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EXPERIMENTAL STUDY OF NUCLEAR EXCITATION WITH
NEUTRON EMISSION, INDUCED BY CASCADING MUONS IN
 Au^{197} , Pb^{206} , Pb^{207} , Pb^{208} , Bi^{209} AND NATURAL Tl *

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September, 1975

* A thesis submitted to the Faculty of Graduate Studies in
partial fulfilment of the requirements for the degree of
Doctor of Philosophy.

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Alessandro M. V. Ricci, 1975

"Experimental Study of Nuclear Excitation with Neutron Emission, Induced by Cascading Muons in Au^{197} , Pb^{206} , Pb^{207} , Pb^{208} , Bi^{209} and Natural Tl^* "

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ABSTRACT

The following work is an experimental investigation of the prompt nuclear emission of neutrons, induced by a cascading negative muon after atomic capture in Au^{197} , Pb^{206} , Pb^{207} , Pb^{208} , Bi^{209} and natural Tl.

The purpose of this study is to confirm and improve upon the results of a previous experiment in which the author has taken part and in the course of which such neutrons were first observed in Bi and Pb^{207} , and to extend the investigation to the other elements mentioned.

The experimental improvements consist in the choice of the geometry, in the flight-path lengthening for better energy resolution, in a drastic reduction of the uncorrelated events simulating nuclear muon capture detection, in the partial automation of collection and analysis of the data and in the improved pulse-shape discrimination between photons and neutrons that has allowed a lowering of the neutron-energy threshold.

The probabilities of prompt neutron emission have been measured for the aforementioned elements together with the neutron-energy spectra. No conclusive proof has been obtained for prompt neutron emission in Au^{197} and natural Tl, while the neutron yield per muon cascade for Pb^{206} , Pb^{207} , Pb^{208} and Bi^{209} are respectively $(1.8 \pm .8)\%$, $(5.3 \pm 2.6) \pm 2.0\%$, $(1.84 \pm .87)\%$ and $(5.4 \pm 2.3)\%$ above .85 MeV neutron energy.

The results are not sufficiently accurate to define

univocally the atomic transitions of the muon inducing the neutron ejection from the nucleus.

The energy spectra and the intensities have been compared with the theoretical predictions available to date and are in substantial agreement with the results of the model used by Srinivasan and Sundaresan.

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CHAPTER 1

INTRODUCTION

1.1 Muonic atoms

A muonic atom is formed when a negative muon is captured in the field of an atomic nucleus.^{1,2} Negligible, at least with the present intensities of muon beams, is the probability that more than one muon be captured in the same atom.

So far, when the weak interaction is neglected, the muon has been proved to be fundamentally nothing else than a heavy electron (its rest mass measuring 105.66 MeV, i.e. 207 times the electron's), but even though no other basic differences exist, the difference in mass has far-reaching consequences. Direct consequences of its larger rest mass with respect to the lighter lepton, are for the muon its instability (its free mean life at rest is 2.2 μ s) and an intrinsic magnetic moment 207 times smaller than the electron's.

Like the electron, the muon interacts with the nucleus mainly electromagnetically, and the residual interaction, neglecting gravity, is of the weak type. The binding of the muon to the nucleus is therefore in practice only electromagnetic, and it can be calculated theoretically.

within the frame of quantum electrodynamics, to a high degree of accuracy. Some discrepancy seems to exist between calculated and experimental transition energies,³ but the theoretical attempts⁴ at finding an explanation appear to be premature, since new experimental data do not show the said discrepancy.⁵ In any case this is small and inconsequential within the context of the present study.

An important feature of muonic atoms, which distinguishes them from electronic atoms and results from the smallness of the muon e/m ratio compared to the electron one, is the reduction of the ratios between the radii of the atomic orbits and the nuclear radius. This entails that, for muon states characterized by a sufficiently small principal quantum number (n), an atomic nucleus of sufficiently large mass number (A) cannot any more be adequately represented, for the purpose of calculating atomic quantities, by a point-like mass endowed with proper electric and magnetic moments. When in the lower orbits, the muon spends a non-negligible fraction of its time within the nuclear volume and the finite extension of the nucleus has to be taken into account explicitly. In the case of lead the 1s Bohr orbit of the muon has a radius of 3.12 fm to be compared with a nuclear radius of 7.7 fm. This means that nuclear structure effects, small in electronic atoms, are quite pronounced in muonic atoms. The smaller muon orbits correspond to higher binding energies and to higher transition energies which make a larger number of nuclear excitations energetically

possible.

Since the magnetic moment of the muon is 207 times smaller than that of the electron, the major hyperfine contributions to the energy levels are not due, as they are in the electronic atoms, to the magnetic dipole interaction but to the electric quadrupole one.

For $n \geq 14$ the muon Bohr orbits are smaller than the smallest electron orbit, so that screening effects are negligible and the states are hydrogen-like.

Systematic descriptions of the properties of the muonic atoms have been made by many authors.⁶

1.2 Formation of muonic atoms

Only a very approximate description is available of the ~~process in which a muon, slowed down mainly by ionization to~~ well below 1 keV kinetic energy, is captured by an atom. The process of slowing-down and atomic capture was studied by Fermi and Teller.⁷ The slowing-down time in condensed matter from 100 MeV to 2 keV kinetic energy takes place in a fraction of a nanosecond. The rest of the slowing-down process and the atomic capture are studied considering the muon as moving in a degenerate electron gas with which it collides until it is captured in the field of the nucleus. The time required for the muon to cascade down to the ground state by Auger transitions and radiative transitions was estimated to be of the order of 10^{-13} s for carbon. In addition they studied the relative probability of capture by the atoms of a compound, arriving at what is called the Fermi-Teller Z-law.

According to this law the capture probability is proportional to the atomic number and to the atomic concentration in the compound. The experiments⁸ show that the Fermi-Teller Z-law is not generally valid. The distribution of states into which the muon is initially captured is not known, but the periodicity as a function of Z of the ratio of atomic capture probabilities in oxides⁹ and its dependence on the chemical composition¹⁰ suggest that the muon is captured with a non-negligible probability into very lightly bound levels, some of these being molecular rather than atomic.

1.3 Muon cascade

As soon as the atomic capture has occurred, a cascading process begins whereby the muonic atom makes transitions to more and more strongly bound states. Fermi and Teller estimated an overall cascading time of the order of 10^{-13} s for solids and liquids, which is only a very small fraction of the free muon lifetime. The muon will spend most of its lifetime in the $1s_{1/2}$ orbit until it either decays or is captured by the nucleus.

For muon atomic states characterized by high principal quantum number n, the cascade transitions occur most likely through the emission of Auger electrons mainly from K and L orbits. The probability of muonic Auger transition has been studied by Burbidge and de Borde.¹¹ The dominant transitions are such that

$$\Delta n = -1, \Delta l = \pm 1 \quad (-1 \text{ favoured})$$

but also S-transitions ($\Delta l = 0$) can take place. However, as Backenstoss¹² points out, in the assumption that the Auger effect occurs preferentially with electrons having about the same Bohr orbital radius and best overlap, the minimum Δn is determined by the binding energy of the electrons. This effect imposes large changes for Δn for the higher values of n . This again has the effect of increasing the population of higher l states. For the higher n -values ($n \approx 20$) the contribution of M Auger electrons is not altogether negligible.¹³ It is still uncertain to what extent Auger electrons are available after a number of preceding Auger transitions. It appears possible that, at least in the case of insulators, electrons are lacking.¹⁴

With decreasing n the radiative transitions become more and more probable until they dominate over the Auger transitions. For radiation transitions

$$\Delta n = -1, -2, \dots \text{ (large } n \text{ preferred)}$$

$$\Delta l = \pm 1 \quad (-1 \text{ favoured})$$

The above selection rules together with the condition $0 < l \leq n-1$ produce a preferential filling of the spherical orbits, i.e. the states with maximum angular momentum $l = n-1$. The fact that the probability of radiative transition is proportional to the cube of the transition energy tends to strongly favour transitions with higher n variation. On the other hand the states for which $l = n-1$ can almost exclusively decay by transitions with $\Delta n = \Delta l = -1$ because of the mainly electric-dipole character of the radiation.

The cascade process has been studied by Eisenberg and Kessler¹⁵ who calculated the populations of the various n levels starting with an original population of the $n=14$ levels given by $e^{al} (2l+1)$. ($l=0, \dots, n-1$) and including radiative dipole transitions and Auger transitions. The initial distribution with $a=-0.2$ gave the best fit of the population ratios to the x-ray intensity ratios in light elements. It has to be remarked here that the error associated with the experimental measurement of intensity of an x-ray is rather large, while much more accurately known is the intensity ratio of two x-rays of not too different energies. For this reason the calculations are compared with the intensity ratios.

A calculation along the same lines by Srinivasan,¹⁶ in which allowance for the fine structure was made, gave the best fit for the intensity ratios for an initial distribution of the above type with $a = -0.25$. However, for this fit, the absolute intensity of the $2s \rightarrow 2p$ transitions is well below the experimental value.

There are two main shortcomings in such a cascade calculation: the artificial forcing of all the muons to pass by the level $n=14$, and the exclusion of radiationless atomic de-excitations occurring by nuclear excitation.

Backe¹⁷ has shown that a cascade calculation starting at $n=14$ reproduces well the x-ray intensities for small n , but is definitely inadequate for reproducing the intensities for higher n . However starting from a statistical

initial population at $n=20$ and including the conversion of M-shell electrons for the low-energy Auger transitions one gets excellent agreement with the experimental values for the higher values of n . One can expect that a cascade calculation using an $n=20$ initial population distribution will not reproduce the experimental intensities for n approaching 20. Only a better knowledge of the atomic-capture process will cure this shortcoming of the cascade calculations.

It is well known that the intensity ratios of the muonic x-rays of many elements do not coincide with the values obtained assuming a statistical filling of the levels involved.¹⁸ All the elements examined in our experiment have anomalous intensity ratios. Even though the assumption of statistical filling is too crude an approximation, and while more complete models reduce the discrepancy,¹⁶ at least a fraction of the discrepancy seems to be due to nuclear excitation.

Another correction to the cascade calculation consists in calculating the Auger coefficients with the inclusion of the penetration effects.¹⁹ These are a consequence of the strong overlap of charge and current densities of the muon and the converted electron in the region between the nuclear surface and the electron K-shell. This correction becomes rather important in magnitude for low energy muonic transitions. The effect consists in a reduction of the Auger coefficients.

The excitation of nuclear rotational levels in muonic

atoms due to a strong E2 interaction was first proposed by Jacobsohn²⁰ and Wilets²¹ in the case of nuclei with large deformation. Its effect on the fine structure of muonic x-rays was predicted together with the probability that the nucleus de-excite emitting a nuclear gamma-ray after the muon reaches the 1s state.

The atomic consequences of the dynamic quadrupole effect were studied by McKee²² who determined with high accuracy the intrinsic quadrupole moments of U^{238} and Th^{232} . Prompt nuclear gamma-rays were observed²³ for Sm^{152} confirming for the first time the prediction. Prompt gamma-rays were subsequently observed for spherical nuclei. (Prompt gamma-rays are so defined as distinct from the delayed gamma-rays that are emitted following the nuclear capture of the muon).

Some of the prompt nuclear gamma-rays observed for the isotopes of lead Pb^{206} , Pb^{207} and Pb^{208} (24), are shown in Table 1.3-1.

The E2 interaction between the muon and the Au^{197} nucleus is strong enough to cause not only mixing of some muonic levels (in particular $2p_{1/2}$ and $2p_{3/2}$) but also excitation of the nucleus into several low-lying states.²⁵ It is generally assumed that for muon levels with $n > 3$ dynamic electric quadrupole effects are negligible.

TABLE 1.3-1. Intensities of the prompt nuclear gamma-rays observed per 100 muon stops in monoisotopic lead targets. The emitted gamma-rays are identified by their energy and by a superscript indicating the emitting lead isotope.

Target Isotope \ E_γ^A (MeV)	$.803^{206}$	$.570^{207}$	$.897^{207}$
Pb ²⁰⁶	5.3 ± 0.2	0	0
Pb ²⁰⁷	5.1 ± 0.3	4.2 ± 0.2	1.2 ± 0.3
Pb ²⁰⁸	0	3.8 ± 0.3	2.2 ± 0.5

Nuclear excited states have also been observed by Hüfner²⁶ in muonic Bi, in which the muon interacts with the magnetic dipole and the electric quadrupole moments of the Bi nucleus (the latter seems to have a larger value than the previous determinations). Hüfner's calculation indicates that the nuclear excitation is the major cause of anomaly in the intensity ratios of Bi muonic x-rays.

The excitation of nuclear states by the cascading muon in muonic Tl^{203,205} has been also observed,¹⁷ and indicates that the discrepancies between the measured and the calculated intensity ratios of the $3d \rightarrow 2p$ and $2p \rightarrow 1s$ transitions in that element are caused by the interaction of the muon with the nucleus.

Dynamic effects occur also as a result of the approximate coincidence between the energy difference of two muonic

levels and the excitation energy of a nuclear level with the proper quantum numbers.²⁸ These effects are not only of E2 type but can be due as well to other multipoles. They were first proposed in the case of Tl^{203,205} by Frati and Rainwater.²⁹

The gamma-rays listed in Table 1.3-1 were interpreted by Hargrove and collaborators²⁴ as a consequence of radiationless muonic transitions that either excite the nucleus Z^A (presence of prompt nuclear gamma-rays of the target element) or provoke the ejection of a neutron that leaves the daughter nucleus Z^{A-1} in an excited state (presence of a prompt Pb²⁰⁶ gamma-ray with a Pb²⁰⁷ target, and of Pb²⁰⁷ gamma-rays with a Pb²⁰⁸ target). The potential of these two processes to affect both the absolute intensities of some muonic x-rays and their ratios was mentioned together with the possibility of studying some properties of the nucleus in some of its states.

While the nuclear excitation by the cascading muon is a non-negligible process common to a large number of heavy muonic atoms, the ejection of neutrons as a result of it is allowed energetically only for a smaller number of heavier muonic atoms. For the muonic atoms of all the elements examined in the present study there is sufficient energy available for neutron ejection from the nucleus for transitions to the 1s state from all the states with $n \geq 3$. It is necessary to understand the mechanisms followed in such a process before it is possible to know its effect on the

cascade.

For the sake of completeness we shall mention another process that is viable for fissionable nuclei and that plays a role in determining the cascade of the muon. This process, foreseen already by Wheeler² in 1949, consists in the fission of the nucleus caused by a radiationless muonic transition. The first indirect evidence for its occurrence was given by Balats and collaborators³⁰ who compared the intensity of the $2p \rightarrow 1s$ x-ray line in muonic U^{238} with the intensity of the same line in muonic Pb. More direct and accurate measurements³¹ have followed since, paralleled by theoretical studies of the process.^{32,33}

1.4 Experimental data on neutron ejection from the nucleus induced by the cascading muon

The first direct observation of prompt neutrons ejected from the nucleus by the cascading muon was made by Hargrove et al..³⁴ The experimental set-up consisted of three basic elements: (I) a muon telescope detecting a muon stop and providing the zero-time signal for the neutron time-of-flight spectrometer, (II) a neutron scintillation counter, N, provided with a pulse-shape discriminator to reject photon pulses, from which the arrival time of the neutrons is derived, and (III) a delayed muon-capture scintillation counter (DC counter) having the function of detecting the occurrence of muon capture by the nucleus within a certain time interval following the muon stop by a time greater than the longest flight accepted. The purpose of the DC counter

was to reduce the background from capture neutrons. Neutrons whose arrival is detected within 50 ns after the stop time are counted, and then only when they are followed by the detection of any kind of nuclear capture radiation by the DC counter. The geometry of the set-up is linear, with the flight path along the axis of the beam.

The target elements used were Bi²⁰⁹ and natural Ag. The silver target was used to check the system, knowing that no prompt neutrons can be ejected by the cascading muon in muonic silver. This is because the energy available in the muon de-excitation is less than the neutron separation energy for both the isotopes. The silver spectrum provided also a suitable evaluation of the background, since the lifetime of the 1s muon is approximately the same in silver as it is in bismuth.

The time resolution obtained was ~ 2.5 ns with a flight-path 18" long.

The main features of the Bi time-of-flight spectrum are:

- 1) A large peak at ~ 1.1 MeV attributed to the $3 \rightarrow 1$ muonic transitions and, because of the location of the peak and of its width compared to that of the photon peak, more likely due to the $3d \rightarrow 1s$ transitions than to the $3p \rightarrow 1s$. It could be also partially due to $4 \rightarrow 1$ muon transitions that leave the daughter nucleus in an excited state in the energy interval 930-1150 keV.
- 2) Another bump in the spectrum corresponding to ~ 1.8 MeV neutron kinetic energy possibly related to the

particle-hole multiplet of Bi^{208} at .635 MeV.

3) A quite sharp rise at a time corresponding to the muon series limit, that indicates that transitions from highly excited muon states contribute significantly to neutron emission.

4) The total prompt neutron yield per muon stop was estimated to be $(7 \pm 2)\%$.

Later the same group repeated the experiment³⁵ for a Pb^{207} target (88% isotopic purity) obtaining a spectrum showing the following features:

- 1) a peak corresponding to the $3 \rightarrow 1$ transitions leaving the daughter nucleus in the ground state;
- 2) the hint of a peak at an energy corresponding to the $3 \rightarrow 1$ transitions leaving the daughter nucleus in the first excited state;
- 3) a sharp rise at the series limit, not as pronounced as for Bi;
- 4) the total prompt neutron yield per muon stop estimated to be approximately 5%.

1.5 Theories of neutron emission from the nucleus induced by cascading muons

Neutron ejection as a result of nuclear conversion of mu-atomic transition energy is a process that is energetically allowed only for heavy atoms for which some transition energies are larger than the neutron separation energy. In the case of the elements here examined, Au, Tl, $\text{Pb}^{206,207,208}$,

Bi, only the mu-atomic transitions ($n=i \rightarrow n=1$) with $i \geq 3$ can provide sufficient energy for neutron ejection (see Table 4.1-2 for transition and neutron separation energies).

The neutrons are ejected as a result of an electromagnetic interaction of the muon with the nucleus so that the ejection can be considered prompt on a nanosecond scale. The probability of emission depends on the muonic wave functions only via the quantum numbers and the energy of the virtual photon mediating the conversion, and, of course, via their spatial overlap with the nucleus.

The process is the inverse of the internal conversion process for nuclear gamma-rays in the sense that the roles of the nucleus and of the atomic atmosphere are interchanged with one important difference, as noted by Hüfner and Sevgen,³⁶ consisting in the presence of nuclear compound levels.

It is to be expected that the higher n , the less the $n \rightarrow 1$ transitions will contribute. In fact, the preferential filling by the cascade of the higher- l states has the effect of making the absolute value of the population of the low- l states of an n "shell" a decreasing function of n , and in general the transition probability decreases rapidly with the increase of its order of multipolarity, so that the low- l states are the ones contributing to the ejection of neutrons.

The energy spectrum of the neutrons is a line spectrum, where the lines are defined by

$$E_{\text{kin}}^n = E_{n1} - E_{1s} - E_{A-1}^* - S_A^n$$

where $E_{nl} - E_{1s}$ is the muon transition energy,

E_{A-1}^* is the excitation energy of the daughter nucleus,

S_A^n is the neutron separation energy.

Due to the dynamical penetration effect,³⁷ the electric monopole muonic transitions ($ns \rightarrow 1s$) can also contribute to the prompt neutron emission. The radiationless muonic $2p \rightarrow 1s$ transition occurring via excitation of the nucleus has been predicted and studied in the case of U^{238} by Zaretsky³² in 1959. In this case the excited nucleus decays via one of the three possible channels, i.e. neutron emission, fission, gamma-ray emission. Zaretsky calculated the dipole transition matrix element in non-relativistic approximation under the assumption that the whole photoexcitation cross-section is due to the absorption of an electric dipole quantum. He used the experimental cross-section for photo-neutron emission -- which is known to be mainly of E1 type -- and estimated the probability of prompt fission by the muon, which he found to be small.

Zaretsky and Novikov³³ repeated the calculation for the same transition in the two limiting cases: $\rho \Gamma_n \ll 1$ and $\rho \Gamma_n \gg 1$, where ρ is the density of nuclear states at an excitation energy equal to the muon transition energy and Γ_n is the average width of the same nuclear states. The first limiting case was considered in their 1961 paper where they obtain $\Gamma_{r1} / \Gamma_x \approx 0.7$ for U^{238} and Th (Γ_{r1} is the partial width for radiationless transition and Γ_x is the

partial width for radiative transition).

The same type of calculation as the one described by Zaretsky has been used by Srinivasan and Sundaresan³⁸ to estimate the probability of radiationless muonic transition for Pb and Bi. Considering only the dipole part of the interaction they obtain $\Gamma_{r1} / \Gamma_x \simeq 0.6$, i.e. the probability of radiationless transition in elements like Pb and Bi is of the same order of magnitude as that of radiative transition.

This estimate for Γ_{r1} / Γ_x has then been checked by calculating the same ratio treating the prompt neutron emission as an inverse internal conversion. The dynamic effect of the finite extension of the nucleus, that is the penetration of the muon in the nucleus, has been taken into account using the surface current model due to Sliv, according to which the nuclear currents are confined to the surface of the nucleus.

Again retaining only the electric dipole term of the interaction, the value obtained for Γ_{r1} / Γ_x is ~ 0.9 , with weak dependence on n . With this value neutron energy spectra were calculated for Pb^{206} and Bi^{209} under the assumption that the daughter nucleus is always produced in its ground state. This assumption was necessary because the excitation spectrum of the daughter nucleus as a result of the radiationless transition is unknown.

The calculated spectra are characterized by peaks corresponding to the $np \rightarrow 1s$ ($n \geq 3$) transitions whose

intensity decreases with increasing n , with a rather high peak corresponding to the high population of the $14p$ level (representing all the np levels with $n \geq 14$). The total neutron yield, calculated with an average value for Γ_{r1} / Γ_x and an initial population $e^{-0.251} (21 + 1)$, is estimated for Bi^{209} to be 5-7 per 100 muon cascades.

That it is unlikely for E1 muonic transitions to account for the entire yield of prompt neutrons experimentally observed in the case of Bi, is shown by Schneuwly et al..³⁹ These authors have observed some E2 radiative transitions in muonic thallium, lead and bismuth. They have determined the intensity ratios and the absolute intensities for some E2 and E1 transitions, and the intensity ratios they fit with a cascade calculation starting with a statistically distributed population at $n=20$. This calculation gives them a better fit to the experimentally determined intensity ratios than they can obtain with an initial population statistically distributed at $n=14$. The effect of starting from a higher n affects particularly the $3p$ level and of course the $14p$ one, since with the higher- n initial population the process of enrichment of the higher angular momentum states is begun earlier in the cascade calculation. From the best-fitting cascade calculation Schneuwly and his colleagues obtain a population of the $3p$ level equal to 2.7% per muon cascade and a total population for the np levels with $n \geq 3$ of 4.7%. With the calculated Γ_{r1} / Γ_x ratio equal to 0.6 (1.0) and the above populations, the $np \rightarrow 1s$ radiationless

transitions can account for a total prompt-neutron yield of 1.7% (2.4%), which is considerably less than the experimental yield.

The above authors obtain 0.5 ± 0.2 as an estimate for the Γ_{r1} / Γ_x ratio from the ratio of measured and calculated $3p_{3/2} \rightarrow 1s_{1/2}$ x-ray intensities ($\approx 2/3$).

Schneuwly et al. conclude that the remaining part of the neutron yield which is not accounted for comes from $nd \rightarrow 1s$ transitions. The lower probability for such transitions (there is no collective E2 state known which might act as a doorway for the compound decay) is compensated by the higher population of the d states. For example, 3d is 30 times more populated by their best-fitting cascade than is 3p. A rough estimate of the Γ_{r1} / Γ_x ratio for the 3d levels is obtained, under the assumption that $\Gamma_{r1} / \Gamma_x = .51$ for the transitions ($3p \rightarrow 1s$), from the comparison of the measured and calculated x-ray intensity ratios

$I(3d_{3/2} \rightarrow 1s_{1/2}) / I(3p_{3/2} \rightarrow 1s_{1/2})$. Allowance is made for the nuclear mixing involving the $3d_{5/2}$ level ($7.3 \pm 0.9\%$ per muon cascade), which reduces the $3d_{5/2} \rightarrow 1s_{1/2}$ transition intensity by a factor of 0.85. Due to the large errors in the intensity values the uncertainty is large. The result is $0.06 < \Gamma_{r1} / \Gamma_x (3d \rightarrow 1s) < 0.4$.

The conclusion as to the $3d \rightarrow 1s$ contribution to the prompt neutron yield should be applicable to the Tl, Au and Pb muonic cascades as well.

The most complete theoretical study up to date of

neutron emission from the nucleus induced by the cascading muon in lead and bismuth has been made by Lohs, Wolschin and Hufner⁴⁰ who refer to the process as nuclear Auger effect. These authors underline the limitations of the phenomenological approach by Srinivasan and Sundaresan, due to the facts that:

- a) the experimental photoneutron emission cross-section at the energy of interest is dominated by the electric dipole, and therefore does not allow for the possible effect of higher multipolarities, and
- b) the partial (γ, n) cross-sections for specific nuclear channels are not available making it thus impossible to calculate the partial conversion coefficients (defined as the ratios of the partial widths for emission of neutrons to the partial width for radiative transition).

Following the general lines of a formal analysis prepared by Hufner and Sevgen,³⁶ Schneuwly and collaborators take a microscopic approach to the study of the nuclear Auger effect. There are three main mechanisms to be considered:

- a) Direct process. This is the result of direct coupling to the neutron which is endowed with an effective electric charge that is a function of the interaction multipolarity. The 50% uncertainty in their results is mainly due to the imprecision with which the value of the effective charge of the neutron can be determined.

b) Semidirect process. In this process a nucleus is virtually excited to a collective state (which is the giant dipole resonance in the case of E1) and then emits a neutron. The authors find this process about equally important as the direct one and in destructive interference with it.

c) Compound nucleus mode. Here a virtual compound nucleus is formed that decays eventually by neutron emission. The authors calculate first the width for the formation of the compound nucleus, then calculate the transmission coefficients into the various neutron channels from an optical potential that describes the elastic scattering of the neutrons. The compound state has been considered potentially excitable both via intermediate simple one-particle configurations and via collective states of simple structure.

The populations of the muonic levels are obtained with a cascade calculation using a statistically distributed initial population at $n=20$. The model used for the nucleus is the independent-particle shell model.

The results of their calculation are given in terms of the partial widths for radiative and radiationless transitions in muonic Bi^{209} , and the total conversion coefficients corresponding to a few representative muonic transitions. They obtain the value of 0.6 for the $3p \rightarrow 1s$ conversion coefficient, in good agreement with the value 0.6 - 0.9 obtained by Srinivasan and Sundaresan.³⁸ For the $3d \rightarrow 1s$

conversion coefficient they obtain 0.9, which doubles the maximum limit of 0.4 estimated from x-ray intensity measurements by Schneuwly and collaborators.³⁹

The conversion neutron spectrum for muonic Pb^{207} is shown explicitly, while the intensities for the various regions of the neutron spectrum are given for Pb^{207} and Bi^{209} . The total yields (% per stopped muon) for these two elements are respectively 8.5 (for $E_n \geq 0.5$ MeV) and 4.7 ($E_n \geq .9$ MeV) or 5.9 ($E_n \geq .5$ MeV). It appears that the probability of excitation of the daughter nucleus (Z^{A-1}) has not been taken into account, while the contribution from the possible creation of higher-energy neutron-holes in the parent nucleus has been included. For $E_n \geq .5$ MeV, this contribution accounts for 60% of the total yield.

1.6 Nuclear capture of the 1s muon

Once the muon has reached the 1s orbit (negligible is the fraction of muons that decay before reaching the ground state), it either decays or is captured by the nucleus.

The lifetime of a free muon, $(2.1999 \pm .0006)\mu\text{s}$,⁴¹ is determined by its decay probability. The decay occurs into two different neutrinos and an electron which is emitted with a continuous spectrum of energy reaching up to several tens of MeV's. It is to be noticed that the decay probability of a bound muon is smaller than the decay probability of a free muon.⁴² This results from (I) the negative binding energy that reduces the phase space available for the decay products, (II) the time dilation consequent to the non-zero speed of

the muon and (III) the effect of the Coulomb field of the nucleus upon the decay electron, that tends to increase the decay rate. The reduction factor in the case of lead has been measured⁴³ to be $.86 \pm .04$. It is close to 1 for the low-Z elements.

The nuclear capture of the muon is a consequence of its weak interaction with the nuclear protons

The neutron so produced can either be directly ejected, and then possibly followed by gamma-rays emitted by the daughter nucleus $(Z-1)^{A-1}$, or transfer its energy to excite collective states of the intermediate nucleus $(Z-1)^A$ *. The intermediate nucleus then decays, mainly by emission of one or more neutrons possibly followed by gamma-rays from the residual nucleus, but also by emitting charged particles like protons and alpha-particles. The probability for emission of charged particles is small and a decreasing function of the nuclear charge.

The direct neutrons ($\sim 10\%$ of the total neutron production) have a high-energy (up to a few tens of MeV's) exponentially decreasing distribution, while a low-energy evaporation type of spectrum corresponds to the other neutrons.⁴⁴ The probability of no neutron emission is at most a few percent.⁴⁵

The average number of neutrons emitted per muon capture increases in general with A. For Ag, Au and Pb the measured values are respectively $1.61 \pm .06$, $1.66 \pm .04$, $1.71 \pm .07$

neutrons per capture.⁴⁶

Following the emission of particles the residual nucleus may emit gamma-rays. The energy spectra of the capture gamma-rays with energy larger than .6 MeV has been measured⁴⁷ for heavy elements. For Bi, radiogenic Pb, Au, and Ag the gamma-ray multiplicities per μ capture are respectively 1.9 ± 0.6 , 1.6 ± 0.5 , 1.9 ± 0.6 and 2.3 ± 0.8 .

The nuclear capture rate of the muon is a function of the nuclear charge and mass. Primakoff⁴⁸ has derived a formula for the total muon capture rate in terms of the nuclear A, Z and effective charge, based on a closure approximation for the states of the daughter nucleus excited by the capture. A good amount of data⁴⁹ on total muon capture rates is available which is rather well fitted by Primakoff's formula. Progress is being made⁵⁰ in the theoretical field toward an interpretation of the total capture rate, that is nearly model-independent and permits one to extract from the total capture rates additional information about the capturing nucleus.

The 1s muon mean lifetimes in Ag, Au, Tl, radiogenic Pb, Bi are reported⁴⁹ to be, in nanoseconds, respectively 88.5 ± 0.7 , 72.6 ± 0.5 , 70.6 ± 0.9 , 71.5 ± 0.4 , 74.6 ± 0.6 , 73.3 ± 0.4 .

CHAPTER 2

THE PRESENT EXPERIMENT

2.1 Aims and methods of the present experiment

The aim of the present experiment was essentially to measure the yield and the energy spectrum of the neutrons emitted by nuclear conversion of the atomic transition energy of the cascading muon. This was intended to be done with an improved energy resolution and with an increased signal-to-noise ratio compared with the previous experiment by Hargrove et al.,²⁴ and was extended beyond the Bi and Pb²⁰⁷ targets to Pb²⁰⁶, Pb²⁰⁸ and to Tl and Au.

The need for such measurements is suggested by the fact that, as seen in the introductory chapter, the theoretical investigation of the process is one step ahead of the experimental data available and should be tested experimentally before further progress can be made without much waste. The theoretical predictions made in particular for Pb²⁰⁷, Pb²⁰⁶ and Bi are valid qualitatively for the other near-spherical heavy nuclei examined.

The basic method of measurement was the same as in the preceding experiment, aiming at measuring the time-of-flight of the neutrons emitted at the time of the muon stop. The latter coincides, for all practical purposes, with the time

of emission of the cascade x-rays.

These time-of-flight spectra should contain as well -- if one neglects the photons of any origin that can be efficiently removed by pulse-shape discrimination -- the neutrons that certainly are emitted after muon or pion capture by the nucleus, and in a heavy atom there is a large probability for the muon to be captured within a few tens of nanoseconds after its stop. In a straightforwardly collected spectrum such neutrons would dominate. It is true that any line structure of the capture-neutron spectrum is efficiently hidden by the fact that the time-of-flight spectrum of the capture neutrons measured from the capture time is folded in with an exponential probability of capture as a function of time measured from the muon stop time. This, together with the smoothness of the energy spectra of the capture neutrons,⁴⁴ precludes the presence of any sharp neutron peaks.

The best way to reduce the capture-neutron background is to accept only those neutrons that, at a time longer than the maximum time of flight, are followed by a capture which is detected through its associated radiation by a counter placed as near as possible to the target.

The first improvement with respect to the earlier experiment consists in the increase of the path length from 18" to 24" to improve the energy resolution. Any increase in path length corresponds to a more than linearly proportional decrease in the efficiency of delayed capture detection, since the capture should be accepted only after a time

greater than the longest neutron time of flight. This last requirement is meant to reduce the background due to detection by both the N and the D counters, of capture neutrons in a multiple neutron emission following nuclear muon capture. From an analysis of the collected events it results that any reduction in capture detection efficiency due to an increase of flight-path length, results in a reduction of the ratio of the number of desired events to the number of capture background events (see Section 3.6).

A major second improvement consists in the choice of a 45° geometry: i.e. a flight path direction at 135° to the beam direction. This choice allows one

- a) to reduce the flat background coming from random N-pulses since the N-counter rates are reduced by moving the counter away from the beam line; this reduced N rate has the other important consequence of improving the photon-neutron discrimination, which deteriorates with increasing counting rates, particularly for low pulse-amplitudes;
- b) to have a more unobstructed and scattering free flight-path by placing the delayed capture counter right against the target and on the opposite side of the neutron counter thus covering with only one scintillator a very large solid angle.

A third improvement consists in viewing the delayed-capture scintillation detector with two photomultipliers in coincidence to reduce the random pulses due to the photomultiplier noise. This is particularly important since the

need for a high capture detection efficiency requires as low a pulse-height threshold as possible.

Another important advantage over the previous experiment consists in the use of a minicomputer interfaced to a 64-k 16-bit-word disk and a digital cassette recorder. These are used

- a) to monitor the data collection and
- b) to store the event data with all the information

(neutron pulse amplitude, cross-over time of the integrated neutron pulse, neutron time-of-flight).

The above information for each individual event is necessary to partially correct the time-of-flight spectra for time-walk as a function of the amplitude of the neutron pulse to improve the time resolution, and to improve the quality of the photon-neutron discrimination.

A different and probably more reliable and stable photon-neutron discrimination circuit⁵² has been used.

2.2 Beam and apparatus

The data were collected in a series of experimental runs performed at the 600 MeV synchrocyclotron of the Space Radiation Effects Laboratory (SREL) in Newport News, Virginia.

An analyzing magnet at the end of the transport system selects particles of momentum (100 ± 6) MeV/c, which should only be muons produced by pions decaying backwards in their own rest frame. Such a "backward" muon beam has, at the

price of some loss in intensity, the advantage of a very low pion contamination ($<1.8\%$). The central momentum of the beam ($100 \text{ MeV}/c$) corresponds to a range in CH_2 of $\sim 4''$. The typical stop rate in a Bi target covering the whole B-counter area and 3.94 g cm^{-2} thick, centered on the beam's axis and at 45° with respect to it, was $2.6 \times 10^4 \text{ muons s}^{-1}$.

Fig. 2.2-1. Counter arrangement (side view). Scale 1:6. Plastic scintillators A, B and C with lucite Cerenkov radiator E, form the muon telescope. Plastic scintillator D is the delayed-capture detector. Liquid scintillator N is the neutron counter. Plastic scintillator V vetoes the detection of charged particles by N. M is the energy moderator.

Table 2.2-1. Counter components.

Counter	Dimensions	Material	Photomult. type ^a
A	8.5"x8.5"x1/8"	NE102 ^b	RCA-8575
B	4 ³ / ₈ "x4 ³ / ₈ "x1/16"	NE102	RCA-91000D
C	5.5"x5.5"x1/16"	NE102	RCA-8575
D	6.5"x6.5"x2"	NE102	RCA-8575
E	6"x6"x1/2"	Lucite	RCA-8575
N	5" diam. x 1/2"	NE213 ^b	Philips 58DVP
V	5.5"x5.5"x1/8"	NE102	RCA-8575

a) All the photomultipliers are shielded from magnetic fields by mu-metal cylinders and iron boxes.

b) Nuclear Enterprises Inc., San Carlos, Cal., U.S.A. .

The experimental set-up is shown in Fig. 2.2-1. It consists essentially of a muon telescope, a time-of-flight measuring system and a delayed-capture detector. In Table 2.2-1 are listed the main components of the counters.

A: Muon telescope

The telescope consists of the large counter A in coincidence with the thin counter B and in anticoincidence with the C counter.

The B counter, which is immediately followed by the target and is parallel to it, provides the time signal for a stop in the target, and defines with its cross-section perpendicular to the direction of the beam, the active area of the same target. This counter is viewed laterally by two

opposite photomultipliers: this is done both to reduce the random coincidence rate, AB , hence the false stop rate, and to reduce the time jitter of the stop event due to variation in path length for collection of photons from the scintillator at the photomultiplier photocathode. The scintillator is NE102 which has one of the highest ratios of light output intensity to luminous decay time of the dominant decay (2.5 ns). The photomultipliers are RCA 8575 with bialkali photocathodes having high quantum efficiencies and therefore good time resolution.

Fast timing by cross-over pick-off method was used, in which the anode current pulse is clipped with a shorted delay line to produce a bipolar pulse with a zero crossing. A fast cross-over discriminator (EG&G T-140) is used as the time pick-off device. The zero-crossing time represents the same phase point on all pulses, and the only walk effect is due to the charge sensitivity of the zero-crossing discriminator. This method gives good walk characteristics over a wide dynamic range such as that provided by the scintillations of the stopping muons (typically the walk is less than .1 ns for a 100:1 dynamic range).

The C counter immediately follows the target and is parallel to it. A general particle stop in the target gives a characteristic signal ABC .

For the purpose of reducing the contribution to the stop rate given by the electrons contaminating the beam, a vetoing threshold Cerenkov counter (E) is placed immediately

after the A counter. It consists of a lucite radiator $1/2$ " thick viewed by one photomultiplier. From now on we shall call "stop" the occurrence of a coincidence $AB\overline{CE}$.

All the counters so far considered are centered on the beam axis, but while A and E are perpendicular to the beam axis, B and C (and the target) are at 45° with respect to it. Between the E counter and the B counter, and perpendicular to the beam axis, is placed a polyethylene energy moderator (whose thickness is determined by the thickness of the target) with the function of centering the muon range peak with respect to the target, so as to maximize the muon stop rate in the target itself.

B: Neutron time-of-flight spectrometer

The neutron time-of-flight spectrometer consists of the counters B, N and V.

The maximum time-of-flight acceptable, as will be justified later, is 60 ns. Since we intend to look at neutrons with minimum energy .6 MeV, the maximum length for the flight-path is 2', which is the value adopted.

The N counter has a liquid scintillator (NE213) sealed inside an aluminium disc-shaped container 5" in diameter and $1/2$ " thick with a glass wall facing opposite to the target.

The cross-section of the N counter perpendicular to the flight path has an area approximately equal to that of the B counter. Its thickness has been chosen so as to have the highest possible efficiency for neutron detection, compatible with the requirement that the uncertainty in flight-path

length be small compared with the relative uncertainty in time-of-flight due to the finite time resolution of the system.

The choice of a circular form for the N counter was dictated by the need to optimize the optical coupling of the scintillator with the photomultiplier, a Philips 56 DVP with the photocathode measuring 5" in diameter. The optimization in question aims at improving both time resolution by reducing the dynamic range of the pulses and by increasing the absolute number of photoelectrons released (the time resolution is inversely proportional to the number of photoelectrons), and the pulse-shape discrimination between the pulses produced by photons and by neutrons.

Immediately in front of the N counter and in anticoincidence with it is the V counter. This is a thin scintillation counter which has the purpose of vetoing all the charged particles, be they either scattered beam particles or charged particles (e.g. protons) emitted from the target nucleus following muon capture. The V counter has been made thin so as to not absorb or scatter noticeably the neutrons heading for the N counter.

The largest contribution to the background in the neutron time-of-flight spectra is given by the capture gamma-rays. It was therefore very important to use a good system for rejection of the photon pulses in the N counter. There is a host of circuits, some of which commercially available, that are sensitive to the shape of the photomultiplier

current pulse produced by the scintillation. They are all based on the fact that for some organic and inorganic scintillators the time dependence of the scintillation is a function of the type and the energy of the particle detected because it is a function of the ionization density.

One of the organic scintillators with high hydrogen content (for efficient detection of neutrons by their recoil protons) which lend themselves best to pulse-shape discrimination between photons and neutrons is NE213. Its scintillation properties have been studied extensively. The decay time constants for the first three components of the scintillation light have been measured⁵¹ to be respectively 3.16 ns, 32.3 ns and 270 ns. The effective overall decay time constant is 3.71 ns, which is shorter than those of the scintillators NE213 M and NE312⁵¹ and therefore more suitable for precise time measurements.

C: Delayed capture detector

The delayed capture detector consists of a 2" thick plastic scintillation counter, D, immediately following the C counter in the direction of the beam and parallel to the target. It covers a solid angle of approximately 2π relative to any atom in the active part of the target.

The efficiency of capture detection, through the detection of any of the particles (massive or not) emitted by the nucleus at the time of capture, should be as high as possible so as to increase the event rate; hence the requirements of a low pulse-height threshold and of the

reduction of the photomultiplier noise that can, by accident, simulate capture events. This noise is virtually eliminated by viewing the scintillator with two photomultipliers and requiring a coincidence between them. To veto the D-counter pulses produced by beam particles, the D counter is vetoed by the A counter with a pulse long enough (20 ns) to eliminate the double pulsing due to the large pulses produced by a fraction of the beam particles. The A veto introduces only a marginal inefficiency in the capture radiation detection.

D: Targets

A list of the targets used, containing their composition, dimensions, purity and mean flight-path is shown in Table 2.2-2. The Pb^{207} target, made up of two square plates $1/4$ " thick, covered only $1/2.85$ of the B-counter area defining the useful cross-section of the beam letting therefore a higher percentage of the stops take place elsewhere with a resulting increase in background.

The Pb^{206} and Pb^{208} targets were nitrates of the respective isotopes. For them a high percentage of stops did not take place in the desired target element, increasing thereby noticeably the relative level of the background events.

The Ag and Tl targets had their natural isotopic compositions, and so did the naturally monoisotopic Bi and Au targets.

Table 2.2-2. Targets and corresponding mean flight-path lengths.

Target	Dimensions (cm x cm x cm ⁻² g)	$\bar{x}$ ^c (cm)	$\sigma_{\bar{x}}$ ^d (cm)	Isotopic purity (%)	Composition
Bi ²⁰⁹ _I	12.7 x 12.7 x 3.98	61.01	0.47	100	metal
Bi ²⁰⁹ _{II}	12.7 x 12.7 x 7.0	60.81	0.51	100	metal
Pb ²⁰⁶ _a	12.7 x 12.7 x 0.76	60.86	0.50	99.4	Pb(NO ₃) ₂
Pb ²⁰⁷ _b	4.04 x 4.04 x 7.2	60.89	0.49	92.4	metal
	4.34 x 4.34 x 7.2				metal
Pb ²⁰⁸ _a	12.7 x 12.7 x 1.52	60.50	0.61	99.14	Pb(NO ₃) ₂
Ag ^{107,109}	12.7 x 12.7 x 4.09	61.01	0.47	natural	metal
Au ¹⁹⁷	9.6 x 9.6 x 4.7	61.06	0.46	100	metal
Tl ^{203,205}	12.7 x 12.7 x 5.6	60.97	0.49	natural	metal

a) The target is contained in one (Pb²⁰⁶) or two (Pb²⁰⁸) boxes having walls perpendicular to the beam, 0.4mm thick (each wall). Only the Pb contribution to the thickness is shown in the table.

b) The two pieces are supported by a thin square of styrofoam and are placed side by side perpendicular to the beam. The other main components of the target are Pb²⁰⁸ (5.48%) and Pb²⁰⁶ (2.16%).

c) Mean flight-path length.

d) Root mean square deviation of the mean flight-path length.

2.3 Electronics and logic

The block diagram of the electronics is shown in Fig. 2.3-1. It is built up mainly from EG&G, Chronetics and ORTEC logic modules in a standard way, the main exception being the pulse-shape discriminator module.

A muon stop is signalled by a coincidence $A \cdot B_R \cdot B_L \cdot \bar{C} \cdot \bar{E}$ (where B_L and B_R are the pulses given by the left and the right photomultipliers viewing the B counter). The stop signal opens (not shown in the block diagram) the gates for the two coincidence modules measuring the time of flight. For each stop the "time of flight" corresponding to any N-pulse appearing within 60 ns after the stop, is measured as half the sum of the time-delays of the N pulse relative to the B_L pulse and the B_R pulse. These are the pulses coming respectively from the left and the right photomultipliers viewing the B counter. The sum is performed in order to compensate for the time delay due to the light signal travelling from the point of scintillation to the photocathode, or, in other words, to make the stop time more independent of the muon stopping position across the target.

The N pulse is derived from the leading edge of the anode pulse. The N pulses are vetoed by coincident pulses of the V counter to avoid counting charged particles.

The time delays between the N pulse and respectively the B_L and the B_R pulses are measured as the time overlaps between properly shaped and timed B_L or B_R and N pulses. After time-to-amplitude conversion the pulses coming from

Fig. 2.3-1 Block diagram of the electronics.

the B_L and the B_R lines are added together, and their sum is read by an analog-to-digital converter, provided that an "event" occurs (see later).

Following a different route, a signal triggered by a stop and delayed by 65 ns with respect to it, looks for a coincidence with a "delayed capture" pulse from the D counter (seen by both upper and lower photomultipliers in coincidence and not vetoed by the A counter). A "delayed capture" pulse is accepted for 100 ns starting from 65 ns after the stop.

A triple coincidence between a stop, a particle detected by the neutron counter N within 60 ns after the stop (and approximately 10 ns before as well), and a "delayed capture" particle detected by the D counter between 65 and 165 ns after the stop is what we call an "event".

Every time an event occurs three pieces of information are collected from the three ADC's gated by the event pulse:

T is an integer of the interval (1, 128), proportional to the time-of-flight, i.e. to the time delay between a stop and an N-counter pulse following it within 60 ns.

H is an integer of the interval (1, 32) proportional to the intensity of the scintillation. It is obtained by suitably integrating and amplifying the output of the 4th last dynode of the N photomultiplier.

X is an integer of the interval (1, 16) containing the pulse-shape information enabling us to distinguish neutrons from gamma-rays.

All three ADC's start converting the data signals only if there is a coincident strobe signal triggered by the occurrence of an event. The stop signal and the "delayed capture" pulse, when the latter is present, respectively start and stop also a time-to-amplitude converter (an EG&G TH200). An output pulse is produced whenever a "delayed capture" pulse appears in the time interval included between 100 ns before and 500 ns after the occurrence of a stop. The TH200 output is fed into an ADC and converted into an integer of the interval (1, 128). This number is recorded when the capture-radiation time-spectra are collected.

2.4 Pulse-shape discrimination (PSD)

Pulse-shape discrimination between the gamma-rays and the neutrons (charged particles are vetoed by the V counter) as seen by the N counter was performed by using the circuit designed by Heistek and Van der Zwan.⁵²

The voltage divider used for the 14 stage photo-multiplier viewing the neutron counter has a fast, an integrated and a linear output. It is a modified version of the type B distribution recommended by Philips, with the main addition of two trimpots for the purpose of adjusting the voltage between the last three electrodes so as to maximize the output without incurring saturation for the high amplitude pulses. Optimizing the distribution to obtain maximum amplification for the large pulses optimizes at the same time the linearity of the amplification over a large range of pulse amplitudes.⁵³

The pulse used for pulse-shape analysis is derived from the 14th dynode of the photomultiplier: it is integrated with a time constant of 700 ns, and then shaped with a shorted delay line 400 nanoseconds long.

The shaped pulse is a bipolar pulse that crosses the zero level at a time when the reflected pulse has reached a fraction 0.75-0.8 of its amplitude at time $t=400$ ns.

Thus the zero cross-over time, measured with respect to the time origin provided by the leading edge of the anode pulse (the initial time of the scintillation), is itself a measure of the rise time of the integrated last dynode pulse. This rise time is, in turn, a function of the time dependence of the scintillation produced in NE213 by the protons scattered by neutrons, or by the electrons scattered or by gamma-rays. The rise time is shorter for gamma-rays than it is for neutrons because a proton scintillation, compared with an electron scintillation having the same total light output, has in NE213 and some other materials a higher percentage of long decay-time components. The zero cross-over time is measured by a high gain differential comparator. The high speed and high gain of the comparator allow for a high dynamic range (500:1) with a relatively small walk.⁵²

The separation between the zero cross-over time distributions of the pulses produced in the photomultiplier by the scintillations induced respectively by photons and by neutrons is a function of the pulse amplitude, that is, of the electron or proton energy. The time separation increases

with decreasing equivalent energies (equivalent with respect to the total scintillation light output). At the same time the statistical fluctuations in cross-over times (and therefore in cross-over time separation) increase with decreasing energy so that at low energies the two cross-over time distributions begin to merge.

The presence of a high radio frequency noise increases the above mentioned fluctuations of cross-over times and worsens the gamma-neutron discrimination. A certain amount of instability in the course of each run (typically 5 hours long) contributes as well to a deterioration of the pulse-shape discrimination.

On the other hand, since the N-counter counting rate is expected to be quite low, the pile-up effect on the pulse-shape discrimination is negligible.

The deterioration of the pulse-shape discrimination caused by its amplitude (H) dependence was partially avoided by the collection of the zero cross-over time (X) spectra for each individual value of the N pulse-amplitude (H) and the selection of X threshold values that were functions of the amplitude H of each event.

Fig. 2.4-1 reproduces a typical series of cross-over time distributions for the first six amplitude intervals. The broken lines marked by x's are the X spectra of the events having time-of-flight greater than 20 ns so to exclude the contribution of the photon-peak events. The continuous lines marked by circles are the X spectra for the

Log N

o : photon-region events
 x : neutron-region events

8

6

4

2

0

6

4

2

0

6

4

2

0

H = 1

H = 2

H = 3

H = 4

H = 5

H = 6

1

3

5

7

9

11

13

15

1

3

5

7

9

11

13

15

Channel no.

Fig. 2.4-1 X spectra for single H intervals (run 4.).

events with $T < 20$ ns (mainly photon-peak events).

2.5 Computer system

The computer system consisted of a PDP-11 computer, having a core memory of 16 k 16-bit words, interfaced with home-made interfaces to

- a) 4 ADC's
- b) a (Tektronix 611) storage display
- c) a Dicom 344 cassette magnetic tape system
- d) a Digital Equipment Corporation RS64 Disk unit having 2^{16} 16-bit words storage capacity.

The data were stored in the disk -- and, at the end of each run, transferred onto the cassette tape -- in the form of a three dimensional array $N(T,H,X)$, with $1 \leq T \leq 128$, $1 \leq H \leq 32$ and $1 \leq X \leq 16$ containing in each point the number of events having the corresponding values for the triplet (T,H,X) .

The core memory was used to:

- 1) Control the data collection either for the time-of-flight spectra or for the time spectra of capture-radiation detection by the D counter (see Appendix).
- 2) Monitor the data collection with the continuous display of the time-of-flight and cross-over-time spectra separately for all the intervals of the N-pulse amplitude. The widths of the first five amplitude intervals could be set arbitrarily at the beginning of each run.

- 3) Display histograms of any selected spectra of the data collected upon interruption or end of the data collection.
- 4) Allow one to perform the pulse-shape discrimination and display the desired spectra after photon rejection.
- 5) Store temporarily the events in a buffer area (to increase the maximum event rate that the system could handle, which was otherwise severely limited by the necessarily slow data transfer to the disk).
- 6) Transfer continuously, when not busy with the recording of events, the data from the buffer area onto the disk by incrementing by 1 for each event (T,H,X) the word $N(T,H,X)$ of the three-dimensional matrix represented by the disk's memory.
- 7) Transfer the contents of the disk at the end of each run onto magnetic tape cassettes for long-term storage. This transfer could also be effected the other way from a cassette to the disk.

2.6 Data collection

Five types of data have been collected.

- 1) Time calibration data. The requirement of a delayed-capture pulse by the D counter was removed, since it is the position of the photon peak to which mainly the cascade x-rays contribute that is used for the time calibration. The N-pulse was then delayed in steps of 10 ns with cables calibrated beforehand, and events were collected for approximately equal times for each delay. The collection of events

was then repeated with delays increased with respect to the previous ones by 5 ns. The spacing between delays was kept at 10 ns to prevent the photon peaks from merging.

After each data collection the disk contents were dumped onto a magnetic tape cassette.

A time calibration run was performed whenever gamma-peak position or gain of the time-of-flight amplifier were changed as a consequence of a change in gains or delays.

2) Delayed capture detection efficiency time spectra.

The detection efficiency of the delayed capture within the chosen time interval was mapped with respect to time, by recording the spectra of the time delay of the D counter pulses with respect to the stop time. Three types of data were collected for each target:

- a) D pulses, gated by the stop pulse (any D pulse accepted)
- b) D pulses vetoed by A, gated by the stop pulse (only non-prompt D-pulses accepted)
- c) D pulses vetoed by A, gated by an "event" pulse (where in this case by event is meant the occurrence of a D pulse within the delayed-capture time interval)

For each spectrum the total number of stops was recorded.

These spectra were recorded on paper tape.

3) Prompt-neutron data. By prompt-neutron data we mean

the collection of sets of (T,H,X) triplets corresponding to events as defined in Section 2.3, i.e. the occurrence of a stop followed by the detection of an N pulse, and a $D \cdot \bar{A}$ pulse within their proper respective time intervals. The

prompt-neutron data were collected in runs typically 5 hours long, for Bi (5 runs), Pb^{207} (4 runs of which one was partially lost), Pb^{206} (7 runs), Pb^{208} (4 runs), Ag (3 runs of which two not usable because of inadequate pulse-shape information due to malfunctioning of the pulse-shape discriminator), Tl (only one 3 hour long run) and natural Pb (1 run).

4) Total neutron spectra. For these time-of-flight

spectra the requirement of delayed capture has been dropped. Prompt and capture neutrons are likewise accepted.

5) Time-of-flight spectra for pion stops in a Bi target.

These spectra are total neutron spectra collected with (mainly) pions stopping in the target. They are used to check the system and its efficiency by comparing the neutron spectra obtained with a previous measurement⁵⁴ of the same spectra.

CHAPTER 3

DATA ANALYSIS

The data recorded on cassettes and subsequently transferred onto IBM magnetic tapes, and the remaining data were analyzed using the Xerox-Corporation Sigma 9 computer of Carleton University.

The analysis consisted of the following steps

- analysis of the channel-to-time calibration runs;
- rejection of the photon events by means of pulse-shape discrimination;
- collection of the photon-free, calibrated time spectra;
- background subtraction and conversion of the time spectra into energy spectra.

3.1 Channel-to-time calibration

A typical time-calibration spectrum is shown in Fig. 3.1-1. For each time calibration run there are 32 spectra of this kind corresponding to the 32 intervals into which the amplitude of the N-pulse has been divided.

The peaks appearing in this spectrum -- prompt gamma-rays and x-rays -- have been obtained in succession by increasing the delay of the N-pulse by 10 ns each time by means of calibrated ($\pm .1$ ns) delay-cables.

Fig. 3.1-1. Channel-to-time calibration spectrum for H=5, run5.

Fig. 3.1-2. Time walk of the N pulse, as a function of (the energy threshold of) H.

The analysis of the time-calibration spectra has been simply performed by calculating the centroids of the peaks and their widths. Centroid positions and their widths were fitted as a function of the time delay for each H interval by a least squares second-degree polynomial. The polynomials' coefficients were then stored in convenient arrays to be used in the analysis of the time-of-flight spectra.

For the analysis of the time-of-flight runs the constant terms of the fitting polynomials were not used: the values obtained from the photon peak of the time-of-flight spectrum to be calibrated were used instead. This was done to allow for changes in timing due to varying and uncontrollable experimental conditions.

Fig. 3.1-2 represents the walk of the N pulse as a function of H for one of the Bi runs.

3.2 Rejection of photon events

The discrimination between the gamma-rays and the neutrons detected by the N-counter was generally performed in two steps.

For each separate run the following X-spectra were collected:

$$N_A(X) = \sum_{T=T_{\min}}^{128} N(T, H, X) \quad \begin{array}{l} 1 \leq X \leq 16 \\ 1 \leq H \leq 32 \end{array}$$

where $T_{\min}$ was chosen to correspond to the maximum energy of the prompt neutrons expected for the target element investigated. This was done to obtain X-spectra in which the otherwise overwhelming presence of the gamma-rays in the distribution was drastically reduced to tolerable levels. This made it possible to differentiate with a higher degree of confidence between the neutron signal cross-over time distribution and that of the gammas.

The X-spectra corresponding to the higher values of H contain fewer events, so that it has been necessary to collect the X-spectra for $H > 6$ summing over two or more values of H, so as to have statistically meaningful spectra.

The second step was necessitated by the fact that the choice of the discriminating threshold for photon rejection was not obvious in all the cases. To help in determining the gamma-ray contamination of the events in a particular X channel for a particular value or interval of values of H, the time spectrum associated with it was displayed or plotted. A comparison of the ratio of the intensity of the existing peaks relative to a certain background, with that found for "good" X-channels -- i.e. those that certainly had little gamma contamination -- could help assessing the amount of photon contamination.

Once the X-discrimination threshold was chosen, the neutron retention efficiency of the discrimination was evaluated. Quite helpful in this process was the fact that the X spectra were displayed with the ordinate in a logarith-

mic scale, which made the extrapolation of the photon distribution easily executable with a straight line. The slope of the straight line was obtained in each case by collecting the X spectrum of the events $N(T,H,X)$ having T comprised in an interval about the photon peak (see continuous lines in Fig. 2.4-1).

The mean values of the neutron retention efficiency and of the photon contamination in the neutron area of the spectra are given in Table 3.2-1.

Table 3.2-1. Mean neutron retention efficiency and mean photon contamination after pulse-shape discrimination.

Target	Bi	Pb ²⁰⁸	Pb ²⁰⁷	Pb ²⁰⁶	Tl	Au
R_n^*	.88	.92	.94	.93	.95	.90
C_T^{**}	1.5%	1.2%	1.3%	1.6%	2.3%	1.0%

*) 5% uncertainty

**) 50% uncertainty

An average neutron retention efficiency of 90% and a minimum gamma-ray rejection ratio of 70 characterize the discriminations performed.

3.3 Time and energy resolution

There are several factors affecting the time resolution of the neutron time-of-flight spectrometer. The more important ones are (a) the finite size of the target and of the N-counter that entails a distribution of flight path

lengths; (b) the finite time resolution of the photo-multipliers involved; (c) the variation of the time of interaction of the radiation with the scintillator; (d) the finite decay time of the scintillator's light-emitting states; (e) walk in the time pick-off modules; and (f) instability of the system.

The collection of our time-of-flight spectra has of necessity been very slow and long (typical "event" rate was 2 events per second; typical duration of each run, 5 hours).

The walk of the discriminator triggering time as a function of the pulse amplitude can be partially corrected because time-of-flight spectra can be obtained separately for each H interval and it is possible to correct for the walk of the various H interval spectra.

It is apparent from the above that the time resolution is a function of, among other things, the shape and amplitude distributions of the triggering pulses, i.e. the B-counter pulses giving the time for the muon stop on one hand, and the N-counter pulses revealing a neutron or a gamma-ray on the other. The width and the shape of the gamma-ray peak can therefore provide only a good first approximation estimate of the time resolution of the neutron time-of-flight measuring apparatus.

When all the events are included, the root mean square deviation from the centroid position of the gamma-ray peak is for all the targets $1.1 \pm .1$ ns (Fig. 3.3-1), but after rejection of the photon events it has slightly higher

values: 1.27 ns for Bi, 1.49 ns for Pb^{208} , 1.40 ns for Pb^{207} , 1.41 ns for Pb^{206} , 1.40 ns for Tl and 1.39 ns for Au.

As to the energy resolution, since the energy is calculated using the non-relativistic formula

$$E(\text{MeV}) = .522 \frac{x(\text{cm})^2}{t(\text{ns})^2} \quad \begin{array}{l} x = \text{flight path length} \\ t = \text{flight duration} \end{array}$$

a simple error propagation formula gives for the root mean square energy-deviation

$$\frac{\sigma_E}{E} = 2 \cdot \sqrt{\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_t}{t}\right)^2}$$

It is possible to obtain a first approximation estimate of the average path length, $\bar{x}$, and of its standard deviation, $\sigma_{\bar{x}}$,

by approximating the square target with a circle having the same centre and the same area, and neglecting the terms of degree higher than the first in t/d , T/d , R/d (see adjacent figure). It turns out that the equivalent radius of the target is approximately equal -- within a few percent -- to the radius of the N-counter. In this approximation

$$\bar{x} = d + \frac{R^2}{2 \cdot d} \quad ; \quad \sigma_{\bar{x}} = \frac{1}{12} \cdot \sqrt{t^2 + T^2 + \frac{2 \cdot R^4}{d^2}}$$

Fig. 3.3-1 Bi prompt-neutron time-of-flight spectrum with all events included.

Fig. 3.3-2 Energy resolution of the prompt neutron spectrum of Bi

The values of $\bar{x}$ and $\sigma_{\bar{x}}$ so obtained for the various targets are reported in Table 2.2-2.

Double peaking is present in the γ -peaks for the runs following and including run 16. The spurious peak is centered at approximately 2 ns before the main peak and it is 20 times less intense. Its origin is not clear.

The energy resolution is given for Bi as a function of the energy in Fig. 3.3-2. This curve is obtained using the calculated $\sigma_{\bar{x}}$ and the σ_t obtained from the width of the calibration photon-peaks. σ_t was, averaging over the pulse amplitude intervals, constant within a 20% uncertainty. This result, together with the negligibility of the contribution of $\sigma_{\bar{x}}$, accounts for the $E^{3/2}$ behaviour of σ_E .

3.4 Neutron detection efficiency

Essential to determine both the absolute and the relative intensities of the prompt neutron lines is the knowledge of the N-counter efficiency for the detection of neutrons. In the present case this knowledge can be derived from two sources: an experimental measurement using a Cf^{252} source, and a calculation based on the cross-sections for neutron scattering from carbon and hydrogen for the scintillating liquid (NE213) used for the detection.

The experimental measurement of the time-of-flight spectrum of the neutrons emitted in the fission of Cf^{252} makes it possible to determine the neutron-detection

efficiency since the energy spectrum and the average yield of the fission neutrons of Cf^{252} is rather well known.⁵⁵ The stop signal in the time-of-flight measurement was given by a fission detector consisting of an argon gas scintillator surrounding the source. The spectrum obtained for 1.76×10^9 fissions for a 6' flight path is shown in Fig. 3.4-1.

However this measurement was not done under the same experimental conditions as for the collection of the prompt-neutron spectra, since only the earth's magnetic field was present and a different (.15 MeV higher) value for the neutron energy threshold was used. However, the effect of the magnetic field is essentially confined to the photomultication process -- and mainly to the collection of the photoelectrons by the first dynode --, and should alter only slightly the counter efficiency once allowance is made for the increase in neutron energy threshold. Therefore the higher neutron energy threshold in the Cf^{252} efficiency calibration has only the effect of preventing the determination of the efficiency at energies around the threshold and below.

As a check on the values of the neutron-detection efficiency determined in the above fashion, and as a means of determining it in the lower energy region just above experimental threshold, a theoretical efficiency curve was obtained, based on the calculations by Toms.⁵⁶ These calculations have been made for stilbene and NE213 scintillators of different shapes and sizes, and for neutron energies up to 24 MeV. The single scattering of neutrons by hydrogen

Fig. 3.4-1. Time-of-flight spectrum of Cf^{252} spontaneous-fission neutrons (photon counts rejected by PSD). Flight distance: 72". No. of fissions: 17.59×10^8 .

and carbon, as well as the effects of the double scatterings and the loss of recoil protons through the wall of the detector have been taken into account.

The agreement between the two determinations of the neutron-detection efficiency is good, as it can be seen in Fig. 3.4-2 where the little circles represent the experimental points whereas the continuous line reproduces the calculated efficiency curve.

In both cases the main source of error associated with the efficiency values is by far the uncertainty in the value of the energy threshold.

3.5 Analysis of the events contributing to the TOF spectra

Each time spectrum is made up of events whose total number per unit time is:

$$E(t) = S \cdot N(t) \cdot \xi_K$$

where

S = total number of stops;

$N(t)$ = probability per unit time for the occurrence of an N-pulse at time t (it is a function of the type of stop);

ξ_K = probability of detection of a $D \cdot \bar{A}$ pulse within the proper time interval (it is a function of the type of stop and a weaker function of the type of N-pulse).

The time correlation of the pulse contributing is shown in

Fig. 3.4-2 N-counter neutron detection efficiency as a function of neutron energy.

the figure:

The time spectra are the distributions of the delay time of the N-pulse with respect to the muon-stop time.

Not all the events recorded correspond to the physical event under study, i.e. the emission of a prompt neutron from the nucleus, because not all S-pulses correspond to muon stop in the chosen target element, not all the K-pulses are produced by nuclear radiation immediately following nuclear capture of the muon, and finally not all the N-pulses identify prompt neutrons.

A. Analysis of the stop pulses.

Let us examine first the various components of the total number of stops. The first information is provided by the differential range spectrum of the beam (Fig. 3.5-1) in polyethylene, measured by recording the stop rate in a target made with 1/4" of polyethylene absorber and increasing in steps of 1/4" the thickness of energy degrader of the same substance placed immediately upstream of the B-counter. The beam was coarsely monitored by the counting rate of the first counter of the "telescope".

It is not possible to determine the amount of pion contamination from the differential absorption curve but

Fig. 3.5-1 Differential range spectrum of the muon beam in polyethylene.

there is no hint of a contamination much higher than the quoted one ($< 1.8\%$).

The electron contamination that can be estimated from the absorption curves in the case of a target thickness equivalent to a polyethylene thickness of .9" (which is the case of the first Bi target) is of the order of 10%. The electron contamination is sufficiently reduced by the presence of the Cerenkov counter that had an electron rejection efficiency of 82%. A higher efficiency would have cost an excessive loss of muon stops. A final electron contamination of the order of 2% was therefore obtained.

The electron-rejection efficiency of the Cerenkov was measured by absorbing practically all the pions and the muons with 7" of polyethylene absorber. The momentum distribution of the electrons was not altered since the Cerenkov counter was preceding the absorber and in fact as close as possible to the concrete wall (Fig. 2.2-1), right against the A-counter.

The maximum CH_2 -equivalent thickness of a target (Bi_{II}) was 1.53". In this case the estimated electron contamination was of the order of 3.5%.

Besides the pion stops and the electron stops there are other possible "false" stops.

-- S_R : uncorrelated chance $A \cdot B_L \cdot B_R$ coincidences. They can be estimated from the recorded singles rates as a fraction of the total stop rate, and their values oscillate between 2% (Bi) and 3.2% (Pb^{206}), with the exception of

Pb^{207} (4.6%) that has the smallest amount of stopping material, i.e. the smallest total stop rate. The effect of S_R consists in contributing little and only to a random and therefore flat background.

-- S_C : apparent stops due to inefficiency of the C-counter. The efficiency of the C-counter has not been recorded, but it was presumably very high (> 99%), hence this contribution to a flat background is very small. The stops resulting from scattering of beam particles outside the solid angle covered by the C-counter are included in this stop rate.

-- S_{NT} : stops of muons occurring not in the target but in the B-counter, in the target container (in the case of the Pb^{206} and Pb^{208} targets), or in the wrong target element (again in the case of the Pb^{206} and Pb^{208} targets containing O, and N atoms as well, and in high percentage). If S_1 and S_2 are the stop rates respectively observed when the target is present and when it has been removed, we have obtained for the various targets:

Target	Bi _I	Bi _{II}	Pb^{208}	Pb^{207}	Pb^{206}	Au	Ag
$\frac{S_1 - S_2}{S_1}$	.912	.975	.957	.852	.795	.890	.919

For metallic targets S_{NT} is not too coarsely approximated by the 1's complements of the above ratios.

In the case of impure targets (Pb^{206} and Pb^{208} nitrates) an additional contribution to S_{NT} is given by those fractions of beam particles that stop in the other elements of the target.

In conclusion we can separate the total number of stops in components according to the formula

$$S = S_T + S_{NT} + S_\pi + S^R$$

where

S_T are the muon stops in the target element under study,

S_{NT} are muon stops taking place elsewhere,

S are pion stops anywhere,

S^R are the remaining types of stops, real and not real.

B. Analysis of the N pulses.

Let us examine next the various main components of the probability of occurrence of an N-pulse per muon stop per unit time. Only neutral particles will be considered since any charged particles detected by the N counter are vetoed by the V counter.

$$N = N^{pn} + N^{p\gamma} + N^{cn} + N^{c\gamma} + N^{c'n} + N^{c'\gamma} + N^{\pi cn} + N^{\pi c\gamma} + N^R \quad (1)$$

with

N^{pn} = prompt neutrons (if any) emitted during non-radiative muon transitions.

$N^{p\gamma}$ = prompt gamma-rays and cascade x-rays following μ -stop

N^{cn} = neutrons emitted following muon-capture in desired target element

$N^{c\gamma}$ = gamma-rays emitted following muon-capture in
desired target element

$N^{c'n}$ = neutrons emitted following muon-capture else-
where

$N^{c'\gamma}$ = gamma-rays emitted following muon-capture else-
where

$N^{\pi cn}$ = neutrons emitted following pion-capture anywhere

$N^{\pi c\gamma}$ = gamma-rays emitted following pion-capture
anywhere

N^R = uncorrelated and noise pulses in the N-counter.

All the terms in this equation are a function of time
measured relative to the stop time, e.g. $N^{pn} = N^{pn}(t)$.

All these N-components are present in the total neutron
spectra, modulated by the N-counter efficiency. However all
the gamma-ray components, that dominate if all the events
are included (compare Fig. 3.5-2 and 3.5-3) are strongly
reduced by the photon rejection following the pulse-shape
discrimination and become almost negligible everywhere but
in the photon-peak region of the spectrum.

It is possible to make an estimate of the various
components of N contributing to the total neutron spectra
availing oneself of the calculated values for the neutron
efficiency of the N-counter, the recorded total number of
stops of each spectrum, the calculated values of neutron
retention efficiency and of photon rejection efficiency of
the pulse-shape discrimination, and the radiation yields
and the lifetimes of the various processes listed in eq. (1),

Fig. 3.5-2 Total neutron spectra for the various targets after rejection of photon events by pulse-shape discrimination.

Fig. 3.5-3 Bi total time spectrum after rejection of the neutron events by PSD.

Fig. 3.5-4 Time-of-flight spectrum of neutrons emitted following nuclear pion capture in Bi (photon events rejected by PSD).

available from the literature.^{45,46,47,49} In Table 3.5-1 we report the calculated estimates for the two cases of Bi and Pb²⁰⁸, typical respectively of the metallic targets and of the composite ones.

Table 3.5-1. Estimates of the main components of the total neutron spectra of Bi²⁰⁹ and of Pb²⁰⁸.

Target	N ^{pn}	N ^{pγ}	N ^{cn}	N ^{cγ}	N ^{c'n}	N ^{c'γ}	N ^{ncn}	N ^{ncγ}	N ^R	Σ N
Bi ²⁰⁹	192	58	1574	10	4	0	90	0	54	1982
Pb ²⁰⁸	34	18	390	5	35	1	97	0	48	628

The totals from the total neutron spectra (Fig. 3.5-2), 1903 for Bi and 541 for Pb²⁰⁸, compare well with the estimates. A yield of conversion neutrons of 5% was assumed for Pb²⁰⁸, while for Bi that yield was assumed to be 7%.

About the time distributions of the various components of the N spectrum we know that

- N^{pn} begins at the time corresponding to the time-of-flight of the prompt neutrons of maximum energy (transition of the muon from a near-free state to the 1s state);
- N^{pγ} is all in the photon peak;
- N^{cn} starts building up at around time 20 ns (relatively few are the capture neutrons with an energy 4-5 MeV and the N-counter efficiency decreases faster beyond that energy) and increases as capture neutrons of lower and lower energy start contributing but then start exponentially decreasing with a time constant of 70 ns; the maximum is reached somewhere around the neutron counter energy threshold,

i.e. .67 MeV corresponding to a time of flight of 54 ns;

-- $N^{c\gamma}$ has an exponential time decay beginning at time zero with a time constant equal to the muon-capture lifetime;

-- $N^{c'n}$ and $N^{c'\gamma}$ follow closely the N^{cn} and $N^{c\gamma}$ distributions but there is no appreciable decay because of the much longer lifetime in the other (light) elements;

-- $N^{\pi cn}$ has the time distribution of Fig. 3.5-4, which is the time spectrum of the neutrons emitted following nuclear pion-capture in Bi, obtained as described in Section 2.6 paragraph 5);

-- $N^{\pi c\gamma}$ is all in the photon peak.

The main contribution to the background of the prompt-neutron time-of-flight spectra comes therefore, in the case of monoelemental targets, from the capture neutrons emitted by the target element; in the case of the Pb^{206} and Pb^{208} targets, which are lead nitrates, proportionally higher is the contribution to the background from neutrons emitted following nuclear pion capture in the other target elements.

C. Analysis of the K pulses.

We shall examine next the main possible contribution to the rate of the nuclear capture pulse $K = D \cdot \bar{A}$. It is any D-counter pulse not coinciding with a beam particle arrival and occurring within a time interval beginning 65 ns after the muon stop. This time interval was (91 ± 9) ns long for the first 26 experimental runs, and (141 ± 5) ns long for the remaining runs.

The D-counter threshold was set, for the first 20 runs, at an energy of 70 keV for photons or 660 keV for neutrons and from run 21 on was set at the lowest possible value of ~40 keV for photons or 430 keV for neutrons.

The components of the K signal are the same as those of the N-counter pulse, with the addition of another component due to charged particles emitted following nuclear muon capture and the exclusion of all the prompt photon components and part of the prompt neutron ones.

We can decompose the K pulses as follows:

$$K(t) = K^C(t) + K^{C'}(t) + K^R(t) + K^{cn}(t) + K^{C'n}(t) + K^{cn}(t)$$

Where

K^C represents the pulses produced by any radiation following nuclear muon capture in the element under study,

$K^{C'}$ represents the $D \cdot \bar{A}$ pulses produced by radiation following nuclear capture in other elements,

K^R represents any type of pulses, including photo-multiplier noise pulses, that are not correlated to the muon stop with which they are associated,

K^{cn} represents the $D \cdot \bar{A}$ pulses produced by radiation following nuclear capture of a neutron emitted following nuclear muon capture in the desired target element within the time interval reserved for the N pulses,

$K^{C'n}$ is the equivalent of K^{cn} , relative to muon capture in any other element,

$K^{\pi en}$ is the equivalent of K^{en} , relative to pion capture anywhere.

Because of the time constraints imposed on the K^- and K^0 -pulses relative to the stop pulse, no other K -pulses can contribute significantly to the background of the time-of-flight spectra.

We have recorded for each "run" the total number of stops and the total ϵ_K efficiency, that is the percentage of the number of stops to which corresponds a K -pulse in the selected time interval. What is necessary to know, to determine the number of stops that has contributed to the measured conversion neutron spectrum in each run, is the ratio $\epsilon_K^c / \epsilon_K$, i.e. the percentage of the total number of K -pulses accepted, corresponding to nuclear muon capture in the proper target element. ϵ_K^c , ϵ_K^R and ϵ_K^{π} are the probabilities that any stop be followed by respectively a K^c pulse, a K^R pulse and a K^{π} pulse, within the proper time interval.

The value of ϵ_K^R will be shown to be needed to obtain an estimate of an important component of the background of the prompt time-of-flight spectra.

To determine the efficiencies ϵ_K , ϵ_K^R , ϵ_K^c and also to measure the nuclear muon-capture lifetime, we have collected the D^- and K -efficiency curves, i.e. the time spectra of the frequency of the D and K pulses measured with respect to the stop time. The spectra have been analysed to derive the ratios $\epsilon_K^R / \epsilon_K$ and $\epsilon_K^c / \epsilon_K$. In

the course of the analysis it was possible to get a rather good estimate of the lifetime of the muon in the 1s orbit of the target element (see Appendix). We list in Table 4.1-1 the values of ξ_K^c and ξ_K / ξ_K^R for each target, those values being the weighted averages over all the runs of interest for each target, each run being weighted with the corresponding total number of stops.

3.6 Background evaluation and subtraction

Taking into account the analysis made in the previous paragraph, the main components of the events appearing in the conversion neutron spectra are given by:

$$\begin{aligned}
 Ev &= S \cdot N \cdot K \\
 &= (S_T + S_{NT} + S_{\pi} + S^{R'}) \\
 &\quad \cdot (N^{pn} + N^{p\bar{p}} + N^{cn} + N^{c\bar{c}} + N^{c'n} + N^{c'\bar{c}} + N^{\pi cn} + N^{\pi c\bar{c}} + N^R) \\
 &\quad \cdot (\xi_K^c + \xi_K^{c'} + \xi_K^{cn} + \xi_K^{c'n} + \xi_K^{\pi cn} + \xi_K^R)
 \end{aligned}$$

By multiplying and omitting the products that are zeroed by the timing constraints and by the physical ones (for instance: S_{π} can only be associated with $N^{\pi cn}$, $N^{\pi c\bar{c}}$ and N^R) we obtain:

$$\begin{aligned}
 Ev &\approx S_T \cdot (N^{pn}(t) + N^{p\bar{p}}(t)) \cdot (\xi_K^c + \xi_K^R + \xi_K^{cn}) & Ev1 \\
 &+ S_T \cdot (N^{cn}(t) + N^{c\bar{c}}(t)) \cdot \xi_K^R & Ev2 \\
 &+ S_{NT} \cdot (N^{c'n}(t) + N^{c'\bar{c}}(t)) \cdot \xi_K^R & Ev3 \\
 &+ S_{\pi} \cdot (N^{\pi cn}(t) + N^{\pi c\bar{c}}(t)) \cdot \xi_K^R & Ev4 \\
 &+ S_T \cdot [N^{cn}(t)(1-f^{cn}) + N^{c\bar{c}}(t)] \cdot \xi_K^{cn} & Ev5
 \end{aligned} \tag{1}$$

$$+ S_{NT} \cdot [N^{c'n}(t)(1-f^{c'n}) + N^{c'\gamma}(t)] \cdot \epsilon_K^{c'n} \quad \text{Ev6}$$

$$+ S_{\pi} \cdot [N^{\pi cn}(t)(1-f^{\pi cn}) + N^{\pi c\gamma}(t)] \cdot \epsilon_K^{\pi cn} \quad \text{Ev7}$$

$$+ S \cdot N^R \cdot \epsilon_K \quad \text{Ev8}$$

where the new symbols f^{cn} , $f^{c'n}$ and $f^{\pi cn}$ represent the fractions of respectively ϵ_K^{cn} , $\epsilon_K^{c'n}$ and $\epsilon_K^{\pi cn}$, due to emission of a single neutron following muon or pion capture. In eq. (1) we have introduced ϵ_K^{cn} and $\epsilon_K^{\pi cn}$ which are the probabilities that respectively a muon stop and a pion stop be followed by nuclear capture producing the emission of two or more neutrons one of which has such an energy ($> \sim .7$ MeV) to be detected by the N counter and the other, after having scattered in the D counter or elsewhere, undergoes nuclear capture with capture radiation being detected in the D counter within the proper time interval. The one just considered is a process involving too many degrees of freedom for us to be able to assess directly the importance of its contribution.

The photon subcomponent of each component is drastically reduced by PSD rejection of the photon events so that its contribution is generally a minor one. The only exception is Ev1 whose photon subcomponent, consisting mainly of cascade x-rays, is quite intense and determines with its position a reference time used for calibrating the time coordinate.

Let us examine briefly the various components.

Ev1 represents the desired prompt events.

Ev2 is the contribution of the neutrons and gamma-rays produced by nuclear muon capture. This capture radiation is only gated, as we shall show later, by random K pulses.

Ev3 is the contribution of capture radiation following a muon stop in any element other than the one under study. Only random K-pulses can gate it. It constitutes a minor component, with respect to Ev2, for metallic targets, both because the corresponding stops are a smaller fraction of the total and because of the much lower efficiency of capture detection within the proper time interval, due to the much longer lifetime of the muon before nuclear capture. Also smaller in the lighter elements is the yield of capture radiation.

Ev4 is the background due to the pion contamination of the beam. It is small both because of the low level of the pion contamination and because the pion capture radiation is gated only by ξ_K^R and $\xi_K^{\pi cn}$. This follows from the prompt capture of the pions and the consequent lack of correlated capture pulses having amplitude above the D-counter threshold.

Ev5 are background events that will appear to be non-negligible and whose contribution is hard to assess. They consist of muon stops followed by nuclear muon capture following which a gamma-ray (N^{γ} term) or a neutron (N^{cn} term) are emitted together with a mainly-low energy neutron that is eventually captured emitting radiation that is detected in the D counter within the proper time interval.

The first gamma-ray or neutron, if it has energy above a threshold which is a function of the time of muon capture, is detected by the N-counter. Only a fraction $(1-f^{cn})$ of the efficiency ξ_K^{cn} operates for the N^{cn} term, since for this term the captures followed by emission of only one neutron are not contributing.

Ev6 are the background events equivalent to Ev5 but due to muon stops in elements other than the one under study.

Ev7 are the background events equivalent to Ev5 but due to pion stops anywhere.

Ev8 is a background that contributes a constant level to the time spectra. It consists of several contributions that can be collectively described as events due to all the effective stops, i.e. to all the stops gated by any K pulse, with which coincides an uncorrelated or random N pulse.

The contribution of Ev5, Ev6, Ev7 to the background is hard to determine both quantitatively and qualitatively from the point of view of its dependence on time. The difficulty consists in the fact that the processes producing them have a large number of degrees of freedom that make it impossible to estimate them with any satisfactory approximation.

The equation (1) shows clearly the convenience of imposing on the event the delayed-capture condition. In fact, while the prompt neutrons are gated by the D counter with efficiency $(\xi_K^c + \xi_K^{cn} + \xi_K^R)$, the capture neutrons are

only gated by it with efficiency $(\xi_K^{cn} + \xi_K^R)$.

Unfortunately, for the reason stated above, it is not possible to estimate ξ_K^{cn} , so that the values of $(\xi_K^c + \xi_K^R) / \xi_K^R$ reported in Table 4.0-1, are only partially indicative of the amount of the "amplification" of the prompt-neutron events relative to the capture-neutron events due to the imposition of the delayed-capture condition.

For the metallic targets it is possible to effect a subtraction of the background due to the events of the type of Ev2, Ev3 and Ev4, using the total neutron spectra collected without the delayed-capture requirement. This subtraction is based on the fact that in the total neutron spectra the background dominates with respect to the signal while the random part of the background is negligible, so that in a first approximation the total neutron spectra coincide with the time spectrum of the capture neutrons. Thus one has to normalize the appropriate total neutron spectrum to the same number of stops as the corresponding prompt-neutron spectrum, multiply it by the value of K^R and finally subtract it from the prompt spectrum.

This subtraction can be refined by proceeding one more step. The prompt spectrum so far obtained can be normalized so that its area corresponds to the estimated contribution of the prompt neutrons to the total N spectrum, or better yet, since this estimate cannot be quite accurate, it can be normalized so that its peaks, if they exist, cancel the

peaks present in the total N spectrum. This is based on the fact that the background cannot have any peaks that be not of statistical nature, since the nuclear capture probability is a continuous (exponentially decreasing with a lifetime long enough compared to the maximum time-of-flight) function of time, and in correspondence to hypothetical peaks -- not found experimentally⁴⁴ -- in the energy distribution of the capture neutrons only steps would appear in the time-of-flight spectra. The capture neutron spectra, on the other hand, are known to have a smooth energy distribution.⁴⁴ The total neutron spectra thus freed of the prompt component can then be used for the final background subtraction in the aforementioned manner.

We list in Table 3.6-1 the factors by which we have to multiply the total neutron spectra to subtract them from the corresponding prompt neutron spectra. They are

$$F = \frac{S^P \cdot K^R}{S^t}$$

where S^P = number of stops of the prompt neutron spectrum

K^R = random K efficiency

S^t = number of stops of the corresponding total neutron spectrum.

The background subtraction would be complete under the assumptions that the ξ_K^{cn} efficiency is zero and that the random N-pulse contribution to the total neutron spectra be negligible. This is not the case. In fact, while the

contribution of the random-N, flat background is rather small for the total neutron spectra, that contribution appears in the prompt spectra magnified by a factor ϵ_K/ϵ_K^R with respect to the rest of the background and is no more negligible.

Table 3.6-1. Normalization factors of the total neutron spectra for partial background subtraction.

Target	Bi	Pb ²⁰⁸	Pb ²⁰⁷	Pb ²⁰⁶	Tl	Au	Ag
F	.86 ⁺ .09		.89 ⁺ .08		.19 ⁺ .02		.10 ⁺ .01
		1.22 ⁺ .12		1.82 ⁺ .18		.20 ⁺ .02	

Also to be noticed is the fact that the background due to the photons that have not been rejected by pulse-shape discrimination is not completely or adequately taken into account by the abovementioned subtraction, since the photon rejection has been performed much more efficiently in the total neutron spectra (proof of this is the reduction of the photon peak: by 330 times in the Bi total neutron spectrum and by only 70 times in the final prompt neutron spectrum for Bi).

The more efficient rejection of photon events for the total neutron spectra entails for the same spectra a lower retention efficiency of the neutron events, i.e. a non-negligible rejection of neutron events. It is to be

expected that the low-H events are more strongly affected by this rejection. Unfortunately the amount of neutron event rejection cannot be estimated for the total neutron spectra, since no record of the photon-neutron pulse-shape discrimination has been kept, although the examination of the X spectra of the Pb^{208} data (run 47) collected just before the total neutron spectra suggests that an increase in the reduction of the photon peak from 70 times to 300 times can entail the rejection of up to 30% of the neutron events. 30% is just the order of magnitude of the upper limit. Because of what has just been said the F-factors used to normalize the total neutron spectra are underestimated.

From Fig. 3.5-3, where the total neutron time spectrum for Bi is shown after rejection of the neutron events by pulse-shape discrimination, we can draw the conclusion that the contribution of the capture gamma-rays to the photon background is not very large, since the negative slope of the spectrum is small and therefore only a small fraction of it follows a spectrum decreasing exponentially with a 70 ns decay constant.

Only a fraction $(\xi_K^R + \xi_K^{cn}) / \xi_K$ of the background component Ev8 (due to random N-pulses) is removed as a consequence of the background subtraction effected by means of the photon-freed total neutron spectra. Unfortunately the value of the Ev8 background level cannot be determined with confidence since the prompt time spectra do not extend far

enough before the photon peak. As the best substitute we shall subtract a flat photon background equal to the average count level of the "valley" region of the prompt neutron spectrum, i.e. that region following the photon peak in which the contribution of the capture neutrons to the background is not yet significant. The part of the spectrum preceding the photon peak before the channel-to-time calibration, leads one to conclude that the levels of counts before the peak and in the "valley" are approximately the same.

After performing the background subtraction following the above criteria (an example of such a subtraction is shown in Fig. 4.2-16 for the Bi prompt neutron spectrum by the curve marked with x's) we notice that, while the shape of the background appears to be rather well followed, its intensity is low, i.e. the background is not totally subtracted. It appears that a proper subtraction could be effected by using an F factor little more than twice as large as the one derived. The reason for the above conclusion is given by the consideration that the energy threshold of the N-counter is set at a neutron energy $(.67 \pm .08)$ MeV corresponding to a time-of-flight of (54 ± 3) ns and that therefore in the final part of the spectrum one would expect that close to 100% of the events be background.

This result brings us to the conclusion that ξ_K^{cn} is not negligible but it is of the same order of magnitude

as ξ_K^R .

There is also a way to determine a minimum level for the background at the long-time end of the spectrum, which is made viable by the presence of neutron peaks near the long-time end of the spectra. The time-of-flight spectra are plotted for each N-pulse height interval H_i ($i = 1, 32$) or, if there are few events present in the individual H interval spectra, for consecutive groups of them. As i increases, the energy threshold for the corresponding time-spectrum increases too, or, in other terms, the maximum time-of-flight limit corresponding to it decreases.

An estimate, however inaccurate, of the energy threshold corresponding to any H_i can be derived from the N-pulse amplitude corresponding to the Compton edge of the Na^{22} gamma-ray used to calibrate the N counter. This estimate can be checked with the single H_i time-of-flight spectra of the neutrons emitted following pion capture in Bi (see Fig. 3.6-1).

The presence of the peaks of the prompt-neutron distribution helps to determine the position of the energy threshold and the level of the background: in fact we can assume that if a certain peak at energy E is not present in the spectrum then all the events corresponding to that energy and to lower energies belong to the background. However this method is of help only in the cases of Bi and Pb^{207} where the presence of strong peaks and good statistics make it practicable. One finds that in the case of Pb^{207}

the minimum background at 55 ns time-of-flight corresponds to a normalization factor for the total neutron spectrum $F=1.36$, while a complete subtraction of the count level at 55 ns entails $F=1.89$. In the case of Bi the corresponding values are respectively 1.4 and 1.8.

The final criterion followed in the background subtraction has been that of subtracting a constant background due to the random N-pulses and then subtracting the corresponding total neutron spectrum normalized to cancel the counts at the end of the spectrum.

The assumption that the total neutron spectra are uniformly gated, as a function of time, by the D-counter efficiencies ξ_K^{cn} and the like is only tenuously based on the background subtraction effected following the above criterion in the case of the Ag "prompt neutron" spectrum. This spectrum, which, as we shall justify later, cannot contain prompt neutrons, produces a good fit of the prompt spectrum (see Fig.4.2-2).

The tenuousness of the argument consists in the unfortunately poor statistics of both the Ag spectra.

It is evident from what has just been said that the background subtraction will be in this experiment one of the main sources of uncertainty for the neutron yields.

Fig. 3.6-1 Time-of-flight spectra for single H intervals of the ~~total following nuclear pion capture in Bi.~~

CHAPTER 4

RESULTS AND DISCUSSION

4.1 The final spectra

The results of the experiment can be completely expressed in terms of the time-of-flight (TOF) spectra obtained with the above described experimental set-up and manipulation of the data, and in terms of the energy spectra derived from the time spectra after subtraction of the background. However the uncertainty in the determination of the background, and hence the arbitrariness in its subtraction, makes of the energy spectra a partially biased, even though more intuitive, presentation of the results.

The energy spectra have been normalized so that the ordinates will represent the probability of emission of conversion neutrons per muon cascade and per unit energy interval (50 keV). This means that the number of neutrons measured in each energy interval has been divided by the number of effective muon stops (those for which a delayed capture has been detected) multiplied by the neutron-detection efficiency of the N counter and a fraction of solid angle subtended by it.

In Table 4.1-1 are reported total running time, total number of stops, D-counter and N-counter rates, and efficien-

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TABLE 4.1-1

COUNTS, RATES AND EFFICIENCIES OF IMPORTANCE FOR THE FINAL SPECTRA OF EACH TARGET

Target	Total time (10^4 s)	Total no. of stops (10^8)	K^C * (%)	Effective no. of stops (10^7)	K^R ** (%)	$(K^C + K^R)/K^R$	D· $\bar{A}$ rate (10^4 s $^{-1}$)	N· $\bar{V}$ rate (s $^{-1}$)
Bi	6.48	17.35	8.3 $\pm$.4	14.4 $\pm$.7	.66	13 $\pm$ 2	5.8	575
Pb ²⁰⁸	7.20	17.54	2.6 $\pm$.3	4.6 $\pm$.5	.99	3 $\pm$ 1	7.0	600
Pb ²⁰⁷	5.58	7.48	7.5 $\pm$.3	5.6 $\pm$.2	.86	9 $\pm$ 2	7.6	640
Pb ²⁰⁶	11.44	21.56	2.4 $\pm$.3	5.2 $\pm$.6	.85	3 $\pm$ 1	6.0	640
Nat. Tl	1.26	1.99	7.4 $\pm$.5	1.47 $\pm$.10	.90	8 $\pm$ 2	6.4	660
Au	1.26	2.34	9.3 $\pm$.3	2.18 $\pm$.07	.81	12 $\pm$ 2	5.8	610
Nat. Ag	.90	2.61	9.9 $\pm$.3	2.58 $\pm$.08	.55	18 $\pm$ 2	6.1	700

*) Percentage of muon stops for which a delayed nuclear capture in the desired target-element has been detected.

**) Random K^R efficiency, i.e. probability of simulation of a delayed capture by a random K pulse in the proper time interval. The uncertainty of the quoted values is of the order of 10%.

TABLE 4.1-2

CONVERSION NEUTRON ENERGIES FOR EACH TARGET ELEMENT Z^A , CORRESPONDING TO SOME $(nlj \rightarrow 1s\frac{1}{2})$ -
MUONIC TRANSITIONS WHEN THE DAUGHTER NUCLEUS Z^{A-1} IS LEFT IN ITS GROUND STATE

n l j	$E_{nlj} - E_{1s\frac{1}{2}} - S^n$ (MeV) *						References
	Bi 209 c	Pb 208 b	Pb 207 b	Pb 206 b	Tl 205 c	Tl 203 c	Au 197 c
3s1/2	1.51	1.46	2.09	0.75	1.15	0.97	0.36
3p1/2	1.12	1.09	1.72	0.38	0.79	0.61	0.01
3p3/2	1.17	1.13	1.77	0.43	0.84	0.66	0.06
3d3/2	1.08	1.05	1.69	0.35	0.76	0.58	0
3d5/2	1.13	1.09	1.73	0.39	0.80	0.62	0.03
4s1/2	2.25	2.19	2.82	1.48	1.86	1.68	1.03
4p1/2	2.08	2.02	2.66	1.32	1.71	1.53	0.90
4p3/2	2.10	2.04	2.68	1.34	1.73	1.55	0.91
4d3/2	2.06	2.01	2.64	1.30	1.69	1.51	0.88
4d5/2	2.08	2.02	2.66	1.32	1.71	1.53	0.89
5s1/2	2.62	2.54	3.18	1.84	2.21	2.03	1.36
5p1/2	2.53	2.46	3.09	1.75	2.14	1.96	1.29
6p3/2	2.76	2.70	3.33	1.99	2.36	2.18	1.50
Series limit	3.31	3.23	3.86	2.52	2.88	2.70	2.00
S^n a	7.453	7.368	6.741	8.081	7.541	7.72	8.08
(MeV)	± 0.11	± 0.02	± 0.02	± 0.04	± 0.04	± 0.02	± 0.01

*) The underlined values have been calculated with the point-nucleus solution of the Dirac equation for the muon, corrected for the nuclear extension by extrapolating the correction factors from the nearby nuclei for which the correct energy levels are known.

cies of delayed capture detection for all the targets examined, while in Table 4.1-2 we report the conversion neutron energies corresponding to some $(nlj \rightarrow 1s\frac{1}{2})$ muonic transitions for the case in which the daughter nucleus Z^{A-1} is left in its ground state.

4.2 Reading the spectra

A. Au¹⁹⁷

The time-of-flight spectrum obtained for gold (Fig. 4.2-1) has few events and a rather high background. To the background belong at least 2/3 of the events. The spectrum seems to show above the background, between approximately 32 and 52 ns, some conversion neutron events in the form of humps. This is suggested also by comparison with the total neutron spectrum for gold (Fig. 3.5-2): this spectrum increases monotonically as a function of time up to 60 ns.

The background subtraction by means of the corresponding prompt neutron spectrum is shown in the figure. We have already argued (Section 3.6) that the goodness of this subtraction is based on the assumption that the muon and pion capture neutron spectra (which account for ~90% of the total neutron spectrum) are gated by the $D \cdot \bar{A}$ pulse with efficiencies ϵ_K^{cn} , $\epsilon_K^{c'n}$ and $\epsilon_K^{\pi cn}$ which are independent of time. A tenuous proof that in a first approximation this is so is given by the examination of the prompt and total neutron spectra obtained for natural Ag (Fig. 4.2-2). The total neutron spectra -- renormalized to the number of stops of

Fig. 4.2-1 Au^{197} prompt neutron TOF spectrum. The circled points identify the background subtraction by means of the corresponding total neutron spectrum, with a normalization factor $F=0.6$.

Fig. 4.2-2 Natural-Ag "prompt" neutron TOF spectrum. The circled points represent the subtraction of the total neutron spectrum, with a normalization factor $F=0.2$.

the prompt spectrum -- is there represented by the level of the little circles relative to the flat background.

For silver the binding energy of the $1s\frac{1}{2}$ muon is only 4.747 keV⁵¹ and therefore no muonic transition can provide sufficient energy for neutron ejection from the nucleus. The neutron separation energy is⁵² in fact 9.55 MeV and 9.19 MeV for Ag^{107} and Ag^{109} respectively. The background subtraction, under the aforementioned assumption, should therefore cancel the entire prompt neutron spectrum. Within the limits of the poor statistics available -- hence the extreme weakness of the argument -- this is what happens.

The weakness of the above argument contributes to increase the degree of uncertainty of the background level (represented by the vertical segments accompanying the background circlets) and therefore the uncertainty in the interpretation of the humps in the prompt time-of-flight spectrum for Au as due to conversion neutrons.

We notice that all the counts above background correspond to energies smaller than the series limit (Table 4.1-2).

The interpretation of the humps in the Au-spectrum distribution as due to conversion neutrons is not quite certain, since the numbers involved are small and the statistical fluctuation relatively large, while the background is relatively high.

The main hump is centered at 45.5 ns, corresponding to an energy $E = .94$ MeV, which could be interpreted as due to the $4 \rightarrow 1$ transitions. It would be imprudent to read in this

spectrum more than the possible existence of conversion neutrons and a maximum limit for their yield. A necessarily approximate estimate of the yield per muon cascade of conversion neutrons gives:

$$y_{\text{Au}}^{\text{En}} \approx .8 \text{ MeV} = (3.0 \pm 2.2)\%.$$

B. Natural Tl

What has been said for the Au spectrum about the poverty of the statistics and the high background, can be repeated here for the thallium prompt spectrum (Fig. 4.2-3 and 4.2-4). Even smaller is in this case the number of effective muon cascades contributing. Hence there is no conclusive proof of existence of conversion neutrons. A very uncertain estimate of the yield per muon cascade gives:

$$y_{\text{Nat. Tl}}^{\text{En} \approx .95 \text{ MeV}} = (1.6 \pm 1.0)\%$$

and

$$y_{\text{Nat. Tl}}^{\text{En} \approx .75 \text{ MeV}} = (5.2 \pm 3.6)\%$$

These yield estimates apply to a thallium target with natural isotopic abundance (70.5% Tl^{205} and 19.5% Tl^{203}).

In this case the spectrum is richer with hints of peaks centered respectively at $(.82 \pm .10) \text{ MeV}$, $(1.08 \pm .06) \text{ MeV}$, $(1.35 \pm .10) \text{ MeV}$, $(1.67 \pm .15) \text{ MeV}$, $(1.90 \pm .16) \text{ MeV}$ and $(2.6 \pm .3) \text{ MeV}$.

No assignments to muonic transitions can safely be made.

Fig. 4-2.3 Natural-Tl prompt neutron TOF spectrum with indication of the background subtraction by means of the total neutron spectrum ($F = 0.38$).

Fig. 4-2.4 Natural-Tl prompt neutron energy spectrum. $P(E_n)$ is probability per hundred muon cascades.

C. Pb²⁰⁶

The prompt time-spectrum for Pb²⁰⁶ (Fig. 4.2-5) has a high background due to the nitrate form of the target: the muon-stops in the other elements (N and O) contribute with a mainly flat background. The time interval in which delayed capture pulses were accepted was too long and therefore accepting an unnecessarily high percentage of muon stops in other elements (see the K-efficiency curve for Pb²⁰⁶) and decreasing the $(\epsilon_K^C + \epsilon_K^R)/\epsilon_K^R$ ratio.

The TOF spectrum shows a sharp rise at a time $t=26.5$ ns, which could be interpreted as a peak centered at 27 ns and corresponding to an energy of $(2.65 \pm .25)$ MeV. From Table 4.1-2 it appears that the maximum energy of the conversion neutrons cannot be larger than $(2.52 \pm .01)$ MeV.

The sharp rise in the number of counts can only be accounted for in terms of conversion neutrons (and in this case statistical fluctuation and uncertainty in the time calibration have to account for the shift in position): in fact it cannot be all due to capture neutrons or to photon events. It cannot be all due to capture neutrons because the experimental capture-neutron energy spectra⁴⁴ do not show any discontinuity at an energy corresponding to the time position of the sharp rise. On the other hand, a hypothetical spurious photon peak cannot seem to contribute to the sharp rise in counts because the peak that is present in the time spectrum containing both photon and neutron events (Fig. 4.2-6) and which is larger than the peak in question, disappears

Fig. 4.2-5 Pb^{206} prompt neutron TOF spectrum. Background estimated by means of the total neutron spectrum, with normalization factor $F = 2.6$.

Fig. 4.2-6 Pb^{206} prompt neutron TOF spectrum with photon events included.

completely with the PSD rejection of the photon events, while the peak in question is affected very little by the PSD. Finally, the sharp rise in counts cannot contain but a small proportion of neutrons produced in the capture of the pions present in the beam, both because the intensity of the peak is far too high relative to the pion contamination of the beam, and because the spectrum of the pion-capture neutrons has a continuous distribution (Fig. 3.5-4).

The prompt spectrum that includes only the neutron events for which $H \geq 5$ (Fig. 4.2-7) has a better signal-to-background ratio for the shorter flight-times, and shows a more regular but still sharp discontinuity at 26.5 ns time.

We conclude therefore -- even though a non-negligible element of doubt persists -- that conversion neutrons are emitted during a fraction of the muon cascades in Pb^{206} .

The estimated yield of conversion neutrons per muon cascade is

$$Y_{\text{Pb}^{206}}^{E_n \geq .95 \text{ MeV}} = (1.8 \pm .8)\%.$$

The energy spectrum (Fig. 4.2-8) hints at a few peaks centered at $(1.24 \pm .08) \text{ MeV}$, $(1.42 \pm .12) \text{ MeV}$, $(1.62 \pm .15) \text{ MeV}$, $(1.92 \pm .16) \text{ MeV}$, $(2.30 \pm .22) \text{ MeV}$ and $(2.65 \pm .25) \text{ MeV}$; but assessing their reality and assigning them to specific muonic transitions can only be guesswork.

D. Pb^{207}

For this isotope of lead the nuclear emission of conversion neutrons induced by the cascading muon is unquestionable

Fig. 4.2-7 Pb^{206} prompt neutron TOF spectrum for $H \geq 5$.

Fig. 4.2-8 Pb^{206} prompt neutron energy spectrum.

since intense and relatively sharp peaks are present in the prompt neutron TOF spectrum (Fig. 4.2-9).

The prompt neutron TOF spectra corresponding to individual H intervals or groups of them (Fig 4.2-10) help to determine a minimum value for the background level as well as allowing for a rough but meaningful check on the proper relative contribution of the various amplitude intervals to the neutron peaks. In other words they constitute a check on the energy of the neutron events contributing to any particular peak of the spectrum.

Another advantage in collecting spectra corresponding to selected groups of values of H, i.e. only those values contributing mostly or exclusively to a certain peak, consists in the maximization of the signal-to-background ratio in selected regions of the TOF spectrum (see Fig. 4.2-11).

We shall now list the main features of the Pb^{207} spectrum.

a) A peak (peak n_1) is centered at $t = (44.7 \pm 1.3)$ ns.

corresponding to $(.97 \pm_{-.05}^{+.06})$ MeV. The position of the peak's centroid has been calculated from the time-of-flight spectrum including only the second and third H-intervals, since it has the highest signal-to-background ratio in the region of the peak and contains $2/3$ of all the events in the peak. The H=3 TOF spectrum -- whose photon peak, unlike those of the contiguous H-intervals, is relatively free of double-peaking -- gives also a time-of-flight value of (44.75 ± 1.3) , corresponding to $(.97 \pm_{-.05}^{+.06})$ MeV.

Fig. 4.2-9 Pb^{207} prompt neutron TOF spectrum. Background estimated by means of the total neutron spectrum ($F = 1.89$).

Fig. 4.2-10 Pb^{207} prompt neutron TOF spectra for individual values of H or small groups of them.

Fig. 4.2-11 Pb^{207} prompt neutron TOF spectra for different (complementary) groups of values of H .

The peak n_1 presents a shoulder on the low-energy side. It may be due, however, to imperfect time calibration, to time walk and to statistical fluctuations.

The yield per muon cascade of the conversion neutrons falling inside this peak is

$$Y_{n_1} = (2.3 \pm 1.1 - 0.7)\%.$$

This peak can be associated with the muonic transitions $3 \rightarrow 1$. On account of the position of the peak, the $3p \rightarrow 1s$ transitions seem to contribute to it more likely than the $3d \rightarrow 1s$ transition that leaves the Pb^{206} nucleus in its first excited state (excitation energy .803 MeV, $J^P=2^+$).⁵² The $4 \rightarrow 1$ transition leaving the Pb^{206} nucleus in an excited state at 1.7 MeV would also give prompt neutrons of the right energy to contribute to the peak n_1 .

b) The second statistically meaningful and also very intense peak (peak n_2), is centered at $t = (33.25 \pm 1.46)$ ns corresponding to an energy of $(1.75 \pm .17 - .14)$ MeV.

The shape of this peak is irregular and -- as the energy spectrum shows -- it suggests a contribution from more than one neutron energy, i.e. from more than one muonic transition.

The yield per muon cascade of the peak n_2 , calculated including the entire broad distribution of events of the peak and after the most drastic background-subtraction possible, is

$$Y_{n_2} = (1.66 \pm .65 - .46)\%.$$

This peak can be associated with the muonic transitions

$P(E_n)$ per 100 muon cascades,
(50 keV)⁻¹

Fig. 4.2-12 Pb^{207} prompt neutron energy spectrum.

3p,d $\rightarrow$ 1s leaving the daughter nucleus in the ground state and, only partially, to the 4p,d $\rightarrow$ 1s transitions leaving the Pb^{206} nucleus in its first excited state.

- c) A third peak (n_3) is present in the spectrum, centered at $t = (27.5 \pm 1.5)$ ns corresponding to $(2.56 \pm_{-.26}^{+.30})$ MeV. It is present in all the individual runs and it constitutes the highest energy part of the spectrum that can be attributed with good confidence to conversion neutrons. Its yield per muon cascade is

$$Y_{n_3} = (.57 \pm_{-.18}^{+.31}) \%$$

It could be associated with the muonic transitions 4p,d $\rightarrow$ 1s leaving the daughter nucleus in its ground state.

- d) The total yield of conversion neutrons per muon cascade, calculated after a background subtraction as described at the end of the third chapter, is

$$Y_{\text{Pb}^{207}}^{E_n = .8 \text{ MeV}} = (5.3 \pm_{-2.0}^{+2.6}) \%$$

The energy spectrum is shown in Fig. 4.2-12.

E. Pb^{208}

It is apparent from the Pb^{208} TOF spectrum including all the values of H (Fig. 4.2-13A), and better still from the TOF spectrum B, which excludes the lower amplitudes (Fig. 4.2-13B), that conversion neutrons are present. The spectrum has very high background components, both a flat, random one and one due to capture neutrons, so that the ratio of conversion events to background events is at best, 1:1. As

Fig. 4.2-13A Pb^{208} prompt neutron TOF spectrum. Background estimated from total neutron TOF spectrum ($F = 3.2$).

Fig. 4.2-13B Pb^{208} prompt neutron TOF spectrum for $H \geq 5$. ($E_n \geq 1.44$ MeV ; or $\text{TOF}_n \leq 36.5$ ns).

Fig. 4.2-14 Pb^{208} prompt neutron TOF spectrum (photon events included).

in the case of Pb^{206} , the time interval within which delayed capture events were accepted was mistakenly chosen too long, with the consequence of greatly increasing the signal-to-noise ratio. The subtraction of the background is consequently not easy to perform, and it is particularly uncertain for flight-times longer than 35 ns. The energy spectrum is therefore most uncertain for energies lower than 1.6 MeV.

The conversion neutrons appear mainly concentrated in two partially merged peaks centered respectively at

$$t_1 = (27.8 \pm 1.5) \text{ ns} \quad \text{--} \quad E_1 = (2.53 \pm .27) \text{ MeV.}$$

$$t_2 = (30.8 \pm 1.5) \text{ ns} \quad \text{--} \quad E_2 = (2.02 \pm .22 \text{ -- } .19) \text{ MeV.}$$

The yields per muon cascade of these two peaks are as uncertain as the background subtraction

$$Y_{n_1} = (.95 \pm .44) \% \quad Y_{n_2} = (.89 \pm .43) \%$$

Conversion neutrons are probably the constituents of a small peak about $t=24.25$, i.e. at energy equal to the maximum allowed in principle to the conversion neutrons (3.25 MeV). However, the smallness of the peak and the presence in the total spectrum with photon events included (Fig. 4.2-14) of a spurious photon peak that even if rejected almost completely might still partially be present, add to the doubts already present on statistical grounds.

Other conversion neutrons might be present around a time of 40 ns, but being that a region of high background and high oscillations, we cannot arrive at any solid conclusion.

$P(E_n)$ per 100 muon cascades

$(50 \text{ keV})^{-1}$

Fig. 4.2-15 Pb^{208} prompt neutron energy spectrum.

The energy spectrum (Fig. 4.2-15) shows clearly the difficulty of the background subtraction at low energies.

F. Bi²⁰⁹

Among the elements studied in the present experiment Bi²⁰⁹ is the one with the best statistics. It is characterized by a very strong peak that -- above the energy threshold of the present experiment -- contributes to more than half of the total yield. Its prompt TOF spectrum that includes the neutron events coming from all the H values of relevance (spectrum A, Fig. 4.2-16, 1 = H = 16) is characterized by a signal-to-noise ratio which is larger than 1 almost everywhere and equal to 4 in correspondence to the largest peak.

The A spectrum can be decomposed into two complementary spectra, spectrum B (Fig. 4.2-17) and spectrum C (Fig. 4.2-18) for which the neutron events accepted belong respectively to the N-pulse-height intervals 1 = H = 3 and 4 = H = 16 (i.e.

$0.67 \text{ MeV} = E_p = 1.0 \text{ MeV}$ and $1.0 \text{ MeV} = E_p = 3.3 \text{ MeV}$, where in a crude approximation E_p is the kinetic energy of the proton scattered by the neutron in the scintillator).

Spectrum B corresponds to the lowest noise-to-signal ratio in the longer times of flight and is used to calculate the centroid of the largest peak. Spectrum C is used to cast some doubts on the adequacy of the background subtraction.

The method followed for the subtraction of the background, consisted as usual in the subtraction of the total neutron spectrum of Bi (Fig. 3.5-2) normalized to cancel the counts

Fig. 4.2-16 Bi²⁰⁹ prompt neutron TOF spectrum. Background estimated from the total neutron TOF spectrum ($P = 1.8$).

Fig. 4.2-17 Bi^{209} prompt neutron TOF spectrum for $1 \leq H \leq 4$.

Fig. 4.2-18 Bi^{209} prompt neutron TOF spectrum for $5 \leq H \leq 16$.

Fig. 4.2-19 Bi²⁰⁹ prompt neutron energy spectrum.

at $t=55$ ns . The normalization factor is then twice as large as the calculated value: this also is a common feature of all the subtractions performed.

The examination of the spectrum C shows that the event counts per time channel rise rather abruptly at $t = 21.5$ ns and increase continuously with increasing time of flight up to $t = 26$ ns where a statistically meaningful discontinuity denotes the almost certain presence of conversion neutrons. On the basis of continuity -- i.e. continuity of the experimental spectrum C starting from time-of-flight values shorter than the value of the series limit, and continuity of the expected capture neutron background -- one could assume that no conversion neutrons or very few of them are present in spectrum C between 21.5 ns and 26 ns . If this is the case the background subtraction performed underestimates the background in the same region by some 20%.

A possible counter argument lies in the fact that the neutron energy threshold for spectrum C is approximately 1 MeV (which the presence of a small fraction of the main peak confirms) combined with the fact that at $t = 26$ ns the background seems to have reached close to its highest levels, which is quite unlikely on the grounds of the energy distribution function of the muon-capture neutrons, which is rapidly decreasing with energy.

This uncertainty has been taken into account in the estimation of the probable error of the yields calculated with the usual background subtraction.

The description of the Bi TOF spectrum follows.

- a) The width of the photon peak, that gives an idea of the time resolution of the system is, expressed as root-mean-square deviation from the centroid position, 1.27 ns. The presence of double-peaking contributes somewhat to the width: the corrected root-mean-square deviation is 1.10 ns. However, due to the negative effect of the double-peaking for which a correction cannot be made for the individual H intervals because of poor statistics after pulse-shape discrimination, the former value of 1.27 ns gives a more realistic estimate of the resolution of the system.

Obviously the presence of double-peaking affects also the determination of the photon-peak centroid. The subtraction of the spurious peak leads to the following value for the photon peak centroid:

$$t = (2.25 \pm 1.31) \text{ ns}$$

From the calculated values of the neutron peaks 0.25 ns will be subtracted, to recalibrate the time spectrum.

- b) The largest neutron peak (n_1) is centered at $t = (41.42 \pm 1.62) \text{ ns}$, which corresponds to an energy of $(1.136 \pm 0.087) \text{ MeV}$. Its width (see Fig. 4.2-19) expressed by the standard deviation from the centroid, is larger than what is expected from the width of the photon peak by approximately 70 keV. Its shape is not symmetric, since its maximum is positioned at $t = (41.0 \pm .2) \text{ ns}$ or 1.154 MeV.

This fact suggests so far that both $3p \rightarrow 1s$ and $3d \rightarrow 1s$ transitions contribute to the peak, and $3p \rightarrow 1s$ carry the

larger weight. Other transitions, as already pointed out by Hargrove et al.,³⁴ can in principle contribute to the peak: that is 4 — 1 transitions leaving the Bi^{208} nucleus in an excited state having excitation energy between 825 and 1120 MeV; and the two overlapping particle-hole multiplets of Bi^{208} in the energy interval 930-1150 keV are good candidates.

The yield per muon cascade of the peak n_1 is

$$Y_{n_1} = (2.8 \pm 1.0) \%$$

- c) Following the peak n_1 there is a broad hump of a rather irregular shape (n_2), that appears to be made up of two peaks almost completely merged. They are centered respectively at $(1.48 \pm .11)$ MeV and $(1.63 \pm .12)$ MeV.

That the events present in this seemingly double peak are due to conversion neutrons is unquestionable, since the peak is evident and strong in all the individual runs of the Bi target. From the energy spectrum it appears that the width of this peak is noticeably wider than the width of the photon peak, and that it is bicuspid. The evaluation of the yield of conversion neutrons per muon cascade of the whole bicuspid peak strongly depends on the background subtraction. Thus the uncertainty is rather high.

$$Y_{n_2} = (.92 \pm .47) \%$$

The assignment of these peaks to muonic transitions cannot be done with any confidence. Several transitions are

possible candidates if excited states for the daughter nucleus are to be considered.

- d) A rather sharp peak (n_3) is centered at $t = (30 \pm 1.25)$ ns, corresponding to an energy of $(2.15 \pm .20 \text{ MeV})$. It is merged with a broad distribution of events that extends in time down to about 26.5 ns (2.8 MeV). The yield per muon cascade of the n_3 peak is

$$Y_{n_3} = (.75 \pm .37) \%$$

It is tempting to assign the n_3 peak to the $4 - 1$ transitions leaving the daughter nucleus in its ground state, even though higher transitions leaving an excited daughter nucleus are also possible candidates.

- e) A final peak (n_4) appears at $t = 24.25$ ns which corresponds to an energy of $(3.4 \pm .4)$ MeV, i.e. to the muon series limit. Its intensity is very strongly dependent on the background subtraction:

$$Y_{n_4} = (.06 \pm .04) \% \text{ neutrons per muon cascade.}$$

- f) The total yield of conversion neutrons per muon cascade is

$$Y_{\text{Bi}209}^{E_n \approx .85 \text{ MeV}} = (5.4 \pm 2.3) \%$$

4.3 Discussion of the results

In this section we shall discuss the results obtained from the point of view of their experimental merits and limitations, and compare them with the previous experimental

results and the theoretical predictions whenever these are available.

In Table 4.3-1 we have summarized what is theoretically and experimentally available in terms of conversion-neutron yields for convenient neutron-energy intervals.

A. Au¹⁹⁷ and Natural Tl

Not much can be added to what has already been said about the time-of-flight spectra of gold and thallium. For these spectra -- whose experimental runs were of necessity (running-time limitation) short ones -- the poverty of the statistics and the uncertainty in background subtraction make uncertain the very presence of conversion neutrons. The presence of high counts in regions where neutron lines can be expected hints at the emission of conversion neutrons. The values shown for Au and natural Tl in Table 4.3-1 can only be used as an order of magnitude of the upper limits of their respective conversion-neutron yields.

B. Pb²⁰⁶

The one striking aspect of the Pb²⁰⁶ conversion-neutron TOF spectrum is the sharp rise at a time-of-flight approximately corresponding to the muon series limit. This should not be surprising since the study by Srinivasan and Sundaresan³⁸

near the series limit, even though the intensity of the line should be expected to be drastically reduced by a more realistic muon cascade model. The experimental intensity at the

Table 4.3-1. Experimental and theoretical conversion-neutron yields (in % per muon cascade) for consecutive energy intervals above threshold.

	E_n range (MeV)	.9-1.3	1.3-1.9	1.9-2.4	2.4-3.1	3.1-3.6	1.3-3.6	Total	^d
Bi ²⁰⁹	$Y_{S\&S}$ ^a	2.3	0	.66	.52	1.7	2.9	5.2	
	Y_{LWH} ^b	2.5	---	---	---	---	2.2	4.7	
	$Y_{exp.}$ ^c	2.8 ⁺ -1.0	1.1 ⁺ -0.5	0.7 ⁺ -0.2	.47 ⁺ -.15	.14 ⁺ -.07	2.4 ⁺ -1.0	5.2 ⁺ -2.0	
Pb ²⁰⁷	E_n range (MeV)	.8-1.1	1.1-1.5	1.5-1.9	1.9-3.3	3.3-3.8	1.9-3.8	Total	^d
	Y_{LWH} ^b	1.43	1.75	3.0	---	---	2.2	8.4	
	$Y_{exp.}$ ^c	2.5 ⁺ -1.2	.52 ⁺ -.35	1.1 ⁺ -0.4	1.1 ⁺ -0.5	.07 ⁺ -.07	1.2 ⁺ -0.5	5.2 ⁺ -2.3	
Pb ²⁰⁶	E_n range (MeV)	1.0-1.5	1.5-2.0	2.0-2.8	---	---	---	Total	^d
	$Y_{S\&S}$ ^a	.85	.87	3.1	---	---	---	4.8	
	$Y_{exp.}$ ^c	.72 ⁺ -.33	.48 ⁺ -.20	.40 ⁺ -.15	---	---	---	1.6 ⁺ -0.7	
Pb ²⁰⁸	E_n range (MeV)	.9-1.3	1.3-1.8	1.8-2.3	2.3-2.9	2.9-3.7	---	Total	^d
	$Y_{exp.}$ ^c	2.1 ⁺ -0.9	.58 ⁺ -.46	.77 ⁺ -.36	.77 ⁺ -.27	.33 ⁺ -.15	---	4.5 ⁺ -2.1	
Au ¹⁹⁷	$(E_n \geq .85 \text{ MeV})$								^c 3.0 ⁺ -2.2 -1.2
Nat.Tl	$(E_n \geq .75 \text{ MeV})$								^c 5.2 ⁺ -3.6

a) Values obtained from Ref.38; uncertainty 40%.

b) Values obtained from Ref.40; uncertainty 40%.

c) Values obtained in the present experiment.

d) For E_n larger than the lower limit of the first energy interval

series limit is nonetheless puzzling because of its higher values compared to the values obtained for the other lead isotopes and for Bi^{209} . The TOF spectra of all these nuclei show the presence of conversion neutrons near the series limit, but their intensity is lower there than it is for Pb^{206} , while their intensities in the lower energy parts of the spectrum are generally much higher than they are in Pb^{206} .

The total yield of conversion neutrons per muon cascade predicted by Srinivasan and Sundaresan for Pb^{206} is 5.2% for neutron energies larger than 1 MeV. However, this value is reduced to 1.7% if one subtracts the artificially high contribution of the $14 \rightarrow 1$ muonic transitions, in good agreement with the experimental value. The general decrease in intensity of the experimental spectrum as a function of increasing energy agrees with the Srinivasan and Sundaresan predictions, even though the high level experimental uncertainty due to the high background makes this statement a rather weak one.

C. Pb^{207}

For this isotope of lead, as well as for Pb^{208} and Bi^{209} which we shall examine later, there can be little doubt about the presence of conversion neutrons, while doubts remain as to the exact value of the conversion-neutron yields, to the muonic transitions inducing the conversions and to the final states of the daughter nuclei.

The Pb^{207} spectra are relatively rich with information due to the definite structure they show and to the possibility of comparison with -- directly or indirectly -- two sets of theoretical predictions. We shall compare first the experimental Pb^{207} TOF spectrum with the one obtained in the previous experiment.³⁵ The background of the new spectrum is considerably reduced with respect to the old one, the main reduction having taken place in the background due to photon events and the random background in general (compare the levels in the "valley" region where correlated capture neutrons cannot contribute much). This reduction is due to an improved pulse-shape discrimination and to the lower N-counting-rate.

We notice at this point that the background estimate for the old spectrum, allegedly obtained from silver data, was incorrect and leading to an overestimation of the total conversion-neutron yield.

Two new peaks appear in the new spectrum, and one of them is by far the strongest in the whole spectrum. Its presence had already been guessed from the previous spectrum though it contained only a hint of it. The only minor presence of this peak in the old spectrum implies that the neutron energy threshold was, in that experiment, very close to the energy of the peak, and that consequently the background had been rather poorly subtracted and the neutron yield overestimated.

The intense n_1 peak is just confirming the intensity

value of the .803 MeV gamma-ray corresponding to the decay of the first excited state of Pb^{206} observed by Hargrove et al.²⁴ with a Pb^{207} target. In fact the n_1 peak is the least energetic peak corresponding to a muonic transition leaving the daughter nucleus (Pb^{206}) in its first excited state. This means that this experiment sets an upper limit for the intensity of the .803 MeV gamma-ray.

To the n_1 peak can conceivably contribute higher-n transitions leaving the Pb^{206} nucleus in higher excited states.

We notice in passing the presence of a small peak of counts centered at an energy of 1.25 MeV. We shall return to it later.

The irregular shape of the n_2 peak is due partly to statistical fluctuations and partly to the fact that more than one transition is contributing to it. The main contributors are the $3 \rightarrow 1$ transitions leaving the daughter nucleus in its ground state (main peak) and the $4 \rightarrow 1$ transitions leaving the daughter nucleus in its first excited state (high-energy-side shoulder).

Mainly the $4 \rightarrow 1$ muonic transitions leaving the daughter nucleus in its ground state contribute to the n_3 peak.

This spectrum shows immediately one of the shortcomings of both the theoretical studies^{38,40} on emission of conversion neutrons by the cascading muon. Neither theory contemplates the possibility for the neutron emission to leave the daughter nucleus in an excited state. To the peak n_1 cannot in

fact contribute any muonic transition leaving the daughter nucleus in an excited state (see Table 4.1-2 and notice that $E_{2p\frac{1}{2}} - E_{1s\frac{1}{2}} - S^n < 0$). The position of the peak and the energy resolution obtained and the Pb^{207} conversion-neutron spectrum calculated by LWH exclude on the other hand that the peak n_1 corresponds to a $3 \rightarrow 1$ muonic transition extracting the third least tightly bound neutron.

The comparison of the experimental conversion-neutron energy spectrum with the one calculated for Pb^{207} by LWH⁴⁰ does not show many similarities. The main peak, n_1 , has not been predicted.

The little peak present in the experimental spectrum -- with little statistical weight -- might be produced by the $3 \rightarrow 1$ transitions ejecting the second least-tightly bound neutron and leaving the daughter nucleus in its ground state.

But it might as well be due to the $4 \rightarrow 1$ transitions leaving the Pb^{206} nucleus in an excited state (1.460 MeV). Its low intensity -- if it is real and its origin the first one suggested -- sets a limit for the ejection of more tightly bound neutrons.

The peak n_2 has been predicted to be about four times more intense than it is experimentally, the $3d - 1s$ transition contributing most of the intensity. It is not possible to estimate from the spectrum the calculated contribution to the peak n_2 .

In general the experimental spectrum points to the extreme inadequacy of the model used by LWH.

The S&S predictions do not extend explicitly to Pb^{207} . The total yield expected should not be too different from the ones obtained for Bi^{209} and Pb^{206} ($3 \rightarrow 1$ transitions included), i.e. respectively 3.5 and 4.1 (the contribution of the $14 \rightarrow 1$ transitions has been subtracted). This value accounts for between $2/3$ and $3/4$ of the experimental value which has a 45% uncertainty. The rest of the intensity could be provided by converted quadrupole muonic transitions. The uncertainties in both experiment and -- perhaps even more -- theory do not allow for more than guesswork in terms of intensities.

Another piece of information given by the experimental spectrum for Pb^{207} is an order of magnitude (3 ± 2) for the ratio of the probabilities of the Pb^{206} final nucleus to be left in its ground state and in its first excited state. This estimate can be made under the assumption that the peak n_1 is mainly due to the $3 \rightarrow 1$ transitions and the peak at 1.75 is due mainly to the $4 \rightarrow 1$ transitions (the contribution of the $3 \rightarrow 1$ transitions leaving the daughter nucleus in its ground state is present as well and produces the high-energy shoulder of the main peak).

There is a qualitative agreement with the S&S model as to the general feature of a contribution from the $np \rightarrow 1s$ transitions that decreases rapidly with n . However the experimental rate of decrease appears to be somewhat higher than the one predicted by S&S.

As to the argument made by Schneuwly et al.³⁹ for the

case of Bi^{209} (and applicable as well to Pb^{207} in view of the approximate equality of the prompt-neutron intensities in the two cases) stating that the cascade population of the p levels is inadequate to account for the observed conversion-neutron intensities, we do not think that it is very stringent since it is based on a cascade calculation that has not been confirmed accurately enough by experimentally measured x-ray intensities.

D. Pb^{208}

For Pb^{208} the total yield of conversion neutrons with energy above .9 MeV is of the same order of magnitude as for Pb^{207} (see Table 4.3-1). The two main peaks of this spectrum can be attributed respectively to the $4 \rightarrow 1$ transitions and to the $5 \rightarrow 1$ ones. The intensity of the two peaks is high if one considers the populations of the $n=4$ and $n=5$ states of angular momentum 0, 1 and 2 (the predictions of Ref. 27 are: 2.3% for the 4p states, 1.1% for the 5p states, 9.2% for the 4d states and 3.1% for the 5d states, while Ref. 39 predicts much lower values than these), and strongly suggests that both p- and d-states contribute to them.

The intensity of the peak located at the maximum energy limit (3.23 MeV) for the conversion neutrons is higher than for the other isotopes and Bi^{209} , and it is probably overestimated because of inadequate background subtraction. It nonetheless seems to be real.

In the lower-energy part of the energy spectrum

(fig. 4.2-15) it is virtually impossible to say what is due to conversion neutrons and what is background. However, it is not possible to find there -- just as it is not possible to find it in the rest of the spectrum -- all the conversion neutrons that have to be emitted if the intensities of the Pb^{207} prompt gamma-rays detected with a Pb^{208} target²⁴ (Table 1.3-1) are correct. It is obviously still possible that these neutrons be found to derive mainly from the $3 \rightarrow 1$ transitions leaving the Pb^{207} nucleus excited, and have therefore energies well below the energy threshold of the present experiment.

There are no previous experimental results or specific theoretical investigations with which to compare our results for Pb^{208} .

E. Bi^{209}

The Bi^{209} conversion-neutron TOF spectrum (Fig. 4.2-16 and 4.2-19) presents with respect to the old one³⁴ a reduced background level, i.e. an increased signal-to-noise ratio due to better photon-neutron pulse-shape discrimination. The time resolution is approximately the same but, because of the better statistics (approximately four times the number of effective muon stops) and because of the longer flight path (24" from 18") the energy resolution has been improved and the spectrum shows more details.

The same remark as for the old Pb^{207} spectrum, has to be made for the old Bi spectrum relatively to the evaluation of the background and its subtraction. The background of

the old spectrum has been underestimated and this is the cause of the difference between new and old spectrum in the estimated conversion neutron yield. Otherwise, all the conclusions drawn^{ibid.} from the old spectrum are confirmed.

The energy resolution has been improved, but not quite enough to resolve its component peaks. Four lines of unknown intensities are present in an energy interval 60 keV wide, while the energy standard deviation expected from the width of the photon peak is itself 87 keV.

The S&S study predicts a good part of the experimental intensity and there is agreement within errors. The intensity argument provided by Schneuwly et al.,³⁹ even within the uncertainty of the level population, induces one to consider probable the contribution of the $nd \rightarrow 1s$ muonic transitions to the ejection of conversion neutrons, although not to the extent predicted by LWH.

What appeared in the old spectrum as a bump at 1.8 MeV energy is now partially resolved into two peaks centered at $(1.48 \pm .11)$ MeV and $(1.63 \pm .12)$ MeV. The $4 \rightarrow 1$ (and $5 \rightarrow 1$) transitions for which the Bi^{208} nucleus is left in an excited state may contribute to these peaks. It is a futile exercise to try to select the excited states that make the more likely candidates since the energy resolution is not good enough for that. Obviously, these peaks are not predicted in either of the theoretical studies mentioned.

The $4 \rightarrow 1$ transitions contribute to the peak centered at 2.2 MeV (EO transition as well?) while the $5 \rightarrow 1$ and

higher- n transitions appear also to be present, even though they are merged due to the finite energy resolution, thus confirming the prediction of Srinivasan and Sundaresan.³⁸

A meaningful comparison of the experimental results for Bi^{209} with the predictions of the study by LWH cannot be made because of the lack of a detailed theoretical spectrum in this case.

A comparison with the S&S prediction for Bi^{209} shows a surprisingly good agreement. The theoretical intensity (3.5% per muon cascade) after subtraction of the artificial contribution of the $14p - 1s$ line accounts for $2/3$ of the experimental intensity.

4.4 Suggestions for a future experiment

As a result of the experience gained in this study, it is possible to make a few suggestions directed at improving the quality of the conversion neutron spectra.

- 1) We reckon that the length of the flight path can be increased, if the present level of neutron energy threshold is maintained, for the purpose of increasing the energy resolution. This is advisable in particular for Bi^{209} and Pb^{207} that have very intense peaks at ≈ 1 MeV and that can sacrifice some intensity for an increase in energy resolution. The flight path can be doubled to study the energy region > 2 MeV, without any increase in the time interval set for the N-pulses, so that the energy resolution can be greatly improved.

The higher intensity beams now available will per se stimulate one to increase the flight path for the purpose of reducing the counting rate of the N-counter since the PSD is very sensitive to it, particularly at low neutron-energy thresholds.

2) Reduction of the size of the N counter should be implemented for improvement of the time resolution (improvement due both to the reduced size of the scintillator and to the better time-characteristics of a smaller phototube).

3) The energy threshold should be lowered for obvious reasons. This is particularly true in the cases of Pb^{208} (prompt Pb^{207} gamma-rays yet unaccounted for by the experimental spectrum) and Pb^{206} (energetically allowed conversion neutron lines from the 3 — 1 muonic transitions at .4 MeV, that are predicted³⁸ to be quite intense).

This might necessitate viewing the N-counter with two photomultipliers put in coincidence and whose outputs containing the PSD information are added together. This is done for the purpose of lowering the level of noise relative to the signal, and maintaining a good pulse-shape discrimination at the higher counting rate that the lowering of the neutron-energy threshold implies.

4) Basing ourselves on the analysis performed in the

Appendix, we can state that it is possible to increase noticeably the $(\xi_K^c + \xi_K^R + \xi_K^{cn})/(\xi_K^R + \xi_K^{cn})$ ratio that determines the preference in detection given by the prompt spectra

to the conversion neutrons with respect to the capture neutrons. This can be accomplished both by reducing the width of the time interval set for detecting a (delayed) capture, and by making the initial time of that interval a function of the time of flight. This means that the opening of the delayed-capture time-interval should be triggered by the N pulse, with the result of greatly increasing both the detection efficiency of capture in the heavy element, and the $(\xi_K^c + \xi_K^R + \xi_K^{cn})/(\xi_K^R + \xi_K^{cn})$ ratio. The analysis of the K-pulse delay spectrum allows one to determine the K efficiencies as a function of the time of flight.

This use of a mobile time "window" for capture detection is equally advantageous for metallic targets and for nitrate ones, while the shortening of the time window is critical only for targets containing low-Z impurities.

The use of a mobile K-pulse time-window will particularly improve the spectrum in the proximity of the time-of-flight corresponding to the muon series limit. We stress in fact that the only part of the spectrum "amplified" by the use of the mobile K-pulse time-window is the prompt part of it (if present).

5) An accurate determination of the N-counter efficiency^o should be alternated with sufficient frequency with the collection of the main data, for the purpose of reducing the uncertainty in the intensity values.

6) The critical gains in the electronics should be digitally stabilized (e.g. gain of the time-of-flight circuit,

gain of the N-counter photomultiplier to improve the quality of the pulse-shape discrimination and keep the neutron-energy threshold constant).

- 7) Statistically richer total neutron spectra should be collected for a more accurate subtraction of the background.
- 8) The data analysis can be almost completely automatized.

4.5 Conclusions

The emission of neutrons induced by the cascading muons has been experimentally studied for Bi²⁰⁹, Pb²⁰⁸, Pb²⁰⁷, Pb²⁰⁶, natural Tl, Au¹⁹⁷.

Conversion neutrons have been observed with certainty in the first three cases, while some doubts persist in the case of Pb²⁰⁶, and no definite conclusions can be drawn from the spectra obtained for Tl and Au in two short exploratory runs.

The uncertainty in the determination of the background level is, followed at a distance by the inaccuracy of the neutron counter efficiency, the main source of uncertainty in the final intensity values measured (Table 4.3-1).

The TOF spectra and the energy spectra derived from them after subtraction of the background have been compared with the predictions contained in theoretical studies by Srinivasan and Sundaresan³⁸ and by Lohs et al..⁴⁰

The model used by LWH is shown to be inadequate because the experimental energy distribution contradicts the

possibility of ejection of neutrons other than the least tightly bound one. Excited final states of the daughter nucleus play an important role certainly in the case of Pb^{207} and quite probably in the case of Bi^{209} .

The agreement of the experimental data with the predictions by S&S is generally good both from the point of view of the total yield and from that of the energy (or n) dependence of the transitions.

The question as to what is the relative value of the contribution to conversion neutron ejection, by respectively $np \rightarrow 1s$ transitions and $nd \rightarrow 1s$ ones is only very partially answered. This is due to insufficient energy resolution as well as to the high level of uncertainty of both experimental measurement and theoretical predictions.

The comparison of the experimental yields of conversion neutrons with their theoretical predictions by S&S suggest the contribution of both the $np \rightarrow 1s$ and the $nd \rightarrow 1s$ transitions, the former probably accounting for $\sim 2/3$ of the yields.

The contribution of higher n transitions to the conversion-neutron spectrum decreases rapidly with increasing n , as predicted by S&S, so that the process of nuclear conversion becomes appreciable in affecting the atomic cascade of the muon only for small values of n . Nonetheless the small peaks present in all the spectra obtained at the series limit show that there is a non-zero contribution even for relatively high values of n .

Some suggestions have been made to improve the quality of future experimental data in order to answer the questions still left open relative to the emission of nuclear conversion neutrons in muonic atoms.

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APPENDIX

EFFICIENCY OF DETECTION OF THE NUCLEAR MUON CAPTURE
AND LIFETIME OF THE BOUND MUON

The nuclear capture of the muon is determined by a 2" thick scintillation counter (D) viewed by two photomultipliers in coincidence and covering a solid angle of 2π relative to the active part of the target. The muon stop signal starts a time-to-amplitude converter (TAC), while a pulse from the D counter stops it (Fig. 2.3-1). D-pulse delay spectra (or D spectra) of this kind, obtained by feeding the output of the TAC into an analog-to-digital converter (ADC), were used to determine the efficiency of capture detection (Fig. A-1).

In the prompt neutron "runs" the capture pulse was accepted between time $t_1 = 65$ ns and time $t_2 = 155$ ns, this condition being imposed by requiring a coincidence (S·D) between a 90 ns long stop-pulse and a properly-timed short D pulse.

To determine the position of t_1 and t_2 on the D spectra, these spectra could be collected by gating the TAC output with the output of the S·D coincidence (Fig. A-2).

The D counter being in the path of the beam, to avoid confusing the detection of beam particles with that of capture radiation it was necessary to veto the D pulses that

Fig. A-1 D spectrum for Bi. 1.93×10^5 muon stops in the 7 g/cm^2 bismuth target.

Fig. A-2 D spectrum for Bi, gated by S.D. $.85 \times 10^5$ muon stops in the 7 g/cm^2 bismuth target.

were simultaneous with pulses of the A counter (the first counter of the telescope). The A veto has the additional effect of eliminating most of the prompt (i.e. non-capture) radiation both correlated and uncorrelated. The $D \cdot \bar{A}$ pulses are called K pulses. The K-pulse delay spectra (simply, K spectra) are used for calculating the various K efficiencies, i.e. the probabilities of having a muon stop followed by a certain type of K pulse (Fig. A-3).

The A veto was removed when collecting D spectra for channel-to-time calibration purposes. This was done because the capture time (the only time reference existing in the spectra) was given by the position of the prompt peak. The prompt peak was shifted in time by a known amount by delaying the D pulse with calibrated delay lines (Fig. A-4).

The purpose of the whole procedure being that of determining the K efficiencies and not the bound-muon lifetime, no care had been spent on the accurate calibration of the spectra. This means that the rise-time increase and the attenuation operated by the coaxial cable on the K pulse were not taken into consideration for correction of the time calibration. Both of the effects just mentioned result in an underestimation of the gain expressed as the ratio of the variation of time delay to the corresponding channel-number variation.

The K spectrum can be decomposed as follows

$$K(t) = K^C(t) + K^{C'}(t) + K^R(t)$$

Fig. A-3 K spectrum for Bi. 1.02×10^6 muon stops in the 7 g/cm^2 bismuth target.

Fig. A-4 Time-to-channel calibration D-spectrum. It is valid for all the elements but bismuth.

where

$K^C(t)$ are pulses produced by nuclear muon-capture radiation in the desired (heavy) target element at time t ;

$K^{C'}(t)$ are pulses produced by muon capture radiation in any other (light) element (mainly C and H for metallic targets, and N and O for nitrate targets);

$K^R(t)$ are any pulses uncorrelated with the stop.

One should notice that $K^C(t)$ and $K^R(t)$ compete with each other, and so do $K^{C'}(t)$ and $K^R(t)$, while $K^C(t)$ and $K^{C'}(t)$ are mutually exclusive and do not interfere. That $K^R(t)$ and $K^C(t)$ compete with each other means that the probability of detecting a K^R pulse at time t , must be multiplied by the probability of not detecting a K^C pulse before time t , and vice-versa. Under the assumption of a statistically constant rate for the K^R pulses, the probability for a random K pulse to occur within a certain time interval $(t, t+\Delta t)$ is given by the probability of a random pulse to occur within Δt , multiplied by the probability that no pulse of any kind occur after muon capture before time t .

The random pulse rate is determined with sufficient accuracy by the level of the K spectrum before the muon-stop time (Fig. A-3), when neither K^C nor $K^{C'}$ can contribute. The probability for no K pulse to occur between stop time (t_0) and a given time $t > t_0$ can be obtained from the K spectrum as the complement to 1 of the ratio of the total number of K pulses recorded between t_0 and t , to the number of stops that have contributed to the spectrum. Thus it is

possible to determine the level of the random background of the K spectrum as a function of time and, knowing t_1 and t_2 , i.e. the extremes of the time interval set for the detection of the (delayed) nuclear capture of the muon, it is possible to calculate the probability that a muon stop be followed by a random K pulse.

Since the mean rate of the K pulses is relatively low, the spectrum of K^C is quite independent of K^R since negligible, in view of the other existing uncertainties, is the total probability that random pulses be detected within the largest of the time intervals in question. The time spectrum of K^C is a negative exponential with a time constant equal to the lifetime of the muon bound in the $1s\frac{1}{2}$ state (τ_c). If, after subtraction of the random background, we can also subtract the rest of the background due to K^C pulses, we can determine the efficiency ξ_K^C , i.e. the probability that any stop be a stop in the desired target element and be detected within the proper time interval by the D counter via the capture radiation. This probability is calculated by adding all the $K^C(t)$ between t_1 and t_2 and then dividing by the total number of stops. Analogously, the efficiency ξ_K^R , i.e. the probability that any stop be followed by an uncorrelated $D\bar{A}$ pulse within the time interval (t_1, t_2) , is calculated by adding all the $K^R(t)$ between t_1 and t_2 and then dividing by the total number of stops.

The time spectrum of K^C is also independent of K^R and has the shape of an exponential with time constant equal to

minus the average lifetime of the muon in the other elements. The fact that these elements are very light makes the decay probability dominate, so that the uncertainty in the K^C time constant is rather small. (For metallic targets it is $(2.10 \pm .05) \mu s$, for nitrate targets it is $(1.85 \pm 0.07) \mu s$).

It is possible to determine the level of $K^C(t)$ at times near the end of the spectrum by availing oneself of the approximate value of the K^C decay constant and of the fact that the spectrum covers a time interval several times larger than τ_c so that $K^C(t)$ is negligible at the end of the spectrum. On the other hand τ_c is much longer than the time span of the K spectra, so that the level of $K^C(t)$ will not be much smaller than $K^C(0)$. $K^C(t)$ near the end of the spectrum will therefore be small compared with $K^C(t)$.

An additional check on the subtraction of the K^C background is given by the semilogarithmic plotting of K^C . The long-time end of the spectrum is much more sensitive to the subtraction than the rest, and will not be on a straight line with the rest of the spectrum if the K^C subtraction is not correct. This is the only use of the long-time end of the spectrum that is otherwise useless because of the large statistical oscillations.

The values of ξ_K^C and ξ_K^R obtained after the K^C subtraction for the various targets are shown in Table 4.1-1 of the text.

The K^C curves can also be used to measure the lifetime of the bound muon. A least-squares fit of the semilogarith-

mic plot of $K^C(t)$ will provide us with the muon lifetime in the target element under study.

The value of the gain for channel-to-time conversion of the spectra is $(5.067 \pm .015)$ ns/channel for the Bi spectrum and $(5.075 \pm .015)$ ns/channel for all the other spectra. The systematic error in gain evaluation caused by cable transmission of one of the triggering pulses has not been taken into account in the evaluation of the uncertainty.

In Table A-1 we list the spectra and the estimated background components for all the elements examined. The corresponding semilogarithmic plots, done after subtraction of the background and least-squares fitted with straight lines, are shown in Fig. A-5 to A-11.

The values obtained for the bound muon lifetimes are displayed in Table A-2 and there compared with the values measured in previous experiments.

The lifetime values for Bi, Au and Ag obtained as a by-product of this experiment appear to be systematically smaller than the previous measurements. Their ratio is, within errors, constant. Thus the difference is deemed to be produced by the incorrect channel-to-time calibration.

For the lead isotopes there is no direct comparison, though the radiogenic lead lifetime value can be approximately compared with our Pb^{206} one.

Table A-2. Bound muon lifetime.

Element	Ch ₁ ^a	Ch ₂ ^a	τ_1 (ns) ^b	τ_2 (ns) ^c	$R = \tau_1/\tau_2$	τ_{1R}^{Bi} (ns)
Bi ²⁰⁹	57	100	69.7 ^{+0.2}	73.3 ^{+0.4}	1.052 ^{+0.008}	---
Nat.Ag	30	90	83.6 ^{+0.3}	88.5 ^{+0.7}	1.059 ^{+0.011}	87.9 ^{+0.9}
Au ¹⁹⁷	35	70	68.6 ^{+0.3}	72.6 ^{+0.5}	1.058 ^{+0.011}	72.2 ^{+0.9}
Pb ²⁰⁷	38	70	71.2 ^{+0.9}	---	---	74.9 ^{+1.5}
Pb ²⁰⁸	35	72	74 ⁺³	---	---	78 ⁺⁴
Pb ²⁰⁶	35	72	67.4 ^{+3.5}	---	---	71 ⁺⁵
Rad.Pb ^d	--	--	---	71.5 ^{+0.4}	---	---
Nat. Pb	--	--	---	74.6 ^{+0.6}	---	---

a) Extremes of the channel interval selected for the least-squares fit.

b) Values obtained in the course of the present experiment.

c) Values obtained in other experiments (see M. Eckhause, R. T. Siegel, R. E. Welsh and T.A.Filippas, Nucl. Phys. 81, 575 (1966)).

d) Radiogenic lead (88% Pb²⁰⁶, 9% Pb²⁰⁷, 3% Pb²⁰⁸).

Unfortunately the muon lifetime values that can be derived from our spectra of the Pb isotopes are rather inaccurate since large statistical fluctuations are present in them, so that no further results can be drawn than a general agreement, within the experimental errors, with the available values.

TABLE A-1

K SPECTRA AND BACKGROUND COMPONENTS FOR EACH TARGET ELEMENT

Ch.	Bi ²⁰⁹		Nat. Ag		Au ¹⁹⁷		Pb ²⁰⁷		Pb ²⁰⁸		Pb ²⁰⁶	
	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R
Ch ₀ *	1817	1817	424	424	317	317	251	251	902	902	422	422
23	--	--	6550	420	3462	315	1760	249	4457	901	1945	421
24	--	--	25542	416	13476	312	7135	248	12545	900	6602	420
25	--	--	17802	412	9218	309	4922	246	8976	899	4610	419
26	--	--	16122	408	8348	306	4436	244	8343	896	4239	418
27	--	--	15145	403	7804	304	4116	242	7527	893	3919	417
28	--	--	14382	400	7371	301	3927	241	7260	890	3870	416
29	--	--	14348	395	7435	298	3916	239	7530	886	3814	414
30	--	--	18665	391	10824	296	5739	237	9769	883	4982	413
31	--	--	17165	387	9899	293	5512	235	9175	880	4753	411
32	--	--	16572	383	9432	290	5369	233	8997	877	4967	409
33	--	--	14657	379	8027	287	4441	231	8107	874	4286	408
34	--	--	14214	376	7593	285	4201	229	7884	868	4204	407
35	--	--	13054	373	6958	282	3878	228	7543	866	4015	406
36	--	--	12683	370	6570	280	3722	226	7339	863	3930	405
37	--	--	11741	367	5939	278	3503	225	7063	860	3688	403
38	--	--	11553	364	5797	276	3392	224	6864	858	3788	402
39	--	--	10498	362	5071	275	3127	222	6186	856	3252	401
40	--	--	9973	359	4964	273	2817	221	6255	854	3150	400
41	--	--	9420	357	4747	272	2622	220	5761	852	2920	399
42	19667	1814	8735	354	4118	271	2548	219	5648	850	2798	398
43	80791	1804	8121	351	3876	270	2153	218	5266	849	2646	398
44	79205	1794	7897	349	3752	268	2162	218	5161	846	2674	397
45	66623	1787	7516	348	3398	267	2069	217	5050	845	2602	396
46	59427	1777	7038	347	3166	267	1973	216	4876	843	2396	396
47	54942	1769	6565	345	2952	265	1917	215	4750	842	2349	395
48	58464	1760	6395	343	2960	264	1775	215	4602	840	2389	394
49	75564	1752	5957	342	2703	264	1593	214	4509	838	2157	394
50	78950	1743	5667	341	2460	263	1525	214	4279	837	2333	393
51	72043	1733	5288	340	2350	262	1464	213	4284	835	2194	393
52	64459	1726	4996	338	2256	262	1341	213	4084	834	2079	392
53	58480	1717	4690	337	2073	261	1267	212	4045	832	2086	392

TABLE A-1 (continued)

Ch.	Bi ²⁰⁹		Nat. Ag		Au ¹⁹⁷		Pb ²⁰⁷		Pb ²⁰⁸		Pb ²⁰⁶	
	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R
54	54215	1711	4524	335	1955	260	1197	212	4012	831	2106	391
55	49970	1705	4422	334	1862	259	1179	211	3849	829	2016	391
56	47413	1699	4166	333	1810	259	1052	211	3851	828	2004	390
57	43932	1692	3854	333	1655	258	1016	210	3610	827	1867	390
58	40330	1687	3598	332	1504	257	1032	210	3682	826	1926	389
59	37553	1681	3387	331	1418	257	930	210	3631	824	1815	388
60	35724	1677	3326	330	1395	257	880	209	3510	823	1876	388
61	33006	1673	3145	329	1299	256	816	209	3507	822	1666	387
62	30171	1669	2852	328	1244	256	807	209	3424	820	1682	387
63	28507	1665	2907	328	1159	255	801	209	3366	819	1749	386
64	26760	1662	2688	327	1105	255	826	208	3482	818	1591	385
65	25092	1659	2510	326	998	255	699	208	3263	817	1647	385
66	23514	1656	2406	326	982	254	707	208	3236	816	1673	384
67	22243	1653	2175	325	975	254	675	207	3318	815	1587	384
68	20510	1650	2178	324	933	254	629	207	3190	814	1569	383
69	19358	1647	2020	324	912	253	558	207	3136	813	1557	383
70	18346	1645	1927	323	807	253	572	207	3033	812	1573	383
71	17043	1642	1869	323	779	253	565	206	3071	810	1530	382
72	16000	1640	1801	323	767	253	527	206	3050	809	1518	382
73	15153	1638	1717	322	756	252	558	206	3031	808	1465	382
74	14104	1637	1636	322	637	252	538	206	3020	807	1439	381
75	13232	1635	1632	322	626	252	502	206	2973	807	1488	381
76	12670	1633	1492	321	667	252	442	205	2834	806	1465	380
77	11904	1632	1379	321	650	252	444	205	2842	805	1514	380
78	10977	1630	1429	321	594	252	469	205	2855	804	1506	379
79	10206	1629	1282	320	633	251	469	205	2773	803	1433	379
80	9952	1628	1254	320	560	251	426	205	2917	802	1466	379
81	9118	1627	1154	320	547	251	392	204	2833	801	1382	378
82	8825	1626	1113	319	551	251	419	204	2843	800	1372	378
83	8285	1625	1102	319	535	250	427	204	2717	799	1459	378
84	7741	1624	1115	319	492	250	409	204	2728	798	1437	377
85	7262	1623	1049	319	457	250	369	204	2777	797	1409	377
86	7061	1622	1054	318	472	250	374	203	2772	796	1340	377

TABLE A-1 (continued)

Ch.	Bi ²⁰⁹		Nat. Ag		Au ¹⁹⁷		Pb ²⁰⁷		Pb ²⁰⁸		Pb ²⁰⁶	
	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R
87	6717	1621	1032	318	449	250	375	203	2729	795	1413	376
88	6403	1620	942	318	439	250	339	203	2789	794	1350	376
89	5965	1619	940	318	425	250	339	203	2656	793	1361	375
90	5788	1619	884	317	405	250	349	203	2655	793	1392	375
91	5337	1618	840	317	417	250	341	203	2583	792	1348	375
92	5301	1617	853	317	423	249	368	203	2669	791	1366	374
93	4895	1617	803	317	364	249	326	202	2629	790	1394	374
94	4689	1616	773	316	388	249	332	202	2611	789	1375	373
95	4668	1615	794	316	418	249	302	202	2682	788	1399	373
96	4441	1615	720	316	357	249	311	202	2635	787	1310	372
97	4127	1615	752	316	401	249	338	202	2566	787	1323	372
98	4187	1614	664	316	378	249	330	202	2482	786	1317	372
99	3842	1613	680	315	381	249	300	202	2578	784	1261	371
100	3681	1613	680	315	362	249	329	202	2591	784	1314	371
101	3611	1612	698	315	410	248	314	202	2443	783	1258	371
102	3466	1612	681	315	368	248	317	202	2523	782	1267	370
103	3321	1612	600	314	345	248	287	201	2487	781	1244	370
104	3322	1611	613	314	382	248	310	201	2526	780	1257	369
105	3318	1611	607	314	387	248	317	201	2589	780	1312	369
106	3070	1610	625	314	394	248	314	201	2502	779	1260	369
107	2864	1610	615	314	361	248	292	201	2488	778	1340	368
108	2960	1610	550	314	374	248	287	201	2483	777	1327	368
109	2850	1609	557	314	346	248	298	201	2491	776	1265	367
110	2764	1609	590	314	338	248	284	201	2443	775	1285	367
111	2690	1609	577	314	341	247	274	201	2482	774	1276	367
112	2725	1608	585	313	369	247	287	200	2503	774	1259	366
113	2690	1608	521	313	330	247	271	200	2404	773	1323	366
114	2627	1608	524	313	353	247	252	200	2374	772	1233	365
115	2442	1607	536	313	337	247	281	200	2339	771	1258	365
116	2445	1607	524	313	346	247	263	200	2417	770	1209	365
117	2341	1607	542	313	325	247	284	200	2462	769	1248	364
118	2405	1606	539	313	352	247	267	200	2444	768	1290	364
119	2376	1606	564	313	358	247	264	200	2461	768	1230	363

TABLE A-1 (continued)

Ch.	Bi ²⁰⁹		Nat.Ag		Au ¹⁹⁷		Pb ²⁰⁷		Pb ²⁰⁸		Pb ²⁰⁶	
	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R	K ^{tot}	K ^R
120	2324	1606	516	312	314	247	270	200	2423	767	1185	363
121	2256	1605	515	312	342	246	288	199	2448	766	1288	363
122	2175	1605	495	312	334	246	233	199	2407	765	1242	362
123	2204	1605	439	312	345	246	261	199	2372	764	1286	362
124	2320	1604	479	312	329	246	279	199	2352	763	1270	362
125	2281	1604	487	311	343	246	285	199	2433	762	1268	361
126	2159	1604	464	311	320	246	262	199	2349	761	1203	361
127	2179	1603	476	311	288	246	274	199	2317	760	1217	360
128	2180	1603	444	311	316	246	273	199	2317	760	1244	360
No. of stops	1.40×10^7		1.69×10^6		1.00×10^6		6.45×10^5		2.6×10^6		1.45×10^6	
**												
K ₀ ^C	440 ± 30		130 ± 15		100 ± 15		82 ± 20		2190 ± 100		1150 ± 90	

*) Ch₀ symbolizes the average over 10 channels, 5 channels before the beginning of the prompt peak.

**) Amplitude at the channel corresponding to capture time (channel 40 for Bi, channel 20 for the other targets) of the negative-exponential background due to capture in other elements than the desired one. The exponential has a time constant of $(2.1 \pm .1)$ s or (414 ± 20) channels for the first four targets, while for Pb²⁰⁸ and Pb²⁰⁶ the time constant has a value of $(1.85 \pm .07)$ s or (365 ± 15) channels.

Fig. A-5 $\text{Bi}^{209} K^C(t)$ spectrum. The straight line is a least-squares fit of the data between channels 57 and 100 (included).

Fig. A-6 Natural Ag $K^C(t)$ spectrum. The straight line is a least-squares fit of the data between channels 30 and 90 (included).

Fig. A-7 $\text{Au}^{197} \text{K}^{\text{C}}(t)$ spectrum. The straight line is a least-squares fit of the data between channels 35 and 70* (included).

Fig. A-8 $\text{Pb}^{207} \text{K}^C(t)$ spectrum. The straight line is a least-squares fit of the data between channels 38 and 70 (included).

Fig. A-9 $\text{Pb}^{208} K^C(t)$ spectrum. The straight line is a least-squares fit of the data between channels 35 and 72.

Fig. A-10. Pb^{206} K^{c} (t) spectrum. The straight line is a least-squares fit of the data between channels 35 and 72 (included).

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