}} F_{A0} - 8 \quad (4.34)$$

$$M_{2A} = -90 \frac{\pi}{2\sqrt{Q}} F_{A1} + 60 \frac{\pi}{2\sqrt{Q}} F_{A0} + 15 \quad (4.35)$$

$$M_{3A} = 60 \frac{\pi}{2\sqrt{Q}} F_{A1} - 42 \frac{\pi}{2\sqrt{Q}} F_{A0} - 8 \quad (4.36)$$

The weight function for the hole surface point A of a corner crack at a fastener hole can be determined directly in the form of Eq. (2.46).

4.2.2 Weight Function for the Surface Point B of a Corner Crack

For the same crack geometry shown in figure 4.1, the weight function for the surface point B was determined by using two reference stress intensity factors under constant and linearly decreasing stress fields acting on the crack face.

Reference Stress Intensity Factors

The converted results for point B performed in section 4.1.2 are used as the reference stress intensity factor solutions under the standard loading system with constant and linearly decreasing stress fields acting on the crack surface.

The stress intensity factors for constant pressure:

$$\sigma(x) = \sigma_0 \quad (4.25)$$

are:

$$K_{r0} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{B0} \quad (4.37)$$

The stress intensity factors for linearly decreasing crack face pressure:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right) \quad (4.27)$$

are:

$$K_{r1} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{B1} \quad (4.38)$$

where F_{B0} and F_{B1} are boundary correction factors for front surface point B with corresponding to standard loading situations, $n=0$ and $n=1$ in equation (3.2). They can be found in tables 4.1 to 4.4.

Based on the results in tables 4.1 to 4.4, the empirical formulas of the boundary correction factors for front surface point B were fitted with accuracy of 1% or better. The expressions are valid for $0.2 \leq \frac{a}{t} \leq 0.8$, $0.5 \leq \left(\frac{a}{c}\right) \leq 1.5$ and $\frac{R}{t} = 0.5$.

Using the curve fitting results shown in figure 4.4 and 4.5, the boundary correction factors F_{B0} and F_{B1} for point B can be expressed as:

$$F_{B0} = B_{10} + B_{20} \left(\frac{a}{t}\right)^2 + B_{30} \left(\frac{a}{t}\right)^4 + B_{40} \left(\frac{a}{t}\right)^6 \quad (4.39)$$

$$B_{10} = 1.0530 - 2.9502\left(\frac{a}{c}\right) + 4.7850\left(\frac{a}{c}\right)^2 - 1.8842\left(\frac{a}{c}\right)^3$$

$$B_{20} = -14.1514 + 58.6303\left(\frac{a}{c}\right) - 68.3265\left(\frac{a}{c}\right)^2 + 23.5413\left(\frac{a}{c}\right)^3$$

$$B_{30} = 60.7437 - 239.2973\left(\frac{a}{c}\right) + 276.4088\left(\frac{a}{c}\right)^2 - 95.0167\left(\frac{a}{c}\right)^3$$

$$B_{40} = -73.2199 + 275.0157\left(\frac{a}{c}\right) - 309.6188\left(\frac{a}{c}\right)^2 + 104.9342\left(\frac{a}{c}\right)^3$$

$$F_{B1} = B_{11} + B_{21}\left(\frac{a}{t}\right)^2 + B_{31}\left(\frac{a}{t}\right)^4 + B_{41}\left(\frac{a}{t}\right)^6 \quad (4.40)$$

$$B_{11} = 0.3896 - 1.4691\left(\frac{a}{c}\right) + 2.2225\left(\frac{a}{c}\right)^2 - 0.8425\left(\frac{a}{c}\right)^3$$

$$B_{21} = -5.4672 + 23.1791\left(\frac{a}{c}\right) - 27.1671\left(\frac{a}{c}\right)^2 + 9.3608\left(\frac{a}{c}\right)^3$$

$$B_{31} = 23.7422 - 93.7886\left(\frac{a}{c}\right) + 108.4131\left(\frac{a}{c}\right)^2 - 37.2388\left(\frac{a}{c}\right)^3$$

$$B_{41} = -28.6249 + 107.4832\left(\frac{a}{c}\right) - 120.9421\left(\frac{a}{c}\right)^2 + 40.9508\left(\frac{a}{c}\right)^3$$

Weight Functions

Knowing two reference stress intensity factor solutions K_{r0} and K_{rI} in Eqs. (4.37) and (4.38), a set of two equations can be found by substituting Eqs (4.25), (4.27) and (2.47) into Eq. (2.41):

$$K_{r0} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{B0} = \int_0^c \sigma_0 \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{1B} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{2B} \left(1 - \frac{x}{c}\right) + M_{3B} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (4.41)$$

$$K_{rI} = \sigma_1 \sqrt{\frac{\pi c}{Q}} F_{B1} = \int_0^c \sigma_1 \left(1 - \frac{x}{c}\right) \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{1B} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{2B} \left(1 - \frac{x}{c}\right) + M_{3B} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] dx \quad (4.42)$$

The third condition necessary to determine parameters M_{1A} , M_{2A} and M_{3A} was derived by double differentiation of the weight function in Eq. (2.47) must vanish for $x=c$, that is:

$$\frac{\partial^2}{\partial x^2} \left\{ \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{1B} \left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{2B} \left(1 - \frac{x}{c}\right) + M_{3B} \left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \right\} \Big|_{x=0} = 0 \quad (4.43)$$

The three unknown parameters M_{1B} , M_{2B} and M_{3B} can be determined by simultaneously solving Eqs. (4.41), (4.42) and (4.43):

$$M_{1B} = 4 \frac{\pi}{\sqrt{2Q}} F_{B0} - 6 \frac{\pi}{\sqrt{2Q}} F_{B1} - \frac{24}{5} \quad (4.44)$$

$$M_{2B} = 3 \quad (4.45)$$

$$M_{3B} = -6 \frac{\pi}{\sqrt{2Q}} F_{B0} + 12 \frac{\pi}{\sqrt{2Q}} F_{B1} + \frac{8}{5} \quad (4.46)$$

The weight function for the front surface point B of a corner crack at a fastener hole can be determined directly in the form of Eq. (2.47).

4.3 Validation of Weight Functions

In order to check the accuracy of the derived functions Eqs. (2.46), (2.47) which the weight function parameters were determined in this chapter, two comparisons were made to verify the derived weight functions presented in this chapter.

4.3.1 Comparisons with Polynomial Stress Distributions

The first was performed between the predictions using weight function method and the converted data in tables 4.1 to 4.4 with standard stress fields listed below.

For $n=2, 3$:

$$\sigma_{s2}(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2 \quad (4.47)$$

$$\sigma_{s3}(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3 \quad (4.48)$$

By substituting the equations (4.47) and (4.48) into equation (2.41) to calculate the stress intensity factors based on the derived weight functions, table 4.7~ 4.10 show

the results of stress intensity factors for points A and B in terms of boundary factors F for the quadratic and cubic stress distributions using weight function method. They also show the accuracy comparing to the converted Grandt results corresponding to stress distribution expressed in Eqs.(4.47) and (4.48).

Within the range of crack depth $0.2 \leq \frac{a}{t} \leq 0.8$, crack shape $0.5 \leq (\frac{a}{c}) \leq 1.5$ and the ratio of hole radius R to plate thickness t , $\frac{R}{t}=0.5$, the difference of the stress intensity factors between these two approaches is less than 7% for point A (figure 4.6 and 4.7) and 8% for point B (figure 4.8 and 4.9).

4.3.2 Comparisons with Remote Tension Loading

The second comparison is made between the predictions using the current weight function approach and the Newman's data [10] based on empirical equations which are specified in section 4.1.3 and listed in tables 4.5 and 4.6 for corner cracks at a fastener hole under uniform remote tension loading as shown in Figure 4.10.

With uniform tensile loading, based on the analysis of Petroski [28], the expression of stress distribution on the crack face can be written as:

$$\sigma(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] \quad (4.49)$$

Where σ_0 is the nominal stress and R is the hole radius shown in Figure 4.10.

By substituting the equation (4.49) into equation (2.41) to calculate the stress intensity factors based on the derived weight functions, the results and comparisons for points A and B were listed in tables 4.11 to 4.14 and plotted in Figures 4.11 to 4.12. The

difference of stress intensity factors between weight function and Newman data is less than 10% for hole surface point A and 8% for front surface point B within for same region of crack depth $0.2 \leq \frac{a}{t} \leq 0.8$, crack shape $0.5 \leq (\frac{a}{c}) \leq 1.5$ and the ratio of hole radius R to plate thickness t , $\frac{R}{t} = 0.5$.

The current stress intensity factor solutions using weight function method is for a single corner crack other than two symmetric corner cracks given by Newman empirical equations. In order to compare these two kinds of results, a conversion factor developed by Shah [36] was used. The relationship between one and two corner cracks was given by:

$$K^I = \sqrt{\frac{\frac{4}{\pi} + \frac{ac}{2tR}}{\frac{4}{\pi} + \frac{ac}{tR}}} K^{II} \quad (4.50)$$

where K^I is stress intensity factor for one corner crack, K^{II} is stress intensity factor for two symmetric corner cracks.

4.4 Discussions and Conclusions

Using two reference stress intensity factors, weight function method are developed to find the stress intensity factors for the surface points of a corner crack at a fastener hole in a finite thickness plate.

The stress intensity factor solutions from Grandt [4] for the crack front points of a corner crack at a hole are converted to the results with the standard stress distributions on the crack face in terms of constant, linearly decreasing, parabolic and cubic stress distributions. The converted stress intensity factor solutions for two surface points of the

corner crack under constant and linearly decreasing stress distributions loading are used as the two reference stress intensity factors for the derivation of the weight functions.

In order to verify the accuracy of the derived weight functions for the surface points of a corner crack, two comparisons are performed between the predictions of weight functions and the existing results. As one of the existing results, the converted stress intensity factor solutions for the surface points of the corner crack under parabolic and cubic stress distributions were used to validate the results from the present weight function method. Another existing result was chosen from Newman's data [10] by empirical equations for these two surface points of two symmetric corner cracks under uniform remote tensile loading.

For the hole surface point A of one corner crack at a fastener hole with the range of a/c , a/t and R/t mentioned above, the maximum difference of the stress intensity factors between the converted results and weight function approach under parabolic and cubic crack face loadings was less than 7%. For two symmetric corner cracks under remote tension, the difference of stress intensity factors at point A between the Newman fitted results and the present weight function approach is less than 10%.

For the front surface point B of one corner crack at a fastener hole within the range of crack depth $0.2 \leq \frac{a}{t} \leq 0.8$, crack shape $0.5 \leq (\frac{a}{c}) \leq 1.5$ and the ratio of hole radius R to plate thickness t , $\frac{R}{t} = 0.5$, the maximum difference of the stress intensity factors between the converted results and weight function approach under parabolic and cubic crack face loadings was less than 8%. For two symmetric corner cracks under remote tension, the difference of stress intensity factors at point B between the Newman fitted results and the present weight function approach is less than 9%.

Considering the complex geometry explored by current research, the accuracy is satisfactory in the views of engineering judgments.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	a/t	A($\Phi=0$)	D($\Phi=\pi/6$)	C($\Phi=\pi/3$)	B($\Phi=\pi/2$)
n=0	0.2	0.6745	0.7234	0.6597	0.5769
	0.4	0.6909	0.7243	0.6907	0.6595
	0.5	0.6968	0.7373	0.7103	0.6897
n=1	0.2	0.5709	0.4218	0.1905	0.1243
	0.4	0.5758	0.4234	0.2082	0.1675
	0.5	0.5764	0.4335	0.224	0.1866
n=2	0.2	0.499	0.2804	0.0741	0.0554
	0.4	0.4976	0.2831	0.0872	0.0828
	0.5	0.4962	0.291	0.0998	0.0965
n=3	0.2	0.4474	0.2004	0.026	0.0355
	0.4	0.4425	0.2037	0.0368	0.0544
	0.5	0.4402	0.2102	0.0473	0.0649

Table 4.1 Converted boundary correction factor F_n for crack front points A~D

with respect to figure 4.1 under standard stress distribution $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ on the

crack face, and $F_n = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c = 0.5$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	a/t	A($\Phi=0$)	D($\Phi=\pi/6$)	C($\Phi=\pi/3$)	B($\Phi=\pi/2$)
n=0	0.2	0.9035	0.9739	0.9726	0.9174
	0.8	0.9245	0.9467	1.0408	1.1769
n=1	0.2	0.762	0.5927	0.3386	0.262
	0.8	0.7538	0.5774	0.3762	0.3896
n=2	0.2	0.6556	0.4072	0.165	0.1409
	0.8	0.6357	0.3982	0.194	0.224
n=3	0.2	0.5811	0.3046	0.0904	0.0975
	0.8	0.5564	0.2985	0.1141	0.1567

Table 4.2 Converted boundary correction factor F_n for crack front points A~D

with respect to figure 4.1 under standard stress distribution $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ on the

crack face, and $F_n = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c = 0.9$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	a/t	A($\Phi=0$)	D($\Phi=\pi/6$)	C($\Phi=\pi/3$)	B($\Phi=\pi/2$)
n=0	0.2	0.9213	1.0099	1.078	1.0582
	0.4	0.9208	0.9511	1.0022	1.0474
	0.6	0.9112	0.9451	1.054	1.1783
	0.8	0.9317	0.9544	1.0675	1.2167
n=1	0.2	0.7763	0.6235	0.3965	0.3295
	0.4	0.7616	0.5871	0.3585	0.3271
	0.6	0.7442	0.5829	0.385	0.3834
	0.8	0.7599	0.5912	0.3954	0.4092
n=2	0.2	0.6671	0.4334	0.204	0.1839
	0.4	0.6467	0.4083	0.1825	0.1849
	0.6	0.6283	0.4051	0.2011	0.2195
	0.8	0.64	0.4121	0.209	0.238
n=3	0.2	0.5935	0.3284	0.1178	0.124
	0.4	0.5695	0.3097	0.106	0.129
	0.6	0.5514	0.3072	0.1207	0.1531
	0.8	0.5607	0.3133	0.1269	0.1673

Table 4.3 Converted boundary correction factor F_n for crack front points A~D

with respect to figure 4.1 under standard stress distribution $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ on the

crack face, and $F_n = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c = 1.1$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	a/t	A($\Phi=0$)	D($\Phi=\pi/6$)	C($\Phi=\pi/3$)	B($\Phi=\pi/2$)
n=0	0.2	0.8151	0.9199	1.0333	1.02
	0.4	0.8096	0.8692	0.9682	1.0213
	0.6	0.7966	0.843	0.9789	1.1036
	0.8	0.8091	0.8499	0.9888	1.1302
n=1	0.2	0.6911	0.5876	0.4057	0.3359
	0.4	0.6747	0.5518	0.3658	0.3342
	0.6	0.6571	0.5359	0.3717	0.3681
	0.8	0.6665	0.542	0.3795	0.387
n=2	0.2	0.5968	0.4193	0.2247	0.1963
	0.4	0.5751	0.3921	0.1963	0.1938
	0.6	0.5573	0.3811	0.2025	0.2148
	0.8	0.5637	0.3864	0.2083	0.2281
n=3	0.2	0.5321	0.3296	0.1399	0.1399
	0.4	0.5098	0.3058	0.1211	0.1357
	0.6	0.4928	0.2973	0.1279	0.151
	0.8	0.4972	0.3021	0.1324	0.1609

Table 4.4 Converted boundary correction factor F_n for crack front points A~D

with respect to figure 4.1 under standard stress distribution $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ on the crack face, and $F_n = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c = 1.5$.

a/c	a/t=0.2	a/t=0.4	a/t=0.5	a/t=0.6	a/t=0.7
0.5	2.6702	2.4422	2.4067	2.3982	2.4021
0.9	2.8752	2.6398	2.561	2.4991	2.4472
1	2.8983	2.667	2.5839	2.5161	2.4582
1.5	2.0472	1.9134	1.8555	1.8032	1.7556

Table 4.5 The boundary correction factors F for point A of two symmetric corner cracks at a fastener hole obtained by Newman empirical equations under remote tension

loading, $F = \frac{K |_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$.

a/c	a/t=0.2	a/t=0.4	a/t=0.5	a/t=0.6	a/t=0.7
0.5	1.2864	1.2348	1.278	1.3451	1.4276
0.9	1.9743	1.7625	1.7528	1.7762	1.8219
1	2.1303	1.8855	1.8636	1.8772	1.9151
1.5	1.9558	1.6905	1.6308	1.5992	1.5875

Table 4.6 The boundary correction factors F for point B of two symmetric corner cracks at a fastener hole obtained by Newman empirical equations under remote tension

loading, $F = \frac{K |_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	Surface Points	a/t=0.2	a/t=0.4	a/t=0.5
n=2	A (WF)	0.5078	0.5083	0.5069
	A (Converted)	0.499	0.4976	0.4962
	A (Accuracy,%)	1.76	2.14	2.16
	B (WF)	0.0593	0.0887	0.1027
	B (Converted)	0.0554	0.0828	0.0965
	B (Accuracy,%)	6.96	7.10	6.46
n=3	A (WF)	0.4637	0.4621	0.4599
	A (Converted)	0.4474	0.4425	0.4402
	A (Accuracy,%)	3.64	4.42	4.48
	B (WF)	0.0359	0.0583	0.0694
	B (Converted)	0.0355	0.0544	0.0649
	B (Accuracy,%)	1.15	7.08	6.91

Table 4.7 Comparisons of boundary correction factors F , for surface points A and B of a corner crack at a fastener hole between weight function method (WF) and the converted results (listed in tables 4.1) under the crack face pressure, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ with $a/c=0.5$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	Surface Points	a/t=0.2	a/t=0.8
n=2	A (WF)	0.6752	0.6571
	A (Converted)	0.6556	0.6357
	A (Accuracy,%)	2.98	3.37
	B (WF)	0.1475	0.2318
	B (Converted)	0.1409	0.224
	B (Accuracy,%)	4.66	3.46
n=3	A (WF)	0.6143	0.5926
	A (Converted)	0.5811	0.5564
	A (Accuracy,%)	5.72	6.50
	B (WF)	0.1009	0.1638
	B (Converted)	0.0975	0.1567
	B (Accuracy,%)	3.53	4.54

Table 4.8 Comparisons of boundary correction factors F , for surface points A and B of a corner crack at a fastener hole between weight function method (WF) and the converted results (listed in tables 4.2) under the crack face pressure, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ with $a/c=0.9$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	Surface Points	a/t=0.2	a/t=0.4	a/t=0.6	a/t=0.8
n=2	A (WF)	0.6874	0.6687	0.6497	0.6626
	A (Converted)	0.6671	0.6467	0.6283	0.64
	A (Accuracy,%)	3.04	3.39	3.41	3.53
	B (WF)	0.1917	0.1911	0.2255	0.2449
	B (Converted)	0.1839	0.1849	0.2195	0.238
	B (Accuracy,%)	4.24	3.35	2.75	2.90
n=3	A (WF)	0.625	0.6053	0.5866	0.5975
	A (Converted)	0.5935	0.5695	0.5514	0.5607
	A (Accuracy,%)	5.31	6.29	6.38	6.57
	B (WF)	0.1338	0.1338	0.1582	0.1737
	B (Converted)	0.124	0.129	0.1531	0.1673
	B (Accuracy,%)	7.88	3.69	3.34	3.83

Table 4.9 Comparisons of boundary correction factors F , for surface points A and B of a corner crack at a fastener hole between weight function method (WF) and the converted results (listed in tables 4.3) under the crack face pressure, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ with $a/c=1.1$.

$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$	Surface Points	a/t=0.2	a/t=0.4	a/t=0.6	a/t=0.8
n=2	A (WF)	0.6137	0.5946	0.5766	0.5842
	A (Converted)	0.5968	0.5751	0.5573	0.5637
	A (Accuracy,%)	2.83	3.38	3.46	3.63
	B (WF)	0.1999	0.198	0.2181	0.2324
	B (Converted)	0.1963	0.1938	0.2148	0.2281
	B (Accuracy,%)	1.84	2.15	1.53	1.90
n=3	A (WF)	0.5588	0.5393	0.522	0.5284
	A (Converted)	0.5321	0.5098	0.4928	0.4972
	A (Accuracy,%)	5.01	5.78	5.92	6.27
	B (WF)	0.1414	0.1396	0.1536	0.1651
	B (Converted)	0.1399	0.1357	0.151	0.1609
	B (Accuracy,%)	1.08	2.87	1.70	2.63

Table 4.10 Comparisons of boundary correction factors F , for surface points A and B of a corner crack at a fastener hole between weight function method (WF) and the converted results (listed in tables 4.4) under the crack face pressure, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n$ with $a/c=1.5$.

a/c	a/t=0.2	a/t=0.4	a/t=0.5	a/t=0.6	a/t=0.7
0.5	2.578	2.5246	2.5251	2.5187	2.4924
0.9	2.6857	2.5909	2.5749	2.5683	2.5659
1	2.617	2.5114	2.4832	2.4668	2.4661
1.5	1.9439	1.8559	1.8212	1.7962	1.7903

Table 4.11 The boundary correction factors F , $F = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, for point A of two

symmetric corner cracks at a fastener hole obtained by weight function method under remote tensile loading .

a/c	a/t=0.2	a/t=0.4	a/t=0.5	a/t=0.6	a/t=0.7
0.5	1.2793	1.3243	1.3844	1.3971	1.3095
0.9	1.8816	1.6824	1.6903	1.7379	1.815
1	2.0122	1.7308	1.736	1.8042	1.8905
1.5	1.8842	1.5972	1.5656	1.5853	1.6127

Table 4.12 The boundary correction factors F , $F = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, for point B of two

symmetric corner cracks at a fastener hole obtained by weight function method under remote tensile loading .

a/c	a/t=0.2	a/t=0.4	a/t=0.5	a/t=0.6	a/t=0.7
0.5	3.4556	3.374	4.9181	5.0248	3.7621
0.9	6.5937	1.8546	0.5399	2.7679	4.8496
1	9.7058	5.832	3.8983	1.9601	0.3234
1.5	5.0452	3.0046	1.8484	0.3855	1.9761

Table 4.13 The percentage difference between boundary correction factor F for point A of two symmetric corner cracks at a fastener hole obtained by weight function method and the Newman fitted results (table 4.5) with remote tension loading.

a/c	a/t=0.2	a/t=0.4	a/t=0.5	a/t=0.6	a/t=0.7
0.5	0.5555	7.2464	8.32	3.8662	8.2723
0.9	4.6951	4.546	3.57	2.1582	0.3799
1	5.5444	8.2046	6.8491	3.8859	1.2843
1.5	3.6603	5.5181	3.9981	0.8692	1.5873

Table 4.14 The percentage difference between boundary correction factor F for point B of two symmetric corner cracks at a fastener hole obtained by weight function method and the Newman fitted results (table 4.6) with remote tension loading.

Figure 4.1 A corner crack at a fastener hole in a finite thickness plate. Point P is arbitrary point along the crack front, points A and B are two surface points, A, hole surface point; B, front surface point.

Figure 4.2 Comparison of reference stress intensity factors from curve fitting and converted results for constant stress distribution, hole surface point A.

Figure 4.3 Comparison of reference stress intensity factors from curve fitting and converted results for linearly decreasing stress distribution, hole surface point A.

Figure 4.4 Comparison of reference stress intensity factors from curve fitting and converted results for constant stress distribution, front surface point B.

Figure 4.5 Comparison of reference stress intensity factors from curve fitting and converted results for linearly decreasing stress distribution, front surface point B.

Figure 4.6 Comparison of reference stress intensity factors from weight function method and converted results for parabolic stress distribution, hole surface point A.

Figure 4.7 Comparison of reference stress intensity factors from weight function method and converted results for cubic stress distribution, hole surface point A.

Figure 4.8 Comparison of reference stress intensity factors from weight function method and converted results for parabolic stress distribution, front surface point B.

Figure 4.9 Comparison of reference stress intensity factors from weight function method and converted results for cubic stress distribution, front surface point B.

Figure 4.10 Symmetric corner cracks at a fastener hole in a plate with uniform remote tension.

Figure 4.11 Comparison of stress intensity factors by present results and Newman fitted results under remote tensile stress distribution loading, hole surface point A.

Figure 4.12 Comparison of stress intensity factors by present results and Newman fitted results under remote tensile stress distribution loading, front surface point B.

Chapter 5

Weight Functions for Local Points in a Corner Crack

Stress intensity factors of the hole surface and front surface points at a fastener hole in a plate are found by weight function approach presented in chapter 4 using Sheng-Glinka [35] general weight function expression forms. However, in practice, the stress intensity factor solutions for points along the crack front may be required for fracture assessment and fatigue analysis. For engineering applications, the most important thing is focusing on finding the maximum stress intensity factor along the crack front. For cracks subjected nonlinear stress distributions, the point which has the maximum stress intensity factor can be any point along the crack front other than the two crack surface points considered in Chapter 4.

As mentioned in chapter 2, Wang and Lambert [8] developed approximate closed form weight functions for any point, $0 < \Phi < \pi/2$, along the crack front of a semi-elliptical surface crack in a finite thickness plate. In the current research, Wang and Lambert approach is applied to the derivation of weight functions for any local points of a corner crack at a fastener hole.

In this chapter, available results from the converted Grandt's data and Newman fitted data are reviewed for the purpose of weight function's derivations and validations. The stress intensity factors determined by weight function method for these local points are valid for the range of relative crack depths, a/t between 0.2 and 0.8, crack depth ratio, a/c between 0.5 and 1.5 and an R/t ratio of 0.5. The weight functions were verified using converted Grandt's results [4] for parabolic and cubic stress distribution on the crack face and the Newman's curve fitting results [10] for uniform remote tension loading.

5.1 Review of Available Results for Corner Cracks

For the purpose of derivation and validation of weight functions presented in this chapter, the converted stress intensity factors from Grandt results [4] discussed in chapter 4, tables 4.1-4.4 are employed as the available results.

5.1.1 Available Results of Grandt

As mentioned in chapter 3, the loading conditions in Grandt's paper [4] is in the form of equation (3.1), that is :

$$\sigma_g(x) = \sigma_0 \left(\frac{x}{R}\right)^n \quad (3.1)$$

where R is hole radius.

The stress intensity factor under the crack face pressure expressed by equation (3.1) is:

$$K_g |_{\sigma_g(x)} = \sigma_0 \sqrt{\pi a} M_n \quad (4.1)$$

The stress intensity factor solution for corner cracks at a fastener hole in a plate given by Grandt is summarized in [4]. The dimensionless stress intensity factor M is presented as a function of crack shape (a/c), size (a/t) position along the crack front angle parameter Φ with R/t ratio of 0.5 as shown in Figure 4.1.

5.1.2 Conversion of Grandt's Result

Referring to Figure 4.1, the standard loading conditions on the crack face expressed by Eq.(3.2) is :

$$\sigma_s(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^n \quad (3.2)$$

Where σ_0 is nominal stress applied on the crack face and c is the crack depth, $n=0, 1, 2, 3$.

The following stress intensity factor expression is adopted corresponding to the standard stress distribution showed in Eq.(3.2), that is:

$$K_s |_{\sigma_s(x)} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_n \quad (4.2)$$

where F_n is the dimensionless stress intensity factor under the standard stress distribution Eq.(3.2) applied on the crack face and Q is the shape factor for an ellipse and is given by equations (4.3a) and (4.3b) discussed in chapter 4.

In chapter 4, all the stress intensity factor solutions from Grandt [4] are converted to the results with the adopted stress intensity factor expression shown by equation (4.2). These converted results are listed in table 4.1 to 4.4 for constant, linearly decreasing, parabolic and cubic stress distributions for $\Phi = 0, \pi/6, \pi/3, \pi/2$. This chapter will employ these results to derive and validate the weight functions along the crack front for corner crack at a fastener hole in a finite thickness plate.

5.1.3 Available Results of Newman

To validate the weight functions derived in present research for corner crack, the stress intensity factor solution obtained from Newman [10] for corner crack under remote tension loading is used. As discussed in chapter 4, the curve fitting equations from Eqs. (4.10) to (4.24) are used to obtain the stress intensity factors along the crack front in which the crack geometry is shown in figure 4.1.

The dimensionless stress intensity factors using these empirical equations are obtained by Matlab programming [39]. The results for local points along the crack front with the angle Φ varying from $\pi/60$ to $29\pi/60$ as shown in figure 4.1 for $0.5 \leq \frac{a}{c} \leq 1.5$, $0.2 \leq \frac{a}{t} \leq 0.8$ and the ratio of $\frac{R}{t} = 0.5$ were presented in tables 5.1-5.4.

5.2 Weight Functions

Using the stress intensity factor results converted from Grandt [4] for uniform and linear stress distributions as reference results, weight functions at any local point P for a corner crack at a fastener hole were derived for the range of $0.5 \leq \frac{a}{c} \leq 1.5$, $0.2 \leq \frac{a}{t} \leq 0.8$ and the ratio of $\frac{R}{t} = 0.5$. The crack geometry is shown in figure 4.1.

For local points on the crack front, $0 < \Phi < \pi/2$, the weight function forms developed in section 2.3.2 using Wang and Lambert approach [8] were used as general weight function expressions.

5.2.1 Reference Stress Intensity Factors

The converted results performed in section 4.1.2 are used for the reference stress intensity factor solutions under the standard loading system corresponding to a constant and linearly decreasing stress field acting on the crack surface.

The stress intensity factors for uniform tension on the crack face:

$$\sigma(x) = \sigma_0 \quad (4.25)$$

are:

$$K_{r0} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{P0} \left(\frac{a}{c}, \frac{a}{t}, \Phi \right) \quad (5.1)$$

The stress intensity factors for linearly decreasing crack face pressure:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c} \right) \quad (4.27)$$

are:

$$K_{r1} = \sigma_0 \sqrt{\frac{\pi c}{Q}} F_{P1} \left(\frac{a}{c}, \frac{a}{t}, \Phi \right) \quad (5.2)$$

where F_{P0} and F_{P1} are boundary correction factors for local point P with corresponding to constant and linearly decreasing stress distributions on the crack face.

Based on the results in tables 4.1 to 4.4, F_{P0} and F_{P1} can be represented by the empirical formulas of the boundary correction factors for points A , D , C and B along the crack front as shown in figure 4.1 as follows:

for $n=0$:

$$F_{P0} = L_{10} + L_{20} \left(\frac{a}{t} \right)^2 + L_{30} \left(\frac{a}{t} \right)^4 + L_{40} \left(\frac{a}{t} \right)^6 \quad (5.3)$$

where

$$L_{10} = K_{10} + K_{20} \left(\frac{a}{c} \right) + K_{30} \left(\frac{a}{c} \right)^2 + K_{40} \left(\frac{a}{c} \right)^3$$

$$K_{10} = -0.1353 + 0.9953 \left(\frac{2\Phi}{\pi} \right) + 3.4411 \left(\frac{2\Phi}{\pi} \right)^2 - 3.2481 \left(\frac{2\Phi}{\pi} \right)^3$$

$$K_{20} = 2.2882 - 1.5372\left(\frac{2\Phi}{\pi}\right) - 18.5886\left(\frac{2\Phi}{\pi}\right)^2 + 14.8873\left(\frac{2\Phi}{\pi}\right)^3$$

$$K_{30} = -1.4998 + 1.2997\left(\frac{2\Phi}{\pi}\right) + 22.7965\left(\frac{2\Phi}{\pi}\right)^2 - 17.8114\left(\frac{2\Phi}{\pi}\right)^3$$

$$K_{40} = 0.2646 - 0.4127\left(\frac{2\Phi}{\pi}\right) - 7.7319\left(\frac{2\Phi}{\pi}\right)^2 + 65.9958\left(\frac{2\Phi}{\pi}\right)^3$$

$$L_{20} = H_{10} + H_{20}\left(\frac{a}{c}\right) + H_{30}\left(\frac{a}{c}\right)^2 + H_{40}\left(\frac{a}{c}\right)^3$$

$$H_{10} = 1.6974 - 39.0985\left(\frac{2\Phi}{\pi}\right) - 19.3720\left(\frac{2\Phi}{\pi}\right)^2 + 42.6217\left(\frac{2\Phi}{\pi}\right)^3$$

$$H_{20} = -5.1522 + 132.2010\left(\frac{2\Phi}{\pi}\right) + 102.5613\left(\frac{2\Phi}{\pi}\right)^2 - 170.9797\left(\frac{2\Phi}{\pi}\right)^3$$

$$H_{30} = 5.1577 - 141.5715\left(\frac{2\Phi}{\pi}\right) - 128.7868\left(\frac{2\Phi}{\pi}\right)^2 + 196.8741\left(\frac{2\Phi}{\pi}\right)^3$$

$$H_{40} = -1.6504 + 46.7940\left(\frac{2\Phi}{\pi}\right) + 45.0117\left(\frac{2\Phi}{\pi}\right)^2 - 66.6139\left(\frac{2\Phi}{\pi}\right)^3$$

$$L_{30} = Q_{10} + Q_{20}\left(\frac{a}{c}\right) + Q_{30}\left(\frac{a}{c}\right)^2 + Q_{40}\left(\frac{a}{c}\right)^3$$

$$Q_{10} = -8.4548 + 125.3356\left(\frac{2\Phi}{\pi}\right) + 96.3405\left(\frac{2\Phi}{\pi}\right)^2 - 152.4775\left(\frac{2\Phi}{\pi}\right)^3$$

$$Q_{20} = 29.4903 - 425.9744\left(\frac{2\Phi}{\pi}\right) - 447.9876\left(\frac{2\Phi}{\pi}\right)^2 + 605.1744\left(\frac{2\Phi}{\pi}\right)^3$$

$$Q_{30} = -31.7494 + 452.9340\left(\frac{2\Phi}{\pi}\right) + 552.6274\left(\frac{2\Phi}{\pi}\right)^2 - 697.4032\left(\frac{2\Phi}{\pi}\right)^3$$

$$Q_{40} = 10.4671 - 149.2936 \left(\frac{2\Phi}{\pi} \right) - 193.1143 \left(\frac{2\Phi}{\pi} \right)^2 + 236.9241 \left(\frac{2\Phi}{\pi} \right)^3$$

$$L_{40} = P_{10} + P_{20} \left(\frac{a}{c} \right) + P_{30} \left(\frac{a}{c} \right)^2 + P_{40} \left(\frac{a}{c} \right)^3$$

$$P_{10} = 8.1383 - 64.7928 \left(\frac{2\Phi}{\pi} \right) - 195.9480 \left(\frac{2\Phi}{\pi} \right)^2 + 179.3826 \left(\frac{2\Phi}{\pi} \right)^3$$

$$P_{20} = -30.5046 + 244.5849 \left(\frac{2\Phi}{\pi} \right) + 732.0744 \left(\frac{2\Phi}{\pi} \right)^2 - 671.1390 \left(\frac{2\Phi}{\pi} \right)^3$$

$$P_{30} = 34.2173 - 277.7847 \left(\frac{2\Phi}{\pi} \right) - 816.4188 \left(\frac{2\Phi}{\pi} \right)^2 + 750.3673 \left(\frac{2\Phi}{\pi} \right)^3$$

$$P_{40} = -11.5229 + 95.5713 \left(\frac{2\Phi}{\pi} \right) + 272.1285 \left(\frac{2\Phi}{\pi} \right)^2 - 251.2426 \left(\frac{2\Phi}{\pi} \right)^3$$

for $n=l$:

$$F_{P1} = L_{11} + L_{21} \left(\frac{a}{t} \right)^2 + L_{31} \left(\frac{a}{t} \right)^4 + L_{41} \left(\frac{a}{t} \right)^6 \quad (5.4)$$

$$L_{11} = K_{11} + K_{21} \left(\frac{a}{c} \right) + K_{31} \left(\frac{a}{c} \right)^2 + K_{41} \left(\frac{a}{c} \right)^3$$

$$K_{11} = -0.0421 + 0.6891 \left(\frac{2\Phi}{\pi} \right) + 0.7335 \left(\frac{2\Phi}{\pi} \right)^2 - 0.9909 \left(\frac{2\Phi}{\pi} \right)^3$$

$$K_{21} = 1.6954 - 2.4522 \left(\frac{2\Phi}{\pi} \right) - 8.4231 \left(\frac{2\Phi}{\pi} \right)^2 + 7.7107 \left(\frac{2\Phi}{\pi} \right)^3$$

$$K_{31} = -1.0235 + 2.3406\left(\frac{2\Phi}{\pi}\right) + 9.0648\left(\frac{2\Phi}{\pi}\right)^2 - 8.1594\left(\frac{2\Phi}{\pi}\right)^3$$

$$K_{41} = 0.1479 - 0.6673\left(\frac{2\Phi}{\pi}\right) - 2.8593\left(\frac{2\Phi}{\pi}\right)^2 + 2.5362\left(\frac{2\Phi}{\pi}\right)^3$$

$$L_{21} = H_{11} + H_{21}\left(\frac{a}{c}\right) + H_{31}\left(\frac{a}{c}\right)^2 + H_{41}\left(\frac{a}{c}\right)^3$$

$$H_{11} = -0.6833 - 20.6316\left(\frac{2\Phi}{\pi}\right) - 3.2098\left(\frac{2\Phi}{\pi}\right)^2 + 19.0575\left(\frac{2\Phi}{\pi}\right)^3$$

$$H_{21} = 3.0473 + 70.4576\left(\frac{2\Phi}{\pi}\right) + 25.2216\left(\frac{2\Phi}{\pi}\right)^2 - 75.5473\left(\frac{2\Phi}{\pi}\right)^3$$

$$H_{31} = -3.7038 - 75.5738\left(\frac{2\Phi}{\pi}\right) - 33.8926\left(\frac{2\Phi}{\pi}\right)^2 + 86.0031\left(\frac{2\Phi}{\pi}\right)^3$$

$$H_{41} = 1.2717 + 24.9808\left(\frac{2\Phi}{\pi}\right) + 11.7774\left(\frac{2\Phi}{\pi}\right)^2 - 28.6690\left(\frac{2\Phi}{\pi}\right)^3$$

$$L_{31} = Q_{11} + Q_{21}\left(\frac{a}{c}\right) + Q_{31}\left(\frac{a}{c}\right)^2 + Q_{41}\left(\frac{a}{c}\right)^3$$

$$Q_{11} = -2.0213 + 76.7697\left(\frac{2\Phi}{\pi}\right) + 20.4678\left(\frac{2\Phi}{\pi}\right)^2 - 71.4740\left(\frac{2\Phi}{\pi}\right)^3$$

$$Q_{21} = 6.5666 - 263.5495\left(\frac{2\Phi}{\pi}\right) - 116.7439\left(\frac{2\Phi}{\pi}\right)^2 + 279.9382\left(\frac{2\Phi}{\pi}\right)^3$$

$$Q_{31} = -6.8213 + 281.9279\left(\frac{2\Phi}{\pi}\right) + 152.1904\left(\frac{2\Phi}{\pi}\right)^2 - 318.8839\left(\frac{2\Phi}{\pi}\right)^3$$

$$Q_{41} = 2.2450 - 93.1918\left(\frac{2\Phi}{\pi}\right) - 53.0478\left(\frac{2\Phi}{\pi}\right)^2 + 106.7557\left(\frac{2\Phi}{\pi}\right)^3$$

$$L_{41} = P_{11} + P_{21}\left(\frac{a}{c}\right) + P_{31}\left(\frac{a}{c}\right)^2 + P_{41}\left(\frac{a}{c}\right)^3$$

$$P_{11} = 4.0382 - 48.6134\left(\frac{2\Phi}{\pi}\right) - 67.5594\left(\frac{2\Phi}{\pi}\right)^2 + 83.5097\left(\frac{2\Phi}{\pi}\right)^3$$

$$P_{21} = -15.1709 + 183.2376\left(\frac{2\Phi}{\pi}\right) + 250.7832\left(\frac{2\Phi}{\pi}\right)^2 - 311.3667\left(\frac{2\Phi}{\pi}\right)^3$$

$$P_{31} = 17.0867 - 207.5759\left(\frac{2\Phi}{\pi}\right) - 276.4336\left(\frac{2\Phi}{\pi}\right)^2 + 345.9807\left(\frac{2\Phi}{\pi}\right)^3$$

$$P_{41} = -5.7950 + 71.1061\left(\frac{2\Phi}{\pi}\right) + 90.2110\left(\frac{2\Phi}{\pi}\right)^2 - 114.5713\left(\frac{2\Phi}{\pi}\right)^3$$

Typical curve fitting results in which the ratios are $a/c=0.5$, $a/t=0.4$; $a/c=0.9$, $a/t=0.2$; $a/c=1.1$, $a/t=0.6$; $a/c=1.5$, $a/t=0.3$ are shown in figures 5.1-5.4.

5.2.2 Weight Functions

Since that two reference stress intensity factor solutions K_{r0} and K_{r1} in Eqs. (5.1) and (5.2) are known, using the weight functions and boundary conditions developed in section 2.3.2, the parameters of M_{1P} , M_{2P} , M_{3P} and M_{4P} can be determined from the following four simultaneous equations:

$$K_{r0} = \sigma \sqrt{\frac{\pi c}{Q}} F_{P0} = \int_0^{\sin \Phi} \sigma_0 \frac{\sqrt{\sin \Phi + 1}}{\sqrt{\pi(c \sin \Phi - x)}} \left[1 + M_{1P} \left(1 - \frac{x}{c \sin \Phi} \right) + M_{2P} \left(1 - \frac{x}{c \sin \Phi} \right)^2 \right] dx +$$

$$\int_{\sin \Phi}^c \sigma_0 \frac{\sqrt{1 - \sin \phi}}{\sqrt{\pi(x - c \sin \phi)}} \left[1 + M_{3P} \left(\frac{x}{c \sin \Phi} - 1 \right)^{\frac{1}{2}} + M_{4P} \left(\frac{x}{c \sin \Phi} - 1 \right) \right] dx \quad (5.5)$$

$$\begin{aligned}
K_{r1} = \sigma_1 \sqrt{\frac{\pi c}{Q}} F_{P1} = \int_0^{c \sin \phi} \sigma_1 \left(1 - \frac{x}{c}\right) \frac{\sqrt{\sin \Phi + 1}}{\sqrt{\pi(c \sin \Phi - x)}} \left[1 + M_{1p} \left(1 - \frac{x}{c \sin \Phi}\right) + M_{2p} \left(1 - \frac{x}{c \sin \Phi}\right)^2\right] dx + \\
\int_{c \sin \phi}^c \sigma_1 \left(1 - \frac{x}{c}\right) \frac{\sqrt{1 - \sin \phi}}{\sqrt{\pi(x - c \sin \phi)}} \left[1 + M_{3p} \left(\frac{x}{c \sin \Phi} - 1\right)^{\frac{1}{2}} + M_{4p} \left(\frac{x}{c \sin \Phi} - 1\right)\right] dx
\end{aligned} \tag{5.6}$$

$$\frac{\partial^2}{\partial x^2} \left\{ \frac{\sqrt{\sin \Phi + 1}}{\sqrt{\pi(c \sin \Phi - x)}} \left[1 + M_{1p} \left(1 - \frac{x}{c \sin \Phi}\right) + M_{2p} \left(1 - \frac{x}{c \sin \Phi}\right)^2\right] \right\} \Big|_{x=0} = 0 \tag{5.7}$$

$$\frac{\sqrt{1 - \sin \phi}}{\sqrt{\pi(x - c \sin \phi)}} \left[1 + M_{3p} \left(\frac{x}{c \sin \Phi} - 1\right)^{\frac{1}{2}} + M_{4p} \left(\frac{x}{c \sin \Phi} - 1\right)\right] \Big|_{x=c} = 0 \tag{5.8}$$

Equations (5.5) and (5.6) come from Eqs. (2.52) and (2.54). The reference stress distributions for reference stress intensity factors, Eqs (4.25) and (4.27), and the general expressions of weight functions for local points in Eqs. (2.52) and (2.54), have been substituted for $\sigma(x)$ and $m(x, c)$ into equation (2.41). The boundary correction factors F_{P0} and F_{P1} is given by Eqs. (5.3) and (5.4). Eqs. (5.7) and (5.8) come from the boundary conditions described in Eqs. (2.53) and (2.55).

Performing the integrations and derivations, the unknown parameters M_{1p} , M_{2p} , M_{3p} and M_{4p} can be obtained by solving the four simultaneous equations from Eqs (5.5) to (5.8).

The weight function for the local point P of a corner crack at a fastener hole can be determined directly in the form of Eq. (2.52) and (2.54).

5.3 Validation of Weight Functions

5.3.1 Comparison between Weight Function predictions and Converted Results

In order to check the accuracy of the derived weight functions in section 5.2.2, two comparisons were made to verify the results of weight function approach presented in this chapter.

The first comparison was performed between the predictions using weight function method and the converted data in tables 4.1 to 4.4 with non-linear standard stress fields listed below:

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2 \quad (5.9)$$

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3 \quad (5.10)$$

By substituting the equations (5.9) and (5.10) into equation (2.41) to calculate the stress intensity factors based on the derived weight functions, tables 5.5 and 5.12 show the results of stress intensity factors for local points in terms of boundary factors F for the parabolic and cubic stress distribution using weight function method and the comparisons between the weight function solutions and the converted Grandt results with corresponding to stress distribution expressed in Eqs. (5.9) and (5.10).

Figures 5.1 to 5.4 show typical stress intensity factor results for any point along the crack front calculated from empirical equations for constant and linear decreasing

stress distributions, and from weight functions for parabolic and cubic stress distributions for the cases of $a/c=0.5$, $a/t=0.4$; $a/c=0.9$, $a/t=0.2$; $a/c=1.1$, $a/t=0.6$; $a/c=1.5$, $a/t=0.8$.

Within the range of crack depth $0.2 \leq \frac{a}{t} \leq 0.8$, crack shape $0.5 \leq (\frac{a}{c}) \leq 1.5$, $0 < \Phi < \pi/2$

and the ratio of hole radius R to plate thickness t , $\frac{R}{t}=0.5$, the differences of the stress intensity factors between these two approaches are generally less than 15%. For the point ($\Phi=87\pi/180$) near the front point B ($\Phi=\pi/2$) with $a/c=0.5$ under parabolic and cubic stress distributions, the percentage of difference is higher than others. It is due to the fact that the stress intensity factor values of these points are very close to zero, even the small change of the absolute value can cause larger percentage difference.

5.3.2 Comparison between Weight Function Predictions and Newman's Fitted Results

Another comparison is made between the predictions using weight function approach and the Newman data [10] based on empirical equations which are specified in section 4.1.3 and listed in tables 5.1 to 5.4 for corner cracks at a fastener hole under uniform remote tension loading as shown in Figure 4.10.

With uniform tensile loading applied on the far end side of plate shown in figure 4.10, based on the analysis of Petroski [28], the expression of stress distribution on the crack face can be written as:

$$\sigma(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] \quad (4.49)$$

The stress intensity factors can be obtained by the integration the product of the derived weight function in this chapter and the stress distribution in equation (4.49). The results of comparison for local points were listed in tables 5.13 to 5.16 and plotted in Figures 5.5 to 5.8. The difference of stress intensity factors between weight function and Newman data is less than 15% for local points following the angle $0 < \Phi < \pi/2$ within for the region of crack depth $0.2 \leq \frac{a}{t} \leq 0.8$, crack shape $0.5 \leq (\frac{a}{c}) \leq 1.5$ and the ratio of hole radius R to plate thickness t , $\frac{R}{t} = 0.5$.

Note that the current stress intensity factor solutions using weight function method is for a single corner crack other than two symmetric corner cracks in Newman empirical equations, in order to compare these two kinds of results, a conversion factor described in chapter 4 was used in this section.

The relationship between one and two corner cracks was given by:

$$K^I = \sqrt{\frac{\frac{4}{\pi} + \frac{ac}{2tR}}{\frac{4}{\pi} + \frac{ac}{tR}}} K^{II} \quad (4.50)$$

where K^I is stress intensity factor for one corner crack, K^{II} is stress intensity factor for two symmetric corner cracks.

5.4 Discussions and Conclusions

Weight functions for any local points, $0 < \Phi < \pi/2$, along the crack front of a corner crack at a fastener hole in a finite thickness plate have been derived and validated using results from Grandt [4] and Newman empirical equations.

The empirical formulas of stress intensity factors are made by curve fitting of the converted results including four points along the crack front listed in tables 4.1 to 4.4 under constant and linearly decreasing stress distributions. These fitted equations are used as the two reference stress intensity factors in the derivation of the weight functions.

Two existing sets of results are employed to validate the derived the weight function solutions for one corner crack or two symmetric corner cracks at a fastener hole. One comes from the converted stress intensity factor of local points along the crack front for one corner crack under parabolic and cubic stress distributions. Another set of result was Newman's [10] empirical equations for two symmetric corner cracks under uniform remote tensile loading.

For one corner crack at a fastener hole, with the range of $0.5 \leq (\frac{a}{c}) \leq 1.5$, $0.2 \leq \frac{a}{t} \leq 0.8$ and $\frac{R}{t} = 0.5$, the comparison of local points along the crack front $0 < \Phi < \pi/2$ between the converted results and weight function approach under parabolic and cubic crack face loading shows that the maximum difference of the stress intensity factors was less than 15% except at the cases of $a/c = 0.5$, $\Phi = 87\pi/180$ both for parabolic and cubic loading situations. Considering the very small value of stress intensity factors (less than 0.01 in terms of dimensionless stress intensity factors F) at points $a/c = 0.5$, $\Phi = 87\pi/180$ both for parabolic and cubic loadings, the large difference in percentage is not significant. Because the weight functions for local points are valid for the range of $0 < \Phi < \pi/2$, for the surface points, the hole surface point $A(\Phi = 0)$ and front surface $B(\Phi = \pi/2)$ as shown in figure 4.1, the comparisons were made by choosing the local points whose the parametric angle Φ are very close to these surface points A as $\Phi = \pi/60$ and B as $\Phi = 87\pi/180$.

For two symmetric corner cracks under remote tension, with the same range of a/c , a/t and R/t mentioned above, the difference of stress intensity factors of local points between the Newman fitted results and the present weight function approach is less than

16% in which the large difference occurred at the points with $\Phi=\pi/60$ and $\Phi=29\pi/60$ very close to the surface points with $\Phi=0$ and $\Phi=\pi/2$.

As discussed in [8], it is likely that the accuracy of the stress intensity factors by present weight function approach could be improved by adding one more term in the weight function formulation, equation (2.52) and (2.54). However, this would require additional reference stress intensity factors, and increase the complexity of the solution.

When used together with weight functions derived for $\Phi=0$ and $\Phi=\pi/2$ in Chapter 4, the weight functions are suitable for the stress intensity factors calculations anywhere along the crack front for corner cracks in fastener hole under any complex stress fields.

a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
0.2	2.5435	2.0013	1.6683	1.4386	1.2759
0.4	2.3032	1.7925	1.5158	1.3307	1.2206
0.5	2.2682	1.7782	1.5175	1.345	1.2603
0.6	2.2616	1.7893	1.5399	1.3782	1.3227
0.7	2.2681	1.8112	1.571	1.4202	1.3995

Table 5.1 The boundary correction factors F for local points of two symmetric corner cracks at a fastener hole obtained by Newman empirical equations under remote tension loading, $F = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c=0.5$.

a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
0.2	2.7725	2.3111	2.0451	1.9179	1.9546
0.4	2.5104	2.0034	1.7583	1.6599	1.739
0.5	2.4261	1.9233	1.695	1.6129	1.7253
0.6	2.3616	1.8694	1.6572	1.5915	1.7435
0.7	2.3093	1.8312	1.6339	1.5847	1.7829

Table 5.2 The boundary correction factors F for local points of two symmetric corner cracks at a fastener hole obtained by Newman empirical equations under remote tension loading, $F = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c=0.9$.

a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
0.2	2.8007	2.3674	2.1306	2.0337	2.1087
0.4	2.5417	2.0505	1.8235	1.7478	1.8601
0.5	2.4523	1.9609	1.7486	1.6885	1.8341
0.6	2.3811	1.8972	1.7001	1.6562	1.8423
0.7	2.322	1.85	1.6676	1.6404	1.8738

Table 5.3 The boundary correction factors F for local points of two symmetric corner cracks at a fastener hole obtained by Newman empirical equations under remote tension loading, $F = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c=1.0$.

a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
0.2	1.9935	1.7998	1.7524	1.7835	1.9359
0.4	1.8402	1.5792	1.5028	1.5198	1.6696
0.5	1.7763	1.5016	1.4245	1.4448	1.608
0.6	1.7199	1.4391	1.3652	1.3914	1.5737
0.7	1.6696	1.3878	1.3191	1.3528	1.5587

Table 5.4 The boundary correction factors F for local points of two symmetric corner cracks at a fastener hole obtained by Newman empirical equations under remote tension loading, $F = \frac{K|_{\sigma(x)}}{\sigma_0 \sqrt{\frac{\pi c}{Q}}}$, $a/c=1.5$.

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.499	0.2804	0.0741	0.0554
	0.4	0.4976	0.2831	0.0872	0.0828
	0.5	0.4962	0.291	0.0998	0.0965
WF	0.2	0.5253	0.261	0.0662	0.0304
	0.4	0.5242	0.2626	0.079	0.0598
	0.5	0.5235	0.2709	0.0911	0.0735
Accuracy (%)	0.2	5.26	6.92	10.72	45.18
	0.4	5.36	7.23	9.38	27.78
	0.5	5.5	6.92	8.75	23.84

Table 5.5 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with parabolic loading on the crack

face, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2$, $a/c=0.5$.

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.6556	0.4072	0.165	0.1409
	0.8	0.6357	0.3982	0.194	0.224
WF	0.2	0.6999	0.3842	0.1584	0.1178
	0.8	0.6774	0.3738	0.1856	0.2032
Accuracy (%)	0.2	6.76	5.65	4.01	16.42
	0.8	6.55	6.14	4.34	9.28

Table 5.6 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with parabolic loading on the crack

face, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2$, $a/c=0.9$.

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.6671	0.4334	0.204	0.1839
	0.4	0.6467	0.4083	0.1825	0.1849
	0.6	0.6283	0.4051	0.2011	0.2195
	0.8	0.64	0.4121	0.209	0.238
WF	0.2	0.7146	0.4102	0.1998	0.1669
	0.4	0.6924	0.3838	0.1729	0.161
	0.6	0.6727	0.3805	0.1918	0.2002
	0.8	0.6851	0.3877	0.1996	0.2173
Accuracy (%)	0.2	7.13	5.35	2.04	9.23
	0.4	7.06	6.01	5.27	12.9
	0.6	7.07	6.07	4.64	8.8
	0.8	7.05	5.93	4.48	8.69

Table 5.7 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with parabolic loading on the crack face, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2$, $a/c=1.1$.

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.5968	0.4193	0.2247	0.1963
	0.4	0.5751	0.3921	0.1963	0.1938
	0.6	0.5573	0.3811	0.2025	0.2148
	0.8	0.5637	0.3864	0.2083	0.2281
WF	0.2	0.644	0.3979	0.2179	0.1821
	0.4	0.6217	0.3699	0.1886	0.1762
	0.6	0.6022	0.3585	0.1929	0.1989
	0.8	0.6097	0.3638	0.1988	0.2114
Accuracy (%)	0.2	7.9	5.1	3.02	7.21
	0.4	8.1	5.65	3.91	9.08
	0.6	8.06	5.92	4.73	7.41
	0.8	8.17	5.86	4.54	7.33

Table 5.8 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with parabolic loading on the crack

face, $\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^2$, $a/c=1.5$.

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.4474	0.2004	0.026	0.0355
	0.4	0.4425	0.2037	0.0368	0.0544
	0.5	0.4402	0.2102	0.0473	0.0649
WF	0.2	0.4712	0.1704	0.0249	0.0043
	0.4	0.4675	0.1719	0.0351	0.0274
	0.5	0.4652	0.1789	0.0447	0.0382
Accuracy (%)	0.2	5.31	14.95	4.09	87.97
	0.4	5.66	15.59	4.55	49.62
	0.5	5.69	14.9	5.43	41.18

Table 5.9 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with cubic loading on the crack face,

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3, a/c=0.5.$$

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.5811	0.3046	0.0904	0.0975
	0.8	0.5564	0.2985	0.1141	0.1567
WF	0.2	0.6297	0.2639	0.0917	0.0684
	0.8	0.6011	0.2559	0.1133	0.1356
Accuracy (%)	0.2	8.36	13.37	1.49	29.84
	0.8	8.04	14.27	0.73	13.47

Table 5.10 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with cubic loading on the crack face,

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3, a/c=0.9.$$

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.5935	0.3284	0.1178	0.124
	0.4	0.5695	0.3097	0.106	0.129
	0.6	0.5514	0.3072	0.1207	0.1531
	0.8	0.5607	0.3133	0.1269	0.1673
WF	0.2	0.6441	0.286	0.1244	0.1069
	0.4	0.619	0.265	0.1031	0.1023
	0.6	0.5982	0.2623	0.118	0.1331
	0.8	0.6091	0.2685	0.1243	0.1466
Accuracy (%)	0.2	8.53	12.9	5.6	13.76
	0.4	8.69	14.42	2.71	20.68
	0.6	8.48	14.6	2.2	13.07
	0.8	8.64	14.29	2.04	12.4

Table 5.11 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with cubic loading on the crack face,

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3, a/c=1.1.$$

Results	a/t	$\Phi=\pi/60(\sim A)$	$\Phi=\pi/6 (D)$	$\Phi=\pi/3 (C)$	$\Phi=87\pi/180(\sim B)$
Converted	0.2	0.5321	0.3296	0.1399	0.1399
	0.4	0.5098	0.3058	0.1211	0.1357
	0.6	0.4928	0.2973	0.1279	0.151
	0.8	0.4972	0.3021	0.1324	0.1609
WF	0.2	0.584	0.2847	0.1429	0.1218
	0.4	0.5595	0.2617	0.1196	0.1171
	0.6	0.5397	0.2527	0.123	0.1349
	0.8	0.5463	0.2572	0.1277	0.1448
Accuracy (%)	0.2	9.75	13.62	2.11	12.96
	0.4	9.74	14.43	1.23	13.7
	0.6	9.52	15.01	3.83	10.63
	0.8	9.88	14.87	3.53	10.02

Table 5.12 The comparisons of weight function solutions based on boundary correction factor F and the converted Grandt's data with cubic loading on the crack face,

$$\sigma(x) = \sigma_0 \left(1 - \frac{x}{c}\right)^3, a/c=1.5.$$

Results	a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
Newman	0.2	2.5435	2.0013	1.6683	1.4386	1.2759
	0.4	2.3032	1.7925	1.5158	1.3307	1.2206
	0.5	2.2682	1.7782	1.5175	1.345	1.2603
	0.6	2.2616	1.7893	1.5399	1.3782	1.3227
	0.7	2.2681	1.8112	1.571	1.4202	1.3995
WF	0.2	2.8122	2.0567	1.7396	1.5536	1.3549
	0.4	2.6359	1.7223	1.5248	1.4454	1.3814
	0.5	2.5909	1.6844	1.5324	1.4775	1.4406
	0.6	2.554	1.7047	1.5866	1.5291	1.4651
	0.7	2.5173	1.7853	1.6899	1.5901	1.41
Accuracy (%)	0.2	10.56	2.77	4.27	7.99	6.19
	0.4	14.44	3.91	0.59	8.62	13.18
	0.5	14.23	5.27	0.98	9.85	14.31
	0.6	12.93	4.73	3.03	10.95	10.77
	0.7	10.99	1.43	7.57	11.96	0.75

Table 5.13 The comparisons of weight function solutions based on boundary correction factor F and the Newman fitted data with uniform remote tensile loading, $a/c=0.5$.

Results	a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
Newman	0.2	2.7725	2.3111	2.0451	1.9179	1.9546
	0.4	2.5104	2.0034	1.7583	1.6599	1.739
	0.5	2.4261	1.9233	1.695	1.6129	1.7253
	0.6	2.3616	1.8694	1.6572	1.5915	1.7435
	0.7	2.3093	1.8312	1.6339	1.5847	1.7829
WF	0.2	2.9516	2.3899	2.1153	1.958	1.7908
	0.4	2.7848	1.9784	1.7282	1.6374	1.5716
	0.5	2.7365	1.8666	1.6501	1.5945	1.5797
	0.6	2.6988	1.7877	1.606	1.5843	1.6242
	0.7	2.6662	1.7296	1.5815	1.5933	1.6936
Accuracy (%)	0.2	6.46	3.41	3.43	2.09	8.38
	0.4	10.93	1.25	1.72	1.35	9.63
	0.5	12.79	2.94	2.65	1.14	8.44
	0.6	14.28	4.37	3.09	0.45	6.84
	0.7	15.46	5.55	3.21	0.54	5.01

Table 5.14 The comparisons of weight function solutions based on boundary correction factor F and the Newman fitted data with uniform remote tensile loading, $a/c=0.9$.

Results	a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
Newman	0.2	2.8007	2.3674	2.1306	2.0337	2.1087
	0.4	2.5417	2.0505	1.8235	1.7478	1.8601
	0.5	2.4523	1.9609	1.7486	1.6885	1.8341
	0.6	2.3811	1.8972	1.7001	1.6562	1.8423
	0.7	2.322	1.85	1.6676	1.6404	1.8738
WF	0.2	2.8898	2.3851	2.1542	2.0309	1.893
	0.4	2.718	1.9523	1.7042	1.623	1.5909
	0.5	2.6616	1.8326	1.6137	1.5679	1.5921
	0.6	2.619	1.7539	1.5748	1.5691	1.6505
	0.7	2.5923	1.7019	1.5613	1.5928	1.7276
Accuracy (%)	0.2	3.18	0.75	1.11	0.14	10.23
	0.4	6.94	4.79	6.55	7.14	14.47
	0.5	8.53	6.54	7.71	7.14	13.19
	0.6	9.99	7.55	7.37	5.26	10.41
	0.7	11.64	8.00	6.38	2.90	7.80

Table 5.15 The comparisons of weight function solutions based on boundary correction factor F and the Newman fitted data with uniform remote tensile loading, $a/c=1.0$.

Results	a/t	$\Phi=\pi/60$	$\Phi=7\pi/60$	$\Phi=13\pi/60$	$\Phi=19\pi/60$	$\Phi=29\pi/60$
Newman	0.2	1.9935	1.7998	1.7524	1.7835	1.9359
	0.4	1.8402	1.5792	1.5028	1.5198	1.6696
	0.5	1.7763	1.5016	1.4245	1.4448	1.608
	0.6	1.7199	1.4391	1.3652	1.3914	1.5737
	0.7	1.6696	1.3878	1.3191	1.3528	1.5587
WF	0.2	2.1471	1.9011	1.8364	1.8228	1.7511
	0.4	2.0264	1.5716	1.4239	1.3951	1.403
	0.5	1.9776	1.4631	1.3172	1.3063	1.3603
	0.6	1.94	1.3859	1.2581	1.2732	1.3683
	0.7	1.922	1.3368	1.2292	1.2671	1.3898
Accuracy (%)	0.2	7.71	5.63	4.79	2.20	9.55
	0.4	10.12	0.48	5.25	8.21	15.97
	0.5	11.33	2.57	7.53	9.58	15.40
	0.6	12.79	3.70	7.85	8.50	13.05
	0.7	15.12	3.68	6.81	6.34	10.84

Table 5.16 The comparisons of weight function solutions based on boundary correction factor F and the Newman fitted data with uniform remote tensile loading, $a/c=1.5$.

Figure 5.1 Comparisons of stress intensity factors from empirical equation (constant and linear stress) and weight function calculation (parabolic and cubic) with the converted results, $a/c=0.5$, $a/t=0.4$.

Figure 5.2 Comparisons of stress intensity factors from empirical equation (constant and linear stress) and weight function calculation (parabolic and cubic) with the converted results, $a/c=0.9$, $a/t=0.2$.

Figure 5.3 Comparisons of stress intensity factors from empirical equation (constant and linear stress) and weight function calculation (parabolic and cubic) with the converted results, $a/c=1.1$, $a/t=0.6$.

Figure 5.4 Comparisons of stress intensity factors from empirical equation (constant and linear stress) and weight function calculation (parabolic and cubic) with the converted results, $a/c=1.5$, $a/t=0.8$.

Figure 5.5 Comparisons of stress intensity factor between weight function solutions and the Newman fitted results (table 5.1) for local points of two symmetric corner cracks at a fastener hole under constant remote tension, $a/c=0.5$, $a/t=0.4$.

Figure 5.6 Comparisons of stress intensity factor between weight function solutions and the Newman fitted results (table 5.2) for local points of two symmetric corner cracks at a fastener hole under constant remote tension, $a/c=0.9$, $a/t=0.2$.

Figure 5.7 Comparisons of stress intensity factor between weight function solutions and the Newman fitted results (table 5.3) for local points of two symmetric corner cracks at a fastener hole under constant remote tension, $a/c=1$, $a/t=0.6$.

Figure 5.8 Comparisons of stress intensity factor between weight function solutions and the Newman fitted results (table 5.4) for local points of two symmetric corner cracks at a fastener hole under constant remote tension, $a/c=1.5$, $a/t=0.6$.

Chapter 6

Applications to Fatigue Crack Propagation Analysis

6.1 Introduction

In previous chapters, the weight function solutions of stress intensity factor are derived for through cracks and corner cracks at a fastener hole in a finite thickness plate. With these given weight function solutions, one can calculate the fatigue life of components containing a initial flaw. In this chapter, as an application of the derived weight function solutions, a fatigue crack propagation analysis is performed to predict the crack growth behavior under remote tension with and without cold working process for a through and corner cracked fastener hole. The stress intensity factors were obtained using weight functions derived in previous chapters.

Cold working is an effective method to extend the fatigue lives of mechanically fastened joints. The process involves introducing residual compressive hoop stresses at the hole edge by pulling an oversize mandrel through the hole. These residual stresses reduce the effective stress concentration when the remote load is applied, such that the fatigue life of the fastener hole is extended. For through crack at a fastener hole, some experimental studies ([40], [41]) were conducted to obtain the fatigue lives of the test specimens with and without cold working. The experiment results show the significant improvement of fatigue life due to the introduced cold working process on the test specimen comparing to the one without cold working.

To quantify the improvement in fatigue crack growth resistance due to cold working, the fatigue life estimations for a through crack and corner crack at a fastener hole in a finite thickness plate are performed using the stress intensity factors obtained by weight function method presented in previous chapters. The loading conditions considered are the remote tension only and the combination of remote tension with the

introduced cold working process. The comparisons of fatigue growth behaviors between these loading situations show that the cold working has a significant improvement on the fatigue life of fastener holes.

6.2 Fatigue Life Estimation of a Through Crack at a Fastener Hole

6.2.1 Experimental Data for a Cold-Worked and Cracked Fastener Hole

In the present chapter, the test specimens listed in the reference [40], [41] is analyzed. They are 2.0in wide and 0.25in thick as shown in figure 6.1. The specimen was machined from a single sheet of 7075-T651 aluminum whose elastic modulus E is 10^4 ksi, yield stress σ_y is 75ksi. Constant amplitude fatigue crack growth experiments conducted on the test specimen are described in reference [41] and stated briefly as bellows.

First the specimen was polished and lightly scribed with small reference lines to facilitate crack length measurements. Next, a 0.125 in diameter pilot hole was drilled on the specimen centerline and notched with a 0.05 in deep jeweler's saw cut. Fatigue cracks were initiated at these notches and grown to the desired length by a number of cycling at a nominal stress of 16ksi. Final drilling and reaming then removed the notch, leaving a through-the-thickness radial crack at the edge of the 0.261 in diameter hole as shown in figure 6.1. The initial crack length c is 0.074in.

The cold working process was introduced by pulling a 0.247in diameter mandrel through the fastener hole and causes a residual stress field around the hole. The specimen was then cycled to failure at the test load corresponding to maximum nominal stress σ_{max} =40ksi, with the R ratio (minimum /maximum load) fixed at 0.05, the minimum nominal stress σ_{min} =2ksi.

The residual stress using elastic plastic analysis induced in the cold-worked test specimen was obtained by Hsu and Forman [43] and was shown in figure 6.2. The experiment results were shown in figure 6.3.

6.2.2 Fatigue Life Calculations

Stress Intensity Factor Analysis

The first step to conduct fatigue crack propagation analysis is to find the range of stress intensity factor ΔK . The stress intensity factor K used in the analysis can be obtained by the weight function approach discussed in previous chapters.

Using the stress intensity factor K expression in terms of weight function form as discussed in chapter 3,

$$K = \int \sigma(x)m(x,c)dx \quad (3.22)$$

where $\sigma(x)$ is the stress distribution at the prospective crack plane crack face, and the weigh function is:

$$m(x,c) = \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{t1}\left(1 - \frac{x}{c}\right)^{\frac{1}{2}} + M_{t2}\left(1 - \frac{x}{c}\right) + M_{t3}\left(1 - \frac{x}{c}\right)^{\frac{3}{2}} \right] \quad (2.42)$$

where M_{t1} , M_{t2} and M_{t3} are determined in chapter 3 with the range of $0.2 \leq c/R \leq 1.5$.

Once knowing the crack face pressure $\sigma(x)$, K can be obtained from equation (3.22) directly. In this chapter, two loading situations which remote tension loading with and without cold working are considered to compare the fatigue life improvement for the same crack geometry.

Crack Face Pressure for Remote Tension without Cold Working

With uniform tensile loading applied on the far end side of plate, the expression of stress distribution on the crack face $\sigma(x)$, based on the present crack geometry shown in figure 3.3, can be written as [28]:

$$\sigma_T(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] \quad (3.29)$$

where σ_0 is the nominal stress on the plate.

For remote tension loading only, the stress intensity factor K can be obtained by substituting equations (3.29) and (2.42) into equation (3.22).

Crack Face Pressure for Remote Tension with a Cold Working Hole

For an infinite plate containing a circular hole that has been cold-worked by an oversized mandrel, the tangential residual stress based on elastic perfectly plastic material property can be written as [42]:

$$\frac{\sigma_R(x)}{\sigma} = \frac{0.5 - \ln(1 + \rho) + \ln(r) - [0.5 + \ln(1 + \rho)] \cdot r^{-2}}{-2 \ln(1 + \rho)} \quad , \quad \frac{x}{R} \leq \rho \quad (6.1a)$$

$$\frac{\sigma_R(x)}{\sigma} = \frac{0.5(1 + \rho)^2 - [0.5 + \ln(1 + \rho)]}{-2r^2 \ln(1 + \rho)} \quad , \quad \frac{x}{R} > \rho \quad (6.1b)$$

and

$$\sigma = \sigma(x)\big|_{x=0} = -\gamma\sigma_y \cdot 2\ln(1+\rho) \quad (6.1c)$$

where

$\sigma_R(x)$: residual stress on the unflawed crack surface caused by cold working,

σ_y : material yield strength,

ρ : dimensionless plastic zone size (plastic zone width over hole radius), it equals to 1.08 obtained from figure 6.2 specified in section 6.2.1.

γ : constant, equals to 1 (Tresca) or $2/\sqrt{3}$ (Von Mises).

Therefore, assuming that the resulting stress from remote stress field can be linearly added together with residual stress. The crack face pressure $\sigma(x)$ of a cold-worked hole under remote tension can be obtained by adding equations (3.29) and (6.1):

$$\sigma(x) = \sigma_T(x) + \sigma_R(x)$$

That is:

$$\sigma(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] + \left[\frac{0.5 - \ln(1+\rho) + \ln(r) - [0.5 + \ln(1+\rho)] \cdot r^{-2}}{-2\ln(1+\rho)} \right] \cdot \sigma$$

$$\frac{x}{R} \leq \rho \quad (6.2a)$$

$$\sigma(x) = \frac{\sigma_0}{2} \left[2 + \frac{1}{\left(1 + \frac{x}{R}\right)^2} + \frac{3}{\left(1 + \frac{x}{R}\right)^4} \right] + \sigma \cdot \left[\frac{0.5(1 + \rho)^2 - [0.5 + \ln(1 + \rho)]}{-2r^2 \ln(1 + \rho)} \right]$$

$$\frac{x}{R} > \rho \quad (6.2b)$$

For remote tension with cold-worked hole, the stress intensity factor K can be obtained by substituting equations (6.2) and (2.42) into equation (3.22).

Calculation of Fatigue Life

Using a power law in the fatigue crack propagation proposed by Paris [44], the fatigue crack growth rate for the present crack geometry shown in figure 6.1 , dc/dN can be expressed by the following equation:

$$\frac{dc}{dN} = C \Delta K^n \quad (6.3)$$

where dc/dN is the crack growth per cycle. C and n are material constants that are determined experimentally.

With the experimental results described in section 6.2 in reference [41], a least squares fit of 340 data points gave a crack growth exponent n equals to 3.444 while the constant C was 9.45×10^{-20} .

By the definition of the range of stress intensity factor, ΔK can be expressed as:

$$\Delta K = K_{\max} - K_{\min} \quad (6.4)$$

where $K_{\max}$ and $K_{\min}$ are the stress intensity factors corresponding to the maximum and minimum nominal stress $\sigma_{\max}$ and $\sigma_{\min}$. They can be found by integrating the equation (3.22) after substituting $m(x, c)$ and $\sigma(x)$ with and without cold working respectively.

Once knowing the ΔK , with the initial crack length value of 0.074in, the fatigue crack propagation of a through crack at a fastener hole can be found by using equation (6.3). The results of crack length vs. number of life cycles with the cold-worked and non cold-worked hole were presented in figure 6.3.

6.2.3 Comparisons

Figure 6.3 shows the results of typical fatigue crack growth curves. Comparisons are made by prediction and experimental data for cold-worked hole while showing prediction for no cold working for the specimens of 7075 T651.

The line labeled “cold work” was for the specimen under remote tension with the introduced cold working process; the line labeled “ no cold work” was for the specimen under remote tension loading only. The experiment data were obtained from fatigue crack growth test in precracked cold-worked hole specimen discussed in section 6.2.1.

As shown in figure 6.3, with the stress intensity factor determined by weight function method, the predicted fatigue crack growth curve for cold-worked hole generally agree quite well with the experimental results in the range of $0.2 \leq c/R \leq 1.5$. This indicates that the stress intensity factor calculations based on weight function approach can provide good estimates of the fatigue crack propagation behavior of cold-worked holes.

By the comparison of fatigue crack growth predictions between the cold-worked and non cold-worked hole, it is proved that the fatigue life can be improved significantly by introducing the cold working process for a through crack at a fastener hole.

6.3 Fatigue Life Estimation of Surface Points for a Corner Crack at a Fastener Hole

6.3.1 Fatigue Life Calculation

To show the improvement of fatigue life with the introduced cold working process, the fatigue life estimation of surface points for a corner crack at a fastener hole is performed in this section. By taking the same material property and geometry dimensions described in section 6.2, the plate with a cold-worked and precracked hole is loaded with a remote uniaxial stress on 40ksi (276MPa). The fatigue life estimations were performed for corner crack at a fastener hole and compared to the loading without cold working process. The initial corner crack is $c=0.09in$, $a=0.09in$.

Stress Intensity Factor Analysis

As shown in figure 4.1, using the stress intensity factor K expression in terms of weight function form as discussed in chapter 3,

$$K = \int \sigma(x)m(x,c)dx \quad (3.22)$$

where $\sigma(x)$ is the tensile loading applied on the uncracked surface.

For hole surface point A shown in figure 4.1, the weight function $m(x,c)$ can be written as follows using the results obtained in chapter 4:

$$m_A(x, c) = \frac{2}{\sqrt{\pi x}} \left[1 + M_{1A} \left(\frac{x}{c} \right)^{\frac{1}{2}} + M_{2A} \left(\frac{x}{c} \right) + M_{3A} \left(\frac{x}{c} \right)^{\frac{3}{2}} \right] \quad (6.5)$$

For front surface point B shown in figure 4.1, the weight function $m(x, c)$ can be written as follows using the results obtained in chapter 4:

$$m_B(x, c) = \frac{2}{\sqrt{2\pi(c-x)}} \left[1 + M_{1B} \left(1 - \frac{x}{c} \right)^{\frac{1}{2}} + M_{2B} \left(1 - \frac{x}{c} \right) + M_{3B} \left(1 - \frac{x}{c} \right)^{\frac{3}{2}} \right] \quad (6.6)$$

where the weight function parameters M_{1A} , M_{2A} and M_{3A} , parameters M_{1B} , M_{2B} and M_{3B} was derived in chapter 4.

The unflawed hoop stress distribution is again given by equations (3.29) and (6.2) as discussed in section 6.2 for remote tension without and with cold working.

Calculation of Fatigue Life

For hole surface point A , the crack growth rate becomes the increment of crack depth “ a ” over a number of life cycle, da/dN , for the front surface point B , the crack growth rate is the increment of crack length “ c ” over a number of life cycle, dc/dN .

Using a power law in the crack propagation proposed by Paris [44],

for point A :

$$\frac{da}{dN} = C \Delta K_A^n \quad (6.7)$$

for point B :

$$\frac{dc}{dN} = C \Delta K_B^n \quad (6.8)$$

where C and n are material constants that are determined experimentally same as in section 6.2.

Corresponding to the maximum and minimum nominal stress σ_{max} and σ_{min} , ΔK can be expressed as,

For hole surface point A :

$$\Delta K_A = K_{Amax} - K_{Amin} \quad (6.9)$$

For hole surface point B :

$$\Delta K_B = K_{Bmax} - K_{Bmin} \quad (6.10)$$

where K_{Amax} , K_{Amin} are the stress intensity factor for hole surface point A corresponding the maximum and minimum nominal stress σ_{max} and σ_{min} . They can be obtained by integrating the equation (3.22) after substituting the weight function at A and the crack face stress distribution $\sigma(x)$ caused by σ_{max} and σ_{min} into equation (3.22). K_{Bmax} , K_{Bmin} are the stress intensity factor for front surface point B corresponding the maximum and minimum nominal stress σ_{max} and σ_{min} , they can be obtained by integrating the equation (3.22) after substituting the weight function at B and the crack face stress distribution $\sigma(x)$ caused by σ_{max} and σ_{min} into equation (3.22).

Knowing the ΔK , by choosing the initial crack length c and crack depth a at the value of 0.09in, the fatigue life of the surface points for a corner crack at a fastener hole can be found by using equations (6.7) and (6.8).

6.3.2 Comparisons

For hole surface point *A*, figure 6.4 shows the results of typical fatigue crack growth predictions under the remote tension with and without cold working process for the specimen of 7075 T651.

For front surface point *B*, figure 6.5 shows the results of typical fatigue crack growth predictions under the remote tension with and without cold working process for the specimen of 7075 T651.

In figures 6.4 and 6.5, the line labeled “ cold work” was for the fatigue life of the specimen under remote tension with the introduced cold working process; The line labeled “ no cold work” was for the fatigue life of the specimen under remote tension loading only.

By the comparison of fatigue crack growth predictions between the cold-worked and non cold-worked hole, it is proved that the fatigue life can be improved significantly by introducing the cold working process.

6.4 Conclusions

As applications of weight function solutions derived in previous chapters, a fatigue crack propagation analysis is performed to predict the crack growth behavior under remote tension with and without cold working process for a through and corner cracked fastener hole.

The weight function solutions for through and corner crack developed in previous chapters was employed to calculate the stress intensity factor for predicting the crack

growth life of the chosen specimen. The stress intensity factor calculation procedure discussed in this chapter is based on the assumption that the effect of remote stress field may be linearly added to the residual stresses.

Both for a through crack and a corner crack at a fastener hole, the comparison of fatigue crack growth curves shows that the introduced cold working process can significantly extend the fatigue lives of mechanically fastener joints. This also indicates that the current weight function approach can be easily applied to the fatigue crack propagation analysis under complex loading conditions.

Nominal Thickness=0.25in

All measurements In in

Figure 6.1 Through crack of 0.25in thick 7075-T6 aluminum specimen used for cold-worked hole experiments.

Figure 6.2 Residual stress distribution due to cold-worked hole by radial expansion u as predicted by Hsu-Forman solution.

Figure 6.3 Fatigue crack growth curve comparing prediction and experimental data with and without cold working.

Figure 6.4 The comparison of fatigue crack growth curve under remote tension with and without cold-working process for the hole surface point A of a corner crack at a fastener hole.

Figure 6.5 The comparison of fatigue crack growth curve under remote tension with and without cold-working process for the front surface point B of a corner crack at a fastener hole.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

The weight function solutions of stress intensity factors for the through-the-thickness cracks and a corner crack at a fastener hole in a finite thickness plate under arbitrary loading conditions have been derived. To facilitate the derivation of weight functions, the existing solutions of stress intensity factor from Grandt ([2][4]) were converted to the standard form of results under standard loading systems. The converted stress intensity factor results with constant and linear stress distributions are used as reference stress intensity factor solutions in the derivations of present weight function solutions. The derived weight functions were verified by other stress distributions.

The stress intensity factors obtained by the present weight function approach for single and double symmetric through cracks are valid for the range of $0.2 \leq c/R \leq 1.5$. The weight function solutions were validated by two dimensional finite element analyses of the same crack geometry under remote tension loading conditions as well as the converted results as mentioned above. The accuracy is within 10 % of these existing data.

These derived weight functions are suitable for fatigue and fracture analysis for through cracks in a fastener hole.

For a corner crack at a fastener hole in a finite thickness plate, weight function solutions of stress intensity factor are obtained for any points along the crack front by combining the weight function solutions at surface points with $\phi = 0$, $\phi = \pi/2$ and local points with $0 < \phi < \pi/2$. These weight functions are valid over a range of aspect ratio, $0.5 \leq a/c \leq 1.5$, and relative crack depth, $0.2 \leq a/t \leq 0.8$ with $R/t = 0.5$.

The weight functions at hole surface point ($\phi = 0$) and front surface point ($\phi = \pi/2$) are verified by the converted results with parabolic and cubic stress distribution on the crack face and Newman data under remote tension loading. The accuracy of weight functions for hole surface point *A* comparing to the converted results were less than 7 % and less than 10 % comparing to the Newman data. The accuracy of weight functions for front surface point *B* comparing to the converted results were less than 8% and less than 9% comparing to the Newman data.

For local points along the crack front of a corner crack, $0 < \phi < \pi/2$, the proposed weight functions were validated by the converted results with parabolic and cubic crack face pressure and Newman's data with remote tension loading. The comparisons show that for a corner crack at a fastener hole, the maximum difference of the stress intensity factors was generally less than 15%. For two symmetric corner cracks under remote tension, the accuracy of weight functions for local point comparing to the Newman data were less than 16%.

When used together, a corner crack with surface points ($\phi = 0$ and $\phi = \pi/2$) and local points ($0 < \phi < \pi/2$) at a fastener hole in a finite thickness plate subjected complex stress fields with aspect ratios in the range, $0.5 \leq a/c \leq 1.5$, and relative depths, $0.2 \leq a/t \leq 0.8$ with $R/t = 0.5$, the weight functions presented in this thesis are suitable for fracture or fatigue analysis for corner cracks.

As the applications of the derived weight function solutions, a fatigue crack propagation analysis is performed in this thesis to predict the fatigue crack growth behavior under remote tension with and without cold working for a cracked fastener hole. Agreement were achieved with experimental data, it also demonstrated that the weight function can be easily applied to the fatigue crack propagation analysis under complex loading conditions.

7.2 Recommendations

For both through and corner cracks, it is possible to improve the accuracy of present weight function solutions by adding one more item in the general weight function expressions. But the complexity of the present solutions will be increased because of the additional reference stress intensity factor required.

Based on the converted results from Grandt ([2], [4]), the weight function solutions of stress intensity factor for a corner crack at a fastener hole in a finite thickness plate were considered for the range of $0.5 \leq a/c \leq 1.5$, $0.2 \leq a/t \leq 0.8$. In order to expand the applications of present weight function approach, as the reference stress intensity factors, more finite element calculations are required to provide the solutions in the region of $0 \leq a/c \leq 2$, and $0 \leq a/t < 1$.

The present research focuses on the stress distributions along x axis (crack depth c direction) only. In practice, more complex stress distributions which change both in x axis and y axis (crack length a) are frequently encountered in applied fracture and fatigue analysis for the estimation of the stress intensity factors. Further approximate weight function analysis is required to calculate the stress intensity factors for these complex stress distributions.

The present results for a corner crack were considered valid only with $R/t = 0.5$, further analysis are recommended to establish the weight function solutions for the more wide region of R/t , such as $R/t=1, 2, 3, \dots, 40$.

References

- [1] **M. D. Coffin** and **D. F. Tiffany**, “New Air Force Requirements for Structural Safety, Durability and Life Measurement”. *J. Aircraft* 13 (2). 93-98 (1976).
- [2] **A. F. Grandt, Jr** and **T. E. Kullgren**, “Tabulated Stress Intensity Factor Solutions for Flawed Fastener Holes”. *Int J. Fracture* 12 (4), 435-451, 1983.
- [3] **Smith, F. W.** and **Kullgren, T. E.**, “Theoretical and Experimental Analysis of Surface Cracks Emanating from Fastener Holes”, *Technical Report AFFDL-TR-76-104, Arir Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, Ohio*, 1977.
- [4] **A. F. Grandt, Jr** and **T. E. Kullgren**, “Stress intensity factors for corner cracked holes under general loading conditions”, *J. Eng. Material and Tech.* Vol.103/171-176, April, 1981.
- [5] **Bueckner, H. F.**, 1970, “A Novel Principle for the Computation of Stress Intensity Factors”, *Z. A gew. Math. Mech.*, Vol. 50, pp. 129-146.
- [6] **Rice, J.**, “Some Remarks on Elastic Crack Tip Field”, 1972, *International Journal of Solids and Structures*, Vol. 8, pp. 751-758.
- [7] **Shen, G.** and **Glinka, G.** 1991b, “Determination of Weight Functions From Reference Stress Intensity Factors”, *Theoretical and Applied Fracture Mechanics*, Vol. 15, pp. 237-245.
- [8] **X. Wang** and **S. B. Lambert**, 1995, “Local Weight Functions for Semi-elliptical Surface Cracks in Finite Thickness Plate”, *Theoretical and Applied Fracture Mechanics*. Vol. 23, pp. 199-208.

- [9] **Shen, G. and Glinka, G.** 1991, "Weight Functions for a Surface Semi-elliptical Crack in a Finite Thickness Plate", *Theoretical and Applied Fracture Mechanics*, Vol. 15, pp 247-255.

- [10] **J. C. Newman, Jr. and I. S. Raju**, "Stress intensity factor equations for cracks in three dimensional finite bodies", *Fracture Mechanics: Fourteenth Symposium—Volume I: Theory and Analysis*, ASTM STP 791, J. C. Lewis and G. Sines, Eds., American Society for Testing and Materials, 1983, pp. I238-I-265.

- [11] **Sih, G. C. and Lee, Y. D.**, 1989, "Review of Triaxial Crack Border Stress and Energy Behavior", *Theoretical and Applied Fracture Mechanics*, Vol. 12, pp. 1-17.

- [12] **Griffith, A. A.**, 1920, "The Phenomena of Rapture and Flow in Solids", *Philosophical Transactions, Series A*, Vol. 221, pp. 163-198.

- [13] **Rice, J.**, "A Path Independent Integral and the Approximate Analysis of Strain Concentration by Niches and Cracks", *Journal of Applied Mechanics*, Vol. 35, pp.379-386.

- [14] **Aliabade, M. H. and Rooke, D.P.** 1991, "Numerical Fracture Mechanics", *Kluwer Academic Publishers*, Boston.

- [15] **Sih, G. C.** 1973, "Handbook of Stress Intensity Factors", *Lehigh University*, Bethlehem, Pennsylvania.

- [16] **Rooke, D. P. and Cartwright, D. J.**, 1976, "Compendium of Stress Intensity Factors", *HMSO*, London.

- [17] **Tada, H., Paris, P. and Irwin, G.**, "The Stress Analysis of Cracks Handbook", *Paris Productions Incorporated and Del Research Corporation*, St. Louis, 1985.

- [18] **Zienkiewicz, O. C. and Taylor, R. L.**, “The Finite Element Method”, 4th Edition, *McGraw-Hill*, London, 1991.
- [19] **Hueber and Thornton**, “The Finite Element Method for Engineers, 2ed Edition”, *John Wiley & Sons*, New York, 1982.
- [20] **Barsoum, R. S.**, “On the Use of Isoperimetric Finite Elements in Linear Fracture Mechanics”, *International Journal for Numerical Methods in Engineering*, Vol. 10, pp.25-37, 1976.
- [21] **Henshell, R. D., and Shaw, K. G.**, 1975, “Crack Tip Finite Elements are Unnecessary”, *International Journal for Numerical Methods in Engineering*, Vol. 9, pp. 495-507.
- [22] **Shih, C. F, Moran, B. and Nakamura, T.**, 1986, “Energy Release Rate along a Three-Dimensional Crack Front in a Thermally Stressed Body”, *International Journal of Fracture*, Vol. 30, pp. 79-102.
- [23] **Chan. S. K., Tuba, I. S. and Wilson W. K.**, 1970, “On the Finite Element Method in Linear Elastic Fracture Mechanics”, *Engineering Fracture Mechanics*, Vol. 2, pp 1-17.
- [24] **Li, F. Z., Shih, C. F. and Needleman. A.**, 1985, “On the J-integral calculation using finite element method”, *Engineering Fracture Mechanics*, Vol. 21, pp. 405-421.
- [25] **Parks, D. M.** 1974, “A Stiffness Derivative Finite Element Technique for Determination of Crack Tip Stress Intensity Factors”, *International Journal of Fracture*, Vol. 10, pp. 487-502.
- [26] **Hellen, T. K.**, 1975, “On the Method of Virtual crack Extensions”, *International Journal for Numerical Methods in Engineering*, Vol.9, pp. 187-297.

- [27] **Banks-Sills, L.**, 1991, "Application of the Finite Element Method to Linear Elastic Fracture Mechanics", *Applied Mechanics Review*, Vol. 44, pp. 447-461.
- [28] **Petroski, H. J. and Achenbach**, 1978, "Computation of the Weight Function from stress intensity factor", *Engineering Fracture Mechanics*, Vol. 10, pp 257-266.
- [29] **Wu, X. R. and Carlsson, A. J.**, 1991, "Weight Functions and Stress Intensity Factor Solutions", *Pergamon Press*.
- [30] **Niu, X and Glinka, G.**, 1987, "The Weld Profile Effect on Stress Intensity Factors in Weldments", *International Journal of Fracture*, Vol. 35, pp. 3-20.
- [31] **Niu, X and Glinka, G.**, 1990, "Weight Function for Edge and Surface Semi-Elliptical Cracks in Flat Plates and Plates with Corners", *Engineering Fracture Mechanics*, Vol. 36, pp. 459-475.
- [32] **Fett, T., Mattheck C. and Munz, Z.**, 1987, "On the Calculation of crack opening displacement from the stress intensity factor", *Engineering Fracture Mechanics*, Vol. 27, pp. 697-715.
- [33] **Fett, T.**, 1988, "Limitations of Petroski-Achenbacg procedure demonstrated for a simple load case", *Engineering Fracture Mechanics*, Vol. 29. pp. 713-716.
- [34] **Wu, X. R.** 1991, "Study of influence of the reference load case on the crack face weight functions", *International Journal of Fracture*, Vol. 48, pp. 3-15.
- [35] **Glinka, G. and Shen, G.**, 1991, "Universal Features of Weight Functions for cracks in Mode I", *Engineering Fracture Mechanics*, Vol. 40, pp. 1135-1146.
- [36] **Shah, R. C.** "Mechanics of Crack Growth", *ASTM STP 590, American Society for Testing and Materials*, 1976, pp. 429-459.

- [37] **Green, A. E. and Sneddon, I. N.**, “Proceeding of the Carbridge Philosophical Society”, Vol. 47, 1950, pp. 159-164.

- [38] **Newman, J. C. Jr. and Raju, I. S.**, “Analysis of Surface Cracks in Finite Plates Under Tension or Bending Loads, NASA TP-1578”, *National Aeronautics and Space Administration, Washington, D. C.*, Aug. 1977.

- [39] **The MathWork, Inc.**, MATLAB-The Language of Technical Computing, Version 5.3. 1999 .

- [40] **Adler, W. F., and Dupree, K. M.**, “Stress Analysis of Cold worked Fastener Holes”, Technical Report AFML-TR-74-44. *Air Force Materials Laboratory, Wright_Patterson Air Force Base*, July 1974.

- [41] **Grandt, A. F., Jr., and Hinnerichs, T. D.**, “Stress Intensity Factor Measurements for Flawed Fastener Holes, Proceedings of the Army Symposium on Solid Mechanics”, 11-12, Sept. 1974, *Technical Report AMMRC MS 74-8*, pp. 161-176.

- [42] **Rich, D. L. and Impellizzeri, L. F.**, “Fatigue Analysis of Cold-Worked and Interference Fit Fastener Holes”, *ASTM STP 637*, 1977, 153.

- [43] **Hsu, Y. C. and Forman, R. G.**, “Elastic-Plastic Analysis of an Infinite sheet Having a Circular Hle Under Pressure”, *ASME Journal of Applied Mechanics*, Vol. 42, No. 2, 1975, pp.347-352.

- [44] **P. C. Paris and F. Erdogan**, “ A Critical Analysis of Crack Propagation Laws”, *Trans. ASME, J. Basic Eng.*, Vol. D85, 1963, pp528-537.