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Short-Block Data Detection Techniques Employing Channel Estimation for Fading Time-Dispersive Channels

by

Stewart N. Crozier, B.Sc.(Eng.), M.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

**Department of Systems and Computer Engineering
Carleton University
Ottawa, Ontario
March 26, 1990**

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ISBN 0-315-58970-1

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The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis,

Short-Block Data Detection Techniques Employing Channel Estimation for Fading Time-Dispersive Channels

submitted by

Stewart N. Crozier, B.Sc.(Eng.), M.Eng.

in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Systems and Computer Engineering

Thesis Supervisor(s)

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Carleton University

Abstract

The problem is that of providing reliable digital communications over fading channels with severe time-dispersion. Three short-block data detection techniques based on linear, nonlinear decision-directed, and maximum likelihood estimation principles are investigated. Short alternating blocks of data and training symbols are used. The training blocks are useful for both channel tracking and data block detection. In contrast to the recursive symbol-by-symbol equalization approaches usually employed, each data block is detected as a unit. Previous restrictions imposed on short-block detection schemes have been removed, resulting in reduced complexity algorithms and higher throughput efficiency. The reduced complexity makes these schemes more practical for higher signaling rates. The higher the signaling rate, the more inherent diversity a time-dispersive fading channel can provide, and the better the error rate performance. The analysis is valid for symbol-spaced, fractionally-spaced, and continuous-time signal models. Both theoretical and simulated performance results are presented. Performance is evaluated for BPSK and QPSK type signaling using a Rayleigh fading channel model with severe time-dispersion. The performance provided by a conventional linear equalizer (LE) and conventional decision feedback equalizer (DFE) is also presented for comparison.

The block detection schemes investigated all require an estimate of the channel impulse response. This is considered an advantage from the adaptation point of view, since channel response estimation is one of the simplest adaptation requirements for any adaptive detection process. A channel estimation algorithm is presented which requires very short training sequences for startup purposes. Optimum training sequences are found and tabulated for different channel and training sequence lengths. A simple formula for the expected degradation in performance caused by channel estimation errors is also derived. The simulation results confirm the accuracy of this approximation for high signal to noise ratios.

Acknowledgements

The author would like to thank professors David Falconer and Samy Mahmoud for their knowledgeable supervision and constructive criticism. I would also like to express my appreciation to William Moreland of the Communications Research Centre (CRC) for his helpful suggestions and insight. Finally, I would especially like to thank my wife, Maureen, for her patience and encouragement.

This research was supported by grants from the Natural Sciences and Engineering Council of Canada (NSERC), the Ontario Graduate Scholarship Program (OGS), and the Telecommunications Research Institute of Ontario (TRIO).

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List of Acronyms and Abbreviations

AC-SSB — amplitude companded single side band
ADPM — adaptive digital phase modulation
AWGN — additive white Gaussian noise
BDFB — block decision feedback
bps — bits per second
BPSK — binary phase shift keying
BW — bandwidth
CDMA — code division multiple access
CPM — continuous phase modulation
CRC — Communications Research Centre, Ottawa
dB — decibel(s)
DFBE — decision feedback block estimation
DFE — decision feedback equalization
DPM — digital phase modulation
EMSE — excess mean squared error
ETA — expanding tree algorithm
fd — Doppler frequency in sampling rates (simulation parameter)
FDMA — frequency division multiple access
FEC — forward error correction
FM — frequency modulation
FSE — fractionally spaced equalization
GHz — 10^9 Hz
GMSK — Gaussian minimum shift keying
HF — high frequency
Hz — cycles per second
I — inphase
Ideal — ideal isolated pulse matched filter detector

ISI — intersymbol interference
 Kbps — 10^3 bits per second
 KHz — 10^3 Hz
 km — kilometers
 Ksps — 10^3 symbols per second
 LBE — linear block estimation
 LE — linear equalization
 LMS — least mean square
 LSSE — least sum of squared errors
 Mbps — 10^6 bits per second
 MHz — 10^6 Hz
 ML — maximum likelihood
 MLBE — maximum likelihood block estimation
 MLSE — maximum likelihood sequence estimation
 MMSE — minimum mean squared error
 MPSK — M-ary phase shift keying
 MSE — mean squared error
 msec — milliseconds
 MSK — minimum shift keying
 Msps — 10^6 symbols per second
 MVA — modified Viterbi algorithm
 N/A — not applicable (simulation parameter)
 nb — number of data blocks per frame (simulation parameter)
 nd — number of symbols per data block (simulation parameter)
 nf — number of frames per simulation run (simulation parameter)
 np — number of symbols per preamble (simulation parameter)
 ns — number of samples per symbol period (simulation parameter)
 nsec — nanoseconds
 NSERC — Natural Sciences and Engineering Council of Canada

nt — number of symbols per training block (simulation parameter)
 nT — number of T-spaced channel tap estimates (simulation parameter)
 OGS — Ontario Graduate Scholarship Program
 prof(sp) — multipath amplitude profile with spacing sp (simulation parameter)
 Q — quadrature
 QPSK — quadrature phase shift keying
 RLS — recursive least squares
 RMS — root mean square
 Rx — receive filter type (simulation parameter)
 sec — seconds
 SER — signal to estimation error ratio
 SER' — signal to estimation error ratio, normalized with respect to SNR
 SG — stochastic gradient
 SHF — super high frequency
 SNR — signal to noise ratio
 sps — symbols per second
 SSE — sum of squared errors
 std. dev. — standard deviation
 TDL — tapped delay line
 TDMA — time division multiple access
 TMSE — total mean squared error
 TRIO — Telecommunications Research Institute of Ontario
 Tx — transmit filter type (simulation parameter)
 u — step-size parameter for SG channel tracking (simulation parameter)
 UB — upper bound
 UHF — ultra high frequency
 usec — microseconds
 VHF — very high frequency

List of Mathematical Symbols and Notation

- A — general matrix
- $\alpha(\tau; t)$ — continuous-time multipath attenuation function
- $\mathbf{a}_1$ — first or left vector of training symbols
- $a_1(t)$ — first or left training signal
- $\mathbf{a}_2$ — second or right vector of training symbols
- $a_2(t)$ — second or right training signal
- $a_{i,j}$ — (i, j) -th element of general matrix A
- a_k — k -th training symbol
- $a_n(t)$ — attenuation factor for the n -th path signal
- $\mathbf{b}$ — vector of data block symbols
- $b(t)$ — data signal
- B_d — Doppler spread or Doppler bandwidth
- b_k — k -th data symbol
- $\hat{\mathbf{b}}_{opt}$ — optimum estimate of $\mathbf{b}$
- $\mathbf{c}$ — discrete vector version of $c(t)$
- $c(t)$ — data block detection signal, with training signals removed
- $c(\tau; t)$ — equivalent low pass channel time-variant impulse response
- $C(f; t)$ — time-varying channel transfer function
- $cov[.]$ — covariance matrix operator
- d — Euclidean distance
- D — set of all Euclidean distances
- $d(\epsilon)$ — Euclidean distance associated with error event ϵ
- Deg_k — k -th symbol degradation from Ideal
- DEG_{lsse} — degradation with LSSE channel estimation
- DEG_{sg} — degradation with SG channel tracking
- $(df)_c$ — coherence bandwidth
- d_{min} — minimum Euclidean distance

$(dt)_c$ — coherence time
 d_{ub} — upper bound on d_{min}
 e — sequence of error symbols for error event ϵ
 $\mathbf{e}$ — symbol estimation error vector
 E — set of all possible error events
 $\mathbf{E}$ — channel response estimation error matrix
 $\tilde{\mathbf{E}}$ — augmented channel response estimation error matrix
 $\mathbf{E}_1$ — precursor channel response estimation error matrix
 $\mathbf{E}_2$ — postcursor channel response estimation error matrix
 $e(t)$ — continuous-time error signal
 $E[.]$ — expected value operator
 $\mathbf{e}_e$ — symbol estimation error vector
 E_b — average energy per signaling bit
 $E_b(\mathbf{f})$ — energy per signaling bit for a given channel response $\mathbf{f}$
 E_c — energy in signal $c(t)$
 E_d — set of all error events with distance d
 E_{db} — average energy per data bit
 $\mathbf{e}_f$ — channel estimation error vector
 E_f — energy in the overall channel response
 e_k — k -th element of $\mathbf{e}$
 e_{min} — error symbol with minimum magnitude
 $emse$ — excess mean squared error
 E_r — energy in the received signal
 $exp(.)$ — natural exponent function
 E_α — set of all error events with distances in the range $[d_{min}^2, \alpha d_{min}^2]$
 f — continuous frequency variable
 $\mathbf{f}$ — discrete overall channel pulse response vector
 $\mathbf{F}$ — channel response matrix
 $\mathbf{F}_1$ — precursor channel response matrix

$\mathbf{F}_2$ — postcursor channel response matrix
 $f(t)$ — continuous-time overall channel impulse response
 $F[.]$ — Fourier transform operator
 $F^{-1}[.]$ — inverse Fourier transform operator
 $F(s_k)$ — maximum J_k over all state sequences ending in state s_k
 f_c — carrier frequency
 f_d — maximum Doppler frequency
 f_l — l -th sample of a discrete channel response
 $\hat{\mathbf{f}}_{opt}$ — optimum estimate of discrete channel response
 $\mathbf{I}$ — identity matrix
 $I_{i,j}$ — (i,j) -th integral
 $I_k(\epsilon)$ — k -th incremental integral for error event ϵ
 $Im[.]$ — imaginary part of a complex term
 j — square root of negative one; also an integer index
 $J(\tilde{\mathbf{b}})$ — objective function for MLBE
 J_k — k -th objective function for MLBE
 K — number of samples per symbol period
 K_d — total probability of all error sequences with distance d
 $K_{d_{min}}$ — total probability of all error sequences with distance d_{min}
 L — channel memory in symbol periods (channel length – 1)
 $\mathbf{L}$ — lower triangular matrix
 $l_{i,j}$ — (i,j) -th element of matrix $\mathbf{L}$
 $\ln(.)$ — natural logarithm function
 $\log(.)$ — base 10 logarithm function
 $lsse$ — least sum of squared errors
 m — length of an m-sequence
 M — alphabet size for MPSK signaling
 $\max_x[.]$ — maximum of $[.]$ over all x

$\max_{x|y} [\cdot]$ — maximum of $[\cdot]$ over all x given y
 M_e — alphabet size for error symbols
 M_{lsse} — misadjustment due to LSSE channel estimation
 $mmse$ — minimum mean squared error
 $mse [\cdot]$ — mean squared error operator
 M_{sg} — misadjustment due to SG channel tracking
 $\mathbf{n}$ — discrete vector of noise samples
 $\tilde{\mathbf{n}}$ — noise vector due to channel estimation errors
 $n(t)$ — continuous-time additive white Gaussian noise
 N — number of symbols per data block
 N_{ct} — number of channel taps
 n_k — k -th noise sample
 N_0 — single-sided noise power spectral density
 N_p — total number of preamble training symbols
 N'_p — number of preamble training symbols, less the precursor
 N_t — number of symbols per training block
 $\binom{N}{n}$ — number of N choose n combinations
 $\mathbf{p}$ — complete preamble training sequence vector
 $\mathbf{P}$ — preamble correlation matrix
 $p(t)$ — transmit filter pulse
 $P(x)$ — probability of x
 $P(x|y)$ — probability of x given y
 $p_{i,j}$ — (i, j) -th element of the preamble correlation matrix $\mathbf{P}$
 p_k — k -th preamble training symbol
 $\mathbf{p}_k$ — k -th partial preamble training sequence vector
 $P_k(\cdot)$ — k -th probability function
 $\mathbf{Q}$ — inverse of matrix $\mathbf{R}^*$
 $Q(x)$ — Q -function; area under the normal distribution from x to infinity
 $q_{i,j}$ — (i, j) -th element of the matrix $\mathbf{Q}$

$\mathbf{r}$ — received signal vector
 $r(t)$ — received signal
 $\tilde{r}(t)$ — received signal without additive noise
 $\mathbf{R}$ — channel autocorrelation matrix
 $\text{Re}[\cdot]$ — real part of a complex term
 $R_f(\cdot)$ — general frequency correlation function
 $R_f(df)$ — spaced-frequency correlation function
 $R_f(df; dt)$ — spaced-frequency spaced-time correlation function
 r_i — i -th off-diagonal element of Toeplitz matrix $\mathbf{R}$
 $r_{i,j}$ — (i, j) -th element of matrix $\mathbf{R}$
 r_k — k -th received signal sample
 $\mathbf{r}_k$ — k -th vector of $\mathbf{R}$ matrix elements
 $R_t(\cdot)$ — general time correlation function
 $R_t(\tau)$ — multipath intensity profile
 $R_t(\tau; dt)$ — general multipath intensity profile
 $\mathbf{s}$ — symbol vector containing both training and data
 $s(t)$ — continuous time signal
 $S(f)$ — power spectrum
 $S(\tau; \lambda)$ — scattering function
 $S_f(\lambda)$ — Doppler power spectrum
 s_k — k -th signaling symbol
 $\mathbf{s}_k$ — k -th state vector
 $\text{sse}(\cdot)$ — sum of squared errors function
 $\text{sum}_k(\cdot)$ — k -th sum function for MLBE
 s_z — scaled sum of the elements of vector $\mathbf{z}$
 t — continuous time variable
 T — symbol period
 T_m — multipath spread of a channel
 tmse — total mean squared error

T_{obs} — observation time for block detection
 $tr(.)$ — trace of a matrix operator
 U — upper triangular matrix
 $u(t)$ — complex baseband equivalent continuous-time signal
 $U(f)$ — frequency domain equivalent of $u(t)$
 $u_{i,j}$ — (i, j) -th element of matrix U
 v — velocity
 V — inverse of the conjugate preamble correlation matrix, P
 $V(.)$ — metric update function for MLBE
 $v_{i,j}$ — (i, j) -th element of matrix V
 v_k — variance of the k -th estimated symbol
 w — vector of filtered noise samples
 $\tilde{w}$ — vector of whitened filtered noise samples
 W — bandwidth
 W — autocorrelation matrix of w
 w_k — k -th element of vector w
 x — intermediate step vector for Cholesky approach
 X_1 — first matrix component of $mse[\hat{b}]$ matrix
 X_2 — second matrix component of $mse[\hat{b}]$ matrix
 $x(t)$ — received bandpass signal
 x_k — k -th element of vector x
 z — vector of (partially) matched filter output samples
 z_k — k -th element of the vector z
 α — squared distance range parameter for MLBE
 $\delta(t)$ — Dirac delta, or unit impulse function
 ϵ — error event for MLBE
 γ_b — average E_b/N_0 ratio
 γ_l — average E_b/N_0 ratio for the l -th path

γ_p — average E_b/N_0 ratio per path
 λ — Doppler frequency; also used for wavelength
 μ — step size parameter for SG channel tracking algorithm
 π — constant 3.141592654....
 ρ — throughput efficiency
 σ — standard deviation
 σ_n — noise standard deviation
 τ — time delay variable
 $\tau_n(t)$ — propagation delay for the n-th path signal
 $|\cdot|$ — absolute value, or magnitude of a complex term
 $\widetilde{(\cdot)}$ — decision based on $(\cdot)$, unless otherwise specified
 $\widehat{(\cdot)}$ — unquantized estimate of $(\cdot)$, unless otherwise specified
 $\lfloor \cdot \rfloor$ — floor function
 $[\cdot]^*$ — complex conjugate
 $[\cdot]^{-1}$ — matrix inverse
 $[\cdot]^r$ — vector or matrix with order of elements reversed
 $[\cdot]^t$ — vector or matrix transpose
 $[\cdot]_{(\cdot)}$ — relevant block size denoted by bracketed subscript
 $\dot{[\cdot]}$ — vector with one edge elements removed
 $\ddot{[\cdot]}$ — vector with both edge elements removed
 $\in$ — element in
 $\int_a^b$ — integral from a to b
 $\prod_a^b$ — product from a to b
 $\sum_a^b$ — summation from a to b
 $\#adds$ — number of complex adds
 $\#comp$ — number of complex comparisons
 $\#mult$ — number of complex multiplies
 $\#recip$ — number of complex reciprocals

Chapter 1

INTRODUCTION

The design objective for most digital communication systems has been to achieve the highest possible transmission rate at a specified reliability, usually given in terms of probability of error. A fundamental problem in trying to achieve high transmission rates is the time dispersion introduced by the channel. When time dispersion causes the energy in adjacent symbols to overlap, the result is what is generally denoted as intersymbol interference (ISI) [Luc68, Pro83, Woz65].

All physical systems exhibit some form of time dispersion, which for digital data transmission results in intersymbol interference. Many examples of time dispersion can be found. In telephone channels, time dispersion is attributed to the restricted bandwidth and imperfect frequency response characteristics of the channel [Pro83]. An ideal channel has a constant amplitude and linear phase across its frequency band. On radio channels, such as short-wave ionospheric propagation or high frequency (HF), mobile radio, tropospheric scatter, and indoor wireless or portable, much of the time dispersion, and hence intersymbol interference, is due to multiple propagation paths with different path delays. The approximate signaling rates, which can result in significant intersymbol interference for these channels, are as follows:

1. greater than 50 sps for HF channels [Bra87, Pro83, Ste87, Wat70],
2. greater than 20,000 sps for mobile radio channels [Bul87, DAv87, Lin87, Lor87, Mas87, Mor87, Ohr88, Rai86, Ste87, Udd87, Udd88],
3. greater than 200,000 sps for tropospheric scatter channels [Pro83, Ste87], and
4. greater than 500,000 sps for indoor wireless and portable [Bul87, Bul89, Ohr88, Sal87].

For many channels, it is not a question of whether significant time dispersion occurs, but at what signaling rate significant time dispersion occurs. It should be pointed out that the more bandwidth efficient the signaling format, the less time-dispersion it will take to cause significant intersymbol interference.

Usually the number of paths and path delays vary with time, giving rise to time varying impulse and frequency responses. Such channels are usually denoted as time-varying multipath channels, and are characterized in terms of a scattering function. The scattering function is a two-dimensional representation of the average received signal power as a function of relative time delay and frequency [Pro75, Pro83, Ste87]. Each type of time-varying multipath channel has its own unique scattering function characteristics. Although the signaling techniques and detectors to be investigated can be applied to all types of fading multipath channels, mobile radio channels will be emphasized in the discussion and examples.

Intersymbol interference has long been recognized as a serious problem when trying to communicate at high transmission rates. Much research has been performed on the problem of mitigating the effects of intersymbol interference. Many of the various techniques and algorithms which have been developed will be reviewed in chapter 2.0.

There are a number of other important aspects of fading time-dispersive channels which need to be mentioned. First, multipath radio channels can exhibit fairly rapid and large variations in their propagation characteristics, and some means of measuring and tracking the channel response is necessary. Such operations are usually performed at the receiver, requiring the receiver to be adaptive in nature.

Second, fading time-dispersive channels can offer improved performance over flat fading (non-time-dispersive) channels due to the inherent diversity available from receiving the signal over multiple paths with different delays. The length of the time-dispersion or delay-spread, measured in symbol periods, is directly related to the signaling rate. The faster the signaling rate the longer the delay-spread and the greater the potential improvement in performance due to inherent diversity. Of course, the longer the delay-spread, the greater the number of significant ISI samples, and the more complex the receiver must be to take advantage of the inherent diversity. The usual trade-off between complexity and performance is more complicated for fading time-dispersive channels, being strongly dependent on the selected signaling rate. The investigated signaling and detection schemes specifically take these aspects into account.

1.1 Outline

The background and literature survey of Chapter 2.0 consists of three major parts. First, the terminology used to describe, and the general nature of, fading time-dispersive channels is reviewed. Second, the specific characteristics of some multipath channels are investigated, with the most attention being paid to the mobile radio channel. Third, the equalization techniques appropriate for mitigating the effects of severe intersymbol interference are investigated. This includes a discussion of adaptive equalization techniques, but is not the main emphasis of the survey. Adaptation is very important for the proposed detection techniques, and is performed indirectly through channel estimation and tracking.

Chapter 3.0 presents the signaling format and proposed detection techniques. The approach taken involves short alternating blocks of data and training symbols. The use of training symbols, to aid in channel estimation and tracking, is not uncommon (e.g [Dav86, Dav88, Ele87, Hsu80, Per82]). The proposed techniques also incorporate the knowledge of the training symbols directly into the detection process. As a result, the detection process is directly tied to the short-block signaling format. Linear and decision directed short-block equalization techniques were first proposed for discrete symbol-spaced HF channel models in [Hsu85]. In contrast to the short-block detection techniques in [Hsu85], for the detectors investigated in this study it is not necessary that the data and training block lengths be at least as long as the channel response length. This results in more practical, reduced complexity algorithms, with higher throughput efficiency. In addition, the detectors investigated in this study are not restricted to symbol-spaced processing, but are extended to include fractionally-spaced or even continuous time processing. A short-block maximum likelihood technique is also presented. The three short-block detection schemes investigated are denoted as:

1. Linear block estimation (LBE),
2. Decision feedback block estimation (DFBE), and
3. Maximum likelihood block estimation (MLBE).

Chapter 4.0 presents performance results based on a theoretical error rate analysis for these three block detectors. Given a specific channel response, it is shown that the performance can be theoretically approximated and/or bounded for each of the

detectors. The theoretical performance for a conventional linear equalizer (LE) and decision feedback equalizer (DFE) is also provided for comparison. A Monte Carlo averaging technique is used to approximate the performance of all the detectors for some simple fading channel models.

Chapter 5.0 addresses the problems of channel estimation and tracking. A least sum of squared errors (LSSE) channel estimation algorithm is presented for estimating an initial channel estimate from a known training sequence. Optimum training sequences are presented for different channel and training sequence lengths. The effect of channel estimation errors on the performance of the detectors is also investigated. A simple approximation for the expected degradation with channel estimation errors is derived. The stochastic gradient (SG) algorithm is also reviewed for channel tracking purposes.

Chapter 6.0 presents extensive simulation results. Binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulation formats are considered. The bit error rate performance is presented for some general multipath fading channels, and for channels specifically tailored to model a mobile radio environment. Results are presented for both symbol-spaced and fractionally-spaced channel models.

Chapter 7.0 contains the summary, conclusions, and suggestions for further research.

1.2 Summary of Thesis Contributions

This thesis makes the following contributions:

1. The extension of previous short-block detector derivations to handle continuous-time and fractionally-spaced signaling and channel models. Previous LBE and DFBE type short-block detectors were derived for symbol-spaced signaling and channel models [Hsu85]. A whitened-matched filter is required to optimally interface these detectors with continuous-time signals. It is shown that the new continuous-time and fractionally-spaced derivations only require matched filtering, as opposed to the more complicated whitened-matched filtering (Sections 3.3 and 3.4.)
2. The removal of the constraint that the number of symbols per training block be greater than or equal to the memory of the channel. This constraint was previously imposed for both the LBE and DFBE type detectors, to ensure that the channel correlation matrices involved in the detection process are Toeplitz. This was necessary because a

Levinson-based approach was used. New LBE and DFBE detection algorithms have been derived. The new algorithms use a Cholesky factorization approach, and can handle the more general non-Toeplitz Hermitian matrices. The new LBE and DFBE algorithms provide a significant reduction in complexity. Generally, this is because much shorter data and training blocks can be used to provide the same throughput efficiency. In fact, even for equal length data blocks, the new Cholesky-based DFBE algorithm is more efficient than the previous Levinson-based DFBE algorithm. Thus, the new Cholesky-based DFBE algorithm can also provide much higher throughput efficiencies, by reducing only the number of symbols per training block, with no penalty in complexity. (Sections 3.3.2 and 3.4.2.)

3. The complexity of the previous Levinson-based DFBE detector was also reduced, by providing a more efficient stage update procedure. The new Cholesky-based DFBE algorithm is still more efficient. (Section 3.4.1.)
4. A maximum likelihood block estimator is also presented. The derivation is similar to that presented in the literature for maximum likelihood sequence estimation (MLSE), using the modified Viterbi algorithm (MVA) [Ung74]. The derivation presented here is more general, in that non-Toeplitz channel correlation matrices are also considered. For very short data blocks, the MVA is not required, only the expanding tree algorithm (ETA) used for initialization. The idiosyncrasies concerning startup and termination are addressed in detail, since they tend to dominate the complexity for short data blocks. It is shown that the short-block format can provide considerable savings in complexity, relative to conventional MLSE. It is also shown that the complexity for the non-Toeplitz case is essentially the same as that for the conventional Toeplitz case. (Section 3.5.)
5. Theoretical probability of error performance results are presented for the LBE, DFBE, and MLBE detectors. Given a specific channel response, it is shown that the probability of error can be theoretically approximated and/or bounded for each of the block detectors. Theoretically based probability of error curves are also presented for various time-dispersive Rayleigh fading channel models. A Monte Carlo averaging technique was used to obtain these results. (Sections 4.2, 4.4, 4.5, and 4.6.)
6. The theoretical performance of the LBE and DFBE detectors is compared to the

theoretical performance of the conventional linear equalizer (LE) and conventional decision feedback equalizer (DFE), for fading time dispersive channels. The theoretical performance for the LE and DFE detectors, for a given channel response, was evaluated using the methods in [Pro83, Qur85]. Monte Carlo averaging was used to obtain the results with fading. (Section 4.9.)

7. A least sum of squared errors (LSSE) channel estimation algorithm is presented. The LSSE algorithm is used to estimate an initial channel response from a short preamble training sequence. Optimum training sequences are found and tabulated for different channel and training sequence lengths. The quality of a training sequence, for a given channel length, is determined by the normalized signal to channel estimation error ratio (SER'), normalized with respect to the signal to noise ratio. A simple expression for SER' is obtained which only depends on the training sequence. Upper bounds on SER' are also derived. (Sections 5.1 through 5.5, and 5.11.)
8. A method of generating "perfect" preamble training sequences is given. A training sequence is said to be perfect if its associated preamble correlation matrix is perfectly diagonal. For these sequences, the upper bound on SER' is achieved, and no matrix calculations are required to implement the channel estimator. Some other "special" training sequences are also presented, which are not perfect, but provide for trivial implementations of the channel estimator. (Sections 5.6, 5.7, and 5.12)
9. A simple approximation has been derived for the expected degradation in performance with channel estimation errors. The approximation is valid for all three block detectors and involves only SER'. The simulation results, with and without channel estimation (without and with perfect knowledge of the channel, respectively), confirm the accuracy of this approximation for high signal to noise ratios. This approximation is useful for determining the appropriate preamble training sequence length to use for different applications. (Sections 5.3, 5.9 and 5.10.)
10. Simulated bit error rate results are presented for all three block detectors, and their performance is compared to the theoretically based probability of error results. Simulation results are presented for both symbol-spaced and fractionally-spaced channel models. (Chapter 6.)

Chapter 2

BACKGROUND AND LITERATURE SURVEY

Fading multipath channels are characterized as having randomly time-varying impulse responses. This representation can be used to model the signal transmission over many radio channels.

Example fading multipath channels are [Pro83, Sch66, Ste87]:

1. shortwave ionospheric (or ionospheric skywave): 3 to 30 MHz (HF);
2. ionospheric forward scatter: 30 to 300 MHz (VHF);
3. tropospheric scatter (beyond-the-horizon): 0.3 to 3 GHz (UHF) and 3 to 30 GHz (SHF);
4. mobile radio: 800 to 900 MHz;
5. indoor wireless and portable.

The background material and literature survey has been divided into three sections. The first section reviews the general nature of fading multipath channels, and the functions and parameters used to describe them. The second section gives a more detailed description of the specific multipath channels listed above, with the most emphasis given to the mobile radio channel. The final section reviews the methods or algorithms that have been devised to handle severe intersymbol interference (ISI). Again, specific attention is paid to the work on mobile radio channels.

2.1 Fading Multipath Channels - General

2.1.1 Channel Characterization

The material presented in this section is well known, and draws heavily on the presentations of Proakis and Stein [Pro83, Ste87]. It is important to review this material, as a proper understanding of the terminology is required in order to describe the characteristic properties of specific fading multipath channels.

Figure 2.1 Illustration of time-varying multipath channel responses.

If a short pulse is transmitted over a time-varying multipath channel, the received signal might appear as a train of pulses, as illustrated in Figure 2.1. Hence, one characteristic of a multipath channel is the time spread introduced in the received signal. A second characteristic is the variation of the multipath with time. The variations with time include changes in the size of the individual pulses, changes in the relative delays among the pulses, and changes in the number of pulses observed. For this reason, time-varying multipath channels are usually characterized statistically.

Let the transmitted signal be represented by

$$s(t) = \text{Re} \left[u(t) e^{j2\pi f_c t} \right] \quad (2.1)$$

Assuming multiple time-varying propagation paths, the received bandpass signal can be expressed in the form

$$x(t) = \sum_n a_n(t) s(t - \tau_n(t)) \quad (2.2)$$

where $a_n(t)$ is the attenuation factor for the signal received on the n -th path and $\tau_n(t)$ is the propagation delay for the n -th path. Substituting (2.1) into (2.2) yields the result

$$x(t) = \text{Re} \left[\left\{ \sum_n a_n(t) e^{-j2\pi f_c \tau_n(t)} u(t - \tau_n(t)) \right\} e^{j2\pi f_c t} \right] \quad (2.3)$$

From (2.3) it is observed that the equivalent low-pass received signal is

$$r(t) = \sum_n a_n(t) e^{-j2\pi f_c \tau_n(t)} u(t - \tau_n(t)) \quad (2.4)$$

and that the equivalent low-pass channel is described by the time-variant impulse response

$$c(\tau; t) = \sum_n a_n(t) e^{-j2\pi f_c \tau_n(t)} \delta(t - \tau_n(t)) \quad (2.5)$$

When the channel can be considered to consist of a continuum of multipath components, the received signal $x(t)$ is expressed in the integral form

$$x(t) = \int_{-\infty}^{\infty} a(\tau; t) s(t - \tau) d\tau \quad (2.6)$$

where $a(\tau; t)$ denotes the attenuation of the signal components at delay τ and at time instant t . Now, substituting (2.1) into (2.6) yields

$$x(t) = \text{Re} \left[\left\{ \int_{-\infty}^{\infty} a(\tau; t) e^{-j2\pi f_c \tau} u(t - \tau) d\tau \right\} e^{j2\pi f_c t} \right] \quad (2.7)$$

Recognizing the convolution integral in (2.7), it follows that the equivalent low-pass time-variant impulse response is given by

$$c(\tau; t) = a(\tau; t) e^{-j2\pi f_c \tau} \quad (2.8)$$

where $c(\tau; t)$ represents the response of the channel at time t due to an impulse applied at time $t - \tau$. Thus (2.5) and (2.8) are the appropriate definitions of the equivalent low-pass impulse response for discrete and continuous multipath, respectively.

When the physical medium consists of a large number of closely time-spaced propagating paths, it is often appropriate to model the impulse response, $c(\tau; t)$, as a complex-valued Gaussian process. When the mean is zero, as is often the case, the envelope $|c(\tau; t)|$ at any delay τ and time t is Rayleigh-distributed, and the channel is said to exhibit *Rayleigh fading*. When the medium includes a line-of-sight path or fixed scatterers or signal reflectors, $c(\tau; t)$ can no longer be modeled as having zero mean at each delay τ . In this case, the envelope $|c(\tau; t)|$ can have a Rice distribution for certain values of delay τ , and the channel is said to exhibit *Ricean fading*. Only the Rayleigh-distributed envelope is considered in this work. This model is widely accepted for many fading multipath channels, as well as representing worst case fading statistics for others. If signaling and detection schemes work well for this channel, they generally yield better performance on other channels with non-fading components.

Fading multipath channels are often characterized by a number of useful correlation functions and power spectral density functions. These will now be reviewed.

The equivalent low-pass impulse response $c(\tau; t)$ is assumed to be a complex valued, zero-mean, Gaussian random process in the t variable, and wide-sense-stationary. The autocorrelation function of $c(\tau; t)$ is then given by

$$\begin{aligned} R_t(\tau_1, \tau_2; dt) &= E[c^*(\tau_1; t) c(\tau_2; t + dt)] \\ &= R_t(\tau; dt) \delta(\tau_1 - \tau_2) \end{aligned} \quad (2.9)$$

Figure 2.2 Typical multipath intensity profile.

where it has been assumed that the scattering at two different delays is uncorrelated, and τ and $\tau + dt$ represent either τ_1 or τ_2 . Letting $dt = 0$, the resulting autocorrelation function $R_t(\tau; 0) = R_t(\tau)$ is simply the average power output of the channel as a function of time delay τ . Thus, $R_t(\tau)$ is called the *multipath intensity profile* of the channel. A typical multipath intensity profile is shown in Figure 2.2. The range over which $R_t(\tau)$ is essentially non-zero is called the *multipath spread*, T_m , of the channel. The shape and duration of the multipath intensity profile play an important role in determining the performance of the signaling and detection schemes to be investigated.

The time-varying multipath channel can also be characterized in the frequency domain. Taking the Fourier transform of $c(\tau; t)$ with respect to the delay τ , gives

$$\begin{aligned} C(f; t) &= F[c(\tau; t)] \\ &= \int_{-\infty}^{\infty} c(\tau; t) e^{-j2\pi f\tau} d\tau \end{aligned} \quad (2.10)$$

which is the time-varying transfer function of the channel. It is straightforward to show that (see[Pro83])

$$\begin{aligned} R_f(f_1, f_2; dt) &= E[C^*(f_1; t) C(f_2; t + dt)] \\ &= F[R_t(\tau; dt)] \\ &= R_f(df; dt) \end{aligned} \quad (2.11)$$

where $df = f_2 - f_1$. In (2.11), $R_f(df; dt)$ is the Fourier transform of the general multipath intensity profile, $R_t(\tau; dt)$, and, because of the uncorrelated scattering assumption, is a function of only the frequency difference df . This function is called the spaced-frequency spaced-time correlation function of the channel.

Setting $dt = 0$ in (2.11), with $R_f(df; 0) = R_f(df)$ and $R_t(\tau; 0) = R_t(\tau)$, the transform relationship is simply

$$R_f(df) = F[R_t(\tau)] \quad (2.12)$$

This relationship is illustrated in Figure 2.3. Autocorrelation function $R_f(df)$ provides a measure of the frequency coherence of the channel. Thus, the reciprocal of the multipath spread, T_m , is a measure of the *coherence bandwidth* of the channel. That is,

$$(df)_c \simeq \frac{1}{T_m} \quad (2.13)$$

Figure 2.3 Relationship between the multipath intensity profile and the spaced frequency correlation function.

where $(df)_c$ denotes the coherence bandwidth. When $(df)_c$ is small relative to the bandwidth of the transmitted signal, the channel is said to be *frequency-selective*, and severe signal distortion occurs. When $(df)_c$ is large relative to the signal bandwidth, the channel is said to be *flat fading* or *frequency-nonselective*.

The time variations of the channel are also very important, because they influence the adaptive requirements and performance of any adaptive detection scheme. The time variations in the channel are evidenced as Doppler shift and/or broadening of the spectral components. The Doppler effect is related to the time variations of the channel, and can be investigated through the function $S_f(df; \lambda)$, the Fourier transform of $R_f(df; dt)$ with respect to the variable dt . That is

$$S_f(df; \lambda) = F[R_f(df; dt)] \quad (2.14)$$

With $df = 0$, $S_f(0; \lambda) = S_f(\lambda)$, and $R_f(0; dt) = R_f(dt)$, we obtain

$$S_f(\lambda) = F[R_f(dt)] \quad (2.15)$$

The function $S_f(\lambda)$ is called the *Doppler power spectrum* of the channel, as it gives the signal intensity as a function of Doppler frequency λ . If the channel is time-invariant, then a signal will not experience any spectral broadening or Doppler shift.

The range over which $S_f(\lambda)$ is essentially non-zero is called the *Doppler spread*, B_d , of the channel. The reciprocal of B_d is a measure of the coherence time of the channel. That is,

$$(dt)_c \simeq \frac{1}{B_d} \quad (2.16)$$

where $(dt)_c$ denotes the *coherence time*. A channel which changes slowly has a large coherence time and, equivalently, a narrow Doppler spread. This condition is highly desirable for adaptive receivers. Figure 2.4 illustrates the relationship between $R_f(dt)$ and $S_f(\lambda)$.

Finally, a particularly revealing function, $S(\tau; \lambda)$, is defined as the Fourier transform of $R_t(\tau; dt)$ with respect to the dt variable. That is,

$$S(\tau; \lambda) = F[R_t(\tau; dt)] \quad (2.17)$$

and is called the *scattering function* of the channel. It provides a measure of the average power output of the channel as a function of time delay τ and Doppler shift λ .

(a) Spaced-time correlation function

(b) Doppler power spectrum

Figure 2.4 Relationship between the spaced-time correlation function and the Doppler power spectrum.

2.1.2 Signaling Considerations

Let $u(t)$ be the equivalent low-pass signal transmitted over the channel and let $U(f)$ denote its frequency content. Then the equivalent low-pass received signal, without noise, is given by

$$r(t) = \int c(\tau; t) u(t - \tau) d\tau \quad (2.18)$$

or

$$r(t) = F^{-1} [C(f; t) U(f)] \quad (2.19)$$

Suppose we are transmitting digital information at a rate $1/T$, where T is the signaling interval. Clearly $C(f; t)$ distorts $U(f)$. If the bandwidth, W , of $U(f)$ is greater than the coherence bandwidth, then $U(f)$ suffers frequency-selective distortion. In addition, $U(f)$ suffers fading distortion due to the time variations of $C(f; t)$. It is important to realize the difference between these two types of distortion. Frequency selectivity depends on the multipath spread, T_m , or, equivalently, on the coherence bandwidth, $(df)_c$. The fading depends on the time variations of the channel, which is characterized by the coherence time, $(dt)_c$, or, equivalently, by the Doppler spread, B_d .

Although both kinds of distortion tend to make successful communications more difficult to achieve, multipath spread, causing frequency selectivity, can actually improve performance. If the fading process is frequency selective, rather than flat, then it is clear that a type of inherent diversity is available to the receiver. Of course, to gain any advantage from this inherent diversity the receiver must be designed to take proper advantage of it. On most multipath fading channels, frequency selectivity can usually be achieved by increasing the signaling rate, which in turn broadens the signal bandwidth. Consequently, whether a fading digital communication system possesses inherent diversity, due to frequency-selective fading, depends not only on the propagating medium, but on the designing engineers and regulatory bodies.

2.2 Fading Multipath Channels - Specific

This section describes some of the specific characteristics and parameters of several multipath fading channels. Most of the emphasis, however, is given to the mobile radio channel.

2.2.1 Ionospheric Skywave (HF)

The ionosphere, which consists of several layers of ionized gas, can provide long distance communication in the frequency range 3 to 30 MHz. A transmitted signal can be refracted by the various layers of the ionosphere. Within each layer there can be high and low rays, and two magnetoionic components for each ray. All these propagation effects, combined with multiple hops between the earth and the ionosphere, result in many possible modes of propagation. Typical channel parameters are described in [Ste87], and are summarized below. (See also [Bra75, Bra87, Fre81, Gup81, Mas82, Pic75, Wat70])

The natural inhomogeneous nature of the layers causes typically 20 to 40 usec of multipath spread. The high and low rays often result in paths spread by 100 to 200 usec. A maximum multipath spread on the order of 100 usec is not unusual for a single-hop link between 800 and 2000 km. The Doppler spread is often under 0.01 Hz for such medium range links, resulting in very slow fading, with coherence times exceeding 100 sec.

The more notorious multipath spreads for HF, usually in the 1 to 2 msec range, occur either for short ranges (under about 800 km), due to repeated earth-ionosphere reflections, or over long paths (over 2000 km) requiring multiple hops. Occasionally even longer multipath spreads, out to 5 msec or more, can be encountered for very long paths. The scattering function is often multiple-lobed in both time-delay and Doppler frequency. This is because the different reflection points often introduce different Doppler shifts. The overall Doppler spread can range up to 1 or 2 Hz. Most HF modems are designed for long skywave paths, assuming multipath spreads out to about 3 msec and Doppler spreads within 2 Hz. However, multipath spreads from 5 to 10 msec and Doppler spreads from 10 to 20 Hz have been observed for paths influenced by the auroral belt.

2.2.2 Tropospheric Scatter

Troposcatter links experience a more continuous distribution of delays [Ste87]. The most common links range from just over the horizon to 200 km long. The multipath spread is usually not greater than 0.1 to 0.2 usec. Multipath spreads of several microseconds have been observed on some longer experimental links. At microwave frequencies, the Doppler spread ranges from 0.1 to 10 Hz. Higher fade rates of 100

Hz or more occur occasionally due to aircraft echoes. For an example tropospheric scattering function see [Pro83].

The multipath intensity profile tends to peak early for tropospheric scatter channels, as it does for most other multipath channels. As will be seen later, this particular characteristic of multipath channels is highly desirable, and is taken advantage of by the proposed signaling and detection schemes.

2.2.3 Mobile Radio

Mobile radio systems suffer from terrain multipath, which can be considerable, especially in urban areas with built-up structures and in mountainous regions. Assuming a stationary transmitter, the result is a kind of stochastic standing wave pattern with a correlation distance on the order of a single wavelength. The motion of a receiver through this pattern results in a fading received signal, often exhibiting Rayleigh fading characteristics [Ste87].

In propagation measurements performed at 1 GHz [Nei77], it was found that multipath spreads can be up to 10 usec with 1 to 3 usec most common, and with 10 or more significant resolvable paths often found. Similar results have been observed or quoted by other authors [Cos86, Cox75, Dam87, DAv87, Gud87, Gup85, Lin87, Lor87, Mor87, Rai86, Tur80]. It is often found that rural and suburban areas tend to have shorter multipath spreads, often under 1 usec. An exception is the 20 usec spreads sometimes observed in the Alps. The one thing that is certain for mobile radio channels is the variability of multipath spread from area to area.

A simple two-path model is often selected when evaluating the performance of candidate detection schemes, e.g. [Rai86, Lin87, Gud87]. This model is useful for showing some fundamental performance trends, but cannot give a clear picture of performance for typical mobile channels. For example, selecting the relative delay of the second path to be one or two symbols can yield significant diversity gain while employing a low complexity maximum likelihood receiver. In reality, however, the receiver must also be able to handle the infrequent longer delay spreads, if significant diversity gain is to be achieved on average.

Figure 2.5 Example echo patterns, redrawn from [Cos86, DAv87, Mor87].

(a) Sequence of impulse responses measured in a Dutch suburban area [Lor87, Van86].

(b) Impulse response measured at the edge of a Swiss suburban area [Deb86, Lor87].

Figure 2.6 Example impulse responses [Deb86, Lor87, Van86].

(a) Approximation from [Lor87] of measurements made in New York city [Cox75].

(b) Approximation from [Lor87] of measurements made on the outskirts of Berne [Zog87].

Figure 2.7 Example multipath intensity profiles [Cox75, Lor87, Zog87].

Some typical echo patterns have been defined in [Cos86], for the specific purpose of comparing system performances. Of these, Figure 2.5 shows the specific echo patterns selected for use in references [DAv87] and [Mor87]. These echo patterns illustrate the variety which can occur. Note that the delay of most echoes is within 3 usec, but that significant echoes with much longer delays are also included. This is consistent with the observations made above. Figure 2.6 shows the impulse responses measured in a Dutch suburban area and a hilly Swiss suburban area [Lor87, Van86, Deb86]. Figure 2.7 shows the average multipath intensity profiles calculated by Lorenz [Lor87] using measurements made in New York city [Cox75] and on the outskirts of Berne [Zog87]. Again, it is seen that the multipath intensity profiles tend to peak early, with most of their energy falling within the first 3 usec, on average. This is not to say that significant echoes don't occur beyond 3 usec, but they tend to be rare and their overall energy contribution is small on average.

The fade rate for terrain multipath is directly related to the vehicle speed. The Doppler shift of each reflection depends on the angle of arrival relative to the direction of travel. For a velocity of $v = 33$ m/sec (≈ 120 km/hr) and a carrier frequency of 900 MHz ($\lambda = .33$ m), the peak Doppler shift (v/λ) is about 100 Hz. A minimum coherence time (0.5 correlation coefficient) of about 1 msec is suggested in [Gup85], and is roughly consistent with these results. The Doppler spread, or equivalently the coherence time, plays an important role in determining the adaptive requirements of a detector. The adaptive requirements also depend on the signaling rate. For signaling rates below 50 Ksps a coherence time of 1 msec, less than 50 symbols, might be considered quite short. In contrast, at 1 Msps a 1 msec coherence time would correspond to 1000 symbols, and might be considered quite long.

2.2.4 Indoor Wireless/Portable

The indoor wireless or portable radio channel is similar to the mobile radio channel in that it also suffers from terrain multipath, but typically on a much smaller scale. One would expect much shorter delay spreads, which is indeed the case. The fading rate is also very slow because of the small Doppler shifts resulting from slow movements of the receiver and reflectors.

Recent measurements at 910 MHz [Bul87, Bul89] and 1.5 GHz [Sal87] indicate delay spreads over a range up to about 200 nsec and root-mean-square (RMS) values up to about 50 nsec. Measurements made at 850 MHz [Dev84], in a much larger office building than used in [Bul87, Bul89] and [Sal87], yielded RMS delay spreads up to 5 times longer. The longer delays usually occurred when the received signal power level was very low. (See also [Dev85, Dev86].) The channels were also observed to be very slowly time varying. Again, as expected, the multipath power profiles tend to peak early, but significant echoes are not uncommon. It is interesting to note that Bultitude [Bul87, Bul89] never observed non-minimum phase characteristics. For attenuation measurements within and around buildings see also [Ale82, Ale83, Cox83, Cox84, Hof82]. Observed distance power-law exponents range anywhere from 1.5 to 6.

2.3 Signaling and Detection Methods

There are three fundamentally different ways to access and share a common channel resource:

1. Frequency division multiple access (FDMA),
2. Time division multiple access (TDMA), and
3. Code division multiple access (CDMA).

Each of these approaches has its advantages and disadvantages. Hybrid schemes are often proposed to make use of the advantages of each. Most mobile radio systems in operation today use analogue FM signaling with FDMA access [Lin87]. Most systems proposed for the future, however, are digital and are well suited to take advantage of TDMA type access schemes. CDMA access is associated with spread spectrum techniques, and has also been proposed for digital radio applications.

2.3.1 Recent Digital Mobile Radio Proposals

Work in Europe is proceeding towards an operating Pan-European cellular system in the early 1990's. Various system solutions have been proposed, which incorporate one or more of the accessing techniques mentioned above. Many of the approaches are fundamentally different, and are often classified according to signaling approach and bandwidth as follows [Lin87, Lor87]:

1. Analogue FM – FDMA (e.g. 25 KHz bandwidth)
2. Analogue ACSSB – FDMA (e.g. 5 KHz bandwidth)
3. Digital FDMA (e.g. 25 KHz bandwidth)
4. Digital narrowband TDMA (e.g. 200-300 KHz bandwidth)
5. Digital wideband TDMA ($\sim$ 1 MHz bandwidth)
6. Spread spectrum CDMA ($>$ 1 MHz bandwidth)

Analogue FM is used today with 25 to 30 KHz channel spacing. Each carrier is modulated by the analogue signal of a single call, with the available bandwidth allocated to a number of carriers.

Digital FDMA is similar to analogue FM except that the modulating signal is a digital representation of the speech, often including some form of forward error correction (FEC) coding. (e.g. see [Udd85].) The bandwidth is still about the same (e.g. 25 KHz in Europe).

In narrowband TDMA systems, different users are allocated different time slots on the same carrier. Each user then transmits and receives short bursts of data packets. The modulation bandwidth is 200 to 300 KHz. These systems also require FDMA for switching between narrowband TDMA carriers. However, the channel bandwidth and spacing is much wider than for narrowband FDMA, and fewer carrier frequencies are required.

Wideband TDMA systems use the same principle as narrowband TDMA but with a higher burst data rate, more users per carrier, and thus a much wider bandwidth. Compared to narrowband TDMA, much more severe intersymbol interference must be handled, but the inherent diversity gain is greater. The coherence time (measured in symbol periods) is also much longer, making adaptation easier as well.

Another proposed technique is code division multiple access (CDMA). In CDMA the users are not separated by different time slots but by different modulation codes. These systems use a type of spread spectrum approach, with some properties similar to that of wideband TDMA. Performance usually degrades gracefully as new users are added. The spectral efficiency is a problem, however, and is often disappointing for full CDMA schemes. To maintain reasonable performance, it is often necessary to accept

bandwidth efficiencies of only 10 to 30 percent, or much less. (See [Kav85, Kav87, Och87, Pur87, Tur80].)

Amplitude companded single sideband (AC-SSB) has also been suggested. This type of system is more bandwidth efficient than the others because of the efficient analogue modulation technique employed. A major disadvantage, however, is that it is not digital. Further, the non-constant envelope nature of the signal, and the many narrowband frequency channels required, are also considered major disadvantages with respect to hardware costs [Lin87].

The signaling rate, and thus the bandwidth, for the narrowband TDMA approaches is partly determined by the complexity of the equalizer. It is interesting to note that pure wideband ($\sim >1$ MHz bandwidth) TDMA schemes have not been proposed. The wideband TDMA approaches which have been considered try to overcome the equalizer complexity problem by employing a hybrid of TDMA and spreading codes. Thus, the penalty for trying to improve the inherent diversity is often a significantly reduced bandwidth efficiency.

2.3.2 Narrowband TDMA versus Wideband TDMA

Digital signaling allows the use of TDMA technology. This permits the elimination of major analogue radio components such as the duplex filter for separating transmit and receive signals. Filter selectivity and frequency stability requirements can also be relaxed, reducing the cost and size of the equipment.

The advantage to each base station is even more significant, since each transceiver can handle several calls at once. In FDMA systems a combiner unit is often necessary to connect a number of transmitters to a single antenna. In the case of full TDMA, no combiner unit is required. In summary, some form of TDMA seems to be preferred over pure FDMA.

Given that TDMA technology is to be employed, it is less clear whether it should be a narrowband or a wideband system. Lindell [Lin87] argues for a narrowband system (e.g. 200 KHz) by making the following points:

1. "Time dispersion is handled with equalization techniques which do not require any spectrum expansion. In wideband systems spectrum wasting methods must be used which reduce capacity.
2. There is no need for excessive peak transmit power nor complex signal processing.
3. Coexistence with current analogue systems is feasible. That is not the case with wideband TDMA."

Concerning the first point, narrowband TDMA systems (200-300 Kbps) tend to introduce just enough time dispersion to be a nuisance. That is, significant delay spreads will occur often enough so that sophisticated equalization techniques will be required, but not often enough to provide a significant diversity gain overall. This is confirmed by the fairly poor performance in field trial tests, and the call for quite sophisticated forward error correction coding (FEC) with interleaving and frequency hopping [Lin87, Mas87, Stj87, Udd87]. The addition of FEC coding, with a fixed modulation format, certainly requires bandwidth expansion and reduces capacity. It also adds to the complexity of the system, mentioned in the second point.

Concerning the second point, the peak transmit power is a major concern, especially for hand-held portables. However, the wideband approaches may not necessarily demand a much higher peak transmit power in a fading time-dispersive environment. For example, if quadrupling the signaling rate, and thus the bandwidth (e.g. 250 KHz compared to 1 MHz), results in an inherent diversity gain on the order of 6 dB, then the peak power requirements would be about the same. Further, because of the shorter burst duration, the average transmit power could actually be less for the faster signaling scheme. In general, even if the peak transmit power needs to be slightly higher, the average transmit power could be less with an inherent diversity gain. As cell sizes get smaller, which seems to be the trend for the future, the peak transmit power requirement may become less of a concern for all signaling schemes.

In addition to the comment made above on FEC complexity, it is noted that the complexity of short-block equalization techniques can be made comparable to that of the equalization techniques proposed for narrowband TDMA schemes by making the blocks short enough and using block decision feedback (BDFB). This is discussed further in Chapter 3.

Concerning the third point, the ability to accommodate existing FM systems may be a serious transitional problem, but depends little on the bandwidth of the TDMA approach, provided the bandwidth of each TDMA carrier is still a small fraction of the overall bandwidth available. (e.g. 1 or 2 MHz per carrier is still small compared to 25 MHz.) Further, frequency hopping clearly requires that more bandwidth be made available than just that required for a single narrowband TDMA carrier.

Certainly, if equalization techniques with reasonable complexity can be used with higher signaling rates, to take advantage of the inherent diversity, and with spectral efficiency comparable to that of FEC coding, then they should be seriously considered.

2.3.3 Adaptive Equalizers for Multipath Channels

There are numerous equalization techniques for mitigating the effects of severe intersymbol interference (ISI), and many have been applied to, or at least proposed for, multipath fading channels. One of the benefits of using adaptive equalizers is that they can take advantage of the inherent diversity. Some techniques are better suited than others for certain specific applications, the appropriateness depending on many factors:

1. information and signaling rate,
2. available channel bandwidth or spectral efficiency,
3. multipath spread or coherence bandwidth,
4. Doppler spread (fading rate) or coherence time,
5. required error rate performance,
6. allowed complexity or cost.

One approach is to avoid significant time-dispersion, and the equalization problem, by using a signal bandwidth narrower than the coherence bandwidth of the channel. This is not always possible or even cost effective, as discussed earlier. The result is a flat fading channel, which provides no inherent diversity. Usually some form of diversity and/or coding will still be necessary to improve performance.

Equalizer design for time-dispersive multipath fading channels basically involves the following two challenges:

1. to handle multiple paths with comparable amplitudes resulting in deep nulls, potentially within the bandwidth of interest, and

2. the necessity for continual and sometimes rapid updating of the equalizer parameters due to changes in the channel.

The first of these challenges is addressed by the type of equalization. The second concerns the adaptive requirements, which depend not only on the type of equalization selected, but on the application. For some channels, such as ionospheric scatter, many symbols are available per fade cycle, allowing a large number of updates for data driven adaptation. At the other extreme, the adaptive equalizers developed for HF operation usually operate with 1200 to 2400 baud transmission, sometimes providing too few symbols for data-directed updates. Mobile radio channels tend to fall between these two cases, depending on the signaling rate.

As pointed out by Monsen [Mon80] and Turin [Tur80], some early multipath receivers were designed to take advantage of the explicit diversity structure of resolvable paths. For example, Price and Green [Pri58] are credited with developing the concept behind the well known *RAKE* receiver. (See also [Bel63, Cha79, Pri56, Tur56].) The goal is to optimally combine the contributions of different paths. Resolving the paths requires bandwidth spreading techniques which are usually very bandwidth inefficient. Typically these approaches also ignore intersymbol interference by ensuring that the signaling period is large compared to the duration of the multipath profile. Turin [Tur80] extended these results to include the effects of intersymbol interference at higher data rates (approaching 1 Mbps for mobile radio), but considerable bandwidth expansion is still employed (a spreading factor of about 100 or more). The effect of ISI is analysed but not directly compensated for in his work.

Equalization techniques originally developed for digital transmission over non-fading band-limited channels, such as telephone lines, have also been applied to the multipath radio channel [Mon71, Mon73, Mon74, Mon77, Mor78]. For most of these techniques the emphasis is on mitigating the effects of intersymbol interference. Even though the diversity combining properties of the receiver are usually only implicit, significant diversity gain has been clearly demonstrated [Mon73, Mon74, Mon77].

Loosely speaking, an "equalizer" is any signal processing device which is designed to deal with intersymbol interference (ISI). Equalizers usually fall into one of the following three classes [Pro83, Qur85]:

1. Linear equalizers;
2. Nonlinear decision directed equalizers;
3. Maximum likelihood sequence estimators (or approximations thereof).

The linear and nonlinear equalizers commonly employ tap delay line (TDL) filters with adjustable tap weights. A linear equalizer linearly combines a number of input samples. These equalizers have been widely employed to combat ISI in telephone channel applications [Luc68]. The linear equalizer can usually be improved by adding a feedback filter which operates on the decided data symbols. This equalizer is known as a decision-feedback equalizer (DFE) and is definitely the most popular of the nonlinear equalizer structures [Bel79, Dut74, Geo71, Mon71, Pri72, Sal73]. A potential problem with DFE equalization is the propagation of errors due to the feedback section. This can be a serious problem for fading channels with severe time-dispersion, and recovery schemes are often necessary for continuous transmission.

Maximum likelihood sequence estimators (MLSE) have also been derived [For72, Ung74] and usually involve some form of the Viterbi algorithm [For73, Hay75, Vit67]. Viterbi algorithm based MLSE's work by assigning a short pattern of source symbols, or state, to each ISI combination. The most likely sequence of source symbols is found by determining the most likely trajectory of states, or path, through a trellis of states representing all possible sequences. The Viterbi algorithm provides a very efficient recursive method for searching the trellis. The problem with MLSE detectors is that the complexity grows exponentially with the number of ISI samples, and thus with the multipath spread. In return for this complexity, MLSE detectors offer a smaller ISI penalty relative to other equalizer structures. Because of the complexity problem with MLSE detectors, much effort has been spent on developing reduced complexity approximations. These will not be discussed further here, but for examples see [And84, Due85, Due89, Eyu88, Fag86, Fal73, Mag75, Mes74, Sim86, Sim90, Ver74].

The other important part of the equalization problem is the adaptation. A common adaptation criterion is to reduce the mean or sum of the squared errors. Simple iterative solutions are possible because the mean squared error (MSE) is a quadratic function of the coefficients. The simplest, and most popular, equalizer adjustment method is commonly referred to as the least mean square (LMS) or stochastic gradient (SG) method. The

stochastic gradient label is appropriate because, instead of the true gradient, a noisy unbiased estimate of the gradient is used [Qur85]. The SG algorithm can be applied to many different equalizer structures. A channel estimator based on the SG algorithm is also described in [Pro83]. This is particularly useful for MLSE detection, which requires an estimate of the channel impulse response.

Other adaptation schemes have been proposed to speed up the convergence of adaptive equalizers over that obtained for the simple stochastic gradient approach. These include orthogonalized LMS algorithms [Cha72, Mue75], periodic or cyclic equalization [Mil83, Mue75, Qur77], and recursive least squares (RLS) algorithms. The RLS algorithms provide the solution to a numerical optimization problem, given the preceding data and associated channel outputs. A number of different RLS algorithms, with different performance/complexity tradeoffs have been developed, including Kalman [Git77, God74], fast Kalman [Fal78], and adaptive lattice [Sat79, Sat81].

The complexity and stability of the fast adapting algorithms is a major concern, but is outside the scope of this survey. Excellent survey papers on adaptive equalization [Qur85] and, more generally, adaptive filtering [Fal80] are available, as well as the following recent texts on adaptive filtering and adaptive signal processing [Cow85, Hay84, Hay86, Hon84, Wid85].

Intermittent training symbols are sometimes introduced, especially for fast fading HF channels (e.g. [Dav86, Dav88, Ele87, Hsu80, Per82]), to aid the adaptation algorithm, or even provide for re-initialization. When training symbols are available, knowledge of them is rarely used to aid the equalization process directly. That is, they are usually provided only to help in the adaptation process. This is because the equalization process is usually designed to work recursively, symbol-by-symbol, and independent of the location of the training symbols. Also, the equalization improvement is usually quite minimal unless a considerable number of training symbols are involved.

Recent exceptions, involving a form of short-block equalization, have been proposed for HF communications [Hsu85, Per87]. These schemes use short alternating blocks of data and training symbols. Each data block is detected as a block using matrix operations. The data blocks must be kept short (less than 30 symbols) for complexity reasons. These schemes require an estimate of the channel impulse response. This is considered an

advantage since channel response estimation is usually one of the simplest adaptation requirements of any equalization process. A simple stochastic gradient algorithm is often sufficient for tracking purposes, assuming uncorrelated data symbols. In fact, it has been shown that the tracking performance of the simple stochastic gradient algorithm is comparable to the more complex RLS algorithms, when applied to tracking the impulse response of a fading multipath channel [Ele86, McL88]. A limitation of these approaches is that the training blocks must be at least as long as the impulse response of the channel. For HF communications this suggests that the training blocks be about the same size as the data blocks. This limitation is removed for the detection techniques investigated in this study, improving the bandwidth efficiency. Even with 50% training, the bandwidth efficiency is still much better than most spread spectrum techniques, even with CDMA.

Finally, it is noted that fractionally spaced equalizers (FSE) can also be employed, and are well suited for equalizing time-dispersive channel responses [Git81, Ung76]. The higher sampling rate at the input allows an FSE to manipulate the spectrum in the roll-off regions. Thus, an FSE can, in effect, perform both matched filtering and symbol rate equalization. In conclusion, FSE equalization can yield improved performance, while at the same time eliminate the need for precise symbol timing.

Chapter 3

SHORT-BLOCK DETECTION TECHNIQUES

3.1 Introduction

This chapter presents three short-block detection techniques based on linear, nonlinear decision-directed, and maximum likelihood estimation principles. Short alternating blocks of data and training symbols are used. In contrast to the recursive symbol-by-symbol equalization approaches usually employed, each data block is detected as a unit. These schemes require an estimate of the channel impulse response. This is considered an advantage from the adaptation point of view since channel response estimation is one of the simplest adaptation requirements of any detection process.

The data detection schemes investigated use a type of least squares estimation technique for dealing with undesired intersymbol interference (ISI). Typical applications are for severe time-dispersive fading channels, such as high signaling rate HF channels (e.g. 2.4 Ksps), mobile radio channels (e.g. 1 Msps) [Ste87], and indoor wireless (e.g. 10 Msps) [Bul87]. The first two schemes are based on direct estimation of the data blocks, followed by decision quantization. The maximum likelihood approach involves a quantized search for the least squares solution. Training data is embedded into the transmission in the form of blocks of training symbols alternated with blocks of source data symbols. The training blocks are useful for both channel tracking and data block detection. They also help reduce error propagation effects, when different forms of decision feedback are employed. The appropriate length of each block depends on the channel parameters expected and the allowed complexity. Typical training and data block sizes might range from 2 to 20 symbols.

Linear and decision-directed short-block equalization techniques have been considered previously in [Hsu85] and [Per87], but the derivations are presented for discrete signal models only, with one sample per symbol period. As a consequence, either a whitened matched filter, as proposed by Fomey [For72], or simple, and less efficient, symbol rate sampling is assumed. In this work the corresponding detectors for continuous-time sig-

nals are derived. These two approaches are denoted as linear block estimation (LBE) and decision feedback block estimation (DFBE). In addition, maximum likelihood block estimation (MLBE) is also considered. The derivations show that these three continuous-time based detectors only require a matched filter, as opposed to the more complicated whitened matched filter. A fractionally spaced formulation is useful for simulation and implementation purposes. Such a formulation is easily deduced from the continuous-time analysis presented. Clarifying comments are made where appropriate.

Another limitation of the short-block detection schemes previously presented in the literature is that the training blocks must be at least as long as the effective memory of the overall channel response. This usually implies that the training blocks be about the same length as the data blocks, resulting in approximately 50% overhead. This high overhead is offset by the fact that higher data rates can be used to take advantage of the inherent diversity provided by time-dispersion, which helps to eliminate the need for powerful coding and interleaving. Even with this consideration, it is still desirable to reduce the overhead even further by eliminating the above mentioned restriction. The detectors derived in this study do allow the training blocks to be shorter than the channel memory, resulting in even greater throughput efficiency. This is partly accomplished by including a form of block decision feedback (BDFB). Another benefit is that the data blocks can also be shorter, which helps reduce the complexity, and makes the complexity less dependent on the channel length.

A channel estimation approach is desirable because channel estimation is generally easier and/or yields faster convergence than similar algorithms for linear and decision feedback equalization techniques. In fact, it has been shown that the tracking performance of the simple stochastic gradient (SG) algorithm is comparable to the more complex recursive least squares (RLS) algorithms, when applied to tracking the impulse response of a fading multipath channel [Ele86] [McL87]. Channel estimation is also directly applicable to maximum likelihood sequence estimation (MLSE) [Ung74], a variation of which is also considered. For simplicity, perfect knowledge of the overall channel response is assumed initially. The issue of data estimation errors due to measurement errors in estimating the channel response is considered in the performance analysis of Chapter 5, and the simulation results of Chapter 6.

One of the more appropriate modulation techniques is single-tone M-ary phase shift keying (MPSK). Results are presented for both BPSK and QPSK signaling. Data rates higher than 1 bps/Hz can easily be achieved by selecting appropriate values for M, and the data and training block lengths.

Section 3.2 describes the signaling and detection format. The linear block estimator (LBE), decision-feedback block estimator (DFBE), and maximum likelihood block estimator (MLBE) are developed in Sections 3.3, 3.4, and 3.5 respectively.

3.2 Signaling and Detection Format

This section lays the groundwork for the derivation of the three continuous-time short block detectors. It is divided into the following three parts:

1. applicable modulation techniques,
2. signaling format, and
3. detection format.

3.2.1 Applicable Modulation Techniques

Constant envelope (e.g. MSK, GMSK, or CPM in general), or close to constant envelope techniques (e.g. MPSK or offset QPSK), are usually favored for the transmission of digital data on mobile radio and HF channels. The detection techniques to be described, with the exception of MLBE, are best suited to linear modulation techniques. Thus the analysis is restricted to linear modulation techniques, such as MSK, MPSK, and offset QPSK.

All signals and filters are written in complex baseband equivalent form. It is assumed that a signaling sequence $\{s_k\}$ modulates a basic transmit filter pulse $p(t)$ at a symbol rate $1/T$. The overall transmitted signal is

$$s(t) = \sum_k s_k p(t - kT) \quad (3.1)$$

where T is the symbol period. The signaling symbols $\{s_k\}$ may be complex-valued in general. It is usually convenient to assume that these symbols are statistically independent, with zero mean and unit variance. Although this formulation can represent many linear modulation schemes, performance results are presented only for BPSK and QPSK signaling.

3.2.2 Signaling Format

The short-block detection schemes require that each block of unknown source data be transmitted between two blocks of known training symbols. To accomplish this the data is transmitted in contiguous signaling blocks, with each signaling block consisting of one source data block and one training block. Thus the transmission consists of alternating data and training blocks. The various kinds of blocks are depicted in Figure 3.1. A detection block consists of three subblocks: one training block with N_t known symbols $\{a_0, \dots, a_{N_t-1}\}$, associated with the end of the previous signaling block; one block with N unknown data symbols $\{b_0, \dots, b_{N-1}\}$; and another training block with N_t known symbols $\{a_{N_t}, \dots, a_{2N_t-1}\}$. The a 's and b 's are all signaling symbols (i.e. s_k).

The throughput efficiency, ρ , is defined as

$$\rho = \frac{N}{N + N_t} \times 100\% \quad (3.2)$$

As an example for mobile radio, if we use QPSK modulation with a source data rate of 1.5 Mbps and select block sizes of $N = 9$ and $N_t = 3$, then the throughput efficiency is $\rho = 75\%$, and the signaling rate is 1.0 Msps. With this signaling rate and fraction of training, it should be no problem to successfully track a fast fading (e.g. 100 Hz one-sided Doppler fading bandwidth) mobile radio channel, even with a simple stochastic gradient algorithm.

For TDMA signaling, each TDMA frame would contain a sequence of signaling blocks, as defined above. In addition, each frame would contain other segments such as an initial preamble and guard times. The training block required prior to the first data block can be part of the known preamble. As with any TDMA scheme, these additional segments will also influence the throughput efficiency. The exact make-up of a TDMA frame is not of interest at this point, only the above definitions of the signaling and detection blocks.

3.2.3 Detection Format

The channel is modeled as a continuous time channel with overall impulse response $f(t)$, which includes pulse shaping and any transmit filtering. The transmit signal is

Signaling and Detection Block Format:

TDMA Frame Format:

Figure 3.1 Signaling block format and definitions

then modeled using

$$s(t) = \sum_k s_k \delta(t - kT) \quad (3.3)$$

where the $\{s_k\}$ are complex transmit symbols and $\delta(t)$ is the standard Dirac delta or unit impulse function. This continuous time model is easily generalized to a discrete time channel model with one or more samples per symbol period, T . The overall impulse response $f(t)$ is assumed to have a finite duration of $(L + 1)T$. The memory of the channel is defined to be L symbol periods long.

The channel will typically vary slowly with time, but it is assumed that the channel is known and fixed for the duration of a detection block. For detecting a data block starting at time $t = 0$, the observed portion of the received signal, without additive noise, is given by

$$\begin{aligned} \tilde{r}(t) &= s(t) * f(t) \quad , \quad t \in T_{obs} \\ &= \sum_{k=-L}^{N+N_t-1} s_k f(t - kT) \quad , \quad t \in T_{obs} \end{aligned} \quad (3.4)$$

where $T_{obs} = [0, (N + N_t)T)$ is the observation interval, and

$$s_k = \begin{cases} a_{N_t+k} & , -L \leq k \leq -1 \\ b_k & , 0 \leq k \leq N-1 \\ a_{N_t+k-N} & , N \leq k \leq N + N_t - 1 \end{cases} \quad (3.5)$$

The overall response, $f(t)$, is assumed to be causal so that there is no interference in the current observation interval from the next data block. In this formulation the number of training symbols, N_t , is not restricted to be greater than or equal to the channel memory, L . If $N_t \geq L$, then only the L training symbols either side of the data block need to be considered. If $N_t < L$, then it is understood that the L training symbols required prior to the current data block include decisions from a previous data block. In other words, the training symbols with negative indices are really previous decisions. With these definitions, block decision feedback (BDFB) is automatically included in the detectors to be defined, if the parameter values so dictate. The corresponding theoretical performance analysis inherently assumes correct BDFB decisions when necessary. The simulation results presented in Chapter 6 include the effects of BDFB decisions errors.

Equation (3.4) can be rewritten as

$$\tilde{r}(t) = a_1(t) + b(t) + a_2(t) \quad , t \in T_{obs} \quad (3.6)$$

where

$$a_1(t) = \sum_{k=-L}^{-1} a_{N_t+k} f(t - kT) \quad , t \in T_{obs} \quad (3.7)$$

$$b(t) = \sum_{k=0}^{N-1} b_k f(t - kT) \quad , t \in T_{obs} \quad (3.8)$$

$$a_2(t) = \sum_{k=N}^{N+N_t-1} a_{N_t+k-N} f(t - kT) \quad , t \in T_{obs} \quad (3.9)$$

With additive white Gaussian noise (AWGN), $n(t)$, the received signal becomes

$$r(t) = a_1(t) + b(t) + a_2(t) + n(t) \quad , t \in T_{obs} \quad (3.10)$$

Assuming the overall impulse response is known and that all feedback decisions from a previous data block are correct (if any), the training signals $a_1(t)$ and $a_2(t)$ can be reconstructed at the receiver. Subtracting the training signals from the received signal gives

$$\begin{aligned} r(t) - a_1(t) - a_2(t) &= b(t) + n(t) \\ &= c(t) \quad , t \in T_{obs} \end{aligned} \quad (3.11)$$

The resulting signal, $c(t)$, is a Gaussian process with mean signal $b(t)$, as defined in (3.8). Data block detection is performed using signal $c(t)$.

3.3 Linear Block Estimation

The source data symbols can be estimated using the least sum of squared errors (LSSE) criterion. The sum of squared errors (SSE) is defined as

$$sse(\hat{\mathbf{b}}) = \int_0^{T_{obs}} |c(t) - \hat{b}(t)|^2 dt \quad (3.12)$$

where

$$\hat{\mathbf{b}} = [\hat{b}_0 \dots \hat{b}_{N-1}]^t \quad (3.13)$$

represents an estimated source data vector, $[]^t$ denotes matrix or vector transpose, and

$$\hat{b}(t) = \sum_{k=0}^{N-1} \hat{b}_k f(t - kT) \quad , \quad t \in T_{obs} \quad (3.14)$$

Note the difference between this LSSE criterion, and that of minimizing the mean squared error (MMSE) at the output of a linear filter and decision device (quantizer). In this case, the unquantized data symbol estimates are to be optimized, rather than a set of filter coefficients for filtering $c(t)$. Substituting (3.14) into (3.12) and rearranging gives

$$sse(\hat{\mathbf{b}}) = E_c - 2Re[\mathbf{z}^t \hat{\mathbf{b}}^*] + \hat{\mathbf{b}}^t \mathbf{R} \hat{\mathbf{b}}^* \quad (3.15)$$

where

$$E_c = \int_0^{T_{obs}} |c(t)|^2 dt \quad (3.16)$$

is the observed energy of the received signal $c(t)$, $\mathbf{z} = [z_0 \dots z_{N-1}]^t$ is an N -dimensional column vector containing the N output samples from a filter matched to the channel, or partially matched to the channel depending on the observation interval. That is, the k -th element of $\mathbf{z}$ is given by

$$z_k = \int_0^{T_{obs}} c(t) f^*(t - kT) dt \quad , \quad k = 0 \dots N-1 \quad (3.17)$$

And, $\mathbf{R}$ is an N -by- N autocorrelation type matrix with (k,l) -th element

$$r_{k,l} = \int_0^{T_{obs}} f(t - kT) f^*(t - lT) dt \quad , \quad k, l = 0 \dots N-1 \quad (3.18)$$

Since $\mathbf{R}$ is an autocorrelation type matrix, it is assumed to be positive definite and thus invertible in what follows. From (3.18) we see that $r_{k,l} = r_{l,k}^*$, and thus that $\mathbf{R}$ is Hermitian. If $f(t - kT)$ is completely contained in the observation interval for $k = 0 \dots N-1$, then $\mathbf{R}$ is also Toeplitz. Thus $\mathbf{R}$ is Toeplitz only if the number of training symbols, N_t , is greater than or equal to the channel memory, L . The Toeplitz and non-Toeplitz cases are handled separately when considering detailed implementation requirements. For now it is sufficient to assume that $\mathbf{R}$ is invertible and proceed with a common derivation.

Assuming unquantized $\hat{\mathbf{b}}$ elements, the objective is to find the $\hat{\mathbf{b}}$ vector which provides the LSSE. Completing the square (in the complex squared-magnitude sense) gives

$$sse(\hat{\mathbf{b}}) = E_c - \mathbf{z}^t \mathbf{R}^{-1} \mathbf{z}^* + (\mathbf{R}^{*-1} \mathbf{z} - \hat{\mathbf{b}})^t \mathbf{R} (\mathbf{R}^{*-1} \mathbf{z} - \hat{\mathbf{b}})^* \quad (3.19)$$

That (3.19) is equivalent to (3.15) is easily checked by multiplying out the completed square and using the property that $\mathbf{R}^t = \mathbf{R}^*$. Since $\mathbf{R}$ is taken to be positive definite, the third term in (3.19) is non-negative and can be minimized (set to zero) by choosing

$$\hat{\mathbf{b}}_{\text{opt}} = \mathbf{R}^{-1} \mathbf{z} \quad (3.20)$$

This choice also yields the LSSE, which can be expressed as

$$lsse = ssc(\hat{\mathbf{b}}_{\text{opt}}) = E_c - \mathbf{z}^t \mathbf{R}^{-1} \mathbf{z}^* = E_c - \mathbf{z}^t \hat{\mathbf{b}}_{\text{opt}}^* \quad (3.21)$$

The data decisions are obtained by thresholding or quantizing the elements of the $\hat{\mathbf{b}}_{\text{opt}}$ vector. The $\hat{\mathbf{b}}_{\text{opt}}$ vector can be viewed as containing the true data symbols plus estimation noise or errors.

A block diagram representation of the linear block estimator (LBE) approach is shown in Figure 3.2. The matched filter is a true matched filter if $N_t \geq L$. It is only partially matched, using restricted integration times for some outputs, if $N_t < L$. The derivation was based on first subtracting off the training signals, $a_1(t)$ and $a_2(t)$, from the received signal, $r(t)$. As shown in Figure 3.2, this interference can alternatively be removed after the matched filter and T -spaced sampler by subtracting off the filtered, and T -spaced sampled, training signals. The resulting input to the LBE box is the same. For time-varying channels the matched filter must be adaptive. A discrete channel model with K samples per symbol period is assumed for channel estimation purposes. Which method of removing the interfering training signals is preferred depends on the value of K . It is generally less work to perform this operation after the matched filter if $K > 1$. This is because many of the calculations can be shared between the different boxes shown. In either case, the input to the linear block estimator box is the N -dimensional vector $\mathbf{z}$, and this box solves equation (3.20). The final step is to quantize the estimated data symbols. An estimate of the overall channel response is required for most of the operations. The received signal used to estimate the channel response must be delayed by one signaling block. Both the known training symbols and past data block decisions can be used for channel estimation. Channel estimation is addressed in Chapter 5.

Figure 3.2 Linear Block Estimator (LBE) approach

From (3.20), the matrix equation to be solved by the LBE box can alternatively be presented as

$$\mathbf{R}^* \hat{\mathbf{b}} = \mathbf{z} \quad (3.22)$$

where the "opt" descriptor on $\hat{\mathbf{b}}$ has been dropped for brevity. This form is used to emphasize that the inverse of $\mathbf{R}^*$ need not be obtained explicitly in order to solve for $\hat{\mathbf{b}}$. A number of different approaches can be used to solve (3.22), depending on the properties of the correlation matrix $\mathbf{R}$.

3.3.1 Toeplitz Case

The correlation matrix, $\mathbf{R}$, is Toeplitz only if the number of training symbols, N_t , is greater than or equal to the channel memory, L . When this is the case $\mathbf{R}$ has the form

$$\mathbf{R} = \begin{bmatrix} r_0 & r_1 & \dots & r_L & 0's \\ r_{-1} & r_0 & & & \ddots \\ \vdots & & \ddots & & r_L \\ r_{-L} & & & \ddots & \vdots \\ & \ddots & & & r_1 \\ 0's & r_{-L} & \dots & r_{-1} & r_0 \end{bmatrix} \quad \begin{matrix} dim \\ N \times N \end{matrix} \quad (3.23)$$

The (k,l)-th element is given by

$$r_{k,l} = r_{l-k} = \int_0^{T_{obs}} f(t - kT) f^*(t - lT) dt \quad (3.24)$$

The finite memory of the channel assures us that

$$r_{k,l} = r_{l-k} = 0 \quad , for \quad |l - k| > L \quad (3.25)$$

The elements of the $\mathbf{R}$ matrix can be evaluated using

$$r_i = \begin{cases} \int f(t) f^*(t - iT) dt & , for \quad 0 \leq i \leq L \\ r_{-i}^* & , for \quad -L \leq i \leq -1 \end{cases} \quad (3.26)$$

If the channel is modeled as being discrete, with K samples per symbol period T , then the integrals in (3.26) become finite summations. That is¹

$$r_i = \sum_{l=0}^{L-i} \sum_{k=0}^{K-1} f_{lK+k} f_{(l-i)K+k}^* \quad , i = 0 \dots L \quad (3.27)$$

¹ Similar formulas and complexities are given in [Rab78] for the autocorrelation method of speech analysis.

The elements of $\mathbf{R}$ are uniquely determined by these $(L + 1)$ values. The total number of complex multiplies and adds required is

$$\begin{aligned}\#mult &= \frac{1}{2}K [L^2 + 3L + 2] \quad , N \geq L + 1 \\ \#adds &= \#mult - (L + 1)\end{aligned}\quad (3.28)$$

The complexity is less if $N \leq L$. For the practical cases of interest in this study, N is on the order of L . Thus, with $K = 2$, the complexity is approximately order N^2 per block of N data symbols.

If $\mathbf{R}$ is Toeplitz then equation (3.22) can be solved recursively using the Levinson algorithm [Hsu85, Lev46]. The Levinson algorithm can be implemented using the following equations:

For $n = 1 \dots N$

$$u_{n-2,n-2} = \frac{r_{n-1} - \sum_{i=0}^{n-3} r_{i+1} u_{n-3,n-3-i}}{D_{n-2}} \quad (3.29)$$

$$u_{n-2,k} = u_{n-3,k} - u_{n-2,n-2} u_{n-3,n-3-k}^* , k = 0 \dots n-3 \quad (3.30)$$

$$D_{n-1} = r_0 - \sum_{i=0}^{n-2} r_{i+1} u_{n-2,i}^* \quad (3.31)$$

$$\hat{b}_{n-1,n-1} = \frac{z_{n-1} - \sum_{i=0}^{n-2} r_{i+1} \hat{b}_{n-2,n-2-i}}{D_{n-1}} \quad (3.32)$$

$$\hat{b}_{n-1,k} = \hat{b}_{n-2,k} - \hat{b}_{n-1,n-1} u_{n-2,n-2-k}^* , k = 0 \dots n-2 \quad (3.33)$$

The algorithm is order recursive in block size n , starting with $n = 1$ and working up to the block size of interest, $n = N$. It is understood that variables with negative indices do not need to be evaluated.

The complexity of the above Levinson algorithm is also only order N^2 per block of N data symbols. Specifically, the number of complex multiplies, adds, and reciprocals is

$$\begin{aligned}\#mult &= \frac{1}{2} [5N^2 - 5N + 2] \\ \#adds &= \frac{1}{2} [5N^2 - 9N + 4] \\ \#recip &= N\end{aligned}\quad (3.34)$$

If $N > L + 1$ then the complexity can be reduced slightly by taking advantage of the zeros in the $\mathbf{R}$ matrix. For the practical cases of interest in this study, N is on the order of L , so the above complexity figures are considered adequate. The complexity is still order N^2 in any case.

3.3.2 Non-Toeplitz Case

The correlation matrix, $\mathbf{R}$, is not Toeplitz if the number of training symbols, N_t , is less than the channel memory, L . While at most N elements of a Toeplitz $\mathbf{R}$ need to be calculated, up to half of the N^2 elements of a non-Toeplitz $\mathbf{R}$ may need to be calculated. As shown below, the complexity of computing the elements of a non-Toeplitz $\mathbf{R}$ can still be kept to order N^2 , by taking advantage of a backward recursion. From (3.18), and using the assumption that $f(t)$ is causal,

$$r_{l,k}^* = r_{k,l} = \int_{kT}^{T_{obs}} f(t - kT) f^*(t - lT) dt \quad , k \geq l \quad (3.35)$$

It follows that

$$r_{k-1,l-1} = r_{k,l} + I_{k,l} \quad (3.36)$$

where

$$I_{k,l} = \int_{T_{obs}}^{T_{obs}+T} f(t - kT) f^*(t - lT) dt \quad (3.37)$$

is an integral over a single symbol period T . Given the N initial values

$$r_{N-1,l} = \int_{(N-1)T}^{T_{obs}} f(t - (N-1)T) f^*(t - lT) dt \quad , l = 0 \dots N-1 \quad (3.38)$$

making up the bottom row of $\mathbf{R}$, all of the other elements can be obtained using the above backwards recursion.² The exact complexity depends on the many possible relationships between N , N_t , and L . In the worst case, all N or $L + 1$ values required for a Toeplitz $\mathbf{R}$ are eventually obtained, with all lower intermediate-step elements obtained for free. Thus, with K samples per symbol period, the complexity is conveniently upper bounded by the number of complex multiplies and adds given in (3.28) for the Toeplitz case. For the practical cases of interest in this study, N is on the order of L , so the complexity can be considered essentially order N^2 . It is worth emphasizing that, given the same values

² A similar recursion is given in [Rab78] for the covariance method of speech analysis.

of N and L , the complexity in computing a non-Toeplitz R is never greater than that for a Toeplitz R . Note also that removing the Toeplitz restriction removes the constraint that N_i be greater than or equal to L .

The problem of solving (3.22) for the non-Toeplitz case is now addressed. The Levinson algorithm is no longer applicable, but a form of Cholesky factorization can be used since R is still Hermitian. The inverse of the R^* matrix still does not need to be computed explicitly.

A positive definite Hermitian matrix, A , containing elements $[a_{i,j}]$, can be factored as [Kre88, Rab78, Wes68]

$$A = LU \quad (3.39)$$

where L and U are lower and upper triangular matrices respectively. The elements of L and U are denoted as $[l_{i,j}]$ and $[u_{i,j}]$ respectively. A number of different factorizations are possible. If square-root Cholesky factorization is used then $L = U^*$. This symmetry offers some minor conveniences, but the solution requires N square-roots, and more multiplies than some other factorizations. A modified Cholesky factorization, requiring no square-roots, and having one's down the diagonal of the L matrix, is easily derived from the square-root Cholesky factorization. The solution is given by the following equations:

for $i = 0, \dots, N-1$

$$u_{i,i} = a_{i,i} - \sum_{k=0}^{i-1} l_{i,k} u_{k,i} \quad (3.40)$$

for $j = i+1, \dots, N-1$

$$u_{i,j} = a_{i,j} - \sum_{k=0}^{i-1} l_{i,k} u_{k,j} \quad (3.41)$$

$$l_{j,i} = \frac{u_{i,j}^*}{u_{i,i}}$$

Using the above Cholesky factorization on R^* in (3.22) gives the two triangular systems

$$U \hat{b} = x \quad (3.42)$$

and

$$L x = z \quad (3.43)$$

where $\mathbf{x}$ is an N -dimensional column vector used as an intermediate step in the solution for $\hat{\mathbf{b}}$. The solution to a triangular system of equations is trivial. The lower triangular system given in (3.43) is solved first for $\mathbf{x}$. The result is

$$x_i = z_i - \sum_{k=0}^{i-1} l_{i,k} x_k, \quad i = 0 \dots N-1 \quad (3.44)$$

Once $\mathbf{x}$ is obtained then the upper triangular system given in (3.42) can be solved for $\hat{\mathbf{b}}$. The result is

$$\hat{b}_i = \frac{1}{u_{i,i}} \left(x_i - \sum_{k=i+1}^{N-1} u_{i,k} \hat{b}_k \right), \quad i = N-1, \dots, 0 \quad (3.45)$$

Note that the unquantized symbol estimates must be computed in reverse order. This is an important consideration for the decision directed detectors derived later.

The complexity of the solution for each of the triangular systems is only order N^2 per block of N data symbols. The complexity of the Cholesky factorization is order N^3 , and thus will dominate the overall complexity for large N . Specifically, the number of complex multiplies, adds, and reciprocals is

$$\begin{aligned} \#mult &= \frac{1}{6} [N^3 + 9N^2 - 4N] \\ \#adds &= \frac{1}{6} [N^3 + 6N^2 - 7N] \\ \#recip &= N \end{aligned} \quad (3.46)$$

As for the Toeplitz case, the complexity can be reduced slightly if $N > L+1$ because of the zeroes in the correlation matrix. The above complexity figures are considered adequate for this study.

Table 3.1 compares the number of complex multiplies required to solve equation (3.22) for the Toeplitz (Levinson) and non-Toeplitz (Cholesky) cases. It is seen that for small N (e.g. $N \leq 6$) the complexities are very close. The added flexibility in being able to choose fewer training symbols for the non-Toeplitz case is an important consideration, and is achieved without any significant complexity penalty for $N \leq 6$.

As an example, suppose the channel memory is $L = 10$, and 50% throughput is acceptable. The minimum complexity, with R Toeplitz, is obtained with $N = N_t = 10$. From Table 3.1, 22.6 multiplies per data symbol are required. If the Toeplitz requirement

Data Block Size (N)	Number of Complex Multiplies			
	Toeplitz (Levinson)		Non-Toeplitz (Cholesky)	
	per block	per symbol	per block	per symbol
1	1	1	1	1
2	6	3	6	3
3	16	5.33	16	5.33
4	31	7.75	32	8
5	51	10.20	55	11
6	76	12.67	86	14.33
7	106	15.14	126	18
8	141	17.63	176	22
9	181	20.11	237	26.33
10	226	22.60	310	31

Table 3.1 Number of complex multiplies per data block and per data symbol for the Toeplitz (Levinson) and non-Toeplitz (Cholesky) LBE approaches.

is removed, then $N = N_t = 5$ could be used for example, requiring only 11 multiplies per data symbol. The error rate performance is even likely to improve with the smaller block size, depending on the channel profile (see the examples in Section 3.3.4).

3.3.3 Mean Squared Error Analysis

From (3.20) the solution for the data symbols is given by

$$\hat{\mathbf{b}} = \mathbf{Q} \mathbf{z} \quad (3.47)$$

where, for convenience, the following definition has been made

$$\mathbf{Q} = \mathbf{R}^*{}^{-1} \quad (3.48)$$

Substituting (3.8) into (3.11), and (3.11) into (3.17) yields

$$z_k = \sum_{l=0}^{N-1} r_{k,l}^* b_l + w_k, \quad k = 0 \dots N-1 \quad (3.49)$$

with w_k defined as

$$w_k = \int_0^{T_{obs}} n(t) f^*(t - kT) dt \quad (3.50)$$

Using vector and matrix notation we obtain

$$\mathbf{z} = \mathbf{R}^* \mathbf{b} + \mathbf{w} \quad (3.51)$$

where $\mathbf{w} = [w_0 \dots w_{N-1}]^t$. The error vector, $\mathbf{e} = [e_0 \dots e_{N-1}]^t$, can be expressed as

$$\mathbf{e} = \hat{\mathbf{b}} - \mathbf{b} = \mathbf{Q} \mathbf{w} \quad (3.52)$$

where equations (3.47), (3.48), and (3.51) have been employed. The covariance matrix for $\hat{\mathbf{b}}$ is then given by

$$\begin{aligned} cov[\hat{\mathbf{b}}] &= E[\mathbf{e} \mathbf{e}^{*t}] \\ &= E[\mathbf{Q} \mathbf{w} \mathbf{w}^{*t} \mathbf{Q}^{*t}] \\ &= \mathbf{Q} \mathbf{W} \mathbf{Q} \end{aligned} \quad (3.53)$$

where, by definition,

$$\mathbf{W} = E[\mathbf{w} \mathbf{w}^{*t}] \quad (3.54)$$

and the fact that $\mathbf{Q}^t = \mathbf{Q}^*$ has been used. Substituting (3.50) into (3.54), it is straightforward to show that

$$\mathbf{W} = N_0 \mathbf{R}^* \quad (3.55)$$

where N_0 is the single-sided noise power spectral density of the white Gaussian noise process $n(t)$. Finally, substituting (3.55) into (3.53) yields the desired result that

$$\text{cov} [\hat{\mathbf{b}}] = N_0 \mathbf{Q} \quad (3.56)$$

Thus the MSE of the k -th symbol is given by

$$\text{mse} [\hat{b}_k] = N_0 q_{k,k}, \quad k = 0 \dots N-1 \quad (3.57)$$

where $q_{k,k}$ is the k -th diagonal element of the $\mathbf{Q}$ matrix. The degradation of the MSE from ideal (i.e. the detection of an isolated pulse with a filter matched to the overall channel response), for the k -th symbol, measured in dB, is easily deduced to be

$$\text{Deg}_k = 10 \log (E_f q_{k,k}), \quad k = 0 \dots N-1 \quad (3.58)$$

where

$$E_f = \int |f(t)|^2 dt \quad (3.59)$$

is the energy in the overall channel response. This result is independent of whether $\mathbf{R}$ is Toeplitz or not. The MSE distribution within the data block does depend on the properties of $\mathbf{R}$, however.

3.3.4 MSE Performance for Example Channels

This section presents the mean squared error (MSE) performance for some example fixed channels with nulls. To keep the examples simple, only short, discrete, T -spaced channel models are considered. More complicated channel models are considered with the simulation results presented in Chapter 6. The formulas for discrete T -spaced channel models are analogous to those presented for the continuous-time model. Specifically, the MSE degradation from Ideal is still given by (3.58), where $\mathbf{Q} = \mathbf{R}^{*-1}$. For the Toeplitz case, the elements of $\mathbf{R}$ are given by (3.27), with $K = 1$. For the non-Toeplitz case, the

analogous discrete versions of (3.35) through (3.38) are required. With these definitions, the appropriate formula for the energy in the overall channel response is

$$E_f = \sum_{l=0}^L |f_l|^2 \quad (3.60)$$

where

$$\mathbf{f} = [f_0 \ f_1 \ \dots \ f_L]^t \quad (3.61)$$

is the discrete channel response.

First, consider the familiar partial response channel

$$\mathbf{f} = [1 \ 1]^t \quad (3.62)$$

Figure 3.3 shows the MSE degradation (in dB) versus the data symbol index (relative to the middle symbol with N odd). Results are presented for different data block lengths, N . The only assumption concerning the number of training symbols, N_t , within a training block, is that it is greater than or equal to the channel memory, $L = 1$. The additional dB shift due to the fraction of training is not taken into account, but is easily calculated given a specific N_t and equation (3.2). It is observed that the MSE is bell shaped, reaching its maximum in the middle of the block, and decreasing towards both edges. Note that the degradation increases as the data block length is increased. This is true even for the edge symbols.

Figure 3.4 shows the MSE performance for the partial response channel

$$\mathbf{f} = [1 \ 2 \ 1]^t \quad (3.63)$$

This channel has a second order null in the frequency domain. The worst case degradation is seen to be much more severe, reaching 18.4 dB for $N = 11$. Again, it is assumed that $N_t \geq L$, where $L = 2$ for this channel. The results presented thus far are for Toeplitz correlation matrices only.

Figure 3.5 shows the MSE performance for the channel defined in (3.63), but with $N_t = 1$. The correlation matrix is not Toeplitz in this case. The MSE performance is no longer symmetric within the data block. The degradation at the right edge is now much worse than that at the left edge. This is because some of the channel energy was

Figure 3.3 MSE degradation (in dB) from Ideal versus symbol index (relative to the middle symbol with N odd) for: LBE detector, channel response (3.62), and $N_t \geq 1$.

Figure 3.4 MSE degradation (in dB) from Ideal versus symbol index (relative to the middle symbol with N odd) for: LBE detector, channel response (3.63), and $N_t \geq 2$.

Figure 3.5 MSE degradation (in dB) from Ideal versus symbol index (relative to the middle symbol with N odd) for: LBE detector, channel response (3.63), and $N_t = 1$.

not collected for the last data symbol. The degradation at the left edge is only slightly worse than that with $N_t = 2$. The performance in the middle of the data block is still much worse than at either edge.

As a comparison example, suppose the channel is as defined in (3.63), and a throughput of 80% is required. One option is to use $N = 8$, $N_t = 2$, and the Toeplitz based LBE algorithm. A second option is to use $N = 4$, $N_t = 1$, and the non-Toeplitz based LBE algorithm. Not only is the performance much better for the second option, because of the shorter data blocks, but the complexity is also considerably less. (See Table 3.1.)

3.4 Decision Feedback Block Estimation

As shown in the examples of Section 3.3.4, the mean squared error (MSE) for each symbol estimate depends on its location within the data block. With LBE detection the MSE typically peaks in the middle and decreases towards both edges. For channels with nulls or near nulls, the MSE of the edge symbols (i.e. $\hat{b}_0$ and $\hat{b}_{N-1}$) can be many times smaller than the MSE of the middle symbol (i.e. $\hat{b}_{N/2}$). For the Toeplitz case ($N_t \geq L$) this distribution is symmetric.

Hsu [Hsu85] proposed a nonlinear decision-directed recursive technique based on the Levinson algorithm, to try and improve the performance for the Toeplitz case. The idea is to make decisions only on $\hat{b}_0$ and $\hat{b}_{N-1}$, then treat these decisions as additional training symbols and solve for the data vector again, reduced in size by 2 symbols. This is repeated until all decisions are made. This nonlinear technique provides performance comparable to a decision feedback equalizer (DFE) detector. A disadvantage, compared to the Toeplitz LBE detector, is that the order of complexity increases by another factor of N , for large data block sizes, N .

This technique can also be applied to the non-Toeplitz case. As will be shown, for the non-Toeplitz case it is more convenient to make decisions from one edge only, and performance is best if decisions are made from the left (or starting) edge. The order of complexity is not increased in this case, compared to the non-Toeplitz LBE detector.

This general approach of feeding back edge decisions will be referred to as decision feedback block estimation (DFBE). A block diagram illustrating the DFBE concept is

Figure 3.6 Decision Feedback Block Estimator (DFBE) approach

shown in Figure 3.6. The structure is similar to that shown in Figure 3.2 for the LBE detector. The main differences are that only edge decisions are made, and the detection process is recursive, working in from the edges.

Section 3.4.1 presents an algorithm for the Toeplitz case. The method of updating the $\mathbf{z}$ vector is order N more efficient than the method used in [Hsu85]. The rest of the solution is essentially the same.

Section 3.4.2 presents the algorithm derived for the non-Toeplitz case.

Sections 3.4.3 and 3.4.4 present the MSE analysis and performance for some example channels.

3.4.1 Toeplitz Case

For the Toeplitz case ($N_t \geq L$) the distribution of MSE's is symmetric within the data block, and the edge symbols ($\hat{b}_0$ and $\hat{b}_{N-1}$) have the smallest MSE. The idea is to make decisions only on $\hat{b}_0$ and $\hat{b}_{N-1}$, then treat these decisions as additional training symbols and solve for the data vector again, reduced in size by 2 symbols. This is repeated until all decisions are made.

Let the size of the data block being considered at each stage be denoted by a bracketed subscript. From (3.22) the first stage matrix equation is

$$\mathbf{R}_{(N)}^* \hat{\mathbf{b}}_{(N)} = \mathbf{z}_{(N)} \quad (3.64)$$

This first stage is solved as before, for the LBE case, except that in the final step of the Levinson algorithm, with $n = N$, only the $k = 0$ case needs to be evaluated in (3.33). The $k = N - 1$ case is solved in (3.32), giving the two edge symbol estimates. (The intermediate results from equations (3.29), (3.30), and (3.31) should be stored for $n < N$ because they can be reused in the later stages.) Assuming 2 correct edge decisions from the first stage, the second stage matrix equation is

$$\mathbf{R}_{(N-2)}^* \hat{\mathbf{b}}_{(N-2)} = \mathbf{z}_{(N-2)} \quad (3.65)$$

Although the dimension of $\mathbf{R}_{(N-2)}$ is two less than $\mathbf{R}_{(N)}$, the elements are the same up to the smaller dimension of $(N-2)$. Thus $\mathbf{R}_{(N)}$ can be expressed as

$$\mathbf{R}_{(N)} = \begin{bmatrix} r_0 & \tilde{\mathbf{r}}_{(N)}^* & r_{N-1} \\ \tilde{\mathbf{r}}_{(N)}^* & \mathbf{R}_{(N-2)} & \tilde{\mathbf{r}}_{(N)}^r \\ r_{N-1}^* & \tilde{\mathbf{r}}_{(N)}^{*r} & r_0 \end{bmatrix} \quad (3.66)$$

where

$$\ddot{\mathbf{r}}_{(N)} = [r_1 \dots r_{N-2}]^t \quad (3.67)$$

and $\mathbf{v}^r$ denotes the elements of vector $\mathbf{v}$ in reverse order. From (3.51), the $\mathbf{z}$ vectors at the first and second stages can be expressed as

$$\mathbf{z}_{(N)} = \mathbf{R}_{(N)}^* \mathbf{b}_{(N)} + \mathbf{w}_{(N)} \quad (3.68)$$

and

$$\mathbf{z}_{(N-2)} = \mathbf{R}_{(N-2)}^* \mathbf{b}_{(N-2)} + \mathbf{w}_{(N-2)} \quad (3.69)$$

where $\mathbf{b}_{(N)}$ is the vector of true data symbols, and $\mathbf{w}_{(N)}$ is derived from the original noise process $n(t)$, as defined in (3.50). By definition,

$$\mathbf{b}_{(N)} = [b_0 \ \mathbf{b}_{(N-2)}^t \ b_{N-1}]^t \quad (3.70)$$

and it follows from (3.50) that

$$\mathbf{w}_{(N)} = [w_0 \ \mathbf{w}_{(N-2)}^t \ w_{N-1}]^t \quad (3.71)$$

Substituting (3.66), (3.70), and (3.71) into (3.68) and comparing with (3.69), gives the following relationship

$$\ddot{\mathbf{z}}_{(N)} = \mathbf{z}_{(N-2)} + b_0 \ddot{\mathbf{r}}_{(N)} + b_{N-1} \ddot{\mathbf{r}}_{(N)}^* \quad (3.72)$$

where $\ddot{\mathbf{z}}_{(N)}$ is the $\mathbf{z}_{(N)}$ vector with its 2 edge elements removed. That is

$$\mathbf{z}_{(N)} = [z_0 \ \ddot{\mathbf{z}}_{(N)}^t \ z_{N-1}]^t \quad (3.73)$$

Rearranging (3.72) gives the result that $\mathbf{z}_{(N-2)}$ can be obtained from $\mathbf{z}_{(N)}$ using the simple backwards recursion

$$\mathbf{z}_{(N-2)} = \ddot{\mathbf{z}}_{(N)} - \tilde{b}_0 \ddot{\mathbf{r}}_{(N)} - \tilde{b}_{N-1} \ddot{\mathbf{r}}_{(N)}^* \quad (3.74)$$

where the previous edge decisions, $\tilde{b}_0$ and $\tilde{b}_{N-1}$, are assumed to equal the corresponding true data symbols.

To update the $\mathbf{z}$ vector for the second stage requires only $2(N-2)$ complex multiplies and adds. This method of updating $\mathbf{z}$ is order N more efficient than the method used in [Hsu85]. Fewer operations are required if $N > L+2$.

The rest of the solution for the second stage is to recursively use equations (3.32) and (3.33) to recalculate the new edge symbols for block size $(N-2)$. Only the $k = 0$ case needs to be evaluated in (3.33) for the last step. Note that (3.29), (3.30), and (3.31) do not need to be recalculated. This procedure is repeated until all decisions are made. The total number of complex multiplies, adds, and reciprocals is

$$\begin{aligned} \#mult &= \begin{cases} \frac{1}{12} [2N^3 + 27N^2 - 32N + 15] & , N \text{ odd} \\ \frac{1}{12} [2N^3 + 27N^2 - 32N + 12] & , N \text{ even} \end{cases} \\ \#adds &= \frac{1}{12} [2N^3 + 24N^2 - 50N + 24] \\ \#recip &= N \end{aligned} \quad (3.75)$$

Note that the complexity is now order N^3 per block of N data symbols, whereas the complexity for the Toeplitz LBE was order N^2 .

3.4.2 Non-Toeplitz Case

For the non-Toeplitz case ($N_t < L$) the distribution of MSE's is not necessarily symmetric within the data block. The estimated edge symbol $\hat{b}_0$ typically has a smaller MSE than $\hat{b}_{N-1}$ because of the additional energy collected by the (partial) matched filter for $\hat{b}_0$. Thus performance is best if decisions are made recursively, one at a time, starting with the first symbol of each data block. However, as will become apparent later, the matrix factorization used for the non-Toeplitz LBE case is more appropriately set up for making decisions recursively from the end of a block. For notational convenience, end decisions will be assumed, instead of start decisions. A simple reversal mapping will then be used to show that decisions can also be made from the start of a block, with no penalty in complexity.

Let the size of the data block being considered at each stage be denoted by a bracketed subscript. From (3.22) the first stage matrix equation is

$$\mathbf{R}_{(N)}^* \hat{\mathbf{b}}_{(N)} = \mathbf{z}_{(N)} \quad (3.76)$$

This first stage is solved as before, for the non-Toeplitz LBE case, up to the solution for the intermediate vector $\mathbf{x}_{(N)}$ given in (3.44). That is, the factorization

$$\mathbf{R}_{(N)}^* = \mathbf{L}_{(N)} \mathbf{U}_{(N)} \quad (3.77)$$

is required, and

$$\mathbf{x}_{(N)} = \mathbf{L}_{(N)}^{-1} \mathbf{z}_{(N)} \quad (3.78)$$

From (3.45), the unquantized end symbol estimate is simply given by

$$\hat{b}_{N-1} = \frac{x_{N-1}}{u_{N-1,N-1}} \quad (3.79)$$

requiring only one complex multiply (the reciprocal having already been calculated for the previous steps). The end decision is denoted as

$$\tilde{b}_{N-1} = \text{decision}(\hat{b}_{N-1}) \quad (3.80)$$

Assuming a correct end decision from the first stage, the second stage matrix equation is

$$\mathbf{R}_{(N-1)}^* \hat{\mathbf{b}}_{(N-1)} = \mathbf{z}_{(N-1)} \quad (3.81)$$

Although the dimension of $\mathbf{R}_{(N-1)}$ is one less than $\mathbf{R}_{(N)}$, the elements are the same up to the smaller dimension of $(N-1)$. Thus $\mathbf{R}_{(N)}$ can be expressed as

$$\mathbf{R}_{(N)} = \begin{bmatrix} \mathbf{R}_{(N-1)} & \hat{\mathbf{r}}_{(N)} \\ \hat{\mathbf{r}}_{(N)}^* & r_{N-1,N-1} \end{bmatrix} \quad (3.82)$$

where

$$\hat{\mathbf{r}}_{(N)} = [r_{0,N-1} \ \dots \ r_{N-2,N-1}]^t \quad (3.83)$$

Upon examination of the Cholesky factorization equations given in (3.40) and (3.41), it follows that

$$\mathbf{L}_{(N)} = \begin{bmatrix} \mathbf{L}_{(N-1)} & 0 \\ \hat{\mathbf{l}}_{(N)} & 1 \end{bmatrix} \quad (3.84)$$

and

$$\mathbf{U}_{(N)} = \begin{bmatrix} \mathbf{U}_{(N-1)} & \hat{\mathbf{u}}_{(N)} \\ 0 & u_{N-1,N-1} \end{bmatrix} \quad (3.85)$$

where

$$\hat{\mathbf{l}}_{(N)} = [l_{N-1,0} \ \dots \ l_{N-1,N-2}] \quad (3.86)$$

and

$$\hat{\mathbf{u}}_{(N)} = [u_{0,N-1} \ \dots \ u_{N-2,N-1}]^t \quad (3.87)$$

From (3.51), the $\mathbf{z}$ vector for the first stage can be expressed as

$$\mathbf{z}_{(N)} = \mathbf{R}_{(N)}^* \mathbf{b}_{(N)} + \mathbf{w}_{(N)} \quad (3.88)$$

where $\mathbf{b}_{(N)}$ is the vector of true data symbols, and $\mathbf{w}_{(N)}$ is derived from the original noise process $n(t)$, as defined in (3.50). Substituting (3.88) into (3.78) and using (3.77) gives

$$\mathbf{x}_{(N)} = \mathbf{U}_{(N)} \mathbf{b}_{(N)} + \tilde{\mathbf{w}}_{(N)} \quad (3.89)$$

where $\tilde{\mathbf{w}}_{(N)}$ is the solution to the lower triangular system

$$\mathbf{L}_{(N)} \tilde{\mathbf{w}}_{(N)} = \mathbf{w}_{(N)} \quad (3.90)$$

Similarly, for the second stage,

$$\mathbf{x}_{(N-1)} = \mathbf{U}_{(N-1)} \mathbf{b}_{(N-1)} + \tilde{\mathbf{w}}_{(N-1)} \quad (3.91)$$

where

$$\mathbf{L}_{(N-1)} \tilde{\mathbf{w}}_{(N-1)} = \mathbf{w}_{(N-1)} \quad (3.92)$$

By definition

$$\mathbf{b}_{(N)} = \begin{bmatrix} \mathbf{b}_{(N-1)}^t & b_{N-1} \end{bmatrix}^t \quad (3.93)$$

and it follows from (3.50) that

$$\mathbf{w}_{(N)} = \begin{bmatrix} \mathbf{w}_{(N-1)}^t & w_{N-1} \end{bmatrix}^t \quad (3.94)$$

Substituting (3.84) and (3.94) into (3.90), and using the fact that $\mathbf{L}_{(N)}$ is lower triangular, gives the important intermediate result that

$$\tilde{\mathbf{w}}_{(N)} = \begin{bmatrix} \tilde{\mathbf{w}}_{(N-1)}^t & \tilde{w}_{N-1} \end{bmatrix}^t \quad (3.95)$$

Now, substituting (3.85), (3.93), and (3.95) into (3.89), and using (3.91), gives the simple relationship

$$\dot{\mathbf{x}}_{(N)} = \mathbf{x}_{(N-1)} + b_{N-1} \dot{\mathbf{u}}_{(N)} \quad (3.96)$$

where $\dot{\mathbf{x}}_{(N)}$ is the $\mathbf{x}_{(N)}$ vector with its end element removed. That is

$$\mathbf{x}_{(N)} = \begin{bmatrix} \dot{\mathbf{x}}_{(N)}^t & x_{N-1} \end{bmatrix}^t \quad (3.97)$$

Rearranging (3.96) gives the result that $x_{(N-1)}$ can be obtained from $x_{(N)}$ using the simple backwards recursion

$$x_{(N-1)} = \dot{x}_{(N)} - \tilde{b}_{N-1} \dot{u}_{(N)} \quad (3.98)$$

where the previous edge decision, $\tilde{b}_{N-1}$, is assumed to equal the corresponding true data symbol.

To update the x vector for the second stage requires only $(N-1)$ complex multiplies and adds. The remainder of the solution for the second stage is simply

$$\hat{b}_{N-2} = \frac{x_{N-2}}{u_{N-2,N-2}} \quad (3.99)$$

In summary, given $x_{(N)}$ from the first stage, the complete solution is given by

$$\begin{aligned} \hat{b}_i &= \frac{1}{u_{i,i}} \left[x_i - \sum_{k=i+1}^{N-1} u_{i,k} \tilde{b}_k \right] \\ \tilde{b}_i &= \text{decision}(\hat{b}_i) \quad , i = N-1, \dots, 0 \end{aligned} \quad (3.100)$$

Comparing (3.100) with the last part of the non-Toeplitz LBE algorithm, given in (3.45), it is seen that these two sets of equations are identical, except that in (3.100) the quantized decisions are used, as obtained, instead of the unquantized symbol estimates. As a result, the complexity of the non-Toeplitz DFBE is exactly the same as that for the non-Toeplitz LBE. (In fact, it can even be considered simpler, since the data symbols are typically made up of small integers such as ± 1 .)

The DFBE derived above makes decisions from the end of a data block. As pointed out earlier, the performance is better if decisions can be made from the start of a data block instead, where the observed energy per symbol is highest. This can easily be accommodated by reversing the order of the elements in the original vectors and matrix of (3.22). The equivalent matrix equation to be solved is

$$R^r \hat{b}^r = z^r \quad (3.101)$$

where

$$R^r = [r_{i,j}^r] = [r_{N-1-i, N-1-j}] \quad , i, j = 0 \dots N-1 \quad (3.102)$$

$$\hat{b}^r = [\hat{b}_i^r] = [\hat{b}_{N-1-i}] \quad , i = 0 \dots N-1 \quad (3.103)$$

$$\mathbf{z}^T = [z_i^T] = [z_{N-1-i}] \quad , i = 0 \dots N-1 \quad (3.104)$$

Applying the DFBE algorithm to (3.101) instead of (3.22) results in decisions being made recursively from the start of a data block, as desired, instead of the end. The complexity is clearly the same for this alternate starting point, and is given by (3.46).

Table 3.2 compares the number of multiplies required for the Toeplitz (Levinson) and non-Toeplitz (Cholesky) DFBE approaches. Recall that the Levinson based algorithm presented here in Section 3.4.1 is more efficient than the Levinson based algorithm presented in [Hsu85], for the Toeplitz case. This time the complexity for the non-Toeplitz case, based on Cholesky factorization, is slightly smaller than the complexity for the Toeplitz case, based on the Levinson algorithm. The reason the factorization approach is more efficient is because it makes use of a two step solution procedure, and the intermediate-step vector can be updated directly and efficiently at each stage of the solution. The Levinson approach is order recursive and does not provide a corresponding intermediate-step vector. This is an important result because it says that no complexity penalty is imposed by eliminating the Toeplitz restriction for the DFBE case. Further, with the Toeplitz restriction ($N_t \geq L$) removed, smaller data and training block sizes can be used, reducing the complexity even further, while maintaining or even increasing the throughput efficiency.

3.4.3 Mean Squared Error Analysis

The mean squared error (MSE) of an edge symbol estimate for the DFBE detectors is computed the same as in Section 3.3.3 for the LBE detectors. For the non-Toeplitz case, it is assumed that decisions are made from the starting edge only, using the first symbol estimate $\hat{b}_0$. For the Toeplitz case, it is assumed that decisions are made from both edges, using the symbol estimates $\hat{b}_0$ and $\hat{b}_{N-1}$. When computing the MSE for the other symbols it is assumed that the previous edge decisions are correct. For the non-Toeplitz case the data block is one symbol smaller at each stage. For the Toeplitz case the data block is two symbols smaller at each stage.

Data Block Size (N)	Number of Complex Multiplies			
	Toeplitz (Levinson)		Non-Toeplitz (Cholesky)	
	per block	per symbol	per block	per symbol
1	1	1	1	1
2	6	3	6	3
3	18	6	16	5.33
4	37	9.25	32	8
5	65	13	55	11
6	102	17	86	14.33
7	150	21.43	126	18
8	209	26.13	176	22
9	281	31.22	237	26.33
10	366	36.60	310	31

Table 3.2 Number of complex multiplies per data block and per data symbol for the Toeplitz (Levinson) and non-Toeplitz (Cholesky) DFBE approaches

From (3.57), the MSE of the k -th symbol estimate, for the non-Toeplitz case, is given by

$$mse [\hat{b}_k] = N_0 q_{0,0,(N-k)} \quad , k = 0 \dots N-1 \quad (3.105)$$

where the bracketed subscript denotes the data block size at each stage of the solution. Thus from (3.58), the corresponding degradation of the MSE from Ideal (i.e. the detection of an isolated pulse with a filter matched to the overall channel response), is given by

$$Deg_k = 10 \log (E_f q_{0,0,(N-k)}) \quad , k = 0 \dots N-1 \quad (3.106)$$

The corresponding MSE degradation formula for the Toeplitz (Levinson) case is

$$Deg_k = Deg_{N-k-1} = 10 \log (E_f q_{0,0,(N-2k)}) \quad , k = 0 \dots \left\lfloor \frac{N-1}{2} \right\rfloor \quad (3.107)$$

3.4.4 MSE Performance for Example Channels

This section presents the mean squared error (MSE) performance for some example fixed channels with nulls. the MSE performance is presented in terms of the dB degradation from Ideal, using formulas (3.106) and (3.107). To keep the examples simple, only short, discrete, T -spaced channel models are considered. More complicated channel models are considered with the simulation results presented in Chapter 6. The results can be compared directly with those presented in Section 3.3.4 for the Toeplitz and non-Toeplitz LBE detectors.

Figure 3.7 shows the MSE degradation (in dB) versus the data symbol index (relative to the middle symbol with N odd) for channels

$$\mathbf{f} = [1 \ 1]^t \quad (3.108)$$

and

$$\mathbf{f} = [1 \ 2 \ 1]^t \quad (3.109)$$

The results are valid for all odd data block lengths up to $N = 11$. The only assumption concerning the number of training symbols, N_t , within a training block, is that it is greater than or equal to the channel memory. The channel memory is $L = 1$ for (3.108), and

Figure 3.7 MSE degradation (in dB) from Ideal versus symbol index (relative to the middle symbol with N odd) for: DFBE detector, channel response (3.108) with $N_t \geq 1$, and channel response (3.109) with $N_t \geq 2$.

$L = 2$ for (3.109). Decisions are assumed to be made from both edges. The additional dB shift due to the fraction of training is not taken into account, but is easily calculated given N_t and equation (3.2). It is observed that the MSE now increases towards the edges of the block. For $N = 11$, the worst case degradation is reduced from 18.4 dB, for the LBE detector, to 6.4 dB, for the DFBE detector. Recall that the MSE in the middle of the data block is computed assuming correct edge decisions. Thus the probability of a decision error in the middle strongly depends on the probability of a decision error at the edges. The results presented thus far are for Toeplitz correlation matrices only.

Figure 3.8 shows the MSE performance for the channel defined in (3.109), with 0, 1, and 2 symbols per training block. The correlation matrix is no longer Toeplitz in general. Decisions are made only from the start or left edge of the data block. The MSE performance is no longer symmetric within the data block. In this case the results are valid for all data block lengths up to $N = 11$. This figure illustrates how the MSE increases as fewer training symbols are used. It should be noted, however, that a higher throughput can be achieved and/or shorter data block lengths can be used, as the number of training symbols is decreased. Note also that the performance with no training symbols is the same as that for a simple DFE detector (operating on the first tap of the channel response), which has a degradation of $10 \log(6) = 7.78$ dB for this channel.

As a comparison example, suppose the channel is as defined in (3.109), and a throughput of 80% is required. One option is to use $N = 8$, $N_t = 2$, and the Toeplitz based DFBE algorithm. A second option is to use $N = 4$, $N_t = 1$, and the non-Toeplitz based DFBE algorithm. Not only is the performance slightly better for the second option, because of the shorter data blocks, but the complexity is also considerably less. (See Table 3.2.)

Figure 3.9 shows the MSE performance for the same case as above, except that decisions are made from the end of the data block instead of from the start. The MSE of the first few symbols to be detected is seen to be significantly degraded for the non-Toeplitz cases. Recall that any feedback errors will affect the performance for the rest of the data block. This illustrates the importance of choosing to make decisions only from the start of a data block, for non-Toeplitz cases. This figure is valid only for $N = 11$.

Figure 3.8 MSE degradation (in dB) from Ideal versus symbol index (relative to the last symbol detected at the end) for: DFBF detector, channel response (3.109), and $N_t = 0, 1$, and 2.

Figure 3.9 MSE degradation (in dB) from Ideal versus symbol index (relative to the last symbol detected at the start) for: DFBE detector, channel response (3.109), and $N_t = 0, 1$, and 2.

3.5 Maximum Likelihood Block Estimation

Assuming additive white Gaussian noise (AWGN) the objective of the maximum likelihood block estimator (MLBE) detector is similar to that of the previous detectors. That is, the MLBE detector should select the data vector $\tilde{\mathbf{b}}$ which minimizes the sum of squared errors (SSE) as defined in (3.12) and (3.15). The difference is that the permissible $\tilde{\mathbf{b}}$ vectors can now only be selected from the set of valid quantized transmit data vectors. For the LBE and DFBE detectors, the elements of $\hat{\mathbf{b}}$ are allowed to take on continuous values in the estimation process, and the symbol estimates are quantized only during the final decision making step. Minimizing the SSE in (3.15), is equivalent to maximizing the objective function

$$J(\tilde{\mathbf{b}}) = \text{Re} [\mathbf{z}^t \tilde{\mathbf{b}}^*] - \frac{1}{2} \tilde{\mathbf{b}}^t \mathbf{R} \tilde{\mathbf{b}}^* \quad (3.110)$$

This is because the constant energy term, E_c , and scale factor of $-1/2$, do not influence the relative importance of the likelihoods.

Figure 3.10 shows a conceptual block diagram for the MLBE approach. The decisions are output in blocks. For short data block lengths, N , the objective function may be evaluated for each possible data block. An efficient way to do this is to use an expanding tree algorithm (ETA). For longer block lengths (longer than the channel length, $L + 1$), a variation of the modified Viterbi algorithm (MVA) proposed by Ungerboeck [Ung74] is applicable. An ETA is also useful for initializing the MVA. The ETA and MVA do not require a whitened matched filter, as defined by Forney [For72], but only a matched filter, as do the LBE and DFBE detectors. Actually, the matched filter is a true matched filter, for all the data symbols, only if the number of training symbols, N_t , is greater than or equal to the channel memory, L . When this is the case, the correlation matrix, $\mathbf{R}$, is Toeplitz. The distinction between the Toeplitz and non-Toeplitz cases is not as important for the MLBE detector as it is for the LBE and DFBE detectors. This is because inverse matrices are not required, either explicitly or implicitly. Thus, the presentation is not organized along these lines. Clarifying comments are made to distinguish the Toeplitz and non-Toeplitz cases only where appropriate.

Section 3.5.1 describes the expanding tree algorithm (ETA).

Section 3.5.2 describes a variation of the modified Viterbi algorithm (MVA), useful only when $N > L + 1$.

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Figure 3.10 Maximum Likelihood Block Estimation (MLBE) approach.

The performance for the MLBE can be approximated and/or bounded using the distance properties of the continuous-time signal defined in (3.8). The performance analysis, and a tree search algorithm used for evaluating the performance, are presented in Sections 3.5.3 and 3.5.4. Section 3.5.5 presents the performance of the MLBE detector for some example channels.

3.5.1 Expanding Tree Algorithm

An expanding tree algorithm is useful both as a complete solution for short data block lengths ($N \leq L+1$), and as a startup algorithm for longer data block lengths. The algorithm will be derived for the general case of a non-Toeplitz correlation matrix, $\mathbf{R}$. Simplifications for the Toeplitz case will be pointed out where appropriate.

Define the set of N -dimensional vectors

$$\tilde{\mathbf{b}}_k = [b_0 \ b_1 \ \dots \ b_k \ 0 \ 0 \ \dots \ 0]^t, \quad k = 0 \dots N-1 \quad (3.111)$$

where the subscript, k , indicates that only the first $k+1$ symbols can be non-zero symbols. From (3.110), the objective function for a data vector of this form can be represented as

$$\begin{aligned} J(\tilde{\mathbf{b}}_k) &= \text{Re} \left[\mathbf{z}^t \tilde{\mathbf{b}}_k^* \right] - \frac{1}{2} \tilde{\mathbf{b}}_k^t \mathbf{R} \tilde{\mathbf{b}}_k^* \\ &= \text{Re} \left[\sum_{l=0}^k z_l \tilde{b}_l^* \right] - \frac{1}{2} \sum_{l=0}^k \sum_{n=0}^k \tilde{b}_l r_{l,n} \tilde{b}_n^* \\ &= J(\tilde{\mathbf{b}}_{k-1}) + \text{Re} \left[z_k \tilde{b}_k^* \right] - \frac{1}{2} |\tilde{b}_k|^2 r_{k,k} - \text{Re} \left[\sum_{l=0}^{k-1} \tilde{b}_l r_{l,k} \tilde{b}_k^* \right] \end{aligned} \quad (3.112)$$

Assuming constant amplitude data symbols, the third term in the last line of (3.112) is a constant and can be dropped, since only relative likelihoods are important. (The constant amplitude assumption is not necessary in general, but is made for convenience, to ensure that the algorithms presented are the most efficient for MPSK signaling types.) Thus the objective function can be computed recursively, for a given block of N data symbols, using

$$J(\tilde{\mathbf{b}}_k) = J(\tilde{\mathbf{b}}_{k-1}) + \text{Re} \left[\left(z_k - \text{sum}_k(\tilde{\mathbf{b}}_{k-1}) \right) \tilde{b}_k^* \right], \quad k = 0 \dots N-1 \quad (3.113)$$

where

$$\text{sum}_k(\tilde{\mathbf{b}}_{k-1}) = \sum_{l=0}^{k-1} \tilde{b}_l r_{l,k}, \quad k = 0 \dots N-1 \quad (3.114)$$

The objective function needs to be evaluated for all possible data vectors $\tilde{\mathbf{b}}_{N-1}$. The number of final data vectors is M^N , where M is the size of the data symbol alphabet (e.g. $M = 2$ for BPSK, and $M = 4$ for QPSK). The value of the objective function can be obtained for all possible data vectors using an expanding tree algorithm (ETA) based on the recursion of (3.113).

Figure 3.11 shows an example expanding tree for BPSK signaling with data block length, $N = 3$. Each node of the tree represents one of the possible data vectors, and expands into M more nodes, as each additional non-zero symbol is added (k is incremented). The most likely data block is the one which provides the maximum value for $J(\tilde{\mathbf{b}}_{N-1})$. Equation (3.113) does not simplify further if $\mathbf{R}$ is Toeplitz. The summations defined in (3.114) can also be evaluated using an ETA. In general, a separate tree calculation is required for each k index because a different column of the $\mathbf{R}$ matrix is used. If $\mathbf{R}$ is Toeplitz, then the summations for $k < N-1$ can be obtained for free while calculating the final set of summations for $k = N-1$. The savings are not significant for the Toeplitz case, however, because including z_k in the root of each tree calculation, and computing the complete tree for each k turns out to be about the same amount of work as adding z_k to a set of all precomputed sums. For the non-Toeplitz case, the total number of complex multiplies, adds, and comparisons is

$$\begin{aligned}\#mult &= \frac{M(M^N - 1)}{M - 1} + \frac{MN(N - 1)}{2} \\ \#adds &= \frac{M^2(M^N - 1)}{(M - 1)^2} - \frac{MN}{M - 1} - M \\ \#comp &= M^N - 1\end{aligned}\tag{3.115}$$

For the Toeplitz case, the number of multiplies can be reduced slightly to

$$\#mult = \frac{M(M^N - 1)}{M - 1} + M(N - 1)\tag{3.116}$$

As expected, the complexity grows exponentially with block length, N . Note, however, that no reciprocals are required and that all the multiplies are trivial (can be absorbed into the adds) for BPSK and QPSK signaling. Although the number of final state metrics is M^N , they are only required once per block, and not once per data symbol.

The number of complex adds per data block and per data symbol for BPSK ($M = 2$) and QPSK ($M = 4$) signaling are provided in Table 3.3. For BPSK, the complexity

Figure 3.11 Expanding Tree Algorithm (ETA) for BPSK signaling with N=3.

Data Block Size (N)	Number of Complex Adds			
	BPSK (M=2)		QPSK (M=4)	
	per block	per symbol	per block	per symbol
1	0	0	0	0
2	6	3	20	10
3	20	6.67	104	34.67
4	50	12.50	444	111
5	112	22.40	1,808	361.60
6	238	39.67	7,268	1,211.33
7	492	70.29	29,112	4,158.86
8	1,002	125.25	116,492	14,561.50
9	2,024	224.89	466,016	51,779.56
10	4,070	407	1,864,116	186,411.60

Table 3.3 Number of complex adds per data block and per data symbol for BPSK and QPSK signaling with MLBE detection. The complexity is essentially the same for the Toeplitz and non-Toeplitz cases.

is reasonable for $N < 10$. For QPSK, the complexity grows very rapidly. The most complex case simulated in Chapter 6 is QPSK with 6 symbols per data block.

For short data block lengths, the objective function may be evaluated for all possible data blocks using the above ETA. For longer data block lengths (longer than the channel length, $L + 1$), a recursive technique based on the modified Viterbi algorithm (MVA) [Ung74] provides a more efficient extension.

3.5.2 Modified Viterbi Algorithm

The following derivation is essentially the same as that presented in [Ung74], except that non-Toeplitz as well as Toeplitz channel correlation matrices are considered. Also, the initialization requirements are explicitly spelled out, since initialization can easily dominate the complexity for short-block detection.

It is useful to define the state vectors

$$\mathbf{s}_{k-1} = [\tilde{b}_{k-1} \ \tilde{b}_{k-2} \ \dots \ \tilde{b}_{k-L}]^t, \quad k = L \dots N \quad (3.117)$$

Assuming constant amplitude data symbols, the recursion formula for the objective function defined in (3.113), for $k \geq L$, can be written as

$$J_k = J_{k-1} + V(z_k, \mathbf{s}_k, \mathbf{s}_{k-1}), \quad k = L \dots N-1 \quad (3.118)$$

where the functional dependence

$$J_k = J(\tilde{\mathbf{b}}_k) = J(\mathbf{s}_{L-1}, \dots, \mathbf{s}_k) \quad (3.119)$$

is to be understood from the subscript k , and

$$V(z_k, \mathbf{s}_k, \mathbf{s}_{k-1}) = \text{Re} \left[(z_k - \text{sum}_k(\mathbf{s}_{k-1})) \tilde{b}_k^* \right] \quad (3.120)$$

where

$$\begin{aligned} \text{sum}_k(\mathbf{s}_{k-1}) &= \sum_{l=1}^L \tilde{b}_{k-l} r_{k-l,k} \\ &= \mathbf{s}_{k-1}^t \mathbf{r}_k \end{aligned} \quad (3.121)$$

and

$$\mathbf{r}_k = [r_{k-1,k} \ \dots \ r_{k-L,k}]^t \quad (3.122)$$

If $\mathbf{R}$ is Toeplitz then

$$\mathbf{r}_k = [r_1 \ r_2 \ \dots \ r_L]^t \quad (3.123)$$

for all k , and the summations in (3.121) only need to be calculated once per data block.

Define

$$F(\mathbf{s}_k) = \max_{\tilde{\mathbf{b}}_k} [J_k] = \max_{\mathbf{s}_{L-1} \dots \mathbf{s}_k} [J_k] \quad (3.124)$$

That is, $F(\mathbf{s}_k)$ is the maximum J_k over all possible data symbol sequences ending in state $\mathbf{s}_k$, or equivalently over all possible valid state sequences ending in state $\mathbf{s}_k$. Bellman's dynamic programming principle can be applied to (3.124), yielding the convenient recursion

$$F(\mathbf{s}_k) = \max_{\mathbf{s}_{k-1} | \mathbf{s}_k} \left[F(\mathbf{s}_{k-1}) + V(z_k, \mathbf{s}_k, \mathbf{s}_{k-1}) \right] \quad (3.125)$$

$k = L \dots N-1$

with initial conditions

$$F(\mathbf{s}_{L-1}) = J(\tilde{\mathbf{b}}_{L-1}) \quad , \text{ for all states } \mathbf{s}_{L-1} \quad (3.126)$$

The initial conditions can be calculated using the ETA based on (3.113) with $N = L$. If the training signal associated with the L -state of previous training symbols (and decisions, if block decision feedback is required) was not subtracted off, then the MVA could alternatively be initialized at time $k = 0$ by simply setting the metric associated with this known L -state to a large number. The MVA could then start operating normally from time $k = 0$, instead of time $k = L$. The ETA method of initialization is recommended instead, because it is more efficient. For short-blocks, efficient initialization and termination are key to reducing the complexity. For example, when $N \leq L + 1$, only the ETA initialization portion of the MVA is required.

Solving (3.125) yields a form of the Viterbi algorithm. The solution can be represented graphically using a trellis [For72, For73, Ung74]. The problem is that of choosing the best path through a trellis representing all possible state sequences, $\mathbf{s}_{L-1}, \dots, \mathbf{s}_{N-1}$.

Figure 3.12 shows a trellis representation for BPSK signaling with channel memory $L = 2$. In this example, only two previous states can lead to each current state, and thus the maximum metric at each state is obtained by choosing between one of two possibilities. There is always one and only one best path through the trellis for each of

Figure 3.12 Viterbi algorithm trellis for BPSK signaling with channel memory $L = 2$.

the current states. These paths are called survivors, and represent the path history for each state. The entire data block is detected by selecting the state at time $k = N - 1$ with the maximum path metric. Note that the removal of the training signal associated with the next N_t training symbols, is what permits the MVA to correctly stop and make its decision at time $k = N - 1$. Without the removal of this training signal, the MVA would have to continue to the end of the training block before making its decision. It is interesting to note the the conventional Viterbi algorithm (VA) [For72] would have to proceed to the end of the training block (i.e observation time) even with the training signal removed. This is one of the reasons the MVA approach was selected over the conventional VA approach. The other reason is that no multiplies are required for simple modulation formats such as BPSK and QPSK.

The surviving paths often merge together as their history is followed back in time. This could allow decisions to be made on past symbols without processing the entire signal, but still with some delay. For performance and practical reasons, a fixed decision delay, typically 3 to 5 times the channel memory, is often selected. This is not useful if the data block lengths of interest are shorter than this delay. One of the advantages of employing short block lengths (e.g $N \simeq L$), with maximum likelihood detection, is that the decisions, which are required by the channel estimator, are relatively up-to-date.

The complexity for the non-Toeplitz case, including ETA set-up of initial conditions, is given by

$$\begin{aligned} \#mult &= \frac{M(M^L - 1)}{M - 1} + \frac{ML(L - 1)}{2} + (N - L) [M^{L+1} + ML] \\ \#adds &= \frac{M^2(M^L - 1)}{(M - 1)^2} - \frac{ML}{M - 1} - M + (N - L) \left[\frac{M(M^{L+1} - 1)}{M - 1} \right] \\ \#comp &= (N - L) [(M - 1)M^L] + M^L - 1 \end{aligned} \quad (3.127)$$

For the Toeplitz case, the number of multiplies can be reduced slightly to

$$\#mult = \frac{M(M^L - 1)}{M - 1} + ML + (N - L)M^{L+1} \quad (3.128)$$

Again, it is emphasized that no reciprocals are required, and that all the multiplies are trivial (can be absorbed into the adds) for BPSK and QPSK signaling. For $N \leq L + 1$, the complexity is the same as that for the ETA alone, and is given in Table 3.3.

3.5.3 Performance Analysis

The mean squared error (MSE) caused by the noise, after matched filtering, and just prior to sequence estimation, is equal to the MSE at the output of an ideal isolated-pulse matched filter detector (Ideal). An estimate of the MSE alone is not sufficient to determine the performance of an MLBE detector. The distance properties of the continuous-time signal defined in (3.8) are also required.

Forney [For72] showed that the probability of a specific error event, ϵ , starting at a specific time, k_1 , is closely approximated (and slightly upper bounded) by

$$P(\epsilon) = Q\left(\frac{d(\epsilon)}{2\sigma}\right) P(e|\epsilon) \quad (3.129)$$

where $d(\epsilon)$ is the Euclidean distance of the error event, σ^2 is the noise variance, and $P(e|\epsilon)$ stands for the probability that the particular error sequence, $e = \{e_k\} = \{\tilde{b}_k - b_k\}$, is valid. That is, $P(e|\epsilon)$ is the probability that the actual data sequence plus the error sequence is another valid data sequence. Assuming independent data symbols

$$P(e|\epsilon) = \prod_k P(e_k|\epsilon) \quad (3.130)$$

As examples, consider BPSK and QPSK signaling with equally likely data symbols for b_k . For BPSK there are 3 possible error symbols, e_k , with values 0, +2, -2, and corresponding probabilities of 1, 1/2, 1/2. For QPSK there are 9 possible error symbols with values 0, +2, -2, +2j, -2j, +2+2j, +2-2j, -2+2j, -2-2j, and corresponding probabilities of 1, 1/2, 1/2, 1/4, 1/4, 1/4, 1/4.

A bound on the overall probability of any error event starting at a specific time is given by the union bound, which states that the probability of the union of a number of events is less than or equal to the sum of their individual probabilities. Let E be the set of all possible error events, ϵ , starting at time k_1 . Then the probability that any error event starts at time k_1 is bounded by [For72]

$$\begin{aligned} P(E) &\leq \sum_{\epsilon \in E} P(\epsilon) \\ &= \sum_{\epsilon \in E} Q\left(\frac{d(\epsilon)}{2\sigma}\right) P(e|\epsilon) \end{aligned} \quad (3.131)$$

An alternate way of expressing $P(E)$ is [For72]

$$P(E) \leq \sum_{d \in D} Q\left(\frac{d}{2\sigma}\right) K_d \quad (3.132)$$

where D is the set of all possible $d(\epsilon)$ and K_d is the total of the probabilities of all error sequences with distance d . That is,

$$K_d = \sum_{e \in E_d} P(e|\epsilon) \quad (3.133)$$

where E_d is the set of all error events with distance d .

The probability of an error event is often approximated by just the single term in (3.132) with minimum distance, so that [For72]

$$P(E) \simeq K_{d_{min}} Q\left(\frac{d_{min}}{2\sigma}\right) \quad (3.134)$$

The accuracy of this estimate depends on the signal-to-noise ratio, how close the other distances are to d_{min} , and the size of the coefficients for the other larger distances. This approximation is usually quite accurate for partial response channels with controlled integer-valued ISI samples. The performance for time-dispersive fading channels, however, cannot be so easily approximated. This is because the complex valued ISI samples usually result in only a few error sequences with distance d_{min} , while there are often many more error sequences with distances very close to d_{min} . For this reason, an approximation has been adopted which includes all error sequences within a given range of distances. That is

$$P(E) \simeq \sum_{e \in E_a} Q\left(\frac{d(e)}{2\sigma}\right) P(e|\epsilon) \quad (3.135)$$

where E_a represents the set of all error events with squared-distances in the range $[d_{min}^2, \alpha d_{min}^2]$, with α being the range parameter. This approximation can be evaluated using a type of tree search algorithm.

3.5.4 Tree Search Algorithm

A tree search algorithm for finding all error sequences with squared Euclidean distance in the range $[d_{min}^2, \alpha d_{min}^2]$ is given below. The algorithm uses many of the ideas

from [For72, Mes73, Aul81]. In contrast to the exhaustive search technique described in [Mes73], DFE coefficients are not used, and an upper and lower bounding technique is not required since the distances sought are defined for finite observation times. The approach taken is more closely related to the continuous-time algorithm presented in [Aul81], for determining the distance properties of continuous phase modulation (CPM) signals. The Toeplitz case ($N_t \geq L$) is assumed initially. The modifications required to handle the non-Toeplitz case are minor, and are addressed at the end of the section.

The algorithm is sequential in the symbol index, k . The sequential property is due to the fact that the squared Euclidean distances are incrementally additive on a symbol by symbol basis. Specifically, for a fixed error event ϵ , and corresponding error sequence e (assumed to start at time $k_1 = 0$), the squared-distance up to time $(k+1)T$ is given by

$$\begin{aligned} d^2(k, \epsilon) &= \int_0^{(k+1)T} |e(t)|^2 dt \\ &= \sum_{i=0}^k \int_{iT}^{(i+1)T} |e(t)|^2 dt \\ &= d^2(k-1, \epsilon) + I_k(\epsilon) \end{aligned} \quad (3.136)$$

where the k -th integral (also the incremental squared-distance) is defined as

$$I_k(\epsilon) = \int_{kT}^{(k+1)T} |e(t)|^2 dt \quad (3.137)$$

and the relevant portion of the error signal is given by

$$e(t) = \sum_{i=k-L}^k e_i f(t - iT) \quad , t \in [kT, (k+1)T) \quad (3.138)$$

Note that in the k -th interval, $e(t)$ is only a function of the current and previous L error symbols. Thus, the incremental squared-distances can be precomputed for all possible combinations of $L+1$ error symbols. The number of possibilities is M_e^{L+1} , where M_e is the number of possible error symbol values. For BPSK $M_e = 3$, and for QPSK $M_e = 9$. Thus the number of integrals to be evaluated can be very large, even for small values of L .

Substituting (3.138) into the integral of (3.137), and arranging appropriately gives

$$I_k(\epsilon) = \sum_{i=-L}^0 \sum_{j=-L}^0 e_{k+i} I_{i,j} e_{k+j}^* \quad (3.139)$$

where

$$I_{i,j} = \int_0^T f(t - iT) f^*(t - jT) dt \quad , i, j = -L \dots 0 \quad (3.140)$$

Note that there are only $(L + 1)^2$ values of $I_{i,j}$ to precompute, which will usually be a much smaller number than the number of $I_k(\epsilon)$ values to be computed. When the integrals are difficult to calculate, method (3.139) is recommended. When the integrals are easy to calculate, as is the case for a discrete channel model, then it is usually more convenient to compute the $I_k(\epsilon)$ directly using (3.137) and (3.138), or analogous discrete versions of (3.137) and (3.138).

Conceptually, the minimum Euclidean distance for a given k can be obtained by computing $d^2(k, \epsilon)$ for all error events, ϵ , and selecting the minimum. This approach would be very inefficient. A more efficient tree search algorithm is described below.

A tree structure is used to potentially represent all possible error sequences. The root node of the tree represents the no-error sequence (or null sequence). Each node can be expanded along M_e branches, one for each possible error symbol value. Each time a node is expanded, it represents an increment in the symbol index k . As the tree expands, the end nodes represent unique error sequences, but not necessarily completed error events. Each node keeps a state variable representing the last L error symbols in this error sequence. If the state variable represents L consecutive no-error symbols, then that node corresponds to a completed error event, and the tree can stop expanding at that node. No attempt is made to collapse the tree into a trellis. This is because the number of nodes requiring expansion at each stage was typically found to be much smaller than the number of states in the equivalent trellis representation.

The first step is to compute the upper bound,

$$d_{ub}^2 = |e_{min}|^2 \int |f(t)|^2 dt \quad (3.141)$$

on the minimum squared Euclidean distance, d_{min}^2 , where e_{min} stands for the non-zero error symbol with minimum magnitude. An upper bound on αd_{min}^2 is given by αd_{ub}^2 . The method is to expand the tree, dropping or saving nodes as the tree expands. Nodes are dropped when the accumulated squared-distance exceeds αd_{ub}^2 , and nodes are saved when the no-error state is reached and the squared-distance is less than or equal to αd_{ub}^2 . Nodes, and their corresponding sequences, can be dropped in this way because the

distance is a non-decreasing function of index k . The upper bound, d_{ub}^2 , depends on the distances found and is updated as the tree expands. Each time a completed error event is encountered, the associated squared-distance is compared to the current upper bound, and if the squared-distance is less than d_{ub}^2 then d_{ub}^2 and αd_{ub}^2 are both updated. Usually all end nodes are dropped or saved in a finite number of steps, even for infinite data block lengths. For short data block lengths, as in this application, all error sequences are forced to terminate in the no-error state after N error symbols. Each saved end node represents a completed error sequence, and a corresponding squared-distance. At the completion of the search d_{min}^2 will have been found, and also all sequences with squared-distances less than or equal to αd_{min}^2 . The list of saved end nodes may now be trimmed of any with squared-distances exceeding αd_{min}^2 .

Also associated with each end node is the running product of the error sequence probability, $P(k, e|\epsilon)$, which can also be computed recursively, because

$$\begin{aligned} P(k, e|\epsilon) &= \prod_{i=0}^k P(e_i|\epsilon) \\ &= P(k-1, e|\epsilon) P(e_k|\epsilon) \end{aligned} \quad (3.142)$$

as defined earlier in (3.130).

The Toeplitz case ($N_t \geq L$) has been assumed for the above analysis and tree search algorithm definition. With this assumption, all error sequences eventually enter the no-error state, forcing only completed error events to occur within the observation interval. Thus, the probability of an error event starting at time $k_1 = 0$ must be the same or greater than the probability of an error event starting at any other time. This is because the short-block nature of the signaling technique forces the set of possible error events for $k_1 \neq 0$ to be a subset of those allowed for $k_1 = 0$. For these reasons, the probability of any error event starting at time $k_1 = 0$ is taken as the relevant performance indicator for the Toeplitz case, and is computed using equation (3.135), which requires the above tree search algorithm.

For the non-Toeplitz case ($N_t < L$), error sequences are allowed to terminate with only N_t no-error symbols. These error sequences are only valid if they terminate at the end of a training block. All other error events are complete. The above tree search algorithm is directly applicable. The tree expands, and nodes are dropped and saved,

in the same way as before. The only change required is that nodes must also be saved when the most recent N_t symbols in the node's state vector are no-error symbols. Such incomplete error events are always valid for some starting index, k_1 . The probability of an error event starting at index $k_1 = 0$ is no longer guaranteed to be the worst case. This is because the set of possible error events for $k_1 \neq 0$ is no longer forced to be a subset of those allowed for $k_1 = 0$. The performance is again approximated using equation (3.135), where all saved error events are included. This approximation is more pessimistic in this case than it is for the Toeplitz case, since some error events are valid only for some starting indices, k_1 .

3.5.5 Performance for Example Channels

As noted above, an estimate of the MSE alone is not sufficient to determine the performance for an MLBE detector. The distance properties of the data signal are also required. The asymptotic (high SNR) degradation from Ideal for the MLBE detector is determined by the minimum distance, d_{min} .

For the discrete T -spaced channel models

$$\mathbf{f} = [1 \ 1]^t \quad (3.143)$$

and

$$\mathbf{f} = [1 \ 2 \ 1]^t \quad (3.144)$$

the asymptotic degradations are 0 dB and $10 \log(6/4) = 1.8$ dB respectively, for the Toeplitz case ($N_t \geq L$). From Section 3.4.4, the corresponding degradations for the DFBE detector are approximately 2.5 dB and 6.5 dB (with $N \approx 10$). The degradations given in Section 3.3.4 for the LBE detector are much worse.

For non-Toeplitz examples, consider channel (3.144) with 1 and 0 symbols per training block. The MLBE degradations from Ideal are now $10 \log(6/3) = 3.0$ dB and $10 \log(6/1) = 7.8$ dB. The corresponding degradations for the DFBE detector are 6.7 dB and 7.8 dB.

More detailed performance examples are presented in Chapter 4.

Chapter 4

PERFORMANCE FOR FADING CHANNELS

This chapter presents the probability of error analysis for a complex baseband Rayleigh fading channel model. Both BPSK and QPSK signaling types are considered. Performance is evaluated for the following 6 detection schemes:

1. Ideal isolated pulse matched filter detection (Ideal),
2. Linear block estimation (LBE),
3. Decision feedback block estimation (DFBE),
4. Maximum likelihood block estimation (MLBE),
5. Conventional linear equalization (LE), and
6. Conventional decision feedback equalization (DFE).

The three block detectors are as described in Chapter 3. The performance for the Ideal detector provides a convenient reference and lower bound on the performance for the other detectors. The performance for the conventional LE and DFE detectors is included for comparison purposes.

4.1 Complex Baseband Channel Model

The channel is modeled as a discrete complex Gaussian (Rayleigh amplitude) fading channel, with one sample per symbol period. The channel response is defined by the complex vector

$$\mathbf{f} = [f_0 \ f_1 \ \dots f_L]^t \quad (4.1)$$

having $L + 1$ significant samples. The real and imaginary components of the l -th complex sample are independent and Gaussian distributed with zero mean and variance σ_f^2 . Fractionally spaced signal models are considered in Chapter 6.

Only binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) type signaling schemes are considered. For BPSK the data symbols have unit amplitude,

and consist of plus and minus ones in the real part only. For QPSK the data symbols have a squared amplitude of 2, and consist of plus and minus ones in both the real and imaginary parts. Assuming that the complex data symbols are uncorrelated, the average energy per bit, for both BPSK and QPSK, is given by

$$E_b = 2 \sum_{l=0}^L \sigma_l^2 \quad (4.2)$$

The noise is modeled as additive white Gaussian noise (AWGN), where the variance of each real and imaginary noise component is $N_0/2$. For convenience, the average E_b/N_0 ratio per bit will sometimes be denoted as γ_b .

4.2 Performance for Fading Channels

For each detection scheme, the average probability of error, averaged over all channel responses, is given by

$$\begin{aligned} P(e) &= E[P(e|f)] \\ &= \sum_{f} P(e|f) P(f) \end{aligned} \quad (4.3)$$

where, for a given detection scheme, $P(e|f)$ is the probability of error given a specific fixed channel response, f . The dependence on the data block length, N , and training block length, N_t , is to be understood. The expectation in (4.3) is with respect to f , and $P(f)$ represents the probability of a specific f , the summation being over all f . The brute force calculation of $P(e)$ is very difficult in general. However, a Monte Carlo calculation is fairly easy to perform and can yield tight confidence intervals with reasonable execution times. What remains is the determination of the appropriate expression for $P(e|f)$ for each of the six detection schemes.

4.3 Ideal Isolated Pulse Matched Filter Detection

For the Ideal detector, and for both BPSK and QPSK signaling, it is well known that the probability of bit error, given a fixed channel f , is given by (e.g. [Pro83])

$$P(e|f) = Q\left(\sqrt{\frac{2E_b(f)}{N_0}}\right) \quad (4.4)$$

where

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\alpha^2}{2}\right) d\alpha \quad (4.5)$$

is the area under the upper tail of the normal distribution, and

$$E_b(f) = \sum_{l=0}^L |f_l|^2 \quad (4.6)$$

is the energy per bit for a given channel, f . This formula for the Ideal detector is independent of the data and training block lengths. Substituting (4.4) into (4.3) gives $P(e)$. For this Ideal detector case the performance is equivalent to that of an ideal diversity combining system with $(L+1)$ channels or paths. Closed form solutions exist. For the case of $(L+1)$ equally energetic paths, the probability of bit error is given by [Pro83, p. 474]

$$P(e) = \left(\frac{1-\mu}{2}\right)^{L+1} \sum_{l=0}^L \binom{L+l}{l} \left(\frac{1+\mu}{2}\right)^l \quad (4.7)$$

with

$$\mu = \sqrt{\frac{\gamma_p}{1+\gamma_p}} \quad (4.8)$$

where γ_p is the average E_b/N_0 ratio per path. The overall average E_b/N_0 ratio per bit is given by

$$\gamma_b = (L+1) \gamma_p \quad (4.9)$$

For the case of $(L+1)$ non-equally energetic paths, the probability of bit error is given by [Pro83, p. 486]

$$P(e) = \frac{1}{2} \sum_{l=0}^L \pi_l \left[1 - \sqrt{\frac{\gamma_l}{1+\gamma_l}} \right] \quad (4.10)$$

with

$$\pi_l = \prod_{\substack{i=0 \\ i \neq l}}^L \frac{\gamma_l}{\gamma_l - \gamma_i} \quad (4.11)$$

where γ_l is the average E_b/N_0 ratio for the l -th path. The overall average E_b/N_0 ratio per bit is given by

$$\gamma_b = \sum_{l=0}^L \gamma_l \quad (4.12)$$

Figure 4.1 shows the probability of bit error versus average E_b/N_0 for 1, 2, 3, 4, 6, 10, 20, and 100 equally energetic paths. A channel with only 1 path exhibits flat fading. The diversity gain with 2 and 3 paths is very significant. The additional diversity gain with more than 4 paths shows diminishing returns, even for equally energetic paths. This observation indicates that most of the diversity gain for a long channel response would be obtained if the energy of the strongest 3 or 4 samples is considered. If most of the channel energy is in the first 3 or 4 symbol periods, then 2 or 3 symbols per training block is sufficient to observe this energy for all data block symbols.

4.4 Linear Block Estimation

The error signal at the output of the LBE is due to linear operations on the original Gaussian noise vector. Consequently, the output error signal is still Gaussian and the real and imaginary components of a given sample are still uncorrelated with equal variance, even though correlation between samples can exist. From (3.57), the variance of the k -th complex symbol estimate, given channel response $\mathbf{f}$, is given by

$$v_k(\mathbf{f}) = N_0 q_{k,k}(\mathbf{f}) \quad , k = 0 \dots N-1 \quad (4.13)$$

where $q_{k,k}(\mathbf{f})$ is the k -th diagonal element of the $\mathbf{Q}$ matrix defined in (3.48) for a specific $\mathbf{f}$. From (3.47) and (3.51), the mean value of each symbol estimate is the true data symbol, which was defined to have a magnitude of one in each dimension contributing a bit. Thus, the probability of bit error for the k -th symbol, given channel response $\mathbf{f}$, is given by

$$P_k(e|\mathbf{f}) = Q\left(\sqrt{\frac{2}{N_0 q_{k,k}(\mathbf{f})}}\right) \quad , k = 0 \dots N-1 \quad (4.14)$$

Averaging over the N data symbols gives the desired result

$$P(e|\mathbf{f}) = \frac{1}{N} \sum_{k=0}^{N-1} P_k(e|\mathbf{f}) \quad (4.15)$$

The above probability of bit error for the LBE detector holds for both BPSK and QPSK signaling. Substituting (4.15) into (4.3) gives $P(e)$. A Monte Carlo calculation is used to evaluate (4.3). The formulas are the same for the Toeplitz and non-Toeplitz cases.

Figure 4.1 Probability of bit error versus average E_b/N_0 for a known, slow fading, multipath channel, with equally energetic T-spaced paths, BPSK or QPSK signaling, and Ideal isolated pulse matched filter detection.

4.5 Decision Feedback Block Estimation

An exact calculation of the average probability of bit error for the DFBE detector is complicated by the presence of feedback decisions. If correct feedback decisions are assumed, as for the previous MSE analysis, the probability of bit error can be evaluated using the same approach as that for the LBE detector. From Section 3.4.3, the variance of the k -th symbol, for the non-Toeplitz (Cholesky) case, is given by

$$v_k(\mathbf{f}) = N_0 q_{0,0,(N-k)}(\mathbf{f}) \quad , k = 0 \dots N-1 \quad (4.16)$$

where decisions are made recursively from the starting edge of the data block. Note that the dimension of the $\mathbf{Q}$ matrix, indicated by the bracketed subscript, is reduced by one each time the symbol index is incremented, and only the first diagonal element of the $\mathbf{Q}$ matrix is required each time. The variance of the k -th symbol, for the Toeplitz (Levinson) case is given by

$$v_k(\mathbf{f}) = v_{N-k-1}(\mathbf{f}) = N_0 q_{0,0,(N-2k)}(\mathbf{f}) \quad , k = 0 \dots \left\lfloor \frac{N-1}{2} \right\rfloor \quad (4.17)$$

where decisions are made from both edges of the data block. Note that the dimension of the $\mathbf{Q}$ matrix is reduced by two each time the symbol index is incremented. Assuming correct early decisions, the variance for the later decisions tends to decrease because of the reduced block size. In practice, the actual probability of error for a later decision is likely to become dominated by the probability of a feedback error from an earlier decision. For this reason, a more appropriate and pessimistic estimate of performance is taken to be the probability of error for the earliest edge decision. That is,

$$P(e|\mathbf{f}) \approx Q \left(\sqrt{\frac{2}{N_0 q_{0,0,(N)}(\mathbf{f})}} \right) \quad (4.18)$$

This approximation is valid for both the Toeplitz and non-Toeplitz cases. Of course the performance still depends on whether the channel correlation matrix is Toeplitz or not. This result for the DFBE detector holds for both BPSK and QPSK signaling.

Substituting (4.18) into (4.3) gives an estimate of $P(e)$ for the DFBE detector. A Monte Carlo calculation is used to evaluate (4.3).

4.6 Maximum Likelihood Block Estimation

For the MLBE detector, an approximation has been adopted which includes all error sequences with distances within a certain range of distances. From (3.135), the probability that any error event starts at a specific symbol is approximately upper bounded by

$$P(e|f) \approx \sum_{\epsilon \in E_\alpha} Q\left(\frac{d(\epsilon)}{N_0}\right) P(\epsilon) \quad (4.19)$$

where E_α represents the set of all error events ϵ with squared-distances in the range $[d_{\min}^2, \alpha d_{\min}^2]$, with α being the range parameter. This approximation is evaluated using the tree search algorithm described in Section 3.5.4. The probability of error event results presented for the MLBE detector are evaluated with $\alpha=1.25$. Performance is not the same for BPSK and QPSK signaling, since the distance properties depend on the signaling format. As discussed at the end of Section 3.5.4, the details of the tree search algorithm, used to evaluate (4.19), also depend on whether the channel correlation matrix is Toeplitz or not.

Substituting (4.19) into (4.3) gives an approximation of $P(e)$ for the MLBE detector. A Monte Carlo calculation is used to evaluate (4.3). It is more appropriate to compare the above result to the symbol error rate (as opposed to the bit error rate) of the Ideal scheme.

4.7 Conventional Linear Equalization

The performance for the LBE detector is compared with that provided by a conventional linear equalizer (LE). Using the minimum mean squared error (MMSE) criterion (not the same as the LSSE criterion used for the block detectors), the probability of bit error for an LE can be approximated as [Pro83, Qur85]

$$P(e|f) = Q\left(\sqrt{2\gamma_d}\right) \quad (4.20)$$

where

$$\gamma_d = \frac{1 - mmse}{mmse} \quad (4.21)$$

is the ratio of signal energy per bit to residual ISI and noise variance at the input to the decision device. The residual ISI is assumed to be Gaussianly distributed for the above approximation. The MMSE for the LE is given by

$$mmse = 1 - \mathbf{h}^* \mathbf{A}^{-1} \mathbf{h} \quad (4.22)$$

where

$$\mathbf{h} = [0 \dots 0 \mathbf{f}^{*r} 0 \dots 0]^t \quad (4.23)$$

is an M_{le} -dimensional column vector containing the conjugate reversed elements of the channel vector $\mathbf{f}$, M_{le} being the total number of symbol-spaced taps for the LE, and

$$\mathbf{A} = \mathbf{R}^* + N_0 \mathbf{I} \quad (4.24)$$

is an M_{le} -by- M_{le} covariance matrix, $\mathbf{I}$ is an M_{le} -by- M_{le} identity matrix, and N_0 is the variance of the complex noise. The M_{le} -by- M_{le} matrix, $\mathbf{R}$, has the same form as the Toeplitz channel correlation matrices required for the three block detectors, with $N_t \geq L$. The elements of $\mathbf{R}$ are given by

$$\begin{aligned} r_{i,j} &= \sum_n f_{n-i} f_{n-j}^* \quad , \quad i, j = 0 \dots M_{le} - 1 \\ &= \begin{cases} r_{j-i} & , \quad |j - i| \leq L \\ 0 & , \quad otherwise \end{cases} \end{aligned} \quad (4.25)$$

For the conventional LE, the dimension of $\mathbf{R}$ depends on the total number of filter taps, M_{le} . For the block detectors, the dimension of $\mathbf{R}$ depends on the block size, N . The performance of the LE depends on the timing reference or location of the main tap, which in turn determines the location of $\mathbf{f}^{*r}$ in the $\mathbf{h}$ vector. For the channel model selected, with equally energetic paths, the mean squared error is minimized by centering $\mathbf{f}^{*r}$ in $\mathbf{h}$.

The above probability of bit error for the LE detector holds for both BPSK and QPSK signaling. Substituting (4.20) into (4.3) gives $P(e)$. A Monte Carlo calculation is used to evaluate (4.3).

4.8 Conventional Decision Feedback Equalization

The performance for the DFBE detector is compared with that provided by a conventional decision feedback equalizer (DFE). Using the minimum mean squared error (MMSE) criterion, and assuming correct feedback decisions, the probability of bit error for a DFE can also be approximated using equations (4.20), (4.21) and (4.22). For the DFE case [Pro83, Qur85]

$$\mathbf{h} = [0 \dots 0 \mathbf{f}^{*r}]^t \quad (4.26)$$

is an M_{fwd} -dimensional column vector containing the conjugate reversed elements of the channel vector $\mathbf{f}$, M_{fwd} being the total number of symbol-spaced forward taps for the DFE, and

$$\mathbf{A} = \mathbf{R}^* + N_0 \mathbf{I} \quad (4.27)$$

is an M_{fwd} -by- M_{fwd} covariance matrix. The M_{fwd} -by- M_{fwd} matrix, $\mathbf{R}$, has a form similar to the non-Toeplitz Hermitian channel correlation matrices required for the three block detectors, with $N_t < L$. The elements of $\mathbf{R}$ are given by

$$r_{i,j} = \sum_{m=j-i}^{\min(M_{fwd}-1, L)} f_m f_{m+i-j}^* \quad , \quad j \geq i = 0 \dots M_{fwd} - 1 \quad (4.28)$$

For the conventional DFE, the dimension of $\mathbf{R}$ depends on the total number of forward filter taps, M_{fwd} . For the block detectors, the dimension of $\mathbf{R}$ depends on the block size, N . The number of feedback taps is assumed to equal the channel memory, L , so that all trailing ISI is completely cancelled by the feedback filter. Thus, the total number of taps for the DFE is $M_{fwd} + L$.

The above probability of bit error for the DFE detector holds for both BPSK and QPSK signaling. Substituting (4.20) into (4.3) gives $P(e)$. A Monte Carlo calculation is used to evaluate (4.3).

4.9 Performance Results

This section presents the fading channel performance results obtained for the six detection schemes considered. The detection schemes are denoted as Ideal, LBE, DFBE, MLBE, LE, and DFE, as defined previously. For the Ideal (isolated pulse matched filter) detector, the exact theoretical formulas given in Section 4.3 are used. Monte Carlo calculations are performed for the other five detectors.

The channel was defined to have a number of independent, T -spaced, equally energetic, Rayleigh fading paths. Results are presented for different numbers of paths. A data block length of $N = 6$ was used. This allows the simulation results presented in Chapter 6 to be directly compared to the theoretically based results presented here. A data block length of 6 was chosen for the simulation results, partly to ensure reasonable

execution times for the MLBE detector with QPSK signaling. The training block size, N_t , although somewhat arbitrary, is assumed to be at least as long as the channel memory, L , to take full advantage of the inherent diversity. Thus, only Toeplitz cases are considered here. The performance for other multipath intensity profiles and non-Toeplitz cases is presented in Chapter 6. The E_b/N_0 ratio is based on the average energy per signaling bit. The average energy per data bit can easily be calculated given a specific throughput efficiency, ρ . For example, with $N = 6$ and $N_t = 3$, so that $\rho = 2/3$, shifting the E_b/N_0 scale $10\log(1/\rho) = 1.76$ dB to the left gives the results in terms of the average energy per data bit. The average probability of error was obtained using 50,000 independent channel responses for the Monte Carlo calculations.

Figure 4.2 shows the performance for a channel with 2 fading paths. The degradations from Ideal, at a bit error rate of 10^{-3} , for the MLBE, DFBE, and LBE detectors are approximately 0.3 dB, 1.5 dB, and 3 dB, respectively. Recall that the theoretical asymptotic (high SNR) degradation from Ideal for the MLBE detector is 0 dB for 2 non-fading paths (see Section 3.5.5). The results for the LBE and DFBE detectors are valid for both BPSK and QPSK signaling. The results for the MLBE detector are appropriate only for BPSK signaling. The QPSK results for the MLBE detector are presented in a separate figure.

Figure 4.3 shows the corresponding performance results with 3 fading paths. The degradations from Ideal for the MLBE, DFBE, and LBE detectors are approximately 0.3 dB, 2 dB, and 3.7 dB respectively.

Figure 4.4 shows the corresponding performance results with 4 fading paths. The respective degradations are now approximately 0.3 dB, 2.2 dB, and 3.7 dB.

Figure 4.5 shows the performance results for the MLBE detector with QPSK signaling, and 2, 3, and 4 equally energetic paths. For comparison, the probability of symbol error (approximated as twice the probability of bit error) for the Ideal detector is also shown. The degradations from Ideal are slightly larger than those presented for BPSK, but are still well within 1 dB.

As expected, the MLBE outperforms the other detectors, and almost takes full advantage of the available inherent diversity as the number of paths is increased. The degradation from Ideal is less than 1 dB for all cases shown. The DFBE detector provides

Figure 4.2 Probability of bit error versus average E_b/N_0 for BPSK and QPSK signaling with: Ideal, LBE, DFBE, and MLBE (BPSK only) detection, a known slow fading multipath channel with 2 equally energetic T -spaced paths, and $N = 6$.

Figure 4.3 Probability of bit error versus average E_b/N_0 for BPSK and QPSK signaling with: Ideal, LBE, DFBE, and MLBE (BPSK only) detection, a known slow fading multipath channel with 3 equally energetic T -spaced paths, and $N = 6$.

Figure 4.4 Probability of bit error versus average E_b/N_0 for BPSK and QPSK signaling with: Ideal, LBE, DFBE, and MLBE (BPSK only) detection, a known slow fading multipath channel with 4 equally energetic T -spaced paths, and $N = 6$.

Figure 4.5 Probability of symbol error versus average E_b/N_0 for QPSK signaling with: Ideal and MLBE detection, known slow fading multipath channels with 2, 3 and 4 equally energetic T -spaced paths, and $N = 6$.

the next best performance, with degradations from Ideal ranging from 1.5 to 2.5 dB. It also makes good use of the inherent diversity as the number of paths is increased. The LBE detector provides the poorest performance, with degradations from Ideal of 3 to 4 dB. Even with these fairly large degradations, performance continues to improve as the number of paths is increased. Therefore, the positive effect of inherent diversity, due to time-dispersive fading paths, is more significant than the negative effect of severe ISI, even for the LBE detector.

These results can be directly compared with the simulation results presented in Section 6.2.1. As expected, the theoretically based approximation for the DFBE detector is shown to be a little pessimistic. Even so, the simulation results are typically within 0.5 dB, for all three block detectors.

Figure 4.6 compares the performance of the LBE and conventional LE detectors. The channel consists of 4 equally energetic T -spaced Rayleigh fading paths. A data block length of 6 was used for the LBE detector. Performance is presented for 6 and 9 tap LE detectors. These detectors are denoted as LBE(6), LE(6), and LE(9), respectively. The LBE(6) detector is observed to provide significantly better performance at high signal to noise ratios. If matrix calculations are used to calculate the LE filter coefficients from a channel estimate, then the complexity of the LE(6) detector is comparable to the complexity of the LBE(6) detector. After solving the matrix equations, the LE must still perform the filtering operation. It is interesting to note further that a fractional T/K -spaced implementation does not increase the dimension of the LBE matrix equation to be solved. The LE detector, however, will require K times as many taps to span the same number of symbol periods.

Figure 4.7 compares the performance of the DFBE and conventional DFE detectors. As for the previous comparison, the channel consists of 4 equally energetic T -spaced Rayleigh fading paths. A data block length of 6 was used for the DFBE detector. Performance is presented for DFE detectors with 4 and 6 forward taps. The number of feedback taps required is 3 in both cases. These detectors are denoted as DFBE(6), DFE(4,3), and DFE(6,3), respectively. The performance of all three detectors is very close. The DFE(6,3) detector is observed to provide slightly better performance than the DFBE(6) detector. The simulation results presented in Chapter 6 show that the

Figure 4.6 Probability of bit error versus average E_b/N_0 for BPSK and QPSK signaling with: Ideal, LBE(6), LE(6), and LE(9) detection, and a known slow fading multipath channel with 4 equally energetic T -spaced paths.

Figure 4.7 Probability of bit error versus average E_b/N_0 for BPSK and QPSK signaling with: Ideal, DFBE(6), DFE(4,3), and DFE(6,3) detection, and a known slow fading multipath channel with 4 equally energetic T -spaced paths.

theoretically approximated performance presented here for the DFBE(6) is about 0.5 dB pessimistic. The simulated performance for the DFBE(6) is found to be essentially the same as that shown for the DFE(6,3) detector. If matrix calculations are used to calculate the DFE filter coefficients from a channel estimate, then the complexity of the DFE(6,3) detector is comparable to the complexity of the DFBE(6) detector. After solving the matrix equations, the DFE must still perform the filtering operation. Again, it is noted that a fractional T/K -spaced implementation does not increase the dimension of the LBE matrix equation to be solved. The conventional DFE detector, however, will require K times as many taps to span the same number of symbol periods with its forward filter.

The above theoretically based performance results do not include the effects of channel estimation errors or decision feedback errors. These effects are included in the simulation results presented in Chapter 6.

Chapter 5

CHANNEL ESTIMATION AND TRACKING

The data detection techniques being investigated in this thesis all require an estimate of the channel response. This is considered an advantage from the adaptation point of view, because channel estimation is usually simpler than estimating or tracking the taps of an equalizer directly.

There are two aspects to the channel estimation problem:

1. Channel estimation from a training sequence (startup), and
2. Channel tracking.

It has been shown that, for channel tracking purposes, the simple least mean square (LMS) algorithm, also known as the stochastic gradient (SG) algorithm, works about as well as the more complicated recursive least squares (RLS) techniques [Ele86, McL87]. The LMS algorithm does not work well for startup purposes. Fortunately, the startup problem can easily be handled by a simple least sum of squared errors (LSSE) approach, similar to that used for the LBE and DFBE detectors. Even though a matrix inverse is involved, the solution is simple because the matrix is only a function of the known training sequence, and a precomputed inverse of the matrix can be stored. For some special training sequences, an inverse matrix is not even required.

The LSSE approach to channel estimation is similar to a correlation method, where a portion of the training sequence is correlated with shifted versions of the received signal. However, the correlation method requires twice as many guard symbols as the LSSE approach to obtain the same approximate processing gain. As will be shown, the LSSE approach requires a precursor of length equal to the channel memory, L , whereas the correlation method requires both a precursor and a postcursor of the same length. For this reason, only the LSSE approach is considered further.

The training sequence does not necessarily have to be a preamble, located at the beginning of a frame. Sometimes it is more advantageous to use a midamble, located in

the middle of the frame. When a midamble is used, the received signal is usually stored and detection is performed in both directions from the middle of the frame. This second choice is often appropriate if the causal nature of the channel is not taken advantage of, or if the detector does not need to perform channel tracking, but estimates the channel once per frame from the training sequence. Since the detectors considered in this study are designed to take advantage of the causal nature of the mobile channel, and it is also desirable to perform channel tracking, only preamble training sequences are considered. This does not place any restrictions on the channel estimation method presented. The simulation results presented in Chapter 6, which include channel tracking, are based on using a training preamble. To avoid confusing the startup training sequence with the much shorter training blocks used for channel tracking and in the detection process itself, the startup training sequence will be referred to as the preamble.

5.1 Channel Estimation from a Known Preamble

This section describes a least sum of squared errors (LSSE) approach to channel estimation. The channel is estimated using the received signal resulting from a known preamble or training sequence being sent from the transmitter. The analysis and results are presented for T -spaced channel models, where T is the symbol period. The results for T/K -spaced channel models are obtained by simply considering K interleaved T -spaced channels. Thus, the results are directly applicable to fractionally spaced channel models as well.

The observed received signal vector

$$\mathbf{r} = [r_0 \ r_1 \ \dots \ r_{N'_p-1}]^t \quad (5.1)$$

containing N'_p observed samples, is modeled as

$$\begin{aligned} r_k &= \sum_{l=0}^L f_l p_{k-l} + n_k \\ &= \mathbf{f}^t \mathbf{p}_k + n_k \quad , \quad k = 0, \dots, N'_p - 1 \end{aligned} \quad (5.2)$$

where

$$\mathbf{f} = [f_0 \ f_1 \ \dots \ f_L]^t \quad (5.3)$$

is the discrete T -spaced channel response,

$$N_{ct} = L + 1 \quad (5.4)$$

is the number of channel taps (L being defined as the channel memory),

$$\mathbf{p} = [p_{-L} \dots p_0 \dots p_{N'_p-1}]^t \quad (5.5)$$

is the complete preamble sequence with

$$\mathbf{p}_k = [p_k \ p_{k-1} \dots p_{k-L}]^t \quad (5.6)$$

and

$$\mathbf{n} = [n_0 \ n_1 \dots n_{N'_p-1}]^t \quad (5.7)$$

is discrete complex additive white Gaussian noise (AWGN) with variance $\sigma_n^2 = N_0$. The set of L symbols required at the start of the preamble, prior to the first observation at time $k = 0$, is defined as the precursor. Note that N'_p samples of the received signal are observed, and that the total length of the preamble is

$$N_p = N'_p + L \quad (5.8)$$

The channel can be estimated using the LSSE criterion. The sum of squared errors (SSE) for a given channel estimate $\hat{\mathbf{f}}$ is defined as

$$sse(\hat{\mathbf{f}}) = |\mathbf{r} - \hat{\mathbf{r}}|^2 = \sum_{k=0}^{N'_p-1} |r_k - \hat{r}_k|^2 \quad (5.9)$$

where

$$\begin{aligned} \hat{r}_k &= \sum_{l=0}^L \hat{f}_l p_{k-l} \\ &= \hat{\mathbf{f}}^t \mathbf{p}_k, \quad k = 0, \dots, N'_p - 1 \end{aligned} \quad (5.10)$$

Substituting (5.10) in (5.9) and rearranging gives

$$sse(\hat{\mathbf{f}}) = E_r - 2 \operatorname{Re} [\mathbf{z}^t \hat{\mathbf{f}}^*] + \hat{\mathbf{f}}^t \mathbf{P} \hat{\mathbf{f}} \quad (5.11)$$

where

$$E_r = |\mathbf{r}|^2 = \sum_{k=0}^{N'_p-1} |r_k|^2 \quad (5.12)$$

is the energy of the observed portion of the received signal r , and z is an N_{ct} -dimensional column vector with l -th element given by

$$z_l = \sum_{k=0}^{N'_p-1} r_k p_{k-l}^* \quad , l = 0, \dots, L \quad (5.13)$$

or, using vector notation,

$$z = \sum_{k=0}^{N'_p-1} r_k p_k^* \quad (5.14)$$

P is an N_{ct} -by- N_{ct} preamble correlation matrix with (i,j) -th element given by

$$p_{i,j} = \sum_{k=0}^{N'_p-1} p_{k-i} p_{k-j}^* \quad , i, j = 0, \dots, L \quad (5.15)$$

or

$$P = \sum_{k=0}^{N'_p-1} p_k p_k^{*t} \quad (5.16)$$

Since P is a selectable correlation matrix, it is assumed to be positive definite, and thus invertible in the analysis which follows. From (5.16) it is seen that $P^t = P^*$, and thus that P is Hermitian. The P matrix can be Toeplitz, but in general is non-Toeplitz. A Toeplitz matrix does not necessarily provide an advantage, since the required matrix inverse can be precomputed and stored.

Assuming the elements of $\hat{f}$ to be continuous variables, the objective is to find the $\hat{f}$ vector which provides the LSSE. Completing the square (in the complex squared-magnitude sense) gives

$$sse(\hat{f}) = E_r - z^t (Vz)^* + (Vz - \hat{f})^t P (Vz - \hat{f})^* \quad (5.17)$$

where, for convenience, the following definition has been made

$$V = P^{*-1} \quad (5.18)$$

That (5.17) is equivalent to (5.11) is easily verified by multiplying out the completed square and using the property that $P^t = P^*$. Since P is taken to be positive definite, the third term in (5.17) is non-negative and can be minimized by choosing

$$\hat{f}_{opt} = Vz \quad (5.19)$$

This choice also yields the LSSE, which can be expressed as

$$lsse = sse(\hat{\mathbf{f}}_{opt}) = E_r - \mathbf{z}^t (\mathbf{V}\mathbf{z})^* = E_r - \mathbf{z}^t \hat{\mathbf{f}}_{opt}^* \quad (5.20)$$

The matrix required in (5.19) depends only on the preamble symbols, and thus can be precomputed and stored. It is desirable to have $\mathbf{P}$, and thus $\mathbf{V}$, as close to diagonal as possible to minimize the SSE's. Of course if $\mathbf{P}$ is diagonal, computing and storing $\mathbf{V}$ is trivial. As will be shown later, this is a fairly rare occurrence, but some special sequences do exist which have these properties.

5.2 Mean Squared Error Analysis

Equation (5.19) gives the solution for the channel estimate which minimizes the sum of squared errors between the received signal and its approximation using the channel estimate. In order to predict the performance of detectors which require an estimate of the channel response, an appropriate measure of the channel estimation error itself must be determined.

Substituting (5.2) into (5.13) gives

$$\mathbf{z} = \mathbf{P}^* \mathbf{f} + \mathbf{w} \quad (5.21)$$

where

$$\mathbf{w} = \sum_{k=0}^{N_p'-1} n_k \mathbf{p}_k^* \quad (5.22)$$

Thus the channel estimation error vector can be expressed as

$$\mathbf{e}_f = \hat{\mathbf{f}} - \mathbf{f} = \mathbf{V} \mathbf{w} \quad (5.23)$$

where equations (5.18), (5.19), and (5.21) have been employed. The covariance matrix for $\hat{\mathbf{f}}$ is then given by

$$\begin{aligned} cov[\hat{\mathbf{f}}] &= E[\mathbf{e}_f \mathbf{e}_f^{*t}] \\ &= E[\mathbf{V} \mathbf{w} \mathbf{w}^{*t} \mathbf{V}^{*t}] \\ &= \mathbf{V} \mathbf{W} \mathbf{V} \end{aligned} \quad (5.24)$$

where $E[\cdot]$ denotes expectation,

$$\mathbf{W} = E[\mathbf{w} \mathbf{w}^{*t}] \quad (5.25)$$

and the fact that $\mathbf{V}^t = \mathbf{V}^*$ has been used. From (5.22), it is straightforward to show that

$$\mathbf{W} = \sigma_n^2 \mathbf{P}^* = N_0 \mathbf{P}^* \quad (5.26)$$

where $\sigma_n^2 = N_0$ is the variance of the complex noise samples in n . Substituting (5.26) into (5.24) yields the desired result that

$$\text{cov} [\hat{\mathbf{f}}] = N_0 \mathbf{V} \quad (5.27)$$

Thus the mean squared error (MSE) of the l -th channel tap estimate is given by

$$\text{mse} [\hat{f}_l] = N_0 v_{l,l}, \quad l = 0, \dots, L \quad (5.28)$$

where $v_{l,l}$ is the l -th diagonal element of the $\mathbf{V}$ matrix. Note that the channel tap estimation errors are approximately independent if $\mathbf{V}$ is close to being diagonal.

5.3 Quality Measure of Channel Estimate

One measure of the quality of a channel estimate is the ratio of total energy in the channel response to total mean squared estimation error. This signal to estimation error ratio (SER) evaluates to

$$\text{SER} = \frac{|\mathbf{f}|^2}{N_0 \text{tr}(\mathbf{V})} \quad (5.29)$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix. It is useful to relate the SER to the signal-to-noise ratio (SNR) during the preamble. Assuming uncorrelated preamble symbols, the SNR is given by

$$\text{SNR} = \frac{|p_k|^2 |\mathbf{f}|^2}{N_0} \quad (5.30)$$

Thus a normalized SER can be defined, and is given by

$$\text{SER}' = \frac{\text{SER}}{\text{SNR}} = \left(|p_k|^2 \text{tr}(\mathbf{V}) \right)^{-1} \quad (5.31)$$

This normalized SER is a convenient measure for comparing the quality of different preamble sequences. Generally, the higher SER' , the better the preamble for channel estimation purposes.

Also of interest is the mean squared error when a given channel estimate is used to predict a future received signal sample. It is shown in Appendix A (Section 5.9) that, as long as the number of estimator taps is greater than L , the minimum mean squared error (MMSE) is the noise variance. That is,

$$mmse = \sigma_n^2 = N_0 \quad (5.32)$$

and the excess mean squared error (EMSE) for a time-invariant channel, due to channel estimation errors, is given by

$$emse = |s_k|^2 \text{tr}(\mathbf{V}) mmse \quad (5.33)$$

where $|s_k|^2$ represents the squared magnitude of the data or training symbols after the preamble. The misadjustment, due to estimating the channel from a preamble using the LSSE algorithm, is defined as

$$M_{lsse} = \frac{emse}{mmse} = |s_k|^2 \text{tr}(\mathbf{V}) \quad (5.34)$$

Thus, the total mean squared error (TMSE) can be expressed as

$$\begin{aligned} tmse &= emse + mmse \\ &= [1 + M_{lsse}] mmse \end{aligned} \quad (5.35)$$

Clearly it is desirable to keep the misadjustment as small as possible, which implies that the trace of $\mathbf{V}$ should be kept small. In general, the longer the preamble, the smaller the misadjustment. For a time varying channel, however, there is also a lag error, and the longer the preamble the worse the lag error becomes. If the channel is slowly time varying then this is not a major concern. The tradeoff between misadjustment and preamble overhead is often a more important consideration. Assuming that the misadjustment, defined in (5.34), translates directly into a corresponding degradation for the detection process involved, the expected degradation is

$$DEG_{lsse} = 10 \log(1 + M_{lsse}) \text{ dB} \quad (5.36)$$

It is shown in Appendix B (Section 5.10) that this degradation formula provides a valid approximation, at high signal to noise ratios, for the block detectors investigated in this study. The simulation results of Chapter 6 confirm the accuracy of this approximation. If

possible, the misadjustment should be kept smaller than 1. For example, misadjustments of 1, 1/2, and 1/4 correspond to degradations of approximately 3 dB, 1.76 dB, and 0.97 dB respectively.

When the amplitude of the preamble and post-preamble symbols is the same, the following simple relationship holds

$$M_{lss} = \frac{1}{SER'} \quad (5.37)$$

That is, the misadjustment is just the reciprocal of the normalized signal to estimation error ratio defined earlier in (5.31).

5.4 SER' Upper Bounds

Upper bounds for SER' are easily obtained. The following two cases are considered:

1. nonzero-amplitude precursor, and
2. zero-amplitude precursor.

All preamble symbols are restricted to have the same amplitude, except for the symbols in a zero-amplitude precursor.

5.4.1 Nonzero-Amplitude Precursor

For a well chosen preamble sequence with nonzero precursor, the $\mathbf{P}$ matrix will be close to diagonal with diagonal elements

$$p_{l,l} = N'_p |p_k|^2, \quad l = 0, \dots, L \quad (5.38)$$

For a diagonal $\mathbf{P}$ matrix, the diagonal elements of the $\mathbf{V}$ matrix are given by

$$v_{l,l} = \frac{1}{p_{l,l}} = \frac{1}{N'_p |p_k|^2}, \quad l = 0, \dots, L \quad (5.39)$$

Incorporating this into (5.31), yields the following simple upper bound on the normalized signal to estimation error ratio

$$SER' \leq \frac{N'_p}{N_{ct}} \quad (5.40)$$

That (5.40) is a true upper bound follows from the observation that a diagonal $\mathbf{V}$ matrix gives the same performance as sounding the channel N'_p times with pulses separated

by more than the channel response length. When sounding the channel with isolated pulses, each tap is measured N'_p times and the measurements can be averaged to give a combined estimate with no interference from the other taps. Assuming that the channel taps are uncorrelated, this is the best estimate for each tap that can be obtained given N'_p measurements. When isolated pulses are not used, each of the N'_p samples can be considered a measurement for a specific tap with interference from the other taps. Since the channel taps are unknown, the interference on each tap measurement from the other taps cannot be perfectly removed. However, the interference on the combined estimate for each tap can be perfectly removed if and only if the interfering taps cancel each other exactly. This is precisely what happens, for all taps, when $\mathbf{P}$, and thus $\mathbf{V}$, is diagonal. A more general and rigorous proof, using only the Hermitian property of $\mathbf{P}$, is given in Appendix C (Section 5.11). Formula (5.40) gives a good indication of the general relationship between the number of observed preamble samples, N'_p , the number of channel taps being estimated, N_{ct} , and SER' . For example, doubling the length of the observed portion of the preamble will typically double SER' . Note, from (5.31), that SER must be less than or equal to SNR with $N'_p = N_{ct}$.

5.4.2 Zero-Amplitude Precursor

When the precursor has zero-amplitude, the diagonal elements of $\mathbf{P}$ are given by

$$p_{l,l} = (N'_p - l) |p_k|^2, \quad l = 0, \dots, L \quad (5.41)$$

When $\mathbf{P}$ is diagonal, the diagonal elements of the $\mathbf{V}$ matrix are given by

$$v_{l,l} = \frac{1}{p_{l,l}} = \frac{1}{(N'_p - l) |p_k|^2}, \quad l = 0, \dots, L \quad (5.42)$$

Incorporating this into (5.31), gives the upper bound

$$SER' \leq \left(\sum_{l=0}^L \frac{1}{N'_p - l} \right)^{-1} \quad (5.43)$$

The proof that (5.43) is a true upper bound is given in Appendix C (Section 5.11). The only difference from the nonzero-amplitude precursor case, is that the number of channel soundings or measurements made for each tap is no longer the same. For $N'_p \gg N_{ct}$, performance approaches that of the nonzero precursor case. For short

preambles, however, performance can be much worse. This is not surprising, given that less energy is actually transmitted, and observed, when zero-amplitude precursor symbols are used.

5.5 Computer Search for Best Preambles

A computer search has been performed to find the best binary preamble sequences for a given number of channel taps and preamble length. The quality measure used is the normalized signal to estimation error ratio (SER'), as defined in (5.31). Two cases have been considered:

1. nonzero-amplitude precursor, and
2. zero-amplitude precursor.

Table 5.1 lists the best preamble sequences found with nonzero-amplitude precursor. The sequences are binary and represented in hexadecimal form for compactness. The bits are transmitted as data symbols using the following mapping:

$$\begin{aligned} 0 &\rightarrow +1 \\ 1 &\rightarrow -1 \end{aligned} \tag{5.44}$$

The least significant bit corresponds to the first symbol of the precursor to be transmitted. The length of the preamble is uniquely determined by its location in the table. The hexadecimal number is assumed to have as many zeros as is necessary (on the most significant bit end) to obtain the correct preamble length. These preamble sequences, and corresponding real preamble correlation matrices, are appropriate for any MPSK system, provided a BPSK signaling characteristic is maintained during the preamble. For QPSK this can be done by simply setting the inphase and quadrature bits identically, and accounting for a fixed 45° rotation in the channel estimate.

In many cases, the same SER' performance is provided by several sequences. Only one representative sequence is listed in each case. The selection could be made more restrictive by considering the spectral shape or other correlation properties useful for synchronization purposes. Such considerations have not been attempted here. It should be noted that a good preamble sequence for LSSE channel estimation does not necessarily look random or have good correlation properties for other purposes. For example, the

Training Sequence (Hex)	Total Preamble Length (including Nct-1 nonzero-amplitude precursor)					
Channel Length (Nct)	2Nct-1	2Nct	2Nct+1	2Nct+2	2Nct+3	2Nct+4
1	0*	0*	0*	0*	0*	0*
2	1*	1	2*	2	5*	5
3	2	4*	4	9	12	24*
4	8*	8	17	27	58*	58
5	10	2E	4F	58	B1	162
6	5C	B0	162	162	45E	530
7	13A	48E	4F5	123B	650	14C1
8	91D	123B	2476	C2EF	2981	2981
9	476E	48ED	8BCE	8D04	5302	CAF9

* The sequences marked with a '*' result in perfectly diagonal preamble correlation matrices, providing SER's equal to the upper bound. They also simplify the channel estimation algorithm, because no matrix inverse or matrix multiplication operations are required.

Table 5.1 Hexadecimal value of best binary training sequences, including a nonzero-amplitude precursor.

minimal length preamble sequence

$$\mathbf{p} = [1 \quad 1 \quad 1 \quad -1 \quad 1 \quad 1 \quad 1] \quad (5.45)$$

with only one negative symbol, is perfect for estimating a 4 tap channel response. That is, the corresponding preamble correlation matrix,

$$\mathbf{P} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad (5.46)$$

is perfectly diagonal, with SER' equal to the upper bound of 0 dB.

Figure 5.1 shows the SER' versus preamble length for the best preambles found with a nonzero-amplitude precursor, and for 2, 3, 4 and 5 channel taps. Figure 5.2 presents the results for 6, 7, 8 and 9 channel taps. These figures also show the upper bound (UB) defined in (5.40). There are a number of cases where the SER' for the best preamble found equals the upper bound. This happens when the preamble correlation matrix is perfectly diagonal. These special cases not only provide optimal performance for the specific combination of preamble length and number of channel taps, but the channel estimator is trivial to implement. Note that, for the preamble lengths considered in the exhaustive search program, no perfect preambles were found for channels with more than 4 taps. In Section 5.7, it is shown that a perfect, but possibly very long preamble, can always be found for any channel response length. In general, the best preambles provide an SER' which is well within 1 dB of the upper bound. With the number of channel taps held fixed, the SER' also tends to get closer to the upper bound as the preamble length increases.

Table 5.2 lists the best preamble sequences found with a zero-amplitude precursor. Again, the sequences are binary and represented in hexadecimal form for compactness. The hexadecimal numbers do not include the precursor in this case. The least significant bit corresponds to the first non-zero symbol transmitted. The bits are transmitted using the mapping defined in (5.44).

Figure 5.3 shows the SER' versus preamble length for the best preambles found with a zero-amplitude precursor, and for 2, 3, 4 and 5 channel taps. Figure 5.4 presents the

Figure 5.1 Normalized SER versus preamble length for best preambles with a nonzero-amplitude precursor, and for 2, 3, 4, and 5 channel taps.

Figure 5.2 Normalized SER versus preamble length for best preambles with a nonzero-amplitude precursor, and for 6, 7, 8, and 9 channel taps.

Training Sequence (Hex)	Total Preamble Length (including Nct-1 zero-amplitude precursor)					
Channel Length (Nct)	2Nct-1	2Nct	2Nct+1	2Nct+2	2Nct+3	2Nct+4
1	0*	0*	0*	0*	0*	0*
2	0	1*	1	2*	2	5*
3	0	1	2	4	9	9
4	0	1	8	11	11	58
5	0	1	1D	3B	68	C2
6	0	1	1D	3B	68	13D
7	0	1	48	91	13D	13D
8	0	1	E2	16E	397	6C2
9	0	1	E2	1C4	4C2	4C2

* The sequences marked with a '*' result in perfectly diagonal preamble correlation matrices, providing SER's equal to the upper bound. They also simplify the channel estimation algorithm, because no matrix inverse or matrix multiplication operations are required.

Table 5.2 Hexadecimal value of best binary training sequences, not including a zero-amplitude precursor.

Figure 5.3 Normalized SER versus preamble length for best preambles with a zero-amplitude precursor, and for 2, 3, 4, and 5 channel taps.

Figure 5.4 Normalized SER versus preamble length for best preambles with a zero-amplitude precursor, and for 6, 7, 8, and 9 channel taps.

results for 6, 7, 8 and 9 channel taps. These figures also show the upper bound (UB) defined in (5.43). There are only a few, but not very useful, cases where the SER' for the best preamble found equals the upper bound, and the preamble correlation matrix is perfectly diagonal. Note that there were no perfect preamble correlation matrices found for channels with more than 2 taps. For short preambles, the performance is much worse than that for the nonzero-amplitude precursor case, given the same total preamble length. However, if one considers only the number of nonzero-amplitude symbols transmitted, then the zero-amplitude precursor case does provide slightly better performance. Note that performance is very poor if the minimum preamble length is used, but that adding only 1 extra preamble symbol improves the performance to within 1 or 2 dB of the upper bound. With the number of channel taps held fixed, the SER' also tends to more closely approach the upper bound as the preamble length is increased.

The best choice of preamble type depends on the signaling structure. It is acceptable to corrupt the signal received during the precursor, because this portion of the signal is not actually used for channel estimation. The only requirement is that the precursor symbols be known. Thus, in some cases, a nonzero-amplitude precursor may be transmitted during part of the guard time without actually increasing the overhead. Nonlinear amplifiers may also be a problem for zero-amplitude precursors, because of the transition from zero-amplitude to nonzero-amplitude in the middle of the preamble.

5.6 Special M-Sequence Preambles

Maximal-length sequences, also known as *m*-sequences, have special correlation properties [Mac76]. The length of an *m*-sequence is

$$m = 2^n - 1 \quad (5.47)$$

where *n* is the order of the *m*-sequence. Also, the *m*-sequence symbols are taken to be +1 and -1. The special property of *m*-sequences which is exploited is the fact that when an *m*-sequence is circularly correlated with itself, the correlation value is *m* with no shift and -1 for every other shift. If a repeated *m*-sequence is used to create a preamble with nonzero precursor, and $N_p' = m$, then the N_{ct} -by- N_{ct} preamble correlation matrix has *m* down the diagonal and -1 everywhere else. This can be written as

$$\mathbf{P} = \mathbf{M}\mathbf{I} - \mathbf{U} \quad (5.48)$$

where

$$M = m + 1 = 2^n \quad (5.49)$$

is a scalar, $\mathbf{I}$ is an identity matrix, and $\mathbf{U}$ is an all ones matrix. As mentioned earlier, an ideal preamble correlation matrix is perfectly diagonal. Therefore, preambles derived from m -sequences, as above, are clearly not perfect. However, the magnitude of all off-diagonal elements is as small as possible with N'_p odd. An even more severe restriction is that N'_p must be the length of an m -sequence. Even with these constraints, m -sequence preambles form an important special case. This is because of the special form of the inverse matrix, which makes the channel estimator trivial to implement. From (5.18) and (5.19), and noting that $\mathbf{P}$ is real for binary m -sequences, the channel estimate can be expressed as

$$\hat{\mathbf{f}} = \mathbf{P}^{-1} \mathbf{z} \quad (5.50)$$

From (5.48) and the result in Appendix D (Section 5.12), the inverse of the channel correlation matrix is given by

$$\begin{aligned} \mathbf{P}^{-1} &= \frac{1}{M} \left(\mathbf{I} - \frac{\mathbf{U}}{M} \right)^{-1} \\ &= \frac{1}{M} \left(\mathbf{I} + \frac{\mathbf{U}}{M - N_{ct}} \right) \end{aligned} \quad (5.51)$$

provided $M > N_{ct}$, or equivalently $N'_p \geq N_{ct}$. Substituting (5.51) into (5.50) gives

$$\hat{\mathbf{f}} = \frac{1}{M} (\mathbf{z} + s_z \mathbf{u}) \quad (5.52)$$

where $\mathbf{u}$ is an all ones N_{ct} -length column vector, and s_z is a scalar defined as

$$s_z = \frac{1}{M - N_{ct}} \sum_{l=0}^L z_l \quad (5.53)$$

Computing $\hat{\mathbf{f}}$ from $\mathbf{z}$ is trivial, requiring only L additions and 1 multiplication to compute the scalar correction term s_z , and N_{ct} additions and multiplications to complete the calculation, for a total of $2N_{ct} - 1$ additions and $N_{ct} + 1$ multiplications. In fact, all of the multiplications can be made trivial (i.e. binary shifts) if M and $M - N_{ct}$ are selected to be powers of 2. Note that, by definition, M is already a power of 2. As

an example, for $M = 16$, one could use 8, 12, 14, or 15 channel taps, and completely eliminate all multiplications. Thus, for m -sequence preambles, most of the work is in computing z , and the complexity of the z calculation is about the same as, or less than, a simple LMS algorithm.

The quality of m -sequence preambles can be seen from SER' . Substituting (5.51) into (5.31), and noting that $|p_k| = 1$, gives

$$\begin{aligned} SER' &= \frac{M(M - N_{ct})}{N_{ct}(M - L)} \\ &= \frac{(m+1)(m+1 - N_{ct})}{N_{ct}(m+2 - N_{ct})} \end{aligned} \quad (5.54)$$

Figure 5.5 shows SER' in dB versus the number of channel taps, N_{ct} , for $m=3, 7$, and 15. Recall that N'_p must equal m , and that the total preamble length is $N_p = N'_p + N_{ct} - 1$. The upper bound (5.40) is also shown in this figure. With $N_{ct} \ll m$, the SER' is very close to the upper bound. The worst case is $N_{ct} = m$ with m large, but SER' is never more than 3 dB worse than the upper bound.

The SER' offered by m -sequence preambles can be compared with that provided by the best nonzero-amplitude preambles found in the previous section. For example, with $N_{ct} = N'_p = 7$, the SER' for an m -sequence preamble is approximately 2.5 dB below the upper bound of 0 dB. The SER' provided by the corresponding best preamble is about 1 dB below this same upper bound. With $N_{ct} = 5$ and $N'_p = 7$ however, both preambles provide an SER' about 0.67 dB below the upper bound value of 1.46 dB.

5.7 Some Perfect Preambles

A perfect preamble is defined as one which yields a perfectly diagonal preamble correlation matrix, as defined in (5.16). Recall that if the preamble correlation matrix is perfectly diagonal, no matrix operations are required for channel estimation and the resulting estimation noise is minimized for the given number of channel taps and preamble symbols. It is shown that, for a given number of channel taps, a perfect preamble can always be found, and a simple method for constructing such preambles is given. The number of preamble symbols required, however, grows exponentially with the number of channel taps. The applicability of such preambles is likely restricted to short channel

Figure 5.5 Normalized SER versus number of channel taps for m -sequence preambles with a nonzero-amplitude precursor

lengths of 6 symbol periods or less, depending on the signal to estimation error ratio desired.

For a channel model with $N_{ct} = L + 1$ T -spaced taps, a perfect preamble can be constructed as shown in Figure 5.6. This figure illustrates the procedure in general, but also gives the result for the specific case of 5 taps.

First, select an m -sequence of order L , so that the length of the sequence is

$$m = 2^L - 1 \quad (5.55)$$

It is a property of m -sequences that each possible state of length L , or L -state, appears in the sequence (in a circular sense) once and only once, except for the all +1's L -state [Mac76].

The next step is to circularly shift the m -sequence so that the all -1's L -state is at the end. Since each L -state occurs only once, the first symbol must now be a +1. After this, $(L + 1)$ -1's are added to the start of the m -sequence, resulting in an overall preamble length of

$$N_p = 2^L + L \quad (5.56)$$

The first L -1's become the precursor, and the observation length is $N'_p = m + 1$. Note that every window of N'_p symbols now consists of a -1 followed by a shifted version of the original m -sequence. If any window of N'_p symbols is correlated with itself, the output is of course N'_p . However, if any two different windows are cross-correlated then the output is exactly zero. This happens because the two shifted m -sequences will correlate to -1, and the two -1's at the start of each window will correlate to +1. Note that the same sequence will work for shorter channel lengths, by removing -1's from the precursor. It cannot be extended to work for longer channel lengths, however, as the starting -1's state becomes too long to continue to provide shifted m -sequences.

The specific example given in Figure 5.6 for a 5-tap channel estimator could be particularly useful for some applications. Note that the total preamble length is 20. The normalized signal to estimation error ratio, as defined earlier in (5.31), evaluates to $SER' = 16/5 \simeq 5$ dB. Equivalently, the misadjustment is $M_{ts,e} = 5/16$, which, from (5.36), implies an approximate degradation in detector performance of 1.2 dB.

m-sequence (e.g. order= $L=4$, $m=15$):

Put -1's L-state at the end:

↓ circular shift

Add $(L+1)$ -1's to the start:

Resulting preamble correlation matrix:

$$P = \begin{bmatrix} 16 & 0 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 16 \end{bmatrix}$$

Figure 5.6 How to construct a perfect preamble for an $(L + 1)$ -tap T -spaced LSSE channel estimator, with 5 taps as an example.

Length of Perfect Preambles, N_p								
m-sequence		Number of T-spaced channel taps						
order	length	1	2	3	4	5	6	7
0	0	1						
1	1	2	3					
2	3	4	5	6				
3	7	8	9	10	11			
4	15	16	17	18	19	20		
5	31	32	33	34	35	36	37	
6	63	64	65	66	67	68	69	70

Table 5.3 Length of perfect preambles which could be constructed using m -sequences.

Table 5.3 lists the lengths of perfect preambles which could be constructed for a given number of T -spaced channel taps, using the method described above. These preambles have the same length as most of the perfect preambles found earlier, using the computer search technique. An exhaustive search technique is not very practical for preamble lengths much greater than 20, given that the execution time for the search grows exponentially with preamble length. Thus, Table 5.3, and its obvious extension, provides a source of many more perfect preambles.

5.8 Stochastic Gradient Channel Tracking

In addition to estimating the channel response from a known preamble, channel tracking during the data and training blocks is also considered. This section reviews the stochastic gradient (SG) channel tracking algorithm and its properties, also known as the least mean square (LMS) algorithm [Pro83] [Wid85].

A T -spaced SG channel tracking filter is shown in Figure 5.7. Note that T/K -spaced channel tracking can be performed by simply using K interleaved T -spaced channel trackers. The estimated channel vector,

$$\hat{\mathbf{f}} = [\hat{f}_0 \hat{f}_1 \dots \hat{f}_L]^t \quad (5.57)$$

is adjusted recursively to minimize the mean squared error (MSE) between the received signal and the estimator output. Specifically

$$\hat{\mathbf{f}}_{k+1} = \hat{\mathbf{f}}_k - 2\mu e_k \tilde{\mathbf{s}}_k^* \quad (5.58)$$

where $\hat{\mathbf{f}}_k$ is the estimated channel vector at time k , μ is a step size parameter,

$$e_k = \hat{r}_k - r_k \quad (5.59)$$

is the error between the estimated channel output and the actual channel output, and

$$\tilde{\mathbf{s}}_k = [\tilde{s}_k \tilde{s}_{k-1} \dots \tilde{s}_{k-L}]^t \quad (5.60)$$

denotes the vector of detected information and training symbols in the channel estimator at time k . The received signal at time k is given by

$$\begin{aligned} r_k &= \sum_{l=0}^L f_l s_{k-l} + n_k \\ &= \mathbf{f}^t \mathbf{s}_k + n_k \end{aligned} \quad (5.61)$$

Figure 5.7 Stochastic Gradient T -spaced channel tracking filter

where L is the channel memory (assumed known), $\mathbf{s}_k$ denotes the vector of actual information and training symbols, and n_k is taken to be complex additive white Gaussian noise (AWGN). It is shown in [Pro83] that, as long as the number of taps is greater than L , the optimum channel estimate $\hat{\mathbf{f}}$ is equal to the actual channel vector $\mathbf{f}$, and the minimum mean squared error (MMSE) is simply given by

$$\begin{aligned} mmse &= E \{n_k n_k^*\} \\ &= \sigma_n^2 \\ &= N_0 \end{aligned} \tag{5.62}$$

where N_0 is the noise variance. This MMSE cannot be achieved in practice, because the MSE is a function of the step size parameter μ , which in turn depends on the desired adaptability. Thus, an excess MSE (EMSE) is usually present.

From [Wid85], the steady state excess mean squared error (EMSE) for an SG channel estimator, assuming a time-invariant channel with $N_{ct} = L + 1$ channel taps, is approximately given by

$$\begin{aligned} emse &\simeq \mu N_{ct} mmse \\ &= \mu N_{ct} N_0 \end{aligned} \tag{5.63}$$

where the training and data symbols are both assumed to have unit amplitude. The misadjustment is defined as

$$M_{sg} = \frac{emse}{mmse} = \mu N_{ct} \tag{5.64}$$

Thus, the total MSE (TMSE) is approximately given by

$$\begin{aligned} tmse &= mmse + emse \\ &= (1 + M_{sg}) mmse \end{aligned} \tag{5.65}$$

Clearly it is desirable to keep the misadjustment as small as possible, which implies that μ should be kept small. When the channel is changing with time there is also a lag error problem, which implies that μ should not be made too small. In practice, the best value for μ depends on many parameters, not the least of which is the Doppler frequency of the fading channel. Good values for μ are presented with the simulation results in Chapter 6. In general, it is hoped that μ can be kept several times smaller than $1/N_{ct}$ so that the misadjustment defined in (5.64) will be less than unity.

Assuming that the misadjustment translates directly into a corresponding degradation for the detection process involved, the expected degradation would be approximately

$$DEG_{sg} = 10 \log (1 + M_{sg}) \quad dB \quad (5.66)$$

For example, misadjustments of 1, 1/2, and 1/4 correspond to degradations of approximately 3 dB, 1.76 dB, and 0.97 dB respectively. It is shown in Appendix B (Section 5.10) that this degradation formula is approximately true for the block detectors investigated in this study.

5.9 Appendix A: Mean Squared Prediction Error

Given a channel estimate, the mean squared error (MSE) that results when this channel estimate is used to predict a future received signal sample is to be determined. The k -th received signal sample is given by

$$r_k = \mathbf{f}^t \mathbf{s}_k + n_k \quad (5.67)$$

where

$$\mathbf{f} = [f_0 \ f_1 \ \dots \ f_L]^t \quad (5.68)$$

is the fixed channel response,

$$\mathbf{s}_k = [s_k \ s_{k-1} \ \dots \ s_{k-L}]^t \quad (5.69)$$

is the assumed known data and training sequence, and n_k is taken to be complex additive white Gaussian noise (AWGN) with variance

$$E \left[|n_k|^2 \right] = \sigma_n^2 = N_0 \quad (5.70)$$

The estimated received signal is given by

$$\hat{r}_k = \hat{\mathbf{f}}^t \mathbf{s}_k \quad (5.71)$$

where

$$\hat{\mathbf{f}} = [\hat{f}_0 \ \hat{f}_1 \ \dots \ \hat{f}_L] \quad (5.72)$$

is the estimated channel response. The error in the estimated received signal sample is given by

$$e_k = \hat{r}_k - r_k = \mathbf{e}_f^t \mathbf{s}_k - n_k \quad (5.73)$$

with the channel estimation error vector defined as

$$\mathbf{e}_f = \hat{\mathbf{f}} - \mathbf{f} \quad (5.74)$$

The additive noise at time k is clearly independent of the channel estimation errors since the noise during the preamble is assumed to be independent of the current noise sample. It is also assumed that the data and training symbols are independent of each other and the noise process. With these assumptions, the total mean squared error (TMSE) is given by

$$\begin{aligned} tmse &= E \left[|e_k|^2 \right] \\ &= E \left[(\mathbf{e}_f^t \mathbf{s}_k - n_k) (\mathbf{e}_f^t \mathbf{s}_k - n_k)^* \right] \\ &= E \left[\mathbf{s}_k^t \mathbf{e}_f \mathbf{e}_f^* \mathbf{s}_k^* \right] + E \left[|n_k|^2 \right] \\ &= E \left[\mathbf{s}_k^t \mathbf{V} \mathbf{s}_k^* \right] N_0 + N_0 \\ &= |\mathbf{s}_k|^2 \text{tr}(\mathbf{V}) N_0 + N_0 \end{aligned} \quad (5.75)$$

where

$$E \left[\mathbf{e}_f \mathbf{e}_f^* \right] = \text{cov} \left[\hat{\mathbf{f}} \right] = N_0 \mathbf{V} \quad (5.76)$$

is the covariance matrix of the channel estimate, as defined earlier in (5.27). When $\hat{\mathbf{f}} = \mathbf{f}$, the minimum mean squared error (MMSE),

$$mmse = N_0 \quad (5.77)$$

is obtained. From (5.75), the excess mean squared error (EMSE) term can be expressed as

$$emse = |\mathbf{s}_k|^2 \text{tr}(\mathbf{V}) mmse \quad (5.78)$$

5.10 Appendix B: Degradation with Channel Estimation Errors

Consider the discrete T -spaced channel vector

$$\mathbf{f} = [f_0 \ f_1 \ \dots \ f_L]^t \quad (5.79)$$

Let the estimated response vector be given by

$$\hat{\mathbf{f}} = \mathbf{f} + \mathbf{e} \quad (5.80)$$

where

$$\mathbf{e} = [e_0 \ e_1 \ \dots \ e_L]^t \quad (5.81)$$

is the channel estimation error vector. The LBE and DFBE detectors estimate the data vector as follows (only the Toeplitz case is considered):

$$\hat{\mathbf{b}} = \hat{\mathbf{Q}} \hat{\mathbf{z}} \quad (5.82)$$

where

$$\hat{\mathbf{Q}} = \hat{\mathbf{R}}^{\star^{-1}} \quad (5.83)$$

$$\hat{\mathbf{R}} = \hat{\mathbf{F}}^t \hat{\mathbf{F}}^{\star} \quad (5.84)$$

$$\hat{\mathbf{z}} = \hat{\mathbf{F}}^{\star t} \hat{\mathbf{c}} \quad (5.85)$$

and

$$\hat{\mathbf{F}} = \mathbf{F} + \mathbf{E} \quad (5.86)$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{f} & 0 & 0's \\ & \mathbf{f} & \\ & & \ddots \\ 0's & & \mathbf{f} \end{bmatrix}, \text{ and } \mathbf{E} = \begin{bmatrix} \mathbf{e} & 0 & 0's \\ & \mathbf{e} & \\ & & \ddots \\ 0's & & \mathbf{e} \end{bmatrix} \quad (5.87)$$

are both $(N + L)$ -by- N dimensioned matrices. The vector $\hat{\mathbf{c}}$ is derived as follows. The observed portion of the received signal is given by

$$\mathbf{r} = \mathbf{F}_1 \mathbf{a}_1 + \mathbf{F} \mathbf{b} + \mathbf{F}_2 \mathbf{a}_2 + \mathbf{n} \quad (5.88)$$

where $\mathbf{a}_1$ is a column vector containing the last L training symbols of the left adjacent training block, $\mathbf{a}_2$ is a column vector containing the first L training symbols of the right adjacent training block, and

$$\mathbf{F}_1 = \begin{bmatrix} f_L & f_{L-1} & \cdots & f_1 \\ & f_L & & f_2 \\ & & f_L & \vdots \\ & & & f_L \\ 0's & & & 0's \end{bmatrix}, \text{ and } \mathbf{F}_2 = \begin{bmatrix} 0's & & & 0's \\ f_0 & & & \\ f_1 & f_0 & & \\ \vdots & & f_0 & \\ f_{L-1} & \cdots & f_1 & f_0 \end{bmatrix} \quad (5.89)$$

are both $(N + L)$ -by- L dimensioned matrices. The estimated versions of matrices $\mathbf{F}_1$ and $\mathbf{F}_2$, required for subtracting off the estimated training signals, are given by

$$\hat{\mathbf{F}}_1 = \mathbf{F}_1 + \mathbf{E}_1, \text{ and } \hat{\mathbf{F}}_2 = \mathbf{F}_2 + \mathbf{E}_2 \quad (5.90)$$

where the corresponding error matrices are

$$\mathbf{E}_1 = \begin{bmatrix} e_L & e_{L-1} & \cdots & e_1 \\ & e_L & & e_2 \\ & & e_L & \vdots \\ & & & e_L \\ 0's & & & 0's \end{bmatrix}, \text{ and } \mathbf{E}_2 = \begin{bmatrix} 0's & & & 0's \\ e_0 & & & \\ e_1 & e_0 & & \\ \vdots & & e_0 & \\ e_{L-1} & \cdots & e_1 & e_0 \end{bmatrix} \quad (5.91)$$

Subtracting the estimated training signals from $\mathbf{r}$ gives

$$\begin{aligned} \hat{\mathbf{c}} &= \mathbf{r} - \hat{\mathbf{F}}_1 \mathbf{a}_1 - \hat{\mathbf{F}}_2 \mathbf{a}_2 \\ &= \mathbf{F} \mathbf{b} + \mathbf{n} - \mathbf{E}_1 \mathbf{a}_1 - \mathbf{E}_2 \mathbf{a}_2 \end{aligned} \quad (5.92)$$

The expected value of $\hat{\mathbf{b}}$, with respect to the noise, the channel estimation errors, and the training symbols, can be expressed as

$$\begin{aligned} E[\hat{\mathbf{b}}] &= E[\hat{\mathbf{Q}} \hat{\mathbf{z}}] \\ &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^* (\mathbf{F} \mathbf{b} + \mathbf{n} - \mathbf{E}_1 \mathbf{a}_1 - \mathbf{E}_2 \mathbf{a}_2)] \\ &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^* \mathbf{F}] \mathbf{b} \\ &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^* (\hat{\mathbf{F}} - \mathbf{E})] \mathbf{b} \\ &= \mathbf{b} - E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^* \mathbf{E}] \mathbf{b} \end{aligned} \quad (5.93)$$

The final expectation in (5.93) is non-zero, in general, showing that, with channel estimation errors, $\hat{\mathbf{b}}$ is no longer an unbiased estimate of the data vector $\mathbf{b}$.

The mean squared error (MSE) matrix for $\hat{\mathbf{b}}$ is defined as

$$mse(\hat{\mathbf{b}}) = E[\mathbf{e}_b \mathbf{e}_b^{*t}] \quad (5.94)$$

where

$$\begin{aligned} \mathbf{e}_b &= \hat{\mathbf{b}} - \mathbf{b} \\ &= \hat{\mathbf{Q}} \hat{\mathbf{z}} - \mathbf{b} \\ &= \hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} \hat{\mathbf{c}} - \mathbf{b} \\ &= \hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} (\mathbf{F} \mathbf{b} + \mathbf{n} - \mathbf{E}_1 \mathbf{a}_1 - \mathbf{E}_2 \mathbf{a}_2) - \mathbf{b} \\ &= \hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} ((\hat{\mathbf{F}} - \mathbf{E}) \mathbf{b} + \mathbf{n} - \mathbf{E}_1 \mathbf{a}_1 - \mathbf{E}_2 \mathbf{a}_2) - \mathbf{b} \\ &= \hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} (\mathbf{n} - \tilde{\mathbf{n}}) \end{aligned} \quad (5.95)$$

with $\tilde{\mathbf{n}}$ defined as

$$\begin{aligned} \tilde{\mathbf{n}} &= \mathbf{E}_1 \mathbf{a}_1 + \mathbf{E} \mathbf{b} + \mathbf{E}_2 \mathbf{a}_2 \\ &= \tilde{\mathbf{E}} \mathbf{s} \end{aligned} \quad (5.96)$$

where

$$\tilde{\mathbf{E}} = [\mathbf{E}_1 \ \mathbf{E} \ \mathbf{E}_2] \quad , \text{ and } \quad \mathbf{s} = [\mathbf{a}_1^t \ \mathbf{b}^t \ \mathbf{a}_2^t]^t \quad (5.97)$$

The noise samples involved in the channel estimation process are assumed to be independent of the noise samples during the detection process. Thus, $\mathbf{n}$ is independent of $\tilde{\mathbf{n}}$, and the MSE matrix evaluates to

$$\begin{aligned} mse(\hat{\mathbf{b}}) &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} (\mathbf{n} - \tilde{\mathbf{n}}) (\mathbf{n} - \tilde{\mathbf{n}})^{*t} \hat{\mathbf{F}} \hat{\mathbf{Q}}^{*t}] \\ &= \mathbf{X}_1 + \mathbf{X}_2 \end{aligned} \quad (5.98)$$

where

$$\begin{aligned} \mathbf{X}_1 &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} \mathbf{n} \mathbf{n}^{*t} \hat{\mathbf{F}} \hat{\mathbf{Q}}^{*t}] \\ &= N_0 E[\hat{\mathbf{Q}}] \end{aligned} \quad (5.99)$$

and

$$\begin{aligned} \mathbf{X}_2 &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} \tilde{\mathbf{n}} \tilde{\mathbf{n}}^{*t} \hat{\mathbf{F}} \hat{\mathbf{Q}}^{*t}] \\ &= E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} \tilde{\mathbf{E}} \mathbf{s} \mathbf{s}^{*t} \tilde{\mathbf{E}}^{*t} \hat{\mathbf{F}} \hat{\mathbf{Q}}^{*t}] \\ &= |s_k|^2 E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} \tilde{\mathbf{E}} \tilde{\mathbf{E}}^{*t} \hat{\mathbf{F}} \hat{\mathbf{Q}}^{*t}] \end{aligned} \quad (5.100)$$

where the data and training symbols in $\mathbf{s}$ are assumed to be independent with amplitude $|s_k|^2$. For small channel estimation errors, $\tilde{\mathbf{E}}$ can be considered approximately independent of $\hat{\mathbf{Q}}$ and $\hat{\mathbf{F}}$, and $E[\hat{\mathbf{Q}}] \approx \mathbf{Q}$. Thus, for small channel estimation errors, $\mathbf{X}_1$ and $\mathbf{X}_2$ can be approximated as follows:

$$\begin{aligned}\mathbf{X}_1 &= N_0 E[\hat{\mathbf{Q}}] \\ &\approx N_0 \mathbf{Q} \\ &= \hat{\mathbf{X}}_1\end{aligned}\tag{5.101}$$

and

$$\begin{aligned}\mathbf{X}_2 &= |s_k|^2 E[\hat{\mathbf{Q}} \hat{\mathbf{F}}^{*t} \tilde{\mathbf{E}} \tilde{\mathbf{E}}^{*t} \hat{\mathbf{F}} \hat{\mathbf{Q}}^{*t}] \\ &\approx |s_k|^2 N_0 \text{tr}(\mathbf{V}) E[\hat{\mathbf{Q}}] \\ &\approx |s_k|^2 N_0 \text{tr}(\mathbf{V}) \mathbf{Q} \\ &= \hat{\mathbf{X}}_2\end{aligned}\tag{5.102}$$

where result (5.28) has been used, and the channel estimation errors are assumed to be approximately independent. The independence approximation is valid only for good training preamble sequence. It is assumed that only good training preamble sequences are to be used. Monte Carlo calculations of $\mathbf{X}_1$ and $\mathbf{X}_2$, for some specific channel responses, confirm that $\hat{\mathbf{X}}_1$ and $\hat{\mathbf{X}}_2$ are good approximations for high signal to channel estimation error ratios. From (3.55), the minimum mean squared error (MMSE) matrix, just prior to making decisions, obtained with perfect knowledge of the channel, is given by

$$\begin{aligned}\text{mmse}(\hat{\mathbf{b}}) &= N_0 \mathbf{Q} \\ &= \hat{\mathbf{X}}_1\end{aligned}\tag{5.103}$$

Using the above approximations, the total mean squared error (TMSE) matrix, obtained using an estimated channel response, is given by

$$\text{tmse}(\hat{\mathbf{b}}) \approx (1 + M) \text{mmse}(\hat{\mathbf{b}})\tag{5.104}$$

where the scalar misadjustment factor, M , is simply given by

$$M = |s_k|^2 \text{tr}(\mathbf{V})\tag{5.105}$$

Thus, the approximate dB degradation for each symbol in the data block is the same, and is given by

$$\text{DEG} = 10 \log(1 + M) \quad \text{dB}\tag{5.106}$$

This result is valid for both the LBE and DFBE detectors. Equations (5.105) and (5.106) agree with (5.34) and (5.36) in the main text.

The approximate degradation for the MLBE detector is the same. The MLBE detector essentially reconstructs all possible transmitted signal sequences, and compares them with the received signal sequence. From Appendix A (Section 5.9), it is seen that signal reconstruction has the same misadjustment factor as that given in (5.105). Thus, the degradation at high signal to channel estimation error ratios is expected to be the same as that given in (5.106). The simulation results presented in Chapter 6 confirm the accuracy of this approximation for all three detectors.

5.11 Appendix C: Proof of Upper Bound for SER¹

The problem is to prove that

$$\text{tr}(\mathbf{A}^{-1}) \geq \text{tr}(\mathbf{A}_d^{-1}) \quad (5.107)$$

where $\mathbf{A}_d$ is a diagonal matrix containing only the diagonal elements of the positive definite Hermitian N -by- N matrix $\mathbf{A} = \{a_{i,j}\}$.

Using Cholesky factorization [Rab78], $\mathbf{A}$ can be factored as¹

$$\mathbf{A} = \mathbf{L} \mathbf{D} \mathbf{L}^{*t} \quad (5.108)$$

where $\mathbf{L} = \{l_{i,j}\}$ is a lower triangular matrix with 1's down the diagonal, and $\mathbf{D} = \{d_i\}$ is a positive definite diagonal matrix. Taking the inverse of $\mathbf{A}$ gives

$$\mathbf{A}^{-1} = \mathbf{U} \mathbf{D}^{-1} \mathbf{U}^{*t} \quad (5.109)$$

where

$$\mathbf{U}^{*t} = \mathbf{L}^{-1} \quad (5.110)$$

It is straightforward to show that $\mathbf{L}^{-1}$ has the same lower triangular form as $\mathbf{L}$, and thus that $\mathbf{U} = \{u_{i,j}\}$ is an upper triangular matrix with 1's down the diagonal. From (5.108), the i -th diagonal element of $\mathbf{A}$ is bounded by

$$\begin{aligned} a_{i,i} &= d_i + \sum_{k=1}^{i-1} |l_{i,k}|^2 d_k \\ &\geq d_i \end{aligned} \quad (5.111)$$

¹ The insight for this proof was provided by William Moreland of the Communications Research Centre, Ottawa.

From (5.109), and using bound (5.111), the i -th diagonal element of $A^{-1} = \{\alpha_{i,j}\}$ is bound by

$$\begin{aligned}\alpha_{i,i} &= \frac{1}{d_i} + \sum_{k=i+1}^N |u_{i,k}|^2 \frac{1}{d_k} \\ &\geq \frac{1}{d_i} \\ &\geq \frac{1}{a_{i,i}}\end{aligned}\tag{5.112}$$

which proves (5.107).

5.12 Appendix D: Inverse of $(I - eU)$

The problem is to compute the inverse of an N -by- N matrix A of the form

$$A = I - eU\tag{5.113}$$

where I is an N -by- N identity matrix, e is a fixed scalar perturbation, and U is an all ones N -by- N matrix. Using the series expansion for A^{-1} gives

$$\begin{aligned}A^{-1} &= (I - eU)^{-1} \\ &= I + eU + (eU)^2 + (eU)^3 + \dots \\ &= I + eU + Ne^2U + N^2e^3U + \dots \\ &= I + eU \sum_{k=0}^{\infty} (Ne)^k \\ &= I + \frac{eU}{1 - Ne}\end{aligned}\tag{5.114}$$

provided that $|Ne| < 1$.

Chapter 6

SIMULATION RESULTS

This chapter presents the results obtained using a computer simulation. The chapter is divided into the following three main sections:

1. Simulation model,
2. General multipath results, and
3. Mobile radio results.

The simulation model is discussed first and appropriate definitions are made. In the general multipath results section, the performance for T-spaced channel models, with equally energetic Rayleigh fading paths, is investigated. In the mobile radio results section, the channel parameters selected are representative of a digital mobile radio environment with signaling rates on the order of 1 Msps. Both T-spaced and T/2-spaced channel models are employed.

6.1 Simulation Model

The simulation program models the transmission of data frames through a fading time-dispersive multipath environment. Figure 6.1 shows the basic simulation model. Data and training bits are generated using a standard pseudo-random number generator. Frames are then constructed using either a BPSK or QPSK format. The simulation is performed using complex baseband equivalent models for each component. The transmit filter not only filters the transmitted symbols, but converts the T-spaced frame signal to a T/K-spaced signal, where K is selectable. This allows the filters, multipath channel, and noise to be modeled using K times the minimum signaling bandwidth. The time-dispersive multipath channel is modeled as a T/K-spaced tap delay line, with independent and Rayleigh distributed tap weights. The details of how the time-varying nature of the tap weights is simulated is presented later. The noise is modeled as additive white Gaussian noise (AWGN), where the variance of each complex noise sample is defined to

Figure 6.1 Basic simulation model

be $\sigma_n^2 = N_0$. This is the appropriate definition of baseband equivalent white Gaussian noise, when the energy of a transmitted pulse, $p(k)$, is defined to be

$$E_p = \sum_k |p(k)|^2 \quad (6.1)$$

These definitions are independent of the number of samples per symbol period and the symbol period. After passing through the receive filter, the signal is detected using three different detectors. The linear block estimator (LBE), decision feedback block estimator (DFBE), and the maximum likelihood block estimator (MLBE) are all implemented as described in Chapter 3. If the memory of the estimated channel response is greater than the number of symbols per training block, then block decision feedback (BDFB) is performed for each of the detectors. No bit-timing recovery is required since each of the three detectors includes a matched filtering front end, which automatically aligns the desired T-spaced samples. No carrier phase recovery is required, since the carrier phase is also handled by the detectors. It is assumed that the symbol rate and carrier frequency are both known and stable enough that any time-variation over the duration of one frame is dominated by the fading channel. The decisions made by the detectors are then compared with the data sent, and error rates and confidence intervals are calculated.

The frames are constructed as shown in Figure 6.2. A binary phase shift keying (BPSK) or quadrature phase shift keying (QPSK) format can be used. The preamble is selected from the list of best preambles presented in Chapter 5, and depends on the number of channel taps to be estimated and the preamble length desired. Only preambles with nonzero precursor were selected.¹ This figure also gives a description of the main simulation parameters. The values selected for most of these parameters are presented with the plotted performance in each case. This is done to ensure that the results are interpreted correctly, and to avoid having to explain the simulation setup in detail each time. Results are presented in terms of the energy per signaling bit to noise power spectral density ratio, E_b/N_0 , as opposed to using the energy per data bit. This is done because in many cases the same performance can be obtained with different frame lengths. For example, when investigating very slow fading channels, only one data block is used per frame, to ensure that as many different channel states as possible are observed for a given

¹ A precursor may be necessary, not to account for data from the previous frame, but rather to allow the power amplifier to come up. Thus, a nonzero precursor may not require any additional symbols to be transmitted.

Symbol	Description
BPSK	Binary Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
ns	Number of samples per symbol period
np	Number of symbols per preamble
nd	Number of symbols per data block
nt	Number of symbols per training block
nb	Number of data blocks per frame
nf	Number of frames per simulation run
prof(sp)	Multipath amplitude profile with spacing sp
fd	Doppler frequency (in sampling rates)
nT	Number of T-spaced channel tap estimates
u	Stochastic gradient step-size parameter
Tx	Transmit filter type
Rx	Receive filter type
N/A	Not applicable

Figure 6.2 Frame format and description of simulation parameters

amount of simulation time. Calculating the energy per data bit for a given or desired frame format is straightforward, and is given by

$$E_{db} = \frac{np + nb(nd + nt)}{nb nd} E_b \quad (6.2)$$

As an example, and as a typical set of parameters used for the mobile radio results, with

$$np = 20, nd = 6, nt = 2, nb = 20 \quad (6.3)$$

we obtain $E_{db} = 1.5E_b$, which corresponds to a shift of 1.76 dB due to overhead. Note that the Doppler frequency is normalized with respect to the sampling rate. Also, nT corresponds to the number of T-spaced taps in the channel estimator. The total number of taps in the channel estimator is always $(ns \times nT)$. The number of taps in the actual channel model is not restricted to be a multiple of ns .

The transmit and receive filters are implemented using a general tapped delay line filter structure, described by the difference equation

$$y_k = b_0 x_k + b_1 x_{k-1} + b_2 x_{k-2} \dots \\ - a_1 y_{k-1} - a_2 y_{k-2} \dots \quad (6.4)$$

For the T-spaced models, the transmit and receive filters are both taken to be ideal all-pass filters, defined simply by $b_0 = 1$. For the mobile radio results with T/2-spacing, a 4-th order Butterworth filter with 3 dB bandwidth equal to half the Nyquist frequency was used. This filter was designed using the digital bilinear transform technique [Opp75]. The appropriate filter coefficients are as follows:

$$[a_0 a_1 a_2 a_3 a_4] = [1.000 \ .0000 \ .4860 \ .0000 \ .0177] \\ [b_0 b_1 b_2 b_3 b_4] = [.0940 \ .3759 \ .5639 \ .3759 \ .0940] \quad (6.5)$$

The impulse response and corresponding frequency response for this filter are shown in Figure 6.3.

The time-dispersive multipath channel is modeled as a tapped delay line filter with independent Rayleigh distributed tap weights. The method used to generate the time-varying tap weights is described in [Jak74] and [Cas88]. Each inphase (I) and quadrature (Q) tap weight is the weighted sum of nine different sinusoids, as shown in Figure 6.4. The amplitude and frequencies of the sinusoids are the same as those recommended in

Figure 6.3 Time and frequency response of 4-th order Butterworth filter with $T/2$ -spacing.

Figure 6.4 Generation of inphase (I) and quadrature (Q) tap weights for a Rayleigh fading mobile channel model. The sinusoidal frequencies are given as a fraction of the maximum Doppler frequency, and the magnitudes are shown along the connecting lines.

[Jak74], and used in [Cas88], to approximate the classic mobile Doppler power spectrum (for an omnidirectional vertically polarized antenna)

$$S(f) \propto \begin{cases} [1 - (f/f_d)^2]^{-1/2} & , f \leq f_d \\ 0 & , f > f_d \end{cases} \quad (6.6)$$

where f_d is the maximum Doppler frequency. The individual T/K-spaced tap weights are made independent by selecting a random and independent set of starting phases for the set of sinusoids associated with each tap weight. The state of the fading channel at the start of each frame is also selected at random, so that the channel is independent from frame to frame. This helps to shrink the confidence intervals for the computed error rates.

6.2 General Multipath Results

This section presents the performance for T-spaced channel models with equally energetic Rayleigh fading paths. Performance is presented in terms of the probability of bit error versus E_b/N_0 in dB, where E_b is the energy per signaling bit. To determine the performance in terms of the energy per data bit, simply use equation (6.2) to calculate the appropriate dB shift in the performance curves. This is not done because the amount of overhead is somewhat arbitrary for many of the results.

6.2.1 Known Slow Fading Channels

Figures 6.5, 6.6, and 6.7 present the probability of bit error versus E_b/N_0 in dB, for 2, 3, and 4 paths, respectively. The paths are equally energetic Rayleigh fading, and T-spaced to match the simulation model. The channel is assumed to be slow fading, meaning that the channel state is fixed for the duration of each frame. A totally independent channel is selected at the start of each frame. The channel state is also perfectly known at the receiver. For these ideal cases, no preamble is necessary. The modulation format is BPSK. The number of symbols per data block is 6. The number of symbols per training block is the minimum for each case, to avoid the need for block decision feedback (BDFB). Specifically, 1, 2, and 3, symbols per training block are required for 2, 3, and 4 T-spaced paths, respectively. Each frame only contains one data block to maximize the number of channel states simulated. The performance would be exactly the same for longer frames since the channel is known perfectly, and there is no BDFB to cause block

Figure 6.5 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with 2 equally energetic T-spaced paths, and BPSK signaling.

Figure 6.6 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with 3 equally energetic T-spaced paths, and BPSK signaling.

Figure 6.7 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with 4 equally energetic T-spaced paths, and BPSK signaling.

error propagation. Included with each plot is the theoretical performance for an ideal system which transmits an isolated pulse and performs perfect matched filtering. The performance for the ideal system is the same as a perfect N-th order diversity system, where N is the number of equally energetic independent channels. This performance provides a convenient lower bound on the performance of the other three detectors. Also included, for reference purposes, is the theoretical performance of an ideal system with only one fading path. This curve corresponds to the performance for a flat fading channel, referring to the channel's frequency spectrum.

In Figure 6.5, for 2 paths, we see that the MLBE detector suffers negligible degradation compared to the ideal lower bound. Note that each simulated curve is plotted with a set of error bars which correspond to plus and minus one standard deviation. This gives a good indication of the accuracy of the simulated results. Naturally the error bars tend to be much larger at the higher signal-to-noise ratios, where fewer errors are counted. Even when the simulation results show large error bars, the relative performance between detectors should still be fairly reliable since the same noise sequence is used for all three detectors, and for all signal-to-noise ratios. The DFBE detector performs about 1 dB worse than the MLBE detector, and the LBE detector about 2.5 dB worse than the MLBE detector, at an error rate of 10^{-3} . This error rate will typically be used when comparing the performance of the detectors. Note that all three detectors make good use of the inherent diversity, and give performance parallel to the ideal 2-nd order diversity curve. The improvement over the flat fading curve is greater than 10 dB for the lower error rates. These simulated performance results are all within half a dB of the theoretically based performance results presented in Figure 4.2.

Figures 6.6 and 6.7, for 3 and 4 paths, show that the performance continues to improve as the number of paths is increased. This is because of the increased inherent diversity provided by the multiple paths. Note that the MLBE is still very close to the ideal lower bound, and thus suffers very little degradation due to the extra intersymbol interference. The DFBE and LBE detectors are seen to be a little more sensitive to the extra intersymbol interference, with their respective degradations now being on the order of 1.5 dB and 3.3 dB. Most of the diversity is provided by the first 3 or 4 paths. Each additional path results in a smaller and smaller incremental gain. It will be shown later that even when

the channel memory is much longer than a training block, as few as 2 or 3 symbols per training block are sufficient to provide a significant diversity gain. Generally, these results also show close agreement with the corresponding theoretical results presented in Figures 4.3 and 4.4. The simulated performance for the DFBE detector shows the largest deviation from the theoretically based results, consistently being about half a dB better. This was expected, since pessimistic approximations were used in determining the theoretical performance for the DFBE detector.

Figures 6.8, 6.9, and 6.10 show the corresponding performance curves for QPSK signaling. As expected, the LBE detector performs about the same for BPSK and QPSK signaling. The DFBE and MLBE detectors both perform a little worse with QPSK signaling. With correct decision feedback (not shown), the theoretical DFBE performance is the same for BPSK and QPSK (see Chapter 4). Thus, the DFBE appears to be a little more sensitive to feedback errors with QPSK signaling. This is not surprising since each data block now contains twice as many bits, and thus twice as many opportunities for feedback errors within a block. The MLBE was also expected to do a little worse with QPSK signaling, given that the worst case minimum distances are more likely to occur with QPSK signaling. In general, however, the relative performance is about the same for the two signaling formats. For 4 paths, the degradations from ideal for the MLBE, DFBE, and LBE detectors are approximately 0.3 dB, 1.8 dB, and 3.3 dB.

6.2.2 Unknown Slow Fading Channel

In practice, the channel response is unknown and must be estimated for each frame. Since slow fading channels are still being considered, it is only necessary to estimate the channel once per frame using an appropriate length preamble. As will be shown later, the addition of channel tracking can improve the performance for slow as well as fast fading channels. It is not considered here, however. Of primary interest at this point is the performance degradation caused by a noisy channel estimate, where the quality of the channel estimate is only a function of the preamble length. The preambles used in each case were selected from the set of best preambles presented in Chapter 5.

Figures 6.11, 6.12, and 6.13 give the bit error rate performance for the LBE, DFBE, and MLBE detectors, respectively. The signaling format is BPSK. The channel consists of 4 equally energetic T-spaced paths, as considered previously. The LSSE channel

Figure 6.8 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with 2 equally energetic T-spaced paths, and QPSK signaling.

Figure 6.9 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with 3 equally energetic T-spaced paths, and QPSK signaling.

Figure 6.10 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with 4 equally energetic T-spaced paths, and QPSK signaling.

Figure 6.11 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with 4 equally energetic T-spaced paths, BPSK signaling, and LBE detection.

Figure 6.12 Probability of bit error versus E_b/N_0 for an unknown slow fading, multipath channel with 4 equally energetic T-spaced paths, BPSK signaling, and DFBE detection.

Figure 6.13 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with 4 equally energetic T-spaced paths, BPSK signaling, and MLBE detection.

estimator was also set to estimate 4 T-spaced channel taps. As before, each frame only contains one data block to maximize the number of channel states simulated. The performance would be exactly the same for longer frames because the channel is not tracked, and there is no BDFB to cause block error propagation. Each figure presents the performance for $n_p=7$, $n_p=12$, and a known channel, where n_p is the number of preamble symbols. Recall that the minimum preamble length for a 4-tap channel estimator is 7. The expected approximate degradations from the known channel case, based on the theoretical analysis of Chapter 5, are 3 dB and 1.6 dB, for 7 and 12 preamble symbols, respectively. The observed degradations for each detector are in close agreement with the expected approximate degradations.

Figures 6.14, 6.15, and 6.16 give the corresponding performance curves for QPSK signaling. Upon inspection, it is easy to see that the same relative performance degradations hold in each case. Thus, the approximate theoretical degradations are not only accurate, but appear to be quite generally applicable.

6.2.3 Unknown Fast Fading Channel

Fast fading, with and without channel tracking, is considered in this section. Again, the channel consists of 4 equally energetic T-spaced paths. In the previous cases, the maximum Doppler frequency was set to zero, for very slow fading. In this case, however, the maximum Doppler frequency is set to 1/1000 of the symbol rate, which is considered to be a reasonably fast fade rate. This is typical of an HF channel with a signaling rate of 2400 sps and a fade rate of 2.4 Hz. It could also correspond to a mobile radio channel with a signaling rate of 100 Ksps, and 100 Hz Doppler. A more realistic multipath profile and fade rate for mobile radio is considered later. Channel tracking was performed using the simple stochastic gradient (SG) algorithm described in Chapter 5. The initial channel estimate was obtained using an LSSE channel estimator with a 12-symbol preamble. With fast fading, the performance is dependent on the frame length, and thus one data block per frame is no longer appropriate for simulation purposes. The number of data blocks per frame was selected to be 15 for these results. With 6 data symbols per data block, this corresponds to 90 and 180 data bits per frame for BPSK and QPSK signaling, respectively.

Figure 6.14 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with 4 equally energetic T-spaced paths, QPSK signaling, and LBE detection.

Figure 6.15 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with 4 equally energetic T-spaced paths, QPSK signaling, and DFBE detection.

Figure 6.16 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with 4 equally energetic T-spaced paths, QPSK signaling, and MLBE detection.

Figure 6.17 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with 4 equally energetic T-spaced paths, BPSK signaling, and no channel tracking.

Figure 6.17 shows the bit error rate performance with BPSK signaling and no channel tracking. The performance without tracking is clearly unacceptable, with each detector exhibiting an error rate threshold above 10^{-2} . The height of the error rate threshold depends on the frame length and the detector. The longer the frame length, the higher the error rate threshold. As expected, the LBE detector has the highest error rate threshold and the MLBE detector has the lowest, for a given frame length. From the presented performance, it is apparent that channel tracking is necessary.

Figure 6.18 shows the bit error rate performance with channel tracking. The signaling format is BPSK. Two performance curves are presented for each detector. The upper curve for each detector corresponds to normal decision directed tracking and the lower curve corresponds to correct data tracking. The performance with correct data tracking is obtained by feeding the channel tracking algorithm the correct data symbols instead of the decisions. This performance cannot be achieved in practice, but is presented for comparison purposes and to assess the severity of the degradation caused by decision errors in the tracking algorithm. With decision directed tracking, the performance for each detector is approximately the same as that presented earlier for the unknown slow fading channel. In fact, the performance for the MLBE detector is even slightly better with fast fading and tracking. It is evident that, for slow to medium fade rates, channel tracking could easily reduce the fundamental degradation imposed by estimating the channel from a short preamble. Recall that with a 12-symbol preamble, the expected degradation for slow fading is 1.6 dB. The degradation due to decision errors in the tracking algorithm is seen to be a small fraction of a dB for BPSK signaling, with the MLBE detector exhibiting the smallest relative degradation. For the above simulation results, the tracking step-size parameter was set to $\mu=0.05$. This value was selected because it provided near optimum performance for all three detectors. The step-size parameter was not optimized for each different detector or signal-to-noise ratio.

Figures 6.19 and 6.20 give the corresponding performance for QPSK signaling. Without channel tracking, performance is unacceptable as before. The threshold error rates are even higher than for BPSK signaling. With correct data tracking, the performance for each detector is just slightly worse than that presented earlier for an unknown slow fading channel. This would indicate that the QPSK format is more sensitive to fast

Figure 6.18 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with 4 equally energetic T-spaced paths, BPSK signaling, and channel tracking.

Figure 6.19 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with 4 equally energetic T-spaced paths, QPSK signaling, and no channel tracking.

Figure 6.20 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with 4 equally energetic T-spaced path , QPSK signaling, and channel tracking.

fading than the BPSK format. This is not surprising, since it is well known that QPSK signaling is much more sensitive to phase jitter as well. For these results, the tracking step-size parameter was set to $u=0.03$. As before, this value was selected because it provided near optimum performance for all three detectors. Note that QPSK signaling is also more sensitive to decision errors in the channel tracking algorithm, and that the MLBE detector again exhibits the smallest relative degradation.

6.3 Mobile Radio Results

This section presents the performance for both T-spaced and T/2-spaced channel models. The multipath profiles considered to this point have all contained equally energetic T-spaced paths, and were of fairly short duration. A multipath profile which is more typical of a mobile radio channel is one that decays exponentially with path delay. The rate of decay per symbol period depends on the carrier frequency, the specific environment, and most importantly, the symbol rate. Results are presented for the specific multipath decay rate of 3 dB per symbol period. The multipath profile is truncated after 8 symbol periods, so that the power in the last contributing symbol period is down 21 dB relative to the first. This approximation should be adequate for all signal-to-noise ratios below 20 dB. Longer multipath profiles would require excessive execution times for the simulation program. Based on the measured multipath profiles found in the literature, and presented in Chapter 2, this profile could be considered typical for symbol rates on the order of 1 Msps, or close to worst case for say 500 Ksps. The maximum Doppler frequency considered is 1/5000 of the symbol rate, which would correspond to 100 Hz and 200 Hz Doppler for symbol rates of 500 Ksps and 1 Msps respectively. As in the previous section, performance is presented in terms of the probability of bit error versus E_b/N_0 in dB, where E_b is the energy per signaling bit. To determine the performance in terms of the energy per data bit, simply use equation (6.2) to calculate the appropriate dB shift in the performance curves. This is not done because the amount of overhead is somewhat arbitrary for many of the results.

3

OF/DE

3

Micro-D

6.3.1 Known Slow Fading Channel

A T-spaced simulation model is considered first. Figure 6.21 shows the performance of the LBE, DFBE, and MLBE detectors for an exponentially decaying channel profile. The channel paths are independent, Rayleigh fading, and T-spaced to match the simulation model. The channel is assumed to be slow fading, meaning that the channel state is fixed for the duration of each frame. The channel state is also perfectly known at the receiver. For these ideal cases, no preamble is necessary. The modulation format is BPSK. The number of symbols per data block is 6. The number of symbols per training block is a parameter, since it is not immediately obvious what the training block length should be. Block decision feedback (BDFB) is automatically performed when the training block length is shorter than the memory of the channel estimate. For these preliminary results, however, each frame only contains one data block to maximize the number of channel states simulated. Included with each plot is the theoretical performance for an ideal system which transmits an isolated pulse and performs perfect matched filtering. The performance for the ideal system is the same as a perfect infinite order exponential diversity system, where the energy of each successive channel is exponentially decreasing. This performance provides a convenient lower bound on the performance of the other three detectors. Also included, for comparison, is the theoretical performance of an ideal system with only one fading path. This curve corresponds to the performance for a flat fading channel, referring to the channel's frequency spectrum. Performance is presented for training block lengths of 2 and 3. It was found that very little improvement in performance could be obtained by considering more than 2 symbols per training block. When the throughput efficiency is taken into account, the performance for the MLBE and DFBE detectors with only 2 symbols per training block is about the same as the performance with 3. The LBE detector shows quite a severe degradation with only 2 symbols per training block.

Figure 6.22 shows the corresponding performance curves for QPSK signaling. In general the results are very similar to the BPSK results, and the same conclusions can be drawn. Since the MLBE and DFBE detectors are of most interest, and it is desirable to keep the overhead as low as possible, the training block length was restricted to 2 for the remainder of the simulation runs.

Figure 6.21 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with exponentially decaying T-spaced paths, and BPSK signaling.

Figure 6.22 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with exponentially decaying T-spaced paths, and QPSK signaling.

6.3.2 Unknown Slow Fading Channel

Figure 6.23 gives the performance for an unknown slow fading multipath channel with exponentially decaying T-spaced paths, and BPSK signaling. The channel is estimated using a preamble of length 20, which is 5 symbols longer than the minimum of 15 for a channel estimator with 8 T-spaced taps. As for the previous results, each frame only contains one data block to maximize the number of channel states simulated. Neglecting any degradation caused by block decision feedback (BDFB) the performance would be exactly the same for longer frames, since the channel is not tracked. The effect of BDFB is included in the results which follow, and is seen to be fairly minor. The expected approximate degradation from the known channel case is 2.1 dB, based on the previous theoretical analysis of Chapter 5. The observed degradations for each detector are seen to be in close agreement with the expected theoretical degradations.

Figure 6.24 gives the corresponding performance for QPSK signaling. The performance is essentially the same for the LBE detector, and a small fraction of a dB worse for the MLBE and DFBE detectors.

6.3.3 Unknown Fast Fading Channel

Fast fading, with and without channel tracking, is considered in this section. Again, the channel consists of exponentially decaying T-spaced paths. In the previous cases, the maximum Doppler frequency was set to zero, for very slow fading. In this case, the maximum Doppler frequency is set to 1/5000 of the symbol rate, which is considered to be a moderately fast fade rate. This is typical of a mobile radio channel with signaling rates of 500 Ksps or 1 Msps, and corresponding maximum Doppler frequencies of 100 Hz and 200 Hz. Channel tracking was performed using the simple stochastic gradient (SG) algorithm described in Chapter 5. The initial channel estimate was obtained using an LSSE channel estimator with a 20-symbol preamble. With fast fading, the performance is dependent on the frame length. Consequently, one data block per frame is no longer appropriate for simulation purposes. The number of data blocks per frame was selected to be 20 for these results. With 6 symbols per data block, this corresponds to 120 and 240 data bits per frame for BPSK and QPSK signaling, respectively.

Figure 6.23 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with exponentially decaying T-spaced paths, and BPSK signaling.

Figure 6.24 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with exponentially decaying T-spaced paths, and QPSK signaling.

Figure 6.25 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying T -spaced paths, BPSK signaling, and no channel tracking.

Figure 6.25 shows the bit error rate performance with BPSK signaling and no channel tracking. The performance without tracking for this fade rate might be acceptable for the MLBE and DFBE detectors. The threshold error rate for the LBE detector, however, is close to 10^{-3} . Recall that the height of the error rate threshold depends on the frame length and the detector. The longer the frame length the higher the error rate threshold.

Figure 6.26 shows the bit error rate performance with channel tracking. The signaling format is still BPSK. Two performance curves are presented for each detector. The upper curve for each detector corresponds to normal decision directed tracking and the lower curve corresponds to correct data tracking. The performance with correct data tracking is obtained by feeding the channel tracking algorithm the correct data symbols instead of the decisions. Of course, this performance cannot be achieved in practice, but is presented for comparison purposes so that the degradation caused by decision errors in the tracking algorithm can be determined. With tracking, the performance for the MLBE and DFBE detectors is considerably better than that presented earlier for the unknown slow fading channel. It is clear then that for slow to medium fade rates, channel tracking can easily reduce the fundamental degradation imposed by estimating the channel from a short preamble. Recall that, with a 20-symbol preamble, the expected degradation for slow fading is 2.1 dB. The degradation due to decision errors in the tracking algorithm is seen to be a small fraction of a dB for BPSK signaling, with the MLBE detector slightly less sensitive than the DFBE detector. For the above results, the tracking step-size parameter was set to $\mu=0.01$. This value was selected because it provided near optimum performance for all three detectors. The step-size parameter was not optimized for each different detector or signal-to-noise ratio.

Figures 6.27 and 6.28 give the corresponding performance curves for QPSK signaling. Without channel tracking, the threshold error rates are much higher than the threshold error rates for BPSK signaling. With decision directed tracking, the performance for the MLBE detector is still slightly better than that presented earlier for an unknown slow fading channel, while the DFBE detector is about the same, and the LBE detector is worse. Again, this indicates that the QPSK format is more sensitive to fast fading than the BPSK format. For these results, the tracking step-size parameter was set to $\mu=0.01$. As before, this value was selected because it provided near optimum performance for

Figure 6.26 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying T-spaced paths, BPSK signaling, and channel tracking.

Figure 6.27 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying T-spaced paths, QPSK signaling, and no channel tracking.

Figure 6.28 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying T-spaced paths, QPSK signaling, and channel tracking.

all three detectors. Note that QPSK signaling is also more sensitive to decision errors in the channel tracking algorithm.

6.3.4 Results With T/2 Spacing

Figures 6.29 through 6.36 present the corresponding performance results for a channel model with T/2-spacing. The results with T/2-spacing are very similar to the T-spaced results just presented. Only the differences are highlighted below.

Besides the T/2-spacing, the major difference in the simulation model is the shape and bandwidth of the transmit and receive filters. The transmit filter is now a 4-th order Butterworth with 3-dB bandwidth equal to the symbol rate. The time and frequency response of this filter were presented in Figure 6.3. The receive filter is again modeled as an ideal antialiasing filter with bandwidth equal to the Nyquist bandwidth. With T/2-spacing, this bandwidth is now doubled to twice the symbol rate. The channel estimator and detectors all perform T/2-spaced processing.

Figures 6.29 and 6.30 present the performance for a known slow fading channel, with BPSK and QPSK signaling, respectively. The performance for the MLBE and DFBE detectors is a fraction of a dB worse than that presented for the T-spaced model, and the LBE detector is much worse. Some degradation is expected since the transmit filter combined with the channel response is now much longer than 8 symbol periods. Only the first 8 symbol periods are considered at the receiver when performing detection.

Figures 6.31 and 6.32 present the performance for an unknown slow fading channel, with BPSK and QPSK signaling, respectively. The expected approximate degradation from the known channel case is still 2.1 dB. For high signal to noise ratios, the observed degradations are again in close agreement with the expected theoretical degradation.

Figures 6.33 and 6.34 present the performance for BPSK signaling with fast fading, without and with channel tracking, respectively. Figures 6.35 and 6.36 present the corresponding results for QPSK signaling. As before, two performance curves are presented for each detector with tracking. The upper curve for each detector corresponds to normal decision directed tracking and the lower curve corresponds to correct data tracking. The relative performance and conclusions are similar to those presented earlier for the T-spaced model.

Figure 6.29 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with exponentially decaying $T/2$ -spaced paths, and BPSK signaling.

Figure 6.30 Probability of bit error versus E_b/N_0 for a known slow fading multipath channel with exponentially decaying $T/2$ -spaced paths, and QPSK signaling.

Figure 6.31 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with exponentially decaying $T/2$ -spaced paths, and BPSK signaling.

Figure 6.32 Probability of bit error versus E_b/N_0 for an unknown slow fading multipath channel with exponentially decaying $T/2$ -spaced paths, and QPSK signaling.

Figure 6.33 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying $T/2$ -spaced paths, BPSK signaling, and no channel tracking.

Figure 6.34 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying $T/2$ -spaced paths, BPSK signaling, and channel tracking.

Figure 6.35 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying $T/2$ -spaced paths, QPSK signaling, and no channel tracking.

Figure 6.36 Probability of bit error versus E_b/N_0 for an unknown fast fading multipath channel with exponentially decaying $T/2$ -spaced paths, QPSK signaling, and channel tracking.

It should be noted that short-block detection with block decision feedback (BDFB) works best if the overall response (transmit filter plus multipath channel plus receiver filter) is fairly causal (minimum phase) in nature. This is why the transmit filter was selected to be a causal type, such as a Butterworth filter, as opposed to a raised-cosine filter, for example. Using causal type filters helps minimize the number of symbols per training block required.

Chapter 7

SUMMARY AND CONCLUSIONS

This research was motivated by the problem of providing reliable digital communications over fading channels with severe time-dispersion, in general, and over mobile radio channels, in particular. Three short-block data detection techniques based on linear, nonlinear decision-directed, and maximum likelihood estimation principles were investigated. The block detection schemes all require an estimate of the channel impulse response. This is considered an advantage from the adaptation point of view, since channel response estimation is one of the simplest adaptation requirements for any adaptive detection process.

The following sections give a summary of:

1. the relevant aspects of fading time-dispersive channels,
2. the three short-block data detection techniques investigated,
3. the channel estimation and tracking methods,
4. the theoretical and simulated performance results,
5. the complexity of the different detection algorithms, and
6. suggestions for further research.

7.1 Fading Time-Dispersive Channels

All physical systems exhibit some form of time dispersion, which for digital data transmission results in intersymbol interference (ISI). On radio channels, such as short-wave ionospheric propagation or high frequency (HF), mobile radio, tropospheric scatter, and indoor wireless or portable, much of the time dispersion, and hence ISI, is due to multiple propagation paths with different path delays. Although the signaling techniques and detectors investigated can be applied to all types of fading multipath channels, mobile radio channels were emphasized in the discussion and examples.

The fade rate for mobile radio applications is directly tied to the carrier frequency and vehicle speed. A maximum Doppler frequency of 100 Hz is often suggested as

being typical of worst case, fast fading, for carrier frequencies on the order of 1 GHz. It is incorrect, however, to consider fast fading to be the worst case. From an adaptation perspective, it may well correspond to a worst case, but from a time diversity perspective, fast fading is a best case. Time diversity, often obtained through coding and interleaving, is only available if the interleaving depth is greater than the coherence time of the channel. The coherence time is directly related to the maximum Doppler frequency, and thus to vehicle speed. Even long interleaving depths can not provide significant diversity gain for very slow moving or motionless vehicles (e.g. emergency situations). Frequency diversity, provided by multipath time-dispersion at high signaling rates, can also be exploited, and is independent of vehicle speed.

Fading time-dispersive channels can offer improved performance over flat fading (non-time-dispersive) channels, due to the inherent diversity available from receiving the signal over multiple paths with different delays. The length of the time-dispersion or delay-spread, measured in symbol periods, is directly related to the signaling rate. The faster the signaling rate, the longer the delay-spread, and the greater the potential improvement in performance due to inherent diversity. Of course, the longer the delay-spread, the greater the number of significant ISI samples, and hence, the more complex the receiver must be to take advantage of the inherent diversity. On the other hand, the higher the signaling rate, the longer the coherence time, and the easier the adaptation problem. For these reasons, the usual trade-off between complexity and performance is much more complicated for fading time-dispersive channels than for static channels, and is strongly dependent on the selected signaling rate.

The multipath intensity profiles for mobile radio channels tend to peak early, with most of the energy falling within the initial 3 usec, on average. This is not to say that significant echoes don't occur beyond 3 usec, but they tend to be rare, and their overall energy contribution is typically small. To provide significant diversity gain, this implies the use of signaling rates on the order of 1 Msps, or higher. Effective detectors must also be able to handle some of the longer path delays when they occur. These aspects have been taken into account for the signaling and detection schemes investigated in this study.

7.2 Short-Block Data Detection Techniques

Three short-block data detection techniques based on linear, nonlinear decision-directed, and maximum likelihood estimation principles were investigated. Short alternating blocks of data and training symbols are used. The training blocks are useful for both channel tracking and data block detection. In contrast to the recursive symbol-by-symbol equalization approaches usually employed, each data block is detected as a unit.

Linear and decision-directed short-block equalization techniques have been considered previously [Hsu85, Per87], but the derivations are presented for discrete T-spaced signal models, where T is the symbol period. As a consequence, either a whitened-matched filter, as defined by Forney [For72], or simple, and less efficient, symbol rate sampling is assumed. In this work, the corresponding detectors for continuous-time signals were derived. These two approaches are denoted as linear block estimation (LBE) and decision feedback block estimation (DFBE). In addition, maximum likelihood block estimation (MLBE) was also considered. The continuous-time derivations show that these three detectors require a matched filter, as opposed to the more complicated whitened-matched filter. A fractionally spaced formulation is useful for simulation and implementation purposes. Such a formulation is easily deduced from the continuous-time analysis presented.

The block detection schemes investigated employ a type of least sum of squared errors (LSSE) estimation for dealing with ISI. The first two schemes are based on direct estimation of the data blocks, followed by decision quantization. The maximum likelihood approach involves a quantized search for the least squares solution. Training data is embedded into the transmission in the form of blocks of training symbols alternated with blocks of source data symbols. The training blocks are useful for both channel tracking and data block detection. They also help reduce error propagation effects, when different forms of decision feedback are employed. The appropriate data and training block lengths depend on the expected channel parameters, the desired throughput efficiency, and the allowed complexity.

A major limitation of the block detection schemes previously presented in the literature, is that the training blocks must be at least as long as the effective memory of the overall channel response. This usually implies, for complexity reasons, that the data

blocks be about the same length as the training blocks, resulting in approximately 50 % throughput efficiency. This restriction has been removed for the detectors derived in this study. Allowing the training blocks to be shorter than the channel memory, results in greater throughput efficiency. This is partly accomplished by including a form of block decision feedback (BDFB). An additional bonus is that the data blocks can also be shorter, which reduces the complexity, and also makes the complexity less dependent on the channel response length. This in turn allows higher signaling rates to be used for a given complexity.

One of the more appropriate modulation techniques is single-tone M-ary phase shift keying (MPSK). Results were presented for both BPSK and QPSK signaling. Data rates higher than 1 bps/Hz can easily be achieved by selecting appropriate values for M, and the data and training block lengths.

7.3 Channel Estimation

The block detection schemes investigated all require an estimate of the channel impulse response. This is considered an advantage from the adaptation point of view, since channel response estimation is one of the simplest adaptation requirements for any adaptive detection process.

There are two aspect to the channel estimation problem: (a) channel estimation from an initial training sequence (startup), and (b) channel tracking. The simple stochastic gradient (SG) algorithm, also known as the least mean square (LMS) algorithm, was employed for channel tracking purposes. Although the SG algorithm works well for channel tracking, it does not work well for startup purposes. The startup problem was solved by using a least sum of squared errors (LSSE) approach, similar to that used for developing the LBE and DFBE detectors. Even though the inverse of a correlation matrix is involved, the solution is simple because the matrix is only a function of a known preamble training sequence. This allows the inverse matrix to be precomputed and stored.

The preamble training sequence lengths required to provide good channel estimates are relatively short, typically only 2 to 3 times the length of the channel response. A computer search technique was used to find optimum preamble training sequences for

different channel and training sequence lengths. These optimum training sequences are tabulated in Chapter 5.

Some special training sequences were presented which result in trivial channel estimator implementations. In some cases, the preamble correlation matrix is perfectly diagonal and the mean squared channel estimation error is minimized. A training sequence is said to be perfect if it provides a perfectly diagonal correlation matrix. It was shown that perfect training sequences can always be obtained for any given channel length, but the length of these sequences grows exponentially with channel response length.

The higher the symbol rate, the more severe the ISI due to time-dispersion, and the longer the channel response to be estimated. The coherence time (measured in symbol periods) is also increased in this case. The longer the coherence time, the larger the averaging (filtering) time constants can be for channel tracking. The longer the memory of the channel tracker, the less sensitive the channel tracker is to individual decision errors. The degradation in performance caused by error propagation was investigated, and was not observed to be a major problem for the simulation results presented.

A simple approximation was derived for translating the mean squared channel estimation error into a corresponding dB degradation for the detector involved, and is given by

$$DEG = 10 \log \left(1 + \frac{1}{SER'} \right) \quad dB \quad (7.1)$$

where SER' is the normalized signal to channel estimation error ratio defined in (5.31). SER' is plotted in Figures 5.1 through 5.4 for different channel lengths, training sequence lengths, and training sequence types (zero-amplitude and nonzero-amplitude precursors). This approximation is valid only for high signal to noise ratios (SNRs). The simulation results confirmed the accuracy of this approximation for high SNRs. The degradations were observed to be slightly larger for low SNRs.

7.4 Performance

Theoretically based probability of error performance results were presented in Chapter 4, for Rayleigh fading channel models with 2, 3, and 4 equally energetic T-spaced paths.

Results were presented for both BPSK and QPSK type signaling. As well as determining the performance for the three block detectors, the performance for a conventional linear equalizer (LE) and a conventional decision feedback equalizer (DFE) was also determined, for comparison. The method of evaluation for the LBE, DFBE, MLBE, LE and DFE detectors, was to use a Monte Carlo calculation over the fading channel process, with theoretical approximations used for each given channel response. The performance for an ideal isolated pulse matched filter (Ideal) detector was also presented for comparison purposes, and to provide a lower bound on the performance for the other detectors. The performance for the Ideal detector is that of a perfect diversity combining system, and exact theoretical formulas were used. Slow fading and perfect knowledge of the channel was assumed.

The performance for the block detectors is discussed first. As expected, the MLBE detector outperformed the LBE and DFBE detectors, and almost takes full advantage of the available inherent diversity, as the number of paths is increased. The degradation from Ideal was less than 1 dB for all the cases considered. The DFBE detector provided the next best performance, with degradations from Ideal ranging from 1.5 to 2.5 dB. It also makes good use of the inherent diversity. The LBE detector provided the poorest performance, with degradations from Ideal of 3 to 4 dB. Even with these fairly large degradations, its performance continued to improve as the number of paths was increased. Thus, the positive effect of inherent diversity, due to time-dispersive fading paths, was demonstrated to be much more significant than the negative impact of severe ISI, even for the LBE detector.

The theoretical performance for the LE and LBE detectors was compared using the channel model with 4 equally-energetic T-spaced paths. A data block length of 6 was used for the LBE detector. LE detectors with 6 and 9 T-spaced taps were considered. These detectors are denoted as LE(6) and LE(9), respectively. The performance for both LE cases was shown to be much worse than the performance for the LBE detector. (See Figure 4.6.)

The theoretical performance for the DFE and DFBE detectors was compared using the same channel model. A data block length of 6 was again used for the DFBE detector. DFE detectors with 4 and 6 forward taps, and 3 feedback taps, were considered.

These detectors are denoted as DFE(4,3) and DFE(6,3), respectively. The theoretical performance curves for the DFE(4,3) and DFE(6,3) detectors are located about half a dB on either side of the theoretical curve for the DFBE detector. (See Figure 4.7.) However, the simulated performance for the DFBE detector was found to be about half a dB better than the theoretical approximation for the DFBE detector. (See Figure 6.7.) Thus, with enough taps for the DFE detector, the DFE and DFBE detectors were observed to provide approximately the same performance. Their relative complexities are compared in the next section.

Extensive simulation results were presented in Chapter 6 for the block detectors. Bit error rate results were obtained for some general multipath channel models, and for channels specifically tailored to model a mobile radio channel with a signaling rate on the order of 1 Msps. The multipath intensity profile for the mobile radio channel was modeled as having an exponential decay of 3 dB per symbol period, for a length of 8 symbol periods. Results were presented for both T-spaced and T/2-spaced simulation models. A fourth order Butterworth transmit filter was used for the T/2-spaced model.

The simulation results, obtained with the same channel models and assumptions used for establishing the theoretical results, were typically within half a dB of the corresponding theoretical approximations, for all three block detectors. The theoretical and simulated performance curves were closest for the LBE detector. The performance curves for the MLBE detector were also in close agreement. The theoretical performance curves for the DFBE detector were observed to be slightly pessimistic, but still within approximately half a dB of the simulated results. This was expected, since pessimistic approximations were used in the derivation of the theoretical performance for the DFBE detector.

The simulation results generally confirmed the expected performance degradations with channel estimation errors. The stochastic gradient (SG) channel tracking algorithm was shown to work well for both slow and moderately fast fading.

For the T/2-spaced mobile radio channel model, it was shown that 2 symbols per training block is sufficient to provide significant diversity gain. A data block length of 6 was used. The length of the estimated channel response was 8 symbol periods. Table 7.1 gives the E_b/N_0 ratios (in dB) required for each of the detectors to achieve a probability of bit error of 10^{-3} . Assuming a signaling rate of 1 Msps, the maximum

Doppler frequency was 200 Hz. Channel estimation and tracking were both performed, and all decision feedback error effects are incorporated. These results underscore the importance of eliminating the restriction that the training blocks be at least as long as the memory of the channel.

It should be noted that short-block detection, with block decision feedback (BDFB), works best if the overall response (transmit filter plus multipath channel plus receive filter) is relatively causal (minimum phase) in nature. This was the reason the transmit filter was selected to be a causal type, such as a Butterworth filter, as opposed to a raised-cosine filter. Using causal type filters helps minimize the number of symbols required per training block.

7.5 Complexity

All three block detectors require the use of a channel correlation type matrix, R . The algorithms depend on the form of this matrix (i.e. whether R is Toeplitz or not). The R matrix is Toeplitz only when the number of symbols per training block is restricted to be greater than or equal to the channel memory. Eliminating the Toeplitz requirement removes this restriction.

The complexity of the Toeplitz LBE detector, using a Levinson approach, is order N per data symbol, where N is the number of symbols per data block. The complexity of the non-Toeplitz LBE detector, using a Cholesky factorization approach, is order N^2

E_b/N_0 (dB)	Data Detection Schemes				
Signaling Format	Ideal (exponential diversity @ 3dB/T)	Maximum Likelihood Block Estimation (MLBE)	Decision Feedback Block Estimation (DFBE)	Linear Block Estimation (LBE)	Ideal (flat fading)
BPSK	10.0	13.0	14.5	19.5	24.0
QPSK	10.0	14.0	16.5	25.0	24.0

Figure 7.1 E_b/N_0 (in dB) required to achieve a bit error rate of 10^{-3} for the T/2-spaced mobile radio channel model. (Ideal flat fading included for comparison.)

per data symbol. For small data blocks ($N < 6$), the complexities are nearly identical. Since the non-Toeplitz LBE detector allows smaller data blocks to be used, for a given throughput efficiency, it is often the least complex of the two options.

It was shown that no complexity penalty is imposed by removing the Toeplitz restriction for the DFBE detector. In fact, the new Cholesky-based DFBE algorithm has a lower complexity than the Levinson-based DFBE algorithm, even for the same Toeplitz case. The complexity is order N^2 per data symbol, and is the same as that for the non-Toeplitz LBE algorithm. Note that the improved performance provided by the DFBE detector (compared to the LBE detector), is obtained without an increase in complexity for the non-Toeplitz case.

The complexity of the MLBE detector was shown to be essentially the same for the Toeplitz and non-Toeplitz cases. The use of small data blocks is even more critical for the MLBE detector, since the complexity grows exponentially with data block length (assuming the data block length is the same as, or less than, the channel response length). The complexity also depends on the size of the data symbol alphabet. For BPSK signaling and small data blocks ($N < 10$), the complexity is reasonable, and comparable to that for the DFBE detector. The complexity with QPSK signaling grows much more rapidly, making it very complicated for block lengths of 6 or more. It should be noted that, for BPSK and QPSK signaling, no multiplies or reciprocals are required to implement the block detection portion of the MLBE detector. This is not the case for the LBE and DFBE detectors.

Table 7.2 gives the total number of complex operations (multiplies, adds, reciprocals, and/or comparisons) per data symbol, for the LBE, DFBE, and MLBE detectors, assuming non-Toeplitz matrices are to be handled. The number of operations for the LBE and DFBE detectors is the same for both BPSK and QPSK signaling. The complexities for the MLBE detector are different for BPSK and QPSK signaling. The channel estimation, tracking, and preprocessing required to setup the matrix equations to be solved, is the same for all three block detectors, and has not been included. The complexity formulas for these additional operations are given in Chapter 3, and depend on such parameters as the channel length, the number of samples per symbol period, and the number of symbols per training block, as well as the number of symbols per data block. Assuming the data

block length is approximately the same as the channel response length, the additional complexity is approximately order N per data symbol.

It is difficult to directly compare the complexity of a block detector to that of a conventional detector. This is because the complexity of a block detector is dominated by the number of symbols per data block, whereas the complexity of a conventional detector is dominated by the number of filter taps. If the span of the LE detector is the same as the data block length, then for T-spaced implementations, the dimensions of the matrix equations to be solved are the same for the LE and LBE detectors. This assumes that the LE tap coefficients are to be calculated from an estimated channel response. It is also assumed that the same update period is required for both detectors. The dimensions of the matrix equations to be solved for the DFE and DFBE detectors are also the same, if the span of the T-spaced forward filter equals the data block length.

Data Block Size (N)	Number of Complex Operations per data symbol		
	LBE and DFBE detectors	MLBE detector	
		BPSK signaling	QPSK signaling
1	2	1	3
2	5.50	4.50	17.50
3	9.67	9	55.67
4	14.50	16.25	174.75
5	20	28.60	566.20
6	26.17	50.17	1,893.83
7	33	88.43	6,499.29
8	40.5	157.13	22,753.38
9	48.67	281.67	80,906.56
10	57.5	509.30	291,269.10

Table 7.1 Total number of complex operations (adds, multiplies, reciprocals and/or comparisons) per data symbol, to implement the block detection portion of the non-Toeplitz LBE, DFBE, and MLBE detectors, with BPSK and QPSK signaling. The channel estimation, tracking, and preprocessing required to setup the matrix equations, is the same for all three block detectors, and has not been included.

With the above assumptions, the complexities for all four detectors are similar. The complexities with fractionally-spaced implementations are quite different, however. If the LE and DFE filters are to span the same number of symbol periods as before, then T/K-spaced implementations require matrices which are K times the size of the previous matrices, for T-spaced implementations. The matrices required for the LBE and DFBE detectors remain the same size, giving the LBE and DFBE detectors a definite complexity advantage in this case.

7.6 Suggestions for Further Research

Much more work could be done using the simulation program, to optimize the numerous algorithm parameters, for each of the different detection methods, and for specific signal to noise ratios. This has only been done in a broad sense. For example, the block sizes and channel tracking constants were always set to the same values, for the different detectors. The DFBE detector was shown to be more sensitive to decision feedback errors in the tracking algorithm, than the other two detectors. Different channel models could also be considered.

Other transmit and receive filters should be investigated. In particular, a Nyquist bandwidth antialiasing filter was always assumed for the receive filter, which results in uncorrelated noise samples. In the absence of adjacent channel interference, this is a reasonable approach, and is near optimum for high signal to noise ratios. With adjacent channel signals present, the receive filter type and bandwidth become important design parameters. In addition to investigating the performance with adjacent channel interference, the detectors could be modified to handle correlated noise samples.

It was noted earlier that the complexities of the conventional DFE and DFBE detectors can be very similar for T-spaced implementations, but that the DFBE detector has a definite complexity advantage for fractionally-spaced implementations. Given that the performance of these two detectors is quite similar, it would be worthwhile to try and reduce the complexity of the conventional DFE detector for fractionally-spaced implementations. One non-conventional DFE approach might be to perform fractionally-spaced matched filtering, followed by a calculated conventional T-spaced

DFE. A potential problem with this approach is the additional delay, introduced by the two-step filtering process, which could adversely affect any channel tracking.

Higher order modulation formats than BPSK and QPSK could also be investigated.

The MLBE detector is a true maximum likelihood detector, given the observation interval for each data block. Reduced-state, or other pseudo-maximum likelihood techniques, can provide a trade-off between performance and complexity. This could be a fruitful area for further investigation. Note that the intermittent training blocks provide a natural reset capability, often required for such schemes.

It would be worthwhile to determine the performance of the detectors for real fading channels, as opposed to simulated channels. This could be done using measured fading channel responses, and/or a real time implementation. Given the determined complexities, a real time implementation, based on current digital signal processing technology, appears quite feasible.

Also worth pursuing is an investigation of overall system capacity (number of users, or number of erlangs, accommodated per unit area), including the effects of combining more users in a wideband TDMA format, frequency re-use, cell structure, adjacent and co-channel interference.

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