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Active Vibration Control of Smart Structures Using Piezoelements

by

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A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

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Ottawa, Ontario

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Faculty of Graduate Studies and Research
acceptance of the thesis,

Active Vibration Control of Smart Structures Using Piezoelements

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Aghil Yousefi-Koma
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Doctor of Philosophy

Chairman, Department of Mechanical and Aerospace Engineering

Thesis Supervisor

Carleton University
September, 1997

Abstract

Active vibration control of flexible structures using piezoelectric sensors and actuators is addressed in this thesis. The objective of this thesis is to find a simple and efficient active controller for vibration suppression of flexible structures, which is to be demonstrated experimentally.

The work begins with a modal method being used to derive the dynamic equations of flexural vibration of a flexible beam. The passive and active influences of the piezoactuators and piezosensors on the flexible beam are also derived in modal form. A state space modal model of the integrated flexible structure with bonded piezotransducers, i.e. *smart structure*, is then used to design the control system. A linear quadratic optimal regulator (*LQR*) is designed and implemented successfully in both simulation and experiment. Since piezoelements are very sensitive to noises, a linear quadratic Gaussian *LQG* compensator is introduced for noise rejection which performed very well. In order to observe and control each dynamic mode individually, an *active modal controller* is designed which can control individual low frequency modes separately using *only rectangular piezotransducers*. To find the optimum shape and location of the piezoactuators in the active control system, an optimization procedure is employed which provides *simultaneous optimal shape and location of the distributed piezoactuators*.

Although the modal modeling and control of the smart structure demonstrated satisfactory performance, this method is very dependent on a good dynamic model and simulation program to obtain the control gains. As an alternative to the modal method, the *wave method* is used in the second part of the thesis to obtain a distributed dynamic model of the system. A novel wave based model of the integrated smart structure is obtained which is then used in the control design. Using the local parameters of the system such as the length of piezotransducers, a local wave absorbing controller (*LWAC*) is designed in the frequency domain to suppress the flexural vibration. Since the

expression of the controller in the frequency domain is nonlinear and very difficult to realize in practice, a time domain approximation of the controller is introduced which results in a parameter based controller, i.e. the control gains are obtained from a parametric formula without the need for any dynamic model or simulation program. To improve the performance of the wave controller, the global parameters of the system such as the length of structure and the positions of the piezoelements with respect to the boundaries are also taken into account, and a novel global wave absorbing controller (GWAC) is derived. The performance of the controller is substantially better than that of the LWAC. This performance is also compared with that based on the conventional optimal approach, and demonstrates performance similar to that of optimal controllers while being much simpler to design. Experimental results match quite well with simulation and the *objective* of this research: “*design and implementation of a simple and efficient active control system for vibration suppression of flexible structures*” is well met.

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I would like to give my heartfelt thanks to my parents for their sincere support, encouragement, and prayers from half a world away. My family deserves more thanks

than I can ever possibly express. While it is true that the life of a graduate student can be highly stressful, it is equally true that living with a student can be a difficult task: I am enormously grateful to my wife for her boundless love, patience, sacrifice, and understanding, to my son for bringing endless joy and inspiration to our life.

Above all, my deepest appreciation to Allah almighty who gave me the strength and opportunity to rise to this stage. I wish for his mercy for the rest of my life.

Dedicated to
my parents, Kafieh and Mohammad-Hosseini,
my wife, Zahra, and my son, Amir-Hosseini

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Nomenclature

Signs

'	Transpose of a matrix or vector
$\cdot$	$\frac{\partial}{\partial t}$
$E(.)$	Expectation operator
sgn	Sign function

Abbreviations

Cte	Constant value
diag	Diagonal matrix
Eq.	Equation
Exp.	Experiment
F	Farad (unit of the electrical capacitance)
Kg	Kilogram
max	Maximum
min	Minimum
m	Meter
mm	Millimeter
nAmp	$10^{-9} \times$ Ampere
Pa	Pascal
sec	Second (time)
Sim.	Simulation
V	Volt

Acronyms

AD	Analog to Digital
AMM	Assumed Mode Method
APB LWAC	Analytical Parameter Based LWAC
CFM LWAC	Curve Fit Model of LWAC
DA	Digital to Analog
DPS	Distributed Parameter Systems
ER	ElectroRheological Fluid
FEM	Finite Element Method
GWAC	Global Wave Absorbing Controller
I	Integral Operator
LQG	Linear Quadratic Gaussian
LQR	Linear Quadratic Regulator
LTR	Loop Transfer Recovery
LWAC	Local Wave Absorbing Controller
MR	MagnetoRheological Fluid
P	Proportional Operator
PAN	PolyAcryloNitrile
PI	Proportional and Integral Operator
PI GWAC	PI approximation of GWAC
PMN	Lead Magnesium Niobate
PZT	Lead Zirconate Titanate
PVDF	Polarized homopolymer of Vinylidene Fluoride
SMA	Shape Memory Alloy
SMP	Shape Memory Polymer
WAC	Wave Absorbing Controller
WAM	Wave Absorbing Method

Latin Symbols

$a_1(x, \omega)$	Traveling wave element in the positive x direction, Eq. (6.10)
$a_1(i)$	$a_1(x_i, \omega)$
$a_2(x, \omega)$	Evanescent wave element in the positive x direction, Eq. (6.10)
$a_2(i)$	$a_2(x_i, \omega)$
$b_1(x, \omega)$	Traveling wave element in the negative x direction, Eq. (6.10)
$b_1(i)$	$b_1(x_i, \omega)$
$b_2(x, \omega)$	Evanescent wave element in the negative x direction, Eq. (6.10)
$b_2(i)$	$b_2(x_i, \omega)$
A	Dynamic matrix of the system, Eq. (3.105)
A1, A2	Partial dynamic matrices of the system, Eq. (3.106)
A_a	Piezoactuator area, $W_a \times l_a$
A_d	$A_d = A - Bk^x$, Eq. (5.6)
A_b	Beam cross section area
A_p	Piezoelement area, Eq. (A.11)
B	Coefficient matrix of control voltages, Eq. (3.108)
BB	Partial coefficient matrix of control voltages, Eq. (3.108)
$c(\omega)$	Wave controller in the frequency domain
C	Coefficient matrix of transverse displacements, Eq. (3.110)
CC	Partial coefficient matrix of transverse displacements, Eq. (3.110)
C^I	Coefficient matrix of piezosensor currents, Eq. (3.114)
CI	Partial coefficient matrix of piezosensor currents, Eq. (3.114)
C^V	Coefficient matrix of piezosensor voltages, Eq. (3.112)
CV	Partial coefficient matrix of piezosensor voltages, Eq. (3.112)
C^{VI}	Coefficient matrix of piezosensor voltages & currents, Eq. (3.116)

cI_i	i^{th} modal coefficient of the piezosensor current, Eq. (3.86)
cQ_i	i^{th} modal coefficient of the piezoactuator force, Eq. (3.31)
cV_i	i^{th} modal coefficient of the piezosensor voltage, Eq. (3.85)
C_p	Piezoelement electrical capacitance, Eq. (A.10)
C_s	Piezosensor electrical capacitance
d	Adiabatic piezoelectric constant matrix
d_{31}	Piezoelectric strain constant
d_{31a}	Strain constant of the piezoactuator
d_{31s}	Strain constant of the piezosensor
D	Electric flux density vector
D	Piezosensor electric flux, Eq. (3.78)
E	Electrical field intensity vector
E_a	Elastic modulus of piezoactuator
E_b	Elastic modulus of beam
E_s	Elastic modulus of piezosensor
E_p	Piezoelectric elastic modulus
$f(x, t)$	Distributed external forces
$F(x, y)$	Piezoelement domain function, Eq. (A.7)
F_l	Elongation force in piezoelement
F_a	Concentrated force induced by the piezoactuators
G_b	Shear elastic modulus of beam
H	Wave coefficient matrix, Eq. (6.20)
i	$i = \sqrt{-1}$
I_b	Beam moment of inertia of the cross section

$I_c(\omega)$	Imaginary part of the wave controller. i.e. $I_c(\omega) = \text{Imag}(c(\omega))$
I_s	Piezosensor current
I_s^j	Output current of the j^{th} piezosensor
J	LQR performance index, Eq. (4.2)
k^x	Control gains for the dynamic states
k	Control gains for the implementation in the experiment
k_c	Controller gain vector
k_e	Estimator gain vector
k_{ij}	$k_{ij} = (x_j - x_i) \mu = (x_j - x_i) \sqrt{\frac{\omega}{\alpha}}$, Eq. (6.25)
k_{ij}^b	Symmetric stiffness coefficient of beam, Eq. (3.6)
k_{ij}^{piezo}	Modal stiffness induced by piezoactuators, Eq. (3.30)
k_{ij}^t	Total modal stiffness induced by piezoelements, Eq. (3.32)
K	Control design parameter for individual mode control, Eq. (4.12)
L	Beam length
L_p	Piezoelement length
m_{ij}^b	Symmetric mass coefficients of beam, Eq. (3.4)
m_a	One half of the piezoactuator length
m_s	One half of the piezosensor length
M	Bending moment
M_a	Total concentrated moment induced by piezoactuators
M_o	$M_o \equiv E_b I_b \mu^2$, Eq. (6.47)
n	Number of assumed modes
ns	Number of piezosensors
na	Number of piezoactuators

$P(x, y)$	Piezoelement poling function, Eq. (A.8)
$q_i(t)$	i^{th} modal (generalized) amplitude
Q	LQR positive semi-definite state weight matrix
$Q_i(t)$	i^{th} modal (generalized) force
Q_p	Piezoelement electric charge
Q_s	Piezosensor electric charge
Q_i^{piezo}	Modal force induced by piezoactuators, Eq. (3.31)
Q_i^t	Total modal force induced by piezoactuators, Eq. (3.33)
R	LQR positive definite input weight matrix
$R_c(\omega)$	Real part of the wave controller, i.e. $R_c(\omega) = Real(c(\omega))$
$s(x, \omega)$	Physical state vector, Eq. (6.17)
s_i	$s(x_i, \omega)$
S	Strain vector
S_1	Eq. (6.60)
S_2	Eq. (6.60)
S^E	Adiabatic elastic compliance constant matrix
S_{11}^E	Piezoelectric elastic compliance, $S_{11}^E = \frac{1}{E_p}$
t	Time
t_a	Piezoactuator thickness
t_b	Beam thickness
t_s	Piezosensor thickness
$\bar{t}$	Thickness ratio, Eq. (3.28)
t_p	Piezoelement thickness
T_{ij}	Transition matrix between waves at positions x_i and x_j , Eq. (6.27)

T	Kinetic energy
T_b	$T_b \equiv (t_b - 2z_o)$, Eq. (3.77)
$\mathbf{T}$	Stress vector
T_{CL}	Closed loop transition matrix between $\mathbf{w}_O$ and $\mathbf{w}_I$, Eq. (7.13)
T_{OL}	Open loop transition matrix between $\mathbf{w}_O$ and $\mathbf{w}_I$, Eq. (6.63)
$\mathbf{u}(t)$	Input voltage vector, Eq. (3.107)
u_a	Induced potential energy per volume of each piezoactuator, Eq. (3.18)
U_b	Potential energy of beam
v	Transverse displacement of beam
$v(x, \omega)$	Wave function, Eq. (6.9)
V	measurement noise covariance
V_3, V	Voltage between upper and lower piezoelement electrodes
V_s	Piezosensor voltage
V_s^j	Output voltage of the j^{th} piezosensor
$\mathbf{w}(x, \omega)$	Wave state vector, Eq. (6.16)
$\mathbf{w}_i$	$\mathbf{w}(x_i, \omega)$
$\mathbf{w}_I$	Input wave vector of the total sensing area, Eq. (6.58)
$\mathbf{w}_O$	Output wave vector of the total sensing area, Eq. (6.58)
W_{TI}	Aggregate wave function input into the sensing area
W_{TO}	Aggregate output wave function from the sensing area
$\mathbf{W}$	process noise covariance
W_a	Piezoactuator width
W_{a1}, W_{a2}	Endpoint positions of the piezoactuator in the Y direction
W_b	Beam width

W_p	Piezoelement width
x	Modal state vector, Eq. (3.104)
$\hat{x}$	Estimate of x
x	Horizontal coordinate
x_a	Midpoint Position of the piezoactuator in the X direction
x_s	Midpoint Position of the piezosensor in the X direction
X	Horizontal coordinate axis
y	Transverse displacement vector, Eq. (3.109)
y^I	Piezosensor current vector, Eq. (3.113)
y^V	Piezosensor voltage vector, Eq. (3.111)
y^{VI}	Piezosensor voltage & current vector, Eq. (3.115)
Y	Vertical coordinate axis
Z	Transverse coordinate axis
z_o	Neutral axis position in the Z direction

Greek Symbols

α	$\alpha = \sqrt{\frac{E_b I_b}{\rho_b}}, \text{ Eq. (6.4)}$
β_{16}	$\beta_{16} = \frac{k_{16}}{\sqrt{\omega}} = \frac{x_6 - x_1}{\sqrt{\alpha}}, \text{ Eq. (7.47)}$
β_s	Resistance to strain rate
$\Gamma(\omega)$	Correlation vector between w and M_a , Eq. (7.5)
γ	Wave eigenvalue, Eq. (6.6)

Δ	Determinant of $(I - c(\omega) \Psi \mu_2)$
ΔL_p	Elongation of Piezoelement
δ	Logarithmic decrement of two consequent vibrational peaks, Eq. (4.1)
δ_1 and δ_2	Eq. (7.8)
ϵ^T	Adiabatic dielectric permittivity constant matrix
$\epsilon_{31}^T = \epsilon_p$	Piezoelectric dielectric permittivity constant
ϵ_a	Piezoactuator strain
ϵ_b^s	Strain at the surface of the beam
ϵ_{tb}^s	Total strain at the surface of the beam, Eq. (3.25)
ϵ_{tb}^{ts}	Total strain at the top surface of the beam, Eq. (3.26)
$\bar{\epsilon}_s$	Average strain of the piezosensor
ζ_i	i^{th} modal damping ratio
η	Eq. (6.22)
$\theta(c)$	LWAC objective function, Eq. (7.31)
θ_1	$\theta_1 = 1 - \frac{x_a}{L}$, Eq. (8.2)
κ_b	Shear coefficient of the beam
Λ	Strain induced by the piezoelectric effect, Eq. (3.16)
Λ_a	Average strain induced by the piezoelectric effect, Eq. (3.22)
λ	Wave length, Eq. (6.12)
λ_i	i^{th} eigenvalue of the flexible beam, Eq. (3.11)
μ	Wave number, Eq. (6.7)
μ_1 and μ_2	Eq. (7.11)

Ξ	Eq. (4.11)
ρ_a	Volume density of the piezoactuator
ρ_b	Mass per length of the beam
ρ_s	Volume density of the piezosensor
ρ_{vb}	Volume density of the beam
σ_a	Piezoactuator normal stress, Eq.(3.17)
σ_b	Beam normal stress, Eq. (3.46)
σ_{tb}	Total normal stress of the beam, Eq. (3.46)
τ	Shear stress
Υ	Stiffness ratio, Eq. (3.48)
Φ	Eq. (6.62)
$\phi_i(x)$	i^{th} mode shape
$\chi(c)$	GWAC objective function, Eq. (8.5)
$\psi(x)$	Piezoelement shape function, Eq. (A.9)
Ψ	Eq. (7.4)
Ω_{ij}	Eq. (6.40)
ω	Frequency
ω_c	Shear mode cut off frequency
ω_i	i^{th} natural frequency
$\mathfrak{R}_i$	Constant, equation (3.69)

Chapter 1

Introduction

Control has been of great interest due to applications requiring low weight, lower energy consumption, ease of handling, safer operation due to reduced inertia, and high speed applications. Flexible structure control is much used in space applications such as potential solar power satellites, large antennas, and large space robots, but is of equal interest for terrestrial applications, such as high speed robots, large bridges, and others. Despite some of the merits listed above, light weight generally implies flimsiness with attendant difficulties such as long decay of vibration, fatigue and instability, and difficulty of accurate position and force tracking which are very important in vibration, position, shape, and force control. The use of highly flexible structures demands sophisticated controllers to overcome these problems.

Control of flexible structures has been an ongoing field of research for the past few decades. Among different control problems, “*vibration control*” is not only intrinsically important, but also is an issue of importance as it relates to many other flexible structural tasks such as position, shape, and force control which usually experience vibrations.

Two major approaches have been reported in the literature for vibration control: passive and active. In the passive method the damping of the structure is increased by using passive dampers or materials with significant viscoelasticity. This method can increase the total weight considerably and is best for high frequency modes. An active method integrates sensors and actuators with the flexible structure, operated by a control scheme. An important issue in active control systems is sensor and actuator selection. In recent years a new generation of sensors and actuators has been developed using so

called “*smart materials*”, such as piezoelectric materials, shape memory alloys, electrorheological fluids, etc., which are capable of automatically and inherently sensing and detecting changes in their environment and responding to those changes with some kind of actuations or actions. Although the definition of smart materials is still fuzzy in the literature, they have some common characteristics which put them in the same category, e.g. distributed nature and capability of integration with the substructure. These transducers can be highly integrated with flexible structures to make “*smart structures*”.

Among the different types of smart transducers, distributed piezoelectric sensors and actuators have received the most attention because of certain features such as: low mass and passive dynamic effect, high bandwidth, low cost, ease of bonding onto or embedding within flexible structures, capability of a wide frequency range of control, etc. . The use of distributed piezotransducers has given rise to new approaches in the active control of flexible structures (which are generally considered to be distributed parameter systems).

Although one of the advantages of use of piezoelements in active control of distributed parameter systems is their distributed nature for control of distributed systems, this issue is not well addressed yet in designing distributed control systems due to analytical complexity. Furthermore the distributed nature of these transducers raises interesting questions of sizing, shaping, and placement in an optimal control system. Despite some results in these areas, there is still a need for a reliable dynamic model, and an easy to design, efficient controller for real implementation, and an optimization of shape and placement of the piezotransducers in the active control of flexible structures using piezoelements.

1.1 Thesis Objective

The main goal of this thesis is to design a simple and effective active controller for vibration suppression of smart structures using piezoelements. En route to the attainment of this goal an accurate model of the integrated system is to be obtained. A flexible beam with bonded piezoelectric transducers is considered as the smart structure. Although the

flexible beam is one of the most common structures studied in the literature. with application to large complex structures, flexible link manipulators, etc., there still remain many challenging problems in finding simple and practical controllers using distributed modeling and control, and optimizing piezotransducer shape and placement in a smart flexible beam. In the following section a brief description of the thesis chapters covering the means of achievement of the research goal is presented.

1.2 Thesis Outline

A literature survey including flexible structure dynamics and control, smart structures, piezoelectric sensors and actuators, dynamics and control of flexible structures with piezoelectric transducers, and transducer size and placement optimization is presented in chapter 2. Two different approaches to dynamic modeling and control are employed and compared in the next chapters. Chapters 3 to 5 deal with a modal model of smart structures while in chapters 6 to 8 a wave method is used for distributed dynamics and control of smart structures.

Modal dynamic modeling of a smart structure consisting of a cantilever flexible beam with bonded piezoelectric sensors and actuators is described in chapter 3. The energy method is used to derive the spatially discretized modal model of the flexible beam. A simple procedure to incorporate modal damping in the dynamic equations is presented. The passive and active effects of the distributed piezoelectric actuators on the dynamic equations for the general case of thick actuators with a linear strain distribution are derived. A *novel concentrated induced force and moment model* of piezoactuators is also presented which is later used for wave dynamic modeling. Distributed modal actuators which in a particular manner decouple the dynamic equations of smart structure and generate individual modal forces are also discussed. General equations of output voltage and current of a thick piezosensor with a linear strain distribution are derived in modal form. The proposed models are checked by comparing them with special cases of thin piezoelements and uniform strain distribution. A brief description of selection of suitable

piezoelectric sensors and actuators is also presented. Finally a state space model of the integrated smart structure with PVDF (Polarized homopolymer of Vinylidene Fluoride) piezofilm sensors and PZT (Lead Zirconate Titanate) piezoceramic actuators is introduced.

In chapter 4 several controllers are applied to the modal model of the smart structure. As the first try an *LQR* optimal controller is implemented on the smart structure with corresponding increase of damping by a factor of twenty. The simulation results are verified by experiment. Since piezoelectric sensors are very sensitive to noises, an *LQG* control system is designed for the smart structure with piezofilm strain and strain rate sensors to overcome the noise problems and results indicate achievement of good noise compensation. A comparison with *LQR* performance demonstrates the capability of the *LQG* compensator for noise rejection. Due to well known problems of shaped modal piezosensors and piezoactuators in active control of individual vibrational modes (see chapter 2), such as the necessity of covering the whole length of the structure, and practical difficulties in shaping and cutting piezoelements, a method of *active individual modal control with only rectangular piezoelectric sensors and actuators* is presented and implemented successfully, with need for neither shaping and cutting of piezoelements nor covering the whole length with them.

Chapter 5 includes a *novel procedure for simultaneous shape and location optimization of piezoactuators* for active control of flexible structures. Three optimization criteria are investigated and compared which can be easily used in the actual experimental design of piezoactuators, and which leads to an optimal configuration of the actuators particularly relevant when weight and geometry limitations are important. Some interesting results have been obtained through this study, i.e. wider and shorter piezoactuators are more effective than longer and narrower ones for active vibration control.

Although certain conventional controllers such as *LQR* performed well for vibration suppression of the smart structure, a good dynamic model and suitable choice of weight matrices are needed to obtain the control gains via simulation; a controller simple to

design but which is still effective is desirable. An interesting approach which leads to such a controller is the wave method in which the flexural vibration can be viewed as distributed traveling waves.

As mentioned earlier, from chapters 6 to 8 the wave method is used for the dynamics and control design of the smart structure. In chapter 6 the distributed dynamic equation of motion of a flexible beam is developed in terms of flexural traveling waves. Next, the effects of the piezoactuators themselves are incorporated into the dynamic equations of traveling waves. Then, novel wave models of output voltage and current of distributed piezosensors (as strain and strain rate sensors) are introduced. Finally *dynamic wave models of the integrated smart structure* with bonded distributed piezoactuators and piezosensors for different collocated and non-collocated configurations are derived.

Using local parameters of the smart structure, such as the length of piezoelements, an analytical *local wave absorbing controller (LWAC)* based on the dynamic wave model is introduced in chapter 7. The control criterion is based on minimization of the ratio of aggregate output to aggregate input wave functions of the sensing area. Use of strain rate as a feedback signal eliminates the need for differentiator terms in the control loop, which usually cause noise problems. In addition, since no mode truncation is used in the control derivation, the controller is free of spillover problems. The LWAC is then derived for different collocated and non-collocated piezotransducer configurations. The natural derivation of the LWAC is in the frequency domain, since the original expression of the waves are frequency dependent, with the final expression of the LWAC in the frequency domain. Although the controller performance appears satisfactory in the frequency domain, its expression is difficult to realize in the time domain for actual implementation. In order to address these realization issues, and also to study the closed loop system in the time domain, a method of approximation of the controller enabling it to be represented as a simple parametric formula is introduced. Therefore, unlike the usual controller design based on a dynamic model (such as LQR) for obtaining the control gains, the LWAC gains depend on only system parameters in a simple formula. Results show that increasing the length of the sensing area improves the control

performance, which is expected due to better observation and control of the traveling waves in a longer area. However the LWAC shows poor performance for very short sensing length.

Incorporating the global parameters of the system, such as the total length of structure and the positions of the transducers with respect to the boundaries, a new control criterion is proposed in chapter 8 to design a *global wave absorbing controller (GWAC)*. Significant improvement over the LWAC is demonstrated and the controller suppresses the vibration quickly. A method of time domain approximation is introduced resulting in a simple parameter based controller. The controller gains are obtained from a parametric formula which includes only dynamic parameters of the system such as elastic modulus, E_b , density, ρ_b , and natural frequencies, while the performance of the controller is similar to that of the optimal controllers. Experimental results verify the simulation results and illustrate that the controller is capable of a quick suppression of vibration. Thus the main advantage of the GWAC over the conventional optimal controllers such as LQR is: easy design with approximately the same performance.

Chapter 9 presents the conclusions, a brief list of the research contributions of the thesis, and suggestions for future work in this area.

A brief description of piezoelectric materials and their constitutive equations is presented in appendix A along with an the electromechanical model of piezoelectric sensors and actuators. Some basic piezoelectric functions and parameters used in the text of the thesis are also introduced here. The characteristics of the smart structure studied in this research including a flexible beam, PZT piezoceramics, PVDF piezofilms are listed in appendix B. Appendix C illustrates the experimental set up and briefly describes its components. Some experimental techniques regarding piezoelectric sensors and actuators are also discussed. Some of the main simulation programs are listed in appendix D. MATLAB and its toolboxes are extensively used to write the simulation programs. A sample of various experimental programs written in C++ is shown in appendix E. Appendix F gives a list of publications reporting various results and contributions of this thesis to date.

Chapter 2

Literature Survey

In this chapter a review of the literature in the area of the thesis is presented. First the dynamics and control of flexible structures are briefly reviewed. Then *Smart Materials and Structures* as new paradigms of control of flexible structures are introduced and reviewed. Piezoelements and their applications as smart sensors and actuators are extensively reviewed. Following this, the dynamics and control of smart structures using piezoelectric sensors and actuators are studied. Finally a survey on the sizing and placement optimization of the sensors and actuators, particularly piezoelectric transducers, in active control of flexible structures is presented.

2.1 Dynamics of Flexible Structures

Flexible structures are often viewed as *Distributed Parameter Systems (DPS)* governed by partial differential equations. Such structures are often constrained to be light-weight and are lightly damped due to low damping inherent in the materials used. They are usually characterized by very low and clustered frequencies and high orders. Typical examples include large flexible robots, solar power satellites, space based robots, large antenna, etc. .There have been numerous investigations on the modeling of flexible structures, particularly the flexible beam, which is a common element of large structures and robotics. Classical analytical techniques can be employed to derive the dynamic equations of motion.

► Canon and Schmitz (1984), Hastings and Book (1986) clearly outlined a

procedure for obtaining the dynamic model of a flexible beam. Application of Euler-Bernoulli and Timoshenko beam theories to flexible beam modeling was examined from several points of view by Spector and Flashner (1989). Their study shows that Euler-Bernoulli model was an accurate model for the low frequency range and/or for very long thin flexible beams. For short thick beams and/or high frequencies, the Timoshenko model provides much more accurate results. The same results have been reported by Aldraihem et al. (1996) in examining the control stability of an active control system of a laminated beam. Morris and Vidyasagar (1990) compared different undamped and damped (viscous and Rayleigh) models of a flexible beam from the stand point of the design of stable controllers.

► Due to the complexity of the equations of motion, discretization techniques are frequently used to construct a finite dimensional system of ordinary differential equations. The two most common techniques used for discretization are: the *Assumed Mode Method (AMM)* and *Finite Element Method (FEM)*. The AMM is employed by many researchers, such as Krishnan and Vidyasagar (1988), Barbieri and Özgüner (1988). The FEM is also much used, e.g. Cardona and Geradin (1988), DiMagio and Bieniek (1993). In a comparative study, Theodore and Ghosal (1995) showed that the FEM was more suitable for complex cross sectional geometries of multilink structures, whereas they recommended AMM for uniform cross sections of single link structures.

► Reducing an infinite dimensional distributed system to a finite dimensional system (discretization) causes some problems such as spillover (Bass and Zes, 1991), and sensitivity to small parameter perturbation of the structure (Miller et al., 1987). There are several studies on methods of reducing the order of dynamic models to prevent these problems. For instance Seto and Mitsuta (1994) introduced a new method for model reduction using unobservability and uncontrollability to prevent spillover. A robust model reduction method was developed by Gawronski and Williams (1991) to cope with the damping uncertainties typical of flexible

structures. A limitation of discretization methods is that a great many modes may need to be considered, especially if it is desired to control the vibrations of large flexible structures or over a wide frequency range (Mace, 1987), which makes the model too complex to handle.

► An alternative to discretization techniques is distributed modeling such as wave modeling of flexural motion. Wave modeling is suitable for many flexible structures such as beams, plates, etc., which can serve as wave guides, thus allowing the response of the flexible system to be viewed as propagating flexural waves. This method employs the exact solution of the partial differential equations, thus it does not suffer from truncation and spillover problems. The wave method in dynamic modeling of flexible structures has been well addressed in some text books, e.g. Graff (1975) and Humar (1990). Dynamic wave modeling of flexible structures has been used by some researchers in recent years, von Flotow (1986), Hagedorn et al. (1989), Fujii et al. (1992), and Mizutani et al. (1996).

► There are several other dynamic modeling methods for flexible structures. For instance continuum modeling for obtaining the transfer function of the flexible structures was presented by Bennett and Kwatny (1989) and control-oriented modeling was introduced by Helmicki et al. (1992).

2.2 Control of Flexible Structures

The control of flexible structures has been an active field of research for the past few decades, due in some part to the advent of large aerospace structure. Two major approaches are used to control flexible structures: passive and active control.

2.2.1 Passive Control Systems

In the passive method desirable behavior is to be achieved by a suitable design of shape and size of the structure as well as a proper choice of material and passive dampers. This method is mostly used in vibration control by using materials with

significant viscoelasticity. The high frequency modes can generally be damped by this method but low frequency modes are more difficult to attenuate. In addition, increase of the total weight of structure, which is a common consequence of the passive method, is not desirable in many applications. Furthermore passive method is not an adaptive and versatile method for different tasks. Thus as an alternative to the passive method, active control of flexible structure, in which an integration of sensors/actuators and a controller is used, is frequently more appropriate.

2.2.2 Active Control Systems

A review of active control of flexible structures is presented in this section, covering active position, vibration, and shape control of flexible structures, and several control schemes.

• Distributed Control Systems

It is worth to mention here that there is considerable confusion about distributed systems. In most literature a controller designed based on the analytical equations of the dynamic distributed parameter system, without any discretization, is called a “distributed controller”, so called since no discretization is used and the final expression of the controller includes distributed parameters of the system. One might call it: “*Distributed Parameter Control System*”. Another definition of distributed control system is: a control system with highly distributed sensors and actuators which allows a spatially varying signal to be measured or applied. Large numbers of distributed sensors and actuators allow an approximation of a truly continuous system (de Luis and Crawley, 1990). To achieve this kind of controller, the continuous system must be discretized to incorporate spatially distributed sensors and actuators, and might therefore be called: “*Discrete Distributed Control System*”. In the following literature the first sense is used by the authors.

➤ Using Lyapunov’s second method Bailey and Hubbard (1985) designed a

distributed parameter controller for vibration suppression of a flexible beam.

➤ An optimal distributed control system using functional Ricatti equations was introduced by Lupi et al. (1992), who used an exact model of an Euler-Bernoulli beam in the control design which eliminated the effects of control spillover, since the mathematical model represented the actual physical structure.

• **Wave Absorbing Controller (WAC)**

Dynamic wave modeling of flexible structures can be used to design controllers based on wave absorption. Application of WAC in flexible structures has been studied by some researchers in recent years.

➤ Using WAC, a velocity feedback system with frequency dependent gains was designed for a flexible beam, in which the reflected energy was minimized (von Flotow and Schäfer, 1986).

➤ The Wiener-Hopf technique was used to derive a causal solution of a wave absorbing controller designed for a long flexible beam (Miller and Hall, 1991).

➤ Tanaka and Kikushima (1992) designed an active wave control system for a flexible beam using an electrodynamic exciter and gap sensors.

➤ Fujii and Ohtsuka (1992) applied this method to a flexible beam with a torque motor actuator and strain gauge sensors. They suppressed the vibration by eliminating outgoing waves at the boundaries of the structure.

• **Spatially Discretized Control Systems**

Using discretized models of flexible structures, e.g. FEM or AMM, controllers have been derived and employed for active control of flexible structures.

➤ Model reference adaptive control (Mufti, 1987) and a method of explicit self tuning adaptive control (Hanagud et al., 1990) were applied to the control of flexible structural systems.

➤ Castelazo and Lee (1990), used a nonlinear feedback system to control the tip position of a single flexible link. This control strategy was implemented as a

nonlinear second order differential equation, in which the nonlinear term represented a time-varying damping. This scheme resulted in faster rise and settling times than those obtained with an optimal linear controller.

► In an excellent paper, Gawronski (1993*b*) introduced a balanced Linear Quadratic Gaussian (LQG) compensator for flexible structures. A new LQG design was introduced and compared with Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) by Tahk and Speyer (1989). The comparison showed the new LQG design to be more robust to parameter variations of flexible structures.

► Coville and Abou-Kandil (1995) presented a robust controller for flexible structures. The controller combined the advantages of the hyperstability and small gain approaches. Computation of the H_∞ norm for flexible structures was addressed by Gawronski (1993*a*), and a formulation and design of H_∞ controllers for flexible beams were presented by Stoughton (1990).

► An optimal modal positive position feedback method for vibration control of flexible structures was introduced by Baz and Poh (1996). The algorithm utilized only modal position signals to suppress the vibration, and was validated experimentally.

► A simultaneous active position and vibration control was employed by Vincent et al. (1990) for flexible beams. The control design was based on open loop positioning followed by closed loop control of damping.

► There have been many experimental studies in active control of flexible structures. Canon and Schmitz (1984) were among the pioneers in experimental research in active control of a flexible beam. An experimental study on position control was performed by Wang et al. (1989). A good survey on experimental active control systems was performed by Sparks and Juang (1992).

Most of the active control literature is in vibration and position control of flexible structures, particularly in vibration control, which is the problem faced in most structural tasks. However, shape control is another important issue in flexible structural control. In

many space applications such as space mirrors and antenna, shape stability is an essential need. Static shape determination and control using Green's functions was performed for flexible beams (Weeks, 1984*a*) and a large antenna (Weeks, 1984*b*). Kashiwase et al. (1991) presented a robust controller for shape control of flexible structures, and using FEM a quasi-static shape controller for large aerospace antenna was introduced by Balas (1985).

2.2.3 Combined Passive and Active Control Systems

In recent years combined passive and active control of flexible structures has attracted much attention, particularly for vibration control. Some benefits of concurrent passive and active control were listed by Gehling et al. (1985). These benefits can include less expensive overall vibration suppression systems, relaxed actuator sensor requirements, more robust closed loop systems, and overall more reliable systems. Several techniques for blending active and passive control of flexible structures were presented by Gaudreault et al. (1993). These techniques were discussed with respect to computation requirements and robustness. Smith et al. (1991) suggested a method for finding the optimal mix of passive and active control of flexible structures. The interactions of active and passive damping of an Euler-Bernoulli beam was studied by Pang et al. (1993) via the beam's exact transfer function model.

2.3 Smart Materials and Structures

Since one of the main applications of smart materials and structures is in the area of active control of flexible structures, which is the core of this research, a review of this subject is presented in the following. This section acts as a bridge to the next sections, i.e. dynamics and active control of smart structures.

Smart materials and structures has opened a new and exciting path in engineering which can be called "dawn of a new material/structure age" as claimed by Rogers (1993). Being new field of research, there is still some confusion about the definitions

and applications of smart materials and structures. In following an attempt is made to provide some clear definitions and applications of smart materials and structures.

2.3.1 Smart Materials

Among many different definitions of smart materials the following might be taken as the most meaningful:

- Takagi (1990) described smart materials, as “materials which respond to environmental changes at the most optimum conditions and manifest their own functions according to the changes”.
- Shahinpoor (1996) defined smart materials as “the materials capable of automatically and inherently sensing or detecting changes in their environment and responding to those changes with some kinds of actuations or actions”. In his definition the smartest materials are biological RiboNucleic Acids (RNA's) and DeoxyriboNucleic Acid (DNA's), and engineering materials are last in the list.

The following materials have already been accepted as smart materials among the smart materials and structures community (refer to the surveys and reviews by: Jang, 1993, Crawley, 1994, Farshad, 1995, Chopra, 1996, Shahinpoor, 1996, Sirkis, 1996).

1. Polarized homopolymer of Vinylidene Fluoride (PVDF)

PVDF is a piezoelectric material (see appendix A) produced by extrusion followed by orientation-stretching below the melting point to cause packing of the unit cells into parallel planes. An extensive literature review on piezoelectric sensors and actuators and their applications in active control of flexible structures will be presented in the next section. PVDF piezofilms are soft, flexible, light weight with high bandwidth. The current uses of PVDF include strain and strain rate sensors, contact sensors, impact sensors, force sensors, biomedical flow meter, thermal radiation sensors, and ultrasonic transducers. They can also be used as actuators, although they cannot produce large forces.

2. Lead Zirconate Titanate (PZT)

PZT is another piezoelectric material with good strength and stiffness and higher strain constant than that of PVDF (see appendix B). It can be used for both sensors and actuators. PZT is the most popular smart material, and has recently entered to industrial applications (Ashley, 1995).

3. Shape Memory ceramics

Lanthanum and Niobium doped piezoceramics as a new material with anti-ferroelectric field-induced phase transition were introduced by Ghandi and Hagood (1995). These materials have higher strain constants than those of PZT's, with shape memory actuation behavior. They are not commercially available yet.

4. Lead Magnesium Niobate (PMN)

PMN is a ceramic electrostrictor which can be used as sensor or actuator with higher strain constant than PZT, but it is very sensitive to temperature.

5. Magnetostrictive Material

One of the best known magnetostrictive materials is Terfenol. Terfenol is a rare earth magnet-like magnetostrictive material with high strain constant. It can be used as an actuator but its application is complicated because of the need for a strong magnetic field.

6. Shape Memory Alloys (SMA)

The most common SMA is Nitinol (Nickel Titanium alloy) which is available in the form of wires. It appears as an attractive actuator because of the large excitation forces and displacements it can provide. When plastically deformed at a low temperature, these alloys recover to their original undeformed conditions if their temperature is raised above the transformation temperature. A major disadvantage of these materials is their very low bandwidth. Applications of SMA in active control of flexible structures have been studied by many researchers (e.g. Rao and Damel, 1994, Baz et al., 1990).

A very good table of comparisons of the above smart materials from strain, stiffness and

bandwidth point of views can be found in Crawley (1994).

7. Shape Memory Polymer (SMP)

SMP are polymers with an elastic memory like SMA. They have the capability of providing large reversible changes in their elastic modulus with temperature. SMP's can be found with base of polynorborene, trans-isopolyprene, and others.

8. ElectroRheological Fluid (ER)

ER's are very fine dielectric particles (1 to 100 μm in diameter) suspended in a carrier medium such as oil or paraffin. In the presence of an electric field, the particles are oriented along the field and a progressive gelling occurs which makes ER stiff enough to carry shear forces. ER's have been employed as actuators in active control of flexible structures (e.g. Ehergott and Masri, 1992, Mahjoob et al., 1993, Choi et al., 1994). A survey of application of ER in vibration control was performed by Stanway et al. (1996).

9. MagnetoRheological Fluid (MR)

MR is very similar to ER except its internal structure change occurs in response to magnetic fields. It also, can be used for actuators in active control systems. MR has much potential of industrial application such as new valves, adaptive automotive dampers, etc. (Kordonsky, 1992, Dyke et al., 1996).

10. Ionic Polymeric Gels

Ionic polymeric gels such as PolyAcryloNitrile (PAN), are soft materials capable of responding to an external stimulus such as chemical reagent, stress, heat, electric current, and others. This is an ion exchange membrane which responds to electrical stimulation by mechanical motion, somewhat resembling muscles (Okuzaki and Osada, 1993, Shahinpoor, 1994). Although these have not generated much interest yet because of their low authority, there is a promising future for these gels since they are the closest smart materials to biological materials, and as such might inspire the next generation of materials and structures. For instance Mojarrad and Shahinpoor (1996) introduced an artificial fish using Nafion polyelectrolyte ion-exchange membrane gel.

11. Fiber Optics

Fiber optics provide an inert, fast, reliable method of sensing changes by transmitting sensor information back to a microprocessor (Jang, 1993). Most optical fibers used today are fused Silica (SiO_2). They are light with high bandwidth. Haung et al. (1996) introduced a strain-distributed technique based on the measurement of the phase spectrum of the reflected fiber optic bragg grating. Application of fiber optic sensors for vibration monitoring of flexible structures was studied by Han et al. (1995). A good survey on fiber optics sensors was presented by Culshaw et al. (1994).

12. Biological Materials and Biomimetics

The emergence of the embryonic field of smart materials has stimulated an interest in biological materials as researchers attempt to mimic the features, characteristics and growth of these naturally-occurring materials. The field of biomimetics is based on the assumption that through evolution, nature has solved an optimization problem (see Gandhi and Thompson, 1992). Some characteristics which a biomimetic material ought to have have been listed by Sriniasan (1995). For instance the fiber spun by spiders can be a good candidate to mimic as the next generation of strong fibers.

2.3.2 Smart Structures

Many different definitions of smart structures have already been presented by researchers:

- An engineering definition of smart structure was given by Crawley (1994) as a subset of active structures that have highly distributed actuator and sensor systems with structural functionality and, in addition, distributed control functions and computing architecture. This definition is very restrictive, and it excludes most of the literature which already exists in the area of smart materials and structures but does not use highly distributed actuators, sensors, and control function.

- Another definition of smart structure was presented by Culshaw et al. (1994): a structure with embedded sensors and actuators within the actual structure which

form an integral part of its behavioral characteristics. Although this definition matches many applications of smart structures, it is not compatible with smart structure literature in which the sensors and actuators are not embedded.

➤ Spillman et al. (1996) defined smart structures as a non-biological structure having the following attributes:

1. A definite purpose.
2. Means and imperative to achieve that purpose.
3. A biological pattern of functioning.

The following two definitions are proposed in this thesis:

• ***Smart structures can be defined as:***

a) Structures integrated with smart material sensors and actuators which have a control function to perform a desired behavior, or

b) Structures with highly distributed sensors and actuators in an active control system.

The above two definitions match literature on smart structures and their applications. For instance, in the following literature the authors used smart structure terminology and their application matched the first case of the definition (*a*).

➤ Lashlee et al. (1994) implemented a mixed H_2 and H_∞ optimal control to a flexible structure equipped by *SMA actuators* and *PVDF piezofilm sensors*.

➤ Measures (1993) used this terminology when referring to control of a flexible structure with *fiber optic sensors* and *piezoceramic actuators*.

➤ Smart structure terminology was used for a flexible structure with integrated *PVDF piezofilms as both sensors and actuators* in an active vibration control system (Charon et al. 1992).

As an example of the second case of the definition (*b*):

➤ Adeli and Saleh (1997) called a bridge a smart bridge structure, which is actively controlled by use of *highly distributed sensors and actuators* in the elements of trusses of the bridge.

Other activities in the smart structures field can be found in the survey by Chandra et al. (1995) and Culshaw (1996). For more detailed information about the general concept of smart structures and materials and their applications, refer to the text books by Gandhi and Thompson (1992), and Culshaw (1996).

Most of the applications of smart structures are in the active control of flexible structures, because flexible structures are distributed parameter systems which naturally allow their integration with distributed sensors and actuators. In the following sections some of the advantages of piezoelectric sensors and actuators in active control of flexible structures are mentioned. Second, a literature survey on the piezosensor and piezoactuator modeling is presented. Third, the literature on control of smart structures using piezoelements is reviewed. Finally, a review on the sizing and placement of the transducers in the active control of flexible structures is presented.

2.4 Smart Structures with Piezoelectric Transducers

The certain features of piezoelectric materials have made them the most popular smart materials in the area of the smart materials and structures. Some of these features are:

- They can be bonded onto or embedded within the structure members and are thus unobtrusive and require no support structure.
- They are commercially available and cheap.
- The dynamics of the direct contact type of distributed piezoelectric transducers permits a wide frequency range of control.
- They have very fast response with high bandwidth.
- Piezoelectric sensors can be used for both strain and *strain rate* measurements.
- Piezoelements can be shaped so as to sense and control individual flexural modes.
- They can be distributed along a structure without greatly increasing the mass or changing the passive dynamic properties, thus an existing plant model may not require radical modification.
- The spatially distributed nature of piezoactuators allows continuously varying force

actuation along the length of the structure.

- Unlike some traditional actuators, piezoactuators can deform the structure directly in the sense that they operate directly on the shape variables, i.e. strain and curvature.

From comparison of the mechanical/electrical characteristics of the two main classes of the piezomaterials, i.e. PVDF and PZT, one concludes that PVDF piezofilm is suitable for sensors, whereas PZT piezoceramic is better for actuators (see chapter 3). As a result, in this thesis PVDF is used for sensors and PZT for actuators for active vibration control of a flexible beam. In the following, different models of piezoelectric sensors and actuators and their interactions with the flexible structure are quickly reviewed.

2.4.1 Piezoelectric Sensors

As mentioned earlier piezoelectric materials are used as both strain and strain rate sensors for structural vibration sensing.

► The feasibility of piezoelectric materials as strain rate sensors for flexible plates and beams was studied by Lee et al. (1989, 1991) who showed that the output current of the piezosensor was proportional to the strain rate. In their study the effect of the bonded piezosensors on the neutral axis position was not considered.

► The equation of a strain rate piezosensor in the finite element analysis of flexural vibration of plates was derived by Hwang and Park (1993).

► Tzou (1992) developed a distributed model of strain piezotransducers for shells.

► Dynamic equations of very thin piezoelectric sensors with uniform strain distribution for strain and strain measurements of flexible beam vibrations were presented by Vaz (1991a, 1996). It was shown that the output voltage and current of a piezosensor were proportional to the strain and strain rate of the flexible beam respectively.

► Miller et al. (1990). and Collins et al. (1994) developed distributed shape piezofilm sensors, whose geometry filtered the spatial harmonics of the structure's shape. Using dynamic wave modeling, they examined several different shaped piezofilm sensors analytically and experimentally.

► Using the orthogonality property of mode shapes of plates and beams, Lee and Moon (1990) developed modal shaped piezosensors to measure specific dynamic modes separately. In similar work, Collins et al. (1992) developed and manufactured modal sensors. They employed these sensors on a two-link planar manipulator to examine their performance experimentally. Zhou et al. (1991) continued the work of Lee et al. and showed that modal piezosensors can provide more precise measurement of natural frequencies than accelerometers. This idea was employed by Callahan and Baruh (1994) for modal analysis of flexible beams and plates. Although modal piezosensors are very promising in full state observation of flexible structures, they do, however, pose some potential problems in practice. For instance, to achieve a modal sensor the whole length of the structure must be covered by a piezosensor. Also, as a result of the shaping and cutting process, piezosensors can suffer such effects as short circuiting. Clark and Burke (1996) addressed some practical limitations of the shaped modal sensors and showed that a true modal sensor could not be realized on a two dimensional structure unless the boundary conditions were all pinned. They also indicated that the positioning of the sensors was critical to the performance of the sensor over a specific bandwidth, specifically in the vicinity of structural resonance.

In this thesis a general formulation for thick piezoelectric strain and strain rate sensors with a strain assumed linear is derived which accounts for the change of the neutral axis position. It is shown that in the special case of neglecting neutral axis change, it matches Lee's model. Also in the case of thin piezosensors, i.e. uniform strain assumption, it also agrees with Vaz's model (see chapter 3).

2.4.2 Piezoelectric Actuators

Piezoactuators can be embedded within or bonded onto the surface of flexible structures. Piezoactuators bonded onto the surface of structures may suffer from delamination or imperfect bonding. Embedding makes for a nice integrated structure. However, it reduces the performance of the actuators since they are closer to the neutral axis and therefore exert less moment than they would on the surface.

- Some of the first literature on application of piezoelectric materials as actuator for flexible structure was published by Bailey and Hubbard (1985) who introduced a distributed linear model of a PVDF piezoactuator bonded on a flexible beam.

- Crawley and de Luis (1987) introduced their well-known model for piezoactuators bonded onto flexible beams. Using equilibrium equations, they modeled the actuation of a pair of rectangular piezoactuators with pin forces at the ends. The derivation was based on a uniform strain distribution in the piezoactuator, but they possibly made some errors in the equilibrium equations and in the final form of the piezoactuator forces (see chapter 3 for more details). Crawley et al. (1988) extended this modeling to plates, and presented a model coupling the electrical and mechanical properties of the piezoactuators. Using the energy method, Crawley and Anderson (1990) improved the previous model (that of Crawley and de Luis (1987)) and extended it to a more realistic case (linear strain distribution assumption) for bonded and embedded rectangular piezoactuators. They obtained an expression for strain but induced pin forces were not discussed. Similar results were reported by Chaudhry and Rogers (1992a) for bonded and embedded piezoactuators.

- The energy method was employed by Vaz (1991a and 1996) to obtain the induced modal stiffnesses and forces of bonded piezoactuators with a uniform strain distribution.

- Modal piezoactuators, which can excite each individual vibrational modes were achieved by shaping piezoelements (Lee and Moon, 1990, Nitzsche, 1993).

This method suffers from the same limitations as modal sensors do, i.e. need for the covering of the whole length of the structure, and technical difficulties in shaping and cutting piezoelements.

- Recently a means of employing a piezoelement as a simultaneous sensor and actuator was presented by Anderson and Hagood (1994). The implementation of the concept involves differencing of a reference charge or current from the charge or current associated with the piezoelement to extract a signal that is proportional to the mechanical strain in the transducer.

- The finite element method has also been used by some researchers to model the dynamic behavior of piezoactuators for flexible structures (Hagood et al., 1990, Mollenhauer et al., 1993)

- Chaudhry and Rogers (1992*b*) introduced a new concept of discretely attached piezoactuators, whereby, piezoelements were attached to a structure at discrete points (as opposed to being bonded). The performance of this actuator was then experimentally compared with the bonded case.

In this study two different approaches, i.e. equilibrium equations and energy method, are used to develop two dynamic models of thick piezoactuators with linear strain distributions. First, using the energy method, the general expressions of the modal stiffness and forces are obtained for a generally shaped piezoactuator. Second, general forms of the strain and concentrated forces and moments induced by rectangular piezoactuators are derived using equilibrium equations. These models have been verified against special case models presented in the literature. For instance, assuming a thin actuator (i.e. uniform strain assumption) the concentrated force/moment converges to Crawley and de Luis's model (1987) and the modal stiffness and force converge to these which Vaz (1991*a*) derived.

2.4.3 Smart Structures with Integrated Piezoelements

In this section a quick review of the modeling of smart structures integrated with

piezoelectric sensors and actuators is presented:

- Distributed modeling of shell and plate structures with integrated piezoelectric sensors and actuators was studied by Tzou and Tseng (1991). Bailey and Hubbard (1985) also introduced a distributed model of a flexible beam with bonded piezofilm actuators.

- An interesting wave modeling of a flexible beam with a bonded piezoceramic actuator was introduced by Pines and von Flotow (1990). Some of the advantages of this method have already been mentioned in the section on the dynamics of flexible structures. Since in the second part of this thesis, the wave method is used for modeling of the integrated smart structure, some of the major differences from the present work are worth mentioning:

- a)* The smart structure studied here has both distributed piezoactuators and piezosensors bonded to it and the wave model of the composite smart structure includes the dynamic effects of both piezotransducers, whereas Pines and von Flotow used point strain sensors, i.e strain gauges.

- b)* Rather than use of strain alone, *strain rate* measured by the distributed piezosensors is integrated in the dynamic model here.

- c)* Pines and von Flotow assumed a uniform strain model for the piezoactuators introduced by Crawley, which was not compatible with the real case of thick piezoceramics in their study. In this thesis, the linear strain assumption model is used for the piezoactuators in the dynamic wave model.

- Modal modeling of the integrated smart structures, i.e. the introduction of modal forces into the modal model of the flexible structure, has been studied by some researches, e.g. Crawley and de Luis (1987), Lee et al. (1991), Vaz (1991b), Akella et al. (1994).

- The FEM has also been employed to incorporate the dynamic effects of the piezosensors and piezoactuators into the dynamic model of the smart structure (Ha and Chang, 1990, Shah et al., 1993, Shieh, 1993, Shen, 1995, Kim et al., 1996).

- Arena, et al. (1994) applied neural networks, fuzzy models, and classical

ARMAX approaches for modeling of a composite smart structure. The comparison between these models showed the suitability of the neural approach.

In addition to the dynamic modeling of the integrated smart structures, there has been work on modeling of bonded and/or embedded piezoelements for static analysis of smart structures (Ray et al., 1993, Chandra and Chopra, 1993).

A good survey on applications and modeling of piezoelectric sensors and actuators for flexible structures was presented by Rao and Sunar (1994).

2.5 Control of Smart Structures with Piezoelements

As mentioned earlier, piezoelements are the most popular smart materials in the control of flexible structures. In the following a survey on some of the activities in this area is presented.

Piezoelectric materials can function not only as active elements, but also can be used as passive dampers for vibration suppression. Hollkamp (1993) connected piezoelements bonded onto a flexible beam, to an electrical resonant shunt to suppress the vibration. Wu (1996) shunted a piezoelement with a parallel resistor and inductor circuit for vibration control. The same idea was used in industrial application of piezoelements in reducing the vibration of skis (Ashley, 1995).

There is much literature on applications of piezoelectric sensors and actuators to flexible robot control, shape, position, noise/sound and vibration control of flexible structures. Piezoelements are mainly used in active control of structures such as, beams, plates, and shells, although there are some papers on applications of them to control of other structures such as trusses (Preumont et al., 1992, Umland and Chen, 1992).

► *Flexible Robotics*: Jiang et al. (1991) presented a discrete-time formulation of the equation of motion for a single flexible arm with bonded piezoelectric actuators so that it tracked the fluctuation of targets precisely. Khorrami et al.

(1993) and Khorrami (1994) employed embedded piezoceramic actuators for active control of flexible link manipulators. Using discretized frequency shaping and self-tuning adaptive controllers, they showed that significant improvement on tip pointing performance could be achieved. Robust regulation and trajectory tracking of a single-flexible-link robot using a piezofilm actuator was studied by Indri and Tornambe (1994).

► *Noise/Sound Control*: Wang et al. (1990) used piezoelements for control of structurally radiated noise. Sound radiation control of a flexible structure using piezoelements was studied by Fuller et al. (1992).

► *Shape Control*: One of the interesting applications of piezoelements is in shape and deflection control of flexible structures. Since a distributed piezoelement can directly affect the strain and curvature of the structure, it is a good candidate for shape control. Recently, this idea was applied to shape and angle control of aircraft and missile wings (Barrett, 1994, Ashley, 1995). Although the conventional methods, i.e. ailerons, have high authority but low bandwidth, they cannot respond fast enough to high frequency flutter and gusts, whereas piezoelements with a high bandwidth are suitable for this task. Although some other literature has been devoted to this area (Pourki, 1993a, Agrawal et al., 1994, Koconis et al. 1994a, 1994b, Ghosh and Batra, 1995, Batra and Ghosh, 1995, Batra et al., 1996), there are still several practical issues such as piezoelectric behavior under a long term steady voltage, and the need for highly discrete distributed piezosensors and piezoactuators, which still need to be solved for real shape observation and control.

• *Vibration Control*

Since the main core of this thesis is active vibration control of flexible structures using piezoelements, an extensive survey of this area follows:

► *Distributed Parameter Control Systems*: The distributed nature of piezoelements suggests the design of distributed control systems. Bailey and

Hubbard (1985), Burk and Hubbard (1986), and Plumb et al. (1987) introduced nonlinear distributed controllers for active vibration control of an Euler-Bernoulli beam using Lyapunov's second method. PVDF piezofilm was used as the actuator, with an accelerometer as the sensor in their study. The stability of this controller was studied by Patnaik et al. (1993). Choi (1994) and Choi et al. (1995) extended the work by Bailey and Hubbard (1985) and modified it to overcome certain practical shortcomings. With this modified control algorithm, undesirable residual oscillations in the settled phase were substantially attenuated or completely eliminated with superior control maintained. Using an energy functional, Pourki (1993*b*) derived a criterion for the design of the shape of the piezoelectric material or its electrode, which allowed distributed control for a flexible beam. The stability of the controller was demonstrated by Lyapunov's method. A distributed linear quadratic regulator (LQR) formulation was used by Banks et al. (1992) to design an optimal controller for an Euler-Bernoulli beam with bonded piezoactuators. They used variational methods to derive the dynamic equations. Lyapunov's method has also been employed by other researchers to design distributed control systems for different flexible structures with piezoelectric sensors and actuators, e.g. Timoshenko beam (Zhaung and Baras, 1992), Plates (Miller et al., 1995), and Shells (Tzou, 1991).

➤ *Wave Absorbing Controller (WAC)*: The WAC was employed by Pines and von Flotow (1990) in active control of a flexible beam with bonded piezoactuators. The controller was designed in the frequency domain. Since the same method of smart structure control is used in the second part of this thesis, a brief comparison follows to note the main advantages and contributions of this work (a comparison from dynamic point of view was made in the previous section):

1. Distributed piezosensors are used in this thesis, whereas Pines and von Flotow used strain gauges.
2. The feedback signals used in this thesis are both strain and strain rate rather than only strain, which eliminates the need for differentiator terms in the control loop.

3. Employing the global parameters of the system, such as the total length of structure and the positions of the transducers with respect to the boundaries, a novel control algorithm is introduced here which has better performance and much faster damping than that in the previous literature.
4. Application of this novel controller for the low frequency vibration control was successfully demonstrated. By contrast, the control methodologies proposed in literature provide very poor performance at low frequencies.
5. Several wave controllers were derived here and compared for different configurations of collocated and non-collocated sensors and actuator.

► *Discrete Distributed Control Systems*: de Luis and Crawley (1990) derived a continuous form of an optimal controller using a functional analysis method, then using a discretization method they transformed it into a discrete distributed control system using several piezoactuators and sensors distributed along the flexible beam.

Using spatial discretization modeling such as the FEM and the AMM, several active control schemes have been developed and employed for active control of flexible structures using piezoelectric sensors and actuators:

► Chiang and Lee (1989) introduced active modal control of a flexible beam using shaped modal piezosensors and piezoactuators. In their study each mode could be sensed and controlled individually. Some of the drawbacks of using shaped piezosensors and actuators have been addressed in the previous sections. A new active modal control system using *only rectangular piezoelements* capable of sensing and controlling certain individual modes, which is free of the drawbacks of the previous literature, is introduced in this thesis.

► Chen et al. (1992) employed the AMM to design an LQG compensator for active vibration control of a flexible structure with a PVDF piezofilm actuator. The authors made no mention of the sensors and their dynamics. Structural damping and passive effects of the transducers on the dynamic system were also not

considered. In this thesis an LQG control system for noise compensation is designed and implemented on a flexible beam with bonded PZT piezoceramic actuators and PVDF piezofilm sensors. Structural damping and passive effects of the piezoelements on the dynamic system are also taken into account. The superiority of this controller over the LQR for active vibration suppression in the presence of noises and disturbances is also demonstrated.

- Using finite element modeling of a flexible shell integrated with piezoelectric sensors and actuators, Tzou and Tseng (1990) designed two different feedback control systems to suppress the flexural vibration of the shell: constant-gain and constant-amplitude.

- The linear quadratic regulator was applied to a finite element model of a flexible beam with bonded piezoelectric sensors and actuators (Hanagud et al., 1992, Gopinathan et al., 1995).

- Gopinathan and Pajunen (1995) introduced a model reference controller for a flexible structure with piezoelectric transducers using a finite element model of the smart structure. An adaptive inverse controller was employed by Cain et al. (1992) for vibration cancellation in a composite beam with embedded piezosensors and piezoactuators.

Several other types of controller have been employed for active control of flexible structures using piezosensors and piezoactuators.

- A control methodology integrating fuzzy control and sliding mode control was applied to a flexible beam with piezoelectric transducers (Meyer et al., 1993). The results demonstrated damping performance superior to velocity feedback alone. Zeinoun and Khorrami (1994) introduced a fuzzy neural based adaptive controller for simultaneous stabilization and robust performance, which was applied to a flexible beam with piezosensors and actuators.

- Dworak et al., 1992 applied H_∞ control to a flexible plate with bonded piezoceramic actuator. μ synthesis was used by Nageshbabu and Hanagud (1991)

to develop a robust controller to account for flaws and damage in a composite smart structure. In their study delamination was assumed to be the damage in a composite beam with piezoelements.

- Nonlinear saturation control and a power flow control of a flexible beam with bonded piezoceramics were introduced respectively by Pai et al. (1997) and Gibbs and Fuller, (1992).

- In an interesting paper, Choi et al. (1996) introduced a hybrid smart structure consisting of a flexible beam, piezoelectric actuator, and electrorheological fluid actuator. A controller incorporating neural networks and sliding mode method was applied to the smart structure, pointing out certain advantages of this hybrid system.

2.6 Sensor/Actuator Sizing and Placement

The effectiveness of an active control system is strongly dependent on actuator and sensor size and location, and an immediate question in active control system is that of the sizing and placement of sensors/actuators for obtaining the best performance of the control system.

2.6.1 Point Sensors and Actuators

Some literature exists on the optimal placement of point actuators and sensors for active control of flexible structures. Since point transducers do not theoretically have any physical extent, there is no question of their sizing.

- Lindberg and Longman (1984) developed a modal space formulation to determine the optimal number and placement of actuators. This approach was aimed primarily at very high dimensional systems.

- Kondoh et al. (1990) introduced a method of determining the optimum positions of sensors and actuators based on the minimization of a quadratic cost functional. They applied this method to a flexible beam and showed the system to

be more sensitive to locations of the actuators than to those of the sensors. The same methodology was used by Xu et al. (1993) for optimal design of placement of actuators and sensors for active control of membrane.

➤ A multiobjective approach was used by Sepulveda and Schmit (1990) to obtain an optimal placement of sensors and actuators for objective functions incorporating contradictory demands such as structural mass, control effort, and number of actuators. The feasibility of the proposed optimal design was demonstrated by a numerical example of a flexible beam.

➤ Baruh and Choe (1990) performed an optimization procedure to determine the most desirable locations of the sensors for extracting modal coordinates in structural control, and this method was applied to a flexible beam.

➤ Smith and Clark (1997) studied the optimal placement of sensors and actuators for structural acoustic radiation control. Employing a cost function including states and control effort, an optimization procedure was performed to minimize the sound transmission in a plate.

➤ One of the interesting applications of actuator placement optimization is in active trusses (Maghami and Joshi, 1990, Sepulveda et al., 1993, Holnicki-Szulc et al., 1993, Baerwald et al., 1996). Optimal placement of piezoelement stacks as point force actuators, i.e. active member, in active trusses was also studied by some researchers, e.g. Lammering (1992) and Menon et al. (1993).

2.6.2 Distributed Piezoelectric Sensors and Actuators

The distributed nature of piezoelectric transducers allows optimization in location, size, and shape of the sensor/actuator for active shape and vibration control of flexible structures. Despite its importance, optimization of embedded and/or bonded piezoelement patches in active control of flexible structures has not been extensively treated.

➤ Lee and Han (1996) used genetic algorithms to find an optimal pattern for

actuator electrodes bonded onto a flexible plate. Location and size optimization were not addressed.

► The effect of the thickness of the piezoactuator bonded onto a flexible beam on the dynamic performance of the smart structure was studied by Chaudhry and Rogers (1993). Although their analysis did not provide an answer to the actuator thickness optimization question, it provided some insights into the parameters involved. Main et al. (1994) studied the optimum thickness of a piezoactuator embedded within or bonded onto a flexible structure, and also studied its optimum distance from structural centre line. They used the induced moment producing a bending curvature as the cost function.

► Hanagud et al. (1988) studied the effect of the placement of collocated piezoelectric sensors and actuators bonded onto a flexible beam on the closed loop performance of smart structures. In their study only certain time domain responses for different cases were compared, without any optimization procedure. In addition there was no discussion of size and shape of piezoelements.

► An interesting optimization study on design of length and placement of bonded piezoactuators in active control of a flexible beam was performed by Devasia et al. (1993). Some of the characteristics of their study compared with those of the optimization studied in this thesis are:

1. Devasia et al. used strain sensors in their analysis, so that differentiators were needed for derivative feedback control. This creates practical problems for the implementation of the method in the presence of noises. Use of strain and strain rate sensors in this study enables the use of a PD feedback control system without the need for differentiators in the methodology.
2. The structural damping and passive effects of the piezoelements are considered in this study, but were neglected by Devasia et al. .
3. In their study the collocated configuration of the piezoelements introduced some geometrical limitations on the physical model, limiting the effectiveness of their method, whereas the non-collocated configuration of the piezoelements in this

thesis eliminates these limitations, and also allows the consideration of as many modes as desired

4. Optimization of piezoelement shape was not addressed in their study, but simultaneous shape and location optimization is performed in this research.

Chapter 3

Modal Modeling of Smart Structures

In this chapter, the dynamic model of a smart structure is developed using the modal approach. Distributed piezoelectric sensors and actuators are chosen as transducers, because of their unique features (see chapter 2). A flexible cantilever beam with bonded piezotransducers is considered as the smart structure. Although dynamics of flexible beams is a well understood problem, inclusion of distributed piezoelements greatly complicates the dynamic equation of the composite structure, which is our interest in this chapter.

An energy method is used to obtain the modal model of the flexible beam. The passive and active effects of the piezosensors and piezoactuators on the dynamic model are then derived, and a new concentrated induced force and moment model of piezoactuators is presented. Distributed modal piezoactuators with the capability of controlling individual modes separately are also discussed. Finally, a state space model of the integrated smart structure with piezoelectric sensors and actuators is presented.

3.1 Flexible Structure without Transducers

The dynamic model of the transverse motion of a cantilever beam with distributed external forces (Figure 3.1) is obtained in this section. For a long, thin flexible beam the shear deformation and rotary inertia of the beam can be assumed to be negligible for static deflections (Popov, 1976). These terms can also be ignored in the low frequency range of dynamic response of long, thin, flexible beams (Spector and Flashner, 1989). They showed that for frequencies much below (by at least a factor of 20) the shear mode

cut off frequency:

$$\omega_c = \left(\frac{A_b^2 \kappa_b G_b}{\rho_b I_b} \right)^{1/2}.$$

the shear and rotary inertias are negligible, and thus an Euler-Bernoulli model can be used to model the dynamic response of the beam. κ_b , G_b , A_b , I_b , and ρ_b represent shear coefficient, shear elastic modulus, cross section area, moment of inertia of the cross section, and mass per length of the beam respectively. For the beam studied here (appendix B) the shear mode cut off frequency is:

$$\omega_c = 12.5 \times 10^6 \text{ rad/sec.}$$

As mentioned, for frequencies beyond $\omega_c/20$, the Euler-Bernoulli model is a suitable model. For the present case:

$$\omega \ll 625.0 \times 10^3 \text{ rad/sec} \Rightarrow \text{Euler-Bernoulli model.}$$

Even the 10th dynamic mode is well within the cut off frequency, i.e.

$$\omega_{10} = 4.1 \times 10^3 \text{ rad/sec} \ll 625.0 \times 10^3 \text{ rad/sec.}$$

Since our interest is just the first few vibrational modes, an Euler-Bernoulli model is suitable for this study.

The partial differential equation of the transverse motion of a flexible beam can be

Figure 3.1 Flexible beam with external forces

given in undamped Euler-Bernoulli form as follows (Meirovitch, 1991a):

$$E_b I_b \frac{\partial^4}{\partial x^4} v(x, t) + \rho_b \frac{\partial^2}{\partial t^2} v(x, t) = f(x, t) . \quad (3.1)$$

E_b represents elastic modulus, $v(x, t)$ is the transverse displacement of the beam in the Z direction, and x and t denote horizontal coordinate and time respectively. $f(x, t)$ denotes external forces.

Using assumed mode method (AMM) the transverse deflection, $v(x, t)$, can be expressed in a separable form (Meirovitch, 1991b):

$$v(x, t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t) , \quad (3.2)$$

where $\phi_i(x)$ and $q_i(t)$ are the i^{th} mode shape and modal amplitude respectively.

The Energy method can be used to derive the dynamic equation of the flexible beam: the kinetic energy of the system is given by:

$$T = \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} m_{ij}^b \dot{q}_i(t) \dot{q}_j(t) , \quad (3.3)$$

where the symmetric mass coefficients of the flexible beam, m_{ij}^b , are given by:

$$m_{ij}^b = \int_0^L \rho_b \phi_i(x) \phi_j(x) dx . \quad (3.4)$$

where L is the length of the flexible beam. The potential energy can be written as:

$$U_b = \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} k_{ij}^b q_i(t) q_j(t) , \quad (3.5)$$

where the symmetric stiffness coefficients, k_{ij}^b are given by:

$$k_{ij}^b = \int_0^L E_b I_b \frac{d^2}{dx^2} \phi_i(x) \frac{d^2}{dx^2} \phi_j(x) dx . \quad (3.6)$$

Using Lagrange's equation, the modal equation of the transverse vibration can now be written as:

$$\sum_{j=1}^{\infty} m_{ij}^b \ddot{q}_j(t) + \sum_{j=1}^{\infty} k_{ij}^b q_j(t) = Q_i(t); \quad i = 1, \dots, \infty. \quad (3.7)$$

where $Q_i(t)$ is the generalized (modal) force given by:

$$Q_i(t) = \int_0^L f(x, t) \phi_i(x) dx. \quad (3.8)$$

With suitable mode shapes, the following orthogonal relations hold:

$$\int_0^L \rho_b \phi_i(x) \phi_j(x) dx = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}. \quad (3.9)$$

$$\int_0^L \frac{d^2}{dx^2} \phi_i(x) \frac{d^2}{dx^2} \phi_j(x) dx = \left(\frac{\lambda_i^4}{\rho_b} \right) \int_0^L \rho_b \phi_i(x) \phi_j(x) dx. \quad (3.10)$$

The eigenvalues of the system, λ_i , are related to the natural frequencies as:

$$\lambda_i^4 = \left(\frac{\rho}{EI} \right)_b \omega_i^2. \quad (3.11)$$

where ω_i is the i^{th} natural frequency. Applying these orthogonal relations, (equations (3.9) and (3.10)), to equation (3.7) results in:

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = Q_i(t); \quad i = 1, \dots, \infty. \quad (3.12)$$

A Rayleigh damping model is used to introduce structural damping to the system (Clough and Penzien, 1975). A proportional modal damping is assumed so that the modal equation of the transverse vibration of the beam can be written as (Krishnan and Vidyasagar, 1988):

$$\ddot{q}_i(t) + 2\zeta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) = Q_i(t); \quad i = 1, \dots, \infty. \quad (3.13)$$

where ζ_i is the i^{th} modal damping ratio. Using a simplified model of Rayleigh damping (i.e. including just the damping force proportional to the strain rate,

$\beta_s I_b \frac{\partial^3}{\partial x^2 \partial t} (v(x, t))$), the damping ratio can be written as (Humar, 1990):

$$\xi_i = \frac{1}{2} \alpha_s \omega_i ; \quad \alpha_s = \frac{\beta_s}{E_b}. \quad (3.14)$$

where β_s is the resistance to the strain rate. Thus, knowing the first modal damping ratio, the damping ratios for the other modes can be obtained from equation (3.14).

3.2 Piezoelectric Actuators

When a piezoelectric material is subjected to an electric field it tends to expand or contract (appendix A). Thus, applying voltage to a piezoelement attached to the surface of a flexible structure induces an actuation. The effect of the piezoactuators on the dynamic model of the flexible structure is studied in this section. Two different models are developed and introduced. The first, via an energy method produces a modal model of piezoactuators. Equilibrium equations are used to introduce the second model as a concentrated moment model. We assume that a pair of identical piezoactuators is bonded

Figure 3.2 Piezoactuator configuration

onto the top and bottom surfaces of the flexible beam (Figure 3.2). The piezoactuators are then connected to a power supply so as to be of opposite polarity causing the top piezoactuator to expand and the bottom to contract. Thus causing the beam to bend.

Since the two actuators are considered identical and attached symmetrically, the neutral axis of the beam does not change. The following assumptions are also considered for simplicity:

1. Temperature effects on the piezoelements are neglected.
2. The thickness and stiffness of the glue/epoxy are neglected.
3. A perfect bond between the flexible structure and piezoelements is assumed. With this assumption, no relative movement between the piezoelement and the beam occurs, so that the strains of the beam and of the piezoelements are equal on the bonding surface.

3.2.1 Modal Model of Piezoactuators

The strain energy induced by the piezoactuators can be used to incorporate the modal forces and stiffnesses caused by the piezoactuators into the dynamic model of the flexible structure. Using equation (A.5) for a rectangular piezoactuator with positive polarization under a voltage V , the strain induced by the piezoelectric effect is:

$$\Lambda = \frac{d_{31a}}{t_a} V, \quad (3.15)$$

where t_a and d_{31a} are piezoactuator thickness and strain constant respectively. Using equations (A.7) and (A.8) for an arbitrarily shaped piezoactuator, Λ can be written as:

$$\Lambda = \left(\frac{d_{31a}}{t_a} V \right) F(x, y) P(x, y). \quad (3.16)$$

where $F(x, y)$ and $P(x, y)$ represent the domain and poling functions respectively (equations (A.7) and (A.8)).

The overall stress-strain relationship of the piezoelement is similar to that of a thermoelastic material (Kagawa and Gladwell, 1970), where the thermal strain term is

replaced by the piezoelectric term. Therefore,

$$\sigma_a = E_a [\epsilon_a - \Lambda(x, y)] , \quad (3.17)$$

where E_a , ϵ_a , and σ_a are the elastic modulus, strain, and stress of the piezoactuator.

The induced potential energy per volume of each piezoelement can be obtained from the following:

$$u_a = \int_0^{\epsilon_a} \sigma_a d\epsilon_a . \quad (3.18)$$

Inserting equation (3.17) into equation (3.18) results in

$$u_a = E_a \left[\frac{1}{2} \epsilon_a^2 - \Lambda(x, y) \epsilon_a \right] . \quad (3.19)$$

The polarity, $P(x, y)$, and consequently the induced strain, $\Lambda(x, y)$, of the bottom piezoactuator have signs opposite to those of the top one (Equation (3.16)). This holds as well for their mechanical strain, therefore multiplication of $\Lambda(x, y)$ and ϵ_a (the second term of u_a) of the bottom and top elements have the same sign and magnitude. On the other hand ϵ_a^2 (the first term of u_a) is always positive. Thus one may consider only ϵ_a and $\Lambda(x, y)$ of the top piezoactuator to calculate the total potential energy induced by the pair of identical actuators which will be twice as that of the top alone. The total induced strain energy of the pair of piezoactuators can then be written as:

$$U_a = 2 \int_{(x_a - m_a)}^{(x_a + m_a)} \int_{-t_b/2}^{t_b/2} \int_{W_{a1}}^{W_{a2}} E_a \left[\frac{1}{2} \epsilon_a^2 - \Lambda(x, y) \epsilon_a \right] dy dz dx , \quad (3.20)$$

where x_a and m_a are the midpoint position and one half of the piezoactuator length in the X direction respectively (Figure 3.2), and t_b is the beam thickness. W_{a1} and W_{a2} are the endpoint positions of the piezoactuators in the Y direction (as described in

section A.3). Using the shape function, $\psi(x)$ defined by equation (A.9), U_a can be written as:

$$U_a = 2 \int_{(x_a - m_a)}^{(x_a + m_a)} \int_{t_b/2}^{-t_b/2} E_a W_a \left[\frac{1}{2} \epsilon_a^2 - \Lambda_a(x, y) \epsilon_a \right] dz dx, \quad (3.21)$$

where the average induced strain due to the piezoelectric effect, Λ_a , is defined by:

$$\Lambda_a \equiv \frac{d_{31a}}{t_a} \psi(x) V. \quad (3.22)$$

As mentioned earlier, with a perfect bond assumption the strains of the beam and piezoelement at the bonding surface are equal. Thus using a linear model of the strain distribution (Figure 3.3), the strain of the piezoactuator can be expressed as:

$$\epsilon_a = \text{sgn}(z) \left(\frac{2}{t_b} \right) \epsilon_{tb}^s z; \quad \frac{t_b}{2} \leq |z| < \frac{t_b}{2} + t_a. \quad (3.23)$$

where ϵ_{tb}^s is the total strain at the surface of the beam caused by the piezoactuator and other external/inertia forces, and

$$\text{sgn}(z) = \begin{cases} +1 & z \geq 0 \\ -1 & z < 0 \end{cases}. \quad (3.24)$$

From beam theory (Popov, 1976), the total surface strain can be written as:

$$\epsilon_{tb}^s = -\text{sgn}(z) \frac{t_b}{2} \frac{\partial^2 v}{\partial x^2}. \quad (3.25)$$

Figure 3.3 Linear strain distribution

Using equation (3.25) the total top surface strain will be:

$$\epsilon_{tb}^{ts} = -\frac{t_b}{2} \frac{\partial^2 v}{\partial x^2}. \quad (3.26)$$

Finally, using equations (3.21)-(3.26), the potential energy induced by the piezoactuators is:

$$U_a = \frac{1}{2} \left(\frac{1}{2} E_a W_a t_a t_b^2 \right) \left(\frac{4}{3} \bar{t}^2 + 2 \bar{t} + 1 \right) \int_{(x_a - m_a)}^{(x_a + m_a)} \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx + \quad (3.27)$$

$$(E_a W_a d_{31a} t_b) (\bar{t} + 1) \int_{(x_a - m_a)}^{(x_a + m_a)} \psi \frac{\partial^2 v}{\partial x^2} V dx,$$

where $\bar{t}$ is the thickness ratio given by

$$\bar{t} \equiv \frac{t_a}{t_b}. \quad (3.28)$$

Using the modal expansion, equation (3.2), U_a can be written as:

$$U_a = \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} q_i q_j k_{ij}^{piezo} - \sum_{i=1}^{\infty} Q_i q_i, \quad (3.29)$$

where the modal stiffness induced by the piezoactuators, k_{ij}^{piezo} , is:

$$k_{ij}^{piezo} = \frac{1}{2} \left(E_a W_a t_a t_b^2 \right) \left(\frac{4}{3} \bar{t}^2 + 2 \bar{t} + 1 \right) \int_{(x_a - m_a)}^{(x_a + m_a)} \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) dx. \quad (3.30)$$

and the modal force induced by the piezoactuators, Q_i^{piezo} , is given by:

$$Q_i^{piezo} = c Q_i \quad V; \quad c Q_i = -(E_a W_a d_{31a} t_b) (\bar{t} + 1) \int_{(x_a - m_a)}^{(x_a + m_a)} \psi \frac{d^2 \phi_i}{dx^2} dx. \quad (3.31)$$

In the case of several pairs of piezoactuators, i.e. na pairs, the total induced modal stiffness and force are:

$$k_{ij}^l = \sum_{l=1}^{na} k_{ij}^{piezo,l} . \quad (3.32)$$

$$Q_i^l = \sum_{l=1}^{na} Q_i^{piezo,l} . \quad (3.33)$$

Finally, adding the induced potential energy of the piezoactuators to the potential energy of the beam, the dynamic modal equation of transverse vibration of the flexible beam can be modified as follows to include the dynamic effects of the bonded piezoactuators:

$$\ddot{q}_i(t) + 2\zeta_i \omega_i \dot{q}_i(t) + \left(\omega_i^2 q_i(t) + \sum_{j=1}^{\infty} k_{ij}^l q_j(t) \right) = Q_i^l(t) ; \quad i = 1, \dots, \infty. \quad (3.34)$$

• Rectangular Piezoactuators

If the piezoactuator is rectangular with a constant positive polarity over its whole area,

$$\psi(x) = 1 . \quad (3.35)$$

the induced modal stiffness and force of the pair of piezoactuators will be:

$$k_{ij}^{piezo} = \frac{1}{2} \left(E_a W_a t_a t_b^2 \right) \left(\frac{4}{3} \bar{t}^2 + 2 \bar{t} + 1 \right) \int_{(x_a - m_a)}^{(x_a + m_a)} \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) dx . \quad (3.36)$$

$$Q_i^{piezo} = -(E_a W_a d_{31a} t_b) (\bar{t} + 1) \int_{(x_a - m_a)}^{(x_a + m_a)} \left(\frac{d^2 \phi_i}{dx^2} \right) dx \quad V . \quad (3.37)$$

Equation (3.37) matches the result which was reported by Crawley and Anderson (1990) for induced modal force, but they gave no corresponding result for the induced modal stiffness.

• Thin Piezoactuators

Assuming that the piezoactuator thickness is much smaller than beam thickness,

$$\bar{t} = \frac{t_a}{t_b} \equiv 0. \quad (3.38)$$

allows the induced modal stiffness and force of the pair of piezoactuators to be written as:

$$k_{ij}^{piezo} = \frac{1}{2} \left(E_a W_a t_a t_b^2 \right) \int_{(x_a - m_a)}^{(x_a + m_a)} \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) dx. \quad (3.39)$$

$$Q_i^{piezo} = -(E_a W_a d_{31a} t_b) \int_{(x_a - m_a)}^{(x_a + m_a)} \psi \left(\frac{d^2 \phi_i}{dx^2} \right) V dx. \quad (3.40)$$

Minor modifications to Vaz's derivations (1991) demonstrates a match of his results with equations (3.39) and (3.40).

When the thickness of the piezoelement is negligible compared to that of the beam, then rather than a linear distribution of the strain in the piezoelement, one can assume a uniform strain in the piezoactuator (Figure 3.4). Using the same methodology as above it can be easily shown that for a uniform distribution of the strain we get the same results

Figure 3.4 Uniform strain distribution in the piezoelement

as equations (3.39) and (3.40) for the induced modal stiffness and force. Thus the assumptions of very thin piezoactuators and uniform strain in the piezoactuators produce the same result.

3.2.2 Concentrated Moment Model of Piezoactuators

The active effect of a pair of rectangular piezoactuators can also be modeled as concentrated moments at the end points of the piezoelements. This model may be more useful for some applications than the modal model, especially when dealing with the original differential equations.

• Uniform Strain Distribution in Piezoactuators

The methodology used here was proposed by Crawley and de Luis (1987), but as there appear to be some errors in their derivations, they are re-done here. Consider a pair of rectangular piezoactuators with constant and opposite polarities attached to the top and bottom surfaces of a flexible beam (Figure 3.2). As previously, we assume that the piezoactuators are much thinner than the beam, so that, as mentioned earlier, a uniform strain distribution in the piezoactuators (Figure 3.4) can be assumed. We examine an

Figure 3.5 Equilibrium forces for an element consisting of the piezoactuators and the beam

element consisting of the piezoactuators and the beam of length dx (Figure 3.5). Since the thicknesses of the piezoelements are small, we can ignore the moment equation for the piezoelements and assume that all moments in the piezoelements are negligible. Thus the only interaction between piezoelements and the beam is the shear stress, τ (Figure 3.5). The force equilibrium equation for each piezoelement can be written as:

$$\sum F_p = 0 \quad \Rightarrow \quad \left(\frac{d\sigma_a}{dx} dx \right) W_a t_a = \tau W_a dx. \quad (3.41)$$

Using equation (3.17), this equation can be rewritten as:

$$\tau = E_a t_a \frac{d\epsilon_a}{dx}. \quad (3.42)$$

The perfect bond assumption between beam and the piezoelement leads to:

$$\epsilon_a = \epsilon_{tb}^s, \quad (3.43)$$

where ϵ_{tb}^s is the total surface strain of the beam. Thus equation (3.42) can be written as:

$$\tau = E_a t_a \frac{d\epsilon_{tb}^s}{dx}. \quad (3.44)$$

We also assume that in addition to the piezoactuator excitation the beam is also under some other external/inertia forces which cause a normal stress σ_b in the beam. The beam forces will be in balance and the moment equilibrium equation for the beam is:

$$\sum M_b = 0 \quad \Rightarrow \quad \int_{-t_b/2}^{t_b/2} \left(\frac{d\sigma_{tb}}{dx} dx \right) W_b z dz + \tau W_a t_b dx - \int_{-t_b/2}^{t_b/2} \left(\frac{d\sigma_b}{dx} dx \right) W_b z dz = 0, \quad (3.45)$$

where σ_{tb} is the total normal stress of the beam. With a linear strain distribution in the beam, the stresses are:

$$\begin{aligned} \sigma_b &= E_b \epsilon_b = E_b \left(\frac{z}{t_b} \right) \epsilon_b^s, \\ \sigma_{tb} &= E_b \epsilon_{tb} = E_b \left(\frac{z}{t_b} \right) \epsilon_{tb}^s. \end{aligned} \quad (3.46)$$

Using equations (3.42) and (3.46), the moment equation. (3.45), can be written as:

$$\frac{d\epsilon_{tb}^s}{dx} + \frac{6}{Y} \frac{d\epsilon_{tb}^s}{dx} = \frac{d\epsilon_b^s}{dx} \quad (3.47)$$

where Y is the stiffness ratio given by:

$$Y \equiv \frac{(EA)_b}{(EA)_a} = \frac{(EWt)_b}{(EWt)_a} \quad (3.48)$$

The total surface strain of the beam is then

$$\epsilon_{tb}^s = \frac{Y}{Y+6} \epsilon_b^s + C \quad (3.49)$$

where C is independent of x . With no other external/inertia forces than the interaction forces by piezoactuators, i.e. $\epsilon_b^s = 0$, we have:

$$\epsilon_{tb}^s = C \quad (3.50)$$

The shear forces induced by the piezoactuators do not depend on x and can be effectively transferred to the concentrated points at the ends of the piezoactuators. Thus a concentrated force model can be used to express the active effect of the piezoelement (Figure 3.6). The cross section AA can be used to express the equilibrium equations (Figure 3.7). Again the moment equation for the piezoelements can be ignored due to

Figure 3.6 Concentrated force model

the thinness of the piezoactuators. In the absence of external/inertia forces, the force equation for each piezoactuator can be written as:

$$\sum F_p = 0 \Rightarrow F_a = -E_a W_a t_a (\epsilon_{tb}^s - \Lambda). \quad (3.51)$$

The beam forces are in balance and the moment equation for the beam alone yields:

$$\sum M_b = 0 \Rightarrow F_a = \frac{1}{6} E_b W_b t_b \epsilon_{tb}^s. \quad (3.52)$$

Solving equations (3.51) and (3.52) for ϵ_{tb}^s produces:

$$\epsilon_{tb}^s = \frac{6}{\Upsilon + 6} \Lambda. \quad (3.53)$$

and, from equation (3.50) C is obtained:

$$C = \frac{6}{\Upsilon + 6} \Lambda. \quad (3.54)$$

Substituting C into equation (3.49) gives the final expression for the total surface strain of the beam:

$$\epsilon_{tb}^s = \frac{1}{\Upsilon + 6} (\Upsilon \epsilon_b^s + 6\Lambda), \quad (3.55)$$

and from equation (3.23) the strain distribution of the piezoactuators in the Z direction

Figure 3.7 AA cross section for equilibrium equations

is:

$$\epsilon_a = \operatorname{sgn}(z) \frac{1}{\Upsilon + 6} \left(\Upsilon \epsilon_b^s + 6\Lambda \right) \left(\frac{2}{t_b} \right) z; \quad \frac{t_b}{2} \leq |z| < \frac{t_b}{2} + t_a. \quad (3.56)$$

Substituting the induced strain ϵ_{tb}^s of the piezoactuators (equation (3.53)) into equation (3.52) gives the final expression of concentrated forces induced by the piezoactuators:

$$F_a = (E_a W_a t_a) \frac{\Upsilon}{\Upsilon + 6} \Lambda. \quad (3.57)$$

The pair of forces at the end points of the piezoactuators can be replaced by concentrated moments, M_a (Figure 3.8):

$$M_a = F_a t_b = (E_a W_a t_a t_b) \frac{\Upsilon}{\Upsilon + 6} \Lambda. \quad (3.58)$$

which provides the final form of the concentrated moment model of the piezoactuators.

• Linear Strain Distribution in Piezoactuators

For thick piezoactuators linear strain distribution (Figure 3.9) should be used for the piezoactuators as well as for the beam. Since in this case the thickness of the piezoactuators cannot be ignored with respect to the beam thickness, the moment

Figure 3.8 Concentrated moment model

equilibrium equations for piezoactuators are to be considered also. For the thick piezoactuators, in addition to the induced forces, induced moments, M_c , also act on the beam. In order to find the concentrated forces and moments caused by the piezoactuators, it is assumed that there are no other external /inertia forces on the beam which is excited by only the piezoactuators. The strain distribution in the beam is as follows:

$$\epsilon_b = \text{sgn}(z) \left(\frac{2}{t_b} \right) \epsilon_{tb}^s z; \quad |z| \leq \frac{t_b}{2}, \quad (3.59)$$

and the strain distribution in the piezoelements is given by equation (3.23).

The force equilibrium equation for each piezoelement yields

$$\sum F_p = 0 \Rightarrow F_a = -E_a W_a t_a \left((1 + \bar{t}) \epsilon_{tb}^s - \Lambda \right). \quad (3.60)$$

And the moment equilibrium for each piezoelement gives

$$\sum M_p = 0 \Rightarrow M_c = -\bar{t} (E_a W_a t_a t_b) \left(\left\{ \frac{1}{2} + \frac{2}{3} \bar{t} \right\} \epsilon_{tb}^s - \frac{1}{2} \Lambda \right). \quad (3.61)$$

Figure 3.9 A cross section for equilibrium equations with a linear strain distribution in the beam and piezoelements

In the beam alone, the forces are in balance and the moment equation yields:

$$\sum M_b = 0 \Rightarrow 2M_c + F_a t_b = \frac{1}{6} E_b W_b t_b \epsilon_{tb}^s. \quad (3.62)$$

Solving equations (3.60), (3.61), and (3.62) for the ϵ_{tb}^s results in:

$$\epsilon_{tb}^s = \frac{6(1 + \tilde{t})}{\Upsilon + 6\left(1 + 2\tilde{t} + \frac{4}{3}\tilde{t}^2\right)} \Lambda. \quad (3.63)$$

This result matches that of Crawley and Anderson (1990) for the strain using the energy method, but they did not provide results for the interaction forces/moments. The interaction force/moment between piezoactuators and the beam is the main contribution of this section. Substituting ϵ_{tb}^s in equations (3.60) and (3.61) gives the pure interaction forces and moments induced by the piezoactuators.

$$F_a = (E_a W_a t_a) \frac{\Upsilon + 2\tilde{t}^2}{\Upsilon + 6\left(1 + 2\tilde{t} + \frac{4}{3}\tilde{t}^2\right)} \Lambda. \quad (3.64)$$

$$M_c = \tilde{t} (E_a W_a t_a t_b) \frac{\frac{1}{2}\Upsilon - \tilde{t}}{\Upsilon + 6\left(1 + 2\tilde{t} + \frac{4}{3}\tilde{t}^2\right)} \Lambda. \quad (3.65)$$

The total concentrated moment at the end points of the pair of piezoactuators is:

$$M_a = 2M_c + t_b F_a. \quad (3.66)$$

Substituting F_a and M_c in the above equation results in:

$$M_a = (E_a W_a t_a t_b) \frac{\Upsilon(1 + \tilde{t})}{\Upsilon + 6\left(1 + 2\tilde{t} + \frac{4}{3}\tilde{t}^2\right)} \Lambda. \quad (3.67)$$

This is the final and general solution for the concentrated moment model of the piezoactuators (Figure 3.8). The strains, forces, and moments of very thin piezoactuators described in the previous section, can also be easily obtained from the

above equations by $\bar{r} \rightarrow 0$:

$$\begin{aligned}\epsilon_{tb}^s &= \frac{6}{\Upsilon+6} \Lambda, \\ F_a &= (E_a W_a t_a) \frac{\Upsilon}{\Upsilon+6} \Lambda, \\ M_c &= 0, \\ M_a &= (E_a W_a t_a t_b) \frac{\Upsilon}{\Upsilon+6} \Lambda.\end{aligned}\tag{3.68}$$

which matches previous results for the uniform strain distribution in the thin piezoactuators.

3.2.3 Modal Piezoactuators

The idea of designing modal actuators, which can excite/control individual dynamic modes, was initiated by Chiang and Lee (1989). In their study one part of the dynamic equation of the piezoactuators, which appears as the modal stiffness (k_{ij}^{piezo}), is ignored. This can be done only in the case of thin and soft piezoactuators, e.g. PVDF piezofilm actuators. In this section we design modal actuators based on the exact model, i.e. linear strain distribution, derived above (equations (3.30) and (3.31)). This allows an improvement over Chiang and Lee's model (1989) by the addition of *modal stiffness* due to the piezoactuators, which was ignored in their study.

In order to design a pair of modal piezoactuators, the whole length of the beam is covered by the piezoelements and the electrodes should be shaped and polarized so that

$$\psi(x) = \frac{1}{W_a} \int_{W_{a1}}^{W_{a2}} P(x, y) F(x, y) dy = \Re_i \frac{d^2 \phi_i}{dx^2}.\tag{3.69}$$

where $\Re_i$ is a constant.

Thus the modal force generated by this pair of piezoelements will be (equation (3.31)):

$$Q_i^{piezo} = -(E_a W_a d_{31a} t_b) (\bar{r} + 1) \int_0^L \Re_i \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) V dx.\tag{3.70}$$

The modal stiffness by these piezoactuators will be:

$$k_{ij}^{piezo} = \frac{1}{2} \left(E_a W_a t_a t_b^2 \right) \left(\frac{4}{3} \bar{t}^2 + 2 \bar{t} + 1 \right) \int_0^L \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) dx. \quad (3.71)$$

Note that the integration is done for the whole length of the beam, i.e. $[0 \ L]$, and that from the orthogonal relations (equations (3.9) and (3.10)) the above equations can be written as:

$$Q_i^{piezo} = -(E_a W_a d_{31a} t_b) (\bar{t} + 1) V \begin{cases} \Re_i \left(\frac{\lambda_i^4}{\rho_b} \right) & i = j \\ 0 & i \neq j \end{cases}. \quad (3.72)$$

$$k_{ij}^{piezo} = \frac{1}{2} \left(E_a W_a t_a t_b^2 \right) \left(\frac{4}{3} \bar{t}^2 + 2 \bar{t} + 1 \right) \begin{cases} \left(\frac{\lambda_i^4}{\rho_b} \right) & i = j \\ 0 & i \neq j \end{cases}. \quad (3.73)$$

The dynamic equations of the integrated smart structure (equation (3.34)) represent a set of modal equations coupled through the modal stiffnesses k_{ij}^{piezo} of equation (3.30) of the piezoelements. However, the modal stiffnesses obtained here (equation (3.73)) decouple the dynamic modal equations (equation (3.34)) as follows:

$$\ddot{q}_i(t) + 2\xi_i \omega_i \dot{q}_i(t) + \left(\omega_i^2 + k_{ii}^t \right) q_i(t) = Q_i^t(t): \quad i = 1, \dots, \infty. \quad (3.74)$$

where k_{ii}^t is the total modal stiffness induced by piezoactuators (equation (3.32)).

Meanwhile, equation (3.72) indicates that these piezoelements produce only the i^{th} modal forces. Thus modal piezoactuators provide two important properties:

1. They decouple the dynamic equation, by eliminating the off-diagonal elements of the induced stiffnesses (this property was not achieved in the previous literature).
2. They generate an individual particular modal force.

As a result, by designing a suitable shape and polarization of the piezoactuator electrodes, we can produce a modal actuator, which by decoupling the dynamic

equations and generating individual modal forces, can excite each individual particular mode separately.

3.2.4 Piezoactuator Selection

Among different piezoelectric materials, PZT piezoceramic, which is easily and commercially available, is one of best and most popular piezoelectric materials for mechanical actuation. From equation (3.40), we see the larger ($E_a d_{31a}$) is, the greater the actuation force generated by the piezoactuators. For instance in Table 3.1 a comparison between PZT piezoceramics and PVDF piezofilms (appendix B) shows that PZT is much more suitable as an actuator. Thus in this study PZT piezoceramic is chosen as the actuator.

Table 3.1 Piezoactuator selection

Piezoactuator material	E_a (Pa)	d_{31a} (m/V)	$E_a d_{31a}$ (Pa.m/V)
PZT piezoceramic	71.4×10^9	200×10^{-12}	14280.0×10^{-3}
PVDF piezofilm	2.0×10^9	23×10^{-12}	46.0×10^{-3}

3.2.5 Piezoactuator Model Selection

In order to select a uniform or a linear strain model for piezoactuators, the thickness ratio, i.e. $\bar{t}$, is needed. Table 3.2 shows the thicknesses of the beam and piezoceramic actuators used in this study (appendix B):

Table 3.2 Thickness comparison

Smart structure element	Thickness (mm)
Flexible beam	0.8
Piezoactuator	0.305

Thus the thickness ratio is: $\bar{t} = \frac{t_a}{t_b} = 0.381$. Since $\bar{t}$ is quite large, we use a linear strain model for piezoactuators in our study.

3.3 Piezoelectric Sensors

Assume that a single piece of a piezoelectric material is bonded onto the surface of the flexible beam (Figure 3.10). When the beam vibrates, the piezoelement expands or contracts. This phenomenon produces an electric charge in the piezoelement (appendix A), and consequently the piezoelement can be used as a sensor. In this section, it is shown how the output voltage and current of a piezosensor can be used to measure the strain and strain rate respectively. The same simplifying assumptions considered above for piezoactuators hold here for piezosensors:

1. Temperature effects on the piezoelements are neglected.
2. The thickness and stiffness of the glue/epoxy are neglected.
3. A perfect bond between the flexible structure and the piezoelements is assumed. With this assumption, no relative movement between the piezoelement and the beam occurs and the strains of the beam and piezoelement are equal at the bonding surface.

3.3.1 Linear Model of Strain Distribution in Piezosensors

In this case we assume that the thickness of the piezosensor is quite large relative to beam thickness, so that a linear model of the strain distribution is suitable for the

Figure 3.10 Piezosensor on the beam

piezosensor. With the bonded piezosensor, the thickness and stiffness of the composite smart structure changes from the original beam and since we consider only one piezosensor, the neutral axis of the integrated structure changes due to the unsymmetrical configuration (Figure 3.11).

Assuming a perfect bond, the neutral axis position can be obtained from solid mechanics (Bickford, 1993):

$$z_o = \frac{t_b + t_s}{2(1 + \Upsilon)} \quad (3.75)$$

where t_s denotes the piezosensor thickness. The perfect bond assumption yields a strain distribution of:

$$\epsilon_s = \left(\frac{2}{T_b} \right) \epsilon_{tb}^s (z - z_o) \quad (3.76)$$

where

$$T_b \equiv (t_b - 2z_o) \quad (3.77)$$

Consider an infinitesimal rectangular element of the piezosensor with constant polarity and dimensions as: dx , dy , t_s , which is under dynamic strain, ϵ_s , caused by beam vibration. The average induced electric flux of the piezosensor is given by (equations (A.2) and (A.3)):

$$D = \frac{Q_s}{dxdy} = E_s d_{31s} \epsilon_s \quad (3.78)$$

Figure 3.11 Neutral axis

Subscript s denotes piezosensor, and Q_s is the electric charge of the piezosensor. The average strain, $\bar{\epsilon}_s$, is given by:

$$\bar{\epsilon}_s = \frac{1}{t_s} \int_{t_b/2}^{(t_b/2 + t_s)} \epsilon_s \, dz = -\frac{2}{T_b} \epsilon_{tb}^s \left(\frac{t_b}{2} (1 + \tilde{t}) - z_o \right). \quad (3.79)$$

where ϵ_{tb}^s is given by equation (3.25). Using equations (A.7) and (A.8) for an arbitrarily shaped and polarized, piezosensor the induced electric flux is:

$$D(x, y) = E_s d_{31s} \bar{\epsilon}_{tb}^s \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) F(x, y) P(x, y). \quad (3.80)$$

Thus the overall electric charge of the piezoelement is:

$$Q_s(x, y) = E_s d_{31s} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_1 - m_1)}^{(x_1 + m_1)} \int_{W_{1s}}^{W_{2s}} \epsilon_{tb}^s F(x, y) P(x, y) \, dy \, dy. \quad (3.81)$$

Using the shape function (ψ) defined by equation (A.9), $Q_s(x, y)$ becomes:

$$Q_s(x, y) = E_s d_{31s} W_s \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_1 - m_1)}^{(x_1 + m_1)} \psi \, \epsilon_{tb}^s \, dx. \quad (3.82)$$

3.3.2 Piezoelectric Strain and Strain Rate Sensors

The open circuit voltage of a piezosensor can be obtained from:

$$V_s = \frac{Q_s(x, y)}{C_s} = \frac{E_s d_{31s} W_s}{C_s} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_1 - m_1)}^{(x_1 + m_1)} \psi \, \epsilon_{tb}^s \, dx. \quad (3.83)$$

where C_s is the piezosensor capacitance given by (A.10). The short circuit current of

the piezosensor is:

$$I_s = \frac{d}{dt} Q_s(x, y) = (E_s d_{31s} W_s) \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_1 - m_1)}^{(x_1 + m_1)} \psi \frac{d\epsilon_{tb}^s}{dt} dx. \quad (3.84)$$

As seen from equations (3.83) and (3.84), the output voltage and current of a piezosensor are proportional to the strain and strain rate respectively. Thus a piezoelement can be used as both strain and strain rate sensors. The differentiation property of a piezosensor current is a unique feature which can be used in the control loop, where the derivative of strain is necessary.

Using (3.25) and (3.2), the output voltage and current of the piezosensor can be written as:

$$V_s = \sum_{i=1}^{\infty} c V_i q_i; \quad (3.85)$$

$$c V_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2 C_s} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_1 - m_1)}^{(x_1 + m_1)} \psi \frac{d^2 \phi_i}{dx^2} dx.$$

$$I_s = \sum_{i=1}^{\infty} c I_i \dot{q}_i; \quad (3.86)$$

$$c I_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_1 - m_1)}^{(x_1 + m_1)} \psi \frac{d^2 \phi_i}{dx^2} dx.$$

3.3.3 Soft Piezosensors

Assume that the piezosensor is very soft, i.e. the stiffness ratio (Υ) is very high. Then from equation (3.75)

$$\Upsilon \rightarrow \infty \quad \Rightarrow \quad z_o \rightarrow 0. \quad (3.87)$$

and consequently from equation (3.77):

$$z_o \rightarrow 0 \Rightarrow T_b \rightarrow t_b. \quad (3.88)$$

The output voltage and current are then simplified to:

$$V_s = \sum_{i=1}^{\infty} cV_i \dot{q}_i; \quad (3.89)$$

$$cV_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2C_s} (1 + \bar{t}) \int_{(x_i - m_i)}^{(x_i + m_i)} \psi \frac{d^2 \phi_i}{dx^2} dx.$$

$$I_s = \sum_{i=1}^{\infty} cI_i \dot{q}_i; \quad (3.90)$$

$$cI_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2} (1 + \bar{t}) \int_{(x_i - m_i)}^{(x_i + m_i)} \psi \frac{d^2 \phi_i}{dx^2} dx.$$

which are compatible with the results of Lee et al. (1991).

3.3.4 Thin Piezosensors

For a very thin piezosensor. i.e. $t_s \rightarrow 0$:

$$\bar{t} \rightarrow 0, \quad \Upsilon \rightarrow \infty, \quad z_o \rightarrow 0. \quad (3.91)$$

Consequently V_s and I_s are simplified to:

$$V_s = \sum_{i=1}^{\infty} cV_i \dot{q}_i; \quad cV_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2C_s} \int_{(x_i - m_i)}^{(x_i + m_i)} \psi \frac{d^2 \phi_i}{dx^2} dx. \quad (3.92)$$

$$I_s = \sum_{i=1}^{\infty} cI_i \dot{q}_i; \quad cI_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2} \int_{(x_i - m_i)}^{(x_i + m_i)} \psi \frac{d^2 \phi_i}{dx^2} dx. \quad (3.93)$$

These results match those reported by Vaz (1991b), except for a sign difference. Equations (3.92) and (3.93) can also be easily derived directly from a uniform strain distribution model which is the model for thin piezoelements as mentioned above for thin piezoactuators.

3.3.5 Thin Rectangular Piezosensors

For a rectangular piezofilm sensor with constant positive polarity:

$$\psi = 1. \quad (3.94)$$

Allowing equations (3.92) and (3.93) to be written as:

$$V_s = \sum_{i=1}^{\infty} cV_i q_i; \quad cV_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2C_s} \int_{(x_i - m_i)}^{(x_i + m_i)} \frac{d^2 \phi_i}{dx^2} dx. \quad (3.95)$$

$$I_s = \sum_{i=1}^{\infty} cI_i \dot{q}_i; \quad cI_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2} \int_{(x_i - m_i)}^{(x_i + m_i)} \frac{d^2 \phi_i}{dx^2} dx. \quad (3.96)$$

3.3.6 Modal Piezosensors

Shaped piezosensors can be designed to sense each individual mode separately, an idea first was used by Lee, et al. (1989). In this section we apply this idea for our model of piezosensors. The main limitation of the modal piezosensors is that they must cover the entire length of the flexible beam in order to use the orthogonality properties of the mode shapes. Assume a piezosensor with the shape function described by equation (3.69), which covers the whole length of the beam. From equations (3.85) and (3.86) the output voltage and current of the piezosensor are

$$V_s = \sum_{i=1}^{\infty} cV_i q_i; \quad (3.97)$$

$$cV_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2C_s} \left(\frac{t_b}{T_b} (1 + \tilde{r}) - z_o \right) \int_0^L \Re_i \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) dx.$$

$$I_s = \sum_{i=1}^{\infty} cI_i \dot{q}_i; \quad (3.98)$$

$$cI_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_0^L \Re_i \left(\frac{d^2 \phi_i}{dx^2} \right) \left(\frac{d^2 \phi_j}{dx^2} \right) dx.$$

Note that the integration range is $[0 \ L]$. From the orthogonality property, equations (3.9) and (3.10), the voltage and current constants, i.e. cV_i and cI_i can be written as:

$$cV_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2C_s} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \begin{cases} \Re_i \left(\frac{\lambda_i^4}{\rho_b} \right) & i = j \\ 0 & i \neq j \end{cases}. \quad (3.99)$$

$$cI_i = (-\text{sgn}(z)) \frac{E_s d_{31s} W_s t_b}{2} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \begin{cases} \Re_i \left(\frac{\lambda_i^4}{\rho_b} \right) & i = j \\ 0 & i \neq j \end{cases}. \quad (3.100)$$

As is seen the output voltage and current of this piezosensor will be proportional to only the i^{th} modal amplitude, q_i , and its derivative, $\dot{q}_i$, respectively. Thus it is called a modal piezosensor, since it is able to sense each individual mode.

3.3.7 Piezosensor Selection

In order to minimize the mechanical and dynamic effects of the sensor on the overall behavior of the integrated smart structure, a rectangular PVDF piezofilm (appendix B) is chosen as the piezosensor. The elastic moduli and thicknesses of the beam and the piezofilm sensor are given in Table 3.3.

Table 3.3 Piezosensor selection

Smart structure element	Elastic modulus (Pa)	Thickness (mm)
Flexible beam	72.0×10^9	0.8
PVDF piezosensor	2.0×10^9	0.028

Assuming an equal width of the piezosensor and the beam:

$$\Upsilon = \frac{(EA)_b}{(EA)_s} = \frac{(EWt)_b}{(EWt)_s} = 1028.57.$$

$$z_o = \frac{t_b + t_s}{2(1 + \Upsilon)} = 0.0004 \text{ mm}.$$

Therefore the effect of the piezosensor on the stiffness and neutral axis of the flexible beam can be ignored.

3.3.8 Piezosensor Model Selection

Selection of a suitable model for the piezosensor requires knowledge of the thickness ratio of the flexible beam and the piezosensor. From Table 3.3:

$$\tilde{t}_s = \frac{t_s}{t_b} = 0.035.$$

Since $\tilde{t}_s$ is quite small it can be ignored. In addition from the above section the PVDF piezosensor is not only a thin but also a soft sensor, so that we can also ignore z_o . As a result, equations (3.95) and (3.96), i.e. thin rectangular piezosensor, can be used for this case of piezosensor selection.

3.4 State Space Model of the Integrated Smart Structure

The state space formulation of the integrated smart structure is described in this section. Assuming a modal model with n dynamic modes, equation (3.2) for transverse vibration can be written as:

$$v(x, t) = \sum_{i=1}^n \phi_i(x) q_i(t). \quad (3.101)$$

The state space equation of motion can then be written as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad ; \quad (3.102)$$

$$\begin{aligned}
y(t) &= Cx(t), \\
y^I(t) &= C^I x(t), \\
y^V(t) &= C^V x(t), \\
y^{VI}(t) &= C^{VI} x(t)
\end{aligned} \quad (3.103)$$

x represents the modal state vector consisting of the modal amplitudes, q_i , and their time derivatives $\dot{q}_i$:

$$x = [q_1 \dots q_n \quad \dot{q}_1 \dots \dot{q}_n]_{1 \times 2n}^T. \quad (3.104)$$

A , can be written as follows using equation (3.34) for n degrees of freedom:

$$A_{2n \times 2n} = \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ A1_{n \times n} & A2_{n \times n} \end{bmatrix}_{2n \times 2n}. \quad (3.105)$$

where

$$A1_{n \times n} = \begin{bmatrix} -(\omega_1^2 + k_{11}^I) & \dots & -k_{1n}^I \\ \vdots & \ddots & \vdots \\ -k_{n1}^I & \dots & -(\omega_n^2 + k_{nn}^I) \end{bmatrix}_{n \times n}; \quad A2_{n \times n} = \begin{bmatrix} -2\zeta_1\omega_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & -2\zeta_n\omega_n \end{bmatrix}_{n \times n}. \quad (3.106)$$

$u(t)$ is the input voltage vector for na piezoactuators, given by:

$$u(t) = [V_1(t) \dots V_{na}(t)]_{1 \times na}^T, \quad (3.107)$$

and B , which contains the related coefficients of the control voltages, can be obtained from equations (3.31) and (3.33):

$$B = \begin{bmatrix} 0_{n \times na} \\ \vdots \\ BB_{n \times na} \end{bmatrix}_{2n \times na}; \quad BB = \begin{bmatrix} cQ_1^1 & \dots & cQ_1^{na} \\ \vdots & \ddots & \vdots \\ cQ_n^1 & \dots & cQ_n^{na} \end{bmatrix}_{n \times na}. \quad (3.108)$$

y represents the transverse displacements of the flexible beam at m different locations along the beam length (i.e. x_i ; $i=1, \dots, m$):

$$y = \begin{bmatrix} v(x_1, t) & \dots & v(x_m, t) \end{bmatrix}'_{1 \times m} \quad (3.109)$$

and from equation (3.101), matrix C can be obtained as follows:

$$C = \begin{bmatrix} CC_{m \times n} & \vdots & 0_{m \times n} \end{bmatrix}; \quad CC = \begin{bmatrix} \phi_1(x_1) & \dots & \phi_n(x_1) \\ \vdots & & \vdots \\ \phi_1(x_m) & \dots & \phi_n(x_m) \end{bmatrix}_{m \times n} \quad (3.110)$$

y^V represents the piezosensor voltages. In case of ns piezosensors:

$$y^V = \begin{bmatrix} V_s^1(t) & \dots & V_s^{ns}(t) \end{bmatrix}'_{1 \times ns} \quad (3.111)$$

where $V_s^j(t)$ is the output voltage of the j^{th} piezosensor, and C^V , which expresses the relation between states and the output voltages of the piezosensors, can be obtained from equation (3.95):

$$C^V = \begin{bmatrix} CV_{ns \times n} & \vdots & 0_{ns \times n} \end{bmatrix}; \quad CV = \begin{bmatrix} cV_1^1 & \dots & cV_n^1 \\ \vdots & & \vdots \\ cV_1^{ns} & \dots & cV_n^{ns} \end{bmatrix}_{ns \times n} \quad (3.112)$$

If the output current of the ns piezosensors are measured, then the output state vector will be:

$$y^I = \begin{bmatrix} I_s^1(t) & \dots & I_s^{ns}(t) \end{bmatrix}'_{1 \times ns} \quad (3.113)$$

where $I_s^j(t)$ is the output current of the j^{th} piezosensor. Using equation (3.96), C^I can be obtained as follows:

$$C^I = \begin{bmatrix} 0_{ns \times n} & \vdots & CI_{ns \times n} \end{bmatrix}; \quad CI = \begin{bmatrix} cI_1^1 & \dots & cI_n^1 \\ \vdots & & \vdots \\ cI_1^{ns} & \dots & cI_n^{ns} \end{bmatrix}_{ns \times n} \quad (3.114)$$

We assume $2 \times ns$ piezosensors of which half are used for voltage, proportional to $q(t)$, measurement and the other half are used for the current, proportional to $\dot{q}(t)$, measurement, so that the output vector, y^{VI} , can be defined as:

$$y^{VI} = \left[V_s^1(t) \dots V_s^{ns}(t) \quad I_s^1(t) \dots I_s^{ns}(t) \right]'_{1 \times (2 \times ns)} \quad (3.115)$$

and matrix C^{VI} is given by:

$$C^{VI} = \left[\begin{array}{c|c} CV_{ns \times n} & 0_{ns \times n} \\ \hline 0_{ns \times n} & CI_{ns \times n} \end{array} \right]_{(2 \times ns) \times (2 \times n)} \quad (3.116)$$

These state space equations are later used in control design and implementation in the following chapters.

Chapter 4

Modal Control of Smart Structures

Piezoelectric transducers can be integrated with the flexible structure to control flexural vibrations. In this chapter active vibration control of the smart structure using the state space model introduced in the previous chapter is studied. A full state space model, in which the number of piezosensors and states are equal, has been used. Although the requirement of equal numbers of sensors and states may seem unduly restrictive, in actuality in many structures we are interested in controlling only a few (generally the lowest) dynamic modes.

4.1 Optimal Control

Application of optimal controllers for smart structures are studied in this section. Piezoelements, as sensors and actuators, are attached to the surface of a flexible beam to form the smart structure as described in the previous chapter. First, the open loop response of the system is studied, and then LQR and LQG controllers are used for vibration control of the beam with an initial tip displacement. A comparison of the open loop and closed loop systems shows that the optimal controllers provide large damping. Experimental results match successfully with the theory.

4.1.1 Physical Model

The physical model in this section consists of a flexible cantilever Aluminum beam and the piezoelectric sensors and actuators. Four SDT1-028K piezofilm sensors

(appendix C) are bonded onto both sides of the beam. The top piezosensors, i.e. sensors #1 and #2, are used for strain, proportional to the aggregate of modal amplitude (q_i), obtained by measuring the output voltages of the piezosensors. The other two piezosensors, i.e. sensors #3 and #4, are used for the strain rate, proportional to the aggregate of modal amplitude rate ($\dot{q}_i$), by measuring the output currents of the piezosensors. Since we are using four piezosensors, a mathematical model consisting of two dynamic modes, i.e. the first two modes in our study, is considered for a full state space model. We measure the strain rates directly, so that we do not need to differentiate signals which is very sensitive to noise and disturbances. PZT piezoceramic actuators (appendix C) are attached to both sides of the beam to make a pair of equal actuators. All sensors and actuators are located along the midline of the beam in the Y direction. Figure 4.1 shows the schematic for this model and Figure C.2 (appendix C) shows the corresponding schematic of the experimental set up. The mechanical properties of the

Figure 4.1 Physical model

smart structure elements are given in appendix B, and the geometric specifications are expressed in the following tables.

Table 4.1 Flexible Aluminium beam

Length: L (mm)	Width: W_b (mm)	Thickness: t_b (mm)
508.0	25.4	0.8

Table 4.2 PZT piezoceramic actuators

No.	Side	Mid point position x_a (mm)	Half of the length m_a (mm)	Width W_a (mm)	Thickness t_a (mm)
1	Top	89.0	38.1	25.4	0.305
2	Bottom	89.0	38.1	25.4	0.305

Table 4.3 PVDF piezofilm sensors

No.	Side	Mid point position x_s (mm)	Half of the length m_s (mm)	Width W_s (mm)	Thickness t_s (mm)
1	Top	35.4	13.5	13.0	0.028
2	Top	142.6	13.5	13.0	0.028
3	Bottom	35.4	13.5	13.0	0.028
4	Bottom	142.6	13.5	13.0	0.028

This model is an actual experimental model which is used as the test bed in this study (appendix C) in order to validate the simulation results.

4.1.2 Open Loop Response

The open loop response of the integrated smart structure is studied in experiment and simulation in this section. The results show that the flexible beam has a long decay of vibration which makes an active control system necessary if vibration is to be effectively damped.

• Experiment

Figure 4.2 shows the output voltage of sensor #1 in free vibration for a 10 mm initial tip displacement. Since the effects of the high frequencies vanish quickly, the dynamic characteristics of the first vibrational mode can be easily obtained from Figure 4.2. For instance the first natural frequency, ω_1 , can be calculated from the number of peaks in a specific time interval. In the time interval of 0.3 to 2.6 seconds, 8 cycles are observed, thus the first natural frequency is:

$$\omega_1 = 2\pi \left(\frac{8}{2.3} \right) = 21.9 \text{ rad/s}$$

The first modal damping ratio, ζ_1 , can be obtained from the following equation (Thomson, 1981):

$$\delta = \frac{2\pi\zeta_1}{\sqrt{1-\zeta_1^2}}, \quad (4.1)$$

Figure 4.2 Open loop response. sensor #1 (experiment)

where δ is the logarithmic decrement of two consecutive vibrational peaks. For the above mentioned time interval, i.e. 0.3 to 2.6 seconds, the logarithmic decrement for 8 cycles is:

$$8\delta = \ln \frac{-0.478}{-0.275} = 0.552.$$

thus

$$\delta = 0.069.$$

Finally from equation (4.1) the first modal damping ratio can be obtained:

$$\zeta_1 = 0.011.$$

• Simulation

The state space model described by equations (3.102) and (3.103) is used to obtain the dynamic response of the system. The natural frequencies, ω_i of the integrated smart structure can be obtained from equation (3.102). Using the first modal damping ratio obtained from experiment, i.e. $\zeta_1=0.011$, the other modal damping ratios can be easily obtained from equation (3.14). The first two natural frequencies and modal damping ratios are given in Table 4.4:

Table 4.4 Modal parameters

mode	ω_i (rad/s)	ζ_i
First	21.45	0.011
Second	106.18	0.055

It is seen that the first natural frequency matches quite well with the experimental one. For the next sections, a tip displacement of 10 mm is the initial excitation. The ratio of the initial values of the second and the first modal amplitudes is obtained from simulation: $q_2(0)/q_1(0) = -0.025$, and shows that the contribution of the second mode is much smaller than that of the first one. As mentioned earlier, the dynamic response of

the system is less affected by the second mode than the first.

Figures 4.3 and 4.4 show the open loop tip displacements of the beam for 3 and 30 seconds respectively. In Figure 4.5 a comparison of the experimental and simulation open loop output voltage of sensor #1 shows good agreement between simulation and experiment.

Figure 4.3 Open loop tip displacement in 3 seconds (simulation)

Figure 4.4 Open loop tip displacement in 30 seconds (simulation)

Figure 4.5 Open loop response, sensor #1

4.1.3 LQR

The linear Quadratic Regulator, LQR, (Kirk, 1970) was used to control the vibration of the physical model, described in section 4.1.1, considering only the first two dynamic modes. The feedback gains are chosen so that the performance index J ,

$$J = \int_0^{\infty} (x'Qx + u'Ru) dt \quad (4.2)$$

is minimized. R is a positive definite input weight matrix and Q is a positive semi-definite state weight matrix. We consider all of the sensor signals to be of equal importance so that Q is chosen as a diagonal matrix with equal elements. R chosen such that the vibration is damped quickly while the control voltage remains below the breakdown voltage of the piezoactuator (i.e. $-120 < V_{\max} < +120$). For the model studied in this thesis Q and R chosen as:

$$Q = \text{diag} [1, 1, 1, 1] \times 10^8,$$

$$R = 1.3.$$

The Matlab control toolbox (Little, 1986) was used in simulation to obtain the control gains for the states, i.e. k^x , ($u = -k^x x$). Since the number of states and piezosensors are equal, the inverse of C^{VI} (equation (3.116)) exists. Thus using equation (3.103), the

control gains for the implementation in the experiment, i.e. k , can be obtained as:

$$k = k^x \left(C^{VI} \right)^{-1}. \quad (4.3)$$

Figure 4.6 shows the LQR control loop of the smart structure where $u = -k y^{VI}$ and $U_o = 0$. In our case:

$$k^x = [0.1826 \quad -5.5481 \quad -1.0087 \quad -3.1534] \times 10^4$$

$$k = [9.9514 \quad -18.3411 \quad -4.1893 \times 10^9 \quad -1.6289 \times 10^9]$$

• Simulation Results

The closed loop response of the beam at the tip, with this controller, is shown in Figure 4.7. The vibration is damped completely after 1.5 seconds which is twenty times faster than the open loop case. Figure 4.8 shows the corresponding control voltage, which has absolute value less than 120 V.

• Experimental Results

The experimental test bed for the physical model, described in section 4.1.1, is illustrated in appendix C. The LQR controller just described for simulation was applied to the real system, and provided substantial damping. In the following, some of the experimental results are compared with simulation. The output voltage of the piezosensor at position $x_s = 35.4$ mm (sensor #1) is shown in Figure 4.9, and the output

Figure 4.6 LQR control loop of the smart structure

current of the piezosensor at the same position (sensor #3) is shown in Figure 4.10. Figures 4.11 and 4.12 show the output voltage and current of sensors at $x_3 = 142.6$ mm. i.e. sensors #2 and #4, respectively.

Figure 4.7 LQR Controlled tip displacement in 3 seconds

Figure 4.8 LQR Control voltage

Figure 4.9 Piezosensor #1 voltage

Figure 4.10 Piezosensor #3 current

Figure 4.11 Piezosensor #2 voltage

Figure 4.12 Piezosensor #4 current

Figure 4.13 LQR control voltage

As we see the experimental results are close to those of the simulation, and the controller introduces great damping in the experiment case as well. A comparison between experimental and simulation control voltages of the piezoactuators is illustrated in Figure 4.13, which shows the control command is in the acceptable range, i.e. $|V_{\max}| < 120$ V.

4.1.4 LQG

Piezoelectric materials are very sensitive to room noise and may absorb acoustic, thermal, mechanical, and electrical disturbances from the environment (Elf Atochem Sensor Inc., 1992*b*). Some shielding techniques have been suggested to reduce the effect of noises in piezoelectric sensors. However, shielded piezoelements cannot fully compensate for all noises (Elf Atochem Sensor Inc., 1992*b*).

In this section we apply the Linear Quadratic Gaussian (LQG) compensator to the physical model, described in section 4.1.1. The LQG compensator feedback gains (i.e. the gains of the controller and the estimator) are obtained from the solutions of the controller Riccati equation and the estimator Riccati equation (Gawronski, 1993*b*). The control loop of the smart structure with the LQG compensator is shown in Figure 4.14.

The process, w , and measurement, v , noises of the control system are modeled as

independent Gaussian random variables with zero mean, which are suitable models for many physical noises (Sage, 1977). Therefore the noises w and v are considered to be uncorrelated, zero mean, Gaussian random variables, where w is the process noise with covariance W , and v is the measurement noise with covariance V . Thus:

$$W = E(w w'), \quad V = E(v v'), \quad E(v w') = 0, \quad E(v) = 0, \quad E(w) = 0. \quad (4.4)$$

where superscript $'$ denotes transpose, and $E(\cdot)$ is an expectation operator.

The task is to determine the controller gain (k_c) and the estimator gain (k_e) such that the performance index J :

$$J = E \left(\int_0^{\infty} (x' Q x + u' R u) dt \right), \quad (4.5)$$

is minimal, where the stationary Kalman filter:

$$\dot{\hat{x}} = A \hat{x} + B e + k_e (y^{VI} - C^{VI} \hat{x}), \quad (4.6)$$

produces an optimal estimate of x .

For simplicity, the process noise w is considered negligible compared to the

Figure 4.14 LQG control loop of the smart structure

measurement noise, therefore:

$$W = 0,$$

and a measurement noise v is assumed as a Gaussian random variable with covariance V as follows:

$$V = Cov(v) = E(vv') = diag [1, 1, 1, 1] \times 10^{-4}.$$

The weight matrices, Q and R , are the same as those in LQR method:

$$Q = diag [1, 1, 1, 1] \times 10^8,$$

$$R = 1.3 \quad .$$

Figures 4.15 and 4.16 show the controlled tip displacement of the smart structure and control voltage respectively. As we see the closed loop response is very smooth and the LQG can compensate the effect of noise completely. A comparison between LQG in the presence of noise and LQR in the absence of noise (Figure 4.17) indicates that LQG can eliminate the effect of the noise so that the control system acts like an LQR controlled system without any noise. On the other hand the comparison of the control voltages (Figure 4.18) shows that the required control voltage is almost identical to that of the LQR control system.

We can also consider noise in the LQR optimal control system without any compensation for it. The closed loop response of the tip of the flexible beam (Figure 4.19) clearly shows the effect of the noise and the vibration cannot be damped. Comparing Figures 4.15 and 4.19, one can see that the LQG method can compensate for the noise successfully. Also from Figure 4.20 it is obvious that the required control voltage for the system without compensator is very high, i.e. $|V_{max}| > 500$ V, which could destroy the piezoactuators, whereas in the LQG control system the required control voltage is much smaller (Figure 4.16) and is in the acceptable range, i.e. $|V_{max}| < 120$ V.

Figure 4.15 LQG controlled tip displacement

Figure 4.16 LQG control voltage

Figure 4.17 LQR and LQG controlled tip displacements

Figure 4.18 LQR and LQG control voltages

Figure 4.19 LQR controlled tip displacement (with noise)

Figure 4.20 LQR control voltage (with noise)

4.2 Individual Mode Control

As mentioned in chapter 3, modal shaped piezoelectric sensors/actuators can be used to sense/actuate individual modes of a flexible beam. Lee et al. (1989), employed this type of piezotransducer for active individual mode control. Shaped modal piezofilms were also used by Zhou et al. (1991), to measure the natural frequencies of a beam and a plate. The use of shaped modal piezoelements introduces two major disadvantages. First, these piezotransducers must cover the entire structure in order to properly sense or actuate a specific mode. Second, construction and shaping of such piezotransducers is difficult. Some of the latter problems were addressed by Lee and Moon (1990). In addition, Clark and Burke (1996) addressed some practical limitations of the shaped modal piezosensors. They also showed that these sensors are sensitive in the vicinity of the structural resonance.

In this section, an alternative approach to the shaping of piezoelements is introduced for active control of specific modes of a flexible beam using *rectangular piezoelectric transducers*. A methodology for choosing suitable input control voltages of actuators is presented to control any specific mode separately. Employing this method, there is no need for shaping piezoelements or covering the whole structure by them, and is therefore much simpler and cheaper. It is shown that each mode can be controlled individually with this modal controller, using rectangular piezoelements.

4.2.1 Physical Model

The dynamic equations of the integrated smart structure consists of a set of coupled modal equations (see chapter 3). This coupling is due to the induced stiffnesses by the piezoelements, k_{ij}^{piezo} , (equation (3.30)). This coupling appears as off-diagonal elements in matrix $\mathbf{AI}$, equation (3.106). In the case of soft piezoelements, e.g. PVDF piezofilms, the induced stiffness caused by the piezoelement is very small and can be ignored, so that matrix $\mathbf{AI}$ can be assumed to be diagonal matrix, and consequently equation (3.102) provides a set of decoupled dynamic equations. This decoupling is a

necessary condition for individual mode control. Therefore to provide this decoupling we use PVDF piezoelements as actuators (appendix B).

As in previous sections, a physical model similar to that described in section 4.1.1 is used, except that the piezoactuators are PVDF piezofilms. A second pair of actuators is added to the model to obtain sufficient control authority for individual mode control (Figure 4.21). All sensors and actuators are located along the midline of the beam in the Y direction. The geometric specifications of the actuators are given in Table 4.5. Since we are interested in flexural vibration control, only the first two dynamic modes are considered in this section.

Table 4.5 PVDF piezofilm actuators

Pair No.	Mid point position x_a (mm)	Half of the length m_a (mm)	Width W_a (mm)	Thickness t_a (mm)
1	89.0	38.1	25.4	0.028
2	196.0	38.1	25.4	0.028

Figure 4.21 Physical model for active individual mode control

4.2.2 Control Design

The model matrix AI of equation (3.106), is:

$$AI = \begin{bmatrix} -263.6 & -0.9 \\ -0.9 & -10330.0 \end{bmatrix}.$$

As we see, the off-diagonal elements are small relative to the others, so that they can be ignored, and with reasonable accuracy we can claim that the dynamic equations are decoupled. Simulation results verify this assumption. Now, if we can design the controller such that the induced voltages excite individual modes separately, then we can control each mode separately through the decoupled dynamic equations.

The piezoactuator equations, (3.31) and (3.33), indicate that the generalized (modal) forces generated by the actuators are linear combinations of the input voltages of the actuators. Using these equations, the first and second induced modal forces are:

$$\begin{aligned} Q_1'(t) &= cQ_1^{piezo-1} V_1(t) + cQ_1^{piezo-2} V_2(t) \\ Q_2'(t) &= cQ_2^{piezo-1} V_1(t) + cQ_2^{piezo-2} V_2(t) \end{aligned} \quad (4.7)$$

Taking the two control voltages to be in the ratio of

$$\frac{V_2}{V_1} = -\frac{cQ_2^{piezo-1}}{cQ_2^{piezo-2}}. \quad (4.8)$$

equation (4.7) indicates that $Q_2'(t) = 0$. Therefore applying voltages with this ratio to the actuators affects only the first mode. This means that the first mode can be controlled individually. For control of the second mode, the first modal force should be zero, i.e.

$Q_1'(t) = 0$, which means that

$$\frac{V_2}{V_1} = -\frac{cQ_1^{piezo-1}}{cQ_1^{piezo-2}}. \quad (4.9)$$

Introducing K :

$$K = V_2/V_1. \quad (4.10)$$

we can write:

$$Bu(t) = B \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = B \begin{bmatrix} 1 \\ K \end{bmatrix} V_1 = B\Xi V_1; \quad \Xi \equiv \begin{bmatrix} 1 \\ K \end{bmatrix}. \quad (4.11)$$

Where K is our control design parameter for active individual mode control:

$$K = \begin{cases} \frac{cQ_2^{piezo-1}}{cQ_2^{piezo-2}} & \left(Q_2'(t) = 0; \quad \text{First mode control} \right) \\ \frac{cQ_1^{piezo-1}}{cQ_1^{piezo-2}} & \left(Q_1'(t) = 0; \quad \text{Second mode control} \right) \end{cases}. \quad (4.12)$$

4.2.3 Control Implementation

Figure 4.22 shows the active individual mode control system of the smart structure model. The LQR method is used to find the optimal control gains, k , which then provide the control commands for the actuators through the vector Ξ . To excite both modes, an excitation voltage of

$$U(t) = 50 \times \sin[(\omega_1+2)t] + 500 \times \sin[(\omega_2+2)t]$$

was applied to the both actuators for 2 seconds, after which the excitation ceased (the

Figure 4.22 Individual mode control of the smart structure

breakdown voltage for the PVDF piezoactuators is ± 1000 V). Figure 4.23 shows the excitation signal. The open loop response of the beam tip displacement for 5 seconds is shown in Figure 4.24. Now by choosing suitable K (equation (4.12)), the first mode control (i.e. $Q_2'(t) = 0$) was operated, after the excitation was off (Figure 4.25). In order to have a better picture of the controller performance, in Figure 4.26 the tip displacement response is shown after 2 seconds (i.e. after the control is on). It is seen that the controller suppressed the low frequency vibration (i.e. first mode), but did not damp the second mode, because the actuators produced the first modal force exclusively while the second modal force was zero. Figure 4.27 illustrates the effect of this controller on the first modal amplitude. As we see the controller quickly suppressed the vibration. On the other hand from Figure 4.28, one can see that the controller did not affect the second modal amplitude, as expected.

The same procedure was followed for the second mode control (i.e. $Q_1'(t) = 0$) to show the performance of the controller for damping the second mode exclusively. In order to better display the second mode damping, the figures are shown in the 2-3 second time interval, since the second mode damps faster. The closed loop tip displacement with this controller is shown in Figure 4.29. As it is seen the effect of the second mode on the tip displacement was damped whereas that of the first mode has remained untouched. Figure 4.30 shows that the second modal amplitude damped quite fast, whereas the controller did not affect the first modal amplitude (Figure 4.31). Finally a comparison between the two controllers, i.e the first mode controller and the second mode controller, is made in figures 4.32 and 4.33. It can be seen from these two figures that the first mode controller suppresses only the effect of the excitation of the first mode and the second mode controller suppresses only that of the second mode.

In summary, it is shown that using the introduced control methodology each individual dynamic mode of the flexible structure can be controlled separately. The controller can be easily implemented and performs well in damping of each individual mode.

Figure 4.23 Excitation signal

Figure 4.24 Open loop tip displacement

Figure 4.25 Closed loop tip displacement (first mode control)

Figure 4.26 Closed loop tip displacement (first mode control)

Figure 4.27 First modal amplitude (first mode control)

Figure 4.28 Second modal amplitude (first mode control)

Figure 4.29 Closed loop tip displacement (second mode control)

Figure 4.30 Second modal amplitude (second mode control)

Figure 4.31 First modal amplitude (second mode control)

Figure 4.32 First modal amplitude (first and second mode control)

Figure 4.33 Second modal amplitude (first and second mode control)

Chapter 5

Shape and Location Optimization of Piezoactuators

The distributed nature of a piezoelectric actuator allows optimization in location and in shape, while point actuators such as torque motors have (ideally) no physical extent. Hanagud et al. (1988) discussed the time domain responses of an optimal control system for smart structures under different locations of piezoelectric sensors and actuators. In their study no optimization criterion was introduced, and there was no discussion of the size and shape of the piezoelements. Sensor and actuator placement using linear quadratic methods was addressed by Kondoh et al. (1990), but shape and size optimization were not addressed. Devasia et al. (1993) presented optimal design of piezoactuator length and placement in which collocated actuators and only strain piezosensors were used. In their study shape optimization was not investigated. The collocated configuration of the piezoelements introduces some geometrical limitations on the physical model, limiting the effectiveness of their method. In addition they used differentiators, which suffer from noise sensitivity, for derivative feedback.

In this chapter the question of what geometry of the piezoactuators, wide or narrow, short or long as well as the optimal location for the active vibration suppression of a smart structure is investigated. PZT piezoceramic is used as the actuators and PVDF piezofilm is employed as the sensors bonded onto a flexible beam. The modal method is used to model the dynamic system, and state space feedback control is employed. In

contrast to the previous literature which has usually dealt with only one dynamic mode (i.e. Crawley and de Luis, 1987), a dynamic modal model, which includes several dynamic modes, is employed. We briefly mention some of the notable features of this study: First we measure the output voltages of piezosensors, which are proportional to the strains, as well as the output currents, which are proportional to the strain rates; thus we can get PD feedback control without the need of differentiators which eliminates the well known problems of differentiation with respect to noises. Second, the non-collocated configuration of the piezoelements in this study allows us to consider as many modes as we desire. Third, shape optimization is also performed simultaneously with the location optimization. In addition, the effect of the passive stiffness of the piezoactuators on the dynamic system is also considered; this was ignored in the previous literature (i.e. Devasia et al. 1993). Finally three different criteria for overall optimization of piezoactuator shape and location are investigated and compared. In the first criterion we seek a configuration of the piezoactuator which maximizes the overall damping ratio of the closed loop system. Minimization of the overall real part of eigenvalues of the controlled system is the second criterion. For the third criterion, we seek the minimum LQR cost function over possible shapes and locations. Simulation provides some interesting results: for instance it is shown that in the case of constant piezoactuator area, wider and shorter actuators are better than longer and narrower ones.

5.1 Physical Model

The flexible structure is the same as the cantilever beam used in section 4.1.1, i.e. an Aluminium beam with mechanical and geometric parameters described in appendix B and Table 4.1 respectively, except the width. A wide beam (Table 5.1) allows variation of dimension, location, and relative positioning of the sensors and actuators. Four PVDF piezofilm sensors are used to measure strains and strain rates. The sensors are also the same as the ones used in chapter 4, except that their locations are different (Table 5.2). These sensors are located on the top surface and at the extreme edges of the beam in the

Y direction, as shown in Figure 5.1. One pair (#1 and #2) is used for strain measurement and the second pair (#3 and #4) is used for strain rate measurement. A pair of PZT piezoceramic actuators is attached on both sides of the flexible beam. The length, width, and location of the piezoactuators are the optimization parameters to study the performance of the control system for different geometries and locations of the actuators.

Table 5.1 Flexible Aluminium beam

Length: L (mm)	Width: W_b (mm)	Thickness: t_b (mm)
508.0	52.0	0.8

Table 5.2 PVDF piezofilm sensors

No.	Side	Mid point position x_s (mm)	Length l_s (mm)	Width W_s (mm)	Thickness t_s (mm)
1 & 3	Top	35.4	13.5	13.0	0.028
2 & 4	Top	142.6	13.5	13.0	0.028

Figure 5.1 Physical model for piezoactuator shape and location optimization

Table 5.3 PZT piezoceramic actuators

No.	Side	Mid point position x_a (mm)	Length l_a (mm)	Width W_a (mm)	Thickness t_a (mm)
1	Top	12.7 → 495.3	25.4 → 508.0	1.27 → 25.4	0.305
2	Bottom	12.7 → 495.3	25.4 → 508.0	1.27 → 25.4	0.305

5.2 Control System

A two mode model of the dynamic system, which consists of the first two vibrational modes, is considered in this chapter. For full observation of the system states, the number of sensors, ns , and the number of assumed states, $2 \times n$ should be equal. For instance, for a two mode model of the dynamic system:

$$ns = 2 \times n = 4. \quad (5.1)$$

Thus four piezosensors can provide a full state observation of the state vector,

$$\mathbf{x} = [q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2]^T, \quad (5.2)$$

In many vibration control tasks, we are interested in controlling only a few low frequency modes, so that the requirement that there be as many sensors as states is easily achievable.

The control block diagram is the same as that of Figure 4.6. The control law for the free vibration case ($U_o=0$) can be written as:

$$\mathbf{u}(t) = -\mathbf{k}^x \mathbf{x}, \quad (5.3)$$

or

$$\mathbf{u}(t) = -\mathbf{k}y^{VI}, \quad (5.4)$$

where $\mathbf{k}$ is the control gain vector of the closed loop system (Figure 4.6). $\mathbf{k}$ and $\mathbf{k}^x$ are related by equation (4.3). Using equations (3.102) and (5.3), the state space model of the closed loop system can be written as:

$$\dot{\mathbf{x}} = \mathbf{A}_d \mathbf{x}, \quad (5.5)$$

where

$$A_d = A - Bk^x \quad (5.6)$$

Equation (5.5) can be used to obtain the dynamic response of the control system. In the following section the criteria which guide selection of configurations of the piezoactuators for optimal dynamic response are presented.

5.3 Optimization Procedure

We seek a geometry of the piezoactuator which in some sense provides optimal performance of the closed loop system. The following three criteria are examined in order to determine the “*optimal*” shape and location of the piezoactuator for the flexible structure. The control gains are obtained from the LQR method for each configuration individually. So that for each configuration the control gains are the optimal gains for that particular case. The weight matrices are the same as those in section 4.1.3:

$$\begin{aligned} Q &= \text{diag} [1, 1, 1, 1] \times 10^8, \\ R &= 1.3 \end{aligned}$$

5.3.1 Overall Damping Ratio

In the first optimization criterion, we maximize the overall modal damping ratio of the closed loop system. In other words we are looking for a configuration of the piezoactuator which maximizes the summation of the damping ratios of the desired modes:

$$\max(\text{Index1}) = \max \left(\sum_{i=1}^n \zeta_i \right), \quad (5.7)$$

where ζ_i is the i^{th} modal damping ratio and n represents the number of modes. When *Index1* increases, the overall damping of the closed loop system increases, thus one can say that the actuator performs more effectively.

5.3.2 Aggregate Real Part of Eigenvalues

The second criterion studied in this chapter is the minimization of the summation of the real parts of the closed loop eigenvalues. This means that we push the closed loop poles leftward in the complex plane to increase the damping of the system:

$$\min(Index2) = \min\left(\sum_{i=1}^n \text{Real}(\lambda_i)\right), \quad (5.8)$$

where λ_i is the i^{th} eigenvalue of the control system, i.e. A_d . Minimization of the *index2* provides greater damping for the closed loop system.

5.3.3 LQR Objective Function

Our third index of optimization is the linear quadratic regulator cost function. This optimization procedure is employed by Kondoh et al. (1990) in positioning of sensors and actuators in vibration control. and Devasia et al. (1993) also used this criterion in piezoelectric actuator design for vibration suppression. In this section this criterion is used for both shape and location optimization of the piezoactuators. The minimum cost function, J , for a particular configuration of piezoactuator is given by (Kwakernaak and Sivan, 1972):

$$index3 = \min(J) = \mathbf{x}'(0) \mathbf{P} \mathbf{x}(0), \quad (5.9)$$

$\mathbf{x}(0)$ is the initial condition of the dynamic system and $\mathbf{P}$ represents the unique non-negative-definite solution to the algebraic Riccati equation of the LQR method. We seek the minimum of *index3* among different possible configurations of the piezoactuator. Thus the third criterion is:

$$\min(index3) = \min(\min(J)). \quad (5.10)$$

In order to consider the effects of all of the dynamic modes equally, a uniform initial condition with $\|\mathbf{x}(0)\| = 10^{-3}$ is assumed, i.e. $\mathbf{x}(0) = [0.5 \ 0.5 \ 0.5 \ 0.5]' \times 10^{-3}$.

5.4 Piezoactuator Shape Optimization

In this section we study the behavior of the dynamic system under different shapes of the piezoactuators to find the optimal shape for a specific location of the actuator. Optimization according to the three criteria is discussed. Rectangular piezoactuators with different lengths and widths are considered for the following two cases.

5.4.1 $W_a = \text{Constant}$ and $x_a = \text{Constant}$

In this case a pair of piezoactuators with 25.4 mm width is bonded in the middle of the flexible beam. The length of the piezoactuator is the optimization variable:

$$W_a = 25.4 \text{ mm}; \quad x_a = 254.0 \text{ mm.}$$

$$l_a = 25.4 \text{ mm} \rightarrow 508.0 \text{ mm.}$$

Figure 5.2 illustrates the three indices with respect to the actuator length. The three indices provide almost the same results (with tolerance of 10%) for the greatest damping configuration:

Table 5.4 Optimal piezoactuator configurations: W_a and $x_a = \text{constant}$

Index No.	Figure No.	l_a/L	l_a (mm)
1	5.2.a	0.60	304.8
2	5.2.b	0.70	462.1
3	5.2.c	0.53	266.7

Thus the optimal piezoactuator is not the one which covers the whole length of the beam, but a pair which covers about 60% of the length.

5.4.2 $A_a = \text{Constant}$ and $x_a = \text{Constant}$

We may wish to maintain the area of the actuator constant. For instance a piezoactuator with a constant area, A_a , which is located in the middle of the beam. The

responses of the smart structure for the following configurations are next examined:

$$x_a = 25.4 \text{ mm}; \quad A_a = 25.4 \times 25.4 = 645.16 \text{ mm}^2.$$

$$l_a = 25.4 \text{ mm} \rightarrow 508.0 \text{ mm}; \quad W_a = 1.27 \text{ mm} \rightarrow 25.4 \text{ mm}.$$

The three indices are shown with respect to the length and width of the piezoactuator in Figures 5.3 and 5.4. As we see, all three criteria indicate that the optimal shape is a short and wide piezoactuator:

Table 5.5 Optimal piezoactuator configurations: A_a and x_a =constant

Index No.	Figure No.	l_a/L W_a/L	l_a (mm) W_a (mm)
1	5.3.a and 5.4.a	0.05	25.4
2	5.3.b and 5.4.b	0.05	25.4
3	5.3.c and 5.4.c	0.05	25.4

In other words this means in order to get the best damping, it is better to concentrate the actuator at an efficient location rather than spreading it along the length of the beam; the shortest and widest possible piezoactuator is the best configuration for this case.

5.5 Piezoactuator Location Optimization

The issue of the best placement of the piezoceramic actuator to produce the greatest damping in the control system is discussed in this section. A specific configuration of the piezoactuator, which was the nominal piezoactuator size of section 4.1.1 is assumed. The actuator is then moved along the length of the beam to study the performance of the control system for different locations:

$$W_a = 25.4 \text{ mm}, \quad l_a = 76.2 \text{ mm}.$$

$$x_a = 38.1 \text{ mm} \rightarrow 469.9 \text{ mm}.$$

In low frequency (e.g. first and second modal frequencies) vibrations, the maximum bending strains and stresses occur at the root of the beam. So that the most effective place for actuators to get the maximum damping is at this location. Figure 5.5 verifies this, since the three indices provide a global optimum location at the root of the beam:

Table 5.6 Optimal piezoactuator configurations: W_a and l_a =constant

Index No.	Figure No.	x_a/L	x_a (mm)
1	5.3.a and 5.4.a	0.075	38.1
2	5.3.b and 5.4.b	0.075	38.1
3	5.3.c and 5.4.c	0.075	38.1

In addition, there is another local optimum case around $x_a/L = 0.5$ with 5% tolerance:

Table 5.7 Optimal piezoactuator configurations: W_a and l_a =constant

Index No.	Figure No.	x_a/L	x_a (mm)
1	5.5.a	0.439	223.16
2	5.5.b	0.536	272.50
3	5.5.c	0.463	235.49

Thus after the root of the beam the second best place for the piezoactuator is somewhere close to the middle of the beam.

Figure 5.2 Piezoactuator shape optimization: $W_a=25.4$ mm, $x_a=254.0$ mm; a) Overall damping ratio, b) Aggregate real part of eigenvalues, c) LQR objective function

Figure 5.3 Piezoactuator shape optimization: $A_a=25.4 \times 25.4$ mm², $x_a=254.0$ mm; a) Overall damping ratio, b) Aggregate real part of eigenvalues, c) LQR objective function

Figure 5.4 Piezoactuator shape optimization: $A_a=25.4 \times 25.4$ mm², $x_a=254$ mm; *a)* Overall damping ratio, *b)* Aggregate real part of eigenvalues, *c)* LQR objective function

Figure 5.5 Piezoactuator location optimization: $W_a=25.4$ mm, $l_a=76.2$ mm; a) Overall damping ratio, b) Aggregate real part of eigenvalues, c) LQR objective function

5.6 Overall Optimization: Shape & Location

Optimization of both shape and location of the piezoactuator is discussed in this section. Combining the ideas presented in sections 5.4 and 5.5, we look for the optimal shape and location of the piezoactuator simultaneously which provides the overall best performance, i.e. highest damping, of the control system under the following two cases.

5.6.1 $W_a = \text{Constant}$

It is assumed that the width of the piezoactuator is constant; the three indices are calculated for the following configurations of the piezoactuator:

$$W_a = 25.4 \text{ mm},$$

$$l_a = 25.4 \text{ mm} \rightarrow 508.0 \text{ mm}; \quad x_a = 12.7 \text{ mm} \rightarrow 495.3 \text{ mm}.$$

A 3D illustration of the indices is shown in Figure 5.6. The overall optimal configurations based on these indices are presented in Table 5.8:

Table 5.8 Overall optimal piezoactuator configurations

Index No.	Figure No.	x_a/L	x_a (mm)	l_a/L	l_a (mm)
1	5.6.a	0.375	190.5	0.400	203.2
2	5.6.b	0.575	292.5	0.750	381.0
3	5.6.c	0.625	317.5	0.750	381.0

As is seen, the second and third criteria ($\sum \text{Real}(\lambda_i)$ and $\min(J)$) match quite well.

5.6.2 $A_a = \text{Constant}$

The area of the piezoactuator is a constraint in this case, and again we look for the best position and shape of the actuators to get the greatest damping for the control

system:

$$A_a = 25.4 \times 25.4 = 645.16 \text{ mm}^2.$$

$$l_a = 25.4 \text{ mm} \rightarrow 508.0 \text{ mm}, W_a = 1.27 \text{ mm} \rightarrow 25.4 \text{ mm}, x_a = 12.7 \text{ mm} \rightarrow 495.3 \text{ mm}.$$

The three indices are illustrated in Figures 5.7 and 5.8. All indices provide the same optimal configurations which are given in Table 5.9.

Table 5.9 Overall optimal piezoactuator optimization

Index No.	Figure No.	x_a/L	x_a (mm)	$\frac{l_a/L}{W_a/L}$	$\frac{l_a}{W_a}$ (mm)
1	5.7.a and 5.8.a	0.025	12.7	0.05	25.4
2	5.7.b and 5.8.b	0.025	12.7	0.05	25.4
3	5.7.c and 5.8.c	0.025	12.7	0.05	25.4

Thus the overall optimum configuration of the piezoactuator is short and wide, located where it can affect the desired dynamic mode.

Results indicate that for actuators of equal width, the optimal length of the piezoactuator is not necessarily the longer one. It is also shown that in general, a shorter and wider actuator produces higher active damping in the system when the piezoactuator area is constant. This means that the overall optimal configuration of the piezoactuator seems to tend to a concentrated actuator located at an effective location. The methodology introduced here can be used for arbitrary smart structures and it leads to an optimal configuration of the actuator which is particularly relevant when the weight and geometry limitations are important factors, such as in long flexible space structures.

Figure 5.6 Piezoactuator overall optimization: $W_a=25.4$ mm, $l_a=25.4$ mm $\rightarrow$ 508.0 mm, $x_a=12.7$ mm $\rightarrow$ 495.3 mm; a) Overall damping ratio, b) Aggregate real part of eigenvalues, c) LQR objective function

Figure 5.7 Piezoactuator overall optimization: $A_a=25.4 \times 25.4$ mm², $l_a=25.4$ mm $\rightarrow$ 508.0 mm, $W_a=1.27$ mm $\rightarrow$ 25.4 mm, $x_a=25.4$ mm $\rightarrow$ 495.3 mm; a) Overall damping ratio, b) Aggregate real part of eigenvalues, c) LQR objective function

Figure 5.8 Piezoactuator overall optimization: $A_a=25.4 \times 25.4$ mm², $l_a=25.4$ mm $\rightarrow$ 508.0 mm, $W_a=1.27$ mm $\rightarrow$ 25.4 mm, $x_a=25.4$ mm $\rightarrow$ 495.3 mm; a) Overall damping ratio, b) Aggregate real part of eigenvalues, c) LQR objective function

Chapter 6

Wave Modeling of Smart Structures

Although the modal method in structural dynamics is a powerful and widely applied technique, it can become numerically cumbersome and inaccurate for high frequency modes. As the response wave length becomes very small relative to the total structural length, the modal description becomes increasingly inefficient (Meirovitch, 1967). Moreover model uncertainty and a very large number of lightly damped structural modes, make a direct approach to full state estimation impractical (von Flotow, 1986). Also the mode superposition method is no longer applicable if the given system is of semi-infinite or infinite extent (Humar, 1990).

In many flexible structures such as beams, trusses, plates, etc. the flexural vibration can be viewed as traveling waves propagating along the length of the structures. Thus as an alternative to modal analysis, a wave model of the structure can be obtained directly from the partial differential equations of the mathematical model of the structural dynamics in the frequency domain. This exact solution can then be used to design a controller which can suppress the flexural waves and consequently the vibration of the flexible structure. The model does not depend on modes and is then free of the crucial truncation effects of modes (Fujii and Ohtsuka, 1992). The continuous wave model of the flexible structure naturally lends itself to implementation with distributed sensors and actuators such as piezotransducers.

In this chapter the dynamic equation of a flexible beam is expressed in wave state space. The distributed equations of the piezoelements such as strain and strain rate sensors are also expressed in the wave state space, and then the wave models of the piezoactuators bonded onto the flexible beam are derived. Finally the dynamic model of

the integrated smart structure is introduced with respect to waves traveling along the flexible structure.

6.1 Flexural Vibration in Wave Form

As mentioned earlier, the flexural vibration of an undamped Euler-Bernoulli beam can be represented by a partial differential equation (equation (3.1)). The Laplace transform of this equation with respect to time yields an ordinary differential equation with respect to the spatial variable x . In the absence of external forces, the Laplace domain equation of the vibration can then be written as the following (to avoid new symbols, the same notation for the transformed variables as their time domain equivalents are used)

$$\frac{d^4}{dx^4}v(x, s) + \frac{\rho_b s^2}{E_b I_b}v(x, s) = 0, \quad (6.1)$$

where s is the Laplace variable. By either direct manipulations on this equation, or by assuming the motion to be represented by (as Graff, 1975):

$$v(x, t) = C e^{\gamma x} e^{i\omega t}, \quad (6.2)$$

we obtain:

$$\frac{d^4}{dx^4}v(x, \omega) - \frac{\omega^2}{\alpha^2}v(x, \omega) = 0. \quad (6.3)$$

where $i = \sqrt{-1}$, ω denotes the radial frequency, γ is a parameter specified below in equation (6.5), and

$$\alpha = \sqrt{\frac{E_b I_b}{\rho_b}}. \quad (6.4)$$

Equation (6.3) describes the motion of the beam in the frequency domain. Using equation (6.2), equation (6.3) leads to an eigenvalue problem:

$$\gamma^4 - \left(\frac{\omega}{\alpha}\right)^2 = 0. \quad (6.5)$$

Solving for γ yields:

$$\gamma = \pm\mu; \quad \pm i\mu. \quad (6.6)$$

where

$$\mu = \sqrt{\frac{\omega}{\alpha}}. \quad (6.7)$$

Therefore the solution of equation (6.3) can be expressed as:

$$v(x, \omega) = A_1 e^{-i\mu x} + A_2 e^{-\mu x} + B_1 e^{i\mu x} + B_2 e^{\mu x}. \quad (6.8)$$

where A_1 , A_2 , B_1 , and B_2 are constants which can be obtained from the boundary conditions.

If we now write

$$v(x, \omega) = a_1(x, \omega) + a_2(x, \omega) + b_1(x, \omega) + b_2(x, \omega). \quad (6.9)$$

where

$$\begin{aligned} a_1(x, \omega) &= A_1 e^{-i\mu x}, \\ a_2(x, \omega) &= A_2 e^{-\mu x}, \\ b_1(x, \omega) &= B_1 e^{i\mu x}, \\ b_2(x, \omega) &= B_2 e^{\mu x}, \end{aligned} \quad (6.10)$$

equation (6.8) can be interpreted as consisting of “*flexural waves*” (von Flotow and Schäfer, 1986), where a_1 , a_2 are waves traveling in the positive x direction, and b_1 , b_2 represent waves in the negative direction (Figure 6.1). Subscript 1 denotes traveling waves and subscript 2 represents evanescent waves for which amplitudes decay exponentially with x (Ghose, 1963). Thus, the flexural motion of the beam can be written in terms of four “*wave elements*”: a_1 , a_2 , b_1 , and b_2 which include the constants, A_1 , A_2 , B_1 , and B_2 . As equation (6.9) is to be dealt with in general terms, it is not necessary to know the values of these constants, and equation (6.9) can be considered without reference to boundary conditions (as the work reported by Pines and von Flotow, 1990). The summation of the wave elements (equation (6.9)) is termed the “*wave function*”.

Reintroducing the time element from equation (6.2) the steady state solution of the system in the time domain can be written as

$$v(x, t) = A_1 e^{-i\mu x + i\omega t} + A_2 e^{-\mu x + i\omega t} + B_1 e^{i\mu x + i\omega t} + B_2 e^{\mu x + i\omega t} \quad (6.11)$$

μ is called the wave number, and the wave length λ is then given by

$$\lambda = \frac{2\pi}{\mu} \quad (6.12)$$

and the wave velocity is (Graff, 1975):

$$c = \frac{\omega}{\mu} \quad (6.13)$$

substituting μ from equation (6.7) into this yields

$$c = \sqrt{\alpha\omega} \quad (6.14)$$

The wave velocity, c , can also be represented with respect to the wave length, λ , by substituting ω and μ from equations (6.7) and (6.12) respectively into equation (6.13):

$$c = \frac{2\pi\alpha}{\lambda} \quad (6.15)$$

As we see from equations (6.13) and (6.15) the velocity is not constant and is dependent on the frequency as well as the wave length. Thus the flexible beam represents a dispersive medium in which the flexural vibration can be represented by dispersive waves with variable velocities (Humar, 1990).

In order to express the dynamic equations in wave state space, i.e. with respect to flexural wave elements, we define a "wave state vector", $\mathbf{w}$, consisting of the four

Figure 6.1 Flexural waves through the flexible beam

elements of equation (6.10) as

$$\mathbf{w}(x, \omega) = \begin{bmatrix} a_1(x, \omega) & a_2(x, \omega) & b_1(x, \omega) & b_2(x, \omega) \end{bmatrix}^T, \quad (6.16)$$

and a so called “physical state vector”, $\mathbf{s}$, as

$$\mathbf{s}(x, \omega) = \begin{bmatrix} v & \theta & M & V \end{bmatrix}^T, \quad (6.17)$$

where

$$\begin{aligned} \theta(x, \omega) &= \frac{\partial}{\partial x} v(x, \omega), \\ M(x, \omega) &= E_b I_b \frac{\partial^2}{\partial x^2} v(x, \omega), \\ V(x, \omega) &= -E_b I_b \frac{\partial^3}{\partial x^3} v(x, \omega). \end{aligned} \quad (6.18)$$

θ , M , and V represent slope, bending moment, and shear force of the flexible beam respectively. Using equations (6.9), (6.10), and (6.18) the physical state vector, $\mathbf{s}$, can then be expressed in terms of the wave state vector, $\mathbf{w}$ as

$$\mathbf{s} = \mathbf{H} \mathbf{w}, \quad (6.19)$$

where the coefficient matrix $\mathbf{H}$, which relates the physical measurable properties of $\mathbf{s}$ to wave properties, is given by:

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i\mu & -\mu & i\mu & \mu \\ -E_b I_b \mu^2 & E_b I_b \mu^2 & -E_b I_b \mu^2 & E_b I_b \mu^2 \\ -iE_b I_b \mu^3 & E_b I_b \mu^3 & iE_b I_b \mu^3 & -E_b I_b \mu^3 \end{bmatrix}. \quad (6.20)$$

Equation (6.19) is later used to derive the dynamic equations of the integrated smart structure in the wave state form, i.e. with respect to flexural wave elements.

Using equations (6.10) and (6.16) the dynamic state space equation of the wave can be expressed as

$$\frac{d\mathbf{w}}{dx} = \boldsymbol{\eta} \mathbf{w} \quad (6.21)$$

where

$$\eta = \mu \begin{bmatrix} -i & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6.22)$$

For simplicity from now on we shall denote the wave state vector at position x_i as:

$$\mathbf{w}_i \equiv [a_1(i) \quad a_2(i) \quad b_1(i) \quad b_2(i)]'. \quad (6.23)$$

where

$$\begin{aligned} a_1(i) &\equiv a_1(x_i, \omega), \\ a_2(i) &\equiv a_2(x_i, \omega), \\ b_1(i) &\equiv b_1(x_i, \omega), \\ b_2(i) &\equiv b_2(x_i, \omega). \end{aligned} \quad (6.24)$$

The above wave elements, i.e. a_1 , a_2 , b_1 , and b_2 , are exponential functions of x . (equation (6.10)). Thus in the absence of external forces and moments each wave element at position x_i of the beam is exponentially related to the wave element at position x_j , e.g.

$$b_2(x_j, \omega) = e^{k_{ij}} b_2(x_i, \omega).$$

where k_{ij} is a dimensionless parameter given by:

$$k_{ij} = (x_j - x_i) \mu = (x_j - x_i) \sqrt{\frac{\omega}{\alpha}}. \quad (6.25)$$

Now using equation (6.16) the wave state vectors at positions x_i and x_j can be related as:

$$\mathbf{w}_j = \mathbf{T}_{ij} \mathbf{w}_i, \quad (6.26)$$

where $\mathbf{T}_{ij}$ is the spatial transition matrix between waves at positions x_i and x_j :

$$\mathbf{T}_{ij} = \begin{bmatrix} e^{-ik_{ij}} & 0 & 0 & 0 \\ 0 & e^{-k_{ij}} & 0 & 0 \\ 0 & 0 & e^{ik_{ij}} & 0 \\ 0 & 0 & 0 & e^{k_{ij}} \end{bmatrix}. \quad (6.27)$$

This diagonal matrix relates the wave elements at different points on the flexible beam.

6.2 Wave Model of Piezoactuators

As mentioned in chapter 3 the actuation induced by a pair of piezoceramics attached to the surface of a flexible beam can be modeled as concentrated moments (Figure 6.2). Equation (3.67) is used for the induced moments, M_a , imposed by the actuators. These actuation moments can then be introduced to the dynamic wave equations as follows.

We define the simplified notations s_i and s_j for the physical state vectors (equation (6.17)) at x_i and x_j respectively as

$$s_i \equiv s(x_i, \omega); \quad s_j \equiv s(x_j, \omega). \quad (6.28)$$

Using the methodology introduced by Pines and von Flotow (1990) and applying the boundary conditions at the end points of the piezoactuators, the physical states vectors at positions x_i and x_j can be expressed as:

$$s_i^- = s_i^+ + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} M_a, \quad (6.29)$$

and

Figure 6.2 Induced moments by piezoactuators

$$s_j^+ = s_j^- + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} M_a. \quad (6.30)$$

Superscript - and + denote left and right neighborhoods at the specific positions respectively. Using equation (6.19), equations (6.29) and (6.30) can be written in terms of the wave state vectors:

$$H_b w_i^- = H_c w_i^+ + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} M_a, \quad (6.31)$$

and

$$H_b w_j^+ = H_c w_j^- + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} M_a. \quad (6.32)$$

where subscript b denotes the beam alone without actuators and subscript c denotes the composite smart structure with the piezoactuators. Coefficient matrix H_b relates the wave state vector to the physical state vector of the beam alone and analogously H_c does the same for the composite smart structure (equations (6.19) and (6.20)):

$$H_b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i\mu_b & -\mu_b & i\mu_b & \mu_b \\ -E_b I_b \mu_b^2 & E_b I_b \mu_b^2 & -E_b I_b \mu_b^2 & E_b I_b \mu_b^2 \\ -iE_b I_b \mu_b^3 & E_b I_b \mu_b^3 & iE_b I_b \mu_b^3 & -E_b I_b \mu_b^3 \end{bmatrix}. \quad (6.33)$$

and

$$H_c = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i\mu_c & -\mu_c & i\mu_c & \mu_c \\ -E_c I_c \mu_c^2 & E_c I_c \mu_c^2 & -E_c I_c \mu_c^2 & E_c I_c \mu_c^2 \\ -iE_c I_c \mu_c^3 & E_c I_c \mu_c^3 & iE_c I_c \mu_c^3 & -E_c I_c \mu_c^3 \end{bmatrix}. \quad (6.34)$$

The waves at positions x_i and x_j (inside the region covered by the piezoactuators) are related as:

$$\mathbf{w}_j^- = \mathbf{T}_{ij} \mathbf{w}_i^+ \quad (6.35)$$

Substituting this expression for $\mathbf{w}_j^-$ into equation (6.32) yields:

$$\mathbf{H}_b \mathbf{w}_j^+ = \mathbf{H}_c \mathbf{T}_{ij} \mathbf{w}_i^+ + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} M_a \quad (6.36)$$

Using equation (6.31), the above equation can be rewritten as

$$\begin{bmatrix} \mathbf{H}_b & -\mathbf{H}_c \mathbf{T}_{ij} \mathbf{H}_c^{-1} \mathbf{H}_b \end{bmatrix} \begin{bmatrix} \mathbf{w}_j^+ \\ \mathbf{w}_i^- \end{bmatrix} = \begin{bmatrix} -\mathbf{H}_c \mathbf{T}_{ij} \mathbf{H}_c^{-1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} M_a \quad (6.37)$$

superscript -1 denotes the inverse of a matrix.

For simplicity we ignore the stiffness and mass of the piezoactuators, then $E_b I_b = E_c I_c$ and $\mu_b = \mu_c$, consequently:

$$\mathbf{H}_b = \mathbf{H}_c = \mathbf{H}. \quad (6.38)$$

Equation (6.37) can then be simplified to:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{T}_{ij} \end{bmatrix} \begin{bmatrix} \mathbf{w}_j^+ \\ \mathbf{w}_i^- \end{bmatrix} = \Omega_{ij} M_a, \quad (6.39)$$

where

$$\Omega_{ij} = (-\mathbf{T}_{ij} + \mathbf{I}) \mathbf{H}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \quad (6.40)$$

This simplification assumption is just used for control design; in control implementation

$$I_s = - \operatorname{sgn}(z) \frac{t_b E_s d_{31s} W_s}{2 E_b I_b} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_i - m_i)}^{(x_i + m_i)} \psi \frac{dM}{dt} dx. \quad (6.43)$$

We apply the Laplace transform to these equations. Then substituting the Laplace parameter, s , by $(i\omega)$ for harmonic motion, the above equations can be written in the frequency domain:

$$V_s = - \operatorname{sgn}(z) \frac{t_b E_s d_{31s} W_s}{2 E_b I_b C_s} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_i - m_i)}^{(x_i + m_i)} \psi M(x, \omega) dx, \quad (6.44)$$

and

$$I_s = - \operatorname{sgn}(z) (i\omega) \frac{t_b E_s d_{31s} W_s}{2 E_b I_b} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \int_{(x_i - m_i)}^{(x_i + m_i)} \psi M(x, \omega) dx. \quad (6.45)$$

Again, to avoid new symbols, the same notation for the transformed variables as their time domain equivalents are used. Equation (6.19), which relates the physical state and wave state vectors, can be used to obtain the expression for bending moment, M , with respect to the wave elements. Using this equation the third row of the physical state vector, bending moment, M , is written as

$$M = \begin{bmatrix} -E_b I_b \mu^2 & E_b I_b \mu^2 & -E_b I_b \mu^2 & E_b I_b \mu^2 \end{bmatrix} \mathbf{w} \quad (6.46)$$

where $\mathbf{w}$ is the wave state vectors consisting of the four traveling wave elements given by equation (6.16). Introducing M_o as

$$M_o \equiv E_b I_b \mu^2, \quad (6.47)$$

equation (6.46) is rewritten as follows

$$M = M_o [-1 \ 1 \ -1 \ 1] \mathbf{w}. \quad (6.48)$$

This equation expresses the bending moment of the beam via wave state vector. Now substituting (6.48) in equations (6.44) and (6.45) and using equation (6.21), the output

voltage and current of the piezosensor can be expressed with respect to the wave state vectors:

$$V_s = -\text{sgn}(z) \frac{t_b E_s d_{31s} W_s}{2E_b I_b C_s} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \left(M_o \sqrt{\frac{\alpha}{\omega}} \right) [-i \ -1 \ i \ 1] \{w_j - w_i\} . \quad (6.49)$$

and

$$I_s = -\text{sgn}(z) \frac{t_b E_s d_{31s} W_s}{2E_b I_b} \left(\frac{t_b}{T_b} (1 + \tilde{t}) - z_o \right) \left(M_o \sqrt{\alpha \omega} \right) [1 \ -i \ -1 \ i] \{w_j - w_i\} . \quad (6.50)$$

These equations indicate that the output voltage and current of the piezosensors in the frequency domain are proportional to the difference of the propagating wave elements at the end points of the piezosensors. Thus, the integral expressions of the distributed piezosensor signals changed to algebraic equations in wave domain. These algebraic equations are much easier to handle than the integral forms in the time domain, which up to now, have been the usual means of dealing with these types of systems.

In the case of thin rectangular piezosensors (refer section 3.3.5), i.e.

$$\tilde{t} \equiv 0, \quad z_o \equiv 0, \quad T_b \equiv t_b .$$

equations (6.49) and (6.50) can be written as

$$V_s = -\text{sgn}(z) \frac{t_b E_s d_{31s} W_s}{2E_b I_b C_s} \left(M_o \sqrt{\frac{\alpha}{\omega}} \right) [-i \ -1 \ i \ 1] \{w_j - w_i\} , \quad (6.51)$$

and

$$I_s = -\text{sgn}(z) \frac{t_b E_s d_{31s} W_s}{2E_b I_b} \left(M_o \sqrt{\alpha \omega} \right) [1 \ -i \ -1 \ i] \{w_j - w_i\} . \quad (6.52)$$

These are the final equations which are used in this study for the piezosensors which are thin rectangular PVDF piezoelements.

6.4 Wave Model of the Integrated Smart Structure

In this section the dynamics of the integrated smart structure with bonded piezotransducers in wave state space is derived. The dynamic equation of the system is

written with respect to the input and output waves of the sensing area. Three different configurations of piezotransducer placement are studied, and are used later on in the control system design.

6.4.1 Non-Collocation

Assume the physical model shown in Figure 6.4 (similar to the model used in chapter 5). Two PZT piezoceramics are used as the actuators and four PVDF piezofilms are employed as sensors. Piezosensors #1 and #2 measure the strain and sensors #3 and #4 measures the strain rates.

For this configuration of the actuators equations (6.39) and (6.40) become:

$$[I \quad -T_{34}] \begin{bmatrix} w_4^+ \\ w_3^- \end{bmatrix} = \Omega_{34} M_a, \quad (6.53)$$

Figure 6.4 Smart structure with non-collocated transducers

where

$$\Omega_{34} = (-T_{34} + I) H^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \quad (6.54)$$

Our region of interest is the total sensing area, i.e. $x_I \rightarrow x_6$. Thus we wish to express equation (6.53) with respect to the waves at positions x_I and x_6 . Using equation (6.26) the wave state vectors at x_3 and x_4 can be expressed with respect to the wave state vectors at the end points of the sensing area, i.e. x_I and x_6 :

$$\mathbf{w}_3^- = T_{13} \mathbf{w}_1; \quad \mathbf{w}_4^+ = T_{46}^{-1} \mathbf{w}_6. \quad (6.55)$$

Thus equation (6.53) can be written as:

$$\begin{bmatrix} I & T_{46}^{-1} & -T_{34} T_{13} \end{bmatrix} \begin{bmatrix} \mathbf{w}_6 \\ \mathbf{w}_1 \end{bmatrix} = \Omega_{34} M_a. \quad (6.56)$$

which written out in full is

$$\begin{bmatrix} e^{ik_{16}} & 0 & 0 & 0 & -e^{-ik_{14}} & 0 & 0 & 0 \\ 0 & e^{k_{16}} & 0 & 0 & 0 & -e^{-k_{14}} & 0 & 0 \\ 0 & 0 & e^{-ik_{16}} & 0 & 0 & 0 & -e^{ik_{14}} & 0 \\ 0 & 0 & 0 & e^{-k_{16}} & 0 & 0 & 0 & -e^{k_{14}} \end{bmatrix} \begin{bmatrix} a_1(6) \\ a_2(6) \\ b_1(6) \\ b_2(6) \\ a_1(1) \\ a_2(1) \\ b_1(1) \\ b_2(1) \end{bmatrix} = \Omega_{34} M_a. \quad (6.57)$$

Defining the “input and output wave vectors” of the total sensing area, i.e. $\mathbf{w}_I$ and $\mathbf{w}_O$ as:

$$\mathbf{w}_I \equiv \begin{bmatrix} a_1(1) \\ a_2(1) \\ b_1(6) \\ b_2(6) \end{bmatrix}; \quad \mathbf{w}_O \equiv \begin{bmatrix} b_1(1) \\ b_2(1) \\ a_1(6) \\ a_2(6) \end{bmatrix}, \quad (6.58)$$

equation (6.57) can be rewritten as:

$$S_1 w_I + S_2 w_O = \Omega_{34} M_a, \quad (6.59)$$

where

$$S_1 = \begin{bmatrix} -e^{-ik_{14}} & 0 & 0 & 0 \\ 0 & -e^{-k_{14}} & 0 & 0 \\ 0 & 0 & -e^{-ik_{46}} & 0 \\ 0 & 0 & 0 & -e^{-k_{46}} \end{bmatrix}; \quad S_2 = \begin{bmatrix} 0 & 0 & e^{ik_{46}} & 0 \\ 0 & 0 & 0 & e^{k_{46}} \\ -e^{ik_{14}} & 0 & 0 & 0 \\ 0 & -e^{k_{14}} & 0 & 0 \end{bmatrix}. \quad (6.60)$$

From equation (6.59) the *final dynamic equation of the integrated smart structure* can be obtained in wave state space:

$$w_O = T_{OL} w_I + \Phi M_a. \quad (6.61)$$

where

$$\Phi = S_2^{-1} \Omega_{34} = \frac{1}{4M_o} \begin{bmatrix} e^{-ik_{14}} & -e^{-ik_{13}} \\ e^{-k_{13}} & -e^{-k_{14}} \\ e^{-ik_{46}} & -e^{-ik_{46}} \\ e^{-k_{46}} & -e^{-k_{46}} \end{bmatrix}, \quad (6.62)$$

and M_o is given by equation (6.47). T_{OL} is the open loop transition matrix between w_O and w_I . It relates the output and input wave vectors in the absence of external moment, M_a :

$$T_{OL} = -S_2^{-1} S_1 = \begin{bmatrix} 0 & 0 & e^{-ik_{46}} & 0 \\ 0 & 0 & 0 & e^{-k_{46}} \\ e^{-ik_{16}} & 0 & 0 & 0 \\ 0 & e^{-k_{16}} & 0 & 0 \end{bmatrix}. \quad (6.63)$$

Assuming $M_a=0$, T_{OL} can be obtained also directly from equation (6.26):

$$\begin{bmatrix} a_1(6) \\ a_2(6) \\ b_1(6) \\ b_2(6) \end{bmatrix} = \begin{bmatrix} e^{-ik_{16}} & 0 & 0 & 0 \\ 0 & e^{-k_{16}} & 0 & 0 \\ 0 & 0 & e^{ik_{16}} & 0 \\ 0 & 0 & 0 & e^{k_{16}} \end{bmatrix} \begin{bmatrix} a_1(1) \\ a_2(1) \\ b_1(1) \\ b_2(1) \end{bmatrix}. \quad (6.64)$$

Rearranging the above equation with respect to the input and output waves of the total sensing area, i.e. w_O and w_I , yields:

$$w_O = \begin{bmatrix} 0 & 0 & e^{-ik_{16}} & 0 \\ 0 & 0 & 0 & e^{-k_{16}} \\ e^{-ik_{16}} & 0 & 0 & 0 \\ 0 & e^{-k_{16}} & 0 & 0 \end{bmatrix} w_I. \quad (6.65)$$

which matches the definition of the open loop transition matrix given by (6.63).

6.4.2 Collocation of Transducers of Unequal Length

In this case instead of four piezosensors we assume two long piezosensors are used. Sensor #1 measures the strain and sensor #2 measures the strain rate. In order to make as few changes as possible in the dynamic equations previously obtained, a similar numbering scheme for the endpoints of the transducers is used for this case (Figure 6.5). The numbers 2 and 5 serve no function except to indicate that these points (2 and 5 of the previous case) are drawn together. Thus equation (6.61), without any change, can express the dynamics of the integrated smart structure for this case.

6.4.3 Collocation for Equal Length Transducers

Now a collocation case with equal length sensors and actuators is assumed. This case is exactly like the previous case except the length of the sensors and actuators are equal.

Figure 6.5 Smart structure with unequal length collocated transducers

Again in order to be able to use the equations obtained in section 6.4.1 we choose a suitable numbering for the end points of the transducers as shown in Figure 6.6. Equation (6.61) can be used again for this case. The two point pairs, 1, 3 and 4, 6 are geometrically superimposed, i.e. $x_1 = x_3$ and $x_4 = x_6$, therefore

$$k_{13} = k_{46} = 0. \quad (6.66)$$

Φ (equation (6.62)) can then be simplified to:

$$\Phi = \frac{1}{4M_o} \begin{bmatrix} e^{-ik_{16}} - 1 \\ 1 - e^{-k_{16}} \\ e^{-ik_{16}} - 1 \\ 1 - e^{-k_{16}} \end{bmatrix}. \quad (6.67)$$

Figure 6.6 Smart structure with equal length collocated transducers

Chapter 7

Local Wave Absorbing Controller (LWAC)

Several researchers have approached the problem of designing “Wave Absorbing Controllers” (WAC) for elastic structures. This is a promising approach for many flexible structures such as beams which can be viewed as flexural wave guides. The controller is designed based on the absorption of as much as possible of flexural waves travelling in the sensing area by minimizing the wave functions (described in chapter 6, equation (6.9)). In the Local Wave Absorbing Controllers (LWAC), which is the primary WAC studied by researchers to date, only the local reflection and transmission behavior of waves around the sensors and actuators need to be modeled (Miller and Hall, 1991). Therefore if control objectives are posed in terms of local behavior, the LWAC may be most appropriate for such applications as local vibration control of very large flexible structures. The LWAC is also suitable for very large structures with unknown boundaries or at positions very far from boundaries.

The most important parameters in the Wave Absorbing Method (WAM), on which LWAC is based, are wave frequency, the length of the sensing area, and the total distance of wave travel. When the frequency of a wave is high or the traveled distance/sensing length are very long, then better observation of the traveled waves is possible. Generally speaking this means the WAM is a good methodology for active control of high frequency vibration, e.g. acoustics in long flexible structures. This concept is investigated in the current literature; in this study the feasibility of the LWAC for low frequency systems is examined.

Thus far in the literature, usually point sensors such as strain gauges have been used to sense strain, which is proportional to the moment, and the actuators have usually been

non-collocated point actuators, such as torque motors (Fujii and Ohtsuka (1992), Miller and Hall (1991)). Pines and von Flotow (1990) employed the WAM to suppress the high frequency acoustic waves along a flexible beam using piezoceramic actuators. In their study point sensors, i.e. strain gauges, were used for strain (i.e. moment) measurement.

In this chapter a new LWAC using only distributed piezoelectric sensors and actuators is introduced. In addition, the unique feature of measuring strain rate, i.e. moment rate, by the piezosensors enables the design of a controller which does not require differentiation terms in the control loop, and thus is more robust to noises and disturbances. The concentrated moment model of the piezoactuators developed in chapter 3 is used. Finally a control criterion based on the absorption of as much of the traveling waves as possible in the sensing area is introduced.

7.1 LWAC Design

An active control system based on the WAM is described in this section, and collocation and non-collocation cases are studied. The controllers are designed in the frequency domain.

In the previous literature (von Flotow, 1986 and Fujii and Ohtsuka, 1992) strain feedback is used to design the controller. This requires differentiation operators in the final form of the controller. As mentioned above, strain rate feedback is used in this study, which eliminates the need for differentiating elements in the controller, thus eliminating one source of sensitivity to noises and disturbances.

The physical and geometric parameters of the smart structure elements used in the following sections are the same as those described in section 4.1.1 except:

$W_b = 52$ mm, $l_a = 150$ mm,

x_a , x_s , and $l_s \equiv$ variable.

The sensors are PVDF piezofilms and the actuators are a pair of PZT piezoceramics attached onto the both sides of the beam.

7.1.1 Non-Collocated LWAC

A physical model of a smart structure with non-collocated transducers as illustrated in Figure 6.4 is the subject of interest here. A feedback control loop in which the control voltage is proportional to the summation of the output currents of piezosensors #3 and #4 is used (Figure 7.1):

$$V_a(\omega) = c(\omega) [I_{s3}(\omega) + I_{s4}(\omega)] . \quad (7.1)$$

The other two sensors (#1 and #2) are used for strain measurements. As mentioned earlier, in chapter 3 (equations (3.83) and (3.84)), the output voltage and current of a piezosensor is proportional to strain and strain rate respectively. Since $x_{s1}=x_{s3}$, $l_{s1}=l_{s3}$ and $x_{s2}=x_{s4}$, $l_{s2}=l_{s4}$ (see Figure 6.4), the output voltages of piezosensors #1 and #2 are proportional to the time integration of output currents of piezosensors #3 and #4 respectively. Thus output voltages of sensors #1 and #2 can be used in the control loop whenever integrations of output currents of sensors #3 and #4 are needed. This eliminates the need for integrating elements in the control loop.

Using equation (6.52) the control command (equation (7.1)) can be written with respect to the wave state vectors (equation (6.16)) at the endpoint of the sensors. For the case of using identical piezosensors:

$$V_a(\omega) = -\text{sgn}(z) c(\omega) \frac{t_b E_s d_{31s} W_s}{2E_b I_b} (M_o \sqrt{\alpha\omega}) [1 \ -i \ -1 \ i] \{ (w_2 - w_1) + (w_6 - w_5) \} . \quad (7.2)$$

Using a concentrated moment model of the piezoactuators with a strain assumed to be linearly distributed, equation (3.67), the moment induced by the piezoactuator pair in

Figure 7.1 Wave absorbing control system

the frequency domain is:

$$M_a(\omega) = c(\omega) \Psi \Gamma(\omega) \{ (\mathbf{w}_2 - \mathbf{w}_1) + (\mathbf{w}_6 - \mathbf{w}_5) \}. \quad (7.3)$$

where

$$\Psi = -\text{sgn}(z) \frac{t_b^2 (E_s d_{31s} W_s) (E_a d_{31a} W_a)}{2E_b I_b} \frac{\Upsilon(1 + \bar{t})}{\Upsilon + 6 \left(1 + 2\bar{t} + \frac{4}{3}\bar{t}^2 \right)}. \quad (7.4)$$

and $\Gamma(\omega)$ is a vector which correlates the wave state vector, $\mathbf{w}$, to the bending moment, M given by:

$$\Gamma(\omega) \equiv [\Gamma_1 \mid \Gamma_2] \equiv (M_o \sqrt{\alpha \omega}) \left[\overbrace{1}^{\Gamma_1} \overbrace{-i}^{\Gamma_2} \mid \overbrace{-1}^{\Gamma_1} \overbrace{i}^{\Gamma_2} \right]. \quad (7.5)$$

Of interest is the closed loop transition matrix between the output and input wave state vectors over the total sensing area, which can then be used in the control design. The total sensing area can be viewed as the entire extent from x_1 to x_6 and the output and input wave vectors of the sensing area, i.e. $\mathbf{w}_O$ and $\mathbf{w}_I$, are defined by equation (6.58). In order to express M_a with respect to $\mathbf{w}_O$ and $\mathbf{w}_I$, equation (7.3) is rewritten as follows (using equation (6.26)):

$$M_a(\omega) = c(\omega) \Psi \Gamma(\omega) \{ (T_{12} \mathbf{w}_1 - \mathbf{w}_1) + (\mathbf{w}_6 - T_{15} \mathbf{w}_1) \}. \quad (7.6)$$

or in expanded form:

$$M_a(\omega) = c(\omega) \Psi [\Gamma_1 \mid \Gamma_2] \left(\begin{bmatrix} a_1(6) \\ a_2(6) \\ b_1(6) \\ b_2(6) \end{bmatrix} - \begin{bmatrix} \delta_1 & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \delta_2 \end{bmatrix} \begin{bmatrix} a_1(1) \\ a_2(1) \\ b_1(1) \\ b_2(1) \end{bmatrix} \right), \quad (7.7)$$

where δ_1 and δ_2 are given by:

$$\delta_1 \equiv \begin{bmatrix} 1 - e^{-ik_{12}} + e^{-ik_{15}} & 0 \\ 0 & 1 - e^{-k_{12}} + e^{-k_{15}} \end{bmatrix}; \quad \delta_2 \equiv \begin{bmatrix} 1 - e^{ik_{12}} + e^{ik_{15}} & 0 \\ 0 & 1 - e^{k_{12}} + e^{k_{15}} \end{bmatrix}. \quad (7.8)$$

Consequently M_a can be expressed in terms of w_I and w_O as follows:

$$M_a(\omega) = c(\omega) \Psi \left(\begin{bmatrix} -\Gamma_1 \delta_1 & \Gamma_2 \end{bmatrix} \underbrace{\begin{bmatrix} a_1(1) \\ a_2(1) \\ b_1(6) \\ b_2(6) \end{bmatrix}}_{w_I} + \begin{bmatrix} -\Gamma_2 \delta_2 & \Gamma_1 \end{bmatrix} \underbrace{\begin{bmatrix} b_1(1) \\ b_2(1) \\ a_1(6) \\ a_2(6) \end{bmatrix}}_{w_O} \right). \quad (7.9)$$

Finally the induced moment, M_a , can be written as follows:

$$M_a(\omega) = c(\omega) \Psi (\mu_1 w_I + \mu_2 w_O). \quad (7.10)$$

where

$$\mu_1 \equiv \begin{bmatrix} -\Gamma_1 \delta_1 & \Gamma_2 \end{bmatrix}; \quad \mu_2 \equiv \begin{bmatrix} -\Gamma_2 \delta_2 & \Gamma_1 \end{bmatrix}; \quad (7.11)$$

Substituting equation (7.10) into the final dynamic equation of the smart structure, i.e. equation (6.61), and arranging it yields:

$$w_O = \left[(I - c(\omega) \Psi \Phi \mu_2)^{-1} (T_{OL} + c(\omega) \Psi \Phi \mu_1) \right] w_I. \quad (7.12)$$

The above equation expresses the output wave vector via the input wave vector of a closed loop system in the wave state space. Thus one can define the *closed loop transition matrix* as

$$T_{CL} = (I - c(\omega) \Psi \Phi \mu_2)^{-1} (T_{OL} + c(\omega) \Psi \Phi \mu_1). \quad (7.13)$$

T_{CL} is a 4×4 matrix, which relates the output and input wave vectors of the total sensing area in a control system.

Figure 7.2 shows the input and output waves of the total sensing region, $x_1 \rightarrow x_6$. Considering the wave elements traveling into the sensing area, i.e. $a_1(1)$, $a_2(1)$, $b_1(6)$, and $b_2(6)$, the “*aggregate wave function input into the sensing area*” is defined as:

$$W_{TI} = a_1(1) + a_2(1) + b_1(6) + b_2(6), \quad (7.14)$$

and considering output wave elements from the sensing area, i.e. $a_1(6)$, $a_2(6)$, $b_1(1)$, and $b_2(1)$, the “aggregate output wave function from the sensing area” is:

$$W_{TO} = a_1(6) + a_2(6) + b_1(1) + b_2(1). \quad (7.15)$$

As mentioned before in chapter 6, the evanescent elements of the wave function, i.e. elements with subscript 2, decay exponentially with x . With a long enough beam, one might expect these evanescent waves to contribute very little to the dynamic response of the structure (von Flotow and Schäfer, 1986). Ignoring these terms, we retain the dominant components of the waves, i.e. the traveling waves, and thus the aggregate input and output wave functions of the sensing region, $x_1 \rightarrow x_6$, will be as follows:

$$W_{TI} = a_1(1) + b_1(6); \quad W_{TO} = a_1(6) + b_1(1). \quad (7.16)$$

Using equation (7.12) and again ignoring the evanescent elements of w_I and w_O , the components of W_{TO} can be written as

$$a_1(6) = T_{CL}(1, 1)a_1(1) + T_{CL}(1, 3)b_1(6), \quad (7.17)$$

and

$$b_1(1) = T_{CL}(3, 1)a_1(1) + T_{CL}(3, 3)b_1(6). \quad (7.18)$$

Figure 7.2 Input and output traveling waves

where the T_{cl} elements are:

$$T_{CL}(1, 1) = -\frac{\sqrt{\alpha\omega}\Psi}{4\Delta} c(\omega) \left(-e^{-ik_{14}} + e^{-ik_{13}} + e^{-ik_{14}} e^{-ik_{12}} - e^{-ik_{14}} e^{-ik_{15}} - \right. \\ \left. e^{-ik_{13}} e^{-ik_{12}} + e^{-ik_{13}} e^{-ik_{15}} + e^{-ik_{14}} e^{-ik_{16}} - e^{-ik_{13}} e^{-ik_{16}} \right) \quad (7.19)$$

$$T_{CL}(3, 3) = \frac{\sqrt{\alpha\omega}\Psi}{4\Delta} c(\omega) \left(-e^{-ik_{36}} e^{ik_{12}} e^{-ik_{16}} + e^{-ik_{36}} e^{-ik_{16}} - e^{-ik_{36}} e^{-ik_{16}} - \right. \\ \left. e^{-ik_{36}} e^{ik_{15}} e^{-ik_{16}} + e^{-ik_{36}} e^{-ik_{36}} - e^{-ik_{36}} e^{-ik_{16}} e^{ik_{12}} + e^{-ik_{36}} e^{ik_{15}} e^{-ik_{16}} \right) \quad (7.20)$$

$$T_{CL}(1, 3) = \frac{1}{\Delta} \left(-e^{-ik_{16}} + \frac{\sqrt{\alpha\omega}\Psi}{4} c(\omega) \left(e^{-ik_{14}} - e^{-ik_{13}} + e^{-ik_{36}} e^{-ik_{16}} - e^{-ik_{36}} e^{-ik_{16}} - \right. \right. \\ \left. ie^{-k_{36}} e^{-ik_{16}} - ie^{-k_{13}} e^{-ik_{16}} + ie^{-k_{36}} e^{-ik_{16}} + ie^{-k_{14}} e^{-ik_{16}} + ie^{-k_{13}} e^{k_{12}} e^{-ik_{16}} - \right. \\ \left. ie^{-k_{13}} e^{k_{15}} e^{-ik_{16}} - ie^{-k_{14}} e^{k_{12}} e^{-ik_{16}} + ie^{-k_{14}} e^{k_{15}} e^{-ik_{16}} \right) \left. \right) \quad (7.21)$$

$$T_{CL}(3, 1) = \frac{1}{\Delta} \left(-e^{-ik_{16}} + \frac{\sqrt{\alpha\omega}\Psi}{4} c(\omega) \left(e^{-ik_{14}} e^{ik_{15}} e^{-ik_{16}} + e^{-ik_{14}} e^{-ik_{16}} - e^{-ik_{13}} e^{-ik_{16}} + \right. \right. \\ \left. e^{-ik_{36}} - e^{-ik_{36}} - e^{-ik_{14}} e^{ik_{12}} e^{-ik_{16}} + e^{-ik_{13}} e^{ik_{12}} e^{-ik_{16}} - e^{-ik_{13}} e^{ik_{15}} e^{-ik_{16}} + \right. \\ \left. e^{-ik_{36}} e^{-ik_{12}} - e^{-ik_{36}} e^{-ik_{15}} - e^{-ik_{36}} e^{-ik_{12}} + e^{-ik_{36}} e^{-ik_{15}} - ie^{-ik_{36}} e^{-ik_{16}} - \right. \\ \left. ie^{-k_{13}} e^{-ik_{16}} + ie^{-k_{36}} e^{-ik_{16}} + ie^{-k_{14}} e^{-ik_{16}} + ie^{-k_{13}} e^{k_{12}} e^{-ik_{16}} - \right. \\ \left. ie^{-k_{13}} e^{k_{15}} e^{-ik_{16}} - ie^{-k_{14}} e^{k_{12}} e^{-ik_{16}} + ie^{-k_{14}} e^{k_{15}} e^{-ik_{16}} \right) \left. \right) \quad (7.22)$$

and Δ is the determinant of $(I - c(\omega) \Psi \mu_2)$.

Using equations (7.16)-(7.18), the aggregate output wave function can be written as:

$$W_{TO} = [T_{CL}(1, 1) + T_{CL}(3, 1)] a_1(1) + [T_{CL}(1, 3) + T_{CL}(3, 3)] b_1(6). \quad (7.23)$$

In order to simplify the complicated elements of the closed loop transition matrix, T_{CL} , the following geometric assumptions are used for the lengths and placements of the transducers:

$$l_{s1} = l_{s2}, \quad (7.24)$$

$$x_a = \frac{x_{s1} + x_{s2}}{2}, \quad (7.25)$$

which means that the piezosensors have the same lengths and the midpoints of the piezoactuators are located exactly at the midpoint of the region between the piezosensors. These reasonable assumptions also introduce some symmetry into some of the T_{CL} elements. The simplified elements of T_{CL} can then be written as:

$$T_{CL}(1, 1) = T_{CL}(3, 3) = -\frac{\sqrt{\alpha\omega}\Psi}{4\Delta} c(\omega) \left(\left(e^{-ik_{13}} - e^{-ik_{14}} \right) \left(1 - e^{-ik_{12}} + e^{-ik_{15}} - e^{-ik_{16}} \right) \right) \quad (7.26)$$

and

$$T_{CL}(1, 3) = T_{CL}(3, 1) = \frac{1}{\Delta} \left(-e^{-ik_{16}} + \frac{\sqrt{\alpha\omega}\Psi}{4} c(\omega) \left(\left(e^{-ik_{14}} - e^{-ik_{13}} \right) \left(1 + e^{-ik_{16}} \right) + i e^{-ik_{16}} \left(2e^{-k_{14}} - 2e^{-k_{13}} + e^{-k_{23}} + e^{k_{23}} - e^{-k_{24}} - e^{k_{24}} \right) \right) \right) \quad (7.27)$$

Using the above symmetric properties equation (7.23) yields

$$W_{TO} = [T_{CL}(1, 1) + T_{CL}(1, 3)] [a_1(1) + b_1(6)] . \quad (7.28)$$

In other words the aggregate output wave function can be expressed with respect to the aggregate input wave function (equation (7.16)) as

$$W_{TO} = [T_{CL}(1, 1) + T_{CL}(1, 3)] w_{TI}. \quad (7.29)$$

• Control Criterion

The control is based on the absorption of as much of the traveling waves as possible in the sensing area. In other words the ratio of aggregate output and input wave functions is to be forced toward zero. Using equation (7.29) this ratio can be written as:

$$\frac{W_{TO}}{W_{TI}} = T_{CL}(1, 1) + T_{CL}(1, 3). \quad (7.30)$$

Substituting $T_{CL}(1, 1)$ and $T_{CL}(1, 3)$ from equations (7.26) and (7.27) yields:

$$\frac{W_{TO}}{W_{TI}} = \frac{\theta(c)}{\Delta}. \quad (7.31)$$

where

$$\begin{aligned} \theta(c) = & -e^{-ik_{16}} + \frac{\sqrt{\alpha\omega}}{4} \Psi_c(\omega) \left(- \left(e^{-ik_{13}} - e^{-ik_{14}} \right) \left(1 - e^{-ik_{12}} + e^{-ik_{15}} - e^{-ik_{16}} \right) + \right. \\ & \left. \left(e^{-ik_{14}} - e^{-ik_{13}} \right) \left(1 - e^{-ik_{16}} \right) + ie^{-ik_{16}} \left(2e^{-k_{14}} - 2e^{-k_{13}} + e^{-k_{23}} + e^{k_{23}} - e^{-k_{24}} - e^{k_{24}} \right) \right) \end{aligned} \quad (7.32)$$

Consequently the control objective is:

$$\theta(c) = 0. \quad (7.33)$$

Applying this criterion to equation (7.32), the final simplified expression of the controller, the LWAC, in the frequency domain is as follows:

$$\begin{aligned} \frac{1}{c(\omega)} = & \frac{\sqrt{\alpha\omega}}{4} \Psi \left(\left(e^{-ik_{14}} - e^{-ik_{13}} \right) \left(2 + e^{-ik_{15}} - e^{-ik_{12}} \right) e^{ik_{16}} + \right. \\ & \left. i \left(2e^{-k_{14}} - 2e^{-k_{13}} + e^{-k_{23}} + e^{k_{23}} - e^{-k_{24}} - e^{k_{24}} \right) \right) \end{aligned} \quad (7.34)$$

7.1.2 Collocated LWAC (Unequal Length Transducers)

We now examine the case of a model with transducers of unequal length, as illustrated in Figure 6.5. In order to use the equations derived in the previous section, we assume this as a special case of non-collocation but with piezosensors extended to reach each other, i.e. $x_2=x_5$. Each of the two piezosensors is the same as the other, the one used for strain rate

measurement (#2) and the other for the strain measurement (#1). The voltage command in this case is:

$$V_a(\omega) = c(\omega) I_{s2}(\omega). \quad (7.35)$$

w_I and w_O will be the same as in the previous case:

$$w_I = \begin{bmatrix} a_1(1) \\ a_2(1) \\ b_1(6) \\ b_2(6) \end{bmatrix}; \quad w_O = \begin{bmatrix} b_1(1) \\ b_2(1) \\ a_1(6) \\ a_2(6) \end{bmatrix}. \quad (7.36)$$

The geometric simplification assumption is:

$$x_a = x_{s1} = x_{s2}. \quad (7.37)$$

So that the elements of the closed loop transition matrix can be written as:

$$T_{CL}(1,1) = T_{CL}(3,3) = -\frac{\sqrt{\alpha\omega}\Psi}{4\Delta} c(\omega) \left(e^{-ik_{13}} - e^{-ik_{14}} \right) \left(1 - e^{-ik_{16}} \right) \quad (7.38)$$

$$T_{CL}(1,3) = T_{CL}(3,1) =$$

$$\begin{aligned} & \frac{1}{\Delta} \left(-e^{-ik_{16}} + \frac{\sqrt{\alpha\omega}\Psi}{4} c(\omega) \left(\left(e^{-ik_{14}} - e^{-ik_{13}} \right) \left(1 + e^{-ik_{16}} \right) + \right. \right. \\ & \left. \left. i e^{-ik_{16}} \left(2e^{-k_{14}} - 2e^{-k_{13}} \right) \right) \right) \end{aligned} \quad (7.39)$$

The control criterion is that of the previous section, i.e. $\theta(c) = 0$, which results in the following LWAC for this case:

$$\frac{1}{c(\omega)} = \frac{\sqrt{\alpha\omega}\Psi}{2} \left(\left(e^{-ik_{14}} - e^{-ik_{13}} \right) e^{ik_{16}} + i \left(e^{-k_{14}} - e^{-k_{13}} \right) \right). \quad (7.40)$$

7.1.3 Collocated LWAC (Equal Length Transducers)

Figure 6.6 illustrates the smart structure model with collocated transducers of equal

length, located at the same position:

$$l_a = l_{s1} = l_{s2} \quad (7.41)$$

$$x_a = x_{s1} = x_{s2} \quad (7.42)$$

In this case the elements of the closed loop transition matrix are simplified as:

$$T_{CL}(1, 1) = T_{CL}(3, 3) = -\frac{\sqrt{\alpha\omega}\Psi}{4\Delta}c(\omega)\left(1 - e^{-ik_{16}}\right)^2 \quad (7.43)$$

$$\begin{aligned} T_{CL}(1, 3) = T_{CL}(3, 1) = & \\ & \frac{1}{\Delta}\left(-e^{-ik_{16}} + \frac{\sqrt{\alpha\omega}\Psi}{4}c(\omega)\left(\left(e^{-ik_{14}} - 1\right)\left(1 - e^{-ik_{16}}\right) + \right. \right. \\ & \left. \left. ie^{-ik_{16}}\left(2e^{-k_{16}} - 2\right)\right)\right) \end{aligned} \quad (7.44)$$

and the control criterion of section 7.1.1 leads to the collocated LWAC for this case:

$$\frac{1}{c(\omega)} = \frac{\sqrt{\alpha\omega}\Psi}{2}\left(\left(1 - e^{ik_{16}}\right) + i\left(e^{-k_{16}} - 1\right)\right). \quad (7.45)$$

7.2 Time Domain LWAC

The LWAC's presented in the previous section (equations (7.34), (7.40), and (7.45)) are difficult to realize in the time domain. These equations represent the exact solutions of the wave absorbing controllers in the frequency domain. In general the wave absorbing method produces controllers which are difficult to realize in practice due, among other things, to the necessity for “half” integrations” and “half” differentiations (Miller and Hall, 1991, von Flotow and Schäfer, 1986, and Fujii and Ohtsuka 1992). In order to more easily implement the LWAC's in the time domain, we use approximations of the controllers in the time domain. In the following sections some techniques to obtain simple time domain versions of the LWAC are introduced.

7.2.1 Analytical Parameter Based LWAC (APB LWAC)

In this section we show that in the case of collocated piezotransducers with equal lengths it is possible to derive a simple analytical approximation of the LWAC, i.e. $c(\omega)$ given by equation (7.45), in the time domain. For low frequencies and for short piezoelements, k_{16} is much less than 1, so that we can ignore the higher power terms of k_{16} which appear in the exponential expansion. This is the case of interest in most flexural vibration control systems. For example in the case of $l_a = l_{s1} = l_{s2} = 76.2 \times 10^{-3} m$, for the model of Figure 6.6 ($\alpha = 1.1904 m^2/sec$) and around the first mode frequency ($\omega_1 = 4.3085 rad/sec$):

$$k_{16} = 0.1450.$$

Ignoring the higher orders of k_{16} , i.e. powers greater than two, the exponential expansion of the local wave absorbing controller, defined by equation (7.45), can be simplified to the following:

$$c(\omega) = \frac{2}{\Psi \sqrt{\alpha}} \frac{1}{\sqrt{\omega}} \left(\frac{\beta_{16}^2 \omega / 2 + i \left(2\beta_{16} \sqrt{\omega} - \beta_{16}^2 \omega / 2 \right)}{4\beta_{16}^2 \omega} \right). \quad (7.46)$$

where

$$\beta_{16} = \frac{k_{16}}{\sqrt{\omega}} = \frac{x_6 - x_1}{\sqrt{\alpha}}. \quad (7.47)$$

This is an expression for the second order of expansion of $c(\omega)$, and can be rewritten as follows:

$$c(\omega) = \left(-\frac{1}{\Psi \sqrt{\alpha} \beta_{16}} \frac{1}{(i\omega)} \right) + \left(\frac{0.25}{\Psi \sqrt{\alpha}} \frac{1}{\sqrt{\omega}} (1 - i) \right). \quad (7.48)$$

The first component is an integration operator ($\frac{1}{(i\omega)}$), which can be simplified as

follows by substituting β_{16} from equation (7.47):

$$\left(-\frac{1}{\Psi l_a} \right) \frac{1}{(i\omega)} . \quad (7.49)$$

where $l_a = x_6 - x_1$ is the piezoactuator length. On substituting α from equation (6.4) the second part, which is a complex variable, can be rewritten as

$$\frac{1}{\Psi} \frac{0.25}{\sqrt{\sqrt{E_b I_b / \rho_b}}} \frac{1}{\sqrt{\omega}} (1 - i) . \quad (7.50)$$

To simplify the realization in the time domain we ignore the imaginary part of the second component (equation (7.50)). This simplification is examined in the next section (section 7.3.3) in which the Bode plots show only a little error. The LWAC matches quite well with its approximation, particularly at low frequencies. In addition, the frequency responses of the closed loop system with the LWAC and this approximation are very close.

Now, if we are interested in vibration control around the first vibrational mode, which is true in many active vibrational control tasks, the second component (equation (7.50)) can be written as

$$\frac{1}{\Psi} \frac{0.25}{\sqrt{\sqrt{E_b I_b / \rho_b}}} \frac{1}{\sqrt{\omega_1}} . \quad (7.51)$$

Finally, from equations (7.48), (7.49), and (7.51) the approximation of the LWAC becomes a simple PI controller, the gains of which can be easily obtained from:

$$\text{Proportional gain} \equiv \frac{0.25}{\Psi \sqrt{\omega_1} \sqrt{E_b I_b / \rho_b}} \quad \text{and} \quad \text{Integral gain} \equiv -\frac{1}{\Psi l_a} . \quad (7.52)$$

Since Ψ depends on only the parameters of the composite smart structure, the control gains can simply be calculated from knowledge of system physical parameters. Hence,

this provides an Analytical Parameter Based LWAC (APB LWAC) design, which can be easily implemented in practice. The controller gains can be obtained from a simple formula (equation (7.52)), which unlike the usual dynamic model based controllers such as LQR, requires no simulation process to obtain the control gains. This is a substantial advantage over conventional controllers. In the following section (section 7.3.3) a comparison between the original LWAC and the corresponding approximate PI version (APB LWAC) is performed. In addition, the performance of the smart structure with the original and approximate controller are compared and discussed.

7.2.2 Curve Fit Model of LWAC (CFM LWAC)

Another approach for approximation of the LWAC is particularly suitable for the cases for which analytical approximations are very difficult to obtain, e.g. non-collocated LWAC. From the Bode plots of the LWAC, it is possible to find a suitable PI realization around the low frequency range by curve matching of the original magnitude and phase plots. This enables us to implement the LWAC in the time domain. In the next section comparisons between the original LWAC and CFM LWAC are made, and their performances for sample structures are compared.

7.3 LWAC Performance

In this section we study the performance of LWAC in both the frequency and time domains. For the smart structure described in section 7.1

$$\Psi = -2.22762 \times 10^{-9} \text{ N/V}^2, \quad \alpha = 1.1904 \text{ m}^2/\text{sec}, \quad .$$

and the first two natural frequencies are:

$$\omega_1 = 4.3085 \text{ rad/sec}, \quad \omega_2 = 30.1398 \text{ rad/sec}, \quad .$$

Approximate LWAC's in the time domain are applied to active vibration control of the flexible beam with an initial tip displacement of 4 mm. The three configurations for

which the LWAC was derived in the previous section are considered here. The first of these is:

7.3.1 Non-collocation

The physical specifications of the smart structure are given in section 7.1, and the following configuration of the piezosensors is considered for this case:

$$l_{s1} = l_{s2} = 200 \text{ mm}, \quad x_{s1} = 200 \text{ mm}, \quad x_{s2} = 800 \text{ mm}.$$

The approximate model for the LWAC obtained from curve fitting the Bode diagrams (CFM LWAC) is:

$$PI \equiv -\frac{7.0}{s} + 0.35.$$

Figure 7.3 shows the bode plot of the controller. It can be seen that in the low frequency range the controller behaves like a PI controller, the approximate, and original controller behavior match quite well. The performance of the closed loop control system is compared with the open loop in Figure 7.4. The LWAC suppresses the vibration effectively, particularly in the low frequency range. The first mode peak is decreased significantly in magnitude by 9 db. Thus the proposed control criterion, which minimizes the ratio of the aggregate output and input wave functions of the sensing area, does indeed also suppress the overall flexural vibration of the structure. This means that although the LWAC was originally designed for acoustic and noise control, i.e. high frequencies, (Pines and von Flotow, 1990), the modified LWAC presented in this study can be used for active control of low frequency vibrations of flexible structures. The PI model of LWAC (CFM LWAC) performs very similarly to the original LWAC, and this simple approximation can be quite effective. The time domain response of the flexible beam is shown in Figure 7.5 with an initial tip displacement of 4 mm with and without control. Comparing the open loop and closed loop responses shows that the controller does indeed significantly damp the vibration.

Figure 7.3 Non-collocated LWAC

Figure 7.4 Frequency response of the smart structure with the non-collocated LWAC

Figure 7.5 Tip displacement of the smart structure with the non-collocated LWAC

7.3.2 Collocation (Unequal Length Transducers)

In this case a long piezosensor is used to sense the traveling waves:

$$l_s = 800 \text{ mm}, \quad x_s = 500 \text{ mm}.$$

The curve fit approximated model of the LWAC (CFM LWAC) is:

$$PI \equiv -\frac{9.0}{s} + 1.0.$$

The magnitude and phase of the controller with respect to the frequency are shown in Figure 7.6. It is seen that for low frequencies, the CFM LWAC is close to the original LWAC. The dynamic response of the control system is compared with the open loop response in the frequency domain in Figure 7.7. The closed loop system performance with the original and approximated controller are very close to each other in the low frequency range. The LWAC provides large damping to the dynamic system. The closed loop response of the smart structure to the initial tip displacement is shown and compared with that of the open loop in Figure 7.9. The controller damps the flexural vibration quickly.

Figure 7.6 Collocated LWAC: unequal length transducers

Figure 7.7 Frequency response of the smart structure with the collocated LWAC: unequal length transducers

Figure 7.8 Tip displacement of the smart structure with the collocated LWAC: unequal length transducers

7.3.3 Collocation (Equal Length Transducers)

A piezosensor with the same length as the actuator length: $l_s = l_a = 150 \text{ mm}$, $x_s = 500 \text{ mm}$, is used for this case of collocation. The analytical parameter based LWAC (APB LWAC), which provides a PI approximation of the LWAC, designed in section 7.2.1 (equation (7.52)) is:

$$PI \equiv -\frac{6.6667}{s} + 0.1104.$$

Figure 7.9 shows the bode plot of the controller. The approximate LWAC (APB LWAC) performance is very close to that of the original LWAC particularly for the frequencies around the first mode. Figure 7.10 shows that the controller shifts the peaks to the left and increases the damping of the system around the second mode, with little attenuation of the first modal amplitude (1 db). The time domain response, Figure 7.11, shows that the controller does not suppress the flexural vibration of the beam effectively. This is due to the short sensing area, i.e. $l_s=150 \text{ mm}$. As mentioned earlier, in order to have an effective wave controller, the sensing area should be long enough to provide good observation and control of waves traveling along the beam. This is also discussed in the next section.

Figure 7.9 Collocated LWAC: equal length transducers

Figure 7.10 Frequency response of the smart structure with the collocated LWAC: equal length transducers

Figure 7.11 Tip displacement of the smart structure with the collocated LWAC: equal length transducers

7.4 Sensing Length Effect on the LWAC

When the length of the sensing region is long, a better observation of the flexural waves traveling along the beam is possible. The performance of the controlled smart structure with different lengths of the piezosensors studied in the previous section shows that the LWAC becomes much more effective in active vibration damping as the length of the sensing area increases. For instance in the case of covering 800 mm of the beam length, the controller provided the highest damping, whereas the lowest damping was for the collocation case with piezosensor of length of 150 mm. Although the performance of the LWAC increases by increasing the length of the piezosensors, there are always some limitation such as geometric, economic, etc. to the use of large sensing length. Improvement over LWAC performance by introduction of a new WAC algorithm is studied in the next chapter.

7.5 LWAC Performance for a Non-Ideal Case

In LWAC design the effects of the boundary conditions are ignored since it is assumed that the structure is of infinite length or the region of interest is very far from boundaries. Thus we ignore the effects of the boundaries and wave reflections on the LWAC and consequently deal with local dynamic behavior of the system. In reality in most cases, these assumptions are not valid, i.e. the structure is not of infinite length or the interest region is not very far from boundaries (non-ideal cases). In this section we perform a case study to test the performance of the LWAC for such non-ideal cases. We consider the physical model described in section 4.1.1, which represents a structure of moderate length, $L=500$ mm, with non-collocated sensors and actuators close to the boundaries, i.e. $x_a=89$ mm, $x_{s1}=35.4$ mm, and $x_{s2}=142.6$ mm. Figure 7.12 illustrates the performance of the LWAC designed for this smart structure as well as that of its curve fit approximate.

i.e. CFM LWAC: $PI \equiv -\frac{14}{s} + 0.02$.

The performance of the non-collocated LWAC in the closed loop system is compared with the open loop dynamic system in Figure 7.13. As is seen the original LWAC and the approximate one provide almost the same performance. The controller does not suppress the vibration, and it has diminished the first modal peak by only 1.5 db without any significant change in the second modal amplitude. This result was expected since the local controller (LWAC) is not suitable for a short structure with transducers close to the boundaries due to the effects of the global factors of the dynamic system, i.e. boundary conditions, structural length, etc. Figure 7.14 shows the time history of the dynamic response of the open loop and closed loop systems with respect to an initial tip displacement of 10 mm. It is seen that the controller only changes the frequency of the response with a little attenuation of the vibrational amplitude.

These results raise a question: is there any way to extend the LWAC to the non-ideal cases such as structures with transducers close to the boundaries and short sensing area? In the next chapter we address this question.

Figure 7.12 Non-collocated LWAC: non-ideal case

Figure 7.13 Frequency response of the non-ideal case with the non-collocated LWAC

Figure 7.14 Tip displacement of the non-ideal case with the non-collocated LWAC

Chapter 8

Global Wave Absorbing Controller (GWAC)

We begin with a note on terminology. “*Global*” here is taken to mean “*overall*”, with respect to the structure, i.e. total length, positions of transducers with respect to the boundaries. This is opposed to the term “*Local*” of the previous chapter, which made no reference to boundaries, and was concerned only with the immediate neighborhood of the transducers.

In the local wave absorbing control (LWAC) design (chapter 7) the boundary conditions of the structure and the positions of the sensors and actuators with respect to the boundaries, i.e. x_s and x_a , are not considered, and only their relative (local) positions are used. In other words this is viewing the response of a flexible structure as a local activity in terms of traveling waves. This makes the LWAC very restrictive in its applications, as it is a local controller. The LWAC is also derived from the point view of minimizing the reflected energy or waves for an ideal infinite dimensional structure with infinite band width, but because of practical limitations this absorption cannot be perfect.

The controller introduced in the previous chapter suffers from poor performance when the structure is small in size or the transducers are very close to the boundaries. In this chapter a new method of WAC design is introduced to account for “*global parameters*” of the system, such as the total length of the structure and transducers positions with respect to the boundaries, and to generalize the WAC to a global controller with better performance for actual structures. In the “*Global WAC*” (GWAC) both the positions of the transducers with respect to the boundaries and the total length of the structure are considered as well as the local parameters of the dynamic system. Unlike the local

version, this method can be effectively used for structures with short sensing area and transducers close to the boundaries.

8.1 Control Design

The control criterion introduced in chapter 7 tends to cause the ratio of aggregate output to aggregate input wave functions (equations (7.14), (7.15), and (7.30)) of the sensing area to approach zero, i.e. $\theta(c) = 0$ (equation (7.33)) for the ideal case, in which structures are very large with transducers very far from boundaries. In this section a new control criterion is introduced to extend the LWAC to the less ideal case. The geometric assumptions for the piezotransducer configurations are the same as equations (7.24) and (7.25), i.e. $l_{s1} = l_{s2}$ and $x_a = \frac{1}{2}(x_{s1} + x_{s2})$. In other words the piezosensor lengths are equal and the position of the midpoint of piezoactuators, x_a , and the position of the centre of total sensing area, $\frac{1}{2}(x_{s1} + x_{s2})$, are identical (Figure 7.2).

When structure is very long, LWAC works well, and LWAC performance improves when the transducers are far from boundaries. Based on these ideas, the following control criterion is proposed rather than equation (7.33), to include structure length and position of the mid point of the transducers, i.e. global parameters, in the GWAC design:

$$\theta(c) = \theta_1 + i\theta_1, \quad (8.1)$$

where

$$\theta_1 = 1 - \frac{x_a}{L}. \quad (8.2)$$

L is the length of the beam and x_a is the position of the centre of the piezoactuators and total sensing area along the beam (equation (7.25)).

We consider the following cases as limiting case checks of the proposed criterion:

- Very long structure:

$[(L \rightarrow \infty) \Rightarrow (\theta(c) \rightarrow 0)]:$ the same as LWAC].

- Piezotransducers far from boundaries:

$[(x_a \rightarrow L) \Rightarrow (\theta(c) \rightarrow 0)]:$ the same as LWAC].

This indicates that GWAC converges to LWAC in certain limiting cases. In other cases,

$\theta(c)$ is a non-zero complex number, which leads to a controller, which is in general non-optimal but is nevertheless capable of addressing the non-ideal cases of WAC.

Defining $\chi(c)$ as:

$$\chi(c) = \theta(c) - \theta_1(1+i). \quad (8.3)$$

the GWAC control criterion defined by equation (8.1) will be equivalent to:

$$\chi(c) = 0. \quad (8.4)$$

Thus the global wave absorbing controller, $c(\omega)$ can be obtained by solving the above equation. Using equations (7.30)-(7.32), $\chi(c)$ can be written as:

$$\chi(c) = [\beta_1 + R_c R_\theta - I_c I_\theta] + i[\beta_2 + R_c I_\theta + I_c R_\theta], \quad (8.5)$$

where

$$\beta_1 = \cos(k_{16}) - \theta_1,$$

$$\beta_2 = -\sin(k_{16}) - \theta_1,$$

$$R_c(\omega) = \text{Real}(c(\omega)),$$

$$I_c(\omega) = \text{Imag}(c(\omega)),$$

$$R_\theta = \text{Real}\left(\frac{1}{c(\omega)}\left(\theta(c) - e^{-ik_{16}}\right)\right),$$

$$I_\theta = \text{Imag}\left(\frac{1}{c(\omega)}\left(\theta(c) - e^{-ik_{16}}\right)\right).$$

Equation (8.5), is valid for all collocated and non-collocated cases with the appropriate $\theta(c)$ for each case.

Solving $\chi(c) = 0$ produces the final expression for $c(\omega)$:

$$c(\omega) = R_c(\omega) + i I_c(\omega), \quad (8.6)$$

where

$$R_c(\omega) = -\frac{R_\theta\beta_1 + I_\theta\beta_2}{R_\theta^2 + I_\theta^2}, \quad (8.7)$$

$$I_c(\omega) = -\frac{R_\theta\beta_2 - I_\theta\beta_1}{R_\theta^2 + I_\theta^2}. \quad (8.8)$$

β_1 , β_1 , I_θ , and R_θ depend on k_{16} , α , and ω . Equations (8.7) and (8.8) provide $c(\omega)$ as a complex function which can be implemented in the frequency domain or approximated as was done for the LWAC.

8.2 Time Domain GWAC

In order to implement the GWAC in practice, we use a simple approximation around the desired frequency, ω_d . Equation (8.6) can be written as:

$$\begin{aligned} c(\omega) &= R_c(\omega) + i \frac{I_c(\omega)(i\omega)}{(i\omega)} \\ &= R_c(\omega) + (-I_c(\omega) \cdot \omega) \frac{1}{(i\omega)} \end{aligned} \quad (8.9)$$

where $(1/i\omega)$ represents the integration operator. Using the same idea of a PI approximation as employed for LWAC in chapter 7, a PI model of GWAC around the desired frequency, ω_d can be used with the following gains:

$$\text{Proportional gain} \equiv R_c(\omega_d),$$

$$\text{Integral gain} \equiv -I_c(\omega_d) \cdot \omega_d.$$

For instance, around the first modal frequency, ω_1 , which is what our primary interest would often be, the gains of approximate GWAC are:

$$P \text{ gain} \equiv R_c(\omega_1), \text{ and } I \text{ gain} \equiv -I_c(\omega_1) \cdot \omega_1. \quad (8.10)$$

$R_c(\omega_1)$ and $I_c(\omega_1)$ depend on k_{16} (l_a , E_b , I_b , and ρ_b), Ψ , and ω_1 . Thus one can obtain the controller gains from parameters of the system. Equation (8.10) provides a simple and straight forward control design, unlike the conventional control algorithms such as LQR which require a dynamic model and simulations to obtain the controller gains. In the next section we examine the performance improvement of GWAC over LWAC and also compare GWAC and LQR performances.

8.3 GWAC Performance

In this section, the results of active control of the smart structure under GWAC, LWAC, and LQR are demonstrated and compared. We consider two of the worst cases, in which the LWAC does not provide good performance, to compare the performance of the global wave absorbing controller (GWAC).

8.3.1 Collocated Transducers

The physical model used in section 7.3.3 is considered to study the smart structure with collocated transducers in this section. As mentioned earlier the LWAC performs poorly for this case. A GWAC is designed and implemented for this case to compare with the LWAC. Using equation (8.2) for this configuration of the transducers, i.e. $x_d=x_s=500$ mm; $\theta_1=0.5$, and applying the control criterion, equation (8.4), a GWAC is obtained in the frequency domain. A PI approximation of GWAC around the first natural frequency of the dynamic system is derived from equation (8.10):

$$PI \equiv -\frac{4.3804}{s} + 1.1139.$$

Figure 8.1 shows the frequency response controller and its PI approximation around the first mode, i.e. $\omega_1=4.3085$ rad/sec. The controller is compatible with the original one around the first modal frequency as desired. The closed loop performances of the system with the GWAC and its PI approximation are compared with the open loop system in the frequency domain in Figure 8.2. It can be seen that the GWAC presented here introduces substantial damping into the system. The performance of the original and the approximate GWAC are very close, particularly in the low frequency range. Figure 8.3 compares the frequency responses of the dynamic system with the GWAC and LWAC. The improvement of wave absorbing performance for active damping of the structure using the GWAC is evident.

In order to study the performance of the controller in the time domain, an initial tip displacement of 4 mm is applied to excite the beam. The closed loop responses under PI

approximate GWAC (PI GWAC), analytical parameter based approximate LWAC (APB LWAC: $PI \equiv -\frac{6.6667}{s} + 0.1104$, from section 7.3.3), and LQR are demonstrated in Figure 8.4. With almost the same control voltages (Figure 8.5), the LWAC shows much poorer damping than does the GWAC. The GWAC performance is also compared with LQR in Figure 8.4. For a meaningful comparison between the GWAC and LQR, equal maximum control voltages are applied to the smart structure in both cases (Figure 8.5). Despite the short sensing length, GWAC provides excellent active control, almost the same as that of the LQR controller.

It should be noted that this performance is obtained by a parameter based design (equation (8.10)) rather than more complicated methods (such as LQR), for which good dynamic models are needed, so that the controller, while providing performance comparable to that provided by the LQR, is much simpler to design, the control gains being obtained from a simple formula.

8.3.2 Non-Ideal Configuration of Smart Structure

As another example in which LWAC performs poorly, we choose the smart structure with non-ideal configuration described in section 7.5. Figure 4.1 shows the physical model of the system. As mentioned before, this is the model of the real experimental structure, in which the beam is not very long, $L=500$ mm, the transducers are non-collocated and close to the boundary ($x_a=89$ mm, $x_{s1}=35.4$ mm, and $x_{s2}=142.6$ mm), and the total sensing length is not long: $l=134.2$ mm. In this case the challenge to the wave controller is increased by reducing the distance of the transducers from the boundary as well as reducing the length of structure, thus rendering one of the key assumptions of the wave absorbing method, i.e. infinite structural length, naturally invalid.

Figure 8.6 shows the bode plot GWAC and its PI approximation around the dominant frequency (first modal frequency) where the approximation is given by:

$$PI \equiv -\frac{9.8439}{s} + 2.6848.$$

The PI model matches the original GWAC in the neighborhood of the first modal

frequency. The frequency responses of the closed loop system under the GWAC and its approximation are compared with the open loop response in Figure 8.7. The GWAC does indeed introduce substantial active damping to the dynamic system. The closed loop response of the smart structure with the PI model of GWAC is very close to that of the original. Figure 8.8 compares the frequency responses of the system with the GWAC and LWAC which demonstrates greater performance of the GWAC over the LWAC.

An initial tip displacement of 10 mm is applied to the beam. The time response of approximate GWAC (PI GWAC) is illustrated in Figure 8.9. Comparing this with the performance of curve fit approximated model of LWAC (CFM LWAC: $PI \equiv -\frac{14}{s} + 0.02$, obtained in section 7.5) it is clear that the GWAC suppresses the vibration very effectively in 2 seconds whereas LWAC almost does not affect the vibration. GWAC performance is also compared with the LQR in Figure 8.9. It can be seen that for the same maximum voltage (Figure 8.10) the two controllers perform similarly. It should be noted again that this performance is obtained by a parameter based design (equation (8.10)) rather than the more complicated LQR design, and knowledge of only the parameters of the system determines the control gains by a simple calculation.

In summary, this novel global wave absorbing controller (GWAC) demonstrates improvement over previous approaches, either superior performance (over LWAC) or simpler design with comparable performance (as with LQR).

Figure 8.1 Collocated GWAC: equal length transducers

Figure 8.2 Frequency response of the smart structure with the collocated GWAC: equal length transducers

Figure 8.3 Frequency response of the smart structure with the collocated GWAC and LWAC: equal length transducers

Figure 8.4 Tip displacement of the smart structure with the collocated GWAC, LWAC, and LQR: equal length transducers

Figure 8.5 Control voltage of the smart structure with the collocated GWAC, LWAC, and LQR: equal length transducers

Figure 8.6 Non-collocated GWAC: non-ideal case

Figure 8.7 Frequency response of the non-ideal case with the non-collocated GWAC

Figure 8.8 Frequency response of the non-ideal case with the non-collocated GWAC and LWAC

Figure 8.9 Tip displacement of the non-ideal case with the non-collocated GWAC, LWAC, and LQR

Figure 8.10 Control voltage of the non-ideal case: non-collocated GWAC, LWAC, and LQR

8.4 Experimental Results

In this section the results of an experimental study to test GWAC performance are presented. The smart structure which we termed the “*non-ideal*” configuration (section 8.3.2) is used as the experimental structure. There were four sensors of which two are used for strain (voltage) and the other two for the strain rate (current) measurements. The PI GWAC introduced in section 8.3.2, $PI \equiv -\frac{9.8439}{s} + 2.6848$, was applied to the structure while it was initially excited by a 10 mm tip displacement.

Figure 8.11 shows the experimental open loop and closed loop output current of piezosensor #3. The controller suppressed the vibration very quickly, i.e. in 2 seconds, whereas in the open loop 30 seconds were required for the vibration to damp to the same level. This is an increase of an order of magnitude in damping. The experimental closed loop response is also compared with that of the simulation with good agreement between them. The same comparison is done for the output current of the second sensor, sensor #4, which again indicates the success of the GWAC, and good agreement between simulation and experimental results (Figure 8.12). The summation of the currents of these piezosensors was used for the proportional term (P) in the control loop. The summation of voltages of the sensors #1 and #2 was used for the integral term (I) of the controller, since the output voltage of the piezosensor is proportional to the time integral of its current. Thus there is no need to use a digital integrator in the control loop. Figures 8.13 and 8.14 show the open loop and closed loop output voltages of piezosensors #2 and #4 respectively. The results illustrate large active damping by the controller, and good agreement between simulation and experiment. The control voltage applied to the actuator is shown in Figure 8.15. The simulation and experimental control voltages are very close to each other, and the control voltage is in the acceptable range: $|V_{\max}| < 120$ V.

We can conclude that the global wave absorbing controller (GWAC) introduced here is efficient for active vibration suppression, and improves the performance of the control system over the conventional local wave absorbing control system (LWAC). The controller is parameter based, easy to design and implement in practice. Using piezosensors as strain and strain rate sensors, integration or differentiation operators in the

control loop are not needed. The experimental results for a non-ideal case of smart structure showed the controller works very well and its performance agrees with simulation.

Figure 8.11 Output current of piezosensor #3

Figure 8.12 Output current of piezosensor #4

Figure 8.13 Output voltage of piezosensor #1

Figure 8.14 Output voltage of piezosensor #2

Figure 8.15 Control voltage

Chapter 9

Conclusions

Active vibration control of flexible structures with piezoelements was studied in this thesis. The main goal of this thesis was to introduce a simple and efficient active control system which could be easily designed and implemented for active vibration suppression of flexible structures. These objective have been successfully achieved through this research, the accomplishment of which is briefly summarized in the following:

Distributed piezoelectric sensors and actuators were chosen as *smart transducers* for the active control system, because of their unique features such as high bandwidth, low cost, no significant passive effects, ease of bonding onto or embedding within flexible structures, etc. (see chapter 2). A *smart structure* was developed by integrating a flexible beam with bonded piezoelements. Some criteria regarding the proper choice of piezosensors and piezoactuators in addition to some handling and bonding techniques were presented. The thesis is in two major parts in which two different dynamic models were used. In the first, the modal method was used to obtain the dynamic equations of the smart structure. By using the energy method a damped modal model of a flexible beam was obtained and two dynamic models of the piezoactuators with a linear strain distribution assumption were developed. The first represented the modal passive and active interactions of the piezoactuators, while the second introduced a concentrated moment model of piezoactuator which was later used in the wave modeling of the smart structure. Next, the equations of the output voltage and currents of the piezosensors, as strain and strain rate sensors respectively, with a linear strain distribution assumption, were derived. Finally incorporating the passive and active effects of the piezoelectric

sensors and actuators, a state space model of the integrated smart structure was introduced, which served as a suitable platform for applying active controllers.

Using a state space model of the integrated smart structure, a linear quadratic optimal regulator (*LQR*) was implemented. Simulation and experimental results showed significant active damping induced into the system, and the vibration was damped quickly. Since piezoelements are very sensitive to the noises and disturbances, a linear quadratic Gaussian (*LQG*) compensator was designed which suppressed the vibrations of the flexible structure effectively in the presence of noises and disturbances. A comparison of *LQR* and *LQG* showed that the *LQG* rejected the noise effectively with a very smooth performance. To be able to observe and control certain individual dynamic modes, a novel *active modal control system* using only rectangular piezoelements was proposed and implemented, which did not suffer from the practical problems which limit the use of shaped modal piezoelements. The results showed that rectangular piezoelements could be used as efficient transducers for controlling individual vibrational modes of flexible structures.

In order to find the *optimum shape and location of the piezoactuators* in the active control system, three optimization criteria were studied and compared. Simultaneous optimization of shape and placement indicated that the best configuration of piezoactuators in active control systems was the shortest and widest.

Although the controllers designed in the modal domain were effective in vibration suppression, they were not easy to design since they need a good dynamic model and their gains were obtained through a simulation program with proper choices of optimal weight matrices. In a search for a simple and efficient control system, in the second part of the thesis a distributed wave model of the smart structure was studied. A *novel dynamic wave model* of the flexible beam with distributed piezoactuators, strain and strain rate piezosensors was developed in the frequency domain. Employing the local parameters of the system such as piezoelement lengths, a *local wave absorbing controller* (*LWAC*) based on the maximum absorption of the traveling waves within the sensing area was designed. Since mode truncation was not used in derivation of the

LWAC, it did not suffer from spillover problems. In addition, use of strain rate as feedback signal eliminated the need for differentiator terms in the control loop, a substantial advantage over the common wave controllers using only strain sensors. Although the LWAC performed satisfactorily in the frequency domain, it was in a complex form difficult to realize in practice. In order to address these realization issues and also to study the behavior of the controller and closed loop system in the time domain, some approximation techniques were used to express this frequency domain controller in the time domain, which finally resulted in a simple parametric form, in which the control gains could be obtained easily from a formula. This controller effectively suppressed the flexural vibration, particularly when the sensing area was long.

To improve the performance of the wave controller a novel wave absorbing control criterion was proposed, which used the *global parameters* of the system, such as the length of the structure and the positions of the piezoelements with respect to the boundaries: *global wave absorbing controller* (GWAC). Results showed that this controller was not only simple in design and implementation, but also was very effective in vibration attenuation. The control gains were obtained from a parametric formula which depended on only the parameters of system such as E_b , I_b , ρ_b , and natural frequencies rather than from a simulation program, while the performance of the active control system was similar to that of optimal controllers. This is a unique advantage over more common dynamic model based controllers such as LQR. Experimental results matched quite well with theory. This concluded the objective of the thesis in finding *a simple and efficient controller for active vibration suppression of the flexible structures* of the type discussed here.

9.1 Research Contributions

The contributions and achievements through the course of this research are listed in the following list as major or minor.

9.1.1 Major Contributions

- Development of a new concentrated force and moment model of piezoactuators bonded onto a flexible beam.
- Implementation of LQG compensator for noise rejection in the active control of smart structures.
- Introduction of a novel active individual mode controller with only rectangular piezotransducers capable of controlling certain dynamic modes individually.
- Simultaneous shape and location optimization of piezoactuators for active control systems.
- Dynamic wave modeling of distributed strain and strain rate piezosensors.
- Development of a new wave absorbing control system with piezosensor output current feedback (strain rate feedback) obviating the need for differentiators with their attendant problems.
- A novel local wave absorbing control (LWAC) criterion based on the minimization of the ratio of aggregate output to aggregate input wave functions.
- Deriving time domain approximations of the local wave absorbing controllers.
- Introducing a new global wave absorbing control (GWAC) criterion to incorporate the global parameters of the dynamic system for better performance achievement.
- Obtaining an efficient and simple time domain global wave absorbing controller for vibration suppression, whose gains can be obtained from a

parametric formula.

- Development of an experimental test bed for active vibration control test, including sensor and actuator technology selection, piezoelement bonding and interfacing, data acquisition set up and programming, experimental control programs.
- Experimental implementation of the modal and wave controllers to verify the simulation results.

9.1.2 Minor Contributions

- Derivation of the passive and active effects of a thick piezoactuator with a linear strain distribution assumption.
- Derivation of the equations of the output voltages and currents of thick piezoelements to be used as strain and strain rate sensors respectively, with a linear strain distribution assumption.
- Proof of the dynamic equation decoupling of the integrated smart structure using modal piezoelements.
- Design and implementation of an LQR control system for vibration suppression using the state space feedback model.
- Employing three optimization criteria for optimal shape and placement of piezoactuators.
- Length optimization of the piezoactuators with fixed width and/or fixed area.
- Location optimization of a piezoactuator with fixed width and area.
- Introducing a novel wave model of the integrated smart structure.
- Deriving local wave absorbing controllers for different collocated and non-

collocated configurations.

- Studying the effect of the total wave sensing length on the performance of the wave controllers.
- Presentation of some experimental techniques in handling, bonding, and interfacing of piezotransducers.

9.2 Suggestions for Future Extension of this Research

The following studies would be appropriate for the continuation of this research to improve the active control systems and to extend them to more complex smart structures.

- Studying other smart sensors such as fiber optic sensors for better sensing and more integrated smart structures. This should be considered with respect to cost, bandwidth, etc. .
- Studying other smart actuators with higher actuation capability such as “shape memory piezoceramics”. This should also be considered with a trade off in cost, nonlinearity, bandwidth, etc. .
- Extending the modal and wave dynamic models and controllers for two and three dimensional flexible structures such as shells, plates, membranes.
- Studying the performance of the modal controllers for high frequency vibrations.
- Studying the performance of wave absorbing controllers for higher frequency vibrations.
- Introducing time domain representations of the wave controllers for high frequency domains.

- Improving the performance of the global wave absorbing controllers by incorporating the control effort into the control design criteria.
- Examining the spillover problem in the active control systems.

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Appendix A

Piezoelectric Materials

The Piezoelectric effect of certain natural crystals was discovered by the Curie brothers in 1880 (Cady, 1964). When a force is applied to a piezoelectric body, surface charges are induced by dielectric displacement and an electric field is built up. This is called the *direct piezoelectric effect*. On the other hand, if an electric field acts on a piezoelement, it becomes distorted. If this distortion is prevented then an elastic tension occurs. This is termed *indirect piezoelectric effect*. In this appendix the electromechanical model of piezoelectric materials is presented. Detailed derivation can be found in Berlincourt (1964) and Vaz (1991a).

A.1 Piezoelectric Constitutive Equation

Consider a piezoelectric block as shown in Figure A.1. The constitutive equation of the piezoelectric block can be expressed as following (Zelenka, 1986, and Tzou and Zhong, 1994):

$$S = S^E T + d' E \quad , \quad (A.1)$$

$$D = d T + \varepsilon^T E \quad , \quad (A.2)$$

where superscript ' denotes transpose of a matrix or a vector, and

$$S' = [\varepsilon_1 \ \varepsilon_2 \ \varepsilon_3 \ \gamma_{23} \ \gamma_{31} \ \gamma_{12}] \equiv \text{Strain vector},$$

$$T' = [\sigma_1 \ \sigma_2 \ \sigma_3 \ \tau_{23} \ \tau_{31} \ \tau_{12}] \equiv \text{Stress vector},$$

$E' = [E_1 \ E_2 \ E_3] \equiv$ Electric field intensity vector.

$D' = [D_1 \ D_2 \ D_3] \equiv$ Electric flux density vector.

$S^E \equiv$ Adiabatic elastic compliance constant matrix.

$d \equiv$ Adiabatic piezoelectric constant matrix.

$\epsilon^T \equiv$ Adiabatic dielectric permittivity constant matrix.

Now assume that the 1 axis is aligned in the direction in which the piezoelement is uniaxially stretched during its manufacturing process, and axis 3 has two properties: first, it is parallel to the direction of the piezoelement thickness and second, the positive direction points in the opposite direction of the electric field used in the manufacturing process to pole the piezoelement. Sessler (1981) showed that in this case the piezoelectric constant matrix, d is:

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix}. \quad (\text{A.3})$$

Figure A.1 Piezoelement configuration

A.2 Electromechanical Model of Piezoelement as Sensor/Actuator

In this section, a simple electromechanical model of a uniaxial rectangular piezoelement is introduced. Assume a piezoelement with length, L_p along 1 axis, width, W_p along 2 axis, and thickness, t_p along 3 axis (Figure A.1). The voltage between upper and lower electrodes is V_3 , and these electrodes have free charges $-Q_p$ and $+Q_p$. Force and elongation along the 1 axis are given by F_1 and ΔL_p respectively. Using this model, we can easily find the action of a piezoelement as a sensor or an actuator.

If a voltage V_3 is applied, the induced polarization can create a tensile force F_1 or an elongation ΔL_p . This phenomenon is termed actuation. On the other hand as a sensor, a tensile force F_1 causes an elongation ΔL_p , and induces a polarization in the piezoelement which causes an increase of free charge on the contacts which generates a voltage V_3 through the capacitive effect of the piezoelement. For simplicity we ignore the strains along the 2 and 3 axes since they are much smaller than those along the 1 axis. Thus, the piezoelectric vectors can be written as:

$$\begin{aligned} S' &= \begin{bmatrix} \frac{\Delta L_p}{L_p} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & T' &= \begin{bmatrix} \frac{F_1}{W_p t_p} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ E' &= \begin{bmatrix} 0 & 0 & -\frac{V_3}{t_p} \end{bmatrix}, & D' &= \begin{bmatrix} 0 & 0 & -\frac{Q_p}{W_p L_p} \end{bmatrix}. \end{aligned} \quad (A.4)$$

Substituting the above vectors in the equations (A.1) and (A.2) yields:

$$\Delta L_p = \left(\frac{S_{11}^E L_p}{W_p t_p} \right) F_1 + \left(-\frac{d_{31} L_p}{t_p} \right) V_3, \quad (A.5)$$

$$Q_p = \left(-\frac{d_{31} L_p}{t_p} \right) F_1 + \left(\frac{\epsilon_{31}^T L_p W_p}{t_p} \right) V_3, \quad (A.6)$$

where

$S_{11}^E = \frac{1}{E_p} \equiv$ Piezoelectric elastic compliance (inverse of the elastic modulus).

$d_{31} \equiv$ Piezoelectric strain constant.

$\epsilon_{31}^T = \epsilon_p \equiv$ Piezoelectric dielectric permittivity constant.

A.3 Shaped Piezoelements

A shaped piezoelement is a general configuration of a piezoelectric material in which either one electrode or both electrodes can be shaped (Figure A.2). Shaped

Figure A.2 Shaped piezoelements: *a*) one electrode is shaped; *b*) both electrodes are shaped.

piezoelements are studied in terms of variation of the poling directions of the electrodes. Therefore to express the properties of a shaped piezoelement, we need the following functions (Lee, 1989). The first is the domain function $F(x, y)$ which defines the region of a shaped piezoelement electrodes:

$$F(x, y) = \begin{cases} 1 & \text{Point } (x, y) \text{ is covered by both electrodes} \\ 0 & \text{Otherwise} \end{cases} \quad (\text{A.7})$$

The second one is the poling function $P(x, y)$ which defines the poling direction of the piezoelement in each point. The poling direction is specified with respect to the electric field intensity associated with a particular electrode polarity. The electric field intensity points from the positive to the negative electrode, therefore:

$$P(x, y) = \begin{cases} \pm 1 & \text{Poling direction toward the } \pm \text{ electrodes} \\ 0 & \text{point } (x, y) \text{ is not covered by electrodes} \end{cases} \quad (\text{A.8})$$

The shape function, $\psi(x)$, is defined as the average of the multiplication of the domain and poling functions over a cross section with a fixed coordinate:

$$\psi(x) = \frac{1}{W_p} \int_{W_{p2}}^{W_{p1}} P(x, y) F(x, y) dy, \quad (\text{A.9})$$

where W_p is the width of the piezoelement, W_{p1} and W_{p2} are the end points of the piezoelement in the Y direction shown in Figure A.2. The shape function, $\psi(x)$, is an important function which is needed to express the operation of piezoelectric sensors and actuators. The electrical capacitance of the shaped piezoelement is:

$$C_p = \frac{\epsilon_p A_p}{t_p}, \quad (\text{A.10})$$

where ϵ_p is the electrical permittivity of piezoelectric material, and A_p is the area of the piezoelement covered by its electrodes:

$$A_p = \iint F(x, y) dx dy. \quad (\text{A.11})$$

Appendix B

Smart Structure Element Properties

In this appendix, the specifications of the elements of the smart structure which are used in this study, are described. Since we have used different lengths and widths of these elements, we do not include these here; they are specified in the text of the thesis where needed.

B.1 Flexible Beam

A flexible Aluminium beam with following specifications is considered as the flexible structure in our study.

Table B.1 Aluminium flexible beam

Description	Cte	Value	Units
Volume density	ρ_{vb}	2710.0	Kg/m ³
Elastic modulus	E_b	72.0×10^9	Pa
Shear elastic modulus	G_b	27.0×10^9	Pa
Shear coefficient	κ_b	5/6	-
Thickness	t_b	0.8×10^{-3}	m

B.2 PZT Piezoceramic

Current piezoceramic materials are mostly oxide materials based on Lead oxide, Zirconate oxide, and Titanate oxide. The physical properties of the resulting Lead Zirconate Titanate (PZT) compound can be controlled by different composition ratios of the basic materials, grinding direction, calcination, shaping, and sintering. PZT piezoceramics (BM532), produced by BM Hi-Tech (Sensor Technology Ltd., 1996), are used in this study with the following properties.

Table B.2 PZT piezoceramic

Description	Cte	Value	Unit
Volume density	ρ_a	7350.0	Kg/m ³
Elastic modulus	E_a	71.4×10^9	Pa
Piezoelectric strain constant	d_{31a}	200.0×10^{-12}	m/V
Electric permittivity	ϵ_p	150.4×10^{-10}	F/m
Thickness	t_a	0.305×10^{-3}	m

B.3 PVDF Piezofilm

The Polarized homopolymer of Vinylidene Fluoride (PVDF) develops a great piezo activity and is greater than other materials in terms of its intensity of piezoelectricity. PVDF is a long chain semi-crystalline polymer of the repeat unit (CH₂-CF₂). The properties of PVDF, including piezoelectricity, depend heavily on the degree and type of its crystal line structure. Table B.3 provides the properties of PVDF piezofilm (Elf Atochem Sensors Inc., 1992a).

Table B.3 PVDF piezofilm

Description	Cte	Value	Unit
Volume density	ρ_s	1780.0	Kg/m ³
Elastic modulus	E_s	2.0×10^9	Pa
Piezoelectric strain constant	d_{31s}	23.0×10^{-12}	m/V
Electric permittivity	ϵ_p	1.06×10^{-10}	F/m
Thickness	t_s	0.028×10^{-3}	m

Appendix C

Experimental Set Up and Techniques

In this appendix a brief description of the experimental test bed used in this study and some techniques are provided. The experimental set up of the smart structure was developed in the Robotics Laboratory at the Canadian Space Agency (CSA) and all of the experiments were conducted there.

C.1 Experimental Set Up

Figures C.1 and C.2 show the experimental set up and its schematic respectively. The experimental set up consists of the following elements:

- **Flexible beam:** A cantilever beam of Aluminium with the parameters described in B.1 is used as the flexible structure to be controlled.
- **Piezoactuator:** A pair of PZT piezoceramic (BM532, Sensor Technology Ltd., 1996) described in B.2 is used as the actuator.
- **Piezosensor:** Two pairs of PVDF piezofilm, SDT1-028K, (Elf Atochem Sensors Inc., 1992) are used to measure the strains and strain rates. The top pair is used to measure the strains and the bottom pair measures the strain rates.
- **Charge amplifier:** Two B&K 2635 charge amplifiers (Brüel & Kjær, 1992) are connected to the top piezosensors, which can amplify and filter the output voltages of these sensors.
- **Current amplifier:** Two Keithley 428 current amplifiers (Keithley, 1992) are used to amplify and filter the output currents of the bottom piezosensors. They can magnify the

amplitude of the current by factor of up to 10^{10} which is then sent to a data acquisition board.

- **Data acquisition board:** An RTI 815 multifunctional AD/DA board (Analog Devices, 1992) is employed for data acquisition. It has 16 analog inputs and 2 analog outputs, with 12 bit, 25 μ s resolution. The output and input voltage range is ± 10 V. Four input channels are used for sensor readings and one output channel is used to send the control command voltage.
- **Voltage amplifier:** A programmable high voltage amplifier, Kepco BOP 1000 (Kepco, 1992), is used to supply the excitation signal for the Piezoactuator. This is a bipolar amplifier with ± 1000 V output voltage and ± 40 mAmps output current, and the amplifier gain is 100. The control command from computer is sent to this amplifier to provide driving control voltage for the piezoactuator.
- **Computer:** A 486DX33 is used to process the data and calculate the control command for the active vibration control. The experimental software was written in C++.

The output voltages and currents of the piezosensors are sent to the RTI AD/DA board through charge amplifiers and current amplifiers respectively. Then these signals are sent to the computer to calculate the control command. Finally the control command is sent to the high voltage amplifier, through the RTI AD/DA board, which in turn amplifies the driving voltage for the piezoactuators.

C.2 Experimental Techniques

Working with piezoelements as the smart elements for the active control system requires some special techniques and skills. In the following we mention some of the important issues regarding these elements.

C.2.1 Handling

Piezoelectric materials should be handled only when wearing vinyl or nylon gloves

because oily finger prints can result in localized oxidation and changes to the surface resistivity. Piezoceramic should be handled with more care than piezofilms because they are very brittle.

C.2.2 Bonding and Electrical Connections

The first step of piezoelement mounting is preparing a smooth and clean surface. Sand papers, a cleaning spray, and isopropyl alcohol are used to clean the surface of the flexible structure. After preparing a proper bonding surface, piezoelements can be bonded onto the structure. The SDT1-028K piezofilm sensor used in this study has shielded wires from electrodes, and it can thus be easily attached to the surface of the structure with a non-conductive epoxy. The piezoceramic actuator has no wire connected to the electrodes. In order to make electrical connection with the electrodes, copper tape was used to attach to the outer electrode and the other electrode is attached to the beam surface. In order to have electrical access to this electrode a small drop of a silver epoxy was applied to the middle of the electrode and the rest covered with a non-conductive epoxy which creates a hard bond to the beam surface. Thus any point of the beam can be used as a connection to this electrode.

- ① Smart beam: flexible beam with bonded piezoelectric sensors and actuators
- ② Charge amplifiers
- ③ Current amplifiers
- ④ Voltage amplifiers
- ⑤ Computer: Controller & Data acquisition

Figure C.1 Experimental set up

Figure C.2 Schematic of the experimental set up

Appendix D

Simulation Programs

In the following, some of the major simulation programs used in the thesis are presented. They are presented in order of their use in the thesis chapters. All simulation programs were written in Matlab with use of various toolboxes.

D.1 LQR

```

% =====
%                                     FILE: \smart\Slqr6.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     Dec. 1994
% This program is used to simulate the closed loop response of smart structure using
%                                     LQR.
% =====

t=0.0:0.02:3;                                % Simulation interval

% ===== Open loop simulation =====
C=eye(n2,n2);
D=zeros(n2,p);
open_xsim=lsim(AA,BB,C,D,0*t,t,Ic');
cy_open=[];
cy_open=CC*xsim';
% ===== Closed loop Simulation =====

[r,n2]=size(CC);
[m,n2]=size(CI);

[n2,p]=size(BB);
Ci=zeros(m,n2);

for index1=1:m,
    for index2=1:n2/2,
        Ci(index1,index2)=CI(index1,n2/2+index2);
    end
end
Dii=zeros(2*m,p);
Cii=zeros(2*m,n2);
Dii1=Ci*BB;
Cii1=Ci*AA;

for j=1:p,
    for i=1:2*m,
        if (i>m),
            Dii(i,j)=Dii1(i-m,j);
        end
    end
end

for j=1:n2,
    for i=1:2*m,
        if (i>m),
            Cii(i,j)=Cii1(i-m,j);

```

```

        else
            Cii(i,j)=Ci(i,j);
        end
    end
end
end

```

```

Q=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1]*10^8;
R=1.3;

```

```

Qii=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1]*10^20;
Rii=5;

```

```

[kk.S.E]=lqry(AA,BB,Cii,Dii,Qii,Rii)
[kk.S.E]=lqr(AA,BB,Q,R)

```

```

a=AA-BB*kk;

```

```

b=[1 1 1 1]';
D=[1 1 1 1]';

```

```

cxsim=lsim(a,b,C,D,0*t,t,lc');

```

```

% =====
%                                     Assign Simulation Outputs
% =====

```

```

cy=[];
cyV=[];
cyI=[];

```

```

cy=CC*cxsim';
cyV=CV*cxsim';
cyI=CI*cxsim';
cV=-kk*cxsim';

```

```

% =====
% =====

```

D.2 LQG

```

%% =====
%%                                     FILE: \smart\Slqg2.m
%% =====
%%                                     Written by: Aghil Yousefi-Koma
%%                                     Dec. 1994
%% This program is used to simulate the closed loop response of smart structure using
%%                                     LQG.
%% =====

clear W;

[r.n2]=size(CC);
[m.n2]=size(CV);
[n2,p]=size(BB);

t=0.0:0.02:3.0;           % Simulation interval

A=AA;
B=BB;
C=eye(4);
G=[1 1 1 1]';
dt=0.02;
xx(1:4,1)=Ic';
xxh(1:4,1)=xx(1:4,1);
e(1)=0;

randn;
rand('seed',1);v=rand(301,1);
rand('seed',1*10e6);W(:,1)=10^(-2)*rand(151,1);
rand('seed',2*10e6);W(:,2)=10^(-2)*rand(151,1);
rand('seed',3*10e6);W(:,3)=10^(-2)*rand(151,1);
rand('seed',4*10e6);W(:,4)=10^(-2)*rand(151,1);

for i=1:151
    v(i,1)=0;
end

covv=1e-8;
covW=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]*10^(-4);
covv=cov(v)
ccovW=cov(W)

Q=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1]*10^8;
R=1.3,

[DA,DB]=c2d(A,B,dt);
[DDA,DG]=c2d(A,G,dt);

```

```

kp=dlqr(DA,DB,Q,R);
ke=dlqe(DA,DG,C,covv,covW);
vt=v';
WT=W';

for i=2:151,
    xx(1:4,i)=DA*xx(1:4,i-1)+DB*(e(i-1)+vt(i-1));
    xxh(1:4,i)=DA*xxh(1:4,i-1)+DB*e(i-1)+ke*((xx(1:4,i-1)+WT(1:4,i-1))-xxh(1:4,i-1));
    e(i)=-kp*xxh(1:4,i);
end

% =====
%                                     Assign Simulation Outputs
% =====
cy=[];
cyV=[];
cyI=[];

cy=CC*xx ;
cyV=CV*xx;
cyI=CI*xx;
cV=e+vt;
% =====
% =====

```

D.3 Individual Mode Control

```

% =====
%                                     FILE: \smart\Modal.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     Dec. 1995
%                                     This program is used for active control of individual dynamic modes.
% =====

[r,n2]=size(CC);
[m,n2]=size(CV);
[n2,p]=size(BB);

for i=1:2500.
    time(i)=(i-1)*0.002;
    if (i<1000)
        U0(i)=(50*sin((modfreq(1)+2)*time(i))+500*sin((modfreq(2)+2)*time(i)));
    else
        U0(i)=0;
    end
end

C=eye(n2,n2);
CVI=[CV;CI];
D_OPEN=zeros(n2,p);
D=zeros(n2,1);
U0_OPEN=[U0 :U0];
alfa1=-BB(3,1)/BB(3,2);
alfa2=-BB(4,1)/BB(4,2);

Q1=[1 0 0 0; 0 5 0 0; 0 0 1 0; 0 0 0 5]*1e8;
Q2=[5 0 0 0; 0 1 0 0; 0 0 5 0; 0 0 0 1]*1e7;
R=1;

%alfa=alfa1; Q=Q1;           % Second mode control
alfa=alfa2; Q=Q2;           % First mode control

% ===== Open loop simulation =====
BBB=(BB(:,1)+alfa*BB(:,2));
visim=lsim(AA,BB,CVI,D_OPEN,U0_OPEN',time);
xsim=lsim(AA,BB,eye(4),D_OPEN,U0_OPEN',time);
[kk,S,E]=lqry(AA,BBB,CVI,D,Q,R);

% ===== Closed loop simulation =====
ax=AA-BBB*kk;
avi=AA-BBB*(kk*inv(CVI));
cvisim(1:999,:)=visim(1:999,:);

```

```

cxsim(1:999,:)=xsim(1:999,:);
cvisim(1000:2500,:)=lsim(avi,BBB,CVI,D.zeros(1,1501),time(1000:2500),visim(999,:));
cxsim(1000:2500,:)=lsim(ax,BBB,eye(4),D.zeros(1,1501),time(1000:2500),xsim(999,:));

```

```

% =====
%                                     Assign Simulation Outputs
% =====
y=[];
yv=[];
yi=[];

y=CC*xsim';
yv=CV*xsim';
yi=CI*xsim';

cy=[];
cyv=[];
cyi=[];

cy=CC*cxsim';
cyv=CV*cxsim';
cyi=CI*cxsim';
cV=-kk*cxsim';
% =====
% =====

```

D.4 Shape and Location Optimization of Piezoactuators

D.4.1 Shape Optimization: W_a =Constant and x_a =Constant

```
% =====
%                                     FILE: \optimiz\Optim2_1.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     2D Shape Optimization of Piezoactuators
% Assumptions:
% Piezoactuator width:  $W_a$ = constant.
% Piezoactuator location:  $x_a$ = constant.
% Piezoactuator length :  $l_a$ = optimization variable.
% =====

global L glam gA ;
iter=39;

piezpar;
aactl=254*1e-3;    % xa=254mm
aact(1)=aactl;
wactl=(1*25.4)*1e-3; % Wa=25.4mm
wact(1)=wactl;

for index1= 1 : iter
    cont= index1-1
    inrement=(index1-1)/2;
    mactl=((inrement+1)*25.4/2)*1e-3;
    mact(1)=mactl;
    optim3;
    optim4;
    optim5_2D;
end

for index2=1: iter,
    inrement=(index2-1)/2;
    Lp_per_L(index2)=((inrement+1)*25.4)/508;
end
% =====
% =====
```

D.4.2 Shape Optimization: $A_a=\text{Constant}$ and $x_a=\text{Constant}$

```

% =====
%                                     FILE: \optimiz\Optim2_2.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     2D Shape Optimization of Piezoactuators
% Assumptions:
% Piezoactuator width:  $A_a = \text{constant}$ .
% Piezoactuator location:  $x_a = \text{constant}$ .
% Piezoactuator length :  $l_a = \text{optimization variable}$ .
% =====

global L glam gA ;
iter=39;

piezpar;
aactl=254*1e-3;
aact(l)=aactl;

% We are assuming a constant area of piezo-actuator of:
%  $A_a = 1'' * 1'' = 645.16 * 1e-6 \text{ m}^2$ 

for index1= 1 : iter
    cont=index1-1
    inrement=(index1-1)/2;
    mactl=((inrement+1)*25.4/2)*1e-3;
    mact(l)=mactl;
    wactl=(645.16*1e-6/(2*mactl));
    wact(l)=wactl;
    Wp_per_L(index1)=wact(l)/0.508;
    optim3;
    optim4;
    optim5_2D;
end

for index2=1: iter,
    inrement=(index2-1)/2;
    Lp_per_L(index2)=((inrement+1)*25.4)/508;
    wactl=(645.16*1e-6/((inrement+1)*25.4e-3));
    Wp_per_L(index1)=wactl/0.508;
end
% =====
% =====

```


D.4.3 Location Optimization: W_a =Constant and l_a =Constant

```

% =====
%                                     FILE: \optimiz\Optim2_0.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     2D Location Optimization of Piezoactuators
% Assumptions:
% Piezoactuator width:  $W_a$ = constant.
% Piezoactuator length:  $l_a$ = constant.
% Piezoactuator location:  $x_a$ = optimization variable.
% =====

global L;

iter=36;

leng_a=3;
incre_a=(508-leng_a*25.4)/(iter-1);

piezpar:
wactl=(1*25.4)*1e-3; %  $W_a=1''$ 
wact(1)=wactl;
mactl=(3*25.4/2)*1e-3; %  $l_a=3''$ 
mact(1)=mactl;

for index1=1:iter
    cont=index1-1
    aactl=((cont)*incre_a+leng_a*25.4/2)*1e-3;
    aact(1)=aactl;
    optim3;
    optim4;
    optim5_2D;
end

for index2=1:iter.
    a_per_L(index2)=((index2-1)*incre_a+leng_a*25.4/2)/508;
end
% =====
% =====

```

D.4.4 2D Shape and Location Optimization Subroutine

```

% =====
%                                     FILE: \optimiz\Optim5_2.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     2D Shape and Location Optimization Subroutine
% =====
[r,n2]=size(CC);
[m,n2]=size(CV);
[n2,p]=size(BB);
C=eye(n2,n2);

Q=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1]*10^8;
R=1.3;

[kk,SS]=lqr(AA,BB,Q,R);

% kk=[2.6311   8.1419  -8.3231   6.8086]*1e3;           % xa=254mm, Wa=1", la=3"
% kk=[0.1906  -1.7454  -0.0988  -0.0818]*1e5;           % xa=38.1(1.5"), Wa=1", la=3"

a=AA-BB*kk;

real_s(cont+1)=sum(real(eig(a)));

[Wn,Z]=damp(a);
zita1(cont+1)=Z(1);
zita2(cont+1)=Z(3);
sum_zita(cont+1)=zita1(cont+1)+zita2(cont+1);

Ic=[.5 .5 .5]*10^(-3);
min_J(cont+1)=Ic'*SS*Ic;
% =====
% =====

```

D.4.5 Overall Optimization: $W_a=\text{Constant}$

```

% =====
%                                     FILE: \optimiz\Optim7.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     Overall shape and Location Optimization of Piezoactuators
%                                      $W_a=\text{Constant}$ 
% =====
nn=20;
mm=20;
for i =1:mm
    for j=1:nn
        if J(i,j)==0
            J(i,j)=500;
        end
        if real_s(i,j)==0
            real_s(i,j)=-550;
        end
        if sum_zita(i,j)==0
            sum_zita(i,j)=0.03;
        end
    end
end
% =====
for index3=1:mm
    Lp_per_L(index3)=(index3*25.4*1e-3)/(508*1e-3);
    Wp(index3)=25.4*1e-3;
    Wp_per_L(index3)=Wp(index3)/(508*1e-3);
end
for index4=1:nn
    a_per_L(index4)=((index4-0.5)*25.4*1e-3)/(508*1e-3);
end
% =====
% =====

```

D.4.6 Overall Optimization: $A_a = \text{Constant}$

```

% =====
%                                     FILE: \optimiz\Optim6.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     Overall shape and Location Optimization of Piezoactuators
%                                      $A_a = \text{Constant}$ 
% =====
nn=20;
mm=20;
for i = 1:mm
    for j = 1:nn
        if J(i,j)==0
            J(i,j)=500;
        end
        if real_s(i,j)==0
            real_s(i,j)=-550;
        end
        if sum_zita(i,j)==0
            sum_zita(i,j)=0.03;
        end
    end
end
% =====
for index3 = 1:mm
    Lp_per_L(index3)=(index3*25.4*1e-3)/(508*1e-3);
    Wp(index3)=645.16*1e-6/(index3*25.4*1e-3);
    Wp_per_L(index3)=Wp(index3)/(508*1e-3);
end
for index4 = 1:nn
    a_per_L(index4)=((index4-0.5)*25.4*1e-3)/(508*1e-3);
end
% =====
% =====

```

D.4.7 3D Shape and Location Optimization Subroutine

```

% =====
%                                     FILE: \optimiz\Optim3_D.m
% =====
%                                     Written by: Aghil Yousefi-Koma
%                                     May 1996
%                                     3D Overall Shape and Location Optimization Subroutine
% =====

Q=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1]*10^8;
R=1.3;

[kk,SS]=lqr(AA,BB,Q,R);

% kk=[2.6311   8.1419  -8.3231   6.8086]*1e3;;

a=AA-BB*kk;

real_s(index1,index2)=sum(real(eig(a)));
[Wn,Z]=damp(a);
zita1(index1,index2)=Z(1);
zita2(index1,index2)=Z(3);
sum_zita(index1,index2)=zita1(index1,index2)+zita2(index1,index2);
Ic=[.5 .5 .5 .5]*10^(-3);
J(index1,index2)=Ic'*SS*Ic;
% =====
% =====

```

D.5 Wave Absorbing Controllers (WAC)

```

%=====
%                                     Wac10.m
%                                     Aghil Yousefi-Koma
%                                     June 1996
%=====
%                                     Local & Global WAC for collocated and non-collocated cases
% Note : Piezoactuators should be always in the middle of the piezosensors
% Note : sensor tech. piezoceramic actuators & PVDF piezofilm sensors are used.
%=====
i=sqrt(-1);

% ----- Simulation Beams -----
la=0.15;      % actuator length

%ls=0.2;      % non-collocation (two sensors)
%ls=0.4;      % collocation (two attached sensors of 0.4m: unequal length transducers)
ls=0.075;     % collocation (two sensors of 0.075m: equal length transducers)
%L=0.8;       % total sensing area: 1-6 (non-collocation)
L=0.15;       % total sensing area: 1-6 (collocation equal length transducers)

x=(L-2*ls-la)/2; % distance between sensor and actuator
a=1.1904;       % sqrt(alpha)
gam=-4.5298e9;  % for a pair of piezoactuators with piezofilm sensor

% ----- Experimental Beam -----
la=0.0762;      % actuator length
ls=27e-3;       % sensor length
x=2e-3;         % distance between sensor and actuator
a=1.1904;       % sqrt(alpha)
gam=-2.2125e9;  % for a pair of piezoactuators with piezofilm sensor

% ----- Simulation Beams -----
tetal=0.5;
teta2=0.5;

% ----- Experimental Beam -----
tetal=0.8248;    % non-collocation
teta2=0.8248;    % non-collocation

% ----- %

% for n=1:1000,      % simulation beams
% omega(n)=n/10;     % simulation beams
for n=1:1000,        % experimental beam
    omega(n)=n;       % experimental beam
    s(n)=i*omega(n);

    kl2(n)=ls/sqrt(a)*sqrt(omega(n));

```

```

k13(n)=(x+ls)/sqrt(a)*sqrt(omega(n));
k23(n)=x/sqrt(a)*sqrt(omega(n));
k14(n)=(x+ls+la)/sqrt(a)*sqrt(omega(n));
k24(n)=(x+la)/sqrt(a)*sqrt(omega(n));
k16(n)=(2*x+2*ls+la)/sqrt(a)*sqrt(omega(n));
k15(n)=(2*x+ls+la)/sqrt(a)*sqrt(omega(n));

alpha=sqrt(omega(n))*sqrt(a)/4;

% ----- GWAC -----
alpha1=cos(k16(n));
beta1=-sin(k16(n));
alpha3=alpha1-teta1;
beta3=beta1-teta2;

S11S33=alpha*((exp(-i*k13(n))-exp(-i*k14(n)))*(1-exp(-i*k16(n))+ ...
exp(-i*k15(n))-exp(-i*k12(n)))- ...
(exp(-i*k14(n))-exp(-i*k13(n)))*(1+exp(-i*k16(n)))- ...
i*exp(-i*k16(n))*(2*exp(-k14(n))-2*exp(-k13(n))+ ...
exp(-k23(n))-exp(k24(n))+exp(k23(n))-exp(-k24(n))));

alpha2=real(S11S33);
beta2=imag(S11S33);

den=alpha2^2+beta2^2;

rcGWAC(n)=-(alpha3*alpha2+beta3*beta2)/den;
icGWAC(n)=-(alpha2*beta3-alpha3*beta2)/den;

cGWAC(n)=rcGWAC(n)+i*icGWAC(n);
magcGWAC(n)=sqrt(rcGWAC(n)^2+icGWAC(n)^2);
logcGWAC(n)=20*log10(magcGWAC(n));
phascGWAC(n)=180/pi*atan(icGWAC(n)/rcGWAC(n));

% ----- LWAC -----
dd1(n)=(exp(-i*k14(n))-exp(-i*k13(n)))*(2+exp(-i*k15(n))-exp(-i*k12(n)))*exp(i*k16(n));
dd2(n)=i*(2*exp(-k14(n))-2*exp(-k13(n))+exp(-k23(n))-exp(k24(n))+exp(k23(n))-exp(-k24(n)));
cvel(n)=1/alpha/(dd1(n)+dd2(n));

% ----- Simulation Beam: Approximate GWAC -----
PI(n)=-4.3804/s(n)+1.1139; % Collocation (equal): Approximated GWAC
% PI(n)=-9.8439/s(n)+2.6848; % Collocation (unequal): Approximated GWAC
% PI(n)=-11.0936/s(n)+1.5450; % Non-Collocation: Approximated GWAC

% ----- Experiment Beam: Approximate GWAC & LWAC -----
PI(n)=-9.4794/s(n)+0.7682; % Approximated GWAC: first modal frequency
% PI(n)=-20.1022/s(n)+0.1484; % Approximated GWAC: second modal frequency
cvel1(n)=-14/s(n)+0.02; % Approximated LWAC

% ----- Simulation Beam: Approximated LWAC -----

```

```

% cvel1(n)=-6.6667/s(n)+0.1104;          % (Collocation (equal length transducers), PI model
% cvel1(n)=-9/s(n)+1;                    % (Collocation (unequal length transducers), PI model
% cvel1(n)=-7/s(n)+0.35;                  % (Non-collocation (two piezosensors), PI model

%-----Simulation Beam: open loop & closed loop transfer functions -----
% Non-collocation: beam length=1.0 m, total sensing length=0.8m, xa=0.5m, ls1=ls2=0.2m, la=0.15m

% num(n)=(0*s(n)^4+3.2207e-09*s(n)^3+7.7898e-10*s(n)^2+1.4815e-07*s(n)-4.3656e-11);
% den(n)=(1*s(n)^4+3.7069*s(n)^3+930.5729*s(n)^2+152.2437*s(n)+1.6926e+04);

% Collocation (unequal): beam length=1.0 m, L=Total sensing length=0.8m, xa=0.5m, xs1=.3, xs2=.7,
ls1=ls2=0.4m,la=0.15m

% num(n)=(0*s(n)^4+2.0330e-08*s(n)^3+2.7720e-09*s(n)^2+5.2872e-07*s(n)+1.4552e-11);
% den(n)=(1*s(n)^4+3.7069*s(n)^3+930.5729*s(n)^2+152.2437*s(n)+1.6926e+04);

% Collocation (equal): beam length=1.0 m, L=Total sensing length=0.15m, xa=0.5m, xs1=.4625,
xs2=.5375, ls1=ls2=.075m, la=0.15m

num(n)=(0*s(n)^4+7.3046e-09*s(n)^3+8.2707e-10*s(n)^2+1.5772e-07*s(n)+4.7294e-11);
den(n)=(1*s(n)^4+3.7069*s(n)^3+930.5729*s(n)^2+152.2437*s(n)+1.6926e+04);

% ----- Experimental Beam -----
% Beam2 (Two modes); Eb=72e9, L=0.508, zeta=.011; actuator_factor=0.2
num(n)=(0*s(n)^4+8.7350e-09*s(n)^3+3.6131e-08*s(n)^2+2.7684e-05*s(n)-6.5193e-09);
den(n)=(1*s(n)^4+14.3651*s(n)^3+1.1788e+04*s(n)^2+1.0630e+04*s(n)+5.2105e+06);

%----- Bode plots -----
gs(n)=num(n)/den(n);                    % open loop
hs(n)=gs(n)/(1-gs(n)*gam*cvel(n));      % closed loop : LWAC
hs1(n)=gs(n)/(1-gs(n)*gam*cvel1(n));    % closed loop : Approximate LWAC
hs2(n)=gs(n)/(1-gs(n)*gam*cGWAC(n));    % closed loop : GWAC
hs3(n)=gs(n)/(1-gs(n)*gam*PI(n));        % closed loop : Approximate GWAC

magPI(n)=sqrt(real(PI(n))^2+imag(PI(n))^2);
logPI(n)=20*log10(magPI(n));
phasPI(n)=180/pi*atan(imag(PI(n))/real(PI(n)));

magcvel(n)=sqrt(real(cvel(n))^2+imag(cvel(n))^2);
logcvel(n)=20*log10(magcvel(n));
phascvel(n)=180/pi*atan(imag(cvel(n))/real(cvel(n)));

magcvel1(n)=sqrt(real(cvel1(n))^2+imag(cvel1(n))^2);
logcvel1(n)=20*log10(magcvel1(n));
phascvel1(n)=180/pi*atan(imag(cvel1(n))/real(cvel1(n)));
maggs(n)=sqrt(real(gs(n))^2+imag(gs(n))^2);
loggs(n)=20*log10(maggs(n));

```



```

phasgs(n)=180/pi*atan(imag(gs(n))/real(gs(n)));

maghs(n)=sqrt(real(hs(n))^2+imag(hs(n))^2);
loghs(n)=20*log10(maghs(n));
phashs(n)=180/pi*atan(imag(hs(n))/real(hs(n)));

maghs1(n)=sqrt(real(hs1(n))^2+imag(hs1(n))^2);
loghs1(n)=20*log10(maghs1(n));
phashs1(n)=180/pi*atan(imag(hs1(n))/real(hs1(n)));

maghs2(n)=sqrt(real(hs2(n))^2+imag(hs2(n))^2);
loghs2(n)=20*log10(maghs2(n));
phashs2(n)=180/pi*atan(imag(hs2(n))/real(hs2(n)));

maghs3(n)=sqrt(real(hs3(n))^2+imag(hs3(n))^2);
loghs3(n)=20*log10(maghs3(n));
phashs3(n)=180/pi*atan(imag(hs3(n))/real(hs3(n)));
end
% =====
% =====

```

Appendix E

Experimental Program

In this appendix one of the main experimental programs is presented. Several other programs were written and used for sensor reading and test, actuator test, and data acquisition communication and test. The program is written in C++ 4.0.

```

/* =====
/*                                     FILE: \experim\Control3.c
/* =====
/*                                     Written by: Aghil Yousefi-Koma
/*                                     Sep. 1996
/* This program is used to apply the LQR and GWAC controllers to the smart structure in
/*                                     Experiment.
/* =====

/*****
This is a project file in Borland C++ as follows:
control3.c           : Main program
LC05K.OBJ           : Logical channel buffer
LLTC.LIB            : Turbo C language library
BL815.LIB           : RTI-815 board library
BLRES.LIB           : Resolving library
*****/

#include <bios.h>
#include <math.h>
#include <process.h>
#include <sys/timeb.h>

#include <dos.h>
#include <conio.h>
#include <sys/types.h>
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include "c:\rti\decl.h"
#include "c:\rti\decl800.h"

#define PI 3.14159265

#define V_limit 1.2           // Makes Power Amp. limit 100*V_limit
#define N 600                // Number of data reading
#define NN 3                 // Data reduction factor
#define N1 N/NN+1            // Dimension of output arrays

#define Offset_I1 0.01        // Offset of current amp. #1
#define Offset_V1 0.0         // Offset of charge amp. #1
#define Offset_I2 0.01        // Offset of current amp. #2
#define Offset_V2 0.0         // Offset of charge amp. #2

#define side_sen0 -1
#define side_sen2 -1
#define side_sen1 1
#define side_sen3 1

```

```

#define Calib_factor0 3.8          // Calibration factor of sensor#0
#define Calib_factor1 0.38         // Calibration factor of sensor#1
#define Calib_factor2 8.2          // Calibration factor of sensor#2
#define Calib_factor3 0.30         // Calibration factor of sensor#3

//define K_p 1.1945/100*side_sen0 //GWAC: P control gain
//define K_i 5.7451/100*side_sen1 //GWAC: I control gain
#define K_p 1.2694/100*side_sen0 //LQR: P control gain
#define K_i 0.0079/100*side_sen1 //LQR: I control gain

#define Psi -2.2125e9 // Psi: defined by equation 7.4
#define Cp 2.6626e-9 // Piezosensor capacitance

int erstat,DA_lchan[2],AD_lchan[4];
int board.AD_gain.AD_ch.DA_ch;
int choice;
int mode;
int chan;
float volt_in[4],vout[2],v_out;
int logchan;
int int_v_out;
float v_out_scaled;

int i,i1,k,k1,m,m1,j,l;
float sens[N1][4];
float actu[N1][2];
float dif_time,step_time,f;
float v_in_old[4],v_in_dot[4];
double f_time,s_time;

struct timeb timebuffer;

FILE *smart1;
FILE *smart2;

char file1[12],file2[12];

void main_initialize();
/*****
Functions
*****/

/*****
Main Program Loop
*****/
void main()
{
main_initialize();
/* Zero the analog outputs on both D/A physical channels */

```

```

for (DA_ch=0 ; DA_ch<=1 ; DA_ch++)
AOT (DA_lchan[DA_ch], 0, &erstat);

for (;;)
{
choice = main_menu();
switch (choice)
{
case 1 :
smart_test();
break;
case 2 :
actuator();
break;
case 3 :
for (DA_ch=0 ; DA_ch<=1 ; DA_ch++)
AOT (DA_lchan[DA_ch], 0, &erstat);
exit(0);
break;
}
}
}

/*****
Main Initialization
*****/
void main_initialize()
{
/* Parameter initialization */

board=1; /* RTI-815 Board reference number */
DA_lchan[0] = 0;
DA_lchan[1] = 1;
AD_lchan[0] = 2;
AD_lchan[1] = 3;
AD_lchan[2] = 4;
AD_lchan[3] = 5;
AD_gain = 1; /* RTI accepts different gains just for inputs*/

/* RTI-815 : initialization */

INITIALIZE (&erstat); /* RTI-815 : D/A initialization */
if(erstat)
{
printf("*****\n");
printf("*** Initialization error: %2d ***\n", erstat);
printf("*****\n\n");
exit(0);
}
}

```

```

    for (AD_ch=0; AD_ch<=3; AD_ch++)
    {
        CLCHAN (AD_lchan[AD_ch],&erstat);
        if(erstat)
        {
            printf("*****\n");
            printf("*** Clear A/D Channel error: %2d **\n", erstat);
            printf("*****\n\n");
            exit(0);
        }
        AIN815 (AD_lchan[AD_ch].board.AD_ch.AD_gain.&erstat);
        if(erstat)
        {
            printf("*****\n");
            printf("*** A/D setup error: %2d **\n", erstat);
            printf("*****\n\n");
            exit(0);
        }
    }

    for (DA_ch=0; DA_ch<=1; DA_ch++)
    {
        CLCHAN (DA_lchan[DA_ch],&erstat);
        if(erstat)
        {
            printf("*****\n");
            printf("*** Clear D/A Channel error: %2d **\n", erstat);
            printf("*****\n\n");
            exit(0);
        }
        AOT815 (DA_lchan[DA_ch], board, DA_ch, &erstat);
        if(erstat)
        {
            printf("*****\n");
            printf("*** D/A setup error: %d **\n", erstat);
            printf("*****\n\n");
            exit(0);
        }
    }
}

/*****

                                main_menu

*****/

int main_menu()
{
    int mode;
    retry: printf("\n***** \n");
    printf(" Select desired test mode for sensors and actuators: \n\n");
    printf("  1. Control of Smart Structure \n");
    printf("  2. Test Actuators\n");

```

```

printf(" 3. Exit.\n");
printf("\nEnter # : ");
scanf("%d", &mode);
if ( !((mode==1)||((mode==2)||((mode==3))) )
goto retry;
printf("\n");
printf("***** \n\n");
return(mode);
}

/*****
smart_test
*****/
smart_test()
{
int i1,k1,m1,j1;
float volt_in[4];
float sens [N1][4];

float vout[2],v_out;
int chan;
float actu [N1][2];

int dummy,dummy1;

vout[1]=0.0;

k1=0;
m1=0;
i1=0;

// printf("\n Enter Filename For Data Storage :");
// scanf("%s",file1);
// smart1=fopen(file1,"w");

printf("\nEnter Output Filename :");
scanf("%s",file2);
smart2=fopen(file2,"w");

ftime(&timebuffer);
s_time= (double)timebuffer.time+timebuffer.millitm/1000.0;
v_in_old[0] =((10.0 / 2048.0 * AIN (AD_lchan[0], &erstat))-Offset_I1)*Calib_factor0;
v_in_old[1] =((10.0 / 2048.0 * AIN (AD_lchan[1], &erstat))-Offset_V1)*Calib_factor1;
v_in_old[2] =((10.0 / 2048.0 * AIN (AD_lchan[2], &erstat))-Offset_I2)*Calib_factor2;
v_in_old[3] =((10.0 / 2048.0 * AIN (AD_lchan[3], &erstat))-Offset_V2)*Calib_factor3;
volt_in[0] =((10.0 / 2048.0 * AIN (AD_lchan[0], &erstat))-Offset_I1)*Calib_factor0;
volt_in[1] =((10.0 / 2048.0 * AIN (AD_lchan[1], &erstat))-Offset_V1)*Calib_factor1;
volt_in[2] =((10.0 / 2048.0 * AIN (AD_lchan[2], &erstat))-Offset_I2)*Calib_factor2;
volt_in[3] =((10.0 / 2048.0 * AIN (AD_lchan[3], &erstat))-Offset_V2)*Calib_factor3;

```

```

while (k1<N)
{

vout[1]=Psi*(K_p/1e8*(volt_in[0]+volt_in[2])+ K_i*Cp*(volt_in[1]+volt_in[3]));

control_output (DA_lchan[1], vout[1]);
if ((m1==NN || m1==0))
{
m1=0;
sens[i1][0]=volt_in[0];
sens[i1][1]=volt_in[1];
sens[i1][2]=volt_in[2];
sens[i1][3]=volt_in[3];
actu[i1][1]=vout[1];
i1=i1+1;
}
v_in_old[0]=volt_in[0];
v_in_old[1]=volt_in[1];
v_in_old[2]=volt_in[2];
v_in_old[3]=volt_in[3];

for (dummy=0; dummy<4500; dummy++)
for (dummy1=0; dummy1<4; dummy1++);

k1=k1+1;
m1=m1+1;
volt_in[0] =((10.0 / 2048.0 * AIN (AD_lchan[0], &erstat))-Offset_I1)*Calib_factor0;
volt_in[1] =((10.0 / 2048.0 * AIN (AD_lchan[1], &erstat))-Offset_V1)*Calib_factor1;
volt_in[2] =((10.0 / 2048.0 * AIN (AD_lchan[2], &erstat))-Offset_I2)*Calib_factor2;
volt_in[3] =((10.0 / 2048.0 * AIN (AD_lchan[3], &erstat))-Offset_V2)*Calib_factor3;

}

ftime(&timebuffer);
f_time=(double)timebuffer.time+timebuffer.millitm/1000.0;
dif_time=f_time-s_time;
step_time=dif_time/k1;
f=1.0/step_time;

//fprintf(smart1,"\\n Total Experiment Time= %.6f [sec]".dif_time);
//fprintf(smart1,"\\n Number of Iterations = %d".k1);
// fprintf(smart1,"\\n Sampling Time      = %.6f".step_time);
// fprintf(smart1,"\\n Sampling Frequency  = %.6f".f);
for (j=0; j<=(i1-1); j++)
{
fprintf(smart2,"\\n %.6f %.6f %.6f %.6f %.6f %.6f".(j*step_time)*NN,
sens[j][0],sens[j][1],sens[j][2],sens[j][3],100*actu[j][1]);
}
fclose (smart1);
fclose (smart2);

```



```

for (DA_ch=0 ; DA_ch<=1 ; DA_ch++)
AOT (DA_lchan[DA_ch], 0, &erstat);

}
/*****

                                actuator
*****/
actuator()
{
float vout[2],v_out;
int  chan;

int  i,j,k,m;
float actu [N1][2];

int dummy,dummy1;

k=0;
m=0;
i=0;

printf("\n Enter Filename For Data Storage :");
scanf("%s",file1);
smart1=fopen(file1,"w");
printf("\n Enter Filename  For Plotting :");
scanf("%s",file2);
smart2=fopen(file2,"w");

ftime(&timebuffer);
s_time= (double)timebuffer.time+timebuffer.millitm/1000.0;

while (!kbhit() && k<N)
{

for (dummy=0; dummy<4500; dummy++)
for (dummy1=0; dummy1<6; dummy1++);

vout[1]=2*sin(2*PI*2*(0.008058*k));
control_output (DA_lchan[1], vout[1]);
if ((m==NN)||(m==0))
{
m=0;
actu[i][1]=vout[1];
i=i+1;
}
k=k+1;
m=m+1;
}
ftime(&timebuffer);

```

```

f_time=(double)timebuffer.time+timebuffer.millitm/1000.0;
dif_time=f_time-s_time;
step_time=dif_time/k;
f=1.0/step_time;

fprintf(smart1, "\n Total Experiment Time= %.6f [sec]", dif_time);
fprintf(smart1, "\n Number of Iterations = %d", k);
fprintf(smart1, "\n Sampling Time= %.6f", step_time);
fprintf(smart1, "\n Sampling Frequency= %.6f", f);
for (j=0; j<=(i-1); j++)
{
    fprintf(smart2, "\n %.6f %.6f", (j*step_time)*NN.actu[j][1]);
}
fclose (smart1);
fclose (smart2);
}

/*****
control_output: Function to Limit and Generate the Controller Output Voltage
*****/
control_output (logchan, v_out)

int    logchan;
float  v_out;
{
    int  int_v_out;
    float v_out_scaled;

    /* printf ("\n\n v_out in control_output= %.2f", v_out);
    printf ("\n logchan in control_output= %d", logchan); */

    if (v_out > V_limit)
        v_out = V_limit;
    if (v_out < -V_limit)
        v_out = -V_limit;

    /* The voltage cmd is changed to an integer value */
    /* in range of +/- 0 to 2048. This corresponds to */
    /* a 12 bit range of possible analog outputs. */
    v_out_scaled = 2048.0/10.0 * v_out;
    /* printf ("\n\n v_out_scaled = %.2f", v_out_scaled); */

    int_v_out = v_out_scaled;

    /* Account for chop-off error */
    if (v_out_scaled > 0.0)
    {
        if ((v_out_scaled - int_v_out) > 0.5)
            int_v_out += 1;
    }
}

```

```
    else
    {
        if ((v_out_scaled - int_v_out) < -0.5)
            int_v_out -= 1;
    }

    /*    printf ("\n\n int_v_out = %d \n\n", int_v_out); */
```

```
    /* Perform the D to A conversion */
```

```
    AOT (logchan, int_v_out, &erstat);
```

```
}
```

```
%c =====
```

```
%c =====
```

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IMAGE EVALUATION TEST TARGET (QA-3)

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