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THE EFFECTS OF AN OIL PRICE RISE ON INFLATION, OUTPUT
AND THE EXCHANGE RATE IN THE CASE OF SUBSIDIZATION POLICY

by

C

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A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
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ABSTRACT

Since the Organization of Petroleum Exporting Countries raised the price of oil by four hundred per cent in 1974, the theory of supply inflation has received a great deal of attention.

This study analyses the short and long run effects of an oil price rise on output, inflation, and the exchange rate. The study also analyses dynamic adjustments to the oil price rise in cases where oil price subsidies are provided and where no subsidies are provided.

In the no subsidy case we show that the oil price rise can be inflationary or deflationary. The implications of the policy of subsidizing the price of oil is highlighted by taking account of a government budget constraint which in turn leads to the possibility of monetization as a source of financing the deficit, and thereby to higher output relative to the no subsidy case. As to the price level we illustrate the possibility that subsidization can actually be more inflationary. The important element giving rise to the above possibility is the subsidy induced increase in the money supply. Exchange rate flexibility is shown not to insulate the domestic price level against an oil price rise.

In the long run the rate of inflation and exchange rate variations are determined by the rate of growth of the money supply.

The dynamic adjustment path of price and output is shown to be determined by the rate of adjustment of inflationary expectations.

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CHAPTER I

INTRODUCTION

In recent years the theory of supply inflation has received a great deal of attention in the literature.¹ The term "supply inflation" refers to increases in the general price level due to increases in price of materials.

During the 1970's the most startling of these increases has been with the price of oil. In 1973-1974 and again in 1979-1980, the world economy faced two sharp oil price increases. These increases were generally seen to cause both inflation and recession in the oil exporting countries.

Several models have been developed to deal with this supply inflation problem. For instance, Gordon (1975) and Phelps (1978) have developed models to analyse supply inflation in a closed economy. Gordon's model consists of two sectors, while Phelps' model includes only one sector. In both of these models, an exogenous decline in supply (of raw materials in Phelps and of farm products in Gordon) leads to higher inflation, and unemployment. An additional feature of Phelps' model is the possibility of monetary accommodation whereby the effect on current unemployment is offset at the expense of future inflation.

While Gordon and Phelps are concerned only with closed economy, the paper by Findlay and Rodriguez (1977) develops the

macroeconomic analysis of supply shocks in an open economy.² Findlay and Rodriguez analyze this problem in context of a one sector model under fixed money wages and flexible exchange rates. The new element introduced in the open economy analysis of a supply shock is that the shock affects the economy not only through aggregate supply but also through aggregate demand (via a balance of trade deficit). For instance, a material price rise would lead to a leftward shift in the aggregate supply as well as aggregate demand. Since both curves shift, the inflationary result of the supply shock can no longer be predicted, a priori, without the knowledge of the relative magnitudes of the two shifts.³

The analysis of Findlay and Rodriguez is not directly applicable to the case of Canada because Canada has kept the domestic price of oil below the world level through a policy of subsidization.⁴

The purpose of this thesis is to study the macroeconomic consequences of this policy. Using an open economy model similar to Findlay and Rodriguez's, we introduce the following new features:

First, we employ a price relationship which allows for the domestic price of oil to adjust incompletely to the world price of oil.

Secondly, we take the government budget constraint into account and explicitly model the possibility of monetization as a source of financing the deficit.

The key results of our model are presented in terms of a simple diagrammatic framework which uses the aggregate supply and demand relationships and can be summarized as follows:

- (a) The direct impact of the oil price rise on aggregate demand and supply depends upon the extent of the subsidization.
- (b) The endogenous changes in the money supply due to the monetization process open up an additional, (indirect), channel on the demand side of the economy. It is generally presumed that holding the oil price below world level will lower the domestic price level.⁵ However, contrary to this presumption, the indirect channel brings out the possibility that the subsidization policy may in fact raise the price level.

This thesis is organized as follows:

Chapter II will provide the basic model which includes the oil price relationship and the government budget constraint.

Chapter III looks at the short run effects of an oil price rise on the price level, output and the exchange rate under the case of the no-subsidy as well as the subsidization policy.

Chapter IV will provide a discussion of long run implications of an oil price rise for inflation and exchange rate depreciation.

In Chapter V, we discuss the dynamic adjustment of the price level and output to an oil price rise in the no-subsidy as well as the subsidization case.

In Chapter VI, we present some evidence to support the following key assumptions of our theoretical model. First, reviewing the Canadian Oil Policy we look at the relationship between the domestic and world price of oil. Secondly, we discuss the empirical importance of monetization as a source of financing the government deficit. And finally, we provide some evidence on the comparison between the no-subsidy and subsidization case.

Chapter VII concludes and summarizes this thesis.

FOOTNOTES TO CHAPTER I

1. See, for example, the articles by R.J. Gordon, 1975; Edmond S. Phelps, 1978; F.M. Gramlich, 1979; and W.B. Buiter 1978..
2. See also Schmid (1976) and (1980) who discusses macroeconomic aspects of materials price rise in a monetary framework, for both a small open economy as well as a large economy, under fixed as well as flexible exchange rate regimes.
3. In reaching this conclusion, Findlay and Rodriguez use slightly different relationships. However an analysis of this question in terms of the aggregate demand and supply relationships is presented in Chapter III below.
4. The Federal Government adopted a one-price oil policy program following the 1973-74 oil price increases. For further details, see Chapter V.
5. See, for instance, Helliwell (1981).

CHAPTER II

THE MODEL

In this chapter, we set up a simple macroeconomic model of an open economy. The model is designed with the purpose of analyzing the macroeconomic effects of increases in the world price of oil under a number of alternative "energy policies".

A. Aggregate Supply

To derive aggregate supply of the economy, we combine the mark up price equation with the expectation augmented Phillips curve.

Beginning with Phillips curve, let $\dot{w}_t (= \frac{dw_t}{w_t})$ be the proportional change in wage rate, $y_t - \bar{y}_t$ the gap between the actual and the potential level of output, and $\dot{p}_t^e (= \frac{dp_t^e}{p_t^e})$ the expected rate of inflation, the wage equation is,

$$\dot{w}_t = b_1 + b_2(y_t - \bar{y}_t) + b_3\dot{p}_t^e, \quad b_2, b_3 > 0. \quad (1)$$

In a macroeconomic model with imported intermediate goods, it is important to distinguish between gross¹ and net output. The net output is defined as gross output less the real cost of imported intermediate inputs. Thus,

$$y = Q - \left(\frac{ep_N^*}{p}\right) \cdot N^*, \quad \frac{\partial y}{\partial p} > 0, \quad \frac{\partial y}{\partial p_N^*} < 0, \quad \frac{\partial y}{\partial e} < 0. \quad (2)$$

where y is net output, e the price of foreign currency in domestic currency units, p_N^* the price of the imported intermediate

input in terms of foreign currency, p , the price of final goods, and N^* the quantity of imported oil.

If the domestic country is itself a producer of oil, as in the case of Canada, the net output can be written as,

$$y = Q - \left(\frac{ep_N^*}{p} \right) (N^* - T) \quad (3)$$

where T is the domestic production of oil per unit of time.

Next, we assume a mark up price equation according to which prices are determined by a proportionate mark up over costs. Since costs depend on the price of oil as well as the wage rate, the mark up relationship can be written as follows:

$$\dot{p}_t = b_4 + b_5 \dot{w}_t + b_6 \dot{p}_{N_t}^*, \quad b_4 < 0. \quad (4)$$

where $\dot{p}_{N_t}^*$ is the proportional rate of change of the domestic price of oil.

By suitable choice of units, the initial values of w , p , and p_N^* can be set equal to unity, that is, $w_t = p_t = p_{N_t}^* = 1$.

Assume that the economy is initially in the long run equilibrium,² so that $p_t = p_t^e$. Using these assumptions and combining equations (1), and (3) above, $\dot{p}(=dp)$ can be written as,

$$dp = (b_4 + b_5 b_1) + b_5 b_2 (y_t - \bar{y}_t) + b_5 b_3 dp^e + b_6 dp_{N_t}^{*3} \quad (5)$$

This equation represents the short run aggregate supply curve of the economy, a relationship between price change and output for a given change in the oil price and the expected price level.⁴

B. Goods Market

The equilibrium in the goods market will be attained when the real net output is equal to real (ex ante) aggregate expenditure. Using lower case letters to denote the real variables, the aggregate real expenditure can be defined as the sum of real consumption c , real investment i , real government expenditure g , net real exports (excluding oil) x , and the real imports of oil m .^{5,6}

$$y = c(y - ty) + i(r) + g + x\left(\frac{ep^*}{p}\right) - \frac{p'_N}{p} m\left(y, \frac{p'_N}{p}\right) \quad (6)$$

$$\frac{\partial i}{\partial r} < 0, \quad \frac{\partial x}{\partial e} > 0, \quad \frac{\partial x}{\partial p} < 0, \quad \frac{\partial m}{\partial y} > 0, \quad \frac{\partial m}{\partial \left(\frac{p'_N}{p}\right)} < 0,$$

The variables inside the brackets represent arguments of each function, where t is the tax rate, $(Y - tY)$ is the disposable income, r the domestic rate of interest,⁷ p^* the foreign price of non-oil imports in terms of the foreign currency, $\frac{ep^*}{p}$ the relative price of exports and non oil imports, p^* and g are assumed to be given.

C. Money Market

Here we consider a very simple model where the money demand, represented by M_d , is specified as a function of real income and the rate of interest as follows:

$$\frac{M_d}{p} = L(r, y) \quad \frac{\partial L}{\partial r} < 0, \quad \frac{\partial L}{\partial y} > 0. \quad (7)$$

The supply of money M_s is assumed to be fully controlled by the Central Bank at $\bar{M}$, so that

$$M_s = \bar{M} \quad (8)$$

The equilibrium in the money market will be attained when the demand for money equals the supply of money,

$$\frac{\bar{M}}{\bar{p}} = L(r, y) \quad (9)$$

D. Government Budget Constraint

In choosing a mix of monetary and fiscal policies, government authorities (including the Central Bank) are bound by a government budget constraint. The constraint states that in each period the budget deficit, in nominal terms, must equal the flow of financing from all sources, that is by borrowing either from the Central Bank or from the private sector. Let α be the proportion of government deficit that is monetized. Therefore, the increase in the money stock dM , will comprise of two parts; an exogenous change du and an endogenous change dictated by the deficit,

$$dM = M - M_{-1} = \alpha(H + g - ty) + du, \quad (10)$$

where M_{-1} is the stock of money in the past period.

The government deficit itself is comprised of two parts: the oil subsidy, and the remaining government account which is assumed to be balanced, as we focus on the oil issue.

The oil deficit, H , is defined by,

$$H = S \cdot m \quad (11)$$

where S is the dollar subsidy per barrel of oil, ($S = ep_N^* - p_N'$), where p_N' is the subsidized (compensated) price of oil which is held below the world level. The government net expenditure (excluding the subsidy) is represented by $g - ty$ which is assumed

to be zero, ($g=ty$), initially.

In Canada, the Federal Government has tended to insulate the domestic economy from the increases in the price of foreign crude oil by subsidizing the domestic price of oil. This type of policy can be captured by the following reaction function,

$$p'_N = \gamma_0 (p_N^*)^{\gamma_1} e^{\gamma_2} \quad (12)$$

where γ_1 and γ_2 can take on values between zero and one, and γ_0 is a positive number.

The above reaction function can be used to represent a wide range of policies. It is interesting to consider the following two special cases.

A) A hands off policy. In this case, there is no subsidization and the domestic price of oil equals the world price of oil. That is,

$$\gamma_0 = \gamma_1 = \gamma_2 = 1, \quad p'_N = ep_N^* \quad (13)$$

when this is satisfied we have, $S = 0$.

The changes in money supply according to the government budget constraint, represented by equation (10), will, therefore, simply equal $dM=du$ (assuming $g=ty$).

B) A subsidization policy which allows for partial adjustment of the domestic price of oil with respect to the foreign price of oil, but no adjustment with respect to the exchange rate. This policy is presented by

$$\gamma_1 < 1, \gamma_2 = 0 \quad (14)$$

As discussed later in Chapter III, this can appear to provide a good approximation to the actual policy pursued in Canada.⁸

E. Balance of Payments

Finally, we close the model assuming that the international capital is perfectly mobile, so that

$$r = r^* + \frac{e_{+1}^* - e}{e} \quad (15)$$

where r^* is the foreign rate of interest, e_{+1}^* is the forecast of the exchange rate one period ahead and hence $\frac{e_{+1}^* - e}{e}$ is the expected rate of change of exchange rate.

FOOTNOTES TO CHAPTER II

1. The gross output, a function of labour, capital and an intermediate good, oil is represented in a production function as follows:

$$Q = f(K, L, N)$$

where K is the capital stock, L the labor input, and N the intermediate input. All three factors of production are assumed to be substitutable.

2. To simplify this analysis we ignore the possible impact of a change in the real price of oil, $(\frac{p_N^i}{p})$, on the full

employment output, $\bar{y}$.

3. According to our empirical evidence, the alternative version of the short run aggregate supply based upon the marginal productivity (as opposed to the mark up price), and the Phillips curve is an inferior fit. See appendix to this chapter for the derivation of this alternative version.

4. For simplicity, the time subscript is omitted throughout the model.

5. In fact, $m.p_N^i$ is nothing but $(\frac{ep_N^*}{p}).N^*$ which is equal to the total/real^P expenditure (nominal expenditure deflated by the price) on the imported oil.

6. The elasticity of the demand for the imported oil is assumed to be less than unity and this is fundamental to our analysis.

7. The nominal rate of interest r is defined as follows:

$r = \rho + p^e$, where ρ is the real rate of interest, and p^e represents the expected rate of inflation. However, the nominal and the real rates will be the same, i.e., $\rho = r$, as the expected rate of inflation is zero.

8. This is supported by our empirical findings that while the domestic price of oil, P_N , is responsive to the world price, it remains significantly unaffected by the exchange rates.

CHAPTER III

THE EFFECTS OF AN INCREASE IN THE PRICE OF OIL IN SHORT RUN

In this chapter we analyse the short run effect of an oil price rise on the price level, output, and the exchange rate, under the two special cases of no-subsidy, and the subsidization policy. These effects are illustrated by two key relationships; the aggregate demand and the aggregate supply.

Case (a) - No Subsidy

In this section, we study the case where the domestic price of oil, p_N^1 , is the same as the world price of oil in terms of the domestic currency so that $p_N^1 = ep_N^*$. This can be obtained from equation (12) by setting $\gamma_0 = \gamma_1 = \gamma_2 = 1$.

We derive aggregate demand and aggregate supply relationships from the model as follows:

To derive the aggregate demand, we first differentiate totally, equations (6), (7), and (15), representing the equilibrium in the commodity market, IS, money market, LM, and the balance of payments, BP, respectively. We then solve these equations for dr and de to obtain the following relationship:

$$dp = B_{01}d(GT) + B_{02}dr^* + B_{02}de_{+1}^* + B_{03}dp_N^* + B_{04}du + B_{05}dy \quad (16)$$

where,

$$B_{01}, B_{02}, B_{04} > 0, \quad B_{03}, B_{05} < 0$$

The equation (16) represents a downward sloping aggregate demand.

Next, we simplify the aggregate supply relationship,

represented by equation (5) in Chapter II, by eliminating the exchange rate from this equation as follows.¹ Solving the differentiated versions of the IS, LM and BP equations for dy , we find the following relationship in terms of de and dp :

$$de = C_0 + C_1 dr^* + C_2 de_{+1}^* + C_2 dp_N^* + C_3 dp \quad (17)$$

where

$$C_0 \geq 0, C_1, C_2 > 0, \text{ and } C_3 \geq 0.$$

The equation (17) is then substituted in equation (5) and solved for the exchange rate obtaining,

$$dp = A_{01} + A_{02} dy + A_{03} dp^e + A_{04} dp_N^* + A_{05} dr^* + A_{05} de_{+1}^* + A_{06} d(GT) + A_{07} dp \quad (18)$$

where

$$A_{01} \geq 0, A_{02}, A_{03}, A_{04}, A_{05} > 0, A_{06} < 0, \text{ and } A_{07} > 0.^2$$

The equation (18) represents an upward sloping aggregate supply function, which like aggregate demand, does not depend on e .

Now, we are in a position to analyse the effect of an oil price rise on the domestic price level, output, and the exchange rate. To focus on the short run, we will treat the expected price level, p^e , and the forecast of the exchange rate one period ahead as given. These effects on price and output are illustrated in terms of AS and AD curves in Diagram I. The effect on the exchange rate is discussed later.

Let the price of imported oil go up. It affects both the AD and AS function, as shown by equations (16) and (18) respectively. The AS will shift up and to the left ($A_{04} > 0$ in

equation (18)), and the AD will shift down and to the left ($B_{03} < 0$ in equation (16)). However, it is convenient to distinguish between the two possibilities: a) The AS shifts by more than AD; and b) The AD shifts by more than AS. These two possibilities are discussed separately in diagrams (I-a) and (I-b).

Consider first the case when the aggregate supply shift is greater than aggregate demand. This will happen if $A_{04} > B_{03}$.³ Let the point A, in Diagram I-a, characterize the initial equilibrium point. At this point, the equilibrium output and price are represented by y_0 and p_0 respectively. Now let us allow the aggregate demand and supply shift due to the oil shock. The new equilibrium, then, will be the intersection of the AS_1 and AD_1 , the point B in diagram (I-a). At this point, the output has fallen and the price level has risen.

Let us examine now the opposite case when the aggregate demand shift is greater than the aggregate supply shift. This will occur if $B_{03} > A_{04}$. This case is illustrated in diagram (I-b). Let us start, again, from the initial equilibrium point A. The new equilibrium point, after both AD and AS have shifted to the left, is represented by point C. This point indicates that both the level of output and price have fallen relative to their initial positions. In this case deflationary effect (price fall) of the oil shock via the demand side outweighs its inflationary effect (price rise) via the supply side.

To examine the effect on the exchange rate we refer

DIAGRAM I-a

DIAGRAM I-b

to equation (17). According to this equation, the exchange rate and price will be positively related if C_3 is positive, and inversely related if C_3 is negative. The sign of coefficient C_3 depends on the relative impact of a price change on the commodity and money markets.⁴ If a fall in price affects the LM by more than the IS, then C_3 will be negative and hence the exchange rate will depreciate. Conversely, if a fall in price has a larger impact on the IS than the LM, the coefficient C_3 will become positive and thereby the exchange rate will depreciate.⁵

Therefore, the price will be inversely related to exchange rate if the relative impact on the LM of a change in p is greater than on the IS. And conversely, they will be positively related if the IS is more sensitive to the price change than LM.⁶

Clearly, our results about the effect on the domestic price level seems to be different from the standard supply shock models of a closed economy,⁷ where a rise in the price of raw materials would only affect the aggregate supply and would cause an unambiguous rise in domestic price. Furthermore, our analysis suggests that, the flexibility of the exchange rate does not insulate the economy against the oil shock. In fact, the recessionary effect of an oil price rise on output and its inflationary or deflationary effects on prices are consistent both with a depreciation or an appreciation of the exchange rate.

Case (b) - Subsidization

This case constitutes the main feature of this dissertation. As discussed below, the study of this case suggests several interesting implications of the Canadian oil policy.

As we saw earlier, this case is characterized by $\gamma_1 < 1$, $\gamma_2 = 0$, in which the oil price equation can be written as $p_N^* = \bar{e}(p_N^*)^{\gamma_1}$. γ_0 is set equal to $\bar{e}$ by the appropriate choice of units. Here $\bar{e}$ is the value of the long term exchange rate to which the domestic price of oil is linked. At the initial equilibrium point, the actual and the long term value of the exchange rates are assumed to be the same, $e = \bar{e}$.

Once again to reduce the system into the aggregate demand and aggregate supply equations, we take the following steps.

In order to derive the aggregate demand, we first differentiate totally equations representing the IS and the LM, the BP, and the budget constraint. Then we solve these for dM , dr and de to obtain the aggregate demand as follows:⁸

$$dp = B'_{01}d(GT) + B'_{02}dr^* + B'_{02}de^*_{+1} + B'_{07}dp_N^* + B'_{05}d\mu + B'_{06}dy \quad (19)$$

where $B'_{01}, B'_{02} > 0$, $B'_{07} < 0$, $B'_{05} > 0$, $B'_{06} < 0$.⁹

The aggregate supply on the other hand can be represented by.

$$dp = A_0 + A_1dy + A_2dp^e + A_3\bar{e}\gamma_1(p_N^*)^{\gamma_1-1}dp_N^* \quad (20)$$

This is an upward sloping relationship in terms of y and p . Note that, while the aggregate demand is not drawn for a given e , i.e., the exchange rate changes from point to point on AD, the aggregate supply is independent of the exchange rate. That is, the exchange rate does not appear in the AS because the government links the oil price to some long term exchange rate, $\bar{e}$.

Before we illustrate the effect of an oil price rise using the key relationships, the AD and AS, it is important to explain some interesting features of this part of the model which arise because of the subsidy program.

First, the impact of the oil shock on the y and p , through the AD and AS, is to some extent absorbed by holding the domestic oil price below the world level.¹⁰

Secondly, some proportion α ¹¹ of government deficit due to larger oil subsidy is monetized whenever rises in p_N^* lead to a larger expenditure on oil by the government. This resulting endogenous change in the money supply will tend to reduce the effect of the oil price shock on AD. The monetization process represents an indirect channel¹² through which the changes in the price of oil is transmitted to the price level and output in the domestic economy.

Note that B_{07} in equation (19) represents the combination of both the direct and indirect channels discussed above. However, the strength of the indirect channel which stems from the extent to which the money supply will have to rise to finance the subsidy depends not only on α , but also γ_1 . This channel

will be stronger, the greater the degree of subsidization and smaller γ_1 , for a given α .

Now we are able to illustrate the effects of an oil price rise on the price, output, and the exchange rate in the AD-AS diagram. Again, the discussion of the effect on the exchange rate will be carried out later.

Let the price of imported oil rise. This will affect both the AD and AS (equations (19) and (20)). Again, the AS will shift up and to the left ($A_3 > 0$), and the AD will shift down and to the left ($B_{07} < 0$). However, since now p_N^* only partially adjusts to p_N^* , the effects of p_N^* on these curves will be smaller than before. It is again convenient to discuss separately the two possibilities of: a) the AS shifts by more than AD; and b) the AD shifts by more than AS. These two cases are represented in diagrams (II-a) and (II-b).

Consider first the case when the aggregate supply shift is greater than the aggregate demand. This case will occur if $(B_{07} > A_3 \epsilon \gamma_1 (p_N^*)^{\gamma_1 - 1})^{13}$. Let point A in diagram (II-a) represent the initial equilibrium point, characterized by $y = y_0$ and $p = p_0$. Now let both the AD and the AS shift to the left due to the oil price rise. The new equilibrium will be represented by the intersection of AD_1 and AS_1 . At this point, point D, in diagram (II-a), the output has fallen whereas the price has risen. Note that AD_1 represents the sum of the direct effect as well as the indirect effect.

It is interesting to compare these results with the no subsidy case. For the purpose of comparison we also show in

DIAGRAM II-a

DIAGRAM II-b

the diagram the shifts in the AD and AS, that would have occurred in the absence of subsidization and these are represented by AD_N and AS_N . Point N, the equilibrium point under the no-subsidy, after the shock, illustrates that the higher output under subsidization is accompanied by a higher price, even though the higher output could have been accompanied by a lower price.

Let us consider now the case when the aggregate demand shift is greater than the aggregate supply. This arises if $(B_{07} < A_3 \bar{e} \gamma_1 (p_N^*)^{\gamma_1 - 1})$. Let us start again from the initial equilibrium, point A, and allow the price of oil to rise. This will lead to a partial adjustment in the domestic price of oil, which will cause both the AD and AS to shift to the left. The new equilibrium point, point E, will be associated to lower output and lower price level, relative to the point A, in diagram (II-b). Once again AD_1 represents the sum of both the direct and indirect effects.

Now comparing this with the no-subsidy case (represented by point N, in diagram (II-b)), we again illustrate the possibility that a higher output is accompanied by a higher price level under subsidization compared to the no-subsidy case.

Now let us consider another interesting possibility. Let $\alpha=0$, i.e., there is no monetization of the deficit. This case is also illustrated in diagrams (II-a) and (II-b). Again, the domestic price of oil adjusts partially to the world price increases, which will cause both the AD and AS to shift to the left. However, in this case there is no indirect channel ($\alpha=0$).

Thus, as compared with the previous case which some proportion α of the deficit was monetized, the equilibrium of the price and output after the shock can be represented by D' and E' on the diagram, the intersection of AD_2 and AS_1 . The point $D'(E')$ will be located somewhere between point N , the equilibrium with no subsidy and the equilibrium of the subsidization $D(E)$.

To summarize, the subsidization policy always leads to a higher level of output in comparison with the no-subsidy case, so that the recessionary effect of the oil price increase is somewhat reduced. With respect to the price the situation is less clear. It is often thought that the policy of subsidizing the price of imported oil can reduce the inflation. However, our diagrammatical analysis brings out the possibility that the inflation can actually be increased.¹⁴ The reason for the different results is that alternative view unlike ours only looks at the aggregate supply shifts. If the effect of the subsidization on the aggregate demand is taken into account as well, as illustrated in the diagram, then it is possible that the subsidization policy will be more inflationary than no subsidy.

The above analysis concerns the case that $\gamma_1 > 0$ (less than one) as for Canada, but it is also interesting to examine the possibility that $\gamma_1 = 0$. In this case, the link between p_N' and p_N^* is completely broken, i.e., p_N' does not respond to the changes in the world price of oil. Therefore, the oil price rise will have no direct impact on the AD and AS . The effect of the oil price rise shock in this case can be readily seen from Diagram III.

DIAGRAM III

In this diagram let the initial equilibrium be represented by point A, and let the price of the imported oil rise. Now, since p_N^* remains intact, the AD and AS will not shift. This, however, necessitates the entire change in p_N^* to be subsidized. That is the indirect channel will be fully in effect.¹⁵ Thus only the AD, due to the indirect effect, will shift to the right,¹⁶ where it intersects the AS_0 at F. The new equilibrium point, F, located on the right hand side of A represents higher income and prices.

Therefore, as demonstrated, a strong subsidizing policy such as our extreme case, leads to a larger output. Nonetheless this policy will also result in higher inflation.

Before concluding this section we discuss briefly the effect on the exchange rate. Note, that again we derive a relationship between p and e . This relationship is as follows:

$$de = D_0 GT + D_1 dr^* + D_1 de_{+1}^* + D_2 dS + D_3 d\mu + D_4 dp_N^* + D_5 dp \quad (21)$$

where $D_0 < 0$, $D_1, D_2, D_3, D_4 > 0$, $D_5 \geq 0$.

If D_5 ¹⁷ is greater than zero then the exchange rate and price will be positively related, and they will be inversely related if D_5 is negative.¹⁸

If a fall in price affects the LM by more than the IS, then D_5 will be negative and the exchange rate will depreciate, and vice versa. Thus, the price will be inversely related to exchange rate if the relative impact on the LM of a change in p is greater than on the IS. And conversely, they

will be positively related if the IS is more sensitive to the price change than LM.¹⁹

To sum the key differences between the subsidization and the no-subsidy cases we have:

i) The effect of an oil price rise on the AD, through the direct channel is minimized since the domestic price of oil only fractionally adjusts to the world price. Thus, the AD shift would be smaller than what it would be in the no-subsidy case.

Also, there is an additional channel in the subsidization case (the indirect channel), generated by the oil subsidy induced changes in the money supply. This effect brings about an upward pressure on the AD, which is in the opposite direction to the direct effect of the oil price rise on AD.

ii) The effect of an oil price rise on the AS is also minimized by allowing the domestic price of oil to adjust only partially to the world price of oil. Thus, the AS shift is also smaller than the previous case.

As a result, the recessionary effect on y of an oil price rise will be minimized by pursuing a subsidization policy. However, as we pointed out, this reduction may be purchased at a cost of higher inflation - a trade off.

FOOTNOTES TO CHAPTER III

1. The derivation and the detailed explanation of these relationships, including their coefficients, are provided in the Appendix II.
2. For the detailed explanation, see Appendix II.
3. The coefficients A_{04} and B_{03} are represented by the following:

$$A_{04} = \frac{A_3 + A_3 \cdot P_N^* \cdot \frac{K_P^1 MR}{B_5}}{1 + A_3 P_N^* \left(\frac{\bar{M}}{P_k^2} - \frac{K_P^1}{B_5} \frac{\partial BT}{\partial e} \right)}$$

where $B_5, A_3 > 0$.

This indicates that the impact of a higher price of oil in the supply side is realized both directly through its negative impact on y , and indirectly through its negative effect on the exchange rate changes on the AS.

$$B_{03} = \frac{\frac{1}{P} MR}{\frac{1}{P} \cdot \frac{\partial BT}{\partial e} - \left(\frac{\partial BT}{\partial e} - 1' e_{+1}^* \right) \frac{\bar{M}}{P_k^2 e_{+1}^*}}$$

indicates the negative impact on the demand side through the increased expenditure on the imported oil.

4.

$$C_3 = - \frac{\frac{\bar{M}}{P_k^2} - K_P^1 \frac{\partial BT}{\partial e}}{B_5}$$

The numerator indicates the relative impact of a price change on the money market and the commodity market. Since $B_5 > 0$, the sign of C_3 is determined by whether $\frac{\bar{M}}{P_k^2}$ is greater or smaller than $K_P^1 \frac{\partial BT}{\partial e}$.

5. The relative impact on the money market and the commodity market of a change in P can be alternatively shown in terms of the relative steepness of the IS-BP, and LM-BP, given by equations (II-A6) and (II-A7) in the Appendix II in a diagram with P on the vertical and y on the horizontal axis.

6. This result has been also noted by Findlay and Rodriguez.
7. Here we are mainly referring to Phelps and Gordon's analysis.
8. Refer to the Appendix II-B.
9. The conditions under which the aggregate demand remains downward sloping are discussed later in the chapter. We will also explore the conditions which may possibly lead to an upward sloping aggregate demand.

10. These are given by $A_3 \bar{e} \gamma_1 (P_N^*)^{\gamma_1 - 1}$, and

$$A_3 \left[-\frac{1}{P} \cdot MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1} \right],$$

where

$$\gamma_1 (P_N^*)^{\gamma_1 - 1} < 1, \text{ when } \gamma_1 < 1.$$

11. According to our empirical evidence, (given in chapter V), some 32 percent of the government budget deficit is monetized in Canada over a period of thirty years, between 1950-1979.

12. This effect is given by,

$$-B_3 \cdot \alpha P_N^* \cdot m \cdot \bar{e} [1 - \gamma_1 (P_N^*)^{\gamma_1 - 1}] > 0,$$

which remains positive as long as $\gamma_1 < 1$, and $\alpha > 0$.

13. The left hand side of the inequality represents the minimized negative impact of the oil shock on the AD. The right hand side represents the minimized impact of the same oil shock on the AS. Note that the keypoint is in the fact that γ_1 is chosen less than one.

14. We will come to this point later in Chapter III.

15. In fact, the coefficient of indirect channel will be at its maximum. That is

$$ds = \bar{e} \frac{(1 - \gamma_1)}{P_N^*} dP_N^*$$

is maximized, when $\gamma_1 = 0$

15. The rightward shift in the AD, at any given price, due to the maximized indirect channel takes place via a multiplier, K_1 , the size of which also depends upon S:

$$K_1 = \frac{1}{1 - \frac{\partial \phi}{\partial y}(1-t) + \left(\frac{P'_N}{P}\right) \frac{\partial m}{\partial y} + \frac{\partial BT}{\partial e} \left(\frac{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m - Pk'}{\partial y}}{Pl e_{+1}} \right) - \frac{1}{Pl} (\alpha \cdot P'_N \cdot s \cdot \frac{\partial m - Pk'}{\partial y})}$$

or

$$K_1 = \frac{1}{\frac{1}{K} + \left(\frac{\partial BT}{\partial e} - i' e_{+1} \right) \left(\frac{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m - Pk'}{\partial y}}{Pl e_{+1}} \right)}$$

$$16. D_5 = \left[-K \frac{\partial BT}{\partial P} + \frac{\alpha \cdot s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial \left(\frac{P_N}{P} \right)} + \frac{\bar{M}}{P}}{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} \right] \begin{matrix} > 0 \\ < 0 \end{matrix}$$

17. So far we have assumed that $\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - P_k' < 0$, i.e., the income induced increase in money supply, $\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y}$, is smaller than the income induced increase in the demand for money, P_k' .
18. When $\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - P_k' > 0$, the relationship between e and P becomes positive, i.e., the effect on the money market of a change in price level will always be in the same direction as that of the commodity market of the same change.

However, when $\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - P_k'$ becomes a large positive, some concern over the stability of the system arises. That is, as long as the denominator of the multiplier K_1 above and hence, the ratio, (multiplier), itself is positive, the system will be stable. That is if,

$$\left(\frac{\partial BT}{\partial e} - 1' e_{+1}^* \right) \left(\frac{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - P_k'}{P_l e_{+1}^*} \right) > - \frac{1}{K},$$

then K_1 will remain positive, which is necessary for the stability condition to be observed.

Furthermore, the stability of the system, $K_1 > 0$, leads to a downward sloping aggregate demand curve. From equation (II-10B) in the appendix, it can be easily seen that when $K_1 < 0$, the aggregate demand will take an upward sloping shape, i.e., the upward sloping AD, is the manifestation of the instability of the system. However, we preclude the possibility of an unstable situation in our analysis.

CHAPTER IV

THE LONG RUN IMPLICATIONS OF AN OIL PRICE RISE

The macroeconomics effects of an oil price rise for the two cases of no subsidy, and subsidization in the short run were discussed in Chapter III. We found out that an oil price rise can be inflationary or deflationary. We also discovered that while in the subsidization case, the recessionary effects of an oil price rise on the output is less than in its absence, the subsidization can be more inflationary.

We should emphasize, however, that our Chapter III is only an analysis of one period effects, and that to the extent that the monetization of the oil subsidy is maintained, there will be some long run implications to be studied. Moreover, it is interesting to examine the dynamic adjustment caused by the policy of subsidizing the price of oil, which will be dictated by the adjustment of the expectations.¹ In this chapter, we will highlight only the long run implications of an oil price rise by focusing on the full employment equilibrium, and in the next chapter, we will look at the dynamic implications of an oil price rise.

We assume that the long run is characterized as a stationary state² with the level of output at its full employment, $\bar{y}$ ³, and that the expected price equals the actual price. In this chapter we consider the effects of a once and for all change in P_N in the long run. To highlight this effect, we assume no change in fiscal variables and in the

exogenous part of the money supply. Also, no change is assumed to take place in domestic and foreign variables. We assume perfect foresight in determining expected changes in exchange rates.

Therefore, the following equations summarize the general characteristics of the long run equilibrium.

$$y = \bar{y}, \text{ which replaces the AS,} \quad (22)$$

$$P = P^e, \quad (23)$$

and

$$dP_N^* = dP^* = d\mu = dGT = dr = 0 \quad (24)$$

Now to derive the long implications of the oil price rise, we again first discuss the no-subsidy and then introduce the subsidization policy.

Case (A) - No Subsidy.

Here, we present the long run implications of an oil price rise in the system of five equations, representing the aggregate supply, the goods market, the money market, budget constraint, and the balance of payments.

After a once for all rise in the price level due to an increase in the price of oil, the rate of inflation will be equal to zero as explained below.

Under the perfect foresight assumption, with the rate of inflation equal to zero we have:

$$P_{+1}^e = P_{+1}, \text{ and } e_{+1}^* = e_{+1} \quad (25)$$

The equilibrium condition in the money market will be presented by:

$$dM = du = \left(\frac{\bar{M}}{P}\right) dP = 0 \quad (26)$$

In growth terms

$$\dot{M} = \dot{P} = 0 \quad (27)$$

where, $\dot{M} = \frac{dM}{M}$, and $\dot{P} = \frac{dP}{P}$.

In the goods market the equilibrium condition will be given by:

$$K \cdot i' \cdot dp + K \frac{\partial BT}{\partial e} de - K \frac{1}{P} \frac{\partial BT}{\partial e} dP = 0 \quad (28)$$

where p , the real rate of interest is equal to $(r - \frac{dP^e}{P^e})$.

Or,

$$K \cdot i' \cdot de - K \cdot i' \cdot dP + K \cdot \frac{\partial BT}{\partial e} de - K \cdot \frac{1}{P} \frac{\partial BT}{\partial e} dP = 0 \quad (29)$$

(given that $dr = dr + de$), where $de = d(\frac{de}{e})$, and $dP = d(\frac{dP}{P})$.

Now, by combining the IS and LM represented by equations (27) and (29),

$$e = \dot{P} = 0, \quad (30)$$

since $dP = de = 0$.

Therefore, the conclusion can be drawn that, in the long run after a once for all change in P_N , the rate of inflation and the rate of exchange rate depreciation will be equal, and set equal to zero.

Case (B) - Subsidization

Now we introduce the case of subsidization policy. As in the previous case, the long run implications will be derived by solving the system of five equations and five unknowns.

The most important feature of this case is that the continuous deficit arising from subsidizing a constant differential between the world and domestic price of oil, leads to a continuous increase in money supply, by a constant dollar amount per period of time, given the subsidization of some α proportion of the oil subsidy.

The equilibrium, in the goods market is as follows:

$$K_1 dp + K \frac{\partial BT}{\partial e} de - K \cdot \frac{1}{P} \cdot \frac{\partial BT}{\partial e} dP = 0 \quad (28)$$

where ρ again represents the real rate of interest.

And again, by substituting for $\rho (=r - \frac{dP^e}{P^e})$:

$$K \cdot i' \cdot d\bar{e} - K \cdot i' \cdot dP + K \cdot \frac{\partial BT}{\partial e} d\bar{e} - K \cdot \frac{1}{P} \cdot \frac{\partial BT}{\partial e} dP = 0 \quad (29)$$

The equilibrium condition in the monetary market will be given as follows:

$$dM - \left(\frac{\bar{M}}{P}\right) dP = P\ell' dr \quad (31)$$

By combining the money market and the budget constraint:

$$dM = \left(\frac{\bar{M}}{P}\right) dP + P\ell' dr = \alpha[S \cdot m] \quad (32)$$

In growth terms, equation (32) will be as follows:

$$\left(\frac{\bar{M}}{P}\right) \dot{P} + \ell' \cdot d\bar{e} = \left(\frac{\bar{M}}{P}\right) \dot{M} \quad (33)$$

However, with the money supply growing by a constant dollar amount over time, the growth of stock of money, $\dot{M}$, will be taking place at a declining rate. That is, with M growing and dM remaining unchanged, $\dot{M}$ will be falling over time.

Two equations (29) and (32) are of differential type, which are solved in the appendix III and the solutions to which are presented here as follows:

$$\dot{P} = \dot{M} + \frac{1}{\bar{K}} (\bar{e}^{\bar{K}M} - 1) \quad (34)$$

and

$$\dot{e} = \dot{M} + \frac{1}{\bar{K}} (\bar{e}^{\bar{K}M} - 1)^4 \quad (35)$$

where $\bar{K}$ is a constant negative term.

According to equations (34) and (35), with the rate of growth of money supply, $\dot{M}$, falling over time, the rate of exchange rate depreciation, and the rate of inflation, will approach zero exponentially. These are illustrated in the diagram I, and II.

Diagram I

Diagram II

FOOTNOTES TO CHAPTER IV

1. The dynamic adjustment of a supply shock for a closed economy has been analysed by Phelps.
2. The assumption of stationary state is solely for the convenience of exposition, and that it can be modified for the trend of full employment.
3. We restate our assumption that the full employment output $\bar{y}$, remains unchanged to the real price of oil rise. However, even if $\bar{y}$ depends on $\frac{P_N}{P}$, there will be only a once and for all change in it.
4. The stability of the system depends on the following condition to be met:

$$(e^{-\dot{K}\bar{M}} - 1) \geq 0,$$

or

$$e^{-\dot{K}\bar{M}} \geq 1$$

By taking logarithm of both sides,

$$-\dot{K}\bar{M} \ln e \geq \ln 1$$

or

$$-\dot{K}\bar{M} \geq 0,$$

as $\ln e = 1$, and $\ln 1 = 0$

Hence,

$$\dot{M} \geq -\frac{1}{K}$$

The rate of growth of money supply must be greater than (or equal to) the negative of the inverse of K . Otherwise,

4. (continued)

with the rate of growth of money supply, $\dot{M}$, falling
 $(e^{-KM} - 1)$ will be negative, and thereby

$$\left(\frac{\bar{M}}{P} \right) \frac{1}{l} (e^{-\bar{K} \dot{M}} - 1)$$

in equation (34), (35) will be positive, meaning that,
 the rate of inflation would rise when $\dot{M}$ declines.

CHAPTER V

The Dynamic Adjustment to an Oil Price Rise

In chapter III, we studied a one period effect of an oil price rise on output and the price level by treating the expected price level as given. The long-run implications of the oil price rise were studied in chapter IV, where we focused on the long-run behavior of the price level, by assuming that output is always at its full employment level and the actual price level equals the expected price level. In this chapter, we tie up the analysis of the short and the long-run together by taking into account the adjustment of inflationary expectations over time.

To make the dynamic analysis tractable, we assume a plausible and yet simple mechanism for the formation of price expectations as follows:

$$dp_t^e = dp_{t-1} \quad (36)$$

The above simple form of price expectations will serve to bring out the key elements of the adjustment process.¹

We simplify the analysis further by assuming static exchange rate expectations,² that is, the current exchange rate is expected to prevail in the next period.

As in chapters III, and IV, we examine and compare the effects of an oil price rise in the no-subsidy with the subsidization case.

Case (A) - No Subsidy

Here we present the dynamic implications of an oil price rise using the system of five equations, the IS, the LM, the BP, the budget constraint, and the aggregate supply. In this case after an initial increase in the price level, caused by oil price rise, the rate of inflation will follow a path which is dictated by the adjustment of inflationary expectations.

The goods market relationship can be written as^{3,4}:

$$dy_t = \alpha_1 d(dp_t^e) + \alpha_2 de_t + \alpha_3 dp_t \quad (37)$$

where $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_3 < 0$.⁵

In the above relationship we assume that $dr = dr^* = 0$,⁶ and that the real rate of interest (ρ) is equal to the nominal rate of interest minus the expected rate of inflation ($r - dp^e$).

The equilibrium condition in the money market will be represented by:

$$dy_t = \beta_1 dp_t \quad (38)$$

where $\beta_1 < 0$.

The aggregate supply is given as follows:

$$dp_t = \theta_0 + \theta_1 dy_t + \theta_2 dp_t^e + \theta_3 de_t + \theta_4 y_{t-1} \quad (39)$$

$\theta_0, \theta_1, \theta_2, \theta_3 > 0$ ⁸

In order to derive the dynamic adjustment equations for the price level and output, we solve equations (37), (38) and (39) by eliminating de and solving for dy and dp to

obtain the reduced forms as follows:⁹

$$dp_t = A_{11} + A_{12}dp_t^e + A_{13}d(dp_t^e) + A_{14}y_{t-1}, \quad (40)$$

$$dy_t = \beta_1 A_{11} + \beta_1 A_{12}dp_t^e + \beta_1 A_{13}d(dp_t^e) + \beta_1 A_{14}y_{t-1}, \quad (41)$$

where $A_{11} > 0$, $A_{12} > 0$, $A_{13} < 0$, $A_{14} > 0$.¹⁰

The equations (40) and (41) highlight the point that the dynamic adjustment of the price level and output critically depend upon price expectations and the way they are formed.

Using equations (36) and (38) to substitute for p^e and y_{t-1} , the equation (40) can be written as:¹¹

$$p_t + Z_1 p_{t-1} + Z_2 p_{t-2} + Z_3 p_{t-3} = Z_4 \quad (42)$$

where $Z_1 < 0$, $Z_2 \geq 0$, $Z_3 > 0$, $Z_4 > 0$.

This is a third order difference equation the general solution to which is of the form as follows:

$$p_t = C_1 \lambda_1^t + C_2 \lambda_2^t + C_3 \lambda_3^t + d \quad d > 0 \quad (43)$$

To define C 's^{13,14} we assume to start from an initial long run equilibrium characterized by $p = p_0$. Furthermore, in this model the price level approaches its long run equilibrium value after having deviated from it during the adjustment period, that is

$$p(0) = p(\infty) = d \quad (44)$$

Next, substituting the solution for the price level, equation (43) in equation (38), we can derive the general solution for the output as follows:¹⁶

$$y_t = \beta_1(C_1\lambda_1^t + C_2\lambda_2^t + C_3\lambda_3^t + d) + \beta_1' \quad (45)$$

where

$$\beta_1' = \bar{y} - \beta_1 p_0$$

The equation (45) highlights the point that, at the initial and final long run equilibrium points, where $p=p_0$, the output will be as follows:

$$y(0) = y(\infty) = \bar{y} \quad (46)$$

Note that since $\beta_1 < 0$, y_t is the mirror image of p_t . Thus, whenever p_t is rising, y_t will be falling and vice versa. Here, we choose to demonstrate the case when the rising p is accompanied by the falling y .

In illustrating the adjustment paths of p and y , where the details of which depend on numerical values of C 's and λ 's, we make the following simplifying assumptions.

Let us assume that λ_1 is the greatest of all the three roots and that it is positive.¹⁷ Furthermore, let us assume $C_1 > (C_2, C_3)$. Hence, two possibilities may arise:

(a) λ_2 and λ_3 are also positive. In this case the paths of adjustment will be smooth, as illustrated in Diagram I.

(b) λ_2 and λ_3 are negative. In this case the path of adjustment will be determined by the dominant root, λ_1 . Nonetheless, the oscillatory behavior of λ_2 and λ_3 ¹⁸ affects the path as illustrated in Diagram II.

DIAGRAM I DIAGRAM II

Case (B) - Subsidization

An important feature of the subsidization policy is that it involves a continuous increase in the money supply by a constant amount over time. Here, we show the interesting implications of this feature for the dynamic adjustment of the price level and output, using the system of five equations.¹⁹

First, by combining the money market and the budget constraint:²⁰

$$dy = M(S) + \mu_1 dp_t \quad (47)$$

where $\mu_1 < 0$, and $M(S) > 0$ ²¹ is the endogenous change in money supply resulting from the monetization of some a proportion of the oil subsidy, and is constant. In this relationship, again, we assume $dr = dr^*$ ²²

Next, using equation (47), and the aggregate supply equation we solve for dy to obtain the reduced form of the price level as follows:

$$dp_t = z'_1 + z'_2 M(S) + z'_3 dp_t^e + z'_4 y_{t-1} \quad (48)$$

where $z'_1, z'_2, z'_3, z'_4 > 0$ ²³

By substituting in equation (48) for dp_t^e from equation (36) and for y_{t-1} from equation (47), we get

$$p_t - \mu_2 p_{t-1} + \mu_3 p_{t-2} = \mu_4 + \mu_5 M(S) \cdot t \quad (49)$$

where

$$\mu_2, \mu_3, \mu_4, \mu_5 > 0. \quad 24, 25$$

The equation (49) is a second order difference equation²⁶ the solution to which can be written as follows:

$$p_t = C_1' \lambda_1^t + C_2' \lambda_2^t + B_0' + B_1' M(S).t \quad (50)$$

where $B_0' > 0$, $B_1' > 0$.²⁷

The equation (50) represents the adjustment path of the price level under the subsidization policy.

The arbitrary constant C_1' and C_2' can be defined by setting the initial conditions as follows:

$$p(0) = B_0' \quad (51)$$

which implies $C_1' = -C_2'$. Thus, the equation (50) can be rearranged and given by:

$$p_t = C_1' (\lambda_1^t - \lambda_2^t) + B_0' + B_1' M(S).t \quad (52)$$

Similarly, the adjustment path of the output can be derived by combining equations (52) and (47),²⁸

$$y_t = \mu_1 \{ C_1' (\lambda_1^t - \lambda_2^t) + B_0' \} + B_0'' \quad (53)$$

where

$$B_0'' = y_0 - \mu_1 B_0' > 0.$$

Note that C 's are defined so that the following relationship holds,

$$y(0) = y(\infty) = \bar{y}. \quad (54)$$

In illustrating the adjustment paths of p and y , we distinguish two possibilities:

(a) The paths are smooth. For this to be the case, the following conditions must hold:

$$\lambda_1 > \lambda_2 > 0, \text{ or } \lambda_2 > \lambda_1 > 0. \quad (55)$$

(b) The paths are oscillatory. This will be the case if

$$\lambda_1 > 0, \lambda_2 < 0, \text{ or } \lambda_2 > 0, \lambda_1 < 0. \quad (56)$$

These possibilities are illustrated in Diagrams III and IV.³⁰ In the presence of the subsidization program, the price level will be continuously rising, at a declining rate. Simultaneously, output, after an initial plunge, will tend to approach its long run value, as illustrated.

At this stage, it is interesting to compare the illustrated paths of price and output level for the alternative energy schemes. This comparison is demonstrated in diagram V, where we superimpose Diagram I on III. Diagram V makes the following interesting points:

- i) In the subsidization case, the level of output is higher than the no-subsidy case, and that the higher level of output has come about at the expense of higher prices.
- ii) The subsidization policy might only lead to a short term fall in the general price level. In the long run the price level will be above the level which would prevail in the no-subsidy case. It is even possible that the price level in the presence of the

DIAGRAM III DIAGRAM IV

DIAGRAM V-a

DIAGRAM V-b

subsidization policy may actually be higher than that, in its absence, even in the short-run. This possibility is illustrated in the diagram by broken lines. Such an outcome, however, is an empirical question which we will consider in the next chapter.

FOOTNOTES TO CHAPTER V

1. We also considered the general form of adaptive expectations. This, however, gave rise to higher order difference equations which changed the details only, without affecting the substance.
2. By making this assumption we abstract from the role of interest rate changes caused by exchange rate changes.
3. In this dynamic framework, we reemploy the time subscripts.
4. The symbol "d" as in dp , dy , is used to represent first difference. However, dp and dy can represent percentage changes if the relationships are assumed to be in the log linear form.
5. Approximated by the above difference equation for very small changes, α_1 , α_2 and α_3 are as follows:

$$\alpha_1 = -K_i', \quad \alpha_2 = K \frac{\partial BT}{\partial e}, \quad \alpha_3 = -\frac{1}{P} \alpha_2 = -\frac{1}{P} K \frac{\partial BT}{\partial e}.$$
6. This is caused by the assumption of inelasticity of exchange rate expectations, ($e_{+1}^* = e$). This assumption is, however, different from that of the price expectations (a similar assumption would instead be represented by $de_{+1}^* = de$), but comes close to it when the exchange rate fluctuates around some stationary value. In the no subsidy case the long run path of the exchange rate is assumed stable.
7. $\beta_1 = -\frac{\bar{M}}{P^2 K'} < 0.$
8. $\theta_0 = A_0$, $\theta_1 = A_1$, $\theta_2 = A_2$, $\theta_3 = A_3 \cdot P_N^*$. These coefficients were introduced earlier in chapter III.
9. For the derivations see the Appendix IV.
10. See the Appendix.
11. See the Appendix.
12. See the Appendix.

13. To see the values of C 's refer to the Appendix.

14. We assume that the roots λ_1 , λ_2 and λ_3 are distinct and real. It is, however, possible that these roots may be of the complex form, in which case the complementary function will be transformed into trigonometric terms.

15. Whether the equilibrium is dynamically stable is a question of whether or not the complementary function will tend to zero as $t \rightarrow \infty$, i.e., $P_\infty = P_0$. The sta-

bility of the system, or the convergence of the time paths depends upon the characteristic roots. More specifically, the dynamic path of adjustment of the price level and output can converge if and only if every root of the characteristic equations is less than one in absolute value. In view of this, the Schur Theory will be applicable:

The roots of the equation (42) (also represented by the polynomial $\lambda^3 + Z_1\lambda^2 + Z_2\lambda + Z_3 = 0$, given by equation (IV-13) in the appendix) will be less than one if and only if the following three determinants are all positive:

$$\Delta_1 = \begin{vmatrix} 1 & Z_3 \\ Z_3 & 1 \end{vmatrix} > 0,$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & Z_3 & Z_2 \\ Z_1 & 1 & 0 & Z_3 \\ Z_3 & 0 & 1 & Z_1 \\ Z_2 & Z_3 & 0 & 1 \end{vmatrix} > 0,$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & 0 & z_3 & z_2 & z_1 \\ z_1 & 1 & 0 & 0 & z_3 & z_2 \\ z_2 & z_1 & 1 & 0 & 0 & z_3 \\ z_3 & 0 & 0 & 1 & z_1 & z_2 \\ z_2 & z_3 & 0 & 0 & 1 & z_1 \\ z_1 & z_2 & z_3 & 0 & 0 & 1 \end{vmatrix} > 0.$$

16. Refer to the Appendix for the derivations.
17. Given the signs of Z's in equation (46), it can be easily shown that at least one of the roots is positive and at most two are negative.
18. The exponential functions of form $y=a^t$ have an oscillatory pattern when $a < 0$.
19. Unlike the no subsidy case, the equilibrium of P and y can be determined independent of the IS equation, and that it, along with AS, is used in the determination of the exchange rate.
20. This relationship which is represented by equation (II-B6) of Appendix I is derived by combining the totally differentiated versions of equations (9) and (10)

$$21. \quad M(S) = - \frac{\alpha \cdot P'_N \cdot m \cdot ds}{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - pk'} > 0$$

$$\mu_1 = \frac{\alpha \cdot s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial (\frac{P_N}{P})} + \frac{\bar{M}}{P}}{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - pk'} < 0$$

22. This is caused again by the assumption of the inelasticity of exchange rate expectations ($e_{+1}^* = e$). However, in the subsidization case

a more appropriate assumption would be the one given by $de^*_{+1} = de$. This assumption, in turn, would bring out an additional factor. The new factor is changes in the real rate of interest which would no longer remain invariant. The variations in the real rate of interest bring about additional effects on the price level.

$$23. \quad z'_1 = \frac{A_0}{1-A_1\mu_1}, \quad z'_2 = \frac{A_1 \cdot M(S)}{1-A_1\mu_1} > 0, \quad z'_3 = \frac{A_2}{1-A_1\mu_1} > 0, \\ z'_4 = \frac{A_1}{1-A_1\mu_1} > 0.$$

24. For the details refer to the Appendix.

25. The difference equation (49) has the following characteristic equation:

$$\lambda^2 - \mu_2\lambda + \mu_3 = 0,$$

which can be solved for two roots λ_1 and λ_2 . Again, the conditions for convergence will be presented in terms of the value of μ_2 and μ_3 . Since the order of the equation is two, in this case, the convergence conditions will be less complicated than that of the no-subsidy case, (footnote 15), as follows:

The two characteristic roots are always related to each other by the following equations:

$$\lambda_1 + \lambda_2 = \mu_2$$

$$\lambda_1 \lambda_2 = \mu_3$$

Since both λ_1 and λ_2 are positive it follows that both λ_1 and λ_2 must be positive. Therefore, the convergence of the time paths of P and y for the case of distinct and real roots will be satisfied by the following condition:

$$0 < \lambda_2 < \lambda_1 < 1 \quad \text{if} \quad \mu_3 (= \lambda_1 \lambda_2) < 1.$$

Note, however, that while the complementary function

tends to zero as $t \rightarrow \infty$, the particular solution of the same equation would be rising over time, since the latter is a function of time. Thus, in the subsidization case the stability of the system can be identified with the price level continuously rising (at a declining rate) over time toward its long run equilibrium value.

26. We must note that the right hand side of the equation is not constant but it is rather a variable in terms of time, in this case.
27. See the appendix.
28. We assume again that the roots λ_1 and λ_2 are distinct and real.
29. Equation (49) clearly unveils that at least one of the roots is greater than zero. Note that we are assuming away the complex roots.
30. Note that as in the no-subsidy case, we choose to illustrate the case where $\lambda_1 > \lambda_2 > 0$, that is, the rising P accompanied by a falling y .

CHAPTER VISOME EVIDENCE ON THE NATURE AND
IMPLICATIONS OF THE CANADIAN OIL POLICY

In this chapter, we present an overview of the pricing history of Canadian Crude Oil, supplemented by some evidence for a decade which preceded the well known oil price rise by OPEC, and a decade which followed it to this date. We divide this chapter into two parts. The first part itself will be divided into three subsections. In the first subsection, a background to the oil policy will be given. In subsection two, we present some empirical evidence regarding the relationship between the domestic and world price of oil. And finally, in subsection three, some budgetary aspects of the Canadian oil policy will be discussed. In part two, we provide some evidence on the comparison between the no-subsidy and subsidization case.

The Canadian domestic price of oil has displayed rather a peculiar trend relative to the price of imported oil over the last two decades. Prior to the 1974 oil price rise, it had exceeded the foreign¹ price of oil, and since then it has been set below the price of imported oil.

During the entire 1950's, the price of Canadian crude oil was set so as to make the Canadian crude competitive with the American. Since American oil was an alternate source for Ontario refineries, the amount of domestic crude oil consumed in Ontario was a function of the relative price of Canadian and American crude oil. Canadian oil was also exported to the U.S. and the volume of this flow also depended upon the relative price of American and Canadian crude oil.

In 1958, however, controlling influence changed from U.S.-markets to offshore markets. The change in the environment facing the Canadian crude oil production industry was mainly the result of two events. First, emerging competition in the world petroleum industry and expanding Middle East supply which led to declining world prices. Secondly, the U.S. imposed a system of protection that served to isolate its domestic market from the downward price trends through a quota system, which led to a rise in the American price. As a result, as the decade progressed, the Canadian price of crude oil, under the increasing influence of the Middle Eastern oil, developed a growing differential with the American price. Nevertheless, it continued to stay above the level that would have equated it to foreign price of oil. This trend was maintained throughout the period ending in 1971 when the differential in crude oil prices in the world as opposed to Canadian started to vanish as OPEC began to exercise its new found power. By late 1971, the increase in the price of the imported delivered to Montreal had outstripped the delivered domestic oil in Toronto price increases. And subsequently, by early-1974, the foreign price had jumped ahead of the domestic price of oil.²

Now, we turn to the episode which is the focus of this thesis.

Since the formation of the organization of petroleum exporting countries (OPEC) in 1973 and the subsequent jump in the price of crude oil by almost four hundred percent, the oil

59.

consuming nations, industrialized as well as developing countries, at least a large number of them, have experienced serious economic disturbances in terms of the growing burden of oil payments, inflation, balance of payments, and unemployment.

However, the sharp oil price increase which revolutionized the international oil industry, which fundamentally changed the economics and politics of oil in the world, caused only a little adjustment on part of some of the industrialized oil importing nations, particularly Canada, toward self-sufficiency and reducing the dependency on the imported oil. It was not until beginning-mid 1979 that the fragile nature of the international oil market preceded by a period characterized by oil glut and eroding the real price of oil made Canada, among other industrialized nations, alter her energy policy decisions in favour of a transition toward a more independent energy situation.

Canada, unlike most of its industrial partners, is a net exporter of energy even though the energy endowment is not evenly distributed. Western Canada is a net exporter while Eastern Canada is a net importer of energy (especially oil). Nonetheless, while the overall energy situation, particularly for natural gas, is promising, it is evident that Canada has a serious problem with oil. For instance, today some 425,000 barrels a day which is 25 per cent of Canadian oil consumption is imported.

The following table provides some information with respect

to the Canada Trade Balance situation for petroleum.

(\$ millions)

Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
Trade Balance	+129	+172	+344	+647	+1045	+171	-625	-1065	-1427	-2103

SOURCE: Statistics Canada, Exports Merchandise Trade, and Imports Merchandise Trade.

The table suggests that Canada has been the net importer of petroleum since 1976.

Following the 1973-1974 OPEC oil price increases, the Federal Government adopted a one price oil policy program for Canada. And in order to soften its uncertain movements, it chose not to tie the domestic price of oil to the world price. Yet, to provide incentives to encourage conservation and to ensure an adequate return to producers to finance needed exploration and development of alternative sources of energy, the government programmed that the domestic price of oil would move towards the world level in stages.

This program known as "Oil Import Compensation Program" has been conducted by the Petroleum Compensation Board established in 1974. The program compensates imports of crude oil for the increases in the price set by OPEC, increases in tax and increases in freight costs. The compensation is implemented on a flat rate basis which is determined as the difference between average costs of crude oil delivered to eastern Canada and the theoretical delivered cost of similar Canadian crude oil.³

However, the government plan with respect to the steady movement of the domestic price of oil towards the world level was never realized. In fact, the wellhead price of oil moved very sluggishly, inadequately and arbitrarily. That is, the wellhead price of oil increased by four dollars in 1974, \$1.50 in 1975, \$1.00 in 1976, \$1.50 in 1977, almost \$2.50 in 1978 and only \$1.00 in 1979. Therefore, the subsequent increases in the oil price by OPEC significantly widened the gap between domestic and the world prices, and hence led to an enlargement of the subsidy per barrel of oil paid to the refiners.

The budget presented to the House of Commons by the Conservative government in 1979 had aimed at reducing this gap and the oil trade deficit by allowing the price of oil to rise by \$4 a barrel in 1980, \$4.50 a year in subsequent years, from January 1, 1981 to January 1, 1984. The amount of these price increases would be revised in January 1983, so that if the prevailing domestic price of oil is less than 75 per cent of the lower "Chicago price", or the international price the amount of the difference would be added to the price increases for January 1, 1983.⁴

Nonetheless, this policy never got off the ground as the Conservative Government was defeated on the presentation of the budget. Instead, the incoming Liberal Government promised that the price of crude oil would not rise by more than a dollar for every six months of 1980.⁵ And subsequently, the wellhead price would rise by \$1 per barrel every six months beginning January 1,

1981. Starting January 1, 1984, the semi-annual increase would be \$2.25 per barrel and in 1986, it would be \$3.50 per barrel and these increases would continue at that pace until the well-head price reaches the "reference price".⁶ Furthermore, a new feature of the current policy manifested itself in a new form of levy named as Petroleum Compensation Charge which would be levied to all domestic refineries to finance the oil compensation payments. Revenue from this charge would be used to pay importing refiners to reduce the average cost of imported oil to the average cost of oil to the Canadian refiners. By the end of 1980, this charge was estimated to be \$2.25 a barrel and was to increase by \$2.50 a barrel on January 1, 1981, 1982, and 1983.

The fact remains that the soft oil price policy carried out during the entire 1979 and 1980 accompanied by a new round of price increases by OPEC which took the current level of price of oil to more than \$40 Canada, caused the federal subsidy on imports to rise from \$3 a barrel in mid 1978 to more than \$20 by the end of 1980.

Now, we discuss the relationship between the domestic price of oil and the world price of oil. The following chart demonstrates the difference between the price of domestic crude oil and the cost of imported crude, and thereby the amount of the federal subsidy on the imported oil. To formulate this relationship we regress the domestic actual price of oil against the foreign price for the period 1970-79. In doing so, we decompose the price of imported oil into two explanatory

SOURCE: Petroleum Compensation Board, the Department of Energy, Mines and Resources.

variables; the price of crude oil in foreign currency, p_N^* , and the exchange rate, e , the result of which is given in line one, Table I. This result suggests that p_N^* is not responsive to the exchange rate fluctuations. Thus, we drop the exchange rate, and run the second regression where the domestic actual price is tested against the world price of crude oil in the domestic currency, p_N .⁷ According to this, the domestic price of oil, on average, shows a degree of responsiveness equal to 20% of the world price of oil.

In terms of the parameters in the reaction function, these results suggest the following:

$$\gamma_0 = .02 ,$$

$$\gamma_1 = .21 ,$$

$$\gamma_2 = 0 .$$

However, it may not be appropriate to regard the parameter γ_1 as constant. These results represent the average response.

In fact, as can be readily seen from the Chart I that γ_1 appears to have changed over time. By the end of 1979 and early 1980, γ_1 seems to be much smaller than it was in the beginning of the decade.

In terms of the budgetary background material on crude oil, the following table provides the payments under the oil export compensation program and revenues from the oil export charges.

TABLE I

REGRESSION RESULTS

(Quarterly Data, 1970 I - 1979 IV)

Dependent Variables	Type of Regression	Coefficient of independent variables (t-value in brackets)					R ²	D-W	SEE
		C	Alog p _N	Alog p _N ¹	Alog e				
Alog p _N ¹	CORC	.027 (2.88)		.219 (2.91)	.54 (.876)		.29	1.99	0.07
Alog p _N ¹	CORC	.029* (3.16)	.206 (2.82)				.28	1.98	0.07

*Significance at 5% level.

¹These regressions are expressed in log linear form.

SOURCE: Statistics Canada, Imports by Commodities and Energy Mines and Resources, OICP.

\$ Millions

	1973-74	1974-75	1975-76	1976-77	1977-78	1978-79	1979-80
Revenue: oil export tax and charge	144	1558	1063	661	432	328	735
Expenditure: oil subsidy	157	1162	1582	945	925	628	1912
Net Expenditure	13	-396*	519	284	493	300	1177

*These figures are net of provincial share of export tax discontinued as of 1975.

SOURCE: Department of Energy, Mines and Resources.

As a proportion of the government budget deficit, the oil deficit constitutes an important and large share.

Year	1975-76	1976-77	1977-78	1978-79	1979-80
Percentage of National Accounts Deficit	13.4	6.5	4.9	3.1	12.7

This information supports the significance that we attach to the role of oil subsidy in the government budget constraint.

Finally, to verify whether and to what extent the increases in the government deficit is financed by a money issue, i.e.,

$$dM = \alpha(D) ,$$

where D = total government deficit, we regressed the changes in money supply against the government surplus, $(T-G)$. The following represents the results of such a regression;

$$\log M - \log M_{-1} = C_0' + C_1'(\log D - \log D_{-1})^{8,9}$$

$$R^2 = .41$$

$$D-W = 2.38$$

$$43.3$$

$$(1.43)$$

$$-.327$$

$$(-4.18)$$

We should note that .327 overstates the degree of monetization if the changes in the money supply has systematically responded to variables that are highly correlated to the government surplus.

Some Evidence on the Comparison Between the No-Subsidy and the Subsidization Case

Chapter III brought out the possibility that the policy of subsidizing the price of imported oil can actually lead to higher prices relative to the no-subsidy case. The analysis of the dynamic adjustments in Chapter V cast more light on this possibility.

Here, focusing on the short run impact effect, we attempt to quantify the effect on the price level of an oil price rise in the subsidization and the no-subsidy case to examine whether the evidence for Canada is consistent with the above mentioned theoretical possibility.

In price setting of the subsidization case, let us assume the possibility that $\gamma_1 = 0$. In this case the link between p_N^i and p_N^* is completely broken, i.e., p_N^i does not respond to the changes in the world price of oil. This assumption is, in fact, supported by our empirical evidence for Canada since mid 1970's.¹⁰

We also make the simplifying assumption that in the no-subsidy case while p_N^i adjusts fully to p_N^* , it makes no adjustment with respect to the exchange rate. The no-subsidy case will hence be characterized by $\gamma_1 = 1$, $\gamma_2 = 0$, in which the oil price equation can be written as $p_N^i = \bar{e} p_N^*$.¹¹

Assuming log linear relationships for the reduced forms, we can show that the oil price rise has a greater impact on price in the subsidization than in no-subsidy case if:

$$x \cdot \eta_1 < \epsilon_1 \cdot \dot{M}$$

(57)

where coefficient $x < 1$, and η_1 is the elasticity of p with respect to p_N^* in the aggregate supply relationship. This represents the direct effect.^{12, 13}

The right hand side of the equality represents the elasticity of p with respect to p_N^* via the monetary channel, the indirect effect, where ϵ_1 is the percentage change in p caused by one percent change in the stock of money, and $\dot{M}$ is the proportional change in money supply resulting from one percent change in p_N^* , $\dot{M} = \alpha \cdot R \cdot \dot{p}_N^*$,¹⁴ where α is the proportion of the monetization of the subsidy; $R = \frac{(p_N^* \cdot m)}{M}$ is the ratio of the value of imported oil to the level of money supply.

To quantify the right hand side of the algebraic expression, let us first calculate $\dot{M}$. In order to do so, we consider the effect of 1% change in p_N^* . The resulting money supply will then depend upon α and R . Here R is the average value of the annual ratios calculated during 1973-79. As to α , we consider two different values: 0.3 which is the actual value of α , estimated in our model; and 1.0, an extreme value, representing the full monetization case.

Using two Canadian large econometric models, CANDID and RDX-F we calculate the effect of one percent change in the money supply on the price level over a specified period of time, in CANDID one year and in RDX-F 9-10 quarters.

Finally, regarding η_1 , we use two estimates: 1) our own estimate which is based on a simple equation, .021.¹⁵ 2) the Statistics Canada estimate which is based on a more detailed model, .016, which is less than ours.

The result of these estimates are summarized in the following table:

The estimated effect of 1% change in P_N^* in subsidization case

	α	
	$\alpha = .3$	$\alpha = 1$
M	.06 - .07	.215 - .24
CANDID ($\epsilon_1 = .14$) M. ϵ_1	.009 - .01	.030 - .034
RDXF ($\epsilon_1 = .3-.5$) M. ϵ_1	.018 - .03	.06 - .10

This table shows the results under different α and ϵ_1 . In the case of full monetization, $\alpha=1$, the indirect effect (M. ϵ_1), given by the third column in the table, is compared with the direct effect (η_1) estimated at .021 in our model and .016 reported by Statistics Canada. This comparison indicates that the right hand side of the equation (57) is greater

than the left hand side. That is, the oil price rise has a greater impact on the price level in the subsidization case than in the no-subsidy case.

The second column in the table presents the indirect effect measured for the case of $\alpha=0.3$, which in turn, is compared with the direct effect. When we use the estimate for ϵ_1 from RDXF, with a period of 9-10 quarters, the right hand side of the equation (the indirect effect) continues to exceed the left hand side (the direct effect). This conclusion, however, does not hold with respect to the estimate from CANDID, with the shorter time span, one year.

Note that according to equation (57) the direct effect is only a fraction of η_1 , since $x < 1$. This, however, has been ignored in the above comparison by treating x as unity which therefore implies that the direct effect has been overestimated.

FOOTNOTES TO CHAPTER VI

1. We particularly refer to the Iranian and Arabian crude oil.
2. State of Competition in the Canadian Petroleum Industry, Volume II.
3. This was done as of July 1975. Prior to July 1, 1975 compensation was based on the lesser of the cost increase sustained by the importer or increases in the host government take and the participation for each cargo of crude oil imported during the program compared to an identical cargo imported in the base year.
4. Budget Paper, presented to the House of Commons, on December 11, 1979.
5. National Energy Program, Department of Energy, Mines and Resources, submitted to the Parliament as part of the 1980 Budget.
6. The "reference price" is a special price which will be offered to synthetic crude from the oil sands, effective January 1, 1981, it will be \$38 per barrel escalated annually by the consumer price index.
7. We also added the lagged value of p_N^t in this regression, but it did not appear to be significant.
8. However, adding the lagged value of the money supply, did not change anything. M_{-1} was not significant.
9. The money supply is defined as M_1 , and the data is a monthly series. The regression is estimated in log-linear form.

10. During the period 1973-1979, we found no significant relationship between p_N' and p_N^* .
11. Any deviation of the actual from the long term exchange rate causes the price of every barrel of imported oil paid by the domestic consumers to diverge from that of paid to the exporters of oil, i.e.,

$ep_N^* \neq \bar{e}p_N^*$. Hence depending upon whether $e > \bar{e}$, or $e < \bar{e}$ the government will wind up subsidizing or collecting the difference, $H=S.m.$ This can be either positive or negative.

Also, in deriving the reduced form of the no-subsidy case, we assume that the subsidy arising from the difference between the actual and the long term exchange rate is temporary. Hence, the resulting changes in the government oil deficit will not have any effect on the money supply.

$$12. \eta_1^* = \frac{\partial p}{\partial p_N^*} \cdot \frac{p_N^*}{p}$$

$$\epsilon_1 = -K_1 \cdot \frac{\eta_3 \cdot \frac{BT}{y} - \eta_4 \cdot \frac{1}{y}}{\eta_5} \cdot \eta_2$$

where K_1 is the multiplier, explained earlier.

$(\eta_3 > 0)$ = the elasticity of the balance of trade with respect to the exchange rate.

$(\eta_4 < 0)$ = the elasticity of the investment with respect to the interest rate.

$(\eta_5 < 0)$ = interest elasticity of the money demand.

$\frac{BT}{y}$ and $\frac{1}{y}$ are the shares of the trade account and the

investment in the real GNP.

$(\eta_2 > 0)$ = elasticity of p with respect to y .

$\dot{M} = \frac{dM}{M} = \frac{\alpha \cdot m}{M} \frac{dp_N^*}{p_N^*}$, where M is the level of money supply

and m , the quantity of imported oil.

Note that η_1 and η_2 are the elasticity versions of A_1 and A_2 in the aggregate supply equation.

13. The net effect of an oil price rise (supply effect - demand effect), in the no-subsidy case is represented by the left hand side of the inequality. Here, we have assumed that this net effect is proportional to the supply effect which is represented by η_1 , i.e., (η_1 - demand effect) = $x \cdot \eta_1$, where $x < 1$.

14. The proportional change in money supply M , crucially depends upon the value of α , the derivation of which can be written as follows:

$$dM = \alpha \cdot m \cdot dS, \text{ or } dm = \alpha \cdot p_N' \cdot mds$$

where $s = \frac{S}{p_N'}$ = the subsidy rate per p_N' .

$$\text{But, } s = (p_N^*)^{1-\gamma_1} - 1$$

and $ds = (1-\gamma_1) dp_N^*$, where $e = \bar{e}$ at the initial equilibrium.

$$\text{Thus } dM = \alpha \cdot m \cdot dp_N^*, \text{ where } \gamma_1 = 0$$

5. This is reported in more detail in Appendix V.

CHAPTER VII
SUMMARY AND CONCLUSIONS

This study has analysed the short and long run effects of an oil price rise on output, inflation and the exchange rate with and without an oil subsidy. This study has also analysed the dynamic adjustment to the oil price rise under alternative energy schemes.

In the short run the important element is that wages and prices are rigid. Here, to analyse the effect of an oil price rise, we considered both aggregate demand and aggregate supply. In the absence of a subsidization policy, the oil price rise causes the aggregate demand and aggregate supply curves to shift to the left leading to an unambiguous fall in output but an ambiguous effect on the price level. It is interesting to note that the exchange rate flexibility does not have the insulating property against the oil price rise, as the exchange rate was shown to have either risen or fallen.

Next, we introduced the effect of the subsidization policy. This policy was shown to have introduced two new features into the analysis of Chapter III. First, by employing a price relationship which allows for the domestic price of oil to adjust incompletely to the world price of oil, the direct impact on aggregate demand and supply was scaled down. Secondly, given that some proportion of the government deficit is to be monetized,

the oil subsidy affects the level of money supply and hence has further effect on the aggregate demand. One clear result is that this effect leads to a higher level of output. However, the rate of inflation could also be higher. This conclusion is in contrast with the popular view that ignores the demand side effect and argues that the subsidization policy can result in lower inflation, by looking only at the supply side.

The analysis showed that the effectiveness of the subsidization policy crucially depended upon two factors; the value of α representing the proportion of the government deficit which is monetized and, the value of γ_1 representing the proportion by which the domestic price of oil responds to the world price of oil.

In Chapter IV, we went on discussing the long run implications of the oil price rise. In the long run wages and prices are flexible and the output is at its full employment level. In the no subsidy case, an increase in the price of oil affects the relative price and not the aggregate price level. After a once for all increase in the price of oil, the price level and the exchange rate remain unaffected, and rate of inflation and the exchange rate depreciation will be zero.

The subsidization policy has an important implication for the long run because of a continuous rise arising from subsidizing a constant differential between the world and domestic price of oil which in turn leads to a continuous

increase in money supply by a constant dollar amount over time. The rate of growth of money supply will be, however, declining and thus would cause the rate of inflation and the exchange rate depreciation to decline over time.

In Chapter V bringing the analysis of the short and long run together, we studied the dynamic adjustment to an oil price rise by taking into account the adjustment of inflationary expectations over time. Here, we provided an illustrative example, and in this we showed that, in the no-subsidy case, after the initial deviation from the long run equilibrium position, due to a once for all rise in price of oil, the price and output approach this equilibrium asymptotically.

As to the subsidization case, the important feature is again that it involves a continuous increase in the money supply by a constant amount over time. Critically depending upon the inflationary expectations and the way they are formed, the dynamic paths of adjustment in the subsidization case were shown to have been different from those of no-subsidy. Here, dictated by a continuous increase in money supply by a constant dollar amount, but at a declining rate, the rate of inflation will be decreasing while the price level is continuously rising at a declining rate. As to output, after an initial plunge it tends to approach its long run value.

In comparing the no-subsidy with the subsidization case, we know that the price level in the subsidization case is

greater than the no-subsidy in the long run, but the interesting question is to focus on the relative positions of the paths of adjustment of price and output in the alternative schemes in the intermediate run, given the information about both short and long run. The result of this comparison between the paths of adjustment can be summarised as 1) in the subsidisation case, level of output is higher than the no-subsidy; 2) the subsidization policy may only lead to a short term decline in the price level relative to the no-subsidy case, and not a long term decline. Yet, we explored the possibility that the subsidization could actually be more inflationary than the no-subsidy even in the short run. Later in Chapter VI we addressed this possibility from an empirical point of view.

In Chapter VI, we moved on to analyse the Canadian oil policy. We showed that the soft energy policy pursued by the Federal Government accompanied by new rounds of OPEC price increases, provided a ground for the government to run a huge deficit on the oil subsidy account, 12% in 1979-80.

In formulating a relationship between the domestic and the world price of oil, we regressed these variables against each other. We found out that the domestic price of oil does not respond to the exchange rate fluctuations. We also found out that even though the Federal Government response to the world price of oil varied from time to time, on the average the domestic price of oil seems to have responded by 20% to the increase in world price of oil during the period of 1970-1979, $\gamma_1 = 0.2$, but no response

to the increase in the world price of oil during the shorter period of 1973-1979, $\gamma_1=0$. Our evidence also suggested that about 32% of the government deficit over a period of thirty years, between 1950-79, has been monetized.

Finally, focussing on the short run impact effect, we quantified the effect on the price level of an oil price rise in the subsidization and the no-subsidy case to find out whether the evidence for Canada has been consistent with the suggested theoretical possibility. Using some plausible estimates to represent the direct and the indirect effects, the above possibility was shown to have been strongly supported by the evidence for the extreme case $\alpha=1$ (full monetization). However, for the actual value of α , 0.3, the results were not as strong as the previous case.

In summary, our analysis suggests that the policy of subsidizing price of oil may have contributed to the domestic inflation. Recently, the government has introduced a levy named as petroleum compensation charge on all domestic refineries. In the light of this, it may well be argued that the government need not finance its deficit by the monetization. However, this way of raising revenue has always been available to the government and even without the subsidy, the government could have reduced the deficit.

In this thesis having rendered a theoretical analysis of an oil price rise, we were content with presenting some

summary evidence. As an area of further research we suggest that, in an analytical framework such as ours, a more complicated reaction function be adopted; one which links the domestic price of oil, in addition to the exchange rate and the world price of oil, to objective functions such as the desired level of unemployment rate or the real GNP.

Appendix I

An Alternative Formulation of the Short Run Aggregate Supply

In this appendix we derive an alternative short run aggregate supply which is based upon marginal productivity equation and Phillips curve

First, we begin by introducing a production function in which the output is a function of labour, capital and the intermediate good, oil.

$$Q = f(K, L, N) \quad (I-1)$$

where all factors of production are substitutable. Given the nominal wage rate w , and the price of final output p , the factor demand, (marginal productivity equation), is derived from the short run profit maximization behavior of the firms when the net marginal product of labour is equated to the wage rate.

$$(p - p_N') f_L = w, \quad p > p_N' \quad (I-2)$$

and $f_L' < 0$ by the diminishing return to labour. By differentiating totally (I-2) we have,

$$(p - p_N') f_L' dL + f_L dp - f_L dp_N' = dw \quad (I-3)$$

From the production function, by allowing L to change and keeping every other factor constant, we have

$$dQ = dy = f_L dL \quad (I-4)$$

Note that y (net output), as we saw earlier, is defined by,

$$y = Q - \left(\frac{p_N'}{p} \right) \cdot N^*$$

By combining (I-3) and (I-4) and solving for dL , dw can be written as follows,

$$dw = (p-p'_N) \frac{f'_L}{f_L} dy + f_L dp - f_L dp'_N \quad (I-5)$$

The equation (I-5) represents our marginal productivity equation.

Now by combining this equation with Phillips curve, represented by equation (1) in Chapter II, we have

$$(p-p'_N) \frac{f'_L}{f_L} dy + f_L dp - f_L dp'_N = b_1 + b_2(y-\bar{y}_t) + b_3 dp_t^e$$

or

$$dp = A_0' + A_1'(y-\bar{y}) + A_2' dp^e + dp'_N + A_3' dy \quad (I-6)$$

where

$$A_0' = \frac{b_1}{f_L} > 0, \quad A_1' = \frac{b_2}{f_L} > 0, \quad A_2' = \frac{b_3}{f_L} > 0, \quad A_3' = (p-p'_N) \frac{f'_L}{f_L^2} < 0.$$

The equation (I-6) represents the alternative short run aggregate supply wherein, the price is a function of short run output gap (the gap between actual and potential output), the price of oil, and the output difference dy , (the difference between the current and past period levels of output); for a given change in the expected price level.

APPENDIX II

Solution and the Short Run Equilibrium of the Model

In this Appendix we derive a relationship between the price of oil, the domestic general level of price, output and the exchange rate. In doing so we follow the pattern of our analysis in Chapter III by dividing this Appendix into two parts, (A) no subsidy, and (B) the subsidization.

CASE (A) - No Subsidy

In order to solve the system of the five equations and five unknowns, we first differentiate totally the equations representing the IS, the LM, the BP and the budget constraint.

Equilibrium in the good market is given by:

$$\begin{aligned}
 dy = & \frac{\partial c}{\partial y} (1-t) dy - \frac{\partial c}{\partial y} y dt + i' dr + dg + \frac{\partial x}{\partial e} de + \frac{\partial x}{\partial p} dp \\
 & - \frac{1}{p} m \cdot dp_N' + \frac{P_N'}{p^2} m \cdot dp - \frac{P_N'}{p} \cdot \frac{\partial m}{\partial y} dy - \frac{P_N'}{p} \cdot \frac{\partial m}{\partial (\frac{P_N}{p})} \cdot \frac{1}{p} dp_N' \\
 & + \frac{P_N'}{p} \cdot \frac{\partial m}{\partial (\frac{P_N}{p})} \cdot \frac{P_N'}{p^2} dp
 \end{aligned}$$

or

$$\begin{aligned}
 (1 - \frac{\partial c}{\partial y}(1-t) + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial y}) dy &= d(GT) + i' dr - \frac{1}{P} (m + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial (\frac{P_N}{P})}) dP'_N \\
 + \frac{\partial x}{\partial e} de + \{ \frac{\partial x}{\partial P} + \frac{P'_N}{P^2} (m + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial (\frac{P_N}{P})}) \} dP
 \end{aligned}$$

or,

$$\begin{aligned}
 (1 - \frac{\partial c}{\partial y}(1-t) + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial y}) dy &= d(GT) + i' dr - \frac{1}{P} MR \cdot dP_N^* \\
 + \{ \frac{\partial x}{\partial e} - \frac{P'_N}{P} MR \} de + \{ \frac{\partial x}{\partial P} + \frac{P'_N}{P^2} MR \} dP
 \end{aligned}$$

From $x = x(\frac{eP^*}{P})$, we have

$$\frac{\partial x}{\partial e} = \frac{P^*}{P}, \quad \frac{\partial x}{\partial P} = \frac{-eP^*}{P^2}$$

Thus,

$$\frac{\partial x}{\partial P} = -\frac{1}{P} \frac{\partial x}{\partial e}$$

We also have,

$$\left(m + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial \left(\frac{P_N}{P}\right)}\right) = \frac{d}{d\left(\frac{P_N}{P}\right)} \left[\frac{P'_N}{P} \cdot m\left(y, \frac{P_N}{P}\right) \right]$$

which represents the derivative of total expenditure on the oil imports with respect to its price. Thus, it is a notion of some kind of marginal expenditure (revenue), MR, associated with the demand for oil where the axes are inverted, and that it (MR) is greater than zero by the virtue of inelasticity of the demand for oil with respect to price.

We further assume that Marshall-Lerner condition holds, and that implies,

$$\left(\frac{\partial x}{\partial e} - \frac{P'_N}{P} \cdot MR\right) = \frac{\partial BT}{\partial e} > 0$$

where BT denotes the balance of trade. With the exchange rate depreciation (e rising), the balance of trade will improve.

Thus, we have,

$$\left(1 - \frac{\partial c}{\partial y}(1-t) + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial y}\right) dy = d(GT) + i' dr - \frac{1}{P} \cdot MR \cdot dP_N^*$$

(II-A1)

$$+ \frac{\partial BT}{\partial e} de - \frac{1}{P} \frac{\partial BT}{\partial e} dP$$

where $dP'_N = e dP_N^* + P_N^* de$, where the domestic price of oil is the same as the world price in terms of the domestic currency, that is $P'_N = e P_N^*$, and where $\frac{\partial c}{\partial y}$ and $\frac{\partial m}{\partial y}$ represent the marginal propensity to consume and marginal propensity to import respectively. Also $d(GT)$ stands for $(dg - \frac{\partial cy}{\partial y} dt)$.

The equilibrium in the money market can be written as,

$$\frac{P \cdot dM - \bar{M} \cdot dP}{P^2} = l' dr + k' dy, \quad ,$$

or

$$dM = \left(\frac{\bar{M}}{P}\right) dP + P l' dr + P k' dy, \quad (II-A2)$$

where $l' = \frac{\partial L}{\partial r}$, and $k' = \frac{\partial L}{\partial y}$.

The equilibrium in the balance of payments is given by,

$$dr = dr^* + \frac{1}{e} de^*_{+1} - \frac{e^*_{+1}}{e^2} de^*,$$

where by the appropriate choice of units the current value of the exchange rate will be unity,

$$dr = dr^* + de^*_{+1} - e^*_{+1} de^*. \quad (II-A3)$$

The aggregate supply of the economy will be as follows:

$$dP = A_0 + A_1 dy + A_2 dP^e + A_3 dP_N^* + A_3 P_N^* de \quad (II-A4)$$

where $A_0 = b_4 + b_5 b_1 + b_5 b_2 Y_{t-1} - b_5 b_2 \bar{Y}_t$,

$$A_1 = b_5 b_2,$$

$$A_2 = b_5 b_3,$$

$$A_3 = b_6$$

From the government budget constraint we have,

$$dM = du \quad (\text{II-A5})$$

Now, by substituting (II-A3) in (II-A1) and (II-A2), and solving for dr , we have:

$$\begin{aligned} dy = & K \cdot d(GT) + K_1' dr^* + K_1' de_{+1}^* - K \frac{1}{P} \cdot MR \cdot dP_N^* \\ & + K \left(\frac{\partial BT}{\partial e} - 1' e_{+1}^* \right) de - K \cdot \frac{1}{P} \cdot \frac{\partial BT}{\partial e} dP \end{aligned} \quad (\text{II-A6})$$

where

$$K = \frac{1}{1 - \frac{\partial c}{\partial y}(1-t) + \frac{P_N}{P} \cdot \frac{\partial m}{\partial y}} > 0$$

and that it represents the notion of a simple multiplier.

The equation (II-A6) represents the combined balance of payments and the IS equations, (IS-BP).

Also, the combined balance of payments and the LM equations, (LM-BP), can be written as follows:

$$dy = \frac{1}{Pk} du - \frac{l}{k} dr^* - \frac{l}{k} de_{+1}^* + \frac{l}{k} e_{+1}^* de - \frac{\bar{M}}{P^2 k} dP. \quad (II-A7)$$

Thus far, we have reduced the system down into three equations (II-A4), (II-A6) and (II-A7) and three unknowns y , e , and P . Now, in order to derive the aggregate demand and the aggregate supply of the economy in terms of two endogenous variables of y , and P , the following steps are taken.

First, regarding the aggregate demand, we combine equations (II-A6) and (II-A7) and solve for de . From (II-A7) we have,

$$de = \frac{k}{l e_{+1}} dy - \frac{1}{P l e_{+1}} du + \frac{1}{e_{+1}} dr^* + \frac{1}{e_{+1}} de_{+1}^* + \frac{\bar{M}}{P^2 l e_{+1}} dP; \quad (II-A8)$$

By substituting (II-A8) in (II-A6), the AD can be written as,

$$dy = K.d(GT) + K l' dr^* + K l' de_{+1}^* - K \frac{1}{P} MR dP_N^* + K \left(\frac{\partial BT}{\partial e} - l' e_{+1}^* \right) \cdot \frac{k}{l e_{+1}} dy$$

$$\begin{aligned}
& - K \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{1}{Pl' e_{+1}^*} d\mu + K \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{1}{e_{+1}^*} dr^* \\
& + K \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{1}{e_{+1}^*} de_{+1}^* + K \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{M}{P^2 l' e_{+1}^*} dP - K \frac{1}{P} \frac{\partial BT}{\partial e} dP,
\end{aligned}$$

or

$$dP = B_{01} d(GT) + B_{02} dr^* + B_{02} de_{+1}^* + B_{03} dP_N^* + B_{04} d\mu + B_{05} dy \quad (II-A9)$$

where

$$B_{01} = \frac{1}{B_{00}} = \frac{1}{\left[\frac{1}{P} \frac{\partial BT}{\partial e} - \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{M}{P^2 l' e_{+1}^*} \right]} > 0,$$

$$B_{02} = \frac{B_1}{B_{00}} > 0, \text{ where } B_1 = \frac{\partial BT}{\partial e} \frac{1}{e_{+1}^*} > 0$$

$$B_{03} = \frac{B_2}{B_{00}} < 0, \text{ where } B_2 = - \frac{1}{P} MR < 0,$$

$$B_{04} = \frac{B_3}{B_{00}} > 0, \text{ where } B_3 = - \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{1}{Pl' e_{+1}^*} > 0$$

and

$$B_{05} = \frac{B_4}{KB_{00}} < 0, \text{ where}$$

$$B_4 = - \left[1 - K \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{k'}{l' e_{+1}} \right] = - [1 + K p k' B_3] < 0$$

The interpretations of these coefficients are as follows:

B_4 :

For a given change in y , dy , $-\frac{k'}{l' e_{+1}}$ is the amount that exchange rate has to rise in order to keep the LM in equilibrium. On the other hand, $\left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right)$ stands for the response of aggregate demand to the exchange rate variations, through the changes in the trade balance (net export), $\frac{\partial BT}{\partial e}$, and investment components, $i' e_{+1}^*$. Therefore, $-k \left(\frac{\partial BT}{\partial e} - i' e_{+1}^* \right) \frac{k'}{l' e_{+1}}$ represents the increase in y via the multiplier of K of an indirect effect that dy exerts on e , which in turn affects the investment and net exports.

B_0 :

A given change in P , equal to dP , affects the AD, by the multiplier, both directly through its impact on the net export $\frac{1}{P} \cdot \frac{\partial BT}{\partial e}$, and indirectly through its effect on the exchange rate which in turn will affect AD.

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$\frac{\bar{M}}{P^2 l' e_{+1}}$ is the effect on e of a given change in P , dP , and i is positive as l' is negative, i.e., it shows a rise in e required to keep LM in equilibrium. Thus, $-(\frac{\partial BT}{\partial e} - i' e_{+1}) \frac{\bar{M}}{P^2 l' e_{+1}}$ represents a rise in AD via an exchange rate depreciation.

In order to derive the aggregate supply AS, we first solve the (IS-BP) and (LM-BP), equations (II-A6) and (II-A8) for dy .

$$\frac{dy}{Pk} - \frac{\bar{M}}{P^2 k'} dP - \frac{l'}{k} dr^* - \frac{l'}{k} \frac{de^*}{e_{+1}} = K \cdot d(GT) + K i' dr^*$$

$$+ K i' de_{+1}^* - K \frac{1}{P} MR \cdot dP_N^* - K \frac{1}{P} \frac{\partial BT}{\partial e} dP + K (\frac{\partial BT}{\partial e} - i' e_{+1}^*) de$$

or

$$de = C_0 + C_1 dr^* + C_1 de_{+1}^* + C_2 dP_N^* + C_3 dP \quad (II-A10)$$

where

$$C_0 = - \frac{K \cdot d(GT) - \frac{dy}{Pk}}{B_5} > 0$$

$$C_1 = - \frac{\frac{\ell}{k} + K_1}{B_5} > 0$$

$$C_2 = \frac{K_P^{1MR}}{B_5} > 0$$

$$C_3 = - \frac{\frac{M}{B_5^2 k} - K_P^{1MR} \frac{\partial BT}{\partial e}}{B_5} > 0$$

and

$$B_5 = [K \frac{\partial BT}{\partial e} - e_{+1}^* (K_1 + \frac{\ell}{k})] > 0$$

Now by combining equations (II-A10) and (II-A4) and solving for de , we have,

$$dP = A_0 + A_1 dy + A_2 dP^e + A_3 dP_N^* + A_3^{PN} \left(\frac{-K.d(GT) + \frac{d\mu}{Pk}}{B_5} \right)$$

$$-A_3^{PN} \left(\frac{\frac{\ell}{k} + K_1}{B_5} \right) dr^* - A_3^{PN} \left(\frac{\frac{\ell}{k} + K_1}{B_5} \right) de_{+1}^* + A_3^{PN} \left(\frac{K_P^{1MR}}{B_5} \right) dP_N^*$$

$$-A_3 P_N^* \left(\frac{\frac{\bar{M}}{P^2 k} - K \frac{1}{P} \frac{\partial BT}{\partial e}}{B_5} \right) dP$$

or

$$dP = A_{01} + A_{02} dy + A_{03} dP^e + A_{04} dP_N^* + A_{05} dr^* + A_{05} de_{+1}^* + A_{06} d(GT) + A_{07} du$$

(II-A11)

where

$$A_{01} = \frac{A_0}{A_{00}} = \frac{A_0}{1 + A_3 P_N^* \left(\frac{\frac{\bar{M}}{P^2 k} - K \frac{1}{P} \frac{\partial BT}{\partial e}}{B_5} \right)} > 0, \quad A_0 > 0,$$

$$A_{00} > 0,$$

$$A_{02} = \frac{A_1}{A_{00}} > 0, \quad (A_1 > 0, \text{ as explained earlier}),$$

$$A_{03} = \frac{A_2}{A_{00}} > 0, \quad (A_2 > 0, \text{ as explained earlier}),$$

$$A_{04} = \frac{A_4}{A_{00}} > 0, \quad \text{where } A_4 = A_3 + A_3 P_N^* \frac{K \frac{1}{P} MR}{B_5} > 0$$

$$A_{05} = \frac{A_5}{A_{00}} > 0, \text{ where } A_5 = -A_3^* P_N \left(\frac{\frac{1}{k} + K1}{B_5} \right) > 0$$

$$A_{06} = \frac{A_6}{A_{00}} < 0, \text{ where } A_6 = -\frac{K A_3^* P_N}{B_5} < 0,$$

and

$$A_{07} = \frac{A_7}{A_{00}} > 0, \text{ where } A_7 = \frac{A_3^* P_N}{B_5} > 0.$$

CASE (B) - Subsidization

In this case, similar to the previous case, we reduce the system down to the two equations of the AD, and AS. In doing so we note that the domestic price of oil is not the same thing as the world price of oil, and also that the budget constraint will be more complicated relative to that of the Case A.

The equilibrium in the goods market is given by,

$$\begin{aligned} \left(1 - \frac{\partial c}{\partial y} (1-t) + \frac{P_N}{P} \cdot \frac{\partial m}{\partial y}\right) dy = d(GT) + i dr - \frac{1}{P} \cdot MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1} dP_N^* \\ + \frac{\partial BT}{\partial e} de + \frac{\partial BT}{\partial P} dP, \end{aligned}$$

$$\text{where } \frac{\partial BT}{\partial P} = \left(\frac{\partial x}{\partial P} + \frac{P_N^*}{P^2} MR \right), \text{ and } \frac{\partial x}{\partial P} < 0, \frac{P_N^*}{P^2} MR > 0. \quad (\text{II-B1})$$

We continue to assume that an increase in P deteriorates the balance of trade through its strong effect on the exports relative to imports (including oil imports), that is,

$$\frac{\partial BT}{\partial P} < 0$$

The money market equilibrium is as follows:

$$dM = \left(\frac{M}{P} \right) dP + P\ell' dr + Pk' dy \quad (\text{II-B2})$$

The balance of payments equilibrium condition can be written as,

$$dr = dr^* + de_{+1}^* - e_{+1}^* de \quad (\text{II-B3})$$

The aggregate supply is given by,

$$dP = A_0 + A_1 dy + A_2 dP^e + A_3 \bar{e} \gamma_1 (P_N^*)^{\gamma_1 - 1} dP_N^* \quad (\text{II-B4})$$

The coefficients in (II-B4) were explained earlier.

Note that from the government reaction function the relationship between P_N' and P_N^* can be written as $P_N' = \bar{e}(P_N^*)^{\gamma_1}$.

From the government budget constraint we have,

$$\begin{aligned}
 dM = & \alpha \cdot s \cdot P'_N \cdot \frac{\partial m}{\partial y} dy + \alpha \cdot P'_N \cdot m \cdot ds + \frac{\alpha \cdot P'_N \cdot s}{P} \cdot \frac{\partial m}{\partial (\frac{P'_N}{P})} \cdot dP'_N \\
 & - \alpha \cdot s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial (\frac{P'_N}{P})} \cdot dP + \alpha \cdot m \cdot s \cdot dP'_N + du \quad (II-B5)
 \end{aligned}$$

where s stands for the subsidy rate per unit of P'_N .

$s = \frac{S}{P'_N}$. Thus, $P'_N \cdot s$ has replaced S in the budget constraint.

Now, by combining the money market and the budget constraint equations, (II-B2) and (II-B5), and solving for dM , we have,

$$\begin{aligned}
 & \alpha \cdot s \cdot P'_N \cdot \frac{\partial m}{\partial y} dy + \alpha \cdot P'_N \cdot m \cdot ds \left\{ \alpha \cdot P'_N \cdot s \cdot \frac{1}{P} \cdot \frac{\partial m}{\partial (\frac{P'_N}{P})} \right\} dP'_N + \alpha \cdot m \cdot s \cdot dP'_N \\
 & - \alpha \cdot s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial (\frac{P'_N}{P})} dP + du = \left(\frac{\bar{M}}{P} \right) dP + P\ell' dr + Pk' dy,
 \end{aligned}$$

or

$$\begin{aligned}
 P\ell' dr = & \left(\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - Pk' \right) dy + \alpha \cdot s \cdot \left(m + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial (\frac{P'_N}{P})} \right) \cdot dP'_N + \alpha \cdot P'_N \cdot m \cdot ds \\
 & - \left[\alpha \cdot s \cdot \frac{P_N'^2}{P^2} + \frac{\partial m}{\partial (\frac{P'_N}{P})} + \frac{\bar{M}}{P} \right] dP + du \quad (II-B6)
 \end{aligned}$$

By substituting the balance of payments equation (II-B3), into the IS and the LM, equations (II-B1) and (II-B6), we will have:

(IS-BP):

$$dy = K \cdot d(GT) + K l' dr^* + K l' de_{+1}^* + K \left(\frac{\partial BT}{\partial e} - l' e_{+1}^* \right) de + K \frac{\partial BT}{\partial P} dP - K \frac{l}{P} MR \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1} dP_N^* \quad (II-B7)$$

where K is defined as before.

(LM-BP):

$$dy = \frac{Pl'}{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} dr^* + \frac{Pl'}{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} de_{+1}^* - \frac{Pl' e_{+1}^*}{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} de - \frac{\alpha \cdot s \cdot MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1}}{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} dP_N^* + \frac{\alpha \cdot s \cdot \frac{P_N^2}{P^2} \cdot \frac{\partial m}{\partial (\frac{P_N}{P})} + \frac{M}{P}}{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} dP - \frac{1}{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} - Pk'} du \quad (II-B8)$$

To derive the aggregate demand, we once again combine the (IS-BP) and (LM-BP), equations (II-B7) and (II-B8). From (II-B8) we can have,

$$\begin{aligned}
 de = & \frac{1}{e_{+1}^*} dr^* + \frac{1}{e_{+1}^*} de_{+1}^* - \frac{\alpha \cdot P_N' \cdot s \cdot \frac{\partial m}{\partial y} \cdot P_k}{P_L' e_{+1}^*} dy - \frac{\alpha \cdot m \cdot P_N'}{P_L' e_{+1}^*} ds \\
 & + \frac{\alpha \cdot s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial (\frac{P_N}{P})} + \frac{M}{P}}{P_L' e_{+1}^*} dP - \frac{1}{P_L' e_{+1}^*} du \\
 & - \frac{\alpha \cdot s \cdot MR \cdot \bar{e}_1 (P_N^*)^{-1}}{P_L' e_{+1}^*} dP_N^* \quad (II-B9)
 \end{aligned}$$

Now, by substituting (II-B9) into (II-B7), we will have:

$$\begin{aligned}
 dP = & B_{01} d(GT) + B_{02} dr^* + B_{02} de_{+1}^* + B_{03} dP_N^* + B_{04} ds \\
 & + B_{05} du + B_{06} dy, \quad (II-B10)
 \end{aligned}$$

where

$$B'_{01} = \frac{1}{B_{00}} > 0, \text{ where } B'_{00} = -\left\{-B_3(\alpha.s.\frac{P_N^2}{P^2} \cdot \frac{\partial m}{\partial(\frac{P_N}{P})} + \frac{\bar{M}}{P}\right\} + \frac{\partial BT}{\partial P} > 0,$$

$$\text{since, } B_3 = \left(\frac{\partial BT}{\partial e} - i' e_{+1}^*\right) \frac{1}{Pl e_{+1}} < 0, \text{ as earlier seen in Section A.}$$

$$B'_{02} = \frac{B_1}{B_{00}} > 0, \text{ where } B_1 = \frac{\partial BT}{\partial e} \cdot \frac{1}{e_{+1}} > 0, \text{ as earlier seen in Section A.}$$

$$\begin{aligned} & -\frac{1}{P} MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1} - B_3(\alpha.s.MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1}) \\ & = \frac{\quad}{B_{00}} < 0 \end{aligned}$$

$$B'_{04} = -\frac{B_3 \cdot \alpha.m.P_N}{B_{00}} > 0,$$

$$B'_{05} = -\frac{B_3}{B_{00}} > 0,$$

$$B'_{06} = -\frac{1}{K_1} \frac{1}{B_{00}} < 0,$$

We also have,

$$s = \frac{S}{P_N^*} = \frac{eP_N^* - P_N^*}{P_N^*} = (P_N^*)^{1-\gamma_1} - 1$$

where $e = \bar{e}$ at the initial equilibrium. Thus

$$ds = \bar{e} \left(\frac{1-\gamma_1}{P_N^*} \right) dP_N^* \quad (\text{II-B11})$$

where $ds > 0$, for all values of γ , except unity. That is the ds approaches zero when γ_1 approaches unity.

Hence, ds in the equation (II-B10) can be expressed in terms of P_N^* , that is,

$$dP = B_{01}' d(GT) + B_{02}' dr^* + B_{02}' de_{+1}^* + B_{07}' dP_N^* + B_{05}' d\mu + B_{06}' dy \quad (\text{II-B10})$$

where,

$$B_{07}' = B_{03}' + B_{04}'$$

B_{07}' represents the combined direct and indirect effect (monetization of the deficit) of an oil price rise.

K_1 is some kind of a multiplier and it is equivalent to,

$$K_1 = \frac{1}{1 - \frac{\partial c}{\partial y}(1-t) + \frac{P'_N}{P} \cdot \frac{\partial m}{\partial y} + \frac{\partial BT}{\partial e} \left(\frac{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} \cdot Pk'}{P_L \cdot e^{+1}} \right) - \frac{1}{P_L} (\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - Pk')}$$

and that it is greater than K.

The interpretation of the denominator is as follows: The second and the third terms are the familiar marginal propensity to consume and marginal propensity to import of oil. The fourth term expresses the effect on the economy through the exchange rate depreciation when the level of real income changes by dy .

As y rises by dy it will bring about two effects on the money market. (a) It directly raises the transaction demand for money, Pk' , and (b) it also causes the money supply to rise through the increase in demand for oil imports and thereby the subsidy to be partially monetized. $\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y}$ is thus, equal to the proportion of the increased subsidy to be financed by a money issue. Thus, $(\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - Pk')$ indicates the net increase in the money supply generated in the procedure. Furthermore, the increase in money supply divided by P_L gives the increase in the interest rate. Also, $\frac{1}{P_L \cdot e^{+1}}$ is the increase in exchange rate to be brought about by a decline in the interest rate to keep the balance of payments equation in equilibrium. Hence, the whole term,

$$\frac{\alpha P_N' s \cdot \frac{\partial m}{\partial y} P_k'}{P l' e_{+1}}$$

represents the depreciation (a rise in e) of the domestic currency, and thus,

$$\frac{\partial BT}{\partial e} \left(\frac{\alpha P_N' s \cdot \frac{\partial m}{\partial y} P_k'}{P l' e_{+1}} \right)$$

gives the increase in the demand via the exchange rate movements.

The last item in the denominator, clearly, provides the increase in investment resulting from rising money supply which in turn is caused by an increase in income by dy .

The interpretation of the coefficients of the AD equation is as follows:

B_{01} is the reciprocal of B_{00} and that the latter consists of three parts. The first term is the representative of a downward effect on the aggregate demand via the exchange rate depreciation caused by a fall in the real money supply and the risen interest rate. The term

$$\frac{\alpha s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial \left(\frac{P_N}{P} \right)} + \frac{M}{P}}{P}$$

indicates the fall in the real money supply (consisting of both the initial level of stock of money $\bar{M}$, and the increased money supply brought about by the money financing of the oil subsidy). When divided by ℓ' , it gives the rise in the interest rate.

From the balance of payment equations, ceteris paribus, a rise in r will be matched by a fall in e . Thus the whole thing multiplied by $\frac{\partial BT}{\partial e}$ would reflect the fall of the net export.

The second term is simply the fall in the investment which again causes a downward pressure on the aggregate demand. The decreased real money supply and thereby risen interest rate,

$$\frac{\alpha.s. \cdot \frac{P_N^2}{P^2} \cdot \frac{\partial m}{\partial \left(\frac{P_N}{P}\right)} + \frac{\bar{M}}{P}}{\ell'}$$

multiplied by i' gives the fall in investment expenditure.

The last term is clearly the fall in the net export due to higher domestic prices.

B'_{02} represents the effect on the balance of trade of a change in e which itself is brought about by a change in r^* and/or e^*_{+1} .

B'_{05} is the monetary policy channel. The other coefficients are already explained.

The aggregate supply is simply the same as the equation (II-B4). Since the domestic price does not respond to the exchange rate fluctuation, the exchange rate does not enter the equation (II-B4). That is, it is independent of e .

To find the relationship between the e and P , we combine (II-B7) and (II-B8) and solve for dy .

$$de = D_0 dGT + D_1 dr^* + D_1 de_{+1}^* + D_2 ds + D_3 du + D_4 dP_N^* + D_5 dP \quad (\text{II-B12})$$

where

$$D_0 = \frac{-K}{D_{00}} < 0, \text{ where } K > 0, \text{ and}$$

$$D_{00} = \left[\left(\frac{\partial BT}{\partial e} - 1' e_{+1}^* \right) + \frac{Pl' e_{+1}^*}{\alpha P_N' s \frac{\partial m}{\partial y} - Pk} \right] > 0,$$

we assume that $\alpha P_N' s \frac{\partial m}{\partial y} - Pk > 0$ unless otherwise mentioned.

$$D_1 = \frac{\frac{Pl'}{\alpha P_N' s \frac{\partial m}{\partial y} - Pk}}{D_{00}} > 0$$

$$D_2 = \frac{\frac{-S.m}{\alpha P_N' s \frac{\partial m}{\partial y} - Pk}}{D_{00}} > 0$$

$$D_3 = \frac{\frac{-1}{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - Pk'}}{D_{00}} > 0$$

$$D_4 = \frac{K \frac{1}{P} \cdot MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1} - \frac{\alpha \cdot s \cdot MR \cdot \bar{e} \cdot \gamma_1 (P_N^*)^{\gamma_1 - 1}}{\alpha \cdot P'_N \cdot s \cdot \frac{\partial m}{\partial y} - Pk'}}{D_{00}} > 0 ;$$

$$D_5 = \frac{\left[-K \frac{\partial BT}{\partial P} + \frac{\alpha \cdot s \cdot \frac{P_N'^2}{P^2} \cdot \frac{\partial m}{\partial \left(\frac{P_N}{P} \right)} + \frac{M}{P} \right]}{D_{00}} \begin{matrix} > 0 \\ < 0 \end{matrix}$$

where $K \frac{\partial BT}{\partial P} < 0$

Appendix III

The Long Run Solution of the Model

The commodity market equilibrium condition can be written as:

$$i' \cdot d\bar{e} - i' \cdot d\bar{p} + \left(\frac{\partial BT}{\partial e}\right) \bar{e} - \left(\frac{\partial BT}{\partial e}\right) \bar{p} = 0 \quad (\text{III-1})$$

The money market equilibrium condition is given by:

$$\left(\frac{\bar{M}}{\bar{P}}\right) \bar{P} + \bar{L}' \cdot d\bar{e} = \left(\frac{\bar{M}}{\bar{P}}\right) \bar{M} \quad (\text{III-2})$$

First we solve the differential equation (III-1). However, due to the special feature of this equation, that the coefficient of the first order derivatives, ($d\bar{e}$, and $d\bar{p}$) on one hand and the coefficient of the unknowns ($\bar{e}$, and $\bar{p}$) on the other hand are equal, the solution can be found as follows:

$$\bar{p} = \bar{e} \quad (\text{III-3})$$

and

$$d\bar{p} = d\bar{e} \quad (\text{III-4})$$

The conditions given by (III-3) and (III-4) are necessary for the equation to be solvable, so that,

$$(i' - i') d\bar{p} + \left(\frac{\partial BT}{\partial e} - \frac{\partial BT}{\partial e}\right) \bar{p} = 0 \quad (\text{III-5})$$

By using the above conditions in equation (III-2):

$$\left(\frac{\bar{M}}{\bar{P}}\right) \bar{P} + \bar{L}' d\bar{p} = \left(\frac{\bar{M}}{\bar{P}}\right) \bar{M} \quad (\text{III-6})$$

or

$$d\bar{P} + \frac{\left(\frac{\bar{M}}{\bar{P}}\right)}{\bar{L}} \bar{P} = \frac{\left(\frac{\bar{M}}{\bar{P}}\right)}{\bar{L}} \bar{M} \quad (\text{III-7})$$

or

$$d\bar{P} + \bar{K} \bar{P} = \bar{K} \bar{M} \quad (\text{III-8})$$

$$\text{where } \bar{K} = \frac{\left(\frac{\bar{M}}{\bar{P}}\right)}{\bar{L}} < 0.$$

To solve this differential equation, we take the following steps:

First we introduce the integrating factor of the equation (III-8). That is

$$I = e^{\int \bar{K} d\bar{M}} = e^{\bar{K}\bar{M}}$$

Now we multiply both sides of the equation by $e^{\bar{K}\bar{M}}$, that is

$$e^{\bar{K}\bar{M}} d\bar{P} + e^{\bar{K}\bar{M}} \bar{K} \bar{P} = \bar{M} \cdot \bar{K} \cdot e^{\bar{K}\bar{M}}$$

or

$$(e^{\bar{K}\bar{M}} \bar{P})' = \bar{M} \cdot \bar{K} \cdot e^{\bar{K}\bar{M}}$$

or

$$\int (\dot{\bar{K}} \cdot \dot{\bar{P}})' = \int \dot{\bar{M}} \cdot \bar{K} \cdot \dot{\bar{K}} \bar{M}$$

or

$$\dot{\bar{K}} \cdot \dot{\bar{P}} = \int \dot{\bar{M}} \cdot \bar{K} \cdot \dot{\bar{K}} \bar{M} + C_1$$

The integral $\int \dot{\bar{M}} \cdot \bar{K} \cdot \dot{\bar{K}} \bar{M}$ can then be solved as follows:

$$\begin{aligned} \int \dot{\bar{M}} \cdot \bar{K} \cdot \dot{\bar{K}} \bar{M} &= \bar{K} \int \dot{\bar{M}} \cdot \dot{\bar{K}} \bar{M} = \dot{\bar{M}} \cdot \dot{\bar{K}} \bar{M} - \bar{K} \int \frac{1}{K} \dot{\bar{K}} \bar{M} d\bar{M} \\ &= \dot{\bar{M}} \cdot \dot{\bar{K}} \bar{M} - \frac{1}{K} \dot{\bar{K}} \bar{M} \end{aligned}$$

Thus,

$$\dot{\bar{K}} \cdot \dot{\bar{P}} = \dot{\bar{M}} \cdot \dot{\bar{K}} \bar{M} - \frac{1}{K} \dot{\bar{K}} \bar{M} + C_1$$

or

$$\dot{\bar{P}} = \dot{\bar{M}} - \frac{1}{K} + C_1 e^{-\bar{K} \bar{M}}$$

(III-9)

In this relationship, $\dot{\bar{P}} = f(\bar{M})$, let us define the initial value of $\dot{\bar{P}}$ equal to zero, when $\bar{M} = 0$,

$$\dot{P}(0) = f(0) = -\frac{1}{K} + C_1 = 0$$

or

$$C_1 = \frac{1}{K} \quad (\text{III-10})$$

By substituting (III-10) into (III-9), we have

$$\dot{P} = \dot{M} + \frac{1}{K} (e^{-KM} - 1) \quad (\text{III-11})$$

From (III-3), it follows:

$$\dot{e} = \dot{M} + \frac{1}{K} (e^{-KM} - 1) \quad (\text{III-12})$$

Equation (III-11) and (III-12) are the solutions to the differential equations of (III-1) and (III-2).

APPENDIX IV

The Dynamic Solutions of the ModelCase (A) No Subsidy

In order to derive the reduced forms of p and y we do as follows. First we combine the money market and goods market equations on one hand, and the money market and aggregate supply on the other hand and solve for dy . From the money market, we obtain:

$$de_t = A'_1 dp + A'_2 d(dp_t^e) \quad (IV-1)$$

where

$$A'_1 = \frac{\frac{K \frac{1}{p} \frac{\partial BT}{\partial e} - \frac{\bar{M}}{P^2 K}}{K \frac{\partial BT}{\partial e}}} > 0,$$

$$A'_2 = \frac{1}{\frac{\partial BT}{\partial e}} < 0.$$

From the money market-aggregate supply we can derive:

$$dp_t = B'_1 + B'_2 dp_t^e + B'_3 de_t + B'_4 y_{t-1} \quad (IV-2)$$

where

$$B'_1 = \frac{\theta_0}{1 - \theta_1 \beta_1} > 0, \quad B'_2 = \frac{\theta_2}{1 - \theta_1 \beta_1} < 0, \quad B'_3 = \frac{\theta_3}{1 - \theta_1 \beta_1} > 0, \quad B'_4 = \frac{\theta_1}{1 - \theta_1 \beta_1} > 0$$

Now by combining equations (IV-1) and (IV-2) and solve for de :

$$dP_t = A_{11} + A_{12} dP_t^e + A_{13} d(dP_t^e) + A_{14} y_{t-1} \quad (IV-3)$$

where

$$A_{11} = \frac{\theta_0}{Z} = \frac{\theta_0}{(1+\theta_1 \frac{\bar{M}}{P^2_k} - \theta_3 \frac{K \frac{1}{P} \frac{\partial BT}{\partial e} - \frac{\bar{M}}{P^2_k}}{K \frac{\partial BT}{\partial e}})} > 0,$$

$$A_{12} = \frac{\theta_2}{Z} > 0, \quad A_{13} = \frac{\theta_3 \frac{1}{P} \frac{\partial BT}{\partial e}}{Z} < 0, \quad A_{14} = \frac{\theta_1}{Z} > 0.$$

By a similar procedure we can derive the reduced form of y , as follows:

$$dy_t = \beta_1 A_{11} + \beta_1 A_{12} dP_t^e + \beta_1 A_{13} d(dP_t^e) + \beta_1 A_{14} y_{t-1} \quad (IV-4)$$

These equations are of non-homogenous second order linear difference type in terms of dP , solution to which can be found by the sum of two components; a particular solution, and a complementary function as follows:

By substituting for the expected price from equation (36) the equation (IV-1) can be written as:

$$P_t - P_{t-1} = A_{11} + A_{12} dP_{t-1} + A_{13} d(dP_{t-1}) + A_{14} y_{t-1}$$

or

$$P_t - (1 + A_{12} + A_{13}) P_{t-1} + (A_{12} + 2A_{13}) P_{t-2} - A_{13} P_{t-3} = A_{14} y_{t-1}$$

(IV-5)

Next, we rearrange the money market equation, given by equation (38), as follows:

$$y_t - y_{t-1} = \beta_1 (P_t - P_{t-1}) \quad (\text{IV-6})$$

By using the telescopic series:

$$y_t = \beta_1 P_t + (\bar{y} - \beta_1 P_0) \quad (\text{IV-7})$$

and

$$y_{t-1} = \beta_1 P_{t-1} + (\bar{y} - \beta_1 P_0) \quad (\text{IV-8})$$

where $\bar{y} = y_0$.

Thus, equation (IV-5) can be written as follows:

$$P_t + Z_1 P_{t-1} + Z_2 P_{t-2} + Z_3 P_{t-3} + Z_4 = 0 \quad (\text{IV-9})$$

where

$$Z_1 = -(1 + A_{12} + A_{13} + A_{14}\beta_1) < 0$$

$$Z_2 = (A_{12} + 2A_{13}) < 0$$

$$Z_3 = -A_{13} > 0$$

$$Z_4 = A_{11} + A_{14}(y_0 - \beta_1 P_0) > 0$$

The particular solution y_p can be found by trying a solution of the form $y_t = d$. By substituting this constant value of y_t in equation (IV-9):

$$d + Z_1 d + Z_2 d + Z_3 d = Z_4 \quad (\text{IV-10})$$

$$d = \frac{Z_4}{1+Z_1+Z_2+Z_3} = -\frac{1}{A_{14}\beta_1} (A_{11} + A_{14}(y_0 - \beta_1 P_0)) > 0 \quad (\text{IV-11})$$

To find the complementary function we do as follows. We assume $y = C\lambda^t$ which implies $y_{t-1} = C\lambda^{t-1}$, and, $y_{t-2} = C\lambda^{t-2}$, and, $y_{t-3} = C\lambda^{t-3}$, thus:

$$C\lambda^t + Z_1 C\lambda^{t-1} + Z_2 C\lambda^{t-2} + Z_3 C\lambda^{t-3} = 0 \quad (\text{IV-12})$$

OR

$$\lambda^3 + Z_1 \lambda^2 + Z_2 \lambda + Z_3 = 0 \quad (\text{IV-13})$$

Assuming that the roots λ_1 , λ_2 and λ_3 are real and distinct, the general solution can be written as follows:

$$P_t = C_1 \lambda_1^t + C_2 \lambda_2^t + C_3 \lambda_3^t = d \quad (\text{IV-14})$$

To define the three arbitrary constants C_1 , C_2 , and C_3 , we specify the critical conditions as follows.

$$P(0) = C_1 + C_2 + C_3 + d = d \quad (\text{IV-15})$$

$$P(1) = C_1 \lambda_1 + C_2 \lambda_2 + C_3 \lambda_3 + d = \epsilon \quad (\text{IV-16})$$

$$P(2) = C_1 \lambda_1^2 + C_2 \lambda_2^2 + C_3 \lambda_3^2 + d = P(1) = \epsilon \quad (\text{IV-17})$$

From these simultaneous equations above, we can obtain the value of C_1 , C_2 , and C_3 as follows:

$$C_1 = \frac{-(\lambda_3 + \lambda_2 - 1)}{(\lambda_1 - \lambda_3)(\lambda_1 - \lambda_2)} > 0$$

$$C_2 = \frac{-(\epsilon - d)(\lambda_3 + \lambda_1 - 1)}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)} > 0$$

$$C_3 = \frac{-(\epsilon - d)(\lambda_1 + \lambda_2 - 1)}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)} > 0$$

$$\epsilon > 0.$$

To derive the general solution of the output, we combine equations (IV-7) and (IV-14) as follows:

$$y_t = \beta_1 (C_1 \lambda_1^t + C_2 \lambda_2^t + C_3 \lambda_3^t + d) + (\bar{y} - \beta_1 P_0) \quad (\text{IV-18})$$

Case (B) - Subsidization

The dynamic path of price is derived as follows:

$$dP_t = Z_1' + Z_2' M(S) + Z_3' dP_t^e + Z_4' y_{t-1}, \quad (\text{IV-19})$$

This is a nonhomogeneous first order linear difference equation in terms of two unknown dP and Y . The equation (IV-19) can be rearranged as follows:

$$P_t - P_{t-1} = Z_1' + Z_2' M(S) + Z_3' dP_{t-1} + Z_4' y_{t-1},$$

or,

$$P_t - (1 + Z_3') P_{t-1} + Z_3' P_{t-2} = Z_1' + Z_2' M(S) + Z_4' y_{t-1} \quad (\text{IV-20})$$

The money market budget constraint relationship given by

$$dy_t = M(S) + \mu_1 dP_t, \quad (\text{IV-21})$$

can be written as follows:

$$y_t - y_{t-1} = M(S) + \mu_1 (P_t - P_{t-1}) \quad (\text{IV-22})$$

The above equation can be easily transformed as follows:

$$y_t = \mu_1 P_t + M(S).t + (y_0 - \mu_1 P_0) \quad (\text{IV-23})$$

Now by substituting (IV-23) in (IV-20) we obtain:

$$P_t - \mu_2 P_{t-1} + \mu_3 P_{t-2} = \mu_4 + \mu_5 M(S).t \quad (\text{IV-24})$$

where

$$\mu_2 = 1 + Z_3' + Z_4' \mu_1 > 0$$

$$\mu_3 = Z_3' > 0$$

$$\mu_4 = Z_1' + (Z_2' - Z_4') M(S) + Z_4' (y_0 - \mu_1 P_0) > 0$$

$$\mu_5 = Z_4' > 0$$

This second order equation is however more complicated than that of the previous case, as the right hand side of the equation is variable and not constant. The particular solution can be found as follows:

The first two differences of $(\mu_4 + \mu_5 M(S).t)$ are:

$$\Delta(\mu_4 + \mu_5 M(S).t) = [\mu_4 + \mu_5 M(S).(t)] - [\mu_4 + \mu_5 M(S).(t-1)] = \mu_5 M(S)$$

(IV-25)

$$\Delta^2(\mu_4 + \mu_5 \cdot M(S) \cdot t) = 0$$

(IV-26)

Therefore let us try the Solution,

$$P_t = B_0 + B_1(\mu_4 + \mu_5 \cdot M(S) \cdot t)$$

for the particular solution. Hence,

$$P_{t-1} = B_0 + B_1[\mu_4 + \mu_5 \cdot M(S) \cdot (t-1)]$$

and

$$P_{t-2} = B_0 + B_1[\mu_4 + \mu_5 \cdot M(S) \cdot (t-2)]$$

This implies

$$B_0 + B_1(\mu_4 + \mu_5 \cdot M(S) \cdot t) - \mu_2 B_0 - \mu_2 B_1[\mu_4 + \mu_5 \cdot M(S) \cdot (t-1)]$$

$$+ \mu_3 B_0 + \mu_3 B_1[\mu_4 + \mu_5 \cdot M(S) \cdot (t-2)] = \mu_4 + \mu_5 \cdot M(S) \cdot t$$

Equating two sides term by term:

$$B_0 + B_1 \mu_4 - \mu_2 B_0 - \mu_2 \mu_4 B_1 + \mu_2 \cdot \mu_5 B_1 M(S) + \mu_3 B_0 + \mu_3 \mu_4 B_1$$

$$- 2\mu_3 \mu_5 \cdot B_1 \cdot M(S) = \mu_4$$

or

$$(1-\mu_2+\mu_3)B_0 + (\mu_4-\mu_2\mu_4+\mu_2\mu_5 \cdot M(S) + \mu_3\mu_4 - 2\mu_3\mu_5 \cdot M(S))B_1 = \mu_4$$

(IV-27)

Also,

$$B_1 \cdot \mu_5 \cdot M(S) \cdot t - B_1 \mu_2 \cdot \mu_5 \cdot M(S) \cdot t + B_1 \mu_3 \mu_5 \cdot M(S) \cdot t = \mu_5 \cdot M(S) \cdot t$$

or

$$\mu_5(1-\mu_2+\mu_3)B_1 = 1$$

(IV-28)

or

$$B_1 = \frac{1}{1-\mu_2+\mu_3}$$

And from equation (IV-25)

$$B_0 = \frac{\mu_4}{1-\mu_2+\mu_3} - (\mu_4-\mu_2\mu_4+\mu_2\mu_5 \cdot M(S) + \mu_3\mu_4 - 2\mu_3\mu_5 \cdot M(S))$$

Thus, the particular solution can be written as follows:

$$P_p = B_0 + B_1 \cdot M(S) \cdot t$$

(IV-29)

where

$$B_0 = B_0 + B_1 \mu_4$$

and

$$B_1' = B_1 \mu_5$$

As to the complementary function:

$$P_t - \mu_2 P_{t-1} + \mu_3 P_{t-2} = 0 \quad (IV-30)$$

the solution can be easily derived, so that

$$P_t = C_1' \lambda_1^t + C_2' \lambda_2^t \quad (IV-31)$$

The general solution will then be represented by

$$P_t = C_1' \lambda_1^t + C_2' \lambda_2^t + B_0' + B_1' M(S)t \quad (IV-32)$$

The adjustment path of the output can be derived by combining equation (IV-32) with (IV-23), as follows:

$$y_t = \mu_1 (C_1' \lambda_1^t + C_2' \lambda_2^t + B_0') + (y_0 - \mu_1 P_0) + (1 + \mu_1 B_1') \cdot M(S) \cdot t$$

(IV-33)

where,

$$1 + \mu_1 B_1' = 1 + \frac{\mu_1 \mu_5}{1 - \mu_2 + \mu_3} = 0$$

Thus,

Appendix V

Some Evidence on the Short Run Aggregate Supply Curve

In this appendix we examine some evidence regarding the short run aggregate supply, by testing the two alternative versions based upon; the mark up price, and the marginal productivity equations.

The former version of the aggregate supply is based upon the following equations:

$$dw = b_1 + b_2(y - \bar{y}) + b_3 dp^e \quad (V-1)$$

$$dp = b_4 + b_5 dw + b_6 dp_N^e \quad (V-2)$$

and it is derived by combining them and solving for dw as follows:

$$dp = (b_4 + b_5 b_1) + b_5 b_2 (y - \bar{y}) + b_5 b_3 dp^e + b_6 dp_N^e \quad (V-3)$$

These equations have been presented earlier as equations (1), (4) and (5) in Chapter II.

In testing the aggregate supply first, the two (structural) equations, (V-1) and (V-2) separately and, and, then the (reduced form) equation (V-3) were estimated by ordinary least squares method.

In overall, the single equation (V-3) was proven a superior fit to the two separate structural equations. In estimating the reduced form the unemployment rate as an alternative to the output gap was also applied. However, the output gap was found a better fit.

$$\begin{aligned}
 (\log p_t - \log p_{t-1}) = & C + C'(\log y_t - \log y_{t-1}) + C''(\log p_{N_t} - \log p_{N_{t-1}}) \\
 & \begin{matrix} .011 \\ (3.92) \end{matrix} \quad \begin{matrix} (1.70) \end{matrix} \quad \begin{matrix} .025 \\ (1.63) \end{matrix} \\
 & + C'''(\log p_t^e - \log p_{t-1}) \\
 & \begin{matrix} .370 \\ (2.73) \end{matrix}
 \end{aligned}$$

$$R^2 = .471$$

$$D-W = 2.29$$

$$SEE = .007$$

The inflation expectation is formed based upon the rational expectations hypothesis.

The data on the expected price in the rational fashion are the estimated price based upon the lagged values which have been generated by regressing the actual price variable on its past values and the past values of money supply.

$$\begin{aligned}
 p_t^e = & a_0 + a_1 p_{t-1} + a_2 p_{t-2} + \dots \\
 & + a_1' M_{t-1} + a_2' M_{t-2} + \dots
 \end{aligned}$$

where p_t^e = expected price

p_t = actual price

and M_t = money supply (M_1).

The inflation expectations, formed in the adaptive fashion, were found equally satisfactory.

The reported regression was obtained using the GNP deflator as the price index and that it was found a better choice than the CPI.

An alternative formulation of aggregate supply was derived from combining the marginal productivity equation and the Phillips curve. Note that in log form the difference between this version of aggregate supply and the alternative one, given by equation (V-3), is in $Dy(y_t - y_{t-1})$, term which appears in the former. When we estimated this version of aggregate supply (given in Appendix I), Dy was found insignificant. In addition, the marginal productivity equation was estimated separately. Again, Dy turned out insignificant. The inclusion of several lagged values of p'_N and p failed to generate any changes.

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END

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