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LA THÈSE A ÉTÉ
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A COMBINED EMPIRICAL AND THEORETICAL STUDY
OF THE DIELECTRIC PROPERTIES OF SEA ICE
OVER THE FREQUENCY RANGE
100 MHz TO 40 GHz

by
MALCOLM R. VANT, B.Eng.

A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

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The undersigned recommend to the Faculty of Graduate
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submitted by Malcolm R. Vant, B.Eng., in partial fulfilment
of the requirements for the degree of Doctor of Philosophy.

Thesis Supervisor

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June, 1976

ABSTRACT

A systematic investigation into the dielectric properties of sea ice is presented. It is shown that the classical dielectric mixture equations yield formulas applicable to the sea ice modelling problem. Included in this discussion is an explanation both of the frequency range of applicability of the equations and of the role played by conductivity effects. A preliminary measurement program, performed at 10 and 30 to 34 GHz, is described. Based on these measurements, an evaluation of several of the classical formulas, and an investigation of the measurement problems involved, is made. The dielectric behaviour of ϵ_r' and ϵ_r'' is described in detail, i.e. they are shown to vary appreciably with age, salinity, and temperature, increasing with temperature and salinity, and decreasing with age. This description is followed by an outline of the design of a novel type of wideband (100 MHz to 7.5 GHz) "coaxial-cage" transmission line. This outline includes an analysis of the errors due both to the line itself and to its integration in the total system. Primary errors are demonstrated to be due to the short electrical length of the sea ice sample. A description is also given of dielectric measurements, employing the "coaxial-cage" line, which were performed on location, in the Beaufort Sea, during the Arctic Ice Dynamics Joint Experiment (AIDJEX). This unique set of measurements incorporates the investigation of a wide range of first-year sea ice samples (salinities between 5.1 and 10.5 ‰, and temperatures between -5°C and -40°C) over a wide range of frequencies (100 MHz to 7.5 GHz). A somewhat less comprehensive set of measurements, made on multiyear sea ice, is also described. It is seen that the AIDJEX measurements allowed the formu-

lation of highly successful empirical and theoretical models. These models are shown to be applicable over the frequency range 100 MHz to 7.5 GHz (for the empirical model) and 400 MHz to 40 GHz (for the theoretical model). The theoretical model is demonstrated to be able to correctly predict the experimental measurements of the author and other workers, over the above frequency range. Also given is a description of artificial sea ice dielectric measurements which were performed in the laboratory. It is shown that both ϵ_r' and ϵ_r'' increase with the deviation of the probing wavefront from normal incidence. These measurements also allow a comparison to be made of the empirical model, based on the artificial sea ice measurements, with the empirical and theoretical models, based on the first-year sea ice measurements. The agreement between the various models is seen to be within the limits of the natural variability of the sea ice. Following this comparison, a discussion of the applicability of the various models to the description of multiyear sea ice is outlined. However, the discussion is demonstrated to be inconclusive due to the small number of samples of multiyear sea ice measured and the difficulties in performing representative sea ice measurements. In addition, various remote sensing applications of the models are discussed briefly, and recommendations for further research are made.

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LIST OF SYMBOLS
(in order of appearance)

Chapter 1

ϵ'	real part of the dielectric constant
ϵ''	imaginary part of the dielectric constant

Chapter 2

ϵ_∞	high frequency dielectric constant
ϵ_0	static dielectric constant
ω	radian frequency
τ	macroscopic relaxation time of the material
$\bar{\epsilon}$	dielectric constant of the zone of interaction
$\epsilon_{AV}, \epsilon_{MIX}$	average dielectric constant of the mixture
A_i	depolarization factor for the i^{th} direction
v_j	volume fraction of j^{th} element
ϵ_j	dielectric constant of the j^{th} element
η, ξ, ζ	ellipsoidal coordinates
$V(u)$	volume of the ellipsoid described by $\xi = u$
a, b, c	axes of the inclusion ellipsoid
ϕ_s	static potential inside the zone of interaction
ϕ_0	static potential outside the zone of interaction
ϕ_1	total static potential outside the inclusion
E_0	mean electric field outside the zone of interaction
$F_1(\xi), F_2(\eta), F_3(\zeta)$	component functions of ϕ_0 when the variables are separated
R_η, R_ξ, R_ζ	radial distance in ellipsoidal coordinates
$G_1(\xi)$	replaces $F_1(\xi)$ (above) in solution for ϕ_s
ϕ_2	static potential inside the inclusion
$F(u)$	the functional dependence of ϕ_1 on u

$\vec{E}$	electric field
$n^a(u)$	depolarization coefficient for the a^{th} direction as defined by Landau and Lifshitz
V_1, V_2	volume fractions of host and inclusion
e	eccentricity of the ellipsoid
λ	wavelength of the electromagnetic radiation
$\vec{E}_{\text{INT}}$	electric field induced by the presence of a polarizable sphere
$\vec{p}$	dipole moment
a	sphere radius (in equation 2.44)
I	intensity of scattered wave
m	complex relative refractive index
C_{sca}	scattering cross section
V	particle volume
Q_{sca}	efficiency factor for scattering
α	particle size parameter relating the radius and wavelength
N	number of scattering particles per unit volume
τ	turbidity (in equation 2.50)
T	transmission coefficient
a'	equivalent radius for ellipsoidal particles
A, B, C	ellipsoid semi-axes
$\alpha'_{A,B,C}$	polarizability (in equation 2.51)
$P_{A,B,C}$	depolarization factors along axes A, B and C
κ^r	relative dielectric constant of a bounded half space
$K(z)$	mean relative dielectric constant expressed as a function of depth
$K'(r)$	random fluctuation imposed on $K(z)$
γ_0	free space propagation constant
μ	permeability of free space

ϵ_0	dielectric constant of free space
σ	ionic conductivity
$\bar{E}$	mean electric field
$\vec{E}^r$	random electric field caused by the perturbation in κ^r
α	correlation length (in equation 2.57)
$\kappa(z)$	the tensor giving the variation in κ^r expressed as a deterministic function of depth (it is composed of κ_1 and κ_2)
ρ, ψ, z	cylindrical coordinates
v	a factor derived from the exponential correlation function
W	the autocorrelation of K' times V
$a_{\psi\psi}, a_{\rho\rho}, a_{zz}$	functions expressing the variation in $\kappa(z)$ with position
γ_1, γ_2	modified propagation constants
$u(r, r')$	exponential correlation function
$K_{1,2}$	relative dielectric constant κ^r with $K'(r)$ replaced by $\kappa_{1,2}$
$\delta(u)$	kronecker delta function

Chapter 3

ρ	density in gm/cm ³
S	salinity in parts per thousand by weight
T	temperature in °C
v	relative volume fraction of brine

Chapter 4

T	transmission coefficient
γ_0	free-space propagation constant
γ	propagation constant in medium
d	sample length
$\epsilon_r = \epsilon_r' - j\epsilon_r''$	relative complex dielectric constant

f_c	cutoff frequency of waveguide
f	operating frequency
λ_0	free-space wavelength of electromagnetic radiation
a	long dimension of waveguide cross-section
b	short dimension of waveguide cross-section
v, v_2	brine volume fraction
f	form factor (in equation 4.14)
ρ_3	density of sea ice in (gm/cm ³)
ρ_{ICE}	density of pure ice in (gm/cm ³)
v_3	volume fraction of air

Chapter 5

VSWR	Voltage Standing Wave Ratio
a	inner conductor radius of coaxial line
b	outer conductor radius of coaxial line
a_1	outer conductor rod radius of "coaxial-cage" line
$\vec{E}_r$	radial electric field
$\vec{H}_\phi$	magnetic field in the ϕ direction
r, z, ϕ	cylindrical coordinates
ω	radian frequency
γ	complex propagation constant
γ'	modified propagation constant
$\epsilon = \epsilon' - j\epsilon''$	complex dielectric constant
ϵ_1, ϵ_2	complex dielectric constants region 1 and 2
a_1, a_2, a_3	radii of concentric cylinders in air gap model
λ_0	free space wavelength
v, v_1, v_2, v_3	potentials in air gap model
ϵ_{rMEAS}	relative complex dielectric constant measured during experiment

ϵ_{rACT}	actual relative complex dielectric constant
$\epsilon_r = \epsilon_r' - j\epsilon''$	relative complex dielectric constant
θ	angle face of sample makes with the probing wavefront (in Table 5.1)
α	imaginary part of the propagation constant in (N/cm)
β	real part of the propagation constant in cm^{-1}
θ	measured phase angle
l	sample length
A	loss measured in db/cm

Chapter 6

ϵ_r'	real part of the relative complex dielectric constant of the sea ice
ϵ_r''	imaginary part of the relative complex dielectric constant of the sea ice
σ_{MIX}	conductivity of the sea ice mixture
$\epsilon_r'' \text{ DIEL-MIX}$	the contribution to ϵ_r'' due to dielectric relaxation

Chapter 7

$\epsilon_r = \epsilon_r' - j\epsilon_r''$	relative complex dielectric of sea ice
r^2	coefficient of determination
$\hat{y} = a_0 + a_1 x$	best straight line estimate of y
s_{yx}	standard error of the estimate $\hat{y}$
n	number of points in the sample
s_0, s_1	standard errors of a_0 and a_1
$\pm \Delta$	estimated measurement error bound
v, v_{br}	relative brine volume fraction of sea ice
$\tan \delta = \epsilon''/\epsilon'$	loss tangent
LOSS	the propagation loss expressed in (db/m)
θ	angle of major axis of ellipsoidal inclusion with respect to the normal to the electric field vector

N	number of scattering particles per unit volume (in equation 7.2)
T	power transmission coefficient (in equation 7.2)
l	propagation length
C_{sca}	scattering cross-section
V	ellipsoid volume
m	ratio of complex refractive index of the scattering particle to that of the medium
a, b, c	ellipsoid dimensions (semi-axes)
λ	wavelength in the medium
a'	equivalent radius for non-spherical particle
P	depolarization coefficient
$\epsilon_{r_{br}}$	complex dielectric constant of brine
$\epsilon_{r_{\infty br}}$	high frequency dielectric constant of brine
$\epsilon_{r_{0 br}}$	static dielectric constant of brine
ω	radian frequency
τ_{br}	relaxation time of brine
σ_{br}	ionic conductivity of brine
ϵ_0	permittivity of free space
N	Normality
$a(N)$	polynomial equation in Normality relating ϵ_{r_0} of pure water to that of brine
T	temperature (in $^{\circ}C$)
$b(N, T)$	polynomial in N and T relating τ of pure water at temperature T to τ of brine
σ_{NaCl}	ionic conductivity of an NaCl solution
S	salinity in parts per thousand by weight
Δ	$25 - T$
XBR	real part of $\epsilon_{r_{br}}$

ϵ_{rbr}	imaginary part of ϵ_{rbr}
τ	relaxation time of pure ice
ϵ_{r0}	static dielectric constant of pure ice
$\epsilon_{r\infty}$	high frequency dielectric constant of pure ice
$\epsilon_{r\text{air}}$	relative dielectric constant of air
v_{air}	relative volume fraction of air
ρ	density of the sea ice
$\epsilon_{\perp}, \epsilon_{\parallel}$	complex dielectric constants of the mixture with the probing field perpendicular and parallel to the major axes of the ellipsoids
$\vec{D}$	electric displacement
$\vec{D}_H, \vec{D}_V$	horizontally and vertically polarized components of $\vec{D}$
ψ	angle between $\vec{D}$ and its horizontally polarized component $\vec{D}_H$
ϵ_H, ϵ_V	complex dielectric constants seen by the horizontally and vertically polarized components of $\vec{E}$
ϵ_{eff}	effective dielectric constant of the mixture seen by the probing field
a/b	axial ratio of the ellipsoids

Chapter 8

$\epsilon_r = \epsilon_r' - j\epsilon_r''$	relative complex dielectric constant of sea ice
λ	wavelength in the medium

Appendix A

$\vec{E}$	electric field
$\vec{H}$	magnetic field
$\epsilon = \epsilon' - j\epsilon''$	complex dielectric constant
$\vec{J}$	conduction current density
$\vec{B}$	magnetic flux density
$\vec{D}$	electric displacement
ρ	charge density

ω	radian frequency
$\vec{P}, \vec{M}$	polarization vectors for $\vec{E}$ and $\vec{H}$
μ	permeability
ϵ_0	permittivity of free space
α	polarization constant
ψ	phase angle between $\vec{P}$ and $\vec{E}$
χ_e	susceptibility
σ	conductivity
k'	complex propagation constant including conductivity
ϕ	scalar potential
$\vec{A}$	vector potential
k	complex propagation constant (excluding σ)
R	radial distance between the source and field points
λ_0	free space wavelength
ϵ_c	complex dielectric constant including conductivity
ϵ_N''	net imaginary part of the dielectric constant which includes contributions from conductivity and dielectric relaxation

Appendix B

G_1	part of the variables separable solution for ϕ_s
R_ξ	radial distance in ξ
ξ	one of the ellipsoidal coordinates
v, w	solutions for G_1
$L(v)$	differentiation operator, operating on v .

Appendix D

$\epsilon_r = \epsilon_r'' - j\epsilon_r'''$	relative complex dielectric constant
T	theoretical transmission coefficient
T_0	measured transmission coefficient
λ_0	free-space wavelength

d	depth of sample
$f(\lambda_0, d, \epsilon_r)$	function relating λ_0 , d and ϵ_r
ANGLE	measured transmission coefficient angle in radians
γ, δ	real and imaginary parts of f
ϵ^*, η	real and imaginary parts of $\partial f / \partial \epsilon_r$

CHAPTER 1. INTRODUCTION

1.1 Background

The lack of adequate information on thickness and type distribution of sea ice in the Arctic has long been a drawback to the development of an adequate understanding of sea ice dynamics and how they affect such factors as climate, oil pollution control and exploration, as well as Arctic transportation. Several international projects such as the US/USSR POLEX-GARP (Polar Experiment - Global Atmospheric Research Program), the US/Canadian AIDJEX (Arctic Ice Dynamics Joint Experiment) and the US/USSR BESEX (Bering Sea Experiment) have been and are being devoted to the study of these problems. For example, the stated objectives of the northern POLEX experiment are: meteorological observations in support of weather forecasting, the study of processes at and above the ice, the study of the heat balance of the Arctic Oceans, the numerical modelling of the aforementioned, and the gathering of a long-term climate record. ⁽¹⁾ AIDJEX, a program begun in 1970 and slated for completion in 1975-1976, has devoted itself to the fundamental understanding of the dynamic and thermodynamic interactions between Arctic sea ice and its local environment. ⁽¹⁾ These studies have the utmost importance to the world population. For instance, there is evidence that an ice-free Arctic Ocean would remain free of ice once that condition had been established by a climatic anomaly (Donn and Shaw ⁽²⁾, and Budyko ⁽³⁾ as cited in Goody ⁽¹⁾). Large variations of the ice boundary in the North Atlantic are associated with fluctuations of the general circulation of the atmosphere (e.g. Scherhag ⁽⁴⁾ as cited in Goody ⁽¹⁾) and temperature anomalies at the surface of the subtropical Pacific Ocean are associated with anomalous conditions in the North Atlantic (Namias ⁽⁵⁾, and

Bjerknes⁽⁶⁾ as cited in Goody⁽¹⁾). It is unthinkable to suggest that the ice cover during previous glaciation did not profoundly affect the earth's heat balance, and hence climate. Therefore, it is reasonable to consider that ice cover may be a valuable form of feedback on overall global climatic variations.⁽¹⁾

A better understanding of the Arctic ice dynamics would allow prediction of the effects on climate of some of man's advertent or inadvertent actions, for example, the alteration of the albedo through oil pollution on ice and water, CO₂ production, and thermal pollution from waste heat.⁽¹⁾ The severe obstacle that ice presents to marine navigation should also be considered since this mode of transport is of fundamental importance as far as resource development and environmental protection are concerned. The technology forecast of the Arctic Institute of North America (as quoted by Goody⁽¹⁾) summarizes the projected development of the Arctic during the next three decades. The years 1976-1979 are seen as a continuation of a period of research with the 1980's heralding a period of rapid development. Specific technological highlights predicted are: Arctic tankers in operation (1981), Arctic ore carriers in operation (1982), first Arctic offshore terminal constructed (1985), and first submarine tanker (1988).

In the development of the necessary parameters for models of ice behaviour, one of the fundamental parameters needed is thickness. Weeks et al⁽⁷⁾ have shown that the independent determination of the ice thickness distribution by using remote sensing techniques is of paramount importance to the successful completion of such programs as AIDJEX and is the foremost problem that must be faced in developing a model of the Arctic ice dynamics. Because of the vast nature of the surface to be sensed, it is apparent that the only feasible means of

obtaining this information is through the use of remote sensing, either airborne, satellite based, or ground based.

An accurate knowledge of the dielectric properties of sea ice forms the basis for any measurement of sea ice properties (especially thickness) using electromagnetic techniques. It is important to know the real part of the dielectric constant ϵ' to determine the velocity of propagation through the ice, so that the transit time measurements obtained with a pulsed radar can be converted to range measurements. It is equally important to know the ratio of the imaginary part of the dielectric constant ϵ'' to the real part ϵ' (commonly termed loss tangent) since it determines the absorption of power in the medium and thus can be used to predict the maximum possible signal penetration of a given system through the ice. Even when the radar signal does not penetrate the ice to any great extent, the complex dielectric constant, $\epsilon = \epsilon' - j\epsilon''$, provides clues as to what the magnitude and angular distribution of the scattered power will be.

Sea ice poses a much more difficult remote sensing problem than does fresh ice, in that it presents a high dielectric loss (determined by the frequency and loss tangent) to any incident electromagnetic radiation. The extent and frequency dependence of such absorption as well as the uncertainty as to the exact values to assign to it has been investigated by several workers. (45,47-50,55,57-64) Most of these measurements have been performed at only a few individual frequencies and selected salinities and temperatures, therefore a complete understanding of the dielectric properties of sea ice does not exist.

Parametric studies on sea ice have also been done in relating such properties as skin depth, salinity, frequency, the correlation length of inhomogeneities, and porosity (Meeks et al⁽⁸⁾), and

Poe et al⁽⁹⁾ to the signal returns from microwave remote sensing instruments. These studies had to rely on somewhat inadequate data for the dielectric properties of sea ice in making their predictions.

The difference in dielectric properties between ice types has only been researched by Vant et al⁽¹⁰⁾ at the frequencies of 10 and 35 GHz, while the effect of surface topography (e.g. melt ponds, surface wetness, hummocks and depressions) on signal returns has yet to be thoroughly investigated.

Models for sea ice scattering behaviour have been presented by Finkel'shteyn et al⁽¹¹⁾ and Parashar.⁽¹²⁾ These models both suffered from a lack of adequate ground truth information, such as the dielectric behaviour of sea ice, on which to base the parameters to be fed into their models. It has been shown by Poe et al⁽⁹⁾, deLoor⁽¹³⁾, van Beek⁽¹⁴⁾, and others that a multi-component dielectric, e.g. sea ice, can be described in terms of its components' properties of relaxation time and concentration, at least over limited frequency ranges. This idea must be extended for sea ice once a firm background of dielectric data is obtained.

1.2 Thesis Development and Objectives

Based on the brief background information given above, it is now possible to examine the specific objectives that were set for this thesis.

It is apparent that little is known about the dielectric behaviour of sea ice, i.e. what the values of ϵ' and ϵ'' are as a function of frequency, temperature, salinity of the ice, age of the ice and orientation of the ice. It is also apparent that given limited time and resources it is not possible due to the complexity of the problem to generate this information for all frequencies, all tempera-

tures, etc. Therefore, it is imperative that certain choices be made as to what narrower range of investigation would optimally use time and resources and solve the problems at hand. Given the limitations imposed by the sea ice, the operating frequencies of the more common remote sensing instruments, and the necessity of making the measurements comprehensive enough so that a generalized model could be developed, the following objectives were set:

- 1) Obtain a preliminary set of dielectric data on both fresh ice and sea ice at 10, 31.4 and 34 GHz to evaluate various measurement techniques and evaluate some basic dielectric mixture equations, as well as determine the effects on the dielectric properties of such sea ice parameters as ice type, age, salinity and temperature.
- 2) Having gained some insight into the measurement problem, perform a series of dielectric measurements on first-year sea ice over as wide a frequency range as possible, utilizing the same specimen, and varying such parameters as temperature (for the same specimen), and salinity and age (between specimens).
- 3) From the measurements in 2) develop both theoretical and empirical models to explain the dielectric behaviour of first-year sea ice.
- 4) Determine the range of applicability of the models and outline how they may, if they can at all, be extended to cover multiyear sea ice as well.
- 5) Outline the usefulness of these models to the in-

vestigation of sea ice properties by remote sensing.

With these objectives in mind, a program of measurements was begun.

Preliminary measurements performed during the summer of 1973 allowed a period of familiarization with dielectric measurement techniques and produced one of the first truly "bulk" (as opposed to measurements performed on small samples in cavity resonators) determinations of the dielectric constant of fresh water ice at 10 GHz. Out of this work developed a series of measurements on sea ice specimens taken from the Bering Sea. These measurements, performed with the sample in a waveguide, at 10 GHz, pointed out the difficulties involved in machining sea ice and allowed evaluation of some basic models for first-year and multiyear sea ice dielectric behaviour at this frequency. This program of measurements was followed by measurements of the "bulk" dielectric constant of artificially prepared sea ice at 31.4 and 34 GHz. Comparison of these measurements with previous work allowed an evaluation of the worth of using "artificial sea ice" to simulate first-year ice (see Vant et al ⁽¹⁰⁾). This work culminated in a comprehensive set of measurements of the dielectric properties of actual sea ice, on location on the Beaufort Sea ice pack, over a wide range of frequencies (100 GHz - 7.5 GHz) and temperatures (-5°C - -40°C), during the AIDJEX-75 experiments. From these measurements, both theoretical and empirical models explaining the complex permittivity of first-year sea ice were derived.

The description of the above experiments and a discussion of the conclusions arising from them forms the body of this thesis.

It should be emphasized from the beginning that sea ice is a very

complex material and that this research tries by no means to formulate a complete treatise on the dielectric behaviour of sea ice, but rather tries to lay a basic foundation upon which further work can be built.

1.3 Thesis Outline

This thesis is divided into chapters roughly according to the chronological progress of the project.

Chapter 2 presents the basic dielectric theory and mixture equations necessary for an understanding of the following material.

Only the most basic dielectric behaviour, such as the Debye equations, is described, with the bulk of the chapter being devoted to an outline of some simple dielectric mixture and scattering theories, and an explanation of how these theories may be used to extend the dielectric mixing equations described, beyond the static case.

Chapter 3 poses the measurement problem presented by sea ice, outlines the physical properties of the various ice types and discusses various available dielectric measurement techniques and their applicability to sea ice. Also included are descriptions of previous dielectric measurement programs by other workers.

Chapter 4 discusses the preliminary results and illustrates some of the problems encountered in the modelling procedure.

Chapter 5 considers the design of the wideband measurement system, discusses the various constraints, details the design of the "coaxial-cage" sample holder and evaluates the estimated measurement errors encountered when using it. The chapter concludes with a description of the final measurement system and an estimate of the total system error.

Chapter 6 presents the dielectric measurements obtained both in the lab and at the AIDJEX site, using both artificial and

natural first-year sea ice. A discussion of measurement procedure and interpretation of the measurements is also included.

Chapter 7 discusses the dielectric models, both theoretical and empirical, that were formulated. Also given is a description of the equations used in the models and a comparison of the model with experimental results. The chapter ends with an evaluation of the applicability of the models and how they could be extended to include the behaviour of multiyear sea ice.

Chapter 8 is the conclusion, and as such summarizes the results of this work, and outlines recommendations for future research.

CHAPTER 2. BASIC DIELECTRIC THEORY

2.1 Introduction

Sea ice is a complex dielectric consisting of several component dielectrics, some of which are lossy. The three principal constituents are liquid brine, pure ice, and air. Also included in the mixture are hydrolyzed salts and impurities. Each of these materials by itself has distinctive dielectric properties and when combined, will produce a dielectric mixture which can be partially described by the theories presented here.

The two major constituents of sea ice, ice and brine, are dipolar and their dielectric dispersion (excluding conductivity) follows the Debye equations reasonably well over the frequency range of interest here.

2.2 The Debye Equations

For substances following the Debye dispersion behaviour pattern, the complex dielectric constant, $\epsilon = \epsilon' - j\epsilon''$, neglecting conductivity (see Appendix A), is given by ⁽⁸³⁾

$$\epsilon' = \epsilon'_{\infty} + \frac{\epsilon'_0 - \epsilon'_{\infty}}{1 + \omega^2 \tau^2} \quad (2.1)$$

and

$$\epsilon'' = \frac{\epsilon'_0 - \epsilon'_{\infty}}{1 + \omega^2 \tau^2} \omega \tau \quad (2.2)$$

where ω is the radian frequency of the incident diagnostic wave, ϵ'_0 the static dielectric constant, ϵ'_{∞} the high frequency dielectric constant, and τ is the macroscopic relaxation time of the material.

The dielectric behaviour of materials such as ice and brine, which follow the Debye equations, can easily be incorporated into the dielectric models for sea ice to be developed in Chapter 7.

2.3

The Static Dielectric Behaviour of Mixtures

It was inferred above that a material's dielectric behaviour can, at least in some cases, be easily obtained from its molecular properties, i.e.: τ , ϵ'_{ω} , ϵ''_{ω} . It is advantageous to try to estimate the properties of various mixtures of dielectrics from the dielectric properties of the constituent particles and the mixture geometry. By making certain tacit assumptions, it has been possible, in some cases, to arrive at mixture formulas which do in fact reflect the behaviour of the composite dielectric.

van Beek⁽¹⁴⁾ has presented a comprehensive historical survey of the development of dielectric mixture equations. Indeed, there is such a vast number of them that one is almost at a loss as to which one is most applicable.

Upon close examination of the formulas, certain similarities emerge: almost all the formulas assume a low concentration of solute by volume (in most cases $< 20\%$) - this allows calculation of the electric field (exact solutions at high concentrations are mathematically untractable); and most of the formulas are only valid for a real dielectric constant, and for a few special shapes and distributions of included particles.

The general mixture geometry is shown in Figure 2.1. The inclusions are shown here as randomly oriented ellipsoids in a host medium of dielectric constant ϵ'_1 . The dashed lines represent a zone of interaction between the particles. The exact value to assign to $\bar{\epsilon}'$, the mean dielectric constant of this zone, is one of the foremost problems of mixture theory.

The solution for $\bar{\epsilon}'$ in this zone does not exist in general, but several expressions for special cases, such as a cubic array of

Figure 2.1 Mixture Geometry. The background material has dielectric constant ϵ_1' , the inclusions have dielectric constant ϵ_j' , and the interaction zones have dielectric constant $\tilde{\epsilon}'$.

Figure 2.2 Interaction effects of a cubic array of perfectly conducting spheres. The mixture dielectric constant ϵ_{AV} relative to the background material ϵ_1 is plotted versus the volume density. Calculated from Kharadly and Jackson. (15)

spheres, have been derived by Kharadly and Jackson.⁽¹⁵⁾ The expressions for up to third order interactions are plotted in Figure 2.2. Note that for low volume concentrations the expressions are identical. In effect, this means that if the particles are far enough apart $\bar{\epsilon}' = \epsilon'_1$, and interaction effects can be neglected. As the concentration increases, $\bar{\epsilon}'$ approaches ϵ'_{AV} , the average mixture dielectric constant.

Polder and van Santen⁽¹⁶⁾ have developed a theory which assumes $\bar{\epsilon}' = \epsilon'_1$, and also, as in most of the other theories, that the mean value of the electric field, in the interior of the particle, provides a good approximation to the actual mean field. For an ellipsoid of any shape, their formula becomes

$$\epsilon'_{MIX} = \epsilon'_1 \left[1 - \frac{1}{3} \sum_j v_j (\epsilon'_j - \epsilon'_1) \sum_{i=1}^3 \frac{1}{\epsilon'_{MIX} + (\epsilon'_j - \epsilon'_{MIX}) A_i} \right]^{-1} \quad (2.3)$$

where ϵ'_1 is the dielectric constant of the host medium, ϵ'_j the dielectric constant of the j th type of particle in the mixture, A_i is depolarization factor for the i th direction, dependent on particle shape, and v_j is the volume fraction of the j th element of the mixture. This equation assumes the particles lie equally in all directions so that the orientation is essentially random.

deLoor⁽¹³⁾ has presented a slightly modified version of equation (2.3)

$$\epsilon'_{MIX} = \epsilon'_1 + \sum_j v_j (\epsilon'_j - \epsilon'_1) \frac{1}{3} \sum_{i=1}^3 \left[1 + ((\epsilon'_j / \bar{\epsilon}') - 1) A_i \right]^{-1} \quad (2.4)$$

where $\bar{\epsilon}'$ lies somewhere between ϵ'_{MIX} and ϵ'_1 . If $\bar{\epsilon}' = \epsilon'_{MIX}$ equation

(2.4) becomes equation (2.3) with some manipulation.

deLoor's equation is of interest because it recognizes that there is interaction between particles, in that the background dielectric is $\bar{\epsilon}$, and not ϵ_1 . However, without some simplification, equation (2.4) is impossible to solve.

Tinga et al⁽¹⁷⁾ have extended the work of deLoor⁽¹³⁾ and Taylor⁽¹⁸⁾ to arrive at a set of self-consistent dielectric equations to describe the mixture, in which the first-order interaction between the particles is taken into account, complex dielectric constants are allowed, a method of avoiding the solution for $\bar{\epsilon}$ is provided, and the high inclusion concentration limit is approached correctly. Their particular solution is of special interest since it most closely approximates what is required for a sea ice model. For this reason, and because it is referenced extensively in Chapter 6, a full derivation of Tinga et al's model, not presented in the literature, is given below.

The geometry shown in Figure 2.3 is assumed. The inclusion is surrounded by a zone of interaction with dielectric constant ϵ_1 , extending to some yet to be determined boundary. Outside of the interaction zone the dielectric constant is that of the mixture, ϵ_{AV} . This technique incorporates deLoor's idea that $\bar{\epsilon}$ should lie between ϵ_1 and ϵ_{AV} (previously denoted ϵ_{MIX}), by allowing a region of transition of undetermined thickness, and of dielectric constant ϵ_1 , to separate the inclusion from the average mixture dielectric constant.

Their solution is found by solving the boundary value equations for the static potentials as outlined below. The solution entails the use of Laplace's equation in ellipsoidal coordinates which is given by

Figure 2.3

Mixture geometry assumed by Tinga et al. (17) The background material has electrostatic potential ϕ_0 and dielectric constant ϵ_{AV} , the inclusion has electrostatic potential ϕ_2 and dielectric constant ϵ_2 , and the zone of interaction has electrostatic potential ϕ_s and dielectric constant ϵ_1 .

$$\begin{aligned}
 & (\eta - \zeta) R_{\xi} \frac{\partial}{\partial \xi} \left(R_{\xi} \frac{\partial \phi}{\partial \xi} \right) + (\zeta - \xi) R_{\eta} \frac{\partial}{\partial \eta} \left(R_{\eta} \frac{\partial \phi}{\partial \eta} \right) + \\
 & (\xi - \eta) R_{\zeta} \frac{\partial}{\partial \zeta} \left(R_{\zeta} \frac{\partial \phi}{\partial \zeta} \right) = 0
 \end{aligned} \tag{2.5}$$

For an inclusion with major and minor axes, a , b and c respectively, the case $\xi = 0$ corresponds to the surface of the inclusion. η , and ζ are confocal hyperboloids which serve to measure position on any ellipsoid $\xi = \text{constant}$. The volume of such an ellipsoid $\xi = u$ is given by

$$V(u) = \frac{4}{3} \pi \left[(u + a^2)(u + b^2)(u + c^2) \right]^{\frac{1}{2}} \tag{2.6}$$

It should be noted that any solution for the potential must satisfy these equations.

The solution for the potential outside the inclusion will consist of two parts: the solution for the potential inside the zone of interaction (ϕ_s), and the solution for the potential outside this zone (ϕ_o). The total potential ϕ_1 will consist of the sum

$$\phi_1 = \phi_o + \phi_s \tag{2.7}$$

Outside of the zone of interaction, the potential is simply

$$\phi_o = -E_o x, \quad u \geq u_1$$

where u_1 is the boundary of the interaction zone. In ellipsoidal coordinates, when the field is parallel to the major axis " a ",

$$x = \left[\frac{(\xi + a^2)(\eta + a^2)(\zeta + a^2)}{(b^2 - a^2)(c^2 - a^2)} \right]^{\frac{1}{2}} \tag{2.8}$$

and

$$\phi_o = C_1 F_1(\xi) F_2(\eta) F_3(\zeta) \tag{2.9}$$

where

$$C_1 = \frac{-E_o}{[(b^2 - a^2)(c^2 - a^2)]^{\frac{1}{2}}} \tag{2.10}$$

and E_o is the electric field outside the zone of interaction, also

$$\begin{aligned}
 F_1(\xi) &= (\xi + a^2)^{\frac{1}{2}} \\
 F_2(\eta) &= (\eta + a^2)^{\frac{1}{2}} \\
 F_3(\zeta) &= (\zeta + a^2)^{\frac{1}{2}}
 \end{aligned}
 \tag{2.11}$$

According to Stratton⁽¹⁹⁾, inside the zone of interaction the potential ϕ_s must vary functionally over every surface $\xi = \text{constant}$ in exactly the same manner as ϕ_o . Therefore, ϕ_s must be given by an equation of the form

$$\phi_s = C_2 G_1(\xi) F_2(\eta) F_3(\zeta) \tag{2.12}$$

Note the similarity to equation (2.9).

To find $G_1(\xi)$, following Stratton⁽¹⁹⁾, one substitutes equation (2.12) in Laplace's equation (2.5) and solves for G_1 . This gives

$$\begin{aligned}
 &(\eta - \zeta) R_{\xi} \frac{\partial}{\partial \xi} \left(R_{\xi} C_2 (\eta + a^2)^{\frac{1}{2}} (\zeta + a^2)^{\frac{1}{2}} \frac{\partial G_1(\xi)}{\partial \xi} \right) \\
 &+ (\zeta - \xi) R_{\zeta} \frac{\partial}{\partial \zeta} (C_2 G_1(\xi) (\eta + a^2)^{\frac{1}{2}} (\zeta + b^2)^{\frac{1}{2}} (\zeta + c^2)^{\frac{1}{2}}) \\
 &+ (\xi - \eta) R_{\eta} \frac{\partial}{\partial \eta} (C_2 G_1(\xi) (\zeta + a^2)^{\frac{1}{2}} (\eta + b^2)^{\frac{1}{2}} (\eta + c^2)^{\frac{1}{2}}) = 0
 \end{aligned}
 \tag{2.13}$$

where

$$\begin{aligned}
 R_{\eta} &= (\eta + a^2)^{\frac{1}{2}} (\eta + b^2)^{\frac{1}{2}} (\eta + c^2)^{\frac{1}{2}} \\
 R_{\xi} &= (\xi + a^2)^{\frac{1}{2}} (\xi + b^2)^{\frac{1}{2}} (\xi + c^2)^{\frac{1}{2}} \\
 R_{\zeta} &= (\zeta + a^2)^{\frac{1}{2}} (\zeta + b^2)^{\frac{1}{2}} (\zeta + c^2)^{\frac{1}{2}}
 \end{aligned}
 \tag{2.14}$$

Simplifying the above and completing the derivatives with respect to ζ and η , one gets

$$R_{\xi} \left(\frac{\partial}{\partial \xi} R_{\xi} \frac{dG_1}{d\xi} \right) - G_1 C_3 = 0 \tag{2.15}$$

Rearranging, it can be seen that

$$\frac{1}{R_{\xi}} \frac{dR_{\xi}}{d\xi} \left(\frac{dG_1}{d\xi} \right) + \frac{d^2 G_1}{d\xi^2} - \frac{C_3}{R_{\xi}^2} G_1 = 0 \tag{2.16}$$

which is a second order differential equation of the form

$$\frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_2(x) y = 0 \tag{2.17}$$

It is already known from equation (2.9) that $G_1 = F_1(\xi)$ is a solution of Laplace's equation. Therefore, the solution for G_1 is (see Appendix B)

$$G_1(\xi) = C_4 F(\xi) \int_u^\infty \frac{1}{R_\xi} \frac{1}{F_1^2(\xi)} d\xi + C_5 F_1(\xi) \quad (2.18)$$

where the limits are chosen so that the first term vanishes at large ξ , i.e. the effect of the interaction is small if the particles are far apart (the concentration is low).

Substitution for G_1 in (2.12) and (2.7) gives

$$\phi_1 = C_1 F_1 F_2 F_3 + C_2 F_2 F_3 \left[F_1 C_4 \int_u^\infty \frac{1}{R_\xi} \frac{1}{F_1^2(\xi)} d\xi + C_5 F_1 \right] \quad (2.19)$$

If the substitutions $C'_1 = C_1 + C_5$ and $C'_2 = C_2 C_4$ are made, then the total potential outside the inclusion is

$$\phi_1 = C'_1 F_1 F_2 F_3 + C'_2 F_1 F_2 F_3 \int_u^\infty \frac{d\xi}{R_\xi (\xi + a^2)} \quad (2.20)$$

which becomes

$$\phi_1 = \frac{C'_1}{C_1} \left[1 + \frac{C'_2}{C'_1} \int_u^\infty \frac{d\xi}{R_\xi (\xi + a^2)} \right] \phi_0 \quad (2.21)$$

The solution for the potential inside the inclusion (ϕ_2) must now be obtained. Inside the inclusion the potential ϕ_2 must have the same form as ϕ_0 ; i.e. it cannot go to infinity at $\xi = -c^2$. This means

$$\phi_2 = C_6 \phi_0 \quad (2.22)$$

where ϕ_0 is given by equation (2.9) and C_6 is a constant.

Tinga et al⁽¹⁷⁾ express the solutions for ϕ_1 and ϕ_2 somewhat differently as

$$\phi_1 = \phi_0 [A_1 + B_1 F(u)] \quad (2.23)$$

and

$$\phi_2 = \phi_0[A_2]$$

where

$$F(u) = \int_u^{\infty} \frac{ds}{(s + a^2)V(s)} \quad (2.24)$$

and $V(s)$ is given by equation (2.6). For the b and c directions $(s + a^2)$ is replaced by $(s + b^2)$ and $(s + c^2)$ respectively.

Following Tinga et al⁽¹⁷⁾ for the remainder of the solution one gets, for the case of $\vec{E}$ (the electric field) parallel to "a",

$$\phi_1 = \phi_0, \text{ and } \epsilon_1 \frac{\partial \phi_1}{\partial u} = \epsilon_{AV} \frac{\partial \phi_0}{\partial u}, \text{ at } u = u_1$$

(2.25)

and

$$\phi_2 = \phi_1, \text{ and } \epsilon_2 \frac{\partial \phi_2}{\partial u} = \epsilon_1 \frac{\partial \phi_1}{\partial u}, \text{ at } u = u_2$$

for the boundary values, where u_2 is the boundary of the inclusion and u_1 is the boundary of the interaction zone. It may be shown that

$$\frac{\partial \phi_0}{\partial u} = \frac{\phi_0}{2(u + a^2)}$$

(2.26)

and

$$\frac{\partial F}{\partial u} = \frac{-F}{2(u + a^2)n^a}$$

where n^a is the depolarization coefficient for the "a" direction (i.e. for the field parallel to the major axis "a"). This is accomplished as follows:

from equation (2.9)

$$\frac{\partial \phi_0}{\partial u} = -E_0 \left[\frac{(\eta + a^2)(\zeta + a^2)}{(b^2 - a^2)(c^2 - a^2)} \right]^{\frac{1}{2}} \frac{\partial}{\partial u} (u + a^2)^{\frac{1}{2}} \quad (2.27)$$

which reduces to

$$\frac{\partial \phi_0}{\partial u} = \frac{\phi_0}{2(u + a^2)}$$

and also, from equation (2.24)

$$\frac{\partial F}{\partial u} = \frac{\partial}{\partial u} \int_u^{\infty} \frac{ds}{(s + a^2)V(s)} \quad (2.28)$$

which becomes

$$\frac{\partial F}{\partial u} = \frac{-1}{(u + a^2)V(u)}$$

Landau and Lifshitz⁽²⁰⁾ define

$$n^a(u) = \frac{1}{2}V(u)F(u) \quad (2.29)$$

which gives

$$\frac{\partial F}{\partial u} = \frac{-F(u)}{(u + a^2)n^a(u)} \quad (2.30)$$

Therefore, reduction of equation (2.25) is now possible.

In the rest of the derivation, the specific reference to the "a" direction can be removed by substitution of $(u + b^2)$ or $(u + c^2)$ for $(u + a^2)$ wherever necessary. Firstly, at $u = u_1$

$$\phi_0[A_1 + B_1F(u_1)] = \phi_0$$

which gives

$$A_1 + (2n_1/V_1)B_1 = 1 \quad (2.31)$$

Also,

$$\epsilon_1 \frac{\partial \phi_1}{\partial u} = \epsilon_1 \phi_0 \left[B_1 \frac{\partial F(u)}{\partial u} \right] = \epsilon_{AV} \frac{\partial \phi_0}{\partial u}$$

gives

$$A_1 \epsilon_1 - \epsilon_1 \left[\frac{2(1 - n_1)}{V_1} \right] B_1 = \epsilon_{AV} \quad (2.32)$$

Secondly, at $u = u_2$

$$\phi_0 A_2 = \phi_0 [A_1 + B_1F(u_2)]$$

which gives

$$A_2 = A_1 + (2n_2/V_2)B_1 \quad (2.33)$$

Also,

$$\epsilon_2 \frac{\partial \phi_2}{\partial u} = \epsilon_2 \frac{\partial}{\partial u} [A_2 \phi_0] = \epsilon_1 \frac{\partial}{\partial u} \left(\phi_0 [A_1 + B_1F(u)] \right)$$

which gives

$$\epsilon_2 A_2 = \epsilon_1 A_1 - \epsilon_1 \left[\frac{2(1 - n_2)}{V_2} \right] B_1 \quad (2.34)$$

In matrix form, equations (2.31) to (2.34) become

$$\begin{bmatrix} 1 & n_1/V_1 & 0 & 0 \\ \epsilon_1 & -\epsilon_1(1-n_1)/V_1 & 0 & -1 \\ -1 & -n_2/V_2 & 1 & 0 \\ -\epsilon & \frac{\epsilon_1(1-n_2)}{V_2} & \epsilon_2 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ 2B_1 \\ A_2 \\ \epsilon_{AV} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.35)$$

Manipulation of the augmented matrix yields the reduced matrix

$$\begin{bmatrix} 0 & T_2 & 0 & 0 & -\epsilon_1 + \epsilon_2 \\ 0 & T_1 & 0 & -1 & 0 \\ 0 & \frac{\epsilon_1 n_2}{V_2} + \frac{\epsilon_1(1-n_2)}{V_2} & (-\epsilon_1 + \epsilon_2) & 0 & 0 \\ -\epsilon_1 & \frac{\epsilon_1(1-n_2)}{V_2} & \epsilon_2 & 0 & 0 \end{bmatrix} \quad (2.36)$$

Therefore,

$$2B_1 T_2 = -\epsilon_1 + \epsilon_2$$

and

$$\epsilon_{AV} = (-\epsilon_1 + \epsilon_2)(T_1/T_2) \quad (2.37)$$

where

$$T_1 = \left[\frac{-\epsilon_2}{\epsilon_1 + \epsilon_2} \left(\frac{\epsilon_1 n_2}{V_2} + \frac{\epsilon_1(1-n_2)}{V_2} \right) - \frac{\epsilon_1(1-n_1)}{V_1} + \frac{\epsilon_1(1-n_2)}{V_2} \right]$$

and

$$T_2 = \left[(-\epsilon_1 + \epsilon_2) \left(\frac{n_1}{V_1} - \frac{n_2}{V_2} \right) - \left(\frac{\epsilon_1 n_2}{V_2} + \frac{\epsilon_1(1-n_2)}{V_2} \right) \right]$$

Solution of the above equations gives the desired self-consistent mixture formula,

$$\frac{\epsilon_{AV} - \epsilon_1}{\epsilon_1} = \frac{V_2}{V_1} \frac{(\epsilon_2 - \epsilon_1)}{\left[-\frac{V_2}{V_1} n_1 (\epsilon_2 - \epsilon_1) + n_2 (\epsilon_2 - \epsilon_1) + \epsilon_1 \right]} \quad (2.38)$$

where V_1 and V_2 are the volume fractions of host and inclusion respectively.

In the case of interest, i.e. ellipsoidal inclusions, the n_1 and n_2 are equal and are given by Kerker⁽²¹⁾ and others as

$$n_1^{b,c} = \left(\frac{b}{a}\right)^2 \left(\frac{1}{4e^3}\right) \left(\frac{2e}{\left(\frac{b}{a}\right)^2} + \log_e \left(\frac{(1-e)}{(1+e)}\right)\right) \quad (2.39)$$

and

$$n_1^a = \left(\frac{b}{a}\right)^2 \left(\frac{1}{2e^3}\right) \left(-2e + \log_e \left(\frac{(1+e)}{(1-e)}\right)\right) \quad (2.40)$$

where the axes b and c are equal and smaller than the " a " axis. e is the eccentricity defined by

$$e = \left(1 - \left(\frac{b}{a}\right)^2\right)^{1/2} \quad (2.41)$$

A plot of n versus a/b is shown in Figure 2.4. It is apparent from the plot that for axial ratios greater than 20 the depolarization coefficient changes very little.

Equation (2.38) now allows calculation of two orthogonal dielectric constants ϵ_{AV_a} and ϵ_{AV_b} , where the " a " subscript denotes that the electric field is applied parallel to the " a " axis and the " b " subscript denotes that the field was applied parallel to the " b " axis. Calculation of each of these is accomplished by using the correct n^a or n^b in equation (2.38). For angles in between, it is assumed that for the horizontally (perpendicular) polarized component of the incident electric field the dielectric constant is always ϵ_{AV_b} irregardless of angle, and that for the vertically (parallel) polarized component of the wave, the equivalent dielectric constant is the same as that for a uniaxial anisotropic crystal and is given by Landau and Lifshitz⁽³⁰⁾ as

$$\frac{1}{\epsilon_{AV}} = \frac{\sin^2 \theta}{\epsilon_{AV_a}} + \frac{\cos^2 \theta}{\epsilon_{AV_b}} \quad (2.42)$$

where θ is the angle between the " a " axis and the normal to the incident

Figure 2.4 Depolarization factors calculated from Kerker ⁽²¹⁾ for ellipsoidal particles, as a function of the ellipsoidal axial ratio.

electric field and ϕ is arbitrary (see Figure 2.5). It is easily seen that when $\theta = 0$, the two cases of vertical and horizontal polarization are equivalent.

A medium composed of such ellipsoids will, provided all the ellipsoids are at the same angle, propagate two modes of waves: extraordinary waves, which see a dielectric constant which varies with angle; and ordinary waves, which see a constant dielectric constant regardless of incident angle.

With the exception of the mention above of the extraordinary and ordinary waves, the extension of the above equation (2.38) to the non-static case has not been discussed. Taylor⁽¹⁸⁾ stipulates that such a quasi-static treatment should be limited to wavelengths λ such that

$$\lambda \gg l$$

where l is the largest dimension of the ellipsoid. A simple substitution of ϵ_1 and ϵ_2 at the frequency concerned yields ϵ_{AV} . A more detailed approach is discussed in the next section under scattering behaviour.

Thus far a method has been developed that allows calculation of ϵ_{AV} for various polarizations of incident electric fields, relatively high concentrations of particles, and provides for two complex constituent dielectrics which can be dispersive with frequency. The extension to multiple dielectrics is made by repeated application of equation (2.38).

deLoor⁽¹³⁾ lists the effects which contribute to the average dielectric constant of mixtures containing water. These are: charged double ion layers ($10^0 - 10^5$ Hz), d.c. conductivity ($10^0 - 10^9$ Hz), ice relaxation ($10^2 - 10^4$ Hz), Maxwell/Wagner losses ($10^2 - 10^6$ Hz), crystal water relaxation ($10^1 - 10^4$ Hz), surface conductivity ($10^3 - 10^9$ Hz),

Figure 2.5

Inclusion orientation, θ is angle with respect to the vertical, ϕ is the azimuthal angle.

Figure 2.6

Geometry for Rayleigh Scattering, θ is the angle the scattered ray makes with the vertical, the incoming ray is incident along the z axis, ϕ is the angle the projection of r makes with the x axis, and ψ is the angle r makes with the x axis (from Kerker (21)).

bound forms of water relaxation ($10^6 - 10^9$ Hz), and water relaxation ($10^9 - 10^{11}$ Hz). In the microwave frequency range 10^8 Hz - 10^{12} Hz only the water relaxation and conductivity effects are important and therefore it can be expected that the mixture dielectric constant will reflect this.

Further discussion of the effect of frequency on the validity of equation (2.38) is given in the following section and in Appendix A.

2.4 Basis for Extension of the Static Theory to Microwave Frequencies

The previous section has given insight into the problems involved in extending the electrostatic theory of mixtures to microwave frequencies. Several assumptions were made. These were: the particles are so small with respect to a wavelength that scattering does not occur; the mixture can be treated as particulate, i.e. the inhomogeneous nature of the medium can be focused in discrete particles; and the extent of the medium has no effect on the observed equivalent dielectric constant. The next two sections examine briefly the truth of these assumptions and outline how the quasi-static theory developed above may be used at microwave frequencies.

2.4.1 Rayleigh Scattering

The following summary of the theory of Rayleigh scattering by small dielectric spheres is taken from Kerker. (21)

The basic premise intrinsic to Rayleigh's treatment of scattering is that when an isolated sphere is illuminated by a parallel beam of light, because of its small size with respect to a wavelength, the instantaneous electric field that it sees is uniform over its extent. Therefore, the treatment reduces to the electrostatic case of finding the perturbation in the electric field due to the presence of

the sphere. This problem is treated by Stratton.⁽¹⁹⁾ The resulting electric field intensity is given by

$$\vec{E}_{INT} = \left[3\epsilon_2 / (\epsilon_1 + 2\epsilon_2) \right] \vec{E}_O \quad (2.43)$$

where $\vec{E}_O$ is the original externally applied field and ϵ_1 and ϵ_2 are the dielectric constants of the sphere and external medium respectively. The external field thus consists of $\vec{E}_O$, the applied field, and $\vec{E}_{INT}$, the induced field. The sphere acts like a dipole oriented parallel to the induced field and possessing a dipole moment

$$\vec{p} = 4\pi\epsilon_2 a^3 \left[(\epsilon_1 - \epsilon_2) / (\epsilon_1 + 2\epsilon_2) \right] \vec{E}_O \quad (2.44)$$

where "a" is the radius of the sphere.

If the incident field $\vec{E}_O$ is varied harmonically, then the induced field follows synchronously and the sphere acts as an oscillating electric dipole which radiates secondary or scattered waves in all directions. For an incident wave of unit intensity, at an angle θ , as shown in Figure 2.6, the intensity of the scattered wave at a distance r from the particle is given by Stratton⁽¹⁹⁾ as

$$I = \frac{16\pi^4 a^6}{r^2 \lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \sin^2 \psi \quad (2.45)$$

where m is the relative complex refractive index $\sqrt{\epsilon_1/\epsilon_2}$ (with the imaginary parts small) and λ is the wavelength.

If equation (2.45) is integrated the total energy scattered by a particle in all directions, or the scattering cross-section, C_{sca} , is obtained as

$$C_{sca} = \int_0^\pi \int_0^{2\pi} I r^2 \sin \psi d\psi d\phi \quad (2.46)$$

where ϕ is the angle the projection of the scattered ray on the x-y plane makes with the x axis (incidence along the z axis is assumed). Completion of the integral yields

$$C_{sca} = \frac{24\pi^3 V^2}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad (2.47)$$

where V is the volume of the particle. The efficiency factor for scattering, Q_{sca} , is obtained by dividing by the actual geometrical cross-section. For a Rayleigh scatterer, this gives

$$Q_{sca} = \frac{128\pi^4 a^4}{3\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad (2.48)$$

If

$$\alpha = \frac{2\pi a}{\lambda}$$

then

$$Q_{sca} = \frac{8}{3} \alpha^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad (2.49)$$

It is particularly important to know the loss due to scattering that a wave experiences when traversing a medium of length l . For N particles per unit volume, the attenuation due to scattering is $NC_{sca}l$. The transmission coefficient is

$$T = \frac{I_l}{I_0} = \exp(-NC_{sca}l) = \exp(-\tau l) \quad (2.50)$$

where I_l is the intensity of the wave emerging at a distance l , I_0 is the incident intensity and τ is the turbidity. In microwave work, the transmission coefficient is normally a ratio of wave amplitudes, therefore this T corresponds to a power transmission coefficient.

Thus far, the theory presented allows the calculation of the losses due to scattering that a wave experiences when traversing a medium containing spherical particles which are limited in size to $a \leq 0.05\lambda$, where λ is the wavelength in the medium.

The development can be further extended to cover the case of a small ellipsoid. This method is of particular importance since most natural particulate material is much better represented by ellipsoids than by spheres. The basic approach is quite simple. All

the radii "a" in the previous equations are replaced by an equivalent radius "a'". This equivalent radius is calculated on the basis of the increase in polarizability caused by the ellipsoidal shape. For the field component along the semi-major axis, A, the polarizability α_A' is

$$\alpha_A' = \frac{V(m^2 - 1)}{4\pi + (m^2 - 1)P_A} \quad (2.51)$$

where

$$P_A = \int_0^\infty \frac{2\pi ABC ds}{(s + A^2)^{3/2} (s + B^2)^{1/2} (s + C^2)^{1/2}} \quad (2.52)$$

and B and C are the semi-minor axes of the ellipsoid. The respective $P_{A,B,C}$ are just 4π times the depolarization factors n in equations (2.39) and (2.40). The equivalent radius a' is now calculated using

$$(a')^3 = \frac{4\pi ABC}{3} \frac{(m^2 + 2)}{4\pi + (m^2 - 1)P_{A,B, \text{ or } C}} \quad (2.53)$$

The equivalent radius in most cases is larger than the actual radius. This results in the ellipsoid being a stronger scatterer in some directions than a sphere of the same volume.

The theory outlined above is limited by the fact that $a < 0.05\lambda$ and that only simple shapes and single scattering are treated. However, it allows one to calculate the actual propagation loss due to scattering in a particular measurement scheme and decide whether or not scattering effects can be neglected. Also, it is a refinement over Taylor's⁽¹⁸⁾ earlier proposal that the quasi-static equations hold for $a \ll \lambda$, in that "a" can now be approximately equal to λ if the ellipsoid is viewed endwise, but "a'" must be less than or equal to 0.05λ if it is viewed broadside.

The results still only pertain to particulate mixtures,

and the effect of the sample size observed, is still only limited to a loss calculation. The next section examines what happens as the inclusions become less centralized and notes some interesting effects in the observed equivalent dielectric constant as the depth of sample increases.

2.4.2 Effects of Random Inhomogeneities

The above theories can be used to treat dielectric irregularities in the volume when they occur in the form of particles contained in a homogeneous substrate. However, small irregularities or fluctuations in the dielectric constant can also give rise to scattering.

Stogryn⁽²²⁾ has presented a technique useful for calculating the scattering from nonhomogeneous media. The dielectric constant, κ^r , of a bounded half space, is assumed to be given as follows

$$\kappa^r = \begin{cases} 1, & z > 0 \\ K(z) + K'(r), & z < 0 \end{cases} \quad (2.54)$$

Note that the mean dielectric constant $K(z)$ is a function of depth only, whereas $K'(r)$ is a random fluctuation superimposed on $K(z)$ and is not a function of depth.

The electric field $\vec{E}(r)$ is determined from the solution of the wave equation

$$\nabla \times \nabla \times \vec{E} - \gamma_0^2 \kappa^r \vec{E} = 0 \quad (2.55)$$

where $\gamma_0 = \sqrt{\omega^2 \mu \epsilon_0}$ is the propagation constant, and the conductivity $\sigma = 0$ (see Appendix A). For an arbitrary $K'(r)$ equation (2.55) has no solution. However, if K' is assumed small, i.e. a perturbation, Karal and Keller⁽²³⁾ have shown that $\vec{E}$ can be decomposed into the sum

$$\vec{E} = \vec{E}^m + \vec{E}^r \quad (2.56)$$

where $\vec{E}^m$ is the mean field and $\vec{E}^r$ is the random field, with expectation

zero, ($\langle E^r \rangle = 0$).

For propagation in an unbounded, statistically homogeneous and isotropic medium with small scale inhomogeneities, a set of validity criteria is

$$\begin{aligned} |(\langle K'^2 \rangle / K^2) \gamma_0^2 \alpha^2| &\ll 1 \\ |\langle K'^2 \rangle / K^2| &\ll 1 \end{aligned} \quad (2.57)$$

where α is the correlation length associated with the inhomogeneities (Ryzhov and Tamoikin. (24) (Note: correlation length is roughly defined as the average distance between fluctuations in the dielectric constant.)

After a series of complicated manipulations on equations (2.56) and (2.55), Stogryn arrives at the equations

$$\begin{aligned} \nabla \times \nabla \times \vec{E}^m - \gamma_0^2 \vec{E}^m &= 0, \quad z > 0 \\ \nabla \times \nabla \times \vec{E}^m - \gamma_0^2 [K(z) + \kappa(z)] \vec{E}^m &= 0, \quad z < 0 \end{aligned} \quad (2.58)$$

where the dielectric constant κ^r has become $K(z) + \kappa(z)$ and the random part of κ^r has been replaced by a deterministic function of depth $\kappa(z)$. Therefore, even though the mean dielectric constant may not vary with depth, the presence of inhomogeneities in a bounded half-space causes the equivalent dielectric constant to be depth dependent.

κ has the form

$$\kappa = \begin{pmatrix} \kappa_1 & 0 & 0 \\ 0 & \kappa_1 & 0 \\ 0 & 0 & \kappa_2 \end{pmatrix} \quad (2.59)$$

where

$$\begin{aligned} \kappa_1(z) &= (4\pi)^{-1} \gamma_0^2 \int_0^\infty \rho d\rho \int_{-\infty}^0 dz' W(\rho, z, z') [a_{\psi\psi} + a_{\rho\rho}] \\ \text{and} \\ \kappa_2(z) &= (2\pi)^{-1} \gamma_0^2 \int_0^\infty \rho d\rho \int_{-\infty}^0 dz' W(\rho, z, z') a_{zz} \end{aligned} \quad (2.60)$$

The $a_{\psi\psi}$, $a_{\rho\rho}$ and a_{zz} are defined as follows

$$\begin{aligned}
 a_{\psi\psi} &= (j/2\gamma_1) \left\{ e^{j\gamma_1 u} + [(\gamma_1 - \gamma_2)/(\gamma_1 + \gamma_2)] e^{-j\gamma_1(z + z')} \right\} \\
 a_{\rho\rho} &= (j\gamma_1/2\gamma_0^2 K) \left\{ e^{j\gamma_1 u} + [(\kappa\gamma_2 - \gamma_1)/(\kappa\gamma_2 + \gamma_1)] e^{-j\gamma_1(z + z')} \right\} \\
 a_{zz} &= -\delta(u)/\gamma_0^2 K + (j\rho^2/2\gamma_0^2 \kappa\gamma_1) \left\{ e^{j\gamma_1 u} - [(\kappa\gamma_2 - \gamma_1)/(\kappa\gamma_2 + \gamma_1)] e^{-j\gamma_1(z + z')} \right\}
 \end{aligned} \quad (2.61)$$

and

$$\gamma_1 = (\gamma_0^2 K - \rho^2)^{1/2}$$

$$\gamma_2 = (\gamma_0^2 - \rho^2)^{1/2}$$

$$u = |z - z'|$$

where ψ , ρ and z are the cylindrical coordinates.

W is defined as

$$W = \langle K'^2 \rangle V$$

where

$$\begin{aligned}
 V &= 2\pi\alpha^2 (1 + \alpha^2 \rho^2)^{-3/2} [1 + (1 + \alpha^2 \rho^2)^{1/2} |z - z'|/\alpha] \\
 &\quad \exp[-(1 + \alpha^2 \rho^2)^{1/2} |z - z'|/\alpha]
 \end{aligned} \quad (2.62)$$

and an exponential correlation function

$$u(r, r') = \exp(-|r - r'|/\alpha) \quad (2.63)$$

has been assumed.

There are accordingly two functions κ_1 and κ_2 such that

$$K_{1,2} = K(z) + \kappa_{1,2}(z) \quad (2.64)$$

For a mixture of two dielectrics $K^{(1)}$ and $K^{(2)}$, with

volume fractions v and $(1 - v)$, the mean dielectric constant is

$$K = vK^{(1)} + (1 - v)K^{(2)} \quad (2.65)$$

using a very simple linear mixture formula. Using the equations given for W and the expressions for κ_1 and κ_2 , K_1 and K_2 can be evaluated.

Figures 2.7 and 2.8 are taken from Stogryn⁽²²⁾ to illustrate the effect of random inhomogeneities on the real and imaginary parts of $K_{1,2}$ for

Figure 2.7 Variation of the $\text{Re}\{K_{1,2}\}$ with z (from Stogryn⁽²²⁾).

Figure 2.8 Variation of the $\text{Im}\{K_{1,2}\}$ with z (from Stogryn⁽²²⁾).

snow of density 0.46 gm/cm^3 . He notes that the asymptotic dielectric losses are higher than expected from the mean dielectric constant given by equation (2.65) and that there is an impedance matching effect at the interface, i.e. a gradual transition in $K_{1,2}$.

The impedance matching effect is probably the most important factor since the difference in loss could be due to the fact that equation (2.65) is a poor choice of mixture equation and holds only for very dilute mixtures. This reduction in equivalent dielectric constant at the interface means that a mixture equation will not give a true value for the equivalent dielectric constant unless the transition region is a small fraction of the sample measured. Therefore, correct choice of sample size can minimize any error introduced in an equivalent dielectric constant calculated by the method of Tinga et al⁽¹⁷⁾ (equation (2.38)).

Stogryn's treatment holds only for correlation lengths small with respect to a wavelength and therefore is somewhat limited in application at high microwave frequencies. Also, it leads to the conclusion (not expounded here) that such a random medium will not depolarize a wave. However, random media do significantly depolarize waves (see Wilhelmi et al⁽²⁵⁾, Leader and Dalton⁽²⁶⁾, Leader⁽²⁷⁾, and Reneau et al⁽²⁸⁾ for a description). Therefore, some of the assumptions made are not entirely correct. Despite this, it demonstrates the effect of random inhomogeneities in producing a deterministic variation in $K_{1,2}$ with depth, a phenomenon not directly evident from the Rayleigh treatment.

Thus, the two techniques covered here allow a calculation of the loss due to scattering for "particles" smaller than a wavelength and permit a quantitative evaluation of the effects of scattering on the quasi-static mixture equations. It may be useful in suspicious

cases to apply a "first-order" Rayleigh scattering analysis to estimate the total energy scattered in the volume before either ignoring the scattering effects as minimal or performing a more complete analysis. Unfortunately, all the techniques except those of Mie⁽²¹⁾ (the exact solution, not covered here) require a small inhomogeneity, or particle, with respect to a wavelength, which may or may not be the case in the material under investigation.

CHAPTER 3. THE SEA ICE MEASUREMENT PROBLEM

3.1 Physical Properties of Sea Ice

3.1.1 First-year Sea Ice

Sea ice is a complex medium composed of a mixture of pure ice, liquid brine, air bubbles, and hydrolyzed salts. In highly simplified terms, first-year sea ice is composed of a layer of primary ice made up of either small, randomly oriented, equiaxed crystals, similar in structure to fresh water frazil ice crystals, or randomly oriented columnar type ice crystals. Beneath this primary layer the ice is composed of long vertical columnar ice crystals with a predominantly horizontal c-axis orientation. The very bottom of the ice layer consists of a short transition region, usually up to 5 cm. in length. This region has a dendritic structure; the ends of the platelets which form the crystal grains are clearly visible when viewed from the bottom. It is usually characterized by a very high salinity and slushy texture.

The presence of salts such as NaCl, Mg_2Cl_2 , etc. in sea water causes sea ice to be very much different in character from ice formed in fresh waters. Sea ice is a multi-phase solution and as such, its properties change drastically with temperature. A simplified phase diagram for sea ice is shown in Figure 3.1.⁽³¹⁾ Above the freezing point, for a particular salinity, the mixture is pure brine; then at the freezing point the brine begins to form ice containing brine inclusions and hydrolyzed salts. Below approximately $-30^{\circ}C$ almost all the brine has formed ice and hydrolyzed salts. If the salinity is increased far enough, the ice will not go through this intermediate stage, but will form an ice plus hydrolyzed salts mixture directly.

The brine is contained in the ice cover in the form of

Figure 3.1 Sea ice phase diagram, Assur⁽³¹⁾.

small inclusions approximately cylindrical or ellipsoidal in shape. They occur primarily between the platelets that form the individual crystal grains, as shown in Figure 3.2. Photomicrograph measurements by Anderson and Weeks⁽²⁹⁾ have shown the platelets to be spaced an average of $a_0 = 0.46$ mm. apart with the spacing between inclusions b_0 being 0.23 mm. The average length and radius of the inclusions are $c_0 = 3$ to 5 mm. (Pee et al⁽⁹⁾) and $r_0 = .025$ mm. (Pounder⁽³⁰⁾) respectively. It is generally assumed that these microscopic brine inclusions are aligned in the growth direction, i.e. they are predominantly vertical. Assur⁽³¹⁾ has formulated a model for the inclusion shape and size as a function of temperature and salinity.

Superimposed on this microstructure is a more diversified system of drainage channels which link the larger inclusions and allow a flow of brine into and out of the ice sheet. Figure 3.3 shows the general shape of this pattern.

Bennington⁽³²⁾ has shown that there is evidence of small inter-platelet brine pockets perpendicular to the c-axis, i.e. predominantly vertical. In Figure 5 of his paper, he shows these small brine pockets with a larger drainage channel clearly visible at the edge of the crystal boundary. Two other figures show angled striations, which he interprets as inter-platelet inclusions and larger drainage channels; and platelets with their corresponding inclusions oriented at approximately 40° to the vertical. Also presented are photographs of the dendritic layer, which is seen to be approximately 2 cm. in length.

In a later paper, Bennington⁽³³⁾ shows further evidence of these drainage networks and attempts to explain the types and origins of the brine drainage features. He divides the drainage networks into

BRINE INCLUSIONS
BETWEEN PLATELETS

Figure 3.2 Brine inclusions between the platelets.

Figure 3.3 Conceptual view of a brine drainage channel.

two principal types: those characteristic of growth, and those characteristic of deterioration.

The first subtype of channel he terms the "inherited channel". Inherited channels are formed during initial growth and are usually straight tubes in excess of 32 cm. in length. The inherited channels are not consistently related to individual channels or crystal boundaries.

Another type of first generation drainage channel is formed by an interconnecting network of brine pockets linking the platelets. These have the characteristic cone-shape, shown in Figure 3.3, and are easily visible to the naked eye as whitish streaks in the ice. These networks join the smaller sub-channels and inclusions to the major drainage tubes.

The second generation drainage channels develop at the interface between the snow overburden layer and the sea ice. They are a result of snow ablation corroding its way down through the ice cover. These features may open up new channels.

Bennington⁽³²⁾ observes that in young ice, the crystal boundaries are commonly sealed and no evidence is observed that channels are preferentially located along the crystal boundaries. In ice 20 to 30 cm. thick, only the first generation channels appear; the second generation channels occur only in ice 100 cm. or more in thickness.

Bennington⁽³²⁾ notes that irregardless of the ambient temperature, freshly cut ice specimens are seen to "bleed" tiny droplets of brine from all the freshly cut surfaces. This process does not seem to be a function of the time of year the samples are collected. For a large specimen, this volume of brine lost repre-

sents only a tiny fraction of the total brine content. He also notes that this expulsion of brine is probably due to the relief of some internal stress.

Lake and Lewis⁽³⁴⁾ have also observed drainage patterns in first-year sea ice. Their measurements indicate that at the ice-water interface the drainage tubes have a diameter of 0.3 to 0.5 mm. They also affirm that the normal placement of the inclusions is between the platelets. The diameter of the large first generation channels is approximately 0.7 cm. with the radial arms being 2 to 3 cm. in length and approximately 2 to 8 mm. in width. The spacing of these major channels is estimated at approximately one every 13 cm. The channels were found to be filled with a porous polycrystalline mass containing trapped brine. The feed arms are comprised of inclusions forming conical patterns with apices on the main channel and semi-angles of 40° to 50° (see Figure 3.3). The largest most well developed channels are found 10 to 20 cm. above the interface with the channels higher up in the ice sheet forming only a fossil remnant of a previously active area. Communication along the brine channels may exist and could be formed by expulsion of brine due to differential thermal expansion, i.e. the partial freezing of brine pockets and subsequent buildup of pressure. Lake and Lewis go so far as to state that "clearly such cracks would tend to follow the lines of natural weakness such as the crystal boundaries".

Eide and Martin⁽³⁵⁾ have further investigated the brine expulsion mechanism using artificial first-year sea ice (see section 3.1.3). They observed vertical channels approximately 1 cm. in diameter, extending approximately 60 cm. into a 1.6 m. ice sheet, and spaced approximately 10 to 20 cm. apart. With the aid of dyed salt

water, they observed migration of brine into and out of the channels during growth of the sheet. Freezing caused reduction in size of the channels and buildup of hydrostatic pressure. This pressure then causes expulsion of some of the brine with a subsequent rushing in of lower salinity sea water to fill the volume vacated, and refreezing to start the cycle anew. This expulsion mechanism leads to a regular pumping action and slow migration of brine out of the ice sheet.

Other mechanisms have been postulated to explain movement of the brine. These are: (1) Brine pocket diffusion by Whitman⁽³⁶⁾ (this mechanism of individual motion of brine inclusions due to slow localized melting in front of the inclusion is probably much too slow to account for drainage rates observed), (2) Gravity drainage by Kingery and Goodnow⁽³⁷⁾ (this method relies solely on the action of gravity to pull the brine out of the ice sheet), (3) Flushing out by surface water suggested by Untersteiner⁽³⁸⁾ (this mechanism is primarily just the formation of second generation channels as suggested by Bennington⁽³²⁾ and is very important in explaining the transition from first-year ice to multiyear ice), and (4) Brine expulsion as explained by Bennington⁽³²⁾ and Eide and Martin.⁽³⁵⁾ Only mechanism (4) really explains the presence of a brine slick or slime on top of very young (or gray) ice. This slick could just be the result of expulsion of brine by the near surface inclusions in the growing ice sheet.

Due to these drainage mechanisms, the ice changes in character with age. Young ice (< 30 cm. thick) has extremely high salinity ($\approx 16\%$) and is commonly known as gray ice or gray-white ice. First-year ice is not more than one year old and is 30 cm. to 2 m. thick. It has a relatively high salinity (5% - 8%). Multi-year ice, ice more than 2 m. thick, will be described in the next

section.

The total ice sheet can be characterized by what is known as a salinity profile, or the amount of salt in parts per thousand by weight plotted versus depth in the ice sheet. Young ice has the characteristic C-shaped profile shown in Figure 3.4. First-year ice has a more gradual C-shape or multiple of C-shapes depending on the history of the ice, i.e. whether or not it has been rafted. The C-shape is caused by the active drainage mechanisms expelling brine from the centre of the ice sheet. The bottom part of the sheet is still in reasonably good communication with the sea below and the top of the sheet is effectively sealed off from further drainage but receives some of the brine actively expelled from the centre of the sheet.

Several workers have attempted to identify basic salinity profiles. Untersteiner⁽³⁸⁾ cautions that a large number of cores (>13) must be taken to have a representative mean salinity for an area, i.e. to predict the true mean with 90 % confidence, and with a $\pm .2$ % standard error, more than thirteen samples must be taken. Weeks and Lee⁽³⁹⁾ have measured the salinity of young pancake ice (ice that has broken up and refrozen forming large disc-like patterns that resemble pancakes) and young sheet ice. The sheet ice gave values of $9.7 \pm .6$ % in the upper third, $6.9 \pm .3$ % in the middle third, and $8.7 \pm .5$ % in the lower third. For pancake ice the average values were: $8.4 \pm .9$ % upper, $6.9 \pm .8$ % middle, and 5.9 ± 1.1 % lower.

It should be noted that these salinity profiles are usually obtained by removing a core or block from the ice and immediately sectioning it, and bagging the samples. Later, the salinity of the melt water is measured with a conductivity bridge, or by titration.

Figure 3.4 Salinity and Temperature profile of young sea ice, taken from the AIDJEX site (spring, 1975).

Typical accuracies of these instruments are $\pm .2 \%$ for commercial conductivity bridges and $\pm .02 \%$ for titration. The variation of salinity from one location to another in close proximity and at the same depth in the sheet is usually large enough ($> .2 \%$) so that more accurate techniques are not required. This variation is probably caused by the presence of the large brine drainage channels. A section directly through a major channel will have a slightly different salinity than its neighbouring section, which does not contain such a channel.

Measurements by the author and others have shown that the ice cores may be stored for long periods of time (many weeks) prior to sectioning with no detectable change in salinity, providing their temperature is kept well below -25°C . It also appears that for reasonably sized specimens the amount of brine lost by bleeding from the freshly cut surfaces is negligible compared to the total brine volume, providing the ambient temperature is low enough to prevent the thawing of major drainage channels.

The relative volume of brine contained in the ice sheet (known as brine volume) has been calculated by Assur.⁽³¹⁾ He assumes an average density for non-porous sea ice of 0.926 gm/cm^3 . The actual density of first-year sea ice has been measured by Zubov⁽⁴⁰⁾ and was found to be a function of temperature and salinity. The values for non-porous ice are plotted in Figure 3.5 and are seen to vary from 0.923 gm/cm^3 to 0.935 gm/cm^3 for the range of salinities found in first-year ice at temperatures above -5°C . Therefore, for the accuracy required in most calculations, this assumption of constant density should not be too critical.

Porous ice has a lower density than bubble-free ice. Its density can be calculated directly on the basis of the volume of air

Figure 3.5 Density of non-porous first-year sea ice, data from Zubov. (40)

introduced. These air bubbles are formed as a result of the escape of dissolved gases from the water as it freezes and are not, as in multiyear ice, a product of recrystallization.

Frankenstein and Garner⁽⁴¹⁾ and Poe et al⁽⁴²⁾ have evaluated curves to fit Assur's brine volume data. Their relations are

$$v = (2.28 + \frac{52.56}{T}) 10^{-3} \cdot S \quad -2.06^{\circ}\text{C} \leq T < -1.5^{\circ}\text{C}$$

$$v = (.930 - \frac{45.917}{T}) 10^{-3} \cdot S \quad -8.2 \leq T < -2.06$$

$$v = (1.189 - \frac{43.795}{T}) 10^{-3} \cdot S \quad -22.9 \leq T < -8.2$$

$$v = (22.8478 + \frac{3.07984 \cdot 10^3}{T} + \frac{1.58402 \cdot 10^5}{T^2} + \frac{3.61615 \cdot 10^6}{T^3} + \frac{3.12862 \cdot 10^7}{T^4}) 10^{-3} \cdot S \quad (3.1)$$

$$-37.8 \leq T < -22.9$$

$$v = (14.145 + \frac{1.6426 \cdot 10^3}{T} + \frac{6.4947 \cdot 10^4}{T^2} + \frac{8.3945 \cdot 10^5}{T^3}) 10^{-3} \cdot S \quad -43.2 \leq T < -37.8$$

where S is salinity in parts per thousand. A plot of these relations is shown in Figure 3.6.

The brine volume model assumes that all the brine is included in finite length cylinders, the shape and size of which were described above.

The salinity of the brine itself is seen to vary as the brine inclusions change shape. Poe et al⁽⁴²⁾ give the following relation for the brine salinity in parts per thousand, computed from Assur's⁽³¹⁾ data

$$\begin{aligned}
 S_D &= 1.725 - 18.756T - .3964T^2 \quad -8.2^\circ\text{C} \leq T \leq -2^\circ\text{C} \\
 S_D &= 57.041 - 9.929T - .16204T^2 - .002396T^3 \\
 &\quad -22.9 \leq T \leq -8.2 \quad (3.2)
 \end{aligned}$$

$$S_D = 242.94 + 1.5299T + .04291T^2 \quad -36.8 \leq T \leq -22.9$$

$$S_D = 508.18 + 14.535T + .2018T^2 \quad -43.2 \leq T \leq -36.8$$

Figure 3.7 compares Zubov's⁽⁴⁰⁾ brine salinity data with that of Assur.⁽³¹⁾ No great difference is seen above temperatures of -12°C .

Figures 3.8(a) and 3.8(b) show representative thick sections of young ice and first-year ice taken by the author and others from the AIDJEX camp area on the Beaufort Sea. Figure 3.9 summarizes the basic features of first-year ice. Also, Figure 3.10 portrays representative salinity and density profiles.

It is apparent that the placement and orientation of the brine inclusions in the ice sheet will have a profound influence on the dielectric properties. The brine has a much higher real part of the dielectric constant ϵ' , and imaginary part ϵ'' , than pure ice, and therefore despite the smaller quantity of brine present its influence will dominate the dielectric behaviour of the mixture.

3.1.2 Multiyear Sea Ice

Multiyear sea ice differs significantly from first-year sea ice in both physical properties and topography. This difference is due to the ice cover having undergone a period of summer melt.

During the summer, the ambient temperatures over the Beaufort Sea are of the order of $+5^\circ\text{C}$. This is sufficient to cause substantial melting of the ice cover and snow overburden. This melting causes the ice cover to take on a gently rolling topography. The elevated areas are known as hummocks and the depressions which fill

Figure 3.6 Brine volume fraction versus temperature for sea ice of density 0.926 gm/cm^3 , and salinity 1‰ . From the data of Assur⁽³¹⁾ and the equations of Poe et al⁽⁴²⁾ and Frankenstein and Garner.⁽⁴¹⁾

Figure 3.7. Brine salinity versus temperature showing data of Assur(31) and Zubov(40) and the equation of Poe et al(42).

Figure 3.8(a) Thick section of Young Ice from the AIDJEX site. (spring, 1975).

Figure 3.8(b) Thick section of First-Year Ice from the AIDJEX site.
(spring, 1975).

Figure 3.9 Simplified geometry of first-year sea ice.

Figure 3.10

Salinity, temperature and density profiles for first-year sea ice from the AIDJEX site (spring, 1975).

with melt water are called meltponds. With the onset of higher temperatures, the ice itself starts to decay. Second generation drainage channels percolate down through the ice cover. The exact processes which take place in the ice sheet during the summer are not well understood, but it is apparent from examination of thick and thin sections of multiyear ice that recrystallization must take place. The ice becomes very porous, with large air bubbles forming where there used to be brine filled inclusions, at least above the water line. The salinity is drastically lower than before, typically $< 1\%$. Below the freeboard the crystal structure appears less altered; the ice is clear and not filled with as many air bubbles. These features are illustrated by Figures 3.11 to 3.14. Figure 3.11 shows a particular set of salinity and density profiles for multiyear ice; Figures 3.12 and 3.13 show typical vertical thick sections (Figure 3.12 is taken through a hummock, and Figure 3.13 is representative of a meltpond). Finally, Figure 3.14 attempts to summarize all the features discussed, with a symbolic reconstruction of multiyear ice.

Cox and Weeks⁽⁴³⁾ have measured the salinity profiles of multiyear ice and have found two distinct profiles. Their profile for a hummock agrees with the one previously presented by Schwarzacher⁽⁴⁴⁾ whereas their depression profile was distinctly different from Schwarzacher's. Unfortunately, the standard error in these average profiles is of the order of 80 to 100 % of the measured values.

The brine inclusion shape, size and distribution in multiyear ice is not well understood, and as far as could be determined, nothing has yet been published on this subject.

This lack of information points out an area where dielectric

Figure 3.11 Salinity, temperature and density profiles for multiyear sea ice from the AIDJEX site (spring, 1975).

Figure 3.12 Thick section through a Multiyear Ice Hummock on the AIDJEX site (spring, 1975).

Figure 3.13 Thick section through a Multiyear Ice Meltpond on the AIDJEX site (fall, 1975).

Figure 3.14 Simplified geometry of multiyear sea ice.

measurements could, through a process of careful modelling, contribute to our knowledge of the shape and distribution of the brine inclusions. This topic will be discussed further in Chapter 7.

3.1.3 Artificial Sea Ice

Artificial sea ice is a laboratory grown facsimile of first-year sea ice. It has been employed by Cox and Weeks⁽⁴³⁾, Eide and Martin⁽³⁵⁾, Addison⁽⁴⁵⁾ and others.

Artificial sea ice is effectively simulated by using the recipe of Kester et al⁽⁴⁶⁾ to make a 35 ‰ solution of saline water, and then freezing this solution in a large stainless steel tank. The freezing is accomplished by having the tank, which is well insulated everywhere except at the open top, in a large walk-in cold room. The ambient temperature is set at approximately -25°C . The growth is effectively isothermal and results in large vertically oriented columnar ice crystals. The salinity profiles and structure have been compared to actual first-year sea ice and shown to be very similar. Crystal size and salinity of the ice are controlled by the ambient temperature during growth and the salinity of the original solution respectively. Unfortunately, only thin sheets may be grown, for there is no provision for recirculation or dilution of the water under the ice sheet during growth.

Eide and Martin⁽³⁵⁾ have verified that this type of artificial sea ice produces the same type of brine drainage networks as does first-year ice, with the spacing and shape of the networks being almost identical (see Figure 3.15). Figure 3.16 shows a thick section of artificial sea ice grown in the author's facility. Note the similarity to Figures 3.8(a) and (b).

This system of growth seems to be effective in replicating

Figure 3.15 Brine drainage channel as envisaged by Eide and Martin. (35)

Figure 3.16

Thick section of laboratory grown Artificial Sea Ice.

first-year sea ice, but so far, no one has succeeded in extending the procedure to multiyear ice or frazil first-year ice. For samples of this type of ice, one must still rely on specimens imported from the Arctic.

It should be added as a postscript that although many of the dimensions and salinities quoted above are given by the authors to reasonably high precision, the inherent accuracy of the measurements is low. This is because the sea ice adds its own element of randomness to the problem. Although one may be able to very precisely and accurately measure a parameter at location A, the same parameter in the same ice layer at a not too distant location B can be quite different. Therefore, many of these quoted figures should be treated with a certain amount of caution so that one is not deluded by a false sense of accuracy.

3.2. Applicable Dielectric Measurement Techniques

It has been demonstrated in the previous section that sea ice is a complex and diverse medium. Certain characteristics of its structure should be re-emphasized before discussing appropriate measurement techniques.

Probably the single most important feature of its structure is the presence of brine inclusions. These inclusions make the dielectric neither homogeneous nor isotropic. Because of the high dielectric constant and high loss of the liquid brine, the sample will present a considerable insertion loss, dependent on orientation, when placed in the measurement system. Therefore, the apparatus should ensure, as much as possible, that the orientation of the electric field with respect to the brine pockets that is encountered in practice, is reproduced during measurement.

The mechanisms of brine drainage also have significant bearing on measurement procedures. As pointed out in Addison^(45,47), and Addison et al⁽⁴⁸⁾, sea ice starts to bleed from the brine pockets immediately after cutting. This process is especially fast at temperatures above -30°C . If brine drainage does occur by the brine expulsion mechanism described by Eide and Martin⁽³⁵⁾, then bleeding will occur to some degree at all temperatures but will be moderated slightly at lower temperatures.

Therefore, it is apparent that the measurement procedure should:

- 1) involve minimal handling of the specimen;
- 2) avoid any handling or cutting at high ambient temperatures;
- 3) avoid long periods of sample storage above temperatures of -30 to -40°C ; and
- 4) where possible, use the same specimen and orientation for a series of measurements, e.g., ϵ' , ϵ'' vs. frequency, or ϵ' , ϵ'' vs. temperature.

Having established the basic set of limitations within which the measurements must be made, it is now possible to review various methods of measurement.

At the lower frequencies d.c. - 1 MHz, the usual method of dielectric measurement is with a capacitance/conductance bridge. This method involves a simple substitution technique in which the sample is compared to a standard admittance. From the measured values, the dielectric constant can be either calculated or obtained from a calibration curve. This technique involves machining only the faces of the specimen, if the electrodes are arranged as a parallel plate

capacitor. Providing this machining is done at reduced temperatures, the sample is not altered greatly. (For an application of this technique see Addison⁽⁴⁵⁾, Cook⁽⁴⁹⁾, and Wentworth and Cohn.⁽⁵⁰⁾) At the higher frequencies, in this range, fringing effects become appreciable and a concentric (coaxial) pair of electrodes is usually used. Unfortunately, it is difficult to grow representative sea ice in such a line, and machining a specimen that will fit inside requires considerable time, which in turn means considerable brine loss. Similarly, an alternate standard technique usually employed in the range 50 KHz to 100 MHz is also unsuitable. This method, named after Hartshorn and Ward⁽⁵¹⁾, is based upon the measurement of the shift of the resonant frequency of a high Q circuit in which the specimen acts as one component of the circuit. Unfortunately, this technique requires that the sample be extremely thin ($< 2 - 3$ mm.) and consequently, is unsuitable for sea ice.

At VHF (30 MHz to 300 MHz), UHF (300 MHz to 3 GHz) and at microwave frequencies (≥ 3 GHz), somewhat different techniques are usually employed. Standard techniques of cavity measurement and measurement of standing wave pattern peak shifts have been described by von Hippel.⁽⁵²⁾ The cavity technique, widely employed for standards measurements (see Bussey et al^(53,54)) is unsuitable for sea ice due to the inordinant amount of machining which is required to accurately fit the specimen to the cavity. The technique of measurement of the position of the first peak after the interface (air-ice) of the standing wave pattern has been used by Hoekstra⁽⁵⁵⁾ for sea ice. This method necessitates being able to precisely determine the peak position accurately and seems to work at least at the higher frequencies, although several sets of apparatus are necessary to cover the

range 100 MHz to 10 GHz.

Another approach applicable to this problem is to treat the specimen as a network and measure the scattering parameter S_{12} where S_{12} is the specimen transmission coefficient (T). The phase and magnitude of T can be measured using a bridge and the theoretical equation for T inverted numerically to obtain the complex permittivity. The T of the sample holder filled with the specimen is compared to a reference, usually the empty sample holder. Various sample configurations are possible. The specimen may be placed in free space between two antennas, or may be inserted in a transmission line. The transmission line sample holder may be more advantageous in certain instances, e.g. below 10 GHz the wideband sample holder has the advantage, due to the difficulty experienced in free space with antenna size, with narrow waveguide bandwidths in the measurement bridge, and with obtaining effective reflection damping at these frequencies. At the higher frequencies (> 10 GHz), a free-space measurement scheme is preferable due to the small size of the waveguide and the difficulty in designing sample holders at these frequencies.

In recent years, time domain techniques have become increasingly popular as a means of obtaining wideband dielectric spectra quickly. With the advent of fast rise time (< 35 ps) pulse generators and fast computer algorithms such as the Fast Fourier Transform, it has been possible to replace laborious point by point frequency domain measurements by fast time domain determinations, at least over the frequency range 100 MHz to 15 GHz. An excellent survey on Time Domain Spectroscopy (TDS) methods is given in van Gemert. (56)

Basic to all published TDS techniques is the use of a coaxial sample holder. This method has proved advantageous to the study

TABLE 3.1 T D S MEASUREMENT TECHNIQUES

METHOD	ANALYSIS	FREQUENCY RANGE	PARAMETERS MEASURED	ACQUISITION TECHNIQUE	COMMENTS
TDR	FFT, or DFT approximations	200 MHz - 15 GHz	ϵ' , ϵ''	X-Y Recorder or automatic	Insufficient time window for $\epsilon'(\omega)$ of pure ice, may be applicable to $\epsilon(\omega)$ of sea ice with suitable sample holder. Fast and easy.
DIRECT TRANSMISSION	"	? - 15 GHz	ϵ' , ϵ''	Automatic	Not sensitive to timing shifts, but uses very long coaxial samples, difficult to make from sea ice.
COMBINATION TRANSMISSION REFLECTION	"	? - 15 GHz	ϵ' , ϵ''	Automatic	As above, very time consuming.
LUMPED CAPACITANCE	"	10 MHz - 10 GHz	ϵ' , ϵ'' , σ	X-Y Recorder Sufficient	Very small sample, impractical with ice.
NICOLSON-ROSS	"	100 MHz - 10 GHz	?	Automatic	Obtain ϵ , and μ . Low frequency limit determined by available time window (sampler-specimen), coaxial specimen.
THIN-CELL	Analytical. behaviour must be known	10 KHz - 1 GHz	ϵ' , ϵ'' , σ	Automatic	Very small specimen, impractical.

of liquids such as polar alcohols and water, as well as polymeric materials such as plexiglass, but as mentioned previously in connection with the concentric electrode sample holder, it is not ideally suited to the investigation of sea ice (see Table 3.1).

3.3 A Survey of Dielectric Measurements on Sea Ice

Unfortunately, comprehensive reviews do not, to the author's knowledge, exist for sea ice. Until recently, very little had been done in this field, partially because of the complexity of the measurement problems and partially because of the difficulty in obtaining and maintaining sea ice specimens. A summary of previously published results is given in Table 3.2 and Figure 3.17. Most of the measurements cited have been done at only a few individual frequencies and selected salinities and temperatures, so that a complete overview of the properties of sea ice does not exist. Certain results should be treated with caution since they were not performed on natural sea ice but only on laboratory artifacts of sea ice. Included in this category is the work of: Addison and Pounder^(57,58), Addison^(45,47), Addison, Stalinski and Pounder⁽⁴⁸⁾, Cook⁽⁴⁹⁾, Hoekstra⁽⁵⁵⁾, and Hoekstra and Cappillino.⁽⁵⁹⁾ The remaining work, that of Bogorodsky and Tripol'nikov⁽⁶⁰⁾, Bogorodsky and Khokhlov⁽⁶¹⁾, Fujino⁽⁶²⁾, Ragle, Blair and Persson⁽⁶³⁾, Wentworth and Cohn⁽⁵⁰⁾, and Sackinger and Byrd⁽⁶⁴⁾, lack a basis of comparison. Either they were performed at differing select values of salinity, or differing select values of temperature or on different ice types.

It should also be noted that Hoekstra and Cappillino⁽⁵⁹⁾ have performed the only measurements in the 1 GHz to 10 GHz range. The values plotted in Figure 3.17 are based on measurements performed on flash frozen sea water. The rise and fall of absorption between

TABLE 3.2 SUMMARY OF PREVIOUSLY PUBLISHED RESULTS FOR SEA ICE

AUTHOR	FREQUENCY	SALINITY (o/oo)	TEMPERATURE (-°C)	COMMENTS
Addison (57,58) & Pounder	20 Hz - 100 MHz	7 - 20	22	Special cell used, artificial sea ice.
Addison (45,47)	20 Hz - 100 MHz	4 - 20	12.5 - 35	Special cell used, artificial sea ice.
Addison, Stalinski & Pounder (48)	1 KHz	From saline water 35 o/oo	25 - 150	Special cell used, artificial sea ice.
Cook (49)	100 MHz	3.5, .7, .35	10 - 70	Artificial sea ice.
Hoekstra (55)	10 GHz	10 - 60	0 - 25	NaCl ice.
Hoekstra (59) & Cappillino	100 MHz - 10 GHz	8	10	Flash frozen sea water, poor fascimile
Bogorodsky (60) & Tripolnikov	30 MHz, 60 MHz, 100 MHz, 200 MHz, 400 MHz	.56, 4.4, 8.6, 17.0	10 - 40	Sea water frozen in coax.
Bogorodsky (61) & Khoklhov	10 GHz	2.5 - 7.5	1 - 13	Good survey of ice types.
Fujino (62)	100 Hz - 50 KHz	11, 13, 15	5 - 70	Natural sea ice.
Ragle, Blair (63) & Persson	150 MHz, 300 MHz, 500 MHz, 1000 MHz	-	1 - 60	Very low salinity.
Wentworth (50) & Cohn	.1 MHz - 30 MHz	.067 - .23	5 - 40	Possibly severe brine drainage.
Sackinger (64) & Byrd	26 GHz - 40 GHz	2.85, 3.40, 7.20	7, 16.5, 21.5 25, 32	Natural sea ice.

Figure 3.17 Summary of previously published Loss data; data are taken over the temperature range (-70°C to -120°C) and the salinity range (5 ‰ to 10 ‰); the density is unspecified in most cases.

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100 MHz and 10 GHz is somewhat suspect since there is no mechanism which would explain it; i.e. the absorption peak at 20 GHz is easily explained by the presence of brine but no such absorption peak occurs in any of the components in the 100 MHz to 10 GHz range.

It is obvious from the data that there was a tremendous gap in our knowledge of the dielectric behaviour of sea ice in the 100 MHz to 10 GHz frequency range and it was with the aim of filling this gap that the present thesis was undertaken.

CHAPTER 4. PRELIMINARY MEASUREMENTS

4.1 Objectives

The preliminary investigation consisted of a series of measurements on fresh and sea ice to: (1) obtain dielectric data on both fresh and sea ice at 10, 31.4 and 34 GHz, (2) evaluate various measurement techniques, and (3) evaluate some basic dielectric mixture equations. It was also designed to determine the effects on the dielectric properties of such sea ice parameters as ice type, age, salinity, and temperature.

These measurements are documented in Vant et al.⁽¹⁰⁾ They consist of: dielectric measurements on fresh ice, employing free-space techniques and using large ice slabs as targets at 10 GHz and over the 26.4 to 40 GHz frequency range; dielectric measurements at 10 GHz on sea ice samples, taken from the Bering Sea; and dielectric measurements at 31.4 and 34 GHz on artificial sea ice sheets.

4.2 Experimental Technique

The experimental apparatus consisted of a CW coherent microwave bridge as shown in Figure 4.1. Certain precautions were taken as outlined by O'Brien⁽⁶⁵⁾ and Jaggard and King.⁽⁶⁶⁾ Measurement of lossy ice samples necessitated the optimization of the sensitivity and dynamic range of the bridge.

Two measurement configurations were used as shown in Figures 4.2(a) and (b). The free-space technique was used at 10 GHz for the large, pure ice slabs and at 31.4 and 34 GHz for the large artificial sea ice slabs. A second technique, in which the ice specimen is fitted inside the waveguide, was used for the natural sea ice measurements at 10 GHz.

Figure 4.1 CW coherent microwave bridge measurement system.

a

b

Figure 4.2

Sample configurations. (a) Slab placed between two horn antennas and free-space transmission coefficient measured. (b) Slab machined to fit in waveguide and transmission coefficient of filled guide relative to empty guide measured.

Scaife⁽⁶⁷⁾ gives a good summary of the techniques for evaluating ϵ_r . In the preliminary investigations, the complex transmission coefficient through the sample was measured relative to an equivalent length of free space. This transmission coefficient is given by equation 4.1 for free space measurement and equation 4.6 for the in-waveguide measurements.

The transmission coefficient T for propagation through a single homogeneous slab is derived to be

$$T = \frac{A \exp[(\gamma_0 - \gamma)d]}{B \exp(2\gamma d) - 1} \quad (4.1)$$

where

$$A = 4\epsilon_r^{1/2} / (1 - \epsilon_r^{1/2})^2 \quad (4.2)$$

$$B = (1 + \epsilon_r^{1/2})^2 / (1 - \epsilon_r^{1/2})^2 \quad (4.3)$$

$$\gamma_0 = j2\pi/\lambda_0 \quad (4.4)$$

$$\gamma = \gamma_0 \epsilon_r^{1/2} \quad (4.5)$$

and ϵ_r is the complex relative dielectric constant, λ_0 is the free space wavelength in cm., and d is the length of the sample in cm.

The solution for propagation through a slab of length d completely filling a waveguide, assumed to be propagating a wave in the dominant mode, TE_{10} , is derived to be

$$T = \frac{C \exp[(\gamma_0 + \gamma)d]}{D \exp(2\gamma d) - E} \quad (4.6)$$

where

$$C = 4(1 - P)^{1/2} / (\epsilon_r - P)^{1/2} \quad (4.7)$$

$$D = [1 + (1 - P)^{1/2} / (\epsilon_r - P)^{1/2}]^2 \quad (4.8)$$

$$E = [1 - (1 - P)^{1/2} / (\epsilon_r - P)^{1/2}]^2 \quad (4.9)$$

$$P = (f_{\omega}/f)^2 \quad (4.10)$$

$$\gamma_0 = j \left| (2\pi/\lambda_0)^2 - (\pi/a)^2 \right|^{1/2} \quad (4.11)$$

$$\gamma = j \left| (2\pi\epsilon_r^{1/2}/\lambda_0)^2 - (\pi/a)^2 \right|^{1/2} \quad (4.12)$$

where f_0 is the cutoff frequency of the empty waveguide, f is the operating frequency, and "a" is the long dimension of the waveguide (see Figure 4.2(b)).

Both these equations assume that the propagation is through a homogeneous and isotropic slab. This assumption is not strictly correct. Sea ice is a mixture of liquid brine and pure ice and therefore is not homogeneous. The brine inclusions have a preferred orientation and therefore the slab is not isotropic. Also, the waveguide is never completely filled; for a 10 cm. long sample and a $\pm 1\%$ variation in filling, the phase error in T is approximately $\pm 5 - 6^\circ$, as shown in Figure 4.3. This causes an error of about $\pm 1\%$ in ϵ_r' .

4.3 Experimental Results

The estimated experimental accuracy in T was $\pm 8 - 9^\circ$ in phase and $\pm .2$ db in amplitude. Examination of the solution to the boundary value problem numerically showed that this should allow determination of the real part of the relative dielectric constant (ϵ_r') to within ± 0.06 and the imaginary part (ϵ_r'') to within $\pm .001$ in most cases.

The value of ϵ_r' for fresh ice at 10 GHz was found to be $3.14 \pm 1.4\%$. This value concurs with Lamb and Turney⁽⁶⁸⁾ who found $\epsilon_r' = 3.18 \pm 2\%$, Cumming⁽⁶⁹⁾ who found $\epsilon_r' = 3.15 \pm 1\%$, and von Hippel⁽⁷⁰⁾ who obtained $3.17 \pm 3\%$.

Measurements were also made on fresh ice in the 26.4 to 40 GHz range. Results indicate a value of $2.92 \pm 4\%$ for pure ice.

Figure 4.3

Variation of propagation constant of dominant mode for a waveguide partially filled in the vertical direction.

This is slightly lower than the $3.08 \pm .02$ result obtained by Perry and Straiton⁽⁷¹⁾ at 96.5 GHz.

Measurements obtained at 10 GHz, using small samples enclosed in a waveguide substantiate the free-space measurements. The results of this determination were $\epsilon_r' = 3.12 \pm 2 \%$, and $\tan \delta$ (loss tangent) $= 10 \times 10^{-4} \pm .001$ (at -35°C).

The results obtained on the Bering Sea ice samples, at 10 GHz, using the in-waveguide technique, are shown in Figures 4.4 to 4.7. It is apparent that at approximately -20°C to -30°C something happens to cause an increase in both ϵ_r' and ϵ_r'' . This is the effect of the hydrolyzed salts separating out as liquid brine. Also apparent is the drastic difference in the magnitudes of ϵ_r'' for first-year and multiyear ice. Some difference is also noted between the frazil and columnar first-year sea ice and for ice samples of different density and salinity.

To estimate the effects of short term temperature cycling on the measurements, all samples, once enclosed in the waveguide, underwent a measurement cycle of decreasing then increasing temperature. The values obtained in either direction agree within the estimated experimental error. Prior to measurement all samples had been stored at temperatures between -35 and -60°C to prevent brine drainage from occurring.

In an attempt to compare these results with those of other workers, the ϵ_r' and ϵ_r'' were correlated with various models. The first model, due to deLoor⁽¹³⁾, assumes spherical brine inclusions and gives ϵ_r' as

$$\epsilon_r' = \epsilon_{r' \text{ ICE}} / (1 - 3v) \quad (4.13)$$

Figure 4.4

Variation of real part of dielectric constant (ϵ_r') of sea ice with temperature at 10 GHz.

o frazil, □ columnar, ▽ multiyear sea ice.

1. S (salinity) = 4.4 ‰, ρ (density) = 0.836 gm/cm³; 2. S = 3.2 ‰, ρ = 0.836 gm/cm³;

3. S = 3.2 ‰, ρ = 0.878 gm/cm³; 4. S = 4.6 ‰, ρ = 0.896 gm/cm³; 5. S = 0.61 ‰, ρ = 0.771 gm/cm³;

6. S = 0.70 ‰, ρ = 0.770 gm/cm³.

Figure 4.5

Variation of imaginary part dielectric constant (ϵ''_r) of frazil sea ice with temperature at 10 GHz.

○ S = 4.4 ‰, $\rho = 0.836 \text{ gm/cm}^3$;

● S = 3.2 ‰, $\rho = 0.836 \text{ gm/cm}^3$.

Figure 4.6 Variation of imaginary part of dielectric constant (ϵ_r'') of multiyear sea ice with temperature at 10 GHz
 ∇ $S = 0.61 \%$, $\rho = 0.771 \text{ gm/cm}^3$;
 ∇ $S = 0.70 \%$, $\rho = 0.770 \text{ gm/cm}^3$.

Figure 4.7 Variation of imaginary part of dielectric constant (ϵ_r'') of columnar sea ice with temperature at 10 GHz
 $\square$ $S = 3.2 \%$, $\rho = 0.878 \text{ gm/cm}^3$;
 $\blacksquare$ $S = 4.6 \%$, $\rho = 0.896 \text{ gm/cm}^3$.

where ϵ_r' ICE is the relative dielectric constant of pure ice and v is the relative brine volume as calculated by Assur.⁽³¹⁾ This equation separates out ϵ_r'' entirely, which is not strictly correct. The results of the correlation are given in Table 4.1. It should be noted that in the case of columnar ice the samples were mounted in the waveguide so as the major axes of the brine inclusions were perpendicular to the electric field vector. The results seem to indicate that ϵ_r' is indeed linearly related to $1/(1 - 3v)$ for first-year ice. The constant of proportionality is not exactly 3.14 however, as found by Hoekstra and Cappillino⁽⁵⁹⁾ for their artificial ice, and there is an additive term. This could be due to the neglect of ϵ_r'' in the formula. The high correlation in the case of frazil sea ice leads to the conclusion that the brine inclusions may indeed be spherical in frazil ice. The high correlation for columnar ice only suggests that the inclusions were not parallel to the field. It may appear at first that this conclusion could also be reached for frazil ice, but it is assumed that frazil ice does not exhibit any elongated brine inclusions and does not have a preferred orientation. The low correlation coefficient for multiyear ice suggests that either our measurements have been confounded by some other mechanism not accounted for, such as scattering from air bubbles, or simply that the shape assumed is not correct.

It was found by correlation of ϵ_r'' with v that in all cases the dielectric loss varied directly with the brine volume. The results of this computation are shown in Table 4.2. Frazil ice exhibited a higher loss for the same brine volume than did columnar ice. This could again be due to the orientation of the brine inclusions with respect to the electric field. Multiyear ice also exhibited a

TABLE 4.1 DIELECTRIC CONSTANT CORRELATIONS WITH SPHERICAL MODEL AT 10 GHz (10)

$$\epsilon_r' = a + bx$$

$$x = 1/(1 - 3v_1)^*$$

Material	a	b	Corr. Coeff.	Std. Error
Columnar Sea Ice	- .377	3.498	.897	.04
Frazil Sea Ice	-1.796	4.863	.812	.16
Multiyear Sea Ice	-4.651	7.172	.565	.05

* v_1 = relative brine volume

TABLE 4.2, DIELECTRIC LOSS CORRELATION WITH MODEL AT 10 GHz (10)

$$\epsilon_r'' = a + bx$$

$$x = v_1^*$$

Material	a	b	Corr. Coeff.	Std. Error
Columnar Sea Ice	.004	7.143	.849	.042 x 10 ⁻²
Frazil Sea Ice	.0316	9.26	.764	.092 x 10 ⁻²
Multiyear Sea Ice	.00633	9.98	.723	.016 x 10 ⁻²

* v_1 = relative brine volume

higher loss for the same brine volume than did columnar ice, providing v was above 0.0355. This value of v seldom occurs in multi-year ice. Furthermore, the brine volume may have little meaning for multiyear ice since a density of .926 gm/cm³ is assumed in its calculation.

To more fully explain the behaviour of multiyear ice, the effect of the air bubbles must be accounted for. In an attempt to do this Weiner's⁽⁷²⁾ dielectric mixing formula (as quoted in Evans⁽⁷³⁾) was employed. In general,

$$\frac{\epsilon_{r_m}' - 1}{\epsilon_{r_m}' + f} = (1 - v_2 - v_3) \frac{\epsilon_{r_1}' - 1}{\epsilon_{r_1}' + f} + v_2 \frac{\epsilon_{r_2}' - 1}{\epsilon_{r_2}' + f} + v_3 \frac{\epsilon_{r_3}' - 1}{\epsilon_{r_3}' + f} \quad (4.14)$$

where ϵ_{r_m}' is the average relative dielectric constant of the mixture, the ϵ_{r_i}' are the respective relative dielectric constants of the constituents, the v_i are the relative volume fractions and f is a form factor. If $\epsilon_{r_3}' = 1$ (air), $\epsilon_{r_1}' = 3.14$ (ice at 10 GHz), and $\epsilon_{r_2}' = 40$ (brine at 10 GHz) (from Stogryn⁽⁷⁴⁾), equation 4.14 becomes

$$\epsilon_{r_m}' = (1 - v_2 - v_3)\epsilon_{r_1}' + v_2\epsilon_{r_2}' \quad (4.15)$$

for v_3 small and f large, which assumes some kind of striations of the included dielectric parallel to the electric field. The volume fraction of air is simply

$$v_3 = 1 - \rho_3/\rho_{ICE} \quad (4.16)$$

where ρ_3 is the sea ice density and $\rho_{ICE} = 0.916$ gm/cm³ (from Cumming⁽⁶⁹⁾; the density of pure ice was assumed). After substitution, equation 4.15 becomes

$$\epsilon_{r_m}' = 36.8v_2 + 3.3\rho_3 \quad (4.17)$$

Table 4.3 gives the measured values of the coefficients

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when ϵ_{r_m}' is correlated against v (the brine volume fraction). The results demonstrate a high variability in the term dependent on the brine volume. This variation could be partly due to the uncertainty in the value of f to employ for each ice type, and to the non-uniformity of the distribution of the material described by f . The results for frazil and multiyear ice indicate that a model with high form factor is applicable. The results for columnar ice indicate that it is not applicable, due to the predominantly perpendicular orientation of the brine inclusions with respect to the electric field.

The experiments with artificial sea ice, at 31.4 and 34.0 GHz, using free space techniques, demonstrated the utility of the laboratory facsimile. The results of the measurements are compared in Table 4.4 with those of Sackinger and Byrd.⁽⁶⁴⁾ The comparison indicates the losses recorded for the laboratory ice are lower than those observed for the natural ice. This discrepancy could be due to the variability in loss of natural sea ice, the orientation of the specimens during measurement, or the greater complexity of natural sea ice.

Despite this minor drawback, it appears that laboratory measurements are a valuable tool for the investigation of the dielectric properties of sea ice.

4.4 Conclusions

The preliminary measurements allowed a period of familiarization with the measurement techniques. They demonstrated that the free space and waveguide measurements have approximately the same accuracies, although different problems were encountered with each. Although not mentioned explicitly above, it was very difficult and time consuming to fit the ice specimens in the waveguide (especially at temperatures below -15°C), and therefore this technique was deemed

TABLE 4.3 CORRELATION OF DIELECTRIC DATA WITH MODIFIED MODEL⁽¹⁰⁾

$$\epsilon_r' = a + bx$$

$$x = v_i^*$$

Sample	a†	a calc. = 3.3p	b†	b calc.	Corr. Coeff.	Std. Error	ρ (g/cm ³)
Frazil	3.04	2.76	15.74	36.8	.98	.05	.836
Sea Ice	2.86	2.76	30.48		.93	.10	.836
Columnar Sea Ice	3.20	2.90	4.11		.15	.12	.878
Multiyear	2.46	2.54	22.39		.96	.01	.771
Sea Ice	2.57	2.54	17.44		.57	.04	.770

* v_i = relative brine volume

† a and b are for best fit of ϵ_r' to v_i

TABLE 4.4 DIELECTRIC LOSS AND CONSTANT OF ARTIFICIAL SEA ICE AT 31.4 AND 34.0 GHz⁽¹⁰⁾

Frequency (GHz)	T (°C)	S _i (ppt)	This Work		Sackinger and Byrd ⁽⁶⁴⁾	
			ϵ_r'	ϵ_r''	ϵ_r'	ϵ_r''
31.4	-7	7.9	3.015 ± 1 %	0.097 ± 1 %	3.09 ± 1 %	0.253 ± 2 %
34.0	-7	8.0	3.035 ± 1 %	0.103 ± 1 %	2.96 ± 1 %	0.254 ± 2 %

rather undesirable. It was also seen that differences in dielectric properties do occur for different ice types. It appears that the dielectric properties are strongly a function of salinity and possibly of orientation and density. The results for frazil sea ice are unique in that they suggest a spherical brine inclusion. Unfortunately, frazil ice occurs in only very thin layers and therefore is not suitable for measurement at lower frequencies where large samples are necessary.

The dielectric mixture formulas do explain the sea ice behaviour to a certain extent, but all of the formulas employed suffered from a lack of ability to predict both ϵ_r' and ϵ_r'' simultaneously, and to realistically model the shape of the inclusions.

The artificial sea ice measurements demonstrated the utility of using laboratory grown sea ice, but were not conclusive enough to determine whether or not the loss for the same salinity and density was lower than that of natural sea ice.

The importance of precisely fitting the sample to the sample holder was discovered for the waveguide measurements. Significant errors were caused by small changes in percentage filling.

The multiyear sea ice measurements suggested that scattering from air bubbles may be a problem at the frequencies employed, i.e. above 10 GHz.

In summary, the preliminary measurements illustrated the importance of knowing the orientation of the brine inclusions with respect to the electric field, and maintaining this orientation for the whole series of measurements. Also, they pointed out the usefulness of the dielectric mixture equation approach, although they did not really succeed in evolving any comprehensive models. The short-

comings of the single frequency approach were also felt in that it was not possible to test the models formulated over a wide frequency range and thus their applicability was seriously limited.

CHAPTER 5. DESIGN OF A SUITABLE WIDEBAND MEASUREMENT SYSTEM

5.1 Measurement Constraints

As established in the previous chapters, especially Chapter 3, several serious measurement problems have to be solved. The first and most serious of these is the creation of a wideband sample holder to cover as much of the frequency range 100 MHz to 10 GHz as possible. The one sample holder should be able to allow measurement of the same specimen over this whole range. This removes some of the variability associated with sampling the sea ice. Also, the sample holder should re-create, as much as possible, both the in-situ conditions of the ice and the in-situ orientation of the ice with respect to the probing radiation. To accomplish this end, the minimum amount of handling and machining of the specimen should be done, and any that is performed should be done at temperatures below -20°C . To re-create the in-situ environment, the minimum possible number of brine pockets should be cut, especially in the region of the sample that will experience the most intense fields. The orientation of the specimen should also be such that it simulates the situation encountered when sensing the ice surface remotely, either from the ground or from the air, i.e. the ice should be sampled by taking a vertical core and arranging the measurement so that the diagnostic waves impinge on the core sideways. Also, the sample holder should be able, over the frequency range of interest, to allow measurement of the transmission coefficient to within the accuracy of commercial wideband coherent bridges, typically $\pm 3^{\circ}$ (in phase) and $\pm .1$ db (in amplitude).

With these constraints established, and with the results of Addison⁽⁷⁵⁾ and the theory of King⁽⁷⁶⁾ in mind, it was decided to

employ a five-wire "coaxial-cage" type transmission line. It has the advantage that the wave propagates in the TEM mode making analysis easy, and also that only five holes need be drilled in the ice core for mounting it on the sample holder. This machining may easily be done in the field and at low temperatures.

The results obtained by Addison⁽⁷⁵⁾, as a function of frequency, for the reflection coefficient and VSWR of a similar line, show that the VSWR can be kept to less than 1.1 for frequencies up to 6 GHz, and that radiation losses are minimal. Encouraged by these measurements, the design of the "coaxial-cage" line was undertaken.

5.2 Design of the "Coaxial-Cage" Transmission Line

5.2.1 Design Procedure

The fundamental idea behind the "coaxial-cage" transmission line is extremely simple. The solid outer conductor of a normal coaxial line is replaced by several equally-spaced rods (King⁽⁷⁶⁾ gives the dimensional relationships). For an original coaxial line with inner conductor of radius "a" and outer conductor of radius "b", the outer conductor rods are $a_1 = b/2N$ in radius, where $2N$ is the number of rods.

It is necessary to make the spacing between outer conductors small enough that higher order modes (other than the dominant TEM mode) do not propagate. To do this, the average circumference of the line must be less than the wavelength in the sample, at the upper operating frequency.

Also, the final device must interface easily with existing commercial connectors such as N type, APC-7 or GR-900. Of these connectors, the GR-900 has the largest inner conductor and probably is the easiest to work with. Therefore, Addison's⁽⁷⁵⁾ lead was followed

and the basic device was built with four outer rods, and connectors that mate with the GR-900 type.

Unfortunately, the exact dimensions of his device and the specific connector kits he used are not available. Therefore, only the basic idea of his line was retained and the dimensions were re-derived. Silver plated precision rods of 0.24425 in. (.62040 cm.) diameter are available from General Radio. These were used for the outer conductor rods. This corresponds to a "b" dimension of 0.9770 in. (2.482 cm.). To get the correct "a" dimension for the prototype, the normal inner conductor to outer conductor radius ratio for 50 Ω was used. The final version of the sample holder is shown in the sketch in Figure 5.1 (plans are given in Appendix C). A prototype was built first to determine the correct location for the shoulder on the inner conductor and the correct inner conductor radius. Addison et al⁽⁴⁸⁾ had determined by experimentation that an abrupt step was the best transition to use for the change in inner conductor radius. This step is necessary to allow mating with the GR-900 connector inner conductor. To avoid having to design beads, the inner conductor was suspended from the mating GR-900 inner conductor, using spring-loaded centering pins.

A prototype model was built that had an inner conductor composed of a sliding bushing over a 0.244 in. (.620 cm.) diameter rod. It had a diameter equal to that given by King's⁽⁷⁶⁾ equations. Tests of this device using a Time Domain Reflectometer (TDR) showed that King's⁽⁷⁶⁾ equations did not produce a 50 Ω line. To rectify this problem the inner conductor was turned down in small increments on the lathe and checked with a TDR until the radius which achieved a 50 Ω

Figure 5.1 "Coaxial-Cage" sample holder.

impedance line was reached. The position of the transition in the inner conductor was determined by sliding the shoulder back and forth until the minimum reflection was observed on the TDR. Having established the final dimensions, an operational model was built with a solid inner conductor and the whole of the inner conductor was silver plated. TDR tests of the final product (see Figure 5.2) showed that the reflections of the device were lower than those due to the APC-7 - GR-900 adapter used to mate the device to the rest of the system and that the performance of the "coaxial-cage" line was equal to an APC-7 air line (see Figure 5.3).

Radiation from the line was not judged to be a problem since a metal object in the vicinity of the line did not affect the measurements at all unless inserted directly between the inner and outer conductor rods.

It was decided to use the line only up to a maximum frequency of 8 GHz since swept frequency measurements, performed with the empty line terminated in an APC-7 short, showed one resonance at about 9.0 GHz and two more between 10 and 12 GHz. The terminated line exhibited a VSWR < 1.5 between 8 and 12 GHz, with the primary excursions above a VSWR = 1.1 being due to the resonances noted above. These resonances were seen to be related to the spacing between the transition in the inner conductor and the brass supporting blocks. This region of the line is always air filled, so the resonant frequencies noted above will not shift as a function of the material filling the line.

The length of the line was determined by the maximum length of sample that could be accurately drilled and mounted using a

Figure 5.2

Time Domain Reflectometer trace of the empty sample holder terminated with an American 3mm 50 Ω load. The arrows denote the GR-900-APC-7mm adapters.

Figure 5.3

Time Domain Reflectometer trace of a Hewlett Packard 20cm long APC-7mm air line terminated with an American 3mm 50 Ω load.

Figure 5.4 Drill jig used to prepare sea ice specimens for mounting on the "coaxial-cage" line.

simple drill jig and a drill press. This sample length was found to be approximately 8 cm. A sketch of the drill jig is shown in Figure 5.4 (plans are given in Appendix C).

5.2.2 Evaluation of the Measurement Errors Due to the "Coaxial-Cage Line

The chief error inherent in the sample holder alone, even when used at low frequencies where the indicated VSWR is low, is that due to an improper fit between the holes drilled in the sample and the conductor rods. The rod sizes are 0.24425 in. (.62040 cm.) diameter for the outer conductors and 0.382 in. (0.970 cm.) diameter for the inner conductor. The minimum standard drill size near to this size but still large enough to allow a sliding fit was 0.386 in. (0.980 cm.) (size W) for the inner hole and 0.250 in. (0.635 cm.) for the outer holes.

The technique employed was to drill, with the aid of the drill jig, five holes parallel to the central axis of the cylindrical specimen. The four equally spaced outer holes are drilled on a 0.977 in. (2.482 cm.) diameter bolt circle concentric with the inner hole. It is obvious that fairly long drill bits are necessary and that the jig will not control the wobble in the drill bit near the bottom of its travel. Therefore, the holes will be slightly non-parallel. If this non-parallel trend is too extreme, the sample will not fit and probably will crack during the process of fitting it to the line. This means the extreme non-parallel errors tend to take care of themselves by not allowing one to fit the specimen to the sample holder.

The holes drilled in the sample must obviously be slightly oversize to allow a sliding fit. It is also apparent that this air gap between the ice and the conductors will cause the most serious problem

around the inner conductor where the fields are highest.

To attempt an analysis of these errors, it was assumed that the line behaved as an ordinary coaxial line, i.e. the air gap was very small. Two regions of interest were defined, as shown in Figure 5.5. Region 1 is $a_1 \leq r \leq a_2$ (the air gap) and Region 2 is $a_2 \leq r \leq a_3$ (the ice).

In a coaxial line propagating in the TEM mode, the electric and magnetic fields are of the same form as in the static case. McIntosh et al⁽⁷⁷⁾ gives

$$\vec{E}_{r_1} = \frac{\gamma}{j\omega\epsilon} \frac{C_1}{r} e^{-\gamma z} \hat{r} \quad (5.1)$$

$$\vec{H}_{\phi_1} = \frac{C_1}{r} e^{-\gamma z} \hat{\phi} \quad (5.2)$$

where γ is the complex propagation constant, $\hat{r}$ and $\hat{\phi}$ are unit vectors, r is the radial distance, ϵ is the complex dielectric constant (see Appendix A) filling the line and C_1 is a constant.

Using the transmission line analogy and the quasi-static assumption, $\vec{E}_r$, for the concentric cylinders, is also given by (see Jordan and Balmain⁽⁷⁸⁾)

$$\vec{E}_{r_1} = \frac{k_1}{r} \hat{r} \quad (5.3)$$

where k_1 is a constant.

For a series of concentric cylinders which delineate the regions shown in Figure 5.5, $\vec{E}_r$ is easily found by the following method. Let the magnitude of the electric fields in the two regions be

$$E_{r_1} = \frac{k_1}{r} \quad \text{and,} \quad E_{r_2} = \frac{k_2}{r} \quad (5.4)$$

respectively. Integration of E_{r_1} with respect to r gives

Figure 5.5

Sample mounted in coaxial line with air gap (between radii a_1 and a_2) shown about inner conductor (a_1). Air gap has dielectric constant ϵ_1 , the rest of the line is filled with material of dielectric constant ϵ_2 . The outer conductor is at radius a_3 . The voltages at radius a_1 , a_2 and a_3 are V_1 , V_2 and V_3 respectively.

$$\int_{a_1}^{a_2} E_{r_1} dr = k_1 \log_e (a_2/a_1) \quad (5.5)$$

$$= V_1 - V_2$$

where V_1 and V_2 are the voltages at a_1 and a_2 respectively. Similarly, integration of E_{r_2} gives

$$\int_{a_2}^{a_3} E_{r_2} dr = k_2 \log_e (a_3/a_2) \quad (5.6)$$

$$= V_2 - V_3$$

Let $V = V_3 - V_1$, then

$$V_2 - V_3 = V_2 - (V + V_1) \quad (5.7)$$

At $r = a_2$, $\vec{D}_{r_1} = \vec{D}_{r_2}$, therefore

$$\epsilon_1 \left(\frac{V_1 - V_2}{\log_e a_2/a_1} \right) = \epsilon_2 \left(\frac{V_2 - V_3}{\log_e a_3/a_2} \right) \quad (5.8)$$

since

$$k_2 = \frac{V_2 - V_3}{\log_e a_3/a_2} \quad (5.9)$$

and

$$k_1 = \frac{V_1 - V_2}{\log_e a_2/a_1} \quad (5.10)$$

from (5.5) and (5.6)

Substitution of (5.7) in (5.8) gives

$$(V_2 - V_1) \left[\frac{-\epsilon_1}{\log_e a_2/a_1} + \frac{-\epsilon_2}{\log_e a_3/a_2} \right] = \frac{-V\epsilon_2}{\log_e a_3/a_2} \quad (5.11)$$

which reduces to

$$V_2 - V_1 = \frac{-V\epsilon_2}{\log_e a_3/a_2} \left[\frac{-\epsilon_1}{\log_e a_2/a_1} + \frac{-\epsilon_2}{\log_e a_3/a_2} \right]^{-1} \quad (5.12)$$

Substitution for $(V_2 - V_1)$ in terms of $V_2 - V_3$ gives

$$V_2 - V_3 = -V + \frac{V\epsilon_2}{\log_e a_3/a_2} \left[\frac{\epsilon_1}{\log_e a_2/a_1} + \frac{\epsilon_2}{\log_e a_3/a_2} \right]^{-1} \quad (5.13)$$

Therefore,

$$E_{r_2} = \frac{-V\epsilon_1}{r} \left[\frac{1}{\epsilon_1 \log_e a_3/a_2 + \epsilon_2 \log_e a_2/a_1} \right] \quad (5.14)$$

and

$$k_2 = \frac{-V\epsilon_1}{\epsilon_1 \log_e a_3/a_2 + \epsilon_2 \log_e a_2/a_1} \quad (5.15)$$

For a line completely filled

$$k_2' = \frac{-V}{\log_e a_3/a_1} \quad (5.16)$$

Therefore,

$$k_2 = k_2' \left[\frac{\epsilon_1 (\log_e a_3/a_1)}{\epsilon_1 \log_e a_3/a_2 + \epsilon_2 \log_e a_2/a_1} \right] \quad (5.17)$$

From equation (5.1) it can be seen that for TEM mode propagation, and with the assumptions made above, (see Jordan and Balmain⁽⁷⁸⁾),

$$k_2' \propto \frac{\gamma C_1}{j\omega\epsilon_2} \quad (5.18)$$

where $\gamma = j(2\pi/\lambda_0)\sqrt{\epsilon_2}$, and ω is the radian frequency.

It is assumed that the conductivity effects have been lumped in ϵ_2 (see Appendix A) and that the air gap is so small that all it does is cause a modification of the propagation constant γ , still allowing pure TEM mode propagation. If the frequency is low enough and the gap small enough, hybrid modes will not propagate, or if they do they will be very low in magnitude⁽⁷⁷⁾ (see McIntosh et al⁽⁷⁷⁾).

Therefore, if

$$k_2 \propto \frac{\gamma' C_1}{j\omega\epsilon_2} \quad (5.19)$$

with the same constant of proportionality as in (5.18), where γ' is the modified propagation constant, then the angle and amplitude measured will be in error due to the fact that

$$\gamma' = \gamma \left[\frac{\epsilon_1 \log_e a_3/a_1}{\epsilon_1 \log_e a_3/a_2 + \epsilon_2 \log_e a_2/a_1} \right] \quad (5.20)$$

If the values for ϵ_2 are derived from the transmission coefficient in the normal way, as given in Chapter 4, then the measured relative complex dielectric constant will be altered and be given by

$$\epsilon_{r_{MEAS}} = \epsilon_{r_{ACT}} \left(\frac{\log_e a_3/a_1}{\log_e a_3/a_2 + \epsilon_{r_{ACT}} \log_e a_2/a_1} \right)^2 \quad (5.21)$$

where $\epsilon_{r_{ACT}} = \epsilon_{r_2}$, and $\epsilon_{r_1} = 1$.

Calculation of this relation for Parowax (a trade name for paraffin wax), using various size air gaps and the exact dimensions of the "coaxial-cage" line yields Figure 5.6. This theoretical curve is compared with measurements made on Parowax at 2.0 and 4.0 GHz. The experimental points were established by successively enlarging the centre conductor hole. The point corresponding to zero gap was obtained independently by filling an APC-7, 8 cm. long air line with Parowax and measuring the dielectric constant using the same technique as for the other points. It is obvious that within the experimental error the theoretical and empirical treatments agree for small air gaps $\leq .0102$ cm. (.004 in.). Beyond this value, they do not agree. This is possibly due to several factors: it has been assumed that the air gap is very small and that its only effect is to slightly modify the propagation constant of the TEM mode. This is not necessarily true for the larger gaps. Also, the theory assumes a symmetrical gap. In practice, the gap is not symmetrical, but the specimen is touching the inner conductor on one side of the gap and not at all on the other. There are also contributions from the four other air gaps around the outer conductors.

However, if the assumed value of a_3 is enlarged so that the theoretical curve fits the experimental one, the points agree very well. If the correct dimensions are employed for all but a_3 which is

Figure 5.6

Comparison of theoretical equation, describing air gap, and experiment; for Parowax at 2.0 and 4.0 GHz.

set equal to 2.54 cm. (1.0 in.), then this new semi-empirical equation predicts the correct measured values.

Further measurements were performed on artificial sea ice samples using the same technique as for Parowax. The results of these measurements, at 4.0 GHz, are shown in Figures 5.7 and 5.8. The shaded area represents the uncertainty in the theoretical calculation due to the possible error in the value that was assumed for $\epsilon_{r,ACT}$. The agreement with the semi-empirical curve is seen to be very good for both ϵ_r' and ϵ_r'' . Only the experimental points at 4.0 GHz are shown since they have the smallest measurement error and represent the most significant test of the theory.

Although the method and assumptions used to obtain this semi-empirical equation leave something to be desired, it has one advantage - it predicts the correct behaviour.

This equation may now be numerically inverted to predict what $\epsilon_{r,ACT}$ is from the measured value of $\epsilon_{r,MEAS}$ and knowledge of the size of the air gap.

The gap size also varies with temperature. Using the linear expansion coefficient for ice, which is $50 \text{ to } 55 \times 10^{-6} \frac{\text{cm}}{\text{cm}}/^\circ\text{C}$ according to the Chemical Rubber Tables⁽⁷⁹⁾ and Zubov⁽⁴⁰⁾, and for brass, which is $18.9 \times 10^{-6} \frac{\text{cm}}{\text{cm}}/^\circ\text{C}$ ⁽⁷⁹⁾, the change in hole circumference can be calculated. The original circumference of the hole in the ice is $(0.980 \text{ cm.})\pi$ at -10°C . The original circumference of the rod is $(0.970 \text{ cm.})\pi$ at -10°C . The difference in circumference at -10°C is then $\Delta l = .010\pi \text{ cm.} = 0.032 \text{ cm.}$ At -40°C the difference is

$$\begin{aligned} \Delta l &= \left[0.980\pi \text{ cm.} (1 + 55 \cdot 10^{-6} \frac{\text{cm}}{\text{cm}}/^\circ\text{C} (-30^\circ\text{C})) \right] \\ &\quad - \left[0.970\pi \text{ cm.} (1 + 18.9 \cdot 10^{-6} \frac{\text{cm}}{\text{cm}}/^\circ\text{C} (-30^\circ\text{C})) \right] \\ &= 0.030488 \text{ cm.} \end{aligned}$$

Figure 5.7

Comparison of semi-empirical equation, describing effect of air gap on ϵ_r' , and experiment; for artificial sea ice at 4.0 GHz.

Figure 5.8 Comparison of semi-empirical equation, describing effect of air gap on ϵ_r' , and experiment; for artificial sea ice at 4.0 GHz.

Therefore, the change in circumference (Δl) is 0.001512 cm. This causes a change in gap of 0.000241 cm. (9.47×10^{-5} in.). Reference to Figures 5.7 and 5.8 shows that this change in gap size causes a very small error in both ϵ_r' and ϵ_r'' . This error is much less than the experimental error due to other causes and can be neglected.

There is a further error involved in the use of the "coaxial-cage" line. This is the effect of the interface of the sample not being parallel to the probing wavefront. To evaluate the magnitude of this error, slabs of Parowax were cut with one face so that it formed an angle, θ , with the impinging diagnostic wave. The results of these measurements are shown in Table 5.1. The attenuation errors were all less than the measurement errors and are therefore not tabulated. The angular errors are given, and it is easily seen that they are also insignificant, for $\theta < 6.1^\circ$. An angle of 3.5° is very obvious to the human eye and it is therefore doubtful any of the angles to be encountered would be more than 1° at most.

That concludes the discussion of errors peculiar to this sample holder. Other errors due to the residual VSWR of the line are present, but since the "coaxial-cage" line performs at least as well as the air lines which make up the rest of the system (as discussed in the previous section) this error will be small compared to the total system error. The question of total system error will be discussed further in Section 5.3.

5.3 Final Measurement Scheme

5.3.1 Instrument Integration

It was decided to use a commercial Hewlett Packard 8410A Network Analyzer as the basis of the measurement system. The Network

TABLE 5.1 ERROR IN MEASURED ANGLE DUE TO NON-PARALLEL SAMPLE FACES

Angular Error ($^{\circ}$)

Frequency (Hz) \ θ ($^{\circ}$)	0	3.5	6.1	11.1	15.0
100 M	$0^{\circ} \pm 1^{\circ}$	$+0.45 \pm 1^{\circ}$	$+0.33 \pm 1^{\circ}$	$+0.40 \pm 1^{\circ}$	$+0.13 \pm 1^{\circ}$
200 M	$0^{\circ} \pm 1^{\circ}$	$+0.00 \pm 1^{\circ}$	$+1.15 \pm 1^{\circ}$	$+0.70 \pm 1^{\circ}$	$+1.08 \pm 1^{\circ}$
400 M	$0^{\circ} \pm 2^{\circ}$	$-0.21 \pm 2^{\circ}$	$+0.97 \pm 2^{\circ}$	$+1.19 \pm 2^{\circ}$	$+1.08 \pm 2^{\circ}$
800 M	$0^{\circ} \pm 2^{\circ}$	$-0.66 \pm 2^{\circ}$	$+2.83 \pm 2^{\circ}$	$+2.87 \pm 2^{\circ}$	$+2.29 \pm 2^{\circ}$
1.0 G	$0^{\circ} \pm 3^{\circ}$	$-2.90 \pm 3^{\circ}$	$+0.06 \pm 3^{\circ}$	$+1.17 \pm 3^{\circ}$	$+0.36 \pm 3^{\circ}$
2.0 G	$0^{\circ} \pm 3^{\circ}$	$+0.48 \pm 3^{\circ}$	$+4.16 \pm 3^{\circ}$	$+3.79 \pm 3^{\circ}$	$+1.87 \pm 3^{\circ}$
4.0 G	$0^{\circ} \pm 3^{\circ}$	$+1.31 \pm 3^{\circ}$	$+1.88 \pm 3^{\circ}$	$+3.82 \pm 3^{\circ}$	$+3.86 \pm 3^{\circ}$

Analyzer consists of the 8410A main frame, the 8411A Harmonic Frequency Converter, the 8414A polar display or the 8413A phase-gain display, plus in this case the 8740A Transmission Test Set. The use of this system allows measurement of the transmission coefficient over the frequency range 100 MHz to 12.4 GHz. A swept frequency source must also be provided. Here, a 8690A mainframe with 8699B (0.1 to 4.0 GHz), 8693A (4.0 to 8.0 GHz), and 8694A (8.0 to 12.4 GHz) plug-ins was employed to cover the entire frequency range.

In practical terms this means that to measure a single specimen at several frequencies, and then at several temperatures without re-calibration, only one plug-in may be used at a time. This limits the most useful range to 0.1 to 4.0 GHz. Separate samples must be prepared for the 4.0 to 8.0 GHz range. The 8.0 to 12.4 GHz range exceeds the capabilities of the "coaxial-cage" line, therefore it was not necessary to use the third plug-in for these experiments. The preparation of separate samples for the 4.0 to 8.0 GHz range will be seen in Section 6.2 not to be a serious problem.

The "coaxial-cage" line must be enclosed in a controlled temperature chamber during the measurements. A Delta design model 6400 chamber was used. Access is provided through an end port for cables and through the front hinged door for the sample holder. The chamber could easily maintain the temperature to within $\pm .1^{\circ}\text{C}$. To connect the sample holder in the chamber to the transmission test set, sections of RG-401/U semi-rigid coaxial cable equipped with Omni-Spectra APC-7 connectors were used. A special section of RG-401/U was used as a delay line in the reference arm of the transmission test set (see Figure 5.9) to equalize the line lengths. Coaxial APC-7 attenuators must be employed in both the reference and test channels

Figure 5.9 Wideband measurement system employing "coaxial-cage" line; bandwidth, 100 MHz - 7.5 GHz.

of the test set so that the signal levels are as specified in the manual.

It should be noted that the attenuation characteristics of RG-401/U do not change significantly with temperature. This makes them easier to use for the connections than air line, since air line is prone to internal water condensation. A water film in the cable causes a considerable error in the attenuation measurements.

The measurement scheme now allows measurement of the transmission coefficient over the frequency ranges 0.1 to 4.0 GHz and 4.0 to 8.0 GHz. The two channel lengths, test and reference, were adjusted to be approximately equal, then measurements on the empty holder were obtained at the frequencies and temperatures of interest during several independent calibration periods. Therefore, it was possible to easily obtain calibration points for the system which, when subtracted from the measured values of angle and amplitude for the specimens, would give the desired transmission coefficient of the full sample holder relative to the empty sample holder.

Now that the measurement system has been described, it is necessary to examine the total errors experienced in the measurement.

5.3.2 Total Estimated System Error

Hewlett Packard⁽⁸⁰⁾ gives the specified amplitude and angular errors for their total instrument system, consisting of: display, transmission test set, harmonic frequency converter and Network Analyzer. These error curves are shown in Figure 5.10. It is apparent that the angular error varies with the absolute value of the angle being measured. The amplitude error quoted is ± 0.1 dB for attenuations less than 10 dB. If the mean value between the poorly mismatched case and well-matched case of reference/test channel amplitudes is

Figure 5.10 Specified errors in measured angle and amplitude for configuration employed (Hewlett Packard (80)).

taken as typical, then the phase errors are given by the broken line.

The effect of the sample and "coaxial-cage" line on these specified errors should be minimal since the "coaxial-cage" line acts very much like the air lines which compose the rest of the system. Tests done with a Parowax sample mounted in an APC-7 air line and the "coaxial-cage" line show the results to agree within the error due to the air gap (See Figure 5.6 and Section 5.22).

Providing the sample's dielectric constant is relatively low, the VSWR produced by the sample-air interface should not produce serious difficulties. It is difficult to estimate the actual additional errors involved in placing the sample in the line unless one has independent, very accurate measurements of a well-known material for which comparison measurements can be made. It will be assumed, therefore, that the specified system errors, apart from those due to the air gaps, are truly indicative of the actual errors. (It will be seen later (Sections 6.3 and 7.1) that the specified errors do truly account for the observed variation in the data.)

An approximate relation between the error in measured angle and amplitude and the error in ϵ_r' and ϵ_r'' can be derived. The relation between ϵ_r' and the real and imaginary parts of the propagation constant β and α is given by Jordan and Balmain⁽⁷⁸⁾ as

$$(\alpha^2 - \beta^2) = -\left(\frac{2\pi}{\lambda_0}\right)^2 \epsilon_r' \quad (5.22)$$

Similarly, ϵ_r'' is related to α and β by

$$\epsilon_r'' = 2\left(\frac{\lambda_0}{2\pi}\right)^2 \beta\alpha \quad (5.23)$$

Differentiation of ϵ' with respect to θ , assuming small α gives

$$\frac{\partial \epsilon'}{\partial \theta} = \frac{-2\lambda_0}{360l} \left(1 - \frac{\theta\lambda_0}{360l}\right) \quad (5.24)$$

where l is the sample length in cm., λ_0 is the free space wavelength in cm., and θ (in degrees) is the measured angle. (In the two extremes of interest for sea ice: $\alpha^2 = .21(N/m)^2$ compared to $\beta^2 = 135.5m^{-2}$ at 200 MHz and $\alpha^2 = 21.2(N/m)^2$, compared to $\beta^2 = 21187m^{-2}$ at 4.0 GHz, and therefore the assumption that $\alpha^2 < \beta^2$ is valid.)

This gives an expression for $\Delta\epsilon_r'$ the error in ϵ_r' due to an error $\Delta\theta$ in θ as

$$\frac{\Delta\epsilon_r'}{\Delta\theta} = \frac{-2\lambda_0}{360l} \left(1 - \frac{\theta\lambda_0}{360l}\right) \quad (5.25)$$

Similarly, an expression for $\partial\epsilon_r''/\partial A$ can be found

$$\frac{\partial\epsilon_r''}{\partial A} = 2\left(\frac{\lambda_0}{2\pi}\right)^2 \left(\frac{2\pi}{\lambda_0}\sqrt{\epsilon_r'}\right) \left(\frac{1}{8.686}\right) \quad (5.26)$$

where A is the loss in db/cm and

$$\frac{\Delta\epsilon_r''}{\Delta A} = 2\left(\frac{\lambda_0}{2\pi}\right)^2 \left(\frac{2\pi}{\lambda_0}\sqrt{\epsilon_r'}\right) \left(\frac{1}{8.686}\right) \quad (5.27)$$

Table 5.2 gives the expected errors in ϵ_r' and ϵ_r'' for the angles and amplitudes measured, based on the Hewlett Packard specifications and including the errors due to subtraction of the measured values of angle and amplitude from the calibration readings.

The error due to the gap is corrected using the semi-empirical equation given in Section 5.2.2. Thus, the total system error, in ϵ_r' and ϵ_r'' , for the corrected values is given by Table 5.2. Errors due to other sources such as the ice itself and the measurement of other ice parameters will be treated in Chapter 6.

TABLE 5.2 ERROR BOUNDS ON ϵ_r' AND ϵ_r''

Frequency	ϵ_r'	ϵ_r''	Sample Length
100 MHz	± 0.60	$\pm .450$	8.0 cm.
200 MHz	± 0.50	$\pm .230$	8.0 cm.
400 MHz	± 0.24	$\pm .100$	8.0 cm.
800 MHz	± 0.17	$\pm .060$	8.0 cm.
1.0 GHz	± 0.13	$\pm .045$	8.0 cm.
2.0 GHz	± 0.07	$\pm .022$	8.0 cm.
4.0 GHz	± 0.03	$\pm .010$	8.0 cm.
4.0 GHz	± 0.12	$\pm .045$	2.0 cm.
7.5 GHz	± 0.07	$\pm .022$	2.0 cm.

CHAPTER 6. SEA ICE DIELECTRIC MEASUREMENTS

6.1 Measurement Procedure

6.1.1 Naturally Occurring Sea Ice

The natural sea ice specimens used in this study were collected from the Beaufort Sea ice cover during the AIDJEX '75 experiment in April 1975. The approximate coordinates of the camp at that time were $76^{\circ}20'N$ $140^{\circ}50'W$. The specimens will, because of the time of sampling, be representative of winter-spring sea ice.

Ice cores were taken with a SIPRE ice corer. It produces approximately vertical cores 1 m. in length and 7.6 cm. in diameter. Salinity and density profiles were constructed by taking cores, sectioning them on a bandsaw into 1 cm. long pieces, then weighing the standard sections to obtain densities, bagging the pieces, melting them and measuring the salinity of the melt with a standard Beckmann conductivity bridge, calibrated in parts per thousand salt content by weight. The accuracy of the density measurements is estimated at $\pm .03$ gm/cm³ and the accuracy of the salinity measurements at $\pm .2$ ‰. More accurate density measurements can be made by an immersion technique.

Temperature profiles of some of the cores were also taken by inserting thermistors in various locations in the core immediately after pulling it out of the ice cover.

The results of these measurements of salinity, temperatures and density of three representative areas, V, J and A are shown in Figures 6.1 to 6.5. The J area represents young sea ice and the samples were taken from a small refrozen lead (crack) approximately 70 cm. thick. The V area represents first-year sea ice and the samples were taken from a large refrozen lead approximately 160 cm. thick. The A area represents multiyear ice which had a thickness of approximately 320 cm.

Figure 6.1 Salinity and temperature profiles of adjacent young sea ice core (J2) showing locations of experimental specimens.

Figure 6.2 Salinity and temperature profiles of adjacent first-year sea ice core (V1) showing locations of experimental specimens.

Figure 6.3

Salinity, temperature, and density profiles of adjacent first-year sea ice core (V6) showing location of experimental specimens.

Figure 6.4 Salinity and density profiles of adjacent first-year sea ice core (V4) showing locations of experimental specimens.

Figure 6.5

Salinity and density profiles of adjacent multiyear sea ice core (A1) showing locations of experimental specimens.

The young sea ice specimens have a bulk structure that is essentially the same as first-year sea ice. The main differences are a higher surface salinity and higher bulk salinity. As the ice grows thicker, it turns into first-year ice. The only major changes are the increase in thickness and decrease in salinity - no recrystallization takes place. Therefore, throughout the rest of this thesis, for the purpose of dielectric measurements, the young sea ice specimens will be treated under the broad category of first-year sea ice.

Adjacent cores V1, V3 and V6 illustrate that there is little variation from location to location in the same first-year ice area. No such lack of variation occurs for multiyear ice; the core is only representative of a very small neighbourhood about itself.

Samples used for both dielectric and physical property measurements were stored in an unheated building at ambient temperature (usually less than -20°C) prior to use.

It is important to try and obtain the dielectric samples from a region of the core that has a relatively flat salinity profile, if any semblance of homogeneity is to be present in the specimen measured. The selected samples that exhibited a reasonable homogeneity are shown on Figures 6.1 to 6.5. Also indicated are the cores the samples were actually taken from. It is apparent that this procedure may still allow samples to be taken from sections of core that were not very homogeneous, since the adjacent cores, while very similar, may have slight offsets in their salinity profiles.

Once the sections of core had been selected they were cut to the desired length and the five holes necessary for mounting on the sample holder were drilled using the drill jig and a hand drill. Unfortunately, a drill press was not available for the Arctic measurements

and this caused considerable difficulty in obtaining specimens with parallel holes. All of the machining and fitting of the specimen to the sample holder was done in the unheated building.

Once the sample was prepared and installed on the "coaxial-cage" sample holder it was taken into the heated building where the Network Analyzer system was situated, and placed in the cold chamber at -40°C .

A preliminary test was performed on a block of ice of the same size and dimensions as the actual samples to determine the thermal time constant of the ice block. The ice block was subjected to a step in temperature by removing it from the -40°C cold chamber and placing it outside at -27°C . The temperature decay curve was measured with the use of a thermistor inserted in the centre of the sample. The time constant for decay to 0.67 of the original temperature was found to be about 28 minutes. Therefore, for a 5°C step, if 3τ have elapsed, the temperature of the block will be within about 0.25°C of the chamber temperature. Therefore, the samples were left 90 to 100 minutes to stabilize at the desired temperature, which would place them almost within the temperature chamber drift of $\pm 0.1^{\circ}\text{C}$.

After the samples had reached the desired temperature they were measured at frequencies of 100 MHz, 200 MHz, 400 MHz, 800 MHz, 1.0 GHz, 2.0 GHz and 4.0 GHz. The frequencies were accurately set to within ± 2 MHz using a digital frequency counter (EIP model 350D). Once the complete set of frequencies was finished, the temperature of the chamber was set to -30°C and the procedure repeated, with the sample being allowed to stabilize again. This procedure was repeated successively throughout the day until the sample had been measured at -25°C , -20°C , -15°C , -10°C and -5°C .

For the higher frequency tests, this procedure was modified somewhat by using shorter samples, approximately 2 cm. in length, and measuring them at 4.0 and 7.5 GHz.

Once the series of measurements was complete, the sample was removed from the chamber, bagged, melted and its salinity measured. The salinity of the sample after measurement agreed within the accuracy of the salinity sampling procedure ($\pm .5 \%$) with the average salinity of the corresponding region in an adjacent core.

Density measurements were not performed on the actual samples themselves. The only worthwhile density measurement that could have been performed on the samples themselves would require use of the immersion technique. It was feared that this technique would contaminate the specimens if done prior to measurement, and may not be representative if done afterwards because of the long period the sample had spent at elevated temperatures during the -5°C measurements. For the above reasons, it was decided to rely solely on density data from adjacent cores. Since the samples were almost all young or first-year ice, this technique should yield reasonably representative figures for density, although it may contribute somewhat to the scatter of the data.

A further complicating factor in the density measurements is the fact that the density changes with temperature. It was decided, for reasons which will become apparent in Chapter 7, that the density at approximately -20°C would be taken to be representative of the density in general (see Chapter 8 for further comments).

The transmission coefficient data obtained from these measurements was first corrected using the calibration equations and then analyzed to a first-order approximation at the site using a graphical procedure outlined in Section 6.2. These calculations indicated that

the measurement results were about what was expected. Detailed measurements involve numerical inversion of equation (4.1) on a computer, using a modified Newton's method in the complex plane (see Appendix D).

6.1.2 Artificial Sea Ice

Artificial sea ice specimens were grown in the laboratory using the method of Section 3.1.3. Vertical slabs were cut and smaller samples were removed from these slabs. The samples were cut at angles of 0° , 30° , 45° , 60° and 90° to the vertical. These samples of ice were machined at approximately -10°C after having been stored below -40°C . The machining, performed with a drill press and the familiar drill jig produced useable specimens much more reliably than before. The resulting specimens were mounted on the "coaxial-cage" line so that the normal vertical direction was at the above-mentioned angles to the direction of wave propagation. This procedure was meant to enable investigation of the effect of incident angle on the measured value. It should be realized, however, that the wave propagating in the "coaxial-cage" line is approximately radially directed, i.e. there is no single electric field direction in the line, so that the wave essentially averages over all the radial directions. Therefore, the process of measuring the sample, which was taken at an angle with respect to the vertical, will not be identical with the process of propagating a vertically-polarized wave through an ice layer at various angles, but rather will represent the effect of averaging over all the possible orientations of the ellipsoids with respect to the electric field vector.

Measurements were performed on the samples at -40°C , -25°C , -15°C , -12°C , -10°C , -8°C and -6°C over the frequencies 100 MHz, 200 MHz, 400 MHz, 800 MHz, 1.0 GHz, 2.0 GHz, and 4.0 GHz exactly as des-

cribed in Section 6.1.1. The analysis of the data was identical.

An attempt was also made to grow a large sheet of artificial sea ice that could be propped up at different angles between two antennas with reasonably linearly polarized fields. The problems involved with the necessarily finite extent of the sheet, its non-parallel sides and the spurious reflections resulting when the sheet is at an angle to the incoming wave were judged to cause the results to be inconclusive. Therefore, this problem of trying to simulate propagation of a linearly-polarized wave was abandoned to a future study, which could possibly use a parallel-plate type transmission line for a sample holder (see Chapter 7 and 8 for further comments).

6.2 Interpretation

As already mentioned above, the dielectric constants ϵ_r' and ϵ_r'' are obtained numerically from equation (4.1). The interpretation is not exactly straightforward, however, as shown in Figures 6.6 to 6.9, which are plots obtained from inversion of equation (4.1). ϵ_r' and ϵ_r'' vary in a non-ambiguous fashion with the measured angle and amplitude of the transmission coefficient. It is necessary to choose a sample length that allows one to find ϵ_r' unambiguously. If a maximum value of 10, for example, is set for ϵ_r' , then it is only necessary to choose the sample length such that the measured angle will only go through one cycle of 360° as ϵ_r' increases from 1 to 10.

If a sample length of 8 cm. is chosen, the interpretation is simple enough at 100 MHz, but at the upper frequency of 4.0 GHz, the measured angle goes through two cycles between $\epsilon_r' = 1$ and 10. Therefore, there are two possible solutions. For example, for a measured angle of -360° , $\epsilon_r' = 3.8$ or $\epsilon_r' = 8.0$. From the preliminary measurements performed at 100 MHz and from a knowledge of the dielectric

Figure 6.6

Variation of ϵ_r' and ϵ_r'' with transmission coefficient for 8 cm. long sample at 100 MHz.

Figure 6.7 Variation of ϵ_r' and ϵ_r'' with transmission coefficient for 8 cm. long sample at 4 GHz.

Figure 6.8 Variation of ϵ_r' and ϵ_r'' with transmission coefficient for 2 cm. long sample at 4 GHz.

Figure 6.9
Variation of ϵ_r' and ϵ_r'' with transmission coefficient for 2 cm. long sample
at 8 GHz.

properties of pure ice, it would seem as though 3.8 is the correct value. To relieve any question of which value to choose, a second set of measurements can be performed at 4.0 GHz and 8.0 GHz using 2.0 cm. long samples. As shown in Figure 6.8 this sample length gives only one value of ϵ_r' between 1 and 25 and it is easy to see what the correct value is. Similarly, in Figure 6.9 it is apparent that as long as it is possible to know the approximate value of ϵ_r' , there is no ambiguity in the interpretation.

The sample length used represents a tradeoff. If a long sample is used, the electrical length is increased and at the lower frequencies it can be seen from Table 5.2 and equations (5.22) to (5.27) that this represents a considerable increase in measurement accuracy for both ϵ_r' and ϵ_r'' . Therefore, a long sample was used to obtain a more accurate measurement of ϵ_r' and ϵ_r'' at the lower frequencies, and a short sample was used at 4.0 GHz and 7.5 GHz to resolve the ambiguity.

It is apparent from Figure 6.7 that as the sample thickness increases, the ambiguity increases at the higher frequencies so that in the GHz frequency range, the measurement of long samples (tens of cm. thick) will cause many ambiguous solutions over a very narrow range of ϵ_r' (e.g. 3 to 4). Therefore, it is desirable to obtain specimens short enough to remove this problem, but long enough to still have an electrical length greater than 100° .

Curves similar to those given in Figures 6.6 to 6.9 were prepared at every frequency of interest and used to obtain approximate values for ϵ_r' and ϵ_r'' , while still in the Arctic. This enabled a rough check to be made on the results before leaving the site.

6.3

Measurement Results

6.3.1 AIDJEX Experiments

6.3.1.1 Results for ϵ_r'

The results of the experiments performed during AIDJEX, on the Beaufort Sea ice cover, are presented in Figures 6.10 to 6.49 and in Table 6.1. The error bounds indicated are those given in Table 5.2, and include both the errors due to the measurement and the errors due to the calibration. The values plotted are already corrected for the error due to the air gap.

The results for ϵ_r' vs. temperature for first-year sea ice are shown in Figures 6.10 to 6.23 and 6.26 to 6.29. It is immediately apparent that the errors are large (almost 10%) for the 100 MHz to 400 MHz points, and become more tolerable (about 1 - 5%) for the 800 MHz to 4.0 GHz values. The solid lines plotted on each curve merely serve to indicate the general trend while demonstrating that in all but a few cases the points can be fitted to a smooth curve, within the estimated experimental errors. In the case of the 100 MHz data, this is not very significant, but for the 4.0 GHz data where the errors are smaller (about 1%) it is significant.

The reason for the large errors at the low frequencies is easily seen from equations (5.22) to (5.27). The electrical length of an 8.0 cm. ice sample at 100 MHz is less than 10° . If the transmission coefficient can measure this angle to within $\pm 1^\circ$, the angular error is then $\pm 10\%$, which from equation (5.25) results in approximately $\pm 10\%$ error in ϵ_r' . To achieve a smaller error at this frequency with the same equipment, a much longer specimen must be used. Longer specimens are impractical for two reasons: the holes for mounting on the sample holder cannot be accurately drilled for samples in excess of 8.0 cm.,

Figure 6.10 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-1 ($S = 8.1 \%$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.11 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-1 ($S = 8.1 \%$, $\rho = 0.91 \text{ gm/cm}^3$), measurement frequencies as shown.

Figure 6.12

ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-2 ($S = 9.1 \text{ ‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.13 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-2 ($S = 9.1 \%$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.14 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-3 ($S = 5.6 \text{ ‰}$, $\bar{\rho} = 0.91 \text{ gm cm}^{-3}$); measurement frequencies as shown.

Figure 6.15 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-3 ($S = 5.6 \%$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.16 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-4 ($S = 5.1 \text{ ‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.17 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-4 ($S = 5.1 \text{ ‰}$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequency shown.

Figure 6.18

ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-9 ($S = 10.5\%$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.19 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-9 ($S = 10.5\text{‰}$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.20

ϵ_r versus temperature for AIDJEX first-year sea ice sample, FYI-10 ($S = 9.3 \%$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.21 ϵ_r' versus temperature for AIDJEX first-year sea ice sample, FYI-10 ($S = 9.3 \text{ ‰}$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.22 ϵ_r' versus temperature for AIDJEX first-year sea ice sample, FYI 14 ($S = 6.1 \text{ ‰}$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.23 ϵ_r' versus temperature for AIDJEX first-year sea ice sample, FYI-14 ($S = 6.1 \text{ ‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.24 ϵ_r' versus temperature, for AIDJEX multiyear sea ice sample, MYI-15 ($S = 1.75\%$, $\bar{\rho} = 0.90 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.25 ϵ_r' versus temperature for AIDJEX multiyear sea ice sample, MYI-13 ($S = 1.32\%$, $\bar{\rho} = 0.90 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.26 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-18 ($S = 7.5\text{‰}$, $\rho = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

Figure 6.27 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-19 ($S = 5.5\text{‰}$, $\rho = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

Figure 6.28 ϵ_r' versus temperature for AIDJEX first-year sea ice sample, FYI-20 ($S = 4.1\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

Figure 6.29 ϵ_r' versus temperature, for AIDJEX first-year sea ice sample, FYI-21 ($S = 4.1\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$);

and the sample is not even approximately homogeneous over lengths greater than 8.0 cm. Therefore, the solution is not found in using longer specimens. This means the measurement accuracy of the equipment must be vastly improved. If an automatic network analyzer system is used the errors can be greatly reduced by performing a detailed calibration of the measurement interfaces at each frequency and numerically solving the necessary scattering equations (see Hackborn⁽⁸¹⁾). Unfortunately, although this method decreases the errors by at least an order of magnitude, it is also much more expensive (approximately \$200 to 300K) and not portable.

The automatic network analyzer technique still maintains the wideband frequency range desired. If one were willing to sacrifice this wideband approach for greater accuracy, a well-tuned single frequency coherent bridge or capacitance-conductance bridge could be employed at each frequency of interest.

If one can overlook the large errors at low frequencies in such a "first-look" study as this, where a large overview is desirable, then certain features become obvious. Examination of the high frequency data shows the measured values follow a specific trend. As was found in the preliminary results, in Chapter 4, the ϵ_r' curves increase with elevated temperature as the effect of the brine coming into solution is felt. The solid lines in Figures 6.10 to 6.29 show this trend. (Where certain frequencies tend to group together the trend for all the frequencies in the group is shown by a single line.) In most of the cases illustrated, it appears that the estimate of errors has been most liberal and that the points follow a smooth curve if only a third or a quarter of the allowed error is assumed. This is especially true of the 100 MHz to 400 MHz data points.

It should also be noted, when examining the results, that all of the points clustered around a particular temperature were taken at -5°C , -10°C , -15°C , -20°C , -25°C , -30°C or -40°C . It was necessary for clarity, when plotting so many points with such large error bounds, to offset the points slightly so they can be seen. Thus, accuracy of representation has been sacrificed for artistic license in the presentation of the data. The interested reader can find the actual values in Table 6.1.

To continue the discussion of the general trend, it is apparent that all the solid lines except those for FYI-2 (Figures 6.12 and 6.13) converge towards some temperature below -40°C . The exception noted for this set of results could be explained by the error bounds on the points in question or by the possibility that something was peculiar about this particular sample. However, as shown in Figure 6.2, FYI-1 and FYI-2 were taken at almost the same location in the V ice layer although they came from different cores. FYI-2 may have been taken from an area with a steep salinity gradient. This gradient in salinity may have distinctly different effects at high and low frequencies, i.e. at low frequencies the gradient is averaged out more due to the longer wavelength, whereas at high frequencies the averaging process is more limited.

Another obvious aspect of these curves is the fact that samples with the same density, within the experimental error, and approximately the same salinity exhibit different asymptotic values of ϵ_r' at low temperatures. This is probably just another demonstration of the variability in the ice, and an indication of the role played, in determining ϵ_r' , by the orientation of the brine inclusions with respect to the electric field. Further discussion of this point

TABLE 6.1 EXPERIMENTAL DATA FROM THE AIDJEX SITE (SPRING, 1975)

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
FYI-1	8.1 ± .2	AVG = 0.91 ± .03	-39.05	0.0046	0.100	3.19 ± .6	0.193 ± .45	0.061	1.0
			-29.44	0.0128		3.42	0.073	0.021	0.4
			-24.55	0.0268		3.86	0.370	0.096	1.7
			-19.82	0.0274		4.04	0.481	0.119	2.2
			-14.55	0.0338		4.09	0.700	0.171	3.1
			-10.19	0.0442		4.25	0.736	0.173	3.2
			-4.94	0.0823		4.40	0.949	0.216	4.1
			-29.44	0.0128	0.200	3.43 ± .5	0.019 ± .23	0.006	0.2
			-24.55	0.0268		3.68	0.029	0.008	0.3
			-19.82	0.0274		3.48	0.302	0.087	2.9
			-14.55	0.0338		3.85	0.479	0.124	4.4
			-10.19	0.0442		3.96	0.545	0.138	5.0
			-4.94	0.0823		4.18	0.581	0.139	5.2
			-24.55	0.0268	0.400	3.68 ± .24	0.260 ± .10	0.071	4.9
			-19.82	0.0274		3.68	0.195	0.053	3.7
			-14.55	0.0338		3.80	0.435	0.114	8.1
			-10.19	0.0442		3.82	0.348	0.091	6.5
			-4.94	0.0823		4.04	0.503	0.125	9.1
			-29.44	0.0128	0.800	3.27 ± .17	0.037 ± .06	0.011	1.5
			-24.55	0.0268		3.39	0.301	0.089	11.9
			-19.82	0.0274		3.44	0.172	0.050	6.8
			-14.55	0.0338		3.50	0.226	0.065	8.8
			-10.19	0.0442		3.56	0.346	0.097	13.3
			-4.94	0.0823		3.72	0.401	0.108	15.1

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-1	8.1 ± .2	AVG = 0.91 ± .03	-39.05	0.0046	1.000	3.20 ± .13	0.027 ± .045	0.008	1.4
			-29.44	0.0128		3.25	0.050	0.015	2.5
			-24.55	0.0268		3.38	0.161	0.048	8.0
			-19.82	0.0274		3.41	0.172	0.051	8.5
			-14.55	0.0338		3.45	0.194	0.056	9.5
			-10.19	0.0442		3.53	0.248	0.070	12.0
			-4.94	0.0823		3.69	0.314	0.085	14.9
			-39.05	0.0046		3.13 ± .07	0.043 ± .022	0.014	4.4
			-29.44	0.0128		3.19	0.060	0.019	6.1
			-24.55	0.0268		3.29	0.118	0.036	11.8
			-19.82	0.0274	2.000	3.33	0.140	0.042	13.9
			-14.55	0.0338		3.37	0.161	0.048	16.0
			-10.19	0.0442		3.43	0.194	0.057	19.0
			-4.94	0.0823		3.59	0.249	0.069	23.8
			-39.05	0.0046		3.12 ± .03	0.067 ± .01	0.022	13.8
			-29.44	0.0128		3.18	0.058	0.018	11.8
			-24.55	0.0268		3.26	0.104	0.032	21.0
			-19.82	0.0274		3.29	0.129	0.039	25.8
			-14.55	0.0338		3.34	0.140	0.042	27.8
			-10.19	0.0442		3.39	0.161	0.048	31.9
FYI-2	9.1 ± .2	AVG = 0.91 ± .03	-39.40	0.0052	0.100	3.17 ± .6	0.546 ± .45	0.172	2.8
			-29.70	0.0141		3.31	0.601	0.182	3.0
			-24.60	0.0300		3.30	0.837	0.254	4.2
			-19.50	0.0313		3.51	0.647	0.184	3.1
			-14.70	0.0379		3.72	0.477	0.128	2.2

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
FYI-2	9.1 ± .2	AVG = 0.91 ± .03	-10.10 -4.60	0.0503 0.9930	0.100	3.94 ± .6 5.08	0.851 ± .45 1.460	0.216 0.287	3.9 5.8
			-39.40 -29.70 -24.60 -19.50 -14.70 -10.10 -4.60	0.0052 0.0141 0.0300 0.0313 0.0379 0.0503 0.9930	0.200	3.20 ± .5 3.28 3.29 3.38 3.58 3.69 4.62	0.095 ± .23 0.225 0.236 0.226 0.357 0.379 1.040	0.030 0.069 0.072 0.067 0.100 0.103 0.226	1.0 2.3 2.4 2.2 3.4 3.6 8.8
			-39.40 -29.70 -24.60 -19.50 -14.70 -10.10 -4.60	0.0052 0.0141 0.0300 0.0313 0.0379 0.0503 0.9930	0.400	3.20 ± .24 3.25 3.24 3.42 3.49 3.70 4.48	0.095 ± .10 0.139 0.300 0.226 0.205 0.499 0.917	0.030 0.043 0.093 0.066 0.059 0.135 0.205	1.9 2.8 6.1 4.4 4.0 9.4 15.7
			-39.40 -29.70 -24.60 -19.50 -14.70 -10.10 -4.60	0.0052 0.0141 0.0300 0.0313 0.0379 0.0503 0.9930	0.800	3.03 ± .17 3.06 2.98 3.21 3.29 3.36 3.99	0.139 ± .06 0.128 0.202 0.225 0.215 0.269 0.557	0.046 0.042 0.068 0.070 0.065 0.080 0.139	5.8 5.3 8.5 9.1 8.6 10.7 20.23
			-39.40 -29.70	0.0052 0.0141	1.000	3.00 ± .13 3.05	0.213 ± .045 0.128	0.071 0.042	11.2 6.7

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
FYI-2	9.1 ± .2	AVG = 0.91 ± .03	-24.60	0.0300	1.000	2.99 ± .13	0.160 ± .045	0.053	8.4
			-19.50	0.0313		3.20	0.171	0.054	8.7
			-14.70	0.0379		3.25	0.171	0.053	8.6
			-10.10	0.0503		3.35	0.236	0.071	11.7
			-4.60	0.9930		3.94	0.491	0.125	22.4
			-39.40	0.0052	2.000	2.98 ± .07	0.079 ± .022	0.026	8.3
			-29.70	0.0141		3.02	0.086	0.029	9.0
			-24.60	0.0300		2.96	0.106	0.036	11.2
			-19.50	0.0313		3.15	0.128	0.041	13.1
			-14.70	0.0379		3.19	0.118	0.037	12.0
			-10.10	0.0503		3.24	0.171	0.053	17.3
			-4.60	0.9930		3.83	0.370	0.097	34.3
			-39.40	0.0052	4.000	2.96 ± .03	0.033 ± .01	0.011	7.0
			-29.70	0.0141		2.89	0.048	0.017	10.2
			-24.60	0.0300		2.95	0.068	0.023	14.4
			-19.50	0.0313		3.13	0.096	0.031	19.7
			-14.70	0.0379		3.16	0.098	0.031	20.1
FYI-3	5.6 ± .2	AVG = 0.91 ± .03	-10.10	0.0503		3.26	0.139	0.043	28.1
			-4.60	0.9930		3.77	0.304	0.081	56.9
			-39.50	0.0032	0.100	3.16 ± .6	0.246 ± .45	0.078	1.3
			-29.40	0.0089		3.23	0.225	0.070	1.1
			-25.00	0.0170		3.55	0.475	0.134	2.3
			-19.60	0.0192		3.82	0.370	0.097	1.7
			-14.70	0.0233		4.22	0.439	0.104	1.9
			-29.40	0.0089	0.200	3.34 ± .5	0.101 ± .23	0.030	1.0
			-25.00	0.0170		3.50	0.194	0.056	1.9

TABLE 6.1

SAMPLE	SALINITY (o/oo)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FY1-3	5.6 ± .2	AVG = 0.91 ± .03	-19.60 -14.70	0.0192 0.0233	0.200	3.67 ± .5 3.82	0.249 ± .23 0.359	0.068 0.094	2.4 3.3
			-39.50 -25.00 -19.60 -14.70	0.0032 0.0170 0.0192 0.0233	0.400	3.39 ± .24 3.52 3.63 3.63	0.096 ± .10 0.129 0.086 0.086	0.028 0.037 0.024 0.024	1.9 2.5 1.6 1.6
			-39.50 -29.40 -25.00 -19.60 -14.70	0.0032 0.0089 0.0170 0.0192 0.0233	0.800	3.22 ± .17 3.28 3.39 3.49 3.56	0.033 ± .06 0.061 0.118 0.108 0.184	0.010 0.019 0.035 0.031 0.052	1.3 2.4 4.7 4.2 7.1
			-39.50 -29.40 -25.00 -19.60 -14.70	0.0032 0.0089 0.0170 0.0192 0.0233	1.000	3.22 ± .13 3.28 3.38 3.45 3.52	0.035 ± .045 0.073 0.129 0.129 0.194	0.011 0.022 0.038 0.037 0.055	1.8 3.7 6.4 6.3 9.4
			-39.50 -29.40 -25.00 -19.60 -14.70	0.0032 0.0089 0.0170 0.0192 0.0233	2.000	3.18 ± .07 3.24 3.31 3.39 3.43	0.028 ± .022 0.051 0.089 0.105 0.129	0.009 0.016 0.027 0.031 0.038	2.8 5.2 8.9 10.4 12.7
			-39.50 -29.40 -25.00	0.0032 0.0089 0.0170	4.000	3.15 ± .03 3.22 3.30	0.003 ± .01 0.029 0.067	0.001 0.009 0.020	0.0 5.8 13.3

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-3	5.6 ± .2	AVG = 0.91 ± .03	-19.60	0.0192	4.000	3.36 ± .03	0.090 ± .01	0.027	17.9
			-14.70	0.0233		3.41	0.055	0.016	10.8
FYI-4	5.1 ± .2	AVG = 0.91 ± .03	-29.70	0.0079	0.100	3.25 ± .6	0.204 ± .45	0.063	1.0
			-24.70	0.0164		3.46	0.323	0.093	1.6
			-19.90	0.0173		3.78	0.500	0.132	2.3
			-14.80	0.0212		3.93	0.578	0.147	2.6
			-10.00	0.0284		4.07	0.645	0.159	2.9
			-5.10	0.0507		4.12	0.861	0.195	3.7
			-39.50	0.0029	0.200	3.09 ± .5	0.139 ± .23	0.045	1.4
			-24.70	0.0164		3.37	0.086	0.026	0.9
			-19.90	0.0173		3.63	0.335	0.093	3.2
			-14.80	0.0212		3.68	0.358	0.097	3.4
			-10.00	0.0284		3.81	0.467	0.123	4.3
			-5.10	0.0507		3.99	0.546	0.137	5.0
			-39.50	0.0029	0.400	3.15 ± .24	0.097 ± .10	0.031	2.0
			-24.70	0.0164		3.38	0.094	0.028	1.8
			-19.90	0.0173		3.57	0.184	0.051	3.5
			-14.80	0.0212		3.69	0.379	0.103	7.2
			-10.00	0.0284		3.77	0.423	0.112	7.9
			-5.10	0.0507		3.87	0.294	0.076	5.4
			-39.50	0.0029	0.600	2.99 ± .17	0.103 ± .06	0.035	4.3
			-29.70	0.0079		3.12	0.058	0.019	2.4
			-24.70	0.0164		3.24	0.076	0.024	3.1
			-19.90	0.0173		3.39	0.183	0.039	7.2
			-14.80	0.0212		3.41	0.215	0.063	8.5

TABLE 6.1

SAMPLE	SALINITY ($^{\circ}/_{\infty}$)	DENSITY (gm/cm ³)	TEMP. ($^{\circ}$ C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-4	5.1 $\pm$.2	AVG = 0.91 $\pm$.03	-10.00	0.0284	0.800	3.46 $\pm$.17	0.259 $\pm$.06	0.075	10.1
			-5.10	0.0507		3.62	0.303	0.084	11.6
			-39.50	0.0029	1.000	3.04 $\pm$.13	0.053 $\pm$.045	0.018	2.8
			-29.70	0.0079		3.14	0.046	0.015	2.4
			-24.70	0.0164		3.24	0.058	0.018	2.9
			-19.90	0.0173		3.37	0.161	0.048	8.0
			-14.80	0.0212		3.40	0.194	0.057	9.5
			-10.00	0.0284		3.43	0.226	0.066	11.1
			-5.10	0.0507		3.58	0.270	0.075	13.0
			-39.50	0.0029	2.000	3.03 $\pm$.07	0.032 $\pm$.022	0.011	3.3
			-29.70	0.0079		3.11	0.032	0.010	3.3
			-24.70	0.0164		3.20	0.064	0.020	6.5
			-19.90	0.0173		3.29	0.118	0.036	11.8
			-14.80	0.0212		3.33	0.129	0.039	12.8
			-10.00	0.0284		3.37	0.150	0.045	14.9
			-5.10	0.0507		3.53	0.194	0.055	18.8
			-39.50	0.0029	4.000	3.02 $\pm$.03	0.009 $\pm$.010	0.003	1.7
			-29.70	0.0079		3.10	0.014	0.005	2.8
			-24.70	0.0164		3.16	0.046	0.015	9.4
			-19.90	0.0173		3.26	0.087	0.027	17.5
			-14.80	0.0212		3.07	0.100	0.033	20.8
			-10.00	0.0284		3.33	0.129	0.039	25.7
			-5.10	0.0507		3.49	0.183	0.053	35.7
FYI-9	10.5 $\pm$.2	AVG = 0.91 $\pm$.03	-39.40	0.0060	0.100	3.18 $\pm$.6	0.075 $\pm$.45	0.024	0.4
			-29.60	0.0164		3.19	0.118	0.037	0.6

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-9	10.5 ± .2	AVG = 0.91 ± .03	-24.80	0.0332	0.100	3.75 ± .6	0.250 ± .45	0.067	1.2
			-20.00	0.0355		4.25	0.714	0.168	3.1
			-14.50	0.0442		4.56	0.897	0.197	3.8
			-9.80	0.0594		4.87	0.993	0.204	4.1
			-5.20	0.1020		5.39	1.470	0.273	5.7
			-29.60	0.0164	0.200	3.47 ± .5	0.016 ± .23	0.005	0.2
			-20.00	0.0355		4.00	0.393	0.098	3.6
			-14.50	0.0442		4.17	0.494	0.118	4.4
			-9.80	0.0594		4.38	0.529	0.121	4.6
			-5.20	0.1020		4.86	0.915	0.188	7.5
			-39.40	0.0060	0.400	3.42 ± .24	0.050 ± .10	0.015	1.0
			-24.80	0.0332		3.63	0.054	0.015	1.0
			-20.00	0.0355		3.90	0.261	0.067	4.8
			-14.50	0.0442		4.04	0.306	0.076	5.5
			-9.80	0.0594		4.33	0.595	0.137	10.4
			-5.20	0.1020		4.77	0.802	0.168	13.3
			-39.40	0.0060	0.800	3.24 ± .17	0.036 ± .06	0.011	1.5
			-29.60	0.0164		3.34	0.204	0.061	8.1
			-24.80	0.0332		3.47	0.119	0.034	4.6
			-20.00	0.0355		3.67	0.292	0.080	11.1
			-14.50	0.0442		3.80	0.369	0.097	13.8
			-9.80	0.0594	1.000	3.95	0.404	0.102	14.8
			-5.20	0.1020		4.28	0.605	0.141	21.2
			-39.40	0.0060		3.26 ± .13	0.047 ± .045	0.015	2.4
			-29.60	0.0164		3.35	0.107	0.032	5.3

TABLE 6.1

SAMPLE	SALINITY ($^{\circ}/_{\infty}$)	DENSITY (gm/cm^3)	TEMP. ($^{\circ}\text{C}$)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
FYI-9	10.5 $\pm$.2	AVG = 0.91 $\pm$.03	-24.80	0.0332	1.000	3.40 $\pm$.13	0.151 $\pm$.045	0.044	7.4
			-20.00	0.0355		3.62	0.292	0.081	14.0
			-14.50	0.0442		3.73	0.358	0.096	16.8
			-9.80	0.0594		3.89	0.446	0.115	20.6
			-5.20	0.1020		4.19	0.593	0.141	26.3
			-39.40	0.0060	2.000	3.21 $\pm$.07	0.044 $\pm$.022	0.014	4.5
			-29.60	0.0164		3.27	0.069	0.021	6.9
			-24.80	0.0332		3.36	0.118	0.035	11.7
			-20.00	0.0355		3.11	0.171	0.055	17.6
			-14.50	0.0442		3.59	0.238	0.066	22.8
			-9.80	0.0594		3.71	0.282	0.076	26.6
			-5.20	0.1020		3.97	0.393	0.099	35.8
			-39.40	0.0060	4.000	3.21 $\pm$.03	0.094 $\pm$.010	0.029	19.1
			-29.60	0.0164		3.26	0.118	0.036	23.7
			-24.80	0.0332		3.34	0.150	0.045	29.9
			-20.00	0.0355		3.46	0.216	0.062	42.1
			-14.50	0.0442		3.54	0.248	0.070	48.0
			-9.80	0.0594		3.64	0.292	0.080	55.6
			-5.20	0.1020		3.88	0.403	0.104	74.3
FYI-10	9.3 $\pm$.2	AVG = 0.91 $\pm$.03	-39.50	0.0053	0.100	3.02 $\pm$.6	0.013 $\pm$.45	0.004	0.1
			-29.80	0.0142		3.23	0.024	0.007	0.1
			-25.00	0.0282		3.49	0.119	0.034	0.6
			-20.10	0.0313		3.80	0.772	0.203	3.6
			-15.10	0.0380		4.07	0.700	0.172	3.1
			-10.30	0.0506		4.29	0.902	0.210	3.9
			-5.50	0.0863		4.81	1.340	0.178	5.5

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-10	9.3 ± .2	AVG = 0.91 ± .03	-25.00	0.0282	0.200	3.34 ± .5	0.014 ± .23	0.004	0.1
			-20.10	0.0313		3.60	0.346	0.096	3.3
			-15.10	0.0380		3.76	0.304	0.081	2.9
			-10.30	0.0506		3.91	0.567	0.145	5.2
			-5.50	0.0863		4.38	0.816	0.186	7.1
			-20.10	0.0313	0.400	3.53 ± .24	0.291 ± .10	0.083	5.6
			-15.10	0.0380		3.70	0.401	0.108	7.6
			-10.30	0.0506		3.81	0.391	0.103	7.3
			-5.50	0.0863		4.17	0.494	0.118	8.8
			-39.50	0.0053	0.800	3.09 ± .17	0.058 ± .06	0.019	2.4
			-29.80	0.0142		3.16	0.010	0.003	0.4
			-25.00	0.0282		3.24	0.042	0.013	1.7
			-20.10	0.0313		3.39	0.215	0.064	8.5
			-15.10	0.0380		3.50	0.270	0.077	10.5
			-10.30	0.0506		3.62	0.335	0.093	12.8
			-5.50	0.0863		3.97	0.469	0.118	17.1
			-39.50	0.0053	1.000	3.13 ± .13	0.014 ± .045	0.004	0.7
			-29.80	0.0142		3.16	0.048	0.015	2.5
			-25.00	0.0282		3.23	0.098	0.030	4.9
			-20.10	0.0313		3.37	0.236	0.070	11.7
			-15.10	0.0380		3.45	0.291	0.084	14.2
			-10.30	0.0506		3.55	0.346	0.097	16.6
			-5.50	0.0863		3.90	0.457	0.117	21.0
			-39.50	0.0053	2.000	3.02 ± .07	0.048 ± .022	0.016	5.0
			-29.80	0.0142		3.07	0.064	0.021	6.6

TABLE 6.1

SAMPLE	SALINITY (°/∞)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-10	9.3 ± .2	AVG = 0.91 ± .03	-25.00	0.0282	2.000	3.10 ± .07	0.085 ± .022	0.028	8.8
			-20.10	0.0313		3.24	0.193	0.060	19.5
			-15.10	0.0380		3.30	0.204	0.062	20.4
			-10.30	0.0506		3.39	0.258	0.076	25.5
			-5.50	0.0863		3.69	0.336	0.091	31.8
			-39.50	0.0053	4.000	2.99 ± .03	0.013 ± .010	0.004	2.6
			-29.80	0.0142		3.05	0.037	0.012	7.7
			-25.00	0.0282		3.08	0.066	0.022	13.7
			-20.10	0.0313		3.22	0.150	0.047	30.4
			-15.10	0.0380		3.26	0.161	0.049	32.4
FYI-14	6.1 ± .2	AVG = 0.91 ± .03	-10.30	0.0506	0.100	3.34	0.204	0.061	40.6
			-5.50	0.0863		3.64	0.281	0.077	53.6
			-24.70	0.0197		3.59 ± .6	0.090 ± .45	0.025	0.4
			-19.80	0.0207		4.08	0.624	0.153	2.8
			-14.80	0.253		4.26	0.901	0.212	4.0
			-10.00	0.0340	0.200	4.38	1.210	0.277	5.2
			-19.80	0.0207		3.72 ± .5	0.184 ± .23	0.050	1.7
			-14.80	0.0253		3.92	0.588	0.150	5.4
			-10.00	0.0340		3.94	0.589	0.150	5.4
			-4.50	0.0679		4.32	0.716	0.166	6.2
			-24.70	0.0197	0.400	3.44 ± .24	0.084 ± .10	0.024	1.6
			-19.80	0.0207		3.62	0.216	0.060	4.1
			-14.80	0.0253		3.74	0.467	0.125	8.8
			-10.00	0.0340		3.79	0.391	0.103	7.3
			-4.50	0.0679		4.08	0.525	0.129	9.4

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-14	6.1 ± .2	AVG = 0.91 ± .03	-24.70	0.0197	0.800	3.30 ± .17	0.056 ± .06	0.017	2.2
			-19.80	0.0207		3.45	0.205	0.059	8.0
			-14.80	0.0253		3.49	0.356	0.102	13.8
			-10.00	0.0340		3.55	0.356	0.100	13.7
			-4.50	0.0679		3.82	0.457	0.120	17.0
			-29.70	0.0094	1.000	3.18 ± .13	0.005 ± .045	0.002	0.2
			-24.70	0.0197		3.25	0.077	0.024	3.9
			-19.80	0.0207		3.40	0.194	0.057	9.5
			-14.80	0.0253		3.42	0.258	0.076	12.7
			-10.00	0.0340		3.51	0.334	0.095	16.2
			-4.50	0.0679		3.76	0.402	0.107	18.8
			-39.40	0.0035	2.000	3.14 ± .07	0.019 ± .022	0.006	1.9
			-29.70	0.0094		3.12	0.050	0.016	5.2
			-24.70	0.0197		3.19	0.073	0.023	7.4
			-19.80	0.0207		3.31	0.172	0.052	17.2
			-14.80	0.0253		3.34	0.215	0.064	21.4
			-10.00	0.0340		3.41	0.258	0.076	25.4
			-4.50	0.0679		3.63	0.314	0.087	29.9
			-39.40	0.0035	4.000	3.08 ± .03	0.028 ± .010	0.009	5.7
			-29.70	0.0094		3.13	0.012	0.004	2.4
			-24.70	0.0197		3.19	0.086	0.027	17.4
			-19.80	0.0207		3.27	0.161	0.049	32.3
			-14.80	0.0253		3.28	0.225	0.069	45.2
			-10.00	0.0340		3.36	0.226	0.067	44.8
			-4.50	0.0679		3.55	0.281	0.079	54.2

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
FYI-18	7.5 ± .2	AVG = 0.91 ± .03	-24.08	0.0258	4.00	2.98 ± .12	0.042 ± .045	0.014	8.7
			-18.83	0.0264		3.09	0.088	0.028	18.1
			-14.15	0.0321		3.09	0.117	0.038	24.3
			-9.21	0.0446		3.20	0.161	0.050	32.6
			-3.99	0.0933		3.30	0.225	0.068	45.1
			-24.08	0.0258	7.50	2.99 ± .07	0.039 ± .022	0.013	15.5
			-18.83	0.0264		3.09	0.107	0.035	41.4
			-14.15	0.0321		3.10	0.139	0.045	53.7
			-9.21	0.0446		3.18	0.182	0.057	69.6
			-3.99	0.0933		3.27	0.322	0.098	121.2
FYI-19	5.5 ± .2	AVG = 0.91 ± .03	-29.37	0.0088	4.00	2.86 ± .12	0.159 ± .045	0.056	34.2
			-24.38	0.0190		2.88	0.127	0.044	27.3
			-19.17	0.0191		2.92	0.096	0.033	20.4
			-14.19	0.0235		2.96	0.128	0.043	27.0
			-9.20	0.0327		2.97	0.138	0.047	29.2
			-3.94	0.0692		3.12	0.192	0.062	39.6
			-29.37	0.0088	7.50	2.92 ± .07	0.007 ± .022	0.002	2.8
			-24.38	0.0190		2.86	0.045	0.016	17.5
			-19.17	0.0191		2.95	0.105	0.036	41.8
			-14.19	0.0235		2.98	0.138	0.046	54.6
FYI-20	4.1 ± .2	AVG = 0.91 ± .03	-9.20	0.0327		2.98	0.170	0.057	67.2
			-3.94	0.0692		2.98	0.234	0.079	92.4
			-24.51	0.0139	4.00	3.08 ± .12	0.011 ± .045	0.003	2.2
			-19.32	0.0143		3.19	0.075	0.024	15.2
			-14.08	0.0178		3.24	0.107	0.033	21.6

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (dB/m)
FYI-20	4.1 ± .2	AVG = 0.91 ± .03	-9.44 -4.06	0.0241 0.0505	4.00	3.31 ± .12 3.42	0.172 ± .045 0.280	0.052 0.082	34.3 55.0
			-24.51 -19.32 -14.08 -9.44 -4.06	0.0139 0.0143 0.0178 0.0241 0.0505	7.50	3.13 ± .07 3.18 3.21 3.25 3.29	0.021 ± .022 0.118 0.128 0.171 0.290	0.007 0.037 0.040 0.053 0.088	8.2 45.0 48.9 64.8 108.8
FYI-21	4.1 ± .2	AVG = 0.91 ± .03	-39.23 -29.50 -24.47 -19.28 -14.10 -9.00 -3.92	0.0023 0.0065 0.0139 0.0142 0.0176 0.0248 0.0518	4.00	2.72 ± .12 2.87 2.94 3.04 2.97 3.11 3.04	0.002 ± .045 0.008 0.029 0.089 0.128 0.171 0.256	0.007 0.003 0.010 0.029 0.043 0.055 0.084	0.0 1.5 6.0 18.4 26.9 35.2 53.4
			-24.47 -19.28 -14.10 -9.00 -3.92	0.0139 0.0142 0.0176 0.0248 0.0518	7.50	2.92 ± .07 2.98 2.90 3.04 2.89	0.032 ± .022 0.096 0.138 0.192 0.297	0.011 0.032 0.045 0.063 0.103	12.7 37.8 55.3 75.1 119.2
MYI-13	1.32 ± .05	0.905 ± .01	-39.30 -29.10 -23.90 -4.00	0.0008 0.0022 0.0051 0.0164	0.100	2.95 ± .6 2.98 2.98 3.29	0.006 ± .45 0.074 0.058 0.101	0.002 0.025 0.019 0.031	0.0 0.4 0.3 0.5
			-4.00	0.0164	0.200	3.20 ± .5	0.067 ± .23	0.021	0.7

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
MYI-13	1.32 ± .05	0.905 ± .01	-23.90 -4.00	0.0051 0.0164	0.400	3.22 ± .24 3.31	0.013 ± .10 0.161	0.004 0.049	0.3 3.2
			-39.30 -29.10 -23.90 -19.30 -14.10 -9.00 -4.00	0.0008 0.0022 0.0051 0.0046 0.0057 0.0080 0.0164	0.800	3.00 ± .17 3.04 3.11 3.18 3.18 3.14 3.10	0.018 ± .06 0.014 0.014 0.014 0.011 0.048 0.084	0.006 0.005 0.004 0.004 0.003 0.015 0.027	0.8 0.6 0.6 0.6 0.4 2.0 3.5
			-39.30 -29.10 -23.90 -19.30 -14.10 -9.00 -4.00	0.0008 0.0022 0.0051 0.0046 0.0057 0.0080 0.0164	1.000	3.04 ± .13 3.04 3.12 3.15 3.16 3.14 3.12	0.009 ± .045 0.008 0.025 0.013 0.011 0.033 0.078	0.003 0.003 0.008 0.004 0.003 0.011 0.025	0.4 0.4 1.3 0.6 0.5 1.7 4.0
			-39.30 -29.10 -23.90 -19.30 -14.10 -9.00 -4.00	0.0008 0.0022 0.0051 0.0046 0.0057 0.0080 0.0164	2.000	3.05 ± .07 3.08 3.15 3.19 3.18 3.11 3.08	0.016 ± .022 0.017 0.026 0.046 0.059 0.077 0.086	0.005 0.006 0.008 0.014 0.019 0.025 0.028	1.6 1.8 2.6 4.7 6.0 7.9 9.0
			-19.30 -14.10	0.0046 0.0057	4.000	3.08 ± .03 3.08	0.006 ± .010 0.010	0.002 0.003	0.9 2.0

TABLE 6.1

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	tan δ	LOSS (db/m)
MYI-13	1.32 ± .05	0.905 ± .01	-9.00 -4.00	0.0080 0.0164	4.000	3.06 ± .03 2.94	0.025 ± .010 0.039	0.008 0.013	5.0 8.3
MYI-15	1.75 ± .05	0.903 ± .01	-3.82	0.0228	0.100	3.27 ± .6	0.193 ± .45	0.059	1.0
			-3.82	0.0228	0.200	3.18 ± .5	0.128 ± .23	0.040	1.3
			-14.78 -9.60 -3.82	0.0073 0.0101 0.0228	0.400	3.15 ± .24 3.24 3.21	0.050 ± .10 0.014 0.022	0.016 0.004 0.067	1.0 0.3 4.3
			-3.82	0.0229	0.800	3.07 ± .17	0.107 ± .06	0.035	4.4
			-14.78 -9.60 -3.82	0.0073 0.0101 0.0228	1.000	3.06 ± .13 3.07 3.07	0.021 ± .05 0.022 0.094	0.007 0.007 0.031	1.1 1.1 4.9
			-29.98 -24.74 -19.56 -14.78 -9.60 -3.82	0.0027 0.0056 0.0060 0.0073 0.0101 0.0222	2.000	2.96 ± .07 2.89 2.98 3.00 3.03 3.02	0.009 ± .022 0.010 0.030 0.052 0.066 0.098	0.003 0.003 0.010 0.017 0.022 0.033	1.0 1.0 3.1 5.5 6.9 10.3
			-24.74 -19.56 -14.78 -9.60 -3.82	0.0055 0.0060 0.0073 0.0101 0.0228	4.000	2.87 ± .03 2.98 3.00 3.03 3.04	0.001 ± .010 0.005 0.021 0.030 0.060	0.000 0.002 0.007 0.010 0.020	0.0 0.9 4.4 6.2 12.4

will be deferred until Chapter 7.

In summary, Figures 6.10 to 6.23 demonstrate that there is a certain amount of variability in the first-year sea ice, even though the basic physical properties of temperature and density have been matched as closely as possible. A general non-linear increase is noted with an increase in temperature, although the exact shape of the curve varies somewhat from sample to sample. Furthermore, the data can be grouped into four basic frequency sets. A large change in ϵ_r' would not be expected between these frequencies. (At the lower salinities where the changes in ϵ_r' are not so manifest, there are only two frequency groups.) The change in ϵ_r' with temperature is always larger at the low frequencies, so that for a given change in salinity and temperature the ϵ_r' at lower frequencies is affected the most. This result is relevant for propagation through slabs of ice containing a salinity profile. An average value of ϵ_r' , which can be used to characterize the slab, is most easily found at the higher frequencies where the changes in ϵ_r' due to the salinity profile will be less severe.

Figures 6.24 and 6.25 show the ϵ_r' vs. temperature trends for the two multiyear ice specimens that were measured. These specimens were taken below freeboard at site A. The ϵ_r' values change very little with temperature and frequency. Almost all the points follow the indicated trend very closely (well within the estimated error). The density of these specimens is high, indicating that they are not very porous, however their salinity is low (approximately 2.0 ‰), indicating their low brine content. The generally low slope of the curve would tend to indicate that for these multiyear ice samples ϵ_r' did not vary appreciably with brine content. Sample MYI-13 shows

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rather poor agreement with the general trend at -5°C and -10°C . This lack of agreement is especially evident at the higher frequencies. This might indicate that the measurement is being complicated by some other factor, not yet considered, such as scattering from air bubbles. Visual examination of the photographs of adjacent cores showed no particularly large air bubbles present, although there were what appeared to be large empty brine drainage channels in the vicinity, which could have been included in this particular specimen. This example illustrates the difficulties encountered when working with such a diverse and complicated medium.

Figures 6.26 to 6.29 illustrate the trend for the samples measured at 4.0 and 7.5 GHz. For salinities, in the previous group of first-year ice samples, that are comparable to the salinities in this group, the same trend is followed with temperature. Once more the asymptotic values for ϵ_r' vary from specimen to specimen. It is immediately obvious that the 4.0 and 7.5 GHz data are closely grouped, and that the trend begun with the 100 MHz to 4.0 GHz data is continued. Sample FYI-21 exhibits some extra scatter not found in FYI-18 to 20. It may have been taken too close to the sharp profile near the bottom of (J-6), as shown in Figure 6.1. Once again, the necessity for careful sampling is illustrated and the need to avoid the inclusion of a salinity profile across the sample is seen.

It should be noted that, although it appears as though some samples were needlessly taken near sections of the core containing steep profiles (see Figures 6.1 to 6.5), this was necessary since it was desirable to obtain samples over as wide a range of salinity as possible, so that the effect of salinity could be fully evaluated. To do this, samples had to be taken from the indicated locations in the

core, since that is the only place where salinities of this order occurred.

6.3.1.2 Results for ϵ_r''

Figures 6.30 to 6.49 show the results obtained for ϵ_r'' in the AIDJEX camp area. There is a notable upward trend in ϵ_r'' with increasing temperature. These trends are shown by the solid lines in the figures. In most cases, the trend allows the experimental points to be fit to a smooth curve within the given error bounds. In general, the lines indicated seem to converge towards an asymptotic value of almost zero (on this scale) at low temperatures. The results for FYI-1, 3, 4, 10 and 14 all indicate roughly the same change in slope with temperature. (Once again, sample FYI-2 exhibits some anomalous behaviour possibly due to a salinity profile in the sample.) The overall pattern indicates that ϵ_r'' is very strongly dependent on the brine contained in the sea ice. Its value changes orders of magnitude over the 40°C temperature range.

As in the case of ϵ_r' , the value of ϵ_r'' tends to decrease with increasing frequency. This indicates a strong contribution to ϵ_r'' from the conductivity σ_{MIX} of the mixture (chiefly of the brine).

σ_{MIX} is included in ϵ_r'' in the following way

$$\epsilon_r'' = \epsilon_r'' \text{ DIEL-MIX} + \frac{\sigma_{MIX}}{j\omega\epsilon_0} \quad (6.1)$$

where ω is the radian frequency, ϵ_0 is the permittivity of free space, and $\epsilon_r'' \text{ DIEL-MIX}$ is the contribution to ϵ_r'' due to dielectric relaxation. If $\epsilon_r'' \text{ DIEL-MIX}$ is the chief contributor to ϵ_r'' , the measured values of ϵ_r'' will gradually increase with frequency, reach a maximum, then decrease. This is obvious from examination of equation (2.1). If instead σ_{MIX} is the main contributor, the measured values of ϵ_r'' will

Figure 6.30

ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-1 ($S = 8.1\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.31 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-1 ($S = 8.1\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.32 ϵ_r'' versus temperature for AIDJEX first-year sea ice sample, FYI-2 ($S = 9.1\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.33

ϵ'' versus temperature, for AIDJEX first-year sea ice sample, FYI-2 ($S = 9.1\%$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.34 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-3 ($S = 5.6\%$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.35 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-3 ($S = 5.6\%$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.36 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-4 ($S = 5.1\%$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.37 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-4 ($S = 5.1\%$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.38 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-9 ($S = 10.5\%$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.39 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-9 ($S = 10.5\%$, $\rho = 0.91 \text{ gm/cm}^3$), measurement frequencies as shown.

Figure 6.40

ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-10 ($S = 9.3\text{‰}$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.41 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-10 ($S = 9.3 \text{ ‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.42 ϵ_r'' versus temperature, for AIDJEX first-year sea ice sample, FYI-14 ($S = 6.1 \text{ ‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.43 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-14 ($S = 6.1\%$, $\rho = 0.91 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.44 ϵ_r'' versus temperature, for AIDJEX multiyear sea ice sample, MYI-13 ($S = 1.32 \text{ ‰}$, $\rho = 0.90 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.45 ϵ_r'' versus temperature, for AIDJEX multiyear sea ice sample, MYI-15 ($S = 1.75 \text{ ‰}$, $\rho = 0.90 \text{ gm/cm}^3$); measurement frequencies as shown.

Figure 6.46 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-18 ($S = 7.5\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

Figure 6.47 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-19 ($S = 5.5\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

Figure 6.48 ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-20 ($S = 4.1\text{‰}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

Figure 6.49

ϵ_r versus temperature, for AIDJEX first-year sea ice sample, FYI-21 ($S = 4.1\text{‰}$, $\rho = 0.91 \text{ gm/cm}^3$); at 4.0 and 7.5 GHz.

steadily decrease with frequency. The measured values of ϵ_r'' do tend to decrease with frequency, hence, the probability of a strong contribution from σ_{MIX} is suggested.

The data for the multiyear sea ice samples, MYI-13 and MYI-15, show that ϵ_r'' follows the same trends as for first-year ice. However, the magnitude of the values is necessarily lower due to the lower salinity of the samples.

In summary, the data for both multiyear and first-year ice indicate that ϵ_r'' is strongly a function of the brine conductivity, that it decreases sharply with temperature, as the brine volume increases and, in general, it decreases with frequency. The exact shape of the indicated trends is not given: this task of evaluating the theoretical behaviour of the sea ice is left to Chapter 7. The samples exhibit much the same sort of variability noted in the ϵ_r' results. The points which seem anomalous are probably once again explained by the effect of variations in the ice and the fact that the samples are not truly homogeneous. The errors in the data vary from approximately $\pm 10\%$ at the higher frequencies to as high as $\pm 200\%$ at the worst cases of the low frequency data. This is not really as horrendous as it seems when the small magnitudes of the ϵ_r'' that were measured, and the relative liberality with which the error bounds were assigned are taken into account.

It can obviously be seen that only the general trends have been obtained for the low frequency data (100 MHz and 200 MHz) but this is still better than what existed before. However, the high frequency data (greater than 200 MHz) represents a useful and reasonably accurate contribution to what is known about sea ice dielectric behaviour.

6.3.2 Artificial Sea Ice

As previously described in Section 6.1.2, artificial sea

ice measurements were performed with the aim of investigating the effect of brine drainage network orientation on the measured dielectric properties. A complete discussion of the results of this investigation and a graphical presentation will be given after the empirical model for first-year sea ice is discussed. Therefore, only a tabular presentation of the data points, Table 6.2, is made here. A more complete discussion is deferred to Section 7.1.2.

TABLE 6.2 EXPERIMENTAL DATA FROM THE ARTIFICIAL SEA ICE MEASUREMENTS

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-A (0°)	3.5 ± .2	AVG = 0.90 ± .03	-40.00	0.0019	0.13	2.60 ± .6	0.015 ± .45	0.006	0.1
			-24.68	0.0113		2.37	1.020	0.429	7.6
			-14.57	0.0147		2.97	0.458	0.154	3.1
			-11.38	0.0176		3.10	0.438	0.141	2.9
			-9.29	0.0207		2.90	0.467	0.161	3.2
			-7.51	0.0247		3.27	0.590	0.181	3.8
			-5.41	0.0330		3.43	0.743	0.217	4.7
			-40.00	0.0019	0.21	2.99 ± .5	0.086 ± .23	0.029	1.0
			-24.68	0.0113		2.94	0.521	0.177	5.8
			-14.57	0.0147		3.32	0.333	0.100	3.5
			-11.38	0.0176		3.40	0.312	0.092	3.2
			-9.29	0.0207		3.46	0.409	0.118	4.2
			-7.51	0.0247		3.54	0.389	0.110	3.9
			-5.41	0.0330		3.65	0.465	0.128	4.6
			-24.68	0.0113	0.45	2.94 ± .24	0.234 ± .10	0.080	5.6
			-14.57	0.0147		3.15	0.150	0.048	3.4
			-11.38	0.0176		3.20	0.139	0.044	3.2
			-9.29	0.0207		3.28	0.236	0.072	5.3
			-7.51	0.0247		3.33	0.215	0.065	4.8
			-5.41	0.0330		3.44	0.312	0.091	6.9
			-24.68	0.0113	0.81	3.13 ± .17	0.214 ± .06	0.068	8.9
			-14.57	0.0147		3.30	0.182	0.055	7.4
			-11.38	0.0176		3.35	0.172	0.051	6.9
			-9.29	0.0207		3.40	0.215	0.063	8.6

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-A (0°)	3.5 ± .2	AVG = 0.90 ± .03	-7.51	0.0247	0.81	3.46 ± .17	0.312 ± .06	0.090	12.4
			-5.41	0.0330		3.54	0.356	0.101	13.9
			-24.64	0.0113	1.00	3.14 ± .13	0.139 ± .045	0.044	7.1
			-14.57	0.0147		3.31	0.140	0.042	7.0
			-11.38	0.0176		3.36	0.140	0.042	6.9
			-9.29	0.0207		3.39	0.172	0.051	8.5
			-7.51	0.0247		3.45	0.215	0.062	10.5
			-5.41	0.0330		3.51	0.270	0.077	13.1
			-40.00	0.0019	2.01	3.12 ± .07	0.015 ± .022	0.005	1.5
			-24.64	0.0113		3.12	0.089	0.028	9.2
			-14.57	0.0147		3.25	0.105	0.032	10.6
			-11.38	0.0176		3.28	0.118	0.036	11.9
			-9.29	0.0207		3.31	0.118	0.036	11.8
			-7.51	0.0247		3.37	0.150	0.045	15.0
			-5.41	0.0330		3.43	0.172	0.050	17.0
			-40.00	0.0019	4.00	3.14 ± .03	0.012 ± .01	0.004	2.4
			-24.64	0.0113		3.12	0.053	0.017	11.0
			-14.57	0.0147		3.22	0.081	0.025	16.5
			-11.38	0.0176		3.24	0.102	0.031	20.6
			-9.29	0.0207		3.26	0.118	0.036	23.7
ASI-B (90°)	3.0 ± .2	AVG = 0.90 ± .03	-7.51	0.0247		3.30	0.140	0.042	27.9
			-5.41	0.0330		3.36	0.161	0.048	32.0
			-40.00	0.0016	0.13	2.25 ± .6	0.012 ± .45	0.005	0.1
			-25.31	0.0084		2.74	0.059	0.022	0.4
			-14.82	0.0122		3.38	0.645	0.191	4.1

TABLE 6.2

SAMPLE	SALINITY ($^{\circ}/_{\infty}$)	DENSITY (gm/cm ³)	TEMP. ($^{\circ}$ C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-B (90 $^{\circ}$)	3.0 $\pm$.2	AVG = 0.90 $\pm$.03	-11.99	0.0143	0.13	3.39 $\pm$.6	0.645 $\pm$.45	0.190	4.1
			-9.46	0.0172		3.52	0.852	0.242	5.3
			-7.53	0.0207		3.58	1.070	0.299	6.6
			-5.36	0.0280		3.65	1.060	0.291	6.5
			-40.69	0.0016	0.21	2.96 $\pm$.5	0.660 $\pm$.23	0.223	7.3
			-25.37	0.0084		3.14	0.224	0.071	2.4
			-14.82	0.0122		3.61	0.541	0.150	5.4
			-11.99	0.0143		3.68	0.672	0.183	6.7
			-9.46	0.0172		3.72	0.662	0.178	6.5
			-7.53	0.0207		3.81	0.641	0.168	6.3
			-5.36	0.0280		3.84	0.783	0.204	7.6
			-40.69	0.0016	0.45	2.96 $\pm$.24	0.234 $\pm$.10	0.079	5.6
			-25.37	0.0084		3.05	0.054	0.018	1.3
			-14.82	0.0122		3.36	0.333	0.099	7.4
			-11.99	0.0143		3.38	0.333	0.099	7.4
			-9.46	0.0172		3.39	0.328	0.095	7.2
			-40.69	0.0016	0.81	3.25 $\pm$.17	0.225 $\pm$.06	0.069	9.2
			-25.37	0.0084		3.15	0.078	0.025	3.2
			-14.82	0.0122		3.51	0.248	0.071	9.8
			-11.99	0.0143		3.53	0.237	0.067	9.3
			-9.46	0.0172		3.55	0.270	0.076	10.5
			-7.53	0.0207		3.57	0.292	0.082	11.4
			-5.36	0.0280		3.63	0.335	0.093	13.0
			-14.82	0.0122	1.00	3.51 $\pm$.13	0.129 $\pm$.045	0.037	6.3
			-11.99	0.0143		3.53	0.151	0.043	7.3

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-B (90°)	3.0 ± .2	AVG = 0.90 ± .03	-9.46	0.0172	1.00	3.55 ± .13	0.173 ± .045	0.049	8.3
			-7.53	0.0207		3.59	0.227	0.063	10.9
			-5.36	0.0280		3.83	0.196	0.051	9.1
			-40.69	0.0016	2.01	3.10 ± .07	0.075 ± .022	0.024	7.8
			-25.37	0.0084		3.15	0.073	0.023	7.5
			-14.82	0.0122		3.30	0.140	0.042	14.0
			-11.99	0.0143	4.00	3.31	0.161	0.049	16.2
			-9.46	0.0172		3.34	0.161	0.048	16.1
			-7.53	0.0207		3.33	0.161	0.048	16.1
			-5.36	0.0280		3.36	0.183	0.054	18.2
			-40.69	0.0016	0.13	3.11 ± .03	0.038 ± .045	0.012	7.9
			-25.37	0.0084		3.14	0.036	0.012	7.5
			-14.82	0.0122		3.28	0.118	0.036	23.7
			-11.99	0.0143		3.29	0.129	0.039	25.8
			-9.46	0.0172		3.29	0.139	0.042	28.0
ASI-C (45°)	3.6 ± .2	AVG = 0.90 ± .03	-7.53	0.0207		3.29	0.150	0.046	30.1
			-5.36	0.0280		3.33	0.161	0.048	32.1
			-40.15	0.0020		2.84 ± .6	0.037 ± .45	0.014	0.3
			-24.89	0.0110		2.89	0.191	0.066	1.3
			-14.64	0.0148		3.48	0.701	0.201	4.4
			-11.69	0.0175		3.60	0.887	0.247	5.5
			-9.76	0.0202		3.62	0.876	0.242	5.4
			-7.71	0.0244		3.65	0.877	0.240	5.4
			-5.71	0.0318		3.73	1.060	0.285	6.4
			-40.15	0.0020	0.21	3.07 ± .5	0.149 ± .23	0.049	1.6

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-C (45°)	3.6 ± .2	AVG = 0.90 ± .03	-24.89	0.0110	0.21	3.24 ± .5	0.236 ± .23	0.073	2.5
			-14.64	0.0148		3.67	0.509	0.139	5.1
			-11.69	0.0175		3.78	0.478	0.127	4.7
			-9.76	0.0202		3.80	0.608	0.160	5.9
			-7.71	0.0244		3.83	0.598	0.156	5.8
			-5.71	0.0318		3.90	0.708	0.181	6.8
			-24.89	0.0110	0.45	3.10 ± .24	0.044 ± .10	0.014	1.0
			-14.64	0.0148		3.37	0.269	0.080	6.0
			-11.69	0.0175		3.44	0.312	0.091	6.9
			-9.76	0.0202		3.45	0.312	0.091	6.9
			-7.71	0.0244		3.51	0.367	0.105	8.0
			-5.71	0.0318		3.55	0.432	0.122	9.4
			-40.15	0.0020	0.81	3.16 ± .17	0.037 ± .06	0.012	1.5
			-24.89	0.0110		3.25	0.063	0.020	2.6
			-14.64	0.0148		3.51	0.248	0.071	9.8
			-11.69	0.0175		3.54	0.259	0.073	10.1
			-9.76	0.0202		3.55	0.292	0.082	11.4
			-7.71	0.0244		3.58	0.313	0.088	12.2
			-5.71	0.0318		3.63	0.368	0.101	14.2
			-14.64	0.0148	1.00	3.54 ± .13	0.173 ± .045	0.049	8.3
			-11.69	0.0175		3.57	0.173	0.048	8.3
			-9.76	0.0202		3.59	0.227	0.063	10.9
			-7.71	0.0244		3.62	0.216	0.060	10.3
			-5.71	0.0318		3.64	0.271	0.074	12.9
			-40.15	0.0020	2.01	3.13 ± .07	0.020 ± .022	0.006	2.1
			-24.89	0.0110		3.19	0.046	0.014	4.7

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-C (45°)	3.6 ± .2	AVG = 0.90 ± .03	-14.64	0.0148	2.01	3.37 ± .07	0.150 ± .022	0.045	15.0
			-11.69	0.0175		3.40	0.183	0.054	18.1
			-9.76	0.0202		3.40	0.183	0.054	18.1
			-7.71	0.0244		3.42	0.194	0.057	19.1
			-5.71	0.0318		3.43	0.194	0.057	19.1
			-24.89	0.0110	4.00	3.22 ± .03	0.004 ± .010	0.001	0.0
			-14.64	0.0148		3.38	0.099	0.029	19.5
			-11.69	0.0175		3.40	0.118	0.035	23.3
			-9.76	0.0202		3.43	0.118	0.035	23.2
			-7.71	0.0244		3.42	0.140	0.041	27.5
			-5.71	0.0318		3.44	0.151	0.044	29.5
ASI-D (60°)	3.2 ± .2	AVG = 0.90 ± .03	-40.41	0.0017	0.13	2.39 ± .6	0.713 ± .45	0.299	5.4
			-24.92	0.0097		2.84	0.329	0.116	2.3
			-14.65	0.0132		3.49	0.949	0.272	6.0
			-10.92	0.0164		3.62	0.930	0.257	5.7
			-8.91	0.0192		3.70	1.080	0.293	6.6
			-6.90	0.0239		3.81	1.090	0.285	6.5
			-40.41	0.0017	0.21	3.02 ± .5	0.543 ± .23	0.180	6.0
			-24.92	0.0097		3.16	0.067	0.021	0.7
			-14.65	0.0132		3.62	0.519	0.143	5.2
			-10.92	0.0164		3.75	0.597	0.159	5.9
			-8.91	0.0192		3.85	0.696	0.181	6.8
			-6.90	0.0239		3.96	0.785	0.199	7.5
			-40.41	0.0017	0.45	2.98 ± .24	0.18 ± .10	0.061	4.3
			-24.92	0.0097		3.08	0.010	0.003	0.2

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-D (60°)	3.2 ± .2	AVG = 0.90 ± .03	-14.65	0.0132	0.45	3.35 ± .24	0.312 ± .10	0.093	7.0
			-10.92	0.0164		3.42	0.355	0.104	7.8
			-6.90	0.0239		3.55	0.454	0.128	9.8
			-40.41	0.0017	0.81	3.21 ± .17	0.139 ± .06	0.043	5.7
			-24.92	0.0097		3.26	0.042	0.013	1.7
			-14.65	0.0132		3.52	0.227	0.064	8.9
			-10.92	0.0164		3.56	0.281	0.079	11.0
			-8.91	0.0192		3.62	0.324	0.090	12.6
			-6.90	0.0239		3.66	0.379	0.104	14.6
			-40.41	0.0017	1.00	3.21 ± .013	0.107 ± .045	0.033	5.4
			-24.92	0.0097		3.28	0.014	0.004	0.7
			-14.65	0.0132		3.50	0.018	0.005	8.9
			-10.92	0.0164		3.54	0.281	0.079	13.6
			-8.91	0.0192		3.58	0.281	0.078	13.5
			-6.90	0.0239		3.60	0.324	0.090	15.5
			-40.41	0.0017	2.01	3.18 ± .07	0.083 ± .022	0.026	8.6
			-24.92	0.0097		3.22	0.058	0.018	5.9
			-14.65	0.0132		3.38	0.172	0.051	17.1
			-8.91	0.0192		3.44	0.215	0.063	21.2
			-6.90	0.0239		3.48	0.237	0.068	23.2
			-40.41	0.0017	4.00	3.11 ± .03	0.050 ± .01	0.016	10.3
			-24.92	0.0097		3.14	0.048	0.015	9.8
			-14.65	0.0132		3.28	0.129	0.039	25.8
			-10.92	0.0164		3.31	0.150	0.045	30.0
			-8.91	0.0192		3.33	0.172	0.052	34.3
			-6.90	0.0239		3.36	0.193	0.058	38.4

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-E (30°)	3.3 ± .2	AVG = 0.90 ± .03	-40.71	0.0018	0.13	2.29 ± .6	0.816 ± .45	0.357	6.3
			-25.46	0.0090		2.74	0.029	0.010	0.2
			-15.47	0.0131		3.31	0.526	0.159	3.4
			-11.91	0.0158		3.46	0.711	0.206	4.5
			-10.04	0.0180		3.54	0.896	0.253	5.6
			-8.22	0.0212		3.54	0.896	0.253	5.6
			-6.70	0.0253		3.62	1.080	0.299	6.6
			-40.71	0.0018	0.21	2.95 ± .5	0.532 ± .23	0.180	5.9
			-25.46	0.0090		3.08	0.149	0.049	1.6
			-15.47	0.0131		3.55	0.540	0.152	5.5
			-11.91	0.0158		3.62	0.519	0.143	5.2
			-10.04	0.0180		3.65	0.520	0.143	5.2
			-6.70	0.0253		3.78	0.739	0.195	7.2
			-40.71	0.0018	0.45	2.95 ± .24	0.202 ± .10	0.069	4.8
			-15.47	0.0131		3.31	0.215	0.065	4.8
			-11.91	0.0158		3.37	0.269	0.080	6.0
			-10.04	0.0180		3.42	0.323	0.094	7.1
			-8.22	0.0212		3.43	0.323	0.094	7.1
			-6.70	0.0253		3.47	0.377	0.109	8.3
			-40.71	0.0018	0.81	3.18 ± .17	0.203 ± .06	0.064	8.4
			-25.46	0.0090		3.19	0.028	0.009	1.1
			-15.47	0.0131		3.42	0.205	0.060	8.1
			-11.91	0.0158		3.45	0.226	0.066	9.0
			-10.04	0.0180		3.48	0.280	0.081	11.1
			-8.22	0.0212		3.50	0.302	0.086	11.9
			-6.70	0.0253		3.55	0.389	0.110	15.2

TABLE 6.2

SAMPLE	SALINITY (o/o)	DENSITY (gm/cm^3)	TEMP. ($^{\circ}\text{C}$)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-E (30 $^{\circ}$)	3.3 $\pm$.2	AVG = 0.90 $\pm$.03	-40.71	0.0018	1.00	3.18 $\pm$.13	0.071 $\pm$.045	0.022	3.6
			-15.47	0.0131		3.48	0.129	0.037	6.3
			-11.91	0.0158		3.50	0.129	0.037	6.3
			-10.04	0.0180		3.53	0.205	0.058	9.9
			-8.22	0.0212		3.54	0.227	0.064	11.0
			-6.70	0.0253		3.58	0.270	0.075	13.0
			-40.71	0.0018	2.01	3.09 $\pm$.07	0.049 $\pm$.022	0.016	5.1
			-15.47	0.0131		3.29	0.087	0.026	8.8
			-11.91	0.0158		3.31	0.102	0.031	10.2
			-10.04	0.0180		3.33	0.118	0.036	11.8
			-8.22	0.0212		3.34	0.118	0.035	11.8
			-6.70	0.0253		3.37	0.118	0.035	11.8
			-40.71	0.0018	4.00	3.09 $\pm$.03	0.068 $\pm$.01	0.022	14.1
			-25.43	0.0090		3.12	0.048	0.015	9.9
			-15.47	0.0131		3.24	0.118	0.036	23.8
			-11.91	0.0158		3.26	0.139	0.043	28.1
			-10.04	0.0180		3.26	0.139	0.043	28.1
			-8.22	0.0212		3.28	0.139	0.043	28.0
			-6.70	0.0253		3.31	0.172	0.052	34.3
ASI-F (45 $^{\circ}$)	3.4 $\pm$.2	AVG = 0.90 $\pm$.03	-42.57	0.0018	0.13	2.36 $\pm$.6	0.765 $\pm$.45	0.324	5.8
			-27.82	0.0066		2.86	0.180	0.063	1.3
			-17.15	0.0127		3.47	0.647	0.186	4.1
			-12.77	0.0157		8.58	0.443	0.124	2.8
			-9.55	0.0196		3.78	0.793	0.210	4.8
			-7.86	0.0230		3.99	0.961	0.241	5.7
			-5.64	0.0308		4.34	1.320	0.305	7.4

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-F (45°)	3.4 ± .2	AVG = 0.90 ± .03	-42.57	0.0018	0.21	2.99 ± .5	0.479 ± .23	0.160	5.3
			-27.82	0.0066		3.19	0.098	0.031	1.1
			-17.15	0.0127		3.70	0.455	0.123	4.5
			-12.77	0.0157		3.83	0.533	0.139	5.2
			-9.55	0.0196		3.96	0.622	0.157	6.0
			-7.86	0.0230		4.11	0.712	0.173	6.7
			-5.64	0.0308		4.43	0.883	0.200	8.0
			-42.57	0.0018	0.45	3.02 ± .24	0.352 ± .10	0.116	8.3
			-17.15	0.0127		3.46	0.323	0.093	7.1
			-12.77	0.0157		3.35	0.226	0.067	5.0
			-9.55	0.0196		3.57	0.400	0.112	8.6
			-7.86	0.0230		3.71	0.499	0.134	10.6
			-5.64	0.0308		3.94	0.611	0.155	12.6
			-42.57	0.0018	0.81	3.18 ± .17	0.225 ± .06	0.071	9.3
			-27.82	0.0066		3.23	0.069	0.021	2.8
			-17.15	0.0127		3.51	0.280	0.080	11.0
			-12.77	0.0157		3.57	0.313	0.088	12.2
			-9.54	0.0196		3.65	0.379	0.104	14.6
			-7.86	0.0230		3.75	0.499	0.133	18.9
			-5.64	0.0308		3.96	0.611	0.154	22.6
			-42.57	0.0018	1.00	2.82 ± .13	0.159 ± .045	0.056	8.6
			-27.82	0.0066		3.25	0.006	0.002	0.2
			-17.15	0.0127		3.50	0.194	0.056	9.4
			-12.77	0.0157		3.55	0.270	0.076	13.0
			-9.54	0.0196		3.63	0.314	0.087	15.0
			-7.86	0.0230		3.69	0.412	0.112	19.5
			-5.64	0.0308		3.87	0.544	0.141	25.1

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-F (45°)	3.4 ± .2	AVG = 0.90 ± .03	-42.57	0.0018	2.01	3.12 ± .07	0.069 ± .022	0.022	7.2
			-27.82	0.0066		3.18	0.041	0.013	4.1
			-17.15	0.0127		3.37	0.150	0.045	15.0
			-12.77	0.0157		3.41	0.172	0.051	17.0
			-9.54	0.0196		3.47	0.205	0.059	20.1
			-7.86	0.0230		3.53	0.205	0.058	19.9
			-5.64	0.0308		3.72	0.271	0.073	25.7
			-42.57	0.0018		3.11 ± .03	0.060 ± .010	0.019	12.3
			-27.82	0.0066		3.15	0.047	0.015	9.6
			-17.15	0.0127		3.30	0.140	0.042	27.9
ASI-G (60°)	3.7 ± .2	AVG = 0.90 ± .03	-12.77	0.0157	4.00	3.35	0.161	0.048	32.0
			-9.54	0.0196		3.40	0.183	0.054	36.1
			-7.86	0.0230		3.46	0.226	0.065	44.2
			-5.64	0.0308		3.64	0.303	0.083	57.8
			-41.73	0.0020		2.39 ± .6	0.829 ± .45	0.346	6.2
			-26.76	0.0828		2.87	0.297	0.104	2.1
			-16.10	0.0145		3.55	0.875	0.246	5.4
			-10.90	0.0193		3.82	1.200	0.313	7.2
			-8.64	0.0232		3.90	1.200	0.307	7.1
			-6.61	0.0291		4.03	1.310	0.326	7.6
			-5.62	0.0337		4.14	1.530	0.371	8.8
			-41.73	0.0020	0.21	3.02 ± .5	0.490 ± .23	0.162	5.4
			-26.76	0.0828		3.21	0.128	0.040	1.4
			-16.10	0.0145		3.75	0.640	0.171	6.3
			-10.90	0.0193		3.89	0.719	0.185	6.9
			-8.64	0.0232		4.00	0.808	0.202	7.7

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'_r	ϵ''_r	$\tan \delta$	LOSS (db/m)
ASI-G (60°)	3.7 ± .2	AVG = 0.90 ± .03	-6.61	0.0291	0.21	4.16 ± .5	1.01 ± .23	0.243	9.4
			-5.62	0.0337		4.24	1.10	0.259	10.1
			-41.73	0.0020	0.45	3.00 ± .24	0.192 ± .10	0.064	4.5
			-26.76	0.0828		3.10	0.028	0.009	0.6
			-16.10	0.0145		3.42	0.344	0.101	7.8
			-10.90	0.0193		3.54	0.443	0.125	9.6
			-8.64	0.0232		3.60	0.487	0.135	10.5
			-6.61	0.0291		3.73	0.586	0.157	12.4
			-5.62	0.0337		3.81	0.674	0.177	14.1
			-16.10	0.0145	0.81	3.56 ± .17	0.281 ± .06	0.079	11.0
			-10.90	0.0193		3.66	0.379	0.104	14.6
			-8.64	0.0232		3.70	0.412	0.111	15.7
			-6.61	0.0291		3.80	0.500	0.132	18.6
			-5.62	0.0337		3.83	0.587	0.153	22.0
			-41.73	0.0020	1.00	3.21 ± .13	0.118 ± .045	0.037	6.0
			-26.76	0.0828		3.29	0.012	0.004	0.6
			-16.10	0.0145		3.54	0.248	0.070	12.0
			-10.90	0.0193		3.63	0.314	0.087	15.0
			-8.64	0.0232		3.66	0.357	0.098	17.0
			-6.61	0.0291		3.74	0.445	0.119	20.9
			-5.62	0.0337		3.75	0.640	0.171	30.0
			-41.73	0.0020	2.01	3.20 ± .07	0.094 ± .022	0.023	9.6
			-26.76	0.0828		3.24	0.088	0.027	8.9
			-16.10	0.0145		3.41	0.204	0.060	20.2
			-10.90	0.0193		3.45	0.036	0.010	3.5

TABLE 6.2

SAMPLE	SALINITY ($^{\circ}/\infty$)	DENSITY (gm/cm ³)	TEMP. ($^{\circ}$ C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-G (60)	3.7 $\pm$.2	AVG = 0.90 $\pm$.03	-8.64	0.0232	2.01	3.52 $\pm$.07	0.281 $\pm$.022	0.080	27.3
			-6.61	0.0291		3.59	0.313	0.087	30.2
			-5.62	0.0337		3.62	0.346	0.096	33.2
ASI-H (0)			-41.73	0.0020	4.00	3.12 $\pm$.03	0.043 $\pm$.01	0.014	8.8
			-26.76	0.0828		3.16	0.048	0.015	9.8
			-16.10	0.0145		3.33	0.140	0.042	27.8
			-10.90	0.0193		3.38	0.172	0.051	34.0
			-8.64	0.0232		3.40	0.194	0.057	38.2
			-6.61	0.0291		3.46	0.226	0.065	44.2
			-5.62	0.0337		3.46	0.259	0.075	50.5
			-25.04	0.0107	0.13	2.83 $\pm$.6	0.002 $\pm$.45	0.001	0.0
			-15.67	0.0141		2.84	1.060	0.373	7.3
			-11.96	0.0172		3.09	0.502	0.163	3.4
			-10.00	0.0198		3.25	0.482	0.149	3.2
			-7.29	0.0257		3.37	0.473	0.141	3.0
			-5.96	0.0307		3.39	0.462	0.137	3.0
			-40.48	0.0019	0.21	3.07 $\pm$.5	0.007 $\pm$.23	0.002	0.1
			-25.04	0.0107		3.20	0.090	0.028	1.0
			-15.67	0.0141		3.29	0.654	0.199	6.9
			-11.96	0.0172		3.39	0.269	0.079	2.8
			-10.00	0.0198		3.54	0.356	0.101	3.6
			-7.29	0.0257		3.59	0.335	0.093	3.4
			-5.96	0.0307		3.64	0.444	0.122	4.4
			-15.67	0.0141	0.45	3.18 $\pm$.24	0.353 $\pm$.10	0.111	8.1
			-11.96	0.0172		3.20	0.139	0.044	3.2

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-H (0°)	3.6 ± .2	AVG = 0.90 ± .03	-10.00	0.0198	0.45	3.29 ± .24	0.161 ± .10	0.049	3.6
			-7.29	0.0257		3.31	0.193	0.058	4.3
			-5.96	0.0307		3.36	0.247	0.074	5.5
			-15.67	0.0141	0.81	3.40 ± .17	0.301 ± .06	0.089	12.0
			-11.96	0.0172		3.43	0.151	0.044	6.0
			-10.00	0.0198		3.49	0.194	0.056	7.7
			-7.29	0.0257		3.54	0.238	0.067	9.3
			-5.96	0.0307		3.56	0.259	0.073	10.1
			-15.67	0.0141	1.00	3.38 ± .13	0.258 ± .045	0.076	12.8
			-11.96	0.0172		3.41	0.140	0.041	6.9
			-10.00	0.0198		3.45	0.215	0.062	10.5
			-7.29	0.0257		3.48	0.259	0.074	12.6
			-5.96	0.0307		3.50	0.302	0.086	14.7
			-40.48	0.0019	2.01	3.19 ± .07	0.039 ± .022	0.012	3.9
			-25.04	0.0107		3.23	0.028	0.009	2.8
			-15.67	0.0141		3.26	0.118	0.036	11.9
			-11.96	0.0172		3.27	0.078	0.024	7.9
			-10.00	0.0198		3.28	0.094	0.029	9.5
			-7.29	0.0257		3.30	0.100	0.030	10.0
			-5.96	0.0307		3.29	0.102	0.031	10.3
			-40.48	0.0019	4.00	3.14 ± .03	0.011 ± .010	0.003	2.2
			-25.04	0.0107		3.19	0.021	0.007	4.3
			-15.67	0.0141		3.22	0.107	0.033	21.7
			-11.96	0.0172		3.23	0.087	0.027	17.6
			-10.00	0.0198		3.23	0.103	0.032	20.8
			-7.29	0.0257		3.25	0.107	0.033	21.6
			-5.96	0.0307		3.26	0.139	0.043	28.1

TABLE 6.2

SAMPLE	SALINITY (°/∞)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-I (90°)	3.9 ± .2	AVG = 0.90 ± .03	-42.49	0.0020	0.13	2.42 ± .6	1.150 ± .45	0.478	8.6
			-27.48	0.0078		2.96	0.351	0.119	2.4
			-15.04	0.0158		4.05	1.310	0.324	7.6
			-11.28	0.0195		4.39	2.100	0.477	11.5
			-9.42	0.0225		4.56	2.220	0.486	11.9
			-7.62	0.0268		4.84	2.900	0.599	15.0
			-42.49	0.0020	0.21	3.03 ± .5	0.597 ± .23	0.197	6.5
			-27.48	0.0078		3.25	0.204	0.063	2.2
			-15.04	0.0158		4.32	2.090	0.485	18.7
			-11.28	0.0195		4.36	1.320	0.303	12.0
			-9.42	0.0225		4.55	1.550	0.341	13.7
			-7.62	0.0268		4.82	2.010	0.417	17.1
			-5.07	0.0384		5.21	2.480	0.476	20.2
			-42.49	0.0020	0.45	3.00 ± .24	0.288 ± .10	0.096	6.8
			-27.48	0.0078		3.11	0.077	0.025	1.8
			-15.04	0.0158		3.65	0.650	0.178	13.9
			-11.28	0.0195		3.79	0.804	0.212	16.8
			-9.42	0.0225		3.98	0.927	0.236	19.0
			-7.62	0.0268		4.12	1.210	0.292	24.0
			-5.07	0.0384		4.41	1.430	0.325	27.6
			-42.49	0.0020	0.81	3.15 ± .17	0.257 ± .06	0.081	10.6
			-27.48	0.0078		3.26	0.069	0.021	2.8
			-15.04	0.0158		3.71	0.445	0.120	17.0
			-11.28	0.0195		3.86	0.533	0.138	19.9
			-9.42	0.0225		3.96	0.524	0.132	19.3
			-7.62	0.0268		4.12	0.822	0.199	29.7
			-5.07	0.0384		4.37	1.040	0.237	36.3

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-I (90°)	3.9 ± .2	AVG = 0.90 ± .03	-42.49	0.0020	1.00	3.16 ± .13	0.128 ± .045	0.041	6.6
			-27.48	0.0078		3.31	0.002	0.000	0.0
			-15.04	0.0158		3.72	0.380	0.102	17.9
			-11.28	0.0195		3.85	0.490	0.127	22.6
			-9.42	0.0225		3.96	0.589	0.149	26.9
			-7.62	0.0268		4.10	0.712	0.174	31.8
			-5.07	0.0384		4.30	0.913	0.212	39.8
			-42.49	0.0020	2.01	3.13 ± .07	0.118 ± .022	0.038	12.1
			-27.48	0.0078		3.22	0.071	0.022	7.2
			-15.04	0.0158		3.52	0.302	0.086	29.4
			-11.28	0.0195		3.63	0.368	0.101	35.3
			-9.42	0.0225		3.70	0.401	0.108	38.1
			-7.62	0.0268		3.81	0.489	0.128	45.7
			-5.07	0.0384		4.03	0.601	0.149	54.6
			-42.49	0.0020	4.00	3.08 ± .03	0.089 ± .010	0.029	18.3
			-27.48	0.0078		3.16	0.074	0.023	15.1
			-15.04	0.0158		3.41	0.237	0.069	46.6
			-11.28	0.0195		3.44	0.291	0.085	57.0
			-9.42	0.0225		3.56	0.335	0.094	64.5
			-7.62	0.0268		3.66	0.422	0.115	80.2
ASI-J (30°)	3.4 ± .2	AVG = 0.90 ± .03	-42.14	0.0018	0.13	2.44 ± .6	1.160 ± .45	0.474	8.5
			-25.42	0.0095		2.90	0.351	0.121	2.4
			-15.28	0.0138		3.37	0.645	0.191	4.1
			-12.88	0.0156		3.42	0.829	0.242	5.3
			-9.59	0.0196		3.54	0.810	0.229	5.1
			-7.78	0.0232		3.23	0.857	0.265	5.6
			-5.56	0.0312		3.79	1.200	0.315	7.2

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ'	ϵ''	$\tan \delta$	LOSS (db/m)
ASI-J (30°)	3.4 ± .2	AVG = 0.90 ± .03	-42.14	0.0018	0.21	3.09 ± .5	0.448 ± .23	0.145	4.9
			-25.42	0.0095		3.23	0.193	0.060	2.1
			-15.28	0.0138		3.63	0.444	0.122	4.4
			-12.88	0.0156		6.66	0.433	0.118	4.3
			-9.59	0.0196		3.74	0.532	0.142	5.2
			-42.14	0.0018	0.45	3.03 ± .24	0.298 ± .10	0.099	7.0
			-25.42	0.0095		3.10	0.063	0.020	1.5
			-15.28	0.0138		3.35	0.247	0.074	5.5
			-12.88	0.0156		3.38	0.269	0.080	6.0
			-9.59	0.0196		3.44	0.323	0.094	7.1
			-7.78	0.0232	0.81	3.44	0.323	0.094	7.1
			-5.56	0.0312		3.59	0.454	0.126	9.8
			-25.42	0.0095		3.28 ± .17	0.107 ± .06	0.033	4.4
			-15.28	0.0138		3.50	0.248	0.071	9.8
			-12.88	0.0156		3.52	0.281	0.080	11.0
			-9.59	0.0196	1.00	3.57	0.324	0.091	12.6
			-7.78	0.0232		3.60	0.378	0.105	14.7
			-5.56	0.0312		3.68	0.477	0.130	18.3
			-42.14	0.0018		3.20 ± .13	0.139 ± .045	0.044	7.1
			-25.42	0.0095		3.29	0.038	0.011	1.9
			-15.28	0.0138		3.48	0.194	0.056	9.5
			-12.88	0.0156		3.48	0.216	0.062	10.5
			-9.59	0.0196		3.51	0.259	0.074	12.6
			-7.78	0.0232		3.53	0.335	0.095	16.2
			-5.56	0.0312		3.58	0.421	0.118	20.2

TABLE 6.2

SAMPLE	SALINITY (‰)	DENSITY (gm/cm ³)	TEMP. (°C)	CALC. BRINE VOLUME FRACTION	FREQ. (GHz)	ϵ_r'	ϵ_r''	$\tan \delta$	LOSS (db/m)
ASI-J (30°)	3.4 ± .2	AVG = 0.90 ± .03	-42.14	0.0018	2.01	3.14 ± .07	0.081 ± .022	0.026	8.4
			-25.42	0.0095		3.20	0.060	0.019	6.1
			-15.28	0.0138		3.35	0.140	0.042	13.9
			-12.88	0.0156		3.34	0.140	0.042	14.0
			-9.59	0.0196		3.38	0.161	0.048	16.0
			-7.78	0.0232		3.37	0.172	0.051	17.1
			-5.56	0.0312		3.42	0.183	0.054	18.1
			-42.14	0.0018	4.00	3.09 ± .03	0.088 ± .010	0.028	18.1
			-25.42	0.0095		3.14	0.077	0.025	15.8
			-15.28	0.0138		3.27	0.139	0.043	28.0
			-12.88	0.0156		3.26	0.150	0.046	30.2
			-9.59	0.0196		3.28	0.172	0.052	34.4
			-7.78	0.0232		3.29	0.193	0.059	38.7
			-5.56	0.0312		3.35	0.226	0.067	44.8

CHAPTER 7. WIDEBAND DIELECTRIC MODELS FOR SEA ICE

7.1 Empirical Models

The most fundamental approach to modelling sea ice dielectric behaviour is to merely correlate the dielectric properties with some significant physical property of the ice. The choice of property will determine the successfulness of this approach. Salinity, temperature and density are the prime candidates for the choice, however some means must be found of normalizing their values so that samples with various combinations of salinity, temperature and density can be compared easily. Equation (3.1) gives the required relation. These are the brine volume fraction equations of Frankenstein and Garner⁽⁴¹⁾ and Poe et al.⁽⁴²⁾ Unfortunately, they assume a fixed density of 0.926 gm/cm^3 so that if the samples to be compared have widely differing densities a certain amount of scatter is to be expected in the results. However, the equations do produce a single quantity, the volume fraction of brine in the ice, which takes into account both salinity and temperature. Figure 3.6 shows the brine volume vs. temperature relation for ice with a salinity of 1 ‰.

7.1.1 Natural Sea Ice

Figures 7.1 to 7.45 show the ϵ_r' , ϵ_r'' and loss correlations with brine volume for all the samples investigated. The coefficient of determination r^2 is given in each figure. A coefficient $r^2 = 1$ is a very good fit, i.e. 100 % of the variation in the data is explained by changes in brine volume. As r^2 drops the "goodness of fit" drops, so that $r^2 = 0.50$ denotes that only 50 % of the variation in the data is explained by changes in the independent variable v . For convenience the independent variable v is taken as ten times the actual brine volume. The best straight line fit to the data points is

indicated by the solid line. In cases where the correlation was very poor the straight line was not shown, since it was felt it was not significant. The equation relating the parameter plotted to v is given in each figure. The standard error of the estimate represented by each straight line is indicated by the error bound given in the figure.

Figures 7.1 to 7.7 show the results of the correlation of ϵ_r' and v for the first-year ice samples over the frequency range 100 MHz to 4.0 GHz. It is apparent that all the r^2 values are between 0.71 and 0.76. This indicates that a reasonably good explanation of the variability of the data is given by v . The method is valuable in that it demonstrates the typical variation to be found in ϵ_r' for first-year sea ice from the same locale. It can also be seen that the standard error decreases with frequency. The values of the standard error s_{yx} , the coefficient of determination r^2 , the equations of the lines and the error bounds (from Table 5.2) are given in Table 7.1. Comparison of the standard errors and the error bounds show that the error bounds are larger than the standard errors at the low frequencies, but smaller than the standard errors at the high frequencies. This implies that the natural variation in the sea ice is much greater than the error due to the measurement procedure, at 4.0 GHz, but much less than the error due to the measurement procedure, at the lower frequencies. Only if the actual measurement accuracy at the low frequencies were improved could the distinction between errors due to measurement and errors due to natural variability be made. Nevertheless, a postulation could be made that from the trends noted in Chapter 6 (i.e. that ϵ_r' for the low frequencies is the most sensitive to temperature varia-

TABLE 7.1 CORRELATION DATA FOR THE EMPIRICAL MODEL
BASED ON THE AIDJEX MEASUREMENTS

$\hat{y} = a_0 + a_1 v$ $\hat{y}$ = estimate of parameter considered
 v = calculated brine volume $\times 10$ n = number of points considered
 a_0 = $\hat{y}$ intercept a_1 = slope
 r^2 = coefficient of determination s_{yx} = standard error of the estimate ($\hat{y}$)
 $s_{0,1}$ = standard errors of $a_{0,1}$ $\pm \Delta$ = error bound on the individual points in the sample

(average density of all samples was $0.91 \pm .03 \text{ gm/cm}^3$)

PARAMETER	FREQ. (GHz)	n	a_0	a_1	r^2	s_{yx}	s_0	s_1	$\pm \Delta$
ϵ'_x (FIRST-YEAR)	0.100	43	3.22	2.06	0.76	0.29	0.07	0.19	0.60
	0.200	43	3.23	1.45	0.74	0.21	0.05	0.12	0.50
	0.400	37	3.26	1.23	0.74	0.18	0.05	0.12	0.24
	0.800	44	3.12	0.99	0.74	0.15	0.03	0.08	0.17
	1.000	46	3.12	0.90	0.72	0.14	0.02	0.07	0.13
	2.000	47	3.07	0.76	0.72	0.12	0.02	0.06	0.07
	4.000	47	3.05	0.72	0.71	0.12	0.02	0.06	0.03
ϵ''_x (FIRST-YEAR)	0.100	43	0.161	1.324	0.70	0.212	0.054	0.135	0.450
	0.200	43	0.043	0.895	0.74	0.132	0.040	0.091	0.230
	0.400	37	0.043	0.715	0.73	0.109	0.033	0.074	0.100
	0.800	44	0.048	0.534	0.80	0.067	0.017	0.041	0.600
	1.000	46	0.039	0.504	0.81	0.061	0.015	0.037	0.045
	2.000	47	0.034	0.356	0.86	0.036	0.008	0.021	0.022
	4.000	47	0.024	0.329	0.77	0.045	0.010	0.027	0.010
	7.50	21	0.032	0.353	0.70	0.051	0.020	0.054	0.022

TABLE 7.1

PARAMETER	FREQ. (GHz)	n	a _o	a ₁	r ²	s _{yx}	s _o	s ₁	±Δ
LOSS (FIRST-YEAR)	0.100	43	0.97	5.11	0.63	0.97	0.25	0.62	
	0.200	43	0.92	7.00	0.69	1.15	0.37	0.81	
	0.400	37	1.24	11.96	0.69	1.99	0.60	1.35	
	0.800	44	2.35	19.00	0.77	2.61	0.66	1.61	
	1.000	46	2.38	22.69	0.79	2.92	0.70	1.77	
	2.000	47	3.97	32.92	0.84	3.55	0.83	2.12	
	4.000	47	6.49	60.46	0.75	8.70	2.06	5.21	
ε _r (MULTIYEAR)	7.50	21	13.37	136.98	0.68	20.02	7.82	21.35	
	0.100	5	0.022	0.666	0.83	0.033	0.022	0.173	0.450
	0.400	5	-0.058	1.201	0.90	0.034	0.033	0.253	0.100
	0.800	8	0.000	0.474	0.92	0.012	0.006	0.058	0.600
	1.000	10	-0.004	0.436	0.91	0.010	0.005	0.048	0.045
	2.000	13	0.013	0.435	0.73	0.016	0.007	0.080	0.022
	4.000	9	-0.007	0.299	0.91	0.006	0.004	0.036	0.010
LOSS (MULTIYEAR)	0.100	5	0.12	3.32	0.83	0.15	0.10	0.74	
	0.400	5	-1.17	24.31	0.90	0.69	0.66	4.74	
	0.800	8	-0.02	19.68	0.92	0.49	0.26	2.42	
	1.000	10	-0.27	22.77	0.91	0.49	0.26	2.50	
	2.000	13	1.32	45.66	0.74	1.68	0.77	8.19	
	4.000	9	-1.65	63.68	0.91	1.33	0.86	7.70	

tions), it should be expected that a greater variability exists from sample to sample for the same ice at the low frequencies than at the high frequencies. Indeed, if the graphs are carefully examined, it can be seen that the points due to any one sample FYI-1 etc. lie more closely along an imaginary straight line at 4.0 GHz than they do at 100 MHz. Thus, these particular curves illustrate that ϵ_r' is strongly affected by the volume fraction of brine in the ice, and that this variation is disturbed more strongly by inter-sample variations at the lower frequencies than at the higher frequencies.

Figure 7.8 would seem to dispute this observation, in that the correlation between ϵ_r' and v is very poor. However, if the trend in s_{yx} is examined in Figures 7.1 to 7.7, it is seen that s_{yx} approaches a limiting value of approximately .15 to .12, above 800 MHz. At these frequencies, s_{yx} probably reflects the inherent variability in the ice. If the points in Figure 7.8 are examined using this value of s_{yx} as a gauge, most of them are seen to lie within one s_{yx} of an imaginary straight line through the points. Acquisition of more points, especially at higher salinities, would probably improve the situation.

Figures 7.9 to 7.15 show the correlation, or lack of it, of ϵ_r' with v , for multiyear sea ice. There were very few points obtained for this type of ice. The points that were obtained tend to illustrate that the correlation with v is not very strong, or any correlation, weak as it is, is masked by the variation in the ϵ_r' points. These observations illustrate an important property of multiyear sea ice that was discussed in Chapter 3, i.e. there is very little brine contained in the multiyear ice, and therefore ϵ_r' is seen to not strongly depend on v , but to be almost independent of it. Furthermore, the equations used to calculate v do not hold for multi-

year ice: the brine is not necessarily distributed as Assur⁽³¹⁾ has assumed; and the density can definitely not be assumed to be a constant 0.926 gm/cm^3 . In the light of these observations, the lack of correlation is not really surprising. Thus, although ϵ_r' for multi-year ice does not correlate well with brine volume, ϵ_r' for first-year ice correlates very well, and v therefore becomes a prime indicator of ϵ_r' for first-year ice only.

Figures 7.16 to 7.30 show the results of the correlation for ϵ_r'' vs. v . As before, the equations relating the two variables, the coefficient of determination, and the standard error of the estimate, are given in each figure. The standard errors once again show a trend of steadily decreasing value as the frequency increases, then a levelling off to about 0.05 at 4.0 GHz. These standard errors compare with the measurement errors, given in Table 5.2, in much the same manner as did the ϵ_r' errors, i.e. the standard errors are smaller than the measurement errors at the low frequencies but they become 3 or 4 times larger than the measurement errors at 4.0 GHz. Statements similar to those made about the variation in ϵ_r' may also be made about the decrease in variability of ϵ_r'' with frequency. In general, ϵ_r'' is higher at the low frequencies, decreases with increasing frequency, and exhibits reasonably linear variation with v . There also appears to be a levelling off of ϵ_r'' , with increasing frequency, to a value of approximately 0.3 to 0.4 at $v \approx 1.0$.

Figure 7.23, unlike its ϵ_r' counterpart (Figure 7.8) shows a good correlation of ϵ_r'' with v . This implies there is a stronger relation between ϵ_r'' and v , than between ϵ_r' and v , in that the data correlates well even under poor conditions of sampling.

In Figures 7.24 to 7.30, the extension of the ϵ_r'' cor-

relations to multiyear ice tend to support this hypothesis. The ϵ_r'' points correlate well with v whereas the ϵ_r' points do not, although the correlations should not be taken too seriously since the number of points is so small. But, on the basis of this data, at least, ϵ_r'' is very strongly related to the properties of the brine. This is probably due to the greater role played by the brine in determining ϵ_r'' compared to that played in determining ϵ_r' .

Figures 7.31 to 7.45 show the correlation of the loss in (db/m) with v , calculated using (see Jordan and Balmain⁽⁷⁸⁾)

$$\text{LOSS (db/m)} = 868.6 (2\pi/\lambda_0) \sqrt{\epsilon_r''/2(\sqrt{1 + \tan^2 \delta} - 1)} \quad (7.1)$$

where λ_0 is the free space wavelength in cm., and $\tan \delta = \epsilon_r''/\epsilon_r'$. The results of the correlation show that the loss increases dramatically with frequency. The loss is only about 5 to 6 db/m for first-year sea ice at 100 MHz, but it increases to approximately 125 db/m at 7.5 GHz, for $v = 0.9$. The slopes relating loss and v also increase dramatically from 5.11 at 100 MHz to 136.98 at 7.5 GHz, so that a large brine volume, say $v = 1.0$ at 100 MHz, may only increase the loss from its $v = 0$ value by approximately 5 db/m, whereas at 7.5 GHz the loss increases by approximately 123 db/m. That is a difference of almost 120 db for propagation through 1 m. of first-year sea ice.

Obviously, this finding has grave implications for the remote sensing of first-year sea ice. For fresh water ice, airborne radar systems operating in the 10 GHz range can easily measure the thickness of ice many meters thick, since the attenuation or loss in the ice layer is only about 5 db/m (Vant⁽⁸²⁾). These same systems face attenuations 120 db/m higher in sea ice and obviously will not operate. For imaging radar systems, or passive microwave radiometer systems,

this means the extent of penetration in first-year sea ice will be very small, a fraction of a wavelength, at frequencies 10 GHz and above.

The standard errors on the loss curves are relatively high. This reflects the combined effect of the errors in ϵ_r' and the errors in ϵ_r'' , both of which must be used to calculate the loss.

Comparison of the results presented in Figures 7.39 to 7.45, for multiyear sea ice, with those in Figures 7.31 to 7.38 shows that although v may not have a physical significance for multiyear ice, the correlations of loss with v produce lines of almost exactly the same slope for both first-year and multiyear sea ice. This infers that although the absolute losses are lower in multiyear ice due to its lower salinity, the rate of variation of loss with brine volume fraction is the same in both cases.

These empirical models provide handy equations that can be used to calculate ϵ_r' , ϵ_r'' and the loss for the different ice types at any salinity and temperature, but they are restricted to the frequencies the experiment was performed at. Beyond indicating trends, the exact value of any of the parameters cannot be calculated at any intermediate frequency. Also, the inherent structures or mechanisms producing these values of ϵ_r' , ϵ_r'' and loss are effectively ignored in this treatment and one is not left with an understanding of the dielectric behaviour of the ice. This limits any projections of the models, to higher or lower frequencies, or to different geometries. Section 7.2 makes an effort to understand the behaviour of the ice in terms of its physical properties and constituents, and thus represents an alternative approach to the modelling effort.

7.1.2 Artificial Sea Ice

Figures 7.46 to 7.59 show the results of a correlation of

Figure 7.1

ϵ_r correlation with brine volume fraction for
 AIDJEX first-year sea ice samples at 100 MHz;
 correlation data given in figure.

Figure 7.2

ϵ' correlation with brine volume fraction for
 AIDJEX first-year sea ice samples at 200 MHz;
 correlation data given in figure.

Figure 7.3

ϵ_r correlation with brine volume fraction for AIDJEX first-year sea ice samples at 400 MHz; correlation data given in figure.

Figure 7.4

ϵ_r correlation with brine volume fraction for
 AIDJEX first-year sea ice samples at 800 MHz;
 correlation data given in figure.

Figure 7.5

ϵ_r correlation with brine volume fraction for AIDJEX first-year sea ice samples at 1.0 GHz; correlation data given in figure.

Figure 7.6 ϵ'_r correlation with brine volume fraction for AIDJEX first-year sea ice samples at 2.0 GHz; correlation data given in figure.

Figure 7.7 ϵ'_r correlation with brine volume fraction for AIDJEX first-year sea ice samples at 4.0 GHz; correlation data given in figure.

Figure 7.8

ϵ_r correlation with brine volume fraction for
 AIDJEX first-year sea ice samples at 7.5 GHz;
 correlation data given in figure.

Figure 7.9 ϵ_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 100 MHz; correlation data given in figure.

Figure 7.10 ϵ_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 200 MHz; correlation data given in figure.

Figure 7.11 ϵ_r' correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 400 MHz; correlation data given in figure.

Figure 7.12 ϵ_r' correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 800 MHz; correlation data given in figure.

Figure 7.13 ϵ_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 1.0 GHz; correlation data given in figure.

Figure 7.14 ϵ_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 2.0 GHz; correlation data given in figure.

Figure 7.15 ϵ_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 4.0 GHz; correlation data given in figure.

Figure 7.16

ϵ_r correlation with brine volume fraction for AIDJEX first-year sea ice samples at 100 MHz; correlation data given in figure.

Figure 7.17

ϵ_r correlation with brine volume fraction for AIDJEX first-year sea ice samples at 200 MHz; correlation data given in figure.

Figure 7.18

ϵ_r'' correlation with brine volume fraction for
 AIDJEX first-year sea ice samples at 400 MHz;
 correlation data given in figure.

Figure 7.19 ϵ_r'' correlation with brine volume fraction for AIDJEX first-year sea ice samples at 800 MHz; correlation data given in figure.

...20. ϵ_r correlation with brine volume fraction for
 AIDJEX first-year sea ice samples at 1.0 GHz;
 correlation data given in figure.

Figure 7.21 ϵ'' correlation with brine volume fraction for AIDJEX first-year sea ice samples at 2.0 GHz; correlation data given in figure.

Figure 7.22 ϵ'' correlation with brine volume fraction for AIDJEX first-year sea ice samples at 4.0 GHz; correlation data given in figure.

Figure 7.23 ϵ_r'' correlation with brine volume fraction for AIDJEX first-year sea ice samples at 7.5 GHz; correlation data given in figure.

Figure 7.24 ϵ_r'' correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 100 MHz; correlation data given in figure.

Figure 7.25 ϵ_r'' correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 200 MHz; correlation data given in figure.

Figure 7.26 ϵ''_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 400 MHz; correlation data given in figure.

Figure 7.27 ϵ''_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 800 MHz; correlation data given in figure.

Figure 7.28 ϵ'' correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 1.0 GHz; correlation data given in figure.

Figure 7.29

ϵ_r'' correlation with brine volume fraction for
 AIDJEX multiyear sea ice samples at 2.0 GHz;
 correlation data given in figure.

Figure 7.30 ϵ''_r correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 4.0 GHz; correlation data given in figure.

Figure 7.31

Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 100 MHz; correlation data given in figure.

Figure 7.32 Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 200 MHz; correlation data given in figure.

Figure 7.33 Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 400 MHz; correlation data given in figure.

Figure 7.34 Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 800 MHz; correlation data given in figure.

Figure 7.35: Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 1.0 GHz; correlation data given in figure.

Figure 7.36 Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 2.0 GHz; correlation data given in figure.

Figure 7.37 Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 4.0 GHz; correlation data given in figure.

Figure 7.38

Loss correlation with brine volume fraction for AIDJEX first-year sea ice samples at 7.5 GHz; correlation data given in figure.

Figure 7.39

Loss correlation with brine volume fraction for
 AIDJEX multiyear sea ice samples at 100 MHz;
 correlation data given in figure.

Figure 7.40. Loss correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 200 MHz; correlation data given in figure.

Figure 7.41 Loss correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 400 MHz; correlation data given in figure.

Figure 7.42 Loss correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 800 MHz; correlation data given in figure.

Figure 7.43 Loss correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 1.0 GHz; correlation data given in figure.

Figure 7.44 Loss correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 2.0 GHz; correlation data given in figure.

Figure 7.45 Loss correlation with brine volume fraction for AIDJEX multiyear sea ice samples at 4.0 GHz; correlation data given in figure.

TABLE 7.2 CORRELATION DATA FOR THE EMPIRICAL MODEL
BASED ON THE ARTIFICIAL SEA ICE MEASUREMENTS

$\hat{y} = a_0 + a_1 v$ $\hat{y}$ = estimate of parameter considered
 v = calculated brine volume $\times 10$ n = number of points considered
 ANGLE = sample taken at this angle, with $a_0 = \hat{y}$ intercept
 respect to vertical
 a_1 = slope r^2 = coefficient of determination
 s_{yx} = standard error of the estimate ($\hat{y}$) $s_{0,1}$ = standard errors of $a_{0,1}$
 $\pm \Delta$ = error bound on the individual points in the sample

(average density of all samples was $0.90 \pm .03$ gm/cm³)

PARAMETER	FREQ. (GHz)	ANGLE (°)	n	a_0	a_1	r^2	s_{yx}	s_0	s_1	$\pm \Delta$
ϵ_r	0.13	0	13	2.48	3.04	0.68	0.19	0.13	0.65	0.60
		30	14	2.46	4.90	0.81	0.21	0.12	0.69	
		45,60,90	42	2.48	5.60	0.85	0.22	0.08	0.42	
	0.21	0	14	2.95	2.31	0.85	0.10	0.06	0.29	0.50
		30	11	2.94	3.91	0.86	0.12	0.08	0.53	
		45,60,90	34	2.99	4.06	0.86	0.15	0.05	0.29	
	0.45	0	11	2.88	1.72	0.81	0.06	0.06	0.28	0.24
		30	13	2.99	2.11	0.91	0.06	0.04	0.20	
		45,60,90	29	2.95	2.71	0.88	0.09	0.04	0.20	
	0.81	0	11	3.10	1.52	0.71	0.07	0.07	0.33	0.17
		30	12	3.15	1.82	0.83	0.07	0.05	0.25	
		45,60,90	32	3.17	2.11	0.82	0.09	0.03	0.18	

TABLE 7.2

PARAMETER	FREQ. GHz	ANGLE (°)	n	a ₀	a ₁	r ²	s _{yx}	s ₀	s ₁	±Δ
ϵ_r'	1.00	0	11	3.14	1.24	0.69	0.06	0.06	0.28	0.13
		30	12	3.21	1.49	0.85	0.06	0.03	0.19	
		45,60,90	29	3.20	1.94	0.81	0.08	0.04	0.18	
	2.01	0	11	3.13	0.78	0.75	0.04	0.03	0.13	0.07
		30	13	3.13	1.06	0.86	0.04	0.02	0.13	
		45,60,90	33	3.12	1.47	0.79	0.07	0.03	0.14	
	4.00	0	14	3.12	0.61	0.80	0.03	0.02	0.09	0.03
		30	14	3.09	0.95	0.88	0.03	0.02	0.10	
		45,60,90	33	3.12	1.16	0.64	0.08	0.03	0.16	
ϵ_r''	0.13	0	13	0.326	1.03	0.08	0.31	0.21	1.03	0.45
		30	14	0.538	1.49	0.16	0.30	0.17	0.99	
		45,60,90	42	0.236	3.35	0.61	0.25	0.09	0.47	
	0.21	0	14	0.141	1.11	0.34	0.15	0.09	0.45	0.23
		30	11	0.329	0.84	0.13	0.17	0.11	0.73	
		45,60,90	34	0.282	1.70	0.49	0.18	0.06	0.31	
	0.45	0	11	0.162	0.26	0.07	0.07	0.07	0.33	0.10
		30	13	0.157	0.78	0.51	0.07	0.04	0.23	
		45,60,90	29	0.139	2.61	0.57	0.22	0.08	0.44	
	0.81	0	11	0.121	0.55	0.35	0.06	0.06	0.25	0.06
		30	13	0.045	1.32	0.73	0.06	0.04	0.24	
		45,60,90	31	0.062	1.34	0.69	0.08	0.03	0.17	
	1.00	0	11	0.068	0.66	0.55	0.04	0.04	0.20	0.045
		30	13	0.030	1.07	0.75	0.06	0.03	0.19	
		45,60,90	30	0.011	1.30	0.61	0.09	0.04	0.20	

TABLE 7.2

PARAMETER	FREQ. (GHz)	ANGLE (°)	n	a ₀	a ₁	r ²	s _{yx}	s ₀	s ₁	±Δ
ε _r ⁿ	2.01	0	14	0.029	0.38	0.65	0.03	0.02	0.08	0.022
		30	13	0.054	0.40	0.66	0.03	0.02	0.09	
		45,60,90	33	0.038	0.73	0.68	0.05	0.02	0.09	
	4.00	0	14	0.005	0.48	0.88	0.02	0.01	0.05	0.010
		30	14	0.050	0.54	0.81	0.02	0.01	0.07	
		45,60,90	33	0.025	0.63	0.72	0.03	0.01	0.07	

the data given in Table 6.2 with the relative brine volume fraction v . Table 7.2 gives the actual correlation data.

The artificial sea ice used here is more closely related to NaCl ice than to the natural first-year sea ice examined in the previous section. Hence, the brine volume fraction equations used to obtain v do not really apply in this case. However, they do provide reasonable estimates of the actual values of v , and therefore are used despite the drawback.

The five angles at which samples were taken from the artificial sea ice sheet have been collected into three groups: 0° , 30° , and $45-60-90^\circ$. The lumping of the 45 , 60 and 90° data was done because there appears to be no significant difference between the data points for each angle, i.e. a 45° point at the same brine volume gives, within the scatter of the data, the same value for ϵ_r' and ϵ_r'' as a 60° or 90° point, and so on. Some of the correlations were not significant, i.e. the r^2 values were very low. This happened only for the ϵ_r'' points, which were relatively small in magnitude, and therefore appear to be more scattered than the ϵ_r' data. When r^2 was low, the line of "best fit" was shown dashed.

The general trend in the data seems to indicate that the measured values of both ϵ_r' and ϵ_r'' increase, with sample offset angle from the vertical, up to a maximum, lying somewhere in the range of values given by the $45-60-90^\circ$ line. It cannot be said for certain from this data whether a decrease subsequently occurs, but it seems from this experiment at least, that the 90° points are generally lower in value than the 60° points.

The rather messy scatter in the data may appear at first to be merely a source of annoyance, but there is an important message

conveyed in it - the ice is not uniform. The general increase, with deviation from the vertical, tends to suggest the brine network has a primarily vertical structure, which is normally aligned almost perpendicular to the electric field. As the samples are tilted, the brine network couples more and more strongly to the field - hence the increase in ϵ_r' and ϵ_r'' .

A comparison of the slopes (a_1 values) given in Table 7.2 with those in Table 7.1 shows that for the same frequency, the vertical (0°) samples of artificial sea ice and the natural first-year ice samples exhibit substantially the same ϵ_r' and ϵ_r'' values. (Differences in the intercept value tend to obscure the similarity in a few cases; see also Section 7.3.) This finding suggests that truly representative studies of the dielectric properties of first-year sea ice may be done using home-grown sea ice sheets.

7.2 Theoretical Model

7.2.1 Scattering Considerations

In Chapter 3 the basic structure of both first-year and multiyear sea ice has been discussed in considerable detail. It was pointed out that sea ice is multiphase and therefore consists of brine scattered throughout the volume in the form of inclusions and drainage channels, as well as randomly scattered air bubbles which, in the case of multiyear sea ice, can be of substantial size. In order to enable one to develop any sort of theoretical model, it is necessary to make some structural simplifications.

In the following discussion, it will be assumed that sea ice consists of a pure ice background medium throughout which are scattered elongated ellipsoidal brine inclusions. It will be assumed also that these inclusions can lie at some angle θ , with respect to the vertical

Figure 7.46 ϵ_r' correlation with brine volume fraction for the artificial sea ice samples, at 130 MHz; correlation data given in figure.

Figure 7.47 ϵ' correlation with brine volume fraction for the artificial sea ice samples, at 210 MHz; correlation data given in figure.

Figure 7.48

ϵ' correlation with brine volume fraction for the artificial sea ice samples, at 400 MHz; correlation data given in figure.

Figure 7.49

ϵ_r' correlation with brine volume fraction for
 the artificial sea ice samples, at 810 MHz;
 correlation data given in figure.

Figure 7.50 ϵ_r' correlation with brine volume fraction for the artificial sea ice samples, at 1.0 GHz; correlation data given in figure.

Figure 7.51 ϵ_r' correlation with brine volume fraction for the artificial sea ice samples, at 2.01 GHz; correlation data given in figure.

Figure 7.52

ϵ' correlation with brine volume fraction for the artificial sea ice samples, at 4.0 GHz; correlation data given in figure.

Figure 7.53

ϵ_r correlation with brine volume fraction for the artificial sea ice samples, at 130 MHz; correlation data given in figure.

Figure 7.54 ϵ' correlation with brine volume fraction for the artificial sea ice samples, at 210 MHz; correlation data given in figure.

Figure 7.55

ϵ' correlation with brine volume fraction for the artificial sea ice samples, at 450 MHz; correlation data given in figure.

Figure 7.56

ϵ_r correlation with brine volume fraction for the artificial sea ice samples, at 810-MHz; correlation data given in figure.

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Figure 7.57

ϵ'' correlation with brine volume fraction for
 the artificial sea ice samples, at 1.0 GHz;
 correlation data given in figure.

Figure 7.58 ϵ_r correlation with brine volume fraction for the artificial sea ice samples, at 2.0 GHz; correlation data given in figure.

Figure 7.59 ϵ'' correlation with brine volume fraction for the artificial sea ice samples, at 4.0 GHz; correlation data given in figure.

direction, but that all the inclusions must lie at the same angle. The azimuthal orientation of the ellipsoids is assumed random (see Figure 7.60 for a conceptual view of the model).

The model will also assume the existence of spherical air bubbles. The spacing of the ellipsoids and spheres must be such that only first order interactions are important. If Figure 2.2 is used as a rough guideline, this will correspond to volume fractions of the included particle of much less than 40 %. In actual practice, the brine volume fractions and air bubble volume fractions measured are less than 10 %. However, these particles are not necessarily evenly distributed, as our model must assume, but are crowded in some areas and thinly spaced in others. The brine inclusions tend to be closely spaced at the bottom of the ice layer, while the air bubbles tend to have a higher concentration at the surface. Most of the ice samples were taken from depths in the ice layer away from these extreme limits and therefore the sampling procedure should minimize this spacing problem.

The model consists, therefore, of equally spaced spherical air bubbles, and equally spaced ellipsoidal brine inclusions oriented at some angle θ with respect to the vertical, but lacking any preferred azimuthal direction. This model must also satisfy the criterion of having scattering centres that are much smaller than a wavelength in size, if a quasi-static field is to be assumed.

Instead of using the largest dimension of the particle compared to a wavelength as a guide, one can instead apply the Rayleigh scattering theory discussed in Chapter 2, and calculate the maximum size particle that can be tolerated if the scattering losses are to be kept below 0.1 db, which is the measurement system amplitude

Figure 7.60 Conceptual view of ellipsoid model; all ellipsoids are assumed to lie at angle θ with respect to the vertical, but can have arbitrary azimuthal orientation.

error. The relevant equations are (2.50), (2.47) and (2.53) which are repeated here below as (7.2) to (7.4)

$$T = \exp(-NC_{sca}l) \quad (7.2)$$

where T is the power transmission coefficient, l is the propagation length, N is the number of particles per unit volume and C_{sca} is

$$C_{sca} = \frac{24\pi^3 V^2}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \quad (7.3)$$

where m is the ratio of the complex refractive index of the particle to that of the medium, λ is the wavelength in the medium, and V is the particle volume, which is given by $V = \frac{4}{3}\pi abc$. a , b , and c are the major and minor semi-axes of the ellipsoid. For an ellipsoid, the equivalent values of the semi-major and minor axes differ depending on whether the ellipsoid is parallel or perpendicular to the electric field. The equivalent value a' is given by

$$(a')^3 = \frac{4\pi abc}{3} \frac{m^2 + 2}{4\pi + (m^2 - 1)P_{a,b, \text{ or } c}} \quad (7.4)$$

where P is the depolarization factor which is given by 4π times the values calculated, using equations (2.39) and (2.40), and plotted in Figure 2.4.

Everything in the above equations is fixed by the sea ice, except for the wavelength λ . These equations were used to calculate limiting values of λ for the various cases of interest. The dimensions of the ellipsoids were taken to be $a = 5.0$ mm., $b = c = 0.25$ mm. The spheres were estimated to be approximately 0.5 mm. in radius for first-year sea ice and 0.25 to 0.50 cm. in radius for multiyear ice (above freeboard). The limiting cases of end-on, and broadside incidence of the electric field with respect to the ellipsoid give maximum tolerable values for λ of 6.1 cm. and 2.0 cm. respectively in first-year

ice. The spherical cases yield limiting values of $\lambda = 0.3$ cm. for first-year ice and 7.0 cm. for multiyear ice. This means that measurements of the dielectric properties of sea ice will not be complicated by scattering and propagation effects at frequencies below approximately 2.8 GHz for broadside incidence, and 8.4 GHz for end-on incidence, for first-year sea ice. For multiyear ice above freeboard, with a density of 0.7 gm/cm^3 and the specified bubble size, the limiting frequency is 1.5 GHz. It should be noted that the frequency calculation for first-year ice has taken into account the combined effect of scattering from brine inclusions plus air bubbles. The total number of particles per cm^3 , N , is based on the total maximum volume fraction (0.11 for brine, 0.015 for air in first year ice, and 0.244 for air in multiyear ice) and the actual size of the particle. Also, the complex relative dielectric constant including conductivity of brine was taken as $70 - j20$. The value assumed for pure ice is $3.14 + 0j$.

From the above, it can be seen that if it is assumed that the incident angle of the probing field is such that the ellipsoids are viewed end-on, then the measurements presented in Chapter 6 for first-year ice will be correct only up to a frequency of 8.4 GHz. If the ellipsoids lie at some angle other than this, then the probability of the interpretation being correct at frequencies above 2.8 GHz drops as one approaches broadside incidence. If an effective radius of 0.707 times the radius at broadside is assumed, for ellipsoids oriented at 45° to the probing field, then the lower frequency bound can be raised to 3.6 GHz. It should be remembered that all these nominal figures were calculated using worst case values. It is probably safe to extend the lower cutoff frequency as high as 4.0 GHz without introducing appreciable error except possibly in the few measurements which involve

large brine volumes. The size of the ellipsoids used in these calculations is 0.25 mm. in radius by 1 cm. in length. Measurements by Poe et al⁽⁹⁾ have shown typical lengths of brine pockets to be only a few mm. If the axial ratio is maintained at 20 and the size reduced to 0.5 cm. in length by 0.125 mm. in radius, then the lower frequency bound for broadside incidence becomes approximately 7.8 GHz. If the angle θ is 35° , then the lower frequency bound rises even higher, to 21.8 GHz.

These figures are only approximate and several assumptions have been made in order to obtain them. The dimensions of the effective brine inclusion, and their precise shape and spacing are not known exactly. Therefore, these numbers are not hard and fast limits, but rather only represent guidelines.

In a reasonably realistic model, it appears, from the above calculations, that to avoid scattering effects, the measurement frequency should be kept to below approximately 4.0 GHz for first-year sea ice, and for multiyear sea ice above freeboard, it should be kept to less than 1.5 GHz. Excursions above these limits will probably result in larger values for ϵ_r'' being measured than are predicted from pure dielectric and conductivity considerations. Furthermore, dielectric mixture equations such as equation (2.38) start to lose their validity above these limits, due to the increased propagation effects and the violation of the quasi-static assumption.

The data obtained at frequencies above the suggested limits have been noted in Chapter 6 to suffer a certain amount of variation in the measured values. This could be due to the inclusion of a new contributor to ϵ_r'' - the scattering loss. This contributor varies greatly from location to location in the ice, especially in the vicin-

ity of major brine drainage networks or large concentrations of air bubbles, and therefore can be expected to add unexpected scatter to the measured data. This scatter is not really an error in the measured values, but merely represents another loss mechanism. It does, however, make comparison with theoretical models especially difficult at frequencies above which the scattering losses become excessive. This aspect of the modelling problem has already been briefly discussed in Chapter^o 4 in connection with the 10 and 35 GHz preliminary measurements.

7.2.2 Dielectric Model for Brine

Central to the sea ice modelling task is the development of adequate models for the constituents. The dielectric behaviour of pure ice is reasonably well understood (see Evans⁽⁷³⁾ for a review). Unfortunately, the behaviour of brine, in particular highly concentrated sea water, is less well understood. Much of the literature available on the subject pertains to simpler solutions consisting only of NaCl and water.

Stogryn⁽⁷⁴⁾ has presented a reasonable dielectric model for brine, for certain combinations of concentration and temperature. His model is primarily an amalgamation of several previously published measurements into a single set of empirical equations.

The model assumes that the dielectric constant of the brine can be adequately represented by an equation of the Debye⁽⁸³⁾ form

$$\epsilon_{r_{br}} = \epsilon_{r_{\infty br}} + \frac{\epsilon_{r_{0br}} - \epsilon_{r_{\infty br}}}{1 + j\omega\tau_{br}} - \frac{j\sigma_{br}}{\epsilon_0\omega} \quad (7.5)$$

where $\epsilon_{r_{0br}}$ and $\epsilon_{r_{\infty br}}$ are, respectively, the static and high frequency complex dielectric constants of the solvent (modified by the solute),

τ_{br} is the relaxation time, ϵ_0 is the permittivity of free space ($= 8.854 \times 10^{-12}$ F/m), σ_{br} is the ionic conductivity of the dissolved salts in mho/m, and ω is the radian frequency.

Stogryn⁽⁷⁴⁾ has fit the tabular data of Lane and Saxton⁽⁸⁴⁾ to simple polynomial equations. He gives $\epsilon_{r_{obr}}$ as

$$\epsilon_{r_{obr}}(T, N) = \epsilon_{r_{obr}}(T, 0) a(N) \quad (7.6)$$

where T is the temperature in $^{\circ}\text{C}$ and N is the normality of the solution. A fit to Lane and Saxton's⁽⁸⁴⁾ NaCl data for $0 \leq T \leq 40^{\circ}\text{C}$ and $0 \leq N \leq 3$ yields

$$a(N) = 1.000 - 0.2551N + 5.151 \cdot 10^{-2}N^2 - 6.889 \cdot 10^{-3}N^3 \quad (7.7)$$

A plot of the resulting relations is given in Figure 7.61. The slight bump in the curve for 3N is a product of the equation, not the plotting. Stogryn⁽⁷⁴⁾ has used the equation for $\epsilon_{r_{obr}}(T, 0)$ given by Malmberg and Maryott⁽⁸⁵⁾, which is

$$\epsilon_{r_{obr}}(T, 0) = 87.74 - 0.4008T + 9.398 \cdot 10^{-4}T^2 - 1.410 \cdot 10^{-6}T^3 \quad (7.8)$$

There is an error in this equation, as originally quoted in Stogryn.⁽⁷⁴⁾ The corrected form is given here.

A similar procedure was followed to obtain the relaxation time equation. It was assumed that

$$2\pi\tau_{br}(T, N) = 2\pi\tau_{br}(T, 0) b(N, T) \quad (7.9)$$

where $\tau_{br}(T, 0)$ is given (from the data of Grant et al⁽⁸⁶⁾) by

$$2\pi\tau_{br}(T, 0) = 1.1109 \cdot 10^{-10} - 3.824 \cdot 10^{-12}T + 6.938 \cdot 10^{-14}T^2 - 5.096 \cdot 10^{-16}T^3 \quad (7.10)$$

Obviously, pure water normally turns to ice below 0°C , therefore this equation is only an extrapolation when used at temperatures below 0°C .

Figure 7.61 Static dielectric constant of NaCl brine solutions versus temperature and normality.⁸

The data of Lane and Saxton⁽⁸⁴⁾ were used to obtain $b(N,T)$ which Stogryn⁽⁷⁴⁾ gives as

$$b(N,T) = 0.1463 \cdot 10^{-2}NT + 1.000 - 0.04896N - 0.0296N^2 + 5.644 \cdot 10^{-3}N^3 \quad (7.11)$$

where $0 \leq T \leq 40^\circ\text{C}$ and $0 \leq N \leq 3$. A plot of $\tau_{br}(N,T)$ is given in Figure 7.62.

The parameter $\epsilon_{r_{\infty br}}$ is taken equal to 4.9. According to Stogryn⁽⁷⁴⁾ this affords the best fit to the published experimental data. He also states that there is no evidence to suggest $\epsilon_{r_{\infty br}}$ depends on salinity. Therefore, the value for pure water is used. It appears that there may be a slight temperature dependence, but this dependence is ignored here.

The main parameter remaining to be determined is that of conductivity. Stogryn⁽⁷⁴⁾ has used the data of Chambers et al⁽⁸⁷⁾ and Chambers⁽⁸⁸⁾ to determine the following relations for the conductivity of NaCl solutions

$$\begin{aligned} \sigma_{\text{NaCl}}(T,N) = & \sigma_{\text{NaCl}}(25,N) [1.000 - 1.962 \cdot 10^{-2} \cdot \Delta \\ & + 8.08 \cdot 10^{-5} \cdot \Delta^2 - \Delta \cdot N(3.020 \cdot 10^{-5} \\ & + 3.922 \cdot 10^{-5} \cdot \Delta + N(1.721 \cdot 10^{-5} \\ & - 6.584 \cdot 10^{-6} \cdot \Delta))] \end{aligned} \quad (7.12)$$

where $\Delta = 25 - T$ and

$$\begin{aligned} \sigma_{\text{NaCl}}(25,N) = & N[10.394 - 2.3776N + 0.68258N^2 \\ & - 0.13538N^3 + 1.0086 \cdot 10^{-2}N^4]; \end{aligned} \quad (7.13)$$

$-20 \leq T \leq 40; 0 \leq N \leq 2.5$

These expressions are plotted in Figure (7.63).

The concentration of sea water is normally measured in parts per thousand of salinity. The relation between salinity and

Figure 7.62 Relaxation time of NaCl brine solutions versus temperature and normality.

$$N = S[1.707 \cdot 10^{-2} + 1.205 \cdot 10^{-5}S + 4.058 \cdot 10^{-9}S^2]; \quad (7.14)$$

$$0 \leq S \leq 260$$

where S is the salinity in parts per thousand. The salinity range restricts N to between 0 and 5.35. A plot of this equation is given in Figure 7.64.

The salinity of the brine contained in sea ice is given by Poe et al⁽⁴²⁾, based on Assur's⁽³¹⁾ computations. These relations have already been listed as equation (3.2). A comparison of Assur's⁽³¹⁾ data with that of Zubov⁽⁴⁰⁾ is given in Figure 3.7. Note the lack of agreement below approximately -12°C .

These equations may now be used to obtain $\epsilon_{r_{br}}$ as given by equation (7.5). The major restrictions on the model are: the conductivity equations only hold up to $N = 2.5$ ($S = 130 \text{ ‰}$), the equations for $\epsilon_{r_{O_{br}}}$, σ_{NaCl} and τ are not valid below 0°C - they must be extrapolated, and the brine salinity equations are suspect below -12°C . If the conductivity equation is taken as the main limitation, then the equations can be used up to $S = 130 \text{ ‰}$, i.e. above approximately -10°C . One can stretch this possibly to -15°C , but below this temperature the reliability of the equations for the high brine concentrations that would be produced at such low temperatures is somewhat suspect.

Stogryn⁽⁷⁴⁾ compares his equations with the measurements of Lane and Saxton.⁽⁸⁴⁾ The agreement is quite good at 9.36 and 48.6 GHz, but an unexplained trend away from the reported measurements was encountered at 24.2 GHz, at temperatures above 25°C . He also compares his predictions with the measurements of Hasted and El Sabeh.⁽⁸⁹⁾ This comparison shows satisfactory agreement.

All the above comparisons are made for temperatures in the range of $0 \leq T \leq 40^{\circ}\text{C}$ and normalities in the range $0 \leq N \leq 3$. There-

Figure 7.63 Ionic conductivity of NaCl brine solutions versus temperature and normality.

Figure 7.64 Normality versus salinity relationship.

fore, the extension of Stogryn's equations to the case of sea ice, where the temperatures are always below 0°C and the normalities can be very high, is not straightforward. However, in the absence of any other measurements on actual brine, one is forced to use Stogryn's⁽⁷⁴⁾ data, at least as a best approximation. In the ensuing model, this is what will be done, although the temperature of the ice will be restricted to temperatures above -15°C ; therefore restricting the brine salinity to less than 130 ‰ ($N \leq 3.0$).

Figures (7.65) and (7.66) show typical variations of the real part of $\epsilon_{r_{br}}$ (here labelled as XBR), and the imaginary part of $\epsilon_{r_{br}}$ (labelled as YBR). Figure (7.65) assumes an ice temperature of -10°C and Figure (7.66), an ice temperature of -5°C . Note that the brine salinity only depends on the temperature of the ice (see Zubov⁽⁴⁰⁾) and not its salinity.

In both figures, there is a strong decrease in YBR with frequency. This denotes the large contribution from $\sigma_{br}/\omega\epsilon_0$. There is also a bump in the curves which occurs at different frequencies depending on the brine salinity. This bump is the contribution to YBR from dielectric origins. Evidence of this behaviour has already been seen in Section 6.3.1.2.

The real part of $\epsilon_{r_{br}}$ changes very little with brine salinity although the frequency at which it starts to drop from $\epsilon_{r_{0br}}$ to $\epsilon_{r_{\infty br}}$ changes slightly. Its behaviour denotes a steady decrease with frequency as was suggested in the results presented in Section 6.3.1.1.

As far as one can determine, the model seems to predict a reasonable type of behaviour for $\epsilon_{r_{br}}$. Further proof of its merit will be seen in the following sections.

Figure 7.65 XBR, YBR versus frequency for $T = -10^{\circ}\text{C}$, $S = 142\%$.

Figure 7.66 XBR, YBR versus frequency for $T = -5^{\circ}\text{C}$, $S = 82\%$.

7.2.3 Dielectric Mixture Equations

Various mixture formulas have been presented in Chapter 2. Also, further references have been given for a source of even more of them. With this staggering array of formulas to choose from, the question arises - which one is best suited to the sea ice modelling task? An extensive literature search was performed with the aim of answering this question. The results of the search indicated that most of the published mixture equations fail to account for particle interaction effects, or do so inadequately. Thus, their application is very limited. In particular, most formulas hold for inclusion volume fractions of less than 0.10.

Also, all of the formulas are only applicable to the static case, and many fail to include conductivity effects. As discussed in Chapter 2 and Appendix A, this need not always be a drawback, since if quasi-static conditions are seen to exist most equations can easily be extended beyond the static case.

Equations which include particle shape and orientation in a "form factor" are also not very desirable, since they have the effect of masking the very behaviour that one is trying to model, by lumping the shape and orientation in a semi-empirical "form factor". This approach does not allow extensions to be made to the theory solely on the basis of a physical understanding of the processes taking place.

Therefore, a set of criteria for a suitable mixture equation are: it includes interaction effects; it is easily extended, using a quasi-static approach, to high frequencies; and it explicitly includes shape and orientation effects in such a way that behavioural predictions can be made.

It was felt that the equation of Tinga et al⁽¹⁷⁾ (given previously as equation (2.38)) best satisfied these criteria. This does not mean that it is the only applicable one. Section 7.2.1 has shown the quasi-static assumptions to be satisfied at frequencies at least as high as 4.0 GHz, and equation (2.38) itself includes first-order interaction effects by the very definition of the boundary value problem. Appendix A has shown that, as long as the quasi-static assumptions are satisfied, the treatment of a material with a complex dielectric constant and non-zero conductivity is easily accomplished. Thus, it appears that no major problems will be encountered.

The dielectric constant and conductivity of the brine is calculated as in Section 7.2.2 above.

The dielectric constant of the pure ice is given by the Debye equations⁽⁸³⁾ (equations (2.1) to (2.2)). The relaxation time used is that due to Ozawa and Kuroiwa⁽⁹⁰⁾

$$\tau_{ICE} = 5.3 \cdot 10^{-6} \exp(756.4/t + 273) \quad (7.15)$$

where T is the temperature in °C. The ϵ_{r_0}' used is given by Powles (in Ozawa and Kuroiwa⁽⁹⁰⁾) as

$$\epsilon_{r_0}' = 20715/(T + 273 - 38) \quad (7.16)$$

For ϵ_{r_∞}' the value of 3.14, obtained by Vant et al⁽¹⁰⁾ is used. The conductivity of the pure ice is ignored as negligible at the frequencies considered here.

The brine volume fraction, for a given ice salinity and temperature, is calculated from the equations of Frankenstein and Garner⁽⁴¹⁾ and Poe et al⁽⁴²⁾ (equation (3.1)). The brine salinity is given by equation (3.2).

The assumed angular variation with ellipsoid orientation for normal, vertically polarized incident waves is included in equation

(2.42).

The effects of the small spherical air bubbles in the ice are included by employing equation (2.38) once more. This time, ϵ_r of the mixture is input as the background material and the inclusions are assumed to be spherical air bubbles with $\epsilon_{r\text{air}} = 1$. The volume fraction of air is calculated using the following equation

$$v_{\text{air}} = 1 - \frac{\rho}{0.926 \text{ gm/cm}^3} \quad (7.17)$$

where ρ is the measured density of the sea ice and 0.926 gm/cm^3 is the density for non-porous sea ice, assumed in the calculation of the sea ice brine volume fraction.

7.2.4 Ellipsoid Orientation and Polarization Considerations

The orientation of the ellipsoids will cause a problem in the comparison of the theory with the experimental results. The experiment uses a TEM mode wave propagating in a "coaxial-cage" transmission line, in which the electric field vector has no one single direction, other than the radial direction. It is desired to simulate the propagation of a linearly polarized wave through the ice. In order to do this, the wave propagating in the "coaxial-cage" line will be thought to act in the same fashion as a "radially polarized" wave propagating through an ice layer. No such wave exists in reality, but it will be seen that conceptually this "radially polarized" wave is a convenient way to compare the behaviour of the wave propagating in the sample holder to the propagation of a linearly polarized wave through the ice.

This relation can be understood by imagining a linearly polarized incident wave vector pivoted on a vertical axis above the ice. As the vector is spun around this vertical axis, it simply maps out all the cases present in the "radially polarized" wave. All the

brine ellipsoids are assumed to be at the same angle θ , with respect to the vertical, but are assumed to possess no intrinsic preferred azimuthal orientation, i.e. there is no horizontal anisotropy. (Campbell and Orange⁽⁹¹⁾ have presented some evidence that such horizontal anisotropy may exist, but it is certainly an anomalous condition and so far has not been observed by any other independent workers.) Figure 7.67 illustrates the two cases. If the medium truly has no preferred azimuthal orientation, then all the cases mapped out by the rotating vector described above will be identical, and all the cases contained in the "radially polarized" wave will in turn be identical. This implies that the "radially polarized" wave will yield the same results as a single linearly polarized wave, as long as both waves are propagating at a normal angle of incidence. It also implies that the propagation of waves through the sample mounted on the "coaxial-cage" line will yield useful information applicable to the propagation of linearly polarized waves through the ice. How does this relate to the propagation of a normally incident, vertically polarized wave through an ordered array of ellipsoids as shown in Figure 7.68?

Equation (2.42), here written as (7.18), gives the value of the effective dielectric constant that will be seen in Figure 7.68(a).

$$\epsilon_v = \left[\frac{\cos^2 \theta}{\epsilon_{\perp}} + \frac{\sin^2 \theta}{\epsilon_{\parallel}} \right]^{-1} \quad (7.18)$$

where θ is the angle the ellipsoids make with respect to the vertical, and $\epsilon_{\perp}$ and $\epsilon_{\parallel}$ are the dielectric constants (including conductivity effects, see Appendix A) for waves propagating with their electric field vectors perpendicular and parallel to the major axis respectively. If one now imagines that each ellipsoid in the random array of ellipsoids

(a) rotating linearly polarized vector

(b) "radially-polarized" wave

Figure 7.67

Relationship of linearly polarized and "radially-polarized" probing fields.

(a) ordered array of ellipsoids all at angle θ with respect to the vertical

(b) random array of ellipsoids all at angle θ with respect to the vertical

Figure 7.68

Generalization from ordered array of ellipsoids to array with random azimuthal orientation.

is oriented at an angle $(\pi/2 - \psi)$ with respect to $\vec{D}$, as shown in Figure 7.69, where $\vec{D}$ is any single linearly polarized vector, then $\vec{D}$ can be decomposed into its horizontally and vertically polarized components $\vec{D}_H$ and $\vec{D}_V$, the magnitudes of which are given as

$$\begin{aligned} D_H &= D \cos \psi = \sqrt{\epsilon_H} E \cos \psi \\ D_V &= D \sin \psi = \sqrt{\epsilon_V} E \sin \psi \end{aligned} \quad (7.19)$$

In equation (7.19) ψ is the angle indicated, ϵ_H and ϵ_V are the dielectric constants seen by the horizontally and vertically polarized components respectively, ϵ_V is given by (7.18), and ϵ_H is simply ϵ_1 . If it can be assumed that the magnitudes of D and its components are related in the following way,

$$\begin{aligned} |D| &= |\sqrt{\epsilon_{\text{eff}}} E| \\ &= \sqrt{(\sqrt{\epsilon_H} E \cos \psi)^2 + (\sqrt{\epsilon_V} E \sin \psi)^2} \end{aligned} \quad (7.20)$$

then ϵ_{eff} , the effective dielectric constant perceived by E at angle ψ , is just

$$\epsilon_{\text{eff}} = \epsilon_H \cos^2 \psi + \epsilon_V \sin^2 \psi \quad (7.21)$$

Equation (7.21) expresses the relation between ϵ_{eff} and ϵ_H and ϵ_V for only one ellipsoid in the random array (see Figure 7.69(b)). To obtain the total effect of the random array of ellipsoids (all at angle θ with respect to the vertical), ϵ_{eff} must be averaged over all possible values of ψ . ψ may take on values between 0 and π depending on the orientation of $\vec{D}$ (angles greater than $\pi/2$ indicate that $\vec{D}_H$ points into the plane shown in Figure 7.69, not out of it).

Integration of equation (7.21) gives

$$\epsilon_{\text{eff}} = \frac{1}{\pi} \int_0^\pi (\epsilon_H \cos^2 \psi + \epsilon_V \sin^2 \psi) d\psi$$

Figure 7.69 Orientation of ellipsoid with respect to $\vec{D}$, ψ is angle between $\vec{D}_H$ and $\vec{D}$, θ is the tilt of the ellipsoid with respect to the vertical.

$$\begin{aligned}
 &= \frac{1}{2\pi} \int_0^{2\pi} \epsilon_H \left(\frac{1 + \cos 2\psi}{2} \right) + \epsilon_V \left(\frac{1 - \cos 2\psi}{2} \right) d2\psi \\
 &= \left(\frac{\epsilon_H}{2} + \frac{\epsilon_V}{2} \right)
 \end{aligned} \tag{7.22}$$

Thus, the effective dielectric constant for random azimuthal orientation of the ellipsoids is just the average between the value expected for horizontal polarization and the value expected for vertical polarization. This result is intuitively acceptable, since in a random medium of this type, there should be no preferred polarization.

Substitution for ϵ_H and ϵ_V gives

$$\epsilon_{\text{eff}} = \frac{1}{2} \epsilon_{\perp} + \frac{1}{2} \left[\frac{\cos^2 \theta}{\epsilon_{\perp}} + \frac{\sin^2 \theta}{\epsilon_{\parallel}} \right]^{-1} \tag{7.23}$$

which is the desired relation between Figures 7.68(a) and (b).

For a non-normal angle of incidence, the problem of finding an effective dielectric constant becomes very involved. In this case, the vertical axis of the cone-like structure formed by the inclusions, and the axis about which the linearly polarized vector is imagined to rotate do not coincide. This means the approach culminating in equation (7.23) is invalid here and that the propagation of a linearly polarized wave and the propagation of the "radially polarized" wave are no longer equivalent. The solutions of each of these cases is difficult to find.

Figure 7.70 shows the geometry involved for the propagation of an originally vertically polarized wave $\vec{D}$ through the sea ice layer. A cone-shaped structure is used to demonstrate all the possible orientations of the ellipsoids.

Unfortunately, most of the ellipsoids are not coplanar with any possible plane containing $\vec{D}$, and therefore the vertical polarization is destroyed, causing depolarization to take place. Every ellipsoid

Figure 7.70 Orientation of cone-like ellipsoid structure with respect to originally vertically polarized $\vec{D}$.

position with respect to the incident wave causes a different degree of depolarization.

A horizontally polarized incident wave acts in the same manner as a wave at normal incidence. Therefore, although depolarization also takes place, the effective dielectric constant is still given by (7.23) (see Figure 7.71).

It is the lack of symmetry in this new problem of non-normal incidence that makes it difficult to solve. Because of the complexity of the solution and the difficulty in applying (7.18) to the random situation, it was felt that the investigation of non-normal incidence should be left to a future study.

The results of the artificial sea ice measurements were given in Section 7.1.2. They illustrate the variation in ϵ_{eff} with measurement angle, for the complex situation of measurement with a radially directed electric field. The propagation of waves in the "co-axial-cage" line ceases to approximate that of linearly polarized waves when the angle of incidence is not normal. Therefore, these results are not directly applicable to the understanding of linearly polarized propagation. The theoretical interpretation of the radially directed case is much more difficult to obtain than that of the linearly polarized one. Indeed, it includes all the possible orientation angles of ellipsoid and electric field encountered in the linearly polarized situation.

As already described in Section 7.1.2, the basic behaviour pattern of ϵ_{eff} at non-normal incidence, seems to be: an increase in ϵ_{eff} with deviation from the normal, a peaking in ϵ_{eff} at approximately 45° to 60° , and a subsequent decrease with further deviation from the normal. This probably indicates that a non-horizontally, linearly polarized wave will experience a small variation in ϵ_{eff} with incidence

Figure 7.71 Orientation of cone-like ellipsoid structure with respect to originally horizontally polarized $\vec{D}$.

angle. The exact form of the variation will have to be determined in a future study.

7.2.5 An Algorithm for Evaluation of the Theoretical Model

An algorithm summarizing the various steps used to arrive at the theoretical value for ϵ_r is given in Figure (7.72). The basic physical properties of the sea ice, that are measured inputs, are shown in circles. These are: the frequency of the probing wave, F ; the salinity of the sea ice, S ; the temperature of the sea ice, T ; and the density of the ice, ρ . Inputs derived from a knowledge of the components in the mixture and from a priori assumptions are shown in squares. These are: ϵ_∞' , ϵ_0' , and τ for pure ice; ϵ_∞' for brine; $\bar{\rho}$, the assumed mean density for non-porous sea ice; a/b , the ellipsoid axial ratio; and θ , the angle of orientation of the ellipsoids. Outputs from the various calculations are shown as boxes with tapered bottoms. These are: the complex relative dielectric constant of pure ice, $\epsilon_{r_{ICE}}'$; the relaxation time of brine, τ_{br} ; the static dielectric constant of brine, $\epsilon_{0_{br}}'$; the brine conductivity, σ_{br} ; the complex relative dielectric constant of brine, $\epsilon_{r_{br}}'$; the brine volume fraction, v_{br} ; the air volume fraction, v_{air} ; the complex relative dielectric constant of sea ice with the ellipsoids oriented perpendicular and parallel to the probing field vector, $\epsilon_{r_{\perp}}$ and $\epsilon_{r_{\parallel}}$, respectively; and the complex relative dielectric constant of the sea ice, as a function of the inclusion angle θ , $\epsilon_{r_{eff}}$. The rectangles denote submodels or sets of equations used to obtain the necessary information.

The information flow is self-evident from the diagram and the processes are the ones already described in detail in this and previous chapters. This model represents the desired product as outlined in Chapter 1. It has certain weaknesses and limitations, which are left

Figure 7.72

SEA ICE DIELECTRIC MODEL

to Section 7.5 for discussion. The main point is that a theoretical model describing the dielectric behaviour of sea ice in terms of its three basic physical properties, salinity, temperature and density, has been formulated. It allows prediction of ϵ_r at any frequency of interest (provided the assumptions made are not violated) and is formulated from a fundamental knowledge of sea ice, and the orientation and distribution of the constituents.

7.2.6 Evaluation of the Theoretical Model for the Cases of Interest

7.2.6.1 Effect of the Various Parameters on ϵ_r' and ϵ_r''

Apart from the obvious parameter of temperature, several other factors must be specified in the model equations. This is apparent from examination of the algorithm shown in Figure 7.72. Of paramount importance are the angle of the ellipsoids with respect to the normal to the electric field and the assumed axial ratio. Figures 7.73 and 7.74 show the variation of ϵ_r' and ϵ_r'' with a/b , as calculated using the theoretical model. The ellipsoid angle θ is plotted as a parameter. Many sets of experimental data were compared with curves like these at several frequencies, to estimate an optimal choice for a/b . In most cases the best overall fit was obtained with $a/b = 20$. As shown in Figure 7.74, ϵ_r'' varies quite a bit with θ for $a/b = 20$, at 100 MHz, but varies much less with θ at 10 GHz. In direct contrast to this, as seen in Figure 7.73, ϵ_r' varies almost not at all with θ at 100 MHz, for any a/b combination, but varies quite strongly with θ for $a/b = 20$ at 10 GHz. Therefore, in order to choose a representative a/b ratio, the fit of ϵ_r'' at 100 MHz and the fit of ϵ_r' at 10 GHz (or some other high frequency) to the experimental points should be used as cri-

Figure 7.73 Variation of calculated ϵ_r' with axial ratio and ellipsoid angle, at 100 MHz and 10 GHz (other parameters as indicated).

Figure 7.74 Variation of calculated ϵ'' with axial ratio and ellipsoid angle, at 100 MHz and 10 GHz (other parameters as indicated).

teria. Unfortunately, the experimental errors at 100 MHz are quite large and this complicates the choice of a/b . No single choice will really be optimum.

The situation assumed, i.e. that the brine is contained in ellipsoids, is fictional. To carry the assumptions further than this and assume that all the ellipsoids have the same a/b ratio or angle θ , seems almost ludicrous; however, it is necessary if even an attempt at modelling the ice is to be made.

The primary importance of Figures 7.73 and 7.74 is to impress upon the naïve reader that for only small changes in a/b or θ , the ϵ_r' and ϵ_r'' predicted by the model can be substantially different. These differences will become apparent in Section 7.2.6.2.

After going through the process of determination of the axial ratio, a value of $a/b = 20$ was arrived at. There is some justification for this assumption in that some brine inclusions may approach ellipsoidal shape with almost this axial ratio. However, it is apparent, from Chapter 3, that not all the brine is in the form of inclusions, but rather exists in the complicated drainage patterns - not in an ellipsoidal shape at all!

Figures 7.75 to 7.77 show the variation in the predicted ϵ_r' and ϵ_r'' with density. The brine is assumed to be in the form of brine inclusions with an axial ratio of 20. Two different angles of orientation of the ellipsoids with respect to the vertical are considered: $\theta = 0^\circ$, and $\theta = 45^\circ$. The model values for ϵ_r' and ϵ_r'' have been evaluated at $T = -4^\circ\text{C}$ (solid line) and $T = -15^\circ\text{C}$ (dashed line) and an ice salinity of 5 ‰, at both 100 MHz and 10 GHz. The density changes are accounted for in the theoretical model by assuming spherical air bubbles ($a/b = 1$) and applying equation (2.38) to the non-porous

(Figure 7.75 Variation of calculated ϵ' with density, at 100 MHz (bottom) and 10 GHz (top), (other parameters as indicated).

Figure 7.76 Variation of calculated ϵ_r'' with density, at 100 MHz (other parameters as indicated).

Figure 7.77 Variation of calculated ϵ_r'' with density, at 10 GHz (other parameters as indicated).

mixture. This produces a linear variation in the predicted values of ϵ_r' and ϵ_r'' . Considering the accuracy with which ϵ_r'' can be measured, changes in ϵ_r'' with density are relatively insignificant, even over the whole range 0.80 to 0.95 gm/cm³. However, the changes in the value of ϵ_r' with density, approximately $\pm .3$, for a $\pm .03$ gm/cm³ change in density, can be significant, and could easily explain the scatter in the experimental data in Figures 7.1 to 7.45. Changes of density of this order occur quite commonly within a short distance (a few cm.) in the ice cover.

The variation in the predicted values of ϵ_r' and ϵ_r'' with salinity is shown in Figures 7.78 and 7.79. Three temperatures, -4°C , -10°C and -15°C , are considered. The density was assumed to be 0.91 gm/cm³ (the average density of the first-year ice in the AIDJEX area), and the axial ratio was taken to be 20. A value of $\theta = 45^\circ$ was used in the calculations, which were performed at 100 MHz and 2 GHz.

The ϵ_r' data show a typical non-linear relationship with salinity, i.e. ϵ_r' increases sharply with salinity at first, then starts to level off at a salinity of about 9 ‰. Generally speaking, the ϵ_r' data follow the same trends at 100 MHz and 2 GHz, although the actual values for ϵ_r' at 100 MHz are higher. The ϵ_r'' data follow the same trend as the ϵ_r' data at 2 GHz, but show little variation at 100 MHz. This is because the conductivity of the brine, which is a very large number at 100 MHz, appears in both the numerator and denominator of the mixture equation (2.38). This tends to cause a cancelling of the effect of variations with salinity, until the conductivity has dropped to a number comparable to the contribution to the brine dielectric constant from other sources, e.g. the Brine relaxation. It appears that at 100 MHz a decrease in ice temperature causes an increase in ϵ_r'' , a totally

Figure 7.78 Variation of calculated ϵ_r' with salinity, at 100 MHz and 2 GHz (other parameters as indicated).

Figure 7.79 Variation of calculated ϵ_r'' with salinity, at 100 MHz and 2 GHz (other parameters as indicated).

anomalous effect in light of the behaviour at higher frequencies. It should be re-emphasized at this time that the theoretical model as postulated here includes: brine ionic conductivity effects, brine dielectric relaxation effects, and ice relaxation effects. The other mechanisms that may contribute to the dielectric constant at frequencies in the range 100 MHz to 1 GHz, such as: inclusion surface conductivity ($10^3 - 10^9$ Hz) and bound forms of water relaxation ($10^6 - 10^9$ Hz), have been ignored due to the excessive complication they would invoke on this first approximation model (see deLoor⁽¹³⁾ and Addison⁽⁴⁷⁾). Whether the neglect of these factors is or is not important in this model remains to be seen. There is very little choice as to whether or not to include them since the information necessary for their inclusion is simply not available at present, and the acquisition of this material comprises a whole study in itself. Therefore, their omission is justified here, but it should be kept in mind that they nonetheless have been neglected.

7.2.6.2 Comparison of the Theoretical Model and the Experimental Data

Figures 7.80 to 7.141 compare the calculated values of ϵ_r' and ϵ_r'' with the experimental values. Each set of curves has been evaluated at the exact temperature and salinity of the experimental points. The density for all the first-year ice samples has been assumed to be 0.91 gm/cm^3 . This was the average density of the first-year ice in the AIDJEX locale. The estimated error of this average density is $\pm .03 \text{ gm/cm}^3$. Most of the sample densities measured fell within this range. Because of this averaging of the density, the theoretical model will not be calculated at exactly the correct density for each of the points. This will lead to some deviation of the individual experimental points for ϵ_r' from the theoretical curves, i.e. the experimental point may be

as much as 0.3 above or below the theory curve. This trend is especially evident in Figures 7.83 to 7.85, 7.99 to 7.104, and 7.108 to 7.110.

The a/b ratio assumed is 20, and the model predictions for ϵ_r' and ϵ_r'' are calculated at six different ellipsoid orientation angles, θ . These angles are 0° , 10° , 20° , 35° , 45° and 90° . The angle θ is defined in Figures 7.80 and 7.111.

It has already been established that the theoretical model holds only at temperatures above -15°C . Therefore, only the experimental points obtained at temperatures above -15°C are compared with the theoretical values.

It should be immediately apparent to the astute observer that certain definite trends exist in the comparison. In most cases all the experimental points for both ϵ_r' and ϵ_r'' lie between the $\theta = 35^\circ$ and $\theta = 45^\circ$ curves. (A notable exception to be discussed shortly exists for the ϵ_r'' data at frequencies less than 400 MHz.)^{*} This general trend of the data tends to suggest that in the theoretical model the fictitious ellipsoids should all lie at angles between 35° and 45° to the vertical. One's judgement on this point, for the ϵ_r' data, is somewhat clouded by the effect of density variations and the close spacing of the contours for the smaller values of θ . Despite this minor complication, it is still clear that such a general trend as suggested above does occur.

The gently dropping curves obtained for ϵ_r' as a function of frequency, are easily understood in the light of the gradually decreasing dielectric constant of the brine.

The more complex ϵ_r'' curves demonstrate the competing brine conductivity and relaxation mechanisms. The relaxation of the brine

causes the bump in the curves between 100 MHz and 5 GHz. Its exact position will depend on the concentration of the brine and therefore the ice temperature. Its magnitude will depend a great deal on the volume of brine present in the ice, which is also a function of temperature and salinity. The role played by the brine ionic conductivity below 500 MHz is somewhat confusing. At these frequencies, the conductivity is rather large, but because it is so large and because it appears in both the numerator and denominator of the mixture equation its effect is partially cancelled, and a comparison of the ϵ_r'' curves for various temperatures and salinities shows that ϵ_r'' is relatively constant. The experimental values show no such constant behaviour - they are seen to diverge in an upward direction from the theoretical curves at the lower frequencies.

It is at this point that one must again recall the assumptions made in the model - certain effects active in the 100 MHz to 1 GHz frequency range have been neglected. These are the inclusion surface conductivity and the relaxation of bound water. If these effects were included better agreement might be obtained. Also the whole substructure of the model, i.e. the brine dielectric model, is based on extrapolations: to lower temperatures, to higher concentrations, and from NaCl solutions to brine. These assumptions definitely have a deleterious effect on the theoretical model, but they are absolutely essential in light of our present understanding of the brine dielectric behaviour.

However, it should be emphasized strongly that despite all the negative aspects and assumptions mentioned above, the theoretical model gives miraculously accurate results over the frequency range 100 MHz to 7.5 GHz, for ϵ_r' , and 400 MHz to 7.5 GHz, for ϵ_r'' . It predicts

the inclusions in first-year ice should lie at angles between 35° and 45° to the vertical - evidence has been presented in Chapter 3 which shows that the brine drainage networks contain a large number of sub-channels at angles between 40° and 54° to the vertical. It suggests an axial ratio of 20 for the inclusions. This may or may not occur in nature, but it is certainly a reasonable figure to assume. It correctly predicts the ϵ_r' and ϵ_r'' values, simultaneously, over the temperature range from -4°C to -15°C , and the salinity range from 5 to 10.5 ‰, at the minimum, and it includes a provision for density changes.

The model also outperforms expectations at the high frequency end of the spectrum by showing good agreement up to 7.5 GHz, without any appreciable scattering effects becoming apparent. The difference in trend between samples can primarily be attributed to natural small scale salinity and density fluctuations both in a particular sample, and between samples. The general diverging trend of the ϵ_r'' data at low frequencies is most likely explained by the aforementioned neglected mechanisms which are contributing to the measured values of ϵ_r'' . All in all, the simple mixture formula, equation (2.38), in the face of innumerable problems and despite the many complex interplays of variables, succeeds remarkably well in predicting the dielectric behaviour of the samples measured during the AIDJEX experiment. The next section explores the success of the theoretical model as a predictor of previously published dielectric measurements.

7.2.6.3 Comparison of the Theoretical Model with Other Published Data

Thus far, the theoretical model has succeeded in predicting the dielectric behaviour of the first-year sea ice measured by the author in the AIDJEX camp area of the Beaufort Sea. To remove any

Figure 7.80 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-1 ($S = 8.1\%$, $T = -4.94^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.81 Comparison of theoretical model (solid line) with experimental data for ϵ'_r , for AIDJEX sample FYI-1 ($S = 8.1\%$, $T = -10.19^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.82 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-1 ($S = 8.1\%$, $T = -14.55^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.83 Comparison of theoretical model (solid line) with experimental data for ϵ' ,
 for AIDJEX sample FYI-2 ($S = 9.1\%$, $T = -4.60^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$);
 angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.84 Comparison of theoretical model (solid line) with experimental data for ϵ'_r , for AIDJEX sample FYI-2 ($S = 9.1\%$, $T = -10.10^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.85 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-2 ($S = 9.1\%$, $T = -14.70^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.86 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-3 ($S = 5.6 \text{ ‰}$, $T = -14.70^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.87 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-4 ($S = 5.1\%$, $T = -5.10^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.88 Comparison of theoretical model (solid line) with experimental data for ϵ' for AIDJEX sample FYI-4 ($S = 5.1\%$, $T = -10.00^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.89 Comparison of theoretical model (solid line) with experimental data for ϵ' for AIDJEX sample FYI-4 ($S = 5.1 \text{ } \%$, $T = -14.80^\circ\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.90 Comparison of theoretical model (solid line) with experimental data for ϵ' for AIDJEX sample FYI-9 ($S = 10.5\%$, $T = -5.20^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.91 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-9 ($S = 10.5\text{‰}$, $T = -9.80^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.92 Comparison of theoretical model (solid line) with experimental data for ϵ'_r , for AIDJEX sample FYI-9 ($S = 10.5\%$, $T = -14.50^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.93 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-10 ($S = 9.3\%$, $T = -5.50^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.94 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-10 ($S = 9.3\%$, $T = -10.30^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.95 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-10 ($S = 9.3\%$, $T = -15.10^\circ\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.96 Comparison of theoretical model (solid line) with experimental data for ϵ''_r for AIDJEX sample FYI-14 ($S = 6.1\%$, $T = -4.50^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.97 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-14 ($S = 6.1\%$, $T = -10.00^\circ\text{C}$, $\rho = 0.91\text{ gm/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.98 Comparison of theoretical model (solid line) with experimental data for ϵ'_r , for AIDJEX sample FYI-14 ($S = 6.1\%$, $T = -14.80^\circ\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.99

Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDJEX sample FYI-18 ($S = 7.5\text{‰}$, $T = -14.15^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.100

Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDJEX sample FYI-18 ($S = 7.5\text{‰}$, $T = -9.21^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.101 Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDJEX sample FYI-18 ($S = 7.5\text{‰}$, $T = -3.99^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.102 Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDJEX sample FYI-19 ($S = 5.5\text{‰}$, $T = -14.19^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.103 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-19 ($S = 5.5\%$, $T = -9.20^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$).

Figure 7.104 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-19 ($S = 5.5\%$, $T = -3.94^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$).

Figure 7.105 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-20 ($S = 4.1\text{‰}$, $T = -14.08^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.106 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-20 ($S = 4.1\text{‰}$, $T = -9.44^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.107 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-20 ($S = 4.1\text{‰}$, $T = -4.06^\circ\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$).

Figure 7.108 Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-21 ($S = 4.1\text{‰}$, $T = -14.10^\circ\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$).

Figure 7.109

Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample. FYI-21 ($S = 4.1\text{‰}$, $T = -9.00^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.110

Comparison of theoretical model (solid line) with experimental data for ϵ' , for AIDJEX sample FYI-21 ($S = 4.1\text{‰}$, $T = -3.82^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$).

Figure 7.111 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-1 ($S = 8.1\%$, $T = -4.94^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.112 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-1 ($S = 8.1\%$, $T = -10.19^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure.7.113 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-1 ($S = 8.1\%$, $T = -14.55^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.114 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-2 ($S = 9.1\%$, $T = -4.60^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.115 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-2 ($S = 9.1\%$, $T = -10.10^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.116 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-2 ($S = 9.1\%$, $T = -14.70^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.117 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-3 ($S = 5.6\%$, $T = -14.70^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.118 Comparison of theoretical model (solid line) with experimental data for ϵ'_r , for AIDJEX sample FYI-4 ($S = 5.1\%$, $T = -5.10^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.119 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-4 ($S = 5.1\text{‰}$, $T = -10.00^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.120 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-4 ($S = 5.1\%$, $T = -14.80^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.121 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-9 ($S = 10.5\%$, $T = -5.20^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.122 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-9 ($S = 10.5\%$, $T = -9.80^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.123, Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDJEX sample FYI-9 ($S = 10.5\%$, $T = -14.50^\circ\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.124- Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-10 ($S = 9.3\%$, $T = -5.50^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.125 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-10 ($S = 9.3\%$, $T = -10.30^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.126 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-10 ($S = 9.3\%$, $T = -15.10^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.127 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-14 ($S = 6.1\%$, $T = -4.50^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.128 Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDVEX sample FYI-14 ($S = 6.1\%$, $T = -10.00^{\circ}\text{C}$, $\rho = 0.91 \text{ g/cm}^3$), angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.129 Comparison of theoretical model (solid line) with experimental data for ϵ_r'' , for AIDJEX sample FYI-14 ($S = 6.1\%$, $T = -14.80^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7-130 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-18 ($S = 7.5\%$, $T = -14.15^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$

Figure 7.131 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-18 ($S = 7.5\%$, $T = -9.21^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.132

Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-18 ($S = 7.5\%$, $T = -3.99^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.133 Comparison of theoretical model (solid line) with experimental data for ϵ_r'' , for AIDJEX sample FYI-19 ($S = 5.5\%$, $T = -14.19^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.134 Comparison of theoretical model (solid line) with experimental data for ϵ'' , for AIDJEX sample FYI-19 ($S = 5.5\%$, $T = -9.20^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.135 Comparison of theoretical model (solid line) with experimental data for ϵ_r'' , for AIDJEX sample FYI-19 ($S = 5.5\%$, $T = -3.94^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

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Figure 7.136 Comparison of theoretical model (solid line) with experimental data for ϵ_r' , for AIDJEX sample FYI-20 ($S = 4.1\text{‰}$, $T = -14.02^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.137 Comparison of theoretical model (solid line) with experimental data for ϵ_r'' , for AIDJEX sample FYI-20 ($S = 4.1\text{‰}$, $T = -9.44^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.138 Comparison of theoretical model (solid line) with experimental data for ϵ_r'' , for AIDJEX sample FYI-20 ($S = 4.1\%$, $T = -4.05^{\circ}\text{C}$, $\rho = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.139 Comparison of theoretical model (solid line) with experimental data for ϵ''_r for AIDJEX sample FYI-21 ($S = 4.1\%$, $T = -14.10^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.140 Comparison of theoretical model (solid line) with experimental data for ϵ''_r , for AIDJEX sample FYI-21 ($S = 4.1\%$, $T = -9.00^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

Figure 7.141 Comparison of theoretical model (solid line) with experimental data for ϵ_r'' , for AIDJEX sample FYI-21 ($S = 4.1\%$, $T = -3.32^{\circ}\text{C}$, $\bar{\rho} = 0.91 \text{ gm/cm}^3$); angle of ellipsoids with respect to vertical shown as a parameter; $a/b = 20$.

doubts about generality and to possibly extend the model further, a comparison was made with the work of Sackinger and Byrd⁽⁶⁴⁾, Vant et al⁽¹⁰⁾, Wentworth and Cohn⁽⁵⁰⁾, Cook⁽⁴⁹⁾, Bogorodsky and Khokhlov⁽⁶¹⁾ and Addison.⁽⁴⁵⁾ Bogorodsky and Tripol'nikov⁽⁶⁰⁾ only provide loss data which makes comparison of their values of ϵ_r' and ϵ_r'' with the model impossible. The data of Hoekstra and Cappillino⁽⁵⁹⁾ were also neglected in the comparison because, as already discussed in Chapter 3, there is considerable doubt as to how representative their measurements, performed on quickly frozen sea water contained in a coaxial line, are of the actual behaviour of natural sea ice.

There is a problem involved in comparing all this data: most of the authors do not give complete density or error bound information, and much of their data must be interpolated from plots of their results. Very few authors give tabular results. This is one of the reasons why the author has undertaken this extensive and systematic study.

Theoretical curves were generated over the frequency range of interest using an axial ratio of 20 and a value for θ of 45° . The density was always assumed to be 0.91 gm/cm^3 . Separate figures were prepared at salinities of 3 ‰, 5 ‰ and 8 ‰. Each contained a set of temperature curves for -4°C , -6°C , -10°C , -12°C and -14°C .

Figures 7.142 to 7.147 compare the measurements of Bogorodsky and Khokhlov⁽⁶¹⁾, Sackinger and Byrd⁽⁶⁴⁾, and Vant et al⁽¹⁰⁾ (the measurements presented in Chapter 4), to the theoretical model. The range of values inherent in the plotted data, from each source, is given in the figure. This range of value in density, salinity and temperature complicates the comparison. In general, the agreement between all sources of data is very good, even up to 40 GHz. In many cases, error

Figure 7.142 Comparison of theoretical model with other published data for ϵ_r , for frequencies between 1 and 40 GHz. Model calculated at $S = 3.0\%$, $\rho = 0.91 \frac{\text{g}}{\text{cm}^3}$, $\theta = 45^\circ$ and $a/b = 20$.

Figure 7.143 Comparison of theoretical model with other published data for ϵ' , for frequencies between 1 and 40 GHz. Model calculated at $S = 5.0\%$, $\rho = 0.91 \text{ gm/cm}^3$, $\theta = 45^\circ$ and $a/b = 20$.

Figure 7.144 Comparison of theoretical model with other published data for ϵ' , for frequencies between 1 and 40 GHz. Model calculated at $S = 8.0\%$, $\rho = 0.91 \text{ gm/cm}^3$, $\theta = 45^{\circ}$ and $a/b = 20$.

Figure 7.145 Comparison of theoretical model with other published data for ϵ_r'' , for frequencies between 1 and 40 GHz. Model calculated at $S = 3.0\%$, $\rho = 0.91 \text{ gm/cm}^3$, $\theta = 45^\circ$ and $a/b = 20$.

Figure 7.146 Comparison of theoretical model with other published data for ϵ_r'' , for frequencies between 1 and 40 GHz. Model calculated at $S = 5.0\%$, $\rho = 0.91 \text{ gm/cm}^3$, $\theta = 45^\circ$ and $a/b = 20$.

Figure 7.147 Comparison of theoretical model with other published data for ϵ_r'' , for frequencies between 1 and 40 GHz. Model calculated at $S = 8.0\%$, $\rho = 0.91 \text{ gm/cm}^3$, $\theta = 45^{\circ}$ and $a/b = 20$.

bounds for ϵ_r were not given so that the true extent of the agreement is hard to estimate. In most cases, especially for the few deviant points, due to Bogorodsky and Khokhlov⁽⁶¹⁾, the disagreement between values was due mostly to the natural scatter in the points (i.e. salinity and density variations), and the fact that the temperatures the experimental points were measured at do not correspond exactly to the values used in the theoretical curve calculation. This is a necessary evil, which cannot be avoided if one is to have a graphical presentation at all.

In summary, the fact that the model showed such good agreement, especially between 34 and 40 GHz, is proof that for normal incidence, in high density first-year sea ice, scattering from brine inclusions is not a major problem even at 40 GHz.

Figures 7.148 and 7.149 show a comparison of the data of Wentworth and Cohn⁽⁵⁰⁾, Cook⁽⁴⁹⁾ and Addison⁽⁴⁵⁾ with the theory (the same parameters as before are employed). Addison⁽⁴⁵⁾ is the only one of the three that does not employ true sea ice. He uses artificial sea ice grown in a 7.6 cm. diameter, 50 cm. long lucite tube, fitted with chambers segmented by gold plated mesh, and submerged in a deep 30 cm. diameter tank. Such a tank, as he himself notes, can cause significant horizontal growth, although he states that the samples were chosen carefully to avoid this problem.

It has been noted that artificial sea ice samples, exceeding 15 cm. in depth, grown in the author's facilities exhibited a high concentration of brine in the lower part of the ice sheet and also showed evidence of horizontal growth. In this facility, the overall growth tank is approximately 150 cm. in diameter by 70 cm. deep, and is well

Figure 7.148 Comparison of theoretical model with other published data for ϵ'' , for frequencies between 100 KHz and 1 GHz (parameters as indicated).

Figure 7.149 Comparison of theoretical model with other published data for ϵ'' , for frequencies between 100 KHz and 1 GHz (parameters as indicated).

insulated to allow isothermal growth. But, as it has already been discussed in Chapter 3, unless the brine under the growing ice sheet is recirculated, it becomes highly concentrated as the ice sheet gets thicker and this concentration changes the ice characteristics considerably.

It could possibly be that Addison's⁽⁴⁵⁾ samples were not representative of first-year sea ice. But instead, because of his growth procedures, they possessed horizontal brine inclusions.

Figure 7.148 shows that the theoretical model correctly predicts the ϵ_r' data of Cook⁽⁴⁹⁾ and Wentworth and Cohn⁽⁵⁰⁾, but gravely underestimates the values obtained by Addison.⁽⁴⁵⁾ A comparison with the theoretical prediction, for ellipsoids at 90° to the vertical, is also shown. It at least approaches Addison's⁽⁴⁵⁾ data. Without more data, little else can be said about the ϵ_r' behaviour at these frequencies.

The data obtained for ϵ_r' in this study, as has already been demonstrated, agrees very well with the theoretical predictions, assuming the ellipsoids are at 45° . This would tend to indicate that Addison's⁽⁴⁵⁾ data, at least at 100 MHz, does not represent true sea ice behaviour.

Figure 7.149 shows the predicted values for ϵ_r'' over the same frequency range as above. It is obvious, both from what was seen in Section 7.2.6.2 and the data of Cook⁽⁴⁹⁾ (and possibly Addison⁽⁴⁵⁾), that the theoretical model falls far short of predicting the correct behaviour for ϵ_r'' below 400 MHz. This is probably due to the aforementioned shortcomings of the brine dielectric model, and the neglect of the other mechanisms contributing to ϵ_r'' . Therefore, the suspicions outlined in Section 7.2.6.2 are simply re-affirmed here, and it is obvious that restrictions must be placed on the low frequency use of the theoretical model.

7.3

Comparison of the Theoretical and Empirical Models

It should already be obvious that the experimental data (of this study) and the theoretical model agree very well over the frequency range 400 MHz to 7.5 GHz, but how do the empirical model, presented in Section 7.1, and the theoretical model compare?

Figures 7.150 to 7.153 attempt to answer this question. The theoretical curves are calculated for $a/b = 20$, $\theta = 45^\circ$, $\rho = .91 \text{ gm/cm}^3$, and $S = 5.1$ or 10.5% . The empirical estimates, based on the linear regression analysis of ϵ_r' and ϵ_r'' versus the brine volume fraction (v) are shown as circles for the natural first-year ice and as squares for the artificial sea ice. The brine volume fraction v has been calculated from the temperature and salinity, and used to obtain ϵ_r' and ϵ_r'' . This technique has the drawback that more than one combination of temperature and salinity produce the same brine volume, and hence ϵ_r' or ϵ_r'' ; whereas the theoretical model produces unique values of ϵ_r' and ϵ_r'' , dependent mostly on the temperature and only to a lesser extent on the salinity. Some of these values may overlap, but not to the same extent as for the empirical models. This slight inconsistency, coupled with the fact that the extreme points in any averaging operation tend to strongly affect the mean value, will cause the empirical estimates to deviate more from the theoretical curves than the actual experimental points did.

It can be seen from Figures 7.150 to 7.153 that the two empirical estimates agree very well, and therefore it can be assumed that the particular type of artificial sea ice used is very representative of natural, high density, first-year sea ice. This implies that this type of laboratory grown sea ice can be used at will, in future dielectric property studies of first-year sea ice.

Figure 7.150 Comparison of theoretical and empirical models for ϵ'_r of sea ice, over the frequency range 100 MHz to 7.5 GHz, for $S = 5.1 \text{ ‰}$. (parameters as indicated).

Figure 7.151 Comparison of theoretical and empirical models for ϵ' of sea ice, over the frequency range 100 MHz to 7.5 GHz, for $S = 10.5 \text{‰}$. (parameters as indicated).

Figure 7.152 Comparison of theoretical and empirical models for ϵ'' of sea ice, over the frequency range 100 MHz to 7.5 GHz, for $S = 5.1 \text{‰}$. (parameters as indicated).

Figure 7.153 Comparison of theoretical and empirical models for ϵ'' of sea ice, over the frequency range 100 MHz to 7.5 GHz, for $S = 10.5\text{‰}$ (parameters as indicated).

It can also be seen from these figures that the ϵ_r' values predicted by the theory are larger than those predicted by the empirical models. This is due to both the choice of $\theta = 45^\circ$, and the density differences between samples. For the detailed curves in Section 7.2.6.2 it was seen that the points lay between 35° and 45° in most cases, but in some cases they were below the 35° line by a value of about .2 to .3. This is more than likely due to the difference between the assumed density and the actual sample density.

The ϵ_r'' values plotted in Figures 7.152 and 7.153 deviate from the theoretical curves below 200 MHz. This is no surprise in view of the discussion presented in the past few sections.

In summary, it has been shown that the artificial and natural first-year sea ice empirical models exhibit substantially the same behaviour within the standard errors of the estimate, and that the theoretical curves predict the same trends as the empirical models, above 400 MHz. The absolute magnitudes predicted by theory are generally larger for ϵ_r' by 0.25 to 0.50, and about the same (within $\pm .1$) for ϵ_r'' .

Considering the fact that natural, localized salinity and density variations (which can be as large as $\pm .2\%$ and $\pm 0.03 \text{ gm/cm}^3$ respectively) can cause changes in ϵ_r' of about $\pm .4$ and in ϵ_r'' of about $\pm .15$, these small differences between the models are not very serious.

7.4. Extension of the Models to Multiyear Sea Ice

Thus far, the entire modelling effort has been directed towards the relatively uncomplicated (compared to multiyear ice) first-year ice. Figures 7.154 to 7.157 attempt to demonstrate the extension of the theoretical model to multiyear ice. Multiyear ice is plagued by rapid variations in density, associated with the large air bubbles it

Figure 7.154 Comparison of theoretical model and experimental data for ϵ' of multiyear sea ice sample MYI-13 ($S = 1.32\%$, $\bar{\rho} = 0.90 \text{ gm/cm}^3$, other parameters as indicated).

Figure 7.155 Comparison of theoretical model and experimental data for ϵ' of multiyear sea ice sample MYI-15 ($S = 1.75\%$, $\rho = 0.90 \text{ gm/cm}^3$, other parameters as indicated).

Figure 7.156 Comparison of theoretical model and experimental data for ϵ_r'' of multiyear sea ice sample MYI-13 ($S = 1.32\%$, $\rho = 0.90 \text{ gm/cm}^3$; other parameters as indicated).

Figure 7.157 Comparison of theoretical model and experimental data for ϵ'' of multiyear sea ice sample MYI-15 ($S = 1.75\%$, $\rho = 0.90 \text{ gm/cm}^3$; other parameters as indicated).

contains. These density fluctuations coupled with its very low brine content tend to make any hard and fast comparisons of theory and experiment difficult. The shape of the brine inclusions in multiyear ice is not well understood. Therefore, for want of a better choice, and based on the conclusions of the preliminary results, the theoretical model assumes they are spherical. This implies there will be no variation in the dielectric properties with angle.

Because of the low brine content, the properties of multiyear ice do not vary appreciably with temperature, and the measured values of ϵ_r' and ϵ_r'' are quite small. Therefore, since the experimental error is still as large as it was for first-year ice, but the values being measured are smaller, the results are more scattered. This is definitely seen to be the case in Figures 7.154 to 7.157. No positive conclusions can be made. It appears that the theoretical model does predict the correct values for both ϵ_r' and ϵ_r'' over the whole 100 MHz to 4 GHz frequency range, but other curves could also be made to fit the data.

It is doubtful whether the theoretical predictions would be accurate above 4 GHz, due to the larger amount of scattering that occurs as the wavelength shrinks. Without more data points, over a wide range of densities, and measured to much higher precision, little more can be said.

The empirical models for multiyear ice given in Section 7.1 also contain too few points for any conclusions to be drawn.

There is some doubt as to whether a theoretical model of the type presented for first-year sea ice would have any application to multiyear sea ice at all, since the natural variation in the ice density and salinity, even from cm. to cm., is so great that such predictive

models would have little use. Rather, a bulk measurement approach based on the scattering from random inhomogeneities, as discussed in Chapter 2, or a bulk backscattering measurement, may be more applicable. In these two cases, the ice would only be described statistically, in terms of the relative deviations in dielectric constant, the bulk mean dielectric constant and the correlation lengths of the inhomogeneities. Conceivably different multiyear ice samples, e.g. hummocks and melt-ponds, would have vastly different statistical descriptions.

It is essential to realize, as well, that any attempt to measure the dielectric properties of multiyear sea ice, at microwave frequencies, is necessarily a scattering measurement, since the porous nature of the ice, and the large air bubbles it contains, prevent the use of simple quasi-static mixture formulas (except below approximately 1.5 GHz). Therefore, it is advisable that any future measurement programs directed toward the evaluation of the effective dielectric constant of multiyear sea ice bear these factors in mind.

7.5

Limitations and Applicability of the Models

The previous sections have described in detail the various models and examined their strengths and weaknesses. It has become clear that the theoretical model performs better than expected, and quite accurately predicts both ϵ_r' and ϵ_r'' for first-year sea ice, over the frequency range 400 MHz to 40 GHz, with no appreciable problems due to scattering. This model incorporates the properties of the constituents of the sea ice and allows prediction of the dielectric properties of the ice from simple physical measurements of salinity, temperature and density. It probably satisfies all requirements for such a sea ice model, since the errors inherent in it are due primarily to the natural variability of the sea ice, which one can never eliminate completely.

A remote sensing ground truth party equipped with such a model has a powerful tool at their disposal. It can be used to calculate such things as: 1) the sea ice brightness temperature for microwave radiometry; 2) the bulk dielectric constant for a profiled layer of first-year sea ice; and 3) the loss expected for propagation through a given thickness of ice at any particular frequency. These calculations can both supplement and complement the usual physical property measurements made by ground truth parties.

The model also allows predictions to be made about the performance of various remote sensing instruments before they are actually used. It may be incorporated into larger models that require dielectric information about the ice. It also provides a further insight into the dielectric behaviour of sea ice for the sole satisfaction of scientific curiosity.

Unfortunately, the model is only as good as the information fed into it, which in most cases, because of the high degree of variability of the ice, is not very good. It fails to predict the correct dielectric behaviour at frequencies below 400 MHz. But this is because not enough is known about the mechanisms that contribute to the dielectric behaviour at these frequencies. Similarly, it is impossible to say how good a predictor the model is for multiyear sea ice, because of the relatively small quantity of data available.

The empirical models provide a good summary of the dielectric information obtained both in the AIDJEX area about first-year sea ice, and in the laboratory about artificial sea ice. They allow predictions to be made about the dielectric behaviour of the ice at the particular frequencies and densities measured, and they illustrate the usefulness of artificial sea ice measurements. Their major drawbacks

are their limited scope, which prevents generalization, and their general masking of the underlying behaviour. Despite these drawbacks, they are still useful for preparing comparisons such as Figure 7.158 (a revised version of Figure 3.17). Figures such as this allow one to predict the absorption loss of sea ice (in db/m) at the frequencies shown.

The information contained in both the theoretical and empirical models is a significant contribution to our knowledge about sea ice dielectric behaviour. Furthermore, this contribution, despite its inherent drawbacks, represents, for most purposes, a sufficiently accurate solution to the measurement objectives outlined in Chapter 1.

Figure 7.158 Summary of loss data (in dB/m) versus frequency, data taken at temperatures between -7°C and -12°C , and salinities between 5 ‰ and 10 ‰.

CHAPTER 8. CONCLUSIONS

8.1 Summary

This thesis has presented both a thorough and systematic investigation into the dielectric behaviour of first-year sea ice, over the frequency range 100 MHz to 40 GHz; and a preliminary investigation into the behaviour of multiyear sea ice. The following summary outlines both of these studies and describes briefly the major highlights.

An investigation into the theory of dielectric mixtures produced some useful formulas, which could be applied to the sea ice mixture problem. The frequency range of applicability and the inherent assumptions about conductivity that are made in the derivation of these formulas were investigated. Modifications were made to the equations to allow for tilted ellipsoids, and arbitrary azimuthal orientation of the ellipsoids. It was determined that the measurement system employed, in particular the "coaxial-cage" line, produced results applicable only to normally incident plane waves.

Preliminary measurements on fresh and sea ice at 10 and 30 to 34 GHz, allowed evaluation of several historical mixture formulas. As well, the basic problems involved in sea ice measurements, such as sample preparation and storage, were explored. It was found that the cruder dielectric mixture formulas allowed a reasonably good prediction of the dielectric behaviour of ϵ_r' and ϵ_r'' to be made on an individual basis, but were not successful in predicting their combined behaviour. The dielectric properties of sea ice were found to vary appreciably with age, salinity and temperature. An increase in temperature and salinity caused a similar increase in the measured value of ϵ_r' and ϵ_r'' , whereas

an increase in age caused a decrease in these values. Also, some hints of possible complications that might arise from scattering were seen in the multiyear sea ice measurements at 10 GHz. In addition, the worth of artificial sea ice measurements was established and a spherical brine inclusion shape for frazil first-year sea ice was suggested.

These preliminary measurements provided the stimulus for the building of a wideband "coaxial-cage" sample holder. This system covered the frequency range 100 MHz to 7.5 GHz. A complete error analysis was performed on the entire system and a semi-empirical correction factor was derived for the "coaxial-cage" sample holder measurements. The primary errors were found to be due to the short electrical length of the sea ice sample, which for practical reasons could not exceed 8 cm. in length. This short electrical length was found to be especially critical at 100 MHz, where the sample length was at most $\lambda/24$.

The total measurement system was transported to the AIDJEX site, on the Beaufort Sea, in the spring of 1975. At this time a comprehensive series of dielectric measurements were performed on first-year sea ice samples over a wide range of salinities (5.1 to 10.5 ‰), temperatures (-5°C to -40°C), and frequencies (100 MHz to 7.5 GHz). A somewhat less comprehensive set of measurements was made on multiyear sea ice samples.

These measurements proved highly successful, and were the first of their kind to be performed at these frequencies. They allowed the formulation of an empirical model for the sea ice dielectric behaviour. This model can be used to calculate ϵ_r' , ϵ_r'' , or the loss in dB/m, at any of the discrete frequencies measured. However, it was found to have two major drawbacks: it was not general enough with respect to density differences, and it only applied to the specific fre-

quencies measured.

The dielectric data obtained in this study and by other authors allowed a check to be made on the theoretical model that has been derived from the dielectric mixture equations. This comparison showed the theoretical model to be accurate enough to correctly predict ϵ_r' and ϵ_r'' over the whole frequency range 400 MHz to 40 GHz. No appreciable deviations from theory due to scattering losses were observed, even at 40 GHz.

Artificial sea ice measurements were also performed in the laboratory, over the 100 MHz to 4 GHz frequency range, on samples taken from the ice layer at angles of 0° , 30° , 45° , 60° and 90° to the vertical. These measurements re-affirmed the belief that the sea ice structure was far from uniform, but failed to predict the effective dielectric constant that would be seen by an impinging plane wave at non-normal incidence. However, they suggested that the effective dielectric constant increases with increasing deviation from non-normal incidence.

Comparison of the empirical models, constructed with the 0° (angle) artificial sea ice data, and the first-year sea ice data, with the theoretical model, showed reasonably good agreement within the limits imposed by the natural variability of the ice. The extension of the models to multiyear sea ice was investigated, but it was found that insufficient data was available to make any real conclusions. Comments were given on the possible methods by which the multiyear ice modelling problem may be solved. In addition, various remote sensing applications of the models were discussed. In particular, it was noted that the strong variations in ϵ' and ϵ'' with temperature and salinity, at the lower frequencies and the large losses at the higher frequencies may

cause difficulties in the interpretation of data obtained with an ice profiling radar.

In summary, it was found that this study, despite the complexity of the problems it faced, resulted in reasonable, and logical models for sea ice dielectric behaviour, which can easily be applied to the investigation of various sea ice/electromagnetic wave interactions.

8.2 Recommendations for Future Research

This study is not a comprehensive treatise on all aspects of sea ice dielectric behaviour - it is only a beginning. As a result there are a multitude of future measurements that can be made. In this section, an attempt will be made to point out the higher priority items.

If this type of measurement is to be applied successfully to the interpretation of remote sensing imagery, it will be necessary to know how ϵ_1' and ϵ_1'' vary with the incidence angle of the probing radiation. It is recommended that this information be obtained by means of wideband measurements employing a transmission line with a linear, not radial, field distribution. One possible candidate for such a line is the parallel-plate transmission line. Samples could be cut at various angles to the vertical and placed in this line, in much the same way as the artificial sea ice samples were placed in the "coaxial-cage" line in this study. There would be one major difference: the generalization from the wave propagating in the sample holder to a linearly polarized plane wave propagating through an actual sea ice sheet would now be possible. However, these measurements may be hampered severely by: 1) problems in sample preparation, and 2) the relatively low, upper frequency limit imposed by the parallel-plate line.

The theoretical model could be improved by the acquisition of higher quality information about the dielectric behaviour of the

brine. It is suggested that a complete series of dielectric measurements be made on highly concentrated brine solutions, at temperatures below 0°C . These measurements may allow further improvement of the model in the 100 MHz to 400 MHz frequency range. More precise dielectric measurements on sea ice in the 100 MHz to 400 MHz range would also aid the modelling procedure. It may be possible to build more precise narrowband bridges in this frequency range, to enable such measurements to be obtained. It may even be desirable to extend the lower frequency limit of the measurements to 100 KHz. This would allow the acquisition of experimental data which could be used to evaluate the role played by interfacial polarization and inclusion surface conductivity effects.

The extension of the model to multiyear sea ice should be attempted. It is recommended that any such attempt be accompanied by a detailed photomicrograph analysis of the microstructure of multiyear sea ice, i.e. the size, shape, and distribution of both air bubbles and brine inclusions should be investigated. Any measurements of multiyear ice should be made so as to encompass a large variety of samples of different densities and salinities. However, it is difficult to see at present just how the measurements on multiyear sea ice could best be accomplished. Perhaps controlled backscattering measurements on bulk samples may contribute more to the understanding of multiyear ice, than detailed dielectric measurements alone.

It should be emphasized that any future study should try, as much as possible, to obtain very accurate density and salinity information on the actual sample measured. If the samples are sectioned after the dielectric measurements are finished, and detailed measurements of

the density and salinity profiles are made, a useful check will be available on the uniformity of the sample.

The suggestions and safeguards outlined here should allow future workers to improve the prediction capability of the theoretical model beyond the "first attempt" model given in this thesis. Very careful density, salinity and temperature control of the samples should remove some of the natural variability noted so often here, but it remains to be seen whether or not the complex problem of extending the model to multiyear sea ice can be solved.

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APPENDIX A. THE QUASI-STATIC SOLUTION FOR NON-ZERO CONDUCTIVITY

The extension of equation (2.38) derived for dielectric mixtures, to include conductivity effects (σ) and to take account for the non-static case is not straightforward.

First, one must examine what is meant by ϵ . Maxwell's equations for sinusoidally time-varying fields are

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad (\text{A.1})$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} + \vec{J} \quad (\text{A.2})$$

$$\nabla \cdot \vec{D} = \rho \quad (\text{A.3})$$

$$\nabla \cdot \vec{B} = 0 \quad (\text{A.4})$$

$$\nabla \cdot \vec{J} = -j\omega\rho \quad (\text{A.5})$$

where ρ is true charge, the effect of the polarization charge is already included in $\vec{D}$. Following the treatment of Plonsey and Collin¹, it is apparent that when time-varying fields are applied to material bodies, the polarization vectors $\vec{P}$ and $\vec{M}$ vary with time at the same frequency as the applied fields. Due to damping forces which are always present to some extent, $\vec{P}$ and $\vec{M}$ will lag $\vec{E}$ and $\vec{H}$. Therefore, in general ϵ and μ must be complex. For example, let

$$\vec{P} = \epsilon_0 \alpha e^{-j\phi} \vec{E} \quad (\text{A.6})$$

where α is a positive real constant and ϕ is the phase angle by which $\vec{P}$ lags $\vec{E}$.

The susceptibility is given by

$$\chi_e = \alpha e^{-j\phi} \quad (\text{A.7})$$

and hence the dielectric permittivity ϵ is given by

$$\begin{aligned} \epsilon &= \epsilon_0 (1 + \chi_e) = \epsilon_0 (1 + \alpha \cos\phi - j\alpha \sin\phi) \\ &= \epsilon' - j\epsilon'' \end{aligned}$$

¹ R. Plonsey and R.E. Collin, Principles and Applications of Electromagnetic Fields, McGraw-Hill Book Co., New York, 1961.

and thus is a complex number.

If in Maxwell's equations ϵ is complex, $\nabla \cdot \vec{H}$ becomes

$$\nabla \times \vec{H} = j\omega\epsilon' \vec{E} + (\omega\epsilon'' + \sigma) \vec{E} \quad (\text{A.9})$$

where $\vec{J}$ has been replaced by $\sigma \vec{E}$ and the imaginary part ϵ'' is equivalent to an increase in conductivity. Frequently, for convenience this net effect is lumped into

$$\epsilon_c = \epsilon' - j\epsilon'' - j\frac{\sigma}{\omega} \quad (\text{A.10})$$

Helmholtz' equation in dielectric and conducting media becomes

$$\nabla^2 \vec{E} - j\omega\mu(j\omega\epsilon + \sigma) \vec{E} = 0 \quad (\text{A.11})$$

where ϵ is in general complex, and k'^2 the propagation constant, is

$$k'^2 = -j\omega\mu(j\omega\epsilon + \sigma) \quad (\text{A.12})$$

In equation (A.11) the term $j\omega\epsilon \vec{E}$ is the displacement current density and $\sigma \vec{E}$ is the conduction current density.

Having established the definitions of the basic constants it is now possible to examine the extension of equation (2.38) to include the effects of conductivity. In the static case ϵ'' is zero since $\sin\phi$, the phase shift in equation (A.8), does not exist. Therefore, the only term contributing to the conduction is σ . The boundary conditions (2.25) assume

$$\begin{aligned} \vec{D} &= \epsilon \vec{E} \\ \text{and} \quad \nabla \phi &= -\vec{E} \end{aligned} \quad (\text{A.13})$$

Plonsey and Collin¹ state that due to the duality of $\vec{J}$ and $\vec{D}$ in the electrostatic case, i.e. they are both proportional to $\vec{E}$, although the constant of proportionality varies, a solution for $\vec{D}$ is also a solution for $\vec{J}$, and therefore also a solution for $\vec{C}_1 \vec{J} + \vec{D}$,

¹ Plonsey and Collin, Principles and Applications of Electromagnetic Fields, 1961.

where C_1 is a constant. This implies that equation (2.38) is also a solution for σ or $\epsilon' + C_1\sigma$. Therefore, it can be seen that equation (2.38) is easily extended to cover conductivity in the static case.

When a sinusoidal variation in the various fields is assumed, Maxwell's equations become (A.1) to (A.5). To solve the boundary value problem in general, one can no longer invoke equation (2.25) as the boundary values. Instead, the scalar and vector potentials are usually used to find $\vec{E}$ and $\vec{H}$. The vector potential $\vec{A}$, is given by

$$\vec{B} = \nabla \times \vec{A} \quad (\text{A.14})$$

Substitution in (A.1) gives

$$\nabla \times \vec{E} = -j\omega \nabla \times \vec{A} \quad (\text{A.15})$$

and

$$\nabla \times (\vec{E} + j\omega \vec{A}) = 0 \quad (\text{A.16})$$

(A.16) is irrotational, therefore it may be derived from the gradient of a scalar potential $\nabla\phi$, i.e.

$$\vec{E} = -j\omega \vec{A} - \nabla\phi \quad (\text{A.17})$$

and $\vec{E} = -\nabla\phi$ is no longer true. Substitution in (A.2) for $\vec{A}$ gives

$$\begin{aligned} \nabla \times \vec{H} &= \frac{1}{\mu} \nabla \times \nabla \times \vec{A} = j\omega\epsilon \vec{E} + \vec{J} \\ &= j\omega\epsilon (-\nabla\phi - j\omega \vec{A}) + \vec{J} \end{aligned} \quad (\text{A.18})$$

Expansion of $\nabla \times \nabla \times \vec{A}$ to give $\nabla\nabla \cdot \vec{A} - \nabla^2 \vec{A}$, and setting of $\nabla \cdot \vec{A} = -j\omega\epsilon\mu\phi$ (the Lorentz condition) gives

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J} \quad (\text{A.19})$$

where $k^2 = \omega^2\mu\epsilon$ and ϵ can be complex. Application of $\nabla \cdot \vec{D} = \rho$ and the replacement of $\vec{E}$ by (A.17) gives

$$\begin{aligned} \nabla \cdot \vec{E} &= \nabla \cdot (-j\omega \vec{A} - \nabla\phi) \\ &= -\nabla^2 \phi - k^2 \phi = \frac{\rho}{\epsilon} \end{aligned} \quad (\text{A.20})$$

This yields

$$\nabla^2 \phi + k^2 \phi = -\frac{\rho}{\epsilon} \quad (\text{A.21})$$

This equation yields ϕ and then the Lorentz condition can be used to get $\vec{A}$. Integration of (A.20) and (A.21) leads to the solutions

$$\vec{A}(x,y,z) = \frac{\mu}{4\pi} \int_V \frac{\vec{J}(x',y',z')}{R} e^{-jkR} dv' \quad (\text{A.22})$$

$$\phi(x,y,z) = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(x',y',z')}{R} e^{-jkR} dv' \quad (\text{A.23})$$

where (x,y,z) are the field points, (x',y',z') are the source points, $R = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}$, and $k^2 = \omega^2 \mu \epsilon$ where ϵ can be complex, but σ , the conductivity, is treated separately in $\vec{J}$.

If kR is constrained such that $kr \ll 1$, or $\frac{2\pi}{\lambda} \sqrt{\epsilon_r} R \ll 1$, (the quasi-static solution, i.e. the sources, dipoles, or inclusions, in the case in mind here, are small compared to a wavelength), which implies R is small, then e^{-jkR} becomes unity and (A.22) and (A.23) reduce to

$$\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(x',y',z')}{R} dv' \quad (\text{A.24})$$

and

$$\phi = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(x',y',z')}{R} dv' \quad (\text{A.25})$$

In other words, when propagation effects are negligible, the potentials reduce to the static potentials. Thus, $\vec{E}$ again becomes $-\nabla\phi$ by definition of the electrostatic field, and one can again apply the boundary conditions (2.25), except with one important difference. In the equation $\vec{D} = \epsilon \vec{E}$, the ϵ now is $\epsilon_c = \epsilon' - j\epsilon'' - j\frac{\sigma}{\omega}$, where the duality of $\vec{J}$ and $\vec{D}$ in the static case has again been invoked to lump ϵ' , ϵ'' and σ together. The ϵ'' term also is non-zero now. But, if the conditions of long wavelength are not met, none of the above is true

and the mixture equations (2.38) do not hold.

It should also be apparent that the result of any application of equation (2.38), or any measurement of ϵ , will yield a complex dielectric constant $\epsilon = \epsilon' - j\epsilon_N''$ (ϵ_N'' may or may not be zero, depending on the constituents). The ϵ_N'' term lumps together the net effect of phase lag due to conductivity and dielectric origins. It is impossible in any mixture scheme of this sort to separate the resulting imaginary part of ϵ into σ and ϵ'' , unless a priori knowledge is obtained about the frequency dependence of one or the other, which is usually not the case. Henceforth, it will be understood that measured values of ϵ will contain σ in ϵ_N'' but the subscript N will be dropped for convenience.

APPENDIX B. SOLUTION FOR $G_1(\xi)$

G_1 is a solution of the second order differential equation

$$\frac{1}{R_\xi} \frac{dR_\xi}{d\xi} \left(\frac{dG_1}{d\xi} \right) + \frac{d^2 G_1}{d\xi^2} - \frac{C_3}{R_\xi^2} G_1 = 0 \quad (B.1)$$

If the substitutions

$$a_1(\xi) = \frac{1}{R_\xi} \frac{dR_\xi}{d\xi} \quad (B.2)$$

and

$$a_2(\xi) = \frac{-C_3}{R_\xi^2} \quad (B.3)$$

are made, then the equation becomes

$$\frac{d^2 G_1}{d\xi^2} + a_1(\xi) \frac{dG_1}{d\xi} + a_2(\xi) G_1 = 0 \quad (B.4)$$

The equation is homogeneous and has one solution $F_1(\xi) = v(\xi)$.

Therefore, the substitution

$$G_1(\xi) = v(\xi)w(\xi) \quad (B.5)$$

can be made.

Differentiation of $G_1(\xi)$ yields

$$\frac{dG_1(\xi)}{d\xi} = v \frac{dw}{d\xi} + w \frac{dv}{d\xi} \quad (B.6)$$

and

$$\frac{d^2 G_1(\xi)}{d\xi^2} = v \frac{d^2 w}{d\xi^2} + 2 \frac{dv}{d\xi} \frac{dw}{d\xi} + \frac{d^2 v}{d\xi^2} w \quad (B.7)$$

If one lets the operator L denote the differentiation in (B.4), then

substitution of (B.6) and (B.7) into (B.4) gives

$$L(v)w + a_1(\xi) v \frac{dw}{d\xi} + v \frac{d^2 w}{d\xi^2} + 2 \frac{dv}{d\xi} \frac{dw}{d\xi} = 0 \quad (B.8)$$

since v is a solution of the homogeneous equation $L(v) = 0$ and

$$a_1(\xi) v \frac{dw}{d\xi} + v \frac{d^2 w}{d\xi^2} + 2 \frac{dv}{d\xi} \frac{dw}{d\xi} = 0 \quad (B.9)$$

This is just

$$\frac{d}{d\xi} \left(v^2 \frac{dw}{d\xi} \right) + a_1(\xi) \left(v^2 \frac{dw}{d\xi} \right) = 0 \quad (B.10)$$

The substitution $z = v^{\frac{2dw}{d\xi}}$ simplifies (B.10) further to

$$\frac{dz}{d\xi} + a_1(\xi)z = 0 \quad (\text{B.11})$$

which is easily solved using an integrating factor. The solution for z is

$$z = C_4 \exp \left(- \int \frac{1}{R_\xi} \frac{dR_\xi}{d\xi} d\xi \right) = \frac{C_4}{R_\xi} \quad (\text{B.12})$$

where $a_1(\xi)$ has been replaced by (B.2). This is just $v^{\frac{2dw}{d\xi}}$. Integration one more time and simplification of the exponential gives w as

$$w = C_4 \int \frac{1}{R_\xi F_1^2(\xi)} d\xi + C_5 \quad (\text{B.13})$$

Therefore, by (B.5), $G_1(\xi)$ is simply

$$G_1(\xi) = C_4 F_1(\xi) \int \frac{1}{R_\xi} \frac{1}{F_1^2(\xi)} d\xi + C_5 F_1(\xi) \quad (\text{B.14})$$

APPENDIX C

PLANS FOR "COAXIAL-CAGE"

LINE AND DRILL JIG

(Plans are also given for the original prototype model of the line which had a two-piece inner conductor (page 417 and 418). This experimental model produced a non-50 Ω impedance line. The final inner conductor structure is given on page 421.)

DEPARTMENT OF
ENERGY, MINES AND RESOURCES
INLAND WATERS BRANCH

PROJECT: COAXIAL CAGE
SAMPLE HOLDER

SHEET
1 OF
FILE NO.

COMPUTED DATE CHECKED DATE

DETAIL: END SUPPORT - I
MAT'L: BRASS

SECTION A-A

NOTES

1. 4 - HOLES TO MAKE CLOSE SLIDING FIT WITH A .24425 ϕ ROD.
2. 3 - 6-32 TAPPED HOLES 0.50 DEEP.
3. ALL DIMENSIONS IN INCHES.
4. ALL SURFACES SMOOTH AND FLAT.

M. YANT

DEPARTMENT OF
ENERGY, MINES AND RESOURCES
INLAND WATERS BRANCH

PROJECT: COAXIAL-CAGE
SAMPLE HOLDER
DETAIL: END SUPPORT-2
MAT'L: BRASS

SHEET
2 OF
FILE NO.

SECTION B-B

NOTES

1. 4 HOLES - .50 DEEP (FROM BACK) TO MAKE CLOSE SLIDING FIT WITH .24125 ϕ ROD, .50 DEEP (FROM FRONT) NO. 6 CLEARANCE HOLES WITH .25 DEEP .266 CORE.
2. 3 - 6-32 TAPPED HOLES .50 DEEP.
3. ALL DIMENSIONS IN INCHES.
4. ALL SURFACES SMOOTH AND FLAT.

QTY-1
MATL- BRASS

M. VANT
JAN. 75

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INLAND WATERS BRANCH

PROJECT: COAXIAL-CAGE
SAMPLE HOLDER
DETAIL: OUTER CONDUCTOR RODS
MAT'L - BRASS

SHEET

3 OF
FILE NO.

COMPUTED	DATE	CHECKED	DATE

NOTES

1. 4 RODS TO BE CUT FROM STOCK PROVIDED AS PER MACHINING INSTRUCTIONS.

M. VANT
JAN. 75

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INLAND WATERS BRANCH

PROJECT:

COAXIAL-CAGE

SHEET

SAMPLE HOLDER

4 OF
FILE NO.

DETAIL:

INNER CONDUCTOR ROD

PT - 1

NOTE

TO BE CUT FROM STOCK
PROVIDED AS PER MACHINING
INSTRUCTIONS.

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PROJECT:	COAXIAL-CAGE
	SAMPLE HOLDER
DETAIL:	INNER CONDUCTOR ROD
	(SLEEVE) PT-2, (SPRING CONTACT)
	PT-3, (SPRING) PT-4.

SPRING CONTACT (QTY - 6)

NOTE

- 1. SPRING FROM .012 STEEL WIRE
30 TURNS / INCH - OR EQUIVALENT

SPRING (QTY-12)

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PROJECT:

COAXIAL - CAGE
SAMPLE HOLDER

DETAIL:

COMPUTED

DATE

CHECKED

DATE

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NOTES
1. INNER CONDUCTOR DERIVES SUPPORT FROM
MATING CONNECTORS

FULL SECTION OF
ASSEMBLED HOLDER

A.	END SUPPORT -1	PT-1
B.	END SUPPORT-2	(SLEEVE) PT-2
C.	OUTER CONDUCTOR ROD	(SPRING CONTACT) PT-3
D.	INNER CONDUCTOR ROD	(SPRING) PT-4
E.	"	"
F.	"	"
G.	"	"

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INLAND WATERS BRANCH

PROJECT: COAXIAL - CAGE
SAMPLE HOLDER

DETAILS:

COMPUTED	DATE	CHECKED	DATE
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ADDITIONAL NOTES

1. MACHINING INSTRUCTIONS FOR
24425 ϕ ROD (GR 900-9507)
WITH STOCK PROVIDED
2. A GR 900-9782 ADAPTOR FLANGE
AND GR 900-QAP7 CONNECTOR
ARE ALSO PROVIDED. THE .5625
INSIDE DIAMETER OF THE
900-QAP7 CONNECTOR SHOULD
MATE EXTREMELY WELL WITH
THE .5625 ϕ HOLE IN THE
END SUPPORTS WHEN THE ADAPTOR
FLANGE IS FIXED TO THE
END SUPPORT.

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PROJECT: COAXIAL-CAGE
SAMPLE HOLDER
DETAIL: INNER CONDUCTOR

COMPUTED

DATE

CHECKED

DATE

.3820
141 DRILL
.9 DEEP,
2 HOLES

.2440

QTY - 1
MAT'L - BRASS
SILVER PLATE ALL
EXTERIOR SURFACES
.0005 THICK

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PROJECT:

DETAIL:

TEFLON SPECIMEN

COMPUTED

DATE

CHECKED

DATE

84
10
FILEM. VANT
JAN. 75

SECTION D-D.

QTY - 1
MAT'L - TEFLON

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PROJECT: *JIG FOR CAGE**TRANSMISSION LINE*DETAIL: *SUPPORT*13 OF
FILE NO

COMPUTED	DATE	CHECKED	DATE

3 HOLES, 6-32 TAP
0.50 DEEP

MAT'L - STEEL
QTY - 1

25φ - 20 UNC
2 HOLES

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INLAND WATERS BRANCH

PROJECT: *JIG FOR CAGE*
TRANSMISSION LINE
DETAIL: *CLAMP*

SHEET
14 OF
FILE NO.

COMPUTED	DATE	CHECKED	DATE

*MAT'L - STEEL
TO BE HARDENED
QTY - 1*

*.250 DRILL
2 HOLES*

M. VANT
JAN. 75

APPENDIX D. NUMERICAL SOLUTION OF TRANSMISSION EQUATION FOR

ϵ_r' AND ϵ_r''

There is no equation that gives ϵ_r directly in terms of the measured transmission coefficient, irregardless of whether the sample is contained in a waveguide or in some other sample holder. However, analytically differentiable transcendental equations do exist linking the transmission coefficient and ϵ_r . These have been given as equations (4.1) and (4.6). The solution for ϵ_r can be found by numerically inverting these equations using one of several techniques. The technique employed here is a modified Newton's Method in the complex plane. (See Purcell¹, Todd² and others for a description of Newton's Method.)

Essentially what one has is a function f dependent on λ_0 (the free-space wavelength), d (the sample depth) and ϵ_r (the complex dielectric constant). This function f is equivalent to the difference between T given by (4.1) or (4.6) and the measured value of T equal to T_0 , i.e.

$$f(\lambda_0, d, \epsilon_r) = T - T_0 \quad (D.1)$$

The goal of the process is to find ϵ_r such that $T = T_0$ and $f = 0$, i.e. one must find the roots of f . To do this an initial estimate of ϵ_r (ϵ_{r1}) is made assuming that $\epsilon_r'' = -.00000001$ (the sign is included with ϵ_r'' in the program) and that $\epsilon_r' = |(\text{ANGLE} - 2\pi n)\lambda_0/2\pi d - 1|^2$ (i.e. the imaginary part of the propagation constant is ignored). ANGLE is the measured angle in radians, n provides the multiple solutions indicated in Section 6.2. This value of ϵ_r is used to calculate

¹ E.J. Purcell, Calculus with Analytic Geometry, Appleton Century Crofts, New York, 1965.

² J. Todd (Ed.), Survey of Numerical Analysis, McGraw-Hill, New York, 1962.

a first approximation to f and a first approximation to $\partial f / \partial \epsilon_r$. The derivative of f with respect to ϵ_r can be found analytically. These first approximations are then used to find the new value for ϵ_r (ϵ_{r_2}) which is given by

$$\epsilon_{r_2} = \epsilon_{r_1} - \left| f / \partial f / \partial \epsilon_r \right| \text{ (At } \epsilon_r = \epsilon_{r_1} \text{)} \quad (D.2)$$

This process is repeated until the magnitude of $|f / \partial f / \partial \epsilon_r|$ is less than a specified limit. If the solution fails to converge for $n = 0$ after a set number of iterations, or if the solution starts to explode, then n is incremented and the process is started anew with a fresh ϵ_{r_1} . To obtain all relevant solutions an upper and lower limit is set on ϵ_r and n is incremented to obtain all values in this range. Solutions that yield a positive imaginary part for ϵ_r (i.e. $\epsilon_r = \epsilon_r' + j\epsilon_r''$) are rejected. Provision is also made for calculation of a series of solutions about the desired one. These solutions reflect the change in ϵ_r caused by the errors in the transmission coefficient. The final values for ϵ_r' and ϵ_r'' are obtained by separating $|f / \partial f / \partial \epsilon_r|$ into $[(\gamma + j\delta) / (\epsilon^* + j\eta)]$ and applying

$$\epsilon_{r_{j+1}}' = \epsilon_{r_j}' - \frac{\epsilon^* \gamma + \eta \cdot \delta}{\epsilon^{*2} + \eta^2} \quad (D.3)$$

and

$$\epsilon_{r_{j+1}}'' = \epsilon_{r_j}'' - \frac{\delta \epsilon^* - \gamma \eta}{\epsilon^{*2} + \eta^2} \quad (D.4)$$

It is obvious from this solution that ϵ_r'' contains contributions from all sources, i.e. it is not due solely to the dielectric relaxation.

A technique similar to the one described above is given by

Somlo.¹

The attached computer program written in Flag (a version of Fortran IV) for a Xerox Sigma 9 computer performs the above calculations for ϵ_r .

¹ P.I. Somlo, The exact numerical evaluation of the complex dielectric constant of a dielectric partially filling a waveguide, IEEE Transactions on Microwave Theory and Techniques, MTT-22, 468 (1974).

11122 JAN-15, 175 10=0711

JOB PVANT

9,3

G3GD

ACCT LAST USED JAN 15 AT 10:10

28,20 RESOURCE UNITS REMAIN

FLAG DR

* FLAG VERSION 001

* AVAILABLE MEMORY

* PROGRAM R INITIALIZED VARIABLES = 2459 (WORDS)

* NON-INITIALIZED VARIABLES = 5732 (WORDS)

* TOTAL = 8191 (WORDS)

0001 C

0002 C

0003 C

0004 C

0005 C

0006 C

0007 C

0008 C

0009 C

0010 C

0011

0012

0013 C

0014

0015

0016

0017

0018 C

0019

0020

0021

0022

0023

0024

0025 C

0026 C

0027 C

0028 C

0029 C

0030 C

0031

THIS PROGRAM IS USED

TO GENERATE SOLUTIONS TO THE TEM MODE

PROPAGATION THROUGH A DIELECTRIC SLAB

USING NEWTONS METHOD IN THE COMPLEX PLANE

READ IN NO. OF SAMPLES , NO. OF FREQ. EACH SAMPLE, AND NO. OF
POINTS EACH FREQ.

READ, NSAMP, NFREQ, NPOINT

```

0033 C READ DEPTH AND SALINITY OF EACH SAMPLE:
0034 C
0035 C
0036 READ,DEPTH,SALIN
0037 JJJ=1
0038 DO 44 JIJ=1,NFREQ
0039 C
0040 C READ IN FREQ IN GHZ
0041 C
0042 READ,FREQ
0043 DO 44 IJI=1,NPOINT
0044 CALL XINPUT (TEMP)
0045 C
0046 C IDENTIFY CONSTANTS
0047 C
0048 CALL CONST(MAX,NINCR,AMINCR,AMINCR,TANG,TAMP,
0049 1 PI,LLIM,ULIM,FREQ,DEPTH,AMPERR)
0050 C
0051 C OUTERS LOOPS ,DO ALL THE ANGLES,THEN THE AMPLITUDES
0052 C
0053 10 DO 1 JJ=1,NUM2
0054 C
0055 C INCREMENT THE AMPLITUDE
0056 C
0057 CALL INITAM(TAMP,JJ,AMINCR,AMP)
0058 C*****
0059 20 DO 2 IJ=1,NUM1
0060 C
0061 C INCREMENT THE ANGLE
0062 C
0063 CALL INITTH(ANG,TANG,PI,IJ,JJ,AMINCR)
0064 C
0065 C IF A DIVIDE BY ZERO ERROR OCCURS DURING THE
0066 SEARCH FOR K CONTROL IS TRANSFERRED TO LINE 4
0067 AND THE INITIAL VALUE OF X IS RECHOSEN
0068 AND THE INITIAL VALUE OF Y IS RESET-THIS
0069 ALSO OCCURS IF SEARCH DOES NOT CONVERGE IN MAX TRIALS
0070 -ONCE A VALUE OF X IS CHOSEN BELOW LLIM OR
0071 ABOVE ULIM OR VINCR IS REACHED ,THE LOOP IS BROKEN
0072 -CONTROL IS TRANSFERRED TO 2 AND A NEW ANGLE IS CHOSEN
0073 C
0074 C*****

```

```

0078 C INITIALIZE X,Y=CHECK AND CALCULATE BETAN0&ZROOT
0079 C
0080 C CALL INIT(IJ,ULIM,LLIM,PI,DEPTH,AMP,ANG,K,&2,&50,&4)
0081 C
0082 C FOR EACH NEW ANGLE AND AMPLITUDE CALCULATE A K
0083 C
0084 C *****
0085 C *****
0086 C *****
0087 C *****
0088 C
0089 50 DO 3 I=1,MAX
0090 C CALCULATE TRANSMISSION COEFF. -ZROOT
0091 C
0092 C CALL XTRANS(PI,K,DEPTH,&70,&4)
0093 60
0094 C CALCULATE THE DERIVATIVE OF NEW VALUE OF TRANS
0095 C WRT K
0096 C
0097 C CALL XOTRAN(PI,K,DT,&80,&4)
0098 70
0099 C FORMULATE NEW ESTIMATE OF K USING NEWTONS RULE
0100 C
0101 C CALL XK(K,DT,&90,&4)
0102 80
0103 C IF THE NEW CORRECTION TO K IS LESS THAN AMPERR
0104 C ACCEPT THIS NEW K
0105 C
0106 C IF(CDABS(TRANS/DT).LT.AMPERR)CALL XPRINT(JJJ,SALIN,TEMP,FREQ,
0107 90 2 DEPTH,AMP,ANG,TANG,TAMP,&4)
0108 C
0109 C IF NOT, CONTINUE
0110 C
0111 C CONTINUE
0112 3
0113 C *****
0114 C *****
0115 C *****
0116 C *****
0117 C

```

```

0119 C*****
0120 C*****
0121 C*****
0122 C*****
0123 C*****
0124 C*****
0125 C*****
0126 C*****
0127 C*****
0128 C*****
0129 C*****
0130 C*****

```

CONTINUE

CONTINUE

CONTINUE

STOP

END

```

0001 C*****
0002 C*****
0003 C*****
0004 C*****
0005 C*****
0006 C*****
0007 C*****
0008 C*****
0009 C*****
0010 C*****
0011 C*****
0012 C*****
0013 C*****
0014 C*****
0015 C*****
0016 C*****
0017 C*****
0018 C*****
0019 C*****
0020 C*****
0021 C*****
0022 C*****
0023 C*****

```

SUBROUTINE XINPUT(TEMP)

REAL*8 TEMP,TANG1,TANG2,TAMP1,TAMP2,TTANG,TTAMP,XERR,YERR
COMMON/4/TANG1,TANG2,TAMP1,TAMP2,NUM1,NUM2

SPECIFY TEMPDEG, CEL.S,ANGLE MEAS,DEG.S,AND AMP,XDBR MEAS.

READ,TEMP,TTAMP,TTANG

SPECIFY ERROR LIMITS ON ANGLE AND AMPLITUDE

XERR=0.0
YERR=0.0
TANG1=TTANG-XERR
TANG2=TTANG+XERR
TAMP1=TTAMP-YERR
TAMP2=TTAMP+YERR
NUM1=1.0
NUM2=1.0
RETURN
END

```

0001 C*****
0002 C*****
0003 C*****

```

SUBROUTINE CONST(MAX,NINCR,ANINCR,AMINCR,TANG,

```

0005
0006 COMPLEX*16 DZERO,DEPT,BETANO,ZROOT
0007 REAL*8 ZERO,PI,C,AMPERR,LAMDA,FREQ,XNUM1,XNUM2,
0008 2 TANG1,TANG2,TAMP1,TAMP2,LLIM,
0009 3 ULIY,TANG,TAMP,DEPTH
0010 REAL*8 ANINCR,AMINCR
0011 C
0012 COMMON/A/ TANG1,TANG2,TAMP1,TAMP2,NUM1,NUM2
0013 COMMON/C/ LAMDA,ZERO,DZERO
0014 COMMON/E/ BETANO,DEPT,ZROOT
0015 C
0016 NINCR=10.0
0017 ZERO=0.0
0018 PI=3.14159
0019 MAX=20
0020 C=30.
0021 AMPERR=.001
0022 DEPT=DCMPLX(DEPTH,ZERO)
0023 LAMDA=C/FREQ
0024 XNUM1=NUM1
0025 XNUM2=NUM2
0026 ANINCR = 0.
0027 IF(NUM1.GT.1) ANINCR = (TANG2-TANG1)/(XNUM1-1.)
0028 TANG=TANG1
0029 AMINCR = 0.
0030 IF(NUM2.GT.1) AMINCR = (TAMP2-TAMP1)/(XNUM2-1.)
0031 TAMP=TAMP1
0032 LLIM=1.0
0033 ULIM=7.0
0034 DZERO=DCMPLX(ZERO,ZERO)
0035 RETURN
0036 END

```

```

0001 C .....
0002 C .....
0003 SUBROUTINE INITAM(TAMP,JJ,AMINCR,AMP)
0004 C .....
0005 REAL*8 TAMP,AMINCR,AMP
0006 C .....
0007 IF(JJ.GT.1) TAMP=TAMP+AMINCR
0008 AMP=10.***(TAMP/20.)

```

END

0010

0001

0002

SUBROUTINE INITTH(ANG,TANG,PI,IJ,JJ,ANINCR)

0003

REAL*8 ANG,ANINCR,TANG,PI

0004

REAL*8 TANG1,TANG2,TAMPI,TAMP2

0005

COMMON/A/TANG1,TANG2,TAMPI,TAMP2,VUM1,NUM2

0006

IF((JJ.GT.1).AND.(IJ.EQ.1))TANG=TANG1

0007

IF(IJ.GT.1) TANG=TANG+ANINCR

0008

IF(TANG.GT.0.) TANG=-360.+TANG

0009

IF(TANG.LT.-360.) TANG=TANG+360.

0010

ANG=TANG/180.*PI

0011

RETURN

0012

END

0001

0002

SUBROUTINE INIT(II,ULIM,LLIM,PI,DEPTH,AMP,ANG,K,*,*,*)

0003

0004

COMPLEX*16 BETANO,DEPT,ZROOT,DZERO,K

0005

REAL*8 ANG,PI,XINCR,LAMDA,DEPTH,ULIM,LLIM,X,Y,AMP,ZERO

0006

COMMON/C/ LAMDA,ZERO,DZERO

0007

COMMON/D/ X,Y

0008

COMMON/E/ BETANO,DEPT,ZROOT

0009

XINCR=(II-1)

0010

Y=-0.00000001

0011

X=((ANG-2.*PI*XINCR)*LAMDA/(2.*PI*DEPTH))-1.)*2

0012

IF(X.LT,LLIM) RETURN 3

0013

IF(X.GT,ULIM) RETURN 1

0014

ZROOT=DCMPLX(AMP*DCOS(ANG),AMP*DSIN(ANG))

0015

IF(LAMDA.EQ.ZERO) RETURN 3

0016

BETANO=DCMPLX(2.*PI/LAMDA,ZERO)

0017

DEPT=DCMPLX(DEPTH,ZERO)

0018

K=DCMPLX(X,Y)

```

0001 C .....
0002 C
0003 SUBROUTINE XTRANS(PI,K,DEPTH,*,*)
0004 C
0005 COMPLEX*16 TRANS,K,JZ2,JZ3,Z2,Z3,ZROOT,BETA,DEPT
0006 2 ,DZERO,BETANO,U,V
0007 REAL*8 LAMDA,PI,ZERO,DEPTH
0008 COMMON/C/ LAMDA,ZERO,DZERO
0009 COMMON/E/ BETANO,DEPT,ZROOT
0010 COMMON/G/ TRANS,BETA
0011 COMMON/F/ U,V,JZ3,JZ2
0012 C
0013 C
0014 BETA=BETANO*CDSQRT(K)
0015 ZP=(BETA+BETANO)*DEPT
0016 Z3=2.*BETA*DEPT
0017 IF((DABS(-DIMAG(Z2)).GT'.150'.).OR.(DABS(-DIMAG(Z3)).GT'.150'.))
0018 RETURN 2
0019 JZ2=DCMPLX(-DIMAG(Z2),DREAL(Z2))
0020 JZ3=DCMPLX(-DIMAG(Z3),DREAL(Z3))
0021 U=4.*CDSQRT(K)*DEXP(JZ2)
0022 V=((CDSQRT(K)+1.)*2)*CDEXP(JZ3)=(CDSQRT(K)-1.)*2
0023 IF(CDABS(V).EQ.ZERO) RETURN 2
0024 TRANS=U/V-ZROOT
0025 RETURN 1
0026 END

```

```

0001 C .....
0002 C
0003 SUBROUTINE XOTRAN(PI,K,DT,*,*)
0004 C
0005 COMPLEX*16 PZ2,PZ3,JZ2,JZ3,DU,OV,U,V,DT,BETANO
0006 COMPLEX*16 DEPT,ZROOT,K
0007 REAL*8 PI
0008 C
0009 COMMON/E/ BETANO,DEPT,ZROOT
0010 COMMON/F/ U,V,JZ3,JZ2
0011 C

```

```

0012 IF (UABS(DREAL(JZ2)) .GT. 1000000) THEN
0013 RETURN 2
0014 PZ2=DCMPLX(DREAL(JZ2),DIMAG(JZ2))+PI/2.)
0015 PZ3=DCMPLX(DREAL(JZ3),DIMAG(JZ3))+PI/2.)
0016 IF (CDABS(K).EQ.0.0) RETURN 2
0017 DU=4.*(CDEXP(JZ2)+CDSORT(K)*BETA+DEPT*CDEXP(PZ2))
0018 DV=2.*(CDSORT(K)+1.)*CDEXP(JZ3)+((CDSORT(K)+
0019 +1.)*2)*2.*BETA+DEPT*CDEXP(PZ3)-2.*(CDSORT(K)-1.)
0020 DT=((V*DU-U*DV)/V**2)*.5/CDSORT(K)
0021 RETURN 1
0022 END

-----
0001 C
0002 C
0003 SUBROUTINE XK (K,DT,**)
0004 C
0005 COMPLEX*16 DT,TRANS,X,BETA
0006 REAL*8 EPSLON,GAMM,DELTA,ETA,DENOM,X,Y
0007 C
0008 COMMON/D/ X,Y
0009 COMMON/G/ TRANS,BETA
0010 EPSLON=DREAL(DT)
0011 GAMM=DREAL(TRANS)
0012 DELTA=DIMAG(TRANS)
0013 ETA=DIMAG(DT)
0014 DENOM=((EPSLON**2+ETA**2)
0015 IF (DENOM.EQ.0.0) RETURN 2
0016 X=X-(EPSLON*GAMM+ETA*DELTA)/DENOM
0017 Y=Y-(DELTA*EPSLON+GAMM*ETA)/DENOM
0018 IF (Y.GT.0.0) RETURN 2
0019 K=DCMPLX(X,Y)
0020 RETURN 1
0021 END

-----
0001 C
0002 C
0003 SUBROUTINE XPRINT(JJ,SALIN,TEMP,FREQ,DEPTH,AMP,ANG,TANG,TAMP,*)
0004 C
0005 REAL*8 SALIN,DEPTH,AMP,ANG,TANG,TAMP,X,Y,TANDEL
0006 REAL*8 BRTN
0007 REAL*8 XLOSS,FREQ,TEMP

```


0008	COMMON/D/X,Y	
0009		
0010	IF(JJ.EQ.1) CALL TITLE(DEPTH,SALIN,FREQ)	
0011	JJ=JJ+1	
0012	CALL LOSS (TANDEL,XLOSS,FREQ)	
0013	CALL XSALIN(SALIN,TEMP,BRIN)	
0014	PRINT 100 ,FREQ,TEMP,BRIN,TAMP,AMP,TANG,ANG,X,Y,TANDEL,XLOSS	
0015	FORMAT(2X,F8.2,3X,F8.2,3X,E9.3,3X,E8.2,3X,F8.2,3X,F8.2,	
0016	2 3X,F8.2,3X,E8.2,3X,E8.2,3X,F8.2/)	
0017	RETURN 1	
0018	END	
0001	-----	
0002		
0003		
0004	SUBROUTINE TITLE(DEPTH,SALIN,FREQ)	
0005	REAL*8 DEPTH,FREQ,SALIN	
0006	PRINT 100,DEPTH,SALIN	
0007	FORMAT(2X,1SAMPLE DEPTH IN CM,=1,F8.2,7/2X,	
0008	2 1SALINITY IN PPT,=1,F8.2,7/)	
0009	PRINT 200	
0010	FORMAT(2X,1FREQ(GHZ),1,2X,1TEMP(C),1,4X,1BRINE VOL,1,1PX,1AMP(DB)	
0011	1,1X,1AMP(ABS),1,3X,1ANG(DEG),1,3X,1ANG(RAD),1,3X,1K-REAL,1,5X,	
0012	3 1K-IMAGI,5X,1LOSS-TGT,1,3X,1LOSS(DB/M),1,7/)	
0013	RETURN	
0014	END	
0001	-----	
0002		
0003		
0004	SUBROUTINE LOSS (TANDEL,XLOSS,FREQ)	
0005	REAL*8 X,Y	
0006	REAL*8 TANDEL,XLOSS,FREQ	
0007	COMMON/D/ X,Y	
0008		
0009	Y=X	
0010	TANDEL=Y/X	
0011	XLOSS=181.9*FREQ*DSORT(X/2.*(DSORT((Y/X)**2+1.))-1.0))	
0012	RETURN	
0013	END	

C

```
0005 TEMP=-TEMP
0006 IF((TEMP.LE.2.06).AND.(TEMP.GT.0.5)) BRIN=10.**(-3)
0007 1 *(-2.28+52.56/TEMP)*SAL
0008 IF((TEMP.LE.8.2).AND.(TEMP.GT.2.06)) BRIN=10.**(-3)*
0009 2 (.930+45.9177TEMP)*SAL
0010 IF((TEMP.LE.22.9).AND.(TEMP.GT.6.2)) BRIN=10.**(-3)*
0011 3 (1.189+43.795/TEMP)*SAL
0012 IF((TEMP.LE.36.8).AND.(TEMP.GT.22.9)) BRIN=10.**(-3)*
0013 4 (22.9478-3.07984*10.**3)/TEMP+1.58402*10.**5/(TEMP**2)
0014 -3.61615*10.**6/(TEMP**3)+3.17862*10.**7/(TEMP**4))*SAL
0015 IF((TEMP.LE.43.2).AND.(TEMP.GT.36.8)) BRIN=10.**(-3)*
0016 6 (14.145-1.6426*10.**3/TEMP+6.4947*10.**4/(TEMP**2)
0017 -8.3945*10.**5/(TEMP**3))*SAL
0018 TEMP=-TEMP
0019 RETURN
0020 END
0021
```

```
* ACTUAL PROGRAM SIZE:
* PROGRAM & INITIALIZED VARIABLES = 1776 (WORDS)
* NON-INITIALIZED VARIABLES = 335 (WORDS)
* TOTAL = 2111 (WORDS)
```

COMPILE TIME = 0002.38

END

FIN