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SEISMIC RESPONSE OF UNANCHORED CYLINDRICAL LIQUID-TANK SYSTEMS

by

Feng Qu

B.Eng., Tsinghua University, 1992

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Department of Civil and Environmental Engineering

The Master of Engineering Program
in Civil and Environmental Engineering
is a joint program with the University of Ottawa,
administered by The Ottawa-Carleton Institute for Civil Engineering

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**SEISMIC RESPONSE OF
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CYLINDRICAL LIQUID-TANK SYSTEMS**

submitted by Feng Qu
in partial fulfillment of the requirements for
the degree of Master of Engineering

A handwritten signature in dark ink, appearing to read "D. T. Lau", is written over a horizontal line.

Thesis Supervisor

D. T. Lau

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May 1997

**To my parents
and
my wife**

Abstract

The “uplift mechanism”, developed in response to large overturning moment, is the dominant response behaviour of unanchored liquid-tank systems under seismic load. Associated with the base plate uplift, significant deformation and intensive stresses are developed in the tank structure. Intense liquid motion, both the impulsive and the convective component, is also induced from the coupling effect. The uplift mechanism is highly nonlinear. Its significance and mechanism in contributing to the seismic behaviour is not fully understood.

In this thesis, the seismic behaviour time history of an unanchored broad liquid-tank system is analyzed with the focus on the uplift mechanism. The 1940 El Centro and the 1994 Northridge earthquakes are used as the ground excitations. The uplift phenomenon of the base plate, its effects on the shell wall deformation and the dynamic characteristics of the liquid-tank system are studied. The liquid motion developed in the partially uplifted liquid-tank system and the corresponding hydrodynamic loads are analyzed. The intensive stress developed in the tank structure and the failure mechanism are discussed.

A parametric study is conducted to define the effects of the properties of the liquid-tank system on the uplift mechanism. The seismic behaviour time history of model liquid-tank systems with variant structural stiffness and liquid height to tank radius ratio are studied. By varying these parameters, the uplift behaviour, liquid motion and structural stress of the liquid-tank system are carefully compared and determined.

Acknowledgement

I would like to express my sincere gratitude to my supervisor Dr. D. T. Lau for his guidance and advice during the course of this research.

Thanks to department of Civil Engineering, Carleton University for the education and technical support.

Particular thanks to my parents. I owe everything to them.

Special thanks to my wife for her encouragement and support during the completion of this thesis.

Thanks to all my colleagues and friends for their help.

Contents

Abstract	i
Acknowledgement	ii
1 INTRODUCTION	1
1.1 General	1
1.2 Typical Cylindrical Tank and Behaviour	2
1.3 Description of Problem	4
1.4 Objectives and Scope	6
1.5 Outline	6
2 LITERATURE REVIEW	10
2.1 General	10
2.2 Dynamic Analysis of Anchored Liquid-Tank Systems	11
2.2.1 Rigid Tank Models	12
2.2.2 Flexible Tank Models	14
2.3 Dynamic Analysis of Unanchored Liquid-Tank Systems	18
2.3.1 Experimental Work	18
2.3.2 Analytical Studies	19
2.4 Design Guidelines	24
2.5 Summary	25

3	ANALYTICAL MODEL	31
3.1	General	31
3.2	Hydrodynamic Behaviour of Liquid Content	32
3.3	Ritz Formulation of Tank Structure	34
3.3.1	Tank Base Plate	35
3.3.2	Tank Shell Wall	40
3.3.3	Other Substructure Elements	41
3.3.4	Assembling the Ritz Shape Matrix	42
3.4	Governing Equation of Motion and Time Step Integration	42
3.5	Summary	45
4	SEISMIC BEHAVIOUR OF UNANCHORED LIQUID-TANK SYSTEMS	53
4.1	General	53
4.2	Resultant Lateral Force And Resultant Overturning Moment	54
4.3	Dynamic Response of Tank Structure	56
4.3.1	Base Plate Uplift	57
4.3.2	Shell Wall Radial displacement	58
4.3.3	Generalized Vibration Period	60
4.4	Dynamic Behaviour of Liquid Content	62
4.4.1	Impulsive Component	62
4.4.2	Convective Component	64
4.5	Stress Analysis and Failure of Tank Structure	67
4.5.1	Tank Shell Wall Stress	67
4.5.2	Tank Base Plate Stress	69
4.5.3	Failure of Tank Structure	69
4.6	Summary	70

5	PARAMETER STUDY	121
5.1	General	121
5.2	Description of Liquid-Tank System and Parameters Considered	122
5.3	Results	124
5.3.1	Broad Liquid-Tank System	125
5.3.2	Intermediate Liquid-Tank System	134
5.3.3	Tall Liquid-Tank System	142
5.4	Summary	147
6	SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK	200
6.1	Summary and Conclusions	200
6.2	Recommendations for Future Work	203
	References	204

List of Tables

4.1	Maximum Resultant Load	72
4.2	Maximum Response of Tank Structure	72
4.3	Maximum Hydrodynamic Pressure	73
4.4	Generalized Vibration Period Corresponding to the Abrupt Change of the Convective Hydrodynamic Behaviour	73
4.5	Maximum Compressive Stress	73
5.1	Dimension and Numbering of Liquid-Tank System	149
5.2	Seismic Behaviour of Broad Liquid-Tank System	150
5.3	Seismic Behaviour of intermediate Liquid-Tank System	151
5.4	Seismic Behaviour of Tall Liquid-Tank System	152

List of Figures

1.1	A Typical Cylindrical Liquid Storage Tank	8
1.2	Behaviour of Unanchored Tank Due to Lateral Load	9
2.1	Liquid-Tank System	27
2.2	Housner's Simplified Model	28
2.3	Clough's Model of Uplift Mechanism	29
2.4	Wozniak's Model of Uplift Mechanism	30
3.1	Coordinate System of Liquid-Tank System	46
3.2	Tank Base Plate With Uplift	47
3.3	Coordinate Mapping of Tank Base Plate	48
3.4	Ritz Shape Function $N(\eta)$ of Uplifted Region	49
3.5	Ritz Shape Function $N(\xi)$ of Uplifted Region (for vertical disp.) . . .	50
3.6	Ritz Shape Function $N(\xi)$ of Uplifted Region (for inplane disp.) . . .	51
3.7	Coordinate System of the Contact Region	52
4.1	Configuration of The Liquid-Tank System	74
4.2	Time History of Ground Acceleration	75
4.3	Spectral Acceleration	76
4.4	Time History of Resultant Lateral Force (El Centro)	77
4.5	Time History of Resultant Overturning Moment (El Centro)	78
4.6	Time History of Resultant Lateral Force (Northridge)	79

4.7	Time History of Resultant Overturning Moiment (Northridge)	80
4.8	Time History of Base Plate Uplift (El Centro)	81
4.9	Time History of Base Plate Uplift (Northridge)	82
4.10	Relationship between Tank Base Plate Uplift and Resultant Overturn- ing Moment	83
4.11	Time History of Tank Shell Wall Radial Displacement ($H_t = 6.096m$, El Centro)	84
4.12	Time History of Tank Shell Wall Radial Displacement ($H_t = 9.144m$, El Centro)	85
4.13	Time History of Tank Shell Wall Radial Displacement ($H_t = 12.192m$, El Centro)	86
4.14	Time History of Tank Shell Wall Radial Displacement ($H_t = 6.096m$, Northridge)	87
4.15	Time History of Tank Shell Wall Radial Displacement ($H_t = 9.144m$, Northridge)	88
4.16	Time History of Tank Shell Wall Radial Displacement ($H_t = 12.192m$, Northridge)	89
4.17	Tank Shell Wall Radial Displacement along Height (El Centro)	90
4.18	Tank Shell Wall Radial Displacement along Height (Northridge) . . .	91
4.19	Example of Tank Shell Wall Out-of-Round Distortion	92
4.20	Time History of Generalized Period (El Centro)	93
4.21	Time History of Generalized Period (Northridge)	94
4.22	Time History of Hydrodynamic Pressure ($H_t = 6.096m$, El Centro) .	95
4.23	Time History of Hydrodynamic Pressure ($H_t = 9.144m$, El Centro) .	96
4.24	Time History of Hydrodynamic Pressure ($H_t = 12.192m$, El Centro) .	97
4.25	Time History of Hydrodynamic Pressure ($H_t = 6.096m$, Northridge) .	98

4.26	Time History of Hydrodynamic Pressure ($H_l = 9.144m$, Northridge) .	99
4.27	Time History of Hydrodynamic Pressure ($H_l = 12.192m$, Northridge)	100
4.28	Profile of Liquid Pressure along Height ($H_l = 6.096m$)	101
4.29	Profile of Liquid Pressure around Circumference ($H_l = 6.096m$) . . .	102
4.30	Profiles of Liquid Pressure along Height ($H_l = 9.144m$)	103
4.31	Profile of Liquid Pressure around Circumference ($H_l = 9.144m$) . . .	104
4.32	Profile of Liquid Pressure along Height ($H_l = 12.192m$)	105
4.33	Profile of Liquid Pressure around Circumference ($H_l = 12.192m$) . . .	106
4.34	Time History of Equivalent Wave Height	107
4.35	Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 6.096m$, El Centro)	108
4.36	Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 9.144m$, El Centro)	109
4.37	Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 12.192m$, El Centro)	110
4.38	Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 6.096m$, Northridge)	111
4.39	Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 9.144m$, Northridge)	112
4.40	Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 12.192m$, Northridge)	113
4.41	Tank Shell Wall Stress along Height ($H_l = 6.096m$, El Centro)	114
4.42	Tank Shell Wall Stress along Height ($H_l = 9.144m$, El Centro)	115
4.43	Tank Shell Wall Stress along Height ($H_l = 12.192m$, El Centro) . . .	116
4.44	Tank Shell Wall Stress along Height ($H_l = 6.096m$, Northridge) . . .	117
4.45	Tank Shell Wall Stress along Height ($H_l = 9.144m$, Northridge) . . .	118

4.46	Tank Shell Wall Stress along Height ($H_t = 12.192m$, Northridge) . . .	119
4.47	Tank Base Plate Stress along Excitation Axis	120
5.1	Configuration of The Liquid-Tank System in the Parameter Study . .	153
5.2	Time History of Ground Acceleration for the Saguenay Earthquake .	154
5.3	Time History of Impulsive Pressure for BT-1 and BT-9 (El Centro) .	155
5.4	Time History of Impulsive Pressure for BT-1 and BT-9 (Saguenay-1)	156
5.5	Maximum Impulsive Hydrodynamic Pressure for Broad Liquid-Tank System	157
5.6	Time History of Wave Height for BT-1 and BT-9 (El Centro)	158
5.7	Time History of Wave Height for BT-1 and BT-9 (Saguenay-1)	159
5.8	Maximum Equivalent Wave Height for Broad Liquid-Tank System . .	160
5.9	Time History of Resultant Overturning Moment for BT-1 and BT-9 (El Centro)	161
5.10	Time History of Resultant Overturning Moment for BT-1 and BT-9 (Saguenay-1)	162
5.11	Normalized Maximum Resultant Overturning Moment for Broad Liquid- Tank System	163
5.12	Time History of Base Plate Uplift for BT-1 (El Centro)	164
5.13	Time History of Base Plate Uplift for BT-1 (Saguenay-1)	165
5.14	Maximum Base Plate Uplift for Broad Liquid-Tank System	166
5.15	Time History of Axial Membrane Stress for BT-1 and BT-9 (El Centro)	167
5.16	Time History of Axial Membrane Stress for BT-1 and BT-9 (Saguenay-1)	168
5.17	Maximum Compressive Axial Stress for Broad Liquid-Tank System .	169
5.18	Time History of Impulsive Pressure for MT-1 and MT-9 (Northridge)	170
5.19	Time History of Impulsive Pressure for MT-1 and MT-9 (Saguenay-2)	171

5.20	Maximum Impulsive Hydrodynamic Pressure for Intermediate Liquid-Tank System	172
5.21	Time History of Wave Height for MT-1 and MT-9 (Northridge) . . .	173
5.22	Time History of Wave Height for MT-1 and MT-9 (Saguenay-2) . . .	174
5.23	Maximum Equivalent Wave Height for Intermediate Liquid-Tank System	175
5.24	Time History of Resultant Overturning Moment for MT-1 and MT-9 (Northridge)	176
5.25	Time History of Resultant Overturning Moment for MT-1 and MT-9 (Saguenay-2)	177
5.26	Normalized Maximum Resultant Overturning Moment for Intermediate Liquid-Tank System	178
5.27	Time History of Base Plate Uplift for MT-1 and MT-9 (Northridge) .	179
5.28	Time History of Base Plate Uplift for MT-1 and MT-9 (Saguenay-2) .	180
5.29	Maximum Base Plate Uplift for Intermediate Liquid-Tank System . .	181
5.30	Time History of Axial Membrane Stress for MT-1 and MT-9 (Northridge)	182
5.31	Time History of Axial Membrane Stress for MT-1 and MT-9 (Saguenay-2)	183
5.32	Maximum Compressive Axial Stress for Intermediate Liquid-Tank System	184
5.33	Time History of Impulsive Pressure for TT-1 and TT-9 (El Centro) .	185
5.34	Time History of Impulsive Pressure for TT-1 and TT-9 (Saguenay-2)	186
5.35	Maximum Impulsive Hydrodynamic Pressure for Tall Liquid-Tank System	187
5.36	Time History of Wave Height for TT-1 and TT-9 (El Centro)	188
5.37	Time History of Wave Height for TT-1 and TT-9 (Saguenay-2)	189
5.38	Maximum Equivalent Wave Height for Tall Liquid-Tank System . . .	190

5.39	Time History of Resultant Overturning Moment for TT-1 and TT-9 (El Centro)	191
5.40	Time History of Resultant Overturning Moment for TT-1 and TT-9 (Saguenay-2)	192
5.41	Normalized Maximum Resultant Overturning Moment for Tall Liquid- Tank System	193
5.42	Time History of Base Plate Uplift for TT-1 and TT-9 (El Centro) . .	194
5.43	Time History of Base Plate Uplift for TT-1 and TT-9 (Saguenay-2) .	195
5.44	Maximum Base Uplift for Tall Liquid-Tank System	196
5.45	Time History of Axial Membrane Stress for TT-1 and TT-9 (El Centro)	197
5.46	Time History of Axial Membrane Stress for TT-1 and TT-9 (Saguenay-2)	198
5.47	Maximum Compressive Axial Stress for Tall Liquid-Tank System . . .	199

Chapter 1

INTRODUCTION

1.1 General

Thin wall, ground supported, cylindrical liquid storage tanks are widely used all over the world in many applications such as water supply systems, fuel transmission systems and other public and industrial facilities. Failure of these tank facilities may cause life threatening fire or explosion, severe environmental damage and significant financial loss. Since the survival and continuous functioning of essential facilities during and after an earthquake is of great importance to public safety, many liquid storage tanks, as a part of complex urban lifeline systems, require special considerations in their design to resist seismic loads.

Since the 1960's, extensive research efforts have resulted in many major advances in the understanding of the seismic behaviour of liquid storage tanks. A number of analytical models have been proposed from the results of these research efforts. Some of these models have been accepted in engineering practice and incorporated into design codes and standards. However, many liquid storage tanks designed in accordance to the provisions of modern design codes still suffered severe damage in recent major earthquakes, for example the 1964 Prince William Sound earthquake in Alaska (Rinne 1967, Hanson 1973), the 1964 Niigata earthquake in Japan (Ueno 1968), the 1971 San Fernando earthquake in California (Jennings 1971), the 1976

Tangshan earthquake in China (Sun 1985), the 1978 Miyagi Ken Oki earthquake in Japan (Kawano *et al.* 1980, Yamada *et al.* 1980), the 1979 Imperial Valley earthquake (Leeds 1980), the 1983 Coalinga earthquake in California (Manos and Clough 1985), the 1985 Chilean earthquake (Butler *et al.* 1987), the 1989 Loma Prieta earthquake (Benuska 1990), and the 1994 Northridge earthquake in California (Lau 1994). The poor performance of liquid storage tanks in these past earthquakes indicates that the seismic behaviour experienced by the liquid storage tanks is much more complex than what is assumed in the analytical models of the design codes, and thus a better understanding on the behaviour and performance of liquid storage tanks is needed.

1.2 Typical Cylindrical Tank and Behaviour

Among the different structural forms of liquid containers, the ground supported upright cylindrical tank is the most commonly used type because it is very efficient in withstanding the hydrostatic load caused by the liquid content.

A typical cylindrical tank is shown in Figure 1.1. It essentially consists of a base plate rested on the ground and a vertically erected thin shell wall. Optionally, there may be a floating or fixed roof to protect the liquid content from the outside air, column(s) inside the tank to support the roof, ring girders to enhance the stiffness of the shell wall and to improve its stability, and anchorage along the border of the base plate to prevent uplift. For a broad tank with its diameter greater than its height, anchorage is usually difficult to design and costly to implement, and therefore the tank is generally left unanchored.

Under the static condition, a cylindrical liquid storage tank is subjected to axisymmetric hydrostatic pressure caused by the liquid content acting radially outwards on the vertical tank shell wall. Hoop tensile force is developed in the tank shell wall

to withstand this pressure. In static design of a cylindrical tank, the required shell thickness of the tank shell wall is determined by the magnitude of this hoop tensile force. Under the static condition, the base plate and other parts of the tank structure bear no significant load and thus can be designed just to comply with the minimum structural requirements.

When a liquid storage tank is subjected to seismic load, vibrational motion of the liquid content is developed due to the ground excitation acting on it via the tank structure. The liquid motion can be divided into two modes. The first one, defined as the “impulsive mode”, consists of the liquid particles moving in unison with the motion of the tank structure. The second one, defined as the “convective mode” or usually called “sloshing”, consists of the liquid particles near the free surface oscillating vertically. The inertia effect of the moving liquid particles induces asymmetric hydrodynamic pressure acting on the tank structure with a magnitude that can reach several times that of the hydrostatic pressure. The resultant load effect of the liquid motion is generally represented by a lateral force and an overturning moment. In response to the hydrodynamic load, axial compressive and tensile stresses as well as additional hoop stress are developed in the tank shell wall. These stresses are asymmetric and much greater than that due to the hydrostatic pressure under the static condition. For an unanchored tank subjected to a sufficiently large resultant overturning moment, a portion of the base plate will separate from the support foundation. Accompanying with the base plate uplift, extensive deformations with significant out-of-round distortion occur in the tank shell wall. This phenomenon is called the “uplift mechanism”. The “uplift mechanism” is highly nonlinear to the applied load. The significant deformation and violent motion of the tank structure in turn greatly affect the liquid motion, and thus the hydrodynamic load. Figure 1.2 illustrates the typical hydrodynamic effect and shell wall distortion of an uplifting tank.

Based on field observations, the typical failure modes of liquid storage tanks can be summarized as follows (Bertero 1982)

1. Failure of the base plate.
2. Buckling of the tank shell wall.
3. Rupture of the tank shell wall at anchorage.
4. Failure of anchorage.
5. Failure of roof.
6. Total collapse.
7. Failure or partial settlement of foundation.
8. Sliding or walk-off of foundation due to rocking.
9. Damages to piping connections or other equipments.

For unanchored liquid storage tanks, buckling of the tank shell wall is the most common type of structure damage due to the narrow concentration and high magnitude of the compressive force developed in the tank shell wall when the tank base plate is partially uplifted from the ground support during an earthquake. Failure of the tank roof, due to the impact from the intense liquid sloshing, and damage to piping connections, due to the large displacement of the tank structure during dynamic response, are also frequently reported for unanchored liquid storage tanks.

1.3 Description of Problem

Because the hydrodynamic behaviour of the liquid content and the dynamic response of the tank structure are closely coupled, the liquid content and the tank structure

must be considered together as an integral system in the analysis. The Laplace's differential equation in terms of a potential velocity function is generally employed to describe the liquid motion. The boundary conditions for the equation describing the liquid motion are determined from the dynamic response of the tank structure.

In previous research efforts (Veletsos and Yang 1976, 1977, Haroun and Housner 1981, Balendra *et al.* 1982, Rammerstorfer *et al.* 1988), it is generally assumed that the tank is fixed in rigid foundation, and the tank shell wall vibrates in specified modes and maintains its cross-sectional shape without any distortion. For unanchored liquid-tank systems, field observations and shaking-table experiments have shown that the base plate uplift is very common and may cause severe out-of-round distortion in the shell wall during an earthquake (Clough 1977, Niwa 1978, Manos and Clough 1982). In some cases, the result obtained from analytical models with the aforementioned assumptions greatly under-estimates the seismic response of unanchored liquid-tank systems (Manos and Clough 1985).

Consequently at present, there are still many uncertainties regarding the significance and mechanism of the uplift phenomenon in contributing to the seismic behaviour of unanchored liquid-tank systems. In particular, the relationships of the deformation of the tank structure, the liquid motion and the stress distribution in the tank structure with the uplift behaviour, the influence of the properties of the liquid-tank system, such as the aspect ratio of liquid height to tank radius and the structure stiffness on the uplift behaviour, are still needed to be defined. A better understanding of the uplift mechanism is essential before a more accurate design procedure for the seismic behaviour of unanchored liquid-tank systems can be developed.

1.4 Objectives and Scope

The main objective of this research is to carry out a parametric study on the seismic behaviour of unanchored liquid-tank systems in order to obtain a better understanding on the uplift mechanism.

In this thesis, the seismic behaviour of unanchored liquid-tank systems is studied through time history analysis with the focus on the uplift mechanism. The relation between the hydrodynamic loads and the base plate uplift, the effect of the base plate uplift on the deformation of the tank structure, the liquid motion and the stresses developed in the tank structure are studied.

A parametric study on the effects of structural stiffness and the geometry of the liquid-tank system on the seismic behaviour of unanchored liquid-tank systems is undertaken. Related to the structural stiffness, the effects of parameters such as the thickness of the shell wall and the base plate are studied. As for the geometry of the liquid-tank system, the influence of the ratio of the liquid height to the tank radius on the uplift behaviour is studied. By varying these parameters, the seismic uplift behaviour of the liquid-tank system is carefully determined.

1.5 Outline

A literature review of previous research on the seismic behaviour of liquid storage tanks is presented in Chapter 2. It includes a review of the existing analytical models for anchored and unanchored liquid-tank systems and observed dynamic behaviour of unanchored liquid-tank systems from experimental studies. The analytical model adopted in the present research is briefly explained in Chapter 3. The formulation of the hydrodynamic loading, the derivation of the Ritz shapes for structural modelling

and discretization, and the iteration procedure used to solve the dynamic equation are presented. A detailed time-history analysis of unanchored liquid-tank systems is presented in Chapter 4. The intent of the time history analysis is to achieve a better understanding on the coupling effect of the uplift mechanism of the tank structure with the liquid motion. Chapter 5 presents the results of a parameter study. The effects of the geometric and structural properties of the liquid-tank system on the uplift mechanism and its unanchored seismic behaviour are discussed. A summary of the present study and recommendations for future work are presented in Chapter 6.

Figure 1.1: A Typical Cylindrical Liquid Storage Tank

(a) Hydrodynamic pressure and resultants

(b) Distortion of tank shell cross section

Figure 1.2: Behaviour of Unanchored Tank Due to Lateral Load

Chapter 2

LITERATURE REVIEW

2.1 General

The analysis of the dynamic behaviour of liquid-tank systems consists of two aspects. One is the modelling of the motion of the liquid content induced by the ground excitation and the resulting hydrodynamic pressure acting on the tank structure. The other is the modelling of the dynamic response of the tank structure due to the ground excitation and the hydrodynamic pressure.

In the formulation of the seismic liquid storage tank problem, the liquid content is generally assumed to be incompressible and inviscid. The flow is assumed to be irrotational. The Laplace's differential equation in terms of a velocity potential function is employed to describe the motion of this ideal fluid. The boundary conditions of the Laplace's equation are defined by the dynamic response of the tank structure, which is the combination of the vibration in response to the ground excitation and the deformation in response to the hydrodynamic load. Thus the motion of the liquid content and the dynamic response of the tank structure are coupled together.

In early research efforts, the tank structure is assumed to be rigid and anchored to the foundation. The coupling effect between the liquid motion and the dynamic response of the tank structure is totally ignored. Later, the flexibility of the tank shell wall and the coupling effect are included in the analysis. Recently, the significant effect

of the base plate uplift on the seismic response of unanchored liquid-tank systems is recognized. Extensive research efforts have been conducted to understand the effect of the uplift mechanism and to include this effect in the analysis procedure.

The analytical models for the dynamic analysis of anchored liquid-tank systems, with the shell wall assumed to be rigid and flexible, are reviewed in Section 2.2. The review of the experimental and analytical work on the seismic response of unanchored liquid-tank systems is presented in Section 2.3. The current design codes and guidelines for liquid storage tanks are briefly reviewed in Section 2.4. A summary of the literature review is presented at the end of the chapter.

2.2 Dynamic Analysis of Anchored Liquid-Tank Systems

The initial studies on the dynamic behaviour of liquid-tank systems focus on anchored liquid-tank systems, which have the tank structure base plate anchored to the foundation thus the corresponding seismic response is simpler compared to that in unanchored liquid-tank systems. Many analytical models have been proposed (Jacobsen 1949, Housner 1957, Veletsos and Yang 1976, 1977, Haroun and Housner 1981, Balendra *et al.* 1982, Rammerstorfer *et al.* 1988), and reliable predictions of the dynamic behaviour of anchored liquid-tank systems can be obtained from these models. Some concepts adopted in these models are useful in the development of the analytical models for unanchored liquid-tank systems.

These analytical models can be categorized into rigid tank models and flexible tank models according to the different assumptions about the dynamic response of the tank shell wall made in the analysis. These two types of analytical models are briefly reviewed in this section.

2.2.1 Rigid Tank Models

Early studies on evaluating the hydrodynamic loads acting on the tank structure were carried out by Hoskins and Jacobsen(1934), Jacobsen(1949) and Housner(1957). In their studies, the tank shell wall is considered to be rigid and experience the same motion as the ground support.

In the analytical procedures proposed by Hoskins and Jacobsen(1934) and Jacobsen(1949), the fluid is assumed to be incompressible and inviscid, the effect of the gravity waves is excluded from the analysis and only the impulsive motion of the liquid content is considered. Hoskins and Jacobsen(1934) employed the Laplace's differential equation in terms of a potential velocity function to express the liquid motion. Jacobsen(1949) later suggested that for a liquid-tank system as shown in Figure 2.1, the boundary conditions of the liquid motion are

- At the base plate, $z = 0$, the vertical velocity of the liquid particles is zero.
- At the tank shell wall, $r = R$, the radial velocity of the liquid particles is the same as that of the tank shell wall, which equals to that of the ground motion.
- At the liquid free surface, $z = H$, the liquid pressure is zero.

The solution of the Laplace's differential equation of the liquid motion is expressed with the modified Bessel function.

In Housner's model(Housner 1957), the effect of the gravity waves is included in the analysis, and the motion of the liquid content is separated into the "impulsive mode", which represents the motion of the liquid particles in unison with the tank shell wall, and the "convective mode", usually called "sloshing", which represents the vertical oscillating motion of the liquid particles. An approach based on direct

physical considerations is employed in the analysis of the liquid motion instead of directly solving the Laplace's differential equation. In the analysis of the impulsive mode, the fluid is assumed to be restrained by two sets of vertical membranes which are thin, rigid and massless, one set fixed along the direction of the ground excitation and the other set free to slide in the direction of the ground excitation. This is shown in Figure 2.2(a). The impulsive pressure acting on the tank shell wall is obtained by applying the Newton's law on the finite volume of liquid content and considering the continuity of the flow. In the analysis of the convective mode, the fluid is assumed to be restrained by horizontal membranes which are also thin, rigid and massless, free to rotate about a diametral axis normal to the direction of the ground excitation. This is shown in Figure 2.2(b). The liquid pressure caused by the convective liquid motion is analyzed by the Hamilton's principle. For the engineering purpose, an equivalent single-degree-of-freedom spring-mass model is used to simplify the analysis and to evaluate the maximum resultant lateral force and overturning moment. In the simplified model, the effect of the impulsive mode is represented by the impulsive mass m_i , attached to the tank shell wall at height H_i by a rigid bar, whereas the effect of the convective mode is represented by the convective mass m_c , attached to the tank shell wall at height H_c by a spring with frequency f_c and damping ratio ξ_c . The height H_i and H_c are determined such that the simplified model has the same overturning effect as the respective liquid motion they represented. The simplified model is shown in Figure 2.2(c). Housner(1957) also suggested that the tanks should be categorized into two types: broad, with liquid depth over tank radius $H/R \leq 1.5$, and tall, with $H/R > 1.5$. For tall tanks, only the portion of the liquid content from the liquid free surface to a depth of $1.5R$ should be analyzed with the above procedure to estimate the effect of the impulsive mode, the rest of the liquid content should be considered as a rigid mass moving in unison with the tank shell wall.

The Housner's model was later modified by Epstein(1976). He suggested that the convective liquid motion is mainly concerned with the upper part of the liquid, thus only the part of the liquid content from the liquid free surface to a depth of $1.5R$ should be considered when evaluating the effect of the convective liquid motion.

2.2.2 Flexible Tank Models

The poor performance of the liquid storage tanks designed with the analytical procedures based on the “rigid-shell-wall” assumption during major earthquakes revealed that the dynamic behaviour experienced by the liquid-tank systems is much more complex than that implied by the assumption. Research efforts(Veletsos 1974, Yang 1976, Veletsos and Yang 1976,1977, Haroun and Housner 1981, Balendra *et al.* 1982, Rammerstorfer *et al.* 1988) were then dedicated to investigate the influence of the flexibility of tank shell wall on the seismic behaviour of liquid-tank systems by considering the coupling effect between the hydrodynamic load and the dynamic response of the tank structure. Different approaches are adopted by different researchers to model and analyze the dynamic behaviour of the liquid-tank system.

To include the effect of the convective liquid motion and the coupling effect between the hydrodynamic load and the dynamic response of the tank structure in the analysis, Veletsos(1974) modified the boundary conditions of the liquid motion as follows

- At the base plate, $z = 0$, the vertical velocity of liquid particles is zero.
- At the tank shell wall, $r = R$, the radial velocity of liquid particles is the same as that of the tank shell wall. Because of its flexibility, the motion of the tank shell wall is no longer the same as that of the ground support, but affected by

the ground excitation, the hydrodynamic load and the vibration of the shell wall itself.

- At the free surface of the liquid content, the liquid pressure is zero. Neglecting the vertical inertia effect of the wave, the hydrodynamic pressure at $z = H$ is equal to the weight of the liquid column above.

In the Veletsos-Yang's procedure(Veletsos 1984), the liquid motion is assumed to satisfy the Laplace's equation. The solution of the Laplace's equation is expressed as the sum of the impulsive and convective components. The impulsive component satisfies the boundary conditions of the liquid motion at the liquid-tank interface and the convective component satisfies the boundary condition at the liquid free surface. In the analysis of the impulsive component, the tank shell wall is assumed to maintain its circular cross-section and vibrate as a uniform cantilever beam with bending and shearing properties. As a result, the deflection of the tank shell wall can be represented by a linear combination of vibration terms, which are defined by the vibration mode shapes of a uniform cantilever beam and the corresponding generalized time-dependent coordinates. The impulsive pressure obtained is expressed by an infinite series as follows

$$p_i(z, \theta, t) = \rho_l \sum_{k=1} c_{ik}(z) A_{ik}(t) R \cos(\theta) \quad (2.1)$$

where $c_{ik}(z)$ is the distribution of the impulsive pressure along the height of the shell wall, and $A_{ik}(t)$ is the pseudoacceleration for a single-degree-of-freedom oscillator, with the same frequency and damping, subjected to the same ground motion, corresponding to the k th vibration mode of the liquid-tank system. Because the principal effect of the tank shell wall flexibility is on the lateral motion of the liquid-tank system, and the convective liquid motion is of much longer period compared to the tank shell wall vibration, it is assumed that the coupling between the convective liquid

motion and the tank shell wall vibration is very weak. For this reason, in the analysis of the convective component of the liquid motion, the tank shell wall is considered to be rigid. The convective pressure obtained is then expressed as follows

$$p_c(z, \theta, t) = \rho_l \sum_{k=1} c_{ck}(z) A_{ck}(t) R \cos(\theta) \quad (2.2)$$

where $c_{ck}(z)$ is the distribution of the convective pressure along the height, and $A_{ck}(t)$ is the pseudoacceleration for a single-degree-of-freedom oscillator, with the same frequency and damping, subjected to the same ground motion, corresponding to the k th sloshing mode of the liquid content.

From a series of research efforts (Yang 1976, Veletsos and Yang 1976, 1977, Veletsos and Kumar 1984), it is concluded that

- The flexibility of the tank shell wall will change the temporal variation and the magnitude of the liquid motion, but has little effect on its spatial distribution.
- The impulsive liquid motion increases from zero at the liquid free surface to the maximum near the bottom of the tank shell wall.
- The convective liquid motion decreases from the maximum at the liquid surface with depth. The effect of the higher modes of the convective liquid motion can be neglected.

As compared to the hydrodynamic effect, the contribution of the mass of the tank structure in the inertia effect is proved to be negligible and is thus usually neglected in the formulation.

In the studies by Rammerstorfer *et al.* (1988), the vibration mode shape of the tank shell wall is obtained by an iterative procedure. The iteration starts by an initial guess of the mode shape. The refined mode shape is obtained by considering the hydrodynamic pressure as “added mass”.

Similar assumptions on the liquid motion as those in the Veletsos-Yang's procedure (Veletsos 1984) are made by Haroun and Housner (1981) in their analytical study on the dynamic behaviour of liquid-tank systems. The vibration of the flexible tank shell wall is not modelled by pre-defined vibration modes but analyzed by the finite element method. The radial displacement of the shell wall, $w(z, \theta, t)$, is determined by the base excitation, the magnitude and distribution of the hydrodynamic pressure and the deformation of the tank shell wall itself. In turn, the hydrodynamic pressure is dependent on the tank shell wall displacement. An energy method is applied to solve the coupling problem between the motion of the liquid content and the dynamic response of the tank structure. The potential and kinetic energy of the tank shell wall is derived in terms of the assembled mass matrix and nodal displacement vectors obtained from the finite element method. The influence of the hydrodynamic pressure on the response of the tank structure is estimated by an additional mass which is evaluated by calculating the work done by the hydrodynamic load through an arbitrary virtual displacement. By applying the Hamilton's principle, a standard eigenproblem is obtained. The frequencies and displacement vectors of the shell wall vibration obtained from the solution of the eigenproblem are used to evaluate the impulsive pressure. Similar to the Veletsos-Yang's model, the convective motion is assumed to be not affected by the tank shell wall flexibility and is evaluated by assuming the tank shell wall is rigid.

Balendra *et al.* modelled and analyzed both the liquid domain and the tank structure with the finite element method (Balendra and Nash 1978, Balendra *et al.* 1982). In the analysis, the liquid domain is discretized by annular ring elements of rectangular cross-section with the hydrodynamic pressure as the nodal degrees-of-freedom. The tank shell wall is discretized by ring elements with four degrees-of-freedom: the axial, circumferential and radial displacements, and the slope of the radial displace-

ment with respect to the axial coordinate. The effect of the liquid sloshing was initially neglected in the analysis(Balendra and Nash 1978) and later included in another study(Balendra *et al.* 1982). In the calculated vibration modes, the high frequency modes are associated with the impulsive effect, whereas the low frequency modes are associated with the convective effect.

2.3 Dynamic Analysis of Unanchored Liquid-Tank Systems

Because of economic and technical reasons, most medium to large liquid storage tanks in the field are unanchored. Compared to anchored liquid-tank systems, the unanchored liquid-tank systems exhibit much more complicated seismic behaviour. This is because of the possibility of base plate uplift, which occurs when the resultant overturning moment generated from the liquid motion is large enough. The uplift mechanism is highly nonlinear to the applied load and causes significant deformations in both the base plate and the shell wall.

Due to the complexity of the problem, few analytical research studies on the dynamic behaviour of unanchored liquid-tank systems has been done. Early understanding of the dynamic behaviour of unanchored liquid-tank systems was mainly obtained from experiments and field observations of tanks damaged in earthquakes.

2.3.1 Experimental Work

Because of the uncertainties in the seismic behaviour of unanchored liquid-tank systems, many experimental studies have been carried out in recent years.

Clough(1977), Niwa(1978), Manos and Clough(1982) conducted a series of ex-

periments on the seismic behaviour of liquid-tank systems. In these studies, scaled models of liquid-tank systems are subjected to dynamic loading as well as static tilt loading on a shaking table. Different conditions, including the base fixity, rigidity of the ground support, ratio of liquid height to tank radius, and type of the tank roof, are considered in the experiments and the results are compared. The general conclusions from these experiments can be summarized as follows

- Uplift mechanism, which is nonlinear to the intensity and somewhat sensitive to the frequency content of the input excitations, is an important phenomenon for the seismic response of unanchored liquid-tank systems.
- Significant out-of-round distortion is developed in the shell wall under seismic loading in both anchored and unanchored liquid-tank systems.
- The base plate uplift and the shell wall out-of-round distortion changes the distribution of the stress, and results in compressive stresses developed in the shell wall of unanchored tanks with much larger magnitude than that in anchored tanks.

Zui *et al.*(1985) examined the influences of the base fixity and the initial irregularities on the seismic behaviour of the cylindrical liquid-tank systems with shaking table tests, and concluded that the base fixity significantly changes the seismic response of liquid-tank systems. Sakai *et al.*(1989) studied the complicated uplift behaviour by a series of static tilt tests using a large scaled model tank. Chiba(1993a, 1993b) examined the nonlinear vibration behaviour of cantilever cylindrical tanks with two polyester test cylinders, and concluded that the nonlinearity depends on the height of the tank as well as the height of the liquid content.

2.3.2 Analytical Studies

The analytical studies on the uplift mechanism have been concentrated on two areas: the relationship between the base plate uplift and the applied load, and the behaviour of the liquid-tank system when the base plate is partially uplifted from the support.

Base Plate Uplift Model

For unanchored liquid-tank systems, a part of the tank base plate will be lifted up in response to large resultant overturning moment until the overturning moment is balanced by the couple formed by the support force and the weight of the uplifted liquid content.

In Clough's model for base plate uplift(Clough 1977), an iterative procedure is employed to achieve the global equilibrium of the liquid-tank system without direct consideration of the deformation of the base plate. The contact region is assumed to have a circular shape with a radius r and an eccentricity e from the center of the base plate. A contact arc of the base plate rim with angle 2β provides the support force to balance the uplifted liquid. The distribution of the compressive force along the contact arc is assumed to be linearly varied from the maximum at the excitation axis to zero at the two ends. The relationship between r and β is given as

$$\tan(\beta) = \frac{r}{R - r}$$

By calculating the overturning moment as if the tank is anchored and equating this moment with the couple of the support force, C , and the weight of the uplifted liquid, w_L , the contact arc angle β and the contact circle radius r can be obtained from equilibrium. This model is shown in Figure 2.3.

To determine w_L , the maximum weight of the uplifted liquid content, Wozniak and Mitchell(1978) developed a simplified procedure to evaluate the uplift mechanism.

The model is shown in Figure 2.4. The uplift region in the base plate is described by an arc of angle $2(\pi - \beta)$ and width L , as shown in Figure 2.4(a). Since both the amount, u , and the extent, L , of the base plate uplift are assumed to be very small compared to the tank radius, the uplift region of the base plate is modeled as a series of beams with unit width and constant length L . In response to the earthquake loading, two plastic hinges, one at the junction of the base plate and the shell wall, and the other some distance inward, are assumed to develop when the base plate reaches its maximum bearing limit. A uniform load equal to the static liquid pressure is applied to the beams. The membrane force developed in the beam is neglected. The weight of the uplifted liquid is assumed to act uniformly downwards on the rim of the uplifted base plate. A corresponding compressive force exists along the contact arc of the base plate, as shown in Figure 2.4(b). The distribution of the compressive force is assumed to be linearly varied from the maximum at the excitation axis to zero at both ends of the contact arc. The contact angle β will decrease from π with an increasing overturning moment to a limit which is $1/8 \pi$. The extent of the uplift region, L , is limited to 7% of the tank radius.

In the analytical work conducted by Auli *et al.*(1985), the uplift region is modelled as one-dimensional beams and the restraining effect of the tank shell wall is modelled by two springs representing the bending and the extensional stiffness of the tank shell wall. It is concluded that the stiffness due to the membrane action is important in resisting the base plate uplift. Also noted from the results, the elasto-plastic material behaviour should be considered in the analysis instead of the pure elastic behaviour in the case of large base plate uplift.

Haroun *et al.*(1987) studied the nonlinear behaviour of the tank base plate uplift by using an energy approach. In this procedure, the base plate is modelled as a circular plate with a crescent-shaped uplifted region instead of one-dimensional-beam

models. The base plate is assumed to be subjected to a hydrostatic loading along the whole plate and uplifting force along the outer edge of the plate. It is concluded that the effect of the membrane action will considerably increase the stiffness of the tank base plate and reduce the amount of uplift.

Lau(1989) analyzed the nonlinear behaviour of the partially uplifted tank base plate with the Von Karman plate theory. The membrane action of the plate and the restraining effect of the tank shell wall are considered in the analysis. Based on the plate solution, a set of Ritz shape functions for the tank base plate are developed. This model was later modified by introducing a set of simplified Ritz shape functions in modelling the uplifted tank base plate(Lau and Zeng 1991). From the results of further studies(Lau and Zeng 1991,1995), it is concluded that the effect of membrane action and the restraining action of the tank shell wall contribute significantly to the resistance of the base plate uplift. The uplift behaviour of a tank with a rigid foundation is also found to be quite different from that with a flexible foundation.

Liquid-Tank System Behaviour Model

The analytical models to describe the behaviour of unanchored liquid-tank systems are categorized into static and dynamic models. Static models give important insight into the characteristics of the uplift mechanism, while dynamic models describe the behaviour of the liquid-tank system under seismic load. Generally, these models are very complex and have limitations on their applications.

Static Models

Peek and Jennings(1988) idealized the tank shell wall as supported by a circular foundation bed of Winkler nonlinear springs. The effect of base plate uplift is simulated by these nonlinear springs. By considering the resisting force at a point in the shell wall as a function of the uplift at that point only, the characteristic of these springs

are determined. The tank base plate is modelled with the Von Karman plate theory and the tank shell wall is analyzed linearly. A finite difference energy method is employed to analyze the response of the tank structure under static lateral load. The displacement of the tank shell wall is expressed by Fourier expansion.

In the model developed by Lau and Clough(1987), the static lateral load is achieved by tilting the support plane with the result that is subjected to the liquid-filled tank an effective lateral acceleration. The tank base plate is divided into the contact and the uplift region. The membrane action as well as the effect of bending are included in the modeling of the uplifted region of the tank base plate, whereas the the contact region of the base plate is analyzed as a plane-stress disc. The tank shell wall is modelled and analyzed by Flügge's thin shell theory. Lau and Zeng(1991) improved this model by introducing a set of simplified Ritz shape functions which greatly reduced the amount of calculation required in the analysis.

Dynamic Models

Barton and Parker(1987) modelled the liquid-tank system with a three-dimensional finite element method. In the model, the effect of the liquid motion is considered as added mass attached to the tank structure.

Ishida and Kobayashi(1988) developed a four-degree-of-freedom model in which the tank shell wall and the liquid content are modelled as a mass-spring system, the support foundation is modelled by elastic springs, and the partially uplifted base plate by a rotational spring.

An analytical procedure based on the Haroun-Housner model for anchored liquid-tank systems(Haroun and Housner 1981) was developed by Natsviavis(1988). In the procedure, the behaviour of the base plate uplift is modeled by a nonlinear rotational spring placed between the base plate and the foundation. The characteristic of the spring is obtained from static analysis. The tank shell wall is assumed to vibrate as

a cantilever beam which maintains its circular cross-section throughout the response during the earthquake. The liquid motion is assumed to be ideal, and is expressed by the Laplace's differential equation. At the base plate, $z = 0$, the boundary condition of the liquid motion is modified such that the vertical motion of the liquid particles is the same as that of the base plate. The generated hydrodynamic behaviour of liquid content is expressed in terms of the tank structure motion. An energy method is employed to derive the differential governing equation of the nonlinear behaviour of the liquid-tank-support system. In a later study, Yi and Natsviavis(1992) modified the previous model by discretizing the tank shell wall with ring-shaped finite elements.

Based on the simplified static model proposed by Lau and Zeng(1991), Zeng(1993) developed a time history analysis procedure to evaluate the nonlinear dynamic behaviour of unanchored liquid-tank systems. Similar assumptions on the liquid motion are made as in the Natsviavis's model(Natsviavis 1988). The tank structure is modelled with Ritz shape functions obtained from an iterative procedure. Further study carried out by Lau and Zeng(1992) using this procedure indicates that the uplift mechanism greatly affects not only the impulsive but also the convective mode of the liquid motion.

2.4 Design Guidelines

AWWA D100-84(American Water Works Association 1984) and API 650(American Petroleum Institute 1988) are the most followed design standards for the design and construction of thin wall metal cylindrical liquid storage tanks in North America. In these two standards, the hydrodynamic load and the effect of the flexible tank shell wall are determined by using the Haroun-Housner model(Haroun and Housner 1982). The effect of the tank base plate uplift is estimated with the Wozniak-Mitchell

model(Wozniak and Mitchell 1978). During the dynamic response, the tank shell wall is assumed to maintain its circular cross-section and behave like a flexible cantilever beam. For anchored liquid-tank systems, the maximum compressive stress, which is the main cause of the buckling of the tank shell wall, is evaluated by considering the tank shell wall as a cantilever beam. For unanchored liquid-tank systems, the maximum compressive stress is evaluated by assuming the compressive stress distributed linearly along the base rim from the maximum at the excitation axis to zero at the two ends of the contact arc.

In the New Zealand recommendations(Priestly *et al.* 1986), the dynamic response of liquid-tank systems is analyzed with the Haroun-Housner model(Haroun and Housner 1982), and the uplift mechanism is estimated with the Clough model(Clough 1977).

In the Austrian recommendations for the design of earthquake resistant liquid storage tanks(Scharf *et al.* 1989), the motion of the liquid content is analyzed with the Laplace's differential equation, and the dynamic response of the tank structure is analyzed with considering the effect of liquid motion as "added mass" of the tank structure(Rammerstorfer *et al.* 1988).

2.5 Summary

In previous studies, the liquid content is generally assumed to be ideal and the liquid motion is analyzed analytically by the Laplace's differential equation. The boundary conditions of the liquid motion are expressed in terms of the motion of the tank structure.

The tank structure is assumed to be rigid and fixed to the foundation in early studies. Then the flexibility of the tank shell wall and the coupling between the

liquid motion and the dynamic response of the tank shell wall are included in the analysis. The shell wall is usually assumed to maintain its circular shape throughout the response and vibrate as a cantilever beam. Finite element method, finite difference method or pre-defined Ritz mode shapes are employed to analyze the shell wall vibration. The effect of liquid motion is either considered as load acting on the vibrating shell wall, or as “added mass”. Many analytical models for anchored tanks have been proposed and these models generally give reliable predictions of the dynamic behaviour of anchored liquid-tank systems. In recent years, the effect of the uplift mechanism on the seismic response of unanchored liquid-tank systems has been recognized, and research effort has been focused on this problem. Most analytical models proposed for unanchored liquid-tank systems are either too simplistic involving many unjustifiable assumptions, as in the modelling of the nonlinear behaviour of unanchored liquid-tank systems by simple mass-spring systems, or too complicated to be of practical use for design purpose.

The current design codes for liquid-storage tanks are based on the analytical models developed for anchored liquid-tank systems, and the code provisions are proved to be not reliable for the design of unanchored liquid storage tanks. A reliable and simple analytical procedure for the design of unanchored liquid storage tanks is not yet available.

Figure 2.1: Liquid-Tank System

a) Impulsive Mode

b) Convective Mode

c) Simplified Model

Figure 2.2: Housner's Simplified Model

Figure 2.3: Clough's Model of Uplift Mechanism

a) Base plate during uplift

b) Assumed load distribution around the rim of base plate

c) Beam model of uplifted plate

Figure 2.4: Wozniak's Model of Uplift Mechanism

Chapter 3

ANALYTICAL MODEL

3.1 General

The analytical model used in the present study is briefly reviewed here. This model is based on the dynamic analysis procedure developed by Zeng (1993). A detailed discussion of this model is presented in the reference (Zeng 1993). In the reference, analytical results are correlated with shaking table test data in a detailed verification study of the model.

The liquid-tank system being analyzed in the model is shown in Figure 3.1. The tank has a radius R , height H and is rested on a rigid foundation. No anchorage is applied to the tank. The liquid content has a free top surface at the height H_l . A cylindrical coordinate system (r, θ, z) is defined with the origin at the center of the base plate, where z is the longitudinal axis and positive upward, r and θ are the radial and tangential coordinates respectively. The mid-plane displacements of the tank structure in radial, tangential and longitudinal direction are denoted by u , v and w respectively. The ground excitation is assumed to be aligned with the horizontal axis $\theta = 0$ and has a known acceleration history $\ddot{x}_g(t)$.

The formulation of the liquid motion is presented in Section 3.2. The liquid motion is analyzed by the Laplace's equation, and expressed in terms of the tank structure motion. The discretization and modelling of the tank structure are presented in Sec-

tion 3.3. The motion of the tank structure is described by Ritz shape functions based on the static analysis procedure developed by Lau and Zeng(1991, 1996). The uplift of the tank base plate and the out-of-round distortion of the shell wall are included in the Ritz shapes. The Flügge's thin shell theory and the Von Karman's plate theory are employed respectively to model the deformed shell wall and the uplifted base plate. The derivation of the governing equation of motion is presented in Section 3.4. The governing equation of motion of the liquid-tank system is derived by an energy method and a step-by-step time integration of the governing equation leads to a detailed time history of the dynamic response of the liquid-tank system. The analytical model is summarized at the end of the chapter.

3.2 Hydrodynamic Behaviour of Liquid Content

Similar to other analytical models, the following assumptions on the liquid motion are made to simplify the formulation

- The liquid is inviscid and incompressible,
- The flow is irrotational and has no source or sink,
- The sloshing is of only small amplitude.

With these assumptions, the liquid is ideal and the velocity potential function ϕ of the liquid flow must satisfy the Laplace's equation.

The liquid motion is categorized into the impulsive and convective mode. The velocity potential function of the liquid motion is expressed as follows

$$\phi(r, \theta, z, t) = \phi_i(r, \theta, z, t) + \phi_c(r, \theta, z, t) \quad (3.1)$$

where ϕ_i denotes the impulsive mode of the liquid motion, i.e., the motion in unison with the dynamic response of the tank structure, and ϕ_c denotes the convective mode.

which represents the sloshing of the liquid content. The boundary conditions that must be satisfied for this problem are

- At $z = H$, the original position of the liquid free surface:

$$\frac{\partial \phi_i}{\partial t} = 0 \quad \frac{\partial^2 \phi_c}{\partial t^2} + g \frac{\partial \phi_c}{\partial z} = -g \frac{\partial \phi_i}{\partial z} \quad (3.2)$$

- At $z = 0$, the liquid-structure interface at the tank base plate:

$$\frac{\partial \phi_i}{\partial z} = 0 \quad \frac{\partial \phi_c}{\partial z} = \dot{b}(r, \theta, t) \quad (3.3)$$

- At $r = R$, the liquid-structure interface at the cylindrical tank shell wall:

$$\frac{\partial \phi_i}{\partial r} = \dot{w}(\theta, z, t) \quad \frac{\partial \phi_c}{\partial r} = 0 \quad (3.4)$$

The complete expressions of the impulsive and convective modes of the liquid velocity potential function are given by the infinite series

$$\phi_i(r, \theta, z, t) = \sum_{n=0}^{\infty} \cos(n\theta) \sum_{i=1}^{\infty} T_{in}(t) I_n\left(\frac{\lambda_i r}{H}\right) \cos\left(\frac{\lambda_i z}{H}\right) \quad (3.5)$$

$$\phi_c(r, \theta, z, t) = \sum_{n=0}^{\infty} \cos(n\theta) \sum_{j=1}^{\infty} J_n\left(\frac{\zeta_{jn} r}{R}\right) \left[\bar{T}_{jn}(t) e^{\frac{\zeta_{jn} z}{R}} + \tilde{T}_{jn}(t) e^{-\frac{\zeta_{jn} z}{R}} \right] \quad (3.6)$$

where

$$\lambda_i = \frac{2i-1}{2} \pi \quad \text{for } i=1, 2, 3 \dots;$$

J_n and I_n are the n th order of Bessel and modified Bessel function respectively;

ζ_{jn} are the zeroes of the first derivatives of the n th order of Bessel function J_n ;

$T_{in}(t)$, $\bar{T}_{jn}(t)$ and $\tilde{T}_{jn}(t)$ are time varying functions in terms of the dynamic response of the tank structure.

By separating the variables, the time varying functions are derived as follows

$$T_{in}(t) = \frac{\kappa_n \int_0^{2\pi} \int_0^H \dot{w}(\theta, z, t) \cos n\theta \cos \frac{\lambda_i z}{H} d\theta dz}{\pi H R_{in}} \quad (3.7)$$

$$\begin{aligned} \bar{T}_{jn}(t) = & a_{jn} \sin \omega_{jn} t + b_{jn} \cos \omega_{jn} t \\ & - \frac{e^{-\frac{2\zeta_{jn}H}{R}}}{1 + e^{-\frac{2\zeta_{jn}H}{R}}} \frac{1}{\omega_{jn}} \int_0^t S''_{jn}(\tau) \sin \omega_{jn}(t - \tau) d\tau \\ & + g \frac{\zeta_{jn}}{R} \frac{e^{-\frac{2\zeta_{jn}H}{R}}}{1 + e^{-\frac{2\zeta_{jn}H}{R}}} \frac{1}{\omega_{jn}} \int_0^t S_{jn}(\tau) \sin \omega_{jn}(t - \tau) d\tau \\ & + \frac{2g \int_0^R \sum_{i=1}^{\infty} \lambda_i I_n(\frac{\lambda_i r}{H}) \sin \lambda_i r J_n(\frac{\zeta_{jn} r}{R}) dr \int_0^t T_{in}(\tau) \sin \omega_{jn}(t - \tau) d\tau}{\omega_{jn} R^2 H (1 - \frac{n^2}{\zeta_{jn}^2}) J_n^2(\zeta_{jn}) (1 + e^{-\frac{2\zeta_{jn}H}{R}})} e^{-\frac{\zeta_{jn}H}{R}} \end{aligned} \quad (3.8)$$

$$\tilde{T}_{jn}(t) = \bar{T}_{jn}(t) + S_{jn} \quad (3.9)$$

where the parameters in the equations are defined as follows

$$\begin{aligned} \kappa_n &= \begin{cases} 2 & \text{for } n = 1, 2, 3, \dots \\ 1 & \text{for } n = 0 \end{cases} \\ R_{in} &= \frac{\lambda_i}{H} I'_n(\frac{\lambda_i R}{H}) \\ \omega_{jn}^2 &= g \frac{\zeta_{jn}}{R} \tanh(\frac{\zeta_{jn} H}{R}) \\ S_{jn} &= - \frac{\kappa_n \int_0^{2\pi} \cos n\theta d\theta \int_0^R r \dot{b}(r, \theta, t) J_n(\frac{\zeta_{jn} r}{R}) dr}{\pi \zeta_{jn} R (1 - \frac{n^2}{\zeta_{jn}^2}) J_n^2(\zeta_{jn})} \end{aligned} \quad (3.10)$$

From the Bernoulli-Euler equation, the liquid pressure acting on the tank structure is given as follows

$$p(r, \theta, z, t) = p_d + p_s = -\rho \frac{\partial \phi}{\partial t} + \rho g(H - z) \quad (3.11)$$

where

$$\begin{aligned} p_d &= -\rho \frac{\partial \phi}{\partial t} && \text{represents the hydrodynamic pressure;} \\ p_s &= \rho g(H - z) && \text{represents the hydrostatic pressure.} \end{aligned}$$

3.3 Ritz Formulation of Tank Structure

In the analytical model, the tank structure is divided into substructure elements of the base plate, the shell wall, the roof, and the ring girders. Each element is described

by an infinite series of Ritz shape functions. The material of the tank structure is assumed to remain linear elastic throughout the entire duration of the earthquake.

3.3.1 Tank Base Plate

During an earthquake, part of the base plate may be lifted up in response to the resultant overturning moment. Results from static tilt tests show that the contact region of the base plate is approximately of a circular shape, and in addition to the separation on the uplift side, a narrow strip of the base plate also separates from the foundation on the contact side, as shown in Figure 3.2, because of the rotation of the shell wall in response to the liquid pressure.

To model the uplift behaviour, the base plate is further divided into a circular contact region and a crescent-shaped uplifted region. The resultant overturning moment is resisted by the couple formed by the weight of the uplifted liquid content and the support force in the contact region. The resultant lateral force is resisted by the friction between the base plate and the ground support. In the formulation, the friction is assumed to be uniformly distributed over the contact region.

Uplifted Region

In the modelling of the uplifted region of the base plate, the nonlinear Von Karman plate theory is employed to account for the coupling effect between the membrane action and the flexural deformation of the uplifted region because the base plate deflection is relatively large when compared to its thickness.

Due to symmetry, only half of the uplifted region of the base plate is considered in the formulation. The crescent-shaped uplifted region is described by a curvilinear natural coordinate system (η, ξ) , as shown in Figure 3.3, where η is approximately the tangential direction and ξ the radial direction. The relationships between the

curvilinear natural coordinate system (η, ξ) and the original polar coordinate system (r, θ) are

$$\begin{aligned}\theta &= \frac{\pi}{2}(\eta + 1) \\ r &= \frac{1}{2}[(R - r_c)\xi + R + r_c] \\ r_c &= b_c \sin \frac{\pi\eta}{2} + \sqrt{b^2 - b_c^2 \cos^2 \frac{\pi\eta}{2}}\end{aligned}\tag{3.12}$$

Separating the variables, the Ritz shape functions defining the displacement of the uplifted base plate have the following general form

$$N(\eta, \xi) = N_\eta(\eta)N_\xi(\xi)\tag{3.13}$$

where $N_\eta(\eta)$ and $N_\xi(\xi)$ are respectively shape functions for the natural coordinates η and ξ only.

Because no concentrated load is applied on the base plate, the deformation and the stress developed correspondingly is assumed to vary smoothly in the uplifted region.

$N_\eta(\eta)$ shape functions

The shape function $N_\eta(\eta)$ describes the displacement of the uplifted region in the tangential direction η . Due to symmetry, it must satisfy the following requirement

$$\frac{dN_\eta(\eta)}{d\eta} = 0 \quad \text{for } \eta = \pm 1, \text{ corresponding to } \theta = 0 \text{ or } \pi$$

Also, it is reasonable to assume that $N_\eta(\eta)$ is monotonic in the region $[-1, 1]$. Typically, the displacement along a curve $\xi = \text{const}$ has the variation shown in Figure 3.4(a).

Instead of using an infinite series of ordinary cosine functions, only two terms, a constant and the modified cosine term shown in Figure 3.4(b), are employed to simulate the displacement in the η direction. The derived $N_\eta(\eta)$ shape functions are

$$N_\eta(\eta) = \left\{ 1, \cos\left[g(x_0, \eta)\frac{\pi}{2}\right] \right\}\tag{3.14}$$

where

$$g(x_0, \eta) = \frac{x_0}{1 - x_0^2}(1 - \eta^2) - \eta + 1$$

The offset parameter x_0 is used to modify the cosine term and is considered as a degree-of-freedom in the analysis.

$N_\xi(\xi)$ shape functions

The shape function $N_\xi(\xi)$ describes the displacement of the uplifted region in the radial direction ξ . It is expressed by an infinite series of deflection shape functions, ψ_i , whose expressions are different for the vertical uplift w and the in-plane displacements u and v .

For the vertical uplift w , the deflection shape functions, ψ_i , are given as follows

$$\begin{aligned}\psi_1 &= (2 - \xi)(\xi + 1)^2/4 \\ \psi_2 &= (1 - \xi)(\xi + 1)^2/4 \\ \psi_i &= (1 - \xi^2)^2 \xi^{i-3} \quad i = 3, 4, \dots\end{aligned}\tag{3.15}$$

The shape functions ψ_1 and ψ_2 correspond to a fixed-end beam with unit displacement and rotation respectively at the end along the base rim. The higher order shape functions correspond to the deflection of a fixed-end beam under distributed loadings of increasing order. This is shown in Figure 3.5.

For the in-plane displacement u in the radial direction and v in the tangential direction, the deflection shape functions, ψ_i , are expressed as follows

$$\begin{aligned}\psi_1 &= (1 - \xi)/2 \\ \psi_2 &= (1 + \xi)/2 \\ \psi_i &= (1 - \xi^2) \xi^{i-3} \quad i = 3, 4, \dots\end{aligned}\tag{3.16}$$

The shape functions ψ_1 and ψ_2 correspond to a fixed-end bar with unit displacement at one of the two ends respectively. The higher order shape functions correspond to

the deflection of a fixed-end bar subjected to distributed axial loadings of increasing order. This is shown in Figure 3.6.

Combining the above shape functions in the two natural coordinates, the Ritz shape functions defining the displacement of the uplifted region are given as the sum of the following terms

$$\begin{aligned} N(\eta, \xi) &= N_\eta(\eta) N_\xi(\xi) \\ &= \{\psi_1(\xi), \psi_2(\xi), \psi_3(\xi), \dots \\ &\quad \cos[g(x_0, \eta)\frac{\pi}{2}]\psi_1(\xi), \cos[g(x_0, \eta)]\psi_2(\xi), \cos[g(x_0, \eta)\frac{\pi}{2}]\psi_3(\xi), \dots\} \end{aligned} \quad (3.17)$$

Using the derived Ritz shape functions as discretization functions and writing in matrix form, the displacements of the uplifted region of the base plate can be expressed as follows

$$U(\eta, \xi) = \begin{Bmatrix} u(\eta, \xi) \\ v(\eta, \xi) \\ w(\eta, \xi) \end{Bmatrix} = \mathbf{N}(\eta, \xi) \mathbf{a} \quad (3.18)$$

where $\mathbf{N}(\eta, \xi)$ is the Ritz shape matrix and $\mathbf{a}$ is the Ritz parameter vector. All the Ritz parameters and the offset parameters are degrees-of-freedom for the uplifted region.

Contact Region

The contact region of the base plate is assumed to be of a circular shape, and is described by a local polar coordinate system (s, β) , as shown in Figure 3.7. The relationships between the local polar coordinate system (s, β) for the contact region and the global polar coordinate system (r, θ) with the origin at the center of the base plate are given as follows

$$\begin{aligned} r \cos \theta &= b_c + s \cos \beta \\ r \sin \theta &= s \sin \beta \end{aligned} \quad (3.19)$$

The contact region is modelled as a two dimensional plane stress disc. As mentioned earlier, the resultant lateral force of the liquid pressure acting on the tank structure is assumed to be resisted by a uniformly distributed friction between the tank base plate and the contact region with a magnitude q . The displacement caused by the friction is taken as the particular solution of the deformation in the contact region. It is determined as follows (Timoshenko 1972)

$$\begin{aligned} u_{sq} &= -\frac{q(1-\mu^2)s^2}{8Et}(3\cos\beta + \cos 3\beta) \\ u_{\beta q} &= \frac{q(1-\mu^2)s^2}{8Et}(\sin\beta + \sin 3\beta) \end{aligned} \quad (3.20)$$

The discrepancies or violations in the continuity of the particular solution at the contact-uplift interface of the base plate are corrected by the homogeneous solution given as follows

$$\begin{aligned} u_{sH} &= b_{01}s + [b_{11}(1-3\mu)s^2 + b_{12}]\cos\beta \\ &\quad + \sum_{n=2}^{\infty} \{b_{n1}s^{n-1} + [-n+2-\mu(n+2)]b_{n2}s^{n+1}\}\cos n\beta \\ u_{\beta H} &= [b_{11}(5+\mu)s^2 - b_{12}]\sin\beta \\ &\quad + \sum_{n=2}^{\infty} \{-b_{n1}s^{n-1} + (n+4+\mu n)b_{n2}s^{n+1}\}\sin n\beta \end{aligned} \quad (3.21)$$

The displacement of the contact region of the tank base plate is then the superposition of the displacement caused by the distributed friction and the homogeneous displacement, and can be expressed as follows

$$\begin{Bmatrix} u_s \\ u_\beta \end{Bmatrix} = \begin{Bmatrix} u_{sq} \\ u_{\beta q} \end{Bmatrix} + \begin{Bmatrix} u_{sH} \\ u_{\beta H} \end{Bmatrix} = \mathbf{N}(s, \beta) \mathbf{b} \quad (3.22)$$

where $\mathbf{N}(s, \beta)$ is the Ritz shape matrix, and $\mathbf{b}$ is the Ritz parameter vector for the contact region which are determined from the displacement continuity conditions at the contact-uplift interface.

3.3.2 Tank Shell Wall

In addition to vibrating like a cantilever beam and being deformed axisymmetrically under the hydrostatic pressure as assumed in previous analytical models, the tank shell wall also undergoes out-of-round distortion in response to the seismic load, as well as significant out-of-plane warping due to the base plate uplift, especially near the bottom. Instead of just considering pure membrane action, the effect of the flexural response should also be included in modelling the behaviour of the tank shell wall. For this reason, the Flügge's linear shell theory (Flügge 1962) is employed in the analysis. The tank shell wall is assumed to have a uniform thickness t_s . For tanks with shell plate thickness varying along the height of the tank, the average thickness is used. The liquid pressure applied on the tank shell wall is normal to the shell surface because the liquid is assumed to be inviscid.

Fixing the two ends of the shell, the displacement of the tank shell wall under the distributed liquid pressure, p_r , is given as follows

$$\begin{aligned} u_q(z, \theta) &= -\frac{1}{R} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\mu \gamma_n^2 - \frac{m^2}{R^2} \right) \gamma_n F_{mn} \cos m\theta \cos \gamma_n z \\ v_q(z, \theta) &= \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left((2 + \mu) \gamma_n^2 + \frac{m^2}{R^2} \right) m F_{mn} \sin m\theta \sin \gamma_n z \\ w_q(z, \theta) &= R \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\gamma_n^2 + \frac{m^2}{R^2} \right)^2 F_{mn} \cos m\theta \sin \gamma_n z \end{aligned} \quad (3.23)$$

where the parameters in the equations are defined as follows

$$\begin{aligned} F_{mn} &= \frac{p_{mn}}{\gamma_n^8 + \frac{4m^2}{R^2} \gamma_n^6 + \left(\frac{6m^4}{R^4} + \frac{Et_s}{R^2 D} \right) \gamma_n^4 + \frac{4m^6}{R^6} \gamma_n^2 + \frac{m^8}{R^8}} \\ p_r &= \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} R D p_{mn} \cos m\theta \sin \frac{n\pi z}{H} \\ \gamma_n &= \frac{n\pi}{H} \quad D = \frac{Et_s^3}{12(1 - \mu^2)} \end{aligned}$$

Correspondingly, the homogeneous solution of the shell wall displacement in order to satisfy the free boundary condition at the top rim or the continuity requirement with

the roof, and the continuity requirement at the bottom with the uplifted base plate. can be determined as follows

$$\begin{aligned} u_H(z, \theta) &= \sum_{m=0} \mathbf{A} e^{\lambda \frac{z}{R}} \cos(m\theta) \\ v_H(z, \theta) &= \sum_{m=1} \mathbf{B} e^{\lambda \frac{z}{R}} \sin(m\theta) \\ w_H(z, \theta) &= \sum_{m=0} \mathbf{C} e^{\lambda \frac{z}{R}} \cos(m\theta) \end{aligned} \quad (3.24)$$

where λ is defined by the solution of the equation

$$\lambda^4 + \frac{k}{1 - \mu^2} (\lambda^2 - m^2)^2 = 0$$

and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are coefficient matrices.

The displacement of the shell wall is given by the combination of the displacement caused by the distributed liquid pressure and the homogeneous displacement.

Accordingly, it can be expressed as follows

$$\begin{Bmatrix} u(\theta, z) \\ v(\theta, z) \\ w(\theta, z) \end{Bmatrix} = \begin{Bmatrix} u_q(\theta, z) \\ v_q(\theta, z) \\ w_q(\theta, z) \end{Bmatrix} + \begin{Bmatrix} u_H(\theta, z) \\ v_H(\theta, z) \\ w_H(\theta, z) \end{Bmatrix} = \mathbf{N}(\theta, z) \mathbf{c} \quad (3.25)$$

where $\mathbf{N}(\theta, z)$ is the Ritz shape matrix, and $\mathbf{c}$ is the Ritz parameter vector for the shell wall.

3.3.3 Other Substructure Elements

Tank Roof

The tank roof, if it is fixed, is modelled as a plane stress disc. The Ritz shape functions for the roof are the same as those for the contact region of the tank base plate. except that the corresponding body force is taken as zero.

Ring Girders

The ring girders of the shell wall are considered to behave as a beam loaded in the in-plane direction, and with no resistance in the out-of-plane direction.

3.3.4 Assembling the Ritz Shape Matrix

Using the derived Ritz shapes as discretization shapes for each substructure element and applying continuity of the displacement at the boundary between adjoining elements, the displacements of the tank structure $\mathbf{U}$, after assembling, can be expressed as follows

$$\mathbf{U} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{N} \mathbf{r} \quad (3.26)$$

where $\mathbf{N}$ is the Ritz shape matrix and $\mathbf{r}$ is the Ritz parameter vector for the tank structure.

3.4 Governing Equation of Motion and Time Step Integration

Within a short time interval, the displacement of the tank structure can be assumed to vary linearly and can be expressed in the following form

$$\mathbf{U}(t) = \Psi(t) v(t) \quad (3.27)$$

where $\Psi(t)$ is the generalized Ritz shape function for the whole tank structure which is assumed to remain unchanged within the short time interval, and $v(t)$ is the corresponding generalized coordinate at that time. Correspondingly, the strain and stress of the tank structure are defined as

$$\epsilon = \partial \mathbf{U} = v \partial \Psi \quad \sigma = D \epsilon = v D \partial \Psi \quad (3.28)$$

where D is the elastic matrix.

With the tank displacement defined as above, the potential energy V and kinetic energy T of the tank structure can be expressed as follows

$$V = \int (\partial \Psi)^T D (\partial \Psi) ds v^2$$

$$\dot{T} = \int (\Psi \dot{v} + \dot{x}_g)^T m (\Psi \dot{v} + \dot{x}_g) ds$$

where m defines the mass distribution of the tank structure, and $\dot{x}_g$ is the velocity of ground motion.

The corresponding generalized liquid pressure is derived as follows

$$Q = \mathbf{F} \Psi \ddot{v}$$

where $\mathbf{F}$ is the coefficient matrix determined from the equations of liquid motion given in Section 3.2.

By applying the Lagrange equation of motion

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{v}} \right) - \frac{\partial T}{\partial v} + \frac{\partial V}{\partial v} = Q$$

the governing equation of motion of the liquid-tank system in terms of the generalized Ritz shape can be expressed in the familiar form as follows

$$M^* \ddot{v} + C^* \dot{v} + K^* v = P^* \quad (3.29)$$

where M^* , C^* , K^* and P^* denote the effective mass, damping, stiffness and load respectively.

The governing equation of motion can be rewritten in the incremental form for the interval from time t_n to time t_{n+1} as follows

$$M_{n+1}^* \Delta \ddot{v} + C_{n+1}^* \Delta \dot{v} + K_{n+1}^* \Delta v = \Delta P^* \quad (3.30)$$

where

$$\Delta \ddot{v} = \ddot{v}_{n+1} - \ddot{v}_n; \quad \Delta \dot{v} = \dot{v}_{n+1} - \dot{v}_n; \quad \Delta v = v_{n+1} - v_n; \quad \Delta P^* = P_{n+1}^* - P_n^*$$

The generalized Ritz shape $\Psi(t)$ varies in time because the dynamic behaviour of the liquid-tank system is highly nonlinear, and the hydrodynamic pressure and the deformation of the tank structure are closely coupled. To obtain accurate results, an

iterative procedure is necessary to determine the proper generalized Ritz shape $\Psi(t)$ for the discretization of the tank structure at time t .

The iterative procedure is summarized as follows

1. At the end of the n th time step t_n , obtain the generalized Ritz shape Ψ_n and the generalized Ritz term v_n ;
2. Take the generalized Ritz shape Ψ_n and the generalized Ritz term v_n as the initial guess for time step t_{n+1} ;
3. Solve Equation 3.29 by the time step integration with the initial guess and obtain the refined generalized coordinate v_{n+1} at the end of time step t_{n+1} ;
4. Determine the dynamic load at the end of t_{n+1} with the initial generalized Ritz shape Ψ_n and the refined generalized coordinate v_{n+1} ;
5. Derive the refined generalized Ritz shape Ψ_{n+1} at the end of time step t_{n+1} with the updated dynamic load;
6. Check convergence. If the difference between the refined generalized Ritz shape Ψ_{n+1} and the initial Ritz shape Ψ_n is within tolerance, go to next time step; otherwise, take the refined generalized Ritz shape Ψ_{n+1} and the refined generalized coordinate v_{n+1} as the new guess, go back to step 3.

With the generalized Ritz shape function $\Psi(t)$, and the coordinate $v(t)$, obtained from the time step integration, the detailed time history of the dynamic behaviour of the liquid-tank system is determined.

3.5 Summary

In the analytical model, the liquid-tank system is described by a cylindrical coordinate system (r, θ, z) with the origin at the center of the base plate. The ground excitation is assumed to be aligned with the horizontal axis $\theta = 0$.

The liquid content is assumed to be ideal, and the liquid motion is categorized into the impulsive and convective mode in the analysis. Analyzed with the Laplace's differential equation, the hydrodynamic behaviour of the liquid content is expressed in terms of the dynamic response of the tank structure.

The tank structure is divided into substructure elements of the uplifted region and the contact region of the base plate, the shell wall and others. Each element is described by a set of Ritz shape functions derived from static analysis, and the Ritz shapes for the tank structure are assembled together by applying the continuity of the displacement at the boundary between the adjoining elements. The material of the tank structure is assumed to remain linear elastic in the analysis.

The response of the tank structure is assumed to be linear during a short time interval and can be expressed by a generalized Ritz shape function and a generalized Ritz coordinate. The governing equation of motion of the liquid-tank system in terms of the generalized Ritz shape function and coordinate is derived by applying the Language's equation. An iterative procedure is employed to achieve the appropriate generalized Ritz shape function and coordinate at a specific time. The time history of the seismic response of the liquid-tank system is obtained from the step-by-step time integration procedure.

Figure 3.1: Coordinate System of Liquid-Tank System

a) plane of uplifted base plate

b) base plate uplift
along the excitation axis

Figure 3.2: Tank Base Plate With Uplift

a) base plate in normal coordinate

b) mapped uplift area

Figure 3.3: Coordinate Mapping of Tank Base Plate

a) typical uplift of base plate in the tangential direction

b) cosine and refined cosine function

Figure 3.4: Ritz Shape Function $N(\eta)$ of Uplifted Region

Figure 3.5: Ritz Shape Function $N(\xi)$ of Uplifted Region (for vertical disp.)

Figure 3.6: Ritz Shape Function $N(\xi)$ of Uplifted Region (for inplane disp.)

Figure 3.7: Coordinate System of the Contact Region

Chapter 4

SEISMIC BEHAVIOUR OF UNANCHORED LIQUID-TANK SYSTEMS

4.1 General

The results of a detailed time history analysis on the seismic behaviour of unanchored liquid-tank systems, including the dynamic response of the tank structure, the liquid motion and the stress developed in the tank structure, are presented in this chapter. The objective of this analysis is to achieve a better understanding on the behaviour of unanchored liquid-tank systems when subjected to seismic loads.

The configuration of the liquid-tank system to be analyzed is shown in Figure 4.1. The tank has an open top. It is unanchored and rested on a rigid foundation. The material of the tank structure is steel with a Young's modulus of elasticity $2 \times 10^5 MPa (29,000 ksi)$. The height of the tank is $12.344m (40.5 ft)$ and the diameter is $30.48m (100 ft)$. The thickness of the shell wall varies along the height from $25.4mm (1 in)$ at the bottom, to $15.9mm (5/8 in)$ in the middle and to $6.4mm (1/4 in)$ at the top. The base plate has a uniform thickness of $25.4mm (1 in)$. The liquid content is water. Three different values of the liquid height are considered in the analysis, which are $6.096m (20 ft)$, $9.144m (30 ft)$ and $12.192m (40 ft)$, corresponding to 50%, 75% and 100% of the tank capacity respectively.

The ground excitations used in the analysis are the 1940 El Centro earthquake (Imperial Valley, N-S component, $a_{max} = 0.348g$ at $2.14sec$) and the 1994 Northridge earthquake (Newhall, LA county, N-S component, $a_{max} = 0.583g$ at $5.36sec$). The time histories of the ground acceleration of the input excitations are shown in Figure 4.2. The corresponding response spectra are shown in Figure 4.3.

In the remaining part of this chapter, the resultant lateral force and the resultant overturning moment of the liquid-tank system are described in Section 4.2. The dynamic response of the tank structure, including the base plate uplift, the shell wall radial displacement and the generalized vibration period of the liquid-tank system, is presented in Section 4.3. The liquid motion and the effect of the coupling between the dynamic response of the tank structure are discussed in Section 4.4. The stresses developed in the tank structure in response to the seismic load and the failure mechanism of the tank structure are analyzed in Section 4.5. Finally, a summary of the seismic behaviour is presented at the end of this chapter.

4.2 Resultant Lateral Force And Resultant Overturning Moment

The time histories of the resultant lateral force and the resultant overturning moment for the liquid-tank system subjected to the El Centro and the Northridge earthquakes are shown in Figures 4.4 to 4.5 and Figures 4.6 to 4.7 respectively. It is observed that the resultant load has two distinct types of behaviour. The first type is observed approximately during the interval from the start of the response to about 5 second in the cases of the liquid-tank system subjected to the El Centro earthquake, as shown in Figures 4.4 and 4.5, and to about 4.5 second in the cases subjected to the Northridge earthquake, as shown in Figures 4.6 and 4.7. During the time interval, the behaviour

of the resultant load appears to be dominated by impulsive modes. The second type of behaviour is observed in the remaining time of the dynamic response. As can be noticed from the plots shown in the figures, the second part is dominated by long period oscillations. Because the resultant loads are calculated from the liquid pressure acting on the shell wall which reflects the overall effect of the liquid motion developed in the liquid-tank system, the observed behaviour of the resultant loads implies that the liquid motion is dominated by the impulsive component in the first part and by the convective component in the second part of the response. From the time history of the resultant load, it is observed that the vibration characteristics of the resultant lateral force, shown in Figures 4.4 and 4.6, and that of the resultant overturning moment, shown in Figures 4.5 and 4.7, are very similar to each other. A good approximation on the resultant overturning moment can be obtained from the product of the resultant lateral force and an effective height of application. This suggests that in the general case, the liquid motion of the liquid-tank system is dominated by the fundamental mode.

Numerical results on the maximum resultant load and the corresponding ground acceleration are listed in Table 4.1. As can be observed from the results presented in the table, the maximum resultant load does not occur at the time of the maximum ground acceleration. Comparing the time history of the input ground excitation with that of the resultant load, it is found that there are three distinct type of behaviour of the maximum resultant load

1. The maximum resultant load occurs at the time when the ground excitation is intense. An example is the liquid-tank system with $H_l = 12.192m$ subjected to the Northridge earthquake, the tank is overturned by the resultant overturning moment at $4.11sec$ with the ground acceleration $a_g > 0.5g$.
2. The maximum resultant load occurs at the time when the superposition of the

effects of the impulsive and the convective liquid motions is at the maximum. An example is the liquid-tank system with $H_l = 9.144m$ subjected to the El Centro earthquake, the maximum resultant load occurs at $9.73sec$ when the peak effect of the impulsive liquid motion occurs at the same time of the peak effect of the convective liquid motion.

3. The maximum resultant load occurs at a time when the ground excitation is moderate but the coupling effect between the liquid motion and the dynamic response of the tank structure is intense. An example is the liquid-tank system with $H_l = 12.192m$ subjected to the El Centro earthquake, the system survives the maximum ground acceleration but collapses at $3.52sec$ with $a_g = 0.11g$. In this case, the effect of the high order modes of the impulsive liquid motion is shown to have significant contribution to the overall response of the liquid-tank system.

4.3 Dynamic Response of Tank Structure

The dynamic uplift behaviour of the base plate, the radial displacement of the shell wall and the generalized vibration period of the liquid-tank system are discussed in this section. When subjected to the hydrodynamic load caused by the liquid motion, the base plate uplift and the shell wall radial displacement are the principal responses of the tank structure. The generalized vibration period is an effective parameter that gives an indication of the nonlinear transient dynamic behaviour of the liquid-tank system. It is greatly influenced by the deformation of the tank structure due to the seismic load and the uplift mechanism.

4.3.1 Base Plate Uplift

The time histories of the vertical uplift at the rim of the base plate subjected to the El Centro and the Northridge earthquakes are shown in Figures 4.8 and 4.9 respectively. In the cases of the liquid-tank system with $H_l = 6.096m$, the tank base plate experiences no uplift when the system is subjected to both the El Centro and the Northridge earthquakes. In the cases with $H_l = 9.144m$, the base plate uplifts once when subjected to the El Centro earthquake, and several times when subjected to the Northridge earthquake. In the cases with $H_l = 12.192m$, the tank collapse at $3.52sec$ subjected to the El Centro earthquake, and at $4.11sec$ subjected to the Northridge earthquake. The results indicate that at the time of collapse, the liquid-tank system experiences significant base plate uplift. Comparing the time histories of the base plate uplift shown in Figures 4.8 and 4.9 with the time histories of the resultant overturning moment shown in Figures 4.5 and 4.7, it is observed that the base plate uplift occurs only when the resultant overturning moment exceeds a certain value, and large base plate uplift occurs at the time of large resultant overturning moment.

The relationship of the amount of base plate uplift with the resultant overturning moment for the liquid-tank system with $H_l = 9.144m$ is plotted in Figure 4.10. The resultant overturning moment is normalized by the product of the total weight of the liquid-tank system G and the liquid height H_l . As shown in Figure 4.10, for the liquid-tank system with $H_l = 9.144m$, the critical value of resultant overturning moment for base plate uplift to occur is observed to be about $0.07 GH_l$. It is also noticed that a clear trend of increase for the amount of base plate uplift with increasing resultant overturning moment is exhibited in the results. In general, a time lag or delay exists between the large overturning moment and the large base plate uplift caused by the moment. Under seismic loads, the resultant overturning moment may have its magnitude and direction significantly changed in a short time interval. The

time for the base plate to lift up in response to the overturning moment is usually insufficient. Therefore, even though the resultant overturning moment is large, the amount of base plate uplift can still be very small, as demonstrated by the case of resultant overturning moment with a magnitude about $0.16 GH_l$ and extent of the uplift region about $1.4m$, the amount of the vertical uplift at the rim of tank base plate is insignificant.

The maximum base plate uplift and the extent of the uplifted region are presented in Table 4.2. From the results presented in the table, it is observed that the extent of the uplifted region can be much larger than 7% of the tank radius R , which is the maximum limit on the extent of the uplifted region specified in the Wozniak and Mitchell model of base plate uplift (Wozniak and Mitchell 1978). For example, the extent of the uplifted region at 5.33sec in the case of the liquid-tank system with $H_l = 9.144m$ subjected to the Northridge earthquake is $2.009m$ as given in Table 4.2, which is about 15% of the tank radius.

4.3.2 Shell Wall Radial displacement

The time histories of the radial displacement at the mid-height level and at the top of the tank shell wall at $\theta = 0$ on the excitation axis are shown in Figures 4.11 to 4.16. The maximum shell wall radial displacement and the height at which it occurs are also presented in Table 4.2.

Because of the effect of the hydrostatic load, the shell wall has different initial radial displacement in different regions. Away from the liquid surface, the shell wall extends in the hoop direction and bulges outward due to the hydrostatic pressure, as can be observed from the results shown in Figures 4.12 and 4.15 for liquid-tank system with $H_l = 9.144m$, and in Figures 4.13 and 4.16 with $H_l = 12.192m$, the initial radial displacement in the mid-height of the shell wall is positive. To be compatible with

the deformation of the shell wall caused by the liquid pressure, the shell wall contacts in the hoop direction near the liquid free surface, as can be observed from the results shown in Figures 4.11 and 4.14 for liquid-tank system with $H_l = 6.096m$ at the mid-height level, and in Figures 4.13 and 4.16 for liquid-tank system with $H_l = 12.192m$ at the top of the shell wall, the initial radial displacement is in the inward direction. Far above the liquid surface, as Figures 4.11 and 4.14 for liquid-tank system with $H_l = 6.096m$, and in Figures 4.12 and 4.15 for system with $H_l = 9.144m$, at the top of the shell wall, the initial radial displacement is close to zero.

During an earthquake, the shell wall vibrates in the lateral direction. This vibration is closely coupled with the effect of the liquid motion. The hydrodynamic loading induced by the liquid motion is usually much greater than the inertia effect of the vibrating shell wall. As the result, the radial displacement of the shell wall is generally dominated by the hydrodynamic effect. The radial displacement at the mid-height of the shell wall is observed to be typically more intense than that at the top because the hydrodynamic effect is more concentrated on the lower part of the tank shell wall rather than on the upper part. It is also observed that significant base plate uplift will greatly affect the shell wall radial displacement, as in the case of the liquid-tank system with $H_l = 9.144m$ subjected to the El Centro earthquake, at 9.73sec shown in Figure 4.12, and also at 4.52, 5.33, 8.60 and 9.46sec subjected to the Northridge earthquake, shown in Figure 4.15, the time of significant base plate uplift, the inward radial displacement at the top of the tank shell wall is larger than that at the mid-height level.

Profiles of the shell wall radial displacement along the height at the time of maximum radial displacement are shown in Figure 4.17 for the cases of the liquid-tank system subjected to the El Centro earthquake and in Figure 4.18 subjected to the Northridge earthquake. For the part of the tank structure which experiences no base

plate uplift, the maximum shell wall radial displacement is outward from the effect of the outward liquid pressure and occurs at the level of about 1/3 the liquid height with the magnitude less than 10mm. For the uplifting part of the tank structure, the maximum shell wall radial displacement is in the inward direction at the top of the shell wall. The large inward radial displacement of the shell wall is caused by the inward rotation of the base-wall joint on the uplift side due to the base plate uplift. As can be observed from Figures 4.16 and 4.17, the inward radial displacement of the shell wall resulting from the base plate uplift is much larger than that associated with the shell wall vibration and liquid pressure. Out-of-round distortion of the shell wall develops because of the significant deformation in the shell wall caused by the hydrodynamic loading effect and the uplift of the base plate, as illustrated by the deformation at the top of the shell wall around 5.33sec in the case of the liquid-tank system with $H_l = 9.144m$ subjected to the Northridge earthquake, as shown in Figure 4.19.

4.3.3 Generalized Vibration Period

In previous studies (Yang 1976, Veletsos and Yang 1976,1977, Haroun and Housner 1981,1982), the dynamic response of anchored liquid-tank systems is analyzed by discretizing the dynamic behaviour of the liquid-tank system with vibration modes. The fundamental vibration period of the linearly behaved anchored liquid-tank system is derived as follows by Veletsos (1984)

$$T = \frac{2\pi}{\frac{C_l}{H_l} \sqrt{\frac{E}{\rho}}} \quad (4.1)$$

where E and ρ are respectively the Young's modulus of elasticity and the density of the tank structure material, H_l is the liquid height, and C_l is a dimensionless parameter determined by the ratio of the liquid height to the tank radius H_l/R and

the ratio of the shell wall thickness to the tank radius t_s/R . Using Equation 4.1 and assuming the tank is anchored, the fundamental vibration periods of the unanchored liquid-tank systems considered in the time history analysis with $H_l = 6.096m, 9.144m$ and $12.192m$ are estimated to be $0.092sec, 0.126sec$ and $0.162sec$, respectively.

Because of the nonlinear uplift mechanism, the concept of “vibration mode” is no longer applicable to the dynamic behaviour of unanchored liquid-tank systems. The concept of the generalized vibration period developed by Zeng (1993) is utilized here to evaluate the transient dynamic behaviour of the unanchored liquid-tank systems. Based on the formulation of liquid motion expressed in Section 3.4, the generalized vibration period of the liquid-tank system at time t is derived as follows

$$T_{(t)} = 2\pi \sqrt{\frac{M_{(t)}}{K_{(t)}}} \quad (4.2)$$

where $M_{(t)}$ and $K_{(t)}$ are the generalized mass and stiffness of the liquid-tank system at time t .

The time histories of the generalized vibration period for the liquid-tank system considered in the analysis are shown in Figures 4.20 and 4.21. As shown in the figures, the lower bound of the generalized vibration periods of the unanchored liquid-tank systems are equal to $0.087sec$ for the liquid-tank system with $H_l = 6.096m$, $0.121sec$ for $H_l = 9.144mm$ and $0.153sec$ for $H_l = 12.192m$, which are similar to the fundamental vibration period for anchored liquid-tank systems. It is observed that due to the contribution from the high order modes in the dynamic response, the generalized vibration period is slightly smaller than the estimated fundamental period given by Equation 4.1 which considers only the fundamental vibration mode. When significant deformation resulted from uplift mechanism occurs in the tank structure, the liquid-tank system becomes more flexible, and thus the generalized vibration period becomes longer, as shown in Figures 4.20 and 4.21. In some cases, the generalized vibration period exceeds $1.0sec$.

4.4 Dynamic Behaviour of Liquid Content

The dynamic behaviour of the liquid content are discussed in this section. As in previous research, the behaviour of the liquid motion is divided into the impulsive component and the convective component. The intensity and distribution of the liquid motion is described by the hydrodynamic pressure which is calculated from the derivatives of the velocity potential function of the liquid content. The coupling effect between the dynamic response of the tank structure and the hydrodynamic pressure generated from the vibrating liquid is also discussed.

The time histories of the impulsive and the convective hydrodynamic pressure acting on the section $\theta = 0$ of the tank shell wall are presented in Figures 4.22 to 4.27. The impulsive pressure at the bottom and at the mid-height of the liquid content are presented, whereas for the convective pressure, the locations being presented include also the top surface of the liquid content. The maximum values of the hydrodynamic pressure are listed in Table 4.4. Profiles of the hydrodynamic pressure acting on the shell wall along the height on the contact side and around the circumference at the mid-height level of the liquid content at the time of the maximum resultant overturning moment are presented in Figures 4.28 to 4.33.

4.4.1 Impulsive Component

From the results shown in Figures 4.22 to 4.27, it is observed that the impulsive hydrodynamic pressure at the mid-height and that at the bottom of the liquid content are of similar vibration pattern. Generally, the magnitude of the impulsive pressure at the bottom is slightly higher than that at the mid-height level. This is because the

impulsive hydrodynamic pressure is generally dominated by the fundamental mode which has a $\cos(z\pi/2H)$ distribution along the height. It is observed that the effect of the high order modes of the impulsive liquid motion may be significant on some occasions, which results in larger pressure at a higher position than that at the bottom. as at 2.26 and 12.71sec in the case of the liquid-tank system with $H_l = 6.096m$ subjected to the El Centro earthquake, as shown in Figure 4.22, and at 9.73sec with $H_l = 9.144m$, as shown in Figure 4.23; and at 7.48sec with $H_l = 6.096m$ subjected to the Northridge earthquake, as shown in Figure 4.25, and at 9.09sec with $H_l = 9.144m$, as shown in Figure 4.26. On these occasions, the high hydrodynamic pressure at both the bottom and the mid-height of the liquid content leads to large resultant lateral force and resultant overturning moment, as can be observed in Figures 4.4 to 4.7. Correspondingly, the radial displacement of the shell wall is also observed to be larger, as shown in Figures 4.11 to 4.16.

As the results presented in Table 4.3 shown, the maximum impulsive hydrodynamic pressure does not occur at the same time of the maximum ground acceleration. It is observed from the time histories of the impulsive hydrodynamic pressure that the impulsive hydrodynamic pressure may be large even though the ground acceleration at the time is relatively moderate, as about 10 and 11.5sec in the cases of the liquid-tank system with $H_l = 6.096m$ subjected to the El Centro earthquake, as shown in Figure 4.22, and at 14.5sec with $H_l = 6.096m$ subjected to the Northridge earthquake, as shown in Figure 4.25, and at 11.3sec with $H_l = 9.144m$, as shown in Figure 4.26. The nonlinear behaviour of the impulsive hydrodynamic pressure is from the coupling effect between the impulsive liquid motion and the dynamic response of the tank structure.

From the profiles of the hydrodynamic pressure along the height, as shown in Figures 4.28, 4.30 and 4.32, it is observed that at the time of the maximum resultant

overturning moment, *i.e.*, the maximum total hydrodynamic effect, the maximum impulsive pressure generally occurs at a point above the bottom due to the contribution from the high order modes. In the cases of the liquid-tank system with $H_l = 6.096m$, no base plate uplift occurs, and the distribution of the impulsive hydrodynamic pressure in the circumferential direction, as shown in Figure 4.29, is approximately of a $\cos(\theta)$ form. For the liquid-tank systems with $H_l = 9.144m$ and $12.192m$, as shown in Figures 4.31 and 4.33, the distribution of the impulsive pressure around the circumference can be significantly different from that of a $\cos(\theta)$ form because of the coupling effect between the hydrodynamic pressure and the vibration motion of the tank structure. For the impulsive hydrodynamic pressure, a positive value represents the case that the pressure is applied on the shell wall by the moving liquid content, whereas a negative value represents the case of the pressure being applied on the liquid content by the vibrating shell wall. The results show that when the base plate uplift occurs, the pressure acting on the liquid content by the tank shell wall is concentrated in a narrow region and has a very large magnitude, because the shell wall on the uplift side rotates inwards violently in response to the base plate uplift and pushes on the liquid content, whereas the pressure acting on the tank shell wall by the liquid content is distributed over a broader region with a much smaller magnitude. It is also shown that the maximum of the liquid pressure around the circumference may occur some distance away from the excitation axis.

4.4.2 Convective Component

From the results as shown in Figures 4.22 to 4.27, it is observed the convective hydrodynamic pressure of the liquid-tank system may change the phase angle abruptly and increase the magnitude significantly during an earthquake. In the first few seconds of the time history, the variation of the convective hydrodynamic pressure is smooth and

the amplitude of the sloshing is negligible compared with the corresponding impulsive hydrodynamic pressure. At a later time as the dynamic response progresses, the phase angle of the oscillation relating to the liquid sloshing can be abruptly changed and the amplitude of the sloshing suddenly increased. Examining the time histories of the convective hydrodynamic pressure and the time histories of the generalized vibration period shown in Figures 4.20 and 4.21, it is observed that the abrupt change of the convective liquid motion always occurs with the peaks of the generalized vibration period. The time of the abrupt change in the convective liquid motion and the corresponding generalized vibration period are presented in Table 4.4. At these occasions, due to the significant deformation experienced by the tank structure, associated with the base plate uplift and the shell wall out-of-round distortion, the liquid-tank system becomes very flexible, which tends to excite the long period convective liquid motion. Although only one abrupt change in the convective liquid motion is observed in the time history analysis of the present study, it is possible that this kind of behaviour in the convective liquid motion can occur more than once during an earthquake if the tank structure experiences significant deformation frequently.

The frequency of the fundamental mode of the convective hydrodynamic behaviour has been given by Veletsos (1984) as follows

$$f = \frac{1}{2\pi} \sqrt{1.842 \frac{g}{R} \tanh(1.842 \frac{H_l}{R})} \quad (4.3)$$

For the liquid-tank system with tank radius $R = 15.24\text{ m}$, the period of the fundamental convective hydrodynamic behaviour is 7.29 sec for liquid height $H_l = 6.096\text{ m}$ and 6.45 sec for $H_l = 9.144\text{ m}$. These values are in good agreement with the oscillations observed in the time history of the convective hydrodynamic pressure in Figures 4.22 to 4.27, which is about 7.0 sec for the liquid-tank systems with liquid height $H_l = 6.096\text{ m}$ and 6.2 sec with $H_l = 9.144\text{ m}$. The results show that although the convective component of the hydrodynamic behaviour may have its phase angle

and amplitude changed abruptly, it is still generally dominated by the fundamental mode. As can be observed and concluded from the results shown in Figures 4.22, 4.23, 4.25 and 4.26, during the first half period of the intensive convective hydrodynamic response, the convective liquid motion at the liquid free surface has higher oscillation frequency and much more intense behaviour than those in the rest time of the duration. This suggests that the high order modes of the convective hydrodynamic behaviour are active in the first half period but attenuate rapidly.

As can be observed from the results shown in Figures 4.32 for the liquid-tank system with $H_l = 12.192m$, it is possible that the maximum convective hydrodynamic pressure occurs at the bottom of the liquid content as a result of the large base plate uplift experienced by the tank structure. For the liquid-tank system with $H_l = 6.096m$ subjected to the Northridge earthquake, the distribution of the convective pressure along the circumferential is slightly different from the $\cos(\theta)$ form, as shown in Figure 4.29. This is because of the the significant contribution to the dynamic response of the tank structure from the high order modes.

The time histories of the equivalent wave height on the excitation axis $\theta = 0$, calculated from the corresponding convective hydrodynamic pressure at the liquid free surface, are shown in Figure 4.34. According to the results, the maximum equivalent wave height is $2.01m$ for the liquid-tank system with $H_l = 6.096m$, and $2.64m$ for $H_l = 9.144m$ when subjected to the El Centro earthquake, and $3.11m$ for $H_l = 6.096m$, and $5.30m$ for $H_l = 9.144m$ when subjected to the Northridge earthquake. The magnitude of the wave height is about $1/2$ of the liquid height. Certainly the “small amplitude sloshing” assumption adopted in the analytical model is not valid anymore under this condition. Still, this result indicates that the sloshing of the liquid content developed during an earthquake may be quite significant which can easily cause damage to the roof and upper part of the tank structure, as has been

observed in many tank damages in previous earthquakes.

4.5 Stress Analysis and Failure of Tank Structure

The compressive and tensile stresses developed in the tank structure in response to the hydrodynamic load, and the failure mechanism of the tank structure are discussed in this section.

4.5.1 Tank Shell Wall Stress

The stress developed in the shell wall during an earthquake is generally dominated by the membrane mechanism. The time histories of the membrane axial stress near the bottom and at the mid-height level of the tank shell wall on the excitation axis $\theta = 0$ are shown in Figures 4.35 to 4.40. Near the bottom of the shell wall, the axial membrane stress is dominated by the effect of the resultant overturning moment. As the result, the variation of the axial membrane stress near the bottom is observed to have similar vibration characteristic as that of the resultant overturning moment shown in Figures 4.4 and 4.6. At the mid-height level of the shell wall, where the effect of the resultant overturning moment is greatly decreased with the height, the axial membrane stress is mainly affected by the shell wall deformation and has a different variation pattern from that near the bottom. It is observed that the axial stress at the mid -height level can be more intense as shown in Figures 4.35 and 4.38 for the liquid-tank system with $H_l = 6.096m$. From the results, it is noticed that the membrane axial stress near the bottom of the shell wall can be greatly affected by the base plate uplift. As in the cases of the liquid-tank system with $H_l = 9.144m$ subjected to the El Centro earthquake approximately from 8 to 12

second, as shown in Figure 4.36, and subjected to the Northridge earthquake from 8 to 11 second, as shown in Figure 4.39, the magnitude of the tensile axial membrane stress is significantly reduced as compared to the compressive stress. The tensile stress may even become compressive due to this reduction from the compression caused by the base plate uplift. It is also observed that the peak axial membrane stress may not occur at the same time as that of the maximum resultant overturning moment due to the effect of the shell wall deformation.

Near the bottom of the tank shell wall where constraints are imposed, and at regions where significant deformation occurs, the effect of the flexural mechanism must be considered. Profile of the axial and hoop stresses along the height of the section of tank shell wall on the excitation axis, at the time of the maximum resultant overturning moment, are shown in Figures 4.41 to 4.46. From the results, it is concluded that in the region from the bottom to about 1.5m height of the shell wall, the axial stress is much more intense than in the upper part. On the contact side, it is noticed that the maximum compressive axial stress usually occurs at some distance above the tank bottom. The explanation of this behaviour is that the shell wall bulges out due to the outward liquid pressure and the bulging-out of shell wall results in tensile stress which compensates some of the compressive stress. On the uplift side, as shown in Figures 4.42 to 4.46, axial stress is observed to be compressive instead of tensile near the bottom of the shell wall because of the compression caused by the base plate uplift. The hoop stress, which is generally tensile because of the outward liquid pressure, is also found to be affected by the deformation of the tank structure. On the contact side, the hoop stress is observed to be compressive near the bottom of the shell wall because the outward radial displacement of the shell wall is restrained. On the uplift side, as shown in Figures 4.42, to 4.46, the hoop stress can be compressive due to the out-of-round distortion of the shell wall resulting from the base plate uplift.

4.5.2 Tank Base Plate Stress

To resist the resultant lateral force from the hydrodynamic pressure, friction develops between the tank base plate and the ground support. In the present study, the friction is assumed to be uniformly distributed among the part of the base plate that remains contact with the ground support. The stress in the base plate engendered by the friction varies linearly from tensile on the contact side to compressive on the uplift side. When the base plate uplifts, the uplifted region of base plate extends and induces intensive tensile stress which overcomes the compressive stress engendered by the friction. The distribution of the axial stress (parallel to $\theta = 0$) along the excitation axis at the time of the maximum resultant overturning moment is shown in Figure 4.47.

4.5.3 Failure of Tank Structure

The buckling of the shell wall, caused by high compressive stress developed in response to the seismic load, is the most commonly reported seismic damage to liquid-tank systems. In Table 4.5, the maximum values of the compressive stress developed in tank shell wall are listed.

The allowable stress for compression with consideration of the structural stability is given as $1.05 \times 10^8 Pa$ (15,000ksi) in AWWA D100-84 (AWWA 1984). According to the results obtained from the analysis, “elephant foot” buckling may occur near the bottom of the tank shell wall in the case of the liquid-tank system with $H_l = 9.144m$ and $12.192m$ subjected to both the El Centro earthquake and the Northridge earthquake due to the high compressive axial stress; and “diamond shape” buckling may occur near the top of the tank shell wall in the case of the liquid-tank system with

$H_l = 12.192m$ subjected to the El Centro earthquake due to the high compressive hoop stress. In other situations, buckling may also occur due to the combination of the compressive axial and hoop stress.

4.6 Summary

From the time history analysis, the dynamic behaviour of the unanchored liquid-tank system during an earthquake can be summarized as

1. The dynamic behaviour of the unanchored system is similar to that of the anchored liquid-tank system when the seismic load is not intense enough to cause base plate uplift. Due to the lack of restraint from the absence of the anchorage system, significant deformation of the tank structure, including uplift of the base plate and out-of-round distortion of the shell wall, will develop in response to intense seismic load. The significant deformation of the tank structure makes the liquid-tank system very flexible.
2. The uplift mechanism is very complicated and highly nonlinear. The base plate uplifts only if the resultant overturning moment exceeds a threshold value. Although there is a clear trend of larger uplift with increasing resultant overturning moment, the behaviour of the base plate uplift may vary significantly within a wide range with the resultant overturning moment.
3. The deformation of the vibrating tank shell wall is generally governed by the hydrodynamic effect. When the base plate uplifts, the shell wall on the uplift side rotates inwards and causes very large deformation and significant out-of-round distortion in the shell wall.
4. The resultant lateral force and resultant overturning moment of the liquid-tank system may exhibit to be impulsive or convective due to the intensity of the

impulsive and the convective component of the liquid motion.

5. The impulsive liquid motion is nonlinearly to the ground excitation. It is closely coupled to the dynamic response of the tank structure. The distribution of the impulsive hydrodynamic pressure is generally dominated by the fundamental mode which has a $\cos(z\pi/2H)$ form along the height and a $\cos(\theta)$ form around the circumference. The significant deformation of the tank structure greatly affects the distribution and the magnitude of the impulsive hydrodynamic pressure.
6. The convective liquid motion is generally dominated by the fundamental mode, but may have its phase angle and amplitude changed abruptly at the occasions when the liquid-tank system becomes flexible due to the significant deformation of the tank structure. The distribution of the convective liquid motion may also be changed by the deformation of the tank structure.
7. The stress developed in the tank shell wall in response to the seismic load is generally dominated by the membrane mechanism. In the regions where restraints are imposed and deformation of tank structure is significant, especially near the bottom of the shell wall, intensive stresses will be caused by the flexural mechanism.
8. High compressive axial stress may be developed near the bottom of the tank shell wall at the contact side which may lead to “elephant foot” buckling failure of the shell wall. The combination of compressive hoop and axial stress that occurs in a high position of the tank shell wall may cause “diamond shape” buckling.

Earthquake	$H_l(m)$	Max Resultant Load		Time(sec)	$a_g(g)$
El Centro (0.348g at 2.14sec)	6.096	L. F. (N)	1.146×10^7	2.26	0.19
		O. M. (N-m)	2.829×10^7	4.51	0.21
	9.144	L. F. (N)	1.662×10^7	9.73	0.04
		O. M. (N-m)	6.909×10^7		
	12.192	L. F. (N)	7.481×10^7	3.52	0.11
		O. M. (N-m)	38.840×10^7		
Northridge (0.583g at 5.36sec)	6.096	L. F. (N)	1.411×10^7	5.44	0.53
		O. M. (N-m)	3.651×10^7		
	9.144	L. F. (N)	2.928×10^7	5.36	0.47
		O. M. (N-m)	13.050×10^7		
	12.192	L. F. (N)	5.449×10^7	4.11	0.51
		O. M. (N-m)	27.810×10^7		

L.F.: Lateral Force, O.M.: Overturning Moment.
 a_g : Ground acceleration.

Table 4.1: Maximum Resultant Load

Earthquake	H_l (m)	Base Plate Uplift			Shell Wall Radial Disp.		
		Value (mm)	Extent (m)	time (sec)	Value (mm)	Height (m)	time (sec)
El Centro	6.096				7.50	2.475	4.40
	9.144	4.66	0.962	9.73	10.09	12.192	9.73
	12.192	145.48	4.458	3.52	144.20	12.192	3.52
Northridge	6.096				7.19	2.475	5.02
	9.144	34.47	2.009	5.36	38.87	12.192	5.36
	12.192	75.83	2.962	4.11	90.00	12.192	4.11

Table 4.2: Maximum Response of Tank Structure

Earthquake	H_l (m)	Impulsive			Convective		
		value (kPa)	time (sec)	height (m)	value (kPa)	time (sec)	height (m)
El Centro	6.096	69.34	4.40	1.95	19.70	6.19	6.096
	9.144	35.48	9.73	2.93	25.89	6.32	9.144
	12.192	71.57	3.52	5.30			
Northridge	6.096	74.41	5.02	1.95	30.99	5.82	6.096
	9.144	86.17	5.45	1.85	51.94	8.75	9.144
	12.192	68.03	4.11	0.00			

Table 4.3: Maximum Hydrodynamic Pressure

Earthquake	H_l (m)	Time (sec)	$T_{(t)}$ (sec)
El Centro	6.096	5.33	3.102
	9.144	2.56	0.723
	12.196	3.52	1.433
Northridge	6.096	5.02	1.882
	9.144	4.66	1.696
	12.196	4.11	1.232

Table 4.4: Generalized Vibration Period Corresponding to the Abrupt Change of the Convective Hydrodynamic Behaviour

Earthquake	H_l (m)	Axial		Hoop	
		Value(10^8 Pa)	height(m)	Value(10^8 Pa)	height(m)
El Centro	6.096	0.314	0.46	0.053	4.46
	9.144	1.096	0.50	0.121	6.44
	12.192	6.628	0.00	1.136	8.42
Northridge	6.096	0.092	0.38	0.247	3.47
	9.144	1.600	0.48	0.218	6.93
	12.192	3.776	0.48	0.719	8.42

Table 4.5: Maximum Compressive Stress

Figure 4.1: Configuration of The Liquid-Tank System

a) El Centro Earthquake

b) Northridge earthquake

Figure 4.2: Time History of Ground Acceleration

a) El Centro Earthquake

b) Northridge Earthquake

Figure 4.3: Spectral Acceleration

Figure 4.4: Time History of Resultant Lateral Force (El Centro Earthquake)

Figure 4.5: Time History of Resultant Overturning Moment (El Centro Earthquake)

Figure 4.6: Time History of Resultant Lateral Force (Northridge Earthquake)

Figure 4.7: Time History of Resultant Overturning Moment (Northridge Earthquake)

Figure 4.8: Time History of Base Plate Uplift (El Centro)

Figure 4.9: Time History of Base Plate Uplift (Northridge)

Figure 4.10: Relationship between Tank Base Plate Uplift and Resultant Overturning Moment

Figure 4.11: Time History of Tank Shell Wall Radial Displacement ($H_t = 6.096m$, El Centro)

Figure 4.12: Time History of Tank Shell Wall Radial Displacement ($H_t = 9.144m$, El Centro)

Figure 4.13: Time History of Tank Shell Wall Radial Displacement ($H_t = 12.192m$, El Centro)

Figure 4.14: Time History of Tank Shell Wall Radial Displacement ($H_t = 6.096m$, Northridge)

Figure 4.15: Time History of Tank Shell Wall Radial Displacement ($H_t = 9.144m$, Northridge)

Figure 4.16: Time History of Tank Shell Wall Radial Displacement ($H_t = 12.192m$, Northridge)

Figure 4.17: Tank Shell Wall Radial Displacement along Height (El Centro)

Figure 4.18: Tank Shell Wall Radial Displacement along Height (Northridge)

5.26 sec

5.33 sec

5.49 sec

Figure 4.19: Example of Tank Shell Wall Out-of-Round Distortion

a) $H_1 = 6.096\text{m}$

b) $H_1 = 9.144\text{m}$

c) $H_1 = 12.192\text{m}$

Figure 4.20: Time History of Generalized Period (El Centro)

a) $H_1 = 6.096\text{m}$

b) $H_1 = 9.144\text{m}$

c) $H_1 = 12.192\text{m}$

Figure 4.21: Time History of Generalized Period (Northridge)

Figure 4.22: Time History of Hydrodynamic Pressure ($H_t = 6.096m$, El Centro)

Figure 4.23: Time History of Hydrodynamic Pressure ($H_t = 9.144m$, El Centro)

Figure 4.24: Time History of Hydrodynamic Pressure ($H_l = 12.192m$, El Centro)

Figure 4.25: Time History of Hydrodynamic Pressure ($H_l = 6.096m$, Northridge)

Figure 4.26: Time History of Hydrodynamic Pressure ($H_l = 9.144m$, Northridge)

Figure 4.27: Time History of Hydrodynamic Pressure ($H_t = 12.192m$, Northridge)

(a) El Centro

(b) Northridge

Figure 4.28: Profile of Liquid Pressure along Height ($H_l = 6.096m$)

(a) El Centro

(b) Northridge

Figure 4.29: Profile of Liquid Pressure around Circumference ($H_l = 6.096m$)

(a) El Centro

(b) Northridge

Figure 4.30: Profiles of Liquid Pressure along Height ($H_l = 9.144m$)

(a) El Centro

(b) Northridge

Figure 4.31: Profile of Liquid Pressure around Circumference ($H_l = 9.144m$)

(a) El Centro

(b) Northridge

Figure 4.32: Profile of Liquid Pressure along Height ($H_t = 12.192m$)

Figure 4.33: Profile of Liquid Pressure around Circumference ($H_l = 12.192m$)

a) El Centro Earthquake

b) Northridge Earthquake

Figure 4.34: Time History of Equivalent Wave Height

Figure 4.35: Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 6.096m$, El Centro)

Figure 4.36: Time History of Tank Shell Wall Axial Membrane Stress ($H_t = 9.144m$, El Centro)

Figure 4.37: Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 12.192m$, El Centro)

Figure 4.38: Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 6.096m$, Northridge)

Figure 4.39: Time History of Tank Shell Wall Axial Membrane Stress ($H_l = 9.144m$, Northridge)

Figure 4.40: Time History of Tank Shell Wall Axial Membrane Stress ($H_t = 12.192m$, Northridge)

a) stress on contact side

b) stress on uplift side

Figure 4.41: Tank Shell Wall Stress along Height ($H_t = 6.096m$, El Centro)

stress (10^8 Pa)

a) stress on contact side

stress (10^8 Pa)

b) stress on uplift side

Figure 4.42: Tank Shell Wall Stress along Height ($H_t = 9.144m$, El Centro)

a) stress on contact side

b) stress on uplift side

Figure 4.43: Tank Shell Wall Stress along Height ($H_t = 12.192m$, El Centro)

a) stress on contact side

b) stress on uplift side

Figure 4.44: Tank Shell Wall Stress along Height ($H_t = 6.096m$, Northridge)

a) stress on contact side

b) stress on uplift side

Figure 4.45: Tank Shell Wall Stress along Height ($H_t = 9.144m$, Northridge)

a) stress on contact side

b) stress on uplift side

Figure 4.46: Tank Shell Wall Stress along Height ($H_l = 12.192m$, Northridge)

a) El Centro

b) Northridge

Figure 4.47: Tank Base Plate Stress along Excitation Axis

Chapter 5

PARAMETER STUDY

5.1 General

Results of a parametric study on the seismic behaviour of unanchored liquid-tank systems are presented in this chapter.

From previous research studies on anchored liquid-tank systems, it is concluded that the liquid height to tank radius ratio characterizes the liquid motion developed in response to the seismic load. The flexibility of the tank structure, which is influenced by the flexibility of the shell wall as well as the type of the tank roof and the stiffness of the ring girders, determines the response behaviour of the tank structure. From the results of static tilt studies (Lau and Clough, 1991), the flexibility of the base plate is shown to be important in the uplift mechanism, which is the dominant structural response for unanchored liquid-tank systems subjected to seismic loads.

In the present study, the liquid height to tank radius ratio and the flexibility of the shell wall and the base plate are taken as the varying parameters to determine the significance of their influence on the seismic behaviour of unanchored liquid-tank systems. The seismic response of unanchored liquid-tank systems subjected to different recorded earthquake ground excitations are analyzed in time history by the analytical model developed by Zeng (1993). The numerical results are analyzed to determine the effect of each parameter on the liquid motion, the base plate uplift and

the axial compressive stress developed in the shell wall. The failure of the thin walled cylindrical shell under dynamic loading conditions is a very complicated problem which is beyond the scope of the present study. In this research, the liquid-tank system is considered “failed” when it is overturned by the resultant overturning moment.

The properties of the liquid-tank system and the varying parameters considered in the parametric study are described in Section 5.2. The results of the time history analysis are discussed in Section 5.3. A summary of the findings of the parametric study is presented in Section 5.4.

5.2 Description of Liquid-Tank System and Parameters Considered

The configuration of the liquid-tank system considered in the parametric study is shown in Figure 5.1. The tank has an open top. It is unanchored and supported by a rigid foundation. The material of the tank structure is assumed to be steel with a Young’s modulus of elasticity equal to $2 \times 10^{11} Pa$. The liquid content is water and the volume is kept constant at $600m^3$.

In general, the seismic behaviour of a liquid-tank system with a larger liquid height to tank radius ratio H_l/R is likely to be more intense, because the resultant of the hydrodynamic load generated from the liquid motion is applied at a higher level which leads to a greater resultant overturning moment and a larger dynamic response of the tank structure. In the present study, the ratio H_l/R is varied from 1.0 for a broad liquid-tank system(BT), to 1.5 for a intermediate liquid-tank system(MT), and to 2.0 for a tall liquid-tank system(TT).

For an anchored liquid-tank system, the dynamic response of the tank structure under seismic loads is primarily governed by the vibration of the shell wall. The

characteristics of the vibration behaviour is significantly influenced by the flexibility of the tank shell wall. For an unanchored liquid-tank system, the dynamic response of the tank structure is dominated by the uplift mechanism, in which a portion of the base plate separates from the foundation in response to a large resultant overturning moment from the effect of the liquid motion. The shell wall undergoes large displacement and rotation because of the interaction between the base plate and the shell wall, which are often neglected in previous studies and current design codes. The uplift mechanism is closely related to the flexibility of the base plate and the retaining effect of the shell wall. In the present study, the shell wall and the base plate are assumed to have uniform thickness. The flexibility of the shell wall is represented by the ratio t_s/R , where t_s is the shell wall thickness, and the flexibility of the base plate is represented by the ratio t_b/R , where t_b is the base plate thickness. Both t_s/R and t_b/R are respectively varied from 0.001 to 0.003. The cases of the liquid-tank system considered in the parametric study are listed in Table 5.1.

Four earthquake ground excitation records are adopted as input excitations in the parametric study. They are the 1940 El Centro earthquake(Imperial Valley, N-S component), the 1988 Saguenay earthquake(St. Ferreol, Longitudinal component, referred as Saguenay-1 in the parametric study; Chicoutimi Nord, Longitudinal component, referred as Saguenay-2), and the 1994 Northridge earthquake(Newhall, N-S component). The ground acceleration time histories of the 1940 El Centro earthquake and the 1994 Northridge earthquake are shown in Figure 4.2. The ground acceleration time histories of the 1988 Saguenay earthquakes are shown in Figure 5.2. These earthquake records are chosen as typical earthquakes to represent ground motions with different ratios of the peak ground acceleration to the peak ground velocity. In order to compare the results, all the earthquake records are scaled to the same maximum ground acceleration of $0.24g$, which is the level of ground excitation assumed

in the API 650 design standard (API 1988) to determine the impulsive effect of the liquid motion.

5.3 Results

In this section, the seismic response of the unanchored liquid-tank system expressed in terms of the impulsive liquid pressure, the equivalent wave height, the resultant overturning moment, the amount of the base plate uplift and the compressive axial stress developed in the shell wall are discussed. The impulsive liquid pressure and the equivalent wave height represent the impulsive and the convective component of the liquid motion respectively. The resultant overturning moment gives the overall effect of the liquid motion acting on the shell wall. In the discussion, the resultant overturning moment is normalized by the product of the total weight of the liquid-tank system and the liquid height GH_l . The amount of the base plate uplift gives an indication on the uplift mechanism in an unanchored liquid-tank system. The compressive axial stress in the shell wall is the principal cause of the shell buckling failure and an important factor to be considered in earthquake resistance design of liquid-tank systems.

The time histories of the seismic response for the liquid-tank system labelled T-1 ($t_b/R = t_s/R = 0.001$) and T-9 ($t_b/R = t_s/R = 0.003$), respectively the most flexible and the most rigid system considered in the parametric study, are presented to illustrate the difference in the behaviour of liquid-tank systems with different structure flexibilities when subjected to seismic loads. The variations of the maximum seismic response of the liquid-tank system are discussed in terms of the varying flexibility of the tank shell wall and the base plate. The results for liquid-tank systems with different liquid height to tank radius ratios are discussed separately because of the

expected different influences on seismic response.

5.3.1 Broad Liquid-Tank System

The time histories of the seismic response of BT-1 and BT-9 subjected to the El Centro and the Saguenay-1 earthquake records are presented in this section. The maximum seismic responses of the broad liquid-tank systems are summarized in Table 5.2. The liquid-tank systems BT-1, BT-2 and BT-3 fail when subjected to the Saguenay-2 earthquake record. The results presented in Table 5.2 for these cases are the maximum seismic response developed in the liquid-tank systems prior to the failure.

Liquid Motion

Impulsive liquid motion

The time histories of the impulsive hydrodynamic pressure near the bottom of the shell wall on the excitation axis $\theta = 0$ for BT-1 and BT-9 subjected to the El Centro and the Saguenay-1 earthquakes are shown in Figures 5.3 and 5.4. From the results, it is observed that the impulsive hydrodynamic pressure in the cases of BT-9 is generally larger than that in BT-1. For example, in the cases when subjected to the El Centro earthquake as shown in Figure 5.3, the impulsive hydrodynamic pressure in BT-9 varies from $-8kPa$ to $10kPa$ while in BT-1 from $-7kPa$ to $8kPa$. In the cases of the Saguenay-1 earthquake shown in Figure 5.4, the impulsive hydrodynamic pressure in BT-9 varies from $-13kPa$ to $10kPa$ while in BT-1 from $-9kPa$ to $7kPa$. It suggests that for a liquid-tank system with a stiffer tank structure, the impulsive component of the liquid motion is more intense. It is also noted that for a flexible liquid-tank system, the large deformation in the tank structure may induce very large impulsive hydrodynamic pressure, as shown in Figure 5.3 in the case of BT-1 subjected to the El Centro earthquake at $1.75sec$, the impulsive hydrodynamic pressure which is

related to the base plate uplift is of a magnitude $-9.81kPa$.

The results of the maximum impulsive hydrodynamic pressure for the cases of the broad liquid-tank system are presented in Figure 5.5. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.5(a), the maximum impulsive hydrodynamic pressure remains constant at about $10.5kPa$ while t_s/R increases from 0.001 to 0.002, and increases to larger than $30kPa$ when t_s/R increases to 0.003. With $t_b/R = 0.002$, as shown in Figure 5.5(b), the maximum impulsive hydrodynamic pressure remains constant at about $11kPa$ except in the cases of BT-5($t_s/R = 0.002$) subjected to the Saguenay-1 earthquake and BT-6($t_s/R = 0.003$) subjected to the Saguenay-2 earthquake, which in both cases the maximum impulsive hydrodynamic pressure exceeds $30kPa$. With $t_b/R = 0.003$, as shown in Figure 5.5(c), the maximum liquid pressure remains constant at about $11kPa$ when t_s/R increases from 0.001 to 0.003. Based on the results shown in Figure 5.5, it is concluded that the base plate has significant influence on the impulsive liquid motion in unanchored broad liquid-tank systems. Unlike an anchored liquid-tank system, of which the tank shell wall is fixed to the ground and whose vibration behaviour is essentially similar to that of a cantilever beam, the tank shell wall of an unanchored liquid-tank system is restrained by the base plate which can be uplifted from the foundation. The base-wall joint is deformed in response to the bending moment from the liquid pressure acting on the tank structure. If the base plate is more flexible than the shell wall, the deformation will be concentrated on the base plate side. For a flexible base plate, the resulting deformation is significant. Associated with the rotation of the base-wall joint, rigid body rotation of the shell wall is generated which results in large shell wall radial displacement. As the result, significant impulsive liquid motions are excited by the motion of the shell wall due to the coupling effect between the liquid motion and the dynamic response of the tank structure. As in the cases of BT-3 with $t_b/R = 0.001$

and $t_s/R = 0.003$, the maximum impulsive hydrodynamic pressure exceeds $30kPa$. If the base plate is stiff, as in the cases of BT-7, BT-8 and BT-9 with $t_b/R = 0.003$, or if the base plate is stiffer than the shell wall, as in the cases of BT-4 with $t_b/R = 0.002$ and $t_s/R = 0.001$, sufficient restraint is provided to the vibrating shell wall, thus the corresponding maximum impulsive hydrodynamic pressure is reduced to about $11kPa$.

Convective Liquid Motion

The time histories of the equivalent wave height at the liquid free surface on the excitation axis $\theta = 0$ for BT-1 and BT-9 subjected to the El Centro and the Saguenay-1 earthquakes are shown in Figures 5.6 and 5.7. In the case of BT-1 subjected to both the El Centro and the Saguenay-1 earthquakes, the oscillation of the liquid free surface wave changes abruptly and the magnitude of the equivalent wave height exceeds $1m$. In the case of BT-9 subjected to the El Centro earthquake, the equivalent wave height varies smoothly, but when BT-9 is subjected to the Saguenay-1 earthquake, the equivalent wave height changes abruptly three times. However, the magnitude of the equivalent wave height for BT-9 is less than $0.5m$. Comparing the time histories of the equivalent wave height with that of the corresponding impulsive hydrodynamic pressure shown in Figures 5.3 and 5.4, it is observed that the abrupt changes in the variation of the equivalent wave height occur at times of large impulsive hydrodynamic pressure. As discussed in Chapter 4, the abrupt change is of similar mechanism as the oscillation of a spring-mass system being abruptly disturbed by an large impulsive load. At these times, the liquid-tank system is also softened because of the base plate uplift and the significant deformation of the shell wall, which excites the long period sloshing of the liquid content. The results also indicate that the convective liquid motion in a more rigid liquid-tank system is less intense because the deformation of the tank structure is more effectively constrained.

The results of the maximum equivalent wave height for the cases of the broad liquid-tank system are shown in Figure 5.8. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.8(a), the maximum equivalent wave height decreases from about $1.5m$ to about $1m$ when t_s/R increases from 0.001 to 0.002 , and increases to more than $3m$ when t_s/R is equal to 0.003 . With $t_b/R = 0.002$, as shown in Figure 5.8(b), the maximum equivalent wave height exhibits different trends of variation with the varying parameter t_s/R when subjected to different earthquake records. With t_s/R increasing from 0.001 to 0.002 , then to 0.003 , the maximum equivalent wave height increases from about $1m$ to more than $5m$, then decreases to less than $0.5m$ when subjected to the El Centro and the Saguenay-1 earthquakes. The same response quantity remains less than $0.5m$ when subjected to the Northridge earthquake. When subjected to the Saguenay-2 earthquake, the equivalent wave height remains about $1.4m$ and jumps to larger than $30m$ when t_s/R increase from 0.001 to 0.002 then to 0.003 . With $t_b/R = 0.003$, as shown in Figure 5.8(c), the maximum equivalent wave height also exhibits different trends of behaviour with varying t_s/R when subjected to different earthquake records. With t_s/R increasing from 0.001 to 0.002 , then to 0.003 , the maximum equivalent wave height increases from $0.421m$ to $2.17m$, and then decreases to $0.523m$ when subjected to the El Centro earthquake. It decreases from more than $1.5m$ to less than $0.6m$ when subjected to the Northridge and the Saguenay-2 earthquakes, and remains less than $0.15m$ when subjected to the Saguenay-1 earthquake. In Figure 5.8, the maximum equivalent wave height shows no clear trend of variation in terms of the varying parameters t_s/R and t_b/R . It implies that the convective liquid motion is greatly affected by the temporary seismic response of the liquid-tank system. However, it is noted that if the base plate is relatively flexible which could not provide sufficient restraint to the shell wall, the liquid-tank system becomes more flexible under the seismic load.

As the result, the convective liquid motion becomes more intense, as in the cases of BT-3($t_b/R = 0.001, t_s/R = 0.003$), the maximum equivalent wave height is larger than $3m$. It is also noted that if the shell wall is flexible, it is likely to experience significant deformation which may cause intense convective liquid motion, as in the cases of BT-1, BT-4 and BT-7($t_s/R = 0.001$), the maximum equivalent wave height generally exceeds $1m$.

Resultant Overturning Moment

The time histories of the resultant overturning moment generated from the liquid motion in the cases of BT-1 and BT-9 subjected to the El Centro and the Saguenay-1 earthquakes are shown in Figures 5.9 and 5.10. In BT-1, the convective effect of the liquid motion is larger than the impulsive effect and thus dominates the resultant overturning moment behaviour. On the other hand, the resultant overturning moment in BT-9 is observed to be dominated by the impulsive effect of the liquid motion.

The results of the maximum resultant overturning moment for the cases of the broad liquid-tank systems are shown in Figure 5.11. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.11(a), the maximum resultant overturning moment decreases from about $0.15GH_l$ to $0.07GH_l$ when subjected to the El Centro and the Saguenay-1 earthquakes, and remains about $0.07GH_l$ when subjected to the Northridge earthquake when t_s/R increases from 0.001 to 0.002. It then increases to larger than $0.20GH_l$ when t_s/R is equal to 0.003. With $t_b/R = 0.002$, as shown in Figure 5.11(b), the maximum resultant overturning moment shows different trends of variation when subjected to different earthquakes. With t_s/R increasing from 0.001 to 0.002, and then to 0.003, the maximum resultant overturning moment increases from $0.054GH_l$ to $0.141GH_l$ and decreases to $0.064GH_l$ when subjected to the El Centro earthquake. It remains constant about $0.06GH_l$ when subjected to

the Northridge earthquake. The same response quantity increases from $0.107GH_l$ to $0.246GH_l$ and decreases to $0.072GH_l$ when subjected to the Saguenay-1 earthquake, and remains about $0.06GH_l$ and increases to $0.492GH_l$ when subjected to the Saguenay-2 earthquake. With $t_b/R = 0.003$, as shown in Figure 5.11(c), the maximum resultant overturning moment stays constant at about $0.07GH_l$ except for the case of BT-8($t_s/R = 0.002$) subjected to the El Centro earthquake with $0.1222GH_l$. From the results, it is noted that the convective liquid motion can be very intense in unanchored broad liquid-tank systems and plays an important role in the seismic behaviour. As listed in Table 5.2, in more than half of the cases analyzed, the convective effect of the liquid motion is the dominant loading effect, with at least equal or greater magnitude than the impulsive effect of the liquid motion. For the cases with the loading dominated by the convective effect of the liquid motion, the maximum resultant overturning moment may reach as high as $0.32GH_l$, as in the case of BT-3 subjected to the Northridge earthquake, which is much higher than that of the cases dominated by the impulsive effect of the liquid motion, whose average maximum resultant overturning moment is about $0.07GH_l$. For the cases of tank failure of BT-1, BT-2 and BT-3 subjected to the Saguenay-2 earthquake, as presented in Table 5.2, the equivalent wave height for these cases is higher than $10m$, and the significant convective effect of the liquid motion results in large resultant overturning moment which leads to the failure.

In the design standard API 650 (API 1988) and AWWA D100-84 (AWWA 1984), the two most commonly used design codes for liquid storage tanks in North America, the maximum resultant overturning moment is calculated by simulating the effect of the liquid motion by the product of effective masses and their height defined in a simplified model. The effective masses and heights which represent the impulsive and convective liquid motions are determined from the liquid height to tank radius

ratio, and are considered to be not affected by the tank structure flexibility. Thus for the broad liquid-tank systems considered here in the present study, the resultant overturning moment is estimated to be a constant of $0.072GH_l$. As can be observed from the results, this value is a good approximation if the seismic load is dominated by the impulsive effect of the liquid motion. For the cases of intense convective liquid motion due to the significant uplift mechanism resulted from the lack of anchorage at the tank base, the resultant overturning moment can be several times larger than the design value calculated in accordance to the provisions of design codes.

Base Plate Uplift

The time histories of the vertical uplift at the rim of the base plate for BT-1 subjected to the El Centro and the Saguenay-1 earthquakes are shown in Figures 5.12 and 5.13. The liquid-tank system BT-9 experiences no base plate uplift. Comparing the time histories of the base plate uplift with that of the resultant overturning moment shown in Figures 5.9 and 5.10, it is observed that the base plate uplifts in response to large resultant overturning moment and the maximum uplift occurs at the same time of the peak resultant overturning moment. As discussed in Chapter 4, the base plate uplift induces rotation of the shell wall and causes intense impulsive hydrodynamic pressure. As shown in Figure 5.3(a) in the case of BT-1 subjected to the El Centro earthquake at 1.75sec and 2.89sec, and in Figure 5.4(a) for BT-1 subjected to the Saguenay-1 earthquake at 12.01sec, the large negative impulsive hydrodynamic pressure is related to the base plate uplift. Because the resultant overturning moments in the cases of BT-1 subjected to the El Centro and the Saguenay-1 earthquakes are dominated by the convective effect of the liquid motion, the base plate uplift occurs at the time when the convective effect of the liquid motion is most significant. Thus the effect of the base plate uplift on the convective liquid motion is shown to be not essential.

The results of the maximum base plate uplift for the cases of the broad liquid-tank systems are shown in Figure 5.14. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.14(a), the maximum base plate uplift decrease from about $15mm$ to less than $3mm$ and increases to more than $20mm$ when t_s/R increases from 0.001 to 0.003 when subjected to the El Centro and the Saguenay-1 earthquake. When subjected to the Northridge earthquake, the amount of base plate uplift remains about $4mm$ with t_s/R increasing from 0.001 to 0.002, and also increases to more than $20mm$ when t_s/R is equal to 0.003. With $t_b/R = 0.002$, as shown in Figure 5.14(b), the maximum base plate uplift increases from $1.1mm$ to $10.2mm$ and decreases to zero when subjected to the El Centro earthquake with t_s/R increasing from 0.001 to 0.002, then to 0.003. The same response quantity remains constant at about $1mm$ when subjected to the Northridge earthquake. When subjected to the Saguenay-1 earthquake, the amount of uplift increases from $9.4mm$ to $30.2mm$ and decreases to zero. It remains less than $2mm$ and increases to $71.7mm$ when subjected to the Saguenay-2 earthquake. With $t_b/R = 0.003$, as shown in Figure 5.14(c), the maximum resultant overturning moment is always less than $3mm$. The variation of the amount of the maximum base plate uplift in terms of the parameters t_b/R and t_s/R is observed to be similar to that of the resultant overturning moment presented in Figure 5.11. This is not surprising because the uplift mechanism is closely coupled with the liquid motion. It is noted here that the thickness of the base plate has important influence on the tank resistance to base plate uplift. If the base plate is thin, it is flexible and offers little resistance to the uplift mechanism. As the result, the deformation of the tank structure is significant, as in the cases of BT-1, BT-2 and BT-3 subjected to the Saguenay-2 earthquake where the base plate uplift exceeds $50mm$ prior to the failure of the tank. It is also observed that with a thicker base plate, the amount of the base plate uplift is greatly reduced, as in the case of BT-2($t_b/R = 0.001$) subjected

to the El Centro earthquake, the maximum base plate uplift is 2.7mm even though the maximum resultant overturning moment is only $0.0614GH_l$, whereas in the case of BT-8($t_b/R = 0.003$) subjected to the same ground excitation, the maximum base plate uplift is 2.9mm with the corresponding maximum resultant overturning moment is $0.1222GH_l$.

Compressive Axial Stress

The time histories of the membrane axial stress near the bottom of the shell wall on the excitation axis $\theta = 0$ for BT-1 and BT-9 subjected to the El Centro and the Saguenay-1 earthquakes are shown in Figures 5.15 and 5.16. It is observed from the results that the membrane axial stress is of similar behaviour to that of the resultant overturning moment, as shown in Figures 5.9 and 5.10. It should be noted from the figures that the compressive stress may have larger magnitude than the tensile stress in response to large resultant overturning moment. As in the cases of BT-9 subjected to the El Centro earthquake, the ratio of the peak negative resultant overturning moment, which is $0.068GH_l$, to the peak positive resultant overturning moment, $0.04GH_l$, is less than $7/4$. On the other hand, the ratio of the peak axial compressive stress, 0.95MPa , to the peak axial tensile stress, 0.4MPa , is greater than $9/4$. This behaviour can be explained that the contact arc of the shell wall, which induces the compressive force in the tank shell wall, becomes narrower in response to large overturning moment.

Near the bottom of the shell wall where there is constraint to the shell wall, the flexural mechanism is significant in the deformation of the tank structure. Thus the flexural mechanism must be considered in the evaluation of the stress at this location. The results of the maximum compressive axial stress, developed in the shell wall for the broad liquid-tank systems, considering also the flexural effect, are shown in Fig-

ure 5.17. For the liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.17(a), the maximum compressive axial stress decreases from larger than $1.5 \times 10^8 Pa$ to about $0.5 \times 10^8 Pa$ when t_s/R increases from 0.001 to 0.002, and increases slightly to about $0.6 \times 10^8 Pa$ when t_s/R increases to 0.003. With $t_b/R = 0.002$ and 0.003, as shown in Figure 5.17(b) and (c) respectively, the maximum compressive axial stress decreases from more than $1.0 \times 10^8 Pa$ to about $0.4 \times 10^8 Pa$ when t_s/R increases from 0.001 to 0.003. However, for the cases of BT-6($t_b/R = 0.002, t_s/R = 0.003$) subjected to the the Saguenay-2 earthquake, the maximum compressive axial stress is $2.5459 \times 10^8 Pa$. It is shown in the figure that although the maximum compressive axial stress is related to the resultant overturning moment, it is greatly influenced by the thickness of the shell wall and exhibits a decreasing trend with increasing t_s/R . One reason for the decrease in the axial stress is the increased thickness of the shell wall. Another reason for the decrease is that with a thicker shell wall, the deformation of the tank structure is constrained and the flexural effect becomes less significant.

5.3.2 Intermediate Liquid-Tank System

The time histories of the seismic response of MT-1 and MT-9 subjected to the Northridge and the Saguenay-2 earthquakes are presented in this section. The maximum seismic response of the intermediate liquid-tank systems are summarized in Table 5.3. Totally, the results of 15 cases indicate that the considered liquid-tank system has been overturned by the resultant overturning moment. The results presented in Table 5.3 for the failed cases are the maximum seismic response developed in the liquid-tank systems prior to the failure.

Liquid Motion

Impulsive liquid motion

The time histories of the impulsive hydrodynamic pressure near the bottom of the shell wall on the excitation axis $\theta = 0$ for MT-1 and MT-9 subjected to the Northridge and the Saguenay-2 earthquakes are shown in Figures 5.18 and 5.19. When subjected to the Northridge earthquake, MT-1 fails at 4.11sec when the impulsive hydrodynamic pressure exceeds $15kPa$ and the corresponding ground acceleration is $0.21g$ (after scaling), whereas MT-9 fails at 4.32sec when the impulsive hydrodynamic pressure exceeds $10kPa$ and the corresponding ground acceleration is $0.12g$ (after scaling). When subjected to the Saguenay-2 earthquake, MT-1 has the peak impulsive hydrodynamic pressure $10kPa$ at 2.015sec with the corresponding ground acceleration $\bar{a}_g = 0.24g$ (after scaling), but fails at 2.550sec with $\bar{a}_g = 0.19g$ (after scaling). On the other hand, MT-9 survives the earthquake even though the peak impulsive pressure is about the same $10kPa$ at 2.015sec. Similar to broad liquid-tank systems, it is observed from the figures that the impulsive hydrodynamic pressure of a more rigid liquid-tank system is larger than that of a more flexible system in general cases. It is also noted that under a large resultant load, liquid-tank systems with a more flexible tank structure are likely to have significant deformation and induces more intense liquid motion.

The results of the maximum impulsive hydrodynamic pressure of the cases that survive the earthquake ground excitations are shown in Figure 5.20. For all the survived cases, the maximum impulsive hydrodynamic pressure is about $10.5kPa$ except the case of MT-5($t_b/R = t_s/R = 0.002$) subjected to the El Centro earthquake which has a maximum impulsive hydrodynamic pressure $17.27kPa$. As presented in Table 5.3, the maximum impulsive hydrodynamic pressure of the failed cases prior to the failure usually has very large magnitude, as in the case of MT-9($t_b/R = t_s/R = 0.003$)

subjected to the Saguenay-1 earthquake with the maximum impulsive hydrodynamic pressure equal to $31.77kPa$. It is observed from the results, the impulsive component of the liquid motion in the failed cases is more intense.

Convective liquid motion

The time histories of the equivalent wave height on the liquid free surface at the excitation axis $\theta = 0$ for MT-1 and MT-9 subjected to the Northridge and the Saguenay-2 earthquakes are shown in Figure 5.21 and 5.22. When the liquid-tank system is subjected to the Northridge earthquake, MT-1 fails at $4.11sec$ with the equivalent wave height larger than $1.2m$, whereas MT-9 develops intense convective liquid motion at $4.11sec$ and fails at $4.32sec$ with the wave height larger than $4.5m$. When subjected to the Saguenay-2 earthquake, MT-1 develops intense convective liquid motion at $2.015sec$ and fails at $2.55sec$ with the wave height larger than $2.5m$, whereas the behaviour of the equivalent wave height varied smoothly in MT-9 and the magnitude less than $0.30m$. The intense convective liquid motion developed in the cases of MT-1 and MT-9 subjected to the Northridge earthquake, and of MT-1 subjected to the Saguenay-2 earthquake are observed to be related to the intense impulsive liquid motion at that time. It is noted that with a stiffer tank structure, the deformation of the structure under seismic load can be much less significant. As the result, the corresponding convective liquid motion is developed gradually and of much smaller magnitude.

The results of the maximum equivalent wave height for the cases that survive the earthquake ground excitations are shown in Figure 5.23. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.23(a), the maximum equivalent wave height remains at constant at about $1.3m$ when subjected to the El Centro earthquake. In the case of MT-3($t_b/R = 0.003$) subjected to the Saguenay-1 earthquake, the maximum equivalent wave height is $3.365m$. With $t_b/R = 0.002$, as shown in Figure 5.23(b).

the maximum equivalent wave height decreases from $3.104m$ to $0.291m$ when subjected to the El Centro earthquake. The same response quantity remains at constant at about $2m$ when subjected to the Saguenay-1 earthquake. With $t_b/R = 0.003$, as shown in Figure 5.23(c), the maximum equivalent wave height, when subjected to both the El Centro and the Saguenay-2 earthquakes, shows similar trend of variation but with different magnitude. In both cases, the wave height remains at the same level when t_s/R increases from 0.001 to 0.002 , and then decreases when t_s/R is equal to 0.003 , with the magnitude of the equivalent wave height of the cases subjected to the El Centro earthquake always about $2m$ higher than that of the cases subjected to the Saguenay-2 earthquake. From the results, it is observed that the maximum equivalent wave height of the survived cases varies in a range from $0.278m$ to $3.365m$ with the impulsive liquid motion remains at approximately the same level. The variation of the maximum equivalent wave height shows no clear relation to the varying parameters t_b/R and t_s/R , nor to the the ground excitations. This implies that the convective liquid motion of these cases is greatly affected by the temporary seismic response of the liquid-tank system. For the failed cases, as presented in Table 5.3, it is noted that the convective liquid motion usually develops very large magnitude of equivalent wave height prior to the failure, as in the cases of MT-6($t_b/R = 0.002, t_s/R = 0.003$) subjected to the Saguenay-2 earthquake with the equivalent wave height as high as $38.432m$.

Resultant overturning moment

The time histories of the resultant overturning moment generated from the liquid motion for MT-1 and MT-9 subjected to the Northridge and the Saguenay-2 earthquakes are shown in Figures 5.24 and 5.25. From the figures, it is observed that the large resultant overturning moment causing the failure in the case of MT-1 subjected to the Northridge earthquake is from the impulsive effect of the liquid motion. On the other

hand. the failure of MT-9 subjected to the Northridge earthquake is observed to be from the convective effect of the liquid motion. In the case of MT-1 subjected to the Saguenay-2 earthquake, the failure is found to be the result of the superposition of the convective liquid motion developed at 2.015sec and the impulsive liquid motion caused by the large ground excitation at 2.55sec. In the case of MT-9 subjected to the Saguenay-2 earthquake, as shown in Figure 5.25(b), the convective effect of the liquid motion is clearly exhibited even though the magnitude of the equivalent wave height is just about 0.2m. It indicates that for intermediate liquid-tank systems, both the impulsive and the convective effect of the liquid motion are able to cause large resultant overturning moment because the effect of the liquid motion is applied at a higher level.

The results of the maximum resultant overturning moment for the cases that survive the earthquake ground excitations are shown in Figure 5.26. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.26(a), the maximum resultant overturning moment increases from $0.07GH_l$ to about $0.12GH_l$, and then remains at the same level, with t_s/R increasing from 0.001 to 0.003, when subjected to the El Centro earthquake. With $t_b/R = 0.002$, as shown in Figure 5.26(b), the maximum resultant overturning moment increases from about $0.10GH_l$ to $0.14GH_l$, then decreases to $0.075GH_l$ when t_s/R increases from 0.001 to 0.003, when the liquid-tank system is subjected to both the El Centro and the Saguenay-2 earthquakes. With $t_b/R = 0.003$, as shown in Figure 5.26(c), the maximum resultant overturning moment remains at about $0.1GH_l$ and $0.07GH_l$ respectively when t_s/R increases from 0.001 to 0.002 when subjected to both the El Centro and the Saguenay-2 earthquakes. In both cases, it decreases to about $0.065GH_l$ when t_s/R is equal to 0.003. For intermediate liquid-tank systems, as presented in Table 5.3, the convective and the impulsive effects of the liquid motion are generally of similar magnitude. Because of the superposition

effect of the different components of the liquid motion, the maximum resultant overturning moment in the cases that survive the ground excitation varies in a range from $0.065GH_l$ to $0.15GH_l$. For the failed cases, as illustrated in Figure 5.24 and 5.25, large resultant overturning moment is generated from the impulsive and the convective effects of the liquid motion. It is observed that for liquid-tank systems with a more flexible structure, the failure is likely from the impulsive effect of the liquid motion because it can not stand the impulsive load. On the other hand, for liquid-tank systems with a more rigid structure, the failure is more likely from the convective effect of the liquid motion.

Because of the amount of the cases failed in the parametric study, detailed discussion on the trend of variation for the liquid motion in terms of the varying parameters t_b/R and t_s/R could not be conducted. However, it is noted from the results that the base plate has an important influence on the liquid motion. With a thicker base plate, the liquid motion is likely to be less intense because the dynamic uplift response of the tank structure is better restrained. As an example, for the cases with $t_b/R = 0.001, 0.002$ and 0.003 , the number of the tank failure due to large resultant overturning moment from the intense liquid motion is 7, 5, and 3 respectively. On the other hand, the thickness of the shell wall is observed to have no significant influence on the liquid motion.

According to the design codes, the maximum resultant overturning moment for the liquid-tank system analyzed in the parametric study is estimated to be $0.083GH_l$. Comparing with the results obtained from the parametric study, the design value greatly underestimates the seismic load experienced by unanchored liquid-tank systems.

Base Plate Uplift

The time histories of the vertical uplift at the rim of the base plate for MT-1 and MT-9 subjected to the Northridge earthquakes, and MT-1 subjected to the Saguenay-2 earthquake are shown in Figures 5.27 and 5.28 respectively. In the case of MT-9 subjected to the Saguenay-2 earthquake, no base plate uplift occurs. Comparing the time histories of the base plate uplift with the time histories of the corresponding impulsive hydrodynamic pressure, it is observed that the large impulsive hydrodynamic pressure in the failed cases is related to the uplift mechanism. It is also found from the results that in the case of MT-1 subjected to the Northridge earthquake, intense convective liquid motion is induced in response to the base plate uplift.

The results of the maximum base plate uplift of the cases that survive the earthquake ground excitations are shown in Figure 5.29. For liquid-tank systems with $t_b/R = 0.001$, as shown in Figure 5.29(a), the maximum base plate uplift increases from $9.5mm$ to $17.1mm$, and then decreases to $10.1mm$ with t_s/R increasing from 0.001 to 0.003 when subjected to the El Centro earthquake. With $t_b/R = 0.002$, as shown in Figure 5.29(b), the maximum base plate uplift remains at constant about $15mm$ when t_s/R increases from 0.001 to 0.002 , and decreases to less than $4mm$ when t_s/R reaches 0.003 when subjected to both the El Centro and the Saguenay-1 earthquakes. With $t_b/R = 0.003$, as shown in Figure 5.29(c), the maximum base plate uplift decreases from $14.2mm$ to zero with t_s/R increasing from 0.001 to 0.003 . For the failed cases, as presented in Table 5.3, the base plate uplift is significant prior to the failure with the magnitude usually larger than $40mm$. As can be observed, the failure typically occurs in liquid-tank systems with $t_b/R = 0.001$, and 0.002 , i.e., the liquid-tank systems with more flexible base plate. It suggests that with thicker base plate, the uplift mechanism is more efficiently restrained, and the liquid-tank system becomes safer.

Compressive Axial Stress

The time histories of the axial membrane stress near the bottom of the shell wall on the excitation axis $\theta = 0$ for MT-1 and MT-9 subjected to the Northridge and the Saguenay-2 earthquakes are shown in Figures 5.30 and 5.31. From the figures, it is observed that prior to the failure, very large compressive axial membrane stress exists in the shell wall. This large compressive stress is a result from the narrowed contact arc due to the significant base plate uplift.

The results of the maximum compressive axial stress, with the effect of flexural mechanism considered, developed in the shell wall of the cases that survive the earthquake ground excitations are shown in Figure 5.32. From the figure, it is noted that the compressive axial stress decreases from larger than $1 \times 10^8 Pa$ to about $0.4 \times 10^8 Pa$ when t_s/R increases from 0.001 to 0.002, then to about $0.3 \times 10^8 Pa$ when t_s/R is equal to 0.003. As discussed previously, the decrease of the compressive axial stress in terms of the increasing t_s/R is due to the increased thickness of the shell wall which provides stronger restraining effect to the deformation of the shell wall.

For the failed cases, as presented in Table 5.3, the compressive axial stress usually has very large magnitude because of the large resultant overturning moment and the significant deformation of the shell wall associated with the base plate uplift. It is noted that in some cases that the tank has thick shell wall and thin base plate. the compressive axial stress is small prior to the failure. For example, in the case of MT-3($t_b/R = 0.001, t_s/R = 0.003$) subjected to the Northridge and the Saguenay-2 earthquakes, the maximum compressive axial stress is $0.35 \times 10^8 Pa$ and $0.51 \times 10^8 Pa$ respectively. It implies that if the shell wall is strong, and the base plate is weak which offers little resistance to base plate uplift, the liquid-tank system could be overturned without buckling failure of the shell wall.

5.3.3 Tall Liquid-Tank System

The time histories of the seismic response of TT-1 and TT-9 subjected to the El Centro and the Saguenay-2 earthquakes are presented in this section. The maximum seismic response of the tall liquid-tank systems are summarized in Table 5.4. Among the cases analyzed, the results of 24 cases indicate failure of the liquid-tank system. The results presented in Table 5.4 for the failed cases are the maximum seismic response developed in the liquid-tank systems prior to the failure.

Liquid Motion

Impulsive liquid motion

The time histories of the impulsive hydrodynamic pressure at the mid-height level of the shell wall for TT-1 and TT-9 subjected to the El Centro and the Saguenay-2 earthquakes are shown in Figures 5.33 and 5.34. When subjected to the El Centro earthquake, TT-1 fails at 2.12sec with the impulsive hydrodynamic pressure larger than 10kPa and the corresponding ground excitation $\ddot{a}_g = 0.24g$ (after scaling). When subjected to the Saguenay-2 earthquake, it fails at 1.510sec with the impulsive hydrodynamic pressure larger than 20kPa and $\ddot{a}_g = 0.146g$ (after scaling). On the other hand, TT-9 survives the earthquake excitation. It is observed from the results that the variation of the impulsive hydrodynamic pressure is similar to the corresponding ground excitations and the peak impulsive hydrodynamic pressure occurs at the same time of the peak ground excitation. It is also noted that liquid-tank systems with a more flexible tank structure is vulnerable to the seismic load because it develops significant deformation and induces intense impulsive liquid motion.

The results of the maximum impulsive hydrodynamic pressure of the cases that survive the earthquake ground excitations are shown in Figure 5.35. For the survived cases, the magnitude of the maximum impulsive hydrodynamic pressure is constant

about $10kPa$. For the failed cases, as presented in Table 5.4, the maximum impulsive hydrodynamic pressure varies from $17.31kPa$ to $33.86kPa$. It indicates that intense impulsive liquid motion is developed in these cases prior to the failure.

Convective liquid motion

The time histories of the equivalent wave height at the liquid free surface on the excitation axis $\theta = 0$ for TT-1 and TT-9 subjected to the El Centro and the Saguenay-2 earthquakes are shown in Figures 5.36 and 5.37. The equivalent wave height of TT-1 subjected to the El Centro earthquake jumps to about $7m$ prior to the failure. When subjected to the Saguenay-2 earthquake, the equivalent wave height of TT-1 varies gradually with the magnitude about $0.5m$ till the failure. The equivalent wave height of TT-9 subjected to both the El Centro and the Saguenay-2 earthquakes varies smoothly with the magnitude less than $0.7m$. It is observed that a liquid-tank system with more flexible tank structure is likely to have intense convective liquid motion developed in response to large seismic load.

The results of the maximum equivalent wave height of the cases that survive the earthquake ground excitations are shown in Figure 5.38. As shown in the figure, the magnitude of the maximum equivalent wave height for the survived cases varies from $0.1m$ to $2m$ with the impulsive liquid motion of similar magnitude. The variation of the maximum equivalent wave height shows no clear relation to the varying parameters t_b/R and t_s/R . It suggests that the convective liquid motion is greatly affected by the temporary seismic response of the liquid-tank system. For the failed cases, as presented in Table 5.4, the magnitude of the equivalent wave height varies from $0.309m$ to $50.173m$. From the results, it is observed that tall liquid-tank systems may be overturned without intense convective liquid motion.

Resultant overturning moment

The time histories of the resultant overturning moment generated from the liquid mo-

tion for TT-1 and TT-9 subjected to the El Centro and the Saguenay-2 earthquakes are shown in Figures 5.39 and 5.40. As shown in the figures, for both TT-1 and TT-9, the resultant overturning moment is observed to be dominated by the impulsive effect of the liquid motion.

The results of the maximum resultant overturning moment of the cases that survive the earthquake ground excitations are shown in Figure 5.41. For the survived cases, the maximum resultant overturning moment varies from $0.06GH_l$ to $0.09GH_l$ except in the case of TT-5 subjected to the El Centro earthquake with $0.174GH_l$. It is the only case that survives the ground excitation and has important convective effect of the liquid motion in the seismic response. For tall liquid-tank systems, as shown in the results, the dominant loading effect is the impulsive effect of the liquid motion. The explanation is that the effect of the impulsive liquid motion is applied at a higher level and generates large overturning moment. The tall liquid-tank system is so unstable, it could not stand the large resultant overturning moment from the intense impulsive liquid motion and is overturned before significant structure deformation is developed. Thus, the convective liquid motion is not developed and shows no significant effect on the seismic response. For tall liquid-tank systems, the failure is so common that discussion about the variation of liquid motion in terms of varying t_s/R and t_b/R is not possible. However, it is observed from the results that for a liquid-tank system with a thicker base plate, the liquid motion is likely to be less significant and the liquid-tank system is safer.

From the design codes, the overturning moment for the tall liquid-tank system considered is estimated to be $0.095GH_l$. As it shows, this value greatly underestimates the seismic load experienced by unanchored liquid-tank systems.

Base Plate Uplift

The time histories of the vertical uplift at the rim of the base plate for TT-1 and TT-9 subjected to the El Centro earthquake, and for TT-1 subjected to the Saguenay-2 earthquake are shown in Figures 5.42 and 5.43. Prior to the failure, the base plate uplift is 180mm and 160mm respectively in the cases of TT-1 subjected to the El Centro and the Saguenay-2 earthquakes. The large impulsive hydrodynamic pressure occurred prior to the failure is observed to be related to the significant coupling effect of the structure response caused by the large base plate uplift. In the cases of TT-9 subjected to the ground excitations, the base plate uplift is very small and shows no significant influence on the liquid motion.

The results of the maximum base plate uplift of the cases that survive the earthquake ground excitations are shown in Figure 5.44. The base plate uplift is less than 14mm for the survived cases. For the failed cases, as presented in Table 5.4, the base plate uplift is larger than 60mm prior to the failure. As discussed, the large base plate uplift causes rotation of the shell wall which results in intense impulsive liquid motion. From the results, the base plate uplift is observed to be very common. It indicates that tall liquid-tank systems are unstable and easy to be lifted up. It should be noted that under similar resultant overturning moment, the base plate uplift experienced by a liquid-tank system with thicker base plate may be even larger than that with thinner base plate, as in the cases subjected to the Saguenay-1 earthquake, TT-6($t_b/R = 0.002$) has maximum base plate uplift 0.8mm with maximum resultant overturning moment $0.0641GH_l$, while TT-9($t_b/R = 0.003$) has maximum uplift 6.8mm with maximum overturning moment $0.0633GH_l$. It implies that for tall liquid-tank systems, the uplift mechanism could not be efficiently resisted by the stiffness of the base plate only. However, it is noted from the results that with thicker base plate, the tank failure is less likely to occur under the seismic load.

Compressive Axial Stress

The time histories of the axial membrane stress near the bottom of the shell wall on the excitation axis $\theta = 0$ for TT-1 and TT-9 subjected to the El Centro and the Saguenay-2 earthquakes are shown in Figures 5.45 and 5.46. It is noted that in response to significant base plate uplift, the compressive axial stress becomes much larger, as in the cases of TT-1 subjected to the El Centro and the Saguenay-2 earthquakes, the resultant overturning moment is $-0.352GH_l$ and $0.35GH_l$ respectively prior to the failure, whereas the corresponding compressive and tensile stress is $45kPa$ and $25kPa$ respectively. In the case of TT-9 subjected to the El Centro earthquake, the compressive axial stress is also observed to be more intensive than the tensile stress because of the narrowed contact arc due to the base plate uplift at the time of large resultant overturning moment.

The results of the maximum compressive axial stress, with the effect of the flexural mechanism considered, developed in the shell wall of the cases survive the earthquake ground excitations are shown in Figure 5.47. In the results, the maximum compressive axial stress shows a trend of decrease with increasing t_s/R for the survived cases. As discussed previously, the decrease of the axial stress is from the increased thickness of the shell wall which provides more efficient restraining effect to the deformation of the shell wall. For the failed cases, large compressive axial stress is usually developed prior to the failure. Nevertheless, it is noted that for some cases, especially for liquid-tank systems with thick tank shell wall, the compressive axial stress could be very small, as in the case of TT-6($t_s/R = 0.003$) subjected to the Saguenay-2 earthquake, the maximum compressive axial stress is just $0.2767 \times 10^8 Pa$ prior to the failure. This suggests that the tall liquid-tank system may be overturned without shell wall buckling.

5.4 Summary

In the parametric study, the seismic response of unanchored liquid-tank systems, with variant structure flexibilities and different liquid height to tank radius ratios, subjected to different ground excitations has been studied. From the results, the following conclusion are derived

1. The dynamic response of the tank structure for unanchored liquid-tank systems is determined by the uplift mechanism, in which the base plate is partially uplifted in response to large resultant overturning moment and the shell wall rotates about the base-wall joint due to the interaction between the base plate and the shell wall.
2. The base plate is observed to have important influence on the dynamic response of an unanchored liquid-tank system. For unanchored liquid-tank systems, as shown in the results, the base plate should not be thinner than the shell wall.
3. The convective hydrodynamic behaviour can be very intense for unanchored liquid-tank systems. Due to the lack of anchorage, unanchored liquid-tank systems may experience significant structure deformation and become very flexible resulting in intense convective hydrodynamic behaviour. The intense convective hydrodynamic behaviour may become the dominant effect of the seismic loading and cause large resultant overturning moment.
4. For intermediate and tall liquid-tank systems with the liquid height equal or larger than the tank radius, the effect of the liquid motion leads to large overturning moment and significant base plate uplift. These kinds of liquid-tank systems are vulnerable to the seismic load.

5. Due to the effect of the uplift mechanism, the liquid motion is much more intense and generates much larger resultant overturning moment for unanchored liquid-tank systems than that defined in the design codes based on anchored liquid-tank system models.
6. Earthquakes with different frequency content but same intensity are observed to cause seismic response with similar magnitude for unanchored liquid-tank systems. The explanation is that the seismic response of unanchored liquid-tank systems is dominated by the uplift mechanism which is mainly affected by the intensity of the seismic load.
7. The compressive axial stress in the shell wall decreases with thicker shell wall because the increased thickness of the shell wall and more efficient restraining effect to the deformation. For liquid-tank systems with thick shell wall, the compressive axial stress can be very small prior to the failure, which indicates that the liquid-tank system may be overturned without shell buckling.

Broad: $R = 5.760m, H_l = 5.760m$				
	t_s/R	0.001	0.002	0.003
t_b/R	$t_b \backslash t_s$ (mm)	5.760	11.520	17.280
0.001	5.760	BT-1	BT-2	BT-3
0.002	11.520	BT-4	BT-5	BT-6
0.003	17.280	BT-7	BT-8	BT-9
intermediate: $R = 5.032m, H_l = 7.548m$				
	t_s/R	0.001	0.002	0.003
t_b/R	$t_b \backslash t_s$ (mm)	5.032	10.064	15.096
0.001	5.032	MT-1	MT-2	MT-3
0.002	10.064	MT-4	MT-5	MT-6
0.003	15.096	MT-7	MT-8	MT-9
Tall: $R = 4.572m, H_l = 9.144m$				
	t_s/R	0.001	0.002	0.003
t_b/R	$t_b \backslash t_s$ (mm)	4.572	9.144	13.716
0.001	4.572	TT-1	TT-2	TT-3
0.002	9.144	TT-4	TT-5	TT-6
0.003	13.716	TT-7	TT-8	TT-9

Table 5.1: Dimension and Numbering of Liquid-Tank System

Case No.	Earthquake	p_i (kPa)	w_h (m)	u (mm)	M_o (G H _l)	σ_c (10 ⁸ Pa)	Type
BT-1	El Centro	12.16	1.812	16.9	0.1553	3.1307	Con.
	Northridge	11.58	1.182	4.1	0.0614	1.5380	Imp.
	Saguenay-1	9.36	1.483	13.0	0.1357	2.6470	Con.
	Saguenay-2	16.83	12.830	76.2	0.4123	5.9789	Fail
BT-2	El Centro	11.12	0.473	2.7	0.0669	0.4086	Imp
	Northridge	11.44	0.456	4.6	0.0781	0.5081	Imp
	Saguenay-1	12.33	1.045	0	0.0702	0.1597	I. & C.
	Saguenay-2	20.78	34.980	171.4	1.4926	2.0106	Fail
BT-3	El Centro	37.53	3.449	22.3	0.2253	0.6492	Con.
	Northridge	31.66	5.906	50.9	0.3188	0.6490	Con.
	Saguenay-1	52.02	4.993	30.4	0.2834	0.4987	Con.
	Saguenay-2	16.09	26.020	52.3	0.3542	0.6673	Fail
BT-4	El Centro	10.09	0.969	1.1	0.0542	1.8533	I. & C.
	Northridge	10.83	0.326	1.1	0.0541	1.8515	Imp.
	Saguenay-1	19.59	1.387	9.4	0.1069	2.1144	Con.
	Saguenay-2	14.80	1.210	1.8	0.0601	1.0010	I. & C.
BT-5	El Centro	9.69	5.793	10.2	0.1408	1.3547	Con.
	Northridge	12.66	0.405	0	0.0644	0.4074	Imp.
	Saguenay-1	35.45	7.540	30.2	0.2458	1.3788	Con.
	Saguenay-2	11.10	1.456	0	0.0707	0.3911	I. & C.
BT-6	El Centro	11.62	0.249	0	0.0640	0.1532	Imp.
	Northridge	11.74	0.267	1.3	0.0590	0.6314	Imp.
	Saguenay-1	12.38	0.380	0	0.0721	0.2707	Imp.
	Saguenay-2	42.40	31.640	71.7	0.4918	2.5200	Con.
BT-7	El Centro	10.45	0.421	0.8	0.0587	2.5459	Imp.
	Northridge	9.41	1.510	0.5	0.0555	1.1504	I. & C.
	Saguenay-1	12.29	0.132	0	0.0663	2.2040	Imp.
	Saguenay-2	10.09	3.008	1.1	0.0755	1.8345	I. & C.
BT-8	El Centro	9.97	2.170	2.9	0.1222	1.2238	Con.
	Northridge	12.35	0.636	0	0.0628	0.5016	Imp.
	Saguenay-1	9.80	0.049	0	0.0597	0.5009	Imp.
	Saguenay-2	10.37	0.861	0	0.0683	0.4669	I. & C.
BT-9	El Centro	10.68	0.523	0	0.0678	0.2690	Imp.
	Northridge	11.89	0.595	0	0.0755	0.2699	Imp.
	Saguenay-1	12.36	0.134	0	0.0701	0.1615	Imp.
	Saguenay-2	13.65	0.139	0	0.0729	0.2618	Imp.

p_i : Maximum impulsive hydrodynamic pressure;

w_h : Maximum equivalent wave height;

u : Maximum vertical uplift at the rim of the tank base plate;

σ_c : Maximum compressive axial stress;

M_o : Maximum resultant overturning moment;

Type: Type of the dominate behaviour of the resultant overturning moment.

Table 5.2: Seismic Behaviour of Broad Liquid-Tank System

Case No.	Earthquake	p_i (kPa)	w_h (m)	u (mm)	M_o (G H _l)	σ_c (10 ⁸ Pa)	Type
MT-1	El Centro	10.23	1.282	9.5	0.0646	1.7310	I. & C.
	Northridge	26.32	1.308	121.4	0.4317	7.7712	Fail
	Saguenay-1	13.01	0.641	18.1	0.0848	2.9911	I. & C.
	Saguenay-2	16.21	3.743	68.3	0.3085	4.5456	Fail
MT-2	El Centro	10.69	1.348	17.1	0.1222	0.3431	I. & C.
	Northridge	24.94	2.192	39.8	0.1636	1.0611	Fail
	Saguenay-1	16.85	6.569	119.7	1.4010	1.0622	Fail
	Saguenay-2	17.97	16.244	99.2	0.4653	1.0613	Fail
MT-3	El Centro	10.65	1.195	10.1	0.1199	0.1603	I. & C.
	Northridge	19.13	24.210	168.0	0.7426	0.3553	Fail
	Saguenay-1	10.47	3.365	25.4	0.1872	0.5820	I. & C.
	Saguenay-2	16.63	4.031	65.2	0.3068	0.5163	Fail
MT-4	El Centro	10.67	3.104	14.2	0.1019	3.9744	I. & C.
	Northridge	17.33	37.560	114.0	0.5299	5.7494	Fail
	Saguenay-1	9.96	1.748	17.5	0.1133	4.7411	I. & C.
	Saguenay-2	18.57	4.300	58.1	0.2501	6.2030	Fail
MT-5	El Centro	17.27	2.177	17.9	0.1431	0.7392	I. & C.
	Northridge	10.10	1.803	8.9	0.0806	0.4365	I. & C.
	Saguenay-1	9.88	2.354	13.2	0.1458	0.5302	I. & C.
	Saguenay-2	17.32	6.568	54.8	0.3047	2.4449	Fail
MT-6	El Centro	10.90	0.291	3.5	0.0725	0.2117	Imp.
	Northridge	30.69	5.065	87.6	0.5933	2.9671	Fail
	Saguenay-1	12.24	2.194	0	0.0820	0.2024	I. & C.
	Saguenay-2	21.14	38.432	109.2	0.5713	2.3172	Fail
MT-7	El Centro	10.03	3.104	14.2	0.1019	3.2475	I. & C.
	Northridge	10.34	0.456	6.4	0.0662	1.1176	Imp.
	Saguenay-1	10.49	1.752	5.6	0.0738	1.5211	I. & C.
	Saguenay-2	10.57	1.283	6.4	0.0656	1.4341	I. & C.
MT-8	El Centro	10.24	3.108	12.2	0.1083	0.5475	I. & C.
	Northridge	11.40	1.035	14.2	0.1101	0.7611	I. & C.
	Saguenay-1	28.41	25.870	63.2	0.5747	1.8420	Fail
	Saguenay-2	9.48	1.364	0	0.0743	0.2317	I. & C.
MT-9	El Centro	10.67	2.032	1.0	0.0680	0.4175	I. & C.
	Northridge	16.41	5.060	122.3	0.6034	10.2840	Fail
	Saguenay-1	31.77	4.363	88.0	0.9987	1.6137	Fail
	Saguenay-2	9.33	0.278	0	0.0619	0.2192	Imp.

p_i : Maximum impulsive hydrodynamic pressure;

w_h : Maximum equivalent wave height;

u : Maximum vertical uplift at the rim of the tank base plate;

σ_c : Maximum compressive axial stress;

M_o : Maximum resultant overturning moment;

Type: Type of the dominate behaviour of the resultant overturning moment.

Table 5.3: Seismic Behaviour of intermediate Liquid-Tank System

Case No.	Earthquake	p_i (kPa)	w_h (m)	u (mm)	M_o (G H _l)	σ_c (10 ⁸ Pa)	Type
TT-1	El Centro	24.49	7.941	175.2	0.3520	5.0687	Fail
	Northridge	20.75	29.653	185.2	0.4449	3.4729	Fail
	Saguenay-1	9.65	0.359	12.6	0.0540	2.9273	Imp.
	Saguenay-2	20.60	0.569	156.1	0.3148	3.1617	Fail
TT-2	El Centro	24.99	11.327	124.3	0.3077	0.5050	Fail
	Northridge	23.96	3.504	146.3	0.2944	2.4717	Fail
	Saguenay-1	17.31	12.592	99.3	0.2792	2.9142	Fail
	Saguenay-2	24.40	2.594	68.4	0.2757	3.1473	Fail
TT-3	El Centro	21.26	8.675	98.1	0.3215	0.3318	Fail
	Northridge	29.04	7.960	133.4	0.4760	1.8916	Fail
	Saguenay-1	20.11	21.796	127.1	0.6653	0.6864	Fail
	Saguenay-2	27.43	0.581	79.1	0.3448	1.1173	Fail
TT-4	El Centro	26.05	6.058	106.2	0.3446	1.0097	Fail
	Northridge	23.16	0.682	95.5	0.2249	2.7638	Fail
	Saguenay-1	18.65	19.000	61.0	0.2098	6.9168	Fail
	Saguenay-2	10.30	0.309	10.7	0.0662	2.3459	Imp.
TT-5	El Centro	12.23	2.063	12.4	0.1739	1.1110	I. & C.
	Northridge	24.32	50.173	139.7	0.6041	2.6054	Fail
	Saguenay-1	11.07	0.101	6.9	0.0901	1.3370	Imp.
	Saguenay-2	27.11	21.260	156.7	0.3795	1.4188	Fail
TT-6	El Centro	33.86	13.765	117.3	0.4368	0.7786	Fail
	Northridge	26.16	8.461	115.6	0.3991	0.4597	Fail
	Saguenay-1	9.40	0.723	0.8	0.0641	0.7056	Imp.
	Saguenay-2	18.40	13.826	139.5	0.3756	0.2767	Fail
TT-7	El Centro	19.43	2.018	102.2	0.2536	6.6283	Fail
	Northridge	10.85	0.387	1.1	0.0878	1.1236	Imp.
	Saguenay-1	9.01	0.389	4.9	0.0642	1.9392	Imp.
	Saguenay-2	19.91	22.900	61.7	0.2951	3.8179	Fail
TT-8	El Centro	17.34	1.439	102.3	0.2986	1.6682	Fail
	Northridge	19.45	8.230	114.5	0.3972	2.8620	Fail
	Saguenay-1	9.02	0.167	6.8	0.0633	0.9812	Imp.
	Saguenay-2	9.41	0.099	1.5	0.0687	0.9710	Imp.
TT-9	El Centro	9.86	1.059	11.8	0.0834	1.9253	Imp.
	Northridge	18.56	6.720	107.9	0.4524	12.7860	Fail
	Saguenay-1	9.01	0.629	13.2	0.0657	0.5506	Imp.
	Saguenay-2	9.38	1.179	0.2	0.0690	0.4943	Imp.

p_i : Maximum impulsive hydrodynamic pressure;

w_h : Maximum equivalent wave height;

u : Maximum vertical uplift at the rim of the tank base plate;

σ_c : Maximum compressive axial stress;

M_o : Maximum resultant overturning moment;

Type: Type of the dominate behaviour of the resultant overturning moment.

Table 5.4: Seismic Behaviour of Tall Liquid-Tank System

Figure 5.1: Configuration of The Liquid-Tank System in the Parameter Study

St. Ferreol, Longitudinal component
(Saguenay-1)

Chicoutimi Nord, Longitudinal component
(Saguenay-2)

Figure 5.2: Time History of Ground Acceleration for the Saguenay Earthquake

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.3: Time History of Impulsive Pressure for BT-1 and BT-9 (El Centro)

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.4: Time History of Impulsive Pressure for BT-1 and BT-9 (Saguenay-1)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.5: Maximum Impulsive Hydrodynamic Pressure for Broad Liquid-Tank System

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.6: Time History of Wave Height for BT-1 and BT-9 (El Centro)

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.7: Time History of Wave Height for BT-1 and BT-9 (Saguenay-1)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.8: Maximum Equivalent Wave Height for Broad Liquid-Tank System

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.9: Time History of Resultant Overturning Moment for BT-1 and BT-9 (El Centro)

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.10: Time History of Resultant Overturning Moment for BT-1 and BT-9 (Saguenay-1)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.11: Normalized Maximum Resultant Overturning Moment for Broad Liquid-Tank System

Figure 5.12: Time History of Base Plate Uplift for BT-1 (El Centro)

Figure 5.13: Time History of Base Plate Uplift for BT-1 (Saguenay-1)

Figure 5.14: Maximum Base Plate Uplift for Broad Liquid-Tank System

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.15: Time History of Axial Membrane Stress for BT-1 and BT-9 (El Centro)

(a) BT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) BT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.16: Time History of Axial Membrane Stress for BT-1 and BT-9 (Saguenay-1)

Figure 5.17: Maximum Compressive Axial Stress for Broad Liquid-Tank System

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.18: Time History of Impulsive Pressure for MT-1 and MT-9 (Northridge)

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-1 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.19: Time History of Impulsive Pressure for MT-1 and MT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.20: Maximum Impulsive Hydrodynamic Pressure for Intermediate Liquid-Tank System

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.21: Time History of Wave Height for MT-1 and MT-9 (Northridge)

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-1 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.22: Time History of Wave Height for MT-1 and MT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.23: Maximum Equivalent Wave Height for Intermediate Liquid-Tank System

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-1 ($t_b/R=0.009$, $t_s/R=0.009$)

Figure 5.24: Time History of Resultant Overturning Moment for MT-1 and MT-9 (Northridge)

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-1 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.25: Time History of Resultant Overturning Moment for MT-1 and MT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.26: Normalized Maximum Resultant Overturning Moment for Intermediate Liquid-Tank System

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.27: Time History of Base Plate Uplift for MT-1 and MT-9 (Northridge)

Figure 5.28: Time History of Base Plate Uplift for MT-1 and MT-9 (Saguenay-2)

Figure 5.29: Maximum Base Plate Uplift for Intermediate Liquid-Tank System

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.30: Time History of Axial Membrane Stress for MT-1 and MT-9 (Northridge)

(a) MT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) MT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.31: Time History of Axial Membrane Stress for MT-1 and MT-9 (Saguenay-2)

Figure 5.32: Maximum Compressive Axial Stress for Intermediate Liquid-Tank System

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.33: Time History of Impulsive Pressure for TT-1 and TT-9 (El Centro)

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.34: Time History of Impulsive Pressure for TT-1 and TT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.35: Maximum Impulsive Hydrodynamic Pressure for Tall Liquid-Tank System

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.36: Time History of Wave Height for TT-1 and TT-9 (El Centro)

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.37: Time History of Wave Height for TT-1 and TT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.38: Maximum Equivalent Wave Height for Tall Liquid-Tank System

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.39: Time History of Resultant Overturning Moment for TT-1 and TT-9 (El Centro)

(a) TT-1 ($t_b/R=0.001$, $t_g/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_g/R=0.003$)

Figure 5.40: Time History of Resultant Overturning Moment for TT-1 and TT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.41: Normalized Maximum Resultant Overturning Moment for Tall Liquid-Tank System

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.42: Time History of Base Plate Uplift for TT-1 and TT-9 (El Centro)

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.43: Time History of Base Plate Uplift for TT-1 and TT-9 (Saguenay-2)

(a) For $t_b/R = 0.001$

(b) For $t_b/R = 0.002$

(c) For $t_b/R = 0.003$

Figure 5.44: Maximum Base Uplift for Tall Liquid-Tank System

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.45: Time History of Axial Membrane Stress for TT-1 and TT-9 (El Centro)

(a) TT-1 ($t_b/R=0.001$, $t_s/R=0.001$)

(b) TT-9 ($t_b/R=0.003$, $t_s/R=0.003$)

Figure 5.46: Time History of Axial Membrane Stress for TT-1 and TT-9 (Saguenay-2)

Figure 5.47: Maximum Compressive Axial Stress for Tall Liquid-Tank System

Chapter 6

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

6.1 Summary and Conclusions

Due to the highly nonlinear uplift mechanism, which develops in response to large seismic load, the seismic behaviour of unanchored liquid-tank systems is quite different from that of anchored systems. Therefore the evaluation of the effect of the uplift mechanism is essential in the seismic resistant design of unanchored liquid-tank systems. Because of the complexity of the problem, the effect of the uplift mechanism is generally estimated by simplified models and the results obtained are not reliable. In the present study, the seismic response of unanchored liquid-tank systems is investigated in detail by the analytical procedure developed by Zeng (1993) in order to obtain a better understanding about the problem.

A detailed time history analysis on the seismic behaviour of unanchored liquid-tank systems is conducted to study the uplift mechanism. The deformation of the tank structure at the time of base plate uplift, the coupling effect between the liquid motion and the dynamic response of the tank structure, and the stresses developed in the partially uplifted tank structure are discussed. The analysis reveals that

1. The base plate uplifts only when the resultant overturning moment from the liquid motion exceeds a critical value, and the uplift mechanism is highly nonlinear to the applied resultant overturning moment.
2. The uplift mechanism dominates the dynamic response of the tank structure. When the tank base plate is partially lifted up, the shell wall rotates about the base-wall joint due to the interaction between the base plate and the shell wall, and the rotation causes much more significant deformation of the shell wall than the shell wall vibration. The liquid-tank systems becomes very flexible at this time due to the significant deformation of the tank structure.
3. The impulsive hydrodynamic behaviour becomes very intense in response to the shell wall rotation resulted from the uplift mechanism because of the coupling effect between the liquid motion and the dynamic response of the tank structure.
4. The convective hydrodynamic behaviour may change its phase angle abruptly and increase its magnitude dramatically in response to intense impulsive seismic load when the tank structure becomes flexible due to the deformation of the tank structure.
5. The uplift mechanism introduces intense stresses in the tank structure. Intense compressive stress exists near the bottom and upper part of the shell wall due to the uplift of the base plate and the out-of-round distortion of the shell wall. In the uplifted region of the base plate, the tensile stress becomes intense because of the membrane mechanism.

A parametric study is conducted to analyze the seismic behaviour of unanchored liquid-tank systems. In the parametric study, unanchored liquid-tank systems with different ratios of the liquid height to the tank radius, variant flexibilities of the shell

wall and the base plate are subjected to ground excitations with different frequency contents. From the results obtained, it is concluded

1. The thickness of the base plate should be considered as an important parameter in the seismic resistant design of unanchored liquid storage tanks, because the base plate is essential in resisting the uplift mechanism and offers restraint to the shell wall. It is recommended that for unanchored liquid storage tanks, the base plate should not be thinner than the shell wall.
2. The ratio of the liquid height to the tank radius is another important parameter in the seismic resistant design of unanchored liquid-tank systems. Because the effect of the uplift mechanism is more significant and the liquid motion causes much larger resultant overturning moment, unanchored tall liquid-tank systems are very unstable and easy to be overturned. It is recommended that liquid-tank systems with the liquid height larger than the tank radius should not be left unanchored.
3. The flexibility of the shell wall is shown to have no essential effect on the seismic response of unanchored liquid-tank systems because the dynamic response of tank structure of unanchored liquid-tank systems is dominated by the uplift mechanism. Nevertheless, the thickness of the shell wall is important in restraining the compressive axial stress, which is the principal cause of shell wall buckling failure.
4. Earthquake ground excitations with different frequency contents but same intensity generally induce liquid motion of similar magnitude for unanchored liquid-tank systems. This is because that the seismic response of unanchored liquid-tank systems is dominated by the uplift mechanism which is influenced by the intensity of the seismic load.

6.2 Recommendations for Future Work

Based on the observations of the present study, future work in the following directions is recommended

1. Develop new analytical procedures for the convective hydrodynamic behaviour.

It is noted in the present study that for unanchored liquid-tank systems, the convective hydrodynamic behaviour can be very intense with the equivalent wave height larger than the liquid height. The “small amplitude sloshing” assumption is no longer applicable for such a case. In the new analytical procedure, the effect of the velocity of the liquid sloshing should be included in the formulation, as well as the effect of the reflection from the tank structure.

2. Study the interaction between the base plate and the shell wall in more detail.

The interaction is essential to the uplift mechanism which is the dominant dynamic response of unanchored liquid-tank systems. It is also essential to the restraining effect of the base plate to the shell wall. In the future work the elasto-plastic behaviour of the base-wall joint should be included in modelling the dynamic response of the tank structure.

3. Develop multi-directional analytical procedure for the time history analysis.

Because the seismic response of unanchored liquid-tank systems is highly non-linear, an analytical procedure capable of analyzing the seismic response of unanchored liquid-tank systems in two directions is needed to get more accurate results.

References

American Petroleum Institute, 1988. Seismic design of storage tanks. *Welded Steel Tanks for Oil Storage*. API 650, Appendix E, Washington, D. C.

American Waterworks Association, 1984. *AWWA Standard for Welded Steel Tanks for Water Storage*. AWWA-D100, Denver, Colorado.

Auli, W., F. D. Fischer and F. G. Rammerstorfer, 1985. Uplifting of Earthquake-loaded liquid-filled tanks. *Proceedings of the Pressure Vessels and Piping Conference, ASME*, **98**, 71-85.

Balendra, T. and W. A. Nash, 1978. *Earthquake analysis of a cylindrical Liquid storage Tank with a Dome by Finite Element Method*. Research report, Department of civil Engineering, University of Massachusetts.

Balendra, T., K. K. Ang, P. Paramasivam and S. L. Lee, 1982. Seismic design of flexible cylindrical liquid storage tanks. *Earthquake Eng. and Structural Dynamics*, **10**, 477-496.

Barton, D. C. and J. V. Parker, 1987. Finite element analysis of the seismic response of anchored and unanchored liquid storage tanks. *Earthquake Eng. and Structural Dynamics*, **15**, 299-322.

Benuska, L. (editor), 1990. *Loma Prieta Earthquake Reconnaissance Report, Earthquake Spectra*. EERI, supplement to Vol. 6.

- Bertero, V. V., 1982. Seismic resist design of cylindrical liquid storage tanks. Civil Engineering, University of California, Berkeley, California. 1982.
- Butler T. A., J. G. Bennett, C. D. Babcock and S. Natsiavas, 1987. Response of Seismic Category I Tanks to Earthquake Excitation. In U. S. Nuclear Regulatory Commission Report no. NUREG/CR-4776.
- Chiba, M., 1993. Nonlinear hydroelastic vibration of a cantilever cylindrical tank-i. Experiment(empty case)*Int. J. Nonlinear Mech.*, **28**, 591-599.
- Chiba, M., 1993. Nonlinear hydroelastic vibration of a cantilever cylindrical tank-ii. Experiment(liquid-filled case)*Int. J. Nonlinear Mech.*, **28**, 601-612.
- Clough, D. P., 1977. *Experimental Evaluation of Seismic Design Methods for Broad Cylindrical Tanks*. Report No. 77/10, Earthquake Engineering Research Center, University of California at Berkeley.
- Epstein, H. I., 1976. Seismic design of liquid storage tanks. *J. of the Structural div. ASCE*, **102**, 1659-1673.
- Hanson, R. D., 1973. Behaviour of Liquid-Storage Tanks. *The Great Alaska Earthquake of 1964*. National Academy of Sciences, Washington, D. C., 331-339.
- Haroun, M. A. and G. W. Housner, 1981. Seismic design of liquid storage tanks. *J. of the Mechanical Councils of ASCE*, **107**, 191-207.
- Haroun, M. A., H. S. Badawi and C. B. Nada, 1987. Nonlinear uplift analysis of crescent-shaped plate. *Proceedings of the Pressure Vessels and Piping Conference, ASME*, 317-324.
- Hoskins, L. M. and L. S. Jacobsen, 1934. Water pressure in a tank caused by a simulated earthquake. *Bulletin of the Seismological society of America*, **34**, 1-32.

Housner, G. W., 1957. Dynamic pressures on accelerated fluid containers. *Bulletin of the Seismological Society of America*, **53**, 381-387.

Ishida, K. and N. Kobayashi, 1988. An effective method of analyzing rocking motion for unanchored cylindrical tanks including uplift. *J. of Pressure Vessel Tech., Trans. of ASME*, **110**, 1087-1100.

Jacobsen, L. S., 1949. Impulsive hydrodynamics of liquid inside a cylindrical tank subjected to transient motions. *Bulletin of the Seismological society of America*, **39**, 189-204.

Jennings, P. C. (editor), 1971. Engineering Features of the San Ferando Earthquake, February 9, 1871. In Earthquake Engineering Research Laboratory Report no. EERL 71-02, California Institute of Technology, Pasadena, California.

Kawano, K., T. Oda, K. Yoshida, S. Yamamoto, S. Shibuya and S. Yamada, 1980. Damages of Oil Storage Tanks for Off Miyagi Prefecture Earthquake of June 12, 1978. in *Proc. of 7th World Conference on Earthquake Engineering*, **8**, 507-510. Istanbul, Turkey.

Lau, D. T. and R. W. Clough, 1989. Static Tilt Behaviour of Unanchored Cylindrical Tanks. Report No. 89/11, Earthquake Engineering Research Center, University of California at Berkeley.

Lau, D. T. and R. W. Clough, 1991. Parameter studies of statically tilted unanchored cylindrical tanks. *6Th Canadian Conf. Earthquake Engineering*, Toronto, 229-236.

Lau, D. T. and X. Zeng, 1991. Simplified uplift analysis of unanchored cylindrical tanks. *Proceedings of the Third U. S. Conference on Lifeline Earthquake Engineering*, 975-984.

- Lau, D. T. and X. Zeng, 1992. Hydrodynamic forces in uplifting cylindrical tanks. *Fluid-Structure vibration and Sloshing, ASME*. **232**, 39-44.
- Lau, D. T., 1994. *Performance of Lifeline Facilities during the January 17, 1994 Northridge Earthquake*, Preliminary Report on the Northridge, California. Earthquake of January 17, 1994. Reconnaissance Team Canadian Association for Earthquake Engineering.
- Lau, D. T. and X. Zeng, 1995. Nonlinear Behaviour of bottom plate in cylindrical liquid storage tanks for seismic applications. *Canadian J. of civil Engineering*. **22**, 180-189.
- Lau, D. T. and X. Zeng, 1996. Generation and use of simplified uplift shape function in unanchored cylindrical tanks. *J. of Pressure Vessel Tech., Trans. of ASME*. **118**, 278-286.
- Leeds, D. J. (editor), 1980. *Imperial County, California Earthquake, October 15, 1979*. Reconnaissance Report, Earthquake Engineering Research Institute, February.
- Manos, G. C. and R. W. Clough, 1982. *Further Study of the Earthquake Response of a Broad Cylindrical Liquid Storage Tank MOdel*. Report No. 82/07, Earthquake Engineering Research Center, University of California at Berkeley.
- Manos, G. C. and R. W. Clough, 1985. Tank Damage During the May 1983 Colinga Earthquake. *Earthquake Engineering and Structure Dynamics*, **13**, 449-466.
- Natsiavas, S., 1988. *Response and Failure of Fluid-Filled Tanks under Base Excitation*. report No. 86/26, California Institute of Technology, Pasadena, California.
- Niwa, A., 1978. *Seismic Behaviour of Tall Liquid-Storage Tanks*. Report NO. 78/04. Earthquake Engineering Research Center, University of California at Berkeley.

- Peek, R., and P. C. Jennings, 1988. Simplified analysis of unanchored tanks. *Earthquake Eng. and Structural Dynamics*, **16**, 1073-1085.
- Priestly, M. J. N., B. J. Davidson, G. D. Honey, D. C. Hopkins, R. J. Martin, G. Ramsey, J. V. Vessey and J. H. Wood, 1986. *Seismic Design of Storage Tanks*. New Zealand National Society for Earthquake Engineering, Wellington.
- Rammerstorfer, F. G., K. Scharf, F. D. Fischer and R. Seeber, 1988. Collapse of earthquake excited tanks. *Res Mechanic J.*, **25**, 129-143.
- Rinne, J. E., 1967. Oil Storage Tanks, The Prince William Sound, Alaska Earthquake of 1964 and Aftershocks. In U. S. Dept. of Commerce, E.S.S.A., U. S. Coast and Geodetic Survey, Washington, D. C., 245-252.
- Sakai, F., A. Isoe, Hi Hirakawa and Y. Mentani, 1989. Large scale tilt tests of seismic behaviour of flat-bottomed cylindrical liquid storage tanks. *ASME Pressure Vessels Piping Div Publ PVP*, **157**, 69-74.
- Scharf, K., F. G. Rammerstorfer and F. D. Fischer, 1989. The scientific background of the Austrian recommendation for earthquake resistant liquid storage tank design. *Structural Dynamics and Soil-Structure Interaction*, 281-313.
- Sun, S., 1985. Seismic Damages of Liquid Tank and Its Analysis of Aseismic Design. In *Proc. of the 1985 Pressure Vessels and Piping Conference*, PVP 98-7 Fluid-Structure Dynamics. pages 61-67. ASME, New Orleans, Louisiana, June 23-26.
- Ueno, M., 1968. Towers, Tanks, Stacks, etc.. *General Report on Niigata Earthquake of 1964*, 345-354. Tokyo Electrical Engineering College Press.
- Veletsos, A. S., 1974. Seismic effects in flexible liquid storage tanks. *Proceedings of the International Association for Earthquake engineering, Fifth World conference*. Rome, **1**, 630-639.

- Veletsos, A. S. and J. Y. Yang, 1976. Dynamics of fixed-based liquid storage tanks. *Proceedings of US-Japan Seminar for Earthquake engineering Research with Emphasis on Lifeline systems*, Tokyo, 317-341.
- Veletsos, A. S. and J. Y. Yang, 1977. Earthquake response of liquid storage tanks. *Advances in Civil Engineering through Engineering Mechanics Division Speciality Conference*, ASCE, North Carolina, 1-24.
- Veletsos, A. S., 1984. Seismic response and design of liquid storage tanks. *Guideline for the Seismic Design of Oil and Gas Pipeline systems*, ASCE Technical Council on Lifeline Earthquake Engineering, 255-365, NY.
- Veletsos, A. S. and A. Kumar, 1984. Numerical response of vertically excited liquid storage tanks. *Proceedings of the Eighth World conference in Earthquake engineering*, San Francisco, California, **7**, 453-460.
- Wozniak, R. S. and W. W. Mitchell, 1978. Basis of seismic design provisions for welded steel oil storage tanks. *Proceedings-Refinings Department of American Petroleum Institute*, **57**, 485-501.
- Yamada, S., T. Oda, K. Yoshida, S. Yamamoto, K. Kawano and S. Shibuya, 1980. Damages Analysis of Oil Storage Tanks for Off Miyagi Prefecture Earthquake of June 12, 1978. in *Proc. of 7th World Conference on Earthquake Engineering*, **8**, 507-510. Istanbul, Turkey.
- Yang, J. Y., 1976. *Dynamic behaviour of fluid-tank systems*. PhD thesis, Rice University, Houston, TX.
- Yi, W. and S. Natsiavas, 1993. Seismic response of unanchored fluid-filled tanks using finite elements. *J. of Pressure Vessel Tech., Trans. of ASME*, **114**, 74-79.

Zeng, X., 1993. *Earthquake Response Analysis of Unanchored Cylindrical Liquid Storage Tanks*. PhD thesis, Carleton University, Ottawa, Canada.

Zui, H., T. Shinke, and A. Nishimure, 1985. Seismic response analysis and dynamic model tests of cylindrical tanks. *Doboku Gakkai Rombun HoKoKushu*, 441-450.