**THE LOCAL BUCKLING RESPONSE OF INTERNALLY
CORRODED STEEL PIPELINES UNDER COMBINED LOADS**

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by

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ABSTRACT

A broad parametric study has been conducted using the finite element analysis (FEA) technique to investigate the local buckling response of pipelines with an axial groove corrosion defect under combined loads. The developed model used in the analyses has been validated against experimental tests. A large number of influential parameters such as the diameter to thickness ratio (D/t), axial load ratio (σ_a/σ_y), internal pressure ratio (p/p_y), defect circumferential length ratio (l_c/D), defect depth ratio (d/t) and material properties (stress-strain curves) have been evaluated. Utilizing results from the parametric study, non-dimensional equations for predicting the critical local moment (M_{lc}), critical compressive strain (ϵ_{lc}) and critical local curvature (Φ_{lc}) for pipes with an internal axial groove corrosion defect as a ratio to that of plain pipes have been developed through a multi-variable non-linear regression. Good agreement has been achieved between the predicted buckling (failure) limits (M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios) using the developed equations and the finite element analyses.

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List of Symbols and Abbreviations

a	-	flow area of pipe
A	-	local area of metal loss in the longitudinal plane
A_0	-	local original metal area
A_{10}	-	geometric parameter related to the wall thickness
C	-	material property parameter
CSA	-	Canadian Standards Association
CR	-	collar region
CV	-	coefficient of variation
D	-	outside pipe diameter
d	-	defect depth
E	-	modulus of elasticity
F	-	design factor
F_a	-	applied axial force
FEA	-	finite element analysis
F_y	-	effective specified minimum yield strength

GEOPIG	-	geometry pigging system
GR	-	general region
imp	-	imperfection expressed as percentage of pipe wall thickness
ILI	-	in-line inspection
J	-	joint factor
L	-	pipe length
l_a	-	defect axial length
l_c	-	defect circumferential length
L_f	-	location factor
M	-	bulging stress magnification factor
MAOP	-	maximum allowable operating pressure
MFL	-	magnetic flux leakage
MOP	-	maximum operating pressure
M_g	-	global moment
M_{gc}	-	critical global moment
M_l	-	local moment
M_{lc}	-	critical local moment

M_{lp}	-	critical local moment for plain pipes
n	-	material property parameter
p	-	internal pressure
p_e	-	maximum external pressure
P_F	-	estimated failure pressure
p_i	-	maximum internal pressure
p_y	-	yield pressure of a pipe
R	-	a specific segment length of the pipe
RR	-	refined region
R_n	-	nonlinear regression
R_{n_m}	-	modified R_n
SF	-	safety factor
$SMYS$	-	specified minimum yield stress
$SMTS$	-	specified minimum tensile strength
stand. dev.	-	standard deviation
S_{flow}	-	flow stress
t	-	pipe wall thickness

T	-	temperature factor
TR	-	transition regoin
U	-	constant related to the nonlinear regression
UT	-	ultrasonic technique
V	-	constant related to the nonlinear regression
y	-	average displacement of the extreme compression and tension faces of the pipe wall
α	-	coefficient of thermal expansion
Δc_D	-	shortening of two points at D distance apart on the extreme compressive face of the pipe
Δ_R	-	the deformation of two points over the segment length R measured in a straight line
Δ_T	-	temperature difference
Δt_D	-	lengthening of two points at D distance apart on the extreme tensile face of the pipe
ϵ_{cD}	-	D local compressive strain
ϵ_{cDc}	-	D critical local compressive strain
ϵ_{cf}	-	factored compressive strain

ϵ_c^{crti}	-	ultimate compressive strain
ϵ_e	-	engineering strain
ϵ_{e0}	-	engineering strain at the proportional limit or end of yield plateau
ϵ_{ep}	-	length of yield plateau
ϵ_{er}	-	relative strain at peak stress
ϵ_{lc}	-	critical compressive strain
ϵ_{lp}	-	critical compressive strain for plain pipes
$\epsilon_R \square$	-	R local compressive strain
ϵ_t	-	true strain
ϵ_{tD}	-	D local tensile strain
ϵ_{tp}	-	true plastic strain
ϵ_{tp0}	-	true plastic strain at the proportional limit or end of yield plateau
ϵ_{tpf}	-	true plastic strain at peak engineering stress
ϵ_y	-	strain corresponding to yield stress
θ_t	-	top end rotation angle
θ_b	-	bottom end rotation angle

ν	-	Poisson's ratio
σ_a	-	axial stress
σ_e	-	engineering stress
σ_{e0}	-	engineering stress at the proportional limit or end of yield plateau
σ_{ef}	-	engineering stress at peak stress
σ_{er}	-	ultimate to yield strength ratio (at the proportional limit)
σ_h	-	hoop stress due to internal pressure
σ_p	-	proportional limit of the material
σ_t	-	true stress
σ_{t0}	-	true stress at the proportional limit or end of yield plateau
σ_{tf}	-	true stress corresponding to peak engineering stress
σ_u	-	ultimate strength
σ_y	-	yield strength
Φ_{ec}	-	resistance factor for compressive strain
Φ_g	-	global curvature
Φ_{gc}	-	critical global curvature
Φ_l	-	local curvature

Φ_{lc}	-	critical local curvature
Φ_{lD}	-	D local curvature
Φ_{lDc}	-	D critical local curvature
Φ_{lp}	-	critical local curvature for plain pipes
0.00_{yp}	-	materials with yield plateau length of 0.0
0.02_{yp}	-	materials with yield plateau length of 0.2

Chapter 1 Introduction

As the demand for energy grows in Canada, the exploration of oil and natural gas resources has extended to remote regions such as Arctic and sub-Arctic regions. Buried pipelines have been efficiently used to transport oil and natural gas from production fields to refineries. These pipelines are subjected to axial load due to an extreme variation between construction and operation temperatures, internal pressure to convey the fluid and imposed curvature due to uneven settlements. Many pipelines have been safely operating for more than 50 years. However, a number of failures caused by corrosion have been reported in aging pipelines.

According to a report published by Alberta Energy Regulator (AER 2013), about 880 ruptures were reported to the Oil and Gas Regulator in Alberta between 1999 and 2012. Over 67% of these incidents were caused by corrosion, mostly internal corruptions (80%). Another report published by BC Oil and Gas Commission (BCOGC 2012) found that internal corrosion is the leading cause of failure in oil and gas pipelines. The report indicates that almost 45% of the total incidents reported between 2009 and 2012 in British Columbia were due to corrosion, most of which internal corrosion (67%).

As corrosion continues to degrade the pipe, failure inevitably will occur as the pipe can no longer sustain the high stresses generated from the loading. For highly pressurized corroded pipes, rupture could occur without any local buckling. However, rupture could also occur after the initiation of local buckling at the corroded area due to the combined effect of internal pressure, axial load and bending. After the initiation of buckling, the deformation is localized to the corroded area in the subsequent loading cycles and

eventually fracture occurs. Thus, the initiation of local buckling can also be considered as a limit state for corroded pipelines.

Most studies have focused on bursting of corroded pipes due to internal pressure. There have been only a few studies on the local buckling of corroded pipes. In addition, no equation has been found to have been developed to predict the buckling (failure) limits of corroded pipes for a wide range of material properties. Thus, it is useful to have a more thorough study of local buckling of corroded pipes and to develop equations to predict the buckling (failure) limits.

1.1 Objective of the Thesis

The objective of this thesis is to develop non-dimensional equations that can predict the buckling limits (critical local moment ratio (M_{lc}/M_{lp}), critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lc})) of a pipe segment with an internal axial groove corrosion defect subjected to a combined loading. These equations will consider the effects of pipe geometry (diameter to thickness ratio), axial load, internal pressure and corrosion defect size for a wide range of material properties (stress-strain curve).

1.2 Methodology Used in the Research

Finite element models are developed based on actual pipe test specimens using the commercial finite element software package ABAQUS (2013). The finite element models are validated against results from full-scale tests and subsequently modified to be used in conducting the parametric study. A preliminary study is carried out to investigate the corrosion defect size ranges and material properties to be used in the parametric study.

Based on this study, the finite element analysis parametric study is conducted for a wide range of the diameter to thickness ratio (D/t), axial load to yield axial load ratio (σ_a/σ_y), internal pressure to yield internal pressure ratio (p/p_y), defect circumferential length to diameter ratio (l_c/D), defect depth to thickness ratio (d/t) and ultimate to yield strength ratio (σ_{er}). The results of the parametric study are utilized in generating the non-dimensional equations for predicting the buckling (failure) limits through a multi-variable non-linear regression. These equations are verified against selective cases of geometries, load combinations, corrosion dimensions and material properties.

1.3 Organization of the Thesis

Chapter 2 provides a brief review on corrosion in pipelines as well as the terms used in describing the initiation of local buckling (failure). Provisions in the standards and specifications to account for pipe rupture due to a corrosion defect and critical strain for plain pipes are discussed. In addition, a brief summary of parameters affecting the pipe buckling, and a number of experimental and finite element analysis studies on local buckling are provided.

Chapter 3 consists of the discussions on the finite element model of the pipe specimen and the mesh refinement study to find the optimum mesh configuration to be used in the study. Results from the proposed finite element model are validated against those from the tests.

Chapter 4 discusses the characterization of the shape of stress-strain curve and the generation of material stress-strain curve to be used in the parametric study. In addition, a

preliminary study on the effects of corrosion dimensions and stress-strain curves is presented.

Chapter 5 consists of a description of the parameters considered in the parametric study and a discussion on the influence of each parameter. A set of buckling equations based on the results of the parametric study is developed and validated.

Chapter 6 comprises a summary of the thesis along with conclusions and recommendations for future work.

Materials used in the validation of the finite element analyses and results of the parametric study are presented in the appendices.

Chapter 2 Literature Review

There have been a number of failures occurred in aging pipelines. These pipelines usually failed because of degradation in the pipe wall due to corrosion. Corrosion can be a dominant cause of rupture and local buckling (wrinkling) in pipelines. Several standards and corrosion assessment criteria have considered rupture in the assessment of the integrity of corroded pipelines. However, none of these standards and criteria takes local buckling into consideration. A better understanding of local buckling characterization technique and the parameters affecting this type of failure is needed to better evaluate the integrity of corroded pipelines beside the consideration of rupture.

2.1 Corrosion in Pipelines

Buried pipelines placed in aggressive environment are inevitably exposed to external corrosion attacks. Internal corrosion can occur in a pipeline transporting corrosive liquids and gases. Corrosion, in general, can be defined as the deterioration of a metal due to chemical or electrochemical reactions with its environment (Revie and Uhlig 2008). The surrounding environment for onshore pipelines is ground water or wet soil, while the pipelines that convey oil and natural gas may contain water (H_2O), carbon dioxide (CO_2) and hydrogen sulfide (H_2S) (Fessler 2008). In both cases, corrosion degrades the wall thickness of the pipeline, which eventually leads to a catastrophic failure.

2.1.1 External Corrosion

Unprotected pipelines are susceptible to external corrosion. The outer surface of the pipe is in direct contact with the corrosive surroundings (water and soil). This form of

corrosion can be mitigated by applying coatings and cathodic protection to the pipeline (CAPP 2009a). Coatings are physical barriers that separate the outer surface of the pipe from water and soil. Beside the physical protection, an electrical protection can also be considered. Cathodic protection electrically provides an effective defense of the pipe against corrosion by changing the electrochemical state of the corroded surface. This is done by connecting the pipe wall to a negative terminal of electrical current, which essentially reduces the corrosion rate (Beavers and Thompson 2006).

2.1.2 Internal Corrosion

The presence of water (H_2O), carbon dioxide (CO_2) and/or hydrogen sulfide (H_2S) can lead to severe internal corruptions in oil and gas pipelines (Fessler 2008). This type of corrosion can form due to different mechanisms depending on the fluid transported in the pipelines. The dominant corrosion mechanism of both oil and natural gas pipelines is pitting corrosion. As can be seen in Figs. 2.1 and 2.2, pitting corrosion initiates along the bottom part of a pipe having a corrosive environment (CAPP 2009b; CAPP 2009c). Different practices have been adopted to protect the inner surface of oil and gas pipelines from corrosion. Dehydration is widely used to chemically remove free water from the gas stream. Another practice is adding chemical inhibitors to gas and oil pipelines, which leads to a decrease in the corrosion rate. Plastic and polyethylene liners can also be placed inside a line pipe to protect the internal surface of the pipe wall from the corrosive transported media (Fessler 2008). Since none of the aforementioned systems are 100% successful, monitoring techniques and corrosion assessments are still needed.

2.1.3 Monitoring Techniques

In-line Inspection (ILI) tools or “Smart Pigs” are the most effective monitoring technique used in oil and gas pipelines. Smart pigs are electronic devices used to collect data about pipeline conditions. They move through the pipeline by the product flow. Several types of smart pigs are available in the market, each of which has different tasks. Magnetic Flux Leakage (MFL) pigs and Ultrasonic (UT) pigs are used to identify the size and location of metal losses in pipelines (POF 2009). Geometry pigs (GEOPIG) can be adapted to measure pipeline deformations (strain and curvature) (Hart et al. 2002). The gathered data by these tools are analyzed and utilized in assessing the integrity of corroded pipes.

2.1.4 Classification of Corrosion

The irregularity of its shape makes corrosion difficult to be characterized. In order to classify corrosion, a simplified geometry that can adequately represent the actual corrosion shape is needed. According to the Pipeline Operators Forum (POF 2009), corrosion can be reasonably categorized in accordance with its geometry shown in Fig. 2.3 in terms of axial length (l_a), circumferential length (l_c) and depth (d), and the parameter A_{10} . The parameter A_{10} is equal to 10 mm when the wall thickness (t) is less than 10 mm, or otherwise equal to t . There are seven geometrical classifications as shown in Table 2.1.

2.1.5 Corrosion Assessment Criteria

A number of assessment criteria from specifications and standards have been used to estimate the maximum operating pressure (MOP) of defective pipelines. The American Society of Mechanical Engineers (ASME) published B31G, the Manual for Determining the Remaining Strength of Corroded Pipelines (ASME 2009). The manual provides four analysis levels in accordance with the data available during evaluation. The levels of analyses are defined as follow

- ❖ **Level 0** can be easily conducted in the field without the need of detailed calculations by simply utilizing the attached tables. These tables have been generated based on the allowable length and depth of corrosion.
- ❖ **Level 1** requires simple calculations which mainly depend on the maximum corrosion depth and the corrosion axial length. The maximum allowable operating pressure (MAOP) or maximum operating pressure (MOP) can be calculated as

$$\text{MAOP or MOP} \leq \frac{P_F}{SF}, \quad (2.1)$$

where P_F is the estimated failure pressure and SF is a safety factor.

The failure pressure (P_F) can be estimated by two methods

1. The Original B31G

$$P_F = \frac{2S_{\text{flow}}t}{D} \left[\frac{1 - \frac{2}{3}\left(\frac{d}{t}\right)}{1 - \frac{2}{3}\left(\frac{d}{t}\right)/M} \right] \quad \text{for } \left(\frac{l_a^2}{Dt}\right) \leq 20 \text{ and} \quad (2.2)$$

$$P_F = \frac{2S_{\text{flow}}t}{D} \left(1 - \frac{d}{t}\right) \quad \text{for } \left(\frac{l_a^2}{Dt}\right) < 20, \quad (2.3)$$

where D is the outside diameter of the pipe and

$$M = \sqrt{1 + 0.8 \left(\frac{l_a^2}{Dt} \right)}. \quad (2.4)$$

2. The Modified B31G (0.85dL)

$$P_F = \frac{2S_{\text{flow}}t}{D} \left[\frac{1 - 0.85 \left(\frac{d}{t} \right)}{1 - 0.85 \left(\frac{d}{t} \right) / M} \right], \quad (2.5)$$

where

$$M = \sqrt{1 + 0.6275 \left(\frac{l_a^2}{Dt} \right) - 0.003375 \left(\frac{l_a^2}{Dt} \right)^2} \quad \text{for} \quad \left(\frac{l_a^2}{Dt} \right) \leq 50 \quad \text{and} \quad (2.6)$$

$$M = \sqrt{0.032 \left(\frac{l_a^2}{Dt} \right) + 3.3} \quad \text{for} \quad \left(\frac{l_a^2}{Dt} \right) < 50. \quad (2.7)$$

The flow stress (S_{flow}) may be taken as

- a) For plain carbon steel operating at temperature below 120°C

$$S_{\text{flow}} = 1.1 \times \text{SMYS} \leq \text{SMTS}. \quad (2.8)$$

- b) For plain carbon and low-alloy steel having SMYS less than 483

MPa and operating at temperature below 120°C

$$S_{\text{flow}} = \text{SMYS} + 69 \text{ MPa} \leq \text{SMTS}. \quad (2.9)$$

- c) For plain carbon and low-alloy steel having SMYS less than 551

MPa

$$S_{\text{flow}} = \frac{(S_{\text{YT}} + S_{\text{UT}})}{2}. \quad (2.10)$$

SMYS is the specified minimum yield strength at ambient conditions, SMTS is the specified minimum tensile strength at ambient conditions, S_{YT} is the specified minimum yield strength at temperature and S_{UT} is the specified minimum ultimate strength at temperature, but may equal SMYS and SMTS, respectively.

- ❖ **Level 2** needs more detailed calculations for measuring the corroded area which leads to a more accurate estimate of the failure pressure (P_F). It requires a precise measurement of the profile surface of the actual corroded area. The method used in this analysis level is called the Effective Area Method and can be utilized as follow

$$P_F = \frac{2S_{flow}t}{D} \left[\frac{1 - \left(\frac{A}{A_0}\right)}{1 - \left(\frac{A}{A_0}\right)/M} \right], \quad (2.11)$$

where A is the local area of metal loss in the longitudinal plane and A_0 is the local original metal area ($l_a \cdot t$).

- ❖ **Level 3** involves a comprehensive analysis which should take into consideration all the effective parameters such as geometry, loading, boundary conditions and material properties. It also allows the user to develop a failure criterion that considers the strain capacity and fracture resistance of the material.

The Canadian Standard of Oil and Gas Pipeline Z662-11 (CSA 2011) has a provision to determine the maximum operating pressure (MOP) in accordance with ASME B31G. The MOP is taken as

$$MOP \leq P_F \times (F \times L_f \times J \times T) . \quad (2.12)$$

The failure pressure (P_F) is determined by either Eq. (2.5) or (2.11), which has been previously described. F is the design factor, L_f is the location factor, J is the joint factor and T is the temperature factor.

All the above-mentioned equations are based on bursting failure. Thus, criteria based on local buckling should also be considered to ensure the integrity of corroded pipelines.

2.2 Local Buckling of Pipes

Fracture of a pipe can be considered as a limit state for buried pipelines. In case of a corroded pipe, fracture can occur either before or after the initiation of local buckling depending on the loading, size of corrosion and material properties. When the loss of material due to corrosion is severe, a highly pressurized buried pipe can rupture without the initiation of a local buckling. However, in most cases, buckling occurs before the pipe fractures as the substantial loss of material due to corrosion does not happen instantaneously. These situations can occur when the corroded region of a pressurized pipe is also subjected to axial compression and bending. Local buckling helps localizing the deformation to this region. This can also accelerate the degradation of the material (low cycle fatigue) when the pipe experiences thermal and pressure loading cycles. All these eventually lead to the fracture of the pipe. Thus, it is reasonable to consider the initiation of local buckling as a limit state for the failure of a pipeline.

The critical moment, critical strain and critical curvature associated with the point of peak moment subjected to a combined loading can be used to define the point of buckling

initiation. A number of tests and studies have been carried out to study the buckling behaviour of steel pipelines. In order to facilitate the discussion, various terms used to discuss the tests are first defined. The critical buckling equations defined in the Canadian Standard Z662-11 (CSA 2011) are also presented.

2.2.1 Critical Buckling Moment

Pipe test specimen subjected to monotonically increasing end rotation experiences a bending moment at each side of the pipe. The average of these end moments is referred to the global moment (M_g). When there is an applied axial load and internal pressure, an additional moment is introduced at the mid-span due to the second order effect, as shown in the schematic of the pipe loads for the tests by Chen et al. (2010) in Fig. 2.4. Thus, the local moment at mid-span about the centre of the pipe cross section can be taken as

$$M_l = M_g + F_a \cdot y, \quad (2.13)$$

where F_a is the applied load and y is the average lateral distance of the extreme tension and compression faces of the pipe wall to the line of the axial force application.

The peak global moment is referred to as the critical global moment (M_{gc}), and the peak local moment at the location of buckling is referred to as the critical local moment (M_{lc}).

2.2.2 Critical Buckling Strain

Since local buckling only involves a short segment of the pipe, the local strain has been used to describe the pipe deformation response. The local strain over a specific length R of a pipe at the location of local buckling is given by

$$\varepsilon_R = \frac{\Delta_R}{R}, \quad (2.14)$$

where Δ_R is the deformation of two points over the length of R measured in a straight line.

For a pipe without corrosion, the local strain up to the initiation of local buckling can be considered to be independent of the length selected. Experimental studies by DelCol et al. (1998), Dorey et al. (2001) and Chen et al. (2010) have shown that the buckling shape of a pipe generally falls within a length equal to the pipe diameter (D). Thus, the average strain over the outside diameter (D), D local strain, has been used in past studies to describe the local strain and post-buckling deformation of the pipe.

For a corroded pipe, the local strain varies with the length selected. Using a length equal to the outside pipe diameter (D) similar to other studies, the D local compressive strain (ε_{cD}) is taken as

$$\varepsilon_{cD} = \frac{\Delta_{cD}}{D}, \quad (2.15)$$

where Δ_{cD} is the shortening of two points at D length apart at the extreme compressive face.

Similarly, the D local tensile strain (ε_{tD}) is given by

$$\varepsilon_{tD} = \frac{\Delta_{tD}}{D}, \quad (2.16)$$

where Δ_{tD} is the lengthening of two points at D length apart at the extreme tensile face of the pipe opposite to the location of buckling.

The D critical local compressive strain (ϵ_{cDc}) is the D local compressive strain corresponding to the critical local moment (M_{lc}).

2.2.3 Critical Buckling Curvature

The third measure that can be used to associate local buckling is the critical curvature. Similar to the critical moment, critical curvature can be separated into local and global curvatures. Referring to Fig. 2.4, the global curvature (Φ_g) is taken as the average rate of change in the rotational angle over the length of the pipe specimen (L) given by

$$\Phi_g = \frac{\theta_t + \theta_b}{L}, \quad (2.17)$$

where θ_t and θ_b are the angular change at the top and bottom ends of the specimen, respectively.

The local curvature (Φ_l) is an average curvature over a segment of the pipe. For the D local curvature, which is the average curvature over a distance equal to the pipe diameter (D) at the location of buckling, is given by

$$\Phi_{lD} = \frac{\epsilon_{cD} + \epsilon_{tD}}{D}. \quad (2.18)$$

The critical global curvature (Φ_{gc}) is defined as the global curvature at the critical global moment. Similarly, the D critical local curvature (Φ_{lDc}) is referred to as the D local curvature at the critical local moment (M_{lc}).

2.2.4 Critical Buckling Strain Equations in the Canadian Standard

Different design standards have developed critical buckling strain equations to ensure the integrity of buried pipelines under combined loads. The Canadian Standard of Oil and Gas Pipeline Z662-11 (CSA 2011) has provisions to determine the ultimate limit states of buried pipelines. In the provision that addresses the local buckling of pipeline subjected to axial force, bending moment and internal pressure, the compressive strain shall be limited to

$$\epsilon_{cf} \leq \Phi_{ec} \epsilon_c^{crit}, \quad (2.19)$$

where Φ_{ec} is the resistance factor for compressive strain taken as 0.8, ϵ_c^{crit} is the ultimate compressive strain of the pipe wall, ϵ_{cf} is the factored compressive strain in the longitudinal or hoop direction.

The compressive strain limit should be taken as

$$\epsilon_c^{crit} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left(\frac{(p_i - p_e)D}{2tE} \right)^2 \quad \text{for } \frac{(p_i - p_e)D}{2tF_y} < 0.4 \text{ and} \quad (2.20)$$

$$\epsilon_c^{crit} = 0.5 \frac{t}{D} - 0.0025 + 3000 \left(\frac{0.4F_y}{E} \right)^2 \quad \text{for } \frac{(p_i - p_e)D}{2tF_y} \geq 0.4, \quad (2.21)$$

where p_i is the maximum internal design pressure, p_e is the maximum external design pressure, E is the modulus of elasticity taken as 207000 MPa and F_y is the effective specified minimum yield strength.

The compressive strain limit in CSA Z662-11 is not applicable to corroded pipes since it does not take into account the effect of metal loss due to corrosion. Thus, it will be

beneficial to have equations to predict the compressive strain at buckling (critical local moment) for corroded pipes to help assessing the integrity of pipelines.

2.3 Effective Parameters on Pipe Buckling

A number of parameters such as geometry, loading and material properties have been found to affect the performance of the pipe. These include pipe diameter to thickness ratio (D/t), axial load, internal pressure (p), corrosion dimensions (l_a , l_c and d) and material properties (stress-strain curve).

2.3.1 Diameter to Thickness Ratio (D/t)

One of the most influential parameters that affect the deformation behaviour of pipeline is the D/t ratio. Tests with different D/t ratios were conducted by Sherman (1976) to study the flexural behaviour of tubes. Under pure bending and a range of D/t ratios (18, 35, 49, 57, 77 and 102), results showed an inverse relationship between D/t ratio and critical local curvature. Another study was carried out by Korol (1979) to investigate the critical buckling strain of circular tubular sections in flexure. The study consisted of eleven tests of various D/t ratios ranging from 29 to 80 that subjected to bending. The critical buckling strain was found to vary inversely with the D/t ratio. Mohareb et al. (1994) conducted seven full-scale tests of plain pipe specimens subjected to a combination of axial load, internal pressure and bending. Specimens were 1690 mm long with the D/t ratio of either 51 or 64. Results showed that the critical global moment and curvature decreased as the D/t ratio increased. In addition, Zimmerman et al. (1995) performed five full-scale tests under a combined loading. Two specimens had a D/t ratio of 41, while the rest had a large D/t ratio of 87. A same inverse relationship was observed

between the D/t ratio and the critical global moment, critical local compressive strain and critical local curvature.

2.3.2 Axial Load

The axial load has a significant effect on the buckling behaviour of line pipes. Two of the five specimens tested by Zimmerman et al. (1995) that were subjected to the same internal pressure but a different net axial force behaved differently. The specimen subjected to a net tensile force had a higher critical global moment, critical local compressive strain and critical local curvature compared to that with a zero net axial force. In addition, the specimen tested by Mohareb et al. (1995) with a zero net axial load had a higher critical global moment and curvature compared to a similar specimen tested under a compressive net axial load. A series of full-scale tests on plain and girth-welded pipes with a D/t ratio of approximately 92 were conducted by Dorey et al. (2001) to determine the critical buckling strain under a combined loading. Comparing the results of tests subjected to a same level of pressure but a different net axial load, lower critical global moment and critical compressive strain were observed in specimens subjected to a compressive net axial load.

2.3.3 Internal Pressure (p)

Another significant parameter affecting the buckling behaviour of pipelines is the internal pressure. Pressurized and non-pressurized specimens subjected to bending were tested by Zimmerman et al. (1995). The tests showed that the internal pressure caused a decrease in the global moment but an increase in the critical local compressive strain and critical local curvature. DelCol et al. (1998) tested four specimens of plain pipes under combined

loads of axial force, internal pressure, and imposed curvature. The specimens had the same D/t ratio of 92, but subjected to a varying level of internal pressure ranging from 0% to 80% of the yield pressure (P_y), the pressure to generate a hoop stress equal to the yield strength. Results showed that the critical strain and critical local curvature increased with internal pressure. In addition, the study conducted by Dorey et al. (2001) also showed a decrease in the critical global moment but an increase in the critical compressive strain with the level of internal pressure.

Another notable observation associated with internal pressure is the buckling shape. Tests by Mohareb et al. (1994), Zimmerman et al. (1995), DelCol et al. (1998) and Chen et al. (2010) have identified two buckling mode shapes that are directly dependent on the level of the internal pressure as shown in Fig. 2.5. In the presence of internal pressure, an outward bulging mode shape of buckling was clearly observed, whereas non-pressurized specimens exhibited an inward-outward “diamond” mode shape.

2.3.4 Material Properties

As with any steel member, material grades play an important role in the response of steel pipes. Several studies have shown that the peak moment capacity is directly proportional to the material strength. Two sets of pipe specimens with the same geometry, material grade and loading combinations were separately tested by DelCol et al. (1998) and Dorey et al. (2001). Comparing the test results, there were minor differences in terms of peak global moment but large differences in terms of the critical strain. Dorey et al. (2001) found that the buckling response of a pipe was significantly affected by the shape of the stress-strain curve, mainly the yield strength (σ_y) and the length of yield plateau (ϵ_{ep}). For

the same material grade but with different yield strengths, the greater plastic deformation was attained before reaching the peak buckling moment with lower yield strength. It was also found that the presence of yield plateau tends to decrease the peak moment capacity but increase the critical buckling strain.

2.3.5 Corrosion Dimensions (l_a , l_c and d)

Beside the parameters that affect the buckling response of plain pipes, the depth, axial and circumferential lengths of a corrosion defect also have a huge influence on the buckling response of corroded pipes. Stephen et al. (1996) performed thirteen full-scale tests of corroded pipes to monitor the failure of rupture under a combined loading. Seven of these specimens failed under wrinkling before rupture. Different corrosion shapes including square patches, axial and circumferential grooves were machined on the tension, compression or both sides of the pipe segments. It was found that the corrosion depth and circumferential length have a much stronger influence than axial length.

Based on these findings, another study was conducted by Smith et al. (1998) to extend the wrinkling investigation of corroded pipes. Four specimens were tested under combinations of axial load, internal pressure and bending moment. Results of this study showed that expanding the corrosion length along the hoop direction reduced the critical moment. Similarly, increasing the corrosion depth led to a decrease in the critical moment.

In addition, Chen et al. (2010) conducted a series of full-scale tests to study the buckling response of corroded pipes. The study consisted of four defect-free specimens and six specimens with machined external corrosion (square patches and circumferential grooves)

for different geometries and load combinations. There were significant differences between results of plain and corroded pipes. A considerable reduction in the critical local moment, critical compressive strain and critical local curvature were observed in the corroded specimens compared to the plain specimens.

2.4 Finite Element Analysis (FEA) Studies

Finite Element Analysis (FEA) can be a powerful and economical technique to simulate experimental tests and conduct parametric studies in order to investigate the buckling response of a line pipe. Many numerical studies have been carried out on plain and corroded pipes using ABAQUS (finite element analysis software), and only some of which will be discussed below.

2.4.1 Finite Element Analyses of Plain Pipes

Mohareb et al. (1994) modeled only half the pipe specimen due to symmetry along the plane of bending. S4RF element (a 4-node shell element with reduced integration and hourglass control using five degrees of freedom at each node) was used in the models. Material properties used in the analyses were based on the results of tension coupon tests taken from the longitudinal direction of the pipes. An isotropic hardening rule was used in the analyses. A good agreement between the experimental and analytical results was achieved. Based on the validated models, a parametric study was also performed to study the buckling modes of pipe segments subjected to different loads and pipe geometry.

DelCol et al. (1998) performed numerical simulations of a full pipe specimen in order to include the effect of initial imperfections. The element used was S4R (a 4-node general-purpose shell element with reduced integration, hourglass control and finite membrane

strain). An elasto-plastic material model with the isotropic hardening rule was utilized in the analyses based on the material properties obtained from the average of the longitudinal and circumferential tension coupon tests. There was a good agreement between the experimental data and analytical results.

Dorey et al. (2001) also modeled an entire pipe specimen using S4R shell element. Material properties used in the analyses were obtained from tension coupon tests taken from the longitudinal direction of the pipe specimens. An isotropic hardening rule was also used in the analyses. Results showed that introducing initial imperfections into the analytical model led to better agreement between experimental and analytical results. In addition, a numerical parametric study consisting of 162 cases was conducted to investigate the effect of parameters, such as D/t ratio, internal pressure to yield pressure ratio (p/p_y), material grades, initial imperfection and circumferential girth weld, on the critical buckling strain of pipe specimens.

Tan (2008) performed an extensive parametric study to examine the local buckling response of pipelines under a combined loading. Utilizing the symmetry along the plane of bending, only half of the pipe was modeled. The pipe was modeled with S4R shell element. An imperfection band was introduced to the model by reducing either 0.05% or 0.1% of the original pipe wall thickness. The model used in the parametric study was validated against test results from Mohareb et al. (1994), DelCol et al. (1998) and Chen et al. (2010). A total of 792 cases with different load combinations, pipe geometries, and a wide range of material properties (with and without yield plateau) were analyzed.

2.4.2 Finite Element Analyses of Corroded Pipes

Smith et al. (1998) successfully used S4R shell element to simulate a defective pipe subjected to a combined loading. Only a quarter of the pipe was modeled assuming symmetry along the plane of bending and across the pipe mid-span. Reduced wall thicknesses were applied to some elements to simulate the artificial corrosion defects. Material properties obtained from the longitudinal and circumferential tension coupon tests were utilized in the analyses. A multi-linear kinematic hardening model was used in the analyses. Comparisons showed a good agreement between experimental and analytical results.

Chen et al. (2010) carried out non-linear finite element analyses using an isotropic hardening rule for one half pipe specimen to study the remaining local buckling resistance of corroded pipelines under a combined loading. The S4R shell element was used and a metal-loss defect was simulated by reducing the element thickness. Material properties based on the results of longitudinal tensile coupon tests were utilized in the analyses. Results obtained from the FEA models agreed well with those measured in the experimental tests. Based on the validated model, a parametric study consisted of more than 150 combinations of geometry, loading and corrosion size and shapes were conducted.

2.5 Critical Buckling Strain Equations of pipelines

When developing critical buckling strain equations for pipelines, the influences of pipe geometry, loading, material properties and corrosion dimensions should be considered. A number of equations that take into consideration these parameters, except the corrosion

defect dimensions, have been proposed to predict the critical buckling strain. Although these equations are not applicable for pipelines with corrosion defects, a brief review of some of them would nevertheless be useful.

An expression was developed by Vitali et al. (1999) based on three parameters of D/t ratio, internal pressure ratio and material properties (stress-strain curve) with no yield plateau. The critical compressive strain equation can be expressed as

$$\epsilon_{lc} = \left(\frac{t}{D} - 0.01 \right) \left(1 + 5 \frac{\sigma_h}{\sigma_y} \right) \left(\frac{\sigma_u}{\sigma_y} \right)^{1.5}, \quad (2.22)$$

where σ_h is the hoop stress, σ_y is the yield strength and σ_u is the ultimate strength.

Dorey et al. (2006) generated a set of critical strain equations associated with plain and girth-welded pipes made from materials with and without yield plateau. The equations were generated based on results from a parametric study investigated for a variety of D/t ratios, internal pressure levels, material properties and initial imperfection patterns. The equation for plain pipe with non-yield plateau material can be expressed as

$$\epsilon_{lc} = \left(\frac{2.94}{D/t} \right)^{1.59} \left(1 - 0.868 \left(\frac{p}{p_y} \right) \right)^{-1} \left(\frac{E}{\sigma_y} \right)^{0.854} \left(1.27 - \left(\frac{\text{imp}}{100} \right)^{0.15} \right), \quad (2.23)$$

where imp is the pipe imperfection defined as a percentage of the pipe wall thickness.

A general critical buckling strain equation was also proposed by Tan (2008). The equation was developed based on results from a finite element analysis parametric study that considered 3 D/t ratios, 12 load combinations, and 12 stress-strain curves (material properties types). The equation takes the form

$$\epsilon_{lc} = k_s \left(\frac{D}{t} \right)^{V_{ds}} \left(1 - \frac{\sigma_a}{\sigma_y} \right)^{V_{as}} \left(1 - V_{ps} \frac{p}{p_y} \right)^{-1}, \quad (2.24)$$

where k_s , V_{ds} , V_{as} and V_{ps} are constants corresponding to each stress-strain curve. In addition to the critical buckling strain equation, Tan (2008) also proposed equations for critical local moment and curvature.

Table 2.1 Classification of Corrosion (modified from POF 2009)

Dimension Class	Definition
General	$\{[l_c \geq 3A_{10}] \text{ and } [l_a \geq 3A_{10}]\}$
Pitting	$\{[(1A_{10} \leq l_c < 6A_{10}) \text{ and } (1A_{10} \leq l_a < 6A_{10}) \text{ and } (0.5 < l_a/l_c < 2)]\}$ $\{ \text{and not } [(l_c \geq 3A_{10}) \text{ and } (l_a \geq 3A_{10})] \}$
Axial Groove	$\{[1A_{10} \leq l_c < 3A_{10}] \text{ and } [l_a/l_c \geq 2]\}$
Circumferential Groove	$\{[l_a/l_c \leq 0.5] \text{ and } [1A_{10} \leq l_a < 3A_{10}]\}$
Pinhole	$\{[0 < l_c < 1A_{10}] \text{ and } [0 < l_a < 1A_{10}]\}$
Axial Slotting	$\{[0 < l_c < 1A_{10}] \text{ and } [l_a \geq 1A_{10}]\}$
Circumferential Slotting	$\{[l_c \geq 1A_{10}] \text{ and } [0 < l_a < 1A_{10}]\}$

Figure 2.1 Pitting corrosion in oil pipelines (modified from CAPP 2009b)

Figure 2.2 Pitting corrosion in natural gas pipelines (modified from CAPP 2009c)

Figure 2.3 Dimensions of corrosion (modified from POF 2009)

Figure 2.4 Pipe loading (modified from Tan 2008)

Figure 2.5 Two typical buckling deformation shapes (adopted from Chen et al. 2010)

Chapter 3 Proposed Model and Validation

Although an experimental test is the most accurate method to investigate the local buckling behaviour of pipelines, it is not economically feasible to test pipe specimens over a wide range of loading combinations, pipe geometries, material properties, and corrosion shapes. Consequently, numerical simulations using finite element analysis (FEA) can be a useful and economical alternative for conducting a wide range of parametric studies. A proposed analytical model has been developed based on the tests setup and validated against test results. In addition, a mesh study has also been conducted to assess the appropriate model to be utilized in performing the parametric study.

Test results from Chen et al. (2010) will be used to validate the finite element model. There were nine full-scale tests of grade X70 (SMYS 483 MPa) pipes, which can be divided into three main groups based on the D/t ratio and load combination. Each group has a defect-free specimen and two specimens with exterior machined corruptions (a circumferential groove and a rectangular patch). All the specimens have an outside diameter (D) of 610 mm and a length of 2300 mm ($3.77 D$).

As shown in Table 3.1, group LB-I has a low D/t ratio of 48, pressurized to 80% SMYS and subjected to 30% SMYS tension load. Group LB-II has a D/t ratio of 69, and subjected to an internal pressure of 40% SMYS with no axial load. The last group, LB-III, has a relatively high D/t ratio of 92, and subjected to a compressive load of 20% SMYS but no internal pressure. Material properties from coupon tests taken from the longitudinal direction of the pipe specimens are shown in Table A.1, and Figs. A1 to A3 in Appendix A.

3.1 Finite Element Model

A number of numerical studies by Mohareb et al. (1994), Dorey et al. (2001) and Tan (2008) have been successfully conducted using the finite element software package ABAQUS. Based on the good agreement between the analytical and experimental results shown in the aforementioned studies, ABAQUS (2013) is also used in the current study. The finite element model is validated against a series of plain and corroded pipe tests that have been conducted by Chen et al. (2010). Therefore, the model similar to that of the test setup is maintained in the numerical model.

3.1.1 Test Setup

As shown in Fig. 3.1, the pipe specimen was welded to 76 mm thick plates at both ends to transfer axial load, apply end rotations and allow the pipe segment to be pressurized. A collar of 150 mm wide was provided at each end to minimize end effects. The specimen was placed vertically under the testing machine where the internal pressure was applied and the axial load was introduced. An increasing curvature was applied through a couple of hydraulic jacks connected to the pivot points by loading arms.

3.1.2 Model Geometry

Similar to the tests, the finite element model consists of a pipe segment wrapped with a collar and welded to a thick plate at each end. The pipe wall is modeled with S4R, a 4-node bi-linear general-purpose shell element with reduced integration. This element is able to account for the finite membrane strain which is suitable for solving large deformation problems. Collars are modeled as twice the pipe thickness using the shell

element S4R. End plates are assumed to be rigid and modeled with STRI3 element, a 3-node triangular facet thin shell. To maintain the rigid body behaviour of the end plate, the nodes in the center of the plate are tied to a coincident reference node. Also, the nodes on the circumference of the plate are linked to the reference node (center node) using a rigid beam, which is then linked to the pivot point by the same constraint type (rigid beam).

Corrosion is simulated by reducing the wall thickness at the defective area. The offset feature of the shell element enables the corroded area to be modeled while maintaining the same reference surface between elements used in this area and those used in the rest of the pipe. The corrosion location is shifted by $0.5D$ along the pipe axial direction in order to avoid the double buckling (two buckling symmetric about mid-span). In addition, a band of a uniform circumferential imperfection of $0.1D$ long and $0.01t$ depth of the wall thickness is introduced to the middle of the defect-free (plain) specimens. This imperfection band would induce the buckling to occur in the middle of the segment rather than close to the end for some cases due to the effects of end boundary conditions.

3.1.3 Model Mesh

The key of attaining a desired level of both accuracy and efficiency is to reduce the number of degrees of freedom of the model without affecting the numerical results. This can be done by modeling only half of the pipe segment due to symmetry and using different element sizes along the longitudinal and circumferential directions. As shown in Fig. 3.2, the model is divided into four main regions. The middle region (refined region) is where the corrosion is located as well as the buckling is expected to initiate. It is $1.5D$ long and has the finest mesh density in order to precisely capture the local deformation. A

transition region is needed to increase the element size used in the refined region to a larger size that is used to model the rest of the pipe wall. Two identical transition regions of $0.5D$ long are added. The rest of the pipe wall is modeled as a general region and a 150 mm long collar region is provided at each end of the general region. Finally, a multi-point constraint (Linear) is applied between the transition and general regions in hoop direction to ensure displacement continuity.

3.1.4 Model Loading

Figure 3.3 shows the schematic of the loading and boundary conditions. The axial load (F_a) is applied to the top pivot point. The pressure load (p) is applied as a follower force which stays normal to the pipe surface during the analysis. The end moments (M_g) are induced by imposing two equal rotations at the top and bottom pivot points.

Loading sequences used in the analysis consist of two main steps. In the first step, a specified axial load is applied and a desired level of internal pressure is maintained. Once the first step is completed, two equal rotations are incrementally applied until the pipe buckles (reaches maximum local moment).

3.1.5 Model Material Properties

The actual material properties of the pipe vary in the longitudinal and circumferential directions, and as well as in compression and tension. It is desirable to use different material properties for the longitudinal and circumferential directions to model the pipe in the finite element analysis. Nevertheless, a good correlation between test and FEA results has been achieved by Mohareb et al (1994), Dorey et al (2001) and Tan (2008) using only

the material properties of the tension coupon test from the longitudinal direction and isotropic hardening. In addition, the material properties in the longitudinal direction will have a more dominant influence on the buckling response of a line pipe that is subjected to a mainly longitudinal combined loading. Thus, with the lack of information on the relationship between the material properties in the longitudinal and circumferential directions and under cycle loading, the pipe material in the study is assumed to be isotropic elasto-plastic with isotropic hardening defined by a multi-linear true stress-true plastic strain curve.

3.2 Mesh Size and Model Length

In order to refine the accuracy and efficiency of the finite element model, mesh size and model length studies have been conducted. Using the material properties of specimens LB-I by Chen et al. (2010) with the Poisson's ratio (ν) of 0.265 and the modulus of elasticity (E) of 200000 MPa, several geometries and load combinations have been considered to examine the effect of element sizes and model lengths on the buckling response of corroded pipes. The study consists of two D/t ratios of 40 and 100, two p/p_y ratios of 0.0 and 0.4, one σ_a/σ_y ratio of 0.2 and an internal axial groove corrosion with $l_a = 1.0D$, $l_c = 0.25D$ and $d = 0.3t$ for a pipe with an outside diameter (D) of 762 mm.

3.2.1 Element Size

Three different mesh densities have been selected based on the model for a specimen at $5.0D$ length. Mesh M1 (the coarsest mesh) has an element size of $0.025D \times 0.02D$ in the refined region, where the first value refers to the circumferential direction and the second value refers to the longitudinal direction. Reducing the mesh size of M1 by half, results

mesh M2 with an element size of $0.0125D \times 0.01D$ in the refined region. The last mesh, M3, has an element size of $0.0062D \times 0.005D$ in the refined region, which is equal to a half size of the M2 and a quarter size of M1. A total of 12 cases of specimens with an axial groove corrosion defect have been evaluated.

To facilitate the discussion, the term critical compressive strain (ϵ_{lc}) henceforth will be used to represent D critical local compressive strain. Similarly, the critical local curvature (Φ_{lc}) for D critical local curvature. A number of measurements at the initiation of buckling can be used to compare the numerical and experimental results. Critical local moment (M_{lc}), critical compressive strain (ϵ_{lc}) and critical local curvature (Φ_{lc}) are compared in the validation.

As presented in Table 3.2, the differences between the predicted critical local moment (M_{lc}) for all three meshes are only about 1%. However, differences between the predicted critical compressive strain (ϵ_{lc}) and critical local curvature (Φ_{lc}) of M1 to that of M2 are up to 5%, but only about 1% between M2 and M3. Consequently, mesh M2 is adopted for the finite element model used in the parametric study. For mesh M2, the size of the elements used in the other regions are $0.0125D \times 0.02D$ for the transition region, $0.025D \times 0.035D$ for the general region and $0.025D \times 0.065D$ for the collar region.

3.2.2 Model Length

A pipe segment having a sufficient length in the modeling can be considered as a good representative of a long line pipe. This length can be described by a length needed to prevent stress distribution in the pipe from being significantly affected by end boundary

conditions. Based on mesh M2, models that are 3.5D, 5.0D and 7.0D long have been evaluated to study the influence of end effects on the pipe buckling response.

Results of the buckling limits (M_{lc} , ϵ_{lc} and Φ_{lc}) comparisons are shown in Table 3.3. Overall, the predicted buckling limits for specimens modeled with 3.5D length show some notable differences from those modeled with 5.0D length. However, only negligible differences are found between results for specimens of 5.0D and 7.0D long. Based on these findings, a pipe model with a length of 5.0D is considered to be sufficiently long enough to eliminate end effects, and hence is adopted for the parametric study.

3.3 Validation

3.3.1 Validated Tests

Results from Chen et al. (2010) for tests shown in Table 3.1 have been used to validate the finite element model. Material properties used in the analyses are presented in Table A.1, and Figs. A1 to A3 in Appendix A. The Poisson's ratio (ν) is taken as 0.265 and the modulus of elasticity (E) as 200000 MPa. The pipe segments are modeled by mesh M2. For specimens with circumferential groove defect, the mesh at the refined region is modified slightly from that of M2 in order to model the defect size exactly. Overall, the element size used at this region is close to that for the refined region of mesh M2.

3.3.2 Result Comparisons

Results of the comparisons are summarized in Table 3.4 and graphically presented in Figs. 3.4 to 3.9. Test and predicted buckling deformation shapes for specimens with a rectangular patch are shown in Fig. 3.10.

Overall, a good agreement between the analytical and experimental results has been achieved. The FEA/Test ratio varies from 1.00 to 1.08 for the critical local moment (M_{lc}), 0.87 to 1.01 for the critical compressive strain (ϵ_{lc}) and 0.87 to 0.99 for the critical local curvature (Φ_{lc}). As expected, the critical local moment is slightly higher (2% to 8%) in most cases. The reason behind this discrepancy is that test results are based on static readings, whereas material properties used in the analyses have been obtained from coupon tests deformed in a low continuous rate (dynamic readings).

Comparisons of the results for specimens LB-I-2 and LB-I-4 show a relatively larger discrepancy in the critical compressive strain (9% and 8%, respectively) and the critical local curvature (13% and 12%) but within 10% for all other specimens except LB-I-1. The main reason for the larger difference is due to the shape of the load-deformation curves (local moment-local curvature and local moment-local compressive strain) of these specimens. As can be seen in Figs. 3.4 and 3.5, the curves are quite flat at close to the peak moment. Hence, making the determination of the point of peak moment for these two specimens is less precise and subjected to larger variability. In general, comparisons of the test and predicted moment-deformation curves also show, in general, a greater difference for defect-free specimens than specimens with corrosion. The imperfection band applied in defect-free specimens may not have accurately captured the effect of pipe imperfection. Nevertheless, the average ratios of FEA/Test for M_{lc} , ϵ_{lc} and Φ_{lc} are 1.03, 0.95 and 0.92, respectively. Thus, the good agreement between the predicted and test results shows that the finite element analyses can be utilized to predict the buckling behaviour of pipelines with and without corrosion.

Z662-11 (CSA 2011) provides equations to predict the compressive strain limit (critical compressive strain). The test to predicted compressive strain limit based on Eqs. (2.20) and (2.21) for LB-I-1, LB-II-1 and LB-III-1 are respectively $(31273) / (10523) = 2.97$, $(8517) / (7326) = 1.16$ and $(4100) / (2904) = 1.41$. It is clear that Eqs. (2.20) and (2.21) from Z662-11 (CSA 2011) can not accurately predict the compressive strain limit of plain pipes as the axial force and the actual stress-strain relationship of the material are not taken into account in these equations. Thus, the critical compressive strain equations from Z662-11 (CSA 2011) will not be used to validate results of plain pipes in the subsequent parametric study.

Table 3.1 Matrix of full-scale tests performed by Chen et al. (2010)

Specimen	Pipe Size (mm)		D/t	Defect Size (mm)			Internal Pressure	Axial Stress
	D	t		Axial	Circumferential	Depth		
LB-I-1	610	12.7	48	No Defect			80% SMYS	-30% SMYS
LB-I-2	610	12.7	48	5.0t	0.5D	0.55t	80% SMYS	-30% SMYS
LB-I-4	610	12.7	48	0.5D	0.5D	0.30t	80% SMYS	-30% SMYS
LB-II-1	610	8.8	69	No Defect			40% SMYS	0
LB-II-2	610	8.8	69	5.0t	0.5D	0.55t	40% SMYS	0
LB-II-4	610	8.8	69	0.5D	0.5D	0.30t	40% SMYS	0
LB-III-1	610	6.6	92	No Defect			0	+20% SMYS
LB-III-2	610	6.6	92	5.0t	0.5D	0.55t	0	+20% SMYS
LB-III-4	610	6.6	92	0.5D	0.5D	0.30t	0	+20% SMYS

Table β.2 Results of the comparisons between meshes M1, M2 and M3

D/t=40 and $p/p_y=0.0$	M1	M2	M3	M2/M1	M3/M2
M_{lc} (kNm)	4985	4970	4968	1.00	1.00
ε_{lc} (mm/mm $\times 10^{-6}$)	13972	13533	13417	0.97	0.99
Φ_{lc} (rad/mm $\times 10^{-6}$)	23.41	22.75	22.57	0.97	0.99
D/t=40 and $p/p_y=0.4$	M1	M2	M3	M2/M1	M3/M2
M_{lc} (kNm)	4239	4225	4223	1.00	1.00
ε_{lc} (mm/mm $\times 10^{-6}$)	18539	18124	18176	0.98	1.00
Φ_{lc} (rad/mm $\times 10^{-6}$)	28.98	28.35	28.40	0.98	1.00
D/t=100 and $p/p_y=0.0$	M1	M2	M3	M2/M1	M3/M2
M_{lc} (kNm)	1468	1452	1447	0.99	1.00
ε_{lc} (mm/mm $\times 10^{-6}$)	4097	3958	3916	0.97	0.99
Φ_{lc} (rad/mm $\times 10^{-6}$)	7.60	7.39	7.32	0.97	0.99
D/t=100 and $p/p_y=0.4$	M1	M2	M3	M2/M1	M3/M2
M_{lc} (kNm)	1211	1196	1194	0.99	1.00
ε_{lc} (mm/mm $\times 10^{-6}$)	5032	4802	4799	0.95	1.00
Φ_{lc} (rad/mm $\times 10^{-6}$)	8.26	7.90	7.88	0.96	1.00

Table 3.3 Results of the comparisons between model lengths 3.5D, 5.0D and 7.0D

D/t=40 and $p/p_y=0.0$	3.5D	5.0D	7.0D	5.0D/3.5D	7.0D/5.0D
M_{lc} (kNm)	4964	4970	4992	1.00	1.00
ϵ_{lc} (mm/mm $\times 10^{-6}$)	13193	13533	13544	1.03	1.00
Φ_{lc} (rad/mm $\times 10^{-6}$)	22.15	22.75	23.01	1.03	1.01
D/t=40 and $p/p_y=0.4$	3.5D	5.0D	7.0D	5.0D/3.5D	7.0D/5.0D
M_{lc} (kNm)	4199	4225	4229	1.01	1.00
ϵ_{lc} (mm/mm $\times 10^{-6}$)	18418	18124	18051	0.98	1.00
Φ_{lc} (rad/mm $\times 10^{-6}$)	28.50	28.35	28.27	0.99	1.00
D/t=100 and $p/p_y=0.0$	3.5D	5.0D	7.0D	5.0D/3.5D	7.0D/5.0D
M_{lc} (kNm)	1484	1452	1419	0.98	0.98
ϵ_{lc} (mm/mm $\times 10^{-6}$)	4021	3958	3926	0.98	0.99
Φ_{lc} (rad/mm $\times 10^{-6}$)	7.62	7.39	7.24	0.97	0.98
D/t=100 and $p/p_y=0.4$	3.5D	5.0D	7.0D	5.0D/3.5D	7.0D/5.0D
M_{lc} (kNm)	1197	1196	1201	1.00	1.00
ϵ_{lc} (mm/mm $\times 10^{-6}$)	4973	4802	4799	0.97	1.00
Φ_{lc} (rad/mm $\times 10^{-6}$)	8.13	7.90	7.91	0.97	1.00

Table 3.4 Comparisons of experimental and numerical results for group tests LB-I, LB-II and LB-III

Specimen	Critical Local Moment (kNm)			Critical Compressive strain (mm/mm $\times 10^{-6}$)			Critical Local Curvature (rad/mm $\times 10^{-6}$)		
	Test	FEA	FEA/Test	Test	FEA	FEA/Test	Test	FEA	FEA/Test
LB-I-1	2009	2160	1.08	31273	31566	1.01	99.98	86.76	0.87
LB-I-2	1765	1766	1.00	41366	37500	0.91	94.37	81.83	0.87
LB-I-4	1722	1747	1.01	39719	36567	0.92	89.37	78.33	0.88
LB-II-1	1383	1427	1.03	8517	8320	0.98	21.34	19.33	0.91
LB-II-2	1004	1057	1.05	17113	15737	0.92	32.91	30.12	0.92
LB-II-4	1073	1093	1.02	12580	12261	0.97	25.76	24.16	0.94
LB-III-1	871	893	1.03	4100	3974	0.97	9.97	9.75	0.98
LB-III-2	516	523	1.01	11900	10379	0.87	21.23	19.11	0.90
LB-III-4	578	584	1.01	3504	3407	0.97	7.50	7.41	0.99
Average			1.03			0.95			0.92

Figure 3.1 Test setup for Chen et al (2010) (modified from DelCol et al. 1998)

Figure 3.2 Perspective view of the FEA model

Figure 3.3 External loading and boundary conditions

Figure 3.4 Test and predicted local moment versus compressive strain curve for specimens group LB-I by Chen et al. (2010)

Figure 3.5 Test and predicted local moment versus local curvature curve for specimens group LB-I by Chen et al. (2010)

Figure 3.6 Test and predicted local moment versus compressive strain curve for specimens group LB-II by Chen et al. (2010)

Figure 3.7 Test and predicted local moment versus local curvature curve for specimens group LB-II by Chen et al. (2010)

Figure 3.8 Test and predicted local moment versus compressive strain curve for specimens group LB-III by Chen et al. (2010)

Figure 3.9 Test and predicted local moment versus local curvature curve for specimens group LB-III by Chen et al. (2010)

Figure 3.10 Test and predicted buckling deformation shapes of corroded pipes

Chapter 4 Material Properties and Corrosion on Pipe Buckling

The local buckling response of corroded line pipes depends on the loading, pipe geometry and material properties, and as well as the size and configuration of the corrosion defect. Mechanical properties of the material can be represented by the stress-strain curve which can be characterized by the ultimate to yield strength (stress) ratio (σ_{er}), the yield strength (stress) (σ_y), the strain corresponding to ultimate stress (ϵ_{ef}) and the length of yield plateau (ϵ_{ep}). Although the corrosion would never have a regular shape, it is nevertheless simplistically assumed to be approximated as a rectangular (square) defect of a uniform depth, which can be described by three simple dimensions: depth, axial and circumferential lengths. A brief discussion on the stress-strain curve and the effect of corrosion on the buckling response is presented below to facilitate the formulation of the parametric study.

4.1 Characterization of Material Properties

A study by Tan and Khoo (2009) showed that the shape of stress-strain curve has a considerable effect on the local buckling response of a defect-free pipe. Therefore, it is expected to have similar influence on pipes with corrosion defects. The shape of a stress-strain curve can be characterized by the ultimate to yield strength (stress) ratio (σ_{er}), the yield strength (stress) (σ_y), the strain corresponding to ultimate stress (ϵ_{ef}) and the length of yield plateau (ϵ_{ep}). However, in order to carry out the finite element analysis of a pipe, the material properties expressed in terms of true stress-true plastic strain data have to be provided.

4.1.1 True Stress-True Strain Equations

A main objective of this research is to study and quantify effects of corrosion on the buckling response for a wide range of material properties (stress-strain curves) of pipelines. These properties have to be represented by true stress-true plastic strain curves in order to be utilized in the finite element analyses. Using only a few parameters, simple analytical equations based on the power-law relationship have been derived by a number of researchers to describe the stress-strain curve. Some of these formulas are described below.

Nadai (1939) proposed a stress-strain curve expression that relates true stress to true strain. It takes the form

$$\varepsilon_t = \frac{\sigma_t}{E} + \varepsilon_y \left(\frac{\sigma_t - \sigma_p}{\sigma_y - \sigma_p} \right)^q \quad \text{for } \sigma_t \geq \sigma_p, \quad (4.1)$$

where σ_t is the true stress, ε_t is the true strain, σ_y is the yield strength, ε_y is the strain corresponding to the yield strength, σ_p is the proportional limit, E is the Young's modulus and q is a constant.

Based on the equation presented by Nadai (1939), Ramberg-Osgood (1943) derived a simpler equation to describe stress-strain curve by only three parameters. This equation has been widely adopted in studies where steel did not exhibit a yield plateau in the stress-strain curve. It can be expressed as

$$\varepsilon_t = \frac{\sigma_t}{E} + K \left(\frac{\sigma_t}{E} \right)^r, \quad (4.2)$$

where K and r are constants.

Following the equations by Nadai (1939) and Ramberg-Osgood (1943), Naqvi and Khoo (2007) proposed a generalized equation taking into consideration the presence of yield plateau. The equation takes the following form with the true plastic strain

$$\varepsilon_{tp} = C \left[\left(\frac{\sigma_t}{\sigma_{t0}} \right)^n - 1 \right] + \varepsilon_{tp0} \quad \text{for } \sigma_t \geq \sigma_{t0} , \quad (4.3)$$

where ε_{tp} is the true strain, ε_{tp0} is the true plastic strain at the end of yield plateau or zero when there is no yield plateau, σ_{t0} is the true stress at ε_{tp0} , C and n are constants.

Equation (4.3) was successfully used by Tan and Khoo (2009) to generate a range of true stress-true plastic strain curves with and without yield plateau for the parametric study on plain pipes. Thus, this equation (Eq. 4.3) is also adopted in the current study.

4.1.2 Generation and Calibration of Material Parameters

The material properties obtained from coupon tests are usually expressed in engineering stress versus engineering strain. These data have to be converted into true stress versus true strain (true plastic strain) in order to be used in ABAQUS (2013). Assuming that there is no volume change throughout deformation, three equations are widely used to relate the engineering and true stress-strain values. These equations can be expressed as

$$\sigma_t = \sigma_e(1 + \varepsilon_e) , \quad (4.4)$$

$$\varepsilon_t = \ln(1 + \varepsilon_e) , \quad (4.5)$$

$$\varepsilon_{tp} = \varepsilon_t - \frac{\sigma_t}{E}, \quad (4.6)$$

where σ_e is the engineering stress and ε_e is the engineering strain.

Utilizing Eqs. (4.3) to (4.6), Tan and Khoo (2009) derived expressions to calculate C and n, which are used to generate the true stress-true plastic strain curves for specific characteristics. These characteristics are the ultimate to yield strength (stress) ratio (σ_{er}), the yield strength (stress) (σ_y), the relative strain at peak stress (ε_{er}), the yield plateau length (ε_{ep}) and the modulus of elasticity (E), which can be obtained from a standard tension coupon test. The equations take the following forms

$$C = \frac{\varepsilon_{tpf} - \varepsilon_{tp0}}{\left(\frac{\sigma_{tf}}{\sigma_{t0}}\right)^n - 1} \text{ and} \quad (4.7)$$

$$\sigma_{tf} + E \left[-1 \frac{n(\varepsilon_{tpf} - \varepsilon_{tp0}) \left(\frac{\sigma_{tf}}{\sigma_{t0}}\right)^n}{\left(\frac{\sigma_{tf}}{\sigma_{t0}}\right)^n - 1} \right] = 0, \quad (4.8)$$

where σ_{tf} and ε_{tpf} are the true stress and true plastic strain, respectively, corresponding to the peak engineering stress (σ_{ef}). These expressions can be used to calculate C and n that can fit the true stress-true plastic strain from the tension coupon test up to the peak engineering stress.

Results of a coupon test for specimens LB-I by Chen et al. (2010) is used to demonstrate and verify how C and n can be calibrated and used to generate a true stress-true plastic strain curve. As shown in Table 4.1, recorded engineering stresses and strains have been transformed into true stresses and strains by using Eqs. (3.4) to (3.6) taking the modulus

of elasticity (E) as 192000 MPa. Measured engineering and calculated true stress-strain curves are shown in Figs 4.1 and 4.2. Since the material did not exhibit a yield plateau, the values of σ_{t0} and ϵ_{tp0} are given by the data at the second row (at the proportional limit) in Table 4.2. The other point of interest is where the engineering stress reaches its peak. In this case, the point of peak engineering stress (σ_{ef}) is difficult to determine precisely. Thus, the values of σ_{tf} and ϵ_{tpf} are varied along the calculated true stress-true plastic strain curve, which simultaneously change the values of C and n from Eqs. (4.7) and (4.8), until the least square error between the generated (using Eq. (4.3)) and calculated curves is achieved. This leads to $\sigma_{tf} = 633.7$ MPa, $\epsilon_{tpf} = 0.05525$, $C = 3.44 \times 10^{-7}$ and $n = 18.04$. Results of the generated true stress-strain curve and the corresponding generated engineering stress-strain curve are reported in Table 4.2. Graphical comparisons of the measured and generated stress-strain curves are presented in Figs. 4.1 and 4.2. The good agreement shows that Eq. (4.3) can be used to generate true stress-true plastic strain data to be used in the finite element analyses.

4.1.3 Stress-Strain Curves Used in the Parametric Study

The Material properties (stress-strain curve) of the pipes vary with the chemical composition and fabrication process. Therefore, a wide range of material properties should be considered in the parametric study to evaluate the buckling behaviour of line pipes. As mentioned earlier, the shape of the engineering stress engineering strain curve can be approximately characterized by the ultimate to yield strength (stress) ratio (σ_{er}), the yield strength (stress) (σ_y), the strain corresponding to the ultimate stress (ϵ_{ef}) and the length of yield plateau (ϵ_{ep}). There are a number of methods to define the yield strength when there is no yield plateau. In order to facilitate the current parametric study, the yield

strength used in generating true stress-true plastic strain curves is taken as the engineering stress at the proportional limit. Thus, the ultimate to yield strength ratio is expressed as

$$\sigma_{er} = \frac{\sigma_{ef}}{\sigma_{e0}}, \quad (4.9)$$

where σ_{ef} is the ultimate strength and σ_{e0} is the engineering stress at the end of yield plateau or the proportional limit when there is no yield plateau.

As noted by Tan and Khoo (2009), the yield strength and the length of yield plateau were found to have only a small effect on the values of n and C parameters. Using Eq. (4.3), the same n and C parameters can be used to generate stress-strain curves for different yield strengths and lengths of yield plateau, but with the same ultimate to yield strength ratio and the strain at the peak stress.

For most steel used in pipelines, the ultimate to yield strength ratio based on the actual yield strength according to testing standards falls within 1.1 to 1.5. This has also been found in test specimens by Dorey et al. (2001) and Chen et al (2010). When there is no yield plateau, the actual yield strength is always higher than the proportional limit. In addition, the ultimate to yield strength ratio based on the proportional limit can reach up to 2.0 since the actual yield strength can be significantly higher than the proportional limit. Thus, three ultimate to yield strength ratios of 1.3, 1.65 and 2.0 are utilized for generating stress-strain curves without yield plateau. An additional stress-strain curve for the material with σ_{er} ratio of 1.3, and a yield plateau length of 0.02 is also considered in a brief study on the influence of yield plateau on the buckling behaviour of corroded pipes.

The strain at peak stress is one of the parameters that can be used to characterize the shape of the stress-strain curve. To facilitate the purpose of generating the stress-strain curve, the relative strain at peak stress (ϵ_{er}) is used instead of the actual strain. The ϵ_{er} is taken as the net difference between the engineering strain at peak stress and the engineering strain at either the end of yield plateau or the proportional limit. Numerical simulations have been carried out for materials with the stress-strain curve with no yield plateau to study the influence of ϵ_{er} and σ_{er} ratio on the buckling limits of corroded pipes as a ratio to that of the plain pipes. Tables 4.3 and 4.4 present the geometries, load combinations and results of the investigation on the buckling limits with respect to the relative strain (ϵ_{er}) and the ultimate to yield ratio (σ_{er}). The yield strength (σ_y) is taken as 480 MPa, the Poisson's ratio (ν) as 0.265 and the modulus of elasticity (E) as 200000 MPa in the numerical simulations. As shown in Table 4.4, the relative strain at peak stress (ϵ_{er}) does not have a significant effect on the critical local moment ratio (M_{lc}/M_{lp}), critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}), but there is a noticeable effect for the ultimate to yield ratio (σ_{er}) on the results shown in Table 4.4. For this reason, only one relative strain at peak stress of 0.12 is considered in the parametric study.

A summary of the material properties utilized in the parametric study is presented in Table 4.5. The modulus of elasticity (E) is taken as 200000 MPa and the Poisson's ratio as 0.265 for the elastic response, according to the value measured by Dorey et al. (2001). The true stress at end of yield plateau or the proportional limit is taken as 480 MPa with the true stress assumed to be constant along the yield plateau.

4.2 Corrosion Dimensions Study

The buckling response of aging pipelines subjected to a combined loading can be significantly affected by corrosion. Since corrosion can form into any shape, it is represented by some more recognizable measurements to simplify the numerical simulations. As mentioned in the literature review, the corrosion defect can be idealized to be represented by the defect axial length (l_a), defect circumferential length (l_c) and defect depth (d). Preliminary analyses on the effects of corrosion size have been carried out in order to examine the significance of these parameters and to design the parametric study. The preliminary analyses have been carried out for D=762 specimens with the generated stress-strain curve for material mt2 ($\sigma_{er}=1.65$), D/t ratio of 70 and σ_a/σ_y ratio of 0.2. Two p/p_y ratios of 0.0 and 0.4 have been considered in investigating all three parameters (l_a/D , l_c/D and d/t ratios). Additional analyses with a p/p_y ratio of 0.8 have also been considered in the study of the effect of the defect depth to investigate the rupture limit.

4.2.1 Defect Axial Length (l_a)

Numerical simulations have been carried out to study the impact of defect axial length, which is defined in relation to the pipe diameter (D) as l_a/D ratio. The study consists of two defect circumferential length ratios (l_c/D) of 0.25 and 1.5, one defect depth ratio (d/t) of 0.3 and a number of defect axial length ratios of 0.1, 0.25, 0.5, 1.0 and 1.5.

Results of the critical local moment ratio (M_{lc}/M_{lp}) versus defect axial length ratio (l_a/D) are shown in Figs. 4.3 and 4.4. Figure 4.3 shows that there is no significant difference between the critical local moment ratios (M_{lc}/M_{lp}) from $l_a/D=0.25$ to 1.5 for

unpressurized pipes. For pressurized pipes in Fig. 4.4, the lowest local moment ratio appears to be at close to $l_a/D=0.25$ regardless of the circumferential length of the corrosion (l_c/D). There is only a small difference in the critical local moment ratio (M_{lc}/M_{lp}) from $l_a/D=0.5$ to 1.5, and it appears to stabilize to a constant value at a defect length ratio (l_a/D) greater than 1.0. Similarly for the critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature (Φ_{lc}/Φ_{lp}) ratios shown in Figs. 4.5 to 4.8 where the minimum appears to occur at $l_a/D=0.25$. There is also a small difference in these ratios from $l_a/D=0.5$ to 1.5, and they tend to stabilize to a constant value when l_a/D ratio increases beyond 1.0. Based on these findings, results for defect axial length ratio of 1.0 can be considered to be representative of an axial groove corrosion of an infinite length. Since the study focuses on the internal axial groove corrosion where the axial length of the defect is at least twice its circumferential length and the difference in the M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios appear to be small for l_a/D from 0.5 to 1.5, only l_a/D ratio of 1.0 will be considered in the finite element analysis parametric study. Not presented, analyses with l_c/D ratios of less than 0.25 show that the change in the buckling limits (M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios) for the l_a/D ratio from twice l_c/D ratio to 1.0 has also been found to be small.

4.2.2 Defect Circumferential Length (l_c)

Similar to the defect axial length (l_a), the defect circumferential length (l_c) is defined in relation to the pipe diameter (D) as l_c/D ratio. Numerical simulations have been conducted to investigate the effect of the defect circumferential length ratio (l_c/D) for l_a/D ratio of 1.0, d/t ratio of 0.3, and l_c/D ratios of 0.125, 0.25, 0.5, 1.0 and 1.5.

As clearly shown in Fig. 4.9, the critical local moment ratio (M_{lc}/M_{lp}) decreases as the defect circumferential length increases. The rate of decrease reduced significantly at a corrosion circumferential length ratio (l_c/D) greater than 1.0. Similarly, the local compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and local curvature (Φ_{lc}/Φ_{lp}) ratios, shown in Figs. 4.10 and 4.11, follow the same trend. Since the circumferential length of an axial groove corrosion defect is not expected to be large relative to the pipe diameter, the finite element analysis parametric study will be limited to the l_c/D ratio of 1.0.

4.2.3 Defect Depth (d)

The corrosion depth (d) of a pipe can be expressed in relation to the wall thickness (t) as the defect depth ratio (d/t). Numerical analyses have been performed to evaluate the influence of the defect depth ratio (d/t) on pipe buckling for l_a/D ratio of 1.0, l_c/D ratio of 0.55 and a range of d/t ratios.

As shown in Fig. 4.12, the corrosion depth significantly affects the critical local moment ratio (M_{lc}/M_{lp}). As the corrosion depth ratio (d/t) increases, the critical local moment ratio (M_{lc}/M_{lp}) decreases dramatically. There appears to be close to a linear relationship between the critical local moment ratio (M_{lc}/M_{lp}) and corrosion depth ratio (d/t) for unpressurized specimens, but a nonlinear relationship between M_{lc}/M_{lp} and d/t ratios for pressurized specimens. Although it did not occur for the selected cases in the study of the effect of defect depth, failure (analysis stopped) has been found to occur before the application of end rotations in the simulation for the specimens with d/t=0.5 and $p/p_y=0.8$ for the ultimate to yield ratio (σ_{er}) of 1.3 as the remaining pipe wall thickness could not

sustain the internal pressure. The rupture limit varies widely with the pressure level, size of corrosion and material properties.

For unpressurized pipes, it can be seen in Figs. 4.13 and 4.14 that both critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature (Φ_{lc}/Φ_{lp}) ratios decrease almost linearly as the depth of corrosion increases. However for pressurize pipes, the $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios initially decrease at a reducing rate as the d/t ratio increases, but increase with the depth at a larger corrosion depth. This increase, shown in Figs. 4.13 and 4.14, is in a direct relation to the pressure level, and will be discussed in greater details in Chapter 5 for a larger range of parameters. The maximum depth of corrosion to be considered in the finite element analysis parametric study will be discussed in conjunction with the pressure level in Chapter 5 as the rupture limit depends on the pressure level and material properties.

Table 4.1 Measured material properties for LB-I specimens by Chen et al. (2010)

σ_e (MPa)	ϵ_e	σ_t (MPa)	ϵ_t	ϵ_{tp}
326	0.0017	326	0.001698	0.000000
417	0.0023	418	0.002297	0.000122
466	0.0028	467	0.002796	0.000364
497	0.0033	498	0.003295	0.000699
517	0.0038	519	0.003793	0.001090
538	0.0048	540	0.004789	0.001974
549	0.0063	553	0.006280	0.003401
556	0.0083	560	0.008266	0.005347
559	0.0103	565	0.010247	0.007305
565	0.0143	573	0.014199	0.011216
569	0.0183	579	0.018135	0.015117
580	0.0263	595	0.025960	0.022861
587	0.0363	609	0.035657	0.032487
593	0.0463	621	0.045260	0.042028
598	0.0603	634	0.058552	0.055252
600	0.0743	644	0.071669	0.068313

Table 4.2 Generated material properties based on LB-I specimen by Chen et al. (2010)

σ_e (MPa)	ϵ_e	σ_t (MPa)	ϵ_t	ϵ_{tp}
0	0.000000	0	0.000000	0.000000
326	0.001700	326	0.001698	0.000000
450	0.002477	452	0.002474	0.000122
478	0.002866	480	0.002862	0.000364
496	0.003295	497	0.003290	0.000699
508	0.003752	510	0.003745	0.001090
524	0.004729	527	0.004718	0.001974
540	0.006248	543	0.006229	0.003401
552	0.008281	557	0.008247	0.005347
561	0.010307	566	0.010255	0.007305
572	0.014339	580	0.014237	0.011216
579	0.018354	590	0.018188	0.015117
588	0.026345	603	0.026004	0.022861
594	0.036336	615	0.035692	0.032487
596	0.046319	624	0.045278	0.042028
598	0.060300	634	0.058552	0.055252
597	0.074281	641	0.071652	0.068313

Table 4.3 Results for materials with the relative strains (ϵ_{er}) of 0.07, 0.12 and 0.17 with σ_{er} ratio of 1.65

D/t	p/p _y	σ_a/σ_y	L _c /D	d/t	Critical Local Moment Ratio (M_{lc}/M_{lp})			Critical Compressive Strain Ratio ($\epsilon_{lc}/\epsilon_{lp}$)			Critical local Curvature Ratio (Φ_{lc}/Φ_{lp})		
					$\epsilon_{er}=0.07$	$\epsilon_{er}=0.12$	$\epsilon_{er}=0.17$	$\epsilon_{er}=0.07$	$\epsilon_{er}=0.12$	$\epsilon_{er}=0.17$	$\epsilon_{er}=0.07$	$\epsilon_{er}=0.12$	$\epsilon_{er}=0.17$
40	0.0	0.2	0.25	0.3	0.89	0.89	0.89	0.57	0.56	0.56	0.55	0.55	0.55
40	0.0	0.2	1.00	0.3	0.68	0.69	0.69	0.40	0.40	0.41	0.37	0.36	0.36
40	0.4	0.2	0.25	0.3	0.84	0.83	0.83	0.60	0.59	0.59	0.55	0.55	0.54
40	0.4	0.2	1.00	0.3	0.59	0.59	0.59	0.46	0.46	0.45	0.39	0.38	0.37
100	0.0	0.2	0.25	0.3	0.74	0.74	0.74	0.58	0.58	0.57	0.58	0.58	0.58
100	0.0	0.2	1.00	0.3	0.55	0.54	0.54	0.49	0.48	0.47	0.46	0.46	0.46
100	0.4	0.2	0.25	0.3	0.74	0.74	0.74	0.69	0.69	0.69	0.66	0.65	0.64
100	0.4	0.2	1.00	0.3	0.47	0.46	0.45	0.54	0.54	0.53	0.47	0.46	0.45

Table 4.4 Results for materials with the ultimate to yield strength ratios (σ_{er}) of 1.3, 1.65 and 2.0 with ε_{er} of 0.12

D/t	p/p _y	σ_a/σ_y	L _c /D	d/t	Critical Local Moment Ratio (M_{lc}/M_{lp})			Critical Compressive Strain Ratio ($\varepsilon_{lc}/\varepsilon_{lp}$)			Critical Local Curvature Ratio (Φ_{lc}/Φ_{lp})		
					$\sigma_{er}=1.3$	$\sigma_{er}=1.65$	$\sigma_{er}=2.0$	$\sigma_{er}=1.3$	$\sigma_{er}=1.65$	$\sigma_{er}=2.0$	$\sigma_{er}=1.3$	$\sigma_{er}=1.65$	$\sigma_{er}=2.0$
40	0.0	0.2	0.25	0.3	0.90	0.89	0.89	0.57	0.56	0.55	0.54	0.55	0.55
40	0.0	0.2	1.00	0.3	0.69	0.69	0.68	0.42	0.40	0.38	0.37	0.36	0.36
40	0.4	0.2	0.25	0.3	0.82	0.83	0.84	0.60	0.59	0.59	0.55	0.55	0.54
40	0.4	0.2	1.00	0.3	0.55	0.59	0.61	0.46	0.46	0.44	0.38	0.38	0.37
100	0.0	0.2	0.25	0.3	0.75	0.74	0.74	0.64	0.58	0.56	0.64	0.58	0.57
100	0.0	0.2	1.00	0.3	0.54	0.54	0.55	0.50	0.48	0.47	0.48	0.46	0.45
100	0.4	0.2	0.25	0.3	0.71	0.74	0.75	0.71	0.69	0.66	0.65	0.65	0.63
100	0.4	0.2	1.00	0.3	0.42	0.46	0.49	0.59	0.54	0.51	0.50	0.46	0.44

Table 4.5 Parameters for the material properties used in the finite element parametric study

Material type	E = 200 000 MPa, $\nu = 0.265$, $\sigma_{t0} = 480$ MPa				
	n	C	σ_{er}	ε_{er}	ε_{tp0}
mt1	8.539	4.729×10^{-3}	1.30	0.12	0.00
mt2	8.930	4.640×10^{-4}	1.65	0.12	0.00
mt3	9.029	7.600×10^{-5}	2.00	0.12	0.00

Figure 4.1 Measured and generated engineering stress-strain curves for LB-I specimen by Chen et al. (2010)

Figure 4.2 Measured (calculated) and generated true stress-strain curves for LB-I specimen by Chen et al. (2010)

Figure 4.3 Critical local moment ratio versus defect axial length ratio for $D/t=70$, $p/p_y=0.0$ and $d/t=0.3$

Figure 4.4 Critical local moment ratio versus defect axial length ratio for $D/t=70$, $p/p_y=0.4$ and $d/t=0.3$

Figure 4.5 Critical compressive strain ratio versus defect axial length ratio for $D/t=70$, $p/p_y=0.0$ and $d/t=0.3$

Figure 4.6 Critical compressive strain ratio versus defect axial length ratio for $D/t=70$, $p/p_y=0.4$ and $d/t=0.3$

Figure #4.7 Critical local curvature ratio versus defect axial length ratio for $D/t=70$, $p/p_y=0.0$ and $d/t=0.3$

Figure #4.8 Critical local curvature ratio versus defect axial length ratio for $D/t=70$, $p/p_y=0.4$ and $d/t=0.3$

Figure 4.9 Critical local moment ratio versus defect circumferential length ratio for $D/t=70$, $l_a/D=100$ and $d/t=0.3$

Figure 4.10 Critical compressive strain ratio versus defect circumferential length ratio for $D/t=70$, $l_a/D=100$ and $d/t=0.3$

Figure 4.11 Critical local curvature ratio versus defect circumferential length ratio for $D/t=70$, $l_a/D=100$ and $d/t=0.3$

Figure 4.12 Critical local moment ratio versus defect depth ratio for $D/t=70$, $l_a/D=1.00$ and $l_c/D=0.55$

Figure 4.13 Critical compressive strain ratio versus defect depth ratio for $D/t=70$, $l_a/D=1.00$ and $l_c/D=0.55$

Figure 4.14 Critical local curvature ratio versus defect depth ratio for $D/t=70$, $l_a/D=1.00$ and $l_c/D=0.55$

Chapter 5 Analysis and Simulation Results

A broad parametric study has been performed to evaluate the behaviour of line pipes with internal axial groove corrosion subjected to a combined loading. It has been found in the preliminary study that the buckling response is directly influenced by many parameters such as D/t , p/p_y , σ_a/σ_y , l_c/D and d/t ratios as well as the ultimate to yield strength ratio (σ_{er}) of the materials. Utilizing the FEA results, non-dimensional equations have been developed to predict the buckling limits for corroded pipelines.

5.1 Parameters Considered in the Parametric Study

One of the objectives of this research is to develop non-dimensional equations that are able to predict the buckling limits of corroded line pipes under a combined loading. Each of these is a function of a number of parameters, and can be expressed as follows:

- The corroded to plain pipe critical local moment ratio (M_{lc}/M_{lp}),

$$\frac{M_{lc}}{M_{lp}} = f \left[\frac{D}{t}, \frac{p}{p_y}, \frac{\sigma_a}{\sigma_y}, \frac{l_c}{D}, \frac{d}{t}, \sigma_{er} \right]. \quad (5.1)$$

- The corroded to plain pipe critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$),

$$\frac{\epsilon_{lc}}{\epsilon_{lp}} = g \left[\frac{D}{t}, \frac{p}{p_y}, \frac{\sigma_a}{\sigma_y}, \frac{l_c}{D}, \frac{d}{t}, \sigma_{er} \right]. \quad (5.2)$$

- The corroded to plain pipe critical local curvature ratio (Φ_{lc}/Φ_{lp}),

$$\frac{\Phi_{lc}}{\Phi_{lp}} = h \left[\frac{D}{t}, \frac{p}{p_y}, \frac{\sigma_a}{\sigma_y}, \frac{l_c}{D}, \frac{d}{t}, \sigma_{er} \right]. \quad (5.3)$$

Numerous combinations of D/t , p/p_y , σ_a/σ_y , l_c/D , d/t and σ_{er} ratios can be considered in the parametric study. However, it is impractical to examine all possible combinations and, thus, only selected ranges of these parameters are evaluated. Based on Chapters 2 and 4, the selected ranges of the parameters considered in the parametric study are presented and discussed below.

For the first parameter considered, the range of diameter to thickness (D/t) ratio found in oil and natural gas pipelines generally varies from 40 to 100 even though it is allowed to go up to 197 according to CAN/CSA-Z662-11 (CSA 2011). As a result, three D/t ratios of 40, 70 and 100 are evaluated in the parametric study.

The second parameter is related to the internal pressure (p). It is usually defined as (p/p_y) ratio in relation to the yield pressure (p_y), the pressure to generate a hoop stress equal to the yield strength. According to CAN/CSA-Z662-11 (CSA 2011), the maximum design pressure for pipelines is 80% of p_y (based on the actual yield strength). Therefore, the p/p_y ratios of 0.0, 0.4 and 0.8 are considered in the parametric study. As previously mentioned, the yield strength used in the parametric study to define the ultimate to yield strength ratio (σ_{er}) is based on the proportional limit. Thus, for a material with a high σ_{er} ratio of 2.0, the p/p_y ratio of 1.0 is also considered as the actual yield strength can be considerably higher than the proportional limit.

The third parameter is the axial load ratio (σ_a/σ_y), which can be defined as the ratio of the average compressive longitudinal stress due to the axial load over the yield strength. In

this study, the yield strength is taken as the stress at the proportional limit. The main contributors for the axial load are the temperature difference between construction and operation, and the stress generated to restrain against axial contraction when the pipe is pressurized. Therefore, the average compressive stress due to temperature difference can be calculated as

$$\sigma_a = E\alpha\Delta_T, \quad (5.4)$$

where α is the coefficient of thermal expansion of the pipe material taken as $11.7 \times 10^{-6}/^\circ\text{C}$ for steel and Δ_T is the difference between construction and operation temperatures.

According to Mohareb et al. (1994), the maximum temperature difference can be as high as 45°C . This approximately leads to a σ_a of 105 MPa and an equivalent σ_a/σ_y ratio of 0.22 based on the yield strength of 480 MPa, or 0.35 for a yield strength of 300 MPa. Thus, three σ_a/σ_y ratios of 0.0, 0.2 and 0.4 are considered in the parametric study. In addition, high internal pressure may induce a significant tensile axial load when the temperature rise is negligible. The induced average stress due to internal pressure, taken as positive in compression, can be calculated as

$$\sigma_a = -\nu \frac{p}{p_y} \sigma_y. \quad (5.5)$$

This leads to $\sigma_a/\sigma_y = -0.212$ for the p/p_y ratio of 0.8 and ν of 0.265. In addition to 0.0, 0.2 and 0.4, analyses with σ_a/σ_y ratio of -0.2 are also being carried out for selective cases of the parametric combinations. These axial stress ratios consider the effect ranging from a maximum temperature rise with no internal pressure to no temperature change with a maximum internal pressure. The effect of temperature drop with the maximum internal

pressure is not considered as the pipe can be expected to reach the temperature at construction during the annual seasonal temperature change, and as well as the buckling (failure) limits tend to improve with a higher tensile axial load.

The fourth parameter is the corrosion size, which can have a significant effect on the pipe performance. Even though the corrosion shape is irregular, it is idealized to be rectangular in shape with sides given by an axial length (l_a), a circumferential width (l_c) and a uniform depth (d). As discussed in the previous chapter, the effect of the corrosion length in the axial direction has been found to be negligible with the corrosion axial length (l_a) longer than $1.0D$. In addition, it has also been found that there is only a small change in the buckling limits from the length of $0.5D$ to $1.0D$. Therefore, the corrosion defect with a constant l_a/D ratio of 1.0 is considered in the parametric study. Similarly, the change in the pipe performance with the defect circumferential length (l_c) is significantly reduced at a length greater than $1.0D$. Since the focus of this study is on axial groove corrosion, the corrosion circumferential length (l_c) considered will be limited to $1.0D$. Thus, the defect circumferential length ratios (l_c/D) of 0.25 and 0.55 and 1.0 are considered to model the effect of corrosion defect size along the circumferential direction. Furthermore, as the corrosion increase in depth, the remaining wall thickness may not be able to sustain the internal pressure. As a result, the pipe may fail under bursting. In case of a pipe having a long axial groove corrosion defect with a depth of $0.5t$, bursting can occur at close to p/p_y ratio of 0.8 for the σ_{er} ratio of 1.65 and p/p_y ratio of 1.0 for the σ_{er} ratio of 2.0 . Thus, the corrosion depth considered in the parametric study will be limited to half the wall thickness, and three corrosion depth ratios (d/t) of 0.15 , 0.3 and 0.5 are assessed in the current study.

The last parameter considered in the parametric study is related to the material properties. As shown in Chapter 4, the relative strain at peak stress (ϵ_{er}) does not have a large effect on the buckling (failure) limits (M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios). Thus, only a single ϵ_{er} of 0.12 is considered. However, the buckling limits are notably affected by the ultimate to yield strength ratio (σ_{er}). Therefore, three σ_{er} ratios of 1.3 1.65 and 2.0 are considered to represent the range of pipe materials used. The modulus of elasticity (E) used in the parametric study is 200000 MPa and the Poisson's ratio (ν) is 0.265, as measured by Dorey et al. (2001).

The finite element model based on the mesh configuration discussed in Chapter 3 is adopted for the parametric study. The model has an outside diameter of 762 mm and a length of 5.0D. As shown in Table 5.1, the principal combinations consist of 3 D/t , 3 p/p_y , 3 σ_a/σ_y , 3 l_c/D , 3 d/t and 3 σ_{er} ratios, with an additional p/p_y ratio of 1.0 for material type mt3 ($\sigma_{er}= 2.0$) and selective combinations for σ_a/σ_y ratio of -0.2 with D/t ratios of 40 and 100, and p/p_y ratio of 0.4 for all three materials. In total, the parametric study consists of over 900 analyses that cover most of the geometries, load combinations and material properties that can be expected for pipelines with internal axial groove corrosion defects. Results of the parametric study are listed in the Appendix C where “Failed” denotes the cases where failure occurred (analyses stopped) before the application of end rotations.

5.2 Effects of Various Parameters

Results of the parametric study are briefly analyzed to get a better understanding of the effects of various parameters (D/t , p/p_y , σ_a/σ_y , l_c/D , d/t and σ_{er}) on the buckling response of corroded line pipes and to help developing the equations for predicting the buckling

(failure) limits. Lists of parameters used in this investigation are shown in Table 5.2. Since the change in the response with various parameters is generally the same for different σ_{er} ratios, only results from material type mt2 with an σ_{er} ratio of 1.65 is utilized in the investigation except on the effect of the length of yield plateau (ϵ_{tp}) where σ_{er} ratio of 1.3 is used. The parameters considered in the study on the effect of yield plateau length are shown in Table 5.3. The following discussions describe the impact of each parameter.

5.2.1 Effects of Geometry (D/t) and Loading (p/p_y , σ_a/σ_y)

Selective results of the analyzed pipe response versus D/t ratio with various p/p_y and σ_a/σ_y ratios for a corrosion defect of $l_c/D=0.55$ and $d/t=0.15$ are shown in Figs. 5.1 to 5.6. It can be seen in Figs 5.1 and 5.2 that the critical local moment ratio (M_{lc}/M_{lp}) decreases as the D/t ratio increases. Figs 5.3 to 5.6 show an increase in the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}) with the D/t ratio. As expected, the D/t ratio appears to have a similar effect on the critical compressive strain and critical local curvature ratios since the critical local curvature is calculated partly from the critical compressive strain. In all cases, there appear to be a relatively smooth change for the decreasing critical local moment ratio (M_{lc}/M_{lp}), and the increasing critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}), with the D/t ratio for various load combinations (p/p_y and σ_a/σ_y ratios). Similar results can be seen in Figs. B.1 to B.12 in Appendix B for the pipe response versus p/p_y and σ_a/σ_y ratios. Thus, equations with a single term of each parameter of D/t , p/p_y and σ_a/σ_y ratios appear to be sufficient to predict the buckling (failure) limits for corroded pipes.

5.2.2 Effects of Corrosion Defect

In current study, the defect axial length ratio (l_a/D) has been taken as 1.0. Therefore, the discussion will focus only on the effects of the defect circumferential length ratio (l_c/D) and the defect depth ratio (d/t).

5.2.2.1 Effects of the Defect Circumferential Length Ratio (l_c/D)

The buckling limits are expected to be significantly influenced by the corrosion defect size. Figures 5.7 to 5.9 show the critical local moment ratio versus defect circumferential length ratio (l_c/D) for selective geometry and load combinations. As expected, the critical local moment ratio (M_{lc}/M_{lp}) changes inversely with the defect circumferential length (l_c/D) ratio, but at a reducing rate of change as the l_c/D ratio increases. The reducing rate of change with the l_c/D ratio is more pronounced for non-pressurized cases. Normally, the size (extent) of the buckling deformation reduces with the increase in the D/t ratio and/or the reduction in the p/p_y ratio. Thus, the effect of increase in the circumferential length of the corrosion defect at a large l_c/D ratio becomes less significant when the local buckling deformation is already within the corrosion defect.

Figures 5.10 to 5.12 show the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) versus l_c/D ratio for selective geometry and load combinations, while Figs. 5.13 to 5.15 show that for critical local curvature ratio (Φ_{lc}/Φ_{lp}). As can be seen in these figures, the overall effect of l_c/D ratio on the $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios is similar. Thus, the discussion here will refer to only the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$), even though it is equally applicable to Φ_{lc}/Φ_{lp} . In general, the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) changes inversely with the l_c/D ratio, but at a reducing rate as the defect circumferential length (l_c) increases. However,

the increase in the defect circumferential length (l_c) will not have any appreciable effect on the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) when the l_c/D is already large, especially for $p/p_y=0.0$. This can be clearly seen in Fig. 5.11 for a non-pressurized pipe with $D/t=70$ and $d/t=0.3$. The extent of buckling deformation is smaller with a lower internal pressure and a higher D/t ratio. When the buckling deformation is already within the corrosion defect, the increase in the l_c/D ratio will not significantly change the extent of buckling deformation, and hence the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) will likely not significantly decrease. This situation can occur at a smaller l_c/D ratio for a non-pressurized pipe with an increase in the effective D/t ratio due to corrosion defect. This may explain the phenomenon observed in Fig. 5.11 where there is a little change in the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) with the increase in the l_c/D ratio after a certain l_c/D ratio.

5.2.2.2 Effects of the Defect Depth Ratio (d/t)

The other parameter for the corrosion defect is the corrosion depth (d/t). Figures 5.16 to 5.18 show the trends of the critical local moment ratio (M_{lc}/M_{lp}) versus the defect depth ratio (d/t) for selective geometry and load combinations. Similar to the l_c/D ratio, the critical local moment ratio (M_{lc}/M_{lp}) changes inversely with the change in the d/t ratio, but at a reducing rate as the d/t ratio increases. However, the critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratios (Φ_{lc}/Φ_{lp}) initially decrease with the increase in the corrosion depth ratio (d/t), but increase at a large defect depth (d). These relations can be seen in Figs. 5.19 to 5.24 across almost all D/t , p/p_y (except for $p/p_y=0.0$) and σ_a/σ_y ratios. The increase of the $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios with the d/t ratio appears to grow with the D/t and p/p_y ratios, but decline with the σ_a/σ_y ratio. This is probably due to the biaxial

stiffening effect in a pressurized pipe with the increase in the effective hoop stress as the corrosion depth increases. For a pressurized pipe, the buckling shape is bulging. Thus, the buckling shape introduces additional axial tension stress as the deformation increases, and hence creates a more favourable stress at crest of the buckle shape that is close to biaxial tension. This will delay the deformation at which the critical local moment is reached.

Nevertheless, in actual situations, buckling (critical local moment) will likely occur at the minimum critical compressive strain and critical local curvature rather than at the higher values at $d/t=0.5$. It is expected that the lower critical compressive strain and critical local curvature at a smaller d/t to be reached during the life of a pipe segment before the corrosion depth of $d/t=0.5$ is reached. Thus for this study, data for cases with the increase in the critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratios (Φ_{lc}/Φ_{lp}) with respect to corrosion depth ratio (d/t) are excluded from the regression analyses to develop the equations for predicting critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratios (Φ_{lc}/Φ_{lp}).

5.2.3 Effects of the Length of the Yield Plateau (ϵ_{tp})

The focus of this study is on the buckling response of line pipes with the material properties that has no yield plateau. Tan (2008) has found that the change in buckling response for plain pipes with varying geometric and loading parameters does not exhibit a smooth trend when there is a yield plateau on the stress-strain curve. Nevertheless, a brief study is conducted to explore if results for the materials without a yield plateau in this study can be extended to that with a yield plateau.

Results of a few cases for materials having yield plateau lengths of 0.00 and 0.02 (0.00_{yp} and 0.02_{yp} , respectively) are compared in Table 5.3. The critical local moment ratio (M_{lc}/M_{lp}), critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}) of 0.02_{yp} specimens in general are higher than that for 0.00_{yp} . This difference is found to be much higher in the $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios than the M_{lc}/M_{lp} ratio. However, there does not appear to be a simple conversion factor that can be used to convert results for 0.00_{yp} to that for 0.02_{yp} . Thus, a more thorough investigation, which is outside the scope of this study, is required on the effect of yield plateau on corroded line pipes.

5.3 Generating the Buckling Equations

A set of equations to predict the buckling (failure) limits of corroded pipes, which include critical local moment ratio (M_{lc}/M_{lp}), critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}), are developed based on the results of the parametric study. As stated earlier, the equations developed in the parametric study are limited to materials with no yield plateau as no clear direct relationship can be found between the response for pipes having materials with and without yield plateau. In addition, results of cases where the specimens failed before the application of end rotations are not included. Data for critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratios (Φ_{lc}/Φ_{lp}) for specimens where the $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios increase with the d/t ratio increase are also excluded.

Single-stage regression has been utilized to generate the non-dimensional equations. The regression is performed to include the effects of D/t , p/p_y , σ_a/σ_y , l_c/D and d/t ratios. With i being the material type, the equations can be expressed as follows:

$$\frac{M_{lc}}{M_{lp}} = f_i \left[\frac{D}{t}, \frac{p}{p_y}, \frac{\sigma_a}{\sigma_y}, \frac{l_c}{D}, \frac{d}{t} \right], i = 1 \text{ to } 3 \quad (5.6)$$

$$\frac{\varepsilon_{lc}}{\varepsilon_{lp}} = g_i \left[\frac{D}{t}, \frac{p}{p_y}, \frac{\sigma_a}{\sigma_y}, \frac{l_c}{D}, \frac{d}{t} \right], i = 1 \text{ to } 3 \quad (5.7)$$

$$\frac{\Phi_{lc}}{\Phi_{lp}} = h_i \left[\frac{D}{t}, \frac{p}{p_y}, \frac{\sigma_a}{\sigma_y}, \frac{l_c}{D}, \frac{d}{t} \right], i = 1 \text{ to } 3 \quad (5.8)$$

5.3.1 Nonlinear Regression (Rn) for Critical Buckling Equations

A nonlinear regression is conducted to generate buckling equations for corroded pipes. Based on the trends observed in Section 5.2, the proposed equations have been constructed to take the following form:

$$\frac{M_{lc}}{M_{lp}} = 1 - \frac{\left[U_m + U_{mDt} \left(\frac{D}{t} \right)^{V_{mDt}} \left(1 + \left(U_{mp} \frac{p}{p_y} \right)^{V_{mp}} \right) \left(1 - U_{ma} \frac{\sigma_a}{\sigma_y} \right)^{V_{ma}} \right]}{\left[\left(\frac{l_c}{D} \right)^{V_{mlc}} \left(\frac{d}{t} \right)^{V_{md}} \right]} \quad (5.9)$$

$$\frac{\varepsilon_{lc}}{\varepsilon_{lp}} = 1 - \left[U_s + U_{sDt} \left(\frac{D}{t} \right)^{V_{sDt}} \left(1 + \left(\frac{p}{p_y} \right)^{V_{sp}} \right) \left(1 - \frac{\sigma_a}{\sigma_y} \right)^{V_{sa}} \right] \left[\left(\frac{l_c}{D} \right)^{V_{slc}} \left(\frac{d}{t} \right)^{V_{sd}} \right] \quad (5.10)$$

$$\frac{\Phi_{lc}}{\Phi_{lp}} = 1 - \left[U_c + U_{cDt} \left(\frac{D}{t} \right)^{V_{cDt}} \left(1 + \left(\frac{p}{p_y} \right)^{V_{cp}} \right) \left(1 - \frac{\sigma_a}{\sigma_y} \right)^{V_{ca}} \right] \left[\left(\frac{l_c}{D} \right)^{V_{clc}} \left(\frac{d}{t} \right)^{V_{cd}} \right] \quad (5.11)$$

where V_{xx} and U_{xx} are parameters that are determined through the nonlinear regression analysis based on the least squares method of the difference between the predicted and FEA ratios of the buckling limits. A set of parameters corresponding to each material

type is calculated and presented in Tables 5.4 to 5.6. Figures 5.25 to 5.42 show the graphical comparisons of the predicted results (R_n), using Eqs. (5.9) to (5.11), to the FEA results for all three materials. The mean, standard deviation and coefficient of variation for R_n /FEA, and the mean and standard deviation for R_n -FEA are shown in Tables 5.7 and 5.8, respectively.

Overall, there is a good agreement between the predicted (R_n) and FEA results for M_{lc}/M_{lp} , $\varepsilon_{lc}/\varepsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios. Figures 5.25 to 5.30 show that the differences between predicted (R_n) and FEA M_{lc}/M_{lp} ratios are mostly within ± 0.1 and the predicted (R_n) to FEA ratios of the M_{lc}/M_{lp} ratio are mostly bounded within 0.85 to 1.15, with a greater deviation from unity as the M_{lc}/M_{lp} ratio decreases. Similar results can be seen in Figs. 5.31 to 5.42 for both $\varepsilon_{lc}/\varepsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios. The differences in the R_n -FEA outside the bound of ± 0.1 are usually associated with unpressurized specimens having a relatively high D/t ratio and a large d/t ratio. As mentioned in the previous section, the effect of the increase in the l_c/D ratio at a large l_c/D ratio is significantly less for an unpressurized pipe. Therefore, there is a smaller difference in the buckling (failure) limits for unpressurized pipe with the same high D/t ratio and large d/t ratio between the l_c/D ratio of 0.55 and 1.00 compared to 0.25 and 0.55. There may also be no change in the $\varepsilon_{lc}/\varepsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios from l_c/D ratio of 0.55 to 1.00, as shown in Figs. 5.11 and 5.14. This phenomenon is not able to be captured by the simple empirical equation, and hence resulted in a larger error in the prediction.

Based on results of the parametric study and Figs. 5.1 to 5.24 and B.1 to B.12 in Appendix B, the d/t ratio appears to be the most important parameter for the critical buckling equations. This is followed by the l_c/D ratio. The other three parameters (D/t ,

p/p_y and σ_a/σ_y ratios) have about the same importance in the critical buckling equations, with the p/p_y ratio to be slightly more important. This is to be followed by the D/t and σ_a/σ_y .

5.3.2 Assessment of Critical Buckling Equations

In order to further examine the accuracy of the proposed equations, two-stage validation process is considered. First, the equations are validated against FEA results of additional geometries, corrosion dimensions and load combinations, which are summarized in Table 5.9. This is followed by the validation using materials (mv1 and mv2) with σ_{er} ratios of 1.5 and 1.8 for the same geometries, corrosion dimensions and load combinations as listed in Table 5.9. The predicted values for materials mv1 and mv2 are calculated using the quadratic Lagrangian interpolation polynomial utilizing the parameters in Tables 5.4 to 5.6.

Good correlation is found between the predicted (R_n) and FEA results. It can be seen in Figs 5.43 to 5.48 that the error is still within the range (± 0.1) for all buckling limits (M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios) and within 10% for almost all cases in the ratios of predicted ratio to FEA ratio (R_n/FEA). These are supported by the calculated mean, standard deviation and coefficient of variation for R_n/FEA , and the mean and standard deviation for R_n -FEA summarized in Tables 5.10 and 5.11, respectively.

Since the equations are generated based on least squares method, it is expected that close to half the cases to be over predicted. A simple correction factor may be used to compensate for these over predictions. A correction factor can be added to the predicted ratios to give a modified ratio as

$$R_{n_m} = R_n - 0.1 \geq 0.0 \quad \text{for } R_n < 0.6 \text{ and} \quad (5.12)$$

$$= 1.25R_n - 0.25 \leq 1.0 \quad \text{for } R_n \geq 0.6, \quad (5.13)$$

where R_n is the ratio calculated using equations (5.9) to (5.11).

Figures 5.49 and 5.51 show the R_n -FEA versus R_n for the buckling (failure) limits data used in the regression analyses, together with the line showing the correction factor to be used at various R_n . It can be seen that the modified R_{n_m} will be conservative in almost all cases except for a few outliers.

Table 5.1 Parameters considered in the parametric study

σ_{cr}	D/t	p/p _y	σ_a/σ_y	l _c /D	d/t
1.30	40, 70, 100	0.0, 0.4, 0.8	-0.2, 0.0, 0.2, 0.4	0.25, 0.55, 1.00	0.15, 0.30, 0.50
1.65	40, 70, 100	0.0, 0.4, 0.8	-0.2, 0.0, 0.2, 0.4	0.25, 0.55, 1.00	0.15, 0.30, 0.50
2.00	40, 70, 100	0.0, 0.4, 0.8, 1.0	-0.2, 0.0, 0.2, 0.4	0.25, 0.55, 1.00	0.15, 0.30, 0.50

Table 5.2 Geometry and load combinations used in the investigation of the effects of the parameters

Investigated parameter : D/t ratio			
p/p _y	σ_a/σ_y	l _c /D	d/t
0.0, 0.4, 0.8	0.0, 0.2, 0.4	0.55	0.15
Investigated parameter : p/p _y ratio			
D/t	σ_a/σ_y	l _c /D	d/t
40, 70, 100	0.0, 0.2, 0.4	0.55	0.15
Investigated parameter : σ_a/σ_y ratio			
D/t	p/p _y	l _c /D	d/t
40, 70, 100	0.0, 0.4, 0.8	0.55	0.15
Investigated parameter : l _c /D ratio			
D/t	p/p _y	σ_a/σ_y	d/t
40, 70, 100	0.0, 0.4, 0.8	-0.2, 0.0, 0.2, 0.4	0.30
Investigated parameter : d/t ratio			
D/t	p/p _y	σ_a/σ_y	l _c /D
40, 70, 100	0.0, 0.4, 0.8	-0.2, 0.0, 0.2, 0.4	0.55

Table 5.3 Comparisons of FEA results for a material having σ_{er} ratio of 1.3 and yield plateau lengths of 0.0 and 0.02

D/t	p/p _y	σ_a/σ_y	l _c /D	d/t	Critical local moment ratio (M _{lc} /M _{lp})			Critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$)			Critical local curvature ratio (Φ_{lc}/Φ_{lp})		
					$\epsilon_{ep}=0.0$ (1)	$\epsilon_{ep}=0.02$ (2)	Ratio (2)/(1)	$\epsilon_{ep}=0.0$ (3)	$\epsilon_{ep}=0.02$ (4)	Ratio (4)/(3)	$\epsilon_{ep}=0.0$ (5)	$\epsilon_{ep}=0.02$ (6)	Ratio (6)/(5)
40	0.0	0.2	0.25	0.3	0.90	0.91	1.01	0.57	0.69	1.21	0.54	0.65	1.21
40	0.0	0.2	1.00	0.3	0.69	0.66	0.96	0.42	0.48	1.15	0.37	0.45	1.22
40	0.4	0.2	0.25	0.3	0.82	0.81	0.99	0.60	0.69	1.16	0.55	0.62	1.14
40	0.4	0.2	1.00	0.3	0.55	0.55	1.00	0.46	0.64	1.40	0.38	0.54	1.42
100	0.0	0.2	0.25	0.3	0.75	0.84	1.13	0.64	0.97	1.52	0.64	0.91	1.42
100	0.0	0.2	1.00	0.3	0.54	0.62	1.14	0.50	0.84	1.66	0.48	0.73	1.52
100	0.4	0.2	0.25	0.3	0.71	0.79	1.10	0.71	0.84	1.17	0.65	0.79	1.22
100	0.4	0.2	1.00	0.3	0.42	0.42	1.00	0.59	0.78	1.31	0.50	0.62	1.24

Table 5.4 Determined parameters for the critical local moment ratio (M_{lc}/M_{lp}) equations

Material	σ_{er}	V_{mDt}	V_{mp}	V_{ma}	V_{mlc}	V_{md}	U_{mDt}	U_m	U_{mp}	U_{ma}
mt1	1.30	0.50	2.54	3.66	0.55	0.75	0.020	0.66	4.886	-1.28
mt2	1.65	0.41	1.67	5.16	0.55	0.75	0.082	0.37	0.909	-0.40
mt3	2.00	0.33	1.68	6.18	0.55	0.75	0.134	0.33	0.499	-0.25

Table 5.5 Determined parameters for the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) equations

Material	σ_{er}	V_{sDt}	V_{sp}	V_{sa}	V_{slc}	V_{sd}	U_{sDt}	U_s
mt1	1.30	0.39	0.75	0.43	0.21	0.35	-0.077	1.22
mt2	1.65	0.24	0.99	0.32	0.21	0.35	-0.165	1.31
mt3	2.00	0.21	1.22	0.29	0.21	0.35	-0.192	1.34

Table 5.6 Determined parameters for the critical local curvature ratio (Φ_{lc}/Φ_{lp}) equations

Material	σ_{er}	V_{cDt}	V_{cp}	V_{ca}	V_{clc}	V_{cd}	U_{cDt}	U_c
mt1	1.30	0.35	1.73	0.28	0.24	0.37	-0.108	1.39
mt2	1.65	0.35	1.73	0.28	0.24	0.37	-0.088	1.331
mt3	2.00	0.35	1.73	0.28	0.24	0.37	-0.078	1.294

Table 5.7 Summary of Rn/FEA ratio comparisons in the parametric study

Material	σ_{er}	M_{lc}/M_{lp}			$\varepsilon_{lc}/\varepsilon_{lp}$			Φ_{lc}/Φ_{lp}		
		Mean	Stand. dev.	C.V.	Mean	Stand. dev.	CV	Mean	Stand. dev.	CV
mt1	1.30	1.005	0.081	8.0%	1.006	0.084	8.3%	1.005	0.082	8.1%
mt2	1.65	1.002	0.073	7.3%	1.008	0.089	8.8%	1.007	0.093	9.2%
mt3	2.00	1.002	0.077	7.7%	1.006	0.082	6.7%	1.005	0.086	8.6%

Table 5.8 Summary of Rn-FEA comparisons in the parametric study

Material	σ_{er}	M_{lc}/M_{lp}		$\varepsilon_{lc}/\varepsilon_{lp}$		Φ_{lc}/Φ_{lp}	
		Mean	Stand. dev.	Mean	Stand. dev.	Mean	Stand. dev.
mt1	1.30	-0.002	0.043	-0.001	0.039	-0.001	0.033
mt2	1.65	0.002	0.036	-0.001	0.038	-0.001	0.037
mt3	2.00	0.000	0.035	0.001	0.037	0.000	0.035

Table 5.9 Parameters used in the validation

D/t	p/p _y	σ_a/σ_y	L _c /D	d/t
55, 85	0.2, 0.6	0.1, 0.3	0.40, 0.75	0.2, 0.4

Table 5.10 Summary of the Rn/FEA ratio comparisons in the validation

Material	σ_{er}	M_{lc}/M_{lp}			$\varepsilon_{lc}/\varepsilon_{lp}$			Φ_{lc}/Φ_{lp}		
		Mean	Stand. dev.	C.V.	Mean	Stand. dev.	CV	Mean	Stand. dev.	CV
mt1	1.30	1.032	0.045	4.3%	1.037	0.027	2.6%	1.043	0.045	4.3%
mv1	1.50	1.018	0.027	2.7%	1.032	0.036	3.5%	1.041	0.036	3.4%
mv2	1.80	1.015	0.016	1.7%	1.018	0.036	3.6%	1.027	0.028	2.7%
mt3	2.00	1.016	0.017	1.7%	0.998	0.038	3.8%	1.014	0.035	3.4%

Table 5.11 Summary of the Rn-FEA comparisons in the validation

Material	σ_{er}	M_{lc}/M_{lp}		$\varepsilon_{lc}/\varepsilon_{lp}$		Φ_{lc}/Φ_{lp}	
		Mean	Stand. dev.	Mean	Stand. dev.	Mean	Stand. dev.
mt1	1.30	0.021	0.031	0.023	0.017	0.023	0.023
mv1	1.50	0.011	0.018	0.020	0.022	0.022	0.018
mv2	1.80	0.010	0.011	0.012	0.022	0.015	0.015
mt3	2.00	0.011	0.011	0.000	0.023	0.007	0.019

Figure 5.1 Critical local moment ratio versus D/t ratio with respect to p/p_y ratio for $\sigma_a/\sigma_y=0.2$, $l_c/D=0.55$ and $d/t=0.15$

Figure 5.2 Critical local moment ratio versus D/t ratio with respect to σ_a/σ_y ratio for $p/p_y=0.4$, $l_c/D=0.55$ and $d/t=0.15$

Figure 5.3 Critical compressive strain ratio versus D/t ratio with respect to p/p_y ratio for $\sigma_a/\sigma_y=0.2$, $l_c/D=0.55$ and $d/t=0.15$

Figure 5.4 Critical compressive strain ratio versus D/t ratio with respect to σ_a/σ_y ratio for $p/p_y=0.4$, $l_c/D=0.55$ and $d/t=0.15$

Figure 5.5 Critical local curvature ratio versus D/t ratio with respect to p/p_y ratio for $\sigma_a/\sigma_y=0.2$, $l_c/D=0.55$ and $d/t=0.15$

Figure 5.6 Critical local curvature ratio versus D/t ratio with respect to σ_a/σ_y ratio for $p/p_y=0.4$, $l_c/D=0.55$ and $d/t=0.15$

Figure 5.7 Critical local moment ratio versus l_c/D ratio with respect to D/t ratio for $p/p_y=0.4$, $\sigma_a/\sigma_y=0.2$ and $d/t=0.3$

Figure 5.8 Critical local moment ratio versus l_c/D ratio with respect to p/p_y ratio for $D/t=70$, $\sigma_a/\sigma_y=0.2$ and $d/t=0.3$

Figure 5.9 Critical local moment ratio versus l_c/D ratio with respect to σ_a/σ_y ratio for $D/t=70$, $p/p_y=0.4$ and $d/t=0.3$

Figure 5.10 Critical compressive strain ratio versus l_c/D ratio with respect to D/t ratio for $p/p_y=0.4$, $\sigma_a/\sigma_y=0.2$ and $d/t=0.3$

Figure 5.11 Critical compressive strain ratio versus l_c/D ratio with respect to p/p_y ratio for $D/t=70$, $\sigma_a/\sigma_y=0.25$ and $d/t=0.3$

Figure 5.12 Critical compressive strain ratio versus l_c/D ratio with respect to σ_a/σ_y ratio for $D/t=70$, $p/p_y=0.4$ and $d/t=0.3$

Figure 5.13 Critical local curvature ratio versus l_c/D ratio with respect to D/t ratio for $p/p_y=0.4$, $\sigma_a/\sigma_y=0.2$ and $d/t=0.3$

Figure 5.14 Critical local curvature ratio versus l_c/D ratio with respect to p/p_y ratio for $D/t=70$, $\sigma_a/\sigma_y=0.2$ and $d/t=0.3$

Figure 5.15 Critical local curvature ratio versus l_c/D ratio with respect to σ_a/σ_y ratio for $D/t=70$, $p/p_y=0.4$ and $d/t=0.3$

Figure 5.16 Critical local moment ratio versus d/t ratio with respect to D/t ratio for $p/p_y=0.4$, $\sigma_a/\sigma_y=0.2$ and $l_c/D=0.55$

Figure 5.17 Critical local moment ratio versus d/t ratio with respect to p/p_y ratio for $D/t=70$, $\sigma_a/\sigma_y=0.2$ and $l_c/D=0.55$

Figure 5.18 Critical local moment ratio versus d/t ratio with respect to σ_a/σ_y ratio for $D/t=70$, $p/p_y=0.4$ and $l_c/D=0.55$

Figure 5.19 Critical compressive strain ratio versus d/t ratio with respect to D/t ratio for $p/p_y=0.4$, $\sigma_a/\sigma_y=0.2$ and $l_c/D=0.55$

Figure 5.20 Critical compressive strain ratio versus d/t ratio with respect to p/p_y ratio for $D/t=70$, $\sigma_a/\sigma_y=0.2$ and $l_c/D=0.55$

Figure 5.21 Critical compressive strain ratio versus d/t ratio with respect to σ_a/σ_y ratio for $D/t=70$, $p/p_y=0.4$ and $l_c/D=0.55$

Figure 5.22 Critical local curvature ratio versus d/t ratio with respect to D/t ratio for $p/p_y=0.4$, $\sigma_a/\sigma_y=0.2$ and $l_c/D=0.55$

Figure 5.23 Critical local curvature ratio versus d/t ratio with respect to p/p_y ratio for $D/t=70$, $\sigma_a/\sigma_y=0.2$ and $l_c/D=0.55$

Figure 5.24 Critical local curvature ratio versus d/t ratio with respect to σ_a/σ_y ratio for $D/t=70$, $p/p_y=0.4$ and $l_c/D=0.55$

Figure 5.25 Rn/FEA of the critical local moment ratio for material mt1 ($\sigma_{cr}=1.3$)

Figure 5.26 Rn-FEA of the critical local moment ratio for material mt1 ($\sigma_{cr}=1.3$)

Figure 5.27 Rn/FEA of the critical local moment ratio for material mt2 ($\sigma_{cr}=1.65$)

Figure 5.28 Rn-FEA of the critical local moment ratio for material mt2 ($\sigma_{cr}=1.65$)

Figure 5.29 Rn/FEA of critical local moment ratio for material mt3 ($\sigma_{er}=2.0$)

Figure 5.30 Rn-FEA of the critical local moment ratio for material mt3 ($\sigma_{er}=2.0$)

Figure 5.31 Rn/FEA of the critical compressive strain ratio for material mt1 ($\sigma_{cr}=1.3$)

Figure 5.32 Rn-FEA of the critical compressive strain ratio for material mt1 ($\sigma_{cr}=1.3$)

Figure 5.33 Rn/FEA of the critical compressive strain ratio for material mt2 ($\sigma_{cr}=1.65$)

Figure 5.34 Rn-FEA of the critical compressive strain ratio for material mt2 ($\sigma_{cr}=1.65$)

Figure 5.35 Rn/FEA of the critical compressive strain ratio for material mt3 ($\sigma_{cr}=2.0$)

Figure 5.36 Rn-FEA of the critical compressive strain ratio for material mt3 ($\sigma_{cr}=2.0$)

Figure 5.37 Rn/FEA of the critical local curvature ratio for material mt1 ($\sigma_{cr}=1.3$)

Figure 5.38 Rn-FEA of the critical local curvature ratio for material mt1 ($\sigma_{cr}=1.3$)

Figure 5.39 Rn/FEA of the critical local curvature ratio for material mt2 ($\sigma_{cr}=1.65$)

Figure 5.40 Rn-FEA of the critical local curvature ratio for material mt2 ($\sigma_{cr}=1.65$)

Figure 5.41 Rn/FEA of the critical local curvature ratio for material mt3 ($\sigma_{cr}=2.0$)

Figure 5.42 Rn-FEA of the critical local curvature ratio for material mt3 ($\sigma_{cr}=2.0$)

Figure 5.43 Rn/FEA of the critical local moment ratio for validation with materials mt1 ($\sigma_{er}=1.3$), mt3 ($\sigma_{er}=2.0$), mv1 ($\sigma_{er}=1.5$) and mv2 ($\sigma_{er}=1.8$)

Figure 5.44 Rn/FEA of the critical local moment ratio for validation with materials mt1 ($\sigma_{er}=1.3$), mt3 ($\sigma_{er}=2.0$), mv1 ($\sigma_{er}=1.5$) and mv2 ($\sigma_{er}=1.8$)

Figure 5.45 Rn/FEA of the critical compressive strain ratio for validation with materials mt1 ($\sigma_{cr}=1.3$), mt3 ($\sigma_{cr}=2.0$), mv1 ($\sigma_{cr}=1.5$) and mv2 ($\sigma_{cr}=1.8$)

Figure 5.46 Rn-FEA of the critical compressive strain ratio for validation with materials mt1 ($\sigma_{cr}=1.3$), mt3 ($\sigma_{cr}=2.0$), mv1 ($\sigma_{cr}=1.5$) and mv2 ($\sigma_{cr}=1.8$)

Figure 5.47 Rn/FEA of the critical local curvature ratio for validation with materials mt1 ($\sigma_{er}=1.3$), mt3 ($\sigma_{er}=2.0$), mv1 ($\sigma_{er}=1.5$) and mv2 ($\sigma_{er}=1.8$)

Figure 5.48 Rn-FEA of the critical local curvature ratio for validation with materials mt1 ($\sigma_{er}=1.3$), mt3 ($\sigma_{er}=2.0$), mv1 ($\sigma_{er}=1.5$) and mv2 ($\sigma_{er}=1.8$)

Figure 5.49 Rn-FEA versus Rn of the critical local moment for all materials with the correction factor

Figure 5.50 Rn-FEA versus Rn of the critical compressive strain for all materials with the correction factor

Figure 5.51 Rn-FEA versus Rn of the critical local curvature for all materials with the correction factor

Chapter 6 Summary, Conclusions and Recommendations

6.1 Summary

A brief literature review on the corrosion types, monitoring techniques and classification of corrosion in pipelines have been provided. Buckling measurements used in quantifying the local buckling (failure) limits for pipelines have also been discussed. Some provisions from standards and specifications accounting for pipe rupture due to corrosion as well as the critical strain limit for plain pipes have been presented. In addition, a number of parameters affecting the buckling response of line pipes along with some experimental and finite element analysis studies on local buckling for both plain and corroded pipes have been summarized.

A number of finite element analysis models have been developed based on actual test specimens. Results from these numerical models have been validated against experimental results from a series of full-scale tests. The validated model has been modified and used in conducting mesh refinement and model length studies. In addition, a preliminary study on the corrosion dimensions has been conducted to investigate the effect of each dimension.

The modified model has been utilized in performing the parametric study for internal groove corrosion. Over 900 cases of different geometries, load combinations, corrosion dimensions and material properties have been evaluated. The parameters ranges considered in the study are from 40 to 100 for D/t ratio, 0.0 to 1.0 for p/p_y ratio, -0.2 to 0.4 for σ_a/σ_y ratio, 0.25 to 1.0 for l_c/D ratio, 0.15 to 0.5 for d/t and 1.3 to 2.0 for σ_{er} ratio.

The effects of those parameters on the buckling response of line pipes have been discussed. Based on the results of the parametric study, three non-dimensional equations associated with the critical local moment ratio (M_{lc}/M_{lp}), critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}) of corroded pipes have been developed through a nonlinear regression. The proposed equations have been verified with FEA results for additional selective geometries, load combinations, corrosion dimensions and material properties. A correction factor to the equations has also been proposed to improve on the level of confidence to give a conservative prediction.

6.2 Conclusions

A number of conclusions can be drawn from this study

- 1) The relative strain at peak stress (ϵ_{er}) of the stress-strain curve does not have a considerable effect on the buckling (failure) limits of corroded pipes in relation to that of plain pipes as in the critical local moment ratio (M_{lc}/M_{lp}), critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}). However, there are greater effects due to the ultimate to yield strength ratio (σ_{er}).
- 2) The change in the buckling (failure) limits due to the increase in the axial length (l_a) of axial groove corrosion beyond a length equal to 1.0D is negligible. In addition, there is only a small difference between the buckling (failure) limits for the l_a/D ratios of 0.5 and 1.5.
- 3) As the circumferential length (l_c) of axial groove corrosion increases, the M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios decrease considerably. However, the rate of decrease is significantly lower at a corrosion circumferential length larger than 1.0D.

- 4) For non-pressurized pipes, the increase in the circumferential length (l_c) of a corrosion defect when l_c is already large causes a small change in the critical local moment ratio (M_{lc}/M_{lp}) and almost no change in the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}). The reason may be due to the smaller extent (size) of the buckling deformation for non-pressurized pipes at a higher D/t ratio.
- 5) The effect of the corrosion depth (d) differs slightly for non-pressurized and pressurized pipes. For non-pressurized pipes, all the buckling limits (M_{lc}/M_{lp} , $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp}) decrease as the corrosion depth increases. Similarly, for the critical local moment ratio (M_{lc}/M_{lp}) of pressurized pipes. However, the critical compressive strain ratio ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratio (Φ_{lc}/Φ_{lp}) for pressurized pipes still generally decrease at a reducing rate as the corrosion depth increases, but may increase when the corrosion exceeds a certain depth depending on the pressure level, corrosion size and material properties. This may be attributed to the stiffening effect due to biaxial tension with the increase in the effective hoop stress in pressurized pipes due to the increase in the corrosion depth (d).
- 6) The critical local moment ratio (M_{lc}/M_{lp}) decreases with the increase of D/t , p/p_y and σ_a/σ_y ratios. On the other hand, both critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratios (Φ_{lc}/Φ_{lp}) decrease with the increase of σ_a/σ_y ratio, but increase with the increase of D/t and p/p_y ratios.
- 7) In general, the buckling (failure) limits for materials with yield plateau are found to be higher compared to those without yield plateau. However, no simple relationship

can be found between buckling (failure) limits for these two types of material stress-strain curves.

- 8) The proposed equations are able to predict the buckling (failure) limit ratios within ± 0.1 for most cases. A correction factor can be added to the equations to improve on the level of confidence to give a conservative prediction.

6.3 Recommendations for Future Study

- 1) As the length of yield plateau can significantly influence the response of corroded pipes, a more detailed study on the effect of yield plateau on corroded line pipes is required. Based on results of this study, buckling (failure) limit equations may then be developed for pipes that have a yield plateau in the material stress-strain curve.
- 2) Further studies are required to investigate the phenomena of increasing critical compressive strain ($\epsilon_{lc}/\epsilon_{lp}$) and critical local curvature ratios (Φ_{lc}/Φ_{lp}) with the increase in the corrosion depth (d) for pressurized pipes, and the small change (or no change) in the $\epsilon_{lc}/\epsilon_{lp}$ and Φ_{lc}/Φ_{lp} ratios for non-pressurized pipes with the increase in the circumferential length (l_c) of the corrosion defect.
- 3) Experimental tests for internal groove corrosion are needed to further validate the proposed equations for the buckling (failure) limits. Currently, only test data for external machined corruptions have been used in the validation.
- 4) The form of the proposed equations may be further simplified without affecting the prediction of the buckling (failure) limits, or further modified to improve on the accuracy of the equations.
- 5) Additional study can be carried out for other corrosion shapes and to investigate the interaction of multiple corrosion defects in close proximity.

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Appendix A Material Properties for Specimen Tests Used in Validations

Table A.1 True stress-true plastic strain relationship for specimen LB-I, LB-II and LB-III by Chen et al. (2010)

LB-I		LB-II		LB-III	
σ_t	ϵ_{tp}	σ_t	ϵ_{tp}	σ_t	ϵ_{tp}
435.08	0.00000	447.86	0.00000	434.27	0.00000
466.84	0.00016	473.65	0.00013	459.01	0.00020
498.14	0.00050	506.13	0.00046	485.55	0.00057
518.76	0.00090	525.82	0.00086	501.07	0.00099
540.21	0.00179	537.04	0.00131	503.73	0.00446
552.58	0.00322	546.29	0.00226	507.40	0.00939
560.15	0.00517	549.98	0.00423	509.45	0.01333
572.43	0.01104	553.12	0.00718	511.79	0.01920
587.94	0.01882	563.67	0.01306	528.99	0.02691
605.94	0.03041	583.69	0.02080	553.83	0.04029
630.09	0.05134	599.11	0.02850	576.23	0.05539
638.27	0.06072	634.34	0.05037	587.54	0.06472
645.62	0.07002	651.42	0.06440	597.76	0.07397
655.77	0.08473	669.39	0.08283	606.61	0.08314
665.58	0.10103	683.87	0.10094	622.36	0.10125
682.22	0.13112	707.77	0.13622	645.52	0.13305

Figure A.1 True stress versus true plastic strain for specimen LB-I by Chen et al. (2010)

Figure A.2 True stress versus true plastic strain for specimen LB-II by Chen et al. (2010)

Figure A.3 True stress versus true plastic strain for specimen LB-III by Chen et al. (2010)

Appendix B Pipe Response Versus p/p_y and σ_a/σ_y Ratios for Selective Cases

Figure B.1 Critical local moment ratio versus p/p_y ratio with respect to D/t ratio for $\sigma_a/\sigma_y=0.2$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.2 Critical local moment ratio versus p/p_y ratio with respect to σ_a/σ_y ratio for $D/t=70$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.3 Critical compressive strain ratio versus p/p_y ratio with respect to D/t ratio for $\sigma_a/\sigma_y=0.2$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.4 Critical compressive strain ratio versus p/p_y ratio with respect to σ_a/σ_y ratio for $D/t=70$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.5 Critical local curvature ratio versus p/p_y ratio with respect to D/t ratio for $\sigma_a/\sigma_y=0.2$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.6 Critical local curvature ratio versus p/p_y ratio with respect to σ_a/σ_y ratio for $D/t=70$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.7 Critical local moment ratio versus σ_a/σ_y ratio with respect to D/t ratio for $p/p_y=0.4$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.8 Critical local moment ratio versus σ_a/σ_y ratio with respect to p/p_y ratio for $D/t=70$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.9 Critical compressive strain ratio versus σ_a/σ_y ratio with respect to D/t ratio for $p/p_y=0.4$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.10 Critical compressive strain ratio versus σ_a/σ_y ratio with respect to p/p_y ratio for $D/t=70$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.11 Critical local curvature ratio versus σ_a/σ_y ratio with respect to D/t ratio for $p/p_y=0.4$, $l_c/D=0.55$ and $d/t=0.15$

Figure B.12 Critical local curvature ratio versus σ_a/σ_y ratio with respect to p/p_y ratio for $D/t=70$, $l_c/D=0.55$ and $d/t=0.15$

Appendix C FEA Results of the Parametric Study

Table C.1 FEA results for specimens with $D/t=40$, $p/p_y=0.0$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	5574	26371	56.95	1.00	1.00	1.00
0.0	0.25	0.15	5409	19133	40.51	0.97	0.73	0.71
0.0	0.55	0.15	5204	16621	33.32	0.93	0.63	0.59
0.0	1.00	0.15	5015	15414	29.46	0.90	0.58	0.52
0.0	0.25	0.30	5215	16886	33.74	0.94	0.64	0.59
0.0	0.55	0.30	4820	13369	24.69	0.86	0.51	0.43
0.0	1.00	0.30	4435	12062	21.18	0.80	0.46	0.37
0.0	0.25	0.50	4953	14119	26.66	0.89	0.54	0.47
0.0	0.55	0.50	4119	8733	15.34	0.74	0.33	0.27
0.0	1.00	0.50	3362	8335	14.03	0.60	0.32	0.25
0.2	0.00	0.00	5445	28695	51.14	1.00	1.00	1.00
0.2	0.25	0.15	5144	19101	33.58	0.94	0.67	0.66
0.2	0.55	0.15	4856	16031	27.09	0.89	0.56	0.53
0.2	1.00	0.15	4587	15210	24.73	0.84	0.53	0.48
0.2	0.25	0.30	4883	16376	27.63	0.90	0.57	0.54
0.2	0.55	0.30	4333	13717	21.81	0.80	0.48	0.43
0.2	1.00	0.30	3765	11963	18.72	0.69	0.42	0.37
0.2	0.25	0.50	4527	14132	22.85	0.83	0.49	0.45
0.2	0.55	0.50	3321	8037	12.93	0.61	0.28	0.25
0.2	1.00	0.50	2433	7448	11.47	0.45	0.26	0.22
0.4	0.00	0.00	4912	30225	47.30	1.00	1.00	1.00
0.4	0.25	0.15	4472	19245	29.85	0.91	0.64	0.63
0.4	0.55	0.15	4070	16080	24.39	0.83	0.53	0.52
0.4	1.00	0.15	3682	14919	22.23	0.75	0.49	0.47
0.4	0.25	0.30	4120	16920	25.59	0.84	0.56	0.54
0.4	0.55	0.30	3369	13986	20.57	0.69	0.46	0.44
0.4	1.00	0.30	2753	11901	17.21	0.56	0.39	0.36
0.4	0.25	0.50	3616	14512	21.56	0.74	0.48	0.46
0.4	0.55	0.50	2386	8238	11.84	0.49	0.27	0.25
0.4	1.00	0.50	1475	7183	9.73	0.30	0.24	0.21

Table C.2 FEA results for specimens with $D/t=40$, $p/p_y=0.4$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	5777	38165	80.21	1.00	1.00	1.00
0.0	0.25	0.15	5281	27209	53.36	0.91	0.71	0.67
0.0	0.55	0.15	4957	23207	43.06	0.86	0.61	0.54
0.0	1.00	0.15	4665	20902	37.11	0.81	0.55	0.46
0.0	0.25	0.30	4970	24087	44.47	0.86	0.63	0.55
0.0	0.55	0.30	4428	21539	36.23	0.77	0.56	0.45
0.0	1.00	0.30	3875	19115	30.52	0.67	0.50	0.38
0.0	0.25	0.50	4545	22538	38.06	0.79	0.59	0.47
0.0	0.55	0.50	3727	20539	31.92	0.65	0.54	0.40
0.0	1.00	0.50	2755	18663	27.72	0.48	0.49	0.35
0.2	0.00	0.00	5185	36778	64.81	1.00	1.00	1.00
0.2	0.25	0.15	4606	24984	41.92	0.89	0.68	0.65
0.2	0.55	0.15	4225	21563	34.69	0.81	0.59	0.54
0.2	1.00	0.15	3859	19660	30.59	0.74	0.53	0.47
0.2	0.25	0.30	4248	22102	35.51	0.82	0.60	0.55
0.2	0.55	0.30	3582	19257	29.06	0.69	0.52	0.45
0.2	1.00	0.30	2844	16798	24.67	0.55	0.46	0.38
0.2	0.25	0.50	3732	19783	30.02	0.72	0.54	0.46
0.2	0.55	0.50	2673	17587	25.49	0.52	0.48	0.39
0.2	1.00	0.50	1586	16075	22.60	0.31	0.44	0.35
0.4	0.00	0.00	4249	35985	55.56	1.00	1.00	1.00
0.4	0.25	0.15	3573	23501	35.15	0.84	0.65	0.63
0.4	0.55	0.15	3107	20601	30.05	0.73	0.57	0.54
0.4	1.00	0.15	2658	18061	25.95	0.63	0.50	0.47
0.4	0.25	0.30	3139	20574	30.04	0.74	0.57	0.54
0.4	0.55	0.30	2366	17662	24.95	0.56	0.49	0.45
0.4	1.00	0.30	1635	15318	21.00	0.38	0.43	0.38
0.4	0.25	0.50	2529	17793	25.24	0.60	0.49	0.45
0.4	0.55	0.50	1467	15709	21.30	0.35	0.44	0.38
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.3 FEA results for specimens with $D/t=40$, $p/p_y=0.8$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	4772	54033	94.63	1.00	1.00	1.00
0.0	0.25	0.15	4093	41242	68.03	0.86	0.76	0.72
0.0	0.55	0.15	3695	39635	62.59	0.77	0.73	0.66
0.0	1.00	0.15	3294	37368	57.34	0.69	0.69	0.61
0.0	0.25	0.30	3698	40716	63.65	0.77	0.75	0.67
0.0	0.55	0.30	3085	49426	73.06	0.65	0.91	0.77
0.0	1.00	0.30	2368	54114	77.94	0.50	1.00	0.82
0.0	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.0	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.0	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	0.00	0.00	3751	53466	81.21	1.00	1.00	1.00
0.2	0.25	0.15	2994	39571	57.81	0.80	0.74	0.71
0.2	0.55	0.15	2535	37454	53.34	0.68	0.70	0.66
0.2	1.00	0.15	2060	35135	49.14	0.55	0.66	0.61
0.2	0.25	0.30	2537	37795	53.59	0.68	0.71	0.66
0.2	0.55	0.30	1823	45658	62.80	0.49	0.85	0.77
0.2	1.00	0.30	990	48777	66.13	0.26	0.91	0.81
0.2	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.00	0.00	2452	52757	69.83	1.00	1.00	1.00
0.4	0.25	0.15	1571	36640	49.41	0.64	0.69	0.71
0.4	0.55	0.15	985	33817	44.86	0.40	0.64	0.64
0.4	1.00	0.15	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.30	918	34435	45.54	0.37	0.65	0.65
0.4	0.55	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.4 FEA results for specimens with $D/t=70$, $p/p_y=0.0$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	3043	10957	22.32	1.00	1.00	1.00
0.0	0.25	0.15	2825	8121	15.54	0.93	0.74	0.70
0.0	0.55	0.15	2657	7122	13.30	0.87	0.65	0.60
0.0	1.00	0.15	2547	6545	12.26	0.84	0.60	0.55
0.0	0.25	0.30	2655	7173	13.36	0.87	0.65	0.60
0.0	0.55	0.30	2356	5510	10.51	0.77	0.50	0.47
0.0	1.00	0.30	2129	5191	9.82	0.70	0.47	0.44
0.0	0.25	0.50	2410	6994	12.58	0.79	0.64	0.56
0.0	0.55	0.50	1743	3751	7.34	0.57	0.34	0.33
0.0	1.00	0.50	1423	3311	6.35	0.47	0.30	0.28
0.2	0.00	0.00	2875	11323	19.45	1.00	1.00	1.00
0.2	0.25	0.15	2523	7883	13.44	0.88	0.70	0.69
0.2	0.55	0.15	2269	6745	11.47	0.79	0.60	0.59
0.2	1.00	0.15	2108	6025	10.28	0.73	0.53	0.53
0.2	0.25	0.30	2273	6703	11.42	0.79	0.59	0.59
0.2	0.55	0.30	1889	5465	9.22	0.66	0.48	0.47
0.2	1.00	0.30	1624	5280	8.65	0.56	0.47	0.44
0.2	0.25	0.50	1955	6408	10.57	0.68	0.57	0.54
0.2	0.55	0.50	1281	3845	6.22	0.45	0.34	0.32
0.2	1.00	0.50	932	3478	5.29	0.32	0.31	0.27
0.4	0.00	0.00	2408	11523	18.06	1.00	1.00	1.00
0.4	0.25	0.15	1979	7790	11.97	0.82	0.68	0.66
0.4	0.55	0.15	1734	6678	10.04	0.72	0.58	0.56
0.4	1.00	0.15	1577	5869	8.73	0.65	0.51	0.48
0.4	0.25	0.30	1744	6573	9.90	0.72	0.57	0.55
0.4	0.55	0.30	1376	5375	7.79	0.57	0.47	0.43
0.4	1.00	0.30	1105	4976	6.91	0.46	0.43	0.38
0.4	0.25	0.50	1444	5904	8.58	0.60	0.51	0.47
0.4	0.55	0.50	784	4022	5.16	0.33	0.35	0.29
0.4	1.00	0.50	447	3806	4.45	0.19	0.33	0.25

Table C.5 FEA results for specimens with $D/t=70$, $p/p_y=0.4$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2982	14205	29.03	1.00	1.00	1.00
0.0	0.25	0.15	2721	11558	21.64	0.91	0.81	0.75
0.0	0.55	0.15	2523	10345	18.16	0.85	0.73	0.63
0.0	1.00	0.15	2335	9980	16.81	0.78	0.70	0.58
0.0	0.25	0.30	2528	10290	18.09	0.85	0.72	0.62
0.0	0.55	0.30	2159	9909	16.25	0.72	0.70	0.56
0.0	1.00	0.30	1791	9134	14.67	0.60	0.64	0.51
0.0	0.25	0.50	2270	11879	19.26	0.76	0.84	0.66
0.0	0.55	0.50	1803	16694	25.10	0.60	1.18	0.86
0.0	1.00	0.50	1273	21235	30.70	0.43	1.49	1.06
0.2	0.00	0.00	2564	13993	23.90	1.00	1.00	1.00
0.2	0.25	0.15	2243	10815	17.56	0.87	0.77	0.73
0.2	0.55	0.15	1959	9384	14.85	0.76	0.67	0.62
0.2	1.00	0.15	1733	8738	13.50	0.68	0.62	0.56
0.2	0.25	0.30	1970	9637	15.21	0.77	0.69	0.64
0.2	0.55	0.30	1556	9145	13.75	0.61	0.65	0.58
0.2	1.00	0.30	1197	8073	11.79	0.47	0.58	0.49
0.2	0.25	0.50	1662	10828	16.23	0.65	0.77	0.68
0.2	0.55	0.50	1157	12533	17.88	0.45	0.90	0.75
0.2	1.00	0.50	590	13529	18.55	0.23	0.97	0.78
0.4	0.00	0.00	1916	13810	20.92	1.00	1.00	1.00
0.4	0.25	0.15	1562	10281	15.17	0.82	0.74	0.72
0.4	0.55	0.15	1310	8956	12.71	0.68	0.65	0.61
0.4	1.00	0.15	1097	8085	11.09	0.57	0.59	0.53
0.4	0.25	0.30	1320	8950	12.72	0.69	0.65	0.61
0.4	0.55	0.30	938	8370	11.22	0.49	0.61	0.54
0.4	1.00	0.30	587	7070	8.96	0.31	0.51	0.43
0.4	0.25	0.50	1024	8269	11.20	0.53	0.60	0.54
0.4	0.55	0.50	520	9666	12.46	0.27	0.70	0.60
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.6 FEA results for specimens with $D/t=70$, $p/p_y=0.8$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2289	23008	39.12	1.00	1.00	1.00
0.0	0.25	0.15	1998	19456	31.46	0.87	0.85	0.80
0.0	0.55	0.15	1790	18536	28.85	0.78	0.81	0.74
0.0	1.00	0.15	1569	18211	27.47	0.69	0.79	0.70
0.0	0.25	0.30	1798	20671	31.86	0.79	0.90	0.81
0.0	0.55	0.30	1489	26134	38.26	0.65	1.14	0.98
0.0	1.00	0.30	1123	31634	45.08	0.49	1.37	1.15
0.0	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.0	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.0	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	0.00	0.00	1639	23615	34.77	1.00	1.00	1.00
0.2	0.25	0.15	1287	18729	26.98	0.79	0.79	0.78
0.2	0.55	0.15	1042	17646	24.93	0.64	0.75	0.72
0.2	1.00	0.15	797	17312	23.98	0.49	0.73	0.69
0.2	0.25	0.30	1053	20553	28.87	0.64	0.87	0.83
0.2	0.55	0.30	707	24531	33.55	0.43	1.04	0.96
0.2	1.00	0.30	298	27172	36.54	0.18	1.15	1.05
0.2	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.00	0.00	824	23602	28.84	1.00	1.00	1.00
0.4	0.25	0.15	425	17208	22.02	0.52	0.73	0.76
0.4	0.55	0.15	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.15	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.55	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.7 FEA results for specimens with $D/t=100$, $p/p_y=0.0$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	1991	6393	12.83	1.00	1.00	1.00
0.0	0.25	0.15	1775	4668	9.55	0.89	0.73	0.74
0.0	0.55	0.15	1668	4252	8.78	0.84	0.67	0.68
0.0	1.00	0.15	1612	3810	8.11	0.81	0.60	0.63
0.0	0.25	0.30	1648	4039	8.46	0.83	0.63	0.66
0.0	0.55	0.30	1425	3327	7.12	0.72	0.52	0.56
0.0	1.00	0.30	1298	3105	6.60	0.65	0.49	0.51
0.0	0.25	0.50	1367	2625	6.07	0.69	0.41	0.47
0.0	0.55	0.50	1035	2341	5.09	0.52	0.37	0.40
0.0	1.00	0.50	919	2357	4.91	0.46	0.37	0.38
0.2	0.00	0.00	1771	6132	11.05	1.00	1.00	1.00
0.2	0.25	0.15	1471	4422	8.03	0.83	0.72	0.73
0.2	0.55	0.15	1349	4031	7.28	0.76	0.66	0.66
0.2	1.00	0.15	1271	3740	6.75	0.72	0.61	0.61
0.2	0.25	0.30	1320	3913	7.07	0.75	0.64	0.64
0.2	0.55	0.30	1091	3252	5.76	0.62	0.53	0.52
0.2	1.00	0.30	964	3080	5.31	0.54	0.50	0.48
0.2	0.25	0.50	1045	2622	4.82	0.59	0.43	0.44
0.2	0.55	0.50	705	2335	3.82	0.40	0.38	0.35
0.2	1.00	0.50	574	2313	3.57	0.32	0.38	0.32
0.4	0.00	0.00	1391	5855	9.34	1.00	1.00	1.00
0.4	0.25	0.15	1132	4232	6.55	0.81	0.72	0.70
0.4	0.55	0.15	998	3839	5.73	0.72	0.66	0.61
0.4	1.00	0.15	922	3618	5.30	0.66	0.62	0.57
0.4	0.25	0.30	991	3798	5.66	0.71	0.65	0.61
0.4	0.55	0.30	749	3236	4.47	0.54	0.55	0.48
0.4	1.00	0.30	613	3117	4.07	0.44	0.53	0.44
0.4	0.25	0.50	720	2600	3.55	0.52	0.44	0.38
0.4	0.55	0.50	376	2313	2.54	0.27	0.39	0.27
0.4	1.00	0.50	226	2281	2.23	0.16	0.39	0.24

Table C.8 FEA results for specimens with $D/t=100$, $p/p_y=0.4$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	1914	7741	15.60	1.00	1.00	1.00
0.0	0.25	0.15	1703	6416	11.93	0.89	0.83	0.76
0.0	0.55	0.15	1521	5970	10.76	0.79	0.77	0.69
0.0	1.00	0.15	1390	5452	9.77	0.73	0.70	0.63
0.0	0.25	0.30	1510	5760	10.44	0.79	0.74	0.67
0.0	0.55	0.30	1251	5235	9.19	0.65	0.68	0.59
0.0	1.00	0.30	1033	4716	8.09	0.54	0.61	0.52
0.0	0.25	0.50	1355	7791	12.91	0.71	1.01	0.83
0.0	0.55	0.50	1073	11800	17.94	0.56	1.52	1.15
0.0	1.00	0.50	755	17217	24.85	0.39	2.22	1.59
0.2	0.00	0.00	1559	7584	12.97	1.00	1.00	1.00
0.2	0.25	0.15	1297	5970	9.84	0.83	0.79	0.76
0.2	0.55	0.15	1119	5520	8.73	0.72	0.73	0.67
0.2	1.00	0.15	995	5147	7.97	0.64	0.68	0.61
0.2	0.25	0.30	1115	5422	8.39	0.71	0.71	0.65
0.2	0.55	0.30	868	5000	7.52	0.56	0.66	0.58
0.2	1.00	0.30	650	4496	6.43	0.42	0.59	0.50
0.2	0.25	0.50	951	6443	9.66	0.61	0.85	0.74
0.2	0.55	0.50	642	8720	12.26	0.41	1.15	0.95
0.2	1.00	0.50	282	10770	14.45	0.18	1.42	1.11
0.4	0.00	0.00	1090	7400	11.13	1.00	1.00	1.00
0.4	0.25	0.15	869	5780	8.12	0.80	0.78	0.73
0.4	0.55	0.15	714	5282	7.01	0.66	0.71	0.63
0.4	1.00	0.15	590	4877	6.20	0.54	0.66	0.56
0.4	0.25	0.30	713	5202	6.89	0.65	0.70	0.62
0.4	0.55	0.30	477	4951	6.08	0.44	0.67	0.55
0.4	1.00	0.30	250	4348	5.25	0.23	0.59	0.47
0.4	0.25	0.50	546	5893	7.51	0.50	0.80	0.67
0.4	0.55	0.50	216	7027	8.50	0.20	0.95	0.76
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.9 FEA results for specimens with $D/t=100$, $p/p_y=0.8$ and material mt1 ($\epsilon_{cr}=1.3$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	1427	13966	23.30	1.00	1.00	1.00
0.0	0.25	0.15	1248	12161	19.51	0.87	0.87	0.84
0.0	0.55	0.15	1099	11787	18.27	0.77	0.84	0.78
0.0	1.00	0.15	940	11243	17.05	0.66	0.80	0.73
0.0	0.25	0.30	1120	12942	19.94	0.78	0.93	0.86
0.0	0.55	0.30	916	17076	25.00	0.64	1.22	1.07
0.0	1.00	0.30	685	21431	30.56	0.48	1.53	1.31
0.0	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.0	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.0	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	0.00	0.00	921	13713	20.11	1.00	1.00	1.00
0.2	0.25	0.15	724	11543	16.53	0.79	0.84	0.82
0.2	0.55	0.15	571	10944	15.25	0.62	0.80	0.76
0.2	1.00	0.15	416	10513	14.24	0.45	0.77	0.71
0.2	0.25	0.30	589	12171	16.97	0.64	0.89	0.84
0.2	0.55	0.30	375	14942	20.19	0.41	1.09	1.00
0.2	1.00	0.30	111	17228	22.85	0.12	1.26	1.14
0.2	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.2	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.00	0.00	362	13673	14.64	1.00	1.00	1.00
0.4	0.25	0.15	132	10442	12.00	0.37	0.76	0.82
0.4	0.55	0.15	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.15	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.55	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.55	0.50	Failed	Failed	Failed	Failed	Failed	Failed
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.10 FEA results for specimens with $D/t=40$, $p/p_y=0.0$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	6866	30088	63.05	1.00	1.00	1.00
0.0	0.25	0.15	6636	20986	44.48	0.97	0.70	0.71
0.0	0.55	0.15	6351	17481	36.19	0.93	0.58	0.57
0.0	1.00	0.15	6100	16829	33.43	0.89	0.56	0.53
0.0	0.25	0.30	6357	18470	37.67	0.93	0.61	0.60
0.0	0.55	0.30	5775	13936	27.14	0.84	0.46	0.43
0.0	1.00	0.30	5270	13142	23.51	0.77	0.44	0.37
0.0	0.25	0.50	5961	15818	30.49	0.87	0.53	0.48
0.0	0.55	0.50	4767	8773	16.34	0.69	0.29	0.26
0.0	1.00	0.50	3983	8512	14.91	0.58	0.28	0.24
0.2	0.00	0.00	6765	32862	59.18	1.00	1.00	1.00
0.2	0.25	0.15	6380	21205	38.77	0.94	0.65	0.66
0.2	0.55	0.15	6008	17799	31.76	0.89	0.54	0.54
0.2	1.00	0.15	5687	16369	28.47	0.84	0.50	0.48
0.2	0.25	0.30	6024	18392	32.62	0.89	0.56	0.55
0.2	0.55	0.30	5310	14574	24.65	0.78	0.44	0.42
0.2	1.00	0.30	4686	12984	21.19	0.69	0.40	0.36
0.2	0.25	0.50	5573	15913	27.24	0.82	0.48	0.46
0.2	0.55	0.50	4082	8780	14.56	0.60	0.27	0.25
0.2	1.00	0.50	3130	8456	13.41	0.46	0.26	0.23
0.4	0.00	0.00	6341	34469	55.66	1.00	1.00	1.00
0.4	0.25	0.15	5813	21072	34.49	0.92	0.61	0.62
0.4	0.55	0.15	5359	17867	28.62	0.85	0.52	0.51
0.4	1.00	0.15	4968	16078	25.32	0.78	0.47	0.45
0.4	0.25	0.30	5387	17924	28.72	0.85	0.52	0.52
0.4	0.55	0.30	4534	14309	22.13	0.72	0.42	0.40
0.4	1.00	0.30	3776	12914	19.46	0.60	0.37	0.35
0.4	0.25	0.50	4854	15637	24.40	0.77	0.45	0.44
0.4	0.55	0.50	3151	8906	13.37	0.50	0.26	0.24
0.4	1.00	0.50	2188	8106	11.57	0.35	0.24	0.21

Table C.11 FEA results for specimens with $D/t=40$, $p/p_y=0.4$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	7394	39771	85.96	1.00	1.00	1.00
0.0	0.25	0.15	6739	27457	55.84	0.91	0.69	0.65
0.0	0.55	0.15	6308	24611	47.27	0.85	0.62	0.55
0.0	1.00	0.15	5950	21172	39.58	0.80	0.53	0.46
0.0	0.25	0.30	6340	24524	47.29	0.86	0.62	0.55
0.0	0.55	0.30	5617	21485	38.10	0.76	0.54	0.44
0.0	1.00	0.30	4927	19336	33.50	0.67	0.49	0.39
0.0	0.25	0.50	5777	22696	40.56	0.78	0.57	0.47
0.0	0.55	0.50	4656	20650	33.42	0.63	0.52	0.39
0.0	1.00	0.50	3480	18462	28.26	0.47	0.46	0.33
0.2	0.00	0.00	6899	38862	72.34	1.00	1.00	1.00
0.2	0.25	0.15	6156	25930	46.21	0.89	0.67	0.64
0.2	0.55	0.15	5668	22978	39.18	0.82	0.59	0.54
0.2	1.00	0.15	5239	19752	32.89	0.76	0.51	0.45
0.2	0.25	0.30	5707	23098	39.46	0.83	0.59	0.55
0.2	0.55	0.30	4878	20045	32.11	0.71	0.52	0.44
0.2	1.00	0.30	4067	17961	27.61	0.59	0.46	0.38
0.2	0.25	0.50	5066	20421	33.06	0.73	0.53	0.46
0.2	0.55	0.50	3754	18572	27.88	0.54	0.48	0.39
0.2	1.00	0.50	2395	16441	23.77	0.35	0.42	0.33
0.4	0.00	0.00	6122	38387	63.47	1.00	1.00	1.00
0.4	0.25	0.15	5299	24648	39.52	0.87	0.64	0.62
0.4	0.55	0.15	4750	21664	33.62	0.78	0.56	0.53
0.4	1.00	0.15	4247	18733	28.50	0.69	0.49	0.45
0.4	0.25	0.30	4797	21957	34.10	0.78	0.57	0.54
0.4	0.55	0.30	3848	19036	28.22	0.63	0.50	0.44
0.4	1.00	0.30	2936	16420	23.69	0.48	0.43	0.37
0.4	0.25	0.50	4067	19258	28.79	0.66	0.50	0.45
0.4	0.55	0.50	2608	16987	24.07	0.43	0.44	0.38
0.4	1.00	0.50	1266	14917	20.15	0.21	0.39	0.32

Table C.12 FEA results for specimens with $D/t=40$, $p/p_y=0.8$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	6687	46703	88.97	1.00	1.00	1.00
0.0	0.25	0.15	5876	34814	62.41	0.88	0.75	0.70
0.0	0.55	0.15	5391	32052	54.83	0.81	0.69	0.62
0.0	1.00	0.15	4928	30662	50.58	0.74	0.66	0.57
0.0	0.25	0.30	5384	32691	55.63	0.81	0.70	0.63
0.0	0.55	0.30	4603	32151	51.04	0.69	0.69	0.57
0.0	1.00	0.30	3759	30670	46.60	0.56	0.66	0.52
0.0	0.25	0.50	4792	42314	65.51	0.72	0.91	0.74
0.0	0.55	0.50	3737	52399	77.00	0.56	1.12	0.87
0.0	1.00	0.50	2488	63353	90.69	0.37	1.36	1.02
0.2	0.00	0.00	5874	46197	77.36	1.00	1.00	1.00
0.2	0.25	0.15	4980	33534	53.64	0.85	0.73	0.69
0.2	0.55	0.15	4435	31190	48.13	0.75	0.68	0.62
0.2	1.00	0.15	3907	29518	44.38	0.67	0.64	0.57
0.2	0.25	0.30	4432	31118	47.94	0.75	0.67	0.62
0.2	0.55	0.30	3553	30134	44.22	0.60	0.65	0.57
0.2	1.00	0.30	2594	28434	40.43	0.44	0.62	0.52
0.2	0.25	0.50	3765	40075	57.96	0.64	0.87	0.75
0.2	0.55	0.50	2567	49607	69.24	0.44	1.07	0.90
0.2	1.00	0.50	1165	55239	75.61	0.20	1.20	0.98
0.4	0.00	0.00	4809	45762	68.93	1.00	1.00	1.00
0.4	0.25	0.15	3842	31766	46.26	0.80	0.69	0.67
0.4	0.55	0.15	3239	29595	42.06	0.67	0.65	0.61
0.4	1.00	0.15	2648	27559	38.52	0.55	0.60	0.56
0.4	0.25	0.30	3239	29643	42.08	0.67	0.65	0.61
0.4	0.55	0.30	2263	28521	39.34	0.47	0.62	0.57
0.4	1.00	0.30	1248	26540	35.68	0.26	0.58	0.52
0.4	0.25	0.50	2493	39395	54.06	0.52	0.86	0.78
0.4	0.55	0.50	1211	45854	61.62	0.25	1.00	0.89
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.13 FEA results for specimens with $D/t=70$, $p/p_y=0.0$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	3675	13661	28.08	1.00	1.00	1.00
0.0	0.25	0.15	3398	9797	20.10	0.92	0.72	0.72
0.0	0.55	0.15	3183	8574	17.11	0.87	0.63	0.61
0.0	1.00	0.15	3054	7740	15.39	0.83	0.57	0.55
0.0	0.25	0.30	3162	7785	15.88	0.86	0.57	0.57
0.0	0.55	0.30	2753	6395	12.50	0.75	0.47	0.45
0.0	1.00	0.30	2536	6202	11.81	0.69	0.45	0.42
0.0	0.25	0.50	2822	7945	14.77	0.77	0.58	0.53
0.0	0.55	0.50	2086	4062	8.23	0.57	0.30	0.29
0.0	1.00	0.50	1761	4166	7.98	0.48	0.30	0.28
0.2	0.00	0.00	3525	14186	25.36	1.00	1.00	1.00
0.2	0.25	0.15	3159	9535	17.18	0.90	0.67	0.68
0.2	0.55	0.15	2885	8648	15.11	0.82	0.61	0.60
0.2	1.00	0.15	2720	7694	13.45	0.77	0.54	0.53
0.2	0.25	0.30	2870	7971	14.23	0.81	0.56	0.56
0.2	0.55	0.30	2353	6370	11.08	0.67	0.45	0.44
0.2	1.00	0.30	2077	6212	10.51	0.59	0.44	0.41
0.2	0.25	0.50	2444	7861	13.19	0.69	0.55	0.52
0.2	0.55	0.50	1618	4118	7.05	0.46	0.29	0.28
0.2	1.00	0.50	1267	4139	6.66	0.36	0.29	0.26
0.4	0.00	0.00	3186	14405	23.26	1.00	1.00	1.00
0.4	0.25	0.15	2734	9393	15.38	0.86	0.65	0.66
0.4	0.55	0.15	2407	8305	13.31	0.76	0.58	0.57
0.4	1.00	0.15	2206	7759	12.24	0.69	0.54	0.53
0.4	0.25	0.30	2385	7935	12.77	0.75	0.55	0.55
0.4	0.55	0.30	1849	6380	9.79	0.58	0.44	0.42
0.4	1.00	0.30	1565	6259	9.27	0.49	0.43	0.40
0.4	0.25	0.50	1918	7308	11.11	0.60	0.51	0.48
0.4	0.55	0.50	1137	4119	5.78	0.36	0.29	0.25
0.4	1.00	0.50	764	4187	5.42	0.24	0.29	0.23

Table C.14 FEA results for specimens with $D/t=70$, $p/p_y=0.4$ and material mt2 ($\epsilon_{er}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	3774	16217	34.30	1.00	1.00	1.00
0.0	0.25	0.15	3407	12546	24.95	0.90	0.77	0.73
0.0	0.55	0.15	3151	11138	21.13	0.83	0.69	0.62
0.0	1.00	0.15	2951	10440	19.18	0.78	0.64	0.56
0.0	0.25	0.30	3156	11029	20.99	0.84	0.68	0.61
0.0	0.55	0.30	2705	9850	17.37	0.72	0.61	0.51
0.0	1.00	0.30	2311	8434	14.45	0.61	0.52	0.42
0.0	0.25	0.50	2816	12081	20.90	0.75	0.74	0.61
0.0	0.55	0.50	2218	17868	27.45	0.59	1.10	0.80
0.0	1.00	0.50	1580	22603	32.97	0.42	1.39	0.96
0.2	0.00	0.00	3414	16231	29.52	1.00	1.00	1.00
0.2	0.25	0.15	3002	12301	21.35	0.88	0.76	0.72
0.2	0.55	0.15	2702	10784	18.10	0.79	0.66	0.61
0.2	1.00	0.15	2453	9847	16.19	0.72	0.61	0.55
0.2	0.25	0.30	2709	10654	17.93	0.79	0.66	0.61
0.2	0.55	0.30	2167	9418	15.06	0.63	0.58	0.51
0.2	1.00	0.30	1734	7958	12.37	0.51	0.49	0.42
0.2	0.25	0.50	2298	11890	18.67	0.67	0.73	0.63
0.2	0.55	0.50	1605	14651	21.38	0.47	0.90	0.72
0.2	1.00	0.50	913	17432	24.34	0.27	1.07	0.82
0.4	0.00	0.00	2897	16330	26.34	1.00	1.00	1.00
0.4	0.25	0.15	2421	12060	18.89	0.84	0.74	0.72
0.4	0.55	0.15	2084	10466	16.01	0.72	0.64	0.61
0.4	1.00	0.15	1828	9604	14.35	0.63	0.59	0.54
0.4	0.25	0.30	2092	10314	15.81	0.72	0.63	0.60
0.4	0.55	0.30	1561	9257	13.38	0.54	0.57	0.51
0.4	1.00	0.30	1140	7745	10.65	0.39	0.47	0.40
0.4	0.25	0.50	1673	9746	14.23	0.58	0.60	0.54
0.4	0.55	0.50	975	12788	17.35	0.34	0.78	0.66
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.15 FEA results for specimens with $D/t=70$, $p/p_y=0.8$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	3274	20711	38.29	1.00	1.00	1.00
0.0	0.25	0.15	2897	17006	29.88	0.88	0.82	0.78
0.0	0.55	0.15	2625	15614	26.36	0.80	0.75	0.69
0.0	1.00	0.15	2373	14858	24.29	0.72	0.72	0.63
0.0	0.25	0.30	2628	16105	27.08	0.80	0.78	0.71
0.0	0.55	0.30	2196	15391	24.43	0.67	0.74	0.64
0.0	1.00	0.30	1731	14583	22.22	0.53	0.70	0.58
0.0	0.25	0.50	2343	19645	30.77	0.72	0.95	0.80
0.0	0.55	0.50	1831	27667	40.66	0.56	1.34	1.06
0.0	1.00	0.50	1218	35523	50.71	0.37	1.72	1.32
0.2	0.00	0.00	2736	20910	33.89	1.00	1.00	1.00
0.2	0.25	0.15	2320	16551	25.92	0.85	0.79	0.76
0.2	0.55	0.15	2011	15120	23.01	0.74	0.72	0.68
0.2	1.00	0.15	1721	14136	21.06	0.63	0.68	0.62
0.2	0.25	0.30	2016	15701	23.83	0.74	0.75	0.70
0.2	0.55	0.30	1528	14828	21.64	0.56	0.71	0.64
0.2	1.00	0.30	1035	14073	19.82	0.38	0.67	0.58
0.2	0.25	0.50	1689	19244	27.89	0.62	0.92	0.82
0.2	0.55	0.50	1117	26815	37.31	0.41	1.28	1.10
0.2	1.00	0.50	431	32203	43.90	0.16	1.54	1.30
0.4	0.00	0.00	2052	20595	29.92	1.00	1.00	1.00
0.4	0.25	0.15	1601	15893	22.53	0.78	0.77	0.75
0.4	0.55	0.15	1281	14365	19.81	0.62	0.70	0.66
0.4	1.00	0.15	991	13182	17.69	0.48	0.64	0.59
0.4	0.25	0.30	1286	14982	20.67	0.63	0.73	0.69
0.4	0.55	0.30	798	14334	18.99	0.39	0.70	0.63
0.4	1.00	0.30	251	13461	17.02	0.12	0.65	0.57
0.4	0.25	0.50	949	19176	25.78	0.46	0.93	0.86
0.4	0.55	0.50	306	25424	33.36	0.15	1.23	1.12
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.16 FEA results for specimens with $D/t=100$, $p/p_y=0.0$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2402	8206	17.58	1.00	1.00	1.00
0.0	0.25	0.15	2156	5988	12.79	0.90	0.73	0.73
0.0	0.55	0.15	1991	5580	11.47	0.83	0.68	0.65
0.0	1.00	0.15	1924	5185	10.72	0.80	0.63	0.61
0.0	0.25	0.30	1939	4822	10.26	0.81	0.59	0.58
0.0	0.55	0.30	1681	4076	8.60	0.70	0.50	0.49
0.0	1.00	0.30	1562	4001	8.31	0.65	0.49	0.47
0.0	0.25	0.50	1562	3123	7.10	0.65	0.38	0.40
0.0	0.55	0.50	1188	2799	5.99	0.49	0.34	0.34
0.0	1.00	0.50	1056	2885	5.88	0.44	0.35	0.33
0.2	0.00	0.00	2257	8273	15.41	1.00	1.00	1.00
0.2	0.25	0.15	1939	5827	10.92	0.86	0.70	0.71
0.2	0.55	0.15	1733	5294	9.75	0.77	0.64	0.63
0.2	1.00	0.15	1631	5078	9.25	0.72	0.61	0.60
0.2	0.25	0.30	1668	4830	8.98	0.74	0.58	0.58
0.2	0.55	0.30	1363	4069	7.37	0.60	0.49	0.48
0.2	1.00	0.30	1226	3998	7.04	0.54	0.48	0.46
0.2	0.25	0.50	1242	3104	5.84	0.55	0.38	0.38
0.2	0.55	0.50	862	2790	4.73	0.38	0.34	0.31
0.2	1.00	0.50	715	2868	4.58	0.32	0.35	0.30
0.4	0.00	0.00	1968	8251	13.90	1.00	1.00	1.00
0.4	0.25	0.15	1593	5732	9.55	0.81	0.69	0.69
0.4	0.55	0.15	1385	5177	8.32	0.70	0.63	0.60
0.4	1.00	0.15	1278	4931	7.76	0.65	0.60	0.56
0.4	0.25	0.30	1325	4791	7.66	0.67	0.58	0.55
0.4	0.55	0.30	1026	4039	6.07	0.52	0.49	0.44
0.4	1.00	0.30	873	3912	5.63	0.44	0.47	0.41
0.4	0.25	0.50	917	3140	4.64	0.47	0.38	0.33
0.4	0.55	0.50	530	2775	3.44	0.27	0.34	0.25
0.4	1.00	0.50	368	2846	3.27	0.19	0.34	0.24

Table C.17 FEA results for specimens with $D/t=100$, $p/p_y=0.4$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2414	9787	20.71	1.00	1.00	1.00
0.0	0.25	0.15	2152	7755	15.57	0.89	0.79	0.75
0.0	0.55	0.15	1957	7280	13.85	0.81	0.74	0.67
0.0	1.00	0.15	1828	6568	12.36	0.76	0.67	0.60
0.0	0.25	0.30	1953	6979	13.39	0.81	0.71	0.65
0.0	0.55	0.30	1611	6398	11.52	0.67	0.65	0.56
0.0	1.00	0.30	1362	5596	9.93	0.56	0.57	0.48
0.0	0.25	0.50	1715	9407	15.93	0.71	0.96	0.77
0.0	0.55	0.50	1333	14010	21.46	0.55	1.43	1.04
0.0	1.00	0.50	938	20917	30.28	0.39	2.14	1.46
0.2	0.00	0.00	2132	9731	17.67	1.00	1.00	1.00
0.2	0.25	0.15	1823	7459	13.10	0.86	0.77	0.74
0.2	0.55	0.15	1593	6604	11.31	0.75	0.68	0.64
0.2	1.00	0.15	1443	6268	10.49	0.68	0.64	0.59
0.2	0.25	0.30	1585	6716	11.44	0.74	0.69	0.65
0.2	0.55	0.30	1227	6147	9.82	0.58	0.63	0.56
0.2	1.00	0.30	982	5268	8.14	0.46	0.54	0.46
0.2	0.25	0.50	1319	8177	12.80	0.62	0.84	0.72
0.2	0.55	0.50	907	11576	16.68	0.43	1.19	0.94
0.2	1.00	0.50	470	14238	19.58	0.22	1.46	1.11
0.4	0.00	0.00	1721	9725	15.70	1.00	1.00	1.00
0.4	0.25	0.15	1392	7318	11.44	0.81	0.75	0.73
0.4	0.55	0.15	1181	6533	9.80	0.69	0.67	0.62
0.4	1.00	0.15	1030	6083	8.82	0.60	0.63	0.56
0.4	0.25	0.30	1169	6596	9.85	0.68	0.68	0.63
0.4	0.55	0.30	834	5925	8.60	0.48	0.61	0.55
0.4	1.00	0.30	591	5097	6.54	0.34	0.52	0.42
0.4	0.25	0.50	910	7641	10.66	0.53	0.79	0.68
0.4	0.55	0.50	482	9388	12.24	0.28	0.97	0.78
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.18 FEA results for specimens with $D/t=100$, $p/p_y=0.8$ and material mt2 ($\epsilon_{cr}=1.65$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2042	13155	23.93	1.00	1.00	1.00
0.0	0.25	0.15	1790	11186	19.45	0.88	0.85	0.81
0.0	0.55	0.15	1605	10345	17.38	0.79	0.79	0.73
0.0	1.00	0.15	1434	9624	15.77	0.70	0.73	0.66
0.0	0.25	0.30	1613	10716	17.94	0.79	0.81	0.75
0.0	0.55	0.30	1332	12688	19.75	0.65	0.96	0.83
0.0	1.00	0.30	1029	14777	21.96	0.50	1.12	0.92
0.0	0.25	0.50	1454	13856	21.74	0.71	1.05	0.91
0.0	0.55	0.50	1130	19556	28.78	0.55	1.49	1.20
0.0	1.00	0.50	750	27363	38.99	0.37	2.08	1.63
0.2	0.00	0.00	1636	13126	20.97	1.00	1.00	1.00
0.2	0.25	0.15	1354	10834	16.82	0.83	0.83	0.80
0.2	0.55	0.15	1147	9759	14.77	0.70	0.74	0.70
0.2	1.00	0.15	967	8915	13.16	0.59	0.68	0.63
0.2	0.25	0.30	1154	9896	14.98	0.71	0.75	0.71
0.2	0.55	0.30	857	10865	15.59	0.52	0.83	0.74
0.2	1.00	0.30	546	11690	16.09	0.33	0.89	0.77
0.2	0.25	0.50	981	12832	18.57	0.60	0.98	0.89
0.2	0.55	0.50	630	18239	25.25	0.39	1.39	1.20
0.2	1.00	0.50	182	22851	30.90	0.11	1.74	1.47
0.4	0.00	0.00	1132	12969	18.42	1.00	1.00	1.00
0.4	0.25	0.15	856	10437	14.20	0.76	0.80	0.77
0.4	0.55	0.15	655	9404	12.29	0.58	0.73	0.67
0.4	1.00	0.15	474	8884	11.18	0.42	0.69	0.61
0.4	0.25	0.30	660	9246	12.10	0.58	0.71	0.66
0.4	0.55	0.30	359	10103	12.67	0.32	0.78	0.69
0.4	1.00	0.30	Failed	Failed	Failed	Failed	Failed	Failed
0.4	0.25	0.50	480	12075	15.61	0.42	0.93	0.85
0.4	0.55	0.50	106	17200	21.94	0.09	1.33	1.19
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.19 FEA results for specimens with $D/t=40$, $p/p_y=0.0$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	8219	30224	62.58	1.00	1.00	1.00
0.0	0.25	0.15	7910	21018	44.20	0.96	0.70	0.71
0.0	0.55	0.15	7546	17526	36.20	0.92	0.58	0.58
0.0	1.00	0.15	7233	16714	33.36	0.88	0.55	0.53
0.0	0.25	0.30	7549	17832	36.61	0.92	0.59	0.59
0.0	0.55	0.30	6790	13823	27.18	0.83	0.46	0.43
0.0	1.00	0.30	6163	13016	24.30	0.75	0.43	0.39
0.0	0.25	0.50	7067	15681	30.93	0.86	0.52	0.49
0.0	0.55	0.50	5441	8425	16.26	0.66	0.28	0.26
0.0	1.00	0.50	4541	8320	15.09	0.55	0.28	0.24
0.2	0.00	0.00	8127	32190	58.88	1.00	1.00	1.00
0.2	0.25	0.15	7652	20739	38.82	0.94	0.64	0.66
0.2	0.55	0.15	7194	17444	32.01	0.89	0.54	0.54
0.2	1.00	0.15	6809	16174	28.99	0.84	0.50	0.49
0.2	0.25	0.30	7207	17557	32.18	0.89	0.55	0.55
0.2	0.55	0.30	6306	13657	24.09	0.78	0.42	0.41
0.2	1.00	0.30	5565	12280	20.94	0.68	0.38	0.36
0.2	0.25	0.50	6637	15209	27.00	0.82	0.47	0.46
0.2	0.55	0.50	4751	8336	14.48	0.58	0.26	0.25
0.2	1.00	0.50	3725	8376	13.75	0.46	0.26	0.23
0.4	0.00	0.00	7768	33891	56.24	1.00	1.00	1.00
0.4	0.25	0.15	7134	20580	35.00	0.92	0.61	0.62
0.4	0.55	0.15	6588	17311	28.94	0.85	0.51	0.51
0.4	1.00	0.15	6133	15390	25.38	0.79	0.45	0.45
0.4	0.25	0.30	6613	17512	29.23	0.85	0.52	0.52
0.4	0.55	0.30	5571	13593	22.00	0.72	0.40	0.39
0.4	1.00	0.30	4726	12244	19.26	0.61	0.36	0.34
0.4	0.25	0.50	5953	14701	24.05	0.77	0.43	0.43
0.4	0.55	0.50	3875	8348	13.15	0.50	0.25	0.23
0.4	1.00	0.50	2830	8289	12.29	0.36	0.24	0.22

Table C.20 FEA results for specimens with $D/t=40$, $p/p_y=0.4$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	8927	38904	84.77	1.00	1.00	1.00
0.0	0.25	0.15	8148	25943	53.78	0.91	0.67	0.63
0.0	0.55	0.15	7631	22637	44.64	0.85	0.58	0.53
0.0	1.00	0.15	7203	20250	38.74	0.81	0.52	0.46
0.0	0.25	0.30	7669	23559	46.24	0.86	0.61	0.55
0.0	0.55	0.30	6788	20442	37.10	0.76	0.53	0.44
0.0	1.00	0.30	5970	18265	31.55	0.67	0.47	0.37
0.0	0.25	0.50	7003	21477	39.35	0.78	0.55	0.46
0.0	0.55	0.50	5566	19416	32.08	0.62	0.50	0.38
0.0	1.00	0.50	4136	17199	26.93	0.46	0.44	0.32
0.2	0.00	0.00	8485	38471	73.63	1.00	1.00	1.00
0.2	0.25	0.15	7619	24475	45.41	0.90	0.64	0.62
0.2	0.55	0.15	7036	21375	38.02	0.83	0.56	0.52
0.2	1.00	0.15	6538	19349	33.45	0.77	0.50	0.45
0.2	0.25	0.30	7088	22646	40.02	0.84	0.59	0.54
0.2	0.55	0.30	6088	19173	31.91	0.72	0.50	0.43
0.2	1.00	0.30	5149	17047	27.27	0.61	0.44	0.37
0.2	0.25	0.50	6336	19841	33.37	0.75	0.52	0.45
0.2	0.55	0.50	4694	18029	27.89	0.55	0.47	0.38
0.2	1.00	0.50	3114	15517	23.01	0.37	0.40	0.31
0.4	0.00	0.00	7817	38610	66.39	1.00	1.00	1.00
0.4	0.25	0.15	6864	24117	40.56	0.88	0.62	0.61
0.4	0.55	0.15	6209	20862	33.95	0.79	0.54	0.51
0.4	1.00	0.15	5643	18993	30.18	0.72	0.49	0.45
0.4	0.25	0.30	6272	21678	35.22	0.80	0.56	0.53
0.4	0.55	0.30	5151	18160	28.15	0.66	0.47	0.42
0.4	1.00	0.30	4111	15934	23.93	0.53	0.41	0.36
0.4	0.25	0.50	5436	18710	29.28	0.70	0.48	0.44
0.4	0.55	0.50	3631	16573	24.27	0.46	0.43	0.37
0.4	1.00	0.50	2052	14030	19.50	0.26	0.36	0.29

Table C.21 FEA results for specimens with $D/t=40$, $p/p_y=0.8$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	8399	41301	82.75	1.00	1.00	1.00
0.0	0.25	0.15	7504	30615	57.88	0.89	0.74	0.70
0.0	0.55	0.15	6933	27940	50.31	0.83	0.68	0.61
0.0	1.00	0.15	6411	25759	44.80	0.76	0.62	0.54
0.0	0.25	0.30	6938	28833	51.55	0.83	0.70	0.62
0.0	0.55	0.30	5992	26886	44.75	0.71	0.65	0.54
0.0	1.00	0.30	5020	25062	39.79	0.60	0.61	0.48
0.0	0.25	0.50	6186	31025	51.12	0.74	0.75	0.62
0.0	0.55	0.50	4918	38663	58.93	0.59	0.94	0.71
0.0	1.00	0.50	3458	45320	66.20	0.41	1.10	0.80
0.2	0.00	0.00	7711	41210	73.51	1.00	1.00	1.00
0.2	0.25	0.15	6728	29380	50.17	0.87	0.71	0.68
0.2	0.55	0.15	6095	26907	44.14	0.79	0.65	0.60
0.2	1.00	0.15	5506	24760	39.50	0.71	0.60	0.54
0.2	0.25	0.30	6107	27809	45.39	0.79	0.67	0.62
0.2	0.55	0.30	5054	26316	40.57	0.66	0.64	0.55
0.2	1.00	0.30	3968	23876	35.53	0.51	0.58	0.48
0.2	0.25	0.50	5268	29975	45.96	0.68	0.73	0.63
0.2	0.55	0.50	3850	36609	52.96	0.50	0.89	0.72
0.2	1.00	0.50	2231	39182	54.96	0.29	0.95	0.75
0.4	0.00	0.00	6795	41032	66.48	1.00	1.00	1.00
0.4	0.25	0.15	5737	28421	44.54	0.84	0.69	0.67
0.4	0.55	0.15	5044	25913	39.38	0.74	0.63	0.59
0.4	1.00	0.15	4393	24351	36.14	0.65	0.59	0.54
0.4	0.25	0.30	5063	26824	40.62	0.75	0.65	0.61
0.4	0.55	0.30	3913	25304	36.71	0.58	0.62	0.55
0.4	1.00	0.30	2760	22582	31.83	0.41	0.55	0.48
0.4	0.25	0.50	4144	29326	42.41	0.61	0.71	0.64
0.4	0.55	0.50	2609	31358	43.60	0.38	0.76	0.66
0.4	1.00	0.50	914	32751	44.24	0.13	0.80	0.67

Table C.22 FEA results for specimens with $D/t=40$, $p/p_y=1.0$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	7919	45339	85.70	1.00	1.00	1.00
0.0	0.25	0.15	6941	34562	61.63	0.88	0.76	0.72
0.0	0.55	0.15	6347	32247	54.98	0.80	0.71	0.64
0.0	1.00	0.15	5775	29930	49.39	0.73	0.66	0.58
0.0	0.25	0.30	6336	33184	56.18	0.80	0.73	0.66
0.0	0.55	0.30	5387	33225	52.67	0.68	0.73	0.61
0.0	1.00	0.30	4352	33503	50.80	0.55	0.74	0.59
0.0	0.25	0.50	5622	44703	68.82	0.71	0.99	0.80
0.0	0.55	0.50	4342	56441	82.67	0.55	1.24	0.96
0.0	1.00	0.50	2813	66771	95.25	0.36	1.47	1.11
0.2	0.00	0.00	7100	44797	75.94	1.00	1.00	1.00
0.2	0.25	0.15	6040	33688	54.48	0.85	0.75	0.72
0.2	0.55	0.15	5385	30993	48.43	0.76	0.69	0.64
0.2	1.00	0.15	4747	28995	44.10	0.67	0.65	0.58
0.2	0.25	0.30	5378	32102	49.87	0.76	0.72	0.66
0.2	0.55	0.30	4328	32375	47.84	0.61	0.72	0.63
0.2	1.00	0.30	3181	32314	46.21	0.45	0.72	0.61
0.2	0.25	0.50	4584	43081	62.60	0.65	0.96	0.82
0.2	0.55	0.50	3162	53601	75.19	0.45	1.20	0.99
0.2	1.00	0.50	1495	63061	86.71	0.21	1.41	1.14
0.4	0.00	0.00	6070	44648	68.86	1.00	1.00	1.00
0.4	0.25	0.15	4942	32326	48.04	0.81	0.72	0.70
0.4	0.55	0.15	4230	30367	43.89	0.70	0.68	0.64
0.4	1.00	0.15	3530	28092	39.81	0.58	0.63	0.58
0.4	0.25	0.30	4227	31257	45.06	0.70	0.70	0.65
0.4	0.55	0.30	3084	30946	43.13	0.51	0.69	0.63
0.4	1.00	0.30	1875	30000	40.78	0.31	0.67	0.59
0.4	0.25	0.50	3354	41911	58.00	0.55	0.94	0.84
0.4	0.55	0.50	1840	51152	69.33	0.30	1.15	1.01
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.23 FEA results for specimens with $D/t=70$, $p/p_y=0.0$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	4343	14292	29.09	1.00	1.00	1.00
0.0	0.25	0.15	3974	10043	20.82	0.92	0.70	0.72
0.0	0.55	0.15	3716	8674	17.80	0.86	0.61	0.61
0.0	1.00	0.15	3567	8128	16.58	0.82	0.57	0.57
0.0	0.25	0.30	3674	8063	16.80	0.85	0.56	0.58
0.0	0.55	0.30	3162	6297	12.97	0.73	0.44	0.45
0.0	1.00	0.30	2914	6101	12.23	0.67	0.43	0.42
0.0	0.25	0.50	3286	7618	14.85	0.76	0.53	0.51
0.0	0.55	0.50	2326	4220	8.75	0.54	0.30	0.30
0.0	1.00	0.50	2018	4385	8.61	0.46	0.31	0.30
0.2	0.00	0.00	4174	14556	26.54	1.00	1.00	1.00
0.2	0.25	0.15	3728	9417	17.77	0.89	0.65	0.67
0.2	0.55	0.15	3413	8458	15.63	0.82	0.58	0.59
0.2	1.00	0.15	3230	8175	14.87	0.77	0.56	0.56
0.2	0.25	0.30	3369	7971	14.85	0.81	0.55	0.56
0.2	0.55	0.30	2774	6297	11.53	0.66	0.43	0.43
0.2	1.00	0.30	2473	6258	11.08	0.59	0.43	0.42
0.2	0.25	0.50	2934	7667	13.45	0.70	0.53	0.51
0.2	0.55	0.50	1866	4228	7.52	0.45	0.29	0.28
0.2	1.00	0.50	1532	4376	7.32	0.37	0.30	0.28
0.4	0.00	0.00	3853	14678	24.54	1.00	1.00	1.00
0.4	0.25	0.15	3331	9397	16.13	0.86	0.64	0.66
0.4	0.55	0.15	2969	8469	14.22	0.77	0.58	0.58
0.4	1.00	0.15	2756	8063	13.36	0.72	0.55	0.54
0.4	0.25	0.30	2923	8135	13.69	0.76	0.55	0.56
0.4	0.55	0.30	2287	6328	10.30	0.59	0.43	0.42
0.4	1.00	0.30	1980	6381	9.97	0.51	0.43	0.41
0.4	0.25	0.50	2449	7654	12.14	0.64	0.52	0.49
0.4	0.55	0.50	1399	4253	6.30	0.36	0.29	0.26
0.4	1.00	0.50	1034	4354	6.00	0.27	0.30	0.24

Table C.24 FEA results for specimens with $D/t=70$, $p/p_y=0.4$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	4521	16387	34.86	1.00	1.00	1.00
0.0	0.25	0.15	4074	12266	25.00	0.90	0.75	0.72
0.0	0.55	0.15	3758	10840	21.28	0.83	0.66	0.61
0.0	1.00	0.15	3527	9983	19.19	0.78	0.61	0.55
0.0	0.25	0.30	3761	10811	21.24	0.83	0.66	0.61
0.0	0.55	0.30	3201	9396	17.36	0.71	0.57	0.50
0.0	1.00	0.30	2766	8321	14.96	0.61	0.51	0.43
0.0	0.25	0.50	3333	12149	21.58	0.74	0.74	0.62
0.0	0.55	0.50	2605	19795	30.64	0.58	1.21	0.88
0.0	1.00	0.50	1851	26276	38.44	0.41	1.60	1.10
0.2	0.00	0.00	4206	16276	30.64	1.00	1.00	1.00
0.2	0.25	0.15	3690	12106	21.97	0.88	0.74	0.72
0.2	0.55	0.15	3327	10449	18.47	0.79	0.64	0.60
0.2	1.00	0.15	3057	9391	16.37	0.73	0.58	0.53
0.2	0.25	0.30	3332	10392	18.39	0.79	0.64	0.60
0.2	0.55	0.30	2701	8976	15.11	0.64	0.55	0.49
0.2	1.00	0.30	2225	8007	13.08	0.53	0.49	0.43
0.2	0.25	0.50	2845	11685	19.08	0.68	0.72	0.62
0.2	0.55	0.50	2018	15917	23.64	0.48	0.98	0.77
0.2	1.00	0.50	1201	18299	25.89	0.29	1.12	0.84
0.4	0.00	0.00	3711	15878	26.92	1.00	1.00	1.00
0.4	0.25	0.15	3167	11522	19.13	0.85	0.73	0.71
0.4	0.55	0.15	2767	9986	16.18	0.75	0.63	0.60
0.4	1.00	0.15	2478	9300	14.74	0.67	0.59	0.55
0.4	0.25	0.30	2771	9981	16.17	0.75	0.63	0.60
0.4	0.55	0.30	2126	8539	13.10	0.57	0.54	0.49
0.4	1.00	0.30	1655	7530	11.03	0.45	0.47	0.41
0.4	0.25	0.50	2264	10602	16.14	0.61	0.67	0.60
0.4	0.55	0.50	1410	15131	21.12	0.38	0.95	0.78
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.25 FEA results for specimens with $D/t=70$, $p/p_y=0.8$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	4117	18838	36.34	1.00	1.00	1.00
0.0	0.25	0.15	3682	15048	27.86	0.89	0.80	0.77
0.0	0.55	0.15	3351	13907	24.69	0.81	0.74	0.68
0.0	1.00	0.15	3061	13299	22.87	0.74	0.71	0.63
0.0	0.25	0.30	3354	14436	25.46	0.81	0.77	0.70
0.0	0.55	0.30	2808	13658	22.66	0.68	0.73	0.62
0.0	1.00	0.30	2272	13007	20.68	0.55	0.69	0.57
0.0	0.25	0.50	2989	16714	27.49	0.73	0.89	0.76
0.0	0.55	0.50	2350	22390	34.03	0.57	1.19	0.94
0.0	1.00	0.50	1628	31190	45.16	0.40	1.66	1.24
0.2	0.00	0.00	3670	18724	32.40	1.00	1.00	1.00
0.2	0.25	0.15	3166	14754	24.64	0.86	0.79	0.76
0.2	0.55	0.15	2796	13256	21.50	0.76	0.71	0.66
0.2	1.00	0.15	2476	12505	19.78	0.67	0.67	0.61
0.2	0.25	0.30	2802	13828	22.31	0.76	0.74	0.69
0.2	0.55	0.30	2204	13047	20.02	0.60	0.70	0.62
0.2	1.00	0.30	1641	11796	17.44	0.45	0.63	0.54
0.2	0.25	0.50	2395	16135	24.65	0.65	0.86	0.76
0.2	0.55	0.50	1692	20515	29.58	0.46	1.10	0.91
0.2	1.00	0.50	899	25027	34.82	0.24	1.34	1.07
0.4	0.00	0.00	3032	18557	29.10	1.00	1.00	1.00
0.4	0.25	0.15	2526	14041	21.54	0.83	0.76	0.74
0.4	0.55	0.15	2138	12605	18.81	0.70	0.68	0.65
0.4	1.00	0.15	1813	11740	17.05	0.60	0.63	0.59
0.4	0.25	0.30	2146	12816	19.11	0.71	0.69	0.66
0.4	0.55	0.30	1543	11992	16.93	0.51	0.65	0.58
0.4	1.00	0.30	986	10517	14.08	0.33	0.57	0.48
0.4	0.25	0.50	1725	15181	21.63	0.57	0.82	0.74
0.4	0.55	0.50	998	18115	24.62	0.33	0.98	0.85
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.26 FEA results for specimens with $D/t=70$, $p/p_y=1.0$ and material mt3 ($\epsilon_{cr}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	3817	20217	37.22	1.00	1.00	1.00
0.0	0.25	0.15	3367	17080	29.99	0.88	0.84	0.81
0.0	0.55	0.15	3032	16301	27.46	0.79	0.81	0.74
0.0	1.00	0.15	2719	15374	25.18	0.71	0.76	0.68
0.0	0.25	0.30	3039	16640	27.98	0.80	0.82	0.75
0.0	0.55	0.30	2529	19441	30.41	0.66	0.96	0.82
0.0	1.00	0.30	1981	21188	31.72	0.52	1.05	0.85
0.0	0.25	0.50	2705	22792	35.41	0.71	1.13	0.95
0.0	0.55	0.50	2096	31970	46.86	0.55	1.58	1.26
0.0	1.00	0.50	1377	42652	60.78	0.36	2.11	1.63
0.2	0.00	0.00	3277	19958	32.87	1.00	1.00	1.00
0.2	0.25	0.15	2787	16417	26.10	0.85	0.82	0.79
0.2	0.55	0.15	2419	15578	23.98	0.74	0.78	0.73
0.2	1.00	0.15	2077	14627	21.99	0.63	0.73	0.67
0.2	0.25	0.30	2425	15340	23.67	0.74	0.77	0.72
0.2	0.55	0.30	1868	16862	24.70	0.57	0.84	0.75
0.2	1.00	0.30	1286	18179	25.66	0.39	0.91	0.78
0.2	0.25	0.50	2055	22002	31.99	0.63	1.10	0.97
0.2	0.55	0.50	1387	29895	41.80	0.42	1.50	1.27
0.2	1.00	0.50	588	38268	52.43	0.18	1.92	1.60
0.4	0.00	0.00	2621	19938	29.64	1.00	1.00	1.00
0.4	0.25	0.15	2096	15947	23.04	0.80	0.80	0.78
0.4	0.55	0.15	1718	14883	20.85	0.66	0.75	0.70
0.4	1.00	0.15	1372	14252	19.40	0.52	0.71	0.65
0.4	0.25	0.30	1722	14762	20.70	0.66	0.74	0.70
0.4	0.55	0.30	1157	15738	21.11	0.44	0.79	0.71
0.4	1.00	0.30	552	16892	21.92	0.21	0.85	0.74
0.4	0.25	0.50	1337	20821	28.28	0.51	1.04	0.95
0.4	0.55	0.50	627	27637	36.72	0.24	1.39	1.24
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.27 FEA results for specimens with $D/t=100$, $p/p_y=0.0$ and material mt3 ($\epsilon_{er}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2812	9162	19.40	1.00	1.00	1.00
0.0	0.25	0.15	2496	6358	13.90	0.89	0.69	0.72
0.0	0.55	0.15	2314	5671	12.30	0.82	0.62	0.63
0.0	1.00	0.15	2236	5464	11.79	0.79	0.60	0.61
0.0	0.25	0.30	2226	5091	11.24	0.79	0.56	0.58
0.0	0.55	0.30	1904	4383	9.46	0.68	0.48	0.49
0.0	1.00	0.30	1787	4348	9.18	0.64	0.47	0.47
0.0	0.25	0.50	1722	3375	7.72	0.61	0.37	0.40
0.0	0.55	0.50	1336	3122	6.69	0.47	0.34	0.35
0.0	1.00	0.50	1204	3214	6.60	0.43	0.35	0.34
0.2	0.00	0.00	2659	9145	17.39	1.00	1.00	1.00
0.2	0.25	0.15	2279	6289	12.23	0.86	0.69	0.70
0.2	0.55	0.15	2061	5622	10.79	0.77	0.61	0.62
0.2	1.00	0.15	1967	5532	10.48	0.74	0.60	0.60
0.2	0.25	0.30	1964	5097	9.88	0.74	0.56	0.57
0.2	0.55	0.30	1596	4376	8.20	0.60	0.48	0.47
0.2	1.00	0.30	1466	4318	7.89	0.55	0.47	0.45
0.2	0.25	0.50	1412	3386	6.52	0.53	0.37	0.37
0.2	0.55	0.50	1014	3122	5.45	0.38	0.34	0.31
0.2	1.00	0.50	873	3265	5.41	0.33	0.36	0.31
0.4	0.00	0.00	2391	9274	16.01	1.00	1.00	1.00
0.4	0.25	0.15	1962	6273	10.93	0.82	0.68	0.68
0.4	0.55	0.15	1730	5662	9.61	0.72	0.61	0.60
0.4	1.00	0.15	1627	5500	9.19	0.68	0.59	0.57
0.4	0.25	0.30	1635	5110	8.66	0.68	0.55	0.54
0.4	0.55	0.30	1269	4402	6.99	0.53	0.47	0.44
0.4	1.00	0.30	1127	4306	6.60	0.47	0.46	0.41
0.4	0.25	0.50	1093	3390	5.29	0.46	0.37	0.33
0.4	0.55	0.50	687	3108	4.18	0.29	0.34	0.26
0.4	1.00	0.50	533	3271	4.14	0.22	0.35	0.26

Table C.28 FEA results for specimens with $D/t=100$, $p/p_y=0.4$ and material mt3 ($\epsilon_{er}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2883	10348	22.05	1.00	1.00	1.00
0.0	0.25	0.15	2545	7832	16.26	0.88	0.76	0.74
0.0	0.55	0.15	2316	6996	14.10	0.80	0.68	0.64
0.0	1.00	0.15	2177	6594	13.10	0.75	0.64	0.59
0.0	0.25	0.30	2299	7089	14.16	0.80	0.69	0.64
0.0	0.55	0.30	1897	5942	11.43	0.66	0.57	0.52
0.0	1.00	0.30	1648	5382	10.16	0.57	0.52	0.46
0.0	0.25	0.50	2008	9580	16.74	0.70	0.93	0.76
0.0	0.55	0.50	1558	15460	23.86	0.54	1.49	1.08
0.0	1.00	0.50	1099	22331	32.50	0.38	2.16	1.47
0.2	0.00	0.00	2616	10375	19.49	1.00	1.00	1.00
0.2	0.25	0.15	2232	7697	14.17	0.85	0.74	0.73
0.2	0.55	0.15	1976	6837	12.28	0.76	0.66	0.63
0.2	1.00	0.15	1813	6562	11.55	0.69	0.63	0.59
0.2	0.25	0.30	1958	6856	12.26	0.75	0.66	0.63
0.2	0.55	0.30	1533	5781	9.87	0.59	0.56	0.51
0.2	1.00	0.30	1278	5252	8.66	0.49	0.51	0.44
0.2	0.25	0.50	1635	9082	14.60	0.62	0.88	0.75
0.2	0.55	0.50	1143	13710	20.02	0.44	1.32	1.03
0.2	1.00	0.50	638	17167	23.91	0.24	1.65	1.23
0.4	0.00	0.00	2245	10294	17.42	1.00	1.00	1.00
0.4	0.25	0.15	1839	7582	12.55	0.82	0.74	0.72
0.4	0.55	0.15	1586	6714	10.75	0.71	0.65	0.62
0.4	1.00	0.15	1428	6133	9.60	0.64	0.60	0.55
0.4	0.25	0.30	1571	6745	10.75	0.70	0.66	0.62
0.4	0.55	0.30	1157	5575	8.25	0.52	0.54	0.47
0.4	1.00	0.30	900	5150	7.18	0.40	0.50	0.41
0.4	0.25	0.50	1244	8243	12.09	0.55	0.80	0.69
0.4	0.55	0.50	727	11202	15.16	0.32	1.09	0.87
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.29 FEA results for specimens with D/t=100, p/p_y=0.8 and material mt3 ($\epsilon_{er}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2572	12091	23.15	1.00	1.00	1.00
0.0	0.25	0.15	2265	9996	18.47	0.88	0.83	0.80
0.0	0.55	0.15	2031	9167	16.39	0.79	0.76	0.71
0.0	1.00	0.15	1841	8788	15.30	0.72	0.73	0.66
0.0	0.25	0.30	2033	9540	16.92	0.79	0.79	0.73
0.0	0.55	0.30	1662	9242	15.46	0.65	0.76	0.67
0.0	1.00	0.30	1314	8414	13.58	0.51	0.70	0.59
0.0	0.25	0.50	1818	11532	19.08	0.71	0.95	0.82
0.0	0.55	0.50	1420	16156	24.62	0.55	1.34	1.06
0.0	1.00	0.50	980	23121	33.48	0.38	1.91	1.45
0.2	0.00	0.00	2198	11978	20.47	1.00	1.00	1.00
0.2	0.25	0.15	1873	9463	15.83	0.85	0.79	0.77
0.2	0.55	0.15	1620	8740	14.16	0.74	0.73	0.69
0.2	1.00	0.15	1422	8144	12.88	0.65	0.68	0.63
0.2	0.25	0.30	1622	9041	14.58	0.74	0.75	0.71
0.2	0.55	0.30	1235	8443	12.87	0.56	0.70	0.63
0.2	1.00	0.30	891	7703	11.15	0.41	0.64	0.54
0.2	0.25	0.50	1390	11029	16.81	0.63	0.92	0.82
0.2	0.55	0.50	963	14203	20.36	0.44	1.19	0.99
0.2	1.00	0.50	477	17924	24.68	0.22	1.50	1.21
0.4	0.00	0.00	1754	11732	18.21	1.00	1.00	1.00
0.4	0.25	0.15	1424	9216	13.86	0.81	0.79	0.76
0.4	0.55	0.15	1178	8414	12.14	0.67	0.72	0.67
0.4	1.00	0.15	982	8036	11.18	0.56	0.68	0.61
0.4	0.25	0.30	1145	8240	11.81	0.65	0.70	0.65
0.4	0.55	0.30	799	8095	10.86	0.46	0.69	0.60
0.4	1.00	0.30	456	7339	9.14	0.26	0.63	0.50
0.4	0.25	0.50	941	10097	13.95	0.54	0.86	0.77
0.4	0.55	0.50	497	12747	16.77	0.28	1.09	0.92
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.30 FEA results for specimens with $D/t=100$, $p/p_y=1.0$ and material mt3 ($\epsilon_{er}=2.0$)

σ_a/σ_y	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
0.0	0.00	0.00	2353	13140	23.89	1.00	1.00	1.00
0.0	0.25	0.15	2048	11510	20.06	0.87	0.88	0.84
0.0	0.55	0.15	1822	10968	18.46	0.77	0.83	0.77
0.0	1.00	0.15	1617	10276	16.89	0.69	0.78	0.71
0.0	0.25	0.30	1835	11877	19.79	0.78	0.90	0.83
0.0	0.55	0.30	1511	13807	21.56	0.64	1.05	0.90
0.0	1.00	0.30	1165	17003	25.23	0.50	1.29	1.06
0.0	0.25	0.50	1651	14594	22.97	0.70	1.11	0.96
0.0	0.55	0.50	1274	21645	31.85	0.54	1.65	1.33
0.0	1.00	0.50	836	31200	44.45	0.36	2.37	1.86
0.2	0.00	0.00	1945	13111	21.24	1.00	1.00	1.00
0.2	0.25	0.15	1617	11179	17.52	0.83	0.85	0.83
0.2	0.55	0.15	1377	10574	16.07	0.71	0.81	0.76
0.2	1.00	0.15	1164	9823	14.54	0.60	0.75	0.68
0.2	0.25	0.30	1390	10909	16.57	0.71	0.83	0.78
0.2	0.55	0.30	1047	12275	17.68	0.54	0.94	0.83
0.2	1.00	0.30	683	13496	18.65	0.35	1.03	0.88
0.2	0.25	0.50	1191	13764	20.02	0.61	1.05	0.94
0.2	0.55	0.50	783	19842	27.59	0.40	1.51	1.30
0.2	1.00	0.50	267	27492	37.37	0.14	2.10	1.76
0.4	0.00	0.00	1463	13043	18.89	1.00	1.00	1.00
0.4	0.25	0.15	1140	10907	15.10	0.78	0.84	0.80
0.4	0.55	0.15	904	10371	13.79	0.62	0.80	0.73
0.4	1.00	0.15	690	9721	12.42	0.47	0.75	0.66
0.4	0.25	0.30	916	10388	13.86	0.63	0.80	0.73
0.4	0.55	0.30	567	11111	14.13	0.39	0.85	0.75
0.4	1.00	0.30	153	12359	15.14	0.10	0.95	0.80
0.4	0.25	0.50	708	13375	17.51	0.48	1.03	0.93
0.4	0.55	0.50	220	18737	24.09	0.15	1.44	1.28
0.4	1.00	0.50	Failed	Failed	Failed	Failed	Failed	Failed

Table C.31 FEA results for tension specimens with $p/p_y=0.4$ and $\sigma_a/\sigma_y=-0.2$

Material	D/t	l_c/D	d/t	M_{lc} (kNm)	ϵ_{lc} (mm/mm $\times 10^{-6}$)	Φ_{lc} (rad/mm $\times 10^{-6}$)	M_{lc}/M_{lp}	$\epsilon_{lc}/\epsilon_{lp}$	Φ_{lc}/Φ_{lp}
mt1	40	0.00	0.00	5935	36492	96.98	1.00	1.00	1.00
mt1	40	0.25	0.30	5333	24308	55.70	0.90	0.67	0.57
mt1	40	1.00	0.30	4448	19604	36.59	0.75	0.54	0.38
mt1	100	0.00	0.00	2069	8001	20.46	1.00	1.00	1.00
mt1	100	0.25	0.30	1798	6720	13.84	0.87	0.84	0.68
mt1	100	1.00	0.30	1417	5457	10.77	0.68	0.68	0.53
mt2	40	0.00	0.00	7529	38194	99.80	1.00	1.00	1.00
mt2	40	0.25	0.30	6698	24831	56.73	0.89	0.65	0.57
mt2	40	1.00	0.30	5502	19901	38.02	0.73	0.52	0.38
mt2	100	0.00	0.00	2556	9704	24.84	1.00	1.00	1.00
mt2	100	0.25	0.30	2178	7171	16.19	0.85	0.74	0.65
mt2	100	1.00	0.30	1704	5988	12.00	0.67	0.62	0.48
mt3	40	0.00	0.00	9096	39033	100.30	1.00	1.00	1.00
mt3	40	0.25	0.30	8019	24860	55.39	0.88	0.64	0.55
mt3	40	1.00	0.30	6553	19567	37.27	0.72	0.50	0.37
mt3	100	0.00	0.00	3053	10226	25.68	1.00	1.00	1.00
mt3	100	0.25	0.30	2537	7457	16.85	0.83	0.73	0.66
mt3	100	1.00	0.30	1978	5730	12.15	0.65	0.56	0.47