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**Numerical Simulation
of Aluminum Sheet Forming
Incorporating Plastic Anisotropy**

by
Chantal G. Lamontagne

B. Eng. (Aerospace Engineering)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering
in Mechanical Engineering

**Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
May, 1996**

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ISBN 0-612-13844-5

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**Numerical Simulation of Aluminum Sheet Forming
Incorporating Plastic Anisotropy**

submitted by Chantal G. Lamontagne
in partial fulfilment of the requirements for
the degree of Master of Engineering
in Mechanical Engineering

Thesis Supervisor M.J. Worswick

Chair, Department of Mechanical and Aerospace Engineering

Carleton University

May, 1996

Abstract

This research examines the influence of constitutive formulation on the numerical prediction of strain distribution in 5754 and 6111 aluminum alloy sheet deformed within a hemispherical punch apparatus and a rectangular deep draw universal forming test. Of particular interest is the comparison of quadratic and non-quadratic anisotropic yield criteria when applied to materials, such as aluminum, that display anisotropy characterized by low values of Lankford's coefficient. Simulations were performed using a commercial, explicit finite element code, LS-DYNA3D. Predicted strain distributions were compared to measurements from supporting experiments using a grid strain analysis technique.

The FEM predictions obtained using the non-quadratic Barlat yield criterion demonstrate reasonably good agreement with experiments, especially in the biaxial stretching and plane strain conditions. These results were superior to those obtained using either Hill's quadratic anisotropic criterion or the von Mises isotropic criterion.

A parametric study of the effects of the Coulomb frictional coefficient, μ , on the predicted strain distributions was undertaken and indicated that the strain distributions were sensitive to variations in μ .

Acknowledgements

To Prof. M.J. Worswick I offer my most sincere thanks. His guidance, patience, and support throughout this research were invaluable. I thank him for sharing his knowledge of the finer points of sheet metal forming and finite element analysis.

I wish to thank the faculty and staff of the Department of Mechanical and Aerospace Engineering for making my stay at Carleton University so memorable. Special thanks to Bruce Johnson and Kris Cooper for answering all my questions.

I would like to express my sincere thanks to Dr. Pierre Martin for his collaboration in this research. His patience in taking the time to assist with the deep drawing experiments is greatly appreciated. I would also like to acknowledge the assistance provided by the staff of the Metal Forming Technology Laboratories of CANMET in the experimental portion of this research.

I would like to express my sincere thanks to Kevin Williams. He passed me the torch, taught me the ins and outs of PATRAN and LS-DYNA3D, and then endured my countless questions with a smile. I also owe a debt of gratitude to Keith Pilkey and John Fowler, whose labours in developing the grid analysis software saved me hours of work. I gratefully acknowledge the assistance provided by Jacques Lemieux and Trevor Yensen in getting the limiting dome height press facility operational.

I would like to acknowledge the assistance of Dr. Mukesh Jane of the ALCAN Kingston Research and Development Centre in providing the aluminum used in the experimental portion of this research. Special thanks also to Mark Finn of ALCAN for the helpful hints on using LS-DYNA3D.

This work could not have been completed without the love and support from my parents and sisters. Their constant encouragement has kept me sane during difficult times. I dedicate this thesis to them, and to the memory of my friend, André Delaney.

Partial financial support for this project was provided by the Natural Sciences and Engineering Research Council of Canada and the Zonta International Foundation.

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List of Symbols

$a, b, c, f, g,$	anisotropic parameters used in Barlat's non-quadratic yield criterion
h, p, K_1, K_2	
A_i	face area of element i
F_f	frictional force
$F, G, H,$	anisotropic parameters used in Hill's quadratic yield criterion
L, M, N	
K	strength constant
K_i	bulk modulus of element i
LDH_0	limiting dome height at plane strain
m	strain rate sensitivity
m, M	exponents in Hill's and Barlat's non-quadratic yield criteria
n	strain hardening exponent
N	normal contact force
r	Lankford's coefficient, normal anisotropic ratio
$\bar{r}$	average anisotropic parameter
R, S, T	principal anisotropic axis shear yield stresses
X, Y, Z	principal anisotropic axis tensile yield stresses
V_i	volume of element i
Δr	planar anisotropic parameter
ϵ_0	strain due to prior coldworking

$\epsilon, \dot{\epsilon}$	strain, strain rate
$\bar{\epsilon}$	effective plastic strain
ξ_i	interface stiffness scaling factor of element i
λ_i	interface spring stiffness of element i
μ	Coulomb friction coefficient
σ	stress
σ_y	yield stress
$\bar{\sigma}$	effective stress
τ	shear stress

Additional Subscripts

0, 45, 90	measured at 0°, 45° and 90° to the sheet rolling direction
1	major principal value
2	minor principal value
x, y, z	corresponding to reference axes or directions

Chapter 1

Introduction

The oil crisis of the late seventies is seen as the precursor to today's world-wide public concern for the environment [1], especially from the effects of global warming and CO₂ emissions [1, 2]. Consumers are now aware of such issues as natural resource saving, lower energy consumption as well as measures against environmental pollution [2, 3]; they are the driving force behind the demand for better fuel economy of vehicles.

There are various methods of improving fuel consumption [1, 2], such as improving the efficiency of parts, by either improving the efficiency of the engine combustion and power transmission or by reducing losses due to friction, and reducing the running resistance of the vehicle, by reducing the air and rolling resistance and making the vehicle more lightweight. Given the current level of economy, the automotive industry has chosen to reduce the weight of their vehicles [1, 2, 3, 4] as the most direct route to achieving conformance with existing safety, emissions and fuel economy regulations. Making vehicles lighter has decidedly improved fuel savings, as clearly indicated in Figure 1.1 from Isogai and Murakami [1]. Weight reduction has also resulted in better braking performance and better vehicle acceleration for the same engine output [1], making these vehicles more attractive to the consumer, both in terms of cost and performance.

Weight reduction in the automotive industry is usually accomplished by sizing down the autobody [2, 5], but driving safety considerations and market demand, (consumers do not want the size or carrying capacity of the vehicle reduced [5]), have precluded further downsizing. Changing the vehicle body structure is another alternative, but replacing conventional body materials with lighter weight materials is more prevalent [2, 6]. In this regard, aluminum, with its high strength-to-weight ratio, is an excellent candidate [2, 4, 6, 7, 8] and has clearly been making progress in replacing the utilization of sheet steel in the automotive industry [3, 6, 7, 9, 10, 11]. Companies such as Rover, Pininfarina, Volvo, Porsche, Mercedes, Audi and Ferrari had already adopted the use of aluminum alloys in the 1980s [10]. In 1982, Miyahara *et al.* [7] estimated that substituting aluminum alloy sheet for mild steel in automotive body panels would result in a 50-60% weight reduction, in spite of the increase in sheet thickness [3, 7] required to obtain the same surface stiffness and static dent resistance as steel panels. The Honda NSX has made the best use yet of aluminum, for both the engine and chassis systems: 31.3% of the car weight is aluminum, compared to 7.2% for conventional cars, which has reduced the weight of the vehicle by about 200 kg compared with a steel design [3], a landmark achievement in the use of aluminum in the automobile industry.

Aluminum not only provides significant weight savings, and hence energy savings, but also has excellent corrosion resistance [10] and conforms to recyclability aims [3, 4, 8]. Unfortunately, the forming characteristics of aluminum are inferior to those of steel [2, 7, 12] and since the previous decades of forming research have mainly dealt with steel, no formability database for aluminum sheet exists [11]. Hence, there is currently a pressing need for research advances into the forming of aluminum sheet metal.

Sheet metal forming has, historically, been regarded as more of an art than a science [11, 13, 14, 15]. Craftsmen utilized their skill and experience trying out various dies in an expensive, time-consuming, trial-and-error method [16-24]. The largely unknown formability behaviour conditions of aluminum, coupled with the complexity of modern automotive parts, has led to the decline of the artisan system of die design [24]. Reduced lead times and the collapse of the long-term apprenticeship training system, with

the early retirement of skilled artisans outdistancing the influx of young people willing to learn the craft, have been cited by Keeler [24] as contributing factors to this breakdown and to the driving need for change to an engineering approach.

The integration of Computer-Aided Design (CAD) and Computer-Aided Manufacturing (CAM) has already benefited the automotive industry in the move towards a cost-effective, knowledge-based, analytical technique [19, 24]. The direct transfer to the shop floor of the latest production and sheet metal forming information [25] has reduced lead times for the production of automotive panels by half [26]. Still more savings, in both time and money, can be obtained by the use of a predictive system [24, 27] which would incorporate all aspects of the forming process and consider the interaction of the process parameters. Though small, personal computer (PC) based simulations are useful, in a limited fashion, to study parametrically the effects of these process variables [28], the results are usually only rough approximations and a significantly more complex simulation is needed. This type of mathematical model has only been making an impact on metal forming processes in the last decade [14], especially in the application of finite element method (FEM) procedures for the simulation of sheet metal forming [23, 29, 30]. Rapid development of computers and software has permitted the FEM to become a powerful tool with tremendous impact, enabling three-dimensional, large-scale numerical simulations of press forming operations to become feasible.

The objective of the current research is to investigate the numerical prediction of laboratory-scale sheet metal forming processes. Of particular interest is the evaluation of material formability with respect to aluminum stretching and deep drawing operations. Formability, the measure of a material's ability to be plastically formed under specified conditions, is usually the controlling factor in most areas of the die design, as shown in Figure 1.2, and is, generally, influenced by the material properties, the failure or formability criterion applied, and the forming conditions. To be accurate, a simulation of a sheet metal forming process must therefore attempt to completely describe the characteristics and behaviour of the material. The current research is concerned with the assessment of various sheet metal constitutive models used in finite element simulations

of sheet forming operations. Of particular interest is the prediction of the effects of normal anisotropy using quadratic and non-quadratic yield surfaces. The sections that follow will review the basic issues, both numerical and experimental, involved in the simulation of sheet metal forming. These include such issues as material formability, the effects of process variables and material properties on sheet metal forming, material constitutive behaviour, and applications of the finite element method to the simulation of sheet metal forming.

1.1 Formability

For many years, the suitability of sheet metal for specific forming operations (i.e., the formability of a sheet metal) has been evaluated by press shops and stamping plants by various forms of simulative cup tests. The diversity of forming operations engendered the advance of a number of cup tests (more than one hundred [31]) which attempt to simulate various modes of deformation, primarily the stretching and drawing mechanisms, in the actual forming of parts from sheet metal. Only a few have become standards upon which to compare material formability. These tests include [32] the Olsen and Erichsen cup tests to assess stretching performance [33, 34, 35, 36], the Swift cup test [37, 38] for deep drawing, and the Fukui cup test [41, 42], which combines both stretching and drawing.

The Swift cup test, shown in Figure 1.3, involves the use of a hemispherical indenter of radius 50.8 to 101.6 mm (2 to 4 inches) to press a circular sheet of metal clamped between two dies. As this test investigates deep-drawing, the clamping force was sufficient only to prevent the formation of wrinkles. The drawing ratio was evaluated from both the punch load-displacement history and the thickness strain. Over time, this test was also used to rank lubricants [25]. The subsequent investigations of Chung and Swift [43] concentrated on the flat-bottomed cup, with a small corner radius, to avoid the complex nature of stretching over the punch nose in the hemispherical case;

the material over the punch nose was thus locked into place through friction and could be neglected in the analysis.

In the Erichsen test (Figure 1.4) a sheet is clamped between two polished flat plates, at a specified clamping load of 1 tonne, and a 20 mm diameter steel ball, as opposed to the larger indenter used in the Swift cup test, is pressed into the sheet through a 27 mm diameter die opening. The sheet metal is mainly deformed over the punch nose by stretching, but some portion of the sheet metal over the blankholder is free to be drawn into the die hole over the die profile. A rough blankholder was employed to reduce this drawing-in, but no attempt was made to fully constrain the blank within the blankholder. The Olsen test, similar to the Erichsen test, is the North American equivalent. It uses a 0.875 inch (22.23 mm) diameter steel ball that presses a blank through a die opening of one inch (25.4 mm) in diameter. The depth of the penetration of the spherical punch into the sheet at the instant where fracture occurs was used as the formability index, known as the Erichsen number. (The Olsen cup test has been standardized in customary English units as the "ball deformation test" as described in ASTM Designation E643-84 [35].)

The Fukui test, which uses a 17.5 mm (11/16 inch) diameter ball-end punch and a conical die with a 20 mm (0.786 inch) diameter throat as shown in Figure 1.5, involves both drawing-in and stretching. A high viscosity lubricating oil and a minimum of blankholding pressure to bring the blank in contact with the conical die are utilized to form a conical cup from a circular sheet of metal. Again, the cup depth at the maximum load is used as the formability index.

The suitability of these tests as measures of formability was the subject of a number of investigations [36, 45], as were the effects of material parameters and test characteristics, comparisons with other formability tests, and the repeatability of the results. In particular, the Erichsen test was generally supported as a method of measuring sheet formability, but the lack of reproducibility [39] between tests has raised a number of questions, including the influence of both stretching and drawing on the formability of the material.

This lack of reproducibility in the Olsen and Erichsen tests, even within the same laboratory due to the experimental scatter being sometimes larger than the material differences to be measured, led Hecker to introduce a new stretch-forming cup test in 1974 [39]. He cited three main problems that precluded success with the Erichsen and Olsen tests: insufficient size of the penetrator, inability to prevent inadvertent drawing-in of the flange, and inconsistent lubrication. Hecker suggested four major improvements to the Erichsen/Olsen cup test. The first consisted of an increase in the size of the penetrator over that used in the Erichsen test since it was doubtful that the 20 mm punch could truly represent a die used to stamp large scale industrial parts. This small indenter overemphasized bending performance of the material and influenced the mode of fracture. A more realistic stretching condition was introduced by the 101.6 mm (4 inch) diameter punch proposed by Hecker. A second modification by Hecker involved the incorporation of a clamping bead on the die plates, as shown in Figure 1.6, to prevent draw-in. The inadvertent drawing-in of the flange produces totally inconsistent results while increasing the cup height at fracture, which in turn gives a false measure of the material's ability to be stretched. Hecker attempted to restrict this previously uncontrollable amount of drawing-in, which was shown to also depend on such variables as, among others, blankholder pressure, lubrication, the surface finish of the dies and specimen size. Another suggestion was to adopt a consistent standard for lubrication between the punch and sheet. All cup tests would be performed dry, or unlubricated, to remove inconsistencies that tend to mask the differences in material properties. A large amount of scatter was noticed by Hecker in the results of lubricated tests, even with careful application, but a dry hemispherical punch test provided the needed reproducibility by allowing an independent measurement of the material's intrinsic ability to distribute strain under an imposed stress state [39, 46]. Finally, a variation in test procedure was deemed necessary due to the difficulties in terminating the test at the appropriate moment to obtain a measure of the cup height. The end-point of the Erichsen test was, to some extent, a matter of operator judgement and was usually left to the discretion of the testing laboratories which, in turn, introduced errors from varying

testing rates and measuring equipment sensitivity. Hecker introduced a load and punch depth monitoring system in an attempt to eliminate these errors.

The relative heights of the failed specimens, argued Hecker, gave an indication of a material's formability. Unfortunately, Hecker's cup test allowed only one strain state in the material to be investigated, that of biaxial stretching. In addition, the strain path was found to be material dependent, and thus the strain state produced in a test could not be repeated between materials, precluding a quantitative comparison of their formability.

Ghosh [40] proposed a further modification to the experimental procedure which would allow the investigation of a wider range of strain conditions. By varying sample width, Ghosh was able to control the amount of lateral draw-in of the blank and thus the strain state at the failure location. This modification led to the observation of a characteristic minimum punch height when the sample failed near or at plane strain, termed the minimum limiting dome height LDH_0 . The limiting dome height (LDH) test thus provides a direct comparison between various types of sheet metals at a common strain state, plane strain, the state at which 85% of production parts and the majority of automotive panels fail [47].

Attempts have been made to improve upon the Hecker/Ghosh cup test, especially in determining the effects of material properties, and in applying numerical and analytical models to the forming process. Such tests as the Ohio State University (OSU) Formability Test [48] reflect these changes, but are far from gaining industry-wide acceptance. The LDH test remains the best simulator yet to evaluate the stretch formability of sheet metals. Recently, box-shaped cup drawing, such as square cup deep drawing (a variation on the Swift cup test) has been added as another formability test for automotive sheet metals [49, 50].

1.1.1 The Forming Limit Diagram

Results of the punch tests mentioned above are usually displayed in terms of the forming limit strains corresponding to the limiting cup height, as first proposed by Gensamer in his investigations of formability [34, 51]. His notion that a critical level of strain for a specific strain state characterized the formability of a material is considered one of the milestones in sheet metal forming research. The loci of the limit strains in the plane of a strained sheet, with the major strain plotted against the minor strain as shown in Figure 1.7, form a curve termed the forming limit curve (FLC), the forming limit diagram (FLD) or the Keeler-Goodwin diagram [27]. Of the various deformation modes in strain space (refer to Figure 1.8), the FLD is usually representative of the limit strains for the most common sheet metal failure mode, that of tearing.

The regions of the FLD also characterize three general strain conditions. The right hand side of the forming limit diagram, where both major and minor strains are tensile, or positive, represents biaxial stretching, or bulging, while the left side of the FLD, where the major strain is positive and the minor strain is negative, represents drawing. The centre line of the FLD, where the minor in-plane strain is zero, represents plane strain deformation. The plane strain intercept, the tip of the FLD (see Figure 1.8), provides the lowest formability limit.

The current configuration of the FLD is based upon the work of Keeler and Goodwin [54, 55]. Keeler first obtained forming limit curves in the biaxial tensile region by measuring the principal strains adjacent to the point of fracture. Goodwin then extended the procedure into the tension-compression region (negative minor strain), but he measured the strains at the actual fracture point. These strains were much greater than those measured by Keeler, as shown in Figure 1.9, or those normally obtained in any pressing operation, resulting in an uncertainty band, known as the Keeler-Goodwin band. In this area between the curves, the material may or may not be considered to have failed, depending on the failure criterion applied.

The forming limit diagram separates the strain combinations which would give a successful pressing from those where the metal has reached its forming limit, leading to weakness or fracture [11, 57]. Since Keeler's pioneering work in 1965, the FLD, in combination with grid strain analysis [54, 58], has been used successfully as a diagnostic tool in press shops [59, 60, 61, 62] for predicting how close to the forming limit a part has been designed.

1.1.2 Grid Strain Analysis

The strains in a sheet metal forming process can be obtained by measuring the shape changes of a grid pattern previously marked on the surface of the sheet metal [63]. This technique of strain analysis, originally proposed with a circle grid by Keeler in 1968 [64], has become one of the most popular methods of determining the strains in deformed parts [65, 66]. It has vastly improved press shop efficiency in die development and tryout [25], providing savings in both time and money, and is used, in conjunction with the previously mentioned forming limit diagram, as a tool to assess the formability of a sheet metal [63].

Grid strain analysis consists of first physically depositing, chemically etching or silk screening a circular or square grid on the surface of the sheet to be formed. Subsequent measurement of both the major and minor axes of the ellipses obtained from circle grids, or the final nodal coordinates from square grids, provides a measure of the principal surface strains when compared to the undeformed grid dimensions. A distribution of the strain is obtained when adjacent grids are measured in series, but the measurement of large areas of a part is prohibitive. Developments in automatic image analysis have greatly reduced the required time and effort in this area by increasing the speed and repeatability of experimental measurements [66]. Digital imaging techniques provide even further improvements through the use of nodal or grid point measurements [67].

Grid strain analysis does have limitations. The overall, or macroscopic, strain distribution is obtained using this technique, but sharp strain gradients are more difficult to identify. This limitation is due to the assumption that there is no strain gradient across the grid [67] and the fact that the strains are derived from the initial and final grid configurations only. Thus the strain path is neglected, which may result in two different deformation processes obtaining the same strain measure through identical grid shapes [67]. The selection of an appropriate grid size is also problematic [65] in grid strain analysis, as is the application of the grid itself [68] and its effects on friction [69].

1.2 Effects of Process Variables on Sheet Metal Forming

Process variables interact with material properties, discussed in Section 1.3 below, in an exceptionally complex manner to affect the formability of a sheet metal [13, 40]. Usually assessed during die tryout, these include punch speed, blankholder pressure, die and punch radii, blank size, blank thickness and lubrication [16, 25, 69]. Some variables, such as press rigidity, die surface finish, and wear, are often neglected in sensitivity analyses due to the prohibitive time and effort required to characterize them and the fact that their significance wanes unless considered for high volume production or large scale presses. Of particular importance to the simulation of sheet metal forming are the lubrication and the binder or blankholder pressure; they both interact to effectively control the drawing of the material through the tooling and, thus, the strain distribution in the part. A brief discussion of the effects of blank size, shape and thickness on formability is also included.

1.2.1 Friction and Lubrication

Friction between the blank and the tool surfaces is considered to be one of the most important boundary conditions in sheet metal forming [70], having a large influence on the material flow. The localization and eventual failure of a sheet in a forming

process is controlled by the amount of friction induced in the process and the lubrication conditions. By modifying the frictional behaviour of the punch-sheet interface, the strain distribution obtained in a pressing can therefore be varied.

In limiting dome height tests, reduced friction at the punch-sheet interface promotes a more uniform strain distribution by allowing the material to stretch or draw over the punch, delaying strain localization and moving localization towards the pole, or the lowest point of the punch. Low friction also moves the strain state away from plane strain, towards biaxial stretching for square sheets, and towards uniaxial tension for narrow sheets. As the friction is increased, the deformation over the punch surface is retarded and the localization moves towards the tangent line of contact, away from the pole. The material constrained beneath the punch also limits lateral draw-in, thereby shifting the strain condition towards that of plane strain. A higher friction coefficient also results in an increased punch force [17].

Lubrication is especially important in deep drawing applications. While it is necessary to allow flange material to flow into the die cavity to prevent excessive thinning or tearing in a drawing operation, this induces compressive minor strains which lead to buckling, or wrinkling. Friction between the blank and the tool surfaces, however, leads to tension or stretching in the sheet, which resists buckling. An appropriate lubrication scheme will control the friction, and therefore the amount of draw-in or stretch in a part, to ensure enough stretching to prevent the formation of wrinkles. A high coefficient of friction at the punch radius serves to lock material in place under the bottom of the punch, while lower friction over the die radius and in the flange permits flow of material into the mouth of the drawing die [71].

The accuracy of finite element analysis simulation results is limited by the accurate modelling of these frictional effects [70, 72]. In FEM analyses, the friction contact problem is commonly described by the Coulomb formulation [13, 17, 46, 70, 72, 73], a linear function relating the frictional force, F_f , to the normal contact force, N , by a constant Coulomb frictional coefficient, μ :

$$F_f = \mu \cdot N \quad (1.1)$$

Though this simple frictional model can give reasonable results, the lack of information regarding the influences of frictional constraints limits the precise modelling of sheet metal forming processes. Very often, a coefficient of friction value is selected to provide the best possible fit between predictions and experimental results [13, 27, 46, 74], despite the various apparatus [46, 75] that exist to experimentally determine the friction coefficient of a material (see Figure 1.10). New friction models are currently under development [70, 72, 76] in an attempt to better address this issue in FEM applications.

1.2.2 Blankholder Force

The blankholder plays a pivotal role in the deep drawing of sheet metal. Through friction, it controls the drawing process by restraining the free inward flow of the blank material through the die cavity while suppressing the formation of wrinkles in the pressed part. Achieving the very delicate balance between wrinkling and thinning or rupture by selecting the correct blankholder force remains one of the most difficult procedures for production engineers when optimizing a forming process. A large blankholder force will cause excessive thinning, leading to tearing and fracture, while a low blankholder force will permit the formation of wrinkles.

Wrinkles, due to compressive stresses developed in the sheet, are usually formed in the area of the blank (the flange) that has not yet been pulled over the radius of the die, or in the unsupported side wall of the stamped part. Figure 1.11 illustrates both types of wrinkling for a deep drawn circular cup and lists the factors that influence wrinkle formation during deep drawing, while Figure 1.12 further illustrates some of the defects that are observed in cup drawing. A pressure plate, or blankholder, serves to prevent this wrinkle formation by holding the flange against the upper face of the die with a clamping force that is sufficient to ensure the material thickens uniformly. A study by Majilessi and Lee [78], on the effects of the blankholder force on drawing, determined that the formation of wrinkles is, in fact, impeded by the blankholder as its application reduces both their wavelength and amplitude, making them less visible. An

increase in the blankholder force would serve to further delay (to larger punch penetrations) the formation of detectable wrinkles, resulting in a smoother flange surface.

There is, however, a limit to the magnitude of the blankholder force that can be applied. The blankholder force must be carefully adjusted so that it does not cause tearing by abruptly raising the frictional resistance between the tooling and the blank. This leads to excessive tensile stresses which the wall of the drawn part has to withstand to permit further drawing. In addition, the material may suffer from what is termed "ironing" [79, 80]. As more of the material is allowed to flow inwards, the flange grows even thicker until it is eventually so thick that it "fouls" the blankholder, attempting to drive it upwards. For a blankholder that is held fixed, the metal flowing inwards is "ironed" by the blankholder, hence the term ironing. The material may subsequently lock between the die and the blankholder, facilitating the occurrence of fracture. Ironing also occurs when the thickness of the blank entering the tooling is greater than the clearance between the punch and the die.

Both modes of failure, tearing and wrinkling, are greatly influenced by the blankholder force, but the inward flow of material can be further modified locally by inserting mating ridges and grooves, commonly termed draw beads, on the faces of the die and the blankholder [39, 79, 81]. The insertion of draw beads can either augment the control of the drawing process or, in the case of a lock bead, completely constrain the material from drawing.

The latter case of a lock bead is a basic requirement for the limiting dome height (LDH) tests mentioned in Section 1.1. The LDH test characterizes the formability of a material by controlling the amount of lateral draw-in of blank materials of different widths. The amount of lateral draw-in is, in turn, controlled by the clamping boundary condition imposed using a circular draw bead (Figure 1.6) around the periphery of the blank which acts to fully bind the blank from drawing. The effects of this clamping force were studied by Schey and Fisher [82] in an attempt to quantify their effects on the LDH. They first investigated LDH variability between ten different laboratories and observed that the resulting sample sizes and LDH values were significantly different, despite the fact that all of the laboratories followed the North American Deep Drawing

Research Group (NADDRG) recommended practice [83]. Variations in test equipment, especially in the available clamping forces, were deemed to be significant as the experimental scatter in the range of LDH values between laboratories was greater than the standard deviation of the individual results. Experiments were carried out using a 101.6 mm punch similar to the Hecker/Ghosh punch to deform a range of specimen widths. From the results shown in Figures 1.13 and 1.14, Schey and Fisher concluded that a minimum clamping force of 250 kN was required to limit the amount of draw-in to less than the 0.25 mm recommended by the NADDRG [83]. A clamping force below this value would cause the measured LDH to be greater than its true value.

1.2.3 Blank Size and Shape

The deep drawing of a rectangular or box-shaped shell does not require a rectangular blank [79]. In fact, the material in the corners of a rectangular-shaped blank presents an obstacle to the deep drawing of square or rectangular sheet metal parts. True deep drawing only occurs at the four corners [78, 79], while the side and end walls are minimally strained through bending over the straight edges of the die, rather like a folding action. This results in severely strained excess material at the corner regions, principally along the diagonal direction of the flange [78]. The relatively undeformed outer boundaries of these diagonals result in the formation of large "ears" at the corners, as shown in Figure 1.15 (see also Figure 1.12). These may become so large that they effectively prevent draw-in, with the ensuing fracture occurring at a particularly shallow punch depth [78].

To prevent earing, and by so doing relieve the constraint in the diagonal directions of the flange, the shape of the blank must be altered [79]. The alteration is usually accomplished by shearing the corners of the blank at an angle of 45 degrees, thereby removing the area resisting draw-in and subjected to severe straining, which then lowers the overall strain distribution in the part [78, 79]. This modification also permits

the drawing of deeper shells before fracture, at substantially lower punch loads [78]. The amount of corner material to be sheared off depends upon the die corner radius as well as the desired pressing depth [79], though it normally does not exceed 30% of the smallest blank dimension [78].

Multiple trials may be required, depending on the depth and geometry of the desired product, before a blank shape and size is selected that best utilizes the available material. For a given punch geometry, there exists an optimal combination of blank size and blankholder force that suppresses wrinkles and avoids fracture. This blank size greatly depends on the blank shape itself, especially when determining the limiting blank size (LBS), the largest possible blank that can be drawn successfully with the given punch [78]. The limiting blank size affects the punch load, and is itself affected by the blankholder force [78] for a given blank shape; the punch load linearly increases as the LBS is increased, while the limiting blank size decreases as blankholder force is raised.

1.2.4 Blank Thickness

Although this research does not delve deeply into the effects of blank thickness variation *per se*, the overall effects of thickness on sheet metal formability must be mentioned. It has been observed that the tendency of a blank to wrinkle is diminished not only with increasing blankholder force, but also with increasing blank thickness [79]. Hence, the thicker the blank, the lesser the required blankholder force will be to prevent wrinkling. The increased forming limit [16, 39, 69] evidenced by sheets of greater thickness also results in an increase in the dome height value of LDH tests [69]. This is due to the fact that more plastic flow can occur at the localization region prior to the development of fracture [34]. For blanks of the same material differing in thickness by more than 0.13 mm, it is recommended that no direct comparisons of the dome test results be made [69].

1.3 Effects of Material Properties on Sheet Metal Forming

The material properties of the blank, and their interaction, are critical to a forming operation and must also be considered in any evaluation of sheet metal formability. The section that follows describes the significant effects of variations in two material properties, that of friction and anisotropy, considered in the current study.

1.3.1 Friction

Section 1.2.1 dealt with friction, and lubrication, as a process variable in the forming of sheet metal, but the friction coefficient, μ , can also be considered a material property, varying with the sheet surface conditions, and the direction in the plane of the sheet. Though μ does not directly affect the actual limit strains in a forming limit diagram, friction influences the strain state in the material and thus has a significant effect on formability. Hecker [39] determined that material property differences are emphasized by a lack of lubrication in formability tests.

In punch tests, the magnitude and position of peak strains varies with the frictional condition [27], thereby affecting the dome height [84]. A high frictional coefficient will delay deformation over the punch surface, shifting the localization away from the pole. A decreased cup height at failure [32] is thus obtained from the resulting lower pole strains. As lubrication over the punch is improved, the strain distribution in the blank also improves, with higher limit strains at failure, resulting in a higher cup [39]. Punch tests are particularly sensitive to friction at low values of μ [32].

1.3.2 Anisotropy

The level of anisotropy in a material is one of the most important material parameters in the formability and drawability of sheet metal [25]. It characterizes the material texture that develops as a result of the crystallographic preferred orientation occurring during the rolling of sheet metal [85, 86, 87], thereby providing valuable information to aid in the determination of a material's deformation characteristics.

The directional internal stress fields created in the sheet by the elongation and alignment of inclusions in the rolling operation leads to mechanical anisotropy, manifested as significant directional differences in the material and fracture behaviour of the sheet [84]. The current research is more concerned with another source of anisotropy, that due to crystallographic texture. Since single crystals have the tendency to elongate in the tensile load direction, through a combination of glide and rigid body rotation, it is evident that, when placed under tension, a polycrystalline material will also elongate [88]. The difference lies in that the grains in the polycrystalline metal tend to also rotate to what is termed the "preferred orientation" [88, 89]. This creates a non-random distribution of crystal orientations [90], as opposed to the random distribution in isotropic materials, thereby giving the metal a crystallographic grain texture, also called crystallographic or plastic anisotropy.

The most common method of characterizing the degree of anisotropy in a sheet metal was introduced by Lankford *et al.* [91] in 1950. They suggested measuring the ratio, r , of the lateral strain to the through-thickness strain in tensile tests oriented parallel, perpendicular, and 45 degrees to the rolling direction, as in Figure 1.16. This ratio, termed the plastic strain ratio, the normal anisotropic ratio or Lankford's coefficient, is commonly referred to as simply the r value.

$$r = \frac{\epsilon_2}{\epsilon_3} \quad (1.2)$$

Increased r values have the general effect of stretching the yield locus of a material into an ellipse along the equibiaxial stretching axis, as shown in Figure 1.17.

The level of anisotropy, or strain ratio, is affected by the number of active slip systems in the material [85, 92]. A material with an infinite number of slip systems will be isotropic, with r equal to 1.0, while materials with a single slip system aligned with the plane of the sheet will have an infinite r value [85]. Face centered cubic (FCC) materials, which have six slip directions, have an average r value of about 0.6 [85], and a maximum r value of 1.0 can be obtained in commercial sheet materials. Though more slip systems exist for body centered cubic (BCC) metals, as such materials have four slip directions for a number of slip planes, these are more restrictive than those in FCC metals and thus higher and lower values of r can be obtained, varying between 0.3 and 2.2 [85]. A single slip direction is active in close packed hexagonal metals in only one of three slip planes. Very high or very low r values can therefore be obtained, depending on the direction of the slip direction with respect to the plane of the sheet; Pearce [85] observed extreme r values such as 0.12 for zinc and 9.0 for zirconium.

The normal or through-thickness anisotropy parameter, r , is a direct measure of material drawability as high values indicate a resistance to both thinning [48, 93], which is the primary failure mode in the deep drawing of metal, and wrinkling [25, 79]. While a high r value is beneficial in the deep drawing of steel, it has a negative effect on stretch formability [85, 94]. Pearce [85], Parmar and Mellor [95], and Sowerby and Duncan [56] observed that steels with r greater than one had increased resistance to yielding, known as texture hardening, while materials with r less than one had decreased resistance to yielding, or texture softening. The yielding resistance evidenced in the stretching of high r materials in Figure 1.17 also indicates that deformation occurs more easily in drawing than in biaxial stretching [94]. A high r value is thus indicative of higher strength in the through-thickness direction, with a greater ability to deform in the plane of the sheet rather than by changes in thickness [71].

The forming limit strains in the stretch regime have the tendency to increase with decreasing r values [96, 97], raising the right hand side of the FLD, while the limit strain in plane strain remains unaffected by anisotropy [61, 97, 98]. Though the level of the left hand side of the FLD is also unaffected by anisotropy, an increasing r tends to shift the limit minor strain further from plane strain, thereby increasing the range of the FLD

towards greater negative minor strains [40, 96, 97]. In terms of punch and LDH tests, an increase in the value of r tends to produce a more localized strain distribution and a decreased limiting dome height [40]. The increase in the frictional forces over the punch head due to higher biaxial yield stresses effectively suppresses the deformation at that location, driving the localization away from the pole, as shown in Figure 1.18.

The level of anisotropy in a sheet material can vary significantly according to the orientation of the sheet with respect to the rolling direction (Figure 1.19). In many metals, the variation is such that r decreases between 0 and 45 degrees (to the rolling direction) and then increases to 90 degrees [101]. This variation in r with the direction of testing is termed planar anisotropy and manifests itself as earing (see Section 1.2.3) in sheet metal forming [25, 60]. Planar anisotropy is characterized by the value Δr ,

$$\Delta r = \frac{r_0 - 2r_{45} + r_{90}}{2} \quad (1.3)$$

where r_0 , r_{45} and r_{90} are the anisotropic values measured along the rolling direction (RD), at 45 degrees to the RD, and transverse to the RD, as shown in Figure 1.16 ($\Delta r=0$ and $r=1$ indicate isotropy). The tendency of a sheet metal to ear is intensified for values of Δr greater than zero, with the ears forming at 0 and 90 degrees to the rolling direction, their size increasing with the value of Δr [80].

Since an analysis of sheet metal forming can grow quite complicated when the variation of the material anisotropy must be considered in every direction, a simpler representation of anisotropy is usually employed which only considers normal or through-thickness anisotropy. The most popular option appears to be that of representing the normal anisotropy by the average strain ratio, $\bar{r}$, in the sheet plane, as suggested by Lankford [91].

$$\bar{r} = \frac{r_0 + 2r_{45} + r_{90}}{4} \quad (1.4)$$

Values of $\bar{r}$ usually lie between 0.8 and 2.40 [80, 102], with earing occurring in sheets exhibiting values substantially different from 1.0 [80]. Materials with $\bar{r}$ values below 1.0

are not usually considered suitable for deep drawing, which requires $\bar{r}$ to lie between 1.5 and 1.8 for good drawability [80]. A value greater than 1.8 for steels is necessary for extra deep drawing properties. As stated by Whiteley [103], the greater the value of $\bar{r}$, the deeper the draw or the greater the drawing ratio before fracture.

Equation 1.4 assumes that the material has rotational symmetry about the sheet normal (i.e., planar isotropy) [104]; that is, the anisotropy is uniform in both magnitude and direction throughout the plane of the sheet, thereby neglecting planar anisotropy. In practice, this approach is technically incorrect [105] but the use of $\bar{r}$ has become common due to the simplicity in measuring the three r values. As previously mentioned, these values are obtained from standard tensile tests on specimens cut along the three directions relative to the rolling direction (Figure 1.16). The lateral and through-thickness strains are then measured, at approximately 15% longitudinal strain as described in ASTM E517 [106], to obtain the strain ratios. The $\bar{r}$ could also be derived from the area underneath an experimentally determined curve such as in Figure 1.19, but this would require enormous amounts of time, effort and material.

As most materials exhibit some degree of planar anisotropy, Equation 1.4 has been challenged as an appropriate representative measure of anisotropy [100, 105]. Although some reasonable correlations have been obtained with the use of $\bar{r}$ [100, 107], if only for very simple loading cases, a better representation has yet to be found to accurately characterize the level of anisotropy in a material [105]. Other problems may lie in assuming a constant state of anisotropy, since anisotropy is known to vary during large deformations [85, 108, 109, 110], and in applying the r value to predict the forming behaviour of aluminum alloys, since the strain ratio in the tensile test for aluminum is not related to the yield surface distortion in the same way as is steel [94]. These simplifications are the usual approach to account for these variables and, though theoretically unjustified, it is nonetheless applied to most current analytical and numerical investigations. Future computational techniques will consider a larger number of the above material and state variables but this investigation, like others before, will accept $\bar{r}$ as a measure of the anisotropy of a material, though an attempt will be made to ascertain the effects of Δr .

Regardless of the difficulties apparent in the representation and modelling of anisotropy, it remains an important parameter to consider in the analysis of sheet metal formability. Controlled anisotropy is advantageous and can be used to considerable advantage in improving deep drawing performance and enhancing mechanical properties [86, 87].

1.4 Material Constitutive Behaviour

1.4.1 Stress-Strain-Strain Rate Measures

There are many methods to describe the relationships between the stress, strain and strain rate of a material undergoing plastic deformation. With the exception of high temperature deformations, metals which are deformed under a uniaxial stress state usually obey a stress-strain-strain rate relation [111] of the form

$$\sigma_1 = f(\epsilon_1) \cdot g(\dot{\epsilon}_1) \quad (1.5)$$

where the strain dependence, $f(\epsilon)$, can be described by many simple expressions, usually in relation to the material strain hardening. Those most frequently used are the expressions proposed by Hollomon [112] in 1945

$$f(\epsilon_1) = K\epsilon_1^n \quad (1.6)$$

and Swift [113] in 1952

$$f(\epsilon_1) = K(\epsilon_0 + \epsilon_1)^n \quad (1.7)$$

where K is the strength constant, ϵ_0 is the strain due to prior coldworking and n is the strain hardening exponent, defined as:

$$n = \left(\frac{\partial \ln \sigma_1}{\partial \ln \dot{\epsilon}_1} \right)_{t, \tau} \quad (1.8)$$

The power-law dependence is most often employed in the strain-rate dependence of stress

$$\sigma(\dot{\epsilon}_1) \propto \dot{\epsilon}_1^m \quad (1.9)$$

where m is the strain rate sensitivity exponent.

The above equations demonstrate the uniaxial stress state material response, but most metal forming operations occur under a much more complicated stress state. The material behaviour of such complex forming operations can be extrapolated from simpler loading conditions, i.e. uniaxial tensile tests, which have the ability to be measured in a more straightforward manner. This concept uses what are termed the "effective stress" and the "effective strain" [114], and their relationship is defined by an analytical description of the yield locus. This yield criterion is a two-dimensional locus, in principal stress space, of all possible combinations of stress states at which a material yields or undergoes plastic flow for a given mechanical or hardness state [13, 111].

1.4.2 von Mises Yield Criterion

One of the most widely accepted yield criteria for isotropic materials is the Huber-Mises or, simply, von Mises criterion. Also known as the shear strain energy criterion, it is based on the fact that yielding occurs in a material when the distortion energy reaches a critical value [115, 116].

The Huber-Mises criterion for yielding, in terms of a general stress state, is defined as:

$$(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = 2\sigma_Y^2 \quad (1.10)$$

and, for a state of plane stress, reduces to:

$$\sigma_1^2 + (\sigma_1 - \sigma_2)^2 + \sigma_2^2 = 2\sigma_Y^2 \quad (1.11)$$

where σ_y is the yield stress in uniaxial tension, and σ_1 and σ_2 are the principal in-plane stresses. The effective stress is then written as:

$$\bar{\sigma} = \sqrt{\frac{1}{2}[\sigma_1^2 + (\sigma_1 - \sigma_2)^2 + \sigma_2^2]} \quad (1.12)$$

Using the Lévy-Mises relation, the effective plastic strain increment can be written for plane stress as:

$$d\bar{\epsilon}^p = \sqrt{\frac{2}{9}[(d\epsilon_1 - d\epsilon_3)^2 + (d\epsilon_1 - d\epsilon_2)^2 + (d\epsilon_2 - d\epsilon_3)^2]} \quad (1.13)$$

from which the effective plastic strain rate differential can be obtained:

$$d\bar{\epsilon}^p = \sqrt{\frac{2}{9}[(d\dot{\epsilon}_1 - d\dot{\epsilon}_3)^2 + (d\dot{\epsilon}_1 - d\dot{\epsilon}_2)^2 + (d\dot{\epsilon}_2 - d\dot{\epsilon}_3)^2]} \quad (1.14)$$

The above relations are valid for any arbitrary loading state. For plane stress states, the effective stress, effective strain, and effective strain-rate, obtained through Equations 1.12, 1.13 and 1.14, can be directly substituted into the stress, strain, strain rate relation given in Equation 1.5. For more general loading states, the differential form of Equation 1.5 is integrated over the strain path, using the differential equations, Equations 1.12, 1.13 and 1.14. These differential or incremental equations, termed constitutive equations, are the type most often employed in non-linear finite element analysis routines to characterize the elastic or plastic behaviour of materials.

1.4.3 Hill's Quadratic Yield Function

The notion that all materials are isotropic and remain so during deformation has been assumed by many traditional yield functions. This is normally not the case as the majority of materials exhibit some form of anisotropy (refer to Section 1.3.2). The most commonly applied anisotropic yield criterion in finite element analyses is that due to Hill [117], a generalization of the von Mises isotropic yield criterion towards anisotropic

materials. Hill's quadratic yield criterion, also known as Hill's 1948 criterion, has been the subject of many investigations [32, 95, 100, 118] and remains, to date, one of the most popular and widely used yield criteria.

Ignoring any internal stresses due to material texture, Hill described a yield criterion with orthotropic anisotropy [117]. That is, the material is assumed to possess three mutually orthogonal planes of symmetry, termed the principal axes of anisotropy, lying parallel to the rolling direction, transverse to the rolling direction, and normal to the plane in a sheet metal. A complete list of the assumptions made by Hill follows [115, 117]:

- the material possesses three orthogonal axes of anisotropy
- there is no Bauschinger effect
- the superposition of a hydrostatic stress does not influence yielding
- the yield criterion reduces to the von Mises isotropic yield criterion
- the yield criterion is quadratic in the stress components
- internal stresses due to grain orientation are ignored
- the anisotropic parameters increase in strict proportion as straining proceeds

From these assumptions, Hill proposed the following yield criterion:

$$2f(\sigma_{ij})^2 = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1 \quad (1.15)$$

where f is a function of the stress tensor, σ_{ij} , and $f(\sigma_{ij})$ is termed the flow stress or plastic potential.

The constants F , G , H , L , M and N in Equation 1.15 are characteristic of the state of anisotropy and are described by the following relations when the selected xyz reference axes coincide with the principal axes of anisotropy (parallel to the rolling direction, transverse to the rolling direction, and normal to the plane of the sheet):

$$\begin{aligned}
\frac{1}{X^2} &= G+H & 2F &= \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \\
\frac{1}{Y^2} &= H+F & 2G &= \frac{1}{Z^2} + \frac{1}{X^2} + \frac{1}{Y^2} \\
\frac{1}{Z^2} &= F+G & 2H &= \frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2} \\
2L &= \frac{1}{R^2} & 2M &= \frac{1}{S^2} & 2N &= \frac{1}{T^2}
\end{aligned} \tag{1.16}$$

In this notation, X, Y and Z are the uniaxial tensile yield stresses in the directions of the principal axes of anisotropy, while R, S and T denote the yield stresses in shear. A transformation of the stress components is required when different axes of reference are chosen.

For rotational symmetry of the anisotropy about the z axis or thickness direction (planar isotropy), then

$$F = G \quad \text{and} \quad N = G + 2H \tag{1.17}$$

For complete spherical symmetry (isotropy),

$$F = G = H \quad \text{and} \quad L = M = N = 3F \tag{1.18}$$

which then reduces the yield function to that of the von Mises criterion. This is consistent with the assumption postulated by Hill that the yield criterion should reduce to the von Mises criterion as the material becomes isotropic.

For the forming of thin sheet metal, it is assumed that the sheet deforms in a state of plane stress, that is, the stress in the normal or z direction is equal to zero ($\sigma_z = 0$). If it is further assumed that the through thickness shear stresses are negligible, Equation 1.15 then reduces to the following equation at yield:

$$2f(\sigma_{ij})^2 = (G+H)\sigma_x^2 - 2H\sigma_x\sigma_y + (H+F)\sigma_y^2 + 2N\tau_{xy}^2 = 1 \tag{1.19}$$

This equation, further simplified by assuming normal anisotropy (Equation 1.17) and negligible in-plane shears, can be rewritten in terms of principal stresses as:

$$\bar{\sigma} = [\sigma_1^2 + \sigma_2^2 - \frac{2r}{1+r} \sigma_1 \sigma_2]^{1/2} \quad (1.20)$$

where r is Lankford's coefficient (Equation 1.2) and $\bar{\sigma}$ is the equivalent stress. This equation is valid only if the principal stresses lie along the principal axes of anisotropy. The equivalent or effective plastic strain increment can then be obtained, using the Lévy-Mises relation, and is written as follows.

$$d\epsilon^p = \frac{1+r}{\sqrt{1+2r}} [d\epsilon_1^2 + d\epsilon_2^2 + \frac{2r}{1+r} d\epsilon_1 d\epsilon_2]^{1/2} \quad (1.21)$$

In his original 1948 [117] and 1949 [119] papers, Hill could only generally substantiate his theory, with evidence of a very qualitative nature. From experimental observations of sheet steel tensile tests, he was able to discern trends in the necking angle with respect to the rolling direction orientation, and thus support the characteristics of the postulated yield function.

Experimental Investigations of Hill's Quadratic Criterion

Since very little experimental work had been carried out to validate Hill's yield theory, Bramley and Mellor [100] undertook an investigation that compared experimental results to predictions obtained using Hill's quadratic yield criterion. As shown in Figure 1.20, their study compared stress/strain results from tensile tests, with specimens cut at sheet orientations of 0, 45 and 90 degrees relative to the rolling direction (curves 1 through 3), to those from biaxial bulge tests (curve 4). These results were obtained for an unspecified steel with $\bar{r} = 1.48$. The theoretical predictions are represented in curves 5 through 7. This figure illustrates the qualitative agreement found by Bramley and Mellor.

In his investigation, Pearce [85] also compared uniaxial stress-strain curves with curves obtained in biaxial tension, but he particularly concentrated on the effects of the level of anisotropy present in the material. In a manner similar to that of Bramley and

Mellor [100], Pearce obtained the curves shown in Figures 1.21 and 1.22, for steel ($\bar{r}=1.62$) and aluminum ($\bar{r}=0.68$ and 0.43), respectively. Again, the predicted curves for materials having r values greater than one were quite good approximations to the determined ones, the predicted curve falling between the uniaxial and biaxial experimental curves. In the case of materials exhibiting very high r values, such as titanium and zirconium, agreement was found to be especially good. Discrepancies were observed, however, for materials having r values less than unity; the biaxial tension curves were actually above the uniaxial stress-strain curves, whereas the theoretical curves fell below. This behaviour, termed "anomalous" by Woodthorpe and Pearce [101], was indicative that Hill's 1948 theory failed to predict the correct behaviour of materials having r values less than unity and that more work was needed in this area. This later prompted Hill to modify his original criterion into a more general formulation, suitable for a wider range of r values.

Analytical Investigations of Hill's Quadratic Criterion

In 1967, Marciniak and Kuczyński (M-K) [120] presented a theory to predict strain localization in the biaxial stretching of sheet metal, based on Hill's quadratic yield criterion [116]. This marked the onset of numerous analytical investigations involving Hill's 1948 criterion and strain localization [56, 121, 122, 123]. The M-K analysis assumed an initial inhomogeneity in the material, due to either thickness variations in the sheet, impurity distributions, or local variations in the grain orientations [120], which developed, under increasing plastic deformation, into a localized neck, perpendicular to the larger principal stress direction. As shown in Figure 1.23, the M-K theory predicts that the parameter r in Hill's 1948 criterion, for high values, has a strong, negative influence on the forming limits for balanced biaxial stretching [120, 124]. This contrasts with the strain localization resistance, and therefore increased drawability, evidenced during the drawing of high r value materials.

A non-mathematical interpretation of the M-K analysis was conducted by Sowerby and Duncan [56] (refer to Figure 1.24). They observed that an increase in the r value

stretches the yield locus along the equibiaxial axis OQ in Figure 1.24, as was previously shown in Figure 1.17. They also observed that the plane strain line OS moves towards the equibiaxial axis OQ for higher r values, indicating that materials with greater r values are more likely to reach failure (plane strain) than materials having an r value of unity. Experimental evidence provided by Sowerby and Duncan contradicts the predicted reverse behaviour for materials having an r value less than unity; they confirmed that the yield locus does not, in fact, shrink towards a circle for such materials.

Another investigation by Venter *et al.* [125] extended the M-K analysis, simulating the initial inhomogeneity with both grooved and ungrooved annealed aluminum sheet specimens, for tensile, torsional, circular bulge and elliptical bulge tests. An average r value of 0.54 was determined from the experiments, but Venter *et al.* demonstrated that the predicted yield locus did not corroborate the results. The inability of Hill's 1948 criterion to correctly predict the behaviour of materials having r values less than unity was cited as a possible explanation for the poor fit.

These results indicate that, as far as yielding behaviour is concerned, the quadratic, anisotropic yield criterion proposed by Hill is reasonably satisfactory for materials having r values greater than unity, such as for steel. For materials with r values less than unity, Hill's 1948 criterion is unable to capture the appropriate material behaviour, precluding its application to aluminum alloys and other materials exhibiting low r values.

1.4.4 Hill's Non-Quadratic Yield Function

In an attempt to encompass the anomalous behaviour of low r materials [85, 101], and account for the deficiencies in his original (1948) theory, Hill proposed a more "flexible" anisotropic yield criterion in 1979 [126] of the form:

$$f|\sigma_2 - \sigma_3|^n - g|\sigma_3 - \sigma_1|^n - h|\sigma_1 - \sigma_2|^n + a|2\sigma_1 - \sigma_2 - \sigma_3|^n + b|2\sigma_2 - \sigma_3 - \sigma_1|^n + c|2\sigma_3 - \sigma_1 - \sigma_2|^n = \bar{\sigma}^n \quad (1.22)$$

where the exponent m is a material parameter, determined by matching the effective stress-effective strain curves from uniaxial and biaxial bulge tests.

For sheet metals exhibiting planar isotropy and undergoing plane stress deformation, Equation 1.22 reduces to:

$$\bar{\sigma} = \left[\frac{1}{2(1+r)} [(1+2r)|\sigma_1 - \sigma_2|^m + |\sigma_1 + \sigma_2|^m] \right]^{1/m} \quad (1.23)$$

and the corresponding effective plastic strain is given by:

$$d\bar{\epsilon}^p = \frac{[2(1+r)]^{1/m}}{2} \cdot \left[\frac{1}{(1+2r)^{1/(m-1)}} |d\epsilon_1 - d\epsilon_2|^{m/(m-1)} + |d\epsilon_1 + d\epsilon_2|^{m/(m-1)} \right]^{(m-1)/m} \quad (1.24)$$

The shape of the yield locus is affected by the exponent M , as shown in Figure 1.25. The yield locus, convex for $m \geq 1$, is elongated in the balanced biaxial tension direction for $m < 2$ [95]. For $m = 2$ and an arbitrary r value, Equation 1.23 and 1.24 reduce to the respective functions of the 1948 Hill quadratic anisotropic yield theory given by Equations 1.20 and 1.21. For $m = 2$ and $r = 1$, the equations reduce to the respective von Mises isotropic yield theory functions, Equations 1.12 and 1.13. Experimental results have shown that m falls within a range of 1.6 to 2.0 for aluminum alloys [127].

Experimental Investigations of Hill's Non-Quadratic Criterion

An investigation by Ragab and Abbas [127] compared experimental uniaxial and biaxial tension stress-strain curves of two commercially pure aluminum alloys, as-received ($\bar{r} = 0.393$, $m = 1.3$) and annealed ($\bar{r} = 0.731$, $m = 1.7$), to curves predicted using Hill's non-quadratic yield criterion. As shown in Figure 1.26, a much better agreement was obtained with this new formulation, especially for the annealed material, than with Hill's old criterion, indicated by the predicted curve with $m = 2$. The predicted biaxial tension curves fall between the experimental and uniaxial curves, a

behaviour similar to that of materials having r values greater than unity in Hill's 1948 criterion.

Kobayashi *et al.* [105] conducted an analytical and experimental investigation on the validity of Hill's new non-quadratic yield criterion. They demonstrated reasonable agreement between experimental stress-strain curves for steel, copper, brass and aluminum and those predicted for each material using the constitutive equations. Kobayashi *et al.* cited two factors that may have contributed to existing variations. The first was the assumption of a constant m value, in their case at an arbitrary strain of 0.1, whereas experiments have shown that the m value varies with strain. The second contributing factor was the assumption of planar isotropy in the material by using an average value of r , given by $\bar{r}$ in Equation 1.4, a common approach when dealing with planar anisotropic materials. Both of these assumptions are necessary in order to considerably simplify any analysis using Hill's non-quadratic yield criterion. A more accurate description of the material behaviour could be obtained by using both a relation for m in terms of the equivalent uniaxial tensile strain and the planar anisotropy coefficients defined in the original formulation (Equation 1.22). Their implementation in finite element codes, however, would be extremely complex and computationally expensive.

Analytical Investigations of Hill's Non-Quadratic Criterion

Parmar and Mellor [95] investigated the application of Hill's non-quadratic criterion to materials with r values less than unity. They first reiterated the fact that, for such materials, the experimental curves for balanced biaxial tension always lie above uniaxial curves [100], while those curves predicted using Hill's quadratic criterion fall well below the uniaxial curves. Preliminary results from their study indicated that a better material description was available for materials having r values less than unity with the use of Hill's new anisotropic criterion.

The implementation of Hill's non-quadratic yield criterion in the Marciniak and Kuczyński model [120] permitted Parmar and Mellor to detect trends in the predictions

that would eventually lead to improvements in the M-K analysis for materials with r less than one. The dependence of the limit strains on variations in the exponent m is shown in Figure 1.27 for a material with $r = 0.5$, where the curve for $m = 2$ is that given by Hill's 1948 criterion. The limit strains near to balanced biaxial tension are significantly reduced by a reduction in m , whereas the limit strain near zero minor strain remains unaffected by variations in m , with the stress ratio for localization (plane strain condition) approaching unity (Figure 1.25) [95]. As shown in Figure 1.27, a reduction in the limit strain over that predicted by using $m = 2$ (the original Sowerby and Duncan M-K analysis) would be obtained for a typical aluminum m value of 1.8 in Parmar and Mellor's analysis, with the yield locus being stretched similar to that observed for increasing values of r (Figure 1.17). Comparisons between experimental observations and numerical predictions from a M-K analysis support this trend [124].

1.4.5 Barlat's Non-Quadratic Yield Function

Another constitutive material model that has gained recent industry popularity is the Barlat criterion, developed in 1989 by Barlat and Lian [128] in response to a lack of yield criteria that could accurately describe the planar anisotropic behaviour of materials [129, 130], especially those with r values less than one. This new model is a generalization of the criterion proposed by Hershey [131] and Hosford [132] and is calculated with a polycrystalline plasticity model [133]. It incorporates shear stresses (i.e., no need for principal stress and material axes to be aligned) and, like Hill's non-quadratic criterion [126], the Barlat criterion also encompasses the anomalous behaviour [101] of aluminum.

The simple non-quadratic yield function developed by Barlat and Lian in 1989 [128] to describe planar anisotropy for a state of plane stress has the form

$$2\bar{\sigma}^m = a |K_1 + K_2|^m + a |K_1 - K_2|^m + c |2K_2|^m \quad (1.25)$$

where the parameters K_1 and K_2 are represented by

$$K_1 = \frac{\sigma_x + h\sigma_y}{2} \quad \text{and} \quad K_2 = \sqrt{\left(\frac{\sigma_x - h\sigma_y}{2}\right)^2 + (p\tau_{xy})^2} \quad (1.26)$$

The material constants a , c , h and p are determined from the Lankford coefficients r_0 , r_{45} and r_{90} obtained in tensile tests of specimens taken at orientations of 0° , 45° and 90° relative to the rolling direction [128]. The parameter M , like in Hill's non-quadratic yield criterion, governs the shape of the yield surface, which is convex for $M \geq 1$ [129]. Barlat [128] has demonstrated, from crystallographic plasticity considerations, that $M=8$ is an appropriate choice for FCC materials, while $M=6$ is used for BCC materials.

For planar isotropy, the constants a , c , h and p are replaced by

$$a = \frac{2}{(1+r)}, \quad c = \frac{2r}{(1+r)} \quad \text{and} \quad h=p=1 \quad (1.27)$$

reducing Equation 1.25 to an in-plane isotropic, non-quadratic yield function. This new equation can then be further reduced to Hill's 1948 criterion (Equation 1.20) by setting $M=2$ for an arbitrary r value, and to the von Mises isotropic yield criterion (Equation 1.11) by setting $M=2$ and $r=1$.

A more general, six-component anisotropic yield function was later proposed by Barlat *et al.* [134] to account for three-dimensional stress states, and to be more consistent with crystallographic-based yield surfaces [135]. It has the form

$$2\bar{\sigma}^M = |S_1 - S_2|^M + |S_2 - S_3|^M + |S_3 - S_1|^M \quad (1.28)$$

where S_1 , S_2 and S_3 are the principal values of the symmetric matrix S , defined with stress components as:

$$\begin{aligned} S_{xx} &= [c(\sigma_{xx} - \sigma_{yy}) - b(\sigma_{zz} - \sigma_{xx})] / 3 \\ S_{yy} &= [a(\sigma_{yy} - \sigma_{zz}) - c(\sigma_{xx} - \sigma_{yy})] / 3 \\ S_{zz} &= [b(\sigma_{zz} - \sigma_{xx}) - a(\sigma_{yy} - \sigma_{zz})] / 3 \end{aligned} \quad (1.29)$$

$$\begin{aligned}S_{yz} &= f \sigma_{yz} \\S_{zx} &= g \sigma_{zx} \\S_{xy} &= h \sigma_{xy}\end{aligned}$$

The material constants a , b , c , f , g and h represent anisotropic properties. When all six constants are equal to one, the material is isotropic and the yield surface reduces to the von. Mises yield surface (Equation 1.11) for M values of 2 or 4. This more general equation does not include shear stresses, therefore restricting its use to stress states where the anisotropy axes and principal stress axes coincide.

The parameter M governs the shape of the yield surface as shown in Figure 1.28 for 2008-T4 aluminum subjected to a state of plane stress. Since the curvature of the yield surface determines the direction of straining [136], a smooth change of strain rate direction is observed for quadratic yield criteria (von Mises and Hill's 1948 criterion) near uniaxial and balanced biaxial tension as stress varies (Figure 1.28), whereas the Barlat formulation exhibits abrupt change in the strain rate direction. These vertices in the yield locus shape are clearly seen in Figure 1.29 for the same aluminum alloy. The plane stress yield loci are normalized by the uniaxial tensile (a) and compressive (b) yield stresses in the rolling direction, and are shown in Figure 1.29 as intersecting loci of constant normalized shear stress, $\sigma_{xy}/\bar{\sigma}$. As mentioned above, a small radius of curvature is exhibited by the yield surfaces in the uniaxial and balanced biaxial tension or compression ranges, similar to yield surfaces calculated from polycrystal models [129].

Investigations of Barlat's Criterion

Due to the recent development of the Barlat non-quadratic anisotropic criterion, the literature contains only a small number of investigations on the validity of this new criterion. The majority of these [128, 133, 134, 137] have been initiated and written by the authors of the Barlat criterion to ascertain that their theory could encompass the anomalous behaviour of aluminum. In addition; Chung and Shah [129], in their implementation of the non-quadratic Barlat criterion in a finite element code, have

obtained predictions for hydraulic bulge and cup drawing tests of 2008-T4 aluminum that are in excellent agreement with experiments. Shah and Dick [135] investigated deformation behaviour, strain localization and wrinkling potential in the drawing of a square cup of 2008-T4 aluminum, examining the effects of various yield criteria (von Mises, Hill's 1948, and Barlat) on the predictions to various yield criteria. They observed a higher strain localization as well as higher forming strains using Barlat's yield criterion, with this criterion providing a better agreement with experimental results. These results are supported by the reasonable agreement obtained between predicted and measured strains for both cup drawing and biaxial stretching [138, 139], where superior results were also observed with the use of Barlat's criterion over that of von Mises and Hill (1948) for low- r materials.

1.4.6 Other Anisotropic Yield Functions

Various other methods exist to characterize the anisotropic behaviour of sheet materials, such as the methods advanced by Hershey in 1954 [131], Hosford in 1972 [132], Gotoh in 1977 [140], Bassani in 1977 [141], Logan and Hosford in 1980 [142], Budanski in 1984 [143], Jones and Gillis in 1984 [144], Montheillet *et al.* in 1985 [145], Chu in 1993 [130], and their extensions. While many of these are generalizations of the criteria described in the previous three sections, none of them has gained the same level of popularity. Hill's 1948 criterion remains the most utilized constitutive equation in industry, but non-quadratic yield criteria such as Hill's 1979 criterion and Barlat's criterion are now gaining wider acceptance. They better capture the plastic behaviour of sheet metal, especially for materials with low values of r in-plane. Problems that remain in their implementation [105] are mainly due to determining the value of the exponent M , which changes with strain, in the respective equations. A more detailed analysis of the effect of M and of planar anisotropy in materials will be a major emphasis in the future of anisotropic constitutive relations.

1.5 Application of FEM to the Simulation of Sheet Forming

Various methods of analysis have been developed and applied to metal forming processes. These include, to name a few, the slab method, the slip-line field method, the viscoplasticity method, and the upper- and lower-bound techniques [77]. While they have been useful in the prediction of general deformation behaviour in metals, it was not until the introduction of the finite element method (FEM) in the 1960s that an accurate determination of the effects of process variables on metal flow could be made [77]. This section will outline previous applications of FEM in investigations and simulations of sheet metal forming. Chapter 3 will describe in detail the numerical models used in this investigation, as well as the finite element code employed in the analysis.

Due to the increasing use of finite element analysis in the modelling of sheet forming processes, especially for producibility analyses and die development of complex automotive parts [23, 72, 135, 146], the literature contains a significant number of papers and other publications pertaining to the application of FEM in the simulation of sheet metal forming. Many of these deal with material constitutive models, element formulations, and the accurate description of boundary conditions; however, the application of FEM techniques to strain and fracture predictions in sheet metal forming is the focus of the current investigation. Hence, this review will mainly focus on the literature concerning such applications, including comparisons with experimental predictions.

1.5.1 Trends in the FEM Simulation of Metal Forming Operations

Continuing advances in this research area makes any review of the state of the research obsolete upon publication [147]. The review by Wagoner and Zhou in 1992 [147], comparing the field at the moment of publication to that in 1989, provides an example of such a phenomenon. In just three years, they observed significant advances

and major improvements in the use of finite element programs to model sheet metal forming.

Although interest in the analysis of sheet metal forming had been great in 1989, no accurate, reliable and consistent methods yet existed to effectively simulate general forming operations [147]. The majority of simulations were limited to simple, basic geometries, relying on membrane elements and based on static, implicit methods [27, 32, 147], but an early study using DYNA3D by Honecker and Mattiasson [21] was cited as one of the very few to use the dynamic, explicit method in a reasonably accurate simulation of sheet wrinkling. The application of sheet forming simulations to die tryout and development was also limited, without the benefit of technical aids such as a CAD/CAM interface. To summarize the field of numerical sheet forming simulations at the time (1989), Wagoner and Zhou [147] arrived upon the following conclusions:

- the scatter among programs is great
- the operator-operator scatter is about equal to the program-program scatter
- general-purpose and custom programs exhibit similar scatters
- single-step and similar programs (geometric, etc.) lack predictive capability
- contact and friction conditions cause the most serious simulation problems
- more experienced groups consistently get better results

At the time of publication three years later, Wagoner and Zhou observed that the field had advanced considerably, with many of the previous problems either solved or reduced to more manageable proportions. Three-dimensional, complex geometries could thus be reliably simulated in industry [21, 22, 148], with greater use of explicit formulations and CAD/CAM linkages. A significant reduction in scatter between solutions was also evidenced in the VDI Benchmark [147] problem, even though its complexity was far greater than that of previous simulations. Recently, application of finite element techniques to the simulation of sheet metal forming has gained widespread popularity in the automotive industry, with the number of industrial and commercial FEM programs growing continuously.

1.5.2 Major Issues

Simulation and Numerical Code Requirements

In their investigations of finite element procedures for sheet forming simulations, Honecker and Mattiasson [21] determined that a successful numerical simulation of a sheet metal stamping must provide information regarding the strain distribution in the material, the occurrence of wrinkles, the draw-in at the edges, as well as springback [21]. To optimize performance, the analyst must also be able to control and vary a number of design parameters, such as blankholder force and lubrication (friction), the use of draw beads and their location, material parameters, the initial blank shape, and the speed of the punch [21]. They concluded that an effective simulation program must have the capability to meet these requirements and, in addition, be robust and reliable, producing results with acceptable accuracy, in a reasonable amount of time and at a reasonable cost. These findings were consistent with other investigations [22, 30, 149].

Although numerical modelling of sheet metal forming has primarily emphasized the analytical aspects, the importance of other aspects, such as the characterization of the material, the development and application of appropriate constitutive yield functions, friction and strain hardening, and the experimental validation process, cannot be underestimated [14]. The use of a finite element model as a design tool can thus be evaluated by a combination of the ease and speed of its preprocessing, analysis and postprocessing capabilities, which include the entry into the database of the geometry, the meshing of the blank, and definitions of constraints and boundary conditions that can faithfully reproduce the process being modelled [14].

Boundary Conditions

The friction boundary condition, characterized by the frictional coefficient, μ , was found to be the most important parameter in a numerical prediction sensitivity study of

limiting dome heights conducted by Burford *et al.* [32]. As one of the least understood, but also one of the most important in formability analyses, the frictional boundary condition has been the subject of a number of other investigations [27, 99]

Knibloe and Wagoner [27] studied the effects of friction on hemispherical punch stretching experiments, comparing experimental results to predictions obtained using Hill's non-quadratic anisotropic criterion, described in Section 1.4.4, assuming normal anisotropy and a Coulomb friction formulation. They observed that the friction and the anisotropic exponent, M , in Hill's criterion had opposite effects on the strain distribution, and that it was necessary to fit μ so that the predicted pole strains matched those experimentally measured, this value further depending on the value of M . Knibloe and Wagoner found that using Hill's criterion with the M value obtained from material characterization led to a lower peak radial strain located nearer to the pole, with a higher strain at the pole, trends characteristic of a lower frictional coefficient. To compensate for this effect, Knibloe and Wagoner had to increase the friction coefficient, to μ values as high as 0.64, to fit the predictions to the experimentally measured strain distributions at the pole. The use of Hill's quadratic criterion provided superior results, with frictional coefficients between 0.094 and 0.351 giving reasonable agreement [27].

In their investigation, Wang and Wenner [99] also used the frictional coefficient to fit experimental results to predictions of hemispherical punch forming, having found that an accurate prediction of the strain distribution at a given punch height was impossible using a single value of μ . This common, though impractical, approach of fitting the frictional coefficient to measured strain distributions indicates a lack of a proper friction model. Mattiasson *et al.* [74], in their study of commercial explicit FEM codes applied to metal forming operations such as cylindrical cup stretch forming and rectangular pan deep drawing, have provided evidence that a value for μ fitted to a relatively simple geometry can be applied to other, more complex problems. Using Hill's quadratic criterion with a Coulomb friction formulation, they obtained close agreement between measured and predicted punch force for both geometries by fitting the μ coefficient to experimental results. Interestingly, the punch force for the more complex rectangular pan was accurately predicted by using the frictional coefficient fitted

to the simpler geometry measurements, though it is unclear if strain predictions obtained for both parts met with similar success.

Constitutive Models

The constitutive equations embodied in numerical models characterize the elastic or plastic behaviour of materials in finite element codes and are vital to an accurate description of deformation. The state of anisotropy in a material is crucial in the simulation and subsequent modelling of sheet metal forming. Section 1.4 described those anisotropic yield criteria most commonly employed with Hill's quadratic yield criterion being the most popular [27, 32, 74, 98, 150]. Hill's non-quadratic yield criterion is also used, though less frequently [32, 151], and a new formulation, the Barlat criterion, is gaining popularity [129, 135, 137].

Burford *et al.* [32] determined that the effect of the anisotropy parameter r was sensitive to the yield function employed. Their sensitivity study employed both Hosford's and Hill's non-quadratic yield criteria (which can reduce to Hill's 1948 criterion and the von Mises criterion), with triangular membrane elements in a 3-dimensional non-linear rigid viscoplastic FEM formulation. A single value of μ in the Coulomb formulation was used to predict friction.

The predictions of the limiting dome heights at failure obtained using Hill's quadratic yield criterion were significantly dependent upon the value of r [32], consistent with results previously discussed [56, 95, 120]. Predictions using Hosford's yield criterion provided better agreement with experimental results, and showed a lower sensitivity to r variations [32]. This agrees with findings [39] that the FLD is not as sensitive to variations in r as predicted using an M-K analysis. The findings by Burford *et al.* [32] demonstrate that r has only a small effect on strain distribution predictions in limiting dome height tests in comparison to friction and strain hardening coefficients.

The quadratic yield theory seems to provide adequate numerical results for steels having r values in the 1.5 to 2.0 range, and remains the most popular despite the discrepancies inherent with experimental r variations. Although predictions obtained

using more general constitutive equations, such as Hill's and Barlat's non-quadratic yield criteria, are in better agreement with experimental results, their application in FEM analyses remains limited. This is most probably due to the difficulties inherent in the determination of the exponent, M , in the respective equations, and its added complexity.

Time Integration Scheme

Dynamic, explicit formulations have only recently replaced the use of implicit, quasi-static formulations in FEM models of sheet metal forming [147], and have already proven to be among the most versatile programs for sheet forming process modelling [15, 74, 152]. Based on direct integration of the equations of motion, explicit time integration uses the central difference scheme to avoid forming and simultaneously solving large sets of equations, a requirement for implicit time integration.

While both methods accurately simulate forming processes, the explicit method is made much more attractive by problems inherent in the implicit method. Rebelo *et al.* [153] and Honecker and Mattiasson [21] outline some of these problems in detail, and determined that wrinkling, a buckling instability which leads to convergence problems, and intermittent contact cannot be dealt with as effectively using implicit codes.

Implicit solvers also require a significant, non-proportional increase in run time and storage capacity for large analyses, while the run time increase for dynamic explicit models is proportional to the model size [21, 153], with increased benefits when parallel processing is considered [152]. Although the implicit formulation offers some efficiency and accuracy advantages for small problems, such as parametric studies on simple geometries [21, 147], these studies concluded that the explicit formulation is significantly more flexible and time efficient for larger, more complex problems [21, 30, 74, 153], such as in the modelling of thin sheet metal forming.

1.6 Current Work

The publications reviewed in the preceding sections indicate the inadequacy of the widely used Hill's quadratic yield criterion in accurately modelling the behaviour of materials having low values of r , such as aluminum. The use of a non-quadratic yield criterion, such as Barlat's criterion, is still quite recent and further investigations are required to assess its performance in capturing the anomalous behaviour of aluminum in various modes of deformation.

The goal of the research is to assess various constitutive models used to predict strain distributions in finite element simulations of two sheet metal forming operations, the limiting dome height or Ghosh formability test, described in Section 1.1, and a laboratory deep drawing test called the Universal Forming Test, described in Section 2.4.1, which simulates a real forming process. Finite element models of the two tests were created employing the commercial three-dimensional non-linear FEM code, LS-DYNA3D, to benefit from the advantages inherent in explicit time integration. The strain distribution histories predicted using the von Mises isotropic criterion, Hill's quadratic anisotropic criterion and Barlat's non-quadratic anisotropic criterion are compared to a series of experimental measurements obtained from two aluminum alloys. A parametric study of the effect of μ , the Coulomb frictional coefficient, was also carried out, as was an investigation of the effect of planar anisotropy, that is, variations in r , with direction in the plane of the sheet.

Chapter 2 outlines the experimental investigation used to provide measured strain distributions from which to assess the numerical predictions, while Chapter 3 describes the numerical models. Results of the experimental investigation for the limiting dome height test and Universal Forming Test are presented in Chapter 4, while the numerical results are presented in Chapter 5. Discussions of the results and conclusions from this research are given in Chapter 6. Future work to be considered is also included in Chapter 6.

Chapter 2

Experimental Programme

Formability and drawability tests were performed on two commercial aluminum alloy sheet materials. The formability experiments consisted of limiting dome height (LDH) experiments, using three different geometries chosen to give a variation in the in-plane strain condition at failure, e.g., biaxial stretching, plane strain, and drawing. Interrupted dome tests were also performed on the same three geometries in order to visualize the development of the strain distribution in the specimen as it is deformed. The drawability experiments consisted of deep drawing flat-bottomed rectangular pans. This second experiment more closely simulates a real stamping operation, with a more complex strain path of bending, plane strain, and drawing deformation. Again, tests were interrupted at varying punch depths to obtain strain history data.

2.1 Material Selection

Three commercial aluminum alloys, AA6111-T4, AA5754-O and AA5182-O, were initially considered for the experimental investigation as they are the leading replacement candidates for sheet steel in the automotive industry (body and panels). The uniaxial properties for each of these materials are given in Table 2.1 [154]. Upon

examination of the similarities between the properties for the 5182 alloy and those for the 6111 and 5754 alloys, it was decided to discard the 5182 alloy for this experimental programme since use of the 5754 and 6111 alloys provided the greatest range in ϵ .

The aluminum sheet was supplied by the ALCAN Kingston Research and Development Laboratories, Kingston, Ontario. Ten 508 mm x 508 mm (20" x 20") sheets of the 6111 aluminum alloy were all cut from the same coil, thus ensuring uniformity between sheets. It was impossible to ascertain if all of the twelve 508 mm x 762 mm (20" x 30") sheets of the 5754 aluminum alloy provided were also cut from the same coil. It was thus determined to carry out tensile testing using specimens taken from different sheets, allowing a check on the level of uniformity between sheets.

2.2 Material Characterization

The chemical compositions of both the 6111 and 5754 alloys are given in Table 2.2. Of particular interest is the high level of magnesium contained in the 5754 alloy as compared to the 6111 alloy. A high level of magnesium in an aluminum alloy contributes to the formation of Lüders bands [8] as the magnesium atoms migrate from vacancy to vacancy, leading to ductile fracture.

The mechanical properties of the two alloys were determined using tensile tests carried out by the mechanical testing laboratory of the Metal Forming Technology Laboratories (MTL) of CANMET. The tests were performed according to ASTM E517 [106] and E646 [156], on a MTS servo-hydraulic 100 kN (22 Kips) testing machine with an Instron 8500 controller, at rates of 3 mm/min. Three tensile test specimens were cut from the as-received aluminum sheets for each of the 0°, 45° and 90° directions, measured with respect to the rolling direction, as shown in Figure 1.16.

The averaged uniaxial tensile data values are shown in Table 2.3 for each of the 0°, 45° and 90° directions. The values represent averages over the three specimens tested for each direction. Table 2.4 summarizes the average tensile properties for both alloys, representing an equivalent anisotropic material exhibiting planar isotropy. Note

that values for $\bar{r}$ were calculated using the weighted average given by Equation 1.4. Table 2.4 also includes values for Δr , planar anisotropy, calculated using Equation 1.3.

The true stress-strain curves for the 6111 and 5754 alloys, derived from the load-extension data recorded during the tensile tests, are shown in Figures 2.1 and 2.2, respectively. Note that the curves for the 5754 alloy indicate serrated flow, which is consistent with an aluminum containing magnesium. Lüders bands were also observed in the failed tensile specimens. An average true stress-strain curve for each alloy, obtained by polynomial fit, is also given in Figures 2.1 and 2.2.

Average sheet thicknesses of 1.0 mm and 0.932 mm were determined for the 5754 and 6111 materials, respectively, by measuring various sheets of each material at different locations. The variation in thickness between locations was very small, less than 0.006 mm.

2.3 Dome Tests

2.3.1 Specimen Preparation

Three specimen geometries were chosen for the dome tests based on previous research [59, 157] to give a wide variation in strain state at failure for three typical forming conditions; biaxial stretching, plane strain, and drawing. These geometries consisted of a 203.2 mm (8 inch) square specimen for biaxial stretching, a 203.2 mm (8 inch) by 127.0 mm (5 inch) rectangular blank for plane strain, and a 203.2 mm (8 inch) by 25.4 mm (1 inch) "dog bone" or notched specimen (Figure 2.3) for drawing. For consistency between tests, the major dimension for all specimens was taken to be in the material rolling direction. The notched specimen was used to provide a larger specimen cross section where the lock bead clamped the specimen. This larger grip at the lock bead prevented tearing at the lock bead, permitting larger minor strains through draw-in for a narrow specimen width of 25.4 mm (1 inch) at the specimen mid-point.

Blanks were cut from various sheets using a powered shear and then trimmed to exact dimensions with a hand operated shear. Care was taken not to scratch or damage the surface of the sheets by always keeping a separator stacked between specimens. This precaution was taken since the surface condition is critical for test repeatability and in preventing the premature failure of specimens. It is also important to have a smooth, unblemished surface for the silk screening procedure.

Due to the poor contrast that chemical etching provides on aluminum, it was decided to use silk screening to apply a grid onto the specimens for the measurement of strains as described in Section 2.5. A square 2.79 mm grid was selected over the circle-grid method as the digital image from a silk screen gives a jagged edge to circular shapes, instead of the smooth, continuous shape desired for image processing. The 2.79 mm grid size corresponds to an internal standard used by ALCAN. The silk screening was performed by Jet Sign, Kingston, Ontario. A deformed grid, shown in Figure 2.4, serves to demonstrate the tenacity of the ink grids.

2.3.2 Experimental Setup

The limiting dome height press facility at Carleton University, shown in Figures 2.5 and 2.6, was used to obtain experimental formability data, in a manner similar to that proposed by Ghosh and Hecker [39, 40] (refer to Section 1.1). The press also permits interrupted dome tests from which the strain history of the localization can be obtained.

The press consists of a four column frame with a die set providing alignment for the clamp, die and punch assembly. For this investigation, a 101.6 mm (4 inch) tool steel hemispherical punch is connected to a 50 kN (11 Kip), 152.4 mm (6 inch) stroke, MTS servo-hydraulic actuator (Model 244.12), and controlled by an MTS 406/436 Controller. Input voltage is supplied to the MTS controller by a custom made triangular wave generator, from which the punch rate can be set by adjusting the rise time or period of the triangular wave.

A 45.5 kN (10 Kip) load cell is mounted between the punch and the actuator to measure punch force. Punch displacement information is provided through a linear variable differential transducer (LVDT), mounted within the actuator. The range of the LVDT is ± 100.0 mm (± 3.937 inches). A Nicolet Pro30 two-channel digital oscilloscope with 12 bit accuracy was used to record the punch load and displacement output voltages. Conversion of the voltage signals to real values is completed using a regular spreadsheet graphics package.

The clamping dies used in this investigation are similar to those shown in Figure 1.6 (due to Ghosh and Hecker [39, 40]), but with inverted lock beads. Blankholder force, which prevents draw-in by clamping the periphery of the blank in a lock bead, is applied through a pair of 30 Ton Enerpac hydraulic cylinders (RCH-306) for a combined clamping force of 533.8 kN (120,000 lbs). The maximum supply pressure is approximately 69.0 MPa (10,000 psi), but the pump flow is typically kept at 75% of maximum, or roughly 52 MPa (7500 psi), for a total clamping force of 400.3 kN (90,000 lbs). The pressure is read from a pressure gauge attached to the hydraulic supply line of the clamping cylinder.

2.3.3 Specimen Test Procedure

Prior to every test, the tooling and the surface of the blank opposite to that of the grids were cleaned using a methanol soaked absorbent cloth. The specimen was then carefully centred atop the draw bead using a transparent scale, with its major dimension (or rolling direction) parallel to the front face of the press.

With the blank centred, the punch was lowered so that it was just touching the specimen. Once this new position had been recorded, the punch was retracted from the blank to permit closure of the clamping die, to a clamping pressure between 48.3 and 55.2 MPa (7000 and 8000 psi), corresponding to loads of 373.6 kN and 427.0 kN (84,000 lbs and 96,000 lbs), respectively. The punch was subsequently returned to the recorded position before triggering the wave generator, set to provide the desired punch

head velocity, usually between 0.2 mm/s and 0.5 mm/s. The lower punch velocity was used to capture the onset of failure, or necking, and to eliminate rate effects through quasi-static deformation. The desired punch depths or dome heights were set by the controller.

The test was terminated either when the punch had reached the specified depth, or when the specimen failed. Failure was determined to occur when necking, or a localization, occurred, as aluminum showed little load drop at failure. The onset of necking was monitored using a video camera mounted directly below the punch cavity. A set of ring lights provided specimen illumination, and the formation of a neck, provided that the punch rate was slow, could easily be identified (see Figure 2.7). This termination method was utilized for tests to failure only, by pressing the retract switch on the wave generator. Otherwise, the wave generator program would automatically retract the punch to the zero position once the desired depth was achieved.

At least one limiting dome height test for each geometry mentioned in Section 2.3.1 was tested in order to validate the two forming limit diagrams provided by ALCAN along with the 6111 and 5754 material. Tests were repeated if an excessively large tear or crack was observed in the failed blank. The same three geometries were also used to form interrupted domes from which the development of the localization in the sheet could be measured as the sheet deformed. These tests were used for comparison with the numerical simulations of the forming process.

A series of tests of varying punch depths was run for each material and geometry. To better capture the development of a localization, small dome height increments were chosen, on the order of 2.5 to 5 mm, based on the LDH from the FLD investigation. An example of such a series of tests is shown in Figure 2.8 for the 127.0 mm (5 inch) blank of the 5754 alloy. Table 2.5 shows the dome heights for each of the deformed specimens. All specimens were tested in an unlubricated condition. Principal strains were then measured (see Section 2.5) from grids closest to the rolling direction line of symmetry. These strains were then plotted against the radial distance from the pole (see Figure 2.9).

2.4 Deep Drawing Rectangular Pans

2.4.1 Experimental Setup

The Universal Forming Test facility at CANMET was used to carry out flat-bottom rectangular pan deep drawing tests for the validation of the corresponding numerical model. The press tooling is shown in Figure 2.10, while a schematic of the facility is provided in Figure 2.11. The press is designed with removable inserts for the die radius and shell corners, as well as various draw beads, which were not used in this investigation. A typical formed pan is shown in Figure 2.12. The cavity has length and width dimensions of 213 mm (8.39 inches) and 110 mm (4.33 inches), respectively, with a die corner radius of 25.4 mm (1 inch).

The Universal Forming Test simulates a real forming process, with stamping process control variables such as lubrication, blankholder force, punch speed and blank surface condition that can be modified to control the strain distribution in the part. The pressing and clamping cylinders have capacities of 80 and 55 metric tons, respectively, and the punch cylinder has a stroke of 150 mm. The pressing and blankholder loads, as well as punch speed and displacement, are monitored and can be controlled through instrumentation connected to a computer. The punch can operate at speeds in the range of 0.1 to 100 mm/s, but is usually run at speeds close to 1.5 mm/s.

2.4.2 Specimen Preparation

Trial tests were performed to determine the geometry of the blank that should be used in this part of the experimental investigation. Due to the limited amount of material available, it was desired to select the smallest possible blank size in order to maximize the number of test specimens. The selection process consisted of testing several blank geometries with various lubricants to obtain a combination which would maximize the pan depth and thus the amount of deep drawing.

The clamping draw beads were first removed from the deep drawing press to reduce the required blank size. This modification served to relax some of the constraint placed on the flow of material in the long wall. Material constraints were further relaxed by cutting off the corners of the blank, as described in Section 1.2.3. After much experimentation, a blank size of 304.8 mm by 254.0 mm (12 inches by 10 inches), with 50.8 mm (2 inches) of the corners removed, was determined optimal for the purposes of this investigation (Figure 2.13). All blanks were cut with the major dimension in the material rolling direction, and a square grid, described in Section 2.3.1, was silk screened onto the specimen surface.

While the blank geometry tests were being performed, tests for the selection of the lubricant to be used on the specimen surface were also being conducted. Any lubricant that damaged or degraded the silk screened grid was rejected from consideration, as this would prevent the future measurement of strains. Initial tests with petroleum jelly (Vaseline) revealed that the grids were partially removed after the specimen was retrieved from the press. Several specimens were then pressed with 0.5 mm thick Teflon sheets placed between the tooling and both surfaces of the material. The Teflon sheets proved to be too thick for the available punch/die clearance, as the pressed specimens failed at the base of the short wall through severe ironing and tearing, instead of through drawing, as shown in Figure 2.14 for both a full size 6111 alloy blank and one with corners cut. Further tests with an industrial lubricant known as Cromac 1049 (provided by CANMET) were more successful, and thus Cromac was chosen as the lubricant for this part of the experimental investigation.

2.4.3 Specimen Test Procedure

The non-gridded surface of the specimens was cleaned with acetone prior to each test, and any material built-up on the die radius was removed using emery paper. The die was then wiped thoroughly, and the Cromac lubricant was applied to the die and the blank with a paint brush. No lubrication was used on the punch. The blank was

carefully centred relative to the die using a scale and a clamping pressure was applied up to the desired blankholder force. For this investigation, a blankholder force of approximately 68 kN was used. Once the specimen was clamped, the punch movement was started using a nominal punch speed of 1.5 mm/s. The punch stopped either when the desired depth had been reached, or when the specimen failed. The clamping pressure was then released and, once the punch and upper die were raised, an ejector was used to remove the pressed specimen from the lower die.

Except for the trial specimens used to determine the blank geometry and lubricant, none of the specimens failed through drawing. The clamping pressure was thus increased to 110 kN, and then to 139 kN, but still no failures occurred. This was probably due to the blank size which did not permit a deeper pan to be drawn as the flange material feeding into the base of the short wall was exhausted. A full-size 5754 alloy blank (corners intact) was also tested at high pressure to try to obtain a failed sample, but with no success, as shown in Figure 2.15. The specimens were thus drawn to a maximum of 60 mm (2.36 inches), with interrupted pans drawn to depths of 50 mm (1.97 inches) and 30 mm (1.18 inches) to obtain strain histories. Table 2.6 lists the pan depth, blankholder force and punch speed for the specimens tested.

2.5 Experimental Strain Measurement

Grid measurements were made using a video-based image acquisition system. The grid image is acquired by a video contact probe, developed at Carleton University, which incorporates a Panasonic micro CCD camera, and digitized using a 480 x 640 pixel, 8-bit grey scale Imaging Technologies PC Vision Plus frame grabber. The corners of the grid are picked manually on the video monitor, and strains at the middle of the silk screened surface are then calculated from the current grid dimensions and the size of the undeformed grid. The equations used to calculate the strain at the centre of the grid were derived by treating each grid as a four-node linear finite element [158].

The original grid size was obtained by calibrating the probe with a sampling of undeformed grids taken from an undeformed blank. About 20 grids were measured, and the average dimension of the grids was entered as the calibrating grid size used to obtain the strains.

Chapter 3

Numerical Models

Numerical simulations to predict the strain distributions in the limiting dome height test and the Universal Forming Test were performed using versions LS-930 and LS-936 of LS-DYNA3D [159], an explicit finite element code for non-linear dynamic analysis. Silicon Graphics Iris (SGI) R4000 Crimson and R8000 Challenge series workstations were employed to run the analyses.

LS-DYNA3D and other explicit dynamic codes are especially suited to sheet metal forming applications [74, 149, 160]. The central difference technique employed in their explicit time integration requires very little storage, as opposed to implicit formulations, where a large number of equations must be solved simultaneously. Other advantages of explicit formulations hinge upon their accurate and efficient implementation of contact surfaces or slide lines, an application for which LS-DYNA3D is particularly adept at, and their great capacity for vectorization and parallelization. The development of efficient and reliable contact algorithms has been an issue of considerable importance in the accurate simulation of thin sheet metal forming with LS-DYNA3D [149].

Although the explicit formulation provides many benefits, these are offset to a degree by the very small time steps required for stability. The stable size of the time step is governed by the Courant criterion [159], with the critical or largest allowable time

step approximated by the smallest time a dilatational or stress wave takes to cross any element in the mesh [153]. The explicit formulation is therefore only conditionally stable, with time step sizes 100 to 1000 times smaller than those required for implicit solvers [14, 21, 30], typically on the order of microseconds, implying that this type of code is well suited for problems of short duration. Sheet metal forming, however, is normally driven by low, quasi-static punch velocities that vary between 0.1 and 10 mm/s, making the application of an explicit code seem completely unsuitable. To achieve acceptable computing times using explicit codes, the speed of the simulated process is artificially increased, with forming times typically reduced by factors of 10-100. Care must be taken in applying this "load factoring" approach as heightened punch speeds may cause undesired (and unphysical) inertial effects, such as oscillation of the binder [74, 160].

3.1 Finite Element Mesh

3.1.1 Mesh Description and Refinements

Numerical models of the hemispherical punch apparatus at Carleton University and of the deep draw Universal Forming press facility at CANMET were constructed with the aid of the PATRAN Mechanical Computer Aided Engineering (MCAE) graphics package [161] and technical drawings of both facilities. The die, clamp and punch tooling surfaces in both simulations were modelled using rigid, four-node quadrilateral shell surface elements and, as such, only a surface representation of the tooling was needed for the model. Due to symmetry considerations, only one quarter of each geometry was modelled, as shown in the exploded views of the finite element meshes used for the 203.2 mm LDH specimen and rectangular pan deep draw models in Figures 3.1 and 3.2, respectively. The boundary conditions required to enforce symmetry are discussed in Section 3.4.1.

Models of the deformable blanks were created using Belytschko-Lin-Tsay [162, 163] four-node quadrilateral shell elements for each of the three LDH geometries described in Section 2.3.1, and for the rectangular deep draw pan geometry described in Section 2.4.2. The number of elements used in each model are listed in Tables 3.1 and 3.2. Although a large LDH blank model, such as the 203.2 mm wide blank, required the largest number of elements, as expected, the computational size of the 127.0 mm model was not reduced substantially. This was due to careful meshing of the area adjacent to the lock bead to prevent hourglassing problems, experienced in initial simulations (discussed in Section 3.1.2), and to better capture the severe deformation evidenced in that area. The relatively small elements that were required had the effect of reducing the associated time step size, adding approximately three hours to the 127.0 mm model run time over that of the 203.2 mm model.

Another area of mesh refinement involved the lock bead, which serves to clamp the material in place, thereby preventing material draw-in. To properly simulate this action without geometric locking of the mesh, and to better capture the sharp lock bead radius, both the binder and die meshes were refined in the area of the lock bead. The mesh area of the blank to be deformed by the lock bead was also refined, in a similar manner, so that the material deformed smoothly into the lock bead. Such modifications were first tested on the 203.2 mm model, and then extended to the 127.0 mm and 25.4 mm models. Figures 3.3, 3.4 and 3.5 show the final meshes adopted in this investigation for the 203.2, 127.0, and 25.4 mm models, respectively.

The larger blank for the deep draw rectangular pan required considerably more elements than the LDH blanks, as summarized in Table 3.2. A larger number of elements was especially needed in the area of the blank flowing into the die corner to better capture the curvature and hence the strain distribution within this region. A fine mesh was also used within the short wall region, to better capture both the ironing phenomenon and material draw-in observed in the deformed specimens (refer to Figure 2.12).

The mesh for the rectangular pan deep draw simulation evolved with changes in the geometry of the blank used in the experimental investigation, described in Section 2.4.2. The removal of the original draw beads and the reduction of the blank size served to significantly decrease the number of elements required in the model. These changes were followed by removing material from the corners of the blank, which again reduced the number of elements required, but also necessitated a major modification of the existing mesh. Due to the substantial amount of time needed to run the simulation (more than 50 hours on an SGI), the model was essentially frozen at this stage to allow adequate time for the completion of any parametric studies. The final mesh discretization used in the current investigation is shown in Figure 3.6. Future work should examine further optimization of this mesh.

3.1.2 Element Formulation Selection and Description

The choice of an appropriate element formulation is crucial to an accurate numerical simulation. In this investigation, shell elements were selected over brick elements largely due to their computational efficiency. The required evaluation of large three-dimensional constitutive equations at each of their integration points makes brick or hexahedral elements expensive in terms of computation. The plane stress assumption in shell elements reduces the state variables, and hence computation, by about half [162], while through-thickness integration points (typically three to seven) are employed to more closely approximate bending and full three-dimensional formulations. Shell elements are especially efficient and accurate in their application to thin sheet metal problems.

Of the thin shell element formulations available in LS-DYNA3D, the Belytschko-Lin-Tsay [162, 163] four-node quadrilateral element was selected for the current investigation. This type of shell element has simplicity, is efficient, and its accuracy compares favourably with that of more complex elements. It employs a corotational description of the local coordinate system to simplify the element formulation. The local element spatial coordinate system is embedded at the reference surface of the element,

defined by the mid-surface containing the four corner nodes. This approach automatically accounts for rigid body rotations [164].

A detailed description of the Belytschko-Lin-Tsay shell element formulation is contained in the original papers by Belytschko, Tsay and Lin [162, 163], while the implementation of the shell formulation in LS-DYNA3D is described in the LS-DYNA3D Theoretical Manual [159]. Its use in LS-DYNA3D had been assessed in a study by Galbraith and Hallquist [164] on the modelling of sheet metal forming with various shell element formulations. They compared the cost-benefit relationship of seven four-node shell elements, with either membrane, 2x2 Gauss point quadrature, or single point quadrature formulations, in the simulation of the Numisheet '93 square benchmark problem [159].

They found that the very accurate results obtained with full integration formulations (2x2 Gauss point quadrature) were severely offset by the high computational times, and hence costs, required to evaluate the constitutive relations at each of the four in-plane Gauss points. Although reduced integration shells were much more efficient, with only one evaluation of these equations required at each through-thickness integration point, the use of a single integration point may lead to the formation of zero energy modes. This effect, termed hourglassing (discussed in greater detail below), can be eliminated by the implementation of hourglassing control routines, with only minimal run time increases. Since membrane shell formulations were limited by their inability to model bending effects, they were only considered as first pass approximations.

Galbraith and Hallquist [164] concluded that, of the three reduced integration shell elements available in LS-DYNA3D, the Belytschko-Tsay formulation provided the best compromise between computational efficiency and accuracy for sheet metal forming applications. As the default shell formulation in DYNA3D, Galbraith and Hallquist [164] indicated that the Belytschko-Tsay shell may also be the more robust formulation. Based on these observations, Belytschko-Lin-Tsay elements were employed in the numerical portion of this investigation, with seven through-thickness integration points.

The tooling components were modelled as rigid, thus neglecting tooling deformation and requiring only a surface discretization of the dies and punch in each

model. Each rigid tooling component was therefore treated as a single lumped mass, with six kinematic degrees of freedom, three for displacements in the x, y and z directions, and three for rotations about the x, y and z axes.

Thickness Update Option

The implementation of a thickness update is critical to any finite element simulation of sheet metal forming processes, especially in predicting sheet thinning effects on strain distributions. LS-DYNA3D uses the thickness update procedure developed by Hughes and Carnoy [165], which updates the shell thickness by adding the mean through-thickness strain component to the nodal fibre lengths. This option, in combination with thickness-based contact algorithms (described in Section 3.3.1), is particularly important in sharp radii locations [74] and in drawing operations, to simulate local variations in clamping pressure due to sheet thickening or ironing in the flange [166].

Hourglass Control

A single integration point in the plane of the element, as in reduced integration shell formulations (i.e., the Belytschko-Lin-Tsay shell element), makes elements susceptible to what is termed hourglassing, as shown in Figure 3.7. These "zero energy" modes of deformation [21, 22, 30, 162, 167] are characterized by zero stress at the central integration point. While the numerical solution is not adversely affected by the rigid body modes represented in the first three displacement modes of Figure 3.7, the fourth, a kinematic deformation mode, may cause the solution to oscillate or, if the mesh design is similar to the modal pattern, permit the formation of singularities which effectively preclude any solution [163, 167]. The hourglass modes receive their name from the kinematic mode shown in Figure 3.7.

Great effort has been spent in the prevention of these kinematic modes through the use of hourglass control techniques [21], which introduce artificial stiffness and

damping terms into the system, either via displacements (stiffness control) or via velocities (viscous control) [152]. The Flanagan and Belytschko routine [163, 167] implemented into LS-DYNA3D [159] is especially efficient in this regard, though simple restructuring of the mesh helps alleviate many hourglassing problems [149].

The default stiffness damping form of hourglass control was employed in the current numerical investigation. Severe hourglassing was observed in initial simulations of the 127.0 mm model, where the combination of the binding and subsequent punch action created large oscillations in the unsupported region of the sheet (Figure 3.8), leading to eventual collapse of the numerical model. Although careful remeshing of the affected region helped alleviate the oscillations, limited hourglassing in the flange area adjacent to the lock bead could not be avoided. This remaining oscillation was attributed to the interface stiffness between the tooling and sheet elements, and disappeared once appropriate corrections were made to both the element hourglass coefficient and interface stiffness scaling factor (refer to Section 3.3.2).

3.2 Constitutive Model

The mechanical properties of the materials selected for this investigation were determined from uniaxial tensile tests, as outlined in Section 2.2. The properties were then incorporated into the material models for the numerical simulations, and are summarized in Table 3.3.

Three constitutive formulations were investigated in the numerical analyses. The first, an isotropic elasto-plastic material, is an implementation of the von Mises yield function for plane stress, given by Equation 1.12. This isotropic model serves as the baseline model from which to compare strain distribution variations with $\bar{\epsilon}$, $\Delta\epsilon$ and μ .

The second constitutive model used in the simulations, an anisotropic elasto-plastic model, is the plane stress implementation of the quadratic yield criterion due to Hill, given by Equation 1.20. This model incorporates effects due to anisotropy, or $\bar{\epsilon}$, derived from tensile tests outlined in Section 2.2.

The third constitutive model used the Barlat non-quadratic yield criterion for plane stress, given by Equation 1.25. This planar anisotropic model allows the effects due to variations in r with direction in the plane of the sheet to be investigated. The exponent M in the yield criterion was given a value of 8, as suggested by Barlat [128] for FCC materials, such as aluminum.

The forming limit diagrams (FLD) adopted in this study for the 5754 and 6111 materials are plotted in Figure 3.9. The effective stress versus effective plastic strain curves shown in Figure 3.10 were employed in both the von Mises and Hill constitutive models. These curves are average, polynomial fits to the experimental curves shown in Figures 2.1 and 2.2 for the 6111 and 5754 materials, respectively. The curves are input as points, and LS-DYNA3D linearly interpolates between these points to acquire strain hardening data. Since the Barlat model implementation in LS-DYNA3D (type 36) does not permit the use of a point-wise plastic hardening curve, power law hardening was adopted (Equation 1.7) and the hardening parameters, K and n , were obtained from regressions on uniaxial tension data [39] for 1 mm thick 5754-O sheet. Only the 5754 material was investigated with the Barlat constitutive model and values of 416.52 MPa and 0.25599 were obtained for K and n , respectively. The power law hardening curve employed in the Barlat model is also shown in Figure 3.10.

3.3 Contact Modelling

3.3.1 Contact Surfaces

Intermittent contact and sliding boundary conditions between the tooling and sheet metal elements are enforced in numerical simulations by special contact algorithms. They assure that no penetrations occur between the tooling and sheet metal interface. The penalty stiffness method used in the contact algorithm implemented in LS-DYNA3D employs penalty function-based contact interfaces, known as slide lines, to define the

elements forming the two surfaces that come into contact. These two surfaces are designated as master and slave surfaces [159], with no distinction being made between the two for the penalty stiffness method.

A contact search routine serves to enforce the impenetrability condition; at every time step, each slave surface node is checked against penetration of element faces on the master surface. Under the penalty stiffness method, the penetration of nodes is prevented by introducing artificial springs into the system. These "interface" springs are placed between the offending nodes and the contact surface, with a stiffness that is scaled to the bulk modulus of the master (or penetrated) surface. The interface spring stiffness of element i , λ_i , is given by [159]

$$\lambda_i = \frac{\xi_i K_i A_i^2}{V_i} \quad (3.1)$$

where K , A , and V are the bulk modulus, the face area, and the volume, respectively, of the element that contains the master segment, and ξ is the user defined interface stiffness scaling factor. The magnitude of the force needed to restore the offending node to a position that does not conflict with the contacting surface is then defined by a combination of three elements: the stiffness of the spring itself, the depth of penetration, and the user defined interface stiffness scale factor.

The spring action is only removed when no further penetration is detected; thus, no contact forces are added to the system, and energy is therefore conserved. When no further penetration of the master surface is detected, the search routine is repeated, to check for any penetration of master surface nodes through slave surface elements.

Of the contact algorithms available in LS-DYNA3D, the penalty method surpasses all others in preventing the initiation of hourglassing. Because of their low through-thickness stiffness, shell elements are especially prone to hourglassing initiated by contact stiffness. This contact can, however, be "softened" by scaling the magnitude of the penalty stiffness down or, inversely, "stiffened" when required.

Two slave/master contact surfaces were defined in each of the two simulation models; one for the die to lower blank surface contact, and another for the punch and binder/blankholder to the upper blank surface contact. The sheet metal mid-plane was adopted as the contact reference surface for the LDH model, corresponding to a membrane contact formulation. Although this permitted penetrations of up to one-half of the sheet thickness, the error was deemed acceptable for the LDH geometry due to the large radius of curvature of the punch (50.8 mm) compared to the sheet thickness.

The thickness-based contact option was required for the deep draw rectangular pan model, where separate reference surfaces for the sheet top and bottom are used to enforce contact with the tooling (Figure 3.11). Although more expensive computationally, this contact option was necessary to properly simulate the sheet thickening and subsequent ironing that occur in drawing operations through local variation of the clamping pressure. The change to a thickness-based contact formulation is supported by a previous analysis of the deep draw model using the membrane contact formulation [166].

3.3.2 Interface Stiffness

As described in Section 3.1.2, severe hourglassing was observed in initial analyses of the 127.0 mm wide LDH model. Large oscillations occurred in the unsupported region of the blank (Figure 3.8), due to the combined binder and punch actions, leading to the eventual collapse of the model. Refinements to the mesh served to lessen these effects, as did setting the element hourglass coefficient to its maximum value of 0.15, but hourglassing still remained and was attributed to the contact stiffness. The default penalty stiffness was therefore reduced from the default value which reduced hourglassing while still avoiding excessive penetration of the contacting surfaces. For consistency, the same modifications were made to stiffness and hourglass parameters in each simulation, and for every material and yield criterion investigated.

3.4 Boundary Conditions

Figure 3.11 is a schematic of the models used in this investigation, showing the tooling and blanks in their initial positions and indicating the direction of punch movement for each simulation. The LDH blank is shown with a thickness in this diagram to distinguish the blank surface from that of the binder, and to illustrate the fact that it is resting upon the male lock bead.

The LDH simulation essentially consists of closing the binder, applying a clamping force, and then actuating punch movement. For the purposes of this numerical investigation, the die/binder closure was realized by translating the die upwards while the binder was held in place. Although this is contrary to the experimental motion, it was shown to have the same overall effect on the blank while decreasing the punch travel and, therefore, the run time. This approach accounts for why the punch is shown in an elevated position in Figure 3.11 (a). The only difference in the deep draw simulation sequence is that the blankholder is already in position, as shown in Figure 3.11 (b), since no lock beads or draw beads were employed in this experiment. The punch is actuated once the desired blankholder force has been reached.

3.4.1 Displacement and Force Boundary Conditions

Displacement boundary conditions control the movement of the rigid tooling. For the LDH model, the punch was constrained from any motion other than translations in the y direction. As described in the preceding section, the die was raised upwards to effect die/binder closure while the binder was initially held motionless. This motion was accomplished by enforcing zero displacement and rotations at all nodes on the binder, while the die velocity in the y direction was prescribed, with all other directions being fixed. Upon closure, the die was fully clamped and the binder restraint in the y direction was removed to permit application of the binder force. The restraints on the punch were only removed once the binder force had reached the desired value. Appropriate

displacement and rotational boundary conditions were enforced at the edges of the deformable blank along the x and z symmetry planes (Figure 3.3).

Similar displacement boundary conditions were applied to the rectangular pan model, except that the punch translated in the z direction, with the symmetry planes being normal to the x and y axes (Figure 3.6). The die was fully clamped throughout the simulation, with the blankholder remaining free to translate in the z direction only.

Load-time histories, commonly termed load curves, control the application of various applied loads in LS-DYNA3D. Consisting of strings of time-value coordinates, load curves effectively define the shape functions (linear, step or sinusoidal) used to apply forces, pressures or prescribed velocities to components in the numerical simulation. Their optimization remains the key to faster and more efficient analyses as binder and especially punch velocities greatly affect the duration of runs. To this end, all velocity load curves in the simulations were of sinusoidal shape, as recommended from previous research [21, 157, 168]. A sinusoidal velocity curve, which provides smooth punch accelerations or decelerations, minimizes inertial effects due to the artificially high velocities required of explicit dynamic simulations to offset the extremely small time step size.

Figure 3.12 shows the load curves used to control the binder displacement/force and punch velocity for the LDH models, for a dome depth of 32 mm. Two stages were employed to simulate the clamping process. The first consisted of translating the die upwards, with an applied sinusoidal velocity curve, to effect closure of the die and binder. Once the velocity of the die had returned to zero, the boundary condition restraining all binder movement was released so that a ramped clamping force could be applied to the rigid binder surface. This clamping force served to complete the motion of closing the die, and to effectively prevent material draw-in during punch motion. As shown in Figure 3.12, the closure of the LDH die and binder is modelled to occur over a 0.25 ms period, with a maximum speed of 30.3 m/s, followed by a 0.05 ms interval in which the clamp force is ramped up to a value of 134 kN. Once this clamping force had been reached, the punch motion was initiated, with a maximum velocity of 25.1 m/s over a 2.0 ms period. These load curves were determined to be effective for the

prevention of hourglassing and dynamic effects, such as bouncing of the die against the binder, and oscillations of the sheet in the unsupported region between the punch and the die. Optimization of the load curves was accomplished using the 203.2 mm wide blank LDH model, and adapted to the two other LDH geometries.

Figure 3.13 shows the load curves used to control the blankholder force and punch velocity for the deep draw model, for a pan depth of 60 mm. The load curves were simpler for this simulation as no draw beads were employed, and the blankholder was already resting atop the die. Therefore, only two load curves were needed. The first consisted of a 0.2 ms ramping force to apply a 17 kN blankholder force. This was followed by a 3.0 ms punch motion, using a sinusoidal velocity curve with a maximum velocity of 31.4 m/s. Analyses were initially conducted with punch displacements occurring over 6.0 ms, but were reduced due to the lengthy run times, without noticeable hourglassing or dynamic effects. Variations in the blankholder force were also analysed, with 27.5 kN and 34.8 kN loads applied over the same 0.2 ms ramp.

3.4.2 Frictional Coefficients

In LS-DYNA3D, frictional boundary conditions are implemented through a linear Coulomb formulation, as given by Equation 1.1. It was decided early in the investigation to make use of realistic frictional coefficients, determined experimentally, instead of fitting the predicted strain distributions to experimental results, as often described in the literature (Section 1.5.2). Friction coefficient measurements were therefore obtained using a draw bead simulator apparatus at the mechanical testing laboratory of the Metal Forming Technology Laboratories (MTL) of CANMET. Since this test is relatively severe for "dry" aluminum, the results for these friction measurements were uncertain. However, a friction coefficient, μ , ranging between 0.18 and 0.25 was determined for the un-lubricated condition for both the 5754 and 6111 materials. The lowest value, 0.18, was employed for all baseline models from which to compare strain distribution variations. A parametric study of friction coefficient was

conducted using Hill's 1948 quadratic yield criterion (Equation 1.20) for the 5754 material. These results are presented in Section 5.4.

Lubricated tests with the Cromac lubricant on the 6111 alloy gave values of μ equal to 0.096 and 0.052 for specimens tested in directions parallel and transverse to the rolling direction, respectively. Although all experimental deep draw specimens were sheared and tested with the major dimension in the direction of rolling, it was decided to again employ the lowest frictional value obtained, 0.052, as the baseline for all numerical simulations. Section 5.4 briefly presents the 5754 predicted strain distributions for both friction coefficients.

Chapter 4

Experimental Results

This chapter presents the results obtained from the experimental formability and drawability investigations described in Chapter 2. Experimental trends in the measured major and minor principal strain distributions are presented for the interrupted dome tests, while comparisons of the major versus minor strain loci are presented for interrupted deep draw pan tests.

4.1 Experimentally Measured Dome Strain Distributions

As described in Section 2.3.3, a series of tests at various punch depths were performed on 203.2, 127.0 and 25.4 mm wide blanks to observe developing strain distributions in 5754 and 6111 aluminum alloys. (See Figures 2.7 and 2.8.) The grid measurement technique was employed to measure the major and minor principal strains from grids lying along the axis of symmetry of the major blank dimension, i.e., along the sheet rolling direction. These measured strains were then plotted against their respective lateral positions in relation to the pole of the dome along the same symmetry axis (Figure 2.9) to obtain strain distributions.

4.1.1 5754 Aluminum

The measured major and minor strain distributions for the 203.2 mm, 127.0 mm and 25.4 mm wide blanks of the 5754 aluminum are shown in Figures 4.1, 4.2 and 4.3, respectively. The depths indicated in these figures correspond to the measured interrupted dome heights. The curve labelled F denotes the limiting dome height (LDH) curve from a specimen tested to the onset of necking. The "up-turn" at the outer periphery of the strain distributions shown in these figures are associated with the strains induced by the formation of the lock bead. Analysis of measurements performed on a series of undeformed grids has predicted, with a 95% confidence interval, an experimental error of ± 0.009 strain on any one measurement [169]. This error can be expected to increase with deformation and deterioration of the grid.

The sharp strain peaks on either side of the pole in Figure 4.1 clearly show the development of strain localizations in the 203.2 mm blank of the 5754 aluminum, with a diffuse localization observed very early in the deformation, at a depth of 15.2 mm, growing more defined as deformation proceeds. The localization process is controlled by the frictional contact forces, with the peak strains occurring very near the punch-blank tangency point (TP) for each of the measured depths. The smaller deformations observed in the region beneath the pole of the punch are due to the frictional contact forces that constrain the material over the punch surface. These constraining forces are removed when the sheet loses contact with the punch, and hence the strains beyond the TP decrease towards the lock bead. The strain localization occurs just inside the TP, where the material is prevented from deforming by frictional contact.

Minor strains reaching a magnitude of 0.12 were measured for the 203.2 mm wide 5754 aluminum specimen, with the largest minor strains occurring over the blank-punch contact area. The magnitude of the minor strains initially increases as deformation proceeds, but is shown to level off once the maximum strain level of 12% is reached across the entire contact region. Lubrication of the punch would lead to larger minor strains. Beyond the contact region, the minor strains decrease as distance from the pole

increases. Strains very close to zero were observed near the lock bead, where the lateral constraint of the lock bead induces a state of plane strain in the material. The larger scatter evidenced in the minor strain measurements is derived from the greater relative error in measuring a smaller strain.

The trends in the measured strains for the 127.0 mm wide blank of 5754 aluminum shown in Figure 4.2 are very similar to those for the 203.2 mm blank discussed above, with a slightly lower peak strain consistent with the lower formability associated with plane strain conditions. As expected, the lower limiting dome height for the 127.0 mm blank indicates that the punch-sheet TP is closer to the pole of the punch, and hence the localizations are located slightly closer to the pole than those of the 203.2 mm blank. The strains under the pole of the punch reach a maximum of approximately 7% very early in the deformation, with no further straining occurring. Very little lateral straining is observed as the minor strains vary between 0.03 and 0.05 at the two localization sites. These minor strains accumulate early in the deformation, with no further variations in strain magnitude occurring over the remainder of the deformation. Although minimal, the existence of small tensile minor strains indicates that a slightly narrower specimen would be required to achieve a true plane strain condition.

The appearance of two necks on either side of the dome for the 203.2 mm and 127.0 mm specimens widths, corresponding to the two peaks on either side of the pole in Figures 4.1 and 4.2, is an indication that the LDH apparatus is well aligned.

The strain distributions for the 25.4 mm blank of the 5754 aluminum, shown in Figure 4.3, do not resemble those for the 127.0 mm and 203.2 mm blank widths. The reduced frictional contact force stemming from the smaller contact area allows material to draw-in laterally and hence deform under the punch, an area where material is usually constrained for larger blank widths. Although the initial major strain seems uniform over the contact area, diffuse localization occurs at a depth of 21.0 mm, and is quickly followed by the onset of necking at 22.5 mm, represented by the very sharp strain peak near the pole of the punch. The single peak observed experimentally is a result of the two localizations being positioned so closely together that only one severely deformed grid, covering the region of interest, is available from which to measure the strains. Due

to the rapid onset of necking in the notched 25.4 mm specimens, the two localizations would often lead to a large diagonal tear joining the two necks across the tapered specimen width before the test could be stopped. The results for the single specimen captured before the specimen split in two are shown in Figure 4.3.

The minor strain distribution for the 25.4 mm blank reflects the lateral material draw-in mentioned above, with minor strains of approximately -0.05 observed in the material under the pole of the punch. These minor strains decrease as the distance from the pole increases to beyond the contact region. Again, larger scatter was evidenced for the minor strains compared to the major strains.

4.1.2 6111 Aluminum

During the experiments with the 6111 alloy, it was observed that the onset of necking was much more sudden than in the 5754 alloy. This often resulted in the formation of very large cracks before the experiment could be interrupted; this was particularly problematic for the 25.4 mm deep draw geometry. Therefore, strain measurements were only taken from the 203.2 mm and 127.0 mm 6111 aluminum geometries.

The measured strain distributions for the 6111 aluminum 203.2 mm dome are shown in Figure 4.4, and are quite similar to those obtained for the 203.2 mm blank of the 5754 aluminum. The localization develops in the same fashion and at a similar location, with slightly lower peak localization strains, consistent of a thinner material. The minor strain distribution again shows large scatter, with a maximum strain across the contact surface of approximately 10%.

The results for the 127.0 mm blank of the 6111 aluminum are shown in Figure 4.5. The behaviour of this material is again quite similar to that of the 5754 aluminum for the same geometry, with similar trends observed in both specimens. The localization occurs at the same location, with the strain peak again slightly lower than that of the

5754 aluminum blank. The only difference lies in the minor strain distributions, which shows some slight draw-in (negative minor strain) at low punch depths.

4.1.3 Comparison of Material Formabilities

A Forming Limit Diagram (FLD) is commonly employed to assess the formability of a material, especially for sheet metal applications, as described in Section 1.1.1. The large amount of material required in their determination has precluded the creation of forming limit curves (FLC) in this investigation, as only a limited quantity of each material was available for testing. It was therefore decided early in the investigation to employ the FLC for the 5754 and 6111 aluminum provided by CANMET, shown in Figure 3.9.

The curves show that the 5754 and 6111 materials have very similar LDH_0 limit strains, of approximately 22% and 20%, respectively. Both materials also behave quite similarly in the biaxial stretching regime (positive major strain), with the curves becoming almost linear as minor strain increases. Their behaviour in the drawing regime (negative minor strain), however, differs significantly. The 5754 aluminum exhibits much greater drawability, or formability in drawing, than does the 6111 aluminum. This difference can, in part, be attributed to the slightly larger initial thickness of the 5754 aluminum (1.0 mm) over that of the 6111 aluminum (0.932 mm); the larger material thickness serves to shift the 5754 curve upwards. The work hardening rate of each material is another consideration, as the strain hardening coefficient, n , tends to shift the FLC downwards as it decreases. In this case, the 5754 aluminum ($n=0.294$) has a greater work hardening rate than the 6111 aluminum ($n=0.259$), hence a further shift of the 6111 curve away from the 5754 curve. The effects of sheet thickness and hardenability are somewhat offset by the lower yield stress of the 5754 aluminum ($\sigma_y = 101.0$ MPa) compared to the 6111 aluminum ($\sigma_y = 141.6$ MPa), which tends to lower the 5754 FLC. The lower formability in drawing of the 6111 aluminum, combined with

the rapidity of the onset of necking exhibited by this material, may explain some of the problems involved in obtaining adequate specimens for the 25.4 mm notched blank geometry. This specific geometry primarily assesses the drawability of a material, and 6111 exhibits lower drawability compared to the 5754 aluminum.

In an attempt to validate the two forming limit curves with experimental results, the strains from grids surrounding the localizations in specimens interrupted at the onset of necking were measured. The limiting dome height specimens described in the previous sections were employed in this area of the investigation, with 50.8 mm notched specimens instead of the 25.4 mm LDH specimens so that results for both materials could be obtained in the negative minor strain region. These three geometries would provide a series of data points in each of the three forming regions of interest, drawing, plane strain and biaxial stretching, from which to compare the forming limit curves.

Figures 4.6 and 4.7 present the measured results for the 5754 and 6111 aluminum, respectively, with the curves provided by CANMET superimposed, showing generally close agreement. Grids were selected for measurement according to their position relative to the localization. Grids labelled "Necked" consist of grids traversed by a localization, or neck (or part thereof), but showing no tearing of the sheet. Grids showing even slight tearing were labelled as "Failed". The results for large failures were also included in the FLD, as the 6111 aluminum was prone to rapid cracking before necking could be visually observed and the test interrupted. Grids labelled "Safe" were measured from grids adjacent to the localization, showing no sign of either necking or failure. In Figure 4.6, the experimental results show good agreement with the FLC provided for the 5754 aluminum, with the exception of the drawing region. The FLC in the negative minor strain region is slightly overestimated when compared to the measured results. Figure 4.7 shows that the FLC is underestimated for the 6111 in all three strain regions, as if the curve provided was developed with initially thinner 6111 aluminum sheet. The measured results, when compared to the curves provided by CANMET, seem to indicate improved drawability for the 6111 aluminum and slightly inferior drawability for the 5754 aluminum. The two materials may therefore exhibit similar behaviours in drawing as they do in stretching.

As the measurement of grids in the development of a forming limit curve is highly subjective, with the interpretation of safe, necked or failed grids varying greatly between laboratories and operators, the validation of the curves provided by CANMET is highly qualitative in nature. The specimens employed in this investigation did not possess a wide enough range of forming conditions to properly assess the shape of a FLC, and more specimens would be needed for adequate repeatability.

4.2 Experimentally Measured Pan Strain Distributions

As described in Section 2.4, strain histories were measured within deep draw specimens of 5754 and 6111 aluminum alloys, at three different punch depths, 30, 50, and 60 mm. The grid measurement technique was again employed to measure the major and minor principal strains so as to obtain the experimental strain variations in three different regions within the deformed pan: the corner wall, the short wall, and the long wall. Measurements were taken along a line of grids in each of these regions, from the flange (A) to the punch bottom (B), as illustrated on a representative FEM mesh of the rectangular pan shown in Figure 4.8.

4.2.1 5754 Aluminum

The deformed 5754 specimens, shown in Figure 4.9 for punch depths of 30, 50 and 60 mm, demonstrate increased material draw-in with punch depth, as expected. A close-up of the deformation within the short wall (Figure 4.10) shows the excessive material draw-in at the bottom of the wall, leading to exhaustion of the flange material feeding into the short wall. Ironing is also detected visually at the bottom of the short wall, and is due to the material thickness exceeding the punch-die clearance in this region. Wrinkling in the flange, induced by biaxial compressive stresses within the specimen, is more clearly demonstrated in Figure 4.11, where a slight misalignment of

the press is indicated by the difference in the magnitude of wrinkles on one side of the specimen compared to the other.

The measured strains for the 5754 aluminum rectangular pans, interrupted at punch depths of 30, 50 and 60 mm, are shown in Figure 4.12 for (a) the corner wall, (b) the short wall, and (c) the long wall. Although failure did not occur in any of the specimens, the FLC is included in these figures for comparison. The experimental trends in the corner and short walls are similar, with little or no strain in the flat bottom region (labelled B), slight stretching (positive minor strain) in the punch radius region, and drawing (negative minor strain) in the walls. Figure 4.12 (a) clearly shows that the largest major strain occurs early in the deformation and at the bottom of the corner draw wall, as expected, with a magnitude of approximately 0.6. A slightly lower strain of approximately 0.45 is observed at the bottom of the short draw wall in Figure 4.12 (b), an area characterized by excessive draw-in (Figure 4.10), reflected by the largest measured minor strains, of approximately -0.65 in magnitude.

The very low strains within the long wall are mainly due to the removal of the draw beads in the experiments, to permit a deeper pan to be drawn. The small strains that do occur are largely due to the bending occurring at the die opening and punch radius. Figure 4.12 (c) shows the flange draw-in and mild wrinkling in the flange, reflected by negative strains; a negative minor strain is indicative of draw-in, while biaxial compressive stresses lead to wrinkling (see also Figure 4.11). The strains in the long wall are small in comparison to those measured in the corner and short walls; thus, further analysis of the strain distribution is therefore limited to these two regions.

The measured strains were affected by severe distortion of the silk screened grids, leading to scatter in the results. This is especially true at the bottom of the short wall, where the grids, already few in number, were so severely elongated that their outlines were no longer uniform, with further degradation induced by the effects of ironing. The lubricant used in the experiments also affected the grid surface; over time, the grid surface would simply rub off.

Scatter in the results for the corner wall can also be attributed to the fact that the grids selected for measurement in this region did not always lie on the specified path

along the deformed specimen contour. A 45° line was drawn onto the specimen, from the centre of the punch corner radius to the flange, with the grids closest to that line selected to be measured. Since the grid measurement technique only provides the strain at the centre of a grid, two grids were often measured, one on either side of the 45° line, to "balance" the strain representation when said line happened to cross directly between two grids. The results from these two grids were not averaged so that a true reflection of the strains in the part could be presented.

Scattering of the measured results for the long wall is due to experimental error when measuring small strains.

4.2.2 6111 Aluminum

The strain distributions for the 6111 aluminum deep draw pan are shown in Figure 4.13, and the trends exhibited are similar to those of the 5754 specimens. Slight stretching was observed in the flat bottom region, while mild drawing and wrinkling were observed in the flange of the long wall. Similar to the 5754 specimens, substantial drawing occurred in the corner and short walls, with the greater (negative) minor strains located at the bottom of the short wall. Slightly smaller strains were observed, however, for the 50 and 30 mm 6111 pan depths compared to the corresponding strains in the 5754 pans. The reduced strains were consistent with the greater strength of the 6111 aluminum in comparison to the 5754 aluminum (see Figure 3.10). While the 6111 aluminum specimens demonstrate increasing corner wall drawing as punch depth increases, further drawing was not exhibited in the 5754 specimens as the pan depth increased from 50 to 60 mm. At the 60 mm pan depth, the strains within the corner wall region are almost identical in both materials. The lack of a further increase in drawing in the 5754 60 mm specimen may be due to the greater material thickness, which caused larger ironing effects.

Significantly larger minor strains were measured in the short-wall region of the 6111 aluminum specimens, as shown in Figure 4.13 (b). This large (negative) minor

strain is consistent with the substantial flange draw-in exhibited at the bottom of the short wall, but the last point on the 50 and 60 mm curves is again suspect due to the severe distortion of the last grid.

Failure, other than due to ironing, was not observed in any of the 5754 or 6111 aluminum specimens, as all of the strain distributions were well below the respective material forming limit curves.

4.2.3 Experimental Investigation of Blankholder Force

In an attempt to force a specimen to fail through drawing, instead of through ironing or through exhaustion of flange material feeding into the short wall, the experimental blankholder forces were increased as described in Section 2.4.3. Figure 4.14 shows the strain distributions measured within two 5754 aluminum specimens, drawn to a 60 mm depth with blankholder forces of 68 kN and 139 kN, respectively. Significant changes in the corner wall and short wall strain distributions due to increased blankholder force were not observed in the figure. The small differences in the short wall strains again suffer an increased level of error due to the distorted grids in this region. No appreciable difference was observed in the long wall strain distributions shown in Figure 4.14 (c) for the high and low blankholder forces. The higher blankholder pressures lead to slightly reduced compressive strains, consistent with reduced wrinkling under larger clamping pressures (see Section 1.2.2). Wrinkling reduction with increasing blankholder pressure can be observed visually in a photograph of the deformed specimens, Figure 4.15. Also, slightly lower (negative) minor strains were measured in the long wall, which is consistent with less material flowing, or being drawn into the die cavity.

Chapter 5

Numerical Results

This chapter presents the results from the numerical investigation discussed in Chapter 3. The predictions obtained from the FEM models are first outlined, and then compared to experimental results. A parametric study of the effects of the frictional coefficient, μ , on the predictions is also presented.

Due to the similarities between the experimental trends for the 5754 and 6111 aluminum for both the dome and deep draw pan tests, further investigation was restricted to only one of the two aluminum alloys, in order to reduce the computational time and effort. The 5754 "deep drawing" aluminum alloy was selected for the purposes of this numerical investigation, and as the basis for comparison with experimental measurements.

5.1 Predicted Dome Strain Distributions

Fringe plots of predicted major and minor principal strains within the 203.2 mm wide blank of the 5754 aluminum are shown in Figures 5.1 and 5.2, for punch depths of 16.0 mm and 30.2 mm, respectively. The 30.2 mm depth corresponds to the numerical state which is closest to the dome height at failure (30.7 mm) determined from

the LDH tests, while the 16.0 mm plot displays the strains within the sheet earlier in the deformation. These predictions were obtained using Barlat's non-quadratic yield criterion (Equation 1.25) with a Coulombic friction coefficient, μ , of 0.18. Figure 5.1 (a) clearly shows the strains induced by the formation of the lock bead, and the early development of the strains within the region of the sheet in contact with the punch. As deformation proceeds, the formation of a fairly symmetric localization near the punch-sheet tangency point (TP) is evidenced (Figure 5.2 (a)). The frictional constraint at the punch-sheet interface has driven the localization away from the pole of the punch and restricted further sheet deformation under the punch.

The minor strain distributions for these punch depths are shown in Figures 5.1 (b) and 5.2 (b). The numerical models, as described in Section 3.1.1, employed quarter symmetry, and thus only the strains on one side of the dome are compared to experimental measurements. These demonstrate that the majority of the lateral stretching in the sheet (tensile strains) occurs at the pole of the punch, the magnitude increasing as deformation continues. Due to the lateral constraint from the lock bead preventing further lateral stretching, these positive minor strains are seen to decrease with increasing distance from the pole. Negative minor strains (drawing), observed early in the flange of the sheet, can be attributed to closure of the clamp. The combined presence of major and minor negative strains in the flange at the axes of symmetry also indicates wrinkling, as occurred experimentally, again during clamp closure.

Figure 5.3 shows the trends in the major and minor strains versus distance from the pole, taken along the symmetry plane. The development of the localization is clearly shown, moving away from the pole of the punch as deformation increases, with minor strains uniformly increasing over the punch-sheet contact area.

The predicted thickness distributions, for both the 16.0 and 30.2 mm punch depths, are shown in Figure 5.4, while Figure 5.5 presents the predicted thickness along the symmetry plane. Although thickness measurements were not performed in the course of the experimental investigation, these distributions are included to demonstrate the capability of the numerical model in capturing thickness variations within the sheet as it deforms. Sheet thinning is predicted in regions of tensile strain and, at the localization,

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a sheet thickness of 0.723 mm is predicted compared to the original sheet thickness of 1.0 mm. In addition, the fringe plots indicate minor thickening in the flange at the edges of the symmetry planes, which is consistent with the wrinkling observed experimentally.

Fringe plots of predicted major and minor principal strains within the 127.0 mm wide blank of the 5754 aluminum are shown in Figure 5.6, for a punch depth of 25.4 mm. The onset of necking in the material is clearly shown by the distinct localization in the predicted major strains, again located very close to the punch-blank tangency point. The development of another localization was also observed within the region of the sheet adjacent to the lock bead and unsupported by punch contact, indicated by the arrow in Figure 5.6. This area of severe deformation is the site where hourglassing problems occurred in initial mesh designs. A slight tensile minor strain is predicted in the region of the neck, which suggests that a perfect plane strain condition is not provided by the 127.0 mm specimen geometry.

Fringe plots of predicted major and minor principal strains within the 25.4 mm wide notched blank of the 5754 aluminum are shown in Figure 5.7, for a punch depth of 23.2 mm. The localization in this geometry occurs very close to the pole of the dome. This location is consistent with the lower frictional constraints imposed by smaller punch-blank contact areas and agrees with the observed location of the neck in the experiments. The predicted localization initiates at the symmetry axis, and then grows outwards, towards the pole of the dome, consistent with the experiments. As a result of the lower frictional constraint for this geometry, material is permitted to draw-in over practically the entire dome region. This is indicated by the negative minor strains in Figure 5.7 (b).

Predicted thickness distributions for both the 127.0 mm and 25.4 mm wide blanks are plotted in Figures 5.8 (a) and (b), at punch depths of 25.4 mm and 23.2 mm, respectively. Minor thickening of the sheet is predicted at the outer edge of the 127.0 specimen, particularly in the severely deformed region at the lock bead. A sheet thickness of 0.826 mm was predicted at the localization for this geometry, while a sheet thickness of 0.833 mm was predicted for the 25.4 mm notched geometry.

It is noted that visual evidence of localizations were exhibited by the meshes in the 127.0 mm and 25.4 mm models at greater punch depths. These strain distributions were not presented as the punch depths exceeded those at which necking normally occurred during the experiments.

5.2 Predicted Pan Strain Distributions

Fringe plots of the predicted principal strains and thickness within the 5754 deep draw pan are shown in Figures 5.9, 5.10 and 5.11, respectively, for a pan depth of 60.0 mm. The predictions were obtained using a thickness-based contact formulation of Barlat's non-quadratic yield criterion (Equation 1.25) with a Coulombic friction coefficient, μ , of 0.052 between the tooling and the sheet, and a blankholder force of 68 kN. The major strains in the specimen are tensile and greatest near the bottom of the corner wall, as expected, and in the flange of the short wall. The flat bottom of the pan shows very little straining.

The minor strain predictions shown in Figure 5.10 more clearly demonstrate the variation in strain state within the pan. The minor strains are predominantly negative, with a maximum of -1.04 occurring in the flange near the center of the short wall, an area characterized by severe material draw-in. Significant drawing was also observed in the corner wall region, with minor strains exceeding -0.67. The change in minor strain, shifting from positive to negative in the center of the long wall, reflects the existence of a state of plane strain in that region, while the positive strains over the punch corner radius are indicative of stretching.

Thinning of the sheet in the stretch region surrounding the punch corner radius is predicted, as shown in Figure 5.11. Although necking did not occur in the experiments for this configuration, fracture in related experiments employing draw beads did occur at this location. Thickening of the sheet is predicted in the flange of the long wall and the induced compressive stresses lead to mild wrinkling, as was observed in the experiments. Considerable thickening of the blank is predicted in the flange of the short

wall, with an approximate thickness increase of 80%. This increase is sufficient to cause ironing in the short wall region, as occurred in the experiments.

Figure 5.12 shows the trends in the predicted major and minor strain variations for increasing punch depth. Plotted are the loci of major versus minor strain along the short wall and corner wall sections, as described in Section 4.2. Minor stretching is observed in the punch radius region, while significant drawing occurs in both the corner and short walls, with increasing magnitudes as punch depth increases. As in the experimental results, very little further deformation occurs between the 50 and 60 mm pan depths, as evidenced by their similar strain distributions in Figure 5.12 (a).

The effect of blankholder force variation on the numerical predictions was not examined as no appreciable differences were observed in the experimental results for the deep draw pans at both low and high blankholder forces.

5.3 Comparison Between Predictions of Numerical and Experimental Strain Distributions

This section compares the experimental results outlined in Chapter 4 to the numerical predictions. The constitutive yield criteria employed to predict the strain distributions are given by Equations 1.12, 1.20 and 1.25, that is, the von Mises, Hill's quadratic (1948) and Barlat's non-quadratic yield functions, respectively.

5.3.1 Dome Results

For the dome strain distributions, only the experimental measurements from one side of the dome were compared to the predicted results. This approach was deemed acceptable since the experimental strain distributions, such as in Figures 4.1 and 4.2, were quite symmetric. In order to show the developing strain state within the part, the

numerical results are presented at three punch depths, corresponding to dome heights at: (i) failure (measured LDH); (ii) at approximately 90-95% of LDH; and, (iii) at a smaller, pre-determined dome height (16.0 mm) occurring earlier in the deformation.

Figure 5.13 serves to compare the measured and predicted major and minor principal strain distributions within the 203.2 mm wide blank of the 5754 aluminum. The predicted curves are for punch depths of 16.0 mm, 28.9 mm and 30.2 mm, while the experimental measurements correspond to dome heights of 15.2 mm, 28.2 mm and 30.7 mm. The early strain distributions predicted by the three yield criteria are practically identical, varying by no more than 2% strain from each other, and from the experimental measurements, as shown in Figure 5.13 (a). As deformation proceeds, however, both the isotropic and Hill's quadratic anisotropic material models have the tendency to over predict the major and minor strains over the punch-sheet contact region, with significantly more deformation over the pole of the punch than indicated by the measured results. In addition, these two formulations predict fairly diffuse strain distributions, instead of the sharp strain peaks exhibited by the measured results, located much closer to the pole of the punch than observed experimentally.

The results using the Barlat formulation, a non-quadratic anisotropic yield criterion, are clearly superior and agree well with the measured results. This material model more accurately predicts the formation of a distinct strain localization within the sheet, as shown in Figures 5.13 (b) and (c), which is positioned slightly further away from the pole than the experimentally measured strain peak. The strains at the pole and within the blank-punch contact region are only slightly under predicted, contrary to significant over predictions obtained using the isotropic and quadratic formulations.

Similar agreement between predictions and experimental results is shown for the 127.0 mm wide blank in Figure 5.14. The general form of the strain distribution, as well as the development and position of a strain localization, are more accurately predicted using the Barlat non-quadratic formulation over that of the von Mises isotropic and Hill's quadratic criteria for each of the three punch depths. For this geometry, the predictions obtained from the von Mises and Hill's quadratic criteria demonstrate much better agreement with experiment than the predictions for the 203.2 mm geometry.

As shown in Figure 1.15, the use of Barlat's criterion did not provide a better agreement with experiment for the 25.4 mm wide notched blank, as it did for the 127.0 mm and 203.2 mm dome models. For this geometry, Hill's criterion was superior, accurately predicting the strain localization near the pole throughout the deformation while the Barlat criterion predicted a more diffuse strain distribution.

Effect of Planar Anisotropy

In order to study the effects of in-plane (planar) anisotropy on the predicted strains, major and minor strain distributions were measured along the rolling and transverse directions of a 203.2 mm wide blank of the 5754 aluminum, at a punch depth of 30.7 mm. The measurements, shown in Figure 5.16, indicate slightly larger strains along the sheet rolling direction. Although a detailed evaluation of μ with direction was not available, the small differences in the experimental results may be attributed to greater frictional restraint in the transverse direction as the strain peak was shifted away from the pole of the punch.

The experimental measurements were compared to numerical predictions obtained using the Barlat yield criterion, with Δr values of 0.0, 0.059 and 0.118. The first value corresponds to planar isotropy, while the second corresponds to planar anisotropy and is based on the measured r_0 , r_{45} and r_{90} values given in Table 2.3. The third value, 0.118, is double the actual Δr value and was obtained by scaling the in-plane values for r about the average $\bar{r}$ value. The almost indiscernible predicted curves in Figure 5.16 for these three calculations, and for both directions, indicate that Δr has only a very small effect on the strain distribution in biaxial stretching. The predictions accurately reflect the occurrence of greater major strains in the parallel direction, although the strain peak is shifted further away from the pole, and only slightly greater than in the transverse direction. The minor strains in both directions are very close to the measurements.

5.3.2 Pan Results

Figures 5.17, 5.18 and 5.19 compare the predictions obtained for the 5754 aluminum 30.0, 50.0 and 60.0 mm pan depths, respectively, to the experimental results. As in the dome predictions, only minor differences were observed between the strains predicted by the von Mises and Hill criteria, throughout the deformation. Although the Barlat criterion seems to more accurately predict the strain variations in the short wall, both the von Mises and Hill criteria better capture the strain variations and magnitudes within the corner wall. The strain distributions predicted for the 30.0 mm depth in Figure 5.17 exhibit the same trend, predicting smaller strains in the corner and short wall compared to the measured results. As deformation proceeds to 60.0 mm depth, the Barlat criterion is seen to over predict the minor strains in the corner wall, while under predicting the major strains in the same region.

Differences between the predicted and measured strains in the deep draw pan can be attributed to the occurrence of ironing, due to both wrinkling and an increase in sheet thickness during deformation to the point where the thickness exceeded the punch-die clearance. The drastically higher friction coefficient resulting from the onset of ironing and subsequent breakdown of the lubrication layer is an effect not accounted for in the friction model. The friction model is also inadequate in that the simulated lubrication conditions between the punch and the sheet did not exactly match the experimental conditions. Prior to the experiment, lubricant was not applied to the punch surface and the area of the sheet that would make contact with the bottom of the punch; only the surfaces in contact with the die and binder were lubricated. The numerical model treated the friction conditions between the sheet and all tooling surfaces with a single value of μ , corresponding to the lubricated condition. This treatment results in a simulated punch-sheet frictional coefficient that is too low (0.052) compared to the dry frictional coefficient (0.18), and would result in lower predicted strains.

The coarseness of the finite element mesh in certain areas, such as the flange, is another source of error in predicted strain. A coarse mesh will typically provide an

overly stiff response, and will not accurately capture steep deformation gradients. The coarseness of the silk-screened grid in these areas also contributed to differences between the predictions and the measurements; this is especially true in the short wall where only a few grids covered the entire region of interest.

From these results, and those for the domes discussed in the previous section, it is clear that the non-quadratic Barlat criterion did not provide significant advantages over Hill's quadratic criterion for draw strain state experiments. This result contrasts those obtained from related studies on the limiting drawing ratio of 5754 aluminum [138, 139] which indicated improved agreement between predictions and measurements with the use of Barlat's non-quadratic yield criterion over Hill's quadratic criterion under drawing conditions. Further refinement of the numerical models considered in the current work to improve the draw strain state predictions was considered beyond the computational resources available for this study. It is important to note that the level of agreement between the model predictions and measured strains is reasonably good, particularly given the complexity of the forming operation.

5.4 Parametric Study of Friction

A limited parametric study was conducted to determine the effects of the friction coefficient, μ , on the predicted strains within dome and deep draw pan specimens of the 5754 aluminum. Calculations were performed using Hill's 1948 quadratic criterion (Equation 1.20), with μ values of 0.052, 0.096, 0.18 and 0.25 for the domes, and μ values of 0.052 and 0.096 for the deep draw pans. The two μ values, 0.052 and 0.096, correspond to frictional coefficients measured in directions transverse and parallel, respectively, to the sheet rolling direction, for a lubricated condition. The μ values 0.18 and 0.25 correspond to frictional coefficients obtained for the un-lubricated condition. These latter values were only employed in the parametric study for the domes in order to reduce computational time and effort. While the parametric study only considers

Hill's 1948 criterion, it is expected that calculations performed using the non-quadratic Barlat criterion would show similar trends.

Predicted strain distributions for the 203.2 mm, 127.0 mm and 25.4 mm wide blanks are shown in Figures 5.20, 5.21 and 5.22, respectively, at punch depths corresponding to the LDH determined for each geometry. Experimentally measured strain distributions are also included in the figures for comparison. The deformation of the material in contact with the punch is suppressed by increasing frictional forces at the blank-punch interface. This effect is clearly shown in Figure 5.20 for the 203.2 mm wide blank, where the strains at the pole of the dome diminish with increasing values of μ . Increasing μ also tends to move the predicted location of the peak strain away from the pole, increasing its magnitude and thereby improving agreement with the experimental results.

Similar, though less severe, trends were observed for the 127.0 mm wide blank (Figure 5.21), indicating that the plane strain model is less sensitive to variations of the friction coefficient.

Increasing the frictional forces in the 25.4 mm wide notched blank lowers the predicted peak strain, as shown in Figure 5.22, thereby providing a more uniform distribution of the major strains. In addition, the frictional forces constraining the material from deforming at the pole move the strain localization away from the pole.

The predicted results for the deep draw pan are shown in Figure 5.23, at a punch depth of 60.0 mm and a blankholder force of 68 kN. Figure 5.23 shows that increasing μ from 0.052 to 0.096 increases the predicted strains within the pan, as expected, particularly in the short wall, where the blankholder force and frictional constraint are intensified by large thickness increases. It is interesting to note that the transverse friction coefficient, $\mu=0.052$, best fits the experimental results.

Chapter 6

Discussion and Conclusions

6.1 Discussion

The simulation of sheet metal forming processes with a commercial finite element code, such as LS-DYNA3D, is supported by the favourable comparisons between the measured and predicted strain distributions. A vast number of variables affect the forming of sheet metal, and its simulation; isolating these variables is important in understanding their individual importance. As such, the current investigation has demonstrated the sensitivity of these processes to the selection of material models and to the frictional boundary conditions.

Under biaxial stretching and plane strain conditions, the strain distributions predicted using the Barlat non-quadratic yield criterion demonstrated good agreement with measured distributions. The Barlat-based predictions were superior to those obtained using either the von Mises isotropic criterion or Hill's quadratic yield criterion, which is known to exhibit anomalous behaviour when applied to low- r materials such as aluminum [101]. The erroneous strain distributions predicted using Hill's 1948 criterion can be attributed to the lower stresses predicted for low r value materials under biaxial stretching conditions. As shown in Figure 1.17, as the r value decreases, the yield locus

draws in towards the origin, thus reducing stresses and the resulting contact force with the punch. This, in turn, allows more deformation over the punch and serves to diffuse the strain localization. This contrasts with the behaviour observed in the experiments, where elevated stress levels induced by the deformation entail higher contact and friction forces, resulting in a sharper localization of strain, located further from the pole.

Under deep drawing conditions, the strain distributions predicted using Hill's quadratic yield criterion, and even the von Mises criterion, were superior to those predicted using the Barlat criterion. This conflicts with the results predicted from current limiting draw ratio investigations at Carleton University [138, 139], where the Barlat non-quadratic yield criterion was found to provide much better agreement with experiment in drawing than Hill's quadratic criterion. It is unclear why the Barlat criterion does not provide a similar improvement over Hill's quadratic criterion in this investigation under drawing conditions. The poor agreement between the Barlat predictions and measurements may be due to problems with the experiments. The rectangular pan test suffered ironing and some misalignment, and is a generally complicated geometry with considerable variation in strain state within the formed part. The LDR geometry, considered in [138, 139], is essentially a right-cylindrical cup and is a much simpler, "cleaner" test to perform. Thus, more work is required to resolve these issues in relation to the problems identified in this research under drawing conditions.

Experimentally determined frictional coefficients were utilized in the current numerical models rather than varying μ to "fit" the predicted strain distributions to the experimental results, a common practice in sheet metal forming analyses [27, 74, 99]. Unfortunately, the uncertainty in the measured friction coefficient for the dry or unlubricated condition complicates comparison of the predicted results to the measurements. The source of this uncertainty lies in the actual friction measurement apparatus, a draw bead simulator (see Figure 1.10). In this friction test, the material is pulled through a draw bead which, for dry conditions, is too severe and damages the surface of the material making the measurement of μ inaccurate. In the current formability experiments, no draw beads were present; thus measured friction coefficients based on

flat or mildly curved tooling surfaces in contact with the workpiece would be desirable in future.

The predicted strain distributions were found to be very sensitive to variations in the frictional boundary conditions, as demonstrated by the parametric study on the effects of μ , with higher frictional coefficients constraining the material deformation in the punch-blank contact region. Although μ evolves with deformation and varies with direction in the plane of the sheet, the friction model employed in the current investigation assumes a constant and uniform frictional coefficient between the tooling and the sheet. In the deep draw simulation, a form of "directional" frictional coefficient would need to be implemented to properly simulate the frictional boundary condition that exists between the sheet and the tooling.

Although the Barlat non-quadratic criterion provides better results in comparison to experiments, it is to be noted that this criterion is computationally more expensive than either the von Mises or Hill criteria, requiring approximately twice the amount of time for completion of numerical runs. An accurate simulation of deep drawing also requires the adoption of a thickness-based contact algorithm, instead of the simpler membrane contact algorithm, with a resulting further increase in computational run time.

6.2 Conclusions

The conclusions that can be drawn from the current investigation are as follows:

1. FEM simulations of sheet metal forming using explicit finite element codes have provided reasonably accurate strain predictions in comparison to experiments. Considerable improvements can be obtained through the use of advanced constitutive models.
2. Anomalous behaviour is observed when Hill's quadratic yield criterion is applied to materials having low values of r , such as aluminum.
3. In biaxial stretching and plane strain conditions, Barlat's non-quadratic yield criterion showed superior agreement with experiments than Hill's quadratic criterion.

4. Uncertainty exists as to which constitutive model is better under drawing conditions, particularly in light of current limiting drawing ratio studies ongoing at Carleton University which have shown the Barlat criterion to be superior.
5. The frictional coefficient significantly affects the predicted strain distributions, with higher values of μ limiting the material deformation in the blank-punch contact region.

6.3 Future Work

The current investigation builds upon previous studies undertaken at Carleton University into the application of finite element analysis to the simulation of sheet metal forming processes. An attempt has been made in this investigation to accurately distinguish the behaviour of various constitutive material models when applied to materials having low values of r .

Future work should investigate the effects of materials exhibiting significant in-plane anisotropy, characterized by large Δr values, on strain states other than biaxial stretching. For these analyses, an average value of the anisotropic parameter, $\bar{r}$, could not be employed as an approximation, as has been the common practice in industry. Although both the non-quadratic Hill and the full Barlat non-quadratic constitutive models have the capability to implement planar anisotropy, published results concerning experimental investigations of this behaviour are quite scarce. Further validation studies using these models are required.

Future work should consider enhancement of the friction models to better reflect the actual lubrication conditions, in particular for the deep draw simulation. A more accurate friction model would need to consider variations in μ within the plane of the sheet, as experimental tests have shown that friction coefficients measured in the longitudinal and transverse directions are quite different. An improved friction model should also consider the evolution of μ with deformation, and be capable of accurately simulating ironing effects on friction.

The current study focused on two commercial aluminum alloys. As such, values of strain hardening, n , were fixed. In addition, the effects of strain rate hardening, m , were not considered but can be important during localization. Future work is needed to characterize the effects of these material properties on the predictions obtained from the material models investigated. This work would require, however, significant experimental testing for the purposes of validation.

To better assess the formability of aluminum in deep drawing conditions, modifications of the rectangular pan tooling would be desirable in order to produce fractured specimens. The punch and die radii would need to be optimized for the forming of aluminum, and the tooling clearances would need to be increased to minimize the effects of ironing. The incorporation of draw beads could be beneficial in promoting fractured specimens, at the cost of a shallower pan.

Finally, future work should also consider optimizing and improving the finite element mesh used to model the deep draw blanks to better capture deformation gradients, especially within the short wall and in the flange where wrinkles appear.

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Table 2.1: Uniaxial material properties of the sheet metals [154].

Property	AA6111-T4	AA5754-O	AA5182-O
Thickness (mm)	0.9	1.0	1.0
Yield Strength (MPa)	135-170	90-110	110-130
UTS (MPa)	270-290	215-230	255-290
Anisotropy Parameter, $\bar{r}$	0.6-0.8	0.7-0.9	0.6-0.7
Uniform Elongation (%)	20-24	18-24	-
Total Elongation (%)	23-27	21-27	28-31
Strain Hardening Exponent, n	0.23-0.27	0.28-0.31	0.25-0.28
Minimum Radius/Thickness	0.4-0.6	0-0.3	0.2-0.5

Table 2.2: Chemical composition of aluminum alloys [155].

Alloy	Chemical Composition (weight %)						
	Mg	Si	Cu	Fe	Mn	Cr	Ti
5754	2.6-3.6	≤0.4	≤0.1	≤0.4	≤0.5	≤0.3	≤0.1
6111	0.5-1.0	0.6-1.1	0.5-0.9	≤0.4	≤0.4	≤0.1	≤0.1

Table 2.3: Uniaxial tensile test data.

Spec.	Yield Stress (0.2%) (MPa)	Ultimate Tensile Stress (MPa)	n Value (4%)	$\bar{r}$ Value (15%)	Unif. Elong. (%)	Elong. (%)
6111-0	145.2	287.8	0.2586	0.6609	22.61	23.04
6111-45	139.2	281.6	0.2584	0.7218	22.82	26.63
6111-90	140.5	284.3	0.2595	0.8414	22.39	26.68
5754-0	100.5	229.6	0.3034	0.7062	20.39	21.19
5754-45	104.7	224.1	0.2795	0.7301	23.21	24.89
5754-90	100.5	225.6	0.2988	0.8719	21.89	23.98

Table 2.4: Summary of average tensile properties.

Spec.	Sheet Thickness (mm)	Yield Stress (0.2%) (MPa)	Ultimate Tensile Stress (MPa)	n Value (4%)	$\bar{r}$ Value (15%) *	Δr Value (15%) **
6111	0.932	141.6	284.6	0.2588	0.7365	0.0294
5754	1.0	101.9	225.6	0.2939	0.7596	0.0590

Uniform Elongation (%)	Elongation (%)	Modulus of Elasticity (GPa)
22.61	25.45	70.5
21.83	23.35	70.0

$$* \bar{r} = (r_0 + 2r_{45} + r_{90})/4$$

$$** \Delta r = (r_0 + r_{90} - 2r_{45})/2$$

Table 2.5: Hemispherical punch test dome heights.

Material	Blank Width (mm)	Specimen Dome Heights (mm)			
		Interrupted			Necked
5754	25.4	-	15.8	21.0	22.5
	127.0	-	16.3	23.9	26.2
	203.2	15.2	21.8	28.2	30.7
6111	127.0	-	15.2	22.6	28.3
	203.2	15.2	22.0	26.0	30.8

Table 2.6: Experimental data for deep draw pan test specimens.

Specimen	Material	Pan Depth (mm)	Blankholder Force (kN)	Punch Speed (mm/s)
P5	5754	60	68	1.28
P8	5754	50	68	1.67
P12	5754	30	68	-
P9	5754	60	110	1.92
P10	5754	60	139	1.45
P7	6111	60	68	1.18
P6	6111	50	68	1.3
P13	6111	30	68	-

Table 3.1: Mesh sizes used in the Limiting Dome Height simulations.

Blank Geometry	Rigid Shell Elements	B-L-T Shell Elements	Total
25.4 mm *	3312	1584	4896
127.0 mm	3312	2784	6096
203.2 mm	3312	3072	6384
Rigid tooling consists of: punch - 432 elements die - 1440 elements binder - 1440 elements			

* notched specimen

Table 3.2: Mesh sizes used in the Universal Forming Test simulations.

Blank or Tooling	Rigid Shell Elements	B-L-T Shell Elements
Punch	2042	-
Blankholder	952	-
Die	1586	-
Blank	-	8032
Total: 12612 elements		

Table 3.3: Material property data used in the numerical material models.

Material	Sheet Thickness (mm)	Density (g/mm³) x10⁻³	Modulus of Elasticity (GPa)	$\bar{r}$ Value	Poisson's Ratio ν
6111	0.932	2.643	70.0	0.7365	0.33*
5754	1.0	2.643	70.5	0.7596	0.33*

* Handbook values [170]

Figure 1.1: Vehicle weight versus fuel consumption [1].

Figure 1.2: Formability in the die face engineering cycle [5].

Figure 1.3: Schematic of the Swift test [38].

Figure 1.4: Schematic of the Erichsen test [35].

Figure 1.5: Schematic of the Fukui test [44].

Figure 1.6: Schematic of the Hecker and Ghosh punch test showing the punch and drawbead (due to Hecker [39]).

Figure 1.7: Loci of limit strains for a simple sheet forming process [52].

Figure 1.8: The forming limit diagram in relation to the representation of various deformation modes in strain space (due to Karima *et al.* [53]).

Figure 1.9: Failure strains predicted at the fracture point (Goodwin curve) compared to those predicted adjacent to the fracture (Keeler curve) [56].

Figure 1.10: Draw bead simulator schematic: assesses the frictional characteristics of a material by drawing a sheet through draw beads [75].

Flange wrinkling

Wall wrinkling

wrinkling

- stress-ratio	- tool-geometry
- moment of resistance	- blank-geometry
	- sheet metal
	- friction-coefficient
	- blankholder pressure

Figure 1.11: Factors that influence wrinkling during deep drawing [71].

Figure 1.12: Some defects that occur in cup deep drawing [77].

(a)

(b)

Figure 1.13: Draw-in and punch depth variations versus sample width for clamping forces of (a) 125 kN and (b) 250 kN [82].

Figure 1.14: Material draw-in variation versus clamping force, shows the minimum clamping force required for a maximum draw-in of 0.25 mm [82].

Figure 1.15: Rectangular deep drawn box with solid corners showing the formation of ears at the corners [79].

Figure 1.16: Schematic showing (a) tensile specimen orientations for the determination of $\bar{r}$ and Δr (b) the axes used to define normal and planar anisotropy [42].

Figure 1.17: Principal stress yield locus showing the predicted effects of r for planar isotropy ($\Delta r=0$), where a is the stress ratio of common loading states [85].

Figure 1.18: Hemispherical dome test strain distribution showing the effect of anisotropy, r [99].

Figure 1.19: Experimentally determined variation of r versus orientation with respect to the rolling direction for four unspecified steels [100].

Figure 1.20: Comparisons of experimental and analytically predicted (Hill's quadratic yield criterion) work hardening characteristics for a steel having $\bar{r}=1.48$, (1-3) simple tension experimental curves at 0° , 45° and 90° to the rolling direction, (4) experimental biaxial test, (5) $\bar{r}=1.48$ theoretical curve, (6) theoretical curve based on 90° tensile curve and the assumption of loading along the anisotropic axes and (7) similar to (6), but based on the 0° curve [100].

Figure 1.21: Uniaxial and biaxial stress-strain curves for annealed copper-containing steel ($r=1.62$) [85].

Figure 1.22: Uniaxial and biaxial stress-strain curves for commercial-purity aluminum in the annealed ($\bar{r}=0.68$) and hard rolled ($\bar{r}=0.43$) conditions [85].

Figure 1.23: Predictions of the effect of r on the FLD based on the Marciniak and Kuczynski theory [95].

Figure 1.24: Sowerby and Duncan qualitative non-mathematical interpretation of the Marciniak and Kuczynski predictions, showing the predicted effect of r on the yield locus (a) isotropic material ($r=1.0$) (b) anisotropic material ($r \geq 1.0$) [56].

Figure 1.25: Yield loci based on Hill's non-quadratic yield criterion showing the effect of the exponent m [95].

Figure 1.26: Comparison of experimental uniaxial (average) and biaxial tension stress-strain curves to predictions using Hill's non-quadratic yield criterion for commercially-pure as-received ($\bar{r}=0.39$, $m=1.3$) and annealed ($\bar{r}=0.731$, $m=1.7$) aluminum. Hill's 1948 quadratic yield criterion is indicated by the $m=2$ predicted curve [127].

Figure 1.27: Predictions of the effect of the exponent, m , in Hill's non-quadratic yield criterion on the FLD, based on the Marciniak and Kuczynski theory [95].

Figure 1.28: Predictions of the plane stress yield loci for 2008-T4 aluminum using von Mises, Hill's 1948 and Barlat yield criteria, based on uniaxial tension tests [129].

(a)

(b)

Figure 1.29: Normalized plane stress yield loci for 2008-T4 aluminum, shown intersecting planes of constant normalized shear stress (a) based on uniaxial tension data (b) based on uniaxial compression data [129].

Figure 2.1: Experimental true stress-strain curves for the 6111 aluminum.

Figure 2.2: Experimental true stress-strain curves for the 5754 aluminum.

Figure 2.3: Notched blank geometry for the 25.4 mm specimen.

Figure 2.4: Necked grid of a 127.0 mm 5754 specimen showing the tenacity of the ink grids.

Figure 2.5: Schematic of the limiting dome height press facility.

Figure 2.6: Limiting dome height press facility at Carleton University.

Figure 2.7: A 5754 25.4 mm notched specimen, showing the visual detection of a neck.

Figure 2.8: Example of a series of interrupted dome test specimens for 127.0 mm blanks of the 5754 alloy. From left to right, the depths are 26.2, 23.9, and 16.3 mm.

Figure 2.9: Schematic of the deformed blank showing the pole of the dome, the blank to punch tangency point (TP) and the coordinate frame for dome height (b) and strain measurement position (a) [due to Williams, 157].

Figure 2.10: CANMET's Universal Forming Test facility - tooling geometry.
(Dimensions given in inches.)

Figure 2.11: Schematic diagram of the Universal Forming Test facility at CANMET.

Figure 2.12: A typical deep drawn pan - 5754 alloy, 60 mm depth.

Figure 2.13: Geometry of the blank used in the Universal Forming Test, showing the removal of the corners.

Figure 2.14: Failed 6111 specimens using 0.5 mm Teflon sheets, full size blank and blank with corners removed.

Figure 2.15: Full-size deformed 5754 specimen tested at high pressure, showing the exhaustion of the flange material at the short wall.

Figure 3.1: Exploded view of the 203.2 mm wide blank LDH numerical model.

Figure 3.2: Exploded view of the rectangular pan deep draw numerical model.

Figure 3.3: FEM mesh plot of the 203.2 mm wide blank LDH model (3072 elements).

Figure 3.4: FEM mesh plot of the 127.0 mm wide blank LDH model (2784 elements).

Figure 3.5: FEM mesh plot of the 25.4 mm wide blank (notched) LDH model (1584 elements).

Figure 3.6: FEM mesh plot of the rectangular deep draw pan blank (8032 elements).

Figure 3.7: Quadrilateral shell element displacement modes (a-c) and hourglass mode (d) [167].

Figure 3.8: Plot of the deformed 127.0 mm blank showing the severe deformation area due to the lock bead and the unsupported region of the sheet where oscillations due to hourglassing occurred.

Figure 3.9: Typical FLD curves for 6111-T4 and 5754-O aluminum [154].

Figure 3.10: Effective stress versus effective plastic strain curves employed in the numerical material models.

Figure 3.11: Schematics of the numerical models, showing the initial component positions and the punch movement direction: (a) LDH model (b) Deep draw rectangular pan model.

Figure 3.12: Load curves used in the LDH numerical simulations.

Figure 3.13: Load curves used in the deep draw rectangular pan simulations.

Figure 4.1: Measured major and minor principal strains for the 203.2 mm wide blank size of 5754 aluminum.

Figure 4.2: Measured major and minor principal strains for the 127.0 mm wide blank size of 5754 aluminum.

Figure 4.3: Measured major and minor principal strains for the 25.4 mm wide blank size of 5754 aluminum.

Figure 4.4: Measured major and minor principal strains for the 203.2 mm wide blank size of 6111 aluminum.

Figure 4.5: Measured major and minor principal strains for the 127.0 mm wide blank size of 6111 aluminum.

Figure 4.6: Comparison of experimentally measured forming limit data for the 5754 aluminum with the FLD curve provided by ALCAN [154].

Figure 4.7: Comparison of experimentally measured forming limit data for the 6111 aluminum with the FLD curve provided by ALCAN [154].

Figure 4.8: Strain path locations for the measurement of strains.

Figure 4.9: Deformed 5754 aluminum specimens, at punch depths of 30, 50, and 60 mm.

Figure 4.10: Close-ups of the deformation within the short wall, showing the excessive material draw-in and ironing at the bottom of the wall. (5754 aluminum specimens, at punch depths of 30, 50 and 60 mm.)

Figure 4.11: Deformed 5754 aluminum specimens, showing wrinkling in the flange, and misalignment of the press. (Punch depths of 30, 50 and 60 mm.)

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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Figure 4.12: Comparison of measured strain distributions for the 5754 aluminum deep draw specimens, with a blankholder force of 68 kN: (a) Corner wall (b) Short wall.

Figure 4.12: Comparison of measured strain distributions for the 5754 aluminum deep draw specimens, with a blankholder force of 68 kN: (c) Long wall.

Figure 4.13: Comparison of measured strain distributions for the 6111 aluminum deep draw specimens, with a blankholder force of 68 kN: (a) Corner wall (b) Short wall.

Figure 4.13: Comparison of measured strain distributions for the 6111 aluminum deep draw specimens, with a blankholder force of 68 kN: (c) Long wall.

Figure 4.14: Comparison of the effects of increased blankholder force on the strain distributions measured within the (a) corner wall and (b) short wall of 5754 aluminum specimens.

Figure 4.14: Comparison of the effects of increased blankholder force on the strain distributions measured within the (c) long wall of 5754 aluminum specimens.

Figure 4.15: 5754 aluminum specimens drawn at low (68 kN), medium (110 kN), and high (139 kN) blankholder forces.

Figure 5.1: Fringe plots of predicted (a) major and (b) minor principal strains for the 203.2 mm wide blank of 5754 aluminum at a punch depth of 16.0 mm.

Figure 5.2: Fringe plots of predicted (a) major and (b) minor principal strains for the 203.2 mm wide blank of 5754 aluminum at a punch depth of 30.2 mm.

Figure 5.3: Predicted major and minor principal strains for the 203.2 mm wide blank of 5754 aluminum.

Figure 5.4: Fringe plots of predicted thickness distribution for the 203.2 mm wide blank of the 5754 aluminum at punch depths of (a) 16.0 and (b) 30.2 mm.

Figure 5.5: Plot of predicted thickness distribution versus position relative to the pole of the dome for the 203.2 mm wide blank of the 5754 aluminum.

Figure 5.6: Fringe plots of predicted (a) major and (b) minor principal strains for the 127.0 mm wide blank of 5754 aluminum at a punch depth of 25.4 mm.

Figure 5.7: Fringe plots of predicted (a) major and (b) minor principal strains for the 25.4 mm wide blank of 5754 aluminum at a punch depth of 23.2 mm.

Figure 5.8: Fringe plot of predicted thickness distribution for 5754 aluminum blanks (a) 127.0 mm wide blank at a punch depth of 25.4 mm and (b) 25.4 mm wide notched blank at a punch depth of 23.2 mm.

Figure 5.9: Fringe plot of predicted major principal strain for the 60.0 mm deep draw pan of the 5754 aluminum, with a blankholder force of 68 kN.

Figure 5.10: Fringe plot of predicted minor principal strain for the 60.0 mm deep draw pan of the 5754 aluminum, with a blankholder force of 68 kN.

Figure 5.11: Fringe plot of predicted thickness distribution for the 60.0 mm deep draw pan of the 5754 aluminum, with a blankholder force of 68 kN.

Figure 5.12: Plot of predicted variations in major and minor principal strains within the 5754 aluminum deep draw pan (a) corner wall and (b) short wall.

Figure 5.13: Comparison of the measured and predicted strain distributions within a 203.2 mm wide 5754 blank for punch depths of (a) 16.0 mm, (b) 28.9 mm and (c) 30.2 mm according to the von Mises, Hill's quadratic and Barlat's non-quadratic yield criteria. (Experimental depths: 15.2, 28.2 and 30.7 mm).

Figure 5.14: Comparison of the measured and predicted strain distributions within a 127.0 mm wide 5754 blank for punch depths of (a) 16.0 mm, (b) 23.2 mm and (c) 25.4 mm according to the von Mises, Hill's quadratic and Barlat's non-quadratic yield criteria. (Experimental depths: 16.3, 23.9 and 26.2 mm).

Figure 5.15: Comparison of the measured and predicted strain distributions within a 25.4 mm wide notched 5754 blank for punch depths of (a) 16.0 mm, (b) 20.9 mm and (c) 23.2 mm according to the von Mises, Hill's quadratic and Barlat's non-quadratic yield criteria. (Experimental depths: 15.8, 21.0 and 22.5 mm).

Figure 5.16: Comparison of the measured and predicted principal strains versus position relative to the pole of the dome for a 203.2 mm wide 5754 blank showing the effects of planar anisotropy ($\Delta r \neq 0$) on the predicted strain distribution in directions (a) parallel to the rolling direction and (b) transverse to the rolling direction.

Figure 5.17: Comparison of the measured and predicted strain distributions within a 5754 deep draw pan according to the von Mises, Hill's quadratic and Barlat's non-quadratic yield criteria for a punch depth of 30.0 mm: (a) Corner wall (b) Short Wall.

Figure 5.18: Comparison of the measured and predicted strain distributions within a 5754 deep draw pan according to the von Mises, Hill's quadratic and Bariat's non-quadratic yield criteria for a punch depth of 50.0 mm: (a) Corner wall (b) Short Wall.

Figure 5.19: Comparison of the measured and predicted strain distributions within a 5754 deep draw pan according to the von Mises, Hill's quadratic and Bariat's non-quadratic yield criteria for a punch depth of 60.0 mm: (a) Corner wall (b) Short Wall.

Figure 5.20: Predicted effect of variations in μ for the 203.2 mm wide blank of the 5754 aluminum for a punch depth of 30.2 mm.

Figure 5.21: Predicted effect of variations in μ for the 127.0 mm wide blank of the 5754 aluminum for a punch depth of 25.4 mm.

Figure 5.22: Predicted effect of variations in μ for the 25.4 mm wide notched blank of the 5754 aluminum for a punch depth of 23.2 mm.

Figure 5.23: Predicted effect of friction coefficients measured parallel and transverse to the sheet rolling direction for strains within the 60.0 mm deep draw pan of the 5754 aluminum (a) corner wall and (b) short wall.

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