INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA

UMI[®]
800-521-0600

HANDLING CHARACTERISTICS OF TRACKED VEHICLES ON NON-DEFORMABLE SURFACES

by

Chi-Feng Chiang, B.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Master of Engineering

Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering
Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
June 1999

© Copyright Chi-Feng Chiang, 1999

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-48517-X

Canada

The undersigned recommend to the Faculty of Graduate Studies
and Research acceptance of the thesis
“HANDLING CHARACTERISTICS OF TRACKED VEHICLES ON
NON-DEFORMABLE SURFACES”

submitted by Chi-Feng Chiang
in partial fulfillment of
the requirements for the degree of
Master of Engineering

Thesis Supervisor

Chair, Department of Mechanical
and Aerospace Engineering

Carleton University

June, 1999

Abstract

Tracked vehicle steering has been a subject of interest to the off-road vehicle industry since it was invented. Over the years a large number of models for tracked vehicle steering have been proposed. In most of the models developed so far, Coulomb's law of friction is used in the prediction of the interacting forces between the tracked vehicle and the ground during steering. This means that the full frictional force will be developed as soon as a small relative motion takes place between the track and the ground. However, experimental evidence has shown that the shear stress developed on the track-ground interface is dependent upon the shear displacement, and the maximum shear stress will be developed only after a certain shear displacement has taken place.

A detailed study of the mechanics of tracked vehicle steering on non-deformable surfaces, taking into account the effects of shear displacement on the development of shear stress has been carried out. Experimental studies on the shear stress-shear displacement relationship for a track link with rubber pad, used in the M113 armoured personnel carrier, on asphalt have been conducted. Based on the results of both analytical and experimental studies, a general theory for the handling of tracked vehicles on non-deformable surfaces has been developed.

The basic features of the general theory are substantiated by available field measured data. It is shown that predictions of sprocket torques and moments of turning resistance as a function of turning radius, based on the general theory, bear a close resemblance to the available experimental data.

With experimental substantiation of its basic features, the general theory has been

employed to evaluate the effects of major design parameters on the maneuverability of tracked vehicles on non-deformable surfaces. It is found that among the design parameters examined, track contact length and tread of the vehicle have significant influence on tracked vehicle steering while track width and location of center of gravity have less significant effects.

It is believed that this research makes a contribution to a better understanding of the handling of tracked vehicles on non-deformable surfaces and that it provides a basis for the further study of tracked vehicle maneuverability on deformable terrain.

Acknowledgements

The author wishes to extend his deepest gratitude to his thesis supervisor, Dr. J.Y. Wong, for his invaluable guidance and suggestions throughout the study, as well as the preparation of the thesis.

The assistance provided by Mr. Y.C. Wu in setting up the apparatus for investigating track link-ground shearing characteristics is also greatly appreciated.

Finally, the author wishes to thank his colleague, Dr. Peijun Xu, for his assistance in conducting the experiments.

Table of Contents

	Page
Acceptance Sheet	i
Abstract	ii
Acknowledgements	iv
Table of Contents	v
List of Tables	viii
List of Figures	xv
List of Symbols	xxvi
 1. Introduction	 1
 2. Review of the State-of-Art on the Study of the Handling of Tracked Vehicles	 3
2.1. Steeds' Model	3
2.2. Crosheck's Model	9
2.3. Kitano's Model	10
2.4. Ehlert's Model	11
 3. A General Theory for the Mechanics of Steering of Tracked Vehicles on Non-deformable Surfaces	 16
3.1. Skid-Steering on Non-deformable Surfaces Including the Effect of Track Width	 16
3.1.1. Shear Displacement of the Outer Track	16
3.1.2. Shear Displacement of the Inner Track	21
3.1.3. Kinetics of Tracked Vehicles during a Steady-State Turning Maneuver	23
3.2. Normal Pressure Distribution Under the Tracks	28

4. Simulation of Handling Behaviour of Tracked Vehicles using the Proposed	
General Theory	30
4.1. Basic Design Parameters of the Vehicle Simulated	30
4.2. Simulation Results	31
4.3. Simulation of Turning Resistance Moment	40
4.4. Summary	41
5. Evaluation of the Effects of Design and Operating Factors on the Handling	
of Tracked Vehicles using the General Theory	46
5.1. Contact Length, L	46
5.2. Tread of Vehicle, B	53
5.3. Longitudinal Offset of Vehicle Center of Gravity, c_y	64
5.4. Track Width, b	73
5.5. Height of Vehicle Center of Gravity, h	73
5.6. Summary	74
6. Experimental Study of the Shearing Characteristics of a Representative	
Track Link on Asphalt	91
6.1. Apparatus for Measuring the Shearing Characteristics	91
6.2. Analysis of Shear Stress-Shear Displacement Data	94
6.3. The Determination of Coefficient of Friction	101
7. Simulation of Steering Behaviour of an M113 Armoured Personnel Carrier	
on Non-deformable Ground Based on Measured Shear Data	102
7.1. Basic Design Parameters of an M113 Armoured Personnel Carrier	102
7.2. Simulation Results	103
7.3. Summary	113

8. Discussion and Conclusions	114
8.1. Conclusions	114
8.2. Future Work	116
Bibliography	117
Appendix	120
A. The Effects of Design and Operating Factors on the Handling of Tracked Vehicles under Continuous Trapezoidal Load Distribution over the Entire Track	120
B. The Effects of Design and Operating Factors on the Handling of Tracked Vehicles for Trapezoidal Load Distribution on the Track Pitch under each Roadwheel	131
C. The Effects of Design and Operating Factors on the Handling of Tracked Vehicles for Concentrated Load under each Roadwheel	142

List of Tables

Table Description	Page
2-1 Comparison of steering models for tracked vehicles	15
4-1 Basic design parameters of the Jaguar tracked vehicle used in the simulation	30
4-2 Sprocket torques of the outer track at various turning radii with a speed of 7.5 km/h	36
4-3 Sprocket torques of the inner track at various turning radii with a speed of 7.5 km/h	36
4-4 Sprocket torques of the outer track at various turning radii with a speed of 14.2 km/h	37
4-5 Sprocket torques of the inner track at various turning radii with a speed of 14.2 km/h	37
4-6 Sprocket torques of the outer track at various turning radii with a speed of 21.3 km/h	38
4-7 Sprocket torques of the inner track at various turning radii with a speed of 21.3 km/h	38
4-8 Sprocket torques of the outer track at various turning radii with a speed of 29 km/h	38
4-9 Sprocket torques of the inner track at various turning radii with a speed of 29 km/h	39
7-1 Basic parameters of the M113 tracked vehicle used in the predictions	102
7-2 Sprocket torques at various turning radii with a vehicle speed of 7.5 km/h for an M113 armoured personnel carrier	103
7-3 Sprocket torques at various turning radii with a vehicle speed of 14.2 km/h for an M113 armoured personnel carrier	103
7-4 Sprocket torques at various turning radii with a vehicle speed of 21.3 km/h for an M113 armoured personnel carrier	104
7-5 Sprocket torques at various turning radii with a vehicle speed of 29 km/h	

	for an M113 armoured personnel carrier	104
A-1	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track	121
A-2	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track	121
A-3	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track	122
A-4	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track	122
A-5	Sprocket torques at various turning radii with different treads at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track	123
A-6	Sprocket torques at various turning radii with different treads at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track	123
A-7	Sprocket torques at various turning radii with different treads at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track	124
A-8	Sprocket torques at various turning radii with different treads at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track	124
A-9	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track	125
A-10	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track	125
A-11	Sprocket torques at various turning radii with different longitudinal CG	

	offsets at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track	126
A-12	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track	126
A-13	Sprocket torques at various turning radii with different track widths at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track	127
A-14	Sprocket torques at various turning radii with different track widths at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track	127
A-15	Sprocket torques at various turning radii with different track widths at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track	128
A-16	Sprocket torques at various turning radii with different track widths at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track	128
A-17	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track	129
A-18	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track	129
A-19	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track	130
A-20	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track	130
B-1	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel	132

B-2	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel	132
B-3	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel	133
B-4	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel	133
B-5	Sprocket torques at various turning radii with different treads at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel	134
B-6	Sprocket torques at various turning radii with different treads at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel	134
B-7	Sprocket torques at various turning radii with different treads at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel	135
B-8	Sprocket torques at various turning radii with different treads at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel	135
B-9	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel	136
B-10	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel	136
B-11	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel	137
B-12	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 29 km/h for trapezoidal load distribution	

	on the track pitch under each roadwheel	137
B-13	Sprocket torques at various turning radii with different track widths at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel	138
B-14	Sprocket torques at various turning radii with different track widths at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel	138
B-15	Sprocket torques at various turning radii with different track widths at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel	139
B-16	Sprocket torques at various turning radii with different track widths at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel	139
B-17	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel	140
B-18	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel	140
B-19	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel	141
B-20	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel	141
C-1	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel	143
C-2	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel	143
C-3	Sprocket torques at various turning radii with different track-ground	

	contact lengths at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel	144
C-4	Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 29 km/h for concentrated load under each roadwheel	144
C-5	Sprocket torques at various turning radii with different treads at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel	145
C-6	Sprocket torques at various turning radii with different treads at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel	145
C-7	Sprocket torques at various turning radii with different treads at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel	146
C-8	Sprocket torques at various turning radii with different treads at a vehicle speed of 29 km/h for concentrated load under each roadwheel	146
C-9	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel	147
C-10	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel	147
C-11	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel	148
C-12	Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 29 km/h for concentrated load under each roadwheel	148
C-13	Sprocket torques at various turning radii with different track widths at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel	149
C-14	Sprocket torques at various turning radii with different track widths at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel	149
C-15	Sprocket torques at various turning radii with different track widths at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel	150
C-16	Sprocket torques at various turning radii with different track widths at a vehicle speed of 29 km/h for concentrated load under each roadwheel	150
C-17	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel	151

C-18	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel	151
C-19	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel	152
C-20	Sprocket torques at various turning radii with different CG heights at a vehicle speed of 29 km/h for concentrated load under each roadwheel	152

List of Figures

Figure Description	Page
2-1 Kinetics of a tracked vehicle during a steady-state turn	5
2-2 Track force ellipse from Micklethwait	6
2-3 Kinetics of tracked vehicles during a steady-state turn.	8
2-4 Fundamentals of tracked vehicle steering considered in Hock's model	13
3-1 Kinematics of a tracked vehicle during a turning maneuver	18
3-2 Kinematics of a tracked vehicle during a steady-state turn	19
3-3 A shear curve of a simple exponential form	23
3-4 Kinetics of a tracked vehicle during a steady-state turn	25
3-5 Normal pressure distribution	28
4-1 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 7.5 km/h with $K = 0.075$ m and $\mu = 0.9$ during a steady-state turn	32
4-2 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 14.2 km/h with $K = 0.075$ m and $\mu = 0.9$ during a steady-state turn	33
4-3 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 21.3 km/h with $K = 0.075$ m and $\mu = 0.9$ during a steady-state turn	34
4-4 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 29 km/h with $K = 0.075$ m and $\mu = 0.9$ during a steady-state turn	35
4-5 Lateral shear stress distribution of the outer track along the longitudinal centerline of the track-ground contact area predicted from the proposed general theory at a vehicle speed of 14.2 km/h with different turning radii	42
4-6 Lateral shear stress distribution of the inner track along the longitudinal centerline of the track-ground contact area predicted from the proposed general theory at a vehicle speed of 14.2 km/h with different turning radii	42
4-7 Lateral shear stress distribution of the outer track along the longitudinal centerline of the track-ground contact area predicted from Steeds' model ($K = 0$ m) at a vehicle speed of 14.2 km/h with different turning radii	43

4-8	Lateral shear stress distribution of the inner track along the longitudinal centerline of the track-ground contact area predicted from Steeds' model ($K = 0$ m) at a vehicle speed of 14.2 km/h with different turning radii	43
4-9	Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track	44
4-10	Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track	44
4-11	Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track	45
4-12	Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track	45
5-1	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	47
5-2	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	47
5-3	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	48
5-4	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	48
5-5	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	49
5-6	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track contact lengths during a steady-state	

	turn under continuous trapezoidal load distribution over the entire track	49
5-7	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	50
5-8	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	50
5-9	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	51
5-10	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	51
5-11	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	52
5-12	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track	52
5-13	Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	54
5-14	Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	54
5-15	Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	55
5-16	Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 29 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	55
5-17	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of	

	7.5 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	56
5-18	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	56
5-19	Sprocket torque vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	57
5-20	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	57
5-21	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	58
5-22	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	58
5-23	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	59
5-24	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	59
5-25	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	60
5-26	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	60
5-27	Moments of turning resistance vs turning radius for Jaguar at a vehicle speed of 21.3 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	61

5-28	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track	61
5-29	Ratio of magnitude of the sprocket toques vs vehicle tread B for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	62
5-30	Ratio of magnitude of the sprocket toques vs vehicle tread B for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	62
5-31	Ratio of magnitude of the sprocket toques vs vehicle tread B for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	63
5-32	Ratio of magnitude of the sprocket toques vs vehicle tread B for a Jaguar at a vehicle speed of 29 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	63
5-33	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	65
5-34	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	65
5-35	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	66
5-36	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	66
5-37	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	67
5-38	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different longitudinal CG offsets during a steady-state turn under	

	continuous trapezoidal load distribution over the entire track	67
5-39	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	68
5-40	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	68
5-41	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	69
5-42	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	69
5-43	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	70
5-44	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track	70
5-45	Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	71
5-46	Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	71
5-47	Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	72
5-48	Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 29 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	72
5-49	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of	

	7.5 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	75
5-50	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	75
5-51	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	76
5-52	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	76
5-53	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	77
5-54	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	77
5-55	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	78
5-56	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track	78
5-57	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track widths during a steady-state turn under trapezoidal load distribution over the entire track	79
5-58	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track widths during a steady-state turn under trapezoidal load distribution over the entire track	79
5-59	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track widths during a steady-state turn under trapezoidal load distribution over the entire track	80

5-60	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track widths during a steady-state turn under trapezoidal load distribution over the entire track	80
5-61	Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	81
5-62	Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	81
5-63	Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	82
5-64	Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 29 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	82
5-65	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	83
5-66	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	83
5-67	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	84
5-68	Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	84
5-69	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	85
5-70	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different CG heights during a steady-state turn under	

	continuous trapezoidal load distribution over the entire track	85
5-71	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	86
5-72	Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	86
5-73	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	87
5-74	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	87
5-75	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	88
5-76	Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track	88
5-77	Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	89
5-78	Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	89
5-79	Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	90
5-80	Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 29 km/h with different turning radii during a steady-state turn under continuous trapezoidal load distribution over the entire track	90
6-1	Schematic of the test setup for the measurement of track shear force and shear	

	displacement on the asphalt	92
6-2	Block diagram of the data acquisition system for track shear force-shear displacement tester	93
6-3	Shear force-shear displacement curve at normal load 890 lb with sliding speed 0.100 m/s	95
6-4	Shear force-shear displacement curve at normal load 890 lb with sliding speed 0.145 m/s	95
6-5	Shear force-shear displacement curve at normal load 1000 lb with sliding speed 0.100 m/s	96
6-6	Shear force-shear displacement curve at normal load 1000 lb with sliding speed 0.145 m/s	96
6-7	Shear force-shear displacement curve at normal load 1000 lb with sliding speed 0.195 m/s	97
6-8	Shear force-shear displacement curve at normal load 1200 lb with sliding speed 0.100 m/s	97
6-9	Shear force-shear displacement curve at normal load 1200 lb with sliding speed 0.145 m/s	98
6-10	Shear force-shear displacement curve at normal load 1200 lb with sliding speed 0.195 m/s	98
6-11	Shear force-shear displacement curve at normal load 1400 lb with sliding speed 0.100 m/s	99
6-12	Shear force-shear displacement curve at normal load 1400 lb with sliding speed 0.145 m/s	99
6-13	Shear force-shear displacement curve at normal load 1400 lb with sliding speed 0.195 m/s	100
6-14	The relation of maximum shear force and normal load	101
7-1	Sprocket torques vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	105
7-2	Sprocket torques vs turning radius for an M113 at a vehicle speed of 14.2 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	105

7-3	Sprocket torques vs turning radius for an M113 at a vehicle speed of 21.3 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	106
7-4	Sprocket torques vs turning radius for an M113 at a vehicle speed of 29 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	106
7-5	Lateral force of the outer track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	107
7-6	Lateral force of the inner track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	107
7-7	Lateral forces vs turning radius for an M113 at a vehicle speed of 14.2 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	108
7-8	Lateral forces vs turning radius for an M113 at a vehicle speed of 21.3 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	108
7-9	Lateral forces vs turning radius for an M113 at a vehicle speed of 29 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	109
7-10	Moment of turning resistance of the outer track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	110
7-11	Moment of turning resistance of the inner track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	110
7-12	Moments of turning resistance vs turning radius for an M113 at a vehicle speed of 14.2 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	111
7-13	Moments of turning resistance vs turning radius for an M113 at a vehicle speed of 21.3 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	111
7-12	Moments of turning resistance vs turning radius for an M113 at a vehicle speed of 29 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn	112

List of Symbols

a_i	slip radius of the inner track
a_o	slip radius of the outer track
b	track width
B	tread of vehicle
c_x	lateral offset of vehicle CG to the geometrical center of the vehicle
c_y	longitudinal offset of vehicle CG to the geometrical center of the vehicle
C_1	cone index of the soil
C_{nj}	track numeric of the j^{th} track
E_1	coefficient of pull-slip equation
E_2	coefficient of pull-slip equation
E_3	coefficient of pull-slip equation
E_4	coefficient defining the external motion resistance
E_5	coefficient defining the external motion resistance
f_r	coefficient of external motion resistance
F_i	inner track longitudinal force
F_o	outer track longitudinal force
F_{xi}	lateral force acting on the inner track
F_{xo}	lateral force acting on the outer track
F_{yi}	longitudinal force acting on the inner track
F_{yo}	longitudinal force acting on the outer track
h	height of vehicle CG
i_y	track slip along the longitudinal direction
j	shear displacement
j_i	shear displacement of the inner track

j_o	shear displacement of the outer track
j_{Xi}	X direction of shear displacement on the inner track in contact with the ground
j_{Xo}	X direction of shear displacement on the outer track in contact with the ground
j_{Yi}	Y direction of shear displacement on the inner track in contact with the ground
j_{Yo}	Y direction of shear displacement on the outer track in contact with the ground
k	track exponent
K	shear deformation parameter
L	track-ground contact length
L_p	track pitch
M_{Li}	turning moment acting on the inner track
M_{Lo}	turning moment acting on the outer track
M_{Ti}	moment of turning resistance acting on the inner track
M_{To}	moment of turning resistance acting on the outer track
M_w	moment of turning resistance
p	normal pressure distribution
p_i	normal pressure distribution on the inner track
p_{ij}	normal load distribution under the i^{th} roadwheel and j^{th} track
p_o	normal pressure distribution on the outer track
r	pitch radius of the sprocket
R	turning radius of the vehicle
R'	lateral distance between the center of turn and vehicle CG
R_j	external motion resistance of j^{th} track
R_k	turning radius at which both track forces are zero
R_t	external motion resistance
R_{th}	theoretical turning radius
R_{ti}	external motion resistance on the inner track
R_{to}	external motion resistance on the outer track

s_0	offset of turning center
t	time
V	mean speed of vehicle center of gravity
V_i	theoretical velocity of the inner track
V_{ji}	sliding velocity of the inner track
V_{jo}	sliding velocity of the outer track
V_{jxi}	lateral sliding velocity of a point (x_2, y_2) with respect to moving frame of reference
V_{jxo}	lateral sliding velocity of a point (x_1, y_1) with respect to moving frame of reference
V_{jyi}	longitudinal sliding velocity a point (x_2, y_2) with respect to moving frame of reference.
V_{jyo}	longitudinal sliding velocity of a point (x_1, y_1) with respect to moving frame of reference
V_{jxi}	X direction of sliding velocity of a point (x_2, y_2) with respect to fixed frame of reference
V_{jxo}	X direction of sliding velocity of a point (x_1, y_1) with respect to fixed frame of reference
V_{jyi}	Y direction of sliding velocity of a point (x_2, y_2) with respect to fixed frame of reference
V_{jyo}	Y direction of sliding velocity of a point (x_1, y_1) with respect to fixed frame of reference
V_o	theoretical velocity of the outer track
$V_{t1/o1}$	relative velocity of point o_{t1} in y_1 direction with respect to o_1
V_{tU}	sliding velocity of point o_{t1} along the longitudinal direction
V_{o1y1}	absolute velocity of point o_1 in y_1 direction
V_{o2y2}	absolute velocity of point o_2 in y_2 direction
W	vehicle weight
W_i	normal load acting on the inner track
W_{ij}	normal load under the i^{th} roadwheel and j^{th} track

W_j	normal load acting on the j^{th} track
W_o	normal load acting on the outer track
β	angle defining the direction of centrifugal force
δ_1	angle between the resulting sliding velocity and the x_1 axis
δ_2	angle between the resulting sliding velocity and the x_2 axis
δ_i	direction angle of shear force acting on the inner track
δ_o	direction angle of shear force acting on the outer track
ε	goodness-of-fit
ϕ	yaw angle
$\dot{\phi}$	angular speed of the vehicle
μ	coefficient of friction
μ_x	coefficient of friction along lateral direction
μ_y	coefficient of friction along longitudinal direction
μ_w	coefficient of turning resistance
μ_{w0}	maximum coefficient of turning resistance
τ	shear stress
τ_c	calculated sprocket torque
τ_i	shear stress on the inner track
τ_m	measured sprocket torque
$\tau_{\max}$	maximum shear stress
τ_o	shear stress on the outer track
ω_i	rotating speed of the sprocket driving the inner track
ω_o	rotating speed of the sprocket driving the outer track

Chapter 1

Introduction

Tracked vehicles are widely used in construction, agriculture and military operations. While many studies (Bekker, 1956, 1960; Wong, 1989, 1993) on the mobility of tracked vehicles have been performed, research on the handling behavior of tracked vehicles, has not yet received the same level of attention.

Handling characteristics of tracked vehicles are quite different from those of wheeled vehicles. Among the various methods that can accomplish the steering of a tracked vehicle, such as skid-steering, steering by articulation, and curved track steering, skid-steering is the most widely used. In skid-steering, the tractive force of the outer track is increased and that of the inner track is reduced to create a turning moment to overcome the moment of turning resistance due to the skidding of the tracks on the ground and the rotational inertia of the vehicle in yaw (Wong, 1993). Since the moment of turning resistance is usually considerable, braking of the inner track is often required in making a turn. This results in a reduction in the forward thrust. Furthermore, skidding of the track on deformable terrain causes additional sinkage. Over weak terrain, these factors combined often lead to immobilization.

Skid-steering has been a subject of research for a considerable period of time. Steeds (1950) examined the mechanics of skid-steering on rigid surfaces. His pioneering work laid the foundation for the subsequent studies of this topic. In recent years, Weiss (1971), Kitano *et al.* (1976, 1977, 1992), and Ehlert *et al.* (1992) have expended considerable effort on the further study of the mechanics of steering of tracked vehicles, and a number of models have

been developed. It should be noted however, that in most of the previous studies on steering, the shear stress developed between the track and the ground is assumed to obey Coulomb's law of friction. This implies that the full shear stress will be developed as soon as a small relative motion between the track and the ground takes place. Experimental evidence (Wong, 1989, 1993) has shown that, the shear stress developed on the tracked vehicle-ground interface is dependent upon the shear displacement. This means that the shear stress will reach its maximum value $\tau_{\max}$ only after a certain shear displacement has been developed.

This research focuses on the detailed examination of the mechanics of vehicle-terrain interaction on non-deformable surfaces during turning maneuvers, taking into account the effects of shear displacement on the development of shear stress. Experimental studies have been carried out to measure the shearing characteristics of representative track links of armoured personnel carriers on asphalt in the laboratory. Based on the results of both analytical and experimental studies, a computer model for simulating the steering behaviour of tracked vehicles under steady-state conditions over non-deformable surfaces has been developed. It is believed that this study makes a contribution to a better understanding of the handling behaviour of tracked vehicles on non-deformable surfaces, and provides a basis for the further study of tracked vehicle maneuverability on deformable terrain.

Chapter 2

Review of the State-of-Art on the Study of the Handling of Tracked Vehicles

As mentioned previously, this research focuses on the handling characteristics of tracked vehicles on non-deformable surfaces. As the forces and moments generated on the track-ground interface control the turning behavior, the study of the interaction between the track and the ground is of importance to the understanding of the handling behavior. Previous studies on tracked vehicle steering will be reviewed.

2.1. Steeds' Model

Steeds (1950) was one of the pioneers who conducted a systematic study of skid-steering of tracked vehicles. Most of the subsequent studies on this topic were based on his approach. A brief outline of his method of approach is given below.

A. Kinematics

Consider a tracked vehicle in a turning maneuver at low speeds with centrifugal force neglected, as shown in Fig.2-1. If the hull is turning about the center O with an angular speed, $\dot{\phi}$, then the mean speed, V, of the vehicle at center of gravity (CG) is given by

$$V = R\dot{\phi} \tag{2-1}$$

where R is turning radius.

Since the tracks are attached to the vehicle hull, the tracks are also rotating with the

same angular speed, $\dot{\phi}$. If a track develops a forward thrust, then it will slip backward with respect to the ground. The magnitude of the sliding velocity of the outer track, V_{jo} , can be expressed by:

$$V_{jo} = a_o \dot{\phi} \quad (2-2)$$

where a_o is called the slip radius of the outer track, which is the distance from the centerline of outer track to the instantaneous center O' of that part of the outer track in contact with the ground. O' must lie on the line passing through the vehicle turning center and perpendicular to the longitudinal centerline of vehicle hull. The instantaneous center O' is in fact the center about which the part of the outer track in contact with the ground rotates. Consequently, the direction and magnitude of the sliding velocity on the ground of a point on the outer track can be determined from the instantaneous center O' . For instance, the direction of the sliding velocity is perpendicular to the line connecting the point in question and O' and its sense is determined by the direction of the angular velocity, $\dot{\phi}$, as shown in Fig.2-1. The magnitude of the sliding velocity is equal to the product of $\dot{\phi}$ and the distance between the point in question and O' . Note that when the track skids forward relative to the ground, the sliding velocity, V_{ji} , is negative (i.e., skid), and the instantaneous center O'' will lie between the centerline of the track and the vehicle turning center, as shown in Fig.2-1. In this case, the inner track develops a braking force.

From the above analysis, it can be seen that the speed of point Q on the outer track in contact with the ground relative to the coincident point on the vehicle hull is $(R + B/2)\dot{\phi} + V_{jo}$, where B is the tread of the vehicle (i.e. the distance between the longitudinal centerlines of the left- and right-hand tracks). This is equal to the theoretical speed of the outer track $r\omega_o$, where r is the pitch radius of the sprocket, and ω_o is the rotating speed of sprocket driving the outer track. This is expressed by

$$r\omega_o = (R + B/2 + a_o) \dot{\phi} \quad (2-3)$$

Similarly, for the inner track, the corresponding equation is

$$r\omega_i = (R - B/2 - a_i)\dot{\phi} \quad (2-4)$$

where a_i is the slip radius of the inner track, assuming that the inner track skids, i.e. sliding forward.

Fig.2-1 Kinematics of a tracked vehicle during a steady-state turn.

B. Kinetics:

In analyzing the forces and moments generated by track-ground interaction, there are three assumptions in Steeds' model:

1. The ground is non-deformable and the interacting forces between the track and the ground obey Coulomb's law of friction; that is, the frictional force is proportional to the normal load and acts in the opposite direction of the relative motion of the track with

respect to the ground as shown in Fig.2-2. If the friction coefficient in the track longitudinal direction, μ_y , is different from that of in the lateral direction, μ_x , that is the friction is anisotropic, then the direction of the resultant shear force is modified as suggested by Mickelthwait (1944) and shown in Fig.2-2.

Fig. 2-2 Track force ellipse from Mickelthwait.

2. Normal pressure distribution on the track-ground contact area is considered to be uniform.
3. The width of the track is neglected.

Based on Coulomb's law of isotropic friction, the forces and moments developed by the track can be formulated as follows (see Fig.2-3). It should be noted that when the effect of centrifugal force is taken into account, the turning center O is at a distance, s_0 , from the center of gravity in the longitudinal direction, as shown in Fig.2-3, in order to satisfy the condition of dynamic equilibrium in the lateral direction.

$$F_{xo} = \frac{\mu W_o}{L} \int_{-(L/2+s_o)}^{L/2-s_o} \sin \delta_o dy = \mu W_o \cdot f(a_o, s_o) \quad (2-5)$$

$$F_{xi} = \mu W_i \cdot f(a_i, s_o) \quad (2-6)$$

$$F_{yo} = \frac{\mu W_o}{L} \int_{-(L/2+s_o)}^{L/2-s_o} \cos \delta_o dy = \mu W_o \cdot \phi(a_o, s_o) \quad (2-7)$$

$$F_{yi} = \mu W_i \cdot \phi(a_i, s_o) \quad (2-8)$$

$$M_{To} = \frac{\mu W_o}{L} \int_{-(L/2+s_o)}^{L/2-s_o} y \sin \delta_o dy = \mu W_o L \cdot \psi(a_o, s_o) \quad (2-9)$$

$$M_{Ti} = \frac{\mu W_i}{L} \int_{-(L/2+s_o)}^{L/2-s_o} y \sin \delta_i dy = \mu W_i L \cdot \psi(a_i, s_o) \quad (2-10)$$

where W_o , W_i , F_{xo} , F_{xi} , F_{yo} , F_{yi} , M_{To} , M_{Ti} , δ_o , and δ_i are the normal loads, lateral forces, longitudinal forces, moments of turning resistance, and direction angles of shear forces acting on the outer and inner tracks, respectively (subscript o indicates outer track, and i indicates inner track). μ is the coefficient of friction, L is the track-ground contact length, and s_o is the offset of turning center from the transverse centerline of the track-ground contact area (the center of gravity is assumed to be located at the center of the track), as shown in Fig.2-3.

As a result, under a steady-state turn the following equilibrium equations can be obtained.

$$F_{yo} + F_{yi} = \frac{WV^2}{gR} \sin \beta + R_t = \frac{WV^2 s_o}{gR^2} + R_t \quad (2-11)$$

$$F_{xo} + F_{xi} = \frac{WV^2}{gR} \cos \beta = \frac{WV^2 \sqrt{R^2 - s_o^2}}{gR^2} \quad (2-12)$$

$$(F_{yo} - F_{yi}) \cdot B + s_o \frac{WV^2}{gR} \cos \beta = M_{To} + M_{Ti} \quad (2-13)$$

where R_t is the external motion resistance, and β is the angle defining the direction of the centrifugal force with respect to the x-axis, as shown in Fig.2-3.

In Steeds' model, a set of monographs was provided, and the iteration method was suggested to obtain the three unknowns a_o , a_i and s_o . Although, the model can be extended to anisotropic case if coefficients of friction along the longitudinal and lateral directions are known, isotropic Coulomb friction was used in the study. It should also be noted that there was no measured data to support his analysis.

Fig.2-3 Kinetics of tracked vehicles during a steady-state turn.

2.2. Croscheck's Model

Based on the study of skid-steering by Steeds, Croscheck (1975) provided a model which is claimed to be applicable to deformable surfaces. In his model, the following assumptions were made:

1. The friction between the track and the ground is anisotropic as suggested by Micklethwait.

The coefficient of friction, μ_y , along the longitudinal direction is expressed by

$$\mu_y = E_1 \left(1 - e^{E_2 C_{nj}} \right) \left(1 - e^{E_3 C_{nj} i_y} \right) \quad (2-14)$$

where E_1 , E_2 and E_3 are coefficients for defining the longitudinal coefficient of friction, derived from the pull-slip test of the track, and are 0.95, -0.1, and -1.0 for the particular track on the specific terrain cited in the reference (Croscheck, 1975), respectively, and i_y is track slip along the longitudinal direction. In the lateral direction, full skid was assumed. Therefore, the lateral coefficient of friction, μ_x , is equal to the value of μ_y at $i_y = 1$. C_{nj} is the track numeric of the j^{th} track and is expressed by

$$C_{nj} = \frac{C_i b L}{W_j} \quad (2-15)$$

where C_i is the cone index of the soil, b is the track width, L is the track contact length, W_j is the normal load acting on the j^{th} track.

2. Triangular normal pressure distribution along track-terrain interface is considered.
3. The motion resistance is expressed by

$$R_j = W_j \left(\frac{E_4 W_j}{C_i b L} + E_5 \right) \quad (2-16)$$

where E_4 and E_5 are 0.45 and 0.045, for the particular track on the specific terrain cited in the reference (Croscheck, 1975), respectively.

4. The width of the track is neglected.

Based on these assumptions and following the same approach to kinetic analysis as that of Steeds, the forces and moments in a steady-state turn can be determined. In this model, bulldozing effect due to skidding of the track on the terrain and actual normal pressure distribution have not been taken into consideration. In addition, no comparison with experimental results was provided.

2.3. Kitano's Model

Similar to Croscheck's analysis, Kitano (1976, 1977, 1992) provided a model for predicting steering behavior of a tracked vehicle on level ground, and was considered to be useful by Ehlert *et al.* (1992). In this model, the following assumptions were made:

1. The friction between the track and the ground is Coulomb friction, and it is anisotropic as suggested by Micklethwait. The coefficients of friction along longitudinal and lateral directions, μ_y and μ_x , respectively, were derived from pull-slip relation for the track as follows:

$$\mu_y = E_1 \left(1 - e^{-E_2 v} \right); \quad \mu_x = E_1 \left(1 - e^{-E_2} \right) \quad (2-17)$$

where $E_1 = 0.44$, and $E_2 = 20.0$ for a representative hard ground.

2. Normal load is concentrated under each road wheel.
3. Transverse distribution of normal load per unit track width is assumed to be of trapezoidal form, which can be expressed by

$$\text{At middle of the track.} \quad p_{ij}(0) = \frac{1.5W_{ij}}{b} \quad (2-18a)$$

and at the edges,
$$p_{ij}\left(-\frac{b}{2}\right) = p_{ij}\left(\frac{b}{2}\right) = \frac{0.5W_{ij}}{b} \quad (2-18b)$$

where p_{ij} and W_{ij} are the normal load per unit width and normal load under the i^{th} roadwheel and j^{th} track, respectively.

4. External motion resistance of the j^{th} track, R_j , is assumed to be proportional to normal load.

$$R_j = f_r \cdot W_j \quad (2-19)$$

where f_r and W_j are the coefficient of external motion resistance and normal load on the j^{th} track, respectively.

Kitano's model is useful in predicting the steering behavior of tracked vehicles on hard ground and is substantiated by Ehlert *et al.* (1992) from the field measurement.

2.4. Ehlert's Model

In order to establish an adequate simulation model for tracked vehicle handling, three different analytical models, including Kitano's model, were investigated by Ehlert *et al.* (1992). After certain modifications and improvements, the results were verified by field measurements, primarily on hard surfaces (Schmid, 1984). Although Kitano's model offers the possibility of predicting the turning radius for a given set of inputs, it is time-consuming and is not suitable for on-line test stand simulation. As a result, a model based on the assumptions similar to those originally proposed by Hock (1970) was developed by Ehlert *et al.* (1992).

1. Center of gravity is located at the center of the vehicle.
2. Ground pressure on the track-ground contact area is uniformly distributed.
3. Outer track longitudinal force, F_o , inner track longitudinal force, F_i , and moment of

turning resistance, M_w , are predicted without considering the effects of centrifugal force, and they are expressed by (as shown in Fig.2-4.)

$$F_o = \frac{W \cdot L}{4B} \cdot \mu_w \quad (2-20a)$$

$$F_i = -\frac{W \cdot L}{4B} \cdot \mu_w \quad (2-20b)$$

$$M_w = \frac{W \cdot L}{4} \cdot \mu_w \quad (2-20c)$$

where μ_w is the coefficient of turning resistance and is a function of theoretical turning radius, R_{th} , which is the turning radius if track slip is equal to zero. The parameters μ_w and R_{th} , are defined as

$$\mu_w = \mu_{w0} \cdot \frac{(1 - R_{th}/R_k)}{(1 + 2R_{th}/B)^k} \quad (2-21a)$$

$$R_{th} = \frac{V_o + V_i}{V_o - V_i} \cdot \frac{B}{2} \quad (2-21b)$$

where μ_{w0} is the maximum coefficient of turning resistance, B is the tread of the vehicle, R_k is the turning radius at which both track forces are approaching zero, k is called track exponent and is a function of R_{th} , and V_o and V_i are theoretical speeds of outer and inner tracks when track slip is equal to zero, respectively. It should be pointed out that k is an empirical parameter and is derived from experimental data.

4. External motion resistance is a function of the theoretical turning radius, R_{th} .
5. The radius enlargement factor, which is the ratio of actual turning radius, R , and theoretical turning radius, R_{th} (as shown in Fig.2-4(c)), is taken to be 1.8.

The basic features of the model have been substantiated by experimental data obtained on non-deformable surfaces and is quite accurate in the prediction of sprocket torques for on-line simulations.

Fig.2-4 Fundamentals of tracked vehicle steering considered in Hock's model (Ehlert *et al.*, 1992).

The basic features of existing models for skid-steering of tracked vehicles are summarized in Table 2-1.

From the brief review given above, it can be said that most of the previous studies focus on tracked vehicle steering on rigid surfaces, and are based on simplifying assumptions, such as the normal pressure on the track being either uniformly distributed or concentrated under the roadwheels, and the negligible effects of track width on the moment of turning resistance. Furthermore, in most of the models developed, it is assumed that during steering the shear forces on the track-ground interface obey Coulomb's law of friction. This implies that full shear force will be developed as soon as a small relative motion between the track and the ground takes place. As noted previously, experimental evidence indicates that the shear force developed on the track-ground interface is related to the shear displacement.

In this research, the effects of shear displacement on the development of shear force will be taken into account. The effect on the handling behaviour of major vehicle design parameters, such as track width, and the location of the center of gravity of tracked vehicles on non-deformable surfaces will be examined in detail.

Table 2-1 Comparison of steering models for tracked vehicles

Model	Features considered	Factors not included	Remarks
Steeds [1950]	<ul style="list-style-type: none"> • Kinematics of steady-state turning on level ground. • Basic mechanics of skid-steering. • Coulomb friction representation of track-ground shearing. • Uniform normal pressure distribution under the tracks. • Centrifugal force and motion resistance. 	<ul style="list-style-type: none"> • Kinematics and kinetics of skid-steering on deformable terrain. • Track width effect. 	<ul style="list-style-type: none"> • No experimental validation of analytical findings. • Iteration method of solution.
Croscheck [1975]	<ul style="list-style-type: none"> • Kinematics of steady-state turning on soft terrain. • Anisotropic Coulomb friction representation of track-ground shearing. • Location of C.G. of the vehicle. • Triangular normal pressure distribution under the tracks. • Centrifugal force and motion resistance. 	<ul style="list-style-type: none"> • Kinetics of skid-steering on deformable surfaces. • Track width effect. 	<ul style="list-style-type: none"> • No experimental validation of analytical findings
Kitano [1975, 1977, 1992]	<ul style="list-style-type: none"> • Kinetics and kinematics of skid-steering on level ground. • Normal pressure concentrated under each roadwheel, and lateral pressure distribution of trapezoidal form. • Anisotropic Coulomb friction representation of track-ground shearing. • Centrifugal force and motion resistance. • Track tension. 	<ul style="list-style-type: none"> • Kinematics and kinetics of skid-steering on deformable terrain. 	
Ehlert [1992]	<ul style="list-style-type: none"> • Uniform ground pressure on track-ground contact area. • Longitudinal tractive forces and turning moments dependent on the turning resistance coefficient μ_w, which was a function of theoretical turning radius. • External motion resistance also a function of theoretical turning radius. • Centrifugal force. 	<ul style="list-style-type: none"> • Kinematics and kinetics of skid-steering on deformable terrain. • Track width effect 	<ul style="list-style-type: none"> • Experimental results on rigid surfaces provided to verify simulation data.

Chapter 3

A General Theory for the Mechanics of Steering of Tracked Vehicles on Non-deformable Surfaces

In most of the models described previously, shear force between the track and the ground is assumed to obey Coulomb's law of friction and the coefficient of friction is either isotropic or anisotropic. As mentioned previously, experimental evidence (Wong, 1989, 1993) indicates that the shear stress developed on the track-ground interface is dependent upon the shear displacement. In addition, the effects of track width on steering have not been considered in most of the previous studies. In this chapter, a general theory is developed based on a detailed analysis of skid-steering on non-deformable surfaces, taking into account the effects of the shear stress-shear displacement relationship and of track width.

3.1. Skid-Steering on Non-deformable Surfaces Including the Effect of Track Width

3.1.1. Shear Displacement of the Outer Track

Consider that a tracked vehicle with track width, b , is in a steady-state turn about O , as shown in Fig.3-1 and Fig.3-2.

Let o_l be the origin of a frame of reference (x_l, y_l) fixed to and moving with the vehicle hull and located on the longitudinal centerline of the outer track and at a distance, s_o , from the center of gravity (CG) of the vehicle, as shown in Fig.3-1. The offset distance, s_o , will be determined later from the dynamic equilibrium of the vehicle in the lateral direction during a

turn. As the vehicle hull is rotating about turning center O with an angular speed, $\dot{\phi}$, the absolute velocity of o_1 , $V_{o_1y_1}$, in the y_1 direction can be expressed by

$$V_{o_1y_1} = (R' + B/2 + c_x) \dot{\phi} \quad (3-1)$$

where R' is the lateral distance between the center of turn O and center of gravity of the vehicle, and is equal to $R \cdot \cos\beta$ (or $\sqrt{R^2 - s_o^2}$), as shown in Fig.3-1, R is the turning radius of the vehicle, c_x is the lateral distance between the vehicle CG and longitudinal centerline of vehicle hull (or the lateral offset of the center of gravity with respect to the geometrical center of the vehicle), and tread, B , is the distance between the centerlines of the outer and inner tracks.

A point o_{t1} on the outer track in contact with the ground coincident with o_1 has a relative velocity, V_{t1/o_1} , with respect to o_1 , which is expressed by

$$V_{t1/o_1} = r\omega_o \quad (3-2)$$

where r is the pitch radius of the sprocket and ω_o is the angular speed of the sprocket of the outer track.

As a result, the sliding velocity, V_{t1j} , of point o_{t1} on the ground along the longitudinal direction of the outer track is expressed by

$$V_{t1j} = V_{o_1y_1} - r\omega_o \quad (3-3)$$

Consider an arbitrary point defined by (x_1, y_1) on the outer track in contact with the ground. Since the track is rotating with the vehicle at angular speed, $\dot{\phi}$, during a steady turn, the relative velocity components of the point (x_1, y_1) with respect to o_{t1} in the longitudinal and lateral directions of the track are given by $x_1\dot{\phi}$ and $y_1\dot{\phi}$ respectively.

18

Fig.3-2 Kinematics of a tracked vehicle during a steady-state turn.

Based on the above analysis, the sliding velocity of a point defined by (x_1, y_1) on the outer track in contact with ground, with respect to the fixed frame of reference (X, Y) can be expressed by (Fig.3-1)

X component of sliding velocity:

$$\begin{aligned} V_{jx_0} &= -V_{o1y1} \sin \phi + r\omega_o \sin \phi - x_1 \dot{\phi} \sin \phi - y_1 \dot{\phi} \cos \phi \\ &= -[(R' + B/2 + c_x + x_1) \dot{\phi} - r\omega_o] \sin \phi - y_1 \dot{\phi} \cos \phi \end{aligned} \quad (3-4)$$

Y component of sliding velocity:

$$\begin{aligned} V_{jy_0} &= V_{o1y1} \cos \phi - r\omega_o \cos \phi + x_1 \dot{\phi} \cos \phi - y_1 \dot{\phi} \sin \phi \\ &= [(R' + B/2 + c_x + x_1) \dot{\phi} - r\omega_o] \cos \phi - y_1 \dot{\phi} \sin \phi \end{aligned} \quad (3-5)$$

The angle, ϕ , as shown in Fig.3-1 is the angular displacement of the vehicle and can be

determined by the integration of the yaw velocity, $\dot{\phi}$, with respect to the time, t , it takes for the point (x_1, y_1) to travel from the initial point of contact at the front of the track (at $y_1 = L/2 + c_y - s_o$), that is

$$\phi = \int_0^t \dot{\phi} dt = \dot{\phi} t \quad (3-6)$$

and

$$t = \int_0^t dt = \int_{L/2+c_y-s_o}^{y_1} \frac{-dy_1}{r\omega_o} = \int_{y_1}^{L/2+c_y-s_o} \frac{dy}{r\omega_o} = \frac{L/2+c_y-s_o-y_1}{r\omega_o} \quad (3-7)$$

where c_y is longitudinal distance between CG and lateral centerline of the vehicle hull, or the longitudinal offset of the center of gravity with respect to the geometric center of the vehicle (see Fig.3-1).

As a result, the shear displacement, j_{x_o} , at a point (x_1, y_1) on the outer track in contact with the ground along the X direction with respect to fixed frame of reference (X, Y) can therefore be determined by

$$\begin{aligned} j_{x_o} &= \int_0^t V_{j_{x_o}} dt = \int_{y_1}^{L/2+c_y-s_o} \left\{ - \left[\left(R' + \frac{B}{2} + c_x + x_1 \right) \dot{\phi} - r\omega_o \right] \sin\phi - y_1 \dot{\phi} \cos\phi \right\} \frac{dy_1}{r\omega_o} \\ &= (R' + B/2 + c_x + x_1) \left\{ \cos \left[\frac{(L/2 + c_y - s_o - y_1) \dot{\phi}}{r\omega_o} \right] - 1 \right\} - \\ &\quad y_1 \sin \left[\frac{(L/2 + c_y - s_o - y_1) \dot{\phi}}{r\omega_o} \right] \end{aligned} \quad (3-8)$$

and shear displacement, j_{y_o} , along the Y direction

$$j_{y_o} = \int_0^t V_{j_{y_o}} dt = \int_{y_1}^{L/2+c_y-s_o} \left\{ \left[\left(R' + \frac{B}{2} + c_x + x_1 \right) \dot{\phi} - r\omega_o \right] \cos\phi - y_1 \dot{\phi} \sin\phi \right\} \frac{dy_1}{r\omega_o}$$

$$\begin{aligned}
&= \left(R' + \frac{B}{2} + c_x + x_1 \right) \sin \left[\frac{(L/2 + c_y - s_o - y_1) \dot{\phi}}{r\omega_o} \right] - \left(\frac{L}{2} + c_y - s_o \right) + \\
&\quad y_1 \cos \left[\frac{(L/2 + c_y - s_o - y_1) \dot{\phi}}{r\omega_o} \right]
\end{aligned} \tag{3-9}$$

The resultant shear displacement, j_o , of a point at (x_1, y_1) on the outer track in contact with the ground is given by

$$j_o = \sqrt{j_{x_o}^2 + j_{y_o}^2} \tag{3-10}$$

3.1.2. Shear Displacement of the Inner Track

Similarly let o_2 be the origin for a frame of reference (x_2, y_2) fixed to and moving with the vehicle hull and be located on the longitudinal centerline of the inner track. The absolute velocity of o_2 , $V_{o_2y_2}$, in the y_2 direction can be expressed by

$$V_{o_2y_2} = (R' - B/2 + c_x) \dot{\phi} \tag{3-11}$$

Following a similar approach described in 3.1.1, the sliding velocity of a point (x_2, y_2) on the inner track in contact with the ground with respect to fixed frame of reference (X, Y) can be expressed by (see Fig.3-1)

X component of sliding velocity:

$$\begin{aligned}
V_{j_{x1}} &= -V_{o_2y_2} \sin \phi + r\omega_i \sin \phi - x_2 \dot{\phi} \sin \phi - y_2 \dot{\phi} \cos \phi \\
&= -[(R' - B/2 + c_x + x_2) \dot{\phi} - r\omega_i] \sin \phi - y_2 \dot{\phi} \cos \phi
\end{aligned} \tag{3-12}$$

Y component of sliding velocity:

$$V_{j_{y1}} = V_{o_2y_2} \cos \phi - r\omega_i \cos \phi + x_2 \dot{\phi} \cos \phi - y_2 \dot{\phi} \sin \phi$$

$$= \left[\left(R' - B/2 + c_x + x_2 \right) \dot{\phi} - r\omega_i \right] \cos\phi - y_2 \dot{\phi} \sin\phi \quad (3-13)$$

where ω_i is the inner track sprocket angular speed.

However, the contact time, t , for the point defined by (x_2, y_2) will be equal to $(L/2 + c_y - s_0 - y_2)/r\omega_i$, and, $dt = -dy_2/r\omega_i$. Therefore, the shear displacement, j_{xi} , on the inner track of the vehicle along the X direction with respect to fixed frame of reference (X, Y) can therefore be determined by

$$\begin{aligned} j_{xi} &= \int_0^t V_{jxi} dt = \int_{y_2}^{L/2 + c_y - s_0} \left\{ - \left[\left(R' - \frac{B}{2} + c_x + x_2 \right) \dot{\phi} - r\omega_i \right] \sin\phi - y_2 \dot{\phi} \cos\phi \right\} \frac{dy_2}{r\omega_i} \\ &= \left(R' - \frac{B}{2} + c_x + x_2 \right) \left\{ \cos \left[\frac{(L/2 + c_y - s_0 - y_2) \dot{\phi}}{r\omega_i} \right] - 1 \right\} - \\ &\quad y_2 \sin \left[\frac{(L/2 + c_y - s_0 - y_2) \dot{\phi}}{r\omega_i} \right] \end{aligned} \quad (3-14)$$

and shear displacement, j_{yi} , along the Y direction

$$\begin{aligned} j_{yi} &= \int_0^t V_{jyi} dt = \int_{y_2}^{L/2 + c_y - s_0} \left\{ \left[\left(R' - \frac{B}{2} + c_x + x_2 \right) \dot{\phi} - r\omega_i \right] \cos\phi - y_2 \dot{\phi} \sin\phi \right\} \frac{dy_2}{r\omega_i} \\ &= \left(R' - \frac{B}{2} + c_x + x_2 \right) \sin \left[\frac{(L/2 + c_y - s_0 - y_2) \dot{\phi}}{r\omega_i} \right] - \left(\frac{L}{2} + c_y - s_0 \right) + \\ &\quad y_2 \cos \left[\frac{(L/2 + c_y - s_0 - y_2) \dot{\phi}}{r\omega_i} \right] \end{aligned} \quad (3-15)$$

The resultant shear displacement, j_i , of a point at (x_2, y_2) on the inner track in contact with the ground is given by

$$j_i = \sqrt{j_{xi}^2 + j_{yi}^2} \quad (3-16)$$

3.1.3. Kinetics of Tracked Vehicles during a Steady-State Turning Maneuver

As noted above, the shear stress developed on the track-ground interface is dependent upon the shear displacement. On a non-deformable surface, in most cases the shear stress, τ , initially increases rapidly with an increase in the shear displacement, j , and then approaches a constant value with a further increase in shear displacement, as shown in Fig.3-3. This type of shear stress-shear displacement relationship may be described by the following exponential equation:

$$\tau = \tau_{\max} (1 - e^{-j/K}) = p \cdot \mu \cdot (1 - e^{-j/K}) \quad (3-17)$$

where K is the shear deformation parameter, and may be considered as a measure of the magnitude of the shear displacement required to develop the maximum shear stress, p is the normal pressure, and μ is the coefficient of friction between the track and the ground. K may be determined by the intersection of the line tangent to the curve at the origin and the horizontal line representing the maximum shear stress, as shown in Fig.3-3.

Fig.3-3 A shear curve of a simple exponential form.

Therefore, the shear force developed on an element dA of the track in contact with the ground can be expressed as:

$$\text{on the outer track} \quad dF_o = \tau_o dA = p_o \mu (1 - e^{-J_o/K}) \cdot dA \quad (3-18)$$

$$\text{on the inner track} \quad dF_i = \tau_i dA = p_i \mu (1 - e^{-J_i/K}) \cdot dA \quad (3-19)$$

where τ_o , τ_i , p_o and p_i are the shear stress and normal pressure on elements of the outer and inner tracks, respectively.

It should be noted that the assumption used in Steeds' model that the shear force between the track and the ground obeys Coulomb's law of friction is a special case of equation (3-17). Coulomb's law assumes that full frictional force is reached as soon as a small relative motion between the track and the ground takes place. This is equivalent to the value of K in equation (3-17) equal to zero. This indicates that equation (3-17) is a more general representation of the characteristics of the shear force between the track and the ground.

As shown in Fig.3-4, the longitudinal forces, F_{yo} and F_{yi} , acting on the outer and inner tracks relative to the moving frame of reference, can be expressed respectively by

$$F_{yo} = \int dF_o \sin(\pi + \delta_1) = - \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} \int p_o \mu (1 - e^{-J_o/K}) \sin \delta_1 dx_1 dy_1 \quad (3-20)$$

$$F_{yi} = \int dF_i \sin(\pi + \delta_2) = - \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} \int p_i \mu (1 - e^{-J_i/K}) \sin \delta_2 dx_2 dy_2 \quad (3-21)$$

where δ_1 and δ_2 are the angles between the resultant sliding velocities of the points on the outer and inner tracks and the lateral directions of the tracks (i.e., x_1 and x_2 axes), respectively. It should be noted that following Coulomb's law of friction, the shear force acting on the track will be in the opposite direction of the resultant sliding velocity.

Fig.3-4 Kinetics of a tracked vehicle during a steady-state turn.

The lateral forces, F_{xo} and F_{xi} , acting on the outer and inner tracks, are given respectively by

$$F_{xo} = \int dF_o \cos(\pi + \delta_1) = - \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} p_o \mu (1 - e^{-J_o/K}) \cos \delta_1 dx_1 dy_1 \quad (3-22)$$

$$F_{xi} = \int dF_i \cos(\pi + \delta_2) = - \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} p_i \mu (1 - e^{-J_i/K}) \cos \delta_2 dx_2 dy_2 \quad (3-23)$$

The turning moments, M_{Lo} and M_{Li} , acting on the outer and inner tracks respectively, due to the longitudinal shear forces acting on the outer and inner tracks respectively with respect to the longitudinal centerline of vehicle hull can be expressed by

$$M_{Lo} = - \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} \int_{-L/2+c_1-s_o-b/2}^{L/2+c_1-s_o-b/2} \left(\frac{B}{2} + x_1 \right) p_o \mu (1 - e^{-J_o/K}) \sin \delta_1 dx_1 dy_1 \quad (3-24)$$

$$M_{Li} = - \int_{-L/2+c_v-s_o}^{L/2+c_v-s_o} \int_{-b/2}^{b/2} \left(\frac{B}{2} - x_2 \right) p_i \mu (1 - e^{-J_i/K}) \sin \delta_2 dx_2 dy_2 \quad (3-25)$$

The moments of turning resistance, M_{To} and M_{Ti} , of the tracks due to the moments of the lateral forces acting on the outer and inner tracks respectively, with respect to the line passing through center of turn O and perpendicular to vehicle longitudinal axis can be expressed by

$$M_{To} = \int dF_o \cos(\pi + \delta_1) y_1 = - \int_{-L/2+c_v-s_o}^{L/2+c_v-s_o} \int_{-b/2}^{b/2} y_1 p_o \mu (1 - e^{-J_o/K}) \cos \delta_1 dx_1 dy_1 \quad (3-26)$$

$$M_{Ti} = \int dF_i \cos(\pi + \delta_2) y_2 = - \int_{-L/2+c_v-s_o}^{L/2+c_v-s_o} \int_{-b/2}^{b/2} y_2 p_i \mu (1 - e^{-J_i/K}) \cos \delta_2 dx_2 dy_2 \quad (3-27)$$

In order to determine δ_1 and δ_2 , the longitudinal sliding velocities, V_{jyo} and V_{jyi} , of the track elements on the outer and inner tracks with respect to the moving frames of reference (x_1, y_1) and (x_2, y_2) , respectively, are first calculated as follows (see Fig.3-2)

$$V_{jyo} = \left(R' + \frac{B}{2} + c_v + x_1 \right) \dot{\phi} - r\omega_o \quad (3-28)$$

$$V_{jyi} = \left(R' - \frac{B}{2} + c_v + x_2 \right) \dot{\phi} - r\omega_i \quad (3-29)$$

The lateral sliding velocities, V_{jxo} and V_{jxi} , of the track elements on the outer and inner tracks can be expressed by

$$V_{jxo} = -y_1 \dot{\phi} \quad (3-30)$$

$$V_{jxi} = -y_2 \dot{\phi} \quad (3-31)$$

Therefore, the δ_1 and δ_2 can be defined, respectively, by the following equations

$$\text{Sin}\delta_1 = \frac{V_{jy0}}{\sqrt{V_{jx0}^2 + V_{jy0}^2}} = \frac{(R' + B/2 + c_x + x_1)\dot{\phi} - r\omega_o}{\sqrt{[(R' + B/2 + c_x + x_1)\dot{\phi} - r\omega_o]^2 + (y_1\dot{\phi})^2}} \quad (3-32)$$

$$\text{Sin}\delta_2 = \frac{V_{jy1}}{\sqrt{V_{jx1}^2 + V_{jy1}^2}} = \frac{(R' - B/2 + c_x + x_2)\dot{\phi} - r\omega_i}{\sqrt{[(R' - B/2 + c_x + x_2)\dot{\phi} - r\omega_i]^2 + (y_2\dot{\phi})^2}} \quad (3-33)$$

$$\text{Cos}\delta_1 = \frac{V_{jx0}}{\sqrt{V_{jx0}^2 + V_{jy0}^2}} = \frac{-y_1\dot{\phi}}{\sqrt{[(R' + B/2 + c_x + x_1)\dot{\phi} - r\omega_o]^2 + (y_1\dot{\phi})^2}} \quad (3-34)$$

$$\text{Cos}\delta_2 = \frac{V_{jx1}}{\sqrt{V_{jx1}^2 + V_{jy1}^2}} = \frac{-y_2\dot{\phi}}{\sqrt{[(R' - B/2 + c_x + x_2)\dot{\phi} - r\omega_i]^2 + (y_2\dot{\phi})^2}} \quad (3-35)$$

From the above analysis, one can derive the equilibrium equations for the tracked vehicle during a steady-state turn as follows (Fig.3-4)

$$\sum F_x = 0 \quad F_{x0} + F_{x1} + \frac{WV^2}{gR} \text{Cos}\beta = 0 \quad (3-36)$$

$$\sum F_y = 0 \quad (F_{y0} + F_{y1}) = \frac{WV^2}{gR} \text{Sin}\beta + (R_{t0} + R_{ti}) \quad (3-37)$$

$$\sum M_{ov} = 0 \quad M_{L0} - M_{L1} - \frac{B}{2}(R_{t0} - R_{ti}) + (s_o \text{Cos}\beta + c_x \text{Sin}\beta) \frac{WV^2}{gR} = M_{T0} + M_{Ti} \quad (3-38)$$

where R_{t0} and R_{ti} are external motion resistance on the outer and inner tracks, respectively.

It should be noted that the forces and moments are functions of the theoretical speeds, $r\omega_o$ and $r\omega_i$, and the offset, s_o . With the other parameters known or given, such as tread, B , shear deformation parameter, K , track-ground contact length, L , forward speed, V , turning radius, R , weight, W , track width, b , longitudinal offset, c_y , and lateral offset, c_x , of the center of gravity with respect to geometrical center of the vehicle, CG height, h , coefficient of motion resistance, f_r , and coefficient of friction, μ , these three unknown parameters, $r\omega_o$, $r\omega_i$ and s_o , can be determined by solving the above three simultaneous equations. Thus all the forces and moments during a given steady-state turn can be completely defined.

3.2. Normal Pressure Distribution under the Tracks

First consider the lateral load transfer of the inner and outer tracks due to the lateral component of the centrifugal force shown in Fig.3-5(a). From force and moment equilibrium, the normal load on the outer track, W_o , and that on the inner track, W_i , can be expressed by:

$$W_o = \frac{W}{2} + \left(\frac{h}{B} \frac{WV^2}{gR} \cos\beta - \frac{c_y W}{B} \right) \quad (3-39)$$

$$W_i = \frac{W}{2} - \left(\frac{h}{B} \frac{WV^2}{gR} \cos\beta - \frac{c_y W}{B} \right) \quad (3-40)$$

Fig.3-5 Normal Pressure distribution.

Then consider the longitudinal load transfer due to the longitudinal component of the centrifugal force and the longitudinal offset of vehicle CG as shown in Fig.3-5(b). Assuming that the normal load distribution under the outer track has the form of a trapezoid, then the normal pressure under the outer track, $p_o(y_1)$, is given by

$$p_o(y_1) = \frac{W_o}{bL} - \frac{12}{bL^3} \left(\frac{W}{2} c_y + h \frac{WV^2}{2gR} \sin\beta \right) (y_1 + s_o - c_y) \quad (3-41)$$

Similarly, normal pressure under the inner track, $p_i(y_2)$, can be expressed as

$$p_i(y_2) = \frac{W_i}{bL} - \frac{12}{bL^3} \left(\frac{W}{2} c_y + h \frac{WV^2}{2gR} \sin\beta \right) (y_2 + s_o - c_y) \quad (3-42)$$

If the normal load is assumed to be supported only by the track link with pitch, L_p , under each roadwheel as shown in Fig.3-5(c), then $p_o(y_1)$ and $p_i(y_2)$ can respectively be expressed by

$$p_o(y_1) = \frac{W_o}{5bL_p} - \frac{24}{bL_p} \frac{\left(\frac{W}{2} c_y + h \frac{WV^2}{2gR} \sin\beta \right) (y_1 + s_o - c_y)}{15L^2 - 24LL_p + 22L_p^2} \quad (3-43)$$

$$p_i(y_2) = \frac{W_i}{5bL_p} - \frac{24}{bL_p} \frac{\left(\frac{W}{2} c_y + h \frac{WV^2}{2gR} \sin\beta \right) (y_2 + s_o - c_y)}{15L^2 - 24LL_p + 22L_p^2} \quad (3-44)$$

Chapter 4

Simulation of Handling Behaviour of Tracked Vehicles using the Proposed General Theory

In order to demonstrate the utility of the general theory described above, the steering behavior of a military tracked vehicle was simulated and the results were compared with available measured data (Ehlert *et al.*, 1992).

4.1. Basic Design Parameters of the Vehicle Simulated

The basic design parameters of a military tracked vehicle, known as a Jaguar, used in the simulations are given in Table 4-1 (Foss, 1993-94).

Table 4-1 Basic design parameters of the Jaguar tracked vehicle used in the simulation.

vehicle weight, kg	25500
Height of CG, m	1.3
Sprocket radius, m (Scaled from the drawing)	0.32
Track-ground contact length, m	3.8
Longitudinal CG location from centerline of vehicle hull, m	0.
Track width, m	0.45
Track pitch, m	0.152
Tread of vehicle, m	2.54
Shearing characteristics	
Shear deformation parameter, m	0.075
Coefficient of friction between the track and the ground,	0.9
Coefficient of external motion resistance,	0.0263

The coefficient of friction, μ , and shear deformation parameter, K , are dependent upon properties of the track-ground interface, such as the material of track pad and the type of ground. Since the values of μ and K were not given in the reference (Ehlert *et al.*, 1992), the values used in the simulations were derived from fitting the simulated results to the measured data using a optimization procedure. The goodness-of fit, ε , achieved is 88.58% and is defined by

$$\varepsilon = \frac{\sqrt{\sum (\tau_m - \tau_c)^2 / (N - 2)}}{\sum \tau_m / N} \quad (4-1)$$

where τ_m and is the measured sprocket torque from the reference (Ehlert *et al.*, 1992), N is the number of data points, and τ_c is the calculated sprocket torque based on the model developed.

4.2. Simulation Results

By solving the three simultaneous equations from (3-36) to (3-38), the circumferential speed of sprocket on the outer track, $r\omega_o$, and that on the inner track, $r\omega_i$, and the offset, s_o , can be obtained. The measured sprocket torque (Ehlert *et al.*, 1992), the predicted data with different types of normal load distribution under the tracks, as well as the predicted results from Steeds' model ($K = 0$ m) are shown in Figs.4-1 to 4-4. Tables 4-2 to 4-4 show a comparison of the predictions obtained from Steeds' model and those obtained from the proposed model with the measured data at different forward speeds.

- : Measured (Speed Range: 5.8 -- 9.2 km/h)
- - - - : Continuous trapezoidal load distribution over the entire track (Goodness-of-fit 90.55%)
- · — · : Trapezoidal load distribution on the track pitch under each roadwheel (Goodness-of-fit 85.67%)
- · — · : Concentrated load under each roadwheel (Goodness-of-fit 82.54%)
- · — · : Steeds' model under trapezoidal load distribution ($K = 0$ m)

Fig.4-1 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 7.5 km/h with $K = 0.075$ m and $\mu = 0.9$ during a steady-state turn.

- : Measured (Speed Range: 11.9 --16.5 km/h)
- - - - : Continuous trapezoidal load distribution over the entire track (Goodness-of-fit 88.65%)
- · — · — : Trapezoidal load distribution on the track pitch under each roadwheel (Goodness-of-fit 86.17%)
- · · · — : Concentrated load under each roadwheel (Goodness-of-fit 84.67%)
- · · · — : Steeds' model under trapezoidal load distribution ($K = 0$ m)

Fig.4-2 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 14.2 km/h with $K = 0.075$ m and $\mu = 0.9$ during during a steady-state turn.

- : Measured (Speed Range: 18.6 – 24.0 km/h)
- - - - : Continuous trapezoidal load distribution over the entire track (Goodness of Fit 88.31%)
- · — · : Trapezoidal load distribution on the track pitch under each roadwheel (Goodness-of-fit 72.51%)
- · — · : Concentrated load under each roadwheel (Goodness-of-fit 73.93%)
- · — · : Steeds' model under trapezoidal load distribution ($K = 0$ m)

Fig.4-3 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 21.3 km/h with $K = 0.075$ m and $\mu = 0.9$ using a steady-state turn.

Fig.4-4 Sprocket torques vs theoretical turning radius for a Jaguar at a vehicle speed of 29 km/h with $K = 0.075$ m and $\mu = 0.9$ during a steady-state turn.

Table 4-2 Sprocket torques of the outer track at various turning radii with a speed of 7.5 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
5	21.808	23.898	+9.58%	20.814	-4.56%	20.612	-5.48%	19.156	-12.2%
10	18.947	23.891	+26.1%	17.875	-5.66%	18.149	-4.21%	17.437	-7.97%
20	12.895	23.881	+85.2%	13.345	+3.59%	14.189	+10.0%	13.926	+7.96%
50	6.711	23.870	+256%	7.682	+15.5%	8.561	+27.6%	8.522	+27.0%
100	3.816	23.866	+525%	4.860	+27.4%	5.462	+43.1%	5.468	+43.3%

Table 4-3 Sprocket torques of the inner track at various turning radii with a speed of 7.5 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
5	-19.079	-21.339	+11.8%	-18.325	-3.95%	-18.237	-4.41%	-16.846	-11.7%
10	-16.053	-21.390	+33.2%	-15.409	-4.01%	-15.714	-2.11%	-15.015	-6.47%
20	-11.053	-21.389	+93.5%	-10.866	-1.69%	-11.718	+6.01%	-11.458	+3.66%
50	-4.211	-21.379	+408%	-5.195	+23.4%	-6.076	+44.3%	-6.037	+43.4%
100	-1.316	-21.375	+1524%	-2.372	+80.2%	-2.974	+126%	-2.980	+126%

Table 4-4 Sprocket torques of the outer track at various turning radii with a speed of 14.2 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
5	21.225	23.566	+11.0%	19.709	-7.14%	20.169	-4.98%	19.060	-10.2%
10	18.947	23.825	+25.7%	17.168	-9.39%	17.956	-5.23%	17.375	-8.30%
20	12.895	23.919	+85.5%	12.796	-0.77%	13.949	+8.17%	13.728	+6.46%
50	6.711	23.925	+256%	7.347	+9.48%	8.348	+24.4%	8.323	+24.0%
100	3.816	23.916	+527%	4.673	+22.5%	5.324	+39.5%	5.339	+39.9%

Table 4-5 Sprocket torques of the inner track at various turning radii with a speed of 14.2 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
5	-17.237	-20.135	+16.6%	-16.369	-5.04%	-17.342	+ 0.61%	-16.353	-5.13%
10	-16.053	-21.135	+31.7%	-14.555	-9.33%	-15.464	-3.67%	-14.914	-7.10%
20	-11.053	-21.332	+93.0%	-10.273	-7.06%	-11.424	+ 3.36%	-11.210	+1.42%
50	-4.211	-21.352	+407%	-4.781	+13.5%	-5.789	+ 37.5%	-5.764	+36.9%
100	-1.316	-21.344	+1522%	-2.105	+60.0%	-2.758	+ 110%	-2.773	+111%

Table 4-6 Sprocket torques of the outer track at various turning radii with a speed of 21.3 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
10	15.789	23.158	+46.7%	15.802	+0.08%	17.254	+9.28%	16.865	+6.81%
20	12.500	23.781	+90.2%	11.895	-4.84%	13.514	+8.11%	13.356	+6.85%
50	6.711	23.963	+257%	6.765	+0.80%	7.974	+18.8%	7.962	+18.6%
100	3.816	23.968	+545%	4.294	+12.5%	5.031	+31.8%	5.055	+32.6%

Table 4-7 Sprocket torques of the inner track at various turning radii with a speed of 21.3 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
10	-10.658	-19.960	+87.3%	-12.605	+18.3%	-14.404	+35.1%	-14.060	+31.9%
20	-8.684	-21.047	+142%	-9.175	+5.65%	-10.884	+25.3%	-10.738	+23.7%
50	-3.947	-21.301	+440%	-4.106	+4.03%	-5.330	+35.0%	-5.321	+34.8%
100	-1.316	-21.310	+1519%	-1.639	+24.5%	-2.380	+80.9%	-2.405	+82.8%

Table 4-8 Sprocket torques of the outer track at various turning radii with a speed of 29 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by Steeds' model		Predicted by the proposed general theory					
				Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
		kN.m	error	kN.m	error	kN.m	error	kN.m	error
20	12.895	23.189	+79.8%	11.444	-11.3%	13.233	+2.62%	13.197	+2.34%
50	6.711	23.932	+257%	6.543	-2.50%	7.777	+15.9%	7.804	+16.3%
100	3.816	24.012	+529%	4.113	+7.78%	4.815	+26.2%	4.836	+26.7%

Table 4-9 Sprocket torque of the inner track at various turning radii with a speed of 29 km/h.

Theoretical turning radius (m)	Measured (kN.m)	Predicted by the proposed general theory							
		Predicted by Steeds' model		Continuous trapezoidal load over the entire track		Trapezoidal load on the track pitch under each roadwheel		Concentrated load under each roadwheel	
				kN.m	error	kN.m	error	kN.m	error
20	-9.367	-20.200	+116%	-8.409	-10.2%	-10.379	+10.8%	-10.362	+10.6%
50	-4.211	-21.165	+403%	-3.764	-10.6%	-5.029	+19.4%	-5.060	+20.2%
100	-1.316	-21.261	+1516%	-1.358	+3.19%	-2.068	+57.1%	-2.090	+58.8%

It can be seen from Figs.4-1 to 4-4 and Tables 4-2 to 4-9 that the relationships between sprocket torques and turning radius at various forward speeds predicted by the proposed general theory bear a close resemblance to those measured, whereas the relationships predicted by Steeds' model are far from the measured data, particularly at larger turning radii. This can be explained by the difference in the lateral shear stress distribution predicted by the proposed general theory and by Steeds' model, as shown in Figs.4-5 to 4-8.

In Steeds' model, the maximum shear stress is developed whenever a small relative motion between the track and the ground takes place. As a result, there is a large lateral shear stress developed even at the front contact point of the track, where a track element just comes into contact with the ground, as shown in Figs.4-7 and 4-8. On the other hand, the proposed general theory takes into account the shear stress-shear displacement relationship. Consequently at the front contact point of the track, the shear displacement and hence the shear stress is zero, as indicated in Figs.4-5 and 4-6. It should also be pointed out that the lateral shear stress distribution predicted by Steeds' model varies little with the increase in turning radius, except at very small turning radius, such as $R = 10$ m shown in Figs.4-7 and 4-8. This is the reason that the sprocket torques predicted by Steeds' model vary little with

turning radius beyond a certain value, as shown in Figs.4-1 to 4-4. On the other hand, the magnitude of the lateral shear stress predicted by the proposed general theory decreases with the increase in turning radius. As a result, the sprocket torques predicted by the proposed general theory also decrease with the increase in turning radius.

It can, therefore, be concluded that the track-ground shearing characteristics, specifically the shear stress-shear displacement relationship, must be taken into account in predicting the handling behaviour of tracked vehicles. The normal pressure distribution is shown to have a certain influence on the relationships between sprocket torques and turning radius. It appears that among the three types of normal pressure distribution examined, the continuous trapezoidal form of normal pressure distribution provides the closest agreement with the measured data.

4.3. Simulation of Turning Resistance Moment

For the computation of the turning resistance moment, several analytical models were developed in the past. The basic model is described in Section 2.4. In this model, the interacting forces and moments between the tracks and the ground can be defined and expressed in terms of coefficient of turning resistance, μ_w , as shown in equations (2-20) and (2-21). It should be noted that μ_w in these equations is considered to be a function of track exponent, k , which varies with turning radius. k is essentially an empirical parameter which can only be derived from experimental data (Ehlert *et al.*, 1992).

As described previously, using the general theory presented in Chapter 3, the moments of turning resistance for the outer and inner tracks can be predicted analytically using equations (3-26) and (3-27), respectively. Consequently, the equivalent coefficient of turning resistance, μ_w , in equation (2-20c) can be derived quantitatively by equating the sum of the moments of turning resistance, M_{To} and M_{Ti} , calculated from equations (3-26) and (3-27) to

M_w in equation (2-20c). The equivalent coefficient of turning resistance, μ_w (for $\mu = 0.9$), using this method as a function of turning radius is shown in Figs.4-9 to 4-12. The equivalent coefficient of turning resistance, μ_w , derived from experimental data reported by Ehlert *et al.* (1992) is also shown in the figures. It was calculated using equation (3-38), with measured data on sprocket torques and external motion resistance and the calculated centrifugal force based on the forward speed and turning radius. It is shown that there is a reasonably close agreement between the measured and predicted results obtained using the general theory presented in Chapter 3.

4.4. Summary

It is shown that there is a reasonably close agreement between the sprocket torques and moments of turning resistance predicted using the general theory and the available measured data. The variations of sprocket torques and equivalent coefficient of turning resistance with turning radius at various forward speeds bear a close resemblance to those measured. On the other hand, the predictions based on Steeds' model are far from measured data. The model proposed by Elhert *et al* includes a number of empirical coefficients, such as track exponent, k , turning radius, R_k , where track forces approaching zero, etc. To obtain these values requires extensively field tests. It can be concluded that the general theory developed offers a significant improvement over all previous models.

Fig.4-5 Lateral shear stress distribution of the outer track along the longitudinal centerline of track-ground contact area predicted from the proposed general theory at a vehicle speed of 14.2 km/h with different turning radii.

Fig.4-6 Lateral shear stress distribution of the inner track along the longitudinal centerline of track-ground contact area predicted from the proposed general theory at a vehicle speed of 14.2 km/h with different turning radii.

Fig.4-7 Lateral shear stress distribution of the outer track along the longitudinal centerline of track-ground contact area predicted from Steeds' model ($K = 0$. m) at a vehicle speed of 14.2 km/h with different turning radii.

Fig.4-8 Lateral shear stress distribution of the inner track along the longitudinal centerline of track-ground contact area predicted from Steeds' model ($K = 0$. m) at a vehicle speed of 14.2 km/h with different turning radii.

+++ : Measured data (Speed range: 5.8 – 9.2 km/h)

— : $\mu = 0.9$

Fig.4-9 Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track.

+++ : Measured data (Speed range: 11.9 – 16.5 km/h)

— : $\mu = 0.9$

Fig.4-10 Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track.

+++ : Measured data (Speed range: 18.6 – 24 km/h)

— : $\mu = 0.9$

Fig.4-11 Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track.

+++ : Measured data (Speed range: 25.8 – 32.2 km/h)

— : $\mu = 0.9$

Fig.4-12 Coefficient of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with $K = 0.075$ m under continuous trapezoidal load distribution over the entire track.

Chapter 5

Evaluation of the Effects of Design and Operating Factors on the Handling of Tracked Vehicles using the General Theory

In the previous chapter, a general theory for tracked vehicle steering on non-deformable surfaces has been established. The predicted results of a particular vehicle during a steady-state turning maneuver compare favourably with available field measurements. With the basic features of the proposed theory substantiated, in this chapter, the effects on handling of major vehicle design parameters, such as track-ground contact length, L , tread, B , track width, b , longitudinal offset, c_y , of vehicle CG with respect to the geometrical center of the vehicle, and CG height, h , will be examined in detail.

5.1. Contact Length, L

Using the tracked vehicle with design parameters given in Table 4-1 as a basis, the effects of track-ground contact length, L , on steering behavior were examined using the simulation model developed. The sprocket torques under continuous trapezoidal normal load distribution over the entire track contact length (as shown in Fig.3-5(b)) increase with the increase of L as shown in Figs.5-1 to 5-4. This is because the moment of turning resistance generated during a turning maneuver also increases with the increase of contact length, L , as shown in Figs.5-9 to 5-12. The lateral force will be more or less the same with different contact lengths as shown in Figs.5-5 to 5-8, since the centrifugal force during the turning maneuver does not change with L . The sprocket torques with different contact lengths and vehicle speeds at various turning radii are tabulated in Tables A-1 to A-4 in Appendix A.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — · : $L = 5.70$ m

Fig.5-1 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — · : $L = 5.70$ m

Fig.5-2 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m — · — : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — : $L = 5.70$ m

Fig.5-3 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m — · — : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — : $L = 5.70$ m

Fig.5-4 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/hr with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — · : $L = 5.70$ m

Fig.5-5 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — · : $L = 5.70$ m

Fig.5-6 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

—: Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 - · - · : $L = 4.75$ m - - - - : $L = 5.70$ m

Fig.5-7 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

—: Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 - · - · : $L = 4.75$ m - - - - : $L = 5.70$ m

Fig.5-8 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 - · - · : $L = 4.75$ m — : $L = 5.70$ m

Fig.5-9 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 - · - · : $L = 4.75$ m — : $L = 5.70$ m

Fig.5-10 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — · : $L = 5.70$ m

Fig.5-11 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Contact Length $L = 3.8$ m)
 - - : $L = 1.90$ m - · - · : $L = 2.85$ m
 — · — : $L = 4.75$ m — · — · : $L = 5.70$ m

Fig.5-12 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track contact lengths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

The effects of track-ground contact length, L , on the moment of turning resistance are summarized in Figs.5-13 to 5-16. It shows the variation of the ratio of the moment of turning resistance (i.e., the sum of M_{To} and M_{Ti} at a particular contact length to that at the reference length of 3.8 m) with the track contact length, L , at various turning radii. At the reference contact length of 3.8 m, the ratio is equal to one.

The same trend is also observed for normal load distributed only on the track pitch ($L_p = 15$ cm) under each roadwheel (as shown in Fig.3-5(c)), as well as those for normal load concentrated at a point ($L_p = 1$ cm) under each roadwheel. These results are shown in Tables B-1 to B-4 in Appendix B, and in Tables C-1 to C-4 in Appendix C, respectively.

5.2. Tread of Vehicle, B

The sprocket torques under continuous trapezoidal normal load distribution over the entire track increase with the decrease of B as shown in Figs.5-17 to 5-20. This is because with the decrease of B , the turning moment formed by the longitudinal shear forces on the outer and inner tracks decreases, due to the decrease in the moment arm. However, the lateral force and moment of turning resistance generated during the turning maneuver remain approximately the same, as shown in Figs.5-21 to 5-28. To balance the moment of turning resistance, the sprocket torques have to be increased with the decrease of B . The sprocket torques with different treads and vehicle speeds at various turning radii are tabulated in Tables A-5 to A-8 in Appendix A.

The effects of vehicle tread, B , on magnitude of the sum of the sprocket torques on the outer and inner tracks are summarized in Figs.5-29 to 5-32. It shows the variation of the ratio of the sum of sprocket torques at a particular vehicle tread, B , to that at the reference tread of 2.54 m. At the reference tread of 2.54 m, the ratio is equal to one.

Fig.5-13 Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-14 Ratios of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-15 Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-16 Ratio of turning resistance moment vs contact length L for a Jaguar at a vehicle speed of 29 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-17 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-18 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Tread B = 2.54 m)
 - - : B = 1.90 m - · - · : B = 2.17 m
 - · - · : B = 3.04 m - - - - : B = 3.80 m

Fig.5-19 Sprocket torques vs turning radius for Jaguar at a vehicle speed of 21.3 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Tread B = 2.54 m)
 - - : B = 1.90 m - · - · : B = 2.17 m
 - · - · : B = 3.04 m - - - - : B = 3.80 m

Fig.5-20 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-21 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-22 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Tread B = 2.54 m)
 - - : B = 1.90 m - · - : B = 2.17 m
 — · — : B = 3.04 m - - - - : B = 3.80 m

Fig.5-23 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Tread B = 2.54 m)
 - - : B = 1.90 m - · - : B = 2.17 m
 — · — : B = 3.04 m - - - - : B = 3.80 m

Fig.5-24 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Tread B = 2.54 m)
 - - : B = 1.90 m — · — : B = 2.17 m
 — · — : B = 3.04 m — · — : B = 3.80 m

Fig.5-26 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Tread B = 2.54 m)
 - - : B = 1.90 m — · — : B = 2.17 m
 — · — : B = 3.04 m — · — : B = 3.80 m

Fig.5-25 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-27 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-28 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different vehicle treads during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-29 Ratio of magnitude of the sprocket torques vs vehicle tread B for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-30 Ratio of magnitude of the sprocket torques vs vehicle tread B for Jaguar at a vehicle speed of 14.2 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-31 Ratio of magnitude of the sprocket torques vs vehicle tread B for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-32 Ratios of magnitude of the sprocket torques vs vehicle tread B for a Jaguar at a vehicle speed of 29 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

The same trend is also observed for normal load distributed only on the track pitch ($L_p = 15$ cm) under each roadwheel (as shown in Fig.3-5(c)), as well as those for normal load concentrated at a point ($L_p = 1$ cm) under each roadwheel. These results are shown in Tables B-5 to B-8 in Appendix B, and in Tables C-5 to C-8 in Appendix C, respectively.

5.3. Longitudinal Offset of Vehicle Center of Gravity, c_y

Track forces are related to the shear displacement as mentioned previously. In general the shear displacement increases from the front to the rear of the track-ground contact length. With the shifting of the center of gravity of the vehicle to the rear or with the increase of c_y (c_y is positive from the center of track contact length to the rear), the normal pressure under the track is higher in the rear than in the front. Therefore, the vehicle will develop higher shear force and as a result, track forces increase with the increase of c_y . As a result, the sprocket torques and moments of turning resistance will also increase with the increase in c_y as shown in Fig.5-33 to Fig.5-44. The sprocket torque at different c_y values and vehicle speeds at various turning radii are tabulated in Table A-9 to Table A-12 in Appendix A.

The same trend is also observed for normal load distributed only on the track pitch ($L_p = 15$ cm) under each roadwheel (as shown in Fig.3-5(c)), as well as those for normal load concentrated at a point ($L_p = 1$ cm) under each roadwheel. These results are shown in Tables B-9 to B-12 in Appendix B, and in Tables C-9 to C-12 in Appendix C, respectively.

The effects of the longitudinal offset of the center of gravity, c_y , on the turning resistance moment are summarized in Figs.5-45 to 5-48. It shows the variation of the ratio of the turning resistance moment (i.e., the turning resistance moment at a particular value of c_y to that at $c_y = 0$ m) with the value of c_y . At $c_y = 0$ m, the ratio is equal to one.

— : Reference (Longitudinal CG offset $c_y = 0$ m)
 - - : $c_y = -0.20$ m — : $c_y = -0.40$ m
 — : $c_y = 0.20$ m — : $c_y = 0.40$ m

Fig.5-33 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Longitudinal CG offset $c_y = 0$ m)
 - - : $c_y = -0.20$ m — : $c_y = -0.40$ m
 — : $c_y = 0.20$ m — : $c_y = 0.40$ m

Fig.5-34 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-35 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-36 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Longitudinal CG offset $c_y = 0$ m)
 - - : $c_y = -0.20$ m — : $c_y = -0.40$ m
 - · - : $c_y = 0.20$ m - · - : $c_y = 0.40$ m

Fig.5-37 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Longitudinal CG offset $c_y = 0$ m)
 - - : $c_y = -0.20$ m — : $c_y = -0.40$ m
 - · - : $c_y = 0.20$ m - · - : $c_y = 0.40$ m

Fig.5-38 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Longitudinal CG offset $c_y = 0$ m)
 - - : $c_y = -0.20$ m — : $c_y = -0.40$ m
 — : $c_y = 0.20$ m — : $c_y = 0.40$ m

Fig.5-39 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Longitudinal CG offset $c_y = 0$ m)
 - - : $c_y = -0.20$ m — : $c_y = -0.40$ m
 — : $c_y = 0.20$ m — : $c_y = 0.40$ m

Fig.5-40 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-41 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-42 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-43 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-44 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different longitudinal CG offsets during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-45 Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-46 Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-47 Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-48 Ratio of turning resistance moment vs longitudinal CG offset c_y for a Jaguar at a vehicle speed of 29 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

5.4. Track Width, b

The effects of track width, b , on the sprocket torque, lateral force and moment of turning resistance during a steady-state turn under continuous trapezoidal normal load distribution over the entire track are shown in Figs.5-49 to 5-60. It is noted that the track width does not have much influence on the tracked vehicle steering on the non-deformable surfaces. The sprocket torques with different track widths and vehicle speeds at various turning radii are tabulated in Tables A-13 to A-16 Appendix A.

The same trend is also observed for normal load distributed only on the track pitch ($L_p = 15$ cm) under each roadwheel (as shown in Fig.3-5(c)), as well as those for normal load concentrated at a point ($L_p = 1$ cm) under each roadwheel. These results are shown in Tables B-13 to B-16 in Appendix B, and in Tables C-13 to C-16 in Appendix C, respectively.

The effects of track width on the turning resistance moment are summarized in Figs.5-61 to 5-64. It shows the variation of the ratio of the turning resistance moment (i.e., the turning resistance moment at a particular track width, b , to that at a track width, b , of 0.45 m) with track width. It appears that the track width has insignificant effect on the turning resistance moment for the vehicle simulated.

5.5. Height of Vehicle Center of Gravity, h

The effects of height of vehicle center of gravity, h , on sprocket torques during a turning maneuver under continuous trapezoidal normal load distribution over the entire track are insignificant, as shown in Figs.5-65 to 5-68. The same observation can be made for both lateral forces and turning resistance moments on both outer and inner tracks of the vehicle, except at a small turning radii, as shown in Figs.5-69 to 5-76.

The sprocket torques at different CG heights and forward speeds at various turning

radii are tabulated in Tables A-17 to A-20 in Appendix A.

The same trend is also observed for normal load distributed only on the track pitch ($L_p = 15$ cm) under each roadwheel (as shown in Fig.3-5(c)), as well as those for normal load concentrated at a point ($L_p = 1$ cm) under each roadwheel. These results are shown in Tables B-17 to B-20 in Appendix B. and in Tables C-17 to C-20 in Appendix C, respectively.

The effects of height of vehicle center of gravity on the turning resistance moments are summarized in Figs.5-77 to 5-80. It appears that the ratio of the turning resistance moment (i.e., the turning resistance moment at a particular CG height to that at a height of 1.3 m) does not vary significantly with the height of center of gravity.

5.6. Summary

Based on the parametric studies described above, it can be concluded that among the vehicle design parameters, the vehicle contact length, L , and tread, B , have significant effects on the steering of tracked vehicles. Increasing the track contact length, L , causes a considerable increase in the turning resistance moment. On the other hand, reducing the tread, B , would cause an increase in the sprocket torques and track longitudinal forces. It is found that track width, b , longitudinal offset, c_y , and CG height, h , of the vehicle have insignificant effects on steering behaviour.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-49 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-50 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-51 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-52 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-53 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-54 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-55 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-56 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45\text{ m}$)
 - - : $b = 0.01\text{ m}$ — : $b = 0.90\text{ m}$
 - · - : $b = 1.35\text{ m}$

Fig.5-58 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45\text{ m}$)
 - - : $b = 0.01\text{ m}$ — : $b = 0.90\text{ m}$
 - · - : $b = 1.35\text{ m}$

Fig.5-57 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-59 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (Track width $b = 0.45$ m)
 - - : $b = 0.01$ m — : $b = 0.90$ m
 - · - : $b = 1.35$ m

Fig.5-60 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different track widths during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-61 Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-62 Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-63 Ratios of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-64 Ratio of turning resistance moment vs track width b for a Jaguar at a vehicle speed of 29 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-65 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-66 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

————: Reference (CG height $h = 1.30$ m)

- - - : $h = 0.80$ — · — : $h = 1.8$ m

Fig.5-67 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

————: Reference (CG height $h = 1.30$ m)

- - - : $h = 0.80$ m — · — : $h = 1.8$ m

Fig.5-68 Sprocket torques vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

————: Reference (CG height $h = 1.30$ m)

- - - : $h = 0.80$ m — · — · — : $h = 1.8$ m

Fig.5-69 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

————: Reference (CG height $h = 1.30$ m)

- - - : $h = 0.80$ m — · — · — : $h = 1.8$ m

Fig.5-70 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (CG height $h = 1.30$ m)
 - - : $h = 0.80$ m — · — : $h = 1.8$ m

Fig.5-71 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

— : Reference (CG height $h = 1.30$ m)
 - - : $h = 0.80$ m — · — : $h = 1.8$ m

Fig.5-72 Lateral forces vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-73 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 7.5 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-74 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 14.2 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

————: Reference (CG height $h = 1.30$ m)

- - - : $h = 0.80$ m — · — : $h = 1.8$ m

Fig.5-75 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 21.3 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

————: Reference (CG height $h = 1.30$ m)

- - - : $h = 0.80$ m — · — : $h = 1.8$ m

Fig.5-76 Moments of turning resistance vs turning radius for a Jaguar at a vehicle speed of 29 km/h with different CG heights during a steady-state turn under continuous trapezoidal load distribution over the entire track.

Fig.5-77 Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 7.5 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-78 Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 14.2 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-79 Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 21.3 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Fig.5-80 Ratio of turning resistance moment vs CG height h for a Jaguar at a vehicle speed of 29 km/h with different turning radii under continuous trapezoidal load distribution over the entire track.

Chapter 6

Experimental Study of the Shearing Characteristics of a Representative Track Link on Asphalt

As mentioned previously, the shearing behaviour of a tracked vehicle is greatly influenced by the shear force developed on the track-ground interface. A detailed experimental study was carried out to measure the shearing characteristics between a representative track link with rubber pad used on a current armoured personnel carrier M113 and asphalt in the laboratory.

6.1. Apparatus for Measuring the Shearing Characteristics

The apparatus used to measure the shearing characteristics, including the relation between shear force and shear displacement, was a modification of an existing tire tester in the Transport Technology and Research Laboratory, Carleton University. The apparatus includes state-of-art data acquisition system, developed by Mr. Y.C. Wu for his Ph.D research on the lateral dynamics of off-road tires.

The schematic of the test apparatus is illustrated in Fig.6-1. The major components of the track link shear force-shear displacement tester include:

- a horizontal motion carrier
- a test bed on which an asphalt block was mounted
- a double parallelogram mechanism on which test track links were mounted
- a data acquisition system

Fig.6-1 Schematic of the test setup for the measurement of track shear force and shear displacement on the asphalt.

- two Lebow 6643 x-y load cells, and one displacement potentiometer.

Two test track links and their supporting frame are connected with a double parallelogram mechanism which is mounted on the horizontal motion carriage. The motion carriage is driven by an electric motor through a chain drive mechanism. When the carriage moves forward, shearing action between track links and the asphalt surface takes place. The horizontal shear force was monitored by two Lebow 6643 x-y load cells located at both left and right hand sides of the carriage, as shown in Fig.6-1. A displacement potentiometer is mounted to measure the horizontal travelling distance of the frame, which is the shear displacement of the track links. These forces as well as horizontal displacement are all recorded by a data acquisition system (Fig.6-2).

Fig.6-2 Block diagram of the data acquisition system for the shear force-shear displacement tester.

Tests were carried out under four normal loads on the track links of 890 , 1000 , 1200,

and 1400 lb. and at three different speeds, 0.195, 0.145, and 0.100 m/sec. Seven test runs were conducted for each forward speed-normal load combination.

6.2. Analysis of Shear Stress-Shear Displacement Data

The most commonly used semi-empirical equation to describe the shear stress-shear displacement relationship on the track-ground interface is equation (3-15). To determine the optimum value of shear deformation parameter, K , which minimizes the error in curve fitting, the following equation is used (Wong, 1983)

$$K = - \frac{\sum (1 - \tau_m / \tau_{max})^2 \cdot j^2}{\sum (1 - \tau_m / \tau_{max})^2 \cdot j \cdot \ln(1 - \tau_m / \tau_{max})} \quad (6-1)$$

where τ_m is measured stress, τ_{max} is maximum shear stress under a particular load, and j is measured shear displacement.

To solve for an appropriate value of K , the following procedure is used.

- 1). The maximum value of shear stress, τ_{max} , in equation (6-1) was first identified from the measured data, as shown in Fig.6-3.
- 2). K is calculated using equation (6-1).
- 3). A goodness-of-fit is calculated using equation (3-43).

Following the above procedure, the average value of the shear deformation parameter, K , for the M113 track link with rubber pad on asphalt is found to be 0.0183 m, and the goodness-of-fit ranges from 86.72% to 93.52%. The measured data and fitted curves using the calculated K value are shown in Figs.6-3 to 6-13.

Fig.6-3 Shear force-shear displacement curve at normal load 890 lb with sliding speed 0.100 m/s.

Fig.6-4 Shear force-shear displacement curve at normal load 890 lb with sliding speed 0.145 m/s.

Fig.6-5 Shear force-shear displacement curve at normal load 1000 lb with sliding speed 0.100 m/s.

Fig.6-6 Shear force-shear displacement curve at normal load 1000 lb with sliding speed 0.145 m/s.

Fig.6-8 Shear force-shear displacement curve at normal load 1200 lb with sliding speed 0.10 m/s.

Fig.6-7 Shear force-shear displacement curve at normal load 1000 lb with sliding speed 0.195 m/s.

Fig.6-9 Shear force-shear displacement curve at normal load 1200 lb with sliding speed 0.145 m/s.

Fig.6-10 Shear force-shear displacement curve at normal load 1200 lb with sliding speed 0.195 m/s.

Fig.6-11 Shear force-shear displacement curve at normal load 1400 lb with sliding speed 0.100 m/s.

Fig.6-12 Shear force-shear displacement curve at normal load 1400 lb with sliding speed 0.145 m/s.

Fig.6-13 Shear force-shear displacement curve at normal load 1400 lb with sliding speed 0.195 m/s.

6.3. The Determination of the Coefficient of Friction

From the measured data shown in Figs.6-3 to 6-13, the maximum shear force developed under various normal loads can be identified. Their relationship is illustrated in Fig.6-14.

Fig.6-14 The relation of maximum shear force and normal load.

The coefficient of friction, μ , between the track link with rubber pad and the asphalt surface, can be found from the slope of the fitted line shown in Fig.6-11.

$$\mu = \tan \phi \quad (6-2)$$

Based on the experimental data shown, it was found the average value of μ was 0.684. It should be mentioned that the shear tests were conducted at three different values of sliding velocity, 0.10, 0.145, and 0.195 m/s. The test results shown that within this range, the sliding velocity has insignificant effect on the coefficient of friction, μ , and the shear deformation parameter, K . Within the range of speeds used, the value of μ varies from 0.664 to 0.737, and K varies in the range of 0.0167 to 0.0225 m.

In the simulation of the steering behaviour of M113 on asphalt described in the next chapter, the average value of the shear deformation parameter $K = 0.0183$ m and $\mu = 0.684$ were used.

Chapter 7

Simulation of Steering Behaviour of an M113 Armoured Personnel Carrier on Non-deformable Ground Based on Measured Shear Data

7.1. Basic Design Parameters of an M113 Armoured Personnel Carrier

Using the measured data on the shearing characteristics of an M113 track link with rubber pad on asphalt described in the previous chapter, the handling behaviour of an M113 armored personnel carrier was predicted. The predictions were made using the general theory described in Chapter 3. The basic vehicle and track parameters used in the predictions are given in Table 7-1.

Table 7-1 Basic parameters of the M113 tracked vehicle used in the predictions

Sprung weight, kN	100.57
Unsprung weight, kN	10.08
Height of C.G., m	0.8733
Sprocket radius, m	0.214
Track-ground contact length, m	2.667
Longitudinal C.G. location from centerline of vehicle hull, m	0.2496
Track width, m	0.381
Track pitch	0.152
Tread of vehicle, m	2.159
Shearing characteristic	
Shear deformation parameter, m	0.0183
Coefficient of friction between track-ground (asphalt) interface,	0.684
Coefficient of motion resistance,	$0.03+0.00015 \cdot V$
*: The unit of forward speed V is in kilometers per hour.	

7.2. Simulation Results

The predicted sprocket torques, lateral forces, and moments of tuning resistance for the M113 during a steady-state turn on an asphalt surface are shown in Figs.7-1 to 7-14. A comparison of sprocket torques predicted by Steeds' model and by the proposed model at different vehicle speeds and normal load distributions are shown in Tables 7-2 to 7-9.

Table 7-2 Sprocket torques at various turning radii with a vehicle speed of 7.5 km/h for an M113 armoured personnel carrier.

Turning radius (m)	Predicted by Steeds' model (kN.m)		Predicted by proposed general theory (kN.m)					
			Continuous trapezoidal load distribution over the entire track		Trapezoidal load distribution on the track pitch under each roadwheel		Concentrated load under each roadwheel	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	4.615	-3.878	4.412	-3.678	4.664	-3.938	4.304	-3.588
20	4.604	-3.868	4.128	-3.394	4.338	-3.607	4.123	-3.393
50	4.594	-3.858	3.222	-2.486	3.496	-2.761	3.427	-2.692
100	4.590	-3.854	2.303	-1.567	2.557	-1.821	2.538	-1.802

Table 7-3 Sprocket torques at various turning radii with a vehicle speed of 14.2 km/h for an M113 armoured personnel carrier.

Turning radius (m)	Predicted by Steeds' model (kN.m)		Predicted by proposed general theory (kN.m)					
			Continuous trapezoidal load distribution over the entire track		Trapezoidal load distribution on the track pitch under each roadwheel		Concentrated load under each roadwheel	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	4.590	-3.748	4.336	-3.504	4.679	-3.854	4.503	-3.700
20	4.630	-3.863	4.102	-3.340	4.386	-3.631	4.256	-3.507
50	4.621	-3.861	3.175	-2.416	3.521	-2.765	3.479	-2.723
100	4.611	-3.851	2.268	-1.508	2.579	-1.820	2.569	-1.810

Table 7-4 Sprocket torques at various turning radii with a vehicle speed of 21.3 km/h for an M113 armoured personnel carrier.

Turning radius (m)	Predicted by Steeds' model (kN.m)		Predicted by proposed general theory (kN.m)					
			Continuous trapezoidal load distribution over the entire track		Trapezoidal load distribution on the track pitch under each roadwheel		Concentrated load under each roadwheel	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	4.192	-2.881	3.769	-2.462	4.380	-3.025	4.518	-3.189
20	4.555	-3.714	3.738	-2.900	4.241	-3.414	4.259	-3.439
50	4.642	-3.855	3.043	-2.257	3.462	-2.680	3.476	-2.696
100	4.636	-3.050	2.162	-1.377	2.564	-1.780	2.575	-1.792

Table 7-5 Sprocket torques at various turning radii with a vehicle speed of 29 km/h for an M113 armoured personnel carrier.

Turning radius (m)	Predicted by Steeds' model (kN.m)		Predicted by proposed general theory (kN.m)					
			Continuous trapezoidal load distribution over the entire track		Trapezoidal load distribution on the track pitch under each roadwheel		Concentrated load under each roadwheel	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	3.732	-2.346	3.144	-1.707	3.836	-2.346	4.193	-2.685
30	4.474	-3.601	2.836	-1.941	3.670	-2.799	3.865	-3.000
50	4.621	-3.797	2.322	-1.489	3.087	-2.264	3.264	-2.443
100	4.656	-3.843	1.851	-1.036	2.353	-1.541	2.441	-1.629

- : Continuous trapezoidal load distribution over the entire track
- - : Concentrated load under each roadwheel
- . - : Trapezoidal load distribution on the track pitch under each roadwheel
- - - : Steeds' model

Fig.7-1 Sprocket torques vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

Fig.7-2 Sprocket torques vs turning radius for an M113 at a vehicle speed of 14.2 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - - - : Concentrated load under each roadwheel
- · — · : Trapezoidal load distribution on the track pitch under each roadwheel
- · - · : Steeds' model

Fig.7-3 Sprocket torques vs turning radius for an M113 at a vehicle speed of 21.3 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

Fig.7-4 Sprocket torques vs turning radius for an M113 at a vehicle speed of 29 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - - - : Concentrated load under each roadwheel
- · — · : Trapezoidal load distribution on the track pitch under each roadwheel
- · - · : Steeds' model

Fig.7-5 Lateral force of the outer track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

Fig.7-6 Lateral force of the inner track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - - - : Concentrated load under each roadwheel
- : Trapezoidal load distribution on the track pitch under each roadwheel
- - - - : Steeds' model

Fig.7-7 Lateral forces vs turning radius for an M113 at a vehicle speed of 14.2 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

Fig.7-8 Lateral forces vs turning radius for an M113 at a vehicle speed of 21.3 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - - - : Concentrated load under each roadwheel
- : Trapezoidal load distribution on the track pitch under each roadwheel
- - - - : Steeds' model

Fig.7-9 Lateral forces vs turning radius for an M113 at a vehicle speed of 29 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - : Concentrated load under each roadwheel
- . - : Trapezoidal load distribution on the track pitch under each roadwheel
- - - : Steeds' model

Fig.7-10 Moment of turning resistance of the outer track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

Fig.7-11 Moment of turning resistance of the inner track vs turning radius for an M113 at a vehicle speed of 7.5 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - - - : Concentrated load under each roadwheel
- . - . : Trapezoidal load distribution on the track pitch under each roadwheel
- - - - : Steeds' model

Fig.7-12 Moments of turning resistance vs turning radius for an M113 at a vehicle speed of 14.2 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

Fig.7-13 Moments of turning resistance vs turning radius for an M113 at a vehicle speed of 21.3 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

- : Continuous trapezoidal load distribution over the entire track
- - - - : Concentrated load under each roadwheel
- · — · : Trapezoidal load distribution on the track pitch under each roadwheel
- · — · : Steeds' model

Fig.7-14 Moments of turning resistance vs turning radius for an M113 at a vehicle speed of 29 km/h with $K = 0.0183$ m and $\mu = 0.684$ during a steady-state turn.

7.3. Summary

It can be seen from Figs.7-1 to 7-4 and Tables 7-2 to 7-5 that the relationships between the sprocket torque and turning radius at various vehicle speeds have similar trend to those predicted for the Jaguar vehicle described in Chapter 4. The predictions using Steeds' model again show that the sprocket torques and moments of turning resistance at higher turning radii remain almost constant. This indicates again that the track-ground shearing characteristics, specifically the shear stress-shear displacement relation, must be taken into consideration in predicting the handling of tracked vehicles. It also appears that normal pressure distribution also has certain influence on the relationships between sprocket torque and turning radius.

Chapter 8

Discussion and Conclusions

This research project focuses on a detailed study of the mechanics of steering of tracked vehicles on non-deformable surfaces, taking into account the effects of shear displacement on the development of shear stress and of track width. Experimental studies have been carried out to identify the shearing characteristics on asphalt of the track link with the rubber pad used on the M113 armoured personnel carrier. A computer simulation model for steering behavior of tracked vehicles under steady-state conditions over non-deformable surfaces has been developed.

8.1. Conclusions

The handling characteristics of tracked vehicles predicted using the model developed in this project bear close resemblance to those measured in the field, as shown in Figs.4-1 to 4-4 and in Tables 4-2 to 4-9. It is also shown that this model represents a significant improvement on Steeds' model. This is because the model developed takes into account the effect of shear displacement on the development of shear stress on the track-ground interface, whereas Steeds' model assumes that full shear stress develops as soon as a small relative motion between the track and the ground takes place. However, Steeds' model may be considered a special case of the general theory presented in this thesis where the shear deformation parameter, K , is zero.

Using the model developed, the effects of design and operating parameters on the

steering of tracked vehicles have been systematically examined. It is found that increasing track-ground contact length, L , the sprocket torques required in a steady-state turning maneuver increase significantly. This is caused by the considerable increase of the turning resistance moment.

It is also found that the sprocket torques required to execute a steady-state turn increase with the decrease of tread, B . This is primarily due to the decrease in the moment arms of the longitudinal track forces, which form the turning moments and initiate the turn of the vehicle.

Longitudinal offset of vehicle center of gravity, c_y , also affects the sprocket torques required during a turning maneuver. General speaking, sprocket torques will increase with the shifting of CG rearward. As a result of shifting CG rearward the normal pressure at the rear is higher than that at the front. This causes an increase in the turning resistance moment and hence sprocket torques.

In general, the height of vehicle CG and track width do not have significant effects on the steering characteristics of tracked vehicles over non-deformable surfaces.

Experimental studies have also been carried out to identify the shearing characteristics between the track link used in the armoured personnel carrier M113 and asphalt. Based on experimental data, the steering characteristics of M113 over asphalt are predicted using the model developed, and compared with those predicted using Steeds' model.

In conclusion, the general theory for tracked vehicle steering on non-deformable surfaces presented in this thesis is a significant improvement on existing models. The predictions based on this model have a very similar trend to measured field data. It is believed that the general theory developed provides a basis for the further study of tracked vehicle maneuverability on deformable terrain.

8.2. Future Work

Based on the experimental and analytical studies mentioned previously, the following recommendations for future work can be made:

- 1). Further study on the factors affecting the shearing characteristics between the track and the ground. such as sliding velocity, temperature, moisture content of the terrain, etc.
- 2). Initiation of systematic study of tracked vehicle steering on deformable terrain based on the studies presented in the thesis, with the following factors included in the analysis:
 - Vehicle sinkage on deformable terrain
 - Shearing characteristics between the track and the deformable terrain
 - Normal pressure distribution under the track on deformable terrain during a turning maneuver
 - Bulldozing effect of the track in lateral skidding on turning resistance

Bibliography

1. Baladi, G.Y. and Rohani, B., 1978, A Mathematical Model of Terrain-Vehicle Interaction for Predicting the Steering Performance of Track-Laying Vehicles, Proceedings of the 6th International Conference of the International Society for Terrain-Vehicle Systems, pp.285-332.
2. Baladi, G.Y. and Rohani, B., 1981. Analysis of Steerability of Tracked Vehicles: Theoretical Predictions Versus Field Measurements, Proceedings of the 7th International Conference of the International Society for Terrain-Vehicle Systems, pp.1175-1220.
3. Bekker, M.G., 1956, Theory of Land Locomotion, Ann Arbor, MI: University of Michigan Press.
4. Bekker, M.G., 1960, Off-the-Road Locomotion. Ann Arbor, MI: University of Michigan Press.
5. Bekker, M.G., 1969, Introduction to Terrain-Vehicle Systems, Ann Arbor, MI: University of Michigan Press.
6. Crosheck, J.E., 1975. Skid-Steering of Crawlers. SAE Paper No.750552, pp.1-15.
7. Ehlert, W. Hug, B. and Schmid, I.C., 1992, Field Measurements and Analytical Models as a Basis of Test Stand Simulation of the Turning Resistance of Tracked Vehicles, Journal of Terramechanics, Vol.29(1), pp.57-69.
8. Eiyo, F. and Kitano, M., 1984, Study on Controllability and Stability of High Speed Tracked Vehicles. Proceedings of the 8th International Conference of the International Society for Terrain-Vehicle Systems, pp.789-819.
9. Foss, C.F., 1993-94, Jane's Armour and Artillery, 14th Ed., Jane's Information Group Inc., Alexandria, USA.
10. Hayashi, I. 1975, Practical Analysis of Tracked Vehicle Steering Depending on Longitudinal Track Slippage, Proceedings of the 5th International Conference of the International Society for Terrain-Vehicle Systems, pp.493-512.

11. Kar, M.K., 1987, Production of Track Forces in Skid Steering of Military Tracked Vehicles, *Journal of Terramechanics*, Vol.24(1), pp.75-86.
12. Karafiath, L.L., 1981, Analytical Model for the Turning of Tracked Vehicles in Soft Soils, *Proceedings of the 6th International Conference of the International Society for Terrain-Vehicle Systems*, pp.1385-1411.
13. Kitano, M. and Jyozaki, H., 1976, A Theoretical Analysis of Steerability of Tracked Vehicles, *Journal of Terramechanics*, Vol.13(2), pp.241-262.
14. Kitano, M. and Kuma, M., 1977, An Analysis of Horizontal Plane Motion of Tracked Vehicles, *Journal of Terramechanics*, Vol.14(4), pp.211-225.
15. Kitano, M. Watanabe, K. Takaba, Y. and Togo, K., 1988, Lane-Change Maneuver of High Speed Tracked Vehicles, *Journal Terramechanics*, Vol.25(2), pp.91-102.
16. K. Kitano, Watanabe, M. and Nagatomo, N., 1990, Stability and Controllability of High-Speed Tracked Vehicles—Linear Model and Vehicle Response, *Proceedings of the 10th International Conference of the International Society for Terrain-Vehicle Systems*, pp.659-670.
17. Merritt, H.E., 1939, Some Considerations Influencing the Design of High Speed Tracked Vehicles, *Proceedings of Institute of Automobile Engineers*, Vol.33, pp.398-430.
18. Micklethwait, E.W.E., 1944, *Soil Mechanics in Relation to Fighting Vehicles*, Military Coll. of Science, Chobham Lane, Chertsey.
19. Murakami, H. Watanabe, K. and Kitano, M., 1992, A Mathematical Model for Spatial Motion of Tracked Vehicles on Soft Ground, *Journal of Terramechanics*, Vol.29(1), pp.71-81.
20. Ogorkiewicz, R.M., 1991, *Technology of Tanks*, London: Jane's Information Group.
21. Schmid, I.C., 1984, A tracked Vehicle Test Plant for the Simulation of Dynamic Operation, *Proceedings of the 8th International Conference of the International Society for Terrain-Vehicle Systems*, pp.835-853.
22. Shiller, Z and Serate, W, 1995, Trajectory Planning of Tracked Vehicles, *Journal of Dynamic Systems, Measurement, and Control*, Vol.117, pp.619-623.
23. Steeds, W., 1950, Tracked Vehicles (in Three Parts), *Automobile Engineer*, pp143-148, pp187-190 and pp.219-222.

24. Stoer, J. and Bulirsch, R., 1980, *Introduction to Numerical Analysis*, Springer-Verlag: New York, Inc.
25. Stroud, A.H., 1966, *Gaussian Quadrature Formulas*, Prentice-Hall, Inc.
26. Sugiyama, N. and Kondo, H., 1984, *Basic Study on the Turning Resistance of Track*, Proceedings of the 8th International Conference of the International Society for Terrain-Vehicle Systems, pp.889-899.
27. Watanabe, K. Kitano, M. and Katahira, T., 1993, *Controllability and Stability of Tracked Vehicles on Inclined Ground*, Proceedings of the 12th International Conference of the International Society for Terrain-Vehicle Systems, pp.426-435.
28. Watanabe, Murakami, H. Kitano, M. and Katahira, T., 1993, *Experimental Characterization of Dynamic Soil-Track Interaction of Dry Sand*, Journal of Terramechanics, Vol.30(2), pp.111-131.
29. Weiss, K.R., 1971, *Skid-Steering*, Automobile Engineer, pp.22-25.
30. Wismer, R.D. and Luth, H.J., 1973, *Off-Road Traction Prediction for Wheeled Vehicles*, Journal of Terramechanics, Vol.10(2), pp.49-61.
31. Wong, J.Y. and Preston-Thomas, J., 1983, *On the Characterization of the Shear Stress-Shear Displacement Relationship of Terrain*, Journal of Terramechanics, Vol.19(4), pp.225-234.
32. Wong, J.Y. and Preston-Thomas, J., 1988, *Investigation into the Effects of Suspension Characteristics and Design Parameters on the Performance of Tracked Vehicles using an Advanced Computer Simulation Model*, Proceedings of Institution of Mechanical Engineers, Vol.202(D3), pp.143-161.
33. Wong, J.Y., 1989, *Terramechanics and Off-Road Vehicles*, Elsevier, Amsterdam.
34. Wong, J.Y., 1993, *Theory of Ground Vehicles*, 2nd Edition, John Wiley & Sons, Inc.
35. Wong, J.Y., 1994, *Computer-Aided Methods for Design Evaluation of Track Systems*, SAE Paper No.941675, pp.1-9.

Appendix A.

The Effects of Design and Operating Factors on the Handling of Tracked Vehicles under Continuous Trapezoidal Load Distribution over the entire track

A.1. Contact Length, L

Table A-1 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	7810	-5191	15598	-13022	21742	-19110	25777	-23043	28519	-25665
10	5460	-2959	12401	-9920	19740	-17273	24532	-22049	27670	-25164
20	3886	-1195	8665	-6181	15762	-13286	21866	-19391	25983	-23505
50	2407	83.70	4980	-2491	9509	-7021	15230	-12744	20712	-18226
100	1947	544.6	3363	-871.8	6012	-3522	9983	-7493	14732	-12242

Table A-2 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m. N.m		L = 3.8 m. (Reference) N.m		L = 4.75 m. N.m		L = 5.7 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	8833	-4010	14837	-9919	20948	-15451	25438	-18997	28654	-20999
10	5483	-2470	11042	-8137	18883	-16047	24033	-21082	27295	-24163
20	3534	-868.8	7431	-4804	15211	-12641	21689	-19125	25902	-23313
50	2291	292.2	4283	-1707	9144	-6578	15153	-12594	20740	-18183
100	1909	664.5	3049	-476.8	5798	-3228	9936	-7368	14765	-12199

Table A-3 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m.)	L = 1.90 m N.m		L = 2.85 m. N.m		L = 3.8 m. (Reference) N.m		L = 4.75 m. N.m		L = 5.7 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	7874	-3096	11896	-7317	17482	-12781	22286	-17078	25615	-19660
20	4720	-1638	7918	-4936	14317	-11440	20887	-18016	25184	-22224
50	2756	-39.98	4315	-1617	8581	-5912	14948	-12300	20688	-18045
100	2085	583.0	2932	-265.9	5368	-2709	9818	-7166	14788	-12140

Table A-4 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m.)	L = 1.90 m . N.m		L = 2.85 m. N.m		L = 3.8 m. (Reference) N.m		L = 4.75 m. N.m		L = 5.7 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	8479	-2763	11489	-6132	15489	-10149	19678	-13972	22952	-16586
30	4919	-1595	7358	-4175	11765	-8684	17881	-14848	22854	-19772
50	3550	-612.0	5231	-2346	8682	-5846	14625	-11833	20410	-17629
100	2494	298.2	3404	-622.3	5396	-2628	9680	-6929	14790	-12048

A.2. Tread of Vehicle, B

Table A-5 Sprocket torques at various turning radii with different treads at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	25877	-23231	24047	-21412	21742	-19110	19003	-16363	15692	-13029
10	23969	-21502	22066	-19601	19740	-17273	17063	-14591	13903	-11419
20	19946	-17473	18010	-15536	15762	-13286	13322	-10843	10554	-8071
50	12497	-10010	11056	-8569	9509	-7021	7968	-5479	6318	-3828
100	7845	-5355	6938	-4448	6012	-3522	5128	-2638	4201	-1711

Table A-6 Sprocket torques at various turning radii with different treads at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	24119	-18255	22898	-17262	20948	-15451	18510	-13057	15633	-10135
10	23081	-20233	21241	-18408	18883	-16047	16088	-13222	12849	-9911
20	19738	-17182	17682	-15122	15211	-12641	12415	-9827	9428	-6813
50	12433	-9882	10880	-8317	9144	-6578	7262	-4692	5587	-3013
100	7818	-5249	6836	-4267	5798	-3228	4706	-2135	3754	-1182

Table A-7 Sprocket torques at various turning radii with different treads at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	20632	-15648	19388	-14588	17482	-12781	15177	-10474	12570	-7763
20	18868	-16024	16829	-13980	14317	-11440	11592	-8662	8980	-5985
50	12333	-9679	10578	-7918	8581	-5912	6628	-3944	5070	-2371
100	7762	-5107	6623	-3967	5368	-2709	4261	-1598	3425	-759.0

Table A-8 Sprocket torques at various turning radii with different treads at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Outer	Outer	Inner	Inner	Inner	Outer	Inner
15	18316	-12546	17225	15489	11356	-5966	-10149	-11743	13490	-8171
30	15786	-12745	13893	11765	7805	-4627	-8684	-10841	9704	-6578
50	12337	-9532	10556	8682	5638	-2757	-5846	-7739	7018	-4159
100	7842	-5085	6621	5396	3669	-889.6	-2628	-3860	4404	-1630

A.3. Longitudinal Offset of Vehicle Center of Gravity, c_y

Table A-9 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	$c_y = -0.4$ m N.m		$c_y = -0.2$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.2$ m N.m		$c_y = 0.4$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	18975	-16300	20747	-18079	21742	-19110	22073	-19495	21722	-19194
10	16110	-13628	18327	-15852	19740	-17273	20450	-17990	20481	-18023
20	11736	-9254	13997	-11519	15762	-13286	16910	-14435	17475	-14999
50	6557	-4068	8040	-5551	9509	-7021	10589	-8101	11371	-8883
100	4168	-1677	5014	-2523	6012	-3522	6724	-4233	7316	-4826

Table A-10 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m.)	$c_y = -0.4$ m N.m		$c_y = -0.2$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.2$ m N.m		$c_y = 0.4$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	19942	-12014	20074	-14598	20948	-15451	21002	-15588	20203	-14986
10	15522	-12659	17554	-14700	18883	-16047	19653	-16851	19975	-17220
20	11665	-9077	13639	-11060	15211	-12641	16423	-13861	17132	-14574
50	6598	-4028	7872	-5304	9144	-6578	10296	-7732	11135	-8571
100	4178	-1607	4932	-2361	5798	-3228	6563	-3993	7191	-4621

Table A-11 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	$c_v = -0.4$ m N.m		$c_v = -0.2$ m N.m		$c_v = 0$ m (Reference) N.m		$c_v = 0.2$ m N.m		$c_v = 0.4$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	15168	-10464	16627	-11916	17482	-12781	17620	-12951	16904	-12277
20	11829	-8929	13350	-10465	14317	-11440	14858	-11990	15469	-12629
50	6989	-4315	7888	-5216	8581	-5912	9386	-6720	10364	-7704
100	4415	-1755	4910	-2250	5368	-2709	6096	-3439	6750	-4094

Table A-12 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m.)	$c_v = -0.4$ m N.m		$c_v = -0.2$ m N.m		$c_v = 0$ m (Reference) N.m		$c_v = 0.2$ m N.m		$c_v = 0.4$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	14156	-8606	15049	-9605	15489	-10149	15356	-10102	14415	-9194
30	10410	-7298	11371	-8283	11765	-8684	11511	-8416	10472	-7339
50	7781	-4940	8462	-5628	8682	-5846	8437	-5591	7729	-4868
100	4967	-2200	5318	-2552	5396	-2628	5248	-2478	5052	-2280

A.4. Track Width, b

Table A-13 Sprocket torques at various turning radii with different track widths at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	21695	-19068	21742	-19110	21879	-19232	22163	-19500
10	19660	-17194	19740	-17273	19975	-17504	20353	-17877
20	15619	-13143	15762	-13286	16190	-13714	16861	-14385
50	9327	-6839	9509	-7021	10037	-7549	10795	-8307
100	5868	-3377	6012	-3522	6406	-3915	6937	-4447

Table A-14 Sprocket torques at various turning radii with different track widths at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	20883	-15432	20948	-15451	21185	-15565	21691	-15931
10	18833	-16007	18883	-16047	19053	-16192	19441	-16547
20	15099	-12530	15211	-12641	15551	-12978	16166	-13591
50	8971	-6405	9144	-6578	9661	-7096	10464	-7900
100	5646	-3076	5798	-3228	6218	-3649	6803	-4234

Table A-15 Sprocket torques at various turning radii with different track widths at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m.)	b = 0.01 m. N.m		b = 0.45 m. (Reference) N.m		b = 0.90 m . N.m		b = 1.35 m . N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	17365	-12694	17482	-12781	17835	-13057	18409	-13539
20	14215	-11342	14317	-11440	14638	-11748	15218	-12309
50	8440	-5771	8581	-5912	9029	-6361	9817	-7151
100	5231	-2572	5368	-2709	5810	-3153	6435	-3778

Table A-16 Sprocket torques at various turning radii with different track widths at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	15332	-10015	15489	-10149	15936	-10532	16608	-11109
30	11622	-8543	11765	-8684	12188	-9099	12868	-9770
50	8555	-5719	8682	-5846	9060	-6223	9680	-6843
100	5301	-2534	5396	-2628	5698	-2931	6243	-3478

A.5. Height of Vehicle Center of Gravity, h

Table A-17 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 7.5 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	$h = 0.8$ m N.m		$h = 1.3$ m (Reference) N.m		$h = 1.8$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
5	21778	-19147	21742	-19110	21681	-19047
10	19747	-17281	19740	-17273	19728	-17261
20	15767	-13281	15762	-13286	15767	-13291
50	9506	-7018	9509	-7021	9513	-7025
100	6010	-3520	6012	-3522	6014	-3523

Table A-18 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 14.2 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	$h = 0.8$ m N.m		$h = 1.3$ m (Reference) N.m		$h = 1.8$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
5	21124	-15640	20948	-15451	20525	-14935
10	19021	-16194	18883	-16047	18671	-15822
20	15196	-12626	15211	-12641	15216	-12645
50	9124	-6558	9144	-6578	9164	-6598
100	5788	-3218	5798	-3228	5808	3238

Table A-19 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 21.3 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
10	17565	-12915	17482	-12781	17199	-12376
20	14301	-11425	14317	-11440	14268	-11385
50	8510	-5840	8581	-5912	8649	-5980
100	5328	2669	5368	-2709	5407	2748

Table A-20 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 29 km/h under continuous trapezoidal load distribution over the entire track.

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
15	15239	-9946	15489	-10149	15505	-9984
30	11648	-8565	11765	-8684	11833	-8749
50	8563	-5725	8682	-5846	8788	-5953
100	5321	-2553	5396	-2628	5470	-2703

Appendix B.

The Effects of Design and Operating Factors on the Handling of Tracked Vehicles for Trapezoidal Load Distribution on the Track Pitch under each Roadwheel

B.1. Contact Length, L

Table B-1 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	8194	-5672	16211	-13787	21698	-19282	25331	-22819	28045	-25376
10	5646	-3159	13627	-11198	19892	-17495	23624	-21231	26139	-23740
20	3807	-1319	10084	-7616	16803	-14349	21643	-19194	24609	-22162
50	2539	-48.48	5959	-3473	10925	-8441	16349	-13867	20927	-18447
100	2070	420.9	3934	-1444	7067	-4577	11244	-8755	15822	-13334

Table B-2 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	9144	-4418	15613	-10946	21206	-15864	25467	-18914	28877	-20776
10	5865	-2926	12311	-9617	19414	-16774	23441	-20697	26105	-23174
20	3724	-1080	8196	-5619	16583	-14083	21727	-19240	24752	-22254
50	2355	223.5	4699	-2128	10730	-8178	16433	-13891	21094	-18555
100	1956	616.7	3258	-686.9	6936	-4370	11294	-8731	15947	-13386

Table B-3 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	7873	-3153	12752	-8325	18226	-13602	22221	-16933	25289	-19068
20	4836	-1794	9060	-6192	16016	-13280	21296	-18548	24368	-21526
50	2825	-116.3	4819	-2141	10337	-7700	16501	-13888	21267	-18661
100	2132	535.1	3137	-476.3	6587	-3936	11344	-8702	16127	-13490

Table B-4 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	8290	-2517	12091	-6725	16390	-10960	20092	-14071	22951	-16016
30	5017	-1716	8224	-5116	13641	-10676	19227	-16291	22869	-19854
50	3639	-712.0	5876	-3024	10428	-7649	16517	-13784	21287	-18560
100	2544	245.2	3735	-962.2	6513	-3762	11436	-8703	16365	-13643

B.2. Tread of Vehicle, B

Table B-5 Sprocket torques of various turning radii with different treads at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	25338	-22874	23725	-21290	21698	-19282	19242	-16828	16062	-13614
10	23506	-21106	21902	-19505	19892	-17495	17497	-15095	14493	-12072
20	20483	-18030	18807	-16354	16803	-14349	14538	-12082	11813	-9351
50	13798	-11315	12426	-9943	10925	-8441	9369	-6885	7646	-5161
100	8925	-6436	8017	-5528	7067	-4577	6121	-3631	5125	-2636

Table B-6 Sprocket torques of various turning radii with different treads at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	24301	-18383	23009	-17396	21206	-15864	18985	-13799	16271	-11105
10	22896	-20197	21405	-18743	19414	-16774	16909	-14269	13622	-10957
20	20515	-18021	18766	-16272	16583	-14083	13904	-11385	9873	-7305
50	13899	-11351	12412	-9863	10730	-8178	8821	-6265	5705	-3135
100	8988	-6423	8002	-5436	6936	-4370	5778	-3211	3912	-1341

Table B-7 Sprocket torques of various turning radii with different treads at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	20678	-15586	19749	-14919	18226	-13602	16189	-11660	13683	-9107
20	19863	-17115	18215	-15482	16016	-13280	13305	-10540	10267	-7451
50	14031	-11406	12355	-9726	10337	-7700	7726	-5070	5499	-2831
100	9082	-6436	7937	-5290	6587	-3936	4719	-2063	3684	-1025

Table B-8 Sprocket torques of various turning radii with different treads at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	18570	-12445	17821	-12100	16390	-10960	14499	-9208	12254	-6973
30	17385	-14416	15726	-12770	13641	-10676	11356	-8367	9066	-6009
50	14255	-11498	12487	-9723	10428	-7649	8330	-5528	6504	-3675
100	9329	-6588	8022	-5277	6513	-3762	5076	-2317	4065	-1299

B.3. Longitudinal Offset of Vehicle Center of Gravity, c_y

Table B-9 Sprocket torques of various turning radii with different longitudinal CG offsets at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	$c_y = -0.40$ m N.m		$c_y = -0.20$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.2$ m N.m		$c_y = 0.4$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	18647	-16239	20430	-18017	21698	-19282	22500	-20090	22851	-20454
10	16755	-14366	18543	-16149	19892	-17495	20812	-18414	21324	-18922
20	13086	-10630	15184	-12729	16803	-14349	17931	-15476	18640	-16183
50	7412	-4926	9449	-6964	10925	-8441	11954	-9470	12633	-10149
100	4502	-2012	5998	-3509	7067	-4577	7779	-5289	8228	-5738

Table B-10 Sprocket torques of various turning radii with different longitudinal CG offsets at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m.)	$c_y = -0.40$ m N.m		$c_y = -0.20$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.20$ m N.m		$c_y = 0.40$ m (N.m)	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	16340	-11241	19190	-13913	21206	-15864	22546	-17229	23273	-18008
10	15671	-13036	17804	-15175	19414	-16774	20644	-17991	21528	-18870
20	12925	-10425	14897	-12397	16583	-14083	17892	-15391	18821	-16318
50	7521	-4967	9166	-6613	10730	-8178	11895	-9344	12716	-10164
100	4553	-1986	5710	-3143	6936	-4370	7755	-5189	8302	-5735

Table B-11 Sprocket torques of various turning radii with different longitudinal CG offsets at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	$c_v = -0.40$ m N.m		$c_v = -0.20$ m N.m		$c_v = 0$ m (Reference) N.m		$c_v = 0.20$ m N.m		$c_v = 0.40$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	13188	-8621	16047	-11446	18226	-13602	19801	-15187	20831	-16251
20	12318	-9551	14405	-11663	16016	-13280	17313	-14574	18404	-15660
50	7852	-5211	9076	-6438	10337	-7700	11538	-8902	12545	-9911
100	4844	-2192	5503	-2851	6587	-3936	7534	-4885	8246	-5597

Table B-12 Sprocket torques of various turning radii with different longitudinal CG offsets at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	$c_v = -0.40$ m N.m		$c_v = -0.20$ m N.m		$c_v = 0$ m (Reference) N.m		$c_v = 0.20$ m N.m		$c_v = 0.40$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	11658	-5955	14245	-8720	16390	-10960	18095	-12727	19354	-14052
30	10380	-7356	12252	-9266	13641	-10676	14610	-11650	15273	-12306
50	8279	-5485	9517	-6735	10428	-7649	11145	-8364	11832	-9048
100	5351	-2598	5990	-3238	6513	-3762	7096	-4345	7778	-5028

B.4. Track Width, b

Table B-13 Sprocket torques of various turning radii with different track widths at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m.)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	21655	-19243	21698	-19282	21818	-19389	21988	-19538
10	19838	-17443	19892	-17495	20049	-17648	20296	-17890
20	16711	-14258	16803	-14349	17069	-14614	17480	-15024
50	10792	-8308	10925	-8441	11299	-8815	11860	-9376
100	6960	-4470	7067	-4577	7366	-4877	7811	-5321

Table B-14 Sprocket torques of various turning radii with different track widths at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m.)	b = 0.01 m . N.m		b = 0.45 m (Reference) N.m		b = 0.90 m. N.m		b = 1.35 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	21142	-15840	21206	-15864	21402	-15941	21736	-16101
10	19387	-16753	19414	-16774	19492	-16834	19620	-16938
20	16517	-14018	16583	-14083	16776	-14271	17084	-14571
50	10604	-8053	10730	-8178	11094	-8543	11659	-9108
100	6826	-4260	6936	-4370	7247	-4681	7715	-5150

Table B-15 Sprocket torques of various turning radii with different track widths at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	18145	-13550	18226	-13602	18466	-13759	18855	-14027
20	15961	-13229	16016	-13280	16183	-13436	16478	-13715
50	10242	-7605	10337	-7700	10623	-7985	11114	-8474
100	6483	-3833	6587	-3936	6903	-4253	7414	-4765

Table B-16 Sprocket torques of various turning radii with different track widths at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	16287	-10881	16390	-10960	16687	-11189	17149	-11543
20	13541	-10579	13641	-10676	13937	-10963	14412	-11425
50	10333	-7554	10428	-7649	10713	-7932	11176	-8394
100	6439	-3688	6513	-3762	6735	-3984	7129	-4378

B.4. Height of Vehicle Center of Gravity, h

Table B-17 Sprocket torques of various turning radii with different CG heights at a vehicle speed of 7.5 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
5	21702	-19287	21698	-19282	21669	-19250
10	19892	-17496	19892	-17495	19887	-17490
20	16798	-14344	16803	-14349	16808	-14354
50	10921	-8438	10925	-8441	10928	-8444
100	7065	-4576	7067	-4577	7068	-4579

Table B-18 Sprocket torques of various turning radii with different CG heights at a vehicle speed of 14.2 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
5	21325	-15991	21206	-15864	20881	-15418
10	19571	-16941	19414	-16774	19162	-16506
20	16573	-14073	16583	-14083	16578	-14077
50	10712	-8161	10730	-8178	10747	-8195
100	6927	-4361	6936	-4370	6944	-4378

Table B-19 Sprocket torques of various turning radii with different CG heights at a vehicle speed of 21.3 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	h = 0.8 m. N.m		h = 1.3 m (Reference) N.m		h = 1.8 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
10	18461	-13922	18226	-13602	17727	-12923
20	16088	-13358	16016	-13280	15845	-13099
50	10266	-7628	10337	-7700	10399	-7762
100	6543	-3893	6587	-3936	6628	-3978

Table B-20 Sprocket torques of various turning radii with different CG heights at a vehicle speed of 29 km/h for trapezoidal load distribution on the track pitch under each roadwheel

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
15	16414	-11078	16390	-10960	16000	-10314
30	13630	-10669	13641	-10676	13572	-10597
50	10325	-7544	10428	-7649	10505	-7727
100	6417	-3666	6513	-3762	6602	-3851

Appendix C.

The Effects of Design and Operating Factors on the Handling of Tracked Vehicles for Concentrated Load under each Roadwheel

C.1. Contact Length, L

Table C-1 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel.

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	7831	-5343	15097	-12764	19638	-17426	22444	-20276	24560	-22382
10	5385	-2899	13277	-10874	18735	-16371	21912	-19571	23928	-21608
20	3658	-1169	10083	-7622	16255	-13806	20702	-18261	23361	-20926
50	2629	-139.1	6032	-3546	10782	-8299	16010	-13529	20394	-17914
100	2121	369.9	3985	-1495	7029	-4540	11124	-8635	15599	-13111

Table C-2 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel.

Turning Radius (m.)	L = 1.90 m. N.m		L = 2.85 m. N.m		L = 3.8 m (Reference) N.m		L = 4.75 m. N.m		L = 5.7 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	6251	-3398	12139	-9504	18732	-16189	22182	-19601	24201	-21522
20	3930	-1311	8112	-5542	16250	-13769	20993	-18532	23696	-21238
50	2470	105.8	4734	-2164	10688	-8139	16169	-13629	20625	-18089
100	2018	-553.9	3364	-793.3	6960	-4394	11215	-8652	15757	-13197

Table C-3 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel.

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	9131	-4542	13696	-9412	18523	-14012	21763	-16561	24207	-18052
20	5514	-2517	9573	-6755	16100	-13405	20923	-18226	23582	-20816
50	3140	-437.4	5052	-2382	10496	-7864	16374	-13766	20913	-18315
100	2277	388.6	3290	-630.8	6752	4104	11342	-8702	15996	-13361

Table C-4 Sprocket torques at various turning radii with different track-ground contact lengths at a vehicle speed of 29 km/h for concentrated load under each roadwheel.

Turning Radius (m)	L = 1.90 m N.m		L = 2.85 m N.m		L = 3.8 m (Reference) N.m		L = 4.75 m N.m		L = 5.7 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	10129	-4349	13665	-8311	17525	-12014	20585	-14331	22905	-15558
30	5900	-2635	9064	-6002	14219	-11289	19344	-16436	22590	-19610
50	4174	-1262	6423	-3589	10877	-8111	16628	-13906	21119	-18405
100	2827	-40.19	4028	-1260	6844	-4097	11567	-8838	16333	-13613

C.2. Tread of Vehicle, B

Table C-5 Sprocket torques at various turning radii with different treads at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel.

Turning Radius (m)	B = 1.90 m. N.m		B = 2.17 m. N.m		B = 2.54 m. (Reference) N.m		B = 3.04 m. N.m		B = 3.80 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	22930	-20703	21494	-19280	19638	-17426	17370	-15135	14402	-12084
10	22147	-19783	20631	-18268	18735	-16371	16500	-14130	13764	-11378
20	19771	-17323	18165	-15717	16255	-13806	14124	-11673	11652	-9198
50	13591	-11108	12245	-9762	10782	-8299	9290	-6807	7710	-5227
100	8871	-6381	7967	-5478	7029	-4540	6111	-3622	5177	-2688

Table C-6 Sprocket torques at various turning radii with different treads at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m. (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	22751	-16940	21919	-16686	20328	-15448	18216	-13557	15510	-10949
10	21994	-19404	20626	-18077	18732	-16189	16275	-13718	12846	-10240
20	20042	-17567	18354	-15879	16250	-13769	13687	-11186	9369	-6803
50	13792	-11246	12330	-9783	10688	-8139	8873	-6320	5490	-2920
100	8992	-6427	8009	-5444	6960	-4394	5857	-3291	3951	-1381

Table C-7 Sprocket torques at various turning radii with different treads at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel.

Turning Radius (m)	B = 1.90 m N.m		B = 2.17 m N.m		B = 2.54 m. (Reference) N.m		B = 3.04 m N.m		B = 3.80 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	20667	-15434	19891	-15062	18523	-14012	16490	-12131	13937	-9566
20	19844	-17132	18251	-15557	16100	-13405	13396	-10671	10240	-7465
50	14121	-11502	12469	-9846	10496	-7864	7921	-5271	5495	-2832
100	9198	-6554	8061	-5415	6752	4104	4825	-2169	3732	-1074

Table C-8 Sprocket torques at various turning radii with different treads at a vehicle speed of 29 km/h for concentrated load under each roadwheel.

Turning Radius (m.)	B = 1.90 m . N.m		B = 2.17 m . N.m		B = 2.54 m. (Reference) N.m		B = 3.04 m. N.m		B = 3.80 m. N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	19974	-13320	18943	-12867	17525	-12014	15478	-10245	13040	-7866
30	17943	-14944	16307	-13379	14219	-11289	11873	-8914	9458	-6445
50	14691	-11944	12935	-10183	10877	-8111	8706	-5919	6745	-3931
100	9639	-6900	8340	-5598	6844	-4097	5302	-2547	4190	-1428

C.3. Longitudinal Offset Vehicle Center of Gravity, c_y

Table C-9 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel.

Turning Radius (m)	$c_y = -0.40$ m N.m		$c_y = -0.20$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.2$ m (N.m)		$c_y = 0.4$ m (N.m)	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	16532	-14390	18244	-16060	19638	-17426	20700	-18464	21472	-19204
10	15440	-13088	17296	-14936	18735	-16371	19809	-17438	20581	-18198
20	12260	-9810	14577	-12129	16255	-13806	17469	-15019	18346	-15890
50	6965	-4480	9305	-6821	10782	-8299	11837	-9354	12604	-10119
100	4318	-1828	5992	-3503	7029	-4540	7745	-5255	8247	-5757

Table C-10 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel.

Turning Radius (m)	$c_y = -0.40$ m N.m		$c_y = -0.20$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.20$ m N.m		$c_y = 0.40$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	15953	-11341	18412	-13637	20328	-15448	21850	-16891	23035	-17964
10	15100	-12610	17094	-14585	18732	-16189	20103	-17526	21204	-18599
20	12449	-9974	14482	-12002	16250	-13769	17642	-15156	18700	-16205
50	7199	-4646	9098	-6547	10688	-8139	11867	-9318	12743	-10192
100	4449	-1882	5780	-3214	6960	-4394	7767	-5202	8345	5779

Table C-11 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel.

Turning Radius (m)	$c_y = -0.40$ m N.m		$c_y = -0.20$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.20$ m N.m		$c_y = 0.40$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	13901	-9516	16484	-12029	18523	-14012	20138	-15597	21391	-16834
20	12513	-9810	14487	-11797	16100	-13405	17471	-14761	18658	-15932
50	7891	-5259	9163	-6531	10496	-7864	11706	-9075	12718	-10085
100	4852	-2202	5612	-2962	6752	4104	7656	-5009	8354	5706

Table C-12 Sprocket torques at various turning radii with different longitudinal CG offsets at a vehicle speed of 29 km/h for concentrated load under each roadwheel.

Turning Radius (m)	$c_y = -0.40$ m N.m		$c_y = -0.20$ m N.m		$c_y = 0$ m (Reference) N.m		$c_y = 0.20$ m N.m		$c_y = 0.40$ m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
15	13248	-7529	15566	-9980	17525	-12014	19170	-13714	20525	-15125
30	11069	-8094	12843	-9899	14219	-11289	15260	-12329	16056	-13112
50	8712	-5938	9923	-7157	10877	-8111	11674	-8905	12415	-9640
100	5595	-2847	6247	-3500	6844	-4097	7474	-4726	8115	-5367

C.4. Track Width, b

Table C-13 Sprocket torques at various turning radii with different track widths at a vehicle speed 7.5 of km/h for concentrated load under each roadwheel.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	19602	-17396	19638	-17426	19743	-17513	19919	-17661
10	18678	-16316	18735	-16371	18900	-16533	19161	-16790
20	16158	-13710	16255	-13806	16531	-14082	16950	-14501
50	10646	-8163	10782	-8299	11161	-8679	11722	-9239
100	6920	-4431	7029	-4540	7331	-4842	7775	-5286

Table C-14 Sprocket torques at various turning radii with different track widths at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
5	20277	-15438	20328	-15448	20486	-15481	20751	-15547
10	18716	-16180	18732	-16189	18766	-16201	18784	-16188
20	16181	-13701	16250	-13769	16453	-13966	16775	-14282
50	10559	-8010	10688	-8139	11059	-8510	11623	-9075
100	6848	-4283	6960	-4394	7271	-4706	7736	-5171

Table C-15 Sprocket torques at various turning radii with different track widths at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	18455	-13974	18523	-14012	18725	-14128	19052	-14326
20	16052	-13361	16100	-13405	16243	-13535	16486	-13763
50	10397	7766	10496	-7864	10791	-8159	11285	-8652
100	6648	-3999	6752	4104	7064	-4416	7555	4908

Table C-16 Sprocket torques at various turning radii with different track widths at a vehicle speed of 29 km/h for concentrated load under each roadwheel.

Turning Radius (m)	b = 0.01 m N.m		b = 0.45 m (Reference) N.m		b = 0.90 m N.m		b = 1.35 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner	Outer	Inner
10	17438	-11955	17525	-12014	17775	-12185	18163	-12447
20	14130	-11203	14219	-11289	14482	-11543	14904	-11952
50	10787	-8022	10877	-8111	11142	-8376	11571	-8802
100	6771	-4024	6844	-4097	7064	-4317	7441	-4693

C.5. Height of Vehicle Center of Gravity, h

Table C-17 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 7.5 km/h for concentrated load under each roadwheel.

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
5	19654	-17447	19638	-17426	19599	-17380
10	18735	-16371	18735	-16371	18731	-16367
20	16250	-13802	16255	-13806	16259	-13810
50	10779	-8296	10782	-8299	10784	-8301
100	7028	-4539	7029	-4540	7031	-4541

Table C-18 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 14.2 km/h for concentrated load under each roadwheel.

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
5	20458	-15664	20328	-15448	20029	-14881
10	18885	-16355	18732	-16189	18473	-15909
20	16242	-13761	16250	-13769	16245	-13762
50	10673	-8125	10688	-8139	10703	-8154
100	6952	-4387	6960	-4394	6967	-4402

Table C-19 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 21.3 km/h for concentrated load under each roadwheel.

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
10	18713	-14327	18523	-14012	17953	-13172
20	16161	-13471	16100	-13405	15935	-13228
50	10432	-7800	10496	-7864	10552	-7921
100	6716	-4067	6752	-4104	6787	-4139

Table C-20 Sprocket torques at various turning radii with different CG heights at a vehicle speed of 29 km/h for concentrated load under each roadwheel.

Turning Radius (m)	h = 0.8 m N.m		h = 1.3 m (Reference) N.m		h = 1.8 m N.m	
	Outer	Inner	Outer	Inner	Outer	Inner
15	17443	-12111	17525	-12014	17017	-11005
30	14188	-11262	14219	-11289	14162	-11220
50	10769	-8002	10877	-8111	10955	-8191
100	6753	-4005	6844	-4097	6929	-4182