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Assessing the Reliability of Joist Hangers in Connections of Wood-Floor Systems

By

Francis O. Berkoh

B.Sc. (Eng.)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

MASTER OF ENGINEERING

**DEPARTMENT OF CIVIL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, CANADA
OCTOBER, 1990.**

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Wood-Floor Systems"**

submitted by

Francis O. Berkoh

in partial fulfilment of the requirements for the degree of

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Abstract

The use of joist hangers in connections of wood-floor systems is rapidly increasing in North America, and elsewhere. However, not much research done to formulate design models for the prediction of their performance consistent with an acceptable level of safety. To date designers of wood-frame structures have been relying only on widely varying specifications provided by the different manufacturers of joist hangers. Of much concern recently, therefore, has been the need to formulate design criteria to provide a common basis for assessing the reliability of such components. Often, design data on joist hangers is limited in uniformity by the diversity of their metal plate geometries and their proprietary nature.

In this study, the performance (strength and serviceability) of joist hanger joints using a probabilistic approach has been examined. In particular, the much needed confidence in their performance under actual field conditions has been measured by the safety or reliability methods. The components reliabilities of the joist hanger joint have been evaluated to determine the material behaviour characteristics which have much more significant influence on the joint performance. This has provided the basis for the formulation of Limit State Design (LSD) equations for light-gauge metal joist hangers. The data obtained from experimental studies conducted for this study have been used to calibrate the design equations so formulated, against a recommended target value of the safety index, β of 2.5.

A simple sampling plan, yet accurate, which incorporates decision theory has also been proposed for quality acceptance of light-gauge metal hangers using the sampling by variable method. Generally, in any given lot of joist hangers, the quality variability of the lot with respect to the mean strength of the hangers is unknown. The approach, however, adopted in formulating the sampling plan has been to develop the sampling parameters on the basis of a fixed variability level for any one

lot, thus facilitating the use of statistics on an assumed Gaussian distribution for the strengths of the joist hangers.

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Notation

a	Damage Model Parameter.
A	Curve-Fitting Constant.
AQL	Acceptance Quality Level.
b	Damage Model Parameter.
b_h	Bearing Length of Joist Hanger.
B	Curve-fitting Constant.
C_N	Modification Factor.
d	Diameter of Nail. Ratio of Actual Dead Load to Nominal Dead Load.
D	Acting Dead Load.
D_n	Nominal Dead Load.
E_i	Probability Event .
E_s	Modulus of Elasticity of Joist Hanger Plate Material.
f_e	Wood Embedding Strength.
f_y	Nail Yield Strength.
F_{cp}	Specified Strength in Compression Perpendicular to Grain.
F_u	Nail Yield Load.
g	Interlayer Gap between Jointed Members.
H, H_o	Section Depth of Lumber.
J_c	Nail Configuration Factor.

J_y	Side Plate Factor.
k	Size Effect Factor.
K^*	Generalized Modification Factor.
K_{01}	Joint Member Elastic Bearing Constant.
K_D	Load Duration Factor.
K_B	Bearing Factor.
K_{SF}	Moisture Content Factor.
K_T	Wood Treatment Factor.
l	Joint Member Thickness.
L	Span.
$LVDTs$	Linear Variable Differential Transformers.
L_s	Acceptance Limit.
m	Number of Wood Grades.
M_y	Nail Yield Moment.
n	Sample Size. Number of Wood Species.
n_1, n_2	Sample Size.
n_f	Number of Nails in a Joint.
$N_{(0.05)}$	5-th Percentile Joint Strength obtained from Test Data.
N_o	Joint Characteristic Strength considered at 3 mm Deformation.
N_u	Lateral Strength Resistance per Nail.
$P(A)$	Probability of Acceptance.
P_{nail}	Joint Load Carried through Supplemental Side Nailing.

p_F, p_F^a	Probability of failure.
p_o, p_1	Parameters for the Foundation Load-Deflection Curve .
q	Load Distribution due to Nail Pressure. Ratio of Actual Live Load to Nominal Live Load.
Q	Demand or Load Effect. Acting Live Load.
Q_n	Nominal Live Load.
r	Number of Defective Units in a Lot.
R	Capacity or Strength.
RQL	Rejection Quality Level.
R_N	Standard Normal Variate.
S_B	Modulus of Rupture of Dowel.
S_H	Wood Embedding Strength.
S_p	Joist Spacing.
t_p	Thickness of Plywood Member. Thickness of Joist Hanger Plate.
t_s	Thickness of Solid Lumber Member.
T_o	Characteristic Strength of Joist Hanger Steel.
u	Total Number of Joist Sizes.
v	Total Number of Loading Cases.
V_N	Standard Normal Variate.
V_p	Quality Variability level for a Lot of Joist Hangers.
V_z	Sample Coefficient of Variation.

w_h	Width of Joist Hanger Seat.
Z	Plastic Modulus.
$\mathbf{X}$	Vector of Basic State variable.
X	System variable.
$\bar{X}$	Mean of Sample size, n .
α	Measure of Damage. Probability of Type I Error.
α_D	Dead Load Factor.
α_Q	Live Load Factor.
β	Reliability Index.
β_d	Reliability Index based on Sample Mean Strength.
β_s	Reliability Index based on the Acceptance Limit Strength.
β_*	Probability of Type II Error.
δ	Joint Deformation. Coefficient of Variation.
δ_s	Slip of Nail due to lateral Loading.
δ_h	Deformation due to Plate Bending and Elongation.
δ_r	Deformation due to Joint Rotation.
δ_d	Total Joint Deformation.
Δ	Joint Slip.
γ_o	Stress Threshold.
γ_s	Strength of Specimen in a "Short-Term" Test.
Γ	Dimensionless Variable.

μ_R, μ_Q	Mean Value of Resistance and Load Effect, respectively.
μ_{maz}	Mean Ultimate Strength per Joist Hanger in a Lot. Acceptance Quality Level.
μ_{min}	Lot Joist Hanger Strength corresponding to Rejection Quality Level.
ω	Penetration of Nail under Wood Foundation.
ϕ	Performance Factor.
ρ	Correlation Coefficient.
σ	Standard Deviation of a Lot.
σ_1, σ_2	Bending Strength of Beams 1 and 2, respectively.
σ_y	Yield Strength of Joist Hanger Steel.
$\sigma_{y(0.05)}$	5-th Percentile Tensile Yield Strength of Joist Hanger Plate.
σ_{ult}	Ultimate Strength of Joist Hanger.
$\sigma_{critical}^*$	Critical Direct Stress in Joist hanger Flange.
$\tau_{critical}^*$	Critical Shear Stress in Joist hanger Plate.

1.1 General

In recent years, technical developments in wood engineering have greatly promoted the use of wood and its derivatives in the housing industry. Modern engineering practices, especially in wood frame construction, combine engineering and quality control to produce safe and reliable design. Though many positive strides have been scored in consequence, considering the variability of wood as a construction material and the dependence of its strength with time, a focus on the overall performance of wood-related structures evaluated in terms of the reliabilities of its structural components is at this stage of immense importance. Of much concern and importance in this regard is the reliability of mechanical fasteners as used in wood to wood connections.

A study of any probabilistic investigation related to the above has always met with the complexities in behaviour between the fastener, often of conventional steel material, and the fastening wood. More subtle still, structural engineers familiar with the design of steel structures have for long extrapolated the design methodologies for steel to the design of wood connections in the more recently appraised wood frame construction. In its wake has resulted the use of plate-type fasteners which combine dowel type actions with metal plates. Of fundamental importance therefore is a confirmation of the appropriateness, through reliability studies, of such metal frame devices as joist (beam) hangers and framing anchors often employed in light frame wood connections to transfer loads from one member to another.

Light metal frame connections used in conjunction with nails have been preferred to the traditional carpentry practices such as toe-nailing, notching and end-

nailing. They are known to provide stronger connections, and are capable of resisting forces in several directions at the same time providing stability in at least two planes. Besides, they provide accuracy in joint fitting and reduce splitting hazards [16].

Published research work on metal framing connections is limited in uniformity by the diversity of metal plate geometries and their proprietary nature. Therefore, any attempt towards the development of more rational design procedures for these products must account for the uncertainties inherent in their production. These methodologies, which should be reliability-based, must also be tailored to assess the strength and stiffness of the connections formed from them. A case in point, which is the subject of this study, has been the recognition, in Canada and elsewhere, of a more codified acceptance of joist hangers as commonly employed in light frame construction. In particular, the need for serviceability evaluation of joist hanger connections has been much more emphasized in view of the uneven and springy floors which have occasionally been reported of floor systems employing them.

1.2 Joist Hanger in Wood Structural Connections

There are a variety of joist (or beam) hangers in use. By far, those employed in light frame constructions such as in residential housing, stand as the most common types. Heavier hangers, often of structural steel plates, have found great use in connections of wood joist to masonry, concrete, steel, and more commonly, in heavy timber constructions employing glulam beams. Whereas the heavier hangers are designed for specific design situations, light gauge metal hangers lack this lustre as they are produced from a common design format adopted by a manufacturer. This design format, however, often differs from one manufacturer to the other, and from one brand to the other. Manufacturers therefore, usually provide tentative design information for 'safe' working loads for which the user is obliged to comply.

Standard manufactured hangers for 2 in (38 mm) nominal thickness members are manufactured from galvanised sheet metal varying in thickness from 20 gauge

(0.95 mm) to 7 gauge (4.76 mm). Hangers for heavy timber connections are made from folded or welded structural steel plates and are usually finished in black bituminous or red oxide paint.

Figure 1.1 shows two light-gauge joist hangers in common use in wood-frame construction. Typically, in almost all hanger types, the load from the wood joist is transferred to the hanger through bearing. In most design situations, the hangers do not have the requisite bearing length to ensure the safe bearing capacity of the joist. In such cases supplementary side nailing through the joist hanger flanges is essential. The reaction load at the hanger is ultimately transferred to the supporting member (called the header) by shear in nails through the hanger flanges. One type of hanger design such as shown in Fig. 1.1(b), has the outstanding legs projected up and over to bear on the top surface of the header, in which case the vertical element of the hanger acts as a member in tension carrying the entire load from the joist [73]. An added advantage of this design type is made use of in connections where two joists bear on each side of a header—the hanger fully bears over the header to provide two bearing seats. In practice, the hanger type selected for a particular job is often dictated by the ease with which it can be installed.

1.3 Scope and Objectives

As pointed out earlier, though the use of joist hangers especially in connections of wood-floor systems is increasing, little however, has been done to formulate design guidelines necessary to ensure their compliance with an acceptable level of performance. To date, designers of wood-frame construction have had to rely on specifications provided by joist hanger manufacturers. Even though some measure of structural adequacy has been scored, in view of the proprietary nature of these products they still leave much to be desired. The purpose of this study, therefore, is to examine the performance (strength and serviceability) of joist hanger joints using a probabilistic approach—the much needed confidence in their performance under actual field conditions measured by safety or reliability methods.

Although lateral stability of joist hangers is known to influence their performance in service, this was, however, not covered under this thesis. Both the American Society for Testing Materials (ASTM) [1] and the International Conference of Building Officials (ICBO) [32] set forward similar test procedures for evaluating the vertical load-carrying capacity of light gauge metal hangers. The results of the testing done under this thesis using these procedures have been compared with those obtained from full-scale tests.

Throughout this study the focus has been on light gauge hanger as against heavy duty hangers. A particular hanger model, the *LUP210*, manufactured by the Simpson Strong-Tie Inc., California, USA, was used as a case study in formulating limit state design equations for joist hangers. This hanger type, of gauge 18 steel (actual thickness is 1.25 mm), is designed to transfer load to the header by shear in nails through the flanges, and has been recommended for 38×235 and 38×295 mm lumber joists.

Based on the results of the experimental studies, an attempt had been made to formulate limit state design (LSD) equations for connections in wood-floor systems employing light gauge metal hangers. This was realized by calibrating certain mathematical formulations against a recommended target value of safety index β which had been decided upon by a code committee. Finally, a sampling plan incorporating decision theories have been proposed to ensure not only quality acceptance of the above and similar joist hanger products, but also their compliance with the minimum standards of safety.

Therefore the objectives of this study were as follows:

- To examine the acceptability of light gauge joist hangers as used in connections of wood-floor systems using reliability methods.
- To propose Limit State Design equations for joist hanger joints consistent with acceptable levels of performance.
- To devise a simple acceptance sampling plan for light gauge metal hangers.

1.4 Structure of the Thesis

An outline of the work in this thesis is set up as follows:

- Chapter 2 presents a literature review of relevant background information. This review includes past predictions of the performance of nail joints based respectively on analytical models and empirical relations. The effect on the performance of nailed joints of such factors as load eccentricity, interface friction, moisture content, species type and partial end fixity of the nail due to steel side plate is also examined. An overview of experimental work conducted by certain investigators and relevant to the study is also presented in this chapter. Mathematical formulations for estimating the probability of failure of a structural system and stochastic modelling of dead and live loads on floor systems are also outlined.
- The details of the experimental work, tests loading facilities and instrumentation are given in Chapter 3.
- Chapter 4 is devoted to the presentation and analysis of experimental results obtained from the tests.
- Chapter 5 focuses on reliability studies on joist hanger joints in wood-floor systems and the calibration of Limit State Design equations for the design of light gauge joist hanger joints.
- An acceptance sampling plan for light gauge joist hanger products using decision theory and reliability analysis is outlined in Chapter 6.
- The conclusions drawn from the test results and a summary of the work done are presented in Chapter 7 together with recommendations for further research in this area.

(a)

(b)

**Fig. 1.1 Typical Light-Gauge Joist Hangers commonly
Used in Wood-Floor Systems**

2.1 General

Three different concerns may provide the design criteria for joist hanger joints. The first concern is the obvious need to provide adequate resistance to the forces transmitted across the joint in order to avoid failure. The second involves meeting serviceability limit states criteria which may demand a thorough knowledge of joint rigidity. The third concern may also necessitate a knowledge of the inevitable changes in the stresses of the joined members due to the presence of the connection. Ordinarily, this concern may be the result of reduced cross-section, eccentrically loaded fasteners, or development of shear or tension perpendicular to grain stress within the joint area [48].

In this chapter a summary of the various research efforts aimed at modelling the performance of nailed wood joints is presented. The initial section is devoted to the summary of some of the more common analytical and empirical models proposed for the prediction of both the strength and load-deformation characteristics of nailed joints. Factors that influence the performance of nailed joints such as joint interface characteristics, joint member composition and variation of moisture content both within the joint and its environment are also discussed. Furthermore, the effect of load eccentricity on the joist hanger joint has been discussed using the results of a case study. Finally, an overview of the basic concepts underlining reliability-based design is presented.

2.2 Strength Modelling and Load-Slip Characteristics of Nailed Joints

Much work, both empirical and analytical, has been carried out to model the strength and load-slip characteristics of nailed joints. In this section, a summary of these models is presented.

2.2.1 Nailed Joint Strength Models

2.2.1.1 Empirical Models

In the formulation of design model for the nailed joint, the strength at some point on the load-deformation curve has been the base for design values in most timber design codes. T. E. McLain [48], in 1983 pointed out that much research in nailed wood joint have been aimed at testing representative joints and developing some empirical functions to predict a particular point on the short-term load deformation relationship. According to McLain [48], such models have been generalized as:

$$P = \alpha G^\beta d^\gamma \quad (2.1)$$

where

P = load at some point on load-deformation curve

G = specific gravity of the wood members

d = diameter of the fastener, and

α, β, γ = curve fitting constants.

Equation 2.1 has been the current basis for the derivation of nail design values in Canada, and elsewhere, and has been found applicable to both lateral and withdrawal loading situations. G. R. Brock [13], in 1957, working with round chequered-head wire nails which were without surface treatment and with the nails driven at right angles to the grain of the wood without prebored holes, obtained the following expressions for the estimation of the ultimate load per nail in softwood. For two-member joint,

$$P = 38000Gd^2 \quad (2.2)$$

and for a three-member joint,

$$P = 70000Gd^2 \quad (2.3)$$

where P , G and d are defined as in Eqn (2.1); the units of P and d being in pounds and inches, respectively.

Little work has been done on the very realistic situation of combined withdrawal and lateral loading on nailed joints. DeBonis and Bodig [20] (1975) had proposed an interaction formula for strength and slip modulus at different degrees of combined loading. Although, their results are of limited application, they point out the significant effect on joint performance of ultimate withdrawal force component even at relatively low load levels [48].

2.2.1.2 Analytical Models

Almost all the analytical models proposed for the strength prediction of dowel connections, and for that matter nailed joints, had been concerned with lateral loaded joints. As early as 1941, and later in 1949, K. W. Johansen [36], originally proposed analytical expressions for the yield strength of dowel joints. The essential features of his models are as follows:

- no axial tension occurs in the fastener and thus no contribution of frictional forces to the lateral load-bearing capacity of the joint.
- the stress-deflection relationships of the dowel in bending and of the wood are ideally plastic as illustrated in Figure 2.1.
- the ends of the fastener are assumed to either be rigid or free to rotate.

For a two-member joint, Johansen [36] predicted the joint ultimate strength as:

$$P = 0.414S_Hld \quad (\text{rigid dowel}) \quad (2.4)$$

and,

$$P = 0.442\sqrt{S_B S_H} d^2 \quad (\text{flexible dowel}) \quad (2.5)$$

in which

S_H = wood embedding strength, MPa

S_B = modulus of rupture of the dowel, MPa

d = diameter of the dowel, mm

l = thickness of each member, mm

The work initiated by Johansen [36] have been modified by several independent researchers to provide a general application. In this regard, therefore, the efforts of T. Möller [51] in 1950, A. Meyer [50] in 1957, F. E. Siimes, P. E. Johanson, and E. Niskanen [69] in 1954 and P. Aune [7] in 1966 have been referenced for nails. Möller [51] applied the yield theory to nailed joints with different member thicknesses and for cases of single and double shear joints. His work also considered the effect of having members of dissimilar embedding strengths.

McLain [48] reported that the assumption of a rigid-plastic joint materials may not always be valid for loads directed at an angle to grain. Work done by T. E. McLain and S. Thangjitham [49] (1983), B. Norén [61] (1961) and Rodd [66] (1973) indicated that end fixity and friction on the dowel may have appreciable influence on the yield load. According to Rodd [66], the effect of friction between the fastener and the surrounding wood fibres on the overall joint strength becomes rather significant when the dowel loads the timber at direction parallel to grain.

Plastic embedding strength of wood under nail has been examined by others (Norén [61]; R. O. Foschi and T. Bonac [27] in 1977). This value is known to be dependent on the strength of the timber, the surface coefficient of sliding friction between the nail and the surrounding wood fibers, the cross-sectional shape and the dimension of the connector, and the orientation of the growth rings of the timber. The real behaviour of the nail bearing in wood is therefore elasto-plastic with yielding in bending occurring in some sections of the nail followed by permanent deformation after some load [27,61].

Following the work of Johnasen [36], and later by Larsen [41] in 1979 , P. Aune

Table 2.1 Yield Strength Models for Two-Member Joints with One Steel Member

(After Aune and Patton-Mallory, [8] 1986)

Failure Geometry	Number of Nail Yield Points	Yield Load F_u (N)	Thickness Conditions
0	$F_u = f_e t_2 (\sqrt{2} - 1)$	$\frac{t_2}{\sqrt{\gamma}} < 2 + \sqrt{2}$
1	$F_u = \sqrt{2 f_e M_y}$	$\frac{t_2}{\sqrt{\gamma}} < 2 + \sqrt{2}$
2	$F_u = \sqrt{2} \sqrt{2 f_e M_y}$	$\frac{t_2}{\sqrt{\gamma}} > 4$

Notes:

1. Member thickness, t_2 , determine mode of failure
2. f_e =wood embedding strength, N/mm
3. M_y =nail yield moment, Nmm
4. $\gamma = M_y / f_e$

and M. Patton-Mallory [8] in 1986, proposed joint strength models derived from test results for dissimilar joint members and varying joint interface characteristics such as the presence of insulating material. Table 2.1 provides a summary of yield strength models for two-member joints employing steel side plate quoted from Ref.[8].

I. Smith [70] in 1982, R. O. Foschi [26] in 1974, Foschi and Bonac [27] and B. Thomas and Malhotra S. K. [74] (1985) have used numerical analytical models to predict nailed joint performance. Though, according to McLain [48], the objective of these efforts have been aimed at estimating the load-deformation relationship, they could well provide estimates for design base capacities if some agreement could be reached as to a rational definition of the point on the load-deformation curve at which joint load capacity is reached.

In an endeavour to investigate the load duration effect on strength of nailed joint, R. O. Foschi [25] in 1979 used a damage accumulation model to predict the mean relationship between load and time to failure using data derived from test results. On the same approach that is contemplated for load-duration effect in solid wood, Foschi [25] proposed that the rate at which damage is accumulated in a nailed joint is specified by the differential equation:

$$\frac{d\alpha}{dt} = ae^{w_1 R_N} \left[\frac{\gamma(t) - \gamma_0}{\gamma_s} \right]^b + \lambda e^{w_2 V_N} \alpha \quad (2.6)$$

where

a, b, λ = constants determined from test results

w_1, w_2 = constants determined by testing and controlling
the variability in the damage rate between specimen

$\gamma(t)$ = applied stress history

γ_0 = stress threshold, i.e. the stress below which no damage is accumulated

γ_s = strength of the specimen in a "short-term" test

R_N, V_N = random numbers from a normalised normal distribution
(mean = 0, standard deviation = 1), and

α = measure of damage, ($\alpha = 0$ corresponds to no damage).

Specifically, Equation 2.6 may be integrated and used for the prediction of time to failure for joint subjected to any arbitrary loading history, and also to estimate the residual strength after a given loading history. It is however, worthy of noting that available test data is still insufficient to calibrate the parametric constants

defined above which largely depend on the properties of the joint members.

2.2.2 Load-Displacement Models

2.2.2.1 General

The importance of the relationship between the applied load and the resulting joint deformation ($P-\delta$) in selecting base capacity values for nailed joints cannot be overemphasized. In recent time, these relationships have been recognised as important input to structural analysis models designed to improve the quality of analysis and, therefore, design of wood structures. J. R. Goodman, M. D. Vanderbilt and M. E. Criswell and J. Bodig [29] in 1974 and A. Polensek [65] in 1976 have shown that the load-slip relationships of nailed joints are important in the development of improved design methods for floor and wall systems. In particular, joint stiffness is known to significantly affect the distribution and magnitude of stresses within the frame system as well as system deformation [29].

Non-linearity of the load-slip relationship of nailed joints, even at relatively low load levels, has been studied by others (T. E. McLain [47], 1976; J. J. Mack [42], 1977); both tangent and secant moduli have been reported in the literature as joint stiffness. In 1983, D. L. Wheat, M. D. Vanderbilt and J. R. Goodman [77], pointed out that as a result of the nonlinearity of the $P-\delta$ curve, the use of critical stiffness moduli may lead to significant error at point load levels found in service.

Various attempts at developing models for the $P-\delta$ relationship for nailed joints have been studied extensively—both analytically and empirically. Several of such efforts are presented in subsequent sections.

2.2.2.2 Load-Slip Analytical Models

Explicit analytical models of the load-slip relationship for laterally loaded nailed joints have been developed by E. W. Kuenzi [39] in 1955, G. B. I. Jasson [34] in 1955, B. Norén [62] in 1951, Wilkinson [79] in 1971, Nicol-Smith [59] in 1972 and Foschi [26]. The following assumptions are common to these researchers.

- There is no tolerance between the nail and the surrounding wood fibres.

- No frictional forces are developed between the interface of the joined members.
- There is no friction between the nail fastener and the surrounding wood fibres.
- The nail connector is not influenced by geometric characteristics such as edge and end distances, or nail spacing.

Using the analysis of beams on elastic foundation first introduced by E. Winkle in 1867, and later by M. Hetényi [30] in 1946, Kuenzi [39] developed mathematical expressions for the pressure, deflection, moment and shear at any point along a nail fastener for single and double shear nailed joints. The nail, embedded in the wood, was modelled as a beam supported on an elastic foundation such that its displacement in the wood foundation is resisted by a pressure proportional to the deflection at any point on the nail, and such that this pressure can be exerted in both upward and downward directions. The expressions obtained by Kuenzi [39], however, involve complicated combinations of hyperbolic and trigonometric functions. Later, Wilkinson [79] provided some simplification to the unwieldy expressions obtained by Kuenzi [39] for the relative displacement of timbers of two piece anti-symmetric and non-symmetric joints.

I. Smith, L. R. J. Whale, C. Anderson, B. O. Hilson and P. D. Rodd [72], in 1986, in a contribution to future revisions to the International Council for Building Research Studies and Documentation (CIB) Code for the design of wood structures, stated that at low loads slip in nailed joints could be estimated from the Wilkinson's linear elastic solutions for joints which are of such construction that formation of plastic hinges in the fastener could occur within the joint members. If such joint has members of similar embedding strength, the load-deformation relationship could be defined as:

$$\frac{P}{\delta} = \frac{0.665668}{4} E^{1/4} K_{01}^{3/4} d^{7/4} \quad (2.7)$$

where

P = load per nail per shear plane at joint slip, δ (N)

P = load per nail per shear plane at joint slip, δ (N)

E = fastener's modulus of elasticity (N/mm²)

K_{01} = joint member elastic bearing constant (N/mm³)

It is pointed out elsewhere in the literature that whilst the theory of beam on elastic foundations can be used to estimate the initial stiffness of nailed joint, it falls short in predicting the maximum load-carrying capacity.

Nicol Smith [59] proposed an expression to describe the load-slip of nailed joints using the mechanical properties of the joint materials within their respective elastic limits. His method was based on an assumed distribution for the force q , within each member. The distribution for q was presented thus as:

$$q = Ae^{Bx} - C \quad (2.8)$$

In equation 2.8, x is measured from the face of the member remote from the joint interface towards the interface; A , B , and C being model constants.

Both Jasson [34] and Norén [62] are of the view that in general, even at low loads, the $P - \delta$ relationship is nonlinear, and this behaviour becomes further accentuated by the creep of the compressed wood under the nail fastener. Foschi [26], using the elasto-plastic behaviour of joint materials under lateral loading, has developed a nonlinear finite element model for the prediction of the load-displacement relationship for single shear nailed joint. In his model, Foschi [26] assumes that the wood foundation is discontinuous and has a load deformation relationship which is of the form:

$$p = (p_0 + p_1\omega)(1 - e^{-\frac{k\omega}{p_0}}) \quad (2.9)$$

in which

ω = nail displacement

p_0, p_1 and k = constants determined by nonlinear least squares fitting of experimental data for any nail type and wood species.

In Foschi's model the slip of the joint at any load level was determined using the principle of virtual work and an iterative solution of the governing equations with

finite element model to investigate the joint behaviour under various interface characteristics such friction and interface gap, for single and multiple fastened joints. The results of their investigation is discussed under section 2.3.2

2.2.2.3 Load-Slip Empirical Models

Similar to analytical models, almost all the empirical models postulated for load-deformation relationships for nailed joints have been concerned with cases of lateral load and short-term loading.

Following successive years of research in nailed wood joints, between 1960 and 1978, J. J. Mack [43] has developed a generalised formula applicable for both the single and double shear joints and for cases of wood to wood and metal to wood joints. His model, which accounts for both the nature of the wood side member(s) and the diameter of the fastener, assumes that the $P - \delta$ relationship is independent of the moisture content of the side member(s). Mack's model is of the form:

$$P = K(A\delta + B)(1 - e^{-C\delta})^D \quad (2.10)$$

in which

P = lateral single shear load at δ

K = constant dependent on fastener's diameter and wood side member(s)

A, B, C, D = curve fitted constants.

Mack's empirical formula is currently the basis for nailed joint load-deformation relationship recommended in the CIB Structural Timber Code (B) [38]. E. N. Morris [52,54,55] also presented a similar model to that of Mack [42] for single and double shear solid to solid timber joints. His model, which accounts for the diameter and the overall length of the nail, assumes that the lateral point load P , is directly proportional to the nominal specific gravity of the timber. The load-slip relationship is thus presented as:

$$P = Ad^B L^C G(D\delta + E)(1 - e^{-F\delta})^H \quad (2.11)$$

where

A, B, C, D, E, F, H = curve fitted constant

d = nail diameter

L = nail length

G = weighted specific gravity.

Morris [53,55] further provides a method where G in equation 2.10 is modified to account for the material characteristics of the joint members. In the case of nailed plywood to solid timber joint, the expression of the weighted specific gravity, G , is given as:

$$G = \frac{t_p}{t_p + t_s} G_p + \frac{t_s}{t_p + t_s} G_s \quad (2.12)$$

in which

t = member thickness

p, s = subscripts denoting plywood and solid timber, respectively

Smith [70] has mentioned that in order to eliminate friction, the model developed by Morris [52,53] was based on test results of joints with initial interface gap. Equation 2.11, according to Smith, is therefore only valid for $\delta \leq 2.54\text{mm}$ (0.1in) which for all joints tested corresponded to a load less than that required to close the gap between joint members. Smith [70] further comments that, though the research findings by Morris [52,53] had been complemented by an earlier work done by Brock [12] to conclude that the $P - \delta$ relationship for nail fastener in either plywood or solid timber is not significantly affected by the angle at which the nail loads the joint members relative to the veneer orientation for plywood or the direction of grain for solid timber, this model lacks general application as its acceptability is basically dependent on the diameter of the nail (Wilkinson [78], 1972; Ehlbeck [23], 1979).

Mack [42], on the other hand, suggests that for $\delta \leq 0.5\text{mm}$ a simplified expression may be used for the $P - \delta$ relationship for nailed solid to solid timber joints. The suggested expression has the form

$$P = \phi G d^{\alpha_1} \delta^{\alpha_2} \quad (2.13)$$

where ϕ, α_1, α_2 are curve-fitting constants.

Investigating a wide range of head and point side member combinations for single shear joints with a single 8d (3.27mm-diameter) common wire nail, McLain [47] obtained expressions relating load and joint displacement and chose an expression;

$$P = A \text{Log}_{10}(1 + B\delta) \quad (2.14)$$

because of its high degree of predictability of the equation parameters. McLain's model is also known to be accurate for joint displacement up to 0.1in (2.54mm) for specimen with an initial gap between the joint members. The parameter A in Eqn. 2.12 is predicted from the nominal specific gravities of the head and point side members using the relationship

$$A = 1105.8 - 187.3/SGS + 184.0(SGS^2 \times SGM) \quad [\text{N,mm}] \quad (2.15)$$

in which SGS and SGM are the nominal specific gravities of the head and point side members, respectively. The constant B is determined by substituting a known combination of P and δ in Equation 2.13.

Commenting on the general application of the $(P - \delta)$ empirical models, Smith [67] states that:

- The models presented by Mack and McLain are of limited application only in joint types where the ratio of the member thickness to fastener diameter is such as to develop plastic hinges of the fastener within any one of the side members (*mode 3* type of failure).
- Morris' model is only applicable for structures where the ratio of the members thickness is insufficient to ensure a *mode 3* type of failure.

2.3 Factors Influencing the Performance of Nailed Joints

Many factors abound, both inherent joint material behaviour and environmental, which influence the performance of the nailed joint. A review of some of these factors of relevance to this study is presented under the following section.

2.3.1 Joint Member Geometry

All the models considered under section 2.2 for the prediction of nail joint performance assume the fastener to be free from geometric influences such as edge and end distances and nail spacing. In actuality, wood has the tendency to split when fastened with nails. This tendency tends to be more significant in hardwood than in softwood, and decreases in wood with pre-bored holes. Many wood design codes such as the CSA code (CAN3-086.1-M84) [14], CP 112 [11] *The Structural Use of Timber*, and the National Design Specification for Wood Construction (NDS) [58] provide design guidelines for joints of various wood species and fastener types.

2.3.2 Interface Characteristics

The effect of interface friction on joint performance (stiffness) have been investigated by others (Meyer [50] in 1957, Larsen [41] in 1979, Malhotra and Thomas [46], and Atherton, Rowe, and Bastendorff [6] in 1980). Malhotra and Thomas [46] recently studied quantitatively this effect on single shear wood-to-wood joints. To determine the effect of interface friction, a comparison of the theoretical behaviour of joints with interface friction and that of joints without friction was made. Figure 2.2(a) shows a graphical representation of the effect of interface friction on the behaviour of joints at proportional limit slip and at ultimate slip observed by Thomas and Malhotra [74]. They pointed out that the effect of interface friction becomes less significant as the number of nails in a row increases. Atherton *et al.* [6] have reported the results of experiments conducted to investigate the effect of friction on slip-modulus of nailed joints. They concluded that joints with interface friction have high initial modulus and that the behaviour of these joints is similar to that of joints without friction once slippage occurs.

Though the effect of friction on nailed joint stiffness becomes significant in newly constructed joint, in service, frictional resistance soon disappears due to cyclic shrinkage in the wood [39,77].

The influence of interlayer gap in wood nailed joints has also been studied

quantitatively by Antonides, Vanderbilt and Goodman [4] (1980), and Maholtra and Thomas [46,75], among others. Antonide *et al.* [5] investigated this effect on the stiffness of two-member, single-nailed specimens and stated that for stiffness evaluated at the NDS [58] load for single nail, the relationship between stiffness and interlayer gap for the sample of joints tested could be expressed as:

$$\text{Log } K = A - Bg \quad (2.16)$$

where

K = joint stiffness

g = interlayer gap

A, B = curve fitted constants.

The physical reason underlying the interlayer gap effect is that nails must bend, even for zero gap, to transfer load between members. Increasing the gap width increases the nail span and hence decreases stiffness. Similar to the effect of interface friction, interface gap has been found to be more pronounced on joint stiffness at proportional limit slip than on ultimate stiffness, and the neglect of this factor in previous research works has accounted in part for the differences in various research results on the prediction of nailed joint performance [5].

2.3.3 Moisture Content

Changing moisture content conditions, both within the joint material and the joint environment, is known to exacerbate deformation as well as reducing joint strength [48]. Creep in nailed joints is said to be very minimal at low load levels if moisture content remains constant. In real construction, however, joint interlayer gaps occur because of inevitable cyclic variation in the moisture content of the wood. Occasionally, when green lumber is used in joint construction, the resulting shrinkage upon drying causes interface gap. McLain [48] pointed out that with some of nail fasteners, changes in moisture content can effectively negate any benefits of coating or threads.

2.3.4 Multiple Nail Fasteners

In accordance with CSA code (CAN3-086.1-M84) [14], the present design value for joints with multiple nail fastenings, P_{joint} , is determined as

$$P_{joint} = N_f(P_{indiv.})K^* \quad (2.17)$$

where

$P_{indiv.}$ = single fastener allowable load

N_f = number of nails per joint

K^* = modification factor accounting for joint geometry,
mode of fastener installation and service conditions.

The above design philosophy has been arrived at on the assumption that all nails share the applied load equally and that each nail in a row of the joint carries a load equivalent to the load capacity of a single nail joint [37].

Several reseachers have developed methods for analysing the distribution of load among fasteners in a row. This distribution has been known to be unequal, hence the need for a modification factor to Equation 2.14. Notable among such pioneer researchers are Lantos [40] (1969), Cramer [19] (1968) and Isyumov [33] (1967), among others. On investigating nail group behaviour on the performance of two-member joints with fasteners loading the wood fibres at parallel to grain direction, Thomas and Malhotra [75] showed that a nonlinear relationship exists between the joint slip, Δ , and a dimensionless variable, Υ . This relationship has been expressed mathematically as:

$$\Upsilon = C_N N_f = \frac{\Delta}{A \cdot \Delta + B} \quad (2.18)$$

where

Υ = a dimensionless variable

Δ = joint slip

A, B = nonlinear parameters

N_f = number of nails in a joint

C_N = modification factor

Figure 2.2(b) illustrates the result of comparative study of modification factors obtained from experimental data [75] and those predicted using equation 2.2 at proportional limit slip (0.4mm) and at ultimate slip (5mm), and for varying number of nails in a row. According to Thomas and Malhotra [75], the present design format leads to an overestimation of the capacity of multiple-fastened nail joints. Based on their test results, the actual joint capacity at proportional limit slip (defined as $\Delta = 0.4$ mm) with 2–8 nails in a row was found to be about 0–10% less than the present design capacity of that joint, and at the ultimate slip, the corresponding reduction in capacity was within the range 0–7%. These researchers therefore proposed that the modification factor, C_N , be taken as 1.0 for joints with up to three nails in a row, and 0.9 for those with more than three in a row [75].

To the author's knowledge no comprehensive studies similar to the work of Thomas and Malhotra [75] have been undertaken for situations where the nail fasteners load the wood fibres at perpendicular to grain direction.

2.3.5 Joint Member Composition

Joint design loads are established for wood-to-wood joints with all members similar. Occasionally, metal side plates are employed in nailed joints and offer the advantage of an increased joint strength. This increase, according to McLain [48], is in the order of 25–50%. Metal side plates provides increased end fixity for the fastener and also, a very stiff bearing surface; the cumulative effect is the overall stiffening of the joint. It is reported elsewhere in the literature that if loads in joints with and without metal side plates are compared at fixed level of deformation, for example at wood-wood joint proportional limit slip, then the former are 1.75 times greater than the latter. McLain [48], however, asserts the fact that the influence of metal side plate on joint performance varies with plate thickness and size.

2.3.6 Load Duration and Load History

The influence of load duration and load history on the performance of nailed

joints have been studied both quantitatively and qualitatively by Bengt N. [9] (1965), Jenkins, Polensek and Bastendorff [35] (1979), A. Polensek [64] (1982), J. J. Mack [43] (1966), B. Norén [60], among others. Bengt [9] is of the view that under constant load the nailed joint slip consists of two components, a load-dependent instantaneous slip and a time-dependent slip. The latter slip component has a rate which depends only on the current load-time level, even though the total slip is also a function of the loading history.

Both Wilkinson [78] and McLain [47] have reported an apparent increase in initial stiffness with cyclic loading, but Polensek [64], has indicated that there is a significant reduction in nailed joint stiffness under long-term loading. Sansalone and White [68] have also noted that for dowels driven parallel to grain, the displacement due to a particular load level increases with repeated application of that load once the proportionality limit of the wood in bearing is exceeded.

2.4 Withdrawal Characteristics of Joist Hanger Connections in Wood-floor Systems

At joist hanger connections in wood-floor systems, load is transferred primarily by shear in the nails; however, due to the end rotation of a beam under gravity loads, the connections become also subjected to tensile (withdrawal) loads. M. Sansalone and R. N. White [68] in 1986 pointed out that in a one-sided connection, ie. where the hanger bears on one side of the header only, some rotation of the header would occur, thus causing a reduction in the induced withdrawal forces resulted from end rotation. On the other hand, since no rotation of the header occurs in a case where two joists are symmetrically supported on each side of the header, the end rotation of each joist must be accommodated by the connection. Induced withdrawal load is therefore maximum in such a connection.

Sansalone and White [68], while investigating the withdrawal effect on the mode of failure of a hanger-type, end-grain, timber-beam multidowel connection made the following observations:

- Due to unequal load distribution among the dowels in the connection coupled with the flexibility of the metal hanger, conventional shear/tension interaction diagrams such as obtained using the procedures by DeBonis and Bodig [20] cannot be used directly to calculate the reduction in capacity of a multi-dowel connection under combined shear and tension.
- Withdrawal forces could not be neglected when the behaviour of the connection beyond the elastic range is considered.
- Highest tensile loads were carried by the dowels at the top of the connection, thus as shown in Figure 2.3, load-slip curves were less stiff for the top dowels as compared to the load-slip curves for the middle dowels, which in turn were less stiff than the load-slip curves for the bottom dowels. However, as the ultimate capacity was approached, the load distribution tended to equalize and the load-slip curves merged.

2.5 Reliability Analysis of Engineering Systems

Traditionally, the reliability of engineering systems has often been achieved through the use of factors of safety and conservative assumptions adopted in the process of design. That the basis of this assessment has long depended on subjective judgement and experiences accrued from past performance of other related systems cannot be overemphasized. This approach, therefore, does not render itself easy for quantification and also lacks the logical basis for addressing uncertainties [3].

In reality, of course, uncertainties are unavoidable in that engineering information is invariably incomplete. On this basis, therefore, the 'supply-demand' formulation of engineering systems are modelled as random variables. This thus paved the way for the reliability of a system to be more realistically measured in terms of probability.

The uncertainties of the imposed load-effect, Q , and the strength or resistance, R , of a structural system is illustrated in Fig. 2.4(b) by means of their respective

probability density functions, $f_R(r)$ and $f_Q(q)$. The objective of reliability analysis in the face of uncertainties is to ensure that throughout the useful life of the system, $R > Q$. The failure probability, $p_F = P(R < Q)$ of such a system may then be expressed in terms of a convolution integral, and for statistically independent R , and Q this is given by

$$\begin{aligned} p_F &= \int_0^\infty f_Q(q) dq \int_0^q f_R(r) dr \\ &= \int_0^\infty F_R(q) f_Q(q) dq \end{aligned} \quad (2.19)$$

It should be pointed out that in most engineering systems, such as wood structural systems, the resistance, R , is dependent on the load history, ie. R and Q correlate. In such cases, the required probability of failure is explicitly defined in terms of the joint probability density functions (PDFs) as:

$$p_F = \int_0^\infty \int_0^q f_{Q,R}(q, r) dr dq \quad (2.20)$$

The reliability of any structural system can now be assessed using the system's probability of failure. This reliability or safety index, β , thus becomes the measure of the performance of the system, and is related to the probability of failure as

$$p_F = 1 - \Phi(\beta) \quad (2.21)$$

where $\Phi(\cdot)$ = the cumulative distributed function (CDF) of the standard normal distribution.

It is evident from Fig. 2.4(b) that the larger the overlapping area, the greater the probability of failure. Indeed the overlap region has been found to depend on the relative positions of the distributions $f_R(r)$ and $f_Q(q)$ [3]. As $f_R(r)$ and $f_Q(q)$ become closer p_F increases, and vice versa. The effect of this "relative degree of closeness" on p_F is measured by the ratio μ_R/μ_Q . The overlap region is also known to depend on the degree of dispersion of $f_R(r)$ and $f_Q(q)$, or simply put, their coefficient of variations δ_Q and δ_R . The probability of failure p_F , therefore, can be expressed functionally as

$$p_F \simeq g(\mu_R/\mu_Q, \delta_R, \delta_Q) \quad (2.22)$$

In most reliability studies it is found that the safety index, β , can be approximated much more easily than the probability of failure. Thus, it is now generally accepted to use the safety index, β , as the indication of the reliability of any engineering system. Once the safety index has been determined for the system, it is then required to determine whether or not this level of safety is "acceptable". This measure of acceptability is often prescribed by code committee which provides target values for the reliability index, β , corresponding to acceptable probabilities of failure. The "Target Reliability Indices" for steel and concrete are given in Canadian Standard Association (CSA) Special Publication S408-1981 [15] and are reproduced as in Table 2.2.

**Table 2.2 Target Reliability Indices for Steel and Concrete Building.
Ultimate Limit States. (Based on 30-yr life)**

Safety Class	Type of Failure	
	Gradual	Sudden
Not Serious	2.5	3.0
Serious (Normal Buildings)	3.5	4.0
Very Serious	4.0	4.5

Notes

1. Not serious—Risk to life negligible & economic consequences are small.
2. Serious—Risk to life exits and/or economic consequences are considerable.
3. Very serious—Risk to life great and/or economic consequences are very great.

2.5.1 Methods For Evaluating the Probability of Failure

The calculation of the probability of failure, as presented in Equations 2.19 and 2.20, requires a thorough knowledge of the distributions $f_R(r)$ and $f_Q(q)$ or the joint distribution $f_{R,Q}(r,q)$. Unfortunately, this information is unavailable or difficult to obtain because of insufficient data. Added to this difficulty, the numerical integration required for the exact evaluation of the probabilities is often intractable. In most engineering problems, the available data may only suffice for the computation of the mean and the variance of the various random variables.

In practice therefore, the reliability may be measured entirely with a function of the first and second moments of the design values [*First/Second Order Reliability Methods (FORM/SORM)*]. Where appropriate forms are prescribed for the distributions, the probability may then be evaluated on the basis of equivalent normal distributions.

Of late, the probability of failure of a structural system has been estimated using the Monte Carlo Simulations. Although this approach has proved simple, the large number of samples run in order to estimate low failure probabilities makes it costly and therefore quite prohibitive [28].

2.5.2 Reliability Index

The reliability of a structural system may be defined as the probability of performing its intended function. Such function, or the *performance function*, invariably depends on the properties of the system. For the simple case of "supply-demand" problem, the performance function could be expressed as

$$\begin{aligned} Z &= \text{Capacity} - \text{Demand} \\ &= R - Q \end{aligned} \quad (2.23)$$

The limiting performance of the system may be defined as $Z = 0$ which defines a state, called the *limit state*, for which there is neither failure nor any 'strength' in the system. "A Failure" state is defined by $Z < 0$, and "a safe" state $Z > 0$.

In general, the limit state variable, G , is a function of several variables, ie.

$$Z = G(\mathbf{X}) = G(X_1, X_2, X_3, \dots, X_n) \quad (2.24)$$

in which,

$G(\mathbf{X})$ = performance function or state function

X_i = design variable.

If the limit state is defined as $G(\mathbf{X}) = 0$, it follows that

$$[G(\mathbf{X}) > 0] = \text{"safe" state}$$

$$[G(\mathbf{X}) < 0] = \text{"failure" state}$$

In Fig. 2.4 (a), the failure surface, $Z = R - Q = 0$, is seen as a line in two-dimensional space which defines the "safe" and "failure" states. If the load, Q , and the resistance, R , can be modelled with Gaussian or log-normal distributions, the probability density function for the performance function, $G(X)$, is then obtained analytically. The reliability index, β , is then given as

$$\beta = \frac{\mu_G}{\sigma_G} \quad (2.25)$$

from which the probability of failure is determined.

CSA Special Publication S408-1981 [15] recommends that for the calculation of reliability index for variables R and Q whose actual pdfs are approximated by the lognormal functions that fit in the significant tails, and for which the coefficient of variations, δ_R and δ_Q , are each not greater than 0.30, the following approximation for the reliability index, β , could be used.

$$\beta = \frac{\ln(\mu_R/\mu_Q)}{\sqrt{\delta_R^2 + \delta_Q^2}} \quad (2.26)$$

In a more general sense, the performance function may not be linear and several of the random variables, $X_1, X_2, \dots, X_n$, may be correlated. The detailed functional description of this situation has been developed in Ref. [3].

2.5.3 System Reliability

Evidently, in most structural systems, failure may be defined by one or a combination of several failure modes (performance functions). Also, from statistical view point, structural systems can be either parallel or series (weakest link) system. Parallel systems are such that total failure of the system requires failure of all the components, ie. if any one of the components survives, the system remains conceptually safe. The probability of failure of an m -component parallel system, therefore, may be represented as

$$p_F = P(E_1 \cap E_2 \cap E_3 \cap \dots \cap E_m) \quad (2.27)$$

where E_i is the event of the occurrence of i potential failure mode. It could be said that since a parallel system is made up of redundant components, its reliability often depends on the ‘activeness’ of the redundancies [3].

In the case of a series system, failure of any one of the system’s components automatically results in failure of the entire system. Being thus of weakest link system they are said to be statically determinate. Thus, for an n -component series system, failure probability is given as

$$p_F = 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \dots \cap \bar{E}_n) \quad (2.28)$$

where $\bar{E}_i$ is the event of non-occurrence of the failure mode i .

In Eqn. 2.28, if the failure modes are statistically independent, p_F is calculated as

$$p_F = 1 - \prod_{i=1}^n P(\bar{E}_i) \quad (2.29)$$

in which $P(\bar{E}_i)$ is the probability of survival of component i .

In reality, estimating the probability of failure of a structural system may involve the consideration of more than two potential failure modes which may prove a difficult task. To circumvent this difficulty, an assumption is often made that for a system with k potential failure modes, say, the reliability index, β , can be assessed as

$$\beta_s = \min [\beta_1, \beta_2, \beta_3, \dots, \beta_k] \quad (2.30)$$

Though the reliability index, β , obtained this way is not very accurate, it is however, accurate for systems for which one potential failure mode is known to be dominant and thus determines the overall reliability of the system. Quite commonly, it is also possible to obtain upper and lower bounds for the overall system reliability either as “first-order/unimodal” bounds or “second-order/bi-modal” bounds.

According to Ang and Tang [3], for any pair of potential failure modes E_i and E_j , which are positively correlated ($\rho_{ij} > 0$), the first-order bounds for the failure

probability p_F , is given as

$$\max p_{F_i} \leq p_F < 1 - \prod_{i=1}^k (1 - p_{F_i})$$

which is approximated as

$$\max p_{F_i} < p_F < \sum_{i=1}^k p_{F_i} \quad (2.31)$$

for small probability, p_{F_i} . The bi-modal lower and upper bound failure probabilities for k potential failure modes have been presented by Kounias, 1968 [38] and Hunter, 1976 [31] as

$$p_{F_1} + \max \left[\sum_{i=2}^k \left\{ p_{F_i} - \sum_{j=1}^{i-1} P(E_i \cap E_j) \right\}; 0 \right] \leq p_F \leq \sum_{i=1}^k p_{F_i} - \sum_{i=2}^k \max P(E_i \cap E_j) \quad (2.32)$$

The various failure modes may at times be correlated and the evaluation of the joint probability $P(E_i \cap E_j)$ is therefore inevitable. This difficulty is however circumvented by the relationships developed by Ditlevsen, 1979 [21] for any pair of positively correlated performance functions which are defined by Gaussian variates. In this way, the joint probability, $P(E_i \cap E_j)$ is defined with upper and lower bounds as

$$\max [P(A); P(B)] \leq P(E_i \cap E_j) \leq P(A) + P(B) \quad (2.33)$$

where

$$P(A) = \Phi(-\beta_i) \Phi \left(-\frac{\beta_j - \rho \beta_i}{\sqrt{1 - \rho^2}} \right) \quad (2.34)$$

and

$$P(B) = \Phi(-\beta_j) \Phi \left(-\frac{\beta_i - \rho \beta_j}{\sqrt{1 - \rho^2}} \right) \quad (2.35)$$

The mutual correlation between the failure modes, according to Ang and Tang [3], is computed as

$$\rho_{g_i, g_j} = \frac{\text{Cov}(g_i, g_j)}{\sigma_{g_i} \sigma_{g_j}}$$

$$\begin{aligned}
&= \frac{\sum_k \left(\frac{\partial g_i}{\partial X_k} \right)_* \left(\frac{\partial g_j}{\partial X_k} \right)_*}{\sqrt{\sum_k \left(\frac{\partial g_i}{\partial X_k} \right)_*^2} \sqrt{\sum_k \left(\frac{\partial g_j}{\partial X_k} \right)_*^2}} \\
&= \sum_{k=1}^n \alpha_{ik}^* \alpha_{jk}^* \tag{2.36}
\end{aligned}$$

where the required correlation coefficients are evaluated using the direction cosines of the tangent planes at the most probable failure points on the failure surface $G_i(\mathbf{X}) = 0$.

2.6 Probabilistic Load Modelling in Wood-Floor Systems

Stochastic characterization of the entire load process in the analysis of wood-floor systems is essential as the strength of such system is load history dependent. The total load on residential floors is made up of a dead load component, classified as permanent load, and a live load component which is a combination of sustained loads and extraordinary or transient loads [57].

2.6.1 Modelling of Dead load

The dead load component of the loads on floor systems consists of the weight of the floor joists and any 'permanent' fixtures upon it such as floor sheathing, covering, ceiling fixtures and any permanent partitions notably non-load bearing walls. As illustrated in Figure 2.5, dead load is assumed to be random in intensity but constant in time, and is modelled simply as a random variable with a normal probability distribution. The mean of the ratio of actual dead load to the design or nominal load $d = \frac{D}{D_n}$, is approximately equal to 1.00 with coefficient of variation 0.10.

Murphy J., Ellingwood B., and Hendrickson (1987) [57] has reported that because of the much larger variation in live load than in dead load, to a first approximation, the dead load can be assumed deterministic. This assumption therefore

enables the formulation of a stochastic load combination analysis simply in terms of the live load distribution.

2.6.2 Modelling of Live Load

The occupancy live load on floor systems is modelled as having two components, sustained live load $Q_s(t)$, and extraordinary load $Q_e(t)$. Thus, as shown in Fig. 2.5, the total live load at any time t , is given as

$$Q(t) = Q_s(t) + Q_e(t) \quad (2.37)$$

The sustained live load component consists primarily of furnishings and people normally present within the structure. This component is modelled probabilistically using results from load surveys [Ellingwood and Culver [24] (1977), Chalk and Corotis [17] (1980)]. Turkstra and Madsen [76] in 1980, captured the essential features of the sustained live load by modelling the occurrence of such load pulses as Poisson point process. For each load pulse its duration was considered as an exponential random variable, and the intensities assumed to be statistically independent and identically distributed random variables modelled by the gamma probability density function.

The extraordinary live load is very rare and occurs for very short periods of time and as a result the duration of which is seldom measured during load surveys. According to Corotis and Jaria (1979) [18], the extraordinary live load can also be separated into two parts; a controllable part and an uncontrollable part. The controllable part results from temporary crowding of floor areas because of parties, remodelling, or emergencies. On the other hand, the uncontrollable part is due to those loads which are associated with structural distress such as those caused by fires or earthquakes.

The occurrences of the extraordinary live load component are also modelled as Poisson point process. The average duration of the extraordinary load event is very short compared with the mean time between any two successive of the load events. Consequently, in earlier live load analyses of the extraordinary load process,

this has been modelled as an impulse when the focus of research interest was on the intensity of the maximum total live load and not its duration. However, with the recent approach to reliability analyses via the damage accumulation model the knowledge of the random duration of the extraordinary live load has become rather essential.

To obtain the distribution of the maximum combined load over 30 year-period, Foschi F. O., Folz B. R. and Yao F. Z (1989) [28], carried out a computer simulation using 5000 replications each of which consisted of different sequence of sustained and extraordinary loads. Using the upper 50% of the simulated data, Foschi *et al.* [28] fitted an Extremal Type I distribution to the data; the resulting distributional parameters associated with each type of occupancy load are given in Table 2.3.

**Table 2.3 Occupancy Loads: $q = Q/Q_n$ Extreme Type I Parameters
(Based on 30-yr Maxima) [28]**

Occupancy Type	Mean	Coefficient of Variation
Residential	0.812	0.272
Office	0.925	0.236
Commercial, First Floor	0.428	0.133

(a) Nail Yield

(b) Wood Embedment

Fig. 2.1 Ideal Plastic Behaviour of (a) Nail Yield and (b) Wood Embedment [8]

(a)

(b)

Fig. 2.2 (a) Study of effects of Interface friction [75]
(b) Comparison of Experimental and Predicted Modification Factors, C_N [75]

Fig 2.3 Load-Slip Curves for Twenty-Four Douglas Fir Connection Made with End-Plate Having 8-mm Diameter Dowel Holes [68]

(a)

(b)

Fig. 2.4 (a) Limit States of Two-Variable Performance Function
(b) Relationship between Load-Effect ($f_Q(q)$)
and Strength/Capacity ($f_R(r)$) [3]

Fig. 2.5 Typical Sample Functions for (a) Dead Load and (b) Live Load [57]

3.1 General

In line with the objectives of this study, the experimental program described under this thesis was carried out in two phases. The first phase was aimed at obtaining relevant material properties for the reliability analysis of the components in joist hanger joints, ie., the steel hanger, the nail fastener, and the wood joist and header.

The properties of these components considered in this category were:

- the respective modulus of elasticity and yield stress of the steel hanger, and the nail,
- parameters describing the load-penetration relationships for wood in bearing under nail, and
- series of different types of joints fabricated with 1, 2, 3, 4 and 5 nail(s) in a row perpendicular to the grain of the wood to determine the modification factor, C_N , to account for group effect of row(s) of nails loading wood fibres at perpendicular to grain direction.

In the second category, samples of joist hanger joints were assembled and tested in accordance with both the ASTM D-1796 [1] and the International Conference of Building Officials (ICBO) [32] test guidelines recommended for the testing and evaluation of joist hangers. The results obtained for under this testing method were classified as Data Set A. In this category, prototype samples of joints were also constructed using joists of 3600 mm span. The latter tests were aimed at providing a comparative study of the performance between joints built and tested

Table 3.1 Allowable Load on LUP210 Joist Hanger
[After Simpson Strong-Tie Company, Inc. Catalog C-89H-1 (1989)]

Nail Diameter (mm)	Floor Load (kN)	Roof Load (kN)
3.75 (10d)	5.229	6.541
4.15 (16d)	5.963	7.454

Note(1) Values quoted above are per reaction of hanger joint

(2) Values have been converted from Pounds to Kilonewtons

under ASTM/ICBO testing procedures and those found in residential construction. The corresponding tests results were also classified as Data Set B.

3.2 Description of Test Materials

3.2.1 Steel Hanger

In this study the LUP210 model joist hanger manufactured by the Simpson Strong-Tie Inc., California, USA, was used as a case study. The joist hanger is of gauge 18 steel material (actual thickness, $t_p = 1.25$ mm) with galvanized finish. A complete detailing of the joist hanger is given in Fig. A.2 in appendix A. The joist hanger, which is designed to support joist of sizes 38×235 and 285 mm, has a total of 10 No. 4.25 mm diameter holes (five on each of the two flanges) provided for the nail connection needed to transfer the load at the joist hanger to the header. To facilitate installation, there are also speed prongs designed to provide both instant location of the hanger onto the wood header and without the use of nails, to secure the joist on the hanger. The joist hanger also has a bearing seat of dimension 40×50 mm, and has been designed for the allowable loads given in Table 3.1 for some recommended nail diameters.

3.2.2 Joist and Header Material

No. 2 SPF lumber available locally in Ottawa was used for the testing reported in this research. The lumber consisted of 38×140 , 38×235 , and 38×285 mm. Preliminary moisture content testing on representative wood samples at the time of assembling the test joints showed average moisture content between 7.0–11.0%.

3.2.3 Nails

The nail type employed for testing were plain shanks, flat head and medium diamond point common wire round nails. The choice of this nail type was based primarily on the specifications governing the use of the test joist hangers. The nails were of average length 62.5 mm and of diameter 3.25 mm.

3.3 Testing Program

3.3.1 Tensile Test of Hanger Steel

Portions of 12×140 mm steel material were obtained from the hanger units from which test coupons were prepared and tested in accordance with the ASTM Standard E8M-87 [2]. (Application of this standard was limited to only the relevant sections which deal with the preparation and testing of sheet-type specimen of maximum nominal width at the reduced section of the specimen not greater than 6 mm).

A typical coupon test consisted of applying a uniaxial tensile load to the test specimen vertically aligned in a 225-kN *Tinius Olsen* universal testing machine at a constant rate of extension of the specimen equivalent to 6 N/s and at a temperature of 20 C°—this rate of loading was meant to facilitate recording which was done manually. By means of two strain gauges, attached to the faces of the test specimen and at its mid-length, and with the gauges coupled to a digital strain recorder, the strains generated in the specimen at load increments were obtained from which stress (load)-strain plots were made. It is to be remarked that the positions of the

strain gauges were such as to minimize error introduced by strains resulting from unknown bending of the specimen.

The modulus of elasticity, E_s , of the steel hanger was obtained as the slope of the initial linear portion of the stress-strain plots (the elastic region), and the yield stress, σ_y , obtained at 1.0% offset—the maximum offset recommended in the ASTM Standard E87-87 [2] is 2.0%. Table 3.2 provides a summary of the parameters E_s and σ_y obtained for 10 replications of above test.

Table 3.2 Material Properties of LUP210 Joist Hanger

Specimen No.	Modulus of Elasticity, E_s (N/mm ²)	Yield Strength σ_y (N/mm ²)	Ultimate Strength σ_{ult} (N/mm ²)
SP-01	185,200	352.47	380.67
SP-02	203,400	346.00	383.29
SP-03	204,800	321.81	367.35
SP-04	186,700	329.75	330.83
SP-05	188,900	339.47	387.29
SP-06	210,400	326.33	377.63
SP-07	198,700	323.59	371.34
SP-08	—	—	—
SP-09	192,100	323.64	364.83
SP-10	185,100	323.37	348.11
No. of data points, n	9	9	9
Mean, m	195,000	331.82	367.93
Standard deviation, s	9,544	11.33	18.227
Coefficient of variation, v	0.049	0.034	0.050
5 th Percentile, X	189,100	324.81	356.63

Notes:

1. Material parameters could not be obtained for specimen no. SP-08.
2. 5th Percentile value is defined as 95% lower confidence limit on m and is given as $m - t_{\alpha, n-1} \frac{s}{\sqrt{n}}$, where $t_{\alpha, n-1} = 1.86$.

3.3.2 Nail Bending Tests

The nail specimens were tested on a *Instron* 100-kN universal testing machine. A typical set up of this test is as shown diagrammatically in Fig. 3.1(a). The nails were tested over a simply supported span of 46.0 mm such that on the application of a load at its mid-span, a three-point bending resulted. For all the test specimens, the load was applied at a constant rate of deformation of 0.5 mm per minute. The deformation at the mid-span of the nail was assumed to be equal to the travel of the cross-head movement of the machine. By means of a chart on the machine, the load-deformation curves were continuously recorded with the speed of the chart's marker set at 50 mm per minute to provide enlarged plots for easy reading and accuracy. This magnification factor was however accounted for in obtaining the elastic moduli of the various nail specimens. A typical nail bending test was terminated when the vertical deformation recorded at the mid-span was 12.5 mm.

Based on the assumptions of fully plastic section and no significant change in the geometry of the cross-section of the nail at yield, the yield stress was calculated as: For fully plastic section,

$$f_y = \frac{M_y}{z} = \frac{M_y}{d^3/6} \quad (3.1)$$

in which

d = nail diameter, mm

f = nail yield strength, N/mm²

M_y = nail yield moment, Nmm

z = plastic section modulus, mm³.

It has long been recognized that as yield stress occurs after excessive deflection, a precise value of the yield stress should be calculated taking into account the geometry of the deformed section [27,61]. In this study, however, compensation for geometry was not made in the estimation of yield stress. For a simply supported beam, therefore, the midspan deflection, δ , assuming that the contribution due to

shear is insignificant, is given as:

$$\delta = \frac{PL^3}{48EI}$$

from which,

$$E = \left(\frac{P}{\delta}\right) \frac{L^3}{48I} \quad (3.2)$$

In the above equation, the P/δ is obtained as the gradient of the linear (elastic) portion of the load-deformation curves recorded with the machine's chart which considers also the magnification of the plots, L is the span of the nail beam, being 46.0 mm in this case, and I is the moment of inertia of the cross-section of the nail shank which is assumed constant in the elastic range.

Table 3.3 provides a summary of the yield stress and modulus of elasticity obtained for 17 replications of the test.

3.3.3 Nail Bearing Tests

Rectangular wooden block specimens of dimensions $50 \times 150 \times 150$ mm with all sides perfectly planed were obtained from the SPF lumber species described under section 3.2.3. These tests were also carried out on a *Instron* 100-kN universal testing machine with the rate of application of the external load, provided via the crosshead movement of the machine which was set at 5 mm/min in this case. The test set up including instrumentation and manner of the applied load is as illustrated in Figure 3.1(b). The nail penetration in the wood specimen in the direction perpendicular to grain was measured by means of *Linear Variable Differential Transformers (LVDTs)* having 150 mm stroke. The *LVDTs'* readings were collected automatically through data acquisition systems and an *HP* micro-computer which also displayed the penetrations during testing.

For each test, the applied load was initially removed when the nail penetration in the wood was about $\frac{1}{2}$ the nail diameter following which the load was applied till full penetration. (This method, as recommended by Foschi and Bonac [27], has been known to modify the initial stiffness of the load-penetration curve). Ten

Table 3.3 Test Nail Yield Strength and Modulus of Elasticity

Specimen No.	Modulus of Elasticity, E (N/mm ²)	Yield Strength σ_y (N/mm ²)
NBT-01	190,600	773.12
NBT-02	186,300	751.34
NBT-03	203,900	766.01
NBT-04	188,700	737.03
NBT-05	197,100	743.76
NBT-06	201,000	739.18
NBT-07	192,200	742.40
NBT-08	196,600	776.06
NBT-09	194,100	758.18
NBT-10	193,600	736.32
NBT-11	195,000	769.23
NBT-12	200,600	751.34
NBT-13	189,700	744.64
NBT-14	198,500	751.79
NBT-15	193,800	749.73
NBT-16	191,100	778.18
NBT-17	200,600	753.31
No. of data points	17	17
Mean, m	194,900	754.21
Standard deviation, s	4915	13.72
Coefficient of variation, v	0.025	0.018
5 th Percentile, X	192,800	748.40

Note:

1. 5th Percentile value is defined as $m - 1.746 \frac{s}{\sqrt{n}}$.

replications of the test were made and the results are as given in Table 3.4 and Fig. 3.2(b). A typical load-penetration curve, such as given in Fig 3.2(a), has three parameters p_0 , p_1 , and k describing it. p_0 is the ordinate intercept of the asymptote to the curve, p_1 indicates the slope of the asymptote, and k the initial tangent modulus of the curve.

Table 3.4 Bearing Parameters of SPF No.2 for Perpendicular to Grain Loading. Diameter of Nail is 3.25 mm.

Specimen no.	Bearing Parameters		
	p_0 (N/mm)	p_1 (N/mm ²)	k (N/mm ²)
NBRG-01	36.23	25.39	88.31
NBRG-02	40.34	19.54	90.21
NBRG-03	46.45	22.18	83.45
NBRG-04	38.67	22.37	89.21
NBRG-05	43.93	20.39	90.74
NBRG-06	39.69	23.09	82.97
NBRG-07	45.97	23.85	91.33
NBRG-08	46.23	21.54	89.72
NBRG-09	38.44	19.66	82.14
NBRG-10	41.26	24.51	87.89
No. of data points	10	10	10
Mean	41.72	22.25	87.60
Coefficient of variation	0.088	0.090	0.039
5 th Percentile	39.59	21.089	86.52

3.3.4 Tests for Determination of Modification Factors, C_N

As pointed out earlier, the objective of these test series was to determine the modification factors, C_N , which would account for the effect of number of nails in a row of metal-timber joint on the overall load-slip behaviour. Towards this objective, therefore, joints with 1, 2, 3, 4 and 5 nails, each of diameter 3.25 mm, in a row parallel to the external load on the joint but perpendicular to the grain direction were assembled. For all the joints tested the timber side members were SPF No.2 species, and as much as was practicable only timber showing insignificant physical defects were used in the joint assembly. The joist hanger flanges served as the metal side plates for the joints with 1, 2, and 3 nails in a row (Figure 3.3(a)).

However, joints with 4 and 5 nails in a row were assembled via full hanger units with the ends required for gripping in the testing machine provided by 6 mm thick metal plates. Figure 3.3(b) illustrates schematically the assembly of this joint type.

There were four replicate specimens of each joint type tested. All tests were performed in a *Instron* 100-kN universal testing machine. As depicted in Figures 3.3(a) and 3.3(b), joint slips were recorded automatically by means of 150-mm stroke *LVDTs*. Through data acquisition systems and a microcomputer, graphical display of joint slip during testing was made.

Following preliminary testing of joints with 1, 2 and 3 nails in a row, joint rotation was prevented by restraining the unloaded ends of the wood side members. This was achieved by attaching at each of the two end of the wood side member, an L-shaped 12.5 mm plywood board which had its shorter leg securely clamped to the seat of the testing equipment.

The modification factor, C_N , for each joint type was computed based on the following procedures. For each of the two-, three-, four-, and five-, nail joints, the load-slip behaviour of the joint and that of a single nailed joint were considered. The ratios of the values of the loads for the two load-slip curves at different levels of slip, arbitrarily selected within the whole loading range, were then computed. Table 3.5 provides a summary of the results from such computations.

To verify the suitability of the empirical model (Equation 2.18) developed by Thomas B. and Malhotra S. K. [75] to accurately model the joint slip-modification factor ($\delta - C_N$) relationship for the case of nails loading the wood fibres at perpendicular to grain, parameters A and F in Equation 2.16 for each joint type were obtained from regression analysis using the load ratios given in Table 3.5 and the selected levels of slip as input data. The modification factors, C_N , (known as the predicted modification factors) at these slip levels were then computed for each joint type.

Table 3.5 Details of Magnification Factor, C_N , For Various Joint Types

Type of Joint	Slip, Δ (mm)	Average Load (kN)	COV %	Ratio, ρ	$C_N = \rho/n_f$	Loss/Nail %
Joint with one nail	0.20	0.3827	21.7	1.00	1.00	—
	0.40	0.5827	20.9	1.00	1.00	—
	1.50	1.2588	19.9	1.00	1.00	—
	2.50	1.5321	15.8	1.00	1.00	—
	3.50	1.7244	16.6	1.00	1.00	—
	5.00	2.0193	15.4	1.00	1.00	—
Joint with two nails	0.20	0.7361	20.8	1.9234	0.96	2.0
	0.40	1.1252	18.7	1.9311	0.96	2.0
	1.50	2.4417	17.2	1.9397	0.97	1.5
	2.50	2.9805	14.3	1.9454	0.99	0.5
	3.50	3.4953	13.0	2.0270	1.01	-0.5
	5.00	4.1678	6.9	2.0610	1.03	-1.5
Joint with three nails	0.20	1.0120	19.1	2.6443	0.88	4.0
	0.40	1.5559	15.1	2.6701	0.89	3.7
	1.50	3.4363	14.3	2.7299	0.91	3.0
	2.50	4.1845	7.9	2.7312	0.91	3.0
	3.50	4.8212	10.3	2.7959	0.93	2.3
	5.00	5.7574	9.5	2.8512	0.95	1.7
Joint with four nails	0.20	1.2713	18.0	3.3219	0.83	4.3
	0.40	1.9365	9.7	3.3233	0.83	4.3
	1.50	4.2789	11.8	3.3992	0.85	3.8
	2.50	5.2777	19.5	3.4448	0.86	3.5
	3.50	6.2073	13.3	3.5997	0.90	2.5
	5.0	7.4769	8.9	3.7207	0.93	1.8
Joint with five nails	0.20	1.4967	14.3	3.9100	0.79	4.2
	0.40	2.3104	19.6	3.9650	0.80	4.0
	1.50	5.0920	13.7	4.0451	0.80	4.0
	2.50	6.2134	11.1	4.0555	0.81	3.8
	3.50	7.0442	10.8	4.0850	0.82	3.6
	5.00	8.2525	16.5	4.0868	0.84	3.2

Notes:

1. For each joint type the average joint load for a specified joint slip is computed from 8 replications (two replications per joint)
2. For any joint type, $\rho = \frac{\text{average load at slip, } \Delta}{\text{average load at the slip, } \Delta \text{ for the single-nailed joint}}$
3. n_f is the number of nail fasteners in a joint
4. COV=coefficient of variation

3.4 Determination of Joist Hanger Vertical Load-Carrying Capacity

3.4.1 ASTM/ICBO Testing Procedure

A typical joist hanger vertical load-carrying capacity test assembly consisted of a joist affixed between two restrained headers by the joist hanger devices. The header restraint was accomplished by blocking between adjacent headers at the ends on both sides of the joist with pieces of 10 mm thick plywood. As dictated by the hanger design, the connections to the headers were provided by 3.25 mm diameter common wire nails for which a total of 10 nails per hanger, five on each of the two available flanges, was required. The nails were hand-driven into the headers using a simple hammer, and were such that they penetrated vertically into the headers.

In accordance with the ASTM D1761 (1977) [1] test procedures for determining joist hanger vertical-load carrying capacity, the actual dimensions of the wood joist and headers were single $38 \times 235 \times 700$ mm and two-layered $38 \times 285 \times 400$ mm, respectively. All the wood species were as described under section 3.2.2, and as much as practicable only samples showing insignificant defects were employed.

To provide continuous support under each header, the latter was placed on a 100 mm I-steel beam. A typical test assembly is as shown in Figure 3.4(a) and Figure 3.5. Following initial consultation with Salinas J. J. [67], it was decided to test all samples at least two weeks after their assembly. Salinas [67] explained that earlier tests on nailed joints conducted at Carleton University showed considerable reduction in joint strength after some initial period of their assembly after which subsequent reduction in strength was very minimal.

Test was conducted in a *Tinius Olsen* 534-kN universal testing machine which provided a vertically-applied load at the mid-span of the joist. The rate of loading was 3 kN/min which resulted in failure times of 8-10 minutes for all the 10 replications of the test. Displacement measurements, taken at the seat of each tested pair of joist hanger and at a distance 25 mm from the headers, were made with

Hewlett Packard displacement transducers each with a stroke of ± 75.0 mm (3.0 in) and a linearity error less than 0.5% of the total stroke range (150 mm/6.0 in). All instrumentation was used in conjunction with a Data Acquisition System run by a *Hewlett Packard* 9836 personal computer. In this way, a continuous record of both the applied load and the joint displacement was kept at all times.

Figure 3.4(b) shows the load-slip curves for the 10 replications, and the corresponding ultimate joint loads are as tabulated in Table 3.5. Immediately after each test, samples of wood specimens, dimensioned $20 \times 20 \times 20$ mm, were taken from both the joist and the header at portions within the joint area for both moisture content and specific gravity tests. The moisture contents computed were based on the oven-dry masses, and the specific gravities on the samples' green volume.

A typical joint exhibited the following behaviour at failure:

- the crushing of the wood fibres of that portion of the joist supported at the hanger seat
- shearing of the top and middle hanger-header fastening nails, and the
- buckling of the flanges which provide the connection of the joist with the hanger, coupled with excessive bending of the hanger seat acting as a loaded plate.

3.4.2 Prototype Testing of Joist Hanger Joints Assembled with Wood Joists of Span 3600 mm

To investigate loading and joist span effects on the performance of joist hangers, sample joints were assembled from long-span joist (lumber size being 38×235 mm). The test assembly shown in Figure 3.6. was employed for this purpose. With this equipment, it was much easier to simulate a uniform floor loads on the joist typical of loading situations in residential housing.

A typical test assembly is as presented in Fig. 3.6. The testing procedures were similar to that described under section 3.5.1. Each header at the end of a joist consisted of a two nailed-layered $38 \times 285 \times 900$ mm SPF No.2 lumber which was

Table 3.6 Summary of ultimate joint load, failure time, moisture content and specific gravity tests—Data Set A

Specimen No.	Failure Time (min)	Ultimate Test Load (kN)	% Moisture Content (average)	Specific Gravity (average)
JHT-01	8.04	24.1105	8.95	0.36
JHT-02	8.32	24.9654	8.51	0.34
JHT-03	7.86	23.5975	7.10	0.32
JHT-04	7.98	23.9395	10.21	0.29
JHT-05	9.97	29.9243	7.77	0.33
JHT-06	7.73	23.2035	9.93	0.35
JHT-07	7.62	22.8615	9.76	0.28
JHT-08	8.72	26.1624	8.47	0.34
JHT-09	8.30	24.9135	7.92	0.37
JHT-10	9.97	29.9243	8.88	0.35

Notes:

1. Ultimate load per joist hanger is assumed one-half the above values.
2. Average ultimate joint load per test is 25.3624 kN with coefficient of variation (COV) 0.10.
3. Values of moisture contents and specific gravities are averages for samples obtained from both the joist and header materials.

Table 3.7 Summary of ultimate joint load, failure time, moisture content and specific gravity tests—Data Set B

Specimen No.	Failure Time (min)	Ultimate Test Load (kN)	% moisture content (average)	Specific gravity (average)
HLJ01	6.46	25.36	9.71	0.35
HLJ02	7.73	26.71	8.57	0.31
HLJ03	6.08	23.57	7.95	0.29
HLJ04	7.49	22.83	8.87	0.27
HLJ05	8.33	26.94	10.91	0.34

restrained against any movement by means of the arrangement shown in Fig. 3.7.

The loads on the joints were applied via the joist through a series of air-actuated jacks having an internal diameter of 100 mm (4 inches) and working strokes varying from 100 mm to 250 mm. The supply of air to the jackets was provided by a 200 psi compressor. The actual pressure to each jack, hence the force transmitted by each jack, was controlled by means of an inline regulator at the inlet to the system. The direction of the pistons was also controlled using the valve at the inlet. Therefore, with both the valve and the regulator, total control of the applied load and for that matter the rate of loading of the joints were in the hands of the operator. As per the test assembly, all the loading jacks were at 600 mm-(2 ft) intervals along the frame. By means of load tree, each jack was able to apply two point loads. This resulted in the application of point loads at 300 mm (1 ft) center to center along the span of the joist which was found adequate to simulate the desired uniform distributed load. Lateral stability of the joists during testing was also provided at quarter-points along the span.

The scope of instrumentation for these test series included the measurements of the reaction (joint) forces and the displacements as indicated by the *LVDTs* positions in Fig. 3.7. The joints loads (the reaction force) were measured using two interface load cells capable of measuring up to 44.5 kN; each with a linearity error of $\pm 0.11\%$ of the full scale. In addition to the instrumentation described under section 3.5.1, the scope of instrumentation for these test series also included the measurement of the pull-out of the nails at the top and middle positions of the flanges with which the test hanger device is connected to header; the test result is given in Fig. 3.9(b). A typical joint failure observed for a prototype joint test is as shown in Fig. 3.8. Aside the much more significant pull-out of the nails witnessed for the joints tested under Data Set B, the failure modes of these joints were, in general, similar to those tested under Data Set A. Figure 3.9(a) depicts the load-deformation curves for five replications of above test, and the corresponding ultimate loads are given in Table 3.7.

Fig. 3.1 Schematic Representation of (a) Nail Bending Test, and (b) Nail Bearing Test

(a)

(b)

Fig. 3.2 (a) Load-Penetration Relationship in Nail-on Wood Bearing Test [75]
 (b) Results of Nail Bearing Tests for 10 Replications

Fig. 3.3 Schematic Representation of (a) Test Set Up for 1, 2 and 3 Nails, and (b) 4 and 5 Nails in a Row of a Single Shear Wood-Metal Joint.

Fig. 3.4 (a) Test Assembly for the ASTM/ICBO Test Method for Joist Hanger Vertical Capacity Rating, (b) Load-Deformation Curves -Data Set A.

Fig. 3.5 Test Assembly (Data Set A) showing the Restraints provided for the Headers.

Fig. 3.6 Test Set Up for Prototype Testing-Data Set B

Fig. 3.7 Restraint Arrangement for the Headers in Prototype Testing, and LVDT locations

Fig. 3.8 Typical Joist Hanger Joint Failure in Prototype Testing.

(a)

(b)

Fig. 3.9 (a) Load-Deformation Curves—Prototype Testing (Data Set B)
(b) Joist Hanger Flanges/Header Separation Curves

4.1 General

In this chapter, the test results reported in Chapter 3 are examined to draw relevant conclusions. The analyses include

- (i) group behaviour of nails loading wood fibres at perpendicular to grain direction,
- (ii) load-deformation characteristics of the joist hanger joint,
- (iii) assessment of the vertical load capacity of test joist hanger,
- (iv) the behaviour of wood joist within the joint area, and
- (v) the performance of the joist hanger steel material under test loads.

4.2 Influence of Number of Nails on Joint Capacity—Modification factors, C_N

The effect of the number of nails on multi-nailed joint is more significant at loading corresponding to the 'apparent' proportional limit slip ($\Delta = 0.4$ mm) than at ultimate limit slip ($\Delta = 5.0$ mm). From the tests results (Table 3.5) the actual joint capacity at the proportional limit slip with joints having 2–5 nails in a row is 0–20% less than the present design capacity stipulated in the CSA code CAN3-086.1-M84 [14]. At the ultimate load, however, the reduction in joint capacity is only in the range of 0–7%. The relatively smaller reduction in the design capacity at the ultimate joint slip can be attributed to the fact that at the load level when the yielding of a nail is impending, redistribution of the joint load occurs in such a manner that the load is more uniformly distributed among the fasteners.

As evident in Figure 4.1(b) and Table 3.5, the magnitude of the magnification factors C_N , is found to vary with the number of nails in joint as well as the level of slip. The above observations seemingly follow the results obtained by Thomas and Malhotra [75] for parallel to grain loading.

4.2.1 Modelling the Magnification Factors, C_N

To verify the suitability of Thomas and Malhotra's empirical model (Equation 2.16) to accurately predict the joint modification factors at various levels of slip and for perpendicular to grain loading, the input parameters A and B to the model for each joint type were obtained from regression analysis using the load ratios given in Table 3.5 and the selected levels of slip as input data. The modification factors, C_N , (termed as the predicted modification factors) at these slip levels were then computed for the various joint types. For each joint type, Table 4.1 and Figure 4.1(b) show a comparison of the predicted modification factors and the experimentally-obtained factors at slip values 0.4 mm (proportional limit slip) and 5.0 mm (ultimate limit slip).

It becomes evident from Fig. 4.1(b) that the model predicts an upper-bound solution of the modification factors at the proportional limit slip but lower-bound solution at the ultimate slip factors. The accuracy of the modification factor predictions is within 2% at the proportional limit slip and 5% at the ultimate limit slip.

4.2.2 Conclusion

Based on the observations made in the preceding section, the following conclusions could be drawn concerning the effect of the number of nails on joint capacity.

- The lateral load carried by a single nail in a multi-nailed joint for loading in the perpendicular to grain direction is not the same in magnitude as that carried by the nail in a single-nailed joint. Thus, for the design of multi-nailed joints CSA code CAN3-086.1-M84 [14] specification for loading perpendicular to the grain should account for the group behaviour of the fasteners.

Table 4.1 Comparative Study of Modification Factors, C_N

Joint type	MODIFICATION FACTORS, C_N					
	Parameter, A (dimensionless)	Parameter, B (mm)	At 0.4 mm Slip		At 5.0 mm Slip	
			Experimental	Predicted	Experimental	Predicted
Two-nailed joint	0.5004	0.00472	0.96	0.98	1.03	1.00
Three-nailed joint	0.3590	0.00439	0.89	0.90	0.95	0.93
Four-nailed joint	0.2820	0.00450	0.83	0.85	0.93	0.88
Five-nailed joint	0.2451	0.00228	0.80	0.80	0.84	0.81

- In later analysis, a modification factor of 0.88 will be employed for the study of the performance of joist hanger connections in wood-floor systems.
- Thomas and Malhotra's model (Eqn. 2.18) could well be used in predicting the modification factors for multi-nailed joints within their elastic range for the case of perpendicular to grain loading.

4.3 Load-Deformation Characteristics of Joist Hanger

The gross vertical deformation, δ_d , of the joist hanger joint measured at the seat of the hanger at any level of loading is made up of the following:

- the slip, δ_s , due to shear transfer of the joint load from joist hanger to the header,
- the deformation, δ_h , resulting from both the elongation of the joist hanger legs under axial loading as well as the plate bending of the hanger seat.
- the deformation, δ_r , resulting from the rotation of the joint in response to the pull-out of the hanger-header nails under the bending influence of the joist. Thus at any load level P , the joint gross vertical displacement may be stated as

$$\delta_d = \delta_s + \delta_h + \delta_r \quad (4.1)$$

It must be pointed out that, in reality it is rather difficult to quantify these deformational components; whereas δ_s is significant at all load levels, both δ_h and δ_r may or may not be significant at low load levels. On the contrary, for joints fabricated using long-span joists, the component δ_r is significant even at low load levels.

4.3.1 Behaviour of Load-Deformation Curves for ASTM/ICBO Test Method—Data Set A

The load-deformation curves of the joints tested under ASTM/ICBO test guidelines are of behaviour typical of nail fasteners under pure shear. As shown in Figures 3.4(b) and 4.1(a), the load-deformation curves contain two major distinct regions, a generally linear region below the apparent proportional limit, exhibiting very high joint stiffness, and a region above the pronounced point of inflection which is linear. This type of curve, it is noted, is very typical of *mode 3* type of failure. The average joint stiffness at the proportional limit slip ($\delta_d = 0.4$ mm) per joist hanger joint for the 10 replications was as high as 16,200 N/mm with coefficient of variation 24.3%. As Figure 4.1(a) indicates, the behaviour of the load-deformation at $\delta_d = 0.4$ mm was not accurately modelled by the beam on elastic foundation analysis—the computed joint stiffness was only 13,016 N/mm. This discrepancy may in part be due to the fact that the elastic constant, k , in the Wilkinson's model [29,79] is sensitive to the variations in magnitude of the parameters describing the nail bearing properties of the wood. In this study, nail bearing tests were carried out on specimens cut arbitrarily from lumber pieces from which joint specimens were also fabricated. To all intents and purposes, these tests do not actually simulate the behaviour of nails in a joint. On the other hand, the ultimate joint load computed using Aune and Patton-Mallory's model predicted well the experimental results.

4.3.2 Behaviour of Load-Deformation Curves for Prototype Joint Tests—Data Set B

Typically, the load-deformation curves obtained for Data Set B exhibit neither a marked region of linearity nor a distinct point of inflection. In contrast with those obtained for the Data Set A, there is also a remarkable decrease in joint stiffness at 0.40 mm joint displacement. The average stiffness for 5 replications was as low as 4250 N/mm with $COV = 17\%$.

The influence of joist span on the performance of joist hanger is evident from Figures 3.9(a) and 4.1(a). Under vertical loading, the end rotation of the joist is accommodated by the joist hanger connection and the cumulative effect of this is to subject the nail fasteners to combined shear and tensile (withdrawal) loading. As further evident in Figure 3.9(b), fasteners at the top of the connection tend to carry the highest load, thus load-deformation curves were less stiff as compared to those obtained for the middle fasteners. It is pertinent to also note that as the ultimate capacity was approached, the load distribution tended to equalize and the curves merged.

4.4 Vertical Capacity of Joist Hanger

In an earlier report by Sansalone and White [68], reduction in joint capacity had been observed for hanger-type, end-grain, timber-beam connection which was subjected to combined lateral and withdrawal loads. In this study, however, no significant difference in failure loads was observed between Data Set A and B (average failure loads per joist hanger for Data Set A was 12.681 kN with $COV = 0.10$, and for Data Set B 12.453 kN with $COV = 0.073$). The allowable load per joist hanger as extrapolated from the manufacturer's catalog number *C-89H-1* (Table 3.1) furnished a factor of safety of 3.2 for the recommended 'normal' loading and 2.8 for 'ultimate' loading based on the 3.25 mm diameter common nail employed as fasteners. The safety factors were calculated as the test ultimate load per hanger divided by the manufacturer's assigned design loads.

As a prelude to estimating the vertical load capacity of this joist hanger type, Aune and Patton-Mallory's model for two-member joint with one steel member was considered. For the two-point nail yield typical of the mode of failure of nail fasteners employed in light-gauge metal hanger connections, this model could be presented as

$$F_u = n_f \sqrt{2} \sqrt{2 f_e M_y} \quad (4.2)$$

in which, n_f is the number of nail fasteners for the joist hanger-header connection. M_y is obtained from the data for the nail bending test (section 3.3.1), and f_e is obtained using Eqn. 2.9 at $\omega = 3.25$ mm, the nail diameter [8].

As Figure 4.1(a) depicts, the load-deformation curves for the two data sets predict well the theoretical capacity of 14.208 kN per joist hanger obtained using the above model. The overconservativeness of Aune and Patton-Mallory model in predicting the joint capacity may be explained as

- Nail yield strength as calculated from Equation 4.2 is always conservative as the influence of geometrical changes in the nail cross-section, at yield, is not considered.
- The bearing behaviour of wood characterized by the parameters described under section 3.3.3 do not in all respect simulate a nail shank driven into a wood member. The actual stress behaviour around the nail shank cannot be simulated with the use of standard nail bearing test.
- In reality, the influence of joint load eccentricity, friction between the nail shank and its surrounding wood fibres cannot be neglected at ultimate joint load.

In the light of the test results it can be concluded for joist hanger capacity rating that

- more testing needs to be done to examine the influence of joist span on the strength of joist hanger connections,
- nail yield theory could be used to provide tentative design capacity for joist hanger types employing the action of nail fasteners under pure shear for the

transfer of joint load to the header.

4.5 Wood Joist Behaviour within the Joint Area

The transfer of loads on the joist to the joist hanger connector, is in most cases accomplished primarily by means of compression perpendicular to grain of the wood against the hanger seat. Accordingly, the ability of the joist to safely transfer this reaction load is largely depended on the bearing length of the hanger seat. As pointed out by Keenan F. J. [37] (1986), the behaviour of wood in compression perpendicular to grain depends on the size of the loaded area. The basic principle underlying this being that the joist's wood fibres in the vicinity of the joint create a localized tension force along the grain which resists the applied load. The magnitude of this induced force increases with the bearing area (or the bearing length).

Figure 3.8 shows a typical bearing failure at test failure load. As Fig. 3.8 depicts, in general, tests for compression perpendicular to grain exhibit no well-defined failure; test specimen usually continues to crush without fracturing.

Bearing failure in wood floor systems in particular, have been known to initiate cracking of ceilings. Therefore, design considerations for joist hanger performance in regard to joist strength properties should be aimed at meeting requirements for serviceability more than the ultimate limit state design. In Chapter 5, the CSA code CAN-086.1-M84 [14] guidelines for the estimation of bearing resistance will be incorporated in reliability analysis to assess the performance of joist hangers.

4.6 Stresses in Joist Hanger Steel

Basically, the stresses developed in the joist hanger material are the interplay of the reaction force from the incoming wood joist and the corresponding load transfer from the joist hanger to the header. As per the joist hanger model selected for this study, this transfer is accomplished primarily by means of nails fasteners bearing in the header foundation. Therefore, to study the stress behaviour in the joist hanger, separate analysis has been carried out for the hanger flanges and the seat.

4.6.1 Finite Element Analysis of Stresses in Joist Hanger Flanges

To ascertain the stress behaviour in the flanges of a joist hanger, finite element technique has been employed. A computer program, known as SP44B, first written by Bowes W. H. and Russell L. T. [10] in 1971 and later revised in 1986, has been used for this analysis. In a composite mesh arrangement of bars and triangular elements [Figure 4.2(a)], this program solves for the stresses in the bar elements, and also, based on the assumption of linear varying stresses within each triangular element [Linear Strain Triangles (LST)], stresses in the plate (flange) elements are correspondingly calculated.

For greater accuracy, the flange has been divided into 396 triangular elements each one of which has been defined by 6 nodes—3 corner nodes (principal nodes) and 3 midpoint nodes (secondary nodes).

In the finite element analysis the following assumptions were made

- The hanger flanges are homogeneous, continuous isotropic linearly elastic material.
- Hanger flanges other than those which directly provide the side plate connection with the header are modelled with bar elements. Such flanges are assumed to provide lateral stability to the incoming joist as well as reducing the joint load on the hanger seat.
- Joint load is equally carried by the fasteners defining the hanger-header connection.
- Load distribution within each nail hole is as shown in Fig. 4.2(b)
- The interface between the hanger flanges and the header is frictionless.

Load levels corresponding to joint gross displacements, $\delta = 0.4, 3.0$ and 5.0 mm are analysed. Figures 4.3 and 4.4 show the stress distributions for the sections indicated in Fig. 4.2(a). It is evident from these stress plots that large stress concentrations are located in the neighbourhood of both nail holes.

4.6.2 Stresses in Hanger Bearing Seat

To examine the structural behaviour of the joist hanger seat under the reaction load from the incoming joist, the seat is modelled as a rectangular plate with two opposite edges fixed and the remaining fixed to the legs of the joist hangers [Stalnaker and Harris [75] (1989), Thadani [74] (1984)]; the latter of which are modelled as infinitely rigid short columns [Figure 4.5(a)]. It is assumed that at any load level P , say, the intensity of the load on the seat is uniform. In reality, due to end-rotation of the wood joist the load intensity on the hanger seat is not evenly distributed. Furthermore, in service, the wood joist might not bear evenly on the entire hanger seat as a result of shrinkage. With further assumption that the entire load from the joist bears at the hanger seat, bending stresses is computed for joint load corresponding to joint gross displacements, $\delta = 0.4, 3.0$ and 5.0 mm.

It is worthwhile pointing out that even at joint load corresponding to $\delta = 0.4$ mm the stresses at the midspan of the seat as obtained from the above analysis exceed the yield strength of the gauge 18 hanger material [Figure 4.5(b)]. However, in practice, the plate deflection decreases the bearing pressure near the center of the seat, concentrating it more near the legs of the hanger. It is noted that with the use of supplemental side nailing, the actual load on the hanger seat is significantly reduced.

(a)

(b)

Fig. 4.1 (a) Comparison of Load-Slip Curves for Data Set A and Data Set B and (b) Comparison of Theoretical (Predicted) and Experimentally-Obtained Modification Factors.

Fig. 4.2 (a) Finite Element Mesh (b) Assumed Load Distribution in a Nail Hole

Fig. 4.3 Direct Stress Distribution in Joist Hanger Flange along Sections 1 - 1, 2 - 2 and 3 - 3 as per Fig. 4.2(a)

Fig. 4.4 Maximum Shear Distribution in Joist Hanger Flange along Sections 1 - 1, 2 - 2 and 3 - 3 as per Fig. 4.2(a)

Fig. 4.5 (a) Structural Modelling of Joist Hanger (b) Bending Stresses in Joist Hanger Seat

RELIABILITY ANALYSIS AND CALIBRATION OF JOIST HANGER

5

5.1 General

In this chapter, the performance of joist hanger connection as employed in wood-floor systems is assessed using reliability methods. This is achieved by first estimating the reliabilities of the joint components (lumber, nail and the joist hanger steel plate) by considering their stochastic nature and uncertainties associated with their strength. The individual reliabilities so obtained then provided the basis for identifying those components which have the most significant effect on the safety of the connection.

The formulation of the limit state design equations for joist hanger has been derived from results obtained above. Both the requirements for strength and serviceability limit states have been defined, and the design equations calibrated using the test results obtained under both Data Set A (ASTM/ICBO Tests) and Data Set B (Prototype Tests).

5.2 Components Reliabilities

The relevant performance functions associated with the structural behaviour of the components in the joist hanger connection are derived from the following

- The yield strength of the nail and the embedding strength of the header material.
- Compression perpendicular to grain strength of the wood joist.
- Bearing, shear, tensile and bending strength of the joist hanger.

The short-term reliabilities of the joint components are computed using the approximate method suggested in the CSA Special Publication S408-1981 [15] which is again stated here as

$$\beta = \frac{\ln(\mu_R/\mu_Q)}{\sqrt{\delta_R^2 + \delta_Q^2}} \quad (5.1)$$

5.2.1 Nail Yielding and Wood Crushing Failure Mode

The nail fasteners, bearing in the header, must provide adequate resistance to ensure the safe transfer of the reaction load on the joist hanger to the header. Failure of this function is characterized by the yielding of the nails and the crushing of the wood under the nail fasteners.

In a probabilistic modelling of the above behaviour, the following assumptions have been made:

- For a typical simply supported joist in wood-floor system, the total load under it is equally shared by the two end connections. The strength of any one of which is statistically independent of the other.
- The ultimate yield load per nail is estimated using the analytical expression developed by Aune and Mallory-Pattory [8] for wood joints employing one metal side plate.
- No load-sharing behaviour of the floor joists.

In this analysis, the joist hanger-header connection consists of five rows of nail fasteners with two nails per row. Failure of the entire joist hanger connection in terms of the reliability of this component is characterized by the yielding of all the nails in bending and bearing failure of the surrounding wood fibres. In cognizance with the composite behaviour of the nail group, failure is assumed to occur when the maximum load-carrying capacity of the group is exceeded. Thus, the performance function G_1 associated with this failure mode is

$$G_1(X) = R_1 - L_1 \quad (5.2)$$

2

of/de

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No 2)

where

$$R_1 = \frac{C_N n_F \sqrt{2} \sqrt{2 f_e M_y}}{(\alpha_D D_n + \alpha_Q Q_n)}$$

and,

$$L_1 = \frac{(D + Q) S_p L}{2(\alpha_D D_n + \alpha_Q Q_n)}$$

and in which

D, Q = acting dead and live load respectively

D_n, Q_n = nominal dead and live load respectively

α_D, α_Q = load factors

f_e = wood embedding strength

n_F = total number of nail fasteners

S_p = spacing of the joists

L = joist span

C_N = Modification factor as outlined in section 4.1 which also represents the model error

The random variables in Equation 5.2 are R_1 and L_1 which are respectively the joint strength and total load effect normalized by the codified design load. The statistical parameters (mean and coefficient of variation) for both R_1 and L_1 are estimated using a *first-order* approximation of the Taylor series expansion of the above expressions. Accordingly, the mean, m_{R_1} of R_1 is obtained as:

$$m_{R_1} = \frac{\bar{C}_N n_F \sqrt{2} \sqrt{2 \bar{f}_e \bar{M}_y}}{(\alpha_D D_n + \alpha_Q Q_n)} \quad (5.3)$$

and the coefficient of variation, δ_{R_1} , assuming the variables C_N , f_e and M_y are statistically independent as

$$\delta_{R_1} = \frac{1}{m_{R_1}} \sqrt{\left[\text{Var}(C_N) \left(\frac{\partial R_1}{\partial C_N} \right)^2 + \text{Var}(f_e) \left(\frac{\partial R_1}{\partial f_e} \right)^2 + \text{Var}(M_y) \left(\frac{\partial R_1}{\partial M_y} \right)^2 \right]} \quad (5.4)$$

where

$$\begin{aligned} \frac{\partial R_1}{\partial C_N}, \frac{\partial R_1}{\partial f_e}, \frac{\partial R_1}{\partial M_y} &= \text{partial derivatives evaluated at} \\ &\text{mean values } \bar{C}_N, \bar{f}_e \text{ and } \bar{M}_y \\ \text{Var}(X) &= \text{variance of } X \end{aligned}$$

The parameters for L_1 are similarly obtained. Based on the results obtained for the perpendicular to grain tests, the correction factor C_N at ultimate joint load is seen to vary between 1.0 and 0.84. In general, this factor tends toward values less than unity as the number of fasteners increases. Thus, C_N may be modelled by the lower triangular distribution, and its parameters obtained accordingly. It is seen from Equations 5.3 and 5.4 that the variabilities and uncertainties inherent in estimation of f_e , M_y and C_N are incorporated in the estimation of the statistical parameters for R_1 .

The pertinent statistical data required for the estimation of the parameters for R_1 and L_1 are as summarised in Table 5.1. The estimated safety index β_1 , for various joist spans and spacings, and for residential and office occupancy types is given in Figure 5.1.

5.2.2 Failure of Joist in Bearing

Joist hangers should have adequate bearing area to support the anticipated joist reaction. Ordinarily, supplemental hanger side nailing helps reduce the reaction load on the hanger seat. However, this effect has not been considered in the analysis of this component reliability. The joist fails in bearing if the bearing stresses at the hanger seat due to the imposed load exceeds the compression perpendicular to grain strength of the joist. The corresponding performance function could be formulated thus as

$$G_2(X) = R_2 - L_2 \quad (5.5)$$

where

$$R_2 = \frac{R}{(\alpha_D D_n + \alpha_Q Q_n)}$$

Table 5.1 Statistical Parameters for Random Variables in Equation 5.1

Variable	Mean	Coefficient of Variation
C_N	0.899	0.040
f_e	95.3 N/mm	0.041
M_y	4313.9 N mm	0.133
$d = \frac{p}{D_n} \uparrow$	1.00	0.10
$q = \frac{Q}{Q_n} \uparrow (Q_n = 1.9 \text{ kN/m}^2 \uparrow)$	0.812	0.272
$(Q_n = 2.4 \text{ kN/m}^2 \uparrow)$	0.925	0.236

$\uparrow$ Occupancy load parameters were obtained from Foschi *et al.* [28].

$\uparrow Q_n = 1.9 \text{ kN/m}^2$ corresponds to residential occupancy type and
 $Q_n = 2.4 \text{ kN/m}^2$ office occupancy.

for no supplemental side nailing, or

$$R_2 = \frac{R + P_{nail}/A_b}{(\alpha_D D_n + \alpha_Q Q_n)}$$

for supplemental side nailing,

and

$$L_2 = \frac{(D + Q)S_p L}{2A_b(\alpha_D D_n + \alpha_Q Q_n)}$$

where R is actual joist bearing strength, A_b is the area of the joist hanger seat and P_{nail} the load reduction due to side nailing.

The statistical parameters for R_2 are obtained from *in-grade* test results reported by Madsen, Hooley and Hall [45] (1982). The mean of R_2 , as given in Ref. [45], is 3.66 with coefficient of variation 25.7%. The statistics on q and d are as provided in Table 5.1. Figure 5.2 shows the effect on the safety index β of the stresses resulting from changes in joist span and spacing.

5.2.3 Failure Modes of Hanger Steel

This section outlines the salient failure modes of the joist hanger steel material

which have decisive influence on the overall joint performance. In this regard, the shear, tensile, bending and bearing strength properties of the hanger steel are of paramount importance. However, in the absence of any standard tests performed under this thesis to determine these properties, the shear, bending and bearing strengths are respectively assumed as 0.55, 1.00 and 2.0 of the tensile yield strength, σ_y . It is worthwhile pointing out that joist hangers are commonly fabricated from scrap metals of varying degree of imperfection, thus the uncertainties associated with the estimated tensile yield strength.

5.2.3.1 Bearing Failure at Positions for Fasteners in the Hanger Flanges

Bearing stresses are generated at the points of contact of the nails with the plate during the process of load transfer. With the assumption that the respective bearing strengths of the portions of the hanger plate within the influence zone of the bearing stresses are perfectly correlated and that each one of these areas is actively providing resistance to these stresses, failure of the connection in this mode is realized if the bearing strength at any of these areas is exceeded. Therefore, under the assumption of equal load sharing among the fasteners, the performance function G_3 describing the above behaviour is presented as

$$G_3(X) = R_3 - L_3 \quad (5.6)$$

in which

$$R_3 = \frac{2.0\sigma_y}{(\alpha_D D_n + \alpha_Q Q_n)}$$

and

$$L_3 = \frac{(D + Q)S_p L}{20td(\alpha_D D_n + \alpha_Q Q_n)}$$

5.2.3.2 Tensile Failure of Joist Hanger Flanges

Tensile stresses set up in the hanger plate should not exceed the tensile yield strength σ_y of the plate in order to ensure high performance of the joist hanger connection. However, such stresses are often difficult to estimate owing partially to

the complexity of the stress distribution within the plate and in part the geometrical discontinuities of the hanger plate. To circumvent this difficulty, therefore, the finite element analysis as described in section 4.4 has been employed to obtain the critical tensile stresses in the hanger plate for various load levels defined by the configurations of the wood-floor.

It is observed that tensile failure of the joist hanger plate could occur in one of the two flanges providing direct shear connection with the header. However, assumption is made that each flange actively supports the joint load and that their tensile strengths are perfectly correlated. Therefore, the failure of one flange automatically leads to the failure of the other. Accordingly, the component failure probability in this mode is given as the failure of one flange. The corresponding performance function, therefore, could be stated as

$$G_4(X) = R_4 - L_4 \quad (5.7)$$

where

$$R_4 = \frac{\sigma_y}{(\alpha_D D_n + \alpha_Q Q_n)}$$

and

$$L_4 = \frac{\sigma_{critical}^*}{(\alpha_D D_n + \alpha_Q Q_n)}$$

The statistical parameters for the random variable R_4 are obtained from Table 3.2. Assuming further that the finite element modelling of the tensile stress behaviour in the hanger plate is 'correct', the critical tensile stress, $\sigma_{critical}^*$, is obtained for each load level, $P = (D + Q)S_p L/4$.

5.2.3.3 Shear Failure of Joist Hanger Flanges

The performance of the joist hanger in terms of the shear stresses developed in the joist hanger could be defined by the relationship

$$G_5(X) = r_5 - L_5 \quad (5.8)$$

in which

$$R_5 = \frac{0.55\sigma_y}{(\alpha_D D_n + \alpha_Q Q_n)}$$

and

$$L_5 = \frac{\tau_{critical}}{(\alpha_D D_n + \alpha_Q Q_n)}$$

These stresses largely result from the tendency of the nail fasteners to shear through the hanger plate. Like the tensile stresses, the shear stresses are difficult to estimate due to the erratic manner of their distribution within the plate. Therefore, the finite element method has been used in estimating the critical shear stress, $\tau_{critical}^*$

Considering the isotropic nature of the hanger plate coupled with the symmetry of the connection, it is assumed, as before, that the shear resistance is uniform for all portions of the plate which are assumed as perfectly correlated. In addition, if all portions of the plate are considered active in resisting the shearing action of the plate due to the applied load, the component failure probability could only be defined by the probability of excessively shearing any portion of the plate, and hence Eqn. 5.8.

5.2.4 Bending Failure at Joist Hanger Seat

The stress behaviour of the hanger seat as discussed in section 4.6.2 can be described by the following performance function:

$$G_6(X) = R_6 - L_6 \quad (5.9)$$

in which

$$R_6 = \frac{\sigma_y t^2}{6(\alpha_D D_n + \alpha_Q Q_n)}$$

for no supplementary side nailing, or

$$R_6 = \frac{\frac{1}{6}\sigma_y t^2 + w_h P_{nail}/(8b_h)}{(\alpha_D D_n + \alpha_Q Q_n)}$$

for supplemental side nailing, and

$$L_6 = \frac{(D + Q)S_p L w_h}{16b_h(\alpha_D D_n + \alpha_Q Q_n)}$$

and where

P_{nail} = Load carried by the side nails

t = thickness of the joist hanger plate

w_h = width of hanger seat

b_h = bearing length of hanger seat

Figures 5.3 through 5.8 show the reliabilities of the strength properties of the hanger plate for various configurations of the joist. From the results of the component reliabilities (Fig. 5.1 to 5.8) it is evident that the performance or the reliability of joist hanger as a system is influenced by

- the strength of the connection of the hanger to the header,
- the compression perpendicular to grain strength of the joist, and
- the bending strength of the joist hanger material. Thus in presenting limit state design equations for joist hanger, the 'Strength Limit State' has been formulated based on the above considerations.

5.3 Calibration of Joist Hanger Design Equations

Joist Hanger connections must satisfy both the *strength* limit state and the *serviceability* limit state criteria. To comply with acceptable standard of performance the limit state equations have been calibrated at the reliability index of $\beta = 2.5$. The Data Set A obtained in accordance with the ASTM/ICBO standard for joist hanger vertical-capacity rating has in this study been used as the design base and pertinent modifications have been proposed for other design situations. As previously pointed out, in this study the emphasis has been on joist hanger connections as employed in wood-floor systems.

According to the Canadian Standard Association Code (CAN3-086.1-M84) [14], the strength limit state of the joist hanger connection, based on the joint ultimate strength could be formulated as follows:

(a) *factored strength resistance of the hanger joint*

$$P_r = \phi_1 P_o K_{SF} K_D K_T K_H \quad (5.10)$$

where

P_o = characteristic strength obtained as the 5-th percentile
of the ultimate loads distribution for the 10 tests (Data Set A)

ϕ_1 = performance factor applied to P_o

K_{SF} = service condition factor which is assumed equal to 1 in this study

K_D = load duration factor

K_T = wood treatment factor also assumed equal to 1

(b) *factored bearing strength of wood joist*

$$F_r = \phi_2 F_{cp} A_b K_D K_{Scp} K_T K_H K_B + \phi_3 N_u n_F (K_D K_{SF} K_T) (J_C J_Y J_P) J_A \quad (5.11)$$

(c) *factored bending resistance of hanger*

$$T_r = \phi_4 T_o + \phi_3 N_u n_F (K_D K_{SF} K_T) (J_C J_Y J_P) J_A \quad (5.12)$$

where

T_o = the characteristic strength which is given as $\frac{4}{3} \sigma_{y(0.05)} t^2 \frac{b_h}{w_h}$

$\sigma_{y(0.005)}$ = the 5-th percentile value of the tensile yield strength distribution

ϕ_4 = performance factor on T_o

ϕ_2, ϕ_3 = 0.75 and 0.70; performance factors applied to F_{cp} and N_u , respectively

F_{cp} = specified strength in compression perpendicular to grain [Ref. 14]

K_B = bearing factor [Ref. 14]

N_u = lateral strength resistance per nail [Ref. 14]

J_C = nail configuration factor [Ref. 14]

J_Y = 1.25, steel side plate factor

J_A = factor for toe nailing [Ref. 14]

J_P = factor for nail penetration [Ref. 14]

The serviceability limit state is formulated by considering, in general, the joint load corresponding to 3 mm vertical gross displacement (ICBOES [32]). On this basis, therefore, the factored joint resistance based on Data Set A is determined as:

$$N_r = \phi_{5A} N_{(0.05)A} K_{SF} \quad (5.13)$$

in which

$N_{(0.05)A}$ = characteristic strength obtained as the 5-th percentile
of the joint load at 3 mm displacement

ϕ_{5A} = performance factor applied on $N_{(0.05)A}$

The corresponding serviceability limit state equation for Data Set B is similarly formulated.

The effect of end rotation of floor joists on the performance of the joist hanger connection has been known to vary with the wood species, grade, span, and depth of wood section, and also the loading pattern on the joist [28]. Based on the results of bending tests of lumber performed at different lengths, Madsen and Buchanan [44] (1986) had used the Weibull's weakest link theory to propose an adjusting factor which accounts for size effects in lumber. To further account for volume dependence which is not accounted for by the Weibull's theory, Foschi *et al* [28] had modified Madsen and Buchanan's proposal as

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{H_2 L_2}{H_1 L_1} \right)^{1/k} \quad (5.14)$$

where σ_1 and σ_2 are the bending strengths respectively of the joist spans L_1 and L_2 , and k size effect parameter.

The above limit state equation (Eqn. 5.13) implies that, at the same reliability level, there would be a multitude of design ϕ_5 factors to cater for species type, grades, lumber sizes, load situations and for various gauges of the joist hanger steel plate. However, the objective of this calibration process is to obtain one value for ϕ_5 so that the resulting variability in β for any design condition is minimal. The approach here recommended for the calibration process, therefore, is to implement a

functional relationship so that for any gauge and type of joist hanger, wood species, grade, and recommended joist sizes for the joist hanger, a common characteristic strength N_o could be found with only significant adjustment in the estimation of the joint capacity made in respect of the variations in joist span. This adjustment factor could then be referenced to a standardized joist member having a depth H_o and length L_o .

A codified limit state serviceability equation based on the foregoing therefore, could be formulated as

$$N_r = \phi_5 N_o K_{SF} \left(\frac{H_o L_o}{H L} \right)^{1/k} \quad (5.15)$$

where

H, L = depth and length respectively of any joist specified for the joist hanger

To incorporate the effect of wood species and grade type, dimensions, loading pattern, and the gauge of the joist hanger, the calibration of the factor ϕ_5 , the characteristic strengths N_o , and the parameter k could be done by performing least square analysis on relevant test data depicting the above effects. Thus, following after Foschi *et al* [28], this necessitates the minimization of the function

$$F = \sum_i \sum_j \sum_l \left\{ \phi_{ijl} N_{(0.05)ij} - \phi_5 N_o \left(\frac{H_o L_o}{H_j L_j} \right)^{1/k} \right\}^2 \quad (5.16)$$

in which

$i = 1, \dots, m + n$ (total of n species and m grades);

$j = 1, \dots, u$ (total of u joist sizes);

$l = 1, \dots, v$ (total of v loading conditions).

In Eqn. 5.16, the product $\phi N_{(0.05)}$, for a given reliability level, is obtained from consideration of several combinations of hanger gauges and types, wood species, sizes, grades, spans and load conditions with the individual performance factors calibrated using Eqn 5.13.

5.3.1 Calibration of ϕ_1

The limit state design equation depicting the capacity of the joist hanger is of the format

$$\frac{S_p L}{2}(\alpha_D D_n + \alpha_Q Q_n) \leq \phi_1 P_o K^* \quad (5.16)$$

where $K^* = K_{SF} K_D K_T K_H$

The performance function, $G(X)$, corresponding to the above can be written as

$$G(X) = R - \frac{S_p L}{2}(D + Q) \quad (5.17)$$

Equation 5.16 could be re-arranged as

$$Q_n = \frac{2}{S_p L} \left(\frac{\phi_1 P_o K^*}{\alpha_D \gamma + \alpha_Q} \right) \quad (5.18)$$

in which $\gamma = \frac{D_n}{Q_n}$

Also, Eqn. (5.17) could be re-written as

$$G = R - \frac{S_p L}{2} Q_n \left(\frac{D}{D_n} \gamma + \frac{Q}{Q_n} \right) \quad (5.19)$$

Combining Equations (5.18) and (5.19), therefore,

$$G = R - \frac{\phi_1 P_o K^*}{(\alpha_D \gamma + \alpha_Q)} (d\gamma + q) \quad (5.20)$$

where

d, q = dead and live loads normalized with respect
to their corresponding design values

The statistical information on the random variables R , d and q are used to calibrate ϕ_1 . The statistical data on d and q are provided in Table 5.1. In this analysis, d is modelled as a normal distribution, q Extreme Type I (Gumbel) distribution [Foschi *et al* [28] (1989)], and R by the lognormal distribution. The assumed statistical distribution for R was verified by the *Kolmogorov-Smirnov* goodness of fit test on Data Set A. Table 5.2 and Fig. 5.9 show the $\phi - \beta$ relationships for two occupancy loads $Q_n = 1.9 \text{ kN/m}^2$ and 2.4 kN/m^2 .

Table 5.2 ϕ, β -Values for Joint Ultimate Load

ϕ	β -Values	
	$Q_n = 1.9 \text{ kN/m}^2$	$Q_n = 2.4 \text{ kN/m}^2$
0.4	6.79	6.89
0.5	5.76	5.77
0.6	4.96	4.93
0.7	4.26	4.18
0.8	3.68	3.54
0.9	3.15	2.98
1.0	2.68	2.45
1.1	2.22	1.98
1.2	1.81	1.52
1.3	1.43	1.11
1.4	1.06	0.72

5.3.2 Calibration of ϕ_4

The design format depicting the bending strength of the joist hanger steel could also be stated as

$$\frac{S_p L}{2} (\alpha_D D_n + \alpha_Q Q_n) \leq \phi_4 T_0 + N_u \quad (5.21)$$

where

$$N_u = \phi_3 N_u n_F (K_D K_{SF} K_T) (J_C J_Y J_P) J_A$$

and

$$T_0 = \frac{4}{3} f_{y(0.05)} t^2 \frac{b_h}{w_h}$$

Following after the derivation carried out in section 5.2.4, the performance function, $G(X)$, corresponding to the above could be formulated as

$$G(X) = R - \left(\frac{\phi_4 \frac{4}{3} f_{y(0.05)} t^2 \frac{b_h}{w_h} + N_u}{\alpha_D D_n + \alpha_Q Q_n} \right) (d\gamma + q) \quad (5.22)$$

The problem, as formulated herein, involves three random variables: R , d and q . The distribution of R was based on the results of the tensile tests of coupons

prepared from samples of joist hanger (section 3.3.1). Statistics on the load variables d and q are as given in Table 5.1. The $\phi - \beta$ relationships based on Equation 5.21 are shown in both Table 5.3 and Figs. 5.10 for supplementary hanger side nailing employing 2No. and 4No. 3.25mm common wire nails respectively. It is worthwhile pointing out that Eqn. 5.20 is satisfied only where supplemental hanger side nailing is employed to reduce the incoming load on the hanger seat.

Table 5.3 ϕ, β -Values for Hanger Bending Strength

ϕ	β -Values			
	Supplemental Side Nailing (2No. 3.25mm Common Nails)		Supplemental Side Nailing (4No. 3.25mm Common Nails)	
	$Q_n = 1.9 \text{ kN/m}^2$	$Q_n = 2.4 \text{ kN/m}^2$	$Q_n = 1.9 \text{ kN/m}^2$	$Q_n = 2.4 \text{ kN/m}^2$
0.4	6.25	6.06	6.43	6.19
0.5	5.35	5.11	5.46	5.19
0.6	4.54	4.34	4.67	4.38
0.7	3.97	3.73	4.01	3.78
0.8	3.43	3.18	3.47	3.22
0.9	2.94	2.69	2.98	2.72
1.0	2.51	2.23	2.54	2.27
1.1	2.10	1.81	2.12	1.85
1.2	1.71	1.40	1.73	1.43
1.3	1.32	1.00	1.34	1.02
1.4	0.95	0.60	0.96	0.61

5.3.3 Calibration of ϕ_5 and k

The serviceability limit state equation of the joist hanger connection supporting joist of depth H and span L is of the format

$$\frac{S_p L}{2} (\alpha_D D_n + \alpha_Q Q_n) \leq \phi_5 N_o K_{SF} \left(\frac{H_o L_o}{H L} \right)^{1/k} \quad (5.23)$$

The performance function corresponding to Eqn. 5.23 could therefore be written

as

$$G(X) = R - \left[\frac{\phi_5 N_o K_{SF} \left(\frac{H_o L_o}{H L} \right)^{1/k}}{\alpha_D D_n + \alpha_Q Q_n} \right] (d\gamma + q) \quad (5.24)$$

Based on Eqn 5.13, the performance function pertaining to Data set A ($H = 235 \text{ mm}$, $L = 725 \text{ mm}$) is

$$G(X) = R_A - \left(\frac{\phi_{5A} N_{(0.05)A} K_{SF}}{\alpha_D D_n + \alpha_Q Q_n} \right) (d\gamma + q) \quad (5.25)$$

and for Data Set B ($H_1 = 235 \text{ mm}$, $L_1 = 3657.6 \text{ mm}$)

where

$N_{(0.05)B}$ =non-parametric 5-th percentile value obtained from Data

ϕ_{5B} =performance factor associated with $N_{(0.05)B}$

In Eqn. 5.25 the random variables are identified as R_A , d and q , and using the Data Set A the statistical parameters for R_A are obtained. Furthermore, with the assumption of lognormal distribution for R_A , ϕ_{5A} is calibrated. The result of this analysis is given in Table 5.4 and Figure 11(a). The calibration of ϕ_{5B} also follows the same approach as ϕ_{5A} , using the statistical data on R_B obtained from Data Set B. Table 5.4 and Figure 11(b) also give the results for ϕ_{5B} , β -values for this calibration.

The calibration of ϕ_5 , N_o and the parameter k has been carried out using as the standardized member defined in Eqn 5.16, the $38 \times 235 \text{ mm}$ No.2 SPF tested under Data Set A. With the Table 5.4, the non-parametric 5-th percentiles obtained from both Data Set A and Data Set B, and setting $m + n = 1$ for the No.2 SPF, $u = 2$ for the two joist spans, and $v = 2$ corresponding to residential and office occupancy types, Eqn. 5.16 becomes

$$F = \sum_{j=1}^2 \sum_{l=1}^2 \left\{ \phi_{1jl} N_{(0.05)1j} - \phi_5 N_o \left(\frac{H_o L_o}{H_j L_j} \right)^{1/k} \right\}^2 \quad (5.26)$$

. The least square minimization of the above function has been performed using the program called BFGS (Pike R. W. [63]) which uses the Broyden, Fletcher, Goldfarb,

Table 5.4 ϕ, β -Values for Serviceability Limit State

β-Values				
ϕ	Data Set A		Data Set B	
	$Q_n = 1.9 \text{ kN/m}^2$	$Q_n = 2.4 \text{ kN/m}^2$	$Q_n = 1.9 \text{ kN/m}^2$	$Q_n = 2.4 \text{ kN/m}^2$
0.4	6.50	6.27	6.37	6.13
0.5	5.59	5.37	5.47	5.24
0.6	4.87	4.63	4.71	4.48
0.7	4.29	4.06	4.13	3.89
0.8	3.77	3.54	3.62	3.38
0.9	3.32	3.09	3.15	2.91
1.0	2.91	2.66	2.74	2.48
1.1	2.53	2.27	2.35	2.08
1.2	2.17	1.90	1.98	1.70
1.3	1.83	1.55	1.63	1.34
1.4	1.51	1.20	1.29	0.97

and Shanno algorithm, at reliability level, $\beta = 2.5$. The characteristic strength N_o is obtained as 10677 N, $\phi_5 = 1.1$ and $k = 2.17$.

5.4 Discussion on the Proposed Joist Hanger Design Equations

The results show that at the same reliability level $\beta = 2.5$, the mean performance factors ϕ_1 , ϕ_4 , and ϕ_5 , corresponding to the design equations which describe the ultimate joint capacity, the hanger plate bending behaviour and the gross joint deformation is respectively given as 1.01, 0.98 and 1.05. These values have been calibrated based on tests results on one species, grade, and a specified gauge material of the joist hanger. In a more general approach, therefore, the factors ϕ_1 and ϕ_4 could be obtained by considering the results obtained from several combinations of wood species and grades, lumber sizes, spans, and other gauges of the hanger material. In conformity with the Canadian Code CSA [CAN3-086.1-M84] [14], however, it is recommended that ϕ_2 and ϕ_3 appearing respectively in the Design Equation

5.11 for the factored bearing resistance and the factored lateral strength resistance of the nail be maintained at 0.75 and 0.70, respectively.

In calibrating the serviceability limit state equation, the factor ϕ_5 , the characteristic strength N_o and the size effect parameter k has been based on Data Set A and Data Set B only, representing one particular gauge of hanger and wood species. As pointed out earlier, the gross vertical displacement of the joist hanger connection is influenced by both the size and span of the joist, the thickness (gauge) of the hanger plate, the species and grade of both the joist and the header material and the loading pattern. Therefore, in a much more codified formulation of the serviceability limit state equation, the calibration of ϕ_5 , N_o and k can be performed by considering, similar to the calibration procedures for ϕ_1 and ϕ_4 , several combinations of wood species and grades, hanger thicknesses, joist spans and sizes.

Fig. 5.1 Safety Index, β vs. Joist Span, L . Capacity of Nailed Joist Hanger-Header Connection.

Fig. 5.2 Safety Index, β vs. Joist Span, L . Wood Joist Perpendicular to Grain Strength. (No Supplemental Flange Side Nailing)

Fig. 5.3 Safety Index, β vs. Joist Span, L , Shear Strength of Joist Hanger Steel Plate

Fig. 5.4 Safety Index, β vs. Joist Span, L . Bearing Strength of Joist Hanger Steel Plate

Fig. 5.5 Safety Index, β vs. Joist Span, L . Joist Hanger Plate Tensile Strength

Fig. 5.6 Safety Index, β vs. Joist Span, L . Bending Strength of Joist Hanger Steel Plate (No Supplemental Side Nailing).

Fig. 5.7 Safety Index, β vs. Joist Span, L . Bending Strength of Joist Hanger Steel Plate (Supplemental Side Nailing 2No. 3.25mm Diameter Common Wire Nails of Length 65.0mm)

Fig. 5.8 Safety Index, β vs. Joist Span, L . Bending Strength of Joist Hanger Steel Plate (Supplemental Side Nailing 4No. 3.25mm Diameter Common Wire Nails of Length 65.0mm)

Fig. 5.9 Joint Ultimate Capacity: Summary of $\phi - \beta$ Relations for Residential and Office Occupancy Types

(i)

(ii)

Fig. 5.10 Joist Hanger Seat Bending Strength Capacity: Summary of $\phi - \beta$ Relations for Residential and Office Occupancy Types

(a)

(b)

Fig. 5.11 Serviceability Limit State: Summary of $\phi - \beta$ Relations for Residential and Office Occupancy Types

ACCEPTANCE SAMPLING PLAN FOR JOIST HANGERS

6

6.1 General

To ensure the design performance capability of engineering materials, it is essential that they comply with the minimum standard of safety in terms of their fabrication, material quality and workmanship; thus, the need for acceptance criteria and acceptance sampling programmes. In certifying products such as joist hangers, the code committee would like all items to be satisfactory. A more realistic situation would be to expect some defective units, and a more ideal and efficient method of assessment would be to test every unit and sentencing it as defective or satisfactory. However, this approach appears impractical as the cost involved is prohibitively high. More still, damage would be incurred during testing even at relatively low load levels. Hence, the use of acceptance sampling plans.

In most engineering practices, sampling plan may often be 'single' or 'double'. In single sampling plan, a batch is considered as being accepted or rejected if the number of defective r found in a sample of n items is less than or equal to the acceptance number, c . Double sampling plan involves drawing a first sample of n_1 items as a result of which one may either accept the batch, reject it, or else take a further sample of n_2 items. If the second sample is taken, a decision to accept or reject the batch is taken upon the combined results. The approach used in this study in formulating sampling plan for joist hangers, therefore, encompasses both the reliability methods and decision-making analysis to reduce risk.

6.2 Operation Characteristic (OC) Curves

In testing engineering products for the purpose of certification, the primary result for each trial testing is always a binary one: survival or failure; the probability of failure (or success) being constant for the independent trials. If in n trials the number of observed failures is x , then the probability of acceptance $P(A)$ could be formulated, in accordance with the binomial distribution as

$$P(A) = P(X = x) = \sum_{r=0}^x \binom{n}{r} p_F^r (1 - p_F)^{n-r} \quad (6.1)$$

where p_F is the probability of failure based on the design load. The graphical representation of Equation 6.1, called the OC-curve, is given in Figure 6.1(a).

Conceptually, when $p_F = 0$, $x = 0$ and the products are accepted ($P_a = 1$), and also they offer a high degree of assurance in terms of safety. As p_F increases P_a decreases until it is asymptotically zero at $p_F = 1$. Figure 6.1(a) thus provide a means of comparing the effectiveness of any given sampling plan.

In reality, one might wish to have an OC-curve such that for batches with $p_F < p_F^a$ (p_F^a corresponding to a reliability level of $\beta = 2.5$, say) are accepted, and all others rejected, ie.,

$$P(A) = \begin{cases} 1 & p_F < p_F^a \\ 0 & p_F > p_F^a \end{cases}.$$

As shown in Fig. 6.1(b), the OC-curve representing the above derivation is impossible to achieve without 100% inspection.

In the discussions so far the inherent error associated with testing and the possibility of the sample not truly representing the entire batch system has not been considered. Practically, errors are unavoidable when decisions are based on the results of random samples. To judge the merits of any decision criterion therefore, it is essential that the probabilities associated with *erroneously rejecting*, called Type I error, or *erroneously accepting* (Type II error) a claim or assumption be known. The probabilities associated with these errors could be stated as

$$\alpha = P(\text{Type I error})$$

$$\beta^* = P(\text{Type II error})$$

The choice of both α and β^* is usually based on the consequences (or risk) involved in committing these errors. However, the assignment of these values has often been subjective. In this study however, α and β^* have been taken as 0.05 and 0.1, respectively. Therefore, any sampling plan must recognize and maximize the chances of accepting high quality samples, ie., $P(A) = 1 - \alpha$ (defined at the Acceptance Quality Level (AQL)), and should also clearly define the lower limit of quality, called the Rejection Quality Level (RQL), such that $P(A) = \beta^*$.

6.3 Acceptance Criteria for Joist Hangers

To device a sampling plan for the certification of joist hangers, it requires that the number of joist hangers (or pairs of joist hangers) to be tested and the associated probability of failure based on the manufacturer's recommended design load be known. As already pointed out, in order to obtain a high probability of acceptance, and at the same time ensuring high degree of safety in terms of performance, one has to test a rather large sample size. Not only is this approach less cost-effective, also it does not lend itself readily to assess each item sampled on its own merit. The methodology, thus adopted in this thesis is the *sampling by variables* in which the values of the variable for each specimen in a chosen sample are measured. The key advantage here is that because the measured data are more fully utilized the sample size required to achieve the same degree of risk (or acceptance) as using *sampling by attributes* is significantly smaller.

6.3.1 Formulation of Sampling Plan for Joist Hangers

In order to derive a sampling plan for joist hangers, a prior knowledge of the following four characteristics of the plan is essential: the probabilities of both Types I and II errors, the Acceptable Quality Level (AQL) and the Rejection Quality Level (RQL). Therefore, such a sampling plan must aim at achieving the following:

- The error of erroneously rejecting high strength joist hangers is minimized.

- Sampling plan must maximize the chances of rejecting samples with mean strengths greater than the *acceptance limit*, L_s , which in actual fact are from population of inferior strength.

In subsequent sections of this chapter, the following assumptions have been made:

- In lots of joist hangers their average ultimate strengths—normally obtained through continuous sampling—is modelled by the normal distribution (Duncan A. J. [22] (1986)).
- For any one lot, the quality variability is unknown. This is, however, estimated on the basis of the manufacturer's supplied data on the mean ultimate strength.
- The strength of the joist hangers forming a test pair are statistically equivalent.

If in accordance with the specifications provided by a manufacturer of joist hangers the average ultimate load per joist hanger, for a particular model, tested under the ASTM/ICBO recommended test guidelines is μ_{max} , then this value may be taken as equivalent to the maximum possible reliability level afforded by the lot of joist hangers. On the other hand if a decision is taken that a lot is to be rejected at quality level of μ_{min} which is equivalent to a reliability level of 2.5 based on the maximum recommended design load, μ_d , then both L_s and n could be computed such that Types I and II errors are optimized. Therefore, for a sample consisting of n hanger pairs with average ultimate strength, $\bar{X}$, and coefficient of variation, V_x , to limit Type I error to α , we should have,

$$P(\bar{X} < L_s | \mu_{max}) = \Phi \left(\frac{L_s - 2\mu_{max}}{\sigma/\sqrt{n}} \right) = \alpha \quad (6.2)$$

and to limit Type II error to β^* ,

$$P(\bar{X} > L_s | \mu_{min}) = 1 - \Phi \left(\frac{L_s - 2\mu_{min}}{\sigma/\sqrt{n}} \right) = \beta^* \quad (6.3)$$

from which,

$$n = \left\{ \frac{\sigma[\Phi^{-1}(1 - \beta^*) - \Phi^{-1}(\alpha)]}{2(\mu_{max} - \mu_{min})} \right\}^2 \quad (6.4)$$

and,

$$L_s = \frac{2\mu_{min}\Phi^{-1}(\alpha) - 2\mu_{max}\Phi^{-1}(1 - \beta^*)}{\Phi^{-1}(\alpha) - \Phi^{-1}(1 - \beta^*)} \quad (6.5)$$

If for the purpose of codification the Rejection quality Level is chosen at the reliability index of $\beta = 2.5$ of the design load, then using Equation 5.1 μ_{min} , is computed as

$$\mu_{min} = \mu_d e^{\beta \sqrt{V_p^2 + V_D^2 + V_Q^2}} \quad (6.6)$$

where V_p is the coefficient of variation of the lot mean strength (or a lot's quality variability), and values of V_D and V_Q are as given in Table 5.1.

The sampling plan for joist hangers as given by Equations 6.4 and 6.5, suggest that the standard deviation of any one lot is known. In most practical situations, however, this is unknown, or at least for the purpose of certification this value needs verification even if it is provided. To circumvent this difficulty, however, the sampling plan should be formulated using reliability methods in order to accommodate the inherent quality variabilities. Therefore if the coefficient of variation of the ultimate strengths, V_p , for the lots is appropriately selected, then the sample size n and L_s are unique parameters depending on V_p and in this wise n and L_s are obtained from Eqns. 6.4 and 6.5 as if the standard deviation of the lots was known. In so doing a stipulation is also placed on the coefficient of variation of the sample mean strength obtained under a sampling plan.

A sampling plan could then be devised as: choose a random sample of n joist hanger pairs and using the ASTM/ICBO test procedure obtain the sample mean, $\bar{X}$ and coefficient of variation, V_x . The joist hanger lot is accepted if $\bar{X} \geq L_s$. It is to be pointed out that in order to control the variability of the sample material, a limit should also be set on V_x . In compliance with the above formulation, therefore, V_x , should not exceed VP , ie, $V_x \leq V_p$.

The above acceptance criteria could also be obtained using reliability method. Thus, on the basis of the foregoing discussion, the reliability index of the acceptance limit strength based on the manufacturer's maximum recommended design load, μ_d ,

could be stated as

$$\beta_s = \frac{\ln(L_s/2\mu_d)}{\sqrt{V_p^2 + V_D^2 + V_Q^2}} \quad (6.7)$$

and that of the sample as

$$\beta_d = \frac{\ln(\bar{X}/2\mu_d)}{\sqrt{V_x^2 + V_D^2 + V_Q^2}} \quad (6.8)$$

The lot is then accepted if $\beta_d \geq \beta_s$, otherwise it is rejected as rated.

6.3.2 Probability of Acceptance of Joist Hangers

As the parameters L_s and n of the proposed sampling plan is dependent on the specified variability level of the lots' ultimate strength distribution, V_p , the corresponding OC curve would in general be unique. Following after Duncan A. J. [22] (1986), the probability of acceptance of a lot based on sample average, $\bar{X}$, and coefficient of variation, V_x could be stated as

$$P(A) = P(\beta_d \geq \beta_s) = \Phi \left[\frac{2\mu_d e^{\beta_d \sqrt{V_x^2 + V_Q^2 + V_D^2}} - L_s}{2V_p \mu_{max} / \sqrt{n}} \right] \quad (6.9)$$

where $V_x \leq 0.2$. The OC curve for one particular joist hanger model is illustrated in Appendix A.

6.4 Discussion

An attempt has been made to formulate acceptance sampling plans for joist hangers using simple decision theory and considerations of structural safety. The sampling plan has been formulated on the basis of assumed sampling characteristics such as the Rejection Quality Level (RQL) and the probabilities of Types I and II errors. For codification purposes a code committee would have to provide data on these parameters. Also, for the purpose of providing unified OC curves for joist hangers, the code committee could specify safety indices corresponding to both AQL and RQL; then, employing tentative design loads estimated on the basis of the

wood joist configurations as provided in the ACNBC Span Tables for Wood Joist , Rafters, Trusses and Beams (1977) [5], OC curves could be constructed relating probability of acceptance to sample safety index on the design load for various standard sizes of wood joists.

The OC curves on the sampling plan as proposed could be used by joist hanger manufacturers as well to specify the maximum joist size, hence the design load, for any joist hanger model once the mean ultimate strength is found from testing and the probability of acceptance specified.

The ICBO Evaluation Service, Inc. (ICBOES) [32] has proposed acceptance criteria for determining the direct load capacity of joist hangers based on minimum of three tests and coefficient of variation of 0.2, and also, on minimum of six tests with unspecified coefficient of variation. It is apparent that the proposed ICBOES sampling plan at the specified coefficient of variation seemingly considers lots' quality variability and can thus be used at lots variability level of 0.2 if consistent AQL and RQL are also specified.

Fig. 6.1 (a) A Typical Operating Characteristic Curve (OC-Curve)
(b) An Ideal Operating Characteristic Curve

SUMMARY, CONCLUSIONS & RECOMMENDATIONS

7

7.1 General

The use of joist hangers in connections of wood-floor systems is on the increase. However, little work has been done to formulate design guidelines necessary to ensure their compliance with acceptable level of safety. Both the strength and serviceability limit states requirements of the joist hanger connection have recently become a subject of concern. To date, designers have had to rely on specifications provided by the manufacturers of joist hangers, and therefore have little or insufficient design data to appraise their performance. As also pointed out in the introductory chapter of this thesis, even the little published research work on joist hangers, like other metal framing connectors, is limited in uniformity ostensibly due to the diversity of the metal plate geometries and their proprietary nature.

The focus of this thesis therefore, has been to formulate limit state design equations for joist hangers to satisfy both the strength and serviceability requirements. In order for new joist hanger products to be accepted, this study has also been aimed at devising a simple acceptance sampling plan based on safety and risk considerations.

A review of the literature showed that though the joist hanger connection functions similarly as the nailed joint, the former, however, often involves the interplay of other joint components behaviour quite different from the nailed connection.

A summary of the important phases of the work performed under this thesis and the relevant conclusions drawn therefrom are given in the following sections.

7.2 Summary and Conclusions

In order to better assess the performance of the joist hanger, the preliminary work done under this thesis has been aimed at understanding the behaviour of the components in the joist hanger connection. Among these included the group performance of the nailed joint, especially of the behaviour of nails loading the wood fibres in the perpendicular to grain direction. This has been studied in view of the increasing number of joist hanger models employing nails as fasteners to effect the transfer of the joint load to the header.

Using the specifications of the ASTM-D 1796 and the ICBO Evaluation Service, Inc. test guidelines for joist hangers, the vertical load carrying capacity of the joist hanger has been evaluated. To further examine the influence of end-rotation on the stiffness of the joist hanger connection due to the bending action of the joist, a prototype joist hanger connection, using joist span of 3600 mm, has also been tested.

Both the strength and serviceability limits states have been formulated for the joist hanger using the results of the reliability analysis on the components of the connection. Finally, simple acceptance criteria for joist hangers have been devised based on the sampling by variables method.

The conclusions drawn from the above studies can be stated as follows:

- The lateral load carried by a single nail in a multi-nailed joint for loading in the perpendicular to grain direction is not the same in magnitude as that carried by the nail in a single-nailed joint. In this study it is found that Thomas and Malhotra's model could be used to obtain the modification factor, C_N , which accounts for the apparent reduction in strength capacity of the multi-nailed joint.
- The effect of end-rotation due to the bending action of the joist under gravity loads is to cause a reduction in the stiffness of the joist hanger connection. This effect is much more significant for the joist hanger models which employ the

shear action of the nail fasteners to transfer the joint load to the header.

- The performance of the joist hanger connection is essentially determined by the joist-header connection.
- The proposed acceptance sampling plan for joist hangers formulated from considerations of safety and decision theory validates the plan proposed earlier on by the ICBO Evaluation Service Inc. When safety indices are assigned to both the RQL and AQL, at a specified variability level of any given lots of joist hangers, unified OC curves could be constructed relating probability of acceptance to sample safety index. These curves could provide the basis of rating joist hanger products or assist the manufacturer in assigning tentative design loads for his products.

7.3 Recommendations

The study has indicated the need for future research in the following areas:

- The behaviour of the multi-nailed joint for the case of perpendicular to grain loading has only been studied for the No.2 SPF, and in fact the results obtained herein have rather a limited application. There is therefore the need to extend this study to other wood species as a means to acquire the data upon which this behaviour could be incorporated in the future formulation of the nailed joint capacity.
- The formulation of the limit state equations for the joist hanger has been based on the No.2 SPF species and 18 gauge steel material. For a codified acceptance of the equations presented, therefore, more tests have to be performed for the combination of other species, grades, joist size and span as well as other gauges of joist hangers.
- With regards to the acceptance sampling plan for the joist hanger, this has been based on an assumed Gaussian distribution of the mean strengths of the lots. Enough data base on some standard approved joist hangers, therefore, need to

be acquired to give a higher level of confidence in the use of the sampling plan proposed under this study.

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Appendix A

Certification of Joist Hangers

Case Study: LUP210 Joist Hanger

Manufacturer's Specification:

Maximum design load per joist hanger, $\mu_d = 7.120$ kN

Preliminary data:

Acceptance Quality Level (AQL)—Lower Limit $\beta = 3.5$ (assumed)

Rejection Quality Level (RQL)—Upper Limit $\beta = 2.5$ (assumed)

Probability of Type I error, $\alpha = 0.05$

Probability of Type II error, $\beta^* = 0.10$

Sampling plan based on coefficient of variation of the population mean of 0.2, ($V_p = 0.2$)

Calculation of sampling parameters:

Considering sample tests conducted under ASTM/ICBO test procedures, if at the load level equivalent to twice the mean ultimate strength of the joist hanger, the test furnishes a reliability level of $\beta = 3.5$ based on the recommended maximum design load, then μ_{maz} could be estimated as

$$\begin{aligned}\mu_{maz} &= (0.5)(2)\mu_d e^{\beta \sqrt{(V_p)^2 + (V_D)^2 + (V_Q)^2}} \\ &= (0.5)(2)(7.120)e^{(3.5)\sqrt{(0.2)^2 + (0.1)^2 + (0.272)^2}} = 24.417 \text{ kN}\end{aligned}$$

The values for V_D and V_Q are obtained from Table 5.1.

From Equations 6.3, 6.4 and 6.5,

$$\mu_{min} = (0.5)(2)(7.12)e^{(2.5)\sqrt{(0.2)^2 + (0.1)^2 + (0.272)^2}} = 11.170 \text{ kN}$$

$$n = \left\{ \frac{2(24.417)(0.2)[\Phi^{-1}(1 - 0.1) - \Phi^{-1}(0.05)]}{2(24.417 - 17.170)} \right\}^2 = 4$$

$$L_s = \frac{2(17.170)\Phi^{-1}(0.05) - 2(24.417)\Phi^{-1}(1 - 0.1)}{\Phi^{-1}(0.05) - \Phi^{-1}(1 - 0.1)} = 40.688 \text{ kN}$$

The reliability level of the acceptance limit value, L_s , based on the design value is estimated as

$$\beta_s = \frac{\ln\left(\frac{40.688}{(2)(7.12)}\right)}{\sqrt{(0.2)^2 + (0.1)^2 + (0.272)^2}} = 3.0$$

A sampling plan based on the above characteristics is as follows:

1. Test four pairs of randomly selected joist hangers from the lot.
2. Obtain the sample mean, $\bar{X}$, and the coefficient of variation, V_x , and calculate the reliability of the sample using the relation:

$$\beta_d = \frac{\ln\left(\frac{\bar{X}}{(2)(7.12)}\right)}{\sqrt{V_x^2 + (0.1)^2 + (0.272)^2}}$$

3. The lot is accepted if $\beta_d \geq \beta_s$, otherwise it is rejected as rated.

The probability of acceptance of the sampling plan is then

$$P(A) = \Phi \left[\frac{(2)(7.12)e^{\beta_d \sqrt{V_x^2 + (0.1)^2 + (0.272)^2}} - 40.688}{(2)(0.2)(24.417)/\sqrt{4}} \right]$$

and the corresponding OC curve for $V_x = 0.2$ is given in Fig. A.1.

Fig. A.1 Operating Characteristic Curve for LUP210 Joist Hanger Model

Fig. A.2 Test Joist Hanger Detailing: (a) General Arrangement (b) Front Elevation (c) Side Elevation (d) Plan

END

1 2 0 4 9 1

FIN