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BOUNDARY ELEMENT ANALYSIS OF BUILDING FRAMES

BY
PEILEI CHEN, B.Eng

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Department of Civil Engineering
Faculty of Engineering
Carleton University
Ottawa, Ontario
6th September

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The undersigned recommend to The Faculty of Graduate Studies
and Research acceptance of the thesis

**BOUNDARY ELEMENT ANALYSIS
OF BUILDING FRAMES**

submitted by Peilei Chen, B. Eng.
in partial fulfillment of the requirements for
the degree of Master of Engineering

A handwritten signature in black ink, appearing to read "S. H. H. H.", is written over a horizontal line.

Thesis Supervisor

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ABSTRACT

The Direct Boundary Element Method (DBEM) has the potential to provide more accurate analysis for design quantities of interest within the domain of simple thin plate subjected to general boundary and load configurations. The method is extended to be used for the structural analysis of flat plate building frames. The DBEM is used to model Flat-Plate floors. The connections of column and slab are modelled using the Influence Coefficient Method (ICM).

The DBEM for plate bending in building frame analysis is reviewed in this thesis. The column patch modelling and the “patch” point configurations are studied by the analysis of several simple building frames. The effects of boundary element refinement and polynomial order are examined. A building frame model with more complicated floor layout is fabricated for generic studies, and the experimental model was tested to demonstrate the suitability of DBEM for building frame analysis.

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2. to study boundary element configurations and column interface models in order to provide recommendations for future users of the program;
3. to analyze and test an experimental building frame model which includes some of the complexity of real building frame;
4. to improve the programs used for the analysis of flat plate building frame.

The building frames may be non-symmetric and have irregular column layout. They may carry both vertical and horizontal load; and output of the programs can give the displacements of each floor, internal reactions of columns and also plate internal actions.

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1. INTRODUCTION

1.1 General

Flat plate structural building frames are of reinforced concrete construction. The frames consist of floor slabs supported directly by columns, without beams and girders, as shown in Fig. 1.1. The "flat plate" system carries loads in the two spanning directions of each slab. In order for the action to be 2-way, it is necessary to restrict each slab dimensional aspect ratio to 2. This is a requirement for use of certain code procedures. The columns frame monolithically into the slab. [12, 25, 31]

Fig. 1.1 Flat Plate Building Frame

Flat plate building frames offer many advantages over conventional beam and girder floors, such as providing larger open floor-space, permitting a given minimum clear height to be maintained with a reduced overall story height, simplifying the formwork and allowing

complete freedom of placement of internal partitions beneath an uninterrupted level soffit. The most important advantages are economy in construction and a clean unbroken appearance. Because of these advantages, the structures are commonly used in hotels, offices and residential buildings. They are therefore a very common structural system. Because the slab itself becomes a grid of two-way shallow beams supported directly on the columns, the columns tend to punch upward through the slab, incurring high shearing stresses which occur at the junction of column and plate. These may require special reinforcement. This means that the structure may be employed generally, when the spans are not large and the live loads are relatively small.

Since the structure is used extensively, the accurate and rapid analysis of the elements of the structure is therefore of great importance for the engineer to ensure safety and to provide an economical design. Two main problems which affect the accuracy of frame analysis have to be studied. They are the analysis of the plate; and the modelling of the column-slab connection. The names of Kirchoff, Neuber, Bubrov, Timoshenko, Galerkin, Vlassov, Kalmanok and Girkmann are inseparably associated with the fundamentals of the classic theory of plates but large numbers of other engineers in all parts of the world have also made significant contributions to the extension and completion of the theory. Until recent time, no methods of analysis had been developed which were suitable for the flat plate building frame with arbitrary geometrical properties and loads. Theoretical solutions exist for flat plates with common layouts and known boundary conditions, however for a plate with unusual boundaries or interacting elements, only approximate techniques can be employed.

The words "building frame" are also used here to define a generic structural type. The analysis approach presented in this thesis is not intended to be exclusive to buildings alone.

The distinguishing features of the present analysis are a super-element (the slab) with "patch" location to which other simpler elements (the columns) are attached. With a building frame stiffness model in mind, the structure ultimately reduces to one having 3 d.o.f (degrees of freedom) per storey and 3 d.o.f per slab-column joint.

1.2 Methods of Analysis for the Flat Plate Building Frame

The flat plate building frame is, as defined above, a flat floor slab with columns fixed to the slab. For the purpose of frame analysis, some assumptions have to be made. The basic assumptions for all analysis methods used for this kind of structure are as follows

- 1) the material of the building frame is assumed to be isotropic and linearly elastic;
- 2) the frame is assumed to undergo small deflections;
- 3) the plate is assumed to be thin.

The column cross section of the frame is considered small compared to the length of the column, so that it can be analyzed as a one-dimensional element. If the column is long in plan (ie. column wall) it is assumed herein that it is analyzed as a one-dimensional element.

With the above assumptions, different methods have been worked out by engineers. These methods include several simplified equivalent frame analysis procedures described in [17], an analytical method for plate analysis, the code methods, the finite element method and the boundary element method. Each method may add additional assumptions relating to the geometry, load conditions and interactions of the structure elements. The accuracy of any method largely depends on how realistically the constituent elements of the frame are mathematically modelled.

1.2.1 Analytical Methods for Plate Analysis

The analytical method assumes that there is a function of the two coordinates in the plane of the plate, which represents the plate deflection w and satisfies a linear partial differential equation and the necessary boundary conditions. All the stress components can then be expressed using the deflection w of the plate. [27, 28, 30]

Using the analytical method, the limitation for the structure is that the flat plates act as continuous plates, with point supports, ie. the plates are supported by rows of columns of negligible flexural rigidity. It is also assumed that the number of spans is large in both

directions. The load acting on the the plate is vertical and uniformly distributed over the whole area. Under these conditions, the bending in all panels which are not close to the boundary of the plate can be considered identical. The problem is reduced to bending in the interior panel only, ie. typical interior span, as shown in Fig. 1.2 [30].

Fig.1.2 Flat Plate Supported by Rows of Equidistant Columns

To simplify the problem further, it is assumed that the cross-sectional dimensions of the columns are small and can be neglected in so far as deflection and moments at the center of the plate are concerned. When the concentrated reactions are acting at the points of support of the plate, the moments calculated from the deflection become infinitely large, therefore the column reactions are further assumed to be uniformly distributed over small area of the column cross section. The assumption of negligible flexural rigidity of the columns can give useable results when the columns carry only vertical reactions but no bending moment. The interaction between column and slab can not be neglected when column spacing is uneven or for alternate span loading. In this case the columns are carrying bending moments and vertical reactions.

It is not an easy task to model the column-slab connection. An approach was introduced by Timoshenko and Woinowsky-Krieger [30] who gave the solution for a circular plate with an internal disk, with a moment acting on the disk (Fig. 1.3). The connection between the disk and the plate is considered absolutely rigid. The rotation of the rigid connection is taken as one of the boundary conditions.

Fig.1.3 Circular Plate with Rigid Disk

This model can be used to describe the column-slab connection in polar coordinates. It is only suitable for circular plates with symmetrical boundary conditions. The exact analytical solutions of deflection are extremely difficult, or in many cases, impossible to obtain for non-symmetric boundary conditions, such as rigid connections which have a square cross section. Infinitely large stresses occur at the corners of the columns in this case. Since the analytical solutions are generally tedious and give results for only a few special cases, it is necessary to introduce a simplified engineering approach.

1.2.2 Code Design Method

The methods in the code [12] follow a more rugged "engineering" approach. The flat plate building frame is divided, for code analysis, into continuous frames centered on the column lines and extending both longitudinally and transversely. Each frame is modeled as a row of columns and a broad continuous beam. A uniformly distributed gravity load acts on the slab which is assumed to be supported by a set of elements in both the transverse and longitudinal directions. The beam elements are located essentially on orthogonal grid lines to form rectangular supported panels with aspect ratios of long to short span not greater than 2.0. ie. this ensures that the structure is restricted to a two-way slab system. A further assumption is that the loading applied to one floor level has a negligible effect on the moments in adjacent floor levels, ie. the different stories can be analyzed separately with the far ends of the column fixed, as shown in Fig. 1.4. The structure is then analyzed as a rigid frame, using one of two methods, namely the Direct Design Method (DDM) and the Equivalent Frame Method (EFM). [12, 17, 25, 31]

Fig. 1.4 Equivalent Frame

Direct Design Method: The word 'direct' here means that, with this method, suitable design moments of the frame elements are obtained by using a set of coefficients without performing detailed analysis. Limitations on continuity and dimensions are required to ensure that the behavior of two-way system is valid for the plates. These limitations and those of the live-load to dead-load ratios ensure validity of the coefficients used in the analysis of the model.

Equivalent Frame Method: As the name implies, the analysis of the "Equivalent Frame" in Fig. 1.4 is performed on a two-dimensional plane frame which is considered equivalent to the actual three-dimensional frame. There are, however, two important differences between a plane frame and a "building frame", which are either neglected or non-existent in the EFM. Although center line dimensions are used to define member lengths, the size of the joints, particularly when beams, capitals or drop panels are used, is significant. The other important difference is that the moments in a building frame can be transferred from one panel to another along their common line of intersection thereby bypassing the column joint, a condition that can not occur in a two-dimensional frame analysis. The loading combination required is dependent on the relative magnitudes of the live and dead load. The usual method used for the frame analysis is the method of moment distribution. Since the resistance of the columns to rotate both above and below of that slab is considered, this method is capable of providing more realistic distribution of moments than the DDM.

The DDM is simple to use, however the method applies too many restrictive conditions to the form of the frame and applied external load. When these conditions cannot be met, the

equivalent frame method(EFM) should be used. In addition, in the EFM the elastic analysis of the equivalent rigid frame is carried out and the maximum positive moment within the span and the negative moment at the supports are determined.

1.2.3 Finite Element Method

A Finite Element model which deals with the building frame as a full three-dimensional structure has been introduced by El-Kafrawy [17]. The loads in this model include horizontal and vertical loads. The structure is composed of two kinds of element, the plate super-element and the column frame type elements. The frame analysis is carried out by using a two-stage finite element process. In this procedure the stiffness coefficients are obtained by calculating the stiffness matrix of the slab super-element first, then combining the stiffness coefficients with corresponding stiffness coefficients of the columns.

For the purpose of calculating the stiffness matrix of the slab super-element, the slab is divided into a mesh of high precision elements with 6 degrees of freedom (d.o.f) per node. The degrees of freedom of the slab super-element are the slab-column interface rotations (θ_x and θ_y). The vertical movements of the columns are restrained ($\delta_z = 0$) because the columns are assumed to be axially rigid. As the slab-column joints are assumed very rigid, the degrees of freedom on the column peripheries can be reduced to 2 d.o.f at the center of each column patch. The slab stiffness matrix is obtained by inverting the flexibility matrix of the slab, in which unit corresponding forces are applied as a set of load cases. i.e.

$$\{f\} = [F]^{-1}\{d\} = [K] \{d\} \quad (1.1)$$

where

- $\{f\}$ is the vector of unknown forces
- $[F]$ is the flexibility matrix
- $\{d\}$ is the vector of known displacements
- $[K]$ is the stiffness matrix

To form the flexibility matrix, a unit moment is applied at each column of d.o.f one at a time, and the rotations under the $2N_{col}$ cases are used. The load case is equal to twice the number

of columns plus the actual applied loads ($2N_{col.} + N_{load}$). The rotation vectors under the actual applied loads are multiplied by the stiffness matrix to produce the fixed end forces.

Three assumptions were made for modelling the column slab interface, They are

- (1) the column cross section remains plane after bending;
- (2) no axial deformations occur in the columns ; and
- (3) the slab is rigidly connected to the columns.

The method introduced by El-Kafrawy does give good results of analysis, but the internal deformations of the slab are generally discontinuous between elements. In addition, Abdel-Akher [2] found drawbacks in the method used to produce the stiffness matrix after he used El-Kafrawy's model to calibrate his space frame model. He found that the flexibility matrix may not exist for some boundary conditions, and the approach can not always be used when the vertical displacements and inplane rotations on the column are permitted. Since vertical displacements of the columns are zero, the model can neither incorporate the effect of foundation settlement nor that of a change in column dimension. Along with the above two disadvantages, the model also requires a large amount of data preparation and computer memory.

1.2.4 Boundary Element Method

In the boundary element method, only the boundary of the domain is discretized into one dimensional elements in the case of plate bending. The primary unknowns are defined in terms of values at a finite number of points which are located only at the boundary of the domain treated. In general in the BEM, the dimension of the problem is reduced by one. The governing partial differential equation (which is valid over the entire solution domain) is transformed into an integral equation relating the functional values at the boundary. The boundary element method includes two approaches. One is the indirect approach (IBEM), which utilizes the principle of superposition, whereby the unknowns appearing in the integral equation are not physical quantities. The other is the direct approach (DBEM), based on the Betti-Maxwell's reciprocal theorem. As the name "direct" implies, the unknowns appearing in the integral equations are real physical quantities of the problem. [10, 29]

As discussed earlier in the finite element analysis of the flat plate frame, the flat slab is taken as a super-element and the column is taken as a space-frame type element. The boundary element method here is used for analyzing the flat plate and calculating the stiffness matrix of the slab super-element. After obtaining the stiffness of the whole frame by combining the stiffness of the slab super-element and columns, the deformation of the frame can then be computed using the stiffness method. [2, 17, 22]

A plate model for calculating the stiffness matrix was introduced by Abdel-Akher [2]. The model is analyzed by the direct boundary element method. The stiffness matrix in this model is obtained in a direct manner, and a column patch in his model is based on the Influence Coefficient method, the details of which will be discussed in next chapter.

1.3 Objectives

The work in this thesis concerns the direct boundary element method for building frame analysis. The method was introduced by Abdel-Akher based on his study of direct boundary element analysis of plate bending. A series of programs have been developed by Abdel-Akher with some additional modification by the present author for building frame analysis. The results obtained from those programs in his thesis only show that the calculations of his method are correct. He did not study such problems as building frame behavior, element configuration and patch modelling in any detail.

Since one of the basic assumptions in direct boundary element analysis is that the material of the frame is linear-elastic, the concrete frame seems not a very good example for analysis according to its material properties. A better example can be a perspex frame. It was determined by the author that this approach to building frame analysis requires validation. The complexity of the building frame model is such that solutions for even simple cases do not exist in the literature. It was also found that the expectations for convergence, inspired by FEM are not always realized in the BEM as boundary discretization is refined. A real model was therefore designed and tested.

The objectives are then as follows:

1. to demonstrate the direct boundary element method for the analysis of flat plate building frame;
2. to study boundary element configurations and column interface models in order to provide recommendations for future users of the program;
3. to analyze and test an experimental building frame model which includes some of the complexity of real building frame;
4. to improve the programs used for the analysis of flat plate building frame.

The building frames may be non-symmetric and have irregular column layout. They may carry both vertical and horizontal load; and output of the programs can give the displacements of each floor, internal reactions of columns and also plate internal actions.

2. DIRECT BOUNDARY ELEMENT METHOD

2.1 Introduction

This thesis is about the direct boundary element analysis of building frames. There are a few texts which deal with BEM generally and research papers which cover plate bending. Since the method has not yet reached a standard achieved by the FEM, it is necessary to give here a general description of the boundary element method, and more specifically a description of the direct boundary element method for the analysis of plate bending .

In this chapter, a short review of the governing differential equations for plates in bending is presented in the first section. In the second section, both the indirect and the direct boundary element approach will be discussed. The boundary integral equations and their fundamental solutions will then be discussed, and finally the procedure for solving the unknown boundary variables for a general plate problem will be given.

2.2 Governing Equations of Plate in Bending

The governing differential equation of the isotropic homogeneous thin plate is well known as

$$\nabla^4 w = \frac{p(x,y)}{D} \quad (2.1)$$

where

w	the lateral displacement of the plate
p	the applied lateral load
x and y	the Cartesian coordinates of a point on the plate
and D	the flexural rigidity of the plate which is defined by

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (2.2)$$

where E and ν are Young's modulus and Poisson's ratio respectively, and h is the plate thickness.

Equation (2.1) is based on the thin plate theory. It is assumed that

1. if deflections w of the plate are small in comparison with its thickness, there is no in plane deformation in the middle plane of the plate.
2. points initially on the normals to the mid-surface remain on the normal (ie. the effect of shear on deformation or deflection w is neglected) and normal stresses transverse to the plate are negligible.

Once the deflection function w is solved from equation (2.1), and the given boundary conditions, the rotations, bending moments, twisting moment and shear forces throughout the plate are defined in terms of the function w [27, 28, 30], ie.

$$\theta_x = \frac{\partial w}{\partial x} \quad (2.3a)$$

$$\theta_y = \frac{\partial w}{\partial y} \quad (2.3b)$$

$$m_{xx} = D \left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right] \quad (2.3c)$$

$$m_{yy} = D \left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right] \quad (2.3d)$$

$$m_{xy} = D (1 - \nu) \frac{\partial^2 w}{\partial x \partial y} \quad (2.3e)$$

$$q_x = -D \frac{\partial}{\partial x} (\nabla^2 w) \quad (2.3f)$$

$$q_y = -D \frac{\partial}{\partial y} (\nabla^2 w) \quad (2.3g)$$

The conventions for the above functions are shown in Fig. 2.1.

In a plate bending problem, there are four boundary variables at each boundary point, two specified and two unspecified. These variables are deflection W , normal slope (rotation) Θ , normal moment M and equivalent shear V . All of these boundary variables are defined in terms of the deflection function w as

Fig. 2.1 Positive Direction of Boundary Variables

$$W = w \quad (2.4a)$$

$$\Theta = \frac{\partial w}{\partial n} = \theta_x n_x + \theta_y n_y \quad (2.4b)$$

$$M = D \left[\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial t^2} \right] \\ = m_{xx} n_x^2 + 2m_{xy} n_x n_y + m_{yy} n_y^2 \quad (2.4c)$$

$$V = -D \frac{\partial}{\partial n} \left[\frac{\partial^2 w}{\partial n^2} + (2-\nu) \frac{\partial^2 w}{\partial t^2} \right] + D \frac{(1-\nu)}{\rho} \left[\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial t^2} \right] \quad (2.4d)$$

where n and t are two Cartesian coordinates normal and parallel to the boundary respectively, n_x and n_y are the direction cosines of the n axis. The moment M and equivalent shear V are given per unit length of the boundary.

In general, the boundary functions are not continuous at a corner because of the discontinuity of the normal to the boundary. This discontinuity results in a discontinuous torsional moment and also a corner force C which is given as

$$C = T^- - T^+ = \llbracket T \rrbracket \quad (2.5)$$

where T^- and T^+ are torsional moments on both sides of the corner, and are defined as

$$T = D (1-\nu) \frac{\partial^2 w}{\partial n \partial t} \\ = (m_{xx} - m_{yy}) n_x n_y + m_{xy} (n_x^2 - n_y^2) \quad (2.6)$$

2.3 Boundary Element Method

The boundary element method, as mentioned in section 1.2.4, includes both the indirect boundary element approach and the direct boundary element approach. The indirect boundary element approach includes the indirect boundary element method and the superposition method. In the past few years, some papers in BEM analysis of thin plates have included these three methods. In this section, only general description of these methods will be given and differences between these methods will be discussed.

2.3.1 Indirect Boundary Element Method (IBEM)

In this method, a thin plate of constant thickness h is considered, the plate has an open region R , boundary B and is loaded transversely by q , as shown in Fig. 2.2. The plate shape is imagined to be embedded in the clamped circular plate of Fig. 2.3. The clamped circular plate serves the same purpose as a Green's function in this development. Fictitious line load P^* and ring normal moment M_n^* are applied along B , as shown in Fig. 2.4. The purpose of using P^* and M_n^* is to force deflections to obey the boundary conditions of the real plate. [6, 34]

The deflection function of the real plate is obtained by superposition.

$$w(x, y) = w_q + w_{P^*} + w_{M_n^*} \quad (2.7)$$

Each sub-solution of w is based on the Influence Surface Method. Where

w_q caused by the actual external load acting on the plate and has the form of

$$w_q = \iint_R w_f(\xi, \eta; x, y) q \partial\xi \partial\eta \quad (2.8)$$

w_{P^*} the deflection caused by the fictitious edge line force P^*

$$w_{P^*} = \oint_B w_f(\xi, \eta; x, y) P^* \partial s(\xi, \eta) \quad (2.9)$$

Fig. 2.2 Problem of Interest

Fig. 2.3 Problem of Which Analytical Solution is known

Fig 2.4 Fictitious Problem

and $w_{M_n^*}$ the deflection caused by the imagined edge line moment M_n^*

$$w_{M_n^*} = \oint_B w^m(\xi, \eta; x, y) M_n^* \partial s(\xi, \eta) \quad (2.10)$$

$$w^m(\xi, \eta; x, y) = - \frac{\partial w^f}{\partial n}(\xi, \eta; x, y) \quad (2.11)$$

where $w^f(\xi, \eta; x, y)$, a Green's function (for example, the clamped circular plate solution), is a deflection function under a unit load, ξ and η are the coordinates of source points, x and y are the coordinates of field points, as shown in Fig. 2.5.

Fig. 2.5 Relationship of Source Point and Field Point

The external edge boundary conditions of the actual plate could be

i) clamped supported edge

$$W = 0, \quad \Theta = 0 \quad (2.12a)$$

ii) simply-supported edge

$$W = 0, \quad M = 0 \quad (2.12b)$$

iii) free edge

$$M = 0, \quad V = 0 \quad (2.12c)$$

Two coupled integral equations can be created by substituting the deflections $w(x, y)$ into the two simultaneous equations for the boundary conditions of the plate in equations (2.12). The integral equations contain the two unknown functions P^* and M_n^* which can be evaluated to satisfy the boundary conditions of the real plate. Once P^* and M_n^* are obtained, the deflections $w(x, y)$ can be calculated using equations (2.7). All other domain forces and displacement quantities can be determined using equations (2.3).

2.3.2 Superposition Method (SUP)

Fundamentally, the SUP method is similar to the IBEM, where the plate in this method is imagined to be embedded in an infinite plate of the same material and thickness, as shown in Fig. 2.6. It differs from the IBEM, in that the fictitious loads are concentrated forces P placed on two concentric boundaries similar in shape to the real plate, so that the deflection function of the plate does not contain any boundary integration.

Fig.2.6 Superposition Method

The force fundamental solution is $w^f(\xi, \eta; x, y)$, which is produced by a unit load, where ξ and η are the coordinates of source points, x and y are the coordinates of field points, as shown in Fig. 2.5.

The deflection function of real plate is

$$w(x, y) = \iint_R q(\xi, \eta) w^f(\xi, \eta; x, y) d\xi d\eta - \sum_{k=1}^{N_n} P_k w^f(\xi, \eta; x, y) \quad (2.13)$$

The boundary conditions are satisfied at a certain number of collocation points N_n , with two boundary conditions specified at each node, so $2N_n$ fictitious concentrated forces are used. The boundary conditions are given in equation (2.12)

The total number of $2N_n$ unknowns are solved from the simultaneous equations, which are obtained by substituting $w(x, y)$ into the corresponding equations for the boundary conditions at each boundary node. All domain values can also be calculated using equation (2.13) for $w(x, y)$ and equations (2.3).

Using the superposition method the boundary conditions are satisfied at the collocation points, however, the satisfaction of boundary conditions between the selected points is not guaranteed, and the convergence to the correct solution is also not guaranteed. Abdel-Akher [2] has presented two examples to show some of the difficulties associated with this method.

2.3.3 Direct Boundary Element Method (DBEM)

Two equilibrium states based on Betti-Maxwell's reciprocal theorem are used in this method. One is the actual equilibrium state for the given plate problem, the other is the virtual equilibrium state for which the load and boundary conditions are chosen so that an analytical solution is known.

The analytical solution is obtained solving the governing partial differential equation for an infinite plate under a single concentrated load (the force fundamental solution). The solution for the same plate under a single couple (the moment fundamental solution) is obtained in the same way to complete the approach. The position, at which the single force or single moment is acting, is called the source point. Two boundary integral equations are created as follows:

$$Kw^f + \oint_S (WV^f + \Theta M^f) ds = \iint_A p w^f dA + \oint_S (W^f V + \Theta^f M) ds \quad (2.14)$$

$$Kw^m + \oint_S (WV^m + \Theta M^m) ds = \iint_A p w^m dA + \oint_S (W^m V + \Theta^m M) ds \quad (2.15)$$

where

K	is a single concentrated load in equation (2.14) or a single couple in equation (2.15)
A	represents the loaded area
s	represents the plate boundary
p	is the applied distributed load
w	is the vertical displacement
W and Θ	are the boundary deflection and rotation respectively
M and V	are the boundary moment and shear respectively
f and m	are superscripts referring to the force and moment fundamental solutions

The boundary of the plate is discretized into elements, each element containing a certain number of nodes. Each node has two unknown values, so that the total number of unknowns is $2N_n$. Applying unit load (source point) at N_n distinct locations, two independent integral equations are set up. These integral equations are then converted to $2N_n$ simultaneous

algebraic equations, containing $2N_n$ boundary unknowns. Once these $2N_n$ unknowns have been determined, the domain deflections and forces at any point inside the plate can be determined. A more detailed discussion of the DBEM will be presented in a subsequent section of this chapter.

2.4 Boundary Integral Equation

As briefly stated in the previous section, the boundary integral equations of the DBEM are created by applying the Betti-Maxwell's reciprocal theorem to the two states of equilibrium. These two equilibrium states, are shown in Fig. 2.7, plate *a* and plate *b*. By the reciprocal theorem, the work done by the forces *a*, if given the displacement *b*, is equal to the work done by the forces *b*, if given the displacement *a*.

Plate *a*, considered as a general plate problem, has an arbitrary boundary *s* with arbitrary boundary conditions, and is subjected to an arbitrary transversely applied load *q*. The plate may be supported by internal point supports at which the reactions are denoted *R*.

Another plate, plate *b*, shown in Fig. 2.7 b), is chosen such that it is larger than plate *a* and loaded by a unit concentrated force applied at point *P*. A portion *b** which has a boundary *s**, identical to *s*, is cut from plate *b*. The internal forces of *b* along *s** become external forces on *b**. These forces are denoted M^f , V^f and C^f , where *C* is the corner force caused by discontinuity of twist moment. The displacements along *s** are denoted W^f and Θ^f . Superscript *f* refers to the force fundamental solution. The boundary integral equation can now be written down directly by applying the reciprocal theorem to plates *a* and *b**. ie.

$$\begin{aligned}
 Kw_P + \oint_S (WV^f + \Theta M^f) ds + \sum_{i=1}^{N_c} W_i C_i^f = \iint_A p w^f dA \\
 + \oint_S (W^f V + \Theta^f M) ds + \sum_{i=1}^{N_c} W_i^f C_i + \sum_{i=1}^{N_r} w_i^f R_i \quad (2.16)
 \end{aligned}$$

When a unit couple is applied instead of the unit force at point *P* (Fig. 2.7c), a similar equation to (2.16) can be written as

Fig. 2.7 Equilibrium States

$$\begin{aligned}
 K\theta_p^\xi + \oint_S (WV^{m\xi} + \Theta M^{m\xi}) ds + \sum_{i=1}^{N_c} W_i C_i^{m\xi} = \iint_A p w^{m\xi} dA \\
 + \oint_S (W^{m\xi} V + \Theta^{m\xi} M) ds + \sum_{i=1}^{N_c} W_i^{m\xi} C_i + \sum_{i=1}^{N_r} w_i^{m\xi} R_i \quad (2.17)
 \end{aligned}$$

where

- R_i the reaction at internal supports
- C_i the corner force caused by sudden change of boundary condition
- ξ the direction of unit moment
- θ_p^ξ the domain rotation at point P in the direction ξ .

$$K = \begin{cases} 1 & \text{if } P \text{ is inside plate a} \\ 0 & \text{if } P \text{ is outside plate a} \end{cases}$$

- N_c the number of corners
- N_r the number of internal reactions

If the function w^f , $w^{m\xi}$ of the plate b is known analytically and accordingly W^f , Θ^f , M^f , V^f , $W^{m\xi}$, $\Theta^{m\xi}$, $M^{m\xi}$ and $V^{m\xi}$, then the only unknowns appearing in equations (2.16) and (2.17) are those of plate a .

2.5 Fundamental Solution

The force fundamental solution given in equation (2.18) is displacement at any point (P) in a plate of infinite extent produced by a unit load acting at the point (Q), which is

$$w^f(P, Q) = \frac{1}{8\pi D} r^2 \ln(r) \quad (2.18)$$

This is the out of plane deflection at field point Q caused by a unit load acting at source point P ; and r is the distance between these two points, as shown in Fig. 2.8.

The boundary force and displacement quantities can be derived from equation (2.18), when Q is a boundary point.

$$\begin{aligned} \Theta^f(P, Q) &= \frac{\partial w^f}{\partial n_Q} \\ &= \frac{1}{8\pi D} (1 + 2\ln(r)) r \cos(\beta) \end{aligned} \quad (2.19a)$$

$$\begin{aligned} M^f(P, Q) &= D \left(\frac{\partial^2 w^f}{\partial n_Q^2} + \nu \frac{\partial^2 w^f}{\partial t_Q^2} \right) \\ &= \frac{1}{8\pi} [2(1 + \nu)(1 + \ln(r)) + (1 - \nu) \cos(2\beta)] \end{aligned} \quad (2.19b)$$

$$\begin{aligned} V^f(P, Q) &= -D \frac{\partial}{\partial n_Q} \left[\frac{\partial^2 w^f}{\partial n_Q^2} + (2 - \nu) \frac{\partial^2 w^f}{\partial t_Q^2} \right] \\ &\quad + D \frac{(1 - \nu)}{\rho} \left[\frac{\partial^2 w^f}{\partial n_Q^2} - \frac{\partial^2 w^f}{\partial t_Q^2} \right] \\ &= -\frac{\cos(\beta)}{4\pi r} [2 + (1 - \nu) \cos(2\beta)] + \frac{(1 - \nu)}{4\pi \rho} \cos(2\beta) \end{aligned} \quad (2.19c)$$

$$\begin{aligned} T^f(P, Q) &= D (1 - \nu) \frac{\partial^2 w^f}{\partial n_Q \partial t_Q} \\ &= \frac{(1 - \nu)}{8\pi} \sin(2\beta) \end{aligned} \quad (2.19d)$$

Fig. 2.8 Boundary Coordinate System

where

- n and t are local Cartesian axes which are normal and parallel to the tangent at boundary point Q ;
- n_Q and t_Q are the n and t coordinates of point Q ;
- β is the angle between the outward normal at Q and PQ ; and
- ρ is the radius of curvature of the boundary at Q .

The moment fundamental solution can be obtained by differentiating equation (2.18), with respect to the coordinate of point P in the direction of the unit couple

$$w^{m\xi}(P, Q) = \frac{\partial w^f(P, Q)}{\partial \xi_P} = \frac{1}{4\pi D} r \ln(r) \cos(\phi) \quad (2.20)$$

As shown in Fig. 2.8, ϕ is the angle between ξ and PQ . The boundary force and displacement quantities will be

$$\Theta^{m\xi}(P, Q) = \frac{1}{4\pi D} [\cos(\phi)\cos(\beta) + \ln(r) \cos(\phi - \beta)] \quad (2.21a)$$

$$M^{m\xi}(P, Q) = \frac{1}{4\pi r} [(1 + \nu)\cos(\phi) + (1 - \nu) \sin(\phi)\sin(2\beta)] \quad (2.21b)$$

$$V^{m\xi}(P, Q) = \frac{1}{4\pi r^2} \{ \cos(\phi + \beta)[2 + (1 - \nu)\cos(2\beta)] - 2(1 - \nu)\sin(2\beta)\sin(\phi)\cos(\beta) \}$$

$$+ \frac{(1-\nu)}{2\pi r \rho} [\sin(\phi) \sin(2\beta)] \quad (2.21c)$$

$$T^{m\pm}(P, Q) = - \frac{(1-\nu)}{4\pi r} \cos(2\beta) \sin(\phi) \quad (2.21d)$$

The simplification of the integration process is given in [2].

2.6 Solution for Unknown Boundary Variable

To solve the boundary integral equation (2.16) and (2.17), the boundary of real plate is divided into elements (Fig.2.9). Each element contains several nodes; the number of the nodes can vary from one to eleven in the DBEM program developed by Abdel-Akher.

Fig.2.9 Boundary Discretisation

The integrals are now evaluated by summing the contributions of each element. For example

$$\begin{aligned} Kw_P + \sum_{i=1}^{Ne} \int_{s_i} (W(s_i)V^f + \Theta(s_i)M^f) ds + \sum_{i=1}^{N_c} W_i C_i^f = \iint_A p w^f dA \\ + \sum_{i=1}^{Ne} \int_{s_i} (W^f V(s_i) + \Theta^f M(s_i)) ds + \sum_{i=1}^{N_c} W_i^f C_i + \sum_{i=1}^{N_r} w_i^f R_i \end{aligned} \quad (2.22)$$

where

N_e the total number of boundary elements.

The boundary functions are usually approximated as polynomial curves within the elements. Thus all the integrations can be reduced to the analytical integration of the product of a single shape function and a known function. The procedure of the approximation is identical to the shape function approach in finite element analysis. The introduced interpolation functions often are of the Lagrange type [2]. Thus for example

$$W(s_i) = \sum_{i=1}^{N_i} N_{ij}(s_i) W_{nij} \quad (2.23)$$

where

- N_i the number of nodes of element i ;
- n_{ij} the global number of the j th node of element i ;
- N_{ij} the shape function for node j ;
- s_i the local boundary coordinate.

The integral of equation (2.22), for example, can be written as

$$\sum_{i=1}^{N_e} \int_{s_i} W(s_i) V^f ds_i = \sum_{i=1}^{N_e} \sum_{j=1}^{N_i} W_{nij} IV^f \quad (2.24)$$

where $IV^f = \int_{s_i} N_{ij}(s_i) V^f ds_i$ which can be expressed in analytical form after some mathematical manipulation.

Equation (2.22) can be simplified further realizing that the boundary functions are continuous except at corners and at points which have sudden change in the boundary conditions. All boundary integral terms are presented in a simple way, for example, $V^f = IV^f$

$$\begin{aligned} Kw_p + \sum_{k=1}^{Nn-Nc} (W_k V_k^f + \Theta_k M_k^f) + \sum_{j=1}^{Nc} \{W_j (V_j^f + C_j^f) + \Theta_j^- M_j^f + \Theta_j^+ M_j^f\} = \\ \iint_A p w^f dA + \sum_{k=1}^{Nn-Nc} (W_k V_k + \Theta_k M_k) + \sum_{j=1}^{Nc} \{ (V_j^- + V_j^+) W_j^f + M_j^- \Theta_j^f + M_j^+ \Theta_j^f \} \\ + \sum_{j=1}^{Nc} W_j^f C_j + \sum_{i=1}^{Nr} w_i^f R_i \end{aligned} \quad (2.25)$$

Similarly

$$\begin{aligned}
K\theta_p^\xi + \sum_{k=1}^{Nn-Nc} (W_k V_k^{m\xi} + \Theta_k M_k^{m\xi}) + \sum_{j=1}^{Nc} \{W_j (V_j^{m\xi-} + C_j^{m\xi}) + \Theta_j^- M_j^{m\xi-} + \Theta_j^+ M_j^{m\xi+}\} = \\
\iint_A p w^{m\xi} dA + \sum_{k=1}^{Nn-Nc} (W_k^{m\xi} V_k + \Theta_k^{m\xi} M_k) + \sum_{j=1}^{Nc} \{ (V_j^- + V_j^+) W_j^{m\xi} + M_j^- \Theta_j^{m\xi-} \\
+ M_j^+ \Theta_j^{m\xi+} \} + \sum_{j=1}^{Nc} W_j^{m\xi} C_j + \sum_{i=1}^{Nr} w_i^{m\xi} R_i
\end{aligned} \quad (2.26)$$

the superscripts - and + refer to the two sides of the corner, and k is simply a boundary node number.

Equations (2.25) and (2.26) are converted to algebraic equations after substituting in the boundary conditions and solving the domain integral of the transverse load $q(x,y)$ [2]. The equations can now be written as

$$Kw_p + \{A^f\}^T_{1 \times 2N_n} \{X\}_{2N_n \times 1} = Y^f + \{B^f\}^T_{1 \times N_r} \{R\}_{N_r \times 1} \quad (2.26)$$

$$K\theta_p^\xi + \{A^{m\xi}\}^T_{1 \times 2N_n} \{X\}_{2N_n \times 1} = Y^{m\xi} + \{B^{m\xi}\}^T_{1 \times N_r} \{R\}_{N_r \times 1} \quad (2.27)$$

where

$\{X\}$	the vector of the nodal unknown variables
Y	the contribution of the domain integral and the boundary conditions
$\{A\}$	vectors containing the results of line integrations corresponding to the unknown nodal values in vector $\{X\}$
$\{B\}\{R\}$	$\sum_{i=1}^{Nr} w_i f R_i$ and $\sum_{i=1}^{Nr} w_i^{m\xi} R_i$ in matrix notation

The number of unknowns in $\{X\}$ and $\{R\}$ is equal to $(2N_n + N_r)$, which can only be obtained by forming $(2N_n + N_r)$ independent equations.

A total of N_r equations are formulated by considering a unit load acting on an inner support each a time, which are

$$\{w\}_{N_r \times 1} + [A_1]_{N_r \times 2N_n} \{X\}_{2N_n \times 1} = \{Y_1\}_{N_r \times 1} + [B_1]_{N_r \times N_r} \{R\}_{N_r \times 1} \quad (2.27)$$

where $\{w\}$ is a vector containing the known displacements of the inner supports and $K=1$ in this case.

A total of $2N_n$ equations are formed employing source point in turn at points outside of the plate but very close to the boundary nodes [2, 3]. The $2N_n$ equations can be represented as

$$[A_2]_{N_r \times 2N_n} \{X\}_{2N_n \times 1} = \{Y_2\}_{N_r \times 1} + [B_2]_{N_r \times N_r} \{R\}_{N_r \times 1} \quad (2.28)$$

Solving the $(2N_n + N_r)$ linear algebraic equations

- i) Multiplying both sides of equation (2.27) by $[B_1]^{-1}$ gives

$$\{R\} = [B_1]^{-1} [A_1] \{X\} + [B_1]^{-1} \{ \{w\} - \{Y_1\} \} \quad (2.29)$$

- ii) Substituting equation (2.29) into equation (2.28) yields

$$[[A_2] - [B_2][B_1]^{-1}[A_1]] \{X\} = \{Y_2\} + [B_2][B_1]^{-1} \{ \{w\} - \{Y_1\} \} \quad (2.30)$$

The DBEM, in this way, is neither a strictly upper- or lower-bound solution. The discrete equations (2.27) and (2.28) are not as simple and straight forward as the pure force or displacement type, they are a mixed stiffness and compatibility type.

3. DIRECT BOUNDARY ELEMENT ANALYSIS OF BUILDING FRAMES

3.1 Introduction

The DBEM for building frame analysis was introduced by Abdel-Akher [2] and is based on his study of the DBEM for plate bending and El-Kafrawy's study of building frame analysis using the FEM [17].

A general description of the DBEM for plate bending has been summarized in chapter 2. In the present chapter, a more detailed discussion relating to building frame analysis by the DBEM will be given. For the typical problem in building frame analysis, the boundary integral equations given in section 2.6 for calculating the stiffness matrix of the plate will be modified such that the boundary condition on the plate edges is "free" condition.

It is believed that this method could eventually be used to analyze full floor layouts and to carry out stiffness analyses of building models, but may attract more general interest in this generic structure type, i.e. thin plate analysis with interconnecting elements. The method may attract particular interest for dynamic and torsional analysis of building frames, and for investigations related to bending moment distributions in flat plates having various configurations of columns and column wall layout [22].

3.2 Frame Model

The building frames to be analyzed consist of flat plate floors without beams, supported directly on columns. A full three-dimensional model of a building structure is shown in Fig. 3.1. The building material is assumed to be isotropic and linearly elastic, and the frame is assumed to undergo small deflections.

Fig. 3.1 Full Three-dimensional Building Frame Model

In general, the global degrees of freedom of three-dimensional frames are six per frame joint. However, since the floor slabs are assumed here to be rigid in their planes, there are two translational and one rotational degree of freedom (Δ_x , Δ_y and Θ_z) per floor and two rotational and one translational degree of freedom (θ_x , θ_y and δ_z) per joint (Fig. 3.2). The degrees of freedom Δ_x , Δ_y and Θ_z can be referred to any arbitrary point in the floor plate.

The building frame is modelled using two kinds of element: the slab super-element consisting of a full floor and the transverse "column" element. Although the term "column" is used, it is assumed by the writer, in accordance with some engineering practise, columns of extended dimension or column walls, also apply. The slab super-element is an arbitrary shaped plate with constant thickness. The slab is assumed to undergo small deflections. The connection areas between the slab and column are considered to displace and rotate rigidly. The degrees of freedom of the slab super-element are the displacements, θ_x , θ_y and δ_z of the slab-column joint, as shown in Fig. 3.2, and the displacements (Δ_x , Δ_y and Θ_z) of the floor slab. The displacements Δ_x , Δ_y and Θ_z are directly related to the stiffness of the column elements only. The description of the model of the connection between slab and column will be given in next section.

Fig. 3.2 Building Frame Stiffness Model

The stiffness matrices of the slabs and the columns are assembled in a global stiffness matrix for the building frame. The details will be discussed later in the chapter.

3.3 Column Patch Model

The method used here for the analysis of the slab column connection is called the Influence Coefficient Method (ICM) used by Wong and Coull [32] for the study of the interaction between the slab and columns or shear walls for the resistance of horizontal loads. The exact distribution of the interface-pressure developed between the wall and slab* is not

* It may be of interest to some readers not fully conversant with building analysis, that the structural mechanics of connecting columns to slab for lateral load analysis, is fundamentally different from the connection of the slab to column for vertical load analysis. This is touched upon in somewhat more detail by El-Kafrawy [17], who also describes some of the historically earliest work done on this type of structure.

described by a continuous function. Wong and Coull used a system of concentrated forces and moments acting at a discrete set of nodes to replace the pressure. The magnitudes of the concentrated forces and moments at the wall-slab nodes are obtained such that the superposition of all forces and moments satisfy the compatibility conditions at the selected nodes. The establishment of compatibility between the displacement produced by the system of unknown forces and linear displacements imposed by an assumed arbitrary parallel wall rotation leads to a system of simultaneous equations

$$[F] \{R\} = \{\delta\} \quad (3.1)$$

where

- $\{R\}$ the unknown forces at node 'i', $\{R\}_i$ in general consist of a point load P_i and two moments M_{x_i} and M_{y_i} .
- $\{\delta\}$ the displacements at node 'i' $\{\delta\}_i$ consist of the transverse deflection w_i and two rotations w_{x_i} and w_{y_i} .
- $[F]$ the flexibility matrix which is defined as

$$[F] = \begin{bmatrix} F_{11} & \dots & F_{1j} & \dots & F_{1n} \\ \vdots & & \vdots & & \vdots \\ F_{i1} & \dots & F_{ij} & \dots & F_{in} \\ \vdots & & \vdots & & \vdots \\ F_{n1} & \dots & F_{nj} & \dots & F_{nn} \end{bmatrix} \quad (3.2)$$

where $[F_{ij}]$ is a 3x3 flexibility submatrix containing the influence coefficients. These influence coefficients may readily be evaluated from the influence function derived from classical plate theory [28, 31].

Equation (3.1) may be solved for the unknown forces to give

$$\{R\} = [F]^{-1} \{\delta\} \quad (3.3)$$

The set of calculated forces at the slab-wall nodes constitutes a resultant moment M about the slab-wall axis in the plane of rotation which is numerically equal to the rotational stiffness of the slab for a unit slab-wall rotation.

For the purpose of frame analysis, Wong and Coull [33] considered the typical interior column slab panel, as found in a regular flat plate structure, to be simply supported along the

edges corresponding to the inflexion lines. The displacement pattern for the slab is symmetric about the x axis, and skew-symmetric about the y axis. Using this symmetry, the computation was considerably reduced by confining the analysis to a typical panel (Fig. 3.3).

Fig. 3.3 Lines of Symmetry and Anti-symmetry and
Typical Interior Floor Panel in Flat Plate Slab

To satisfy a more general requirement, the basic idea of the Influence Coefficient Method was extended by Abdel-Akher [2]. The method can now be applied to a model with arbitrary shaped floor slabs and arbitrary boundary conditions of the slab. The plates are supported by internal columns. The number of degrees of freedom of each slab-column joint is 3 (ie. two rotational and one translational). A system of discrete point forces at selected nodes inside the column patch is used to replace the continuous function (Fig.3.4). The magnitudes of these forces are obtained by satisfying the compatibility of the displacements, and unknown forces are solved as redundant forces in flexibility analysis.

A unit displacement is given at each of the total of $3N_{col}$ degrees of freedom related to the slab-column patch one at a time. Once the unknown forces are determined, the stiffness coefficients corresponding to the given displacement, ie. the reactions at each column, can be calculated by the superposition of these forces as

Fig. 3.4 Discrete Forces Inside Column Patch

$$R_i = \sum_{j=1}^{N_{f_i}} f_{n_j} \quad (3.4a)$$

$$(M_x)_i = \sum_{j=1}^{N_{f_i}} (y_{n_j} - \bar{y}_i) f_{n_j} \quad (3.4b)$$

$$(M_y)_i = - \sum_{j=1}^{N_{f_i}} (x_{n_j} - \bar{x}_i) f_{n_j} \quad (3.4c)$$

where

n_j the global number of the j th node of the i th column

$\bar{x}_i$ and $\bar{y}_i$ the coordinates of the center of the i th column

N_{f_i} the number of forces in the i th column patch

In the application of the ICM, the influence surface should be known for the given boundary conditions. In building frame analysis, the Direct Boundary Element Method is used as a numerical technique to generate the influence surface.

3.4 Stiffness Matrix of Building Frame

The stiffness matrix of the building frame is obtained by assembling the slab stiffness matrix and the column stiffness matrix in the global coordinate system.

The stiffness matrix of the floor slab element is obtained in a joint application of ICM and DBEM. Assume first that one of the degrees of freedom, say a single rotation of one column patch, is given a unit value. The prescribed displacements are fully defined, the interface pressures at all column patches are at this stage unknown. These pressures are assumed to be replaced by discrete unknown forces within each column patch. These forces can be treated as unknown (or unprescribed) internal boundary conditions and solved using the DBEM as outlined earlier. Finally the required stiffness coefficients are equivalent statically to the discrete forces.

For calculating the stiffness matrix, the slab is analyzed for multiple load cases. The number of "load" cases is equal to $3N_{col}$, where N_{col} is the number of joint patches. The boundaries of the floor slab are considered as free edges, i.e. $V = 0$ and $M = 0$ along the boundaries of slab. Under these conditions, equation (2.27) and (2.28) take the form of

$$K w_p + \sum_{k=1}^{N_n-N_c} (W_k V_k^f + \Theta_k M_k^f) + \sum_{j=1}^{N_c} \{ W_j (V_j^f - C_j^f) + \Theta_j^- M_j^{f-} + \Theta_j^+ M_j^{f+} \} = \iint_A p w^f dA + \sum_{i=1}^{N_r} w_i^f R_i \quad (3.5)$$

and

$$K \theta_p^\xi + \sum_{k=1}^{N_n-N_c} (W_k V_k^{m\xi} + \Theta_k M_k^{m\xi}) + \sum_{j=1}^{N_c} \{ W_j (V_j^{m\xi} - C_j^{m\xi}) + \Theta_j^- M_j^{m\xi-} + \Theta_j^+ M_j^{m\xi+} \} = \iint_A p w^{m\xi} dA + \sum_{i=1}^{N_r} w_i^{m\xi} R_i \quad (3.6)$$

The boundary unknowns at the corners of the floor slab are W , Θ^- and Θ^+ for this case. But Θ^- and Θ^+ can be replaced by the rotation in the direction of the bisector of the corner angle Θ^b [2]. The boundary unknown variables are now reduced to W and Θ^b .

To form the stiffness matrix, a unit generalized displacement is applied in the direction of the slab degree of freedom in turn for each case of the $3N_{col}$ loads. Using the same method used to form equations (2.27) and (2.28), equations (3.5) and (3.6) take the matrix form

$$- [Y_1] + [w_r] + [A_1] [X] = [B_1] [R] \quad (3.7)$$

$$[A_2] [X] = [Y_2] + [B_2] [R] \quad (3.8)$$

where

$[Y_1]$ and $[Y_2]$ the contribution of the domain integral, but $[Y_1] = [0]_{N_r \times 3N_{col.}}$ and $[Y_2] = [0]_{2N_n \times 3N_{col.}}$ because with the domain integral $3N_{col.}$ load cases, there is no actual external load; and

$[w_r]$ a $(N_r \times 3N_{col.})$ matrix containing the value of the known displacements at each position of the column point forces, It can be rewritten as

$$[w_r] = \begin{bmatrix} [w_1] & \dots & [0] & \dots & [0] \\ \vdots & & \vdots & & \vdots \\ [0] & \dots & [w_j] & \dots & [0] \\ \vdots & & \vdots & & \vdots \\ [0] & \dots & [0] & \dots & [w_{N_{col.}}] \end{bmatrix}_{N_r \times 3N_{col.}} \quad (3.9)$$

where

$$[w_j] = \begin{bmatrix} 1 & y_{njl} - \bar{y}_j & -(x_{njl} - \bar{x}_j) \\ \vdots & \vdots & \vdots \\ 1 & y_{njk} - \bar{y}_j & -(x_{njk} - \bar{x}_j) \\ \vdots & \vdots & \vdots \\ 1 & y_{n_j N_{fj}} - \bar{y}_j & -(x_{n_j N_{fj}} - \bar{x}_j) \end{bmatrix}_{N_{fj} \times 3} \quad (3.10)$$

$[X]$ a $(2N_n \times 3N_{col.})$ matrix of unknown boundary variables, which is in the form

$$[X] = \begin{bmatrix} [X_{11}] & \dots & [X_{1j}] & \dots & [X_{13N_{col}}] \\ \vdots & & \vdots & & \vdots \\ [X_{i1}] & \dots & [X_{ij}] & \dots & [X_{i3N_{col}}] \\ \vdots & & \vdots & & \vdots \\ [X_{N_n 1}] & \dots & [X_{N_n j}] & \dots & [X_{N_n 3N_{col}}] \end{bmatrix}_{2N_n \times 3N_{col}} \quad (3.11)$$

where

$$[X_{ij}] = \begin{Bmatrix} W_{ij} \\ \Theta_{ij} \end{Bmatrix} \quad (3.12)$$

$[R]$ a $(N_r \times 3N_{col})$ matrix of unknown point forces caused by unit displacement or rotation in each degree of freedom.

$[A_1]$ and $[A_2]$ matrices containing the results of line integrals corresponding to the unknown nodal values in matrix $[X]$. They are presented as

$$[A_1] = \begin{bmatrix} [A_{111}] & \dots & [A_{11j}] & \dots & [A_{11N_n}] \\ \vdots & & \vdots & & \vdots \\ [A_{1i1}] & \dots & [A_{1ij}] & \dots & [A_{1iN_n}] \\ \vdots & & \vdots & & \vdots \\ [A_{1N_r 1}] & \dots & [A_{1N_r j}] & \dots & [A_{1N_r N_n}] \end{bmatrix}_{N_r \times 2N_n} \quad (3.13)$$

where

$$[A_{1ij}] = [V_{ij}^f \quad M_{ij}^f]_{1 \times 2} \quad (3.14)$$

and

$$[A_2] = \begin{bmatrix} [A_{211}] & \dots & [A_{21j}] & \dots & [A_{21N_n}] \\ \vdots & & \vdots & & \vdots \\ [A_{2i1}] & \dots & [A_{2ij}] & \dots & [A_{2iN_n}] \\ \vdots & & \vdots & & \vdots \\ [A_{2N_n 1}] & \dots & [A_{2N_n j}] & \dots & [A_{2N_n N_n}] \end{bmatrix}_{2N_n \times 2N_n} \quad (3.15)$$

where

$$[A_{2ij}] = \begin{bmatrix} \mathbf{V}_{ij}^f & \mathbf{M}_{ij}^f \\ \mathbf{V}_{ij}^{m\xi} & \mathbf{M}_{ij}^{m\xi} \end{bmatrix}_{2 \times 2} \quad (3.16)$$

$[B_1]$ and $[B_2]$ matrices containing the results of the fundamental solution corresponding to the unknown point forces within the column patch $[R]$. They are expressed as

$$[B_1] = \begin{bmatrix} [B_{111}] & \dots & [B_{11j}] & \dots & [B_{11N_r}] \\ \vdots & & \vdots & & \vdots \\ [B_{1i1}] & \dots & [B_{1ij}] & \dots & [B_{1iN_r}] \\ \vdots & & \vdots & & \vdots \\ [B_{1N_r1}] & \dots & [B_{1N_rj}] & \dots & [B_{1N_rN_r}] \end{bmatrix}_{N_r \times N_r} \quad (3.17)$$

where

$$[B_{1ij}] = \frac{1}{8\pi D} r_{ij}^2 \ln r_{ij} \quad (3.18)$$

and

$$[B_2] = \begin{bmatrix} [B_{211}] & \dots & [B_{21j}] & \dots & [B_{21N_r}] \\ \vdots & & \vdots & & \vdots \\ [B_{2i1}] & \dots & [B_{2ij}] & \dots & [B_{2iN_r}] \\ \vdots & & \vdots & & \vdots \\ [B_{2N_r1}] & \dots & [B_{2N_rj}] & \dots & [B_{2N_rN_r}] \end{bmatrix}_{2N_r \times N_r} \quad (3.19)$$

where

$$[B_{2ij}] = \left\{ \begin{array}{l} \frac{1}{8\pi D} r_{ij}^2 \ln r_{ij} \\ \frac{1}{4\pi D} r_{ij} \ln(r_{ij}) \cos(\phi) \end{array} \right\}_{2 \times 1} \quad (3.20)$$

Equations (3.7) and (3.8) can be further simplified to the form

$$- [A_1] [X] + [B_1] [R] = [w_r] \quad (3.21)$$

$$[A_2] [X] - [B_2] [R] = [0] \quad (3.22)$$

Equations (3.21) and (3.22) are solved in the same way as equation (2.27) and (2.28),

ie.

$$[R] = [B_1]^{-1}[w_r] + [B_1]^{-1}[A_1][X] \quad (3.23)$$

$$[A_2] + [B_2][B_1]^{-1}[A_1][X] = [B_2][B_1]^{-1}[w_r] \quad (3.24)$$

Equation (3.24) is solved for $[X]$, then equation (3.23) is used to calculate $[R]$.

The K -coefficients for the slab super-element are statically equivalent to the forces in $[R]$. The slab stiffness matrix is calculated as

$$[K_s] = [T][R] \quad (3.25)$$

where

$$[T] = [w_r]^T \quad (3.26)$$

The stiffness coefficients of the column elements are available from standard textbook on matrix frame analysis [14, 17]. However, the additional terms, corresponding to Δ_x , Δ_y and Θ_z , have to be added into column stiffness matrix when those coefficients are considered to be assembled to frame stiffness matrix. These terms are

$$\begin{aligned} k'_{c6,5} &= \frac{12EI_x}{L^3} x_i \\ &= k'_{c5,6} = k'_{c12,11} = k'_{c11,12} \end{aligned} \quad (3.27a)$$

$$\begin{aligned} k'_{c6,4} &= \frac{12EI_y}{L^3} y_i \\ &= k'_{c4,6} = k'_{c12,10} = k'_{c10,12} \end{aligned} \quad (3.27b)$$

$$\begin{aligned} k'_{c6,6} &= \frac{12EI_x}{L^3} x_i^2 + \frac{12EI_y}{L^3} y_i^2 \\ &= k'_{c12,12} \end{aligned} \quad (3.27c)$$

where x_i and y_i are coordinates of i th column cross section center.

The building frame stiffness matrix is formed by adding the column element stiffness coefficients to the corresponding coefficients of the slab stiffness matrix in the global coordinate system.

3.5 Fixed End Forces

The fixed end forces are column reactions caused by the actual applied loads, while all degrees of freedom of the slab are locked. The number of load cases is equal to N_{load} , where N_{load} is the number of actual applied loads. The boundaries of the floor slab are also considered as free edges. Equations (2.27) and (2.28) have forms similar to equations (3.7) and (3.8)

$$-[Y_1^F] + [w_r^F] + [A_1] [X^F] = [B_1] [R^F] \quad (3.28)$$

$$[A_2] [X^F] = [Y_2^F] + [B_2] [R^F] \quad (3.29)$$

where

$[Y_1]$ and $[Y_2]$ the contribution of the domain integral

$$[Y_1^F] = \begin{bmatrix} [Y_1^F]_{11} & \dots & [Y_1^F]_{1j} & \dots & [Y_1^F]_{1N_{load}} \\ \vdots & & \vdots & & \vdots \\ [Y_1^F]_{i1} & \dots & [Y_1^F]_{ij} & \dots & [Y_1^F]_{iN_{load}} \\ \vdots & & \vdots & & \vdots \\ [Y_1^F]_{Nr1} & \dots & [Y_1^F]_{Nrj} & \dots & [Y_1^F]_{NrN_{load}} \end{bmatrix}_{Nr \times N_{load}} \quad (3.30)$$

$$[Y_2^F] = \begin{bmatrix} [Y_2^F]_{11} & \dots & [Y_2^F]_{1j} & \dots & [Y_2^F]_{1N_{load}} \\ \vdots & & \vdots & & \vdots \\ [Y_2^F]_{i1} & \dots & [Y_2^F]_{ij} & \dots & [Y_2^F]_{iN_{load}} \\ \vdots & & \vdots & & \vdots \\ [Y_2^F]_{Nr1} & \dots & [Y_2^F]_{Nrj} & \dots & [Y_2^F]_{NrN_{load}} \end{bmatrix}_{Nr \times N_{load}} \quad (3.31)$$

where

$$[Y_1^F]_{ij} = \iint_A p_j w^{f_i} dA \quad (3.32)$$

$$[Y_2^F]_{ij} = \iint_A p_j w^{m_{f_i}} dA \quad (3.33)$$

i is the field point and j is the actual external load. The total of N_r is the

number of unknown point forces, column patch forces, acting on the slab

$[w_r^F]$ a $(N_r \times N_{load})$ matrix containing the value of the known displacements at the positions of the column point forces. $[w_r^F] = [0]_{N_r \times N_{load}}$ because all of the floor degrees of freedom are locked for the calculation of the fixed end forces

$[X^F]$ a $(2N_n \times N_{load})$ matrix of unknown boundary variables

$$[X^F] = \begin{bmatrix} [X_{11}^F] & \dots & [X_{1j}^F] & \dots & [X_{1N_{load}}^F] \\ \vdots & & \vdots & & \vdots \\ [X_{i1}^F] & \dots & [X_{ij}^F] & \dots & [X_{iN_{load}}^F] \\ \vdots & & \vdots & & \vdots \\ [X_{N_n 1}^F] & \dots & [X_{N_n j}^F] & \dots & [X_{N_n N_{load}}^F] \end{bmatrix}_{2N_n \times N_{load}} \quad (3.34)$$

where

$$[X_{ij}^F] = \begin{Bmatrix} W_{ij}^F \\ \Theta_{ij}^F \end{Bmatrix} \quad (3.35)$$

$[R^F]$ a $(N_r \times N_{load})$ matrix of unknown point forces caused by the actual external loads acting on the floor slab.

$[A_1]$ and $[A_2]$, $[B_1]$ and $[B_2]$ are the same matrices as those given in equations (3.13) to (3.20). Equations (3.28) and (3.29) can now be rewritten as

$$-[A_1] [X^F] + [B_1] [R^F] = -[Y_1^F] \quad (3.36)$$

$$[A_2] [X^F] - [B_2] [R^F] = [Y_2^F] \quad (3.37)$$

Solving for $[X^F]$ and $[R^F]$, the fixed end force matrix is obtain

$$[F_f] = [T] [R^F] \quad (3.38)$$

where

$$[T] = [w_r]^T \quad (3.26)$$

$[F_f]$ represents the fixed end forces caused by the actual external transverse load acting directly on the floor slab. Lateral loads acting on the building frame can be dealt with in a manner similar to that used for direct joint forces in the matrix frame analysis.

3.6 Building Frame Analysis

Once the stiffness matrix and fixed end forces are determined, the displacements of the frame can be calculated by the stiffness method, that is

$$[K] [\Delta] = [F] \quad (3.39)$$

$$[\Delta] = [K]^{-1} [F] \quad (3.40)$$

where $[\Delta]$ is a $\{N_{Storey}(3N_{col}+3) \times 1\}$ matrix, which contains the column-slab joint displacements and two translational and one rotational displacement for each floor.

The column-slab joint displacement matrix for a floor is denoted as $[\delta]_{3N_{col} \times N_{load}}$. The column internal forces are easy to calculate by matrix multiplication, ie. $[F_c] = [K_c] [\Delta]$. Subscript c refer to the column elements. The final magnitudes of the point forces, due to both the applied loads and the corresponding column patch displacements, are as follows

$$[R_f]_{N_f \times N_{load}} = [R] [\delta] + [R^F] \quad (3.41)$$

The final values of boundary nodal variables are calculated as follows

$$[X_f]_{N_f \times N_{load}} = [X] [\delta] + [X^F] \quad (3.42)$$

The internal actions in the floor slab can be determined following the standard procedure of the DBEM for plate bending, for which the details are given in reference [2].

4. SIMPLE BUILDING FRAME STUDIES

4.1 Introduction

To demonstrate the method of building frame analysis by the DBEM, examples of building frame analysis are provided in this chapter. Results are compared for different column patch models, and various boundary element discretisations and polynomial orders of boundary element. The analyses include building frames under both vertical and horizontal loads, and column patch modelling.

4.2 Floor Slab Analysis of Simple Building Frame Example

This section will present the results of analysis of both a one-storey and a multi-storey building frame examples. Floor slab dimensions and column layout are identical for each. The Young's modulus for each is assumed to be 480,000 psi (3300 MPa), the Poisson's ratio is 0.35.

4.2.1 Analysis of Floor Slab under Vertical Loading

One-storey building frame structure.

The dimensions of the one-storey building frame are shown in Fig. 4.1. A uniform load of $p=10 \text{ lb/ft}^2$ is assumed to act on the floor slab.

The column-slab joints will be modelled in four different ways. The first two models consider column patches with zero displacements. The difference in the first two models is the way the rigid patches are treated. In the first model, the rigid patches are modelled by the Influence Coefficient Method (ICM) using discrete point forces to replace the pressure between column and slab. In the second model, boundary elements are employed around the boundary of each patch. This method is superior from a theoretical point of view, and the author is

Fig. 4.1 Simple Flat Plate Building Frame

interested in comparing the results of this with model one. It is, however, very limited in practical terms. In the models three and four, the patches, which are constrained to remain rigid can rotate and displace (as rigid bodies). In summary the models are described as follows:

- 1) The ICM is used for the column patches. The interface pressure between each column and plate is approximated by discrete forces. The number of forces is equal to 5 (Fig.4.2). The displacements at the positions of the point forces are zero.
- 2) The common boundaries between the plate and columns are modelled by boundary elements. The common boundaries are assumed to be clamped in this example. Each side of each column is modelled by a 2nd order boundary element (Fig. 4.3).

Fig.4.2 Discrete Forces within the Column Patch
(for Simple Building Frame)

Fig. 4.3 Clamped Boundary between Column and Slab

- 3) The ICM is used for the column patches. The columns are assumed fixed at their remote ends. The stiffness, in the directions shown in Fig. 4.4a, of each column are taken into account. The frame is shown in Fig. 4.4b. This model was used for frame analysis under vertical loads in Abdel-Akher's thesis [2].
- 4) The ICM is used for the column patches. The degrees of freedom of the column number 12 (Fig.4.5a). The analyzed frame is shown in Fig. 4.5b.

Fig.4.4 Column Patch Model Three

Fig. 4.5 Column Patch Model Four

Model four was initially used only for calculation of building frame sways; it has now been extended to frame analysis and floor slab bending analysis under vertical loads as well as horizontal loads.

The floor slab is assumed to have free edges. For the boundary element method, the number of boundary elements along each plate external edge is set equal to 6, and each is taken as a 2nd order boundary element (Fig.4.6).

Fig.4.6 Boundary Element

The deflection and moment distributions along $y=0$ and $y=9.0$ in. are provided in Figs. 4.7 to 4.9. The differences between the deflections and moments of models with four column patches can be observed. The deflections and moments calculated using model one and model two are smaller than those calculated using model three and model four at most of the internal points except at points near the edge of the floor slab along the line $y=9$ in. (Fig. 4.7). As expected, model one is a bit more flexible than model two. Differences between the first two models and second two are obvious and relate to the higher flexibility of frames in model three and four. Model three and model four produce greater deflections and positive moments since the column patch is able to move rigidly. The smaller deflection values of the model three and

model four, along the line $y=9$ in. near the floor slab edge(Fig. 4.7), are caused by the rotations of the column patch.

Model one and model two provide stiffer column support than model three and model four do. Model two produces the smallest deflections and positive moments at the mid span and greatest negative moments near the column region because of the clamped boundary elements used for the column patch, these boundary elements keep the column patch from displacing and rotating.

The column stiffness in model three is double of that in model four for the same column in this typical one-storey building frame structure (Figs. 4.4 and 4.5). The different column stiffness results in the differences in deflection and moment distributions. For multi-storey building frames or structures with no plane of symmetry, the deflections of the adjacent floor slab or the frame sway and twist would cause additional deflections and moments in the analyzed floor slab. The effects of these kinds of structure will be discussed later in this sections or next chapter.

Fig. 4.7 Deflection along $y=0$ and $y=9$

Fig. 4.8 Moment along $y=0$

Fig. 4.9 Moment along $y=9$

Multi-storey building frame structure.

For further study, a three-story building frame example (Fig. 4.10) has been developed. The model has an identical side plan, that is

Fig. 4.10 Multi Storey Building Frame

The dimensions of the floor slabs are the same as those shown in Fig. 4.1. The frame carries a uniform vertical load on each floor ($p=10 \text{ lb/ft}^2$). The deflection and moment distributions for the second floor slab were calculated using the aforementioned column-slab joint models, three and four, in turn. The results can thus provide some information relating to single storey vertical load analysis compared to full frame analysis. The results are given in Figs. 4.11 to 4.13.

Fig. 4.11 Deflection along $y=0$ and $y=9$
(for the Second Floor Slab of the Multi-Storey Building Frame)

Fig. 4.12 Moments along $y=0$
(for the Second Floor Slab of the Multi-Storey Building Frame)

Fig. 4.13 Moment along $y=9$
(for the Second Floor Slab of the Multi-Storey Building Frame)

For this typical building frame structure, there is no frame sway and twist under uniform vertical loads. The deformations of the two kinds of frames, as shown in Figs. 4.14 and 4.15, have been studied to explain the differences of the results calculated using model three and model four, one for model three which just considers the floor and connected columns; the other for model four which considers the full building frame.

Fig. 4.14 Equivalent Plane Frame

Fig. 4.15 Three Storey Plane Frame

The deflected shapes, sketched in Figs. 4.14 and 4.15, illustrate that for these configurations of structure/load, the full three-storey model is actually stiffer than the single one-storey frame.

For practical purposes, model three can be used to simplify the analysis, and it gives conservative results. However, this model has some restrictions on the layout of the structure and the load conditions.

The remainder of this thesis will deal with models of type four.

Boundary element study.

The one storey building frame example is again used to show the variation of results for different numbers of boundary elements and for different orders of elements. Figs. 4.16 to 4.19 provided the calculated distributions of deflection, moments and shear forces along the x -axis and the line $y=9.0$ in. for 2, 4, 6, 8 and 12 boundary elements per side, respectively. A 2nd order element is used in all of these cases. The deflection and moment distributions give very good agreement with each other, except near the the edge of the slab, the distributions of shear force are not as good. From the deflection and moment distributions, it has been found that the increase of the number of boundary elements does not provide a consistent improvement of the accuracy of final results, but does of course increase the number of unknown variables and consume more c.p.u. time.

The theoretical reason for the poorer results for equivalent shear calculations have been described by Abdel-Akher [2] and by Abdel-Akher and Hartley [3]. The poorer results for this quantity near the boundary and moments very close to the edge relate to the highly singular nature of the kernels of boundary integral and also in the numerical process itself which places unit impulses along boundary locations and integrates through them. Where two different elements meet along a line of symmetry the numerical process is unstable enough to disturb symmetry. Abdel-Akher and Hartley considered high accuracy for shear to be of less practical importance than moments. The author, herself considers equivalent shear to be of less interest here since her main concern is with the stiffness modelling of building frame.

Figs. 4.20 to 4.21 show the results of calculations for one boundary element per side; the order of the boundary element being varied from 2 to 8. The curves of deflections and moments along the x -axis seem to have good agreement with each other. Along the line $y=9$ in., the deflections, obtained using a 2nd order and 4th order element, are also reasonable. However the curves of deflections obtained using order greater than 6 and moments M_{xx} are not symmetrical for this symmetrical building frame under a symmetrical uniform vertical load. This means that increase of polynomial order of the boundary element could cause a decrease of the accuracy. Choosing a high order boundary element will also be more c.p.u. intensive.

Fig. 4.16 Deflections at $y=0$ and $y=9$
(Different Number of Boundary Element)

Fig. 4.17 Moment M_{xx} at $y=0$ and $y=9$
(Different Number of Boundary Element)

Fig. 4.18 Moment M_{yy} at $y=0$ and $y=9$
(Different Number of Boundary Element)

Fig. 4.19 Shear Forces in x-direction at $y=0$ and $y=9$
(Different Number of Boundary Element)

Fig. 4.20 Deflection and Moment Distributions along y=0

Fig. 4.21 Deflection and Moment Distributions along $y=9$

It can be concluded from this simple building frame example, that dividing the floor slab boundary into several 2nd order elements per side may be more reliable than using one high order element per side.

4.2.2 Analysis of Floor Slab under Horizontal Loading

The one-storey building frame example (Fig. 4.1) is used to analyze the bending of a floor slab under a horizontal load of 10 lb. A concentrated force is acting in x -direction on the floor level.

The results of the calculations of deflection and moment are provided in Figs. 4.23 and 4.24. Comparing the results calculated by the DBEM with the the deflection and moment diagram of a simple plane frame under a concentrated horizontal load (Fig. 4.22), the deflection and moment distributions appear to be correct.

Fig. 4.22 Simple Plane Frame

Fig. 4.23 Deflections at $y=0$ and $y=9$
(under Lateral Load)

Fig. 4.24 Moments at $y=0$ and $y=9$
(under Lateral Load)

4.3 Arrangement of Point Forces within the Column Patch

In the ICM, a set of selected point forces are used to replace the pressure between the column and the floor slab. The preliminary studies of arrangement were done by Abdel-Akher [2], and some suggested arrangements were given for square and elongated columns. However, further study is still needed to find the ideal column patch pattern, which incorporates a reasonable number of discrete point force and acceptable accuracy.

A building frame, which has been shown in Fig. 4.1, was used to study the arrangement of point forces within the square column patch. Discrete point force layouts 4, 5, 8 and more than 8 are shown in Fig. 4.25, and the maximum values of deflection and moment in middle span of the floor slab and the moment of the column are given in the same figure.

Fig. 4.25 Square Column Point Force Arrangements

Comparing arrangements, it is seen that the deflection and moment in the floor slab decreased when the discrete point forces increase in the column patch. This means that the rigidity of the column patch can be increased using more point forces. However, the internal reactions of the floor slab and columns were not changed as much as expected when the number of point forces

increased from 8 to 20, and more point forces produce more unknown variables. It is interesting that the rigidity of the column patch increased when the second layer of point forces move close to the first layer; and once the two layers of point forces were located on the column patch, the effect of the subsequent layers of point forces could be omitted (Fig. 4.26).

$N = 8$

Defl. = -0.02028 in.

$M(\text{slab})_{\max} = 1.842 \text{ lb-in./in.}$

$M(\text{col.}) = 16.9 \text{ lb-in.}$

$N = 9$

Defl. = -0.02033 in.

$M(\text{slab})_{\max} = 1.845 \text{ lb-in./in.}$

$M(\text{col.}) = 16.9 \text{ lb-in.}$

$N = 12$

Defl. = -0.02028 in.

$M(\text{slab})_{\max} = 1.842 \text{ lb-in./in.}$

$M(\text{col.}) = 16.9 \text{ lb-in.}$

$N = 24$

Defl. = -0.02024 in.

$M(\text{slab})_{\max} = 1.840 \text{ lb-in./in.}$

$M(\text{col.}) = 17.0 \text{ lb-in.}$

Fig. 4.26 Additional Arrangement of Point Forces
(for Square Column)

Using the same frame as that used for the square column, the point force arrangements in a rectangular column are investigated. The effects of discrete point forces are similar to those observed in square column, the results are given in Fig. 4.27.

Fig. 4.27 Arrangement of Point Forces and Analysis Results
(for Rectangular Column)

Another one-storey building frame was used to study the column patch of an elongated column. The dimensions of the frame are shown in Fig. 4.28.

Fig. 4.28 Dimensions of Building Frame for Elongated Column

The arrangement suggested by the previous example and some other patterns were studied, results of the analysis are given in Fig. 4.29. The results show that the rigidity of column patch are primarily affected by the point forces on the edge. It is noted that the results do not change when four of the twelve point forces are moved from the first layer to the second layer (Fig. 4.29).

Comparing all the results of the tests, it can be concluded that the 8 point forces located on two layers, as shown in Fig. 4.30a) and 4.30b), is a reasonable point force arrangement for the square and rectangular columns given the limited investigation presented here. For the elongated column, the arrangement suggested in Abdel-Akher's thesis and shown in Fig. 4.30c) is recommended.

Fig. 4.29 Arrangement of Point Forces and Analysis Results
(for Elongated Column)

Fig. 4.30 The Suggested Arrangement of Point Forces

4.4 Conclusion

Different building frame examples are analyzed by the Direct Boundary Element Method. An acceptable column patch model is chosen by the study of column patch models. Different boundary mesh and different boundary element are studied by comparing the deflection, moment and shear force distributions in the floor slabs. The method does give very good continuous deflection and moment curves. However, the equivalent shear forces in the floor slab are still not satisfactory.

It is suggested that a reasonable number of boundary elements and a 2nd order element are superior to ensure reasonably accurate results for the floor slab bending analysis in the DBEM of building frame analysis.

5. ANALYSIS AND EXPERIMENTAL INVESTIGATION OF BUILDING FRAME

5.1 Introduction

This chapter considers the numerical and experimental study of a structure which includes some of the complexity of real building frames. The earlier definition given of a building frame continues to hold. The structural type described here is perfectly elastic; the slabs are perfect thin plates which follow Kirchhoff's hypothesis; and the columns behave like space frame elements based on simple beam theory, in which plane sections remain plane throughout the member even at points of contact with the slabs.

For the experimental portion the author has designed and built two plexiglass models. In the work that is described in this chapter, it is intended that these models are physical analogues of the corresponding numerical models created by the DBEM.

In two models which are considered, one is a three-storey building frame with an asymmetrical floor slab plane, the other is a one-storey version of the first model. The one-storey version is not typical of a one-storey building frame in that it has a shaft whose walls are thick elongated columns. The purpose of the one-storey model was to study stiffness influence coefficients and also to have a more detailed instrumentation of a single floor. Due to delays in being able to schedule laboratory work, the three story model could not be tested. The results of an analysis are given in appendix along with all of the stiffness matrix terms. The static and dynamic testing of this model are recommended for future work, perhaps as an undergraduate project. The results given in appendix B are intended to relieve the future researcher from the burden of using the computer program.

The experimental testing for influence coefficients has also not been done. The results of a computer run for a single rotational stiffness coefficient are given in appendix C. The features of the structure include asymmetry in plan, and elongated columns including a type of shear core. The layout is not strictly rational in the one-storey model, but again, it was originally intended that this simple one-storey version be used to obtain influence coefficients,

plus more detailed strain measurements in the slab than would be possible in the three-storey version. In other words the model is not meant to represent a true building, but does incorporate many of its complexities.

The Young's modulus is assumed to be 480,000 psi(3300MPa) in accordance with published literature and confirmed by experiment. The Poisson's ratio was measured to be 0.35.

5.2 Analysis of One-Storey Building Frame Example

The full three dimensional one-storey building frame is given in Fig. 5.1. The floor layout of the frames is shown in Fig. 5.2. The thickness of the floor slab is 0.5 inch, ie. $l_n(\text{max.})/38$, where l_n is the clear span. The base slab thickness is 1.0 inch. The height of storey is 10 inches.

Fig. 5.1 Three Dimensional One-Storey Building Frame Model

Column patch model four (see section 4.2.1) is used for calculating the stiffness of the slab super-element and the internal actions of the slab. Within the column patch, eight discrete point forces are used for the 1.0x1.0 (inch) and 1.0x2.0 (inch) column sections, twelve discrete point forces are used for the 1.0x5.0 (inch) column section .

Fig. 5.2 The Floor Slabs Dimensions

In the analysis, a uniform vertical load of $p=10 \text{ lb/ft}^2$ is assumed to act on the whole area of the top floor. Three types of mesh are used to study the effect of boundary mesh refinement. These are shown in Fig. 5.3. A 2nd order element is used for all of these types. The results of the frame analysis are provided in Fig. 5.4, giving the calculated frame sway and rotation as well as column internal forces. It is found that there are transverse displacements in the x - and y - directions and about the z -axis, although only vertical loads act on the floor

Fig. 5.3 Boundary Element Mesh Types

```

.....
*   ANALYSIS OF BUILDING FRAME   *
.....

NO OF COLUMNS PER FLOOR = 9
NO OF STOREYS             = 1
YOUNG MODULUS             = 480000.0
POISSONS RATIO            = 0.35
SLAB THICKNESS            = 0.50

STOREY HEIGHT
-----
STOREY NO.    HEIGHT
      1      0.1000E+02

APPLIED LATERAL LOADS
-----
FLOOR NO.    LOAD
      1      0.0000E+00 0.0000E+00 0.0000E+00

THE STOREY SWAY
-----
STOREY NO.    X-TRANS    Y-TRANS    Z-ROT
      1      0.2795E-03    0.1084E-03    0.1235E-04

***** THE COLUMNS MOMENTS *****

STOREY NO = 1
-----
COLUMN NO.    MXX          MYY          MT          N          VX          VY
-----
          LOWER    UPPER    LOWER    UPPER    ---    ---    ---    ---

1 -0.653E+01 -0.121E+02  0.507E+01  0.104E+02  0.285E-01 -0.920E+01  0.155E+01  0.186E+01
2 -0.164E+02 -0.350E+02 -0.596E+00  0.802E+00  0.106E+00 -0.191E+02  0.206E-01  0.514E+01
3 -0.717E+01 -0.145E+02 -0.771E+01 -0.134E+02  0.285E-01 -0.998E+01 -0.211E+01  0.217E+01
4  0.904E+01  0.157E+02 -0.675E+01 -0.122E+02  0.285E-01 -0.974E+01 -0.189E+01 -0.247E+01
5  0.645E+01  0.108E+02  0.170E+01  0.616E+01  0.106E+00 -0.202E+02  0.786E+00 -0.173E+01
6  0.525E+01  0.922E+01 -0.614E+01 -0.116E+02  0.285E-01 -0.897E+01 -0.178E+01 -0.145E+01
7  0.356E+01  0.666E+01  0.633E+01  0.116E+02  0.285E-01 -0.823E+01  0.179E+01 -0.102E+01
8  0.132E+02  0.129E+01  0.567E+01  0.121E+02  0.337E+00 -0.176E+02  0.178E-01 -0.145E+01
9  0.207E+01  0.839E+01 -0.106E+01 -0.390E+00  0.337E+00 -0.751E+01 -0.145E+00 -0.105E+01

```

Fig. 5.4 Results of Frame Analysis (under Uniform Vertical Load)

slab of the frame. The results of the floor slab bending analysis are given in Figs. 5.5 to 5.9. The frame sways are about two orders of magnitude smaller than slab displacement (see Figs. 5.4 and 5.5). This frame is obviously very stiff horizontally. It may be of interest to the reader, however, to note that the calculation of horizontal displacements under vertical loading is quite routine in the procedure being used here.

The deflection distributions for the three types of mesh along the x - and y - axes and the line $x=9.0$ (inch) are almost identical (Fig. 5.5 and Fig. 5.8). The values of deflection along the line $y=12.5$ (inch) for the three mesh types (Fig. 5.9) do not agree with each other since most of the internal points are near the boundary nodes or discrete point force within the column-slab patch. It is found generally in this type of analysis that results are superior at points remote from edge boundary nodes. Even with discrepancies such as those shown in Fig. 5.9, it is felt, that the mesh refinement is adequate for the purpose of frame analysis. The distributions of moments shown in Figs. 5.6 to 5.8 along lines $y=0$, $x=0$, $x=9.0$ and $y=12.5$ show good agreement between the three mesh types

The deflection and moment distributions for the case of 40 boundary elements are also compared with the results obtained using column patch model three with a 40 boundary element mesh. The curves are shown in Fig. 5.10. The differences of deflection and moment distributions reflect the effects of frame sway, providing a comparison with one particular assumption made in code procedures. There are of course several other simplifications in the code procedures. These results are provided only for the interest of the reader, and are not meant to criticize current practise.

Fig. 5.5 Deflections along x and y axes

Fig. 5.6 Moments along x-axis

Fig. 5.7 Moments along y-axis

Fig. 5.8 Deflections and Moments at $x=9$

Fig. 5.9 Deflections and Moments at $y=12.5$

Fig. 5.10 Comparison of Column Patch Model Three and Model Four

5.3 Experimental Study

5.3.1 Introduction

Historically, experimental modelling of structures has been very important in structural engineering, both research and design. For a complex structure, such as a flat plate building frame structure with an irregular floor layout, there is no analytical solution. It is expected that the numerical model is capable of producing excellent results. However, beyond the initial idealization of the structure, approximations to the rigid joint displacements, in addition to the approximation of slab boundary function, have been made. A plexiglass model has the potential of being a good physical analogue of the numerical model. Those who are not familiar with theoretical basis for the DBEM will have to be convinced of its potential for high accuracy, even at points of singularity on the slab surfaces. This is the main reason for the experimentation described in this chapter.

The model building frame structure was designed and instrumented for both vertical and horizontal loads. In the tests described here, the one-storey frame was subjected to vertical loads only.

5.3.2 Construction of the Test Models

Material.

As discussed in chapter one, the calculations for flat plate building frame structures are limited to the elastic range in the DBEM analysis. Models used in this experimental study are made from rectangular sheet plastic. All material properties except Poisson's ratio were provided by Chemieryl Plastics Limited. Some flexure tests and tensile tests were carried out to obtain Poisson's ratio and to verify the modulus of elasticity. All specimens used for these tests were cut from the same sheets used for the actual models. Two beam specimens were used for the flexural test. Four tensile specimens were tested according to ASTM standard [7].

Based on the flexure and tensile tests and referring to the study of plastic structural models by J. S. Gero and H. J. Cowan's [19] and physical properties of Acrylite FF sheet provided by Chemicryl Plastics Limited, the final results are

Modulus of Elasticity	0.48×10^6 psi
Poisson's Ratio	0.350

Geometry and Construction.

Usually, experimental models are scale models of prototype structures. The purpose of this analysis, however, is to verify the DBEM for building frame analysis. The geometry of the two models being used in the experimental study are the same as those used in the DBEM analysis in section 5.2. One of the models is a three-storey building frame with an asymmetrical floor plan discussed further in appendix B. The other is a one-storey version of the first one, as mentioned earlier, having for example an elevator core and dimensions identical to those given earlier in this chapter (Figs. 5.1 and 5.2).

All floor slabs of two models were cut from 0.5 inch thick sheets, all columns and the base slabs were cut from 1 inch sheet. Floor slabs and base slabs were cut into the dimensions as shown in Fig. 5.2. All columns were cut into the length of 9.5 inches, hence, the floor slab thickness of 0.5 inch adding to the column length maintains the storey height of 10 inches.

Before the construction, all column locations were marked on the floor slabs and base slabs, the dimensions of all columns were carefully measured to ensure the columns would be connected at right angles to the slabs. The models were assembled on a flat table with a steel frame fixed to the table. The steel frame was used to align the slabs during the construction. A chlorinated hydrocarbon solvent, Dichloromethane, was used to "weld" columns to the base slab and the floor slab to obtain rigid column-slab joints. Only 30 seconds of working time are permitted once the solvent is applied on the plexiglass. The columns were glued to the slab one by one with pressure applied on the top of the column in the first five minutes to assure perfect contact, then weights were put on the column top until the next slab was ready to be glued to the tops of the columns. The top slab was glued to the columns in two stages, in the first stage gluing the slab to the columns located on the B-axis (Fig. 5.2) and applying the weights directly on those glued columns for four hours; and in the second stage lifting the unglued parts

of the slab slightly and applying the solvent between the rest of the column slab interfaces; and finally adding the weights on the locations of each column for one day. Photographs are provided in appendix D to show: the model during the assembling; vertical alignment members; data acquisition; and the supporting table. During testing the base slab was clamped near its outside edges to prevent displacement.

5.3.3 Instrumentation

Strain measurements were taken using electrical resistance strain gauges for the floor slab, A System 4000, data acquisition, was used to record strain values. Although, due to the characteristics of the material, this part of the experimentation was less conclusive, valuable experience has been gained, and therefore the details will be reported here. Forty linear gages were used. Thirty five gages were attached to the top surface of the slab, as shown in Fig. 5.11, though companion gages were attached on the bottom surface at three points (see Fig. 5.11) to determine the presence of axial stresses in the slab. At two locations, the gauges were mounted on the bottom of the slab only. Most gauges were located along the x - and y - axes and the line of y equals 12.5 inch. Some gauges were located on the column or at representative points near the boundary.

The deflections of the floor slab of the one storey model were measured with a 0.0001-in. dial gauge, as shown in Fig. 5.12. A Letraline tape grid was provided on the base slab, as shown in Fig. 5.13. The deflections of the top slab, along the x - and y - axes, along the lines x equals -6 in., 6 in. 9 in. and 22 in., and along the lines y equals -6 in. 6 in. and 22 in., were obtained by moving the dial gauge along the mesh.

5.3.4 Methods of Loading

A uniform vertical load was used to test the one-storey frame model. About 120 small sand bags were used to produce three layers of uniform distributed load over the whole area of floor slab, each layer is 5 lb/ft² (0.24 KN/m²). The purposes of using three layers of load are

Fig. 5.11 Locations of Strain Gauges on the Floor Slab

to reduce the error which is caused by the gaps between each small bags, and to observe how the structure behaves under each 5 lb/ft^2 such that a more realistic test result can be obtained. Two loading cycles were undertaken in the test to assure obtaining useful test results and to compare the test results for different load durations. In the first cycles, the loading time was

2

of/de

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Fig. 5.12 Dial Gauge

Fig. 5.13 Grid for Measuring the Deflection

about forty eight hours between $p = 0 \text{ lb/ft}^2$ and $p = 15 \text{ lb/ft}^2$. Six hours were used for unloading from $p = 15 \text{ lb/ft}^2$ to $p = 10 \text{ lb/ft}^2$, sixteen hours were used from $p = 10 \text{ lb/ft}^2$ to p

= 5 lb/ft² and eleven hours were $p = 5 \text{ lb/ft}^2$ to $p = 0 \text{ lb/ft}^2$. In the second cycles, twelve hours loading time was used for each layer of loading and unloading. The procedure of operations is listed in detail as follows:

1. Initialize strain gauge readings
2. Record strain gauge readings and measure floor slab deflections for zero loading
3. Add first layer of load (5 lb/ft²), record strain gauge readings and measure floor slab deflections for load of 5 lb/ft² just after loading
4. Record strain gauge readings and measure floor slab deflections for load of 5 lb/ft² before add next layer of load
5. Add second layer of load (5 lb/ft²), record strain gauge readings and measure floor slab deflections for load of 10 lb/ft² just after loading
6. Record strain gauge readings and measure floor slab deflections for load of 10 lb/ft² before adding next layer of load
7. Add third layer of load (5 lb/ft²), record strain gauge readings and measure floor slab deflections for load of 15 lb/ft² just after loading
8. Record strain gauge readings and measure floor slab deflections for load of 15 lb/ft² before unloading, unload one layer of load (5 lb/ft²)
9. Record strain gauge readings and measure floor slab deflections for load of 10 lb/ft² just after unloading
10. Record strain gauge readings and measure floor slab deflections for load of 10 lb/ft² before unloading, unload one layer of load (5 lb/ft²)
11. Record strain gauge readings and measure floor slab deflections for load of 5 lb/ft² just after unloading
12. Record strain gauge readings and measure floor slab deflections for load of 5 lb/ft² before unloading, unload last layer of load (5 lb/ft²)
13. Record strain gauge readings and measure floor slab deflections for zero loading just after unloading
14. Record strain gauge readings and measure floor slab deflections for zero loading 12 hours after unloading

5.3.5 Results

Deflections.

All the deflection readings were obtained for an equivalent uniformly distributed vertical load of 10 lb/ft² (0.479 KN/m²), applied to the floor slab of the building frame structure. Since the deflections were measured by moving one dial gage over the base slab, the initial deflection readings were generally different from each other. The deflection values were obtained by calculating the differences in deflection readings of increments of 10 lb/ft². For instance, the initial deflection was subtracted from the deflection readings under a load of 10 lb/ft², and the deflection readings under a load of 5 lb/ft² was subtracted from those under a load of 15 lb/ft². The calculated deflections of each load of 10 lb/ft² can be used to compare with the deflection distributions calculated by the DBEM. The deflection values presented in this study are those calculated from values read just before loading and unloading.

Figs. 5.14 to 5.19 show the deflection curves along the *x*- and *y*- axes and along the lines described in section 5.3.3. In these figures, load1 means loading from 0 lb/ft² to 10 lb/ft² and load2 means loading from 5 lb/ft² to 15 lb/ft², the plotting is deflection values of differences between two load level, as mentioned above; unload1 means unloading from 15 lb/ft² to 5 lb/ft², unload2 means unloading from 10 lb/ft² to 0 lb/ft², the plotting is deflection values of differences between two load level, which are obtained by subtracting deflection readings of 5 lb/ft² from those of 15 lb/ft² or subtracting deflection readings of 0 lb/ft² from those of 10 lb/ft², such that the deflections to be plotted are all negative, these values can be compared with the deflections obtained during the loading; c1 means the first load cycle and c2 means the second load cycle. The calculated deflection distributions using the DBEM of 40 boundary element mesh are given on the same figures for purposes of comparison.

(a)

(b)

Fig. 5.14 Deflections along x-axis

(a)

(b)

Fig. 5.15 Deflections along y-axis

Fig. 5.16 Deflections at $x = -6$ and $x = 6$

Fig. 5.17 Deflections at $x=9$ and $x=22$

Fig. 5.18 Deflections at $y=-6$ and $y=6$

Fig. 5.19 Deflections at $y=22$

Strain Gauge Readings.

Strains measured on the floor slab were not stable during the loading duration. Choosing a typical set of strain gauge readings under a certain load, it was found that the directions and the differences in magnitudes of the moment in the floor slab could be seen from the strain values. Examining all the test data, however, the strains were so different under the same load level that the stress distributions of the floor slab were difficult to explain. The variation in strain readings is thought to be caused by material property variations and by thermal expansion due to temperature variations. These are discussed in the following section.

5.3.6 Discussion of the Results

Deflections.

As an ideal linear-elastic structure, the deflections of the floor slab should be basically the same for any 10 lb/ft² uniform vertical load applied to the floor slab. However, it can be observed from Figs. 5.14 to 5.19 that although the deflection distributions along certain lines, obtained from different loading cycles, are similar in curve shape and slope, the maximum magnitude of the deflections were not in mutual agreement. The deflections of most points measured from longer loading duration were greater than those measured from shorter loading duration. For loading times of 10 lb/ft² longer than four days in first load cycle, the deflection distributions were close to those obtained using the DBEM along the x -axis (see Fig. 5.14a). The deflection distributions along the y -axis, however, were quite different from the results of the DBEM analysis (see Fig. 5.15a). For the second load cycle, the loading and unloading durations were restricted to 12 hours. The deflection readings were more stable (see Fig. 5.14b and 5.15b).

Comparing the tested results with those calculated using the DBEM demonstrates that the DBEM does indeed "model" the building frame. All curves are similar, and some of the measured values are very close. It should be noted that most of the deflections obtained by experiment were greater than those calculated for the same internal points of the floor slab (see Figs. 5.14 to 5.19).

Sources of the error could be the change of modulus of elasticity of the material, the creep of the material, and to a lesser extent the effect of deflections in base slab and the deviation of measuring points.

The deflections of the floor slab in the present experiments increase with the decrease of the modulus of elasticity of the material. A flexure test was done by J. S. Gero and H. J. Cowan [19], the modulus of elasticity, by measuring the deflection and the strain values of strain gauges, decreased from 438,00 psi to 397,00 psi (from deflection measurements) and from 450,00 psi to 424,00 psi (from strain measurements on tensile face) when loading time changed from 1 minute to 60 minutes.

Since the deflection readings were obtained using only one dial gauge which was moved along the grid mesh on the base slab, small deflections on the base slab may have affected the readings, although the flexural rigidity of the base slab was 8 times of the that of the floor slab. The dial gauge could not be exactly at the given points at each measuring time.

Temperature changes in the test region of the laboratory could also produce error in the deflections. The thermal expansion of this material, provided by Chemcryl Plastics Limited, is 64×10^{-6} in/in/ $^{\circ}\text{C}$ (in acrylic sheet of 0.250" thickness). The load levels were not large enough to reduce the relative importance of thermal expansion error to negligible limits

Strain Gauge Readings.

All strain gauges on the model were checked thoroughly before the test to ensure that they were properly applied. It was found, however, that the maximum strain gauge reading for the load of 10 lb/ft² could be double that of calculated result. The differences in strain readings were significant between two sets of readings at the same load level, ie. the strain gauge readings recorded just after loading or unloading and just before adding another 5 lb/ft² loading or unloading. Two sets of test data are given in Table 5.1. It can be noticed from the table, all strain gauge readings increased in certain positive values within twelve hours, the maximum increased value is 75×10^{-6} in/in, which is at the point right on the corner column.

Table 5.1 Strain Gauge Readings under Load of 10 lb/ft²
(in Second Load Cycle)

x	y	Strain ($\times 10^{-6}$) (9:29 pm 05/22/90)	Strain ($\times 10^{-6}$) (9:38 am 05/23/90)
-12.5	-12.5	-23	9
0.0	-12.5	-91	-64
10.0	-12.5	31	46
-9.5	-10.0	189	218
-10.0	-9.5	293	368
-10.5	-10.0	-61	-44
-10.0	-10.5	-98	-93
0.0	-10.0	6	33
0.0	-1.0	-74	-44
* 0.0	-1.0	7	49
-12.5	0.0	-5	33
-10.0	0.0	11	43
* 0.0	0.0	35	68
-1.0	0.0	-38	-7
* -1.0	0.0	45	70
-10.0	0.0	-48	-30
20.0	0.0	28	36
* 20.0	0.0	40	50
30.0	0.0	6	27
32.5	0.0	63	81
-3.0	10.0	-5	52
0.0	10.0	-39	-9
10.5	10.5	65	101
11.5	10.0	209	245
10.5	9.5	162	175
9.5	10.0	144	169
-12.5	12.5	-26	13
-2.5	12.5	-12	19
0.0	12.5	15	43
5.0	12.5	-38	-11
10.0	12.5	42	73
-12.5	12.5	67	89
** -12.5	12.5	120	154
-3.0	14.5	13	68
-10.0	20.0	-8	22
* 0.0	20.0	33	68
10.0	20.0	19	50
0.0	24.0	-41	-13
0.0	30.0	-2	30
0.0	32.5	5	45

* at these points the strain gauges are located at the bottom of the plate

** the strain gauge located on No.9 axis (see Fig. 5.11)

The differences might be caused by a decrease in modulus of elasticity with time, which would result in a stress re-distribution in the floor slab. Temperature change in the laboratory is another source which might affect the strain gauge readings. It was observed by initializing the strain gauges on all of the channels, then scanning the strain gauge readings for two days without any applied load, that the maximum strain gauge reading during this period was 25×10^{-6} in/in.

5.3.7 Conclusion

The DBEM for analysis of building frame applies to structures within the elastic range. The scale model described here is a rigid building frame structure constructed from plastic, to provide a physical analogue. The test gives good deflection distributions for the tested plate, although the deflections at equal levels of loading were not the same. As indicated by the difference of the deflection distribution, the loading duration for a plastic model test may require careful consideration.

Results from the strain gauges provided limited information due to the small load levels applied, the change of room temperature in the laboratory and the possible change in plastic properties with loading time. To calculate the moment distributions on the floor slab directly from the strain gauge readings, rosette strain gauges might be superior to the linear type at those points which are not close to the column region and to the edges of the floor slab.

The material used in this experiment was quite brittle compared to other plastics. In fact there were problems with the reentrant cut corner, as shown in Fig. 5.20a. In one plate a crack had to be repaired after it formed during strain gauge mounting. Small round corners were then applied to the others. The relative brittleness of this material indicates that it probably has some superior characteristics compared to other plastics which might be used for experimental modelling.

Fig. 5.20 Shape Corner in the Plate

6 CONCLUSION AND FUTURE WORK

6.1 Summary

In this thesis, the methods for the analysis of flat plate building frames were reviewed in the first chapter, and the DBEM(boundary element method) for the analysis of building frame slabs was discussed in chapter two and chapter three. In chapter four, some simple frames have been analyzed in order to provide comparisons of different approaches and simplifications in modelling. The study of point configurations in the column patches was limited but reasonably conclusive. The same might also be said of edge element configuration and order. For at least the foreseeable future it is felt that for the purpose of building frame analysis further work in studying other configurations is not warranted. The approach given herein is adequate. Chapter 5 described the analysis of a more complex building model and compared this with experimental studies of a plexiglass frame.

The thesis has featured the following approach to the numerical analysis of building frames. The work of Abdel-Akher on the DBEM of slab with finite rigidly displacing patch locations has been incorporated in the analysis of the building frame slabs. This includes the use of the approach similar to the Influence Coefficient Method to represent uniform patch pressures by discrete forces and connecting the two systems through compatibility requirements on the patch displacements.

The stiffness method was employed in the building frame analyses with columns and column walls modelled as frame elements.

The contributions of this thesis may be summarized as follows:

- 1) It provides the first test of the procedures described for building frame analysis. In doing this the author has corrected several bugs and one theoretical error in programs which had been developed earlier. The documentation described in Appendix A was prepared by the author.

- 2) Recommendations have been made relating to boundary element configuration and column patch point layout.
- 3) The thesis supervisor has encouraged the author to add here that the theoretical descriptions which are provided in the first few chapters have further clarified a mathematically complex new numerical method.
- 4) The plexiglass model studies were not fully satisfactory, but were done with care. The description of this work will provide some guidance to those doing similar work in the future. Plexiglass studies have been used by engineering structural designs in the past in order to confirm the appropriate actions in complex structures. The author attempted initially in this thesis to validate a numerical procedure. While it has been an adequate demonstration of structural action, the data was too imprecise for calibration or validation.

6.2 Conclusion

The direct boundary element method (DBEM) is an attractive method for the analyzing a relatively complex structure - the flat plate building frame. The building frames can be analyzed as full three-dimensional structures with interaction between floors and adjacent spans and deformation which include frame sway and rotation. The flat plate can have arbitrary shape and boundary conditions, and different configurations of columns and column wall layouts can be dealt with.

A computer program, based on the DBEM and stiffness method, can be used for both the analysis of frame sway and rotation and for the investigations of bending moment distributions in the flat plate.

The arrangement of discrete point forces within the column patch affects the column stiffness in frame analysis. An arrangement of a reasonable number of point forces has been suggested to obtain an acceptable rigid column patch.

The results of some examples show that the refinement of the boundary elements do not always help to increase the accuracy, and using high order polynomials for the boundary elements may cause significant error in the results. Increasing the number of boundary elements or using high order polynomials will also increase c.p.u. time. It is suggested that a reasonable number of 2nd order boundary element be used to obtain acceptable accuracy.

Some further work on the experimental study should be done prior to its continuation.

Several fixed dial gauges for the measurement of deflections should be mounted on a fixed frame. This would provide some improvement over the single dial gauge used in the present work.

Unless a material with superior properties can be employed, further work should be done on both loading cycle and the timing of the actual measurements.

It is believed that small fluctuations in the temperature were an important factor in the experimental work. Either a temperature controlled experiment or a change in instrumentation should be employed in future work of this type. In order to estimate moment distributions in the floor slab directly from strain readings, it is necessary to use rosette strain gauges at the points which are not close to the column region or to the edge of the plate.

6.3 Future Work

The DBEM for building frame analysis has been discussed and investigated. In order to improve this method which will eventually be used in practical design, further investigation and some modification are recommended, especially in the following areas:

- 1) The three-storey model should be tested statically and comparisons with numerical analysis made. This should be done to prove the power and versatility of the numerical analysis approach.
- 2) Dynamic testing of the model should also be done. If this could be extended to include dynamic vibration measurements taken on a real building with subsequent

numerical analysis, then the results would prove conclusively that this approach is ready for use in structural engineering.

- 3) As of this time the programs described here have not been extended to cover the fully general load condition. This would involve the full integration of techniques described in the present thesis; Abdel-Akher's [2] and El-Kafrawy's [15]. This work could be described as program development which is probably a different level of expertise than that practiced by engineering graduate students, but could be mentioned in order to point out that the procedures here are practical as tools, though perhaps more complicated to describe than the usual structural engineering approach.
- 4) Something has already been said about the accuracy of internal shear forces. It could be mentioned here for the record, that if it should eventually be required to improve the values of shear force -even at points near the boundary -and even at points close to singularity, the answer lies in increasing the level of continuity between elements. This has been beyond the scope of the present thesis, but is introduced well in Abdel-Akher [2].

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APPENDIX A

INSTRUCTION FOR RUNNING "DBEMFA"

DBEBFA is a computer program for the building frame analysis by the DBEM, the program consists of three parts, the first part is the program for calculating the boundary values and slab stiffness matrix, named **FPLATE** using the DBEM; the second part is for frame analysis and calculating the column internal reaction, named **FRAME_BE**, the slab stiffness matrix and column stiffness matrix are assembled in the global coordinate system, the stiffness method is used to solve the displacement and rotation in each degree of freedom; the third part is for calculating the floor slab internal reactions, named **FSTRESS**.

Data Flow Chat.

Data Format of "indata_1"

----- For program FPLATE and FSTRESS

1. Title of the problem
2. **MTDATA** ---- code (material data)
3. Material data **E, PR, TH**

E	=	Young's Modulus
PR	=	Possion's ratio
TH	=	Plate thickness (m)
4. **BGDATA** ---- code (Data of boundary geometry)
5. The number of boundary nodes **NBN**
6. One line for each node **I, M, S(J)** (**J=1,2*M**)

I	=	node number
M	=	node condition
M = 1		node on the edge which parallel to x- or y-coordinate
2		node on regular edge
3		node on the corner
S(1) , S(2)	=	x-, y- coordinates of the node
S(J), S(J+1) (J=3, 2*M)	=	direction cosines of the normal
7. The number of boundary elements **NBE**

8. One line for each element **I**, **NP(I)**, **JBE(I,1)**, **JBE1**, **IG(I)**, **ICON(I)**

I = element number
NP(I) = number of inner nodes in the element
JBE(I,1) = first node number of the element
JBE1 = last node number of the element
IG(I) = element condition
 IG(I) = 1 straight element
 2 curved element
ICON(I) = boundary condition of the element
 ICON(I) = 1 fixed boundary
 2 simply supported boundary
 3 free boundary

9. **NVDATA** ---- code (the known value)10. The number of known value **NNV**11. One line for each known value **I**, **NV**

I = boundary node number
NV = column number

12. **CLDATA** ---- code (the concentrated force data)13. The number of concentrated forces **NCF**14. One line for each force **I**, **XCF(I)**, **YCF(I)**, **FCF(I)**

I = force number
XCF(I),YCF(I) = x-, y- coordinates of the point on which the force is acting
FCF(I) = the concentrated force

15. **CMDATA** ---- code (the concentrated force data)16. The number of concentrated moments **NCM**17. One line for each force **I**, **XCM(I)**, **YCM(I)**, **MCM(I)**

I = moment number
XCM(I),YCM(I) = x-, y- coordinates of the point on which the moment is acting
MCM(I) = the concentrated moment

18. **ISDATA** ---- code (internal support data)19. The number of column **NCL**20. One line for each support **I**, **XC(I)**, **YC(I)**, **NF**

I = support number

- XC(I), YC(I) =** x-, y- coordinates of the internal support
NF = number of force point within the column
 One line for each force point **II, XC(II), YC(II)**
II = point number
XC(II), YC(II) = x-, y- coordinates of the force point (local)
21. **LDDATA ----** code (applied uniform load data)
 22. The number of loaded node **NLN**
 23. One line for each loaded node **I, XLN(I), YLN(I)**
I = loaded node number
XLN(I), YLN(I) = x-, y- coordinates
 24. The number of loaded element **NLE**
 25. One line for each loaded element **I, JLE(I,J) (J=1,4), W(I)**
I = loaded element number
JLE(I,J) = node number of the element
W(I) = uniform load value
 26. **IPDATA ----** code (internal point data)
 27. The floor number and number of internal points **NFLOOR, NIP**
 28. One line for each internal point **I, XIN(I), YIN(I)**
I = internal point number
XIN(I), YIN(I) = x-, y-coordinates of the internal point
 29. **ENDATA ----** code

Data Format of "indata_2"

----- for program FRAME_BE

1. **M, NS**
M = number of column
NS = number of storey
2. **E, PR, TH**
E = Young's Modulus
PR = Possion's ratio
TH = Plate thickness (m)
3. **I, X(I), Y(I), A(I), B(I), THETA**
I = column number

- $X(I), Y(I)$ = x-, and y-coordinates of the column center
 $A(I), B(I)$ = column dimension in x- and y-direction
 $THETA$ = angle between column and global coordinate
4. One line for each kind of height **I1, I2, HH**
- I1** = first floor number
I2 = last floor number
HH = column height
5. One line for each floor **I, F1, F2, F3**
- I** = floor number
F1, F2 = lateral force in x- and y-direction
F3 = twist moment (in z-direction)
6. Two lines of **0, 0.0**

Example.

A simple one-storey building frame example is used to illustrate how to prepare the input data. The floor slab of the frame is directly supported by four columns located at four corners. The dimensions and material properties are given in the following figure. The external edge of the floor slab is divided in to eight 2nd order boundary element (two per side).

$$E = 480,000 \text{ psi}$$

$$\nu = 0.35$$

$$\text{Floor Slab Thickness} = 0.5''$$

$$\text{Storey Height} = 10''$$

$$p = 10 \text{ lb/ft}^2 (0.0694 \text{ lb/in}^2)$$

Dimensions of One-Storey Building Frame Example

"indata_1"**MTDATA**

480000,0.35,0.5

BGDATA

8

1,1,10.,-10.

3,1,10.,0

5,1,10.,10.

7,1,0,10.

9,1,-10.,10.

11,1,-10.,0

13,1,-10.,-10.

15,1,0,-10.

8

1,2,1,3,1,3

2,2,3,5,1,3

3,2,5,7,1,3

4,2,7,9,1,3

5,2,9,11,1,3

6,2,11,13,1,3

7,2,13,15,1,3

8,2,15,1,1,3

NVDATA

4

1,1

5,2

9,3

13,4

ISDATA

4

1,9.5,-9.5,8

1,0.5,-0.5

2,0.5,0.5

3,-0.5,0.5

4,-0.5,-0.5

5,0.25,-0.25

6,0.25,0.25

7,-0.25,0.25

8,-0.25,-0.25

2,9.5,9.5,8

1,0.5,-0.5

2,0.5,0.5

3,-0.5,0.5

4,-0.5,-0.5

5,0.25,-0.25

6,0.25,0.25

7,-0.25,0.25

8,-0.25,-0.25

3,-9.5,9.5,8

1,0.5,-0.5

2,0.5,0.5

3,-0.5,0.5

4,-0.5,-0.5

```
5,0.25,-0.25
6,0.25,0.25
7,-0.25,0.25
8,-0.25,-0.25
4,-9.5,-9.5,8
1,0.5,-0.5
2,0.5,0.5
3,-0.5,0.5
4,-0.5,-0.5
5,0.25,-0.25
6,0.25,0.25
7,-0.25,0.25
8,-0.25,-0.25
LDDATA
4
1,10.,-10.
2,10.,10.
3,-10.,10.
4,-10.,-10.
1
1,1,2,3,4,-0.0694
IPDATA
1,5
1,-9.9995,0
2,-5.,0
3,0,0
4,5.,0
5,9.9995,0
ENDATA
```

"indata_2"

```
4,1
480000,0.35,0.5
1,9.5,-9.5,1.,1.,0
2,9.5,9.5,1.,1.,0
3,-9.5,9.5,1.,1.,0
4,-9.5,-9.5,1.,1.,0
1,1,10.
0,0.0
0,0.0
```

APPENDIX B

Analysis of Three-Storey Building Frame Example

The three-storey building frame model, as shown in Fig. B.1, is used mainly for sidesway analysis.

Fig. B.1 Three Storey Building Frame Example

The floor layouts, dimensions of floor slabs and base slab are the same as those of the one-storey example (Fig. 5.2). All storey heights are 10 inches.

Three horizontal concentrated forces are applied, one to each floor level of the building frame. The location and direction of the concentrated force of 10 lb is shown in Fig. B.2.

Fig. B.2 Location of the Concentrated Force on Each Floor

The results of an analysis of the three-storey frame using the DBEM are shown in Figs. B.3 and B.4. The calculated frame sway and rotation are given in Fig. B.3. The deflections of each floor slab are provided in Fig. B.4.

The horizontal loads, acting in the y -direction, are not just produce the frame sway in y -direction, they also cause the frame sway in x -direction and rotation in z -direction, since the applied loads are located on the line which pass through the stiffness center of the building frame. Fig. B.4 shows the deflection distributions on each floor slab are affected by the transverse displacements and rotation on each floor of the frame, it also shows that the internal actions of the floor slab, in the frame analysis under horizontal loads, are not negligible.

Fig. B.3 Frame Sway and Rotation
(under Lateral Load)

Fig. B.4 Deflections of Floor Slabs in Sideway Analysis

The stiffness matrix terms of slab super-element are obtained using the DBEM and the ICM, which is a 27 by 27 symmetrical matrix. In this symmetrical matrix, only the terms on the principal diagonal and on one side of the diagonal, say to the right and above, are stored in rows beginning from each term located on the principal diagonal. These terms are

STIFFNESS MATRIX

162.285	761.037	-781.440
-96.468	-101.282	-651.098
3.562	11.135	17.131
2.453	-10.241	11.862
23.804	-3.878	-14.891
0.549	-2.601	1.467
-0.341	0.925	0.086
-26.653	405.360	36.695
-69.187	618.961	0.049
10417.974	-1941.382	-85.247
-1094.992	-255.495	9.837
22.416	40.757	7.803
-18.069	32.772	8.908
252.700	-125.200	3.727
-18.188	6.427	-1.633
4.760	0.394	-194.457
2098.917	236.078	-509.963
3720.594	-135.877	10541.249
681.130	383.538	3519.904
-16.549	-43.916	-78.173
-11.021	39.665	-52.577
-33.125	-109.537	193.865
-3.033	14.525	-5.710
0.382	-0.368	2.686
257.256	-1639.485	-54.194
-93.612	299.806	-322.151
337.380	1529.057	14.005
-101.413	-108.025	-699.948
18.170	49.428	-55.876
-159.644	892.574	-35.597
4.261	-22.331	2.189
-0.429	-0.679	0.337
-34.018	423.289	43.237
32.158	-113.869	16.612
20154.214	81.351	-125.576
-1209.288	-493.148	-78.724
721.655	-462.474	-1018.953
4491.676	-25.933	21.738
-112.310	11.406	-2.011
-3.783	0.629	-272.589
2428.346	331.556	48.337
54.366	50.282	20140.680

662.560	319.921	3578.537
6.528	-198.661	428.946
79.315	-316.400	-856.555
2.514	-7.753	11.936
0.629	-2.308	-4.024
-205.793	1385.783	239.787
91.344	-291.004	88.591
161.898	759.570	768.201
-87.839	634.214	-63.955
17.177	79.663	9.035
0.364	-2.621	-1.716
-0.158	0.196	0.629
7.015	-40.671	-10.871
-0.604	-1.197	-0.745
10410.988	1869.957	-636.705
3461.350	-302.748	-47.080
635.396	185.524	2.961
-18.036	-6.964	-0.745
0.489	2.659	19.047
-71.253	-31.134	-0.148
-7.904	-2.027	10465.912
-55.485	292.307	-825.464
-62.444	435.299	442.220
1.862	-13.052	-4.000
-0.532	0.126	1.904
33.750	-163.111	-55.308
-2.519	-6.171	-3.408
173.164	-757.672	830.055
-118.952	101.611	720.304
1.660	-8.174	1.502
-0.105	-0.254	-0.084
12.401	-35.148	-27.068
-0.951	-1.326	-0.113
10415.634	-1856.155	107.003
-1129.262	-328.977	-5.976
37.696	8.512	1.656
-1.050	-6.745	-18.477
71.584	33.376	0.093
9.034	1.683	10803.467
-783.781	383.491	3797.163
6.335	-25.100	13.561
-0.001	-2.901	-2.237
59.276	-132.852	-129.925
-3.878	-7.331	-0.751
774.262	-514.669	1675.003
-79.414	583.259	52.592
33.011	-30.843	-93.434
-549.332	599.425	2127.423
59.084	-56.483	-57.806
25824.233	1995.629	-525.394
2913.121	74.472	75.406
32.221	-183.559	-97.692
-5359.282	1099.979	-7.635

322.528	-39.731	33307.391
-15.176	8.796	-525.329
11.889	-1.173	51.854
-2505.257	2400.614	7414.552
154.706	-95.581	-119.535
176.557	-835.239	804.373
-80.027	30.861	580.757
-65.583	-640.443	189.465
41.634	143.669	0.158
10793.284	-2061.092	1.508
-575.799	-55.714	347.112
2718.971	-873.988	-60.951
-88.769	17.107	10633.062
-592.548	120.099	3206.590
-408.134	-2329.134	712.643
140.247	436.781	55.081
220.432	-1064.198	-809.934
-53.870	-608.409	53.558
-118.511	-961.704	4.208
11965.821	2071.315	295.047
2453.143	-299.309	768.930
4953.602	215.714	10739.672
468.117	2703.299	-99.367
-146.483	-507.795	-599.436
1898.606	-233.212	1041.435
-1188.564	65.803	3937.391
49749.529	991.000	129.798
-11663.469	-58.096	41513.209
-3453.868	-143.147	8365.515
1244.937	306.157	-3899.742
29569.947	104.169	24395.470

APPENDIX C

One-Storey Building Frame under Concentrated Moment

A concentrated moment of 100 lb-in in the x direction is assumed to act on the floor slab of one-storey building frame (as shown in Fig.5.1), at the location shown in Fig. C.1. This type of loading is closely related to a stiffness influence coefficient.

Fig. C.1 Location of the Concentrated Moment

The distributions of deflection and moment along the line $x=0$ and $x=9$ are given in Figs. C.2 and C.3. The deflection distributions show smooth curves and reasonable slope. However, the moment distributions are not smooth, especially near the column regions.

Fig. C.2 Deflections along $x=0$ and $y=9$
(under a Concentrated Moment)

Fig. C.3 Moments along $x=0$ and $x=9$
(under a Concentrated Moment)

APPENDIX D

Photographs of Experimental Work

Fig. D.1 Building Frame Model During the Assembling (A)

Fig. D.2 Building Frame Model During the Assembling (B)

Fig. D.3 One-Storey Building Frame Model

Fig. D.4 Data Acquisition - System 4000

END

1 2 0 4 9 1

FIN