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An Investigation of Spectrally Efficient Turbo Coded Modulation

by

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in partial fulfilment of the requirements for the degree of**

Master of Applied Science

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Dedication

To my father
The greatest man I know
“Te quiero mucho babbo”

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Abstract

Turbo codes have been widely considered to be the most significant error control coding technique of practical importance over the past decade. They are powerful error correcting codes, which offer impressive performance at energy efficiencies close to the limits predicted by information theory. Limited availability of spectrum and the ever increasing demands in wireless services for higher data rates and larger number of users ultimately calls for high spectral efficiency in communication systems. This thesis presents a study of two state-of-the-art spectrally efficient coded modulation schemes based on turbo codes, namely, turbo trellis coded modulation (TTCM) and bit interleaved turbo coded modulation (BITCM) and introduces an alternative bandwidth efficient iteratively decoded technique which combines serially concatenated non-binary cyclic (NBC) codes with optimally designed multidimensional signal constellations. This turbo coding technique is addressed in this work as turbo non-binary coded modulation (T-NBCM). In this thesis, it is shown that T-NBCM outperforms TTCM and BITCM for the spectral efficiencies and message sizes of practical interest that have been investigated.

Table of Contents

ABSTRACT.....	ii
TABLE OF CONTENTS.....	iii
LIST OF FIGURES.....	iv
LIST OF TABLES.....	vi
LIST OF ACRONYMS.....	vii
CHAPTER 1 INTRODUCTION.....	1
1.1 BACKGROUND.....	1
1.2 RESEARCH CONTRIBUTIONS.....	3
1.3 ORGANIZATION OF THESIS.....	4
CHAPTER 2 TURBO TRELLIS CODED MODULATION (TTCM) AND TURBO CODE BASED BIT INTERLEAVED TURBO CODED MODULATION (BITCM)	6
2.1 CHAPTER OVERVIEW	6
2.2 TRANSMITTER.....	7
2.2.1 ENCODER FOR TTCM	8
2.2.2 ENCODER FOR BITCM.....	11
2.2.3 SIGNAL MAPPING AND MODULATION	13
2.3 CHANNEL MODEL	15
2.4 RECEIVER	16
2.4.1 DECODER FOR TTCM	16
2.4.2 DECODER FOR BITCM.....	18
CHAPTER 3 TURBO NON-BINARY CODED MODULATION.....	20
3.1 CHAPTER OVERVIEW	20
3.2 MULTIDIMENSIONAL SIGNAL CONSTELLATIONS ON DENSE LATTICES.....	21
3.3 TURBO NON-BINARY CODED MODULATION (T-NBCM).....	27
3.3.1 ENCODER	28
3.3.2 DECODER	30
CHAPTER 4 SIMULATION RESULTS AND DISCUSSIONS.....	32
4.1 CHAPTER OVERVIEW	32
4.2 TNBCM SYSTEMS COMPARISON.....	33
4.3 SYSTEMS WITH SPECTRAL EFFICIENCY OF 2 BITS/s/Hz	34
4.4 SYSTEMS WITH SPECTRAL EFFICIENCY OF 3 BITS/s/Hz.....	41
CHAPTER 5 CONCLUSIONS	48
SUGGESTIONS FOR FUTURE WORK.....	50
REFERENCES	52

List of Figures

Figure 2.1	Elements of a turbo coded digital communication system model.	7
Figure 2.2	TTCM encoder structure.	9
Figure 2.3	Rate 2/3 TCM encoder with generator $(21,10,2)_8$ for the 2 bps/Hz TTCM..	9
Figure 2.4	Rate 3/4 TCM encoder with generator $(21,10,4,2)_8$ for the 3 bps/Hz TTCM	10
Figure 2.5	Symbol puncturing pattern for TTCM.	11
Figure 2.6	BITCM encoder structure.	12
Figure 2.7	RSC encoder with generator $(23,31)_8$ for the BITCM schemes.....	12
Figure 2.8	Bit puncturing pattern for 2 bps/Hz BITCM.	13
Figure 2.9	Bit puncturing pattern for 3 bps/Hz BITCM.	13
Figure 2.10	8-PSK signal constellation. Signal labels in boxes are for TTCM and in ovals are for BITCM.....	14
Figure 2.11	16-QAM signal constellation. Signal labels in boxes are for TTCM and in ovals are for BITCM.	14
Figure 2.12	The additive noise channel.	15
Figure 2.13	Decoding structure of the TTCM scheme.	17
Figure 2.14	Decoding structure of the BITCM scheme.	19
Figure 3.1	Two-dimensional dense hexagonal lattice.....	22
Figure 3.2	Optimal 16-ary hexagonal signal constellation.	23
Figure 3.3	Upper and lateral-diagonal view of an 8 point/3d signal constellation.	26
Figure 3.4	System overview of T-NBCM scheme.	28
Figure 3.6	Decoder for T-NBCM scheme.....	31
Figure 4.1	BER performance comparison over an AWGN channel for T-NBCM using 16QAM and near-optimal 16 points in 2 dimensions signal set, with spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 2000 bits.....	33
Figure 4.2	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 2000 bits.	36
Figure 4.3	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 200 bits.	38
Figure 4.4	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 4000 bits.	39

Figure 4.5	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 10000 bits.	40
Figure 4.6	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 2000 bits.	43
Figure 4.7	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 200 bits.	44
Figure 4.8	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 6000 bits.	45
Figure 4.9	BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 15000 bits.	46

List of Tables

Table 3.1	Basis vectors for the construction of multidimensional signal constellations.....	25
Table 3.2	Theoretical and practical gains of 16-2d, 256-4d and 4096-6d over 16-QAM.....	27
Table 4.1	System parameters for $\eta = 2$ bps/Hz TTCM, BITCM and T-NBCM schemes.....	36
Table 4.2	Code gains at a BER of 10^{-5} for $\eta = 2$ bits/sec/Hz T-NBCM and TTCM schemes over an AWGN channel.....	41
Table 4.3	System parameters for $\eta = 3$ bps/Hz TTCM, BITCM and T-NBCM schemes.....	42
Table 4.4	Code gains at a BER of 10^{-5} for $\eta = 3$ bits/sec/Hz T-NBCM and TTCM schemes over an AWGN channel.....	47

List of Acronyms

<i>M</i> -QAM:	<i>M</i> -ary quadrature amplitude modulation
<i>M</i> -PSK:	<i>M</i> -ary phase shift keying
BER:	Bit error rate
SER:	Symbol error rate
bps/Hz:	Bits per second per Hertz
AWGN:	Additive white Gaussian noise
SNR:	Signal-to-noise ratio
MAP:	Maximum a posteriori
APP:	<i>A posteriori</i> probability
BCJR:	Bahl, Cocke, Jelinik and Raviv
NBC:	Non-binary cyclic
RSC:	Recursive systematic convolutional
GF:	Galois field
TCM:	Trellis coded modulation
TTCM:	Turbo trellis coded modulation
BITCM :	Bit interleaved turbo coded modulation
T-NBCM:	Turbo non-binary coded modulation
SISO:	Soft-input / soft-output

1.1 Background

In the past several years we have experienced an explosion in demand for digital communication systems, fueled by growth of the Internet and wireless personal communication systems. Although demand has increased, available transmission bandwidth has not expanded as rapidly. Satisfying demand requires spectrally efficient coding and modulation schemes, capable of delivering higher throughput for fixed bandwidth and transmit power. Turbo codes, one of the most promising breakthroughs in communication theory, may help achieve this goal.

Claude Berrou and his colleagues originally proposed turbo codes in 1993[1]. These codes are generated by the parallel concatenation of two (or possibly more) constituent recursive systematic convolutional (RSC) codes with non-uniform interleaving. Not only are these codes extremely powerful error correcting codes, they can be decoded using a relatively simple iterative decoding algorithm, whereby information is repeatedly exchanged between two constituent decoder units. Although iterative decoding is sub-optimal, the bit error rate (BER) performance is nonetheless very impressive because of the great strength of turbo codes. Over the past decade turbo and turbo-like codes have been widely considered to be the most significant error control coding technique of practical importance [2].

The original turbo codes were designed to operate at spectral efficiencies of 1 bit per second per Hertz (bps/Hz) or less, and at these spectral efficiencies turbo codes have been shown to operate within a fraction of a dB of the Shannon limit.

Almost immediately after the introduction of turbo codes, researchers began exploring ways to apply these codes, and the associated iterative decoding algorithm, to higher spectral efficiencies. The first proposal was Le Goff's "Pragmatic Approach", which directly combined standard turbo codes with bandwidth efficient high order modulation and Gray mapping [34]. Later, Robertson and Worz proposed trellis turbo coded modulation (TTCM), which combines two Ungerboeck TCM codes [32] as component codes with signal mapping using set partitioning [43,4]. Benedetto et al. proposed a similar, but less flexible approach [42,8]. More recently Le Goff applied bit interleaver coded modulation techniques (BICM) [17] to turbo codes, when he proposed bit interleaved turbo coded modulation (BITCM) [3].

Of these approaches, Robertson's TTCM [4] and Le Goff's BITCM [3] proved to be the most effective and flexible. These state-of-the-art coded modulation techniques can provide significantly better performance than traditional, non-iteratively decoded techniques. However, unlike the original turbo codes, these new spectrally efficient techniques often operate at several dB higher than the Shannon limit. Operating close to the Shannon limit requires extremely large interleaver sizes (message sizes), which is undesirable because doing so leads to a large decoding delay¹. Increasing the strength (and complexity) of the constituent codes, which would normally improve performance of traditional non-iteratively decoded codes, is not effective for turbo codes [10].

¹ Decoding cannot commence until the entire code word has been received, and the codeword size depends on the interleaver size.

1.2 Research Contributions

In an attempt to improve the performance of spectrally efficient coded modulation, this thesis proposes a novel coded modulation technique that exploits multidimensional signal constellations on dense lattices [5]. When choosing a signal constellation for a communication system, it is desirable to space the signal points as far apart as possible to reduce the likelihood of decoding errors. On the other hand, to minimize the average transmitted signal energy it is necessary to have the points close together. For a fixed transmit energy, it is therefore desirable to have as large a distance as possible between adjacent points in the signal constellation, while still maintaining that fixed energy.

Both TTCM and BITCM use two-dimensional signal constellations, where signal points are either arranged in a circle (for M -ary PSK), or on an rectangular grid (for QAM). Neither of these constellations is very efficient in terms of the minimum separation between adjacent points for a fixed energy. A better solution is to place the points on a dense lattice, such as a hexagonal layout in two dimensions. Even better results can be achieved by using a multidimensional constellation. For example, for a fixed energy per bit, it is better to find the best placement of 256 points in four dimensions than 16 points in two dimensions, because larger inter-point separations are possible. This follows from the fact that if one used the same 16-point/2d constellation twice during two consecutive symbol intervals, the result is a 256-point/4d constellation, but not necessarily the best 256-point/4d constellation.

By using energy-efficient multidimensional signal constellations, better performance should be possible. Unfortunately, this is not necessarily the case. Although the separation between neighbouring points can be increased by using dense lattices, the number of neighbours of a given point also increases. This prevents the proper use of set partitioning required by TTCM when mapping code bits to signals, as well as the use of Gray mapping

required by BITCM. The performance of neither TTCM nor BITCM can be improved by using dense constellations [3]. To exploit the advantage of using dense lattices, a different coding scheme must be used, one that is insensitive to the mapping of the code bits to signal points.

Motivated by Reed-Solomon codes, for which the performance depends on the probability of the code symbol errors instead of code bit errors, we propose a novel coding method based on the serial concatenation of two non-binary cyclic codes. Non-binary cyclic codes are a large class of linear block codes that include Reed-Solomon codes as a sub-class. These codes operate on non-binary symbols instead of bits, and, provided the number of bits per symbol is not too large, can be decoded using soft-input / soft-output (SISO) decoding algorithms with acceptable complexity. SISO decoding is necessary for iterative decoding of concatenated codes.

Performance evaluation results presented in this thesis obtained by means of computer simulation, show that the concatenated coding scheme, which we call turbo non-binary coded modulation (T-NBCM), delivers better results than the current state-of-the-art TTCM and BITCM. Furthermore, it is shown that T-NBCM is less sensitive to block size than other schemes, providing good performance even with small block sizes.

1.3 Organization of Thesis

This thesis is organized into three major sections. Chapter 2 describes the two existing spectrally efficient turbo coded modulation schemes considered in this thesis. The digital communication system model for TTCM and BITCM are presented, including the encoders, channel model and decoders.

Chapter 3 presents several error control techniques based on non-binary cyclic (NBC) codes in which modulation is made over optimally designed multidimensional signal constellations. Emphasis is placed on the serial concatenation of two NBC codes with symbol interleaving, yielding the novel T-NBCM scheme. The iterative decoding structure for T-NBCM is described and illustrated. In addition, a construction method for optimum multi-dimensional signal sets is presented.

Chapter 4 provides performance evaluation results obtained by means of computer simulation. A comparison of the performance of TTCM, BITCM and several different T-NBCM schemes is presented for the additive white Gaussian noise (AWGN) channel model, for spectral efficiencies of both 2 bps/Hz and 3 bps/Hz.

Conclusions and topics for future research are provided in Chapter 5.

Turbo Trellis Coded Modulation (TTCM) and Turbo Code based Bit Interleaved Turbo Coded Modulation (BITCM)

2.1 Chapter Overview

One of the most important considerations in wireless communications is the efficient use of the available bandwidth. This goal is only attainable by digital communication systems capable of operating close to the information theoretical limits, such as those that make use of turbo codes.

Claude Berrou and his colleagues originally introduced turbo codes in 1993 [1]. Turbo codes combine error-correcting codes with interleaving and are capable of achieving excellent error performance provided that iterative decoding at the receiver is implemented. The impressive behaviour of turbo codes compared to other conventional coding schemes makes them suitable for low SNR applications, leading to applications with improved quality of service.

This chapter describes, within the context of spectrally efficient modulation based on turbo codes, the elements and functions of a digital communication systems model that are considered relevant to this research. Specifically, Robertson and Worz's TTCM [4] and Le Goff's BITCM [3] are described.

Figure 2.1 Elements of a turbo coded digital communication system model.

Figure 2.1 illustrates the functional diagram of a turbo coded digital communication system model. It consists of three basic elements: the transmitter, a transmission medium and a receiver. The transmitter employs a turbo encoder, which includes two convolutional encoders, an interleaver, and a modulator. The transmission medium is the communication channel where the transmitted signal is corrupted in a random manner. The receiver consists of a demodulator, and two constituent decoders with interleavers and de-interleavers.

2.2 Transmitter

The purpose of the transmitter is to convert digital messages into analogue signals suitable for transmission over the communication channel. The transmitter is composed of two parts, an encoder and a modulator. The encoder introduces, in a controlled manner, redundancy that can be exploited at the receiver, and the modulator modulates the amplitude and phase of a carrier wave which will convey the encoded data across the channel.

The encoder used for TTCM is described in Section 2.2.1 and for BITCM in Section 2.2.2. The symbol to amplitude/phase mapping schemes used by the modulator is described in Section 2.2.3.

2.2.1 *Encoder for TTCM*

Robertson and Worz's TTCM scheme employs two Ungerboeck TCM encoders [32] connected in parallel as shown in Fig 2.2. The message word is partitioned into a sequence of symbols, each containing k_c bits. These symbols are encoded with a rate k_c/n_c TCM code, such as the one produced by the rate $2/3$ encoder shown in Figure 2.3 or the rate $3/4$ encoder shown in Figure 2.4. At the same time, the message word is interleaved using a symbol interleaver and the interleaved sequence is also encoded, using an identical encoder.

The symbol interleaver rearranges the order of the message symbols, but keeps each message symbol intact. The pseudo-random permutation performed by the interleaver is known to both the transmitter and the receiver, and is constructed according to the S-random interleaver principle [6], with the additional constraint that the symbols in even-numbered positions be mapped to even positions, and the odd to odd [4]. The purpose of the interleaver is to increase the strength of the overall turbo code [6][7], and to permit iterative decoding at the receiver by ensuring local independence between the simultaneous outputs of both encoders [1].

Figure 2.4 Rate 3/4 TCM encoder with generator $(21,10,4,2)_8$ for the 3 bps/Hz TTCM.

Error performance of turbo codes is not primarily determined by the strength of the component codes, but rather by the size of interleaver. For larger interleaver sizes the performance of turbo codes improves significantly. However, larger interleavers create a major disadvantage, since they require longer decoding time and additional complexity, thus limiting the use of turbo codes to non-time-sensitive digital data applications.

The output of the second encoder is de-interleaved, using the inverse permutation used by the interleaver, and the de-interleaved symbols are passed along with the coded symbols from the first encoder, to the symbol puncturer. The purpose of the puncturer is to adjust the overall code rate. Prior to puncturing, the overall code rate is $k_c / (2 \cdot n_c)$, so to achieve the desired rate of k_c / n_c half of the symbols from each encoder must be punctured.

	X
X	

Figure 2.5 Symbol puncturing pattern for TTCM.

Puncturing is performed according to the mask shown in Figure 2.5. The ‘**X**’s in the mask indicate the positions where symbols are to be punctured. This mask therefore shows that the first symbol from the first encoder and the second symbol from the second encoder are to be transmitted, and the first symbol from the second encoder and the second symbol from the first encoder are to be punctured. The mask is then reused, so the third symbol from the second encoder and the fourth symbol from the first encoder are also punctured, and so on. The un-punctured symbols are passed to the symbol mapper and modulator for transmission.

2.22 *Encoder for BITCM*

In contrast to Robertson’s TTCM scheme where TCM encoders are used, Le Goff’s BITCM scheme employs two recursive systematic convolutional (RSC) encoders, concatenated in parallel as shown in Figure 2.6. Two different S-random interleavers are used prior to the RSC encoders. Both encoders are identical, with a structure similar to the example shown in Figure 2.7. Puncturing is used on the rate 1/3 output of the encoders. To achieve an overall rate of 2/3 the puncturing pattern shown Figure 2.8 is used and the pattern shown in Figure 2.9 yields an overall rate of 3/4.

	X	X	X
X	X		X

Figure 2.8 Bit puncturing pattern for 2 bps/Hz BITCM.

	X	X	X	X	X
X	X	X		X	X

Figure 2.9 Bit puncturing pattern for 3 bps/Hz BITCM.

2.2.3 *Signal Mapping and Modulation*

To convey the encoded data across the communication channel, the amplitude and phase of the carrier wave are modulated according to the modulation scheme being used. The amplitude / phase corresponding to a given symbol are determined by the signal constellation, examples of which are given in Figure 2.10 and 2.11 for 8-PSK and 16-QAM, respectively. The mapping between symbols and points in the signal constellation are also shown. For TTCM systems, mapping is done using set partitioning, so the last bit in symbols at adjacent points is different. For BITCM systems, Gray mapping is used, so that adjacent points differ in only a single bit position.

Figure 2.10 8-PSK signal constellation. Signal labels in boxes are for TTCM and in ovals are for BITCM.

Figure 2.11 16-QAM signal constellation. Signal labels in boxes are for TTCM and in ovals are for BITCM.

2.3 Channel Model

The physical medium that is used to send the signal from the transmitter to the receiver is defined as the communication channel. The transmission channel may include a variety of physical media, including the atmosphere, wire lines, optical fibre and microwave radio. Noise is an essential feature present in all channels. It is mainly introduced and amplified by the receiver's front-end and it corrupts in a random manner the transmitted signal. Therefore, to ensure reliable communication and counteract the effects of the channel, modeling of the transmission medium is essential.

The model of interest for this work is the common additive white Gaussian noise (AWGN) process, since it applies to a broad class of physical communication channels and because of its mathematical tractability. Figure 2.13 illustrates the basic model for a digital communication additive noise channel. The noise samples are statistically independent with a variance of $E[|w_n|^2] = N_0$, where N_0 is the single-sided noise spectral density of the AWGN.

Figure 2.12 The additive noise channel.

2.4 Receiver

The basic purpose of the receiver structure is to determine the transmitted message. The receiver must be designed so that, despite a hostile channel, enough information can arrive to allow the receiver to overcome the effects of noise encountered in the transmission, thus an extraction of the intended message from the received symbol sequence can be performed.

The receiver consists basically of two parts, the demodulator and the decoder. The demodulator demodulates the amplitude / phase of the carrier wave in which the encoded data symbols were mapped to and the decoder exploits the redundancy introduced by the encoder. The overall purpose is to reduce the probability of incorrectly identifying the transmitted message. Specifically for this research the criteria used to quantify the results for analysis is the bit error rate (BER) at a given signal to noise ratio (SNR). The decoder used for TTCM is described in Section 2.4.1 and for BITCM in Section 2.4.2.

2.4.1 Decoder for TTCM

The decoding structure for the TTCM scheme is illustrated in Figure 2.16. At its core there are two maximum *a posteriori* (MAP) decoders for decoding the two constituent TCM codes. One decodes the codeword for the non-interleaved message symbols and the other decodes the codeword based on the interleaved version of the message. Both calculate the *a posteriori* probabilities (APP's) of the message symbols using the algorithm developed by Bahl, Cocke, Jelinik and Raviv (the BCJR algorithm) [41]. The APP output from each decoder is passed (after appropriate interleaving) to the other, where it is used as estimates of the *a priori* probabilities of the message symbols. Thus each decoder has two sources of

information about the transmitted data: the received channel samples and the *a priori* probabilities. These two sources are combined by the BCJR algorithm to provide more reliable decoding. Each decoder operates in turn and the process is repeated for several iterations, with reliability increasing with each iteration. Of course, eventually additional improvements become negligible after a sufficiently large number of iterations. The metric calculation unit (MCU) shown in Figure 2.16 calculates the branch metrics based on the received samples that are used by the MAP decoders. They are calculated by evaluating the likelihood function of the received samples, conditioned on each of the M possible hypothetical code symbols. The un-puncturing device compensates for the effect of puncturing at the transmitter. In place of punctured symbols, for which there is no received sample since the corresponding code symbol was not transmitted, the un-puncturer inserts a set of M metrics, all of equal value. The decoder then automatically treats these symbols as “lost” so they do not affect the decoder output in any way.

Figure 2.13 Decoding structure of the TTCM scheme.

2.4.2 Decoder for BITCM

The decoding structure of the BITCM scheme is shown in Figure 2.15. It is very similar to the decoder for the TTCM in that two constituent MAP decoders are employed. The first decoder, denoted as MAP_1 , decodes the code of the first encoder at the transmitter, which is based on a RSC code. The first decoder uses the BCJR algorithm and its input are the de-interleaved branch metrics based on the received bit sequence and the respective un-punctured (and interleaved) samples that correspond to the punctured coded bits at the output of the first encoder. This interleaving is performed bit-wise and according to the interleaving mapping rule of the first interleaver at the transmitter. The first decoder estimates the message bit APP's, which are de-interleaved and interleaved according to de-interleaving and interleaving mapping rules of the first and second interleavers at the transmitter, respectively. This output is fed back as input to the second decoder (denoted as MAP_2), along with the interleaved branch metrics based on the received sequence and the respective un-punctured (and interleaved) samples that correspond the punctured coded bits at the output of the second RSC encoder. This interleaving is performed bit-wise and according to the interleaving mapping rule of the second interleaver at the transmitter.

The second decoder then estimates the APP's for coded message bits corresponding to the second code and hard decisions are made based on this output after appropriate de-interleaving (according to second interleaver mapping rule) to estimate the desired information.

The second decoder also produces the corresponding code bits APP's, which are de-interleaved and interleaved (according to the second de-interleaver and the first interleaver mapping rules, respectively), to be fed back into the first decoder, where are used as

estimates of the first code message bit *a priori* probabilities, improving the reliability of these bit probabilities and the decoder output.

Figure 2.14 Decoding structure of the BITCM scheme.

3.1 Chapter Overview

The proper combination of powerful error correcting codes and optimally designed signal constellations is of significant importance. It has been proven that those signal constellations which display optimal error performance in the absence of coding are generally not of interest for the design of commonly known turbo coding techniques, such as TTCM and BITCM schemes [4][3]. The most attractive multilevel signal constellation for BITCM and TTCM are 8-PSK and 16-QAM sets [3].

This chapter reviews efforts to improve the performance of efficient signal constellations for band limited channels through non-binary turbo encoding. It presents several error control techniques based on turbo decoding of serially concatenated non-binary cyclic (NBC) codes in which modulation is made over optimally designed multidimensional signal constellations. A construction method for near-optimal multidimensional signal constellations based on dense lattices is presented in Section 3.2, along with a comparative analysis of several signal sets in terms of uncoded symbol error rate performance. Section 3.3 describes the digital communications systems model, including the encoding and decoding processes performed at the transmitter and receiver respectively.

3.2 *Multidimensional Signal Constellations on Dense Lattices*

Traditional QAM signal constellations, which are built on rectangular lattices, are inefficient in the sense that the average signal energy required to ensure a given minimum separation between adjacent signal points is higher than it need be, and it is easy to construct constellations to prove this. Ideally, one would like to find optimal constellations by finding the positions of M arbitrary points in the carrier signal space that provide the lowest average energy while ensuring that no two points have a separation of less than some desired value, D_1 . Unfortunately, finding truly optimal constellations is difficult. Search algorithms based on non-linear programming, developed in the course of research for this thesis, proved to be computationally expensive and ultimately ineffective. As a result, we focused on searching for near-optimal constellations drawn from the set of vertices of dense lattices.

A two dimensional dense hexagonal lattice is defined as the set of points defined by all integral linear combinations of the two basis vectors, $(D_1, 0)$ and $(D_1/2, D_2)$, where D_1 is the minimum separation between points in the lattice, and $D_2 = \frac{\sqrt{3}}{2} D_1$. This resulting hexagonal lattice is shown in Figure 3.1.

A minimum-energy M -ary signal constellation based on this lattice can be constructed by finding the smallest possible circle that contains exactly M points. These M points, translated so that their centre of mass is coincidental with the signal space origin, define the constellation. Figure 3.2 shows a minimal energy constellation found with $M = 16$ points for this lattice. Note that the energy of this constellation is $2.1875 \cdot D_1^2$, as compared with $2.5 \cdot D_1^2$ for regular rectangular 16-QAM. This corresponds to an energy savings of 0.58 dB.

Figure 3.1 Two-dimensional dense hexagonal lattice.

That is, a communication system employing the constellation shown in Figure 3.2 should provide the same symbol error rate performance as rectangular 16-QAM but with 0.58 dB less energy per transmitted symbol.

Further energy savings are possible by extending this concept to higher dimension signal spaces. Table 3.1 lists the basis vectors for dense lattices in 2, 3, 4 and 6 dimensions. Extensions to other dimensions are straightforward. The same search algorithm can be used, except that the smallest multidimensional sphere containing M points must be found.

Figure 3.2 Optimal 16-ary hexagonal signal constellation.

Figure 3.3 shows an upper and side view of an 8-point constellation in three dimensions, which gives a 0.12 dB gain over 4-QAM, which gives the same throughput and has the same minimum separation.

By increasing the number of points and dimensions, it is possible to achieve additional gains while maintaining the same throughput. Table 3.2 shows the advantages of using higher dimensional signal constellations in terms of the symbol error rate performance for uncoded systems with an AWGN channel. All systems have a throughput of 2 bps/Hz/dimension, but since two dimensions can be transmitted simultaneously (one on the in-phase channel and one on the quadrature-phase channel), the throughput is 4 bps/Hz. The theoretical gains reported in Table 3.2 are based solely on the energy savings over rectangular 16-QAM. The practical gains are determined by computer simulation. The

difference between the two results is due to the fact that not only the separation between adjacent points is important, but also the number of nearest neighbours. The nearest decision regions for the dense-lattice constellations tend towards a multidimensional spherical shape as the number of dimensions increases, as opposed to a hypercube as assumed by theoretical analysis. Since the volume of a sphere is less than that of a cube with the same width, the probability of error is higher. To compensate, the transmit energy must be increased slightly.

In the next section we will present suitable coding methods for near-optimal multidimensional signal constellations that can exploit the energy gain over techniques that employ rectangular M -QAM.

Number of dimensions	Basis vectors
2	$(D_1, 0)$
	$\left(\frac{D_1}{2}, D_2\right)$
3	$(D_1, 0, 0)$
	$\left(\frac{D_1}{2}, D_2, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, D_3\right)$
4	$(D_1, 0, 0, 0)$
	$\left(\frac{D_1}{2}, D_2, 0, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, D_3, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, \frac{D_3}{4}, D_4\right)$
6	$(D_1, 0, 0, 0, 0, 0)$
	$\left(\frac{D_1}{2}, D_2, 0, 0, 0, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, D_3, 0, 0, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, \frac{D_3}{4}, D_4, 0, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, \frac{D_3}{4}, \frac{D_4}{5}, D_5, 0\right)$
	$\left(\frac{D_1}{2}, \frac{D_2}{3}, \frac{D_3}{4}, \frac{D_4}{5}, \frac{D_5}{6}, D_6\right)$

Where D_1 is the distance between adjacent signal points and $D_n = D_1 \sqrt{\frac{n+1}{2 \cdot n}}$

Table 3.1 Basis vectors for the construction of multidimensional signal constellations.

Figure 3.3 Upper and side view of an 8 point/3d signal constellation.

Modulation	Bits per Symbol	Energy per Symbol	Energy per Bit	Theoretical gain [dB]	Practical gain [dB]
16/2d	4	2.1875	0.5469	0.58	0.50
256/4d	8	3.6056	0.4507	1.42	1.15
4096/6d	12	4.7980	0.3998	1.94	1.60

Table 3.2 Theoretical and practical gains of 16-2d, 256-4d and 4096-6d over 16-QAM.

3.3 *Turbo Non-Binary Coded Modulation (T-NBCM)*

Multidimensional signal constellations on dense lattices can provide a given minimum separation between signal points with lower energy than is possible with constellations based on rectangular lattices, such as M -QAM. This does not, however, necessarily lead to a reduction in the bit error rate (BER) of coded systems. Because each signal point has a larger number of immediate neighbours than when a rectangular lattice is used, Gray mapping of code symbols to points in the constellation is not possible. As a result the performance of BITCM is degraded, more than cancelling any energy gains achieved through the use of a dense signal constellation. Similarly, appropriate set partitioning is also not possible, preventing the use of TTCM. These observations were confirmed by Le Goff [3].

To exploit the energy gains of dense lattices it is necessary to use a coding scheme that is not sensitive to the mapping between code symbols and signal points, such as non-binary codes. In this section we described a novel coding scheme based on the serial concatenation of two non-binary cyclic (NBC) codes separated by a symbol interleaver.

Figure 3.4 System overview of T-NBCM scheme.

The code symbols of this code are mapped to points in a dense multidimensional constellation, and an iterative decoding structure is used at the receiver. A schematic of this system, which we call turbo non-binary coded modulation (T-NBCM), is shown in Figure 3.4.

3.3.1 Encoder

The source information is first partitioned into a sequence of m -bit symbols, which are encoded by the outer NBC encoder, then interleaved on a symbol-by-symbol basis and re-encoded by the inner NBC encoder. Figure 3.5 illustrates a block diagram of a generic encoder for NBC codes. All adders, multipliers, and shift registers operate on m -bit symbols, and arithmetic is implemented over $\text{GF}(2^m)$ [11-14]. A message word of K symbols is fed into the encoder, one symbol at a time. As each symbol is processed the encoder state (as defined by the shift register contents) is updated according to the input and the code

generator coefficients, $g_0, g_1, \dots, g_{n-k-1}$, which are also elements of $\text{GF}(2^m)$. When all the message symbols have entered the encoder, the $N - K$ symbols in the shift register provide parity symbols which are appended to the K message symbols, given the N -symbol code word. The resulting code rate is K/N .

The concatenated encoder shown in Figure 3.4 consists of two different NBC encoders; an outer encoder with rate K_o/N_o and an inner encoder with rate K_I/N_I . Both encoders, and the intervening symbol interleaver, operate on symbols of m bits. A source message block of K_T symbols is partitioned into K_T/K_o message words which are independently encoded by the outer NBC encoder. The resulting $N_o K_T/K_o$ symbols are interleaved using a pseudo random permutation. The interleaved symbols are then partitioned into $N_o K_T/(K_o K_I)$ message words for the inner encoder, which provides a total of $N_T = N_o N_I K_T/(K_o K_I)$ code symbols. Each of these symbols is mapped to a point in an M -ary multidimensional signal constellation, where $M = 2^m$, and transmitted over the channel.

Figure 3.5 Block diagram of a generic encoder for non-binary cyclic codes.

3.3.2 Decoder

The decoder structure for the T-NBCM scheme is shown in Figure 3.8. This structure is essentially identical to the one proposed by Benedetto et al. for decoding serially concatenated codes [28]. The primary difference is that our constituent decoders operate on the trellis for NBC codes instead of convolutional codes.

Since the NBC encoders, as shown in Figure 3.5 are finite state machines, soft-input / soft-output maximum *a posteriori* probability (MAP) decoding can be implemented using the BCJR algorithm [40]. The inner decoder decodes the inner NBC code, and is implemented using the BCJR algorithm. Its input are the branch metrics which are based on the samples of the received signal. The output of the inner decoder are the *a posteriori* probabilities (APP's) of the message symbol.

The symbol de-interleaver rearranges the message symbol APP's according to the inverse of the interleaver mapping rule used at the transmitter.

The outer decoder decodes the outer code and is also implemented using the BCJR algorithm. Its input are the de-interleaved symbol probabilities, which are used as branch metrics and its output are the estimated APP's of the outer code message symbols. Hard decisions are made based on this output to obtain an estimate of the original message. The outer decoder also produces the corresponding code symbol APP's, which are de-interleaved and fed back into the inner decoder, where they are used as estimates of the inner code message symbol *a priori* probabilities. The inner decoder then reprocesses the data to further improve the reliability of the symbol probabilities.

Figure 3.6 Decoder for T-NBCM scheme

4.1 Chapter Overview

This chapter investigates and compares the BER performance for several different configurations of the proposed T-NBCM scheme, against its equivalents Robertson's turbo trellis coded modulation (TTCM) [4] and Le Goff's bit interleaved turbo coded modulation (BITCM) [3] schemes, in terms of message block size and spectral efficiency.

It provides BER performance under an additive white Gaussian noise (AWGN) channel for various message block sizes and for rates $2/3$ and $3/4$ turbo codes, providing spectral efficiencies of $\eta = 2$ and 3 bps/Hz, respectively.

The following performance evaluation is based on computer simulation, using simulators written in the C++ programming language. The number of iterations is set to 12. After the last iteration hard decisions are made based on the output of the outer decoder, estimating the most likely original information. Both original and decoded information sequences are compared and bit errors are accumulated to provide an accurate assessment of the bit error performance of the coded at the end of the simulation. To ensure reliability of the results, a sufficiently large number of message blocks were transmitted for each simulation point to guarantee that a minimum of 100 message words with errors were detected.

4.2 TNBCM systems comparison

Determining signal sets that perform optimally when combined with powerful error-correcting codes is of significant importance. The performance of two TNBCM systems is shown in Figure 4.1. One uses a near-optimal 16-point signal constellation in two dimensions and the second uses 16QAM.

Figure 4.1 BER performance comparison over an AWGN channel for T-NBCM using 16QAM and near-optimal 16 points in 2 dimensions signal set, with spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 2000 bits.

It can be seen that TNBCM provides superior performance when combined with near-optimally designed signal constellations. Since the TNBCM schemes employs NBC codes, larger improvements can be achieved by exploiting the advantages of near optimally designed signal constellations, which offer a lower uncoded probability of symbol errors than other less efficiently designed signal sets, namely M -ary QAM and PSK. Le Goff [3] also proves that signal constellations that display optimal error performance in the absence of coding are not very attractive for BITCM schemes, and obviously neither they would be for TTCM schemes, where signal mapping through set partitioning is required. Near optimally designed signal sets are best suited to be combined with TNBCM schemes.

4.3 *Systems with spectral efficiency of 2 bits/s/Hz*

To investigate the system performance at a spectral efficiency of 2 bps/Hz, the TTCM and BITCM systems were simulated using 8-PSK. The TTCM system was based on the parallel concatenation of two rate-2/3 TCM encoders, both with generators $(21,10,2)_8$, as shown in Figure 2.3, with puncturing according to Figure 2.5 to give an overall code rate of 2/3. The BITCM system used two rate-1/2 RSC encoders with generators $(23,31)_8$ as shown in Figure 2.7, punctured according to Figure 2.8 to also give an overall code rate of 2/3. These choices of encoders/puncturing patterns were the best ones found by Robertson and Worz [2] and Le Goff [3].

Two different T-NBCM schemes are considered, one using a near-optimal 16-point constellation in two dimensions and the second using a 32-point constellation in three dimensions. In both cases the code rate was adjusted to yield a spectral efficiency of 2 bps/Hz. In this work the code generation parameters are determined by selecting the ones that best enhance the error correcting capabilities of the code. The first scheme uses a rate-4/6 NBC outer code and a rate-6/8 NBC inner code, both defined over $\text{GF}(2^4)$ with generator $(14,7)_8$. The second scheme uses a rate-6/8 NBC outer code and a rate 8/10 NBC inner code, defined over $\text{GF}(2^5)$ with generator $(21,21)_8$. Table 4.1 shows the system parameters for the 2 bps/Hz spectral efficient schemes under consideration. Figure 4.2 shows the BER performance of these three schemes with a message block length of 2000 bits¹. For comparative purposes, the message block lengths in all schemes were chosen to be similar in size. The number of decoding iterations was set to twelve.

All systems present exceptional performance, although the BITCM scheme presents an error floor imposed by its error free feedback bound [16], indicated by the “levelling off” of the BER curve at high E_b/N_0 . The corresponding TTCM and T-NBCM error floor convergences do not become an issue unless BERs below 10^{-5} are to be considered.

¹ For the T-NBCM scheme 2, the message length, which must be a multiple of 30 was selected to be of 2010 bits.

Scheme	Code Rate	Signal Set	Encoders
TTCM	2/3	8-PSK	TCM(23,2,10) ₈
BITCM	2/3	8-PSK	RSC(23,31) ₈
T-NBCM 1	1/2	16/2d	NBC(14,7) ₈
T-NBCM 2	3/5	32/3d	NBC(21,21) ₈

Table 4.1 System parameters for $\eta = 2$ bps/Hz TTCM, BITCM and T-NBCM schemes.

Figure 4.2 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 2000 bits.

This research along with others [9][10] have revealed that variations in constraint length of the constituent codes in on the TTCM and BITCM schemes do not significantly influence overall performance and in fact performance degrades if the constraint lengths are too large. This implies that there is no way to improve the performance of these schemes, other than by increasing the block size. In T-NBCM, as the constraint length is increased, larger signal constellations of higher number of dimensions are allowed and although this results in an increase in the systems complexity, there is a slight improvement of the overall performance.

Figures 4.3 through 4.5 show the BER performance of these three schemes for several message block lengths of 200, 4000 and 10000 bits². The performance of all schemes improves as block lengths are increased. It is worth noting that as larger interleavers are examined, the BITCM systems displays an error performance at $\sim 10^{-4}$ close to that of the turbo TCM based on the same signal set. This result is not surprising since as it is shown by Caire, Taricco, and Biglieri, BICM schemes can achieve coding gains very close to those obtained with bandwidth efficient coding systems designed according to TCM principles, provided that very powerful codes are employed [17].

The TTCM schemes have been taken as benchmarks for comparison with the T-NBCM schemes, since they present in most cases, a superior performance over the BITCM schemes. The code gains in dB at a BER of 10^{-5} are reported in Table 4.2 for the different block lengths under consideration and at 2 bit/s/Hz of spectral efficiency.

As shown Table 4.2, the BITCM scheme provides similar performance as the TTCM one, particularly if larger interleaver sizes are considered.

² For the T-NBCM scheme 2, the message lengths were selected to be of 210, 3990 and 9990 bits.

Figure 4.3 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 200 bits.

When smaller block lengths are employed, the T-NBCM scheme are less affected than the TTCM and BITCM schemes, both of which suffer a higher degradation in performance in such cases.

The TTCM scheme displays an error performance close to that of BITCM when 10,000 information bits per block length is considered. The SNR advantage at a BER of 10^{-5} for TTCM over BITCM in this case is merely a 0.05 dB. As smaller block lengths of 4000, 2000 and 200 bits are employed, code gain of TTCM over BITCM increases approximately to 0.12, 0.20 and 0.60 dBs respectively.

Figure 4.4 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 4000 bits.

The T-NBCM schemes provide far superior code gains than those of TTCM and BITCM. At a BER of 10^{-5} a 10000 bits message block length the T-NBCM first scheme provides 2.85 dB, which is about 0.72 dB and 0.77 dB superior in SNR terms when compared to its counterparts TTCM and BITCM schemes. As the message blocks lengths are reduced to 4000, 2000 bits and 192 bits, the respective 10^{-5} of BER is reached approximately at 3.08 dB, 3.22 dB and 5.06 dB, where as shown in Table 4.2, they are still well over of what TTCM and BITCM has to offer, providing code gains over TTCM of .72 dB, .85 dB, .74 dB, respectively.

Figure 4.5 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 2$ bits/sec/Hz, and a message word size of 10000 bits.

By implementing a 3 dimensional signal constellation in the second T-NBCM scheme, the overall complexity is increased and even though the complexity is larger in this case, the benefits are not very significant. It is worth noting that both T-NBMC systems in this case are already providing excellent results that are very close to the theoretical limits [38], any further improvement beyond this point represents a very demanding task.

As shown in Table 4.2, further improvement is possible when T-NBCM is employed in combination with near-optimally designed signal constellations, in 2 or more dimensions.

Aprox. Block Length (bits)	SNR in dB at BER of 10^{-5}		Code Gains	
	BITCM	TTCM	T-NBCM 1	T-NBCM 2
~200	6.40	5.80	0.74	0.85
~2000	4.27	4.07	0.85	0.92
~4000	3.92	3.80	0.72	0.78
~10000	3.62	3.57	0.72	0.74

Table 4.2 Code gains at a BER of 10^{-5} for $\eta = 2$ bits/sec/Hz T-NBCM and TTCM schemes over an AWGN channel.

4.4 Systems with spectral efficiency of 3 bits/s/Hz

The performance of the TTCM and BITCM systems at a spectral efficiency of 3 bps/Hz was simulated using a 16-QAM signal set. The TTCM system, based on the parallel concatenation of two rate-3/4 TCM encoders with generator $(21,10,4,2)_8$ and puncturing pattern according to Figures 2.4 and 2.5, provides an overall code rate of 3/4. The BITCM system employed two RSC encoders with generator $(23,31)_8$ and puncturing according to Figures 2.7 and 2.9, to also provide an overall code rate of 3/4.

Similarly, two different T-NBCM schemes were considered, both using a near – optimally designed 16 point constellation in two dimensions. The code rate for both schemes was adjusted to provide a spectral efficiency of 3 bps/Hz. Again, the code generation parameters were determined by selecting the ones that best enhance the error correcting capabilities of the code. The T-NBCM 3 scheme uses a rate-6/8 NBC outer code defined over $GF(2^4)$ with generator $(14,5)_8$. The inner code is a rate-1 accumulator code

over $GF(2^4)$, which is a NBC code with generator $(1)_8$, which operates on a message word size that is equal to the length of the entire block. In this case the parity symbol is not transmitted, so the inner code introduces no redundancy, but does create a correlation between codewords of the outer code. The T-NBCM 4 scheme employs a rate-12/14 NBC outer code and a rate-14/16 NBC inner code, both also defined over $GF(2^4)$ but with generator $(14,7)_8$. The simulation parameters for all schemes are presented in Table 4.3.

Figure 4.6 shows the BER performance of these three schemes with a message block length of approximately to 2000 bits³. It can be seen that superior performance is achieved by T-NBCM schemes combined with near-optimally designed signal constellations, with gains of up to 0.45 dB possible.

T-NBCM may demonstrate to be a preferable alternative as technology advances and limitations imposed by complexity diminish, making of these more complex but enhanced systems, a feasible reality.

Scheme	Code Rate	Signal Set	Encoders
TTCM	3/4	16-QAM	TCM(21,2,4,10) ₈
BITCM	3/4	16-QAM	RSC(23,31) ₈
T-NBCM 3	3/4	16/2d	NBC(14,5) ₈
T-NBCM 4	~3/4	16/2d	NBC(14,7) ₈

Table 4.3 System parameters for $\eta = 3$ bps/Hz TTCM, BITCM and T-NBCM schemes.

³ For the T-NBCM scheme 3, the message length, which must be a multiple of 6 was selected to be of 2040 bits. For the T-NBCM scheme 4, the message length, which must be a multiple of 12 was selected to be of 2016 bits.

Figure 4.6 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 2000 bits.

Figures 4.7 through 4.9 show the BER performance of these three schemes for several other message block lengths of size 198, 6000 and 15000 bits⁴.

It can also be appreciated that as the message block size becomes larger the behaviour of the TTCM and BITCM systems becomes more alike but always outperformed in every case by both the T-NBCM schemes. Once more, performance in all schemes is improved through block length increases. The EFF bound becomes again more of an issue in the BITCM systems with smaller interleaver size.

⁴

For the T-NBCM scheme 3, the message lengths were selected to be of 216, 6000 and 15000 bits. For the T-NBCM scheme 4, the message lengths were selected to be of 192, 6000 and 14976 bits.

Figure 4.7 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 200 bits.

Table 4.4 shows for different message block sizes, the code gains achieved over the TTCM schemes at a BER of 10^{-5} for the third and fourth 3bps/Hz spectral efficient T-NBCM schemes. At a BER of 10^{-5} the TTCM and BITCM schemes with a ~ 200 bits block length are about 0.20 dB and 0.95 dB inferior in SNR terms when compared with the T-NBCM third scheme. Even larger code gains of 0.40 dB and 1.15 dB are achieved when compared to the fourth T-NBCM scheme. For 2000 and 6000 bits of block length the third T-NBCM scheme provides 0.33 dB and 0.20 dB over the TTCM scheme.

Figure 4.8 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 6000 bits.

The fourth T-NBCM scheme provides 0.45 dB and 0.25 dB of code gain against TTCM. At the largest block size of 15000 information bits, both TTCM and BITCM systems provide similar benefits in performance. In this case, BITCM displays a SNR of approximately 5.26 dB at 10^{-5} of BER, slightly outperforming TTCM by merely 0.08 dB. and in this case the third and fourth T-NBCM schemes provide code gains of .23 dB and 0.27 dB over the TTCM scheme, respectively.

Figure 4.9 BER performance comparison over an AWGN channel for T-NBCM, BITCM and TTCM schemes, with a spectral efficiency of $\eta = 3$ bits/sec/Hz, and a message word size of 15000 bits.

It should be pointed out that although the T-NBCM schemes can deliver better performance than TTCM and BITCM, the decoder is more complex in the sense that the Trellis has more states. However, the performance of the TTCM and BITCM schemes cannot be improved for a same block size. Increasing the number of trellis states in these systems leads only to an increase of complexity, but does not result in an increase of performance [10]. This indicates that under these circumstances, both of these techniques cannot be exploited any further in order to provide additional improvements.

Aprox. Block Length (bits)	SNR in dB at BER of 10^{-5}		Code Gains	
	BITCM	TTCM	T-NBCM 3	T-NBCM 4
~200	8.50	7.75	0.20	0.40
~2000	5.93	5.95	0.33	0.45
~6000	5.43	5.55	0.20	0.25
~15000	5.26	5.34	0.23	0.27

Table 4.4   Code gains at a BER of 10^{-5} for $\eta = 3$ bits/sec/Hz T-NBCM and TTCM schemes over an AWGN channel.

This work has introduced a novel spectrally efficient turbo coded modulation technique for band limited channels, referred to as turbo non-binary coded modulation (T-NBCM). In an attempt to reduce the dependency of turbo codes on the size of the interleaver, which typically needs to be very large in order to achieve satisfactory results, this work shows that T-NBCM schemes offer superior performance that is less dependent on the message word length, than other turbo coding schemes of similar spectral efficiency, including turbo trellis coded modulation (TTCM) and bit interleaved turbo coded modulation (BITCM). This improvement in performance is obtained through an appropriate combination of serially concatenated non-binary cyclic codes and multidimensional signal constellations on dense lattices.

This thesis presented a description of the digital communication systems model and its elements considered relevant within the context of turbo codes. It studied two state of the art turbo coded technologies, specifically Robertson's TTCM and Le Goff's BITCM. It provided a performance analysis under AWGN channel, for 2 and 3 bps/Hz spectral efficient systems, within the context of 8-PSK and 16-QAM signal sets and turbo coding rates of $\frac{1}{2}$ and $\frac{3}{4}$ respectively, focusing on their behavior and predominant dependency on the interleaver size.

Emphasis was placed on the advantages of implementing optimally designed multidimensional signal constellations and a comprehensive research was conducted on several T-NBCM schemes, in which modulation was performed over 16 and 32 level optimum signal constellations in 2 and 3 dimensions respectively. The results demonstrate

that this technique outperforms BITCM and TTCM for the spectral efficiencies and interleaver sizes that are of practical interest that are considered in this work.

Since T-NBCM becomes affected to a lesser degree by interleaver size reductions, it is possible to consider smaller block lengths while obtaining notable code gains and therefore better performance, thus allowing them to be implemented in time sensitive digital data applications

Results show that T-NBCM gives better performance than TTCM and BITCM for a fixed block size and spectral efficiency. Although, T-NBCM is more complex, the other schemes could not match its performance even if they could be made more complex.

A performance evaluation of BITCM and TTCM schemes shows that block length increases cause BER to improve, but only up to certain level, after which further enhancement cannot be accomplished by the decoding process. Based on the same signal set, as the message block lengths become larger these techniques display similar performances.

All systems display performance improvements through message word length increases. However, the performance in BITCM and TTCM is limited by their channel capacity, as the most attractive signal sets for these techniques are the standards circular 8-PSK and rectangular 16-QAM signal sets [18][3][4]. This is a physical restriction and an additional drawback that it is not present in the T-NBCM scheme, which has the flexibility of implementing optimum and multidimensional signal constellations at the modulation stage. In addition, in the BITCM schemes, when its block length decreases, the BER tends to converge to its Error Floor Feedback bound, where its performance does not improve with higher SNRs, like it does in the TTCM and T-NBCM schemes.

It is clear that T-NBCM offers superior performance over the state of the art BITCM and TTCM turbo coding techniques. Though nominally more complex, T-NBCM allows for complexity alleviation techniques to be implemented to reduce the difference. Thereby, making T-NBCM techniques more applicable to an even wider range of applications.

Suggestions for Future Work

The findings of this work raise a number of interesting proposals for future work and research. These are mentioned as follows:

- The implementation of other high spectral efficient systems through the vast number of possibilities and combinations of different T-NBCM schemes and multidimensional signal constellations.
- The verification and analysis results of the complexity alleviation techniques of valid code words identification and analysis, which may vary depending on the choice of T-NBCM scheme and signal constellation.
- Since the T-NBCM schemes make use of non-binary cyclic codes, several specifically designed soft decision decoding algorithm for the Reed-Solomon codes may be worthwhile looking into, instead of BCJR algorithm considered in this work. As in [19][20][21] and [22], further complexity alleviation may be obtained along with an additional improvement in the overall performance.
- In this work no efforts are made towards studying the benefits of employing a more appropriate interleaver. An interleaver designed with specific rules to maximize the

performance of a T-NBCM scheme, rather than employing a pseudo random generated interleaver mapping rule, should provide even better performance.

- One interesting observation about T-NBCM schemes, is that their frame error rates appeared to be considerably lower, which is an attractive feature digital communication systems using automatic repeat request protocols. It would be worthwhile to formally study this behaviour.
- Finally, all simulations were performed under AWGN and even though the Gaussian noise process applies to a broad class of physical communication channels, further analysis of the T-NBCM turbo coding technology should be performed under different types of environments, such as Rayleigh and Ricean fading channels.

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