1

CANADIAN THESES ON MICROFICHE

I.S.B.N.

THESES CANADIENNES SUR MICROFICHE

National Library of Canada
Collections Development Branch

Canadian Theses on
Microfiche Service

Ottawa, Canada
K1A 0N4

Bibliothèque nationale du Canada
Direction du développement des collections

Service des thèses canadiennes
sur microfiche

NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us a poor photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30. Please read the authorization forms which accompany this thesis.

THIS DISSERTATION
HAS BEEN MICROFILMED
EXACTLY AS RECEIVED

AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de mauvaise qualité.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de ce microfilm est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30. Veuillez prendre connaissance des formules d'autorisation qui accompagnent cette thèse.

LA THÈSE A ÉTÉ
MICROFILMÉE TELLE QUE
NOUS L'AVONS REÇUE

National Library
of CanadaBibliothèque nationale
du Canada

Canadian Theses Division

Division des thèses canadiennes

Ottawa, Canada
K1A 0N4

59983

PERMISSION TO MICROFILM — AUTORISATION DE MICROFILMER

• Please print or type — Écrire en lettres moulées ou dactylographier

Full Name of Author — Nom complet de l'auteur

Abdelwahab Mohamed Awad

Date of Birth — Date de naissance

Sept 12, 1947

Country of Birth — Lieu de naissance

Egypt

Permanent Address — Résidence fixe

Faculty of Engineering
Alexandria, Egypt

Title of Thesis — Titre de la thèse

Torsional Response of A Single Storey
Building Model to Ground Motion.

University — Université

Carleton University

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

Ph.D

Year this degree conferred — Année d'obtention de ce grade

1982

Name of Supervisor — Nom du directeur de thèse

J. L. Humar

Permission is hereby granted to the NATIONAL LIBRARY OF CANADA to microfilm this thesis and to lend or sell copies of the film.

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission.

L'autorisation est, par la présente, accordée à la BIBLIOTHÈQUE NATIONALE DU CANADA de microfilmer cette thèse et de prêter ou de vendre des exemplaires du film.

L'auteur se réserve les autres droits de publication; ni la thèse ni de longs extraits de celle-ci ne doivent être imprimés ou autrement reproduits sans l'autorisation écrite de l'auteur.

Date

Sept. 20, 1982

Signature

A. M. Awad

TORSIONAL RESPONSE OF A SINGLE STOREY
BUILDING MODEL TO GROUND MOTION

by

© Abdelwahab M. Awad
B.Sc. Eng., M.Sc. Eng.

A Thesis Submitted to
the Faculty of Graduate Studies and Research
of Carleton University in partial
fulfillment of the degree of
DOCTOR OF PHILOSOPHY IN CIVIL ENGINEERING

Carleton University

Ottawa, Canada

/ August 1982

The Undersigned recommend to the Faculty of Graduate
Studies and Research acceptance of the thesis:

"TORSIONAL RESPONSE OF A SINGLE STOREY
BUILDING MODEL TO GROUND MOTION"

by

Abdelwahab M. Awad,

in partial fulfillment of the requirement of the degree of
DOCTOR OF PHILOSOPHY

Thesis Supervisor

Chairman,

Department of Civil Engineering

Carleton University

ABSTRACT

A three-dimensional analysis of a single storey building model is performed using a modal superposition technique to evaluate the elastic response of systems to the translational and rotational components of the ground motion. The effect of the rotational component of the ground motion on the response is investigated. The rotational motion is found to cause a very significant increase in the value of the torsional moment and sometimes in the shear as well.

A rational method of establishing the rotational earthquake histories from records of translational histories is proposed. A mean rotational response spectrum as well as a mean plus one standard deviation rotational response spectrum are derived from historical earthquake records for the purpose of seismic design of buildings.

Elastic and inelastic responses of single storey building models to the translational and rotational components of an earthquake are evaluated using a time series analysis. Inelasticity is found to cause a reduction in the rotational response of systems.

For systems all of whose lateral resisting elements contributing to the torsional stiffness have the potential to develop inelasticity, the rotational ground motion causes a considerable increase in the ductility demand of the lateral resisting elements. However, for systems some of whose lateral resisting elements contributing to the torsional stiffness always remain elastic, the effect of the ground rotation on the ductility demand of the lateral resisting elements that yield is not uniform.

The seismic torsional provisions of U.S.A.-UBC, Canada, Mexico, New Zealand and German codes are examined with particular reference to the effect of rotational ground motion on response, and it is observed that the torsional moments specified by the codes are, in many cases, significantly lower than those obtained in the analytical studies presented here.

ACKNOWLEDGEMENT

The author wishes to express his gratitude and appreciation to his supervisor, Professor J.L. Bumar, for the invaluable advice and suggestions received from him in carrying out this research work.

The author wishes to thank Mr. A. Abdul Acher for his help in plotting some of the curves in the thesis.

Thanks are also extended to Mrs. Sandra Ryan for her care in typing the manuscript.

TABLE OF CONTENTS

	PAGE
ABSTRACT	i
ACKNOWLEDGEMENTS	iii
LIST OF FIGURES	vi
LIST OF TABLES	xi
LIST OF SYMBOLS	xii
 CHAPTER	
1. INTRODUCTION	1
1.1 Background	1
1.2 Literature Review	2
1.3 Objectives and Scope of the Present Work	5
1.4 Layout of the Thesis	6
2. EFFECT OF TORSIONAL GROUND MOTION ON ELASTIC RESPONSE	8
2.1 Introduction	8
2.2 Torsional Response Spectra	10
2.3 Equations of Motion of the System	13
2.4 Presentation and Analysis of Results	20
2.4.1 One-Way Torsionally Coupled Systems	21
2.4.2 General Torsionally Coupled Systems	24
2.5 Summary and Conclusions	25
3. DESIGN SPECTRA FOR ROTATIONAL EARTHQUAKE GROUND MOTION	45
3.1 Introduction	45
3.2 Analytical Background	46
3.3 Details of Earthquake Records Used	50
3.4 Comparison of Results with Previous Work	51
3.5 Presentation of Results	52
3.6 Summary and Conclusions	53
4. INELASTIC RESPONSE TO EARTHQUAKE GROUND MOTION	60
4.1 Introduction	60
4.2 Description of Model	62
4.3 Equations of Motion	64
4.4 Step-by-Step Integration	68
4.5 Presentation and Analysis of Results	70

CHAPTER	PAGE
4.6 Elastic and Inelastic Response of One-Way Torsionally Coupled System to Translational Ground Motion	72
4.6.1 Response of Elastic Systems	72
4.6.2 Comparison of Elastic and Inelastic Responses	73
4.7 Inelastic Response of One-Way Torsionally Coupled Systems to Translational and Torsional Ground Motion	73
4.8 Inelastic Response and Ductility Demand	77
4.9 Summary and Conclusions	78
4.10 Concluding Remarks	80
5. AN EVALUATION OF SEISMIC TORSIONAL PROVISIONS IN BUILDING CODES	102
5.1 Background	102
5.2 Torsional Provisions in Building Codes	103
5.3 Comparison of Code Seismic Torques with the Results of Dynamic Analysis	105
5.4 Comparison of Edge Displacements According to Codes and Dynamic Analysis	108
5.5 Evaluation of Seismic Torsional Provisions in NBCC 1980	109
5.6 Summary and Conclusions	113
6. SUMMARY AND CONCLUSIONS	125
6.1 Summary	125
6.2 Conclusions	126
6.3 Future Research	129
REFERENCES	130
APPENDIX I. EXPRESSIONS FOR THE NORMALIZED FORCES	134
APPENDIX II. RESPONSE OF TORSIONALLY COUPLED SYSTEMS - ADDITIONAL RESULTS	139
APPENDIX III. DETERMINATION OF THE PRINCIPAL DIRECTIONS OF RECORDED EARTHQUAKE MOTIONS	154

LIST OF FIGURES

FIGURE		PAGE
2.1	Idealized Response Spectrum	28
2.2	Structural Model of a One Storey Building	29
2.3	Response of One Way Torsionally Coupled Systems to Flat Acceleration Spectrum, $e_y/a=0.025$ and $e_y/a=0$	30
2.4	Response of One Way Torsionally Coupled Systems to Flat Acceleration Spectrum, $e_y/a=0.1$ and $e_y/a=0$	31
2.5	Response of One Way Torsionally Coupled Systems to Flat Acceleration Spectrum, $e_y/a=0.4$ and $e_y/a=0$	32
2.6	Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of One Way Torsionally Coupled Systems to Flat Acceleration Spectrum	33
2.7	Effect of the Aspect Ratio b/a on the Response of One Way Torsionally Coupled Systems to Flat Acceleration Spectrum	34
2.8	Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.025$) and ($e_x/b=e_y/a=0$)	35
2.9	Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.1$) and ($e_x/b=e_y/a=0$)	36
2.10	Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.4$) and ($e_x/b=e_y/a=0$)	37
2.11	Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum	38
2.12	Effect of the Aspect Ratio b/a on the Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum	39
2.13	Variation of the Response with ω_y/ω_x to Flat Acceleration Spectrum, $\omega_\theta/\omega_x=0.8$	40

FIGURE		PAGE
2.14	Variation of the Response with ω_y/ω_x to Flat Acceleration Spectrum, $\omega_\theta/\omega_x=1.0$	41
2.15	Variation of the Response with ω_y/ω_x to Flat Acceleration Spectrum, $\omega_\theta/\omega_x=1.2$	42
2.16	Variation of the Response with ω_y/ω_x to Flat Acceleration Spectrum, $\omega_\theta/\omega_x=1.5$	43
2.17	Variation of the Response with ω_y/ω_x to Flat Acceleration Spectrum, $\omega_\theta/\omega_x=2.0$	44
3.1	Directions of Earthquake Shear Waves	55
3.2	El Centro 1940 Rotational Response Spectrum ($C=304.8$ m/sec., $\xi=0.07$)	56
3.3	El Centro 1940 Rotational Response Spectrum ($C=304.8$ m/sec., $\xi=0.1$)	57
3.4	Mean Rotational Response Spectrum ($c=173.74$ m/sec.)	58
3.5	Mean Plus One Standard Deviation Rotational Response Spectrum ($c=173.74$ m/sec.)	59
4.1	Idealized One Storey Structure	81
4.2	Force-Deformation Relations	82
4.3	1940 El Centro Earthquake Translational History	83
4.4	1940 El Centro Earthquake Rotational History ($c=300$ m/sec.)	84
4.5	Response of Elastic Systems, at the Centre of Mass, to Translational Excitation	85
4.6	Response of Lateral Resisting Elements of Elastic Systems to Translational Excitation	86
4.7	Comparison of Displacements at the Centres of Mass of Elastic and Inelastic Systems (Translational Excitation)	87
4.8	Comparison of Rotations at the Centres of Mass of Elastic and Inelastic Systems (Translation Excitation)	88

FIGURE		PAGE
4.9	Response of the Centre of Mass of Inelastic Systems ($\omega_\theta/\omega_x=0.8$)	89
4.10	Response of the Centre of Mass of Inelastic Systems ($\omega_\theta/\omega_x=1.0$)	90
4.11	Response of the Centre of Mass of Inelastic Systems ($\omega_\theta/\omega_x=1.25$)	91
4.12	Response of the Centre of Mass of Inelastic Systems ($\omega_\theta/\omega_x=1.5$)	92
4.13	Response of the Centre of Mass of Inelastic Systems ($\omega_\theta/\omega_x=2.0$)	93
4.14	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=0.8$)	94
4.15	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=1.0$)	95
4.16	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=1.25$)	96
4.17	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=1.5$)	97
4.18	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=2.0$)	98
4.19	Comparison of Response of Lateral Resisting Elements in the y Direction with Reference to the Effect of Ground Rotation	99
4.20	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=1.5$, $e_y/a=0.1$)	100
4.21	Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=2.0$, $e_y/a=0.1$)	101

FIGURE		PAGE
5.1	Comparison of Design Eccentricities in Building Codes with Dynamic Analysis Results (Flat Acceleration Spectrum)	115
5.2	Comparison of Design Eccentricities in Building Codes with Dynamic Analysis Results (Hyperbolic Acceleration Spectrum)	116
5.3	Comparison of Edge Displacement in Building Codes with Dynamic Analysis Results ($\omega_\theta/\omega_x=1$)	117
5.4	Comparison of Edge Displacement in Building Codes with Dynamic Analysis Results ($\omega_\theta/\omega_x=1.5$)	118
5.5	Comparison of Edge Displacement in Building Codes with Dynamic Analysis Results ($\omega_\theta/\omega_x=2.0$)	119
5.6	Response of Single Storey Building with Small Eccentricities to Flat Acceleration Spectrum	120
5.7	Response of Single Storey Buildings with Moderate Eccentricities to Flat Acceleration Spectrum	121
5.8	Response of Single Storey Building with Large Eccentricities to Flat Acceleration Spectrum	122
5.9	Response of Single Storey Buildings with Small Eccentricities to Hyperbolic Acceleration Spectrum	123
5.10	Response of Single Storey Buildings with Moderate Eccentricities to Hyperbolic Acceleration Spectrum	123a
5.11	Response of Single Storey Buildings with Large Eccentricities to Hyperbolic Acceleration Spectrum	124
II.1	Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum, $e_y/a=0.025$ and $e_y/a=0$	139
II.2	Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum, $e_y/a=0.1$ and $e_y/a=0$	140
II.3	Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum, $e_y/a=0.4$ and $e_y/a=0$	141

FIGURE		PAGE
II.4	Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum	142
II.5	Effect of the Aspect Ratio b/a on the Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum	143
II.6	Response of General Torsionally Coupled System to Hyperbolic Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.025$) and ($e_x/b=e_y/a=0$)	144
II.7	Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.01$) and ($e_x/b=e_y/a=0$)	145
II.8	Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.4$) and ($e_x/b=e_y/a=0$)	146
II.9	Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum	147
II.10	Effect of the Aspect Ratio b/a on the Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum	148
II.11	Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x=0.8$	149
II.12	Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x=1$	150
II.13	Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x=1.2$	151
II.14	Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x=1.5$	152
II.15	Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x=2$	153
III.1	Definition of the Earthquake Epicentre A and the Instrument Location B by Longitude and Latitude at Each Location	154

LIST OF TABLES

TABLE

PAGE

3.1 Details of Earthquake Records

54

LIST OF SYMBOLS

A	amplification factor for constant translational acceleration bound
A'	amplification factor for constant rotational acceleration bound
a	plan dimension of building or side of a spherical triangle
a_0, a_1	coefficients used in Eqn. (4.5)
b	plan dimension of building perpendicular to 'a' or side of a spherical triangle
$[C]$	damping matrix
c	shear wave velocity or side of a spherical triangle
D	amplification factor for constant translational displacement bound
D'	amplification factor for constant rotational displacement bound
D_a	spectral pseudo acceleration bound
D_d	spectral relative displacement bound
D_v	spectral pseudo relative velocity bound
d_a	maximum translational ground acceleration
$\dot{d}_a$	maximum value of time derivative of ground acceleration
d_d	maximum value of ground displacement
$\dot{d}_v$	maximum value of ground velocity
e	static eccentricity of building
e_c	design eccentricity
e_d	dynamic eccentricity
e_x	static eccentricity of building in the x direction
e_y	static eccentricity of building in the y direction

F_y	yield shear level of a structural element
g	acceleration of gravity
I_o	rotatory moment of inertia of the deck about the centre of mass
$[K]$	stiffness matrix
K_x	translational stiffness of building in the x direction
K_y	translational stiffness of building in the y direction
K_θ	torsional stiffness of building about a vertical axis at the centre of mass
$K_{\theta R}$	torsional stiffness of building about a vertical axis at the centre of resistance
k_{ix}	translational stiffness of the i-th structural element in the x direction
k_{iy}	translational stiffness of the i-th structural element in the y direction
$[M]$	mass matrix of building
M_n	generalized mass for the n-th mode
m	total mass of the deck
$P(t)$	load vector
$P^t(t)$	equivalent total load vector
Q	resultant response quantity
Q_n	response quantity due to the n-th mode
r	radius of gyration of the deck
S_{ax}	translational spectral acceleration in the x direction
S_{ay}	translational spectral acceleration in the y direction
$S_{a\theta}$	rotational spectral acceleration about the vertical direction
S_v	translational spectral pseudo velocity
S_θ	rotational spectral displacement

S_θ	rotational spectral pseudo velocity
S_θ	rotational spectral pseudo acceleration
T_n^x	modal torque at the centre of mass due to x excitation
T_n^y	modal torque at the centre of mass due to y excitation
T_n^θ	modal torque at the centre of mass due to θ excitation
$\bar{T}_n = \frac{T_n}{rV_{xo}}$	normalized torque defined at the centre of mass due to the n-th mode
T_R	base torque defined at the centre of resistance
T_{Rn}	torque defined at the centre of resistance due to the n-th mode
$\bar{T}_{Rn} = \frac{T_{Rn}}{rV_{xo}}$	normalized torque defined at the centre of resistance due to the n-th mode
t	time
$\{u_g\}$	displacement vector of the ground motion
$\{\dot{u}_g\}$	velocity vector of the ground motion
$\{\ddot{u}_g\}$	acceleration vector of the ground motion
u_{gx}	translational displacement of the ground in the x direction
$\dot{u}_{gx}$	translational velocity of the ground in the x direction
$\ddot{u}_{gx}$	translational acceleration of the ground in the x direction
$\dot{\ddot{u}}_{gx}$	time derivative of $\ddot{u}_{gx}$
u_{gy}	translational displacement of the ground in the y direction
$\dot{u}_{gy}$	translational velocity of the ground in the y direction
$\ddot{u}_{gy}$	translational acceleration of the ground in the y direction
$\dot{\ddot{u}}_{gy}$	time derivative of $\ddot{u}_{gy}$
$u_{g\theta}$	rotational displacement of the ground about the vertical direction

$\dot{u}_{g\theta}$	rotational velocity of the ground about the vertical direction
$\ddot{u}_{g\theta}$	rotational acceleration of the ground about the vertical direction
u_x, u_m	translational displacement of the centre of mass in x direction relative to the ground
$\dot{u}_x$	translational velocity of the centre of mass in x direction, relative to the ground
$\ddot{u}_x$	translational acceleration of the centre of mass in x direction, relative to the ground
u_x^t	total translational displacement of the centre of mass in x direction
U_x, U_y	translational displacement of the ground along the principal directions, X and Y respectively, of the shear waves
u_y	translational displacement of the centre of mass in y direction, relative to the ground
$\dot{u}_y$	translational velocity of the centre of mass in y direction, relative to the ground
$\ddot{u}_y$	translational acceleration of the centre of mass in y direction, relative to the ground
$\{u^r\}$	relative displacement vector at the centre of mass
$\{\dot{u}^r\}$	relative velocity vector at the centre of mass
$\{\ddot{u}^r\}$	relative acceleration vector at the centre of mass
$\{u^t\}$	total displacement vector at the centre of mass
$\{\dot{u}^t\}$	total velocity vector at the centre of mass
$\{\ddot{u}^t\}$	total acceleration vector at the centre of mass
u_θ	rotational displacement of the centre of mass, relative to the ground
$\dot{u}_\theta$	rotational velocity of the centre of mass, relative to the ground
$\ddot{u}_\theta$	rotational acceleration of the centre of mass, relative to the ground

V	amplification factor for constant translational velocity bound
V'	amplification factor for constant rotational velocity bound
V_x	base shear in the x direction
V_{x0}	shear in the x direction of uncoupled system
V_{xn}^x	modal shear in the x direction of coupled system due to x excitation
V_{xn}^y	modal shear in the x direction of coupled system due to y excitation
V_{xn}^θ	modal shear in the x direction of coupled system due to θ excitation
$\bar{V}_x = \frac{V_x}{V_{x0}}$	normalized base shear in the x direction
V_y	base shear in the y direction
V_{yn}^x	modal shear in the y direction of coupled system due to x excitation
V_{yn}^y	modal shear in the y direction of coupled system due to y excitation
V_{yn}^{θ}	modal shear in the y direction of coupled system due to θ excitation
$\bar{V}_y = \frac{V_y}{V_{y0}}$	normalized base shear in the y direction
X, Y	principal directions of shear waves at the instrument site
x, y	principal axes of resistance of building or recording directions of earthquake motions
x_i, y_i	distance of the i-th resisting element from the centre of mass
Y_n	modal coordinates
α, β, γ	angles of a spherical triangle
α_n	mode shape vector
$\alpha_{xn}, \alpha_{yn}, \alpha_{\theta n}$	mode shape coefficients
Δ_c	edge displacement of building due to static torque

Δ_d	edge displacement of building due to dynamic torque
Δ_o	edge displacement of symmetric building due to shear
ε_{nm}	parameter defined by Eqn. (2.15)
ζ_{xn}	modal earthquake-excitation factor due to x excitation
ζ_n	modal earthquake-excitation factor due to y excitation
θ, θ'	angles define the principal directions of shear waves or angles define the longitude of a location
μ	ductility demand of a resisting element
ξ	viscous damping factor
$\tau = \frac{a}{c}$	travel time of the shear wave, second
ϕ	rotational displacement of the ground or angle defines the latitude at a location
$\dot{\phi}$	rotational velocity of the ground
$\ddot{\phi}$	rotational acceleration of the ground
ϕ_a	maximum rotational ground acceleration
ϕ_d	maximum rotational ground displacement
ϕ_v	maximum rotational ground velocity
ψ	rotational displacement of the torsional oscillator relative to the ground
$\dot{\psi}$	rotational velocity of the torsional oscillator relative to the ground
$\ddot{\psi}$	rotational acceleration of the torsional oscillator relative to the ground
ψ_o	total rotational displacement of the torsional oscillator
ω_n	natural circular frequency
ω_x	uncoupled translational frequency in x direction
ω_y	uncoupled translational frequency in y direction
ω_θ	uncoupled torsional frequency, about a vertical axis, at the centre of mass

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

A building subjected to an earthquake undergoes a torsional motion about the vertical axis in addition to the translational motions in three orthogonal directions. Observations of damage to buildings in past earthquakes have revealed that in a number of instances the damage could be attributed to an unexpected overall torsional motion of buildings (1,2). Experimental studies on real buildings have further confirmed the presence of such torsional motion (3). The torsional response of buildings subjected to earthquakes is therefore a subject of considerable practical interest.

The torsional motion of a building may arise due to an eccentricity between the centres of mass and resistance of the building. However, even a linear elastic symmetric building can be excited into torsional motion when its translational and torsional frequencies are close to each other. Further, a non-linear force-deformation characteristic of the lateral force resisting structural elements can also induce torsion in symmetric buildings.

Torsional vibrations that arise from one or more of the sources mentioned above take place even when the earthquake ground motion is purely translational. A more direct cause of torsional response of buildings is the rotational component of the ground motion about a vertical axis.

This work is related to the study of the torsional motions arising from dynamic coupling between translation and torsion as well as from the effect of rotational component of the ground motion.

1.2 LITERATURE REVIEW

Early studies (4,5,6,7,8) as well as the more recent ones (9,10,11, 12) have shown that the torsional response of buildings subjected to earthquakes can be ascribed to a lack of symmetry in the buildings. However, as shown by Hoerner (13) and Keintzel (14), even a linear elastic symmetric building can be excited into a torsional motion when its translational and torsional natural frequencies are close to each other. A similar conclusion has been reached from vibration tests of actual buildings (15,16). Further, as shown by Tso (17,18), a non-linear force-deformation characteristic of the lateral resisting elements can also induce torsional response in symmetric buildings.

Another source of torsional response of a building subjected to an earthquake is the rotational component of the ground motion about a vertical axis. Newmark (19) has drawn attention to the fact that rotational ground motion about a vertical axis occurs in addition to the translational motions in three orthogonal directions. He established this from theoretical considerations. Hart et al (3) have confirmed the presence of rotational ground motion by measurements of the responses of several buildings during the 1971 San Fernando earthquake. The effect of the rotational component of the ground motion about vertical axis on the elastic response of symmetric buildings has been studied by Newmark (19) and Veletsos, Erdik and Kuo (20).

The study of the effect of rotational components of the ground motion on the elastic response of unsymmetric buildings is one of the objectives of the present work. Ideally, measured rotational acceleration histories should be used in such response studies. Up to this time, however, no measurements have been made of the actual rotational motions of the ground during earthquakes.

Assuming the earthquake motions to be caused by horizontal plane shear waves propagating in two orthogonal directions, Newmark (19) has derived expressions which relate the bounds of ground rotational acceleration, velocity and displacement to similar bounds for translational motion. The torsional response spectrum is then given by three straight lines drawn parallel to the ground motion bounds on a tripartite logarithmic graph by using appropriate amplification factors.

Other investigators have used the assumption of propagating plane shear waves to generate rotational ground motion histories from actual records of translational ground motions. The derived rotational motion depends on the assumed directions of the propagating shear waves, the manner in which they are superposed and the value of the shear wave velocity.

Nathan and Mackenzie (21) used the instrument directions as the direction of the two propagating waves. Hart et al (3) assumed that the earthquake motion was comprised of a single shear wave propagating along the major principal direction of the recorded earthquake motions, which was the direction joining the epicentre of the earthquake and the instrument site.

Tso and Hsu (22) assumed the earthquake motion to be comprised of shear waves propagating in two mutually perpendicular principal directions, one of them pointing towards the earthquake epicentre. They obtained the torsional spectra for each of these two waves and then superposed them by taking their absolute sum.

One of the objectives of the present work is to establish a design torsional response spectrum based on rotational ground motion histories derived from the records of translational motions. As in the study carried out by Tso and Hsu (22), it is assumed that the earthquake motion is comprised of shear waves propagating in two mutually perpendicular directions, but rather than computing two separate spectra and superposing them, the shear waves are assumed to be propagating simultaneously and the resultant spectrum is computed directly from the combined motion.

The effect of the rotational component of the ground motion on the inelastic response of buildings has not been investigated before. A part of this work is devoted to the study of the effect of inelasticity on the response and of the effect of the rotational component on such response.

Most seismic codes (23,24,25,26,27) contain provisions for seismic torsional design forces to account for the different causes of the torsional response.

In this study, the seismic torsional provisions in the codes of U.S.A.-UBC, Canada, Mexico, New Zealand and Germany are examined with reference to the effect of the rotational component of the ground motion on the response. Particular attention is given to the National Building Codes of Canada (NBC 1980).

1.3 OBJECTIVES AND SCOPE OF THE PRESENT WORK

The objectives of the present work can be summarized as follows:

1. To investigate the effect of the rotational component of the ground motion about a vertical axis on the response of linearly elastic unsymmetric single storey buildings.
2. To establish a reliable torsional response spectrum for the purpose of building design.
3. To investigate the effect of the rotational component of the ground motion about vertical axis on the response of inelastic systems as well as the effect of inelasticity on the response.
4. To examine the seismic torsional provisions for building design in the Codes of U.S.A.- UBC, Canada, Mexico, New Zealand and Germany with particular reference to the effect of the rotational ground motion on response.

The work is based on analytical studies of the response of single storey buildings that are rectangular in plan and have principal directions parallel to those of their lateral resisting elements. The effect of the following parameters on response is considered: the uncoupled translational frequency ratio $\frac{\omega_y}{\omega_x}$, the ratio between the uncoupled torsional frequency and one of the uncoupled translational frequency $\frac{\omega_\theta}{\omega_x}$, the eccentricity ratios $\frac{e_x}{b}$ and $\frac{e_y}{a}$, the aspect ratio of the building plan $\frac{b}{a}$, and the travel time of the shear wave across one of the sides of the building τ .

1.4 LAYOUT OF THE THESIS

The thesis consists of six chapters followed by a list of references and three appendices.

Chapter 1 provides a general introduction to the phenomenon of torsional response under earthquakes. It points out the importance and causes of the torsional motion in buildings, presents a brief review of the existing literature on the subject and describes the objectives, scope and layout of the thesis.

Chapter 2 is devoted to a study of the effect of rotational ground motion about vertical axis on the response of linearly elastic unsymmetric systems. A modal superposition technique is used for the analysis and the building model used in the study represents a single storey building having three degrees of freedom, two translations in the horizontal plane and one rotational about a vertical axis.

Chapter 3 presents rational method of computing the rotational response spectrum from the translational records of an earthquake ground motion. Rotational response spectra for the purpose of building design are derived and compared with similar spectra derived by other researchers.

Chapter 4 presents the results of elastic and inelastic analyses of single storey building models subjected to earthquake motion. A time series method of dynamic analysis is used and elastic and inelastic responses are computed for a purely translational ground motion as well as for a combined translational and rotational motion of the ground.

A comparison is made between elastic and inelastic response results for the purpose of investigating the effect of inelasticity on the response.

Chapter 5 is devoted to an examination of the seismic torsional provisions for the design of buildings in the Codes of U.S.A.-UBC, Canada, Mexico, New Zealand and Germany with particular reference to the effect of the rotational component of the ground motion on the response of buildings. The provisions of the National Building Code of Canada (NBCC 1980) are examined in somewhat greater detail.

Chapter 6 presents a summary of the entire work as well as a list of the conclusions drawn.

CHAPTER 2

EFFECT OF TORSIONAL GROUND MOTION
ON ELASTIC RESPONSE2.1 INTRODUCTION

Translational earthquake ground motion generally induces a torsional response in an unsymmetric building, that is a building whose centres of mass and resistance are not coincident. Early research attempted to predict the response of such an unsymmetric building to a translational ground motion on the basis of a planar analysis. The method employed was to analyze a symmetric system with total masses and stiffnesses equal to those of an unsymmetric system. For such a symmetric system, the degrees of freedom are only translational in the direction of the excitation. Such an analysis gave the maximum shears induced in the system. The torsional moments were then taken to be equal to the product of these maximum shears and the eccentricities between the centres of mass and resistance (perpendicular to the direction of the maximum shears. The values of the torsional moments calculated by this method are generally referred to as the static torques.

However, as pointed out by Ayre (4,5), this method is not accurate and the coupling between the translational and rotational motions of the building should be considered in analysis if the centres of mass and resistance are not coincident.

More recent studies have considered the three-dimensional nature of the problem (10,11,12,13). It has been observed that strong modal

coupling can occur even for buildings with small eccentricities if the lower natural frequencies are close together (28,29,30,31). In evaluating the response by the modal analysis method, the modal maxima are obtained and combined by a rational method. One such method is to take the square root of the sum of the squares of the modal maxima. The other method is to use the sum of the absolute values of the modal maxima. In both these methods, the signs of the modal values are lost. The absolute sum method is very conservative and is useful only for obtaining an upper bound on the response. The root sum square method gives better results but may significantly underestimate or overestimate the response when two or more frequencies of the system are coincident or near coincident. Rosenblueth and Elorduy (31) have suggested a rational method of combining the modal maxima, which accounts for the closeness of the natural frequencies (see Eqns. (2.14)). Since the translational and torsional frequencies of a system can, at times, be close to each other, Rosenblueth's method is more appropriate in studying the torsional response of buildings.

Studies carried out by other researchers have led to the following important conclusions:

1. Coupling between translational and rotational motions of a system may result in a dynamic torque which is considerably higher than the static torque (8,9,28,29,30,31,32).
2. Torsional coupling results in a reduction in the horizontal storey shear when compared to an uncoupled system (9,31,32); however, a study made by Hoerner (13) has reached an opposite conclusion.

In this chapter, a three-dimensional analysis is employed to evaluate the response of single storey buildings. A modal superposition method rather than a time series method is used. In combining the modal maxima, Rosenblueth's method that accounts for the closeness of natural frequencies is used (31).

The response of buildings to a translational ground motion is computed first. Then the response to a translational motion combined with a rotational motion about the vertical axis is computed and compared with the response to a purely translational motion to evaluate the effect of the rotational ground motion. Also, the effect of several parameters controlling the response of such buildings is investigated.

2.2 TORSIONAL RESPONSE SPECTRA

The seismic response of an elastic system can be estimated from a response spectrum of the ground motion. However, because measurements of ground rotation do not exist, torsional response spectra must be derived from records of translational motions in two mutually perpendicular horizontal directions.

Considering the earthquake motion to consist of shear waves propagating in two mutually perpendicular directions, x and y , the transient displacements of the ground surface u_{gx} and u_{gy} in the x and y directions are given by (19):

$$u_{gx} = f(y-ct) \quad (2.1a)$$

$$u_{gy} = f(x-ct) \quad (2.1b)$$

where c is the shear wave velocity and t is the time, and f represents a function.

The rotation of the ground surface about a vertical axis, ϕ , can be derived from the theory of elasticity and is given by:

$$\phi = \frac{1}{2} \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right) \quad (2.2)$$

Assuming that the two terms in Eqn. (2.2) are similar in magnitude but opposite in sign (19), Eqn. (2.2) yields:

$$\phi = - \frac{\partial u}{\partial y} = -f'(y-ct) \quad (2.3)$$

From Eqn. (2.1a), the ground surface velocity $\dot{u}_{gx}$ is given by:

$$\dot{u}_{gx} = -cf'(y-ct) \quad (2.4)$$

Comparing Eqns. (2.3) and (2.4), the ground rotation about a vertical axis can be expressed as:

$$\phi = \frac{\dot{u}_{gx}}{c} \quad (2.5)$$

Using Eqn. (2.5) with its first and second derivatives with respect to time, the maximum ground torsional displacement ϕ_d , the maximum torsional velocity ϕ_v and the maximum torsional acceleration ϕ_a are related to the maximum ground translational velocity d_v , the maximum translational acceleration d_a and the maximum time derivative of acceleration $\dot{d}_a$ by the following equations:

$$\phi_d = \frac{d_v}{c}$$

$$\phi_v = \frac{d_a}{c} \quad (2.6)$$

$$\phi_a = \frac{d_a}{c}$$

It is well known that when plotted on a tripartite logarithmic graph the response spectrum for translational motion can be approximated by a line of constant displacement D_d , a line of constant velocity D_v and a line of constant acceleration D_a where:

$$\begin{aligned} D_d &= D d_d \\ D_v &= V d_v \end{aligned} \quad (2.7)$$

$$D_a = A d_a$$

and D , V and A are amplification factors whose values depend on the damping.

The response spectrum for torsional motion is derived in a similar manner from ϕ_d , ϕ_v and ϕ_a . The rotational displacement bound θ_d , the velocity bound θ_v and the acceleration bound θ_a are given by:

$$\begin{aligned} \theta_d &= D' \phi_d \\ \theta_v &= V' \phi_v \\ \theta_a &= A' \phi_a \end{aligned} \quad (2.8)$$

where D' , V' and A' are the amplification factors.

The analytical results presented in this chapter are based on the response of a single storey building to an earthquake motion represented by one of the following two types of spectra:

1. A translational response spectrum, known as the flat response spectrum, given by D_a with the associated torsional response spectrum given by θ_v (Fig. 21(a)).
2. A translation response spectrum known as the hyperbolic response spectrum, given by D_v , with the associated torsional response spectrum given by θ_d (Fig. 21(b)).

2.3 EQUATIONS OF MOTION OF THE SYSTEM

The single storey building shown in Fig. 2.2 is comprised of a horizontal deck supported on massless axially inextensible columns and/or walls. The deck is assumed to move in a horizontal plane as a rigid body. The building, idealized as above, has three degrees of freedom: two translations u_x and u_y in the horizontal x and y directions respectively, and the rotation u_θ about the vertical direction z at the centre of mass of the deck, all measured relative to the ground.

The translational stiffnesses of the system along the x and y direction K_x and K_y are defined by:

$$K_x = \sum_i k_{ix} \quad (2.9)$$

$$K_y = \sum_i k_{iy}$$

where k_{ix} and k_{iy} are the translational stiffnesses of the i -th resisting

element along the principal axis of resistance x and y respectively.

The torsional stiffness of the system about the z axis, K_θ , is defined by:

$$K_\theta = \sum_i k_{ix} y_i^2 + \sum_i k_{iy} x_i^2 \quad (2.10)$$

where x_i and y_i are coordinates of the i -th resisting element measured along x and y axes respectively, and the local torsional stiffnesses of the individual elements are neglected.

The centre of resistance of the system is located by the static eccentricities e_x and e_y measured from the centre of mass along the x and y directions, and defined by:

$$e_x = \frac{1}{K_y} \sum_i x_i k_{iy} \quad (2.11)$$

$$e_y = \frac{1}{K_x} \sum_i y_i k_{ix}$$

The equations of motion of the system are given by (10):

$$\begin{bmatrix} \ddot{u}_x \\ r\ddot{u}_\theta \\ \ddot{u}_y \end{bmatrix} + \begin{bmatrix} \omega_x^2 & \frac{-e_y}{r} \omega_x^2 & 0 \\ -\frac{e_y}{r} \omega_x^2 & \omega_\theta^2 & \frac{e_x}{r} \omega_y^2 \\ 0 & \frac{e_x}{r} \omega_y^2 & \omega_y^2 \end{bmatrix} \begin{bmatrix} u_x \\ r u_\theta \\ u_y \end{bmatrix} = - \begin{bmatrix} \ddot{u}_{gx} \\ r\ddot{u}_{g\theta} \\ \ddot{u}_{gy} \end{bmatrix} \quad (2.12)$$

where $\omega_x = \sqrt{K_x/m}$ and $\omega_y = \sqrt{K_y/m}$ are the uncoupled translational frequencies in the x and y directions respectively, $\omega_\theta = \sqrt{K_\theta/mr^2}$ is the uncoupled torsional frequency, m is the mass of the deck, r is the radius of gyration, $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ are the translational ground accelerations

in the x and y directions respectively, and $\ddot{u}_{g\theta}$ is the rotational acceleration of the ground motion about the z direction.

Equations (2.12) are for an undamped system. Damping is defined later directly as a fraction of the critical in each of the three modes of vibration.

Figure 2.2 represents a three degree of freedom system. The three vibration mode shapes of the system can be obtained by solving Eqns. (2.12) with the right-hand side zero.

The displacement components are then defined in terms of the modal coordinates by the following equations:

$$\begin{bmatrix} u_x \\ ru_\theta \\ u_y \end{bmatrix} = \sum_{n=1}^3 Y_n \begin{bmatrix} \alpha_{xn} \\ \alpha_{\theta n} \\ \alpha_{yn} \end{bmatrix} = \sum_{n=1}^3 Y_n \{\alpha_n\} \quad (2.13)$$

where Y_n are the modal coordinates.

The maximum value of a response quantity in each natural mode of vibration is determined from the modal equations and the acceleration spectrum. The modal maxima Q_n are then superimposed in an appropriate manner to obtain the resultant response quantity.

The normal method of combining the modal values is to take the square root of the sum of their squares. This method (referred to as the SRSS method) in general gives good approximation of the dynamic response in structures with well-separated frequencies. However, in structures with closely spaced modes, the fact that the SRSS method

neglects the signs of modal components may cause the resultant response to be overestimated in some cases while being significantly underestimated in others. Since such a situation is likely when the torsional and translational motions are coupled, the following equation is used in combining the modal response (31):

$$Q^2 = \sum_{n=1}^3 Q_n^2 + \sum_{n=1}^3 \sum_{\substack{m=1 \\ n \neq m}}^3 \frac{Q_n Q_m}{1 + \epsilon_{nm}} \quad (2.14)$$

in which

$$\epsilon_{nm} = \frac{\sqrt{1 - \xi^2}}{\xi} \frac{\omega_n - \omega_m}{\omega_n + \omega_m} \quad (2.15)$$

and ξ is the damping factor.

Equation (2.14) takes the signs of the modal component into account and is likely to give more reliable results.

The response quantities to be determined are: the base shear V_x in the x direction, the base shear V_y in the y direction, and the base torque T about a vertical axis at the centre of mass or T_R about a vertical axis at the centre of resistance.

For a translational ground motion in the x direction only, the modal maxima are given by (see Appendix I):

$$\begin{aligned} V_{xn}^x &= m \alpha_{xn}^2 S_{ax}(\omega_n, \xi) \\ T_n^x &= m r \alpha_{\theta n} \alpha_{xn} S_{ax}(\omega_n, \xi) \quad n = 1, 2, 3 \\ V_{yn}^x &= m \alpha_{xn} \alpha_{yn} S_{ax}(\omega_n, \theta) \end{aligned} \quad (2.16)$$

where $S_{ax}(\omega_n, \xi)$ is the spectral acceleration in the x direction, and ω_n is the n-th natural frequency.

Similarly, for a translational ground motion in the y direction only, the modal maxima are:

$$\begin{aligned} V_{xn}^y &= m \alpha_{xn} \alpha_{yn} S_{ay}(\omega_n, \xi) \\ T_n^y &= m r \alpha_{\theta n} \alpha_{yn} S_{ay}(\omega_n, \xi) \quad n=1,2,3 \\ V_{yn}^y &= m \alpha_{yn}^2 S_{ay}(\omega_n, \xi) \end{aligned} \quad (2.17)$$

For a rotational ground motion about the z direction, the modal maxima are given by (see Appendix I):

$$\begin{aligned} V_{xn}^\theta &= m r \alpha_{xn} \alpha_{\theta n} S_{a\theta}(\omega_n, \xi) \\ T_n^\theta &= m r^2 \alpha_{\theta n}^2 S_{a\theta}(\omega_n, \xi) \quad n=1,2,3 \\ V_{yn}^\theta &= m r \alpha_{yn} \alpha_{\theta n} S_{a\theta}(\omega_n, \xi) \end{aligned} \quad (2.18)$$

where $S_{a\theta}(\omega_n, \xi)$ is the spectral rotational acceleration about the vertical direction z. Equations (2.16), (2.17) and (2.18) are valid provided the modes have been normalized so that $\{\alpha_n\}^T \{\alpha_n\} = 1$.

It is convenient to normalize the modal maxima for base shears V_x and V_y by the base shear $V_{xo} = m S_{ax}(\omega_x, \xi)$ for a symmetric building, and to normalize the torque T by the value of rV_{xo} . The normalized values of V_x , V_y and T are denoted by $\bar{V}_x$, $\bar{V}_y$ and $\bar{T}$ respectively.

For a translational flat response spectrum given by D_a , shown in Fig. 2.1(a), the spectral accelerations are:

$$\begin{aligned}
 S_{ax}(\omega_x, \xi) &= A d_a \\
 S_{ax}(\omega_n, \xi) &= A d_a \\
 S_{ay}(\omega_n, \xi) &= A d_a \\
 S_{a\theta}(\omega_n, \xi) &= V' \omega_n \frac{d_a}{c}
 \end{aligned} \tag{2.19}$$

Substitution of Eqs. (2.19) into Eqs. (2.16), (2.17) and (2.18) and normalization gives

$$\begin{aligned}
 \bar{V}_{xn}^x &= \alpha_{xn}^2 \\
 \bar{T}_n^x &= \alpha_{xn} \alpha_{\theta n}
 \end{aligned} \tag{2.20}$$

$$\bar{V}_{yx}^n = \alpha_{xn} \alpha_{yn},$$

and

$$\begin{aligned}
 \bar{V}_{yn}^y &= \alpha_{xn} \alpha_{yn} \\
 \bar{T}_n^y &= \alpha_{yn} \alpha_{\theta n}
 \end{aligned} \tag{2.21}$$

$$\bar{V}_{yy}^y = \alpha_{yn}^2.$$

and

$$\bar{V}_{xn}^{\theta} = \frac{V'}{A} \alpha_{xn} \alpha_{\theta n} \left(\frac{\omega_n}{\omega_x} \right) \omega_x \frac{r}{a} \tau$$

$$\bar{T}_n^\theta = \frac{V'}{A} \alpha_{\theta n}^2 \left(\frac{\omega_n}{\omega_x}\right) \omega_x \frac{r}{a} \tau \quad (2.22)$$

$$\bar{V}_{yn}^\theta = \frac{V'}{A} \alpha_{yn} \alpha_{\theta n} \left(\frac{\omega_n}{\omega_x}\right) \omega_x \frac{r}{a} \tau$$

where 'a' is a plan dimension of the building and $\tau=a/c$ is the time that it takes for an earthquake wave to travel a distance 'a'.

For a translational hyperbolic response spectrum given by D_v , shown in Fig. 2.1(b), the spectral accelerations are:

$$S_{ax}(\omega_x, \xi) = \omega_x V d_v$$

$$S_{ax}(\omega_n, \xi) = \omega_n V d_v$$

$$S_{ay}(\omega_n, \xi) = \omega_n V d_v$$

$$S_{a\theta}(\omega_n, \xi) = \omega_n^2 D' \frac{dv}{c}$$

(2.23)

Substitution of Eqns. (2.23) into Eqns. (2.16), (2.17) and (2.18) and normalization gives:

$$\bar{V}_{xn}^x = \alpha_{xn}^2 \left(\frac{\omega_n}{\omega_x}\right)$$

$$\bar{T}_n^x = \alpha_{xn} \alpha_{\theta n} \left(\frac{\omega_n}{\omega_x}\right)$$

(2.24)

$$\bar{V}_{yn}^n = \alpha_{xn} \alpha_{yn} \left(\frac{\omega_n}{\omega_x}\right)$$

$$\bar{V}_{xn}^y = \alpha_{xn} \alpha_{yn} \left(\frac{\omega_n}{\omega_x}\right)$$

$$\bar{T}_n^y = \alpha_{yn} \alpha_{\theta n} \left(\frac{\omega_n}{\omega_x}\right)$$

(2.25)

$$\bar{V}_{yn}^y = \alpha_{yn}^2 \left(\frac{\omega_n}{\omega_x} \right)$$

and

$$\bar{V}_{xn}^\theta = \frac{D'}{V} \alpha_{xn} \alpha_{\theta n} \left(\frac{\omega_n}{\omega_x} \right)^2 \omega_x \frac{r}{a} \tau$$

$$\bar{T}_n^\theta = \frac{D'}{V} \alpha_{\theta n}^2 \left(\frac{\omega_n}{\omega_x} \right)^2 \omega_x \frac{r}{a} \tau \quad (2.26)$$

$$\bar{V}_{yn}^\theta = \frac{D'}{V} \alpha_{\theta n} \alpha_{yn} \left(\frac{\omega_n}{\omega_x} \right)^2 \omega_x \frac{r}{a} \tau$$

The normalized modal maximum of the torque about the centre of resistance, $\bar{T}_{Rn}$, is then given by the following equation:

$$\bar{T}_{Rn} = \bar{T}_n + \frac{e_y}{r} \bar{V}_{xn} - \frac{e_x}{r} \bar{V}_{yn} \quad (2.27)$$

Equations (2.20) to (2.22) or (2.24) to (2.26) give the modal response quantities for the x, y and θ ground excitations. The modal responses for any one type of excitation can be combined by using Eqn. (2.14) to give the total response due to that excitation.

When the building is subjected to an excitation which has more than one component, the total responses due to each individual component can be further superimposed by taking the square root of the sum of their squares.

2.4 PRESENTATION AND ANALYSIS OF RESULTS

The results presented in this section are for a flat acceleration response spectrum.

2.4.1 One-Way Torsionally Coupled Systems

The system eccentricity is considered to be in the y direction. The damping in each mode is assumed to be 5% of the critical. For this value of damping, the following amplification factors for the spectral response can be considered reasonable (33,19):

$$D = 2, \quad V = 2, \quad A = 3$$

and

$$D' = 1.1D, \quad V' = 1.05D$$

Numerical results are presented for the normalized base shear $\bar{V}_x$ and the normalized dynamic eccentricity e_d/a . The normalized dynamic eccentricity, a measure of the torque, is defined as follows:

$$\frac{e_d}{a} = \frac{T_{Ry}}{a V_{x0}} = \frac{T_R}{r V_{x0}} \left(\frac{r}{a} \right) = \bar{T}_R \left(\frac{r}{a} \right) \quad (2.28)$$

The normalized base shear $\bar{V}_x$ and the normalized eccentricity e_d/a are presented in Figs. 2.3, 2.4 and 2.5 for three different values of plan eccentricity ratio e_y/a representing small, moderate and large eccentricities. For the purpose of comparison, plots of normalized base shear and normalized dynamic eccentricity for a symmetric building are presented alongside the curves for unsymmetric building. Also shown are plots of normalized base shear and normalized dynamic eccentricity for unsymmetric buildings with the effect of ground rotational motion neglected ($\tau=0$).

The results for the response of unsymmetric buildings with the effect of the ground rotational motion neglected are similar to those given in Reference (10). Dynamic coupling causes a translational ground motion to excite the torsional mode of vibration, the effect being very pronounced for coincident or near coincident translational and torsional uncoupled frequencies. At the same time, dynamic coupling results in a reduction of the base shear when compared to that in an uncoupled system. The effect of dynamic coupling increases with the plan eccentricity.

The response values obtained by taking ground rotation into consideration show that the torsional response caused by the rotational component of the ground motion is much more significant than that resulting purely from dynamic coupling between translation and rotation. This is particularly so for buildings with small and moderate eccentricities but with uncoupled torsional frequency ω_θ higher than the uncoupled translational frequency ω_x (see Figs. 2.3(b) and 2.4(b)).

For unsymmetric buildings, the rotational component of the ground motion causes an increase in the base shear. The base shear resulting from the combined translational and rotational ground motion is, however, lower than that of uncoupled systems except for buildings which possess both a high eccentricity e_y and a high uncoupled translational frequency ω_x (see Fig. 2.5(a)).

The effect of the rotational component of the ground motion on the response of symmetric buildings is similar to that observed by Newmark (19). For systems with small and moderate eccentricities and

having $\frac{\omega_\theta}{\omega_x}$ equal or near to 1, the combined effect of the rotational component of the ground motion and dynamic coupling of translation and rotation increases the response beyond that in a symmetric building. For regions of well separated ω_θ and ω_x and/or high ω_x , the response of an unsymmetric building is almost the same as that of a symmetric building.

Higher plan eccentricities accentuate the response over a wider range of the frequency ratio $\frac{\omega_\theta}{\omega_x}$. However, the torsional response of a building with a high uncoupled translational frequency ω_x is not affected much (see Fig. 2.5(b)).

As seen from Figs. 2.3(b), 2.4(b) and 2.5(b), the effect of the ground rotation on the base torque increases with the increase of both the uncoupled translational frequency ω_x and the uncoupled frequency ratio $\frac{\omega_\theta}{\omega_x}$. Figure 2.6 shows that, as would be expected, an increase in the transit time τ increases both the base shear and the torque.

Figure 2.6 also shows that for a higher eccentricity the rotational component of the ground motion significantly increases the base shear while the effect of rotational motion on the base torque is more pronounced for lower eccentricities.

The response also increases with the increase in the aspect ratio $\frac{b}{a}$ as shown in Fig. 2.7.

Results for a hyperbolic acceleration response spectrum are similar and lead to similar conclusions (see Appendix II, Figs. II.1 to II.5).

2.4.2 General Torsionally Coupled Systems

General torsionally coupled systems or two-way torsionally coupled systems are those that are eccentric both in the x and y directions. The corresponding eccentricities being denoted by e_x and e_y , respectively.

When compared to a single symmetric system, the existence of additional eccentricity e_x in the doubly eccentric system results in the introduction of an additional parameter $\frac{\omega_y}{\omega_x}$, which is the ratio between the uncoupled translational frequencies in the y and x directions respectively.

In the results presented here, it is assumed that the translational component of the ground motion in the y direction is the same as that in the x direction.

The conclusions arrived at in Section 2.4.1 for one-way torsionally coupled systems are, in general, also applicable to two-way torsionally coupled systems (see Figs. 2.8 to 2.12).

The following additional observations can be made:

The effect of the uncoupled translational frequency ratio $\frac{\omega_y}{\omega_x}$ on the base shear V_x of two-way torsionally coupled systems, with high values of uncoupled torsional to translation frequency ratio $\frac{\omega_\theta}{\omega_x}$, is almost negligible as it could be seen from Figs. 2.16(a) to 2.17(a). The effect is more significant for low values of $\frac{\omega_\theta}{\omega_x}$ but is still quite small as shown in Figs. 2.13(a) to 2.15(a).

The effect of $\frac{\omega_y}{\omega_x}$ on the base torque is also quite significant as seen in Figs. 2.13(c) to 2.17(c).

For purely translational excitations in the x and y directions, the base shear V_x and the base torque in a two-way torsionally coupled system are higher than their corresponding values in a one-way torsionally coupled system (compare Figs. 2.3(a) to 2.5(a) with Figs. 2.8(a) to 2.10(a) and Figs. 2.3(b) to 2.5(b) with 2.8(c) to 2.10(c)).

It can also be seen from the figures mentioned in the foregoing paragraph that for translational excitations in the x and y directions together with the rotational excitation about the z direction, the base shear V_x in a two-way torsionally coupled system is higher than its corresponding value in a one-way torsionally coupled system. In regions where $\frac{\omega_\theta}{\omega_x}$ is close or equal to 1, the torsional response of a general torsionally coupled system is higher than that of a one-way torsionally coupled system; this leads one to the conclusion that in relation to a symmetric building the torsional response of general torsionally coupled systems is accentuated over a wider region compared with the response of a one-way torsionally coupled system.

For substantially high values of $\frac{\omega_\theta}{\omega_x}$, say $\frac{\omega_\theta}{\omega_x} > 2$, the response of a two-way torsionally coupled system and a one-way torsionally coupled system to x, y and θ excitations are almost the same.

Results presented in this section are for flat acceleration response spectrum. Results for hyperbolic acceleration response spectrum lead to similar conclusions (see Appendix II, Figs. II.6 to II.14).

2.5 SUMMARY AND CONCLUSIONS

A three-dimensional analysis of a single storey building model is performed to evaluate its elastic response to translational and

rotational components of earthquake excitation. A modal superposition method is used in the analysis.

For the special cases of a ground motion with no rotational component, the response results are found to be similar to those obtained by other investigators (10).

For the other special case of a ground motion that has a rotational component but which acts on a building that has no unsymmetry, the results are found to be in agreement with those obtained by Newmark (19).

The torsional response of a symmetric building is compared with that of an unsymmetric building. It is found that for a building with small and moderate eccentricities and with $\frac{\omega_\theta}{\omega_x}$ equal or near to 1, the torsional response of the unsymmetric building is considerably higher than that of the corresponding symmetric building; the difference in the response however decreases as the uncoupled translation frequency ω_x increases. For well-separated uncoupled torsional and translational frequencies, ω_θ and ω_x , the difference is very small.

Higher plan eccentricities accentuate the torsional response of unsymmetric buildings over a wider range of the value of uncoupled frequency ratio $\frac{\omega_\theta}{\omega_x}$. However, buildings with a high uncoupled translational frequency ω_x are not affected much.

It is found that the rotational component of the ground motion about vertical axis has a very significant effect on the torsional response of buildings and that this effect is much more significant than that resulting from a purely dynamic coupling between the translational and rotational motions of the building. This is particularly so

for buildings that have an uncoupled torsional frequency ω_θ substantially higher than the uncoupled translational frequency ω_x .

The rotational component of the ground motion results in an increase in the value of the base shear and this effect is particularly significant for buildings that possess a high eccentricity and a high uncoupled translational frequency ω_x .

Although the results presented here are for simple structures, it is believed that the conclusions will apply to complex structures as well.

Figure 2.1. Idealized Response Spectra

Figure 2.2. Structural Model of a One Storey Building

Figure 2.3. Response of One-Way Torsionally Coupled Systems to Flat Acceleration Spectrum, $e_y/a=0.025$ and $e_y/a=0.0$

Figure 2.4. Response of One-Way Torsionally Coupled System to Flat Acceleration Spectrum, $e_y/a=0.1$ and $e_y/a=0.0$

Figure 2.5. Response of One-Way Torsionally Coupled Systems to Flat Acceleration Spectrum, $e_y/a=0.4$ and $e_y/a=0.0$

Figure 2.6. Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of One-Way Torsionally Coupled Systems to Flat Acceleration Spectrum

Figure 2.7. Effect of the Aspect Ratio $\frac{b}{a}$ on the Response of One-Way Torsionally Coupled Systems to Flat Acceleration Spectrum

Figure 2.8. Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum ($e_x/b=0.1, e_y/a=0.025$) and ($e_x/b=e_y/a=0$)

Figure 2.9. Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum ($e_x/b=0.1, e_y/a=0.1$) and ($e_x/b=e_y/a=0$).

Figure 2.10. Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum ($e_x/b=0.1, e_y/a=0.4$) and ($e_x/b=e_y/a=0$)

Figure 2.11. Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum

Figure 2.12. Effect of the Aspect Ratio $\frac{b}{a}$ on the Response of General Torsionally Coupled Systems to Flat Acceleration Spectrum

Figure 2.13. Variation of the Response with $\frac{\omega_y}{\omega_x}$ to Flat Acceleration Spectrum, $\omega_\theta/\omega_x = 0.8$

Figure 2.14. Variation of the Response with $\frac{\omega_y}{\omega_x}$ to Flat Acceleration Spectrum, $\omega_\theta/\omega_x = 1.2$

Figure 2.15. Variation of the Response with $\frac{\omega_y}{\omega_x}$ to Flat Acceleration Spectrum, $\omega_\theta/\omega_x = 1.0$

Figure 2.16. Variation of the Response with $\frac{\omega_y}{\omega_x}$ to Flat Acceleration Spectrum, $\omega_\theta/\omega_x = 1.5$

Figure 2.17. Variation of the Response with $\frac{\omega_y}{\omega_x}$ to Flat Acceleration Spectrum, $\omega_\theta/\omega_x = 2.0$

CHAPTER 3

DESIGN SPECTRA FOR ROTATIONAL
EARTHQUAKE GROUND MOTION3.1 INTRODUCTION

In Chapter 2, it was pointed out that the rotational component of ground motion about a vertical axis causes a very significant torsional response in both symmetric and unsymmetric buildings and that such response may, at times, overshadow the torsion caused in an unsymmetric building purely due to dynamic coupling.

It is evident that the effect of ground rotation must be accounted for explicitly in the design of buildings for earthquake, and as in the case of translational motion, design response spectra should be established for rotational motion as well. As stated earlier, no direct records exist of the actual ground rotation. Attempts have therefore been made to derive the torsional spectrum from translational ground motion records.

Assuming the earthquake motion to be caused by plane shear waves in two orthogonal directions, Newmark (19) derived relations by which the bounds of ground rotational acceleration, velocity and displacement could be related to similar bounds for translation. The torsional response spectrum could then be approximated by three straight lines drawn parallel to the ground motion bounds on a tripartite logarithmic graph using appropriate amplification factors.

Other investigators have used the assumption of propagating plane waves to generate rotational ground motion histories from actual records of translational motions. The derived motion depends on the assumed direction of propagating waves, the manner in which they are superimposed, and the value of shear wave velocity.

Nathan and Mackenzie (21) used the instrument directions as the directions of the two propagating waves. Hart et al (3) assumed that the earthquake motion was comprised of a single shear wave coming from the epicentre of the earthquake.

Tso and Hsu (22) assumed the motion to be comprised of shear waves propagating in two mutually perpendicular principal directions, one of them pointing towards the earthquake epicentre. They obtained the torsional spectra for each of these two waves and then superposed them by taking their absolute sum.

The work presented here also uses the assumption of shear waves propagating in two principal directions, but rather than computing two separate spectra and superposing them, the shear waves are assumed to be propagating simultaneously and the resultant spectrum is computed directly from the combined motion.

3.2 ANALYTICAL BACKGROUND

The method proposed here is based on the assumption that the rotational component of the earthquake ground motion about the vertical axis is a result of two plane horizontal shear waves propagating simultaneously along the principal axes of the ground motions recorded

at the instrument site. Penzien and Watabe (34) have derived such principal directions for several recorded earthquakes and have shown that the major principal direction of the earthquake is determined by joining the instrument site to the epicentre of the earthquake and that the other principal direction is perpendicular to it. They have also shown that the principal directions stay practically constant over the duration of the earthquake.

Referring to Fig. 3.1(a), let x and y axes be the two orthogonal horizontal directions in which the instrument located at 0 records the translational motions. Let X and Y axes be the horizontal principal directions of the recorded ground motions.

The displacements U_X and U_Y in the principal directions X and Y respectively are related to the displacements u_{gx} and u_{gy} recorded in the instrument directions x and y by the following equations:

$$U_X(t) = u_{gx}(t) \cos \theta + u_{gy}(t) \sin \theta \quad (3.1a)$$

$$U_Y(t) = -u_{gx}(t) \sin \theta + u_{gy}(t) \cos \theta \quad (3.1b)$$

where θ is the angle between the x and X directions.

The shear wave propagating in the X direction causes a translational motion U_Y in the Y direction as well as a rotational motion about the vertical axis, while the shear wave propagating in the Y direction causes a translational motion U_X in the X direction as well as a rotational motion about the vertical axis.

The plane wave assumption as given by Newmark (19) yields:

$$U_Y = f(X-ct) \quad (3.2a)$$

$$U_X = g(Y-ct) \quad (3.2b)$$

where c is the shear wave velocity at the instrument site.

The resultant of rotational displacement component of the ground motion about the vertical axis, $\phi(t)$, is given by:

$$\phi(t) = \frac{1}{2} \left(\frac{\partial U_Y}{\partial X} - \frac{\partial U_X}{\partial Y} \right) \quad (3.3)$$

On differentiating Eqn. (3.2a) with respect to X and Eqn. (3.2b) with respect to Y and substituting the result into Eqn. (3.3)

$$\phi(t) = \frac{1}{2} [f'(X-ct) - g'(Y-ct)] \quad (3.4a)$$

$$\phi(t) = -\frac{1}{2c} [\dot{U}_Y(t) - \dot{U}_X(t)] \quad (3.4b)$$

where the overdot denotes differentiation with respect to time t .

Differentiation of Eqns. (3.1) with respect to time t and substitution of the results in Eqn. (3.4b) gives:

$$\phi(t) = \frac{1}{2c} [\dot{u}_{gx}(t)(\sin \theta + \cos \theta) + \dot{u}_{gy}(t)(\sin \theta - \cos \theta)] \quad (3.5a)$$

$$\phi(t) = \frac{1}{2c} [\ddot{u}_{gx}(t)(\sin \theta + \cos \theta) + \ddot{u}_{gy}(t)(\sin \theta - \cos \theta)] \quad (3.5b)$$

$$\ddot{\phi}(t) = \frac{1}{2c} [\ddot{u}_{gx}(t)(\sin \theta + \cos \theta) + \ddot{u}_{gy}(t)(\sin \theta - \cos \theta)] \quad (3.5c)$$

The rotational response spectrum can be evaluated by solving the equation governing the response of a single degree of freedom torsional oscillator, subjected to the rotational acceleration of the ground motion $\ddot{\phi}(t)$. This equation is given by:

$$\ddot{\psi}(t) + 2\xi \omega_n \dot{\psi}(t) + \omega_n^2 \psi(t) = -\ddot{\phi}(t) \quad (3.6)$$

where $\psi(t)$, ξ and ω_n are respectively the rotational displacement relative to ground, the damping factor and the natural frequency of the torsional oscillator.

The use of Eqn. (3.6) is likely to involve inaccuracies arising from the numerical differentiation of the translational acceleration records, $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$, required for evaluating the rotational acceleration of the ground motion $\ddot{\phi}(t)$, especially if the time increment used is not small enough. In order to circumvent such inaccuracies, the following approach can be used.

The total rotational displacement of the torsional oscillator $\psi_o(t)$ is given by:

$$\psi_o(t) = \psi(t) + \phi(t) \quad (3.7)$$

Substitution of Eqn. (3.7) together with its first and second derivatives with respect to time in Eqn. (3.6) gives:

$$\ddot{\psi}_o(t) + 2\xi \omega_n \dot{\psi}_o(t) + \omega_n^2 \psi_o(t) = 2\xi \omega_n \dot{\phi}(t) + \omega_n^2 \phi(t) \quad (3.8)$$

In order to evaluate the right-hand side of Eqn. (3.8), one needs to know the values of $\phi(t)$ and $\dot{\phi}(t)$ which are in turn dependent on the

velocity and acceleration records of the ground motion.

Such records are now available for a large number of historical earthquakes (35).

Equation (3.8) can be solved by any one of the standard methods. The solution will give the total rotational displacement $\psi_0(t)$ of the torsional oscillator. The relative rotational displacement $\psi(t)$ can be obtained by substituting the value of $\psi_0(t)$ in Eqn. (3.7).

The rotational spectral displacement is then given by:

$$S_\theta = |\psi(t)|_{\max} \quad (3.9a)$$

and the rotational spectral pseudo-velocity and acceleration are respectively

$$S_{\dot{\theta}} = \omega_n S_\theta \quad (3.9b)$$

$$S_{\ddot{\theta}} = \omega_n^2 S_\theta \quad (3.9c)$$

3.3 DETAILS OF EARTHQUAKE RECORDS USED

Rotational response spectra are computed for ten historical earthquakes whose details are shown in Table 3.1. An appropriate scale factor is applied to each earthquake record so that the maximum translational acceleration is, in each case, equal to the acceleration of gravity ($1g$). These scale factors are also shown in Table 3.1.

As mentioned in the previous section, computation of the torsional response spectrum requires a knowledge of the principal directions of the recorded ground motion. Since the locations of the instrument and the earthquake epicentre are both known (35), the major principal

direction X can be determined by using spherical trigonometry (36).

The computations for a typical case are shown in Appendix III.

The principal directions of the earthquake records used in this study were obtained by the foregoing procedure and are listed in Table 3.1. The major principal directions of the 1940 El Centro earthquake and the 1952 Taft earthquake are found to be in agreement with those obtained by Penzien and Watabe (34). It should be noted here that Tso and Hsu (22) have also obtained these principal directions but their values are different. The author however believes that the values calculated by him are correct.

The shear wave velocities measured at the instrument site are also listed in Table 3.1 (3,37).

The direction of the shear wave propagating along the major principal axis X is determined uniquely, since the wave is assumed to travel from the earthquake epicentre E to the instrument site O as shown in Figs. 3.1(a) and (b); however, the shear wave propagating in the other principal direction could have the direction shown in Fig. 3.1(a) or the alternative direction shown in Fig. 3.1(b). In this study, both of these possibilities are considered and the larger of the two values of calculated response is used in determining the response spectrum.

3.4 COMPARISON OF RESULTS WITH PREVIOUS WORK

The rotational response spectrum of El Centro 1940 earthquake has been computed by several authors and forms a good basis for comparison of the results. It has been calculated in this study for a damping

factor of 0.07, shear wave velocity of 304.8 m/sec and a scale factor of 1. Figure 3.2 compares the rotational response spectrum obtained by the present method with that obtained by the absolute sum method proposed by Tso and Hsu (22). The response spectrum obtained by Newmark (19) is also shown.

As would be expected, the absolute sum method invariably gives higher values compared with the present method. For low periods, the maximum difference is about 40%, while for high periods it is about 60%. Newmark's method (19) gives higher values for periods less than 0.5 seconds, the maximum difference being more than 100% for comparatively low periods. For periods greater than 0.5 seconds, Newmark's values are at times lower than those of the present work, the maximum difference being of the order of 40%.

Figure 3.3 compares the values obtained by the present method with those obtained by Nathan and Mackenzie (21). The response values obtained by the latter are considerably lower for low periods but are significantly higher for high periods.

3.5 PRESENTATION OF RESULTS

Rotational response spectra are calculated for several values of damping for each of the ten earthquakes listed in Table 3.1. A shear wave velocity of 173.74 m/sec is used in the computations. The mean of the ten response spectra are plotted in Fig. 3.4. Since the number of earthquakes used in establishing the mean rotational spectrum is not statistically large enough, it may be more appropriate to use a mean

plus one standard deviation rotational response spectrum for the purpose of building design. Figure 3.5 shows such spectra for different values of damping.

3.6 SUMMARY AND CONCLUSION

Because rotational component of earthquake ground motion about vertical axis can cause a significant torsional response in structures, it may be advisable to account for the effect of ground rotation more directly and explicitly. This, however, requires that torsional ground motion records or torsional response spectra be available to the designers.

Unfortunately, no direct records exist of the actual ground rotation during earthquake, and torsional response spectra must therefore be derived from the records of translational motions.

A method of obtaining the torsional response spectra from translational records is presented in this chapter. The procedure is based on the assumption that the earthquake motion is caused by two plane propagating shear waves. Data from ten recorded earthquakes is used to obtain a mean spectrum and a mean plus one standard deviation torsional response spectrum.

It is believed that within the limitation of the assumption of earthquake motion as comprising of two plane propagating shear waves, the method used in this study is a rational method of obtaining the design response spectra for torsional earthquake response.

Table 3.1. Details of Earthquake Records.

No.	Earthquake	x-direction θ^* degree	x'-direction θ'^* degree	Scale Factor	Shear Wave Velocity (m/sec)
1	El Centro 1934	E-W -86.05	N-S -03.95	5.476	173.74
2	El Centro 1940	E-W -36.75	N-S -53.25	2.871	173.74
3	El Centro 1956	E-W +73.32	N-S +16.68	19.587	173.74
4	El Centro 1968	E-W -36.03	N-S -53.97	7.678	174.74
5	Taft 1952	S69E -02.62	N21E -87.38	5.576	163.07
6	Taft 1954	S69E -01.80	N21E -88.20	14.684	163.07
7	Olympia 1949	N86E +22.16	N04W +67.84	3.572	182.88
8	Olympia 1965	S86W +38.20	S04E +51.80	5.048	182.88
9	Ferndale 1951	N46W +77.90	S44W +12.10	8.958	188.37
10	Ferndale 1954	N44E +28.07	N46W +71.93	4.933	188.37

* Refer to Fig. 3.1 for definitions of x, x', θ , θ' .

Figure 3.1. Directions of Earthquake Shear Waves.

Figure 3.2. El Centro 1940 Rotational Response Spectrum
($c=304.8$ m/sec, $\xi=0.07$)

Figure 3.3. El Centro 1940 Rotational Response Spectrum
($c=304.8$ m/sec, $\xi=0.1$)

Figure 3.4. Mean Rotational Response Spectrum ($c=173.74$ m/sec)

Figure 3.5. Mean Plus One Standard Deviation Rotational Response Spectrum ($c=173.74$ m/sec)

CHAPTER 4

INELASTIC RESPONSE TO EARTHQUAKE GROUND MOTION4.1 INTRODUCTION

The results of an analytical study of the response of a single storey building model to earthquake ground motion including a ground rotation were presented in Chapter 2. A basic assumption made in these studies was that the building model remained elastic. Buildings subjected to moderate and strong earthquakes may, however, deform significantly beyond the elastic range. A more realistic estimate of the response can therefore be obtained only through an inelastic analysis.

Shibata, Onose, and Shiga (38) have studied the inelastic response of a three degrees of freedom one storey building model to ground motion in two mutually perpendicular horizontal directions. In that study, each component of motion was represented by a sine acceleration pulse, and the lateral force-resisting elements of the building were idealized by horizontal springs. Each spring was assumed to yield independently of each other and to have elastic-perfectly plastic force-deformation relationship. The horizontal deck was considered to move in the horizontal plane as a rigid body and the mass of the deck was considered to be uniformly distributed. Kan and Chopra (39) have studied the response of a two degrees of freedom one storey building model to the N-S translational component of 1940 El Centro earthquake. They used the concept of yielding surface to define yielding and plasticity of the lateral resisting elements.

When yielding occurs in columns, the concept of yield surface must be used because moments in the two directions as well as the axial load will affect the yield phenomenon. When yielding occurs in beams, there is no coupling of moments. Yielding is affected by moment about only one axis and the concept of yield surface is not applicable.

Since the aim in design is generally to restrict yielding to beams, the model used by Shipata, Onose and Shiga (38) is quite relevant.

The previously mentioned studies have provided valuable information by identifying the various parameters that affect the inelastic response. However, it has not been possible to generalize the results and conclusions reached. This could be attributed to the complex nature of inelastic analysis.

The effect of the ground rotational motion on inelastic response has not been investigated before.

In this chapter, elastic and inelastic responses of a one storey building to purely translational horizontal ground motion are calculated using a time series analysis. A comparison is made of the two responses to investigate the effect of inelasticity on the response. The inelastic response of a one storey building to a translational horizontal ground motion accompanied by a rotational ground motion about vertical axis is then computed and compared with the inelastic response of the same building to a purely translational horizontal ground motion in order to investigate the effect of the rotational component of the ground motion on the response of inelastic systems.

4.2 DESCRIPTION OF MODELS

Two different models representing a one storey building are used in this chapter.

Model No. 1

This model, shown in Fig. 4.1, is assumed to have all its lateral resisting elements located on the periphery, in both x and y directions. The lateral resisting elements in the direction of the translational excitation, x direction, are assumed to yield, while those in the y direction remain elastic. The lateral resisting elements located in both the x and y directions contribute towards the torsional stiffness of the building.

Model No. 2

This model is assumed to have all its lateral resisting elements that contribute towards the torsional stiffness oriented in the x direction and located at the periphery as well as at or close to the core. The lateral resisting elements in the y direction are located along the centre line of the building so that they do not contribute towards the torsional stiffness of the building. As in Model No. 1, the lateral resisting elements located in the x direction are assumed to yield, while those located along the y direction remain elastic.

The buildings represented by Model No. 1 have an uncoupled torsional to translational frequency ratio $\frac{\omega_{\theta}}{\omega_x}$ greater than 1.225, while those represented by Model No. 2 have an uncoupled torsional to translational frequency ratio $\frac{\omega_{\theta}}{\omega_x}$ less than or equal to 1.225. In both models,

the mass of the deck is assumed to be uniformly distributed and the deck is assumed to move in the horizontal plan as a rigid body. The lateral force resisting elements of the system are assumed to possess stiffness along one direction only, their stiffness in the perpendicular direction being negligible. The models are assumed to be excited in the x direction by the N-S component of 1940 El Centro earthquake. The E-W component acts along the y axis but its effect is not taken into account in the calculation of the response. Since the models being investigated are symmetric about the y axis and the elements with stiffness in the y direction are assumed to remain elastic, the ground motion in the y direction has no effect on components oriented in the x direction.

If desired, the total response of components in the y direction can be obtained by adding the effect of E-W excitation to the response calculated.

When the effect of ground rotation is to be taken into account, the ground rotation history to be used in computation would be the one resulting from the simultaneous action of N-S and E-W components. This ground rotation history is computed by the method described in Chapter 3.

The force-displacement relationship of the lateral resisting elements which have their stiffness in the direction of the translational excitation are of the elastic-perfectly plastic type as shown in Fig. 4.2(a), while the resisting elements which have their stiffness in the other perpendicular direction are assumed to remain linearly elastic (Fig. 4.2(b)). For the elastic-plastic elements, the yield level is determined as follows:

The elastic model is analyzed first to obtain the maximum lateral force in a resisting element. The yield level for that element in the inelastic model is then taken as the value of the maximum force in the corresponding elastic model.

The models used in this chapter (Fig. 4.1) are representative of a system in which the yielding is assumed to be confined to the beams, the columns remaining elastic.

4.3 EQUATIONS OF MOTION

The structural model shown in Fig. 4.1(b) has three degrees of freedom, the translations u_x and u_y at the centre of mass and rotation u_θ about a vertical axis, also through the centre of mass. The displacements are measured relative to the ground.

The translation u_y is uncoupled from u_x and u_θ . It can therefore be derived separately and superimposed on the response of the components in y direction resulting from x and θ excitations.

Let k_{ix} be the stiffness of the i -th lateral force resisting element along the principal axis of resistance x and k_{iy} its stiffness along the principal axis y . It should be noted that for the model being considered, each element has only one non-zero component of stiffness, either k_{ix} or k_{iy} . The translational stiffness of the system K_x along the x axis and the torsional stiffness K_θ about the vertical axis at the centre of mass are respectively given by

$$K_x = \sum_i k_{ix} \quad (4.1)$$

$$K_\theta = \sum_i k_{ix} y_i^2 + \sum_i k_{iy} x_i^2 \quad (4.2)$$

where x_i and y_i define the location of the i -th force resisting element with respect to the centre of mass.

The static eccentricity e_y measured from the centre of mass along the y axis is given by

$$e_y = \frac{1}{K_x} \sum_i y_i k_{ix} \quad (4.3)$$

The system is assumed to be symmetric about the y axis, that is e_x is equal to zero.

Let m denote the total uniformly distributed mass of the deck and I_0 the rotatory moment of inertia about the vertical axis at the centre of mass.

The equations of motion of the system are then given by:

$$[M]\{\ddot{u}^T\} + [C]\{\dot{u}^T\} + [K]\{u^T\} = \{P(t)\} \quad (4.4)$$

where

$$[M] = \begin{bmatrix} m & 0 \\ 0 & I_0 \end{bmatrix} \text{ is the mass matrix,}$$

$$[K] = \begin{bmatrix} K_x & -e_y K_x \\ -e_y K_x & K_\theta \end{bmatrix} \text{ is the stiffness matrix,}$$

$[C]$ is the damping matrix,

$\{P(t)\}$ is the load vector,

$$\{u^T\} = \begin{bmatrix} u_x \\ u_\theta \end{bmatrix} \text{ is vector of displacement relative to the ground,}$$

$\{\dot{u}^I\}$ and $\{\ddot{u}^I\}$ are respectively the relative velocity and acceleration vectors.

In an inelastic analysis, the damping matrix must be defined explicitly. A reasonable method of expressing the damping is in terms of Rayleigh damping matrix which is defined by (40):

$$[C] = a_0 [M] + a_1 [K] \quad (4.5)$$

in which a_0 and a_1 are coefficients chosen to give some specified values for damping in the first and second modes of the equivalent elastic system.

It should be noted that the stiffness matrix $[K]$ used in Eqn. (4.5) is the stiffness matrix of the elastic system. The mass matrix and the damping matrix are assumed to be unchanged during the inelastic analysis, while the stiffness matrix in Eqn. (4.4) varies in value because the lateral resisting elements oriented in the x direction undergo yielding.

Consider the system to be excited by the translational acceleration of ground motion $\ddot{u}_{gx}$ in the x direction accompanied by the rotational acceleration of ground motion $\ddot{u}_{g\theta}$. Then, the load vector in Eqn. (4.4) can be expressed as:

$$\{P(t)\} = - [M] \begin{bmatrix} \ddot{u}_{gx} \\ \ddot{u}_{g\theta} \end{bmatrix} \quad (4.6)$$

As mentioned in Chapter 3, the evaluation of the rotational acceleration $\ddot{u}_{g\theta}$ requires that the first derivative of the translational acceleration $\ddot{u}_{gx}$ be obtained. This might lead to inaccuracies resulting

from the numerical differentiation. The following alternative formulation is used to circumvent such inaccuracies. Let the relative displacement vector of the centre of mass $\{u^r\}$ be defined by:

$$\{u^r\} = \{u^t\} - \{u_g\} \quad (4.7)$$

then

$$\{\dot{u}^r\} = \{\dot{u}^t\} - \{\dot{u}_g\} \quad (4.8)$$

$$\{\ddot{u}^r\} = \{\ddot{u}^t\} - \{\ddot{u}_g\} \quad (4.9)$$

where

$$\{u^t\} = \begin{bmatrix} u_x^t \\ u_\theta^t \end{bmatrix} \text{ is the total displacement vector at the centre of mass,}$$

$\{\dot{u}^t\}$ and $\{\ddot{u}^t\}$ are respectively the first and second derivatives, with respect to time, of the total displacement vector $\{u^t\}$,

$$\{u_g\} = \begin{bmatrix} u_{gx} \\ u_{g\theta} \end{bmatrix} \text{ is the displacement vector of the ground motion,}$$

and

$\{\dot{u}_g\}$ and $\{\ddot{u}_g\}$ are respectively the first and second derivatives, with respect to time of the displacement vector $\{u_g\}$.

Substitution of Eqs. (4.7) to (4.9) into Eqn. (4.4) and rearrangement gives:

$$[M]\{\ddot{u}^t\} + [C]\{\dot{u}^t\} + [K]\{u^t\} = [C]\{\dot{u}_g\} + [K]\{u_g\} \quad (4.10a)$$

or

$$[M]\{\ddot{u}^t\} + [C]\{\dot{u}^t\} + [K]\{u^t\} = P^t(t) \quad (4.10b)$$

Since the stiffness matrix $[K]$ in the inelastic analysis is not constant, there is no unique set of frequencies and mode shapes for the system, and the modal superposition method cannot be used in the analysis. The analysis is therefore performed by a step-by-step integration of the coupled equations of motion (4.10b).

4.4 STEP-BY-STEP INTEGRATION

For a step-by-step integration, Eqns. (4.10b) are first expressed in the incremental form as follows:

$$[M]\{\Delta\ddot{u}^t\} + [C]\{\Delta\dot{u}^t\} + [K]\{\Delta u^t\} = \{\Delta P^t\} \quad (4.11)$$

The stiffness matrix $[K]$ is assumed to remain constant during a small time increment Δt . Equation (4.11) can now be solved by using a numerical integration technique such as the linear acceleration method (40). In this method, the acceleration vector is assumed to vary linearly during a time increment. This assumption implies a quadratic variation of the velocity vector and a cubic variation of the displacement vector. It can be shown that these assumptions lead to the following expressions for the incremental acceleration and velocity vectors:

$$\{\Delta\ddot{u}^t\} = \frac{6}{\Delta t^2} \{\Delta u_n^t\} - \frac{6}{\Delta t} \{\dot{u}^t\} - 3\{\ddot{u}_n^t\} \quad (4.12)$$

$$\{\Delta\dot{u}^t\} = \frac{3}{\Delta t} \{\Delta u_n^t\} - 3\{\dot{u}_n^t\} = \frac{\Delta t}{2} \{\ddot{u}_n^t\} \quad (4.13)$$

Substitution of Eqns. (4.12) and (4.13) into Eqns. (4.11) and rearrangement gives:

$$[\bar{K}_n] \{\Delta u_n^t\} = \{\Delta P_n^t\} \quad (4.14)$$

where

$$[\bar{K}_n] = [K] + \frac{6}{\Delta t^2} [M] + \frac{3}{\Delta t} [C] \quad (4.15)$$

and

$$\{\Delta P_n^t\} = \{P_n^t\} + [M] \left[\frac{6}{\Delta t} \{\dot{u}_n^t\} + 3\{\ddot{u}_n^t\} \right] + [C] \left[3\{\dot{u}_n^t\} + \frac{\Delta t}{2} \{\ddot{u}_n^t\} \right] \quad (4.16)$$

If the state of the system displacements, velocities and accelerations at the time $t=n\Delta t$ are known, matrix $[\bar{K}_n]$ and vector $\{\Delta P_n^t\}$ can be computed. The incremental displacement vector $\{\Delta u_n^t\}$ is then obtained by solving the system of the linear Equations (4.14). Equations (4.12) and (4.13) are used next to compute the incremental acceleration vector $\{\Delta \ddot{u}_n^t\}$ and the incremental velocity vector $\{\dot{u}_n^t\}$.

The displacement, velocity and acceleration vector at time t_{n+1} are then given by:

$$\{u_{n+1}^t\} = \{u_n^t\} + \{\Delta u_n^t\}$$

$$\{\dot{u}_{n+1}^t\} = \{\dot{u}_n^t\} + \{\Delta \dot{u}_n^t\}$$

$$\{\ddot{u}_{n+1}^t\} = [M]^{-1} [P_{n+1}^t - [C] \{\dot{u}_{n+1}^t\} + [K] \{u_{n+1}^t\}]$$

The step-by-step integration is started with the known values of the displacement and velocity vectors, $\{u_0^t\}$ and $\{\dot{u}_0^t\}$, at time $t=0$.

By repeating the procedure described here for successive time intervals, a complete response history of the system can be obtained for any specified duration of time. At the end of such an analysis, the maximum response values during the entire history can also be obtained.

When the shear force in a resisting element reaches the yield plateau, the stiffness of the element would immediately change to zero. However, if such a transition occurs during an integration time interval rather than at the end of it, an error will be involved in the computation, because the stiffness is assumed to remain constant during the entire time interval. To minimize such error, every time a transition occurs during a time interval, the integration is retraced over that period using a smaller time increment of $\frac{\Delta t}{5}$.

In the idealized force-deformation relationship, it is assumed that as soon as unloading occurs in an element the state of the element changes from inelastic to elastic. The transition from the inelastic to the elastic state is detected by noting a change of sign in the incremental displacement of the element during a time step. Since the transition will normally occur within a time step rather than at the end of it, the procedure is subject to computational error. The strategy of retracing the integration using a smaller time increment is used in this case as well to minimize the errors.

4.5 PRESENTATION AND ANALYSIS OF RESULTS

The results of the elastic and inelastic response of a single storey building model are presented in this and the succeeding sections. The models used in the analysis have a square plan of 30m x 30m plan dimension. The rigid deck has a uniform mass, but the distribution of stiffness is not uniform. The centre of rigidity is eccentric in the y direction with respect to the centre of mass, the eccentricity being e_y . The model is, however, symmetric in the

The excitation consists of the N-S translational component of the 1940 El Centro earthquake acting parallel to the x direction, accompanied by a ground rotational motion about the vertical axis. The rotational motion is generated by considering that the earthquake consists of two simultaneously propagating horizontal shear waves along the principal directions shown in Fig. 3.1(a), Chapter 3. This method of evaluating the torsional ground motion implies that the 1940 El Centro E-W component is acting in the y direction. Because the model is symmetric about y axis and the stiffness components parallel to y direction do not yield, the ground motion in the y direction does not affect the response in the x direction. For the foregoing reason, the presence of y component of motion is ignored in the response calculations while it is taken into account in the evaluation of ground rotational motion.

Figure 4.3 shows the N-S translational history of the 1940 El Centro earthquake, while Fig. 4.4 shows the history of rotational motion about the vertical axis.

The responses of both the elastic and inelastic systems are evaluated by performing a time-series analysis. The time increment used is 0.01 seconds which is believed to be small enough for minimizing the truncation errors and ensuring the stability of the integration procedure. A shear wave velocity of 300 m/second is used in this chapter, which implies a travel time $\tau=0.1$ for the used models, is thought to be a reasonable value for rocky soils. This value has also been used by several other researchers including Newmark (19) and Nathan and Mackenzie (21).

4.6 ELASTIC AND INELASTIC RESPONSE OF ONE-WAY TORSIONALLY COUPLED SYSTEMS TO TRANSLATIONAL GROUND MOTION

4.6.1 Response of Elastic Systems

The elastic response of a single storey system to the N-S translational component of the 1940 El Centro earthquake is presented in this section.

Figures 4.5 show the maximum displacements at the centre of mass of the system plotted against the uncoupled translational period T_x along the direction of excitation for an uncoupled torsional to translational frequency ratio $\frac{\omega_\theta}{\omega_x} = 1.5$.

As seen from Fig. 4.5(a), the translational displacement of the centre of mass u_{xm} does not necessarily increase with the eccentricity, in fact, in some cases, the system with a higher eccentricity has a lower displacement. Figure 4.5(b) shows that the rotation of the centre of mass u_θ increases with the eccentricity e_y . These conclusions agree with the results obtained in Chapter 2 using a modal analysis.

Figures 4.6 show that, as the eccentricity e_y increases, the displacement u_{x2} of the lateral resisting element far from the centre of resistance of the system increases while the displacement u_{x1} of the lateral resisting element close to the centre of resistance decreases.

4.6.2 Comparison of Elastic and Inelastic Response

Figures 4.7 present a comparison between the displacements at the centre of mass for an elastic and an inelastic system. The systems are excited by the N-S translational component of the 1940 El Centro earth-

quake. The elastic system has an uncoupled torsional to translational frequency ratio $\frac{\omega_{\theta}}{\omega_x} = 1.5$ (Model No. 1). The inelastic system also has the same frequency ratio before inelasticity sets in.

The comparison shows that, for high uncoupled translational periods, the displacement u_{xm} at the centre of mass of the elastic system is higher than that of the inelastic system, while for low uncoupled translational periods, the displacement at the centre of mass of the inelastic system is higher than in the elastic system. For moderate uncoupled translational periods, the displacement u_{xm} of the elastic system is at times higher than that of the inelastic system, and at other times lower.

Figures 4.8 show a comparison between the rotations u_{θ} at the centre of mass for an elastic and an inelastic system for uncoupled frequency ratio $\frac{\omega_{\theta}}{\omega_x} = 1.5$ (Model No. 1).

The comparison shows that the rotation at the centre of mass of the elastic system is higher than that of the inelastic system, i.e.,

the inelasticity results in a reduction in the rotational response. A similar conclusion has been obtained by Kan and Chopra (39) from analytical studies of their building model which is somewhat different from the models used in this work.

4.7 INELASTIC RESPONSE OF ONE-WAY TORSIONALLY COUPLED SYSTEMS TO TRANSLATIONAL AND TORSIONAL GROUND MOTION

In this section, the inelastic response of a one storey building is computed first for the N-S translational excitation of the 1940 El Centro earthquake and then for the N-S translational component accompanied by the rotational component about the vertical axis.

The two responses are compared to evaluate the effect of the rotational component of the ground motion on the response of an inelastic system.

Figures 4.9 to 4.13 show the displacements and rotations of an inelastic system at the centre of mass plotted against the uncoupled translational period T_x . The system has a small eccentricity. Results are presented for the two different models and for several values of the uncoupled frequency ratio $\frac{\omega_\theta}{\omega_x}$.

Figures 4.9 and 4.10 show that, for systems represented by Model No. 2 and having an uncoupled torsional to translational frequency ratio $\frac{\omega_\theta}{\omega_x} < 1.225$, the effect of the ground rotation on the displacement u_{xm} at the centre of mass is quite small. For the building represented by Model No. 1 and having an uncoupled frequency ratio $\frac{\omega_\theta}{\omega_x} > 1.25$ (Figs. 4.12, 4.13) the ground rotation at times results in a significant reduction in the displacement u_{xm} at the centre of mass, and as the uncoupled frequency ratio $\frac{\omega_\theta}{\omega_x}$ increases the reduction in the displacement u_{xm} at the centre of mass becomes more significant over a wider range of uncoupled frequency ratio T_x .

As seen from Figs. 4.9 to 4.13, the ground rotation results in a significant increase in the rotation u_θ at the centre of mass. As the uncoupled frequency ratio $\frac{\omega_\theta}{\omega_x}$ increases, the effect of the ground rotation on the rotation $u_{\theta m}$ at the centre of mass also increases.

Figures 4.14 to 4.16 show the response of the lateral force resisting elements parallel to the x direction and located at the periphery plotted against the uncoupled translational period T_x . The

response of the lateral resisting elements is expressed in terms of the ductility demand μ which is the ratio of the maximum plastic displacement to the yield displacement. The yield displacement is chosen to be one-fourth the maximum displacement of the element obtained from the elastic response of the system to the translational excitation.

Figures 4.14 to 4.16 show that the rotational component of the ground motion results in considerable increase in the ductility demands of the lateral resisting elements in the direction of the translational excitation, for systems represented by Model No. 2. It should be noted that these systems have their lateral resisting elements in the x-direction located at the periphery and at the centre of mass while the resisting elements in the y direction, if any, are located at the centre of mass. The elements in the x direction contributing to the torsional excitation undergo yielding during the excitation; the elements in the y direction that do not contribute to the torsional stiffness remain elastic.

Figures 4.17 show the effect of the rotational component of the ground motion on the ductility demands of the lateral resisting elements for systems represented by Model No. 1. For a wide range of uncoupled translational periods T_x , the ground rotation causes in a considerable increase in the ductilities of the lateral resisting, while in a small range of uncoupled translational frequencies, it causes a decrease in the ductility demands of the elements.

Figures 4.18 show that for systems with $\frac{\omega_\theta}{\omega_x} = 2$, the rotational component of the ground motion about vertical axis in general results in a decrease in the ductility demands of the lateral resisting elements.

The effect of the rotational component of the ground motion on the ductility demands of the lateral resisting elements of systems with $\frac{\omega_{\theta}}{\omega_x} = 1.5$ and 2 as described in the foregoing paragraphs is interesting and needs a closer scrutiny.

It is observed that the rotational component of the ground motion has decreased the ductility demands of the lateral resisting elements for systems with $\frac{\omega_{\theta}}{\omega_x} = 2$ and also for systems with $\frac{\omega_{\theta}}{\omega_x} = 1.5$ although only over a limited range of uncoupled translational frequencies, in spite of the fact that it has increased the rotation u_{θ} at the centre of mass of both systems as seen in Figs. 4.12 and 4.13. This can be explained by noting the fact that the rotational component has caused a decrease in the displacement u_{xm} at the centre of mass of these systems as shown in Figs. 4.12(a) and 4.13(a). When the decrease in u_{xm} is substantial, the combined effect of decreased u_{xm} and increased u_{θ} is still a reduction in the displacements of the elements on the periphery.

This can be further appreciated by observing the behaviour of the lateral resisting elements located in the y direction at the periphery, which are assumed to remain elastic.

Figure 4.19 shows the response of the lateral resisting element in the y direction caused by the translational and rotational excitation acting on the system in the elements parallel to the x axis which become inelastic. The response is expressed as a ratio of the displacement of the lateral resisting element caused by a combined translational and torsional excitation to that caused by a purely translational excitation ($\tau=0$). For comparison, the response of a fully elastic system is also shown.

22

OF/DE

22

OF / DE

The following important observation can be made. The rotational component of the ground motion has resulted in a very significant increase in the displacement of the elements located on the periphery in the y direction in both the elastic and inelastic systems. However, the increase is much more significant for inelastic systems. Thus, while the ground rotation might result in a reduction in the ductility demands of the lateral resisting elements in the x direction, it very significantly increases the response of the lateral resisting elements at the periphery in the y direction.

4.8 INELASTIC RESPONSE AND DUCTILITY DEMAND

As has been mentioned previously, the yield level of the lateral resisting element is fixed at one-fourth of its maximum response obtained from an elastic analysis. In other words, if the total displacement in an elastic and inelastic system were to be of the same order, a ductility factor of 4 would be obtained in the inelastic analysis. A correspondence of this nature between the force reduction factor and ductility demands would enable one to use the results of an elastic analysis in the design of inelastic systems. Analytical results show that the ductility demand is not always close to the force reduction factor (4 in this case).

Figures 4.14 to 4.18 show that for low values of uncoupled translational period T_x , the ductility demands of the lateral resisting elements are significantly higher than the force reduction factor of 4. For high uncoupled translational periods, the ductility demands of the lateral resisting elements are lower than the force reduction factor. For

moderate uncoupled periods, the ductility demands are at times lower than the force reduction factor and at other times higher.

Figures 4.17, 4.20 and 4.21 show the effect of the eccentricity e_y on the ductility of the lateral resisting elements for systems with $\frac{\omega_y}{\omega_x} = 1.5$. As the eccentricity e_y increases, the ductility demand of the lateral resisting element close to the centre of resistance of the system μ_1 increases, and the ductility demand of the lateral resisting element far from the centre of resistance of the system μ_2 decreases.

4.9 SUMMARY AND CONCLUSIONS

The elastic and inelastic responses of a one-way torsionally coupled single storey building, represented by two different models, to a purely translational excitation as well as to a combined translational and rotational excitation are evaluated using time series analysis. The exciting input used is the N-S translational component and the corresponding rotational component of the 1940 El Centro earthquake.

In the inelastic analysis, it is assumed that the lateral resisting elements located in the direction of translational excitation undergo inelastic deformation, while those perpendicular to the direction of translational excitation remain elastic.

For high uncoupled translational period T_x , the translational response u_{xm} at the centre of mass of an elastic system is higher than that of the corresponding inelastic system, the opposite is true for low uncoupled translational period T_x . For moderate uncoupled translational period T_x , the response u_{xm} of elastic system is at times higher than that of the inelastic system and at other times it is lower.

The rotational response u_θ of the elastic system is higher than that of the inelastic system. In other words, inelasticity causes a reduction in the rotational response u_θ .

For systems represented by Model No. 2 where all resisting elements contributing to the torsional stiffness may yield during excitation, the rotational component of the ground motion causes considerable increase in the ductility demand of the lateral resisting elements, contributing to the torsional stiffness. For systems represented by Model No. 1 where some of the resisting elements contributing to the torsional stiffness always remain elastic, the effect of the ground rotation on the ductilities of elements that yield is not uniform.

For inelastic systems represented by Model No. 1, the rotational component of the ground motion causes a significant increase in the response of the lateral resisting elements that remain elastic, and the effect of the ground rotation on such elements is significantly higher than that when the entire system is elastic.

The ductility demands of inelastic systems do not match the force reduction factor, used in defining the yield level of the shear. For systems with low uncoupled translational periods, the ductility demands are higher than the force reduction factor while for systems with high uncoupled translational periods, they are lower. For systems with moderate uncoupled translational periods, the ductility demands are, at times, higher than the force reduction factor and at other times they are lower.

4.10 CONCLUDING REMARKS

It is apparent from the results presented in this chapter that the inelastic behaviour of buildings subjected to earthquakes is very complex and the complexity increases when the building is unsymmetric and is subjected to ground rotation. The combined translational and torsional behaviour is complicated due to the presence of simultaneous excitation in two mutually perpendicular directions and the fact that the maximum displacements of an element caused by translation and torsion may not peak at the same time, in fact, they may not even be in the same direction. It is thus possible that while the elements oriented in say the x direction are yielding, causing a reduction in the value of ω_x , the elements oriented in the y direction are still elastic. Still, in general, elements in both x and y directions contribute to the torsional stiffness, the reduction in the value of ω_θ may not be as large as in the value of ω_x . The ratio $\frac{\omega_\theta}{\omega_x}$ may thus vary in an unpredictable manner depending on the actual ground motion. This will, of course, affect the response.

The foregoing discussion will show that the response will be strongly affected by the particular ground motion used as the input. The analytical results based on a single or a smaller number of ground motions cannot therefore be expected to lead to very general conclusions. However, based on the present study and those carried out by others, the following certain trends are apparent:

1. Inelasticity causes a reduction in the rotational response.
2. The ductility demand in inelastic systems is not the same as the force reduction factor used in determining the strength and may at times be much higher.

(a) Plan View

(b) Structural Model

Figure 4.1. Idealized One Storey Structure

(a)

(b)

Figure 4.2. Force-Deformation Relations

Displacement (cm)

Velocity (cm/sec.)

Figure 4.3. - 1940 El Centro Earthquake Translational History

Displacement (radian)

Velocity (radian/sec.)

Figure 4.4. 1940 El Centro Earthquake Rotational History
($c=300$ m/sec.)

Figure 4.5. Response of Elastic Systems at the Centre of Mass to Translational Excitation.

Figure 4.6. Response of Lateral Resisting Elements of Elastic Systems to Translational Excitation

Figure 4.7. Comparison of Displacements at the Centres of Mass of Elastic and Inelastic Systems (Translational Excitation).

Figure 4.8. Comparison of Rotations at the Centres of Mass of Elastic and Inelastic Systems (Translational Excitation)

Figure 4.9. Response of the Centre of Mass of Inelastic Systems. ($\frac{\omega_{\theta}}{\omega_x} = 0.8$)

Figure 4.10. Response of the Centre of Mass of Inelastic Systems ($\frac{\omega_{\theta}}{\omega_x} = 1.0$)

Figure 4.11. Response of the Centre of Mass of Inelastic Systems ($\frac{\omega_{\theta}}{\omega_x} = 1.25$)

Figure 4.1.2. Response of the Centre of Mass of Inelastic Systems ($\frac{\omega_\theta}{\omega_x} = 1.5$)

Figure 4.13. Response of the Centre of Mass of Inelastic Systems ($\frac{\omega_\theta}{\omega_x} = 2.0$)

Figure 4.14. Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x=0.8$)

Figure 4.15. Response of Lateral Resisting Elements in the Direction of Translational Excitation ($w_0/w_x = 1.0$)

Figure 4.16. Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x = 1.25$)

Figure 4.17. Response of Lateral Resisting Elements in the Direction of Translational Excitation $(\omega_\theta/\omega_x = 1.5)$

Figure 4.18. Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x = 2.0$)

Figure 4.19. Comparison of Response of Lateral Resisting Elements in the y-Direction with Reference to the Effect of Ground Rotation

Figure 4.20. Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x = 1.5$, $e_y/a = 0.1$)

Figure 4.21. Response of Lateral Resisting Elements in the Direction of Translational Excitation ($\omega_\theta/\omega_x = 1.5$, $e/a = 0.2$)

CHAPTER 5

AN EVALUATION OF SEISMIC TORSIONAL
PROVISIONS IN BUILDING CODES5.1 BACKGROUND

Most building codes, including the National Building Code of Canada (NBOC 1980), specify methods for the evaluation of design forces and torques arising from the seismic torsional response of buildings (23,24,25,26,27).

The seismic torsional provisions are supposed to account for the torsional forces that arise from modal coupling between the translational and rotational motions, caused by an eccentricity between the centres of mass and resistance. The design eccentricity is assumed to consist of two parts: a computed eccentricity and an accidental eccentricity.

An accidental eccentricity may result from differences between the actual eccentricity and the one computed on the basis of estimates of mass and stiffness of the structure. In addition, the accidental eccentricity is also supposed to account for the effect of torsional ground motion.

Tso and Dempsey (41) and Tsicnias and Hutchinson (42,43) have examined the torsional provisions of U.S.A.-ACT3 and four codes (Canada, Mexico, New Zealand and Germany) and have concluded that, with the exception of the German code, the other three codes and U.S.A.-ACT3 underestimate the torsional moment and the edge displacement in systems with small eccentricities. The underestimation is particularly significant when the uncoupled torsional and translational frequencies are close to one another.

In particular, the National Building Code of Canada (NBCC 1980) stipulates that the design storey torque be taken as a product of the storey shear and the design eccentricity. The Code also provides expressions for computing the design eccentricity.

It has been shown (44) that, for buildings with design eccentricities e_c less than 25 percent of the plan dimension D parallel to the eccentricity, the design torque computed by NBCC 1980 is lower than the dynamic torque arising from earthquake excitation. For buildings with design eccentricities exceeding 25 percent of the plan dimension D , the Code stipulates that the computed design torque be doubled. This is a very conservative provision.

In previous studies, the effect of torsional ground motion has not been explicitly accounted for but is supposed to be taken care of by the stipulated accidental eccentricity.

In this chapter, the seismic torsional provisions in the codes of U.S.A.-UBC, Canada, Mexico, New Zealand and Germany are examined with specific reference to the effect of the torsional ground motion about vertical axis on the response. The expressions for design eccentricity specified by the Uniform Building Code (UBC) is the same as that in the recommendation of the Advanced Technology Council (ATC3) (45). The provisions of the NBCC 1980 are examined in greater detail.

5.2 TORSIONAL PROVISIONS IN BUILDING CODES

Building codes recognize the static torque as an adequate estimate of the design storey torque. The static torque is defined as a product of the storey shear V and the design eccentricity e_c .

Design eccentricities in different building codes are generally expressed in the following form:

$$e_c = e_d + \beta D \quad (5.1)$$

where e_d is the dynamic eccentricity, and

βD is the accidental eccentricity in which β is a coefficient and

D is the plan dimension of the building parallel to the direction in which the structural eccentricity is measured.

Design eccentricities in different building codes are given by the following expressions:

U.S.A. - UBC

$$e_c = e + 0.05D \quad (5.2a)$$

or

$$e_c = e - 0.05D \quad (5.2b)$$

Canada

$$e_c = 1.5e + 0.05D \quad (5.3a)$$

or

$$e_c = 0.5e - 0.05D \quad (5.3b)$$

Mexico

$$e_c = 1.5e + 0.1D \quad (5.4a)$$

or

$$e_c = 0.5e - 0.1D \quad (5.4b)$$

New Zealand

$$e_c = 1.7e - \frac{e^2}{D} + 0.1D \quad (5.5a)$$

or

$$e_c = e - 0.1D \quad (5.5b)$$

Germany

$$e_c = e + e_1 + 0.05D \quad (5.6a)$$

or

$$e_c = e - 0.05D \quad (5.6b)$$

where

e is the structural eccentricity, and

e_1 is a supplementary eccentricity which accounts for sympathetic resonance.

In each of the five building codes considered in this study, two alternative expressions are given for the evaluation of the design eccentricity. In the design of a lateral resisting element, that value of eccentricity should be used which will give the maximum value of shear in the element.

5.3 COMPARISON OF CODE SEISMIC TORQUES WITH THE RESULTS OF DYNAMIC ANALYSIS

The elastic response of a singly symmetric one storey building model to the combined action of the translational and rotational excitation is evaluated using the analysis procedure described in Chapter 2. The travel time of the shear wave across the building side D , parallel to the structural eccentricity, is considered to be $\tau=0.1$ second. This is a representative value and has also been used by other researchers (19). For a building dimension of 30 m, it implies that the shear wave velocity is 300 m/s. The torsional response is expressed by the normalized dynamic eccentricity $\frac{e_d}{D}$.

It should be noted that in the dynamic analysis, the effects of both modal coupling and the torsional ground motion are accounted for

but the possibility of a difference between the actual and computed eccentricities is not considered.

The design torques in the single storey building are computed according to the provision of the five codes mentioned earlier. The torques are expressed in terms of the normalized design eccentricity $\frac{e_c}{D}$. Figures 5.1 and 5.2 depict the variation of the normalized code-derived design eccentricity $\frac{e_c}{D}$ with the normalized structural eccentricity $\frac{e}{D}$. The normalized dynamic eccentricity $\frac{e_d}{D}$ for a flat and a hyperbolic acceleration spectrum is also shown on the same figures for the purpose of comparison.

Since most actual buildings have torsional to translational frequency ratio $\frac{\omega_\theta}{\omega_x}$ higher than 1 (3,19), in general between 1 and 2, the normalized dynamic eccentricity $\frac{e_d}{D}$ is plotted for different values of $\frac{\omega_\theta}{\omega_x}$: 1, 1.5 and 2.0 which are believed to represent the practical situation.

From Figs. 5.1 and 5.2, it can be shown that the torsional moments specified by the UBC are significantly smaller than those obtained in the analysis presented here for systems with small and moderate eccentricities. A similar observation can also be made about several systems with large eccentricities.

In comparison to the results of a dynamic analysis, the NBCC 1980 gives lower torsional moments for systems with small eccentricity ratio $\frac{e}{D} < 0.133$, while it gives generally significantly higher torsional moments for systems with eccentricity ratio $\frac{e}{D} > 0.133$.

The Mexico and New Zealand codes give lower torsional moments for systems with small eccentricities. These codes also give lower torsional moments for systems with moderate eccentricities, but lead to higher moments for systems with large eccentricities.

Although the German code gives torsional moments that are higher than the other codes in the range of low and moderate eccentricities, it still gives lower torsional moments for systems with small eccentricities when compared to a dynamic analysis. It also gives lower torsional moments for systems with moderate eccentricities but high uncoupled torsional to translational frequency ratio $\frac{\omega_\theta}{\omega_x}$, particularly for hyperbolic acceleration spectrum, as it can be seen from Fig. 5.2. The German code, however, gives higher torsional moments for systems with large eccentricities.

The codes prescribed values of $\frac{e_c}{D}$ are independent of the fundamental frequency of the system. The dynamic eccentricity $\frac{e_d}{D}$, however, increases with the uncoupled translational frequency ω_x . The effect is that with increasing values of ω_x provisions become even more non-conservative where they are already so, while for situations for which the code provisions are normally conservative, the conservatism decreases.

5.4 COMPARISON OF EDGE DISPLACEMENTS ACCORDING TO CODES AND DYNAMIC ANALYSIS

The torsional response of a building induces the maximum displacements in the resisting elements located at the periphery of the building. Therefore, the true measure of the effect of torsion can be obtained by finding the edge displacements of the building.

The displacement of a lateral resisting element located at the edge of the building model, far from the centre of resistance, resulting from the torsional moment is evaluated and normalized with respect to the displacement Δ_0 of the uncoupled system due to the base shear. The normalized displacement can be formulated as follows.

For building codes:

$$\Delta_c = \frac{V_x e_c}{K_{\theta R}} \left(e + \frac{D}{2} \right) \quad (5.7)$$

and

$$\Delta_o = \frac{V_x}{K_x} \quad (5.8)$$

where

V_x is the base shear in the direction of translational excitation perpendicular to the design eccentricity e_c ,

K_x is the translational stiffness of the system in the direction of translational excitation, and

$K_{\theta R}$ is the torsional stiffness of the system at the centre of resistance.

From Eqs. (5.7) and (5.8), one can get:

$$\frac{\Delta_c}{\Delta_o} = \frac{e_c \left(e + \frac{D}{2} \right)}{\left(\frac{\omega_{\theta}}{\omega_x} \right)^2 r^2 - e^2} \quad (5.9)$$

where

$\frac{\omega_{\theta}}{\omega_x}$ is the uncoupled torsional to translational frequency at the centre of mass, and

r is the radius of gyration of the deck of the system.

For dynamic analysis:

$$\Delta_d = \frac{T_R}{K_{\theta R}} \left(e + \frac{D}{2} \right) \quad (5.10)$$

From Eqns. (5,8) and (5.10), the following equation can be obtained:

$$\frac{\Delta_d}{\Delta_o} = \frac{e_d (e + \frac{D}{2})}{(\frac{\omega_\theta}{\omega_x})^2 r^2 - e^2} \quad (5.11)$$

where e_d is the dynamic eccentricity.

Figures 5.3 to 5.5 show the relationships between the normalized edge displacements Δ_c/Δ_o or Δ_d/Δ_o , against e/D for the five building codes and also as obtained from a dynamic analysis.

The observations made with reference to Figs. 5.1 and 5.2 apply in this case too; the UBC, Mexico and New Zealand codes give lower edge displacements than those obtained in the analysis presented here for systems with small and moderate eccentricities. The German code specifies a higher value of edge displacement when compared to the other codes but may still result in smaller edge displacements than those predicted by a dynamic analysis particularly for smaller eccentricities. In general, the UBC code gives lower edge displacements for systems of high eccentricities while the other three codes give higher values when compared to the results of a dynamic analysis.

The NBCC 1980 gives low edge displacements for eccentricity ratio $\frac{e}{D} < 0.133$ but it gives significantly higher edge displacements for eccentricity ratio $\frac{e}{D} > 0.133$.

5.5 EVALUATION OF SEISMIC TORSIONAL PROVISIONS IN NBCC 1980

The National Building Code of Canada 1980 specifies an equivalent static load method of calculating the design lateral forces arising from the effect of earthquakes. The code recognizes that the lateral forces may cause torsion in the building because of asymmetrical arrangements of structural elements or uneven distribution of mass.

Additional torsional moments may also arise from variations in the estimates of relative rigidities, uncertain estimates of dead and live loads and the effect of possible torsional ground motions. The Code stipulates that to account for these torsional effects, the lateral forces be assumed to act through points that are eccentric with respect to the centres of rigidities and that the building be designed for the resulting static torques. Expressions are given for computing the design eccentricities.

The equivalent static load method of calculating the lateral forces and the resulting torques is applicable to buildings that are regular in plan and uniform in the distribution of stiffness and mass. The commentaries to the code recommend that for buildings with major setbacks, large torsional eccentricities, unusual mass or stiffness distributions, unusual foundation conditions, and in general with unusual or complex structural configurations, a dynamic method of analysis be used to estimate the earthquake design forces.

The type of analysis to be used that is either a two-dimensional analysis in which the lateral modes along the two principal axes are assumed to be de-coupled or a three-dimensional analysis in which the torsional and translational modes are assumed to be coupled, depends upon the magnitudes of the computed eccentricities between the centres of mass and the centres of resistance.

When the computed eccentricity, e , between the centre of mass and the centre of resistance is less than $0.05D_n$ in all storeys, and the fundamental torsional frequency is removed at least ± 20 percent from

the fundamental lateral frequency, the code permits a two-dimensional de-coupled analysis. For this case, the torques are calculated by using the design eccentricities specified in the code.

It is shown in the previous section that the design torques calculated as above are lower than the torsional moment obtained from a dynamic analysis with the effect of ground rotation included for systems with eccentricities $e < 0.133 D$, while they give significantly higher values for systems with eccentricities, $e > 0.133 D$. Tso (44) has arrived at similar conclusions.

In commentary K to the NBCC 1980, it has been stated that: if the eccentricity ratio $\frac{e}{D}$ exceeds 0.1, a dynamic analysis should be performed in which the translational and torsional modes are assumed to be coupled.

It is further stated that when $\frac{e}{D}$ lies between 0.05 and 0.1 either a dynamic analysis or an equivalent static analysis may be used to evaluate the torsional moment.

When the torsional moment is obtained from a dynamic analysis, the calculated torque should be increased by 25 percent to account for accidental eccentricity and torsional ground motion.

When an uncoupled analysis is used and the difference between the torsional and translational natural period is within ± 20 percent, the static torque should be doubled to account for sympathetic resonance.

In this section, the seismic torsional provisions of NBCC 1980 are further examined with reference to the effect of the torsional ground motion on the response. Tso's proposed expression for the NBCC 1985 is also evaluated.

Figures 5.6b to 5.9b show the dynamic eccentricity $\frac{e_d}{D}$ against the ratio of the uncoupled torsional and translational frequencies $\frac{\omega_\theta}{\omega_x}$ for systems with small, moderate and large eccentricities and a flat acceleration spectrum.

It is noted that for systems with small eccentricities the NBCC 1980 gives significantly lower torsional moment in comparison to a dynamic analysis except when the uncoupled torsional to translational frequency ratio $\frac{\omega_\theta}{\omega_x}$ is small. Further, as $\frac{\omega_\theta}{\omega_x}$ increases, the difference between the results of the code and a dynamic analysis become more and more significant.

For systems with moderate eccentricities and having an uncoupled torsional frequency ω_θ considerably higher than the uncoupled translational frequency ω_x , the NBCC 1980 gives significantly lower torsional moment.

For systems with large eccentricities, the NBCC 1980 gives estimates of the torsional moment which are slightly higher than those obtained by a dynamic analysis.

It may be of interest to compare the values obtained from the Mexican code expression (Eqn. 5.4(a)) with those obtained from NBCC 1980 and a dynamic analysis. For systems with small and moderate eccentricities, expression 5.4(a) gives higher torsional moments than NBCC 1980 except in a narrow band of frequency ratio $\frac{\omega_\theta}{\omega_x}$ around 1. However, for these same systems, expression 5.4(a) gives torsional moments that are lower than those obtained from a dynamic analysis except when $\frac{\omega_\theta}{\omega_x}$ is small. For systems with high eccentricities, the Mexican code expression gives torsional moments higher than those obtained by a dynamic analysis.

The results presented for the case of flat acceleration spectrum are also applicable to hyperbolic acceleration spectrum as shown in Figs. 5.10b to 5.12b.

5.6 SUMMARY AND CONCLUSIONS

The seismic torsional provisions in five building codes (U.S.A.-UBC, Canada, Mexico, New Zealand and Germany) have been studied and examined with reference to the effect of the torsional ground motion on the response and with particular emphasis on the National Building Code of Canada (NBCC 1980).

The study is based on the analysis of a singly symmetric one storey building model. The effect of the ground rotation is included in the analysis. It should, however, be noted that the estimates of ground rotation are obtained indirectly from translational records and thus involve approximating inherent in the assumption of plane propagating shear waves. However, at the current state of art, this is only rational method of estimating ground rotations. Comparisons are made between the values of torsional moments obtained from the code specifications and those given by a dynamic analysis.

In general, the five codes give significantly lower values for torsional moments for systems with small eccentricities.

For systems with moderate eccentricities, UBC, Mexican and New Zealand codes give considerably lower torsional moments. When compared to the above codes, the German code gives higher torsional moments which may still be less than the torsional moments obtained from a dynamic analysis.

For systems with large eccentricities, UBC gives generally lower torsional moments while the other three codes give higher values.

The static torques obtained according to the provisions of the NBCC 1980 results in significantly lower estimates of the torsional moment for systems with eccentricities $e < 0.133 D$. The code requirements, however, give significantly higher torsional moments for systems with eccentricities e considerably higher than $0.133 D$.

For systems with small and moderate eccentricities and with ω_θ considerably higher than ω_x , the NBCC 1980 gives lower values of torsional moment whether an uncoupled or a coupled analysis is used.

For systems with large eccentricities, the torsional moment computed from a coupled analysis specified in the NBCC 1980 are in the same range as those obtained from a dynamic analysis.

Figure 5.1. Comparison of Design Eccentricities in Building Codes with Dynamic Analysis Results (Flat Acceleration Spectrum)

Figure 5.2. Comparison of Design Eccentricities in Building Codes with Dynamic Analysis Results (Hyperbolic Acceleration Spectrum)

Figure 5.3. Comparison of Edge Displacement in Building Codes with Dynamic Analysis Results ($\omega_\theta/\omega_x = 1$)

Figure 5.4. Comparison of Edge Displacement in Building Codes with Dynamic Analysis Results ($\omega_\theta/\omega_x = 1.5$)

Figure 5.5. Comparison of Edge Displacement in Building Codes with Dynamic Analysis Results ($\omega_{\theta}/\omega_x = 2.0$)

Figure 5.6. Response of Single Storey Building with Small Eccentricities to Flat Acceleration Spectrum.

Figure 5.7. Response of Single Storey Buildings with Moderate Eccentricities to Flat Acceleration Spectrum

Figure 5.8. Response of Single Storey Building with Large Eccentricities to Flat Acceleration Spectrum

Figure 5.9. Response of Single Storey Buildings with Small Eccentricities to Hyperbolic Acceleration Spectrum

Figure 5.10. Response of Single Storey Buildings with Moderate Eccentricities to Hyperbolic Acceleration Spectrum

Figure 5.11. Response of Single Storey Buildings with Large Eccentricities to Hyperbolic Acceleration Spectrum

CHAPTER 6

SUMMARY AND CONCLUSIONS6.1 SUMMARY

The torsional response of buildings under earthquakes is studied in this work, with particular reference to the effect of ground rotation. It is noted in theoretical investigations by several researchers, field observations of damage to buildings as well as experimental studies on real buildings have shown that a direct cause of the torsional response could be the rotational component of the ground motion.

A three-dimensional analysis of a single storey building model is performed to evaluate its elastic response to translational and rotational components of the ground motion.

The ground motion is represented by two idealized response spectra, a translational flat acceleration spectrum and its associated rotational response spectrum and a translational hyperbolic response spectrum and its associated rotational spectrum. A modal superposition technique is used in the computation of response.

Expressions are derived for the shears and torques induced by translational as well as rotational excitations, and the effect of several parameters on the response is studied.

It is noted that to include the effect of ground rotation in a dynamic analysis for an earthquake motion, it would be essential to have

either an earthquake ground rotation history or alternatively a response spectrum for the rotational ground motion. Since recorded earthquake ground rotation histories do not exist, they must be derived from the associated translational motion records. This is achieved by assuming that the earthquake ground motion consists of plane shear waves propagating in two mutually perpendicular principal directions. The derived ground rotation histories are used in establishing rotational response spectra. A mean rotational response spectrum and a mean plus one standard deviation rotational response spectrum are obtained using ten historical earthquake records.

Elastic and inelastic responses of two single storey building models to the translational and rotational components of 1940 El Centro earthquake are obtained by using a time series analysis to investigate the effect of inelasticity as well as the effect of the rotational component of the ground motion on the response of inelastic systems.

Seismic torsional provisions in five building codes are examined and evaluated with particular reference to the effect of the rotational ground motion on the response.

6.2 CONCLUSIONS

The study of the elastic response of a single storey building leads to the following main conclusions.

Dynamic coupling between translational and rotational motions generally results in a magnification of the torsional moments over the static torques. However, the base shear in a coupled system is lower than that in the corresponding uncoupled system.

The rotational component of the ground motion has a very significant effect on the torsional response of linearly elastic systems. At times, this effect may be much more significant than that resulting purely from a dynamic coupling between the translational and rotational motions. This is particularly so for systems with uncoupled torsional frequency ω_θ higher than the uncoupled translational frequency ω_x .

The rotational component of the ground motion results in an increase in the value of the base shear, in a coupled system, and this increase is particularly significant for systems with large eccentricities and having high uncoupled translational frequency ω_x . However, even the increased base shear is, in general, lower than that in the corresponding uncoupled systems.

A rational method of computing the rotational ground motion histories from the associated translational histories is proposed. It is observed that such derivation is possible if the earthquake motion is assumed to result from plane shear waves propagating simultaneously in two mutually perpendicular principal directions which remain invariant during the entire duration of an earthquake.

Once the rotational ground motion histories have been derived, it is possible to obtain the corresponding response spectra for specified values of damping.

Elastic and inelastic time series analysis carried out with a view to studying the effect of inelasticity as well as the effect of the ground rotation on the response of two single storey building models to translational and rotational ground motions lead to the following conclusions.

The inelasticity results in a reduction of the rotational response u_θ of systems.

The rotational component of the ground motion, in general, causes a considerable increase in the ductility demand of the lateral resisting elements located on the periphery in the direction of the translational excitation, for systems whose all lateral resisting elements contributing in resisting the torques become inelastic during the excitation.

For systems where some of the lateral resisting elements contributing in resisting the torques remain elastic, the effect of the ground rotation on the ductility demand of those elements located in the direction of translational excitation is not uniform.

A study of the code provisions on torsional forces induced in buildings by earthquake motions with the result of dynamic analyses presented in this work shows that:

The seismic building codes of U.S.A.-UBC, Mexico, New Zealand and Germany, generally give lower torsional moments in buildings with small and moderate eccentricities. For buildings with large eccentricities, the U.S.A.-UBC code still gives lower values of torsional moments while the other three codes give higher values.

The design static torques specified by the NBCC 1980 give lower torsional moment for buildings with structural eccentricities e less than $0.133 D$, while it gives significantly higher values for systems with eccentricities e considerably higher than $0.133 D$.

The NBCC 1980 coupled method of analysis generally gives lower values of torsional moments for buildings with small and moderate eccentricities, but for systems with high eccentricities, provides values that are in the same range as those given by a dynamic analysis.

6.3 FUTURE WORK

The analytical study of the response of single storey building models has shown that in many situations the effect of the rotational ground motion on the response is significant. Because records of torsional ground motion do not exist, estimates of such ground motion can only be obtained from recorded translational motions. The procedure used for obtaining such estimates is based on the assumption that the earthquake motion consists of plane propagating shear waves. The errors involved in this assumption due to the presence of other waves such as compression and surface waves, in the recorded motion, can not be ascertained at this stage. A more realistic evaluation of the importance of ground motion is therefore possible only if actual records of ground rotational seismographs be developed and located at several stations to obtain records of ground rotation during earthquakes.

The conclusions drawn in this study are based on the model of a single storey building. Although they would also apply to multi-storey buildings which have a certain regularity in plan and in the distribution of stiffness, the response of buildings with combination of shear walls and frames, buildings with off-sets and other irregular structures can not be predicted on the basis of this study. There is therefore the need for carrying out additional studies on the response of multi-storey buildings with different framing system to earthquake motion and, in particular, to ground rotational motion.

REFERENCES

1. Esteva, L., Rascon, O.A. and Gutierey, A., "Lessons from Some Recent Earthquakes in Latin America", Proceedings of Fourth World Conference on Earthquake Engineering, Vol. 3, Santiago, Chile, 1969, pp. J2-58 to J2-73.
2. Shiga, T., Shibata, A., Onose, J. and Hagio, K., "Torsional Response of Structures to Earthquake Motion", Proceedings of U.S.-Japan Seminar on Earthquake Engineering, Sept. 1970, pp. 156-171.
3. Hart, G.C., DiJulio, R.M., Jr. and Lew, M., "Torsional Response of High-Rise Buildings", Journal of the Structural Division, ASCE, Vol. 101, No. ST2, Proc. Paper 11126, Feb. 1975, pp. 397-416.
4. Ayre, R.S., "Interconnection of Translational and Torsional Vibrations in Buildings", Bulletin of the Seismological Society of America, Vol. 28, No. 2, Apr. 1938, pp. 89-130.
5. Ayre, R.S., "Experimental Response of an Asymmetric, One-Story Building Model to an Idealized Transient Ground Motion", Bulletin of the Seismological Society of America, Vol. 33, No. 2, Apr. 1943, pp. 91-119.
6. Ayre, R.S., "Methods for Calculating the Earthquake Response of 'Shear' Buildings", Proceedings of the World Conference on Earthquake Engineering, Berkeley, California, June 1956, pp. 13-1 to 13-24.
7. Jacobsen, L.S. and Ayre, R.S., "Engineering Vibrations", McGraw-Hill Book Company, 1958, pp. 335-344.
8. Housner, G.W. and Outinen, H., "The Effects of Torsional Oscillations on Earthquake Stresses", Bulletin of the Seismological Society of America, Vol. 48, No. 2, Jul. 1958, pp. 221-229.
9. Bustamante, J.I. and Rosenblueth, E., "Building Code Provisions on Torsional Oscillations", Proceedings of the Second World Conference on Earthquake Engineering, Vol. 2, Tokyo, Japan, 1960, pp. 879-894.
10. Kan, C.L. and Chopra, A.K., "Coupled Lateral-Torsional Response of Buildings to Ground Shaking", Report No. EERC 76-13, Earthquake Engineering Research Center, University of California, Berkeley, California, May 1976.
11. Kan, C.L. and Chopra, A.K., "Effects of Torsional Coupling on Earthquake Forces in Buildings", Journal of the Structural Division, ASCE, Vol. 103, ST4, Proc. Paper 12876, Apr. 1977, pp. 805-819.

12. Kan, C.L. and Chopra, A.K., "Elastic Earthquake Analysis of a Class of Torsionally Coupled Buildings", Journal of the Structural Division, ASCE, Vol. 103, ST4, Proc. Paper 12877, Apr. 1977, pp. 821-838.
13. Hoerner, J.B., "Modal Coupling and Earthquake Response of Tall Buildings", Report No. EERL 71-07, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, May 1971.
14. Keintzel, E., "On the Seismic Analysis of Unsymmetric Storied Buildings", Proceedings of the Fifth World Conference on Earthquake Engineering, Paper 10, Rome, 1973.
15. Bouwkamp, J.G. and Blohm, J.K., "Dynamic Response of a Two-Story Steel Frame Structure", Bulletin of the Seismological Society of America, Dec. 1966, pp. 1289-1303.
16. Jennings, P.C., Mathiesen, R.B. and Hoerner, J.B., "Forced Vibrations of a 22-Storey Steel Frame Building", Earthquake Engineering Research Laboratory, California Institute of Technology of California at Los Angeles, Feb. 1971.
17. Tso, W.K. and Asmis, K.G., "Torsional Vibration of Symmetrical Structures", Proceedings of the First Canadian Conference on Earthquake Engineering, Vancouver, Canada, 1971, pp. 178-186.
18. Tso, W.K., "Induced Torsional Oscillations in Symmetrical Structures", International Journal of Earthquake Engineering and Structural Dynamics, Vol. 3, 1975, pp. 337-346.
19. Newmark, N.M., "Torsion in Symmetrical Buildings", Proceedings of the Fourth World Conference on Earthquake Engineering, Vol. 2, Santiago, Chile, 1969, pp. A3-19 to A3-32.
20. Veletsos, A.S., Erdik, M.O. and Kuo, P.T., "Response of Structures to Propagating Ground Motions", Structural Research Report No. 22, Department of Civil Engineering, Rice University, Houston, Texas, Apr. 1975.
21. Nathan, N.D. and Mackenzie, J.R., "Rotational Components of Earthquake Motion", Canadian Journal of Civil Engineering, Vol. 2, 1975, pp. 430-436.
22. Tso, W.K. and Hsu, T.C., "Torsional Spectrum for Earthquake Motion", Earthquake Engineering and Structural Dynamics, Vol. 6, 1978, pp. 375-382.
23. International Conference of Building Officials, "Uniform Building Code", Whittier, California, 1976.

24. Associate Committee on the National Building Code, "National Building Code of Canada", National Research Council of Canada, Ottawa, Ont., 1980.
25. Rosenblueth, E., "Seismic Design Requirements in the 1976 Mexican Code", Journal of Earthquake Engineering and Structural Dynamics, Vol. 7, 1979, pp. 49-61.
26. New Zealand Standard NZS 4203:1976, "Code of Practice for General Structural Design Loading for Buildings", Wellington, New Zealand, 1976.
27. Muller, F.P. and Keintzel, E., "Approximate Analysis of Torsional Effects in the New Seismic Code DIN 4149", Proceedings of the Sixth European Conference on Earthquake Engineering, Dubrovnik, Yugoslavia, II, 1978, pp. 101-108.
28. Skinner, R.I., Skilton, D.W.C., and Laws, D.A., "Unbalanced Buildings and Buildings with Light Towers Under Earthquake Forces", Proceedings of the Third World Conference on Earthquake Engineering, Vol. 2, Auckland and Wellington, New Zealand, 1965, pp. 586-602.
29. Shepherd, R. and Donald, R.A.H., "Seismic Response of Torsionally Unbalanced Buildings", Journal of Sound and Vibration, Vol. 6, No. 1, 1967, pp. 20-37.
30. Penzien, J., "Earthquake Response of Irregularly Shaped Buildings", Proceedings of the Fourth World Conference on Earthquake Engineering, Vol. 2, Santiago, Chile, 1969, pp. A3-75 to A3-89.
31. Rosenblueth, E. and Elorduy, J., "Response of Linear Systems to Certain Transient Disturbances", Proceedings Fourth World Conference on Earthquake Engineering, Vol. 1, Santiago, 1969, pp. A-1, 185-196.
32. Jhaveri, D.P., "Three-Dimensional Dynamic Analysis of High-Rise Buildings", Report JAB-99-36, John A. Blume and Associates Research Division, San Francisco, California, Aug. 1968.
33. Associate Committee on the National Building Code, "The Supplement of the National Building Code of Canada", National Research Council of Canada, Ottawa, Ont., 1980, Commentary K.
34. Penzin, J. and Watabe, M., "Characteristics of Three-Dimensional Earthquake Ground Motions", Earthquake Engineering and Structural Dynamics, Vol. 3, 1975, pp. 365-367.
35. Hudson, D.E. and Brady, A.G., "Strong Motion Earthquake Accelerograms", Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, Vol. II, 1971.

36. Juma, J.J., "Engineering Mathematics Handbook", Second Edition, McGraw-Hill Book Company, 1979, pp. 34-36.
37. Shanon and Wilson, Inc. and Aghabian Associates for U.S. Nuclear Regulatory Commission, "Geotechnical and Strong Motion Earthquake Data from U.S. Accelerograph Stations", Vol. 1, 1975.
38. Shibata, A., Onose, J. and Shiga, T., "Torsional Response of Buildings to Strong Earthquake Motions", Proceedings of the Fourth World Conference on Earthquake Engineering, Vol. 2, Santiago, Chile, 1969, pp. A4-123 to A4-138.
39. Kan, C.L. and Chopra, A.K., "Linear and Nonlinear Earthquake Responses of Simple Torsionally Coupled Systems", Report No. UCB/EERC-79/03, Earthquake Engineering Research Centre, University of California, Berkeley, California, Feb. 1979.
40. Clough, R.W. and Penzien, J., "Dynamics of Structures", McGraw-Hill Book Company, 1975.
41. Tso, W.K. and Dempsey, K.M., "Seismic Torsional Provisions for Dynamic Eccentricity", Earthquake Engineering and Structural Dynamics, Vol. 8, 1980, pp. 275-289.
42. Tsicnias, T.G. and Hutchinson, G.L., "Evaluation of Code Requirement for the Earthquake Design of Torsionally Coupled Buildings", Institute of Civil Engineers, Part 2, 71, Sept. 1981, pp. 821-843.
43. Tsicnias, T.G. and Hutchinson, G.L., "Evaluation of Code Torsional Provisions for Earthquake Resistant Design of Single Storey Buildings with Two Eccentricity Components", Institute of Civil Engineers, Part 2, 73, June 1982, pp. 455-464.
44. Tso, W.K. and Meng, V., "Torsional Provisions in Building Codes", Proceedings of the Third Canadian Conference on Earthquake Engineering, Montreal, Canada, Vol. I, 1979, pp. 663-684.
45. Applied Technology Council, "Tentative Provisions for the Development of Seismic Regulations for Buildings", ATC3-06.

APPENDIX I

EXPRESSIONS FOR THE NORMALIZED FORCESI.1 DERIVATIONS OF THE NORMALIZED FORCES

The uncoupled equations of motion of the system of Eqns. (2.12), Chapter 2, are given by (40):

$$\ddot{Y}_n(t) + 2\xi \omega_n \dot{Y}_n(t) + \omega_n^2 Y_n(t) = \frac{P_n(t)}{M_n} \quad (I.1)$$

where

$Y_n(t)$ is the modal coordinate for the n-th mode, at time (t)

ξ is the damping factor, assumed to be the same for all modes

ω_n is the natural frequency for the n-th mode

$M_n = \alpha_n^T [M] \alpha_n$ is the generalized mass for the n-th mode, and $[M]$ is the mass matrix of the system

and

$P_n(t) = \alpha_n^T P(t)$ is the generalized load for the n-th mode.

I.1.1 For Ground Excitation in the x Direction Only

$$P_n(t) = -\alpha_n^T [M] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \ddot{u}_{gx}$$

$$= -\alpha_n^T m[I] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \ddot{u}_{gx}$$

$$= -m[\alpha_{xn} \alpha_{\theta n} \alpha_{yn}] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \ddot{u}_{gx}$$

$$= -m \alpha_{xn} \ddot{u}_{gx}$$

and

$$M_n = \alpha_n^T M \alpha_n = \alpha_n^T m [I] \alpha_n$$

$$= m \alpha_n^T \alpha_n$$

If the mode shapes are normalized such that $\alpha_n^T \alpha_n = 1$, the generalized mass is given by:

$$M_n = m$$

and

$$\frac{P_n(t)}{M_n} = -\alpha_{xn} \ddot{u}_{gx}$$

Using response spectrum, the absolute maximum value of the modal coordinate for the n -th mode is given by:

$$Y_n(t)_{\max} = \frac{\zeta_{xn}}{M_n \omega_n^2} S_{ax}(\omega_n, \xi) \quad (I.2)$$

where

$\zeta_{xn} = \alpha_{xn} m$ is the modal earthquake-excitation factor due to the x -excitation, and

$S_{ax}(\omega_n, \xi)$ is the translational spectral acceleration.

Substitution of α_{xn} for ζ_{xn} in Eqn. (I.2) gives:

$$Y_n(t)_{\max} = \frac{\alpha_{xn}}{\omega_n^2} S_{ax}(\omega_n, \xi) \quad (I.3)$$

The maximum base forces due to the n-th mode are given by (40).

$$\begin{bmatrix} V_{xn} \\ T_n \\ V_{yn} \end{bmatrix} = m \omega_n^2 \begin{bmatrix} \alpha_{xn} \\ r \alpha_{\theta n} \\ \alpha_{yn} \end{bmatrix} Y_n(t)_{\max} \quad (I.4)$$

Substituting Eqns. (I.3) in Eqns. (I.4):

$$V_{xn} = m \alpha_{xn}^2 S_{ax}(\omega_n, \xi)$$

$$T_n = m r \alpha_{xn} \alpha_{\theta n} S_{ax}(\omega_n, \xi) \quad (I.5)$$

$$V_{yn} = m \alpha_{xn} \alpha_{yn} S_{ax}(\omega_n, \xi)$$

The normalized forces are then given by:

$$\frac{V_{xn}}{V_{xo}} = \alpha_{xn}^2 S_{ax}(\omega_n, \xi) / S_{ax}(\omega_x, \xi)$$

$$\frac{T_n}{V_{xo} r} = \alpha_{xn} \alpha_{\theta n} S_{ax}(\omega_n, \xi) / S_{ax}(\omega_x, \xi) \quad (I.6)$$

$$\frac{V_{yn}}{V_{xo}} = \alpha_{xn} \alpha_{yn} S_{ax}(\omega_n, \xi) / S_{ax}(\omega_x, \xi)$$

where $V_{xo} = m S_{ax}(\omega_x, \xi)$ is the base shear for the uncoupled system.

I.1.2 For Rotational Ground Excitation About a Vertical Direction Only

$$P_n(t) = -\alpha_n^T [M] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \ddot{u}_{g\theta}$$

$$= -m \alpha_{\theta n} \ddot{u}_{g\theta}$$

and

$$M_n = \alpha_n^T [M] \alpha_n = \alpha_n^T m [I] \alpha_n$$

$$= m \alpha_n^T \alpha_n = m$$

where

$$\alpha_n^T \alpha_n = 1$$

$$\frac{P_n(t)}{M_n} = -\alpha_{\theta n} \ddot{u}_{g\theta}$$

Using response spectrum, $Y_n(t)_{\max}$ is given by:

$$Y_n(t)_{\max} = \frac{\zeta_{\theta n}}{M_n \omega_n^2} S_{a\theta}(\omega_n, \xi) \quad (I.7)$$

where

$\zeta_{\theta n} = \alpha_{\theta n} m$ is the modal earthquake-excitation factor due to θ -excitation, and

$S_{a\theta}(\omega_n, \xi)$ is the rotational spectral acceleration.

Substitution of $\alpha_{\theta n} m$ for $\zeta_{\theta n}$ in Eqns. (I.7) gives:

$$Y_n(t)_{\max} = \frac{\alpha_{\theta n}}{\omega_n^2} S_{a\theta}(\omega_n, \xi) \quad (I.8)$$

The maximum base forces due to the n-th mode are given by:

$$\begin{bmatrix} V_{xn} \\ T_n \\ V_{yn} \end{bmatrix} = m \omega_n^2 \begin{bmatrix} \alpha_{xn} \\ r \alpha_{\theta n} \\ \alpha_{yn} \end{bmatrix} Y_n(t)_{\max} \quad (I.9)$$

Substituting Eqn. (I.8) into Eqn. (I.9):

$$\begin{aligned} V_{xn} &= m \alpha_{xn} \alpha_{\theta n} S_{a\theta}(\omega_n, \xi) \\ T_n &= m r \alpha_{\theta n}^2 S_{a\theta}(\omega_n, \xi) \end{aligned} \quad (I.10)$$

$$V_{yn} = m \alpha_{yn} \alpha_{\theta n} S_{a\theta}(\omega_n, \xi)$$

The normalized forces are then given by:

$$\begin{aligned} \bar{V}_{xn} &= \frac{V_{xn}}{V_{xo}} = \alpha_{xn} \alpha_{\theta n} S_{a\theta}(\omega_n, \xi) / S_{ax}(\omega_x, \xi) \\ \bar{T}_n &= \frac{T_n}{V_{xo} r} = \alpha_{\theta n}^2 S_{a\theta}(\omega_n, \xi) / S_{ax}(\omega_n, \xi) \\ \bar{V}_{yn} &= \frac{V_{yn}}{V_{xo}} = \alpha_{yn} \alpha_{\theta n} S_{a\theta}(\omega_n, \xi) / S_{ax}(\omega_n, \xi) \end{aligned} \quad (I.11)$$

where $V_{xo} = m S_{ax}(\omega_x, \xi)$ is the base shear for the uncoupled system due to the x-excitation.

Figure II.1. Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum, $e_y/a = 0.025$ and $e_y/a = 0$

Figure II.2. Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum, $e_y/a=0.1$ and $e_y/a=0$

Figure II.3. Response of One-Way Torsionally Coupled System to Hyperbolic Acceleration Spectrum, $e_y/a=0.4$ and $e_y/a=0$

Figure II.4. Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum

Figure II.5. Effect of the Aspect Ratio $\frac{b}{a}$ on the Response of One-Way Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum

Figure II.6. Response of General Torsionally Coupled System to Hyperbolic Acceleration Spectrum ($e_x/b=0.1, e_y/a=0.025$) and ($e_x/b=e_y/a=0$).

Figure II.7. Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum ($e_x/b=0.1, e_y/a=0.1$) and ($e_x/b=e_y/a=0$)

Fig. II.8. Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum ($e_x/b=0.1$, $e_y/a=0.4$) and ($e_x/b=e_y/a=0$)

Figure II.9. Effect of the Plan Eccentricity e_y and the Transit Time τ on the Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum

Figure II.10. Effect of the Aspect Ratio $\frac{b}{a}$ on the Response of General Torsionally Coupled Systems to Hyperbolic Acceleration Spectrum

Figure II.11. Variation of the Response with $\frac{\omega_y}{\omega_x}$ to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x = 0.8$

Figure II.12. Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x = 1$

Figure II.13. Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x = 1.2$

Figure II.14. Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x = 1.5$

Figure II.13. Variation of the Response with ω_y/ω_x to Hyperbolic Acceleration Spectrum, $\omega_\theta/\omega_x = 2$

Let A and B be defined respectively by (ϕ_1, θ_1) and (ϕ_2, θ_2) , where ϕ and θ denote respectively the longitude and the latitude of a point on the surface of the earth.

From Fig. III.1, the angular measures of sides a and b and the sustained angle γ of the spherical triangle ABC are given by:

$$\begin{aligned} a &= 90^\circ - \phi_2 \\ b &= 90^\circ - \phi_1 \\ \gamma &= \theta_2 - \theta_1 \end{aligned} \quad (\text{III.1})$$

The major principal direction X is defined by the angle β which is given by (36):

$$\beta = X_1 - Y_1 \quad (\text{III.2})$$

in which

$$X_1 = \tan^{-1} \frac{\cos(\frac{\gamma}{2}) \cos(\frac{a-b}{2})}{\sin(\frac{\gamma}{2}) \cos(\frac{a+b}{2})} \quad (\text{III.3})$$

$$Y_1 = \tan^{-1} \frac{\cos(\frac{\gamma}{2}) \sin(\frac{a-b}{2})}{\sin(\frac{\gamma}{2}) \sin(\frac{a+b}{2})} \quad (\text{III.4})$$

Example:

For 1940 El Centro earthquake, the epicentre A and the instrument location B are respectively given by (35):

$$(\phi_1, \theta_1) = (32.733^\circ \text{N}, 115.45^\circ \text{W})$$

$$(\phi_2, \theta_2) = (32.795^\circ \text{N}, 115.548^\circ \text{W})$$

Substitution of the coordinates of A and B in Eqn. (III.1) gives:

$$a = 57.205^\circ$$

$$b = 57.267^\circ$$

$$\gamma = 0.0986^\circ$$

Substituting the values of a , b and γ in Eqns. (III.3) and (III.4):

$$X_1 = 89.973^\circ$$

$$Y_1 = -36.780^\circ$$

Equation III.2 now gives:

$$\beta = 126.75^\circ$$

which means that the major principal direction X makes an angle of -53.25° with the North-South direction or an angle of -36.75° with the East-West direction. The other principal direction Y should be perpendicular to X .

END

0	5	H	0	9	H	8	4
---	---	---	---	---	---	---	---

FIN