**Phase Noise Estimation and Suppression for a SDMA
DFT-Precoded OFDM System**

By

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Abstract

The main focus of this thesis is to investigate the impact and compensation of the independent phase noises(PHN) caused by the free running oscillators in the single carrier uplink(UL) of a typical wireless communication system from a group of independent users. The random behavior of the PHN is studied and is modeled as a Wiener process. A phase factor for phase noise compensation for single input single output (SISO) channels is extended to the case of space division multiple access (SDMA) channels by adopting a parallel interference cancellation strategy. Starting with the relationship between the time varying phase noise and the signal expression at the transmitter output, a single carrier iterative block based frequency domain equalization with joint recursive second order phase lock loop algorithm (SCIBFDE-RSPLL) is proposed and evaluated. The IBFDE equalization receiver structure is created based on parallel interference cancellation (PIC) and minimum mean squared error (MMSE) criterion. For each received data stream, the user data is simultaneously detected and equalized. The signals from other users are subtracted before the equalization process. In each iteration, the proposed RSPLL phase tracking method is derived using the maximum *a posteriori* probability (MAP) criterion and the density function of decoded data is replaced by the *posterior* density function of estimated data from the previous iteration given the equalizer outputs . The phase tracking can be generalized as a second order PLL with two tracking parameters. Simulation results show that overall, the proposed SCIBFDE-RSPLL algorithm demonstrates a noticeable gain in the urban wide area

fading (UWAF) channel and enjoys more diversity gain with SDMA channel configurations.

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List Of Acronyms

- **AWGN:** Additive White Gaussian Noise
- **BER:** Bit Error Rate
- **BS:** Base Station
- **BPSK:** Binary Phase Shift Keying
- **CP:** Cyclic Prefix
- **CPE:** Common Phase Error
- **DDC:** Digital Up Converter
- **DFT:** Discrete Fourier Transform
- **DL:** Down-Link
- **DSP:** Digital Signal Processing
- **DUC:** Digital Down Converter
- **E_b/N_0 :** Energy Per Bit to Noise Power Spectral Density Ratio
- **EM:** Expectation Maximization
- **FD:** Frequency Domain
- **FDE:** Frequency Domain Equalization
- **FD-DFE:** Frequency Domain Decision Feedback Equalizer
- **FFT:** Fast Fourier Transform

- **HPA:** High-Power Amplifier
- **IBFDE:** Iterative Block Based Frequency Domain Equalization
- **ICI:** Inter Carrier Interference
- **IFDMA:** Interleaved Frequency Domain Multiple Access
- **IFFT:** Inverse Fast Fourier Transform
- **ISI:** Inter Symbol Interference
- **LLR:** Log Likelihood Ration
- **LOS:** Line Of Sight
- **LTE:** Long-Term Evolution
- **MAP:** Maximum A Posteriori Probability
- **MIMO:** Multiple Input, Multiple Output
- **ML:** Maximum Likelihood
- **MMSE:** Minimum Mean Square Error
- **MS:** Mobile Station
- **MUD:** Multiple User Detection
- **NLOS:** No Line Of Sight
- **OFDM:** Orthogonal Frequency Division Multiplexing
- **OFDMA:** Orthogonal Frequency Division Multiplexing Access
- **PAPR:** Peak to Average Power Ratio

- **PHN:** Phase Noise
- **PIC:** Parallel Interference Cancellation
- **PLL:** Phase Lock Loop
- **PSD:** Power Spectrum Density
- **QPSK:** Quadrature Phase Shift Keying
- **QAM:** Quadrature Amplitude Modulation
- **RF:** Radio Frequency
- **RSPLL:** Recursive Second order Phase Lock Loop
- **SC:** Single Carrier
- **SCFDE:** Single Carrier Frequency Domain Equalization
- **SCIBFDE:** Single Carrier Iterative Block based Frequency Domain Equalization
- **SC-FDMA:** Single Carrier Frequency Division Multiple Access
- **SDMA:** Space-Division Multiple Access
- **SISO:** Single Input ,Single Output (SISO)
- **SNR:** Signal to Noise Ratio
- **S/P:** Serial to Parallel
- **TD:** Time Domain
- **UL:** Uplink
- **UWAF :** Urban Wide Area Fading Channel

- **WINNER:** Wireless World Initiative New Radio
- **3GPP :** 3rd Generation Partnership Project
- **3GPP-LTE:** 3rd Generation Partnership Project Long Term Evolution
- **4G:** Fourth Generation

Chapter 1 Introduction

1.1 Background and Scope

In wireless communication systems, there are many sources of interference and noise such as multi-path channel interference which leads to fading and intersymbol interference (ISI). Other kinds of interference and noise sources are caused by the system itself, and include the phase noise caused by imperfect oscillators which are used as up-converters in the transmitters and down-converters in the receivers. The oscillator also provides the crucial clock reference as the sampling clock in the base-band system. So the impact of inaccurate oscillators/clocks and the analysis of such an impact under different wireless communication systems are important focus of wireless communication system design and implementation. Commonly known as phase jitter, the phase noise of the free running oscillator is related to the external perturbation which depends on the physical nature of each oscillator. There are profound studies of the cause and nature of the phase noise and the different analytical methods in [1] [2] [3]. Since it is not the main focus of this thesis, we only characterize the random behavior of the phase noise as the starting point of this thesis. The focus of this study is to answer the question of how this random phase noise source or multiple independent random phase noise sources impact the uplink (UL) between mobile station (MS) to base station (BS) of a wireless communication system in an urban wide area fading channel (UWAF) single input, single output (SISO) and space division multiple access (SDMA) antenna configurations,

and how to reduce and compensate for the interference subsequently imposed on this wireless communication system.

DFT pre-coded OFDM or single carrier modulation has shown a better Peak to Average Power Ratio (PAPR) trait[4] [5] compared to orthogonal frequency division multiplexing (OFDM) which has the potential for very high PAPR. OFDM is formed as the summation of N sinusoidal waveforms, so by chance, its amplitude could range to a maximum of N times the amplitude of a single waveform. For single carrier (SC) or DFT-precoded OFDM, this is not true since for each time domain symbol, the amplitude is mainly determined by the current symbol waveform itself. We can see this more clearly in the later chapters. This puts single carrier with frequency domain equalization(SCFDE) in the position as the preferred candidate used for the uplink since lower PAPR means less power consumption which is crucial for mobile devices.

The main challenge for the next generation of wireless communication system is to provide high capacity, high reliability communications solutions. In order to obtain high throughput while maintaining high reliability, an accurate phase noise estimation and compensation method becomes mandatory and is more and more challenging with higher data rates.

By exploiting wide spectrum bandwidth and more DSP power from hardware, the requirements for next generation wireless communication system design are pushed to

the limit. OFDM in the down-link (DL) from base station (BS) to mobile station (MS) combined with SCFDE for the UL [6] is an attractive combination for future wide area cellular wireless systems. This combination provides good immunity to channel ISI [7] [8] in the multi-path fading channel and appealing implementation simplicity by using the fast Fourier transform (FFT) as the main digital signal processing (DSP) component. It also provides a good solution for the oscillator imperfections- the BS transmitter can afford more stable higher quality oscillators, while the MS, with obvious cost and power constraints, can use lower quality oscillators.

Challenge No.1: The sensitivity to phase noise.

Phase noise becomes the major concern, because introducing even small noise into an oscillator leads to dramatic changes in the frequency spectrum and timing property. Since an OFDM based system is very sensitive to the phase noise and clock offset [9]-[12], any timing difference caused by unsynchronized transmitter and receiver clock leads to the loss of orthogonality of the OFDM signal, that consequently brings inter carrier interference (ICI) [13] [14]. SC on the other hand, is less sensitive to phase noise.

Challenge No.2: High level modulation has limited tolerance to phase noise.

Besides the sensitivity to the phase noise, in order to get higher throughput, higher M-ary modulation types such as 16 up to 256-quadrature amplitude modulation (QAM) can be considered, and that in turn again puts more demanding pressure on phase noise estimation and compensation capability.

Challenge No.3: Time-varying random phase noise requires a maximum a posteriori (MAP) based phase noise estimation.

The time-varying Phase noise can be expressed as $\varphi(t) = \theta_0 + 2\pi\Delta_f t + \alpha(t)$ [15] where θ_0 represents a fixed phase shift; Δ_f a fixed frequency offset, and $\alpha(t)$ represents slow time-varying random process which is the manifestation of phase noise. For a multicarrier signal, such as OFDM, the impact of $\alpha(t)$ can be further divided into common phase error (CPE) which is the common phase rotation for all carriers, and inter-carrier interference (ICI) which contains the random time-varying interferences in terms of summation of convolutions from all the other carriers[16]-[18][19]. For analytical simplicity and practical system implementation only Δ_f and the common phase error (CPE) are taken into consideration in [19]. That leads to the assumption of deterministic nature of the phase noise. Maximum likelihood (ML) based phase estimation and compensation therefore becomes the optimal solution [19] [20]. In order to more accurately reflect the random nature of the phase noise, a maximum a posteriori(MAP) based phase noise estimation algorithm is to be considered.

Challenge No.4: Multiple independent phase noise sources:

One of the real breakthroughs in recent wireless communication history is the usage of the antenna array and spatial coding. It dramatically increases the channel capacity and frequency usage efficiency [21] by exploiting the uncorrelated nature of the space diversity which makes it possible to transmit and receive different user information at the

same time-frequency slot. So in a typical SDMA wireless RF environment in which Z_t active users simultaneously communicate with the base station which is equipped with Z_r receiving antennas, the receiver antenna will receive Z_r users at the same time-frequency slot pictured in Figure 1. Each user's transmitter injects its own independent phase noise. The phase noise becomes the summation of Z_t independent random phase noise sources. This will further complicate the phase noise estimation.

Figure 1: Uplink, with Multiple Independent User Transmitter Antennas, Multiple Base Station Receiver Antennas. DC:Down Converter; UC:Up Converter.

1.2 Research Contributions

Phase noise analysis and estimation become crucial for next generation wireless systems and at the same time there is a relative lack of research considering random phase effects in SCFDE systems under SDMA channel conditions. Therefore it becomes the main motivation and focus of this thesis. In this thesis the contributions can be summarized in four areas:

Contribution No.1: extension of SISO phase noise analysis and compensation into SDMA with parallel interference cancellation (PIC). A very important conclusion we reach in Chapter 3 says: for a single carrier (SC) system, the phase noise due to the transmitter free running oscillator could be approximated as a time domain phase modulation given the nature of the slow varying phase noise (the bandwidth of the time varying phase process is relatively small comparing to the communication channel bandwidth) . This notion has been extended into the SDMA scenario by adopting PIC. It makes the recursive second order phase lock loop (RSPLL) random phase estimation for individual phase tracking possible. Previous research has been focusing on the phase tracking under SISO. This extension will give more powerful tools to deal with phase estimation with multiple independent active users.

Contribution No. 2: In order to accurately estimate and compensate for phase noise, single carrier iterative block based frequency domain equalization (SCIBFDE) with a joint recursive second-order phase lock loop (RSPLL) under PIC is proposed. For each

received user data stream, an iterative minimum mean square error (MMSE) based frequency domain equalization is performed. The forward filter coefficients and backward filter coefficients are updated in each iteration using the MMSE criterion to combat the ICI and ISI caused by the multi-user and multi-path channel condition. The second-order phase locked loop (PLL) is also jointly applied for the output signal of the equalizer in each iteration while using the a posteriori density function calculated from previous iteration equalizer output as the feedback prior density function in the MAP analysis.

Contribution No.3: dealing with independent phase noise sources. Analysis of the multiple independent phase noise behavior and impact under SC SDMA is the main contribution of this thesis, since most previous research has assumed either it is a single user system [22][23] or assumed a perfect frequency and phase synchronization shared common noise source even under SDMA. The proposed method takes the multiple independent phase noise sources as a fundamental assumption.

Contribution No. 4: The simulation results show the proposed SCIBFDE-RSPLL has better performance compared to a typical OFDM with minimum mean squared error criterion and multi-user detection (OFDM-MMSE-MUD) [19] with CPE phase compensation.

1.3 Thesis Organization

In chapter 2, a general introduction of DFT pre-coded OFDM, also called single carrier with frequency division multiple access(SC-FDMA) is given followed by mathematical expressions and signal generation processes. Comparisons between the traditional OFDM and SC-FDMA are also given, focusing on the advantages of SC-FDMA over OFDM such as low PAPR as well as robustness to frequency offset and phase noise given similar channel capacity.

In chapter 3, the impact of time-varying phase noise caused by the user's free running oscillator is analyzed. First, a mathematical phase noise model is given which links the random phase difference during t_s to a Gaussian random process with zero mean and variance proportional to t_s . Then the phase noise impacts under SISO channel condition are analyzed, focusing on ICI and CPE in the frequency domain. Due to the slow varying nature of phase noise, an important conclusion is derived at the end of the chapter which says: For a SC system, the phase noise due to the transmitter free running oscillator could be approximated as a time domain phase modulation on the data symbols, given the nature of the slow varying phase noise (the bandwidth of the time varying phase process is relatively small comparing to the communication channel bandwidth). This conclusion is later extended and used in the proposed SCIBFDE-RSPLL algorithm under SDMA channel conditions.

In chapter 4, a typical UWAF channel model and its channel parameters are given. Later,

the channel model is used in the simulation to generate system performance curves. Chapter 4 displays the SDMA transceiver structure with independent phase noise sources. The CPE and ICI interference analysis is extended in this chapter under a SDMA scenario. The channel model and the transceiver structure later become the basis of the proposed SCIBFDE-RSPLL algorithm.

In chapter 5, we derive the proposed the IBFDE-PLL algorithm. First the iterative block based FDE is derived using the MMSE criterion. The PIC receiver structure is given to show the simultaneous independent user data stream detection and FDE process. For each block based FDE iteration, a second order PLL phase estimation and compensation process is derived based on the MAP criterion. Later, a similar second order phase noise (PHN) estimation and compensation method based on Expectation Maximization (EM) algorithm is briefly studied as a comparison and it is concluded that the two methods are essentially the same. For comparison reasons, a OFDM-MMSE-MUD receiver is introduced to be later simulated side by side with the proposed SCIBFDE-RSPLL equalization and phase tracking algorithm. At the end of chapter 5, the proposed IBFDE-RSPLL algorithm complexity is estimated.

In chapter 6, the simulation results and simulation comparison between SC and OFDM based system are shown and discussed. The curves in the plots show the different performance under different channel conditions and different levels of phase noise impairment.

In chapter 7, conclusions and the possible future work beyond this thesis are discussed. It includes further work needed to be done in an extended wireless channel conditions such as the multi-cellular case, and the possibility of better PIC- based FDE methods such as turbo equalization. Some optimization work is also needed to be considered for simplifying the implementation.

Chapter 2 DFT pre-coded OFDM

A big advantage of OFDM is its robustness in the presence of multi-path signal propagation [24]. This is due to the fact that an OFDM system transmits with a relatively long symbol time period. Thus there is less impact from channel ISI. However, the OFDM waveform exhibits a high PAPR [5] [25]-[27]. The result is low power efficiency (measured by the ratio of transmitted power to DC power dissipated). Also, OFDM systems are very sensitive to offset in frequency references [9]-[12] [28]. Frequency offset destroys the orthogonality of the transmissions, thus introducing ICI [13] [14].

To overcome these disadvantages, the 3GPP long-term evolution (LTE) [30] and WINNER project [29] [31] have investigated a modified form of OFDM for uplink transmissions in the cellular systems. The modified version of OFDM is referred to as single carrier FDMA (SC-FDMA) in LTE and DFT pre-coded OFDM in WINNER. As in OFDM, the transmitters in a SC-FDMA system use different orthogonal frequencies (subcarriers) to transmit information symbols. However, they transmit the symbols sequentially, rather than in parallel. Relative to OFDM, this arrangement reduces considerably the envelope fluctuations in the transmitted waveform [5]. Therefore, SC signals have inherently lower PAPR than OFDM signals.

2.1 Signal Generation Using DFT

As will be seen in the analysis below, the signal carrier (SC) or DFT-precoded OFDM is recognized as a block of data symbols serially transmitted at intervals of N/M samples, so that in each time interval only one symbol is transmitted. The sampled pulse waveform given by equation (3) and (5). It is a circular version of a *sinc* pulse with zero excess bandwidth. An example is shown in Figure 3. SC signals generated in this way are called DFT-precoded OFDM signals by the WINNER project [29], SC-FDMA by the 3GPP-LTE standards body [30]. For equation (3) and (5) the DFT-precoded OFDM waveform at data symbol intervals depends only on a single data symbol, and therefore has a significantly lower PAPR than that of a corresponding OFDM waveform in which each sample is a linear combination of M data symbols, equally weighted in magnitude. Therefore, its maximum possible magnitude is M times the average data symbol magnitude.

Generation of a SC signal block proceeds by using the discrete Fourier transform (DFT), normally implemented as a fast Fourier transform (FFT) as follows: The coded data source d_n is first mapped into complex constellation points representing a certain modulation (e.g. QPSK, BPSK, QAM), then after serial to parallel (S/P) conversion, a block of M coded data symbols is converted into the frequency domain (FD) by a M -point FFT. The resulting FD data components are zero-padded, implemented by a Pre-matrix frequency-space selector to a set of N ($N > M$) data-carrying subcarriers, and

then processed by an N -point inverse FFT to convert back to the time domain (TD). The resulting samples are parallel-to-serial (P/S) converted and appended with a CP for the transmission of TD sample x_n . The procedure is illustrated in Figure 2.

Figure 2: DFT pre-coded OFDM Transmitter

2.2 General Description and Mathematical Expression

OFDM has long been considered as one of the candidate technologies used by the next generation wireless systems which has been proposed both by 4G long term evolution and WINNER project[29][30][31] due to its efficiency and adaptability in frequency selected channel. However, due to the high PAPR nature of the OFDM signal and its sensitivity to the frequency offset and time-varying phase noise, it will be flexibly combined with SCFDE. SC is preferred for the uplink of cellular systems because of its low PAPR, thus increasing power amplifier efficiency, which means less power consumption in the user terminals. Using the generalized multi-carrier approach [32], a SC signal can be generated by DFT, followed by the same procedure to generate an OFDM signal. Hence, a SC system can also be called a DFT-precoded OFDM. In this case, the output samples in the base-band digital time domain are expressed in equation

(1) where M represents the number of data symbols per block, and N represents the number of time- and frequency domain samples per block.

$$\begin{aligned}
x(n) &= \frac{1}{N} \sum_{l=0}^{M-1} s_l \sum_{p=0}^{M-1} e^{j2\pi p(\frac{n}{N} - \frac{l}{M})} \\
&= \frac{1}{N} \sum_{l=0}^{M-1} \frac{1 - e^{j2\pi(n-l\frac{N}{M})M}}{1 - e^{j2\pi(n-l\frac{N}{M})}} s_l, \quad n=0,1,\dots,N-1, \\
&= \frac{1}{N} \sum_{l=0}^{M-1} e^{j\pi \frac{(M-1)(n-l\frac{N}{M})}{N}} \frac{\sin(\frac{\pi M(n-l\frac{N}{M})}{N})}{\sin(\frac{\pi(n-l\frac{N}{M})}{N})} s_l, \quad n=0,1,\dots,N-1,
\end{aligned} \tag{1}$$

where $\{s_l\}$ are complex data symbol from data mapping block. If we let

$$g(n) = \frac{1}{N} e^{j\pi \frac{(M-1)n}{N}} \frac{\sin(\frac{\pi Mn}{N})}{\sin(\frac{\pi n}{N})} \tag{2}$$

The above equation can be rewritten as :

$$x(n) = \sum_{l=0}^{M-1} g\left(n - l\frac{N}{M}\right) s_l, \quad n=0,1,\dots,N-1, \tag{3}$$

While $\{x(n)\}, n=-L, -L+1, \dots, -1$, contains the CP where L is the length of the CP. The above two equations are based on the simplest frequency mapping which contains N contiguous sub-carrier frequencies, with the remaining $M-N$ being padded with zeros. This is recognized as a block of data symbols serially transmitted at intervals of N/M

samples. An example of DFT-precoded OFDM time domain waveform is shown in Figure 3, in which, a BPSK($s_l = [\pm 1]$) with 16 random generated data symbol case is shown, and N is equal to 512. In the context of 3GPP-LTE, this type of signal is called single carrier-frequency division multiple access (SC-FDMA). It is unlike an OFDM signal which can be viewed as M orthogonal sinusoidal waveform added together in time domain which is given below:

$$x(n) = \frac{1}{N} \sum_{l=0}^{M-1} s_l \exp\left(j \frac{2\pi l n}{N}\right), \quad n = 0, 1, \dots, N-1 \quad (4)$$

Where M representing the number of data symbols, and N the number of samples in one TD data block.

Figure 3: DFT pre-coded OFDM Periodic sinc Like Signal Waveform with $M=16, N=512$ for BPSK.

Besides padding contiguous zeros in the frequency domain to create M complex samples, the DFT of M data symbols could also be mapped to a set of non-contiguous frequencies for diversity implementation or inserting pilot tones without interfering with data. The below equation shows an example of mapping M point DFT outputs from each of up to Q different independent data sources onto M frequencies which are spaced at intervals of Q subcarriers. This is a non-contiguous data symbol mapping(zeros insertion) access scheme sometimes called interleaved frequency domain multiple access (IFDMA) [33]. Each data source takes a unique set of M frequencies in an interleaved pattern. For

$N > Q \times M$, for the user i , we have:

$$x(n, i) = e^{j\pi \frac{inM-1}{N}} \sum_{l=0}^{M-1} g\left(n-l\frac{N}{QM}\right) s_{l,i}, \quad n=0,1,\dots,N-1 \quad (5)$$

Where

$$g(n) = \frac{1}{N} e^{j\pi \frac{Q(M-1)n}{N}} \frac{\sin\left(\frac{\pi QMn}{N}\right)}{\sin\left(\frac{\pi QM}{N}\right)}, \quad n=0,1,\dots,N-1. \quad (6)$$

The signal expressed by the equation above, also bears the same low peak power to average ratio feature like the SC signal generated by contiguous data mapping. We will see this feature more clearly in the following sections. In this thesis we do not consider IFDMA further, but assume that the signal occupies M contiguous frequencies.

2.3 Some Benefits of DFT pre-coded OFDM

Radio transmitters which have nonlinear characteristics can cause significant distortion to signals whose instantaneous power fluctuations come too close to the high-power amplifier (HPA) output saturation power. Even small amounts of nonlinear distortion can cause undesirable spectral re-growth, which can interfere with signals in adjacent frequency channels. It has impact in term of in-band interference as well. The research [5] done on the benefits of combination of lower PAPR with clipping HPA show a noticeable out-band interference reduction. Usually, HPA is operated with a certain “power back-off” which can be defined as the ratio of maximum saturation output power

to average output power to compensate the nonlinearity. This means by reducing the PAPR, for a given transmitted power, it operates on a smaller power back-off and boots the HPA efficiency and increases battery life.

Frequency offset and phase noise will introduce CPE and ICI. Later in Chapter 4 we will analyze the phase noise generated by the transmitter free running oscillator, and compare it between OFDM and DFT pre-coded systems. We will see that the SC system enjoys an advantage over OFDM due to its robustness to phase noise.

By carefully choosing frequency domain decision feedback equalizer (FD-DFE) for single carrier modulation which makes use of a data block transmission format similar to that of OFDM, the feed-forward part is implemented in the frequency domain, while the feedback signal is generated by time-domain filtering. [34] shows this FD-DFE yields a capacity very close to that of OFDM. When no channel adaptation is considered, FD-DFE performs closely to OFDM for the same averaged frame error rate in a coded transmission. Considering implementation complexity, FD-DFE and OFDM could have a similar computational complexity in signal processing.

Chapter 3 Time-varying Phase Shift Analysis

3.1 Phase Noise due to Free Running Local Oscillator

In a wireless communication system, oscillators are used for frequency translation of information signals and for channel selection. Oscillators are also used in baseband wired digital electronic systems which require the same reference clocks for both transmitting and receiving nodes. Noise is the major concern for oscillators, since even small amounts of noise will result in changes to the frequency spectrum and timing properties. The oscillator noise is often manifested as a slow time-varying frequency offset and time-varying phase and amplitude. The output of the free running oscillator is given by

$$V_{out}(t) = A(t) \cdot e^{j[\omega_0(t) + \phi(t)]} \quad (7)$$

where $A(t)$, $\omega_0(t)$ and $\phi(t)$ are time-varying amplitude, frequency and phase respectively, and they are functions of time. Some form of amplitude limitation mechanism is essential for stable oscillator action, and so we can assume that $A(t)$ stays constant during the period of oscillation[1]. Also for certain applications, there is a frequency tracking loop implemented on the receiver side, and the frequency offset is a small fraction of the transmitted signal's bandwidth. In that case we can also assume that the frequency offset stays constant during the one communication period (between frequency tracking updates). So the time varying phase can be rewritten as (8)[15].

$$\varphi(t) = \theta_0 + 2\pi \Delta_f t + \alpha(t) \quad (8)$$

Where θ_0 Represents a fixed phase shift, Δ_f a fixed frequency offset, and $\alpha(t)$ a random process that is manifestation of phase noise.

Reference[2] characterizes the internal structure and common features of different oscillators and also the nature of phase noise stochastic process $\alpha(t)$. In the literature, phase noise is either characterized by the power spectral density (PSD) found by measurement [35] or more conveniently for purpose of analysis, the time-varying phase noise process $\alpha(t)$ can be modeled as a Brownian motion or Wiener process [41] resulting in a Lorentzian power density spectrum where the added phase to the n -th symbol is related to the previous one [42] in the time domain as

$$\alpha(n) = \alpha(n-1) + \varepsilon_n \quad (9)$$

where ε_n is a Gaussian distribution random variable with zero mean and variance of $\sigma^2 = 4\pi\beta T$. We assume that $\alpha(0) = 0$. Here, β is the 3dB frequency of the Lorentzian PSD function (i.e. half the bandwidth of the phase noise process), and T is the data symbol period. Thus the phase added to the n -th symbol is a Gaussian with zero mean and a variance increasing linearly with time. Since it is a Wiener process, the non-overlapping time incremental phases are independent.

Note that, the time-domain multiplication of the amplified signal with the time-varying

noisy carrier is equivalent to convolving the amplified signal's spectrum with the phase noise spectrum in the frequency domain. As a consequence, the wider the $3dB$ bandwidth of the phase noise spectrum compared to the carrier spacing, i.e., the larger Δ_f , the higher the signal energy of the single sub-channel spilled into other sub-channels, resulting in ICI. This effect will be mathematically described in next section.

3.2 Phase Noise Modeling

Under SISO, there is only one transmitting source. For simplicity, we can assume that the phase noise is generated by the free running oscillators, the receiver has implemented frequency tracking, and it works perfectly to compensate Δ_f in (8). Furthermore, we can also assume that the phase noise mainly comes from the transmitter's free running oscillator. This assumption is realistic for the uplink due to the cost factor regarding the user terminal.

The transmitted signal waveform samples, from equation (3) in the absence of frequency offset and phase noise, can be written as

$$x(nt_s) = \sum_{l=0}^{M-1} s_l g((n-l\frac{N}{M})t_s) \quad \text{for } n=0,1,\dots,N-1 \quad (10)$$

where t_s is the sampling time interval. Note that the data symbol interval $T = (\frac{N}{M})t_s$.

Due to frequency offset Δ_f and phase noise $\alpha(t)$, $x(nt_s)$ is multiplied by the factor $e^{j(2\pi\Delta_f nt_s + \alpha(nt_s))}$ assuming $\theta_0=0$. After some rearrangement, we can express the transmitted signal, including transmitter oscillator phase noise and frequency offset as

$$x(nt_s) = \sum_{l=0}^{M-1} s_l e^{j(2\pi\Delta_f l \frac{N}{M} t_s + \alpha(l \frac{N}{M} t_s))} g'((n-l \frac{N}{M}) t_s) e^{j(\alpha(nt_s) - \alpha(l \frac{N}{M} t_s))} \quad (11)$$

for $n=0,1,\dots,N-1$

where $g'(nt_s) = g(nt_s) e^{j2\pi\Delta_f nt_s}$.

Since the bandwidth of the phase noise process is much less than the bandwidth of the transmitted signal, we can assume that

$\alpha(nt_s) - \alpha(l \frac{N}{M} t_s) \approx 0$ for $n=0,1,\dots,N-1$ and $l=0,1,\dots,M-1$, and therefore the

factor $e^{j(\alpha(nt_s) - \alpha(l \frac{N}{M} t_s))} \approx 1$. Thus the sampled transmitted signal waveform, including the effects of phase noise and frequency offset, can be modeled as

$$x(nt_s) = \sum_{l=0}^{M-1} s_l e^{j(2\pi\Delta_f l \frac{N}{M} t_s + \alpha(l \frac{N}{M} t_s))} g'((n-l \frac{N}{M}) t_s) \quad \text{for } n=0,1,\dots,N-1 \quad (12)$$

where $g'(nt_s)$ is defined above. Note that the sampled baseband response $g(nt_s)$ results solely from the rectangular frequency domain windowing in generating the DFT-precoded OFDM signal. After the signal passes through a channel with sampled impulse response $h(nt_s)$, the resulting sampled waveform at the channel output, exclusive of

additive noise is

$$y(nt_s) = \sum_{l=0}^{M-1} c_l h'((n-l\frac{N}{M})t_s) \text{ for } n=0,1,\dots,N-1 \quad (13)$$

where $c_l = s_l e^{j(2\pi\Delta_f l \frac{N}{M} t_s + \alpha(l \frac{N}{M} t_s))}$ and $h'(nt_s) = h(nt_s) e^{j2\pi\Delta_f nt_s}$. Note that the frequency offset and phase noise are now modeled as factors that affect the received data symbols

(transforming the $\{s_l\}$ into $\{c_l\}$). Note again that $l \frac{N}{M} t_s = lT$, where T is the serial data symbol interval. We can see that each received symbol phase is rotated by the

$$\varphi(l) = 2\pi\Delta_f l \frac{N}{M} t_s + \alpha(l \frac{N}{M} t_s). \text{ In other words, each SC data symbol gets modulated}$$

by a slow changing phase factor resulting as a phase rotation.

For an OFDM system if we take the DFT operation of the output from (11) and rearrange it into three terms we can see the equivalent detected data symbol at the receiver after the DFT is given as

$$Y(k) = S_k I(0) + \sum_{\substack{n=0 \\ n \neq k}}^{M-1} S_k H_k I(k-n) \quad (14)$$

where

$$\begin{aligned}
I(i) &= \frac{1}{N} \sum_{n=0}^{M-1} e^{-j2\pi n i / M} e^{j\phi(n)} \\
I(0) &\approx e^{j\phi(0)}
\end{aligned} \tag{15}$$

Comparing (13) (14) we can see the received symbols in OFDM system not only have a common phase error $I(0)$ (CPE) as for SC, but also the second term in the equation (14) shows the impairment from ICI which is a noise-like error, dependent on $M-1$ other data symbols. We can see this clearly in Figure 4 which is generated under AWGN channel with $E_b/N_o = 20(\text{dB})$ and phase noise impairment $\gamma^2 = 0.002$ which is about 6.366KHz for phase noise bandwidth, that SC data symbols mostly have only phase rotation due to $\phi(l)$, while OFDM suffers not only phase rotation, but also noise-like ICI.

Figure 4: SC vs OFDM received Constellations for QPSK with Same Phase Noise Variance and Same Data Rate.

The important conclusion we draw from above is: For a SC system, the phase noise due to the transmitter free running oscillator could be approximated as a time domain phase modulation given the nature of the slow varying phase noise (the bandwidth of the time varying phase process is relatively small compared with that of the communication channel bandwidth) .

We will propose an iterative joint FDE coupled with a time domain (TD) phase tracking

method in later chapters. Since we take the joint FDE approach of simultaneously detection and phase tracking, we can extend the conclusion we reached in the single transmitter and receiver antenna case to the SDMA scenario. So we still can do phase noise estimation in the time domain after joint channel equalization and before data decoding. Next we will look at the transceiver structure which is described in following chapter.

Chapter 4 Channel Model and Transceiver Structure

In this Chapter we will give the channel model and transceiver structure under SDMA, more specifically the uplink of the cellular communication system. We assume that the system is confined in one macro cell in which Z_t independent active MS using a single transmitting antennas to communication with one BS which is equipped with Z_r receiving antennas. The BS simultaneously receives, detects and decodes the Z_t data streams. Therefore inter-cell interference is not considered here. That could be addressed as a straight forward extension in the future work. First some notation has to be given in Table 1:

Table 1: Notation Table of SDMA Transceiver Model

$[]^T$	$[]^T$ Denotes the transpose
Z_t	The number of active users which are sharing the same uplink channel.
Z_r	The number of antenna in base station receiver antenna array.
N	Transmitted data block length.
M	FFT block length(number of samples per block, with $N > M$).
$d_m(k)$	m^{th} Data symbol for k^{th} user where $m=0,1,2,\dots,M-1$, $k=0,1,2,\dots,Z_t$.for zero padding, $d_m(k)=0$, when $N > m > M$
$d(k)=[d_0(k), d_1(k), \dots, d_{M-1}(k)]$	M -dimensional column vector, it's the representing M data symbols of k^{th} user, where $k=1,2,\dots,Z_t$

$\mathbf{d}_m = [d_m(1), d_m(2), \dots, d_m(Z_t)]^T$	Z_t -dimensional column vector, is the representation of Z_t user data sources at m^{th} symbol, where $m=0,1,2,\dots,M-1$
$\mathbf{D}(k) = [D_0(k), D_1(k), \dots, D_{M-1}(k)]$	FFT of $d_k, k=1,2,\dots,Z_t$, is the representation of k^{th} user data sources in frequency where $k=1,2,\dots,Z_t$
$\mathbf{D}_f = [D_f(1), D_f(2), \dots, D_f(Z_t)]$	Column wise FFT of $\mathbf{d}_m, m=0,1,2,\dots,M-1$, it's the representation of Z_t user data sources at f^{th} frequency, where $f=0,1,2,\dots,M-1$.
$D_f(k)$	k^{th} element of vector $\mathbf{D}_f$, representing k^{th} user's data where $f=0,1,2,\dots,M-1$, $k=1,2,\dots,Z_t$, can also be expressed as $D_f(k) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} d_m(k) e^{-\frac{j2\pi fm}{M}}$
$c_l(k) = e^{j\varphi_l(k)}$	Random phase error at symbol l of user k , where $k=1,2,\dots,Z_t$, $l=0,1,2,\dots,M-1$, $\varphi_l(k) = 2\pi \Delta_f(k)l + \alpha_l(k)$.
$C_f(k)$	FFT of $c_l(k) = e^{j\varphi_l(k)}$ over $l=0,1,2,\dots,M-1$. $C_f(k) = \sum_{l=0}^{M-1} e^{j\varphi_l(k) - \frac{j2\pi lf}{M}}, \quad k=1,2,\dots,Z_t$
$\mathbf{c}_l = [c_l(1), c_l(2), \dots, c_l(Z_t)]^T$	Z_t -dimensional column vector, it represents Z_t independent random phase source at symbol l where $l=0,1,2,\dots,M-1$.
$g_l(k)$	User k baseband output of transmitter at symbol l , where $k=1,2,\dots,Z_t$, $l=0,1,2,\dots,M-1$.

$G_f(k)$	DFT of $g_l(k)$,where $k=1,2,\dots,Z_t$, $f=0,1,2,\dots,M-1$.
$\mathbf{H}_f$	Z_r by Z_t matrix whose n^{th} column is the FFT of the vector channel impulse response from the n^{th} transmitting source where $f=0,1,2,\dots,M-1$. $\mathbf{H}_f = \begin{bmatrix} H_{11}(f) & H_{12}(f) & \dots & H_{1Z_t}(f) \\ H_{21}(f) & H_{22}(f) & \dots & H_{2Z_t}(f) \\ \vdots & \vdots & \ddots & \vdots \\ H_{Z_r1}(f) & H_{Z_r2}(f) & \dots & H_{Z_rZ_t}(f) \end{bmatrix}$
$\mathbf{H}_f(k)$	k^{th} Column of $\mathbf{H}_f$, $f=0,1,2,\dots,M-1$. It represents channel impulse response from the k^{th} transmitting sources.
$H_f(k, j)$	$\langle k, j \rangle^{th}$ element of $\mathbf{H}_f$. It represents channel impulse response between user k and receiver antenna j where $k=1,2,\dots,Z_t$, $j=1,2,\dots,Z_r$.
$\mathbf{N}_f$	Z_r By 1 vector representing noise after FFT at Z_r receiving antenna outputs, $f=0,1,2,\dots,M-1$
$N_f(j)$	Element of vector $\mathbf{N}_f$ representing FFT outputs of noise source of antenna j , $j=1,2,\dots,Z_r$
$\mathbf{R}_f$	Z_r by 1 vector representing FFT of the Z_r received antenna output sample blocks(one sample per data symbol) $f=0,1,2,\dots,M-1$
$R_f(j)$	Element of vector $\mathbf{R}_f$ representing FFT outputs

	of received data stream from antenna j , $j=1,2,\dots,Z_r$
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4.1 SDMA Transceiver Models

Let us now introduce the transceiver model under SDMA upon which we base the following derivations of the proposed joint FD equalization and phase synchronization. We consider an uplink discrete complex baseband transceiver model for multiple MSs and the BS equipped with multiple receiving antennas.

The received signal from each antenna is sampled at N/M times the symbol rate, processed in blocks of N samples by N -point DFT, and the M frequency domain samples corresponding to the original M -point DFT of the data symbols, are selected. It is assumed that the channel is stationary over one decoding block of data for an unsynchronized system as well as for a synchronized system. In other words we are assuming a multi-path fading channel which is approximately time invariant within the whole data burst. We also assume that for an unsynchronized system, each data burst is precoded with channel friendly sync-symbols for synchronization and channel estimation purposes and for synchronized system, training symbol or carriers are inserted for each data frame, so synchronization and channel estimation can be done before hand. So we can treat the receiver as part of the synchronized system with known estimated channel responses between each transceiver. The SDMA transceiver structure is shown in Figure 5,

Figure 5: SDMA Transceiver Model, with Independent Phase Noise Sources, With $Z_r > Z_t$. CP Represents CP Appending Operation, CP* Represents CP Removal Operation.

The cyclic prefix(CP) is assumed to be longer than the worst channel delays between any pair of transmitting and receiving antennas. Under this condition, linear convolution between channel impulse response and transmitted signals becomes circular convolution which is equivalent to frequency domain multiplication of the two . Without considering phase noise from all the active users, the receiving signal can be expressed as in (16).

$$R_f = H_f D_f + N_f = \sum_{k=1}^{Z_t} H_f(k) D_f(k) + N_f \quad (16)$$

where $f=0,1,2,\dots,M-1$.

The expression of each individual receiving antenna data stream can be written as:

$$R_f(j)=R_f(j)=\sum_{k=1}^{Z_r} H_f(k, j)D_f(k)+N_f(j) \quad (17)$$

where $j=1,2,\dots,Z_r$, $f=0,1,2,\dots,M-1$.

4.2 Channel Model and Parameters

The channel model considered in this thesis is a typical urban macro-cell in a wide area fading channel (UWAF). Buildings height and density in typical urban macro-cell are mostly homogeneous with either line of sight (LOS) or no line of sight (NLOS) between any pairs of MS and BS antenna. In this highly clustered environment with NLOS, the power envelope will show as a Rayleigh distribution. So, in this thesis, the signal fading characteristics will be assumed to have a Rayleigh distribution. The channel delay profile parameters are listed in Table 2. Each channel impulse response sample is an independent complex Gaussian random variable with zero mean and with variance given by the delay profile power at the corresponding delay.

Table 2: Urban Macro-cell:NLOS Clustered Delay Line Model [31]

Cluster #	Delay[ns]	Power[dB]
1	0	-6.4
2	60	-3.4
3	75	-2.0
4	145,150,155	-3.0,-5.2,-7.0
5	150	-1.9
6	190	-3.4
7	220,225,230	-3.4,-5.6,-7.4
8	335	-4.6
9	370	-7.8
10	430	-7.8
11	510	-9.3
12	685	-12.0
13	725	-8.5
14	735	-13.2
15	800	-11.2
16	960	-20.8
17	1020	-14.5
18	1100	-11.7
19	1210	-17.2
20	1845	-16.7

4.3 The Effects of Multiple Phase Noise

As the model indicates, the random phase noise due to the free running oscillator embedded in each mobile user transmitter is modeled as a multiplication factor in the time domain. In the frequency domain the phase factor will be shown in the form of a convolution which causes inter carrier interference(ICI) as well as common phase error(CPE). To reflect the multiple independent phase noise sources effects, the expression of equation (17) is subsequently extended to SDMA as

$$R_f(j) = \sum_{k=1}^{Z_t} \sum_{l=0}^{M-1} H_l(k, j) D_l(k) C_{f-l}(k) + N_f(j) \quad (18)$$

where $j=1,2,\dots,Z_r$, $f=0,1,2,\dots,M-1$.

$$R_f(j) = \sum_{k=1}^{Z_t} H_l(k, j) D_l(k) C_0(k) + \sum_{k=1}^{Z_t} \sum_{l=0, l \neq f}^{M-1} H_l(k, j) D_l(k) C_{f-l}(k) + N_f(j) \quad (19)$$

where $j=1,2,\dots,Z_r$, $f=0,1,2,\dots,M-1$.

In equation (18), the $C_0(k)$ represents the common phase error(CPE) from each user, and the second term represents the ICI which We can see its the natural extension of equation (3) in chapter 3 for SDMA.

Chapter 5 Joint SDMA FDE and TD PHN Estimation

In this chapter, we propose an iterative block based FDE with recursive second order PLL for a SDMA system with Z_t simultaneous independent transmitters and with Z_r receiving antenna elements. The equalization process and random phase estimation on the current data block are done in an iterative way with feedback generated from soft decision data symbol estimation. In each iteration, the FD forward filter coefficients and FD backward coefficients are updated based on the minimum mean squared error (MMSE) criterion. The outputs of equalization for each transmitter data stream is coupled with a second order TD PLL and for each iteration the phase noise is estimated using the maximum *a posteriori* probability (MAP) criterion based on the estimated data symbols of the current iteration.

5.1 IBFDE-RSPLL Structure and Notation table

The proposed iterative FDE and recursive second order PLL(RSPLL) algorithm is based on parallel interference cancellation(PIC). The detection consists of Z_t parallel detection and equalization data paths, where symbols of all streams at a given time instant are detected simultaneously by linear processing of the received signal and partial cancellation of interference from the other streams and residual ISI using previously detected data symbols. The receiver system consists Z_t simultaneous parallel data stream equalization and random phase estimation and compensation paths. First, We give

the notation used throughout this chapter in Table 3.

Table 3: Notation Table of IBFDE-RSPLL

M	FFT block length
$E^c(a)$	Conditional expectation of a , given equalization outputs from previous iteration.
$[A]^*$	Complex conjugate transpose of A .
$\Re(a), \Im(a)$	Real, Imaginary part of a .
Z^{-1}	Time delay
$W_f^{(i)}(k)$	Frequency domain Z_r by 1 vector which represents the k^{th} transmitter's forward filter coefficients vector at i^{th} iteration. $k=1,2,\dots,Z_t$, $f=0,1,2,\dots,M-1$.
$Q_f^{(i)}(k)$	Frequency domain , backward filter coefficient for user k at i^{th} iteration. $k=1,2,\dots,Z_t$, at frequency $f=0,1,2,\dots,M-1$.
$S_f^{(i)}(k)$	Scalar equalizer output at i^{th} iteration. $k=1,2,\dots,Z_t$, $f=0,1,2,\dots,M-1$.
$s_l^{(i)}(k)$	IDFT of scalar equalizer output at i^{th} iteration. $k=1,2,\dots,Z_t$, at time instant $l=0,1,2,\dots,M-1$.
$\mathbf{s}^{(i)}(k)=[s_0^{(i)}(k), s_1^{(i)}(k), \dots, s_{M-1}^{(i)}(k)]^T$	M by 1 column vector representing IDFT of equalizer output at i^{th} iteration range from 0 to $M-1$ of user k . $k=1,2,\dots,Z_t$.

$u_l^{(i)}(k)$	$u_l^{(i)}(k) = s_l^{(i)}(k) e^{-j\phi_l^{(i)}(k)}$ scalar equalizer output at i^{th} iteration with phase correction of user k at time instant $l=0,1,2,\dots,M-1$ and $k=1,2,\dots,Z_t$.
$d_l^{(i)}(k)$	l^{th} data symbol of user k . $l=0,1,2,\dots,M-1$, $k=1,2,\dots,Z_t$
$\mathbf{d}(k) = [d_0(k), d_1(k), \dots, d_{M-1}(k)]^T$	M -dimensional column vector, it's the representation of M data symbols of k^{th} user, where $k=0,1,2,\dots,Z_t$
$\bar{d}_l^{(i)}(k)$	Conditional expectation of $d_l(k)$ from equalizer given $u_l^{(i)}(k)$ at i^{th} iteration of user k at time instant $l=0,1,2,\dots,M-1$ and $k=1,2,\dots,Z_t$. $\bar{d}_l^{(i)}(k) = E^c(d_l(k))$
$\hat{c}_l^{(i)}(k)$	Estimated phase noise factor at i^{th} iteration of user k at time instant $l=0,1,2,\dots,M-1$ and $k=1,2,\dots,Z_t$. $\hat{c}_l^{(i)}(k) = e^{j\phi_l^{(i)}(k)}$
$\bar{G}_f^{(i)}(k)$	FFT of $\bar{d}_l^{(i)}(k)$ multiplied by estimated phase noise factor at i^{th} iteration. $\bar{G}_f^{(i)}(k) = \sum_{m=0}^{M-1} \bar{d}_m^{(i)}(k) \hat{c}_m^{(i)}(k) e^{-\frac{j2\pi fm}{M}}$
$A_f^{(i)}(k)$	$A_f^{(i)}(k) = G_f(k) - \bar{G}_f^{(i)}(k)$
$\mu_f^{(i)}(k)$	Scalar Lagrange multiplier used for transmitter k at i^{th} iteration. $k=1,2,\dots,Z_t$, at frequency $f=0,1,2,\dots,M-1$.

$X_f^{(i)}(k)$	$X_f^{(i)}(k) = R_f - \sum_{n=1}^{Z_i} H_f(n) \bar{G}_f^{(i-1)}(n) \text{ for}$ <p>transmitter k at i^{th} iteration. $k=1,2,\dots,Z_i$, at frequency $f=0,1,2,\dots,M-1$.</p>
σ^2	Channel noise variance per complex sample.
$\sigma^{(i)2}(k)$	<p>Conditional minimum mean squared error at i^{th} iteration for user k.</p> $\sigma^{(i)2}(k) = \frac{1}{M} \sum_{f=0}^{M-1} E^c \left(\left S_f^{(i-1)}(k) - G_f(k) \right ^2 \right)$
$P_f^{(i)}(k)$	$P_f^{(i)}(k) = E^c \left(\left G_f^{(i)}(k) \right ^2 \right)$, for transmitter k at i^{th} iteration. $k=1,2,\dots,Z_i$, at frequency $f=0,1,2,\dots,M-1$
$\mathcal{Q}_f^{(i)}(k)$	$\mathcal{Q}_f^{(i)}(k) = E^c \left(\left A_f^{(i)}(k) \right ^2 \right)$, for transmitter k at i^{th} iteration. $k=1,2,\dots,Z_i$, at frequency $f=0,1,2,\dots,M-1$.
$\phi_l^i(k)$	<p>PHN estimation based on RS-PLL at l^{th} data symbol of user k at i^{th} iteration.</p> <p>$k=1,2,\dots,Z_i$, at time instant $l=0,1,2,\dots,M-1$.</p>
$\boldsymbol{\phi}^i(k) = [\phi_0^i(k), \phi_1^i(k), \dots, \phi_{M-1}^i(k)]^T$	<p>M by 1 column vector representing PHN estimation for data symbol from 0 to $M-1$ of user k at i^{th} iteration. $k=1,2,\dots,Z_i$.</p>
$\lambda_l^i(k)$	<p>LLR of $d_l(k)$ at i^{th} iteration of user k.</p> $\lambda_l^i(k) = \ln \left[\frac{p[d_l(k)=+1 u_l^{(i-1)}(k)]}{p[d_l(k)=-1 u_l^{(i-1)}(k)]} \right]$, at time

	instant $l=0,1,2,\dots,M-1$, for user $k=1,2,\dots,Z_t$.
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5.2 SDMA-PIC-MMSE-IBFDE

The proposed SCIBFDE algorithm equalizer outputs for user k can be written as,

$$S_f^{(i)}(k) = \mathbf{W}_f^{(i)}(k)^T \left[\mathbf{R}_f - \sum_{n=1, n \neq k}^{Z_f} \mathbf{H}_f(n) \bar{G}_f^{(i-1)}(n) \right] - Q_f^{(i)}(k)^T \bar{G}_f^{(i-1)}(k) \quad (20)$$

The Minimum Mean Square Error (MMSE) criterion is to minimize the mean square of the absolute value of error at i^{th} iteration which is represented as $|S_f^{(i)}(k) - G_f(k)|$, conditioned on the equalizer outputs at the previous iteration, represented as $S_f^{(i-1)}(k)$. Here we also introduce another constraint: at the equalizer output, the coefficient multiplying the k^{th} user's data symbols is 1 for all k . It is expressed in equation (21).

$$\frac{1}{M} \sum_{f=0}^{M-1} \mathbf{W}_f^{(i)}(k)^T \mathbf{H}_f(k) = 1 \quad (21)$$

Now the MMSE value of Equalizer coefficients is given by the following expression (22) which includes a Lagrange multiplier term for the constraint of (21).

$$[\mathbf{W}_f^{(i)}(k), Q_f^{(i)}(k)] = \underset{\mathbf{W}_f^{(i)}(k), Q_f^{(i)}(k)}{\text{argmin}} \left[\frac{1}{M} \sum_{f=0}^{M-1} E^c \left(|S_f^{(i)}(k) - G_f(k)|^2 \right) + \frac{2}{M} \mu_f^{(i)}(k) \sum_{f=0}^{M-1} \mathbf{W}_f^{(i)}(k)^T \mathbf{H}_f(k) \right] \quad (22)$$

Equalization above gives a pair of equations, (23) and (24) which satisfy the MMSE criterion.

$$\left[E^c \left(X_f^{(i-1)}(k) X_f^{(i-1)}(k)^* \right) \right] \mathbf{W}_f^{(i)}(k) = E^c \left(X_f^{(i-1)}(k) G_f(k)^* \right) + \mathbf{H}_f(k) \left[\bar{G}_f^{(i-1)}(k) \right]^2 Q_f^{(i)}(k) \quad (23)$$

$$Q_f^{(i)}(k) = H_f(k)^* W_f^{(i)}(k) - 1 \quad (24)$$

Now taking (24) into (20) we get

$$S_f^{(i)}(k) = W_f^{(i)}(k)^T X_f^{(i-1)}(k) + \bar{G}_f^{(i-1)}(k) \quad (25)$$

Then, taking (25) into (22) and invoking constraint (21), we get the solution for

$W_f^{(i)}(k)$ which is

$$W_f^{(i)}(k) = \left(Q_f^{(i-1)}(k) - \mu_f^{(i)}(k) \right) \left[\sum_{n=1}^{Z_r} H_f(n) H_f(n)^* Q_f^{(i-1)}(n) + \sigma^2 I_{Z_r} \right]^{-1} H_f(k) \quad (26)$$

where

$$\begin{aligned} Q_f^{(i)}(k) &= E^c \left(\left| A_f^{(i)}(k) \right|^2 \right) \\ &= P_f^{(i)}(k) - \frac{1}{M} \sum_{l=0}^{M-1} \left(\left| \bar{d}_l^{(i)}(k) \right|^2 \right) \end{aligned} \quad (27)$$

where $A_f^{(i)}(k) = G_f(k) - \bar{G}_f^{(i)}(k)$.

Then take (21) into (26) to get the solution for $\mu_f^{(i)}(k)$ as:

$$\mu_f^{(i)}(k) = \Omega_f^{(i-1)}(k) - \frac{1}{\frac{1}{M} \sum_{f=0}^{M-1} \mathbf{H}_f(k)^* \left[\sum_{n=1}^{Z_i} \mathbf{H}_f(n) \mathbf{H}_f(n)^* \Omega_f^{(i-1)}(k) + \sigma^2 I_{Z_i} \right]^{-1} \mathbf{H}_f(k)} \quad (28)$$

And by the definition of $\sigma^{(i)2}(k)$, from (28) we also can prove that

$$\mu_f^{(i)}(k) = -\sigma^{(i)2}(k) = -\frac{1}{M} \sum_{f=0}^{M-1} E^c \left(\left| S_f^{(i-1)}(k) - G_f(k) \right|^2 \right) \quad (29)$$

Next, take (26) back to (24) to get the solution for $Q_f^{(i)}(k)$ as:

$$Q_f^{(i)}(k) = \mathbf{H}_f(k)^* \left(\Omega_f^{(i-1)}(k) - \mu_f^{(i)}(k) \right) \left[\sum_{n=1}^{Z_i} \mathbf{H}_f(n) \mathbf{H}_f(n)^* \Omega_f^{(i-1)}(k) + \sigma^2 I_{Z_i} \right]^{-1} \mathbf{H}_f(k) - 1 \quad (30)$$

Figure 6: IBFDE Convergence Indicator $\Omega(K)$ with 16QAM SDMA(4x4) with 5 Iterations

So, up to this point, equations (28), (26), and (30) give the solution for (22) under the MMSE criterion. Finally the calculation of $\bar{d}_i^{(i)}(k) = E^c(d_i(k))$ and $P = E^c(|G_f^{(i)}(k)|^2)$ for different modulation types are given in section 5.6 .

If we look at the definition of $\Omega_f^{(i)}(k)$ in Table 3, we can see that $\Omega_f^{(i)}(k)$ is the mean square error(MSE), a good indicator of the convergence in each iteration. The smaller $\Omega_f^{(i)}(k)$ gets in each iteration, the closer the equalizer output to the real expected value.

In Figure 6 and Figure 7 we show how the values of $\Omega_f^{(i)}(k)$ vary with each iteration for 4x4 and 8x8 16QAM SDMA systems for the independent urban wide area fading

channel model with $E_b/N_o=10\text{ (dB)}$ to demonstrate that the IBFDE improves the performance of the equalizer outputs in each iteration. We can also see there is negligible further improvement after several iterations.

Figure 7: IBFDE Convergence Indicator $\Omega(K)$ with 16QAM SDMA(8x8) with 11 Iterations.

5.3 Second Order PLL using Iterative MAP Estimation

The obvious advantage of the PIC approach is the assumption that knowledge of the current received data symbols from all the active users, makes it possible to reduce or remove interference to each user's data stream so the RSPLL can be applied to each transmitter stream. The time domain random phase estimation we have studied for the SISO scenario and the conclusion we have reached for SISO can be used here for each data stream. It makes the time domain phase noise estimation for each independent noise source possible.

First let's recall the random phase definition in chapter 4 where the random phase caused by the free running user terminal oscillator in baseband model is given as:

$$\varphi_l = \theta_0 + 2\pi \Delta_f l T + \alpha_l \quad (31)$$

where l is the data symbol index. The nature of phase noise stochastic process of α_l is modeled as a Brownian motion or Wiener process [3] where the added phase to the l^{th} symbol is related to the previous one in digital domain as

$$\alpha_l = \alpha_{l-1} + \varepsilon_l \quad (32)$$

where ε_l is a Gaussian distribution random variable with zero mean and variance of $\gamma^2 = 4\pi\beta T$ and assume that $\alpha(0) = 0$. Here, β is the 3dB frequency (i.e. half the

bandwidth) of the Lorentzian power spectrum density (PSD) function, and T is data symbol period. Thus the phase added to the n -th symbol is a Gaussian with zero mean and a variance increasing linearly with time. Combining the two equations above we can write the PHN at symbol l for transmitter k as

$$\varphi_l(k) = \varphi_{l-1}(k) + 2\pi \Delta_f(k)T + \varepsilon_l(k) \quad (33)$$

For the MAP estimation, the *posterior* density function of PHN (with the observation of equalizer outputs) for user k at the i^{th} iteration which is written as

$$p[\boldsymbol{\varphi}^i(k) | \mathbf{s}^{(i)}(k)] = \frac{\sum_{\mathbf{d}(k)} p[\mathbf{s}^{(i)}(k) | \boldsymbol{\varphi}^i(k), \mathbf{d}(k)] p[\mathbf{d}(k)] p[\boldsymbol{\varphi}^i(k)]}{p[\mathbf{s}^{(i)}(k)]} \quad (34)$$

should be maximized. In equation (34) $\mathbf{s}^{(i)}(k)$ denotes the k^{th} user time domain equalizer outputs for current data symbol block at i^{th} iteration.

Since $\varepsilon_l(k)$ is Gaussian with $\sim N(0, \gamma^2)$, $p[\boldsymbol{\varphi}^i(k)]$ is written as:

$$p[\boldsymbol{\varphi}^i(k)] = \frac{\exp\left[-\frac{1}{2\gamma^2} \sum_{l=1}^{M-1} (\varphi_l^i(k) - \varphi_{l-1}^i(k) - 2\pi \Delta_f(k)T)^2\right]}{(2\pi\gamma^2)^{M/2}} \quad (35)$$

Based on the conclusion we have reached in previous chapter, which is that the phase

noise due to the transmitter free running oscillator could be approximated as a time domain phase modulation on the receiver equalized output given the nature of the slow varying PHN, the equalizer outputs in time domain can be portrayed as

$$\mathbf{s}^{(i)}(k) = \mathbf{d}(k) e^{j\phi^{(i)}(k)} + \mathbf{n}^{(i)}(k) \quad (36)$$

where $\mathbf{n}^{(i)}(k) \sim N(0, \sigma^{(i)2}(k))$. so $p[\mathbf{s}^{(i)}(k) | \phi^i(k), \mathbf{d}(k)]$ can be written as:

$$p[\mathbf{s}^{(i)}(k) | \phi^i(k), \mathbf{d}(k)] = \frac{1}{(2\pi\sigma^{(i)2}(k))^{M/2}} \times \exp\left[-\frac{1}{\sigma^{(i)2}(k)} \|\mathbf{s}^{(i)}(k) - \mathbf{d}(k) e^{j\phi^i(k)}\|^2\right] \quad (37)$$

Taking (35),(37) into (34), the posterior density function of PHN becomes:

$$\begin{aligned} p[\phi^i(k) | \mathbf{s}^{(i)}(k)] &= \frac{\exp\left[\frac{\|\mathbf{s}^{(i)}(k)\|^2 + \|\mathbf{d}(k)\|^2}{-\sigma^{(i)2}(k)}\right]}{(4\pi^2 \sigma^{(i)2}(k) \gamma^2)^{M/2}} \times \\ &\exp\left[-\frac{1}{2\gamma^2} \sum_{l=1}^{M-1} (\phi_l^i(k) - \phi_{l-1}^i(k) - 2\pi \Delta_f(k) T)^2\right] \times \\ &\sum_{\mathbf{d}(k)} p[\mathbf{d}(k)] \exp\left[\frac{2}{\sigma^{(i)2}(k)} \Re(\mathbf{s}^{(i)*}(k) \mathbf{d}(k) e^{j\phi^i(k)})\right] \end{aligned} \quad (38)$$

where $p[\mathbf{d}(k)]$ can be replaced by the *posterior* density function of $\mathbf{d}(k)$ which is defined in (39), from the previous iteration given the observation of the equalizer output with phase correction $u_l^{i-1}(k)$, where for BPSK, it is derived as:

$$p[d_i(k)=d | u_i^i(k)] = \frac{\exp[d \times \lambda_i^i(k)]}{1 + \exp[\lambda_i^i(k)]}, \quad d = \pm 1 \quad (39)$$

where λ represents the log likelihood ration(LLR) of MAP decoder output.

By using Bayes' theorem and with $u_i^{(i)}(k) = s_i^{(i)}(k) \exp(-j \varphi_i^{(i)}(k))$, we can rewrite

$\lambda_i^i(k)$ as:

$$\begin{aligned} \lambda_i^i(k) &= \ln \left[\frac{p[d_i(k)=+1 | u_i^{(i)}(k)]}{p[d_i(k)=-1 | u_i^{(i)}(k)]} \right] \\ &= \ln \left[\frac{p[u_i^i(k) | d_i(k)=+1] \times p[d_i(k)=+1]}{p[u_i^i(k) | d_i(k)=-1] \times p[d_i(k)=-1]} \right] \\ &= \ln \left[\frac{\frac{1}{\sqrt{2\pi\sigma^{(i)2}(k)}} \times \exp\left[-\frac{|1-u_i^{(i)}(k)|^2}{2\sigma^{(i)2}(k)}\right]}{\frac{1}{\sqrt{2\pi\sigma^{(i)2}(k)}} \times \exp\left[-\frac{|1+u_i^{(i)}(k)|^2}{2\sigma^{(i)2}(k)}\right]} \right] \\ &= \frac{|1+u_i^{(i)}(k)|^2}{2\sigma^{(i)2}(k)} - \frac{|1-u_i^{(i)}(k)|^2}{2\sigma^{(i)2}(k)} \\ &= \frac{2\Re(u_i^{(i)}(k))}{\sigma^{(i)2}(k)} \end{aligned} \quad (40)$$

Then substituting (39) (40) into (38), we have:

$$\begin{aligned}
p[\boldsymbol{\varphi}^i(k) | s^{(i)}(k)] = & \frac{\exp\left[\frac{|s^{(i)}(k)|^2 + |d(k)|^2}{-\sigma^{(i)2}(k)}\right]}{(4\pi^2 \sigma^{(i)2}(k) \gamma^2)^{M/2}} \times \\
& \exp\left[-\frac{1}{2\gamma^2} \sum_{l=1}^{M-1} (\varphi_l^i(k) - \varphi_{l-1}^i(k) - 2\pi \Delta_f(k) T)^2\right] \times \\
& \exp\left[\sum_{l=1}^M \ln \left(\frac{\cosh\left(\frac{\Re(u_l^{(i)}(k))}{\sigma^{(i)2}(k)} + \frac{2}{\sigma^{(i)2}(k)} \Re(s_l^{(i)*}(k) e^{j\varphi_l^i(k)})\right)}{\cosh\left(\frac{\Re(u_l^{(i)}(k))}{\sigma^{(i)2}(k)}\right)} \right)\right]
\end{aligned} \tag{41}$$

If $d(k)$ is selected from a BPSK or QPSK constellation types, the first term in (41) will keep constant so maximizing the *posterior* density function will be reduced to minimizing J , where for BPSK, J is derived as:

$$\begin{aligned}
J = & -\frac{1}{2\gamma^2} \sum_{l=1}^{M-1} (\varphi_l^i(k) - \varphi_{l-1}^i(k) - 2\pi \Delta_f(k) T)^2 + \\
& \sum_{l=1}^M \ln \left(\frac{\cosh\left(\frac{\Re(u_l^{(i)}(k))}{\sigma^{(i)2}(k)} + \frac{2}{\sigma^{(i)2}(k)} \Re(s_l^{(i)*}(k) e^{j\varphi_l^i(k)})\right)}{\cosh\left(\frac{\Re(u_l^{(i)}(k))}{\sigma^{(i)2}(k)}\right)} \right)
\end{aligned} \tag{42}$$

The minimizing criteria will lead to a set of $\varphi_l(k)$ which is formulated at the i^{th} iteration for BPSK as:

$$\begin{aligned} \phi_i^i(k) = & 2\phi_{i-1}^i(k) - \phi_{i-2}^i(k) - 2\frac{\gamma^2}{\sigma^{(i)2}(k)} \Im(s_{i-1}^{(i)*}(k) e^{j\phi_{i-1}^i(k)}) \times \\ & \tanh\left(\frac{\Re(u_{i-1}^{(i)}(k))}{\sigma^{(i)2}(k)} + \frac{2}{\sigma^{(i)2}(k)} \Re(s_{i-1}^{(i)*}(k) e^{j\phi_{i-1}^i(k)})\right) \end{aligned} \quad (43)$$

(43) Can be further written in a more general form as:

$$\begin{aligned} \phi_i^i(k) = & \phi_{i-1}^i(k) + \alpha_1(\phi_{i-1}^i(k) - \phi_{i-2}^i(k)) - \alpha_2 \Im(s_{i-1}^{(i)*}(k) e^{j\phi_{i-1}^i(k)}) \times \\ & \tanh\left(\frac{\Re(u_{i-1}^{(i)}(k))}{\sigma^{(i)2}(k)} + \frac{2}{\sigma^{(i)2}(k)} \Re(s_{i-1}^{(i)*}(k) e^{j\phi_{i-1}^i(k)})\right) \end{aligned} \quad (44)$$

where α_1 , α_2 are design coefficients which yield convergence speed with stability and which will depend on β . Equation (44) represents a second order phase locked loop, driven by the soft equalizer outputs .

Following the same deriving process for QPSK which $d(k) = \frac{1}{\sqrt{2}}((\pm 1) + j(\pm 1))$,

(40) becomes

$$\begin{aligned} \lambda_{i,r}^i(k) &= \frac{\sqrt{2} \Re(u_i^{(i)}(k))}{\sigma^{(i)2}(k)} \\ \lambda_{i,i}^i(k) &= \frac{\sqrt{2} \Im(u_i^{(i)}(k))}{\sigma^{(i)2}(k)} \end{aligned} \quad (45)$$

Where $\lambda_{i,r}^i(k)$ $\lambda_{i,i}^i(k)$ are LLR values for real and imaginary part of equalizer outputs respectively. Then the iterative phase estimation for QPSK becomes:

$$\begin{aligned}
\phi_i^j(k) &= \phi_{i-1}^j(k) + \alpha_1 (\phi_{i-1}^j(k) - \phi_{i-2}^j(k)) \\
&\quad - \alpha_2 \frac{\Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) + \pi/4)}) \sinh(\rho_1)}{\cosh(\rho_1) + \cosh(\rho_2)} \\
&\quad - \alpha_2 \frac{\Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) - \pi/4)}) \sinh(\rho_2)}{\cosh(\rho_1) + \cosh(\rho_2)}
\end{aligned} \tag{46}$$

where

$$\begin{aligned}
\rho_1 &= \frac{\Re(u_i^{(i)}(k))}{\sqrt{(2)}\sigma^{(i)2}(k)} + \frac{\Im(u_i^{(i)}(k))}{\sqrt{(2)}\sigma^{(i)2}(k)} + \frac{1}{\sigma^{(i)2}(k)} \Re\left(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) + \frac{\pi}{4})}\right) \\
\rho_2 &= \frac{\Re(u_i^{(i)}(k))}{\sqrt{(2)}\sigma^{(i)2}(k)} - \frac{\Im(u_i^{(i)}(k))}{\sqrt{(2)}\sigma^{(i)2}(k)} + \frac{1}{\sigma^{(i)2}(k)} \Re\left(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) - \frac{\pi}{4})}\right)
\end{aligned} \tag{47}$$

for 16QAM which $d(k) = \frac{1}{\sqrt{(10)}} ((\pm 1, \pm 3) + j(\pm 1, \pm 3))$, it becomes:

$$\begin{aligned}
\phi_i^j(k) &= \phi_{i-1}^j(k) + \alpha_1 (\phi_{i-1}^j(k) - \phi_{i-2}^j(k)) \\
&\quad - \frac{1}{\sqrt{(5)}} [\Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) + \frac{\pi}{4})}) \sinh(\rho_1) + \Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) - \frac{\pi}{4})}) \sinh(\rho_2)] \beta_1 \\
&\quad - \alpha_2 \frac{[\cosh(\rho_1) + \cosh(\rho_2)] \beta_1 + [\cosh(\rho_3) + \cosh(\rho_4) + \cosh(\rho_5) + \cosh(\rho_6)] \beta_2 + [\cosh(\rho_7) + \cosh(\rho_8)] \beta_3}{[\cosh(\rho_1) + \cosh(\rho_2)] \beta_1 + [\cosh(\rho_3) + \cosh(\rho_4) + \cosh(\rho_5) + \cosh(\rho_6)] \beta_2 + [\cosh(\rho_7) + \cosh(\rho_8)] \beta_3} \\
&\quad - \alpha_2 \frac{[\Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) + \arctan(3))}) \sinh(\rho_3) + \Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) - \arctan(3))}) \sinh(\rho_4)] \beta_2}{[\cosh(\rho_1) + \cosh(\rho_2)] \beta_1 + [\cosh(\rho_3) + \cosh(\rho_4) + \cosh(\rho_5) + \cosh(\rho_6)] \beta_2 + [\cosh(\rho_7) + \cosh(\rho_8)] \beta_3} \\
&\quad - \alpha_2 \frac{[\Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) + \arctan(\frac{1}{3}))}) \sinh(\rho_5) + \Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) - \arctan(\frac{1}{3}))}) \sinh(\rho_6)] \beta_2}{[\cosh(\rho_1) + \cosh(\rho_2)] \beta_1 + [\cosh(\rho_3) + \cosh(\rho_4) + \cosh(\rho_5) + \cosh(\rho_6)] \beta_2 + [\cosh(\rho_7) + \cosh(\rho_8)] \beta_3} \\
&\quad - \alpha_2 \frac{\frac{3}{\sqrt{(5)}} [\Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) + \frac{\pi}{4})}) \sinh(\rho_7) + \Im(s_{i-1}^{(i)*}(k) e^{j(\phi_{i-1}^j(k) - \frac{\pi}{4})}) \sinh(\rho_8)] \beta_3}{[\cosh(\rho_1) + \cosh(\rho_2)] \beta_1 + [\cosh(\rho_3) + \cosh(\rho_4) + \cosh(\rho_5) + \cosh(\rho_6)] \beta_2 + [\cosh(\rho_7) + \cosh(\rho_8)] \beta_3}
\end{aligned} \tag{48}$$

where

$$\beta_1 = e^{-\frac{|u_i^{(i)}(k)|^2 + \frac{1}{5}}{\sigma^{(i)2}(k)}}, \beta_2 = e^{-\frac{|u_i^{(i)}(k)|^2 + 1}{\sigma^{(i)2}(k)}}, \beta_3 = e^{-\frac{|u_i^{(i)}(k)|^2 + \frac{9}{5}}{\sigma^{(i)2}(k)}} \tag{49}$$

and

$$\begin{aligned}
\rho_1 &= \frac{\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{1}{\sqrt{(5)}\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) + \frac{\pi}{4})}\right) \\
\rho_2 &= \frac{\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} - \frac{\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{1}{\sqrt{(5)}\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) - \frac{\pi}{4})}\right) \\
\rho_3 &= \frac{\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{3\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{1}{\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) + \arctan(3))}\right) \\
\rho_4 &= \frac{\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} - \frac{3\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{1}{\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) - \arctan(3))}\right) \\
\rho_5 &= \frac{3\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{1}{\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) + \arctan(\frac{1}{3}))}\right) \\
\rho_6 &= \frac{3\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} - \frac{\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{1}{\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) - \arctan(\frac{1}{3}))}\right) \\
\rho_7 &= \frac{3\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{3\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{3}{\sqrt{(5)}\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) + \frac{\pi}{4})}\right) \\
\rho_8 &= \frac{3\Re(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} - \frac{3\Im(u_l^{(i)}(k))}{\sqrt{(10)}\sigma^{(i)2}(k)} + \frac{3}{\sqrt{(5)}\sigma^{(i)2}(k)} \Re\left(s_{l-1}^{(i)*}(k)e^{j(\phi_{l-1}^i(k) - \frac{\pi}{4})}\right)
\end{aligned} \tag{50}$$

We can further generalize (44) (46) (48) as a second order decision directed PLL with the general form of

$$\phi_l^i(k) = \phi_{l-1}^i(k) + \alpha_1(\phi_{l-1}^i(k) - \phi_{l-2}^i(k)) + \alpha_2\psi_{l-1} \tag{51}$$

where ψ_{l-1} is a correction term. α_1 and α_2 are chosen in such a way as to satisfied

the stability requirements [36].

In Figure 8 we show the RSPLL PHN tracking in an urban wide area fading(UWAF) SISO scenario, with $E_b/N_o=10(dB)$, $\alpha=[0.9,0.0027]$, and $\gamma^2=0.002$ (i.e. 6.4KHz in bandwidth). In Figure 9 under the same channel condition with $\alpha=[0.9,0.0027]$ and $\gamma^2=0.004$ (i.e. 12.7KHz in bandwidth) we can see the phase tracking in SDMA scenario with independent phase noise sources for each user. Comparing the common phase error(CPE) phase estimation method in which the common phase error is estimated based on the averaged metrics over each data blocks, the proposed RSPLL phase tracking method is data symbol based, that means, the phase estimation and compensation is calculated and applied on a symbol by symbol basis, thus generating much more accurate over-all phase noise estimation and better performance. We will see this in the simulation results in chapter 6.

The linearity range of the proposed RSPLL phase tracking method for different modulation types is presented in Figure 10. It shows that 16QAM has a linearity range between about $[-0.5, 0.5]$ and QPSK has a linearity range between about $[-1.4, 1.4]$. For BPSK, it has a fairly large linearity range beyond $[-2\pi, 2\pi]$. These measurements are taken under AWGN channel with $E_b/N_o=20(dB)$.

Figure 8: RSPLL SISO ,16QAM Time Varying Phase Tracking with PHN: $\gamma^2=0.002$.

Figure 9: RSPLL SDMA(4x4) ,QPSK Time Varying Phase Tracking with Independent PHNs: $\gamma_2=0.004$.

Figure 10: Linearity range (radian) of RSPLL Method for BPSK, QPSK, 16QAM

5.4 Algorithm Summation of the Proposed SCIBFDE-RSPLL

Here we summarize the proposed equalization and jointed phase estimation algorithm starting from initial condition at iteration $i=0$:

$$\Omega_f^{(-1)}(k)=1 \quad , \quad \varphi_i^{(0)}(k)=\varphi_i^{(-1)}(k)=0$$

$$\mu_f^{(0)}(k)=\Omega_f^{(-1)}(k)-\frac{1}{\frac{1}{M} \sum_{f=0}^{M-1} \mathbf{H}_f(k)^* \left[\sum_{n=1}^{Z_t} \mathbf{H}_f(n) \mathbf{H}_f(n)^* + \sigma^2 I_{Z_r} \right]^{-1} \mathbf{H}_f(k)}$$

$$\mathbf{W}_f^{(0)}(k)=\left(1-\mu_f^{(0)}(k)\right) \left[\sum_{n=1}^{Z_t} \mathbf{H}_f(n) \mathbf{H}_f(n)^* + \sigma^2 I_{Z_r} \right]^{-1} \mathbf{H}_f(k)$$

$$\mathbf{S}_f^{(0)}(k)=\mathbf{W}_f^{(0)}(k) \mathbf{R}_f \quad , \quad s_l^{(0)}(k)=IFFT(\mathbf{S}_f^{(0)}(k))$$

$$g_l^{(0)}(k)=s_l^{(0)}(k) e^{-j \hat{\varphi}_l^{-1}(k)} \quad , \quad \bar{d}_l^{(0)}(k)=E^c(d_l(k))$$

$$\sigma^{(0)2}(k)=-\mu_f^{(0)}(k)=\frac{1}{\frac{1}{M} \sum_{f=0}^{M-1} \mathbf{H}_f(k)^* \left[\sum_{n=1}^{Z_t} \mathbf{H}_f(n) \mathbf{H}_f(n)^* + \sigma^2 I_{Z_r} \right]^{-1} \mathbf{H}_f(k)} - \Omega_f^{(-1)}(k)$$

$$\bar{G}_f^{(0)}(k)=FFT(\bar{d}_m^{(0)}(k) \hat{c}_m^{(0)}(k))$$

at iteration $i > 0$:

$$\Omega_f^{(i-1)}(k) = P_f^{(i-1)}(k) - \frac{1}{M} \sum_{l=0}^{M-1} (|\bar{d}_l^{(i-1)}(k)|^2) \quad \text{for } k=1,2,\dots,Z_t$$

$$\mu_f^{(i)}(k) = -\sigma^{(i)2}(k) = \Omega_f^{(i-1)}(k) - \frac{1}{\frac{1}{M} \sum_{f=0}^{M-1} \mathbf{H}_f(k)^* \left[\sum_{n=1}^{Z_t} \mathbf{H}_f(n) \mathbf{H}_f(n)^* \Omega_f^{(i-1)}(n) + \sigma^2 I_{Z_t} \right]^{-1} \mathbf{H}_f(k)}$$

$$\mathbf{W}_f^{(i)}(k) = \left(\Omega_f^{(i-1)}(k) - \mu_f^{(i)}(k) \right) \left[\sum_{n=1}^{Z_t} \mathbf{H}_f(n) \mathbf{H}_f(n)^* \Omega_f^{(i-1)}(n) + \sigma^2 I_{Z_t} \right]^{-1} \mathbf{H}_f(k)$$

$$\mathbf{S}_f^{(i)}(k) = \mathbf{W}_f^{(i)}(k)^T \left(\mathbf{R}_f - \sum_{n=1}^{Z_t} \mathbf{H}_f(n) \bar{\mathbf{G}}_f^{(i-1)}(n) \right) + \bar{\mathbf{G}}_f^{(i-1)}(k) \quad , \quad s_l^{(i)}(k) = \text{IFFT}(\mathbf{S}_f^{(i)}(k))$$

$$\phi_l^{i-1}(k) = \phi_{l-1}^{i-1}(k) + \alpha_1 (\phi_{l-1}^{i-1}(k) - \phi_{l-2}^{i-1}(k)) + \alpha_2 \psi_{l-1}$$

$$g_l^{(i)}(k) = s_l^{(i)}(k) e^{-j\phi_l^{i-1}(k)} \quad , \quad \bar{d}_l^{(i)}(k) = E^c(d_l(k))$$

$$\bar{\mathbf{G}}_f^{(i)}(k) = \text{FFT}(\bar{\mathbf{d}}_m^{(i)}(k) \hat{\mathbf{c}}_m^{(i)}(k))$$

5.5 Calculation of Conditional Expectation

Here we give the conditional expectation expressions for $\bar{d}_i^{(i)}(k)$ and $P_f^{(i)}(k)$ from the equalizer output in different modulation types.

For BPSK: $d_i(k) = \pm 1$

$$\bar{d}_i^{(i)}(k) = \tanh\left(\frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right), P_f^{(i)}(k) = 1 \quad (52)$$

For QPSK: $d_i(k) = \frac{1}{\sqrt{2}}(\pm 1 \pm j)$

$$\bar{d}_i^{(i)}(k) = \frac{1}{\sqrt{2}} \left[\tanh\left(\frac{\sqrt{2}\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right) + j \tanh\left(\frac{\sqrt{2}\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right) \right], P_f^{(i)}(k) = 1 \quad (53)$$

For 16QAM: $d_i(k) = \frac{1}{\sqrt{10}}(\pm 1, \pm 3) + j(\pm 1, \pm 3), s = \frac{1}{\sqrt{10}}$

$$\begin{aligned} \bar{d}_i^{(i)}(k) = & s \left[\frac{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)}\right)} \sinh\left(2s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right) + 3e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)}\right)} \sinh\left(6s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right)}{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)}\right)} \cosh\left(2s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right) + e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)}\right)} \cosh\left(6s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right)} \right] \\ & + j s \left[\frac{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)}\right)} \sinh\left(2s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right) + 3e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)}\right)} \sinh\left(6s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right)}{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)}\right)} \cosh\left(2s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right) + e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)}\right)} \cosh\left(6s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)}\right)} \right] \end{aligned} \quad (54)$$

$$\begin{aligned}
P_f^{(i)}(k) &= E^c \left(\left| G_f^{(i)}(k) \right|^2 \right) \\
&= \frac{1}{M} \sum_{i=0}^{M-1} E_c \left(\left| u_i^{(i-1)}(k) \right|^2 \right)
\end{aligned} \tag{55}$$

where

$$\begin{aligned}
E_c \left(\left| u_i^{(i-1)}(k) \right|^2 \right) &= s^2 \left[\frac{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(2s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right) + 9 e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(6s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right)}{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(2s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right) + e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(6s \frac{\Re(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right)} \right. \\
&\quad \left. + s^2 \left[\frac{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(2s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right) + 9 e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(6s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right)}{e^{\left(\frac{-s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(2s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right) + e^{\left(\frac{-9s^2}{\sigma^{(i-1)2}(k)} \right)} \cosh \left(6s \frac{\Im(u_i^{(i-1)}(k))}{\sigma^{(i-1)2}(k)} \right)} \right] \right] \tag{56}
\end{aligned}$$

5.6 Hyperbolic Function Approximation

In RS-PLL the phase updating equation (46), and IBFDE conditional expectation calculation (54) and (56). It involves the calculation of hyperbolic functions in the form of:

$$p(x, y) = \frac{A \times \sinh(x) + B \times \sinh(y)}{\cosh(x) + \cosh(y)} \quad \text{or} \quad q(x, y) = \frac{A \times \cosh(x) + B \times \cosh(y)}{\cosh(x) + \cosh(y)}$$

In a real implementation, there will be overflow risk for calculating $\sinh(x)$ and $\cosh(x)$ when x becomes a large value, for example when the channel signal to noise ratio (SNR) is above certain value and same for y . So a proper approximation is needed to address this overflow issue.

For large $|x|$; e.g. $|x| \geq 10$, $\cosh(x)$ can be replaced by $\frac{1}{2}e^{|x|}$, and $\sinh(x)$ by $\frac{1}{2}\text{sign}(x)e^{|x|}$,

and same for y . So Suppose both $|x|$ and $|y|$ are greater than 10. Then

$$p(x, y) \approx A \frac{\text{sign}(x)}{1 + e^{|y|-|x|}} + B \frac{\text{sign}(y)}{1 + e^{|x|-|y|}} \quad (57)$$

Or, suppose $|x| > 10$ and $|y| < 10$ Then

$$p(x, y) \approx A \frac{\text{sign}(x)}{1 + 2e^{-|x|}\cosh(y)} + 2B \frac{\sinh(y)e^{-|x|}}{1 + 2e^{-|x|}\cosh(y)} \quad (58)$$

We can take similar approach for $q(x, y)$.

5.7 Complexity Analysis of SCIBFDE-RSPLL

Table 4: Complexity Analysis of SCIBFDE per Iteration per Subcarrier:

Constellation	# of operation per iteration (addition,multiply)	#of Matrix multiply with complexity of $O(Z_t \times Z_r \times Z_t)$ for each matrix multiply	# of FFT/IFFT with complexity of $O(M \times \log(M))$ for each FFT/IFFT operation	# of Matrix inversion with complexity of $O(Z_r^3)$ for each Matrix inversion operation using Gaussian Elimination
BPSK	$4 \times Z_t + 2 \times Z_r$	$2 \times Z_t$	$2 \times Z_t$	$2 \times Z_t$
QPSK	$6 \times Z_t + 2 \times Z_r$	$2 \times Z_t$	$2 \times Z_t$	$2 \times Z_t$
16QAM	$41 \times Z_t + 2 \times Z_r$	$2 \times Z_t$	$2 \times Z_t$	$2 \times Z_t$

Table 5: Complexity Analysis of RSPLL per Iteration per Subcarrier:

Constellation	# of additions per iteration	# of multiplication per iteration
BPSK	$Z_t \times 4 \times M$	$Z_t \times 5 \times M$
QPSK	$Z_t \times 11 \times M$	$Z_t \times 6 \times M$
16QAM	$Z_t \times 40 \times M$	$Z_t \times 18 \times M$

5.8 PHN Estimation using Expectation-Maximization(EM) Algorithm

The discussions we have up to now on random PHN estimation, are based on MAP criteria. Here we propose another approach using the Expectation-Maximization(EM) Algorithm. The EM Algorithm iteratively optimizes the estimation of the unknown random variable $\Theta(\varphi_l(k)$ in our case), given observation data U (the equalizer output $s_l(k)$) and some hidden variables $J(d_l(k)$ in our case)[39]. The EM Algorithm can be viewed as: first, find the lower bound of the estimation then instead of optimizing the estimation, maximize the bound [39].

The EM Algorithm can be divided into “E” step and “M” step which calculate the estimation in terms of expectation and bound maximization respectively. In the “E”(expectation) step, the estimation of $p'(a)$ can be found from the equalization and decoding during the i^{th} iteration, and the “M” (maximization) step is written as:

$$\varphi^i(k) = \arg \max_{\varphi(k)} \{ E[\ln(p(s^{(i-1)}(k) | \varphi^{(i-1)}(k), d(k))) + \ln(p(\varphi(k)))] \} \quad (59)$$

If we assume that the data symbol estimation are independent, (59) can be further written as:

$$\boldsymbol{\varphi}^i(k) = \underset{\boldsymbol{\varphi}(k)}{\arg \max} \left\{ \sum_{l=1}^M E[\ln(p(s_l^{(i-1)}(k) | \varphi_l^{(i-1)}(k), d_l(k))) + \ln(p(\boldsymbol{\varphi}(k)))] \right\} \quad (60)$$

and we know from previous sections that the posterior density function

$p(s_l^{(i-1)}(k) | \varphi_l^{(i-1)}(k), d_l(k))$ and the PHN increments at each data symbol are

independent Gaussian density functions. (60) becomes:

$$\boldsymbol{\varphi}^i(k) = \underset{\boldsymbol{\varphi}(k)}{\arg \max} \left\{ \frac{2}{\sigma^{(i-1)2}(k)} \sum_{l=1}^M E[\Re(s_l^{(i)*}(k) d_l(k) e^{j\varphi_l^i(k)})] - \frac{1}{2\gamma^2} \sum_{l=1}^{M-1} (\varphi_l(k) - \varphi_{l-1}(k) - 2\pi A_f(k)T)^2 \right\} \quad (61)$$

Taking the derivative with respect to $\varphi_l(k)$ and setting the initial PHN to zero from

(61) we get the optimal updates for $\varphi_l(k)$ for BPSK as:

$$\varphi_l^i(k) = 2\varphi_l^{i-1}(k) - \varphi_l^{i-2}(k) + 2 \frac{\gamma^2}{\sigma^{(i)2}(k)} E[\Im(s_l^{(i)*}(k) d_l(k) e^{j\varphi_l^i(k)})] \quad (62)$$

Comparing EM based (62) to MAP based (44), they both can be viewed as the second order phase tracking process. The difference between the two is, for EM based phase tracking, the correction is done between adjacent iterations through the iteration index i , however, for MAP based phase tracking, the update is done between adjacent data symbols at the same iteration through the time index l . By looking at the nature of the updates, the two methods are essentially the same.

5.9 MMSE-MUD-CPE Algorithm for OFDM

Beside the proposed SCIBFDE-RSPLL algorithm, an OFDM receiver equalizer and phase synchronization method proposed in [19] is also evaluated and the simulation results are compared with the proposed SCIBFDE-RSPLL algorithm to see the performance differences under different channel circumstances.

MMSE-MUD has been known to have good performance by keeping lower computational complexity as well as maintaining a good trade off between multiuser interference and noise amplification. The basic idea of MMSE-MUD is to separate the signals of the simultaneous users by MMSE linear filtering which is similar to the first iteration of SCIBFDE.

Specifically, with the channel estimation $\hat{H}_f$ (same definition as H_f in Table 1) which is acquired by using preamble[19], the MMSE filter [40] and the linear frequency domain equalizer outputs are shown in equation (63) (64).

$$F = \hat{H}_f^* (\hat{H}_f \hat{H}_f^* + \sigma^2 I)^{-1} \quad (63)$$

$$\begin{aligned} X_f &= F R_f \\ &= \hat{H}_f^* (\hat{H}_f \hat{H}_f^* + \sigma^2 I)^{-1} Y_f \end{aligned} \quad (64)$$

where Y_f and X_f represent the MMSE-MUD receiver input and equalizer output

vector respectively. The proposed CPE phase synchronization method is based on the maximum likelihood (ML) criterion which is calculated by analyzing the inserted pilot symbols thus the CPE phase estimation is derived as

$$\begin{aligned}\hat{c} &= (\Theta_p^* \hat{H}_p^* \hat{H}_p \Theta_p)^{-1} \Theta_p^* \hat{H}_p^* Y_p \\ &= \Theta_p^{-1} (\hat{H}_p^* \hat{H}_p)^{-1} \hat{H}_p^* Y_p\end{aligned}\quad (65)$$

where Θ_p is $\text{diag}(X_p)$ which is the pilot symbol vector and Y_p is the equalizer output of the received pilot symbol vector. The CPE phase estimation can be further improved by averaging over multiple data blocks. With the phase synchronization in place equation (64) becomes

$$X_f = [\text{diag}(\hat{c})]^{-1} \hat{H}_f^* (\hat{H}_f \hat{H}_f^* + \sigma^2 I)^{-1} Y_f \quad (66)$$

For a coded system, in order to generate data metrics (LLR approximation) for a soft Viterbi decoder, the variances of the equalizer outputs have to be calculated. In the case of SC systems, generally all noise samples are assumed to have the same variance σ^2 as we see in equation (29). Since the noise variance is the same for all samples, its value can be ignored in computing and maximizing the metric in the Viterbi decoding algorithm. However in the case of OFDM transmitted over a frequency selective channel, where coding is across multiple subcarriers, each received coded sample will have a different SNR, since each subchannel will experience a different gain. Thus the variance for each sample will be different. σ^2 becomes σ_f^2 for each subcarrier where σ_f^2 is

the minimum mean squared error of the f^{th} subchannel. From (63), σ_f^2 can be derived as

$$\sigma_f^2 = \frac{\sigma^2}{Z_r} \text{tr} \{ [\hat{\mathbf{H}}_f \hat{\mathbf{H}}_f^* + \sigma^2 \mathbf{I}]^{-1} \} \quad (67)$$

where $\text{tr}\{\mathbf{A}\}$ represents the trace (sum of diagonal elements) of the matrix $\mathbf{A}$ and $\mathbf{I}$ is the Z_r by Z_r identity matrix. When the phase compensation algorithm CPE is in place,

$\hat{\mathbf{H}}_f$ in the above equation is replaced by $\hat{\mathbf{H}}_f \text{diag}(\hat{\mathbf{c}})$.

Chapter 6 Simulation Results

In this chapter, we show the simulation results. The simulation is done using MATLAB as the modeling tool and simulation environment. The model is designed based on the proposed IBFDE-RSPLL algorithm and for the purpose of performance comparison, a SDMA OFDM based multiple independent phase noise correction algorithm described in section 5.9 is also simulated. These two models are tested under the same channel conditions with the same phase noise impairments and the performance curves are also generated under the same data and coding rate.

6.1 System Parameters

The system uses the WINNER urban system parameters [30]. FFT blocks are 25.6 microsecond long (not including the 3.2 microsecond cyclic prefix). The bandwidth used is 45MHz, which means that the single carrier symbol rate is 45MHz. There are a total of 1152 carriers per FFT block. For SC or DFT-precoding to generate the transmitted signal, an extra 3968 zeros are added to make a total block of 5120 samples, and the output time domain samples are generated by taking the inverse FFT of this block of 5120 and adding 640 samples from the end as the CP. The result is time samples of the transmitted waveform at intervals of 5 nanoseconds. For the OFDM system, 1152 complex data symbols and 3968 zeros are fed into the 5120 point IFFT and the output follows the same procedure to add the CP extension. As a result, both systems will have the same

bandwidth and same data rate (coded and uncoded).

6.2 Channel Coding

For comparison between coded and uncoded system, a typical half rate convolution code with constraint length $k=7$, and generators given by $(171,133)$ is chosen as the channel encoder and a simple random permutation based interleaver is used to interleave the coded data bits after the channel encoder. At the receiver side, a soft Viterbi decoder with 8 quantization levels and trace back depth of 128 samples is used as the channel decoder for both SC and OFDM. The initial and end state of the encoder status register is set to be zero.

6.3 Urban Wide Area Fading Channel (UWAF) Generation

The urban macro-cell with non line of sight (NLOS) Clustered delay line model [24] is selected in this thesis as the fading channel reference. The impulse response samples in Table 2 are at intervals of multiples of 5 nanoseconds. So the channel response generated for each channel is sampled at 200 MHz. In Table 2, first we convert the components in each cluster from dB to a relative power, then generate two independent Gaussian random variables with zero mean and variance equal to half the impulse power. We are assuming the channels stay invariant during one transmitted data block(28.8 microsecond), So in

order to get a good distribution over fading channels, a total of 2500 to 10,000 independent channels are generated in this way used for each simulation.

6.4 E_b/N_o and Channel Noise Injection

The energy per bit to noise power spectral density ratio (E_b/N_o) instead of signal to noise ratio (SNR) is used here to compare the bit error rate (BER) performance of different digital modulation schemes without taking bandwidth into account. The channel noise is also generated based on a given E_b/N_o value and the averaged signal power C at the output of the receiver antenna. The relation between SNR in the 45MHz signal bandwidth and E_b/N_o can be expressed as

$$SNR = \frac{C}{N_o W} = \frac{E_b}{N_o} \frac{R}{W} \quad (68)$$

where $W = 45MHz$ is the signal bandwidth, R is the information bit rate ($R = 45$ Mega symbols/s $\times \log_2 M \times$ code rate). The code rate is 1 for uncoded systems and $1/2$ for the coded systems considered in this thesis. The sampling bandwidth is $B = 200MHz$, so the total noise power in the sampling bandwidth is $N_p = N_o B$.

For the SISO channel setup, first the average received signal power C is measured over the ensemble of channels at the receiver antenna output point and then the noise power

N_p is calculated from the relation in (68) and $N_p = N_o B$. for any given E_b/N_o value. And finally, the $N_p/2$ is used as the variance of the random Gaussian signal generator to generate the complex channel noise samples which are added to the 200MHz-sampled signal.

In the SDMA case, we simulate the added noise at the output to each antenna exactly the same as we do in the SISO case. This is because the noise at each antenna does not come from any transmitter, but originates primarily in the low noise amplifier and other elements at each antenna. The additive noises at all the antennas are independently generated. The average signal power C at each antenna is the average signal power received from any one of the transmitters.

6.5 Phase Noise Impairment

The PHN is described in two terms as PHN: ($\sigma f, \gamma^2$), where σf is specified as a certain fraction of the sub-carrier spacing representing the frequency offset caused by the transmitter free running oscillator, which is (45MHz/1152 subcarriers), and γ^2 which is defined in equation (9). The generated phase noise samples are then applied to the transmitted signals at the input of the transmitter antenna.

6.6 Test Cases

In general, the results shown in the following plots will be categorized into two groups, one group is performance comparison (E_b/N_o vs BER) between SC and OFDM systems with the same channel setups, which include channel responses, noise impairments, number of transceiver antennas and different modulation types. It gives a check on how well the SC performs compared to OFDM given same data rate and same channel conditions. The other group covers other performance aspects like performance comparison with different PHNs and iteration MSE comparison SDMA diversity gain etc. This is helpful to see how the system behaves under different scenarios. Besides the coded BER results, the uncoded BER is also plotted in order to get better looks at the SDMA scenarios.

All the simulation is done using MATLAB Version 7.5.0.342 (R2007b) as the simulation tool and WINDOWS as the working environment. All the MATLAB scripts are available upon request.

There are a total of six test cases, each focusing on a different aspect of the performance of the system. In general, each test runs 9 times to cover three modulation types (BPSK, QPSK, 16QAM) and three antenna setups (SISO, SDMA(2x2, 4x4)). As mentioned before, it also brings up the equivalent OFDM system simulation results side by side for comparison.

Test case 1 shows the system baseline performance under AWGN without any phase noise impairments. It will give us a general idea about the system performance under SISO. Test case No. 2 shows the system performance for SC and OFDM based system under SISO and SDMA scenarios in urban wide area multi-path fading NLOS channel environments. Test case No.3 shows the impact of the PHN impairments for AWGN channel for different modulation type . It shows the performance for both coded and uncoded BER. Test case No.4, shows the impact of the PHN impairments under the UWAF channel for different modulation types under SISO and SDMA scenarios . Test case No.5 shows the system performance under UWAF with different PHNs for SCIBFDE-RSPLL. There are no corresponding OFDM curves for comparison. Test case No.6 shows the MSE of SCIBFDE in each iteration under SISO and SDMA scenarios.

6.7 Simulation Results

First, let's explain the names used for each simulation scenarios:

AWGN: Additive white Gaussian noise channel scenario

UWAF: Urban wide area fading Channel scenario.

PHN : $(\sigma f, \gamma^2)$: Phase noise with frequency shift , random time varying phase noise.

SISO: SISO channel setup.

SDMA $(Z_t \times Z_r)$: SDMA channel setup with independent Z_t transmitting antennas and

Z_r receiving antennas.

OFDM-MMSE-MUD: OFDM using MMSE-MUD based frequency domain equalization method.

SCIBFDE: SC using iterative block based frequency domain equalization method.

SCIBFDE-RSPLL: SC-IBFDE jointed with recursive second order PLL phase tracking method.

OFDM-MMSE-MUD-CPE: OFDM-MMSE-MUD jointed with common phase error correction method.

Figure 11 shows the BER of QPSK and BPSK for an AWGN channel. Figure 12 shows the BER of 16QAM for an AWGN channel. Both coded and uncoded BER are presented along with the theoretical curves for uncoded BER for QPSK/BPSK and 16QAM respectively. Figure 11 and Figure 12 show that under AWGN, SCIBFDE and OFDM-MMSE-MUD based systems have the same performance in both coded and uncoded cases. At $E_b/N_o=3dB$, the BER of coded BPSK/QPSK is about $4dB$ better than uncoded cases. For 16QAM, at $E_b/N_o=7dB$, the BER of coded 16QAM is about $4dB$ better than uncoded results.

Figure 11: AWGN channel , SCIBFDE and OFDM-MMSE-MUD ,BPSK,QPSK [TEST CASE 1, SISO]

Figure 12: AWGN Channel, SCIBFDE and OFDM-MMSE-MUD, 16QAM [TEST CASE 1, SISO]

Figure 13 to Figure 15, show the coded BER under UWAF channel in both SISO and SDMA(2x2, 4x4) scenarios. First we can see in the SISO case, SCIBFDE has better performance at higher E_b/N_o in all different modulation types (QPSK and 16QAM, BPSK) and that is very noticeable in the SDMA cases, across different modulation type and channel configurations. The probable reason for this is that the SCIBFDE algorithm described in Chapter 5 iteratively estimates and subtracts cochannel interference, whereas the OFDM SDMA system described in Section 5.9 can only suppress cochannel interference using linear frequency domain filter approach characterized by equations (63) and (64). Thus our results do not imply that SDMA OFDM has inherently worse performance than that of single carrier. To prove this point, in Figure 16 we set the SCIBFDE iteration time to be 1 under SDMA UWAF for QPSK and BPSK, so that the SCIBFDE becomes only a linear equalizer, a fairer comparison with the SDMA OFDM MUD. It shows the SC and OFDM have similar performance curves. Secondly, we can see from Figure 13 to Figure 15, for the proposed SCIBFDE, there is about $2dB$ diversity gain for QPSK SDMA(2x2,4x4). For BPSK the diversity gain is about $3dB$, and the diversity gain for 16QAM is about $2dB$. Thirdly, the performance differences between SC and OFDM are more noticeable in the SDMA channel configuration than SISO.

Figure 13: UWAF Channel, SCIBFDE and OFDM-MMSE-MUD, QPSK [TEST CASE 2]

Figure 14: UWAF Channel, SCIBFDE and OFDM-MMSE-MUD , BPSK [TEST CASE 2]

Figure 15: UWAF Channel, SCIBFDE and OFDM-MMSE-MUD, 16QAM [TEST CASE 2]

Figure 16: UWAF SDMA Channel, SCIBFDE at Iteration 1 and OFDM-MMSE-MUD, QPSK,BPSK [TEST CASE 2]

Figure 17 to Figure 19, show the BER under AWGN with time varying phase noise. It's much easier to see the different performance without taking the frequency selectivity of the channel into consideration. It demonstrates the phase tracking performance of SC based RSPLL and OFDM based CPE. For QPSK SCIBFDE-RSPLL with frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about 781.25Hz , and phase noise variance $\gamma^2 = 0.003$ which is about 9.6KHz for phase noise bandwidth, it has similar performance as it does in the AWGN without PHN. It's about 0.2dB better than OFDM-CPE at $E_b/N_o=6\text{dB}$ in the uncoded case and offers no noticeable gain for coded case. This performance difference becomes more obvious for BPSK with higher PHN impairment: frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about 781.25Hz , and phase noise variance $\gamma^2 = 0.006$ which is about 19.1KHz in terms of phase noise bandwidth. It's about 1.5dB better than OFDM-CPE at $E_b/N_o=6\text{dB}$ in the uncoded case and 0.7dB better than at $E_b/N_o=3\text{dB}$ for the coded case. For 16QAM, it's about 0.3 dB better than OFDM-CPE at $E_b/N_o=10\text{dB}$ in the uncoded case and offers no noticeable gain for coded case with frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about 781.25Hz , and phase noise variance $\gamma^2 = 0.0015$ which is about 4.8KHz in terms of phase noise bandwidth.

Figure 17: AWGN Channel, SCIBFDE and OFDM-MMSE-MUD with PHN(0.02,0.003) , QPSK [TEST CASE 3 SISO]

Figure 18: AWGN Channel, SCIBFDE and OFDM-MMSE-MUD with PHN(0.02,0.006) , BPSK [TEST CASE 3 SISO]

Figure 19: AWGN Channel, SCIBFDE and OFDM-MMSE-MUD with PHN(0.02,0.0015), 16QAM [TEST CASE 3 SISO]

Figure 20 to Figure 22, show the BER under UWAF with time varying phase noise in both SISO and SDMA(2x2, 4x4) scenarios. We see a similar trend as we saw in Figure 13 to Figure 15 regarding the diversity gain, and also a similar trend as we saw in Figure 17 to Figure 19 regarding phase tracking performance in each individual channel configuration. The diversity gain is about 2.0dB, 3.0dB, and 2.0dB for QPSK, BPSK and 16QAM respectively. The phase tracking performance by using RSPLL compared to that of the CPE approach becomes clear for BPSK when the PHN is high with frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about 781.25Hz, and phase noise variance $\gamma^2 = 0.006$ which is about 19.1KHz in terms of phase noise bandwidth. For BPSK, the proposed RSPLL has similar performance curves as those of BER under UWAF without PHN, while the OFDM CPE is about 0.5 to 1.0dB worse than the BER curve under UWAF without PHN. For all the modulation types, the proposed RSPLL gives performance almost as good as that for the curves without PHN impairments. Again in Figure 23 we show a similar plot as that of Figure 16 this time with PHN to bring a fairer comparison between SC and OFDM. The PHN set for QPSK is with frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about 781.25Hz, and phase noise variance $\gamma^2 = 0.003$ which is about 9.6KHz in terms of phase noise bandwidth and the PHN for BPSK is with frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about 781.25Hz, and phase noise variance $\gamma^2 = 0.006$ which is about 19.1KHz in terms of phase noise bandwidth

Figure 20: UWAF Channel, SCIBFDE and OFDM-MMSE-MUD with PHN(0.02,0.003) , QPSK [TEST CASE 4]

Figure 21: UWAF Channel, SCIBFDE and OFDM-MMSE-MUD with PHN(0.02,0.006) , BPSK [TEST CASE 4]

Figure 22: UWAF Channel, SCIBFDE and OFDM-MMSE-MUD with PHN(0.02,0.0015), 16QAM [TEST CASE 4]

Figure 23: UWAF MIMO Channel with PHN, SCIBFDE at Iteration 1 and OFDM-MMSE-MUD, QPSK PHN[0.02,0.003],BPSK PHN[0.02,0.006] [TEST CASE 4]

Figure 24 to Figure 26, show the BER for SCIBFDE-PLL only under UWAF with different PHNs in both SISO and SDMA scenarios. For QPSK, the PHN is set to be $\sigma f = 2\%$ of the subcarrier spacing which is about 781.25Hz and $\gamma^2 = [0.003, 0.0045, 0.006]$ which is about $9.6\text{KHz}, 14.3\text{KHz}, 19.1\text{KHz}$ respectively in terms of phase noise bandwidth. For BPSK, the PHN is set to be $\sigma f = 2\%$ of the subcarrier spacing which is about 781.25Hz and $\gamma^2 = [0.006, 0.012, 0.018]$ which is about $19.1\text{KHz}, 38.2\text{KHz}, 57.3\text{KHz}$ respectively in terms of phase noise bandwidth. For 16QAM, the PHN is set to be $\sigma f = 2\%$ of the subcarrier spacing which is about 781.25Hz and $\gamma^2 = [0.0015, 0.003, 0.0045]$ which is about $4.8\text{KHz}, 9.6\text{KHz}, 14.3\text{KHz}$ respectively in terms of phase noise bandwidth. Besides the performance degradation we see in each curve with the increase of the PHN, it also shows the sensitivity to PHN for each modulation type in which BPSK shows the least PHN sensitivity and the 16QAM the most. Figure 24 to Figure 26, show good complementary aspects to Figure 10 in Chapter 5 in term of real system performance.

Figure 24: SCIBFDE-RSPLL Coded QPSK, UWAF Channel with Different PHNs, [TEST CASE 5]

Figure 25: SCIBFDE-RSPLL Coded BPSK,UWAF Channel with Different PHNs,[TEST CASE 5]

Figure 26: SCIBFDE-RSPLL Coded 16QAM, UWAF Channel with Different PHNs, [TEST CASE 5]

Figure 27 to Figure 29, show the MSE of SCIBFDE under UWAF with and without time varying phase noise in both SISO and SDMA(2x2 ,4x4) scenarios. We can see a similar trend regarding the diversity gain we saw in previous results. For each channel configuration, improvements are seen from iteration to iteration. For QPSK/BPSK the result is very close to that of the case with PHN in comparison to cases without PHN. For 16QAM the case without PHN is about $0.1dB$ better than the case with PHN[0.02,0.0015], i.e. with frequency offset $\sigma f = 2\%$ of the inter-subcarrier spacing which is about $781.25Hz$, and phase noise variance $\gamma^2 = 0.0015$ which is about $4.8KHz$ in terms of phase noise bandwidth.

Figure 27: SCIBFDE MSE in Each Iteration, QPSK [TEST CASE 6]

Figure 28: SCIBFDE MSE in Each Iteration, BPSK [TEST CASE 6]

Figure 29: SCIBFDE MSE in Each Iteration, 16QAM [TEST CASE 6]

Chapter 7 Conclusions and Suggestions for Future Work

By comparing the proposed SCIBFDE with OFDM-MMSE-MUD under AWGN and UWAF channel with SISO and SDMA channel configurations, we can conclude that SCIBFDE has better BER performance. While both systems show improved performance with diversity gains in SDMA channel configurations, SCIBFDE performs even better in SDMA cases. This is because in SCIBFDE the signals from other users are subtracted first before the FD equalization and with each iteration it can improve even further, while in OFDM MMSE MUD, the signals from other users are treated as noise. As mentioned in the previous chapter, this does not imply that OFDM SDMA is inherently worse than single carrier in this respect. The OFDM SDMA performance would undoubtedly have improved if we had applied iterative interference cancellation to it, analogous to that employed in the SCIBFDE algorithm. The superior phase tracking performance of the proposed RSPLL is observed in both AWGN and UWAF channels under SISO and SDMA channel configurations. It becomes more obvious in AWGN channel or when the PHN is higher. Note that the OFDM system we have used for comparison in this thesis uses only common phase error (CPE) compensation. OFDM receivers can also compensate for phase noise-caused inter-carrier interference (ICI), but the algorithm is quite complex[37].

Overall the proposed SCIBFDE-RSPLL algorithm demonstrates a similar performance in AWGN compared to a typical OFDM system and noticeable gain in UWAF channel at

higher E_b/N_o and enjoyed more diversity gain in SDMA channel configurations. For PHN tracking and compensation, the proposed SCIBFDE-RSPLL algorithm gives similar BER performance compared to the same case without PHN while the OFDM based CPE method suffers performance loss especially when the PHE is higher.

The assumption we made for both the proposed SC-IBFDE equalization algorithm and the RSPLL phase tracking algorithm is that the fed back soft data symbols are uniformly distributed. However this assumption is not very accurate when coding and decoding is taken into account. That could be compensated by including the decoder module into the equalization iteration loop. For example, a turbo equalizer [20] could be considered in the further works to get better performance.

The future work can include the investigation of a calculation of the *a priori* probability estimation which is not based on Gaussian distribution. The assumption we made for the MMSE based IBFDE and MAP based RS-PLL is the noise distribution of the equalizer outputs can be treated as Gaussian like white noise. For example, one possible way to improve the estimation calculation for the feedback loop involves the iteration of the Viterbi decoding. By saving multiple trace back paths, when the current path failed, a second or third path could be used to decode the data which could lead to a better data estimation when the inputs is not Gaussian.

For future work, we could to use the proposed equalization schemes and phase tracking method in the multi-cellular case which involves more complicated interference, for example the channel responses from out of cell users are unknown[38]. Also for future work, we may consider the cases where received signals at antenna elements are correlated.

Finally, for the proposed SC-IBFDE-RSPLL, further work is need for the purpose of simplicity and optimization, especially when it involves matrix inversion and hyperbolic functions. Seeing the advantage and superior performance of the proposed SC-IBFDE-RSPLL in the case of SISO and SDMA random time varying phase noise, practical simplification of the algorithm would be desirable.

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