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Adjacent Channel Interference Reduction by Adaptive Equalizers

by

Shahram Golestaneh

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University
Ottawa, Ontario, Canada, K1S 5B6

May 22, 1992

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submitted by Shahram Golestaneh, B.Sc., in partial fulfillment of the requirements for the degree of Master of Engineering.

Chair,

Department of Systems and
Computer Engineering

Thesis Supervisor

Thesis Supervisor

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University
May 22, 1992

Abstract

One of the major limiting factors in an efficient utilization of the frequency spectrum is adjacent channel interference (ACI). This thesis investigates the performance of a finite length digitally implemented decision feedback equalizer (DFE) in an adjacent channel interference environment. Quadrature phase shift keying (QPSK) is employed as the modulation scheme. To evaluate system performance, analytical method and direct computer simulation are used. In the analytical approach, minimum mean square error (MMSE) is used as the performance criterion. Mathematical expression for MMSE is derived and the effect of different channel spacing as well as different signal to interference ratio (SIR) on MMSE is discussed. In the simulation, error rate is considered as the performance criterion. A comparison between analytical approach and simulation method is made.

It is shown that deployment of DFE leads to significant improvement of the system performance under ACI condition. It is also shown that when one interferer is dominant, the performance of the system is far better than when two interferers are present. For the investigated range of channel spacing in this thesis, which is less than the baud rate, it is shown that for the same SIR, a larger channel spacing always leads to a better system performance.

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List of Abbreviations and Acronyms

ACI	Adjacent channel interference
AWGN	Additive White Gaussian Noise
BER	Bit error rate
BOSS	Block Oriented Systems Simulator™
CCI	Cochannel interference
db	decibel
DFE	Decision feedback equalizer
GMSK	Gaussian filtered minimum shift keying
GSM	Groupe Spécial mobile
Hz	Hertz (cycle per second)
ISI	Intersymbol interference
LMS	Least mean square
MMSE	Minimum mean square error
MSE	Mean square error
MSK	Minimum shift keying
QPSK	Quadrature phase shift keying
RLS	Recursive least square
SER	Symbol error rate
SIR	Signal to interference ratio
SNOI	Signal not of interest
SNR	Signal to noise ratio
SOI	Signal of interest
TCM	Trellis coded modulation
TDAF	Time dependent adaptive filter

Lists of Mathematical Symbols and Notation

A	Autocorrelation matrix
A^{-1}	Inverse of A
A^T	Transpose of A
A^*	Complex conjugate of A
a_k	Transmitted symbol of the main channel at time $t = kT$
$a_k^{(i)}$	Transmitted symbol of the i^{th} adjacent channel at time $t = kT$
$\hat{a}_n$	detected symbol
c_m	m^{th} forward tap coefficient
C	Vector of the equalizer tap weight coefficients
D	Delay in the equalizer
E_b	Energy per bit
$E[\cdot]$	expectation of the argument
f_c	Carrier frequency
f_m	m^{th} feedback tap coefficient
$F(nT)$	Sampled output of the forward filter
FB	Output of the feedback filter
$h_{SRC}(t)$	Impulse response of the square root raised cosine filter
$H_{SRC}(f)$	Frequency response of the square root raised cosine filter
$h_t(t)$	Transmitter filter impulse response
$h_r(t)$	Receiver filter impulse response
I	Identity matrix
L_f	Number of forward taps
L_R	Number of feedback taps
$n(t)$	Filtered complex valued AWGN

N_0	Two sided power spectral density of complex AWGN
$\mathbf{q}$	Cross correlation vector
$r(t)$	Received signal at the input of the equalizer
$R(t)$	Autocorrelation of filtered AWGN
$s(t)$	Impulse response of the overall main channel
$s^{(i)}(t)$	Impulse response of the overall i^{th} adjacent channel
$\text{sinc}(x)$	$\sin(\pi x)/(\pi x)$
T	Symbol period
$x^{(i)}(t)$	i^{th} adjacent channel signal at the input of the equalizer
$z(t)$	Additive White Gaussian Noise
α	Roll-off factor of square root raised cosine filter
δf	Channel spacing
$\delta(n)$	Kronecker delta function
$\delta f T$	Normalized channel spacing
Δ	Tap spacing of forward filter
μ	LMS adaptation step size
σ_a^2	Variance of the data
π	constant 3.1415...
θ	Sampling phase
τ_i	Timing misalignment of the i^{th} adjacent channel

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Chapter 1

Introduction

1.1 Introduction

The extraordinary expansion of different radio communication systems specially in the past decade has motivated researchers to look for new techniques to cope with the ever increasing demand for frequency bands. While the trend is to move to higher frequencies where more spectrum is available, the efficient utilization of the spectrum is always a key factor to accommodate more users in lower frequency bands. The ultimate aim of using different techniques in digital radio communications is to develop reliable links with a given required bit error rate. In many applications, this error rate performance is dominated by impairments other than thermal noise, such as multipath fading, cochannel and adjacent channel interferences. Extensive research has been done to mitigate the effects of each of these impairments in communication systems.

It is clear that the type of impairment that should be dealt with more carefully depends on the specific application of the system. In many cases, although one major impairment may be more dominant, combined techniques should be used to mitigate the effects of that impairment and simultaneously suppress other impairments as much as possible.

As far as the spectrum utilization is concerned and regardless of the type of transmission media (wired or wireless), the adjacent channel interference (ACI) and cochannel interference (CCI) are dominant limiting factors.

One approach to utilize the frequency spectrum more efficiently is based on the concept of frequency reuse which refers to the use of radio channels on the same carrier frequency. This technique is used in cellular radio systems to cover different areas which are separated from each other by a sufficient distance. The limiting factor here is the increased amount of cochannel interference.

In the overall scenario of mobile and portable communications, signal fading is also a major impairment in radio channels. To mitigate such impairment diversity techniques are required. To combat signal shadowing, macroscopic diversity may be used (macroscopic diversity is termed so due to the large spatial separation between receiver antennas). To combat signal fading, microscopic space diversity can be used where signal decorrelation occurs in less than half a wavelength. Combining techniques such as maximal ratio, equal gain, selection, switched or optimum combining may be used for space diversity either in pre-detection or post-detection (baseband) configuration [16].

1.2 Problem definition

In many applications the problem of adjacent channels interference is not considered to be a major impairment since it is assumed that it can be reduced by filtering and by avoiding the allocation of adjacent channels to neighboring radio stations. In other applications, however, these assumptions may place unnecessary constraints on the network design, and could ultimately result in a less efficient utilization of the valuable spectral capacity. The aim of this work is to investigate the degradation of a given signal called Signal Of Interest (SOI) in the presence of adjacent channels signals called Signal

Not Of Interest (SNOI) as a function of channel spacing and receiver structure. The problems addressed in this study include the determination of :

- (1)—Minimum channel spacing required to achieve certain bit error rate .
- (2)—The difference between the increased interference power due to increased magnitude of SNOI and reduced channel spacing on the system performance.
- (3)—A suitable receiver structure to attain minimum channel spacing.

1.3 Thesis Objectives

The objectives of this work are to determine the extent to which channel spacing between adjacent channels can be reduced and to investigate the proper way to reach this point without increasing channel distortion and the intersymbol interference (ISI). Achieving these objectives will ultimately improve the utilization of the radio spectrum.

1.4 Proposed Approach

Different equalization techniques and equalizers structures have been widely used in digital communication systems primarily to combat signal fading due to multipath propagation . One class of equalization techniques, known as Decision Feedback Equalizers (DFE), proved to be efficient in combating both Continuous Wave (CW) interference and frequency selective fading [13] as well as some other types of cyclostationary interferences [17,18]. The impact of DFE's on adjacent channel interference (ACI) has also been studied in previous research work [3,8,9]. However, in none of the previous ACI

related research work, the effect of channel spacing and different signal to interference ratio (SIR) on the performance of a DFE based receiver has been investigated.

The use of DFE to combat ACI and attaining minimum channel spacing is suggested. Both baud rate and fractionally spaced equalizers are investigated. Since DFE's are non-linear devices, mathematical evaluation of error rate when the received signal contains ACI, AWGN and ISI components is a complex task and could be mathematically intractable [7]. This suggests direct computer simulations as a means to obtain the error rate performance of the system which is the most desirable criteria in a digital communication system. The computer simulation is performed using a simulation package called BOSSTM (Block Oriented System Simulator).¹

1.5 Thesis outline

The remaining chapters of this thesis are organized as follows :

Chapter 2 describes the system under investigation in more details. Different stages of the system are explained. A survey of literature publications addressing the same type of problems is presented. Basic mathematical equations are written and formulas for Minimum Mean Square Error (MMSE) are derived.

Chapter 3 is divided into two major parts : (1)- Mathematical analysis of the system via MMSE calculations, and (2)- Simulation results for error rate performance of the system under different conditions. An introduction to BOSS simulation package and its parameters is given in this chapter. The results obtained from computer simulations

¹ BOSS and Block Oriented System Simulator are trademarks of COMDISCO Inc.

are compared with the mathematical analysis. Various parameters which play a role in obtaining the results are also examined.

Finally Chapter 4 summarizes the conclusions of the work and presents recommendations for further research.

Chapter 2

System Description

2.1 Introduction

In this chapter, a brief survey of literature publications addressing similar problems is presented first. The survey justifies the need for the work reported in this thesis. The overall system model and its components are then explained in more details. More emphasis will be given to decision feedback equalizer (DFE) and its importance in solving the problem addressed here. The structures of different types of equalizers are presented next. The performance criteria are explained and the basic mathematical expressions are developed accordingly.

2.2 Previous works

The problem of Adjacent Channel Interference (ACI) has been addressed in many papers[2,6,7,8]. The term Intersymbol Interference (ISI) is known as the interference due to symbols which precede and succeed the symbol under detection and do not have zero crossing at sampling time. This situation happens due to either overall non-ideal Nyquist¹ pulse shapes or timing misalignment at sampling instant. These interfering

¹ Nyquist criteria to have zero intersymbol interference is

$$h(nT) = \begin{cases} 1 & ; n = 0 \\ 0 & ; n \neq 0 \end{cases} \quad (2.1)$$

where $h(t)$ is the overall pulse shape.

symbols come from the same source as the symbol under detection. On the other hand, Adjacent Channel Interference (ACI) is the interference due to symbols which come from other independent sources which transmit over adjacent channels 'leaking' to the main channel. Figure 1 depicts the spectrum of the main and adjacent channels.

In Figure 2 the general block diagram of the system is shown. In this figure all impairments (including ACI) are assumed to be parts of the channel block which will be explained in more details in the forthcoming sections. Clearly, the task of the receive filter is to limit the noise and ACI. However, since signals can not be both time limited and frequency limited [14], the use of brickwall filters results in a lengthy impulse response and increased ISI. Consequently, conventional receivers are based on a trade off between higher out of band noise and interference or higher ISI. On the other hand, keeping the adjacent channels and the main channel as separate as possible reduces spectrum efficiency. This is an important consideration in view of today's congested spectrum. Our objective is to suppress or reduce the effect of the ACI without increasing ISI by implementing DFE as part of the receiver. This will relax the filtering requirements.

Despinic and Taylor studied the effect of ACI and Trellis Coded Modulation (TCM) in Digital Microwave Radio [1]. In their work, the performance criterion is based on Signal to Distortion Ratio (SDR) and not the error rate. They used only linear baud rate or fractionally spaced equalizers and DFE receivers were not considered. The ACI is not considered as a major impairment ($SIR > 30$ db) and only its effect on determining the required number of equalizer taps has been considered.

The performance of Minimum Shift Keying (MSK) and Sinusoidal Frequency Shift

Keying (SFSK) in presence of ACI and frequency selective fading was investigated by Tahermezghi and Cruz [2]. Bit error rate curves were given for different channel spacing but no effort was made to cancel or suppress the ACI. Simple matched filter or sampling detector has been used in the receiver.

Hedstrom, Kirlin and Driessen proposed an injection method to cancel ACI but their method showed only some degree of improvement in the ACI scenario and they concluded that the receiver should be a multi-channel receiver [6].

Niger and Vandamme studied the performance of equalization techniques for digital radio systems in the presence of Continuous Wave (CW) interference and ACI [7]. Their work indicates that T/2 spaced and Decision Feedback Equalizers (DFE) can improve ACI margin significantly, and their results are given in terms of degradation of CNR due to ACI for fixed channel spacing. The potential effect on system performance when ACI power is changed due to different channel spacing or due to its level for fixed channel spacing has not been investigated.

Gardner looked at the problem in a more general way and proposed the FREquency SHift (FRESH) filtering theory for the removal of cyclostationary interferences. This approach exploits the cyclostationarity nature of the interfering signal. His work suggests that by properly shifting and weighting the frequency components of the received signal one may cancel the interference [4]. Reed and Hsia in [5] gave several examples on how a Time Dependent Adaptive Filter (TDAF) can cancel cyclostationary interfering signals. They showed that fractionally spaced equalizer is a simple TDAF.

Figure 1 Spectrum of the main and adjacent channels

Figure 2 General block diagram of the main system

Uesugi et al. used computer simulation to show that distortion of the signal caused by frequency selective fading and ACI can be removed by fractionally spaced DFE [3]. The channel spacing is assumed to be fixed and the effect of ACI on BER is presented for the case of GSM¹. The effect of channel spacing has not been considered in this work.

Also a large number of papers may be found on various properties of equalizers, different algorithms, structures, estimates of the BER in DFE-based receivers and different methods to eliminate cochannel interference. Many of these papers are directly or indirectly related to our work and will be referred to when required.

2.3 System model

The overall baseband model of the system under investigation is shown in Figure 3. The modulation scheme is assumed to be quadrature phase-shift keying (QPSK). Since complex baseband representation of the signals and the system is used, the main channel is centered at 0 Hz. The two immediate adjacent channels on the left and right hand side of the main channel in the frequency domain are considered as the source for ACI and the effect of other adjacent channels is considered to be negligible. As shown in Figure 3, the carrier frequencies of adjacent channels are symmetrically located with respect to the main channel and δf is the frequency separation between the main and adjacent channels. Note that the receiver is one dimensional single input receiver. This

¹ GSM (Groupe Spécial Mobile) is the standard for the Pan-European digital mobile telephone system in which the modulation scheme is Gaussian filtered Minimum Shift Keying (GMSK). More details about GSM can be found in [19].

means that the receiver has to make decisions only on data of one transmitter and that no diversity is considered in this model. However, the advantages and disadvantages of multi-dimensional or multi-input receivers will be briefly explained later.

The transmitted symbols $\{a_k ; k = \dots, -2, -1, 0, 1, 2, \dots\}$ are complex, zero mean and mutually uncorrelated:

$$E[a_k] = 0 \quad (2.2)$$

$$E[a_k a_n^*] = \sigma_a^2 \delta(k - n) \quad (2.3)$$

where $\delta(n)$ denotes the *Kronecker delta* function :

$$\delta(n) = \begin{cases} 1 & ; n = 0 \\ 0 & ; n \neq 0 \end{cases} \quad (2.4)$$

$E[\cdot]$ is the expectation and σ_a^2 is the variance of the data which is set to unity without loss of generality. Symbols are sent every T seconds or equivalently the symbol rate is 1/T.

2.3.1 The ACI model

The data sequence of the adjacent channels $\{a_k^{(i)} ; k = \dots, -2, -1, 0, 1, 2, \dots\}$ are assumed to be of the same form (i.e. QPSK modulated) and the same rate as that of the main channel but independent of the main channel's data and of each other. Thus they have the following properties :

$$E[a_k^{(i)}] = 0 \quad (2.5)$$

$$E[a_k^{(i)} a_m^{(i)*}] = \sigma_a^2 \delta(k - m) \quad (2.6)$$

$$E[a_k a_m^{(i)}] = 0 \quad (2.7)$$

$$E[a_k^1 a_m^{-1}] = 0 \quad (2.8)$$

Superscript (i) refers to the i th channel and since we are considering only two immediate adjacent channels $i = \pm 1$. Each of the adjacent channel signals at the input of the equalizer can be written in its complex-valued low pass equivalent form as

$$x^{(i)}(t) = \sum_{k=-\infty}^{\infty} a_k^{(i)} s^{(i)}(t - kT - \tau_i) \quad (2.9)$$

where $s^{(i)}(t)$ is the impulse response of the overall filtering and can be written as

$$s^{(i)}(t) = A_i h_t(t) e^{j(i2\pi \delta f t + \varphi_i)} \otimes h_r(t) \quad (2.10)$$

in which A_i corresponds to the ACI level, $h_t(t)$ is the impulse response of the transmit filter, $h_r(t)$ is the receive filter impulse response, $\otimes$ denotes the convolution and φ_i and τ_i represent the phase and timing misalignment of the adjacent channels with respect to the main channel and are assumed to be uniformly distributed on $[0, 2\pi]$ and $[0, T]$ respectively.

The received signal at the input of the equalizer then can be written as

$$r(t) = \sum_{k=-\infty}^{\infty} a_k s(t - kT) + \sum_i x^{(i)}(t) + n(t) \quad (2.11)$$

where a_k is the transmitted symbol on the main channel, $s(t)$ is the impulse response of the overall main channel and $n(t)$ is the complex-valued AWGN, after filtering by the receive filter.

2.3.2 Transmit and Receive filters

Square root raised cosine filters were used as transmit and receive filters. It was assumed that the overall filtering is equally split between the transmitter and the receiver thus both transmit and receive filters are identical :

$$|H_{transmit}(f)| = |H_{receive}(f)| \quad (2.12)$$

This configuration may not guarantee the best achievable performance of the system. Specifically, allowing wider bandwidth receive filter may lead to a better performance under ACI conditions [17]. This issue will be revisited in section 3.2. The same type of filtering is applied to adjacent channels. For square root raised cosine filters, $H_{SRC}(f)$ is given by :

$$H_{SRC}(f) = \begin{cases} 1 & ; \quad |f| \leq \frac{1}{2T}(1 - \alpha) \\ \sin \left\{ \frac{T}{4\alpha} [2\pi|f| - \frac{\pi}{T}(1 + \alpha)] \right\} & ; \quad \frac{1}{2T}(1 - \alpha) < |f| \leq \frac{1}{2T}(1 + \alpha) \\ 0 & ; \quad |f| > \frac{1}{2T}(1 + \alpha) \end{cases} \quad (2.13)$$

or equivalently $h_{SRC}(t)$ is [1]

$$h_{SRC}(t) = \begin{cases} \frac{(1-\alpha)\text{sinc}\left[\frac{t}{T}(1-\alpha)\right] + \frac{4\alpha}{\pi} \cos\left[\frac{\pi t}{T}(1+\alpha)\right]}{\left[\frac{4\alpha}{\pi} + (1-\alpha)\right] \left[1 - \left(\frac{4\alpha t}{T}\right)^2\right]} & ; \quad t \neq \frac{T}{4\alpha} \\ \frac{\frac{\alpha}{\sqrt{2}} \left[\left(1 - \frac{2}{\pi}\right) \cos\left(\frac{\pi}{4\alpha}\right) + \left(1 + \frac{2}{\pi}\right) \sin\left(\frac{\pi}{4\alpha}\right) \right]}{\left[\frac{4\alpha}{\pi} + (1-\alpha)\right]} & ; \quad t = \frac{T}{4\alpha} \end{cases} \quad (2.14)$$

Figure 3 System

in which $\text{sinc}(\bullet)$ is defined as

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \quad (2.15)$$

and α is the roll-off factor of the filter which is between 0 and 1. The overall transfer function of two square root raised cosine filters will have the raised cosine characteristic

$$H_{RC}(f) = \begin{cases} 1 & ; \quad |f| \leq \frac{1}{2T}(1 - \alpha) \\ \frac{1}{2} \{ 1 - \sin [\frac{T}{2\alpha} (2\pi|f| - \frac{\pi}{T})] \} & ; \quad \frac{1}{2T}(1 - \alpha) < |f| \leq \frac{1}{2T}(1 + \alpha) \\ 0 & ; \quad |f| > \frac{1}{2T}(1 + \alpha) \end{cases} \quad (2.16)$$

which meets the Nyquist criteria for zero intersymbol interference [15]. Figure 4 shows the frequency response of the raised cosine pulse for different roll-off factors.

2.3.3 AWGN

Additive White Gaussian Noise, $z(t)$, is considered as another impairment, but, the major impairment in our system is ACI. AWGN has a two sided power spectral density of N_0 and

$$E[z(t)] = 0 \quad (2.17)$$

$$E[z(t_1)z^*(t_2)] = N_0 \delta(t_1 - t_2) \quad (2.18)$$

The variance of the AWGN is determined by the receive filter and the Signal to Noise Ratio at the input of the equalizer is considered as a parameter. Noise power at the input

of the equalizer can be written as $N_0|H_{receive}|^2$ which is equal to N_0/T for square root raised cosine filter. $z(t)$ is independent of all the data of the main and adjacent channels

$$E[a_k z(t)] = 0 \quad (2.19)$$

$$E[a_k^{(i)} z(t)] = 0 \quad ; \quad i \in \{1, -1\} \quad (2.20)$$

2.3.4 The Equalizer structure

A digital realization of DFE is used. This means that the received signal at the input of the equalizer is sampled at a rate of M/T where M is an integer greater than or equal to one. The tap spacing of the forward filter is T/M . In an analog implementation this tap spacing can be any real number less than or equal to T and no sampling is required at the input of the equalizer. The general structure of the equalizer is shown in Figure 5. The forward filter is shown in Figure 6. For the T -spaced equalizer $M=1$ and for the $T/2$ -spaced equalizer $M=2$. The sampled output of the forward filter can be written as

$$F(nT) = \sum_{m=0}^{L_f-1} c_m r(nT - m\Delta - \theta) \quad (2.21)$$

where $\Delta = \frac{T}{M}$, c_m is the m th forward tap coefficient at $t=nT$ and θ is the time shift in sampling. The total number of forward taps will be L_f . Figure 7 illustrates the feedback filter. The tap spacing of the feedback filter is always T since decisions are made every T second. In case of linear equalizers, feedback filter simply does not exist. In all the

Figure 4 Raised cosine pulse spectrum

cases, the number of forward taps is split to L_1 future taps operating on precursor ISI, L_2 past taps operating on postcursor ISI and one central reference tap. In this configuration the L_2 taps operating on postcursor ISI work in parallel with the feedback filter which improves the rejection of in-band interference [10]. The output of the feedback filter is

$$FB = \sum_{m=1}^{L_R} f_m \hat{a}_{n-m-D} \quad (2.22)$$

where f_m is the m th feedback tap coefficient and D is the delay in the equalizer which also determines the position of the central reference tap. The input of the decision device will then be

$$Y_n = \sum_{m=0}^{L_f-1} c_m r(nT - m\Delta - \theta) + \sum_{m=1}^{L_R} f_m \hat{a}_{n-m-D} \quad (2.23)$$

Based on Y_n , the decision device will decide which one of the four possible symbols was transmitted

$$\hat{a}_n = \frac{1}{\sqrt{2}} \left\{ \text{sign}[\text{Re}(Y_n)] + j \text{sign}[\text{Im}(Y_n)] \right\} \quad (2.24)$$

where

$$\text{sign}(x) = \begin{cases} 1 & ; x \geq 0 \\ -1 & ; x < 0 \end{cases} \quad (2.25)$$

and $\text{Re}(\bullet)$ is the real part of $(\bullet)$ and $\text{Im}(\bullet)$ is the imaginary part of $(\bullet)$. The scaling factor of $1/\sqrt{2}$ is used to comply with the assumption of having a unity variance for the data. If we assume that there is no decision errors, which is a reasonable assumption when the probability of error is low, then

$$\hat{a}_n = a_n \quad (2.26)$$

Using matrix notation, the above equations can be written as

$$Y_n = \mathbf{C}^T \mathbf{V}_n \quad (2.27)$$

in which superscript T denotes the transpose operator and

$$\mathbf{C}^T = [c_0 \quad c_1 \quad \cdots \quad c_{L_f-1} \quad f_1 \quad f_2 \quad \cdots \quad f_{L_R}] \quad (2.28)$$

$$\mathbf{V}_n^T = [\mathbf{R}_n^T \quad \mathbf{a}_{n-1-D}^T] \quad (2.29)$$

$$\mathbf{R}_n^T = [r(nT-\theta) \quad r(nT-\Delta-\theta) \quad \cdots \quad r(nT-(L_f-1)\Delta-\theta)] \quad (2.30)$$

$$\mathbf{a}_{n-1-D}^T = [a_{n-1-D} \quad a_{n-2-D} \quad \cdots \quad a_{n-L_R-D}] \quad (2.31)$$

The mean square error MSE would then be :

$$MSE = E \left[\left| (Y_n - a_{n-D}) \right|^2 \right] \quad (2.32)$$

which can be written as

$$MSE = \mathbf{C}^{*T} \mathbf{A} \mathbf{C} - 2\mathbf{C}^{*T} \mathbf{q} + 1 \quad (2.33)$$

in which

$$\mathbf{A} = E [\mathbf{V}_n^* \mathbf{V}_n^T] \quad (2.34)$$

$$\mathbf{q} = E [\mathbf{V}_n^* a_{n-D}] \quad (2.35)$$

Figure 5 General structure of the equalizer

Figure 6 Forward filter

Figure 7 Feedback filter

and $*$ T denotes transpose conjugate. The error between the input and the output of the decision device, $e(t)$, is used to update the tap coefficients of the equalizer. The optimization is based on Minimum Mean Square Error (MMSE) criterion. Although this criterion may not lead to minimum error rate, it was chosen because of three major reasons. First, it makes the analysis more tractable. Second, it is easy to implement the equalizer based on this criterion and finally it gives an upper bound on the probability of error [17]. For MMSE the derivative of MSE should be set to zero and we will have

$$\mathbf{C}_{\text{opt}} = \mathbf{A}^{-1} \mathbf{q} \quad (2.36)$$

and

$$MMSE = 1 - \mathbf{q}^* \mathbf{T} \mathbf{C}_{\text{opt}} \quad (2.37)$$

The next step is to find $\mathbf{q}$ and $\mathbf{A}$. By assuming that

$$E(a_k) = 0 \quad (2.38)$$

$$E(|a_k|^2) = 1 \quad (2.39)$$

$$E(a_k a_m^{(i)}) = 0 \quad (2.40)$$

we obtain

$$\mathbf{q} = E[\mathbf{V}_n^* a_{n-D}] = E \begin{bmatrix} r^*(nT - \theta) a_{n-D} \\ r^*(nT - \Delta - \theta) a_{n-D} \\ \vdots \\ r^*(nT - (L_f - 1)\Delta - \theta) a_{n-D} \\ a_{n-1-D}^* a_{n-D} \\ a_{n-2-D}^* a_{n-D} \\ \vdots \\ a_{n-L_f-D}^* a_{n-D} \end{bmatrix} \quad (2.41)$$

or

$$\mathbf{q} = \begin{bmatrix} s(DT - \theta) \\ s(DT - \Delta - \theta) \\ \vdots \\ s(DT - (L_f - 1)\Delta - \theta) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2.42)$$

$$\mathbf{A} = E \begin{bmatrix} r^*(nT - \theta) \\ r^*(nT - \Delta - \theta) \\ \vdots \\ r^*(nT - (L_f - 1)\Delta - \theta) \\ a_{n-1-D}^* \\ \vdots \\ a_{n-L_K-D}^* \end{bmatrix} \begin{bmatrix} r(nT - \theta) \\ r(nT - \Delta - \theta) \\ \vdots \\ r(nT - (L_f - 1)\Delta - \theta) \\ a_{n-1-D} \\ \vdots \\ a_{n-L_K-D} \end{bmatrix}^T \quad (2.43)$$

Matrix $\mathbf{A}$ can be subdivided into :

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_2^{*T} & \mathbf{I} \end{bmatrix} \quad (2.44)$$

where

$$\mathbf{A}_1 = E \begin{bmatrix} r^*(nT - \theta)r(nT - \theta) & \dots & r^*(nT - \theta)r(nT - (L_f - 1)\Delta - \theta) \\ \vdots & \ddots & \vdots \\ r^*(nT - (L_f - 1)\Delta - \theta)r(nT - \theta) & \dots & r^*(nT - (L_f - 1)\Delta - \theta)r(nT - \theta) \end{bmatrix} \quad (2.45)$$

Each element of $\mathbf{A}_1$ can be written as

$$A_1(i, j) = E[r^*(nT + (1 - i)\Delta - \theta)r(nT + (1 - j)\Delta - \theta)] \quad (2.46)$$

thus

$$\begin{aligned} A_1(i, j) = & \sum_{k=-\infty}^{\infty} s^*(kT + (1 - i)\Delta - \theta)s(kT + (1 - j)\Delta - \theta) + \\ & \sum_{k=-\infty}^{\infty} s^{(1)*}(kT + (1 - i)\Delta - \theta - \tau_1)s^{(1)}(kT + (1 - j)\Delta - \theta - \tau_1) + \\ & \sum_{k=-\infty}^{\infty} s^{(-1)*}(kT + (1 - i)\Delta - \theta - \tau_{-1})s^{(-1)}(kT + (1 - j)\Delta - \theta - \tau_{-1}) \\ & + R((i - j)\Delta) \end{aligned} \quad (2.47)$$

where $R(t)$ is the autocorrelation of AWGN at the output of the receive filter and equals

$$R(t) = N_0 \cdot h_r(t) \quad (2.48)$$

Matrix $\mathbf{A}_2$ is given by :

$$\mathbf{A}_2 = E \begin{bmatrix} r^*(nT - \theta)a_{n-1-D} & \dots & r^*(nT - \theta)a_{n-L_k-D} \\ \vdots & \ddots & \vdots \\ r^*(nT - (L_f - 1)\Delta - \theta)a_{n-1-D} & \dots & r^*(nT - (L_f - 1)\Delta - \theta)a_{n-L_k-D} \end{bmatrix} \quad (2.49)$$

$$\mathbf{A}_2 = \begin{bmatrix} s^*((D+1)T - \theta) & \dots & s^*((D+L_R)T - \theta) \\ \vdots & \ddots & \vdots \\ s^*((D+1)T - (L_f-1)\Delta - \theta) & \dots & s^*((D+L_R)T - (L_f-1)\Delta - \theta) \end{bmatrix} \quad (2.50)$$

and $\mathbf{I}$ is a $(L_R \times L_R)$ identity matrix.

Since the interfering signals under the condition that $\delta f > \alpha/T$, are second order stationary signals and their mean and variance are independent of sampling time [7], and in MMSE criterion the second order moments of ACI and noise are involved, therefore the random variables τ_i and ϕ_i have no effects on the optimum equalizer tap calculation and may be set to zero for this purpose. Calculation of error rate under ACI remains a tedious and analytically intractable task [7]. Although some researchers derived equations for the probability of error in PSK systems under ISI and cochannel interference [11] or gave some bound on the probability of error in systems using DFE's [12], exact error rate calculation under ACI in a system in which an equalizer is used seems to be intractable and even an approximation for the error rate needs the calculation of the moments of ACI samples [7].

It is worthwhile to notice that for a matched filter receiver, specially when the channel spacing is small (corresponding to a large ACI), ACI can be treated as an equivalent Gaussian noise [2] and the error rate performance is directly related to the Signal to

Interference Ratio (SIR). The results for such an approximation in a matched filter receiver are given in chapter 3. However, this is not true when we employ an equalizer in the receiver since the equalizer tends to exploit the cyclostationary nature of the interference and cancel it. As we will see later the equalizer can cancel this interference to some extent but it can not suppress the AWGN.

Chapter 3

Simulations and results

3.1 Introduction

The system model and the MMSE equations were presented in Chapter 2. In this chapter, the MMSE performance of the system is evaluated for different scenarios of ACI levels and different parameters. However, since our objective is to evaluate the error rate performance of the system, and since there is no direct relation between the probability of error and the mean square error, a bit by bit simulation had to be performed to evaluate the error rate performance of DFE in presence of ACI and the results are presented and discussed in this chapter. The simulations are performed using BOSSTM (Block Oriented System Simulator)TM. BOSSTM is a software package in which systems are described by block diagrams. Each block performs a signal processing operation. A hierarchical block diagram can be constructed to represent more complex systems. The main advantage of BOSSTM is that it includes many of the modules required to simulate a complete communication system. Other required modules can be constructed either from BOSS supplied modules or by writing a Fortran subroutine which performs the function of the module. In Appendix A, some examples of modules created to perform the simulations and parameter settings are given. Each block may have different parameters that can be set to desired values for different simulations. The user may define new parameters in supplied modules or user's created modules. Of major importance in dealing with BOSS

is the processing delay of each module that should be taken into consideration. As is common in the analysis of many communication systems, complex lowpass equivalent of the signals and systems are used in order to relax the sampling rate requirements for the real signals. The sampling rate was chosen as eight sample per symbol interval for all simulations. Stop time is the duration of the simulation run in seconds.

3.2 Basic simulation parameters

In all the simulations, the symbol duration is normalized to unity ($T=1$). As explained in the system model description, two immediate adjacent channels located symmetrically with respect to the main channel are the sources for ACI. To make some qualitative comparisons, the case of only one adjacent channel interferer is also considered and the results are given respectively. Channel spacing δf (normalized with respect to the symbol duration T) is considered the major parameter. The interference power coming from adjacent channels to the main channel depends on the frequency separation δf , interference level and filter type. Consequently for a given interference level and δf , we may have different SIR depending on the type of filtering. In the equalizer, the number of forward taps L_f and the number of feedback taps L_R are the parameters. The channel is considered ideal (unfaded) and perfect carrier recovery at the receiver is assumed. Signal level on the main channel is held fixed. AWGN power and ACI levels are other variables such that Signal to Noise Ratio (SNR) and Signal to Interference Ratio (SIR) at the input of the equalizer can be considered as parameters. Since the modulation scheme is QPSK, there is a 3db difference between SNR and E_b/N_0 which is considered in all the

results. Square root raised cosine filters were used for transmit and receive filters. The roll-off factor of these filters was set to 0.35 (i.e. 35 percent excess bandwidth) for all the simulations. Table 3.1 summarizes all the fixed and variable simulation parameters.

$T=1$	Symbol duration
(L_f, L_R)	L_f forward and L_R feedback taps
$\delta f T$	Normalized frequency separation
SIR	Signal to Interference Ratio
SNR	Signal to Noise Ratio
τ_i	Timing misalignment of ACI
α	Roll off factor for square root raised cosine filter

Table 3.1 Basic simulation parameters

3.3 MMSE Calculations

The basic equations for MMSE of a finite length decision feedback equalizer were derived in chapter 2. Here we present the results for various parameters. The aim of presenting these results is to demonstrate the effect of different parameters on MMSE which can be used roughly to select a good combination of parameters for the simulations. Moreover, as we will see later, although there is no direct relation between the MMSE and the error rate, usually a lower MMSE corresponds to a better error rate performance. Since the simulations are very time consuming, whenever a comparison is to be made between the performance of the system under different conditions, it can be done based on the MMSE, specially when the error rate performance is also available at least for one of them.

3.3.1 Number of forward and feedback taps

The required number of taps in an equalizer to achieve a certain performance is highly dependent on the type of impairment that the equalizer is used for [1,3,18]. Although one would expect that an infinite length equalizer is the best equalizer that might be used, it has its own drawbacks. For an infinite length DFE, the effect of decision errors, usually called error propagation [13], may be considerable while this effect is reduced when the number of feedback taps is limited. Moreover, increasing the number of forward taps beyond some levels, would usually result in a very negligible improvement of the system performance while it makes the equalizer more complex. Convergence properties of the

equalizer is also affected by the number of taps and increasing the number of taps would result in a slower convergence.

Figure 8 shows MMSE of a $T/2$ spaced DFE. The number of forward taps, L_f , and the number of feedback taps, L_R , are considered as variables. The results shown in Figure 8 are for a normalized channel spacing of $\delta f=0.8$ with equal power left and right ACI. This corresponds to a signal to interference ratio (SIR) of 4.9 db. Signal to noise ratio (SNR) is fixed at 30 db. Based on these results, it can be seen that when only one feedback tap is used ($L_R=1$), increasing the number of forward taps beyond 6 results in a negligible improvement of MMSE. Based on these results and considering other limitations specially the convergence properties, a (9,4) equalizer was chosen for the rest of this thesis unless otherwise stated. The results obtained here are based on the best sampling phase, Θ , and delay, d , to have minimum MMSE for each case.

3.3.2 T-spaced versus $T/2$ -spaced Equalizer

The advantages of using fractionally spaced equalizer over baud rate (T-spaced) equalizer have been studied by many researchers for various applications [3,7,13,18]. Since the sampling rate of $T/2$ -spaced equalizer is twice as that of T-spaced equalizer, the periodicity of the spectrum of $T/2$ -spaced equalizer is $2/T$ while that of T-spaced equalizer is $1/T$ [13]. This is specially important for timing phase errors. Figure 9 shows the effect of sampling phase for a (9,4) T-spaced and (9,4) $T/2$ -spaced equalizer on MMSE. Signal to Noise ratio (SNR) is 30 db and channel spacing is $\delta f=0.9$ which corresponds to SIR=8.2 db. The sensitivity of T-spaced DFE on sampling phase compared

Figure 8 Effect of the number of forward and feedback taps on MMSE, SNR=30 db, $\delta f=0.8$ (SIR=4.9 db)

to T/2-spaced DFE is quite clear. It is also seen that for this case, MMSE of T/2-spaced DFE is always better than that of T-spaced DFE. It must be emphasized that the timing span of T/2-spaced DFE is half of timing span of T-spaced DFE.

In Figure 10 channel spacing is $\delta f=0.75$ (corresponding to SIR=3.64 db) and other variables are the same as for Figure 9. Here, slightly different scenario may be observed. There are instances where the T-spaced DFE performs better than T/2-spaced DFE when their number of taps is equal. However, the basic results and discussion made earlier about the advantages of fractionally spaced equalizer over baud rate equalizer still hold. First, the sensitivity of T/2-spaced DFE to sampling phase is less than T-spaced DFE. Second, when the timing span of the T/2-spaced DFE and T-spaced DFE are equal, the MMSE of T/2-spaced DFE is less than that of T-spaced DFE for all the cases. This can be seen from Figure 10 by comparing a (5,4) T-spaced DFE and a (9,4) T/2-spaced DFE. The results shown in Figures 9 and 10 are based on an excess bandwidth of 35 percent ($\alpha=0.35$ for both transmit and receive filters). The effect of a different excess bandwidth will be briefly discussed in section 3.3.4.

It may appear from the above argument that having a fractionally spaced equalizer with a tap spacing of T/N where N is greater than 2, would improve the system performance and result in lower MMSE. This argument is generally true provided that :

- 1- Receive filter bandwidth be increased. Otherwise increasing N which is equivalent to increasing the equalizer bandwidth is not beneficial.
- 2- Same timing span is used for T/N-spaced DFE. This requires more number of taps.

Figure 9 Effect of sampling phase for T-spaced and T/2-spaced DFE, SNR=30 db, $\delta f=0.9$ (SIR=8.2 db)

Figure 10 Effect of sampling phase for T-spaced and T/2-spaced DFE, SNR=30 db, $\delta f=0.75$ (SIR=3.64 db)

Clearly, the main disadvantage of having smaller tap spacing is the increased complexity. Having more number of taps also would affect the convergence of the equalizer. Considering the above mentioned limitations we restrict ourselves to $T/2$ -spaced DFEs. It is worthwhile to note that for a variety of channel spacing and bandwidths, $T/2$ -spaced DFE was reported to be optimum (in MSE sense) when one ACI is considered [8]. It is also reported that increasing N beyond 4, does not improve ACI rejection [8].

3.3.3 One interferer versus two interferers

The MMSE performance of the equalizer is presented here when there are two adjacent channel interferers (left and right) and when there is only one adjacent channel interferer. In Figure 11, the MMSE of (9,4) $T/2$ -spaced equalizer with best sampling phase and delay is shown.

It can be seen from Figure 11 that the MMSE depends on both channel spacing and ACI level. It is observed that for a fixed SIR, a larger channel spacing corresponds to a better MMSE. It is also seen that for larger channel spacing, the effect of SIR on MMSE becomes smaller. For example when SIR is changed from 3.64 db to 8.2 db at $\delta f=0.65$, MMSE has a change of 3.63 db while for the same change in SIR at $\delta f=0.95$, MMSE has a change of 1.83 db.

Figure 11 MMSE of (9,4) T/2-spaced equalizer with two ACL, SNR=30 db

Figure 12 MMSE of (9,4) T/2-spaced equalizer with one ACI, SNR=30 db

In Figure 12 results are presented when only one ACI is present. In this case, the level of ACI is adjusted to have same SIR as in the case of two ACI for a given channel spacing. The difference in system performance when one interferer is present compared to the case when two interferers are present is quite considerable. It can be seen that for $SIR=3.64$ db at $\delta f=0.65$, MMSE is reduced about 12.8 db. This highlights the superiority of the equalizer to deal with one interferer as opposed to two or more interferers. It must be noted that we have chosen channel spacings which are less than the theoretical limit of $1/T$. This means that we are working in a region where ACI can not be totally suppressed [17]. However, the results shown here suggest that based on the application and the required performance we can choose channel spacings of less than $1/T$. This will be emphasized later when the error rate performance of the system is given in coming sections.

5.4 Receive filter bandwidth

The MMSE of the equalizer is examined here when the receive filter bandwidth is increased from 35 percent to 100 percent ($\alpha=1$). Figure 13 shows the results for $\alpha=0.35$ and $\alpha=1$. The transmit filter bandwidth is kept at 35 percent excess bandwidth. In Figure 14, the effect of receive filter bandwidth is examined when only one interferer is present. It can be seen from both Figures that MMSE performance of the equalizer is better when the receive filter excess bandwidth is increased only when the channel spacing is less than about 0.82. In the best case, the improvement of MMSE is about 1.5 db for the shown

**Figure 13 Effect of receive filter excess
bandwidth (α) on MMSE, two ACI, SNR=30 db**

**Figure 14 Effect of receive filter excess
bandwidth (α) on MMSE, one ACL, SNR=30 db**

channel spacing and SIR. However, as observed from both Figures, MMSE performance degrades when the channel spacing becomes larger for a receive filter with an excess bandwidth of 100 percent. Before giving any explanation for the above, an important thing about the SIR should be mentioned here. The signal to interference (SIR) shown in Figures 13 and 14, is calculated based on the amount of adjacent channel power spilled over the main channel when a 35 percent excess bandwidth square root raised cosine filter is used at the front end of the receiver. To have a constant SIR, the level of the interfering signals are adjusted to compensate for the channel spacing (this simply means that when the channel spacing is larger, the ACI level has been increased properly to have constant SIR).

The results obtained here may be explained as follows. When the channel spacing is small, larger filter bandwidth would allow more ACI to come into the main channel. Since many frequency components of this SNOI are present in SOI spectrum, it can be recognized by the equalizer and be suppressed. However, when the channel spacing is relatively large, only some tail of SNOI spectrum is present in SOI and since we are not making separate decision on ACI's data, using a tighter filter will eliminate the undesired interference. Large filter bandwidth in this case would not help to estimate and suppress the SNOI tail which is present in our SOI spectrum unless other techniques which use spectral correlation between different part of the spectrum are used [4].

The results shown in Figures 13 and 14 suggest that for small channel spacing it is not advisable to reduce the filter bandwidth while for relatively larger channel spacings, it is better to use a tighter filter. These results are in compliance with those reported in [8].

3.4 System performance without equalizer

We begin to discuss the results of our simulations first by looking at the system performance when no equalizer is used in the receiver. The model for the system is as described in section 2.1 except that instead of an equalizer, a matched filter receiver is used. Figure 15 shows symbol error rate curves for different channel spacing. In this case the magnitude of the adjacent channel signals are equal to the main channel's signal. Figure 16 shows error rate curves when ACI is treated as an equivalent Gaussian white noise with the same power. As it can be seen from the results, the difference in error rate performance of the matched filter receiver between Gaussian approximation of the ACI and the actual ACI when the channel spacing is small (up to $\delta f T=0.95$), is so small that can not be distinguished from the graphs. For QPSK signal symbol error rate is given by [15]

$$P_M = \text{erfc}(\sqrt{\gamma_b}) \left(1 - \frac{1}{4} \text{erfc}(\sqrt{\gamma_b}) \right) \quad (3.1)$$

where $\text{erfc}(\cdot)$ is the complementary error function defined as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2/2} dt \quad (3.2)$$

and $\gamma_b = \frac{E_b}{N_0}$ is the received Signal to Noise Ratio per bit. The results of the simulations are also in close agreement with theoretical results obtained from the above equation when ACI is treated as an equivalent Gaussian Noise. These results show that when there is no equalizer in the receiver, specially when the channel spacing is relatively small, there is no need to go to computationally expensive methods such as Helstrom's one [11] to find

the error rate and the approximation given above leads to a sufficiently close agreement with the actual error rate. Same observations were reported in [2] but for minimum shift keying (MSK) and sinusoidal frequency shift keying (SFSK) modulation schemes. For small channel spacing and relatively high SNR this error rate is solely determined by SIR.

$S/I=4.9$ db $\delta f T=0.80$

$S/I=6.38$ db $\delta f T=0.85$

$S/I=8.2$ db $\delta f T=0.90$

$S/I= 10.3$ db $\delta f T=0.95$

$S/I= 13$ db $\delta f T=1.0$

Figure 15 Symbol error rate of matched filter receiver for different channel spacing

Figure 16 Symbol error rate of matched filter receiver for different ACI (Treated as WGN)

3.5 System Performance with equalizer

Based on the results from the MMSE calculation of DFE in presence of ACI, a (9,4) DFE was chosen for the simulations. The equalizer tap weight coefficients are updated once in every symbol interval according to complex valued LMS (least-mean-square) gradient algorithm [20]. In this algorithm the tap coefficients are adjusted according to

$$\mathbf{C}_{K+1} = \mathbf{C}_K + \mu e_K \mathbf{V}_K^* \quad (3.3)$$

where $\mathbf{C}$ and $\mathbf{V}$ are defined in (2.28) and (2.29) respectively. e_K is the error between the input and the output of the decision device as illustrated in Figure 5. Subscript K in the above equation refers to the value of the parameter at $t=KT$. μ is the adaptation step size. In Appendix B, the derivation of the complex valued LMS algorithm is given.

Since the objective of this work is primarily to investigate the impact of DFE in ACI environment, other equalizer structures (such as lattice) or other alternatives for adaptation algorithm were not considered here. Specifically in a time varying channel, the convergence and tracking properties of the equalizer may become crucial. It is well known that RLS (recursive-least-square) and Fast RLS Kalman algorithms have much better convergence rate than LMS algorithm [15]. However, the price paid for this faster convergence is the computational complexity and the required number of complex operations for each iteration. A comparison among various algorithms and their properties may be found in [15].

The adaptation step size, μ is set to 0.06 during training period and 0.002 during normal data transmission. Since the channel is an ideal unfaded channel (time invariant),

this combination was found to be reasonable although a choice of $\mu=0.02$ for training period and $\mu=0.002$ for normal data transmission worked as well.

Most of the simulations were run for 2×10^5 symbols but for the points with an error rate between 10^{-4} and 10^{-5} the simulations were run for 10^6 symbols. Since the results shown in the graphs are based on counting the number of symbols which are in error, they may not be very accurate whenever the probability of error is low (in the range of 10^{-4} and 10^{-5}). There is no specific criteria in a bit by bit simulation about the accuracy of the results but it is common to count about 100 errors for a reasonable error rate estimate. In view of execution time constraints which is the most prohibitive factor in a bit by bit simulation, we limited ourselves to the above mentioned simulation run time and whenever no error was observed during this simulation run time, the graphs were terminated.

3.5.1 System Performance with two ACI

In Figure 17 error rate results are shown for two ACI. In this Figure, the magnitude of both right and left adjacent channel signals are equal to that of the main channel's signal. Thus, in this case SIR is solely determined by channel spacing. A quick comparison between this Figure and Figure 15 (no equalizer) highlights the substantial improvement of the error rate performance of the system due to implementation of DFE. This is more evident for higher SNR. For instance at $E_b/N_0 = 15$ db and $\delta f = 0.85$ (SIR=6.38 db), two order of magnitude reduction in SER is observed. This difference reaches three order of magnitude for $\delta f = 0.8$ at $E_b/N_0 = 20$ db.

$S/I=3.6$ db $\delta f T=0.75$

$S/I=4.9$ db $\delta f T=0.80$

$S/I=6.38$ db $\delta f T=0.85$

$S/I=8.2$ db $\delta f T=0.90$

Figure 17 Symbol error rate for different channel spacing

In Figures 18 through 21, the performance of the system is examined for fixed SIR and different channel spacings. It is assumed that left and right ACI are equal power. In all of these four figures, it is seen that there are some irreducible error floors specially for smaller channel spacing. This is expected since a one dimensional single input receiver is unable to remove ACI completely when the channel spacing is less than the fundamental value of the symbol rate. This error floor is reduced by an increase in SIR. It is also seen that regardless of the amount of the interference, choosing larger channel spacing always leads to a better performance. However, a comparison between these figures shows that the gain obtained from larger channel spacing is less for higher SIR specially for relatively large E_b/N_0 .

Figures 22 through 28 show the error rates for fixed channel spacing and different SIR. The observations made earlier about the error floor is more clear here. Figures 22 and 23 specially show this error floor for $\delta f=0.65$ and $\delta f=0.7$. As shown in Figures 27 and 28, which correspond to $\delta f=0.9$ and $\delta f=0.95$ respectively, no errors were observed at $E_b/N_0=20$ db even when $SIR=3.64$ db.

The results obtained from simulations agree in general with the mathematical analysis of MMSE done in section 3.2. By referring back to Figure 11, it is seen that the impact of SIR for small channel spacing is higher than for larger channel spacing. Same thing can be seen here by comparing Figures 22 and 28. A comparison between Figures 22 and 28 indicates that for $\delta f=0.95$ changing SIR from 3.64 db to 8.2 db does not improve the error rate as much as it does for $\delta f=0.65$.

Figure 18 Symbol error rate for different channel spacing at fixed SIR=3.64 db

Figure 19 Symbol error rate for different channel spacing at fixed SIR=4.9 db

Figure 20 Symbol error rate for different channel spacing at fixed SIR=6.38 db

Figure 21 Symbol error rate for different channel spacing at fixed SIR=8.2 db

Figure 22 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.65$

Figure 23 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.7$

Figure 24 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.75$

Figure 25 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.8$

Figure 26 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.85$

Figure 27 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.9$

Figure 28 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.95$

3.5.2 System Performance with one ACI

Now, the error rate results are presented when only one ACI is present. Figures 29 to 32 show the results for fixed SIR and different channel spacing. Results for fixed channel spacing and different SIR are shown in Figures 33 through 35.

It can be seen from all these figures that in this case too, same kind of results are observed as for the case of two ACI. Specifically it is observed from Figures 29 to 32 that larger channel spacing provides better error rate performance. As for the effect of SIR, it can be seen from Figures 33 to 35 that the impact of SIR for smaller channel spacing is more than larger channel spacing. For instance, at $E_b/N_0 = 15$ db, changing SIR from 3.64 db to 8.2 db causes SER to drop from $1e-2$ to $2.1e-4$ (about two order of magnitude) for $\delta f = 0.65$ (Figure 33), while for the same E_b/N_0 and same change in SIR, SER is dropped from $8e-5$ to $6e-6$ (about one order of magnitude) when $\delta f = 0.95$ (Figure 35).

Although the results obtained here agree in general with the MMSE analysis for one ACI, their differences are mentioned here. By stating general agreement, we mean that a lower MMSE for each individual case (either a fixed SIR or a fixed channel spacing) corresponds to a better error rate performance. However, by referring back to Figure 12, we can see that the impact of SIR on MMSE is almost the same for different channel spacings while as mentioned above, SIR has more impact on smaller channel spacing from the error rate point of view. We will emphasize the differences between the MMSE analysis and the error rate in the next section.

Figure 29 Symbol error rate for different channel spacing at fixed SIR=3.64 db, One ACI present

Figure 30 Symbol error rate for different channel spacing at fixed SIR=4.9 db, One ACI present

Figure 31 Symbol error rate for different channel spacing at fixed SIR=6.38 db, One ACI present

Figure 32 Symbol error rate for different channel spacing at fixed SIR=8.2 db, One ACI present

Figure 33 Symbol error rate for different SIR at fixed channel spacing of $\delta f=0.65$, One ACI present

Figure 34 Symbol error rate for different SIR at fixed channel spacing of $\delta f = 0.8$, One ACI present

Figure 35 Symbol error rate for different SIR at fixed channel spacing of $\delta f=0.95$, One ACI present

3.5.3 Comparison of one ACI and two ACI

Although we mentioned earlier that as far as signal to interference ratio (SIR) and channel spacing are concerned, same kind of results are observed for one ACI and two ACI, we highlight here their differences and compare the simulation results with the mathematical analysis of MMSE for both cases. Figures 36 and 37 compare the error rate performance of the system between one ACI and two ACI, for $\delta f=0.65$ and $\delta f=0.95$ respectively.

It is seen from Figure 36 that the difference between one ACI and two ACI for the same SIR is quite considerable. This difference is more evident for higher SNR. At $E_b/N_0 = 15$ db the difference is more than three order of magnitude in SER for $SIR=3.64$ db. This difference is even higher for $SIR=8.2$ db. This difference in SER between one ACI and two ACI was expected since in the MMSE analysis too, the difference between these two cases was noticeable (12.8 db at $\delta f=0.65$).

However, as shown in Figure 37, almost no difference can be seen between one ACI and two ACI cases when $\delta f=0.95$, although for the same channel spacing, the MMSE of one ACI case is about 5 db lower than for two ACI case at $SIR=8.2$ db. The difference in MMSE is about 6.7 db for $SIR=3.64$ db. This observation emphasizes the very first thing we said that there is no specific relation between MMSE and SER, although usually we expect to have a lower error rate when the MMSE is lower. The above mentioned differences also emphasize the need for direct computer simulation of the system in order to have a better view of the error rate performance of the system.

Figure 36 Comparison of SER between one ACI and two ACI for $\delta f=0.65$

Figure 37 Comparison of SER between one ACI and two ACI for $\delta f=0.95$

3.5.4 Effect of timing misalignment of ACI (τ_i)

It was explained earlier in Chapter 2 that as long as the condition of $\delta f > \alpha/T$ holds, the timing misalignment of the adjacent channel interferers (τ_i) have no effect on MMSE. Here, we examine the effect of τ_i on error rate for only few cases. Interestingly, we found that the error rate performance of the system is independent of τ_i and in almost none of the examined cases, the error rate of the system having ACI timing misalignment was worse than that of one having no ACI timing misalignment. This conclusion is important when one tries to compare a traditional MMSE DFE with a multi-dimensional DFE. Such a comparison is made in [21] where a multi-dimensional DFE is reported to be far superior than a traditional one. However, as mentioned in [21], a major inherent assumption for the multi-dimensional DFE is the timing alignment of ACI. Under normal operating conditions, multi-dimensional DFE is completely unable to work if this assumption does not hold. On the contrary, as we mentioned here, one dimensional DFE is insensitive to timing misalignment of ACI. It should be noted that we *may not* be able to extend our argument about the timing misalignment of ACI to other scenarios such as non ideal channels, although the argument about the effect of τ_i on MMSE holds for the cases. The cases simulated are as follows :

- Two ACI, $\delta f=0.75$, $\tau_1=0.25$, $\tau_{-1}=0.75$
- Two ACI, $\delta f=0.80$, $\tau_1=0.25$, $\tau_{-1}=0.75$
- Two ACI, $\delta f=0.85$, $\tau_1=0.25$, $\tau_{-1}=0.75$
- Two ACI, $\delta f=0.90$, $\tau_1=0.25$, $\tau_{-1}=0.75$

- Two ACI, $\delta f=0.75$, $\tau_1=0.5$, $\tau_{-1}=0.5$
- Two ACI, $\delta f=0.80$, $\tau_1=0.5$, $\tau_{-1}=0.5$
- One ACI, $\delta f=0.65$, $\tau_1=0.25$ and 0.5
- One ACI, $\delta f=0.95$, $\tau_1=0.25$ and 0.5

Since the results obtained for the above two ACI cases are extremely close to those of similar cases but without any timing misalignment, we do not duplicate the results here. Slightly different results are observed for one ACI cases for large SNR. However, the differences are not statistically significant. Table 3.2 shows and compares the results for $\delta f=0.65$. Results for $\delta f=0.95$ are shown in Table 3.3.

	$E_b/N_0 = 10\text{db}$	$E_b/N_0 = 12.5\text{db}$	$E_b/N_0 = 15\text{db}$	$E_b/N_0 = 20\text{ db}$
Misalignment = 0	5.83 e-2	2.6 e-2	9.9 e-3	6.9 e-4
Misalignment = 0.25	5.94 e-2	2.76 e-2	8.97 e-3	4.4 e-4
Misalignment = 0.5	5.9 e-2	3.28 e-2	8.45 e-3	3.8 e-4

Table 3.2 SER for one ACI, $\delta f = 0.65$

	$E_b/N_0 = 10 \text{ db}$	$E_b/N_0 = 12.5 \text{ db}$	$E_b/N_0 = 15 \text{ db}$
Misalignment = 0	$1.01 \text{ e } -2$	$1.52 \text{ e } -3$	$1.06 \text{ e } -4$
Misalignment = 0.25	$1.08 \text{ e } -2$	$1.17 \text{ e } -3$	$8.5 \text{ e } -5$
Misalignment = 0.5	$1.01 \text{ e } -2$	$1.34 \text{ e } -3$	$7.1 \text{ e } -5$

Table 3.3 SER for one ACI, $\delta f \approx 0.95$

Chapter 4

Conclusion and Recommendations

4.1 Conclusion

This thesis was motivated by the highly congested spectrum in many frequency bands and by the growing need for a more efficient spectrum utilization. A decision feedback equalizer was used as a countermeasure to combat excessive adjacent channel interference. Both cases of one interferer and two interferers were considered. Square root raised cosine filters were used as transmit and receive filters.

To study the system performance, an analytical method as well as direct bit-by-bit computer simulations were used. Since an analytical approach to find the error rate (which is the most desirable criteria in digital communications) was intractable, minimum mean square error (MMSE) criterion was chosen for analytical study of the system performance.

It was found that in all the cases, the equalizer can suppress one ACI better than two ACI for the same SIR. However, the difference in system performance between one ACI and two ACI is more noticeable for smaller channel spacings than for larger ones. This agrees with previous research reported in [17] that for the equalizer to be able to suppress more interferers, more degrees of freedom are required (such as multiple antenna receiver).

It was also found that for the same SIR, a larger channel spacing always leads to a better MMSE. The impact of SIR on MMSE was found to be more profound for smaller

channel spacings in two ACI environment. For one ACI, SIR had almost the same effect on MMSE for $\delta f=0.65$ to $\delta f=0.95$. It was also noticed that for the system model presented in Chapter 2, timing misalignment of the ACI had no effects on MMSE.

In the evaluation of system performance via simulations, most of the results of the analytical method were justified. The equalizer superiority to deal with one ACI rather than two was highlighted in Figure 36. The general conclusion that for fixed SIR, a larger channel spacing always leads to a better system performance was also justified here.

In the case of two ACI, some error floors were observed which were due to the fact that we worked with channel spacings of less than the fundamental value of baud rate. We may expect to have similar situations for one ACI. However, since the simulations run time were limited, we were unable to make same observations. We found that the system performance is insensitive to timing misalignment of ACI (this *may not* be extended to fading time variant channels).

4.2 Recommendations for future research

Throughout this thesis, an effort was made to point out all the assumptions leading us to the reported results and the potential differences that may lead to other types of results and conclusions. In this section these points are discussed.

Fading is a major impairment in all radio links which was not addressed in this thesis. Besides introducing difficulties in maintaining carrier and symbol synchronization, fading would decrease the tracking capability of the equalizer in a time-varying channel. It is thus proposed that to observe the system performance under fading, fading countermeasures

such as diversity techniques combined with DFE based receiver be studied. Specifically, it appears that a combination of multiple antenna receiver followed by DFE is a feasible way to combat ACI. On the contrary, the practicality of a multi dimensional DFE seems to be questionable.

Since a time invariant channel was assumed in this thesis, the tracking capability of the equalizer was not of concern. However, since this issue is crucial in time variant channels, a study of other equalizer structures such as lattice and faster convergent algorithms such as RLS would be beneficial. Practical problems associated with these approaches such as sensitivity of convergence to quantization error in a real time situation are also recommended for further research.

Appendix A

Simulations documentation

In this Appendix, some examples of BOSS modules created to perform the simulations are shown. Our purpose here is only to show how the simulations were performed in BOSS and not to describe the whole simulation procedure. Figure A.1 shows the main system under investigation consisting of all the stages previously described in section 2.2. In Figure A.2 the equalizer structure is shown. Figure A.3 is a sample parameter settings for the simulation and Figure A.4 is a sample set of simulation results.

Figure A.1 Base system

2 1 2
of/de

PM-1 3½" x 4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

PIONEERS IN METHYLENE BLUE TESTING SINCE 1974

1000 COUNTY ROAD 8, BAINBRIDGE, GA 30607 USA
TEL. 812 486 7887 FAX 812 486 7887 TLA 240000000

Figure A.2 The equalizer

```
STOP-TIME = 1000000
DT = 0.125
GAIN IN TRAINING MODE = (10 , 0.0)
LMS STEP SIZE = 0.002
NUMBER OF FORWARD TAPS = 9
NUMBER OF FEEDBACK TAPS = 4
DELAY SAMPLES = 228
RSEED_ACI_2 = 8761235
ISEED_ACI_2 = 8731299
RSEED_ACI_1 = 90882765
ISEED_ACI_1 = 87615523
RSEED_SIGNAL = 11897765
ISEED_SIGNAL = 87625545
RSEED_NOISE = 87879993
ISEED_NOISE = 22535541
SNR = ITERATION: from 13, by 2.5, for 5 steps
CHANNEL SPACING = 0.8
SIR_2 (DB) = 6.64
SIR_1(DB) = 6.64
MISALISN-1 = 0
MISALISN-2 = 0
TIME DELAY TO INPUT (SEC) = 24
NO.OF TRAINING SYMBOLS = 1000
NUMBER OF POINTS FOR FFT PROCESSING = 64
PRINT ESTIMATE MODULO = 10000
T-ON = 1000
SYMBOL RATE (HZ) = 1
```

Figure A.3 A sample simulation parameter settings

ITERATION STEP = 1 SNR = 18 db

Number of Symbols Counted = (10000)
 Probability of Symbol Error = (0.)
 Number of Symbols Counted = (20000)
 Probability of Symbol Error = (8.00000E-04)
 Number of Symbols Counted = (30000)
 Probability of Symbol Error = (2.66667E-03)
 Number of Symbols Counted = (40000)
 Probability of Symbol Error = (4.40000E-03)
 Number of Symbols Counted = (50000)
 Probability of Symbol Error = (4.16000E-03)
 Number of Symbols Counted = (60000)
 Probability of Symbol Error = (5.60000E-03)
 Number of Symbols Counted = (70000)
 Probability of Symbol Error = (7.42857E-03)
 Number of Symbols Counted = (80000)
 Probability of Symbol Error = (8.90000E-03)

Number of Symbols Counted = (800000)
 Probability of Symbol Error = (1.10800E-02)

ITERATION STEP = 2 SNR = 23 db

Number of Symbols Counted = (10000)
 Probability of Symbol Error = (0.)
 Number of Symbols Counted = (20000)
 Probability of Symbol Error = (0.)
 Number of Symbols Counted = (30000)
 Probability of Symbol Error = (1.60000E-03)
 Number of Symbols Counted = (40000)
 Probability of Symbol Error = (1.60000E-03)
 Number of Symbols Counted = (50000)
 Probability of Symbol Error = (1.28000E-03)
 Number of Symbols Counted = (60000)
 Probability of Symbol Error = (1.06667E-03)
 Number of Symbols Counted = (70000)
 Probability of Symbol Error = (1.14286E-03)
 Number of Symbols Counted = (80000)
 Probability of Symbol Error = (1.00000E-03)

Number of Symbols Counted = (800000)
 Probability of Symbol Error = (2.35000E-03)

Figure A.4 A sample simulation results

Appendix B

Complex valued LMS algorithm

The complex valued LMS algorithm is based on the stochastic steepest descent method [5]. To achieve the minimum of the squared error surface (which is a quadratic function), the tap weight coefficients of the equalizer are updated once every symbol in the opposite direction of the gradient of the squared error. The LMS update equation can be written as

$$\mathbf{C}_{n+1} = \mathbf{C}_n - \frac{1}{2}\mu_n \nabla_{\mathbf{C}_n} E_n \quad (\text{B.1})$$

where

$$E_n = |e_n|^2 = |Y_n - a_{n-D}|^2 \quad (\text{B.2})$$

$$= |\mathbf{C}^T \mathbf{V}_n - \hat{a}_{n-D}|^2 \quad (\text{B.3})$$

In the above equations E_n is the squared error, μ_n is the LMS step size, ∇ is the gradient operator and Y_n , $\hat{a}_{n-D}$, $\mathbf{C}_n$ are as described in chapter 2. The values of μ for which the convergence of the algorithm is guaranteed depends on the overall channel characteristics and eigen value spread of the correlation matrix $\mathbf{A}$ [5,12]. Squared error is real but $\mathbf{C}_n$ are complex valued tap weight coefficients, thus

$$\nabla_{\mathbf{C}_n} E_n = \frac{\partial E_n}{\partial \text{Re}(\mathbf{C}_n)} + j \frac{\partial E_n}{\partial \text{Im}(\mathbf{C}_n)} \quad (\text{B.4})$$

Since $E_n = e_n \cdot e_n^*$, the first component of the above equation may be written as

$$\frac{\partial E_n}{\partial \text{Re}(\mathbf{C}_n)} = \frac{\partial e_n}{\partial \text{Re}(\mathbf{C}_n)} e_n^* + \frac{\partial e_n^*}{\partial \text{Re}(\mathbf{C}_n)} e_n \quad (\text{B.5})$$

$$= -\mathbf{V}_n e_n^* - \mathbf{V}_n^* e_n \quad (\text{B.6})$$

$$= -2\text{Re}(e_n \mathbf{V}_n^*) \quad (\text{B.7})$$

The second component of the equation can be written as

$$\frac{\partial E_n}{\partial \text{Im}(\mathbf{C}_n)} = \frac{\partial e_n}{\partial \text{Im}(\mathbf{C}_n)} e_n^* + \frac{\partial e_n^*}{\partial \text{Im}(\mathbf{C}_n)} e_n \quad (\text{B.8})$$

$$= -j\mathbf{V}_n e_n^* - j\mathbf{V}_n^* e_n \quad (\text{B.9})$$

$$= -2\text{Im}(e_n \mathbf{V}_n^*) \quad (\text{B.10})$$

Combining equations 10 and 11 yields the complex valued LMS tap weight coefficients update for DFE

$$\mathbf{C}_{n+1} = \mathbf{C}_n + \mu_n e_n \mathbf{V}_n^* \quad (\text{B.11})$$

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