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Code Combining of Reed-Muller Codes in An Indoor Wireless Environment

submitted by

Welly T. Firmanto, B. S. E. E.

Purdue University

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering

Faculty of Engineering

Department of Systems and Computer Engineering

Carleton University

Ottawa, Ontario, Canada K1S 5B6

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**Code Combining of Reed-Muller Codes
in An Indoor Wireless Environment**

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Welly T. Firmanto, B. S. E. E.

In partial fulfillment of the requirements
for the degree of Master of Engineering

Thesis supervisor

Chair, Department of Systems and Computer Engineering

Carleton University

December 14, 1995

Abstract

Radio communications are desirable for data and voice communications in an indoor environment. Often, a user wishes to move around while communicating. Wireless technology offers an improved mobility since the users are not confined to any particular location by wires.

This thesis investigates the use of Reed-Muller codes in an indoor wireless environment. Equal gain code combining is used to exploit the multiple received copies of the transmitted signal, resulting from the inherent frequency diversity of spread spectrum. The performance of Reed-Muller codes will be compared with that of convolutional codes of similar rates and comparable decoding complexity.

The complete weight distributions of various Reed-Muller codes will be presented. Using these results, it will be shown that the performance of Reed-Muller codes is inferior to that of convolutional codes. This loss in performance may be compensated by using a lower rate code, since maximum likelihood decoding of Reed-Muller codes is generally computationally manageable.

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My family in Indonesia deserve a special thanks for their warm love and support, and for making it possible for me to get the finest of education abroad.

I reserve my final acknowledgement for Jesus Christ, my Lord and Savior, to whom all good things in my life can be traced. He whose mercies are new every morning has carried and will carry me when I am too weak to carry on.

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List of Acronyms

AWGN	Additive White Gaussian Noise
BCH	Bose Chaudhury Hocquenghem
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CC	Convolutional Code
CDMA	Code Division Multiple Access
CPU	Central Processing Unit
dB	Decibel
DS/SS	Direct Sequence/Spread Spectrum
FDMA	Frequency Division Multiple Access
FEC	Forward Error Control
GF	Galois Field
IWC	Indoor Wireless Channel
LOS	Line of Sight
ML	Maximum Likelihood
MRC	Maximum Ratio Combining
PBX	Private Branch Exchange

PCC	Punctured Convolutional Code
PG	Processing Gain
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
RM	Reed-Muller
WSSUS	Wide Sense Stationary Uncorrelated Scattering

List of Symbols

L	the number of resolvable paths
t	observation time instant
$h(\tau; t)$	time varying impulse response of a multipath channel
$h(t)$	time-invariant impulse response of a multipath channel
α_l	amplitude of the l 'th path
τ_l	time delay of the l 'th path
θ_l	phase of the l 'th path
$\delta(t)$	delta function
$s(t)$	transmitted signal
$r(t)$	received signal
$n(t)$	additive white Gaussian noise
$I_0(x)$	the zeroth order Bessel function of the first kind
$\varphi_h(\tau_1, \tau_2; \Delta t)$	autocorrelation of the channel impulse response
$E[x]$	statistical average of x
$\varphi_h(\tau)$	delay power spectrum
T_m	multipath delay spread
$\varphi_h(\Delta f; \Delta t)$	spaced-frequency spaced-time correlation function of a multipath channel
B_c	coherence bandwidth
$S_h(\lambda)$	Doppler power spectrum
B_d	Doppler spread
t_c	coherence time
T_b	bit period

N	the length of a signature sequence
T_c	chip period
n	code length
k	code dimension
u_k	k 'th information bit
$\mathbf{G}$	generator matrix
$\mathbf{u}$	information sequence
$\mathbf{v}$	codeword
$\mathbf{H}$	parity check matrix
$d_{\min}$	minimum distance
d_H	Hamming distance
m	memory of a convolutional code
d_{free}	free distance of a convolutional code
r	the order of a Reed-Muller code
R	code rate
K	number of users
l_A	number of receiving antennas
$c^k(t)$	the signature sequence for the k 'th user
c_n^k	the amplitude of the n 'th chip in $c^k(t)$
$p_{T_c}(t)$	unit pulse of duration T_c seconds
$a^k(t)$	information sequence
a_m^k	m 'th data bit in the information sequence
$p_{T_b}(t)$	unit pulse of duration T_b seconds
ω_c	carrier frequency in rad/s
f_c	carrier frequency in Hertz
A	signal amplitude
θ^k	signal phase
$h^k(t)$	lowpass equivalent of the passband channel impulse

	response for the k 'th user
U_i	i 'th decision variable
M	diversity order
t_i	branch metric of the i 'th bit in a codeword
$a_{\mathbf{v}_i}$	i 'th symbol/bit in a codeword sequence for the detector output $\mathbf{v}$
$\mathbf{v}_M$	code-combined version of $\mathbf{v}$
$P_c(d)$	pairwise probability that the transmitted codeword is decoded into another codeword which is Hamming distance d away
$F_{\mathbf{v}}$	cumulative path metric leading to decoder output $\mathbf{v}$
$\text{erfc}(x)$	the complementary error function
P_b	bit error rate
c_d	the total number of erroneous bits in all the incorrect paths of Hamming weight d
A_d	the number of codewords of weight d

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Chapter 1

Introduction

An ever increasing demand for communication services has been the major catalyst in the rapid development of wireless communication systems. A major shift from analog to digital modulation systems has also provided a boost to this development. Digital modulation allows for error control coding, diversity combining, and spread spectrum, all of which can be incorporated in a wireless communication system. With these techniques, wireless technology has found its place in numerous applications, providing a multitude of communication services [29].

One application which has become increasingly of interest is indoor wireless communications. Radio communications are often desirable for data and voice communications in office buildings, hospitals, and factories. Wireless technology offers an improved mobility, since the users are not confined to any particular location by wires. Demand on communication services fluctuates over time, so that an expansion or reduction of services is often needed. Wireless systems are capable of adapting to accommodate this dynamic behavior, as little rewiring is involved.

The wireless channel is a harsh environment to communicate through. Reflections from the building's structure, room furniture, and people moving in

the room cause the transmitted signal to arrive in multiple paths. Spread spectrum has been used in digital radio systems to combat the deteriorating effects of multipath channels [38], [39]. To achieve desirable performance, forward error control coding and diversity techniques are usually incorporated into these wireless systems.

1. 1 Thesis Motivation and Objectives

A number of research efforts have been devoted to investigating the performance of various wireless communication systems. Some of these systems make use of spread spectrum technique to provide multiple access capability [3], [14], [21]. Multiple copies, resulting from the inherent frequency diversity of spread spectrum, are often combined using maximum ratio combining or selection diversity [22]. A method whose complexity lies in between those of maximum ratio combining and selection diversity is code combining which uses equal gains. It is instructive to determine the performance loss incurred in using this method instead of maximum ratio combining.

Almost exclusively, convolutional codes with maximum likelihood decoding are used for error control purposes [1], [11], [12], [28], [43], [44]. However, block codes may be more desirable in certain applications, such as packet voice or data communications, in which the length of the packet may be made equal to a multiple of the length of the code. In these applications, block codes are more attractive than convolutional codes, since the memory of the convolutional codes must be flushed each time. To reduce decoding complexity, hard decision Viterbi decoding is often done. The use of soft information in maximum likelihood decoding is expected to raise the level of performance substantially.

Bit error rate evaluation of a coding scheme with maximum likelihood decoding requires knowledge of the number of erroneous information bits when decoding error occurs. This information is readily available for convolutional codes, making the performance straightforward to evaluate [8], [15], [41]. On the other hand, such information is not generally available for block codes. The weight distribution of a block code, which is available from literature, lists only the number of codewords of a given weight, allowing for the evaluation of the word error rate only [24], [26], [42]. For this reason, convolutional coding have gained greater popularity than block codes. This lack of information also prohibits any useful comparison between various block codes and convolutional codes with maximum likelihood decoding.

A lack of interest in block codes may also be attributed to the high level of complexity of their soft decision decoding [27]. Several authors have suggested some innovative ways to reduce the decoding complexity of block codes with little loss in performance [6], [25], [40]. Reed-Muller codes, being investigated in this thesis, have simple structures, allowing true soft decision maximum likelihood decoding [13], [19]. A literature research found that this class of block codes had not been widely used in wireless applications, if at all.

For these reasons, this thesis attempts to achieve the following objectives:

- i. to obtain the weight distributions of Reed-Muller codes which have simple trellis structures.
- ii. to analytically evaluate the performance of these codes in a wireless CDMA environment, and to compare their performance with that of convolutional codes of similar rates and comparable decoding complexity.
- iii. to incorporate soft information in the maximum likelihood decoding of block and convolutional codes.

- iv. to investigate the performance loss of code combining when equal gain combining is used rather than maximum ratio combining.

1. 2. Thesis Organization

This thesis is organized in the following manner. Chapter 2 describes the characteristics of indoor wireless communication (IWC) channels. A brief review of the mathematical and statistical models of multipath channels is presented. This chapter also provides an overview of forward error control coding, and relevant aspects of code division multiple access techniques (spread spectrum).

Chapter 3 gives a detailed description of the system under consideration, including the network structure, the transmitter, the channel, and the receiver. The theoretical performance of the system in terms of the bit error rate is derived.

In Chapter 4, new results on the weight distributions of various Reed-Muller codes are presented. The decoding complexity of various codes is assessed, and the performance of various codes is evaluated and compared, using the results from Chapter 3.

Chapter 5 summarizes the results in Chapter 4, draws conclusions, and provides suggestions for further research.

Chapter 2

Background

In order to obtain an acceptable performance despite severe channel impairments, a robust communication system needs to be developed. The selection of modulation, coding, and diversity schemes will be influenced a great deal by the characteristics of the channel. The proper system design, therefore, requires a sufficient characterization and statistical knowledge of the channel. A number of publications have characterized wireless channel and its propagation model [5], [10], [17], [18], [34], [37].

This chapter first provides a brief review of indoor wireless channel characterizations. Important and relevant aspects of forward error control coding and code division multiple access are described in later sections of this chapter.

2. 1. Indoor Wireless Communication Channels

While propagating through its medium, a radio wave often encounters obstructions blocking its propagation path. In an indoor environment, these obstructions may be the building's walls, ceilings, floors, room partitions, furniture, or people moving in the room. Due to these obstructions, radio wave propagation is usually governed by three general mechanisms; namely reflection,

diffraction, and scattering [2].

When a propagating electromagnetic wave encounters an obstruction with a dimension which is very large compared to the wavelength of the radio wave, reflection occurs. Reflected waves will arrive at a receiver with different phases due to different path lengths, and they will combine destructively or constructively. Meanwhile, impenetrable bodies, such as hard walls, cause diffraction on the transmitted signal. Diffraction allows radio frequency energy to propagate in the absence of a direct (LOS) path by forming secondary waves behind the obstructing body. This phenomenon is often termed shadowing. The third mechanism, scattering, is due to an obstruction with a dimension on the order of the wavelength of the propagating wave or less. Physically, by scattering, the energy is reradiated or scattered in many different directions. In the absence of a LOS path, radio wave propagation is usually dominated by scattering and diffraction, and conversely, they are unlikely to dominate the propagation when LOS is present. These propagation mechanisms are illustrated in Figure 2.1 which shows a simple propagation model in a room.

The three mechanisms explained above cause the transmitted signal to arrive at a receiver through more than one path, a phenomenon which is called multipath fading. The received signal is also corrupted by AWGN noise and interference from other users. In wide-band pulse transmission, the multipath channel produces a series of delayed and attenuated echoes of each transmitted pulse. With proper techniques, these echoes may be exploited to overcome the deteriorating effects of multipath channels.

Figure 2.1

A simple propagation mechanism in an indoor environment.

2. 2. Mathematical Modelling of a Multipath Channel

A time varying indoor wireless channel may be modeled as a linear time-varying filter, with an impulse response given by [17],

$$h(\tau; t) = \sum_{l=1}^L \alpha_l(t) \delta[t - \tau_l(t)] \exp[j\theta_l(t)] \quad (2.1)$$

where it has been assumed that the transmitted energy arrives in L discrete paths; t and τ are the observation time instant and the application time of the impulse, respectively. The variables $\{\alpha_l(t)\}$, $\{\tau_l(t)\}$, $\{\theta_l(t)\}$ are time-varying random variables, representing the amplitude, the arrival time, and the phase of the l 'th path, and $\delta(t)$ is the delta function. If time-variation of these quantities is slow compared to the data rate, i.e. they are constant over a few symbol intervals, the time-invariant version of this model may be introduced, which is given in (2.2).

$$h(t) = \sum_{l=1}^L \alpha_l \delta(t - \tau_l) e^{j\theta_l} \quad (2.2)$$

If the signal $s(t)$ is transmitted, the output $r(t)$ of the multipath channel can be obtained by convolving the transmitted signal and the channel's impulse response. Hence,

$$r(t) = \int_{-\infty}^{\infty} h(\tau) \tilde{s}(t - \tau) d\tau + n(t) \quad (2.3)$$

where $s(t) = \text{Re} \{ \tilde{s}(t) e^{j\omega_c t} \}$, ω_c is the carrier frequency in rad/s, and $n(t)$ is the lowpass equivalent of the complex-valued additive white Gaussian noise.

2. 3. Statistical Modelling of a Multipath Channel

In section 2.2, a multipath channel was completely characterized by its three components; the path amplitudes, the arrival times, and the corresponding phases. Since these components are random, for a complete channel characterization, it is necessary to specify their probability distributions. Furthermore, some correlation properties exist between different paths. These properties will be explained later in sub-section 2.3.2.

2. 3. 1. Amplitude Distributions

Consider the channel impulse response given in (2.2). If an unmodulated carrier is sent, the received signal $r(t)$ may be expressed in a discrete model as,

$$r(t) = \sum_{l=1}^L \alpha_l e^{j\theta_l} \quad (2.4)$$

In complex form, $r(t)$ may be written as,

$$r(t) = X_1(t) + jX_2(t) \quad (2.5)$$

where $j = \sqrt{-1}$.

The envelope R of the received waveform is given by,

$$R = \sqrt{X_1^2 + X_2^2} \quad (2.6)$$

By virtue of the central limit theorem, X_1 and X_2 approximate complex valued Gaussian random processes. In the absence of a strong LOS component, they both have zero mean, and the small scale rapid amplitude fluctuation is Rayleigh dis-

tributed with mean $\sqrt{\pi/2}\sigma$, and variance $(2 - \pi/2)\sigma^2$. The probability distribution of the envelope R , therefore, may be written as,

$$p_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \quad (2.7)$$

On the other hand, if a strong direct path exists in addition to the lower level scattered paths, X_1 and X_2 have non-zero means, and the envelope R follows a Rician distribution. In this case, the received signal consists of random, Rayleigh distributed scattered paths with random amplitude and phase, and a deterministic path with deterministic amplitude and phase. The envelope of the received waveform may be written as,

$$p_R(r) = \frac{r}{\sigma^2} e^{-(s^2 + r^2)/2\sigma^2} I_0\left(\frac{rs}{\sigma^2}\right) \quad (2.8)$$

where $I_0(x)$ is the zeroth order modified Bessel function of the first kind, s is the amplitude of the deterministic path, and σ^2 is proportional to the power of the scattered Rayleigh component.

2.3.2. Correlation Functions

As mentioned at the beginning of this section, several correlation properties are also important in characterizing a multipath channel. In analysis, the channel impulse response $h(\tau; t)$, given by (2.1), is usually taken to be wide sense stationary. This assumption implies that the averages for different sets of samples taken from the same area are statistically stationary. The autocorrelation $\phi_h(\tau_1, \tau_2; \Delta t)$ of the channel impulse response is defined as [31],

$$\varphi_h(\tau_1, \tau_2; \Delta t) = \frac{1}{2} E [h^*(\tau_1; t) h(\tau_2; t + \Delta t)] \quad (2.9)$$

where $E[x]$ indicates statistical average, and x^* () denotes complex conjugation. Another common assumption in radio transmission is that the scattering at two different delays is uncorrelated. This assumption is incorporated in (2.11).

$$\frac{1}{2} E [h^*(\tau_1; t) h(\tau_2; t + \Delta t)] = \varphi_h(\tau_1; \Delta t) \delta(\tau_1 - \tau_2) \quad (2.10)$$

The stationarity and uncorrelatedness assumptions together form the wide sense stationary uncorrelated scattering (WSSUS) model. The delay power spectrum $\varphi_h(\tau)$ is obtained when $\Delta t = 0$. The range of τ values over which $\varphi_h(\tau)$ is essentially non-zero is called the multipath spread of the channel, denoted by T_m .

By taking the Fourier transform of (2.11), the spaced-frequency spaced-time correlation function of the channel, $\varphi_h(\Delta f; \Delta t)$, is obtained,

$$\varphi_h(\Delta f; \Delta t) = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E [h^*(\tau_1; t) h(\tau_2; t + \Delta t)] e^{j2\pi(f_1\tau_1 - f_2\tau_2)} d\tau_1 d\tau_2 \quad (2.11)$$

where $\Delta f = f_2 - f_1$. If Δt is set to zero, then $\varphi_h(\Delta f; 0) \equiv \varphi_h(\Delta f)$. $\varphi_h(\Delta f)$ is a measure of the frequency coherence of the channel. The coherence bandwidth B_c is related to the multipath spread by the following relation,

$$B_c \approx \frac{1}{T_m} \quad (2.12)$$

Coherence bandwidth determines the frequency selectivity of the channel. If the coherence bandwidth is comparably smaller than the bandwidth of the transmitted signal, the channel distorts the signal severely and the channel is said

to be frequency selective. Otherwise, it is frequency non-selective.

Another important parameter in mobile radio links is the coherence time which will now be described. Let $S_h(\Delta f; \lambda)$ be the Fourier transform of $\varphi_h(\Delta f; \Delta t)$ with respect to the time variable Δt . Thus,

$$S_h(\Delta f; \lambda) = \int_{-\infty}^{\infty} \varphi_h(\Delta f; \Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t \quad (2.13)$$

The Doppler power spectrum of the channel, $S_h(\lambda)$, is obtained when Δf is set to zero, and $S_h(0; \lambda) \equiv S_h(\lambda)$. The Doppler spread, B_d , of the channel is subsequently defined as the range of λ over which the Doppler power spectrum is essentially non-zero. The coherence time t_c is defined as the reciprocal of B_d . A small Doppler spread implies a slowly changing channel, and vice versa. With the uncorrelated scattering assumption, the Doppler spectrum B_d is independent of the delay, which can be expected in a wireless environment. Figure 2.2 illustrates channel classifications based on the bit period T_b and bandwidth B_w . Channels are classified into different categories depending on their bit periods and bandwidths relative to the coherence time t_c and the coherence bandwidth B_c .

Figure 2.2
Channel classification

2. 4. Forward Error Control

In 1948, Claude Shannon stated a theorem which has inspired much of the subsequent research in error control coding. Shannon stated that with proper encoding and decoding techniques, reliable communications are achievable as long as the transmission rate is less than the channel capacity. Based on his theorem, a great deal of effort has been dedicated to developing coding techniques to ensure reliable and fast data transmission, as the transmission rate approaches the channel capacity.

Error correcting codes are divided into two major divisions, block codes and convolutional codes [9]. In a (n, k) block code C , a block of k information symbols $\mathbf{u} = (u_1, u_2, \dots, u_k)$ is fed into an encoder which will output a block of n encoded symbols. In the binary case, there are 2^k distinct information sequences, corresponding to 2^k distinct codewords. The ratio k/n is called the code rate, which must be less than one to ensure a one to one correspondence between the set of information sequences and the set of codewords.

A linear block code is specified by its generator and parity check matrices. The generator matrix of a (n, k) linear block code is a k by n matrix, whose rows are linearly independent and span the (n, k) linear code C . The rows and columns of a generator matrix may be rearranged by linear row or column operations into a systematic form, as shown in Figure 2.3. An alternative format places the redundant or parity bits at the first $n-k$ bit positions, followed by the information bits.

Figure 2.3

The format of a systematic block code.

The general form of a generator matrix of a systematic block code is given by,

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & \dots & 0 & g_{00} & g_{01} & \dots & g_{0,n-k-1} \\ 0 & 1 & \dots & 0 & g_{10} & g_{11} & \dots & g_{1,n-k-1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & g_{k-1,0} & g_{k-1,1} & \dots & g_{k-1,n-k-1} \end{bmatrix} \quad (2.14)$$

In this form, the generator matrix of a systematic block code consists of a k by k identity matrix $\mathbf{I}_k$, and a k by $(n-k)$ matrix $\mathbf{P}$. Having the generator matrix $\mathbf{G}$, the codeword $\mathbf{v}$, corresponding to a block of k information symbols $\mathbf{u}$, may be defined as,

$$\mathbf{v} = \mathbf{u} \cdot \mathbf{G} \quad (2.15)$$

Another important matrix associated with a linear block code is the parity check matrix $\mathbf{H}$. The parity check matrix of a (n, k) linear block code is a $(n-k)$ by n matrix whose rows are linearly independent. The parity check matrix $\mathbf{H}$ of a code $\mathbf{C}$ has the property that a vector $\mathbf{v}$ is a codeword in $\mathbf{C}$, if and only if $\mathbf{v} \cdot \mathbf{H}^T = \mathbf{0}$, where the superscript T indicates the transpose of a matrix. Referring to the generator matrix in expression (2.14), the corresponding parity check matrix may be written as follows,

$$\mathbf{H} = \begin{bmatrix} g_{00} & g_{10} & \dots & g_{k-1,0} & 1 & 0 & \dots & 0 \\ g_{01} & g_{11} & \dots & g_{k-1,1} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_{0,n-k-1} & g_{1,n-k-1} & \dots & g_{k-1,n-k-1} & 0 & 0 & \dots & 1 \end{bmatrix} \quad (2.16)$$

Therefore, the parity check matrix consists of the transpose of the $\mathbf{P}$ matrix, followed by an identity matrix $\mathbf{I}_{n-k}$ of size $(n-k)$.

An important parameter that determines the performance of a linear block code is the minimum distance $d_{\min}$. The minimum distance of a linear block code C is defined as the minimum Hamming distance between any two codewords in the code. The Hamming distance of two codewords is the number of bit positions at which they differ. Therefore, the minimum distance $d_{\min}$ may be defined more formally as,

$$d_{\min} = \min \{d_H(\mathbf{v}, \mathbf{v}'): \mathbf{v}, \mathbf{v}' \in C, \mathbf{v} \neq \mathbf{v}'\} \quad (2.17)$$

where $d_H(\mathbf{v}, \mathbf{v}')$ is the Hamming distance between codewords $\mathbf{v}$ and $\mathbf{v}'$. Equivalently, the minimum distance may also be defined as the minimum weight codeword in a linear block code C , excluding the all-zero codeword. A linear block code with minimum distance $d_{\min}$ is capable of detecting all error patterns containing $d_{\min} - 1$ errors or less, and a large percentage of error patterns containing $d_{\min}$ or more errors. This code guarantees that all error patterns containing $\lfloor (d_{\min} - 1)/2 \rfloor$ or fewer errors, where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x , can be corrected.

While linear block codes are memoryless, convolutional codes have memory. The n -tuple output block of a (n, k, m) convolutional code depends not only on the present k -tuple input block, but also on the m previous input blocks. Thus, it is said that a (n, k, m) convolutional code has a memory of order m . A parameter that determines the performance of convolutional codes is the minimum free distance d_{free} , defined as follows,

$$d_{free} = \min \{ d_H(\mathbf{v}, \mathbf{v}') : \mathbf{u} \neq \mathbf{u}' \} \quad (2.18)$$

where $\mathbf{v}$ and $\mathbf{v}'$ are the codewords corresponding to information sequence $\mathbf{u}$ and $\mathbf{u}'$, respectively. Since convolutional codes are linear, the free distance can alternatively be defined as the minimum-weight codeword of any length, produced by a non-zero information sequence.

2. 4. 1. Reed-Muller Codes

Reed-Muller codes are well known for their simplicity and reasonably easy decoding using majority logic. This class of codes, however, does not have a good minimum distance, except for the first order Reed-Muller codes and codes of modest lengths [24].

For any integers p and r such that $0 \leq r \leq p$, there exists an r 'th order Reed-Muller code $\mathcal{R}(r, p)$ of length $n = 2^p$, and rate k/n . The number of information bits per codeword is given by the following expression,

$$k = 1 + \binom{p}{1} + \dots + \binom{p}{r} \quad (2.19)$$

where $\binom{a}{b} = \frac{a!}{b!(a-b)!}$.

The r 'th order Reed-Muller code $\mathcal{R}(r, p)$ has minimum distance $d_{min} = 2^{p-r}$. In the following, an alternate notation for a Reed-Muller code, (n, k, d_{min}) , will also be used.

Reed-Muller codes can be most easily understood in terms of a non-systematic generator matrix G . The generator matrix G of $\mathcal{R}(r, p)$ is defined as follows [4],

$$G = [g_0, g_1, \dots, g_r]^T \quad (2.20)$$

where g_0 is the all-ones vector of length n ; g_1 is an p by n matrix, having each binary p -tuple appearing once in increasing order from 0 to $n-1$ as its columns from left to right; and g_i is a $\binom{p}{i}$ by n matrix, whose rows are all possible componentwise product in GF(2) of i rows of g_1 . An example, given below, for the (8,4,4) Reed-Muller code will clarify this procedure.

For the first order (8,4,4) Reed-Muller code,

$$g_0 = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \quad (2.21a)$$

$$g_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (2.21b)$$

Therefore,

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (2.22)$$

Table 2.1 tabulates the parameters of various Reed-Muller codes of short and moderate lengths.

p	r	n	k	$d_{\min}$
3	2	8	7	2
3	1	8	4	4
4	3	16	15	2
4	2	16	11	4
4	1	16	5	8
5	4	32	31	2
5	3	32	26	4
5	2	32	16	8
5	1	32	6	16
6	5	64	63	2
6	4	64	57	4
6	3	64	42	8
6	2	64	22	16
6	1	64	7	32

Table 2.1

Parameters of various Reed-Muller codes of short and moderate lengths

2. 4. 2. Code Combining

Code combining was introduced by Chase in 1985 [7]. This is a technique which combines multiple noisy copies of a transmitted codeword to obtain a low rate code-combined codeword. L repeated symbols of rate R and minimum distance d will combine into a new codeword of rate R/L , with minimum distance dL . This ability to vary the code rate, by combining any number of noisy copies, is the main advantage of code combining, and this is desirable in numerous applications. Code combining is readily applicable in an adaptive coding scheme, in which the code rate is varied depending on the channel condition.

Code combining adds a dimension to conventional diversity concepts which are confined to combining individual symbols. With conventional diversity combining, a single error caused by interference, for example, may cause the entire sum of L repeated symbols to be incorrect, or less reliable. With code combining, however, L repeated symbols will lead to a code-combined codeword which is effectively a low rate code, and thus has a more powerful error correcting capability. Hence, a single error caused by interference is usually correctable.

Consider a codeword $\mathbf{v} = [v_k], k = 1, \dots, n$ of rate R , and length n . Assume that the receiver has L copies of $\mathbf{v}$, obtained through diversity reception. Let $\mathbf{v}^{(i)} = [v_1^{(i)}, v_2^{(i)}, \dots, v_n^{(i)}], i = 1, \dots, L$, denote the i 'th copy of the codeword $\mathbf{v}$, where $v_j^{(i)}$ indicates the j 'th bit of the i 'th copy. After code combining, the new code-combined codeword $\mathbf{v}_L$ is obtained, and can be formally defined as,

$$\mathbf{v}_L = [v_{Lk}] = [v_1^{(1)}, v_1^{(2)}, \dots, v_1^{(L)}, v_2^{(1)}, \dots, v_2^{(L)}, \dots, v_n^{(L)}, \dots, v_n^{(L)}] \quad (2.23)$$

In order to gain the full advantage of code combining, maximum likeli-

hood decoding, instead of bounded distance decoding, is necessary. Unlike a bounded distance decoder, a maximum likelihood decoder will invariably make the best possible estimate of the transmitted codeword, regardless of the channel condition. Bounded distance decoding, on the other hand, will result in a correct decoded output only if the number of erroneous bits is less than or equal to the error correcting capability of the code. For this reason, a maximum likelihood decoder is more robust, since it can still operate in a hostile environment or in an environment with a high level of noise and interference. Because of random effects, such as noise and interference, each received copy will have a different level of reliability. For example, one copy may be affected by less noise than the others, and hence is more reliable. Therefore, a soft information on each received bit is also critical in a code combining scheme.

Maximum likelihood decoding of convolutional codes and block codes is typically done using the code trellis. Note that $\mathbf{v}$ and $\mathbf{v}_L$ share the same code structure in terms of their trellis, and so can be decoded using the same decoder.

2.4.3. Maximum Likelihood Soft Decision Decoding

As mentioned in section 2.4.2, maximum likelihood soft decision decoding is crucial to gain the full advantage of code combining. The trellis structure of block codes can be quite complex. The level of complexity involved usually limits soft decision decoding of block codes to codes of short or moderate lengths, otherwise the required computation becomes prohibitive. Numerous papers have been published on soft decision decoding of block codes [6], [25], [27]. In 1978, Wolf introduced the concept of maximum likelihood decoding of block codes using the Viterbi algorithm [40]. In finding the most likely codeword, the Viterbi algorithm

searches through the code trellis and compares the likelihood of competing branches in the trellis. This algorithm can be extended by making use of reliability information on each received bit.

A trellis, defined in terms of the parity check matrix $\mathbf{H}$, is a set of nodes interconnected by unidirectional edges. These nodes, sometimes called the states of the trellis, are grouped into different sets, and each set is indexed according to their level. A code of length n has a trellis with $n + 1$ levels. The first level consists of only one state, the all-zero state; and each successive level corresponds to the successive bit in the codeword. Since $\mathbf{G} \cdot \mathbf{H}^T = 0$, the parity check matrix is the null-space of the generator matrix $\mathbf{G}$. Therefore, in terms of the trellis, all codewords generated by the generator matrix $\mathbf{G}$ must reach the all-zero state at the final level. Figure 2.4 shows the trellis for the (8,4,4) Reed-Muller code.

In finding the most likely codeword, the decoder assigns a branch metric to each branch in the trellis. As it moves to the next level, the metrics from previous branches are summed up into the cumulative path metric. When the decoder encounters a node reached by more than one path, it will compare their path metrics, keep the one with the largest metric value, and discard the rest. This process continues until it reaches the last level. At the last level, the decoder will be left with one path which has the largest path metric. This path is selected as the transmitted codeword.

Figure 2.4

The trellis for the (8,4,4) Reed-Muller code

2. 5. Code Division Multiple Access

Code Division Multiple Access (CDMA) is an alternative to FDMA and TDMA as a multiple access technique [23]. Unlike its counterparts, CDMA does not generally maintain orthogonality among users. While FDMA maintains orthogonality among users by allocating different portions of the spectrum to different users, TDMA does so by assigning different time slots to different users.

CDMA, on the other hand, allows all users to share the same allocated spectrum and to transmit simultaneously. The basis of CDMA technology is spread spectrum communications [36]. Spread spectrum has been used for military purposes for many years, due to its ability to resist jamming and its low probability of intercept [33]. As the name suggests, with spread spectrum signaling the narrowband information signal is spread over a much larger bandwidth before transmission. The spectrum spreading is achieved via frequency hopping or direct sequence.

In a frequency hopping spread spectrum system, the narrowband information signal is pseudo-randomly hopped into one of N carriers. The hopping pattern is pseudo-random, so that an unintended receiver cannot easily track the signal transmission. However, knowing the code, the intended receiver will be able to generate the same hopping pattern quite easily. Direct sequence spread spectrum systems achieve spectrum spreading by multiplying the narrowband information signal with a periodic pseudo-random signature sequence. The signature sequence is pseudo-random for the same reason the hopping pattern is pseudo-random. Multiplication by a pseudo-random sequence of length N results in a bit rate which is N times higher, and so requires a bandwidth which is larger by the same factor. Figure 2.5 illustrates the multiplication of information bits

with a user's signature sequence with $N = 8$ in this example, T_b is the information rate and T_c is the chip rate, so that $T_b = NT_c$.

2. 5. 1. Advantages of CDMA

As mentioned previously, CDMA is based on spread spectrum signaling. This section is devoted to describing the distinctive nature of CDMA [30], [35].

Upon comparing the three multiple access methods, one will find that one method may be superior in some ways but not in the others. There seems to be no method which is superior in all aspects. With FDMA, each user is assigned a portion of allocated bandwidth for transmission. Since the transmitted signal is relatively narrowband, it does not experience frequency selective fading. However, since the available bandwidth is finite, carrier frequencies have to be reused to accommodate more users, causing cochannel interference. Fixed bandwidth allocation makes it difficult for an FDMA system to support variable data rates. The spectrum gets crowded rather quickly as the number of users increases. A TDMA system, on the other hand, assigns to each user a time slot in a frame at a given carrier frequency. The time slot approach allows TDMA to accommodate users with different data rates. K time slots in a frame will lead to a bit rate which is K times higher, requiring a bandwidth expansion by the same factor. In a TDMA system, equalization is usually required to mitigate the effects of frequency selective fading experienced by the transmitted signal. Furthermore, intricate synchronization and frame timing are necessary to avoid overlap among users. TDMA may be combined with FDMA to improve the spectral efficiency of FDMA. CDMA has a distinctive nature which may lead to a greater capacity in terms of the number of users.

Signature sequence

Data bit

Data bit ready for transmission

Figure 2.5

Multiplication between information bits and a user's signature sequence, $N = 8$

While the performance of the two non-spread techniques is limited by the worst case interference, CDMA's performance is limited by the average interference. Another interesting characteristic of CDMA is a graceful degradation in performance as the number of users increases, or as the amount of interference increases. This is in contrast to the performance of non-spread techniques which degrades drastically when a certain point is exceeded.

Any reduction in the amount of interference will boost CDMA performance. The inherent frequency diversity introduced by the wide bandwidth gives CDMA its efficiency in combatting interference due to multipath. With CDMA, the deteriorating effects of multipath interference may be reduced in one of the following ways. First, multipath mitigation may be done by rejecting the multipath which falls outside of the correlation interval of the spread spectrum waveform. Second, it can be done by clever combining of the multipath signals. This method requires a receiver structure which is capable of detecting the delayed multipath components. A Rake receiver is usually used for this purpose. Figure 2.6 depicts a simple Rake receiver structure. With maximum ratio combining, the multiplier coefficients are the path gains or their approximations. On the other hand, with equal gain combining, or code combining, as will be defined in Chapter 3, these coefficients are all equal to one.

Figure 2.6
A Rake receiver structure

Chapter 3

System Analysis

In this chapter, the details of the system under consideration and an analysis of its performance in terms of the bit error rate (BER) are presented. In depth treatments on the transmitter, channel, and receiver models can be found in different sections of this chapter.

3. 1. System Description

Figure 3.1 depicts the structure of an indoor wireless communication network in which a user communicates to another user through a base station. The base station is subsequently connected to a public branch exchange (PBX), allowing communication to and from other networks. Data transmission between a subscriber and the base station is done through direct sequence spread spectrum signalling. Spread spectrum is chosen for its multiple access capability and its resistance to multipath fading. With spread spectrum signalling, K subscribers share the same transmission bandwidth. Each of the K subscribers has its own transmitting and receiving unit, and is distinguished from other users by a unique signature sequence.

Figure 3.1
An indoor wireless network

The base station has a bank of spread spectrum receivers. This bank of receivers will detect active subscribers and the data being sent. It is assumed that, on average all active users's signals arrive at the base station with the same power. This scenario may also be applied to a cellular system in order to increase capacity. In a cellular environment, a network is dedicated for each cell, and networks from different cells are all interconnected through a separate communication link. In what follows, the analysis of the system's performance in terms of the bit error rate for the link between a user and its receiving unit at the base station is presented.

Figure 3.2 illustrates the model of the system under consideration. The binary input stream is encoded using a code of rate R . Two types of codes, Reed-Muller and convolutional codes, are considered in this work. Prior to modulation, the coded data bits are interleaved to break the correlation between adjacent bits which is introduced by the fading channel. Bit interleaving is especially critical if convolutional codes are used, because of the error propagation caused by their memory. The amount of interleaving needed depends on the length of the fade experienced in the channel. In what follows, sufficient interleaving is assumed, so that the fading channel appears memoryless and received bits are independent of each other. After interleaving, the data sequence is multiplied by the user's signature sequence to achieve spectrum spreading, and is then BPSK modulated. The spread spectrum signal is subsequently transmitted, degraded by multipath fading, and corrupted by multiple user interference and thermal noise.

At the other end, the base station receives the distorted signal plus interference from other users and additive white Gaussian noise (AWGN). Multiple antennas, if used, are assumed to be sufficiently spaced apart so that they experience independent fading. Furthermore, each receiving antenna is assumed to receive K signals (one desired signal and $K-1$ interferers), each with L discrete

paths. The total received signal is despread by multiplying it with the reference user's signature sequence.

Using an equal gain Rake receiver, a branch is assumed to have L noisy copies of the transmitted bit which, together with copies from the other branch, are ready to be code-combined into a new codeword of rate $R/(l_A L)$, where l_A is the number of receiving antennas. In the analysis, it is assumed that the number of noisy copies, L , is equal to the number of paths. These copies are just the integrator outputs. This method is simpler than maximum ratio combining, since it does not require an accurate knowledge of each path gain. The receiver needs only knowledge of whether the paths exist. After code-combining, the codewords are de-interleaved before maximum likelihood decoding is performed. The decoder employs the Viterbi algorithm in determining the most likely codeword.

Figure 3.2
System model

3. 2. Transmission and Channel Models

Each user is assigned a distinct signature sequence which consists of a periodic sequence of unit amplitude, positive and negative, rectangular pulses of duration T_c . These rectangular waveforms are called chips. Let $c^k(t)$ be the signature sequence for the k 'th user, and c_n^k be the amplitude of the n 'th chip in $c^k(t)$ such that $c_n^k(t) \in \{-1, 1\}$. Thus,

$$c^k(t) = \sum_{n=-\infty}^{\infty} c_n^k p_{T_c}(t - nT_c), \quad (3.1)$$

where $p_{T_c}(t)$ is a unit pulse of duration T_c seconds. Similarly, if $a_m^k \in \{-1, 1\}$ is the m 'th data bit to be transmitted by the k 'th user, then the information sequence $a^k(t)$ may be expressed as,

$$a^k(t) = \sum_{m=-\infty}^{\infty} a_m^k p_{T_b}(t - mT_b), \quad (3.2)$$

where $p_{T_b}(t)$ is a unit pulse of duration T_b seconds. It is assumed that each user's signature sequence has period $N = T_b/T_c$ so that each data symbol a_m^k contains one period of $c_0^k, c_1^k, \dots, c_{N-1}^k$. In spread spectrum terminology, the quantity N is often called the processing gain.

Based on the information sequence $a^k(t)$, the signature sequence $c^k(t)$, and assuming BPSK modulation, the transmitted signal $s^k(t)$ of the k 'th user may be written as,

$$s^k(t) = A a^k(t) c^k(t) \cos(\omega_c t + \theta^k) \quad (3.3)$$

where $\omega_c = 2\pi f_c$ is the carrier frequency in rad/s, A is the signal amplitude, and

θ^k is the signal phase.

Let $h^k(t)$ be the lowpass equivalent of the passband channel impulse response, for the link between the k 'th user's transmitter and the base station's receiver. Assuming that L discrete paths exist between a user and each receiving antenna, $h^k(t)$ may be expressed as [21],

$$h^k(t) = \sum_{l=1}^L \alpha_l^k \delta(t - \tau_l^k) e^{j\phi_l^k} \quad (3.4)$$

In the above equation, α_l^k is the path gain, τ_l^k is the path time delay, and ϕ_l^k is the path phase, of the l 'th path of the k 'th user, and $\delta(t)$ is the delta function. Furthermore, it is assumed that the path gain is a Rayleigh distributed random variable, the time delay is uniformly distributed between $[0, T)$, and the phase is uniformly distributed in the range $[0, 2\pi)$. Under the slowly fading assumption, these variables are constant over a symbol period. Therefore, they may be appropriately modeled as time invariant random variables.

3.3. Receiver Model

Assuming K active users, and based on the expressions of the transmitted signal and the channel response, the waveform received by the base station's receiver may be obtained by performing the convolution operation,

$$r(t) = \text{Re} \left\{ \sum_{k=1}^K \int_{-\infty}^{\infty} h^k(\tau) \tilde{s}^k(t - \tau) \exp(j\omega_c t) d\tau \right\} + n(t) \quad (3.5)$$

where $\tilde{s}^k(t)$ is the complex envelope of $s^k(t)$ given by,

$$\tilde{s}^k(t) = A a^k(t) c^k(t) e^{j\theta^k} \quad (3.6)$$

Assuming $\theta^k = 0$, upon substituting (3.4) for $h^k(t)$ in (3.5), the received waveform may be expressed as,

$$\begin{aligned} r(t) &= A \operatorname{Re} \left\{ \sum_{l=1}^L \sum_{k=2}^K \int_{-\infty}^{\infty} \alpha_l^k a^k(t-\tau) c^k(t-\tau) \delta(\tau-\tau_l^k) \exp[j(\omega_c t + \phi_l^k)] d\tau \right\} + n(t) \\ &= A \sum_{l=1}^L \alpha_l^1 a^1(t-\tau_l^1) c^1(t-\tau_l^1) \operatorname{Re} \{ \exp[j(\omega_c t + \phi_l^1)] \} \\ &\quad + A \sum_{l=1}^L \sum_{k=2}^K \alpha_l^k a^k(t-\tau_l^k) c^k(t-\tau_l^k) \operatorname{Re} \{ \exp[j(\omega_c t + \phi_l^k)] \} + n(t) \quad (3.7) \end{aligned}$$

in which $\operatorname{Re}\{x\}$ indicates the real part of x . Upon taking the real part of the exponential terms, the received waveform $r(t)$ may be written as,

$$\begin{aligned} r(t) &= A \sum_{l=1}^L \alpha_l^1 a^1(t-\tau_l^1) c^1(t-\tau_l^1) \cos(\omega_c t + \phi_l^1) \\ &\quad + A \sum_{l=1}^L \sum_{k=2}^K \alpha_l^k a^k(t-\tau_l^k) c^k(t-\tau_l^k) \cos(\omega_c t + \phi_l^k) + n(t) \quad (3.8) \end{aligned}$$

where $n(t)$ is a Gaussian noise process with two sided spectral density of $\frac{N_o}{2}$ watt/Hz.

Without loss of generality, it is assumed that user 1 is the reference user, and that the coherent detector can lock to the reference user's delays and the corresponding phases. With the received waveform $r(t)$ as the input, the i 'th decision variable U_i , corresponding to the i 'th path, is the output of the correlation receiver filter,

$$U_i = \int_0^{T_b} r(t) c^1(t - \tau_i^1) \cos(\omega_c t + \phi_i^1) dt \quad (3.9)$$

Upon substituting (3.9) for $r(t)$ in (3.7), the following expression for the decision variable U_i is obtained,

$$\begin{aligned} U_i = & A \int_0^{T_b} \sum_{l=1}^L \alpha_l^1 c^1(t - \tau_l^1) a^1(t - \tau_l^1) \cos(\omega_c t + \phi_l^1) c^1(t - \tau_i^1) \cos(\omega_c t + \phi_i^1) dt \\ & + A \int_0^{T_b} \sum_{k=2}^K \sum_{l=1}^L \alpha_l^k c^k(t - \tau_l^k) a^k(t - \tau_l^k) \cos(\omega_c t + \phi_l^k) \\ & \cdot c^1(t - \tau_i^1) \cos(\omega_c t + \phi_i^1) dt \\ & + \int_0^{T_b} n(t) c^1(t - \tau_i^1) \cos(\omega_c t + \phi_i^1) dt \end{aligned} \quad (3.10)$$

Ignoring the double frequency terms, the above expression may be simplified to,

$$\begin{aligned} U_i = & a_0^1 \frac{AT}{2} \alpha_i^1 + \frac{A}{2} \sum_{l=1, l \neq i}^L \alpha_l^1 \cos(\phi_l^1 - \phi_i^1) \\ & \cdot \int_0^{T_b} c^1(t - \tau_l^1) a^1(t - \tau_l^1) c^1(t - \tau_i^1) dt \\ & + \frac{A}{2} \sum_{l=1}^L \sum_{k=2}^K \alpha_l^k \cos(\phi_l^k - \phi_i^1) \cdot \int_0^{T_b} c^1(t - \tau_l^k) a^k(t - \tau_l^k) c^1(t - \tau_i^1) dt \\ & + \int_0^{T_b} n(t) c^1(t - \tau_i^1) \cos(\phi_i^1) dt \end{aligned} \quad (3.11)$$

The continuous partial cross-correlation functions between the k 'th user's

signature sequence in (3.1), and the reference user's may be defined as follows,

$$R_{k1}(\tau) = \int_{\tau}^{\tau+T_b} c^k(t-\tau) c^1(t) dt \quad (3.12a)$$

$$\hat{R}_{k1}(\tau) = \int_{\tau}^{\tau+T_b} c^k(t-\tau) \hat{c}^1(t) dt \quad (3.12b)$$

By virtue of the partial cross correlation functions above, (3.11) simplifies to,

$$\begin{aligned} U_i = & \alpha_i^1 \frac{AT}{2} a_0^1 \\ & + \frac{A}{2} \sum_{l=1, l \neq i}^L \alpha_l^1 \cos(\phi_l^1 - \phi_i^1) \cdot [a_{-1}^1 R_{11}(t_l^1) + a_0^1 \hat{R}_{11}(t_l^1)] \\ & + \frac{A}{2} \sum_{l=1}^L \sum_{k=2}^K \alpha_l^k \cos(\phi_l^k - \phi_i^1) \cdot [a_{-1}^k R_{k1}(t_l^k) + a_0^k \hat{R}_{k1}(t_l^k)] + \hat{n} \end{aligned} \quad (3.13)$$

where $t_l^k = \tau_l^k - \tau_i^1$ is the time difference between the i 'th path of the reference user and the l 'th path of the k 'th user. The last term $\hat{n}$ is a Gaussian noise process with zero mean and variance $N_0 T/4$. In (3.13), a_0^1 is the current data bit of the reference user to be detected, and a_{-1}^1 is the previous bit of the same user.

In general, the decision variable U_i may be viewed as the sum of the following four terms,

$$U_i = Z_{si} + I_{mpi} + I_{mai} + \hat{n} \quad (3.14)$$

with the desired signal Z_{si} , self interference term due to multipath I_{mpi} , and multiple access interference term I_{mai} , given by,

$$Z_{si} = \alpha_i^1 \frac{AT}{2} a_0^1 \quad (3.14a)$$

$$I_{mpi} = \frac{A}{2} \sum_{l=1, l \neq i}^L \alpha_l^1 \cos(\phi_l^1 - \phi_i^1) \cdot [a_{-1}^1 R_{11}(t_l^1) + a_0^1 \dot{R}_{11}(t_l^1)] \quad (3.14b)$$

$$I_{mai} = \frac{A}{2} \sum_{l=1}^L \sum_{k=2}^K \alpha_l^k \cos(\phi_l^k - \phi_i^1) \cdot [a_{-1}^k R_{k1}(t_l^k) + a_0^k \dot{R}_{k1}(t_l^k)] \quad (3.14c)$$

respectively.

3.4. Gaussian Assumption

Under the Gaussian assumption, all interfering terms are taken as Gaussian noise. Although it underestimates the amount of interfering noise, this assumption simplifies the analysis considerably. Earlier research has found that the Gaussian assumption underestimates the interference noise by at most 20% of the actual amount [21].

With the Gaussian assumption, for fixed α_i^1 and a_0^1 , the quantity Z_{si} is deterministic and contributes to the mean of U_i . The interfering components I_{mpi} and I_{mai} are random variables whose collective variance is given in the following. Define a new variable v to be the sum of the interfering components I_{mpi} and I_{mai} . Hence,

$$\begin{aligned} v = & \frac{A}{2} \sum_{l=1, l \neq i}^L \alpha_l^1 \cos(\phi_l^1 - \phi_i^1) \cdot [a_{-1}^1 R_{11}(t_l^1) + a_0^1 \dot{R}_{11}(t_l^1)] \\ & + \frac{A}{2} \sum_{l=1}^L \sum_{k=2}^K \alpha_l^k \cos(\phi_l^k - \phi_i^1) \cdot [a_{-1}^k R_{k1}(t_l^k) + a_0^k \dot{R}_{k1}(t_l^k)] \end{aligned} \quad (3.15)$$

From (3.15), it is clear that v consists of $L(K-1)$ independent, but similar terms. If K

is large, as is usually the case, $L(K-1) \approx LK$. Therefore, the variance of v is approximately LK times the variance of one of these terms. Now, let $v_{l,k}$ be the interfering component due to the l 'th path of the k 'th user. The variance σ_v^2 of $v_{l,k}$ is defined as,

$$\sigma_v^2 = E[v_{l,k}^2] \quad (3.16)$$

since $v_{l,k}$ has zero mean. For Gold codes, the following result was presented by Pursley [32],

$$E \left[\frac{[b_{-1}R(t_1) + b_0\dot{R}(t_1)]^2}{T^2} \right] = \frac{2}{3N} \quad (3.17)$$

where $b_i = \{-1, 1\}$, $i = -1, 0$, are independent binary random variables. Thus, the variance may be expressed as,

$$\sigma_v^2 = \frac{A^2 T^2 E[\alpha^2]}{4} \frac{2}{3N} \quad (3.18)$$

Therefore, σ_v^2 , the variance of v , is,

$$\sigma_v^2 = LK \frac{A^2 T^2 E[\alpha^2]}{4} \frac{2}{3N} \quad (3.19)$$

where $\frac{E[\alpha^2]}{2}$ is the average path power.

If there are L resolvable paths, each branch of the receiving antenna will have L decision variables U_i , $i = 1, \dots, L$ available for detection purposes. These multiple copies may be exploited to mitigate the effects of multipath fading. Depending on the combining algorithm, these copies can be exploited in different ways. With L noisy copies from each of the L_A receiving branches, diversity of

order M is obtained, where,

$$M = l_1 \cdot L \quad (3.20)$$

3. 5. Code Combining

With code combining, all copies are retained and combined into a new codeword of lower rate. In subsection 2.2.4, it was emphasized that soft decision decoding is crucial for code combining to be effective. In order to perform soft decision decoding, the branch metric for each received bit needs to be determined. A practical means of obtaining the branch metric is to multiply the detector output/decision variable by the trellis branch value. Formally, the branch metric f_i of the i 'th bit in a codeword may be defined as,

$$f_i = -a_{vi} U_i \quad (3.21)$$

where a_{vi} denotes the i 'th symbol/bit in a codeword sequence for the detector output v .

Consider the codeword v , corresponding to the all-zero codeword. Let v' be another codeword such that the Hamming distance between them is $d_H(v, v') = d$. The code-combined version of v and v' are v_M and v'_M , respectively, where M denotes the order of diversity. Hence, $d_H(v_M, v'_M) = Md$.

Let v be the transmitted codeword. Because of channel errors, the receiver may incorrectly decode the received codeword into v' . With code-combining of order M , the following relation is true,

$$P_e(d) \equiv Pr\{v \rightarrow v'\} = Pr\{v_M \rightarrow v'_M\} \quad (3.22)$$

$Pr \{P \rightarrow Q\}$ denotes the probability that the transmitted codeword P is incorrectly decoded into another codeword Q by the receiver, and $P_e(d)$ denotes the pairwise probability that the transmitted codeword is decoded into another codeword which is Hamming distance d away.

Let F_v be the cumulative path metric leading to decoder output v , and $F_{v'}$ be that leading to decoder output v' . With maximum likelihood decoding, v will be decoded into v' if $F_v < F_{v'}$. Thus,

$$P_e(d) = Pr \{F_v - F_{v'} < 0\} \quad (3.23)$$

Since $d_H(v_M, v_{M'}) = Md$, the expression for F_v will differ from that for $F_{v'}$ in Md bit positions. It may be assumed, furthermore, that these bit disagreements occur in the first Md bits without loss of generality. $P_e(d)$, therefore, may be written as,

$$P_e(d) = Pr \left\{ -\sum_{i=1}^{Md} a_{vi} U_i + \sum_{i=1}^{Md} a_{v'i} U_i < 0 \right\} \quad (3.23a)$$

where U_i is the decision variable of the i 'th bit in the code-combined codeword. Since it is assumed that v , corresponding to the all-zero codeword, is transmitted, then $a_{vi} = 1$ for all i , and $a_{v'i} = -1$ for the first Md bits, and is 1 otherwise. Hence,

$$P_e(d) = Pr \left\{ -2 \sum_{i=1}^{Md} U_i < 0 \right\} \quad (3.23b)$$

if $Y = \sum_{i=1}^{Md} U_i$, then

$$P_e(d) = \Pr\{Y > 0\} \quad (3.23c)$$

With the Gaussian assumption for interference and sufficient interleaving, Y is comprised of Md independent Gaussian random variables with known mean and variance. Hence, Y is also a Gaussian random variable. Therefore, $P_e(d)$ may be expressed in terms of its conditional probability as follows,

$$P_e(d|\gamma_o) = \frac{1}{2} \operatorname{erfc} \left[\frac{\frac{AT}{2} \gamma_o}{\sigma_Y \sqrt{2}} \right] \quad (3.24)$$

$$\gamma_o = \sum_{i=1}^{Md} \alpha_i \quad (3.25)$$

in which $\operatorname{erfc}(x)$, the complementary error function and σ_Y^2 , the variance of Y , are as specified below,

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2) dt \quad (3.26)$$

$$\sigma_Y^2 = Md \left(LK \frac{A^2 T^2}{4} \frac{2}{3N} \frac{E[\alpha^2]}{2} + \frac{N_o T}{4} \right) \quad (3.27)$$

If $p_{\Gamma_o}(\gamma_o)$ is the probability density function of γ_o , then

$$P_e(d) = \int_{-\infty}^{\infty} P_e(d|\gamma_o) p_{\Gamma_o}(\gamma_o) d\gamma_o \quad (3.28)$$

3. 6. Bit Error Rate

Having obtained the pairwise error probability $P_e(d)$, the weight distribution or the weight spectrum of the codes used for error correction are required to obtain the bit error rate.

If convolutional codes of rate β/v are used for error correction purposes, the bit error rate P_b is upper bounded by [9],

$$P_b \leq \frac{1}{\beta} \sum_{d=d_{free}}^{\infty} c_d P_e(d) \quad (3.29)$$

in which c_d is the total number of erroneous bits in all the incorrect paths of Hamming weight d , and d_{free} is the free distance of the codes. $P_e(d)$ is the pairwise error probability that, given the all-zero codeword is transmitted, another codeword which is Hamming distance d away is selected by the decoder. The calculation of (3.28) converges quickly after summation of the first few terms.

Similarly, if Reed-Muller codes, or other types of block codes of rate k/n are used, the bit error rate can be upper bounded by,

$$P_b \leq \frac{1}{k} \sum_{d=d_{min}}^n c_d P_e(d) \quad (3.30)$$

where d_{min} is the minimum distance of the code, and c_d and $P_e(d)$ are as defined previously for (3.29).

Chapter 4

Performance Results

In this chapter, theoretical performance results of the system proposed in Chapter 3 are presented. Results on the structure of various Reed-Muller and convolutional codes are presented. In presenting these results, this chapter is divided into two major sections. In section 4.1, their trellises and decoding complexity are compared, and new results on the weight distributions of Reed-Muller codes are presented. In addition, the weight distributions of various convolutional codes are also listed. In section 4.2, the system performance is evaluated in terms of the bit error rate, the bandwidth efficiency, and the additional coding gain. The performance of different codes are compared. Reed-Muller codes are compared with convolutional codes of similar rates and comparable decoding complexity. The performance results in section 4.2 are generated by making use of the results on code structures in section 4.1.

4. 1. Reed-Muller and Convolutional Codes

This section will assess the structure of the codes considered in this thesis. Specifically, the structures of the (8,4,4), (16,11,4), (16,5,8), and (32,16,8) Reed-Muller codes will be examined and compared with those of rate $2/3$, $1/2$, and $1/3$

convolutional codes, with memory order of 2 and 3. The rate $2/3$ punctured convolutional code is derived from the rate $1/2$ convolutional code by eliminating some of its symbols. This method results in a punctured convolutional code which is explained briefly in sub-section 4.1.1.

4. 1. 1. Punctured Convolutional Codes

A punctured convolutional code (PCC) is obtained by performing periodic symbol elimination on an original code, which is generally a low rate convolutional code. Symbol puncturing is performed by following a perforation pattern consisting of 0's and 1's which represent eliminating and keeping the corresponding symbol in the original code, respectively. Therefore, an encoder for a punctured convolutional code may be thought of as a conventional convolutional encoder producing the low rate original code, followed by a symbol selector which periodically eliminates symbols according to a given perforation pattern. Maximum likelihood decoding of punctured convolutional codes can be done using the trellis of the original code with simple modifications to accommodate the fact that some symbols have been eliminated. Arbitrary data is inserted in the symbol positions which have been eliminated, and the same path metric value is assigned to this symbol position regardless of the code symbol.

A number of research efforts have focused on finding good punctured convolutional codes, some of which have compatible rates which are desirable for certain applications [16]. In this work, the rate $2/3$ punctured convolutional code, originating from the rate $1/2$ convolutional code, reported by Haccoun and Begin, is used [15]. This paper also tabulates the weight distributions of other con-

volutional codes.

Convolutional codes derive their name from the shift register structure generating the code. The shift registers perform a convolution operation between the input sequence and the generating sequence. The shift register structures generating the rate $1/2$ and the rate $1/3$ convolutional codes are given in Figures 4.1 and 4.2, respectively. The information sequence u is shifted into the shift register which produces the output sequences v^1 , v^2 , and also v^3 in the case of the rate $1/3$ convolutional code. It is important to note that the adders in both figures operate in $GF(2)$ for the case of binary codes. Addition in $GF(2)$ is equivalent to XOR operation. Alternating between these output sequences, a switch will form the encoder output sequence V , which may subsequently be fed into a symbol selector operating on a specific perforation matrix to obtain a punctured convolutional code.

Figure 4.1

Shift register structure generating the rate 1/3, memory 2 convolutional code.

Figure 4.2

Shift register structure generating the rate 1/2, memory 3 convolutional code.

4. 1. 2. Weight Distributions

Equation (3.30), which is used to generate the performance curves, requires the weight distributions of Reed-Muller codes. All publications which have been found on the weight distributions of Reed-Muller codes only list $\{A_d\}$, the number of codewords with Hamming weight d . This information only allows one to determine the word error rate, while to determine the bit error rate one needs $\{c_d\}$, the total number of erroneous information bits in all incorrect codewords of weight d , assuming the all-zero codeword is transmitted. The $\{A_d\}$ of Reed-Muller codes may be determined by direct enumeration, or by making use of the MacWilliams identity and the property that the $\mathcal{R}(r, p)$ Reed-Muller code is the dual of the $\mathcal{R}(p-r-1, p)$ Reed-Muller code [24]. On the other hand, to determine $\{c_d\}$, the complete set of codewords has to be generated.

Finding the weight distribution of a Reed-Muller code entails determining all linear combinations of the rows of the generator matrix G . The $\{A_d\}$ are then determined simply by finding the weight of each codeword. On the other hand, $\{c_d\}$ is determined by calculating the total number of erroneous information bits in the event that a codeword of weight d is selected, assuming the all-zero codeword has been transmitted. The amount of computation required to generate the complete set of codewords and to determine $\{c_d\}$ increases exponentially as the dimension of the code increases. This calculation becomes prohibitive for Reed-Muller codes with dimensions greater than 16. Results on the weight distributions of various Reed-Muller codes are tabulated in Table 4.1. A closed form formula for the first order Reed-Muller codes was obtained empirically. From these results, a new closed form formula for $\{c_d\}$ of the first order Reed-Muller code is determined, and can be stated as,

The weight distribution of a first order $\mathcal{R}(1, p)$ Reed-Muller code whose length $n = 2^p$, and dimension $k = 1 + p$, is given by,

$$A_0 = 1, A_{n/2} = 2^k - 2, A_n = 1$$

$$A_d = 0, \quad \text{for } d \notin \{0, n/2, n\}$$

$$c_0 = 0, c_{n/2} = (n-1)k, A_n = k$$

$$c_d = 0, \quad \text{for } d \notin \{0, n/2, n\}$$

Code	$\{A_d\}, d = 0, 1, 2, \dots$	$\{c_d\}, d = 0, 1, 2, \dots$
(8,4,4)	$A_0 = 1, A_4 = 14, A_8 = 1$	$c_0 = 0, c_4 = 28, c_8 = 4$
(16,5,8)	$A_0 = 1, A_8 = 30, A_{16} = 1$	$c_0 = 0, c_8 = 75, c_{16} = 5$
(16,11,4)	$A_0 = 1, A_4 = 140, A_6 = 448, A_8 = 870, A_{10} = 448, A_{12} = 140, A_{16} = 1$	$c_0 = 0, c_4 = 410, c_6 = 1980, c_8 = 4731, c_{10} = 3056, c_{12} = 1114, c_{16} = 11$
(32,6,16)	$A_0 = 1, A_{16} = 62, A_{32} = 1$	$c_0 = 0, c_{16} = 186, c_{32} = 6$
(32,16,8)	$A_0 = 1, A_8 = 620, A_{12} = 13888, A_{16} = 36518, A_{20} = 13888, A_{24} = 620, A_{32} = 1$	$c_0 = 0, c_8 = 2480, c_{12} = 83328, c_{16} = 292144, c_{20} = 138880, c_{24} = 7440, c_{32} = 16$
(64,7,32)	$A_0 = 1, A_{32} = 126, A_{64} = 1$	$c_0 = 0, c_{32} = 441, c_{64} = 7$
(128,8,64)	$A_0 = 1, A_{64} = 254, A_{128} = 1$	$c_0 = 0, c_{64} = 1016, c_{128} = 8$

Table 4.1

Weight distributions of various Reed-Muller codes.

Similar tables for convolutional codes may be found in [15], [8]. Table 4.2 lists the weight distributions of the convolutional codes whose performance will be compared with that of the Reed-Muller codes listed in Table 4.1. In Table 4.2, R is the code rate, m is the memory, and d_{free} is the free distance. The complete weight distribution of a convolutional code is an infinite sequence. Nevertheless, the partial weight distributions in Table 4.2 are sufficient, since the bit error rate is dominated by the first few terms, and contributions from later terms are typically negligible.

4. 1. 3. Decoding Complexity

In order to assess the decoding complexity of a code, its trellis needs to be determined. A parameter which gives a measure of the decoding complexity is the maximum number of states in a section of the trellis. Another parameter which is a better complexity indicator is the maximum number of branches in a section of the trellis. This parameter determines the memory requirements and the number of comparisons the decoder has to perform in finding the most likely codeword. Figures 4.3 - 4.7 show the trellises for the codes under investigation. In these figures, except Figure 4.7, the first and last few sections have been omitted to avoid clutter, and only the middle sections are shown. The structures for Reed-Muller codes are given in several publications [13], [19]. On the other hand, those for convolutional codes have to be derived. Table 4.3 presents a summary of the decoding complexity for these codes.

R	m	d_{free}	$\{A_d\},$ $d = d_{free} + l, l = 0, 1, \dots$	$\{c_d\},$ $d = d_{free} + l, l = 0, 1, \dots$
1/3	2	8	{2, 0, 5, 0, 13, 0, 34, 0, 89, 0, 610, 0, 1597, 0, 4181, 0}	{3, 0, 15, 0, 58, 0, 201, 0, 655, 0, 2052, 0, 6255, 0, 18687, 0, 54974, 0}
1/3	3	10	{3, 0, 2, 0, 15, 0, 24, 0, 87, 0, 188, 0, 557, 0, 1354, 0, 3713, 0}	{6, 0, 6, 0, 58, 0, 118, 0, 507, 0, 1284, 0, 4323, 0, 11846, 0, 36009, 0}
1/2	2	5	{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536, 131072}	{1, 4, 12, 32, 80, 192, 448, 1024, 2304, 5120, 11264, 24576, 53248, 114688, 245760, 524288, 1114112, 2359296}
1/2	3	6	{1, 3, 5, 11, 25, 55, 121, 267, 589, 1299, 2865, 6319, 13937, 30739, 67797, 149531, 329801, 727399}	{2, 7, 18, 49, 130, 333, 836, 2069, 5060, 12255, 29444, 70267, 166726, 393635, 925334, 2166925, 5057286, 11767305}
1/2	4	7	{2, 3, 4, 16, 37, 68, 176, 432, 925, 2156, 5153, 11696, 26868, 62885, 145085, 334024, 774966, 1793363}	{4, 12, 20, 72, 225, 500, 1324, 3680, 8967, 22270, 57403, 142234, 348830, 867106, 2134239, 5205290, 12724352, 31022962}
2/3	2	3	{1, 4, 14, 40, 116, 339, 991, 2897, 8468, 24752}	{1, 10, 54, 226, 856, 3072, 10647, 35998, 119478, 390904}
2/3	3	4	{3, 11, 35, 114, 381, 1276, 4257, 14208, 47413, 158245}	{10, 43, 200, 826, 3336, 13032, 49836, 187480, 696290, 2559521}

Table 4.2

Weight distributions of various convolutional codes.

Code	memory	max. # of states	max. # of branches
(8,4,4) Reed-Muller	-	4	8
(16,5,8) Reed-Muller	-	8	16
(16,11,4) Reed-Muller	-	8	32
(32,16,8) Reed-Muller	-	64	512
rate 1/3 conv. code	2	4	8
rate 1/3 conv. code	3	8	16
rate 1/2 conv. code	2	4	8
rate 1/2 conv. code	3	8	16
rate 2/3 conv. code	2	4	16
rate 2/3 conv. code	3	8	32

Table 4.3

Decoding complexity of various Reed-Muller and convolutional codes

Figure 4.3

The trellis for the rate 1/2 and rate 1/3, memory 2 convolutional codes

Figure 4.4

The trellis for the rate 2/3, memory 2 punctured convolutional code

Figure 4.5

The trellis for the rate 1/2 and rate 1/3, memory 3 convolutional codes

Figure 4.6

The trellis for the rate 2/3, memory 3 punctured convolutional code

Figure 4.7

The trellis for the (8,4,4) and the (16,5,8) Reed-Muller codes

In Figure 4.7, the dark lines indicate the trellis for the (8,4,4) Reed-Muller code. The dark and the grey lines altogether form the trellis for the (16,5,8) Reed-Muller code. This pattern may be continued to form the trellises for other Reed-Muller codes such as the (32,6,16) and the (32,16,8) Reed-Muller codes.

4. 2. System Performance

In this section, results on the weight distributions and decoding complexity of various codes presented in the previous section are used to evaluate the theoretical performance of the system. In sub-section 4.2.2, a system employing equal gain code combining is compared to another system employing maximum ratio combining; both using soft decision maximum likelihood decoding. The performance degradation when equal gain code combining is used rather than maximum ratio combining is determined. In sub-section 4.2.3, the performance of various Reed-Muller codes and convolutional codes of similar rates and comparable decoding complexity are evaluated and compared. To facilitate a fair comparison, the decoding complexity and the bandwidth efficiency of these codes are also considered.

The performance curves presented in this chapter are generated using (3.28), (3.29), and (3.30). Note that the integral equation in (3.28) has no known closed form solution since an analytic expression for the distribution function of γ_0 is unknown. Therefore, to obtain the bit error rate, it is necessary to approximate (3.28) with its discrete form as follows,

$$P_e(d) = \sum_{n=0}^{\gamma} P_e(d|n\Delta) p_{\gamma_0}(n\Delta) \quad (4.1)$$

where,

$$\gamma = \max(\gamma_0) \cdot \Delta \quad (4.2)$$

and $\lceil x \rceil$ is the smallest integer larger than or equal to x . In (4.1), $P_{\gamma_0}(d/n\Delta)$ is the probability that, given the all-zero codeword is transmitted, another codeword Hamming distance d away is selected, on the condition that $\gamma_0 = n\Delta$. Here, $p_{\gamma_0}(n\Delta)$ is the probability that $\gamma_0 = n\Delta$. During preliminary investigations, it was found that a value of 0.01 for Δ was sufficient to obtain an accurate approximation. Smaller values of Δ resulted in the same performance curves as when a value of 0.01 was used. Numerically, when a value of 0.001 was chosen for Δ , the bit error rate differed by less than 0.04% from that when Δ was 0.01, while taking a substantially greater CPU time. Therefore, it was decided to use a value of 0.01 for Δ . Evaluation of (4.1) requires a knowledge of the distribution of γ_0 which is unknown. A small simulation program was run to generate this distribution function. It was found that 35,000 samples was sufficient to generate a distribution function leading to an accurate approximation of (4.1).

Figure 4.8 shows the distribution of γ_0 when $M = 2$ and $d = 8$. This figure suggests that the distribution resembles that of a Gaussian process with mean and variance which depend on the value of Md . This agrees with the central limit theorem, since γ_0 is a sum of Md Rayleigh distributed random variables. In generating the results to be presented, it is assumed that each Rayleigh path arrives at the receiver with the same average power. Although this assumption does not usually hold in a real channel, it is sufficiently valid for the purpose of comparing the performances of different techniques. Unless specified otherwise, it is also assumed that the number of physical paths in the multipath channel and the number of resolvable paths are the same and are given by [21],

$$L = \lceil T_m / T_c \rceil + 1 \quad (4.3)$$

where $\lceil x \rceil$ is the largest integer smaller than or equal to x , T_m is the maximum delay spread and T_c is the chip period.

Figure 4.8

Distribution function of γ_0 when $M = 2$ and $d = 8$.

4. 2. 1. Code Combining of Reed-Muller Codes

Having obtained the weight distributions, (4.1) can be evaluated to determine the bit error rate. In every result presented, the value of E_b/N_0 on the x-axis only includes thermal noise, but not the multi-user interference. The effects of multi-user interference set a lower limit on achievable bit error rate which is usually called the irreducible bit error rate or the noise floor.

In this sub-section, the performances of two Reed-Muller codes are evaluated. They are the first order (8,4,4), and the second order (16,11,4) Reed-Muller codes. Both codes have a minimum distance of 4, and are capable of correcting all single error patterns and detecting all double error patterns. The inherent frequency diversity of spread spectrum modulation and the use of multiple receiving antennas result in multiple copies which can be exploited to improve the bit error rate performance. Multiple copies of a symbol are code-combined and a maximum likelihood soft decision decoder is used to recover the symbol.

Figures 4.9 and 4.10 present, for the (8,4,4) Reed-Muller code, the bit error rate as a function of E_b/N_0 when multiple users are active in the system. In what follows, an encoded data rate of 32,000 kbps is assumed. This rate will result in a chip period of 122.07 ns and 61.035 ns when the processing gain is 256 and 512, respectively. Assuming a multipath delay spread of 200 ns, and using (4.3), these gains result in two and four resolvable paths. Both figures show that after a certain point, any increase in the E_b/N_0 does not result in a lower bit error rate because of the effects of multi-user interference. They also show that as the number of active users grows, the irreducible bit error rate increases since the noise due to multiple user interference increases proportionally.

Figure 4.9

Bit error rate as a function of E_b/N_0 and the number of active users in the system. $PG = 256$, $T_m = 200$ ns, diversity order = 2, (8,4,4) Reed-Muller code.

Figure 4.10

Bit error rate as a function of E_b/N_o and the number of active users in the system. PG = 512, $T_m = 200$ ns, diversity order = 4, (8,4,4) Reed-Muller code.

The system with the larger processing gain (Figure 4.10) is able to accommodate more users at any given bit error rate, since a larger processing gain results in a lower level of noise. As a result, more users can be accommodated before the noise level limit is exceeded. Moreover, doubling the processing gain doubles the order of diversity. Figure 4.10 clearly shows that doubling the processing gain allows the number of users to be increased by a factor greater than two. Both figures also suggest that as the number of users decreases, a greater improvement in the irreducible bit error rate is gained. In general, a drop in the processing gain limits the maximum number of active users in the system at any given target bit error rate.

Figure 4.11 shows the irreducible bit error rate of the same system as a function of the number of users when the E_b/N_0 approaches infinity, i.e. ignoring thermal noise and only considering multi-user interference. The solid line in this figure indicates the irreducible bit error rate when the processing gain is 512 (fourth order diversity) and the dotted line when it is 256 (second order diversity). This figure shows the obvious advantage of using a larger processing gain. This improvement, however, requires a larger bandwidth which may or may not be readily available. Both curves also clearly illustrates the graceful degradation feature of the system under consideration. As the number of active users increases, the irreducible bit error rate increases gradually and no dramatic drop in performance is observed.

In Figure 4.12, a performance comparison between the (8,4,4) and the (16,11,4) Reed-Muller codes with a diversity order of two is illustrated. Although both codes have the same minimum distance of 4, the (8,4,4) Reed-Muller code offers substantial improvement over the (16,11,4) Reed-Muller code in terms of the bit error rate.

Figure 4.11

The graceful degradation feature of the system under consideration. The (8,4,4) Reed-Muller code with $T_m = 200$ ns, and second and fourth order diversity.

Figure 4.12

Bit error rate as a function of E_b/N_0 for the (8,4,4) and (16,11,4) Reed-Muller codes. PG = 256, T_m = 200 ns, second order diversity

In fact, the difference in irreducible bit error rate is almost an order of a magnitude. This improvement comes from the fact that the (8,4,4) Reed-Muller code has a lower rate than the (16,11,4) Reed-Muller code, an issue that is not captured by Figure 4.12. Because of this lower rate, the (8,4,4) Reed-Muller code has a weight distribution which leads to a lower bit error rate. From Table 4.1, in total there are only 28 erroneous information bits in a set of codewords of Hamming weight 4 in the (8,4,4) Reed-Muller code. On the other hand, the same set in the (16,11,4) Reed-Muller code contains 410 erroneous information bits. Furthermore, the (8,4,4) Reed-Muller code has no codeword of weight 6, while the (16,11,4) Reed-Muller code has 448 such codewords, resulting in 1980 erroneous information bits.

In order to take into account the fact that they have different rates, a parameter called the bandwidth efficiency Bw_{eff} is defined as follows,

$$Bw_{eff} = \frac{KR}{N} \quad (4.3)$$

where R is the code rate, N is the length of the signature sequence, and K is the maximum number of active users in the system before the target bit error rate is exceeded. Of codes from the same class, a lower rate code has a more powerful error correcting capability at the expense of a wider bandwidth, leading to a system capable of accommodating more active users. The bandwidth efficiency parameter takes this factor into account, and thus provides a fairer comparison among codes of different rates.

Table 4.4 tabulates the bandwidth efficiency of the (8,4,4) and the (16,11,4) Reed-Muller codes in achieving a target bit error rate of less than 10^{-5} at an E_b/N_0 of 20 dB. In this table, K is the number of active users in the system. A graphical

representation of these results can be found in Figure 4.13. Along with Table 4.4, Figures 4.12 and 4.13 indicate that although the (8,4,4) Reed-Muller code results in a lower bit error rate than the (16,11,4) Reed-Muller code, the latter is spectrally more efficient. Without diversity, the bandwidth efficiency of the two codes is essentially the same. However, the use of multiple copies accentuates the superiority of the (16,11,4) Reed-Muller code in bandwidth efficiency. Furthermore, for each class of codes investigated, doubling the bandwidth results in a system that is more bandwidth-efficient as long as the required bandwidth is available.

Code	PG	Diversity	K	BER	Bw_{eff}
(8,4,4) RM	256	1	69	0.0820e-4	0.0117
(8,4,4) RM	256	2	69	0.0931e-4	0.1348
(8,4,4) RM	256	3	150	0.0984e-4	0.2930
(8,4,4) RM	512	2	69	0.0931e-4	0.0674
(8,4,4) RM	512	4	235	0.0998e-4	0.2295
(8,4,4) RM	512	6	415	0.0989e-4	0.4053
(16,11,4) RM	256	1	2	0.0997e-4	0.0054
(16,11,4) RM	256	2	51	0.0996e-4	0.1370
(16,11,4) RM	256	3	118	0.0988e-4	0.3169
(16,11,4) RM	512	2	51	0.0996e-4	0.0685
(16,11,4) RM	512	4	189	0.0944e-4	0.2538
(16,11,4) RM	512	6	345	0.0987e-4	0.4633

Table 4.4

Bandwidth efficiency of the (8,4,4) and the (16,11,4) Reed-Muller codes.
Target $P_b \leq 10^{-5}$, for $E_b/N_0 = 20$ dB.

Figure 4.13

Bandwidth efficiency of the (8,4,4) and (16,11,4) Reed-Muller codes.
Target $P_h \leq 10^{-5}$

4. 2. 2. Code Combining and Maximum Ratio Combining

In this sub-section, the performance of code combining will be compared to that of maximum ratio combining. Intuitively, it is expected that maximum ratio combining will outperform code combining. With maximum ratio combining, each sample is multiplied by the corresponding channel gain. In effect, this multiplicative factor accentuates contributions from stronger signals, and reduces those from weaker ones which are more likely to cause errors. In order to facilitate the combining process, the receiver must have knowledge of the signal strengths and phases, or of their approximations. On the other hand, code combining, as it is defined, simply combines multiple copies available regardless of the signal strengths. Essentially, in this manner code combining is equivalent to equal gain combining [20]. This approach is much simpler and is easier to implement since the receiver does not require knowledge of the signal strengths nor the phases.

Figure 4.14 illustrates the bit error rate performance of a system with 50 and 75 simultaneous active users in a building with a multipath delay spread of $T_m = 200$ ns. The processing gain of the system is 256, resulting in a second order diversity, and the (8,4,4) Reed-Muller code is used for error correction. The solid lines show the performance curves when multiple copies are code combined, while the dotted lines denote the bit error rate performance when maximum ratio combining is used. This figure demonstrates the superiority of maximum ratio combining over code combining. For a target bit error rate of 10^{-3} , the maximum ratio combining offers coding gain of 2 dB in the presence of 75 active users, and 3 dB when only 50 simultaneous users are active.

Figure 4.14

Bit error rate as a function of E_b/N_0 , PG = 256, $T_m = 200$ ns, 50 and 75 users, second order diversity, the (8,4,4) Reed-Muller codes.

Figure 4.15

Bit error rate as a function of E_b/N_0 , PG = 256, $T_m = 200$ ns, 75 users, second and fourth order diversity, the (8,4,4) Reed-Muller codes.

Maximum ratio combining also reduces the irreducible bit error rate by about 70% and 75% in the presence of 50 and 75 active users, respectively.

Figure 4.15 confirms the results presented in Figure 4.14. It is assumed that there are 75 users in the system. This figure reveals to us that as the order of diversity increases, in this case by doubling the bandwidth, the advantage of maximum ratio combining is more pronounced. At a target bit error rate of 10^{-3} , maximum ratio combining offers 1.5 dB of additional coding gain when the processing gain is 512, while it gains about 3 dB with a processing gain of 256. As E_b/N_0 increases, the maximum ratio combining offers a greater improvement when the order of diversity is higher. With a processing gain of 512, maximum ratio combining provides a greater improvement in terms of the irreducible bit error rate.

4. 2. 3. Performance Comparison of Various Codes

Figure 4.16 illustrates the performance of the (16,11,4) Reed-Muller code in comparison with the rate 2/3, memory 2 punctured convolutional code. Because of their similar rates, these codes require the same amount of bandwidth expansion. The bit error rates in this figure are derived for the case when there are 75 simultaneous active users requiring service in a building with a multipath delay spread of 200 ns. This maximum delay spread results in a second and fourth order diversity when the processing gains are 256 and 512, respectively. Multiple copies are code combined and then detected using a maximum likelihood soft decision decoder. The curves show that at any given E_b/N_0 , the (16,11,4) Reed-Muller code offers a lower bit error rate. The benefit of using the Reed-Muller code is greater as the order of diversity increases.

Figure 4.16

BER performance of the (16,11,4) Reed-Muller code and the rate 2/3, memory 2 PCC. $T_m = 200$ ns, 75 active users in the system, second and fourth order diversity.

Figure 4.17

BER performance of the (16,11,4) Reed-Muller code and the rate 2/3, memory 3 PCC. $T_m = 200$ ns, 75 active users in the system, second and fourth order diversity.

Nevertheless, Table 4.3 reveals that the trellis of the (16,11,4) Reed-Muller code has twice as many branches in a section than that of the rate 2/3, memory 2 punctured convolutional code. Therefore, the additional coding gain of the (16,11,4) Reed-Muller code comes at the expense of a more complex, but still manageable, decoding trellis.

Figure 4.17 compares the performance of the rate 2/3, memory 3 punctured convolutional code with the (16,11,4) Reed-Muller code. The trellises for these codes have the same level of complexity as given in Table 4.3. Since these codes have comparable rates, and are of the same decoding complexity, Figure 4.17 illustrates a fair comparison. The performance curves show that the rate 2/3 punctured convolutional code with memory 3 outperforms the Reed-Muller code. As before, 75 active users are assumed in a building with a multipath delay spread of 200 ns. Furthermore, Figure 4.18 shows that the performance curve of the (16,11,4) Reed-Muller code falls midway between that of the rate 2/3, memory 2 and memory 3 punctured convolutional codes. Table 4.5 and Figure 4.19 illustrate the performance of these three codes in terms of their bandwidth efficiency. The asterisks in Table 4.5 indicate codes of similar decoding complexity.

2 of/de 2

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Code	Diversity	K	BER	B_{eff}
rate 2/3, mem.2 PCC	1	-	-	-
(16,11,4) Reed-Muller*	1	2	0.0997e-4	0.0054
rate 2/3, mem.3 PCC*	1	6	0.0998e-4	0.0156
rate 2/3, mem.2 PCC	2	29	0.0929e-4	0.0755
(16,11,4) Reed-Muller*	2	51	0.0996e-4	0.1370
rate 2/3, mem.3 PCC*	2	70	0.0987e-4	0.1823
rate 2/3, mem.2 PCC	3	84	0.0954e-4	0.2188
(16,11,4) Reed-Muller*	3	118	0.0988e-4	0.3169
rate 2/3, mem.3 PCC*	3	149	0.0926e-4	0.3880

Table 4.5

Bandwidth efficiency comparison of the (16,11,4) Reed-Muller code and the rate 1/2, memory 2 and 3 convolutional codes

Figure 4.18

BER performance of the (16,11,4) Reed-Muller, rate 2/3, memory 2 and 3 PCCs. $PG = 256$, $T_m = 200$ ns, 75 active users, fourth order diversity.

Figure 4. 19

Bandwidth efficiency of the (16,11,4) Reed-Muller code and the rate 2/3, memory 2 and 3 convolutional codes

Note that with no diversity, the rate $2/3$, memory 2 punctured convolutional code is unable to achieve the target bit error rate. Figure 4.19 shows that the performance of the (16,11,4) Reed-Muller code is closer to that of the rate $2/3$, memory 3 punctured convolutional code than the corresponding memory 2 code.

Another useful comparison is between the (16,5,8) and (16,11,4) Reed-Muller codes. Although it has a lower rate, and thus is more powerful, the (16,5,8) Reed-Muller code requires a trellis which has a smaller number of branches in a section than the (16,11,4) code. Since the (16,5,8) Reed-Muller code has a lower rate and a larger minimum distance, it is expected that it will outperform the (16,11,4) Reed-Muller code. This is indeed the case, as shown in Figure 4.20. This figure shows that the (16,5,8) Reed-Muller code offers a tremendous improvement over the (16,11,4) Reed-Muller code in terms of the coding gain and the bit error rate. However, since they have different rates, it is necessary to compare their performance in terms of the bandwidth efficiency.

Figure 4.21 shows the bandwidth efficiency of the two codes in attaining a target bit error rate of less than 10^{-5} , when E_b/N_0 is 20 dB and the processing gain is 256. It is clearly shown that despite its lower rate, the (16,5,8) Reed-Muller code is more bandwidth efficient than the (16,11,4) Reed-Muller code. In addition, the trellis of the (16,5,8) Reed-Muller code has a smaller number of branches which makes decoding simpler. Therefore, whenever the bandwidth required is available, one should choose the (16,5,8) Reed-Muller code over the (16,11,4) Reed-Muller code for the reasons mentioned above.

Figure 4.20

BER performance of the (16,11,4) and the (16,5,8) Reed-Muller codes.

PG = 256, T_m = 200 ns, 75 active users, second order diversity.

Figure 4.21

BER performance of the (16,11,4) and the (16,5,8) Reed-Muller codes.

PG = 256, $T_m = 200$ ns, 75 active users, second order diversity

It is also of a considerable interest to compare the performance of the rate 1/2 memory 3 punctured convolutional code with the (16,5,8) Reed-Muller code. Although they have different rates, the trellises for both codes require the same maximum number of states in a section, and a section in each trellis contains the same maximum number of branches. The bit error rate performance of both codes was generated for 100 users requiring service in a room with a multipath delay spread of 200 ns, and a processing gain of 256, resulting in a second order diversity. Figure 4.22 shows that the (16,5,8) Reed-Muller code offers some improvement in terms of the bit error rate at any given E_b/N_0 . At a target bit error rate of 10^{-3} , the (16,5,8) Reed-Muller code offers almost 3 dB in coding gain. It also reduces the irreducible bit error rate by about 87%. Nevertheless, since the two codes are of different rates, we should look further into their bandwidth efficiency before drawing further conclusions.

Figure 4.23 and Table 4.6 show the bandwidth efficiency of the two codes at a target bit error rate of less than 10^{-5} , and an E_b/N_0 of 15 dB. The multipath delay spread of the room is assumed to be 100 ns and the system has a processing gain of 256. The figure shows that when multiple copies are available, the rate 1/2, memory 3 punctured convolutional code is more bandwidth efficient than the (16,5,8) Reed-Muller code.

Figure 4.22

BER of the (16,5,8) Reed-Muller code and the rate 1/2 memory 3 PCC.

$PG = 256$, $T_m = 200$ ns, 100 active users, second order diversity.

Figure 4.23

Bandwidth efficiency of the (16,5,8) Reed-Muller code and the rate 1/2 memory 3 PCC. PG = 256, $T_m = 100$ ns.

Code	Diversity	K	BER	Bw_{eff}
(16,5,8) Reed-Muller	1	32	0.937e-5	0.0391
(16,5,8) Reed-Muller	2	184	0.949e-5	0.2246
(16,5,8) Reed-Muller	3	353	0.989e-5	0.4309
rate 1/2, mem. 3 PCC	1	10	0.975e-5	0.0195
rate 1/2, mem. 3 PCC	2	134	0.949e-5	0.2617
rate 1/2, mem. 3 PCC	3	272	0.999e-5	0.5312

Table 4.6

Bandwidth efficiency of the (16,5,8) Reed-Muller code and the rate 1/2, memory 3 convolutional code.

Other pairs of codes whose performance comparisons may be worthwhile to look into are the (16,5,8) Reed-Muller code with the rate 1/3, memory 3 convolutional code, and the (8,4,4) Reed-Muller code with the rate 1/2, memory 2 convolutional code. Both codes in each pair are of similar or same rates, and require the same level of decoding complexity. Figure 4.24 shows that the rate 1/3, memory 3 convolutional code offers tremendous improvement in terms of the bit error rate performance over the (16,5,8) Reed-Muller code. Even the rate 1/3, memory 2 convolutional code, whose trellis contains a half as many branches, still offers some improvement.

Figure 4.25 shows the performance of the (8,4,4) Reed-Muller code in comparison with that of other codes of the same rate. With the same level of decoding complexity, the rate 1/2, memory 2 convolutional code offers a better bit error rate at medium to high levels of E_b/N_0 . If the memory order is increased by one, the performance is even better, as shown by the dashed and dotted curve.

Figure 4.24

BER performance of the (16,5,8) Reed-Muller code, and memory 2 and 3, rate 1/3 convolutional codes. $PG = 256$, $T_m = 200$ ns, 100 active users, second order diversity.

Figure 4.25

Bit error rate performance of the $(8,4,4)$ and $(32,16,8)$ Reed-Muller codes, and the rate 1/2, memory 2, 3 and 4 convolutional codes. $PG = 256$, $T_m = 200$ ns, 75 active users.

The (32,16,8) Reed-Muller code, which is also of the same rate, is able to match the performance of the rate 1/2, memory 3 convolutional code. However, this is achieved at the expense of increased decoding complexity. While the trellis of the rate 1/2, memory 3 convolutional code has only 16 branches in a section, that of the (32,16,8) Reed-Muller requires 512 branches. Figure 4.26 and Table 4.7 show the bandwidth efficiency of these codes.

Code	Diversity	K	BER	Bw_{eff}
(8,4,4) Reed-Muller*	1	6	0.0820e-4	0.0117
rate 1/2, mem. 2 CC*	1	26	0.0946e-4	0.0508
rate 1/2, mem. 3 CC	1	38	0.0976e-4	0.0742
(32,16,8) Reed-Muller	1	39	0.0809e-4	0.0762
(8,4,4) Reed-Muller*	2	69	0.0931e-4	0.1348
rate 1/2, mem. 2 CC*	2	135	0.0994e-4	0.2637
rate 1/2, mem. 3 CC	2	162	0.0987e-4	0.3164
(32,16,8) Reed-Muller	2	160	0.0973e-4	0.3125
(8,4,4) Reed-Muller*	3	150	0.0984e-4	0.2930
rate 1/2, mem. 2 CC*	3	255	0.0997e-4	0.4980
rate 1/2, mem. 3 CC	3	299	0.0973e-4	0.5840
(32,16,8) Reed-Muller	3	299	0.0955e-4	0.5840

Table 4.7

Bandwidth efficiency of various rate 1/2 codes.

Figure 4.26

Bandwidth efficiency of various rate 1/2 codes as a function of diversity order.

Up until this point, the performance of various Reed-Muller codes have been evaluated in comparison with convolutional codes of comparable rates and similar levels of decoding complexity. Most of the convolutional codes investigated have been shown to offer improvements in terms of the bit error rate and the irreducible bit error rate. In what follows, the improvements in terms of the coding gain obtained by convolutional codes of comparable rates at a practical E_b/N_0 to achieve a given target bit error rate will be presented.

Figures 4.27 illustrates the additional coding gain obtained by the rate 1/3, memory 2 and 3 convolutional codes with respect to the (16,5,8) Reed-Muller code at target bit error rates of 10^{-5} , 10^{-4} , and 10^{-3} . The processing gain is assumed to be 256, the multipath delay spread is 200 ns, and 100 active users are requiring service in the building. Figure 4.28 shows the additional coding gain obtained by the rate 1/2, memory 2 and 3 convolutional codes with respect to the (8,4,4) Reed-Muller code. In addition, the additional coding gain of the (32,16,8) Reed-Muller code is indicated by the dotted curve. In generating the results in this figure, 100 active users are assumed, and the processing gain is 512. In the next figure, the additional coding gain of the rate 2/3, memory 2 and 3 convolutional codes with respect to the (16,11,4) Reed-Muller code is presented. In this case, 75 active users are assumed, and the processing gain is 512. This figure shows that the rate 2/3, memory 2 convolutional code has a negative coding gain. A non-positive gain means that the reference code, in this case the (16,11,4) Reed-Muller code, outperforms the code in question.

All three figures show that at a target bit error rate of 10^{-3} , which may be suitable for voice communications, the convolutional codes of comparable rates and similar decoding complexity offer additional coding gains in the range of 0.5 dB to 3 dB. At higher target bit error rates, however, they offer greater additional

coding gains. This is due to the noise caused by multipath and multi-user interference. Because of this noise, each performance curve levels off at a certain level of bit error rate. A code with a higher irreducible bit error rate levels off before any code with a lower irreducible bit error rate. If the irreducible bit error rate is just past the target bit error rate, this will lead to a large additional coding gain for any code whose performance curve has not leveled off yet.

Code	10^{-3}	10^{-4}	10^{-5}
rate 1/2, mem. 2 CC	2.2353 dB	3.5294 dB	5.6897 dB
rate 1/2, mem. 3 CC	2.9412 dB	4.3529 dB	6.5517 dB
(32,16,8) Reed-Muller	2.4706 dB	4.1176 dB	6.5510 dB

Table 4.8a

Additional coding gain of various rate 1/2 codes over the (8,4,4) Reed-Muller code

Code	10^{-3}	10^{-4}	10^{-5}
rate 1/3, mem. 2 CC	0.8235 dB	1.7241 dB	3.7931 dB
rate 1/3, mem. 3 CC	2.3529 dB	3.6207 dB	5.1724 dB

Table 4.8b

Additional coding gain of rate 1/3 convolutional codes over the (16,5,8) Reed-Muller code

Code	10^{-3}	10^{-4}	10^{-5}
rate 2/3, mem. 2 CC	-1.7442 dB	-2.5581 dB	-8.2759 dB
rate 2/3, mem. 3 CC	1.1628 dB	1.8605 dB	2.5862 dB

Table 4.8c

Additional coding gain of rate 2/3 convolutional codes over the (16,11,4) Reed-Muller code

Figure 4.27

Coding gain of the rate 1/3, memory 2 and 3 convolutional codes over the (16,5,8) Reed-Muller code.

Figure 4.28

Additional coding gain of the rate 1/2, memory 2 and 3 convolutional codes and the (32,16,8) Reed-Muller code with respect to the (8,4,4) Reed-Muller code.

Figure 4.29

Additional coding gain of the rate 2/3, memory 2 and 3 convolutional codes with respect to the (16,11,4) Reed-Muller code.

Chapter 5

Conclusions

In this thesis, an indoor wireless CDMA system is investigated. Code combining with equal gain is used to exploit the inherent frequency diversity of spread spectrum system. Reed-Muller and convolutional codes are used for error control purposes and soft decision maximum likelihood decoding is employed. The theoretical performance of the system is derived.

5. 1. Summary of Results and Contributions

In the first section of Chapter 4, new results on the weight distributions of various Reed-Muller codes are presented. With these new results, the bit error rate of a system using a Reed-Muller code with maximum likelihood decoding can be determined. The new closed form formula for the complete weight distributions of the first order Reed-Muller codes was also found. Previous results available in the literature only allow for the evaluation of the word error rate. This prohibited any useful performance comparisons between Reed-Muller codes and convolutional codes.

The second half of Chapter 4 contains performance comparisons of various codes of interest. It also shows the performance loss of equal gain code combining

if used instead of maximum ratio combining. Maximum ratio combining offers an additional coding gain of up to 3 dB over equal gain code combining. Maximum ratio combining, however, involves a more complex combining algorithm, requiring knowledge of the path gains. In the case where accurate approximations of path gains are not feasible, equal gain code combining may be desirable. The loss in performance may be compensated by using a more powerful code.

The remainder of Chapter 4 contains performance comparisons between various Reed-Muller codes and convolutional codes of similar rates and comparable decoding complexities. Overall, convolutional codes provide a better performance than Reed-Muller codes. At a target bit error rate of 10^{-3} , which may be suitable for voice applications, the additional coding gain offered by convolutional codes ranges from 0.5 dB to 3 dB. The results show that simple block codes, such as Reed-Muller codes, provide an alternative to convolutional codes. In a situation where a Reed-Muller code is more desirable than a convolutional code, performance loss may be compensated by using a lower rate code. This can be done since the soft decision decoding of many Reed-Muller codes of short or moderate lengths is computationally manageable.

5. 2. Recommendations for Further Research

In this thesis, it is assumed that the number of paths in the multipath channel is fixed, and each path has the same average power. It would be interesting to see how the performance of equal gain code combining will be affected when the number of paths is a random variable, and the average power of each path is not constant.

This thesis only considered one class of block codes, Reed-Muller codes

which are simple block codes and do not have the best distance properties. It would be desirable to extend these results to other block codes which have more powerful error correcting capabilities, such as the BCH codes. Before this can be done, however, their complete weight distributions have to be determined.

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