INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI[®]

Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

Model-Reduction Techniques for High-Speed Interconnect Analysis

by

Ramachandra Achar, B. E., M. E.,

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada

© Ramachandra Achar, 1998

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-37040-2

The undersigned recommend to the Faculty of Graduate Studies and Research
acceptance of this thesis

“Model-Reduction Techniques for High-Speed Interconnect Analysis”

submitted by Ramachandra Achar (B.E., M. E.) in partial fulfillment of the
requirements for the degree of Doctor of Philosophy

Chairman, Department of Electronics
Professor Jim Wight

Thesis Supervisor
Professor Michel Nakhla

External Examiner
Professor Nicholas Rumin
Department of Electrical and Computer Engineering, McGill University

Carleton University

September 24, 1998

Abstract

During recent years the intense drive for signal integrity has been at the forefront of rapid and new developments in CAD algorithms focussed on microelectronics. As signal frequencies are approaching the GHz range, the interconnect effects such as delay, crosstalk, ringing and distortion become the dominant factors limiting the overall performance of VLSI systems. At relatively higher frequencies, interconnects can no longer be treated as lumped components; instead distributed, full-wave and measured models become necessary. However, simulation of large number of interconnects with high-frequency models using conventional simulators such as SPICE becomes prohibitively CPU expensive and also suffers from mixed frequency/time difficulties. Recently, several model-reduction algorithms, such as asymptotic waveform evaluation (AWE) and complex frequency hopping (CFH) have been introduced to enable rapid simulation of large interconnect networks. However, in the case of multiport subnetworks, the macromodels obtained using these techniques become inefficient for the purpose of nonlinear simulation. In addition, the reduction algorithms available in the literature can neither handle full-wave and measured subnetworks efficiently nor guarantee the passivity of reduced-order models for such networks.

In this thesis, a novel algorithm is presented for efficient model-reduction of multiport interconnect subnetworks. The proposed technique enables the generation of time-domain macromodels with fewer states and facilitates efficient simulation of large interconnect networks in the presence of nonlinear elements. In addition, a new scheme to preserve the passivity of macromodels is presented to ensure stable nonlinear simulation. Also new techniques for simulation of full-wave and measured subnetworks using model-reduction algorithms are described. The proposed algorithm provides 1 - 3 orders of speed up compared to SPICE and is useful for accurate and efficient transient response estimation involving linear subnetworks in VLSI chip packaging, multi chip modules, printed circuit boards and in VLSI systems design.

Acknowledgments

I would like to thank Professor Michel Nakhla for the lively discussions, helpful suggestions and the general encouragement. His friendly manner, ability to motivate when the things are seeming difficult, patience and the good judgement made the pursuit of this thesis an enjoyable and stimulating experience. I feel privileged to have him as my thesis supervisor.

Professor Q. J. Zhang contributed continuously to this research. His comments, ideas and reviews were very helpful in producing this thesis. Also, my special thanks are due to Dr. Eli Chiprout of IBM research labs, Austin, Texas for useful discussions and getting this thesis started on right tracks.

I would like to acknowledge and thank several of my colleagues, R. Khazaka, R. Griffith, M. Srinivasan, D. G. Liu, F. Wang, I. Erdin, P. Kumar, X. Li, A. Dounavis, M. Kolbehdari and G. Zheng for helpful discussions, contributions and relaxing/refreshing chats. I would also like to express my gratitude to the professors, staff and students at the department of electronics for making my work an enjoyable experience. I also thank specially some of my friends, Srinath, Venkatesh, Roopa, Anil, Brenda, Satish, Sheela, Subodh, Sonali, Praveen, Ashutosh and Govind for making my stay in Ottawa cheerful.

For financial support, thanks to NSERC, MICRONET, CMC, SMC, CITO, NORTEL and GENNUM, without which I wouldn't have enjoyed those conferences and publications. Thank you for recognizing this research through 1996 CMC, 1997 SMC and 1998 MICRONET awards.

I leave a special note of appreciation for my wife, Sumana. Without her continued loving support, encouragement, patience and good humor, working on this thesis would have been very hard. Hope to compensate for the late working hours with the completion of this thesis. Finally it is my parents, for years of patient backing in everything, I am grateful to you. I believe your patience finally paid off.

Dedicated to my mother

Jambavati Raghavendra Achar

for showering boundless patience, love and affection on her kids

Table of Contents

<i>Abstract</i>	<i>i</i>
<i>Acknowledgments</i>	<i>ii</i>
<i>Glossary of Terms</i>	<i>xii</i>
 CHAPTER 1	
Introduction	1
1.1 Background and Motivation.	1
1.2 Contributions	7
1.3 Organization of the Thesis	10
 CHAPTER 2	
Interconnect Modeling and Simulation	11
2.1 Interconnect Models.	11
2.1.1 Lumped Models	14
2.1.2 Distributed Transmission Line Models.	15
2.1.3 Full-Wave Models.	17
2.1.4 Measured Subnetworks.	18
2.2 Interconnect Simulation Issues	19
2.2.1 Background on Circuit Simulation	19
2.2.1 Discussion of CPU Cost	20
 CHAPTER 3	
Unified Formulation of Circuit Equations	22
3.1 General Formulation of Network Equations	22
3.2 Formulation of Linear Subnetwork	25
3.2.1 Lumped Components	25
3.2.2 Distributed Elements	26
3.2.3 Full-Wave Models.	27
3.2.4 Measured Subnetworks.	28
3.3 Example	30

CHAPTER 4	Review of Moment-Matching Techniques	33
4.1	Principles of Model-Reduction Techniques	33
4.2	<i>Padé Approximations</i>	35
4.3	<i>Computation of Moments</i>	38
4.3.1	<i>Generalized Computation of Moments</i>	39
4.4	<i>Limitations of Single Expansion MMT Algorithms</i>	41
4.5	<i>Complex Frequency Hopping</i>	42
4.5.1	Selection and Minimization of Hops in CFH	43
4.5.2	Selection of f_{max}	46
4.6	<i>Reduced-Order Models of Multiport Linear Networks</i>	48
4.5	<i>Limitations of CFH and Other MMT Algorithms</i>	50
CHAPTER 5	Block Complex Frequency Hopping for Multiport Networks ..	52
5.1	New Residue Computation Algorithm	52
5.2	Selection of Dominant Pole-Set	55
5.3	Proposed Block CFH Algorithm	58
5.4	Passivity Preservation	59
5.4.1	Review of Passivity Properties	59
5.4.2	Proposed Passivity Preservation Algorithm	64
5.4.3	Selection of $\Psi(s)$	67
5.5	<i>Macromodel Synthesis</i>	69
5.5.1	Including Complex Poles in the State Matrix	72
5.6	<i>Unified Transient simulation</i>	74
5.6.1	Conversion of Macromodels to Equivalent Subcircuits	75
5.6.2	Summary of Computational Steps	77
5.7	Numerical Examples	77
5.8	CPU and Accuracy Analysis	90

CHAPTER 6	Efficient Simulation of Full-Wave Models Using Block CFH ..	93
6.1	Background	93
6.2	Generalized Stencil for Full-Wave Subnetworks	95
6.2.1	Current-Eigenvector Matrix	96
6.2.1	Modal Characteristic Impedance	97
6.2.1	Generalized Full-Wave Stencil	98
6.3	Block CFH Based Simulation of Full-Wave Models	98
6.2.1	Full-Wave Transmission Line Moments	99
6.2.1	Summary of Computational Steps	102
6.4	Numerical Examples	103
CHAPTER 7	New Simulation Technique for Measured Subnetworks	117
7.1	Background	117
7.2	Generalized Stencil for Measured Subnetworks	118
7.3	Computation of Measured Subnetwork Moments	126
7.3.1	Comparison of Moment-Generation Algorithms	126
7.3.1	Proposed Algorithm for Moments of Measured Data.	128
7.3.1	Time-Limited Responses	132
7.3.1	Summary of Computational Steps	135
7.4	Numerical Examples	135
CHAPTER 8	Conclusions and Future Work	146
8.1	Summary	146
8.2	Future Research	147
<i>Appendix A</i>		<i>151</i>
<i>Appendix B</i>		<i>157</i>
<i>References</i>		<i>161</i>

List of Figures

Fig. 1.1.	High-speed interconnect effects	2
Fig. 1.2.	Transient response illustrating Gibbs' phenomenon.	5
Fig. 2.1.	Interconnect hierarchy	12
Fig. 2.2.	Top and cross-sectional view of an example interconnect system	12
Fig. 2.3.	Lumped component models	14
Fig. 2.4.	Distributed transmission line model	16
Fig. 2.5.	Cross-sectional view of a multiconductor dispersive system	17
Fig. 2.6.	Example circuit for MNA formulation	20
Fig. 3.1.	Nonlinear network containing linear subnetwork	23
Fig. 3.2.	Subnetwork with distributed, full-wave and measured components . .	25
Fig. 3.3.	Crcuit with lumped/distributed/measured & nonlinear devices	30
Fig. 4.1.	Illustration of dominant poles	34
Fig. 4.2.	Summary of the steps involved in the MMT algorithm	40
Fig. 4.3.	Illustration of CFH	42
Fig. 4.4.	Transfer function based binary search algorithm	44
Fig. 4.5.	Graphical illustration of transfer function based search algorithm. . . .	44
Fig. 4.6.	Pole-matching based binary search algorithm	45
Fig. 4.7.	Frequency spectrum of trapezoidal pulse	47
Fig. 4.8.	Multiport linear subnetwork	48
Fig. 5.1.	Illustration of the new residue computation algorithm	53
Fig. 5.2.	Pole distribution for 3-port transmission line circuit	57
Fig. 5.3.	Accurate frequency response (matched up to 4GHz)	61
Fig. 5.4.	Time response with oscillations due to passivity violation.	61
Fig. 5.5.	Illustration of passivity violation	62
Fig. 5.6.	Frequency response matched up to the highest pole (5.22 GHz)	62
Fig. 5.7.	Passivity conditions satisfied	63

Fig. 5.8.	Time response	63
Fig. 5.9.	Illustration of the general behavior of $\Psi(s)$	65
Fig. 5.10.	Illustration of possible passivity violation	65
Fig. 5.11.	Illustration of the new passivity algorithm	66
Fig. 5.12.	Graphical illustration of restriction on residues	69
Fig. 5.13.	Illustration of macromodel synthesis for nonlinear simulation	70
Fig. 5.14.	Illustration of equivalent subcircuit generation from macromodels . . .	76
Fig. 5.15.	Nonlinear circuit containing RLC lumped network	78
Fig. 5.16.	Frequency responses for Example 1	80
Fig. 5.17.	Eigenvalues for Example 1	80
Fig. 5.18.	Time responses for Example 1	81
Fig. 5.19.	Seven distributed transmission line circuit (Example 2).	84
Fig. 5.20.	Frequency responses (magnitude) for Example 2.	85
Fig. 5.21.	Frequency responses (real-part) for Example 2	86
Fig. 5.22.	Eigenvalues for Example 2	87
Fig. 5.23.	Time responses for Example 2	88
Fig. 5.24.	Time responses for Example 2	89
Fig. 6.1.	Cross-sectional view of a multiconductor dispersive system	94
Fig. 6.2.	Relationship between modal and circuit parameters	97
Fig. 6.3.	Illustration of polynomial break points.	101
Fig. 6.4.	Circuit with seven full-wave modeled interconnects (Example 1) . . .	104
Fig. 6.5.	Modal parameters for Example 1	105
Fig. 6.6.	Frequency responses for Example 1	106
Fig. 6.7.	Transient response (Example 1)	106
Fig. 6.8.	Transient response - nonlinear termination (Example 1)	107
Fig. 6.9.	Transient response (Example 2)	107
Fig. 6.10.	Circuit for Example 3	108
Fig. 6.11.	Modal parameters for Example 3.A ($h_1 = 5\text{mm}$)	109
Fig. 6.12.	Frequency responses for Example 3.A ($h_1 = 5\text{mm}$)	110

Fig. 6.13.	Frequency responses for Example 3.A-1 ($h_1 = 10\text{mm}$)	112
Fig. 6.14.	Frequency responses for Example 3.A-2 ($w = 2.5\text{mm}$)	113
Fig. 6.15.	Frequency responses for Example 3.A-3 ($s = 2.5\text{mm}$)	114
Fig. 6.16.	Transient responses at node V_{out1} for Example 3.A-3 ($s = 2.5\text{mm}$) . .	114
Fig. 6.17.	Transient responses at node V_{out2} for Example 3.A-3 ($s = 2.5\text{mm}$) . .	115
Fig. 6.18.	Frequency responses for Example 3.B	116
Fig. 7.1.	Illustration of a general measurement setup	119
Fig. 7.2.	Circuit for illustration of formulation of measured subnetworks	123
Fig. 7.3.	Comparison of derivatives for $h(t)$	128
Fig. 7.4.	Time response for $h(t)$	133
Fig. 7.5.	Comparison of derivatives of truncated time response	134
Fig. 7.6.	Comparison of frequency-responses	134
Fig. 7.7.	Circuit for Example 1	136
Fig. 7.8.	Time-domain measurements at nodes v_1 and v_2 for Example 1	137
Fig. 7.9.	Time responses for Example 1	137
Fig. 7.10.	Circuit for Example 2 - mixed simulation environment	138
Fig. 7.11.	Time responses for Example 2	139
Fig. 7.12.	Time responses for Example 3	140
Fig. 7.12.	Circuit for Example 4	141
Fig. 7.14.	Time responses for Example 4 (V_{out1})	142
Fig. 7.15.	Time responses for Example 4 (V_{out2})	142
Fig. 7.16.	Frequency responses for Example 5 (with noise of type (a))	144
Fig. 7.17.	Time responses for Example 5 (with noise of type (a))	144
Fig. 7.18.	Time responses for Example 5 (with noise of type (b))	145
Fig. A.1.	Effective dielectric constant v/s number of terms (P)	155
Fig. B.1.	Scattering system for a N -conductor structure for mode i excitation .	157

List of Tables

Table. 2.1.	Interconnect Technologies	13
Table. 5.1.	Number of poles required for each Y_{ij} using conventional techniques .	79
Table. 5.2.	Poles and residues obtained using the BCFH (Example 1)	79
Table. 5.3.	Number of poles required for each Y_{ij} using conventional techniques .	83
Table. 5.4.	CPU comparison (Sun SPARC10)	83
Table. 6.1.	CPU comparison of conventional and BCFH (in secs, SPARC10) ..	105
Table. 7.1.	Comparison of derivatives for $h(t)$	127
Table. 7.2.	Derivatives of truncated time response	133
Table. 7.3.	CPU comparison of FFT and the proposed BCFH (in seconds).	140

Glossary of Terms

AWE	Asymptotic Waveform Evaluation
BCFH	Block Complex Frequency Hopping
CAD	Computer Aided Design
CPU	Central Processing Unit
FEM	Finite element Method
FD	Frequency Dependent
FDTD	Finite-Difference Time-Domain
EMI	Electromagnetic Interference
LMS	Least-Mean Squares
LU	Lower-Upper
KCL	Kirchoff's Current Law
MCM	Multichip Module
MMT	Moment Matching Technique
MM	Macromodel
MNA	Modified Nodal Analysis
PACT	Pole Analysis via Congruent Transformation
PCB	Printed Circuit Board
PR	Positive Real
PRIMA	Passive Reduced Order Interconnect Macromodel Algorithm
PVL	Padé via Lanczos
SDT	Spectral Domain Technique
SPICE	Simulation Program with Integrated Circuits Emphasis
TDR	Time-Domain Reflectometry
TEM	Transverse Electromagnetic
TF	Transfer Function

CHAPTER 1

Introduction

“Any sufficiently advanced technology is indistinguishable from magic.” - Arthur Clarke

1.1 Background and Motivation

With the rapid developments in the VLSI technology, design and CAD techniques, at both the chip and package level, the central processor cycle times are reaching the vicinity of 1ns and communication switches are being designed to transmit data that have bit rates faster than 1Gb/s. The ever increasing quest for high-speed applications is placing higher demands on interconnect performance and highlighted the previously negligible effects of interconnects (Fig. 1.1), such as ringing, signal delay, distortion, reflections and crosstalk [1] - [27]. In addition, the trend in the VLSI industry towards miniature designs, low power consumption and increased integration of analog circuits with digital blocks has further complicated the issue of signal integrity analysis. Thousands of engineers, intent on best design possible, use SPICE [5] on a daily basis for analog simulation and general purpose circuit analysis. However, the high-speed interconnect problems are not always handled appropriately by the present levels of SPICE. If not considered during the design stage, these interconnect effects can cause logic glitches which render a fabricated digital circuit inoperable, or they can distort an analog signal such that it fails to meet specifications. Since extra iterations in the design cycle are costly, accurate

prediction of these effects is a necessity in high-speed designs. Hence it becomes extremely important for designers to simulate the entire design along with interconnect subcircuits as efficiently as possible while retaining the accuracy of simulation [10], [25], [20] - [22], [28] - [88].

Fig. 1.1. High-speed interconnect effects

Interconnections can be from various levels of design hierarchy, such as on-chip, packaging structures, multichip modules, printed circuit boards and backplanes. Depending on the operating frequency and nature of the structure, these interconnects can be modeled as *lumped components* (RC or RLC) [1] - [3], *distributed elements* (frequency independent/dependent RLCG parameters, lossy, coupled) [1] - [3], [18], [22], *full-wave* [10] - [16], [25], [68], [69] or *measured* linear subnetworks [20], [21], [24], [79] - [88]. However, transient simulation of interconnects is associated with two major bottlenecks. First, interconnect networks can sometimes consist of thousands of components, such as resistors, capacitors, inductors, transmission lines and other levels of interconnect models, which make the simulation CPU expensive. Secondly, at the terminations there

generally exist some nonlinear elements such as drivers and receivers, which are described only in the time-domain, while the high-frequency models such as distributed/full-wave elements are best described in the frequency-domain. Such a simultaneous (*mixed frequency/time*) formulation can't be handled by a traditional ordinary differential equation solver such as SPICE. These two hurdles lead to intense research in the CAD area during the past several years to develop efficient transient simulation algorithms for accurate signal integrity analysis [10], [25], [20] - [22], [28] - [88].

These techniques are based on the logic that, generally interconnect models tend to add large number of nodes to the circuitry, due to which the nonlinear simulation becomes practically difficult under the given design-time and memory constraints. Hence in order to make interconnect simulation CPU efficient it was found useful to reduce the order of the linear subnetwork prior to performing nonlinear simulation. The process of finding such a *reduced-order frequency-domain model* characterizing the terminal (*port*) behavior of the original large linear subnetwork is known as *model-reduction*.

Model-reduction techniques available in the literature are based on *moment-matching* and *Padé approximation* [34] - [79], [84] - [91]. The crux of moment-matching techniques (*MMT*) is the extraction of a small set of dominant poles and residues of a large network. The lower-order approximation is obtained using the Taylor series coefficients (*moments*) of the system and performing Padé approximation on the Laplace-domain transfer function. Such reduced-order models can be used to estimate the transient response of a linear network over a predetermined range of excitation frequencies. Reduced-order frequency-domain models can be easily translated into a set of ordinary differential equations, thus enabling a mixed frequency/time simulation, [39] - [44], [61]. Derivation of

differential equations from reduced-order models is referred as *macromodel synthesis*. Another prime advantage of MMTs is the speed improvement which ranges from 100 to 1000 times faster compared to the SPICE. Recently developed important transient simulation techniques based on moment-matching are *asymptotic waveform evaluation (AWE)* [34], [35], *complex frequency hopping (CFH)* [35], [47] - [49], [61], *Padé via Lanczos (PVL)* [51], [52], *pole analysis via congruent transformation (PACT)* [55], *Arnoldi* [53], [54], [58], [59] and *passive reduced order interconnect macromodeling algorithm (PRIMA)* [57].

AWE is based on the Taylor expansion and Padé approximation at $s = 0$. However, since Padé approximation is accurate only near the point of expansion, reduced-order models obtained through AWE suffers from accuracy and stability problems. This problem was addressed by the introduction of CFH [47]. CFH ensures the accuracy of an approximation for a desired frequency range using multiple expansion points (*hops*) and corresponding Padé approximations at the frequencies of interest. Additional hops are generated at an incremental CPU expense above the cost of the first. CFH can directly handle distributed interconnect models (without discretization) [22], [25], [47] - [49]. However, in the case of multiport distributed networks, *the reduced-order models obtained using these techniques become inefficient for the purpose of nonlinear simulation*. This is because the macromodels for such networks from these techniques tend to contain large number of states, which makes the transient simulation CPU expensive.

The other alternative approaches, PVL and Arnoldi are based on the Krylov-subspace methods. These techniques overcome the ill-conditioning associated with the direct Padé approximation and are very efficient for simulating large *lumped linear networks* in the presence of nonlinear components. But one of the limitations of these methods is that, in order to handle distributed transmission

lines, Telegrapher's equations (partial differential equations) are discretized into a set of ordinary differential equations. However, such an approach suffers from several disadvantages: (a) Large number of sections are required in order to ensure reasonable accuracy, which leads to large circuit matrices. This problem deteriorates with higher frequencies or sharper rise/fall times. (b) Discretization of transmission lines leads to Gibbs' phenomenon, which becomes more pronounced at higher frequencies [6]. (c) The method cannot directly handle transmission lines with frequency-dependent parameters. The difficulties (a) and (b) are illustrated through a numerical example below.

(a) MCM interconnect network

(b) Transient response

Fig. 1.2. Transient response illustrating Gibbs' phenomenon

The network under consideration is given in Fig. 1.2(a) and it consists a distributed MCM interconnect [48]. The network is excited with a trapezoidal pulse having rise/fall times of 0.1ns and pulse width of 1ns. The transient responses at node V_{out} using both the distributed and lumped models are given in Fig. 1.2(b). In this experiment 300 lumped π sections were considered (which leads to a MNA size of approximately 900×900) whereas the distributed stamp was treated with a 2×2 Telegrapher's stamp. As seen, in spite of using such a high-number of sections, the lumped approximation exhibits spurious oscillations due to the Gibbs' phenomenon. In addition, a major limitation of all the above mentioned model-reduction techniques is that they require analytical models for interconnects to compute system moments [20], [21], [84] - [88] and *they can't handle full-wave models or measured subnetworks* [25].

Also, it is demonstrated that in addition to macromodel stability, macromodel passivity is required to guarantee the overall circuit stability during unified transient simulation of the macromodel and rest of the circuitry [55] - [61]. Passivity implies that a network cannot generate more energy than it absorbs, and no passive termination of the network will cause the system to go unstable. The loss of passivity can be a serious problem because transient simulations of reduced networks may encounter artificial oscillations. Although, it is possible to obtain an asymptotically stable model by simply discarding the unstable poles during AWE/CFH, passivity is not guaranteed. The PACT algorithm [55], which is based on congruent transformation, introduced a new direction for obtaining passive reduced-order models of lumped RC networks. Efficient schemes based on congruent transformation for preservation of passivity during the reduction of RLC networks is available in the literature [56] - [59]. *However, none of the model-reduction algorithms which currently exist, preserve passivity for distributed/full-wave/measured interconnect models.*

1.2 Contributions

The subject of this thesis is the simulation of multiport high-speed interconnect networks in the presence of nonlinear elements. The simulated interconnect models can include diverse elements such as lumped RLC elements, distributed transmission lines with frequency independent/dependent parameters, full-wave models, EMI subnetworks and measured subnetworks with time or frequency-domain characterized data. The work presented here provides an easy means to handle high-speed interconnect models in industrial grade simulators such as SPICE. Specifically, in view of the above-mentioned problems, the main contributions of this thesis are as follows.

- 1) An efficient multiport algorithm, namely block complex frequency hopping (BCFH), is developed for reduction of high-speed interconnect subnetworks (chapter 5). The proposed algorithm generates time-domain macromodels with fewer states and facilitates fast and accurate simulation of interconnect subnetworks in the presence of nonlinear elements [49], [61], [64]. The prime advantage of the proposed technique is that it can handle diverse interconnect models, such as *distributed*, *full-wave* and *measured* subnetworks, without any restriction on the number of interconnects. In addition, the new algorithm puts no restrictions on the highest frequency range and provides a scheme to preserve the frequency-spectrum of the original network up to the maximum frequency of interest. The proposed technique provides 1 - 3 orders of speed-up compared to existing simulation techniques.
 - 2) A new residue computation algorithm has been developed so as to improve the accuracy of reduced-order models (chapter 5) [49], [61]. Also a new scheme for selection of dominant poles of multiport networks is developed
-

(chapter 5). This helps to reduce the number of poles in the lower-order model and leads to compact macromodels [49], [61].

- 3) A novel passivity preservation algorithm is developed for networks containing diverse interconnect models such as *distributed transmission lines with frequency independent/dependent parameters, full-wave and measured* subnetworks (chapter 5) [60], [61]. Thus far there is no known algorithm for preserving the passivity during the reduction of networks containing above mentioned type of interconnect models. The proposed method is based on compensating for any loss of passivity during the network reduction. It is to be noted that the new passivity preservation algorithm is not just limited to the block CFH technique but can be easily used in conjunction with any other model-reduction techniques which can generate accurate pole-residue models in the frequency domain.
 - 4) An efficient technique for macromodel synthesis from frequency-domain reduced-order models is presented (chapter 5). The method also discusses the inclusion of complex poles during state-space synthesis. In addition, a new technique for converting macromodels described in terms ordinary differential equations to an equivalent subcircuit is presented.
 - 5) A generalized technique to include full-wave and measured subnetworks in modified nodal analysis is presented. The proposed technique is useful in extending the model-reduction algorithms to full-wave/measured models (chapters 3, 6, 7) [10], [68], [69], [20], [21], [87], [88].
 - 6) A new algorithm is presented for computation of full-wave moments (chapter 6). The techniques available in the literature are limited to quasi-TEM
-

assumptions and none of these techniques can handle full-wave models. In addition, a comparison of quasi-TEM and full-wave models at higher frequencies is presented (chapter 6). The study investigates the need for a full-wave analysis in general interconnect circuits [10], [69].

- 7) A new technique based on time-domain integration is presented for efficient and accurate computation of measured subnetwork moments (chapter 7). The proposed method overcomes the ill-conditioning problem associated with the previous moment-computation techniques, which are based on the numerical differentiation [20], [21], [87], [88]. The proposed method can handle devices characterized by both frequency and time-domain measurements. In addition, a new scheme based on least mean square optimization is developed to handle subnetworks with overdetermined measurements.
 - 8) An algorithm for minimum-realization of given reduced-order models is developed. This further helps to reduce the number of states in a macro-model [43], [44], [75] and leads to efficient simulation.
 - 9) A new algorithm based on CFH is developed for efficient solution of finite element formulated electromagnetic problems. The proposed method gives 2-3 orders of speed-up compared to conventional EM solution techniques [50], [78], [76], [74]. Also, a novel algorithm is developed for mixed simulation of EM devices in a general circuit environment [25], [77].
 - 10) A new technique based on matrix-Padé is developed for discretization of transmission line models. This technique yields better accuracy compared to conventional lumped discretization [27].
-

1.3 Organization of the Thesis

This thesis is organized as follows. In chapter 2, a brief discussion of interconnection issues and various possible models is given. This chapter also provides some background on circuit simulation and highlights the difficulties encountered by conventional simulators while handling high-speed interconnect models. Chapter 3 presents a new unified formulation scheme for circuits containing nonlinear elements and various high-speed interconnects such as distributed, measured and full-wave models. Chapter 4 outlines the principles of model-reduction techniques used in interconnect simulation algorithms. In addition, this chapter lists the limitations of current model-reduction techniques which limit their application in industrial level simulators. Chapter 5 presents the new algorithm for efficient simulation of multiport distributed interconnect networks in the presence of nonlinear elements. This chapter also presents the new residue computation and dominant pole-search algorithms. It also details the new passivity preservation algorithm and unified transient simulation scheme. It also discusses macromodel synthesis from reduced-order models. Chapter 6 describes a new technique to include full-wave models in SPICE-like simulators. It also presents the new full-wave moment-computation algorithm and a comparison of quasi-TEM and full-wave models. Chapter 7 describes a new technique to include measured subnetworks in a MNA analysis and also a new algorithm for computation of moments of measured subnetworks. Conclusions are presented in chapter 8.

"I invented the cordless extension cord." - Steven Wright

As discussed in chapter 1, issues related to interconnect analysis has been very challenging to the next generation of circuit simulators. Interconnect analysis is two fold; finding an appropriate model for the interconnect and the development of a solution algorithm. Depending on the desired accuracy, frequency of operation and nature of the structure, different models could be used. Each model presents different degrees of challenges for the simulator. In this chapter, various interconnect models are discussed, and also a background on solution algorithms is presented.

2.1 Interconnect Models

Designers are reducing the wiring cross-section and trying to pack the lines together, while at the same time innovating on signals with faster rise times. Reducing the wiring dimensions result in appreciably resistive lines. In addition, all these interconnections may have nonuniform cross-sections caused by discontinuities such as connectors, vias, wire bonds, flip-chip solder balls, redistribution leads and orthogonal lines. Interconnections can be from various levels of design hierarchy (Fig. 2.1) such as on-chip, packaging structures, multichip modules, printed circuit boards and backplanes. Fig. 2.2 shows the top

and cross sectional view of an example interconnect system. On a broader perspective, interconnection technology can be classified into five categories [4], as shown in Table 2.1, namely, on-chip wiring, thin-film wiring, ceramic carriers, thin-film wiring, printed circuit boards and shielded cables.

Fig. 2.1. Interconnect hierarchy

Fig. 2.2. Top and cross-sectional view of an example interconnect system

Table. 2.1. Interconnect Technologies

<i>Interconnection type</i>	<i>Line Width (μm)</i>	<i>Line Thickness (μm)</i>	<i>Line Resistance (Ohm/cm)</i>	<i>Maximum Length (cm)</i>
<i>On-chip</i>	<i>0.5 - 2</i>	<i>0.7 - 2</i>	<i>100 - 1000</i>	<i>0.3 - 1.5</i>
<i>Thin-film</i>	<i>10 - 25</i>	<i>5 - 8</i>	<i>1.25 - 4</i>	<i>20 - 45</i>
<i>Ceramic</i>	<i>75 - 100</i>	<i>16 - 25</i>	<i>0.4 - 0.7</i>	<i>20 - 50</i>
<i>Printed-circuit-board</i>	<i>60 - 100</i>	<i>30 - 50</i>	<i>0.06 - 0.08</i>	<i>40 - 70</i>
<i>Shielded cables</i>	<i>100 - 450</i>	<i>35 - 450</i>	<i>0.0013 - 0.033</i>	<i>150 - 500</i>

An important criterion used for classifying the interconnects is the *electrical length* of an interconnect. An interconnect is considered to be “*electrically short*”, if at the highest operating frequency of interest, the interconnect length is physically shorter than one-tenth of the wave-length (i.e., $length\ of\ the\ interconnect/\lambda < 0.1$, $\lambda = v/f$). Else the interconnect is referred as “*electrically long*” [1], [6]. Electrically short interconnects can be represented by lumped models whereas electrically long interconnects need quasi-TEM distributed or full-wave models. For the categories shown in Table 1, the wave-length is in the order of 1 - 10cm. The propagated signal rise times are in the range 100-1000ps. Hence the line lengths are either comparable or much longer than the signal wavelengths. Depending on the operating frequency, signal rise times and nature of the structure, these interconnects can be modeled as lumped (RC or RLC), distributed (frequency independent/dependent RLCG parameters, lossy, coupled), full-wave models or measured linear subnetworks.

2.1.1 Lumped Models

In the past, interconnect models have been generally restricted to RC tree models. RC trees are RC circuits with capacitors from all nodes to ground, no floating capacitors, no resistors to ground. The signal delay through RC trees were often estimated using a form of the Elmore delay [6], [34], [35] which provided a dominant time constant approximation for monotonic step responses. However, with increasing signal speeds, and in diverse technologies such as Bipolar, BiCMOS or MCMs, RC tree models are no longer adequate. In bipolar circuits, lumped-element interconnect models may require the use of inductors or grounded resistors, which are not compatible with RC trees. Even for MOS circuits operating at higher-frequencies, the effects of coupling capacitances may need to be included in the delay estimate. RLC circuits with non-equilibrium initial conditions may have responses which are non-monotonic. This typically results in visible signal ringing in the waveform. A single time constant approximation with Elmore delay is not generally sufficient for such circuits. Usually lumped interconnect circuits extracted from layouts contain large number of nodes which make the simulation highly CPU intensive. Fig. 2.3 shows a general lumped model where r , l , c and g corresponds to the resistance, inductance, capacitance and conductance of the interconnect, respectively.

Fig. 2.3. Lumped component models

2.1.2 Distributed Transmission Line Models

At relatively higher signal-speeds, electrical length of interconnects becomes a significant fraction of the wavelength, giving rise to signal distorting effects that do not exist at lower frequencies. Consequently, the conventional lumped impedance interconnect models become inadequate and transmission line models based on quasi-TEM assumptions are needed. The TEM (Transverse Electromagnetic) approximation represents the ideal case, where both E and H fields are perpendicular to the direction of propagation and it is valid under the condition that the line cross-section is much smaller than the wavelength. However, in practical wiring configurations, the structure has all the inhomogeneities mentioned previously. Such effects give rise to E or H fields in the direction of propagation. If the line cross-section or the extent of these nonuniformities remain a small fraction of the wavelength for the frequency range of interest, the solution to Maxwell's equations are given by the so called quasi-TEM modes and are characterized by distributed R, L, C, G per unit length parameters (Fig. 2.4).

The basic quasi-TEM model is the simple “*delay*” line or lossless line ($R = G = 0$). In this case a signal travelling along a line has the same amplitude at all the points, but is shifted in time with a propagation delay per unit length (τ) given by

$$\tau = \sqrt{LC} = \sqrt{\epsilon\mu} \quad (2.1)$$

where ϵ, μ are the dielectric permittivity and permeability of the medium, respectively. The characteristic impedance for the lossless case is given by

$$Z = \sqrt{L/C} \quad (2.2)$$

More complicated models include per-unit-length loss (either in the direction of travelling wave or due to dielectric substrate loss) or coupling between adjacent transmission lines, where the coupling may be resistive, inductive, capacitive or a combination of these [6]. In such cases, the propagation constants (Υ) and the characteristic impedances (Z) are given by

$$\Upsilon = \sqrt{(R + j\omega L)(G + j\omega C)}; \quad Z = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad (2.3)$$

Fig. 2.4. Distributed transmission line model

The models may even require frequency dependent parameters (for addressing high-frequency effects such as skin effect, edge effect and proximity effect) [18] or nonuniform per-unit-length parameters to accurately model different geometries [35]. In addition, as operating frequencies increase, electrically long interconnects function as efficient antennas and pick up emissions from nearby electronic systems. This aspect of EMI (susceptibility of interconnects to emissions), is therefore becoming an important design requirement, and including it in the analysis at an early stage of the design can result in reduced costs, improved product reliability and reduced time to market [7], [73].

2.1.3 Full-Wave Models

At further sub-nano second rise times, the line cross-section or the nonuniformities become a significant fraction of the wavelength and under these conditions, the field components in the direction of propagation can no longer be neglected (Fig. 2.5). Consequently, even the distributed models based on quasi-TEM approximations become inaccurate in describing the interconnect performance [10] - [25]. In such situations full-wave models which take into account all possible field components and satisfies all boundary conditions are required to give an accurate estimation of high-frequency effects.

Fig. 2.5. Cross-sectional view of a multiconductor dispersive system

However, circuit simulation of full-wave models is highly involved. The information that is obtained through a full-wave analysis is in terms of field parameters such as propagation constant, characteristic impedance etc., whereas a circuit simulator requires the information in terms of currents, voltages and circuit impedances. This demands a generalized method to combine modal results into circuit simulators in terms of a full-wave stencil. Another important

issue involved here is the cost of a full-wave analysis associated with each interconnect subnetwork at each frequency point of analysis. For typical high-speed interconnect circuits which need thousands of frequency point solutions to get accurate responses, it would become prohibitively CPU expensive to simulate because of the combined cost involved, i.e, evaluation of a full-wave model to obtain modal parameters and computation of circuit response at each point.

2.1.4 Measured Subnetworks

In practice, it may not be possible to obtain accurate analytical models for interconnects because of the geometric inhomogeneity and associated discontinuities. To handle such situations modeling techniques based on measured data have been proposed in the literature [20], [21], [23], [24]. In general, the behavior of high-speed interconnects can easily be represented by measured frequency-dependent scattering parameters or time-domain terminal measurements. Time-domain data could be obtained by time-domain reflectometry (TDR) measurements [79] or electromagnetic techniques [23], [24], [82]. The tabulated data, regardless of how they are obtained will be called *measured data* in this thesis. One important factor to note here is that the subnetwork may be characterized by large sets of time-domain data, making the system overdetermined. This kind of measurement data in large number of sets is essential in a practical environment as the measurements are usually contaminated by noise and the use of only single set of measurements may lead to inaccurate results. Including all available sets of measurements in the simulation helps to reduce the impact of noise on network responses. However, handling such overdetermined situations in circuit simulation is a tedious and a computationally expensive process [20], [21], [79] - [88].

2.2 *Interconnect Simulation Issues*

As pointed out earlier, simulation of interconnects is associated with two major bottlenecks. First, interconnect networks consist of hundreds or thousands of components, such as resistors, capacitors, inductors, transmission lines and other levels of interconnect models. At the terminations there generally exist some nonlinear elements such as drivers and receivers. If only lumped RLC models are considered, ordinary differential equation solvers such as SPICE may be used for simulation purposes. However, the CPU cost may be large, owing to the fact that SPICE is mainly a nonlinear simulator and it does not handle large RLC networks efficiently (at least one LU decomposition is required at every time step as discussed in Section 2.2.1). Secondly, the major difficulty in simulating these high-frequency models lies in the fact that distributed/full-wave elements, while formulated in terms of partial differential equations, are best described in the frequency-domain. Nonlinear terminations on the other hand can only be given in the time-domain. These simultaneous formulations can't be handled by a traditional ordinary differential equation solver such as SPICE [6], [35].

2.2.1 Background on Circuit Simulation

Prior to introducing interconnect simulation algorithms it would be useful to have a glimpse at the basic circuit simulation techniques. Conventional circuit simulators are based on the simultaneous solution of linear equations which are obtained by applying Kirchoff's current law (KCL) to each node in the network. Either for frequency or time-domain analysis, the first step is to set up the *modified nodal analysis matrix* (MNA) [92]. For example consider the small circuit in Fig. 2.6. Its MNA equations are

Fig. 2.6. Example circuit for MNA formulation

$$\begin{pmatrix} G_1 & 0 & 1 & 1 \\ 0 & G_2 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + s \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & -L & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ I_L \\ I_E \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ E \end{pmatrix}$$

The above equation representing a simple two node circuit, has the same form as any other MNA matrix representing a large linear lumped network. Hence MNA equations in general for lumped linear networks can be written as

$$(G + sW)X(s) - b = 0 \quad (2.4)$$

or, in the time-domain it can be written as

$$Gx(t) + W\dot{x}(t) - b(t) = 0 \quad (2.5)$$

2.2.2 Discussion of CPU Cost in Conventional Simulation Techniques

Frequency-domain simulation is conventionally done by solving (2.4) at each frequency point through LU decomposition and forward-backward substitution. For time-domain simulation, linear multi-step techniques [93] are used. The most common of these integration formulae is the trapezoidal rule, which gives the following difference formula

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \Delta t \left(\frac{\dot{\mathbf{x}}(t) + \dot{\mathbf{x}}(t + \Delta t)}{2} \right) \quad (2.6)$$

$$\left(\mathbf{G} + \frac{2}{\Delta t} \mathbf{W} \right) \mathbf{x}(t + \Delta t) = \left(\frac{2}{\Delta t} \mathbf{W} - \mathbf{G} \right) \mathbf{x}(t) + (\mathbf{b}(t) + \mathbf{b}(t + \Delta t)) \quad (2.7)$$

Note that the trapezoidal rule is an integration formula of order of 2, which is the highest possible order that could ensure absolute stability [93]. Due to such a low-order, simulators are forced to use small step sizes to ensure the accuracy during transient simulation. Transient response computation requires the LU decomposition of (2.7) at every time step. It gets further complicated in the presence of nonlinear elements, in which case, the (2.5) gets modified as

$$\mathbf{G}\mathbf{x}(t) + \mathbf{W}\dot{\mathbf{x}}(t) + \phi(\mathbf{x}(t)) - \mathbf{b}(t) = \mathbf{0} \quad (2.8)$$

where $\phi(\mathbf{x}(t))$ is a nonlinear function of $\mathbf{x}$. The difference equation based on trapezoidal rule therefore becomes

$$\left(\mathbf{G} + \frac{2}{\Delta t} \mathbf{W} \right) \mathbf{x}(t + \Delta t) + \phi(\mathbf{x}(t + \Delta t)) = \left(\frac{2}{\Delta t} \mathbf{W} - \mathbf{G} \right) \mathbf{x}(t) + (\mathbf{b}(t) + \mathbf{b}(t + \Delta t)) - \phi(\mathbf{x}(t)) \quad (2.9)$$

In the case of nonlinear elements Newton iterations are used to solve (2.9) at each time step, which requires at least one LU decomposition. However, due to the low order of integration formula, generally a large number of LU decompositions are required at each time step. This causes (note that $\mathbf{W}$ and $\mathbf{G}$ matrices for interconnect networks are usually very large) the CPU cost of a time-domain analysis to be very expensive. This lead to the development of model-reduction algorithms, which effects a reduction in the order of the linear subnetwork before performing a nonlinear analysis so as to yield fast simulation results. A detailed discussion of model-reduction algorithms is given in chapter 4. However, since the formulation of circuit equations in the MNA form is the first step even for model-reduction techniques, details of the same is given in the next chapter.

“Every concept is vague to a degree you don’t realize, till you have tried to make it precise.”

- Bertrand Russell

In the previous chapter we looked at various interconnect simulation issues, which suggested that there is a need to develop a unified scheme for formulation of circuit equations for the application of model-reduction algorithms. This chapter presents a technique to extend the modified nodal analysis (MNA) to transmission line networks, measured and full-wave interconnect subnetworks. In order to make the formulation suitable for model-reduction, a given network is divided into nonlinear and linear subnetworks and the corresponding formulation is presented in the first part of this section. The linear subnetwork can contain lumped components, distributed elements based on quasi-TEM formulations as well as full-wave/measured subnetworks and the corresponding formulation is given in the second part of this section. An illustrative example of the proposed formulation scheme is given at the end of this chapter.

3.1 General Formulation of Network Equations

Consider a general network containing arbitrary number of nonlinear and linear (lumped/distributed/measured/full-wave) components. The linear lumped components may be described by equations in either time-domain or frequency-

domain, whereas the nonlinear components may only be described by time-domain equations. For simplicity, let all the linear components be grouped into a single linear subnetwork π as shown in Fig. 3.1.

Fig. 3.1. Nonlinear network ϕ containing linear subnetwork π

Without loss of generality, the MNA matrix [92] for the network ϕ can be written as

$$W_{\phi} \frac{d}{dt} v_{\phi}(t) + G_{\phi} v_{\phi}(t) + L_{\pi} i_{\pi}(t) + F(v_{\phi}(t)) - b_{\phi}(t) = 0, \quad t \in [0, T] \quad (3.1)$$

where

- $v_{\phi}(t) \in \mathcal{R}^{N_{\phi}}$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,
- $W_{\phi} \in \mathcal{R}^{N_{\phi} \times N_{\phi}}$ and $G_{\phi} \in \mathcal{R}^{N_{\phi} \times N_{\phi}}$ are constant matrices describing the lumped memory and memoryless elements of network ϕ , respectively,
- $b_{\phi} \in \mathcal{R}^{N_{\phi}}$ is a constant vector with entries determined by the independent voltage and current sources,

- $F(v_\phi)$ is a function describing the nonlinear elements of the circuit,
- $L_\pi = [l_{i,j}]$ with elements $l_{i,j} \in \{0, 1\}$ where $i \in \{1, \dots, N_\phi\}, j \in \{1, \dots, n_\pi\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_\pi(t) \in \mathbb{R}^{n_\pi}$ the vector of currents entering the linear subnetwork π , into the node space $\mathbb{R}^{N_\phi}$ of the network ϕ ,
- N_ϕ is the total number of variables in the MNA formulation and n_π is the total number ports associated with the linear subnetwork π .

The first, second and fourth terms in (3.1) cover network's lumped components and independent sources. The third term describes currents at linear subnetwork terminals and then maps them into rest of the network through L_π matrix. The linear multi-terminal subnetwork π can be characterized in the frequency-domain by its terminal behavior. Without loss of generality, the terminal relations for subnetwork π can be represented by frequency-domain equations in the form

$$Y_\pi(s)V_\pi(s) = I_\pi(s) \quad (3.2)$$

where

- $Y_\pi(s)$ is the complex frequency-domain admittance representation of subnetwork π
- $V_\pi(s)$ is the vector of the terminal voltage nodes that connect the subnetwork to the network ϕ
- $I_\pi(s) = L(i_\pi(t))$; L denotes the Laplace transform, s is the complex frequency.

3.2 Formulation of Linear Subnetwork π

In this section formulation of linear subnetwork π suitable for model-reduction is presented. For this purpose, a systematic extension of MNA analysis is described to build the circuit matrix including lumped/distributed/full-wave and measured components. For the purpose of illustration (Fig. 3.2) consider one distributed element, one measured subnetwork and arbitrary number of linear lumped components in the linear subnetwork π . Generalization of the proposed technique to include multiple distributed/measured elements in a given subnetwork is straight-forward.

Fig. 3.2. Subnetwork π with distributed, full-wave and measured components

3.2.1 Lumped Components

Consider only the linear lumped components and independent voltage/current sources in the linear subnetwork π . MNA equations in this case can be written as

$$(sW_{\pi} + G_{\pi})V(s) - b = 0 \quad (3.3)$$

where

- $W_\pi, G_\pi \in \mathfrak{R}^{N_\pi \times N_\pi}$ are constant matrices describing the lumped memory and memoryless elements of subnetwork π , respectively, $\mathfrak{R}^{N_\pi}$ is the node-space of subnetwork π ,
- $b \in \mathfrak{R}^{N_\pi}$ is a constant vector with entries determined by independent voltage/current sources of subnetwork π ,
- $V(s) \in \mathfrak{R}^{N_\pi}$ is the vector of Laplace-domain node voltage waveforms appended by independent voltage source currents, linear inductor current waveforms of linear subnetwork π .

3.2.2 Distributed Elements Based on Quasi-TEM Assumption

Let the given linear subnetwork include distributed elements based on quasi-TEM assumption. The frequency-domain equations of a distributed subnetwork consisting of n_d coupled conductors can be written in the form [35], [38], [39]

$$A_d(s)V_d(s) + B_d(s)I_d(s) = 0 \quad (3.4)$$

where

- $V_d(s)$ and $I_d(s)$ represent the Laplace-domain terminal voltages and currents of the distributed element, respectively, (subscript 'd' is used to denote the variables belonging to distributed elements),
- A_d and B_d are matrices having complex dependency on frequency, which are described in terms of line parameters [35].

Equation (3.4) representing distributed elements can be combined with (3.3) as

$$\begin{bmatrix} sW_\pi + G_\pi & L_d \\ A_{d(s)}L_d^t & B_{d(s)} \end{bmatrix} \begin{bmatrix} V(s) \\ I_{d(s)} \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix} \quad (3.5)$$

where

- $L_d = [l_{i,j}]$ with elements $l_{i,j} \in \{0, 1\}$ where $i \in \{1, \dots, N_\pi\}$, $j \in \{1, \dots, 2n_d\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_d(t) \in \Re^{2n_d}$, the vector of currents entering the distributed subnetwork, into the node space $\Re^{N_\pi}$ of subnetwork π .

3.2.3 Full-Wave Models

Let the given linear subnetwork include elements characterized by full-wave analysis. The frequency-domain equations of a full-wave subnetwork consisting of n_f coupled conductors can be derived in the form [10], [68], [69] (a detailed derivation is presented in chapter 6)

$$A_f(s)V_f(s) + B_f(s)I_f(s) = 0 \quad (3.6)$$

where

- $V_f(s)$ and $I_f(s)$ represent the Laplace-domain terminal voltages and currents of the distributed element, respectively, (subscript ' f ' is used to denote the variables belonging to full-wave subnetworks),
- A_f and B_f are matrices having complex dependency on frequency, which are obtained from full-wave analysis (a detailed derivation is given in chapter 6).

Equation (3.6) representing distributed elements can be combined with (3.5) as

$$\begin{bmatrix} sW_\pi + G_\pi & L_d & L_f \\ A_{d(s)}L_d^t & B_{d(s)} & \mathbf{0} \\ A_{f(s)}L_f^t & \mathbf{0} & B_{f(s)} \end{bmatrix} \begin{bmatrix} V(s) \\ I_d(s) \\ I_f(s) \end{bmatrix} = \begin{bmatrix} b \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (3.7)$$

where

- $L_f = [l_{i,j}]$ with elements $l_{i,j} \in \{0, 1\}$ where $i \in \{1, \dots, N_\pi\}$, $j \in \{1, \dots, 2n_d\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_f(t) \in \mathbb{R}^{2n_d}$, the vector of currents entering the distributed subnetwork, into the node space $\mathbb{R}^{N_\pi}$ of subnetwork π .

3.2.4 Measured Subnetworks

Consider a measured subnetwork with n_m terminals included in the given linear subnetwork π . Let the measured subnetwork be characterized by several sets of measurements and the stencil of such a measured subnetwork can be represented in the frequency-domain as [20], [21]

$$I_m(s) = Y_m(s)V_m(s) \quad (3.8)$$

where $I_m(s)$ and $V_m(s)$ are the terminal current and voltage vectors for the measured subnetwork and $Y_m(s)$ is the y- parameter matrix of the measured subnetwork, of dimension $n_m \times n_m$ (a detailed derivation for $Y_m(s)$ is given in chapter 7). Next, MNA represented by (3.7) can be expanded to include measured subnetwork stencil as

$$\begin{bmatrix} sW_{\pi} + G_{\pi} & L_d & L_f & L_m \\ A_d(s)L_d^t & B_d(s) & 0 & 0 \\ A_f(s)L_f^t & 0 & B_f(s) & 0 \\ Y_m(s)L_m^t & 0 & 0 & -U \end{bmatrix} \begin{bmatrix} V(s) \\ I_d(s) \\ I_f(s) \\ I_m(s) \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.9)$$

where

- $L_m = [l_{i,j}]$ with elements $l_{i,j} \in \{0, 1\}$ where $i \in \{1, \dots, N_{\pi}\}, j \in \{1, \dots, n_m\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_m(t) \in \mathbb{R}^{n_m}$ the vector of currents entering the measured subnetwork, into the node space $\mathbb{R}^{N_{\pi}}$ of the subnetwork π ,
- $U \in n_m \times n_m$ represents the unity matrix.

Next, (3.9) can be concisely written as

$$\Psi(s)X(s) = E \quad (3.10)$$

where

$$X(s) = \begin{bmatrix} V^t(s) & I_d^t(s) & I_f^t(s) & I_m^t(s) \end{bmatrix}^t \quad (3.11)$$

$$E = \begin{bmatrix} b^t & 0^t & \dots & 0^t \end{bmatrix}^t \quad (3.12)$$

3.3 Example

To illustrate the formulation scheme described in this section, an example is given below. Consider the circuit shown in Fig. 3.3. The network equations for this circuit can be represented by (3.1) and (3.2) as

Fig. 3.3. Example circuit with lumped/distributed/measured & nonlinear devices

$$G_\phi = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \quad W_\phi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C & -C & 0 & 0 \\ 0 & 0 & -C & C & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F(v_\phi(t)) = \begin{bmatrix} -f_1(v_2 - v_1) \\ f_1(v_2 - v_1) \\ f_2(v_8) \\ 0 \\ 0 \\ f_3(v_9) \end{bmatrix}; \quad L_\pi = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad v_\phi = \begin{bmatrix} v_1 \\ v_2 \\ v_8 \\ v_9 \\ i_e \\ q \end{bmatrix}; \quad b_\phi = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ e \\ 0 \end{bmatrix}$$

$$i_\pi(t) = \begin{bmatrix} i_2(t) \\ i_3(t) \end{bmatrix}; \quad I_\pi(s) = \begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix}; \quad V_\pi(s) = \begin{bmatrix} V_2(s) \\ V_3(s) \end{bmatrix}$$

$$Y_\pi(s) = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix}$$

Next, the parameters for the linear subnetwork π can be written as (3.7)

$$G + sC = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 0 & 0 \\ -G_1 & G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sC_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & sC_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_4 \end{bmatrix}$$

$$L_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad L_{m1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad L_{m2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X(s) = \begin{bmatrix} V(s) \\ I_d(s) \\ I_{m1}(s) \\ I_{m2}(s) \end{bmatrix}; \quad V(s) = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}; \quad I_d(s) = \begin{bmatrix} I_{d1} \\ I_{d2} \\ I_{d3} \\ I_{d4} \end{bmatrix}$$

$$I_{m1}(s) = \begin{bmatrix} I_{m11} \\ I_{m12} \end{bmatrix}; \quad I_{m2}(s) = \begin{bmatrix} I_{m21} \\ I_{m22} \end{bmatrix}$$

CHAPTER 4

Review of Moment-Matching Techniques

"The more you know, the less you need to explain." - Mahathma Gandhi

This chapter concerns itself with the simultaneous solution of network equations described by (3.1) and (3.2). As it was pointed out in Section 2.2.1, such a simultaneous (mixed frequency/time) formulation is difficult to handle in conventional general purpose simulators. In addition, due to the low-order of integration formula a large number of LU decompositions are required at each time step which makes the time-domain analysis to be CPU expensive. These two difficulties lead to the development of model-reduction (MR) algorithms which provide convenient means to link the networks characterized in the frequency-domain with nonlinear simulators as well as yield faster simulations.

4.1 Principles of Model-Reduction Techniques

Interconnect networks generally tend to have large number of poles, spread over a wide frequency range. Even though majority of these poles would normally have very little effect on simulation results, they cause the linear multi-step algorithms to significantly reduce the step size in order to stay within the stability region. The small step size would normally warrant many time point solutions, which in turn forces large number of LU decompositions and consequently increasing the CPU cost. The moment-matching techniques (MMTs) capitalize on

the fact that irrespective of the presence of large number of poles in a system, only a few of those poles mainly govern the behavior of the system transfer function and such poles are referred to as *dominant poles*. This effect is demonstrated in Fig. 4.1, where it is clear that pole P_2 will have little effect on the final transient result. A brief mathematical description of the underlying concepts of moment-matching techniques is given below.

Fig. 4.1. Illustration of dominant poles

Consider a single input/single output system and let $H(s)$ be the transfer function. $H(s)$ can be represented in a rational form as

$$H(s) = \frac{P(s)}{Q(s)} \quad (4.1)$$

where $P(s)$ and $Q(s)$ are polynomials in s . Equivalently, (4.1) can be written as

$$H(s) = c + \sum_{i=0}^{N_p} \frac{k_i}{s - p_i} \quad (4.2)$$

where p_i and k_i are the i^{th} pole-residue pair, N_p is the total number system poles and c is the direct coupling constant. Next, the time-domain impulse response can be computed in a closed form using inverse Laplace transform as

$$h(t) = c\delta t + \sum_{i=0}^{N_p} k_i e^{p_i t} \quad (4.3)$$

In case of large networks, N_p , the total number of poles can be of the order of thousands. Generating all the N_p poles will be highly CPU intensive even for a small network and for large networks it is completely impractical. MMTs address the above issue by deriving a reduced-order approximation $\hat{H}(s)$ in terms of dominant poles, instead of trying to compute all the poles of a system. Assuming that only L dominant poles were extracted which give a reasonably good approximation to the original system, equation (4.1) and the corresponding approximate frequency and time responses can be written as

$$H(s) \approx \hat{H}(s) = \frac{\hat{P}(s)}{\hat{Q}(s)} = \hat{c} + \sum_{i=0}^L \frac{\hat{k}_i}{s - \hat{p}_i} \quad (4.4)$$

$$h(t) \approx \hat{h}(t) = \hat{c}\delta t + \sum_{i=0}^L \hat{k}_i e^{\hat{p}_i t} \quad (4.5)$$

MMTs are based on Padé approximation and the steps involved in generating a reduced-order model using Padé approximation are outlined below.

4.2 Padé Approximations

Consider a system-transfer function $H(s)$ which is approximated by a rational function $\hat{H}(s)$ as

$$H(s) \approx \hat{H}(s) = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_L s^L}{1 + b_1 s + \dots + b_M s^M} = \frac{P_L(s)}{Q_M(s)} \quad (4.6)$$

where $a_0, \dots, a_L, b_1, \dots, b_M$ are the unknowns (total of $L + M$ variables). Next, expanding $H(s)$ using Taylor series coefficients M_i (*moments*) we have

$$H(s) \approx \hat{H}(s) = m_0 + m_1 s + m_2 s^2 + \dots + m_n s^n = \sum_{i=0}^n s^i m_i \quad (4.7)$$

The series (4.7) is then matched to the lower-order rational polynomial given by (4.6) (*hence is the name, moment-matching techniques (MMTs); which is also known as Padé approximation*). There are $L + M$ unknowns and hence we need to match only the first $L + M$ moments as

$$\frac{a_0 + a_1 s + a_2 s^2 + \dots + a_L s^L}{1 + b_1 s + \dots + b_M s^M} = m_0 + m_1 s + m_2 s^2 + \dots + m_{(L+M)} s^{L+M} \quad (4.8)$$

Cross multiplying and equating the powers of s starting from s^{L+1} to s^{L+M} on both sides of (4.8), we can evaluate the denominator polynomial coefficients as

$$\begin{bmatrix} m_{L-M+1} & m_{L-M+2} & \dots & m_L \\ m_{L-M+2} & \dots & \dots & m_{L+1} \\ \dots & \dots & \dots & \dots \\ m_L & m_{L+1} & \dots & m_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-2} \\ \dots \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ \dots \\ m_{L+M} \end{bmatrix} \quad (4.9)$$

The numerator coefficients can then be found by equating the remaining powers of s , starting from s^0 to s^L as

$$\begin{aligned} a_0 &= m_0 \\ a_1 &= m_1 + b_1 m_0 \\ &\dots \\ a_L &= m_L + \sum_{i=1}^{\min(L, M)} b_i m_{L-i} \end{aligned} \quad (4.10)$$

Equations (4.9) and (4.10) yield an approximate transfer function in terms of rational polynomials. Alternatively, an equivalent pole-residue model can be found as follows. Poles p_i are obtained by applying a root-solving algorithm on denominator polynomial $\hat{Q}(s)$. In order to obtain k_i , expand the approximate transfer function given by (4.4) using Maclaurin series as

$$\hat{H}(s) = \hat{c} - \sum_{n=0}^{\infty} s^n \left(\sum_{i=0}^L \frac{\hat{k}_i}{\hat{p}_i^{n+1}} \right) \quad (4.11)$$

Comparing $\hat{H}(s)$ from equations (4.7) and (4.11), we note that

$$\begin{aligned} m_0 &= \hat{c} - \sum_{i=0}^L \frac{\hat{k}_i}{\hat{p}_i} \\ &\dots \quad (0 < i < 2L) \\ m_i &= - \sum_{i=0}^L \frac{\hat{k}_i}{\hat{p}_i^{i+1}} \end{aligned} \quad (4.12)$$

Residues can be evaluated by writing the equations in (4.12) in a matrix form as

$$\begin{bmatrix} \hat{p}_1^{-1} & \hat{p}_2^{-1} & & \hat{p}_L^{-1} & -1 \\ \hat{p}_1^{-2} & \hat{p}_2^{-2} & & \hat{p}_L^{-2} & 0 \\ & & \dots & & \\ & & & \dots & \\ \hat{p}_1^{-L-1} & \hat{p}_2^{-L-1} & & \hat{p}_L^{-L-1} & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_1 \\ \hat{k}_2 \\ \dots \\ \hat{k}_L \\ \hat{c} \end{bmatrix} = - \begin{bmatrix} M_0 \\ M_1 \\ \dots \\ M_{L-1} \\ M_L \end{bmatrix} \quad (4.13)$$

In the above equations $\hat{c}$ represents the direct coupling between input and output. There are more exact ways to compute $\hat{c}$ which are not detailed here, interested readers can refer to [35].

4.3 Computation of Moments

Having outlined the concept of MMTs, to proceed further, we need to evaluate the moments of the system. Consider the circuit equation represented by (3.10) and expand the response vector $X(s)$ using Taylor series as

$$X(s) = M_0 + M_1s + M_2s^2 + \dots + M_ns^n = \sum_{i=0}^n s^i M_i \quad (4.14)$$

where M_i represents the i^{th} moment-vector. For the purpose of illustration consider the simple case of network with only lumped models. In this case the network can be represented in the form (3.10) as

$$[G + sC][X(s)] = [E] \quad (4.15)$$

or

$$[G + sC][M_0 + M_1s + M_2s^2 + \dots] = [E] \quad (4.16)$$

Equating powers of s on both sides of (4.16) we obtain the following relationships

$$GM_0 = E \quad (4.17)$$

$$GM_i = -CM_{i-1} \quad ; i > 0$$

Above equations give a closed form relationship for the computation of moments. The moments of a particular output node of interest (which are represented by m_i in equations (4.7) - (4.13)), are picked from moment-vectors M_i . As seen, (4.17) requires only one LU decomposition and few forward/backward substitutions during the recursive computation of higher order moments. Since the major cost involved in circuit simulation is due to LU decomposition, MMTs yield very high speed advantage (100 to 1000 times) compared to conventional simulators.

4.3.1 Generalized Computation of Moments

In the case of networks containing transmission lines and measured subnetworks moment-computation is not straight-forward. A generalized relation for recursive computation of higher order moments at a expansion point $s = \alpha$ can be derived using (3.10) as [47]

$$\begin{aligned} [\psi(\alpha)]M_0 &= E \\ [\psi(\alpha)]M_n &= - \sum_{r=1}^n \frac{(\psi^{(r)}|_{s=\alpha})M_{n-r}}{r!} \end{aligned} \quad (4.18)$$

where superscript r denotes the r^{th} derivative at $s = \alpha$. It can be seen that the coefficient on the left hand side does not change during higher-order moment computation and hence only one LU decomposition would suffice. It is also to be noted that the lumped networks are a special case of (4.18) (where $\psi^{(r)} = 0$ for $r \geq 2$ in which case (4.18) reduces to the form given by (4.17)).

Having obtained a recursive relationship (4.18) for higher-order moments, in order to proceed further, we need the derivatives of (ψ) . These can be obtained as

$$[\psi]^{(1)} = \begin{bmatrix} W_\pi & 0 & 0 & 0 \\ A_d^{(1)} L_d^t & B_d^{(1)}(s) & & \\ A_f^{(1)} L_f^t & 0 & B_f^{(1)} & \\ Y_m^{(1)} L_m^t & 0 & 0 & 0 \end{bmatrix}; \quad [\psi]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ A_d^{(r)} L_d^t & B_d^{(r)}(s) & & \\ A_f^{(r)} L_f^t & 0 & B_f^{(r)} & \\ Y_m^{(r)} L_m^t & 0 & 0 & 0 \end{bmatrix} \quad (4.19)$$

$(r \geq 2)$

The derivatives $A_d^{(r)}$ and $B_d^{(r)}$ corresponding to transmission lines can be computed using the matrix exponential based method [35], [47]. However, computation of full-wave/measured subnetworks is not previously addressed in

the literature. New techniques for obtaining these moments are presented in chapters 6 and 7. Fig. 4.2 summarizes the computational steps involved in the MMT algorithm.

Fig. 4.2. Summary of the steps involved in the MMT algorithm

4.4 *Limitations of Single Expansion MMT Algorithms*

Obtaining a lower-order approximation of the network transfer function using a single Padé expansion is commonly referred as *Asymptotic Waveform Evaluation* (AWE) in the literature. However, due to the inherent limitations of Padé approximants, MMTs based on single expansion often give inaccurate results. The following is a list of those properties which have the most impact on MMTs.

- The matrix in (4.9) (which is known as Toeplitz matrix) is ill-conditioned if its size is large. This implies that one can only expect to detect 6 to 8 accurate poles from a single expansion.
- Padé often produces unstable poles on the right hand side of the complex plane.
- Padé accuracy deteriorates as we move away from the expansion point.
- Padé provides no estimates for error bounds.

Generally AWE gives accurate results for RC networks, but often fails for non-RC networks. This is due to the fact that the poles of an RC network are all on the real axis (refer to Fig. 4.1) and therefore an expansion at the origin could clearly determine which ones are dominant and which ones are not. However, in the case of general RLC networks it is possible to have some of the dominant poles outside the radius of convergence of the Padé expansion (Fig. 4.3(a)). In systems containing distributed elements the number of dominant poles will be significantly higher, and it is very difficult to capture all of them with a single Padé expansion. This lead to the development of multi-point expansion techniques such as *complex frequency hopping* (CFH), which are summarized in the next section.

4.5 Complex Frequency Hopping

CFH extends the process of moment Matching to multiple expansion points (*hops*) in the complex plane near or on the imaginary axis using a binary search algorithm. With a minimized number of frequency point expansions, enough information is obtained to enable the generation of an approximate transfer function that matches the original function up to a predefined highest frequency of interest. Using the information from all the expansion points, CFH extracts a dominant pole set as illustrated in Fig. 4.3(b). In addition, CFH provides an error criterion for the selection of accurate poles and transfer functions.

Fig. 4.3. Illustration of CFH

4.5.1 Selection and Minimization of Hops in CFH

A Padé approximation is accurate only near the point of expansion and its accuracy decreases as we move away from the *point of expansion (hop)*. In order to validate the accuracy of such an approximation, at least two expansion points are necessary. Accuracies of these two expansions can be verified by matching the poles generated at these two hops [47] (referred as *pole-matching based approach*). Alternatively, the two hops can be verified for their accuracy by comparing the value of the transfer functions due to both these hops at a point intermediate to them (referred as *transfer-function based approach*) [48]. CFH relies on a binary search algorithm to determine the expansion points and to minimize the number of expansions. The steps involved in the binary search algorithm for both the above approaches are summarized below.

A. Transfer Function Based Approach

In this approach the transfer functions obtained at various hops (expansions) are used to ensure the accuracy of the reduced-order model up to the highest frequency of interest. Steps involved in the algorithm are given in Fig. 4.4 and Fig. 4.5. It is to be noted that the computational effort needed for a comparison as required by *Step 5* is trivial as the responses can be computed in a closed-form using the transfer functions generated in *Steps 2 & 3*. Here ϵ_{th} is a per-defined threshold for the relative error in the transfer functions. At the completion of the binary search algorithm, a set of transfer functions are generated. When evaluating the frequency response at a frequency point α , only the transfer function which is valid in the region containing α is used. This is repeated for all other frequency points to obtain the frequency response of the system.

- *Step 1: Set $f_L = 0$ and $f_H = f_{max}$ (Fig. 4.6).*
- *Step 2: Expand system's response at frequency $f_L = 0$. Determine the coefficients of the corresponding transfer function $H_L(s)$ using (4.9) and (4.10).*
- *Step 3: Expand system's response at $f_H = f_{max}$. Determine the coefficients of the corresponding transfer function $H_H(s)$.*
- *Step 4: Set $f_{mid} = \frac{1}{2}(f_L + f_H)$. Calculate $H_L(j2\pi f_{mid})$ and $H_H(j2\pi f_{mid})$ using the transfer function coefficients obtained in Steps 2 and 3.*
- *Step 5: If $|H_H(j2\pi f_{mid}) - H_L(j2\pi f_{mid})| < \epsilon_{th}$, Go To Step 6. Otherwise expand at f_{mid} and obtain $H_{mid}(s)$.*
- *Step 6: If the threshold condition specified by step 5 is satisfied, STOP. ELSE repeat steps 2-5 between every two consecutive frequency points (e.g., between f_L & f_{mid} and f_{mid} & f_H).*

Fig. 4.4. Transfer function based binary search algorithm

Fig. 4.5. Graphical illustration of transfer function based search algorithm

B. Pole-Matching Based Approach

In this approach poles of the transfer function are explicitly evaluated at each hop and the hops are verified for their accuracy by comparing the poles from two adjacent hops using a binary search algorithm. If a matching pole is found between two adjacent expansions, then the binary search is stopped. The distance between the matching pole and the expansion point under consideration defines the radius of accuracy for the corresponding expansion. All the poles which are within the radius of accuracy are treated as accurate poles and are retained in the final pole-set. The poles which are outside the radius of accuracy are considered as inaccurate poles and are discarded. Fig. 4.6 illustrates the idea of the pole search. Steps involved in this approach are identical to the one given in Fig. 4.4, except for the matching criterion; which is based on poles here. Once a set of dominant poles are obtained, residues of the system are obtained using (4.13). Further details of CFH and its search algorithms can be found in [47] and [48].

Fig. 4.6. Pole-matching based binary search algorithm

4.5.2 Selection of f_{max}

An important issue in constructing the reduced-order model of a linear subnetwork is to determine an approximate bandwidth of interest. In most digital applications, the minimum wavelength of interest of the propagating signal can be chosen to determine the required bandwidth. For example, the frequency spectrum of a trapezoidal pulse having unit amplitude is given by

$$S(w) = \frac{1}{s^2 t_r} (1 - e^{-st_{pw}}) \quad (4.20)$$

where t_r is the rise/fall time and t_{pw} represents the pulse-width of the signal. $S(w)$ has an energy spectrum spread over an infinite frequency range, however, most of the signal energy is concentrated near the low frequency region and decreases rapidly with increase in frequency (this is illustrated in Fig. 4.7 for two different instances of rise times, 1ns and 0.1ns). Hence ignoring the high-frequency components of the spectrum above a maximum frequency, f_{max} , will not seriously alter the overall signal shape. Consequently, for all practical purposes, the width of the spectrum can be assumed to be finite. In other words, the signal energy of interest is assumed to be contained in the major lobe of the spectrum. Hence f_{max} can be defined as the 3-db bandwidth point of $S(w)$ [26], [42], i.e.,

$$f_{max} \approx \frac{0.35}{t_r} \quad (4.21)$$

or for high-frequency applications, the limit can be more conservatively set as [42]

$$f_{max} \approx \frac{1}{t_r} \quad (4.22)$$

(a) Network with two transmission lines

(b) Frequency spectrum for $t_r = 1\text{ns}$; $t_{pw} = 5\text{ns}$ (c) Frequency spectrum for $t_r = 0.1\text{ns}$; $t_{pw} = 5\text{ns}$ **Fig. 4.7. Frequency spectrum of trapezoidal pulse**

4.6 Reduced-Order Models of Multiport Linear Networks

So far we looked into the model-reduction of single input-single output systems. In this section a discussion pertaining to the multiport networks is given. Consider the general multiport interconnect linear subnetwork shown in Fig. 4.8 (this corresponds to the linear subnetwork π which is described during the general formulation in Section 3.1; Fig. 3.1). Such a network can be characterized in terms of y (admittance), z (impedance), h (hybrid) or s (scattering) parameters. For the purpose of simplicity of presentation, only the discussion with respect to y parameters is considered in this thesis. (the idea presented here can be easily extended to other types of characterization). The network can be characterized using y -parameters (3.2) as

Fig. 4.8. Multiport linear subnetwork π

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n_\pi} \\ \dots & \dots & \dots & \dots \\ Y_{n_\pi 1} & Y_{n_\pi 2} & \dots & Y_{n_\pi n_\pi} \end{bmatrix} \begin{bmatrix} V_1 \\ \dots \\ V_{n_\pi} \end{bmatrix} = \begin{bmatrix} I_1 \\ \dots \\ I_{n_\pi} \end{bmatrix} \quad (4.23)$$

Using model-reduction techniques, each entry Y_{jk} in (4.23) can be approximated by a q -pole lower-order model having a general form as shown below.

$$Y_{ij}(s) = c^{i,j} + \sum_{a=1}^{q^{i,j}} \frac{k_a^{i,j}}{s - p_a^{i,j}}; \quad 1 \leq (i, j) \leq n_\pi \quad (4.24)$$

where $p_a^{i,j}$ and $k_a^{i,j}$ are the a^{th} dominant pole-residue pair at a port j due to an input excitation at port i . $c^{i,j}$ is the direct coupling constant. $q^{i,j}$ is the number of dominant poles used for approximating Y_{ij} . Using (4.24) a general form for multi-port reduced-order frequency-domain matrix-transfer function is represented by a general notation $Y_\pi(s)$ as

$$Y_\pi(s) = \begin{bmatrix} c^{1,1} + \sum_{a=1}^{q^{1,1}} \frac{k_a^{1,1}}{s - p_a^{1,1}} & c^{1,2} + \sum_{a=1}^{q^{1,2}} \frac{k_a^{1,2}}{s - p_a^{1,2}} & \dots & \dots & c^{1,n_\pi} + \sum_{a=1}^{q^{1,n_\pi}} \frac{k_a^{1,n_\pi}}{s - p_a^{1,n_\pi}} \\ \dots & \dots & \dots & \dots & \dots \\ c^{n_\pi,1} + \sum_{a=1}^{q^{n_\pi,1}} \frac{k_a^{n_\pi,1}}{s - p_a^{n_\pi,1}} & c^{n_\pi,2} + \sum_{a=1}^{q^{n_\pi,2}} \frac{k_a^{n_\pi,2}}{s - p_a^{n_\pi,2}} & \dots & \dots & c^{n_\pi,n_\pi} + \sum_{a=1}^{q^{n_\pi,n_\pi}} \frac{k_a^{n_\pi,n_\pi}}{s - p_a^{n_\pi,n_\pi}} \end{bmatrix} \quad (4.25)$$

There are two main approaches available in the literature to obtain the matrix-transfer function represented by (4.25). In the first approach a common set of poles obtained for any one of the entries Y_{ij} are used for the computation of residues of all other entries [39], [41], [45]. In the second approach self poles are used for the computation of residues of each entry [40]. In this approach the total number of poles involved is approximately n_π (number of ports) times higher compared to the first approach.

However, it is to be noted that the nonlinear simulation time using general purpose simulators is superlinearly proportional to the number of states involved. Hence the efficiency of the transient simulation in the presence of

nonlinear elements is dependent on the number of states introduced by the reduced-order models. The total number of states from both the above approaches is given by

$$\text{Total number of states} = \text{Total number of poles} \times \text{Total number of ports} \quad (4.26)$$

Hence the reduced-order from the second approach becomes very inefficient for the purpose of nonlinear simulation as the number of poles involved in this approach is directly proportional to the number of ports. On the other hand, although the reduced-order models from the first approach are very efficient, but they generally lead to inaccurate results. This is because *the residue computation using (4.13) is very sensitive to the location of poles of each Y_{jk}* . In the next section, a summary of the limitations associated with the current moment-matching techniques is given.

4.7 Limitations of CFH and Other MMT Algorithms

As mentioned in previous sections, there are several limitations associated with the current model-reduction techniques, which are hindering their usage as general purpose, industrial grade circuit simulation tools. These drawbacks are

1. Residues obtained using (4.13) are not guaranteed to be accurate. This is due to the difficulty in choosing moments; from which expansion should they be picked for the RHS of (4.13) during the computation of residues. In case of single expansion methods this was trivial as poles and moments belong to the same expansion. However, accuracy of the residues is extremely important since the frequency response as well as time responses are very sensitive to their values.

-
2. In the case of multiport subnetworks, the reduced-order models obtained using the above techniques become either inefficient for the purpose of non-linear simulation or yield inaccurate simulation results.
 3. There is no guarantee that the reduced-order model obtained from the CFH is passive, and hence the final transient simulation can be unstable. In addition, none of the techniques available in the literature guarantee the passivity during the model-reduction of networks containing distributed, full-wave and measured subnetworks.
 4. CFH requires analytical models for interconnects to compute system moments and it can not handle full-wave models or measured subnetworks. Also it is to be noted that the other techniques, PACT, PRIMA and Arnoldi based algorithms can be applied only to the networks described by first order differential equations, and hence they are limited to lumped linear networks.

It is the goal of this thesis to address the above issues so that all levels of high-speed interconnect networks can be handled easily in general purpose simulators such as SPICE. The next chapter presents a new algorithm to improve the accuracy of residues and also to efficiently simulate multiport interconnect networks in the presence of nonlinear elements. Also a new technique to preserve the passivity of reduced-order models of networks containing distributed, full-wave and measured subnetworks is presented in the next chapter. Chapter VI and VII present efficient algorithms to extend the model-reduction techniques to include full-wave models and measured subnetworks, respectively.

CHAPTER 5

Block Complex Frequency Hopping for Multiport Interconnect Networks

“Imagination is more important than knowledge.” - Albert Einstein

So far, we have looked at the theory and advantages of moment-matching techniques used for model-reduction of high-speed interconnect networks. Also, we looked into the disadvantages associated with the current moment-matching techniques in Section 4.8. It is pointed out that the residues from CFH are not guaranteed to be accurate which can result in faulty frequency/time simulation results. Also we noted that the moment-matching based reduced-order models either become inefficient for the purpose of nonlinear simulation or may lead to inaccurate/unstable transient simulations due to passivity violation. In this chapter we present a new technique called ‘*Block Complex Frequency Hopping (BCFH)*’ to overcome both these difficulties. As will be shown in subsequent chapters, block CFH can easily handle all levels of high-speed interconnect hierarchy, including distributed, full-wave as well as measured subnetworks.

5.1 New Residue Computation Algorithm

In this section, a new technique for improving the accuracy of residues is presented [49], [61]. This technique is an important constituent of the proposed block CFH algorithm. It is to be noted that the inaccuracy of residues is mainly due to the use of Vandermond matrix and the difficulty to determine which

moments to use from various hops in (4.13). This difficulty is addressed in the proposed *residue computation algorithm* which combines the merits of two different approaches of CFH, namely, *transfer-function based approach* and *pole-matching based approach*. The transfer-function based approach collects a set of transfer functions which *accurately matches the frequency response up to the highest frequency of interest*. On the other hand pole-matching based approach *collects all the dominant poles accurately up to the highest frequency of interest*. In the proposed algorithm, the relationship between frequency response and pole-residue model of the system is used to compute the residues (this idea is illustrated in Fig. 5.1) as

$$c + \sum_{i=0}^q \frac{k_i}{s - p_i} = H(s) \quad (5.1)$$

where q is the total number of dominant poles (p_i) extracted using CFH and (k_i) are the corresponding unknown residues. c is the direct coupling constant. $H(s)$ represents the frequency response obtained using the transfer-function based approach.

Fig. 5.1. Illustration of the new residue computation algorithm

In order to compute the unknown residues, a set of linear equations can be formulated using (5.1), spanning many frequency points in the region of interest as shown below

$$c + \frac{k_1}{s_m - p_1} + \frac{k_2}{s_m - p_2} + \dots + \frac{k_q}{s_m - p_q} = H(s_m) \quad (5.2)$$

where s_m represents the m^{th} frequency point. In the case of complex poles, both the pole as well as its conjugate should be used while formulating (5.2). Let N_m be the total number of frequency points matched. Next, equating both the real and imaginary parts of (5.2) separately, we obtain a new set of linear equations as

$$AK = H \quad (5.3)$$

where $K \in \mathbb{R}^{q \times 1}$ is the column vector containing the unknowns corresponding to the real and imaginary parts of residues. $H \in \mathbb{R}^{2N_m \times 1}$ is the column vector containing real and imaginary parts of the frequency responses represented by $H(s_m)$. $A \in \mathbb{R}^{2N_m \times q}$ consists of entries contributed by the LHS of (5.2). In case the direct coupling constant is computed by adding it as one more variable in (5.3), then the order of matrix/vectors indicated above will change from q to $q + 1$.

Next, the solution of (5.3) is needed to evaluate unknown residues K . However, it is important to note that the transient response is sensitive to the accuracy of residues. Hence, in order to improve the accuracy of residues, it is necessary to match as many frequency points as possible using (5.2). However, in the case of choosing number of frequency points higher than the number of unknown residues, matrix A will not be square. To overcome this difficulty, (5.3) is solved using the least mean square error algorithm and the corresponding solution is given by a new set of normal equations

$$A_s K = A^t H \quad (5.4)$$

where

$$A_s = A^t A \quad (5.5)$$

is a square matrix of order $q \times q$. A^t represents the transpose of A . Formulating the residue equations in the form given by (5.4) leads to additional computational savings as the *number of equations to be solved remains equal to the number of unknown residues despite the increase in matched frequency points*. Alternatively, (5.3) can be solved using QR decomposition $K = R^{-1} Q^t H$ (where $A = QR$) [99]. The dominant set of poles along with the residues computed using (5.4) accurately characterizes the given linear subnetwork up to the maximum frequency of interest.

5.2 Selection of Dominant Pole-Set

In order to achieve efficient nonlinear simulation, it is essential to have macromodels with fewer states. However, as discussed in section III.2, when CFH is applied on each of the Y_{ij} , the resulting total number of poles can be very large. This in turn will drastically increase the total number of states needed in the macromodel, as it is proportional to the product of total number of poles and ports. Hence it is essential that the number of poles in the matrix transfer-function (Y_π) is reduced [61]. In order to minimize the number of poles in the matrix-transfer function, the following two propositions are used.

- 1) In general, the pole set corresponding to any individual transfer impedances (Y_{ij}) is a subset of the union of all driving point impedances (Y_{ii}) [97].

- 2) CFH accurately computes the dominant poles of a system. Generally in a system with large number of dominant poles (20 - 40), pole sets belonging to different driving point admittances (Y_{ii}) obtained using CFH contain mostly identical poles and with only a very small number of poles differing among these sets (experience based on running several practical examples shows that the maximum range for these differing poles is around 5-10%).

These two issues are demonstrated through a numerical example below. The data presented here pertains to seven transmission line circuit with three ports (circuit is given in example 2, Fig. 5.19). Fig. 5.2 gives the pole distribution corresponding to various Y_{ii} (note that the real axis in these figures is much more enlarged compared to the imaginary axis). Fig. 5.2(a) shows the distribution of poles belonging to Y_{11} and Y_{22} . Pole-set of Y_{11} has 21 poles (2 real poles and 19 complex poles; complex conjugate poles are excluded in the count) whereas the pole-set of Y_{22} contains 20 poles (2 real poles and 18 complex poles). As seen in Fig. 5.2(a) 19 poles in each of these are identical and only three poles (which are explicitly marked on the plot indicating which pole-set they belong to) differ among these sets. Fig. 5.2(b) shows the distribution of poles belonging to Y_{11} and Y_{33} . Pole-set of Y_{33} has 20 poles, of which 19 are identical to those in Y_{11} .

Hence in case of networks having large number of ports, the following strategy is used for picking a good dominant pole-set: choose the common pole set for the entire matrix-transfer function as the union of all the driving-point pole-sets. For the example under consideration this will result in 22 poles (which is just one more than the number of poles in the pole set of Y_{11}). It is to be noted that, use of the new residue compensation algorithm helps to match the response accurately even if few of the poles (not so dominant) are missing in the final common pole-set.

(a): Pole distribution for Y_{11} and Y_{22} (b): Pole distribution for Y_{11} and Y_{33}

Fig. 5.2. Pole distribution for 3-port transmission line circuit

5.3 Proposed Block CFH Algorithm

In the proposed algorithm, we make use of the residue compensation algorithm presented in section 5.1 and the common dominant pole-set which is obtained as discussed in section 5.2, to arrive at compact reduced-order models for multiport networks. In the proposed technique, an accurate frequency-response for each of the entries in the Y matrix is obtained using complex frequency hopping based on transfer-function approach. Next, an accurate pole-set is obtained as the union of driving point impedances. Using these accurate poles and frequency responses, residues for each Y_{ij} is obtained through the new residue computation algorithm. For this purpose equation (5.2) and (5.3) are modified as

$$c^{i,j} + \frac{k_1^{i,j}}{s_m - p_1} + \frac{k_2^{i,j}}{s_m - p_2} + \dots + \frac{k_q^{i,j}}{s_m - p_q} = H^{i,j}(s_m) \quad (5.6)$$

$$A_s K^{i,j} = A^t H^{i,j} \quad (5.7)$$

with $A_s \in \mathbb{R}^{q \times q}$, $A \in \mathbb{R}^{2N_m \times q}$, $K^{i,j} \in \mathbb{R}^{q \times 1}$ and $H^{i,j} \in \mathbb{R}^{2N_m \times 1}$. N_m represents the total number of frequency points used during the residue computation. Using (5.7) residues for all Y_{ij} in the matrix-transfer function (Y_π) are computed and the reduced-order model for the linear subnetwork is obtained as

$$Y_\pi(s) = \begin{bmatrix} c^{1,1} + \sum_{i=1}^q \frac{k_i^{1,1}}{s-p_i} & c^{1,2} + \sum_{i=1}^q \frac{k_i^{1,2}}{s-p_i} & \dots & \dots & c^{1,n_\pi} + \sum_{i=1}^q \frac{k_i^{1,n_\pi}}{s-p_i} \\ \dots & \dots & \dots & \dots & \dots \\ c^{n_\pi,1} + \sum_{i=1}^q \frac{k_i^{n_\pi,1}}{s-p_i} & c^{n_\pi,2} + \sum_{i=1}^q \frac{k_i^{n_\pi,2}}{s-p_i} & \dots & \dots & c^{n_\pi,n_\pi} + \sum_{i=1}^q \frac{k_i^{n_\pi,n_\pi}}{s-p_i} \end{bmatrix} \quad (5.8)$$

where q is the number of poles in the common pole-set P . $k^{i,j}$ and $c^{i,j}$ represents residue and the direct coupling constant for the parameter Y_{ij} , respectively.

5.4 Passivity Preservation

As discussed in the introduction, one of the key issues during model-reduction is the preservation of passivity. Passivity implies that a network cannot generate more energy than it absorbs, and no passive termination of the network will cause the system to go unstable [94] - [96]. The loss of passivity can be a serious problem because transient simulations of reduced networks may encounter artificial oscillations. Efficient schemes based on congruent transformation for preservation of passivity during the reduction of RLC networks is available in the literature [56] - [59]. *However, none of the reduction algorithms which currently exist preserve passivity for general high-speed interconnects such as distributed transmission lines, frequency dependent parameters, measured or full-wave subnetworks.* In this section we present a new algorithm to preserve the passivity during the reduction of general high-speed interconnect networks.

5.4.1 Review of Passivity Properties

A passive network is asymptotically stable. (i.e. the real parts of the poles are less than or equal to zero), but asymptotic stability does not imply passivity. A network with admittance matrix represented by $Y(s)$ is passive iff [94] - [96]

1) $Y(s^*) = Y^*(s)$ where $*$ is the complex conjugate operator.

2) $Y(s)$ is a positive real (PR) matrix, that is the product $z^{*t}[Y^t(s^*) + Y(s)]z \geq 0$ for all complex values of s satisfying $Re(s) > 0$ and any arbitrary vector z .

The condition 2 also implies analyticity of $Y(s)$ for $Re(s) > 0$, since $Y(s)$ is a rational function of s , and hence separate test for analyticity is not necessary.

Also, the condition 2 implies that the $Real(Y(s)) = \frac{1}{2}[Y'(s^*) + Y(s)]$ has to be a nonnegative matrix for $Re(s) > 0$.

The poles obtained through CFH belong only to the left half plane (-ve real parts) of the j -axis and hence the asymptotic stability is easily ensured. Condition 1 is automatically satisfied since complex poles/residues are always considered along with their conjugates in the formulation of (5.7). However, the condition 2 is not easily ensured and may lead to passivity violation.

An illustration of possible passivity violation due to not satisfying condition 2 is given below (for details of the network please see example 1 in section 5.7). For the network under consideration, a set of common poles are obtained using the block CFH and responses are accurately matched up to 4GHz. The corresponding frequency and the time responses are given in Fig. 5.3 and Fig. 5.4. As seen, though the reduced-order model matches accurately with the original response of the network up to the highest frequency of interest, the transient simulation suffered due to spurious oscillations. The reason why the transient simulation failed can be deciphered by observing the frequency spectrum of the reduced-order model beyond 4GHz, which is shown in Fig. 5.5. Here the response $Re(Y_{22}(s))$ is negative beyond the highest frequency of interest, making the system nonpassive. A closer examination of this problem revealed that the common pole set contained two poles collected by the CFH, which are larger than the highest frequency up to which the response was matched (4GHz). This extra degree of freedom (which was lying beyond the highest matched frequency) in (5.7) severely affected the passivity property of the reduced-order system. When the response was matched up to the highest pole in the common pole-set (or poles

beyond the highest frequency matched are removed from the pole-set), the passivity in the resulting reduced-order system was restored, as shown in Fig. 5.6 - Fig. 5.8. In the following section a new algorithm for passivity preservation during reduction of high-speed interconnect networks is presented.

Fig. 5.3. Accurate frequency response (matched up to 4GHz)

Fig. 5.4. Time response with oscillations due to passivity violation

Fig. 5.5. Illustration of passivity violation

Fig. 5.6. Frequency response matched up to the highest pole (5.22 GHz)

Fig. 5.7. Passivity conditions satisfied **Fig. 5.8. Time response**

5.4.2 Proposed Passivity Preservation Algorithm

In this section we present a new algorithm to ensure passivity during the model-reduction of networks containing high-frequency interconnect models such as distributed transmission lines, frequency dependent parameters, measured or full-wave subnetworks. The proposed algorithm is based on compensating for any loss of passivity during model reduction. In this context, the following assumptions are made during the development of the new algorithm.

- 1) The original network/data conforms to the passivity conditions.
- 2) The dominant pole-set doesn't contain any pole that is larger than the highest matched frequency.
- 3) The reduced-order model is such that it matches the original network accurately up to the highest frequency of interest and is passive at $s = \infty$.

Since the reduced-order model is matched accurately up to the highest frequency, we have $z^{*t}[Re(Y(s))]z \geq 0$ for $Re(s) > 0$, so that the network is passive in this region. Also, since we have the network passive at $s = \infty$, and also the system contains no poles in the region $\forall (\|p_{max}\| \leq w \leq \infty)$, for most of the practical cases, the product $z^{*t}[Re(Y(s))]z$ gradually settles down (shown by dotted lines in Fig. 5.9) to a constant positive value (≥ 0) for large values of w , without violating the passivity conditions. This is illustrated in Fig. 5.9 and such a behavior holds good for most of the cases, wherein the response near p_{max} is mainly dominated by p_{max} and last 1-2 poles.

Fig. 5.9. Illustration of the general behavior of $z^{*t}[Re(Y(s))]z$

However, the behavior depicted in Fig. 5.9 may not always be guaranteed and there is a possibility that product $z^{*t}[Re(Y(s))]z$ can reach a negative minimum immediately after p_{max} as shown in Fig. 5.10, leading to passivity violation [96]. In order to overcome this difficulty, a new technique is described in the rest of this section, which compensates for loss of passivity in those regions where the PR conditions are violated.

Fig. 5.10. Illustration of possible passivity violation

Proposition: If $Y(s)$ is PR $\forall (0 \leq w \leq \|p_{max}\|)$, and if $\Psi(s)$, a matrix that is PR $\forall (0 \leq w \leq \infty)$ and $\Psi(s)$ is such that $Y(s) + \Psi(s)$ is PR $\forall (\|p_{max}\| \leq w \leq \infty)$ then the sum $Y(s) + \Psi(s)$ is PR $\forall (0 \leq w \leq \infty)$.

Proof: Since $Y(s)$ is nonnegative definite in the region $\forall (0 \leq w \leq \|p_{max}\|)$ and $\Psi(s)$ is such that $Y(s) + \Psi(s)$ is nonnegative definite $\forall (\|p_{max}\| \leq w \leq \infty)$ then the sum $Y(s) + \Psi(s)$ will be nonnegative definite for the entire region $\forall (0 \leq w \leq \infty)$ subject to the condition that the addition of $\Psi(s)$ didn't spoil the PR property of the resulting sum in the region $\forall (0 \leq w \leq \infty)$. However, this condition is always satisfied since the sum $(Y(s) + \Psi(s))$ of two PR matrices that are PR in the region $\forall (0 \leq w \leq \|p_{max}\|)$ is also PR in the same region ($\forall (0 \leq w \leq \|p_{max}\|)$). Graphical interpretation of the above proposition is given in Fig. 5.11.

Fig. 5.11. Illustration of the new passivity algorithm

5.4.3 Selection of $\Psi(s)$

In order to proceed with the new algorithm, we need to compute $\Psi(s)$ so as to compensate for any loss of passivity in the reduced-order model beyond the highest matched frequency. For this purpose the following theorem [98] is used. The theorem states that “the necessary and sufficient conditions for a real symmetric matrix to be positive real is all its eigenvalues have to be positive”. As discussed in the previous section, passivity violation can occur in the region beyond the p_{max} . Consequently, in the case of passivity violation, few of the eigenvalues can be negative in this region. Hence in the proposed algorithm, a fine frequency-domain analysis is performed using the reduced-order model in the region immediately next to the p_{max} . Next, at each of these frequency points, $Real(Y(s))$ and subsequently its eigenvalues are computed. If none of the eigenvalues are negative, then there is no need for passivity compensation. If any of the eigenvalues are to be negative then passivity compensation is performed using $\Psi(s)$ according to the proposition given in the previous section. $\Psi(s)$ is constructed as a diagonal matrix having the order same as that of $Y(s)$ as follows

$$\Phi(s) = Y(s) + \Psi(s) = \begin{bmatrix} y_{11}(s) & y_{12}(s) & \dots & y_{1n}(s) \\ y_{21}(s) & y_{22}(s) & \dots & y_{2n}(s) \\ \dots & \dots & \dots & \dots \\ y_{n1}(s) & y_{n2}(s) & \dots & y_{nn}(s) \end{bmatrix} + \begin{bmatrix} \Psi_a(s) & 0 & \dots & 0 \\ 0 & \Psi_a(s) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \Psi_a(s) \end{bmatrix} \quad (5.9)$$

in which $\Psi_a(s)$ is constructed as the response due to an artificial pole whose imaginary part is determined by the frequency point at which the negative maximum occurs among all the eigenvalues. Next, the real part of the pole and its residues are determined such that the real-part response of this pole-residue pair offsets the maximum negative value as well as the maximum negative spectrum

experienced by any of the eigenvalues. Let $\Psi_a(s)$ be represented by the pole, $p = -\alpha + j\beta$ and residue, $k = d_1 + jd_2$. The real part response of $\Psi_a(s)$ can be expressed as

$$\text{Real}(\Psi_a(jw)) = \frac{1}{2}[\Psi_a(jw) + \Psi_a(-jw)] = \frac{(d_1\alpha + d_2\beta)w^2 + (d_1\alpha - d_2\beta)(\alpha^2 + w^2)}{(\alpha^2 + \beta^2 - w^2)^2 + 4\alpha^2\beta^2} \quad (5.10)$$

In addition $\Psi_a(s)$ is required to be PR in the region $\forall (0 \leq w \leq \infty)$. This leads to the condition [96]

$$\frac{d_1}{|d_2|} \geq \frac{\beta}{\alpha} \quad (5.11)$$

Equation (5.11) gives the restrictions on residue values for a given complex pole, if the component admittance representing this pole-pair were to be a PR function. Graphical interpretation of (5.11) is given in Fig. 5.12, in which the limit expressed by (5.11) is shown in the lighter shaded region.

Next, since β is fixed, α , d_1 and d_2 are computed subject to (5.11) and such that $\text{Real}(\Psi_a(jw))$ represented by (5.10) is greater than the maximum negative value suffered by any eigenvalue. The new function, $\Phi(s)$ in which the real part of all the diagonal entries boosted by a positive real value represented by $\text{Real}(\Psi(s))$ (which takes into effect in the vicinity of p_{max}), has all its eigenvalues incremented by the same factor. This means that none of its eigenvalues are negative in the region $\forall (0 \leq w \leq \infty)$ making the sum $Y(s) + \Psi(s)$ to be PR in the entire region $\forall (0 \leq w \leq \infty)$.

However, it is to be noted that the addition of $\Psi(s)$ should be such that it shouldn't effect the frequency spectrum below the maximum frequency of interest. In order to achieve this, w_{max} (and also the corresponding dominant pole-set) should be chosen slightly higher than the desired maximum frequency of interest. For most practical cases, choosing w_{max} to be 10% higher than the maximum frequency of interest would be sufficient.

Fig. 5.12. Graphical illustration of restriction on residue values (5.11)

5.5 Macromodel Synthesis

It is evident that the twin goals of model-reduction techniques is to reduce CPU cost as well as to achieve mixed frequency/time simulation for circuits with large interconnect subnetworks containing high-frequency models as well as nonlinear elements. In this section an efficient technique for linking reduced-order models to nonlinear SPICE-like simulators is described. Derivation of differential equations from reduced-order interconnect models is referred as *macromodel synthesis*. The differential equations can be easily linked to nonlinear simulators as they are described in time-domain. This process is illustrated in Fig. 5.13.

Fig. 5.13. Illustration of macromodel synthesis for nonlinear simulation

In general, a set of first-order differential equations in the state-space domain can be described as

$$\begin{aligned} [\dot{X}] &= [A][X] + [B][U] \\ [Y] &= [C][X] + [D][U] \end{aligned} \quad (5.12)$$

where $A \in \mathcal{R}^{n \times n}$ is a state-matrix, $B \in \mathcal{R}^{n \times n_\pi}$ is a matrix that relates the inputs to state-variables, $C \in \mathcal{R}^{n_\pi \times n}$ is a matrix relating state variables to the outputs (Y) and $D \in \mathcal{R}^{n_\pi \times n_\pi}$ is a matrix relating inputs directly to the output. X is the state vector of length n and U is the input vector of length n_π (where n_π is the number of ports).

Given a matrix-transfer function described by (5.8), several forms of time-domain realization can be obtained. For the purpose of illustration, macromodel synthesis using Jordan-canonical [100] - [102] form of realization is given below.

Example: Macromodel Synthesis

Consider a two-port network containing two common poles, whose transfer function is described by

$$\begin{bmatrix} c^{1,1} + \sum_{a=1}^2 \frac{k_a^{1,1}}{s-p_a} & c^{1,2} + \sum_{a=1}^2 \frac{k_a^{1,2}}{s-p_a} \\ c^{2,1} + \sum_{a=1}^2 \frac{k_a^{2,1}}{s-p_a} & c^{2,2} + \sum_{a=1}^2 \frac{k_a^{2,2}}{s-p_a} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

A Jordan-canonical [100], [102] form of realization for this case would need four state variables and it can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 \\ 0 & 0 & p_2 & 0 \\ 0 & 0 & 0 & p_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} k_1^{1,1} & k_1^{1,2} & k_2^{1,1} & k_2^{1,2} \\ k_1^{2,1} & k_1^{2,2} & k_2^{2,1} & k_2^{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} c^{1,1} & c^{1,2} \\ c^{2,1} & c^{2,2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

5.5.1 Including Complex Poles in the State Matrix

In case the matrix-transfer function $Y_\pi(s)$ has complex poles, then they need to be treated differently as they do not have a direct meaning in the time-domain. However, since all the coefficients of the denominator polynomials of $Y_\pi(s)$ (which are obtained through Padé approximation) are real, if a complex number λ is a root of $Y_\pi(s)$ then its complex conjugate λ^* is also a root of $Y_\pi(s)$. Hence a Jordan-form of state-space realization for a complex pole-pair would be [102]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_1^* \end{bmatrix} u \quad (5.13)$$

$$y = \begin{bmatrix} C_1 & C_2^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (5.14)$$

where A_1 is a diagonal matrix constructed using poles and A_1^* is complex conjugate of A_1 . Next, introducing an equivalence transformation defined by

$$\begin{bmatrix} \hat{x} \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} x \end{bmatrix}; \quad T = \begin{bmatrix} I & I \\ iI & -iI \end{bmatrix} \quad (5.15)$$

Equations represented by (5.13) and (5.14) can now be easily transformed into

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} Re(A_1) & Im(A_1) \\ -Im(A_1) & Re(A_1) \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 2Re(b_1) \\ -2Im(b_1) \end{bmatrix} u \quad (5.16)$$

$$y = \begin{bmatrix} Re(C_1) & Im(C_1) \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \quad (5.17)$$

An illustrative example of the above steps is given below.

Example: Inclusion of Complex Poles in the State Matrix

Consider a two port network containing one pair of complex poles $\lambda_{1,2} = w \pm iz$.

Let the corresponding residues at different ports be $c_{i,j} = (r \pm iv)_{i,j}$. Original Gilbert's realization will yield

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} w+iz & 0 & 0 & 0 \\ 0 & w+iz & 0 & 0 \\ 0 & 0 & w-iz & 0 \\ 0 & 0 & 0 & w-iz \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (r+iv)_{11} & (r+iv)_{12} & (r-iv)_{11} & (r-iv)_{12} \\ (r+iv)_{21} & (r+iv)_{22} & (r-iv)_{21} & (r-iv)_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Define

$$P = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ i & 0 & -i & 0 \\ 0 & i & 0 & -i \end{bmatrix}$$

Next, the realization represented by (5.16) and (5.17) can be obtained as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{bmatrix} = \begin{bmatrix} w & 0 & z & 0 \\ 0 & w & 0 & z \\ -z & 0 & w & 0 \\ 0 & -z & 0 & w \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & v_{11} & v_{12} \\ r_{21} & r_{22} & v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix}$$

5.6 Unified Transient simulation

Once a matrix-transfer function describing the multiport interconnect network is obtained, a time-domain realization in the form of state-space equations can be obtained using the techniques described in section 5.5 as

$$\begin{aligned}\frac{d}{dt}[z_\pi(t)] - [A_\pi][z_\pi(t)] - [B_\pi][i_\pi(t)] &= 0 \\ [v_\pi(t)] - [C_\pi][z_\pi(t)] + [D_\pi][i_\pi(t)] &= 0\end{aligned}\tag{5.18}$$

where i_π and v_π are the vector of terminal currents and voltages of the linear subnetwork π (described by (3.2)). The differential equations represented by the macromodel (5.18) can be combined with (3.1) using the relation $v_\pi = (L_\pi)^t v_\phi$ as

$$\begin{aligned}\frac{d}{dt}z_\pi(t) - A_\pi z_\pi(t) - B_\pi i_\pi(t) &= 0 \\ (L_\pi)^t v_\phi(t) - C_\pi z_\pi(t) - D_\pi i_\pi(t) &= 0 \\ C_\phi \frac{d}{dt}v_\phi(t) + G_\phi v_\phi(t) + L_\pi i_\pi(t) + F(v_\phi(t)) - b_\phi(t) &= 0\end{aligned}\tag{5.19}$$

Using standard nonlinear solvers or any of the general-purpose circuit simulators the unified set of differential equations represented by (5.19) can be solved to yield unified transient solutions for the entire nonlinear circuit consisting of high-frequency interconnect subnetworks. For those simulators (such as HSPICE) which do not directly accept the differential equations as input, the macromodel represented by (5.12) can be converted to an equivalent subcircuit, and is described in the next section.

5.6.1 Conversion of Macromodels to Equivalent Subcircuits

Conversion of differential equations to equivalent subcircuits can be accomplished in several ways. For this purpose a new technique which is easy to understand as well as to implement, is developed. For the purpose of illustration, consider a simple case of two port network with two states represented in the form of (5.12)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (5.20)$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (5.21)$$

Next, (5.20) and (5.21) can be rearranged as

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_{11}v_1 + b_{12}v_2 \quad (5.22)$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_{21}v_1 + b_{22}v_2 \quad (5.23)$$

$$i_1 = c_{11}x_1 + c_{12}x_2 + d_{11}v_1 + d_{12}v_2 \quad (5.24)$$

$$i_2 = c_{21}x_1 + c_{22}x_2 + d_{21}v_1 + d_{22}v_2 \quad (5.25)$$

In the above equations, the port voltages and currents are represented by v_1, v_2 and i_1, i_2 , respectively. An equivalent network representing (5.22) - (5.25) can be constructed as shown in Fig. 5.14. Each state in the macromodel requires a separate node in the equivalent circuit and are represented by nodes n_1, n_2 . The state variables x_1, x_2 can be represented by the capacitor voltages. These

capacitors are denoted by C_{n1} , C_{n2} and the corresponding voltages by v_{n1} , v_{n2} . Next, the terms such as $a_{11}x_1$ in (5.22) - (5.25) can be represented by voltage controlled current sources. (5.22) and (5.23) are fully represented by Fig. 5.14(c) and Fig. 5.14(d). Output equations represented by (5.24) and (5.25) are realized through equivalent circuits shown in Fig. 5.14(a) and Fig. 5.14(b). Generalization of the above discussion in the presence of more number of states or ports is straightforward.

Fig. 5.14. Illustration of equivalent subcircuit generation from macromodels

5.6.2 Summary of Computational Steps

A brief summary of the computational steps involved in simulating nonlinear networks comprising distributed, full-wave and measured subnetworks using the proposed block CFH algorithm is given below.

-
- *Step 1: Formulate the MNA and obtain equations (3.1) and (3.2).*
 - *Step 2: Consider the linear subnetwork π and choose the maximum frequency of interest w_{max} .*
 - *STEP 3: Compute the moments of the system M_n using (4.18).*
 - *Step 4: Using the block CFH algorithm obtain the dominant pole-set and dominant residues at all the ports using (5.7). Obtain the reduced-order model for $Y(s)$ represented by (5.8).*
 - *Step 5: Check for any negative eigenvalues in $Re(Y(s))$, in the vicinity of w_{max} . If any negative eigenvalues are found, choose $\Psi(s)$ so as to compensate for the loss of passivity and satisfying (5.11).*
 - *Step 6: Obtain the macromodel represented by (5.12).*
 - *Step 7: Repeat steps 2 - 6 for every linear subnetwork in the system.*
 - *Step 8: Perform unified nonlinear transient simulation using (5.19).*
-

5.7 Numerical Examples

In this section, two numerical examples are presented to demonstrate the efficiency and accuracy of the proposed Block CFH algorithm. The discussions are presented with respect to both frequency-domain and time-domain results. In addition, CPU comparison of the proposed method with the commercially available HSPICE simulation tool is presented.

Example 1: Two Port RLC Network

In this example, a relatively small network containing 4 resistors, 4 inductors, 3 capacitors and two nonlinear elements is analyzed using the proposed block CFH technique. The network is shown in Fig. 5.15. In order to validate the accuracy and efficiency of the proposed method, the network is analyzed using three different approaches. In the first approach, a regular HSPICE analysis is performed. In the second approach a conventional CFH analysis is performed in which each Y_{ij} is treated with different sets of dominant poles. Table 5.1 gives the number of poles required to represent each Y_{ij} . A total of 14 dominant poles were needed in this case to accurately represent the lower-order frequency-domain model. The macromodel in this case contains $25 \times 2 = 50$ states. In the third approach, the proposed BCFH is performed (highest frequency being 5GHz) and a common set containing five poles is extracted. The poles, and the corresponding residues which are obtained using the new residue computation algorithm, are given in Table 5.2 (the values given here are scaled by a factor of $1e9$ and the conjugate values are not given). The macromodel in this case consists of $9 \times 2 = 18$ states. Hence, in this example, a savings of 64% in the macromodel states is achieved using the block CFH technique compared to conventional model-reduction techniques.

Fig. 5.15. Nonlinear circuit containing RLC lumped network

Table. 5.1. Number of poles required for each Y_{ij} using conventional techniques

Y_{ij}	Number of poles (# real poles + # complex poles)
Y_{11}	(1+8)
Y_{12}	(1+6)
Y_{22}	(1+8)
Total # of poles	(3+22)

Table. 5.2. Poles and residues obtained using block CFH (Example 1)

Poles		R^{11}	$R^{12} \text{ \& } R^{21}$	R^{22}
P1	-3.3385e+00	6.0952e-02	-6.3611e-02	6.6433e-02
P2	-1.9889e+00 + 1.7803e+01i	1.2621e+00 - 5.3765e-02i	1.9258e-01 + 3.8040e-02i	2.7693e-02 + 1.2788e-02i
P3	-1.6406e+00 + 2.4315e+01i	9.3386e-01 + 2.2660e-01i	-1.5918e-01 - 4.1521e-02i	2.7124e-02 + 7.5771e-03i
P4	-4.0573e-01 + 3.2744e+01i	2.4328e-01 + 1.1005e-01i	3.0243e-02 + 1.2547e-02i	3.7551e-03 + 1.4208e-03i
P5	-8.0196e+00 + 2.9485e+01i	-2.1525e-05 + 2.0716e-04i	-1.4051e-05 + 2.4123e-05i	-3.7035e-06 + 2.3108e-06i

Fig. 5.16 shows the comparison of frequency responses obtained using the proposed block CFH and HSPICE, and they match accurately. The real-part response, the behavior of principle minor and the eigenvalues v/s frequency are given in Fig. 5.6, Fig. 5.7 and Fig. 5.17, respectively. As seen from Fig. 5.17, since none of the eigenvalues are negative passivity compensation is not required. Next, the macromodel from the block CFH is directly linked to HSPICE and a nonlinear analysis is performed. The network is excited with an input having a rise/fall time of 0.1ns and a pulse width of 4ns. Responses at nodes P₁, P₂, and V_{out} are compared with the HSPICE simulation of the original network (Fig. 5.18). As seen, the results from both the methods match accurately.

Fig. 5.16. Frequency responses for Example 1

Fig. 5.17. Eigenvalues of $Re(Y(s))$ for Example 1

Fig. 5.18. Time responses for Example 1

Example 2: Distributed Transmission Line Network

In this example, a simulation consisting of seven lossy/distributed transmission lines along with nonlinear terminations is performed using the proposed BCFH algorithm. The linear subnetwork contains three ports and is shown in Fig. 5.19. It is to be noted that this network can't be directly handled by model-reduction techniques such as Arnoldi, PVL and PACT as it contains distributed elements. However, during the model-reduction using block CFH, transmission line stamps are used directly (without discretization).

As in Example 1, in order to validate the accuracy and efficiency of the proposed algorithm, the network is analyzed using three different approaches. In the first approach, a regular HSPICE analysis using the original circuit is performed. In the second approach, a conventional CFH analysis is performed in which each Y_{ij} is treated with different sets of dominant poles. Table 5.3 gives the number of poles required to represent each Y_{ij} . A total of 226 dominant poles were needed in this case to accurately represent the lower-order frequency-domain model. The macromodel in this case contains $226 \times 3 = 668$ states. In the third approach, the proposed BCFH is performed (highest frequency being 5GHz) using a common set containing 42 poles (for details please refer to section 5.2). The macromodel in this case contained $42 \times 3 = 126$ states. *Hence, in this example, a savings of 82% in the macromodel states is achieved using the block CFH technique compared to conventional model-reduction techniques.*

Fig. 5.20 shows the comparison of frequency responses obtained using the proposed BCFH and HSPICE, and they match accurately. Real-part responses and the behavior of eigenvalues of $Re(Y(s))$ are given Fig. 5.21 and Fig. 5.22, respectively. Next, the macromodel from the BCFH is directly linked to HSPICE and a nonlinear simulation is performed with an input pulse having a rise/fall

time of 0.1ns and a pulse width of 5ns. Responses at node P_1 , P_2 , P_3 , V_{out2} and V_{out3} are compared with the HSPICE simulation of the original network and (Fig. 5.23 and Fig. 5.24). As seen, the two responses match accurately. (responses from approach 2 are not given for the purpose of clarity).

A CPU comparison of all the three approaches is given in Table 5.4. A speed up of 52 is achieved using BCFH compared to HSPICE. It is to be noted that the speed up ratio increases significantly in the presence of large number of interconnects as well as with the increase in the number of ports for the linear subnetwork.

Table. 5.3. Number of poles required for each Y_{ij} using conventional techniques

Y_{ij}	Number of poles
Y_{11}	$2+38 = 40$
Y_{12}	$3+34 = 37$
Y_{13}	$2+36 = 38$
Y_{22}	$2+36 = 38$
Y_{23}	$3+32 = 35$
Y_{33}	$2+36 = 38$
Total number of poles	226

Table. 5.4. CPU comparison (Sun SPARC10)

	<i>HSPICE</i>	Previous model-reduction techniques	Proposed Block CFH
<i>CPU time (seconds)</i>	1510	380	29
<i>Speed-up</i>	**	4	52

Fig. 5.19. Seven distributed transmission line circuit (Example 2)

Fig. 5.20. Frequency responses (magnitude) for Example 2

Fig. 5.21. Frequency responses (real-part) for Example 2

Fig. 5.22. Eigenvalues of $\text{Re}(Y(s))$ for Example 2

(a): Time response at node P_1 (b): Time response at node P_2 **Fig. 5.23. Time responses for Example 2**

(a): Time response at node V_{out2} (b): Time response at node V_{out3} **Fig. 5.24. Time responses for Example 2**

As seen from frequency and time responses in Fig. 5.20 - Fig. 5.24, the proposed BCFH yields results comparable to that of HSPICE, however, with significant CPU speedups. In addition, as we see in subsequent chapters, block CFH also provides efficient means to simulate networks with measured data, frequency dependent RLCG parameters as well as full-wave models.

5.8 CPU and Accuracy Analysis

The Speed-up achieved through the proposed block CFH algorithm is expected to increase considerably in the presence of large number of nonlinear elements as well as large interconnect subnetworks containing high-frequency interconnect models. The main reason behind the efficiency of the proposed block CFH algorithm is that it requires one LU decomposition of circuit matrix per frequency hop during model-reduction, *whereas the SPICE requires one LU decomposition of the large circuit matrix during each Newton iteration at each time point*. The number of states in the macromodel which is used during nonlinear simulation is much smaller than the case of linking the original network directly to SPICE. Hence the nonlinear simulation using macromodels is significantly faster than using the original large interconnect networks. Also another point worth mentioning here is that, in cases of distributed/full-wave/measured subnetworks, direct linking of these subnetworks to SPICE is not possible whereas the block CFH algorithm provides efficient solution for even such cases. A detailed discussion of the CPU and the accuracy analysis of the new algorithm is given below.

Formulating the residue equations in the form given by (5.7) leads to two important advantages which contributes to additional computational savings.

- 1) *Due to the form used in (5.7), the number of equations to be solved remains equal to*

the number of unknown residues despite increase in the matched frequency points. However, increasing the number of points (N_m) will give better accuracy for residues.

2) Furthermore, irrespective of the number of ports, LHS of equation (5.7) needs to be formulated only once (requires only one LU decomposition)

The CPU cost involved in the proposed algorithm is mainly due to the transfer-function based approach to obtain frequency responses of each entry in the matrix Y_π . Assume that on average N_H hops are required to obtain frequency responses of each entry (in general, N_H ranges between 2 to 6 hops depending on the nature of the response and the highest frequency of interest). This means that for the entire matrix-transfer function we need $N_H \times \left[\frac{n_\pi(n_\pi + 1)}{2} \right]$ LU decompositions (n_π is the number of ports; since matrix Y_π is symmetric, we need to compute data for elements of only the upper-half matrix). The CPU cost needed for evaluating the poles is negligible as they are obtained by directly applying a root-solving algorithm on the denominator polynomial of each of the transfer-functions during transfer-function based CFH approach.

For a sparse matrix of order L the CPU time for LU decomposition is proportional to L^α where α is problem dependent and ranges from 1.2 to 1.7 [105]. Using this information, the total CPU cost involved during the transient simulation with the block CFH algorithm is as follows:

$$CPU_{BCFH} = k_1 \times [N_T \times N_{NI}] \times S^\alpha + k_2 \times \left(N_H \times \left[\frac{n_\pi(n_\pi + 1)}{2} \right] \right) \times L^\alpha \quad (5.26)$$

where k_1, k_2 are constants which are dependent on the problem. L represents the size of the large circuit matrix before model-reduction and S represents the circuit size with macromodel ($S \ll L$), which is obtained after model-reduction. N_T is the total number of time-points needed during the transient simulation, N_{NI} is the average number of Newton iterations required per time point (usually $N_T \times N_{NI}$ is in the range of few thousands).

However, the total time needed for SPICE-like conventional simulation is

$$CPU_{SPICE} = k_1 \times [N_T \times N_{NI}] \times L^\alpha \quad (5.27)$$

It is to be noted that during conventional simulation which uses the original large network, the maximum step size during nonlinear transient analysis is very much limited due to the widely spread pole distribution. Hence in general, the total number of time points required during conventional circuit simulation is much higher than using the BCFH). Since $S \ll L$, we gain a net approximate speed up of

$$Speed\ up \approx \frac{[N_T \times N_{NI}]}{N_H \times \left[\frac{n_\pi(n_\pi + 1)}{2} \right]} \quad (5.28)$$

which is in the order of 100 - 1000, depending on the nature of the circuit.

***Efficient Simulation of
Full-Wave Models
Using Block CFH***

"A new idea is delicate. It can be killed by a sneer or a yawn; however, it needs all the perseverance and the optimism to blossom."
- Benjamin Franklin

We noted in chapter two that the major limitation of all the model-reduction techniques is that they are limited to interconnect networks described by quasi-TEM approximations. However, if wavelengths of propagation become comparable to that of line cross sections, they result in *substantial electric (E) or magnetic (H) fields in the direction of propagation*. In such situations quasi-TEM based models become inadequate and full-wave models become necessary. In this chapter, a new technique is presented to extend the block CFH algorithm to include full-wave based subnetworks.

6.1 Background

Consider a dispersive system of interconnects embedded in an inhomogeneous dielectric medium, as shown in Fig. 6.1. The main problem in the analysis of such dispersive systems is that these structures involve mixed boundaries. Therefore, in a full-wave analysis not only must all the six components of the field be taken into account, but due to the presence of strips at some interfaces, the final equations must be solved numerically, since they are rather complicated and the solution can't be given in a closed-form. The complexity of the solution increases if the substrates/superstrates are anisotropic materials.

Fig. 6.1. Cross-sectional view of a multiconductor dispersive system

Combining circuit simulation and full-wave analysis techniques is highly involved. The information that is obtained through a general full-wave analysis is in terms of modal parameters such as propagation constants and characteristic impedances. However, a circuit simulator requires the information in the form of terminal currents and voltages. This demands a generalized technique to transform modal results into a full-wave stencil, which can be included in a nodal analysis for the purpose of circuit simulation. Another important issue involved here is the cost of a full-wave analysis at each frequency point. For a typical high-speed interconnect circuits which need thousands of frequency point solutions to get accurate responses, it would become prohibitively CPU expensive to simulate because of the combined cost involved, i.e, evaluation of a full-wave model to obtain modal parameters and computation of circuit response at each point.

It is evident by now that the model-reduction techniques enable efficient simulation of high-speed interconnects. However, the MR techniques available in the literature are based on quasi-TEM approximation and have no mechanism to

address full-wave interconnect simulation. In this chapter, we describe a new method to extend the block CFH technique for efficient simulation of full-wave interconnects. The proposed method overcomes the major limitation of existing model-reduction techniques, which are based on quasi-TEM assumptions.

Three main contributions are presented in this chapter [10], [68], [69]. First, the modified nodal admittance (MNA) [92] formulation has been extended to include full-wave stencils. Second, a new algorithm for moment-generation involving full-wave models has been developed. Finally, using the new tool, need for a full-wave analysis in general interconnect circuits is investigated.

6.2 *Generalized Stencil for Full-Wave Subnetworks*

In this section, a brief summary of the derivation of a generalized full-wave stencil suitable for inclusion in circuit simulators is presented. Consider the multiconductor interconnect system with N coupled conductors as shown in Fig. 6.1. The system contains hybrid modes (TE and TM). The objective is to evaluate N dominant modes of propagation present in the system using a full-wave technique and subsequently to obtain a relationship between terminal voltages and currents. In this work, spectral-domain technique (SDT) is used for full-wave analysis, which is widely used because of its computational superiority over other full-wave methods. A brief description of the computational steps associated with the SDT is given in Appendix A. However, it is to be noted that the proposed algorithm is not limited to a particular type of full-wave analysis method, and can be easily integrated with general full-wave techniques such as FEM or FDTD. In the rest of this section, several definitions of interest concerning modal parameters is given, and also an expression relating terminal voltages/currents with full-wave parameters is derived.

6.2.1 Current-Eigenvector Matrix

Every mode propagating in the structure contributes to the modal currents flowing through each of the lines in the system. In order to characterize the modal current contribution by a mode i to each of the lines (illustrated in Fig. 6.2(a)), we define a current-eigenvector, M_i , as

$$M_i = [m_{1i} \ m_{2i} \ m_{3i} \ \dots \ m_{Ni}]^T \quad (6.1)$$

where m_{ki} is the longitudinal current amplitude on line k normalized to the current amplitude on line 1 (both currents are due to mode i), and is computed as

$$m_{ki} = \frac{\sum_{j=1}^P \int_{x_k}^{(x_k + w_k)} c_{ij} J_z^{ij}(x) dx}{\sum_{j=1}^P \int_{x_1}^{(x_1 + w_1)} d_{ij} J_z^{ij}(x) dx} \quad (6.2)$$

where x_k and w_k are defined in Fig. 6.1. Next, define a global current-eigenvector matrix, M , which characterizes the relative modal current contributions on different lines due to all the dominant modes as

$$M = [M_1 \ M_2 \ \dots \ M_N] \quad (6.3)$$

Fig. 6.2. Relationship between modal parameters and terminal voltages/currents

6.2.2 Modal Characteristic Impedance (Z_{ki})

The modal currents obtained from a full-wave analysis as mentioned above do not necessarily correspond to the physical/observable currents on a signal line. Therefore, it is necessary to transform the modal description of the system into a new form suitable for nodal analysis. The nodal analysis needs an explicit relationship between the terminal voltages and currents of a subnetwork (Fig. 6.2(b)). Generally, modal characteristic impedances are used to relate the modal currents to the nodal (terminal) voltages/currents of the system. However, because of the non-TEM nature of dispersive systems, the definition of modal characteristic impedance is not unique. In this work, modal characteristic impedances are evaluated based on the total power transported on a line k due to a mode i (Fig. 6.2(a)) as

$$Z_{ki} = \frac{\iint (E_{ki} \times H_{ki}^*) \cdot \hat{z} (dx)(dy)}{|I_{ki}|^2} \quad (6.4)$$

where E_{ki} and H_{ki} are the fields generated by mode i on line k . The current I_{ki} can be calculated by an one dimensional integration of longitudinal current distribution on line k due to mode i .

6.2.3 Generalized Full-Wave Stencil

Consider the multiconductor full-wave interconnect system shown in Fig. 6.2(b). Let V_I , I_I be the input and V_O , I_O be the output voltage and current vectors, respectively. It is now necessary to link these physical voltages and currents to the modal parameters described by (6.1) - (6.4). Such a relationship can be derived as

$$A_f \begin{bmatrix} V_O(s) \\ V_I(s) \end{bmatrix}_f + B_f \begin{bmatrix} I_O(s) \\ I_I(s) \end{bmatrix}_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (6.5)$$

where matrices A_f and B_f are complex functions of modal parameters and the subscript ' f ' indicates that the subnetwork under consideration is based on full-wave analysis. Results are summarized in Appendix B.

6.3 Block CFH Based Simulation of Full-Wave Models

Having obtained a generalized stencil for full-wave subnetworks, we will present in this section a new algorithm for efficient transient simulation of full-wave subnetworks. The proposed algorithm is based on the block CFH technique presented in chapter 5. However, in order to apply the BCFH algorithm on

networks containing full-wave subnetworks, moments of the full-wave subnetwork as indicated by equations (4.18) - (4.19) are needed. In the case of quasi-TEM models for interconnect subnetworks, efficient and accurate methods for the computation of moments $A^{(r)}(s)$ and $B^{(r)}(s)$ in terms of R, L, C and G distributed parameters are discussed extensively in the literature [33], [38] - [42], [46] - [48].

6.3.1 Full-Wave Transmission Line Moments

As seen from (4.19), in order to compute full-wave moments, the derivatives represented by $A^{(r)}(s)$ and $B^{(r)}(s)$ are needed (the subscript f is dropped for simplicity). These derivatives can be obtained using (B.13) (appendix B) as

$$A^{(r)} = \begin{bmatrix} \varphi^{(r)} & \mathbf{0} \\ \mathbf{0} & \varphi^{(r)} \end{bmatrix}; \quad B^{(r)} = \begin{bmatrix} -[\varphi Z_c]^{(r)} & -Z_c^{(r)} \\ -Z_c^{(r)} & [\varphi Z_c]^{(r)} \end{bmatrix} \quad (6.6)$$

The derivatives of $[\varphi Z_c]$ can be obtained recursively as

$$[\varphi Z_c]^{(n)} = \sum_{k=1}^n \binom{n}{k} \varphi^{(n-k)} Z_c^{(k)} + \varphi^{(n)} Z_c \quad (6.7)$$

The derivatives of φ and Z_c are calculated using equations (B.4), (B.8), (B.10) and (B.12). Let $E = e^{-\Gamma L}$ and using (A.15) we can write

$$\varphi Z_m = Z_m E \quad (6.8)$$

Also re-writing equation (B.10), we have

$$\mathbf{Z}_c \mathbf{M} = \mathbf{Z}_m \quad (6.9)$$

Differentiating (6.8) and (6.9) we get

$$\varphi^{(n)} \mathbf{Z}_m = - \sum_{k=1}^n \binom{n}{k} \varphi^{(n-k)} \mathbf{Z}_m^{(k)} + \sum_{k=1}^n \binom{n}{k} \mathbf{Z}_m^{(n-k)} \mathbf{E}^{(k)} + \mathbf{Z}_m^{(n)} \mathbf{E} \quad (6.10)$$

$$\mathbf{Z}_c^{(n)} \mathbf{M} = - \sum_{k=1}^n \binom{n}{k} \mathbf{Z}_c^{(n-k)} \mathbf{M}^{(k)} + \mathbf{Z}_m^{(n)} \quad (6.11)$$

(6.10) and (6.11) give the derivatives of φ and $\mathbf{Z}_c$ in terms of derivatives of $\mathbf{E}$, $\mathbf{M}$ and $\mathbf{Z}_m$. The derivatives of $\mathbf{E}$, $\mathbf{M}$ and $\mathbf{Z}_m$ are found from (B.4), (6.3) and (B.8) as

$$\mathbf{E}^{(n+1)} = \text{diagonal} \left[-L(e^{-\Upsilon_i L})^{(n)} \Upsilon_i^{(1)} - L \sum_{k=1}^n \binom{n}{k} (e^{-\Upsilon_i L})^{(n-k)} \Upsilon_i^{(k+1)} \right], i = 1, \dots, N \quad (6.12)$$

$$\mathbf{M}^{(n)} = \begin{bmatrix} m_{11}^{(n)} & \dots & m_{1N}^{(n)} \\ \dots & \dots & \dots \\ m_{N1}^{(n)} & \dots & m_{NN}^{(n)} \end{bmatrix} \quad (6.13)$$

and

$$\mathbf{Z}_m^{(n)} = \begin{bmatrix} \sum_{k=0}^n \binom{n}{k} m_{11}^{(n-k)} \mathbf{Z}_{11}^{(k)} & \dots & \sum_{k=0}^n \binom{n}{k} m_{1N}^{(n-k)} \mathbf{Z}_{1N}^{(k)} \\ \dots & \dots & \dots \\ \sum_{k=0}^n \binom{n}{k} m_{N1}^{(n-k)} \mathbf{Z}_{N1}^{(k)} & \dots & \sum_{k=0}^n \binom{n}{k} m_{NN}^{(n-k)} \mathbf{Z}_{NN}^{(k)} \end{bmatrix} \quad (6.14)$$

Next, to evaluate the expressions given by (6.12) - (6.13), we need the derivatives of individual entries represented by Υ_i , m_{ij} and, $\mathbf{Z}_{ij}$ where $i, j = 1, \dots, N$.

However, all these entries are frequency dependent and they do not have a closed form expression with respect to frequency. To overcome this difficulty, full-wave analysis is performed on each subnetwork; and a piecewise polynomial approximation based on least squares is obtained for each of the above parameters. However, it is to be noted that due to the complex frequency dependence, approximation by a single polynomial may not be possible and more than one polynomial may be needed. For example, the variation of modal characteristic impedance shown in Fig. 6.3, needs three polynomials (P_1 , P_2 and P_3) to fit accurately over a frequency range between 0 to 15 GHz. Due care should be taken in CFH implementation to handle any crossover between CFH expansion points and polynomial break points.

Fig. 6.3. Illustration of polynomial break points

Next, we need to compute the moments of polynomials. For the purpose of illustration, consider the case of propagation constants ($\Upsilon_i(s)$), which can be expressed in terms of a polynomial as

$$\Upsilon_i(s) = (\Upsilon_0)_i + (\Upsilon_1)_i s^2 + (\Upsilon_2)_i s^4 + (\Upsilon_3)_i s^6 + \dots + (\Upsilon_n)_i s^{2n} \quad (6.15)$$

Only even powers of s are used so that the resulting polynomial coefficients are real when an equivalence $s = j2\pi f$ is used. The r^{th} derivative of the above polynomial is given by

$$Y_i^{(r)}(s) = \sum_{k=l}^n \frac{(2k)!}{(2k-r)!} (\Upsilon_k)_i s^{(2k-r)} \quad (6.16)$$

where, $l = \frac{r}{2}$ if r is even and $l = \frac{r+1}{2}$ if r is odd. Similarly the derivatives of m_{ij} and Z_{ij} can be computed. It can be seen that the CPU cost is significantly reduced since the moments are evaluated in a closed form. Also it is to be noted that the modal parameters do not necessarily be represented by piecewise polynomials; other forms such as rational functions [104] can also be used for this purpose.

6.3.2 Summary of Computational Steps

A summary of the steps involved in the computation of moments for circuits containing full-wave subnetworks is given below.

-
- *Step 9: Obtain polynomial approximations for matrices E , M and Z_m , represented by equations (B.4), (6.2) & (B.8).*
 - *Step 10: Chose polynomials for E , M and Z_m belonging to a particular frequency range containing the current frequency.*
 - *Step 11: Compute the derivatives of E , M and Z_m by evaluating the derivatives of polynomial entries using (6.16), (6.12) - (6.14).*
 - *Step 12: Calculate the derivatives of φ , Z_c and φZ_c using (6.7) - (6.11).*
 - *Step 13: Find $A^{(r)}$ and $B^{(r)}$ using (6.6).*
 - *Step 14: Repeat steps 1 through 5 for every full-wave system in the network.*
 - *Step 15: Calculate $Y^{(r)}$ using (4.18) - (4.19).*
-

6.4 Numerical Examples

In this section three examples are provided: Examples 1 and 2 demonstrates the accuracy and efficiency of the new algorithm as compared to conventional FFT simulation. Example 3 provides a comparison between full-wave and TEM models at higher frequencies and analyzes the impact of cross-sectional dimensions and dielectric materials of interconnect structures on network responses.

Example 1

In this example, a circuit containing 7 full-wave modeled transmission lines (Fig. 6.4) is analyzed using the proposed full-wave CFH simulation technique. The microstrip under consideration is chosen from the literature [5], [6] and has the following dimensions: $h_1 = 0$, $h_2 = 1.27mm$, $h_3 = 10.43mm$, $h_4 = 1mm$, $s = 0$, $w = 0.635mm$, $a = 6.35mm$, $\epsilon_{r2} = 8.875$ and, $\epsilon_{r3}, \epsilon_{r4} = 1$ (as per the symbols shown in Fig. 6.1). The frequency dependent variations of propagation constant and characteristic impedances (obtained using the spectral-domain full-wave analysis) are shown in Fig. 6.5.

Next, the network (with $R_{in} = 75\Omega$) is excited with a 1V pulse having 0.1ns rise/fall time and 10ns duration. Fig. 6.6. compares the frequency responses from both the proposed full-wave CFH simulation and conventional simulation where the response is obtained by solving ((3.10) directly at 1024 frequency points. As seen, the two responses match accurately up to the pre-defined highest frequency (13 GHz). Fig. 6.7 gives the transient responses using the new method, conventional FFT approach, and also using the commercial simulator 'HP EEsof - libra'. As seen, all three responses are almost indistinguishable.

Fig. 6.4. Circuit with seven full-wave modeled interconnects (Example 1)

Next, the same circuit is analyzed with a nonlinear termination (Load (b)). The network (with $R_{in} = 5\Omega$) is excited with an input pulse of 5V having 1ns rise/fall time and 10ns duration. Fig. 6.8. shows the transient responses using both the proposed full-wave simulation and HP EEsosf. In the proposed method, a SPICE simulation is performed using the time-domain macromodel obtained through block CFH. As seen, both responses match accurately.

Example 2

A relatively large circuit having 243 nodes and containing 160 resistors, 162 inductors, 162 capacitors and 12 full-wave modeled transmission lines is simulated using the proposed algorithm. The input signal is a 1V pulse with 0.1ns rise/fall time and 10ns duration. The response is taken near the source node.

Transient responses obtained from the new method and conventional approach are given in Fig. 6.9 and they match accurately.

A CPU comparison of conventional simulation and the proposed technique for examples 1 (linear load) and 2 is given in Table 6.1.

Table. 6.1. CPU comparison of conventional and BCFH (in secs, SPARC10)

	Conventional method	Proposed Method	Speedup Ratio
Example 1	259.11	9.02	29
Example 2	793.51	22.46	35

Fig. 6.5. Modal parameters for Example 1

Fig. 6.6. Frequency responses for Example 1

Fig. 6.7. Transient response (Example 1)

Fig. 6.8. Transient response - nonlinear termination (Example 1)

Fig. 6.9. Transient response (Example 2)

Example 3

In this example we investigate the need for full-wave models in signal integrity analysis involving high-speed interconnect structures, using the full-wave simulator proposed in this paper. The network shown in Fig. 6.10(a) is used as a test circuit. The interconnect structure chosen is a two conductor coupled system (shown in Fig. 6.10(b)). An emphasis is given during the analyses to study the impact of variation in cross-sectional dimensions and dielectric materials associated with the interconnect structures on network responses with respect to the performance of both quasi-TEM and full-wave models. The results are reported in terms of operating wavelength.

Fig. 6.10. Circuit for Example 3

A. Effect of cross-sectional dimensions

Let the interconnect configuration be as shown in Fig. 6.10(b). The structure is first analyzed with quasi-TEM approximation using the software package developed in [19]. The L and C matrices are found to be:

$$L = \begin{bmatrix} 297.9 & 25.3 \\ 25.3 & 297.9 \end{bmatrix} (nH/m) \quad C = \begin{bmatrix} 263.7 & -0.0476 \\ -0.0476 & 263.7 \end{bmatrix} (pF/m)$$

Next, the structure is analyzed using full-wave model. The modal parameters for both even and odd modes are shown in Fig. 6.11.

Fig. 6.11. Modal parameters for Example 3.A ($h_1 = 5\text{mm}$)

The frequency responses at node $V_{\text{out}1}$, for both quasi-TEM and full-wave models are shown in Fig. 6.12(a). It is observed that the response corresponding to quasi-TEM approximation starts deviating from full-wave results when the frequency approaches 650 MHz. The effective dielectric constants at this frequency for both even and odd modes are found to be 7.77 and 6.71, respectively. The *effective wavelength** [6], which we refer henceforth as $\lambda_{\text{effective}}$ or λ_e in short, is found to be

* The effective wavelength of a wave propagating in a dielectric medium at a certain frequency is given by

$$\lambda_e = \frac{\text{velocity of the wave}}{\text{frequency}} = \frac{1/\sqrt{\mu_r \epsilon_r(\text{effective})}}{f}$$

178mm, for odd mode. The dielectric thickness here being 5mm, the above result implies that a full-wave analysis becomes essential when the vertical cross-section approaches a ratio of $\lambda_e/35$ or more. Fig. 6.12(b) shows the frequency responses at node V_{out1} when both the input nodes are simultaneously excited by voltage sources (in Fig. 6.10(a) R_2 is also connected to a voltage source).

(a) Frequency responses at node V_{out1} for circuit in Fig. 6.10(a)

(b) Frequency responses at node V_{out1} when both the lines are excited by voltage sources

Fig. 6.12. Frequency responses for Example 3.A ($h_l = 5\text{mm}$)

A.1 Effect of dielectric height

In this case the dielectric height (h_1) is changed from 5mm to 10mm, keeping all other parameters constant. Here, quasi-TEM based analysis yields

$$L = 1e^{-7} x \begin{bmatrix} 4.195 & 0.7365 \\ 0.7365 & 4.195 \end{bmatrix} (H/m) \quad C = 1e^{-11} x \begin{bmatrix} 17.57 & -0.147 \\ -0.147 & 17.57 \end{bmatrix} (F/m)$$

The new frequency responses are given in Fig. 6.13. It can be observed that the frequency response from quasi-TEM approximation starts deviating at a frequency 520MHz, which is relatively lower than the case with $h_1=5\text{mm}$ (Fig. 6.12). Also it can be noted from the responses that the amount of deviation suffered is relatively higher than the previous case. This is expected as the dielectric height is increased, more number of higher order modes start propagating [6] and quasi-TEM assumptions become inaccurate even at relatively lower frequencies. The effective dielectric constants corresponding to the deviating frequency (520MHz) are 7.46 and 6.02 for even and odd modes, respectively. Here we obtain $\lambda_e = 235\text{mm}$ for odd mode, which implies that a full-wave analysis becomes essential if the cross-sectional dimensions are larger than $\lambda_e/23$.

Next, the dielectric height (h_1) is reduced from 5mm to 1mm. The deviation between quasi-TEM and full-wave models in this case is observed at a higher frequency of 3.2 GHz. The effective dielectric constants corresponding to this frequency are 8.88 and 8.81 for even and odd modes, respectively. Here we obtain $\lambda_e = 32\text{mm}$ for the odd mode, which implies that a full-wave analysis becomes essential if the dielectric height is larger than $\lambda_e/32$.

Fig. 6.13. Frequency responses for Example 3.A-1 ($h_l = 10\text{mm}$)

A.2 Effect of conductor width

In this case the conductor width (w) in Fig. 6.10(b) is changed from 5mm to 2.5mm, keeping all other parameters constant. The quasi-TEM approximation for this configuration yields

$$L = 1e^{-7} x \begin{bmatrix} 4.232 & 0.3823 \\ 0.3823 & 4.232 \end{bmatrix} (H/m) \quad C = 1e^{-11} x \begin{bmatrix} 17.37 & -0.4083 \\ -0.4083 & 17.37 \end{bmatrix} (F/m)$$

Corresponding new frequency responses are given Fig. 6.14. The frequency response corresponding to the quasi-TEM approximation starts deviating full-wave results at 620MHz. Also it is seen that the deviation between the two responses behaved almost identical to the case when $w=5\text{mm}$ (Fig. 6.12).

Fig. 6.14. Frequency responses for Example 3.A-2 ($w = 2.5\text{mm}$)

A.3 Effect of separation width

Now vary the separation width (s) in Fig. 6.10(b) from 5mm to 2.5mm, keeping all other parameters constant. The quasi-TEM analysis for this configuration yields the following L and C matrices:

$$L = 1e^{-7} x \begin{bmatrix} 2.96 & 0.459 \\ 0.459 & 2.96 \end{bmatrix} (H/m) \quad C = 1e^{-11} x \begin{bmatrix} 26.53 & -1.585 \\ -1.585 & 26.53 \end{bmatrix} (F/m)$$

Corresponding new frequency responses are given in Fig. 6.15. It is observed that the frequency response corresponding to the quasi-TEM model starts deviating at a relatively lower frequency (400MHz) compared to the previous case with larger s ($s=5\text{mm}$, Fig. 6.12); and the deviations are relatively larger. The time response for

an input excitation of 0.01ns rise/fall time and 5ns pulse width, at node V_{out1} is given in Fig. 6.16. The crosstalk output V_{out2} is given in Fig. 6.17. Observing the two outputs, it is evident that the TEM responses differ from full-wave responses significantly. In addition, a shift in the time-related events can be observed in the responses. The deviations are more pronounced in the crosstalk output.

Fig. 6.15. Frequency responses for Example 3.A-3 ($s = 2.5\text{mm}$)

Fig. 6.16. Transient responses at node V_{out1} for Example 3.A-3 ($s = 2.5\text{mm}$)

Fig. 6.17. Transient responses at node V_{out2} for Example 3.A-3 ($s = 2.5\text{mm}$)

B. Effect of dielectric constant

Consider the interconnect configuration described in Fig. 6.10(b) and vary the dielectric value from 9.9 to 4.0, to study the effect of variation in dielectric constant. The new L and C matrices from a quasi-TEM approximation are found to be

$$L = 1e^{-7} x \begin{bmatrix} 2.978 & 0.2523 \\ 0.2523 & 2.978 \end{bmatrix} (H/m) \quad C = 1e^{-11} x \begin{bmatrix} 1.143 & -3.488 \\ -3.488 & 1.143 \end{bmatrix} (F/m)$$

Corresponding new frequency responses are given in Fig. 6.14. It can be observed that the frequency response corresponding to the quasi-TEM approximation starts deviating at a much higher frequency (1.3GHz) compared to the Case A (where the deviation started to appear at 0.65GHz).

Fig. 6.18. Frequency responses for Example 3.B ($\epsilon_{r1} = 4$)

The results and inferences from the example 3 can be summarized as follows. An increase in the dielectric thickness causes quasi-TEM models to become inaccurate at relatively lower frequencies. This implies that a full-wave analysis becomes necessary as we move up in the integration hierarchy, from the chip to PCB/system level. It is found that depending on the interconnect structure, when the cross-sectional dimensions approach $1/40$ to $1/10$ of the effective wavelength, a quasi-TEM approximation deviates considerably from full-wave results. Also a reduction in the separation width between adjacent conductors makes the full-wave analysis more essential, specially for crosstalk evaluation. The same is true with an increase in the dielectric constant values.

In conclusion, the new contributions presented in this chapter are as follows. First, the modified nodal admittance formulation has been extended to include full-wave stencils. Second, a new algorithm for moment-generation involving full-wave models has been developed. Finally, using the new tool, need for a full-wave analysis in general interconnect circuits is investigated. The proposed algorithm can handle networks with arbitrary number of measured subnetworks.

“When you aim for perfection, you discover it’s a moving target.” - Epicurus

Network-reduction techniques available in the literature require analytical models for interconnects to compute system moments. In practice, however, it may not always be possible to obtain accurate analytical models for interconnects because of the geometric inhomogeneity and associated discontinuities. In this section we present a new technique to extend the block CFH algorithm to measured subnetworks.

7.1 Background

There has been a limited number of attempts in the literature to simulate linear subnetworks with measured data. In [83] a technique is described based on the rational function approximation of the measured data and recursive convolution to analyze transmission lines characterized by tabulated s -parameters. A method based on the macromodeling of each measured subnetwork and recursive convolution for SPICE simulation has been described in [79]. Another method was described in [84] for including measured data into moment-matching simulation. In this method, the circuit, excluding the measured subnetworks, is analyzed using moment-matching techniques; the measured and component-based subcircuits are then combined as one set of frequency-domain stencils and

solved in the frequency-domain. The time-domain impulse response is obtained using inverse fast Fourier transform (IFFT). An alternative method described in [85] computes the measured subnetwork moments based on numerical differentiation in frequency-domain. However, due to numerical differentiation, the accuracy of higher-order moments may not be guaranteed in this case [85].

In this chapter we present a new moment-generation algorithm that is based on the time-domain integration of measured data. The moments thus generated are more accurate compared to the ones obtained through differentiation in the frequency-domain [20], [21], [87], [88]. In addition, a new stencil for measured subnetworks has been developed for inclusion in modified nodal analysis. This method is general in nature and can handle an arbitrary number of measured subnetworks. Also, in order to address the case of overdetermined measurements, an algorithm based on least mean square optimization has been developed. This feature helps to reduce the impact of noise in measurements on network responses.

7.2 *Generalized Stencil for Measured Subnetworks*

In this section, a generalized stencil for measured subnetworks is derived. Such a stencil would be useful while extending the model-reduction based analysis to measured subnetworks. Consider a measured subnetwork k with N_k terminals (Fig. 7.1), characterized by m sets of measurements. Let the i^{th} set of excitation (e.g. voltage sources) and the corresponding measurements at the terminals of subnetwork k (e.g. voltage) be represented by column vectors $J^{(i)}$ and $V^{(i)}$, respectively, where

$$\mathbf{J}^{(i)} = \begin{bmatrix} J_1^{(i)} \\ \vdots \\ J_{N_k}^{(i)} \end{bmatrix} ; \quad \mathbf{V}^{(i)} = \begin{bmatrix} V_1^{(i)} \\ \vdots \\ V_{N_k}^{(i)} \end{bmatrix} \quad (7.1)$$

Let the corresponding terminating impedances be represented by an impedance matrix $\mathbf{Z}_L^{(i)}$ as

$$\mathbf{Z}_L^{(i)} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N_k} \\ \dots & \dots & \dots & \dots \\ Z_{N_k1} & Z_{N_k2} & \dots & Z_{N_kN_k} \end{bmatrix} \quad (7.2)$$

Each set of measurement can have a different set of excitation sources and terminating impedances. For the purpose of illustration, consider the circuit shown in Fig. 7.1(b). Here $\mathbf{Z}_L^{(i)}$ is represented by a diagonal matrix given by $\mathbf{Z}_L^{(i)} = \text{diag}[Z_1 \dots Z_{N_k}]$ and the excitation vector is given by $\mathbf{J}^{(i)} = [v_s \dots 0 \ 0 \ 0]^t$.

Fig. 7.1. Illustration of a general measurement setup

With these m sets of measurements, the subnetwork k can be characterized in frequency-domain as

$$Y_k \begin{bmatrix} V^{(1)} & \dots & V^{(m)} \end{bmatrix}_{N_k \times m} = \begin{bmatrix} Z_L^{-1(1)} (J^{(1)} - V^{(1)}) & \dots & Z_L^{-1(m)} (J^{(m)} - V^{(m)}) \end{bmatrix}_{N_k \times m} \quad (7.3)$$

or, (7.3) can be concisely written as

$$Y_k(s) \tilde{V}_k(s) = \tilde{I}_k(s) \quad (7.4)$$

where Y_k is the unknown $N_k \times N_k$ y-parameter matrix of the measured subnetwork k , represented by

$$Y_k = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N_k} \\ \dots & \dots & \dots & \dots \\ Y_{N_{k1}} & Y_{N_{k2}} & \dots & Y_{N_k N_k} \end{bmatrix} \quad (7.5)$$

Next, we need to evaluate the unknown y-parameters from the given measurements (so as to stencil the measured subnetwork k into MNA as represented by (3.8)). These values can be obtained as follows. Let the elements of the matrix Y_k be represented by a column vector Y_{v_k} of length N_k^2 as

$$Y_{v_k} = [Y_{11} \ Y_{12} \ \dots \ Y_{1N_k} \ Y_{21} \ Y_{22} \ \dots \ Y_{2N_k} \ \dots \dots \ Y_{N_{k1}} \ Y_{N_{k2}} \ \dots \ Y_{N_k N_k}]^t \quad (7.6)$$

Next, taking the transpose of (7.4) we have

$$\tilde{V}_k^t Y_k^t = \tilde{I}_k^t \quad (7.7)$$

Expressing Y_k in terms of its vector notation Y_{v_k} , and with little mathematical manipulations on both RHS and LHS, (7.7) can be expressed as

$$\begin{bmatrix}
 V^{(1)t} & 0 & \dots & \dots & \dots & 0 \\
 0 & V^{(1)t} & 0 & \dots & \dots & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots \\
 0 & 0 & \dots & 0 & \dots & 0 & V^{(1)t} \\
 V^{(2)t} & 0 & \dots & \dots & \dots & \dots & 0 \\
 0 & V^{(2)t} & 0 & \dots & \dots & \dots & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 0 & 0 & \dots & 0 & \dots & 0 & V^{(2)t} \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 V^{(m)t} & 0 & \dots & \dots & \dots & \dots & 0 \\
 0 & V^{(m)t} & 0 & \dots & \dots & \dots & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 0 & 0 & \dots & 0 & \dots & 0 & V^{(m)t}
 \end{bmatrix}_{(N_k \times m) \times N_k^2}
 \begin{bmatrix}
 Y_{11} \\
 Y_{12} \\
 \dots \\
 Y_{1N_k} \\
 Y_{21} \\
 Y_{22} \\
 \dots \\
 Y_{2N_k} \\
 \dots \\
 \dots \\
 Y_{N_k 1} \\
 Y_{N_k 2} \\
 \dots \\
 Y_{N_k N_k}
 \end{bmatrix}_{N_k^2 \times 1}
 =
 \begin{bmatrix}
 Z_L^{-1(1)}(J^{(1)} - V^{(1)}) \\
 \dots \\
 Z_L^{-1(2)}(J^{(2)} - V^{(2)}) \\
 \dots \\
 \dots \\
 Z_L^{-1(m)}(J^{(m)} - V^{(m)})
 \end{bmatrix}_{(N_k \times m) \times 1} \quad (7.8)$$

or

$$W_k Y_{v_k} = B_k \quad (7.9)$$

where

$$W_k = \begin{bmatrix} W^{(1)} \\ W^{(2)} \\ \dots \\ W^{(m)} \end{bmatrix}_{(N_k \times m) \times N_k^2} ; \quad W^{(i)} = \begin{bmatrix} V^{(i)t} & & & \\ & V^{(i)t} & & \\ & & \dots & \\ & & & V^{(i)t} \end{bmatrix}_{N_k \times N_k^2} \quad (7.10)$$

and

$$B_k = \begin{bmatrix} B^{(1)} \\ B^{(2)} \\ \dots \\ B^{(m)} \end{bmatrix}_{(N_k \times m) \times 1}; \quad B^{(i)} = Z_L^{-1(i)} (J^{(i)} - V^{(i)}) \quad (7.11)$$

Example: Formulation of Measured Data

Consider a measured subnetwork with two terminals. Let the subnetwork be characterized using three sets of measurements in the set up shown in Fig. 7.2. These measurements can be used to obtain the stencil of the subnetwork using (7.9) as follows.

$$J^{(i)} = \begin{bmatrix} V_s^{(i)} \\ 0 \end{bmatrix}; \quad V^{(i)} = \begin{bmatrix} V_1^{(i)} \\ V_2^{(i)} \end{bmatrix}; \quad Z_L^{(i)} = \begin{bmatrix} R_1^{(i)} & 0 \\ 0 & R_2^{(i)} \end{bmatrix} \quad \text{where } i = 1, 2, 3$$

$$W^{(i)} = \begin{bmatrix} V^{(i)'} & \mathbf{0} \\ \mathbf{0} & V^{(i)'} \end{bmatrix} = \begin{bmatrix} V_1^{(i)} & V_2^{(i)} & 0 & 0 \\ 0 & 0 & V_1^{(i)} & V_2^{(i)} \end{bmatrix}$$

$$B^{(i)} = Z_L^{-1(i)} (J^{(i)} - V^{(i)}) = \begin{bmatrix} -(V_1^{(i)} - V_s^{(i)})/R_1^{(i)} \\ -V_2^{(i)}/R_2^{(i)} \end{bmatrix}$$

$$W_k = \begin{bmatrix} W^{(1)} \\ W^{(2)} \\ W^{(3)} \end{bmatrix}; \quad Y_{v_k} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \end{bmatrix}; \quad B_k = \begin{bmatrix} B^{(1)} \\ B^{(2)} \\ B^{(3)} \end{bmatrix}$$

Fig. 7.2. Circuit for illustration of formulation of measured subnetworks

It is to be noted that in general N_k^2 variables are needed to determine the behavior of a N_k terminal network. However, if some prior information about the topology of the measured subnetwork (such as network is passive or symmetric etc.) is available, then the number of independent variables required to determine network's characteristics can be made less than N_k^2 . Let this number be denoted by n_{f_k} and in general $n_{f_k} \leq N_k^2$. This fact could be utilized to reduce the computational cost involved in solving (7.9). Let Y_{f_k} be the vector of independent variables which is a subset of vector Y_{v_k} . In order to establish a relation between Y_{v_k} and Y_{f_k} , define F_k such that

$$Y_{v_k} = F_k Y_{f_k} \quad (7.12)$$

where $F_k = [f_{i,j}]$, $i \in \{1, \dots, N_k^2\}$, $j \in \{1, \dots, n_{f_k}\}$, is a $N_k^2 \times n_{f_k}$ mapping matrix with elements $f_{i,j} \in \{0, 1\}$. Using (7.12), (7.9) can be rewritten in terms of vector of independent variables Y_{f_k} as

$$A_k Y_{f_k} = B_k \quad (7.13)$$

where

$$A_k = W_k F_k \quad (7.14)$$

Example: Formulation of Measured Data

To illustrate the above steps, consider the two terminal, passive and symmetric measured subnetwork shown in Fig. 7.2. Let the subnetwork have $Y_{11} = Y_{22}$ and $Y_{12} = Y_{21}$. Hence Y_{v_k} , Y_{f_k} and F_k for this case can be written as

$$Y_{v_k} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \end{bmatrix}; \quad Y_{f_k} = \begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix}; \quad F_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Correspondingly (7.13) can be formulated as

$$\begin{bmatrix} V_1^{(1)} & V_2^{(1)} \\ V_2^{(1)} & V_1^{(1)} \\ V_1^{(2)} & V_2^{(2)} \\ V_2^{(2)} & V_1^{(2)} \\ V_1^{(3)} & V_2^{(3)} \\ V_2^{(3)} & V_1^{(3)} \end{bmatrix} \begin{bmatrix} Y_{11} \\ Y_{12} \end{bmatrix} = \begin{bmatrix} -(V_1^{(1)} - V_s^{(1)})/R_1^{(1)} \\ -V_2^{(1)}/R_2^{(1)} \\ -(V_1^{(2)} - V_s^{(2)})/R_1^{(2)} \\ -V_2^{(2)}/R_2^{(2)} \\ -(V_1^{(3)} - V_s^{(3)})/R_1^{(3)} \\ -V_2^{(3)}/R_2^{(3)} \end{bmatrix}$$

Next, solution of (7.13) is needed for obtaining Y_{f_k} . However, in the presence of m different sets of measurements the subnetwork k can be of either types, determined ($N_k \times m = n_{f_k}$) or overdetermined ($N_k \times m > n_{f_k}$). Overdetermined measurements are of practical importance as measurements usually contain noise and hence the use of single set of measurements may yield inaccurate results. Including all available sets of measurements in the simulation helps to reduce the impact of noise on network responses. If the system is overdetermined then the matrix A_k which has a dimension $(N_k \times m) \times n_{f_k}$ may not be square, in which case the LHS and RHS of (7.13) may not be equal (i.e., the error $A_k Y_{f_k} - B_k \neq 0$). To overcome this difficulty, (7.13) is formulated as an *optimization problem based on the least square approximation* [103], [105] such that the least mean square error of (7.13), i.e., $(A_k Y_{f_k} - B_k)^t (A_k Y_{f_k} - B_k)$ is minimized. The solution to this formulation is given by a new set of equations

$$A_{s_k} Y_{f_k} = A_k^t B_k \quad (7.15)$$

where

$$A_{s_k} = A_k^t A_k \quad (7.16)$$

is a square matrix of order $n_{f_k} \times n_{f_k}$. (7.15) can be solved for Y_{f_k} and (7.12) in turn yields Y_{v_k} . The stencil of the measured subnetwork which is then in (3.8) can then be obtained using (7.5).

7.3 Computation of Measured Subnetwork Moments

One of the critical issues during the process of model-reduction is the accuracy of the higher-order moments. *However, the derivatives of the measured data have to be computed numerically as they do not have a closed-form solution. Hence the accuracy of the higher-order derivatives, which is essential to obtain a good Padé approximation, entirely depends on the numerical technique used for the computation of these derivatives.* A straightforward way to compute these moments is to perform direct numerical differentiation or to compute these derivatives through local rational function approximations [85]. However, the difficulties in such approaches are obtaining an accurate local approximation, selection of the appropriate step size and choosing an optimum bandwidth for such an approximation. Choosing a bandwidth much smaller or larger than the modulus of the nearest pole results in rapidly deteriorating derivative values. In order to overcome these difficulties, an accurate and efficient algorithm for computing these derivatives is presented in this thesis and is based on performing integration in the time-domain [20], [21], [87], [88]. The following example illustrates the accuracy associated with both the frequency-domain numerical differentiation and time-domain integration based schemes.

7.3.1 Comparison of Moment-Generation Algorithms

This example demonstrates the accuracy of the proposed moment-generation algorithm. For this purpose, a test function, $h(t) = e^{-t} - 2e^{-2t} + e^{-3t}$ is chosen (its frequency-domain equivalent is: $H(s) = 1/(s+1) - 2/(s+2) + 1/(s+3)$). Fig. 7.3 and Table 7.1 give the comparison of derivatives computed using both numerical differentiation and time-domain integration based schemes against the analytically computed derivatives, at $s = 0$. A step size of 0.001 (highest frequency

being 0.1) is used so as to get best possible results during the numerical differentiation. As seen, the numerical differentiation and analytical derivatives differ significantly, as early as from the 8th derivative. This error may deteriorate further in the presence of complex and fast varying waveforms. On the contrary, as seen from Table 7.1, the derivatives from the integration based scheme follows the analytical derivatives very closely.

Table. 7.1. Comparison of derivatives for $h(t) = e^{-t} - 2e^{-2t} + e^{-3t}$

Derivative number	Analytical derivatives	Integration based derivatives	Numerical-differentiation based derivatives
0	3.3333e-01	3.3333e-01	3.3333e-01
1	-6.1111e-01	-6.1111e-01	-6.1110e-01 + 2.4725e-03i
2	1.5741e+00	1.5741e+00	1.5739e+00 - 1.6725e-02i
3	-5.3241e+00	-5.3242e+00	-5.3226e+00 + 1.0648e-01i
4	2.2599e+01	2.2599e+01	2.2584e+01 - 7.3120e-01i
5	-1.1641e+02	-1.1642e+02	-1.1625e+02 + 5.5745e+00i
6	7.0908e+02	7.0910e+02	7.0137e+02 - 4.6997e+01i
7	-5.0014e+03	-5.0016e+03	-5.0304e+03 - 3.1061e+03i
8	4.0165e+04	4.0166e+04	2.1354e+06 - 3.5102e+04i
9	-3.6218e+05	-3.6219e+05	1.6113e+07 + 1.1862e+09i
10	3.6253e+06	3.6254e+06	-6.3426e+11 + 7.7615e+09i
11	-3.9897e+07	-3.9899e+07	-2.7794e+12 - 3.1661e+14i
12	4.7888e+08	4.7890e+08	1.4661e+17 - 2.7214e+14i
13	-6.2263e+09	-6.2265e+09	-7.1579e+17 + 6.3336e+19i
14	8.7173e+10	8.7176e+10	-2.6550e+22 - 8.8883e+20i

Fig. 7.3. Comparison of derivatives for $h(t) = e^{-t} - 2e^{-2t} + e^{-3t}$

7.3.2 Proposed Algorithm for Computing Moments of Measured Data

In this section, the details of the proposed algorithm for computing measured subnetwork moments, and also to compute the derivatives of the unknown y -parameter matrix, $Y_k^{(r)}$ is given. In the ensuing discussion, it is assumed that the time-domain measurements ($v_k(t)$) for the given subnetwork are available.

From (7.5), derivatives $Y^{(r)}$ (the subscript k is dropped in the rest of the discussion for simplicity) can be expressed as

$$Y^{(r)} = \begin{bmatrix} Y_{11}^{(r)} & Y_{12}^{(r)} & \dots & Y_{1N}^{(r)} \\ Y_{21}^{(r)} & Y_{22}^{(r)} & \dots & Y_{2N}^{(r)} \\ \dots & \dots & \dots & \dots \\ Y_{N1}^{(r)} & Y_{N2}^{(r)} & \dots & Y_{NN}^{(r)} \end{bmatrix} \quad (7.17)$$

Next, the derivatives represented by individual entries in the RHS matrix of (7.17) are needed. From (7.6), these can be expressed as the derivatives in the vector Y_v as

$$Y_v^{(r)} = [Y_{11}^{(r)} \ Y_{12}^{(r)} \ \dots \ Y_{1N}^{(r)} \ Y_{21}^{(r)} \ Y_{22}^{(r)} \ \dots \ Y_{2N}^{(r)} \ \dots \dots \ Y_{N1}^{(r)} \ Y_{N2}^{(r)} \ \dots \ Y_{NN}^{(r)}]^t \quad (7.18)$$

Derivatives $Y_v^{(r)}$ can be evaluated using the relation (7.12) as

$$Y_v^{(r)} = F Y_f^{(r)} \quad (7.19)$$

Next, derivatives $Y_f^{(r)}$ can be obtained using (7.15) as follows

$$\sum_{r=1}^n \binom{n}{r} A_s^{(r)} Y_f^{(n-r)} + A_s Y_f^{(n)} = \sum_{r=1}^n \binom{n}{r} A^{t(r)} B^{(n-r)} + A^t B^{(n)} \quad (7.20)$$

or

$$A_s Y_f^{(n)} = - \sum_{r=1}^n \binom{n}{r} A_s^{(r)} Y_f^{(n-r)} + \sum_{r=1}^n \binom{n}{r} A^{t(r)} B^{(n-r)} + A^t B^{(n)} \quad (7.21)$$

(7.21) gives a recursive relationship for the computation of derivatives of vector Y_f in terms of the derivatives of A_s , A and B . Further, the derivatives of A_s can be obtained recursively using (7.16) as

$$A_s^{(n)} = \sum_{r=1}^n \binom{n}{r} A^{t(r)} A^{(n-r)} + A^t A^{(n)} \quad (7.22)$$

To proceed further, the derivatives of A matrix and B vector are needed. From (7.14) the r^{th} derivatives of A can be computed as

$$A^{(r)} = W^{(r)} F \quad (7.23)$$

Using (7.10) and (7.11) the derivatives $W^{(r)}$ and $B^{(r)}$ can be computed as

$$W^{(r)} = \begin{bmatrix} W^{(1)(r)} \\ W^{(2)(r)} \\ \dots \\ W^{(m)(r)} \end{bmatrix}; \quad B^{(r)} = \begin{bmatrix} B^{(1)(r)} \\ B^{(2)(r)} \\ \dots \\ B^{(m)(r)} \end{bmatrix} \quad (7.24)$$

Next, r^{th} derivatives of $W^{(i)}$ and $B^{(i)}$, represented by $W^{(i)(r)}$ and $B^{(i)(r)}$, respectively, are needed. Again, from (7.10) and (7.11) these can be computed as

$$W^{(i)(r)} = \text{diag} \left[(V^{(i)t})^{(r)} (V^{(i)t})^{(r)} \dots (V^{(i)t})^{(r)} \right]_{N \times N^2}^t \quad (7.25)$$

and

$$B^{(i)(r)} = (Z_L^{-1(i)} (J^{(i)} - V^{(i)}))^{(r)} \quad (7.26)$$

Derivatives of $B^{(i)}$ can be recursively computed as

$$Z_L^{(i)} B^{(i)(n)} = J^{(i)(n)} - V^{(i)(n)} - \sum_{r=1}^n \binom{n}{r} Z_L^{(i)(r)} B^{(i)(n-r)} \quad (7.27)$$

Next, to proceed further, (7.25) and (7.27) require derivatives of the measurement vector $\mathbf{v}^{(i)}$. For the purpose of illustration, consider a particular node j with time-domain measurements represented by $v_j^{(i)}(t)$. Using integration in time-domain, V_j and its derivatives in frequency-domain can be computed as (superscript i is dropped for simplicity)

$$\begin{aligned}
 V_j(s) &= \int_0^\infty v_j(t) e^{-st} dt \\
 \frac{d}{ds} V_j(s) &= -\int_0^\infty t v_j(t) e^{-st} dt \\
 &\dots \\
 &\dots \\
 \frac{d^r}{ds^r} V_j(s) &= \int_0^\infty (-t)^r v_j(t) e^{-st} dt
 \end{aligned} \tag{7.28}$$

As seen, required moments corresponding to any individual node in the measurement vector $\mathbf{v}^{(i)}$ at a specific hop, $s = \alpha$, can be computed using the set of equations (7.28). Since the derivatives are obtained by time-domain integration of measured data, the algorithm is stable and more accurate compared to the methods based on numerical differentiation in frequency-domain. This fact is substantiated by the computational results reported in the next section.

Several methods could be used for evaluation of integrals in equations (7.28) [104], [105]. Two important points under consideration here are the efficiency and accuracy of the method used for evaluating the integrals in (7.28). For practical purposes it has been found that even simpler methods like Simpson's rule would suffice. For further accuracy, higher order methods such as Romberg's method, Bode's rule [105] or adaptive integration schemes [106] could be used.

7.3.3 Time-Limited Responses

It is to be noted that the accuracy of the moments from the proposed time-domain integration based scheme is dependent on time-limiting in the time-domain. If the time-domain data is slightly truncated (time-limited) near the tail of the response, then the resulting moments deviate slightly from the original moments. However, it is observed that the new set of moments are correlated and the transfer function generated through these moments follows the original function closely. This is demonstrated in the following example. Fig. 7.4 shows the time response for the function under consideration, $h(t) = e^{-t} - 2e^{-2t} + e^{-3t}$ (which is discussed earlier). Next, this response was time-limited up to 6 seconds, and the new moments are computed by integrating the data up to 6 seconds. Fig. 7.5 and Table 7.2 give the new moments, which deviate slightly from the analytical moments. However, it is observed that the new set of moments are correlated, and the transfer function generated through these moments follows the original function closely. This is demonstrated in Fig. 7.6, in which frequency responses from the new set of moments and the original function $H(s) = 1/(s+1) - 2/(s+2) + 1/(s+3)$ are compared. As seen, these responses are indistinguishable. Hence, as long as band-limiting does not cause substantial loss of time-domain information, the original system function can be safely retrieved from the moments of such truncated time-domain measurements.

Fig. 7.4. Time response for $h(t) = e^{-t} - 2e^{-2t} + e^{-3t}$

Table. 7.2. Derivatives of truncated time response

Derivative number	Integration based derivatives
0	3.3086e-01
1	-5.9380e-01
2	1.4504e+00
3	-4.4186e+00
4	1.5769e+01
5	-6.3009e+01
6	2.7306e+02
7	-1.2553e+03
8	6.0283e+03
9	-2.9923e+04
10	1.5237e+05
11	-7.9158e+05
12	4.1791e+06
13	-2.2354e+07
14	1.2087e+08

Fig. 7.5. Comparison of derivatives of truncated time response

Fig. 7.6. Comparison of frequency-responses

7.3.4 Summary of Computational Steps

A summary of the steps involved in the computation of moments for circuits containing linear subnetworks characterized by measured data is given below.

-
- *STEP 1: Obtain time-domain measurements $v_k(t)$ for a given subnetwork k , and corresponding excitations $j_k(t)$ and terminations Z_L .*
 - *STEP 2: Determine the expansion point and the number of moments required using block CFH algorithm.*
 - *STEP 3: Compute the derivatives of measurement vector $V^{(i)}$ using (7.28).*
 - *STEP 4: Compute the derivatives $W^{(r)}$ and $B^{(r)}$ using (7.25) - (7.27).*
 - *STEP 5: Compute the derivatives $A^{(r)}$ and $A_s^{(r)}$ using (7.23) and (7.22), respectively.*
 - *STEP 6: Compute the derivatives of vector of independent variables characterizing the given measured subnetwork, $Y_f^{(r)}$, using (7.21).*
 - *STEP 7: Find $Y_v^{(r)}$, and hence $Y_k^{(r)}$, the moments of the unknown Y parameter matrix of the measured subnetwork under consideration, using (7.19) and (7.17).*
 - *STEP 8: Repeat steps 1 through 5 for every measured subnetwork in the network.*
 - *STEP 9: Calculate $\Psi^{(r)}$ using (4.19).*
 - *STEP 10: Compute the moments of the system M_n using (4.18).*
-

7.4 Numerical Examples

In this section, several examples are presented to demonstrate the efficiency and accuracy of the proposed method. The networks include mixed simulation environment consisting of lumped, distributed and measured subnetworks.

Example 1

An example circuit shown in Fig. 7.7(a) containing a single measured subnetwork is simulated using the proposed technique. The measured subnetwork under

consideration is a single transmission line whose quasi-TEM model parameters are known ($L = 10\text{nH/cm}$, and $C = 1\text{pF/cm}$ and $\text{Length} = 1\text{cm}$). The measured subnetwork is characterized in a test environment shown in Fig. 7.7(b) using HSPICE and the time-domain measurements at nodes v_1 and v_2 are given in Fig. 7.8. These tabulated measurements are then used to model the measured subnetwork and the circuit is simulated using the proposed technique. The input signal was a 1V pulse with 0.1ns rise/fall time and 1ns duration. The results were compared with the HSPICE simulations in which the subnetwork is treated with its original quasi-TEM parameters. The time responses at node V_{out} from both simulations are given in Fig. 7.9 and they match accurately.

(a) Single measured subnetwork circuit for Example 1.

(b) Measurement setup

Fig. 7.7. Circuit for Example 1

Fig. 7.8. Time-domain measurements at nodes v_1 and v_2 for Example 1

Fig. 7.9. Time responses for Example 1

Example 2

Fig. 7.10 represents a mixed simulation of measured subnetworks and transmission lines with quasi-TEM models. The time-domain measurements for the measured subnetworks are given in Fig. 7.8. The circuit is driven by a source of 1V pulse with 0.1ns rise/fall time and 3ns duration. Responses are taken at node V_{out} . Comparison with conventional simulation where the response was obtained by solving (3.10) directly at 1024 frequency points were made. Transient responses from both the new method and conventional approaches are given in Fig. 7.11 and they are indistinguishable.

Fig. 7.10. Circuit for Example 2 - mixed simulation environment

Fig. 7.11. Time responses for Example 2

Example 3

A relatively large circuit having 240 nodes and containing 160 resistors, 162 inductors, 162 capacitors and 12 transmission lines modeled as measured subnetworks was simulated. The input signal was a 1V pulse with 0.1ns rise/fall time and 10ns duration. The response was taken near the source node. 9 hops were required for convergence, for a maximum search frequency of 2GHz. Transient responses obtained from the new method and conventional approach are given in Fig. 7.12.

A CPU comparison of conventional FFT and the proposed BCFH algorithm for Examples 5 and 6 is given in Table 7.3. It is to be noted that for large circuits major cost involved in the simulation is due to the LU decomposition of circuit matrix and hence the speed up achieved by the proposed algorithm increases substantially in the presence of larger circuits.

Fig. 7.12. Time responses for Example 3

Table. 7.3. CPU comparison of FFT and the proposed BCFH (in seconds, SPARC10)

Example	Method	Maximum Frequency (GHz)	CPU time (seconds)	# of solutions (Hops)	Speed up
Example 2 (Seven Tells)	FFT	10	98.0	1024	22.03
	CFH	10	4.45	10	
Example 3 (240 nodes, 420x420 matrix)	FFT	2	452.3	1024	36.47
	CFH	2	12.3	9	

Example 4

Fig. 18 shows a circuit containing measured subnetwork representing a coupled line structure. The coupled line structure shown in Fig. 7.13(b) is treated as a measured subnetwork and time-domain measurements were obtained at all the four terminals using terminations of 10 Ohms. The subnetwork is then simulated in the environment of Fig. 7.13(a) using the proposed technique and the conventional approach. The input signal is a source of 1V pulse with 1ns rise/fall time and 5ns duration. Transient responses from both conventional approach and the new method at direct node V_{out1} and at coupled node V_{out2} are given in Fig. 7.14 and Fig. 7.15, respectively, and they match accurately.

(a) Four port measured subnetwork circuit for Example 4.

(b) Measurement setup

Fig. 7.13. Circuit for Example 4

Fig. 7.14. Time responses for Example 4 (V_{out1})

Fig. 7.15. Time responses for Example 4 (V_{out2})

Example 5

This example demonstrates the case of overdetermined measurements, noise in measurements and also the advantage of using LMS based optimization for such cases. Circuit chosen is the one described in Example 2 (Fig. 7.10, seven transmission line example). Each transmission line is analyzed with 40 sets of measurements. To emulate a practical environment, all the measurements; V_1 , V_2 , and V_{source} were perturbed ($v_{perturbed} = (V_{unperturbed} + V_{noise})$) by adding following random noises:

- (a) $-0.2(\text{Max}(|V_{unperturbed}|, 5\text{mV})) < V_{noise} < 0.2(\text{Max}(|V_{unperturbed}|, 5\text{mV}))$
- (b) $0.1(|V_{\text{Max}(unperturbed)}|) < V_{noise} < 0.1(|V_{\text{Max}(unperturbed)}|)$

In both cases noise is uniformly distributed with the mean at zero. Fig. 7.16 shows the frequency response in the presence of noise described by (a). Solid line indicates HSPICE simulation with quasi-TEM parameters (as in Example 2), in the absence of noise. Broken line represent the simulation with the new approach using a single set of measurement data containing noise. Dotted line, which match reasonably well with the HSPICE simulations, is obtained using the proposed CFH algorithm with LMS optimization performed on 40 sets of noise affected measurements. It can be seen that the effect of noise is filtered out by LMS optimization. Corresponding time-domain responses are given in Fig. 7.17. It is evident from the responses that, in the absence of LMS optimization, noise affected measurements may yield inaccurate results. Fig. 7.18 shows the time-domain responses in the presence of noise described by (b). As seen from the responses, in spite of the presence of relatively large noise in different sets of measurements, LMS based optimization was able extract the original response fairly well.

Fig. 7.16. Frequency responses for Example 5 (with noise of type (a))

Fig. 7.17. Time responses for Example 5 (with noise of type (a))

Fig. 7.18. Time responses for Example 5 (with noise of type (b))

In conclusion, the new contributions presented in this chapter are as follows. (1) A generalized approach to include measured or tabulated results characterizing linear subnetworks into circuit simulators. (2) A new algorithm based on time-domain integration for computation of moments of measured subnetworks. The proposed method is stable and more accurate compared to methods based on numerical-differentiation in frequency-domain. (3) Also, a new technique based on LMS optimization to address subnetworks characterized by overdetermined measurements as well as measurements with noise. The proposed algorithm can handle networks with arbitrary number of measured subnetworks.

“If it weren’t for the last minute, nothing would get done.” - Charles H. Duell

This chapter contains a summary of the work that was presented in this thesis. In addition, the directions for future work are discussed.

8.1 Summary

In this thesis an efficient multiport algorithm, namely, block complex frequency hopping, is presented for the simulation of high-speed interconnect networks in the presence of nonlinear elements. In addition, a new residue computation algorithm is developed to improve the accuracy of reduced-order models. Also a new scheme for selection of dominant poles of multiport networks is developed.

A new passivity preservation algorithm is developed for networks containing diverse interconnect models such as *distributed transmission lines with frequency independent/dependent parameters, full-wave and measured* subnetworks. In addition, efficient techniques for macromodel synthesis from frequency-domain reduced order models is presented in this thesis. Also a new technique for minimum-realization of given reduced-order frequency-domain models is developed. This further helps to reduce the number of states to be used during nonlinear analysis and efficient simulation.

In addition, techniques are presented to include full-wave and measured subnetworks in a model-reduction based analysis. New moment-computation algorithms for networks containing measured/full-wave devices are presented and they overcome the ill-conditioning problem associated with the previous moment-computation techniques. Also a comparison of quasi-TEM approximations and full-wave models at higher frequencies is given in this thesis.

The new algorithm, BCFH, presented in this thesis provides an easy means to handle high-speed interconnect models in industrial grade simulators such as SPICE. The method is based on the moment-matching techniques which has been proven to provide a significant improvement in CPU time. The advantages of using the new technique for interconnect simulation are listed below.

- 1) The method overcomes the mixed frequency/time difficulties encountered by SPICE-like conventional simulators and provides an efficient means to combine the frequency-domain characterized interconnect networks with nonlinear elements for transient simulation purposes. The simulated interconnect models can include diverse elements such as distributed transmission lines with frequency independent/dependent parameters, full-wave models, EMI devices and measured subnetworks with time or frequency-domain characterized data.
 - 2) The method gives 2 - 3 orders of speed-ups compared to conventional simulation techniques for a comparable accuracy.
 - 3) The approach presented in this thesis for the analysis of multiport interconnects presents no theoretical restrictions on the number of subnetworks. In
-

addition, the new algorithm puts no restrictions on the highest frequency and can preserve the frequency-spectrum of the original network up to the maximum frequency of interest. The proposed technique is useful for accurate and efficient transient response estimation involving linear subnetworks in VLSI chip packaging, multi chip modules, printed circuit boards and in VLSI systems design.

- 4) Unlike the previous passivity preservation algorithms, the new scheme presented in this thesis can handle diverse interconnect models such as *distributed transmission lines, full-wave and measured* subnetworks. Also it is to be noted that the new passivity preservation algorithm is not just limited to the block CFH technique but can be easily used in conjunction with any other model-reduction techniques which can generate accurate reduced-order pole-residue models in the frequency domain.

8.2 *Future Research*

- 1) Combining BCFH and Krylov-subspace based techniques: Generally, CFH based algorithms require more hops compared to the Krylov-subspace based model-reduction techniques such as Arnoldi. This is because a direct Padé approximation is attempted at every hop during the CFH, whereas the Krylov-subspace based techniques derive the required information through a well-conditioned process (using QR decomposition and orthonormalization) and thus avoid the ill-conditioning experienced during the inversion of the moment-matrix. However, one of the bottlenecks with the Krylov-subspace
-

based techniques is that they can't handle distributed, full-wave or measured subnetworks. Hence further research has to be done, if somehow the merits of the Krylov-subspace based techniques and the CFH can be combined so as to reduce the number of hops required during the BCFH.

- 2) **Passivity by Formulation:** The passivity preservation algorithm presented in this paper is one step ahead from the results reported in the literature which are limited to RLC lumped models. However, these algorithms preserve the passivity by formulation and there is no need to check for the passivity, once the reduced-order models are obtained. On the other hand the passivity preservation presented in this thesis is based on compensating for any loss of passivity. This requires checking of passivity violations near the highest frequency and the task may become expensive if the number of ports are very large (in the range of hundreds). Hence this area is further open to research if the passivity can be ensured by formulation of the model-reduction problem.
 - 3) **Nonlinear Simulation of EMI networks using BCFH:** BCFH has been successfully applied for simulating distributed, full-wave and measured subnetworks. CFH has been applied for linear-domain analysis of EMI and susceptibility problems. It is of immense interest to macromodel the EMI networks along with the distributed sources for the purpose of nonlinear simulation. This area of applications is drawing attention with the growth of the wireless industry.
-

- 4) Application to other areas: The proposed algorithm has a lot of applications concerning high-speed and microwave designs. The algorithm can be used for simulation of linear microwave devices and solution of EM problems. In addition it provides many options for bridging the gap between EM and circuit applications.
-

In this section a brief summary of full-wave analysis using spectral domain technique is given. Detailed information can be found in [11] - [14].

A.1. Formulation Of Field Equations

Consider a dispersive system with N coupled conductors embedded in an inhomogeneous dielectric medium, as shown in Fig. 6.1. The system contains hybrid modes (TE and TM) and it is required to determine N dominant modes of propagation. The fields associated with any i^{th} dielectric can be written as

$$E_{z,i} = (\omega^2 \mu_i \epsilon_i - \Upsilon^2) \phi_i e^{-j\Upsilon z} \quad (\text{A.1})$$

$$H_{z,i} = (\omega^2 \mu_i \epsilon_i - \Upsilon^2) \psi_i e^{-j\Upsilon z} \quad (\text{A.2})$$

$$E_{t,i} = -j\Upsilon \left(\nabla_t \phi_i - \left(\frac{\omega \mu_i}{\Upsilon} \right) \hat{k} \times \nabla_t \psi_i \right) e^{-j\Upsilon z} \quad (\text{A.3})$$

$$H_{t,i} = -j\Upsilon \left(\nabla_t \psi_i + \left(\frac{\omega \mu_i}{\Upsilon} \right) \hat{k} \times \nabla_t \phi_i \right) e^{-j\Upsilon z} \quad (\text{A.4})$$

where,

- $\phi_i = \phi_i(x, y)$, $\psi_i = \psi_i(x, y)$ are the scalar electric and magnetic potential functions,
- Υ is the complex propagation constant of waves moving in the z direction,
- ϵ_i and μ_i are the permittivity and permeability of the i^{th} dielectric medium, and
- $\nabla_t = (\partial/\partial x)\hat{i} + (\partial/\partial y)\hat{j}$ where $\hat{i}$, $\hat{j}$ and $\hat{k}$ represent the directed unit vectors.

Scalar potentials represented by ϕ_i and ψ_i (henceforth denoted by a general notation u_i) satisfy the Helmholtz wave equation [12]

$$\nabla_t^2 u_i + (\omega^2 \mu_i \epsilon_i - \Upsilon^2) u_i = 0 \quad (\text{A.5})$$

A.2. Spectral Domain Technique (SDT)

The solution of the wave equation (A.5) which is needed for the computation of dominant modes is highly involved. In SDT, partial differential equations governing the behavior of dispersive structures are transformed into Fourier domain. In this domain the resulting equations have easy solutions. In order to apply SDT, it is assumed that the metal strips have negligible thickness and the entire system is enclosed in a metal box of infinite conductivity (Fig. 6.1). Using a finite Fourier transform in the x direction, u_i can be written as

$$\tilde{u}_i(\alpha_n, y) = \frac{1}{2a} \left(\int_{-a}^a u_i(x, y) e^{j\alpha_n x} dx \right) \quad (\text{A.6})$$

A Fourier series expansion of u_i is now given by

$$u_i(x, y) = \sum_{-\infty}^{\infty} \tilde{u}_i(\alpha_n, y) e^{j\alpha_n x} \quad (\text{A.7})$$

where α_n is a spectral parameter, defined as $\alpha_n = (2n-1)\pi/2a$ for even modes and $\alpha_n = (n-1)\pi/a$ for odd modes. Next, the transformed Helmholtz's equation can be written as

$$\left(\Upsilon^2 - \alpha_n^2 + \frac{\partial^2}{\partial y^2} \right) \tilde{u}_i + k_i^2 \tilde{u}_i = 0 \quad (\text{A.8})$$

where $k_i^2 = \omega^2 \mu_i \epsilon_i$. Next, the general solution of homogeneous differential equations represented by (A.8) is given by

$$\tilde{u}_i = a_i \cosh(\hat{\Upsilon}_i y) + b_i \sinh(\hat{\Upsilon}_i y) \quad (\text{A.9})$$

where $\hat{\Upsilon}_i = \sqrt{\alpha_n^2 - \Upsilon^2 - k_i^2}$, is a function of the propagation constant Υ . Equation (A.9) gives the solution of scalar potentials in terms of propagation constants, and physical parameters. Next, for each dielectric region, each field component is expressed in the Fourier domain. For example, $E_{z,i}$ can be expressed as

$$\tilde{E}_{z,i} = (k_i^2 - \Upsilon^2) \tilde{\phi}_i(\alpha_n, y) e^{-j\Upsilon z} \quad (\text{A.10})$$

Imposing boundary conditions at each of the consecutive dielectric interfaces and also noting that the tangential electric fields vanish on the strips [11] - [15], we get a set of homogeneous linear equations relating the spectral electric fields to spectral current distributions on the surface of the strips as

$$\begin{bmatrix} \tilde{E}_x \\ \tilde{E}_z \end{bmatrix} = \begin{bmatrix} G_{11}(\alpha_n, \Upsilon) & G_{12}(\alpha_n, \Upsilon) \\ G_{21}(\alpha_n, \Upsilon) & G_{22}(\alpha_n, \Upsilon) \end{bmatrix} \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix} \quad (\text{A.11})$$

where $G_{11}(\alpha_n, \Upsilon)$, etc. represent spectral domain Green's functions. Next, the unknown current distribution functions $\tilde{J}_x$ and $\tilde{J}_z$ are expanded in terms of known basis functions, $\tilde{J}_x^{pq}$ and $\tilde{J}_z^{pq}$ as

$$\tilde{J}_x = \sum_{p=1}^P \sum_{q=1}^N c_{pq} \tilde{J}_x^{pq} ; \quad \tilde{J}_z = \sum_{p=1}^P \sum_{q=1}^N d_{pq} \tilde{J}_z^{pq} \quad (\text{A.12})$$

where c_{pq} and d_{pq} are the unknown coefficients, P is the number of terms needed in the expansion and N is the number of striplines in the system. The basis functions $\tilde{J}_x^{pq}$ and $\tilde{J}_z^{pq}$ are the spectral-domain counter parts of the spatial domain basis functions, which are chosen to approximate the true but unknown current distributions. Increasing the number of coefficients (P) in (A.12) would yield better accuracy, however, at the cost of CPU speed. It is be noted that the computing time can be significantly reduced if the basis functions have closed-form Fourier transforms. Rate of convergence may be speeded up by employing basis functions whose behavior resemble the physical distribution of the expanded current. For our analysis the basis functions chosen are, Legendre polynomials for $J_z^{pq}(x)$ and trigonometric functions for $J_x^{pq}(x)$. For instance for the even mode of microstrip line, the chosen basis functions look as follows [12]

$$\begin{aligned} J_x^{pq}(x) &= \sin\left\{p\pi\left(\frac{x-x_q}{w_q}\right)\right\}; & x_q < x < x_q + w_q \\ J_z^{pq}(x) &= L_{2(p-1)}\left(\frac{x-x_q}{w_q}\right) \end{aligned} \quad (\text{A.13})$$

where $L_{2(p-1)}\{(x-x_q)/w_q\}$ represents the Legendre polynomials of even degree. The choice of Legendre polynomials is justified on the ground that they have closed-form solution in the spectral domain and the behavior of these functions resemble the longitudinal current distributions of discrete modes of an isolated conducting strip. With the choice of Legendre polynomials as basis functions, it is found that for most cases P can be chosen very small without compromising the accuracy. This is illustrated in Fig. A.1, which gives a typical plot of ϵ_{eff} versus P (computed for the interconnect structure described in example 1 at 10GHz). As seen from the graph, increasing P from 1 to 2 changes the result from 6.8 to 6.67,

an improvement of 1.9% towards convergence. Further increase of P doesn't change ε_{eff} significantly, and the result converges very quickly.

Fig. A.1. Effective dielectric constant v/s number of terms in the expansion (P)

Substituting (A.12) in (A.11), and applying Galerkin's method [12] - [15], a set of homogeneous linear equations can be derived as

$$A(\Upsilon, \omega)C = 0 \quad (\text{A.14})$$

where A is a $(2P \times N) \times (2P \times N)$ matrix and $C = [c_{pq} \ d_{pq}]^t$ is a $(2P \times N)$ column vector containing the unknown coefficients of basis functions. In order to have non-trivial solutions for the unknown coefficients in (A.14), we must have

$$\det(A(\Upsilon, \omega)) = 0 \quad (\text{A.15})$$

Equation (A.15) is solved for N propagation constants $(\Upsilon_i, i = 1, 2 \dots N)$, at each frequency of interest. Once the value of Υ_i for each dominant mode is obtained,

the solution for the unknown coefficients can be found by setting $C_{11} = 1$ and solving the following equations (which are obtained by rearranging (A.14)) as

$$\begin{bmatrix} A_{12} & A_{13} & \dots & \dots & A_{1,2PN} \\ A_{22} & A_{23} & \dots & \dots & A_{2,2PN} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ A_{2PN,2} & A_{2PN,3} & \dots & \dots & A_{2PN,2PN} \end{bmatrix} \begin{bmatrix} c_{12} \\ \dots \\ c_{PN} \\ d_{11} \\ \dots \\ d_{PN} \end{bmatrix} = - \begin{bmatrix} A_{11} \\ A_{21} \\ \dots \\ \dots \\ A_{2PN,1} \end{bmatrix} \quad (\text{A.16})$$

Now that the mode i propagating constant Υ_i , and the corresponding coefficients c_{ij} and d_{ij} , $j = 1, 2, \dots, P$, are known, the surface current distributions, scalar potentials and the corresponding field distributions can be computed. This gives us the complete description of the modal waves propagating in the structure.

As a note of caution, although the SDT has many attractive features, one shortcoming of the technique is its inability to deal with the structures employing thick conductors. Therefore, at frequencies where the wavelengths become comparable with the metallization thickness, the results from SDT must be treated cautiously.

In this appendix, a brief summary of the steps in developing a scattering matrix which relates the incident and reflected waves originating both at the input and at the output ports, is given. In addition, the derivation of a full-wave stencil from a scattering matrix is outlined (details can be found in [10]). Consider the scattering system for mode i , shown in Fig. B.1.

Fig. B.1. Scattering system for a N -conductor structure for mode i excitation

Let $A_{iI} = [A_{iI}^1 \ A_{iI}^2 \ \dots \ A_{iI}^N]^T$, be the mode i incident wave at the input, where A_{iI}^k represents the amplitude of the mode i incident wave on line k . Similarly define B_{iI} which corresponds to the reflected component at the input. Let A_{iO} and B_{iO} be incident and reflected waves at the output port. Next, using (6.1), we can write

$$A_{iI} = A_{iI}^1 M_i; \quad B_{iI} = B_{iI}^1 M_i; \quad A_{iO} = A_{iO}^1 M_i; \quad B_{iO} = B_{iO}^1 M_i \quad (\text{B.1})$$

Using (B.1) and also noting that the reflected wave at the output is due to the wave incident at the input port and travelling through the conductor (and vice versa), we can write,

$$\begin{bmatrix} B_I \\ B_O \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \text{diag}(e^{\gamma_i L}) \\ \text{diag}(e^{\gamma_i L}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} A_I \\ A_O \end{bmatrix} \quad (\text{B.2})$$

where $A_I = [A_{1I}^1 \dots A_{NI}^1]^T$, $A_O = [A_{1O}^1 \dots A_{NO}^1]^T$, $B_I = [B_{1I}^1 \dots B_{NI}^1]^T$ and $B_O = [B_{1O}^1 \dots B_{NO}^1]^T$. $\text{diag}(e^{-\gamma_i L})$; $i = 1, \dots, N$ is the modal propagation matrix.

Next, consider an arbitrary incident wave at the input port, $a_I = [a_I^1 \ a_I^2 \ \dots \ a_I^N]^T$. This can be expressed as a linear combination of dominant modal waves as, $a_I = [A_{1I}^1 M_1 + A_{2I}^1 M_2 + \dots + A_{NI}^1 M_N]$, or, $a_I = M A_I$. Correspondingly, an arbitrary reflected wave at the input port can be defined by b_I , where $b_I = M B_I$. Similarly, $a_O = M A_O$ and $b_O = M B_O$ can be defined for arbitrary incident and reflected waves at output port. Using these and (B.2), we can write

$$\begin{bmatrix} b_I \\ b_O \end{bmatrix} = \begin{bmatrix} \mathbf{0} & M e^{-\Gamma L} M^{-1} \\ M e^{-\Gamma L} M^{-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} a_I \\ a_O \end{bmatrix} \quad (\text{B.3})$$

where

$$e^{-\Gamma L} = \text{diag}(e^{-\gamma_i L}), i = 1, \dots, N \quad (\text{B.4})$$

Equation (B.3) represents the modal scattering matrix for the dispersive multiconductor interconnect system and it relates the reflected/incident waves at

the input/output ports. Next, a relationship to link the terminal voltages and propagating waves needs to be developed. Consider the terminal parameters shown in Fig. 6.2(b). Let I_{il} be the input terminal current vector for mode i , which can be expressed in terms of incident and reflected waves at the input as

$$I_{il} = \begin{bmatrix} I_{il}^1 \\ \dots \\ I_{il}^N \end{bmatrix} = \begin{bmatrix} A_{il}^1 \\ \dots \\ A_{il}^N \end{bmatrix} - \begin{bmatrix} B_{il}^1 \\ \dots \\ B_{il}^N \end{bmatrix} = (A_{il}^1 - B_{il}^1) \begin{bmatrix} m_{1i} \\ \dots \\ m_{Ni} \end{bmatrix} = (A_{il}^1 - B_{il}^1) M_i \quad (\text{B.5})$$

The corresponding terminal voltage vector V_{il} can be expressed as

$$V_{il} = \begin{bmatrix} V_{il}^1 \\ \dots \\ V_{il}^N \end{bmatrix} = \begin{bmatrix} Z_{1i}(A_{il}^1 + B_{il}^1) \\ \dots \\ Z_{Ni}(A_{il}^N + B_{il}^N) \end{bmatrix} = (A_{il}^1 + B_{il}^1) \begin{bmatrix} Z_{1i}m_{1i} \\ \dots \\ Z_{Ni}m_{Ni} \end{bmatrix} \quad (\text{B.6})$$

The total terminal currents I_I and voltages V_I at the input port are computed taking into account of contributions from all the N dominant propagating modes

$$\begin{aligned} I_I &= \sum_{i=1}^N I_{il} = M(A_I - B_I) = a_I - b_I \\ V_I &= \sum_{i=1}^N V_{il} = Z_m(A_I - B_I) = Z_m M^{-1}(a_I + b_I) \end{aligned} \quad (\text{B.7})$$

where Z_m is a $N \times N$ matrix defined as

$$Z_m = \begin{bmatrix} m_{11}Z_{11} & \dots & m_{1N}Z_{1N} \\ \dots & \dots & \dots \\ m_{N1}Z_{N1} & \dots & m_{NN}Z_{NN} \end{bmatrix} \quad (\text{B.8})$$

Similarly we can define for the output port

$$\begin{aligned} I_O &= a_O - b_O \\ V_O &= Z_m M^{-1} (a_O + b_O) \end{aligned} \quad (\text{B.9})$$

Next, to develop a relationship between terminal voltages and currents, define a system characteristic impedance matrix, Z_c as

$$Z_c = Z_m M^{-1} \quad (\text{B.10})$$

Eliminating the wave terms (a_I , b_I , etc.) from the above, we get a set of equations describing the multiconductor structure in terms of line voltages and currents

$$\begin{aligned} V_I - Z_c I_I &= Z_m e^{-\Gamma L} Z_m^{-1} (V_O + Z_c I_O) \\ V_O - Z_c I_O &= Z_m e^{-\Gamma L} Z_m^{-1} (V_I + Z_c I_I) \end{aligned} \quad (\text{B.11})$$

Next, a relationship in the form (6.5) is required for linking full-wave models to general purpose circuit simulators. For this reason define a variable matrix, φ as

$$\varphi = Z_m e^{-\Gamma L} Z_m^{-1} \quad (\text{B.12})$$

Using (B.12), Equations (B.10) and (B.11) can be rearranged to obtain the stencil for a k^{th} full-wave modeled interconnect subnetwork as

$$\begin{bmatrix} \varphi & -U \\ -U & \varphi \end{bmatrix} \begin{bmatrix} V_O(s) \\ V_I(s) \end{bmatrix} + \begin{bmatrix} -\varphi Z_c & -Z_c \\ -Z_c & \varphi Z_c \end{bmatrix} \begin{bmatrix} I_O(s) \\ I_I(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\text{B.13})$$

where U represents the unity matrix. Next, (B.14) can be concisely written as

$$A_f(s) V_f(s) + B_f(s) I_f(s) = 0 \quad (\text{B.14})$$

Here subscript ' f ' is used to identify the variables belonging to full-wave subnetworks. (B.15) represents the general full-wave stencil which is in the form represented by (6.5).

References

- [1] H. B. Bakoglu, *Circuits, Interconnections and packaging for VLSI*, Addison-Wesley, Reading MA, 1990.
- [2] W. W. M. Dai (Guest Editor), "Special issue on simulation, modeling, and electrical design of high-speed and high-density interconnects," *IEEE Transactions on Circuits and Systems*, vol. 39, no. 11, pp. 857-982, Nov. 1992.
- [3] M. Nakhla, R. Achar and R. Khazaka, *Modeling and Simulation of High-Speed VLSI Interconnects*, CHAPTER IV: *Circuits And Systems In The Information Age*, New Jersey: IEEE Publishers, pp. 187-215, 1997.
- [4] A. Deustsch, "Electrical characteristics of interconnections for high-performance systems," *Proceedings of the IEEE*, vol. 86, no. 2, pp. 315-355, Feb. 1998.
- [5] T. L. Quarles, *The SPICE3 Implementation Guide*, Technical Report, ERL-M89/44, University of California, Berkeley, 1989.
- [6] C. Paul, *Analysis of Multiconductor Transmission Lines*, New York: John Wiley and Sons Inc., 1994.
- [7] C. Paul, *Introduction to Electromagnetic Compatibility*, New York: John Wiley, 92.
- [8] S. S Gao, A. Y. Yang and S. M. Kang, "Modeling and simulation of interconnection delays and crosstalks in high-speed integrated circuits," *IEEE Transactions on Circuits and Systems*, vol 37, pp. 1-9, Jan. 1990.
- [9] H. Hasegawa and S. Seki, "Analysis of interconnection delay on very high-speed LSI/VLSI chips using a microstrip line model," *IEEE Trans. Electron Devices*, vol. ED-31, pp. 1954-1960, Dec 1984.
- [10] R. Achar, M. Nakhla and Q. J. Zhang, "Full-wave analysis of high-speed interconnects with complex frequency hopping", To appear in *IEEE Trans. Computer-Aided Design*.
- [11] T. Itoh and R. Mittra, "Spectral Domain approach for calculating the dispersion Characteristics of microstrip lines", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496-499, Feb. 1973.

-
- [12] D. Mirshekar-Syahkal, *Spectral Domain Method for Microwave Integrated Circuits*, Joinery & Sons Inc., 1990.
 - [13] D. Mirshekar-Syahkal and J. B. Davis, "Accurate solution of microstrip and coplanar structures for dielectric and conductor losses", *IEEE Trans MTT*, vol. MTT-27, pp. 694-699, Feb. 79.
 - [14] R. H. Jansen, "Spectral Domain Approach for microwave integrated circuits", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1043-1056, Feb. 1985.
 - [15] R. Wang, and O. Wing, "A circuit model of a system of VLSI interconnects for time response computation", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 688-693, April 1991.
 - [16] A. E. Ruehli, "Equivalent circuit models for three dimensional multiconductor systems", *IEEE Trans. Microwave Theory Tech.*, vol. 22, no. 3, pp. 216-224, Mar. 1974.
 - [17] A. E. Ruehli and H. Heeb, "Circuit models for three dimensional geometries including dielectrics", *IEEE Trans. Microwave Theory Tech.*, pp. 1507-1516, Mar. 1992.
 - [18] A. R. Djordjević and T. K. Sarkar, "Closed-form formulas for frequency-dependent resistance and inductance per unit length of microstrip and strip transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-42, pp. 241-248, Feb. 1994
 - [19] A. R. Djordjević, R. F. Harrington, T. K. Sarkar and M. Bazdar, *Matrix Parameters for Multiconductor Transmission Lines: Software and Users Manual*: Boston: Re-teach House, 1989
 - [20] R. Achar, Q. J. Zhang, M. Nakhla and G. Zheng, "Efficient and accurate moment-matching simulation of linear subnetworks characterized by measured data", Accepted for second review in *IEEE Trans. Computer-Aided Design*.
 - [21] R. Achar and M. Nakhla, "Efficient transient Simulation of embedded subnetworks characterized by S-parameters in the presence of nonlinear elements", To appear in *IEEE Transactions on Microwave Theory and Techniques*, Dec. 1998.
 - [22] R. Khazaka, J. Poltz, M. Nakhla, and Q. J. Zhang, "A fast method for the simulation of lossy interconnects with frequency dependent parameters," *Proceedings IEEE Multi-Chip Module Conference*, (Santa Cruz, CA), pp. 95-98, February 1996.
-

-
- [23] Y. Tsuei, A. C. Cangellaris and J. L. Prince, "Rigorous electromagnetic modeling of chip-to-package (first-level) interconnections," *IEEE Trans. Components Hybrids Manufacturing Technology*, vol. 16, no. 8, pp. 876-883, Aug. 1993.
 - [24] M. Picket-May, A. Taflove and J. Baron, "FD-TD modeling of digital signal propagation in 3-D circuits with passive and active loads," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 8, pp. 1514-1523, Aug. 1994.
 - [25] R. Achar, M. Nakhla, and M. A. Kolbehdari "An efficient global transient simulation algorithm for mixed EM/circuit analysis using model-reduction techniques", submitted for publication in *IEEE Transactions on Circuits and Systems*.
 - [26] T. Dhane and D. D. Zutter, "Selection of lumped element models for coupled lossy transmission lines", *IEEE Trans. Computer-Aided Design*, vol. 11, July 1992.
 - [27] X. Li, M. Nakhla and R. Achar, "A universal closed-loop high-speed interconnect model for general purpose circuit simulators", *Proc. IEEE International Symposium on Circuits and Systems (ISCAS)*, pp. 66 - 69, June, 1998, Monterey, CA.
 - [28] F. Y. Chang, "The generalized method of characteristics for waveform relaxation analysis of lossy coupled transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 2028-2038, Dec. 1989.
 - [29] N. Orhanovic, P. Wang, and V. K. Tripathi, "Generalized method of characteristics for time domain simulation of multiconductor lossy transmission lines," *Proceedings. IEEE Symposium on Circuits and Systems*, May 1990.
 - [30] E. C. Chang and S. M. Kang, "Computationally efficient simulation of a lossy transmission line with skin effect by using numerical inversion of Laplace transform," *IEEE Transactions on Circuits and Systems*, vol. 39, pp. 861-868, July 1992.
 - [31] R. Griffith and M. Nakhla, "Mixed frequency/time domain analysis on nonlinear circuits," *IEEE Trans. Computer-Aided Design*, vol. 10, no. 8, pp. 1032-1043, Aug. 1992.
 - [32] R. Wang and O. Wing, "Transient analysis of dispersive VLSI interconnects terminated in nonlinear loads," *IEEE Trans. Computer-Aided Design*, vol. 11, pp. 1258-1277, Oct. 1992.
-

-
- [33] S. Lin and E. S. Kuh, "Transient simulation of lossy interconnects based on the recursive convolution formulation," *IEEE Transactions on Circuits and Systems*, vol. 39, no. 11, pp. 879-892.
 - [34] L. T. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. Computer-Aided Design*, vol. 9, pp. 352-366, Apr. 1990.
 - [35] E. Chiprout and M. Nakhla, *Asymptotic Waveform Evaluation and Moment Matching for Interconnect Analysis*, Boston: Kluwer Academic Publishers, 1993.
 - [36] S. Kumashiro, R. A. Rohrer, A. J. Strojwas, "Asymptotic waveform evaluation for transient analysis of 3-D interconnect structures," *IEEE Trans. Computer-Aided Design*, vol. 12, no. 7, pp. 988-996, 1993.
 - [37] X. Huang, "Padé Approximation of linear(ized) circuit responses," Ph.D. dissertation, Carnegie Mellon Univ., Nov 1990.
 - [38] T. Tang and M. Nakhla, "Analysis of high-speed VLSI interconnect using the asymptotic waveform evaluation technique," *IEEE Trans. Computer-Aided Design*, vol. 11, pp. 2107-2116 Mar. 1992.
 - [39] D. Xie and M. Nakhla, "Delay and crosstalk simulation of high speed VLSI interconnects with nonlinear terminations," *IEEE Trans. Computer-Aided Design*, vol. 12, no. 11, pp. 1798-1811, Nov. 1993.
 - [40] V. Raghavan, J. E. Bracken, and R. A. Rohrer, "AWESpice: A general tool for accurate and efficient simulation of interconnect problems," in *Proc. ACM/IEEE Design Automation Conf.*, pp. 87-92, June 1992.
 - [41] S. Y. Kim, N. Gopal, and L. T. Pillage, "AWE macromodels of VLSI interconnect for circuit simulation," in *Proc. ACM/IEEE Design Automation Conf.*, pp. 64-70, June 1992.
 - [42] S. Y. Kim, N. Gopal, and L. T. Pillage, "Time-domain macromodels for VLSI interconnect analysis," *IEEE Trans. Computer-Aided Design*, pp. 1257-1270, Oct. 1994.
 - [43] R. Achar and M. Nakhla, *Minimum Realization of Reduced-Order Models of High-Speed Interconnect Macromodels*, To appear as a chapter in the book *Signal Propagation on Interconnects*, Boston: Kluwer Academic Publishers, 1998.
-

-
- [44] R. Achar and M. Nakhla, "A novel technique for minimum-order macromodel synthesis of high-speed interconnect subnetworks", *Proc. IEEE International Symposium on Circuits and Systems (ISCAS)*, pp. 70 - 73. June, 1998, Monterey, CA.
 - [45] R. Griffith, E. Chiprout, Q. J. Zhang, and M. Nakhla, "A CAD framework for simulation and optimization of high-speed VLSI interconnections," *IEEE Transactions on Circuits and Systems*, vol. 39, no. 1, pp. 893-906.
 - [46] J. E. Bracken, V. Raghavan, and R. A. Rohrer, "Interconnect simulation with asymptotic waveform evaluation (AWE)," *IEEE Transactions on Circuits and Systems*, vol. 39, no. 11, pp. 869-878, Nov. 1992.
 - [47] E. Chiprout, M. Nakhla, "Analysis of interconnect networks using complex frequency hopping," *IEEE Trans. Computer-Aided Design*, vol. 14, pp.186-199, Feb. 1995.
 - [48] R. Sanaie, E. Chiprout, M. Nakhla, and Q. J. Zhang, "A fast method for frequency and time domain simulation of high-speed VLSI interconnects," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 12, pp. 2562-2571, Dec. 1994.
 - [49] R. Achar, M. Nakhla and E. Chiprout, "Block CFH: A model-reduction technique for distributed interconnect networks", *Proc. IEEE European Conference on Circuits Theory and Design (ECCTD)*, pp. 396-401, Sept. 1997, Budapest, Hungary.
 - [50] M. A. Kolbehdari, M. Srinivasan, M. Nakhla, Q. J. Zhang and R. Achar, "Simultaneous time and frequency domain solution of EM problems using finite element and CFH techniques", *IEEE Trans on Microwave Theory and Techniques*, vol. 44, pp. 1526-1534, Sept. 1996.
 - [51] P. Feldmann and R. W. Freund, "Efficient linear circuit analysis by Padé via Lanczos process", *IEEE Transactions on. Computer-Aided Design*, vol. 14, pp. 639-649, May 1995.
 - [52] P. Feldmann and R. W. Freund, "Reduced order modeling of large linear subcircuits via a block Lanczos algorithm", in *Proc. ACM/IEEE Design Automation Conf.*, pp. 474-479, June 1995.
 - [53] M. Silveria, M. Kamon, I. Elfadel and J. White, "A coordinate-transformed Arnoldi algorithm for generating guaranteed stable reduced-order models of arbitrary RLC
-

-
- circuits", in *Proc. IEEE ICCAD*, Nov. 96.
- [54] M. Chou and J. White, "Efficient reduced order modeling for the transient simulation of three dimensional interconnect", in *Proc. ICCAD*, pp. 40-44, Nov. 1995.
- [55] K. J. Kerns and A. T. Yang, "Stable and efficient reduction of large, multiport RC networks by pole analysis via congruence transformations," in *Proc. ACM/IEEE DAC.*, pp. 280-285, June 1995.
- [56] K. J. Kerns and A. T. Yang, "*Preservation of passivity during RLC network reduction via split congruence transformations,*" *IEEE Transactions on. Computer-Aided Design*, vol. 17, pp. 582-591, July 1998.
- [57] A. Odabasioglu, M. Celik and L. T. Pillage, "PRIMA: Passive Reduced - Order Interconnect Macromodeling Algorithm", *Proceedings IEEE ICCAD*, pp. 58-65, Nov. 1997.
- [58] I. M. Elfadel and D. D. Ling, "A block rational Arnoldi algorithm for multiport passive model-order reduction of multiport RLC networks," *Proc. of ICCAD-97*, pp. 66-71, Nov. 1997.
- [59] Q. Yu, J. M. L. Wang and E. S.Kuh, "Multipoint multiport algorithm for passive reduced-order model of interconnect networks," *Proc. of ISCAS-98*, vol. 6, pp. 74-77, Jun. 1998.
- [60] R. Achar and M. Nakhla, "Addressing passivity issues in CFH", *Proceedings 2nd IEEE International Workshop on Signal Propagation on Interconnects*, pp 33-34, May 1998, Hannover, Germany.
- [61] R. Achar, M. Nakhla, P. K. Gunupudi and E. Chiprout, "Multiport distributed interconnect network reduction with passivity preservation using block CFH", *Submitted for publication in IEEE Trans. Computer-Aided Design*.
- [62] P. K. Gunupudi, M. Nakhla and R. Achar, "Efficient simulation of high-Speed interconnects using Krylov-space techniques", *Submitted for publication in IEEE 7th Topical Meeting on Electrical Performance of Electronic Packaging*, Oct 98, USA
- [63] E. Chiprout and M. Nakhla, "Addressing high-speed interconnect issues in asymptotic waveform evaluation," in *Proc. ACM/IEEE Design Automation Conf.*, pp. 732-
-

736, June 1993.

- [64] R. Achar and M. Nakhla "A fast method for nonlinear simulation of transmission line networks", *Proc. IEEE International Symposium on Nonlinear Theory and Its Applications (NOLTA)*, pp. 837-840, Nov 1997, Hawaii, USA.
 - [65] D. F Anastasakis, N. Gopal, S. Y. Kim, and L. T. Pillage, "On the stability of moment-matching approximations in asymptotic waveform evaluation," in *Proc. ACM/IEEE Design Automation Conf.*, pp. 207-212, June 1992,
 - [66] E. Chiprout and M. Nakhla, "Transient waveform estimation of high-speed MCM networks using complex frequency hopping," *Proc. MCMC*, pp. 134-139, March 1993.
 - [67] E. Chiprout and M. Nakhla, "Fast nonlinear waveform estimation of large distributed networks," in *Proc. IEEE MTT-S Int. Microwave Symp.*, 1992, pp. 1341-1344.
 - [68] R. Achar, M. Nakhla and Q. J. Zhang, "Addressing high-frequency issues in VLSI interconnects with full-wave models and CFH", *Proc. IEEE/ACM Int. conference on Computer Aided Design*, pp. 53-57, Nov. 1995, San Jose, CA, USA
 - [69] R. Achar, M. Nakhla and Q. J. Zhang, "Moment-matching techniques for transient analysis of high-speed VLSI interconnects with full-wave models", *Proc. IEEE 25th European Microwave Conference, Sept. 1995*, pp. 820-823, Bolongo, Italy.
 - [70] E. Chiprout, H. Heeb, M. Nakhla and A.E. Ruehli, "Simulating 3D retarded interconnect models using Complex Frequency Hopping (CFH)", *Proc. IEEE International Conference on Computer-Aided Design (ICCAD)*, pp. 66-72, November 1993.
 - [71] R. Khazaka, J. Poltz, M. Nakhla, and Q. J. Zhang, "A fast method for the simulation of lossy interconnects with frequency dependent parameters," *Proc. IEEE Multi-Chip Module Conference*, (Santa Cruz, CA), pp. 95-98, February 1996.
 - [72] R. Khazaka, E. Chiprout, M. Nakhla and Q. J. Zhang, "Analysis of high-speed interconnects with frequency dependent parameters," *Proc. Intl. Symposium on Electromagnetic Compatibility*, pp.203-208, Zurich, March 1995.
 - [73] R. Khazaka and M. Nakhla, "Analysis of high-speed interconnects in the presence of electromagnetic interference," *IEEE MTT-S International Microwave Symposium*
-

-
- Digest*, (San Francisco, CA) pp. 1811-1814, June 1996.
- [74] M. Nakhla, E. Chiprout, R. Achar and R. Khazaka, "Recent progress in simulation of high speed interconnects using moment matching techniques", *Proc. IEEE 4th Topical Meeting on Electrical Performance of Electronic Packaging*, Oct 1995 (Portland, Oregon, USA).
- [75] M. Nakhla, R. Khazaka and R. Achar, "A universal macromodeling technique for high-speed VLSI interconnects", *1st IEEE International Workshop on Signal Propagation on Interconnects*, Hannover, Germany, May 1997.
- [76] M. Nakhla, R. Achar and M. A. Kolbehdari, "Model-reduction techniques for EM and mixed EM/circuit simulation", *1997 IEEE URSI North American Radio Science Meeting*, July 1997, Montreal, PQ, pp. 151.
- [77] R. Achar, M. A. Kolbehdari and M. Nakhla, "A unified approach for mixed EM/circuit simulation using model-reduction techniques", *IEEE International Microwave Symposium Digest*, June 1997, pp. 1017-1021, Denver, USA.
- [78] M. A. Kolbehdari, M. Nakhla, R. Achar and M. Srinivasan, "Solution of EM problems using reduced-order models by complex frequency hopping", *13th IEEE Annual Review of Progress in Applied Computational Electromagnetics*, Monterey, CA, March, 1997.
- [79] S. D. Corey and A. T. Yang, "Interconnect characterization using time-domain reflectometry," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2151-2156, Sep. 95.
- [80] B. J. Cooke, J. L. prince and A. C. Cangellaris "S-parameter analysis of multiconductor integrated circuit interconnect systems," *IEEE Trans. Computer-Aided Design*, vol. 11, No. 3, pp. 353-360, Mar. 1992.
- [81] J. E. Schutt-Aine and R. Mittra, "Scattering parameter transient analysis of transmission lines loaded with nonlinear terminations," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 529-536, 1988.
- [82] P. C. Cherry and M. F. Iskander, "FDTD analysis of high frequency electronic interconnection effects," *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 10, pp. 2445-2451, Oct. 1995.
-

-
- [83] L. M. Silveria, I. M. Elfadel, J. K. White, M. Chilukuri and K. S. Kundert, "An efficient approach to transmission line simulation using measured or tabulated S-parameter data," in *Proc. ACM/IEEE Design Automation conf.*, pp. 634-639, June 1994.
 - [84] R. Sanaie, E. Chiprout, M. S. Nakhla, and Q. J. Zhang, "Integrating subnetworks characterized by measured data into moment-matching simulations," *Proc. Multi-Chip Module Conf.*, pp. 114-119, Mar. 1994.
 - [85] M. Celik, A. C. Cangellaris and A. Deutsch, "A new moment generation technique for interconnects characterized by measured or calculated S-parameters," *IEEE Intl. Microwave Symposium Digest*, pp. 196-201, June 1996.
 - [86] W. T. Beyene, and J. E. Schutt-Aine, "Efficient transient simulation of high-speed interconnects characterized by sampled data," *IEEE Transactions on CPMT*, Part B, Vol. 21, pp. 105 - 113, Feb. 1998.
 - [87] G. Zheng, Q. J. Zhang, M. Nakhla and R. Achar "An efficient approach for simulation of measured subnetworks with complex frequency hopping", *Proceedings IEEE/ACM In. Conf. Computer Aided Design*, pp. 23-26, Nov 1996, San Jose, CA.
 - [88] R. Achar, M. Nakhla and E. Ahmed "Nonlinear transient simulation of embedded subnetworks characterized by s-parameters using complex frequency hopping", *Proceedings 1998 IEEE MTT-S International Microwave Symposium*, pp. 1219 - 1223 June 1998, Baltimore, USA.
 - [89] W. B. Jones and A. Magnus, "Computation of poles of two-point Padé approximants and their limits," *J. Computational App. Math.*, vol. 6, no. 2, pp. 105-119, 1980.
 - [90] G. A. Baker Jr., *Essential of Padé Approximants*. New York: Academic, 1975.
 - [91] J. H. McCabe, "A formal extension of the Padé table to include two point Padé quotients," *J. Inst. Math. Applic.*, vol. 15, pp. 363-372, 1975.
 - [92] C. W. Ho, A. E. Ruehli and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Trans. Circuits and Systems*, vol. CAS-22, pp. 504-509, June 1975.
 - [93] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*, New York: Van Nostrand Reinhold, 1983.
 - [94] L. Weinberg, *Network Analysis and Synthesis*, New York: McGraw-Hill Book Com-
-

-
- pany Inc., 1962.
- [95] E. S. Kuh and R. Rohrer, *Theory of Active Linear Networks*, San Francisco: Holden-day Inc., 1967.
- [96] E. A. Guillemin, *Synthesis of Passive Networks*, New York: John Wiley and Sons Inc., 1957.
- [97] M. E. V. Valkenburg, *Introduction to Modern Network Synthesis*. New York: John Wiley and Sons Inc., 1960.
- [98] R. Bellman, *Introduction to Matrix Analysis*, Philadelphia, PA: SIAM, 1997.
- [99] J. W. Demmel, *Applied Numerical Linear Algebra*, Philadelphia, PA: SIAM Publishers, 1997.
- [100] T. Kailath, *Linear Systems*. Toronto: Printce-Hall Inc., 1980.
- [101] E. G. Gilbert, "Controllability and observability in multivariable control systems", SIAM J. Control, pp. 128-151, 1963.
- [102] C. T. Chen, *Linear system theory and design*. New York: Holt, Rinehart and Winston, 1984
- [103] J. W. Bandler and Q. J. Zhang, *Optimization techniques for modeling, diagnosis and tuning*, Chapter 14: *Analog Methods for Computer Aided Circuit Analysis and Diagnosis*, T. Ozawa Ed, New York: Marcel Dekker, 1988.
- [104] W. H. Press, S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, *Numerical Recipes in C*, 2nd ed., Cambridge University Press, 1992.
- [105] J. N. Lyness and T. J. Kaper, in *Numerical Integration*, P. Keast and G. Fairweather, eds. Reidl: Dordrecht, 1987.
- [106] MATLAB, The MathWorks Inc., Version 5.0, Natick, MA.
-

IMAGE EVALUATION TEST TARGET (QA-3)

APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved