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Transient Analysis of Nonuniform High-Speed Interconnects

by

Sanjay Manney, B. Tech.

**This thesis is submitted to the Faculty of Graduate Studies and Research in
partial fulfillment of the requirements for the degree of**

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering

Department of Electronics

Faculty of Engineering

Carleton University

Ottawa, Canada, K1S 5B6

September, 1992

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Abstract

In high-speed circuits with sharp signal rise and fall times interconnects exhibit transmission line effects causing unforeseen circuit behavior. Owing to the presence of complex geometries of interconnects and the coupling between various layers on the chip interconnects are in practice nonuniform and distributed in nature. Two methods are proposed in the thesis for the transient analysis of nonuniform high-speed interconnects.

- One method is based on the numerical inversion of the Laplace transform (NILT). Transmission line equations are solved in the frequency domain by Runge-Kutta numerical integration. A new and efficient algorithm is proposed to overcome numerical instabilities.
- The NILT based method can also handle frequency dependence of transmission line parameters at no additional cost.
- The other method is based on the Asymptotic Waveform Evaluation (AWE) technique. The moments for the nonuniform transmission lines are solved by Runge-Kutta numerical integration.
- The AWE based method is shown to be one or two orders of magnitude faster than published techniques.

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List Of Symbols

C_{π}	MNA matrix representing lumped capacitive and inductive elements.
G_{π}	MNA matrix representing lumped resistive elements.
v_{π}	vector of node voltages appended by independent voltage source currents and inductor currents
D_k	a selector matrix
i_k	terminal currents entering the linear subnetwork.
N_{π}	size of the MNA matrix
e_{π}	independent sources
Y_k	STAMP
Z_p	per unit length impedance of the lines
Y_p	per unit length admittance of the lines
R	per unit length resistance of the lines
L	per unit length inductance of the lines
C	per unit length capacitance of the lines
G	per unit length conductance of the lines
N	number of conductors
$A(x, s)$	system matrix
$H(x)$	State Transition Matrix
h_{fij}	complex NxN H parameter submatrix evaluated from forward integration.
h_{bij}	complex NxN H parameter submatrix evaluated from forward integration
x_1	forward "singular" distance
x_2	backward "singular" distance
$\{\alpha_i, \beta_i, \delta_i\}$	Y parameter submatrices
$\{A_k, B_k\}$	complex matrices representing the MNA stamp of the nonuniform

	transmission line network
$\{p_j, k_j\}$	complex approximating poles and residues.
M_n	moment vectors
G_T	per unit length resistance and conductance matrix
C_T	per unit length capacitance and inductance matrix

Chapter 1

Introduction

1.1 Problem Description

With the advent of VLSI technologies and their increasing popularity as the design methodology for the future, immense efforts have been directed towards research and development in these areas. Computer Aided Design (CAD) is an imperative and crucial stage in the VLSI design process. Due to the high cost and time involved in fabrication and testing, designers are driven to produce designs that have extremely high success rates. As VLSI technology advances new or previously dormant effects begin to gain significance in deciding circuit performance. There is an important need to foresee design

issues for the future to provide reliable and efficient CAD tools for the circuit designer. Consequently, CAD professionals constantly endeavour to upgrade their tools to meet the rapidly advancing changes in VLSI technology.

With VLSI technology trends towards finer feature sizes, faster devices, increased density of circuits, and higher clock rates with sharper rise and fall edges, interconnect delays have become longer than device switching times. The number of interconnects has also grown rapidly. In many cases interconnect performance is the dominant effect deciding circuit performance. At sub-nano second rise times the electrical length of the interconnects is a significant fraction of the operating wavelength, consequently lumped circuit modelling of interconnects is inadequate. Models which represent the distributed nature of interconnects such as transmission line models are to be employed. Owing to the presence of complex geometries of interconnects and the coupling between various layers on the chip, the electromagnetic coupling between interconnects is usually nonuniform in nature. In such cases interconnects are modelled as nonuniform transmission lines. In many practical cases they are more accurately described by frequency dependent parameters. It is necessary to simulate the transmission line effects of interconnects along with other circuit elements in order to detect performance violations at the design stage.

During the design stage circuit designers work within the constraints of design rules. The undesirable effects of high speed interconnects such as delay, crosstalk, ringing, reflections and attenuation of signals can, perhaps, be avoided by formulating appropriate design rules. However there are many reasons for the designer to have access to a full analog simulation capability rather than merely relying in design rules. To formulate the design rules requires rigorous experimentation and simulation. In many cases design rules tend to be overspecified for the particular design and it is not uncommon for designers to

intentionally break design rules in order to improve the quality of design. In such cases the design needs to be verified by a full simulation. Also, design rules tend to remain unchanged for long periods of time during which technology may have changed considerably and they need to be reformulated or upgraded. Therefore, any successful VLSI design environment for high-speed circuits must provide the means to simulate circuit performance including interconnects. This requires accurate modelling of interconnect performance, reliable and efficient transient analysis methods.

For many years conventional circuit simulation programs such as SPICE have gained wide acceptance. With growing demands on circuit simulators such as higher efficiency, lossy multiconductor transmission line components, mixed frequency/time domain formulations, very large scale systems etc., has prompted alternative simulation methods. The research community, in the past few years, has been actively engaged in developing better techniques. The numerical inversion of the Laplace transform method (NILT) [26,27] and the Asymptotic Waveform Estimation (AWE) [31,32] method are two of the more recent techniques which are also found to be very useful in addressing the issue of high-speed interconnects.

Transmission line effects of interconnects are visible at different frequencies at different levels of the design. In present CMOS systems, transmission line properties are important only at the module and board levels; bipolar circuits require transmission line analysis at the chip carrier level and beyond; GaAs technology may require transmission line calculations even for on-chip interconnections [49].

1.2 Thesis Objectives

The objectives of this thesis are as follows:

- I. Nonuniform transmission line equations are formulated as linear ordinary

differential equations (ODEs) with nonconstant coefficients and are solved as an initial value problem (IVP) using numerical integration. (Sections 3.4.1, 4.4.1). This allows the R, L, C, G parameters to have an arbitrary variation along the length of the lines.

II. The transient response of nonuniform high-speed interconnects is analyzed based on the NILT technique. (Section 3.5). Frequency dependence of transmission line parameters is accounted for.

III. A new and efficient algorithm is proposed for overcoming the inherent initial value instability encountered while numerically integrating nonuniform transmission line equations. Theoretical arguments are presented for the stability of the new algorithm. (Section 3.4.3)

IV. Transient response of nonuniform high-speed interconnects is analyzed based on the highly efficient AWE technique. (Section 4.5)

V. A theoretical discussion is presented for the transient analysis of nonlinear networks containing nonuniform transmission line components. It is shown that the proposed methods are completely compatible with the Piecewise Decomposition technique (PDT). (Chapter 5)

In this it is assumed that parameters describing the nonuniform transmission lines are available either from other numerical procedures or from measurements.

An important criterion for the selection of the methods employed is their applicability to large scale systems with a large number of interconnects. Hence efficiency and accuracy are of vital importance. Also the case of nonlinear networks showing nonuniform high-speed interconnect effects is addressed.

The objectives of this thesis were designed to incorporate the nonuniform

transmission line component into a modern CAD framework, SYSCAD, for the design and optimization of high-speed interconnects [47]. With reference to Figure 1.1, the PDT method effectively partitions the network into a linear and a nonlinear portion. The linear portion is analyzed using either one of the two linear circuit simulators based on NILT or AWE. The linear solution is then coupled with the nonlinear portion and solved using Newton-Raphson iterations to obtain the time domain response. A sensitivity analysis is then performed which enables the design to be optimized. By providing the two linear circuit simulators with the capability of analyzing nonuniform transmission line components the nonuniform high-speed interconnect issue is incorporated into this framework.

1.3 Thesis Organization

“Chapter 2: Literature Survey” published techniques addressing the issue of nonuniform high-speed interconnects are reviewed. A summary of research in the NILT, AWE and PDT methods is presented.

“Chapter 3: NILT Based Analysis” the formulation and solution of nonuniform transmission line components in a linear network using the NILT technique is presented. A new algorithm is proposed to overcome numerical instabilities.

“Chapter 4: AWE Based Analysis” the formulation and solution of nonuniform transmission line components in a linear network using the AWE technique is presented. Some stability issues are discussed.

“Chapter 5: Analysis Of Transmission Line Networks with Nonlinear Terminations” discusses the PDT technique for the solution of nonlinear networks containing nonuniform transmission line components.

“Chapter 6: Conclusions And Future Research” summarizes the results of the thesis and suggests directions for future research.

The High-Speed Interconnect Design Problem

Figure 1.1: SYSCAD

Chapter 2

Literature Survey

2.1 Introduction

High-speed interconnections have been modelled as lumped RC tree circuits in the past. With increasing signal speeds these models have become inaccurate. A large amount of literature deals with the high-speed interconnects modelled as uniform transmission line elements. Bends and simple curved structures in interconnect geometries were approximated by a cascade of uniform sections. Because of the complex geometries, such as the presence of crossing wires at different heights, the coupling between two parallel wires is in general nonuniform. Accordingly we need a more complete transient anal-

ysis of nonuniformly coupled transmission lines.

2.2 Nonuniform Transmission Lines

Transmission lines are linear distributed multi-port elements characterized by hyperbolic partial differential equations in the time domain. These equations are also referred to as Telegraphers Equations. Parameters of transmission line equations are the per unit length resistive and dielectric loss matrices, R and G respectively, and the per unit length inductance and capacitance matrices, L and C respectively. Uniform transmission lines are characterized by constant R, L, C, G parameters. When the R, L, C, G parameters vary along the length of the lines they represent nonuniform transmission lines.

Nonuniform transmission lines have been studied since the early 1930's [1-15]. Early research was aimed at obtaining analytical solutions for special classes of lines for the application of nonuniform lines as impedance transformers, resonators, filters, directional couplers and other microwave components. In a series of papers [16-18] Protonotarios and Wing have studied interesting theoretical properties of the nonuniform line. Although their results are derived for a pair of RC or LC nonuniform lines, they provide a good insight into nonuniform transmission lines. Recently there has been a renewed interest in nonuniform transmission lines in the context of high-speed interconnects. Researchers in the area began to investigate numerical techniques to solve nonuniformities of a general nature. A wide range of numerical techniques have been investigated.

In [19] a method based on perturbational techniques for the analysis of a pair of nonuniform lines is presented. By a suitable transformation and a combination of both the method of characteristics and perturbational series the transient response is obtained from a set of iterative integration formulas. Transmission line equations of a pair of dispersionless nonuniform lines are decoupled by a linear combination of the current and voltage variables scaled by the self characteristic impedance. The resulting equations are a

pair of hyperbolic partial differential equations which are solved by the method of characteristics. The ordinary differential equations that hold true over a set of characteristic curves are coupled. In order to get around this a perturbational series is resorted to. Several limiting assumptions are imposed and it is shown that each term in the perturbational series represents repetitive reflections along the lines due to nonuniform coupling.

The spectral technique often used in the numerical solution of partial differential equations is applied to nonuniform lines in [20]. Spatial derivatives are approximated by constructing a global interpolant through discreet data points. The interpolants used are Chebychev polynomials. Using Chebychev polynomials to represent the variation in space, the currents i and voltages v are expressed as

$$v = \sum_{n=0}^{\infty} a_n(t) T_n(x)$$

$$i = \sum_{n=0}^{\infty} b_n(t) T_n(x)$$

and the derivatives are expressed as

$$\frac{\partial v}{\partial x} = \sum_{n=0}^{\infty} a_n^*(t) T_n(x)$$

$$\frac{\partial i}{\partial x} = \sum_{n=0}^{\infty} b_n^*(t) T_n(x)$$

where

$T_n(x)$ are the Chebychev polynomials and $a_n(t)$, $b_n(t)$, $a_n^*(t)$, $b_n^*(t)$ are the expansion coefficients. Substituting these expressions into transmission line equa-

tions, the combined equations are expressed as ordinary differential equations in time with constant coefficients. The variables to be solved are now the expansion coefficients which is accomplished by BDF numerical integration. The advantage of spectral methods is that they are more accurate than finite difference or finite element schemes. However, stability theory for spectral techniques is not well developed. The method when applied to nonuniform lines involves a convolution of a system of equations the size of which grows rapidly as the number of coupled lines is increased. This could make the method computationally expensive. In its current form frequency dependent transmission line parameters may not be handled.

A frequency domain scattering parameter approach is used in [21]. The technique is very practical in the sense that it assumes only a knowledge of the physical dimensions of the interconnects. Effects of frequency on the effective dielectric constants are accounted for. The method approximates nonuniform lines by a cascade of uniform sections therefore its applicability is limited to simple geometries such as a taper. The method involves the calculation of S parameters for each uniform section by determining Green's functions and potentials. This is done by solving integral equations of charge and current distributions. S parameters of individual sections are then cascaded to give an overall S parameter representation in the frequency domain. The time domain response is obtained by inverting the frequency domain S parameters using inverse Fast Fourier Transform. The results are then convolved with the I-V characteristics of the terminations to obtain the time domain response.

The simulation method in [22] consists of iterative waveform relaxation analysis. Nonuniform lines are assumed to be decoupled and are treated as asymmetric, disjoint two port networks constructed with a characteristic impedances in series with voltage generators. The voltage generator waveforms are computed by a Fast Fourier Transform

(FFT) analysis. The method accounts for frequency dependent transmission line parameters. However, the method assumes that the ABCD parameters of the lines are available which is not trivial assumption. Usually transmission lines are characterized by R, L, C, G parameters. Analytic expressions for the ABCD parameters are possible for special classes of lines. Practical lines however require numerical methods that can be computationally intensive.

The method of characteristics is used in [23]. The L and C parameters of the coupled nonuniform lines are diagonalized. Transmission line equations in the time domain are transformed into ordinary differential equations that hold true over a family of characteristic curves. The resulting equations are solved by a finite difference scheme. Diagonalizing the L and C matrices is equivalent to computing the kinematic eigen values (eigen values as a function of the distance) of the corresponding matrices which is CPU intensive operation. Also, frequency dependence of transmission line parameters is difficult to incorporate into this scheme.

Very recently,[24] the method of convolution characteristics was proposed. This method combines the advantages of the method of characteristics in analyzing non-uniform transmission lines while the frequency dependence of line parameters is analyzed using convolution. The same authors have used a cascaded network chain formulation [25] to address the nonuniform transmission line problem. Coupled lines are treated as a cascade of uniform multiport networks each characterized by ABCD parameters. The ABCD parameters are evaluated by analytic series expansion. An FFT procedure is then applied to obtain the time domain response.

2.3 Numerical Inversion Of The Laplace Transform

The Pade method of numerical inversion of the Laplace transform (NILT) was proposed in [26,27]. Recently, the NILT technique has been applied to multiconductor uni-

form transmission lines [28]. NILT has several advantages over the conventional FFT analysis techniques. In transmission line circuits the transients last for several transit times of the transmission line network. In such cases several computational points need to be added to the FFT analysis in order to avoid aliasing problems. In addition aliasing errors are difficult to quantify. NILT has been shown to be absolutely stable. It exactly inverts a certain number of terms in the Taylor series expansion of the response and is therefore equivalent to numerical integration. The equivalent order of integration may be changed between 1 and 46 without affecting stability properties. Also the response at a given time point is independent of the response from all other time points hence the problem of aliasing is completely avoided. The number of computational points required by NILT are far fewer than a FFT analysis, hence the method is efficient.

The accuracy of NILT deteriorates for time points further away from the origin. Two improvements have been proposed to the NILT. In [29] derivative information is obtained in a relatively inexpensive way and is used to improve accuracy. In [30] the principle of reinitialization is applied. The time response at a time point t_1 is used as initial conditions of the circuit and NILT is used to evaluate the circuit response at $t_2 > t_1$ improving accuracy.

2.4 Asymptotic Waveform Evaluation

The Asymptotic Waveform Evaluation (AWE) [31] technique was developed at Carnegie Mellon University. The AWE technique attempts to approximate the dominant poles of the network in an efficient manner. It is an extremely efficient transient simulation method with a compute cost equivalent to a DC solution or a single LU decomposition. Recently the AWE technique was extended to linear distributed circuits applied to multi-conductor uniform transmission lines [32,33]. CPU speed up of 2 to 3 orders of magnitude over SPICE have been reported. In [32,33] an eigen analysis approach is used for the anal-

ysis of uniform transmission lines. In [34,35] the uniform multiconductor transmission line problem is addressed using the AWE technique.

Although AWE is an extremely efficient estimation technique it poses some numerical problems. Positive approximating poles may be generated for stable circuits and only the poles close to the origin are accurately extracted. Transmission line circuits usually have a large number of poles close to the imaginary axis but away from the origin. These poles are difficult to approximate by the original AWE formulation but they have significant contributions to the final response. Some interesting improvement and generalizations to the AWE have been proposed addressing the above issues in [36,37] and will be discussed in section 4.3.2.

2.5 Nonlinear Analysis

Nonlinear networks are defined and may be analyzed only in the time domain whereas transmission line networks are best described in the frequency domain. The difficulty in the analysis of transmission line circuits with nonlinear terminations is due to this mixed time and frequency domain formulations. This issue has been addressed in [38,39] where the Piecewise Decomposition Technique (PDT) is proposed. The method relies on partitioning the network in to linear and nonlinear portions. The linear partition is solved in the frequency domain using a linear circuit simulator. The results of the linear network are coupled to the nonlinear partition which is then solved in the time domain using Newton-Raphson iterations. Using PDT both the NILT [39] and the AWE [33] methods have been successfully used to analyze nonlinear networks containing uniform transmission line components.

Chapter 3

NILT Based Analysis

3.1 Introduction

The conventional method for analyzing time domain response of transmission line circuits is based on solving circuit equations formulated in the frequency domain. Voltages in the time domain are obtained by applying the inverse Fast Fourier Transform (FFT). This approach is plagued by aliasing problems due to the long transients present in transmission line circuits. The numerical inversion of the Laplace transform (NILT) adopted for this research is based on a Pade approximation scheme [26,27] which makes

the time response at any given time point independent of all other time points thus avoiding the problem of aliasing. Additionally it requires far fewer frequency domain computations than the FFT analysis. Several improvements to the NILT have appeared in literature which will be discussed in section 3.3. NILT has been shown to be accurate and efficient for the transient analysis of uniform lossy multiconductor transmission lines [28]. In this chapter the nonuniform lossy multiconductor transmission line is analyzed using the NILT inversion technique.

3.2 Formulation Of Linear Network Equations

Consider a linear network π which contains linear lumped components and arbitrary linear subnetworks. The linear lumped components can be described by equations in either the frequency or the time domain. The arbitrary linear subnetworks may contain frequency dependent lumped and distributed (uniform or nonuniform) elements that are best described in the frequency domain. Without loss of generality the modified nodal admittance (MNA) matrix equations of the network π can be written as [40,47]

$$C_{\pi} \frac{d}{dt} v_{\pi}(t) + G_{\pi} v_{\pi}(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e_{\pi}(t) = 0 \quad (1)$$

where,

$C_{\pi}(t) \in \mathcal{R}^{N_s \times N_s}$, $G_{\pi}(t) \in \mathcal{R}^{N_s \times N_s}$ are constant matrices with entries determined by the lumped linear components and are assumed to be frequency independent.

$v_{\pi}(t) \in \mathcal{R}^{N_s}$ is the vector of node voltage waveforms appended by independent voltage source current and inductor current waveforms.

$D_k = [d_{i,j}]$, $d_{i,j} \in \{0, 1\}$, $i \in \{1, 2, \dots, N_\pi\}$, $j \in \{1, 2, \dots, n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $i_k(t) \in \mathcal{R}^{n_k}$ the vector of currents entering the linear subnetwork k , into the node space $\mathcal{R}^{N_\pi}$ of the network π , N_π is the number of linear subnetworks and $e_\pi(t) \in \mathcal{R}^{N_\pi}$ is the vector of source waveforms.

The frequency domain representation can be obtained by taking the Laplace Transform of (1):

$$[sC_\pi + G_\pi] V_\pi(s) + \sum_{k=1}^{N_\pi} D_k I_k(s) = E_\pi(s) + C_\pi v_\pi(0) \quad (2)$$

Assume the frequency domain equations of the linear subnetwork to be in the form

$$Y_k V_k(s) = I_k(s) \quad (3)$$

where $V_k(s)$ and $I_k(s)$ represent the frequency domain terminal voltages and currents of the subnetwork k and Y_k represents the MNA matrix of the subnetwork.

Combining (2) and (3) gives

$$Y_\pi V_\pi = E_\pi + C_\pi v_\pi(0) \quad (4)$$

where,

$$Y_\pi = sC_\pi + G_\pi + \sum_{k=1}^{N_\pi} D_k Y_k D_k^T \quad (5)$$

Figure 3.1: An example circuit containing linear lumped and distributed elements.

If the subnetwork k is a set of coupled lines then the matrix Y_k in (3) will be referred to as the "stamp" of the transmission line system.

Example

To illustrate the formulation method, consider the network shown in Fig.

3.1. The network can be described by (2) where,

$$C_{\pi} = \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_{\pi} = \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & -G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G_3 & 0 & G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The entries of the matrix Y_1 are evaluated by solving transmission line equations as described in the following sections.

3.3 Computation Of The Time Domain Response

The method for numerical inversion of the laplace transform (NILT) is

described in [26,27]. Recently [28] this method has been applied to multiconductor uniform transmission lines. The method involves computing the frequency domain function at preassigned complex frequency points and forming a weighted sum. It exactly inverts a certain number of terms of the Taylor series expansion of the time response and is thus equivalent to the methods used for the integration of differential equations. It has been shown that the method is absolutely stable and that the equivalent order of integration can be changed between 1 and 46 without affecting the stability properties. The inversion formula is

$$\vartheta(t) = -\left(\frac{1}{t}\right) \sum_{i=1}^M K_i V\left(\frac{z_i}{t}\right)$$

where, t is the independent variable, time, $\vartheta(\cdot)$ is the time domain response and $V(\cdot)$ is the frequency domain response. K_i and z_i are the residues and poles of the Pade approximation to the exponential function in the right half plane. Real time functions can be evaluated using only the poles z_i in the upper half plane, this reduces the computation by one half. Recently two new techniques were introduced to enhance the accuracy of NILT. In [29] derivative information of $V(s)$ with respect to s is obtained in a relatively inexpensive way and is used to improve accuracy of NILT. In [30] the reinitialization principle of NILT is used. The time response at time t_1 is used as initial conditions of the circuit and NILT is used to evaluate the circuit response at $t_2 > t_1$ improving accuracy.

Inversion formulas based on Laguerre polynomial expansion [22] or the improved IFFT [41] method also require frequency domain responses of the circuit at preassigned complex frequency points. Hence the time domain response may be alternately obtained from the frequency domain response by either one of these schemes.

3.4 Evaluation Of The Transmission Line Stamp

3.4.1 Basis Of The Method

Transmission line equations in the frequency domain may be represented in the matrix form as

$$\frac{d}{dx} \begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix} = \begin{bmatrix} 0 & -Z_p \\ -Y_p & 0 \end{bmatrix} \begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix} \quad (6)$$

where,

$$Z_p = R(x, s) + sL(x, s) \quad Y_p = G(x, s) + sC(x, s)$$

$R(x, s)$, $L(x, s)$, $C(x, s)$, $G(x, s)$ are the per unit length parameters of the transmission lines that are nonuniform and frequency dependent, $x \in [0, d]$ is the distance along the lines and s is the complex frequency. If N is the number of coupled lines then,

$$\{R(x, s), L(x, s), C(x, s), G(x, s)\} \in C^{N \times N}$$

$$\{V(x, s), I(x, s)\} \in C^N$$

The transmission line parameters may also be frequency dependent. We define the system matrix,

$$A(x, s) \equiv \begin{bmatrix} 0 & -Z_p \\ -Y_p & 0 \end{bmatrix}$$

We assume $A(x, s)$ is piecewise continuous in x , which is a practical assumption. Using the property of linear systems, that the State Transition Matrix (STM) for the system (4) satisfies the system equations with unity initial conditions, the forward or backward transmission matrices can be evaluated by numerical integration. Hence, we have,

$$\frac{d}{dx}H(x) = A(x)H(x) \quad (7)$$

where, $A(x)$ is the system matrix $A(x, s)$ at a preassigned frequency point $s = z_i/t$ as defined in the previous section. $H(x) \in \mathbb{C}^{2N \times 2N}$, is the corresponding transmission matrix. In this research a 4th/5th order explicit Runge-Kutta numerical integrator is used, however, any other integration scheme may be employed.

3.4.2 Stability Considerations

The construction of a fundamental matrix for a system of linear ODEs has been one of the chief problems in carrying out numerical procedures [42]. If the solution space of the transmission matrix (which is a fundamental matrix with initial conditions taken to be the unity matrix) can be partitioned into dominant and dominated solution sub-spaces then the dominant solutions lead to instabilities in numerical integration resulting in numerical "singularities". The existence and characterization of such sub-spaces is reported in [43,44]. The presence of dominant and dominated solution sub-spaces is termed as *dichotomy*. Consequently, stable integration of linear ODEs is possible either in the absence of dichotomy or over a short interval of change of the independent variable, x .

Various methods have been proposed in literature to overcome this inherent initial value instability [45]. These methods may be broadly classified into two categories. One set of methods aims at transforming the ODEs before discretizing the equations for numerical integration so as to decouple the increasing and decreasing solutions. The Riccati transform is one such transformation. The second set of methods solves the ODEs up to the point when the dominant solutions tend to significantly deteriorate accuracy. This point is determined by monitoring the linear dependence of the solution on the dominant solution subspace. At this point an orthogonalizing transform such as QU or LU factoriza-

tion is preformed to restore linear independence in the solutions. Usually these methods are fairly compute intensive.

In this research it is shown that, for the purposes of describing terminal electrical behavior, numerical integrations up to the onset of instability is adequate to derive the Y parameter stamp as defined in (3).

3.4.3 The Proposed Algorithm

To illustrate the idea of the proposed algorithm consider a set of coupled multiconductor nonuniform transmission lines extending between $x = 0$ and $x = d$. For the purpose of argument let the system be divided into three nonuniform cascaded sections Fig. 3.2(a). Associated with each section is its H parameter representation denoted by $H1$, $H2$ and $H3$. The product of the individual H parameters gives the H parameters of the system which may then be converted to a Y parameter representation for the purposes of circuit simulation. Procedure A in Fig. 3.2(b) is a flow diagram of the above method. The individual H parameters are to be evaluated by numerical integration. In the presence of instabilities in numerical integration the H parameters are relatively large numbers hence their product leads to large round off errors. A large number of sections would reduce the round off errors at the expense of increased matrix multiplications. Hence procedure A is either numerically ill conditioned or inefficient.

Theoretically equivalent to procedure A is procedure B, Fig. 3.2(c) which is adopted by the proposed algorithm. In the 1st stage the individual H parameters are converted to Y parameters in a stable manner. In the 2nd stage the operator " Θ " is introduced, which operates on the Y parameters of the individual sections to produce the Y parameters of the cascade. The operator Θ is defined such that procedure A and procedure B are theoretically equivalent. It is shown that stage 1 and stage 2 may be combined to develop a sta-

(a)

PROCEDURE A **Ill-conditioned**

(b)

PROCEDURE B **Well conditioned**

(c)

Figure 3.2: Illustrating the idea of the proposed algorithm

ble and efficient method to build the Y parameter stamp.

STAGE 1

In Stage 1, H parameters of individual sections are evaluated by numerically integrating (7) over the length of each section. Individual H parameters are then converted to Y parameters for the purpose of formulating circuit equations.

Let $U \in C^{2N \times 2N}$, be the identity matrix. We use the subscript '1' to denote the near-end and '2' to denote the far-end of a set of coupled transmission lines. Numerically integrating (7) from $x = 0$ to $x = d$ (forward recursions) with initial conditions $H(0) = U$ gives the forward transmission matrix

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{f11} & h_{f12} \\ h_{f21} & h_{f22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad (8)$$

The submatrices $\{h_{fij}\}, i \in \{1, 2\}, j \in \{1, 2\}$ are a partition of the forward transmission matrix where each $h_{fij} \in C^{N \times N}$. Similarly, numerically integrating (7) from $x = d$ to $x = 0$ (backward recursions) with initial conditions $H(d) = U$ gives the backward transmission matrix

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} h_{b11} & h_{b12} \\ h_{b21} & h_{b22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad (9)$$

The submatrices $\{h_{bij}\}, i \in \{1, 2\}, j \in \{1, 2\}$ are a partition of the backward transmission matrix where each $h_{bij} \in C^{N \times N}$.

Rearranging (8) the Y parameter stamp in terms of the forward transmission submatrices is

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -h_{f12}^{-1} h_{f11} & h_{f12}^{-1} \\ (h_{f21} - h_{f22} h_{f12}^{-1} h_{f11}) & h_{f22} h_{f12}^{-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (10)$$

It is noted from (10) that the submatrix relating far-end currents to near-end voltages involves the subtraction of large numbers if the H parameters are relatively large numbers. This could lead to large round off errors.

Rearranging (9) the Y parameter stamp in terms of the backward transmission submatrices is

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{b22} h_{b12}^{-1} & (h_{b21} - h_{b22} h_{b12}^{-1} h_{b11}) \\ h_{b12}^{-1} & -h_{b12}^{-1} h_{b11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (11)$$

It is noted from (11) that the submatrix relating near-end currents to far-end voltages involves the subtraction of large numbers if the H parameters are relatively large numbers. This could lead to large round off errors.

Rearranging (10) and (11) the Y parameter stamp is built from a combination of both forward and backward transmission matrices to avoid large round off errors

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{b22} h_{b12}^{-1} & h_{f12}^{-1} \\ h_{b12}^{-1} & h_{f22} h_{f12}^{-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (12)$$

Equation (10) corresponds to equation (3).

Let $H(x) = \begin{bmatrix} H_{11}(x) & H_{12}(x) \\ H_{21}(x) & H_{22}(x) \end{bmatrix}$ be a partition of the transmission matrix

into four $N \times N$ matrices. Then according to (7) the following matrix equation also holds true

$$\frac{d}{dx} \begin{bmatrix} H_{12}(x) \\ H_{22}(x) \end{bmatrix} = A(x) \begin{bmatrix} H_{12}(x) \\ H_{22}(x) \end{bmatrix} \quad (13)$$

Let $u \in \mathbb{C}^{N \times N}$ be the identity matrix. In the case that the system of equations (13) is stable, solving (13) using forward recursions with initial conditions

$$\begin{bmatrix} H_{12}(0) \\ H_{22}(0) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (14a)$$

we get,

$$\begin{bmatrix} H_{12}(d) \\ H_{22}(d) \end{bmatrix} = \begin{bmatrix} h_{f12} \\ h_{f22} \end{bmatrix} \quad (14b)$$

Solving (13) using backward recursions with initial conditions

$$\begin{bmatrix} H_{12}(d) \\ H_{22}(d) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} \quad (15a)$$

we get,

$$\begin{bmatrix} H_{12}(0) \\ H_{22}(0) \end{bmatrix} = \begin{bmatrix} h_{b12} \\ h_{b22} \end{bmatrix} \quad (15b)$$

Using the submatrices of (14b) and (15b) the stamp defined by (12) may be built.

We define the forward "singular" distance, x_1 , as the distance over which the forward integration solution exceeds a prespecified solution growth limit, GROWTOL ($\sim 10^8$ to $\sim 10^{20}$). Similarly, the backward "singular" distance, x_2 , is defined as the distance over which the backward integration solution exceeds a prespecified solution growth limit, GROWTOL. x_1 and x_2 indicate the distances over which the forward and backward

integration solutions respectively, approach the onset of numerical instability.

It is shown in Appendix A that the Y parameter stamps associated with the section of transmission lines either over x_1 or x_2 may be approximated as block diagonal matrices without loss of accuracy in the frequency response. This important result will be referred to as Result 1.

STAGE 2

The definition and relevant properties of the operator Θ are discussed in Appendix B. In the event that transmission line equations lead to instabilities in numerical integration two possible cases arise.

Case 1: $x_1 < x_2$

Consider the lines as a cascade of three sections, between $x = 0$ and $x = x_1$, between $x = x_1$ and $x = x_2$, between $x = x_2$ and $x = d$. The corresponding Y parameters of the three sections are denoted by

$$Y_i = \begin{bmatrix} \alpha_i & \beta_i \\ \beta_i & \delta_i \end{bmatrix} \quad i = 1, 2, 3 \quad \{\alpha_i, \beta_i, \delta_i\} \in C^{N \times N}$$

Since Y_1 and Y_3 are evaluated over the forward and backward “singular” distances respectively, from Result 1 we have

$$Y_i = \begin{bmatrix} \alpha_i & \beta_i \\ \beta_i & \delta_i \end{bmatrix} \equiv \begin{bmatrix} \alpha_i & 0 \\ 0 & \delta_i \end{bmatrix} \quad i = 1, 3$$

From property 2, the Y parameter representation of the cascade is

$$Y = Y_1 \Theta Y_2 \Theta Y_3 \equiv Y_1 \Theta Y_3$$

From property 1

$$Y \equiv Y_1 \ominus Y_3 \equiv \begin{bmatrix} \alpha_1 & 0 \\ 0 & \delta_3 \end{bmatrix}$$

α_1 is evaluated by solving (13) using backward recursions from $x = x_1$ to $x = 0$ with

$$\text{initial conditions } \begin{bmatrix} H_{12}(x_1) \\ H_{22}(x_1) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}.$$

δ_3 is evaluated by solving (13) using forward recursions from $x = x_2$ to $x = d$ with

$$\text{initial conditions } \begin{bmatrix} H_{12}(x_2) \\ H_{22}(x_2) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}.$$

Case 2:

$$x_1 > x_2$$

Consider the lines as cascade of two sections, between $x = 0$ and $x = x_1$, between $x = x_1$ and $x = d$. The corresponding Y parameters of the two sections are denoted by

$$Y_i = \begin{bmatrix} \alpha_i & \beta_i \\ \beta_i & \delta_i \end{bmatrix} \quad i = 4, 5 \quad \{\alpha_i, \beta_i, \delta_i\} \in \mathbb{C}^{N \times N}$$

Since Y_4 is evaluated over the forward "singular" distance, from Result 1 we have

$$Y_4 = \begin{bmatrix} \alpha_4 & \beta_4 \\ \beta_4 & \delta_4 \end{bmatrix} \equiv \begin{bmatrix} \alpha_4 & 0 \\ 0 & \delta_4 \end{bmatrix}$$

Hence the Y parameters of the cascade is

$$Y = Y_4 \ominus Y_5$$

Next, consider the same set of lines as a cascade of two sections, one between $x = 0$ and $x = x_2$ and the other between $x = x_2$ and $x = d$. The corresponding Y parameters of the two sections are denoted by

$$Y_i = \begin{bmatrix} \alpha_i & \beta_i \\ \beta_i & \delta_i \end{bmatrix} \quad i = 6, 7 \quad \{\alpha_i, \beta_i, \delta_i\} \in \mathbb{C}^{N \times N}$$

Since Y_7 is evaluated over the backward "singular" distance, from Result 1 we have

$$Y_7 = \begin{bmatrix} \alpha_7 & \beta_7 \\ \beta_7 & \delta_7 \end{bmatrix} \equiv \begin{bmatrix} \alpha_7 & 0 \\ 0 & \delta_7 \end{bmatrix}$$

Hence, the Y parameters of the cascade are

$$Y = Y_6 \Theta Y_7$$

Using property 3

$$Y = Y_4 \Theta Y_5 = Y_6 \Theta Y_7 \equiv Y_4 \Theta Y_7$$

From property 1

$$Y \equiv Y_4 \Theta Y_7 = \begin{bmatrix} \alpha_4 & 0 \\ 0 & \delta_7 \end{bmatrix}$$

α_4 is evaluated by solving (13) using backward recursions from $x = x_1$ to $x = 0$ with

$$\text{initial conditions } \begin{bmatrix} H_{12}(x_1) \\ H_{22}(x_1) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}.$$

δ_7 is evaluated by solving (13) using forward recursions from $x = x_2$ to $x = d$ with

initial conditions
$$\begin{bmatrix} H_{12}(x_2) \\ H_{22}(x_2) \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}.$$

Special Case:

Stable Equations

In the event that the solution of transmission line equations is stable the algorithm sets $x_1 = d$ and $x_2 = 0$. This case reduces to integrating transmission line equations from end to end.

The algorithm for all the above cases may be summarized as below.

At a complex frequency point $s = \frac{z_i}{t}$, as defined in section 3.3:

STEP 1: Do one forward recursion of equations (13) and (14a) from $x = 0$ to

$$x_1 = \min_x \{h_{f12}(x) \geq GROWTOL, d\}. \text{ Hence, determine } x_1.$$

STEP 2: Do one backward recursion of equations (13) and (15a) from $x = d$ to

$$x_2 = \max_x \{h_{b12}(x) \geq GROWTOL, 0\}. \text{ Hence, determine } x_2.$$

In steps 1 & 2 above transmission line equations are solved as vector equations to determine forward and backward "singular" distances, the idea being that the vector equation has the same stability properties as the matrix equation. Hence, this is only a marginal increase in computation.

STEP 3: Do N forward recursions of equations (13) and (14a) from $x = x_2$ to $x = d$.

Hence, evaluate h_{f12}, h_{f22}

STEP 4: Do N backward recursions of equations (13) and (15a) from $x = x_1$ to $x = 0$.

Hence, evaluate h_{b12}, h_{b22} .

STEP 5: Using h_{f12}, h_{f22} and h_{b12}, h_{b22} build the stamp as defined by (12).

3.5 Simulation Results

EXAMPLE 1:

A parabolic lossy frequency dependent nonuniform transmission line is described by the following equations [22]

$$\frac{d}{dx}V(x, s) = -Z(s) (1 + ax)^2 I(x, s)$$

$$\frac{d}{dx}I(x, s) = -Y(s) (1 + ax)^{-2} V(x, s)$$

where, $Z(s)$ and $Y(s)$ are the per unit length series impedance and shunt admittance of the parabolic line at $x = 0$. The line has a length of $d = 96.8\text{mm}$ and tapering factor $1 + ad = \sqrt{2}$. $Z(s) = sL + R + K\sqrt{s}$, $Y(s) = sC$ are defined by the skin-effect model [22]. The line parameters are given by

$$Z_0 = \sqrt{\frac{L}{C}} = 53\Omega, \quad R = 0.35\Omega/\text{mm}, \quad K = 25(10^{-5})\Omega\sqrt{s}/\text{mm},$$

$\frac{1}{\sqrt{LC}} = 2(10^8)\text{mm/sec}$. Closed form expressions for the transmission matrix of the parabolic line in the frequency domain are available [22] which is used for comparison with the proposed method. The circuit configuration is shown in Fig. 3.3. The response due to a unit step is shown in Fig 3.4 and Fig. 3.5. There is perfect agreement between the proposed solution and that from the closed form expression. The CPU time on 4/75 SUN SPARC 2 is 203 seconds.

Figure 3.3: Interconnect model containing a frequency dependent parabolic line

Figure 3.4: Response of the Frequency Dependent Parabolic Line Using NILT

Figure 3.5: Response of the Frequency Dependent Parabolic Line Using NILT

EXAMPLE 2:

A double line prototype chip interconnect is shown in Fig. 3.6 [20]. The line parameters are as listed in Table 3.1. The circuit configuration is shown in Fig. 3.6(c). The transient response due to the input voltage of Fig. 3.6(d) computed using the proposed method is shown in Fig. 3.7 and Fig. 3.8. The results compare favorably with the spectral technique [20], the CPU time of the proposed solution is 97 seconds on 4/75 SUN SPARC 2 and the CPU time of the solution from the spectral method is 199 seconds on a VAX-11/750 VMS [20]. The spectral method uses a convolution of a system of equations the size of which grows rapidly as the number of coupled lines is increased. Therefore the CPU gain of the proposed NILT based method over the spectral method is expected to be higher as the number of coupled lines is increased.

In order to reduce the accumulation of round off errors the error tolerance on the numerical integration procedure could be increased, however this results in excess CPU time. Splitting the line into two nonuniform sections prevents large round off error accumulation with marginal increase in CPU time. The response shown is evaluated using 40 equally spaced time points. Without loss of accuracy the response may be obtained by fewer time points if the points are spaced at short interval over rapidly varying portions of the response and at larger intervals over the smooth portions of the response. This will further reduce CPU time.

Table 3.1: Parameters for the prototype chip interconnect

x (cm)	C11 = C12 (pF/cm)	C12 = C21 (pF/cm)	L11 = L22 (nH/cm)	L12 = L21 (nH/cm)
0.0 - 1.0	1.84	-0.090	1.96	0.23
1.14	1.76	-0.073	2.04	0.22
1.29	1.60	-0.050	2.22	0.20
1.43	1.44	-0.035	2.43	0.19
1.57	1.28	-0.024	2.69	0.17
1.71	1.12	-0.017	3.01	0.17
1.86	0.96	-0.012	3.44	0.16
2.0 - 3.0	0.88	-0.009	3.71	0.16

Prototype Chip Interconnect

(a)

(b)

(c)

(d)

Fig. 3.6: (a) Top view of metallization (b) Cross section (c) Circuit configuration
(d) Driving voltage

Figure 3.7: Response of Prototype Chip Interconnect Using NILT

Figure 3.8: Response of Prototype Chip Interconnect Using NILT

EXAMPLE 3:

A linear circuit consisting of seven nonuniform lossless transmission lines is shown in Fig. 3.9. The circuit configuration is adapted from [32]. The parameters of the nonuniform transmission lines are tabulated in Tables 3.2 - 3.6 and they represent tapered interconnection geometries. An input pulse of 4 nano second pulse width and rise and fall times of 20 pico seconds is applied. The transient response of the circuit is analysed using 50 points on the time scale using the NILT analysis technique. The response is shown in Fig. 3.10 and the CPU time is 564.62 seconds on 4/75 SUN SPARC 2.

Figure 3.9: Circuit for example 3. Interconnect model with nonuniform transmission lines

Table 3.2: Parameters for TL1

x (m)	C (pF/m)	L (nH/m)
0.0	130	80.1
0.008	124	84.1
0.017	112	91.4
0.025	100	100
0.030	88	111
0.042	76	124
0.05	70	134

Table 3.3: Parameters for TL2

x (m)	C (pF/m)	L (nH/m)
0.0	156.0	48.4
0.005	148.8	50.4
0.010	134.4	54.8
0.015	120.0	60.0
0.020	105.6	66.6
0.025	91.2	74.6
0.030	84.0	80.6

Table 3.4: Parameters for TL3

x (m)	C (pF/m)	L (nH/m)
0.0	130	48.4
0.005	124	50.4
0.010	112	54.8
0.015	100	60.0
0.020	88	66.6
0.025	76	74.6
0.030	70	80.6

Table 3.5: Parameters for TL4

x (m)	C (pF/m)	L (nH/m)
0.0	130	48.4
0.007	124	50.4
0.013	112	54.8
0.020	100	60.0
0.027	88	66.6
0.030	76	74.6
0.04	70	80.6

Table 3.6: Parameters for TL5

x (m)	C (pF/m)	L (nH/m)
0.0	195	80.1
0.003	186	84.1
0.007	168	91.4
0.010	150	100
0.013	132	111
0.017	114	124
0.020	105	134

Figure 3.10: Response of the 7 Nonuniform Transmission Line Model Using NILT

EXAMPLE 4:

A large circuit consisting of 35 nonuniform transmission lines is formed by cascading five identical sections of the circuit in Fig. 3.9. The 35 transmission line interconnect model is shown in Fig. 3.11. The response due to an input pulse of 1 Volt with rise/fall times of 20 pico seconds and pulse width 4 nano seconds is evaluated at nodes A, B and C, the responses are plotted in Fig. 3.12, 3.13 and 3.14 respectively.

The NILT based analysis used 50 time points. The CPU cost associated with this solution is 2798.46 seconds on 4/75 SUN SPARC 2.

Figure 3.11: Interconnect model with 35 nonuniform transmission lines

Figure 3.12: Response at Node A of the 35 Nonuniform Transmission Line Circuit Using NILT

Figure 3.13: Response at Node B of the 35 Nonuniform Transmission Line Circuit Using NILT

Figure 3.14: Response at Node C of the 35 Nonuniform Transmission Line Circuit Using NILT

3.6 Summary

- I. In this chapter nonuniform transmission line equations are formulated as linear ODEs and are solved as an initial value problem using Runge-Kutta numerical integration. Transmission line equations are formulated in the frequency domain at a preassigned complex frequency as required by the NILT analysis.
- II. Stability issues associated with the nonuniform transmission line equations are discussed. An efficient algorithm is proposed to overcome the inherent initial value instability encountered in the solution of transmission line equations. It is shown that integrations up to the onset of numerical instability is adequate to derive a Y parameter representation for terminal electrical behavior.
- III. The proposed method is applicable to nonuniform lines with frequency dependent parameters. There is no additional computational cost involved due to the frequency dependence of line parameters.
- IV. Simulation results are compared with published numerical methods and examples show that the proposed method is accurate and efficient. Greater saving in CPU time is expected with the application of parallel processing techniques [47].

Chapter 4

AWE Based Analysis

4.1 Introduction

The Asymptotic Waveform Evaluation (AWE) technique is a waveform estimation technique developed at Carnegie Mellon University for the timing analysis of linear lumped RLC circuits [31]. The method was later extended to include linear distributed elements [32,33]. The clear advantage of the AWE simulation scheme over other numerical techniques is its extremely low CPU cost equivalent to a single DC solution. The method relies on computing the time moments of the circuit response. These moments are then matched via Pade approximation to a reduced order rational function model. The moment computations are usually accomplished more rapidly than a complete solution of

the network, this allows AWE to compute responses several orders faster than conventional simulation methods. The idea of applying the AWE technique for the analysis of large scale circuits is gaining wide popularity. The AWE has been applied to uniform lossy multiconductor transmission line components [32,33]. In this chapter the AWE is applied to nonuniform lossy multiconductor transmission line networks. The moments are evaluated by numerical integration and some examples are provided to demonstrate the advantages of the AWE based analysis over other simulation methods.

4.2 Formulation Of Linear Network Equations

Consider a linear network π which contains linear lumped components and arbitrary linear subnetworks. The linear lumped components can be described by equations in either the frequency or the time domain. The arbitrary linear subnetworks may contain uniform or nonuniform distributed elements that are best described in the frequency domain. Without loss of generality the modified nodal admittance (MNA) matrix equations of the network π with an impulse input excitation may be represented as [32]

$$C_{\pi} \frac{d}{dt} v_{\pi}(t) + G_{\pi} v_{\pi}(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e_{\pi}(t) = 0 \quad (16)$$

where,

$(G_{\pi}, C_{\pi}) \in \mathcal{R}^{N_s \times N_s}$, are constant matrices with entries determined by the lumped linear components.

$v_{\pi}(s) \in \mathcal{R}^{N_s}$ is the vector of node voltage waveforms appended by independent voltage source current and linear inductor current waveforms.

$b \in \mathcal{R}^{N_s}$ is a constant vector representing the independent sources.

N_s is the number of linear subnetworks.

The terminal behavior of the k^{th} linear subnetwork is described in the frequency domain as

$$A_k V_k + B_k I_k = 0 \quad (17)$$

When the k^{th} subnetwork is a system of coupled transmission lines the entries of the matrices $(A_k, B_k) \in \mathcal{R}^{n_k \times n_k}$ are determined by solving transmission line equations.

$(V_k, I_k) \in \mathcal{R}^{n_k}$ are the terminal voltages and currents respectively, of the k^{th} subnetwork.

$D_k = [d_{i,j}]$, $d_{i,j} \in \{0, 1\}$, $i \in \{1, 2, \dots, N_s\}$, $j \in \{1, 2, \dots, n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $I_k \in \mathcal{R}^{n_k}$ the vector of currents entering the linear subnetwork k , into the node space $\mathcal{R}^{N_s}$ of the network π , N_s is the number of linear subnetworks.

In the frequency domain (16) may be represented in a matrix form as [32]

$$\begin{bmatrix} (sC_\pi + G_\pi) & D_1 & D_2 & \dots & D_{N_s} \\ A_1 D_1^t & B_1 & 0 & \dots & 0 \\ A_2 D_2^t & 0 & B_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{N_s} D_{N_s}^t & 0 & 0 & \dots & B_{N_s} \end{bmatrix} \begin{bmatrix} V_\pi(s) \\ I_1(s) \\ I_2(s) \\ \vdots \\ I_{N_s}(s) \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (18)$$

Equation (18) may be written as

$$Z(s) = [Y_\pi^{-1}(s)] E \quad (19)$$

where,

$$Z(s) = [V_{\pi}(s) \ I_1(s) \ I_2(s) \ \dots \ I_{N_j}(s)]^t \quad (20)$$

$$E = [b \ 0 \ 0 \ \dots \ 0]^t \quad (21)$$

and the superscript t denotes transpose.

Example

To illustrate the formulation method consider the network shown in Fig. 3.1. The network can be described by (16) where the matrices C_{π} , G_{π} and D_1 are identical to the example in section 3.2.

$$Z(s) = [V_{\pi}(s) \ I_1(s)]^t$$

$$I_1(s) = [I_a \ I_b \ I_c \ I_d]^t$$

$$V_{\pi}(s) = [V_1 \ V_2 \ V_3 \ V_4]^t$$

$$b = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^t$$

A_1 and B_1 are 4×4 matrices such that

$$A_1 \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + B_1 \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} = 0$$

A_1 and B_1 are evaluated by solving transmission line equations which is described in the following sections.

4.3 Asymptotic Waveform Evaluation (AWE)

4.3.1 Basic Method

The Asymptotic Waveform Evaluation (AWE) [31] method approximates the transient response of a network by a Maclaurin's series [32]

$$Z(s) = \sum_{n=0}^{\infty} M_n s^n \quad (22)$$

where the moment vectors are given by

$$M_n = \frac{\frac{\partial^n}{\partial s^n} [Y_{\pi}^{-1}] \big|_{s=0}}{n!} E \quad (23)$$

where, E represents the inputs. The moments $m_n = [M_n]_{(i)}$; $n = 0, 1, 2, \dots, 2q-1$ of an output i are then matched to a lower order q -pole frequency domain function

$$[Z^*(s)]_{(i)} = \sum_{j=1}^q \frac{k_j}{s-p_j} \quad (24)$$

where, p_1 through p_q are the complex approximating poles and k_1 through k_q are their corresponding residues. Hence, the time domain response is approximated as

$$[z^*(t)]_{(i)} = \sum_{j=1}^q k_j e^{p_j t} \quad (25)$$

The reduced model characteristic polynomial of the system is represented as

$$a_0 + a_1 p^{-1} + a_2 p^{-2} + \dots + a_{q-1} p^{-q+1} + p^{-q} = 0 \quad (26)$$

where the coefficients in (26) are related to the moments by

$$\begin{bmatrix} m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ m_2 & m_3 & m_4 & \dots & m_{q+1} \\ \dots & \dots & \dots & \dots & \dots \\ m_{q-1} & m_q & m_{q+1} & \dots & m_{2q-2} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \dots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_q \\ m_{q+1} \\ m_{q+2} \\ \dots \\ m_{2q-1} \end{bmatrix} \quad (27)$$

and the roots of (26) are the reciprocal poles. The residues k_i are related to the poles and the moments by

$$\begin{bmatrix} p_1^{-1} & p_2^{-1} & p_3^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & p_3^{-2} & \dots & p_q^{-2} \\ p_1^{-3} & p_2^{-3} & p_3^{-3} & \dots & p_q^{-3} \\ \dots & \dots & \dots & \dots & \dots \\ p_1^{-q} & p_2^{-q} & p_3^{-q} & \dots & p_q^{-q} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \dots \\ k_q \end{bmatrix} = - \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ \dots \\ m_{q-1} \end{bmatrix} \quad (28)$$

4.3.2 Generalized AWE

Recently the AWE technique was extended to include distributed networks [32]. The method involves the application of Leibnitz's rule to express the moments in a recursive relation as

$$[Y_\pi]^0 M_n = - \sum_{r=1}^n \frac{Y_\pi^{(r)} M_{n-r}}{r!} \quad (29)$$

$$[Y_\pi]^0 M_0 = E \quad (30)$$

where the superscript (r) indicates the r^{th} derivative at $s = 0$ and $[Y_\pi]^0 = [Y_\pi]_{s=0}$.

Next, the derivatives of Y_π are obtained as

$$[Y_\pi]^{(1)} = \begin{bmatrix} C_\pi & 0 & 0 & \dots & 0 \\ A_1^{(1)} D_1^t & B_1^{(1)} & 0 & \dots & 0 \\ A_2^{(1)} D_2^t & 0 & B_2^{(1)} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ A_{N_s}^{(1)} D_{N_s}^t & 0 & 0 & \dots & B_{N_s}^{(1)} \end{bmatrix} \quad (31)$$

and

$$[Y_\pi]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ A_1^{(r)} D_1^t & B_1^{(r)} & 0 & \dots & 0 \\ A_2^{(r)} D_2^t & 0 & B_2^{(r)} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ A_{N_s}^{(r)} D_{N_s}^t & 0 & 0 & \dots & B_{N_s}^{(r)} \end{bmatrix} \quad (32)$$

When the subnetwork k consists of a multiconductor nonuniform transmission line system the derivatives $A_k^{(r)}$ and $B_k^{(r)}$ are obtained as described in the following section.

Some drawbacks of AWE have been identified in recent research [36,37]. The AWE method approximates the network poles and the corresponding residues by a moment-matching technique and the Maclaurin's series expansion of the output frequency response function. Large variations of errors is reported for different networks at different outputs. Increasing the order of approximation may generate right-half plane poles for stable circuits. In [36] it is suggested that the most accurate set of approximating poles for a particular output are not necessarily generated at that same output. A criterion based on moment skew is proposed for selecting the optimal set of poles corresponding to a particular output. In [37] a stable method of generalized moment matching is presented. A set of moment matching approximations are generated in the form of a Pade table. Based on the

smallest pole and the overall moment skew a stable selection criterion is established for selecting an optimal moment matching set.

4.4 Nonuniform Transmission Line Moments

4.4.1 Basis Of The Method

Transmission line equations in the frequency domain may be represented in the matrix form as,

$$\frac{d}{dx} \begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix} = \left\{ \begin{bmatrix} 0 & -R(x) \\ -G(x) & 0 \end{bmatrix} + s \begin{bmatrix} 0 & -L(x) \\ -C(x) & 0 \end{bmatrix} \right\} \begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix} \quad (33)$$

where, $R(x, s)$, $L(x, s)$, $C(x, s)$, $G(x, s)$ are the per unit length parameters of the transmission lines, $x \in [0, d]$ is the distance along the lines and s is the frequency. If N is the number of coupled lines then,

$$\{R(x), L(x), C(x), G(x)\} \in \mathfrak{R}^{N \times N}$$

$$\{V(x, s), I(x, s)\} \in \mathfrak{R}^N$$

We define the system matrices,

$$G_T(x) = \begin{bmatrix} 0 & -R(x) \\ -G(x) & 0 \end{bmatrix} \quad (34)$$

$$C_T(x) = \begin{bmatrix} 0 & -L(x) \\ -C(x) & 0 \end{bmatrix} \quad (35)$$

We assume $G_T(x)$ and $C_T(x)$ are piecewise continuous in x , which is a practical

assumption. The solution of (33) is the State Transition Matrix (STM) of the system which describes terminal behavior and is given by

$$\begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix} = H(x, s) \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} \quad (36)$$

Since the STM satisfies its own system equations with unity initial condition, we have

$$\frac{d}{dx} H(x, s) = \{ G_T(x) + s C_T(x) \} H(x, s) \quad (37)$$

$$H(0, s) = U \quad (38)$$

To obtain the nonuniform transmission line moments we evaluate moments of the STM by using a Maclaurin's series expansion

$$H(x, s) = H_0(x) + s H_1(x) + s^2 H_2(x) + \dots + s^{2q-1} H_{(2q-1)}(x) \quad (39)$$

Substituting (39) into (37) and (38) and equating coefficients of like powers of s we get the following sets of equations. The variable x has been dropped for convenience.

$$H'_0 = G_T H_0 \quad H_0(0) = U \quad (40a)$$

$$H'_1 = G_T H_1 + C_T H_0 \quad H_1(0) = 0 \quad (40b)$$

$$H'_2 = G_T H_2 + C_T H_1 \quad H_2(0) = 0 \quad (40c)$$

$$\dots\dots \quad \dots\dots \quad \dots\dots$$

$$H'_{(2q-1)} = G_T H_{(2q-1)} + C_T H_{(2q-2)} \quad H_{(2q-1)}(0) = 0$$

The primed quantities represent derivatives with respect to x .

4.4.2 The Proposed Method

The sets of equations (40) are solved by numerical integration. In this

research a 4th/5th order explicit Runge-Kutta numerical integrator is used, however, any other integration scheme may be employed. In the absence of instabilities straight forward numerical integration of (18) from $x = 0$ to $x = d$ is performed to obtain the required derivatives H_r , $r = 0, 1, 2, \dots, 2q - 1$. Let,

$$H_r = \begin{bmatrix} h_1^{(r)} & h_2^{(r)} \\ h_3^{(r)} & h_4^{(r)} \end{bmatrix} \quad (41)$$

be a partition of the r^{th} derivative into four equal square submatrices. By a simple reordering of terms we have, for the k^{th} multiconductor transmission line subnetwork.

$$A^{(r)} = \begin{bmatrix} h_1^{(r)} & -U^{(r)} \\ h_3^{(r)} & 0 \end{bmatrix} \quad B^{(r)} = \begin{bmatrix} h_2^{(r)} & 0 \\ h_4^{(r)} & -U^{(r)} \end{bmatrix}$$

$$r = 0, 1, 2, \dots, (2q - 1)$$

The subscript k has been dropped for simplicity.

4.4.3 Stability Considerations For Lossless Lines

When resistive and dielectric losses in the interconnects are negligible it can be shown that the system (40) is stable. It is well known that the stability properties of a set of linear ODEs is governed by the homogenous equation. It is shown in [45] that a system of linear ODEs is stable if the corresponding fundamental solution matrix is bounded by a constant. For lossless lines the system matrix $G_T(x) = 0$. Hence the fundamental matrix corresponding to the homogenous equation is the unity matrix. Since all derivatives H_r , $r = 0, 1, 2, \dots, 2q - 1$ satisfy the same homogenous equations it is there-

fore proved that the system (40) is stable.

4.4.4 Stability Considerations For Practical Interconnects

Experiments with single uniform transmission lines have shown that it is very unlikely for practical interconnects to present instabilities in numerical integration. Practical lines have finite lengths which means that only rapidly increasing and rapidly decreasing solution modes could lead to instabilities. It is noticed that the system matrix $G_T(x)$ decides the stability properties of the system (40). The resistive and dielectric loss parameters are decided by the specific VLSI process employed. Usually resistive losses are of the order of $m\Omega/(unit\ length)$ and dielectric losses are of the order of $nS/(unit\ length)$ which suggests that the solution modes (eigen values in the case of uniform lines and kinematic eigen values in the case of nonuniform lines) are of order of $\pm 10^{-3}$ to $\pm 10^{-6}$. This implies that the length of the lines needs to be several orders of the *unit length* before instabilities are encountered. Because of the finite lengths and small dimensions of VLSI interconnects it is fair to say that instabilities in (40) are unlikely.

When circuit equations are formulated for the NILT based simulation as (2) the system matrix is a function of the complex frequency $s = z_i/t$ as defined in section 3.3. At time points close to the origin the order of magnitude of the entries in the system matrix could be very large. Hence instabilities in numerical integration under the NILT based simulation are expected to set in over short intervals along the lines. When circuit equations are formulated for the AWE based simulation as (40) the system matrix is

dependent on the resistive and dielectric losses alone and as discussed above instabilities are unlikely.

4.5 Simulation Results

EXAMPLE 1: A prototypical chip -interconnect The prototype chip interconnect of Fig. 3.6 is analyzed using the AWE based technique. The response is shown in Fig. 4.1 and Fig. 4.2. The results compare accurately with the spectral method [20] and the proposed NILT based analysis, Fig. 3.7 and Fig. 3.8. CPU times are listed in Table 4.1. The advantage of the AWE based analysis is its high efficiency in terms of CPU time saving.

Figure 4.1: Response of Prototype Chip Interconnect Using AWE

Figure 4.2: Response of Prototype Chip Interconnect Using AWE

EXAMPLE 2: Uniform lines

Shown in Fig. 4.3 is an interconnect model containing two lossy coupled uniform transmission lines. Both sets of lines are 0.1m long. The parameters for the two conductor lines are

$$L = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} nH/m$$

$$C = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} pF/m$$

$$R = \begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \Omega/m$$

$$G = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} S/m$$

The parameters for the four conductor lines are

$$L = \begin{bmatrix} 494.6 & 63.3 & 7.8 & 0.0 \\ 63.3 & 494.6 & 63.3 & 7.8 \\ 7.8 & 63.3 & 494.6 & 63.3 \\ 0.0 & 7.8 & 63.3 & 494.6 \end{bmatrix} nH/m$$

$$C = \begin{bmatrix} 62.8 & -4.9 & -0.3 & 0.0 \\ -4.9 & 62.8 & -4.9 & -0.3 \\ -0.3 & -4.9 & 62.8 & -4.9 \\ 0.0 & -0.3 & -4.9 & 62.8 \end{bmatrix} pF/m$$

$$R = \begin{bmatrix} 50 & 10 & 1 & 0 \\ 10 & 50 & 10 & 1 \\ 1 & 10 & 50 & 10 \\ 0 & 1 & 10 & 50 \end{bmatrix} \Omega/m$$

$$G = \begin{bmatrix} 0.1 & -0.01 & -0.001 & 0.0 \\ -0.01 & 0.1 & -0.01 & -0.001 \\ -0.001 & -0.01 & 0.1 & -0.01 \\ 0.0 & -0.001 & -0.01 & 0.1 \end{bmatrix}$$

The applied voltage is a pulse with 0.1 nanosecond rise/fall time and 4 nanoseconds pulse width. The results from the proposed AWE based analysis are compared with the eigen analysis method of [32] which is applicable to uniform lines only. The response is shown in Fig. 4.4. There is perfect agreement between the two solutions. The CPU times are listed in Table 4.1. The computational cost of the proposed method is expected to be higher than the eigen analysis solution which is a specialized technique applicable to uniform lines only.

Figure 4.4: Response of Lossy Uniform Multiconductor Transmission Line Model Using AWE.

EXAMPLE 3:

The circuit of Fig. 3.9 is analyzed using the AWE based analysis of order 6. The response of the circuit to a pulse input as described in Section 3.5, Example 3, is shown in Fig. 4.5.

The response matches with the response from the NILT based analysis. The CPU comparisons in Table 4.1 indicate the low CPU cost associated with the AWE based analysis.

EXAMPLE 4: 35 transmission line circuit

A large circuit containing 35 nonuniform transmission lines shown in Fig. 3.11 is analyzed using the AWE based analysis of order 6. The response to a pulse input as defined in section 3.5, example 4, is evaluated at node A, B and C (Fig. 3.11) is shown in Fig. 4.6, 4.7 and 4.8 respectively. The CPU cost associated with this example is listed in Table 4.1. The results from the AWE based simulation is compared with those from the NILT based simulation from example 4, section 3.5. The low CPU cost associated with this example demonstrates the applicability of the AWE based analysis to large scale circuits.

It is noticed that the response has higher frequency components for nodes closer to the source. For nodes further away from the source the response is smoother. This may be attributed to the low pass filtering effects of the transmission lines and the lumped components in the circuit. It is also noticed that the AWE based response matches the NILT based response more closely as the response becomes smoother. In order to improve accuracy of the AWE based analysis, improvements to the AWE suggested in [36,37] may be employed.

Figure 4.5: Response of the 7 Nonuniform Transmission Line Circuit Using AWE.

Figure 4.6: Response at Node A of the 35 Nonuniform Transmission Line Circuit Using AWE.

Figure 4.7: Response at Node B of the 35 Nonuniform Transmission Line Circuit Using AWE.

Figure 4.8: Response at Node C of the 35 Nonuniform Transmission Line Circuit Using AWE.

Table 4.1: CPU seconds on 4/75 SUN SPARC 2

(seconds)	NILT	AWE
Circuit 1	203	not applica- ble
Circuit 2	97	2.9
Circuit 3	564.62	4.03
Circuit 4	2798.46	17.77
Circuit 5	X	2.99

Circuit 1: Nonuniform parabolic frequency dependent line (Figure. 3.3)

Circuit 2: Prototype chip interconnect (Figure. 3.6)

Circuit 3: Seven nonuniform transmission line circuit (Figure. 3.9)

Circuit 4: Large circuit with 35 nonuniform transmission lines (Figure. 3.11)

Circuit 5: Coupled uniform transmission lines (Figure. 4.3)

4.6 Summary

- I. In this chapter the linear ODEs for the derivatives of the STM are derived and are solved by Runge-Kutta numerical integration. The nonuniform transmission line moments are evaluated from the derivatives using recursive relations.
- II. The case when transmission line equations are numerically stable is addressed. An argument based on the order of magnitude of the eigen values is presented to show that instabilities in formulation (40) are unlikely.
- III. Simulation results are compared with published numerical techniques. CPU measurements show that the AWE based analysis not only provides excellent accuracy for transient response but also has a large CPU advantage over most other numerical methods and is therefore ideal for large scale systems.

Chapter 5

Analysis of Transmission Line Networks With Nonlinear Terminations

5.1 Introduction

Recently [38,39] the transient simulation of nonlinear networks containing linear lumped components, nonlinear components and arbitrary linear subnetworks was described. The method is based on the Piecewise Decomposition Technique (PDT) where the nonlinear elements of the network are replaced by time-dependent sources. The resulting linear network is analyzed in the frequency domain and its transient response is computed using the Asymptotic Waveform Evaluation technique. The parameters defining the time-dependent sources are evaluated iteratively such that the topological and constitutive relations of the nonlinear network are satisfied. A piecewise representation of the time-

dependent sources is used to avoid the need for reevaluating the transient response of the transformed linear network at each iteration.

The PDT method effectively partitions the network, Fig. 1.1, into a purely linear part and a purely nonlinear part. The linear portion of the network is analyzed using either the formulation of (1) or (16) appended by the time-dependent linear sources. Hence the proposed methods discussed in chapter 3 and chapter 4 are fully compatible with the PDT method which allows nonlinear networks containing nonuniform transmission lines to be analyzed.

5.2 The Piecewise Decomposition Technique

5.2.1 Formulation Of Nonlinear Network Equations

Consider a nonlinear network π containing lumped components, nonlinear elements and arbitrary linear subnetworks. The linear subnetworks may contain distributed elements such as nonuniform transmission lines. The modified nodal admittance matrix equations of the network may be written as [33]

$$C_{\pi} \frac{d}{dt} v_{\pi}(t) + G_{\pi} v_{\pi}(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e_{\pi}(t) - f(v_{\pi}(t)) = 0 \quad (42)$$

where, $f(v_{\pi}): \mathcal{R}^{N_s} \rightarrow \mathcal{R}^{N_s}$ is a function describing the nonlinear elements of the network.

The Other entries of (42) are as defined in (1).

The nonzero entries of $f(v_{\pi}(t))$ are replaced by a set of piecewise linear independent waveforms $x(t)$, Fig. 5.2, such that [39]

$$f(v_{\pi}(t)) = D_f x(t) \quad (43)$$

where, $x(t) \in \mathcal{R}^{N_f}$, $D_f = [d_{ij}]$, $d_{ij} \in \{0, 1\}$, $i \in \{1, 2, \dots, N_f\}$, $j \in \{1, 2, \dots, N_{\pi}\}$ is

a selector matrix, and N_f is the number of nonzero entries in $f(v_\pi(t))$. Using (43), (42) is reduced to a set linear differential equations of the form

$$C_\pi \frac{d}{dt} v_\pi(t) + G_\pi v_\pi(t) + \sum_{k=1}^{N_f} D_k i_k(t) - e_\pi(t) - D_f x(t) = 0 \quad (44)$$

It can be seen that (44) has a form similar to (1) and (16) except for the additional term due to the piecewise linear time dependent sources.

5.2.2 Solution Of The Linear Subnetwork

With reference to Fig. 5.2, divide the interval of analysis $t \in [0, T]$ into L equal intervals such that $t_k - t_{k-1} = \Delta$ and $L\Delta = T$. The piecewise linear function $x(t)$ may be expressed as a scaled and shifted sum of the functions $u_h(t)$ and $u_g(t)$, where

$$u_h(t) = \text{diagonal} \{ u(t) - u(t - \Delta) \} \in \mathcal{R}^{N_f \times N_f} \quad (45)$$

$$u_g(t) = \text{diagonal} \{ t(u(t) - u(t - \Delta)) \} \in \mathcal{R}^{N_f \times N_f} \quad (46)$$

$u(t)$ is the unit step function.

The response of the linear network to an input $x(t)$ may then be constructed using the response to the inputs $u_h(t)$ and $u_g(t)$ and superposition theory as [39]

$$v_\pi(t) = \sum_{k=1}^L H(t - t_{k-1}) x_{k-1} + \sum_{k=1}^L G(t - t_{k-1}) \frac{x_k - x_{k-1}}{\Delta} + F(t) \quad (47)$$

where

$$x_k = x(t_k)$$

$F(t)$ is the response due to the independent inputs, $G(t)$ and $H(t)$ are the matrices

Figure 5.1: Piecewise linear waveform.

$$G(t) = \{g_{ij}\} ; i \in \{1, 2, \dots, N_\pi\} \quad j \in \{1, 2, \dots, N_f\} \quad (48)$$

$$g_{ij}(t) = [v_\pi(t)]_i \begin{cases} x_j(t) = u_g(t) \\ x_k(t) = 0, \quad k \neq j \\ e(t) = 0 \end{cases} \quad (49)$$

$$H(t) = \{h_{ij}\} ; i \in \{1, 2, \dots, N_\pi\} \quad j \in \{1, 2, \dots, N_f\} \quad (50)$$

$$h_{ij}(t) = [v_\pi(t)]_i \begin{cases} x_j(t) = u_h(t) \\ x_k(t) = 0, \quad k \neq j \\ e(t) = 0 \end{cases} \quad (51)$$

$F(t)$, $G(t)$ and $H(t)$ can be obtained by analyzing the linear network using either the NILT analysis [39] or the AWE analysis [33] as described in the previous sections and need only be solved once for a given network.

5.2.3 Solution Of The Nonlinear Subnetwork

The augmenting waveforms $x(t)$ are computed starting with an initial guess $x^0(t)$ and an iterative technique [33] based on the Newton-Raphson method. Iteration continues until the waveforms $x(t)$ satisfy the linear network constraints (44) and the nonlinear constraints (43).

To use Newton-Raphson iterations the nonlinear equations (43) are discretized at L time points resulting in

$$f(v_\pi(t_r^*)) - D_f x(t_r^*) = 0 \quad (52)$$

where, $t_{r-1} < t_r^* \leq t_r$ and $r \in \{1, 2, \dots, L\}$. Let $x_r = x(t_r)$ and $v_r^* = v_\pi(t_r^*)$. Using (47) v_r^* may be represented explicitly as

$$v_r^* = v_r(t_r^*) = R_r x_r + S_r \quad (53)$$

where,

$$R_r = \frac{G(t_r^* - t_{r-1})}{\Delta} \quad (54)$$

$$S = \sum_{k=1}^r \left[H(t_r^* - t_{k-1}) - \frac{G(t_r^* - t_{k-1})}{\Delta} \right] x_{k-1} + \sum_{k=1}^{r-1} \left[\frac{G(t_r^* - t_{k-1})}{\Delta} \right] x_k + F(t_r^*) \quad (55)$$

At each time point Newton-Raphson iterations are formulated as

$$J^{(i)} [x_r^{(i+1)} - x_r^{(i)}] = -[f(v_r^{*(i)}) - D_f x_r^{(i)}] \quad (56)$$

$$J = \left[\frac{df}{dv} \right] R_r - \frac{t_r^* - t_{r-1}}{\Delta} D_f \quad (57)$$

5.3 Summary

In this chapter The PDT method is discussed. It is shown that formulation of linear circuit equations in the PDT method as (44) have a form similar to the linear network equations in the NILT formulation as (1) and the AWE formulation as (16). Hence, the proposed methods discussed in chapters 3 and 4 are fully compatible with the PDT method.

By choosing a piecewise linear representation for the augmenting time dependent sources, the linear network need only be analyzed once. Hence, the method is efficient and the desired accuracy may be obtained by choosing shorter time intervals.

Chapter 6

Conclusion And Future Research

6.1 Summary

Two promising methods are proposed for the transient time domain analysis of nonuniform high-speed interconnects. Both methods do not impose any restrictions on the shape or the nature of coupling between the lines. Lossy multiconductor uniform and nonuniform transmission lines may be analyzed. The NILT based method also accounts for the frequency dependence of transmission line parameters. Transmission line equations are formulated as linear ordinary differential equations and are solved using numerical integration. Stability issues associated with the solution of transmission line equations are considered. For the NILT based analysis an efficient algorithm is proposed to overcome initial value instability. The accuracy of the proposed algorithm is verified by

comparisons with closed form expressions and published techniques. The efficiency of the NILT based technique may be improved by the use of parallel processors [47].

The AWE based method is shown to provide a large saving in the computational cost and is also accurate. Examples show that the AWE based method is well suited for transient analysis of large scale systems containing a large number of nonuniform interconnects. Only the case when transmission line equations are stable is addressed. However, it is argued that lossless lines and most practical interconnects fall into this category.

A theoretical discussion is presented to show that both the NILT and AWE based methods are compatible with the Piecewise Decomposition Technique (PDT). Hence, transmission line networks containing nonlinear terminations may be analyzed without any modifications to the proposed methods. These methods have promising applications in VLSI chip design, packaging, MCM design, PCB design and overall system design.

6.2 Future Research

- I. It is shown in [48] that diffusion equations in nuclear physics have the same form as nonuniform transmission line equations. The idea of "critical lengths" reported in [48] has a close resemblance to the definitions of the forward and backward "singular" distances defined in the thesis. Theoretical studies in this direction would provide an insight into nonuniform transmission lines.
- II. It is desirable to employ other numerical integration schemes such as backward difference formulas, in the event that nonuniformities lead to stiff systems.
- III. A boundary value approach suggested in [45] may be applied for the solution of numerically unstable equations under the AWE formulation.
- IV. Several improvements to the AWE [36,37] and NILT [29,30,47] may be applied to enhance the capabilities of the proposed methods.

- V. Alternative inversion schemes such as the improved inverse FFT method [41] and the numerical inversion of Laplace transform using Laguerre polynomials may be considered.
- VI. Alternative solution methods to transmission line equations using spectral methods may be considered [20].
- VII. Analysis of nonuniform lines with frequency dependent parameters in the AWE formulation may be attempted.

2 OF/DE 2

PM-1 3 1/2" x 4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

APPENDIX A: Integrations over the singular distances

RESULT 1:

A Y parameter stamp evaluated either over the forward “singular” distance x_1 or the backward “singular” distance x_2 may be approximated as a block diagonal matrix without loss of accuracy in the final response.

Proof:

Forward integrations over the forward “singular” distance x_1 gives an estimate of the norm of the forward transmission submatrix as

$$\begin{aligned}\|h_{f12}(x_1)\| &\cong GROWTOL \\ \Rightarrow \|h_{f12}^{-1}(x_1)\| &\cong \frac{1}{GROWTOL} \\ \Rightarrow \|h_{f12}^{-1}(x_1)\| &\ll \|h_{f22}(x_1)h_{f12}^{-1}(x_1)\|\end{aligned}$$

By choosing GROWTOL to be sufficiently large the upper diagonal submatrix in (12) will be several orders of magnitude smaller than the diagonal submatrices. By approximating the upper off diagonal submatrix by a zero matrix there is no significant loss of accuracy in the final response. Since the nonuniform line is a reciprocal network we may also approximate the lower off diagonal submatrix by a zero matrix.

Similarly, backward integrations over the backward “singular” distance x_2 gives an estimate of the norm of the backward transmission submatrix as

$$\|h_{b12}(x_2)\| \cong GROWTOL$$

$$\Rightarrow \|h_{b12}^{-1}(x_2)\| \cong \frac{1}{GROWTOL}$$

$$\Rightarrow \|h_{b12}^{-1}(x_2)\| \ll \|h_{b22}(x_2) h_{b12}^{-1}(x_2)\|$$

Following the same argument as before the Y parameter stamp of (12) may be approximated by zero matrices in the off diagonal blocks.

APPENDIX B: The Theta Operator

DEFINITION OF THE Θ OPERATOR:

Let subscripts 'a' and 'b' denote the two cascaded sections respectively, and 'c' denotes the combined 2-section cascade. The Θ operator over Y parameters of the individual sections produces the Y parameters of the cascade and is defined as

$$\begin{bmatrix} y_{a11} & y_{a12} \\ y_{a21} & y_{a22} \end{bmatrix} \Theta \begin{bmatrix} y_{b11} & y_{b12} \\ y_{b21} & y_{b22} \end{bmatrix} = \begin{bmatrix} y_{c11} & y_{c12} \\ y_{c21} & y_{c22} \end{bmatrix}$$

where,

$$y_{c11} = y_{a11} - y_{a12} (y_{a22} + y_{b11})^{-1} y_{a21}$$

$$y_{c12} = -y_{a12} (y_{a22} + y_{b11})^{-1} y_{b12}$$

$$y_{c21} = -y_{b21} (y_{a22} + y_{b11})^{-1} y_{a21}$$

$$y_{c22} = y_{b22} - y_{b21} (y_{a22} + y_{b11})^{-1} y_{b12}$$

$\{y_{aij}, y_{bij}, y_{cij}\}$ are complex square submatrices of equal dimension.

RELEVANT PROPERTIES

Some relevant properties of the operator Θ are listed below. All the properties listed may be easily verified from the definition of the operator. In the following

$\{\alpha_i, \beta_i, \delta_i\} \in \mathbb{C}^{N \times N}$ and $\{\alpha_i, \beta_i, \delta_i\} \neq 0$ for all i .

PROPERTY 1

If $Y_1 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \delta_1 \end{bmatrix}$ and $Y_2 = \begin{bmatrix} \alpha_2 & 0 \\ 0 & \delta_2 \end{bmatrix}$

Then,

$$Y_1 \ominus Y_2 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \delta_2 \end{bmatrix}$$

PROPERTY 2

If $Y_1 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \delta_1 \end{bmatrix}$ $Y_2 = \begin{bmatrix} \alpha_2 & \beta_2 \\ \beta_2 & \delta_2 \end{bmatrix}$ $Y_3 = \begin{bmatrix} \alpha_3 & 0 \\ 0 & \delta_3 \end{bmatrix}$

Then,

$$Y_1 \ominus Y_2 \ominus Y_3 = Y_1 \ominus Y_3$$

PROPERTY 3

If $Y_1 = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \delta_1 \end{bmatrix}$ $Y_2 = \begin{bmatrix} \alpha_2 & \beta_2 \\ \beta_2 & \delta_2 \end{bmatrix}$

and $Y_3 = \begin{bmatrix} \alpha_3 & \beta_3 \\ \beta_3 & \delta_3 \end{bmatrix}$ $Y_4 = \begin{bmatrix} \alpha_4 & 0 \\ 0 & \delta_4 \end{bmatrix}$

and if, $Y_1 \ominus Y_2 = Y_3 \ominus Y_4$

Then,

$$Y_1 \ominus Y_2 = Y_3 \ominus Y_4 = Y_1 \ominus Y_4$$

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