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SYNTHETIC DOPPLER FOR PRECISE INDOOR GEOLOCATION

by

Adrian Taylor

A Thesis Submitted to the School of Graduate Studies in Partial

Fulfillment of the Requirements for the Degree

Master of Applied Science

in Electrical Engineering

Carleton University

Department of Electronics

Faculty of Engineering

Carleton University

April 2005

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Abstract

The potential applications for accurate in-building location sensing are numerous. Hospitals and warehouses could track assets, or emergency crews could be tracked as their operatives fulfil their missions. These varied applications have different requirements and challenges. This thesis investigates a novel signal transmission method for location estimation, called Synthetic Doppler. The synthetic Doppler signals provide information from which the Angle Of Transmission (AOT) and Time of Arrival (TOA) from the transmitter may be determined. With multiple beacons, a receiver can determine its position using each beacon's AOT the TOA. This research includes an analysis of the synthetic Doppler system, and estimation techniques suited to solving the positioning problem. System performance is evaluated by simulating the indoor channel, and comparing methods of mitigating multipath and other errors. The simulation results show the limitations and effectiveness of combining multiple types of location metrics as afforded by synthetic Doppler.

Acknowledgments

I would like to thank Dr. Jim Wight, who's advice and suggestions were invaluable in completing this work. I would also like to thank Dr. Nicole Brousseau and Geoff Colman, who along with Dr. Wight first developed the idea for this research. Their innumerable suggestions and ideas were very helpful over the course of the work.

I owe a great debt to Defence R&D Canada – Ottawa, for supporting this research. I would like to thank my many colleagues at DRDC for their suggestions and feedback. In particular I am indebted to Andrew Mudry, both for his support for this thesis and his many and varied motivational techniques.

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List of Acronyms

Acronym	Description
AOA	Angle Of Arrival
AOT	Angle Of Transmission
AWGN	Additive White Gaussian Noise
CEP	Circular Error Probable
CRLB	Cramér-Rao Lower Bound
dB	Decibel
DLL	Delay Lock Loop
DSSS	Direct Sequence Spread Spectrum
FM	Frequency Modulation
GDOP	Geometric Dilution Of Precision
GPS	Global Positioning System
LOS	Line Of Site

MSE	Mean Squared Error
NLOS	Non Line Of Site
PSD	Power Spectral Density
RMS	Root Mean Square
RMSE	Root Mean Square Error
RWGH	Residual Weighting Algorithm
RSS	Received Signal Strength
SNR	Signal to Noise Ratio
TDOA	Time Difference of Arrival
TOA	Time Of Arrival
WLAN	Wireless Local Area Network
WGN	White Gaussian Noise

List of Symbols

Symbol	Description
π	constant: 3.141592654 ...
c	speed of light: $3E8$
R_i	range from target to i^{th} beacon
$R_{i,j}$	range difference to user between i^{th} and j^{th} beacon
(X_i, Y_i)	coordinates of i^{th} beacons
(x, y)	coordinates of user
t_i	TOA from i^{th} beacon
θ_i	azimuth bearing from station i to user
ν_t	tangential velocity of rotating antenna
r	radius of rotation for synthetic Doppler antenna
ω	radial velocity of rotating antenna
ν	velocity of antenna in direction of receiver

Δf	instantaneous frequency shift of synthetic Doppler signal
f_c	carrier frequency of synthetic Doppler signal
λ	wavelength of synthetic Doppler carrier
Ω	instantaneous angle of rotating antenna
π	constant: 3.141592654 ...
A_z	azimuth from beacon to user, referenced to north
A_m	FM modulating signal amplitude
ω_m	FM modulating signal sinusoidal frequency
$x(t)$	FM modulating signal
$\phi(t)$	Instantaneous FM modulating signal phase
k_f	FM frequency deviation constant
A_c	Carrier signal magnitude
β	FM modulation index
f_m	frequency of antenna rotation
B	bandwidth
$s(t)$	transmitted signal
$c(t)$	direct sequence spreading code waveform
T_c	spreading symbol period
$p(t)$	spreading symbol pulse shape

$r(t)$	received signal
τ_d	transmission delay
θ_r	unknown received signal phase
$y(t)$	despread signal
$y'(t)$	demodulated FM signal
$x_1(t)$	FM phase detector pulse signal
$x_2(t)$	spreading code phase detector pulse signal
$\delta(t)$	Dirac delta function
$\hat{\tau}_d$	transmission delay estimate
ϵ_t	transmission delay estimate error
N_0	noise power spectral density
P	signal power
σ_ϵ	code tracking error standard deviation
γ	signal to noise ratio
B_t	receiver input noise bandwidth
σ_t	TOA measurement variance
E	signal energy
N_c	number of chips
$n(t)$	receiver input noise process

$n'(t)$	spread noise process
$S_N(\omega)$	PSD of a spread noise process
$n''(t)$	despread, FM-demodulated noise
$G_n(f)$	PSD of recovered FM modulating signal
$G_{n_p}(f)$	PSD of noise term in phase-demodulated signal
$G_{n_s}(f)$	quadrature PSD of noise at FM demodulation output
f_x	maximum frequency deviation of FM modulating signal
k_d	FM discriminator sensitivity
$E[X]$	expected value of random variable X
$n_s(t)$	quadrature noise process
$n_p(t)$	noise process at output of phase demodulator
$n_o(t)$	demodulation output noise
t_c	TOA based on spreading code tracking
n_t	TOA error
$\hat{t}_c$	TOA estimate
t_d	Doppler phase zero-crossing time
n_d	Doppler phase zero-crossing time error
$\hat{t}_d$	Doppler phase zero-crossing time estimate
n_{A_z}	AOT estimate error

$\hat{A}_z$	AOT estimate
σ_{A_z}	standard deviation of AOT error
σ_{n_d}	standard deviation of Doppler phase error
σ_{n_c}	standard deviation of code phase error
N_s	number of stations (beacons)
N_m	number of measurement types
$\epsilon_{i,k}$	error associated with each measurement
(x_i, y_i)	coordinates of station i
$m_{i,k}$	a measurement of type k from station i useful for positioning
$f_{i,k}(x, y, x_i, y_i)$	a function relating the user position to station i based on a measurement $m_{i,k}$
$n_{i,k}$	noise associated with measurement of type k from station i
$\mathbf{x}$	vector of beacon and user positions
$\mathbf{m}$	vector of measurements
$\mathbf{f}(\mathbf{x})$	vector of measurement functions
$\mathbf{n}$	vector of measurement noise terms
$\mathbf{N}$	covariance matrix of noise vector $\mathbf{n}$

$\hat{\mathbf{x}}$	maximum likelihood estimate of user position
$Q(\mathbf{x})$	least-squares estimation cost function
$\hat{\mathbf{x}}$	user position estimate
$\mathbf{x}_0$	user position initial guess
$G(\mathbf{x})$	gradient matrix of $\mathbf{f}(\mathbf{x})$
δ	increment of solution in nonlinear least-squares estimation
(x_ν, y_ν)	intermediate user position estimate
r_i, r_j	range measurement from user to beacon i and j , respectively
t_0	time of transmission (common to all beacons)
$\hat{t}_i$	TOA measurement for beacon i
$n_{t,i}$	error sample associated with TOA measurement i
$R_i(x, y)$	range from user to beacon i
$\mathbf{m}_t$	vector of TOA measurements
$\mathbf{t}_0$	vector of transmission times
$\mathbf{f}_t(\mathbf{x})$	vector of time-position equations
$\mathbf{n}_t$	vector of time measurement errors
τ_i	TDOA between stations i and $i + 1$

$\hat{\tau}_i$	noise-corrupted estimate of τ_i
$n_{\tau,i}$	error associated with τ_i
$\mathbf{m}_{\tau}$	vector of TDOA measurements
$\mathbf{f}_{\tau}(\mathbf{x})$	vector of TDOA equations
$\mathbf{n}_{\tau}$	vector of TDOA noise values
$\mathbf{H}$	transform matrix to convert TOA matrices to TDOA matrices
$\mathbf{G}_{\tau}$	gradient matrix of TDOA equations
$\mathbf{N}_{\tau}$	TDOA error covariance matrix
$\sigma_{\tau,i}$	standard deviation of TDOA measurement i
$\text{COV}(x, y)$	covariance of random variables x and y
θ_i	angle of arrival from station i
$n_{\theta,i}$	error associated with angle of arrival measurement i
$\boldsymbol{\theta}$	vector of AOT measurements
$\mathbf{f}_{\theta}$	vector of AOA-position equations
$\mathbf{n}_{\theta}$	vector of AOA measurement noise
$\mathbf{G}_{\theta}$	gradient matrix of $\mathbf{f}_{\theta}$
$\mathbf{N}_{\theta}$	covariance matrix of AOA noise variables
$\sigma_{\theta,i}$	standard deviation of AOA noise variable i

$f(\mathbf{x}, \boldsymbol{\theta})$	alternate formulation of angle-position functions
θ_e	noise associated with i^{th} AOA
$\mathbf{n}_{\theta_i}$	vector of AOA noise terms
$\mathbf{G}_{\mathbf{x}, \boldsymbol{\theta}}$	gradient matrix of $f(\mathbf{x}, \boldsymbol{\theta})$
$\mathbf{N}_{\tau, \boldsymbol{\theta}}$	covariance matrix of TDOA and AOA noise variables
σ_x	standard deviation of x -component of position estimate
σ_y	standard deviation of y -component of position estimate
σ_{xy}	covariance of x and y components of position estimate
$\mathbf{P}$	position estimate covariance matrix
σ_d	mean-square ranging error
S	set of stations
R_{es}	Residual of position estimate $\hat{\mathbf{x}}$
$\widetilde{R}_{es}$	normalized residual
$\hat{\mathbf{x}}_k$	position estimate from station set k
$R_{\text{total}, i}$	normalized mean of all residuals associated with station i
R_{total}	Overall normalized mean of residuals
S_k	subset of stations k
Ψ	angle of incidence

ϵ	complex permittivity of material
P	power
T	coefficient of transmission
R	coefficient of reflection
$P_r(d)$	function of received power at distance d
P_t	transmitted power
G_t	transmitter antenna gain
G_r	receiver antenna gain
d	distance from transmitter to receiver
L	system loss factor
PL	path loss
D	largest linear dimension of antenna
d_f	far field distance of antenna

Chapter 1

Introduction

Radio-based positioning services have become an important research area in recent years. The phenomenal utility of the Global Positioning System (GPS) has demonstrated the power of positioning for hundreds (if not thousands!) of applications. However GPS is not the solution for every problem; it has the distinct disadvantage of not working inside buildings.

In-building radio location sensing is a difficult problem, but the potential applications are numerous. Hospitals could track portable assets, from wheelchairs to expensive equipment. Prisons and care-giving institutions could track their wards, and warehouses could track their inventory. Emergency crews (fire fighters, police, military) could be tracked as their agents fulfil their missions. These varied applications all require a solution for inside buildings, but have different requirements and challenges.

This chapter includes a review the current state of the art in indoor positioning. A novel improvement for dealing with indoor location is presented, followed by an outline of this investigation of its utility.

1.1 Indoor Location Systems

Indoor location systems are chiefly characterized by the indoor propagation environment. The man-made structure is no friend to radio propagation – the indoor radio channel is dominated by high attenuation and countless sources of multipath. The varied approaches to overcoming these difficulties have a lot to do with the constraints of the problem they address. Indoor positioning systems may be roughly categorized into two camps: systems for asset tracking in a controlled environment, and systems for people tracking in ad hoc environments.

1.1.1 Asset Tracking Location Systems

Asset tracking systems are characterized by a long planning stage and a customized site installation (the term asset is used loosely in this context – persons can be assets too). The equipment assigned to assets to be tracked is usually designed with economy as the chief concern. Accuracy requirements for these systems tend to be low – finding the particular room where the asset is located is often sufficient. Propagation issues are mitigated by embedding

sensors throughout the area of interest. Due to the large market for inventory tracking, commercial off-the-shelf indoor location systems are usually of this sort.

Prasithsangaree et al describe an asset tracking system [1]. It makes use of existing Wireless Local Area Network (WLAN) signals to locate assets. Received signal strength measurements of the WLAN network are made throughout the building to build a database of Received Signal Strength (RSS) responses by location. User location is determined by matching the spectral signal strength fingerprint from a set of measurements to the database. While they report a relatively low accuracy using this method, it does have the advantage of taking into account the existing radio infrastructure.

The PinPoint Local Positioning System is outlined in [2]. It is based on a network of transponders at a site. At regular intervals, they send short spread spectrum signal bursts into the environment. Asset tags respond by repeating the signal with their ID information phase-modulated onto the signal. The network measures the round trip time from various sensors to tags to determine each tag's location.

1.1.2 People Tracking Location Systems

People tracking indoor location systems are generally designed for emergency response situations, to be used by firefighters, police assault teams, or soldiers. The system must be capable of being deployed in short notice, and

detailed advance knowledge of the building may not be available. Accuracy is of higher concern for this application; it can make a big difference knowing which side of a door the user is on. While the expense of the system components is still a concern, higher accuracy requirements allow more flexibility in their design than for asset tracking systems.

An example of such a system is described by Peterson et al [3]. The infrastructure for this system consists of a number of portable receivers that are placed outside a building. The user carries a transmitter that sends a Direct Sequence Spread Spectrum (DSSS) signal with a 23.5 MHz chip rate. User position is determined by comparing the time of arrival measurements at the receivers. The team was able to obtain fixes within three metres of the true location in ninety percent of their tests.

1.2 Indoor Position Location with Synthetic Doppler

Wight et al have developed a novel signal transmission method to improve position location estimation, called Synthetic Doppler [4]. Synthetic Doppler beacons are similar to the concept used in [3] in that they use a DSSS modulated signal to provide time of arrival information, but they also provide Angle Of Transmission (AOT) measurements. This additional information from each beacon can be used to improve location estimates and mitigate

multipath.

The idea is based on the traditional pseudo-Doppler direction-finding receiver (see Peavey [5] or Poisel [6]). In a pseudo-Doppler receiver, the receiver antenna rotates in the horizontal plane, as shown in Figure 1.1. The rotation can be either mechanical, or electrical via cycling through an array of antennas arranged in a circle. The antenna rotation imparts a sinusoidal FM modulation on the received signal, the phase of which being proportional to the Angle Of Arrival (AOA) of the signal. By comparing the phase of the FM modulating signal to the azimuth-referenced phase of the antenna rotation, the receiver can determine the AOA of the signal.

The synthetic Doppler transmitter operates on the same principle. The motion of the antenna imparts a sinusoidal frequency modulation to the transmitted signal carrier. The receiver must also know the instantaneous position of the antenna in order to determine the AOT from the transmitter. This information is modulated onto the transmission by way of a DSSS code synchronized with the antenna position. The receiver compares the phase of the spreading code with the phase of the FM modulating signal to obtain an AOT as well as a TOA, as shown in Figure 1.2.

Thus the receiver obtains two kinds of information from the signal – a time of arrival based on the spreading code, and an AOT based on the relative phases of the spreading code and the FM modulation.

This technology is designed to be used in a cooperative position location system. There are a variety of ways to use the combined AOT/TOA mea-

Figure 1.1: Synthetic Doppler Antenna.

surements from beacons to improve the location estimates vs. a strict time of arrival-based system. They are the subject of this research.

Figure 1.2: Synthetic Doppler.

1.3 Research Outline

1.3.1 Thesis Objectives

The primary goal of this research is to determine whether the AOT measurements afforded by the synthetic Doppler concept can improve position location estimates in high-multipath environments. Since the system is similar to DSSS-based TOA systems, these will be the main basis for comparison.

1.3.2 Thesis Organization

This chapter provides a general outline of indoor position location systems. Chapter 2 explains the common methods of locating things with radio signals, and identifies position location algorithms best suited to the synthetic Doppler system. In Chapter 3, the synthetic Doppler transmitter and receiver are fully characterized and evaluated. Chapter 4 more fully develops the applicable position location algorithms, including those for mitigating multipath effects, and presents theoretical bounds for accuracy. Chapter 5 describes the simulation model used to evaluate the synthetic Doppler concept. In Chapter 6, the performance of the different algorithms is compared under different channel conditions. Chapter 7 concludes the thesis with discussion of the results.

Chapter 2

Background: Geolocation

Methods

There are many ways to estimate the position of a radio emitter. Many of the choices in implementation have to do with the environment (e.g. indoor vs. outdoor) and level of user cooperation in the system. Users may range from being completely uncooperative (e.g. locating an enemy transmitter in combat) to completely integrated within the system (e.g. tracking employee badges as they move through a building). Each scenario will have different constraints and requirements.

In this chapter we discuss the methods and challenges for position location estimation. These include the type of infrastructure for the system, the radio signal metrics with which to estimate position, and the propagation

environment.

2.1 Location System Infrastructure

The constraints on infrastructure are a major issue in the design of a position location system. There are two broadly-defined categories of systems, those with a network-based architecture and those with a mobile-based architecture [2].

2.1.1 Network-Based Architecture

In a network-based architecture, mobiles transmit a signal and a fixed receiver infrastructure determines the user position based on characteristics of that transmission. These sorts of systems are characterized by a large infrastructure installed ahead of time at the facility where location tracking will take place. Because these systems rely on the extent of the sensor network for fidelity in tracking users, the user equipment can be very simple, and also inexpensive and low-power. Typically simple location metrics like Received Signal Strength (RSS) are the basis of the location estimates. Commercial systems like PinPoint [7] have network-based architectures.

The advantages of this approach are that the user equipment can be very cheap, small, and lightweight. The network-centric architecture is more focused on the problem of tracking many things at once. Research on these

systems focuses on fusing different kinds of estimates [8], [9], or tracking users with statistical methods [10].

The disadvantages of this approach include the expense of designing and installing the network infrastructure. Network nodes need to communicate with a central tracking authority, typically over a network.

These issues make the network-centric architecture appropriate for asset tracking in a fixed site, for example in a warehouse or hospital.

2.1.2 Mobile-Based Architecture

A mobile-based architecture focuses the processing at the user terminal. Usually, the user equipment processes signals from emitters at known locations and determines its own location based on characteristics of those signals. The Global Positioning System (GPS) is an example of this kind of system.¹ Mobile-based systems are less driven by cost, and can make use of more sophisticated signal processing yielding higher accuracy location estimates. They are more flexible, since their infrastructure may be global in scope, as is the case for GPS, or portable (see Peterson [3]). Existing signals may be co-opted for location use; for example, there has been a great deal of interest in recent years in meeting the E-911 standards for locating cell phone callers.

This kind of system is the only choice when it is impossible to install

¹Of course, GPS only works outdoors. The market for indoor mobile-based positioning systems is very limited, and there are no known commercial offerings.

an infrastructure in the region of interest ahead of time, for example for firefighters or a tactical assault team. The obvious downside to this approach is the increased expense of the equipment. The user terminal may need to be a multi-channel receiver, with the ability to perform non-trivial signal processing.

These kinds of systems tend to be more focused on sensor and implementation issues. The synthetic Doppler concept described in this thesis is a mobile-centric system.

2.2 Sensor Location Metrics

Various parameters can be extracted from radio signals. Common features used for positioning are Received Signal Strength (RSS), Time Of Arrival (TOA), Time Difference Of Arrival (TDOA), and Angle Of Arrival (AOA). For each of these parameters, equations may be formed that relate the position of the user to the position of the infrastructure sites and the measured radio metrics. With enough measurements, the user position may be estimated by solving the system of equations.

The chief difficulties with these methods are that the equations are generally nonlinear, and the radio metrics are corrupted by noise. This means the equations are difficult or impossible to solve analytically, and due to errors in the measurements they have no exact solution. Thus statistical methods are brought to bear in order to estimate a ‘best fit’ solution [11] [12]. Each

type of measurement and its mathematical details are described below.

2.2.1 Received Signal Strength

Received Signal Strength (RSS) measures the range from a transmitter by comparing the received signal strength to the known transmitted signal strength using a propagation model appropriate for the scenario. This is the simplest of the metrics, but it is also generally the most inaccurate [13]. RSS can be extremely unpredictable if there is shadowing or multipath in the environment. On the other hand, for asset tracking network-centric systems this is not a concern. A simple tracking system could be designed with sensors installed in all of the doors in a building. The system would log the passage of user equipment through every door; if the goal is only to determine the room containing the equipment, this is sufficient. However in a building with large rooms, e.g. a warehouse, this is obviously insufficient. For mobile-based architectures such as the Synthetic Doppler location system, RSS is too unpredictable for high accuracy and is not used.

2.2.2 Time of Arrival

In Time of Arrival (TOA) positioning systems, the time from a signal being transmitted to its reception is measured. Consider a system comprised of a number of listening stations, and a user terminal transmitting a signal. Each receiving station i measures the TOA from the source, converting it to a

range:

$$R_i = c t_i. \quad (2.1)$$

Here R_i is the i^{th} range estimate, and t_i is the i^{th} TOA. TOA is typically measured using Direct Sequence Spread Spectrum (DSSS) signals. The range measurements can be used to form a system of equations relating the target position to the receiver positions as follows:

$$R_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2} \quad (2.2)$$

The position of station i is (X_i, Y_i) . These equations define circles around each receiver; the target lies on the intersection of all the circles (see Figure 2.1). Three measurements are required to avoid any solution ambiguities, although other information may be available to determine the correct point for fewer measurements. In practice there will be errors associated with each range measurement, so there will be no one intersection point. An estimator must be used to find the best answer given the available information. Measurements from additional sensors can be used to improve the estimate.

TOA requires time synchronization between transmitter and receiver. This is normally quite difficult to achieve; a simpler way of eliminating the transmission time unknown is to employ time difference of arrival.

2.2.3 Time Difference of Arrival

Time Difference Of Arrival (TDOA) is the most common approach to mobile-based positioning systems [3]. Time difference measurements are obtained

Figure 2.1: Time of Arrival.

between measurement stations, so the corresponding range differences will have the form

$$R_{i,j} = R_i - R_j \quad (2.3)$$

for the range difference between stations i and j . These are related to the station positions (X_i, Y_i) and (X_j, Y_j) and the target position (x, y) by the equation

$$R_{i,j} = \sqrt{(X_i - x)^2 + (Y_i - y)^2} - \sqrt{(X_j - x)^2 + (Y_j - y)^2} \quad (2.4)$$

This equation has two unknowns and requires two station measurements.

Figure 2.2: Time Difference of Arrival.

When there are at least three stations, there is a system of equations of the form of equation 2.4 that defines two or more hyperbolas. The target lies on the intersection of these hyperbolas (see Figure 2.2). In practice there will be errors associated with each of the measurements, so an estimate of the most likely user position must be made. Taylor-Series expansion and least-squares estimation is a straightforward way to find the solution [12] [11].

A simple way to implement TDOA is to use a DSSS signal. TDOA measurements can be formed directly by cross-correlating received signals

at different stations. Alternatively, TDOA measurements can be formed by taking differences of TOA measurements from different stations. This kind of system eliminates the need for synchronization between all elements of the system, but the stations (or beacons) must still be synchronized with one another. If the beacons are outdoors, GPS can be used for this purpose.

Solving these equations is normally achieved using least-squares optimization. Due to the non-linearity of the equations, the Taylor-series expansion method is a simple way to solve the matrix equation. This method's main shortcoming is that it relies on a good choice for the initial point in order to converge on the correct answer. For example, Caffery [14] points out that when the target is near an edge of the polygon with the stations as its vertices, the Taylor series expansion LS method has convergence problems. Newer methods have been developed that address practical shortcomings of the Taylor-series expansion method without sacrificing its accuracy. Chan's method is frequently cited [15] as a closed-form solution that is nearly as accurate as the Taylor Series method.

2.2.4 Angle of Arrival

Angle Of Arrival (AOA) can be used to determine a target's position by simple triangulation. Two AOA measurements from different stations will intersect at the user position. More than two measurements will can increase the accuracy of the position fix, but they will generally intersect at different

Figure 2.3: Angel of Arrival.

points due to errors.

Formally, for each station i and bearing measurement θ_i , the user position (x, y) can be related to the station position (X_i, Y_i) by

$$\theta_i = \arctan \left(\frac{y - Y_i}{x - X_i} \right). \quad (2.5)$$

The system of equations resulting from multiple measurements is illustrated in Figure 2.3.

To deal with the inevitable measurement errors, there are a variety of

methods available. Poisel [6] describes several methods, all based on maximizing the joint probability of the target position. The Stansfield fix algorithm maximizes the exponent in the joint probability density function. The mean-squared distance algorithm minimizes the total squared distance from the target position estimate to the various lines of bearing (this is similar to the method used for TDOA and TOA). Closed-form solutions are available [16] that approximate the maximum-likelihood one.

AOA can be determined using an antenna array, a directional antenna, a rotating antenna (interferometry), or using other methods; Poisel has a good overview of the popular methods [6]. This thesis considers a new method, synthetic doppler, which provides angle of transmission (AOT) information. While the roles of transmitter and receiver are reversed, in terms of the mathematics the situation is exactly the same. This is fully described in Chapter 3.

Obtaining AOA measurements is easier to do with a fixed infrastructure, since it doesn't move (and doesn't need to be man-portable) and can support a large directional antenna. User terminals are more difficult to outfit with direction-sensing antennas, for obvious reasons.

2.2.5 Combinations of Methods

Combinations of these methods can be employed to improve the overall accuracy of the position fix. This can combine the benefits of different approaches.

The simplest method for combining is to determine position estimates for each method, and then take some weighted average of the solutions. A more general method is to consider all the measurements at once and determine the overall best answer for the available information. This approach is described in Chapter 4.

2.3 Propagation Environment

The propagation environment in which the location metrics are measured is an important element to be considered in a positioning system, because algorithms must be designed to account for the kinds of errors that will be observed. While a large system like GPS must contend with the shadowing of signals on the ground, a terrestrial wideband location system is dominated by multipath effects. For indoor systems, multipath is by far the largest contributor to position estimate errors.

Multipath arises due to the large number paths for radio transmission in a reflective environment such as a building or urban center. In addition to the Line Of Site LOS path, which may be attenuated beyond the threshold of detection, buildings, vehicles, and other objects provide reflected paths between transmitter and receiver. The three main effects of the reflected paths are [17]:

- Fading: Fading causes rapid variations in signal strength due to signals arriving on different paths very close together in time. The random

combinations of signal phase sometimes combine constructively, and at other times destructively. The result is rapid variations in signal strength.

- Doppler: If the transmitter, receiver, or reflectors in the channel are moving, random frequency modulation of the transmitted signal will be observed at the receiver due to the Doppler effect.
- Multipath: When copies of the transmission signal arrive far enough apart in time that they may be distinguished from one another, the effect is the appearance of echoes in the received signal.

Deterministically predicting these reflections is virtually impossible, as the mobile channel is a dynamic, complex system where the transmitter, receiver, and scatterers are often moving. At high frequencies, the movement of leaves in a tree or even rain can create unpredictable and volatile channel profiles.

Of the three effects above, coherent multipath is the most important in location estimation. Fading is ignored in this study. It is assumed that some useful percentage of the time, the signal of interest can be received and its location-relevant metrics extracted. The Doppler effect is also ignored, since we are tracking people (who move relatively slowly) and the Synthetic Doppler beacons are stationary (although we of course make use of the intentional Doppler modulation due to their rotation).

Multipath is a problem when the LOS path is undetectable. Every metric

discussed above will suffer when it is derived from a NLOS path incorrectly identified as LOS. Various approaches have been suggested to deal with these sorts of errors. Woo et al [18] use a Kalman filter to model the NLOS behavior of corrupted measurements – this assumes knowledge of the distribution of NLOS errors, and of the motion of the target. Chen [19] uses extra measurements to reduce the impact of those containing NLOS errors. This method has the advantage of not requiring knowledge of the distribution of the errors, but requires redundant measurements with a high percentage of LOS paths to work. This approach is used in this study, and it is fully described in Chapter 4.

2.4 Chapter Summary

In this chapter an overview of the issues for indoor location have been presented. The system architecture, positioning metrics, and channel impairments are reviewed.

There are two general categories of indoor positioning systems. The infrastructure-based system is based on the installation of sensors embedded in the area of interest. This extensive sensor network allows for reasonably high accuracy with simple and inexpensive sensors, and is well-suited for inventory tracking applications. Mobile-based systems are geared for flexibility and mobility, and typically employ a portable infrastructure. The reduced number of sensors requires more complicated user equipment to obtain high

accuracy, but for some applications the tradeoff in flexibility is worth the cost. The Synthetic Doppler positioning system is a mobile-based system.

There are a number of parameters of a radio signal that can be used to relate transmitter and receiver positions. RSS uses propagation power loss to calculate distances from sensors. TOA compares a received signal time to a known transmission time to determine distance. TDOA compares time of arrival differences at multiple sites to determine position. AOA and AOT can be used to determine the bearing of the target. Each of these metrics define systems of equations, generally nonlinear, that can be solved with well-known methods. Methods can be combined if multiple types of information is available to improve the estimate or reduce the required number of sensors.

The use of any of these metrics is impaired by the imperfect channel. The indoor channel is dominated by NLOS effects. Fading, caused by reflections of the signal combining destructively and constructively over time, result in severe signal level fluctuations. Doppler, caused by a moving transmitter, receiver, or reflector, imparts a frequency shift to the signal. Multipath is the effect of signal echoes is caused by reflected copies with delays greater than the signal symbol duration. Each of these can cause problem for geolocation, but the focus in this research is dealing with the multipath problem.

Following chapters focus on how these elements of the problem apply to the synthetic Doppler concept.

Chapter 3

Synthetic Doppler System

Analysis

In this chapter the transmission and demodulation of the synthetic Doppler waveform are described. The analysis of the Doppler modulation is nearly identical to that of a synthetic Doppler direction sensing antenna – reversing the roles of transmitter and receiver makes no difference to the geometry of the situation. Components of the signal modulation and demodulation process are approached in turn.

3.1 Synthetic Doppler Signal

We begin by describing the modulation and transmission of the synthetic Doppler signal. There are two components of modulation: the Frequency Modulation (FM), and the DSSS code modulation.

3.1.1 FM Doppler Modulation

The rotation of the synthetic Doppler antenna imparts an FM modulation on its carrier. As mentioned above, the relevant geometry is identical to the receiver case. This analysis follows the Synthetic Doppler direction finding system analysis presented by Poisel [6].

Consider figure 3.1. This shows a top-down view of the transmitting antenna, rotating about the origin, and a stationary receiver antenna at position $(0, y)$. The transmitting antenna has radius r and rotates with radial velocity ω . Define the tangential velocity of the antenna as ν_t , which is given by

$$\nu_t = r\omega. \quad (3.1)$$

The component of ν_t in the direction of transmission to the receiver of the signal is

$$\nu = \nu_t \cos(\pi/2 - \theta) \quad (3.2)$$

$$= r\omega \sin \theta \quad (3.3)$$

The approximate instantaneous Doppler shift (assuming $\nu \ll c$, where c

Figure 3.1: Synthetic Doppler Geometry.

is the speed of light) is

$$\Delta f = \frac{v}{c} f_c \quad (3.4)$$

$$= \frac{r\omega \sin \theta}{c} f_c \quad (3.5)$$

$$= \frac{r\omega}{\lambda} \sin \theta \quad (3.6)$$

where f_c is the carrier frequency of the signal, and $\lambda = \frac{c}{f_c}$ is its corresponding wavelength. We can relate the instantaneous azimuth angle of the antenna

Ω to the angle θ using the law of sines:

$$\frac{\sin \theta}{y} = \frac{\sin(\pi/2 - \Omega)}{\sqrt{r^2 \cos^2 \Omega + (y - r \sin \Omega)^2}} \quad (3.7)$$

Simplifying and solving for θ , we obtain

$$\theta = \arcsin \left(\frac{\sin(\pi/2 - \Omega)}{\sqrt{1 - \frac{2r}{y} \sin \Omega + \frac{r^2}{y^2}}} \right). \quad (3.8)$$

For the case where $y \gg r$, the denominator is approximately 1, and θ simplifies to

$$\theta \approx (\pi/2 - \Omega). \quad (3.9)$$

Substituting this into equation 3.6, we find the instantaneous Doppler shift to be

$$\Delta f = \frac{r\omega}{\lambda} \sin(\pi/2 - \Omega) = \frac{r\omega}{\lambda} \cos \Omega. \quad (3.10)$$

Now, Ω is the instantaneous position of the antenna arm. It is changing at a constant rate of ω . If at time zero $\Omega = 0$, then $\Omega(t) = \omega t$. Thus the instantaneous frequency as a function of time is

$$\Delta f(t) = \frac{r\omega}{\lambda} \cos(\omega t). \quad (3.11)$$

We have assumed thus far that the receiver lies directly north of the transmitter. If it is at any other azimuth A_z , the entire picture can be rotated by that same angle to obtain a frequency shift of

$$\Delta f(t) = \frac{r\omega}{\lambda} \cos(\omega t - A_z). \quad (3.12)$$

At this point it is helpful to couch the Doppler modulation in terms of standard FM analysis. We can make use of this later in analyzing the performance of the system. In a standard analysis of FM (for example see Shanmugam [20]), a sinusoidal FM modulating signal $x(t)$ is transformed into a phase modulation $\phi(t)$ on a carrier signal $x_c(t)$ as follows:

$$x(t) = A_m \cos \omega_m t \quad (3.13)$$

$$\phi(t) = k_f \int_{t_0}^t x(\lambda) d\lambda + \phi(t_0) \quad (3.14)$$

$$= \frac{k_f A_m}{\omega_m} \sin \omega_m t \quad (3.15)$$

$$x_c(t) = A_c \cos(\omega_c t + \phi(t)) \quad (3.16)$$

$$= A_c \cos(\omega_c t + \beta \sin(\omega_m t)) \quad (3.17)$$

Here the standard angle modulation parameters are employed:

- A_m is the modulating signal's magnitude in volts.
- ω_m is the modulating signal's frequency in radians.
- k_f is the frequency deviation constant in radian/sec/volt.
- A_c is the carrier signal's magnitude in volts.
- β is the modulation index, defined as $\beta = k_f A_m / \omega_m$ (Hertz).

By comparing these equations with equation 3.12, we can rewrite the synthetic Doppler modulating signal to have the form

$$x(t) = \cos(\omega_m t - A_z) \quad (3.18)$$

where ω_m is the frequency of rotation of the antenna in radians. Then the FM modulation parameters are

$$A_m = 1 \quad (3.19)$$

$$k_f = \frac{r\omega_m}{\lambda} \quad (3.20)$$

$$\beta = \frac{r}{\lambda} \quad (3.21)$$

With these and equation 3.17, we obtain the equation of a carrier tone transmitted by the rotating antenna:

$$s(t) = A \cos \left[\omega_c t + \frac{r}{\lambda} \sin(\omega_m t - A_z) \right] \quad (3.22)$$

Now, the bandwidth of an FM signal modulated by a sine wave is given by Carson's rule:

$$B \approx 2(\beta + 1)f_m \quad (3.23)$$

so for the synthetic Doppler system the bandwidth of the FM modulation is

$$B \approx 2 \left(\frac{r}{\lambda} + 1 \right) f_m \quad (3.24)$$

where f_m is the rotation frequency of the antenna in Hz.

3.1.2 DSSS Modulation

The carrier signal is modulated by a binary direct sequence spreading code. Direct sequence spread spectrum modulation is described in many references (see Proakis [21] for an introduction, or Holmes [22] for a detailed analysis),

and the details will not be belabored here. The DSSS waveform $c(t)$ has the form

$$c(t) = \sum_k a_k p(t - kT_c) \quad (3.25)$$

where a_k is the k^{th} spreading symbol ($a_k \in \{-1, 1\}$), $p(t)$ is the chip pulse waveform, and T_c is the chip duration. This is a binary phase shift keying (BPSK) signal, which is multiplied with the carrier (equation 3.22) to produce the signal:

$$s(t) = A_c c(t) \cos(\omega_c t + \beta \sin(\omega_m t - A_z)) \quad (3.26)$$

The FM modulation is imparted by the rotation of the antenna, so in fact it will modulate the DSSS part of the signal. However the chip rate will normally be orders of magnitude faster than the antenna rotation rate, so we can safely ignore this effect.

Finally we note that the phase of the spreading code $c(t)$ is aligned with the rotation of the antenna, so the receiver can determine the angle of transmission by comparing the code phase with the phase of the FM modulating signal.

3.2 Synthetic Doppler Demodulation

The demodulation of the synthetic Doppler signal is a two-stage process. First the receiver must synchronize with and remove the spreading code, and

Figure 3.2: Synthetic Doppler Demodulator.

then it must demodulate the FM signal to obtain its phase. The AOT is determined by comparing the phase of the FM signal with that of the spreading code. The spreading code may also be used to determine an unreferenced TOA from the transmitter. A block diagram of this process is shown in Figure 3.2. In this section we cover the basic demodulation path; following sections address the signal processing details and the determination of TOA and AOT in detail.

In the absence of noise, multipath, and other interference, the input to the receiver is a time delayed and phase-shifted version of the transmitted signal:

$$r(t) = A_c c(t - \tau_d) \cos(\omega_c t + \beta \sin(\omega_m t - A_z) + \theta_r) \quad (3.27)$$

where τ_d is the transmission delay referenced to an unknown time t_0 , and θ_r is the unknown carrier phase. The Code Synchronizer determines the state of the spreading code, and produces a local reference code with identical timing, $c_r(t - \tau_d)$. The value of τ_d is also noted to obtain a TOA for the received signal. The despread block correlates $r(t)$ with the local code $c(t - \tau_d)$ to obtain the FM-modulated carrier:

$$y(t) = c(t - \tau_d) r(t) \quad (3.28)$$

$$y(t) \approx A_c \cos(\omega_c t + \beta \sin(\omega_m t - A_z) + \theta) \quad (3.29)$$

The FM Demodulation block pulls out the Doppler signal (as described in section 3.2.3):

$$y'(t) = \cos(\omega_m t - A_z). \quad (3.30)$$

The Positive Zero-Crossing Detector emits a pulse train synchronized with the phase of this signal, with a pulse echoed every time t satisfies

$$t \bmod 2\pi = \frac{A_z}{\omega_m}. \quad (3.31)$$

We can describe the detector's output using the Dirac delta function:

$$x_1(t) = \sum_{n=0}^{\infty} \delta(t - 2\pi n - A_z). \quad (3.32)$$

Note that the transmission delay has been ignored in the description of the FM signal phase. It is assumed these timing details are accounted for in the code synchronization block.

Meanwhile the Code Synchronizer produces its own pulse train, synchronized with the spreading code phase. Assuming that the spreading code

Figure 3.3: Pulse Train Comparison.

duration is equal to the period of FM modulating signal, the code pulse train has the form

$$x_2(t) = \sum_{n=0}^{\infty} \delta(t - 2\pi n). \quad (3.33)$$

The final block computes the difference in pulse times between $x_1(t)$ and $x_2(t)$ during one antenna rotation period to obtain an estimate of A_z , i.e. the AOT, as shown in Figure 3.3.

We have considered this demodulation process in the absence of noise and multipath. Multipath copies of the signal are assumed to be dealt with by the code synchronization block – if multiple delayed copies of the signal

are present, it will synchronize with the first arriving detectable signal. It is assumed that delayed copies will be rejected by the despreading process. Errors caused by the code synchronizer locking to a multipath reflection (e.g. if the LOS path is undetectable) will be dealt with in the following chapters. The analysis to characterize the effect of Additive White Gaussian Noise (AWGN) on the TOA and AOT estimates follows.

3.2.1 TOA Estimation

In the receiver architecture described above, estimation of the TOA is the responsibility of the Code Synchronization block. Note that the TOA estimate as described here is unreferenced; that is, the transmission time is unknown. This can be estimated using multiple beacons in later stages, so long as the beacons are synchronized with one another.

DSSS synchronization takes place in two steps: coarse acquisition to within half a chip, and fine synchronization to a much higher (but still limited) accuracy. In the position location literature, some studies assume only coarse synchronization, and some consider fine synchronization [14]. Others have utilized special lead-edge-detector circuits to achieve their results [3]. Our goal in this section is to arrive at a reasonable estimate of the expected TOA accuracy as a function of the code chip rate and the received SNR, without going to the trouble of designing a particular algorithm to perform

the task. We assume the time estimate will take the form of

$$\hat{t}_c = t_c + n_t, \quad (3.34)$$

where t_c is the true TOA and n_t is a zero-mean Gaussian variable representing the uncertainty associated with the estimate. The goal of this section is then to determine the parameters of n_t .

In order to evaluate the various methods described, they will all be compared with the Cramér-Rao Lower Bound (CRLB) for TOA estimation.

Coarse Code Acquisition

The simplest method of DSSS TOA estimation is to use the synchronized code phase. Coarse code acquisition provides synchronization to within half a chip [22]. If this is the ultimate resolution for estimating TOA, the error will clearly be dominated by the uncertainty of exactly where in the chip the synchronizer has locked on to. The error is thus uniformly distributed over half a chip, so the estimate has the form

$$\hat{\tau}_d = \tau_d + \epsilon_t \quad (3.35)$$

where ϵ_t is a uniform random variable with mean zero over the range $[-T_c/2, T_c/2]$.

Fine Code Tracking

Once coarse code acquisition has been achieved, the next step is to track the code. We apply the results presented in [22] for a non-coherent, full-time,

1-delta Delay Locked Loop (DLL). This analysis concludes the timing error variance in *chips*² is

$$\left(\frac{\sigma_\epsilon}{T_c}\right)^2 = \frac{N_0 B_L}{2\alpha P} \left(\frac{\alpha'}{\alpha} + \frac{2N_0 B'}{\alpha P}\right) \quad (3.36)$$

where σ_ϵ is the tracking variance in seconds, T_c is the chip period, and N_0 is the noise PSD. α is the time-average, ensemble average of the data signal. Since there is no data in the synthetic Doppler signal, $\alpha = 1$. With sharp-cutoff filters, $\alpha' \approx \alpha$, so $\alpha' = 1$ also for the no-data case. P is the signal power. B' is related to the loop input filter (see [22], p.480), and is approximately equal to B , the bandpass filter bandwidth on each arm of the DLL. Generally, B is chosen to be approximately twice the data bandwidth.

B_L is the one-sided tracking loop bandwidth, i.e. the one-sided bandwidth of the closed-loop transfer function $H(s)$ (pp. 472-473, [22]):

$$H(s) = \frac{2\alpha P K F(s)/s}{1 + 2\alpha P K F(s)/s} \quad (3.37)$$

Here $F(s)$ is the loop filter transfer function, and K is the loop gain. For a first-order loop, B_L is given by ([22], pp. 530-531)

$$B_L = \frac{\alpha P K_{VCO}}{2} \quad (3.38)$$

This is the gain of the error signal (K_{VCO} is the gain of the code 'oscillator'). Ultimately the value of B_L will be determined by the exact design of the DLL.

Taking into account that $\alpha = 1$, $\alpha' = 1$, and $B' \approx B$, the tracking error

expression simplifies to

$$\left(\frac{\sigma_\epsilon}{T_c}\right)^2 = \frac{N_0 B_L}{2P} \left(1 + \frac{2N_0 B}{P}\right) \quad (3.39)$$

This expression holds when the tracking error is less than half a chip (since we are considering a half-chip-delta DLL). We can use this expression as the variance of a zero-mean Gaussian variable to simulate TOA accuracy in the receiver.

If we define the signal to noise ratio as

$$\gamma = \frac{P}{N_0 B_i}, \quad (3.40)$$

where B_i is the input noise bandwidth of the receiver (and for our DSSS system, $B_i \approx 1/T_c$), we can write the tracking error as a function of the SNR and the chip rate:

$$\sigma_\epsilon = T_c^2 \frac{B_L}{B_i 2\gamma} \left(1 + \frac{2B}{B_i \gamma}\right) \quad (3.41)$$

This is a difficult expression to use for determination of TOA, because it depends so much on the specific design of the DLL.

TOA Cramér-Rao Lower Bound

To verify this expression we compare it with the Cramér-Rao Lower Bound (CRLB) for estimating the TOA of any signal. This is given in [6] as

$$\sigma_t^2 \geq \left(\frac{2E}{N_0} \beta^2\right)^{-1} \quad (3.42)$$

where σ_t^2 is the variance of the measurement in seconds squared, E is the energy of the signal in Watt-seconds, N_0 is the noise density (Watts per Hertz), and β is the root-mean-squared bandwidth of the signal in radians:

$$\beta = 2\pi \left[\frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2 df}{\int_{-\infty}^{\infty} |S(f)|^2 df} \right]^{\frac{1}{2}} \quad (3.43)$$

For an ideal signal with a rectangular envelope, β simplifies to

$$\beta = \frac{2\pi B}{\sqrt{3}} \quad (3.44)$$

where B is the bandwidth of the signal in Hertz.

Noting that the energy is given by $E = PT_i$, (where T_i is the integration time), and the noise power is $N_0 B$, where B is the input noise bandwidth of the receiver, we can rewrite the CRLB in terms of the SNR γ as

$$\sigma_t = \frac{\sqrt{3}}{2\pi B \sqrt{\gamma T_i B}} \quad (3.45)$$

Furthermore, for a DSSS system we may assume the input noise bandwidth will be about $1/T_c$, and if we write the integration time in terms of the number of chips ($N_c = T_i/T_c$), then we can write equation 3.45 in terms of the chip rate:

$$\sigma_t = T_c \frac{\sqrt{3}}{2\pi \sqrt{\gamma N_c}} \quad (3.46)$$

It is difficult to compare expressions 3.41 and 3.46 for the tracking error, because they each encapsulate different kinds of information. Furthermore the DLL expression contains terms that are difficult to estimate without designing and testing a particular DLL.

3.2.2 TOA Estimation Comparison and Summary

The accuracy of TOA estimation of DSSS is potentially quite high. Figure 3.4 shows a comparison of the expected DLL jitter and the CRLB for TOA accuracy. This is based on equations 3.41 and 3.46, assuming a spread-spectrum code signal with a chip rate of 1.25 Mcps, and a 1000-chip dwell time. This is clearly an approximation, as the DLL is shown to beat the CRLB for high SNR cases. In any case the achievable performance appears to be quite good, but comparisons with the literature are in order to determine whether these expectations are realistic.

TOA Accuracies Reported in the Literature

Due to the imprecision of the accuracy estimates reported in the previous section, comparisons with the literature are in order.

In [14], a DLL is simulated using an m sequence of length 127 and chip rate 1.2288 Mcps; using slices of 120th of a chip, the ranging resolution is about 2 m. For various δ values, and in a microcell, the standard deviation of the ranging error was 15.5 m ($\delta = 1/2$), 12.1 m ($\delta = 1/4$), and 6.1 m ($\delta = 1/8$). The CRLB for these parameters is 37.8 metres divided by the square root of the integration length in chips. E.g. for 100 chips integration time, the standard deviation is 3.78 metres.

In [23], Gutowski et al report being able to achieve a time estimate error of the order (mean absolute error) $0.1T_c$ with a dwell time of 0.1 second,

Figure 3.4: TOA Accuracy.

with a minimum SNR of about -30 dB. This corresponds to a ranging error of about 25 metres.

In Peterson's paper [3], with 5.5 ms of samples, a 23.5 MHz chipping rate, with 3.2 to 400 mW transmissions, and using least-squares fitting to the leading edge of the pulse, they obtain a timing standard deviation of 8 ns (2.4 metres).

It is assumed in later sections that obtaining TOA variance of 2 metres squared is achievable. This may be ambitious to implement, but it is certainly

possible.

3.2.3 AOT Estimation

Here we will determine the error statistics of the AOT estimate. The AOT estimation accuracy is based primarily on the accuracy of estimating the phase of the FM modulating signal. To find this, we must first find the statistics of the noise term at the output of the FM demodulator.

Once the spreading code timing has been obtained, the receiver multiplies the received signal with an aligned local copy of the spreading code, to obtain equation 3.29 (repeated here):

$$y(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t - A_z)) + n'(t) \quad (3.47)$$

Here $n'(t)$ is the original band-limited White Gaussian Noise (WGN) process $n(t)$ that has been multiplied by the spreading code $c(t)$:

$$n'(t) = c(t) n(t); \quad (3.48)$$

It can be shown (see [22], p.366) that the output of a band-limited noise process multiplied by a pseudo-random spreading code has a power spectral density (PSD) at the carrier frequency of

$$S_N(\omega_0) = \frac{N_0}{2} \int_{-BT_c/2}^{BT_c/2} \left(\frac{\sin \pi x}{\pi x} \right)^2 dx \quad (3.49)$$

For a filter with bandwidth $B = 1/T_c$ the reduction in $N_0/2$ is about 1.02 dB, and we note that the character of the noise is still band-limited Gaussian.

Optimal FM demodulation (see [20]) will recover the modulating (Doppler generated) signal $\cos(\omega_m t - A_z)$, accompanied by a noise term $n''(t)$ with PSD

$$G_n(f) = \begin{cases} \frac{k_d^2}{A_c^2} (2\pi f)^2 N_0 & \text{for } |f| < f_x \\ 0 & \text{otherwise} \end{cases} \quad (3.50)$$

Here, k_d is the discriminator sensitivity, and $2f_x$ is the modulating signal bandwidth.

The task remains to determine the effect of this noise on the measurement of the demodulated signal's phase. This can be found in any noise analysis of phase modulation. Following the analysis of phase demodulation in [20], p. 353, the PSD of the noise term in the output is

$$G_{n_p}(f) = \frac{k_d^2}{A_c^2} G_{n_s}(f) \quad (3.51)$$

where $k_d = 1$ and $A_c = 1$ and $G_{n_s}(f)$ is the quadrature component of the baseband noise that accompanies the output signal of the FM demodulator. Assuming the quadrature noise n_s contains half the energy of the complete noise signal n_p , then

$$G_{n_p}(f) = \frac{G_{n_s}(f)}{2} \quad (3.52)$$

After the post-detection filter (which rejects components above f_x) the final output noise power is

$$E[n_o^2(t)] = \int_{-f_x}^{f_x} \frac{1}{2} \frac{k_d^2}{A_c^2} (2\pi f)^2 N_0 df = \frac{k_d^2}{3A_c^2} (2\pi)^2 N_0 f_x^3 \quad (3.53)$$

Making the substitution $k_d = 1/k_f = \lambda/r\omega_m$, and $f_x \approx f_m$ the noise power is (in radians²)

$$P_N = \frac{\lambda^2}{3A_c^2 r^2 (2\pi)^2 f_m^2} (2\pi)^2 N_0 f_m^3 = \frac{\lambda^2}{3A_c^2 r^2} N_0 f_m, \quad (3.54)$$

or in terms of the input SNR ($(SNR)_{input} = \frac{A_c^2}{4f_m N_0}$)

$$P_N = \left(\frac{\lambda}{r}\right)^2 \frac{1}{12 SNR_{input}} \quad (3.55)$$

Note that this expression only holds for high SNR. We may take this as the variance of a zero-mean, Gaussian r.v. representing the error term of the phase estimate.

Finally we must determine the resulting AOA error, which is based on the phase noise measurement error coupled with the TOA estimate error.

The actual measurements will be made in the time domain (as opposed to the phase domain). The time of arrival estimate (determined from the spreading code state) is

$$\hat{t}_c = t_c + n_t \quad (3.56)$$

and the Doppler phase zero-crossing time is

$$\hat{t}_d = t_d + n_d. \quad (3.57)$$

Recall that the code phase is synchronized with the antenna rotation phase. The code TOA measurement t_c then marks the time that the antenna was at, say, 0 degrees azimuth. The doppler time measurement t_d marks the time of peak doppler shift, so a receiver at 0 degrees azimuth from

the transmitter would find $t_c = t_d$. The angle of transmission A_z is then proportional to the difference in time of these two measurements, so

$$\hat{A}_z = \alpha(\hat{t}_d - \hat{t}_c) \quad (3.58)$$

$$= \alpha(t_d - t_c) + \alpha(n_d - n_c) \quad (3.59)$$

$$= A_z + n_{A_z} \quad (3.60)$$

The difference in times will range from 0 to the rotation period $1/f_m$. This time difference can be related to the angle of transmission by dividing the time by the antenna rotation period, $1/f_m$ to obtain a fraction between 0 and 1, and multiplying by 2π ; thus to determine the azimuth angle of transmission in radians, α must be

$$\alpha = 2\pi f_m \quad (3.61)$$

So we've defined

$$A_z = 2\pi f_m(t_d - t_c) \quad (3.62)$$

and the noise term

$$n_{A_z} = 2\pi f_m(n_d - n_c) \quad (3.63)$$

If n_d and n_c are zero-mean, independent Gaussian random variables, the

measurement noise n_{A_z} is also zero-mean with variance:

$$\sigma_{A_z} = \text{Var}(n_{A_z}) = \text{E}[\alpha^2(n_{A_z} - \text{E}[n_{A_z}])^2] \quad (3.64)$$

$$= \alpha^2 \text{E}[(n_d - n_c - \text{E}[n_d] + \text{E}[n_c])^2] \quad (3.65)$$

$$= \alpha^2 \text{E}[(n_d - \text{E}[n_d])^2 + (n_c - \text{E}[n_c])^2] \quad (3.66)$$

$$- 2(n_d - \text{E}[n_d])(n_c - \text{E}[n_c]) \quad (3.67)$$

$$= \alpha^2(\text{Var}(n_d) + \text{Var}(n_c) - 2\text{Cov}(n_d, n_c)) \quad (3.68)$$

Since the two random variables are independent, this simplifies to

$$\sigma_{A_z} = \alpha^2(\sigma_{n_d}^2 + \sigma_{n_c}^2). \quad (3.69)$$

Equation 3.69 is plotted in Figure 3.5 for a range of SNR values. The parameters used for the plot are a carrier frequency $f_c = 900$ MHz, rotation frequency $f_r = 10$ Hz, and antenna radius $r = 1$ m. It appears that a high SNR at the receiver input is necessary in order to achieve reasonable AOT estimation accuracies.

3.3 Chapter Summary

In this chapter we have analyzed the modulation and demodulation of the synthetic Doppler signal. The two modulation types embedded in the signal are a frequency modulation, imparted physically by the motion of the antenna, and a DSSS component, modulated by the transmitter. A system for demodulating this signal and extracting the location metrics was evaluated.

Figure 3.5: AOT Accuracy.

First the DSSS code is removed, and then the FM signal is extracted. The relative timing of these two signals can be used to obtain TOA and AOT measurements suitable for positioning. Although predicting the practically achievable accuracy of these measurements is difficult, the analysis shows that a ranging variance of two metres and an angle variance of two to five degrees is possible.

Chapter 4

Positioning Algorithms

In the previous chapter we derived expressions for the accuracy of several metrics that could be obtained from a synthetic Doppler signal. Here we introduce the theory of converting several of those measurements from multiple beacons into a position estimate of the receiver.

Envision a scenario with two or more synthetic doppler beacons surrounding an area of interest. Assume each beacon's position is known exactly (e.g. using GPS receivers), and all the beacons are synchronized with a common clock (again, possibly using GPS). Then the receiver will obtain an AOT and an unreferenced TOA measurement from each beacon. Taking time differences between beacons yields time difference measurements. The goal is to estimate the position using this information.

A system of equations can be formulated relating the positions of the bea-

cons and their TDOA and AOT measurements to the the position of the receiver. These equations are non-linear, and the measurements are corrupted by noise, so solving for the receiver position is not entirely straightforward. Happily this type of problem is well-understood, and the theory of linearized least-squares estimation may be applied to find a solution.

In this section two-dimensional position estimates are derived. Extending the math to three dimensions is straightforward, but two dimensions are sufficient and more transparent.

The goal of this thesis is to determine the improvement made possible by using AOT instead of, or in addition to, TDOA information. To this end, we develop methods for positioning using TDOA measurements only, AOT measurements only, and combined measurements. We also present several variations of an algorithm designed to mitigate multipath that is effective for any measurement type. Finally bounds are derived for the position location errors as a function of the measurement errors.

4.1 Taylor-Series Estimator Theory

The problem as outlined above may be solved using the Taylor-series linearization method, as described in [11], [12]. We begin by describing the information available to the estimator:

- N_s stations, located at coordinates $(x_i, y_i), i = 1, 2, \dots, N_s$

- The receiver located at coordinates (x, y)
- A collection of measurements $m_{i,k}$ each from station i and of type $k, k = 1, \dots, N_m$, where N_m is the number of measurement types (for our case the types are TOA and AOT)
- A random error $\epsilon_{i,k}$ associated with each measurement

For every type of measurement, there exists a function $f_{i,k}(x, y, x_i, y_i)$ relating each measurement to the receiver and beacon positions, so that

$$m_{i,k} = f_{i,k}(x, y, x_i, y_i) + n_{i,k}, i = 1, 2, \dots, N_s, k = 1, 2, \dots, N_m \quad (4.1)$$

These equations can be written in matrix form:

$$\mathbf{m} = \mathbf{f}(\mathbf{x}) + \mathbf{n} \quad (4.2)$$

and we note that the error vector $\mathbf{n}$ has covariance matrix

$$\mathbf{N} = E[(\mathbf{n} - E[\mathbf{n}])(\mathbf{n} - E[\mathbf{n}])^T] \quad (4.3)$$

If the measurement noise is Gaussian, the maximum likelihood estimator for (x, y) minimizes the expression

$$Q(\mathbf{x}) = [\mathbf{m} - \mathbf{f}(\mathbf{x})]^T \mathbf{N}^{-1} [\mathbf{m} - \mathbf{f}(\mathbf{x})] \quad (4.4)$$

or in other words, it is solving for $\hat{\mathbf{x}}$:

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} ([\mathbf{m} - \mathbf{f}(\mathbf{x})]^T \mathbf{N}^{-1} [\mathbf{m} - \mathbf{f}(\mathbf{x})]) . \quad (4.5)$$

If the noise processes in these equations are Gaussian, this is the maximum-likelihood estimator. If the noise is not Gaussian in nature, then it is a least-squares estimator.

Now since $\mathbf{f}(\mathbf{x})$ is usually a non-linear function of $\mathbf{x}$, it is rarely possible to find a general solution. Instead we seek a numeric one, by linearizing the system at selected points. We linearize $\mathbf{f}(\mathbf{x})$ by using the Taylor series expansion at some reference point $\mathbf{x}_0$ and retaining the first two terms:

$$\mathbf{f}(\mathbf{x}) \approx \mathbf{f}(\mathbf{x}_0) + \mathbf{G}(\mathbf{x} - \mathbf{x}_0) \quad (4.6)$$

$\mathbf{G}$ is the gradient matrix of $\mathbf{f}(\mathbf{x})$, evaluated at $\mathbf{x}_0$:

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} \left. \frac{\partial f_1}{\partial x} \right|_{\mathbf{x}=\mathbf{x}_0} & \left. \frac{\partial f_1}{\partial y} \right|_{\mathbf{x}=\mathbf{x}_0} \\ \vdots & \vdots \\ \left. \frac{\partial f_N}{\partial x} \right|_{\mathbf{x}=\mathbf{x}_0} & \left. \frac{\partial f_N}{\partial y} \right|_{\mathbf{x}=\mathbf{x}_0} \end{bmatrix} \quad (4.7)$$

We iteratively solve for $\hat{\mathbf{x}}$ by choosing an initial point and calculating successive estimates of the best solution. The choice of increment $\delta = (\delta_x, \delta_y)$ that gives the best estimate in the least-square sense is

$$\delta = [\mathbf{G}^T \mathbf{N}^{-1} \mathbf{G}]^{-1} \mathbf{G}^T \mathbf{N}^{-1} \mathbf{x}_\nu \quad (4.8)$$

The algorithm to determine the solution is then:

1. Choose an initial guess of the solution, $\mathbf{x}_\nu$.
2. Calculate δ .

3. Replace

$$x_\nu \leftarrow x_\nu + \delta_x, y_\nu \leftarrow y_\nu + \delta_y \quad (4.9)$$

4. Repeat until the difference between successive estimates is lower than some threshold

One downside is that this method requires an initial guess. The choice of starting point may affect the convergence of the algorithm, though in practice nearly any initial point will lead to the same solution. Excessively noisy measurements may also prevent convergence. Furthermore, if there are multiple solutions to the equations, some other criteria will be required to evaluate each answer to see if it is the correct one.

In the following sections this theory is applied to the geometry associated with TDOA and AOT measurements. Once the equations for these particular measurements have been properly formulated, the above theory can be applied and we can develop their specific solutions.

4.1.1 TDOA Estimator

Here we apply the general Taylor-Series estimator theory to the problem of estimating a user position using TDOA measurements. We need simply calculate the $\mathbf{f}(\mathbf{x})$, $\mathbf{G}$, and $\mathbf{N}$ matrices to obtain the estimator. Before jumping into the algorithm derivation, it is worthwhile to formulate the problem specifically for TDOA. Given the set of stations S , each with position X_i and

range measurement to the receiver r_i , the best least-squares estimate is

$$\hat{x} = \arg \min_x \sum_{\substack{i,j \in S \\ i \neq j}} (|r_i - r_j| - ||x - X_i|| - ||x - X_j||)^2 \quad (4.10)$$

The solution to this equation is found starting with the measurement information.

We begin with a time of arrival measurement $\hat{t}_i$ for each beacon i

$$\hat{t}_i = t_0 + \frac{r_i}{c} + n_{t,i} \quad (4.11)$$

where c is the speed of light, t_0 is the time of transmission (common to each measurement), and $n_{t,i}$ is the time error associated with the i^{th} TOA measurement. r_i is defined as the range from the user position (x, y) to the i^{th} station:

$$r_i = R_i(x, y) \quad (4.12)$$

where

$$R_i(x, y) \triangleq \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad (4.13)$$

and (x_i, y_i) is the location of the i^{th} beacon.

The complete system is described by the collection of equations 4.11, with one for each beacon. These are summarized in the matrix equation

$$m_t = t_0 + f_t(x) + n_t \quad (4.14)$$

Now, generally we don't know the time of transmission t_0 , so we can't directly solve for the user location using this information. However t_0 can be

removed by subtracting the TOA values in pairs (assuming the beacons are synchronized with each other, t_0 is common to each measurement). This yields the relation

$$\hat{\tau}_i = \hat{t}_i - \hat{t}_{i+1} = (R_i(x, y) - R_{i+1}(x, y))/c + n_{\tau,i} \quad (4.15)$$

where $n_{\tau,i} = n_{t,i} - n_{t,i+1}$ for $i = 1, 2, \dots, N_s - 1$. In matrix form, this becomes

$$\mathbf{m}_{\tau} = \mathbf{f}_{\tau}(\mathbf{x}) + \mathbf{n}_{\tau} \quad (4.16)$$

As an aside, these terms can be found using the $(N_s - 1) \times N$ transform matrix $\mathbf{H}$

$$\mathbf{H} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \quad (4.17)$$

so that

$$\mathbf{m}_{\tau} = \mathbf{H}\mathbf{m}_t \quad (4.18)$$

$$\mathbf{f}_{\tau} = \mathbf{H}\mathbf{f}_t \quad (4.19)$$

$$\mathbf{n}_{\tau} = \mathbf{H}\mathbf{n}_t \quad (4.20)$$

$$(4.21)$$

We can use $\mathbf{H}$ to try different combinations of measurements; for exam-

ple,

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & -1 \\ 0 & 1 & 0 & \cdots & 0 & -1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \quad (4.22)$$

references each measurement to the last one available. Using this $\mathbf{H}$ matrix makes it simple to try different combinations if the need arises.

The gradient matrix $\mathbf{G}_\tau$ is the first derivative of $\mathbf{f}_\tau$ evaluated at some point $\mathbf{x}$. Since $\mathbf{H}$ is a constant, it can also be used in the calculation of $\mathbf{G}$. We first calculate a gradient matrix for $\mathbf{f}_t$:

$$\mathbf{G}_t = \frac{1}{c} \begin{bmatrix} \frac{x-x_1}{R_1(x,y)} & \frac{y-y_1}{R_1(x,y)} \\ \vdots & \vdots \\ \frac{x-x_{N_s}}{R_1(x,y)} & \frac{y-y_{N_s}}{R_1(x,y)} \end{bmatrix} \quad (4.23)$$

Since $\mathbf{f}_\tau = \mathbf{H}\mathbf{f}_t$,

$$\mathbf{G}_\tau = \mathbf{H}\mathbf{G}_t \quad (4.24)$$

All that remains to determine is the covariance matrix. We assume the TOA errors are independent, so the TOA covariance matrix $\mathbf{N}_t$ will be diagonal with each station's measurement variance on the diagonal. The TDOA covariance matrix $\mathbf{N}_\tau$ is related to $\mathbf{N}_t$ by the relation

$$\mathbf{N}_\tau = \mathbf{H}\mathbf{N}_t\mathbf{H}^T \quad (4.25)$$

If TOA measurement error i is zero-mean Gaussian with variance $\sigma_{t,i}^2$, then the TDOA measurements will also be zero-mean Gaussian with variance

$$\sigma_{\tau,i} = \sigma_{t,i}^2 + \sigma_{t,i+1}^2 \quad (4.26)$$

The covariance of different TDOA measurements is then

$$\text{COV}(n_{\tau,i}, n_{\tau,j}) = \text{E}[n_{\tau,i}n_{\tau,j}] \quad (4.27)$$

$$= \text{E}[(n_{t,i} - n_{t,i+1})(n_{t,j} - n_{t,j+1})] \quad (4.28)$$

$$= \text{E}[n_{t,i}n_{t,j} - n_{t,i}n_{t,j+1} - n_{t,i+1}, n_{t,j} + n_{t,i+1}, n_{t,j+1}] \quad (4.29)$$

Clearly then for $i \neq j$

$$\text{COV}(n_{\tau,i}, n_{\tau,j}) = \begin{cases} -\sigma_{t,i}^2 & \text{for } i = j + 1, \\ -\sigma_{t,i+1}^2 & \text{for } i = j - 1, \\ 0 & \text{otherwise} \end{cases} \quad (4.30)$$

Having derived the necessary matrices, the algorithm at the end of section 4.1 may be applied to solve the TDOA estimation problem.

4.1.2 Angle of Transmission Estimator

The geometry of the synthetic Doppler AOT system is identical to that of an AOA system with the transmitters and receivers reversed. The position

estimator derivation for AOT then follows the AOA case in [11] nearly identically. The AOT estimator attempts to solve the equation

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \sum_{i \in S} (|-\sin \theta_i(x - x_i) + \cos \theta_i(y - y_i)|)^2 \quad (4.31)$$

where θ_i is the AOT measurement from station i .

The measurements θ_i relate the angle of transmission from the beacon at (x_i, y_i) to the receiver (at (x, y) , (see figure 4.1) yielding the expression

$$\theta_i = \arctan \left(\frac{y - y_i}{x - x_i} \right) + n_{\theta,i} \quad (4.32)$$

or in matrix form

$$\boldsymbol{\theta} = \mathbf{f}_{\theta} + \mathbf{n}_{\theta} \quad (4.33)$$

with

$$\mathbf{f}_{\theta}(x, y) = \begin{bmatrix} \arctan \left(\frac{y - y_1}{x - x_1} \right) \\ \vdots \\ \arctan \left(\frac{y - y_{N_s}}{x - x_{N_s}} \right) \end{bmatrix} \quad (4.34)$$

The gradient matrix can be shown to be

$$\mathbf{G}_{\theta} = \begin{bmatrix} \frac{-(y - y_1)}{R_1^2(x, y)} & \frac{(x - x_1)}{R_1^2(x, y)} \\ \vdots & \vdots \\ \frac{-(y - y_{N_s})}{R_{N_s}^2(x, y)} & \frac{(x - x_{N_s})}{R_{N_s}^2(x, y)} \end{bmatrix} \quad (4.35)$$

with $R_i(x, y)$ as defined in equation 4.13

Figure 4.1: AOT Geometry.

The noise covariance matrix for the AOT measurements is diagonal, since the measurement errors are independent:

$$N_{\theta} = \begin{bmatrix} \sigma_{\theta,1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\theta,2}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{\theta,N_s}^2 \end{bmatrix} \quad (4.36)$$

This derivation results in a non-linear description of the problem. There is an alternative formulation that is linear in $\mathbf{x}$ (nicely presented by Cong [24]). This problem formulation avoids the arctan term in the system of equations. Refer to Figure 4.2. Given the beacon at (x_i, y_i) , a measurement

Figure 4.2: Alternative AOT Geometry.

θ_i associated with that beacon, and an angular error θ_e associated with that measurement, we can write the position estimate equation:

$$r_i \sin \theta_e = (x - x_i) \sin \theta_i - (y - y_i) \cos \theta_i \quad (4.37)$$

Rearranging terms in order to isolate the unknown error variable $\theta_e(x, y)$, and using the approximation $\sin \theta_{e,i} \approx \theta_{e,i}$, we have

$$0 = \frac{-(x - x_i) \sin \theta_i}{r_i} + \frac{(y - y_i) \cos \theta_i}{r_i} + \theta_{e,i} \quad (4.38)$$

In terms of the position location problem, equation 4.38 for all beacons can

be summarized in a matrix equation:

$$\mathbf{0} = \mathbf{f}(\mathbf{x}, \boldsymbol{\theta}) + \mathbf{n}_{\theta_e} \quad (4.39)$$

where

$$\mathbf{m}_{\theta} = \mathbf{0} \quad \mathbf{f}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \frac{-(x-x_1)\sin\theta_1}{r_1} + \frac{(y-y_1)\cos\theta_1}{r_1} \\ \frac{-(x-x_1)\sin\theta_2}{r_2} + \frac{(y-y_2)\cos\theta_2}{r_2} \\ \vdots \\ \frac{-(x-x_N)\sin\theta_N}{r_N} + \frac{(y-y_N)\cos\theta_N}{r_N} \end{bmatrix} \quad \mathbf{n}_{\theta_e} = \begin{bmatrix} \theta_{e,1} \\ \theta_{e,2} \\ \vdots \\ \theta_{e,N} \end{bmatrix}$$

The system is now in the desired form for the Taylor-Series solution.

Taking the gradient of $\mathbf{f}(\mathbf{x}, \boldsymbol{\theta})$ yields

$$\mathbf{G}_{\mathbf{x},\boldsymbol{\theta}} = \begin{bmatrix} -\sin\theta_1/r_1 & \cos\theta_1/r_1 \\ -\sin\theta_2/r_2 & \cos\theta_2/r_2 \\ \vdots & \vdots \\ -\sin\theta_N/r_N & \cos\theta_N/r_N \end{bmatrix} \quad (4.40)$$

The covariance matrix for this system is identical to the one in equation 4.36.

As with the TDOA case, using these matrices in the algorithm from section 4.1 will yield a position fix.

4.1.3 Combined Estimator

Combining different kinds of estimates is very simple with the Taylor-Series estimator; we can simply stack the matrices from the individual estimators

to form the combined estimator.

The combined equation relating measurements and positions is

$$\mathbf{m} = \mathbf{f} + \mathbf{n} \quad (4.41)$$

$$\begin{bmatrix} \mathbf{m}_\tau \\ \mathbf{m}_\theta \end{bmatrix} = \begin{bmatrix} \mathbf{f}_\tau \\ \mathbf{f}_\theta \end{bmatrix} + \begin{bmatrix} \mathbf{n}_\tau \\ \mathbf{n}_\theta \end{bmatrix} \quad (4.42)$$

and the combined $\mathbf{G}$ matrix is

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_\tau \\ \mathbf{G}_\theta \end{bmatrix} \quad (4.43)$$

In practice, the arctan derivation of the AOT solution causes a high number of trials to fail to converge. The alternative AOT problem formulation described in the previous section was found to be much more stable (no doubt owing to its linearity).

The covariance matrix for the combined estimator has the form:

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_\tau & \mathbf{N}_{\tau,\theta} \\ \mathbf{N}_{\tau,\theta}^T & \mathbf{N}_\theta \end{bmatrix}. \quad (4.44)$$

The $\mathbf{N}_\tau$ matrix is defined in Equation 4.25, and $\mathbf{N}_\theta$ is defined in 4.36. The derivation of $\mathbf{N}_{\tau,\theta}$ is a little more complicated, because of the dependence of the AOT measurements on the TOA measurements. The AOT errors will normally be dominated by the phase measurement errors, since they are orders of magnitude larger than the TOA errors. Nevertheless, the covariance of the AOT and TOA is computed below for completeness.

We assume each FM modulation phase estimate is independent from its corresponding time of arrival estimate (because the chip duration is much lower than the frequency modulation period). Both types of measurement errors are assumed to be Gaussian random variables.

Define the TDOA estimate noise values $n_{\tau,i}$ and the AOT estimate noise values $n_{\theta,i}$. From equation 3.63, we have the noise term in the AOT measurement as $n_{\theta,i} = (2\pi f_m)(n_{t,i} - n_{d,i})$, or in vector form,

$$\mathbf{n}_{\theta} = (2\pi f_m)(\mathbf{n}_t - \mathbf{n}_d) \quad (4.45)$$

In matrix form, the covariance matrix $\mathbf{N}_{\tau,\theta}$ can be calculated directly:

$$\mathbf{N}_{\tau,\theta} = \mathbf{E}[\mathbf{n}_{\tau}\mathbf{n}_{\theta}^T] \quad (4.46)$$

$$= \mathbf{E}[\mathbf{H}\mathbf{n}_t(2\pi f_m)(\mathbf{n}_t - \mathbf{n}_d)^T] \quad (4.47)$$

$$= (2\pi f_m)\mathbf{H}\mathbf{E}[\mathbf{n}_t(\mathbf{n}_t - \mathbf{n}_d)^T] \quad (4.48)$$

$$= (2\pi f_m)\mathbf{H}\mathbf{E}[\mathbf{n}_t\mathbf{n}_t^T - \mathbf{n}_t\mathbf{n}_d^T] \quad (4.49)$$

$$= (2\pi f_m)\mathbf{H}\mathbf{N}_t \quad (4.50)$$

where we have used the fact that the doppler errors $\mathbf{n}_d$ and the TOA errors $\mathbf{n}_t$ are independent. The $(N_s - 1 \times N_s)$ covariance matrix for the TDOA and AOT measurements has the resulting form:

$$\mathbf{N}_{\tau,\theta} = 2\pi f_m \begin{bmatrix} \sigma_{t,1}^2 & -\sigma_{t,2}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{t,2}^2 & -\sigma_{t,3}^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \sigma_{t,N_s-1}^2 & -\sigma_{t,N_s}^2 \end{bmatrix} \quad (4.51)$$

As with TDOA and AOT, these matrices may be used to solve for the user position using the estimator algorithm described at the beginning of this chapter.

4.1.4 Closed Form Estimators

The Taylor-series estimators are not guaranteed to converge to the correct answer. Convergence behavior is improved by using the actual target position as the initial point (this approach was chosen for the experiments presented in Chapter 6). In practice, this is of course unknown. Sub-optimal closed-form estimators have been developed for position estimation that can be used to obtain an initial estimate close to the true position.

A closed-form estimator for TDOA is given by Chan [15]. It can be extended for combined angle/TDOA measurements, as described in [25]. In the simulations presented in Chapter 6, the TDOA and combined Taylor-series estimators performed reasonably well, so the closed-form estimators were not used.

The AOT measurements do have convergence problems, especially when there are NLOS errors present or there are only two or three measurements. For some cases a closed-form estimator was employed; it is developed here. There is a simple solution for angle-based position estimation, given in Poisel ([6], p.386). The AOT miss distance for each station i is (see Figure 4.1)

$$d_i = a_i x_T + b_i y_T - c_i \quad (4.52)$$

where i is the i^{th} AOT measurement, and a_i , b_i , and c_i are given by

$$a_i = \sin \theta_i \quad (4.53)$$

$$b_i = -\cos \theta_i \quad (4.54)$$

$$c_i = x_i \sin \theta_i - y_i \cos \theta_i \quad (4.55)$$

Combining the measurements for all i into matrices, we get

$$\mathbf{D} = \mathbf{H}\mathbf{P} - \mathbf{C} \quad (4.56)$$

where

$$\mathbf{C} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} x_T \\ y_T \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} a_0 & b_0 \\ a_1 & b_1 \\ a_2 & b_2 \\ \vdots & \vdots \\ a_N & b_N \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}$$

The least-squared estimator for $\mathbf{P}$ is

$$\hat{\mathbf{P}} = [\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}]^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{C} \quad (4.57)$$

$\mathbf{R}$ is a weighting matrix for the estimates; the covariance matrix of the AOT measurements may be used here.

A simplified closed-form AOT solution was used in trials with only two measurements. If both estimates have identical zero-mean noise variables, the ideal solution is the intersection of the two rays.

4.2 Measures of Accuracy

To evaluate the performance of these algorithms, some measure of accuracy is required. A simple method is to calculate the mean squared error of the fix, defined as

$$\text{MSE} = \text{E}[(x - \hat{x})^2 + (y - \hat{y})^2]. \quad (4.58)$$

Usually the Root Mean Square Error (RMSE) is used:

$$\text{RMSE} = \sqrt{\text{E}[(x - \hat{x})^2 + (y - \hat{y})^2]} \quad (4.59)$$

The RMSE can also be used for comparison with a theoretical bound for the Taylor-series algorithm. It is shown in [11] that for the least-squares estimator, the covariance matrix of the 2-D position vector is

$$\mathbf{P} = (\mathbf{G}^T \mathbf{N}^{-1} \mathbf{G})^{-1} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}, \quad (4.60)$$

where $\mathbf{G}$ is calculated using the actual position of the receiver. Now by definition the RMSE is

$$\text{RMSE} = \sqrt{\sigma_x^2 + \sigma_y^2} \quad (4.61)$$

where σ_x and σ_y are the variances of the position estimate. Thus the sum of the diagonal elements of $\mathbf{P}$ is the lower bound on the squared error of the estimator. Note that this bound only takes into account the Gaussian errors described by the error covariance matrices in the estimators.

There are various other measures of accuracy for position location. Circular Error Probable (CEP) is defined as the radius of a circle, with the position fix mean at its center and having a 50 percent probability of containing the actual position. The CEP can be approximated within about 10 percent by the formula [11]

$$CEP \approx 0.75\sqrt{\sigma_x^2 + \sigma_y^2} \quad (4.62)$$

The Geometric Dilution of Precision (GDOP) is a measure of the how the various measurement errors are affected by the geometry of the station positions. It is defined as the ratio of RMS position error to the RMS measurement error. For time measurements, the mean-squared ranging error is defined as $c^2\sigma_s^2$, where

$$\sigma_s^2 = \frac{1}{N} \sum_{i=1}^N \sigma_{t,i}^2 \quad (4.63)$$

Then the GDOP for timing-based systems is

$$GDOP = \sqrt{\text{trace}\mathbf{P}}/c\sigma_s. \quad (4.64)$$

For angle measurements, the mean-square ranging error is defined as

$$\sigma_d^2 = \frac{1}{N} \sum_{i=1}^N R_i(x, y) \sigma_{\theta,i}^2 \quad (4.65)$$

The GDOP for angle-based systems is

$$GDOP = \sqrt{\text{trace}\mathbf{P}}/\sigma_d. \quad (4.66)$$

The evaluation of simulation results in Chapter 6 uses the RMSE.

4.3 Multipath Mitigation

Multipath is the chief source of unpredictable errors in radio location systems. The location-finding algorithms described above only account for Gaussian-type errors in the measurements. But in the highly cluttered environment of in-building propagation, multipath will be an ever-present issue. The general problem is that it is difficult to detect which measurements have been corrupted by multipath.

Various approaches have been tried to mitigate errors. Wylie and Holtzman [26] use a time history of the measurements to identify and remove NLOS errors. Woo et al [18] use a Kalman filter to smooth their results. We will consider very simple method to remove badly corrupted measurements is called the Residual Weighting Algorithm (RWGH), presented by Chen in [19] and applied to TDOA by Cong and Zhuang [27]. A very similar algorithm for AOA NLOS suppression is given by Xiong in [28]. The RWGH method uses extra measurements to reduce the impact of those containing NLOS errors; it has the advantage of not requiring knowledge of the distribution of the errors, but requires additional measurements with some LOS paths to work.

The abundance of measurements available in the synthetic doppler concept makes RWGH an attractive choice.

The RWGH method makes use of redundant measurements to reduce the impact of NLOS-corrupted data. For a given type of measurement, there are a minimum number of stations required to calculate an unambiguous fix. For a two-dimensional position estimate, four stations are needed for a TDOA-based fix, and at least two are required for an AOT fix. For the sake of this discussion we concentrate on the TDOA case. If more than the minimum required measurements are available, every possible combination of stations can be combined to produce a collection of location estimates. For example (following the argument in Chen's paper [19]), if there are five available stations and three measurements are required, we can choose to use all five, four, or three measurements, resulting in a total of 16 combinations:

$$5 \text{ out of } 5 : \binom{5}{5} = 1 \text{ combination} \quad (4.67)$$

$$4 \text{ out of } 5 : \binom{5}{4} = 5 \text{ combinations} \quad (4.68)$$

$$3 \text{ out of } 5 : \binom{5}{3} = 10 \text{ combinations} \quad (4.69)$$

The fitness of each estimate can be measured by the extent to which the position estimate agrees from the provided data. For TDOA, the least-squares estimator attempts to solve equation 4.10. The residual of a fix is defined as the value of this equation with the estimate $\hat{x}$ as the argument:

$$R_{es}(\hat{x}, S) = \sum_{\substack{i,j \in S \\ i \neq j}} (|r_i - r_j| - ||\hat{x} - X_i|| - ||\hat{x} - X_j||)^2 \quad (4.70)$$

where $\hat{x}$ is the location solution, S is the set of range measurements, X_i is the position of station i , and r_i is the i^{th} range measurement.

To remove the dependance of the size of the set S from the measurement, the normalized residual is defined as

$$\widetilde{R}_{es}(x, S) = \frac{R_{es}(x, S)}{|S|} \quad (4.71)$$

where $|S|$ is the size of S . The lower the residual, the closer the position fix agrees with the data.

The RWGH algorithm is as follows:

1. Starting with N_s measurements, form $N = \sum_{i=3}^{N_s} \binom{N_s}{i}$ measurement combinations.
2. For each combination of stations S_k , calculate a position fix $\hat{x}_k$ and its normalized residual $\widetilde{R}_{es}(\hat{x}_k, S_k)$.
3. Compute the final estimate as a weighted linear combination of all the estimates:

$$\hat{x} = \frac{\sum_{k=1}^N \hat{x}_k \left(\widetilde{R}_{es}(\hat{x}_k, S_k) \right)^{-1}}{\sum_{k=1}^N \left(\widetilde{R}_{es}(\hat{x}_k, S_k) \right)^{-1}} \quad (4.72)$$

This method can also be applied to AOT and combined AOT/TDOA fixes. From equation 4.31, the residual is defined as

$$R_{es}(x, S) = \sum_{i \in S} (|-\sin \theta_i(x - x_i) + \cos \theta_i(y - y_i)|)^2 \quad (4.73)$$

For the combined estimator, we simply combine the two residual types:

$$R_{es}(x, S) = \sum_{i \in S} (|-\sin \theta_i(x - x_i) + \cos \theta_i(y - y_i)|)^2 \quad (4.74)$$

$$+ \sum_{\substack{i, j \in S \\ i \neq j}} (|r_i - r_j| - \|\|x - X_i\| - \|x - X_j\|\|)^2 \quad (4.75)$$

Using these definitions for R_{es} , the RWGH algorithm is otherwise unchanged for AOT and combined position estimators.

4.3.1 Additional NLOS Mitigation Strategies

Different researchers have published various studies using variations of the RWGH algorithm described above. It's not obvious which strategy is the best, so a number of variations were tested in simulations. These are described below.

Cong [27] applies the RWGH to TDOA, with a couple of changes: the LS estimator takes into account the NLOS errors and their distribution (exponential), and the residuals are calculated in order to remove the worst measurements entirely. The procedure for detecting the worst measurement is as follows:

- Calculate a BS goodness quality as the sum of the residuals of sets containing that BS.
- Remove the BS with the highest weight.

In Cong's paper, the effect of using of different position estimates in the residual calculations is explored. They are calculated using either the current estimate found with that measurement set, or the overall estimate found with all the measurements. They find using the current estimate in the calculation tends to work better for a large number of stations (five or more), and the overall one (i.e. the estimate using the full set) works better for fewer measurements.

Xiong [28] presents yet another approach for NLOS mitigation with AOA measurements. First the overall location estimate is found by normal means. Then for each station/measurement, the individual error residual is calculated:

$$R_{es}(x, S) = (|-\sin \theta_i(x - x_i) + \cos \theta_i(y - y_i)|)^2 \quad (4.76)$$

Each residual is compared to the RMS value of all the estimates, and any that have a higher value than some threshold (e.g. $1.5 \text{ RMS}(R_{es})$) are removed. The difficulty with both these approaches is deciding how to set the threshold to remove measurements.

In the experiments presented in Chapter 6, the following variations of RWGH were considered in NLOS mitigation trials:

- RWGH-C: RWGH as implemented by Chen. Intermediate estimates for each set are used to evaluate R_{es} , and the final answer is the weighted combination of individual solutions.
- RWGH-O: RWGH as described above, but using the all-station position

estimate to evaluate the residues.

- RWGH-C-R: RWGH as described above, but instead of forming a weighted combination, stations with exceptionally bad residuals are removed from the system. The residuals are calculated for each set, and then for each station, a mean is formed of the residuals associated with that station:

$$R_{\text{total},i} = \frac{\sum_{i \in S} R_{es}(x, S)}{|S_i|} \quad (4.77)$$

where $|S_i|$ is defined as the number of sets with station i . This is compared to the overall mean of the residuals:

$$R_{\text{total}} = \frac{\sum_S R_{es}(x, S)}{|S|} \quad (4.78)$$

Stations with a mean residual greater than $1.4R_{\text{total}}$ are removed from consideration, until the minimum number of stations is reached or the remaining mean residuals lower than the threshold

- RWGH-O-R: bad stations are removed in the same manner as RWGH-C-R, but the residuals are calculated using the all-station estimate.
- good: the ideal case in which NLOS-corrupted measurements are always correctly identified and discarded.

The performance of each of these methods is shown in Chapter 6 for a variety of measurement types and environmental conditions.

4.4 Chapter Summary

In this chapter the algorithms for measuring position with the synthetic Doppler system were presented and evaluated. The basic scenario is this: a number of beacons arranged around an area of interest and a target, roughly situated somewhere in the middle of all the beacons. Noise-corrupted AOT and TDOA measurements are available from each of the beacons. The combination of station locations and measurement data are combined into a system of nonlinear equations, that may be solved using least-squares estimation and Taylor-series linearization. Some closed-form estimators were also presented, in particular one for badly-behaved AOT situations. Measures of accuracy are discussed, and the CRLB is presented as a bound on performance.

Algorithms for the mitigation of multipath errors were also presented. Several variations of a simple method, RWGH, are described. The RWGH methods use extra sensor information to identify multipath-corrupted measurements, and either reduce their bearing on the position fix or remove them altogether.

Chapter 5

Simulation Model

5.1 Introduction

Testing the effectiveness of the position estimation algorithms presented in the previous chapter requires a suitable channel model. Specifically, it must account for the geography of the test scenarios (i.e. placement of transmitters and receiver), and provide realistic TOA and AOT measurements from each beacon. It must also provide multipath-corrupted estimates for TOA and AOT. Since any AOT value is dependant upon the geometry of the scenario, using a ray-tracing model is an obvious approach.

There are various approaches to ray tracing to predict radio propagation; the motivation for different techniques is based on the computational complexity of the method, and the type of information that is desired. Many ray

tracing models have been built in recent years (see [29], [30], [31], [32]), but they are all designed to predict indoor radio coverage as a function of signal strength. In fact, most only use their results as parameters for a statistical model of radio coverage. The requirements for this thesis demand a more flexible model, albeit one that had to be created from scratch.

A deterministic, 2-dimensional ray-tracing channel model was developed specifically for the synthetic Doppler evaluation. The goal for this model was to produce channel responses similar in character to what one might expect from the real world, but not necessarily to accurately predict coverage. The basis for this model is the image method, as described in McKown and Hamilton's paper [33]. This approach has the benefits of being relatively straightforward to implement.

Using the image method solves the specific problem we're considering: finding all the paths a radio signal can take from a transmitter to a receiver, and reporting their path delays and AOTs. The simulator calculates a snapshot of the propagation channel between a single transmitter and a single receiver. The scenario is represented on a 2-D plane (extending the model to three dimensions adds significant complexity without adding much value). Structure walls are represented by line segments; they have coefficients of transmission and reflection loss that vary by material type. Antennas are considered to be single points – we assume that the rotation of the synthetic doppler antenna would be about a small radius that wouldn't drastically alter the AOT and TOA elements of the ray paths. Given this information,

the algorithm calculates all paths from transmitter to receiver with up to an arbitrary number of reflections. This data represents the site-specific channel profile for the given scenario. TOA and AOT values are extracted from this profile based on system parameters.

5.2 The Image Method

The image method is a way of calculating possible paths between a particular source and destination. The model described below ignores some components of radio propagation, such as diffraction (diffraction is not thought to be of great importance in small-scale propagation [30]). As mentioned above, it is only the basic character of the channel that we are trying to capture.

The basic concept of the image method is illustrated in Figure 5.1. The figure shows a transmitter position t , receiver position r , and line A . To find the one-bounce path from the transmitter to the receiver, the receiver is reflected off the surface A to obtain a virtual receiver r_A . A line is drawn from t to r_A . If the line intersects A , it is a valid reflection. The intersection point x is calculated, and the complete ray is stored as the two lines $t\vec{x}$ and $x\vec{r}$. If the line from t to r_A did not intersect A , then no one-bounce path off of A would exist.

As additional walls are added to the model, the process is repeated recursively, as shown in Figure 5.2. Here the receiver is first reflected through line B . Regardless of whether there was a valid one-bounce reflection from this

Figure 5.1: Reflection with one bounce.

surface, the virtual receiver is reflected by line A to obtain a second virtual receiver at r_{AB} . A line is drawn from t to r_{AB} . If it intersects line A at point x , a second line is drawn from x to r_B . If this line intersects B at y , the path is valid and the complete ray path is composed of lines $\vec{tx}$, $\vec{xy}$, and $\vec{yr}$. Note that the technique is valid for walls at any angle, but some optimizations can be employed if all walls are at right angles to each other, as in the figure.

Either the transmitter or the receiver can be reflected off each wall; in this implementation, the transmitter is reflected. Each reflected transmitter can

Figure 5.2: Reflection with two bounces.

be reflected again off of every surface. To see if any of these yield valid paths, the chain of virtual transmitters must be checked for intersections with their corresponding lines until the actual transmitter is reached.

This can be implemented with a tree structure, where each node on the tree is a virtual transmitter, and all of its children are reflections from all walls in the model. The size of the tree (and the running time of the algo-

rithm) grows exponentially with the number of reflections. The root node, representing the transmitter, has N children (one for each reflection). Each child beyond that has $N - 1$ children, representing their virtual transmitter reflections. The depth of the tree is equal to the desired number of reflections.

The algorithm may be summarized as follows. For each wall (i.e. line):

1. Find the reflection of the transmitter off the wall.
2. Find the intersection point between the line from the virtual transmitter to the receiver, and the wall (if it exists).
3. If there is a valid intersection, add this component to the ray path and check the parent for a valid intersection. Repeat recursively until the parent is the root node (i.e. the actual transmitter)
4. Repeat the above steps for every virtual transmitter, until the desired number of reflections is calculated.

Once all valid paths have been calculated, they must be searched for transmissions through walls. For each path:

1. Check every segment of the ray path for intersection with every wall element.
2. For every detected transmission, add the transmission loss to that ray path's profile

This algorithm was implemented in a C++ program. Scenario inputs – wall vertices and characteristics, transmitter and receiver locations – are generated in Matlab and passed to the program in a text file. The program runs the algorithm for a given number of reflections, and saves the results to a file, containing the line segment vertices, reflection and transmission losses, path losses, AOTs, and path lengths for that particular channel. Follow-on processing (e.g. position estimation) and display of this information is performed in Matlab.

5.3 Propagation Power Losses

Three sources of propagation loss are accounted for in this implementation of the image method: transmission and reflection losses, and free-space path loss. The methods of calculating these factors are described below.

5.3.1 Transmission and Reflection

A ray encountering an obstacle has three outcomes; the transmitted ray, the reflected ray, and the energy absorbed by the material. The energy of these three effects must total the energy of the incident ray. It follows that by determining any two of the effects, the third may be calculated directly.

The amount of energy in a ray reflected off a material is dependant on the electrical properties of the material. Pahlavan [29] and Rappaport [17]

give functions for reflection coefficients:

$$R_h = \frac{\sin \Psi - \sqrt{\epsilon - \cos^2 \Psi}}{\sin \Psi + \sqrt{\epsilon - \cos^2 \Psi}} \quad (5.1)$$

for horizontal polarization and

$$R_v = \frac{\epsilon \sin \Psi - \sqrt{\epsilon - \cos^2 \Psi}}{\epsilon \sin \Psi + \sqrt{\epsilon - \cos^2 \Psi}} \quad (5.2)$$

for vertical polarization, where Ψ is the angle of incidence and ϵ is the complex permittivity of the material. This is related to the dielectric and conductive properties of the reflecting material, and the signal wavelength. More complex models can take into account the depth of the reflecting surface (e.g. reflection and transmission through a uniform slab).

In our model, a simplified method was used. Reflection losses are fixed for all materials, regardless of the incident angle and other properties. This approach is used in [33], where they assign 3 dB loss to each transmission and reflection. For our model, different materials are assigned transmission and reflection coefficients based loosely on tables of transmission loss for various building materials in [17], pp. 124-125.

Transmission ray energy can be calculated by subtracting the reflected and absorbed energy from the incident ray energy. Again for simplicity we assume no energy is absorbed, so the reflection coefficient is determined by ensuring that power is conserved in the system.

So for an incident ray with power P , the reflected ray has power $R^2 P$ and

the transmitted ray T^2P , so to conserve energy we have:

$$\begin{aligned} P &= R^2P + T^2P \\ 1 &= R^2 + T^2 \end{aligned}$$

Given a transmission coefficient based on the tables in [17], the reflection coefficient is

$$R = \sqrt{1 - T^2} \quad (5.3)$$

Note that phase changes due to reflection and transmission are also ignored. Since for synthetic Doppler the important modulation attributes are frequency- and pulse-modulated, fixed phase shifts may be ignored.

5.3.2 Free Space Path Loss

In addition to transmission and reflection losses, radio signals weaken as they travel through space. This channel simulator calculates path loss as described in [17]. The path distance is calculated using the total length of all path components of each signal.

The received power at an antenna d metres from the transmitter is given by

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \quad (5.4)$$

where P_t is the transmitted power, $P_r(d)$ is the received power, G_t is the transmitter antenna gain, G_r is the receiver antenna gain, d is the distance

between transmitter and receiver, and L is the system loss factor. For simplicity we assume the antennas have unity gain ($G_t = G_r = 1$), and there is no system loss ($L = 1$).

The propagation loss is expressed as Path Loss (PL), which is a positive quantity representing the signal attenuation from free space propagation. Taking into account the assumptions already mentioned, the path loss as a function of distance in dB is

$$PL = 10 \log \frac{P_t}{P_r} = -20 \log \frac{\lambda}{4\pi d} \quad (5.5)$$

This is valid in the far-field distance d_f of the transmission antenna, defined as

$$d_f = \frac{2D^2}{\lambda} = \frac{2D^2 f}{c} \quad (5.6)$$

where D is the largest linear dimension of the antenna and λ is the transmission wavelength (and c is the speed of light). Also, d_f must satisfy

$$d_f \gg D$$

$$d_f \gg \lambda$$

To check the validity of using this model, we must ensure that we are operating in the far field of the antenna. Consider these examples with reasonable parameters for the synthetic Doppler transmitter. For a $D = 1$ m antenna operating in the 900 MHz frequency range, the far field distance is

$$d_f = \frac{2(1)^2 900 \times 10^6 \text{ Hz}}{3 \times 10^8 \text{ m/s}} \text{ m} = 6 \text{ m} \quad (5.7)$$

For a $D = 0.1$ m antenna operating at 2 GHz range, the far field distance is

$$d_f = \frac{2(0.1)^2 2 \times 10^9 \text{ Hz}}{3 \times 10^8 \text{ m/s}} \text{ m} = 0.1\bar{3} \text{ m} \quad (5.8)$$

In these simulations the beacons are placed tens of metres away from the buildings, and the receivers are inside buildings, so the far-field propagation requirement holds in most instances.

5.4 TOA and AOT estimation

This model produces a power-delay channel profile representing the channel as seen from the receiver (see Figure 5.3 for an example). The problem remains to obtain TOA and AOT estimates from this deterministic profile. There are two issues to deal with: choosing the first detectable arriving path, and determining the variance of the AOT and TOA errors associated with that path (aside from multipath error, which is deterministic and related to the geometry of the players in the scenario).

The channel profile contains deterministic information for all the possible arriving paths – a perfect receiver in the absence of noise would obtain the exact AOT and TOA from first detectable path. In reality the time resolution of the receiver is limited by the bandwidth of the synthetic Doppler signal (which is based on the chip rate), and the arriving signals would be subject to fading. The effect of signal bandwidth can be found by filtering the received profile with the transmission pulse shape (see Alavi and Pahlavan [34]).

However this simulation does not extend to modeling the receiver, that is, generating a synthetic Doppler signal, filtering it with the channel profile, and demodulating it as described in Chapter 3. This would be more realistic, but it would make analysis of the location algorithms far more complicated – it would be difficult to control the AWGN error statistics of the TOA and AOT measurements. The goal of the channel simulator is to produce multipath-corrupted AOT and TOA measurements that are consistent with one another. We are not particularly concerned about accurately predicting propagation or the effect of fading due to intra-symbol multipath.

With this in mind, the TOA and AOT values are calculated as follows. For a particular receiver\transmitter set, a maximum attenuation value is chosen. Any paths with an attenuation greater than this value are removed from the channel profile. The first arriving path in the remaining set is assumed to be demodulated by the receiver, and the AOT and TOA of that path are recorded. For a particular simulation, zero-mean Gaussian random errors are added to these values. The variance of each error can be determined using the SNR of the chosen path and the values derived in Chapter 3, or chosen arbitrarily to be a particular value. The second method is used for the simulations, so the PL algorithm performance may be evaluated with respect to the number and magnitude of multipath-corrupted measurements.

5.5 Channel Simulation Results

For a particular scenario, the program is run with a fixed receiver position and repeated for each beacon position. This is repeated for various receiver positions to amass a catalogue of channel profiles for each beacon-receiver pair. An imaginary building was designed with various material types, with the aim of generating a range of multipath-corrupted situations. Figure 5.3 shows a top-down view of a scenario, in which several paths between a single beacon and receiver are shown. Figure 5.4 shows the related channel power-delay profile; only those rays arriving with power above the threshold are shown in the figure.

In order to make meaningful comparisons of the algorithm performance for various combinations of multipath errors, the receiver position data is sorted according to the number of visible beacons, and number of ULOS measurements. Figure 5.5 shows the breakdown of multipath-corrupted measurements as the maximum attenuation is changed. Armed with a large number of cases, it is clear that there are many examples of every desirable combination of multipath.

5.5.1 Computational Complexity and Running Time

The image method algorithm grows exponentially in complexity with the number of reflections, but signals tend to fade below the detectable threshold very quickly as the number of reflections increases. For the building geometry

Figure 5.3: Propagation results for a receiver-beacon pair.

shown in Figure 5.3, the dominant rays tend to be among those with the shortest paths and the fewest reflections. Four reflections was found to be sufficient to generate a large range of combinations of multipath errors.

Running times are reasonable on a Pentium IV 2.4 GHz computer – the data generated for Figure 5.3 required about ten hours to calculate all paths with five reflections for five beacon-receiver pairs.

Figure 5.4: Channel profile for scene in Figure 5.3.

5.6 Chapter Summary

In this chapter a simple channel model for predicting indoor radio propagation is presented. The model is based on ray-tracing, and uses the image method to calculate possible paths between a transmitter and receiver. Calculation of power losses is based on simplified models of transmission, reflection, and free-space propagation loss. Channel profiles are produced for

Figure 5.5: Sorting of measurement error types.

each particular transmitter-receiver position pair, from which AOT and TOA values can be extracted. Results from this model are used in the simulations presented in Chapter 6.

Chapter 6

Simulation Results

In this chapter the results of the simulation experiments are presented. The experiments compare positioning algorithms using TDOA, AOT, and combined measurements as described in Chapter 4, under controlled conditions and using the channel model of Chapter 5. The simulations are based on the Synthetic Doppler beacon system of Chapter 3, but the results from that analysis are not directly applied here – the focus is on the performance of the positioning algorithms.

The algorithms are compared under a variety of conditions. The following variables are investigated in terms of their effect on positioning accuracy:

- the number of beacons,
- the measurement estimate accuracy,

- the contribution of AOT and TOA accuracy to the combined algorithm's performance,
- the performance of the RWGH algorithm variations for controlled NLOS errors, and
- the performance of the RWGH algorithms for the ray-tracing channel model.

A number of factors are common to most of the experiments. All trials results are based on 10,000 runs for each scenario. The AOT and TOA measurements in each trial are calculated for the scenario's geometry, and random errors are added for each run. TOA and AOT measurement errors are simulated as zero-mean, Gaussian random variables. For all trials except those based on the channel model, the target position was varied randomly for each run inside a bounded region to reduce the overall impact of GDOP. This introduces a problem for comparing with the CRLB, since it is dependant on the geometry of each case. The reported CRLB is the RMS CRLB for all the runs. Positions are reported in metres, and position estimate accuracy is given by the RMSE in metres.

In all simulations, the true position of the target is used as the initial point for the positioning algorithms to improve the odds of convergence. In trials with only Gaussian random errors, this is sufficient to guarantee convergence on the correct solution in nearly all of the cases. Occasionally situations with no converging solution arise; these cases are discarded. This

happens on the order of about 0.1 percent of the trials, and generally arise from unfavorable geometry. More precisely, the non-convergent cases are isolated to a small number of geometries of beacons and target that exhibit a high-non-convergence rate.

Convergence has an impact on the RWGH algorithms. The RWGH-C algorithms require an intermediate estimate for each combination of stations. The non-convergence rate can be somewhat high for some combinations, so this is a problem – for many of the scenarios, every run contains some non-convergent situations. The solution chosen here is to ignore combinations that did not provide an answer. This does not affect the ‘overall’ algorithms but it may impact on the performance of the ‘current’ algorithms.

6.1 Trial 1: Effect of Number of Beacons

In the first experiment the goals are twofold. First, we wish to verify that the algorithms are performing as well as expected. This is determined by comparing the experimental RMSE to the CRLB for each case. The second goal is to determine the effect of the number of beacons on the positioning error. To this end, simulations were run using two to eight beacons. The arrangement of the beacons and the region in which the receiver was located are shown in Figure 6.1. The results for N beacons include every combination of size N from the entire set, which averages the effect of GDOP. The TOA and AOT error variances in all trials were fixed at 2 metres and 2 degrees,

respectively.

The results from the trial are collected in Table 6.1. Note the RMSE is very close to the CRLB for each case. In some cases the RMSE is actually slightly lower than the CRLB. The reason for this is that the simulation takes an additional factor into account – it is taken as given that the receiver is within a bounding box (defined as a box slightly larger than the receiver position selection box). Solutions found outside of this box are discarded. For some receiver-beacon geometries, the GDOP is exceptionally bad, causing RMS errors orders of magnitude larger than in the majority of cases. The bounding box affects the selection of results and invalidates the CRLB for those cases.

One case deserves special mention; the two-beacon AOT CRLB is extremely poor, yet the RMSE is an order of magnitude lower. In fact for two beacons with zero-mean errors, the best solution for positioning is to compute the intersection of the two lines of bearing from the beacons. Again due to the bounding box, only cases with favorable results are reported, invalidating the statistical significance of the CRLB in this case.

Overall this shows the algorithms are working as well as expected. This table also illustrates the effect of the number of beacons. More measurements improve the estimates, but the gains drop off rather quickly. There are dramatic differences going from two to four measurements, but five and above don't make a huge difference. For further trials, five beacons were used as a baseline.

Table 6.1: Basic Algorithm Results for 2 to 8 Beacons. All values are reported in meters.

# Stations	Algorithm Type					
	TDOA		AOT		Combined	
	CRLB	RMSE	CRLB	RMSE	CRLB	RMSE
2	N/A	N/A	221.2136	12.7880	4.7088	4.3720
3	4.5051	4.5784	4.4150	4.4379	2.1559	2.1553
4	2.1587	2.1652	3.2666	3.2658	1.5104	1.5091
5	1.5415	1.5404	2.7257	2.7267	1.2375	1.2386
6	1.2772	1.2778	2.3846	2.3820	1.0827	1.0848
7	1.1251	1.1299	2.1428	2.1495	0.9784	0.9801
8	1.0223	1.0218	1.9577	1.9589	0.9014	0.9000

Figure 6.1: Trial 1 Geometry.

6.2 Trial 2: Effect of AOT and TOA accuracy

The aim in trial 2 is to look at the effects of measurement accuracy on the position estimate accuracy. Following the conclusions from trial 1, five beacons were used for these simulations. The beacons were positioned at (0,0), (75,-50), (70, 20), (25, 15), and (100, -15) – see Figure 6.2. TOA accuracy was varied over the range $\{0.1, 0.5, 1, 2, 3, \dots, 20\}$ metres squared, and AOT accuracy was varied over the range $\{0.1, 0.5, 1, 2, 3, \dots, 20\}$ degrees squared. Using these values, all three estimate algorithms were run 10,000 times for each measurement value. The combo method was run over both complete ranges.

The results for TDOA and AOT performance are shown in Figure 6.3. Note that again the algorithm performance is very close to the CRLB. To compare the techniques fairly, we must keep in mind that the positioning errors for angle-based estimation are proportional not only to the measurement variance, but also to the distance from the beacon to the target. The mean distance from each station to the centre of the box of valid receiver positions in this trial is about 124 metres. Thus for this trial, the AOT measurement variance in metres squared is roughly equal to five times the variance in degrees squared. Nevertheless, we can see that TDOA is much more robust in dealing with larger errors. For example, a 10 meter squared measurement variance for TDOA corresponds roughly with a 2.5 degree squared angle error variance in terms of fix accuracy (for the geometry of this trial). Angular

Figure 6.2: Trial 2 Geometry.

Figure 6.3: Trial 2 TDOA vs. AOT Performance.

error adds up quickly as the distance increases; this is a relatively small area with a high number of stations, but the AOT positioning performance is already rather poor.

Figure 6.4 shows the simulation RMSE and RMS CRLB performance bound for the combo algorithm as a function of the TOA estimate error variance and the AOT error variance. As with the TOA and AOT cases, the CRLB is very close to the actual performance. While this graph is more difficult to interpret than for the other cases, we can see that even the worst-case error here is actually better than most of the single-measurement-

type fix cases. One might suppose that the combined estimator would only perform as well as it's best component, but this trial clearly illustrates that it performs better than the sum of its parts.

6.3 Trial 3: Effect of multipath and NLOS mitigation

In trial 3, the effect of adding one multipath-corrupted measurement to the simulation data was investigated. We also look at the ability of the RWGH algorithm variations to deal with this error. For each trial, the unmodified Taylor-Series solution was calculated, as were the results for each of the RWGH variations (RWGH-C, RWGH-O, RWGH-C-R, and RWGH-O-R). For comparison, a 'good' case is also presented – this is the ideal case where the corrupted measurement is correctly identified and removed.

A simplified channel model was used for this trial, in order to control the character of the NLOS errors. The geometry of the trial is the same as trial 2. A user position and beacon positions were chosen arbitrarily, and the expected TOA and AOT values were calculated for each beacon-receiver pair. For each run, one measurement was randomly selected for NLOS corruption.

Figure 6.4: Trial 2 Combo Performance.

6.3.1 Trial 3a: 5/1 TDOA NLOS mitigation testing

For the TDOA algorithm, one in five measurements was corrupted with a NLOS error ranging from 0 to 80 meters, in 10 meter increments. In each run a different beacon measurement was chosen randomly to be corrupted. Figure 6.5 shows the results for all algorithms for a range of NLOS errors. Note the naive algorithm's RMSE is linearly proportional to the magnitude of NLOS error.

In the top graph, the RWGH-O and RWGH-C perform much worse than the naive algorithm. This highlights a problem with the RWGH algorithm; the fixes using the minimum number of station measurements often produce answers that agree very well with the measurement data, but that are also extremely poor. The lower graph shows results using a minimum of four beacons for the measurement combinations. In this case, RWGH-C leaps to the top. Both removal algorithms fare poorly in this case (their results are identical so one is obscured).

Oddly, RWGH-C-R works extremely well in the min-3-station case, but is worse than the naive algorithm in the min-4-station case.

6.3.2 Trial 3b: 5/1 AOT NLOS mitigation testing

For the AOT trials, one in five measurements was corrupted with an angular error of -90 to 90 degrees, in steps of 10 degrees. For each run, one of the beacons was randomly selected for the corrupted measurement. Figure 6.6

Figure 6.5: Trial 3 TDOA NLOS Mitigation.

shows all algorithms for a range of NLOS errors. The top graph uses min 2 stations, and bottom uses min 3 stations.

As with TDOA, the RMSE of the naive algorithm is more or less linearly related to the magnitude of the NLOS error. Note also that the graphs are symmetric about the 0-error case. As with TDOA, RWGH-C improves dramatically when the min-station case is not the absolute minimum required. In the min-2 case there is no redundant information to identify the bad station combinations as bad. Interestingly, the RWGH-C-R performed nearly as well as the good case when the minimum number of measurements was three. RWGH-O-R isn't very good in either case, and RWGH-O is mediocre in both.

6.3.3 Trial 3c: 5/1 Combo NLOS mitigation

In this trial AOT and TOA errors were added to a single beacon's measurement set. The beacon to be corrupted was chosen randomly for each trial, and both types of NLOS errors were varied over the same range in the previous two trials (in real channels, the AOA and TOA NLOS errors for a signal are uncorrelated, so we must compare the effect of NLOS impairments independently [35]). Of course, comparison of the effects of errors becomes more difficult due to the number of dimensions of the problem.

Figure 6.7 shows the performance of all algorithms for a range of TOA and AOT NLOS errors. Performance becomes more difficult to read since there

Figure 6.6: Trial 3 AOT NLOS Mitigation.

are two dimensions of inputs. Overall, RWGH-C-R appears to provide the best and most uniform performance. Figure 6.8 shows all algorithms using a minimum of three measurements. There are 10 measurements available for positioning, which means there's a lot of redundant information that the RWGH-C can make use of.

RWGH-C does a great job here, as with the AOT and TDOA cases. The removal algorithms do not fare as well. The results here are consistent with the TDOA and AOT methods. All the NLOS mitigation algorithms perform much better for the combined case; however only the combining algorithm comes close to the good case, and even then only when there are a minimum of three station measurements used.

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Figure 6.7: Trial 3 Combo NLOS Mitigation, minimum 2 measurements.

6.3.4 Trial 3: All Algorithm Comparison

Figure 6.9 shows the performance for all measurement types, and all algorithms. This demonstrates the massive improvement afforded by combining AOT and TOA, evidenced by the improvement in the naive algorithm. The RWGH variations all do quite well for the combo case, although RWGH-C with min 3 is the best.

6.4 Trial 4: One NLOS Error in Four Measurements

For trial 4, the same procedure was followed as in the previous trial. The only difference is that there are only four beacons with one NLOS-corrupted error.

6.4.1 Trial 4a: TDOA 4/1 NLOS Mitigation

Figure 6.10 shows the RWGH algorithm performance when one in four measurements is corrupted by multipath. The algorithms all fare worse than the naive algorithm in this case. Either additional adjustments of the algorithms are required, or they just can't cope without additional redundant data. The results from trial 3a suggest the latter is the case.

Figure 6.8: Trial 3 Combo NLOS Mitigation, minimum 3 measurements.

Figure 6.9: Trial 3 All Algorithm NLOS Mitigation.

Figure 6.10: Trial 4 TDOA NLOS Mitigation.

Figure 6.11: Trial 4 AOT NLOS Mitigation.

6.4.2 Trial 4: AOT 4/1 NLOS Mitigation

Figure 6.11 shows the RWGH performance for AOT measurements. Since we still have redundant measurements, both min 2 and min 3 results are shown. The removal algorithms perform well even in the min 2 case, although they clearly fail for 3 beacons. As in trial 3, RWGH-C improves remarkably for a minimum of three beacons.

6.4.3 Trial 4: All Algorithm Comparison

Figure 6.12 shows a bar plot of the overall performance for each algorithm type, using a minimum number of 2 and 3 stations. The combo performance for this trial is also shown here. Comparisons are difficult, because the combo case comprises a larger number of situations with only one highly corrupted measurement. Nevertheless, even the combo naive algorithm performance is better than two of the three single-measurement type cases. Furthermore, all the mitigation algorithms perform quite well, nearly meeting the ‘good’ performance in the min 3 case.

6.5 Trial 5: Position estimation with data from model

In trial five we focus on NLOS mitigation for data from the ray-tracing channel model. The channel simulation was run for a number of positions in a simulated building. From all those cases, receiver positions with five visible beacons and one NLOS path (5/1) and four visible beacons with one NLOS path (4/1) were selected for comparison with the previous trials. The scale of this simulation was somewhat smaller than the previous trials, so the AOT variance was set at 2 degrees squared, and the TOA variance at 1 meter squared.

Based on the results of trials 3 and 4, the RWGH algorithms used one

Figure 6.12: Trial 4 All Algorithm Comparison.

more than the minimum number of stations (except for the TDOA 4/1 case, for which 3 is necessarily the minimum). The 5/1 cases are shown in depth, and all results are shown in the final bar graph.

6.5.1 Trial 5a: TDOA 5/1 results from model

Figure 6.13 shows results from 5/1 cases. There are a number of NLOS errors that are actually smaller than the variance of the TOA measurements, and other values are all over the place (more or less exponentially distributed, as expected). The graph is much less smooth than the earlier trials because each data point corresponds to just one target position, as opposed to the earlier trials where the target was moved for each run. Nevertheless, there is a general linearly increasing trend for the naive algorithm's RMSE with the increase in NLOS error magnitude.

As with other cases, the only clear winner here is the RWGH-C when the NLOS errors are relatively large. However, the overall improvement is not very impressive, and nothing approaches the 'good' case.

6.5.2 Trial 5b: AOT 5/1 results from model

Figure 6.14 shows AOT positioning with data from the channel model. As with the TDOA picture, the graph is not as smooth as earlier trials because the target position could not be varied. There is a general trend for the naive algorithm's performance to decrease linearly with the magnitude of

Figure 6.13: Trial 5 TDOA, channel model data.

the NLOS error.

Even with three redundant measurements, the RWGH-C algorithm performs worse than the naive one for this case. The RWGH-O however beats it everywhere. This is a little surprising, given the excellent performance of RWGH-C-R in the previous trials. It could be that the threshold for NLOS identification is badly tuned for the geometry of this trial. Due to the smaller size of the scenario, the residual errors would tend to be smaller, so the NLOS identification threshold probably needs adjustment. This reinforces the idea that the NLOS mitigation algorithms need to be tuned for the specific geometry of the scenario.

6.5.3 Trial 5: All 5/1 and 4/1 results from model

Figure 6.15 shows all the algorithm results together. The RWGH-C again proves to be the most reliable, and the RWGH removal algorithms are the worst across the board. The combo beats the other two overall, and as with the previous trial, its RWGH-C performance nearly matches the good algorithm when there is enough extra information.

Figure 6.14: Trial 5 AOT, channel model data.

Figure 6.15: Trial 5 All Algorithms Comparison, channel model data.

6.6 Trial 6: Overall position estimation performance with model

In trial 6, we extend the simulation to cover every receiver point in the channel simulation run for trial 5. The TDOA, AOT, and combo position location algorithms are run with all RWGH variations for each position from the model. The TOA measurement variance is set at 1 meter, and the AOT variance is set at 1 degree. In the following graphs, the overall CRLB for each scenario is also shown (the CRLB is based on all-LOS measurements).

6.6.1 Trial 6a: TDOA Performance with Channel

Model

Figure 6.16 shows TDOA performance for the various combinations of stations/NLOS corrupted situations. This is the first time we've seen more than one NLOS error in a single measurement set. We can see that when there is enough extra information, i.e. the 5/2 and 5/1 cases, the RWGH-C comes out ahead. In most other cases, the naive algorithm works the best. In the 0-NLOS cases, all the algorithms are worse than the naive one. Clearly there isn't enough extra information to mitigate the multipath with these methods in the TDOA case.

Figure 6.16: Trial 6 TDOA NLOS Mitigation, channel model data.

6.6.2 Trial 6b: AOT Performance with Channel Model

Figure 6.17 shows AOT performance for the various cases that arose from the channel model. The pattern is similar to the TDOA cases, although in general the performance is much better. The distances from beacons to target are quite short, so even bad angular errors should not affect the location estimates as badly as the TDOA estimates.

When there is a enough extra information, the RWGH-C algorithm outperforms the naive one. When the measurements are all uncorrupted, it is worse. This is clearly a problem with all the algorithms and measurement types. The 3/1 case is interesting – the naive and combining algorithms are exceptionally bad, but the removal algorithms match the good performance. This again suggests the best strategy for NLOS mitigation might depend on the number of measurements available.

6.6.3 Trial 6c: Combo Performance with Channel Model

Figure 6.18 shows the combo performance for the various cases in the channel model. RWGH-C is clearly the winner here, beating the naive algorithm in every case with NLOS errors. It also nearly meets the naive algorithm performance for the uncorrupted cases.

Figure 6.17: Trial 6 AOT NLOS Mitigation, channel model data.

Figure 6.18: Trial 6 Combo NLOS Mitigation, channel model data.

Figure 6.19: Trial 6 All Algorithm NLOS Mitigation, channel model data.

6.6.4 Trial 6: All Algorithm Comparison with Channel Model

Figure 6.19 shows the overall performance of all algorithms together. It's obvious that the combo is the best, and also that the NLOS mitigation algorithms work best with the large amount of redundant information used in the combo algorithm.

6.7 Chapter Summary

In this chapter results from simulations of the TDOA, AOT, and combined positioning algorithms were presented. These algorithms were tested for the effects of the number of measurements, and the accuracy of the measurements. Algorithms for mitigation NLOS multipath errors were also evaluated. These were tested using a simplified channel model, as well as the somewhat more realistic channel model of Chapter 5. Recommendations and conclusions from these results are presented in the final chapter.

Chapter 7

Conclusions and Future Work

This thesis presents a novel communications system designed for precise positioning of users in indoor environments, called synthetic Doppler. This is a mobile-based positioning system, geared for flexibility, mobility, and high accuracy. The concept uses a moving antenna to generate a Doppler FM modulation; by encoding the antenna position on the signal with a DSSS spreading code, the receiver can determine the AOT of the signal as well as estimate an unreferenced TOA. If multiple beacons are synchronized, the TOA's can be combined to form a system of TDOA's. By surrounding the area of interest with two or more beacons, enough measurements can be made by the receiver to estimate its position.

The system's performance is limited by the environment in which it will be used. The indoor channel is dominated by NLOS effects, multipath being

the dominant source of error. Testing the basic location algorithm requires a channel model that can generate TOA and AOT values similar in character to the real environment. A channel model was developed for this thesis that employs ray tracing and the so-called image method to obtain this data. A simpler model was also used in the simulations in order to isolate the effects of particular degrees of multipath.

Evaluation of positioning algorithms included methods for mitigating multipath errors. RWGH is a simple algorithm that attempts to identify and reduce the impact of multipath-corrupted measurements. Four variations of RWGH were tested in simulation.

In the first trial the effect of the number of beacons on positioning accuracy was demonstrated for each of the measurement types. Increasing from the minimum number required for each type was shown to improve performance, although gains drop off after a few beacons are added. Five was chosen (somewhat arbitrarily) as a baseline for further simulations. In reality not all beacons will be received with the same signal strength (or even at all), and many of their measurements will be corrupted by multipath. In order to choose the ideal number for a given performance level, a study of the detectability of beacons in the target environment would be required. These results also showed that the least-squares estimators are performing as well as their performance bound predicts. Finally the power of combining all the measurements for making estimates was demonstrated – the combo performance was significantly better than either of the single-measurement

cases with otherwise identical parameters.

The performance of each algorithm by measurement variance was investigated. The TDOA- and AOT-based positioning performance was compared for measurement variances of similar magnitude. TDOA was demonstrated to be the more robust of the two techniques, at least for the geometry of the simulations. Once again in this trial the combined algorithm performance dramatically surpassed either of the two single-measurement algorithms. The contribution of AOT and TOA measurement variance to the combo performance was compared. For the geometry of the simulation, TOA errors were shown to have less impact on the position estimate. The worst performance, corresponding to a 20 m^2 ranging variance and a 20 degrees^2 angular variance, is equivalent to a 7 m^2 variance for TDOA alone and a 2 degrees^2 angular variance for AOT alone.

The effect of corrupting one of the five measurements with varying degrees of multipath on the naive algorithm was shown. The performance of each RWGH algorithm in dealing with the error was demonstrated. For RWGH-C, and with all three measurement combinations, the importance of using more than the minimum number of measurements for the sub-estimates is clearly demonstrated. In fact with just one multipath-corrupted measurement, RWGH-C nearly met the ideal case in performance. Again the combo algorithm was much more robust for all cases measurement. The removal algorithms show promise, but are unreliable at identifying the bad measurement.

The resulting performance from reducing the total number of measurements and including some multipath-corrupted data was demonstrated. A similar trend to the previous trial is evident; the RWGH methods require extra good measurements to be effective. The advantage of the AOT-based approach over TDOA is that it has a lower minimum number of measurements, which gives RWGH-C enough information to outperform the naive algorithm. However the the combo was again shown to have dramatically better performance than the other two cases.

In order to evaluate these algorithms under more realistic conditions, the algorithms were applied to the ray-tracing channel model data. In the first trial, the 5/1 and 4/1 cases were isolated from the channel model data for comparison with previous trials. Direct comparisons are somewhat difficult because not all parameters of the simulations are identical – in particular, the receiver position was not varied over all the runs, and the scale was somewhat smaller. Accordingly, this trial's results were not completely consistent with previous trials. The RWGH algorithms tended to perform worse than the naive one except for the largest multipath error cases. These problems did not prevent the combo method from outperforming the two single-measurement methods.

Finally the positioning algorithms were applied to all the channel profiles calculated from the model. Results were shown broken down by base stations seen and number of multipath corrupted paths. Here the results for the individual algorithms were disappointing. The RWGH variations often

perform worse than the naive algorithm, especially in multipath-free situations. The combined algorithm using the RWGH-C multipath mitigation algorithm showed its utility here, outperforming even the best cases of the single-measurement algorithms by fifty percent.

7.1 Future Research

This thesis covers three broad and distinct (though obviously related) topics. The first major component is the analysis of the synthetic Doppler system, including the prediction of positioning metrics (AOT, TOA) accuracy. The second is the development of an indoor channel model. The third is research into the positioning algorithms, in which the AOT and TOA metrics and knowledge of the channel impairments are used to estimate the user's position. Each of these areas could be more fully developed.

The system analysis of Chapter 3 is very high-level. In particular, the analysis assumes a mechanically rotating antenna is employed. This is not the most practical approach; for synthetic Doppler receivers an antenna array approach is normally used. Instead of rotating one antenna, the signal is switched electrically through a circular array of antennas. The resulting FM signal is then a series of pulsed sinusoids rather than a continuous sinusoid. This raises practical issues that would need to be addressed in an implementation of the beacon, and would require further analysis to determine its effect on the positioning metrics.

The channel model used for simulations is very simplistic. At the very least, it would be worthwhile to consider the channel bandwidth in determining the received TOA metrics. The SNR of the received signal could also be used to estimate the accuracy of the TOA and AOT measurements for a given receiver position (simulations in the previous chapter assumed the same statistics for all trials, regardless of the received SNR). Another factor not considered is how the channel changes as the antenna rotates. In reality the channel profile will be constantly changing; knowledge of this could be incorporated into a channel estimation algorithm that could consider a time history of the channel impulse response, the motion of the user (perhaps using inertial sensors), and using this, estimate the line of site path.

Even more ambitious approaches are possible. Given knowledge of the structure, the channel model could be used as part of the channel estimator. If the receiver could simultaneously track all detectable multipath components with their associated AOT values, these rays could be cast from the beacon. The positioning algorithm would look for positions in the building that match the received channel profile. It is questionable that a channel model with sufficient fidelity could be developed for this purpose, and doing so would require accurate building blueprints (and furniture, duct work, and plumbing layouts!) and extensive experimental verification.

Solving for location using least-squares estimation is a well-understood problem. However the classic methods pursued in this research ignore a lot of the available information. As discussed above, the time-history of the channel

contains valuable information. Other research has focused on using Kalman or Bayesian filters to utilize this, with varying degrees of success. These models can be used to incorporate the statistical distribution of multipath errors and the motion of the user (and any other information that might be available) into the position estimate.

The RWGH multipath mitigation algorithm used in this research has not been proven to be optimal. For example, the versions of RWGH that attempt to identify and remove multipath-corrupted measurements do so in an ad-hoc manner; a more sound approach would be to develop a statistically significant threshold, such as an hypothesis test. Another factor is ignored in the calculation of the residuals. All possible combinations of stations are considered, and weighted only by their agreement with the measurement data. However, many of those combinations have extremely poor GDOP. Considering this in the weighting of the combined solution might improve the overall estimate.

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