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Design of a Digital Vision System for Measuring Surface Tension

by

YunZhen (Larry) Zhou, B. Eng

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirement for the degree of
Master of Engineering

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of
Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
August, 2000

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acceptance of the thesis**

Design of a Digital Vision System for Measuring Surface Tension

**Submitted by
Yunzhen (Larry) Zhou, B. Eng.**

**In partial fulfillment of the requirements for
the degree of Master of Engineering**

Thesis Supervisor

Chair, Department of Mechanical & Aerospace Engineering

Carleton University

September 2000

Abstract

Experiment techniques for measuring surface tension using the shape of either axisymmetric sessile or pendant drops have existed for many years. Recent developments in digital image acquisition and processing have permitted the computerization of the process by which the coordinates of the drop's edge profile are obtained. Algorithms like the ADSA-P program developed by Rotenberg et. al.(1,2) use the edge profile coordinates to estimate quantities like the surface tension, drop volume and contact angle. The precision of these estimated quantities depends on various effects which all influence the accuracy to which the edge profile coordinates are acquired. We have modeled this uncertainty in coordinate information as a perturbation effect and related the size of the perturbation to the surface tension accuracy. Two analogous relations are used to set regions of surface tension accuracy; e.g. $\pm 0.01 \text{ mJ/m}^2$ or $\pm 0.1 \text{ mJ/m}^2$ as functions of the magnification of the drop, CCD camera array size, pixel size, drop shape and drop edge precision. An algorithm for the design of various vision systems based on these criteria will be discussed.

According to above criteria, a new digital vision system is designed, integrated and tested. The vision system includes optical subsystem, flexible optical-mechanical subsystem, digital data acquisition hardware, image processing and analysis software and vibration isolation subsystem, etc. The vision system automatically determines liquid-fluid interfacial tensions and contacts from the shape of axisymmetric menisci, i.e. from

sessile drop or pendant drop. The vision system is have good resolution, fast computer rate speed, its flexible and stable. It can capture static or dynamical digital images of sessile drops or pendant drops, the spreading of liquid and the impact of a drop.

A software package named WinDrop was developed to integrate GUI application software to access complete data processing, analysis and graphics plotting using Visual C++/MFC/OCX Control on Windows NT 4.0.

Key Words

Surface Tension, Accuracy, Digital Vision System, Perturbation, WinDrop, Design, Axisymmetric Drop.

Reference

1. Y. Rotenberg, L. Boruvka and A.W. Neumann, "Determination of Surface Tension and Contact Angle from the Shape of Axisymmetric Fluid Interfaces", J. Colloid Interface Sci., Vol 93, p. 169, 1983.
2. S. Lahooti, O. I. Del Rio, P. Cheng and A. W. Neumann, "Axisymmetric Drop Shape Analysis (ADSA)" in " Applied Surface Thermodynamics", (A. W. Neumann and J.K. Spelt, eds.), pp. 441-507, Marcel Dekker, New York, 1996.

Acknowledgements

I would like to thank my lovely wife Wang LingLing for giving me bravery, strength and wisdom to finish this thesis work. It was She who would not let me have gave up, even in my darkest time of my life, and who helped me know that love is so beautiful.

A very special thanks to my supervisor – Professor John Gaydos, for all his guidance and help throughout my study and life. Without him, I would not have continued my research after this personal tragedy.

I would like to thank my family, especially my brother Norman Zhou, and Lingling's family, for their support and help.

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List of Symbols

- γ : Surface tension; units of mJ/m^2 .
- $\Delta\gamma$: Surface tension precision; units of mJ/m^2 .
- λ : Perturbation length as measured in normal distance from original profile edge to perturbed profile of drop; units of m .
- λ_i : Perturbation length measured from i^{th} point along drop profile. Points selected by integration scheme (4th order Runge-Kutta).
- λ_{eff} : Effective maximum perturbation length; $\lambda_{eff} = \max\{\lambda; \text{for } 0 \leq \hat{s} \leq \hat{s}_{final}\}$
- λ_{max} : Absolute Maximum perturbation length; $\lambda_{max} = \max\{\lambda_{eff}; \text{for } \hat{s}_{final} = 1 \text{ or } \theta_i = \pi\}$.
- $\hat{\lambda}_i$: Dimensionless ratio of perturbation lengths; $\hat{\lambda}_i = \frac{\lambda_i}{\lambda_{max}}$.
- θ_i : Contact angle of sessile drop on supporting substrate.
- $\Delta\rho$: Density difference across interface; units of kg/m^3 .
- c : Capillary constant, units of m^2 .
- g : Gravity; units of m/s^2 .
- s : Arc-length coordinate from drop apex along original drop profile; units of m .
- s_i : Arc-length measured from apex to i^{th} profile point; units of m .
- s_{max} : Maximum arc-length such that, $s_{max} = \max\{s_i; \text{for } \hat{s}_{final} = 1 \text{ or } \theta_i = \pi\}$.
- s_{final} : Final arc-length position

$\hat{s}_i$: Dimensionless ratio of arc-lengths; $\hat{s}_i = \frac{s_i}{s_{\max}}$.

B_0 : Dimensionless radius of curvature at the origin of either an axisymmetric pendant drop or a sessile drop; $B_0 = R_0 \sqrt{c}$.

M : Optical magnification.

N : Minimum number of pixels in either row or column of a CCD area array camera of dimension $N_1 \times N_2$ such that, $N = \min(N_1, N_2)$.

N_v : Vision system precision, measured in units of pixels.

P : Maximum actual pixel size for a pixel dimension $P_1 \times P_2$ such that,

$$P = \max(P_1, P_2) ; \text{ units of } m.$$

R_0 : Radius of curvature at apex point of drop; units of m .

$2X_{\max}$: Actual maximum diameter of physical drop; units of m .

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$$\text{as } Error = \left| \left(\frac{(\tilde{X}_i - X_i^1)}{X_i^1} \right) \times 100 \right| \text{ in units of percentage, all other quantities are}$$

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$$Error = \left| \left(\frac{(\tilde{Z}_i - Z_i^1)}{Z_i^1} \right) \times 100 \right| \text{ in units of percentage, all other quantities are}$$

dimensionless including the turning angle θ (in degree) measured from the horizontal; see Fig. 4. 1. 68

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Chapter 1

Introduction

1.1 Objective of the Thesis

Interfacial tensions are negligible in many engineering problems, but they may be predominant in the cases such as: the capillary rise of liquids in narrow spaces, the mechanics of bubble formation, and the formation of liquid drops. The accurate measurement of interfacial tension is essential in many scientific as well as industrial areas, for example, in the cleanliness industry, oil recovery, etc.

Many techniques have been developed to measure interfacial tensions. Among all the methods which have been developed, the Wilhelmy plate method and the drop shape method (2) are used most often. Recently, digital vision system technology has advanced considerably with improvements in the resolution, speed, and cost effectiveness, so that acquisition of high resolution drop shapes images are possible. Thus, drop shape methods, such as the Axisymmetric Drop Shape Analysis Profile (ADSA-P) (2-4), have become more popular as a technique for using the drop shape information to measure surface tension.

Basically, the drop shape method uses the drop's profile, i.e., some particle parameters of the shape profile or the coordinates of the drop profiles, to determine the surface tension. The accuracy of surface tension is determined by the precision of drop edge coordinates. Figure 1.1 and 1.2 simply show this relation.

Fig. 1.1 Using Digital Image Technology to Determine the Surface Tension from the Drop's Edge Profile Coordinates.

Fig.1.2 The Precision of a Surface Tension Value is Decided by the Accuracy of the Edge Profile Coordinates.

Software packages, such as ADSA-P, use profile coordinates to determine γ in a least-squares sense, so that any change in coordinate value will create a change in the ADSA-P based estimate of γ . We may interpret this coordinate change as drop profile uncertainty

and the change in γ ; that is, $\Delta\gamma$, as a surface tension error associated with this uncertainty. Many factors may influence the precision or uncertainty of the drop profile coordinates, such as:

- From the optical system
 - Blur
 - Noise
- From the image processing
 - The precision by which the edge detection algorithm determines profile coordinates.
- From the environment
 - Light
 - Vibration

For a long time, it has been known that many complicated factors affect the accuracy of the surface tension and many papers have been written on these influences. Lahooti and A. W. Neumann (3, 4) considered the errors in individual data points and the number of points used. P.W.P Cheng considered errors associated with the optical distortion and vertical alignment etc. (5). Song and Springer mentioned that accuracy of a surface tension estimation is sensitive with the illumination conditions, the shape of drop, the contact angle, magnification and the edge location, etc. (6, 7). Lin et al. mentioned the accuracy of surface tension was size dependent (8). Much effort has been expended to improve the standard minimization algorithms for fitting the theoretical and experimental drop profiles (6, 7, 9). Despite these efforts, the problem of how the accuracy of a drop

profile influences the precision of a surface tension value was still not answered clearly. This thesis will address this question. Furthermore, when we design a new vision system, we face a monetary problem: How can we spend our budget optimally and which components are the most important or what is the relation among those components to achieve the designed vision system. Two key questions: (1) What is the surface tension precision level for a particular experimental system, e.g. a vision system. and (2) The inverse, if we require a certain surface tension precision, how do we select various components to achieve suitable parameters and satisfy the requirements. For industry, sometimes, the highest accuracy with the highest cost may not be needed, or they may ask for a coarser measurement with a simpler vision system. Thus, we need to answer the question “ what is the surface tension precision level if we just use a coarse vision?” or “ how can we design a cheap vision system to match the lower level requirement for surface tension precision?”

These considerations motivated us to find a way whereby we can simulate the uncertainty of a vision system and we can show the relationship between the precision of surface tension and the uncertainty of vision system. To achieve these ends we developed a perturbation model and a suitable criteria. (see Chap. 4)

1.2 The Layout of the Thesis

This thesis is composed of the following chapters:

- Chapter 1 - Introduction

Chapter 1 outlines the current status and problems using digital vision system for surface tension measurement. Also it outlines the objective and layout of the thesis.

- **Chapter 2 - Fundamentals of Vision systems**

Chapter 2 introduces basic concepts for vision systems, i.e., the definitions, history and applications. After introducing the system structure of a vision system, it discusses some of the component technologies of a vision system, their characteristics and performance in the integrated system. Consequently, it outlines the fundamentals of the vision system design procedure, from identification of requirements, budget, detailed design, integration and testing.

- **Chapter 3 - Image Analysis and Measurement**

Chapter 3 presents a number of algorithms commonly used in image vision systems. Most of those algorithms have only been described in outline and, in a sense, provide guidance only on the techniques that the vision engineer might implement when analyzing an image. In the chapter, I will not describe everything using mathematical details, rather I will outline only the most important concepts of image processing and measurement, such as the general steps and basic image processing operations. In the drop shape measurement technique, the most important image processing operation will be edge detection, so Chap. 3 describes edge detection and other related methodologies. The ultimate aim of Chap. 2 and 3 is to provide suitable background material for Chap. 4.

- **Chapter 4 - Criteria for Evaluating the Accuracy of Surface Tension Values from Digital Vision Systems**

Digital image techniques for measuring surface tension from the shape of either axisymmetric sessile or pendant drops are used widely. After a drop image is obtained, algorithms like the ADSA-P program developed by Rotenberg et al. use the edge profile coordinates to estimate quantities like the surface tension, drop volume and contact angle.

The precision of these estimated quantities depends on various effects which all influence the accuracy to which the edge profile coordinates are acquired. In this chapter, the uncertainty in coordinate information is modeled as a perturbation effect and we relate the size of the perturbation to the surface tension accuracy. Two analogous relations are used to set regions of surface tension accuracy; e.g. $\pm 0.01 \text{ mJ/m}^2$ or $\pm 0.1 \text{ mJ/m}^2$ as functions of the magnification of the drop, CCD camera array size, pixel size, drop shape and drop edge precision. An algorithm for the design of various vision systems based on these criteria will be discussed.

- Chapter 5 - Design a Vision System for Surface Science

According to the criteria of Chap. 4, a new digital vision system was designed, integrated and tested using the procedure that was described in Chap. 2. The vision system includes optical subsystem, flexible optical-mechanical subsystem, digital data acquisition hardware, image processing and analysis software and vibration isolation subsystem, etc. Every subsystem was described by its function and structure.

- Chapter 6 - Error-estimation of Vision System

In Chap. 4, interference problems might corrupt an image, so the information extracted from that image can be misleading. The interference problems may come from: the position of the camera relative to the object, the type of lighting used, the presence of certain electromagnetic waves, etc. Obviously, a designer of a vision system must try to avoid these interference problems; however, in practice, it is impossible to develop a vision system that is free of all errors. Thus, it is important to estimate the error of the vision system designed in Chap. 5, for the measurement of surface tension. The

perturbation modeling technique, discussed in Chap. 4, provides us with a useful tool to simulate some of these errors.

- Chapter 7 - Conclusion and Future Work

Chapter 7 provides a conclusion and a brief discussion of possible future work.

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Chapter 2

Fundamentals of Vision Systems

2.1 Basic concept of vision system

2.1.1 Definitions

Vision systems, sometimes called machine vision systems or image vision systems, exist everywhere in industry and scientific laboratories.

Ballard and Brown (1982) suggested that "Computer vision is the enterprise of automating and integrating a wide range of processing and representation for visual perception." (1) In 1985, the Society of Manufacturing Engineers (SME), defined machine vision as "The use of devices for non-contact sensing to automatically receive and interpret an image of a real scene in order to obtain information and/or control machines or processes."

As a new technology, vision systems enable complex phenomena to be evaluated, which could not be adequately accessed with conventional techniques. Vision system research was a branch of systems engineering. It was not a subset of either computer science, image processing or pattern recognition artificial intelligence. To many people, vision systems consist of just the (PC)+(Frame grabber)+(Camera)+(Image processing software). However, a multi-disciplinary approach to design of the vision systems is necessary. The primary skill needed by a vision systems engineer is the ability to integrate a complex electro-opto-mechanical system, not to manipulate mathematical equations, or to program a computer. Image processing is just one element of a vision system. Figure 2.1 provides a schematic of a typical vision system.

Figure 2.1 An example of vision system

An important feature of the above diagram is that it emphasizes the inter-disciplinary nature of vision systems. For this reason, it is imperative that vision systems are designed and installed by a team having a broad range of skills, including: (1-5)

- (a) mechanical engineering,
- (b) illumination engineering,
- (c) optical engineering,
- (d) sensor (camera) technology,
- (e) electronic signal processing hardware technology,
- (f) software engineering, and
- (g) mathematics.

2.1.2 History & Application

Ten years ago, it was common to ask the question: Does this vision system really work?

Primarily this question arose because the available technology had too little resolution and too little speed. Image acquisition was often a serious bottleneck. In the 1980's, IBM's compatible PC system provided the computer industry with a standard but for vision systems, it was not configured to be easily connected at the application level to other subsystems. There were different languages, operating systems and communication protocols, etc. In that period, a general vision system was not sufficiently cheap, reliable and easy to setup; in otherwords, vision system technology provided low performance at high cost while the customers wanted low cost and high performance.

Recent developments are much different. The key driving forces were the sharp reduction in computer hardware costs and the increase in powerful CPUs in personal computers. Today, vision systems are more adaptable than optical mechanical sensors. They offer the versatility, flexibility and reconfigurability. When alterations are necessary, "tool changes" for vision system often involve only software changes rather than replacement of costly hardware. Vision systems can often be re-used easily.

The earliest application of vision systems can be traced to the mid 1960's, when NASA sent unmanned spacecraft into orbit. Hundreds of images of the moon and planets were sent back to earth. Most images required processing by a computer before information could be extracted. After that, vision systems found a wide range of uses in medical science. For example, in the late 1960's, these systems were used in X-ray imagery, in 1980's, Computer Tomography (CT), Magnetic Resonance Imaging (MRI), Positron Emission Tomography (PET), and Diagnostic Ultrasound Imagery (DUI). Today, vision

systems are used in a variety of fields, for a variety of purposes. For example, they are used in factory automation to look for defective parts along an assembly line, law enforcement forensics to analyze fingerprints and match DNA, and in cinematography to create special effects for movies such as morphing, warping and blending.

At the University of Missouri School of Medicine, Professor Timothy Liem's group used a vision system to gain better understanding of blood flow in an effort to combat the arterial disease arteriosclerosis. They used a vision system to record images at a rate of 50 to 1000 frame/s. They accurately controlled the fluid pressure and flow rate, measured the velocity, viscosity, and density of the fluid, and the computer vision system was used to calculate the shear stress at selected locations from the data, since area of low shear stress and high shear stress are thought to be related to arteriosclerosis. Other examples occur in dental image systems. Dentists were able to use vision systems to view patients' teeth directly instead of using X-ray systems, which needs a big machine and regular safety maintenance. The data stored in a vision system can be easily operated and managed (6, 7).

As another example, European countries are adapting industrial vision systems to classify plastic shapes. In order to protect and sustain natural resources and environments, dumping different garbage types into separate waste disposals is now very popular.

However in the plastic industry, we still need man-hours to grade and sort incoming waste products for recycling. A vision system, called a garbage processing system, can distinguish and recognize common plastic materials, such as bottles and milk cartons from the mixed, moving stream (8).

In the military, vision systems are used to capture signatures of complex targets. Instead of traditional pattern-recognition techniques, object identification is performed by estimating the configuration of objects within an infrared image. After transformation and computing the best match, objects within each image can be automatically target-tracked (9).

2.1.3 Components technologies of vision system

The actual components that comprise a vision system vary greatly from one implementation to the next. There is, however, a common structure that applies to most general purpose integrated vision systems. In this section, we discuss some of the component technologies of vision systems, their characteristics and performance in the integrated system (10-14).

1. Opto-Mechanical system

The type of activity and environment factors surrounding a vision system can have a significant impact on the success of the system. End-users need access to the system to maintain operations, such as changing position, moving objects etc.

Opto-Mechanical system includes mounting, enclosures and handling, etc. The following issues will be the main function of opto-mechanical system:

- **Mechanical stability**

Mechanical stability is very important. The optics must view the proper scene from the proper angle and at the proper focus setting. When optics are used for alignment of some processing, this requirement is even more critical. If the optics move, the process goes out of control.

In practice, it is very important to minimize the number of degrees of the freedom in the optical system because the adjustable elements are inherently less stable than fixed elements. A large set of interacting adjustments is a technician's nightmare. However, on the other hand, in the laboratory, flexible adjustment is a goal of a vision system's design. Thus, hardware is designed to be more adjustable. Many motions have no locks. Stages have delicate micrometer adjustments and posts allow hardware rotation about the post, as well as translation along the post. It is a challenge to maintain mechanical stability because adjustable elements are less stable than fixed elements and locked adjustments may come loose, or a delicate adjusting rod may bend.

- Vibration isolation

Vibration isolation is another challenge for the designer. Precision optical experiments depend upon reliable position stability. Vibration around the work area can produce relative motions between optical elements, resulting in unacceptable misalignment and time variant aberrations such as image blur. Vibration may come from the building itself or from the ground beneath the building through the table mount. Vacuum pumps, passing traffic, earth tremors, local construction, elevators, even people passing in the hall can contribute to the dynamics of vibration.

Also, in order to avoid dynamic instability and oscillation due to excessive rocking, the total table system's center of mass should be arranged to be within the volume defined by connecting each center point of the isolator (base plane) with the pyramid apex, whose vertical height equals $\frac{1}{2}$ the shortest distance between the isolators. (12)

2. Lighting

Lighting is still the most critical aspect of a vision system application. It is also a big challenge for the designer because the lighting is never constant in either time or in space and colour variations in objects and fluctuating ambient lighting conditions can produce imaging distortions. Choosing the proper lighting scheme can result in increases in accuracy, system reliability, and response time.

It is a big mistake for the vision system designers to assume that they can use software to compensate for a poor lighting system. Poor lighting may lead to non cost effective and poor system performance. It is also not reasonable to view a vision system as a machine-like version of human vision. Humans can use a wide range of techniques to extract information from an image, such as colour, perspective, shading, parallax, and individual experiences. A vision system, by contrast, still is a machine and it is dependent on filtering, image subtraction, and histogram-equalization algorithm to gain information from image. It will ease the burden on the software development if the vision system is provided with an image where the areas of interest stand out distinctly against the background.

Most lighting pathways may also be polarized or filtered with absorption or interference filters. Light sources used to accomplish a given technique are varied. Some sources work better than others in a given environment and each has its own set of integration characteristics.

3. Optics

In general, real optical design is beyond the scope of any machine vision application.

Unless you want to build a product and support it, avoid implementing a long, complex optic train in applications. These optics are difficult to adjust, and nearly impossible to repair or replace. Optical lenses are used to either magnify or de-magnify an object with respect to the objective and the Charge-Coupled Device (CCD) sensor. With off-the-shelf lenses, most considerations will be correct magnification, lens resolution and depth of field.

4. Video Cameras

A first choice for a two-dimensional image input device might be a television camera whose output is a video signal. A TV camera targets scanned line by line horizontally by an electron beam and the current proportional to the intensity of light at each point.

However this form of device has several disadvantages, such as limited resolution and distortion.

The most popular two-dimensional imaging device is the CCD camera. Advances in the application of CCD sensors have produced cameras with other powerful features like progressive scanning, asynchronous reset, and on-board image storage. These more expensive CCDs can be useful in certain more demanding applications.

5. Software

Software is very important to any vision system because once a vision system is setup with every component tested to work properly, the end-user will use image processing software to control the vision system, get the final result, or perform image analysis. The

advantage of a vision system, flexibility, is mainly demonstrated by the imaging processing software.

In the past, although C-language based callable libraries are available from several vendors, they were often tied to that company's image-processing hardware, thereby locking in the designer to both hardware and software products from a particular vendor. In other words, the vision system developer had few choices when selecting image-processing software-development tools. Recently, easy-to-use tools have sped up imaging software design and development. There are two tendencies, one is based on the icon-based data flow, another is ActiveX controls (also called OCX controls).

In the icon-based data flow development the designer first selects the icons that represent the functions to be performed on the source image. By linking a series of these functions, the designer can produce a visual data graph that shows the flow of the image-processing program. These software flows provide real-time performance in a heterogeneous, multiprocessing environment, that automatically address real time issues such as data queuing, synchronization, and load balancing. For further development, the operators, such as filtering, thresholding, and rotation, are implemented as C++ files. They can be combined, modified, or re-written in a C++ editing window within the application.

OCX controls are software components based on Microsoft's 32-bit component object module interface. These modules can be easily dropped into a development environment, such as Visual Basic or Visual C++. Using this approach, system developers can implement controls from companies to write their own application code. Working in such an environment, developers gain access to powerful features such as standard graphical property pages, context-sensitive help, and standard conventions for

programming the controls. Using OCX controls simplify the development process by taking advantage of object-oriented architectures and graphical developments. A developer can just drag and drop OCX components and configure them with graphics property sheets. At run time, the tools are executed and examined using a standard interface of properties, methods, and events, and as the result, vision applications can be created more rapidly using fewer lines of code.

6. Frame grabber

The frame grabber is the center part of a vision system. The frame grabber acquires the image from the sensor, such as a CCD camera, scanner, in the form of an electrical signal. These signals can be digitized into sequence of number which can then be processed by the image processor or host computer. Thus, the frame grabber gets input from optical components and provides output to the host computer system. The major components of the frame grabber are video input, the frame buffer and the video output.

2. 2 Fundamentals of vision systems design

2.2.1 General guide line of vision system design

The image is everything. No one will fail to address the importance of designing an application around a quality image. On the otherhand, when you design one must let the technology drive the application to some degree. As we mentioned before, a vision system is not human-like in its performance so one may not assume that a part or feature that can be seen with the human eye, or measured using hard tooling can automatically be successfully inspected using vision systems.

The following section describes a set of guidelines that can be applied to the vision application. However, before we design a vision system, keep in our mind the following points.

- fixed application vs. general-purpose,
- software vs. hardware,
- cost-driven vs. performance-drive, and
- programmed vs. automatically generated

In addition to the special concepts associated exclusively with vision system, the integrator must deal with all of the machinery, components and technology needed to complete a turnkey system.

2.2.1.1 Identification of requirements

A) General identification

The design processing begins with the identification of requirements or system/application specifications. The development of a full specification for the application is the most important and yet often ignored step in the vision design process. We use the specification and design process described herein to insure that a particular application is suited for vision systems.

The specification should list, in detail, everything that the inspection system is expected to do, and to some extent, how it should be done. In any case, all aspects of a system or application specification should be considered. For example, the measurements it is required to make, the environment in which it will work, system with which it will interface, people who will support it, time available for development, cost target, etc.

Flexibility is another factor that must be considered when designing vision systems. For instance, one of the strengths of using a vision systems for an inspection stage is the inherent ability of most systems to be configured for a variety of part types. Care must be taken, however, to insure that an application is constrained to a reasonable set of part types, features, or variations.

B) Identify part variations

All parts have variations that will affect the vision system task. The application specification at least should identify these variables, and at best should address how the system will handle them.

Part to part variations can be subtle or severe. For example, to take the case of an assembly that has housing that can be either white or black. This has an obvious impact on any inspection that has the housing as a feature. However, less drastic colour variations, such as differences in the surface colour and texture of a smooth piece of metal can also wreak havoc in an onspection.

C) Identify process variations

Sometimes a more difficult set of variables to track down are those caused by the automation process. A few, however, seem to appear quite often. For instance, foreign materials on the inspection surface cause lighting schemes to fail and images to be unrecognizable. Things like fluids or dust can come from the air around the inspection station so that they may cause problems. In addition, part handling and placement in front of the imaging system often contributes to errors. As flexible as a vision system is, some

positional variations can not be handled programmatically. In particular, this occurs in gauging applications when a measurement depends upon the orientation of a planar surface.

2.2.1.2 System considerations

Sometimes the biggest challenge in vision system design/integration is not the system itself, but rather the seamless integration of a wide variety of components and tasks into a single, controlled system. These components can range from specialized sensors to robots, and the tasks could be as straightforward as providing a custom GUI performing in-process communications over wireless Ethernet. Whatever the case, one needs to examine these issues as part of the system specification. Particular attention needs to be paid to machine control, external devices, and communications with the outside world.

2.2.1.3 Unit consideration

It seems funny that one should need to consider the basic unit of length measurement but this must be done. In North America, most vendors use the English version of length as a unit. Outside of North America, most vendors stay with the Metric version. If the parts are ordered from the different vendors who use different units, seamless integration will be a problem.

2.2.1.4 User consideration

Every vision system eventually will be used by someone. The end-user must be included in the specification step of the integration process.

2.2.1.5 Budgeting for vision system

There is a tendency in vision system design to overestimate technology capability and understate hardware and system costs. Vision systems are not overly expensive, however they often cost more than expected. As in our experience, a frame grabber in a PC or a light duty general-purpose machine vision processor might be available for \$5,000 to \$9,000. However, small peripheral components, lighting, cameras, optics, enclosures and mounting hardware will quickly push the hardware costs to about \$15,000 or more. Special care must be taken to insure that the capability of such a system would match the proposed application.

2.2.2 Vision system detailed design

“Vision systems integration is the incorporation of components into a complete system. Unless you want to be a manufacturer of vision products, avoid designing your own devices from the ground up. This does not apply to specialized mechanics or part handling as you may be a product manufacturer. However, don’t let your zeal for the project, cost effectiveness or personal glory tempt you to build a frame grabber, camera, lighting system or other component unless you are absolutely ready to support it as a product. Designers come and go, but an integrated vision system will be around for a long time and must be serviceable”. (12)

The same argument holds true for software design. Where possible, it is much better to use available libraries for vision primitive functions. You may have the “ultimate” algorithm for edge detection. Unfortunately, it probably is not quantifiably better than the one that is available from your machine vision processor manufacturer. And even if it is, you will find that any difference in the two is quickly outweighed by the extensive number of man hours it takes to code, debug, implement, document and support your custom algorithm. Again, there are exceptions to the off-the-shelf rule when applied to software design. In many cases libraries or particular algorithms are just not available from any source. Also, system performance requirements may dictate that you develop customized inspections to meet cycle times.

2.2.3 Vision systems integration

A vision system truly is not an “off-the-shelf”, or “end-user” technology. In order to achieve a successful vision system, it requires competent integration within the target process or system.

Integration requires one to make into a whole by bringing all parts together. Systems integration is the part of a vision system design where we actually have to make the application work. This is sometimes simple, but often more difficult than it would seem. Successful systems integration requires comprehensive technological expertise in a wide range of engineering and management disciplines. Vision system integration is the process where significant value is added to a vision processor or imaging engine by the incorporation of software, peripheral hardware, mechanical devices, materials and

engineering. The end product is a complete inspection system or cell for use in an automated environment.

2.2.4 Test and condition

One needs to test the process constantly throughout the building phase. One should not wait for testing until the design is completely fabricated, assembled, painted and polished. Too often one will find that something didn't work quite the way one wanted. A rebuild is much more expensive from the point of view of a completely finished system. Classic examples include camera mis-positioning or position adjustments too limited for the application, lighting that doesn't work the way it did in the laboratory and inadequacy of part handling mechanisms.

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Chapter 3

Image Analysis and Measurement

One approach adopted within this chapter has been to present to the reader a number of algorithms commonly used in image vision systems. Most algorithms have only been described in outline and, in a sense, form only a guide to the techniques that the vision engineers might implement on an image. For most scientists or engineers, only the barest of knowledge about how to use the computer vision systems, but the greater their understanding of the various image analysis operations which are possible, their advantages, and limitations, the greater the likelihood of success in their applications. We are talking about application of image analysis and measurement, such as general steps, basic image processing operators, not about the theory of image analysis. In the drop shape measurement, the most important image processing operation will be edge detection, so this section will mainly discuss the edge detection and other related procedures. More detail information can be found in references (1-11).

3.1 Introduction

Image measurement is tied to recognition, counting and measuring the size, shape, position, density, and other similar properties of particular objects in an image. An image is digitized to convert it to a form, which can be stored in a computer's memory or on some form of storage media such as a hard disk or CD-ROM. This digitization procedure can be done by a scanner or by a video camera connected to a frame grabber board in a

computer. Once the image has been digitized, it can be operated upon by various image-processing operations.

Image processing operations can be roughly divided into three major categories: Image improvement, image enhancement and transformation and measurement extraction.

image defects which could be caused by the digitization process or by faults in the imaging set-up (e.g., bad lighting) can be corrected using image enhancement techniques.

Once the image is in good condition, the Measurement extraction operations can be used to obtain useful information from the image. These operations can be extended to operate on colour images. The examples below represent only a few of the many techniques available for operating on images. Details about the inner workings of the operations have not been given, but references to appropriate books containing this information are given at the end of this chapter.

3.1.1 Data Representation

The image, we consider herein will be limited to 2-D images as the default because our experiments will only concern 2D images.

The basic two-dimensional image is a monochrome (grayscale) image, which has been digitized. We let i and j denote two integers where $1 \leq i \leq m$ and $1 \leq j \leq n$. In addition, we let $f(i,j)$ denote an integer function such that $1 \leq f(i, j) \leq w$. An array F , defined as

$$F = \begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,n) \\ \dots & & & \\ f(m,1) & f(m,2) & \dots & f(m,n) \end{bmatrix}$$

is called digital image. An address (i,j) defines a positions in F , called a pixel. The elements of F denote the intensities within a number of small rectangular regions within a real image. F is proportional to the brightness or gray value of the image at that point.

Figure 21. Shows a gray scale image. The black box shown in the image (Fig. 3.1) has its corresponding intensity values shown in Table 3.1.

Fig. 3.1 Greyscale image and highlighted square region

99	71	61	51	49	40	35	53	86	99
93	74	53	56	48	46	48	72	85	102
101	69	57	53	54	52	64	82	88	101
107	82	64	63	59	60	81	90	93	100
114	93	76	69	72	85	94	99	95	99
117	108	94	92	97	101	100	108	105	99
116	114	109	106	105	108	108	102	107	110
115	113	109	114	111	111	113	108	111	115
110	113	111	109	106	108	110	115	120	122
103	107	106	108	109	114	120	124	124	132

Table 3.1 Pixel values in highlighted square region

An image can have pixel values that can be of several data types:

In a black-and-white image (also known as a monochrome or grayscale image) there are different shades of gray between black and white; these shades of gray are called gray-levels. The number of possible gray-levels in an image depends on how many bits are

available for each pixel. If each pixel use 8 bits, for example, then there are $2^8 = 256$ different gray-levels possible in an image.

A binary image refers to an image in which each pixel can have one of only two values, 0 or 1. This binary choice allows the image to be represented in a very compact way because it is easier to process and display a image in this representation. The more pixels present in an image, the more details or resolution that is possible. In addition, the greater the range of values in an image, the more contrast that is possible.

3.1.2 Template/Filter

In image processing, a very important algorithm is convolution. A template is an array of values. This temple is placed step-by-step over the image, at each step creating a new window in the image the same size as the template, and then associating with each element in the template a corresponding pixel in the image. Typically, the template element is multiplied by the corresponding image pixel gray level and the sum of these results, across the whole template, is recorded as a pixel gray level in a new image. This "shift, add, multiply" operation is termed the "convolution" of the template with the image. For example, applying the following template or kernel results in a sharpening of the image:

$$T = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Whereas, applying the following kernel smoothes an image (it also increases the intensity of the image by a factor of 16, so one needs to normalize the convolution result):

$$T = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

3.2 General Image Processing Steps

The basic steps consist of : (1) acquired an image, (2) improve image data, (3)enhance/transform the image, and (4) analyst image or image measurement.

3.3 Image Improvement

The first stage of any vision system is the image acquisition stage. After the image has been obtained, various methods of processing can be applied to the image to perform the many different vision tasks that are required. When grabbing an image, there are many factors that degrade the quality of the resulting image: the position of the camera relative to the object, the type of lighting used, the presence of certain electromagnetic waves, etc. All of these physical problems are called interface problems. Because interference problems corrupt an image, the information extracted from that image can be misleading. There are, however, various techniques that you can use to eliminate or reduce the effect of these problems.

3.3.1 Correcting image defects

Noise may either be random or periodic. Random noise is most familiar as “snow” in a TV image. Pixels are either randomly increased or decreased in brightness compared to

their proper value. Hardware often treats dropped-out or missing pixels by resetting the pixel intensity value to either the darkest or brightest value available.

When it is practical, averaging through several identical image acquired by the system can solve most random noise problems. This kind of signal averaging depends upon the randomness of the noise itself so that the problem pixel occurs in different locations on each subsequent image frame. If this is the case, then by acquiring many images and adding them together, the effects of the noise will be averaged out and it will effectively vanish.

On the otherhand, in many cases, averaging of many successive frames from a live image source is not possible because the image is not constant or stationary, or it may already have been acquired and stored. If random noise is still present, then the image may be directly processed to improve it. In this case, smoothing would be a useful tool. Within the image, each pixel has a brightness value; it also has 8 neighbours in the most common case of a rectilinear raster scan. The information in the images is usually of significantly greater size than the individual pixels, so it may be expected that neighbouring pixels would usually be a part of the same surface or object, so the averaging can also be performed using the neighbours.

we use the following diagram to identify a central pixel and its eight nearest neighbours those pixels. The central pixel is called P0, and the neighbours will be P1, ...P8. ``

$$\begin{bmatrix} 1 & 2 & 3 \\ 8 & 0 & 4 \\ 7 & 6 & 5 \end{bmatrix}$$

With this notation, the simplest way to average the information in neighbouring pixels to smooth out noise in the image would be to replace the value of each pixel by the average of it and its neighbours, or

$$P_0 = \frac{1}{9} \sum P_i (i = 0,8)$$

which is a simple but not entirely satisfactory method for averaging. Other weighting factors can also be devised. These schemes allow for making the original pixel value more important, and neighbouring pixels less important in proportion to their distance from the central pixel. Using a 3*3 kernel of weighting factors would usually be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

gives an example. However, the smoothing with these weighted functions is rarely used, because it degrades the sharpness of real edges or discontinuities in the image. In addition to averaging noisy values, smoothing will average together values that really are different where they are adjacent. This means that the boundaries of surfaces and objects will be “smattered out” and the brightness step across them will be reduced in magnitude. Unfortunately, both of these effects are exactly contradictory to our purpose, which is to find those boundaries for measurement or identification purposes that represent the edge. Therefore, another approach to smoothing is preferred.

One of the best ways to remove random noise from an image is to apply a “median filter.” This also works with each pixel and its neighbours, usually in 3*3 pattern. The brightness values of the 9 pixels are ranked in order, and the median value (the fifth

brightest in this example) is used to replace the original brightness of the central pixel. Again, the process is carried out simultaneously for all pixels in the images so that the values that are ranked are the original ones. Median filters may be applied repeatedly to an image till there are no further changes. They will smooth out noisy but grossly uniform areas, because isolated random noise pixels will rarely have a median brightness value and will be replaced by one of the neighbouring values. Median filters will even correct images for dropouts - entirely missing pixels that sometimes result from uneven or noisy scan generators. At the same time, true discontinuities such as surface edges or object boundaries will not be broadened, shifted or reduced in contrast.

3.3.2 Image distortions

Image distortions can affect application results. For example, in a medical application that analyzes blood cells, if the camera does not have a one-to-one aspect ratio and no correction is performed, the cells appear distorted and elongated and incorrect diagnostic interpretations might result. Rotating such an image causes even more serious blob distortion. There are some approaches that can be used to solve these problems, such as resizing and rotating.

3.4 Image Enhancement and transformation

Thresholding is the basic approach for image enhancement and transformation.

Thresholding images means reducing each pixel to a certain range of values. Some operations can be performed more efficiently on threshold images. Images with full grayscale levels are useful for some tasks, but have redundant information for others.

3.4.1 Thresholding errors

Rarely is it possible to identify a perfect gray-level break. Normally errors are made in the classification of pixels as background or foreground as soon as any thresholding is done. When classifying a range of gray levels into one set, two types of error can be made:

Type1. Not all pixels are caught that should be included.

Type2. Some pixels are caught that should not be in the group.

Also, those two types of errors always happen in the other image procedures.

The choice of threshold level may aim to balance these two types error.

3.5 Measurement extraction

Once you have improved and transformed your image using the previously discussed techniques, you can begin to analyze your image. In otherwords, you can begin to extract useful information from your image. The information to extract can be basic, such as distribution of pixel values in an image, or a line profile. The information also can be of a more advanced nature, such as pattern recognition.

3.5.1 Calibration

Digitization divides images into a two dimensional grid of small units called pixels.

When you analyze images, however, you usually want results in real world units (inches, cm, etc.). To accomplish this, you have to calibrate your image; that is, assign a specific number of pixels to a specific number of real-world units in the x and y directions.

3.5.2 Image statistics

Many applications need to obtain some type of image statistic to condense a frame of pixels into a smaller, more functional set of values for analysis. The statistic might be required to perform some subsequent operation and/or might be used to summarize the effect of some image operation.

One common statistic is the histogram; See Fig. 3.2. A histogram is the intensity distribution of pixel values in an image and is generated by counting the number of times each pixel intensity occurs. This information is very useful for several applications. In particular, it is useful to select a threshold level when binarizing an image and to change the image intensity distribution when trying to increase the image contrast.

It should be realized that since a histogram summarizes the distribution of pixel values in an image, it can reveal the degree of contrast or brightness in the image. For example, if an image has low contrast, its histogram will occupy just a fraction of the horizontal axis; if an image is too bright or too dark, its histogram will mostly occupy one end of the horizontal axis; See Fig. 3.2

A histogram can also be used when thresholding the image. For example, when object has different colour from background in one color, the histogram of the image will contain two distinct regions. Therefore, you easily select a threshold by picking any value between these regions; See Fig. 3.2 and 3.3.

Fig. 3.2 Histogram of Fig. 3.3

Fig. 3.3 An image of pendant drop

3.5 Edge Detection and Segmentation

3.5.1 Edge

In image processing, an edge is a sharp change in intensity value between adjacent pixels. Since an edge can be caused by a transition between an object and the background, or by a cluster of noise pixels, many operations are designed to emphasize or de-emphasize the edge of an image.

An edge can be orientated at any angle (that is, in the horizontal direction, the vertical direction, or some direction in between). When a group of connected pixels has values that are very different from those pixels above or below it, a horizontal edge is said to be created. When a group of connected pixels has values that are very different from those pixel to its right or left, a vertical edge is said to be created. An edge may be regarded as a boundary between two dissimilar regions in an image. These may be different surfaces of the object, or perhaps a boundary between light and shadow falling on a single surface. Thus, edge detection, ideally, identifies all the lines that outline the objects in an image. In principle an edge is easy to find since differences in pixel values between regions are relatively easy to calculate by considering gradients. However, rarely does it work that well. Edges caused by poor lighting or poor capture equipment frequently appear where they are the errors as we mentioned when discussing Type 2 errors. On the otherhand, edges of objects are often incomplete because the background is too like the foreground intensity of the object and a clearly defined line or edge is not visible as we mentioned Type 1 errors.

Many edge extraction techniques can be broken up into two distinct phases:

- Finding pixels in the image where edges are likely to occur by looking for discontinuities in gradients. Candidate points for edges in the image are usually referred to as edge points, edge pixels, or edgels.
- Linking these edge points in some way to produce descriptions of edges in terms of lines, curves etc.

Each phase in turn will be discussed in the following Sections.

3.5.2 Detecting Edge Points

1. Gradient based methods

An edge point can be regarded as a point in an image where a discontinuity (in gradient) occurs across some line. A discontinuity may be classified as one of five types (see Fig. 3.4):

Fig. 3.4 discontinuity type

The gradient is a vector, whose components measure how rapidly pixel values are changing with distance in the x and y directions. Thus, the components of the gradient

may be found using the following approximation: where dx and dy measure distance along the x and y directions respectively.

$$\begin{aligned}\frac{\partial f(x, y)}{\partial x} &= \Delta_x = \frac{f(x + dx, y) - f(x, y)}{dx} \\ \frac{\partial f(x, y)}{\partial y} &= \Delta_y = \frac{f(x, y + dy) - f(x, y)}{dy}\end{aligned}\quad [3-1]$$

In (discrete) images we can consider dx and dy in terms of numbers of pixels between two points. Thus, when $dx = dy = 1$ (pixel spacing) and we are at the point whose pixel coordinates are (i, j) we have

$$\begin{aligned}\Delta_x &= f(i + 1, j) - f(i - 1, j) / 2d_x \\ \Delta_y &= f(i, j + 1) - f(i, j - 1) / 2d_y\end{aligned}\quad [3-2]$$

In order to detect the presence of a gradient discontinuity we must calculate the change in gradient at (i, j) . We can do this by finding the following gradient magnitude M , and the gradient direction, θ , given by

$$\begin{aligned}M &= \sqrt{\Delta_x^2 + \Delta_y^2} \\ \theta &= \tan^{-1} \left[\frac{\Delta_y}{\Delta_x} \right]\end{aligned}\quad [3-3]$$

The difference operators in Eq. [3-3] correspond to convolving the image with the two masks as shown in in Fig. 3.5.

Fig. 3.5 Edge operator convolution masks.

This is easy to compute: the top left-hand corner of the appropriate mask is superimposed over each pixel of the image in turn, a value is calculated for or by using the mask

coefficients in a weighted sum of the value of pixel (i,j) and its neighbours. These masks are referred to as convolution masks or sometimes convolution kernels.

Instead of finding approximate gradient components along the x and y directions we can also approximate gradient components along directions at 45 degree and 135 degree to the axes respectively. In this case the following equations are used:

$$\begin{aligned}\Delta_1 &= f(i+1, j+1) - f(i, j) \\ \Delta_2 &= f(i, j+1) - f(i+1, j)\end{aligned}\quad [3-4]$$

This form of operator is known as the Roberts edge operator and was one of the first operators used to detect edges in images. The corresponding convolution masks are given by:

$$\begin{array}{|c|c|} \hline 0 & 1 \\ \hline -1 & 0 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & -1 \\ \hline \end{array}$$

Δ_1 Δ_2

Fig. 3.6 Roberts edge operator

Many edge detectors have been designed using convolution mask techniques, often using 3*3 mask sizes or even larger.

An advantage of using a larger mask size is that errors due to the effects of noise are reduced by local averaging within the neighbourhood of the mask.

An advantage of using a mask of odd size is that the operators are centered and can therefore provide an estimate that is biased towards the center pixel (i,j).

One important edge operator of this type is the Sobel edge operator. The Sobel edge operator masks are given in Fig 3.7.

$$\begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

Δ_x Δ_y

Fig. 3.7 Sobel edge operator convolution masks

2. Second Order Methods

All of the previous edge detectors have approximated the first order derivatives of pixel values in an image. It is also possible to use second order derivatives to detect edges.

A very popular second order operator is the Laplacian operator. The Laplacian of a function $f(x,y)$ is defined by:

$$\nabla^2 f(x_i, y_j) = \frac{[f(x_{i-1}, y_j) + f(x_i, y_{j-1}) + f(x_{i+1}, y_j) + f(x_i, y_{j+1}) - 4f(x_i, y_j)]}{h^2} \quad [3-5]$$

Once more we can use discrete finite difference approximations to estimate the derivatives and represent the Laplacian operator with the convolution mask shown in Fig 3.8.

0	1	0
1	-4	1
0	1	0

Fig. 3.8 Laplacian operator convolution mask

However, there are disadvantages to the use of second order derivatives. Since second-order exaggerate signal noise significantly more than first-order derivative operations.

Finally, no directional information about the edge is given. The problems that the presence of noise causes when using edge detectors means we should try to reduce the noise in an image prior to or in conjunction with the edge detection process.

3.5.3 Segmentation

One way of extracting and representing information from an image is to group pixels together into regions of similarity. This process is commonly called segmentation.

Segmentation is concerned with splitting an image up into segments (also called regions and areas) where each holds some property distinct from their neighbour. This is an

essential part of image analysis - in answering questions like where and how large is the object, where is the background, how many objects exist where and how many surface exist, and so on.

Segmentation can be approached from two points of view: by identify edges(or lines) or by identifying regions(or areas) within an image. In 2D, we would group pixels together according to the rate of change of their intensity over a region.

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Chapter 4

Criteria for Evaluating the Accuracy of Surface Tension

Values from Digital Vision Systems

4.1 Introduction

There are numerous methodologies for the measurement of contact angles and surface tensions. One of the most common methods for obtaining the interfacial tension and/or contact angle is based on the shape of a sessile or pendant drop. For example, the ADSA-P algorithm of Rotenberg et. al (1,2) uses the edge profile coordinates of the drop to estimate quantities like surface tension, drop volume and contact angle. Obviously, the precision of these estimated quantities depends on the accuracy of the edge profile coordinates acquired by the ADSA-P program. One method for acquiring the edge profile coordinates involves digital image acquisition. The drop image is recorded by a CCD camera and the light intensity distribution recorded on the CCD is used to determine the drop's edge profile and coordinates. Any uncertainty in the vision system will effect the accuracy of the drop's profile coordinates and the precision of surface tension. Thus, to quantify the uncertainty between the digital vision system and the precision of the surface tension we will (1) estimate the experimental surface tension precision for an arbitrary vision system and (2) detail the methodology for the design of a vision system. The Axisymmetric Drop Shape Analysis (ADSA-P) methodology has been used in this thesis because software source code is available in the public domain (3) and because the ADSA-P program has been used by a number of researchers (4-8).

As is well known, the numerical integration of the Laplace equation of capillary permits one to determine the shape of a drop when given values of surface tension and initial parameters (e.g. radius of curvature of the drop at the origin). This integration ability permits one to create a theoretical/numerical drop edge profile or set of profile coordinates without experimental errors. As shown below, the magnitude of typical numerical errors are orders of magnitude below the magnitude of typical experimental errors (e.g. for pure water at 20°C typical experimental errors for the surface tension range from $\pm 0.08 \text{ mJ/m}^2$ to $\pm 0.5 \text{ mJ/m}^2$). If this initial theoretical/numerical profile is perturbed (as described below) and the set of perturbed profile coordinates are used as input to the ADSA-P program, (or any similarly designed software package that uses drop edge or profile points), then it is possible to estimate the relation between size of profile perturbation (detailed in several ways below) and the surface tension deviation, $\Delta\gamma$, that results from the perturbation. For area array CCD cameras with 10^6 pixels or more the magnitude of the perturbation maybe characterized in terms of pixels so that there is a direct connection between the CCD pixel array size and the smallest possible surface tension uncertainty, $\Delta\gamma$.

In this thesis, by combining the numerical solution of the Laplace equation of capillary and the ADSA-P method, we were able to develop (1) a perturbation model to simulate the uncertainty in the edge profile coordinates and to relate the size of the perturbation to the precision of surface tension and (2) two analogous relations that set regions of surface tension accuracy; e.g. $\pm 0.01 \text{ mJ/m}^2$ or $\pm 0.1 \text{ mJ/m}^2$, as functions of the magnification of the drop, CCD camera array size, pixel size, drop shape and drop edge precision. Finally,

an algorithm for the design of various vision systems based on these criteria will be discussed.

4.2 Numerical Integration of Laplace Equation, ADSA-P Algorithm and Definitions

In essence, the shape of a drop is influenced physically by a combination of surface tension and gravity. When gravitational and surface tension effects are in mechanical equilibrium, the Laplace equation of capillary reflects this balance through the equation

$$\gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \Delta P \quad [4-1]$$

where γ is the interfacial tension, R_1 and R_2 are the two principal radii of curvature and ΔP is the pressure difference across the interface. This pressure difference may be expressed as a linear function of the elevation, z , using the expression

$$\Delta P = \Delta P_0 + \Delta \rho g z \quad [4-2]$$

where ΔP_0 is the pressure difference at the reference plane, $\Delta \rho$ is the density difference between the two bulk phases, g is the gravitational acceleration, and z is the vertical elevation of the drop measured from the reference plane. Thus, for a given value of γ and $\Delta P_0 = 2\gamma/R_0$, the shape of a drop may be determined (via R_1 and R_2). The inverse, determination of the interfacial tension, γ , from the shape, is also possible.

Figure 4.1 illustrates the coordinate system used for the numerical integration of Eq. [4-1] with the apex point of the drop placed at the origin of the coordinate system. The interface is symmetric about the Z-axis and the principal radius of curvature, R_1 , is

related to the arc length, s , and the angle of inclination of the interface to the horizontal, θ . This first radius of curvature is given by the relation

$$\frac{1}{R_1} = \frac{d\theta}{ds} \quad [4-3]$$

while the second radius of curvature is given by the relation

$$\frac{1}{R_2} = \frac{\sin \theta}{x} \quad [4-4]$$

At the origin, $s=0$, both radii are equal so

$$\frac{1}{R_0} = \frac{1}{R_1} = \frac{1}{R_2} \quad [4-5]$$

and Eq.[4-1] can be written as

$$\Delta P_0 = \frac{2\gamma}{R_0} \quad [4-6]$$

Substituting Eqs. [4-3], [4-4] and [4-6] into Eq. [4-1] permits one to write Eq. [4-1] as

$$\gamma \left[\frac{d\theta}{ds} + \frac{\sin \theta}{x} \right] = \frac{2\gamma}{R_0} + \Delta \rho g z \quad [4-7]$$

This form of the Laplace equation may be integrated along the profile curve and through the region where the slope is vertical without any difficulty since the arc-length is a monotonic function.

In order to reduce the number of parameters present in the Laplace equation we combine all physical quantities into one factor using the capillary constant, c , defined as

$$c = \Delta \rho g / \gamma \quad [4-8]$$

The capillary constant is positive in value for sessile drops (supported) and negative for pendant drops (suspended). Multiplying all length scales by the factor $\sqrt{c}$, we write the dimensionless lengths as

$$S = sc^{1/2}; X = xc^{1/2}; Z = zc^{1/2}; B_0 = R_0c^{1/2}; \quad [4-9]$$

Substituting these dimensionless variables into Eq. [4-7] yields

$$\frac{d\theta}{dS} = \frac{2}{B_0} + Z - \frac{\sin \theta}{X} \quad [4-10]$$

The variables X , Z , S and θ are related geometrically via the relations (10, page 7),

$$\frac{dX}{dS} = \sin \theta \quad [4-11]$$

$$\frac{dZ}{dS} = \cos \theta \quad [4-12]$$

The natural boundary conditions (defined at the apex of the drop) are

$$\frac{d\theta}{dS} = \frac{\sin \theta}{X} = \frac{1}{B_0} \quad [4-13]$$

where X , Z , S and θ are all zero at this location. Expressions, [4-10], [4-11] and [4-12] in combination with the natural boundary conditions at the apex of the drop constitute a complete specification of the problem. A procedure to perform this numerical integration using the dimensionless arc-length S was established by Hartland and Hartley (10).

If we construct an objective function E ,

$$E = \frac{1}{2} \sum_{i=1}^N d(u_n, v)^2 \quad [4-14]$$

which is a measure of the absolute normal distance between the N input coordinates (either experimental or numerical) u_n and the corresponding Laplacian curve v determined

uniquely by the parameters R_0 and c (see Fig. 4.2). We may use the minimization of E as the criteria for determining a suitable fit between the input profile data points and the Laplacian curve (i.e. an acceptable solution to Laplace's equation of capillarity). In the ADSA-P algorithm, the objective function, E , is minimized, subject to the differential equations given above, by an incremental loading method and the Newton-Raphson method, with the implicit Euler method of integration, to yield the desired optimized curve from which R_0 and γ may be obtained. In essence, the Axisymmetric Drop Shape Analysis (ADSA) scheme employs the coordinates of the edge profile, u_r , to determine surface tension and contact angle (1).

4.3 Modelling for the Distribution of Drop Edge Perturbation Length

A model algorithm was developed to perturb an ideal drop profile and to then determine the distribution of normal distances between the two profiles. In a physical sense, a perturbation of the edge profile may come from any uncertainty within the experiment. This perturbation modelling was a tool to simulate these experimental uncertainties so that their influence on other parameters like the surface tension could be quantified. This may help in the design of an experimental system or with the estimation of the precision by which various parameters can be determined by the experimental system.

The Laplace equation of capillarity in the form given by Eq. [4-10] permits one to determine the theoretical edge profile of an axisymmetric sessile or pendant drop. An extended precision program based on the algorithm of Hartland and Hartley (10, pp. 696-701) was developed to obtain a numerical solution to Laplace equation of capillarity. This program will be referred to herein as the Laplace program. The Laplace program

was then used to create an accurate initial drop profile (as illustrated in Fig. 4. 3 for a sessile drop). This initial profile served as a reference profile (denoted herein as Profile_0) for comparison with the perturbed profile. Profile_0 was a function of the density difference across the interface, $\Delta\rho$, surface tension, γ , and the radius of curvature of the apex of the drop, R_0 . This apex point occurs at the top of a sessile drop or the bottom of a pendant drop. Alternatively, one may determine a capillary constant, c , whereupon Profile_0 becomes a function of c , γ and the dimensionless apex radius $B_0 = R_0\sqrt{c}$.

A weakly perturbed profile was obtained using the Laplace program to alter B_0 and γ according to the expressions

$$\tilde{B}_0 = B_0 + \delta B_0, \text{ provided } \left| \frac{\delta B_0}{B_0} \right| \ll 1 \quad [4-15]$$

$$\tilde{\gamma} = \gamma + \delta\gamma, \text{ provided } \left| \frac{\delta\gamma}{\gamma} \right| \ll 1 \quad [4-16]$$

This perturbed profile will be referred to as Profile_1. As a result of the numerical integration procedure, the initial and the perturbed profiles (i.e. Profile_0 and Profile_1) do not have the same termination point or final end point (see Fig. 4. 3). Consequently, to permit a comparison between the two profiles created via Hartland and Hartley's algorithm it was necessary to shift Profile_1 by a small vertical displacement to insure that the end points of both profiles occur at the same horizontal elevation (see Zshift in Fig. 4. 3). In a physical sense, the shift insures that one compared two drops supported by the same horizontal substrate (sessile drop situation) or suspended by the same vertical tube (pendant drop situation). We denote Profile_1 after the suitable vertical shift as

Profile_2: see Fig. 4. 4. A comparison of Profile_0 with Profile_2 yields the distribution of normal perturbation lengths, λ , as a function of B_0 , γ and the arc-length, s , or the turning angle θ ; that is, $\lambda = f(B_0, \gamma, \theta \text{ or } s)$ see Fig. 4.11.

If we denote $\lambda_{\max}$ as the absolute maximum actual physical perturbation length for the whole arc length of the drop, then the dimensionless perturbation length ratio, $\hat{\lambda}$, maybe defined as

$$\hat{\lambda} = \frac{\lambda}{\lambda_{\max}} \quad [4-17]$$

where $0 \leq \hat{\lambda} \leq 1$ and $\lambda_{\max} = \max\{\lambda_{eff}; \text{ for } \hat{s} = 1 \text{ or } \theta_i = \pi\}$. The dimensionless arc-length, $\hat{s}$, defined as

$$\hat{s} = \frac{s}{s_{\max}} \quad [4-18]$$

where $s_{\max}$ represents the maximum arc-length defined as $s_{\max} = \max\{s_i, \text{ for all } i\}$ and

$0 \leq \hat{s} \leq 1$. Subsequently $\hat{\lambda} = f(B_0, \gamma, \theta \text{ or } \hat{s})$. This distribution, $\hat{\lambda} = f(B_0, \gamma, \theta \text{ or } \hat{s})$,

provides us with a new way to predict the displacement of the edge profile of a drop

based on the original edge profile. For instance, if a set of edge profile coordinates

(X_i^0, Z_i^0) is given, then it is possible to create a new set of edge profile coordinates

$(\tilde{X}_i, \tilde{Z}_i)$ which are perturbed geometrically from (X_i^0, Z_i^0) via the relations

$$\tilde{X}_i = X_i^0 + \delta X_i \text{ with } \delta X_i = \lambda_i \cos \theta \quad [4-19]$$

$$\tilde{Z}_i = Z_i^0 + \delta Z_i \text{ with } \delta Z_i = \lambda_i \sin \theta \quad [4-20]$$

where $\lambda_i = \hat{\lambda}_i \lambda_{\max}$. These new edge profile coordinates, $(\tilde{X}_i, \tilde{Z}_i)$, will still satisfy the Laplace equation of capillary. The original edge profile (X_i^0, Z_i^0) may be created from a numerical integration of the Laplace equation of capillary, or obtained from the image representation of the drop's edge. This perturbation modelling provides us with an accurate means of moving from an arbitrary but known edge profile to an adjacent profile that is either an expansion or contraction of the drop's size. Figures 4.6 through 4.11 illustrate various λ -distributions that will be discussed below. In the next section we evaluate the accuracy or precision of the perturbation modelling technique.

4.3.1 Evaluation of the Perturbation Modelling Approach

An original or unperturbed set of profile coordinates maybe created by the Laplace program as the edge profile coordinates for the ADSA-P program. ADSA-P uses these coordinates to estimate quantities like the surface tension. When the original set of coordinates are used as input to ADSA-P, one should and one does recover the same value of surface tension as used as input to the Laplace program. Thus, if we geometrically perturbed the original profile coordinates and use this perturbed set of coordinates as input to ADSA-P we will be able to estimate the effect of profile coordinate uncertainty on the estimated surface tension value. We use this procedure to establish a connection between edge detection uncertainty and surface tension accuracy as discussed below. It is important to recognize that our methodology is completely general and can be applied to any software package (not just ADSA-P). First, we evaluate the consistency of the Laplace program and the ADSA-P program using the algorithm diagrammed below.

$B_0|_{Laplace}^0$: Input value to Laplace program

$\gamma|_{Laplace}^0$: Input value to Laplace program

$P^0(X_i^0, Z_i^0)$: profile output from Laplace program based on
input values of $B_0|_{Laplace}^0$ and $\gamma|_{Laplace}^0$

$B_0|_{ADSA}^0$: Output from the ADSA – P program based on input profile $P^0(X_i^0, Z_i^0)$

$\gamma|_{ADSA}^0$: Output from the ADSA – P program based on input profile $P^0(X_i^0, Z_i^0)$

In theory, the following equalities should be satisfied:

$$B_0|_{ADSA}^0 = B_0|_{Laplace}^0 \quad [4-21]$$

$$\gamma|_{ADSA}^0 = \gamma|_{Laplace}^0 \quad [4-22]$$

Table 4.1 illustrates this comparison.¹ Values of B_0 and γ were used as input to the Laplace program which created drop profile coordinates (X_i^0, Z_i^0) . These profile points were used as input to the ADSA-P program which determined a set of B_0 and γ values from the profile coordinates (X_i^0, Z_i^0) . We used 36 points plus one point at the drop's apex in all profiles. As may be seen from Table 4.1, the ADSA-P program correctly determines both B_0 and γ values to five digits from the numerically accurate profile coordinates. Several Fortran compilers on different OS platform were used to establish this accuracy. The compilers included: Absoft Fortran v.5.1 from Absoft Inc., G77 Fortran from the Free Software Foundation (GNU) on a WinNt 4.0 O.S. and F77 on SUN UNIX. The accuracy would be improved if the number of points in the profile input were increased beyond 37. Typically, experimental values of surface tension have either 2 or 3 digits of accuracy so that a surface tension value of 72.1 mJ/m^2 is not experimentally different than a value of 72.0 mJ/m^2 . Table 4.1 indicates that the numerical integration of the Laplace program of capillarity (Laplace program) was able to produce a sufficiently accurate edge profile, (X_i^0, Z_i^0) , so that the ADSA-P program was able to obtain consistent value of B_0 and γ .

¹ In this paper, all example data is based on a pure water sessile drop, $\Delta\rho = 1000 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$, $\gamma = 72 \text{ mJ/m}^2$.

Table 4.1. Comparison of the ADSA-P program estimates B_0 and γ with a numerically accurate integration of the Laplace equation of capillary (Laplace program). Surface tension γ in units of mJ/m^2 with B_0 dimensionless

Case	Parameters	Laplace Program	ADSA Program
1	B_0	0.4	0.400103
	γ	72	72.016975
2	B_0	0.8	0.800010
	γ	72	72.008894
3	B_0	2	2.000102
	γ	72	72.004280
4	B_0	10	10.000207
	γ	72	72.000683
5	B_0	20	20.001356
	γ	72	71.997939

If the original values of B_0 and γ were altered by small increments δB_0 and $\delta\gamma$ so that altered values $\tilde{B}_0 = B_0 + \delta B_0$ and $\tilde{\gamma} = \gamma + \delta\gamma$ were used as input to the Laplace program, then one may obtain a new set of altered edge profile coordinates, $(X_i, Z_i)_{new}$. If this set of coordinates is used as input to ADSA-P, then the ADSA-P program will produce consistent values of $\tilde{B}_0$ and $\tilde{\gamma}$ to the suggested numerical accuracy in Table 4.1. The following flow chart illustrates the evaluate procedure.

Defining:

$B_0|_{Laplace}^0, \gamma|_{Laplace}^0$: Value input to the Laplace program to generate original profile.

$P^0(X_i^0, Z_i^0)$: Original profile based on $B_0|_{Laplace}^0, \gamma|_{Laplace}^0$.

$\tilde{B}_0|_{Laplace}, \tilde{\gamma}|_{Laplace}$: Value input to the Laplace program with small perturbation.

$\tilde{P}|_{Laplace}(\tilde{X}_i|_{Laplace}, \tilde{Z}_i|_{Laplace})$: New drop profile coordinates output from the Laplace

program based on perturbed values of $\tilde{B}_0|_{Laplace}, \tilde{\gamma}|_{Laplace}$.

$\tilde{B}_0|_{L^{n-perturbed}}^{ADSA}, \tilde{\gamma}|_{L^{n-perturbed}}^{ADSA}$: Value output from ADSA-P based on input profile

$\tilde{P}|_{Laplace}(\tilde{X}_i|_{Laplace}, \tilde{Z}_i|_{Laplace})$. Subscript “un-perturbed” means that the coordinates were

obtained without invoking a small perturbation on the original profile.

$\tilde{P}|_{ADSA}(\tilde{X}_i|_{ADSA}, \tilde{Z}_i|_{ADSA})$: New perturbed drop profile coordinates obtained by

geometrically perturbing the original drop profile, $P^0(X_i^0, Z_i^0)$ using the perturbation

length distribution $\hat{\lambda} = f(B_0, \gamma, \theta \text{ or } \hat{s})$

$\tilde{B}_0|_{Perturbed}^{ADSA}, \tilde{\gamma}|_{Perturbed}^{ADSA}$: Value output from ADSA-P based on input profile

$\tilde{P}|_{ADSA}(\tilde{X}_i|_{ADSA}, \tilde{Z}_i|_{ADSA})$.

As we know already, based on the any original profile, $\tilde{P}|_{Laplace}(\tilde{X}_i|_{Laplace}, \tilde{Z}_i|_{Laplace})$.

ADSA-P was able to obtain values of $\tilde{B}_0|_{Un-perturbed}^{ADSA}$ and $\tilde{\gamma}|_{Un-perturbed}^{ADSA}$ that were the same

numerically as the values of $\tilde{B}_0|_{Laplace}$ and $\tilde{\gamma}|_{Laplace}$ used as input to the numerical

integration of the Laplace equation of capillarity. Next, we geometrically perturbed an

original profile, $P^0(X_i^0, Z_i^0)$, based on original values of $B_0|_{Laplace}$ and $\gamma|_{Laplace}$ to obtain a

perturbed profile, $\tilde{P}|_{ADSA}(\tilde{X}_i|_{ADSA}, \tilde{Z}_i|_{ADSA})$, using the perturbation length distribution

$\hat{\lambda} = f(B_0, \gamma, \theta \text{ or } \hat{s})$. After these perturbed profile coordinates were used as input to

ADSA-P we obtained from ADSA-P estimated values of $\tilde{B}_0|_{Perturbed}^{ADSA}$ and $\tilde{\gamma}|_{Perturbed}^{ADSA}$. If the

perturbation length distribution procedure was correct, then ADSA-P should predict

values of $\tilde{B}_0|_{Perturbed}^{ADSA}$ and $\tilde{\gamma}|_{Perturbed}^{ADSA}$ based on the input edge profile,

$\tilde{P}|_{ADSA}(\tilde{X}_i|_{ADSA}, \tilde{Z}_i|_{ADSA})$ that were numerically equal to $\tilde{B}_0|_{Un-perturbed}^{ADSA}$ and $\tilde{\gamma}|_{Un-perturbed}^{ADSA}$. The

perturbation length distribution, λ , and its absolute maximum value, in particular λ_{max} ,

provides the connection between the edge detection uncertainty and a robust estimation

of the surface tension uncertainty. Table 4.2 through 4.4 illustrate this consistency

between $(\tilde{B}_0|_{Laplace}, \tilde{\gamma}|_{Laplace})$, $(\tilde{B}_0|_{Un-perturbed}^{ADSA}, \tilde{\gamma}|_{Un-perturbed}^{ADSA})$ and $(\tilde{B}_0|_{Perturbed}^{ADSA}, \tilde{\gamma}|_{Perturbed}^{ADSA})$.

Tables 4.2 through 4.4 were based on a fixed value of surface tension, $\gamma = 72 \text{ mJ/m}^2$, and

various value of B_0 , from 0.4 to 20. All examples are based on a sessile drop

configuration. An analogous comparison procedure was possible for the pendant drop configuration or for significantly different values of the surface tension.

Table 4.2 Perturbed original profile $P^0(X_i^0, Z_i^0)$, based on $B_0 = 0.4$ and $\gamma = 72 \text{ mJ/m}^2$, to

$\tilde{P}|_{\text{ADSA}}(\tilde{X}_i|_{\text{ADSA}}, \tilde{Z}_i|_{\text{ADSA}})$. Table shows that ADSA-P provides consistent values of

$(\tilde{B}_0|_{\text{Perturbed}}^{\text{ADSA}}, \tilde{\gamma}|_{\text{Perturbed}}^{\text{ADSA}}), (\tilde{B}_0|_{\text{Un-perturbed}}^{\text{ADSA}}, \tilde{\gamma}|_{\text{Un-perturbed}}^{\text{ADSA}})$ with $(\tilde{B}_0|_{\text{Laplace}}, \tilde{\gamma}|_{\text{Laplace}})$

		Laplace Program $\tilde{B}_0 _{\text{Laplace}}, \tilde{\gamma} _{\text{Laplace}}$	ADSA-P Program $\tilde{B}_0 _{\text{Un-perturbed}}^{\text{ADSA}}, \tilde{\gamma} _{\text{Un-perturbed}}^{\text{ADSA}}$	ADSA-P Program $\tilde{B}_0 _{\text{Perturbed}}^{\text{ADSA}}, \tilde{\gamma} _{\text{Perturbed}}^{\text{ADSA}}$
1	B_0	0.4001	0.4002028308450388	0.4002034460833402
	γ	71.9	71.91779821241126	71.91727139129841
2	B_0	0.401	0.4011028340561549	0.4011069137236579
	γ	71.9	71.91775201866739	71.92049399573635
3	B_0	0.402	0.4021030482759037	0.4021108838794188
	γ	71.8	71.81772983237832	71.82478875765604
4	B_0	0.404	0.4041034822211440	0.4041183843962203
	γ	71.6	71.61766286161283	71.63576040273969

Table 4.3 Perturbed original profile $P^0(X_i^0, Z_i^0)$, based on $B_0 = 2$ and $\gamma = 72 \text{ mJ/m}^2$, to

$\tilde{P}|_{\text{ADSA}}(\tilde{X}|_{\text{ADSA}}, \tilde{Z}|_{\text{ADSA}})$. Table shows that ADSA-P provides consistent values of

$(\tilde{B}_0|_{\text{Perturbed}}^{\text{ADSA}}, \tilde{\gamma}|_{\text{Perturbed}}^{\text{ADSA}})$, $(\tilde{B}_0|_{\text{Ln-perturbed}}^{\text{ADSA}}, \tilde{\gamma}|_{\text{Ln-perturbed}}^{\text{ADSA}})$ with $(\tilde{B}_0|_{\text{Laplace}}, \tilde{\gamma}|_{\text{Laplace}})$.

		Laplace Program $\tilde{B}_0 _{\text{Laplace}}, \tilde{\gamma} _{\text{Laplace}}$	ADSA-P Program $\tilde{B}_0 _{\text{Ln-perturbed}}^{\text{ADSA}}, \tilde{\gamma} _{\text{Ln-perturbed}}^{\text{ADSA}}$	ADSA-P Program $\tilde{B}_0 _{\text{Perturbed}}^{\text{ADSA}}, \tilde{\gamma} _{\text{Perturbed}}^{\text{ADSA}}$
1	B_0	2.0001	2.000201489880077	2.000201252373759
	γ	71.9	71.90444751921750	71.90414031120217
2	B_0	2.01	2.010103386825651	2.010062201152535
	γ	71	71.00446296549320	70.97769203576324
3	B_0	2.02	2.020105574858692	2.020002800288979
	γ	70	70.00448160364786	69.95684736329190
4	B_0	1.999	1.999101080140750	1.998991426425036
	γ	72.1	72.10444226869738	72.10149955890701
5	B_0	1.99	1.990099252420538	1.988924961990333
	γ	73	73.00442576085958	72.97694248266409
6	B_0	1.98	1.980097267652052	1.977578929023627
	γ	74	74.00440981453589	74.36147866387952

Table 4.4 Perturbed original profile $P^0(X_i^0, Z_i^0)$ based on $B_0 = 20$ and $\gamma = 72 \text{ mJ/m}^2$

to $\tilde{P}|_{\text{ADSA}}(\tilde{X}_i|_{\text{ADSA}}, \tilde{Z}_i|_{\text{ADSA}})$. Table shows that ADSA-P provides consistent values of

$(\tilde{B}_0|_{\text{Perturbed}}^{\text{ADSA}}, \tilde{\gamma}|_{\text{Perturbed}}^{\text{ADSA}}), (\tilde{B}_0|_{\text{Un-perturbed}}^{\text{ADSA}}, \tilde{\gamma}|_{\text{Un-perturbed}}^{\text{ADSA}})$ with $(\tilde{B}_0|_{\text{Laplace}}, \tilde{\gamma}|_{\text{Laplace}})$.

		Laplace Program $\tilde{B}_0 _{\text{Laplace}}, \tilde{\gamma} _{\text{Laplace}}$	ADSA-P Program $\tilde{B}_0 _{\text{Un-perturbed}}^{\text{ADSA}}, \tilde{\gamma} _{\text{Un-perturbed}}^{\text{ADSA}}$	ADSA-P Program $\tilde{B}_0 _{\text{Perturbed}}^{\text{ADSA}}, \tilde{\gamma} _{\text{Perturbed}}^{\text{ADSA}}$
1	B_0	20.1	20.10085656930746	20.09883527090825
	γ	71.9	71.89921351906830	71.90603208845079
2	B_0	20.2	20.20086918868996	20.19498120400785
	γ	71.8	71.79919858616212	71.81811387644353
3	B_0	20.4	20.40094311882747	20.38312012154941
	γ	71.6	71.59905121726018	71.65419018933392
4	B_0	21	21.00102500580084	20.91291302679198
	γ	71	70.99896288950340	71.25374988119756

These tabulated results demonstrate that the ADSA-P program yields essentially the same (i.e. results depends on the actual number of points) set of $\tilde{B}_0$ and $\tilde{\gamma}$ values irregardless of the method used to create the input edge profile coordinates. Therefore, it would seem reasonable to conclude that the method used to create a perturbing length distribution, λ , is reasonable when combined with an initial unperturbed set of profile coordinates to generate a new set of perturbed profile coordinates. When the sessile drop is either too small (nearly spherical) or too large (nearly flat) the utility of the ADSA-P routine suffers and we must obtain the perturbation length distribution from limiting analytical solutions. These two limiting situations are discussed below.

4.4 Limiting Analytical Solutions for the Situation of either Extremely Large or Extremely Small Drops

Two possible types of solutions exist and will be discussed below. One solution relates to extremely large, flat drops with large values of B_0 while the analytical solution relates to extremely small, nearly spherical drops with small values of B_0 . Our purpose is to derive analytical solutions for $\hat{\lambda} = f(B_0, \gamma, \theta \text{ or } \hat{s})$ for these situations.

4.4.1 Limiting Solution for Large Curvature Drop

If we rewrite the Eq. [4-10] as

$$\frac{1}{X} \frac{d}{dX} (X \sin \theta) = \frac{2}{B_0} \pm Z \quad [4-23]$$

then the integration of Eq [4-23] yields

$$X \sin \theta = \frac{X^2}{B_0} \pm \int_0^X ZX dX \quad [4-24]$$

Very small (large curvature) sessile and pendent drops are very nearly spherical and the gravitation term in the pressure balance may be neglected without substantial error when B_0 is small. The term $\frac{2}{B_0}$ in Eq. [4-23] is much greater than Z and the drop is essentially spherical (or circular in a plane that includes the axis of symmetric with radius B_0). Therefore, the equation governing the shape of this circular cross-section may be written as

$$X^2 + Z^2 = 2B_0Z \quad [4-25]$$

so that

$$Z = B_0 \pm (B_0^2 - X^2)^{1/2} \quad [4-26]$$

where the + sign applies when $\theta > 90^\circ$ and the – sign applies when $\theta < 90^\circ$ for both sessile and pendant drops. Consequently, the solution to Eq. [4-24] is

$$\begin{aligned} X \sin \theta &= \frac{X^2}{B_0} \pm \int_0^X Z X dX \\ &= \frac{X^2}{B_0} \pm \int_0^X [B_0 \pm (B_0^2 - X^2)^{1/2}] X dX \\ &= \frac{X^2}{B_0} \pm \frac{B_0 X^2}{2} \pm \int_0^X X (B_0^2 - X^2)^{1/2} dX \\ &= \frac{X^2}{B} \pm \left\{ \frac{B_0 X^2}{2} \pm \left[\frac{(B_0^2 - X^2)^{3/2}}{3X} - \frac{B_0^3}{3X} \right] \right\} \end{aligned} \quad [4-27]$$

where the first $\pm$ sign refers to sessile or pendant drops respectively and the second $\pm$ sign to the situation $\theta > 90^\circ$ or $\theta < 90^\circ$ respectively.

Two restrictive situations arise: When θ and X are small, Eq. [4-27] may be approximated further as

$$X \sin \theta \rightarrow \frac{X^2}{B_0} \quad [4-28]$$

and, when θ is large and X is small, to

$$X \sin \theta \rightarrow \frac{X^2}{B_0} \pm \frac{2}{3} \frac{B_0^3}{X} \quad [4-29]$$

where the – sign applies to sessile drops and the + sign to pendant drops. This occurs since the $B_0 X^2$ term is small compared to the X^2/B_0 term. For the case of pendant drops the point of inflection may be obtained by differentiation (10, p. 637).

The relation between X and Z for the situation of a nearly spherical drop, described by Eq. [4-25] and subjected to a small perturbation in drop profile shape is obtained by varying Eq. [4-25] according to the scheme

$$(X + \delta X)^2 + (Z + \delta Z)^2 = 2(B_0 + \delta B_0)(Z + \delta Z) \quad [4-30]$$

so that

$$\delta X \approx \frac{Z}{X} \delta B_0 - \frac{Z - B_0}{X} \delta Z \quad [4-31]$$

and

$$\delta Z \approx \frac{Z}{Z - B_0} \delta B_0 - \frac{X}{Z - B_0} \delta X \quad [4-32]$$

If we have two similar axisymmetric profiles, one denoted by $P_0(X_i^0, Z_i^0)$ based on B_0 , and another profile, denoted by $P_1(X_i^1, Z_i^1)$ based on $\tilde{B}_0$ such that

$$\delta B_0 = \tilde{B}_0 - B_0 \text{ where } \frac{|\delta B_0|}{|B_0|} \ll 1 \quad [4-33]$$

then the vertical distance difference

$$\delta Z_i = Z_i^1 - Z_i^0 \quad [4-34]$$

in combination with Eq. [4-31] maybe used to estimate the radial variation, δX_i , via the relation

$$\delta X_i \approx \frac{Z_i^0}{X_i^0} \delta B_0 - \frac{Z_i^0 - B_0}{X_i^0} \delta Z_i \quad [4-35]$$

Analogous comments apply to the use of Eq. [4-32] to estimate δZ_i .

Tables 4.5 and 4.6 illustrate the accuracy of using Eqs. [4-31] and [4-32] to estimate the change in a coordinate's value when a nearly spherical drop was subjected to a small perturbation. The original edge profile of the sessile drop was created by the Laplace program with $B_0=0.01$. Equations [4-31] and [4-32] were then used to predict the perturbed edge profile when $\tilde{B}_0=0.0112202$ and $\delta B_0=0.0012202$. Table 5 shows representational data.

Table 4.5 The edge profile coordinates (X_i^0, Z_i^0) based on $B_0 = 0.01$ and (X_i^1, Z_i^1) based on $B_0 = 0.0112202$ calculated using the Laplace program. All quantities are

dimensionless including the turning angle θ (in degree) measured from the horizontal;

see Fig. 4. 1.

θ	X_i^0	Z_i^0	X_i^1	Z_i^1	$\delta X_i = X_i^1 - X_i^0$	$\delta Z_i = Z_i^1 - Z_i^0$
10	1.73648E-03	1.51922E-04	1.94836E-03	1.70460E-04	2.118800E-04	1.853800E-05
20	3.42020E-03	6.03072E-04	3.83752E-03	6.76658E-04	4.173201E-04	7.358601E-05
30	4.99998E-03	1.33974E-03	5.61007E-03	1.50321E-03	6.100903E-04	1.634700E-04
40	6.42784E-03	2.33953E-03	7.21214E-03	2.62500E-03	7.843003E-04	2.854699E-04
50	7.66038E-03	3.57208E-03	8.59507E-03	4.00792E-03	9.346898E-04	4.358401E-04
60	8.66016E-03	4.99990E-03	9.71683E-03	5.60996E-03	1.056670E-03	6.100601E-04
70	9.39680E-03	6.57963E-03	1.05433E-02	7.38242E-03	1.146500E-03	8.027898E-04
80	9.84792E-03	8.26325E-03	1.10495E-02	9.27144E-03	1.201579E-03	1.008190E-03
90	9.99983E-03	9.99960E-03	1.12199E-02	1.12196E-02	1.220070E-03	1.220000E-03
100	9.84793E-03	1.17359E-02	1.10495E-02	1.31678E-02	1.201570E-03	1.431900E-03
110	9.39683E-03	1.34195E-02	1.05434E-02	1.50567E-02	1.146571E-03	1.637201E-03
120	8.66025E-03	1.49990E-02	9.71696E-03	1.68289E-02	1.056709E-03	1.829900E-03
130	7.66061E-03	1.64267E-02	8.59540E-03	1.84307E-02	9.347894E-04	2.004001E-03
140	6.42831E-03	1.76590E-02	7.21280E-03	1.98133E-02	7.844898E-04	2.154300E-03
150	5.00085E-03	1.86585E-02	5.61129E-03	2.09347E-02	6.104400E-04	2.276199E-03
160	3.42181E-03	1.93948E-02	3.83980E-03	2.17607E-02	4.179899E-04	2.365900E-03
170	1.74014E-03	1.98455E-02	1.95353E-03	2.22662E-02	2.133899E-04	2.420699E-03
180	8.16415E-05	1.99957E-02	1.02778E-04	2.24344E-02	2.113650E-05	2.438700E-03

Table 4.6 Values of X_i^0 were obtained from the Laplace program using a value of

$B_0 = 0.01$. The values of X_i^0 when $\tilde{B}_0 = 0.0112202$ were also obtained via the Laplace

program. A perturbation estimate of $\tilde{X}_i$ that is denoted by $\tilde{X}_i = X_i^0 + \delta X_i$ when δX_i was

evaluated using Eq. [4-31] is tabulated. The relative error was calculated as

$$Error = \left| \left(\frac{\tilde{X}_i - X_i^1}{X_i^1} \right) \times 100 \right| \text{ in units of percentage, all other quantities are}$$

dimensionless including the turning angle θ (in degree) measured from the horizontal:

see Fig. 4. 1.

θ	X_i^0	δX_i (Eq. [4-31])	$\tilde{X}_i = X_i^0 + \delta X_i$	X_i^1	Error (%)
10	1.736480E-03	2.118878E-04	1.948368E-03	1.948360E-03	4.023008E-04
20	3.420200E-03	4.173297E-04	3.837530E-03	3.837520E-03	2.500021E-04
30	4.999980E-03	6.100912E-04	5.610071E-03	5.610070E-03	1.594107E-05
40	6.427840E-03	7.843270E-04	7.212167E-03	7.212140E-03	3.705353E-04
50	7.660380E-03	9.347052E-04	8.595085E-03	8.595070E-03	1.798048E-04
60	8.660160E-03	1.056706E-03	9.716866E-03	9.716830E-03	3.635524E-04
70	9.396800E-03	1.146593E-03	1.054339E-02	1.054330E-02	8.797589E-04
80	9.847920E-03	1.201654E-03	1.104957E-02	1.104950E-02	6.745847E-04
90	9.999830E-03	1.220221E-03	1.122005E-02	1.121990E-02	1.344526E-03
100	9.847930E-03	1.201726E-03	1.104966E-02	1.104950E-02	1.408163E-03
110	9.396830E-03	1.146777E-03	1.054361E-02	1.054340E-02	1.953332E-03
120	8.660250E-03	1.057026E-03	9.717276E-03	9.716960E-03	3.259135E-03
130	7.660610E-03	9.352707E-04	8.595881E-03	8.595400E-03	5.599674E-03
140	6.428310E-03	7.852341E-04	7.213544E-03	7.212800E-03	1.031968E-02
150	5.000850E-03	6.116225E-04	5.612473E-03	5.611290E-03	2.107331E-02
160	3.421810E-03	4.203551E-04	3.842165E-03	3.839800E-03	6.159623E-02
170	1.740140E-03	2.198006E-04	1.959941E-03	1.953530E-03	3.281593E-01
180*	8.164150E-05	2.724123E-04	3.540538E-04	1.027780E-04	2.444841E+02

*When $\theta = 180$, $\left| \frac{\delta X_i}{X_i^0} \right| > 1$, and Eq. [4-31] is not satisfied.

Table 4.7 Values of Z_i^0 were obtained from the Laplace program using a value of $B_0=0.01$. The values of Z_i^0 when $\bar{B}_0=0.0112202$ were also obtained via the Laplace program. A perturbation estimate of $\tilde{Z}_i$ that is denoted by $\tilde{Z}_i = Z_i^0 + \delta Z_i$ when δZ_i was evaluated using Eq. [4-32] is tabulated. The relative error was calculated as

$$Error = \left| \left(\frac{\tilde{Z}_i - Z_i^1}{Z_i^1} \right) \times 100 \right| \text{ in units of percentage, all other quantities are dimensionless}$$

including the turning angle θ (in degree) measured from the horizontal; see Fig. 4. 1.

θ	Z_i^0	δZ_i (Eq. [4-32])	$\tilde{Z}_i = Z_i^0 + \delta Z_i$	Z_i^1	Error (%)
10	1.51922E-04	1.853662E-05	1.704586E-04	1.704600E-04	8.108040E-04
20	6.03072E-04	7.358252E-05	6.766545E-04	6.766580E-04	5.160486E-04
30	1.33974E-03	1.634695E-04	1.503210E-03	1.503210E-03	3.434816E-05
40	2.33953E-03	2.854475E-04	2.624978E-03	2.625000E-03	8.542285E-04
50	3.57208E-03	4.358217E-04	4.007902E-03	4.007920E-03	4.595274E-04
60	4.99990E-03	6.099989E-04	5.609899E-03	5.609960E-03	1.090634E-03
70	6.57963E-03	8.025350E-04	7.382165E-03	7.382420E-03	3.451822E-03
80	8.26325E-03	1.007768E-03	9.271017E-03	9.271440E-03	4.558680E-03
90°	9.99960E-03	-2.553553E-03	7.446048E-03	1.121960E-02	3.363357E+01
100	1.17359E-02	1.432783E-03	1.316868E-02	1.316780E-02	6.703511E-03
110	1.34195E-02	1.637767E-03	1.505727E-02	1.505670E-02	3.758771E-03
120	1.49990E-02	1.830448E-03	1.682945E-02	1.682890E-02	3.260051E-03
130	1.64267E-02	2.004575E-03	1.843127E-02	1.843070E-02	3.112880E-03
140	1.76590E-02	2.154925E-03	1.981392E-02	1.981330E-02	3.153102E-03
150	1.86585E-02	2.276882E-03	2.093538E-02	2.093470E-02	3.262345E-03
160	1.93948E-02	2.366762E-03	2.176156E-02	2.176070E-02	3.958751E-03
170	1.98455E-02	2.421832E-03	2.226733E-02	2.226620E-02	5.088680E-03
180	1.99957E-02	2.440752E-03	2.243645E-02	2.243440E-02	9.148165E-03

Tables 4.6 and 4.7 demonstrates that variational expressions Eqs. [4-31] and [4-32] provide acceptable accuracy when modelling the perturbation of small, nearly spherical drop profile.

Figure 4.5 illustrates, for very small (large curvature) sessile and pendant drop, a simplified method for calculating the normal perturbation length, λ_n , between the original profile and the perturbed profile. Point P_0 is a profile point for the original profile with $P_0 = f(\theta)$ and $\tilde{P}_0$ is a profile point on the profile of the perturbed drop shape in a direction that is normal to the profile curve at P_0 ; that is, in direction $\hat{n}(P_0)$. We denote the original radius of the profile curve by R_0 and the perturbed profile radius by $\tilde{R}$. The distance between O and O' is $\tilde{R} - R_0$ and distance between P_0 and $\tilde{P}_0$ in the direction $\hat{n}(P_0)$ is λ_n . θ is turning angle measured from the horizontal, see Fig. 4. 5.

Employing the law of cosines for a planar triangle, we have that

$$(\tilde{R} - R_0)^2 + (R_0 + \lambda_n)^2 - 2(R_0 - \tilde{R})(R_0 + \lambda_n)\cos\left(\frac{\pi}{2} - \theta\right) = \tilde{R}^2 \quad [4-36]$$

which may, upon rearranging, be written as a quadratic relation in λ_n

$$(\lambda_n)^2 + 2(\lambda_n)[R_0(1 + \sin \theta) - \tilde{R} \sin \theta] - 2R_0(\tilde{R} - R_0)(1 + \sin \theta) = 0 \quad [4-37]$$

Figure 4.6 plots the dimensionless normal perturbation length distribution using definition [4-17], for small, spherical drop using positive solutions to Eq. [4-37].

4.4.2 Limiting Solution for Flat Drops of Small Contact Angle and Large Curvature

The governing differential equation for the shape, i.e., edge profile, of an axisymmetric drop maybe written as

$$\frac{1}{X} \frac{d}{dX} (X \sin \theta) = \frac{2}{B_0} \pm Z \quad [4-38]$$

Differentiating Eq. [4-38] with respect to X and using the approximation that at small angles $\sin \theta$ is equivalent to $\tan \theta$ so that

$$\frac{dZ}{dX} = \tan \theta \approx \sin \theta \quad [4-39]$$

gives

$$\frac{d^2(\sin \theta)}{dX^2} + \frac{1}{X} \frac{d(\sin \theta)}{dX} - \frac{\sin \theta}{X^2} - \sin \theta = 0 \quad [4-40]$$

which is intrinsically valid only at small angles of declination from the horizontal where $\delta X \approx \delta S$ and $\sin \theta \approx \theta$. Numerical results show that these are “reasonable” approximations over a surprising large range of contact angle (10, pp. 638-640).

Equation [40] is an ordinary differential equation is of the form (11)

$$\eta^2 \frac{d^2 \xi}{d\eta^2} + \eta \frac{d\xi}{d\eta} - (\eta^2 + m^2) \xi = 0 \quad [4-41]$$

which has the general relation

$$\xi(\eta) = D_1 I_m(\eta) + D_2 K_m(\eta) \text{ for all } m \quad [4-42]$$

where $I_m(\eta)$ and $K_m(\eta)$ denote modified Bessel functions of the first and second kind respectively, of order m and D_1, D_2 are arbitrary constants.

Therefore, Eq. [4-40] has the solution

$$\sin \theta = D_1 I_1(X) + D_2 K_1(X) \quad [4-43]$$

where $I_1(X)$ and $K_1(X)$ are modified Bessel functions of the first and second kind respectively, of order one. The constant of integration, D_2 , must be zero because $\sin \theta$ is

zero when X is zero and where $K_1(X)$ is infinite. The remaining constant of integration,

D_1 , maybe evaluated from the properly that

$$\frac{\sin \theta}{X} \rightarrow \frac{1}{B_0} \text{ as } X, \theta \rightarrow 0 \quad [4-44]$$

and

$$\frac{I_1(X)}{X} \rightarrow 2 \text{ as } X, \theta \rightarrow 0 \quad [4-45]$$

This yields the approximate solution

$$\sin \theta \approx \frac{2}{B_0} I_1(X) \quad [4-46]$$

The expression for Z may then be obtained by writing, from Eq. [4-39], that

$$Z = \int_0^X \tan \theta dX \approx \int_0^X \sin \theta dX \approx 2 \int_0^X \frac{1}{B_0} I_1(X) dX \quad [4-47]$$

whereupon

$$Z(X) \approx \frac{2}{B_0} [I_0(X) - 1] \quad [4-48]$$

The curvature, B_0 , maybe eliminated from Eq. [4-48], using Eq. [4-46], to obtain

$$Z(X) \approx \frac{I_0(X) - 1}{I_1(X)} \sin \theta \quad [4-49]$$

which relates, Z , X , and θ with no arbitrary constants.

Polynomial approximation for $I_0(X)$ and $I_1(X)$ are given by

$$I_0(X) = 1 + \frac{X^2}{2^2} + \frac{X^4}{2^2 \cdot 4^2} + \frac{X^6}{2^2 \cdot 4^2} + \frac{X^8}{2^2 \cdot 4^2 \cdot 6^2} \cdots \quad [4-50]$$

and

$$I_1(X) = \frac{X}{2^2} + \frac{X^3}{2^2 \cdot 4} + \frac{X^5}{2^2 \cdot 4^2 \cdot 6} + \frac{X^7}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8} \dots \quad [4-51]$$

Recognizing that

$$I_0'(X) = I_1(X) \quad [4-52]$$

because

$$I_{m+1}(X) = I_{m-1}(X) - \frac{2m}{X} I_m(X) \quad [4-53]$$

and

$$I_m'(X) = \frac{1}{2} \{I_{m-1}(X) + I_{m+1}(X)\} \quad [4-54]$$

permits one to write

$$I_1'(X) = \frac{1}{2} \{I_0(X) + I_2(X)\} \quad [4-55]$$

$$I_2(X) = I_0(X) - \frac{2}{X} I_1(X) \quad [4-56]$$

$$I_1'(X) = I_0(X) - \frac{1}{X} I_1(X) \quad [4-57]$$

It is now possible to obtain an analytical solution for a small perturbation of a large, flat sessile drop whose profile shape is given by Eq. [4-48]. Using Eq. [4-48], a vertical perturbation or variation, δZ , will be connected to variations in curvature, δB_0 , and radial displacement, δX , via the expression

$$\delta Z \approx \frac{2}{B_0} I_1(X) \delta X - 2 \frac{I_0 - 1}{B_0^2} \delta B_0 \quad [4-58]$$

The curvature, B_0 , maybe replaced in Eq. [4-58], using Eq. [4-46], to obtain

$$\begin{aligned}\delta Z &\approx \frac{I_0' I_1 - I_0 I_1' + I'}{(I_1)^2} \sin \theta \delta X \\ &\approx \sin \theta \left[1 - \left(\frac{I_0}{I_1} \right)^2 + \frac{I_0}{X I_1} + \frac{I_0}{I_1^2} - \frac{1}{X I_1} \right] \delta X\end{aligned}\quad [4-59]$$

At any point on the profile edge (i.e. axisymmetric surface of the sessile drop) the normal distance, $\lambda_{\bar{n}}$, between two similar profiles (one profile slightly perturbed with respect to the other profile) will be given by

$$\lambda_{\bar{n}} = \delta X \sin \theta + \delta Z \cos \theta \quad [4-60]$$

This relation may be used in collaboration with Eq. [4-58] to obtain an estimate of δZ when δX is varied. A comparison between this analytical approach and a strictly numerical approach using the Laplace program demonstrates the consistency of the two approaches. This comparison is illustrated in Table 4.9 (with $B_0 = 100$) and Table 4.11 (with $B_0 = 1000$). Tables 4.9 to 4.12 illustrate the accuracy of Eq. [4-59]. The original edge profile was created by the Laplace program.

Table 4.8 The edge profile coordinates (X_i^0, Z_i^0) where determined using the Laplace program with $B_0 = 100$ and the coordinates (X_i^1, Z_i^1) where determined similarly with $\tilde{B}_0 = 105$. The difference $\delta X_i = X_i^1 - X_i^0$ was substituted into Eq. [4-59] to determine δZ_i . All quantities are dimensionless including the turning angle θ in degree measured from the horizontal.

θ_i	X_i^0	Z_i^0	X_i^1	Z_i^1	$\delta X_i = X_i^1 - X_i^0$
10	3.868812E+00	1.830021E-01	3.922605E+00	1.834381E-01	5.379272E-02
20	4.606206E+00	3.726210E-01	4.659513E+00	3.729320E-01	5.330706E-02
30	5.009408E+00	5.577508E-01	5.062461E+00	5.579455E-01	5.305290E-02
40	5.268285E+00	7.374265E-01	5.321188E+00	7.375173E-01	5.290270E-02
50	5.442418E+00	9.104621E-01	5.495229E+00	9.104613E-01	5.281067E-02
60	5.558694E+00	1.075630E+00	5.611448E+00	1.075549E+00	5.275440E-02
70	5.631881E+00	1.231728E+00	5.684603E+00	1.231580E+00	5.272198E-02
80	5.671247E+00	1.377621E+00	5.723954E+00	1.377416E+00	5.270672E-02
90	5.683229E+00	1.512259E+00	5.735932E+00	1.512009E+00	5.270290E-02
100	5.672676E+00	1.634696E+00	5.725381E+00	1.634412E+00	5.270529E-02
110	5.643487E+00	1.744110E+00	5.696199E+00	1.743803E+00	5.271149E-02
120	5.598973E+00	1.839809E+00	5.651690E+00	1.839489E+00	5.271721E-02
130	5.542051E+00	1.921242E+00	5.594772E+00	1.920917E+00	5.272102E-02
140	5.475375E+00	1.988010E+00	5.528094E+00	1.987687E+00	5.271864E-02
150	5.401407E+00	2.039867E+00	5.454116E+00	2.039551E+00	5.270863E-02
160	5.322462E+00	2.076725E+00	5.375150E+00	2.076418E+00	5.268860E-02
170	5.240732E+00	2.098658E+00	5.293390E+00	2.098359E+00	5.265808E-02
180	5.158308E+00	2.105892E+00	5.210924E+00	2.105597E+00	5.261612E-02

Table 4.9 Coordinates (X_i^0, Z_i^0) were calculated numerically using the Laplace program based on $B_0 = 100$. Slightly perturbed coordinates, (X_i^1, Z_i^1) were calculated numerically using the Laplace program based on $\tilde{B}_0 = 105$. Values of $\tilde{Z}_i = Z_i^0 + \delta Z_i$ were determined using Eq. [4-59] to calculate δZ_i . A comparison of $\tilde{Z}_i$ (analytical Eq. [59]) with Z_i^1 (from the Laplace program with $\tilde{B}_0 = 105$) shows that results are not significantly different. $Error = \left| \left(\frac{\tilde{Z}_i - Z_i^1}{Z_i^1} \right) \times 100 \right|$ in units of percentage, all other quantities are dimensionless including the turning angle θ measured from the horizontal.

θ_i	Z_i^0	δZ_i	$\tilde{Z}_i = Z_i^0 + \delta Z_i$	Z_i^1	Error (%)
10	1.830021E-01	4.511449E-04	1.834532E-01	1.834381E-01	8.219289E-03
20	3.726210E-01	3.323153E-04	3.729533E-01	3.729320E-01	5.711031E-03
30	5.577508E-01	2.264479E-04	5.579773E-01	5.579455E-01	5.695740E-03
40	7.374265E-01	1.382043E-04	7.375647E-01	7.375173E-01	6.422463E-03
50	9.104621E-01	6.739189E-05	9.105295E-01	9.104613E-01	7.487056E-03
60	1.075630E+00	1.359895E-05	1.075643E+00	1.075549E+00	8.745785E-03
70	1.231728E+00	-2.365102E-05	1.231705E+00	1.231580E+00	1.014013E-02
80	1.377621E+00	-4.508345E-05	1.377576E+00	1.377416E+00	1.160414E-02
90	1.512259E+00	-5.186920E-05	1.512207E+00	1.512009E+00	1.310262E-02
100	1.634696E+00	-4.580184E-05	1.634650E+00	1.634412E+00	1.455669E-02
110	1.744110E+00	-2.944908E-05	1.744081E+00	1.743803E+00	1.591434E-02
120	1.839809E+00	-6.243262E-06	1.839803E+00	1.839489E+00	1.707388E-02
130	1.921242E+00	1.950348E-05	1.921262E+00	1.920917E+00	1.794489E-02
140	1.988010E+00	4.270394E-05	1.988053E+00	1.987687E+00	1.840134E-02
150	2.039867E+00	5.774879E-05	2.039925E+00	2.039551E+00	1.833207E-02
160	2.076725E+00	5.892731E-05	2.076784E+00	2.076418E+00	1.762701E-02
170	2.098658E+00	4.101757E-05	2.098699E+00	2.098359E+00	1.620288E-02
180	2.105892E+00	1.641347E-11	2.105892E+00	2.105597E+00	1.402931E-02

Table 4.10 The edge profile coordinates (X_i^0, Z_i^0) where determined using the Laplace

program with $B_0 = 1000$ and the coordinates (X_i^1, Z_i^1) where determined similarly

with $\tilde{B}_0 = 1100$. The difference $\delta X_i = X_i^1 - X_i^0$ was substituted into Eq. [4-59] to determine

δZ_i . All quantities are dimensionless including the turning angle θ measured from the

horizontal.

θ	X_i^0	Z_i^0	X_i^1	Z_i^1	$\delta X_i = X_i^1 - X_i^0$
10	6.370899E+00	1.878913E-01	6.473048E+00	1.877906E-01	1.021495E-01
20	7.088938E+00	3.725727E-01	7.190491E+00	3.723202E-01	1.015530E-01
30	7.482803E+00	5.534333E-01	7.584074E+00	5.530519E-01	1.012707E-01
40	7.736273E+00	7.293671E-01	7.837381E+00	7.288730E-01	1.011076E-01
50	7.907096E+00	8.991226E-01	8.008104E+00	8.985295E-01	1.010079E-01
60	8.021362E+00	1.061444E+00	8.122310E+00	1.060765E+00	1.009474E-01
70	8.093408E+00	1.215118E+00	8.194321E+00	1.214365E+00	1.009130E-01
80	8.132228E+00	1.359000E+00	8.233124E+00	1.358187E+00	1.008959E-01
90	8.144063E+00	1.492035E+00	8.244955E+00	1.491173E+00	1.008921E-01
100	8.133610E+00	1.613264E+00	8.234505E+00	1.612365E+00	1.008949E-01
110	8.104640E+00	1.721843E+00	8.205542E+00	1.720919E+00	1.009016E-01
120	8.060350E+00	1.817048E+00	8.161259E+00	1.816109E+00	1.009083E-01
130	8.003560E+00	1.898287E+00	8.104473E+00	1.897341E+00	1.009130E-01
140	7.936833E+00	1.965099E+00	8.037745E+00	1.964154E+00	1.009121E-01
150	7.862556E+00	2.017167E+00	7.963460E+00	2.016227E+00	1.009040E-01
160	7.782981E+00	2.054314E+00	7.883867E+00	2.053382E+00	1.008863E-01
170	7.700258E+00	2.076507E+00	7.801117E+00	2.075582E+00	1.008592E-01
180	7.616453E+00	2.083857E+00	7.717274E+00	2.082935E+00	1.008205E-01

Table 4.11 Coordinates (X_i^0, Z_i^0) were calculated numerically using the Laplace program based on $B_0 = 1000$. Slightly perturbed profile, (X_i^1, Z_i^1) were calculated numerically using the Laplace program based on $\tilde{B}_0 = 105$. Values of $\tilde{Z}_i = Z_i^0 + \delta Z_i$ were determined using Eq. [4-59] to calculate δZ_i . A comparison of $\tilde{Z}_i$ (analytical Eq. [59]) with Z_i^1 (from the Laplace program with $\tilde{B}_0 = 1100$) shows that results are not significantly.

$$Error = \left| \left(\frac{\tilde{Z}_i - Z_i^1}{Z_i^1} \right) \times 100 \right| \text{ in units of percentage, all other quantities are dimensionless}$$

including the turning angle θ measured from the horizontal.

θ_i	Z_i^0	δZ_i	$\tilde{Z}_i = Z_i^0 + \delta Z_i$	Z_i^1	Error (%)
10	1.87891E-01	-9.66627E-05	1.87795E-01	1.87791E-01	2.12717E-03
20	3.72573E-01	-2.47930E-04	3.72325E-01	3.72320E-01	1.22354E-03
30	5.53433E-01	-3.73280E-04	5.53060E-01	5.53052E-01	1.45932E-03
40	7.29367E-01	-4.78892E-04	7.28888E-01	7.28873E-01	2.08964E-03
50	8.99123E-01	-5.66218E-04	8.98556E-01	8.98530E-01	2.98802E-03
60	1.06144E+00	-6.35238E-04	1.06081E+00	1.06077E+00	4.12706E-03
70	1.21512E+00	-6.85385E-04	1.21443E+00	1.21437E+00	5.51274E-03
80	1.35900E+00	-7.15930E-04	1.35829E+00	1.35819E+00	7.16510E-03
90	1.49204E+00	-7.26231E-04	1.49131E+00	1.49117E+00	9.09702E-03
100	1.61326E+00	-7.15842E-04	1.61255E+00	1.61237E+00	1.13274E-02
110	1.72184E+00	-6.84701E-04	1.72116E+00	1.72092E+00	1.39187E-02
120	1.81705E+00	-6.33217E-04	1.81642E+00	1.81611E+00	1.68642E-02
130	1.89829E+00	-5.62392E-04	1.89772E+00	1.89734E+00	2.02206E-02
140	1.96510E+00	-4.73874E-04	1.96463E+00	1.96415E+00	2.40152E-02
150	2.01717E+00	-3.70024E-04	2.01680E+00	2.01623E+00	2.82618E-02
160	2.05431E+00	-2.53893E-04	2.05406E+00	2.05338E+00	3.30229E-02
170	2.07651E+00	-1.29162E-04	2.07638E+00	2.07558E+00	3.83230E-02
180	2.08386E+00	-3.98860E-11	2.08386E+00	2.08294E+00	4.42399E-02

Table 4.9 and 4.11 demonstrates that the expression Eq. [4-59] acceptable accuracy when modelling the perturbation of large drop profile.

Figure 4.7 shows the dimensionless normal perturbation length distribution for large, flat drops of small contact angle and small curvature with B_0 ranging from 50 to 1000 as calculated using Eq. [4-59] and Eq. [4-60].

4.5 Discussion of the Normal Perturbation Distribution, λ

The analytical solution approach (applicable for small and large drops) permits us to obtain the normal perturbation length distribution, $\hat{\lambda}(B_0, \gamma, \theta)$, over a large range of B_0 values; for example $10^{-2} \leq B_0 \leq 10^3$. Investigating the variables of this function, shows that B_0, γ and θ have different influence on the perturbation length distribution.

Figure 4.8 illustrates the weak influence of γ when using a sessile drop with $\gamma = 72 \text{ mJ/m}^2$ and $B_0 = 2$ as the reference (unperturbed) original configuration. When γ is perturbed to 71, 60, 40, 30, 20, the corresponding $\Delta\gamma$ change to 1, 12, 32, 42, 52 but the perturbation length distribution, $\hat{\lambda}$, is not changed significantly. In contrast, Fig. 4.9 (using the same reference configuration as in Fig. 4.8) illustrates the strong influence of B_0 . When B_0 is perturbed to 2.01, 3, 10, 20 from $B_0 = 2$, the perturbation length distribution, $\hat{\lambda}$, has a very different shape in each case. Figure 4.10 illustrates the symmetric effect on the normalized perturbation length distribution $\hat{\lambda}$ of slightly increasing and slightly decreasing B_0 . Essentially one recovers similar distributions for

the both expanding and contracting drop situations. One may perturb a drop shape outward (expansion) by adding the $\hat{\lambda}$ distribution to the drop's profile or inward (contraction) by subtracting the same $\hat{\lambda}$ distribution from the drop's profile. Also, the $\hat{\lambda}$ value is strongly influenced by the turning angle θ or the arc-length position s . When B_0 is small, e.g. $0 < B_0 < 1$, a perturbation in B_0 causes the drop profile to move in a near-radial fashion, see Figs. 4.6 and 4.11. As B_0 increases, e.g., $1 < B_0 < 5$, small perturbations induce complicated profile shapes that are a combination of vertical and horizontal shifts, see Fig. 4.4 and 4.11. When B_0 is still larger, e.g. $B_0 > 5$, small perturbations cause very small vertical motion (against gravity) and mostly horizontal movement, see Figs. 4.7, 4.11.

In conclusion, the perturbation length distribution $\hat{\lambda}$ was strongly influenced by the size and shape of the drop, as characterized by the variables B_0 and θ or s .

4.6 Predicted Precision of Surface Tension using Perturbation

Distribution

The perturbation length distribution $\lambda = \lambda(B_0, \gamma, \theta \text{ or } s)$ may be used to predict the surface tension precision ability of numerical software packages like ADSA-P.

The ADSA-P program uses the coordinates of a drop profile as input to calculate the surface tension in a least-square sense. One theoretical profile is provided by the Laplace program, discussed above, which yields a set of profile coordinates (X_i^0, Z_i^0) . For the original profile, we know $B_0, \gamma, \theta_i \text{ or } s_{final}$. If the original profile is perturbed according to the scheme:

$$\tilde{X}_i = X_i^0 + \delta X \text{ with } \left| \frac{\delta X_i}{X_i^0} \right| \ll 1 \quad [4-61]$$

$$\tilde{Z}_i = Z_i^0 + \delta Z \text{ with } \left| \frac{\delta Z_i}{Z_i^0} \right| \ll 1 \quad [4-62]$$

boundary conditions apply such that $\delta X = 0$ at the apex point (to keep the profile axisymmetric) and $\delta Z = 0$ at the contact point (to keep the sessile drop in contact with the substrate). The perturbations δX_i and δZ_i are related to the normal (to the original profile) perturbation distance, $\delta \lambda_i$ via the relations

$$\delta X_i = \lambda_i \cos \theta_i \text{ and } \delta Z_i = \lambda_i \sin \theta_i \quad [4-63]$$

where $\lambda_i = \hat{\lambda}_i \lambda_{\max}$ and λ_i is obtained from a calculation that yields figures like Fig. 4.11.

Using the new coordinates for perturbed drop profile $(\tilde{X}_i, \tilde{Z}_i)$, we insert these values into the ADSA-P program and obtain a new ADSP-P based estimated of $\tilde{\gamma}$ for the perturbed profile. The surface tension change will be denoted as

$$\Delta \gamma = \tilde{\gamma} - \gamma \quad [4-64]$$

Thus, if we select an original drop shape profile by selecting B_0 , γ , and θ_i or s_{final} , then we may use the appropriate perturbation length profile $\lambda = \lambda(B_0, \gamma, \theta \text{ or } s)$ to perturb the profile coordinates and obtain a corresponding perturbation, $\tilde{\gamma}$, for the drop's surface tension. A sequence of λ functions corresponding to sequence of larger $\lambda_{\max}$ values will produce a sequence of $\tilde{\gamma}$ value and a corresponding range of $\Delta \gamma$. This relation between $\lambda_{\max}$ (the absolute maximum size of the drop's profile perturbation) and $\Delta \gamma$ (corresponding shift in surface tension) permits one to access surface tension precision as a function of profile coordinates precision. Figure 4.12 plots four sets of

surface tension precision, $\Delta\gamma$; that is $\pm 1 \text{ mJ/m}^2$, $\pm 0.5 \text{ mJ/m}^2$, $\pm 0.1 \text{ mJ/m}^2$ and $\pm 0.01 \text{ mJ/m}^2$ with corresponding absolute maximum perturbation length, $\lambda_{\max}$ as the function of the dimensionless radius of curvature B_0 .

4.7 The Relation between the Precision of the Surface Tension, $\Delta\gamma$, the Maximum Perturbation length, $\lambda_{\max}$, and the Drop Shape Dimensionless Radius, B_0 .

As stated in the previous sections, the surface tension change, $\Delta\gamma$, is a function of various parameters, such as B_0 , γ and the perturbation length distribution,

$\hat{\lambda} = \hat{\lambda}(B_0, \gamma, \theta \text{ or } \hat{s})$. The $\hat{\lambda}$ values in Fig. 4. 11 include the full range of possible sessile drop configurations up to and including a contact angle $\theta_t = \pi$ (or $\hat{s}_{final} = 1$). The value of the effective maximum physical perturbation length, λ_{eff} , occurs depends on the shape of the drop; see Fig. 4. 11. As most sessile drop systems occur for situations where $\theta_t < \pi$ it is necessary to define a factor, f , defined as

$$f = \max \left\{ \hat{\lambda} = \frac{\lambda}{\lambda_{\max}}; \text{ for } 0 \leq \hat{s} \leq \hat{s}_{final} \leq 1 \right\} \quad [4-65]$$

which addresses this situation. Factor f is restricted to the range $0 \leq f \leq 1$ because $0 \leq \hat{\lambda} \leq 1$ and it changes drastically with the location of the final arc-length position, $\hat{s}_{final}$ and the shape of drop, B_0 . The final arc-length value, $\hat{s}_{final}$ will correspond to a

$\theta_i < \pi$ and the effective maximum perturbation length for a particular situation will equal $f\hat{\lambda}_{\max}$; that is

$$\lambda_{eff} = \max\{\lambda, \text{for } 0 \leq s \leq s_{final} \leq 1\} = f\hat{\lambda}_{\max} \quad [4-66]$$

If we consider a sessile drop situation with $B_0 = 2$, then we may contrast the effect on factor f of changing the contact angle θ_i . When $\theta_i = \pi$ or $\hat{s}_{final} = 1$ (full theoretically possible range), then $f=1$, since λ_{eff} will equal $\lambda_{\max}$ somewhere within the range $0 \leq \hat{s} \leq 1$. In fact, $\lambda_{eff} = \lambda_{\max}$ or $\hat{\lambda} = 1$ occurs at $\hat{s} = 0.69$; see Fig. 4.11. In contrast, when $\theta_i = \pi/18$ for example, the λ_{eff} value (corresponding to a value of $\hat{\lambda} = 0.12 < 1$ occurs at $\hat{s} = 0.506$ where $f=0.12$. Consequently for this small contact angle drop the maximum perturbation length displacement of the drop's profile will occur at the contact line and it will be approximately $1/8$ th of the size that would occur for a similarly shaped drop (same B_0) with a much large contact angle. Alternatively, for a drop of a particular shape (B_0 fixed) we may interpret these results another way. Since each $\hat{\lambda}$ curve distribution, for fixed B_0 , is also a reflection of the surface tension change, $\Delta\gamma$, we may also conclude that it would be necessary to devise an experimental system that is eight times more sensitive to drop edge profile coordinate position when using small angle sessile drop ($\theta_i \approx \pi/18$) than large angle sessile drops ($\theta_i > 2/3\pi$); see Fig. 4. 13.

When B_0 is not fixed but the dimensionless arc-length point is fixed, say $\hat{s}_{final} = 0.1$, factor f changes. For example, when $B_0 = 5$, $f = 0.22$ whereas when $B_0 = 0.8$, $f = 0.96$.

These considerations would seem to imply that we may use λ_{eff} as a simple variable to show the relationship between the surface tension change or precision, $\Delta\gamma$, the maximum perturbation length, λ_{max} , and the drop shape, B_0 .

4.8 Criteria for Evaluating the Precision of the Optical System for the Measurement of Surface Tension

In previous sections, the perturbation model was shown to provide us with an ability to estimate the precision of numerical packages like ADSA-P for determining surface tension. As software packages, such as ADSA-P use drop profile coordinates to determine surface tension values, any uncertainties in those profile coordinates may cause surface tension error; that is, $\Delta\gamma$. In a physical sense, those uncertainties can be influenced by any experimental factor. Thus, the perturbation length of a drop profile, λ , may be used to relate experimental uncertainties to the surface tension precision, $\Delta\gamma$, see Fig. 4. 12.

When we speak about an experimental system or digital vision system with specified surface tension precision there are two questions that must be considered: (1) What is the surface tension precision level for a particular experimental system, e.g. a vision system, and (2) The inverse, if we require a certain surface tension precision, how do we select various components to achieve suitable parameters and satisfy the requirement that $\Delta\gamma$ be sufficiently small.

Recently, digital image technology has become very important in the study of interfacial tensions. In this section, we develop a criteria that may be used to assess the relationship

between the various parameters of a vision system, such as magnification, pixel size and CCD sensor array, etc. and the level of the surface tension precision.

In order to assist the reader, five possible situations are provided as concrete illustrations:

- (1) Situation 1: If we know the requirement for surface tension precision, $(\Delta\gamma)_{req}$, and the drop information, such as the maximum size of the drop $2X_{max}$ and either θ_l or s_{final} , then we determine the CCD sensor parameters, P (pixel size) and N (area array size), and the magnification of optical lens M to achieve $(\Delta\gamma)_{req}$.
- (2) Situation 2: If we know the camera information P and N , the magnification of the optical lens M , the drop information, such as $2X_{max}$ and either θ_l or s_{final} , then we can determine the best surface tension precision, $\Delta\gamma$, that is theoretically possible.
- (3) Situation 3: If given camera information P and N , and the requirement, $(\Delta\gamma)_{req}$, then we can select the magnification of optical lens M to satisfy the surface tension precision, $(\Delta\gamma)_{req}$.
- (4) Situation 4: If given camera information P and N , and the requirement, $(\Delta\gamma)_{req}$, then we select approximate drop size range to satisfy this requirement.
- (5) Situation 5: If given magnification of the optical lens, M , drop shape and the requirement of surface tension precision, $(\Delta\gamma)_{req}$, then we can select camera parameters, P and N , to achieve $(\Delta\gamma)_{req}$.

In each situation, a detailed algorithm is given whereby readers can easily use the criteria to select various parameters of a vision system to meet their requirements to estimate the level of surface tension precision that is possible. After each situation, practical examples (based on pure water sessile drop, see footnote 1, page 54) were provided to illustrate the design procedure.

There are six types of related parameters when using vision systems to determine surface tension.

- [1] The drop shape information, $2X_{\max}$, θ_l or s_{final} . Variable $2X_{\max}$ is the maximum potential length for the drop. It is anticipated that a sessile drop will have its maximum length in the horizontal direction (perpendicular to the gravitational field vector) while a pendant drop will have its maximum length in the vertical direction (parallel to the gravitational field vector). In addition, we need an estimate for either θ_l or the maximum axisymmetric arc-length distance, s_{final} , measured from the apex origin.
- [2] The level of surface tension precision that is required, $(\Delta\gamma)_{req}$.
- [3] The magnification of the optical lens system, M .
- [4] CCD sensor parameters, P and N ; where P represents the maximum actual pixel size for a pixel dimension $P_1 \times P_2$ such that $P = \max(P_1, P_2)$. The variable N represents the minimum number of pixels in either row or column of CCD area array of dimension $N_1 \times N_2$ such that $N = \min(N_1, N_2)$.
- [5] Drop edge precision, $|\pm N_v|$, in units of pixels.

[6] The perturbation length distribution, λ and the factor f to characterize the extent of the distribution since $\theta_i < \pi$ (or $\hat{s} < 1$) in most case. In the situation considered below we set λ_{eff} to the largest effective maximum using the relation $\lambda_{eff} = f\lambda_{max}$. SI units for λ_{max} and λ_{eff} are the meter.

The precision of a vision system can be described by following equations

$$N_c = \frac{\lambda_{eff} M}{P} \quad [4-68]$$

where the lateral or transverse magnification M is defined by (12)

$$M = \text{Image Size} / \text{Object Size} = \frac{PN}{2X_{max}} \quad [4-69]$$

and N_c is the criteria of precision for the surface tension (discussed below).

After substituting Eq. [4-69] into Eq. [4-68], we obtain

$$N_c = \frac{\lambda_{eff} N}{2X_{max}} \quad [4-70]$$

Thus, if the precision of the drop's edge is given as $|\pm N_v|$, then the criteria for evaluating the precision of the optical system for the surface tension measurement will be

$$N_c \geq |\pm N_v| \quad [4-71]$$

Discussion of Criteria

As we know, a CCD area array camera has a photosensitive surface composed of tiny light sensitive regions called pixels. These pixels maybe square ($P_1 = P_2$) or they may

have an aspect ratio P_1/P_2 that is not equal to unity. Common CCD cameras have area arrays of 640 pixels by 480 pixels with a typical pixel size of $13\ \mu m$. Cameras with greater resolution are possible and many research level cameras have array sizes of 2K by 2K with individual pixels around $6.5\ \mu m$. An individual pixel is the smallest unit of resolution on a CCD array and all image recorded by the CCD camera are binned into a two dimensional array of pixels.

Consequently, the determination of the edge of a drop is achieved subject to this pixel array and the precision to which the pixels permit the edge to be positioned. In many cases the size of the edge crosses several pixels and the edge is set as the location (or pixel) closest to the point where the intensity gradient is a maximum. While some authors have suggested that the intensity information may be used to achieve sub-pixel resolution, we feel that it is more prudent to set the lower limit of resolution at one pixel. During image capture by the CCD camera, frame grabber, etc. every component of vision system may introduce distortions to image. These may include, for example, lens distortion, blur, blooming, CCD sensor noise and frame grabber edge detection error. In general, the edge detection precision of a vision system, $|\pm N_v|$, will be influenced by many complicated effects and parameter $|\pm N_v|$ must include all the errors for every component of the vision system. Figure 4.14 shows a simplified vision system. Possible error sources are:

1. Software sub-system

- Error from operating system and related software

- Error due to the precision of edge detection algorithms that is used by image processing software.

2. Optical sub-system

- Blur
- System noise
- Random noise
- Aspect ratio
- Scaling

3. Vibration control sub-system

- Error due to vibration

4. Handling and position control sub-system

- Leveling
- Vertical misalignment of the camera

5. Frame grabber in PC-subsystem

- Precision of frame grabber

6. Light sub-system

- Type of lighting

Fig. 4. 14 A simplified vision system

Thus, it will be assumed that $\min(|\pm N_v|) = 1$ where $|\pm N_v|$ is the detectable edge precision in number of pixels for the vision system. If the precision to which the edge of a drop can be estimated is 2 pixels, then we set $|\pm N_v| = 2$ pixels as the typical vision system position uncertainty in pixels.

The criteria, N_c , for evaluating the precision of the optical system for a surface tension measurement will then be determined by the requirement that $N_c \geq |\pm N_v|$ for all possible combinations of P , N , M , $(\Delta\gamma)_{req}$ and drop shape.

We may understand the criteria N_c in a physical sense by considering the ratio

$\lambda_{eff} M / P$ when $M=1$ (no magnification), and the total vision system error $|\pm N_v| = 1$. In this situation, $N_c = \lambda_{eff} / P$. Recognizing that the smallest CCD unit in the sensor array is the pixel of size P , we conclude that the optical system will not distinguish any dimension smaller than the unit pixel size and that the criteria for detecting a perturbation of the drop's edge must be $\lambda_{eff} / P \geq 1$ or $N_c \geq 1$. If $\lambda_{eff} / P < 1$, then the vision system will

not sense the perturbation to the drop's edge. The following section illustrates five possible situations.

- Situation 1: If we know the requirement for surface tension precision, $\Delta\gamma$, and the drop shape information, such as $2X_{\max}$ and either θ_i or s_{final} , then to determine the CCD sensor parameters, P and N , and the magnification of optical lens M , we proceed as follow:

- Example 1

If given the requirement for precision of the surface tension precision as

$(\Delta\gamma)_{req} \approx 0.1 \frac{mJ}{m^2}$, drop shape information as $2X_{max} \approx 8 mm$ with $\hat{s}_{final} = 1$ (physically,

this means the contact angle $\theta_i = \pi$ if the system is a sessile drop), and the precision of

the vision system as $|\pm N_v| = 3$, then we determine the parameters of the optics system. P ,

N and M as follows:

Step 1. Based on $2X_{max} \approx 8 mm$, we estimated $B_0 = 2$

Step 2. Based on Fig. 4.12, for given $\Delta\gamma$ and B_0 , obtain $\lambda_{max} = 3.46 \times 10^{-5} m$

Step 3. Based on Fig. 4.11 with given $\hat{s} = 1$, we obtain $f = \max[\hat{\lambda}] = 1$, so

$$\lambda_{eff} = f\lambda_{max} = 3.46 \times 10^{-5} m$$

Step 4. Using Eq. [4-70] and Eq. [4-71]

$$N \geq \frac{2X_{max}}{\lambda_{eff}} \times |\pm N_v| \approx 690$$

Step 5. Using Eq. [4-68] and Eq. [4-71]

$$M \geq \frac{P}{\lambda_{eff}} \times |\pm N_v|$$

which yields $M \geq 0.87$ for the most common pixel size $P \approx 10 \mu m$. If $P \approx 20 \mu m$

then $M \geq 1.73$.

In conclusion, we see that a surface tension precision of $(\Delta\gamma)_{req} \approx 0.1 \text{ mJ/m}^2$ is possible for a droplet (either sessile drop or pendant drop) with a maximum size of $2X_{\max} \approx 8 \text{ mm}$ when the uncertainty is $|\pm N_v| \leq 3$ pixels provided the magnification, $M \geq 0.87$, and the CCD camera has both array dimension greater than 690 pixels and the pixel size is $P \approx 10 \mu\text{m}$.

- Situation 2.

If we know the camera information P and N , and the magnification of optical lens M , the drop information, such as $2X_{\max}$ and either θ_l or s_{final} , then we can determine the best surface tension precision, $\Delta\gamma$, possible as follow:

- Example 2

If given $P = 10\mu m$, $N = 1000$, $M = 5$, and the precision of the vision system as $|\pm N_v| \leq 3$.

then for a droplet representing water at room temperature we can determine the best possible surface tension precision, $\Delta\gamma$, as follows:

Step 1. Using Eq. [4-69], the minimum drop size would be estimated as

$$\min(2X_{\max}) = \frac{PN}{\max(M)} = \frac{10^{-5} \times 1000}{5} = 2 \times 10^{-3} m$$

For $\min(2X_{\max}) \approx 2 \times 10^{-3} m$, we obtain $R_0 \approx 1 \times 10^{-3} m$ and $B_0 = \sqrt{c} R_0 \approx 0.4$

Step 2. Using Eq. [4-70] and Eq. [4-71]

$$\lambda_{eff} \geq \frac{2X_{\max} |\pm N_v|}{N} \approx 6 \times 10^{-6} m$$

Step 3. Once again, if we assume that $\hat{s}_{final} = 1$, and consequently $f=1$, then based on Fig.

4.11,

$$\lambda_{\max} = \frac{\lambda_{eff}}{f} = \lambda_{eff}$$

Step 4. Based on Fig. 4.12, for given $B_0 \approx 0.4$ and $\lambda_{\max} \approx 6 \times 10^{-6} m$, we estimate the surface tension precision is to be in the range $0.05 \frac{mJ}{m^2} < \Delta\gamma < 0.1 \frac{mJ}{m^2}$.

- Situation 3

If given camera information P and N , and the requirement of $(\Delta\gamma)_{req}$, then we can select the magnification of the optical lens M to satisfy the requirement of surface tension precision, $(\Delta\gamma)_{req}$ as follow:

- Situation 4

If given camera information P and N , and the requirement of $(\Delta\gamma)_{req}$, then we select approximate drop size to satisfy the surface tension precision requirement $(\Delta\gamma)_{req}$.

- Example 3

If the camera information is already known: for example $P = 10\mu m$ and $N=500$, with the requirement for surface tension precision set at $(\Delta\gamma)_{req} \approx 0.1 \frac{mJ}{m^2}$, initial drop shape information, $2X_{max} \approx 8mm$, $\hat{s}_{final} = 0.14(\theta_i = 10^0)$ for sessile drop, and the precision of the vision system $|\pm N_v| \leq 2$, then we may determine the parameters of the optical system; that is, M as follows:

Step 1. Based on $2X_{max} \approx 8mm$, we estimated $B_0=2$

Step 2. Using Eq. [4-70] and Eq. [4-71], we obtain estimate

$$\lambda_{eff} \geq \frac{2X_{max}|\pm N_v|}{N} \approx 3.2 \times 10^{-5} m$$

Step 3. Based on Fig. 4.11, given $\hat{s}_{final} = 0.14$, obtain $f = \max[\hat{\lambda}] = 0.485$, so

$$\lambda_{max} = \frac{\lambda_{eff}}{f} = 6.6 \times 10^{-5} m$$

Step 4. Based on Fig. 4.12, for given λ_{max} and B_0 , we obtain inequality for

$\Delta\gamma > 0.1 \frac{mJ}{m^2} > (\Delta\gamma)_{req}$ which is does NOT satisfy criteria. Because the criteria

$\lambda_{max} \leq 3.46 \times 10^{-5} m$. Therefore, this CCD vision system will NOT be able to

deliver the required surface tension precision, $\Delta\gamma \leq 0.1 \frac{mJ}{m^2}$. The reason this

occurs may be seen by using the criteria. From Fig. 4.12, if

required $(\Delta\gamma)_{req} \approx 0.1 \frac{mJ}{m^2}$ when $B_0 = 2$, then $\lambda_{max} \approx 3.46 \times 10^{-5} m$, so

$\lambda_{eff} = f\lambda_{max} = 1.68 \times 10^{-5} m$. If we use Eq. [4-70] and Eq. [4-71], we obtain

$$N \geq \frac{2X_{\max}}{\lambda_{eff}} \times |\pm N_v| \approx 480|\pm N_v|$$

so that $N \geq 960$, when $|\pm N_v| = 2$

Consequently, a digital CCD camera with an array dimension of just 500 pixels is too coarse to permit surface tension measurements to be performed at the level of precision required. One possible solution is to increase N to 1200 to satisfy the criteria. If we repeat Steps 1 to Step 4 with $N=1200$, we obtain

$$\lambda_{eff} \geq \frac{2X_{\max}|\pm N_v|}{N} \approx 1.33 \times 10^{-5} m$$

whereupon

$$\lambda_{\max} = \frac{\lambda_{eff}}{f} = 2.74 \times 10^{-5} m$$

Thus, based on Fig. 4.12, for given $\lambda_{\max} = 2.74 \times 10^{-5} m \leq 3.46 \times 10^{-5}$ and $B_0 = 2$, we

obtain $\Delta\gamma < 0.1 \frac{mJ}{m^2} = (\Delta\gamma)_{req}$ which satisfies the surface tension precision requirement.

For this $N=1200$ situation, the magnification M will be:

$$M \approx \frac{P}{\lambda_{eff}} \times |\pm N_v| \approx 1.5$$

Another possible solution is to change the final arc-length position of the pendant drop or the contact angle of sessile drop. This alternative is a more difficult to achieve in most practical situations but is included here to illustrate the effect of the f parameters. If

given $\hat{s}_{final} = 0.56[\theta_l = 50]$ for sessile drop, then $f = \max[\hat{\lambda}] = 0.92$ and

$$\lambda_{eff} \geq \frac{2X_{\max}|\pm N_v|}{N} \approx 3.2 \times 10^{-5} m$$

and

$$\lambda_{\max} = \frac{\lambda_{\text{eff}}}{f} = 3.48 \times 10^{-5} m$$

Using Fig 4.12, for given $\lambda_{\max} = 3.48 \times 10^{-5} m$ and $B_0 = 2$, we obtain a $\Delta\gamma$ that is very close to $0.1 \frac{mJ}{m^2} = (\Delta\gamma)_{\text{req}}$. Because the criteria of $\lambda_{\max}$ for $\Delta\gamma \approx 0.1 \frac{mJ}{m^2}$ is $3.46 \times 10^{-5} m$. Thus, the minimum CCD array size, N , may be estimated as

$$N \approx \frac{2X_{\max}}{\lambda_{\text{eff}}} \times |\pm N_v| = 500$$

whereupon, a CCD camera with a minimum array of 500 pixels will be able to yield surface tension measurements at the precision $\Delta\gamma \leq 0.1 \frac{mJ}{m^2}$ when the pixel uncertainty is $|\pm N_v| \leq 2 \text{ pixels}$. Subsequently, the resulting best magnification may be estimated as,

$$M \approx \frac{P}{\lambda_{\text{eff}}} \times |\pm N_v| \approx 0.63$$

If we improve the vision system precision $|\pm N_v|$ to one pixel, then

$$\begin{aligned} N &= 500 \\ \lambda_{\text{eff}} &\geq \frac{2X_{\max} |\pm N_v|}{N} \approx 1.6 \times 10^{-5} m \\ \lambda_{\max} &= \lambda_{\text{eff}} / f = 3.3 \times 10^{-5} m \end{aligned}$$

Thus, based on Fig. 4.12, for given $\lambda_{\max} = 3.3 \times 10^{-5} m \leq 3.45 \times 10^{-5}$ and $B_0 = 2$, we obtain

$\Delta\gamma < 0.1 \frac{mJ}{m^2} = (\Delta\gamma)_{\text{req}}$ which satisfies the surface tension precision requirement.

- Example 4

If the camera information is already known; for example $P = 10\mu m$ and $N=1200$, with the requirement of surface tension precision set at $(\Delta\gamma)_{req} \approx 0.01 mJ/m^2$, drop shape information, $2X_{max} \approx 8mm$, $\hat{s}_{final} = 0.27(\theta_l \approx 30)$ for sessile drops, and the precision of the vision system set at $|\pm N_v| \leq 2$ pixels, then we may determine the parameters of the optical system.

Step 1. Based on $2X_{max} \approx 8mm$, we estimated $B_0 = 2$

Step 2. Using Eq. [4-70] and Eq. [4-71], we obtain

$$\lambda_{eff} \geq \frac{2X_{max}|\pm N_v|}{N} \approx 1.33 \times 10^{-5} m$$

Step 3. Based on Fig. 4.11, if given $\hat{s} = 0.27$, we obtain $f = \max[\hat{\lambda}] = 0.6$, so

$$\lambda_{max} = \frac{\lambda_{eff}}{f} = 2.22 \times 10^{-5} m$$

Step 4. Based on Fig. 4.12, for given λ_{max} and B_0 , we obtain

$$\Delta\gamma > 0.01 mJ/m^2 = (\Delta\gamma)_{req} \text{ which indicates that this system will not satisfy the}$$

criteria. Because the criteria of λ_{max} for $\Delta\gamma \approx 0.1 mJ/m^2$ is $7.0 \times 10^{-6} m$.

As illustrated above, one solution is to determine the minimum N required. From Fig.

4.12, for given $\delta\gamma$ and B_0 , so obtain $\lambda_{max} = 7 \times 10^{-6} m$, So

$$\lambda_{eff} = f\lambda_{max} = 4.2 \times 10^{-6} m$$

If we use Eq. [4-70] and Eq. [4-71], we can estimate this value as

$$N \geq \frac{2X_{\max}}{\lambda_{eff}} \times |\pm N_v| = 3800$$

which indicates that $N=1200$ is too small to satisfy the criteria when $|\pm N_v| \leq 2$ pixels.

In the current CCD sensor market, a large CCD sensor array of this size at least

3800×3800 is expensive. In order to get high surface tension precision,

$(\Delta\gamma)_{req} \approx 0.01 \text{ mJ/m}^2$, one possible solution is to reduce $|\pm N_v|$. For example,

when $|\pm N_v| = 1$, then the criteria for N is reduced to $N \approx 2000$ and it is possible to get a

$2K \times 2K$ CCD sensor array. When $|\pm N_v| = 0.5$, then the criteria for N is reduced to

$N \approx 1000$. If using a $2K \times 2K$ CCD sensor array with $|\pm N_v| = 1$, based on Eq. [4-70] and

Eq. [4-71], we obtain

$$\lambda_{eff} \geq \frac{2X_{\max} |\pm N_v|}{N} = 4.0 \times 10^{-6} \text{ m}$$

$$M \approx \frac{P}{\lambda_{eff}} \times |\pm N_v| \approx 2.5$$

as the required magnification that is necessary to achieve the surface tension precision.

The conclusion is the vision system precision influences the parameters of optical system dramatically.

- Example 5

If the camera information is already known, for example $P = 10\mu m$ and $N = 1200$, with the requirement of surface tension precision set at $(\Delta\gamma)_{req} \approx 0.01 \frac{mJ}{m^2}$, drop shape information, $2X_{max} \approx 13mm$, $\hat{s}_{final} = 0.5$, the precision of vision system set at $|\pm N_v| \leq 2$, we may determine the parameters of the optical system as follows:

Step 1. Based on $2X_{max} \approx 13mm$, we obtain $B_0 = 5$

Step 2. Based on Fig. 4.12, for given $\Delta\gamma$ and B_0 , we obtain $\lambda_{max} = 1.45 \times 10^{-5} m$

Step 3. Based on Fig. 4.11, if given $\hat{s} = 0.5$, obtain $f = \max[\hat{\lambda}] = 0.52$, so

$$\lambda_{eff} = f\lambda_{max} = 0.754 \times 10^{-5} m$$

Step 4. Using Eq. [4-70] and Eq. [4-71], we estimate

$$N \geq \frac{2X_{max}}{\lambda_{eff}} \times |\pm N_v| \approx 3450$$

Consequently, a digital CCD camera with an array dimension of just 1200 pixels is too coarse to permit surface tension measurements to be performed at the level of precision required.

But if $|\pm N_v| = 1$ and $\hat{s} = 1$. Thus,

$$\lambda_{eff} = f\lambda_{max} = 1.45 \times 10^{-5} m$$

$$N \geq \frac{2X_{max}}{\lambda_{eff}} |\pm N_v| \approx 900$$

It is possible for a digital CCD camera with an array dimension of 1200 pixels to achieve

$(\Delta\gamma)_{req} \approx 0.01 \frac{mJ}{m^2}$. This also illustrated the vision system precision influences the parameters of optical system dramatically.

- Situation 5

If given the magnification of the optical lens, M , drop shape and the requirement for the surface tension precision, $(\Delta\gamma)_{req}$, then we can select camera parameters, P and N , to achieve $(\Delta\gamma)_{req}$ as follows:

- Example 6

If given the requirement for the surface tension precision as $\Delta\gamma \approx 0.1 \text{ mJ/m}^2$, drop information, $2X_{\max} \approx 8 \text{ mm}$, $\hat{s}_{final} = 0.27[\theta_t \approx 27.5^\circ]$ for a sessile drop, $M=5$, the precision of the vision system $|\pm N_v| \leq 3$, we can determine the parameters of the optical system, P and N

Step 1. Based on $2X_{\max} = 7.96 \text{ mm}$, we estimate $B_0 = 2$

Step 2. Using Fig. 4.12, for given $\Delta\gamma$ and B_0 , we obtain $\lambda_{\max} = 3.46 \times 10^{-5} \text{ m}$

Step 3. Using on Fig. 4.11, if given $\hat{s}_{final} = 0.27$, we obtain $f = \max[\hat{\lambda}] = 0.6$, so

$$\lambda_{eff} = f\lambda_{\max} = 2.1 \times 10^{-5} \text{ m}$$

Step 4. Using Eq. [4-70] and Eq. [4-71], we obtain

$$N \geq \frac{2X_{\max}}{\lambda_{eff}} \times |\pm N_v| \approx 1140$$

Step 5. Using Eq. [4-68] and Eq. [4-71], we estimate the pixel size as

$$P \leq \frac{M\lambda_{eff}}{|\pm N_v|} \approx 35 \mu\text{m}$$

Inclusion, for these requirements: that is, $(\Delta\gamma)_{req} \leq 0.1 \text{ mJ/m}^2$; $M=5$; $2X_{\max} \approx 8 \text{ mm}$,

$\hat{s} = 0.27$ and $|\pm N_v| \leq 3$, we can see that $N \geq 1140$ and $P \geq 35 \mu\text{m}$ is indicates that this

system will satisfy the criteria.

If we compared with, if $\hat{s}_{final} = 1[\theta_i = 180]$ for a sessile drop. Based on Fig. 4.11, if

given $\hat{s}_{final} = 1$, obtain $f = \max[\hat{\lambda}] = 1$, then $\lambda_{eff} = f\lambda_{max} = 3.46 \times 10^{-5} m$. Using Eq. [4-70]

and Eq. [4-71], we obtain

$$N \geq \frac{2X_{max}}{\lambda_{eff}} \times |\pm N_v| = 230|\pm N_v| \approx 690$$

Then using Eq. [4-68] and Eq. [4-71], we obtain

$$P \leq \frac{M\lambda_{eff}}{|\pm N_v|} \approx 58 \mu m$$

Consequently, we can compare the two answers at different situation; when $\hat{s}_{final} = 1$,

$N \geq 690$ and $P \leq 58 \mu m$; when $\hat{s}_{final} = 0.27$, $N \geq 1140$ and $P \leq 35 \mu m$. The conclusion is the drop size influences the parameters of optical system dramatically.

4.9 Conclusions

The concept of a normal perturbation length distribution, $\lambda = f(B_0, \gamma, \theta \text{ or } s)$, was developed to relate experiment uncertainties to the surface tension precision. This normal perturbation distribution is strongly influenced by the drop shape; that is, the initial B_0 value for the drop and the arc-length position s or tuning angle θ . Based on this normal perturbation length distribution, a criteria was developed to evaluate the accuracy of surface tension values obtained for axisymmetric drop systems (e.g. either sessile or pendant drops) using a digital vision system. The criteria included the drop shape information, $2X_{max}$, θ_i or s_{final} , the level of surface tension precision that is required,

$(\Delta\gamma)_{req}$, the magnification of the optical lens system, M , the CCD sensor parameters, P and N , the drop edge precision, $|\pm N_e|$ and the perturbation length distribution, λ . Using this criteria, we can either estimate the surface tension precision level for a particular experimental system or we can select various system components to achieve suitable parameters and satisfy a pre-defined surface tension accuracy.

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Chapter 5

Design of a Digital Vision System for Surface Science

As we know, the image processing technique is a very useful tool for drop analysis. In this Chapter, a new digital vision system was designed for our surface tension experiment.

Again, from Chap. 2, we know the requirements for a vision system is the first step of the design process. However, everything is not just a simply one step process, rather, it is an iteration. It is hard to list all detailed requirements at once because any original idea may be reasonable or it may not. Thus, in the design processing, we repeatedly find the meeting point between the requirement of our experiment with the vision system. This Chapter gives the outline of design, emphasis the **SYSTEM** concept of vision system.

5.1 General goal

Our primary interesting is to set-up a digital vision system to measure surface tension, and subsequently to measure fluid flow features of surface systems. As we know, we can use the geometric profile of the drop to get the value of liquid's surface tension. In Chap. 3, we described related theory and information on the use of image processing techniques in drop analysis. The following are key requirements:

- **High speed**

To capture a sequence images of drop movement, the system should have enough speed for taking pictures at the required rate.

- **High resolution**

It should also have enough resolution. In Chap. 4, the relation between the accuracy of surface tension and the precision of the vision system was described.

- Flexible

The vision system should be flexible for different purposes.

- Stable

It is obvious that a liquid drop is very sensitive to any vibration, so that to ensure the precision of experimental results, stability of vision system is critical.

- User friendly

Experimental surface science is very complex and delicate, so the vision system was designed to be easy to operate.

5.2 Identify variation

In this section, we will define several requirements for any experiments.

5.2.1 Image

The quality of the image is central to the vision system's performance. Thus, initial requirements involve the image.

A) Image representation

To obtain a surface tension value from a drop our first concern is the shape of drop.

The colour of the drop is not interesting, so a monochrome image is enough. In addition, monochrome images will require less computer resources, such as memory, CPU time, and storage space at equal resolutions compared with colour images. Thus, monochrome images can yield higher resolutions for the same computer resources.

B) Image resolution

Of course, one wishes to achieve the highest possible resolution but, from the Chap. 4, error analysis, we know, for a certain precision of surface tension, a certain image resolution will adequately meet the requirement.

C) Image acquisition speed

As the resolution, higher speed is better.

D) Image format

After we process the image file and acquire a drop edge profile file, the coordinates of every point on the profile will be output to the ADSA-P program. Consequently, the profile of image needs to be output as an ASCII text file format.

E) Image processing

Edge detection will be one of the most important image processing operations. Other process will be to smooth out image noise, calibration and to generate an intensity histogram, etc.

5.2.2 Opto-mechanical system

1. Fine position control

The experiment system was required to be highly flexible and movable. Comparing with vision systems in industry, our experiment has multiple missions. As pendant drops and sessile drops will be used in this experiment system the vision system was asked to be flexible and to change position to meet several requirements. Pendant drops will be photographed from the side view, while the sessile drops will be from either top view or side view.

From Chap. 4. we know, the accuracy of position control, such as vertical alignment is very important in surface tension measurement so the vision system should be able to insure very precise position.

2. Vibration isolation

Vibration is one of the critical points in most experiments, the drop is very small, 0.1 mm ~ 1 cm, and drop will spread on the surface when forced by vibration. Thus, vibration will effect the stability of the drop and the velocity of drop flow (spreading).

3. Leveling

For the sessile drop, if the platform that supports the drop is inclined, then the drop will flow due to gravity. Thus, a leveling adjustment was a basic requirement. For the pendant or hanging drop the tube supplying the drop's liquid must also be oriented vertically so that the drop hangs evenly from the tube.

5.2.3 Software

I. Development

It is very important that users are able to develop their own user-friendly application software. The goal of thesis was to develop an integrated software package that was specially designed for our whole experiment. This software integrated all functions and processing of experiment.

a) Image capturing and storage

- Configure vision system.
- Capture image.
- Save data.

- Image improvement, such as noise reduction.

b) Image analysis and measurement

- Edge detection.
- Output profile to ADSA-P program.
- Get surface tension or any required features.
- Output graphics that users wanted.

It should also be able to be re-usable, to use the windows GUI resource and easy to learn how to develop. The software should also have a long life-circle and be easy to update.

II. Easy to use

The Software should be easy to use with regards to:

- Modify the configuration of vision system in real time.
- User-friendly GUI to do all operations, such as image acquisition, image processing and image output

III. Operating system

OS system has following requirements.

- OS should be stable.
- Meet requirements for development.
- Suitable to support most popular software and support most computer language such as FORTRAN, C, etc.

5.3 Protocol design

1. System picture

Figure 5.1 is a prototype of a digital vision system that is based on the requirements of vision system and our experimental requirements.

Fig. 5.1

5.4 Detailed Design

5.4.1 Optical sub-system

1. Camera - CA-D4, Dalsa Inc., Canada

The CA-D4 is a non-standard, high speed and high resolution CCD digital video camera.

The main specifications are as following:

Resolution	1024 * 1024
Pixel dimension	12 μm
Pixel ratio	1:1
Frame rate	40 frame/sec (max.)
Data output	8 bits, 12 bits

2. Lens- AF Zoom-Micro Nikkor 70-180mm f/4.5-5.6 D ED, Nikon Corp., Japan

This micro-lens provide close-up photograph without compromise. The ability of zooming into an object will permit easy adjustment of perspective and reproduction ratio.

3. Microscope, SZ-STU2, Olympus, USA

The maximum magnification with the current lens is 70.

4. Evaluate optical sub-system

As we know, the parameters of the Dalsa CA-D4 CCD camera are 1024×1024 pixel resolution with $12\mu\text{m}$ square pixels. We assume the precision of the vision system as

$|\pm N_v| \leq 3$ (coarse) and a drop size of about $R_0 \approx 1 \times 10^{-3} \text{ m}$. Thus, for a droplet

representing water at room temperature we can estimate the best possible surface tension precision, $\Delta\gamma$.

Step 1. For $R_0 \approx 1 \times 10^{-3} \text{ m}$, we may estimate $2X_{\max} \approx 2 \times 10^{-3} \text{ m}$, and $B_0 = \sqrt{c}R_0 \approx 0.4$

Using Eq. [4-69], the magnification would be estimated as

$$\max(M) = \frac{PN}{2X_{\max}} = \frac{12 \times 10^{-6} \times 1024}{1.0 \times 10^{-3}} \approx 1.23$$

Step 2. Using Eq. [4-70] and Eq. [4-71]

$$\lambda_{\text{eff}} \geq \frac{2X_{\max} |\pm N_v|}{N} \approx 6 \times 10^{-6} \text{ m}$$

Step 3. Once again, if we assume that $\theta_l = 120^\circ$, then based on Fig. 4-11, $f=0.71$,

$$\lambda_{\max} = \frac{\lambda_{eff}}{f} \approx 8.6 \times 10^{-6} m$$

Step 4. Based on Fig. 4.12, for given $B_0 \approx 0.4$ and $\lambda_{\max} \approx 8.6 \times 10^{-6} m$, we estimate the surface tension precision to be in the range $0.05 mJ/m^2 < \Delta\gamma < 0.1 mJ/m^2$.

If the precision of the vision system $|\pm N_v|$ is better, meaning $|\pm N_v| \leq 2$ pixels, or worse, meaning $|\pm N_v| \geq 3$ pixels, then we would need to repeat step 2 through 4. For example, if $|\pm N_v| \approx 1$ pixels, then by using Eq. [4-70] and Eq. [4-71], we obtain

$$\lambda_{eff} \geq \frac{2X_{\max}|\pm N_v|}{N} \approx 2 \times 10^{-6} m$$

when $\theta_l = 120^\circ$, based on Fig. 4-11, $f=0.71$ and

$$\lambda_{\max} = \frac{\lambda_{eff}}{f} \approx 3 \times 10^{-6} m$$

Based on Fig. 4.12, for given $B_0 \approx 0.4$ and $\lambda_{\max} \approx 3 \times 10^{-6} m$, we estimate the surface tension precision to be in the range $0.01 mJ/m^2 < \Delta\gamma < 0.05 mJ/m^2$.

If the precision of the vision system $|\pm N_v|$ was increased further to one-half pixel,

$|\pm N_v| = 0.5$, we repeat Steps 2 to 4 to obtain

$$\lambda_{eff} \geq \frac{2X_{\max}|\pm N_v|}{N} \approx 1 \times 10^{-6} m$$

and with $\theta_l = 120^\circ$, based on Fig. 4.11, $f=0.71$, so

$$\lambda_{\max} = \frac{\lambda_{eff}}{f} \approx 1.4 \times 10^{-6} m$$

Whereupon, based on Fig. 4.12, for given $B_0 \approx 0.4$ and $\lambda_{\max} \approx 1.4 \times 10^{-6} m$, we

estimate the surface tension precision as close to $\Delta\gamma \approx 0.01 mJ/m^2$.

5.4.2 PC sub-system

The PC subsystem is an IBM's compatible Desktop Computer with Intel Pentium II processor. In order to obtain a higher data storage rate at reasonable cost, a SCSI system was installed.

Table 5.1 Outline of PC-based subsystem

Component Name	Type	Speed	Feature	Vendor
CCD Camera	CA-D4	20-40 frame/sec	Resolution 1024*1024	Dalsa, CA
Frame grabber	Genesis-LC	130MHz(Data transformation)	Resolution 1600*1200	Matrox, CA
Mother board	PC4L	100MHz(Local PCI bus)		Ausa, Taiwan
CPU	PII	300MHz		Intel, USA
S C S I	Backup	18 GB HD	20-40MHz	Seagate, USA
		Zip		Iomega, USA
		LS 120M	120 M floppy	
	HD	HD 18GB	20-40MHz	Seagate, USA
Memory	SDRAM	100MHz	256M	

In the design of a PC-based subsystem, one challenge is with the balancing of bus speed with the data transfer rates of the other devices (e.g. CCD camera, etc.). As we mentioned before, the vision system should have enough speed to capture sequences of images. The camera generate data at a high rate to meet this requirement. For instance, if the rate is a maximum of 30 frame/second at resolution of 1024*1024 with 8 bit depth, then for every second of real time the system will have 30 M of image data generated and ready for storage. It is impossible to use a conventional PC for real time image processing, or to save data to the hard drive. Even for a SCSI system, the maximum speed of storing data to hard drive is at 25 MHz which is less than the 30MHz data stream generated by the CCD camera. Of course, there are some techniques to solve this problem, such as array processor, concurrent processing, etc. These approaches are expensive and not usable as a solution for a moderately priced vision system. To partially solve this problem, based on a PC-system, we installed 256 M of SDRAM.

Fig 5.3 Solution for PC system speed balance

From Fig. 5.3 which shows the data pathway, it is clear that the bus speed between the components are unequal. If every image is at resolution of 1024×1024 with 8 bit data depth, then the 256 M SDRAM can acquire approximately $200/30 = 8$ seconds of continuous images. The reason the usable RAM is less than 256M is that some memory is needed for operating system.

- Frame grabber – Genesis-LC, Matrox, CA

Genesis-LC is a frame grabber with a programmable processor that includes its own graphics, signal or general-purpose processor to enhance image-processing capabilities. Standard tools such as C-compilers and development tool kits are available to program the boards in a much more flexible and portable way.

Matrox Genesis-LC writes image data directly to on-board memory and, in parallel, transfers it in real-time over the PCI bus to system RAM. This results in simultaneous display and processing of the image. Two monitors are used in the system, one controlled by the Genesis-LC to display the captured image and one controlled by the Matrox Millenium II to display the control system. The Matrox Genesis-LC takes full advantage of Matrox's image processing software package as detailed below.

5.4.3 Software sub-system – Matrox Image Processing Package, Matrox, CA

Software comes from Matrox, Montreal, Canada. The package includes IntellCam 2.0, Inspector 2.2, MIL 5.01 and ActiveMilLite 1.0 and Genesis Native Library 2.0.

- **Structure**

Figure 5.4 demonstrated the structure of the software package.

Fig. 5.4 Software Structure

- **Design**

In general, the software is mainly for vision system configuration and image processing and measurement. It should meet all requirements as mentioned above. However, it is certain that different levels of users will have different purpose. For instance, a new user will be more concerned with a easy-to-use GUI interface and they will spend less time to learn how to use the image processing software itself. A specialist will pay more attention to control the hardware features, such as graphic processor.

The MIL image processing software package will meet most of our requirements. We divide MIL image processing software package into 5 levels which every level is for a different purpose. Every level is able to used individually.

Level 1. To setup new configuration of the vision system and to test a system –
IntellCam.

IntellCam is a Windows 95/NT-based utility that allows the user to interface any Matrox frame grabber or vision processor to a standard or non-standard video device. It provides the following features:

1. Set up configuration of frame grabber or video device.
2. Experiment with different digitizer configuration. The adjustment takes effect immediately so there is no need to compile and execute.

3. IntellCam script – Matrox imaging library interpreter (MILINTER)

MILINTER provides a flexible way to control the vision system because it is programmable, users can use a certain program to test the performance of vision system within a certain environment.

Level 2.Normal user – Inspector

1. Inspector provides an integrated GUI to access complete image processing, analysis and measurement functions. This interface is easy-to-use and all operations involve a windows-based graphic interface.
2. Inspector also provides the ability to reconfigure the vision system, such as frame grabber and camera. Users can use Matrox Inspector as a prototyping tool to quickly build proof-of-concept demonstrations for their own usage.

3. Automate procedure by programming with script. After users find a suitable configuration and image processing procedures for a certain experiment, he can use Inspector script to set-up a program to execute this procedure automatically. It will improve efficiency.
4. Using OLE to access powerful GUI and window library. Using OLE (Object Linking and Embedding) automation, users can integrate Inspector's functionality into another Windows-based application.

Level 3. Windows-based developer –ActiveMIL-Lite

As we mentioned before, OCX control is a very powerful technique for GUI programming, users can easily use OCX control for programming and to develop their own software. ActiveMIL-Lite is a collection of ActiveX controls for managing image capture, transfer, processing, analysis and display. Fully integrating into Microsoft Visual Basic or C++ RAD environments, ActiveMIL lets you quickly and easily put together an imaging application with a custom, professional-looking, Windows user interface.

Level 4 - MIL library

MIL is a high-level C++ code library with an extensive set of optimized functions for image processing. We can learn the core of image processing software and modify it for our own purpose.

Level 5 –Genesis Native library

The Genesis Native Library is a board-specific library. Genesis Native Library (GNL) supports some of the more specialized features of the Genesis-LC module that MIL does not support. Thus, MIL is hardware-independent, but GNL is not. Since GNL is a board-

specific code, it supports requirements that need more real time control or processing.

GNL can also control the core of Genesis-LC - image processor C 80, that give the user a more power tool to access the resource of the computer.

5.4.4 Mechanical handling and position control sub-system

- Rail sub-system

Figure 5.5 demonstrate the layout of rail sub-system. The main function is described as following.

Function:

- Provide the mobility of optical system and platform on the vibration table
- Provide fine position adjustment on the table

Fig. 5.5 Optical rail system

Table 5.2 shows the function of the individual components of rail sub-system.

Table 5.2 The function of the individual components of rail sub-system.

	Name	Function
1	100 mm rail	Handling and position control for camera sub-system
2	50 mm rail	
3	100 mm carrier	Handling and position control for platform of sessile drop sub-system
4	50 mm carrier	

- Camera handling sub-system

Figure 5.6 demonstrated the camera handling sub-system.

Function:

- Provide a safety handling for CCD camera and lens.
- Provide fine position control of camera and lens.

Fig. 5.6 Camera handling sub-system

5.4.5 Platform for sessile drop

Function:

1. To provide a plate to support sessile drop and substrate.
2. To provide fine position control of sessile drop.
3. To provide the leveling adjustment.

This sessile drop platform is very flexible, it can move in 3 axis directions and 2 rotational directions. It has 5-degrees of freedom. These freedoms are:

- Z-Rotation: Using rotator, we can rotate the sessile drop a full 360 degrees around the Z-axis
- Y-Rotation: Using angle mounting, we can let sessile drop rotate completely around Y-axis.
- X-transformation: Using translator, we can let sessile drop move along X direction; Maximum travel distance is 2 inches.
- Y-transformation: Using rail system, we can let sessile drop move along Y-direction; Maximum travel distance is 30 cm.
- Z-transformation: Adjusting the height of the right angle bracket, we can let sessile drop move along Z-direction.

The respective five resolution adjustments are:

Table 5.3 Freedom of Sessile drop platform (1-4)

		Resolution
X	Translation	<0.5 μ m
Y	Translation	<1mm
Z	Translation	<1mm
X	Rotation	<0.015"/turn
Y	Rotation	<12 μ Rad/turn

Fig 5.7 Outline of Sessile Drop Platform

Fig. 5.8 Sessile Drop Platform

5.4.6 Optical table subsystem

Fig. 5.9 Outline of Optical Table

Most components of the vision system were placed on the top of the breadboard(vibration table), such as the optical subsystem, opto-mechanical subsystem, the platform for sessile

drop and microscope etc. The optical table subsystem isolates these components from the outside environment.

Functions:

1. To provide vibration isolation.
2. To level the whole system.

If, during the exposure time, every point of the optical system (illuminator, sample, microscope optics, camera optics, and film plane) exactly move together so that there is no relative motion, the image will be clear. If, on the other hand, there is relative motion of the sample with respect to the objective lens, the image will be blurred. Since it is impossible to create a perfectly rigid structure an effective vibration isolation system must take into consideration these factors:

- **Dynamic Forces (Vibration)**

Dynamic forces cause structural deformations that vary with the frequency of the driving force. Structural resonance can amplify the relative motion between optical components.

1. Connect all of the critical elements together in a dynamically rigid structure that is designed to eliminate (damp) structural resonances.
2. Isolate the system from vibration with mechanical filters or active cancellation technology.

- **Static Forces**

Static forces cause deformations that are constant in time. However, the addition or movement of equipment in the system will change the static forces and cause misalignment of system elements.

1. The optical breadboard - HighPerformance optical breadboards, Melles Griot. USA.

Size is large at 4 feet by 6 feet because space must be available for a microscope on the optical breadboard. According to the principle of mass arrangement, the maximum vertical height should be less than $\frac{1}{2}$ the shortest distance between isolators. The height of microscope is 1.5 feet which is less than half of 6 feet.

This breadboard is honeycomb structure. In simple terms a honeycomb table is a honeycomb core sandwiched between two faces of skin material and finished with side panels whose primary function is aesthetic. The goal is to design a structure that approaches a perfectly rigid body that is extremely stiff.

Damping is necessary to dissipate vibration energy in the table. Higher damping efficiency lowers the dynamic deflection coefficient, resulting in less table top motion.

With enhanced damping, this breadboard provides improved performance.

2. Level Mounts - SLM-1A, Melles Griot. USA.

Those level mounts were used as level adjustment. It also is used as vibration isolator.

Fig5.10 demonstrated the SLM-1A level mounts.

Fig. 5.10 Air feet

Fig. 11 Whole system

5.4.7 Light subsystem

Some techniques for illumination include:

- Front lighting and Back lighting - both may be directional or diffusion. Front lighting techniques are also categorized as bright field and dark field.
- Structured lighting - generally a linear stripe or collection stripes. Often a collimated laser light source.
- Coaxial lighting - using a beam splitter to place the light source directly on axis with the image.

Reference:

1. The Book of Photon Tools, Oriel Instruments, 2000.
2. The Practical Application of Light, Melles Griot, 2000.
3. Optics and Mechanics, 1999/2000 Catalog, Newport Instrument, 2000.
4. Optics and Optical Instruments Catalog 2000, Edmund Industrial Optics, 2000.
5. Matrox Software and hardware documentation, Matrox Electronic Systems Ltd. 1999.

Chapter 6

Error-estimation of vision system

6.1 Error-estimation of vision system

A vision system is a complicated system that includes many sub-systems. For example, because ADSA-P uses the coordinates of a drop profile to calculate the drop's surface tension, any effect on profile accuracy will be a possible error source. Some possible error sources are:

1. Software sub-system

- Error from operating system and related software
- Error due to the precision of the edge detection algorithms that is used by image processing software.

2. Optical sub-system

- Blur
- System noise
- Random noise
- Aspect ratio
- Scaling

3. Vibration control sub-system

- Error due to vibration

4. Handling and position control sub-system

- Leveling

- Vertical misalignment of the camera
5. Frame grabber in PC-subsystem
 - Precision of frame grabber
 6. Light sub-system
 - Type of lighting
 - Uncertainty of intensity

As we see, these possible error sources are not independent and maybe related. We also can re-categorize the possible error resource based on the ADSA-P program that will finally be used to calculate the surface tension. The ADSA-P program and related parameters are:

1. The precision of the edge detection

The possible sources that may effect the precision of the edge detection are:

- The image processing software that is used for edge detection
- Blur or blooming in the optical system
- The accuracy of focusing either manually or by the auto-focus tool
- Error due to system noise
- Error due to random noise
- Error due to vertical mis-alignment

2. The numerical algorithms related to ADSA-P

- The compilers and operating systems

3. Related parameters

- The precision of the gravity constant
- The precision of the density difference across the fluid interface

In Chap. 4, the Laplace program was used to generate a theoretical Laplacian curve. In this Chapter, this Laplacian curve was used as a reference profile to simulate possible experimental errors.

In order to estimate the final error on the surface tension, the following variables were adopted in both the ADSA-P and the Laplace programs. Surface tension,

$\gamma = 72.000 \text{ mJ/m}^2$; Density difference, $\Delta\rho = 1.000 \text{ g/cm}^3$; Gravitational acceleration,

$g = 981.00 \text{ cm/s}^2$; Yielding a capillary constant, $\sqrt{c} = 369.12 \text{ 1/m}$.

6.2 The Numerical Algorithms Related to ADSA-P

6.2.1 Influence of the different FORTRAN compiler

The ADSA-P program was compiled by different FORTRAN compilers to compare their results. One compiler was from Absoft Inc, USA. Another compiler was the G77 that is a Fortran compiler from the Free Software Foundation (GNU). The reference profile for the ADSA-P program was generated by the Laplace program with $\gamma = 72.000 \text{ mJ/m}^2$ and $B_0 = 10.000$.

Table 6.1 Influence of different FORTRAN compilers on the ADSA-P results

	Reference	Absoft Fortran	G77
$\gamma(\text{mJ/m}^2)$	72.000	72.000490897918	72.0004908979
$B_0(\text{Dimensionless})$	10.000	10.0002120021106	10.0002339295
$R_0(\text{m})$	0.0027091461	0.002709199280293	0.0027092052207

6.2.2 Influence of the different operating system

ADSA-P program was compiled on different operating systems. One was Sun UNIX with F77 compiler. Another was WinNT 4.0 with G77 compiler. Once again, the reference profile for the ADSA-P program was generated by the Laplace program with

$$\gamma = 72.000 \text{ mJ/m}^2 \text{ and } B_0 = 10.000.$$

Table 6.2 Influence of different operating systems on the ADSA-P results

	Reference	G77 on WinNT 4.0	F77 on UNIX sun
$\gamma(\text{mJ/m}^2)$	72.000	72.0004908979	72.0004908979
$B_0(\text{Dimensionless})$	10.000	10.0002339295	10.0002339295
$R_0(m)$	0.027091461	0.0027092052207	0.0027092052207

The conclusion was the any difference between Fortran compilers and operating systems will not influence the acceptable precision of the surface tension.

6.3 Related Parameters

The Laplace equation of capillary was represented by the expression

$$\gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \Delta p \quad [6-1]$$

where γ is the interfacial tension, R_1 and R_2 are the two principal radii of curvature and Δp is the pressure difference across the interface. The pressure difference may be expressed as a linear function of the elevation:

$$\Delta P = \Delta P_0 + \Delta \rho g Z \quad [6-2]$$

which implies that the accuracy of value of density difference and gravity will effect the error on the surface tension.

6.3.1 Influence of the error from the variable of density difference

Different $\Delta \rho$ values were input into the ADSA-P program and compard with the reference profile for ADSA-P generated by the Laplace program with

$$\gamma = 72.000 \text{ mJ/m}^2 \text{ and } B_0 = 2.000.$$

Table 6.3 Effect on surface tension error due to the error in the density difference

$\Delta \rho (\text{g/cm}^3)$	$B_0 (\text{Dimensionless})$	$\gamma (\text{mJ/m}^2)$	$\Delta \gamma (\text{mJ/m}^2)$
1.00000(reference)	2.00010277	72.00428	0.0
1.00001	2.00010277	72.00501	0.00073
1.0001	2.00010277	72.01149	0.00721
1.001	2.00010277	72.07630	0.07202
1.01	2.00010277	72.72433	0.72005
1.05	2.00010277	75.60450	3.60022

6.3.2 Influence of gravitational acceleration constant

Different g values were input into the ADSA-P program and compare with the reference profile for ADSA-P generated by the Laplace program with $\gamma = 72.000 \text{ mJ/m}^2$ and

$$B_0 = 2.000.$$

Table 6.4 Effect on surface tension error due to the error in the gravitational acceleration

$g (\text{cm/s}^2)$	$B_0 (\text{Dimensionless})$	$\gamma (\text{mJ/m}^2)$	$\Delta \gamma (\text{mJ/m}^2)$
981.00(reference)	2.00010277	72.00428	0.0
981.001	2.00010277	72.00436	0.00008
981.01	2.00010277	72.00502	0.00074
981.1	2.00010277	72.01163	0.00735
981.5	2.00010277	72.04099	0.03671
982	2.00010277	72.07769	0.07341

Thus, the accuracy of $\Delta\rho$ and g have a small influence on the error in the surface tension. In most case, this will not be significant as both g and $\Delta\rho$ may be determined with good accuracy.

6.4 Influence of Drop Shape

In Chap. 4, the perturbation model illustrated that the drop shape has a very strong influence on the precision of the surface tension. For more details please see Chap. 4 and related examples.

6.5 Parameters Related to Edge Coordinate Precision

6.5.1 Random noise point that effect in the local point of profile

There are various possibilities to generate random noise so as to investigate the influence of the noise on local profile points. These random influences may occur from semi-unstable electrical power suppliers that affect the digitizing of the CCD sensor or they may occur from unstable lighting. Irregardless of the source of this randomness one expects that the measured position of points along the drop's edge will be randomly altered. The purpose of this section is to investigate this influence on the surface tension accuracy and to quantify (or weight) the influence of different profile points along the drop's edge.

To estimate the effect of random noise on a single local point, one single point in the perfect profile generated by the Laplace program was perturbed individually in the normal direction. Thus, one point at a time was perturbed in the direction normal to profile by the distance, δ , whereupon,

$$\begin{aligned}\tilde{X}_i^0 &= X_i^0 + \delta \cos \theta \\ \tilde{Z}_i^0 &= Z_i^0 + \delta \sin \theta\end{aligned}\tag{6-3}$$

$$\begin{aligned}\tilde{X}_{j \neq i}^0 &= X_j^0 \\ \tilde{Z}_{j \neq i}^0 &= Z_j^0\end{aligned}$$

Thus, a new profile $(\tilde{X}, \tilde{Z})$ was generated with only one perturbed outwards point in each case. In order to have an equal number of points on each side of the apex point of the profile, an odd number of points were used.

In connection with Fig. 6.1, we defined two quantities:

$$\hat{\gamma} = \frac{\gamma_i - 72.00}{\max(\gamma_i) - 72.00}\tag{6-4}$$

$$\hat{s} = \frac{s_i}{\max(s_i)}\tag{6-5}$$

where γ_i was the surface tension value where the i^{th} individual point was perturbed. Only one-half of the profile was shown ($s \geq 0.0$ cm) with the point at $s=0.0$ representing the apex of the drop. The results for the other half of the profile ($s \leq 0.0$ cm) were identical. The same results were obtained when perturbing each point inward but with the opposite sign for the errors on the surface tension.

Using a sessile drop as an example, Fig. 6.1 shows that each point has a different effect on the error, the stronger effect being from points around the points that is a maximum distance from the axisymmetric axis, the apex point and the point that contacts the substrate. Some regions of the profile that have little influence on the result and two specific points occur with no influence at all.

Once a point was perturbed, it behaves as an outlier and the least-squares fit tends to shift the Laplacian profile slightly in the direction of the perturbation. As a result, small errors in the location of the origin of the drop (X_0, Z_0) were generated by ADSA-P. The errors in the coordinates of the origin have a strong influence on the results. The larger the perturbation, the larger the errors. These results were consistent with those of Cheng et. al. (Ref. 1, page 476 and Fig. 6.2).

6.5.2 Blooming and the effect on the global profile point

When the light intensity of either of the object or background is too large the intensity registers or pixels of the CCD area array will become saturated. At saturation the intensity level of the pixel will be set to its maximum value (e.g. 255 for an 8 bit register with 2^8 levels from 0 to 255). Depending on the intensity of the light, adjacent pixels to the saturated pixels may experience current leakage with a corresponding increase in their intensity level. As a consequence of this leakage, the brightly lit zone of the image will appear larger. All CCD cameras attempt to control this effect but it will occur to some degree in all cameras and is known as blooming (2). In the case of a back-lit pendant or sessile drop a too intense background will cause the drop to appear smaller. This blooming effect may be simulated using the perturbation technique described in Chap. 4 by treating a bloom as a uniform inward displacement of the drop's edge profile. The λ -distribution is constant and may be set equal to increments that correspond to integer pixel numbers, that is $\lambda = j$ (pixel length) where $j \in I^+$. Additional details may be found in Section 4.5.

6.5.3 Random noise point that effects on the global point of profile

Section 6.5.1 described the effect of individually perturbing one profile point due to random noise, at a time. There is another possibility that random noise will effect all points along the profile. For example, in image processing operators, a smoothing operator or segment operator may assign each pixel point of the profile an uncertain direction and an uncertain value. As a result, all points of the profile will be influenced. As we showed earlier, the contribution to the surface tension error depends on the location of the profile point and the shape of the drop or B_0 . Thus, to estimate the global influence of any particular random profile point a weight factor was introduced to weight the influence of each point. If we define the weight factor as

$$w_i = f(B_0) = \frac{\lambda_i}{\lambda_{\max}} \quad [6-6]$$

then we may create a randomly perturbed set of profile coordinates $(\tilde{X}_i^0, \tilde{Z}_i^0)$ from an exact numerical set of points (X_i^0, Z_i^0) generated by the Laplace program if we use the relations

$$\begin{aligned} \tilde{X}_i^0 &= X_i^0 + (-1)^{xd_i} \lambda \delta x_i w_i \\ \tilde{Z}_i^0 &= Z_i^0 + (-1)^{zd_i} \lambda \delta z_i w_i \end{aligned} \quad [6-7]$$

where, xd_i and zd_i are random integer numbers that are uniformly and randomly distributed from 1 to 10 (5 even and 5 odd numbers). Quantities δx_i and δz_i are random real numbers that are uniformly distributed from 0 to 1. Parameter λ acquires a value to represent the maximum perturbation distance, e.g. 2 pixels or $2 \times 10 \mu m$ in length. Thus, a new drop profile $(\tilde{X}_i^0, \tilde{Z}_i^0)$ may be generated by randomly perturbing every point according to the above scheme.

Figure 6.3 plots the results of randomly perturbing a sessile water drop

($\gamma = 72.00 \text{ mJ/m}^2$; $B_0 = 2$) using different constant values of λ with different random values of xd_i , zd_i , δx_i and δz_i . Twenty random examples were generated for each λ value and the associated surface tension error was calculated. The random number, xd_i , zd_i , δx_i and δz_i were generated by a portable random number generator (3, page 278) and the surface tension error was determined by adjusting the λ value. Random errors have a strong effect on the surface tension error. Most errors will occur in a certain region. Thus, it is possible to estimate the maximum surface tension error due to random effects on the global profile.

6.5.4 Vertical mis-alignment and effects in the global profile point

Fig. 6.4 Vertical misalignment

In practice, it is difficult to achieve a perfect vertical alignment of the camera when acquiring the image of a drop. The profile was rotated through a specified angle α to test the effect of vertical mis-alignment of the camera.

If we denoted (X, Z) as the reference profile set of coordinates generated by the Laplace program, then $(\tilde{X}, \tilde{Z})$ represents the rotated profile where α is the angle of rotation of the coordinate system (see Fig. 6.4). The coordinates were related by

$$\begin{aligned}\tilde{X} &= X \cos \alpha + Z \sin \alpha \\ \tilde{Z} &= Z \cos \alpha - X \sin \alpha\end{aligned}\quad [6-8]$$

Table 6.5 tabulated the surface tension error due to different vertical mis-alignment α angles.

Table 6.5 Effect on surface tension error of vertical mis-alignment

α (degrees)	B_0	$\gamma(mJ/m^2)$	$\Delta\gamma(mJ/m^2)$
0	2.0001027	72.00428	
0.01	2.0001024	72.00432	0.00004
0.1	2.0000644	72.00751	0.00323
0.2	1.9999615	72.01716	0.01288
0.4	1.9995376	72.05582	0.05154
0.6	1.9988311	72.12036	0.11608
0.8	1.9978416	72.21099	0.20671
1.0	1.9965693	72.32793	0.32386

Reference:

1. S. Lahooti, O. I. Del Rio, P. Cheng and A. W. Neumann, "Axisymmetric Drop Shape Analysis (ADSA)" in " Applied Surface Thermodynamics", (A. W. Neumann and J.K. Spelt, eds.), pp. 441-507, Marcel Dekker, New York, 1996.
2. 1996-1997 DATABOOK, DALSA manual, p. 20.
3. William H. Press, Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery, " Numerical Recipes in C, The Art of Scientific Computing" Second Edition, Cambridge University Press, 1992

Chapter 7

Conclusions and Future Work

7.1 Conclusions

One of the important aspects of our research was that we were first ones to provide a physical/numerical model for simulating the uncertainty in a digital vision system. Based on the perturbation model developed in Chap. 4, a criteria was presented to answer following questions: (1) What is the surface tension precision level for a particular experimental system, e.g. a vision system. (2) The inverse, if we require a certain surface tension precision, how do we select various components to achieve suitable parameters and satisfy the requirements.

The results show that accuracy of a surface tension value from a digital vision system is determined by various complicated, related parameters: the drop shape information, such as size of drop and the final position of arc-length or contact angle, the magnification of the optical lens system, CCD sensor parameters, the drop edge precision, etc. my viewpoint is that the accuracy of the surface tension values from a digital vision system are determined by the whole system quality, and that simply mentioning any single subsystem, such as the software for edge detection, is not to prove the precision of surface tension significantly.

One of the most challenging aspects of this thesis was how to simulate the uncertainty of a vision system. For a long time, science and industry knew that various factors influenced the accuracy of the surface tension measurement, but there is still no a simply

numerical/physical modeling to simulate experimental uncertainty. The perturbation modeling provides us an effective way to do this simulation.

7.2 Future Work

As I mentioned, the accuracy of surface tension from digital vision system is influenced by various complicated factors. The modeling and criteria described in this thesis can be improved and used in many ways. In this section of the thesis, some possible future work is mentioned:

- The distribution of normal perturbation length can be calculated to greater accuracy.
- Use perturbation modeling technique to simulate more situations.
- Use perturbation modeling to simulate the drop spreading at low Reynold's number.
- Integrate the distribution of normal perturbation length into algorithms that calculate the surface tension and contact angle.
- Integrate the distribution of normal perturbation length to digital vision system as a calibration to estimate the accuracy of surface tension value in real-time.
- Improve the criteria so that it includes more reasonable parameters and consider more complex situations.

Appendix A

Two Experimental Examples

Fig. A-1 A pendant drop example

Fig. A-2 The edge of a pendant drop.

Fig. A-3 A sessile drop example

Fig. A-4 The edge of a sessile drop

Appendix B

Application Software

WinDrop 2000

WinDrop

Program Highlights

- Flexible and friend interface
- Developed using VC++ 6.0 Matrox Mil 2.0, MilActiveX, MFC/ActiveX on WinNT 4.0.
- Integrated the Edge Detection program, Surface Tension Calculation program, Data Processing program, Image Display program into one GUI application software.
- Edge detection using Sobel Edge Filter.
- Dynamic Memory Management.

Fig. B-1 Simple GUI interface

Fig. B-2 Opening an image file

Fig. B-3 After an image file is loaded, the size of image is automatically detected, at the same time, user can select the calibration factor and output points for ADSA-P program

Fig. B-4 After edge detection, the profile edge coordinates of the drop are obtained. These coordinates will be used by ADSA-P subprogram.

Fig. B-5 The coordinates may to be processed by MS Excel.

Appendix C

Program – Normal Perturbation Length Distribution, λ

```
C=====
C Larry Zhou
C Date: 1999
C Supervisor : Professor John Gaydos
C=====
```

```
C=====
C Function:
C The program compares original profile (unperturbed) with perturbed
C profile, then generates the normal perturbation length distribution.
C=====
```

```
C=====
C Input :
C     1. Slightly shifted dimensionless radius of curvature at the origin of either an
C         axisymmetric pendant drop or a sessile drop; Bo
C     2. Slightly shifted surface tension, Gamma
C     3. The original profile (unperturbed) is input from "B0=1-1800.dat"
C Output :
C     1. The normal perturbation length distribution and related parameters (on screen)
C=====
```

program main

```
implicit integer (i-n)
implicit real*8 (a-h,o-z)
real*8 da(3),dx(3),dz(3),dv(3),dar(3)
real*8 Xpertur(19),Ypertur(19)
real*8 xoriginal1800(1801),yoriginal1800(1801)
real*8 dmin(19), dist,DistPerturbation,smax
real*8 disV(100),gammaV(100), bV(100),sl(19)
```

```
C=====
C Input the shifted gamma and B0
C=====
```

```
OPEN(UNIT=9,STATUS='OLD', FILE='xz.dat')
write(*,*) "INPUT GAMMA = "
```

```
read(*,*) gamma
gamma = gamma/1000.0d00
```

```
write(*,*) "INPUT B0 = "
```

```
read(*,*) epp
ep = (1.0d00)/(epp*1.0d00)
```

```

write(*,*)"Start at:="
write(*,*) gamma, ep
write(*,*)"-----"

```

C===== Finish Input

C Initial angle and the step of angle

```

ang=10.00d00*b
deltaangle=10.00000d00
Npt = 180.00 / deltaangle

```

C===== >>

C=====

```

delrho=1000.d00
grav=9.81d00

```

```

bendj=0.1d-07
bendk=0.0d-09
cc=(delrho*grav)/gamma
cca=dsqrt(cc)
ccj=cca*bendj/gamma
cck=cc*bendk/gamma

```

c Set main variables

```

pi=4.0d00*datan(1.0d00)
b=pi/180.0d00
bf=0.1d00*b
error=1.0d-07+0.1d00*dsqrt(bendj)+0.1*dsqrt(bendk)
error=error*1.0d+01
aa=1.0d00

```

c aa=-1.0d00

c Set aa=+1 for sessile and -1 for pendant

```

ab=1.0d00

```

c May fit do-loop on bbc=log 10 (b) with statement 27

```

210 bbc=1.0d00
c 210 read 521,bbc
c 521 format(f5.2)

```

C=====

c Output cca, Number of points tp xz.dat

C=====

```

write(*,*) cca, Npt
write(*,*) " C constant is = ", cca

```

c Define problem and stating conditions 10,2,69

```

c write(*,*) ep
c ep=10.0d00
c ep=0.1d00

```

```

a=0.0d00
z=0.0d00
x=0.0d00
v=0.0d00
ar=0.0d00
ss=0.0d00
c  b1=2.0d00/ep
   if(aa) 200,200,201
200 continue
c      write(6,202)
   goto 203
201 continue
c write(6,78)
203 continue
78 format(1H1/,33X,33HSessile Drop or Internal Meniscus,/)
202 format(1H1/,44X,12HPendant Drop,/)
c  write(6,800) delrho,gamma,bendj,bendk
800 format(31x,7Hdelrho=,e11.5,2x,6Hgamma=,e11.5/,
+ 32x,6Hbendj=,e11.5,2x,6Hbendk=,e11.5,/)

c  Set switches

n=1
i=0
l=1
m=0
mm=2

c  write(6,80)bbc,b1
c 80 format(29X,7Hlog10b=,f5.2,2X,2Hb=,e11.5,/)
c  write(6,79)
79 format(7X,5HAngle,5X,10HArc-length,11X,1HX,15X,1HZ,15X,
+4HArea,13X,6HVolume,/)

c  Set step length

ds=0.1d-07*b/ep

c  Fourth-order Runge-Kutta Integration
c  Initial conditions with switch iii

61 continue
s=0.0d00
dal=ep*ds
iii=1
goto 1

c  Runge-Kutta step 1

2 s=dsin(a)
xda1=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z-s/x
yda1=1.0d00+2.0d00*ccj*s/x
da1=(xda1/yda1)*ds
1 dz1=ds*s*ab
dx1=dcos(a)*ds
dv1=dz1*x*x

```

```
dar1=2.0d00*x*ds
```

c Runge-Kutta step 2

```
s=dsin(a+0.5d00*da1)
xda2=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*(z+0.5d00*dz1)-
+ s/(x+0.5d00*dx1)
yda2=1.0d00+2.0d00*ccj*s/(x+0.5d00*dx1)
da2=(xda2/yda2)*ds
dz2=ds*s*ab
dx2=ds*dcos(a+0.5d00*da1)
dv2=(x+0.5d00*dx1)*(x+0.5d00*dx1)*dz2
dar2=ds*(x+x+dx1)
```

c Runge-Kutta step 3

```
s=dsin(a+0.5d00*da2)
xda3=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*(z+0.5d00*dz2)-
+ s/(x+0.5d00*dx2)
yda3=1.0d00+2.0d00*ccj*s/(x+0.5d00*dx2)
da3=(xda3/yda3)*ds
dz3=ds*s*ab
dx3=ds*dcos(a+0.5d00*da2)
dv3=(x+0.5d00*dx2)*(x+0.5d00*dx2)*dz3
dar3=ds*(x+x+dx2)
```

c Runge-Kutta step 4

```
s=dsin(a+da3)
xda4=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*(z+dz3)-s/(x+dx3)
yda4=1.0d00+2.0d00*ccj*s/(x+dx3)
da4=(xda4/yda4)*ds
dz4=ds*s*ab
dx4=ds*dcos(a+da3)
dv4=(x+dx3)*(x+dx3)*dz4
dar4=2.0d00*(x+dx3)*ds
```

c Variable mm is truncation error control (tec) switch

```
goto (3,4,74,76) mm
3 continue
```

c mm=1 and using a precise step length to locate a special point

```
a1=a+(da1+da4+2.0d00*(da2+da3))/6.0d00
z1=z+(dz1+dz4+2.0d00*(dz2+dz3))/6.0d00
x1=x+(dx1+dx4+2.0d00*(dx2+dx3))/6.0d00
dv5=(dv1+dv4+2.0d00*(dv2+dv3))/6.0d00
v1=v+dv5
ar1=ar+(dar1+dar4+2.0d00*(dar2+dar3))/6.0d00
ss1=ss+ds
goto 8
```

c Check on truncation error (statements 4 to 77)

```
4 mm=3
```

c mm=3 step=h, have delta(1)

```

da(1)=(da1+da4+2.0d00*(da2+da3))/6.0d00
dz(1)=(dz1+dz4+2.0d00*(dz2+dz3))/6.0d00
dx(1)=(dx1+dx4+2.0d00*(dx2+dx3))/6.0d00
dv(1)=(dv1+dv4+2.0d00*(dv2+dv3))/6.0d00
dar(1)=(dar1+dar4+2.0d00*(dar2+dar3))/6.0d00
ds=0.5d00*ds
if(iii.eq.1) goto 61

```

c Step=h/2 from original point

```

goto 2
74 mm=4

```

c mm=4 have delta(2) at step h/2

```

da(2)=(da1+da4+2.0d00*(da2+da3))/6.0d00
dz(2)=(dz1+dz4+2.0d00*(dz2+dz3))/6.0d00
dx(2)=(dx1+dx4+2.0d00*(dx2+dx3))/6.0d00
dv(2)=(dv1+dv4+2.0d00*(dv2+dv3))/6.0d00
dar(2)=(dar1+dar4+2.0d00*(dar2+dar3))/6.0d00

```

c Save starting point

```

a2=a
z2=z
x2=x
v2=v
ar2=ar
ss2=ss

```

c Start+delta(2)

```

a=a+da(2)
z=z+dz(2)
x=x+dx(2)
v=v+dv(2)
ar=ar+dar(2)
ss=ss+ds

```

c Step h/2

```

goto 2

```

c Hopefully, delta(1)=delta(2)+delta(3) with small error

```

76 da(3)=da(2)+(da1+da4+2.0d00*(da2+da3))/6.0d00
dz(3)=dz(2)+(dz1+dz4+2.0d00*(dz2+dz3))/6.0d00
dx(3)=dx(2)+(dx1+dx4+2.0d00*(dx2+dx3))/6.0d00
dv(3)=dv(2)+(dv1+dv4+2.0d00*(dv2+dv3))/6.0d00
dar(3)=dar(2)+(dar1+dar4+2.0d00*(dar2+dar3))/6.0d00

```

c Variable mm is tec comparison control

```

if(dabs(da(3)-da(1)).gt.dabs(da(3))*error) goto 77
if(dabs(dz(3)-dz(1)).gt.dabs(dz(3))*error) goto 77
if(dabs(dx(3)-dx(1)).gt.dabs(dx(3))*error) goto 77
if(dabs(dv(3)-dv(1)).gt.dabs(dv(3))*error) goto 77
if(dabs(dar(3)-dar(1)).gt.dabs(dar(3))*error) goto 77

```

- c Advances using the best estimate.

```

a1=a2+da(3)
z1=z2+dz(3)
x1=x2+dx(3)
v1=v2+dv(3)
ar1=ar2+dar(3)
ss1=ss2+2.0d00*ds
dsb=ds

```

- c Variable dsb is the step length to satisfy tec control

```
ds=ds*2.2d00
```

- c Step grows by 10%

```
iii=2
```

- c Maximum step length limiter

```

if(dabs(da(3)).gt.bf) ds=0.5d00*ds
dsa=ds
mm=2
goto 8
77 mm=3

```

- c Failed tec, Halving step size and returning to last good point

```

ds=0.5d00*ds
dsa=ds
da(1)=da(2)
dz(1)=dz(2)
dx(1)=dx(2)
dv(1)=dv(2)
dar(1)=dar(2)
a=a2
z=z2
x=x2
v=v2
ar=ar2
ss=ss2
if(iii.eq.1) goto 61
goto 2

```

- c Tests for answers

```

8 if(i) 22,52,71
52 if(m) 22,10,49
10 if(l) 15,22,16
16 if(a1-a) 17,17,19

```

```

19 if(a1-ang) 7,18,21
15 if(a1-a) 23,24,24
23 if(a1-ang) 21,18,7
17 l=-1
    ang=ang-deltaangle*b
    goto 25
18 goto 21

c   l=+1 Angle increasing; l=-1 Angle decreasing

24 l=1
    ang=ang+deltaangle*b
25 continue

c   Search for point of inflection (25-21), switch m=l
c   Looks for good starting point for search

xdd2=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z2-dsin(a2)/x2
ydd2=1.0d00+2.0d00*ccj*dsin(a2)/x2
dd=dabs(xdd2/ydd2)
a3=a2
z3=z2
x3=x2
v3=v2
ar3=ar2
ss3=ss2
xdd=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z-dsin(a)/x
ydd=1.0d00+2.0d00*ccj*dsin(a)/x
dc=dabs(xdd/ydd)
xdd1=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z1-dsin(a1)/x1
ydd1=1.0d00+2.0d00*ccj*dsin(a1)/x1
db=dabs(xdd1/ydd1)

c   Stores most advanced point

48 a2=a1
    z2=z1
    x2=x1
    v2=v1
    ar2=ar1
    ss2=ss1
    if(db.le.dc.and.db.le.dd) goto 43
    if(dc.le.db.and.dc.le.dd) goto 44
    a=a3
    z=z3
    x=x3
    v=v3
    ar=ar3
    ss=ss3
    goto 44
49 xdd=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z-dsin(a)/x
    ydd=1.0d00+2.0d00*ccj*dsin(a)/x
    dc=dabs(xdd/ydd)
    xdd1=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z1-dsin(a1)/x1
    ydd1=1.0d00+2.0d00*ccj*dsin(a1)/x1
    db=dabs(xdd1/ydd1)

```

```

        if(db-dc) 43,44,44
43 a=a1
   z=z1
   x=x1
   v=v1
   ar=ar1
   ss=ss1

```

c Locating answers

```

44 if(dabs(ds).lt.dabs(dsb)*0.1d-07) goto 45
47 s=dsin(a)

```

c Variable da1 is first derivative, da2 is second derivative

```

zda1n=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z-s/x
zda1d=1.0d00+2.0d00*ccj*s/x
da1=zda1n/zda1d
zz=dcos(a)/x*da1-s*dcos(a)/x/x
zda1np=s-zz
dda1dp=2.0d00*ccj*zz
da2=(zda1np*zda1d-zda1dp*zda1n)/zda1d/zda1d
ds=-da1/da2
m=1
mm=1

```

c Variable dsb is step length to satisfy tec

```

        if(dabs(ds).gt.dsb) goto 90
        goto 2
90 ds=dabs(ds)/ds*0.9d00*dsb
        goto 2
45 continue
a=a/b
c   if(ang.lt.0.173d00) goto 81
c   if(ang.lt.1.744d00) goto 82
c   write(6,*)a,x,z
c   goto 86
c 82 write(6,*)a,x,z
c   goto 86
c 81 write(6,*)a,x,z
c   goto 86
c 83 format(9X,1H*,e11.5,5X,2e13.5)
c 84 format(10X,1H*,e11.5,4X,2e13.5)
c 85 format(11X,1H*,e11.5,3X,2e13.5)
c 86 continue
        ij=-5
c   punch 500,bbc,ij,x,z
c 500 format(f6.2,i5,2e13.5)

```

c Reloading most advanced point and resetting switches

```

m=0
mm=2
ds=dsa
a=a2

```



```

z=z2
x=x2
v=v2
ar=ar2
ss=ss2

```

c Returning to main integration

```

goto 2
21 continue

```

c Search for definite angle, switch i=1 (21-37)

```

a2=a1
z2=z1
x2=x1
v2=v1
ar2=ar1
ss2=ss1

```

c Test for good starting point

```

ax=dabs(a1-ang)
ay=dabs(a-ang)
if(ay.lt.ax) goto 6
goto 71
6 a1=a
z1=z
x1=x
v1=v
ar1=ar
ss1=ss
71 if(dabs(ang-a1)-0.1d-07) 56,72,72
72 xt1n=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z1-dsin(a1)/x1
xt1d=1.0d00+2.0d00*ccj*dsin(a1)/x1
term1=xt1n/xt1d
ds=(ang-a1)/term1

```

c Locating answers

```

mm=1
i=1
if(dabs(ds).gt.dsb) goto 92
goto 12
92 ds=dabs(ds)/ds*0.9d00*dsb
goto 12
56 dth=ang-a1

```

c Final estimation by Euler's method

```

s=dsin(a1)
xt2n=2.0d00*ep*(1-ccj*ep-cck*ep*ep)+aa*z1-s/x1
xt2d=1.0d00+2.0d00*ccj*s/x1
term2=xt2n/xt2d
ds=dth/term2
ddz=s*ab*ds

```

```

ddx=dcos(al)*ds
ddv=x1*x1*ddz
ddar=2.0d00*x1*ds
a=(al+dh)/b
z=z1+ddz
x=x1+ddx

```

c Scaling of volume and areasa

```

av=(v1+ddv)*pi
ar=(ar1+ddar)*pi
ss=ss1+ds
sa=(a+0.001d00)*10.0d00
ij=sa

```

```

c =====OUTPUT=====
      write(9,999)a,ss,x,-z
999 format(4(f16.8,2x))
c =====OUTPUT=====

```

```

c      write(6,51)a,ss,x,-z,ar,av
51 format(3X,f8.2,2x,5(e15.8,2x))
c      punch 500,bbc,ij,x,z
      if(ij.eq.900) n=n+5
      if(n.gt.9) goto 22

```

c Reloading most advanced point

```

mm=2
i=0
ds=dsa
al=a2
z1=z2
x1=x2
v1=v2
ar1=ar2
ss1=ss2
if(l) 36,22,37
37 ang=ang+deltaangle*b
goto 38
36 ang=ang-deltaangle*b
38 continue
      if(ang+0.01d00) 22,35,35
c 35 continue
35 if(ang-3.142d00) 7,7,22

```

c Update variables

```

7 continue
12 a=a1
z=z1
x=x1

```

```

v=v1
ar=ar1
ss=ss1
goto 2
c 22 if(bbc.gt.0.58d00) goto 27
c   goto 210
22 continue

```

```

C =====
C Read the reference profile which is not perturbed
C =====

```

```

      OPEN(UNIT=18,STATUS='OLD', FILE='b0=1-1800.dat')

      do 1003 j=1, 1801
      read(18,*) t11, t12, xoriginal1800(j), yoriginal1800(j)
1003   continue
      close(unit=18)

      OPEN(UNIT=21,STATUS='OLD', FILE='xz.dat')

      do 1001 i=1, 19
      read(21,*) tt1, tt2, Xpertur(i), Ypertur(i)
1001   continue
      CLOSE(UNIT=21)

      write(*,*) "two end point", yoriginal1800(1801), Ypertur(19)
      write(*,*) "two first point", yoriginal1800(1), Ypertur(1)
      DistPerturbation= yoriginal1800(1801)- Ypertur(19)
      write(*,*) "shift with Z=", DistPerturbation
      write(*,*) "shift with x=", xoriginal1800(1801)- xpertur(19)

      write(*,*) "----- X and Z conordinate -----"
      do 1002 i=1, 19
      if(DistPerturbation.gt.0) then
        Ypertur(i) = Ypertur(i) + abs(DistPerturbation)
      else if(DistPerturbation.lt.0) then
        Ypertur(i) = Ypertur(i) - abs(DistPerturbation)
      end if
      write(*,*) Xpertur(i), Ypertur(i)
1002   continue

```

```

C =====
C Comparing two profiles' every points and search the shortest distance
C =====

```

```

      do 1005 m = 1, 19

        if(m .eq. 1) then
          iup = 1
          idown = m + 30
          end if

        if(m.gt.1 .and. m.lt.19) then
          iup = (m-1)*100 - 30
          idown = (m-1)*100 + 30
          end if

```

```

        if(m.eq. 19) then
            iup = 1770
            idown = 1801
        end if
dmin(m) = 100
        do 1004 i2=iup, idown

            dx1 = Xpertur(m) - xoriginal1800(i2)
            dy1 = Ypertur(m) - yoriginal1800(i2)

            dist = sqrt( dx1*dx1 + dy1*dy1 )

            if(dist .lt. dmin(m)) then
                dmin(m) = dist
            end if

1004      continue
1005  continue

      OPEN(UNIT=10,STATUS='OLD', FILE='b0=1-18.dat')
      do 1006 i3 = 1, 19
      read(10,*)t1, sl(i3), t2, t3
1006  continue
      close(unit=10)

      write(*,*) dmin(1), dmin(19)

      dmin(1) = abs(DistPerturbation)
      dmin(19)= 0
      Vmax = -1.
      Smax = -1.
      do 1008 i44=1,19
      if(dmin(i44) .gt. Vmax) then
          Vmax = dmin(i44)
      end if
      if(sl(i44) .gt. smax) then
          smax = sl(i44)
      end if
1008  continue
C ===== OUTPUT =====
      write(*,*) "-----"
      write(*,*) " Length maximum distance is = ", Vmax
      write(*,*) " delta_gama          is = ", gamma*1000.00 - 72.0000
      write(*,*) " delta_B0              is = ", 1.0/ep -0.400000
      write(*,*) "-----"
      do 1009 i55 =1,19
1009  write(*,*) dmin(i55)/Vmax, sl(i55)/Smax

      write(*,*) "-----"
      write(*,*)(xoriginal1800(1801)- xpertur(19))/Vmax
      write(*,*) "-----"

      stop
      end

```

C Input File : xz.dat - that is come from Lal.f
 C Data: Nov. 15, 199

```
C  PROGRAM SURTE
  IMPLICIT REAL*8(A-H,O-Z)
  Real*8 delx, delz, Gama0,q30, B0NEW, R0NEW, delLengthLess
  real*8 delLength_max
  CHARACTER*1 NAME*60
  COMMON/INPUT/ X(100),Z(100),GRAVITY,DELRHO,GAMA.Q(4),ZMAX.
+      NPTS,NCYCL,INCRNT,NAME,delLength,cca,cc,cc_new
+      ,XendDmax
  DATA GRAVITY/981.00/, DELRHO/1.000/, GAMA/00.0/,
+  ZMAX/0.56/, NCYCL/3/, INCRNT/300/
```

```
C-----
c      write(*,*) "-----"
c      Write(*,*) "input delLength "
c      read(*,*) delLengthLess_max
C      write(*,*) " please inpu new cc "
C      read(*,*) cc_new
C      write(*,*) " input X-end/D_max "
C      read(*,*) XendDmax

      XendDmax = 0.363
      cc_new = 369.1205765058350
      write(*,*) "-----"
```

```
c===== Keep oringal Gama and R0 =====
C q(3) is a dimensionless R0, it is B0 symbol called
C cc is sqrt(c) . c is capillary constant = delta_density
C * gravity / surface_tension
C so cc is 1/m
C-----
```

```
      delLength=0.0
      Gama=0.0
      CALL ADSA
      gama0 = gama
```

```
C-----
C CC is 1/m
C q(3) is cm
C q(3)/100 is 1/m
C q(3)/100 * cc is dimensionless
C-----
```

```
      B0 = (q(3)/100.d00)*cc
      ccR0 = q(3)
      write(*,*) "Original gama = ", gama0
      write(*,*) "Original B0 = ", B0
      write(*,*) "Original R0 = ", R0
      write(*,*) "Original sqrt_C ", cc
      write(*,*) "--- End of output original varibales "
```

```
C ----- End of output original varibales -----
```


Appendix D

Program – Predicted Precision of Surface Tension using Perturbation Distribution, λ

```
C=====
C Larry Zhou
C Date: 2000
C Supervisor : Professor John Gaydos
C=====
```

```
C=====
C Function:
C The program predicts the precision of the surface tension using the
C perturbation distribution generated in Appendix C
C=====
```

```
C=====
C Input:
C 1. The original profile data, from "b0=0.4-18ADSA.dat"
C 2. The perturbation distribution, from "ratio.dat"
C Output :
C 1. The surface tension error.
C 2. The maximum perturbation length.
C=====
```

SUBROUTINE ADSA

```
IMPLICIT REAL*8(A-H,O-Z)
CHARACTER*1 NAME*60
DIMENSION Q0(4),EQ0(4),EQQ0(4,4),F0(3), zr(100), xr(100)
+.xb(100),zb(100), SitaTan(100), DeltaX(100), DeltaZ(100)
+.SitaTan0(100)
COMMON/INPUT/ X(100),Z(100),GRAVITY,DELRHO,GAMA,Q(4),ZMAX,
+ NPTS,NCYCL,INCRNT,NAME,delLength,cca, cc,cc_new
+ .XendDmax
COMMON/BLOCK1/ F(3),G(3),H(3),ERR,EQ(4),EQQ(4,4),SOLAST
+ ,SMAX,DELS
COMMON/INPUTAT/xi(20),yi(20),ai(10),dti(3),mAt,nAt
COMMON/OUTPUTAT/zAtAt,tanAngle
real*8 ratio(20)
```

```
C=====
C===== Transfer Data from Laplace Program =====
C=====
Do 888 i=1,100
xr(i)=0.0
zr(i)=0.0
```

```

      xb(i)=0.0
      zb(i)=0.0
      x(i)=0.0
888  z(i)=0.0

```

```

      Open(Unit=2, Status='Old', File='b0=0.4-18ADSA.dat')
      Open(Unit=3, Status='Old', File='xzless.out')
      Open(Unit=5, Status='Old', File='xzd.out')
      Open(Unit=4, Status='Old', File='profile.out')

```

```

      Read(2,*) cc, NPT

```

```

      DO 99 i=1,NPT
      READ(2,*) SitaTan0(i),stemp,xr(i), zr(i)
99  continue
      close(unit=2)

```

```

      j=1

```

```

      cca=100.00d00*(1.00D00/cc)

```

```

C=====
C cca now is cm
C because input cc 369 l/m
C 1/cc -> 1./369 (m)
C 100. * (1/cc) = 100./369 ( cm)
C Input X and Z from Laplace is dimensionLess coordinates
C so after X*100./cc, x becomes cm
C X with cm is input for ADSA program
C=====

```

```

      x(1)=0.0
      z(1)=0.0
      SitaTan(1) =0.0

```

```

      Do 999 i=1,NPT
      j=2*i
      x(j)=xr(i)
      x(j+1)=-xr(i)
      z(j)=-zr(i)
      z(j+1)=zr(i)
      SitaTan(j)=SitaTan0(i)
      SitaTan(j+1)=SitaTan0(i)
999  continue

```

```

      NPTs=2*NPT+1

```

```

c===== Ini Xb, Zb =====

```

```

      do 990 jj=1,NPTs
      xb(jj)=x(jj)*1.0d00
990  zb(jj)=z(jj)*1.0d00

      do 991 jj=NPTs+1,100
      xb(jj)=x(jj)*0.0d00
991  zb(jj)=z(jj)*0.0d00

```



```

c=====
c Perturbed X and Z with small step
c=====

```

```

      PI=DACOS(-1.D0)
      Do 600 i=1, NPTS
        SitaNew=SitaTan(i)*PI/180.0
        DeltaX(i)=dabs(delLength)*sin(sitaNew)
        DeltaZ(i)=dabs(delLength)*cos(sitaNew)
600  continue

```

```

c=====
c                               Input the perturbation distribution
c=====

```

```

      Open(Unit=22, Status='Old', File='ratio.dat')
      do 669 ira= 1,19
        read(22,*)ratio(ira), temp003
C      ratio(ira) = 1.0
669  continue
      close(unit=22)

```

```

c=====
c                               Boundary Conditions
c=====

```

```

      do 880 i1=1, 18
        i2 = i1*2
        DeltaX(i2)=DeltaX(i2)*ratio(i1+1)
        DeltaX(i2+1)=DeltaX(i2+1)*ratio(i1+1)
        DeltaZ(i2)=DeltaZ(i2)*ratio(i1+1)
        DeltaZ(i2+1)=DeltaZ(i2+1)*ratio(i1+1)
880  continue

```

```

      DeltaX(1)=DeltaX(1)*ratio(1)
      DeltaZ(1)=DeltaZ(1)*ratio(1)
      DeltaX(36) = DelLength * XendDmax
      DeltaZ(36) = 0.0
      DeltaX(37) = DelLength * XendDmax
      DeltaZ(37) = 0.0

```

```

      do 997 j=1,NPTs

```

```

c=====
c According to the position of point, to decide how to perturb
c=====

```

```

      if(delLength.gt.0.0) then

        if(xb(j).gt.0.0) then
          x(j)=xb(j)+DeltaX(j)
        end if

```

```

if(xb(j).le.0.0) then
x(j)=xb(j)-DeltaX(j)
end if

if(DeltaZ(j).lt.0.0) then
z(j)=zb(j)+abs(DeltaZ(j))
end if

if(DeltaZ(j).ge.0.0) then
z(j)=zb(j)-DeltaZ(j)
end if

End if

```

```

if(delLength.lt.0.0) then

if(xb(j).gt.0.0) then
x(j)=xb(j)-DeltaX(j)
end if

if(xb(j).lt.0.0) then
x(j)=xb(j)+DeltaX(j)
end if

if(DeltaZ(j).le.0.0) then
z(j)=zb(j)-abs(DeltaZ(j))
end if

if(DeltaZ(j).ge.0.0) then
z(j)=zb(j)+DeltaZ(j)
end if

End if

```

997 continue

```

C      Boundary Condition:
C      Move profile back to (0,0) point

```

```

      Vmove_zero = DeltaZ(1)
      Do 801 i=1, NPTs
      if(delLength.gt.0.0) z(i)=z(i)+dabs(Vmove_zero)
      if(delLength.lt.0.0) z(i)=z(i)-dabs(Vmove_zero)
801   continue

```

```

c      if(delLength .ne. 0.0 ) then
      write(*,*) "cc_new is ",cc_new

```

```

      cca = 100.00d00*(1.00D00/cc_new)
      end if

      do 802 i802=1, npts
c       if(x(i802) .ge. 0.0) then
c       write(*,*) x(i802), -z(i802)
c
c       x(i802) = x(i802) *cca
c       z(i802) = z(i802) *cca

802      continue
=====
c Above is a program to generate a perturbing profile according to perturbation distribution
c Following code is original ADSA-P code.
=====

      KSOL=1
      IF (GAMA.NE.0.D0) KSOL=2
      CALL INIT(KSOL,PI)
      IF (KSOL.EQ.1) THEN
        K=-5
        ALFA=0.005D0
        M=0
        A=DABS(GRAVTY)/GRAVTY
      ELSE
        ALFA=DELRHO*GRAVTY/GAMA
      END IF

C
C .....
C
C . Implementing the combined optimization method. Q(4) is the
C . loading parameter: applicable only when KSOL=1.
C .....
C

      DO 16 L=1,NCYCL
        DELS=DELS/(2**(L-1))
c      WRITE (6,28) L,DELS
        ERR0=1.0D10
        ER0=ERR0
        GO TO (1,12),KSOL
1      MJOB=3
        CALL SYSEQ(KSOL,MJOB)
        CALL PRINT
        IF (DABS(ERR0-ERR)/ERR.LT.1.0D-9) GO TO 3
        ERR0=ERR
        DO 2 I=1,3
          IF (DABS(EQ(I)).GT.1.0D-9) GO TO 9
2      CONTINUE
3      IF (DABS(ER0-ERR)/ERR.LT.1.0D-9.OR.
+        DABS(EQ(4)).LT.1.0D-9) GO TO 16
        IF (ERR.GT.ER0) GO TO 10
        ER0=ERR
        IF (K.LE.3) ALFA=ALFA*2
        IF (ALFA.GT.1) ALFA=1
        DO 4 I=1,4
          Q0(I)=Q(I)

```

```

      EQ0(I)=EQ(I)
      DO 4 J=1,4
4      EQQ0(I,J)=EQQ(I,J)
      CALL SOLVE(4)
      DEL4=EQ(4)
      M=0
5      DEL=DEL4*ALFA
c      WRITE (6,100) M,K,ALFA
100    FORMAT(2X,'*** 'I2,2X,I2,2X,D12.5,'***')
      Q(4)=Q0(4)-DEL
      IF ((A-DABS(Q(4))/Q(4)).NE.0.D0) GO TO 27
      DO 6 I=1,3
        Q(I)=Q0(I)
        EQ(I)=EQ0(I)-EQQ0(I,4)*DEL
      DO 6 J=1,3
6      EQQ(I,J)=EQQ0(I,J)
      K=1
7      CALL SOLVE(3)
      DO 8 I=1,3
8      Q(I)=Q(I)-EQ(I)
      GO TO 1
9      K=K+1
      IF (K.LE.5) GO TO 7
10     ALFA=ALFA/2
      M=M+1
      IF (M.NE.9) GO TO 5
      DO 11 I=1,4
        IF (EQ0(I).GT.1.D-5) GO TO 27
11     Q(I)=Q0(I)
C
C .....
C . Solving for the case when GAMA is specified .
C .....
C
12 CONTINUE
13 MJOB=3
      Q(4)=ALFA*Q(3)*Q(3)
      CALL SYSEQ(KSOL,MJOB)
      CALL PRINT
      IF (DABS(ERR0-ERR)/ERR.LT.1.0D-9) GO TO 16
      ERR0=ERR
      DO 15 I=1,3
        IF (DABS(EQ(I)).GT.1.0D-9) THEN
          CALL SOLVE(3)
          DO 14 J=1,3
14         Q(J)=Q(J)-EQ(J)
          GO TO 13
        END IF
15 CONTINUE
16 CONTINUE
C
C .....
C .
C . Integrating the Laplacian curve to determine the value
C . of the contact angle at  $Z+Q(2)=Z_{MAX}$ , ( $Z=Q(3)*F(2)$ ), and
C . the values of the surface area and the volume of the drop.

```

```

C .....
C
  IF (ZMAX.EQ.0) GO TO 24
  A=0
  V=0
  A1=0
  V1=0
  S0=0
  J=1
  MJOB=1
  DO 17 I=1,3
17   F(I)=0
18   S1=S0+DELS
      CALL I2EULR(S0,S1,Q(4),MJOB)
      SS=Q(3)*F(2)+Q(2)-ZMAX
      IF (SS. GT. 0) THEN
19     DLS=(Q(3)*F(2)+Q(2)-ZMAX)/(Q(3)*DSIN(F(3)))
        IF (DABS(DLS/SMAX). LT. 1.D-9) J=2
        S1=S0-DLS
        S0=S0LAST
        DELS=S1-S0
        DO 20 I=1,3
20     F(I)=F0(I)
        CALL I2EULR(S0,S1,Q(4),MJOB)
        GO TO (19,21),J
      END IF
21   A2=F(1)*DSIN(F(3))
      A=A+0.5D0*DELS*(A1+A2)
      A1=A2
      V2=F(1)*A2
      V=V+0.5D0*DELS*(V1+V2)
      V1=V2
      S0LAST=S0
      DO 22 I=1,3
22   F0(I)=F(I)
      GO TO (18,23),J
C --> Surface area
23   A=2*PI*Q(3)*Q(3)*A
C --> Volume
      V=PI*Q(3)*Q(3)*Q(3)*V
C --> Contact radius
      XC=Q(3)*F(1)

24   IF (Q(4).NE.0) GO TO 25
      GAMA=-0.0D0
      GO TO 26
C --> Surface tension

C -----
C -----

C -----
C -----
C .....
C .
C . Printing the computational results. .

```

```

C .....
C -----
25  Gama= DELRHO*GRAVITY*Q(3)*Q(3)/Q(4)

C --> Contact angle
26  PHI=F(3)*180/PI
c    WRITE (6,29) NAME,Q(1),Q(2),Q(3),GAMA,A,V,PHI,XC
c    WRITE (6,*) NAME,Q(1),Q(2),Q(3),GAMA,A,V,PHI,XC
C
c    WRITE(2,29) NAME,Q(1),Q(2),Q(3),GAMA,A,V,PHI,XC

C -----
C -----

      return
27  Continue

c    WRITE (6,30)
C
C
28  FORMAT(' ',20X,I2,' OPTIMIZATION CYCLE.'/
+ 20X,INCREMENTAL ARC-LENGTH, DELS :',D15.8/)
29  FORMAT(' ',5X,///15X,COMPUTATIONAL RESULTS:',14X,24('.'/
+ 8X,SAMPLE: ',A60//
+ 10X,'A. Origin of the drop coordinate system :'/
+ 40X,X0 (cm) ',10('.'/),D13.6/40X,Z0 (cm)',10('.'/),D13.6/
+ 10X,'B. Radius of curvature at origin (cm)',10('.'/),D13.6/
+ 10X,'C. Surface tension (ergs/cm sq.) ',15('.'/),D13.6/
+ 10X,'D. Surface area (cm sq.) ',23('.'/),D13.6/
+ 10X,'E. Volume (cm cu.) ',29('.'/),D13.6/
+ 10X,'F. Contact angle (degrees) ',21('.'/),D13.6/
+ 10X,'G. Contact radius (cm) ',25('.'/),D13.6/)
30  FORMAT(' ',3X,** TERMINAL ERROR. CHECK THE INPUT DATA. **)
      close(4)
      RETURN
      END

C
C
      SUBROUTINE INIT(KSOL,PI)
C .....
C .
C . In subroutine INIT the first values of array Q are determined. .
C .....
      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*1 NAME*60
      COMMON/INPUT/ X(100),Z(100),GRAVITY,DELRHO,GAMA,Q(4),ZMAX,
+ NPTS,NCYCL,INCRNT,NAME,delLength,cca,cc,cc_new
      COMMON/BLOCK1/ R(6),E(6),A,B,C,D,F,G,H,O,DEL,DEL1,E0,E00,ALFA,
+ Q1,Q2,E3,E6,E5,DUM,SMAX,DELS
      COMMON/BLOCK2/ T(124),P(124)

C
C
      PI=DACOS(-1.D0)
      A=0

```

```

      B=0
      DO 2 I=1,NPTS
        IF (Z(I).LT.A) GO TO 1
        M=I
        A=Z(I)
1      IF (DABS(X(I)).LT.B) GO TO 2
        K=I
        B=DABS(X(I))
2      CONTINUE
C
C .....
C .
C . Constructing the merit function E(1) and its first and second
C . partial derivatives E(I) I=2,...,6. with respect to Q(1) and
C . Q(2).
C .....
C
      A=0
      B=0
      DO 3 I=1,NPTS
        A=A+X(I)
3      B=B+Z(I)

=====
c larry zhou
=====
      Q(1)=A/NPTS
      Q2=4*DABS(X(K)-Q(1))
      DEL=Q2/12
      Q(2)=B/NPTS-Q2
      Q2=Q2+B/NPTS

=====
c larry zhou
=====

      ALFA=0.05D0
      E0=1.D10
      E00=E0
      L=-5
      K=0
      INI=1
4      CONTINUE
      DO 5 I=1,6
        R(I)=0
5      E(I)=0
      DO 6 I=1,NPTS
        A=X(I)-Q(1)
        B=Z(I)-Q(2)
        D=DSQRT(A*A+B*B)
        C=1/D
        F=C*C*C
        R(1)=R(1)+D
        R(2)=R(2)+A*C
        R(3)=R(3)+B*C

```

```

        R(4)=R(4)+A*A*F-C
        R(5)=R(5)+A*B*F
6      R(6)=R(6)+B*B*F-C
        DO 7 I=1,6
7      R(I)=R(I)/NPTS
        DO 8 I=1,NPTS
            A=X(I)-Q(1)
            B=Z(I)-Q(2)
            D=DSQRT(A*A+B*B)
            C=1/D
            F=C*C*C
            G=R(2)-A*C
            H=R(3)-B*C
            O=D-R(1)
            E(1)=E(1)+O*O
            E(2)=E(2)+O*G
            E(3)=E(3)+O*H
            E(4)=E(4)+G*G+O*(R(4)+C-A*A*F)
            E(5)=E(5)+G*H+O*(R(5)-A*B*F)
8      E(6)=E(6)+H*H+O*(R(6)+C-B*B*F)
C
C .....
C .
C . Locating the initial position of the center .
C . of the fitted circle.
C .....
C
IF (INI.EQ.0) GO TO 11
IF (Q(2).GT.Q2) GO TO 9
K=K+1
T(K)=E(1)
P(K)=Q(2)
Q(2)=Q(2)+DEL
GO TO 4
9  A=1.D10
    DO 10 I=1,K
        IF (T(I).GT.A) GO TO 10
        J=I
        A=T(I)
10  CONTINUE
    Q(2)=P(J)
    INI=0
    GO TO 4
C
C .....
C .
C . Optimization method using the combined INCREMENTAL LOADING
C . method and the NEWTON-RAPHSON method. The loading parameter is
C . Q(1).
C .....
C
11 IF (DABS(E0-E(1))/E(1).LT.1.D-9) GO TO 12
    E0=E(1)
    IF (DABS(E(3)).GT.1.D-9) GO TO 15
12 IF (DABS(E0-E(1))/E(1).LT.1.D-9. OR .DABS(E(2)).LT.1.D-9)
    + GO TO 16

```



```

E00=E(1)
IF (L.LE.3) ALFA=ALFA*2
IF (ALFA.GT.1) ALFA=1
E3=E(3)
E5=E(5)
E6=E(6)
DEL1=(E3-E6*E(2)/E5)/(E5-E6*E(4)/E5)
Q1=Q(1)
Q2=Q(2)
13 DEL=DEL1*ALFA
   Q(1)=Q1-DEL
   Q(2)=Q2

   Q(1)=Q1
   Q(2)=Q2

C=====
C=====
E(3)=E3-DEL*E5
E(6)=E6
L=1
14 Q(2)=Q(2)-E(3)/E(6)
   GO TO 4
15 L=L+1
   IF (L.LE.5) GO TO 14
   ALFA=ALFA/2
   GO TO 13
C
C .....
C .
C . Establishing the first values of Q(I) and DELS. .
C .....
C
16 Q(2)=Q(2)-R(1)
   Q(3)=R(1)
   Q(4)=0
   IF (KSOL.EQ.2) Q(4)=DELRHO*GRAVITY*Q(3)*Q(3)/GAMA
   SMAX=PI-2*DATAN(DABS((X(M)-Q(1))/(Z(M)-Q(2))))
   DELS=SMAX/INCRNT
   RETURN
   END
C
C
C
SUBROUTINE SYSEQ(KSOL.MJOB)
C .....
C .
C . In subroutine SYSEQ the objective function and its 1-st and
C . 2-nd partial derivatives are constructed. For every datum point,.
C . the routine locates initially a point on the calculated curve
C . which is closest to the datum point and then calculates the
C . function value and the derivatives at that point. The global set.
C . of equations is obtained by summing up these individual
C . expressions.
C . Subroutine SYSEQ calls subroutine I2EULR.
C .....

```

```

      IMPLICIT REAL*8(A-H,O-Z)
      CHARACTER*1 NAME*60
      COMMON/INPUT/ X(100),Z(100),GRAVITY,DELRHO,GAMA,Q(4),ZMAX,
+       NPTS,NCYCL,INCRNT,NAME,delLength,cca,cc,cc_new
+       .XendDmax
      COMMON/BLOCK1/ F(3),G(3),H(3),ERR,EQ(4),EQQ(4,4),S0LAST,
+       SMAX,DELS
      COMMON/BLOCK2/ F0(3),G0(3),H0(3),ERRSQ(100),ESQ(4),QQ(10),
+       L(100),LP(100),IK(25),JK(25)
C
      Q1=Q(1)
      Q2=Q(2)
      Q3=Q(3)
      Q4=Q(4)
      ERR=0
      DO 1 I=1,4
          EQ(I)=0
          DO 1 J=1,4
1          EQQ(I,J)=0
      S0LAST=0
      DO 2 I=1,3
          F(I)=0
          F0(I)=0
          G0(I)=0
2          H0(I)=0
      DO 3 I=1,NPTS
          L(I)=I
          LP(I)=1
          A=X(I)
          B=Z(I)
          IF (A.LT.Q1) LP(I)=-1
3          ERRSQ(I)=(Q1-A)*(Q1-A)+(Q2-B)*(Q2-B)
      M0=NPTS
      KL=0
C
C .....
C
C . Locating the point Sm on the curve X=X(S), Z=Z(S)
C . ( X=Q3*F(1), Z=Q3*F(2) ), which is closest to a datum
C . point (X(NPIV),Z(NPIV)).
C .....
C
      DO 22 NP=1,NPTS
          KJOB=1
          S0=S0LAST
4          S1=S0+DELS
5          CALL I2EULR(S0,S1,Q4,MJOB)
          IF (KJOB.NE.1) GO TO 11
          DO 8 N=1,M0
              M=L(N)
              A=Q3*F(1)+LP(M)*(Q1-X(M))
              B=Q3*F(2)+Q2-Z(M)
              SQI=A*A+B*B
              IF (SQI-ERRSQ(M)) 6,6,7
6          ERRSQ(M)=SQI
              GO TO 8

```

```

7   KL=KL+1
    IK(KL)=M
    JK(KL)=N
8   CONTINUE
    IF (KL.NE.0) GO TO 10
    S0LAST=S0
    DO 9 I=1,3
        F0(I)=F(I)
        G0(I)=G(I)
9   H0(I)=H(I)
    GO TO 4
10  NPIV=IK(KL)
    K=LP(NPIV)
    MJOB=1
    KJOB=2
11  A=Q3*F(1)+K*(Q1-X(NPIV))
    B=Q3*F(2)+Q2-Z(NPIV)
    COS=DCOS(F(3))
    SIN=DSIN(F(3))
    APB=A*COS+B*SIN
    BMA=B*COS-A*SIN
    IF (DABS(F(2)).LT.1.0D-12) GO TO 14
    ESS=BMA*(2+Q4*F(2)-SIN/F(1))+Q3
    IF (KJOB.EQ.3) GO TO 15
    SQI=APB/ESS
    S1=S0-SQI
    IF (DABS(SQI/SMAX).GT.1.0D-9) GO TO 12
    MJOB=3
    KJOB=3
12  S0=S0LAST
    DO 13 I=1,3
        F(I)=F0(I)
        G(I)=G0(I)
13  H(I)=H0(I)
    GO TO 5

C
C .....
C
C . Constructing the objective function ERR, and its 1-st and
C . 2-nd partial derivatives EQ(I) and EQQ(I,J), respectively.
C .....
C
14  MJOB=3
    ESS=Q3+Q2-Z(NPIV)
15  CONTINUE
    ESQ(1)=K*COS
    ESQ(2)=SIN
    ESQ(3)=APB/Q3+F(1)*COS+F(2)*SIN
    ESQ(4)=BMA*G(3)+Q3*(G(1)*COS+G(2)*SIN)
    QQ(1)=1
    QQ(2)=0
    QQ(3)=K*F(1)
    QQ(4)=K*Q3*G(1)
    QQ(5)=1
    QQ(6)=F(2)
    QQ(7)=Q3*G(2)

```

```

      QQ(8)=F(1)*F(1)+F(2)*F(2)
      QQ(9)=A*G(1)+B*G(2)+Q3*(F(1)*G(1)+F(2)*G(2))
      QQ(10)=Q3*(A*H(1)+B*H(2)+Q3*(G(1)*G(1)+G(2)*G(2)))
      ERR=ERR+A*A+B*B
      EQ(1)=EQ(1)+K*A
      EQ(2)=EQ(2)+B
      GO TO (16,17),KSOL
C
C --> Solving with four variables
16  IV=4
      EQ(3)=EQ(3)+A*F(1)+B*F(2)
      EQ(4)=EQ(4)+Q3*(A*G(1)+B*G(2))
      GO TO 18
C
C --> Solving with three variables
17  IV=3
      Q4Q3=(Q4+Q4)/Q3
      EQ4=Q3*(A*G(1)+B*G(2))
      ESQ(3)=ESQ(3)+ESQ(4)*Q4Q3
      QQ(3)=QQ(3)+QQ(4)*Q4Q3
      QQ(4)=QQ(5)
      QQ(5)=QQ(6)+QQ(7)*Q4Q3
      QQ(6)=QQ(8)+(QQ(9)+QQ(9)+QQ(10)*Q4Q3+EQ4/Q3)*Q4Q3
      EQ(3)=EQ(3)+A*F(1)+B*F(2)+EQ4*Q4Q3
C
18  CONTINUE
      IJ=0
      ESS=Q3/ESS
      DO 19 I=1,IV
        SQI=-ESQ(I)*ESS
        DO 19 J=I,IV
          IJ=IJ+1
19  EQQ(I,J)=EQQ(I,J)+ESQ(J)*SQI+QQ(IJ)
C
      M0=M0-1
      IF (M0.EQ.0) GO TO 23
      DO 20 I=1,M0
        IF (I.LT.JK(KL)) GO TO 20
        L(I)=L(I+1)
20  CONTINUE
      KL=KL-1
      IF (KL.GT.0) GO TO 10
      DO 21 I=1,3
        F(I)=F0(I)
        G(I)=G0(I)
21  H(I)=H0(I)
22  CONTINUE
23  CONTINUE
      DO 24 I=1,IV
        DO 24 J=I,IV
24  EQQ(J,I)=EQQ(I,J)
      RETURN
C
C
      ENTRY PRINT
c  WRITE (6,100) ERR,(Q(I),I=1,4),(EQ(I),I=1,4)

```

```

100 FORMAT(' ',2X, Value of error function:75X,D19.12/
+ 22X, Value of Q(I), I=1,4:7,4(25X,D15.8/),
+ 42X, Value of EQ(I), I=1,4:7,4(45X,D15.8/)/)
RETURN
END
C
C
C
SUBROUTINE I2EULR(S0,S1,Q4,K)
C .....
C .
C . I2EULR is a second order implicit EULER METHOD of integration.
C . This subroutine will do a single step integration of the system
C . of first order ordinary differential equations:
C .  $V(S)=U(S,V(S)), V=(F(3),G(3),H(3))$ 
C . given the initial condition  $V0=V(S0)$ .
C .....
IMPLICIT REAL*8(A-H,*O-Z)
DIMENSION DELF(3),DELG(3),DELH(3)
COMMON/BLOCK1/ F(3),G(3),H(3),DUM(24)
C
C
DELS=S1-S0
DEL=DELS/2
IF (F(1).EQ.0) THEN
DO I=1,3
F(I)=DELS
G(I)=0
1 H(I)=0
F(2)=F(2)*DEL
S0=S1
RETURN
END IF
C
C .....
C .
C . Calculating the increments DELF(3), DELG(3) and DELH(3).
C .....
C
SIN=DSIN(F(3))
COS=DCOS(F(3))
PS1=SIN/F(1)
PC1=COS/F(1)
PS2=PS1/F(1)
A1=DEL*SIN
A2=-DEL*COS
A3=DEL*PS2
A4=DEL*Q4
A5=1+DEL*PC1+A1*A3-A2*A4
B1=DELS*COS
B2=DELS*SIN
B3=DELS*(2+Q4*F(2)-PS2)+B1*A3+B2*A4
DELF(3)=B3/A5
DELF(2)=B2-A2*DELF(3)
DELF(1)=B1-A1*DELF(3)
GO TO (7,2,2),K

```

```

C
2  PS3=PS2/F(1)
   PC2=PC1/F(1)
   B1=-(DELS*SIN+DEL*COS*DELF(3))*G(3)
   B2=(DELS*COS-DEL*SIN*DELF(3))*G(3)
   B3=DELS*(G(1)*(PS2-PS3*DELF(1))+F(2)+Q4*G(2)-PC1*G(3))+
+ DEL*(PC2*G(3)*DELF(1)+DELF(2)+(PC2*G(1)+PS1*G(3))*DELF(3))
   B3=B3+B1*A3+B2*A4
   DELG(3)=B3/A5
   DELG(2)=B2-A2*DELG(3)
   DELG(1)=B1-A1*DELG(3)
   GO TO (5,5,3),K
C
3  G3G3=G(3)*G(3)
   G1G1=G(1)*G(1)
   G1G3=G(1)*G(3)
   PC3=PC2/F(1)
   B2=(G3G3*COS+H(3)*SIN)
   B3=G3G3*SIN-H(3)*COS
   B1=-(DELS*B2+DEL*(B3*DELF(3)-2*COS*G(3)*DELG(3)))
   B2=-(DELS*B3+DEL*(B2*DELF(3)+2*SIN*G(3)*DELG(3)))
   B3=DELS*(PS3*(G1G1*(3/F(1)-2)-H(1))+G1G3*(2*(PC2-PC3)
+ -PS2*DELF(3))+2*(G(2)-PS3*G(1)*DELG(1))+PC2*(G(3)*DELG(1)
+ +G(1)*DELG(3))+PS1*(G3G3+G(3)*DELG(3))+PS2*H(1)+Q4*H(2)
+ -PC1*H(3)-PC3*G1G1*DELF(3)+DELG(2))+DEL*(PC2*(H(3)*DELF(1)
+ +H(1))+G3G3*(PC1-PS2*DELF(1))+PS1*H(3)))
   B3=B3+B1*A3+B2*A4
   DELH(3)=B3/A5
   DELH(2)=B2-A2*DELH(3)
   DELH(1)=B1-A1*DELH(3)
C
C .....
C
C . Performing the integration step: V(I+1)=V(I)+DEL V(I).
C .....
C
DO 4 I=1,3
4  H(I)=H(I)+DELH(I)
5  CONTINUE
DO 6 I=1,3
6  G(I)=G(I)+DELG(I)
7  CONTINUE
DO 8 I=1,3
8  F(I)=F(I)+DELF(I)
   S0=S1
   RETURN
   END
C
C
C
C SUBROUTINE SOLVE(N)
C .....
C
C . This subroutine does a gaussian elimination on the
C . augmented matrix of the linear system of equations:
C . [A(N,N)]x{X(N)}={B(N)},

```

```

C . and it does the backward substitution in the solution .
C . of the above system of equations. The solution vector .
C . is returned in B(N).
C .....
  IMPLICIT REAL*8(A-H,O-Z)
  COMMON/BLOCK1/ DUM(10),B(4),A(4,4),DUMY(3)
C
C
  M=N-1
  DO 3 I=1,M
    L=I+1
    DO 2 J=L,N
      S=A(J,I)/A(I,I)
      DO 1 K=L,N
1      A(J,K)=A(J,K)-S*A(I,K)
2      B(J)=B(J)-S*B(I)
3    CONTINUE
C
C
  B(N)=B(N)/A(N,N)
  K=N
  DO 5 I=2,N
    L=K
    K=K-1
    S=B(K)
    DO 4 J=L,N
4      S=S-A(K,J)*B(J)
5    B(K)=S/A(K,K)
  RETURN
  END

```

Fig. 4.1

Definition of coordinate system for two immiscible homogeneous fluids separated by an interface. The arc length, s , is measured along the drop from the apex. The turning angle, θ , is defined as the angle between the vertical axis of symmetry and the line through the point (x_i, z_i) on the drop's profile. Radii R_1 and R_2 are the two principal radii of curvature; R_1 turns in the plane of the paper and R_2 rotates in the plane perpendicular to the plane of paper.

Fig. 4.2

Comparison of the observed vs. calculated drop profile. The ■ symbol represents the observed interfacial profile, while the solid curve represents the calculated Laplacian curve. $d(u_n, v)$ represents the normal distance.

Fig. 4.3
Computer-generated water drop profile, based on $\gamma=72\text{mJ/m}^2$

Fig. 4.4
Definition of Related Parameters

Fig. 4.5 Very Small Drop's Perturbation

P_0 is a profile coordinates point on the original sessile drop profile for a small, nearly-spherical drop. Point $\tilde{P}_0$ is a corresponding profile coordinates point on the perturbed drop sphere in a direction that is normal to the profile curve at P_0 ; that is, in direction $\hat{n}(P_0)$. We denote the original radius of the profile curve by R_0 and the perturbed profile radius by $\tilde{R}$. The distance between O and O' is $\tilde{R} - R_0$ and the distance between P_0 and $\tilde{P}_0$ in the direction $\hat{n}(P_0)$ as $\lambda_{\hat{n}}$. In both cases shown the drop contact the substrate at a contact angle of $\theta_t = 180^\circ$. Angle θ is the turning angle measured from horizontal.

Fig. 4.6
The Normal Perturbation Length Distribution for Very Small Drops

Fig. 4.7
The Normal Perturbation Length Distribution for Very Large Drops

Fig. 4.8
Influence of γ on the magnitude and shape of the perturbation length λ curve using a sessile drop with $\gamma=72\text{mJ/m}^2$ and $B_0=2$ as the reference (unperturbed) configuration.

Fig. 4.9
Influence of B_0 on the magnitude and shape of the perturbation length λ curve using a sessile drop with $\gamma=72\text{mj/m}^2$ and $B_0=2$ as reference (unperturbed) configuration.

Fig. 4.10
Influence of increasing and decreasing B_0 on the magnitude of the perturbation λ , based on a sessile drop with $\gamma=72\text{mJ/m}^2$ and $B_0=2$ as reference (unperturbed) configuration

Fig. 4.11

Normal Perturbation Length Distribution (λ) for various sessile drops characterized by B0 value with $\gamma=72 \text{ mJ/m}^2$ used as reference (unperturbed)

Fig 4.12
Precision of surface tension measurement, $\Delta\gamma$, as a function of the absolute maximum
perturbation length, λ , and initial B_0

Fig. 4.13 Normal perturbation λ_{eff} is changed dramatically with contact angle θ_i or final arc-length s_{final} . For example, in the figure (not to scale),

$\lambda_{eff}(\theta \approx \frac{1}{18}\pi)$ is about $\frac{1}{8} \times \lambda_{eff}(\theta > \frac{2}{3}\pi)$ when $B_0=2$.

Fig. 6.1
Effect on surface tension of random errors by individual profile point, using a sessile drop
with $\gamma = 72.00 \text{ mJ/m}^2$ and $Bo = 2$. The constant normal perturbation distance $\delta = 0.0025 \text{ cm}$

Fig. 6.2 Effect on surface tension of error in the local of position (Ref. 1, page 472)

Fig. 6.3
Effect on the Surface Tension of Randomly Perturbing the Sessile Drop's Profile